



POLITECNICO
MILANO 1863

SCUOLA DI INGEGNERIA INDUSTRIALE
E DELL'INFORMAZIONE

Safe Guidance Under Uncertainty via Stochastic Optimal Control for Autonomous Small-Body Proximity Operations

TESI DI LAUREA MAGISTRALE IN
SPACE ENGINEERING - INGEGNERIA SPAZIALE

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Academic Year: 2024-25

*“Mystery creates wonder and wonder is the basis
of man’s desire to understand.”*

— ***Neil Armstrong***

Abstract

Close-proximity operations around small bodies require high levels of autonomy and robustness under weak gravity, highly nonlinear gravity fields and significant dynamics modeling, navigation and maneuver execution uncertainties. In such environments, traditional trajectory design based on heuristic safety margins becomes inadequate, as small uncertainties can rapidly amplify and compromise mission objectives. This challenge becomes particularly critical in autonomous science-orbit transfer scenarios, where off-nominal events such as missed-thrust events can lead to loss of control authority and jeopardize safety.

This thesis addresses the problem of designing safe and autonomous orbit transfers around small bodies by applying chance-constrained linear covariance steering with sequential convex programming. Within this framework, robust trajectories and feedback control policies are simultaneously designed to probabilistically guarantee safety under navigation uncertainty, maneuver execution errors and unmodeled accelerations. Moreover, the proposed framework ensures safety in off-nominal scenarios caused by temporary loss of control authority during the transfer, guaranteeing that trajectories following a missed-thrust event remain outside a predefined safety region with high probability.

Two stochastic optimization strategies are compared. The first minimizes the statistical fuel cost through the ΔV_{99} metric. The second introduces a convex bound on nonlinear dynamical effects, combining State Transition Tensors with semi-analytical measures of nonlinearity to mitigate the inconsistencies between the linearized predictions and the actual nonlinear behavior of the covariance controller.

The methodology is applied to representative Apophis PTO–RTO transfers and validated via nonlinear Monte Carlo simulations. Results show that explicitly minimizing nonlinear effects significantly improves robustness in highly nonlinear regimes, with limited impact on fuel consumption.

The proposed methods present a unified risk-aware framework to design safe orbit transfers under uncertainty for autonomous small-body proximity operations.

Keywords: Stochastic Optimal Control, Linear Covariance Steering, Sequential Convex Programming, Small-Body Proximity Operations, Autonomous Spacecraft Guidance, Risk-Aware Trajectory Optimization

Sommario in lingua italiana

Le operazioni di prossimità attorno a piccoli corpi richiedono elevati livelli di autonomia e robustezza in presenza di gravità debole, forte non linearità dinamica e significative incertezze di modellazione, navigazione ed esecuzione delle manovre. In tali condizioni, approcci tradizionali basati su margini di sicurezza euristici risultano inadeguati, poiché piccole incertezze possono amplificarsi rapidamente e compromettere la missione, soprattutto nei trasferimenti autonomi tra orbite scientifiche, dove eventi quali mancate accensioni possono causare perdita di controllo e sicurezza.

Questa tesi affronta il problema della progettazione di trasferimenti orbitali sicuri e autonomi attorno a piccoli corpi applicando tecniche di *linear covariance steering* con vincoli probabilistici (*chance-constraints*) combinate con *sequential convex programming*. Traiettorie robuste e politiche di controllo in retroazione vengono progettate simultaneamente al fine di garantire, in senso probabilistico, la sicurezza in presenza di incertezze di navigazione, errori di esecuzione delle manovre e accelerazioni non modellate. Inoltre, il metodo proposto garantisce la sicurezza anche in scenari fuori nominale dovuti a temporanea perdita di autorità di controllo durante il trasferimento, garantendo che le traiettorie successive a una mancata accensione rimangano, con alta probabilità, al di fuori di una regione di sicurezza predefinita.

Sono confrontate due strategie di ottimizzazione stocastica: minimizzazione del costo statistico di propellente tramite ΔV_{99} e minimizzazione degli effetti non lineari mediante vincoli convessi basati su State Transition Tensors.

La metodologia è applicata a trasferimenti PTO–RTO attorno ad Apophis e validata tramite simulazioni Monte Carlo non lineari. I risultati mostrano che la minimizzazione esplicita della non linearità migliora significativamente la robustezza nei regimi fortemente non lineari, con limitato incremento di consumo.

Il lavoro propone un metodo unificato e *risk-aware* per la progettazione di trasferimenti sicuri in operazioni autonome attorno a piccoli corpi.

Parole chiave: Controllo Ottimo Stocastico, Linear Covariance Steering, Program-

mazione Convessa Sequenziale, Operazioni di Prossimità attorno a Corpi Minori, Guida Autonoma di Veicoli Spaziali, Ottimizzazione di Traiettorie Risk-Aware

Acknowledgements

First and foremost, I want to thank my parents, Francesca and Claudio. Thank you for your continuous and unconditional support, for the education that you gave me and for everything that you taught me. You have always believed in me, and you taught me to always believe in myself.

Thanks to my family, my sister Giulia, and my grandparents Grazia, Gianni, Silvia and Roberto, for always being a place of comfort, warmth and simplicity.

A special thanks goes to Politecnico di Milano, for giving me unforgettable years in Milan, and to Purdue University, for welcoming me and providing all the support and resources that made my stay in the United States possible.

I would like to thank my advisor, Professor Francesco Topputo, for your feedback and support during the development of this thesis, and for making me passionate about astrodynamics through your lectures.

I am deeply grateful to Professor Kenshiro Oguri for welcoming me into your research group, for trusting me, and for treating me as one of your own students. Your drive and passion guided me throughout these months, and what I achieved in this work would not have been possible without your mentorship. You have been an outstanding advisor and showed me a side of academia that I had not experienced before.

I would also like to thank all my labmates for making my experience in the U.S. truly unique. In particular, thanks to Naoya, Dan, and Yanis for the discussions and advice on my work. More generally, thank you all for the unforgettable experiences we shared in the Midwest, from chasing the northern lights to trips to the shooting range, and countless Saturday nights playing Catan: Aaron, Abdullah, Adi, Andres, Dan, Elliot, Hiro, Kaushik, Mikayla, Naoya, Nick, Ryo, Sae, Shaurya, and Yanis. Thank you for welcoming me and making me feel part of your group.

I also want to thank all my Italian friends. The guys from Polimi: Edo, Gian, Davi, and many others. Thank you for sharing these last two and a half years. Through exams, late night project deliveries, and hard work, we still managed to find time to enjoy ourselves.

A special thanks to Ale and Gabri: it was wonderful to be able to count on friends even on the other side of the world and to share these months together. I will miss losing to you at pool and, above all, always winning at basketball.

To my friends from la veranda: thank you for always being such a special and stimulating place to come back to whenever we returned home.

Last but not least, I want to thank you, Anna. Simply put, you are my favourite person. You always know how to be there for me, even on my toughest days. Thank you for always reminding me that I have everything I need, and for making me look at life through a warm and shiny filter.

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1 | Introduction

Small bodies are defined by NASA's Jet Propulsion Laboratory (JPL) as "all natural bodies that are not a planet or natural satellite". This definition includes all asteroids and comets, but also dwarf planets (e.g., Ceres) under the small body category. Other definitions are also present, for example the International Astronomical Union (IAU) defines a slightly different term "Small Solar System Bodies" (SSSBs) which excludes dwarf planets. In general, small bodies constitute a broad and heterogeneous class of celestial objects within the Solar System.

Small bodies are commonly grouped into asteroids and comets, although the distinction is not always sharp. Asteroids are generally rocky objects that do not exhibit sustained volatile activity, while comets contain significant amounts of ice and may develop a coma or tail when approaching the Sun. Dynamically, most small bodies follow heliocentric orbits primarily governed by solar gravity, with important perturbations arising from planetary encounters and, in some cases, non-gravitational effects. Among them, Near-Earth objects (NEOs) are of particular interest due to their accessibility for exploration and their relevance for planetary defense.

From a dynamical and physical point of view, small bodies are typically characterized by limited mass, irregular shapes, weak and highly non-uniform gravitational fields, and often complex rotational states. As a result, the term small body does not only refer to size, but rather to a distinct class of objects whose physical and dynamical properties are fundamentally different from those of larger primary bodies.

Historically, comets have been observed by mankind for thousands of years, but asteroids became widely known only in the past few centuries. The first discovery of asteroids was rather incidental, arising from late eighteenth-century efforts to identify a hypothesized "missing planet" between the orbits of Mars and Jupiter. In 1801, the first ever small body was identified, named Ceres by Giuseppe Piazzi. Subsequent discoveries of similar objects in the same orbital region revealed the presence of a population of small bodies rather than a single planet, leading to the recognition of what is now known as the main asteroid belt. Today, almost 1.5 millions of these small celestial bodies have been discovered and

are recorded in the IAU Minor Planet Center database*.

Over the years, these objects have progressively transitioned from purely observational targets to destinations of in-situ space missions. Unlike traditional planetary environments, the proximity operations required to explore small bodies expose spacecraft to highly irregular and weak gravitational fields, strong nonlinearities, and limited navigational observability. As a consequence, space missions to small bodies demand a higher level of care in both trajectory design and guidance, navigation, and control strategies, in order to ensure safety and mission performance within these challenging dynamical regimes.

1.1. Context and Motivations

The exploration of small bodies has become a central theme in modern planetary science and space engineering, driven by a combination of scientific, economic, and strategic motivations. Unlike major planetary bodies, asteroids and comets preserve records of the early Solar System, making them invaluable targets for understanding planetary formation and evolution. At the same time, the growing interest in sustained space exploration and planetary defense has further increased the interest in small bodies.

Small bodies are considered some of the oldest objects in the Solar System. Since they have undergone fewer changes than planets, they preserve valuable information about the conditions that existed when the Solar System was forming. Their composition, internal structure, and surface features help gather insight into how planets began to form and how material was distributed in the early Solar System. In particular, studying asteroids and comets allows to investigate how water and other volatile materials were transported, how complex molecules developed, and how repeated impacts influenced the evolution of these bodies over time.

Beyond their scientific value, small bodies have attracted significant attention for their potential role in in-situ resource utilization (ISRU). Many asteroids are believed to contain substantial quantities of water, metals, and other valuable materials that could support future space activities. The ability to extract and process resources directly from small bodies could substantially reduce the cost and logistical complexity of deep-space missions, enabling more sustainable exploration architectures. As a result, the characterization of small-body composition, internal structure, and surface mechanics has become a prerequisite for assessing the feasibility of resource extraction and utilization. Moreover, interest

*"Latest Published Data", IAU Minor Planet Center. Retrieved January 9th, 2026. <https://minorplanetcenter.net/mpc/summary>

in ISRU is no longer limited to governmental space agencies and scientific institutions. In recent years, private companies such as AstroForge and TransAstra have also begun pursuing asteroid mining as a potential commercial opportunity.

A further motivation for small-body exploration arises from planetary defense considerations. Near-Earth objects (NEOs), particularly potentially hazardous asteroids (PHAs), pose a of impact with Earth that cannot be neglected. To assess and mitigate this risk, a detailed understanding of small-body dynamics, physical properties, and response to external perturbations is required. In this context, space missions to NEOs play a crucial role in refining orbital predictions, validating deflection strategies, and improving models of asteroid structure and behavior under applied forces.

1.1.1. Overview of Small Bodies Exploration Missions

The exploration of small bodies has progressively evolved from remote observations to increasingly complex in-situ missions. Early studies relied on ground-based optical and radar observations, which enabled discovery and orbit determination but provided limited insight into physical properties. The advent of space missions marked a turning point, allowing direct characterization of surface morphology, composition, and dynamical behavior. Figure 1.1 shows a visual overview of the small bodies visited by spacecraft as of 2022 (except Ceres and Vesta), highlighting the diversity in size and shape.

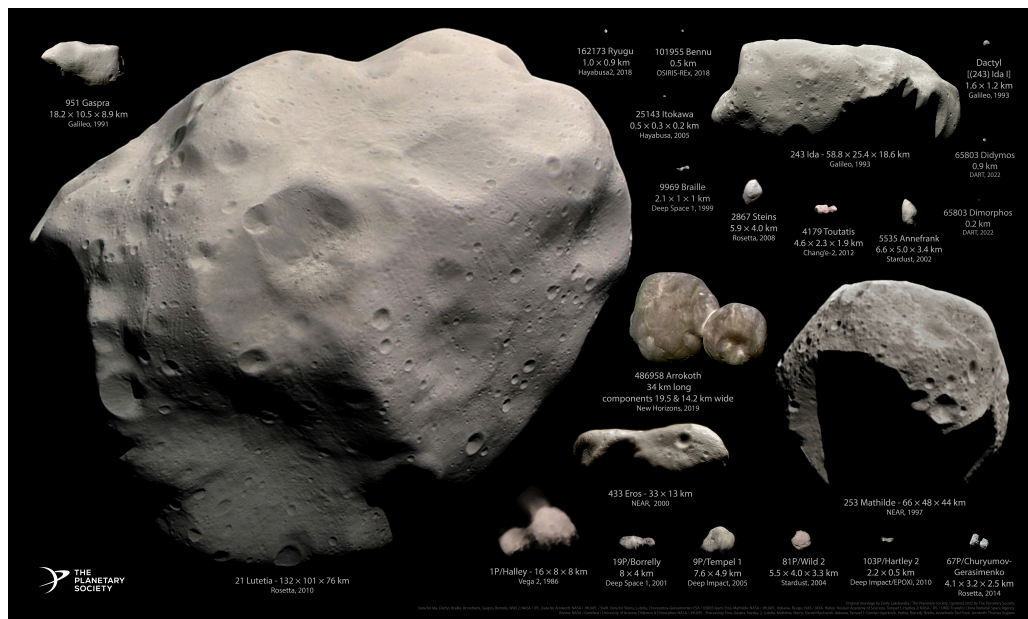


Figure 1.1: Small bodies visited by spacecraft as of September 2022. [†]

Initial in-situ missions primarily consisted of flybys, offering brief encounters at relatively low cost. Notable milestones include Galileo’s flybys of Gaspra and Ida, Deep Space 1’s encounter with Braille, NEAR Shoemaker’s flyby of Mathilde [1], and Rosetta’s flybys of Steins and Lutetia [2]. Similar strategies were applied to comets, including Giotto, Stardust, and Deep Space 1. While flybys significantly improved knowledge of surface properties, their short duration limited precise estimation of gravity fields and mass properties.

To overcome these limitations, rendezvous missions enabled prolonged close-proximity operations. NEAR Shoemaker successfully orbited and landed on Eros [1], Dawn orbited Vesta and Ceres [3], and Rosetta conducted the first long-term rendezvous with a comet and deployed the Philae lander [2]. These missions allowed detailed mapping, improved gravity estimation, and refined understanding of rotational dynamics.

Sample return missions further advanced exploration by enabling laboratory analysis of pristine material. JAXA’s Hayabusa and Hayabusa2 [4, 5] and NASA’s OSIRIS-REx [6] provided unprecedented insight into composition and internal structure.

More recently, planetary defense has emerged as a central objective of small-body exploration. NASA’s DART mission demonstrated the feasibility of kinetic impact deflection by successfully altering the orbit of Dimorphos [7], while ESA’s Hera mission will provide a detailed post-impact characterization to refine models of asteroid response to external perturbations [8].

A particularly significant upcoming event in this context is the 2029 close Earth flyby of asteroid (99942) Apophis, which will pass within geosynchronous orbit and offer a unique opportunity to observe tidal effects on a near-Earth asteroid in real time. NASA’s OSIRIS-REx spacecraft will transition into the OSIRIS-APEX extended mission to rendezvous with Apophis after its encounter [9], while ESA is developing the RAMSES mission to accompany the asteroid during its Earth flyby [10].

In parallel, future missions such as Lucy and Psyche continue expanding exploration toward Trojan and metallic asteroids, broadening both the scientific scope and operational complexity of small-body missions.

These missions represent a new generation of operationally demanding close-proximity campaigns around small bodies. Since NEAR Shoemaker [1], close proximity operations (CPOs) have become a central theme in small-body exploration. Historically, such missions relied on partial onboard autonomy, but advanced autonomous strategies were

[†]Image credit: The Planetary Society. Retrieved January 2026. <https://www.planetary.org/space-images/asteroids-and-comets-visited-by-spacecraft>

conservatively adopted due to mission cost and limited trust in fully autonomous systems [11].

In recent years, a paradigm shift has emerged with the growing use of CubeSats and nanosatellites in deep-space exploration [12–14]. The lower development cost of these platforms enables higher operational risk tolerance and encourages autonomous experimentation. Several ongoing and forthcoming missions, including M-ARGO [15], Hera (with Milani and Juventas) [11, 16], and RAMSES [10], rely on nanosatellite architectures.

However, the limited resources of CubeSats and their reliance on miniaturized off-the-shelf components introduce higher levels of uncertainty, including thrust misalignment and actuation errors. The absence of continuous ground support further increases the need for robust onboard autonomy. As a result, the rise of deep-space CubeSats calls for a paradigm shift in trajectory design and GNC strategies for small-body close proximity operations.

1.1.2. Technical Challenges of Small-Body Proximity Operations

Across all major domains of small-body exploration, the capability to operate spacecraft in close proximity to the target body is essential. Close-proximity operations (CPOs) enable high-resolution surface mapping, gravitational field reconstruction, morphology characterization, and precise relative navigation. However, the dynamical and operational environment around small bodies differs fundamentally from classical planetary orbits and poses unique challenges for guidance, navigation, and control (GNC).

Spacecraft motion in the vicinity of a small body is governed by weak and highly irregular gravity fields, often poorly characterized prior to rendezvous with the target. The gravitational potential is strongly influenced by irregular shape and uneven mass distribution, leading to complex and nonlinear dynamics. In addition, perturbations such as solar radiation pressure (SRP), third-body effects, and outgassing phenomena for comets, are non-negligible and can even dominate the local dynamics [17].

For these reasons, there is non-negligible intrinsic level of uncertainty in small-body CPOs. Dynamical mismodeling and uncertainties related to navigation errors and maneuver execution inaccuracies must all be taken into account. As a consequence, the spacecraft state relative to the small body should be treated through a probabilistic approach. Moreover, communication delays and constrained visibility windows make robust onboard autonomy not only advantageous, but also necessary for mission success.

These considerations naturally call for a risk-aware mission design paradigm, in which uncertainty is treated as an integral component of the design process rather than addressed through a posteriori validation of nominal trajectories. This requires explicit uncertainty quantification (UQ) and probabilistic assessment of constraint violations during proximity operations.

These considerations first require the explicit quantification and propagation of uncertainty along the trajectory, and several uncertainty quantification (UQ) methods have been proposed in the astrodynamics literature to model the evolution of state uncertainty under nonlinear dynamics. While these UQ techniques enable increasingly accurate prediction of uncertainty evolution, they do not provide alone a framework to actively control the uncertainty distribution or enforce safety requirements during trajectory design. To achieve this, uncertainty propagation has been embedded in stochastic optimal control (SOC) frameworks.

Stochastic optimal control formulations incorporate the probability distribution of the state directly into the optimization process. Rather than enforcing constraints deterministically on a nominal trajectory, SOC enables safety and performance requirements to be expressed probabilistically, for example through chance constraints that bound the allowable probability of violation.

Finally, it is important to emphasize that proximity operations around small bodies increasingly demand for on-board trajectory generation and decision-making. However, spacecraft computational capabilities remain limited, especially in harsh radiation environments [18]. Therefore, robust trajectory design methods must not only account for uncertainty, but also remain computationally efficient and suitable for on-board implementation.

These requirements motivate the adoption in this work of recently developed stochastic optimal control problems solved via convex programming methods, which aim to combine robustness and safety guarantees with computational efficiency suitable for autonomous close-proximity operations around small bodies.

1.2. State of the Art

The previous sections presented the motivations behind the growing interest in the exploration of small celestial bodies, as well as some of the unique challenges associated with close-proximity operations. As spacecraft must operate with limited knowledge of the target, complex and highly nonlinear dynamical environments, and under significant

model uncertainties and communication delays, the Guidance, Navigation, and Control (GNC) system is required to possess a certain degree of onboard autonomy [19].

In particular, GNC algorithms must be both computationally efficient and robust to disturbances and uncertainties, in order to enable reliable and timely onboard decision-making [18]. To address these requirements, recent advances in convex optimization and stochastic optimal control provide a promising theoretical and algorithmic foundation for the design of safe and autonomous GNC frameworks around small bodies.

This section reviews the relevant literature along three main research directions: convex programming for trajectory optimization, stochastic optimal control and chance-constrained formulations, and autonomous GNC for small-body proximity operations.

Convex Programming

Techniques for solving trajectory optimization problems can be broadly classified into indirect methods, direct methods, and dynamic programming [20]. Among these, direct methods transcribe the infinite-dimensional optimal control problem into a finite-dimensional nonlinear programming (NLP) problem via collocation or single/multiple shooting [21, 22]. Although widely used, NLP problems are computationally demanding and generally lack guarantees on the existence of the solution, convergence, or global optimality [18, 23].

In contrast, convex optimization problems can be solved efficiently and reliably in polynomial time, with deterministic bounds on the number of arithmetic operations needed to solve the problem [18]. Moreover, any local minimum of a convex problem is also a global optimum [23]. These properties make convex programming particularly attractive for autonomous GNC applications, where computational efficiency and reliability are critical.

However, most trajectory optimization problems are inherently non-convex due to nonlinear dynamics and non-convex state or control constraints. When possible, such problems can be reformulated via *lossless convexification*, using change of variables or lifting techniques that augment the control input into a higher-dimensional space [24]. Pontryagin's Maximum Principle can be used to prove that the solution of the convexified problem is optimal also for the original one.

For general nonlinear problems where lossless convexification is not applicable, *sequential convex programming* (SCP) provides a practical alternative. SCP iteratively linearizes the non-convex elements about a reference trajectory, solves the resulting convex subproblem, and updates the reference until convergence [18].

Among recent SCP frameworks, Successive Convexification (SCvx [25, 26]) addresses non-linear dynamics and non-convex constraints while mitigating artificial unboundedness and artificial infeasibility through trust-region and virtual control mechanisms. Nevertheless, SCvx only guarantees convergence to a KKT point of a penalized problem and not necessarily to a feasible solution of the original non-convex problem. For control-affine systems, the Guaranteed Sequential Trajectory Optimization (GuSTO [27]) framework provides convergence guarantees to a stationary point of the original problem. More recently, SCvx has been incorporated within an augmented Lagrangian framework (SCvx* [28]), which guarantees convergence to a local optimum of the original non-convex problem.

More broadly, advances in numerical optimization have further improved SCP methodologies, incorporating techniques such as indirect methods for continuous-time problems with manifold constraints [29] and strategies based on line search and trust regions [30].

Stochastic Optimal Control

Stochastic optimal control is an optimization framework widely used in control theory and robotics. Robust trajectory design under uncertainty can be formulated as either hard-constrained or chance-constrained [31]. Figure 1.2 conceptually shows the difference between the two approaches: the hard-constrained approach considers bounded stochastic disturbances and controls all the state realizations to stay within the feasible region [31, 32], while the chance-constrained approach deals with unbounded distributions and imposes probabilistic constraints on stochastic variables, ensuring that the system remains within the feasible region with a probability greater than a user-defined risk level [31, 33]. Although the hard-constrained approach guarantees feasibility with 100% probability for all possible realizations, most astrodynamics applications operate with unbounded distributions (e.g., Gaussian), making chance constrained approaches more suitable.

Chance constrained optimization has been applied across various domains, including control theory, robotics, economics, and finance [34]. Particularly relevant to aerospace and astrodynamics are recent advances in *optimal covariance steering* [33, 35]. Originally introduced by Hotz and Skelton in the 1980s [36], covariance steering aims to control both the mean and covariance of the system while minimizing a statistical cost and satisfying chance constraints. Recent developments have addressed both continuous-time [37] and discrete-time formulations [33].

Among discrete-time variants, several extensions of covariance steering have been proposed to address different structural assumptions and feedback architectures. Input-constrained formulations were considered by Bakolas [38], while alternative feedback

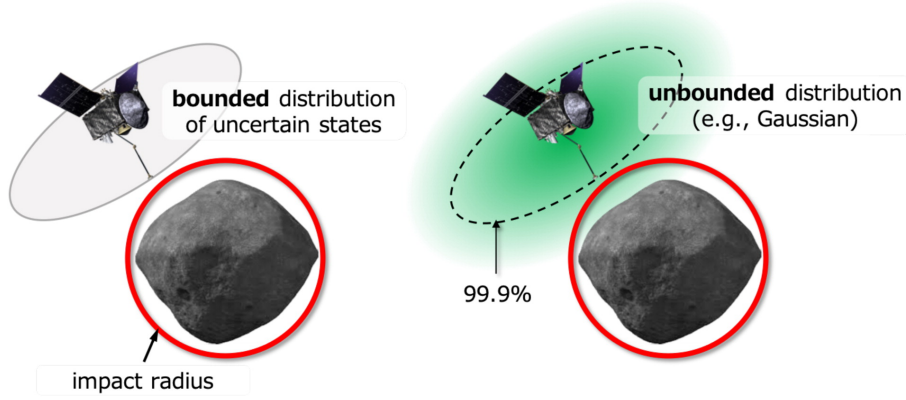


Figure 1.2: Hard-constrained (left) vs Chance-constrained (right) approaches for stochastic optimal control. Credits: [31]

structures, including formulations without history dependence and output-feedback implementations, were developed in [35, 39, 40]. More recently, chance-constrained covariance steering has been extended to Markov Jump Linear Systems [41].

Covariance steering has also been combined with sequential convex programming (SCP), leveraging dynamics linearization and linear covariance propagation [42]. These approaches can be broadly classified into two categories: *full covariance* formulations, where state covariances are explicit decision variables [42–45], and *block-Cholesky* formulations, which implicitly optimize the covariance by constructing a large block system and using the Cholesky factor of covariance matrices [43, 46–48].

The *block-Cholesky* formulation is attractive due to the inherent convex properties of the Cholesky decomposition. However, its computational complexity scales approximately as $\mathcal{O}(n_x N^2)$, where n_x is the state dimension and N the number of discretization nodes [42, 45, 47, 48].

The *full covariance* formulation, in contrast, exhibits computational complexity proportional to $N(n_x + n_u)$, where n_u is the control dimension. A key contribution of this approach is the exact covariance propagation obtained via lossless convexification using Schur complements, KKT conditions, and linear matrix inequalities (LMIs), as proved by Liu et al. in [44]. This makes the formulation attractive for large-scale trajectory optimization problems with large N . However, compared to the block-Cholesky formulation, the full covariance approach presents three main drawbacks: numerical ill-conditioning, non-convex chance constraints, and limited flexibility in the cost function.

First, numerical ill-conditioning arises from the large-scale separation typical of space trajectories. As shown by Oguri and Lantoine in [48], the disparity between state magni-

tudes and covariance magnitudes can lead to extremely large condition numbers, degrading solver reliability. Typically, scaling is performed to address this issue.

Second, the use of full covariance matrices as decision variables makes several chance constraints non-convex. As shown in [33, 35, 39, 40, 48, 49], convex reformulations are naturally obtained when the Cholesky factor is used instead. In contrast, directly optimizing over covariance matrices makes control magnitude and other chance constraints nonlinear, typically requiring linearization or convex-concave procedures [42, 43, 45].

Finally, the lossless convexification at the foundations of full-covariance approaches imposes restrictive assumptions on the objective function. As shown in [44, 45], this prevents the exact inclusion of the ΔV_{99} cost metric, commonly used in space mission design. Kumagai and Oguri proposed approximate relaxations in [42], partially alleviating this limitation.

More recently, further advancements have been proposed. Within the block-Cholesky framework, Qi and Oguri introduced a statistical cost aimed at minimizing nonlinear propagation errors by leveraging State Transition Tensor (STT) information and measures of nonlinearity [50, 51]. This method was successfully applied to a cislunar periodic orbit stationkeeping scenario, demonstrating improved robustness of the resulting linear covariance controller.

Moreover, Qi et al. addressed a key limitation of classical covariance steering, namely the Gaussian assumption, which may become inaccurate in strongly nonlinear astrodynamics environments [52, 53]. They introduced a non-Gaussian distribution steering framework based on the Conjugate Unscented Transformation (CUT), enabling control of higher-order statistical moments beyond mean and covariance within an SCvx*-based framework. Finally, Kumagai and Oguri proposed a square-root factorized covariance steering method [54]. This formulation improves numerical reliability and scalability compared to full covariance approaches, while retaining the cost function flexibility typical of block-Cholesky formulations. Its solution requires sequential convex programming and, although not yet applied to astrodynamics and mission design problems, represents a promising research direction to combine the strengths of previous formulations.

In general, the integration of chance-constrained covariance steering with sequential convex programming is often referred to in the recent literature as *sequential covariance steering*. This framework has been successfully applied to trajectory design in cislunar space [42], as well as to interplanetary multi-phase mission design with gravity assists [48]. Despite these advances, applications of sequential covariance steering to small-body close proximity operations remain unexplored in the literature.

Close Proximity Operations around Small Bodies and the Role of Autonomy

Autonomous GNC algorithms for small-body close proximity operations have been studied extensively in the literature.

Scheeres and McMahon [55] propose different mission architectures for autonomous small-body exploration, demonstrating mission feasibility with minimal instrumentation. Takahashi and Scheeres [56] show the feasibility of an autonomous exploration campaign of a small near-earth asteroid under current technology constraints. Several autonomous navigation schemes [57–59] have been developed for CPOs and applied to various mission scenarios.

Other works focus more specifically on autonomous and robust guidance and control laws to deal with the highly uncertain environment around small bodies. In this context, Furfaro [60, 61] develops real-time algorithms for hovering and landing scenarios. Negri et al. [62] formulate robust G&C algorithms that directly incorporate uncertainties without needing extensive navigation campaigns. Bottiglieri et al. [11] evaluate the flyability of close-proximity trajectories of the Milani CubeSat under navigation and control uncertainties via knowledge analysis (KA) and dispersion analysis (DA), highlighting the need to re-iterate the trajectory design process when a posteriori verification reveals unsafe conditions.

In the context of trajectory design under uncertainty around small bodies, some relevant contributions have emerged in recent years. Jenson and Scheeres [63] design Terminator orbit transfers under uncertainty aimed at minimizing both error covariance and energy using indirect methods. Oguri and McMahon propose a stochastic optimal control framework for robust spacecraft guidance around small bodies, solving chance-constrained optimization problems via a convex programming stage followed by NLP refinement [31]. They further extend classical primer vector theory to a stochastic setting for robust low-thrust trajectory design under uncertainty [64]. Rizza et al. [65] propose a goal-oriented trajectory refinement for autonomous mapping campaigns, leveraging SCP and accounting for navigation and control uncertainties to re-plan online the trajectory.

Despite these advances, to best of the author’s knowledge no prior work has integrated optimal covariance steering via chance-constrained control and sequential convex programming for the design of transfer trajectories in small-body close proximity operations.

1.3. Research Questions and Objectives

The primary objective of this thesis is to develop and assess a guidance and trajectory design framework for spacecraft proximity operations around small celestial bodies that explicitly accounts for the presence and evolution of uncertainty. The proposed approach aims to provide probabilistic safety guarantees while remaining computationally tractable for realistic mission scenarios, with asteroid (99942) Apophis serving as a representative and practically motivated case study.

In addition to this overarching goal, secondary objectives are also pursued. These include the integration of probabilistic safety guarantees in off-nominal situations, namely loss of control authority after a missed-thrust event, within the chance-constrained framework. Finally, another objective of this work is the validation and evaluation of the resulting trajectories in terms of safety, robustness, and performance in a high-fidelity nonlinear and stochastic setting through Monte Carlo simulations.

Within this context, the central research question addressed by this thesis is formulated as follows:

Research question

"How can *safe* and *efficient* spacecraft proximity operations around small bodies be designed by explicitly accounting for state and operational uncertainty in highly nonlinear and weak-gravity dynamical environments?"

This primary research question is further detailed through the following subordinate questions:

- How can a stochastic optimal control framework based on linear covariance steering guarantee robust and efficient performance during proximity operations in strongly nonlinear dynamical environments, such as those encountered around asteroid (99942) Apophis?
- How can probabilistic safety requirements, including collision avoidance and keep-out-zone constraints under missed-thrust events and loss of control authority, be incorporated into trajectory optimization frameworks while avoiding overly conservative deterministic design margins?
- How does the explicit treatment of uncertainty influence the feasibility, safety, and performance of proximity trajectories when compared to conventional deterministic approaches?

1.4. Thesis Outline

The primary objective of this thesis is to design safe and efficient guidance and control policies for autonomous science-orbit transfers around asteroid (99942) Apophis. To this end, operational and dynamical uncertainties are explicitly incorporated in the stochastic trajectory optimization stage, and probabilistic safety guarantees are imposed, including robustness to missed-thrust events. The thesis is organized as follows:

- Chapter 2 introduces the theoretical concepts that serve as foundations for the mathematical and computational tools adopted in this thesis for deterministic and stochastic trajectory optimization around small bodies.
- Chapter 3 presents the sequential convex programming algorithm employed throughout the thesis to solve both deterministic and stochastic optimal control problems.
- Chapter 4 describes the methodology used to design periodic orbits around asteroid (99942) Apophis, namely Periodic and Resonant Terminator Orbits. This chapter also defines the operational transfer scenario investigated in this work.
- Chapter 5 formulates and solves the deterministic impulsive transfer problem between the selected departure and arrival terminator orbits. The results serve both as a performance baseline for a comparison under uncertainty between deterministic and risk-aware solutions, and also as initial reference trajectories for the stochastic optimization problem.
- Chapter 6 formulates the stochastic optimal control problem under uncertainty and presents the solution framework based on sequential convex programming. Two optimization strategies are introduced and the corresponding convex subproblems are derived. This chapter also develops the probabilistic safety formulation for missed-thrust events, modeled as a continuous-time chance constraint.
- Chapter 7 presents the numerical results of the chance-constrained optimization problems applied to two representative case studies. A comparative analysis highlights the strengths and limitations of the proposed strategies, and the safety guarantees are quantitatively assessed.
- Chapter 8 concludes the dissertation by summarizing the main findings, answers the research questions and outlines the directions for future research.

2 | Theoretical Background

This chapter provides a comprehensive overview of the theoretical background of the primary topics employed in this thesis. In particular, Section 2.1 introduces the dynamical model adopted in this thesis in terms of reference frames, general form of equations of motion under impulsive control and natural dynamics, as well as the theoretical foundations for the design of periodic orbit families. Section 2.2 introduces the mathematical tools to describe the dynamics with a variational approach, as well as the theoretical foundations to quantify the nonlinearities in a nonlinear dynamical system. It also provides the necessary tensor notation and operations used throughout this thesis. Section 2.3 reviews the theoretical background of optimal control theory for the solution of trajectory optimization problems via indirect methods, while Section 2.4 reviews the fundamentals of convex programming. Finally, Section 2.5 provides an overview of stochastic optimal control, presenting the necessary framework for risk-aware guidance and control design.

2.1. Dynamical Model Background

This section introduces the theoretical foundations required to model the dynamics encountered during close proximity operations around small celestial bodies. As discussed in Section 1.1.2, spacecraft operating in these regimes are subject to weak and highly irregular gravitational fields, which give rise to strongly nonlinear dynamics [66]. In addition to gravitational effects, non-gravitational perturbations play a critical role, with solar radiation pressure (SRP) often becoming a dominant force acting on the spacecraft.

2.1.1. Reference Frame

This work uses the asteroid-fixed rotating Hill frame [59, 67]. The origin of the reference frame is at the center of mass of the asteroid. The x-axis is aligned with the sun-body vector; the z-axis is in the direction of the orbit normal of the asteroid's heliocentric orbit; the y-axis completes the right-handed frame. Figure 2.1 shows a schematic overview of the asteroid-fixed rotating Hill frame.

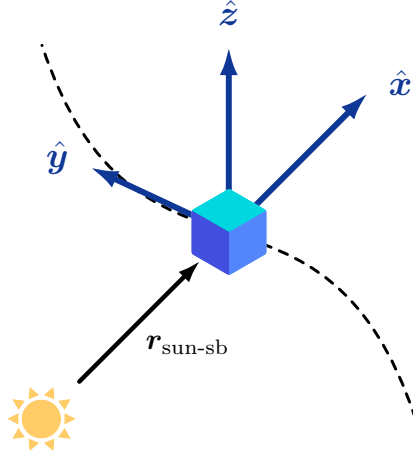


Figure 2.1: Asteroid-fixed Rotating Hill Frame

2.1.2. Generic Equations of Motion Under Impulsive Control

In this work, the spacecraft orbital dynamics under control are modeled as a control-affine system of the form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) = \mathbf{f}_0(\mathbf{x}, t) + B \mathbf{u}(t), \quad B = \begin{bmatrix} \mathbf{0}_{n_u \times n_u} \\ \mathbf{I}_{n_u} \end{bmatrix}, \quad (2.1)$$

where $\mathbf{x} \in \mathbb{R}^{n_x}$ and $\mathbf{u} \in \mathbb{R}^{n_u}$ denote the state and control vectors, respectively. The vector field $\mathbf{f}_0$ represents the natural (uncontrolled) dynamics of the system, while the input matrix B maps the control action directly into the velocity components of the state. To model the natural dynamics $\mathbf{f}_0$, this work utilizes the Augmented Normalized Hill Three Body Problem (ANH3BP), which will be explained in detail in the following section.

Throughout this thesis, the state is expressed in Cartesian coordinates as $\mathbf{x} = [\mathbf{r}^\top, \mathbf{v}^\top]^\top \in \mathbb{R}^6$, where $\mathbf{r}, \mathbf{v} \in \mathbb{R}^3$ denote the spacecraft position and velocity vectors in the rotating Hill frame, which is defined in Section 2.1.1.

An impulsive control model is assumed. Under this assumption, the control input $\mathbf{u}(t)$ is represented as a finite sequence of impulsive Δv maneuvers $\mathbf{u}_k \in \mathbb{R}^{n_u}$ applied at prescribed maneuver epochs t_k , $k = 0, \dots, N-1$:

$$\mathbf{u}(t) = \sum_{k=0}^{N-1} \mathbf{u}_k \delta(t - t_k), \quad (2.2)$$

where $\delta(\cdot)$ denotes the Dirac delta function and N is the number of control opportunities over the planning horizon [49].

As a result, the state trajectory exhibits instantaneous discontinuities at each maneuver epoch, corresponding to jumps in the velocity components:

$$\mathbf{x}(t_k^+) = \mathbf{x}(t_k^-) + B \mathbf{u}_k, \quad k = 0, \dots, N-1, \quad (2.3)$$

where $\mathbf{x}(t_k^-)$ and $\mathbf{x}(t_k^+)$ denote the state immediately before and after the k -th impulse, respectively.

This impulsive formulation is naturally aligned with the trajectory optimization framework adopted in the following chapters, in which decision variables are defined at a finite number of nodes and the dynamics are enforced through discrete-time propagation between successive maneuver epochs.

2.1.3. Augmented Normalized Hill Three-Body Problem

To model the dynamical environment around asteroids in the presence of strong SRP perturbations, this thesis utilizes the Augmented Normalized Hill Three Body Problem (ANH3BP) [17]. The underlying assumptions of the ANH3BP are the following:

- the small body is modeled as a point-mass with gravitational parameter μ_{sb} ;
- the small body is assumed to be in a circular heliocentric orbit, resulting in a constant sun-asteroid distance;
- the influence of SRP is modeled using a cannon-ball approximation, resulting in a constant acceleration directed away from the Sun;
- the spacecraft mass is negligible with respect to the ones of both primaries (Sun and asteroid);
- higher-order gravitational harmonics and non-uniform rotation of the small body are neglected.

The dynamical system is normalized in terms of characteristic length and time, given by:

$$LU = \left(\frac{\mu_{\text{sb}}}{\mu_{\text{sun}}} \right)^{1/3} d_{\text{sun-sb}}, \quad TU = \frac{1}{N_{\text{sb}}} \quad (2.4)$$

Where μ_{sb} and μ_{sun} are the gravitational parameters of the asteroid and the sun, respectively, $d_{\text{sun-sb}}$ is the constant sun-asteroid distance and N_{sb} is the mean motion of the

small body in its heliocentric orbit.

Let $\mathbf{r} = [x, y, z]^T$ denote the spacecraft position in the normalized Hill frame. The equations of motion for the ANH3BP under the assumption of primaries in mutually circular orbits are given by [17]:

$$\mathbf{f}_0(\mathbf{x}) = \begin{bmatrix} \mathbf{v} \\ 2\dot{y} + 3x - \frac{x}{r^3} + \beta, \\ -2\dot{x} - \frac{y}{r^3}, \\ \dot{z} \\ -z - \frac{z}{r^3}, \end{bmatrix}, \quad (2.5)$$

where $r = \|\mathbf{r}\|$ and β is a non-dimensional parameter that represents the strength of SRP in the system with respect to the gravitational attraction of the small body and is defined as:

$$\beta = \frac{G_1}{(m/A)\mu_{\text{sun}}^{2/3}\mu_{\text{sb}}^{1/3}} \quad (2.6)$$

where G_1 is the solar flux constant ($\approx 10^{14}$ (kg km)/s²) and m/A is the spacecraft mass-to-area ratio.

Dynamical properties

The ANH3BP is an autonomous dynamical system and admits a conserved integral of motion known as the Jacobi constant. This quantity is given by

$$J_c = \frac{1}{2}\|\mathbf{v}\|^2 - \frac{1}{r} - \frac{3}{2}x^2 + \frac{1}{2}z^2 - \beta x, \quad (2.7)$$

and defines the allowable regions of motion in the five-dimensional phase space known as zero-velocity-curves (ZVC) [17].

The system also exhibits several symmetries, namely symmetry with respect to the z and $\dot{z}$ components, as well as time-reversal symmetry [68, 69].

Moreover, the ANH3BP admits a rich set of bounded solutions, including equilibrium points, frozen orbits, and continuous families of periodic orbits [17, 66, 68]. Of particular interest are the solutions associated with the terminator plane, where the spacecraft motion remains stable due to the balancing effect between solar radiation pressure, the weak gravitational attraction of the small body, and inertial effects in the rotating Hill frame. These solutions are often referred to as Sun-Stabilized Terminator Orbits (SSTOs) [70, 71].

Periodic Orbit Fundamentals

This section presents the theoretical background behind the existence of periodic solutions. Further detail on the computational methods used for designing periodic orbits in the ANH3BP can be found in Section 4.1.

A periodic orbit is a trajectory such that, for all times t , the following expression holds true [68, 72]:

$$\mathbf{X}(t) - \varphi_T(\mathbf{X}(t)) = 0 \quad (2.8)$$

Where T is the orbit period and φ_T is the flow of the dynamics after a period.

By integrating the state transition matrix Φ together with the equations of motion of the ANH3BP Eq. (2.5) with a variational approach for exactly one period, one obtains the Monodromy matrix M [68, 73]. Due to Hamiltonian nature of the ANH3BP dynamics, M is guaranteed to be symplectic and its eigenvalues are always in reciprocal pairs [72]. For a periodic orbit in the ANH3BP, the eigenvalues of M are in the form: $(1, 1, \lambda_1, 1/\lambda_1, \lambda_2, 1/\lambda_2)$, where λ_i may be complex. Moreover, for a periodic orbit to be linearly stable all eigenvalues of M must lie on the unit circle, meaning their magnitude must be equal to one. Complex and stable eigenvalues can be conveniently written using Euler's form $e^{i\theta}$, where θ represents the angle between the eigenvalue and the positive real axis in the complex plane and is called *phase angle*.

The Monodromy matrix is utilized in this work to detect the intersections of the many periodic orbit families that exist in the ANH3BP, called *bifurcations* [68]. A bifurcation point occurs when a small parameter variation can cause qualitative changes in the structure of the periodic solutions [72].

Bifurcation theory is useful to detect orbits that 'grow' into new periodic families with structure that is unique with respect to the originating family. Among the various types of bifurcations, those of particular relevance to this thesis are the $m:n$ period-multiplying bifurcations, which occur when a pair of eigenvalues of the monodromy matrix reaches a characteristic phase angle $\theta = 2\pi n/m$ [72].

In the following paragraphs the main families of solutions utilized in this thesis are briefly introduced.

Frozen Terminator Orbits: Frozen Terminator Orbits (FTOs) are a type of quasi-periodic orbits existing in the ANH3BP. They arise from an averaged formulation of the perturbed dynamics in close-proximity of a small body, which is explained in detail by Scheeres in [66]. By expressing the dynamics in orbital elements through the Lagrange Planetary Equations (LPEs) and averaging the perturbations over one heliocentric period

of the small body, it is possible to derive equations that describe the secular evolution for the orbital elements. The frozen conditions are then found by computing the values of the orbital elements for which this secular evolution is zero.

The frozen conditions are found in terms of the classical orbital parameters:

$$\sin(\Omega) \sin(\omega) = 1, \quad e = \cos(\Lambda), \quad i = \frac{\pi}{2}, \quad (2.9)$$

where Λ is the ratio between the time rate of change of the true anomaly $\dot{f}$ and the SRP acceleration. This ratio is a time-invariant constant for a given mission and is described by:

$$\tan(\Lambda) = \frac{3\sigma}{2} \sqrt{\frac{a}{\mu_{sb}\mu_{sun}p}} \quad (2.10)$$

where $\sigma = (1 + \rho)G_1/(A/m)$, ρ is the spacecraft's reflectance parameter, and a and p are the semi-major axis and the semi-latus rectum of the asteroid's heliocentric orbit.

Physically, FTOs represent a particular class of sun-stabilized orbits, in which the spacecraft remains confined to the plane perpendicular to the Sun–asteroid direction, commonly referred to as the *terminator plane*. Operationally, they are particularly attractive for their long-term boundedness and stability, as well as their ability to avoid eclipses. These configurations have been successfully exploited by the OSIRIS-REx mission and are expected to play a central role in the extended OSIRIS-APEX mission to (99942) Apophis [9, 74].

In this thesis, FTOs serve as physically meaningful initial guesses for the computation of fully periodic solutions within the ANH3BP framework.

Periodic Terminator Orbits: Periodic Terminator Orbits (PTOs) are a class of periodic solutions within the ANH3BP framework that lie on the terminator plane. Starting from frozen terminator configurations, FTOs can be corrected into fully periodic solutions (PTOs) using a single-shooting differential corrector by enforcing periodicity conditions on the flow map over one orbital period [59, 66, 68, 70]. PTO families undergo multiple bifurcations, leading to the emergence of additional branches of periodic solutions called Resonant Terminator Orbits (RTOs) [68, 70].

Resonant Terminator Orbits: Resonant Terminator Orbits (RTOs) are a class of terminator orbits that emerge from period-multiplying bifurcations of the PTO family [71] and are characterized by increased geometric complexity and enhanced spatial coverage around the small body [68]. In fact, while PTOs exhibit Sun-body-spacecraft angles, ϕ , of about 90° , RTOs allow for much lower angles, with ϕ as low as 30° . For these reasons,

they are particularly attractive for global mapping and reconnaissance operations, and are in fact extensively studied and being considered for current and future missions [9, 75].

2.1.4. Elliptic Augmented Normalized Hill Three Body Problem

In this thesis, a more realistic dynamical model which accounts for the asteroid's elliptic heliocentric orbit is used in Chapter 4 to simulate the designed science orbits. Their equations of motion are obtained by defining e_{sb} and f as the heliocentric asteroid's orbit eccentricity and true anomaly, respectively [17]:

$$\begin{aligned} x'' &= 2y' + \frac{1}{1 + e_{sb} \cos f} \left(-\frac{x}{r^3} + 3x + \beta \right), \\ y'' &= -2x' + \frac{1}{1 + e_{sb} \cos f} \left(-\frac{y}{r^3} \right), \\ z'' &= \frac{1}{1 + e_{sb} \cos f} \left(-\frac{z}{r^3} \right) - z, \end{aligned} \tag{2.11}$$

where the superscript $'$ denotes the derivatives with respect to the asteroid's true anomaly, which is here utilized as the independent variable. The time evolution of the state elements is retrieved by applying the chain rule.

2.2. Variational Dynamics and Nonlinearity Quantification

This section provides a concise theoretical recap of the mathematical and dynamical tools required to describe the dynamics with a variational approach and to quantify dynamical nonlinearities in nonlinear systems. In particular, it introduces the concepts of State Transition Matrices (STMs) and higher-order State Transition Tensors (STTs), together with the tensorial framework necessary to characterize higher-order nonlinear effects through norms and tensor eigenpairs.

These concepts form the theoretical foundation for the quantification of dynamical nonlinearities adopted in this thesis and are instrumental to the formulation of the minimum nonlinearity cost function used in Chapter 6. By enabling an analytic characterization of nonlinear error growth along a reference trajectory, the tools reviewed in this section allow nonlinear effects to be explicitly accounted for and minimized during trajectory optimization.

2.2.1. State Transition Matrices and Tensors

The local dynamics of a nonlinear dynamical system in the neighborhood of a reference trajectory can be approximated by a series expansion of the system flow, and using the state transition matrix (STM) and state transition tensors (STTs) [76]. Consider the spacecraft dynamics described in canonical form as a first-order ODE:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), t), \quad \mathbf{x}(t_0) = \mathbf{x}_0, \quad (2.12)$$

where $\mathbf{x} \in \mathbb{R}^n$ denotes the state vector and $\mathbf{f}(\cdot)$ the equations of motion. For a given initial condition, the solution of the system can be expressed through its solution flow [77]:

$$\mathbf{x}(t) = \varphi(t, t_0; \mathbf{x}_0), \quad (2.13)$$

which maps the initial state at t_0 to the state at time t .

Let $\bar{\mathbf{x}}(t) = \varphi(t, t_0; \bar{\mathbf{x}}_0)$ denote a reference trajectory generated from the nominal initial condition $\bar{\mathbf{x}}_0$, and define the state deviation as $\delta\mathbf{x}(t) = \mathbf{x}(t) - \bar{\mathbf{x}}(t)$. The evolution of trajectories near the reference can be described by performing a Taylor series expansion (TSE) of the flow with respect to the initial deviation $\delta\mathbf{x}_0 = \mathbf{x}_0 - \bar{\mathbf{x}}_0$. Truncating the expansion at order m yields:

$$\delta\mathbf{x}_i(t) \approx \sum_{p=1}^m \frac{1}{p!} \Phi_{j_1 \dots j_p}^i(t, t_0) \delta x_{0,j_1} \dots \delta x_{0,j_p}, \quad (2.14)$$

where Einstein summation is implied and $\Phi_{j_1 \dots j_p}^i$ denotes the p -th order partial derivative of the flow with respect to the initial state,

$$\Phi_{j_1 \dots j_p}^i(t, t_0) = \frac{\partial^p \varphi_i(t, t_0; \mathbf{x}_0)}{\partial x_{0,j_1} \dots \partial x_{0,j_p}} \quad (2.15)$$

The first-order term of (2.14) corresponds to the classical State Transition Matrix (STM),

$$\delta\mathbf{x}(t) \approx \Phi(t, t_0) \delta\mathbf{x}_0, \quad (2.16)$$

which provides a linear mapping between state deviations at different times. Higher-order partial derivatives of the flow define the State Transition Tensors (STTs), which generalize the STM by capturing nonlinear contributions to the state deviation. Following the convention adopted in the literature, the STM can be interpreted as the first-order

STT, while the p -th order STT is a tensor of order $p + 1$ [76, 78].

The time evolution of the STM and STTs can be obtained by differentiating the Taylor-expanded solution with respect to time and equating terms of equal order in the initial deviation, as done by Park and Scheeres in [76]. Following this procedure it is possible to obtain the variational equations governing the dynamics of the STM and STTs:

$$\frac{d}{dt} \Phi_{ji_1} = \mathcal{A}_{jk_1} \Phi_{k_1 i_1}, \quad (2.17)$$

$$\frac{d}{dt} \Phi_{ji_1 i_2} = \mathcal{A}_{jk_1} \Phi_{k_1 i_1 i_2} + \mathcal{A}_{jk_1 k_2} \Phi_{k_1 i_1} \Phi_{k_2 i_2}, \quad (2.18)$$

$$\begin{aligned} \frac{d}{dt} \Phi_{ji_1 i_2 i_3} = & \mathcal{A}_{jk_1} \Phi_{k_1 i_1 i_2 i_3} + \mathcal{A}_{jk_1 k_2} (\Phi_{k_1 i_1} \Phi_{k_2 i_2 i_3} + \Phi_{k_1 i_1 i_2} \Phi_{k_2 i_3} + \Phi_{k_1 i_1 i_3} \Phi_{k_2 i_2}) \\ & + \mathcal{A}_{jk_1 k_2 k_3} \Phi_{k_1 i_1} \Phi_{k_2 i_2} \Phi_{k_3 i_3}. \end{aligned} \quad (2.19)$$

Here, $\mathcal{A}_{jk_1} = \partial f_j / \partial x_{k_1}$ denotes the Jacobian of the dynamics evaluated along the reference trajectory, while $\mathcal{A}_{jk_1 k_2}$ and $\mathcal{A}_{jk_1 k_2 k_3}$ denote the higher-order partial derivatives of the vector field with respect to the state. All derivatives are evaluated along the reference trajectory $\bar{\mathbf{x}}(t)$. Similar expressions can be derived for higher-order STTs following the same recursive structure [76].

2.2.2. Tensor Fundamentals

In this section, some fundamental notions about tensors are outlined. Tensors are the generalization of scalars, vectors, and matrices to m -th order arrays, where the order (m) is defined as the number of indices needed to define the tensor.

Tensor Notation and Multilinear Products

The tensor notation adopted in this thesis follows [79]. Let $\mathcal{T}^{(\ell)}$ denote a real ℓ -th order tensor with dimension n along each mode, whose elements are indexed as

$$\mathcal{T}_{i_1 i_2 \dots i_\ell}, \quad 1 \leq i_1, i_2, \dots, i_\ell \leq n. \quad (2.20)$$

Repeated indices imply summation according to Einstein notation.

Given a vector $\mathbf{v} \in \mathbb{R}^n$, the contraction of $\mathcal{T}^{(\ell)}$ with $\mathbf{v}$ along $\ell - 1$ modes yields a vector in $\mathbb{R}^n$, whose j -th component is

$$(\mathcal{T}^{(\ell)} \cdot \mathbf{v}^{\ell-1})_j = \mathcal{T}_{ji_2 \dots i_\ell} v_{i_2} \cdots v_{i_\ell}. \quad (2.21)$$

This operation generalizes the matrix–vector product and reduces to it when $\ell = 2$, with an $n \times n$ matrix interpreted as a second-order tensor.

More generally, contracting an ℓ -th order tensor with ℓ copies of $\mathbf{v}$ produces a scalar,

$$\mathcal{T}^{(\ell)} \mathbf{v}^\ell = \mathcal{T}_{i_1 i_2 \dots i_\ell} v_{i_1} v_{i_2} \cdots v_{i_\ell}. \quad (2.22)$$

Supersymmetric Tensors

A tensor is said to be *supersymmetric* if it is invariant under any permutation of its indices [79].

State Transition Tensors (STTs) associated with autonomous dynamics are supersymmetric, as they are defined by repeated partial derivatives of the system flow with respect to the state.

However, not all tensors encountered in this thesis are supersymmetric by construction and require symmetrization. For a generic nonsymmetric tensor $\mathcal{T}^{(\ell)}$, an equivalent supersymmetric tensor $\mathcal{S}^{(\ell)}$ can be constructed through symmetrization such that

$$\mathcal{S}^{(\ell)} \mathbf{v}^\ell = \mathcal{T}^{(\ell)} \mathbf{v}^\ell, \quad (2.23)$$

for all $\mathbf{v} \in \mathbb{R}^n$. In this thesis, tensor symmetrization is performed numerically using the `syntensor` function from MATLAB’s Tensor Toolbox [80].

Induced Tensor Norms

Given a supersymmetric ℓ -th order tensor $\mathcal{T}^{(\ell)}$, its induced 2-norm is defined as the maximum value of the associated homogeneous polynomial over the unit sphere, namely

$$\|\mathcal{T}^{(\ell)}\|_2 = \max_{\|\mathbf{v}\|_2=1} \mathcal{T}^{(\ell)} \mathbf{v}^\ell. \quad (2.24)$$

This definition generalizes the concept of the induced matrix norm and provides a scalar measure of the worst-case nonlinear amplification captured by the tensor [81].

For even-order supersymmetric tensors, the induced 2-norm is equal to the largest Z-eigenvalue of the tensor, where a Z-eigenpair $(\lambda, \mathbf{v})$ satisfies

$$\mathcal{T}^{(\ell)} \mathbf{v}^{\ell-1} = \lambda \mathbf{v}, \quad \|\mathbf{v}\|_2 = 1. \quad (2.25)$$

As shown in [81], this property allows the computation of the induced norm through the

dominant Z-eigenvalue, providing a direct and computationally tractable link between tensor representations of nonlinear dynamics and scalar measures of nonlinearity.

In this thesis, induced tensor norms are used to quantify the magnitude of higher-order nonlinear effects included in the State Transition Tensors. The resulting scalar measure, obtained from the dominant Z-eigenvalue, forms the basis of the Tensor Eigenpair Measure of Nonlinearity and is directly embedded in the minimum nonlinearity stochastic optimization framework.

2.2.3. Measures of Nonlinearity

Measures of Nonlinearity (MoNs) aim at quantifying the degree to which nonlinear effects invalidate local linear approximations of the dynamics. Several classes of MoNs have been proposed in the literature, each targeting different aspects of nonlinear behavior. Sampling-based and Monte Carlo MoNs estimate nonlinearity through statistical dispersion or loss of Gaussianity [82]. Curvature-based indicators characterize local deviations from linearity using higher-order derivatives of the flow [76]. Other approaches rely on reachability analyses to identify regions where linear models remain valid [83]. While these methods provide valuable diagnostic insight, most are either empirical, computationally intensive, or not readily integrable within optimization-based control frameworks.

A distinct class of MoNs is formed by measures that leverage STT information, exploiting the higher-order terms of the Taylor expansion of the system dynamics to analytically characterize the growth of nonlinear errors. These measures directly relate nonlinear effects to State Transition Tensors and admit closed-form scalar quantities that can be embedded into optimization problems. Among these, measures based on tensor eigenpairs are of particular interest for this work.

Tensor Eigenpair Measures of Nonlinearity

The Tensor Eigenpair Measure of Nonlinearity (TEMoN), introduced by Jenson and Scheeres [79], is a semi-analytical MoN that quantifies how nonlinear a dynamical system is in the neighborhood of a nominal state or reference trajectory. TEMoN is derived from the higher-order terms of the Taylor expansion of the system flow, which capture deviations between the true nonlinear dynamics and their linear approximation.

For a given expansion order $k \geq 2$, TEMoN is defined as the induced norm of the corre-

sponding k -th order State Transition Tensor,

$$\mathcal{T}_k = \|\Phi^{(k)}\|_2 = \max_{\|\mathbf{v}\|_2=1} \Phi^{(k)}\mathbf{v}^k, \quad (2.26)$$

which provides an upper bound on the worst-case nonlinear error associated with the k -th order term of the Taylor expansion. Large values of TEMoN indicate that higher-order terms dominate the local dynamics, implying that propagation of uncertainty based on linearization is unreliable.

In the context of stochastic optimal control, TEMoN is particularly attractive as it provides an analytic upper bound on nonlinear error growth that can be directly minimized. This property establishes a direct connection with the minimum nonlinearity formulation proposed by Qi and Oguri [51], where nonlinear error accumulation is explicitly minimized in a linear covariance steering framework. As a result, TEMoN enables nonlinear effects to be accounted for during optimization, rather than treated a posteriori through re-linearization or heuristic corrections.

Z-eigenpairs and Numerical Computation

As introduced in the previous sections, for a supersymmetric ℓ -th order tensor $\mathcal{T}^{(\ell)}$, a Z-eigenpair $(\lambda, \mathbf{v})$ is defined as a scalar $\lambda \in \mathbb{R}$ and a unit-norm vector $\mathbf{v} \in \mathbb{R}^n$ satisfying Eq. (2.25).

For even-order supersymmetric tensors, the largest Z-eigenvalue coincides with the induced tensor 2-norm and therefore characterizes the worst-case nonlinear amplification captured by the tensor [81, 84].

The dominant Z-eigenpair is computed numerically using the Symmetric Shifted Higher-Order Power Method (SS-HOPM), originally introduced by Kolda and Mayo [85] and later analyzed in the context of Z-eigenvalue problems [86]. SS-HOPM is an iterative fixed-point algorithm that exploits tensor supersymmetry to guarantee convergence to a real Z-eigenpair associated with a locally maximum eigenvalue.

In this work, SS-HOPM is used with a multi-start strategy to efficiently compute an estimate of the dominant Z-eigenvalue of the State Transition Tensors underlying the Tensor Eigenpair Measure of Nonlinearity.

2.3. Optimal Control Theory

This section reviews the theoretical background of Optimal Control Theory, following the classical results of Lawden's primer vector theory [87].

2.3.1. The Optimal Control Problem

Optimal control involves minimizing a chosen objective function while obeying to the system's dynamic. In this work, deterministic optimal control will be employed to design orbit transfers departing from the initial orbital state $\mathbf{x}_0$ and arriving at the final state $\mathbf{x}_f$. The most generic optimal control problem with a cost function in Lagrange form [88] can be stated as:

$$\begin{aligned} \min_{\mathbf{x}, m, \mathbf{u}, t_0, t_f} \quad & J = \int_{t_0}^{t_f} L \, dt \\ \text{s.t.} \quad & \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, m, \mathbf{u}, t), \quad \dot{m} = -\Gamma \dot{m}_{\max}, \\ & \boldsymbol{\psi}_0(\mathbf{x}_0, t_0) = 0, \quad \boldsymbol{\psi}_f(\mathbf{x}_f, t_f) = 0. \end{aligned} \quad (2.27)$$

where J is the objective function to be minimized, L is the lagrangian cost, $\mathbf{x}$ and $\mathbf{u}$ are the state and control vector, respectively, $\mathbf{f}$ indicates the equations of motion that determine the system's dynamics and $\boldsymbol{\psi}$ are the boundary conditions that constrain the initial and final state.

The constrained problem is converted into an unconstrained problem by leveraging the use of Lagrange multipliers associated with both the dynamics and the terminal constraint. The lagrange multipliers associated with the dynamics denoted with $\boldsymbol{\lambda} = [\boldsymbol{\lambda}_r^\top, \boldsymbol{\lambda}_v^\top, \lambda_m]^\top$ are also called costates, while the ones associated with the boundary conditions are denoted with $\boldsymbol{\nu} = [\boldsymbol{\nu}_0^\top, \boldsymbol{\nu}_f^\top]^\top$.

The unconstrained optimal control problem is obtained as:

$$\begin{aligned} \min \quad & J_A(\mathbf{x}, m, \mathbf{u}, \boldsymbol{\lambda}, \boldsymbol{\nu}_0, \boldsymbol{\nu}_f, t_0, t_f), \\ J_A(\cdot) = & \boldsymbol{\nu}_0^\top \boldsymbol{\psi}_0(\mathbf{x}_0, t_0) + \boldsymbol{\nu}_f^\top \boldsymbol{\psi}_f(\mathbf{x}_f, t_f) + \int_{t_0}^{t_f} [H(\mathbf{x}, m, \mathbf{u}, \boldsymbol{\lambda}, t) - \boldsymbol{\lambda}_x^\top \dot{\mathbf{x}} - \lambda_m \dot{m}] \, dt, \end{aligned}$$

where $J_A(\cdot)$ is the augmented cost functional and $H(\cdot)$ is a scalar quantity called "Control Hamiltonian" introduced to solve the optimal control problem.

$$H(\cdot) \triangleq L + \boldsymbol{\lambda}_x^\top \mathbf{f} - \lambda_m \Gamma \dot{m}_{\max}. \quad (2.28)$$

The necessary conditions for a minimum of $J_A(\cdot)$ are found through the Calculus of Variations [89], by imposing $\partial J_A = 0$ with respect to all the variables of the problem. The necessary conditions lead to the Euler-Lagrange equations:

$$\dot{\boldsymbol{\lambda}}^\top = -\frac{\partial H}{\partial \mathbf{x}}, \quad \frac{\partial H}{\partial \mathbf{u}} = 0, \quad \mathbf{f}(\mathbf{x}, m, \mathbf{u}, t) - \dot{\mathbf{x}} = 0, \quad \Gamma \dot{m}_{\max} + \dot{m} = 0. \quad (2.29)$$

and the transversality conditions:

$$\begin{aligned} & \left\{ \nu_0^\top \frac{\partial \psi_0}{\partial \mathbf{x}_0} + \boldsymbol{\lambda}_x^\top(t_0) = 0 \text{ or } \delta \mathbf{x}_0 = 0 \right\}, \quad \left\{ \nu_f^\top \frac{\partial \psi_f}{\partial \mathbf{x}_f} - \boldsymbol{\lambda}_x^\top(t_f) = 0 \text{ or } \delta \mathbf{x}_f = 0 \right\}, \\ & \{ \psi_0 = 0 \text{ or } \delta \nu_0 = 0 \}, \quad \{ \psi_f = 0 \text{ or } \delta \nu_f = 0 \}, \\ & \left\{ \nu_0^\top \frac{\partial \psi_0}{\partial t_0} - H^*(\cdot, t_0) = 0 \text{ or } \delta t_0 = 0 \right\}, \quad \left\{ \nu_f^\top \frac{\partial \psi_f}{\partial t_f} + H^*(\cdot, t_f) = 0 \text{ or } \delta t_f = 0 \right\}, \\ & \lambda_m(t_f) = 0 \end{aligned}$$

Pontryagin's Minimum Principle (PMP) states that the optimal control $\mathbf{u}^*$ must minimize the hamiltonian H throughout the trajectory in order to minimize J on an optimal trajectory $\mathbf{x}^*$:

$$H(\mathbf{x}^*, \mathbf{u}^*, \boldsymbol{\lambda}, t) \leq H(\mathbf{x}^*, \mathbf{u}, \boldsymbol{\lambda}, t) \quad (2.30)$$

which implies that the generic necessary condition for $\mathbf{u}^*$ is the following:

$$\mathbf{u}^* = \arg \min_{\mathbf{u} \in \Omega} H(\mathbf{x}, \mathbf{u}, \boldsymbol{\lambda}, t) \quad (2.31)$$

Given that the control is within all the admissible sets of control inputs Ω .

2.3.2. Minimum Fuel Problem

While the previous section laid out the generic forms of optimality, in this section the specific conditions needed to solve a minimum fuel, fixed time and fixed initial and final states problem are detailed.

The lagrangian cost for the fuel optimal problem is defined as $L = \|\mathbf{u}\|_2$, yielding the objective function:

$$\min J = \int_0^T \|\mathbf{u}\|_2 dt \quad (2.32)$$

The control Hamiltonian is:

$$H = \|\mathbf{u}\|_2 + \boldsymbol{\lambda}^\top \mathbf{f} \quad (2.33)$$

Let the controller take the following form:

$$\mathbf{u}^* = \Gamma^* \hat{\boldsymbol{\alpha}} \quad (2.34)$$

where Γ^* is the throttling parameter ($\Gamma^* \in [0, 1]$) and $\hat{\boldsymbol{\alpha}}$ is the optimal control direction. Let $u_{\max}$ be the maximum admitted control magnitude and applying PMP Eq. (2.31) yields:

$$\hat{\boldsymbol{\alpha}} = \frac{\mathbf{p}}{\|\mathbf{p}\|_2} \quad (2.35)$$

$$\Gamma^* = \begin{cases} u_{\max}, & \text{if } \|\mathbf{p}\|_2 > 1 \rightarrow \mathcal{S} > 0, \\ 0, & \text{if } \|\mathbf{p}\|_2 < 1 \rightarrow \mathcal{S} < 0. \\ \in [0, u_{\max}], & \text{if } \|\mathbf{p}\|_2 = 1 \rightarrow \mathcal{S} = 0. \end{cases} \quad (2.36)$$

where $\mathbf{p}$ is the so-called primer vector and $\mathcal{S}$ is the switching function, defined as:

$$\mathbf{p} = -\boldsymbol{\lambda}_v, \quad \mathcal{S} = \|\mathbf{p}\|_2 - 1 \quad (2.37)$$

It is possible to verify that the condition $\mathcal{S} = 0$ happens only instantaneously [90] and therefore there are no single arcs and the optimal control profile is bang-bang, meaning that the throttling is either at its minimum or its maximum value.

It is important to notice that the optimal control conditions require the use of a piecewise function which is not continuous nor smooth. This can lead to several challenges for numerical solvers, classically addressed in the literature either via switching detection [91] or smoothing techniques [92]. The present work adopts smoothing particularly through an hyperbolic tangent function to approximate the bang-bang profile [92] which has been shown to be more robust and more computationally efficient [93]:

$$\Gamma^* \approx \frac{u_{\max}}{2} \left[1 + \tanh \left(\frac{\mathcal{S}}{\rho} \right) \right] \quad (2.38)$$

given a small value of the smoothing parameter $\rho > 0$. The approximation gets better as $\rho \rightarrow 0$, but numerical robustness decreases if the parameter is too small. For these

reasons, this thesis uses $\rho = 10^{-4}$ for the approximation.

Embedding Γ^* and $\hat{\alpha}$ into the control Hamiltonian, the Euler-Lagrange equations are retrieved:

$$\dot{\mathbf{x}} = \frac{\partial H}{\partial \boldsymbol{\lambda}} \quad \dot{\boldsymbol{\lambda}} = -\frac{\partial H}{\partial \mathbf{x}} \quad (2.39)$$

Indirect optimization always yields a Two Point Boundary Value Problem (TPBVP) [87], which is typically solved via the shooting method. For a more comprehensive description of the solution methods to solve the indirect optimization problems in this thesis, readers are referred to Chapter 5 and specifically Section 5.1.2.

2.4. Convex Programming

Convex programming refers to a class of direct optimization methods characterized by a convex objective function, affine equality constraints, and a convex feasible set defined by inequality constraints. This section introduces the fundamental concepts of convex programming and discusses how such techniques can be systematically exploited to address non-convex optimal control problems in astrodynamics. Particular focus is put on sequential convex programming (SCP) methods, which are extensively employed throughout this thesis.

2.4.1. Fundamental Definitions

Definition 1 (Convex Set): A set C is convex if the line segment between any two points in C lies in C , i.e., if [23]:

$$\forall x_1, x_2 \in C, \text{ and } \forall \theta \in [0, 1] \quad \Rightarrow \quad \theta x_1 + (1 - \theta)x_2 \in C \quad (2.40)$$

Figure 2.2 shows some examples of convex and non-convex sets.

An important property of convex sets is that convexity is preserved under set intersection [23], so it is possible to build more complicated convex sets by intersecting simpler ones.

Definition 2 (Convex Function): A function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is convex if f lies below the line which connects any of its two points, i.e., if $\text{dom}(f)$ is a convex set and [23]:

$$\forall x, y \in \text{dom}(f), \forall \theta \in [0, 1] \quad \Rightarrow \quad f(\theta x + (1 - \theta)y) \leq \theta f(x) + (1 - \theta)f(y). \quad (2.41)$$

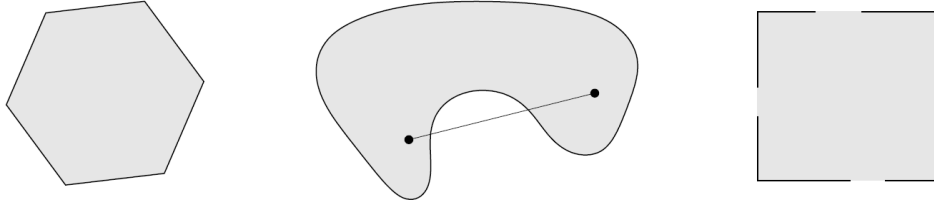


Figure 2.2: Example of convex and nonconvex sets. *Left.* Hexagon (including its boundaries) is convex. *Middle.* Nonconvex set, as the line segment the two points is not contained in the set. *Right.* Square is nonconvex as some boundary points are not contained. Figures from [23]

This means that the region above the curve the set must be a convex set.

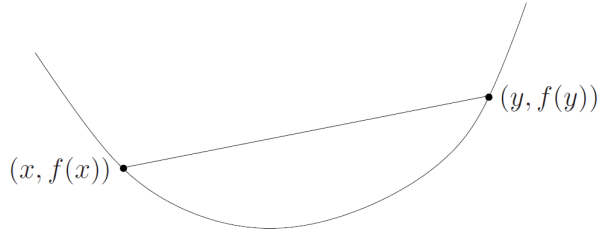


Figure 2.3: Example of a convex function. Figure from [23]

Definition 3 (Convex Problem): A mathematical optimization problem is convex if it can be written in the standard form [18, 23]:

$$\begin{aligned}
 \min_{\mathbf{x}} \quad & f_0(\mathbf{x}) \\
 \text{s.t.} \quad & g_j(\mathbf{x}) = 0, \quad j = 1, \dots, n_{eq}, \\
 & h_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, n_{ineq},
 \end{aligned} \tag{2.42}$$

and the following conditions are satisfied:

1. $f_0 : \mathbb{R}^n \rightarrow \mathbb{R}$ is a convex objective function;
2. $g_j : \mathbb{R}^n \rightarrow \mathbb{R}$, $j = 1, \dots, n_{eq}$, are affine equality constraints, i.e., $g_j(x) = a_j^\top x + b_j$;
3. $h_i : \mathbb{R}^n \rightarrow \mathbb{R}$, $i = 1, \dots, n_{ineq}$, are convex inequality constraints.

Under these conditions, the feasible set of solutions is convex and any local minimum of the problem is also a global minimum. In this thesis, all convex problems are solved in MATLAB with YALMIP [94] and MOSEK [95].

2.4.2. Convexification Techniques

Convexification refers to a set of techniques aimed at transforming a non-convex optimal control problem into an equivalent or approximate convex formulation. Whenever possible, *lossless convexification* is preferable, as it is a transformation that preserves the original feasible set and optimal solution without introducing approximations. Lossless convexification is highly problem-dependent and typically relies on suitable reformulations of variables or constraints. When these transformations are not possible, approximations must be employed, most notably successive linearization of the constraints via Taylor series expansions (TSEs).

A. Change of variables: A first class of lossless convexification techniques consists of introducing a change of variables that removes non-convexities from the problem formulation. By redefining nonlinear expressions in terms of new decision variables, it is sometimes possible to make the equations of motions or constraints affine or convex [96].

B. *Lifting* via slack variables: An alternative lossless convexification strategy is *lifting*, where auxiliary variables are introduced to expand the feasible set of the original problem and transform it into a higher-dimensional convex set. This approach is commonly implemented through the introduction of slack variables. Although the operation of *lifting* convexifies the constraint, it alone does not guarantee feasibility with respect to the original problem. A separate analytical proof is therefore required to show that the optimal solution of the lifted problem also satisfies the original constraints, as shown in [96].

C. Successive linearization via TSEs: When lossless convexification is not possible, non-convex dynamics and constraints can be approximated via first-order Taylor series expansions (TSEs) about a reference trajectory. This procedure yields affine equality constraints and convex inequality constraints that are valid locally. In this thesis, where the equations of motion are inherently nonlinear, linearization about a nominal trajectory combined with time discretization is systematically employed. Since this approach introduces approximation errors, it is adopted within an iterative sequential convex programming (SCP) framework.

2.4.3. Sequential Convex Programming

Sequential Convex Programming (SCP) is an iterative optimization framework designed to solve nonlinear and non-convex optimal control problems by successively approximating

them with a sequence of convex subproblems.

The original optimal control problem with Lagrange cost is formulated as [97]:

$$\begin{aligned}
\min_{\mathbf{x}, \mathbf{u}} \quad & J_0 = \int_{t_0}^{t_f} L(\mathbf{x}(t), \mathbf{u}(t), t) dt \\
\text{s.t.} \quad & \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) \\
& \mathbf{g}_i(\mathbf{x}(t), \mathbf{u}(t), t) = 0 \quad \forall i, \quad \mathbf{h}_j(\mathbf{x}(t), \mathbf{u}(t), t) \leq 0 \quad \forall j, \\
& \mathbf{g}_{\text{affine}}(\mathbf{x}(t), \mathbf{u}(t), t) = 0 \quad \forall i, \quad \mathbf{h}_{\text{cvx}}(\mathbf{x}(t), \mathbf{u}(t), t) \leq 0 \quad \forall j
\end{aligned} \tag{2.43}$$

where the Lagrangian cost L is assumed to be convex in $\mathbf{x}, \mathbf{u}$. Without loss of generality, a nonconvex cost can always be convexified by introducing an auxiliary variable and an inequality constraint [23]. Here, the equations of motion are explicitly expressed and not included as nonlinear equality constraints, and $\mathbf{g}_{\text{affine}}, \mathbf{g}_i, \mathbf{h}_{\text{cvx}}$ and $\mathbf{h}_j$ denote the affine equality, non-affine equality, convex inequality and non-convex inequality constraint functions, respectively.

Starting from the generic nonlinear, the SCP methodology systematically convexifies all the non-convex constraints $\mathbf{g}_i \forall i, \mathbf{h}_j \forall j$ using the techniques presented in Section 2.4.2. Let $\tilde{\mathbf{g}}_i(\cdot)$ and $\tilde{\mathbf{h}}_j(\cdot)$ denote their convexified counterparts. By applying linearization and discretization, the convex subproblem is obtained as:

$$\begin{aligned}
\min_{\mathbf{x}_k, \mathbf{u}_k} \quad & J_0 = \sum_{k=0}^N L(\mathbf{x}_k, \mathbf{u}_k) \\
\text{s.t.} \quad & \tilde{\mathbf{g}}_i(\mathbf{x}_k, \mathbf{u}_k) = 0 \quad \forall i, \quad \tilde{\mathbf{h}}_j(\mathbf{x}_k, \mathbf{u}_k) \leq 0 \quad \forall j, \\
& \mathbf{g}_{\text{affine}}(\mathbf{x}_k, \mathbf{u}_k) = 0 \quad \forall i, \quad \mathbf{h}_{\text{cvx}}(\mathbf{x}_k, \mathbf{u}_k) \leq 0 \quad \forall j
\end{aligned} \tag{2.44}$$

where $\tilde{\mathbf{g}}_i(\mathbf{x}_k, \mathbf{u}_k) = 0$ also contains the linearized and discretized EOMs. In the next sections the linearization and discretization steps for a generic problem are outlined, while the specific steps adopted in this thesis for the adaptive mesh formulation and the stochastic formulation are described in their respective Chapters 5 and 6.

After time discretization, the resulting finite-dimensional convex subproblem is solved to obtain an updated trajectory, which is subsequently used as the new linearization reference. This process is repeated until convergence criteria are satisfied. A conceptual overview of the SCP iteration is provided in Fig. 2.4. It can be noted that a reference trajectory must be given to the algorithm as a first guess. This guess can be infeasible with respect to both the dynamics and the nonconvex constraints but it must be feasible with respect to the convex path constraints [18].

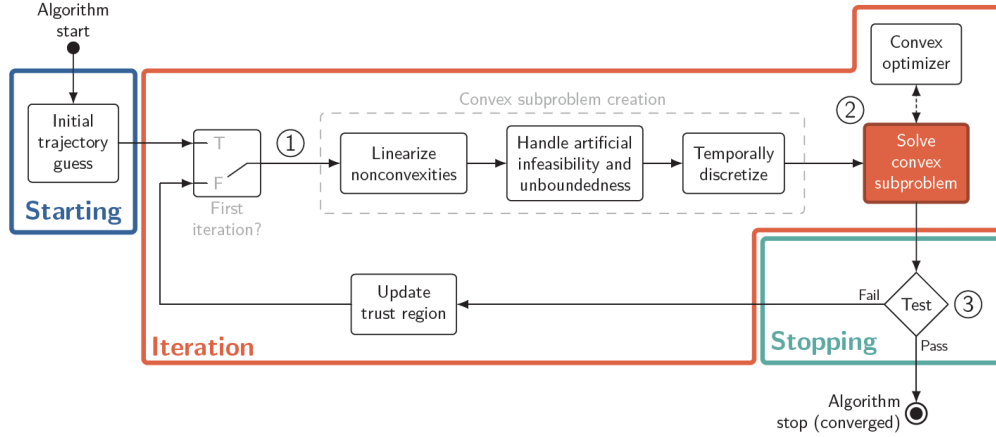


Figure 2.4: Block diagram illustration of a typical SCP algorithm. Credits: [18]

Despite its effectiveness, two major issues are common when dealing with sequential convex programming: *artificial unboundedness* and *artificial infeasibility* [18, 19, 25, 28].

Artificial unboundedness: Artificial unboundedness occurs when the linearized cost or constraints allow the optimizer to exploit directions that are not valid for the original nonlinear problem. The accuracy of linear approximations is limited to the vicinity of the reference solution. Therefore, the convex subproblem must be maintained sufficiently close to the linearization reference point at each iteration. Furthermore, the solution might become unbounded as well, as a result of the linearization, pushing the convex cost to infinity.

This issue is typically addressed by including *trust regions* constraints on the state and control variables, constraining these variables to stay arbitrarily close to the reference trajectory. This retains the validity of the linearization and prevents the problem from becoming unbounded. At every iteration, the linearized approximation of the original non-convex elements is measured by the update rule linked to the trust region. This allows the algorithm decide whether to maintain, expand, or shrink the trust region radius.

Artificial infeasibility: Artificial infeasibility arises when the linearization overly restricts the feasible set, making the convex subproblem infeasible even though the original problem is feasible. This can happen when the trust region radius is too small and it confines the solution variables to a portion of the solution space that is outside of the feasible set, or when the intersection of the convexified path constraints is empty [18].

This is typically addressed in practice by either introducing for each linearized constraint an unconstrained but penalized slack variable, or by adding soft penalty terms to the

original cost function to penalize constraint violations. Ultimately, both strategies result in the subproblem being guaranteed to be feasible.

Linearization and Discretization

In SCP algorithms, non-affine system dynamics constitute a primary source of non-convexity and must be linearized in order to obtain affine equality constraints. In this paragraph, the standard procedure for linearizing and discretizing a nonlinear controlled dynamical system is briefly reviewed.

Given a controlled nonlinear system in the generic form Eq. (2.1), it can be linearized about a reference trajectory and control $\{\mathbf{x}^*(t), \mathbf{u}^*(t)\}$, to yield:

$$\dot{\mathbf{x}} \approx A(t)\mathbf{x}(t) + B(t)\mathbf{u}(t) + \mathbf{c}(t) \quad (\text{or } \delta\dot{\mathbf{x}}(t) \approx A(t)\delta\mathbf{x}(t) + B(t)\delta\mathbf{u}(t)) \quad (2.45)$$

where $\delta\mathbf{x}(t) = \mathbf{x}(t) - \mathbf{x}^*(t)$, $\delta\mathbf{u}(t) = \mathbf{u}(t) - \mathbf{u}^*(t)$, and $A \in \mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times n_u}$, $\mathbf{c} \in \mathbb{R}^{n_x}$ are given by:

$$\begin{aligned} A(t) &= \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{\mathbf{x}^*(t), \mathbf{u}^*(t)}, \quad B(t) = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right|_{\mathbf{x}^*(t), \mathbf{u}^*(t)}, \\ \mathbf{c}(t) &= \mathbf{f}(\mathbf{x}^*(t), \mathbf{u}^*(t), t) - A(t)\mathbf{x}^*(t) - B(t)\mathbf{u}^*(t). \end{aligned} \quad (2.46)$$

The resulting Linear Time-Varying (LTV) system is discretized by partitioning the time interval $[t_0, t_f]$ into N subintervals such that $t_0 < t_1 < \dots < t_N = t_f$. Integrating Eq. (2.45) over a time interval $[t_k, t_{k+1})$ with impulsive control Eq. (2.2) it is possible to obtain the discrete-time form of the LTV system:

$$\mathbf{x}_{k+1} = A_k \mathbf{x}_k + B_k \mathbf{u}_k + \mathbf{c}_k, \quad k = 0, 1, \dots, N-1, \quad (2.47)$$

where:

$$A_k = \Phi(t_{k+1}, t_k), \quad B_k = A_k B, \quad \mathbf{c}_k = A_k \int_{t_k}^{t_{k+1}} \Phi^{-1}(t, t_k) \mathbf{c}(t) dt \quad (2.48)$$

where $B_k = A_k B$ follows from the impulsive control assumption, and Φ is the state transition matrix (STM) that satisfies the differential equation Eq. (2.17).

The numerical procedure adopted to compute the discrete-time matrices A_k , B_k , and $\mathbf{c}_k$ is detailed in Appendix A.

2.5. Stochastic Optimal Control

Spacecraft operations are inherently subject to various sources of uncertainty, such as imperfect state knowledge, unmodeled accelerations, maneuver execution errors, and missed thrust events, among others. Neglecting these uncertainties in trajectory optimization problems may yield solutions that are optimal in a deterministic setting, but unsafe or even infeasible when applied in practice, especially in dynamically challenging environments such as small-body close-proximity operations. Stochastic optimal control provides a framework to design control strategies that systematically address these uncertainties. However, general stochastic formulations are often computationally intractable for complex nonlinear systems.

This thesis adopts a chance-constrained formulation that ensures probabilistic safety guarantees while remaining computationally tractable through convex optimization [49]. Although the complete stochastic problem formulation is presented in detail in Chapter 6, this section introduces the mathematical preliminaries necessary to model the system in a stochastic setting and reformulate it into computationally tractable convex forms.

2.5.1. Stochastic Differential Equations

To model spacecraft motion under uncertainty, a powerful mathematical tool that can be employed are stochastic differential equations (SDEs) [98]. The main sources of uncertainty in this work are in the orbital state $\mathbf{x}$ and unmodeled accelerations. The disturbing accelerations are modeled as a continuous-time Brownian motion, and the stochastic system can be expressed by using the following set of SDEs:

$$d\mathbf{x} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t)dt + G(\mathbf{x}, \mathbf{u}, t)d\mathbf{w}(t) \quad (2.49)$$

where $\mathbf{f}(\mathbf{x}, \mathbf{u}, t)$ denotes the system's deterministic EOM under control, $d\mathbf{w}(t) \in \mathbb{R}^{n_w}$ is standard Brownian motion ($\mathbb{E}[d\mathbf{w}] = \mathbf{0}_{n_w}$ and $\mathbb{E}[d\mathbf{w}(t)d\mathbf{w}^\top(t)] = I_{n_w}dt$) and $G(\mathbf{x}, \mathbf{u}, t)$ is the covariance matrix of the noise term, mapping the Brownian motion to the dynamics of the system. This matrix is also commonly referred to in SDEs as the diffusion matrix, and defines how random disturbances enter the system dynamics, in terms of both the intensity and the directionality of the stochastic perturbations.

Numerically, Eq. (2.49) is difficult to solve because Brownian motion is not differentiable. Hence, to integrate SDEs this thesis approximates it with a zero-order hold white noise

over a small integration step Δt_k [64, 99]:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) + G_k(\mathbf{x}, \mathbf{u}, t) \frac{1}{\sqrt{\Delta t_k}} \mathbf{w}_k, \quad (2.50)$$

where $\mathbf{w}_k$ is a discrete-time standard gaussian white noise, i.e., $\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, I_{n_w})$.

In this thesis, the unmodeled accelerations are assumed to be affecting only the acceleration component of the dynamics, and the stochastic process is also assumed to fit a constant Gaussian distribution which is independent on state, control input, or time. The G matrix is then defined as follows:

$$G = \begin{bmatrix} 0_{3 \times 3} \\ \sigma_G I_3 \end{bmatrix} \quad (2.51)$$

where σ_G is the standard deviation of the stochastic process. The value of σ_G must be related to a physical value, since the quantity $G/\sqrt{\Delta t_k}$ in Eq. (2.49) is the standard deviation of the unmodeled accelerations. For simplicity, in this thesis the value of σ_G is related to the value of σ_{stoch} which corresponds to the magnitude of the stochastic unmodeled accelerations:

$$\sigma_G = \sigma_{\text{stoch}} \sqrt{\Delta t_k} \quad (2.52)$$

where, in the context of small-body proximity operations, the unmodeled accelerations are in the order of $\sigma_{\text{stoch}} \approx 10^{-12} \div 10^{13} \text{ km/s}^2$ [67, 74].

2.5.2. Chance Constraints

Chance constraints are the stochastic counterparts of deterministic inequality constraints [67].

Let x denote a random variable with a known probability distribution. Classical deterministic inequality constraints may be imposed when the distribution of x is bounded (e.g., $x_{\max} < a$, where $x_{\max}$ denotes the largest possible value of x). However, such constraints cannot be directly enforced when x follows an unbounded distribution.

In this thesis, the state, control, and other uncertainty sources are assumed to follow Gaussian distributions, which are inherently unbounded. As a result, deterministic inequality constraints are no longer suitable, and chance constraints are instead employed. Chance constraints probabilistically restrict unbounded random variables by enforcing that a given constraint is satisfied with a prescribed minimum probability. This modeling choice is well suited to astrodynamics applications, where uncertainties in state estimation, actuation, and environmental disturbances are typically modeled as unbounded random processes.

This section provides a mathematical definition of *individual* chance constraints and introduces the fundamental theoretical tools required to convert them into deterministic inequalities that are both computationally tractable and convex. While *joint* chance constraints are also widely studied in the literature, this work focuses exclusively on individual chance constraints. Joint chance constraints are commonly approximated as a conjunction of individual chance constraints through the use of Boole's inequality. For a more detailed discussion of joint chance constraints and their conservative approximations, readers are referred to [31, 100].

Definition (Individual Chance Constraints) *A generic chance constraint is mathematically defined as [67]:*

$$\mathbb{P}[\mathbf{x} \in \mathcal{S}] \geq 1 - \epsilon \quad (2.53)$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathcal{S}$ is the feasible region and ϵ is a user-defined risk bound.

For instance, setting $\epsilon = 0.001$ allows the state to be outside the feasible region with 0.1% probability. Let $f_{\text{pdf}}(\mathbf{x})$ denote the probability density function of $\mathbf{x}$, the chance constraint is computed as:

$$\mathbb{P}[\mathbf{x} \in \mathcal{S}] = \int_{\mathbf{x} \in \mathcal{S}} f_{\text{pdf}}(\mathbf{x}) d\mathbf{x} \geq 1 - \epsilon. \quad (2.54)$$

This equation is, in general, intractable to compute. However, this thesis makes the following assumptions, which enable to convert Eq. (2.54) into a deterministic form:

1. State vectors are subject to multivariate Gaussian distributions, i.e.:

$$\mathbf{x} \sim \mathcal{N}(\bar{\mathbf{x}}, P_x) \quad (2.55)$$

where $\bar{\mathbf{x}}$ and P_x are the mean and covariance of $\mathbf{x}$.

2. Feasible regions are affine in the state vectors. i.e.:

$$\mathcal{S} = \{\mathbf{x} | \mathbf{a}^\top \mathbf{x} + b \leq 0\}, \quad \mathbf{a} \in \mathbb{R}^{n_x}, b \in \mathbb{R} \quad (2.56)$$

with $\mathbf{a} \in \mathbb{R}^{n_x}$ and $b \in \mathbb{R}$.

Chance constraints also exist in more general settings, such as with non-Gaussian distributions and non-convex feasible regions [101, 102], but their use is out of the scope of this thesis.

Tractable Reformulation of Chance Constraints

As previously mentioned, the intractable Eq. (2.54) to compute chance constraints can be reformulated into deterministic inequalities that are convex and tractable, under the aforementioned assumptions.

This section defines a set of lemmas, originally introduced by Oguri in [49], that provide the fundamental tools used throughout this thesis to obtain convex formulations of chance-constrained problems. These lemmas will be used in Chapter 6 to construct convex deterministic approximations of chance-constrained trajectory optimization problems.

Lemma 1. *Let $\boldsymbol{\xi} \sim \mathcal{N}(\bar{\boldsymbol{\xi}}, \mathbf{P}_\xi) \in \mathbb{R}^{n_\xi}$. Then, the chance constraint*

$$\mathbb{P}\left[\sum_j c_j(\boldsymbol{\xi}) \leq 0\right] \geq 1 - \varepsilon \quad (2.57)$$

is implied by

$$\mathbb{P}[c_j(\boldsymbol{\xi}) \leq 0] \geq 1 - \varepsilon_j, \quad \forall j, \quad \sum_j \varepsilon_j \leq \varepsilon. \quad (2.58)$$

Lemma 2. *Let $\boldsymbol{\xi} \sim \mathcal{N}(\bar{\boldsymbol{\xi}}, \mathbf{P}_\xi) \in \mathbb{R}^{n_\xi}$ and $\varepsilon \in (0, 1)$. Then, the chance constraint*

$$\mathbb{P}[a_j^\top \boldsymbol{\xi} + b_j \leq 0, \quad \forall j] \geq 1 - \varepsilon \quad (2.59)$$

is implied by

$$a_j^\top \bar{\boldsymbol{\xi}} + b_j + m_{\mathcal{N}}(\varepsilon_j) \sqrt{a_j^\top \mathbf{P}_\xi a_j} \leq 0, \quad \forall j, \quad \sum_j \varepsilon_j \leq \varepsilon, \quad (2.60)$$

where $m_{\mathcal{N}}(\varepsilon) = Q_{\mathcal{N} \sim \mathcal{N}(0,1)}(1 - \varepsilon)$ denotes the quantile function of the standard normal distribution evaluated at probability $(1 - \varepsilon)$.

Lemma 3. *Let $\boldsymbol{\xi} \sim \mathcal{N}(\bar{\boldsymbol{\xi}}, \mathbf{P}_\xi) \in \mathbb{R}^{n_\xi}$, $\varepsilon \in (0, 1)$, and $\gamma > 0$. Then, the chance constraint*

$$\mathbb{P}[\|\boldsymbol{\xi}\|_2 \leq \gamma] \geq 1 - \varepsilon \quad (2.61)$$

is implied by

$$\|\bar{\boldsymbol{\xi}}\|_2 + m_{\chi^2}(\varepsilon, n_\xi) \sqrt{\lambda_{\max}(\mathbf{P}_\xi)} \leq \gamma, \quad (2.62)$$

where

$$m_{\chi^2}(\varepsilon, n_\xi) = \sqrt{Q_{\chi^2(n_\xi)}(1 - \varepsilon)} \quad (2.63)$$

and $Q_{\chi^2(n_\xi)}(1 - \varepsilon)$ denotes the quantile function of the chi-squared distribution with n_ξ degrees of freedom evaluated at probability $(1 - \varepsilon)$.

Lemma 4. *Suppose $\mathbf{u}_k \sim \mathcal{N}(\bar{\mathbf{u}}_k, \mathbf{P}_{u_k})$. Then, the probabilistic cost term*

$$Q_{\chi \sim \|\mathbf{u}_k\|_2}(p) \tag{2.64}$$

is upper bounded as

$$Q_{\chi \sim \|\mathbf{u}_k\|_2}(p) \leq \|\bar{\mathbf{u}}_k\|_2 + m_{\chi^2}(1 - p, n_u) \sqrt{\lambda_{\max}(\mathbf{P}_{u_k})}, \tag{2.65}$$

where n_u denotes the dimension of the control vector.

In MATLAB, $m_{\mathcal{N}}$ and m_{χ^2} are computed with `norminv`($1 - \varepsilon$) and `chi2inv`($1 - \varepsilon, n_\xi$), respectively.

3 | Sequential Convex Programming Framework (SCvx*)

To solve non-convex optimal control problems, this thesis adopts solution methods based on sequential convex programming (SCP). Among the various SCP algorithms, the successive convexification (SCvx [25, 26]) and the guaranteed sequential trajectory optimization (GuSTO [27]) algorithms are arguably the most notable ones, due to their rigorous theoretical foundations. However, SCvx guarantees convergence to a KKT point of the penalized problem, but not of the original nonconvex problem [26], while GuSTO requires the dynamical system to be control-affine [27].

For these reasons, this thesis adopts the SCvx* algorithm, originally formulated by Oguri in [28]. This algorithm integrates the SCvx iteration into a framework based on the augmented Lagrangian (AL) method [103], iterating on the optimization variables while algorithmically improving the Lagrange multiplier estimates. In this way, the resulting SCP algorithm provides a feasibility guarantee for the original nonconvex problem, while retaining SCvx’s minimal requirements on the dynamical system.

This chapter introduces the SCvx* algorithm and presents the numerical framework adopted throughout this thesis to solve all trajectory optimization problems based on sequential convex programming. It is worth emphasizing that this chapter is deliberately limited to SCP-based methods and, in particular, to the SCvx* framework. Other numerical approaches are present in this thesis, such as indirect methods based on optimality conditions and shooting techniques, but they are introduced and discussed separately in the relevant sections. This separation reflects the different conceptual roles played by the numerical methods considered: while SCvx* serves as a general solution framework reused across multiple problem formulations, other solution methods are applied to specific case studies.

3.1. Successive convexification with augmented Lagrangian method

This section states the problem of successive convexification within the augmented Lagrangian method, summarizing the steps originally presented in the original paper [28] to define the algorithmic framework of SCvx*.

Original Problem Statement

A generic discrete-time non-convex optimal control problem can be written as Problem 1.

Problem 1 (Non-convex Optimal Control Problem).

$$\begin{aligned}
 \min_{\mathbf{X}, \mathbf{U}} \quad & f_0(\mathbf{x}, \mathbf{u}) \\
 \text{s.t.} \quad & \mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{u}_k), \quad k = 1, 2, \dots, N-1, \\
 & g_i(\mathbf{x}, \mathbf{u}) = 0, \quad i = 1, 2, \dots, n_{eq}, \\
 & h_j(\mathbf{x}, \mathbf{u}) \leq 0, \quad j = 1, 2, \dots, n_{ineq}, \\
 & \mathbf{g}_{\text{affine}}(\mathbf{x}, \mathbf{u}) = 0, \quad \mathbf{h}_{\text{cvx}}(\mathbf{x}, \mathbf{u}) \leq 0.
 \end{aligned}$$

where $\mathbf{X} = [\mathbf{x}_0^\top, \mathbf{x}_1^\top, \dots, \mathbf{x}_N^\top]^\top$, $\mathbf{U} = [\mathbf{u}_0^\top, \mathbf{u}_1^\top, \dots, \mathbf{u}_{N-1}^\top]^\top$, $\mathbf{g}_{\text{affine}}(\cdot)$ and $g_i(\cdot)$ denote the affine and non-affine equality constraints, and $\mathbf{h}_{\text{cvx}}(\cdot)$ and $h_j(\cdot)$ denote the convex and non-convex inequality constraints.

Defining $\mathbf{z} = [\mathbf{X}^\top, \mathbf{U}^\top]^\top \in \mathbb{R}^{n_z}$ with $n_z = (N+1)n_x + Nn_u$, Problem 1 can be rewritten as a non-convex optimization problem.

Problem 2 (Non-convex Optimization Problem).

$$\begin{aligned}
 \min_{\mathbf{X}, \mathbf{U}} \quad & f_0(\mathbf{z}) \\
 \text{s.t.} \quad & g_i(\mathbf{z}) = 0, \quad i = 1, 2, \dots, n_{eq} + Nn_x, \\
 & h_j(\mathbf{x}, \mathbf{u}) \leq 0, \quad j = 1, 2, \dots, n_{ineq}, \\
 & \mathbf{g}_{\text{affine}}(\mathbf{x}, \mathbf{u}) = 0, \quad \mathbf{h}_{\text{cvx}}(\mathbf{x}, \mathbf{u}) \leq 0.
 \end{aligned}$$

where $g_i(\cdot)$ incorporates also the dynamical constraints.

Defining $\mathbf{g}_{\text{all}}(\mathbf{z}) = [\mathbf{g}(\mathbf{z})^\top, \mathbf{g}_{\text{affine}}(\mathbf{z})^\top]^\top$ and $\mathbf{h}_{\text{all}}(\mathbf{z}) = [\mathbf{h}(\mathbf{z})^\top, \mathbf{h}_{\text{cvx}}(\mathbf{z})^\top]^\top$, the Lagrangian function $\mathcal{L}(\cdot)$ with vectors of Lagrange multipliers of the same size as $\mathbf{g}_{\text{all}}(\cdot)$ and $\mathbf{h}_{\text{all}}(\cdot)$ can be defined as:

$$\mathcal{L}(\mathbf{z}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = f_0(\mathbf{z}) + \boldsymbol{\lambda} \cdot \mathbf{g}_{\text{all}} + \boldsymbol{\mu} \cdot \mathbf{h}_{\text{all}} \quad (3.1)$$

Convex Penalty Problem with Augmented Lagrangian

The augmented Lagrangian method [103], augments Eq. (3.1) as:

$$\mathcal{L}_{AL}(\mathbf{z}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \mathcal{L}(\cdot) + \frac{w}{2} \mathbf{g}_{all} \cdot \mathbf{g}_{all} + \frac{w}{2} [\mathbf{h}_{all}]_+ \cdot [\mathbf{h}_{all}]_+ \quad (3.2)$$

where $w \in \mathbb{R}$ is a scalar weight parameter and $[\cdot]_+$ extract the maximum element of a vector. The augmented Lagrangian cost can be rewritten in a penalty form as

$\mathcal{L}_{AL}(\mathbf{z}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = f_0(\mathbf{z}) + P(\mathbf{g}_{all}, \mathbf{h}_{all}, w, \boldsymbol{\lambda}, \boldsymbol{\mu})$, where $P(\mathbf{g}, \mathbf{h}, w, \boldsymbol{\lambda}, \boldsymbol{\mu})$ is a penalty function.

Different forms of the penalty function can be used. For instance, [104] includes in the original penalty [28] an additional term that penalizes the 1-norm of the constraints, while [48] uses a penalty form which approximates the ℓ_1 -norm penalty while retaining the smoothness by introducing a smoothing parameter τ . As it has been shown to result in convergence in fewer iterations [42, 104], this thesis utilizes the former penalty function:

$$P(\cdot) = \boldsymbol{\lambda} \cdot \mathbf{g} + \frac{w}{2} \mathbf{g} \cdot \mathbf{g} + \sqrt{w} \|\mathbf{g}\|_1 + \boldsymbol{\mu} \cdot [\mathbf{h}]_+ + \frac{w}{2} [\mathbf{h}]_+ \cdot [\mathbf{h}]_+ + \sqrt{w} \|\mathbf{h}\|_1 \quad (3.3)$$

This augmented Lagrangian formulation guarantees convergence to a feasible local solution of the original Problem 2, provided that the Lagrange multipliers and penalty parameters are updated according to the augmented Lagrangian scheme. As demonstrated in Lemma 1 of [28], the converged solution satisfies the first- and second-order optimality conditions of the original problem.

It is now possible to define the non-convex penalty problem with the augmented Lagrangian cost.

Problem 3 (Non-convex penalty problem with AL).

$$\begin{aligned} \min_{\mathbf{z}} \quad & J(\mathbf{z}) = f_0(\mathbf{z}) + P(\mathbf{g}(\mathbf{z}), \mathbf{h}(\mathbf{z}), w, \boldsymbol{\lambda}, \boldsymbol{\mu}) \\ \text{s.t.} \quad & \mathbf{g}_{\text{affine}}(\mathbf{x}, \mathbf{u}) = 0, \quad \mathbf{h}_{\text{cvx}}(\mathbf{x}, \mathbf{u}) \leq 0. \end{aligned}$$

This problem is non-convex due to the non-affine and non-convex constraints $\mathbf{g}(\cdot)$ and $\mathbf{h}(\cdot)$. This issue is addressed by linearizing them about a reference variable $\bar{\mathbf{z}}$, obtaining:

$$\begin{aligned} \tilde{\mathbf{g}}(\mathbf{z}) &= \mathbf{g}(\bar{\mathbf{z}}) + \nabla_{\mathbf{z}} \mathbf{g}(\bar{\mathbf{z}}) \cdot (\mathbf{z} - \bar{\mathbf{z}}), \\ \tilde{\mathbf{h}}(\mathbf{z}) &= \mathbf{h}(\bar{\mathbf{z}}) + \nabla_{\mathbf{z}} \mathbf{h}(\bar{\mathbf{z}}) \cdot (\mathbf{z} - \bar{\mathbf{z}}) \end{aligned} \quad (3.4)$$

As mentioned in the previous chapter, imposing $\tilde{\mathbf{g}}(\cdot) = 0$ and $\tilde{\mathbf{h}}(\cdot) \leq 0$ in a convex

subproblem may lead to *artificial infeasibility*, so the constraints are relaxed introducing slack variables:

$$\tilde{\mathbf{g}}(\mathbf{z}) = \boldsymbol{\xi}, \quad \tilde{\mathbf{h}}(\mathbf{z}) \leq \boldsymbol{\zeta} \quad (3.5)$$

where $\boldsymbol{\xi} \in \mathbb{R}^{n_{eq}+Nn_x}$ and $\boldsymbol{\zeta} \in \mathbb{R}^{n_{ineq}}$.

Linearization and constraint relaxation may also lead to *artificial unboundedness*, which is addressed by imposing a trust region constraint on the optimization variable:

$$\|\mathbf{z} - \bar{\mathbf{z}}\|_{\infty} \leq \Delta_{TR} \quad (3.6)$$

where Δ_{TR} is the trust region radius and $\|\cdot\|_{\infty}$ extracts the maximum element of a vector.

It is now possible to reformulate Problem 3 in its convex form.

Problem 4 (Convex penalty problem with AL).

$$\begin{aligned} \min_{\mathbf{z}} \quad & L(\mathbf{z}, \boldsymbol{\xi}, \boldsymbol{\zeta}) = f_0(\mathbf{z}) + P(\boldsymbol{\xi}, \boldsymbol{\zeta}, w, \boldsymbol{\lambda}, \boldsymbol{\mu}) \\ \text{s.t.} \quad & \tilde{\mathbf{g}}(\mathbf{z}) = \boldsymbol{\xi}, \quad \tilde{\mathbf{h}}(\mathbf{z}) \leq \boldsymbol{\zeta}, \quad \boldsymbol{\zeta} \geq 0 \\ & \mathbf{g}_{\text{affine}}(\mathbf{z}) = 0, \quad \mathbf{h}_{\text{cvx}}(\mathbf{z}) \leq 0 \\ & \|\mathbf{z} - \bar{\mathbf{z}}\|_{\infty} \leq \Delta_{TR} \end{aligned}$$

3.2. SCvx* algorithm

This section outlines the SCvx* algorithm [28], which solves the non-convex penalty Problem 4 through successive convexification within an augmented Lagrangian framework. The complete procedure is summarized in Algorithm 3.1, while its key components are detailed below.

1. Successive linearization:

At iteration k , the penalty function is defined as:

$$P^{(k)}(\mathbf{g}, \mathbf{h}) \triangleq P(\mathbf{g}, \mathbf{h}, w^{(k)}, \boldsymbol{\lambda}^{(k)}, \boldsymbol{\mu}^{(k)}) . \quad (3.7)$$

Accordingly, the penalized objectives of Problems 3 and 4 at iteration k are:

$$J^{(k)}(\mathbf{z}) \triangleq f_0(\mathbf{z}) + P^{(k)}(\mathbf{g}(\mathbf{z}), \mathbf{h}(\mathbf{z})) , \quad (3.8)$$

$$L^{(k)}(\mathbf{z}, \boldsymbol{\xi}, \boldsymbol{\zeta}) \triangleq f_0(\mathbf{z}) + P^{(k)}(\boldsymbol{\xi}, \boldsymbol{\zeta}). \quad (3.9)$$

Given a reference point $\bar{\mathbf{z}}^{(k)}$, the non-convex constraints are linearized as described in the previous section, yielding the convex subproblem Problem 4. This problem is then solved, yielding the solution at iteration k : $\{\mathbf{z}_\star^{(k)}, \boldsymbol{\xi}_\star^{(k)}, \boldsymbol{\zeta}_\star^{(k)}\}$.

After solving Problem 4, SCvx* computes the following quantities:

$$\begin{aligned}\Delta J^{(k)} &= J^{(k)}(\bar{\mathbf{z}}^{(k)}) - J^{(k)}(\mathbf{z}_\star^{(k)}), \\ \Delta L^{(k)} &= J^{(k)}(\bar{\mathbf{z}}^{(k)}) - L^{(k)}(\mathbf{z}_\star^{(k)}, \boldsymbol{\xi}_\star^{(k)}, \boldsymbol{\zeta}_\star^{(k)}), \\ \chi^{(k)} &= \left\| \begin{bmatrix} \mathbf{g}(\mathbf{z}_\star^{(k)}), [\mathbf{h}(\mathbf{z}_\star^{(k)})]_+ \end{bmatrix}^\top \right\|_2.\end{aligned}\tag{3.10}$$

where $\Delta J^{(k)}$, $\Delta L^{(k)}$ denote the actual nonlinear cost reduction and the predicted cost reduction, respectively; $\chi^{(k)}$ denotes the infeasibility of the problem.

2. Step acceptance:

The solution of Problem 4 is accepted if:

$$\rho^{(k)} \triangleq \frac{\Delta J^{(k)}}{\Delta L^{(k)}} \geq \rho_0,\tag{3.11}$$

where $\rho_0 \in (0, 1)$ is a user-defined threshold and $\rho^{(k)}$ is a ratio that measures the relative decrease of the objective. The solution is accepted only if this ratio is greater than ρ_0 , which helps in discarding solutions that do not improve the non-convex cost. If the condition is satisfied, the reference point is updated for the next iteration as $\bar{\mathbf{z}}^{(k+1)} = \mathbf{z}_\star^{(k)}$, otherwise the step is rejected and $\bar{\mathbf{z}}^{(k+1)} = \bar{\mathbf{z}}^{(k)}$.

3. Lagrange multiplier update:

The Lagrange multipliers are constant in each convex subproblem, but must be updated to effectively march towards the convergence of the original non-convex Problem 2. When the solution is accepted and the following condition is satisfied:

$$|\Delta J^{(k)}| < \delta^{(k)},\tag{3.12}$$

the augmented Lagrangian parameters are updated as:

$$\begin{aligned}\boldsymbol{\lambda}^{(k+1)} &= \boldsymbol{\lambda}^{(k)} + w^{(k)} \mathbf{g}(\mathbf{z}_\star^{(k)}), \\ \boldsymbol{\mu}^{(k+1)} &= [\boldsymbol{\mu}^{(k)} + w^{(k)} \mathbf{h}(\mathbf{z}_\star^{(k)})]_+, \\ w^{(k+1)} &= \beta w^{(k)}, \quad \beta > 1.\end{aligned}\tag{3.13}$$

The threshold $\delta^{(k)}$ is updated according to:

$$\delta^{(k+1)} = \begin{cases} |\Delta J^{(k)}|, & \delta^{(k)} = \infty, \\ \gamma \delta^{(k)}, & \text{otherwise,} \end{cases} \quad \gamma \in (0, 1), \quad (3.14)$$

ensuring $\delta^{(k)} \rightarrow 0$ as $k \rightarrow \infty$.

4. Trust region update:

The trust region radius $\Delta_{TR}^{(k)}$ is updated as:

$$\Delta_{TR}^{(k+1)} = \begin{cases} \max\left\{\Delta_{TR}^{(k)}/\alpha_1, \Delta_{TR}^{\min}\right\}, & \text{if } \rho^{(k)} < \rho_1, \\ \Delta_{TR}^{(k)}, & \text{elseif } \rho^{(k)} < \rho_2, \\ \min\left\{\alpha_2 \Delta_{TR}^{(k)}, \Delta_{TR}^{\max}\right\}, & \text{else} \end{cases} \quad (3.15)$$

where $\alpha_1 > 1$, $\alpha_2 > 1$, and $0 < \Delta_{TR}^{\min} < \Delta_{TR}^{\max}$.

5. Convergence check:

SCvx* detects the convergence when these two conditions are simultaneously satisfied:

$$|\Delta J^{(k)}| \leq \epsilon_{\text{opt}} \quad \wedge \quad \chi^{(k)} \leq \epsilon_{\text{feas}}, \quad (3.16)$$

where $\epsilon_{\text{opt}} > 0$ and $\epsilon_{\text{feas}} > 0$ are user-defined optimality and feasibility tolerances.

Algorithm 3.1 SCvx*

Input: $\bar{\mathbf{z}}^{(1)}, \Delta^{(1)}, w^{(1)}, \epsilon_{\text{opt}}, \epsilon_{\text{feas}}, \eta_0, \eta_1, \eta_2, \alpha_1, \alpha_2, \beta, \gamma, \Delta_{\min}, \Delta_{\max}$
 Initialize: $k = 1, \Delta J^{(0)} = \chi^{(0)} = \delta^{(1)} = \infty, \boldsymbol{\lambda}^{(1)} = \boldsymbol{\mu}^{(1)} = 0$
while $|\Delta J^{(k-1)}| > \epsilon_{\text{opt}}$ **or** $\chi^{(k-1)} > \epsilon_{\text{feas}}$ **do**
 $\{\tilde{\mathbf{g}}^{(k)}, \tilde{\mathbf{h}}^{(k)}\} \leftarrow \text{Eq. (3.4) at } \bar{\mathbf{z}}^{(k)}$
 $\{\mathbf{z}_*^{(k)}, \boldsymbol{\xi}_*^{(k)}, \boldsymbol{\zeta}_*^{(k)}\} \leftarrow \text{solve Problem 5}$
 $\{\Delta J^{(k)}, \Delta L^{(k)}, \chi^{(k)}\} \leftarrow \text{Eq. (3.10)}$
 $\rho^{(k)} \leftarrow \Delta J^{(k)} / \Delta L^{(k)}$
 Default: $\{\bar{\mathbf{z}}, w, \boldsymbol{\lambda}, \boldsymbol{\mu}, \delta\}^{(k+1)} \leftarrow \{\bar{\mathbf{z}}, w, \boldsymbol{\lambda}, \boldsymbol{\mu}, \delta\}^{(k)}$
 if acceptance conditions Eq. (3.11) met **then**
 $\bar{\mathbf{z}}^{(k+1)} \leftarrow \mathbf{z}_*^{(k)}$
 if Eq. (3.12) is satisfied **then**
 $\{\boldsymbol{\lambda}, \boldsymbol{\mu}, w\}^{(k+1)} \leftarrow \text{Eq. (3.13)}$
 $\delta^{(k+1)} \leftarrow \text{Eq. (3.14)}$
 end if
 end if
 $\Delta_{\text{TR}}^{(k+1)} \leftarrow \text{Eq. (3.15)}$
 $k \leftarrow k + 1$
end while
return $(\mathbf{z}_*^{(k)}, \boldsymbol{\lambda}^{(k)}, \boldsymbol{\mu}^{(k)})$

4 | Science Orbit Design

This chapter describes the methodology adopted in this thesis to compute periodic orbit families around asteroid (99942) Apophis. From the resulting families, the departure and arrival orbits are selected and the scenario for the deterministic and stochastic trajectory optimization problems is defined. Finally, the selected terminator orbits are simulated in a perturbed ANH3BP model that accounts for the true heliocentric elliptic orbit of Apophis.

As a baseline scenario, the close-proximity operations planned for NASA's OSIRIS-APEX mission at asteroid (99942) Apophis are considered in this thesis [9, 75]. The dynamical and physical parameters adopted throughout this work are summarized in Table 4.1. The orbital elements reported in the table refer to Apophis' heliocentric orbit after the 2029 Earth encounter, while the remaining parameters describe the physical properties of the asteroid and the spacecraft [9, 75]. The spacecraft reflectance coefficient is assumed to be $\rho = 0$.

Table 4.1: Orbital, physical, and spacecraft parameters.

Apophis Post-Encounter Orbital Parameters	
Semi-major axis, a_S (m)	$1.650227501552187 \times 10^{11}$
Eccentricity, e_S	0.1890650444165436
Inclination, i_S (deg)	2.221099203220031
Longitude of ascending node, Ω_S (deg)	203.5628891952976
Argument of periapsis, ω_S (deg)	71.43287204673284
Physical and Spacecraft Parameters	
Gravitational parameter of Apophis, μ (m^3/s^2)	2.68
Spacecraft mass-to-area ratio, B (kg/m^2)	81.119
Spacecraft reflectance coefficient, ρ	0

The resulting SRP parameter by applying Eq. (2.6) is equal to $\beta = 34.1129$.

4.1. Periodic Orbit Families

The science orbit design process is applied sequentially. First, Frozen Terminator Orbits (FTOs) are generated by applying the frozen conditions Eq. (2.9). These frozen solutions are then corrected into Periodic Terminator Orbits (PTOs) with a single-shooting differential corrector. Subsequently, numerical continuation is exploited to generate the PTO family, identifying several period-multiplying bifurcations. These bifurcations give rise to distinct families of Resonant Terminator Orbits (RTOs), which are computed using a multiple-shooting differential correction scheme.

4.1.1. Frozen Terminator Orbits

Frozen Terminator Orbits (FTOs) are generated by enforcing the frozen conditions Eq. (2.9), which are used to reconstruct the initial spacecraft state vector from the corresponding orbital parameters, which is then propagated under ANH3BP dynamics.

Figure 4.1 illustrates a representative Frozen Terminator Orbit with a semi-major axis of 1.37 km, closely matching the terminator science orbit planned for OSIRIS-APEX during its Terminator Orbit Phase [9].

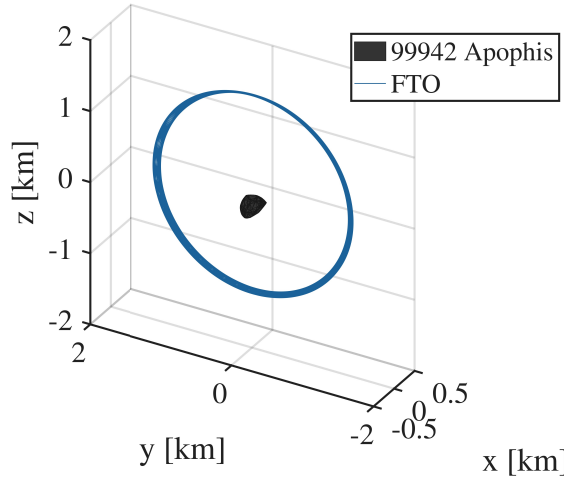


Figure 4.1: Representative Frozen Terminator Orbit

4.1.2. Periodic Terminator Orbits

Periodic Terminator Orbits (PTOs) are obtained by correcting Frozen Terminator Orbits into fully periodic solutions using a single-shooting differential correction scheme. The procedure follows the methodology proposed in [68].

Due to the intrinsic symmetries of the ANH3BP, PTOs are symmetric with respect to the x - z plane. As a result, the initial state of a PTO can be parametrized as

$$\mathbf{X}_0 = \begin{bmatrix} x_0 & 0 & z_0 & 0 & \dot{y}_0 & 0 \end{bmatrix}^\top, \quad (4.1)$$

which reduces the dimensionality of the problem and improves numerical efficiency [68].

Let T denote the orbital period. The unknown vector to be corrected is defined as

$$\mathbf{Z} = \begin{bmatrix} x_0 & z_0 & \dot{y}_0 & T \end{bmatrix}^\top. \quad (4.2)$$

Periodic solutions are enforced by defining a shooting function $\mathbf{F}(\mathbf{Z})$ that imposes the symmetry and periodicity conditions at time T :

$$\mathbf{F}(\mathbf{Z}) = \begin{bmatrix} y(T) \\ \dot{x}(T) \\ \dot{z}(T) \\ J_c(\mathbf{X}(T)) - J_{c,0} \end{bmatrix} = \mathbf{0}, \quad (4.3)$$

where J_c denotes the Jacobi constant of the ANH3BP and $J_{c,0}$ is a prescribed target value. The first three constraints enforce the symmetry conditions at half-period crossing of the x - z plane, while the Jacobi constraint allows to find the period of the orbit.

The correction step requires the Jacobian of the shooting function with respect to $\mathbf{Z}$, which is computed analytically using the State Transition Matrix (STM) Φ . By integrating the variational equations alongside the equations of motion, the Jacobian matrix is assembled:

$$\mathbf{J} = \begin{bmatrix} \Phi_{21} & \Phi_{23} & \Phi_{25} & \dot{y}(T) \\ \Phi_{41} & \Phi_{43} & \Phi_{45} & \ddot{x}(T) \\ \Phi_{61} & \Phi_{63} & \Phi_{65} & \ddot{z}(T) \\ \nabla_{x_0} J & \nabla_{z_0} J & \nabla_{\dot{y}_0} J & 0 \end{bmatrix}, \quad (4.4)$$

where Φ_{ij} denotes the (i, j) element of the STM evaluated at time T .

The nonlinear system $\mathbf{F}(\mathbf{Z}) = \mathbf{0}$ is solved iteratively using MATLAB's `fsolve`, with `FunctionTolerance` and `StepTolerance` values set to 10^{-12} .

To find periodic orbit families, numerical continuation over the Jacobi Constant J_c of individual solutions is exploited. In particular, the new guess for a given level of J_c at

iteration n is achieved using a linear prediction based on the two previous solutions [68]:

$$\mathbf{Z}_{\text{guess}}^n = \mathbf{Z}^{n-1} + \frac{\mathbf{Z}^{n-1} - \mathbf{Z}^{n-2}}{J_c(\mathbf{X}_0^{n-1}) - J_c(\mathbf{X}_0^{n-2})}(J_{c,0}^n - J_c(\mathbf{X}_0^{n-1})) \quad (4.5)$$

where the first two solutions $\mathbf{Z}^1$ and $\mathbf{Z}^2$ are found by correcting the initial FTO for two close levels of Jacobi Constant.

More advanced continuation strategies, such as pseudo-arclength continuation [72, 105], are known to improve robustness and efficiency particularly in the vicinity of bifurcation points. This method is particularly useful for a more systematic exploration of the solution space because it can automatically follow new solutions at bifurcation points, without the need to explicitly identify the bifurcation directions in advance. However, since the objective of this work is the generation of representative science orbit families rather than the complete discovery of all possible periodic solutions, a simpler linear prediction was deemed sufficient.

Figure 4.2 shows the resulting family of PTOs around Apophis, together with the corresponding phase-space plot highlighting the bifurcations associated with the Sun-side and Dark-side resonances.

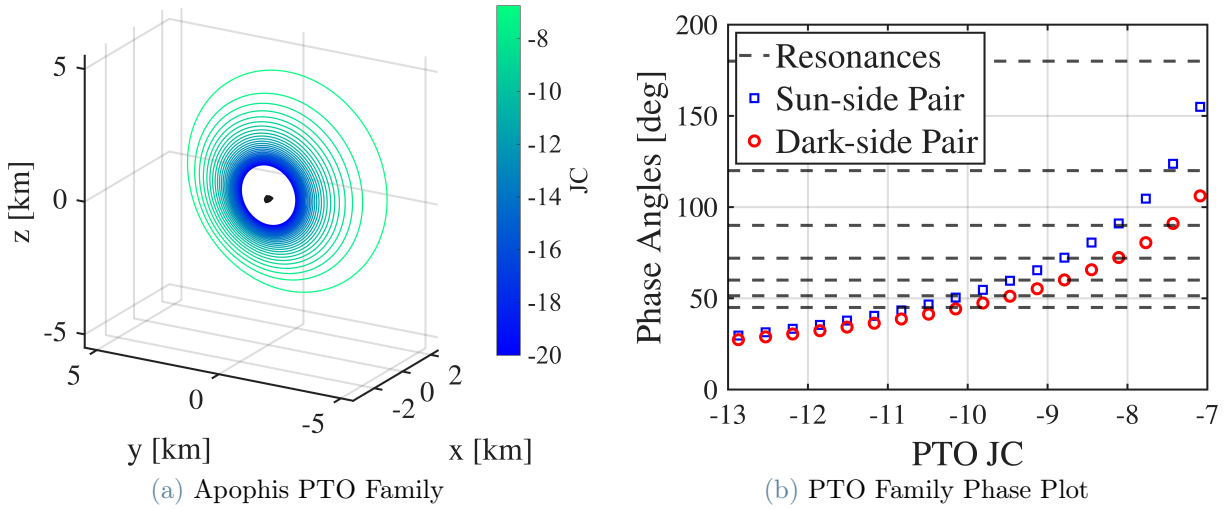


Figure 4.2: Apophis PTO Family and Corresponding Phase Plot with Bifurcations

4.1.3. Resonant Terminator Orbits

Resonant Terminator Orbits (RTOs) are obtained as secondary periodic solutions that bifurcate from the family of Periodic Terminator Orbits (PTOs) at period-multiplying bifurcation points. These bifurcations occur when a pair of complex-conjugate eigenvalues

of the Monodromy matrix associated with a PTO reaches a phase angle

$$\theta = \frac{2\pi n}{m}, \quad (4.6)$$

corresponding to an $m : n$ resonance between the fundamental frequencies of motion [68, 71]. At these points, new families of periodic solutions with period mT emerge from the parent PTO family. Numerically, precise bifurcation points are detected by using a bisection method with absolute and relative tolerances set to 10^{-12} .

Once a period-multiplying bifurcation is detected along a PTO family, an initial guess for the corresponding RTO can be constructed by perturbing the initial condition of the parent PTO along the eigenvector associated with the critical eigenvalue pair. Following the approach proposed in [59, 68], the perturbed initial condition is defined as:

$$\mathbf{x}_0^{\text{RTO}} = \mathbf{x}_0^{\text{PTO}} + \varepsilon \begin{bmatrix} \Im(\xi_1) & 0 & \Im(\xi_3) & 0 & \Im(\xi_5) & 0 \end{bmatrix}^T, \quad (4.7)$$

where ξ is the eigenvector associated with the bifurcating eigenvalue, $\Im(\cdot)$ denotes the imaginary part, and ε is a small scalar perturbation parameter, set to $\varepsilon = 0.3$. The structure of the perturbation enforces symmetry with respect to the x - z plane, consistently with the symmetry properties of terminator orbits, and ensures that the perturbed trajectory crosses the $y = 0$ plane perpendicularly.

The resulting perturbed trajectory generally does not satisfy periodicity and is therefore corrected using a multiple-shooting differential correction scheme. In this work, the trajectory over one period is divided into q segments, and the initial state of each segment, together with the segment integration time, is included in the correction vector:

$$\mathbf{Z} = \begin{bmatrix} \mathbf{X}_1^T & \mathbf{X}_2^T & \dots & \mathbf{X}_q^T & t_{\text{seg}} \end{bmatrix}^T. \quad (4.8)$$

Periodicity is enforced by requiring continuity between consecutive segments and closure over the full period, together with symmetry and Jacobi constant constraints. The correction is obtained iteratively using a Newton-type update:

$$\mathbf{Z}^{(k+1)} = \mathbf{Z}^{(k)} - (\mathbf{J}^T \mathbf{J})^{-1} \mathbf{J}^T \mathbf{F}(\mathbf{Z}^{(k)}), \quad (4.9)$$

where $\mathbf{F}(\mathbf{Z})$ is the vector of periodicity constraints and $\mathbf{J}$ is its Jacobian, assembled using the State Transition Matrices of each segment [31, 68].

Once two converged RTO solutions are obtained, numerical continuation along the Ja-

cobi Constant is employed to generate full RTO families. In particular, the same linear prediction based on the two previously converged solutions Eq. (4.5) is used to construct an initial guess for nearby orbits [68].

Among the various RTO families, the 6:1 resonance is selected in this work as the baseline science orbit family for subsequent transfer design, and is showcased in Figure 4.3.

Figure 4.4 shows representative families of Resonant Terminator Orbits (RTOs) obtained from period-multiplying bifurcations of the PTO family. In particular, the 3:1, 5:1, and 8:1 resonant families are illustrated, highlighting the progressive increase in geometric complexity and spatial coverage as the resonance order increases.

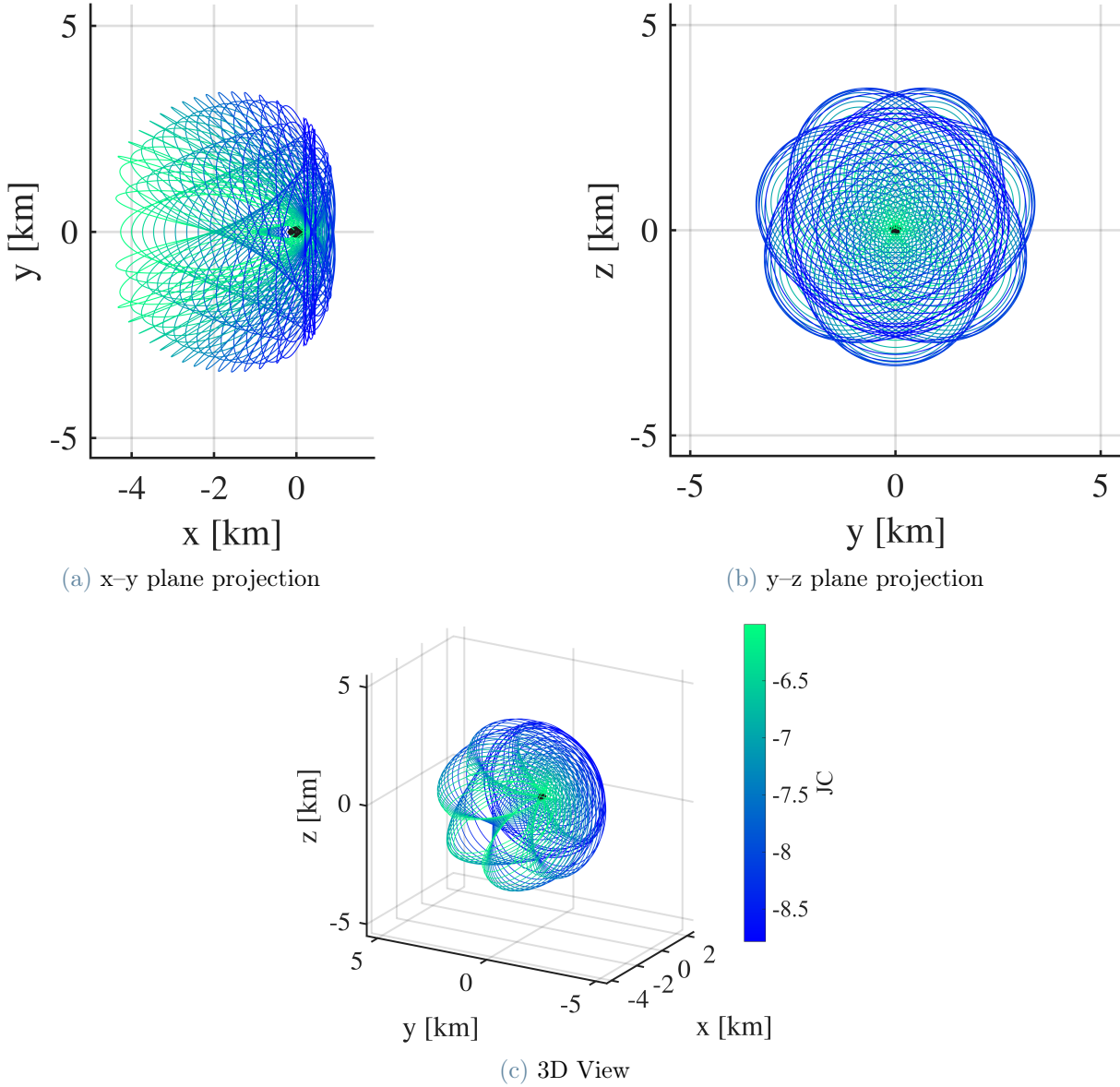


Figure 4.3: 6:1 Apophis RTO Family

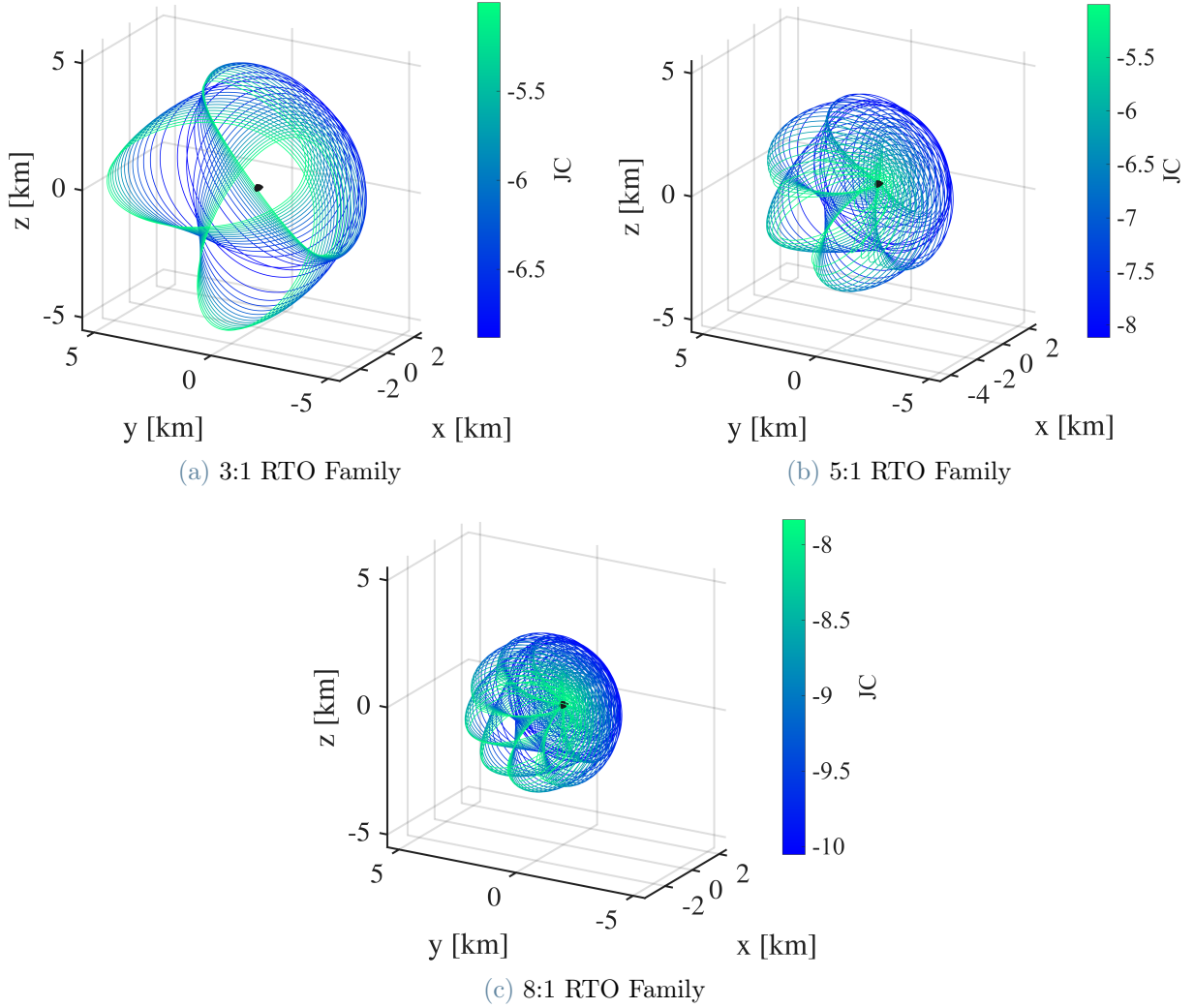


Figure 4.4: Representative Resonant Terminator Orbit families

4.2. Science Orbit Transfer Scenario Definition

The autonomous science orbit transfer considered in this thesis connects two bounded solutions of the ANH3BP representing realistic operational configurations for proximity operations around asteroid (99942) Apophis. The departure orbit is selected as a Periodic Terminator Orbit (PTO) at an average altitude of approximately 4.6 km. The arrival orbit is chosen as a 6:1 Resonant Terminator Orbit (RTO) similar to the one that will be used by OSIRIS-APEX at (99942) Apophis for the Variable Phase Orbit (VPO). This orbit is characterized by a minimum radius $R_{\min} \approx 0.8$ km, a maximum radius $R_{\max} \approx 5$ km, and a minimum phase angle of approximately 50° .

The selected transfer therefore represents a transition from a higher stable Periodic Ter-

minator Orbit to a lower Resonant Terminator Orbit with enhanced geometric diversity and observation capability.

Table 4.2: Selected Departure and Arrival Science Orbits

	Departure Orbit	Arrival Orbit
Orbit Type	PTO	6:1 RTO
Average Radius, R (km)	~ 4.6	–
Closest distance, $R_{\min}$ (km)	–	~ 0.8
Farthest distance, $R_{\max}$ (km)	–	~ 5.0
Minimum Phase Angle, $\phi_{\min}$ (deg)	–	~ 50

Science Orbit Simulation via Elliptic ANH3BP

The periodic departure and arrival orbits introduced above are computed within the circular ANH3BP framework. While this simplified autonomous model admits conserved quantities and well-defined periodic families, a more realistic dynamical model must account for the ellipticity of Apophis’ heliocentric orbit, especially when assessing long-term boundedness and robustness of science orbits. In this thesis, the elliptic ANH3BP introduced in Chapter 2 is used to validate the robustness of the science orbits designed within the simplified ANH3BP framework. Periodic solutions obtained in the circular model are propagated under the perturbed dynamics using the actual heliocentric motion of asteroid (99942) Apophis during OSIRIS-APEX proximity operations.

The results show that, despite the loss of strict periodicity, the orbits remain bounded and retain their geometric structure over several weeks, supporting their use as departure and arrival orbits in the subsequent trajectory optimization problems. Designing periodic orbits directly within the elliptic framework is possible [75], but it is beyond the scope of this work.

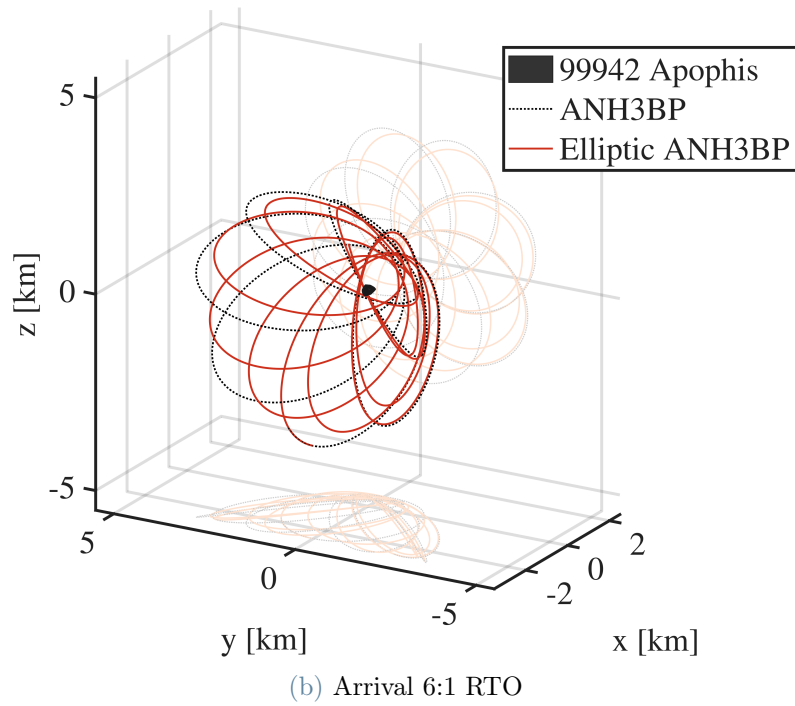
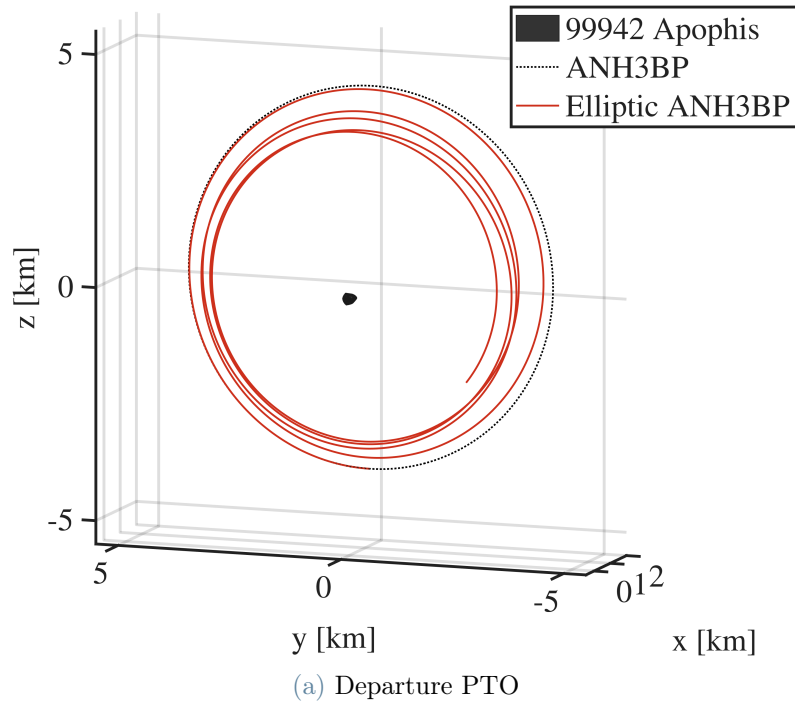


Figure 4.5: Comparison of selected science orbits propagated with circular and elliptic ANH3BP dynamics.

5 | Deterministic Trajectory Optimization

This chapter presents the methodology and the solutions of deterministic impulsive transfers between the selected departure and arrival terminator orbits. Deterministic optimization is addressed first for two distinct reasons. First, it provides a baseline solution that enables a comparison between purely deterministic and risk-aware trajectories solved through stochastic optimization. Second, a deterministic reference trajectory is required as an initial guess for the subsequent chance-constrained stochastic optimal control problem via SCvx* [18].

Among the possible optimization techniques for trajectory optimization, indirect methods are employed first to obtain a reference solution that is as close as possible to the theoretically optimal trajectory, and the corresponding formulation is described in Section 5.1. In fact, indirect methods rely on the calculus of variations and Pontryagin's Minimum Principle (PMP) to solve the optimal control problem in its continuous, infinite-dimensional form, providing strong optimality guarantees. In contrast, direct methods discretize the problem at a finite set of nodes and epochs and, while generally more numerically robust, lack the same level of optimality guarantee.

Since indirect methods are typically used for solving continuous low-thrust control problems, an impulsification strategy is adopted in this work to retrieve the optimal placement and magnitude of impulsive maneuvers [106]. However, indirect methods are also known to be computationally expensive, highly sensitive to the initial guess, and often characterized by limited convergence robustness [107].

For these reasons, a second strategy for deterministic trajectory design strategy is introduced in Section 5.2, which aims to combine the robustness and computational efficiency of direct methods with solutions that remain close to the optimal trajectory of PMP. Specifically, the Adaptive Mesh Sequential Convex Programming (AMSCP) approach [108] is adopted, which simultaneously optimizes both the trajectory and the discretization mesh. This allows the method to optimally place the impulsive maneuvers at a significantly re-

duced computational cost and with improved convergence properties compared to indirect methods.

The science orbit transfer problem is formulated as a free-phase and free final time trajectory optimization problem, in which the departure and arrival phases along the respective terminator orbits are treated as optimization variables, enabling the computation of locally optimal transfers that optimize both the impulsive maneuvers and the phasing.

It is important to note that the impulsified indirect solutions were not directly used as initial reference trajectories in the subsequent stochastic optimization framework. Instead, they were employed as initial guesses for the Adaptive Mesh Sequential Convex Programming (AMSCP) stage, which ultimately generated the deterministic reference trajectory used in the chance-constrained stochastic optimization. This approach ensures that the stochastic solver is initialized from a dynamically feasible deterministic transfer; therefore, the trajectories computed via indirect methods still play a fundamental role in the design of the deterministic reference solution.

5.1. Indirect Methods via Impulsification of Low-Thrust Trajectories

The indirect methods adopted in this thesis are based on the $a_{\max}$ method formulated in [109]. Within this framework, acceleration-based switching surfaces corresponding to near-impulsive solutions are obtained via numerical continuation on the maximum admissible acceleration [110]. The $a_{\max}$ Method provides a purely acceleration-based formulation of the optimal control problem, which removes the explicit dependency on spacecraft mass, specific impulse, and thrust magnitude. As a consequence, both the spacecraft mass and the associated mass costate are eliminated from the problem formulation.

This assumption is particularly well suited for the scenario of impulsive control around small bodies. Indeed, impulsive maneuvers in close-proximity operations typically involve magnitudes of only a few cm/s and occur sparsely along the transfer. Under these conditions, neglecting the variation of the spacecraft mass does not represent a strong modeling assumption. In addition to the reduced dimensionality of the problem, the $a_{\max}$ method offers several practical advantages. In particular, it removes the need to select the exhaust velocity as a control parameter and allows for relatively large continuation steps in $a_{\max}$ during the impulsification process, without significantly degrading the convergence properties of the solution [106, 109].

5.1.1. Problem Formulation via the $a_{\max}$ Method

The transfer is initially formulated as a fuel optimal fixed-phase, fixed-time problem, where the cost function to be minimized is the total ΔV required to transfer the spacecraft between the departure and arrival states. Instead of minimizing the propellant mass, in the $a_{\max}$ method the cost functional is expressed in Lagrange form as the time integral of the acceleration magnitude [110]:

$$\min_{\mathbf{a}(t)} J = \int_{t_0}^{t_f} \|\mathbf{a}(t)\| dt, \quad (5.1)$$

where t_0 and t_f denote the initial and final times, respectively, and $\mathbf{a}(t)$ is the control acceleration. The acceleration magnitude is upper bounded by its maximum allowable value, as $\|\mathbf{a}(t)\| \leq a_{\max}$, and the control is rewritten as

$$\mathbf{a}(t) = a_{\max} \delta(t) \hat{\boldsymbol{\alpha}}(t), \quad (5.2)$$

where $\delta(t) \in [0, 1]$ is the throttling parameter and $\hat{\boldsymbol{\alpha}}(t)$ is the thrust direction.

Following the procedure outlined in Chapter 2, the Hamiltonian associated with the problem is written as:

$$H = \delta a_{\max} + \boldsymbol{\lambda}^\top (\mathbf{f}_0(\mathbf{x}) + a_{\max} \delta \hat{\boldsymbol{\alpha}}), \quad (5.3)$$

where $\boldsymbol{\lambda}$ denotes the costate vector and $\mathbf{f}_0(\mathbf{x})$ represents the uncontrolled spacecraft dynamics. Applying Pontryagin's Minimum Principle, the optimal thrust direction is obtained by minimizing the Hamiltonian with respect to $\hat{\boldsymbol{\alpha}}$. The throttling parameter δ is governed by the switching function, defined as:

$$\mathcal{S}(t) = \|\boldsymbol{\lambda}_v(t)\| - 1, \quad (5.4)$$

where $\boldsymbol{\lambda}_v$ denotes the velocity costates. To approximate the bang–bang structure of the optimal control, the throttling parameter is approximated using the hyperbolic tangent smoothing method [92]:

$$\delta(t) \approx \frac{1}{2} \left[1 + \tanh\left(\frac{\mathcal{S}(t)}{\rho}\right) \right], \quad (5.5)$$

with ρ being the smoothing parameter.

The state and costate dynamics are finally obtained from the Euler–Lagrange equations,

$$\dot{\mathbf{x}} = \frac{\partial H}{\partial \boldsymbol{\lambda}}, \quad \dot{\boldsymbol{\lambda}} = -\frac{\partial H}{\partial \mathbf{x}}. \quad (5.6)$$

5.1.2. Solution via Indirect Forward-Backward Shooting

The optimal control problem is solved via the indirect forward–backward shooting method presented by Sidhoum et al. [111]. This approach is adopted to improve the numerical robustness of indirect methods, which are known to be highly sensitive to the initial guess of the costates in strongly nonlinear dynamical environments. The method transforms the original Two-Point Boundary Value Problem (TPBVP) into a Multi-Point Boundary Value Problem (MPBVP) by splitting the trajectory into two arcs that are patched together at an interior matching point.

The indirect forward–backward shooting framework has been shown to provide increased convergence robustness compared to classical single shooting for problems involving complex dynamics and high nonlinearities, both for two [111] and multiple shooting segments [112].

The classical indirect shooting method for TPBVPs consists in determining the initial costates λ_0 such that, when the system dynamics are propagated forward in time while enforcing the optimal control law derived from Pontryagin’s Minimum Principle, the terminal boundary conditions are satisfied.

In contrast, the indirect forward–backward shooting method introduces an interior matching point at time t_m , at which the trajectory is required to be continuous in both states and costates. The continuity conditions enforced at the matching point replace the terminal constraints of the classical shooting formulation. Figure 5.1 presents a schematic overview of the adopted shooting method.

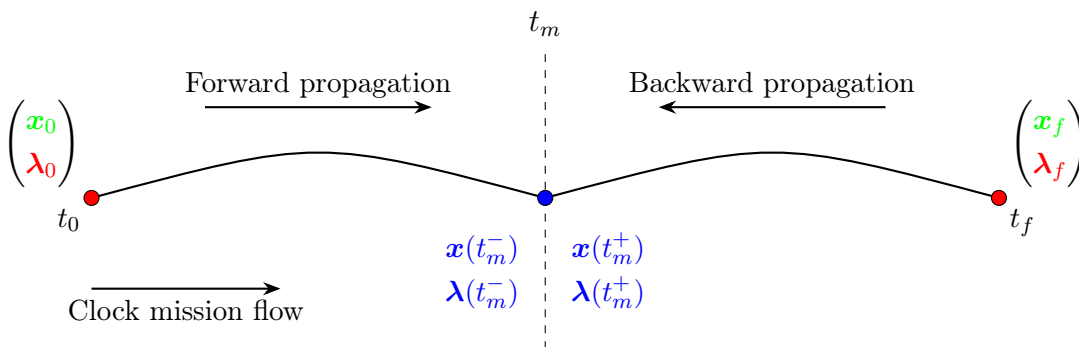


Figure 5.1: Indirect forward–backward shooting scheme. Green denotes the known initial and final states, red the unknown initial and final costates, while blue indicates the boundary conditions enforced at t_m to guarantee trajectory continuity.

Let λ_0 and λ_f denote the unknown initial and final costates, respectively. The vector of shooting unknowns and the shooting function to enforce continuity of both the state and

costate vectors at the matching time t_m are defined as:

$$\mathbf{Z} = \begin{bmatrix} \boldsymbol{\lambda}_0 \\ \boldsymbol{\lambda}_f \end{bmatrix}, \quad \boldsymbol{\Psi}(\mathbf{Z}) = \begin{bmatrix} \Delta \mathbf{x}_{\text{mp}} \\ \Delta \boldsymbol{\lambda}_{\text{mp}} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{\text{for}}(t_m^+) - \mathbf{x}_{\text{back}}(t_m^-) \\ \boldsymbol{\lambda}_{\text{for}}(t_m^+) - \boldsymbol{\lambda}_{\text{back}}(t_m^-) \end{bmatrix} = \mathbf{0} \quad (5.7)$$

To efficiently solve the nonlinear system $\boldsymbol{\Psi}(\mathbf{Z}) = \mathbf{0}$, analytic derivatives of the shooting function are employed. These derivatives are computed using the State Transition Matrix (STM), which is propagated along with the trajectory with the variational equation Eq. (2.17).

By propagating the STM forward and backward together with the state and costate dynamics, the Jacobian of the shooting function can be assembled in closed form as:

$$\mathbf{J}_{\boldsymbol{\Psi}}(\mathbf{Z}) = \begin{bmatrix} \frac{\partial \Delta \mathbf{x}_{\text{mp}}}{\partial \boldsymbol{\lambda}_0} & \frac{\partial \Delta \mathbf{x}_{\text{mp}}}{\partial \boldsymbol{\lambda}_f} \\ \frac{\partial \Delta \boldsymbol{\lambda}_{\text{mp}}}{\partial \boldsymbol{\lambda}_0} & \frac{\partial \Delta \boldsymbol{\lambda}_{\text{mp}}}{\partial \boldsymbol{\lambda}_f} \end{bmatrix} = \begin{bmatrix} -\Phi_{\text{for}}^{(1:6, 7:12)} & \Phi_{\text{back}}^{(1:6, 7:12)} \\ -\Phi_{\text{for}}^{(7:12, 7:12)} & \Phi_{\text{back}}^{(7:12, 7:12)} \end{bmatrix} \quad (5.8)$$

where Φ_{for} and Φ_{back} denote the STMs of the forward and backward propagation arc, respectively, and the superscripts denote the appropriate block extraction depending on the ordering of the state and costate variables. The use of analytic Jacobians obtained through STM propagation significantly improves convergence speed and robustness compared to finite-difference approximations.

5.1.3. Impulsification Procedure

The ultimate goal of this chapter is to compute deterministic, impulsive science-orbit transfers between a PTO and a 6:1 RTO around Apophis. Directly solving a minimum-fuel impulsive transfer is challenging, because the amount, epochs and magnitude of the maneuvers are unknown a priori. However, this problem is closely related to the fixed-time, minimum fuel optimal control problem for a low-thrust spacecraft.

For minimum-fuel low-thrust transfers, Pontryagin's Minimum Principle shows that the optimal control structure is given by the primer vector, as it determines both the control direction and the throttle profile through the switching function $\mathcal{S}$. For a fuel-optimal low-thrust trajectory, $\mathcal{S}$ is unbounded on an optimal trajectory, as long as the necessary optimality conditions are satisfied [113].

In contrast, impulsive optimal solutions impose stricter requirements on the behavior of the switching function. For an impulsive optimal trajectory, optimality requires that

$\mathcal{S} \leq 0$ over the entire transfer, with $\mathcal{S} = 0$ only when an impulsive maneuver is applied [114]. Previous studies [106, 110, 113] have shown that, as the maximum allowable acceleration increases ($a_{\max} \rightarrow \infty$), the duration of $\mathcal{S}$ being positive approaches zero, converging to the switching function profile of a purely impulsive trajectory.

Numerically, this is achieved through an impulsification procedure based on continuation with respect to the maximum acceleration $a_{\max}$. The continuation is carried out iteratively until the switching function satisfies $\mathcal{S} \leq 0$ for nearly the entire time of flight.

Figure 5.2 illustrates the impulsification procedure adopted in this thesis. Following [110], a two-level continuation scheme is employed: a continuation on the parameter ρ , used to generate near bang–bang throttle profiles via a hyperbolic tangent smoothing, and a continuation on the maximum acceleration $a_{\max}$.

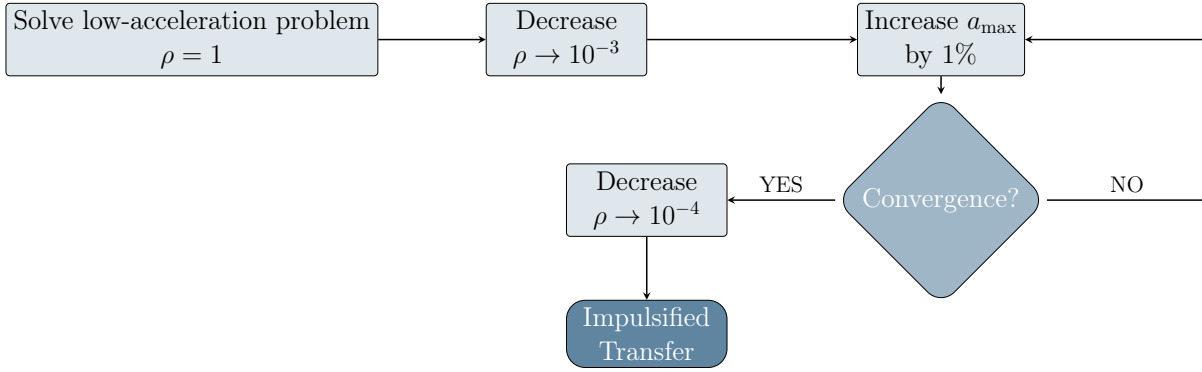


Figure 5.2: Impulsification procedure based on continuation on the smoothing parameter ρ and maximum acceleration $a_{\max}$.

First, a low-acceleration problem is solved with $\rho = 1$. After obtaining a solution, the level of the smoothing parameter is reduced to $\rho = 10^{-3}$ in 100 algorithmic steps. Then, the acceleration level is increased by 1% and a check is performed on the fraction of the time of flight during which the switching function is strictly positive. The continuation on $a_{\max}$ is terminated when this fraction falls below a prescribed threshold, set in this work to 2% of the total time of flight. Once the highest $a_{\max}$ solution is obtained, the smoothing parameter is reduced to $\rho = 10^{-4}$.

All resulting MPBVPs are solved using the indirect forward-backward shooting method via `fsolve` in MATLAB. During the continuation procedure, convergence tolerances are intentionally relaxed to 10^{-6} to promote robust convergence, and subsequently tightened to 10^{-10} in the final step to accurately recover the near-impulsive solution with $\rho = 10^{-4}$.

5.1.4. Fixed-Phase Impulsified Transfers

In this section some representative fixed-phase, fixed-time, impulsified transfers obtained using the previously explained procedure are showcased, together with the acceleration-based switching surface plots generated by concatenating the $\mathcal{S}(t)$ curve for each level of $a_{\max}$. For the purpose of this analysis, three different arrival phases have been selected on the 6:1 RTO, departing from the same phase on the departure PTO.

Applying the aforementioned procedure, a low-acceleration solution is found first, as shown in Figure 5.3. Then, the impulsified transfers are found following Figure 5.2 and are shown in Figure 5.4.

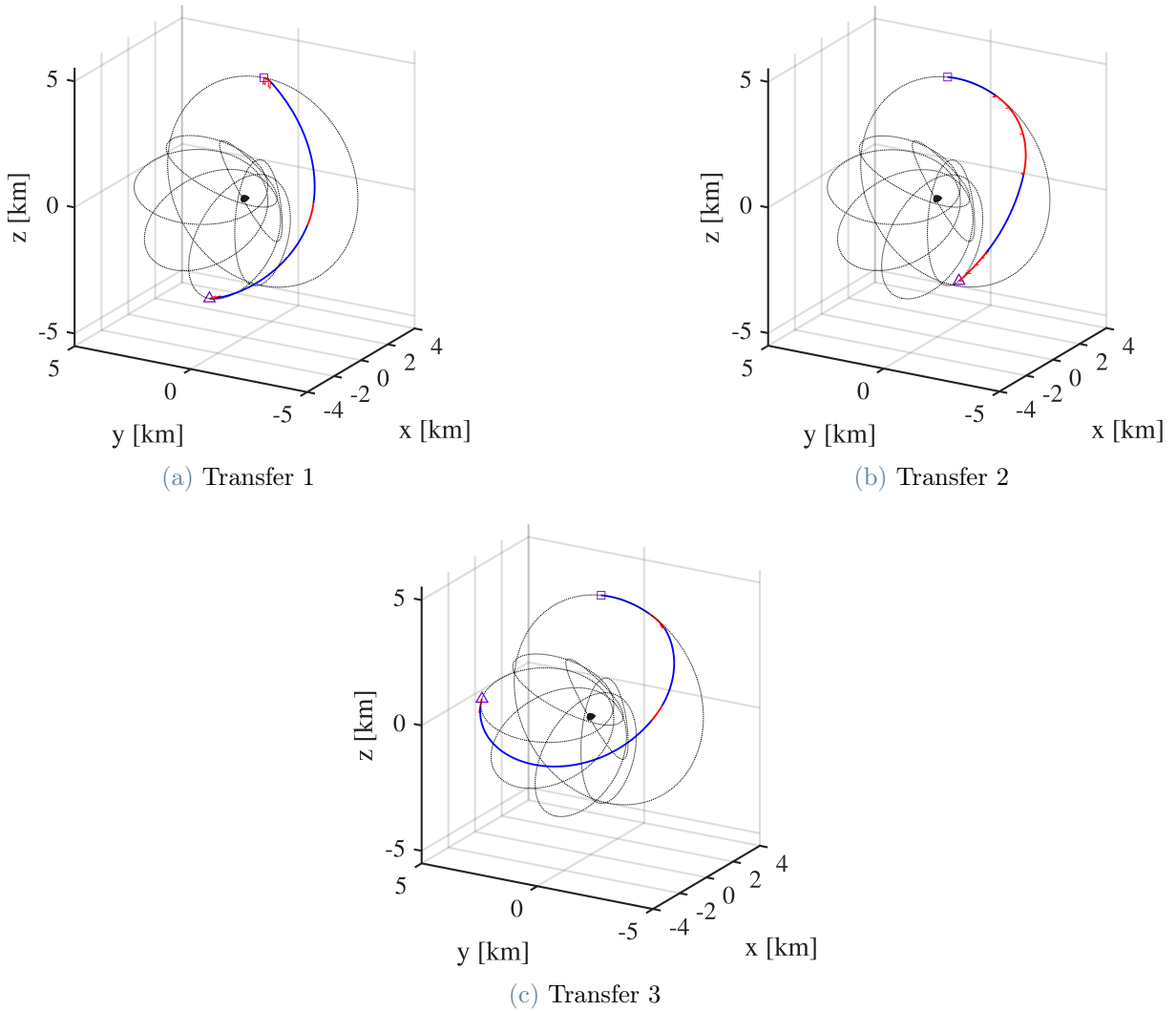


Figure 5.3: Representative Low-Acceleration PTO-RTO transfers with Burn Arc in red, Coast Arc in blue, Departure and Arrival Locations denoted with purple square and triangle.

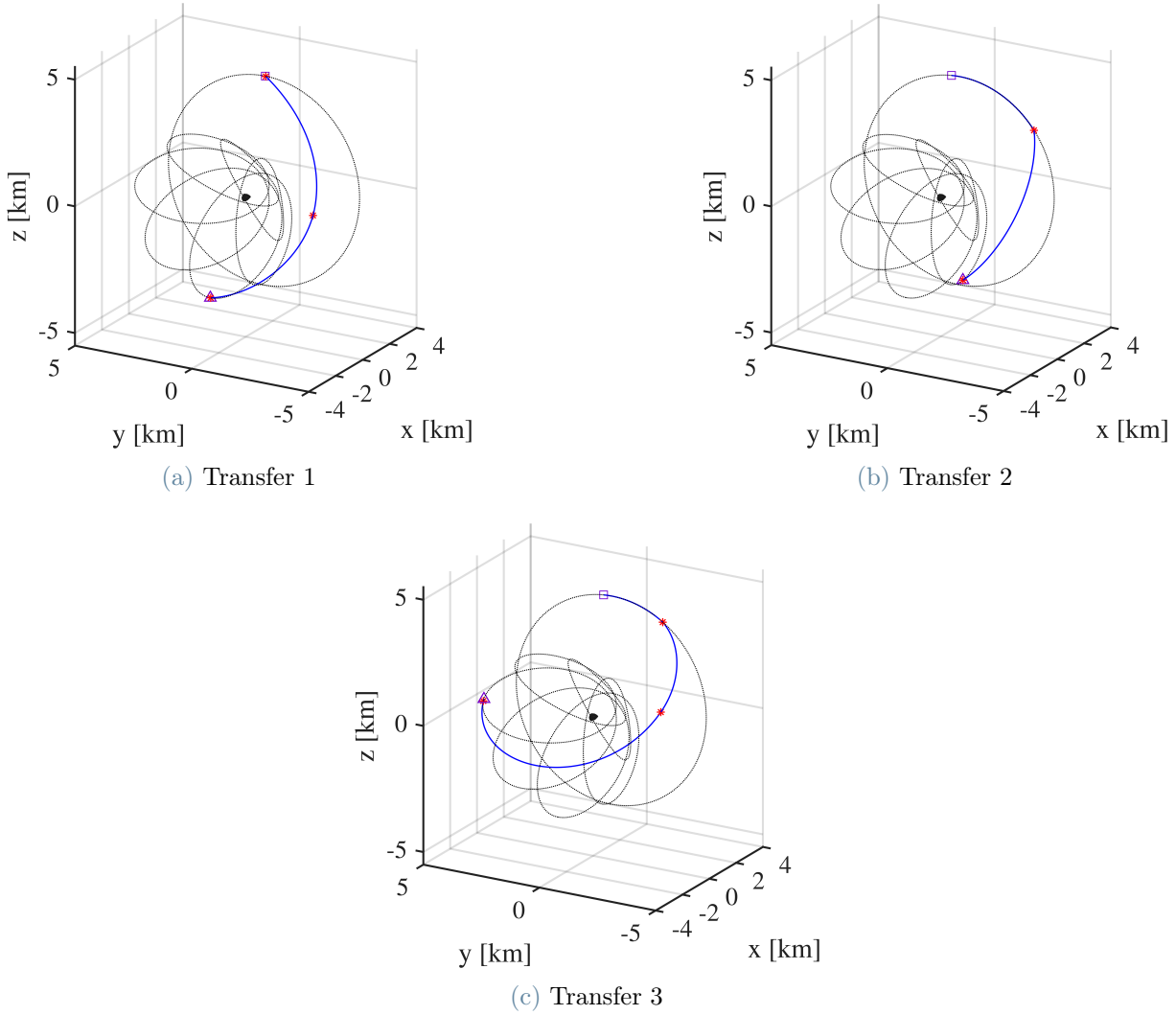


Figure 5.4: Representative Impulsified PTO-RTO Transfers with Maneuvers marked with red asterisks.

A summary of the maneuver and overall transfer cost for each transfer in terms of total ΔV is reported in Table 5.1.

Table 5.1: ΔV (cm/s) Cost for representative impulsified PTO-RTO transfers.

	$\Delta V_{\text{Initial}}$	$\Delta V_{\text{Midcourse}}$	ΔV_{Final}	ΔV_{Total}
Transfer 1	1.38	2.31	1.04	4.73
Transfer 2	—	2.36	1.06	3.42
Transfer 3	1.45	0.62	0.68	2.75

Comparing the low-acceleration and the impulsified trajectories, it can be observed that

the shape of the transfer does not change, but rather the main difference is the duration of each burn which decreases for increasing levels of $a_{\max}$. This shows also that impulsifying the trajectory does not reduce the number of the maneuvers, but only the duration of each burn.

A clear consequence of this phenomenon is the inherent bias of the impulsified trajectory towards the original low-acceleration one, as also acknowledged in previous works [113]. As any continuation method, the impulsification procedure does not search the entire domain of all possible solutions for a given level of $a_{\max}$, but rather it simply extends the previous solution with the new acceleration condition. Although the problem of being susceptible to local optimality is true for almost any optimization problem and especially holds for any continuation procedure, it is still important to mention this inherent drawback.

To appreciate the successful impulsification of trajectories, Figure 5.5 shows the switching function during the transfers for increasing levels of $a_{\max}$, while Figure 5.7 shows the switching surfaces. Recall that for an impulsive trajectory, optimality requires that $\mathcal{S} \leq 0$ over the entire transfer, with $\mathcal{S} = 0$ only when an impulsive maneuver is executed. As the maximum allowable acceleration increases, the $\mathcal{S}(t)$ curve is progressively shifted towards zero, resembling the switching function for impulsive maneuvers.

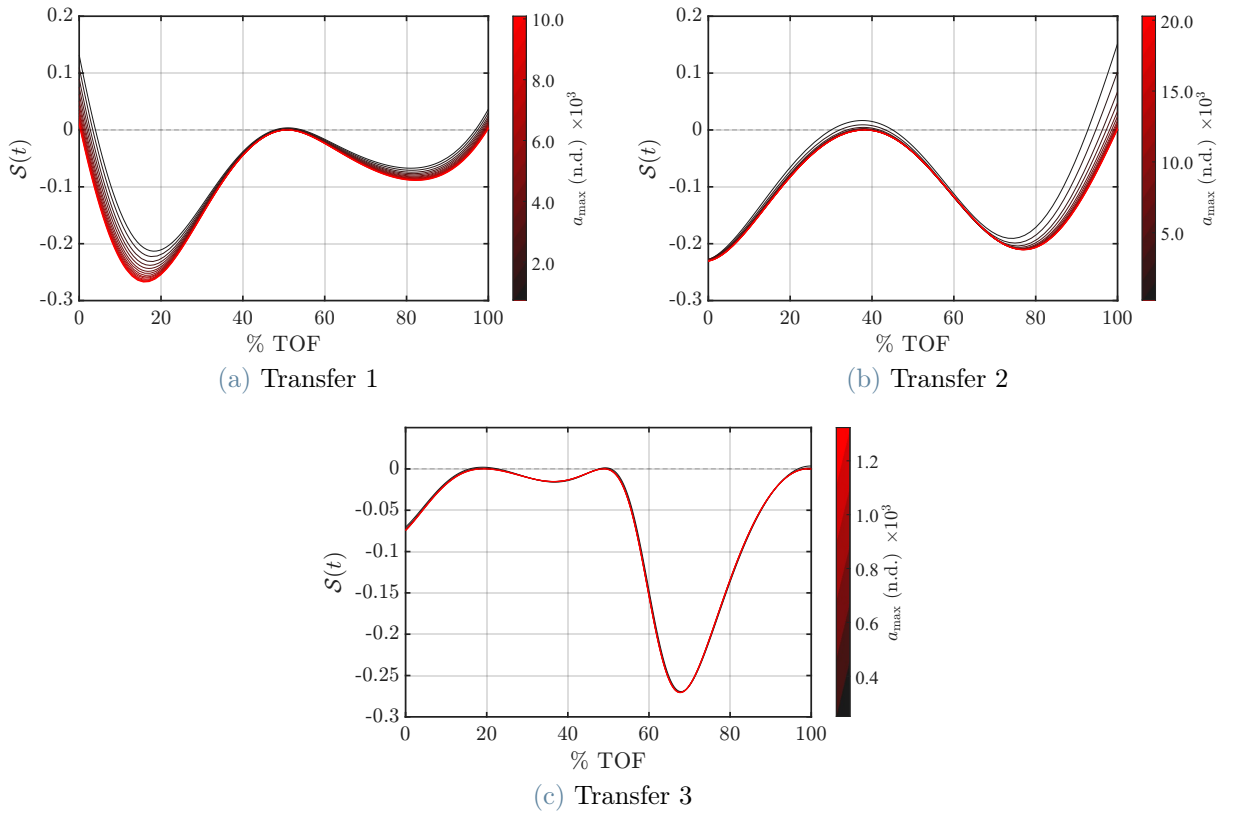


Figure 5.5: Switching Function throughout the Impulsification Procedure

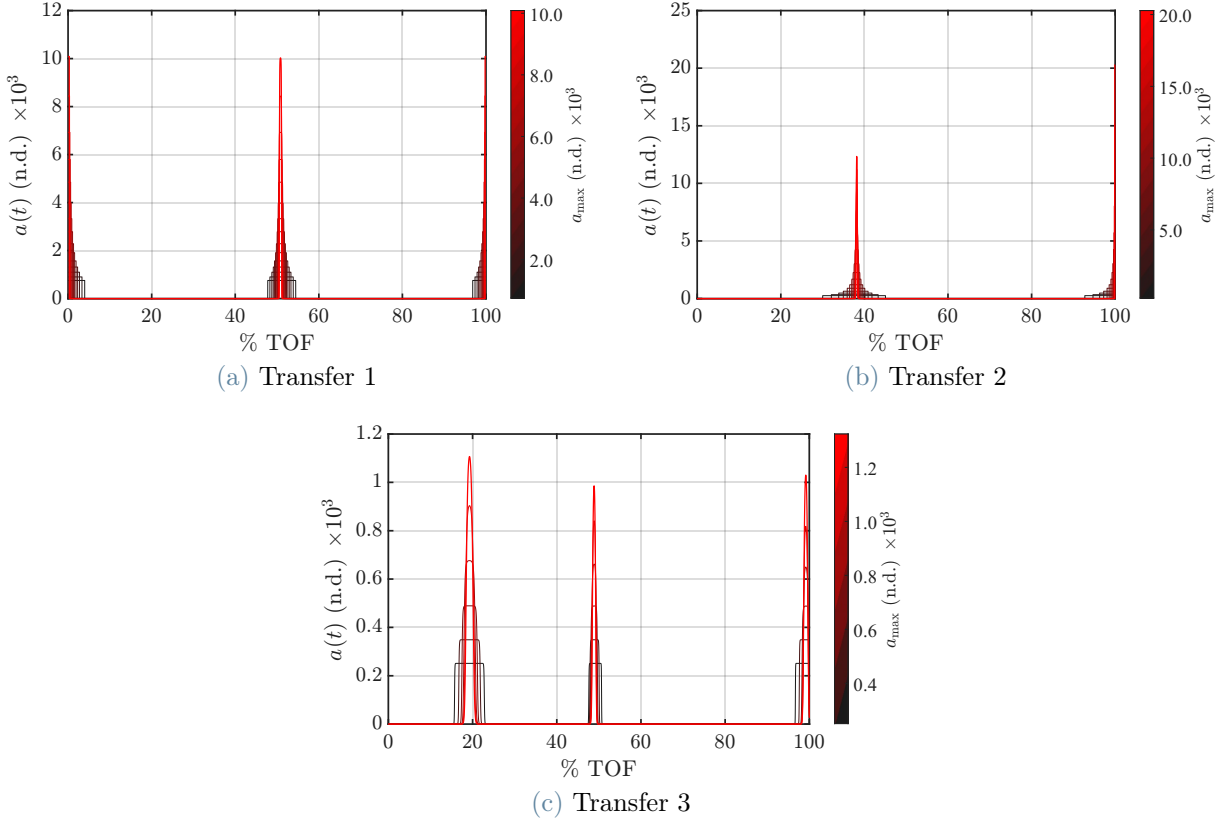


Figure 5.6: Control Accelerations throughout the Impulsification Procedure

It can be observed that, for some of the mid-course maneuvers, the control magnitude does not reach the maximum allowable value even as the impulsification procedure progresses. This behavior has already been reported and discussed by Qi [113]. This phenomenon can primarily be attributed to two effects.

First, it could be due to the hyperbolic tangent approximation of bang–bang control, which introduces small nonzero control values before the actual optimal switching instant. These residual values become more significant as the maximum allowable acceleration increases and the maneuver duration shrinks. As a result, the control may not have sufficient time to fully saturate and reach its maximum value.

A second possible explanation is the presence of singular control. According to Pontryagin’s Minimum Principle, the optimal control is bang–bang when the switching function is strictly positive or negative, but the control magnitude is not uniquely defined when the switching function remains identically zero over a finite time interval. Resolving this case would require higher-order optimality conditions [115], but this analysis is beyond the scope of this thesis.

Nevertheless, the resulting trajectories remain near-optimal, as the necessary optimality

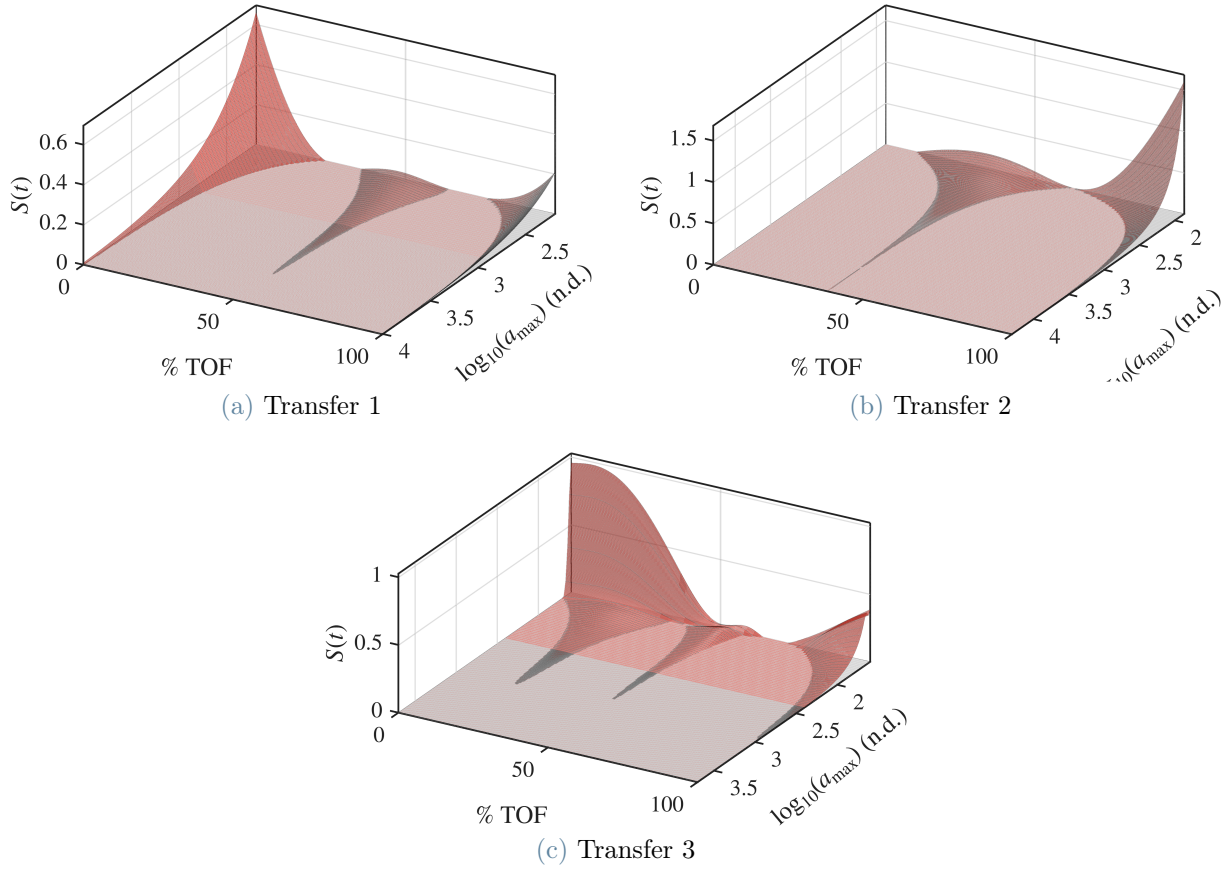


Figure 5.7: Switching Surface Plots

conditions are satisfied everywhere else along the transfer, and the impulsification process still yields near-impulsive maneuvers.

5.1.5. Free-Phase Transfer

The impulsification procedure presented in the previous section was applied to transfers with fixed departure and arrival phases along the terminator orbits. While this assumption allowed for a clear assessment of the impulsification process and its effects on the control structure, it also restricts the solution space by predefining the epochs at which the transfer must begin and terminate.

In many practical mission scenarios, however, the departure and arrival phases the departure and arrival orbits are not fixed a priori, and allowing them to vary can significantly improve transfer feasibility and fuel efficiency. For this reason, this section extends the indirect optimal control framework to the computation of free-phase transfers, in which both the departure and arrival phases are treated as decision variables.

The extension requires the introduction of additional optimization parameters, new transver-

sality conditions derived via a change-of-variables formulation, and a corresponding modification of the shooting function and its Jacobian.

Let $\tau_0, \tau_f \in [0, 1]$ denote the phases along the departure and arrival orbit, respectively, corresponding to the fraction of propagation time along the orbit. In this new formulation, let $\mathbf{x}_0$ and $\mathbf{x}_f$ be free but constrained to be equal to the periodic orbit evaluated at the phases τ_0, τ_f . This yields a new set of boundary conditions:

$$\Psi_0(\mathbf{x}_0, \tau_0) = \mathbf{x}_0 - \bar{\mathbf{x}}_{\text{dep}}(\tau_0) = 0, \quad \Psi_f(\mathbf{x}_f, \tau_f) = \mathbf{x}_f - \bar{\mathbf{x}}_{\text{arr}}(\tau_f) = 0 \quad (5.9)$$

where $\bar{\mathbf{x}}_{\text{dep}}(\tau_0)$ and $\bar{\mathbf{x}}_{\text{arr}}(\tau_f)$ indicate the departure and arrival trajectories evaluated at the initial and final phases, respectively.

For this new problem, the new set of necessary conditions of optimality are derived by using Calculus of Variations. By inspecting the first variation of J :

$$dJ = (\dots) + (\boldsymbol{\nu}^\top \Psi_{0,\mathbf{x}_0} + \boldsymbol{\lambda}_0^\top) d\mathbf{x}_0 + (\boldsymbol{\nu}^\top \Psi_{f,\mathbf{x}_f} - \boldsymbol{\lambda}_f^\top) d\mathbf{x}_f + \boldsymbol{\nu}^\top \Psi_{0,\tau_0} d\tau_0 + \boldsymbol{\nu}^\top \Psi_{f,\tau_f} d\tau_f \quad (5.10)$$

Since $\{\mathbf{x}_0, \mathbf{x}_f, \tau_0, \tau_f\}$ are all allowed to vary, the quantities that multiply their derivative must be zero in order to satisfy the condition $dJ = 0$. The partial derivative of the boundary conditions with respect to each variables are:

$$\Psi_{0,\mathbf{x}_0} = \Psi_{f,\mathbf{x}_f} = \mathbf{I}_{6 \times 6}, \quad \Psi_{0,\tau_0} = \dot{\bar{\mathbf{x}}}_{\text{dep}}(\tau_0) T_{\text{dep}}, \quad \Psi_{f,\tau_f} = \dot{\bar{\mathbf{x}}}_{\text{arr}}(\tau_f) T_{\text{arr}} \quad (5.11)$$

where T denotes the period of the departure and arrival orbits and $\dot{\bar{\mathbf{x}}}$ denotes the equations of motion evaluated along the periodic orbits at the phases τ_0, τ_f .

Enforcing the quantities that multiply the derivative of the variables to be zero and combining all equations together yields the following transversality conditions:

$$\boldsymbol{\nu}^\top = -\boldsymbol{\lambda}_0^\top, \quad \boldsymbol{\nu}^\top = \boldsymbol{\lambda}_f^\top, \quad \boldsymbol{\lambda}_0^\top \dot{\bar{\mathbf{x}}}_{\text{dep}}(\tau_0) = 0, \quad \boldsymbol{\lambda}_f^\top \dot{\bar{\mathbf{x}}}_{\text{arr}}(\tau_f) = 0 \quad (5.12)$$

The problem is converted into a MPBVP using the indirect forward-backward shooting method described in the previous sections. With respect to the previous fixed-phase case, the vector of unknowns is augmented with the unknown orbital phases $\{\tau_0, \tau_f\}$ and the

shooting function is augmented with the newly derived transversality conditions.

$$\mathbf{Z} = \begin{bmatrix} \boldsymbol{\lambda}_0 \\ \boldsymbol{\lambda}_f \\ \tau_0 \\ \tau_f \end{bmatrix}, \quad \boldsymbol{\Psi}(\mathbf{Z}) = \begin{bmatrix} \Delta \mathbf{x}_{\text{mp}} \\ \Delta \boldsymbol{\lambda}_{\text{mp}} \\ \boldsymbol{\lambda}_0^\top \dot{\mathbf{x}}_{\text{dep}}(\tau_0) \\ \boldsymbol{\lambda}_f^\top \dot{\mathbf{x}}_{\text{arr}}(\tau_f) \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{\text{for}}(t_m^+) - \mathbf{x}_{\text{back}}(t_m^-) \\ \boldsymbol{\lambda}_{\text{for}}(t_m^+) - \boldsymbol{\lambda}_{\text{back}}(t_m^-) \\ \boldsymbol{\lambda}_0^\top \dot{\mathbf{x}}_{\text{dep}}(\tau_0) \\ \boldsymbol{\lambda}_f^\top \dot{\mathbf{x}}_{\text{arr}}(\tau_f) \end{bmatrix} = \mathbf{0} \quad (5.13)$$

Analytical gradients are computed to assemble the Jacobian of the free-phase shooting function for improved numerical robustness.

$$\mathbf{J}_{\boldsymbol{\Psi}}(\mathbf{Z}) = \begin{bmatrix} -\Phi_{\text{for}}^{(1:6, 7:12)} & \Phi_{\text{back}}^{(1:6, 7:12)} & -\Phi_{\text{for}}^{(1:6, 1:6)} \dot{\mathbf{x}}_0 T_{\text{dep}} & \Phi_{\text{back}}^{(1:6, 1:6)} \dot{\mathbf{x}}_f T_{\text{arr}} \\ -\Phi_{\text{for}}^{(7:12, 7:12)} & \Phi_{\text{back}}^{(7:12, 7:12)} & -\Phi_{\text{for}}^{(7:12, 1:6)} \dot{\boldsymbol{\lambda}}_0 T_{\text{dep}} & \Phi_{\text{back}}^{(7:12, 1:6)} \dot{\boldsymbol{\lambda}}_f T_{\text{arr}} \\ \dot{\mathbf{x}}_0 + \boldsymbol{\lambda}_0^\top \frac{\partial \dot{\mathbf{x}}_0}{\partial \boldsymbol{\lambda}_0} & \mathbf{0}_{1 \times 6} & \dot{\boldsymbol{\lambda}}_0^\top \dot{\mathbf{x}}_0 + \boldsymbol{\lambda}_0^\top \ddot{\mathbf{x}}_0 & 0 \\ \mathbf{0}_{1 \times 6} & \dot{\mathbf{x}}_f + \boldsymbol{\lambda}_f^\top \frac{\partial \dot{\mathbf{x}}_f}{\partial \boldsymbol{\lambda}_f} & 0 & \dot{\boldsymbol{\lambda}}_f^\top \dot{\mathbf{x}}_f + \boldsymbol{\lambda}_f^\top \ddot{\mathbf{x}}_f \end{bmatrix} \quad (5.14)$$

Numerical Challenges and Role of Impulsification

Extending the formulation with indirect methods to free-phase transfers significantly introduced several numerical challenges. In particular, convergence of the shooting method has become extremely slow, and in many cases full convergence could not be achieved within reasonable computational time. Moreover, the optimal values of the departure and arrival phase parameters, τ_0 and τ_f , have been observed to remain very close to their initial guesses, suggesting the presence of multiple local minima and a highly nonconvex solution domain.

When combined with the impulsification procedure outlined in the previous sections, these numerical challenges became even more pronounced. Indeed, the continuation strategy showed prohibitively slow convergence, if convergence was achieved at all. This behavior is likely due to the increased sensitivity of the shooting function to variations in both the phase parameters and the acceleration bound a_{max} .

Despite these challenges, the free-phase indirect solutions obtained prior to full convergence remain highly valuable and were still utilized in this thesis. These preliminary solutions exploit the natural dynamics of the ANH3BP much more effectively, yielding longer-duration transfers that incorporate features multiple-revolutions and flyby of Apophis. Figure 5.8 shows some representative examples of these partially converged solutions, which are used as initial guesses for the subsequent AMSCP optimization stage.

As demonstrated in the following section, initializing AMSCP with these free-phase indirect solutions enables the computation of fully impulsive, bounded-time, free-phase

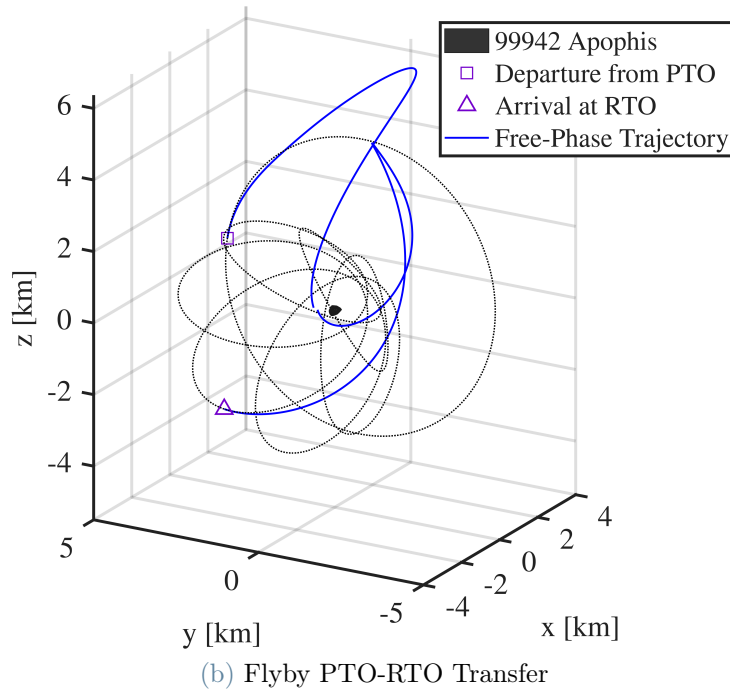
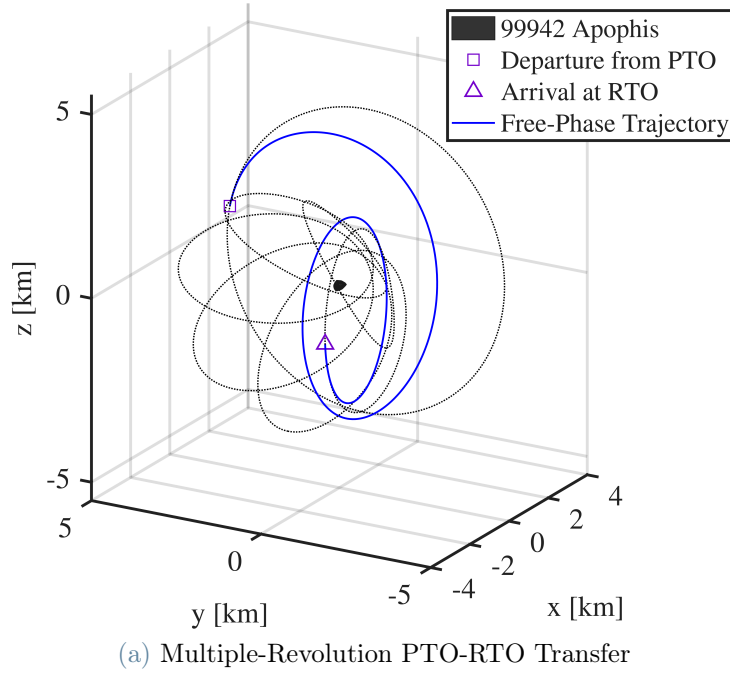


Figure 5.8: Free-Phase transfers obtained with the indirect shooting method (not fully converged) used as initial guess for the subsequent AMSCP optimization.

transfers that effectively exploit the natural dynamics of the ANH3BP and would have been difficult to obtain otherwise.

5.2. Adaptive Mesh Sequential Convex Programming

The limitations and numerical challenges encountered when implementing the indirect forward–backward shooting approach for free–phase transfers motivate the adoption of a different optimization framework. Although indirect methods guarantee satisfaction of the theoretical optimality conditions, their practical performance has proven inadequate when dealing with free-phase trajectories, especially when dealing with highly nonlinear dynamics. In particular, the shooting formulation has been shown to be extremely sensitive to the initial guess, and convergence has proven unreliable even when continuation strategies are applied, as explained in the previous sections.

For these reasons, Sequential Convex Programming (SCP) represents an attractive alternative for the design of impulsive, free–phase transfers between science orbits. SCP addresses the original non-convex problem by solving a sequence of convex subproblems obtained through local linearization and convexification. However, as with any other direct method, an inherent drawback stems from time discretization. Direct methods divide the time horizon into a finite number of nodes, usually equispaced in time. In the context of impulsive trajectory optimization, this implies that the control actions can only occur at these predefined epochs. As a result, the resulting control solution is inherently suboptimal, and recovering the truly optimal profile predicted by Pontryagin’s Minimum Principle would require an infinite number of discretization nodes.

To address this limitation, this thesis adopts the Adaptive Mesh Sequential Convex Programming (AMSCP) formulation proposed by Kumagai and Oguri [108], combined with a free-phase approach inspired by the work of Hiraiwa [116]. By allowing both the temporal distribution of the discretization nodes and the departure and arrival phases to be optimized, this method enables impulsive maneuvers to emerge naturally during the optimization process, without prescribing their location a priori.

Although the AMSCP formulation does not provide formal guarantees of global optimality, it has been shown in previous works to recover control structures that more closely resemble the optimal profiles predicted by PMP [108, 117]. As a result, this approach allows the computation of fully impulsive, free-phase, bounded-time transfers while maintaining numerical robustness.

5.2.1. Original Free-Phase Non-Convex Problem

This section formulates the general free-phase fuel optimal control problem to be solved via AMSCP. As previously introduced in Chapter 2, a generic form of the optimal control

problem, expressed in Lagrange-form, can be written as:

$$\begin{aligned}
\min_{\mathbf{x}, \mathbf{u}} \quad & J_0 = \int_{t_0}^{t_f} L(\mathbf{x}(t), \mathbf{u}(t), t) dt \\
\text{s.t.} \quad & \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) \\
& \mathbf{g}_i(\mathbf{x}(t), \mathbf{u}(t), t) = 0 \quad \forall i, & \mathbf{h}_j(\mathbf{x}(t), \mathbf{u}(t), t) \leq 0 \quad \forall j, \\
& \mathbf{g}_{\text{affine}}(\mathbf{x}(t), \mathbf{u}(t), t) = 0 \quad \forall i, & \mathbf{h}_{\text{cvx}}(\mathbf{x}(t), \mathbf{u}(t), t) \leq 0 \quad \forall j
\end{aligned}$$

where $\mathbf{x} \in \mathbb{R}^{n_x}$ represents the state and $\mathbf{u} \in \mathbb{R}^{n_u}$ is the control input. The function $\mathbf{f}$ denotes the system dynamics, which are generally non-convex in $\mathbf{x}$ and $\mathbf{u}$. The functions $\mathbf{g}_{\text{affine}}$ and $\mathbf{g}_i$, $\forall i$, represent the affine and non-affine equality constraint functions, respectively. $\mathbf{h}_{\text{cvx}}$ and $\mathbf{h}_j$, $\forall j$, denote the convex and non-convex inequality constraint functions, respectively.

For fuel optimal problems, the running cost takes the form $L(\mathbf{x}(t), \mathbf{u}(t), t) = \|\mathbf{u}(t)\|_2$. In the case of free-phase transfers, the boundary conditions at the initial and final times are constrained to lie on the departure and arrival orbits, respectively, while allowing the corresponding phases τ_0 and τ_f to vary freely, as defined in Section 5.1.5. These boundary constraints are expressed as:

$$\mathbf{x}(t_0) = \bar{\mathbf{x}}_{\text{dep}}(\tau_0), \quad \mathbf{x}(t_f) = \bar{\mathbf{x}}_{\text{arr}}(\tau_f) \quad (5.15)$$

Accordingly, the original optimal control problem for a free-phase transfer can be stated as follows.

Problem 1 (*Original non-convex Free-Phase fuel-optimal control problem*)

$$\min_{\mathbf{x}, \mathbf{u}, \tau_0, \tau_f} \quad J_0 = \int_{t_0}^{t_f} \|\mathbf{u}(t)\|_2 dt \quad (5.16)$$

$$\text{s.t.} \quad \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) \quad (5.17)$$

$$\mathbf{x}(t_0) = \bar{\mathbf{x}}_{\text{dep}}(\tau_0), \quad (5.18)$$

$$\mathbf{x}(t_f) = \bar{\mathbf{x}}_{\text{arr}}(\tau_f) \quad (5.19)$$

The resulting optimization problem is non-convex due to the presence of nonlinear dynamics and non-affine boundary conditions, which depend nonlinearly on the phase variables through the orbit parametrization. As a consequence, Problem 1 must be reformulated into a sequence of convex subproblems in order to be tractable within a SCP framework.

5.2.2. Convex Subproblem via Adaptive Mesh Formulation

Having formulated the original free-phase optimal control problem, this section describes how it is reformulated into a sequence of convex subproblems using Adaptive Mesh Sequential Convex Programming (AMSCP). The adopted method linearizes and discretizes the nonlinear equations of motion by including the time between each node as a decision variable.

Linearization and Discretization with Adaptive Mesh

The process of linearization and discretization builds upon the classical procedure introduced in Chapter 2, modifying it to include the time variables as decision variables.

First, normalized time $\sigma \in [0, 1]$ is introduced with N equally spaced partitions, such that:

$$\sigma_k = \frac{k}{N}, \quad k = 0, 1, \dots, N \quad (5.20)$$

A mapping from normalized time to non-normalized time is defined, $s : \mathbb{R} \rightarrow \mathbb{R}$, such that:

$$s(\sigma) = \frac{dt}{d\sigma} \quad (5.21)$$

This quantity is also referred to in previous works as time delation [108, 118]. A zero-order-hold formulation is considered for s , i.e., the time dilation is considered constant on each time segment.

The dynamics in normalized time is expressed as [108]:

$$\frac{d\mathbf{x}}{d\sigma} = \frac{d\mathbf{x}}{dt} \frac{dt}{d\sigma} = \dot{\mathbf{x}}s = s(\sigma)\mathbf{f}(\mathbf{x}, \mathbf{u}, t) := \mathbf{F}(\mathbf{x}, \mathbf{u}, \sigma) \quad (5.22)$$

Or, by writing $\mathbf{x}' = \frac{d\mathbf{x}}{d\sigma}$:

$$\mathbf{x}' = \mathbf{F}(\mathbf{x}, \mathbf{u}, \sigma) \quad (5.23)$$

The dynamics are linearized about the reference $\{\mathbf{x}^*, \mathbf{u}^*, s^*\}$ to obtain the LTV system in normalized time:

$$\mathbf{x}' = A(\sigma)\mathbf{x}(\sigma) + B(\sigma)\mathbf{u}(\sigma) + \mathbf{d}(\sigma)s(\sigma) + \mathbf{c}(\sigma) \quad (5.24)$$

where:

$$A(\sigma) = s^* \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{\mathbf{x}^*, \mathbf{u}^*}, \quad B(\sigma) = s^* \left. \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right|_{\mathbf{x}^*, \mathbf{u}^*}, \quad \mathbf{d}(\sigma) = \mathbf{f}(\mathbf{x}^*, \mathbf{u}^*, \sigma),$$

$$\mathbf{c}(\sigma) = s^* \mathbf{f}(\mathbf{x}^*, \mathbf{u}^*, \sigma) - A(\sigma) \mathbf{x}^*(\sigma) - B(\sigma) \mathbf{u}^*(\sigma) - s^* \mathbf{d}(\sigma) = -A(\sigma) \mathbf{x}^*(\sigma) - B(\sigma) \mathbf{u}^*(\sigma)$$

Since the time delation parameters are constant on each time interval $[t_k, t_{k+1})$, they are also constant on each normalized time interval $[\sigma_k, \sigma_{k+1})$. Finally, the discrete-time LTV system is obtained:

$$\mathbf{x}_{k+1} = A_k \mathbf{x}_k + B_k \mathbf{u}_k + \mathbf{d}_k s_k + \mathbf{c}_k \quad k = 0, 1, \dots, N-1 \quad (5.25)$$

where:

$$A_k = s^* \Phi(\sigma_{k+1}, \sigma_k), \quad B_k = A_k \int_{\sigma_k}^{\sigma_{k+1}} \Phi^{-1}(\sigma, \sigma_k) B(\sigma) d\sigma,$$

$$\mathbf{d}_k = \int_{\sigma_k}^{\sigma_{k+1}} \mathbf{f}(\mathbf{x}^*, \mathbf{u}^*, \sigma) d\sigma, \quad \mathbf{c}_k = c_k \int_{\sigma_k}^{\sigma_{k+1}} \Phi^{-1}(\sigma, \sigma_k) \mathbf{c}(\sigma) d\sigma \quad (5.26)$$

This formulation allows the discretization grid to deform during the optimization process, enabling the algorithm to automatically allocate finer temporal resolution in regions where the dynamics or control vary rapidly, while coarsening the grid along ballistic or slowly varying arcs.

Each time delation parameter is constrained to be non-negative [108], and after obtaining s_k , the non-normalized epochs can be retrieved with the following:

$$s_k \geq 0 \quad \forall k, \quad t_k = \frac{1}{N} \sum_{i=0}^{N-1} s_i \quad (5.27)$$

Convexification of Other Non-Convex Constraints

In addition to the nonlinear dynamics constraints, the free-phase boundary conditions are also non-convex as they depend nonlinearly on the phase variables through the orbit parametrization. To retrieve convexity, the boundary conditions are linearized about a reference phase vector $\boldsymbol{\tau}^* = [\tau_0^*, \tau_f^*]^\top$, as done by Hiraiwa [116].

$$\mathbf{x}_0 = \bar{\mathbf{x}}_{\text{dep}}(\tau_0^*) + \left. \frac{\partial \bar{\mathbf{x}}_{\text{dep}}}{\partial \boldsymbol{\tau}} \right|_{\tau_0^*} (\tau_0 - \tau_0^*) \quad \mathbf{x}_f = \bar{\mathbf{x}}_{\text{arr}}(\tau_f^*) + \left. \frac{\partial \bar{\mathbf{x}}_{\text{arr}}}{\partial \boldsymbol{\tau}} \right|_{\tau_f^*} (\tau_f - \tau_f^*) \quad (5.28)$$

Where $\bar{\mathbf{x}}_{(\cdot)}(\tau^*)$ denote the departure and arrival states obtained by evaluating the periodic

trajectory at the reference phase parameter τ^* .

Recall that, from the phase parameter along the periodic orbit, the propagation time can be retrieved as $t = \tau T$, where T is the period of the science orbit. It follows that $\partial t / \partial \tau = T$. As a consequence, by using the chain rule:

$$\left. \frac{\partial \bar{\mathbf{x}}_{(\cdot)}}{\partial \boldsymbol{\tau}} \right|_{\tau^*} = \left. \frac{\partial \bar{\mathbf{x}}_{(\cdot)}}{\partial t} \frac{\partial t}{\partial \boldsymbol{\tau}} \right|_{\tau^*} = T_{(\cdot)} \dot{\bar{\mathbf{x}}}_{(\cdot)}(\tau^*) \quad (5.29)$$

where $\dot{\bar{\mathbf{x}}}_{(\cdot)}(\tau^*)$ denote the equations of motion evaluated at the reference phase along the departure and arrival orbit.

Finally, the linearized boundary conditions can be written as:

$$\begin{aligned} \mathbf{x}_0 &= \bar{\mathbf{x}}_{\text{dep}}(\tau_0^*) + T_{\text{dep}} \dot{\bar{\mathbf{x}}}_{\text{dep}}(\tau_0^*) \cdot (\tau_0 - \tau_0^*) \\ \mathbf{x}_f &= \bar{\mathbf{x}}_{\text{arr}}(\tau_f^*) + T_{\text{arr}} \dot{\bar{\mathbf{x}}}_{\text{arr}}(\tau_f^*) \cdot (\tau_f - \tau_f^*) \end{aligned} \quad (5.30)$$

Trust Region Constraints and Augmented Lagrangian Method

The AMSCP problem is solved by adopting the SCvx* algorithm, proposed in [28] and presented in Chapter 3. Recall that, to address the common issues of *artificial unboundedness* and *artificial infeasibility* [25], additional constraints are introduced in the convex subproblem.

To avoid *artificial unboundedness*, trust regions are imposed on the decision variables in order to prevent excessive deviation from the linearization point around the reference trajectory, following the approaches of [25, 28]. This results in the following convex constraint:

$$\|\mathbf{z} - \mathbf{z}^*\|_\infty \leq \Delta^{(j)} \quad (5.31)$$

where $\Delta^{(j)}$ is the trust region radius at the j -th iteration of the SCP algorithm, and the vector of decision variables is defined as $\mathbf{z} := [\mathbf{X}^\top, \mathbf{U}^\top, \mathbf{s}^\top, \boldsymbol{\tau}^\top]^\top$, with $\mathbf{X} := [\mathbf{x}_0^\top, \mathbf{x}_1^\top, \dots, \mathbf{x}_N^\top]^\top$, $\mathbf{U} := [\mathbf{u}_0^\top, \mathbf{u}_1^\top, \dots, \mathbf{u}_{N-1}^\top]^\top$, $\mathbf{s} := [s_0, s_1, \dots, s_{N-1}]^\top$, $\boldsymbol{\tau} := [\tau_0, \tau_f]^\top$.

To avoid *artificial infeasibility*, some constraints are relaxed and their violation is penalized inside the augmented Lagrangian penalty of the SCVx* algorithm [28]. In this work, the convexified constraints that are relaxed are the linearized boundary conditions of Eq. (5.30). A slack variable vector $\boldsymbol{\xi}$ is introduced, as well as the vector of lagrangian multipliers $\boldsymbol{\lambda}$. The original cost function is augmented with the penalty function defined

by:

$$J_{\text{AL}}(\mathbf{z}, \boldsymbol{\lambda}) = J_0(\mathbf{z}) + J_{\text{pen}}(\boldsymbol{\xi}) = \sum_{k=0}^{N-1} \|\mathbf{u}_k\|_2 + \boldsymbol{\lambda}^{(j)} \cdot \boldsymbol{\xi} + \frac{1}{2} w^{(j)} \boldsymbol{\xi} \cdot \boldsymbol{\xi} + \sqrt{w^{(j)}} \|\boldsymbol{\xi}\|_1 \quad (5.32)$$

Finally, the convex subproblem solved at each iteration of SCvx* is outlined in the following.

Problem 2 (*Augmented Convex Subproblem for Adaptive Mesh Formulation*)

$$\begin{aligned}
& \min_{\mathbf{x}_k, \mathbf{u}_k, s_k, \tau, \boldsymbol{\xi}} & J_{\text{AL}}(\mathbf{z}, \boldsymbol{\lambda}) = J_0(\mathbf{z}) + J_{\text{pen}}(\boldsymbol{\xi}) & \triangleright \text{Augmented Cost Function} \\
\text{subject to} & \mathbf{x}_{k+1} = A_k \mathbf{x}_k + B_k \mathbf{u}_k + \mathbf{d}_k s_k + \mathbf{c}_k & \triangleright \text{Linearized EoM} \\
& \mathbf{x}_0 - (\bar{\mathbf{x}}_{\text{dep}}(\tau_0^*) + T_{\text{dep}} \dot{\bar{\mathbf{x}}}_{\text{dep}}(\tau_0^*) \cdot (\tau_0 - \tau_0^*)) = \boldsymbol{\xi}^{1:6} & \triangleright \text{Departure Free-Phase State} \\
& \mathbf{x}_N - (\bar{\mathbf{x}}_{\text{arr}}(\tau_f^*) + T_{\text{arr}} \dot{\bar{\mathbf{x}}}_{\text{arr}}(\tau_f^*) \cdot (\tau_f - \tau_f^*)) = \boldsymbol{\xi}^{7:12} & \triangleright \text{Arrival Free-Phase State} \\
& \frac{1}{N} \sum_{k=0}^{N-1} s_k - \text{tof}_{\text{max}} \leq 0 & \triangleright \text{Bounded T.o.F.} \\
& -\mathbf{s} \leq 0, \quad -\tau_0 \leq 0, \quad \tau_0 \leq 1, \quad -\tau_f \leq 0, \quad \tau_f \leq 1 & \triangleright \text{Additional Constraints} \\
& \|\mathbf{z} - \mathbf{z}^*\|_{\infty} \leq \Delta^{(j)} & \triangleright \text{Trust Region Constraints}
\end{aligned}$$

5.2.3. Solution Method via SCvx*

The AMSCP problem is solved by using the SCvx* algorithm [28], using as initial guess the non fully converged free-phase indirect trajectories presented in Section 5.1.5. For $\mathbf{x}_k$, the states are extracted from the trajectories. The controls $\mathbf{u}_k$ are set to zero for all k , and the initial mesh is assumed uniform to initialize s_k .

Table 5.2 shows the parameters of the SCP algorithm.

Table 5.2: SCvx* parameters

Parameter	$\{\epsilon_{\text{opt}}, \epsilon_{\text{feas}}\}$	$\{\rho_1, \rho_2, \rho_3\}$	$\{\alpha_1, \alpha_2\}$	β	γ	$\{\Delta_{\text{min}}, \Delta_{\text{max}}\}$	$\Delta^{(1)}$	w_{max}	$w^{(1)}$
Value	$\{10^{-4}, 10^{-6}\}$	$\{0, 0.25, 0.75\}$	$\{2, 3\}$	2	0.9	$\{10^{-10}, 1\}$	0.1	10^8	10^3

For a schematic overview of the algorithm, readers are referred to Algorithm 5.1. All convex subproblems are modeled in MATLAB using YALMIP [94] and solved with MOSEK [95].

Algorithm 5.1 Deterministic Adaptive Mesh Trajectory Optimization via SCvx*

Initialize SCvx* parameters
 $\Delta_{\text{tr}}^{(0)}, w^{(0)}, \rho_0, \rho_1, \rho_2, \alpha_1, \alpha_2, \beta, \gamma, \varepsilon_{\text{opt}}, \varepsilon_{\text{feas}}$
 $k = 1, \Delta J = \chi = \delta = \infty, \boldsymbol{\lambda} = \mathbf{0}$

- 1: **while** $\Delta J > \varepsilon_{\text{opt}} \vee \chi > \varepsilon_{\text{feas}}$ **do**
Obtain $\{\mathbf{A}_k, \mathbf{B}_k, \mathbf{d}_k, \mathbf{h}_k\}$ from $\mathbf{Z}^{\text{ref}}$
 $\mathbf{Z}^{\text{sol}} \leftarrow$ Solve convex subproblem
Evaluate $\rho \leftarrow \Delta J / \Delta L$
- 2: **if** acceptance conditions met **then**
- 3: $\mathbf{Z}^{\text{ref}} \leftarrow \mathbf{Z}^{\text{sol}}$
- 4: **if** $|\Delta J| < \delta$ **then**
- 5: Update Lagrange multipliers: $\delta^{(i+1)}, w^{(i+1)}, \boldsymbol{\lambda}^{(i+1)}$
- 6: **end if**
- 7: **end if**
- 8: $\Delta_{\text{tr}} \leftarrow$ Update trust region
- 9: $k \leftarrow k + 1$
- 10: **end while**
- 11: **return** $\mathbf{Z}^*$

5.2.4. Numerical Results

In this section, the numerical results obtained by solving the formulated free-phase, bounded-time impulsive transfer via Adaptive Mesh Sequential Convex Programming (AMSCP) are presented. The two representative transfers considered correspond to the free-phase cases that did not fully converge when the previously presented indirect forward–backward shooting method was employed. This choice is motivated by the need to highlight both the qualitative structure of the resulting trajectories and the numerical performance of the SCP algorithm. Particular emphasis is placed on the role of adaptive time discretization and bounded free final time formulation, which enable the optimizer to determine the optimal departure and arrival phases as well as the overall time of flight.

Case Study 1: Multi-Revolution Transfer

The first case study considers an impulsive multi-revolution transfer from the departing PTO to the arrival 6:1 RTO. The number of discretization nodes was set to $N = 16$ after testing different values and selecting the configuration that minimized the overall fuel consumption. Figure 5.9 shows the obtained free-phase transfer, which results in a total ΔV of about 2.8 cm/s and a total time-of-flight of approximately 15.5 days.

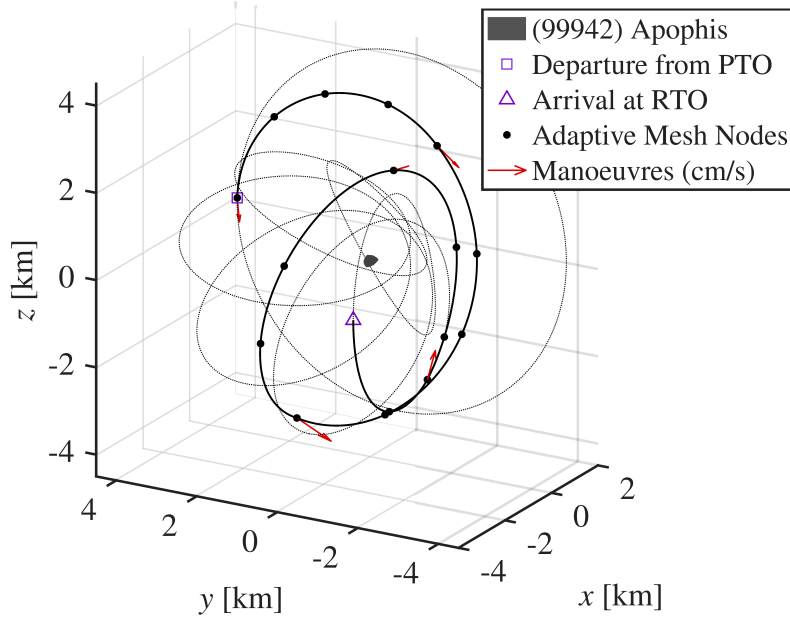
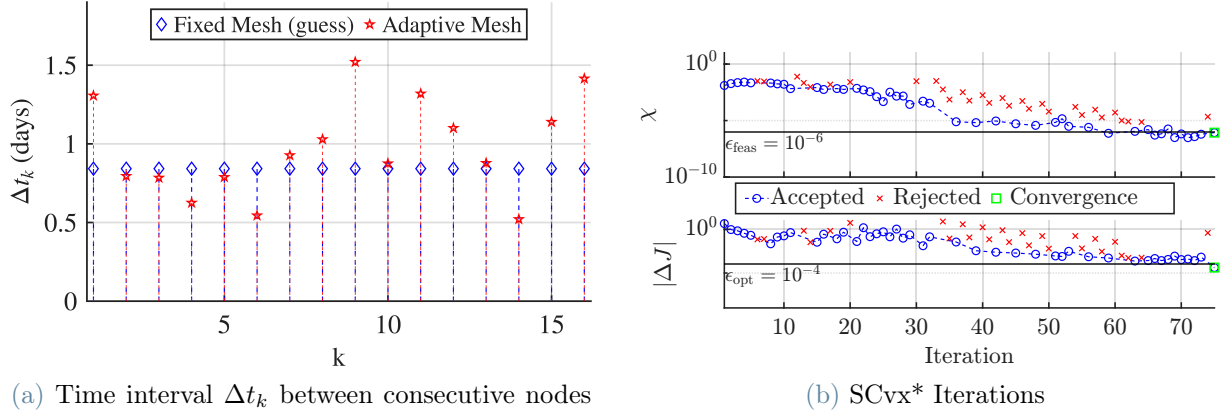


Figure 5.9: Free-Phase, Impulsive Multi-Revolution PTO–6:1 RTO Transfer

Figure 5.10a shows the evolution of the time discretization from the initial uniform mesh to the converged adaptive mesh. While the initial fixed mesh features constant time steps, the final solution provided by AMSCP exhibits non-uniform Δt_k values. The optimizer automatically redistributes the nodes, concentrating them in regions characterized by stronger nonlinear effects and allowing larger intervals where the trajectory evolves smoothly under the natural dynamics. This, in turn, also helps the optimizer in placing the impulsive maneuvers at optimal epochs.

From a numerical standpoint, the solution converges in 75 SCvx* iterations, with a total computational time of approximately 32 seconds. The convergence profile of the algorithm is reported in Figure 5.10b.

Table 5.3 reports the numerical results of the first transfer obtained by AMSCP. On the left, a comparison between the initial fixed mesh (F.M., guess) and the adaptive mesh (A.M., converged) time of flight, showcasing the ability of the algorithm to increase the duration of the transfer to optimality. On the right, a summary of the impulsive maneuvers of the trajectory is reported, with the total cost of the transfer.



(a) Time interval Δt_k between consecutive nodes for the initial fixed mesh and the converged adaptive mesh.

(b) SCvx* Iterations

Figure 5.10: Mesh redistribution and convergence behavior of AMSCP for Multi-Revolution transfer.

Table 5.3: Multi-Revolution transfer AMSCP Results

(a) Time of Flight (days)		(b) Maneuvers (cm/s)					
F.M. (guess)	A.M. (converged)	ΔV_1	ΔV_2	ΔV_3	ΔV_4	ΔV_5	ΔV_{Total}
13.47	15.57	0.43	0.49	0.79	0.39	0.66	2.76

Case Study 2: Flyby Transfer

The second case study considers an impulsive flyby transfer from the departing PTO to the arrival 6:1 RTO. The number of discretization nodes was set to $N = 20$. Figure 5.11 shows the obtained free-phase transfer, resulting in a total ΔV of about 2.97 cm/s and a total time of flight of approximately 14 days.

Figure 5.12a shows the evolution of the comparison of the time intervals between discretization nodes of the initial uniform mesh to the converged adaptive mesh. In contrast to the previous case, the bounded free final time formulation leads to a reduction of the total time of flight with respect to the initial guess, as also shown in Section 5.2.4. This indicates that the optimizer is able to identify a dynamically favorable configuration in which the flyby geometry allows the transfer to be completed in a shorter time.

The solution converges in 84 SCvx* iterations, with a total computational time of approximately 41 seconds. The convergence profile of the algorithm is reported in Figure 5.12b.

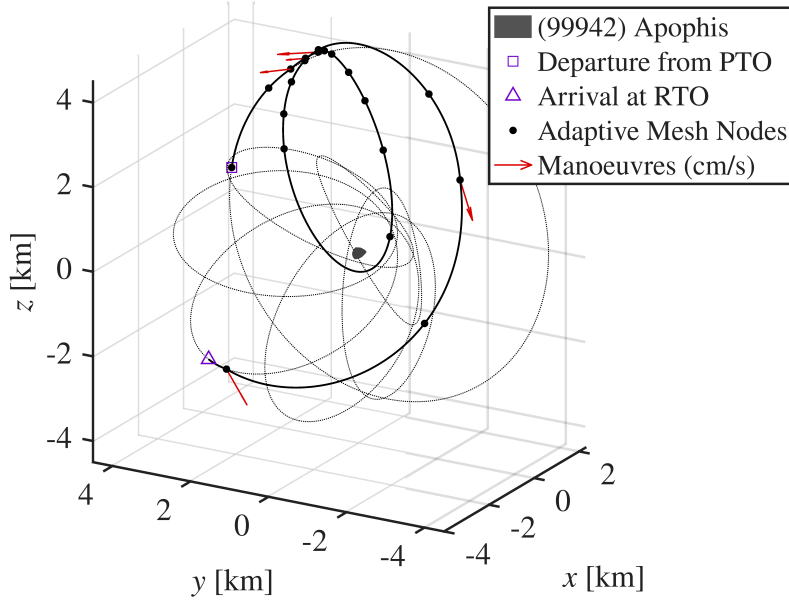
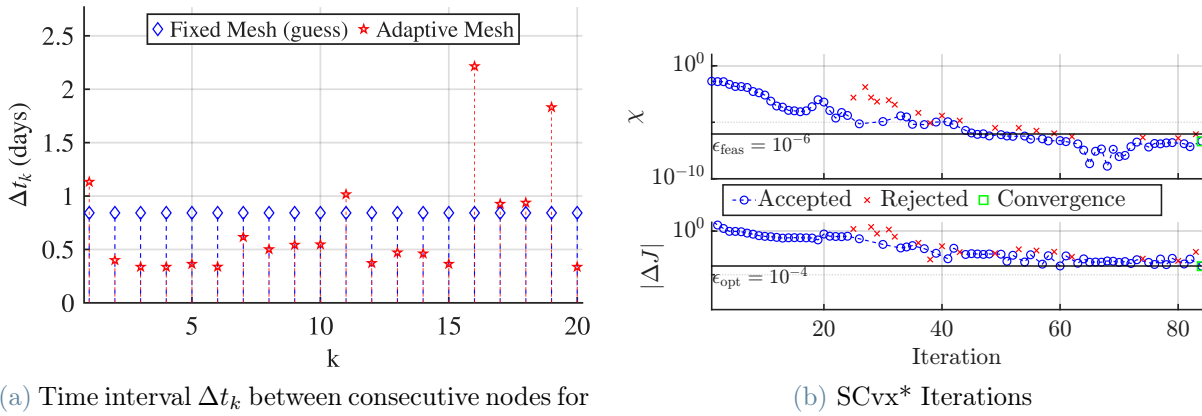


Figure 5.11: Free-Phase, Impulsive Flyby PTO–6:1 RTO Transfer



(a) Time interval Δt_k between consecutive nodes for the initial fixed mesh and the converged adaptive mesh.

(b) SCvx* Iterations

Figure 5.12: Mesh redistribution and convergence behavior of AMSCP for Flyby transfer.

Section 5.2.4 reports the numerical results for the flyby transfer obtained by AMSCP.

Table 5.4: Flyby transfer AMSCP Results

(a) Time of Flight (days)		(b) Maneuvers (cm/s)						
F.M. (guess)	A.M. (converged)	ΔV_1	ΔV_2	ΔV_3	ΔV_4	ΔV_5	ΔV_6	ΔV_{Total}
16.84	14.05	0.51	0.32	0.67	0.05	0.70	0.73	2.97

Compared to the indirect forward–backward shooting approach discussed in the previous

section, the proposed AMSCP method demonstrates significantly improved robustness and convergence reliability, successfully handling the free-phase and impulsive nature of the problem. This improvement is particularly evident in the second case study, where, despite the increased nonlinearity associated with the flyby dynamics, the convergence behavior remains stable.

The adaptive mesh formulation also enables the convex solver to determine the optimal time of flight without constraining it *a priori*, showing the ability to both increase or decrease the transfer duration.

Overall, these results demonstrate that the proposed AMSCP formulation enables a significantly more efficient and robust computation of fully impulsive, free-phase, bounded-time transfers compared to indirect shooting approaches. The converged deterministic trajectories obtained in this section are subsequently used as reference solutions for the chance-constrained stochastic optimization presented in the next chapter.

6 | Stochastic Trajectory Optimization Problem Formulation

This chapter presents the methodological framework adopted for the formulation and solution of a stochastic optimal control problem under uncertainty, with particular emphasis on chance-constrained control and covariance steering techniques. The objective of this chapter is to optimize guidance and control strategies for the design of robust science orbit transfers around Apophis, explicitly accounting for uncertainty sources affecting spacecraft dynamics, navigation, and maneuver execution, while guaranteeing safety and performance with a prescribed confidence level.

6.1. Fundamentals of Chance-Constrained Control

Among the existing approaches for solving statistical trajectory optimization problems under uncertainty, this thesis adopts *optimal covariance steering* techniques. Covariance control is a class of stochastic optimal control problems that focuses on the design of control policies for systems characterized by Gaussian state distributions [33, 36]. By explicitly controlling the evolution of the state mean and covariance, this approach aims at minimizing a statistical cost while satisfying a set of probabilistic constraints, commonly referred to as *chance constraints*.

Chance-constrained control allows to handle *unbounded* uncertainty distributions by enforcing probabilistic constraints on the stochastic state, ensuring that constraint violations occur with a probability lower than a user-defined value [31]. Chance constraints generalize the deterministic constraints $f_{\text{safe}} \leq 0$ to be probabilistic, and are defined as [33, 39, 42, 49]:

$$\mathbb{P}(f_{\text{safe}} \leq 0) \geq 1 - \epsilon, \quad (6.1)$$

where ϵ is a user-defined risk bound that represents the probability of chance-constraint

violation (e.g., $\epsilon = 0.01$ for 99% confidence).

The framework adopted in this thesis follows the safe autonomy paradigm proposed by Oguri in [49]. Chance-constrained control policies are computed on ground via convex programming and subsequently uploaded to the spacecraft. On board, the spacecraft runs a navigation filter to estimate its state and executes the control actions whenever a new state estimate becomes available. The resulting maneuvers are safe by design with respect to the uncertainties explicitly modeled in the planner. The overall safe autonomy architecture is illustrated in Figure 6.1.

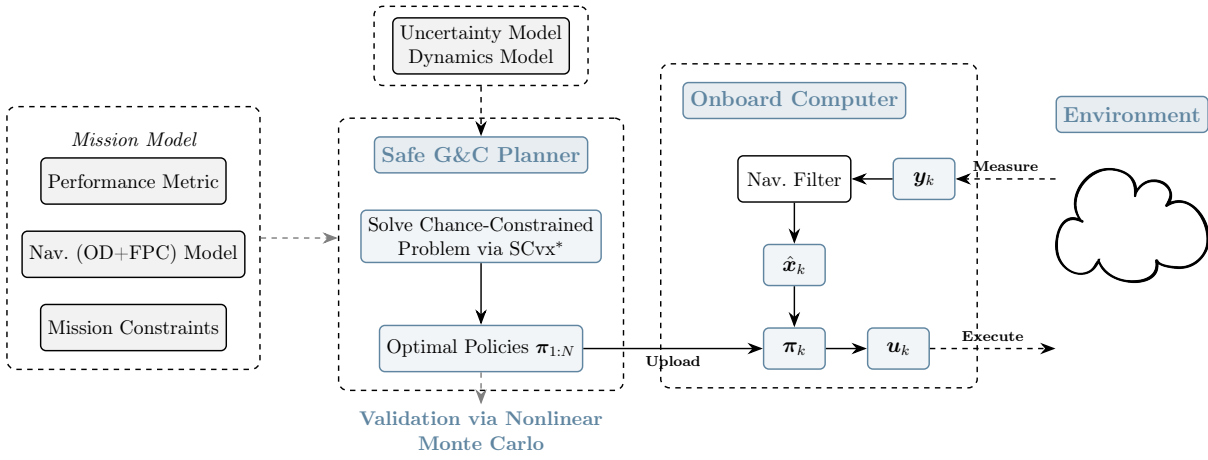


Figure 6.1: Safe autonomy framework based on chance-constrained control

Credits: [49]

The methodology used to solve the statistical trajectory optimization problem under uncertainty follows the fundamental idea proposed by Oguri and Lantoine in [48]. The overall formulation is illustrated in Figure 6.2, where operational uncertainty models (e.g., measurement functions and noise characteristics, orbit determination update frequency) and statistical chance constraints are directly incorporated into the optimization problem while minimizing a selected statistical cost.

The algorithm computes statistically optimal trajectories under the modeled uncertainties by exploiting the mean and covariance information provided by the Kalman filtering process. Both the nominal trajectory and the flight-path control (FPC) policies are optimized through a sequential convex programming (SCP) scheme. The resulting solution is finally validated via nonlinear Monte Carlo simulations.

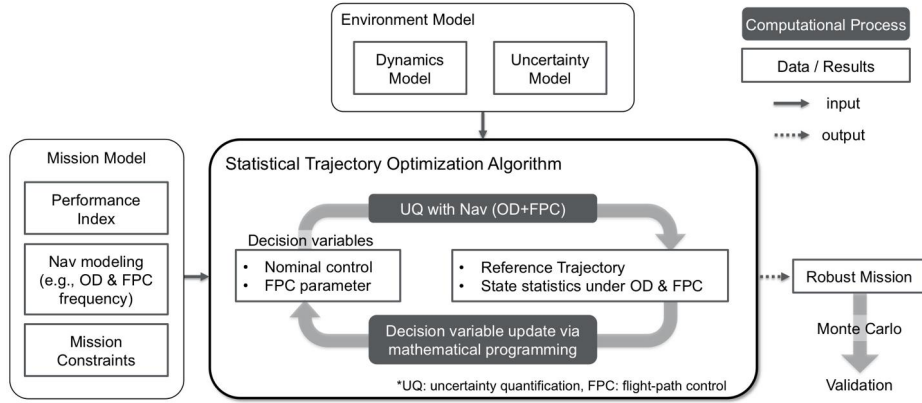


Figure 6.2: Fundamental idea of statistical trajectory optimization under uncertainty. Credits: [48]

6.1.1. Original Chance-Constrained Nonlinear Stochastic Optimal Control Problem

This section formulates the problem of safe trajectory optimization under uncertainty as a chance-constrained optimal control problem. The original formulation, introduced by Oguri and Lantoine [48] in the context of low-thrust control, is here adapted to an impulsive control framework. The formulation explicitly accounts for the major sources of uncertainty encountered in small-body proximity operations, including initial state dispersion, maneuver execution errors, orbit determination (OD) errors, and unmodeled accelerations, as well as for the corrective effects introduced by flight-path control (FPC).

Nonlinear Stochastic System

1. Initial State Dispersion

The distribution of the system state $\mathbf{x}$ at the initial time t_0 is assumed to be known and characterized by a prior state estimate $\hat{\mathbf{x}}_0^-$ and an associated estimation error $\tilde{\mathbf{x}}_0^-$. These quantities are modeled as independent Gaussian random variables:

$$\hat{\mathbf{x}}_0^- \sim \mathcal{N}(\bar{\mathbf{x}}_0, \hat{P}_0^-), \quad \tilde{\mathbf{x}}_0^- \sim \mathcal{N}(\mathbf{0}, \tilde{P}_0^-), \quad (6.2)$$

where $\bar{\mathbf{x}}_0$ is the known mean initial state and $\hat{P}_0^-, \tilde{P}_0^- \succeq 0$ are known covariance matrices.

It follows that

$$\mathbf{x}_0 \sim \mathcal{N}(\bar{\mathbf{x}}_0, \hat{P}_0^- + \tilde{P}_0^-). \quad (6.3)$$

2. Maneuver Execution Error

Any impulsive maneuver is inherently affected by uncertainties given by thruster misalignment, under or over burn, and other propulsion-related errors. To model such errors, this work uses the Gates model [119], which captures both fixed errors and errors proportional to the applied control magnitude.

The model is parameterized by four coefficients, $\{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$, which represent fixed magnitude [km/s], proportional magnitude [%], fixed pointing [km/s], and proportional pointing [deg] errors, respectively, for a spacecraft equipped with an impulsive propulsion system.

For a planned control input $\mathbf{u}_k$, the maneuver execution error $\tilde{\mathbf{u}}_k$ is modeled as:

$$\tilde{\mathbf{u}}_k = G_{exe}(\mathbf{u}_k)\mathbf{w}_{exe}, \quad G_{exe}(\mathbf{u}_k) = T(\mathbf{u}_k)P_{\text{gates}}^{1/2}(\mathbf{u}_k), \quad \mathbf{w}_{exe} \sim \mathcal{N}(\mathbf{0}, I_3), \quad (6.4)$$

where $\mathbf{w}_{exe}$ is an independent and identically distributed (i.i.d.) standard Gaussian random vectors, such that $\mathbb{E}[\mathbf{w}_{exe}] = 0$ and $\mathbb{E}[\mathbf{w}_{exe}(t_i)\mathbf{w}_{exe}(t_j)^\top] = \delta_{i,j}I$, where $\delta_{i,j}$ is the Kronecker delta. The matrix $P_{\text{gates}}(\cdot)$ represents the execution error covariance expressed in a local coordinate frame aligned with the thrust direction, while $T(\cdot)$ is a rotation matrix that maps this local frame to the reference frame in which the planned control input is represented, here the ANH3BP rotating frame. These matrices are defined as:

$$P_{\text{gates}}(\mathbf{u}_k) = \text{diag}(\sigma_p^2, \sigma_p^2, \sigma_m^2), \quad T(\mathbf{u}_k) = [\hat{\mathbf{s}}, \hat{\mathbf{e}}, \hat{\mathbf{z}}], \quad (6.5)$$

where the orthonormal frame associated with the Gates model is constructed by selecting the planned thrust direction $\hat{\mathbf{z}}$ as the third axis, while the $\hat{\mathbf{s}}$ and $\hat{\mathbf{e}}$ axes span the plane orthogonal to the thrust direction.

$$\hat{\mathbf{z}} = \frac{\mathbf{u}_k}{\|\mathbf{u}_k\|_2}, \quad \hat{\mathbf{e}} = \frac{[0, 0, 1]^\top \wedge \hat{\mathbf{z}}}{\|[0, 0, 1]^\top \wedge \hat{\mathbf{z}}\|_2}, \quad \hat{\mathbf{s}} = \hat{\mathbf{e}} \wedge \hat{\mathbf{z}} \quad (6.6)$$

To account for the control-dependent nature of the Gates model, the iterative structure of the sequential convex programming (SCP) algorithm is exploited [42]. At each iteration, the reference feedforward control $\bar{\mathbf{u}}_k^*$ is used to evaluate the execution error covariance $G_{exe}(\mathbf{u}_k^*)$. This approach neglects the contribution of the feedback component of the control policy defined in to the execution error. However, since the feedback term is typically small compared to the feedforward component, this simplifying assumption is numerically justified.

3. Unmodeled Accelerations

In this work, unknown and unmodeled accelerations are accounted for by modeling them as a Brownian motion which perturbs the spacecraft's motion as an additional term in the dynamics. The Brownian is modeled as:

$$G(\mathbf{x})d\mathbf{w}(t) \quad (6.7)$$

where the matrix $G(\cdot) : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_x \times n_w}$ represents the intensity of the disturbances and maps $d\mathbf{w}(t)$ to the variation of the state $d\mathbf{x}$. $d\mathbf{w}(t) : \mathbb{R} \rightarrow \mathbb{R}^{n_w}$ is a standard Brownian motion vector, such that $\mathbb{E}[d\mathbf{w}] = 0$ and $\mathbb{E}[d\mathbf{w}(t)d\mathbf{w}^\top(t)] = Idt$.

4. Orbit Determination Model

In this framework, the orbit determination (OD) process is modeled as a sequence of filtering processes with discrete-time observations. When a set of data becomes available at time t_k , the observation process is given by:

$$\mathbf{y}_k = \mathbf{f}_{obs}(\mathbf{x}_k, t_k) + G_{obs}(\mathbf{x}_k)\mathbf{w}_{obs,k}, \quad k = 0, 1, \dots, N \quad (6.8)$$

where $\mathbf{y}_k \in \mathbb{R}^{n_y}$ are the measurements; $\mathbf{f}_{obs}(\cdot, \cdot) : \mathbb{R}^{n_x} \times \mathbb{R} \rightarrow \mathbb{R}^{n_y}$, $G_{obs}(\cdot) : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_x \times n_y}$ model the observation process and $\mathbf{w}_{obs,k} \in \mathbb{R}^{n_y}$ is the measurement noise modeled as a sequence of i.i.d. standard Gaussian random vectors.

The solution $\hat{\mathbf{x}}_k$ of the OD process at time t_k is obtained by a filtering process with the past observations $\mathbf{y}_i$ ($i = 0, 1, \dots, k$):

$$\hat{\mathbf{x}}_k = \mathcal{F}_k(\hat{\mathbf{x}}_0^-, \mathbf{y}_i : i = 0, 1, \dots, k) \quad (6.9)$$

where the "-" superscript indicates a quantity right before the measurement update and $\hat{\mathbf{x}}_0^-$ denotes the initial state estimate.

The filtering process utilizes the innovation process, defined as:

$$\tilde{\mathbf{y}}_k^- = \mathbf{y}_k - \mathbf{f}_{obs}(\hat{\mathbf{x}}_k^-) \quad (6.10)$$

5. Flight-Path Control Model

The control policy adopted in this work is defined as a function of the state estimate provided by the onboard navigation process. It consists of a nominal feedforward control component and a sequence of feedback actions designed to regulate state deviations with

respect to the nominal trajectory, based on orbit determination (OD) solutions.

This thesis employs the flight-path control (FPC) model introduced in [48], which optimizes a sequence of feedback control policies that compute corrective maneuvers using the current state estimate. The control policies are expressed as:

$$\mathbf{u}_k = \pi_k(\hat{\mathbf{x}}_k, \Omega_k) \quad (6.11)$$

where π_k denotes the control policy at time step k , and Ω_k represents the set of parameters defining the corresponding feedback law used to compute the control input $\mathbf{u}_k$.

6. Stochastic Differential Equations

The spacecraft dynamics are then expressed as a set of nonlinear stochastic differential equations (SDEs), in order to incorporate the uncertain errors and FPCs in the trajectory optimization process. Based on the classical form of equations of motion under control expressed in Eq. (2.1), the stochastic system is expressed in the following set of nonlinear SDEs:

$$d\mathbf{x} = [\mathbf{f}(\mathbf{x}, \mathbf{u}, t) + B\tilde{\mathbf{u}}] dt + G(\mathbf{x})d\mathbf{w}(t) \quad (6.12)$$

where the matrix $G(\cdot) : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_x \times n_w}$ represents the intensity of the disturbances and maps $d\mathbf{w}(t)$ to the variation of the state $d\mathbf{x}$; $d\mathbf{w}(t) : \mathbb{R} \rightarrow \mathbb{R}^{n_w}$ is a standard Brownian motion vector, such that $\mathbb{E}[d\mathbf{w}] = 0$ and $\mathbb{E}[d\mathbf{w}(t)d\mathbf{w}^\top(t)] = Idt$. The term $B\tilde{\mathbf{u}}$ models the effects of maneuver execution errors to the variation of the state.

Original Problem

1. Statistical Cost Metric

In deterministic trajectory optimization, a commonly adopted performance index is the total control effort, typically expressed as the integral of the control norm over the maneuver duration. In a stochastic setting, however, the control input becomes a random variable due to the presence of feedback policies and state uncertainty. As a result, the classical deterministic definition of control effort is no longer well-defined.

To address this issue, the performance metric is formulated in a statistical sense by considering suitable risk measures of random variables induced by the closed-loop control policy. In particular, this work adopts a general statistical cost formulation based on the

minimization of a quantile function of a random variable X evaluated at probability p :

$$J = \int_{t_0}^{t_f} \mathcal{Q}_X(p) \quad (6.13)$$

Where, given a random variable X , its p -quantile $\mathcal{Q}_X(p)$ is defined as

$$Q_X(p) \triangleq \min \{x \in \mathbb{R} \mid \mathbb{P}[X \leq x] \geq p\}. \quad (6.14)$$

This definition provides a probabilistic characterization of worst-case performance at a predefined confidence level p .

Specific choices of the underlying performance index and confidence level are introduced in the following sections.

2. Chance Constraints

This thesis considers both path chance constraints and distributional terminal constraints.

Path chance constraints imposed on discrete-time state and control variables are mathematically expressed as [48]:

$$\mathbb{P}[c_j(\mathbf{x}, \mathbf{u}, t) \leq 0] \geq 1 - \epsilon_j \quad (6.15)$$

where $c_j(\cdot) : \mathbb{R}^{n_x} \times \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}$ represents the j -th nonlinear constraint function.

In particular, in this work ℓ^2 -norm control chance constraints are imposed at discrete epochs:

$$\mathbb{P}[\|\mathbf{u}_k\| \leq u_{max}] \geq 1 - \epsilon_u, \quad k = 0, \dots, N-1 \quad (6.16)$$

where u_{max} is a scalar upper bound on the control input ℓ^2 -norm.

Additionally, this work considers *continuous-time path chance constraints*, which require probabilistic feasibility to be guaranteed over an entire time interval, rather than at a finite set of discretized epochs. Formally, following [120], a continuous-time chance constraint is defined as:

$$\mathbb{P}[\mathbf{x}(\tau) \in \mathcal{S}(\tau), \forall \tau \in (t_0, t_f)] \geq 1 - \epsilon_{ct} \quad (6.17)$$

where $\mathcal{S}(\tau) \subset \mathbb{R}^{n_x}$ denotes a time-varying feasible set and $\epsilon_{ct} \in (0, 1)$ is a user-defined risk bound over the entire continuous-time interval. Eq. (6.17) is stronger than its discrete-time counterpart Eq. (6.15), as it enforces probabilistic safety over an infinite set of time instants, rather than discrete time instants. The use of continuous-time chance constraints

is specific to the formulation of passive safety guarantees in case of loss of control authority during the transfer, and a more in-depth explanation will be presented in the relevant section.

Similar to the chance constraints, this work also models the terminal constraints as distributional constraints in order to ensure that the final state is within a prescribed region. In particular, at the final time t_f the mean must match a desired target state $\mathbf{x}_f$ and the covariance must be within a prescribed target covariance matrix $P_f \succeq 0$:

$$\mathbb{E}[\mathbf{x}(t_f)] = \mathbf{x}_f, \quad \text{Cov}[\mathbf{x}(t_f)] \preceq P_f \quad (6.18)$$

where $\mathbb{E}[\cdot]$ and $\text{Cov}[\cdot]$ denote the expectation and covariance operators, respectively.

Let Θ denote the vector of relevant parameters to optimize, which can take different forms depending on the specific problem. The original chance-constrained nonlinear stochastic optimal control problem can be expressed as Problem 1 [48].

Problem 1 (Original Problem). *Find $\mathbf{x}^*(t), \pi_k, \forall k$ and Θ that minimize the cost metric Eq. (6.13) subject to the nonlinear stochastic dynamics Eq. (6.12), path chance constraints Eq. (6.15, 6.17) and terminal constraints Eq. (6.18) under a sequence of FPCs determined by the policy Eq. (6.11) with state estimates Eq. (6.9) based on the observation process Eq. (6.8).*

6.2. Impulsive Linear Covariance Controller

An exact solution of Problem 1 is, in general, computationally intractable. A primary source of complexity arises from the propagation of the state probability distribution through nonlinear dynamics, which, in principle, requires solving the associated Fokker–Planck equation [42]. This formulation provides an exact description of uncertainty evolution but becomes computationally prohibitive for high-dimensional systems.

Consequently, a set of simplifying assumptions is introduced on the uncertainty propagation model, control policy structure, and navigation filtering process. These assumptions are justified under the condition that the state uncertainty remains sufficiently small over the considered time horizon.

To efficiently solve this problem, this thesis employs a solution method based on sequential covariance steering. This method combines optimal covariance steering and sequential convex programming (SCP) to reformulate and solve Problem 1, and is based on previ-

ous work [42, 48]. This approach approximately reformulates the original intractable stochastic problem into a convex and deterministic form, iteratively solved with guaranteed convergence to the global optimum of the approximated subproblem due to its convexity.

6.2.1. Convex Block-Cholesky Covariance Steering Problem

This thesis uses a block-Cholesky formulation introduced by Oguri in [49] to design a linear covariance controller, with the additional assumption of only impulsive control actions.

In this section, the assumptions on dynamics linearization, filtered covariance propagation model and control policy are outlined. These models allow to tractably design a closed-loop linear covariance controller that explicitly takes into account the evolution of the state distribution under uncertainty.

Dynamics Linearization and Discretization

To obtain the convex subproblem, first the set of nonlinear SDEs (Eq. 6.12) is linearized and discretized around a reference trajectory and control $\{\mathbf{x}^*(t), \mathbf{u}^*(t)\}$ using a first-order truncation of the Taylor expansion:

$$d\mathbf{x} \approx [A(t)\mathbf{x} + B(t)\mathbf{u} + \mathbf{c}(t)] dt + G(t)d\mathbf{w} \quad (6.19)$$

where:

$$A(t) = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{\mathbf{x}^*, \mathbf{u}^*}, \quad B(t) = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right|_{\mathbf{x}^*, \mathbf{u}^*}, \quad \mathbf{c}(t) = \mathbf{f}(\mathbf{x}^*, \mathbf{u}^*) - A\mathbf{x}^* - B\mathbf{u}^* \quad (6.20)$$

By discretizing the time into intervals and considering impulsive control, yields [49, 51]:

$$\mathbf{x}_k^+ = \mathbf{x}_k + B\mathbf{u}_k, \quad \mathbf{x}_{k+1} = A_k\mathbf{x}_k^+ + \mathbf{c}_k + G_k\mathbf{w}_k \quad (6.21)$$

where $\mathbf{x}_k$ and $\mathbf{x}_k^+$ are respectively the pre- and post- maneuver states at a given epoch t_k , $\mathbf{u}_k$ is an impulsive control action and B is the control mapping matrix.

The discrete-time matrices are evaluated by simultaneously integrating the following set of ODEs from time t_k to t_{k+1} [42, 121]:

$$\begin{aligned}
\dot{\mathbf{x}}(t) &= \mathbf{f}_0(\mathbf{x}(t)), & \mathbf{x}(t_k) &= \mathbf{x}_k^* + B\mathbf{u}_k^* \\
\dot{\Phi}_A(t, t_k) &= A(t)\Phi_A(t, t_k), & \Phi_A(t_k, t_k) &= I_{n_x} \\
\dot{\Phi}_P(t, t_k) &= A(t)\Phi_P(t, t_k) + \Phi_P(t, t_k)A(t)^\top + G(t)G(t)^\top, & \Phi_P(t_k, t_k) &= \mathbf{0}_{n_x \times n_x}
\end{aligned}$$

where $\mathbf{f}_0(\cdot)$ denotes the ballistic, uncontrolled, dynamics; Φ_A is the state transition matrix and Φ_P is the covariance matrix of the Brownian motion, where its differential equation is derived in [42]. Let $\bar{\boldsymbol{\varphi}}(t_{k+1})$ denote the result of integrating the set of ODEs from t_k to t_{k+1} , then:

$$A_k = \Phi_A(t_{k+1}, t_k), \quad B_k = A_k B, \quad \mathbf{c}_k = \bar{\boldsymbol{\varphi}}(t_{k+1}) - A_k \mathbf{x}_k^* - B_k \mathbf{u}_k^* \quad (6.22)$$

and G_k is any matrix that satisfies the relationship $G_k G_k^\top = \Phi_P(t_{k+1}, t_k)$, obtained by Cholesky decomposition.

Filtered State Dynamics

Since the derived systems follows linear dynamics in a convex subproblem and is initially Gaussian distributed, this work assumes that the filtering process is given by a Kalman filter, as previously done in [42, 48, 49]. This choice stems from the fact that the Kalman filter is a minimum-variance estimator for linear systems with Gaussian noise (Eq. 6.21) [121]. The nonlinear observation process is also linearized about a reference state [49]:

$$\mathbf{y}_k = C_k \mathbf{x}_k + D_k \mathbf{w}_{obs,k} + \mathbf{c}_{obs,k} \quad (6.23)$$

where:

$$C_k = \left. \frac{\partial \mathbf{f}_{obs}}{\partial \mathbf{x}} \right|_{\mathbf{x}_k^*}, \quad D_k = G_{obs}(\mathbf{x}_k^*), \quad \mathbf{c}_{obs,k} = \mathbf{f}_{obs}(\mathbf{x}_k^*) - C_k \mathbf{x}_k^* \quad (6.24)$$

A brief notation recap useful for the sake of clarity in the derivation follows. $\mathbf{x}_k$ is the true state with mean denoted as $\bar{\mathbf{x}}_k$, which is also referred to as the nominal state. $\hat{\mathbf{x}}_k$ is the state estimate, while $\hat{\mathbf{x}}_k^-$ is the prior state estimate; $\tilde{\mathbf{x}}_k \triangleq \mathbf{x}_k - \hat{\mathbf{x}}_k$ is the estimation error, while $\tilde{\mathbf{x}}_k^- \triangleq \mathbf{x}_k - \hat{\mathbf{x}}_k^-$ is the prior estimation error. The respective associated covariance matrices are denoted by $P_k, \hat{P}_k, \hat{P}_k^-, \tilde{P}_k, \tilde{P}_k^-$ [42]. Figure 6.3 offers a visualization of the uncertainty-related terms.

Between each step, the Kalman filter, considering impulsive control actions, updates the state estimates as follows [49, 51]:

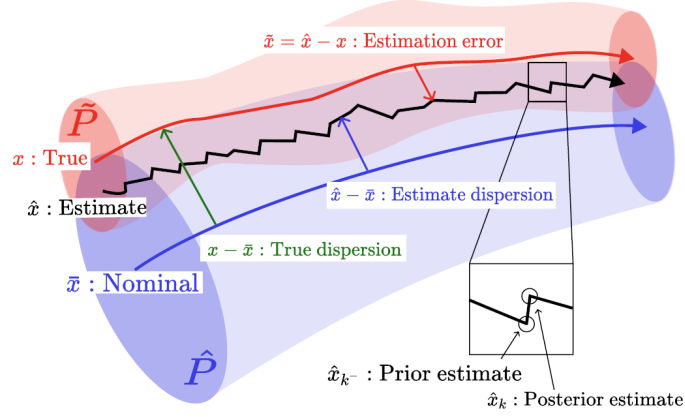


Figure 6.3: Schematic of uncertainty-related terms

Credits: [42]

$$\begin{aligned}
\hat{\mathbf{x}}_k^+ &= \hat{\mathbf{x}}_k + B\mathbf{u}_k &> \text{Impulsive Control Input} \\
\hat{\mathbf{x}}_{k+1}^- &= A_k \hat{\mathbf{x}}_k^+ + \mathbf{c}_k &> \text{Time Update} \\
\tilde{P}_{k+1}^- &= A_k \tilde{P}_k A_k^\top + G_{exe,k} G_{exe,k}^\top + G_k G_k^\top \\
\hat{\mathbf{x}}_{k+1} &= \hat{\mathbf{x}}_{k+1}^- + L_{k+1} \tilde{\mathbf{y}}_{k+1}^- &> \text{Measurement Update} \\
\tilde{P}_{k+1} &= (I - L_{k+1} C_{k+1}) \tilde{P}_{k+1}^- (I - L_{k+1} C_{k+1})^\top \\
&\quad + L_{k+1} D_{k+1} D_{k+1}^\top L_{k+1}^\top
\end{aligned} \tag{6.25}$$

where L_k denotes the Kalman gain:

$$L_k \triangleq \tilde{P}_k^- C_k^\top \left(C_k \tilde{P}_k^- C_k^\top + D_k D_k^\top \right)^{-1} \tag{6.26}$$

and $\tilde{\mathbf{y}}_k^-$ is the innovation process, whose linearized form and covariance $P_{\tilde{\mathbf{y}}_k^-}$ are [49]:

$$\tilde{\mathbf{y}}_k^- = C_k \tilde{\mathbf{x}}_k^- + D_k \mathbf{w}_{\text{obs},k}, \quad P_{\tilde{\mathbf{y}}_k^-} \triangleq \text{Cov}[\tilde{\mathbf{y}}_k^-] = C_k \tilde{P}_k^- C_k^\top + D_k D_k^\top. \tag{6.27}$$

A classic notable property known in estimation theory is that, since the system is linear, $\tilde{P}_k$ and L_k can be computed *a priori* before each SCP iteration [121].

Linear Flight-Path Control Policy

The class of control policy employed in this work is restricted to the class of affine controllers, as done in previous work [42, 48, 49]. The policy π is made of an impulsive control input at each discrete instant t_k , $\mathbf{u}_{0:N-1}$, which is a linear function of the state estimate

$\hat{\mathbf{x}}_k$ [42]:

$$\mathbf{u}_k = \bar{\mathbf{u}}_k + K_k(\hat{\mathbf{x}}_k - \bar{\mathbf{x}}_k) \quad (6.28)$$

where $\bar{\mathbf{u}}_k$ is the nominal, feedforward, control action, and $K_k \in \mathbb{R}^{n_u \times n_x}$ is the feedback gain matrix. The term $K_k(\hat{\mathbf{x}}_k - \bar{\mathbf{x}}_k)$ represents the trajectory correction maneuver (TCM) executed to correct the nominal control based on the deviation between the estimated state and the nominal state.

By defining $\mathbf{z}_k$ as a stochastic process given by:

$$\mathbf{z}_{k+1} = A_k \mathbf{z}_k + L_{k+1} \tilde{\mathbf{y}}_{k+1}^-, \quad \mathbf{z}_0 = \hat{\mathbf{x}}_0 - \bar{\mathbf{x}}_0 \quad (6.29)$$

it is possible to reformulate Eq. 6.28 in the following form, as originally done in [40]:

$$\mathbf{u}_k = \bar{\mathbf{u}}_k + K_k \mathbf{z}_k \quad (6.30)$$

Pre-Maneuver Statistics

In this section, the statistics of the state and control is derived, particularly in terms of their mean and covariance. By combining together the equations of Eq. 6.25, the filtered state process under stochastic error and Kalman filtering can be defined as [48]:

$$\hat{\mathbf{x}}_{k+1} = A_k \hat{\mathbf{x}}_k + B_k \mathbf{u}_k + \mathbf{c}_k + L_{k+1} \tilde{\mathbf{y}}_{k+1}^- \quad (6.31)$$

where $B_k = A_k B$ for impulsive control. This equation can then be expressed in a block-matrix form [49]:

$$\begin{bmatrix} \hat{\mathbf{x}}_0^- \\ \hat{\mathbf{x}}_1^- \\ \hat{\mathbf{x}}_2^- \\ \vdots \end{bmatrix} = \begin{bmatrix} I_{n_x} \\ A_0 \\ A_1 A_0 \\ \vdots \end{bmatrix} \hat{\mathbf{x}}_0^- + \begin{bmatrix} 0 & 0 & 0 \\ B_0 & 0 & 0 \\ A_1 B_0 & B_1 & 0 \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \mathbf{u}_2 \\ \vdots \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ I_{n_x} & 0 & 0 \\ A_1 & I_{n_x} & 0 \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \mathbf{c}_0 \\ \mathbf{c}_1 \\ \mathbf{c}_2 \\ \vdots \end{bmatrix} + \begin{bmatrix} L_0 & 0 & 0 \\ A_0 L_0 & L_1 & 0 \\ A_1 A_0 L_0 & A_1 L_1 & L_2 \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{y}}_0^- \\ \tilde{\mathbf{y}}_1^- \\ \tilde{\mathbf{y}}_2^- \\ \vdots \end{bmatrix} \quad (6.32)$$

This can be expressed in a more compact form:

$$\hat{\mathbf{X}} = \mathbf{A} \hat{\mathbf{x}}_0^- + \mathbf{B} \mathbf{U} + \mathbf{C} + \mathbf{L} \mathbf{Y}, \quad (6.33)$$

where $\hat{\mathbf{X}} = [\hat{\mathbf{x}}_0^\top, \hat{\mathbf{x}}_1^\top, \dots, \hat{\mathbf{x}}_N^\top]^\top$, $\mathbf{U} = [\mathbf{u}_0^\top, \mathbf{u}_1^\top, \dots, \mathbf{u}_{N-1}^\top]^\top$, $\hat{\mathbf{Y}} = [\tilde{\mathbf{y}}_0^\top, \tilde{\mathbf{y}}_1^\top, \dots, \tilde{\mathbf{y}}_{N-}^\top]^\top$, and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{L}$ are defined accordingly.

Also the control policy Eq. 6.30 can be expressed in block-matrix form:

$$\mathbf{U} = \bar{\mathbf{U}} + \mathbf{K}\mathbf{Z}, \quad \mathbf{Z} = \mathbf{A}(\hat{\mathbf{x}}_0^- - \bar{\mathbf{x}}_0) + \mathbf{L}\mathbf{Y} \quad (6.34)$$

and:

$$\mathbf{K} = \begin{bmatrix} K_0 & 0 & 0 & \dots & 0 \\ 0 & K_1 & \ddots & \dots & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & \dots & K_{N-1} & 0 \end{bmatrix} \quad (6.35)$$

Combining Eqs. 6.33 and 6.34 in a single expression yields:

$$\hat{\mathbf{X}} = \mathbf{A}\hat{\mathbf{x}}_0^- + \mathbf{B}\bar{\mathbf{U}} + \mathbf{B}\mathbf{K}\mathbf{Z} + \mathbf{C} + \mathbf{L}\mathbf{Y} \quad (6.36)$$

Recalling that $\mathbb{E}[\mathbf{Y}] = \mathbf{0}$, $\mathbb{E}[\mathbf{Z}] = \mathbf{0}$, and $\mathbb{E}[\mathbf{U}] = \bar{\mathbf{U}}$, it is possible to derive the mean and covariance of the estimated state and control [49]:

$$\begin{aligned} \bar{\mathbf{X}} &\triangleq \mathbb{E}[\hat{\mathbf{X}}] = \mathbf{A}\bar{\mathbf{x}}_0 + \mathbf{B}\bar{\mathbf{U}} + \mathbf{C}, \\ \hat{\mathbf{P}} &\triangleq \text{Cov}[\hat{\mathbf{X}}] = (\mathbf{I} + \mathbf{B}\mathbf{K})\mathbf{S}(\mathbf{I} + \mathbf{B}\mathbf{K})^\top \\ \mathbf{P}_U &\triangleq \text{Cov}[\mathbf{U}] = \mathbf{K}\mathbf{S}\mathbf{K}^\top \end{aligned} \quad (6.37)$$

where, since the initial states and measurements are independent, $\mathbf{S}$ is defined as,

$$\mathbf{S} \triangleq \text{Cov}[\mathbf{S}(\hat{\mathbf{x}}_0^- - \bar{\mathbf{x}}_0)] + \text{Cov}[\mathbf{L}\mathbf{Y}] = \mathbf{A}\mathbf{P}_0^- \mathbf{A}^\top + \mathbf{L}\mathbf{P}_y \mathbf{L}^\top \quad (6.38)$$

with $\mathbf{P}_y = \text{Cov}[\mathbf{Y}] = \text{blkdiag}(P_{\tilde{y}_0^-}, P_{\tilde{y}_1^-}, \dots)$ and $P_{\tilde{y}_k^-} = C_k \tilde{P}_k^- C_k^\top + D_k D_k^\top$.

The Cholesky decomposition of the covariance matrices is then applied:

$$\mathbf{P}_k^{1/2} = [\hat{P}_k^{1/2}, \tilde{P}_k^{1/2}], \hat{P}_k^{1/2} = E_{x_k}(\mathbf{I} + \mathbf{B}\mathbf{K})\mathbf{S}^{1/2}, P_{u_k}^{1/2} = E_{u_k}\mathbf{K}\mathbf{S}^{1/2}, \mathbf{S}^{1/2} = [\mathbf{A}\hat{P}_{0-}^{1/2}, \mathbf{L}\mathbf{P}_y^{1/2}] \quad (6.39)$$

where E_{x_k} and E_{u_k} are defined to extract $\mathbf{x}_k$ and $\mathbf{u}_k$ from $\mathbf{X}$ and $\mathbf{U}$, as: $\mathbf{x}_k = E_{x_k}\mathbf{X}$ and $\mathbf{u}_k = E_{u_k}\mathbf{U}$ [48].

As stated in Proposition 1 of [49], applying the Cholesky decomposition results in $\bar{\mathbf{x}}_k$, $P_k^{1/2}$,

$\hat{P}_k^{1/2}$, $P_{u_k}^{1/2}$ being all affine in the control variables $\bar{\mathbf{u}}_k$ and K_k . These inherent convex properties are particularly useful to formulate the convex subproblem solved at each SCP iteration.

Post-Maneuver Statistics

In the case of impulsive control actions, it is convenient to derive the statistics of the estimated state, in terms of mean and covariance, after a maneuver is executed [51].

The estimated post-maneuver state at t_k is $\hat{\mathbf{x}}_k^+$. Combining the equations of Eq. 6.25 yields:

$$\hat{\mathbf{x}}_{k+1}^+ = A_k \hat{\mathbf{x}}_k^+ + B \mathbf{u}_{k+1} + \mathbf{c}_k + L_{k+1} \tilde{\mathbf{y}}_{k+1}^- \quad (6.40)$$

Similarly to the process done previously, it is convenient to express this in block-matrix form throughout the entire time horizon:

$$\begin{bmatrix} \hat{\mathbf{x}}_0^+ \\ \hat{\mathbf{x}}_1^+ \\ \hat{\mathbf{x}}_2^+ \\ \vdots \end{bmatrix} = \begin{bmatrix} I_{n_x} \\ A_0 \\ A_1 A_0 \\ \vdots \end{bmatrix} \hat{\mathbf{x}}_0^- + \begin{bmatrix} B & 0 & 0 & \\ A_0 B & B & 0 & \dots \\ A_1 A_0 B & A_1 B & B & \\ & \vdots & & \ddots \end{bmatrix} \begin{bmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \mathbf{u}_2 \\ \vdots \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & \\ I_{n_x} & 0 & 0 & \dots \\ A_1 & I_{n_x} & 0 & \\ & \vdots & & \ddots \end{bmatrix} \begin{bmatrix} \mathbf{c}_0 \\ \mathbf{c}_1 \\ \mathbf{c}_2 \\ \vdots \end{bmatrix} + \begin{bmatrix} L_0 & 0 & 0 & \\ A_0 L_0 & L_1 & 0 & \dots \\ A_1 A_0 L_0 & A_1 L_1 & L_2 & \\ & \vdots & & \ddots \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{y}}_0^- \\ \tilde{\mathbf{y}}_1^- \\ \tilde{\mathbf{y}}_2^- \\ \vdots \end{bmatrix} \quad (6.41)$$

In compact form:

$$\hat{\mathbf{X}}^+ = \mathbf{A} \hat{\mathbf{x}}_0^- + \mathbf{B}^+ \mathbf{U} + \mathbf{C} + \mathbf{L} \mathbf{Y} \quad (6.42)$$

Then, the mean and covariance of the estimated state post-impulsive control are derived analogously as before:

$$\bar{\mathbf{X}}^+ \triangleq \mathbb{E}[\hat{\mathbf{X}}^+] = \mathbf{A} \bar{\mathbf{x}}_0 + \mathbf{B}^+ \bar{\mathbf{U}} + \mathbf{C}, \quad (\hat{\mathbf{P}}^+)^{1/2} = (\mathbf{I} + \mathbf{B}^+ \mathbf{K}) \mathbf{S}^{1/2}. \quad (6.43)$$

Building upon the block-Cholesky formulation of the pre- and post-maneuver state statistics derived in the previous sections, the following sections introduce two alternative optimization strategies within the same chance-constrained control framework.

First, the original cost function proposed by Oguri [49], which aims at minimizing the 99%-quantile of the total control effort, commonly referred to in mission design as ΔV_{99} , is presented. Subsequently, an alternative formulation is considered, in which the cost function is designed to mitigate nonlinear effects by minimizing the spacecraft's exposure to nonlinearities based on the Tensor Eigenpair Measure of Nonlinearities (TEMoN), following the recent developments introduced by Qi [51].

6.3. Problem Statement: Minimization of ΔV_{99}

This section provides the statement of the chance-constrained covariance steering subproblem solved at each iteration of SCvx*. This first problem focuses on the minimization of the 99% quantile of the statistical fuel cost, i.e., ΔV_{99} .

6.3.1. Cost Function and Constraints

Cost Function

The original stochastic cost function in lagrangian form in order to minimize the integral of the p -quantile of $L(\cdot)$ has been previously defined in Eq. 6.13. In this formulation, $L(\mathbf{u}) = \|\mathbf{u}\|_2$ with $p = 0.99$. The original cost function in discrete-time, assuming impulsive control, is [49]:

$$J = \sum_{k=0}^{N-1} \mathcal{Q}_{\|\mathbf{u}_k\|_2}(p) \quad (6.44)$$

By applying Lemma 4 of [49], the discrete-time cost function can be bounded from above. For better tractability of the problem, $J_{ub} \geq J$ is minimized instead:

$$J_{ub} = \sum_{k=0}^{N-1} \|\mathbf{u}_k\|_2 + m_{\chi^2}(1 - p, n_u) \|P_{u_k}^{1/2}\|_2 \quad (6.45)$$

The upper-bounded cost function is then augmented to penalize the relaxed equality constraints that will be introduced in the following paragraphs. These virtual buffers are introduced in each SCP problem to facilitate the constraint satisfaction.

Specifically, in this work the final mean state and the continuous-time passive safety equality constraints are relaxed by introducing the slack variables $\boldsymbol{\xi}$, as well as the vector of Lagrange multipliers $\boldsymbol{\lambda}$. Then the augmented cost function is defined by using the 1-norm penalty introduced in [42]:

$$J_{aug}(\bar{\mathbf{X}}, \bar{\mathbf{U}}, \mathbf{K}, \boldsymbol{\xi}) = J_{ub}(\bar{\mathbf{U}}, \mathbf{K}) + J_{pen}(\boldsymbol{\xi}), \quad J_{pen}(\boldsymbol{\xi}) = \boldsymbol{\lambda}^{(j)} \cdot \boldsymbol{\xi} + \frac{1}{2} w^{(j)} \boldsymbol{\xi} \cdot \boldsymbol{\xi} + \sqrt{w^{(j)}} \|\boldsymbol{\xi}\|_1 \quad (6.46)$$

Constraints

In this framework, the different constraints considered are: thrust magnitude constraint, maximum covariance constraint, terminal constraints and continuous-time passive safety chance constraints against missed-thrust events (MTEs).

Maximum Control Chance Constraints:

The original thrust magnitude chance constraint of Eq. 6.16 is converted in convex form by applying Lemma 3 of [49], obtaining:

$$\|\bar{\mathbf{u}}_k\|_2 + m_{\chi^2}(\epsilon_u, n_u)\|P_{u_k}^{1/2}\|_2 \leq u_{\max}, \quad k = 0, 1, \dots, N-1 \quad (6.47)$$

where ϵ_u is a risk tolerance parameter for the constraint violation and $u_{\max} > 0$ is the maximum allowed thrust acceleration.

Maximum Covariance Constraint:

To limit the growth of the state uncertainty, a maximum state covariance constraint is imposed in this formulation at any time step t_k .

$$\text{Cov}[\mathbf{x}(t_k)] \preceq P_{\max} \quad (6.48)$$

where $P_{\max} \succ 0$ is a user-specified positive-definite matrix. This constraint is reformulated in convex form following the formulations in [39, 49]:

$$\|(P_{\max} - \tilde{P}_k)^{-1/2} E_{x_k}(I + \mathbf{B}\mathbf{K})\mathbf{S}^{1/2}\|_2 - 1 \leq 0 \quad (6.49)$$

It is worth noting that this maximum covariance constraint is imposed only in the ΔV_{99} minimization problem, although it is not explicitly included in the original formulation by Oguri and Lantoiné [48]. In this case, the objective function does not directly regulate the growth of state uncertainty, making an explicit upper bound on the covariance necessary to preserve the validity of the linearized Gaussian framework. Conversely, in the formulation aimed at minimizing the nonlinear errors based on TEMoN, the cost function itself implicitly limits the growth of uncertainty, and no additional covariance constraint is imposed.

Terminal Constraints:

The terminal constraints Eq. 6.18 in terms of final mean and covariance are formulated in convex form as [39, 49]:

$$E_{x_N}(\mathbf{A}\bar{\mathbf{x}}_0 + \mathbf{B}\bar{\mathbf{U}} + \mathbf{C}) - \mathbf{x}_f = \boldsymbol{\xi}, \quad \|(P_f - \tilde{P}_N)^{-1/2} E_{x_N}(I + \mathbf{B}\mathbf{K})\mathbf{S}^{1/2}\|_2 - 1 \leq 0 \quad (6.50)$$

where the final mean constraint is relaxed to facilitate the constraint satisfaction by introducing the vector of slack variables $\boldsymbol{\xi}$, which is penalized in the augmented cost function.

Other Constraints:

Apart from the discussed constraints, the dynamical continuity inside each convex subproblem is enforced by imposing the filtered state dynamic constraints in Eq. 6.37. Moreover, trust region

constraints are imposed on the decision variables to avoid *artificial unboundedness*:

$$\|D_x(\mathbf{X} - \mathbf{X}^*)\|_\infty \leq \Delta_{TR}, \quad \|D_u(\bar{\mathbf{U}} - \bar{\mathbf{U}}^*)\|_\infty \leq \Delta_{TR} \quad (6.51)$$

where Δ_{TR} is the trust region radius and D_x, D_u are scaling diagonal matrices.

6.3.2. Continuous-Time Passive Safety Chance Constraint Against MTEs

This section is solely devoted to the formal definition and methodology for the implementation of the passive safety guarantees in case of loss of control authority during the autonomous science-orbit transfers. Indeed, in the second case study, an additional passive safety constraint is introduced to explicitly account for missed thrust events (MTEs).

Specifically, at each discretization node, a missed thrust event is assumed to occur immediately after the execution of the impulsive maneuver. Starting from the corresponding post-maneuver state distribution, the spacecraft is then propagated ballistically over a prescribed safety horizon t_{safety} . The resulting off-nominal trajectory distribution is required to remain outside a predefined safety keep-out zone (KOZ) surrounding Apophis. An overview of the proposed passive safety framework is provided in Figure 6.4a.

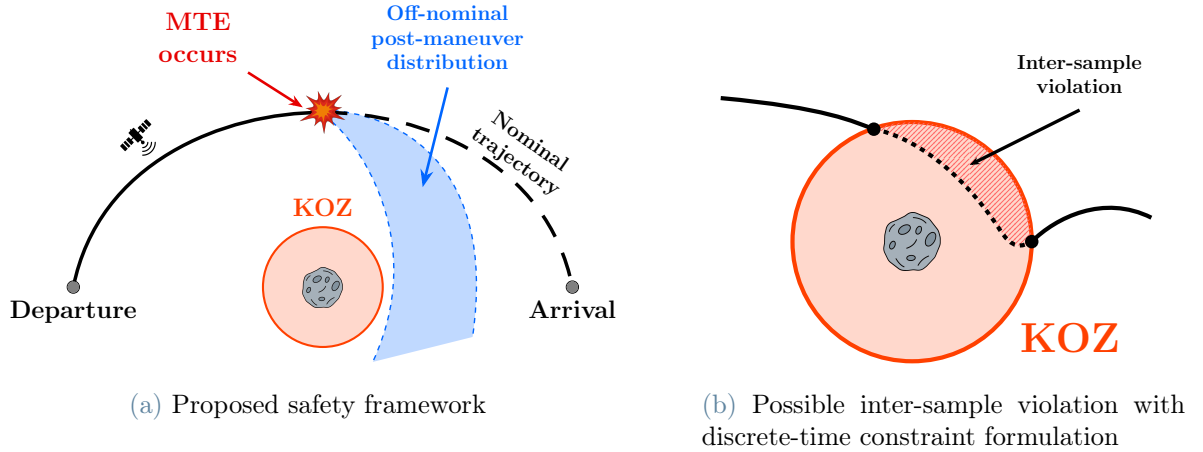


Figure 6.4: Conceptual illustration of continuous-time safety guarantees against MTEs

Several different approaches could be followed to guarantee safety against missed-thrust events. One possible approach would be to constrain the distribution after an MTE (in terms of mean and covariance) to remain outside the prescribed KOZ at discrete-time nodes. However, enforcing safety constraints solely at discrete epochs cannot guarantee the satisfaction between nodes. This limitation is particularly critical for collision avoidance and probability-of-impact (PoI) constraints, where constraint violations may occur between discretization points, as illustrated in Figure 6.4b and as extensively addressed in the literature [120, 122, 123]. For this reason, the

present work formulates the safety guarantee in case of loss of control authority as a continuous-time chance constraint, in order to guarantee the safety of the spacecraft over the entire safety time horizon in case a missed-thrust event occurs.

The practical implementation of the continuous-time constraint in the SCvx* framework follows the procedure outlined by Elango et al. [122], where the system dynamics are augmented with an auxiliary state that accumulates constraint violations over time. Safety is then guaranteed by enforcing a terminal equality constraint on this auxiliary state.

Let $\mathbf{r}(t) \in \mathbb{R}^3$ denote the spacecraft position at time t with Gaussian distribution, i.e. $\mathbf{r}(t) \sim \mathcal{N}(\bar{\mathbf{r}}(t), P_r(t))$, where $\bar{\mathbf{r}}(t)$ is the mean position and $P_r(t)$ is the positional covariance. Passive safety with respect to a spherical keep-out zone (KOZ) of radius r_{safe} centered at the asteroid is expressed by the continuous-time chance constraint in the form:

$$\mathbb{P}(\|\mathbf{r}(t)\|_2 \geq r_{\text{safe}}) \geq 1 - \epsilon_{\text{safe}}, \quad \forall t \in \mathcal{T}_s(t_k), \quad (6.52)$$

where $\epsilon_{\text{safe}} \in (0, 1)$ is a user-defined risk parameter (here, $\epsilon_{\text{safe}} = 0.01$ for 99% safety) and $\mathcal{T}_s(t_k) \triangleq [t_k, t_k + t_{\text{safety}}]$ denotes the safety horizon associated with node k , i.e., the continuous-time interval over which passive safety must be guaranteed following a missed thrust event.

Following the continuous-time convexification approach of [122], the nonconvex and infinite-dimensional chance constraint Eq. (6.52) is converted into an equivalent continuous-time penalty formulation. First, an *instantaneous penalty violation function* $\Lambda(t)$ is defined as

$$\Lambda(t) \triangleq \max\left(0, r_{\text{safe}} - [\|\bar{\mathbf{r}}(t)\|_2 - \sqrt{m_{\chi^2}(\epsilon_{\text{safe}}, 3)} \|P_r^{1/2}(t)\|_2]\right)^n, \quad n > 1, \quad (6.53)$$

where $m_{\chi^2}(\epsilon_{\text{safe}}, 3)$ is the chi-square quantile with three degrees of freedom evaluated at probability $1 - \epsilon_{\text{safe}}$. Then, an auxiliary scalar state $y(t)$, introduced to accumulate violations of the passive safety condition:

$$\dot{\mathbf{y}}(t) = \Lambda(t), \quad \mathbf{y}(t_k) = 0. \quad (6.54)$$

The system is augmented with the additional scalar state y , which collects the time integral of the instantaneous penalty:

$$\dot{\mathbf{x}}_{\text{aug}}(t) = \begin{bmatrix} \mathbf{f}_0(t) \\ \Lambda(t) \end{bmatrix}, \quad \mathbf{x}_{\text{aug}}(t) = \begin{bmatrix} \mathbf{x}(t) \\ y(t) \end{bmatrix} \quad (6.55)$$

where $\mathbf{f}_0$ denotes the natural dynamics of the ANH3BP Eq. (2.5). Since $\Lambda(t) \geq 0$ and $y(t_k) = 0 \forall k$ by construction, enforcing

$$\mathbf{y}(t_k + t_{\text{safety}}) = 0 \quad (6.56)$$

is equivalent to requiring $\int_{\mathcal{T}_s(t_k)} \Lambda(t) dt = 0$, guaranteeing that the passive safety constraint is

satisfied for all $t \in \mathcal{T}_s(t_k)$ [122].

At each SCvx* iteration, the post-maneuver distribution for each node, initially given by $\mathbf{x}_k^+$ and $(P_k^+)^{1/2}$, is propagated ballistically from t_k^+ to the end of the safety horizon, using the linearized and discretized dynamics:

$$\mathbf{x}_{\text{aug},k,j+1}^{(\text{bal})} = A_j^{(\text{bal})} \mathbf{x}_{\text{aug},k,j}^{(\text{bal})} + \mathbf{c}_j^{(\text{bal})}, \quad (6.57)$$

where a secondary discretization index $j = 0, \dots, N_{\text{bal}}$ is introduced to describe the ballistic propagation, with $j = 0$ corresponding to t_k^+ .

The derivation of $A_j^{(\text{bal})}$ and $\mathbf{c}_j^{(\text{bal})}$ follows the usual linearization and discretization procedure outlined in Chapter 2. The comprehensive derivation for the augmented linear matrices is reported in Appendix B. Finally, the passive safety constraint is enforced as:

$$\mathbf{y}_{k, N_{\text{bal}}}^{(\text{bal})} = \boldsymbol{\xi}_k, \quad k = 0, \dots, N, \quad (6.58)$$

where $\boldsymbol{\xi}_k$ denotes the k -th vector of slack variables introduced due to the linearization about the previous SCP solution, and is penalized in the augmented cost function Eq. (6.77).

This relaxation ensures feasibility of each convex subproblem while preserving convergence of the SCvx* algorithm. Although only one terminal constraint is imposed per SCP node k , the construction guarantees passive safety over the entire continuous-time interval following each node.

6.3.3. Convex Subproblem

It is now possible to formally state the convex subproblem for ΔV_{99} minimization solved at each iteration of SCvx*.

Problem 2 (Convex ΔV_{99} Subproblem). *Find $\bar{\mathbf{X}}^*, \bar{\mathbf{U}}^*, \mathbf{K}^*, \boldsymbol{\xi}^*$ that minimize the augmented cost Eq.(6.46) subject to the filtered state dynamics constraints Eq. (6.37), control magnitude constraints Eq. (6.47), maximum covariance constraints Eq. (6.49), terminal constraints Eq. (6.50) and trust region constraints Eq. (6.51).*

$$\begin{aligned}
& \min_{\bar{\mathbf{X}}, \bar{\mathbf{U}}, \mathbf{K}, \boldsymbol{\xi}} & J_{aug}(\bar{\mathbf{X}}, \bar{\mathbf{U}}, \mathbf{K}, \boldsymbol{\xi}) &= J_{ub}(\bar{\mathbf{U}}, \mathbf{K}) + J_{pen}(\boldsymbol{\xi}) & \triangleright \text{Augmented Cost Function} \\
\text{subject to} & \bar{\mathbf{X}} &= \mathbf{A}\bar{\mathbf{x}}_0 + \mathbf{B}\bar{\mathbf{U}} + \mathbf{C} & \triangleright \text{Filtered State Dynamics} \\
& \|\bar{\mathbf{u}}_k\|_2 + m_{\chi^2}(\epsilon_u, n_u) \|P_{u_k}^{1/2}\|_2 &\leq u_{\max} & \triangleright \text{Maximum Control} \\
& \|(P_{\max} - \tilde{P}_k)^{-1/2} E_{x_k} (I + \mathbf{B}\mathbf{K}) \mathbf{S}^{1/2}\|_2 - 1 &\leq 0 & \triangleright \text{Maximum Covariance} \\
& \mathbf{y}_{k, N_{\text{bal}}}^{(\text{bal})} &= \boldsymbol{\xi}_k & \triangleright \text{CT-Passive Safety (relaxed)} \\
& E_{x_N} (\mathbf{A}\bar{\mathbf{x}}_0 + \mathbf{B}\bar{\mathbf{U}} + \mathbf{C}) - \mathbf{x}_f &= \boldsymbol{\xi} & \triangleright \text{Terminal Mean (relaxed)} \\
& \|(P_f - \tilde{P}_N)^{-1/2} E_{x_N} (I + \mathbf{B}\mathbf{K}) \mathbf{S}^{1/2}\|_2 - 1 &\leq 0 & \triangleright \text{Terminal Covariance} \\
& \|D_x(\mathbf{X} - \mathbf{X}^*)\|_{\infty} \leq \Delta_{TR}, \|D_u(\bar{\mathbf{U}} - \bar{\mathbf{U}}^*)\|_{\infty} &\leq \Delta_{TR} & \triangleright \text{Trust Region}
\end{aligned}$$

6.4. Problem Statement: Minimization of Nonlinear Errors

While linear covariance steering provides an efficient and convex framework for stochastic optimal control under uncertainty, its validity fundamentally relies on the assumption that the state distribution remains approximately Gaussian and that the linearized dynamics accurately represent the underlying nonlinear system. As illustrated in Fig. 6.5, this assumption can progressively deteriorate when the state distribution evolves through regions of strong nonlinearity. In such cases, the predicted covariance obtained from linear propagation may significantly deviate from the true distribution, leading to a loss of consistency between the optimized solution and the actual nonlinear behavior.

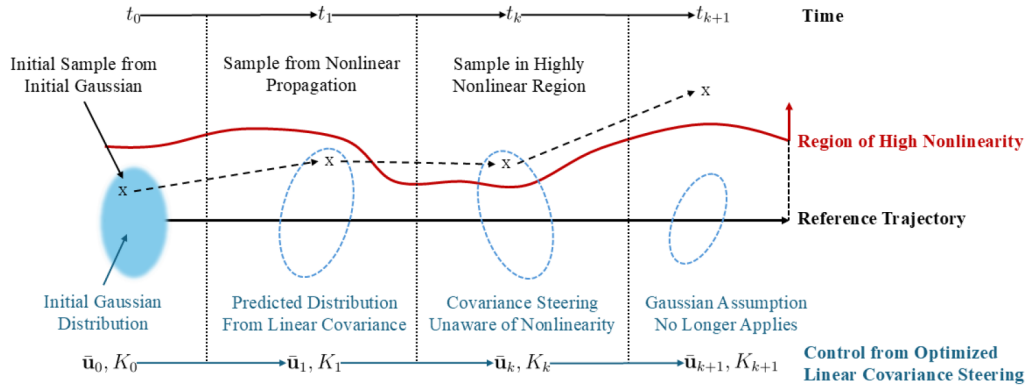


Figure 6.5: Effects of Nonlinear Errors in Linear Covariance Steering

Credits: [51]

This limitation motivates the introduction of optimization objectives that explicitly account for nonlinear effects. Rather than indirectly controlling nonlinearity through ad hoc constraints on the covariance magnitude, a more principled approach consists in directly quantifying and minimizing the accumulation of nonlinear errors along the trajectory. Measures of Nonlinearity (MoNs) provide a systematic way to assess how strongly higher-order dynamics contribute to the deviation between the true nonlinear evolution and its linear approximation. In particular, the Tensor Eigenpair Measure of Nonlinearities (TEMoN) enables a semi-analytical characterization of nonlinear effects based on higher-order state transition tensors, without resorting to Monte Carlo sampling.

The formulation adopted in this work builds upon the recent developments by Qi [51], where nonlinear dynamical errors are upper-bounded by a convex function of the state deviation and incorporated directly into the optimization objective. In that work, the proposed minimum-nonlinearity formulation is applied to a station-keeping problem, for which the reference trajectory is known a priori. Under this assumption, the problem can be solved through a single convex optimization, without the need for iterative re-linearization.

In contrast, the present work considers trajectory optimization problems in which the nominal trajectory itself is an optimization variable. To address this added level of complexity, the minimum-nonlinearity formulation is embedded within a Sequential Convex Programming (SCP) framework. At each SCP iteration, the nonlinear dynamics are linearized about the current reference trajectory, higher-order state transition tensors are computed offline, and a convex subproblem is formulated to minimize an upper bound on the accumulated nonlinear errors. This approach generalizes the minimum-nonlinearity covariance steering framework to fully nonlinear trajectory optimization problems, while retaining convexity and computational tractability for each subproblem.

A complete theoretical treatment of the minimum-nonlinearity formulation is given in [51], while the following sections adapt this framework to the SCvx* setting.

6.4.1. Accumulation of Nonlinear Errors

This section presents how local nonlinear errors at each discretization epoch propagate and accumulate over the trajectory, following the formulation introduced in [51].

Recall that, as introduced in Chapter 2, by performing a series expansion of the dynamics about the reference trajectory $\mathbf{x}^*$ and integrating from t_k to t_{k+1} , the state deviation at time t_{k+1} , denoted by $\delta\mathbf{x}_{k+1}$, can be expressed as a function of the state deviation at t_k , $\delta\mathbf{x}_k$, through the state transition matrix (STM) and the higher-order state transition tensors (STTs):

$$\delta\mathbf{x}_{k+1} = \Phi(t_{k+1}, t_k) \delta\mathbf{x}_k + \sum_{m=2}^{\infty} \frac{1}{m!} \Phi^{(m)}(t_{k+1}, t_k) (\delta\mathbf{x}_k)^m \quad (6.59)$$

with Φ and $\Phi^{(m)}$ denoting the STM and the m -th order STT, respectively.

Let $\mathbf{x}_k^*$ denote the reference trajectory at time t_k , about which the dynamics are linearized. Let $\delta\mathbf{x}_k$ represent the actual state deviation and $\delta\tilde{\mathbf{x}}_k$ the state deviation propagated through the linearized dynamics. At the initial epoch t_0 , the discrepancy between these two deviations is null, hence no nonlinear effects are present, or $\delta\mathbf{x}_0 = \delta\tilde{\mathbf{x}}_0$.

To compute the deviations at the next epoch, t_1 , define $\delta\mathbf{x}_k^+ = \delta\mathbf{x}_k + B\mathbf{u}_k$ as the actual state deviation after applying an impulsive control action, and analogously $\delta\tilde{\mathbf{x}}_k^+ = \delta\tilde{\mathbf{x}}_k + B\mathbf{u}_k$. Then,

$$\delta\mathbf{x}_1 = A_0\delta\mathbf{x}_0^+ + \mathcal{R}(\delta\mathbf{x}_0^+), \quad \delta\tilde{\mathbf{x}}_1 = A_0\delta\tilde{\mathbf{x}}_0^+ = A_0\delta\mathbf{x}_0^+ \quad (6.60)$$

where A_k denotes the linearized dynamics from t_k to t_{k+1} , and $\mathcal{R}(\cdot)$ represents the *remainder* term neglected by the first-order truncation of the Taylor expansion. Indeed, it follows directly

from Eq. (6.59) that the remainder corresponds to the sum of all higher-order terms:

$$\mathcal{R}(\delta \mathbf{x}_k^+) = \sum_{m=2}^{\infty} \frac{1}{m!} \Phi^{(m)}(t_{k+1}, t_k) (\delta \mathbf{x}_k^+)^m \quad (6.61)$$

At this point, the error introduced by the linearization process at t_1 can be defined as:

$$\boldsymbol{\epsilon}_1 = \delta \mathbf{x}_1 - \delta \tilde{\mathbf{x}}_1 = \mathcal{R}(\delta \mathbf{x}_0^+) \quad (6.62)$$

By following the same procedure, the nonlinear errors at subsequent epochs can be derived. In particular, at t_2 and t_3 [51]:

$$\boldsymbol{\epsilon}_2 = A_1 \mathcal{R}(\delta \mathbf{x}_0^+) + \mathcal{R}(\delta \mathbf{x}_1^+), \quad \boldsymbol{\epsilon}_3 = A_2 A_1 \mathcal{R}(\delta \mathbf{x}_0^+) + A_2 \mathcal{R}(\delta \mathbf{x}_1^+) + \mathcal{R}(\delta \mathbf{x}_2^+) \quad (6.63)$$

By extending this process to any future epoch t_k , it is clear that the nonlinear error $\boldsymbol{\epsilon}_k$ depends on the remainder $\mathcal{R}(\cdot)$ at the current step, as well as all the accumulated remainders from all previous steps. As a consequence, nonlinear effects progressively accumulate along the trajectory. Accurately quantifying and minimizing these errors requires the knowledge of all higher-order contributions over the entire time horizon, which can be retrieved by exploiting the higher-order dynamical information provided by state transition tensors (STTs).

6.4.2. Convex Bound of Nonlinear Errors

As discussed in the previous section, nonlinear errors accumulate over the time horizon as a consequence of repeated linearization. A natural way to quantify these effects is through the use of state transition tensors (STTs). However, tensor operations introduce nonconvexities that are incompatible with the optimal covariance steering and sequential convex programming framework adopted in this thesis. Following the approach proposed by Qi [51], this section derives a convex upper bound on the nonlinear errors, enabling a tractable formulation suitable for SCP.

As done by Qi [51], this thesis considers the minimization of the ℓ^2 -norm of the nonlinear errors. Then, by using the triangle inequality, the remainders at each time step can be isolated within its own norm:

$$\begin{aligned} \|\boldsymbol{\epsilon}_1\|_2 &= \|I_n \mathcal{R}(\delta \mathbf{x}_0^+)\|_2 \\ \|\boldsymbol{\epsilon}_2\|_2 &\leq \|A_1 \mathcal{R}(\delta \mathbf{x}_0^+)\|_2 + \|\mathcal{R}(\delta \mathbf{x}_1^+)\|_2 \\ \|\boldsymbol{\epsilon}_3\|_2 &\leq \|A_2 A_1 \mathcal{R}(\delta \mathbf{x}_0^+)\|_2 + \|A_2 \mathcal{R}(\delta \mathbf{x}_1^+)\|_2 + \|\mathcal{R}(\delta \mathbf{x}_2^+)\|_2 \end{aligned} \quad (6.64)$$

The right-hand sides of Eq. 6.64 are a summation of functions in the form $\|\mathcal{AR}(\delta\mathbf{x}_k^+)\|_2$, where $\mathcal{A} = \prod_i A_i$. By considering only the STTs up to order $m^* \geq 2$ and using again the triangle inequality, each m -th order STT is separated into its norm, yielding:

$$\|\mathcal{AR}(\delta\mathbf{x}_k^+)\|_2 \approx \left\| \mathcal{A} \sum_{m=2}^{m^*} \frac{1}{m!} \Phi^{(m)}(t_{k+1}, t_k) (\delta\mathbf{x}_k^+)^m \right\|_2 \leq \sum_{m=2}^{m^*} \frac{1}{m!} \left\| \mathcal{A} \Phi^{(m)}(t_{k+1}, t_k) (\delta\mathbf{x}_k^+)^m \right\|_2 \quad (6.65)$$

The product between $\mathcal{A}$ and $\Phi^{(m)}(t_{k+1}, t_k)$ can be expressed by defining a tensor $\mathcal{B}^{(m)}$, such that:

$$\mathcal{B}_{ji_1i_2\dots i_m}^{(m)}(t_{k+1}, t_k) = \mathcal{A}_{jk} \Phi_{ki_1i_2\dots i_m}^{(m)}(t_{k+1}, t_k) \quad (6.66)$$

Eq. (6.65) can be upper bounded by using the 2-norm definition provided by Kulik et al. in [81]:

$$\left\| \mathcal{B}^{(m)}(t_{k+1}, t_k) (\delta\mathbf{x}_k^+)^m \right\|_2 \leq \left\| \mathcal{B}^{(m)}(t_{k+1}, t_k) \right\|_2 \|\delta\mathbf{x}_k^+\|_2^m \quad (6.67)$$

This bound is selected because it does not include any tensor products, eliminating the nonconvexities in the problem. Moreover, it provides a tighter bound than the one proposed in previous work by Qi in [50], which uses the Cauchy-Schwarz and triangle inequalities.

6.4.3. Cost Function and Constraints

This section introduces the cost function and the set of constraints adopted in the second optimization problem aimed at minimizing nonlinear dynamical errors.

Cost Function

Having derived the theoretical convex upper bound to the nonlinear errors, this section presents the necessary steps to define the cost function to be minimized inside each convex subproblem [51].

Let $g_k(\mathcal{A}, m)$ denote the 2-norm function of the $\mathcal{B}^{(m)}$ tensor, i.e., $g_k(\mathcal{A}, m) = \|\mathcal{B}^{(m)}(t_{k+1}, t_k)\|_2$. Then,

$$\|\mathcal{AR}(\delta\mathbf{x}_k^+)\|_2 \leq \sum_{m=2}^{m^*} \frac{1}{m!} \left\| \mathcal{B}^{(m)}(t_{k+1}, t_k) \right\|_2 \|\delta\mathbf{x}_k^+\|_2^m = \sum_{m=2}^{m^*} \frac{1}{m!} g_k(\mathcal{A}, m) \|\delta\mathbf{x}_k^+\|_2^m \quad (6.68)$$

It is now possible to rewrite the bounds of the nonlinear errors Eq. 6.64 as:

$$\begin{aligned}
 \|\epsilon_1\|_2 &\leq \sum_{m=2}^{m^*} \frac{1}{m!} g_0(I_{n_x}, m) \|\delta \mathbf{x}_0^+\|_2^m = \tilde{\epsilon}_1 \\
 \|\epsilon_2\|_2 &\leq \sum_{m=2}^{m^*} \frac{1}{m!} g_0(A_1, m) \|\delta \mathbf{x}_0^+\|_2^m + \sum_{m=2}^{m^*} \frac{1}{m!} g_1(I_{n_x}, m) \|\delta \mathbf{x}_1^+\|_2^m = \tilde{\epsilon}_2 \\
 (\dots)
 \end{aligned} \tag{6.69}$$

In block matrix form:

$$\tilde{\epsilon} = \begin{bmatrix} \tilde{\epsilon}_0 \\ \tilde{\epsilon}_1 \\ \tilde{\epsilon}_2 \\ \vdots \end{bmatrix} = \sum_{m=2}^{m^*} \frac{1}{m!} \begin{bmatrix} 0 & 0 & 0 & \dots \\ g_0(I_{n_x}, m) & 0 & 0 & \dots \\ g_0(A_1, m) & g_1(I_{n_x}, m) & 0 & \dots \\ g_0(A_2 A_1, m) & g_1(A_2, m) & g_2(I_{n_x}, m) & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \|\delta \mathbf{x}_0^+\|_2^m \\ \|\delta \mathbf{x}_1^+\|_2^m \\ \|\delta \mathbf{x}_2^+\|_2^m \\ \vdots \end{bmatrix} \tag{6.70}$$

where $\tilde{\epsilon} = [\tilde{\epsilon}_0, \tilde{\epsilon}_1, \dots]^\top$ is the vector containing the upper bounds to $\|\epsilon_k\|_2 \leq \tilde{\epsilon}_k$ at each t_k .

Eq. (6.70) are convex functions of the optimization variables $\delta \mathbf{x}_k^+$, and provide an upper bound of the nonlinear dynamical errors [51]. Similarly to the linearization and discretization step, the computation of the STTs can be done prior to the optimization process, alleviating most of the computationally expensive steps from the convex solver. This means that $g_k(\cdot) \geq 0$ is a constant in each convex subproblem, and is computed beforehand.

Since the state vector is made of quantities with different units, it is convenient to split the contributions from position and velocity [51, 81]. As $B = [0_{3 \times 3}, I_3]^\top$, the post-maneuver state deviation is $(\delta \mathbf{x}_k^+)^\top = [\delta \mathbf{r}_k^\top, (\delta \mathbf{v}_k^+)^\top]^\top$. The nonlinear errors for position $\tilde{\epsilon}_r$ are computed with $g_{r,k} = \|\mathcal{B}_r^{(m)}(t_{k+1}, t_k)\|_2$. Similarly for the velocity nonlinear errors $\tilde{\epsilon}_v$ with $g_{v,k} = \|\mathcal{B}_v^{(m)}(t_{k+1}, t_k)\|_2$. The tensor $\mathcal{B}^{(m)}$ is partitioned into submatrices that contain exclusively either positional or velocity units:

$$\left. \begin{aligned} \left(\mathcal{B}_r^{(m)} \right)_{j i_1 \dots i_m} &= \left(\mathcal{B}^{(m)} \right)_{j i_1 \dots i_m} \\ \left(\mathcal{B}_v^{(m)} \right)_{j i_1 \dots i_m} &= \left(\mathcal{B}^{(m)} \right)_{(j+3)(i_1+3) \dots (i_m+3)} \end{aligned} \right\} \text{ for } 1 \leq j, i_1, \dots, i_m \leq 3 \tag{6.71}$$

Then:

$$\begin{aligned}
\tilde{\epsilon}_r &= \sum_{m=2}^{m^*} \frac{1}{m!} \begin{bmatrix} 0 & 0 & 0 & \cdots \\ g_{r,0}(I_{n_x}, m) & 0 & 0 & \cdots \\ g_{r,0}(A_1, m) & g_{r,1}(I_{n_x}, m) & 0 & \cdots \\ g_{r,0}(A_2 A_1, m) & g_{r,1}(A_2, m) & g_{r,2}(I_{n_x}, m) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \|\delta \mathbf{r}_0\|_2 \\ \|\delta \mathbf{r}_1\|_2 \\ \|\delta \mathbf{r}_2\|_2 \\ \vdots \end{bmatrix} \\
\tilde{\epsilon}_v &= \sum_{m=2}^{m^*} \frac{1}{m!} \begin{bmatrix} 0 & 0 & 0 & \cdots \\ g_{v,0}(I_{n_x}, m) & 0 & 0 & \cdots \\ g_{v,0}(A_1, m) & g_{v,1}(I_{n_x}, m) & 0 & \cdots \\ g_{v,0}(A_2 A_1, m) & g_{v,1}(A_2, m) & g_{v,2}(I_{n_x}, m) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \|\delta \mathbf{v}_0^+\|_2 \\ \|\delta \mathbf{v}_1^+\|_2 \\ \|\delta \mathbf{v}_2^+\|_2 \\ \vdots \end{bmatrix}
\end{aligned} \tag{6.72}$$

As done in [51], the stochastic quantities involved in the nonlinear error bounds ($\|\delta \mathbf{r}_k\|_2$ and $\|\delta \mathbf{v}_k^+\|_2$) are replaced by deterministic quantities that characterize the region within which nonlinearities are evaluated with a prescribed confidence level. In particular, for a Gaussian random variable $\mathbf{x} \sim \mathcal{N}(\bar{\mathbf{x}}, P) \in \mathbb{R}^n$, the quantile of the ℓ^2 -norm of $\mathbf{x} - \mathbf{a}$ evaluated at probability $1 - \epsilon$ admits an upper bound that depends on the mean and covariance of the distribution [51]. Specifically,

$$\text{Quant}(1 - \epsilon) \leq \text{Quant}_{ub}(1 - \epsilon) = \|\mathbf{x} - \mathbf{a}\|_2 + \sqrt{m_{\chi^2}(\epsilon, n)} \left\| P^{1/2} \right\|_2. \tag{6.73}$$

Adopting this upper bound, and following the chance-constrained formulation in [49], the stochastic norms appearing in the nonlinear error bounds are substituted by their corresponding quantile-based deterministic surrogates:

$$\begin{aligned}
\|H_r(\hat{\mathbf{x}}_k^+ - \mathbf{x}_k^*)\|_2 + \sqrt{m_{\chi^2}(\epsilon_x, 3)} \left\| H_r(P_k^+)^{1/2} \right\|_2 &= \tilde{r}_k \\
\|H_v(\hat{\mathbf{x}}_k^+ - \mathbf{x}_k^*)\|_2 + \sqrt{m_{\chi^2}(\epsilon_x, 3)} \left\| H_v(P_k^+)^{1/2} \right\|_2 &= \tilde{v}_k
\end{aligned} \tag{6.74}$$

where H is a matrix that extracts either the position or velocity components from $\mathbf{x}_k$, $(P_k^+)^{1/2}$ denotes the state covariance matrix *post* impulsive maneuver, and ϵ_x is the risk parameter that specifies the confidence level associated with the quantile bound.

Eq. (6.72) can be rewritten as:

$$\begin{aligned} \tilde{\epsilon}_r &= \sum_{m=2}^{m^*} \frac{1}{m!} \begin{bmatrix} 0 & 0 & 0 & \cdots \\ g_{r,0}(I_{n_x}, m) & 0 & 0 & \cdots \\ g_{r,0}(A_1, m) & g_{r,1}(I_{n_x}, m) & 0 & \cdots \\ g_{r,0}(A_2 A_1, m) & g_{r,1}(A_2, m) & g_{r,2}(I_{n_x}, m) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \tilde{r}_0 \\ \tilde{r}_1 \\ \tilde{r}_2 \\ \vdots \end{bmatrix} \\ \tilde{\epsilon}_v &= \sum_{m=2}^{m^*} \frac{1}{m!} \begin{bmatrix} 0 & 0 & 0 & \cdots \\ g_{v,0}(I_{n_x}, m) & 0 & 0 & \cdots \\ g_{v,0}(A_1, m) & g_{v,1}(I_{n_x}, m) & 0 & \cdots \\ g_{v,0}(A_2 A_1, m) & g_{v,1}(A_2, m) & g_{v,2}(I_{n_x}, m) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \tilde{v}_0 \\ \tilde{v}_1 \\ \tilde{v}_2 \\ \vdots \end{bmatrix} \end{aligned} \quad (6.75)$$

Finally, it is possible to write the MoN objective function in the following form:

$$J^{(\text{NL})} = \max_k [(1 - \lambda)\tilde{\epsilon}_{r,k} + \lambda\tilde{\epsilon}_{v,k}] \quad (6.76)$$

where $\max(\cdot)$ is the pointwise maximum of the inputs, and a tuning parameter $\lambda \in [0, 1]$ is introduced to emphasize more the minimization of nonlinear effects from positional or velocity and control deviations.

As previously done by Qi in [51], this thesis uses $m^* = 2$, i.e., only the second order STT is used to construct the objective function.

Also in this optimization framework, the original cost function $J^{(\text{NL})}$ is augmented with the same penalty function of Eq. (6.46), to penalize the slack variables ξ introduced for the relaxed equality constraints.

$$J_{\text{aug}} = J^{(\text{NL})} + J_{\text{pen}}(\xi) \quad (6.77)$$

Constraints

In this second optimization framework, most of the constraints adopted in the ΔV_{99} minimization problem are retained. In particular, the *thrust magnitude constraint*, the *terminal constraints* and the *continous-time passive safety constraints* against MTEs remain enforced.

The *maximum covariance constraint*, however, is deliberately removed in this formulation. Unlike the ΔV_{99} objective, the minimum-nonlinearity cost function inherently regulates the growth of state uncertainty by penalizing trajectories that evolve through highly nonlinear regions of the state space. As a result, the optimization naturally favors solutions for which the linearized dynamics and Gaussian assumptions remain valid, without the need for an explicit upper bound on the state covariance.

Moreover, avoiding artificial constraints on the covariance is desirable from a practical standpoint. Selecting an appropriate maximum admissible covariance *a priori* is highly scenario-dependent and often requires extensive tuning, typically involving repeated Monte Carlo simulations to assess the performance of the designed linear covariance controller. By contrast, the proposed minimum-nonlinearity formulation provides a more systematic and adaptive mechanism to control uncertainty growth, automatically adjusting to different mission scenarios.

6.4.4. Convex Subproblem

The formal problem statement for the nonlinear dynamical error minimization solved at each iteration of SCvx* is now presented.

Problem 3 (Convex Min. NL Subproblem). *Find $\bar{\mathbf{X}}^*, \bar{\mathbf{U}}^*, \hat{\mathbf{P}}^{1/2,*}, \mathbf{K}^*, \boldsymbol{\xi}^*$ that minimize the augmented cost Eq. (6.77) subject to the filtered state dynamics constraints Eq. (6.37) and covariance dynamics Eq. (6.39), MoN constraints Eq. (6.74) and (6.75), control magnitude constraints Eq. (6.47), continuous-time safety constraints Eq. (6.58), terminal constraints Eq. (6.50) and trust region constraints Eq. (6.51).*

$$\begin{array}{ll}
\min_{\bar{\mathbf{X}}, \bar{\mathbf{U}}, \mathbf{K}, \boldsymbol{\xi}} & J_{aug}(\bar{\mathbf{X}}, \bar{\mathbf{U}}, \mathbf{K}, \boldsymbol{\xi}) = J^{(NL)} + J_{pen}(\boldsymbol{\xi}) & \triangleright \text{Augmented Cost Function} \\
\text{subject to} & \bar{\mathbf{X}} = \mathbf{A}\bar{\mathbf{x}}_0 + \mathbf{B}\bar{\mathbf{U}} + \mathbf{C} & \triangleright \text{Filtered State Dynamics} \\
& \hat{\mathbf{P}}^{1/2} = (\mathbf{I} + \mathbf{BK})\mathbf{S}^{1/2} & \triangleright \text{Covariance Dynamics} \\
& \tilde{\mathbf{r}}, \tilde{\mathbf{v}} \leftarrow \text{Eq. (6.74)} & \triangleright \text{Computing MoNs} \\
& \tilde{\boldsymbol{\epsilon}}_r, \tilde{\boldsymbol{\epsilon}}_v \leftarrow \text{Eq. (6.75)} & \\
& \|\bar{\mathbf{u}}_k\|_2 + m_{\chi^2}(\epsilon_u, n_u)\|\mathbf{P}_{u_k}^{1/2}\|_2 \leq u_{\max} & \triangleright \text{Maximum Control} \\
& \mathbf{y}_{k, N_{\text{bal}}}^{(\text{bal})} = \boldsymbol{\xi}_k & \triangleright \text{CT-Passive Safety (relaxed)} \\
& E_{x_N}(\mathbf{A}\bar{\mathbf{x}}_0 + \mathbf{B}\bar{\mathbf{U}} + \mathbf{C}) - \mathbf{x}_f = \boldsymbol{\xi} & \triangleright \text{Terminal Mean (relaxed)} \\
& \|(P_f - \tilde{P}_N)^{-1/2} E_{x_N}(\mathbf{I} + \mathbf{BK})\mathbf{S}^{1/2}\|_2 - 1 \leq 0 & \triangleright \text{Terminal Covariance} \\
& \|D_x(\mathbf{X} - \mathbf{X}^*)\|_{\infty} \leq \Delta_{TR}, & \triangleright \text{Trust Region} \\
& \|D_u(\bar{\mathbf{U}} - \bar{\mathbf{U}}^*)\|_{\infty} \leq \Delta_{TR} &
\end{array}$$

6.4.5. Numerical Implementation

The numerical evaluation of the convex bound on nonlinear errors introduced in the previous sections presents nontrivial computational challenges. In particular, the computation of the coefficients $g_{r,k}(\cdot)$ and $g_{v,k}(\cdot)$ in Eq. (6.75) requires evaluating the induced two-norm of a multilinear tensor obtained from the combination of the state transition tensors (STTs) and the linearized dynamics. As introduced in Chapter 2, this induced norm is computed from the

maximum Z -eigenvalue of the associated tensor [51].

Z -eigenvalues are defined only for real supersymmetric tensors [84], while the tensors in the present formulation are generally nonsymmetric due to the contraction of the STTs with the linearized dynamics. Consequently, a symmetrization step is required prior to eigenvalue computation.

In this work, symmetrization is performed using the `symtensor` function of MATLAB's Tensor Toolbox, which averages the tensor over all permutations of its three indices.

After symmetrization, the maximum Z -eigenpair is computed using the adaptive *shifted symmetric higher-order power method* (SS-HOPM [85]) implemented in the Tensor Toolbox. Since this method is not guaranteed to converge to the global maximum eigenpair, a multi-start strategy with random initializations and restarts in case of non-convergence is adopted, following the approach suggested in [81]. While not mathematically rigorous, this procedure provides a reliable and numerically stable estimate of the dominant Z -eigenvalue, which is sufficient for the purpose of bounding nonlinear errors.

Most notably, the above procedure can be carried out offline, prior to the optimization, to construct the constant block-matrices in Eq. (6.75) used at each iteration. This significantly alleviates the computational burden of the proposed framework.

6.5. Solution Method via Sequential Convex Programming

Once the original stochastic optimal control problem has been reformulated in its block-Cholesky convex form, the resulting optimization problems can be efficiently solved via Sequential Convex Programming (SCP). In particular, two convexified problems are obtained: Problem 2, corresponding to the minimization of ΔV_{99} , and Problem 3, corresponding to the minimization of nonlinear dynamical errors.

Similarly to the deterministic adaptive-mesh trajectory optimization discussed in Section 5.2, this thesis employs the SCvx* algorithm to solve both stochastic problem formulations. A detailed description of the SCvx* algorithm is provided in Chapter 3.

The implementation closely follows the methodology described in [42, 48]. A key difference with respect to the deterministic AMSCP implementation described in Section 5.2 lies in the step acceptance criterion and in the update of the trust region radius.

As in the deterministic case, the ratio between the actual and predicted cost reductions is computed as

$$\rho^{(k)} = \frac{\Delta J^{(k)}}{\Delta L^{(k)}}, \quad (6.78)$$

where $\Delta J^{(k)}$ and $\Delta L^{(k)}$ denote the actual and predicted reductions of the augmented Lagrangian at iteration k , respectively.

Unlike the AMSCP rationale, a symmetric acceptance rule centered around $\rho = 1$ is adopted. The step is accepted if

$$\rho^{(k)} \in [1 - \eta_0, 1 + \eta_0], \quad (6.79)$$

where $\eta_0 > 0$ is a user-defined tolerance parameter.

Similarly, the trust region radius Δ_{TR} is updated according to

$$\Delta_{\text{TR}}^{(k+1)} = \begin{cases} \min\{\alpha_2 \Delta_{\text{TR}}^{(k)}, \Delta_{\text{TR,max}}\}, & \text{if } \rho^{(k)} \in [1 - \eta_2, 1 + \eta_2], \\ \Delta_{\text{TR}}^{(k)}, & \text{if } \rho^{(k)} \in [1 - \eta_1, 1 + \eta_1], \\ \max\{\Delta_{\text{TR}}^{(k)} / \alpha_1, \Delta_{\text{TR,min}}\}, & \text{otherwise,} \end{cases} \quad (6.80)$$

where $0 < \eta_2 < \eta_1 < \eta_0$ and $\alpha_1, \alpha_2 > 1$ define the contraction and expansion factors, respectively. This symmetric trust-region strategy enlarges the trust region only when the convex model accurately predicts the nonlinear cost variation, i.e., $\rho^{(k)} \approx 1$, while it contracts the region when significant model mismatch is detected.

In the stochastic setting, the relaxed equality constraints entering the augmented Lagrangian include the terminal mean state constraint and the continuous-time passive safety constraint against missed thrust events. At each SCP iteration, the convex subproblem enforces linearized versions of these constraints, while the augmented penalty term requires evaluating their original nonlinear forms.

Following [48], the nonconvex terminal mean state violation is computed as

$$\mathbf{g}^{\text{mean,relaxed}}(\bar{\mathbf{X}}, \bar{\mathbf{U}}, \mathbf{K}) = \bar{\mathbf{x}}_0 + \sum_{k=0}^{N-1} \int_{t_k}^{t_{k+1}} \mathbf{f}(\bar{\mathbf{x}}, \bar{\mathbf{u}}_k, t) dt - \mathbf{x}_f, \quad (6.81)$$

which measures the deviation of the nonlinear mean trajectory from the desired terminal state.

The continuous-time passive safety constraint is evaluated by propagating the post-maneuver mean and covariance ballistically over the safety horizon associated with each discretization node. This propagation is performed under the augmented dynamics introduced in Eq. (6.55), and the corresponding relaxed constraint is obtained by extracting the auxiliary scalar state at the end of the safety horizon:

$$g_k^{\text{safety,relaxed}} = y(t_k + t_{\text{safety}}), \quad k = 0, \dots, N. \quad (6.82)$$

These nonlinear constraint violations are incorporated into the augmented Lagrangian and used

to update the penalty parameters and Lagrange multipliers within the SCvx* iteration. The resulting sequence of convex subproblems converges to a solution that is locally optimal and feasible with respect to the original stochastic optimal control problem.

Algorithm 6.1 summarizes the SCvx* procedure applied to the ΔV_{99} minimization problem, while Algorithm 6.2 presents the corresponding algorithm for the minimum–nonlinearity formulation, including the precomputation of the MoN coefficients.

Algorithm 6.1 Stochastic min- ΔV_{99} Optimization under Uncertainty via SCvx*

```

Generate the initial reference trajectory  $\{\mathbf{X}^{\text{ref}}, \mathbf{U}^{\text{ref}}, \mathbf{K}^{\text{ref}} = \mathbf{0}, \Theta^{\text{ref}}\}$ 
Initialize SCvx* parameters
while the convergence criteria not met do
    Obtain linearized block system matrices  $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{L}, \mathbf{Y}$ 
     $\{\mathbf{X}^*, \mathbf{U}^*, \mathbf{K}^*, \Theta^*, \xi^*, \zeta^*\} \leftarrow$  solve Convex Subproblem
    if acceptance conditions met then
         $\{\mathbf{X}^{\text{ref}}, \mathbf{U}^{\text{ref}}, \mathbf{K}^{\text{ref}}, \Theta^{\text{ref}}\} \leftarrow \{\mathbf{X}^*, \mathbf{U}^*, \mathbf{K}^*, \Theta^*\}$ 
        Update Lagrange multipliers
    end if
    Update trust region
end while
return  $\{\mathbf{X}^{\text{ref}}, \mathbf{U}^{\text{ref}}, \mathbf{K}^{\text{ref}}, \Theta^{\text{ref}}\}$ 

```

Algorithm 6.2 Stochastic min-NL Optimization under Uncertainty via SCvx*

```

Generate the initial reference trajectory  $\{\mathbf{X}^{\text{ref}}, \mathbf{U}^{\text{ref}}, \mathbf{K}^{\text{ref}} = \mathbf{0}\}$ 
Initialize SCvx* parameters
while the convergence criteria not met do
    Obtain linearized block system matrices  $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{L}, \mathbf{Y}$  and STTs
    Compute MoNs  $g_r, g_v$  with SS-HOPM
     $\{\mathbf{X}^*, \mathbf{U}^*, \mathbf{K}^*, \xi^*\} \leftarrow$  solve Problem 3
    if acceptance conditions met then
         $\{\mathbf{X}^{\text{ref}}, \mathbf{U}^{\text{ref}}, \mathbf{K}^{\text{ref}}\} \leftarrow \{\mathbf{X}^*, \mathbf{U}^*, \mathbf{K}^*\}$ 
        Update Lagrange multipliers
    end if
    Update trust region
end while
return  $\{\mathbf{X}^{\text{ref}}, \mathbf{U}^{\text{ref}}, \mathbf{K}^{\text{ref}}\}$ 

```

6.6. Nonlinear Monte Carlo Validation

After obtaining a solution via SCvx*, nonlinear Monte Carlo (MC) simulations are performed to verify that the approximations introduced to obtain a tractable formulation are reasonable, and that the resulting linear covariance controller remains effective when applied to the original

nonlinear stochastic dynamics. The validation procedure adopted in this thesis follows the methodology originally proposed by Oguri and Lantoiné [48].

An Extended Kalman Filter (EKF) is employed in place of the Kalman Filter (KF) defined in Eq. (6.25). In particular, the time update step is replaced by a nonlinear propagation of the mean state. The linear system matrices $A_k, B_k, c_k, G_{exe,k}, G_k$ are obtained by linearizing the dynamics about the nonlinear mean trajectory, while the innovation process exploits the nonlinear measurement model Eq. (6.10).

For state propagation, the full nonlinear set of stochastic differential equations (SDEs) in Eq. (6.12) is integrated. In the numerical implementation, the Brownian motion increment $d\mathbf{w}(t)$ is approximated by a discrete-time white noise process $\mathbf{w}_k$ with zero-order hold over a small time interval $t \in [t_k, t_k + \Delta t_k)$, as described in [64]:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) + G_k(\mathbf{x}, t) \frac{1}{\sqrt{\Delta t_k}} \mathbf{w}_k. \quad (6.83)$$

Here, Δt_k denotes the discretization time step used to approximate the continuous-time SDE with a discrete-time white noise process.

The original control policy defined in Eq. (6.30) is converted into an equivalent formulation better suited for nonlinear Monte Carlo simulations. Since it is not straightforward to compute the nonlinear counterpart of the stochastic process $\mathbf{z}_k$, such a conversion is desirable in order to facilitate implementation without affecting the optimal solution. Following Lemma 2 of [48], Eq. (6.30) can be rewritten as

$$\mathbf{u}_k = \bar{\mathbf{u}}_k + \sum_{i=0}^k \hat{K}_{k,i}(\hat{\mathbf{x}}_i - \bar{\mathbf{x}}_i), \quad (6.84)$$

which computes the control input based on the deviation of the estimated state $\hat{\mathbf{x}}$ from the nominal trajectory. With this formulation, the state estimate provided by the EKF is directly used to compute the control input $\mathbf{u}$ during the nonlinear Monte Carlo simulations.

The matrices $\hat{K}_{k,i}$ correspond to submatrices of $\hat{\mathbf{K}}$, defined as

$$\hat{\mathbf{K}} = \begin{bmatrix} \hat{K}_{0,0} & 0 & 0 & \cdots & 0 \\ \hat{K}_{1,0} & \hat{K}_{1,1} & \ddots & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 & 0 \\ \hat{K}_{N-1,0} & \hat{K}_{N-1,1} & \cdots & \hat{K}_{N-1,N-1} & 0 \end{bmatrix}, \quad \hat{\mathbf{K}} = \mathbf{K}(\mathbf{I} + \mathbf{BK})^{-1}, \quad (6.85)$$

where $(\mathbf{I} + \mathbf{BK})$ is assumed to be invertible.

Figure 6.6 illustrates the overall flow of the nonlinear Monte Carlo simulations implemented in this study. A total of 500 nonlinear trajectories are simulated by sampling the initial state

according to $\mathbf{x}_0 \sim \mathcal{N}(\bar{\mathbf{x}}_0, \hat{P}_0^- + \tilde{P}_0^-)$ and propagating each realization under the closed-loop FPC defined in Eq. (6.84). For each sample, two trajectories are propagated: the true trajectory $\mathbf{x}(t)$ in the physical space and the estimated trajectory $\hat{\mathbf{x}}(t)$ in the belief space.

For the true trajectory $\mathbf{x}(t)$, stochastic perturbations due to unmodeled accelerations are included in the nonlinear propagation, together with maneuver execution errors modeled through the Gates model. In the belief space, the planned control inputs $\mathbf{u}_k$ incorporate statistical corrections through the optimized feedback policy. The estimated state $\hat{\mathbf{x}}(t)$ is propagated nonlinearly, and orbit determination is performed via EKF updates to obtain $\hat{\mathbf{x}}_{k+1}$. The computation of the control inputs $\mathbf{u}_k$ is based on the FPC policy in Eq. (6.84), which exploits the history of state estimates.

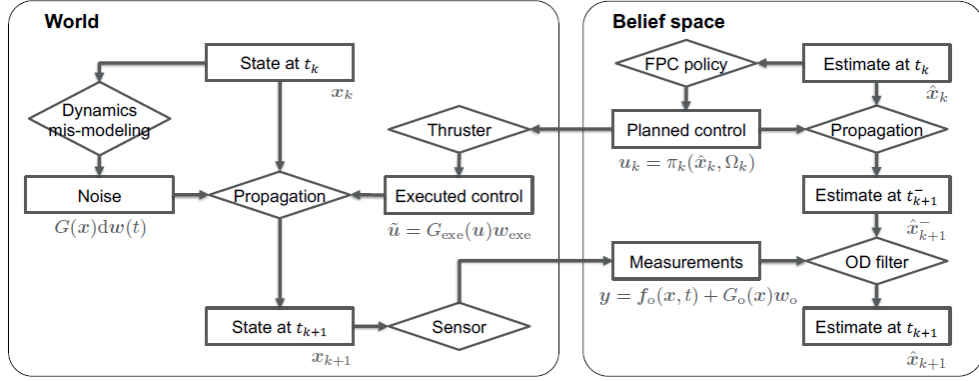


Figure 6.6: Nonlinear Monte Carlo flow chart, illustrating a simulation flow from t_k to t_{k+1} . Credits: [48]

7 | Numerical Results

After describing the sequential covariance steering problem formulations in Chapter 6, this chapter applies Algorithms 6.1 and 6.2 to two transfers in the ANH3BP between a PTO and a 6:1 RTO, extending the deterministic transfers obtained via AMSCP in Chapter 5 to the stochastic setting under uncertainty.

The values of the uncertainty parameters for the two scenarios are reported in Table 7.1.

Table 7.1: Uncertainty Parameters for Science-Orbit Transfer

Category	Error Type	Symbol	Value
Initial state	Position std. dev.	$\tilde{\sigma}_{r,0}$	25 m
	Velocity std. dev.	$\tilde{\sigma}_{v,0}$	0.05 cm/s
	Covariance matrix	$\tilde{P}_0$	$\text{blkdiag}(\tilde{\sigma}_{r,0}^2 I_3, \tilde{\sigma}_{v,0}^2 I_3)$
Final state	Position std. dev.	$\sigma_{r,f}$	10 m
	Velocity std. dev.	$\sigma_{v,f}$	0.05 cm/s
	Covariance matrix	P_f	$\text{blkdiag}(\sigma_{r,f}^2 I_3, \sigma_{v,f}^2 I_3)$
Max. Covariance ¹	Position std. dev.	$\sigma_{r,\max}$	150 m
	Velocity std. dev.	$\sigma_{v,\max}$	0.25 cm/s
	Covariance matrix	$P_{\max}$	$\text{blkdiag}(\sigma_{r,\max}^2 I_3, \sigma_{v,\max}^2 I_3)$
Gates model	Fixed mag. error	σ_1	10^{-2} mm/s ²
	Proportional mag. error	σ_2	0.33%
	Fixed pointing err.	σ_3	10^{-2} mm/s ²
	Proportional pointing err.	σ_4	0.2 deg
Nav. Uncertainty	Position std. dev.	σ_r^{nav}	0.5 m
	Velocity std. dev.	σ_v^{nav}	0.1 mm/s
Max. control	Upper bound	$u_{\max}$	1.5 cm/s
Unmodeled acc.	Stochastic acceleration	σ_{stoch}	10^{-13} km/s ²

The stochastic acceleration is modeled as a state-independent Gaussian white noise process acting only on the acceleration component of the dynamics, as introduced in the theoretical background. Its continuous-time intensity is characterized by the parameter $\sigma_{\text{stoch}} = 10^{-13}$

¹Only imposed for min- ΔV_{99} optimization.

km/s², representing the standard deviation of the unmodeled accelerations [74]. The associated diffusion matrix in the nonlinear SDEs Eq. (6.12) is therefore given by:

$$G = \begin{bmatrix} \mathbf{0}_{3 \times 3} \\ \sigma_{\text{stoch}} \sqrt{\Delta t} \cdot I_3 \end{bmatrix}. \quad (7.1)$$

The factor $\sqrt{\Delta t}$ arises from the Itô discretization of the Wiener process. For a standard Brownian motion increment, the variance scales linearly with the time step, which implies that the standard deviation of the increment scales as $\sqrt{\Delta t}$. Therefore, when discretizing the continuous-time stochastic acceleration over a time interval Δt , the corresponding discrete-time noise term must be scaled by $\sqrt{\Delta t}$ in order to remain consistent with the underlying continuous-time SDE.

In this work, two different discretization steps are employed depending on the computational framework. During the SCP optimization phase, the time grid is defined by the adaptive mesh, with time intervals between consecutive nodes of approximately $\Delta t_k \approx 24$ hours.

Conversely, in the nonlinear Monte Carlo validation, the SDEs are integrated using a much finer time discretization. Each adaptive mesh interval is subdivided into 1000 smaller steps, resulting in a discretization interval of approximately $\Delta t_{\text{WN}} \approx 1.5$ minutes. Over each subinterval, the stochastic acceleration is modeled as a zero-order-hold Gaussian white noise process with standard deviation σ_{stoch} , scaled by $\sqrt{\Delta t_{\text{WN}}}$.

Although the discretization step differs between the SCP phase and the Monte Carlo simulations, both implementations are consistent with the same underlying continuous-time stochastic model.

The filtering process in this thesis is assumed to be a full-state observation with Gaussian noise:

$$\mathbf{f}_{\text{obs}}(\mathbf{x}) = \mathbf{x}, \quad G_{\text{obs}}(\mathbf{x}) = \text{blkdiag}(\sigma_r^{\text{nav}} I_3, \sigma_v^{\text{nav}} I_3) \quad (7.2)$$

Finally, the risk-parameter for the maximum control magnitude constraint violation is set to $\epsilon_u = 0.001$, i.e. 99.9% satisfaction guarantee.

The parameters for the SCvx* algorithm are provided in Table 7.2. The optimality threshold is set to $\epsilon_{\text{opt}} = 10^{-6}$ for the minimum ΔV_{99} optimization [48], while to $\epsilon_{\text{opt}} = 10^{-4}$ for the minimum nonlinearity case.

The different choice in optimality tolerances stems from the numerical characteristics of the two objective functions. The minimum ΔV_{99} cost is primarily quadratic and well-conditioned, allowing tighter convergence tolerances without excessive computational burden. Conversely, the minimum nonlinearity formulation involves higher-order nonlinear metrics and repeated computations of convex upper bounds via SS-HOPM, which are more expensive. Moreover, reducing the tolerance below 10^{-4} did not produce meaningful changes in the terminal covariance or control profiles. Therefore, the chosen threshold was selected to balance numerical accuracy

and efficiency.

Table 7.2: SCvx* parameters

Parameter	ϵ_{feas}	$\{\eta_0, \eta_1, \eta_2\}$	$\{\alpha_1, \alpha_2\}$	β	γ	$\{\Delta_{\min}, \Delta_{\max}\}$	$\Delta^{(1)}$	$w_{\max}$	$w^{(1)}$
Value	10^{-6}	$\{1, 0.5, 0.1\}$	$\{2, 3\}$	2	0.9	$\{10^{-8}, 1\}$	0.2	10^8	10^2

For all numerical integration in MATLAB, the `ode45` propagator is used with absolute and relative tolerances set to 10^{-12} . Convex optimization subproblems are solved using YALMIP [94] and MOSEK [95]. All computations are performed in MATLAB with a standard personal laptop.

7.1. Case Study 1: Multiple-Revolution PTO-RTO Transfer

Algorithms 6.1 and 6.2 are applied using as initial reference trajectory the deterministic multiple-revolution transfer computed via AMSCP in Chapter 5. The objective is to design statistically optimal control policies capable of steering the initial Gaussian distribution to the prescribed terminal distribution in the presence of operational and dynamical uncertainties.

Figure 7.1 compares the nominal trajectories obtained with the two stochastic optimization strategies. It can be observed that the resulting nominal paths are very similar and remain close to the deterministic reference transfer. Indeed, in this first case study the primary difference between the two solutions does not lie in the geometric shape of the nominal trajectory, but rather in the associated control structure and feedback gains. In particular, the different cost metrics lead to different maneuver magnitudes and covariance steering behavior, which will be analyzed in detail in the subsequent control comparison plots.

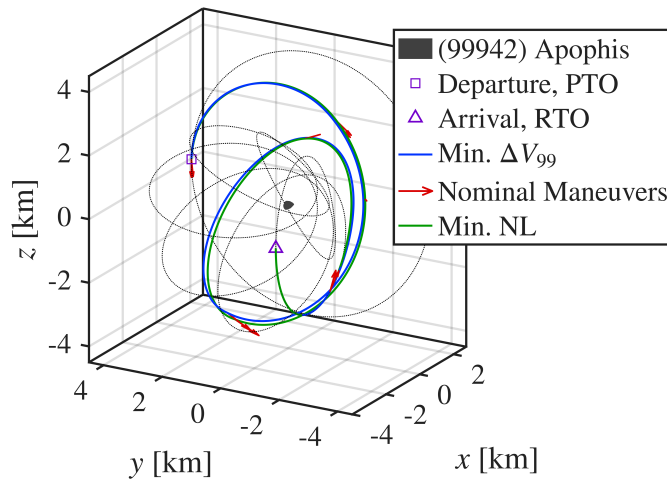


Figure 7.1: Nominal robust trajectories for Multi-Revolution transfer

Figure 7.2 shows the convergence profiles of SCvx* for both formulations. The minimum ΔV_{99} problem converges in 20 iterations (approximately 7.5 minutes), with no rejected steps. In contrast, the minimum nonlinearity formulation requires 59 iterations and approximately 16 minutes to converge. Although each convex subproblem is solved slightly faster in this latter case, the overall computational time increases due to the larger number of SCvx* iterations and the additional cost of computing the convex bound on nonlinear errors via SS-HOPM at each step. Furthermore, more candidate steps are rejected because the trust-region mechanism, shown in Figure 7.3, reacts more conservatively to variations in the nonlinear metric.

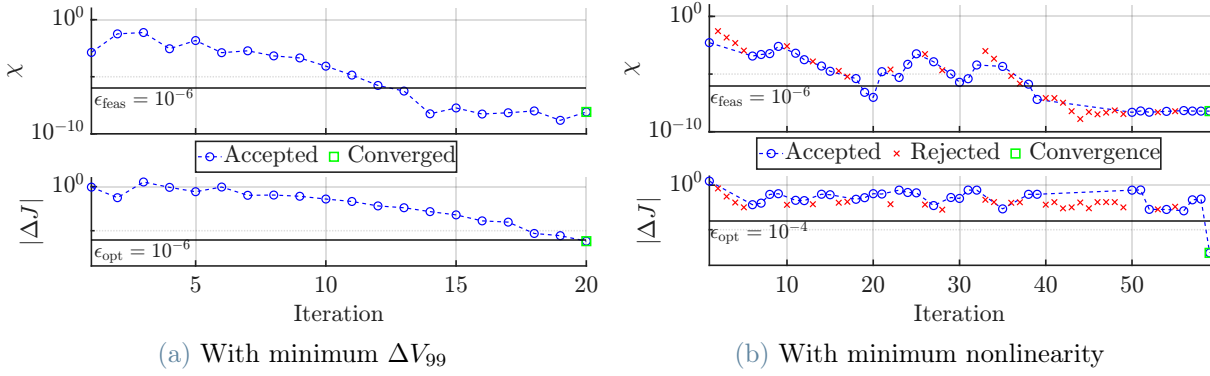


Figure 7.2: Convergence profile of SCvx* for Multi-Revolution transfer.

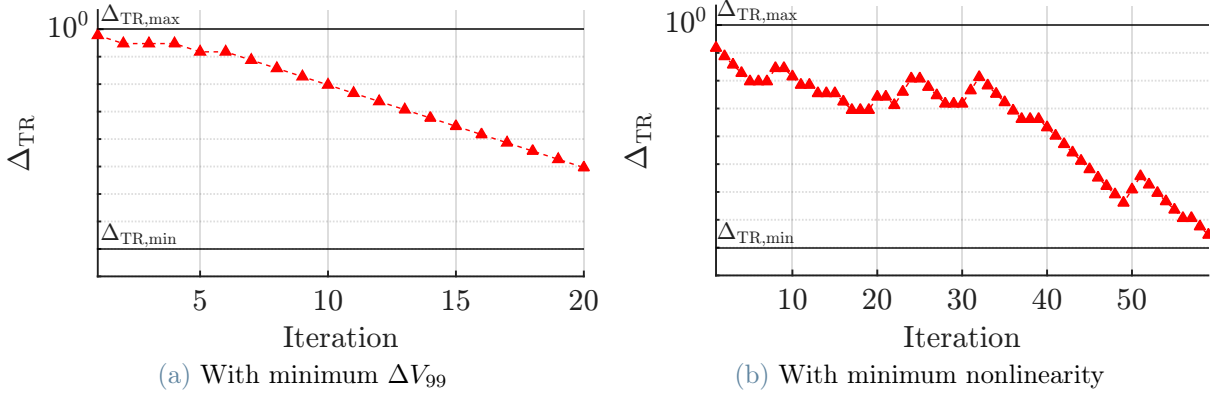


Figure 7.3: Trust region radius evolution in SCvx* for Multi-Revolution transfer.

To assess the robustness of the obtained policies, a nonlinear Monte Carlo simulation with 500 samples is performed following the procedure described in Section 6.6. The open-loop propagation of the deterministic optimal trajectory under uncertainties are shown in Figure 7.4. As expected, the deterministic solution fails to satisfy the terminal covariance constraint: the dispersion grows significantly and the final state distribution largely exceeds the target bounds. The dominant dispersion direction aligns with the velocity components, consistent with the

theoretical propagation of uncertainties in orbital dynamics and in agreement with previous studies [42]. This highlights the necessity of feedback-based stochastic optimization.

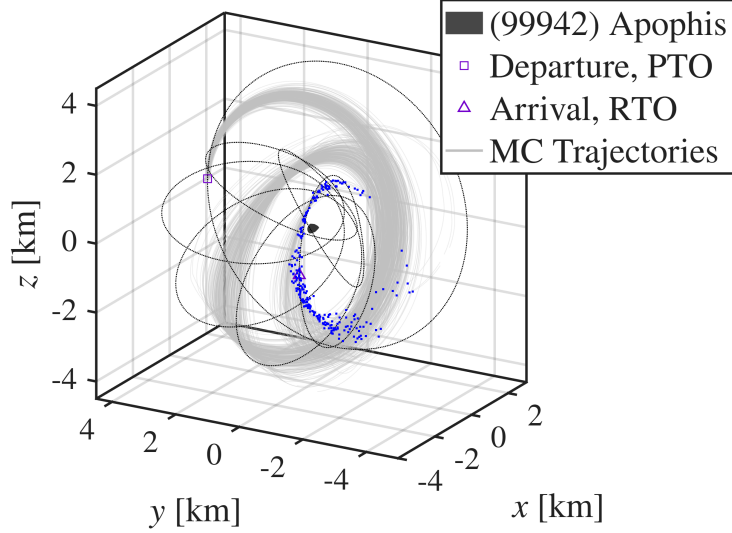


Figure 7.4: Nonlinear Monte Carlo trajectories (open-loop)

The closed-loop Monte Carlo results are shown in Figure 7.5. Both optimization strategies are effective in controlling the spacecraft during the transfer by applying feedback trajectory correction maneuvers (TCMs), keeping the dispersion bounded throughout the trajectory. However, a significant difference emerges near the final phase of the transfer.

In the minimum ΔV_{99} case, the sample dispersion increases toward the end, and a noticeable fraction of trajectories violates the terminal covariance constraint. This degradation is attributable to the accumulation of nonlinear effects along the trajectory: while the linear covariance steering formulation predicts acceptable dispersion levels, the actual nonlinear propagation exceeds the validity region of the linear approximation, reducing the effectiveness of the feedback control law.

Conversely, the minimum nonlinearity strategy maintains the trajectory within a regime where the linear approximation remains accurate. As a result, the Monte Carlo samples remain tightly bounded and fully contained within the target covariance at the final node. Moreover, this improved robustness is achieved without explicitly imposing intermediate maximum covariance constraints, and instead emerges directly from minimizing the nonlinear dynamical errors within the optimization problem.

Figure 7.6 further quantifies this behavior by comparing the predicted terminal distribution with the empirical Monte Carlo distribution, for each combination of state variables. On the diagonals, the dispersion of each state is showcased in terms of a histogram.

In the minimum ΔV_{99} case, a clear mismatch is observed between the predicted covariance

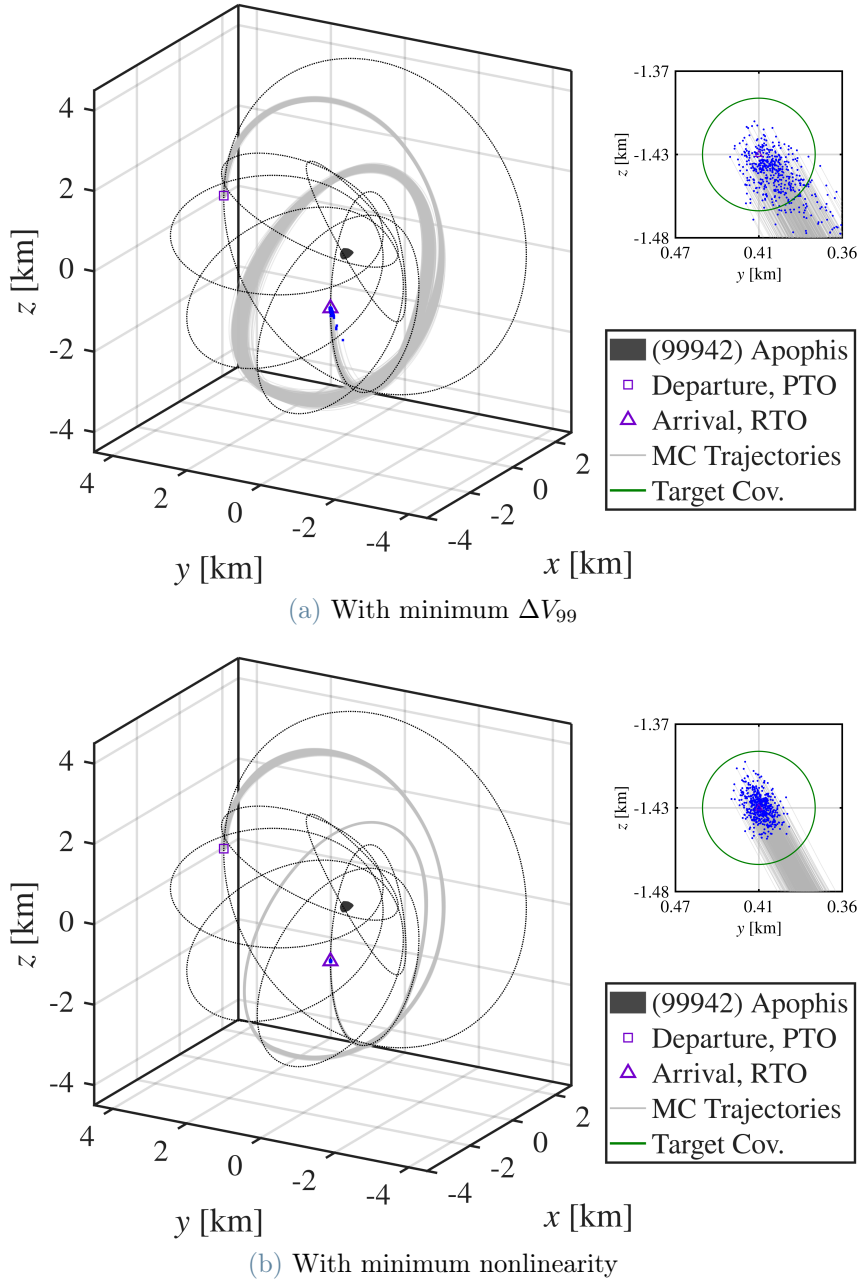
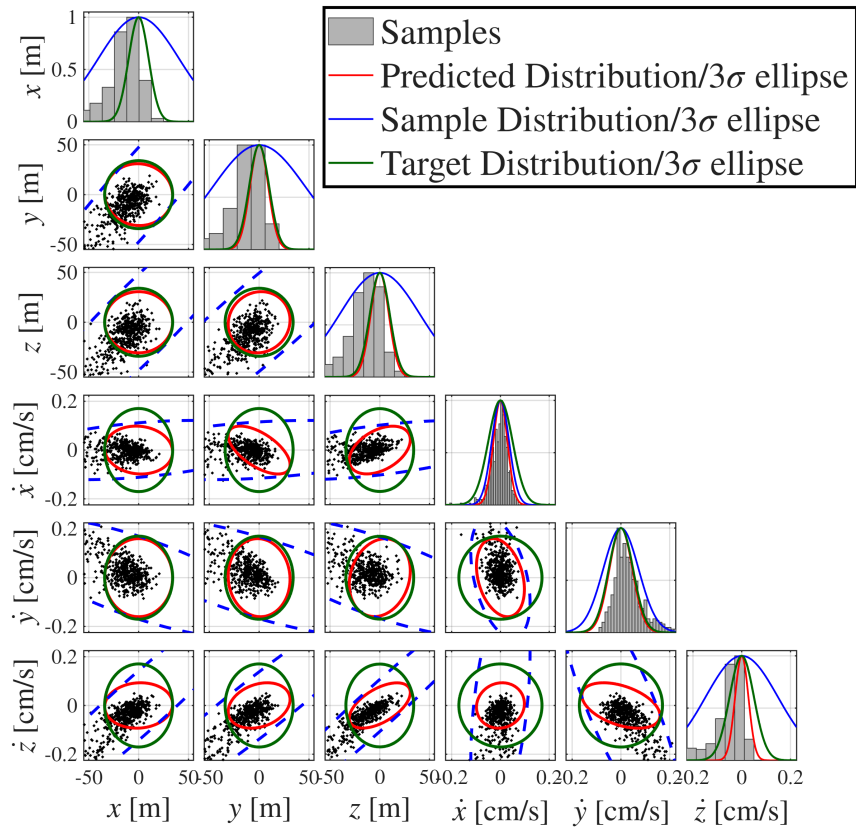
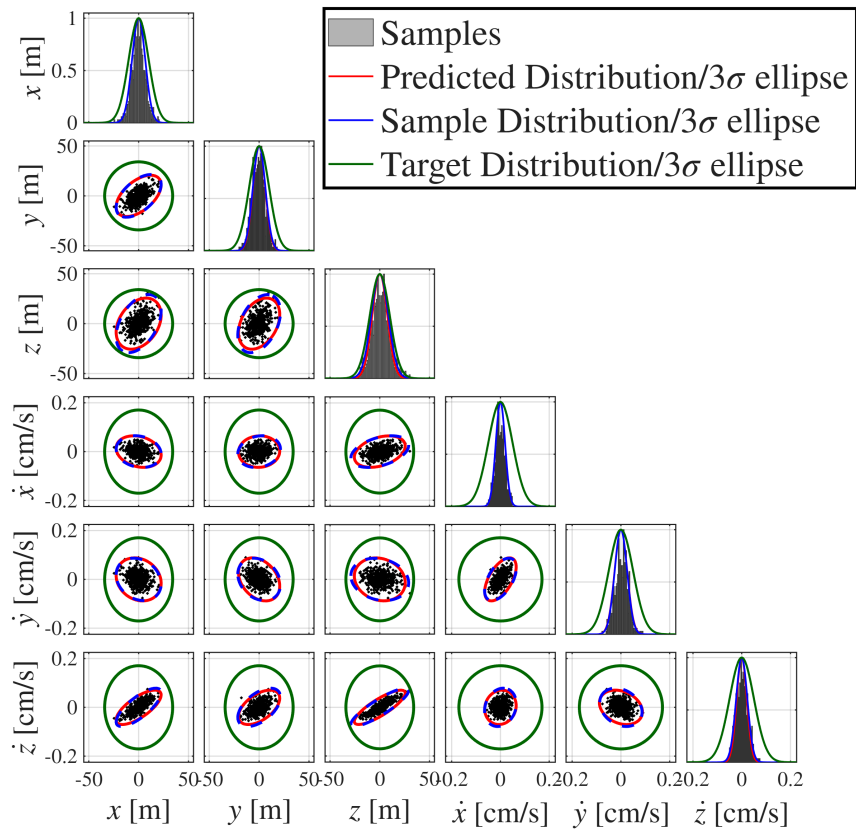


Figure 7.5: Multi-Revolution transfer Nonlinear Monte Carlo trajectories (closed-loop)

(red) and the empirical sample distribution (blue), particularly in the positional components. This discrepancy confirms that the linearized uncertainty propagation model becomes inaccurate along the transfer.

In contrast, for the minimum nonlinearity case, the predicted and empirical distributions are in close agreement. The 3σ sample ellipses are entirely contained within the prescribed terminal covariance bounds (green). This indicates that the final covariance constraint is satisfied in the Monte Carlo samples.

(a) With minimum ΔV_{99} 

(b) With minimum nonlinearity

Figure 7.6: State dispersion at the final node

The time evolution of state deviations from the nominal trajectory are showcased in Figure 7.7. The red vertical segments represent the predicted 99.9% confidence bounds obtained from the linear covariance optimization. For the minimum ΔV_{99} formulation, the Monte Carlo samples progressively violate toward the end of the transfer, especially in the position components. In contrast, the minimum nonlinearity solution maintains consistency between predicted and empirical deviations, indicating that the linear covariance steering model remains valid throughout the trajectory. Moreover, the velocity deviations are sharply reduced at each TCM epoch in the minimum nonlinearity case, while the minimum ΔV_{99} formulation exhibits a more gradual and diffuse reduction. This difference in the temporal structure of dispersion evolution plays a crucial role in the overall robustness of the solution, where the minimum nonlinearity formulation reduces error accumulation and limits the amplification of nonlinear effects between correction epochs.

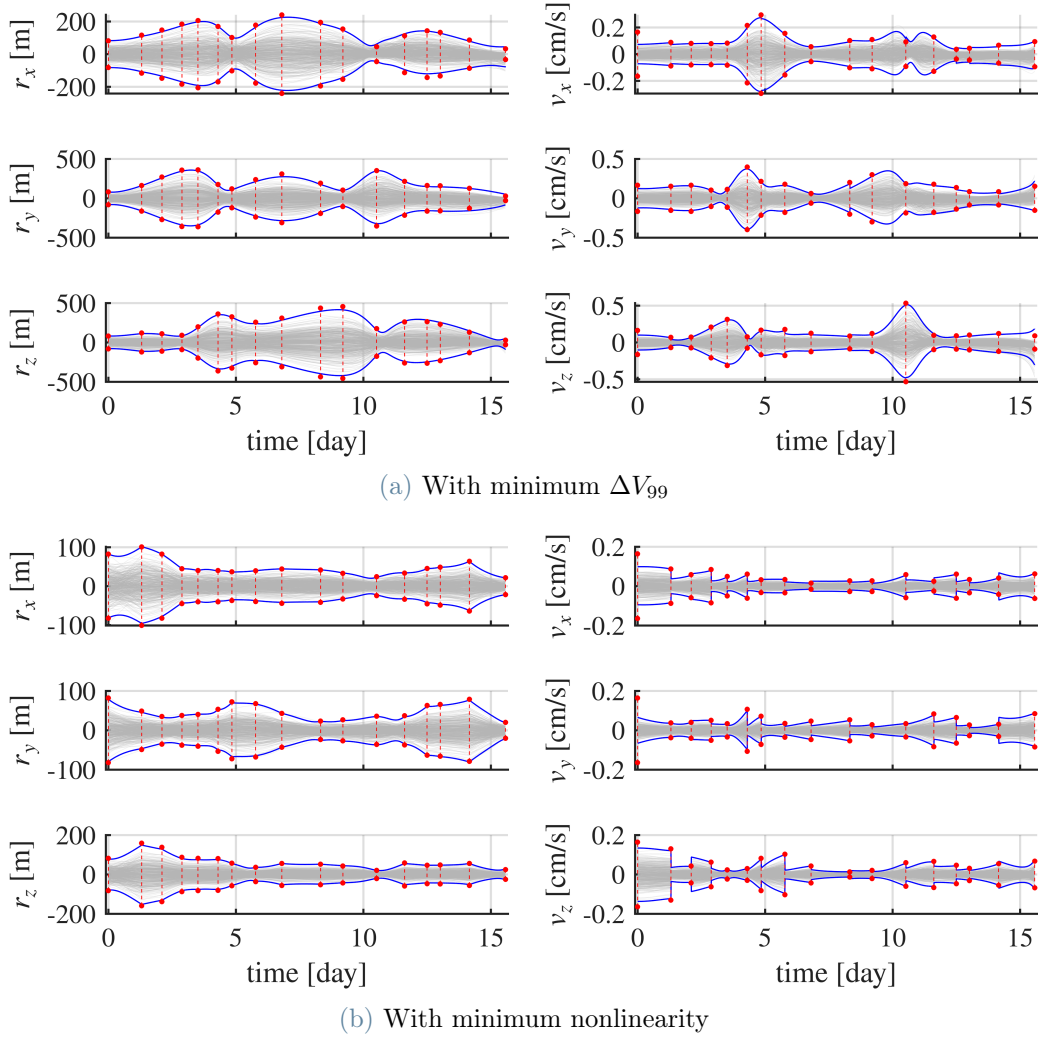


Figure 7.7: State deviations: MC samples (gray), empirical 99.9% confidence (blue), predicted deviations (red, 99.9% confidence)

The structural difference observed in the dispersion evolution can be directly correlated to the control architecture of the two optimization strategies, as shown in Figure 7.8, which reports the feedforward and feedback magnitudes at each maneuver opportunity. In the minimum ΔV_{99} case, the feedback effort is negligible throughout most of the transfer, with near-zero TCM magnitudes at the majority of maneuver epochs. This behavior is a direct consequence of the objective: once the linear model predicts acceptable covariance evolution and terminal distribution satisfaction, additional corrective actions are avoided. The resulting policy prioritizes statistical fuel efficiency over dynamical correction.

In contrast, the minimum nonlinearity strategy allocates feedback maneuvers more evenly along the trajectory, including near the final phase of the transfer. The additional corrective maneuvers mitigate the mismatch between the linear covariance propagation model and the true nonlinear dynamics, improving the consistency between predicted and effective distribution. This results in improved dynamical coherence, albeit with a slightly higher fuel expenditure.

Figure 7.9 reports the distribution of total ΔV obtained from Monte Carlo simulations. The deterministic ΔV , the nominal ΔV , the empirical ΔV_{99} , and the analytical upper bound J_{ub} are shown. As expected, J_{ub} is a conservative upper bound of the empirical ΔV_{99} . This result consistent with previous work [42, 48], and considering different formulations that have smaller conservativeness is an important direction for future improvements.

The minimum nonlinearity strategy showcases a marginally higher fuel cost, on the order of a few mm/s. However, in small-body proximity operations absolute velocities are extremely small, and robustness and reliability are often more critical than marginal fuel optimality [65]. These results suggest that explicitly accounting for nonlinear propagation effects can significantly enhance robustness with minimal impact on fuel consumption.

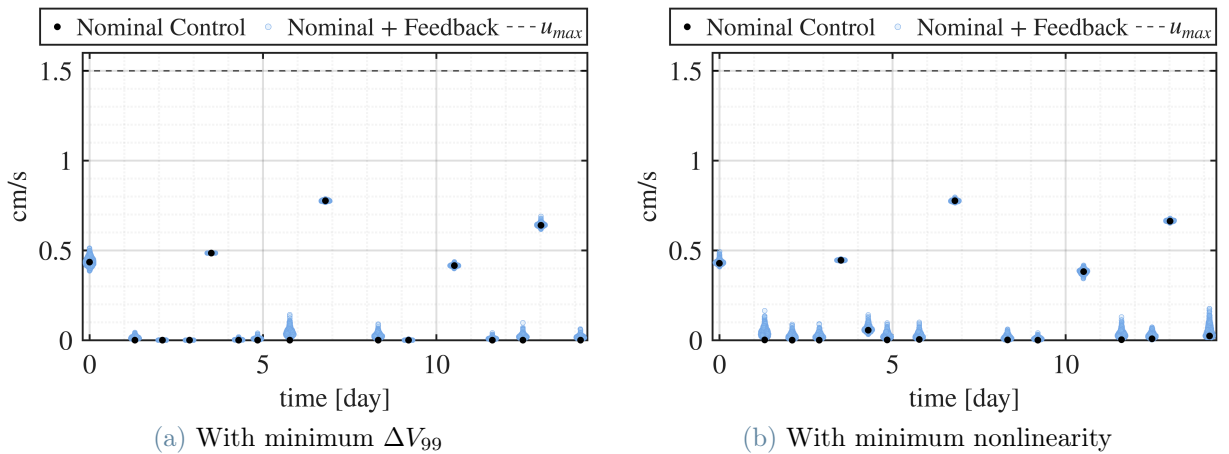


Figure 7.8: Control magnitudes at discrete maneuver epochs for Multi-Revolution transfer.

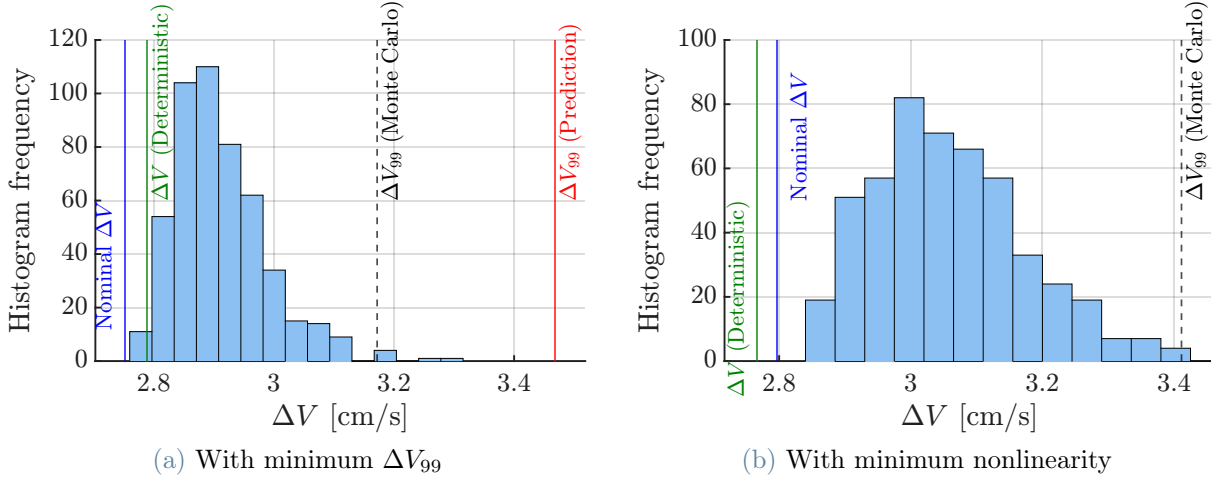


Figure 7.9: Monte Carlo ΔV distribution for Multi-Revolution transfer.

7.2. Case Study 2: Flyby PTO-RTO Transfer

Algorithms 6.1 and 6.2 are again applied using as initial reference trajectory the deterministic flyby transfer computed via AMSCP in Chapter 5. With respect to the previous multiple-revolution scenario, the flyby transfer introduces a much closer passage around Apophis, resulting in significantly stronger nonlinear dynamical effects. Moreover, this case explicitly enforces continuous-time passive safety guarantees in case of missed-thrust events (MTEs).

The parameters defining the passive safety requirement are summarized in Table 7.3.

Table 7.3: Parameters for continuous-time passive safety guarantee

Parameter	r_{safe}	t_{safety}	ϵ_{safe}
Value	400 m	7 days	0.01

After a missed-thrust event, the uncontrolled trajectory distribution must remain outside a keep-out-zone of radius $r_{\text{safe}} = 400$ m for a safety horizon of $t_{\text{safety}} = 7$ days, with confidence level $1 - \epsilon_{\text{safe}} = 99\%$. The selected safety horizon represents a practical time window within which nominal operations are expected to be restored. The keep-out radius corresponds approximately to twice the average radius of Apophis, ensuring conservative protection against surface impact.

Figure 7.10 compares the resulting nominal trajectories. The minimum ΔV_{99} solution shows a noticeably higher flyby altitude compared to the minimum nonlinearity solution. This behavior is directly linked to the interaction between uncertainty growth and the passive safety constraint. Since the minimum ΔV_{99} formulation does not explicitly control nonlinear propagation effects, the predicted uncertainty growth during the flyby is larger, so to maintain the passive safety guarantee under a missed-thrust scenario the optimizer compensates by increasing the nominal flyby distance, reducing the risk of violating the keep-out-zone.

Conversely, the minimum nonlinearity solution maintains lower nonlinear exposure along the trajectory. As a result, the predicted dispersion during a missed-thrust event is smaller, allowing a closer and less conservative flyby while still satisfying the passive safety requirement.

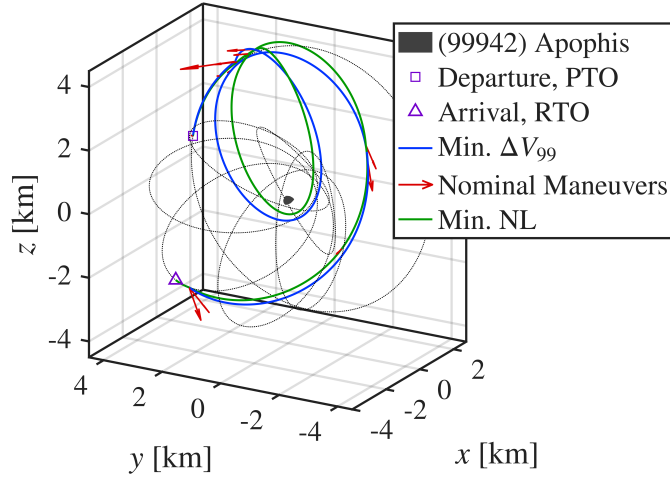


Figure 7.10: Comparison of the robust Flyby trajectories (min- ΔV_{99} and min-NL)

Figure 7.11 reports the SCvx* convergence behavior. The minimum ΔV_{99} formulation converges in 56 iterations (approximately 22 minutes), significantly more than in the previous case study. This increase is attributable to both the stronger nonlinearities induced by the close flyby and the substantial deviation from the deterministic reference trajectory required to satisfy passive safety constraints under uncertainty.

In contrast, the minimum nonlinearity formulation converges in 23 iterations (approximately 13 minutes). In this formulation, the cost function directly penalizes the mismatch between linearized and nonlinear propagation. This makes the convex subproblems more representative of the true nonlinear dynamics during the flyby, leading to more effective search directions and faster convergence compared to the ΔV_{99} formulation.

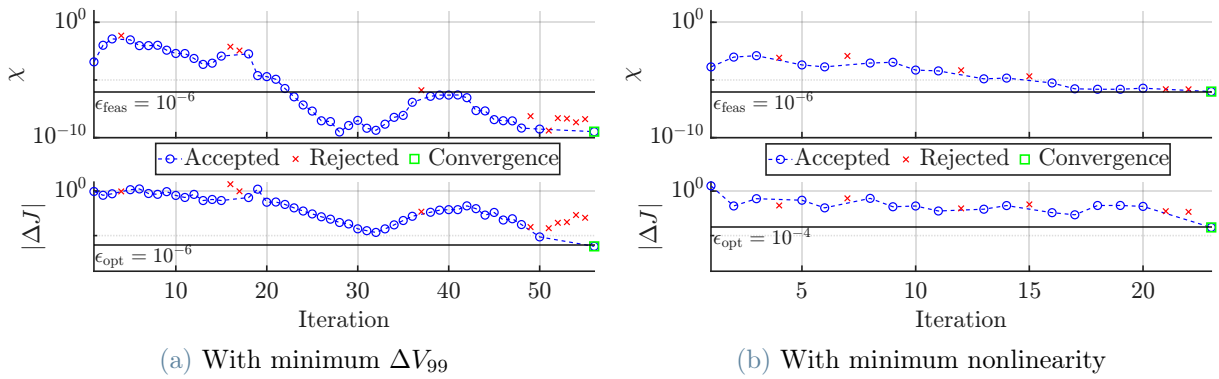


Figure 7.11: Convergence profile of SCvx* for Flyby Transfer.

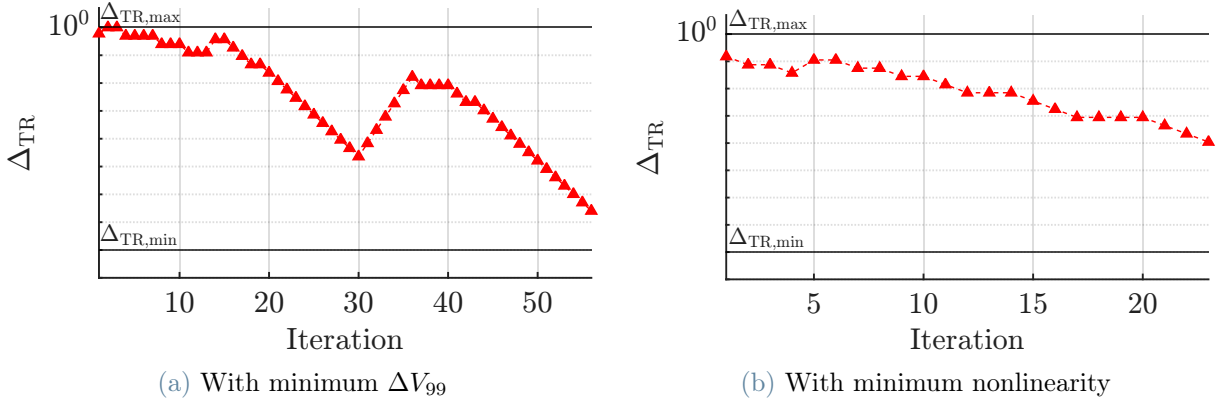


Figure 7.12: Trust region radius evolution in SCvx* for Flyby Transfer.

The solutions are verified once again through a nonlinear Monte Carlo simulation with 500 samples. As shown in Figure 7.13, the deterministic open-loop trajectories fail to satisfy the terminal covariance constraints once uncertainties are included. The dispersion is even more pronounced than the previous Multi-Revolution scenario due to the nonlinear effects introduced by the close asteroid approach, further emphasizing the necessity of statistical TCMs.

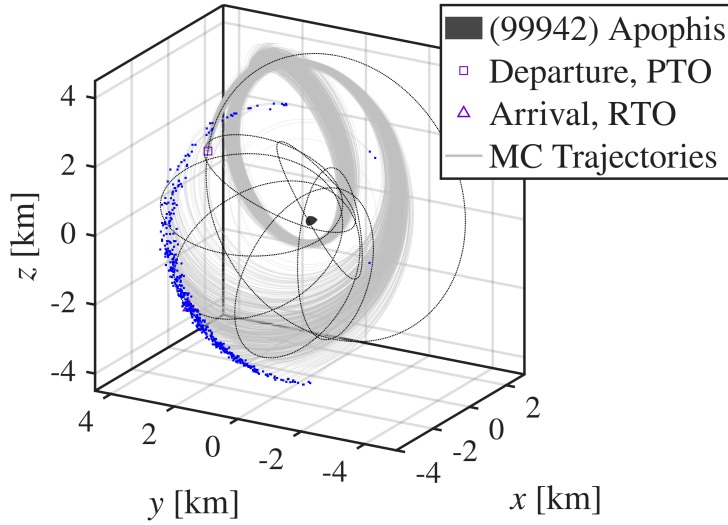


Figure 7.13: Flyby Transfer: Nonlinear MC trajectories (open-loop)

Figure 7.14 shows the closed-loop Monte Carlo results. Both strategies maintain bounded trajectories through feedback TCMs, but the degradation observed for the minimum ΔV_{99} formulation is still present, and even more pronounced than in Case Study 1. The nonlinear amplification of uncertainties during the flyby causes the linear covariance prediction to underestimate the true nonlinear dispersion. Consequently, the terminal covariance constraint is partially violated once the samples are propagated through nonlinear and stochastic dynamics.

The minimum nonlinearity formulation significantly mitigates this effect, as the predicted and empirical behaviors remain largely consistent. Minor violations of the terminal covariance are observed in the positional components, reflecting the intrinsic difficulty of accurately linearizing dynamics in the immediate vicinity of the asteroid during the flyby. Nonetheless, the violations are substantially reduced compared to the ΔV_{99} case.

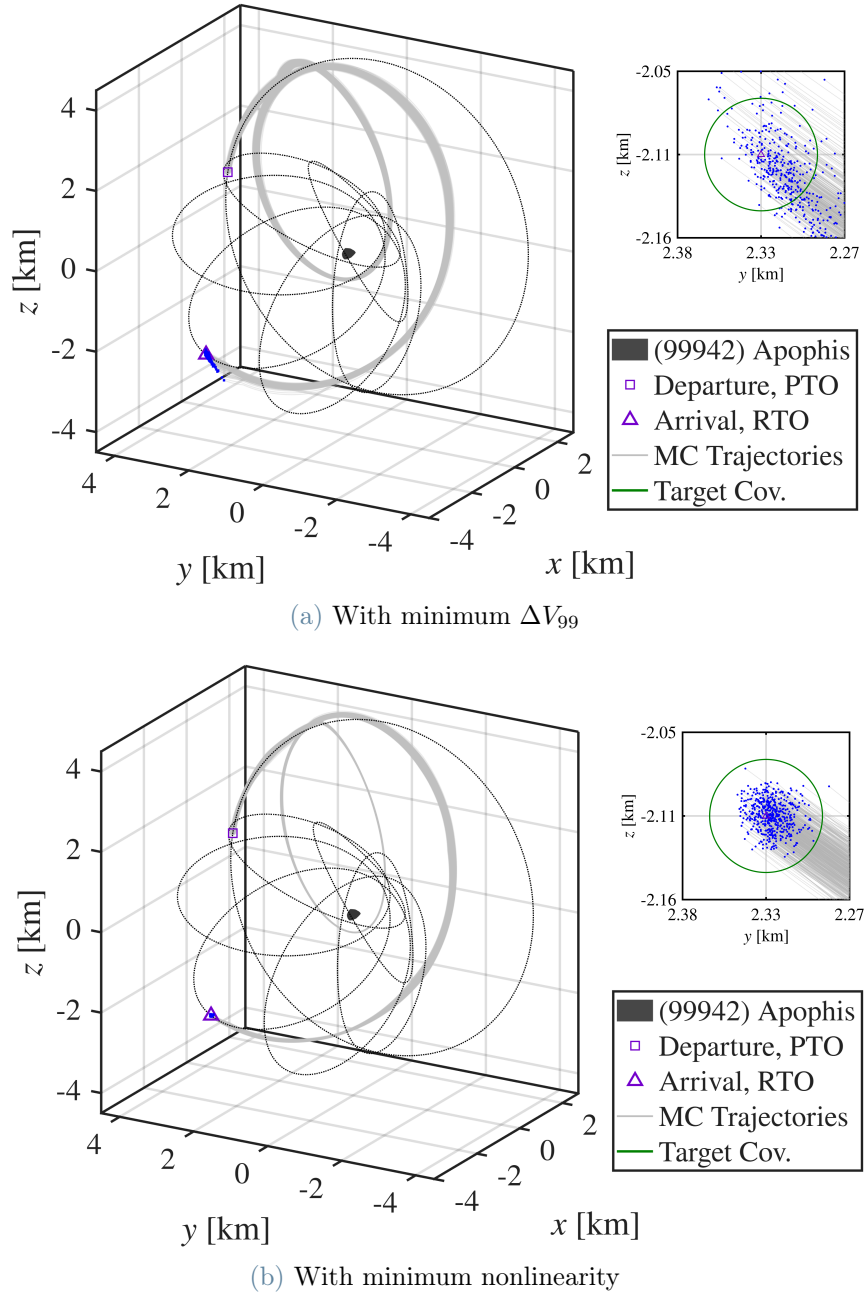


Figure 7.14: Flyby transfer nonlinear MC trajectories (closed-loop)

Figure 7.15 shows the statistics at the final epoch, for each combination of state variables, compares predicted and empirical terminal distributions.

For the minimum ΔV_{99} solution, the mismatch between predicted (red) and empirical (blue) distributions is amplified with respect to Case Study 1, particularly in position components, showing that the Gaussian assumption is no longer valid. This confirms that linear covariance steering becomes increasingly inaccurate when strong local nonlinearities are present, which are introduced during the flyby.

The minimum nonlinearity solution instead shows good consistency between prediction and Monte Carlo samples. Slight non-Gaussianity can be observed in the positional components due to the intrinsic nonlinearity of the flyby region. It is important to note that the presented results correspond to $\lambda = 0.5$ in the composite minimum nonlinearity cost function Eq. (6.76), which gives equal weight to position and velocity nonlinear errors. A higher λ value could further reduce positional violations at the expense of increased control effort, but this is not explored here to maintain the comparability across case studies.

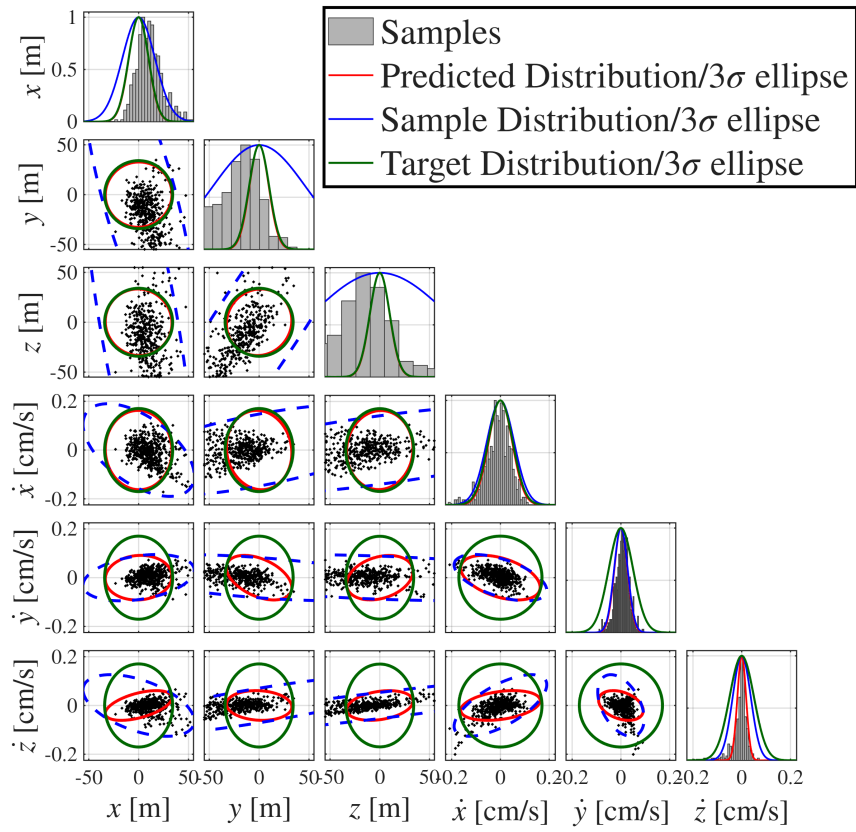
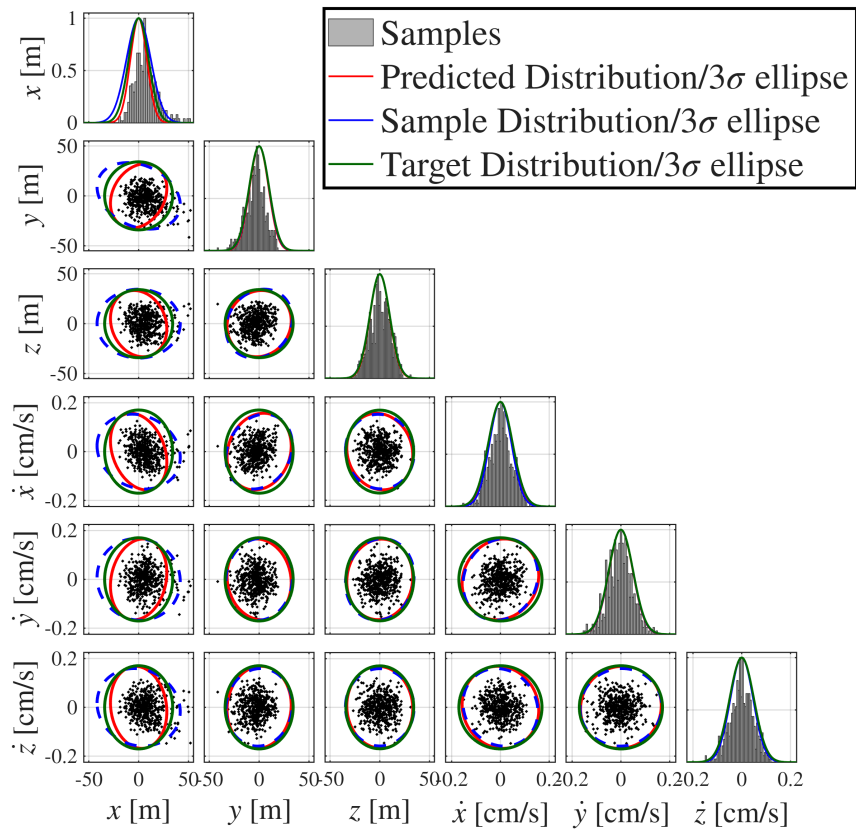
Figure 7.16 illustrates the time evolution of state deviations. In the minimum ΔV_{99} case, predicted confidence bounds are progressively violated after the flyby, reflecting the breakdown of the linear covariance assumption.

In contrast, the minimum nonlinearity solution maintains consistency between predicted and empirical deviations. A transient peak in dispersion is observed during the close approach, but uncertainties contract rapidly afterwards.

Figure 7.17 and Figure 7.18 report the control effort and total ΔV distributions.

Consistent with Case Study 1, the minimum nonlinearity formulation employs more distributed feedback corrections, leading to a marginally higher fuel consumption.

However, the additional fuel expenditure remains on the order of a few mm/s and results in substantially improved robustness with respect to both terminal covariance satisfaction and passive safety guarantees. These results further support the conclusion that explicitly controlling nonlinear propagation effects enhances reliability in proximity operations without significantly increasing fuel cost.

(a) With minimum ΔV_{99} 

(b) With minimum nonlinearity

Figure 7.15: State dispersion at the final node

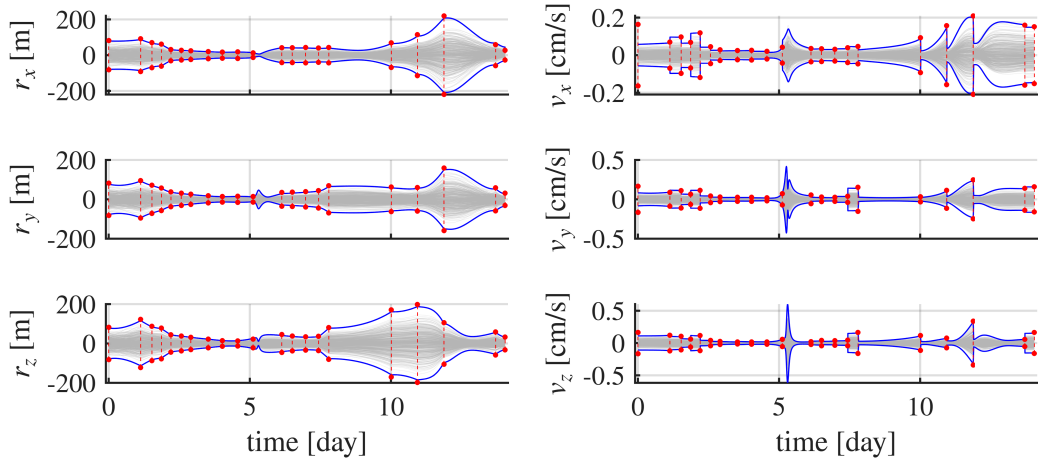
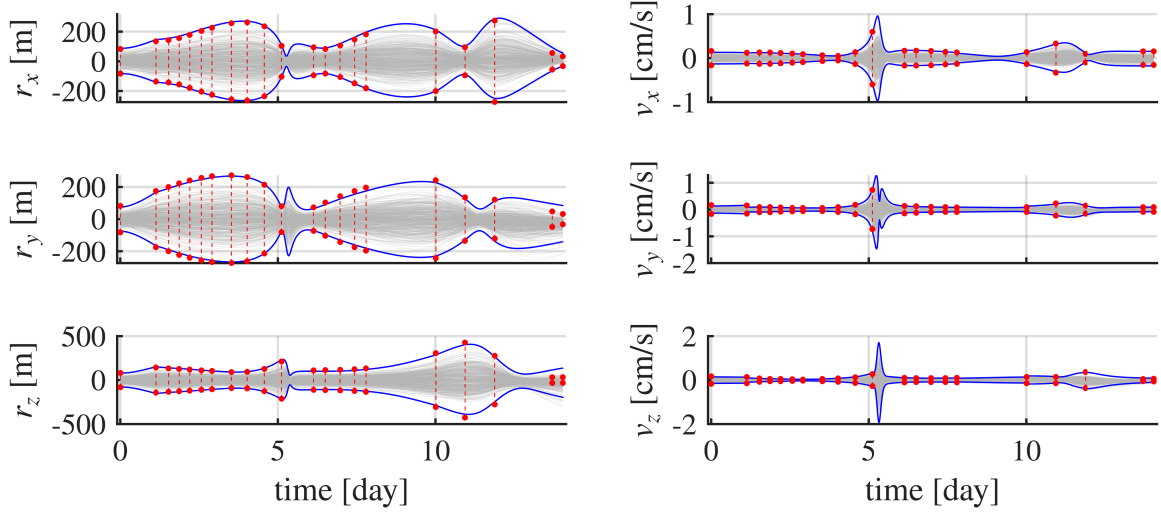


Figure 7.16: State deviations: MC samples (gray), empirical 99.9% confidence (blue), predicted deviations (red, 99.9% confidence)

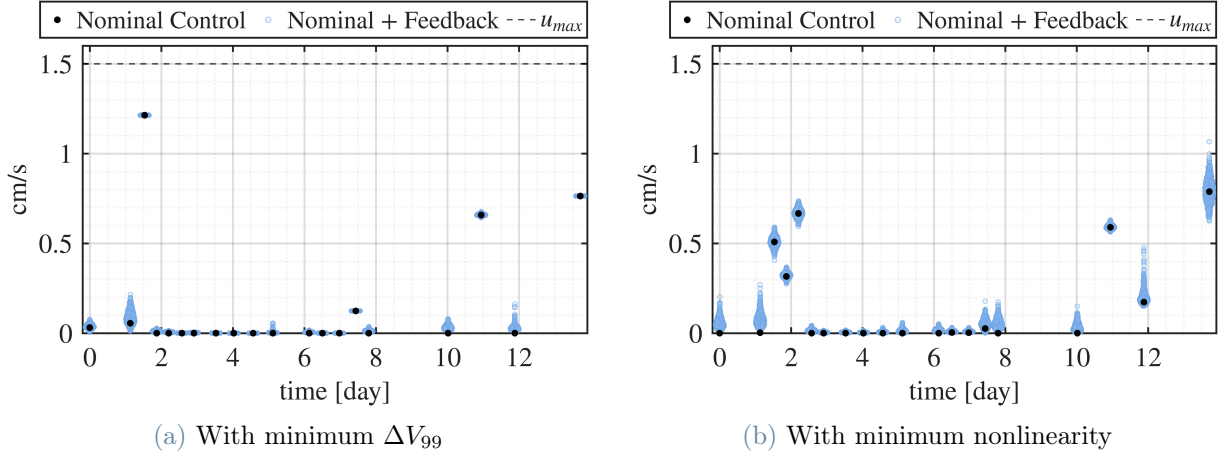


Figure 7.17: Control magnitudes at discrete maneuver epochs for Flyby transfer.

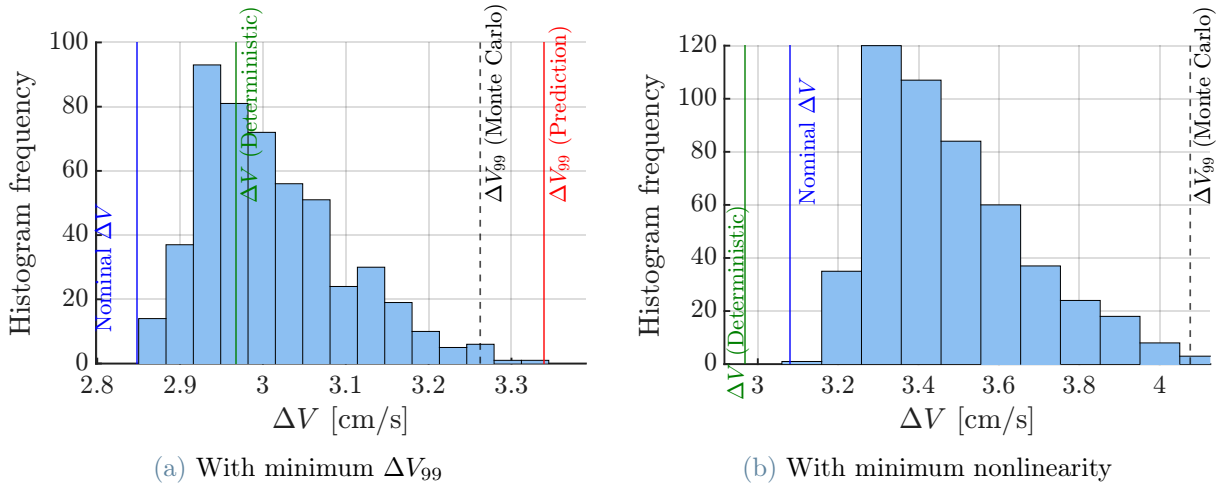


Figure 7.18: Monte Carlo ΔV distribution for Flyby transfer.

7.2.1. Passive safety guarantees against MTEs

To assess the validity of the continuous-time passive safety guarantees, a dedicated Monte Carlo simulation is performed to simulate missed-thrust events (MTEs).

For each discretized node along the trajectory, a missed-thrust event is assumed to occur immediately after the maneuver execution. From that epoch, the spacecraft is propagated ballistically, i.e., without further control authority, for the full safety horizon of 7 days. For each MTE epoch, 100 ballistic trajectories are propagated. The resulting nonlinear Monte Carlo analysis evaluates the safety guarantees of the entire stochastic transfer under possible scenarios of loss of control.

Figure 7.19 shows the ballistic Monte Carlo trajectories following MTEs for both optimization strategies. For the minimum ΔV_{99} solution, all ballistic trajectories remain clearly outside

the prescribed keep-out-zone (KOZ). The passive safety requirement is therefore fully satisfied with visible margin. This is consistent with the higher nominal flyby altitude selected by the optimizer, which introduces additional separation from the asteroid in case of loss of control. Conversely, the minimum nonlinearity solution exhibits a less conservative flyby geometry. As shown in the zoomed view in Figure 7.20, the ballistic trajectories pass significantly closer to the KOZ, and a small number of samples exhibit marginal violations of the boundary, occurring at distances only slightly below the prescribed safety radius. These violations are localized in time and remain limited in magnitude.

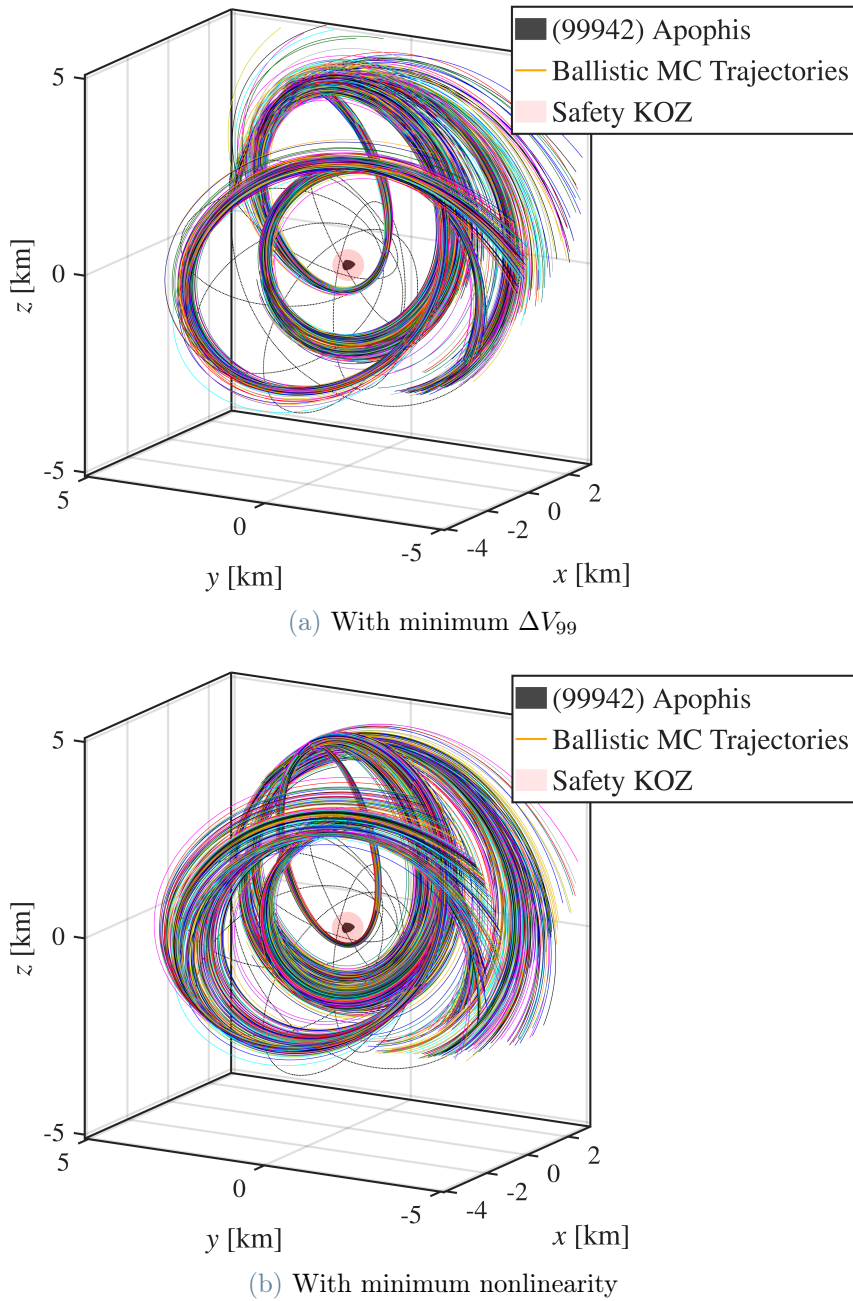


Figure 7.19: Ballistic Monte Carlo trajectories after MTEs

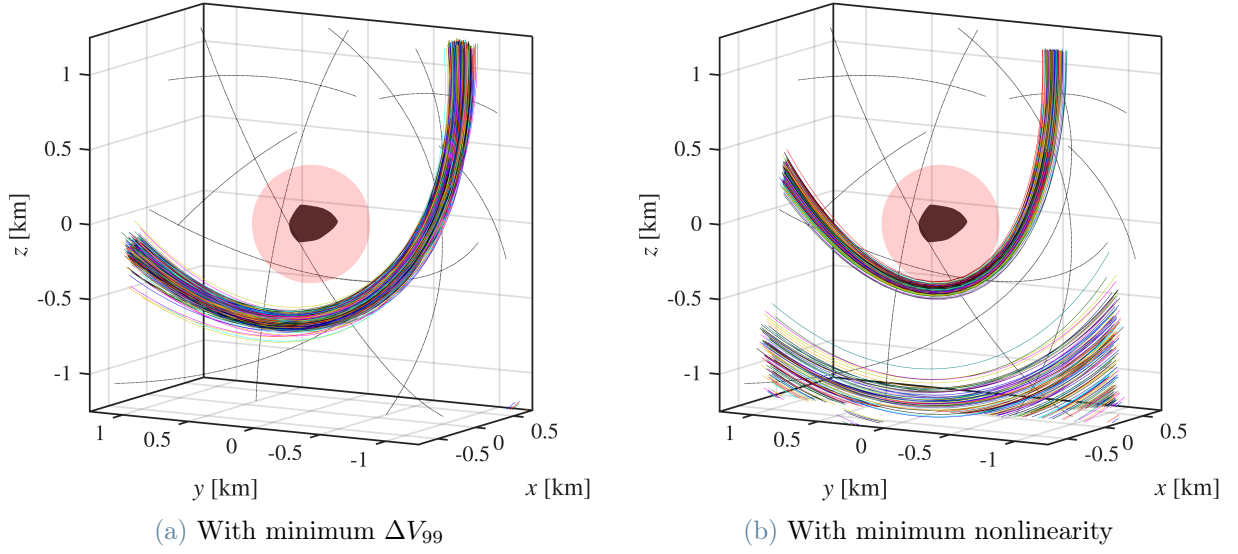


Figure 7.20: Zoomed view. Flyby ballistic passages after MTEs

It is worth noting that, although the minimum nonlinearity solution starts from smaller post-maneuver distributions, the resulting ballistic dispersion appears larger in the Monte Carlo plots. This effect is not due to higher uncertainty levels, but rather to the structure of the control profile. In the minimum ΔV_{99} solution, several intermediate nodes are already essentially ballistic, with both nominal and feedback controls close to zero. As a result, simulating a missed-thrust event (MTE) at those nodes has limited impact. Conversely, in the minimum nonlinearity solution, a larger number of nodes involve active control. When an MTE is introduced at these controlled nodes, the loss of the corresponding corrective action leads to more pronounced ballistic deviations, even if the initial covariance is smaller.

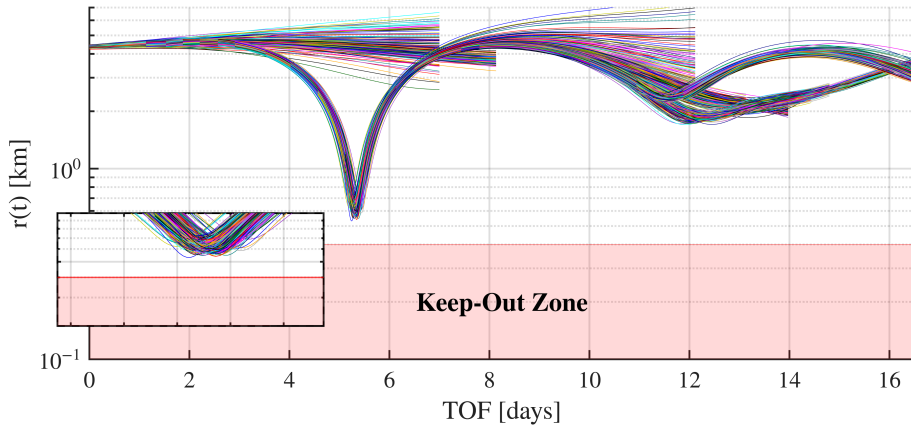
This observation suggests that robustness to missed-thrust events is influenced not only by the magnitude of the post-maneuver covariance, but also by the temporal distribution of corrective actions along the trajectory. While the minimum nonlinearity formulation improves consistency by actively reducing the uncertainty, it also relies on a larger number of TCMs, which increases the number of potential vulnerability points under MTE scenarios. An improved trade-off could be achieved by simultaneously minimizing nonlinear effects and promoting sparsity in the feedback policy. In this context, sparsity-promoting covariance steering approaches, such as the hands-off covariance steering framework proposed by Kumagai and Oguri [124], which induces feedback sparsity via iteratively reweighted $\ell_{1,p}$ regularization, could provide a promising direction for further improving passive safety while preserving nonlinear robustness.

The time evolution of the position norm $r(t)$ after MTEs is reported in Figure 7.21. For the minimum ΔV_{99} case, the minimum radial distance always remains above the KOZ threshold throughout the entire safety horizon. The radial profiles exhibit a clear separation margin, confirming strict passive safety compliance.

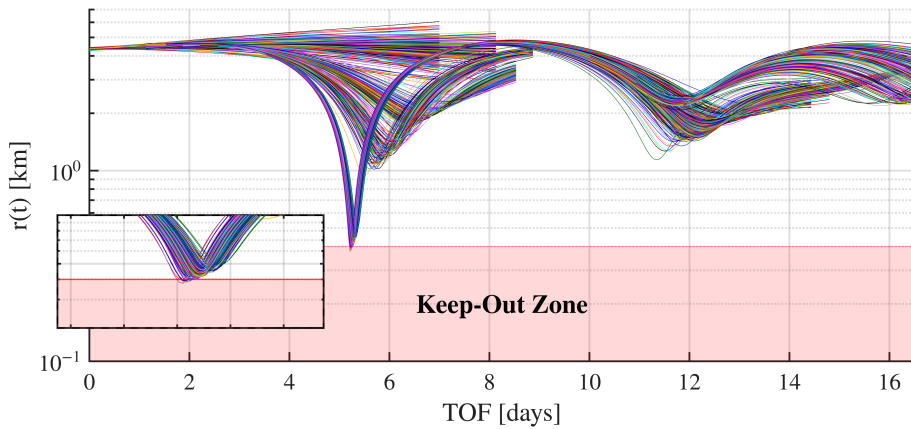
For the minimum nonlinearity case, the minimum radial distance approaches the KOZ boundary more closely, particularly during the flyby passage. The zoomed inset highlights that a small subset of trajectories briefly crosses the KOZ threshold. However, the majority of samples remain outside the safety region, and the violation amplitude is limited.

It is worth noting that, for the minimum nonlinearity formulation, the introduction of multiple path constraints may slightly degrade the controller performance in highly nonlinear regimes, as already discussed in [51]. In the present flyby scenario, the simultaneous presence of strong local nonlinearities and passive safety constraints likely contributes to the small KOZ violations observed in a few ballistic samples.

Overall, both formulations demonstrate strong passive safety characteristics. The minimum ΔV_{99} solution provides stricter geometric separation at the expense of higher conservativeness in nominal trajectory design. The minimum nonlinearity solution achieves a closer flyby and improved covariance steering performance, while maintaining passive safety compliance with only marginal violations.



(a) With minimum ΔV_{99}



(b) With minimum nonlinearity

Figure 7.21: Time evolution of position norm after MTEs

7.3. Summary of Results

This chapter evaluated two stochastic optimal control strategies for autonomous science-orbit transfers in the ANH3BP: a minimum ΔV_{99} strategy and a minimum nonlinearity approach. The two formulations were tested on a multiple-revolution transfer and a flyby transfer, which also included continuous-time passive safety guarantees in case of missed-thrust events (MTEs).

Across both case studies, the minimum ΔV_{99} formulation proved to be effective in achieving strong statistical fuel efficiency, with empirical ΔV only marginally larger than that of the deterministically optimal solution. However, in both scenarios, the predicted linear covariance evolution and the nonlinear Monte Carlo propagation showcased significant discrepancies, particularly near the final phase of the transfers. These mismatches resulted in partial violations of the target covariance, indicating that the linear and Gaussian assumptions broke down during the nonlinear trajectories and the designed linear covariance controller was not effective in achieving the terminal distribution constraint.

Conversely, the minimum nonlinearity formulation consistently improved the cohesion between predicted and empirical distributions. By explicitly penalizing nonlinear propagation effects, the resulting trajectories remained within the validity region of the linear covariance steering model. In both case studies, the empirical Monte Carlo distributions closely matched the predicted terminal covariance, and the degradation observed in the ΔV_{99} formulation was significantly mitigated. This robustness was achieved at the expenses of a marginal increase in total ΔV , on the order of a few mm/s.

In the flyby transfer scenario, where nonlinear effects are amplified, the benefits of the minimum nonlinearity objective became even more pronounced. The minimum nonlinearity solution enabled a closer nominal flyby while maintaining acceptable passive safety performance. Although minor keep-out-zone (KOZ) violations were observed in a limited number of ballistic samples after MTEs, these were localized and of small magnitude. The ΔV_{99} formulation instead enforced safety through increased geometric separation, resulting in a more conservative nominal trajectory.

Overall, the results demonstrate that explicitly controlling nonlinear dynamical exposure within a sequential covariance steering framework enhances distribution steering accuracy, terminal covariance satisfaction, and robustness in proximity operations. While the minimum ΔV_{99} optimization achieves lower fuel cost, incorporating measures of nonlinearity in the cost function provides improved reliability under realistic nonlinear and stochastic dynamics, with limited impact on fuel expenditure.

8 | Conclusions and Future Developments

This chapter presents a concise overview of the work conducted in this thesis and a summary of the main results. The overarching research questions are addressed, and the limitations of the proposed framework are discussed to outline directions for future research.

8.1. Dissertation Summary

This thesis investigated the design of guidance and control policies for safe and autonomous impulsive transfers in small-body proximity operations. In particular, the problem of transfers between Periodic and Resonant Terminator Orbits around asteroid (99942) Apophis was considered as a representative scenario, characterized by weak-gravity dynamics, strong nonlinearities, and significant operational uncertainty.

To address these challenges, the adopted framework leverages recent developments in stochastic optimal control and convex optimization techniques to explicitly account for uncertainty in navigation solutions, maneuver execution, and unmodeled accelerations. By leveraging sequential covariance steering, chance-constrained guidance policies are computed in order to guarantee constraint satisfaction with a user-defined confidence level.

The resulting control policies take the form of linear state-feedback mappings around a nominal trajectory, making them computationally efficient and suitable with onboard spacecraft autonomy.

After constructing families of Periodic and Resonant Terminator Orbits around Apophis through continuation methods and bifurcation analysis in Chapter 4, a deterministic free-phase impulsive transfer between selected science orbits was designed in Chapter 5. The deterministic problem was solved using both indirect methods with impulsification techniques and a direct adaptive mesh sequential convex programming (AMSCP) approach, enabling optimal magnitude and timing of impulsive maneuvers in a computationally efficient framework. These deterministic trajectories provided the initial guess for the subsequent stochastic covariance steering formulation.

The stochastic optimal control problem is formulated in a block-Cholesky formulation, and two

different optimization strategies are implemented and compared. The first objective minimizes the statistical fuel cost in terms of ΔV_{99} . The second objective minimizes nonlinear dynamical errors by incorporating higher-order State Transition Tensor information and constructing a convex upper bound on the resulting nonlinear propagation errors.

Both formulations are solved iteratively using the recently introduced SCvx* algorithm, which guarantees convergence to a feasible local optimum of the original nonconvex problem while retaining numerical robustness.

The two optimization methods are applied to two representative scenarios: the first consists of a multiple-revolution transfer between the departure PTO and the arrival RTO, while the second involves a close flyby of Apophis. In this second case study, particular emphasis is placed on ensuring safety even in the event of loss of control authority during the transfer. In practice, the spacecraft is constrained to remain outside a predefined keep-out-zone around the asteroid after the missed-thrust event occurs and until nominal operations are restored. This safety requirement is formulated as a continuous-time chance constraint, constraining the dispersion of the resulting ballistic trajectory to remain outside the keep-out zone over a seven-day time horizon. This formulation significantly strengthens the autonomy and fault-tolerance of the obtained guidance policies.

Finally, the validity and reliability of the obtained solutions are assessed through nonlinear Monte Carlo simulations. The results highlight a clear trade-off between fuel optimality and robustness to nonlinear effects. The minimum ΔV_{99} formulation exhibits greater flexibility and is more capable of generating solutions structurally distinct from the deterministic reference. However, in the strongly nonlinear small-body environment, the assumptions of linearity and Gaussianity underlying linear covariance steering tend to deteriorate, leading to discrepancies between predicted and actual state dispersion. Conversely, the minimum nonlinearity formulation effectively limits higher-order dynamical effects, keeping the closed-loop system within regimes where the linear and Gaussian assumptions remain valid. This results in improved consistency between covariance predictions and Monte Carlo dispersion, as well as more reliable satisfaction of terminal distribution constraints.

Overall, both strategies achieve probabilistic safety guarantees, demonstrating the capability of the adopted framework to design robust and fault-tolerant control policies that ensure mission safety even in off-nominal situations.

Answers to the Research Questions

In this section, the overarching research questions are explicitly addressed succinctly. The main research question stated:

MRQ: How can *safe* and *efficient* spacecraft proximity operations around small bodies be designed by explicitly accounting for state and operational uncertainty

in highly nonlinear and weak-gravity dynamical environments?

The results of this work show that safe and efficient spacecraft proximity operations around small bodies can be achieved by integrating deterministic trajectory design with chance-constrained covariance steering, in order to explicitly control the state dispersion under uncertainty. Furthermore, the findings highlight the necessity of employing optimization strategies that are aware of the level of dynamical nonlinearity, in order to preserve the validity of the linear and Gaussian assumptions underlying linear covariance steering.

To extensively address the MRQ, secondary research questions have been answered throughout this thesis.

RQ1: How can a stochastic optimal control framework based on linear covariance steering guarantee robust and efficient performance during proximity operations in strongly nonlinear dynamical environments, such as those encountered around asteroid (99942) Apophis?

This thesis demonstrates that linear covariance steering remains effective in strongly nonlinear environments only if higher order dynamical effects are explicitly and directly minimized. Without this mitigation, the Gaussian and linear assumptions degrade, compromising the reliability of the controller. The numerical results show that the minimum nonlinearity formulation is more effective for close-proximity operations than a strategy that focuses solely on minimizing the statistical fuel cost. Although its solutions are not optimal from a ΔV_{99} point of view, the improved robustness justifies the moderate increase in fuel expenditure.

RQ2: How can probabilistic safety requirements, including collision avoidance and keep-out-zone constraints under missed-thrust events and loss of control authority, be incorporated into trajectory optimization frameworks while avoiding overly conservative deterministic design margins?

This thesis shows that probabilistic safety requirements, in terms of keep-out-zone constraints under missed-thrust events, can be enforced rigorously by formulating continuous-time chance constraints to avoid inter-sample violations. This approach guarantees safety with high confidence levels without resorting to conservative deterministic margins, improving safe autonomy and mission efficiency.

RQ3: How does the explicit treatment of uncertainty influence the feasibility, safety, and performance of proximity trajectories when compared to conventional deterministic approaches?

The comparison between robust and deterministic solutions, when simulated in a stochastic setting under the effect of operational and dynamical uncertainty, reveals that explicitly incorporating uncertainty during the optimization stage is fundamental for designing guidance and control policies that ensure mission success. Findings show that the deterministic open-loop

solutions satisfy nominal constraints, but fail to provide reliable performance under realistic uncertainty. Conversely, the stochastic feedback policies maintain constraint satisfaction with high probability throughout the entire transfers.

8.2. Directions for Future Developments

Although the proposed framework demonstrates that a unified probabilistic formulation enables the computation of safe and efficient guidance policies for small-body close proximity operations, some theoretical, computational and modeling limitations remain. Addressing these limitations allows to provide meaningful directions for future research.

The most important limitation of the approach adopted in this thesis lies in the linearity and Gaussianity assumptions underlying linear covariance steering. Although the minimum nonlinearity formulation has been shown to improve the consistency between the linearized predictions and the actual nonlinear performance of the controller, the proposed framework does not directly address the problem of distribution steering in strongly nonlinear regimes. In nonlinear dynamical environments such as those encountered around small bodies, the Gaussian assumption eventually breaks down. The minimum nonlinearity approach does not eliminate this limitation, but rather mitigates its effects.

For these reasons, extending the framework toward non-Gaussian distribution steering techniques is a natural and necessary development. One promising approach that retains compatibility with a sequential convex programming framework is outlined in [125]. This method uses the Conjugate Unscented Transform (CUT) to quantify higher-order statistical moments of non-Gaussian distributions and aims at directly controlling the associated sigma points. Adopting this framework and extending it to account for navigational errors and process noise, which are not accommodated in the original formulation, represents an important direction for future research.

In parallel, recent work by Qi et al. [126] proposes the imposition of non-Gaussian chance constraints within a covariance steering framework, together with a less conservative reformulation of the ΔV_{99} metric. Such an approach directly addresses the limitations of Gaussian-based risk metrics and could further reduce conservatism while preserving probabilistic safety guarantees. Integrating non-Gaussian chance constraints into the proposed framework would constitute a natural extension of this thesis and could significantly enhance robustness in strongly nonlinear regimes.

A second limitation of this work concerns the computational complexity of the adopted block-Cholesky formulation. Although the time horizons associated with small-body proximity operations are generally shorter than those of interplanetary transfers, the block-Cholesky approach scales poorly as the number of discretization nodes increases. Alternative covariance steering formulations exist, such as the full covariance and square-root factorized methods introduced

in Section 1.2, that offer improved computational efficiency. The full covariance formulation combined with the classical minimum ΔV_{99} objective was implemented and tested following the procedure outlined in [42]. This new approach demonstrated convergence of the SCvx* algorithm in less than one minute, yielding very promising results from a computational standpoint. However, nonlinear Monte Carlo simulations revealed that neglecting nonlinear dynamical effects in the full covariance formulation leads to an even more pronounced inconsistencies between linearized covariance predictions and actual nonlinear dispersion than those observed with the block-Cholesky min- ΔV_{99} approach. For this reason, integrating the minimum nonlinearity objective within computationally efficient full covariance or square-root formulations represents a promising direction for future research, potentially enabling reliable on-board implementation of stochastic guidance and control policies.

Another limitation of the present study lies in the simplified modeling of the navigation process. The adopted framework assumes full state observations, while in real operational scenarios spacecraft around small bodies typically rely on optical or angles-only navigation. Incorporating realistic state estimation processes within the chance-constrained guidance framework is an important future research direction that would enable to close the GNC loop. This is a particularly necessary step toward the operational readiness of the proposed methodology.

Finally, the fidelity of the dynamical model constitutes another limitation of this thesis, that relies on the Augmented Normalized Hill Three-Body Problem (ANH3BP) to model the dynamics in close-proximity of Apophis. Although this dynamical framework is extensively used in the literature to model CPOs around small bodies, some of its simplifying assumptions could be surpassed. In particular, the assumption of a circular heliocentric orbit for the asteroid should be replaced by a fully elliptical heliocentric model. Furthermore, incorporating higher-order spherical harmonic models would allow more accurate representation of the local dynamics than that provided by the point mass assumption of the ANH3BP, thereby improving the reliability of the obtained control policies.

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A | Appendix A

This appendix describes the numerical procedure adopted to compute the discrete-time linearized system matrices resulting from the linearization and discretization of a nonlinear controlled dynamical system.

Consider the linear time-varying system obtained by linearizing the nonlinear dynamics about a reference trajectory:

$$\dot{\mathbf{x}}(t) \approx A(t)\mathbf{x}(t) + B\mathbf{u}(t) + \mathbf{c}(t). \quad (\text{A.1})$$

The discrete-time matrices A_k and $\mathbf{c}_k$ over an interval $[t_k, t_{k+1})$ are given by:

$$A_k = \Phi(t_{k+1}, t_k), \quad B_k = A_k B, \quad \mathbf{c}_k = A_k \int_{t_k}^{t_{k+1}} \Phi^{-1}(t, t_k) \mathbf{c}(t) dt. \quad (\text{A.2})$$

where $\Phi(t, t_k)$ is the state transition matrix which is integrated following its variational equations.

Defining an augmented state vector

$$\mathbf{Y}(t) \in \mathbb{R}^{n_x + n_x^2 + n_x}, \quad \text{with: } \mathbf{Y}(t_k) := \begin{bmatrix} \mathbf{x}(t_k) \\ \text{vec}(I_{n_x}) \\ \mathbf{0}_{n_x \times 1} \end{bmatrix}, \quad (\text{A.3})$$

the augmented EoM for computation of discrete-time system matrices over $[t_k, t_{k+1})$ is given by:

$$\dot{\mathbf{Y}}(t) = \begin{bmatrix} \mathbf{f}(\mathbf{x}(t), \mathbf{u}_k, t) \\ \text{vec}(A(t)\Phi(t, t_k)) \\ \Phi^{-1}(t, t_k) \mathbf{c}(t) \end{bmatrix}, \quad (\text{A.4})$$

At the end of the integration interval, the quantities are extracted from $\mathbf{Y}_{k+1} = \mathbf{Y}(t_{k+1})$ as:

$$\begin{aligned} \mathbf{x}_{k+1} &\leftarrow \mathbf{Y}_{k+1}(1 : n_x), & A_k &\leftarrow \text{reshape}(\mathbf{Y}_{k+1}(n_x + 1 : n_x + n_x^2), n_x, n_x), \\ B_k &\leftarrow A_k B, & \mathbf{c}_k &\leftarrow A_k \mathbf{Y}_{k+1}(n_x + n_x^2 + 1 : n_x + n_x^2 + n_x). \end{aligned} \quad (\text{A.5})$$

The procedure for computing the discrete-time system matrices is summarized in Algorithms A.1 and A.2.

Algorithm A.1 Discrete-time system matrix computation (impulsive control)

Require: $t_k, t_{k+1}, \mathbf{Y}_k = [\mathbf{x}_k^\top, \text{vec}(I_{n_x})^\top, \mathbf{0}_{n_x}^\top]^\top$

- 1: $\text{traj} \leftarrow \text{ode45}(@(\mathbf{t}; \mathbf{Y}, \mathbf{u}_k)\text{EoMwithDiscreteMatrix}(\mathbf{t}; \mathbf{Y}; \mathbf{u}_k); [t_k; t_{k+1}]; \mathbf{Y}_k)$
 - 2: $\mathbf{Y}_{k+1} \leftarrow \text{deval}(\text{traj}, t_{k+1})$
 - 3: $\mathbf{x}_{k+1} \leftarrow \mathbf{Y}_{k+1}(1 : n_x)$
 - 4: $A_k \leftarrow \text{reshape}(\mathbf{Y}_{k+1}(n_x + 1 : n_x + n_x^2), n_x, n_x)$
 - 5: $B_k \leftarrow A_k B$
 - 6: $\mathbf{c}_k \leftarrow A_k \mathbf{Y}_{k+1}(n_x + n_x^2 + 1 : n_x + n_x^2 + n_x)$
-

Algorithm A.2 EoMwithDiscreteMatrix

Require: $t, \mathbf{Y}, \mathbf{u}_k$

- 1: $\mathbf{x} \leftarrow \mathbf{Y}(1 : n_x)$
 - 2: $\Phi \leftarrow \text{reshape}(\mathbf{Y}(n_x + 1 : n_x + n_x^2), n_x, n_x)$
 - 3: $A \leftarrow \frac{\partial \mathbf{f}}{\partial \mathbf{x}}|_{\mathbf{x}, \mathbf{u}_k}, \mathbf{c} \leftarrow \mathbf{f}(\mathbf{x}, \mathbf{u}_k, t) - A\mathbf{x} - B\mathbf{u}_k$
 - 4: $\dot{\mathbf{Y}} \leftarrow \begin{bmatrix} \mathbf{f}(\mathbf{x}, \mathbf{u}_k, t) \\ \text{vec}(A\Phi) \\ \text{vec}(\Phi^{-1}\mathbf{c}) \end{bmatrix}$
-

B | Appendix B

This appendix derives the augmented Jacobian matrix used for the ballistic propagation of the post-maneuver distribution under the continuous-time passive safety constraint introduced in Eq. (6.58).

The linearization and discretization procedure follows exactly the methodology presented in Chapter 2. The only additional step consists in defining an augmented dynamical system including the passive safety scalar state.

Augmented Continuous-Time Dynamics

Let the nominal ballistic dynamics be governed by the ANH3BP:

$$\dot{\mathbf{x}} = \mathbf{f}_0(\mathbf{x}), \quad (\text{B.1})$$

with state $\mathbf{x} = [\mathbf{r}^\top \ \mathbf{v}^\top]^\top \in \mathbb{R}^6$.

To enforce passive safety, a scalar state Λ is introduced:

$$\Lambda(t) = \max \left(0, r_{\min} + m_\chi \|P_r^{1/2}(t)\|_2 - \|\mathbf{r}(t)\|_2 \right)^n, \quad (\text{B.2})$$

where $r_{\min}$ is the safety radius and m_χ defines the confidence margin.

Define:

$$\phi(\mathbf{r}) = r_{\min} + m_\chi \|P_r^{1/2}(t)\|_2 - \|\mathbf{r}\|_2. \quad (\text{B.3})$$

The augmented state is:

$$\mathbf{x}_{\text{aug}} = \begin{bmatrix} \mathbf{x} \\ \Lambda \end{bmatrix} \in \mathbb{R}^7. \quad (\text{B.4})$$

The augmented continuous-time dynamics are:

$$\dot{\mathbf{x}}_{\text{aug}} = \mathbf{f}_{\text{aug}}(\mathbf{x}_{\text{aug}}) = \begin{bmatrix} \mathbf{f}_0(\mathbf{x}) \\ \Lambda(\mathbf{r}) \end{bmatrix}. \quad (\text{B.5})$$

Augmented Jacobian Matrix

The augmented Jacobian is defined as:

$$A_{\text{aug}} = \frac{\partial \mathbf{f}_{\text{aug}}}{\partial \mathbf{x}_{\text{aug}}}. \quad (\text{B.6})$$

Its block structure is:

$$A_{\text{aug}} = \begin{bmatrix} \frac{\partial \mathbf{f}_0}{\partial \mathbf{x}} & \mathbf{0}_{6 \times 1} \\ \frac{\partial \Lambda}{\partial \mathbf{x}} & 0 \end{bmatrix}. \quad (\text{B.7})$$

The upper-left block is simply the standard ANH3BP Jacobian:

$$A = \frac{\partial \mathbf{f}_0}{\partial \mathbf{x}} \in \mathbb{R}^{6 \times 6}. \quad (\text{B.8})$$

The last column is zero because $\mathbf{f}_0$ does not depend on Λ .

The last row contains:

$$\frac{\partial \Lambda}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial \Lambda}{\partial \mathbf{r}} & \frac{\partial \Lambda}{\partial \mathbf{v}} \end{bmatrix}. \quad (\text{B.9})$$

Since Λ depends only on $\mathbf{r}$:

$$\frac{\partial \Lambda}{\partial \mathbf{v}} = \mathbf{0}_{1 \times 3}. \quad (\text{B.10})$$

Derivative of Λ with respect to $\mathbf{r}$

If $\phi(\mathbf{r}) \leq 0$, then:

$$\Lambda = 0, \quad \frac{\partial \Lambda}{\partial \mathbf{r}} = \mathbf{0}. \quad (\text{B.11})$$

If $\phi(\mathbf{r}) > 0$, then:

$$\Lambda = \phi(\mathbf{r})^n. \quad (\text{B.12})$$

Applying the chain rule:

$$\frac{\partial \Lambda}{\partial \mathbf{r}} = n\phi(\mathbf{r})^{n-1} \frac{\partial \phi}{\partial \mathbf{r}}. \quad (\text{B.13})$$

Since the only term in ϕ depending on $\mathbf{r}$ is $-\|\mathbf{r}\|_2$:

$$\frac{\partial \phi}{\partial \mathbf{r}} = -\frac{\mathbf{r}^\top}{\|\mathbf{r}\|_2}. \quad (\text{B.14})$$

Therefore,

$$\frac{\partial \Lambda}{\partial \mathbf{r}} = -n\phi(\mathbf{r})^{n-1} \frac{\mathbf{r}^\top}{\|\mathbf{r}\|_2}, \quad \text{if } \phi(\mathbf{r}) > 0. \quad (\text{B.15})$$

Discrete-Time Linearization

Once A_{aug} is defined, the discrete-time matrices $A_j^{(\text{bal})}$ and $\mathbf{c}_j^{(\text{bal})}$ are obtained through the same integration of the linearized system described in Chapter 2, by propagating the augmented state transition matrix and affine term over each ballistic subinterval.

No additional modifications to the discretization procedure are required beyond the definition of the augmented Jacobian presented above.

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List of Symbols

Variable	Description	SI unit
$\mathbf{x}$	State vector $[\mathbf{r}^\top \mathbf{v}^\top]^\top$	-
$\mathbf{r}$	Spacecraft position vector	m
$\mathbf{v}$	Spacecraft velocity vector	m/s
$\mathbf{u}$	Control input (impulsive maneuver)	m/s
$\mathbf{x}_{\text{aug}}$	Augmented state vector	-
μ_{sb}	Gravitational parameter of the small body	m^3/s^2
μ_{sun}	Gravitational parameter of the Sun	m^3/s^2
β	Non-dimensional SRP parameter	-
LU	Characteristic length unit	m
TU	Characteristic time unit	s
J_c	Jacobi constant	-
P	State covariance matrix	-
$\Phi(t, t_0)$	State Transition Matrix (STM)	-
$\Phi(t, t_0)^{(m)}$	m -th order State Transition Tensor (STT)	-
$\mathbf{M}$	Monodromy matrix	-
Λ	Continuous-time passive safety function	-
r_{safe}	Safety radius (KOZ boundary)	m
ΔV_{99}	99% statistical fuel cost	m/s
N	Number of discretization nodes	-
N_{bal}	Number of ballistic sub-intervals	-
Δt_k	Time interval between nodes	s

