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EXECUTIVE SUMMARY OF THE THESIS

A New Model for Bitcoin Mining Costs: An Econometric Analysis of the Bitcoin Price and Cost Dynamics

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1. Introduction

Bitcoin, born in January 2009, is the new technology replacing the function of money. The key innovation was the use of a distributed computational system, and the *Proof-of-Work* that allows the decentralized network to substitute the central authorities. In the Bitcoin System, the validation and verification of transactions are conducted by miners by adding the transactions to a decentralized ledger of transactions, the *blockchain*. Miners use dedicated computer machines, application-specific integrated circuits (ASICs), to perform this function. Every time a block containing transactions is added to the blockchain a reward is given to the miner who solves the complex problem of the Proof-of-Work. The reward is a determined number of new bitcoins and the transaction fees associated with the block. The supply of bitcoin is predetermined by an algorithm that limits the number of blocks to be added to the blockchain. The algorithm changes the difficulty for producing a new block every 2016 blocks. This feature denies miners from increasing the block pace even if they increase their computational power. Every 210,000 blocks or approximately four years the bitcoins rewarded per block halves.

This work answers a central theme discussed in the bitcoin mining literature concerning the dynamics between the price and the costs of producing bitcoins. For this purpose, a new model describing the operational costs of miners was developed. This model also includes the miners' investments in ASIC machines; therefore the mining costs are divided into energy and investment costs, and they both require specific assumptions and variables constructions. After the creation of a dedicated dataset, cointegration and causality tests are conducted to analyze the bitcoin price and costs dynamics. The results show a strong directionality from the price toward costs in accordance with the literature and the economic theory since an increase in price will cause an increase in profitability, new miners entering the business will increase the computational power and lead the excess profits to zero.

2. Literature

The debated theme between price and costs was firstly originated from studies on bitcoin price formation (Ciaian et al. 2016). After a first approach, Hayes (2017) presented a model to determine the fair value of a bitcoin based on a formalized cost of production model (CPM). By assuming that the bitcoin value is explained by the cost of producing bitcoins, the CPM attempts to derive the bitcoins cost of production for an individual miner, the break-even cost of mining. The break-even cost is used to determine whether the miner should be involved in mining bitcoin. According to established microeconomic theory, in a competitive market, the selling price is equal to the marginal cost. The main cost of bitcoin mining is energy consumption, and the other costs can be regarded as negligible.

Hayes (2019) tested empirically the model proposed using data from June 2013 to April 2018. After estimating the average energy efficiency of mining hardware, the results show that the price of bitcoin tends to fluctuate around the model price, and the model price predicts the market price in a statistically significant manner. Fantazzini and Kolodin (2020) investigate the relationship between the bitcoin price, its costs, and the hashrate[1]. The purpose of the work is to explain the contradiction in the literature about the dynamics of bitcoin price using econometrics models and different sets of explanatory variables. In particular, the conflict between the significance of the hashrate and the costs for producing bitcoin in predicting the bitcoin price. The results clearly contradict Hayes (2019), where there is found evidence of unidirectional Granger causality going from the bitcoin price to the hashrate or to the CPMs but not *vice versa*.

2.1. A New Development of The Cost of Production Model

The new CPM includes the investments in mining hardware, this new model provides a complete overview of the profit function for miners and describes with greater precision the costs of miners, so the estimated break-even price and other costs variables can be analyzed with the bitcoin price.

Considering only the cost of energy is an assumption that captures only marginal costs. As the ASIC rigs become more efficient and miners use renewable energy, the cost of energy becomes less relevant, while the cost of the machines and buildings takes the lead. In this model miners' costs are simplified into two major components: energy costs and investment costs.

The CPM is based on the daily cost of energy, but this time-frequency is not matching the Bitcoin frequency of changing the difficulty[2]: the time for producing 2016 blocks. The time-frequency is not fixed but it is fixed by the number of blocks produced (2016). This change allows a clear estimation of the average hashrate, instead of estimating it day by day based on the theoretical daily production. The equation estimating the cost of energy for miners of previous work (Hayes, 2019) is adjusted with:

$$E_t = \rho_t \cdot k_t \cdot EEF_t \cdot n_t \cdot 24h \tag{1}$$

Equation (1) expresses the costs of energy required for producing 2016 blocks. Where ρ_t is the average hashes per second of the network. n_t is the number of days for producing 2016 (it also expresses the fractional part of the last day). k_t is the price per kilowatt-hour, and EEF_t is the energy efficiency of the mining hardware.

On the other hand, investment costs require different assumptions:

- an increase in the hashrate that surpasses its maximum level is caused by the investments in new ASICs;
- investment costs are divided according to the useful life of the machines;
- at the end of the machines' useful life their hashrate is replaced by new machines' hashrate.

Following these assumptions, the investment costs are like a leasing, they represent the costs sustained by miners for buying the ASICs. Therefore, what miners are paying is comparable with the usage of the machines in the period of producing 2016 blocks, after that miners have invested, they cannot disinvest anymore.

[1] The hashrate is a measure of the computational power of the network. It expresses the number of hashes computed in a second (H/s).

[2] The difficulty is a measure of how difficult is to mine a Bitcoin block. It is updated every 2016 blocks added to the blockchain.

For capturing the effect that new machines are purchased only when a new hashrate maximum is reached, the hashrate function used for the estimation of the investment costs is expressed as:

$$\Delta\vartheta_t = \Delta\rho_{t+l}^{max} \tag{2}$$

Where the term $\Delta\rho_{t+l}^{max}$ represent the increase in hashrate of the max function of the hashrate (ρ_t^{max}), l expresses the lag between the purchase of the machines and their installation and full deployment of their hashrate in the network. Given the cost per TH/s (c_t), the investments costs of the period can be calculated as:

$$I_t = \sum_{j=-r}^{t} (\Delta\vartheta_j c_j + \Delta\vartheta_{j-r} c_j) \frac{n_t}{d_t} \tag{3}$$

Equation (3) expresses the amount of investment spent by miners during one period. The parameter c_t is the average cost per TH/s at time t. The term $\Delta\vartheta_j c_j$ reflect the investment in new machines caused by the increase in hashrate. The term $\Delta\vartheta_{j-r} c_j$ describes the amount of hashrate replaced by new machines. In particular, the r index is used to express the lag of the useful life of the machines, meaning that the old machines purchased at time *t-r* are replaced at time *t* with new machines paying a cost per TH equal to c_t. The parameter d_t is the number of deprecation days expected for the machines. This value starts at 730 and changes after June 2016 in 1095.

Miners' revenues are the product between price and the awarded bitcoin with fees, by assuming that all the bitcoins mined during the period are exchanged and solving for the Price the equation where revenues is equal to costs, we obtain the break-even price:

$$P_t^* = \frac{E_t + I_t}{(2016\beta_t + F_t)} \tag{4}$$

Where β_t is the number of bitcoin rewarded per block and F_t the transaction fees paid to miners.

3. Dataset

Starting from the hashrate, the CPM described requires specific data construction and assumption to estimate the key parameters necessary to calculate the Investment and Energy costs.

The hashrate is estimated using the difficulty for mining a block and because the time-frequency is the time for producing 2016 blocks, meaning that the difficulty is constant, the hashrate is the average hashrate of the period.

3.1. Energy Efficiency (EEF)

One of the biggest challenges of the model is the correct estimation of the mining energy efficiency (EEF_t) of the network. The history of bitcoin mining hardware evolved until ASICs came into the market, their arrival completely outperformed the other hardware used for mining, in fact, this work does not cover the mining energy efficiency before ASICs.

For the EEF_t calculation, the hardware used as a reference are the ASICs produced by the company Bitmain, which has proven to be the market leader in this sector. According to Fantazzini (2019), a reasonable time used for the deployment in the market of a new ASIC goes from 2 to 3 months, this behavior is described by assuming a delay of approximately 90 days between the purchase date and the time of actual increase of the hashrate. The overall EEF_t is built using a linear interpolation of the Bitmain models, however, due to the better efficiency of these Bitmain models compared with competitors, the efficiency was adjusted by increasing the EEF_t by 10%[3]. The EEF_t of the network changes with the purchase of the new machines, the EEF_t of the network is the weighted average energy efficiency ($WEEF_t$) between the old machines and the new machines that are providing additional computational power. The model assumes that ASICs are replaced after 3 years with the last generation machines, and because of the first ASIC generation was enormously more inefficient the replacement is after only 2 years. The first machines with a useful life of 3 years start from the Antminer S9 ASIC models which shows a great efficiency performance. Finally, the replacement of the machines affects the $WEEF_t$ by decreasing it faster.

[3] The weighted average of EEF considering the Bitmain ASIC market share and the other competitors is approximately 10% higher than the EEF of the Bitmain models.

3.2. Cost of Hashrate

The cost per hashrate is expressed by the parameter c_t ($/ TH/s) which indicates the dollars required for buying the computational power of one TH/s. Similar reasoning and logics applied for the EEF are also being applied for cost per hashrate and the difference in this approach is that the cost per hashrate is not weighted and investments follow Equation (3).

As explained in Section 3.1 for the construction of the EEF_t, c_t uses a linear interpolation as well, it is based on the price of ASICs and their hashrate. The price of the ASICs was obtained by announced transactions done by mining companies or when it was possible the release price announced by Bitmain. Moreover, there are some limits to this estimation, the ASICs' price can be influenced by: the demand and offer dynamics of machines. Bullish Bitcoin periods can increase ASICs price, nevertheless mining companies usually preorder machines a long time in advance fixing the price of the rigs. Moreover, the cost per TH is referred to the best hardware available at that time, but the cost can be lower if miners are willing to accept low-efficiency machines hence spending more money on energy.

Finally, the price is not considering taxation or tariffs, they may be subject to different rules based on where ASICs are bought, shipped, and sold. These limitations can create estimation errors in the investment costs sustained by miners, but most of the prices come from the empirical world and real transactions.

3.3. Electricity Price

The price of a kilowatt-hour (k_t) is kept constant at a value of 0.046 $/kWh. Considering that the cost of the electricity of 8 public mining companies (Table 1) represent approximately 21% of the total hashrate during the last quarter of 2021, using the average value would provide a lower bound for the cost of energy. The high energy cost efficiency achieved by these companies is extraordinarily low. By taking the least energy price efficiency company (0.04 $/kWh) and the maintenance cost of $0.006 per kilowatt-hour declared by Marathon Digital Holdings, the value of 0.046 $/kWh will provide an upper bound cost of energy estimation. Therefore, assuming higher electricity costs can capture other operational expenses.

3.4. CPM Variables

The parameters hashrate, energy efficiency, cost per hashrate are estimated for calculating the energy costs with Equation (1) and the investments with (3). With energy and investment costs, the break-even price can be calculated with Equation (4). However, the analysis between bitcoin price and break-even can be extended by taking separately the energy cost and the investment for producing one bitcoin. Instead of estimating the break-even price with Equation (4) the energy cost for a single bitcoin can be expressed as:

$$e_t = \frac{E_t}{2016 \cdot \beta_t} \tag{5}$$

Similar to the cost of energy for producing one bitcoin, the investment costs for producing a single bitcoin can be expressed by replacing the energy cost E_t with the investment cost I_t. This variable represents the cost of machines for producing one bitcoin. In addition, the sum of energy cost and investment cost can be expressed by the Total costs for miners. Finally, a variable representing the profitability for miners can be expressed by the ratio between the price of bitcoin and its break-even price, called margin. If the ratio is higher than one miners are profitable, if not, miners are taking losses.

Companies	EH/s	$/kWh
Argo Blockchain PLC	1.7	0.029
Bitfarms Technologies Ltd.	2.6	0.040
Cipher Mining	19.5	0.027
Greenidge	1.4	0.022
Hive Blockchain Technologies	1	0.040
Hut 8 Mining Corp.	1.17	0.0274
Marathon Digital Holdings, Inc.	3.5	0.028
Riot Blockchain	4.1	0.025

Table 1: Mining companies energy cost.

4. Methodology and Results

The purpose of the analysis is to investigate the dynamics between price, hashrate, and costs of production of Bitcoin. The results of the directionality from price to costs can be interpreted by an increase in the Bitcoin prices, this will increase the miners' profit, *ceteris paribus* of miners' costs, an increase in price will increase profits. The presence of profits causes the entrance of new miners in the business and the increase of the hashrate, hence it distributes the profit between miners. By considering all the miners present in the network, an increase in the hashrate increases the energy and investment costs for all the network and clearly it decreases the profit. Therefore, a growth in hashrate also contribute to an additional reduction of profits caused by the adjustment mechanism of difficulty. Since there was an increase in the hashrate, in the next period the difficulty will increase meaning that for producing the same number of bitcoins the hashes required are higher than before. Miners entering the market have a double effect in decreasing the profits:

1. Division of profits between miners;
2. Decrease of bitcoin productivity.

The same inverse mechanism is applied when the price decreases and miners leave the market and increase miners' profits.

4.1. Methodology

Tests are conducted to bivariate models formed by the Bitcoin price and a variable expressing the production costs. After a log transformation, variables are tested for stationary using the Augment Dickey-Fuller test (ADF). Tests are performed in two samples: the first from 24/01/2014 to 03/03/2022 including the presence of halving events, whereas the second from 18/07/2016 to 21/04/2020 excludes them. The second step is to test for cointegration using the Engle and Granger approach. After finding the optimal VAR lag length, models are tested for cointegration using the Johansen approach. If any cointegrating vector is found, then a VECM model is analyzed for further information. Finally, the last step is to test the causality using the Granger and Toda and Yamamoto tests to check the causality relationship between the variables.

4.2. Results

All the variables, except for margin, are integrated of the first order which is a required condition for testing the cointegration. Margin is the only variable that is stationary in sample 1. For the Engle-Granger cointegration, the closest relevant finding is the cointegration between the price of bitcoin and the investment cost for a single bitcoin, but the null hypothesis has a p-value of 0.126 and concludes the absence of cointegration. The Johansen cointegration results show in the first sample (Table 2) that only bivariate models price and hashrate, revenues and total costs have one cointegrated equation (C.E.). For the other sample the C.Es. are present in more bivariate models, the exclusion of the halving events from the sample seems to improve the cointegration results. The Granger causality tests (Table 3 and Table 4) show a causality direction going from the price to the costs but not vice versa. In the second sample, some variables show bidirectional causality, however the Toda and Yamamoto tests reject the bidirectionality of energy costs validating the results obtained with the sample 1 Granger causality tests.

Johansen Cointegration				
	Sample 1		Sample 2	
Variable Pair	C.E.	Lag	C.E.	Lag
Log (Price), Log (Price_BE)	0	1	0	1
Log (Price), Log (BTC_EnCost)	0	2	**1**	1
Log (Price), Log (BTC_InCost)	0	1	**1**	2
Log (Price), Log (Margin)	/	/	0	1
Log (Price), Log (Hash)	**1**	1	**1**	1
Log (Rev), Log (TotCost)	**1**	2	**1**	3

Table 2: Johnsen cointegration.

Granger Causality Test – Sample 1				
Y_1	Y_2	Y_1 *does not cause* Y_2	Y_2 *does not cause* Y_1	VAR Lags
Log (Price)	Log (Price_BE)	**0.0192**	0.8429	1
Log (Price)	Log(BTC_EnCost)	0.0972	0.3882	2
Log (Price)	Log (BTC_InCost)	0.0907	0.2819	1
Log (Price)	Log (Margin)	0.5585	0.8429	2
Log (Price)	Log (Hash)	**0.0000**	0.9140	1
Log (Rev)	Log (TotCost)	**0.0004**	0.9797	2

Table 3: Granger Causality test sample 1.

Granger Causality Test – Sample 2				
Y_1	Y_2	Y_1 *does not cause* Y_2	Y_2 *does not cause* Y_1	VAR Lags
Log (Price)	Log (Price_BE)	**0.0003**	0.1289	1
Log (Price)	Log(BTC_EnCost)	**0.0001**	**0.0004**	1
Log (Price)	Log (BTC_InCost)	**0.0109**	0.5296	2
Log (Price)	Log (Margin)	0.3887	0.1289	1
Log (Price)	Log (Hash)	**0.0000**	**0.0269**	1
Log (Rev)	Log (TotCost)	**0.0002**	0.9439	3

Table 4: Granger Causality test sample 2.

The results from the empirical analysis clearly states that the CPM variables and hashrate do not Granger cause the price. Moreover, the cointegration between the bitcoin price and production costs has limited significance, there is no real reason why the bitcoin price should depend on its production costs and why there should be a significant ECM that binds the variables. In the period from 24/01/2014 to 03/03/2022, the hashrate has lower p-values in Granger causality and Toda and Yamamoto tests compared with the variables constructed with the CPM. A possible explanation of these results relies on the impacts of halving events: the impossibility to reflect the anticipation of this event by miners may cause defects in the CPM but not in the hashrate. Excluding them by the period of the analysis is not enough to describe the relationship between price and costs. In addition, the velocity of the adjustment of the hashrate with the price is faster compared with the break-even price that considers both energy and investments costs. Investment costs reflect fixed costs, while the hashrate and energy costs have a quick reaction.

In the second sample the Granger tests highlight bidirectionality from price to hashrate or from price to energy, however, Toda and Yamamoto tests suggest the unidirectionality from price to costs. This evidence and the lower p-values of the price causing the costs reject the hypothesis that costs granger cause the price, contrary to what was found by Hayes (2019). The increase in costs, even during halving, seems to marginally affect the margin, whereas the bitcoin price strongly causes margin. Finally, periods of high volatility suggest favoring miners' profitability rather than a period where the price is more stable.

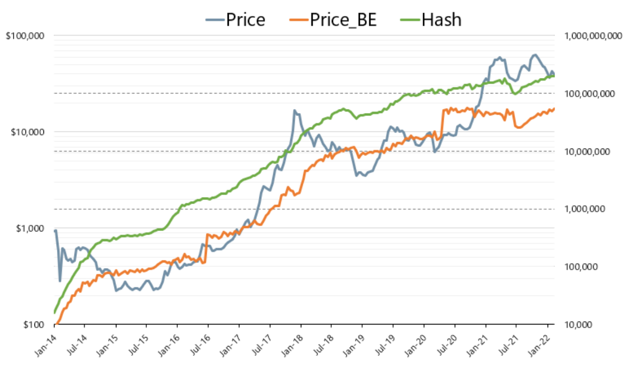

Figure 1: Plot of Bitcoin Price, Break-even Price, and Hashrate.

5. Conclusions

Proving that the bitcoin price is affecting the hashrate and the bitcoin production costs verifies that costs are useless in explaining the bitcoin price. In particular, the sample analyzed shows a significant unidirectionality going from the price to hashrate or CPM variables. After a price drop follows a hashrate or a cost drop, this particular causality is more evident when price falls (i.e. December 2018, May 2021). For this dynamic, the hashrate has superior results if compared with the other variables. However, the development of the CPM highlights the consistency of the economic theory because an increase in price will increase the miners' profitability, therefore new miners will enter the market and will increase the hashrate. The overall effect is that profits will be shared by a higher number of miners and costs will rise bringing profit to zero.

This work contributes to the development of a new model for describing the estimates of the investments on ASIC rigs. Although it may have limitations, this can settle the basis for new studies and removing part of the limits of the cost per TH can be a possible starting point. Finally, in this work the CPM was completely transformed into a new model including the miners' investments, but it would be interesting to improve it by removing the effect of halving events.

6. References

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