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Optimal management of distribution networks in the presence of renewable and distributed resources flexibility and uncertainty

TESI DI LAUREA MAGISTRALE IN
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Abstract

The increasing penetration of renewable energy sources and distributed energy resources (DERs) is profoundly transforming the operation of power distribution networks. While they represent a key driver towards decarbonization, they also introduce new challenges related to forecast uncertainty and the need to ensure grid stability and security. In this context, flexibility emerges as a crucial resource, capable of providing ancillary services at both transmission (TSO) and distribution (DSO) levels, while requiring a rethinking of market structures and coordination schemes among system operators.

This thesis analyzes the European and Italian regulatory framework for flexibility markets, with particular attention to pilot projects and TSO–DSO coordination mechanisms. Subsequently, a mathematical model of a Local Flexibility Market (LFM) is developed, explicitly incorporating the uncertainty associated with renewable generation and flexible load profiles. To address this, Robust Optimization (RO) is introduced as the proposed methodology, ensuring reliable solutions even under worst-case variability scenarios.

The model has been validated on reference distribution grids, highlighting how the explicit consideration of uncertainty affects flexibility activation. The results confirm that the robust approach, although more conservative than deterministic optimization, enhances operational reliability and enables effective management of congestion and voltage limits.

In conclusion, this thesis contributes to the design of local flexibility markets that are more resilient and secure, thus supporting the increasing role of DERs in the ongoing energy transition.

Keywords: flexibility, distribution networks, distributed energy resources, robust optimization, local flexibility markets, uncertainty.

Abstract in italiano

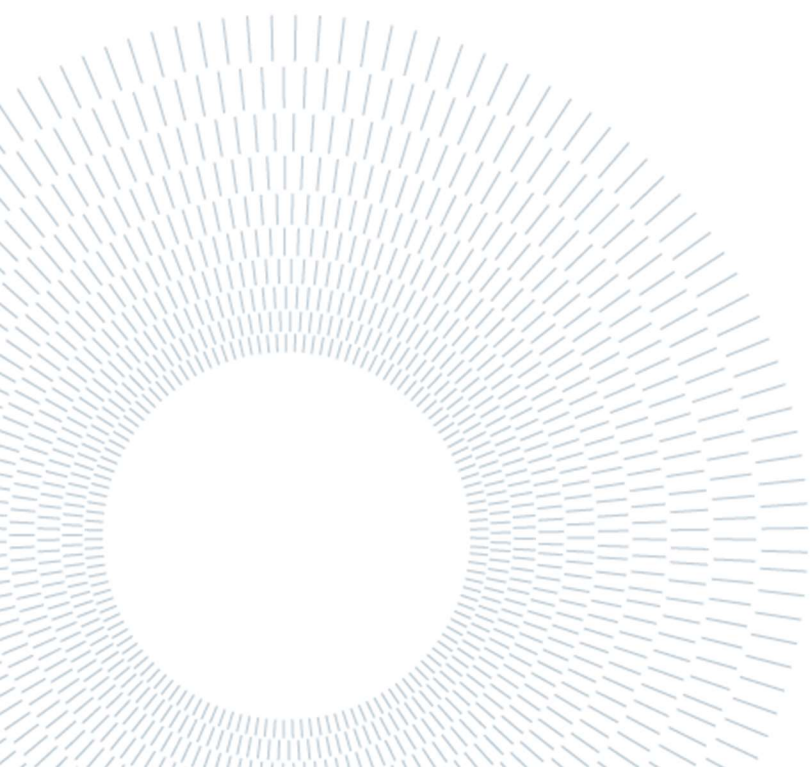
La crescente penetrazione delle fonti rinnovabili e delle risorse energetiche distribuite (Distributed Energy Resources, DER) sta trasformando profondamente il funzionamento delle reti elettriche. Se da un lato esse rappresentano un'opportunità per accelerare il processo di decarbonizzazione, dall'altro introducono nuove sfide legate all'incertezza delle previsioni e alla necessità di garantire stabilità e sicurezza operativa. In questo contesto, la flessibilità emerge come risorsa chiave, capace di fornire servizi ancillari sia a livello di trasmissione (TSO) sia di distribuzione (DSO), richiedendo al tempo stesso un ripensamento dei modelli di mercato e delle modalità di coordinamento tra gli operatori.

La presente tesi analizza il quadro regolatorio europeo e italiano sui mercati della flessibilità, ponendo particolare attenzione ai progetti sperimentali e ai meccanismi di coordinamento TSO-DSO. Successivamente, viene sviluppato un modello matematico di Local Flexibility Market (LFM) che integra l'incertezza derivante dal comportamento delle fonti rinnovabili e dei carichi flessibili. A tal fine, si introduce la Robust Optimization (RO) come approccio risolutivo, in grado di garantire soluzioni affidabili anche in scenari di massima variabilità.

Il modello proposto è stato validato su reti di distribuzione di riferimento, mostrando come l'inclusione dei termini di incertezza influenzi l'attivazione della flessibilità e il rispetto dei vincoli di rete. I risultati confermano che l'approccio robusto, pur comportando soluzioni più conservative rispetto all'ottimizzazione deterministica, permette di incrementare l'affidabilità operativa e di gestire efficacemente congestioni e limiti di tensione.

In conclusione, la tesi fornisce un contributo alla progettazione di mercati locali della flessibilità più resilienti e sicuri, supportando il ruolo crescente dei DER nella transizione energetica.

Parole chiave: flessibilità, reti di distribuzione, risorse energetiche distribuite, robust optimization, mercati locali della flessibilità, incertezza.



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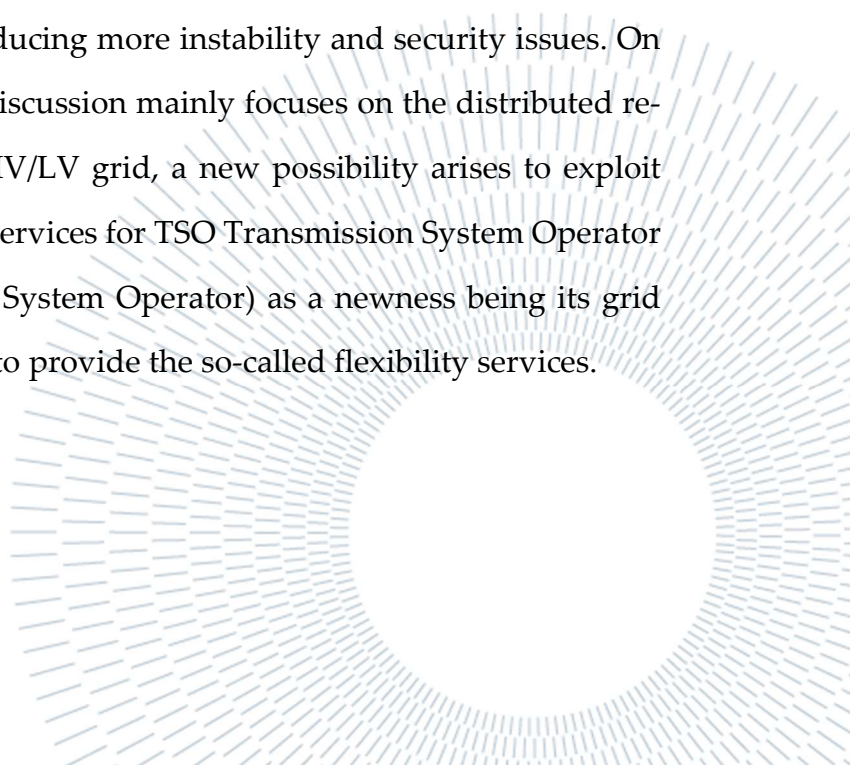
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1. CHAPTER 1

1.1 Context

Climate change represents one of the most critical challenges of our time, threatening the stability of natural ecosystems, human health, and global economies. In the recent year this issue is gaining more importance and countless initiatives that strive to reduce its impact are introduced in all the sectors. In the electricity sector the need to integrate more renewable energy sources into the power grid has become increasingly urgent to support global decarbonization efforts. The objective of having a grid sustained by 100% of RES is unreached up to now and a strong effort is required to achieve this objective but the starting point is delighted and this direction starts to be explored.

This challenge introduces a new paradigm for the power grid: the grid has to host a lot of renewable energy sources that are subject to a strong uncertainty regarding the forecasting of the power output, introducing more instability and security issues. On the other hand, considering that this discussion mainly focuses on the distributed resources that are connected into the MV/LV grid, a new possibility arises to exploit these resources and provide different services for TSO Transmission System Operator (TSO) and also for DSO (Distribution System Operator) as a newness being its grid directly involved: the DG can be used to provide the so-called flexibility services.



The importance of the topic is strongly impacting also on the preexistent markets used to trade the electricity; the aim of this work is to describe the different possibilities to exploit the potentiality of the RES focusing on what's intended as flexibility, which is the best way of providing these services analyzing the existing ones and possibly exploiting new possibilities and what's the new role of TSO and DSO in terms of coordination to use equally the resources.

1.1.1 Climate issues

The climate change is one of the big challenges of our time and it's strictly related by the effect of the human activity on the Earth ecosystem.

The graph published by NOAA/Climate.gov clearly illustrates the correlation between rising global temperatures and atmospheric carbon dioxide (CO₂) concentrations from 1880 to 2021.

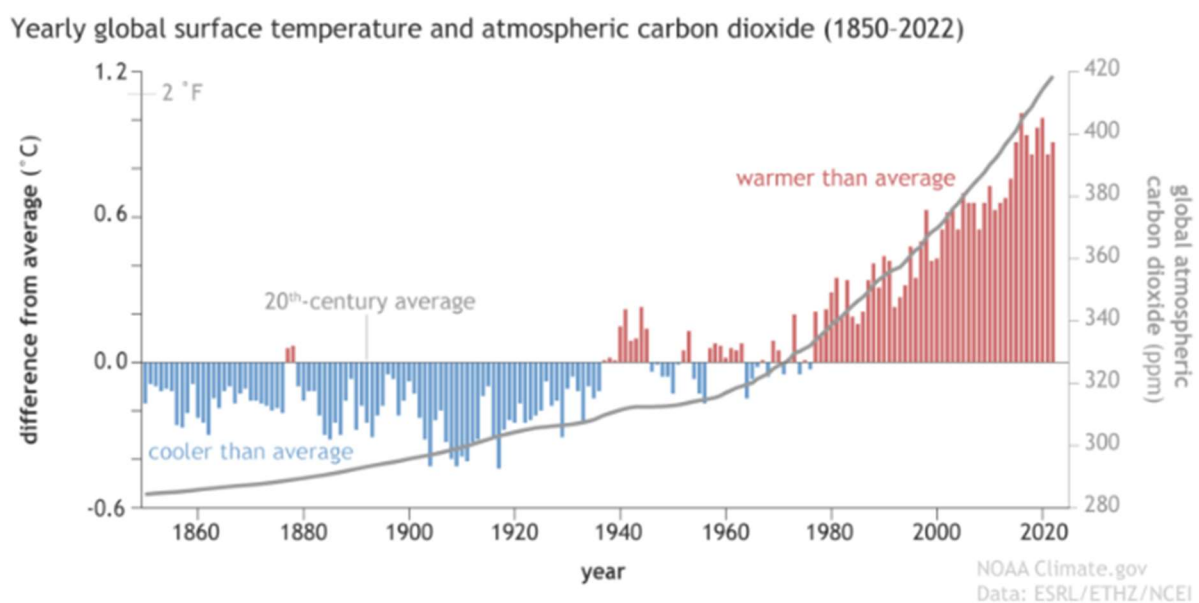


Figure 1-1: Global temperatures related to CO₂ emissions [1]

The Figure 1-1 highlights how, especially since the 1950s, global temperatures have risen in parallel with CO₂ concentrations, emphasizing a strong correlation between human activity, primarily fossil fuel emissions, and observed climate change.

Depicting instead the global GHG emission by sector in the years 1990-2020 in the Figure 1-2 , it's evident how the electricity sector gives a not negligible contribute all over the years even if the trend is decreasing in the last years.

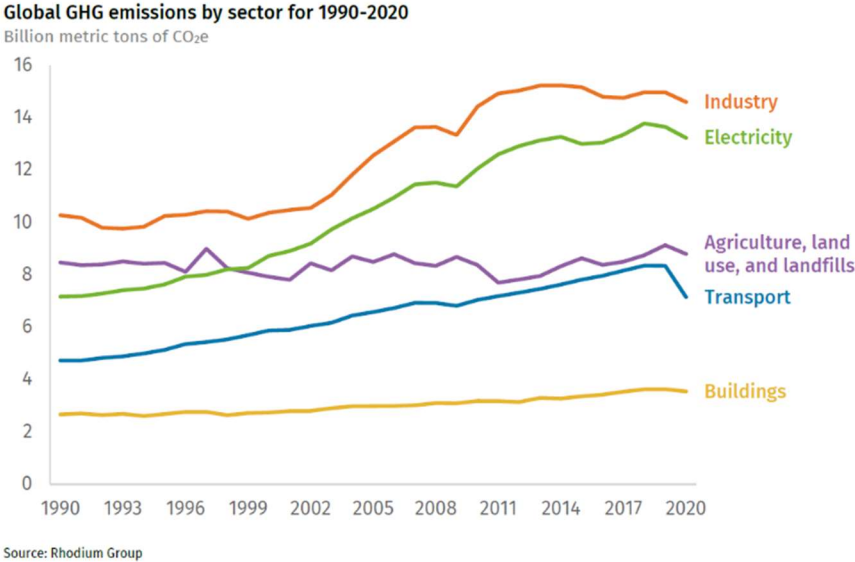


Figure 1-2: Global GHG emissions by sector for 1990-2020 [2]

Considering that the importance of the problem involves directly the electricity sectors, also the EU regulatory framework has changed to embrace it.

1.1.2 European regulatory framework

The European Union has put in place a lot of directives to push in the direction of achieving the decarbonization objectives. This strong regulation favors the introduction of RES and in particular of DER (distributed energy resources).

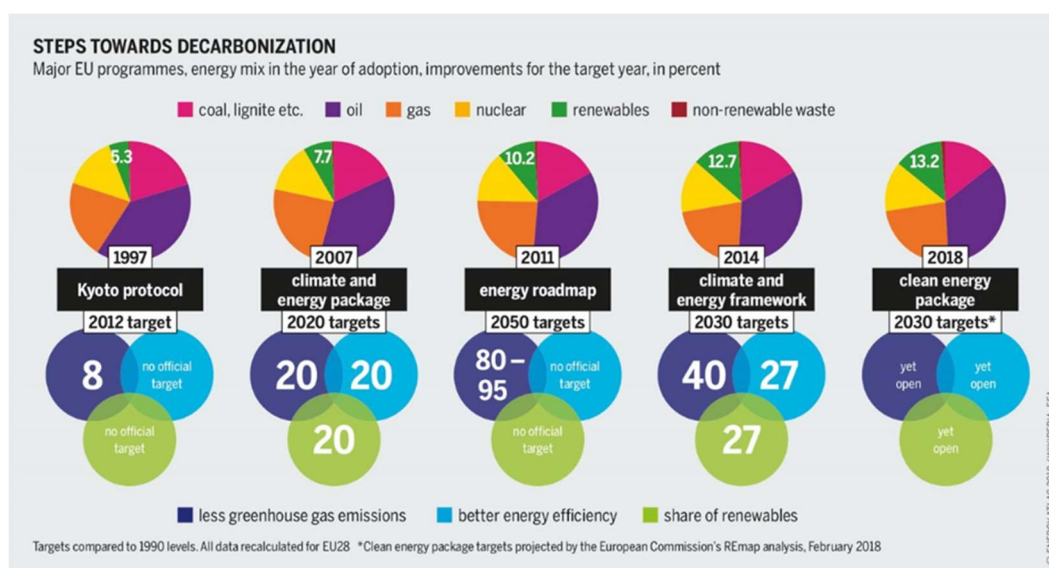


Figure 1-3: Regulatory steps toward decarbonization in EU [3]

In this Figure 1-3 the steps towards decarbonization are highlighted. In the 1997 Kyoto Protocol, the EU committed to an 8 percent cut in greenhouse gas emissions by 2012 compared to 1990 levels. In the same year, the Amsterdam Treaty included sustainable development as a cross-cutting objective. An energy and climate package agreed in 2007 set binding sustainable energy targets for 2020. These are a 20 percent cut in greenhouse gas emissions, a 20 percent share of renewables in final energy consumption, and an indicative target of 20 percent improvement in energy efficiency

In 2014, the EU adopted its 2030 energy and climate framework that called for a greenhouse gas reduction goal of at least 40 percent, at least a 27 percent share for renewables in the energy sector, and at least a 27 percent improvement in energy efficiency.

These targets form a basis for the Clean Energy Package which lays out the legal groundwork for future energy policy and are represented in the Figure 1-4.

The third energy package of 2007 provides quite good results in particular in terms of renewable energy.

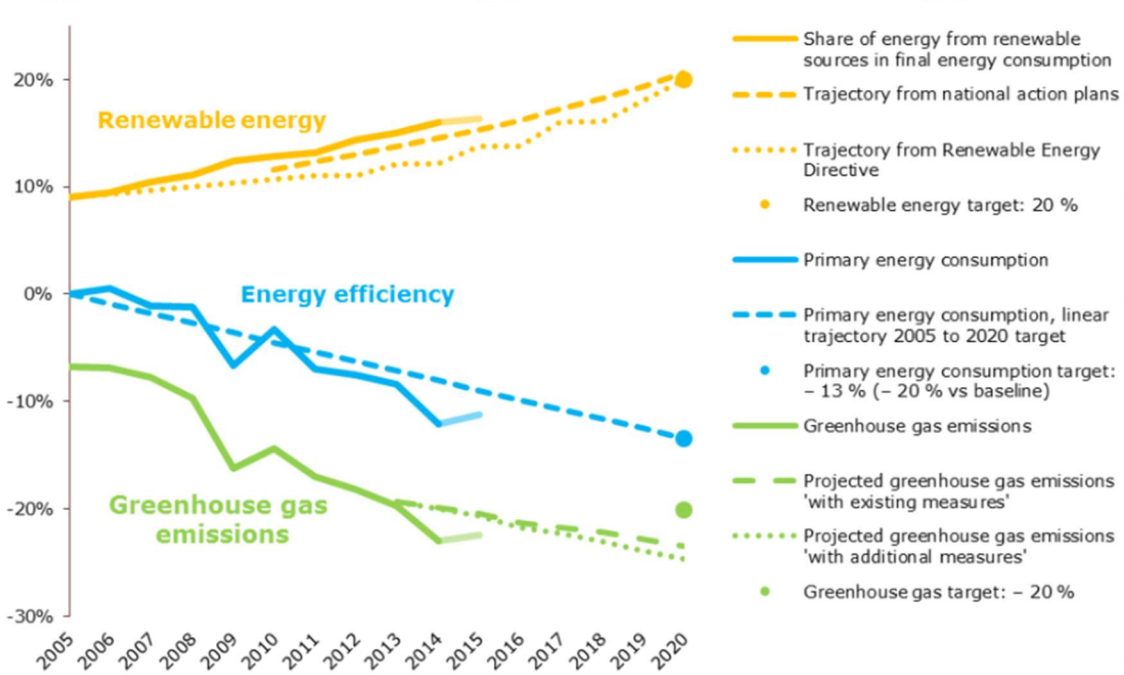


Figure 1-4: EU progress towards 2020 climate and energy targets (20-20-20 package) [4]

The clean energy package (fourth energy package) is the package that promotes the change in the regulatory framework of the member states in terms of energy legislation in order to integrate the renewable energy sources to reach the decarbonization objective in 2030. This package determines an increase in the process of installing more RES to be integrated in the market activities. This package was introduced because of the good results of the third energy package reported above (2007).

1.1.3 Clean energy package for all Europeans

The Clean Energy Package for All Europeans is a comprehensive set of directives and regulations developed by the European Commission with the aim of updating the Union's political and regulatory framework to align with the energy transition. This initiative is essential for meeting the commitments undertaken under the Paris Agreement (COP21). The legislative package, consisting of eight proposals, was published for consultation in 2016, officially adopted by the European Commission in 2019, and subsequently transposed into national law by Member States before 2021. According to the ACER [5] it addresses five key dimensions:

1. Energy security
2. Internal energy market
3. Energy efficiency
4. Decarbonisation of the economy
5. Fostering research, innovation and competitiveness

In particular it addresses five points [6]:

1. Energy efficiency: new target is set at 32.5 % lower for the primary energy consumption with respect to 2007 PRIMES scenarios, in addition to the target of cut of 40% in GHG (greenhouse gases emission). Furthermore also an electricity interconnection is required at 15%.
2. Renewables: 32% of total energy consumption has to be supplied by the renewable energy. This point goes into the direction of accelerating the decarbonization process. These two points are highlighted in Figure 1-5.

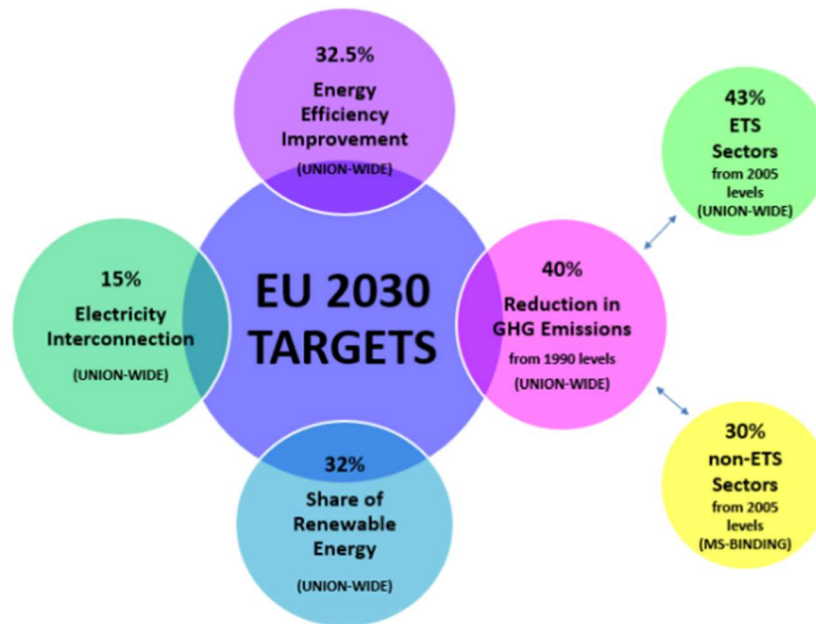


Figure 1-5: Clean Energy Package (CEP) Targets [7]

3. A new energy rulebook: the target are fixed at european level but each Member State has to contribute for achieving of the goal in the way that more fits the exigencies of the State putting in place a National Energy and Climate Plan (NECP) for 2021-2030.
4. More right for the consumers: the consumer will have more transparency in household bills, greater choice and more flexibility to change supplier and also the possibility to produce its own energy and sell it onto the grid in enhanced by this new regulation.
5. A smarter and more efficient electricity market: new laws that will increase security of supply by helping integrate renewables into the grid and manage risks, improving cross-border cooperation.

The general framework introduced by the Clean Energy Package is described by the five pillars described above. The legislative acts of which the CEP is composed are the ones described by Figure 1-6.



Figure 1-6: CEP packages [7]

From these legislative acts, it can be useful to highlight some aspects related to the integration of the RES in the market and in particular how the market should change in order to accommodate these new resources.

1.1.3.1 Recast Electricity Directive: Directive (EU) 2019/944

The legislative act Recast Electricity Directive: Directive (EU) 2019/944 strives to highlight the importance of a new market framework. To give an example, in the following extract

‘The Union would most effectively meet its renewable energy targets through the creation of a market framework that rewards flexibility and innovation. A well-functioning electricity market design is the key factor enabling the uptake of renewable energy’ [8].

Also the importance of the participation of users, that more than ever can be both consumers and producers, is a key point in this directive:

‘Consumers have an essential role to play in achieving the flexibility necessary to adapt the electricity system to variable and distributed renewable electricity generation.

Technological progress in grid management and the generation of renewable electricity has unlocked many opportunities for consumers. Healthy competition in retail markets is essential to ensuring the market-driven deployment of innovative new services that address consumers' changing needs and abilities, while increasing system flexibility. However, the lack of real-time or near real-time information provided to consumers about their energy consumption has prevented them from being active participants in the energy market and the energy transition. By empowering consumers and providing them with the tools to participate more in the energy market, including participating in new ways, it is intended that citizens in the Union benefit from the internal market for electricity and that the Union's renewable energy targets are attained.

It's clear that new requirements arise from this new framework and the users acquire a very central role; they need to be equipped with new tools to participate in the energy market and more data are needed to be collected in order to let them participate actively in the market itself; this last attempts clearly refers to the reinforcement of the smart metering presence in the grid.

Furthermore the role of DSO and TSO are described deeply in different articles of the law.

Role of the Transmission System Operator (TSO)

As per Article 40 [8], TSOs are responsible for:

- **System Operation:** Ensuring a secure, reliable, and efficient electricity transmission system over the long term.
- **System Development:** Maintaining and developing the grid under economic and environmental considerations.

- **Market Facilitation:** Guaranteeing non-discriminatory access, providing necessary information to system users, managing cross-border exchanges, and facilitating market integration.
- **System Reliability and Security:** Procuring ancillary services (including demand response and storage), managing electricity flows, and ensuring system adequacy and digitalization.
- **Coordination:** Cooperating with neighboring TSOs, DSOs, and Regional Coordination Centers.
- **Information Sharing:** Providing other system operators with the data needed for secure and efficient operations

Role of the Distribution System Operator (DSO)

According to Article 31 [8], DSOs are responsible for:

- **Local Network Management:** Operating, maintaining, and developing the distribution grid in a secure, reliable, efficient, and environmentally conscious way.
- **Non-Discrimination:** Ensuring equal treatment of all system users.
- **Transparency:** Providing users with the information needed for efficient access and usage of the network.
- **Renewables Integration:** Giving priority to renewable or high-efficiency cogeneration in dispatching, where required.
- **Neutral Market Facilitation:** Procuring energy to cover losses using transparent and market-based methods, and enabling participation of all market actors in service provision.
- **Coordination:** Collaborating with TSOs and market participants for system planning and flexibility service procurement.

- **Data Management:** Acting neutrally in handling metering and consumer data, ensuring open access and avoiding competitive distortions

As shown, the coordination between TSOs and DSOs is a key principle of Directive (EU) 2019/944. Their collaboration ensures a more efficient, flexible, and reliable energy system capable of integrating a growing share of distributed and renewable energy resources. One of the main areas of coordination involves the joint procurement of flexibility services, where DSOs and TSOs must work together to define common technical specifications and standardized market products, enabling the participation of all market actors on fair and non-discriminatory terms. In addition, balancing services provided by distributed resources require agreement between DSOs and TSOs to ensure safe and efficient system operation. Another key element is the coordinated network planning process, in which DSOs are required to consult TSOs and relevant stakeholders when drafting their development plans, particularly regarding future infrastructure investments and flexibility needs. Finally, both operators must engage in continuous operational information exchange, sharing real-time and planning data to enhance system reliability, interoperability, and the integration of decentralized resources into wholesale and balancing markets. This multi-level coordination framework is essential for achieving the Union's energy transition and decarbonization goals, thus it strongly enters into the discussion of reformulating the market in order to accommodate the RES.

Overall this regulation highlights several key aspects: the market must be correctly adjusted to reward flexible services, such as energy storage and demand response, which are essential for managing the variability of renewable sources like wind and solar. This could include incentives to ensure that all the plurality of actors such as aggregators, active consumers and renewable energy communities can actively

participate in the energy system. This process requires a strong coordination between TSO and DSO that are strongly involved in such a big challenge.

This regulation can be seen complementary with respect to the Regulation (EU) 2019/943 [9] in which the operative principles of the market are defined and the role of TSO and DSO is clarified in this new context. It is also in synergy with the Renewable Energy Directive (EU 2018/2001) about the target on the RES.

1.1.3.2 Renewable Energy Directive: Directive (EU) 2018/2001

The Renewable Energy Directive (RED II), formally Directive (EU) 2018/2001, is the definitive legislative act within the Clean Energy Package that focuses specifically on renewable energy sources [10].

In essence, RED II provides a comprehensive framework for the EU to transition towards a more sustainable energy system by promoting the widespread adoption of renewable energy sources. RED II has been revised and updated as part of the "Fit for 55" package, with the aim of achieving climate neutrality in the EU by 2050.

The Renewable Energy Directive (EU) 2018/2001 (RED II) and the Electricity Market Directive (EU) 2019/944, as said, operate in close synergy to support the European Union's energy transition and decarbonization objectives.

While RED II establishes the strategic framework for scaling up the share of renewables, setting binding targets such as a minimum 32% share of renewable energy in gross final consumption by 2030, it relies on a reformed electricity market capable of accommodating an increasing penetration of variable renewable energy sources. This role is fulfilled by Directive 2019/944, which redesigns the market structure to promote flexibility, efficiency, and real-time price signals.

Both directives converge notably on the empowerment of consumers and communities: RED II introduces the legal recognition of renewable energy communities and self-consumers (Articles 2, 21, 22), whereas Directive 2019/944 provides these actors with the operational means to participate in electricity markets, including access to flexibility and aggregation mechanisms. Furthermore, RED II encourages market-based support schemes and system flexibility, objectives which are operationalized through the electricity market reforms laid out in Directive 2019/944, such as the removal of price regulation, the facilitation of demand response, and the coordination between TSOs and DSOs.

Together, these two legislative instruments establish a coherent framework where strategic ambition (RED II) is matched with regulatory and technical implementation (2019/944), ensuring not only the deployment of renewables, but their effective integration into a modern, consumer-driven energy system.

To cite an extract from the text of the law:

“Member States shall ensure that renewable energy communities are entitled to:

(a) produce, consume, store and sell renewable energy, including through renewables power purchase agreements;

(b) share, within the renewable energy community, renewable energy that is produced by the production units owned by that renewable energy community [...]

(c) access all suitable energy markets both directly or through aggregation in a non-discriminatory manner.”

(Article 22, RED II)

This paragraph demonstrates how RED II legally empowers renewable energy communities to engage in generation, consumption, and market participation—rights that

are operationally enabled by the market reforms and flexibility mechanisms introduced in Directive (EU) 2019/944.

1.1.3.3 Electricity Regulation: Regulation (EU) 2019/943

This Regulation is intended to be complementary to the Recast Directive, as it's clearly expressed in the text of the Directive (EU) 2018/2001. In particular the Regulation implements operatively the objectives posed and declared in the Directive.

'This Directive should be read together with Regulation (EU) 2019/943, which lays down the key principles of the new market design for electricity which will enable better rewards for flexibility, provide adequate price signals, and ensure the development of functioning integrated short-term markets. Regulation (EU) 2019/943 also sets out new rules in various areas, including on capacity mechanisms and cooperation between transmission system operators.

In this context the difference between a directive and a regulation is crucial: the directive sets binding objectives that each Member State must transpose into its national legal system within a specified deadline but gives Member States freedom in choosing the means to achieve those objectives (e.g., laws, decrees, national regulations). In this case the *Renewable Energy Directive (RED II)* establishes that by 2030 32% of energy must come from renewable sources and each Member State is free to choose how to achieve this target (through incentives, obligations, etc.). Instead the regulation is applied directly in each Member State without an explicit reception of the national law: it has an immediate effect. It has to be respected and applied integrally by each State, thus uniforming the law all over the EU. In the specific case the *Regulation (EU) 2019/943* directly reforms the functioning of electricity markets across all EU Member

States. Operators and authorities must comply with it without the need for additional national legislation.

This regulation strongly turns the need of reforming the markets to host the RES in an obligation, trying in this way to reach efficiently and quickly the objectives poned in the directive. In this sense it's clear why the two legislative acts are in a strong synergy.

Regulation (EU) 2019/943 contains also several explicit provisions that define and support the coordination between Transmission System Operators (TSOs) and Distribution System Operators (DSOs).

“Distribution system operators and transmission system operators shall cooperate with each other in planning and operating their networks. In particular, they shall exchange all necessary information and data regarding the performance of generation assets and demand side response, the daily operation of their networks and the long-term planning of network investments, with the view to ensure the cost-efficient, secure and reliable development and operation of their networks.”

Regulation (EU) 2019/943 explicitly recognizes the central role of coordination between Transmission System Operators (TSOs) and Distribution System Operators (DSOs) in ensuring a secure, efficient, and flexible operation of the European electricity system. In particular, Article 57 establishes that TSOs and DSOs must cooperate in the planning and daily operation of their networks, sharing all necessary information regarding the performance of generation assets, demand-side response, long-term network planning, and real-time operations. This cooperation aims to guarantee the cost-efficient, reliable, and secure development of grid infrastructure. Furthermore, the Regulation introduces additional mechanisms to reinforce this collaboration, such as the joint assessment of flexibility needs, coordinated by ENTSO-E and the EU DSO Entity, in alignment with the Union's decarbonization objectives. The Regulation also promotes transparency regarding grid connection capacity data and foresees the joint involvement of TSOs and DSOs in the development of network codes. These legal

provisions confirm the necessity of an integrated and coordinated approach between transmission and distribution, especially in a context increasingly marked by the rise of decentralized energy resources and the growing demand for flexibility in electricity markets.

Regulation (EU) 2019/943 devotes specific articles to the organization and principles of the day-ahead, intraday, and balancing markets, which together form the core of the EU's short-term electricity trading architecture. According to Article 7, the day-ahead and intraday markets must be managed jointly by Transmission System Operators (TSOs) and Nominated Electricity Market Operators (NEMOs), ensuring maximum efficiency and cross-zonal liquidity across the Union. These markets must be non-discriminatory, transparent, and organized to allow market participants to manage imbalances and trade electricity as close to real time as possible.

Article 6 outlines that balancing markets must provide non-discriminatory access to all technologies, including demand response, energy storage, and renewable generation. Balancing energy must be settled based on marginal pricing (pay-as-cleared), and the procurement of balancing capacity must be market-based and inclusive of aggregators. These provisions ensure that system imbalances are managed cost-effectively while accommodating an increasing share of variable renewable energy sources, thus maintaining system stability and security of supply.

Analyzing three out of eight legislative acts of the clean energy package, it's evident how the penetration of the RES in the grid has to be strongly regulated on one hand shaping correctly the market and its products and on the other hand encouraging the participation of the users. The DSO TSO coordination reveals to be crucial and it will be source of discussion in the following steps.

1.2 Definition of flexibility

In these legislative acts the importance of having flexibility resources is highlighted; however it's still missing an uniform definition of how flexibility is intended. In this paragraph the article 'characterizing flexibility in power market and system' (2022) is used as key reference [11].

In this article the flexibility intended as a service is described using four dimensions:

1. Spatial dimension: where it's needed (transmission or distribution grid)
2. Time dimension :
 - Duration time: for how much time it's needed;
 - Activation time: how fast the flexibility resource can be available for usage;
 - Ramping rate: how fast a resource can ramp up or down;
 - Market time unit resolution: which is the duration of one market window for which the flexibility is traded.
3. Resource dimension: which is the flexibility resource? (supply-side, demand-side, grid-side) .
4. Risk dimension: uncertainties due to unexpected events f.e. error in forecasting the meteo and so error in RES generation and others.

In synergy with the dimension subdivision it's important to analyse the structures of the existing markets to understand which are the existent flexibility services on the current electricity trading platforms and which are the ones instead that need to be implemented with a different scheme.

Structure of flexibility trading in current power systems and market designs.

Time interval	Market mechanism	Product	Flexibility provider	Spatiality	Connection to grid	Uncertainty
Real time	Direct control	Power	Household appliances	Local	Distribution	Resource duration, Demand, Congestion
	Indirect control		Household appliances, EVs	Local	Distribution	
Within day	Balancing markets	Energy and power	EVs, Industrial DS, Aggregators	Local and non-local	Transmission, Distribution	Resource duration, Demand, Congestion, Fuel availability and cost, Wholesale market price, VRES generation
	Ancillary services		Aggregators, Conventional, Renewable			
Short term	Intraday	Energy	Aggregators, Conventional, Renewable			VRES generation
	Day-ahead	Energy	Aggregators, Conventional, Renewable, Storage			
	Forward markets	Energy and power	Conventional, Renewable, Storage			Demand, Fuel availability and cost, Wholesale market price, VRES generation
Medium term	Capacity markets	Capacity	Conventional, Renewable			Demand, Fuel availability and cost, Wholesale market price
Long term	Network expansion and investments	Capacity	Network reconfiguration, Grid expansion, Capacity expansion			Network investments, Policy and regulation

Figure 1-7: Structure of flexibility trading in current power systems and market designs [11]

Figure 1-7 is the result of a systematization of experiences from existing markets, both operational (such as CAISO, DS3, MISO) and experimental or emerging (e.g., Local Flexibility Markets in Europe). It is enhanced with a new risk dimension and aims to provide an integrated view for the analysis and design of future markets.

The authors analyze existing markets, such as those in the U.S. (e.g., CAISO) and Ireland (e.g., DS3), which already implement flexibility products to manage supply–demand imbalances, network congestion, and voltage issues. In these cases, flexibility is valued along the dimensions of time, spatiality, and resource, demonstrating a good capacity of markets to adapt.

However it acknowledges that some barriers are still present, especially related to the lack of efficient price signals for certain resources (such as demand-side flexibility), thus the introduction or enhancement of Local Flexibility Markets (LFMs) sounds to be a good evolution of current market structures.

The study emphasizes that efficient coordination between TSOs (Transmission System Operators) and DSOs (Distribution System Operators) is essential to optimally use flexibility considering that the resources to be integrated are both local and system-wide.

1.3 Options to implement flexibility

As briefly anticipated, there are different ways to implement the flexibility services, starting from the preexistent market structures up to the introduction of new local flexibility markets.

To make a concrete example, the flexibility services are already bought by the TSO for solving TSO issues in the ASM (ancillary services market) but in these markets only the traditional generators were included up to these very last years.

Figure 1-8 depicts a general framework that comprehends the different possibilities of exchanging flexibility:

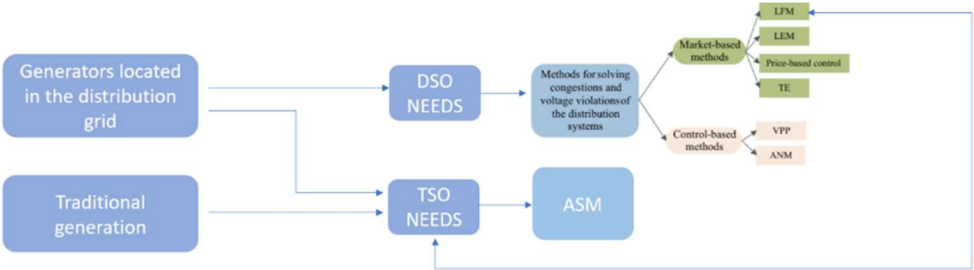


Figure 1-8: Flexibility management general framework [11]

The new entrance of the DSO changes the framework, in which the methods for procuring the flexibility of the DSO are depicted [11].

The two different approaches are market-based methods and control-based methods. Market-based methods provide trading platforms where the distribution system

operator (DSO) and prosumers can benefit from trading flexibility services, resolving congestion and voltage issues through market transactions. In contrast, control-based methods rely on active control strategies (such as Active Network Management - ANM), where the DSO sends direct commands or price signals to network components to manage power flows and operating conditions. While market-based methods promote a competitive and collaborative environment, control-based ones are more centralized and focused on the direct technical management of the grid.

Market-based methods (such as Local Flexibility Markets) are favored when the goal is to actively engage prosumers and create a competitive environment that shares economic benefits. They are also more transparent and adaptable, but they require advanced ICT infrastructure and a solid market design to avoid instability.

Control-based methods, on the other hand, are better suited for contexts where the DSO needs direct and fast control of the grid. They are technically effective, but less incentive-driven for participants, and may lead to uncertainty or inefficiencies if coordination is lacking.

Highlighting that it's important to engage actively the users to enhance awareness, in this analysis the focus is on one of the market-based methods that sounds to be more promising: the Local flexibility market.

Keeping this as statement, let's analyse the branches of the plot considering as option for the DSO the LFM.

1. The TSO can procure its flexibility in the ASM from the traditional generation
→ the new challenge is to integrate the RES in the existing ASM;
2. The TSO can procure its flexibility from the distributed generation in the ASM
→ the same challenge is in place;
3. The TSO procures its flexibility from the DG in a LFM → the new challenge is to shape accordingly the new architecture of the LFM:

- a. The DSO procures its own flexibility from the distributed resources (mainly RES) dispatched in the LFM and the part of flexibility that's not dispatched is aggregated and sent to the TSO;
- b. The DSO and TSO compete in a market (with priority to the TSO, with priority to the DSO, with the same priority for both) →TSO and DSO need to communicate (coordination schemes).

In the above-mentioned review a strong contrast is in place between the willingness to integrate the flexibility resources into the existing markets and the option to create new forms to exchange flexibility. This dualism strongly depends on the different exigencies that DSO and TSO are facing and consequently which type of flexibility is needed, as shown in Figure 1-9.

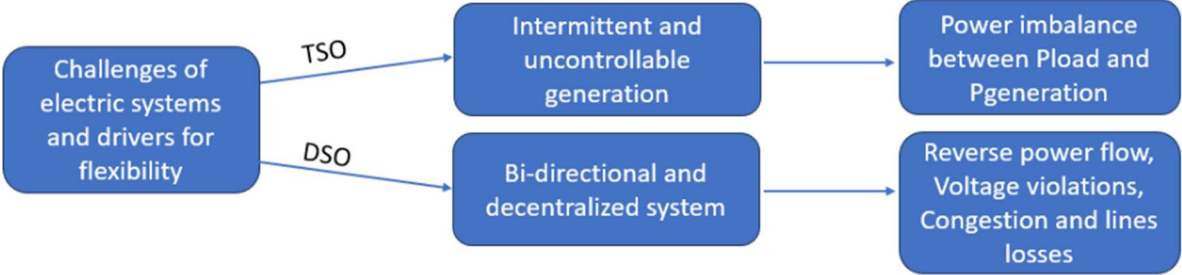


Figure 1-9: Challenges of electric power systems and drivers for flexibility [11]

For the different exigencies different types of flexibility services are traded and they are described in Figure 1-10.

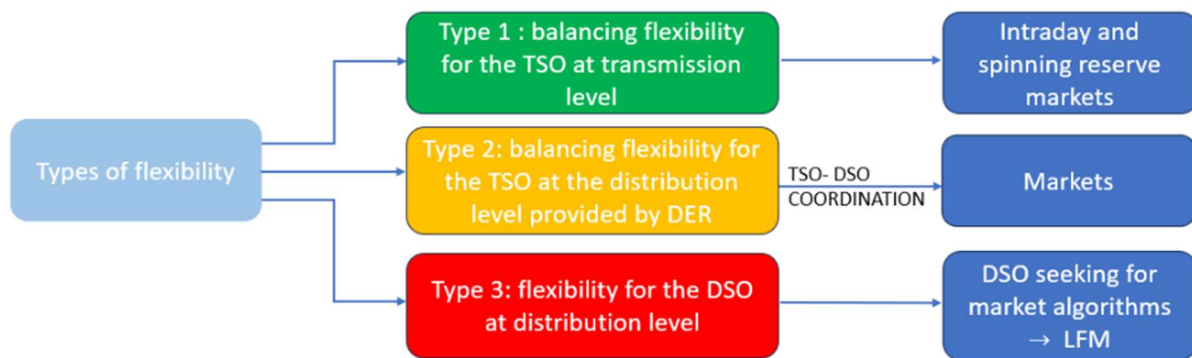


Figure 1-10: Types of flexibility [11]

The type 3 flexibility is the type of service that needs to be characterized so that the LFM can be shaped accordingly.

In Figure 1-11 some important attributes of the flexibility are highlighted representing them in the location node in which the flexibility resource is placed. In particular each flexibility product is defined by five key attributes: the direction (up or down), power capacity, start time, duration, and location within the distribution system.

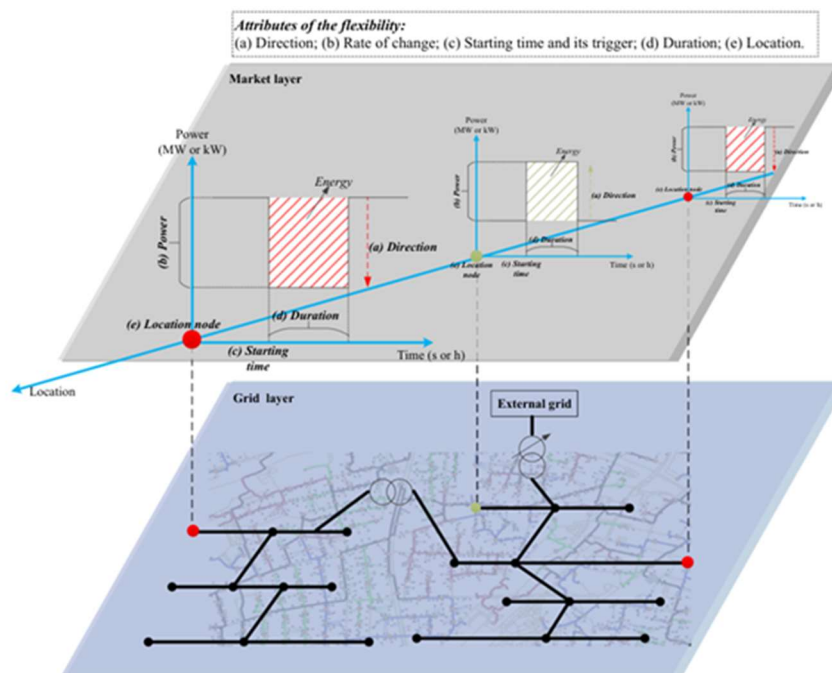


Figure 1-11: Attributes of the flexibility [12]

Flexibility resources in distribution networks can be grouped into three main categories which are highlighted in Figure 1-12. *Supply-side flexibility* refers to generation and storage units, such as distributed generators and batteries, that can adjust their output to support grid balance. *Demand-side flexibility* comes from end users and prosumers who can modify their energy consumption in response to price signals or control actions, using technologies like heat pumps, electric vehicles, or HVAC systems. Finally, *grid-side flexibility* involves the grid's own ability to reconfigure or adjust its operation through devices such as tap-changing transformers, reactive power compensators, or topology control, enabling better management of voltage levels and network congestion.

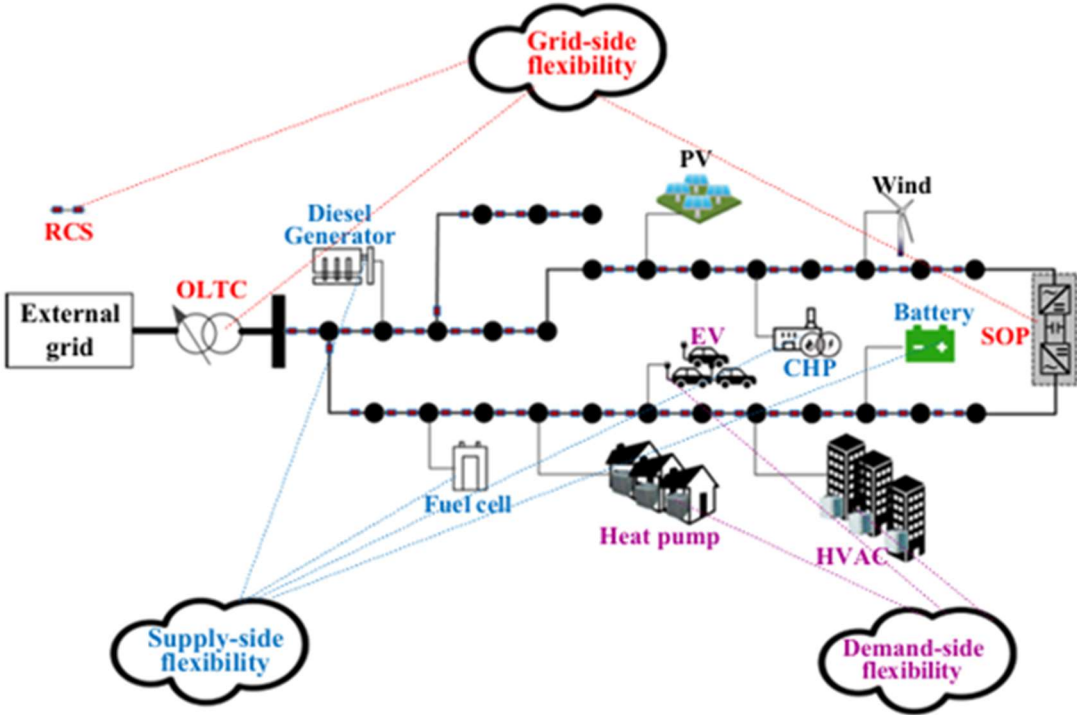


Figure 1-12: Sources of flexibility in an electric distribution system [12]

The Table 1-1 provides a complete overview of the main types of sources of flexibilities.

Supply-side flexibility	Demand-side flexibility	Grid-side flexibility
<ul style="list-style-type: none"> ➤ CHP (Combined Heat and Power) ➤ Diesel generators ➤ Fuel cells ➤ Battery storage systems 	<ul style="list-style-type: none"> ➤ Heat pumps ➤ HVAC systems ➤ Electric Vehicles (EVs) ➤ Smart homes and buildings, microgrids ➤ Aggregators ➤ Demand Response ➤ VESS (Virtual Energy storage systems) ➤ DSM 	<ul style="list-style-type: none"> ➤ OLTC ➤ SVR ➤ RCS (Remotely Controllable Switches) ➤ Capacitor banks ➤ SVC (Static Var Compensator) ➤ DSTATCOM (Distribution Static Compensator) ➤ Network reconfiguration for loss reduction and voltage control

Table 1-1: Sources of flexibility

1.3.1 Design of LFM

Building upon the concepts discussed above, the following section is devoted to the definition of a possible general structure of a LFM.

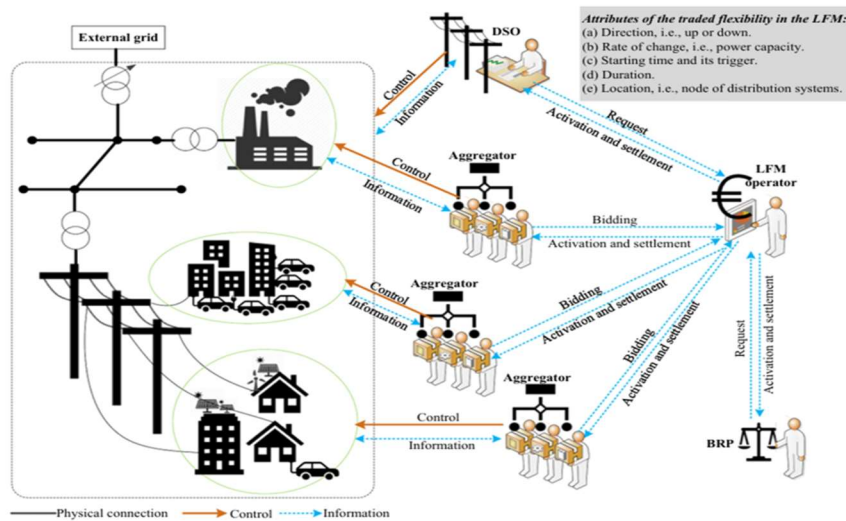


Figure 1-13: Schematic overview of a LFM [12]

First of all a Local Flexibility Market is defined as a geographically bounded trading platform, such as at the level of neighborhoods or cities, where flexibility is exchanged between buyers (mainly the DSO and Balance Responsible Parties, or BRPs) and sellers (typically aggregators managing distributed resources and prosumers).

The LFM model involves several key market participants described in Figure 1-13:

- The DSO, acting as the main buyer of flexibility for operational purposes;
- Aggregators, who gather flexibility from distributed energy resources and prosumers and act as intermediaries in the market;
- BRPs, who may also procure flexibility to manage imbalances in their portfolios;
- And an independent LFM operator, responsible for coordinating the market, collecting offers and requests, and performing the market clearing process.

The market follows a structured timeline illustrated in Figure 1-14, which includes three main phases:

First, the *contracting and bidding* phase, where the DSO identifies upcoming network issues and sends flexibility requests, while aggregators submit bids based on their available resources.

Second, the *activation* phase, during which the DSO triggers the contracted flexibility services.

Third, the *settlement* phase, where transactions are validated and payments are processed.

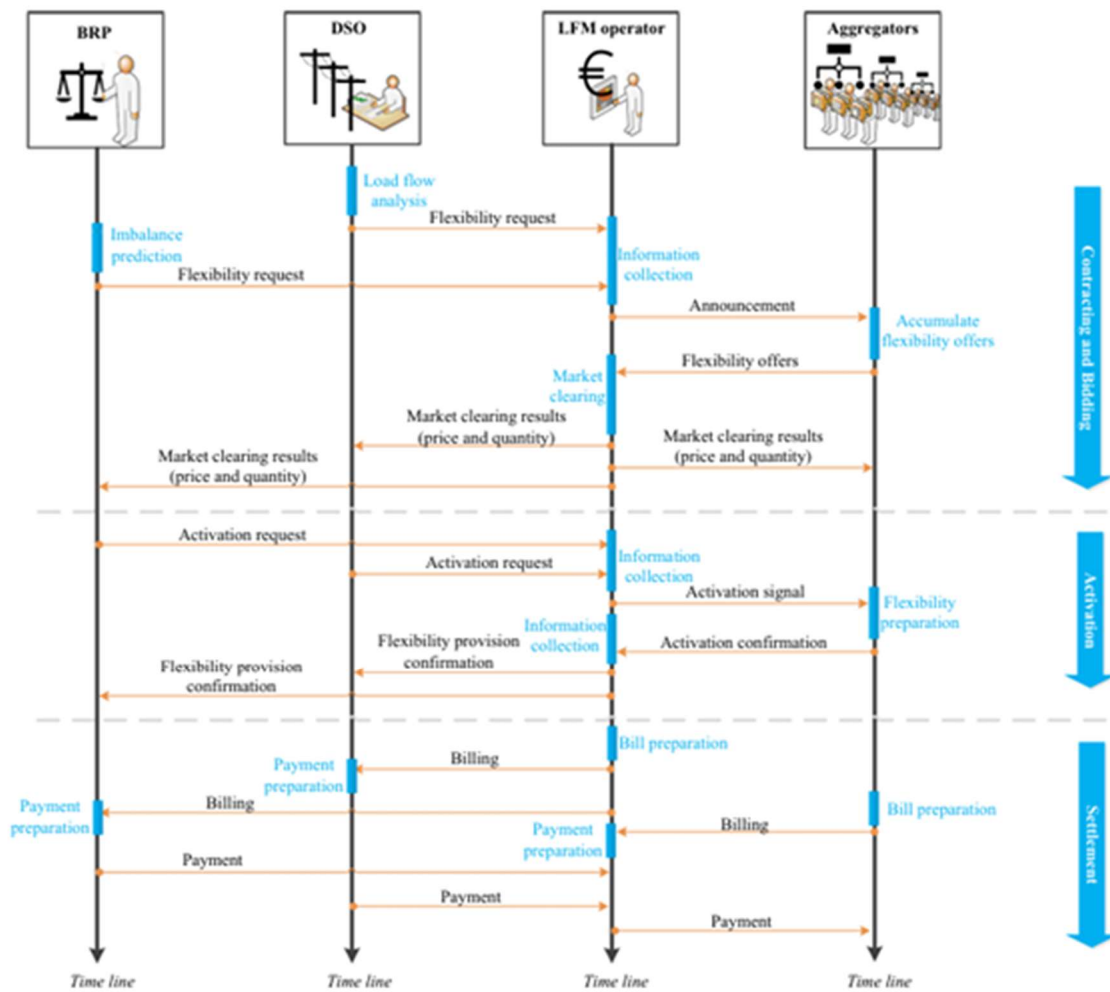


Figure 1-14: An example of the timeline of a LFM [12]

From an architectural perspective, the LFM is structured in four layers, as shown in Figure 1-15.

1. The power grid layer includes all physical components—generation units, storage, controllable loads.
2. The ICT layer supports data communication and monitoring.

- 3. The control layer governs the operation of resources through various control strategies.
- 4. Lastly, the market layer defines the trading mechanisms, participant roles, and clearing rules.

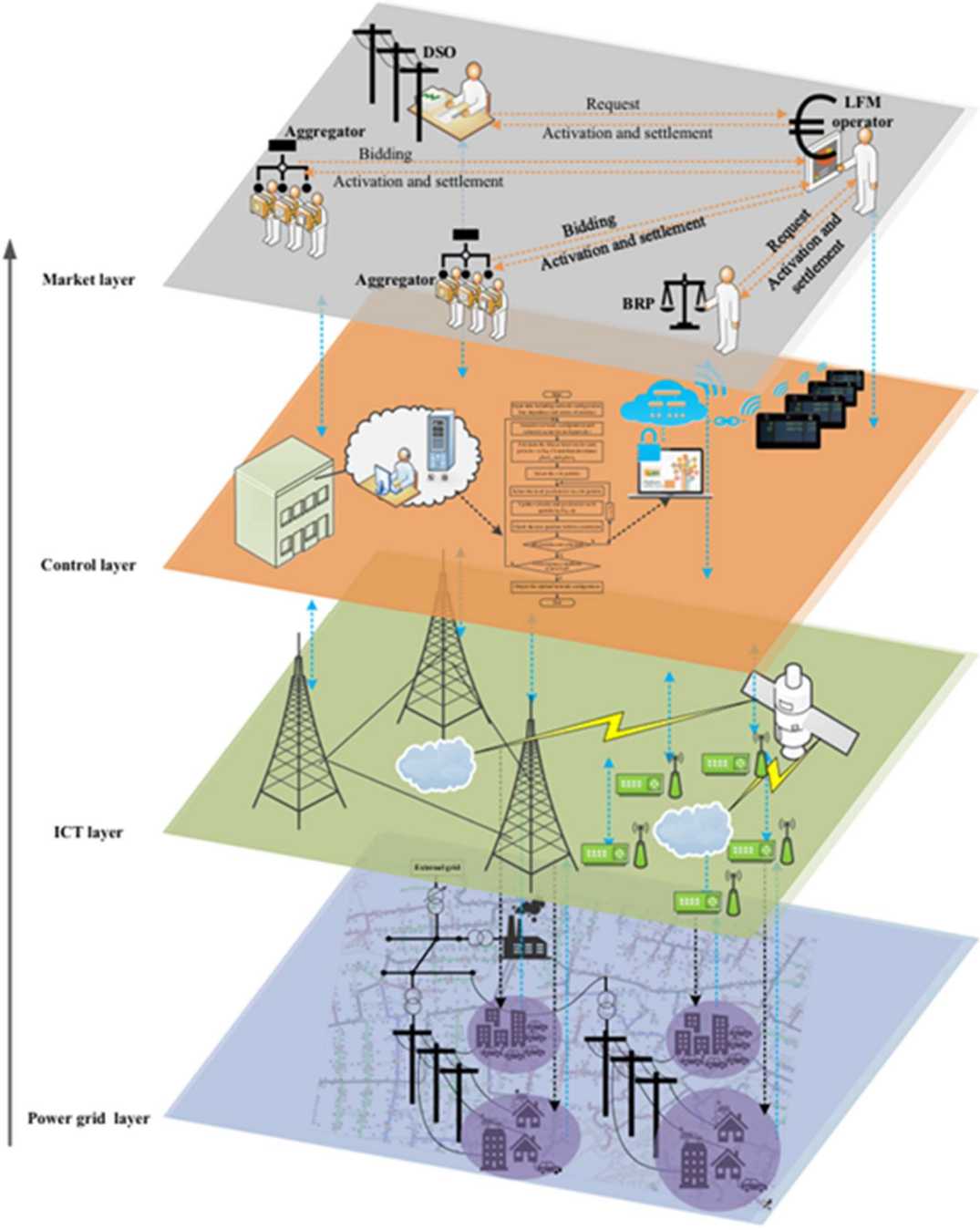


Figure 1-15: A four-layer architecture of local flexibility trading [12]

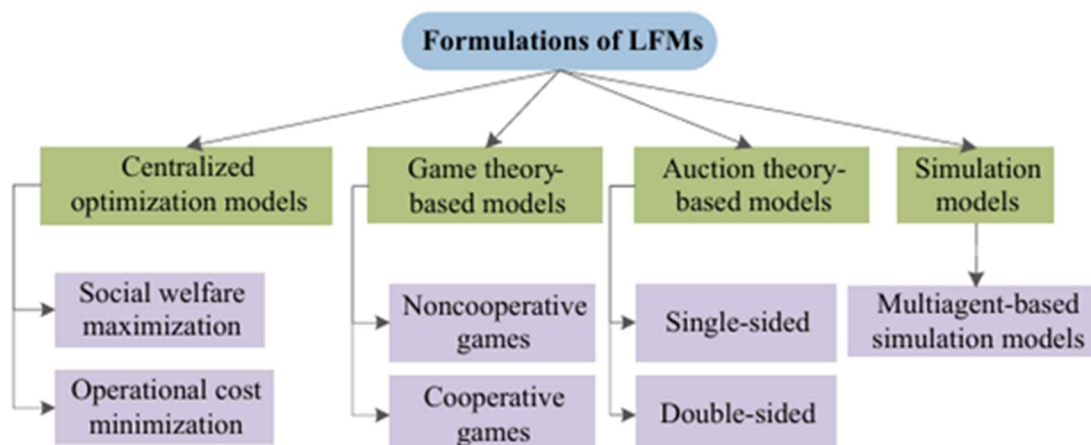


Figure 1-16: Summary of formulations of LFMs [12]

In terms of mathematical formulation (Figure 1-16), the LFM is often modeled through centralized optimization approaches. These typically aim to either maximize social welfare, the net benefit for all market participants, or to minimize operational costs, particularly for the DSO. The optimization is commonly formulated as a Mixed Integer Linear Programming (MILP) problem. In addition, the literature also explores game-theoretic, auction-based, and multi-agent simulation models, depending on the desired market dynamics.

Clearing methods vary depending on the formulation, ranging from centralized algorithms to distributed and decentralized approaches, such as ADMM, ATC, and PMP. In cases where a hierarchical structure is involved, such as in Stackelberg-type interactions between DSOs and aggregators, bi-level optimization methods are employed. The plurality of methods here discussed is a key point faced.

1.4 Projects and market platforms overview

A lot of European Projects are put in place and financed by the European Institutions in order to achieve different goals in terms of developing flexibility. In particular the HORIZON 2020 is an European Financial Program financed by the EU from 2014 to 2020 to develop projects whose aim is to enhance the technological research and innovation and to let the economics grow. In this program a lot of projects are developed to achieve decarbonization objective poned by the Fourth Energy Package.

Also outside of this programs a lot of market platforms were developed to trade flexibility for exploiting the potentialities of the RES introduced in the grid (picloflex, ERERA, NODES).

To have an exhaustive overview, let's use the article 'Analysis of New Flexibility Market Models in Europe' as reference [13].

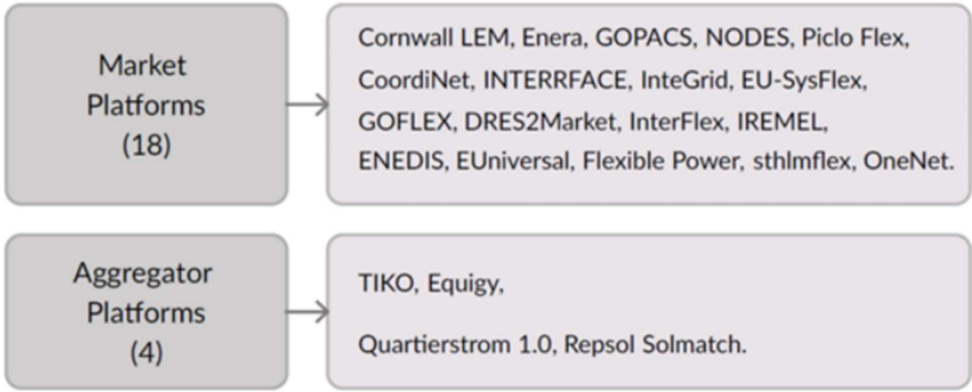


Figure 1-17: New flexibility market models in Europe [13]

Figure 1-17 shows a group of projects developed to face the flexibility challenge, both for market platforms and aggregator platforms.

The article builds a complete table (Figure 1-18 and Figure 1-19) to give a brief description of each of the market platform involved.

Market Platforms	Ownership	Countries of Application	Network Scope	Definitions and Motivations	Main Functions
Cornwall LEM, Pilot ¹ : 2016–2020.	Centrica	The United Kingdom	ESO-DSO Level	To develop a local energy market platform to facilitate WPD (Western Power Distribution) and the National Grid ESO (Electric System Operator) in the procurement of critical flexibility services that support their business activities in the electricity industry. Hence, enabling other market participants to offer their flexibility [32].	Product enrolment, activation of flexibility, bid collection, market clearing, aggregation activities, and settlement [33].
Enera, Pilot: 2018–2020.	EPEX SPOT, EWE AG, TenneT DE (TSO), Avacon Netz and EWE NETZ (DSOs).	Germany	TSO-DSO Level	This flexibility platform coordinates flexibility demand and supply, supports DSOs with congestion management, and reduces the overall curtailment of renewable energy. ENERA continuously matches supply and demand and offers DSOs order books with specific area identifiers, such that these flexibilities can be used for network operation purposes [11].	Bid collection, market clearing, market monitoring (verification platform), settlement, developing aggregation activities (unclear if platform or external), computing network impact (by grid operators), and individual flexibility option/activation.
GOPACS, In operation ² : 2019–to date.	Tennet (TSO), Stedin, Liander, Enexis Groep, and Westland Infra (DSOs).	The Netherlands	TSO-DSO Level	GOPACS is not a market platform itself, but it is connected to other market platforms. Currently, ETPA (Energy Trading Platform of Amsterdam) [34] is the first market platform that has joined GOPACS. It manages congestion at all voltage levels, increasing the available flexibility for re-dispatch and improving DSO/TSO coordination.	Congestion management needs are forecasted and announced via GOPACS by grid operators. The flexibility providers make offers to solve this congestion through the market platform, which acts as a gateway to GOPACS. A flexibility offer is placed as an IDCONS (Intraday Congestion Spread) if it meets specific conditions. It is also necessary to add a location tag [35].
NODES, Pilot: 2018–to date.	Nord Pool (European power exchange)	Norway and Germany	TSO-DSO Level	NODES aims to identify and give value to local flexibility [36] and link the NODES marketplace with the existing platforms that operate ID and balancing markets. Moreover, to increase value for flexibility providers and reduce costs for the DSO, flexibility not used locally could be sold to the TSO or to Balancing Responsible Parties (BRPs). It can solve imbalances in transmission.	To procure both LongFlex (Availability) and ShortFlex (Activation). Flexibility can be used for voltage control, frequency regulation, and congestion management.
Piclo Flex, In operation: 2019–to date.	Piclo	The United Kingdom	DSO Level	To develop a marketplace to standardize and facilitate DSO flexibility procurement, make more efficient use of the existing grid, and reduce the need for grid reinforcement [5].	Piclo Flex provides an independent platform to publish flexibility needs based on the demand location [37]. DSOs can see qualifying assets in the constraint management zones. The resulting map of competitors enables them to source flexibility with precise locational, technical, and temporal requirements.

Figure 1-18: Description of the market platform [13]

Table 1. Cont.

Market Platforms	Ownership	Countries of Application	Network Scope	Definitions and Motivations	Main Functions
GOFLEX, Pilot: 2017–2020	During the project time, the main actor carrying out the flexibility use cases are local energy suppliers/utilities and smaller DSOs [42].	Demos in Cyprus, Germany, and Switzerland.	TSO-DSO Level	The main objective of GOFLEX is to make a set of technology solutions for distributed flexibilities and automated dynamic pricing market-ready, enabling consumers, generators, and prosumers to aggregate and trade flexibilities. By taking a bottom-up approach (flexibility is harvested from the prosumer level and procured to higher levels in the electricity grid), GOFLEX makes DR more cost-effective and increases the level of DR available [19].	An Automatic Trading Platform (ATP) is developed as three independent core sub-systems: Flex-Order Agent (FOA), Flex-Order Manager (FMAN), and Flex-Order Market (FMAR) [43]. The functionalities include Forecast grid congestions and grid needs, automatic issue of Flex-Offers depending on needs, collect Flex-Offers, market clearing, send activation signal, and settlement.
DRES2Market, Pilot: 2020–2023	To be defined	Demos in Austria, France, Greece, Norway, Poland, and Spain.	DSO Level	To develop a comprehensive and affordable approach to facilitate the effective participation of DG based on renewable energies in the electricity markets and provide balancing and reserve services according to market criteria [20].	The project will be validated at two levels: (1) simulating the impact of the promising solutions considering the evolution and variability of market prices with an increase in the share of renewables, (2) simulating an electricity market and system to identify technical and regulatory solutions, effective grid codes and market rules.
InterFlex, Pilot: 2017–2019	In both Dutch and French demos, the market platform is operated by the DSOs.	Demos in Czech Republic, France, Germany, Netherlands, and Sweden	DSO Level	The project investigated the potential of local flexibility to relieve grid constraints at a local and regional level. Therefore, the project contributed to enhance the development of new distributed energy resources and prepare the electric system for new uses. InterFlex mainly focuses on the interactions between DSO and market players.	In the Dutch and French demos (flexibility markets between DSO and aggregators), the flex markets were tested with the following functions: Forecast grid congestions and grid need, collect flexibility offers, market clearing, send activation signal, and settlement.
IREMEL, Pilot: 2019–to date	OMIE (Iberian Electricity Market Operator)	Spain	DSO Level	To facilitate the participation of RES and consumption connected to distribution networks in the markets, promoting the proper management of their discharged and consumed energy allows them to benefit from better market prices due to their flexibility and management [22].	IREMEL proposes to the DSO market mechanisms to solve eventual problems of congestion or supply. The market operator can activate local products to solve congestions.

¹ Pilot means that a proof of the concept was tested with limited impacts (in terms of agents participating, network scope, or functionalities). Although products are delivered, there may not be economic consequences. Pilots are trials that aim to test certain functionalities before the market starts operating. ² In operation means that the market is fully operational. This means that already the products are traded, bids are submitted, cleared, and economically settled. ³ Under development means that the initiative is in an initial phase where the market concept has been defined and preliminary developments are implemented.

Figure 1-19: Cont. [13]

1.5 Focus: TSO-DSO coordination schemes

Within this framework, some projects are of major interests for the particular attention in the discussion of TSO-DSO coordination that, as anticipated, is one of the pillars of the flexibility market construction.

1.5.1 SmartNet project

To introduce the topic, it is appropriate to begin with the key findings of the Smart Net Project, with the complete title “Smart TSODSO interaction schemes, market architectures and ICT Solutions for the integration of ancillary services from demand side management and distributed generation”.

The project proposes an Integrated Reserve market architecture that leverages DER flexibility to provide ancillary services not only to the transmission grid but also local services to the distribution grid. It aims to procure such services optimally, e.g., by minimizing activation costs or maximizing welfare, while enabling DERs to compete on equal footing with centralized flexibility sources.

Various coordination schemes for the procurement of ancillary services between the Transmission System Operator (TSO) and the Distribution System Operator (DSO) have been defined. These schemes (Figure 1-20) are based on either centralized or decentralized market approaches in which the flexibility provided by different Distributed Energy Resources (DERs) is utilized through local and/or common markets operating at both the DSO and TSO levels.

Centralized architecture	Decentralized market architecture
Centralized AS market	Local AS market
Common TSO-DSO AS market (centralized)	Common TSO-DSO AS market (decentralized)
Integrated flexibility market	Shared balancing responsibility model

Figure 1-20: Different TSO-DSO coordination schemes according to their centralized/decentralized nature [14]

1. Centralized AS Market Model:

The TSO operates a single ancillary services market covering both transmission- and distribution-connected resources. The DSO plays a minimal or no role in the process.

2. Local AS Market Model:

The DSO manages a local market aimed at solving distribution grid issues. After addressing local needs, the DSO aggregates the remaining flexibility and submits bids to the TSO market.

3. Shared Balancing Responsibility Model:

Balancing responsibilities are divided between the TSO and DSOs according to a predefined scheme. Each system operator (SO) runs its own market independently, and the TSO does not have access to flexibility from DERs within the DSO domain.

4. Common TSO-DSO AS Market Model: both TSO and DSO collaborate to minimize the total cost of fulfilling their respective service requirements. This coordination can take two forms:

- A centralized market jointly managed for both TSO and DSO needs.
- A decentralized market, where a local DSO market dynamically interacts with the TSO level.

5. Integrated Flexibility Market Model: a unified, centralized market platform is established, accessible to TSOs, DSOs, and also commercial market participants (e.g., Balance Responsible Parties – BRPs), facilitating broad participation in flexibility procurement.

The centralized and decentralized subdivision of the schemes relies on two different hypothesized market architectures.

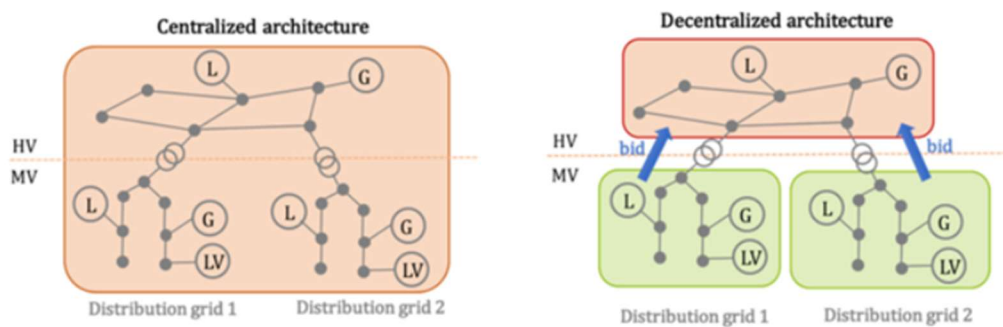


Figure 1-21: Centralized and Decentralized market architectures [15]

Figure 1-21 illustrates the conceptual difference between centralized and decentralized market architectures for the procurement of ancillary services. In the centralized architecture, the Transmission System Operator (TSO) manages the entire ancillary services market, collecting bids from both transmission-connected and distribution-connected resources. In this configuration, Distribution System Operators (DSOs) play little or no active role in market operations. While this approach simplifies coordination and provides the TSO with a system-wide view, it may overlook local network constraints within distribution grids.

Conversely, the decentralized architecture assigns a more active role to DSOs, who operate local markets to address their own grid needs. After satisfying local requirements, DSOs aggregate any remaining flexibility and submit bids to the TSO-level market. This model improves the management of local grid conditions and enables a

more efficient use of distributed energy resources (DERs), but it also introduces greater complexity and requires close coordination between TSOs and DSOs.

For these two structures the mathematical models are presented within the project.

1.5.2 CoordiNet project

For the same discussion the CoordiNet project builds upon the coordination scheme typologies developed in SmartNet, using them as a baseline for classifying and comparing existing approaches in literature, regulation, and field demonstrations.

CoordiNet's use of SmartNet as the foundational reference for classifying TSO–DSO coordination schemes highlights the central role that SmartNet has played in shaping the conceptual framework for TSO–DSO interactions in Europe.

However, CoordiNet goes beyond merely building on SmartNet's results: it expands and refines them in light of more recent scientific literature, evolving regulatory frameworks, and practical needs emerging from real-world system operation.

Notably, the project introduced a new, shared taxonomy aimed at facilitating comparison across different coordination approaches while reducing terminological ambiguity.

Indeed, unlike the more fixed architecture-based typologies used in earlier projects such as SmartNet, CoordiNet's approach is structured around four classification layers:

- (i) **the “Need”**, which identifies whether the flexibility service addresses local (DSO), central (TSO), or combined system needs;
- (ii) **the “Buyer”**, indicating who procures the service, TSO, DSO, commercial aggregators, or end-users in peer-to-peer settings;

- (iii) **the “Markets”**, distinguishing between single-market and multi-market frameworks;
- (iv) **the “Resources”**, specifying whether the TSO has direct access to distributed energy resources (DERs) or must coordinate through the DSO.

Combining these layers CoordiNet identifies 7 principal categories of coordination schemes: Local Market, Central Market, Common Market, Integrated Market, Multi-level Market, Fragmented Market, Distributed Market (Figure 1-22 and Figure 1-23).

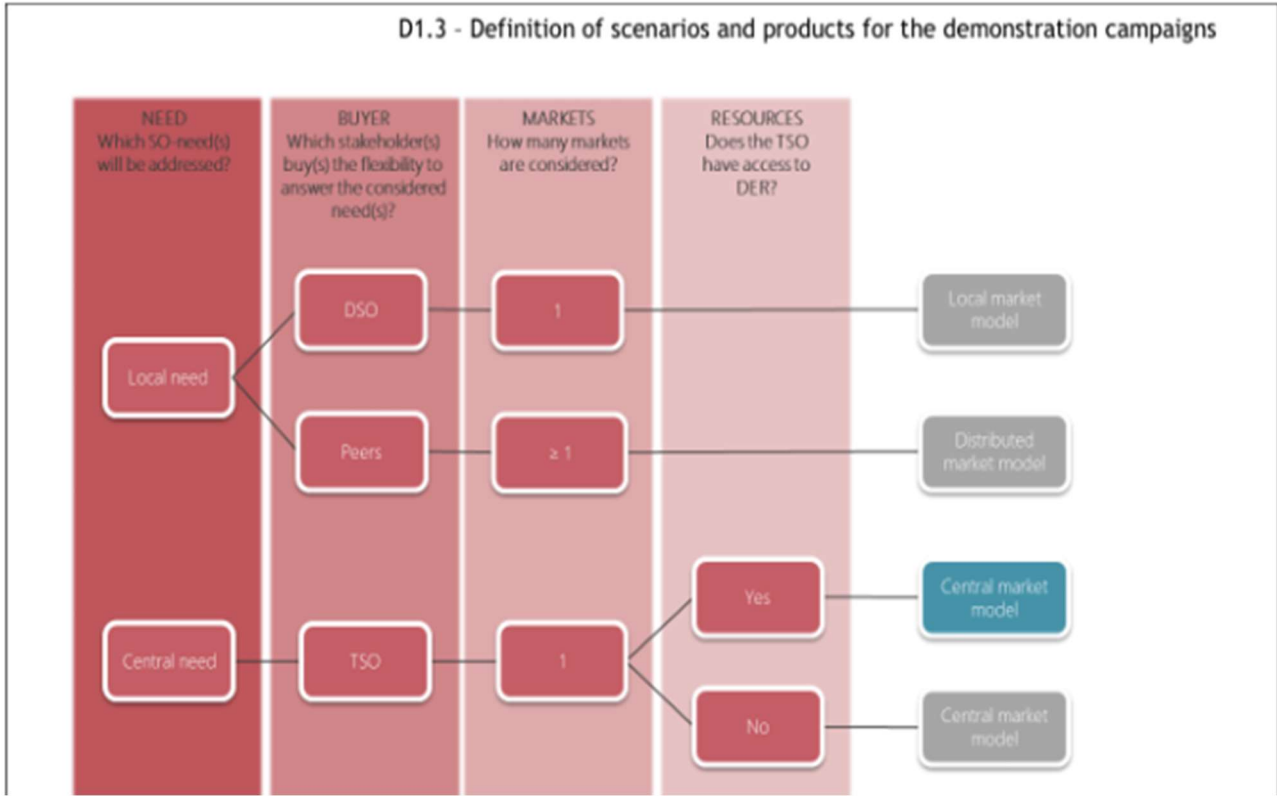


Figure 1-22: Categorization structure of coordination schemes considered within the CoordiNet project [16]

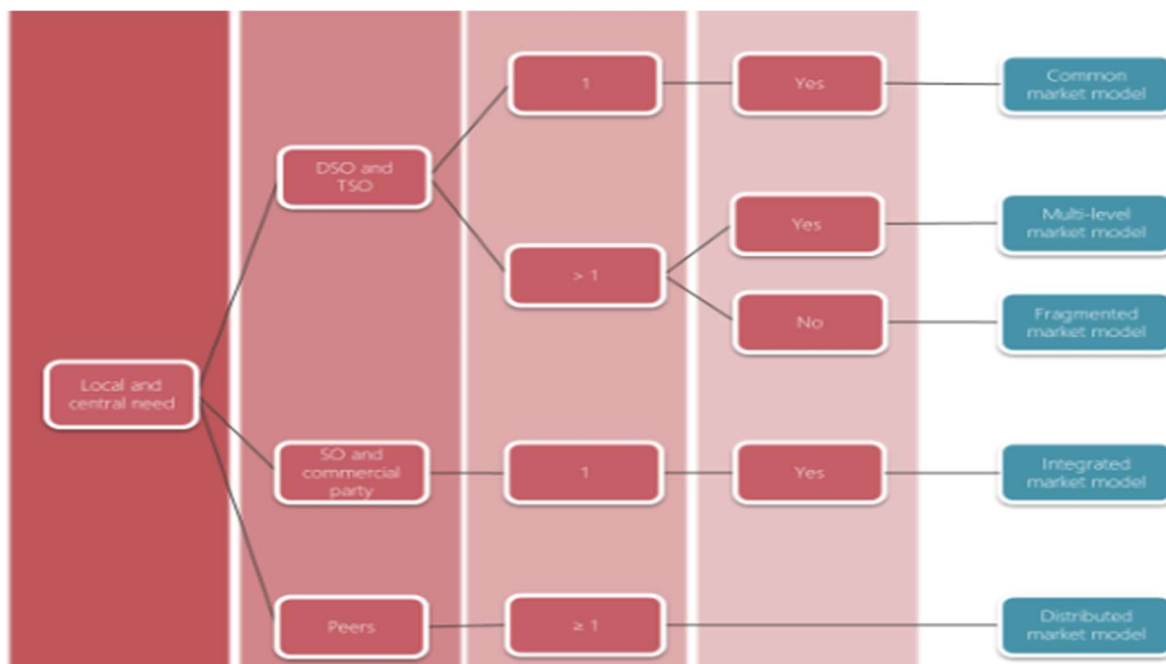


Figure 1-23: Categorization structure of coordinaton schemes considered within the CoordiNet project [16]

Furthermore, CoordiNet addresses several aspects that remained underexplored in SmartNet, such as operational data handling, ICT infrastructure costs, and the integration of distributed and peer-to-peer market models.

As a final point the most significant innovation lies in the transition from theory to practice: through large-scale field demonstrations conducted in Greece, Spain, and Sweden, CoordiNet has tested the conceptual models under real operational conditions, contributing to an increase in their Technology Readiness Level (TRL) up to levels 7–9 with respect to SmartNet that instead lies between the levels 4-5.

1.5.3 State of Art

Among the academic contributions that have shaped the current understanding of TSO–DSO coordination, the article “*A Review on TSO–DSO Coordination Models and Solution Techniques*” [17] holds particular relevance.

This review serves as a comprehensive synthesis of the coordination schemes proposed in recent literature, with a strong conceptual foundation drawn from the SmartNet project. In fact, the authors explicitly adopt the five coordination models defined within SmartNet, namely the centralized, local, shared balancing responsibility, common, and integrated market models, as the primary classification framework for their analysis.

The article consolidates the five coordination models introduced in SmartNet into three broader categories, TSO-managed, DSO-managed, and joint coordination, by focusing on the actor responsible for market operation and decision-making.

Table 1-2 shows the correspondence found by the article for the SmartNet models:

SmartNet model	Mapped to
Centralized AS market	TSO-Managed Coordination
Local AS market	DSO-Managed Coordination
Shared balancing responsibility	Joint TSO–DSO Coordination
Common TSO–DSO AS market (centralized)	Joint TSO–DSO Coordination
Common TSO–DSO AS market (decentral.)	Joint TSO–DSO Coordination
Integrated flexibility market	Joint TSO–DSO Coordination

Table 1-2: SmartNet models transposition

TSO MANAGED

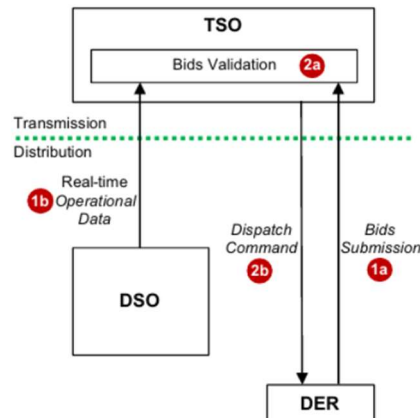


Figure 1-24: TSO managed model [17]

1. DER send their bids directly to the TSO for a given service period (1a). The TSO also receives real-time operational data from the DSO (1b).
2. The TSO performs a centralised validation of DER services bids (2a) and computes the dispatch of TER and DER. The corresponding dispatch commands are sent to DER (2b) (Figure 1-24).

TSO-DSO HYBRID MANAGED MODEL

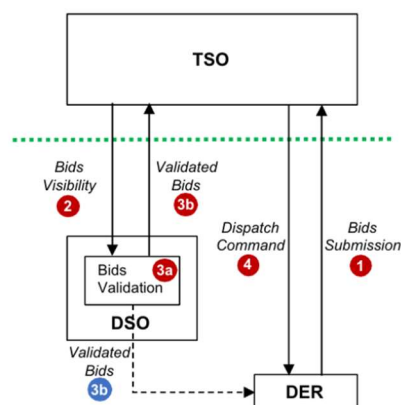


Figure 1-25: TSO-DSO hybrid managed model [17]

1. DER send their bids directly to the TSO for a given service period.
2. The DSO is informed about the bids.

3. The DSO performs the validation of DER services bids (3a). Then, it transmits the validated bids and requests to the TSO (3b-red) and/or to DER (3b-blue).
4. The TSO performs the dispatch and sends the dispatch command to DER (4) (Figure 1-25).

DSO MANAGED MODEL

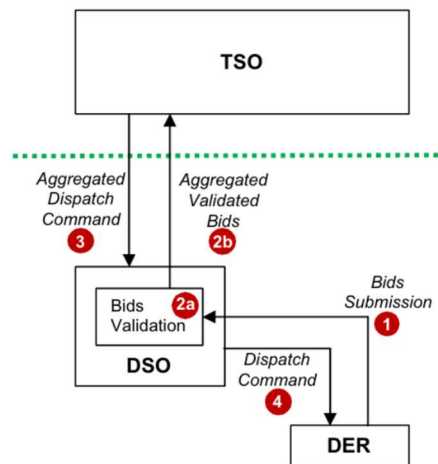


Figure 1-26: DSO managed model [17]

1. DER send their bids to the DSO for a given service period.
2. The DSO performs validation of DER services bids and aggregates them (2a). Aggregated bids are submitted to the TSO (2b).
3. The TSO performs the dispatch and sends the aggregated dispatch command to the DSO.
4. The DSO sends the dispatch command to DER to match the aggregated requirement (Figure 1-26)

To wrap up all the advantages and disadvantages of the methods the following table (Figure 1-27) is reported in the article:

TSO-DSO coordination models characteristics

	TSO-Managed Model	TSO-DSO Hybrid-Managed Model	DSO-Managed Model
Advantages	<ul style="list-style-type: none"> • Simplifies the TSO-DSO coordination process • TSO know-how on dispatch • TSO has no conflict of interest when pushing distribution network assets to the limits 	<ul style="list-style-type: none"> • Likely to have a more efficient facilitation of DER services • DSO know-how on distribution networks • Less computational and modeling requirements (restricted to each DSO) • No need for operational data transfer from DSO to TSO 	<ul style="list-style-type: none"> • Likely to have the most efficient facilitation of DER services • DSO know-how on distribution networks • Less computational and modeling requirements (restricted to each DSO) • No need for operational data transfer from DSO to TSO and less data exchanges between TSO and DSO/DER
Disadvantages and Challenges	<ul style="list-style-type: none"> • Likely to have a less efficient facilitation of DER services • Huge computational and modeling challenges for the TSO (transmission and distribution) • High amounts of operational data transfer between TSO and DSOs • TSO lacks know-how on distribution networks 	<ul style="list-style-type: none"> • Potential DSO conflict of interest • Each DSO still has computational and modeling challenges • More complex TSO-DSO coordination processes 	<ul style="list-style-type: none"> • Potential DSO conflict of interest • Each DSO still has computational and modeling challenges with further complexity with distribution-level markets • DSO lacks know-how on markets

Figure 1-27: TSO-DSO coordination model characteristics [17]

1.5.4 Recent approaches

1.5.4.1 “A novel two-stage TSO-DSO coordination approach for managing congestion and voltages”

This article [18] proposes a novel approach to TSO–DSO coordination in day-ahead operation planning for the procurement of ancillary services (AS) such as congestion management and voltage control.

The main novelty lies into the decomposition of the overall AS problem into two sequential stages, which seek the procurement of:

- (i) “active power related” AS (i.e., for congestion management)
- (ii) the “reactive power related” AS (i.e., for voltage support), respectively as well as a fast approximation of the cost of aggregated active and reactive power flexibilities of active distribution systems (ADSs).

The process of decomposition and the consequent coordination TSO-DSO is represented in Figure 1-28:

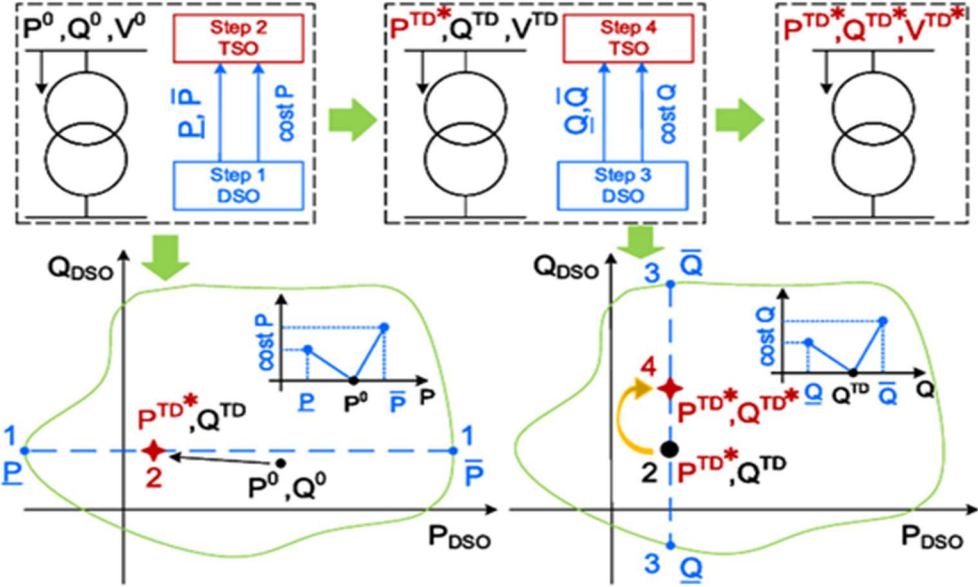


Figure 1-28: Conceptual illustration of the proposed AS procurement methodology at a given time step and difference compared to PQ charts [18]

Five-Step TSO–DSO Coordination Process

1. **Assessment of Active Power Flexibility by the DSO**
 Each DSO determines the feasible range of active power it can safely exchange with the TSO at the interface $[\underline{P}_{TD}, \overline{P}_{TD}]$ along with the corresponding cost of providing flexibility. This defines how much the DSO can increase or decrease the active power flow P_{TD} at the interface point.
2. **Optimal Active Power Selection by the TSO**
 If congestion is detected in the transmission system, the TSO procures the required active power flexibility—potentially including bids from DSOs—by minimizing

the total cost. This establishes the committed active power exchange P_{TD}^* between TSO and each DSO.

3. Assessment of Reactive Power Flexibility by the DSO

Given the fixed active power P_{TD}^* , the DSO computes the feasible range of reactive power at the interface $[\overline{Q_{TD}}, \underline{Q_{TD}}]$ and the associated cost. This flexibility is essential for voltage regulation.

4. Optimal Reactive Power Selection by the TSO

If voltage issues are identified in the TS, the TSO procures the required reactive power flexibility—again possibly including offers from DSOs—to ensure voltage stability and minimize cost.

This sets the agreed reactive power exchange Q_{TD}^* , and voltage level V_{TD}^* at the TSO–DSO interface.

5. Redispatch of Resources by the DSO

The DSO optimally redispatches its internal flexible resources (DER, storage, flexible loads) to meet the agreed exchange values: $Q_{TD}^*, V_{TD}^*, P_{TD}^*$. The DSO ensures safe operation of the distribution network while minimizing its local redispatch cost.

This scheme proposes a novel interaction between TSO and DSO considering the decomposition of the problem of active and reactive power. Furthermore it integrates in the mathematical formulation with respect to previous studies N-1 constraints and AC non linear model.

The results, in terms of costs of activation in two cases of activated flexibility and not activated flexibility (referred to the DSO flexibility), are promising (Figure 1-29).

	Without activated Flex.	Activated Flex.
Operation	151,145	141,437
Normal Gen. curt.	18,425	18,375
Post-contin. Gen. curt.	29,784	29,753
Flexibility	–	3268
Total cost	199,354	192,833
Elapsed time (s)	2445	2449

Figure 1-29: Cost (€) of activation of ADSs flexibility by TSO [18]

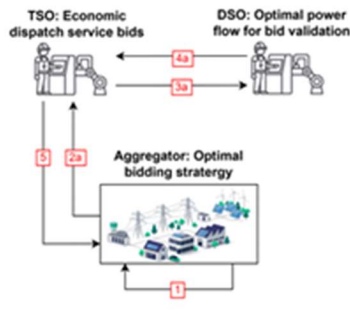
Furthermore the results demonstrate that DSOs are capable of providing significant flexibility at the TSO–DSO interface, up to 450 MW of active power and 252 MVar of reactive power.

The conclusion is that activating this flexibility enables the TSO to reduce operational costs, relieve network congestion, and manage voltage profiles effectively, even in scenarios where, without DSO support, the transmission system would become infeasible to operate. All of this is achieved with a moderate computational burden, making the proposed coordination approach practical for real-world day-ahead operational planning.

1.5.4.2 “A comparative study of different TSO-DSO coordination scheme in the reserve market”

Taking the article “a review on TSO-DSO coordination models and solution techniques” [17] as reference, this new article analyses “coordination in the offering process for aFRR provision proposed with two schemes: DSO-managed model and TSO-DSO hybrid-managed model” providing respectively the results of a comparison described in Figure 1-30 and Figure 1-31.

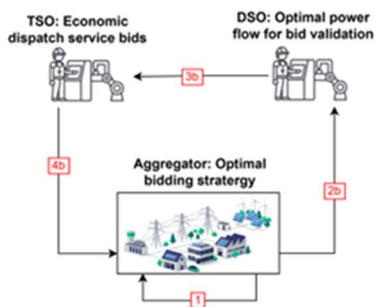
TSO-DSO hybrid-managed



- 1) Aggregator optimizes their bidding strategy and defines the volume they can bid for regulation service.
- 2a) Aggregator sends the volume they can bid for upward and downward regulation to TSO.
- 3a) TSO sends the MOL to DSO per ISP.
- 4a) DSO updates the boundary from the MOL and sends the new boundary to TSO.
- 5) TSO activates service in real-time.

Figure 1-30: TSO-DSO hybrid-managed coordinations scheme [19]

DSO-managed



- 1) Aggregator optimal bidding strategy and define the volume they can bid for upward and downward regulation.
- 2b) Aggregators send the volume they can bid for upward and downward regulation to **DSO**.
- 3b) DSO sends the new boundary of the upward/downward volume that the aggregator can provide to TSO.
- 4) TSO defines the MOL and sends activation requests to the aggregator in real-time.

Figure 1-31: DSO-managed coordination scheme [19]

Looking at the results f.e. at branch loading in Figure 1-32

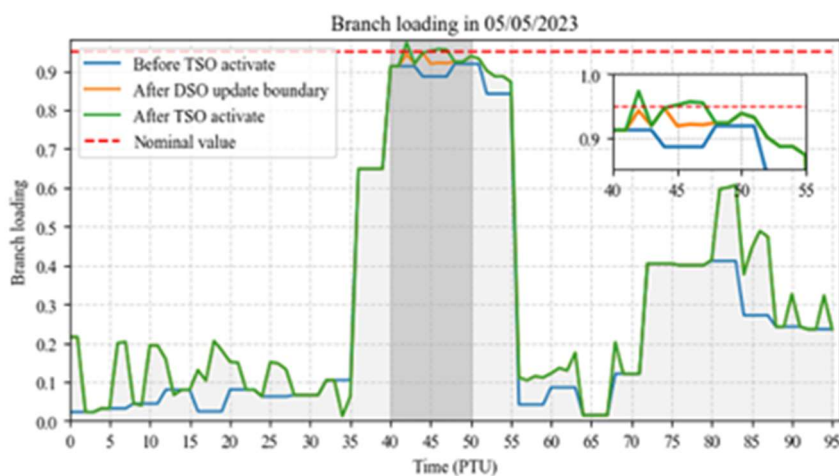


Figure 1-32: TSO-DSO hybrid-managed [19]

In the TSO-DSO hybrid-managed scheme, the hybrid-managed scheme allows the TSO to select the most cost-effective flexibility offers and then requests validation from the DSO. This ensures optimal use of aggregator resources, but more importantly, guarantees that activations comply with network constraints. The graph shows that the activation of the service by the TSO would have caused an overload (green line), exceeding the 95% loading threshold. However, thanks to the intervention of the DSO, the flexibility boundaries are updated, and the new loading level (orange line) remains below the critical threshold, thus avoiding congestion.

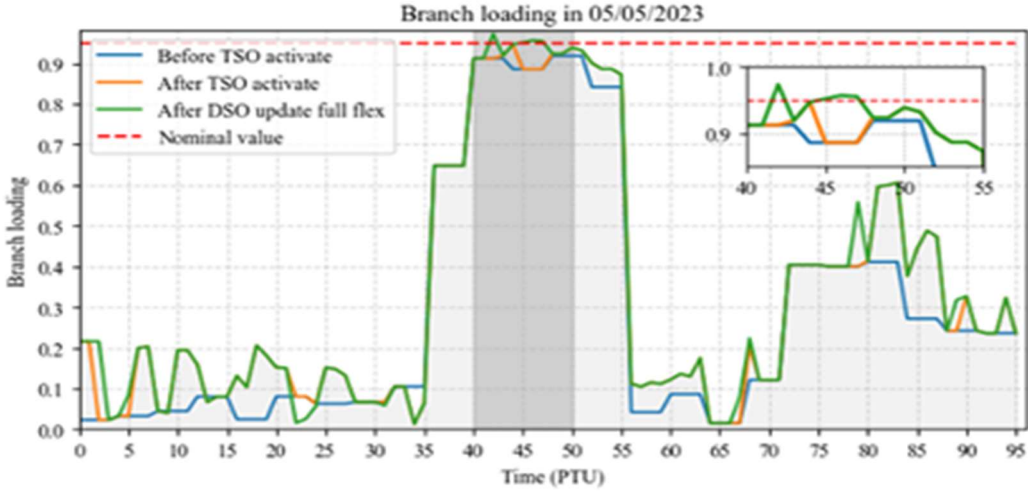


Figure 1-33: DSO-managed [19]

In the DSO-managed scheme case shown in Figure 1-33, the DSO receives flexibility proposals directly from the aggregators and updates the boundaries before the TSO selects the offers. Also in this case a potential overload (green line) is observed at Imbalance Settlement Period (ISP) 44, which is then prevented after the DSO updates the boundaries (orange line). However, in this approach, the DSO, not knowing the TSO's exact intentions, tends to act more conservatively: it preemptively reduces the limits for all aggregators to avoid congestion, thereby limiting the total available flexibility volume.

After performing these simulations the article concludes that the TSO-DSO hybrid-managed coordination scheme is superior to the DSO-managed model for several key reasons. First, it allows for a greater volume of flexibility to be activated, since the TSO selects the bids before the DSO validates them, thereby preserving more valid offers. Second, aggregators benefit from increased profits, as more resources are activated and sold, especially for upward regulation services. Third, the hybrid model enables a more efficient use of resources, with the DSO updating constraints based on already selected bids rather than preemptively restricting the entire flexibility pool. Lastly, the grid constraints are still effectively managed, but through a more targeted intervention that is less detrimental to aggregators' participation and potential earnings.

1.6 Focus: Flexibility market design

The complexity of the problem is evident: a lot of pillars has to be taken into account in the design of a flexibility market structure.

For this reason the authors of the article "Market-Based TSO-DSO Coordination: A Comprehensive Theoretical Market Framework and Lessons from Real-World Implementations" [20] tried to map a TMF useful to study the actual market design solutions highlighting important innovations and conclusions.

Using this TMF (Figure 1-34), the study maps 13 European demonstrators' TSO-DSO coordination solutions.

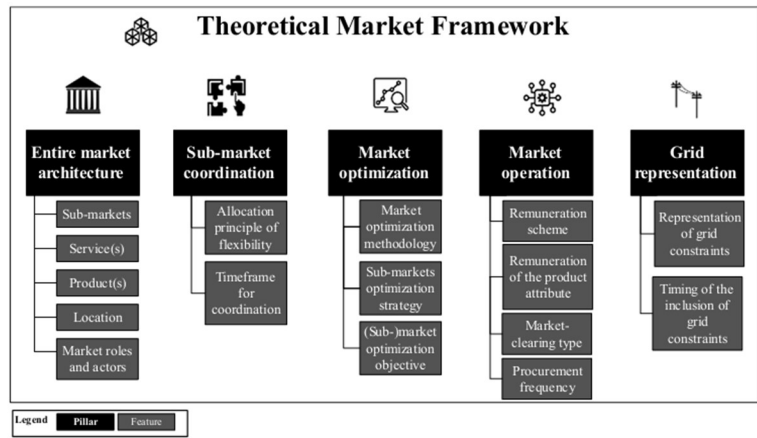


Figure 1-34: Key elements of the theoretical market framework [20]

The interaction between provider and beneficiary can be classified as technical-based or market-based. As shown in Figure 1-35, in market-based interactions, the actors interact through a market architecture which may include the interaction with the actor that plays the market operator and, eventually, the market platform. The technical-based interaction is characterized by information exchange and control actions. The paper focuses on the set of the market-based interactions tested in the OneNet project.

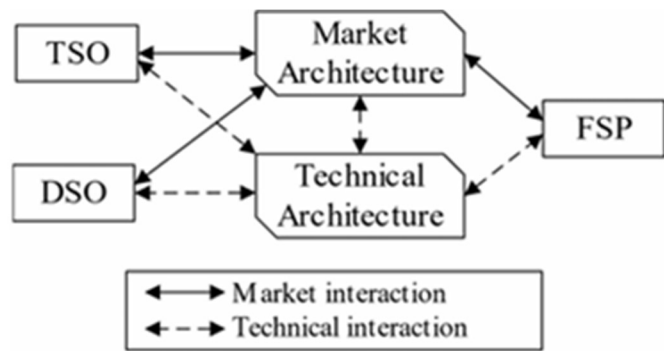


Figure 1-35: Scheme of the possible interactions between the actors involved in system service procurement and provision [20]

In this article the TMF proposed is useful to understand the common features of the different existent projects about the implementation of a flexibility market structure. This allows to have a complete overview of the theme.

In particular it provides the entire market architecture in which it describes the sub-market types, the services, the products, location and market roles. In the submarket coordination it exploits the different types of coordination between market actors.

The market optimization depicts the different possibilities to solve the algorithm of the market based on the different objectives to be reached.

The market operation defines the type of market that has to be implemented with the relative remuneration scheme and economics-related aspects; finally the grid representation defines how the grid is modeled in terms of the type of parameters used, the considered constraints and the timing inclusion of the grid constraint.

Within this TMF as said 13 European projects are mapped and tested; furthermore it's importance to notice that other projects in place about the flexibility market development could be classified in their specifications using this general framework.

On the way of defining the contribute of this thesis work it's important to define which are the contributes of other precedent Italian projects that can be used as a good starting point to completely understand the key features of the problem.

1.7 Italian flexibility projects: “Analisi dell'applicazione al contesto italiano dei meccanismi di approvvigionamento di flessibilità locale”

1.7.1 Italian regulatory context

The Regulation (EU) 2019/943 and the Directive (EU) 2019/944, both part of the Clean Energy Package, have significantly influenced Italian legislation regarding the

integration of renewable energy sources (RES) into electricity markets and the introduction of new market actors. In line with the principles set out by these EU measures, Italy has adopted national regulatory frameworks aimed at increasing market participation and flexibility. Notably, ARERA's Resolution 300/2017/R/eel introduced the experimental framework for Mixed Enabled Virtual Units (UVAM), allowing heterogeneous aggregations of distributed resources, such as small-scale generation, storage systems, and flexible loads, to participate in the dispatching services market. Later, Resolution 352/2021/R/eel established the regulatory foundation for the role of the independent aggregator, a new market participant who aggregates consumption or generation units without being the user's energy supplier.

These resolutions represent crucial steps in the implementation of the Clean Energy Package at the national level, fostering a more open and dynamic energy market structure in Italy.

At Italian level, thanks to the EU new legislations, efforts are currently being made on two distinct yet interconnected fronts. First, a coordination mechanism between the Distribution System Operator (DSO) and the Transmission System Operator (TSO) is being developed to ensure that users connected to the distribution grid can also participate in the provision of system-wide flexibility services, without compromising the reliable operation of the distribution network. Second, local flexibility markets are being designed to leverage distributed flexibility as an alternative or complement to traditional grid reinforcement measures. In this evolving context, the DSO takes on two new roles: acting as a neutral facilitator for enabling distributed resources to provide services to the TSO, and as a buyer of local flexibility to meet specific needs within the distribution grid.

The general context in which the Italian legislation is changed is depicted in Figure 1-36:

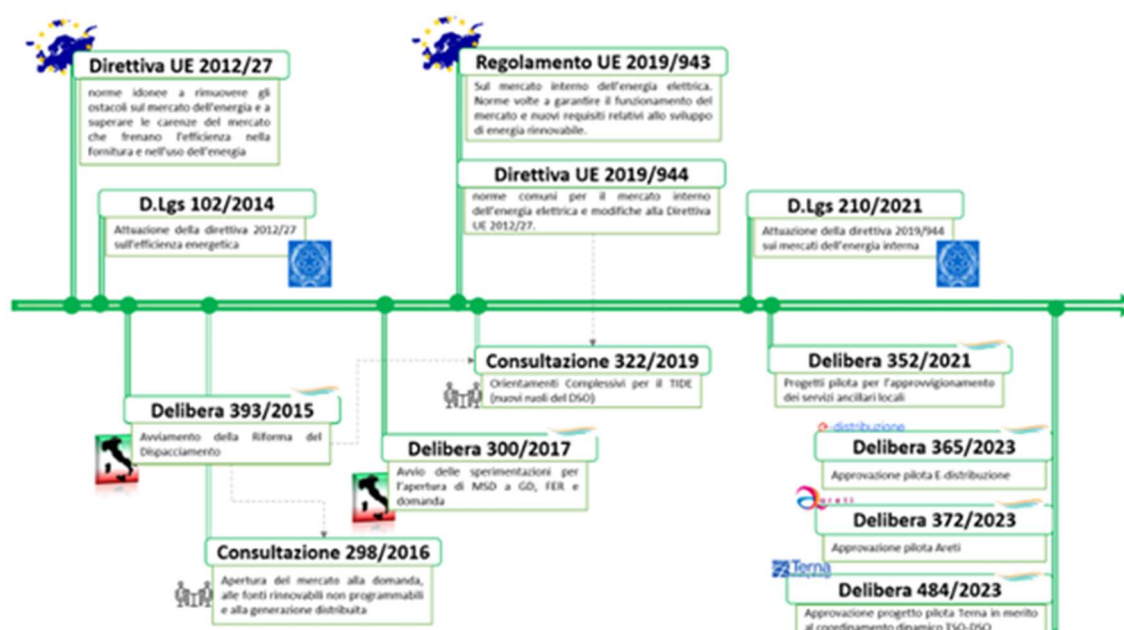


Figure 1-36: Development of the European and Italian regulation with respect to new roles of DSO [21]

Going into the detail, with Resolution 300/2017/R/eel, the Italian Regulatory Authority for Electricity, Gas and Water Services established the criteria to enable demand-side resources, non-enabled generation units, such as those powered by non-dispatchable renewable sources and distributed generation, and storage systems to participate in the Ancillary Services Market (MSD) through pilot projects.

Pending the adoption of a new Integrated Text on Electricity Dispatching, and following the public consultation launched with Document 298/2016/R/eel, Resolution 300/2017/R/eel initiated a first pilot phase aimed at gathering practical insights to support a comprehensive reform of the dispatching system. This initial phase also sought to immediately make new dispatchable resources available. Specifically, the pilot projects were to be conducted according to harmonized procedural criteria defined by Terna, the Italian TSO, potentially also based on proposals submitted by market participants.

Regarding the new two roles that the DSO assumes with respect to its presence in the TSO-managed markets, different pilot projects highlight this double mechanism.

1.7.2 DSO as buyer of services: local flexibility markets

At the beginning of this new process, flexibility procurement has been approached through pilot projects, starting with the identification of required services and corresponding system needs. Given the lack of well-established best practices, ARERA has adopted a pragmatic and adaptive strategy: rather than imposing rigid procedures, the Authority has allowed Distribution System Operators (DSOs) to define the methods and implementation strategies for acquiring local flexibility services. This bottom-up approach is intended to test different models and help assess which services are most effective, how they should be procured, and what remuneration mechanisms are appropriate.

By the end of 2023, two major pilot initiatives had been approved in Italy:

- the EDGE project, led by E-distribuzione, and
- the RomeFlex project, launched by Areti, which also serves as a regulatory framework reference for MindFlex, a related initiative by Unareti.

The EDGE project is structured around long-term contractual arrangements for flexibility procurement. In contrast, the RomeFlex project adopts a more diversified approach, combining long-term contracts with short-term market mechanisms that enable procurement closer to real-time operation.

The procurement of flexibility services begins with a preliminary phase in which the service and its technical characteristics are clearly defined, such as the direction of modulation (upward or downward), the required infrastructure (e.g., metering equipment), and activation conditions (e.g., response times). This is followed by a qualification phase, during which individual resources or aggregated units are technically

assessed against the service requirements. Once these steps are completed, flexibility is procured through either the forward market, the spot market, or a combination of both, depending on the specific design of the procurement mechanism.

1.7.2.1 Forward Market and Spot Market

The forward market is based on long-term contracts between the Distribution System Operator (DSO) and flexibility providers (Service Providers – SPs). These contracts ensure the availability of flexibility services over predefined time windows.

Contracts include:

- Availability price (€/kW): paid for the committed capacity made available.
- Utilization price (€/kWh): paid for the energy actually dispatched.

SPs commit to being available within specific timeframes, even if not activated. Activation is carried out directly by the DSO based on system needs.

This model offers several advantages: it provides economic certainty for participants, encourages the growth of flexible resources by supporting long-term investment planning, and enables DSOs to perform cost-benefit analyses comparing flexibility procurement to traditional grid reinforcement.

On the other hand, the spot market (Mercato a Pronti) allows DSOs to procure flexibility in the short term, closer to real-time operations. In this model, only pre-qualified resources can participate by submitting offers containing a single utilization price (€/kWh) for the energy to be delivered. SPs are activated only if their offers are selected. This mechanism promotes market transparency and competitiveness, as prices reflect real-time supply and demand dynamics. It also gives DSOs more operational flexibility to respond quickly to emerging needs.

An increasingly common solution is the hybrid approach, which combines the advantages of both mechanisms. For example, in the RomeFlex project, long-term contracts are used to guarantee a minimum level of available flexibility, while spot market sessions are held to optimize costs and expand participation. This dual system supports both the stability needed for investment and the responsiveness required for efficient grid management.

1.7.3 DSO as neutral facilitator

With the increase in distributed resources (such as renewable generation and storage), the TSO (Transmission System Operator) has begun to consider these resources for dispatching services. However, since these resources are connected to the distribution grid, managed by the DSO (Distribution System Operator), it is the DSO's responsibility to ensure operational security. For this reason, the DSO takes on a new role as a neutral facilitator, meaning it enables or restricts the participation of distributed resources in the already existing markets managed by the TSO (MSD), ensuring that this does not compromise the local network.

Since these resources are connected to the distribution networks, their activation by the TSO could generate unexpected power flows, potentially leading to overloads or operational violations on the local grid. The DSO, being responsible for the safety and reliability of its network, must therefore be able to assess the grid's conditions in real time and communicate clearly and efficiently to the TSO whether and to what extent resources can be activated.

To enable a reliable exchange, a color-coded mechanism has been introduced by Terna to identify critical areas of the distribution network. Specifically:

- **Green** indicates that the network is operating normally, and distributed resources can participate without restrictions in the provision of services to the TSO.
- **Yellow** means that potential issues may arise; participation is partially allowed, but subject to certain technical limitations (e.g., reduced available capacity).
- **Red** signals that the network is in a critical state, and therefore the participation of distributed resources must be completely blocked to avoid compromising the safety and reliability of the grid.

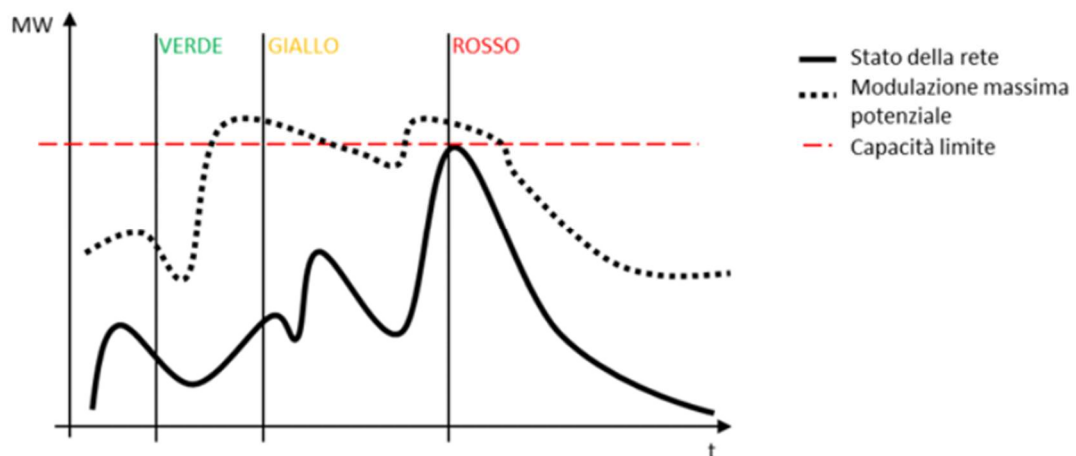


Figure 1-37: Concept of Traffic Light [21]

To effectively apply the Traffic Light mechanism (Figure 1-37), it was necessary to introduce the concept of DSO perimeters, defined portions of the distribution network on which the local operator (DSO) can evaluate and communicate technical constraints. Each perimeter represents an area of the grid that may include multiple connection points (PODs) and distributed resources. A color (green, yellow, or red) is assigned to each perimeter based on local operating conditions, indicating whether and to what extent the connected resources can participate in providing services to the TSO (Figure 1-38).

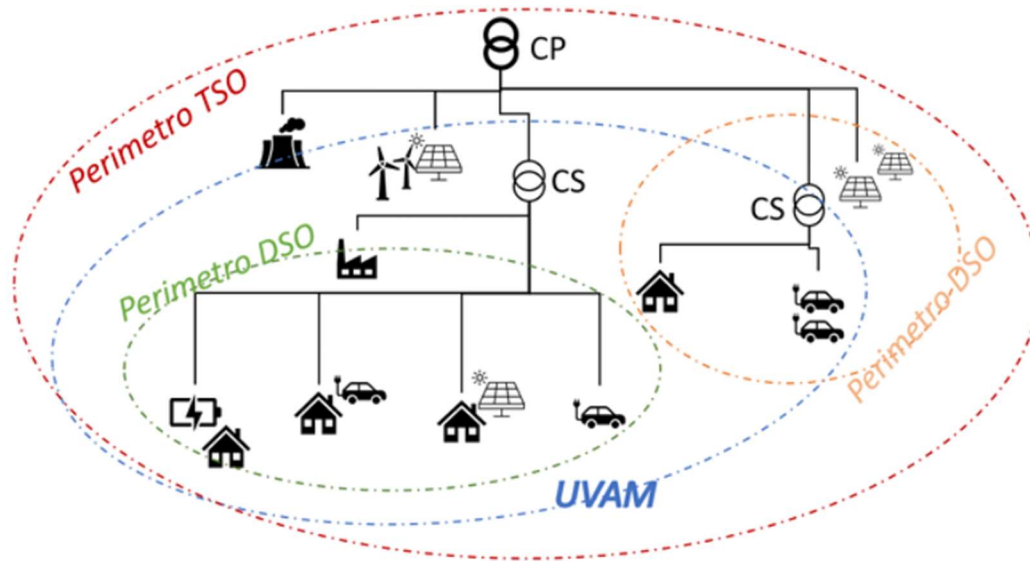


Figure 1-38: Example of generic configuration UVAM [21]

Using perimeters allows the DSO to avoid placing constraints directly on individual resources, simplifying communication and leaving it to the TSO to select which resources to activate in compliance with the stated limits. However, defining perimeters requires a balance between technical granularity and operational complexity: perimeters that are too broad may overlook local bottlenecks, while highly detailed perimeters increase coordination challenges.

The Traffic Light mechanism can be implemented in two ways: static or dynamic, each with different characteristics and impacts on system operation.

In its initial implementation, the static Traffic Light served as a prequalification process: during the registration phase of flexibility aggregations (e.g., UVAM), the DSO would perform a one-time assessment of the network's technical conditions to determine whether a given resource could participate in the Balancing Market (MSD) without endangering the stability of the distribution grid. This assessment was based on static parameters such as contractual limits, voltage level, and nominal capacity, and it remained valid unless a significant change occurred (e.g., a reconfiguration or upgrade). While operationally simple, the static approach proved overly conservative, as

it forced DSOs to reject or constrain resources preemptively, even during periods when the network was actually capable of accommodating flexibility.

To address this limitation, Terna introduced the dynamic Traffic Light mechanism, officially regulated in 2023, allowing DSOs to update the operational status of the grid in near-real time.

The Topological Matrix (MaT) is a key tool for enabling operational coordination between the TSO (Terna) and DSOs in managing distributed flexibility resources. It defines, in a structured and standardized way, the correspondence between market-registered resources and the actual configuration of the distribution network, allowing each resource (or aggregated group) to be linked to a specific DSO perimeter and its corresponding TSO market perimeter. To build the MaT, the DSO is required to provide a set of technical parameters for each resource connected to its grid, as shown in Figure 1-39. These include the voltage level (low or medium), the exact connection node, the electrical topology, and the available modulation capacity in both upward and downward directions. The DSO must also report any structural or operational limitations, such as current-carrying constraints on lines, transformer capacity, or network conditions affected by scheduled maintenance or degraded states (e.g., N-1 operation).

This information enables Terna to accurately map distributed resources within its dispatch model and to embed local constraints into the clearing process of the Balancing Market (MSD), ensuring that only technically viable resources are activated.

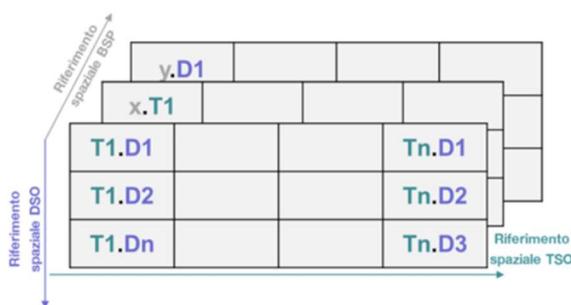


Figure 1-39: Qualitative representation of Topological Matrix [21]

1.7.4 Coordination of local and global markets

The topic of TSO-DSO coordination is widely discussed in technical literature, not only in relation to the DSO's role as a neutral facilitator, but also regarding the market design to be adopted when the DSO acts as a buyer of local ancillary services. The European project SmartNet explored several coordination schemes, showing that operational models are heavily influenced by the type of market structure adopted: whether a common market (where TSO and DSO operate jointly) or separate markets (with independent procedures); whether shared or distinct services are offered; and whether there is a single counterparty or multiple counterparties. Depending on the chosen configuration, the coordination logic changes, as does the nature of the information exchange required to ensure a competitive, transparent, and efficient procurement of flexibility resources.

In the Italian pilot phase launched in 2023, ARERA Resolution 352/2021 introduced a simplification by requiring that, for each resource (POD), the aggregator offering global services must also be the one offering local services. Nonetheless, even under this constraint, it remains essential to define priority mechanisms for resource selection, balancing economic efficiency objectives with the technical needs of both grid operators. This challenge becomes even more complex in the presence of long-term contracts, which impose offer obligations and thus influence the order in which resources are selected across different markets.

Overall, the coordination between local and global markets can be structured around two key dimensions:

- the definition of the information to be exchanged between TSO and DSO throughout the various operational phases (prequalification, forecasting, clearing, activation, settlement);

- the establishment of prioritization rules to determine which market should have precedence when multiple offers are submitted for the same resource.

To introduce the entire framework, it's important to highlight that in general the flexibility procurement mechanisms can be divided in five steps:

- Preparatory phase: definition of flexibility products, technical specifications, participation requirements, and prequalification of resources.
- Forecasting phase: analysis of expected demand and generation, identification of potential network issues, and estimation of flexibility needs.
- Market phase: opening of auctions (either spot or long-term), collection of bids, and selection of the required flexibility volumes.
- Activation and monitoring phase: activation commands are issued, service execution is monitored, and any necessary corrective actions are taken.
- Measurement and settlement phase: verification of the service delivered based on certified measurements, validation against baselines, and financial settlement including payments and penalties.

Information exchange

Stated that this subdivision plays a crucial role with respect to the type of interaction between TSO and DSO, let's use this base to analyze the first dimension: information exchange

The type of market model adopted, whether a common market or separate markets, has a direct impact on the nature, complexity, and structure of information exchange between the TSO and the DSO.

In a common market model, where TSO and DSO operate within a shared platform or coordinated process, the information exchange is essentially embedded in the market operation itself. While tight synchronization is required, it can be more naturally integrated through a joint infrastructure.

In contrast, in a separate market model, where TSO and DSO manage independent procurement processes, information exchange becomes more critical and structured, since it must prevent conflicts or overlaps in the activation of the same resources and enable the operator that runs the second market phase (either TSO or DSO) to factor in the results of the other.

In summary, the more separated the market processes are, the greater the volume and granularity of information that needs to be exchanged. This, in turn, demands a more advanced and reliable ICT infrastructure to ensure effective coordination and transparency.

The following schedule provides examples of key information required to ensure effective coordination between the TSO and DSO. As said, the selection and scope of these data exchanges are closely tied to the chosen market model, as different configurations affect both the type and level of detail of the information that needs to be shared.

Preparatory Phase $\xrightarrow{\text{relevant info}}$ enabled resources, product definition, required services, requests for qualification.

Forecasting Phase $\xrightarrow{\text{relevant info}}$ expected constraints, flexibility needs, temporary requirements

Market Phase $\xrightarrow{\text{relevant info}}$ selected volumes, selected resources

Activation and Monitoring $\xrightarrow{\text{relevant info}}$ congestion resolution checks, activation monitoring

Control and Settlement $\xrightarrow{\text{relevant info}}$ measurements, baseline

Procurement priority

The second dimension is the procurement priority; in the document three well known possibilities are described; these schemas are derived by different international projects such as SmartNet and BeFlexible.

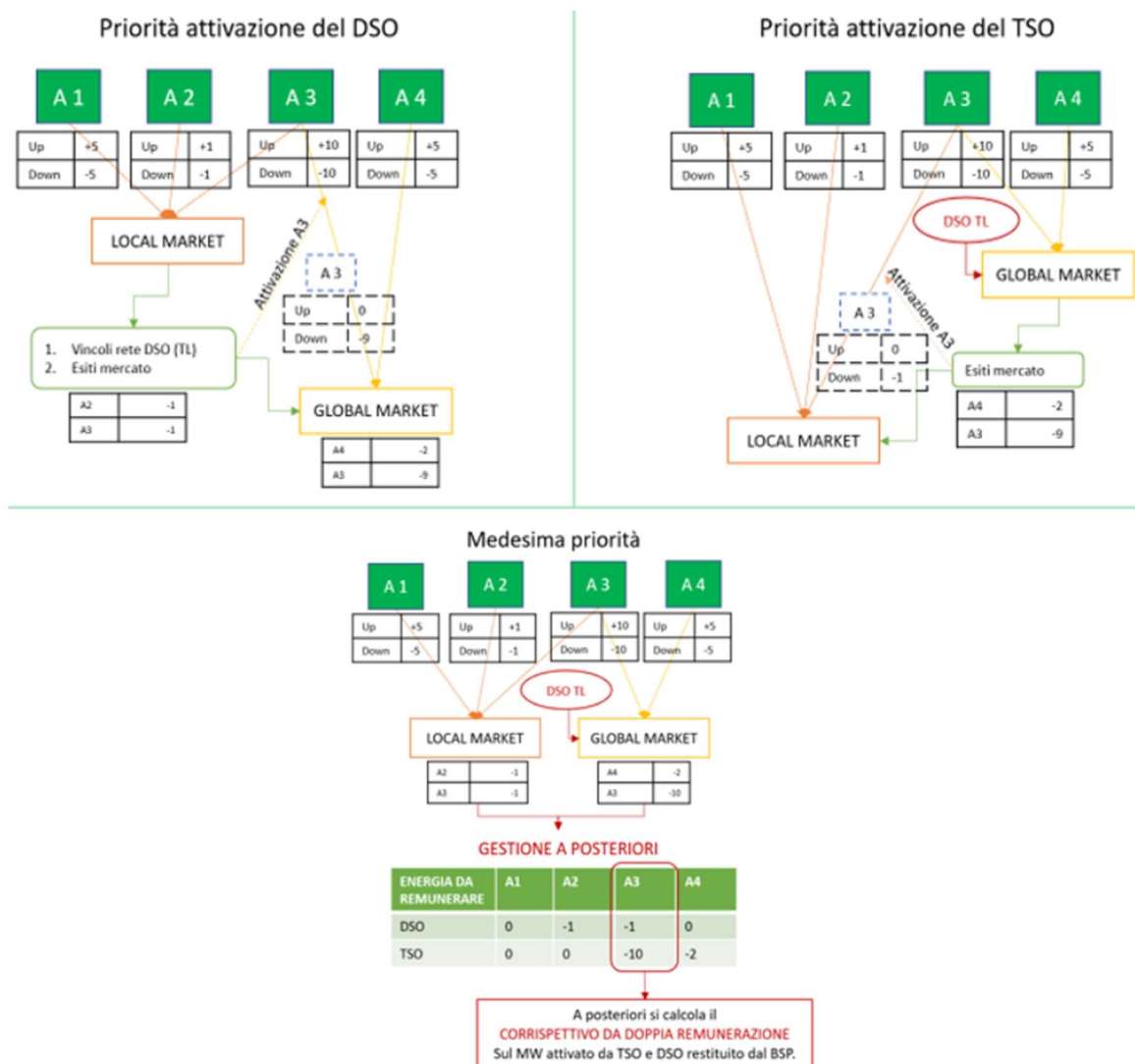


Figure 1-40: Priority schemes of bids selection [21]

The DSO-priority model represented in Figure 1-40, where the local market closes before the global one, is the approach currently adopted in Italy during the experimental phase, as established by ARERA Resolution 352/2021. Therefore, it is being implemented in practice in this context.

In particular the separate market model was chosen also based on the past experiences of Italian DSOs in European projects, including:

The Platone project, with the participation of Areti, the DSO of the municipality of Rome, whose involvement enabled the launch of the RomeFlex pilot.

The CoordiNet project, with the participation of E-Distribuzione, which served as the basis for the experimental activities carried out in the EDGE pilot project. Within this model, the DSO publishes its local flexibility needs, and market operators submit their bids, which may be identical to those submitted to the national Balancing Market (MSD) managed by Terna. However, local offers are processed independently, via a parallel channel. Once the local market session closes, the DSO communicates the results to Terna, enabling the TSO to take them into account during the MSD clearing process and to exclude resources already activated at the local level. This mechanism ensures that the needs of the distribution grid are addressed first, helps avoid activation conflicts between TSO and DSO, and maximizes the efficient use of distributed flexibility in a technically sound and coordinated manner.

To have a clearer idea of the coordination between resources in MSL and MSD, the temporal pianification of MSL and MSD is highlighted in Figure 1-41.

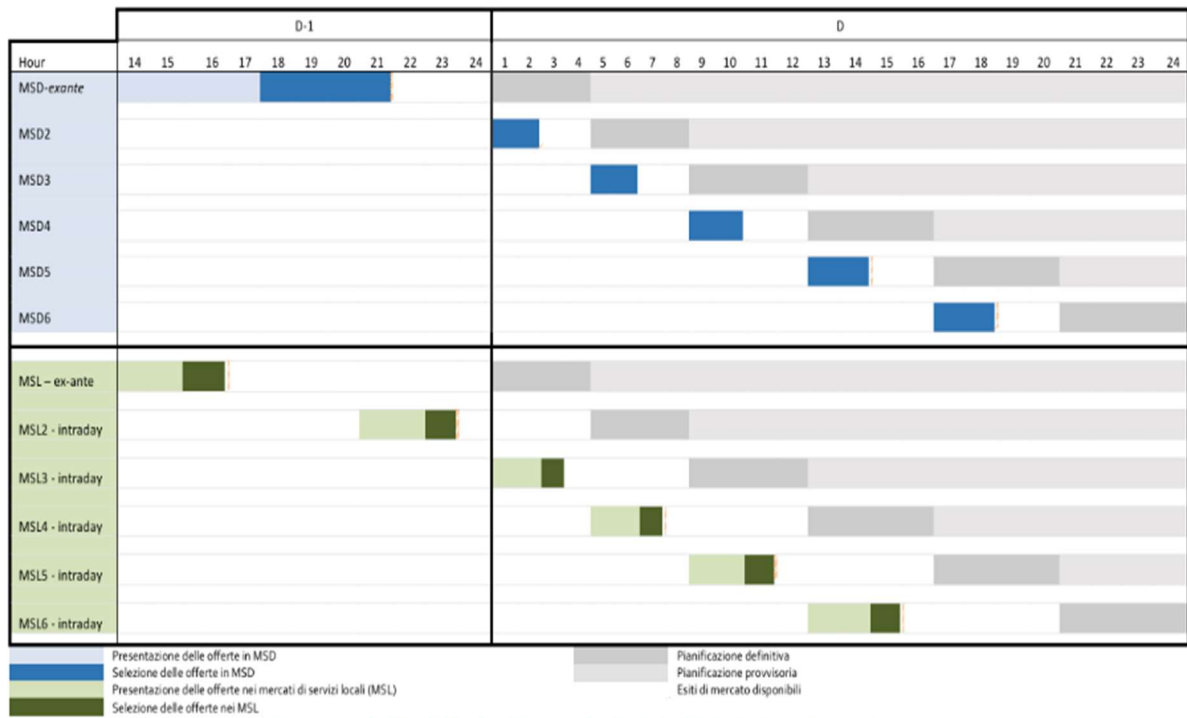


Figure 1-41: Temporal scheme of local and global market pianification for comparison [21]

The other two schemes proposed (TSO priority and co-optimization) are considered theoretical or alternative models, which have been studied and tested in other countries or pilot projects, but have not yet been adopted within the Italian regulatory framework.

2. CHAPTER 2

The first chapter is used to clarify that the introduction of the RES has the double effect of increasing the number of flexibility resources that can be effectively used to solve congestion on both transmission and distribution network and on the other hand of the urgent need of flexibility adjustment considering that the source of uncertainty introduced by the usage of more RES is increasing.

Different methodologies are exploited to correctly implement the participation of RES in existing markets or to introduce new different markets whose objective are clarified above.

With that settled, this second part is dedicated to the mathematical formulation of a possible market structure including in the model the uncertainty introduced by the power output both of the RES and of the possible flexible loads; first the existing literature is analyzed and then the introduction of the uncertainty as a key element is introduced in the model.

The paper taken as reference to start the analysis is 'Optimal management of flexibility services in LV distribution grids' [22] and it's the basis to introduce the mathematical formulation for the model of the local flexibility market.

In the paper two possibilities are exploited:

- 1) the DSO solves its congestions in the local market without unbalancing the HV grid
- 2) First case completed by the remaining flexibility that's not dispatched in the local market is aggregated to be offered in the traditional ASM.

The goal is reached with an optimization model with two different objective functions based on the 1) and 2) approaches. If the market operates in 1) logic, then the total cost of activated flexibility is minimized; in the case in which the market operates in 2) and the total available flexibility from the network is required, then the total cost of activated flexibility needs to be maximized in one of the two directions, upwards (UP) or downwards (DW).

Together with the LV network model to identify which are the classical constraints to be satisfied with the flexibility, the flexibility model is formalized. In this article the flexibility is intended to be on the real power output of the PV and loads (and the reactive power is regulated through a constant power factor); specifically, the power output of the PV is defined as a constant value $P_{i,POD}^0$ on which it's possible to activate flexibility UP and the power output of the demand is regulated to activate DW flexibility.

The choice of modeling the power output of the PV or power absorbed by the demand as constant is not correct in principle because of a certain variability of the prediction to which the forecasting is subject. This introduces a considerable error in the resolution of the optimization model due to the fact that the solutions could be too optimistic with respect to the real power output with its characteristic variability.

The aim of this thesis is to deepen the analysis of the uncertainty introduced both by forecasting of PV and flexible load profiles and to understand their effect on the solution of the optimization model that represents the local flexibility market.

2.1 Overview on the optimization methods

Introducing the uncertainty in the model, the mathematical formulation must evolve from the Linear Programming (LP) into Robust Optimization, changing also the way of resolution of the algorithm.

In [23] the possible modeling techniques of uncertainty and risk are explored (the risk is defined as a likelihood of a loss, while uncertainty is the lack of knowledge about the likelihood of a loss); focusing on the uncertainty part the following uncertainty handling techniques are grouped. As depicted in Figure 2-42 the Robust optimization part is one of the exploitable branches.

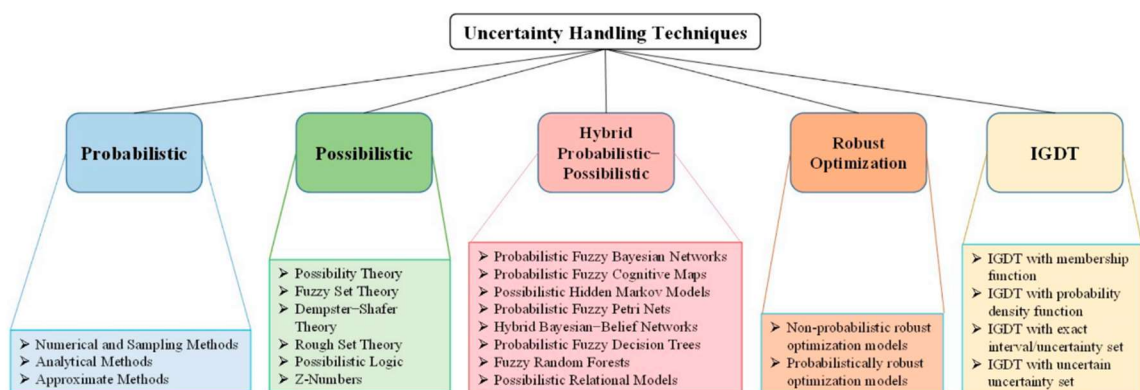


Figure 2-42: Uncertainty Handling Techniques [23]

Considering that robust optimization can be one of the right approaches to be used when the optimality of the solution must be found, it's important to make a clear comparison between the uncertainty optimization techniques (including RO) and the classical deterministic optimization (Figure 2-43).

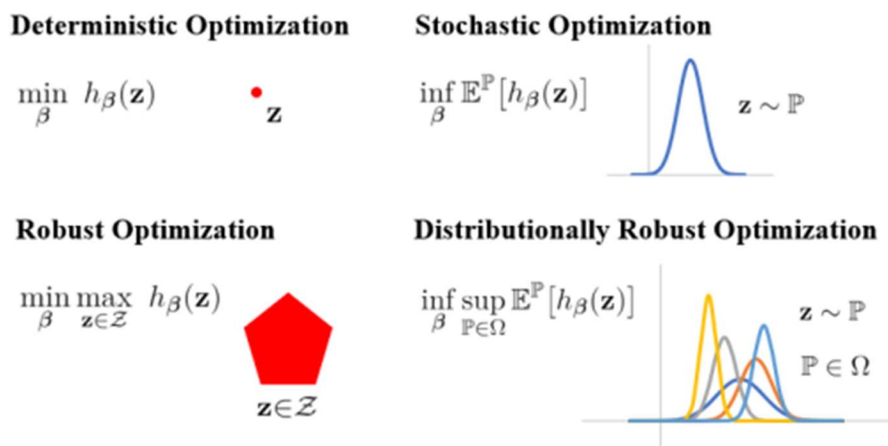


Figure 2-43: Comparison of robust optimization with distributionally robust optimization [24]

Deterministic optimization assumes that the data z is known exactly, and thus directly minimizes the objective function $h_\beta(z)$ with respect to the decision variable β .

In contrast, stochastic optimization models z as a random variable distributed according to a known probability distribution P and aims to minimize the expected value of the objective, $E^P[h_\beta(z)]$.

Robust optimization adopts a conservative perspective by considering that z may lie anywhere within an uncertainty set Z and seeks to minimize the worst-case realization of the objective function over this set.

Finally, distributionally robust optimization (DRO) generalizes the stochastic approach by acknowledging ambiguity in the underlying probability distribution itself. Rather than assuming a fixed distribution, DRO considers a family Ω of plausible distributions and minimizes the worst-case expected value of the objective function across all $P \in \Omega$.

Choosing worst-case robust optimization is preferable in this context where safety is paramount: guaranteeing constraint feasibility for all realizations within a specified uncertainty set, with no distributional assumptions, makes the method 100% reliable exploiting its conservativeness.

To design a qualitative framework before introducing the complete mathematical modeling, let's compare graphically solutions of deterministic optimization and RO.

Comparing the deterministic optimization (such as the MILP) with the robust optimization (Figure 2-44), it's clear that the approaches of the two methods are different because they tend to two different perspectives.

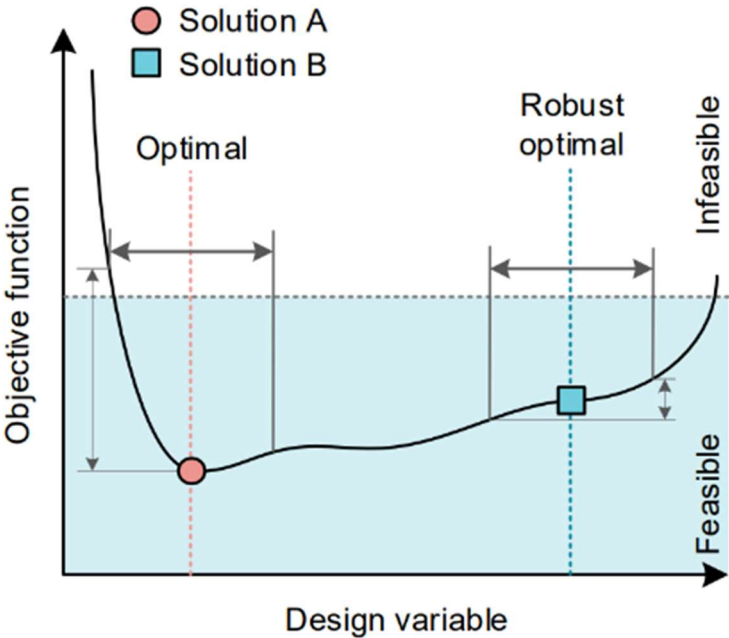


Figure 2-44: Comparison of deterministic optimization and RO [23]

Solution A, derived from deterministic optimization, excels in optimality but shows considerable variation in the objective function relative to changes in the design variable, potentially extending into infeasible ranges. On the other hand, Solution B, achieved through RO, demonstrates moderate optimality and excels in robustness, maintaining a narrow dispersion in the objective function as the design variable varies.

2.2 Robust optimization focus

The capability of managing uncertainty and having a correct balance between optimality and robustness is a key element of the success of Robust Optimization in different fields of the power system sector.

This relies on the fact that the operation and planning of electric power systems require continuous decision-making across various time horizons. At the operational level (from one week to real-time), decisions involve how to schedule and dispatch generation units to meet electricity demand. At the operation-planning level (from several months to one week), decisions concern fuel procurement (nuclear, coal, natural gas, biomass, etc.) and preventive maintenance of system components. Finally, at the strategic planning level (over several years), decisions focus on the expansion of the grid (both transmission and distribution) and the construction of new generation facilities.

In all these cases, decisions must be made under significant uncertainty, which has become even more pronounced due to the growing integration of non-dispatchable renewable energy sources (such as wind and solar), making the available generation capacity highly variable.

The increasing adoption of distributed energy resources (e.g., rooftop solar panels) further amplifies the uncertainty on the demand side as seen by the system operator.

Robust Optimization seems to be a good choice to introduce the uncertainty in this context for different reasons. First and foremost, RO provides strong guarantees for operational reliability because it enables the design of decisions that remain feasible and effective even under the worst-case realizations within a defined uncertainty set, thereby ensuring secure system behavior.

Secondly, RO can manage the substantial variability in wind and solar generation without requiring precise knowledge of the underlying probability distributions

which are often difficult to estimate in practice, offering greater flexibility in handling renewable energy sources.

Another key advantage is its ability to efficiently model the trade-off between cost and security. RO allows system planners to explicitly balance risk and cost, often without the need to generate a large number of scenarios, as required in Stochastic Programming (SP). This makes RO particularly attractive in operational and planning contexts where computational efficiency matters.

Finally, RO stands out for its scalability and practical applicability. Compared to multi-stage SP models, RO is generally easier to implement, less computationally intensive, and more readily integrated into existing planning software used in the power sector. These features make Robust Optimization a powerful and pragmatic approach for decision-making under uncertainty in modern and increasingly complex electric energy systems.

Figure 2-45 [25] is introduced to provide a more detailed information on the use cases in which the RO, in particular the two stage one, is used. UC stands for Unit commitment, ED stands for economic dispatch, PQ stands for active/reactive power coordination, RD stands for resilient dispatch.

Comparison analysis of TSRO scheduling problems.

Problems	Similarities	Highlighting issues	Existing solutions	Effects	Limitations
UC	Two-stage decision framework (fixed/flexible unit plans before/after uncertainty)	<ul style="list-style-type: none"> Power flow of transmission network Nonanticipativity issue Multi-area coordinated scheduling 	<ul style="list-style-type: none"> Distribution-factor based transmission flow or DC power flow Affine/implicit decision rule Distributed optimization SOCP or linearized Dis-flow Game-theory based scheduling method 	<ul style="list-style-type: none"> Transmission capacity involved All-scenario feasibility Multi-area tie-line scheduling Approximate power flow included Coordination of multi-interests 	<ul style="list-style-type: none"> Power loss and phase angle neglected Dimension disaster caused by auxiliary variables Computation burden for large-scale systems Inaccurate power flow Non-guaranteed solution optimality and convergence for nonconvex robust game model
ED		<ul style="list-style-type: none"> Power flow of distribution or MG network Multi-stakeholder coordinated operation 			
PQ		<ul style="list-style-type: none"> Coupling of active and reactive power 	<ul style="list-style-type: none"> Linearized Dis-flow or SOCP for networks Fixed power factors or piecewise droop-control model for DGs 	<ul style="list-style-type: none"> Power loss, voltage limit and unit capacity included 	<ul style="list-style-type: none"> Inaccurate PQ flow Low computational efficiency for equivalent MIQP or MISOCP model
RD		<ul style="list-style-type: none"> Non-uniform objectives before and after uncertainty Multi-dimensional uncertainty of disturbance in object, time and space 	<ul style="list-style-type: none"> Objective-decoupled TSRO model Discrete-state set for disturbance 	<ul style="list-style-type: none"> Robust plans that meet differentiated operational requirements under uncertainty 	<ul style="list-style-type: none"> Simple disturbance uncertainty sets that may be unrealistic No uniform index for characterizing system resiliency

Figure 2-45: Comparison analysis of TSRO scheduling problems [25]

Nowadays the RO is being used also in some applications that involve the flexibility resources. For example, the paper [26] proposes an innovative approach to analyze the local flexibility of Virtual Power Plants (VPPs) using robust optimization models, taking into account the uncertainty associated with renewable energy production (such as photovoltaic) and electricity demand. VPPs are distributed systems that aggregate local energy resources (generation, storage, demand) to provide flexibility services to the market, improving the efficiency and stability of the grid. The proposed model is structured in two phases: a first phase of Demand Side Management (DSM) to shift the demand based on renewable energy production forecasts, and a second phase in which the optimized demand is met using local resources (such as CHP generators, batteries, or the grid). The goal is to evaluate, for different local VPP configurations, the offered flexibility and the related costs, considering two possible objective functions: minimizing operational costs and minimizing the energy exchange with the external grid, thereby increasing local self-consumption.

Another example is represented by [27]. The paper investigates the integration of renewable energy sources (RES) and electric vehicles (EVs) into microgrids through Vehicle-to-Grid operations, representing both an example of generation and load. The authors emphasize the uncertainty associated with wind power generation and the state of charge of EVs, proposing a robust optimization framework to ensure reliable system operation even under worst-case scenarios. Simulation results show that while robust optimization entails higher operating costs, it significantly improves system resilience, reduces reserve violations, and guarantees secure scheduling. The study further introduces the possibility of using adjustable “dispatch interval factor” to balance conservatism and economic performance, thereby offering decision-makers a flexible trade-off between robustness and cost-efficiency. However the conservatism of the robust optimization process is more promising with respect to the stochastic approach simplification, giving the opportunity to exploit it in the flexibility market context.

2.3 Mathematical model of RO

A typical optimization problem with uncertainty can be written as

$$\min_{x \in X} \{f(x) : g(x, u) \leq 0, \forall u \in U\} \quad (1)$$

Where $x \in X$ is the vector of decision variables, $f(x)$ is the objective function that should be minimized in this example, $g(x, u) \leq 0$ represents the inequality constraints of the problem, where $u \in U$ are the uncertain parameters, and U is the uncertainty set, which describes the possible realizations of the uncertain parameters.

In RO, the goal is to ensure that the solution x is feasible for all the possible values of the uncertain parameter u within the uncertain set U . The general RO formulation of a min-max problem is as follows:

$$\min_{x \in X} \max_{u \in U} f(x, u) \quad (2)$$

The lower problem ensures that the worst-case scenario is considered for the uncertain parameter u and the upper problem tries to find the best decision x considering the worst-case scenario.

2.4 Uncertainty modeling

About the uncertainty, it is generally modeled using scenarios in stochastic programming and using either robust sets or distribution bounds in robust optimization.

A scenario is a given realization of the uncertain parameter vector u that is represented by u^ω , with an occurrence probability of p^ω . Scenarios should describe all possible uncertainty realizations and thus $\sum_{\omega} p^\omega = 1$.

Robust sets are generally based on boxes or polyhedral constraints. The uncertain parameter $u_i \in R$ in the vector $u \in U$ is considered to be centered into a reference value u_i^{ref} and it can vary at most $\pm u_i^\Delta$. Thus

$$u_i \in [u_i^{ref} - u_i^\Delta, u_i^{ref} + u_i^\Delta] \quad (3)$$

An uncertainty budget can be imposed on the robust set define above by the constraint:

$$\sum_{i=1}^n \frac{|u_i - u_i^{ref}|}{u_i^\Delta} \leq \Gamma \quad (4)$$

Where Γ varies between 0 and n with n the total number of uncertain variables considered in the model. $\Gamma = 0$ implies no uncertainty since each uncertain parameter equals its reference value, while $\Gamma = n$ implies maximum uncertainty, as each uncertain parameter deviates the maximum from its reference value.

The importance of the uncertainty budget is that the intermediate values of Γ are used to introduce different levels of uncertainty based on the compromise between the conservativeness and the optimality of the solution. It's a way to give a measure of how much the constraint imposed has to impact on the problem and consequently on the solution, providing a more or less robust solution (the Γ more or less lets the constraint weight).

2.5 Conversion of the constraints

The introduction of an uncertainty in the general linear constraint with the form $a^T x \leq b$ has to be treated carefully. The basic objective of the robust optimization is to take a decision that's optimally valid for all the possible realizations of the uncertain parameter that can vary within a bounded set, the so-called uncertainty set.

To give an example if a is the variable parameter and the uncertainty set is an hypercube the uncertain parameter is centered around a fixed value a_0 and can vary in a range of $\pm\Delta$, so the $a \in [a_0 - \Delta, a_0 + \Delta]$.

This means that the objective now is to find a robust solution that satisfies the linear constraint with the form $a^T x \leq b$ for all the realizations of the parameter a that varies in the range defined before, that has a form of a hypercube.

To do so, it's important to put the optimization problem in the correct mathematical formulation. Some steps have to be done in order to achieve this goal.

Starting from the linear constraint with the form $a^T x \leq b$ and adding the uncertainty term,

the new equivalent form becomes

$$(A_0 + \xi\Delta) * x \leq b \quad (5)$$

$$A_0 x + \xi\Delta x \leq b \quad (6)$$

The term Δ is the uncertainty term controlled by the factor $\xi \in [-1; 1]$, that makes the uncertain parameter vary between $[A_0 - \Delta, A_0 + \Delta]$.

This constraint given as example is a lower bound constraint, meaning that the worst case to be treated in the robust optimization solution is the one in which the uncertain term is maximized; this guarantees that the solution found is the most robust possible. It covers even the worst case: if such a case happens, the solution will be good to solve the worst contingency.

Clearly this approach implies an overestimation of the problems that could arise in the grid; at the same time it guarantees a 100% level of security.

The introduction of an uncertain parameter produces an infinite set of new constraints due to the infinite number of realizations that the uncertain parameter can introduce.

To solve the problem the key is to reconvert this infinite set of boundaries (in particular upper and lower bound) in a unique constraint.

To apply this logic, the linear constraint over a box uncertainty set can be reformulated in the following steps[28]. [28]

If each uncertain coefficient a_i takes value in an interval around a nominal value \bar{a}_i with width Δ_i , i.e. $a_i \in [\bar{a}_i - \Delta_i, \bar{a}_i + \Delta_i]$ for $i = 1, 2, \dots, n$ the uncertain vector a is in a box uncertainty set U

$$U_{box} = \{a \in R^n: \exists \xi \text{ s. t. } a = \bar{a} + \Delta \xi, \|\xi\|_\infty \leq 1\} \quad (7)$$

Where \bar{a} is the vector of nominal value a_i 's and Δ is a diagonal matrix with Δ_i 's on the diagonal.

The l_∞ norm $\|\xi\|_\infty := \max_{i=1,2,\dots,n} \|\xi_i\|$.

Considering the linear constraint

$$a^T x \leq b, \forall a \in U_{box} \quad (8)$$

$$\Leftrightarrow \max_{\|\xi\|_\infty \leq 1} \{\bar{a}^T x + \xi^T (\Delta x)\} \leq b \quad (9) \Leftrightarrow$$

$$\bar{a}^T x + \|\Delta x\|_1 \leq b \quad (10) \Leftrightarrow \bar{a}^T x + \sum_{i=1}^n |\Delta_i x_i| \leq b \quad (11)$$

$$\Leftrightarrow \begin{cases} \bar{a}^T x + \sum_{i=1}^n t_i \leq b \\ -t_i \leq \Delta_i x_i \leq t_i, \forall i = 1, 2, \dots, n \end{cases} \quad (12)$$

where (10) comes from the fact that the l_∞ norm and the l_1 norm are dual to each other. Constraint (12) replaces the absolute value function in (11) using additional variables t_i 's. Notice that the ξ that achieves the maximum in (9), i.e., the worst-case scenario ξ ,

has the form $\xi_i = \pm 1$ for each i . In other words, the worst-case scenario takes the largest deviation from the nominal value.

This logic has to be applied as said on upper and lower bounds of the mathematical formulation; thus, the strong duality has to be applied in a problem in which the uncertainty term appears only in the upper and lower bounds to correctly convert the robust problem in a new deterministic problem and to solve it with well-known techniques. So, the mathematical model has to be accordingly shaped, transposing the uncertainty term introduced in the equality constraints in the bounds.

2.6 From deterministic to robust model

The deterministic model in place has the objective of minimizing the flexibility activated both in downward and upward direction such to solve congestions and voltage violations in the grid. It is composed by the following equations:

$$\min \sum_{POD} \pi_{POD}^{\uparrow} f_{POD}^{\uparrow} + \sum_{POD} \pi_{POD}^{\downarrow} f_{POD}^{\downarrow} \quad (13)$$

s. t.

$$A_{pod,b}^0 + f_{POD}^{\uparrow} - f_{POD}^{\downarrow} = v_b * i_{POD,b}^* \cong \widetilde{v}_b * i_{POD,b}^* \quad (14)$$

$$A_{D,b} = v_b * i_{D,b}^* \cong \widetilde{v}_b * i_{D,b}^* \quad (15)$$

$$A_{G,b} = v_b * i_{G,b}^* \cong \widetilde{v}_b * i_{G,b}^* \quad (16)$$

$$\forall b \quad \sum_G i_G + \sum_{POD} i_{pod,b} - \sum_D i_{D,b} = i_b \quad (17)$$

$$\forall l \quad i_l = Y_l * v_b \quad (18)$$

$$\forall b \quad i_b = \sum_{l \in b} i_l \quad (19)$$

$$V_b^{LB} \leq |v_b| \leq V_b^{UB} \quad (20)$$

$$|i_l| \leq I_l^{UB} \quad (21)$$

Where (14) is the equation that describes the apparent power at the bus b for flexible resources, composed by a fixed term $A_{pod,b}^0$ and the two terms related to the flexibility both in upward f_{POD}^\uparrow and downward f_{POD}^\downarrow direction; for linearization purposes the nodal voltage in (14) is fixed to the latest value \widetilde{v}_b available from an initial AC PF computation or a previous optimization. The index * on the currents stands for the conjugate being the model in complex numbers.

Constraints (15) and (16) describe the apparent power available from the non-flexible resources. Constraint (17) is the equation related to the nodal current injection at the bus b i_b that is the sum of currents coming from generators, loads and PODs.

Constraint (18) relates the current of the branches with the voltages through the matrix Y_l , whilst (19) expresses the current of the bus i_b as the sum of the current of all the branches that are connected to the bus b , i_l .

Constraints (20) and (21) are the upper and lower bounds constraints on voltage and line currents.

In the Robust Optimization model the $A_{pod,b}^0$ becomes uncertain and its possible value can be constrained by a box.

Any deviation from $A_{pod,b}^0$ value within the box will change the power flow solution and hence affect the values of v and i_l that are upper /lower bounded.

If violation occurs, then flexibility is activated to solve it; the objective of introducing the RO approach is to minimize the flexibility activated in the worst-case scenario that is verified in correspondence of the maximization of the objective function with respect to the uncertain parameter ξ .

The new objective function takes the form of a max-min problem (the dual of (2)):

$$\max_{\xi} \left(\min \sum_{POD} \pi_{POD}^{\uparrow} f_{POD}^{\uparrow} + \sum_{POD} \pi_{POD}^{\downarrow} f_{POD}^{\downarrow} \right) \quad (22)$$

s. t.

To correctly formulate the RO the objective function has to be subject to only inequality constraints. So, we should compact all the constraints (14)-(19) to obtain an explicit expression of the (20)-(21) with respect to the uncertainty introduced on $A_{pod,b}^0$.

However, this approach is not feasible in terms of model formulation and most likely numerical computations could be hard to process efficiently by optimization solvers since the model will be described by full matrices.

To avoid this issue the decomposition of the variables involved in (20)-(21) in a deterministic part related to the median value of $A_{pod,b}$ ($A_{pod,b}^0$) and a part related to the uncertainty associated to $A_{pod,b}$ ($A_{pod,b}^{\xi}$) is proposed .

The deterministic part will be given by the (14)-(19) while the uncertain part will be added only in the (20)-(21) constraints to the deterministic values of the voltages and current calculated through (14)-(19). The new constraints are:

$$V^{LB} \leq |v_b| + v_b(\xi) \leq V^{UB} \quad (23)$$

$$|i_l| + i_l(\xi) \leq I_l^{UB} \quad (24)$$

2.7 Model derivation

In this paragraph we focus on obtaining the terms $v_b(\xi)$ and $i_l(\xi)$ from (23) and (24). Let's consider a generic grid (Figure 2-46) for which it's possible to define the following parameters.

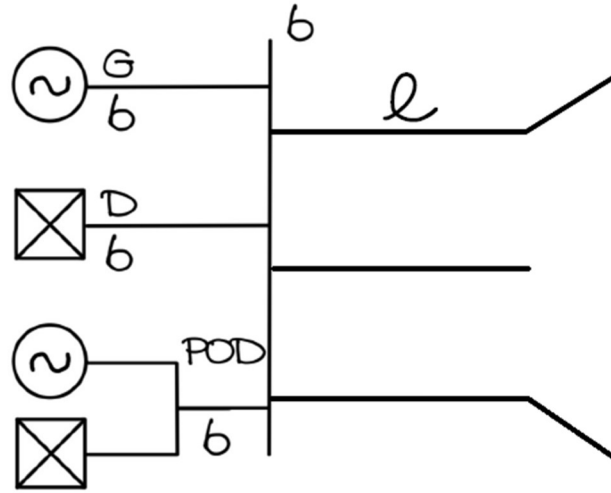


Figure 2-46: Generic grid structure

Sets:

$$b \in B \rightarrow \text{buses with } B = \{1, 2, \dots, N_B\}$$

$$g \in G \rightarrow \text{generators, without flexibility } G = \{1, 2, \dots, N_G\}$$

$$d \in D \rightarrow \text{demands, without flexibility } D = \{1, 2, \dots, N_D\}$$

$$pod \in POD \rightarrow \text{PODs, with flexibility } POD = \{1, 2, \dots, N_{POD}\}$$

$$M_G^b \rightarrow \text{subset of } G \text{ containing the generators connected to bus } b$$

$$M_D^b \rightarrow \text{subset of } D \text{ containing the demands connected to bus } b$$

$$M_{POD}^b \rightarrow \text{subset of } G \text{ containing the PODs connected to bus } b$$

Thus, considering these new sets the equation (17) becomes

$$b \rightarrow \sum_{g \in M_G^b} I_g - \sum_{d \in M_D^b} I_d + \sum_{pod \in M_{POD}^b} I_{pod} = I_b \quad (25)$$

Regarding the line currents, defining the set

$$l \in L \text{ with } L = \{1, 2, \dots, N_L\}$$

It's possible to customize equations (18)-(19).

Let's first explore equation (18); in the deterministic model the matrices Y_f and Y_t are introduced to calculate the line currents: if multiplied by a complex voltage vector, they yield the vector currents injected into each line from the "from" and "to" buses respectively of each line. Y_l is the concatenation of Y_f and Y_t .

→ Case 1: b is at the bus from (f) as shown in Figure 2-47

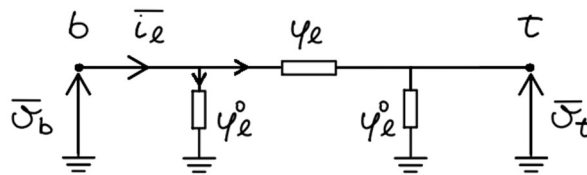


Figure 2-47: Grid equivalent scheme-bus from

According to the Figure 2, we get

$$i_l = \bar{v}_b * y_l^0 + (\bar{v}_b - \bar{v}_t) * y_l \quad (26)$$

In matrix form (26) becomes

1	2	...	b	...	t	...	N_L	
...	$y_l + y_l^0$...	$-y_l$			

v_1	
v_2	
...	
v_b	$= Y_l^f * v_b$
...	
v_t	
...	
v_{N_L}	

Where the vector row is the l_{th} row of Y_f matrix.

→ Case 2: b is at the bus to (t) represented in Figure 2-48

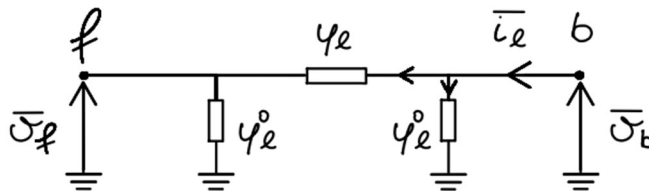


Figure 2-48: Grid equivalent scheme-bus to

$$i_l = v_b * y_l^0 + (\bar{v}_b - \bar{v}_f) * y_l \tag{27}$$

In matrix form (27) becomes

1	2	...	f	...	B	...	N_l	
...	$-y_l$...	$y_l + y_l^0$	

v_1
v_2
...
v_f
...
v_b
...
v_{N_l}

$= Y_l^t * v_b$

Where the vector row is the l_{th} row of the Y_t matrix.

Then let's customize the nodal balance current equations (19).

Defining the two subsets:

Subset 1: $l \in M_L^{b,f}$ – subset of lines connected at bus b which is the ‘from’ bus for l

Subset 2: $l \in M_L^{b,t}$ – subset of lines connected at bus b which is the ‘to’ bus for l .

The (19) is shaped as

$$i_b = \sum_{l \in \{M_L^{b,f} \cup M_L^{b,t}\}} i_l \quad (28)$$

For further development of the formal optimization model, we need to express this equation as a function of voltage phasors of the buses instead of i_l .

We introduce C_l^b as the corresponding matrix between a bus b and the lines connected to it; it resembles an incidence matrix in which the l_{th} element of this row vector is 1 if l is connected to b and 0 if l is not connected to b .

This matrix implies:

$$i_b = \sum_{l \in \{M_L^{b,f} \cup M_L^{b,t}\}} i_l = C_l^b * I_l \quad (29) =$$

1	...	l	...	N_l
	

i_1
...
i_l
...
i_{N_l}

Substituting i_l with (26) or (27)

$$\sum_{l \in \{M_L^{b,f} \cup M_L^{b,t}\}} i_l = [C_l^b * Y_l^b] * v_b \quad (30)$$

From (30) calling the product $[C_l^b * Y_l^b]$ as CY_l^b it's easy to derive that:

$$I_b = CY_l^b * v_b \quad (31)$$

From which the following comes:

$$v_b = [CY_l^b]^{-1} * I_b \quad (32)$$

The third equation we need to consider is the one defining i_l with respect to v_b .

Considering the matrix form of (26) and (27) and using (32)

$$i_l = [Y_l] * [CY_l^b]^{-1} * I_b \quad (33)$$

Finally for each element connected to the bus (G, D, POD) we need to link the power injected to the current injected, to understand how the $A_{pod,b}^*$ influences the currents and voltages upper and lower bounds. Focusing on the PODs elements and considering that the procedure is equivalent for both generators G and loads D , we have:

$\forall pod \in POD :$

$$A_{pod,b}^u + f_{pod,b}^+ - f_{pod,b}^- = v_b * i_{pod}^{\xi * } \cong \widetilde{v}_b * i_{pod}^{\xi * } \quad (34)$$

with

$$A_{pod,b}^u = A_{pod,b}^0 + A_{pod,b}^{\xi} = A_{pod,b}^0 + \xi * \Delta_{pod} \quad (35)$$

where Δ_{pod} is a complex number related to the uncertainty in P and in Q of POD; however, since Q is related to P , for example through constant power factor assumption, Δ_{pod} depends only on P profile uncertainty. $\xi \in [-1,1]$ is the box constraint.

Let's elaborate (34) by substituting (35) and considering the total current divided into two components: one related to base value of $A_{pod,b}^u$ over imposed by the worst case flexibility activation $\rightarrow i_{pod}^0$ and the one related to uncertainty part of $A_{pod,b}^u$, i.e. $\xi * \Delta_{pod} \rightarrow i_{pod}^{\xi}$. The dependency of this component i_{pod}^{ξ} on ξ will be determined, and, using the other equality constraints of the model, its impact on the inequality constraints will be obtained.

Expanding all the terms, (34) becomes:

$$A_{pod,b}^0 + f_{pod,b}^+ - f_{pod,b}^- + \xi * \Delta_{pod} = \widetilde{v}_b * i_{pod}^* \quad (36)$$

Defining

$$i_{pod} = i_{pod}^0 + i_{pod}^\xi \quad (37)$$

and substituting in (36)

$$A_{pod,b}^0 + f_{pod,b}^+ - f_{pod,b}^- + \xi * \Delta_{pod} = \widetilde{v}_b * (i_{pod}^{0*} + i_{pod}^{\xi*}) \quad (38)$$

Splitting the equation (38) in the two separate parts:

$$A_{pod,b}^0 + f_{pod,b}^+ - f_{pod,b}^- = \widetilde{v}_b * i_{pod}^{0*} \quad (39)$$

$$\xi * \Delta_{pod} = \widetilde{v}_b * i_{pod}^{\xi*} \quad (40)$$

where (39) is related to the deterministic part of the apparent power and flexibility activation, while (40) is related the uncertainty part of the apparent power, i.e. $\xi * \Delta_{pod}$.

Next, from (40) we get:

$$i_{pod}^{\xi*} = \xi * \left(\frac{\Delta_{pod}}{\widetilde{v}_b} \right) = \xi * \Delta_{A,pod} \quad (41)$$

Putting (41) in vector form:

$$I_{pod}^\xi = D_{A,pod} * X_{pod} \quad (42)$$

Where, in the simplifying hypothesis there is one POD per node, $D_{A,pod}$ is the square diagonal matrix with the $\left(\frac{\Delta_{pod}}{\widetilde{v}_b} \right)$ on the diagonal and X_{pod} a vector column containing ξ variables.

Thus (37) in matrix form becomes:

$$I_{POD} = I_{POD}^0 + D_{A,pod} * X_{pod} \quad (43)$$

In these conditions, considering the uncertainty present only at the level of POD (but this can be easily extended to G, D the only, difference being the activation of flexibility), then I_b can be formally decomposed into a part related to the deterministic profiles (I_b^0) and a part related to uncertainty component (I_b^ξ). Clearly I_{POD}^0 is included in I_b^0 and $I_b^\xi = D_{A,pod} * X_{pod}$.

Therefore, considering that now $I_b = I_b^0 + I_b^\xi$, constraint (32) becomes:

$$v_b = [CY_l^b]^{-1} * I_b = [CY_l^b]^{-1} * I_b^0 + [CY_l^b]^{-1} * I_b^\xi = [CY_l^b]^{-1} * I_b^0 + [CY_l^b]^{-1} * D_{A,pod} * X_{pod}. \quad (44)$$

Let's define $v_b^0 = [CY_l^b]^{-1} * I_b^0$, this vector includes the complex voltage phasors resulting from power flow calculated for A_{pod}^0 over imposing the effect of activated flexibility. The term $[CY_l^b]^{-1} * D_{A,pod} * X_{pod}$ represents thus the uncertainty that is propagating in the equations of the model starting from the apparent power definition.

Thus, the new equation for the vector of voltages is:

$$v_b = v_b^0 + [CY_l^b]^{-1} * D_{A,pod} * X_{pod} \quad (45)$$

Substituting I_b expression in (33) gives:

$$i_l = [Y_l] * [CY_l^b]^{-1} * I_b^0 + [Y_l] * [CY_l^b]^{-1} * D_{A,pod} * X_{pod} \quad (46)$$

Let's define $i_l^0 = [Y_l] * [CY_l^b]^{-1} * I_b^0$, this vector includes the complex currents phasors resulting from power flow calculated for A_{pod}^0 over imposing the effect of activated flexibility. The term $[Y_l] * [CY_l^b]^{-1} * D_{A,pod} * X_{pod}$ represents thus the uncertainty that is propagating in the equations of the model starting from the apparent power definition.

Equation (45) is constrained for the upper and lower bounds in the form:

$$V^{LB} \leq |v_b| \leq V^{UB} \quad (47)$$

Substituting (45) in (47) gives

$$V^{LB} \leq \left| v_b^0 + [CY_l^b]^{-1} * D_{A,pod} * X_{pod} \right| \leq V^{UB} \quad (48)$$

Due to the presence of the uncertain parameter here represented by the vector X_{pod} , equation (48) translates into an infinite set of constraints as ξ varies within the box interval. The goal is to reformulate this constraint into a deterministic form by applying the strong duality theory introduced earlier, while separating the UB and LB conditions:

$$|v_b^0 + [CY_l^b]^{-1} * D_{A,pod} * X_{pod}| \leq V^{UB} \quad (49)$$

$$|v_b^0 + [CY_l^b]^{-1} * D_{A,pod} * X_{pod}| \geq V^{LB} \quad (50)$$

In particular, the uncertain term realizes the worst-case for the upper bound limit with its maximum value over $\xi \in [-1,1]$ and it behaves dually for the lower bound limit.

Thus

$$|v_b^0 + \max_{\xi \in [-1,1]} ([CY_l^b]^{-1} * D_{A,pod} * X_{pod})| \leq V^{UB} \quad (51)$$

$$|v_b^0 + \min_{\xi \in [-1,1]} ([CY_l^b]^{-1} * D_{A,pod} * X_{pod})| \geq V^{LB} \quad (52)$$

In the absence of optimization decision variables in the term subject to maximization or minimization i.e. $v(\xi)$, it's easy to deduce that the minimum of $v(\xi)$ is attained by taking the sum of the absolute values of $[CY_l^b]^{-1} * D_{A,pod}$ with a negative sign-sum performed over its columns, whereas the maximum is attained by the same without the negative sign.

Rearranging

$$|v_b^0 + \Delta V^{\xi,max}| \leq V^{UB} \quad (53)$$

$$|v_b^0 + \Delta V^{\xi,min}| \geq V^{LB} \quad (54)$$

with $\Delta V^{\xi,max} = \sum |[CY_l^b]^{-1} * D_{A,pod}|$ and $\Delta V^{\xi,min} = \sum |-[CY_l^b]^{-1} * D_{A,pod}|$; these terms are constant and can be easily calculated from the input data.

Making the same consideration for the current constraints

$$|i_l^0 + [Y_l] * [CY_l^b]^{-1} * D_{A,pod} * X_{pod}| \leq i_l^{UB} \quad (55)$$

Due to the presence of the uncertain parameters represented by the vector X_{pod} , equation (55) translates into an infinite set of constraints as ξ varies within the box interval. The goal is to reformulate this constraint into a deterministic form by applying the strong duality theory introduced earlier.

In particular the uncertain term realizes the worst-case for the upper bound case with its maximum value for $\xi \in [-1,1]$. Thus

$$|i_l^0 + \max_{\xi \in [-1,1]} \left([Y_l] * [CY_l^b]^{-1} * D_{A,pod} * X_{pod} \right)| \leq i_l^{UB} \quad (56)$$

In the absence of optimization decision variables within the term subject to maximization or minimization i.e. $i(\xi)$, it's easy to deduce that the maximum of $i(\xi)$ is attained by taking the sum of the absolute values of $[Y_l] * [CY_l^b]^{-1} * D_{A,pod}$, sum performed over its columns.

Rearranging

$$|i_l^0 + \Delta I^{\xi,max}| \leq i_l^{UB} \quad (57)$$

with $\Delta I^{\xi,max} = \sum [Y_l] * [CY_l^b]^{-1} * D_{A,pod}$. These terms are constant and can be calculated from the input data.

With these deductions, the model reverts to a deterministic formulation, which can be efficiently handled by standard optimization solvers.

The formal definition of the complete model is the following:

$$\min \sum_{POD} \pi_{POD}^{\uparrow} f_{POD}^{\uparrow} + \sum_{POD} \pi_{POD}^{\downarrow} f_{POD}^{\downarrow} \quad (58)$$

s. t.

$$A_{pod,b}^0 + f_{POD}^{\uparrow} - f_{POD}^{\downarrow} = v_b^0 * i_{POD,b}^* \cong \widetilde{v}_b * i_{POD,b}^{0*} \quad (59)$$

$$A_{D,b} = v_b^0 * i_{D,b}^* \cong \widetilde{v}_b * i_{D,b}^* \quad (60)$$

$$A_{G,b} = v_b^0 * i_{G,b}^* \cong \widetilde{v}_b * i_{G,b}^* \quad (61)$$

$$\forall b \quad \sum_G i_G + \sum_{POD} i_{POD,b}^0 - \sum_D i_{D,b} = i_b^0 \quad (62)$$

$$\forall l \quad i_l^0 = Y_l * v_b^0 \quad (63)$$

$$\forall b \quad i_b^0 = \sum_{l \in b} i_l^0 \quad (64)$$

$$|v_b^0 + \Delta V^{\xi,max}| \leq V^{UB} \quad (65.a)$$

$$|v_b^0 + \Delta V^{\xi, min}| \geq V^{LB} \quad (65.b)$$

$$|i_l^0 + \Delta I^{\xi, max}| \leq i_l^{UB} \quad (66)$$

In which $f_{POD}^\uparrow, f_{POD}^\downarrow, v_b^0, i_G, i_{POD,b}^0, i_{D,b}, i_b^0, i_l^0$ are the variables of the problem.

2.8 Model linearization

The model just formalized needs to be linear in order to be handled by solvers based on Linear Programming. In particular, the UB and LB constraints pose a critical issue due to the presence of absolute value terms.

In particular the argument of the absolute value in (65)-(66) gives rise to the phasors

$$v_b = v_b^0 \pm \Delta V^{\xi, max} \quad (67)$$

$$i_l = i_l^0 + \Delta I^{\xi, max} \quad (68)$$

Thus the real and imaginary parts of the phasor v_b and i_l are subject to the linearization. Nonlinear constraints, such as these involving the Euclidean norm of a vector, often make robust optimization models significantly more challenging. While linear constraints allow the worst-case realization of uncertainty to be derived in closed form or reformulated as an equivalent linear program, nonlinear terms destroy this property and usually lead to more complex and computationally demanding counterparts. In particular, constraints based on vector magnitudes or quadratic expressions may require conic reformulations (e.g., second-order cone programming) or local linearizations to remain tractable. These approaches, however, either increase the computational burden or introduce conservatism, since approximations are needed to guarantee feasibility across all possible uncertainty realizations. Thus, the linearization of the

equations results to be a fair solution that both considers computational and approximation burdens.

The linearization of the currents upper bounds is performed taking as reference [29]. In particular the current i_l , being a complex number, can be represented on the complex plane using the phasor representation and the Euclidean norm, which is more simply called as magnitude, is the measure of its length in the space. In the complex plane, the Euclidean norm of a vector corresponds to a circular constraint, since the magnitude of a complex number defines its distance from the origin. To preserve tractability, the circular shape can be locally or globally approximated by linear hyperplanes. Geometrically, this means replacing the curved boundary of the circle with tangent or intersecting lines, so that the feasible region is approximated by a polyhedral set. As the number of linear segments (or intersecting planes) increases, the approximation becomes tighter, asymptotically approaching the original circular constraint. This idea is precisely exploited in [29], where the magnitude of branch currents, naturally expressed as a Euclidean norm, is linearized in this case using a set of intersecting planes that provide a conservative and computationally efficient representation of the current limits.

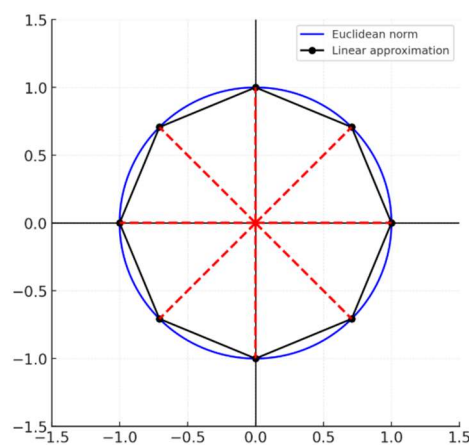


Figure 2-49: Intersecting Plane Approximation ($N=8$)

For a preliminary qualitative approach, Figure 2-49 represents the intersecting plane approximation that defines the 8 regions of the complex plane in which the branch

current magnitude is approximated with the segment comprised between two vertices of a plane. It's intuitive to note that if the intersecting planes are increased, the number of regions increases implying a better approximation of the circle with a more adherent segment, as shown in Figure 2-50 for the case of 32 planes.

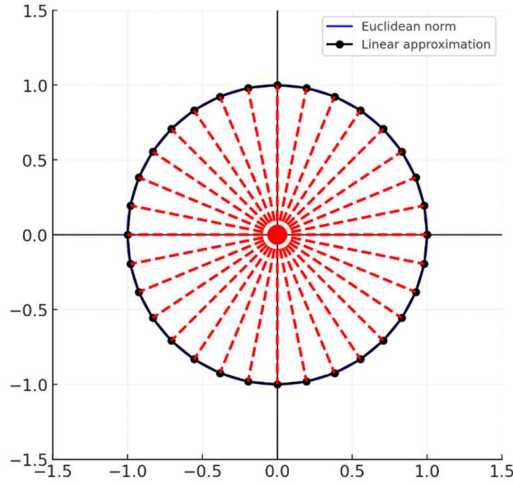


Figure 2-50: Intersecting Planes Approximation (N=32)

For the mathematical formulation, let's take $z = \{x, y\}$ and $f(x, y) = \sqrt{x^2 + y^2}$. Three noncollinear points,

$$A = \{0,0,0\}$$

$$B_n = \{\cos(\theta n - \theta), \sin(\theta n - \theta), 1\}$$

$$C_n = \{\cos(n\theta), \sin(n\theta), 1\}$$

are defined such as they belong to a set of intersecting planes with $f(x, y)$ for each plane $n \in \Omega_N$, where $\Omega_N = \{1, 2, \dots, N\}$ is the set of intersecting planes used to perform the approximation. A general representation of the approximation is shown in Figure 2-51 where $\theta = \frac{2\pi}{N}$ represents the angle between vectors \vec{b} and \vec{c} . With \vec{b} and \vec{c} it is possible to obtain planes $Z_n = AB_nC_n \forall n \in \Omega_N$, such as

$$Z_n = (A_n x + B_n y) / C_n$$

$$\text{With } A_n = \sin(\theta n - \theta) - \sin(n\theta),$$

$$B_n = \cos(n\theta) - \cos(\theta n - \theta)$$

$$C_n = \cos(n\theta)\sin(\theta n - \theta) - \cos(\theta n - \theta)\sin(n\theta).$$

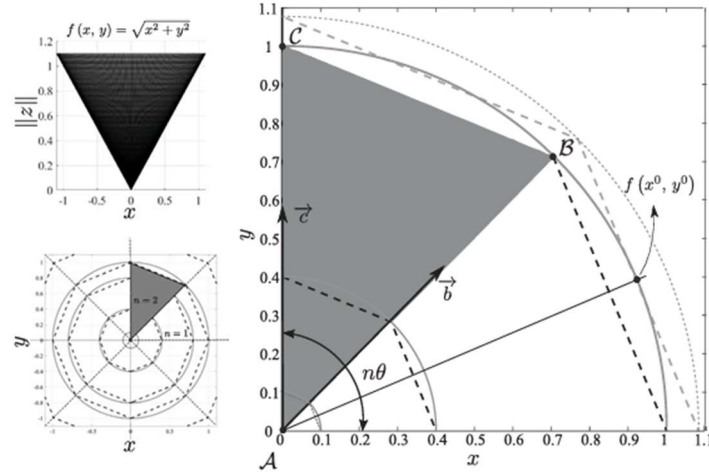


Figure 2-51: Euclidean norm approximation using intersecting planes [29]

Thus, the objective is using this approximation on the branch current i_l . Let's split the branch current in real and imaginary part

$$i_l = i_l^r + j * i_l^i \quad (69)$$

Defining the following indexes and sets:

$k, j \in B$ with B the set of buses

$n \in \Omega_N$ with Ω_N as the set of intersecting planes for the current magnitude approximation,

$\varphi \in \Omega_\varphi$ with Ω_φ the set of phases

The branch's upper bound current linearization, defined as $\Gamma_{k,j,\varphi,n}$, becomes

$$\Gamma_{k,j,\varphi,n} = \frac{i_l^r * A_n + i_l^i * B_n}{C_n} \quad \forall k, j \in B, n \in \Omega_N, \varphi \in \Omega_\varphi : k = j \quad (70)$$

$$\Gamma_{k,j,\varphi,n} \leq I_{k,j} \quad \forall k, j \in B, n \in \Omega_N, \varphi \in \Omega_\varphi : k = j \quad (71)$$

It's important to highlight that the goodness of the approximation increases as the number of planes increases (Figure 2-52[29]).

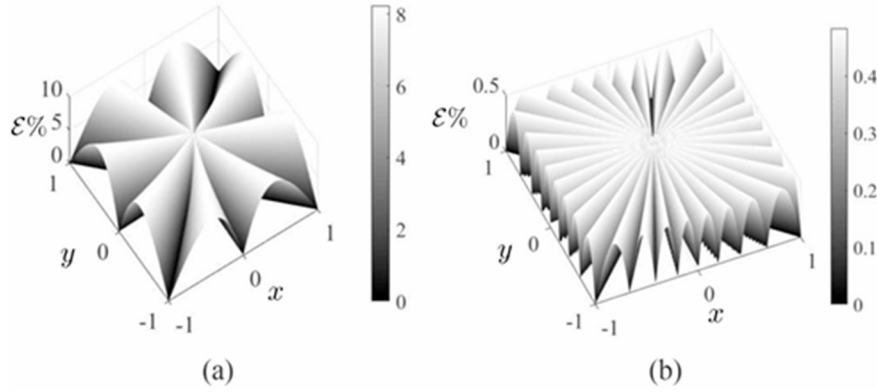


Figure 2-52: Error of the Euclidean norm approximation. (a) $N=8$. (b) $N=32$ [29]

Figure 2-52 is a graphical representation of the error of the Euclidean norm performed both in the case of 8 and 32 semiplanes confirming that the error reduces as the N increases. Thus choosing 32 semiplane is the choice that constitutes a good compromise between approximation and computational complexity.

The linearization of the voltages bounds is also treated in [29]. The linearization approach starts from the statement that any norm on R^n can be approximated as a positive linear combination of other norms.

The linearization approximates the Euclidean norm, $\|z\|_2$, as a linear combination of the $\|z\|_1$ norm and the $\|z\|_\infty$ norm. By definition, $\|z\|_\infty \leq \|z\|_2 \leq \|z\|_1$. Hence, it is possible to approximate $\|z\|_2$ as a linear combination of the other two

$$\|z\|_2 \approx \Psi(z) = \lambda \|z\|_\infty + \beta \|z\|_1 \quad \lambda, \beta \geq 0 \quad (72)$$

where $\|z\|_\infty \triangleq \max\{|x|, |y|\}$ and $\|z\|_1 \triangleq |x| + |y|$.

Using (72) and expliciting $v_b = v_b^r + j * v_b^i$ the voltage magnitudes are approximated as

$$\Psi(v_b) = \lambda_\varphi \|v_b\|_\infty + \beta_\varphi \|v_b\|_1 \quad \forall k \in \Omega_B, t \in \Omega_T, \varphi \in \Omega_\varphi. \quad (73)$$

Due to power quality standards, distribution systems are designed such as voltages are close to nominal values under normal operation, i.e., magnitudes within 0.90–1.10 p.u., phase angles deviations as small as possible, and a voltage unbalance factor lower than 5%. Thus, a nonnegative angle range, namely Θ , is introduced to tighten up the approximation.

Noticing that $v_b^r \geq 0, |\{v_b\}| = \{v_b\}$. Furthermore, since $v_b^r > v_b^i$ for $\theta < 45^\circ$, then $\|v_b\|_\infty = v_b^r$. Therefore, (73) is rewritten as

$$\Psi(v_b) = \lambda v_b^r + \beta (v_b^r + |v_b^i|) \quad \forall b \in \Omega_B \quad (74)$$

Thus, the final definition of the model becomes the following:

$$\min \sum_{POD} \pi_{POD}^\uparrow f_{POD}^\uparrow + \sum_{POD} \pi_{POD}^\downarrow f_{POD}^\downarrow \quad (75)$$

s. t.

$$A_{pod,b}^0 + f_{POD}^\uparrow - f_{POD}^\downarrow = v_b^0 * i_{POD,b}^* \cong \tilde{v}_b * i_{POD,b}^{0*} \quad (76)$$

$$A_{D,b} = v_b^0 * i_{D,b}^* \cong \tilde{v}_b * i_{D,b}^* \quad (77)$$

$$A_{G,b} = v_b^0 * i_{G,b}^* \cong \tilde{v}_b * i_{G,b}^* \quad (78)$$

$$\forall b \quad \sum_G i_G + \sum_{POD} i_{POD,b}^0 - \sum_D i_{D,b} = i_b^0 \quad (79)$$

$$\forall l \quad i_l^0 = Y_l * v_b^0 \quad (80)$$

$$\forall b \quad i_b^0 = \sum_{l \in b} i_l^0 \quad (81)$$

$$V^{LB} \leq \lambda v_b^r + \beta (v_b^r + |v_b^i|) \leq V^{UB} \quad (82)$$

$$\frac{i_l^r * A_n + i_l^i * B_n}{c_n} \leq i_l^{UB} \quad (83)$$

With these linearizations, the model is full linear and ready for implementation: it is now MILP-tractable and directly solvable with standard solvers.

2.9 Results

2.9.1 Simulations settings

The MILP optimization problem is implemented in GAMS 38.3.0 and solved using CPLEX 12.6, while the complete network model and datasets (topology, parameters) reside in MATLAB; to not jeopardize the accuracy of the linearizations introduced in the MILP models, the relative gap set-up is 0%.

2.9.2 Test grid

The test grid is a radial distribution network with 12 buses and 11 branches represented in Figure 2-53.

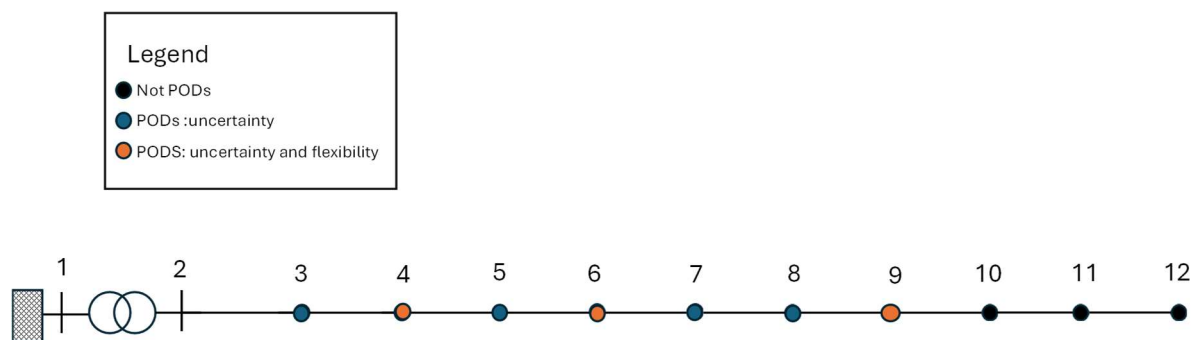


Figure 2-53: Test grid: 12-bus radial distribution network

The buses are characterized by aggregated nodal loads at the POD level, which incorporate the effect of renewable generation. By convention, every resource, whether load or generation, is modeled as a load. Blue circles represent the PODs characterized by uncertainty. These correspond to PODs 1 through 7, connected respectively to nodes 3 through 9. In particular, the blue PODs, labeled “PODs: uncertainty”, are not associated with flexible resources but only with an assigned level of uncertainty. The orange circles, labeled “PODs: uncertainty and flexibility”, denote PODs that combine the presence of flexible resources with an assigned uncertainty. These are the POD2, POD4, POD7 connected to the buses 4,6,9. Finally, the remaining nodes, which do not belong

to PODs and are not affected by either uncertainty or flexibility, are shown in black and are referred to as “not PODs”.

The baseline profile of the loads is reported in Table 2-3.

#Bus	P [kW]	Q [kVar]	cosφ
3	200	115.44	0.866
4	250	144.30	0.866
5	300	173.16	0.866
6	150	89.88	0.858
7	140.25	67.32	0.902
8	100	59.11	0.861
9	250	126.52	0.892

Table 2-3: Baseline profiles of PODs

The flexible PODs are the ones that offer flexibility to the market. To model the flexibility the hypothesis made is that the flexibility is available only in the direction to reduce the equivalent load until 0, being the scenario mainly composed by loads. Thus each PODs participating in the market makes an offer of upward flexibility, respecting the convention adopted, that is equivalent to its capacity presented in the Table.

The prices at which the flexibility is offered are considered equal for each POD in order to evaluate the technical efficiency of the model; the objective is thus to verify that the model selects the resources that are technically most effective, namely those that can best resolve network issues while minimizing the overall amount of flexibility to be activated.

These previously described data constitute the starting point for computing the estimated profile $A_{pod,b}^0$ without accounting for estimation uncertainty.

At this stage, it's necessary to introduce the $D_{A,pod}$. In particular let's define the half-band of uncertainty, in % with respect to the corresponding average values P and Q defined in the Table 2-3, corresponding to half of the total uncertainty interval.

To do so, the assumption is that the network users with flexibility acting as prosumers are subject to the regulatory framework that enables them to provide flexibility services to the grid. Thus, prosumers are equipped with accurate smart meters or, more generally, with technologically advanced measurement devices, allowing for accurate monitoring of power consumption and generation. Consequently, their power profiles estimation can be considered more accurate and reliable compared to those of conventional consumers. The latter, in fact, are not involved in dedicated smart metering upgrade programs and thus lack the same level of precision in the estimation of their power profiles.

Building on the previous considerations, for prosumers (flexible loads and flexible generators) the corresponding uncertainty should be significantly lower.

As a proof of concept, several studies show that the availability of advanced metering and high temporal resolution data significantly improves the accuracy of load/generation profile estimation, and reduces forecast error. Article [30] highlights that uncertainties in energy flexibility are largely driven by external factors such as occupant behavior, building characteristics, and the availability of monitoring technologies, thereby suggesting that the absence of advanced metering infrastructures can significantly increase the variability of demand profiles. In line with this, article [31] argues that the deployment of advanced metering infrastructures at the prosumer level enables higher temporal resolution and bidirectional data communication, substantially reducing the uncertainty associated with the estimation of load and generation profiles. Similarly, article [32] demonstrates that smart-meter data improve load forecasting accuracy in networks with distributed PV, thereby showing that users without such technologies are subject to higher levels of forecast error and uncertainty. Finally, article [33] emphasizes that the precision of prosumer flexibility assessments strongly depends on the availability of detailed consumption and generation data, which smart meters and advanced monitoring infrastructures can provide, leading to lower uncertainty compared to conventional consumers.

These observations align with the assumption that non-flexible loads (or non-prosumers) will exhibit greater uncertainty in power estimation than flexible loads or prosumers.

Building upon the previous considerations, three distinct levels of flexibility are defined in Table 2-4.

$D_{pod,p}$ [%]	CASE 1	CASE 2	CASE 3
FLEXIBLE SOURCES	2.5	5	10
NON-FLEXIBLE SOURCES	7.5	15	30

Table 2-4: CASES definition

We hypothesize three uncertainty scenarios: CASE 1 represents low uncertainty, CASE 2 medium uncertainty and CASE 3 is high uncertainty. $D_{pod,p}$ is the half-band of uncertainty defined in % with respect to the corresponding average values of P and Q defined in Table 2-3.

2.9.3 Results

The three terms $\Delta V^{\xi,max}$, $\Delta V^{\xi,min}$, $\Delta I^{\xi,max}$ of constraints (65) and (66), easily computed from the model, constitute linear approximations of the uncertain term introduced in the voltage and current equations. For validating the results it's meaningful to evaluate the accuracy of these values, which can be directly computed from the model.

In particular their expressions derive from the application of strong duality, whereby an infinite set of possibilities is reduced to the solution of a maximization or minimization problem. To confirm that this duality-based approach indeed leads to the worst-case realization, we generated random load profiles within their respective uncertainty intervals and combined them to create multiple scenarios. For each scenario, power flow simulations were carried out and the resulting values were compared with those obtained from the base profile. For every branch, the maximum deviation observed across all simulations was then reported alongside the uncertain term previously derived from the model. This procedure makes it possible to verify that the dual

formulation effectively identifies the worst-case outcome, thus confirming the accuracy of the approximations.

Table 2-5, Table 2-6 and Table 2-7 include for CASE 1,2,3:

- $i_{i, re}^0$: this column reports the real part of the branch current corresponding to the deterministic power profile i_l^0 , which serves as the reference without uncertainty.
- $\Delta I_{re}^{\xi, max, ACPF}$: This indicates the maximum deviation of the real part of the current obtained from the MATLAB simulations. Specifically, it represents the worst-case difference between the deterministic power flow results and those computed over the randomly generated profiles within the uncertainty intervals.
- $\Delta I_{re}^{\xi, max}$: This column shows the real part of the uncertain term directly calculated from the model (through the duality-based formulation). It is the theoretical value to be compared against the simulation-based result ($\Delta I_{re}^{\xi, max, ACPF}$).
- $i_{i, imag}^0$: This represents the imaginary part of the branch current corresponding to the deterministic power profile.
- $\Delta I_{imag}^{\xi, max, ACPF}$: This is the maximum deviation of the imaginary part of the current obtained from MATLAB simulations, following the same worst-case criterion described for the real part.
- $\Delta I_{imag}^{\xi, max}$: This column reports the imaginary part of the uncertain term calculated by the model, to be compared with the corresponding simulation-based value ($\Delta I_{imag}^{\xi, max, ACPF}$).

#Branch	$i_{l,re}^0 [A]$	$\Delta I_{re}^{\xi,max,ACPF} [A]$	$\Delta I_{re}^{\xi,max} [A]$	$i_{l,imag}^0 [A]$	$\Delta I_{imag}^{\xi,max,ACPF} [A]$	$\Delta I_{imag}^{\xi,max} [A]$
2-1	-95.673	-4.9437	-4.9378	53.2097	2.8128	2.7845
2-3	95.673	4.9437	4.9378	-53.2097	-2.8128	-2.7845
3-4	81.9176	3.9118	3.9065	-45.2716	-2.2092	-2.1849
4-5	64.7188	3.4812	3.4766	-35.3274	-1.9545	-1.9349
5-6	44.0759	1.9319	1.9287	-23.3763	-1.0475	-1.0343
6-7	33.7537	1.6734	1.6707	-17.2024	-0.8886	-0.8784
7-8	24.0972	0.9484	0.9465	-12.5854	-0.5348	-0.5276
8-9	17.2142	0.4317	0.4304	-8.5442	-0.2248	-0.2198
9-10	0.0007	0	0	0.1867	0	0
10-11	0.0005	0	0	0.1245	0	0
11-12	0.0002	0	0	0.0622	0	0

Table 2-5: $\Delta I_{re}^{\xi,max,ACPF}$ and $\Delta I_{re}^{\xi,max}$ comparison-CASE 1

#Branch	$i_{l,re}^0 [A]$	$\Delta I_{re}^{\xi,max,ACPF} [A]$	$\Delta I_{re}^{\xi,max} [A]$	$i_{l,imag}^0 [A]$	$\Delta I_{imag}^{\xi,max,ACPF} [A]$	$\Delta I_{imag}^{\xi,max} [A]$
2-1	-95.673	-9.8879	-9.8756	53.2097	5.6283	5.569
2-3	95.673	9.8879	9.8756	-53.2097	-5.6283	-5.569
3-4	81.9176	7.8239	7.8129	-45.2716	-4.4206	-4.3698
4-5	64.7188	6.9627	6.9532	-35.3274	-3.911	-3.8697
5-6	44.0759	3.8641	3.8574	-23.3763	-2.096	-2.0686
6-7	33.7537	3.347	3.3414	-17.2024	-1.7781	-1.7568
7-8	24.0972	1.8968	1.8931	-12.5854	-1.0701	-1.0551
8-9	17.2142	0.8635	0.8607	-8.5442	-0.4499	-0.4397
9-10	0.0007	0.0001	0.0001	0.1867	-0.0001	-0.0001
10-11	0.0005	0	0	0.1245	0	0
11-12	0.0002	0	0	0.0622	0	0

Table 2-6: $\Delta I_{re}^{\xi,max,ACPF}$ and $\Delta I_{re}^{\xi,max}$ comparison-CASE 2

#Branch	$i_{l,re}^0 [A]$	$\Delta I_{re}^{\xi,max,ACPF} [A]$	$\Delta I_{re}^{\xi,max} [A]$	$i_{l,imag}^0 [A]$	$\Delta I_{imag}^{\xi,max,ACPF} [A]$	$\Delta I_{imag}^{\xi,max} [A]$
2-1	-95.673	-19.7777	-19.7511	53.2097	11.2674	11.138
2-3	95.673	19.7777	19.7511	-53.2097	-11.2673	-11.138
3-4	81.9176	15.6496	15.6258	-45.2716	-8.85	-8.7396
4-5	64.7188	13.9271	13.9064	-35.3274	-7.8298	-7.7395
5-6	44.0759	7.7293	7.7149	-23.3763	-4.1965	-4.1371
6-7	33.7537	6.6949	6.6828	-17.2024	-3.5599	-3.5137
7-8	24.0972	3.7942	3.7861	-12.5854	-2.1424	-2.1102
8-9	17.2142	1.7272	1.7214	-8.5442	-0.9008	-0.8794
9-10	0.0007	0.0001	0.0001	0.1867	-0.0001	-0.0001
10-11	0.0005	0.0001	0.0001	0.1245	-0.0001	-0.0001
11-12	0.0002	0	0	0.0622	0	0

Table 2-7: $\Delta I_{re}^{\xi,max,ACPF}$ and $\Delta I_{re}^{\xi,max}$ comparison-CASE 3

It is worth noting that in Columns 3 and 4 some terms appear with positive and others with negative values. This is due to the fact that the objective is to maximize the magnitude of the branch current. Consequently, if $i_{l,re}^0$ is negative, the direction in which the magnitude increases is also negative. In such cases, a negative deviation must be added, since the sum of two negative terms results in a larger (in absolute value) current magnitude.

As shown in Table 2-5, Table 2-6 and Table 2-7, the last three branches (9–10, 10–11, and 11–12) exhibit very small $\Delta I^{\xi,max}$, in some cases equal to zero. This can be explained by the fact that the last flexible node is node 9, whose flexibility can influence all upstream branches (i.e., those preceding node 9) but not the downstream ones, considering the radial structure of the grid. As a result, the three lines immediately downstream of node 9 (9–10, 10–11, and 11–12) remain unaffected by the flexibility.

By computing the differences $\Delta I_{real} = \Delta I_{re}^{\xi,max,ACPF} - \Delta I_{re}^{\xi,max}$ and $\Delta I_{imag} = \Delta I_{imag}^{\xi,max,ACPF} - \Delta I_{imag}^{\xi,max}$ the largest values in magnitude can be identified as $\Delta I_{max} = \Delta I_{real,max} + j * \Delta I_{real,imag}$ and subsequently plotted in Figure 2-54.

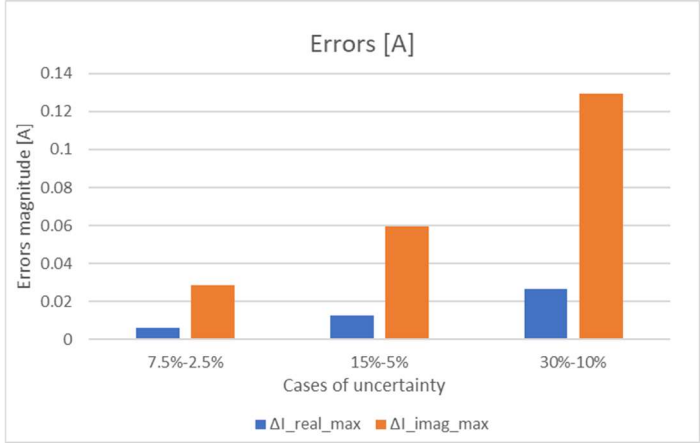


Figure 2-54: Current magnitude errors-CASE 1-2-3

As can be observed, the error in terms of magnitude increases with the level of uncertainty applied to all loads. The greater the uncertainty of the loads, the larger the

uncertain term becomes, and the approximation between the power flow and the uncertain term itself worsens.

It is worth emphasizing that the maximum load scenario is considered, as it represents the most critical operating condition. In this case, the overall passivity of the network leads to an increase in branch currents and a reduction in nodal voltages.

However, the most concerning aspect is the current limit violation, since despite the voltage reduction, the values of voltages $v_b = |v_b^o + \Delta V^{\xi, max}|$ remain above the reasonably defined lower threshold of 0.95 p.u. in all the three different CASES considered, as represented in the Table 2-8.

buses	$ v_b^o $ [p.u.]	CASE 1 $ v_b^o + \Delta V^{\xi, max} $ [p.u.]	CASE 2 $ v_b^o + \Delta V^{\xi, max} $ [p.u.]	CASE 3 $ v_b^o + \Delta V^{\xi, max} $ [p.u.]
1	1	1	1	1
2	0.9979	0.9978	0.9977	0.9975
3	0.9974	0.9973	0.9972	0.997
4	0.9971	0.997	0.9968	0.9965
5	0.9968	0.9967	0.9965	0.9962
6	0.9966	0.9965	0.9963	0.996
7	0.9965	0.9963	0.9962	0.9958
8	0.9964	0.9962	0.996	0.9957
9	0.9963	0.9961	0.996	0.9956
10	0.9963	0.9961	0.996	0.9956
11	0.9963	0.9961	0.996	0.9956
12	0.9963	0.9961	0.996	0.9956

Table 2-8: Worst case scenario voltage comparison-CASES 1-2-3

Voltage $|v_b^o + \Delta V^{\xi, max}|$ decreases with the increasing of the % of uncertainty considered; this implies more power absorbed by the line thus a bigger voltage drop is verified. However, the values are completely reasonable so no flexibility is activated for compensating voltage issues.

By inspecting Table 2-6, Table 2-7 and Table 2-8, it emerges that under the three CASES reported in Table 2-4 $\Delta I^{\xi, max}$ is considerably higher in the first branches 2-1, 2-3 and 3-4 than the remaining branches, thus the resulting $i_l = i_l^o + \Delta I^{\xi, max}$ would be higher. Hence, let's plot i_l^o and $i_l = i_l^o + \Delta I^{\xi, max}$ for branches 2-3 and 3-4 in the three CASES.

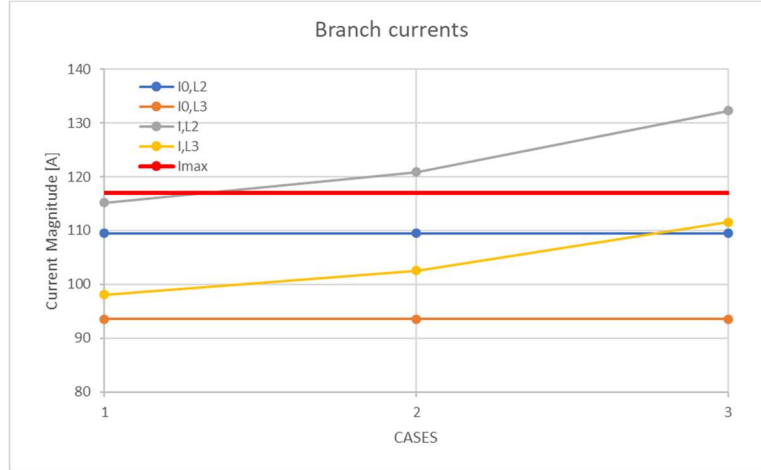


Figure 2-55: Branch currents comparison-CASE 1-2-3

I_{max} is a maximum branch current corresponding to the thermal limit.

Figure 2-55 clearly highlights the relevance of adopting a robust optimization approach to explicitly account for uncertainty within the model.

Indeed, the values of i_l , branch current under worst case, consistently exceed the corresponding i_l^0 , branch current corresponding to the deterministic power profile, across all lines. This demonstrates that relying solely on the deterministic model is insufficient, as it fails to ensure an adequate level of operational security. The deterministic formulation does not capture the range of possible uncertainties that may arise in the grid, which in this analysis are assumed to take the form considered here. Consequently, neglecting uncertainty makes the solution provided by the deterministic model much less reliable and potentially unsafe for real grid operation.

By observing the i_l values in the graph, it is possible to identify a current limit such that the proposed RO model activates zero flexibility in CASE 1 and increasing levels of flexibility in CASE 2 and CASE 3, identified as I_{max} . Accordingly, in the first case, the defined I_{max} is higher than both i_{l2} and i_{l3} , no flexibility is activated; in the second case, i_{l2} slightly exceeds I_{max} leading to a small activation of flexibility; in the third case, i_{l2} is consistently larger than I_{max} , thus requiring a significant amount of flexibility to be activated.

Once I_{max} has been identified (117 A in this case), it is important to represent the previous graph in percentage terms in order to highlight the saturation of the current limit on branch 2-3 in CASES 2 and 3 of Table 2-4. This representation is performed in Figure 2-56.

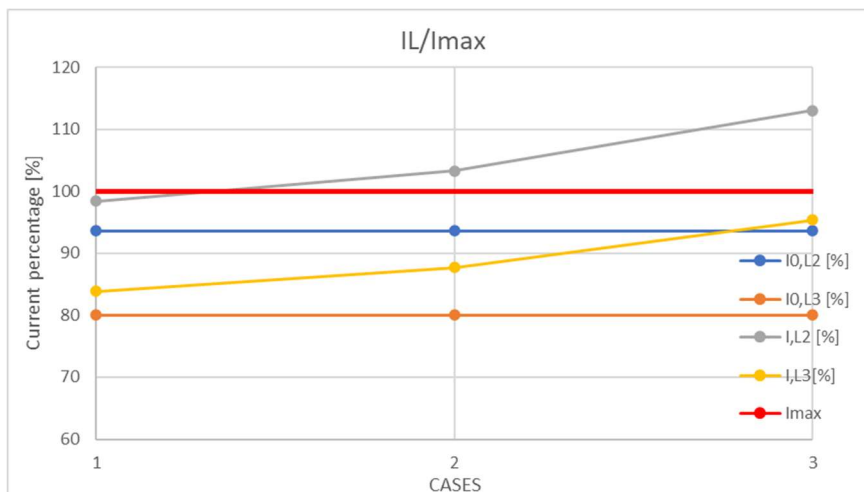


Figure 2-56: Branch currents comparison-CASE 1-2-3 in percentage

Consequently, the proposed RO model activates flexibility in CASE 2 and CASE 3 in order to mitigate the current violation occurring on this branch.

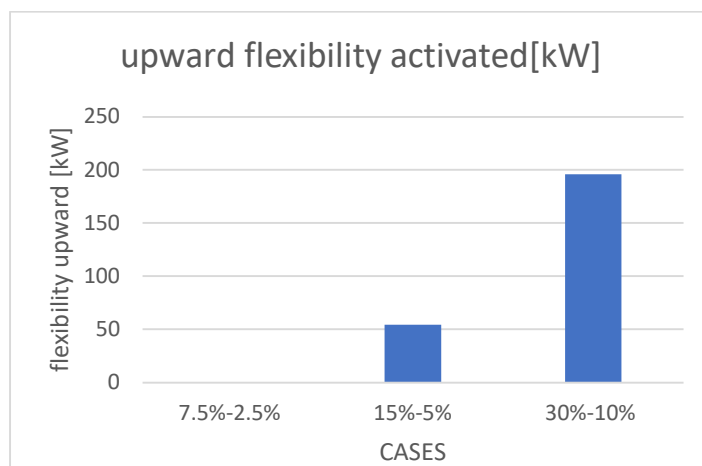


Figure 2-57: upward flexibility activated CASE 1-2-3

As described in Figure 2-57, upward flexibility f_{POD}^{\uparrow} is triggered in both CASE 2 and CASE 3, with the activated flexibility being clearly larger in CASE 3.

In CASE2 the flexible source activated is the POD2, whilst in CASE 3 the flexible sources activated are POD2 and POD4 for procuring the total amount of power, as indicated in the Table 2-9.

	POD	f_{POD}^{\uparrow} [kW]
CASE 2	2	0
	4	54.268
	7	0
CASE 3	2	45.87
	4	150
	7	0

Table 2-9: upward flexibility activated per POD- CASES 1-2-3

An important technical consideration concerns the order in which the generators are activated. It can be observed that the first activated POD is POD 4, located at node 6, and once its available power is saturated, the activation shifts to POD 2 at node 4. At first glance, this sequence may appear counterintuitive, since one might expect the farthest generator to be prioritized in order to minimize power losses for the same circulating power. However, in this relatively small network, the actual priority is determined by the contribution of apparent power to the current magnitude. When flexibility is activated in terms of active power, each node also injects reactive power according to its power factor. For the same upward flexibility activated f_{POD}^{\uparrow} —for instance, 1 kW—node 6 produces more reactive power because its power factor is lower, as demonstrated in the Table 2-10.

POD	f_{POD}^{\uparrow} [kW]	$\text{Cos}\varphi$	Q[kvar]	S [kVA]
1	1	0.866	0.5772	1.155
2	1	0.866	0.5772	1.155
3	1	0.866	0.5772	1.155
4	1	0.858	0.5992	1.166
5	1	0.902	0.4799	1.109
6	1	0.861	0.5911	1.162
7	1	0.892	0.5061	1.121

Table 2-10: Active and reactive power per POD

Since a larger Q leads to a larger magnitude of S , the contribution to the current is higher for the same active power. This explains why the activation order is POD 4, POD 2, and then POD 7.

To reinforce the concept, the plot of the upward flexibility activated per POD in the CASE 2 and CASE3 is reported respectively in Figure 2-58 and Figure 2-59. The graph is reported in percentage with respect to the maximum flexibility of each POD to highlight the saturation of flexibility power.

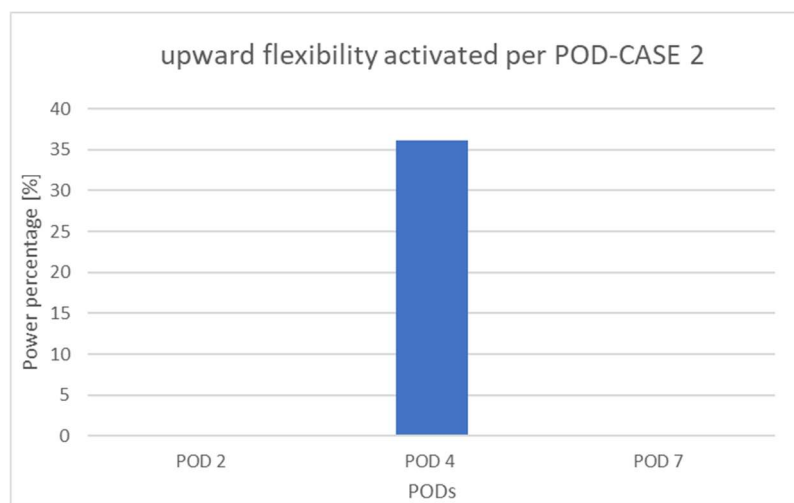


Figure 2-58: Upward flexibility activated per POD-CASE 2

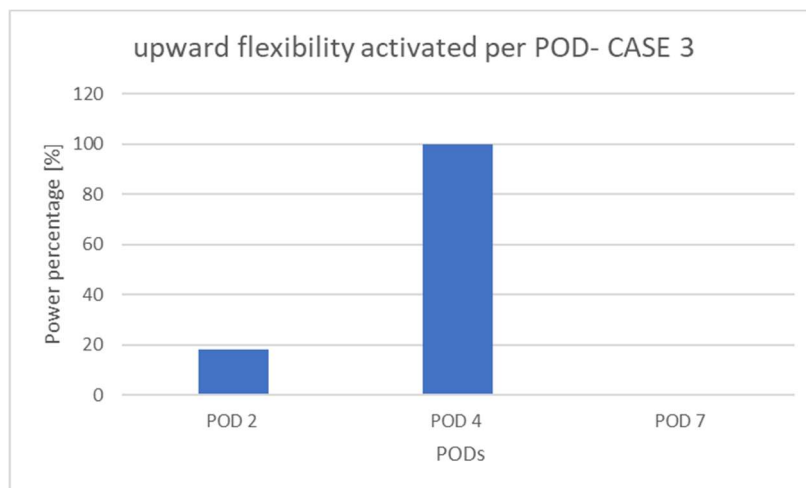


Figure 2-59: Upward flexibility activated per POD-CASE 3

As a validation step, in order to verify that optimization model correctly reduce i_{l2} below the limit of 117 A, an AC power flow can be performed imposing the upward flexibility obtained from the optimization f_{POD}^\uparrow .

The branch current obtained executing the AC power flows i_{l2}^{ACPF} is reported in the Table 2-11:

	i_{l2}^{ACPF} [A]
CASE 2	116.47
CASE 3	116.46

Table 2-11: $i_{l,2}$ - CASES 2-3

Therefore, the optimization has consistently activated flexibility resources to resolve the congestion and bring $i_{l,2}$ back within the prescribed limits.

To illustrate that the RO optimization model proposed activates only the flexibility strictly required (namely, just enough to bring the current very close to the imposed maximum limit), it is useful to plot the obtained $i_{l,2}$ in the worst-case scenario after flexibility has been activated.

Specifically, $i_{l,2} = i_{l,2}^0 + \Delta I_2^{\xi,max}$: $i_{l,2}^0$ is computed in proposed RO model as part of the variables of the optimization model, whilst $\Delta I_2^{\xi,max}$ is constant and can be easily calculated from the input data. By summing these two contributions, the resulting current matches the expected value. Referring the obtained value to $I_{max} = 117$ A:

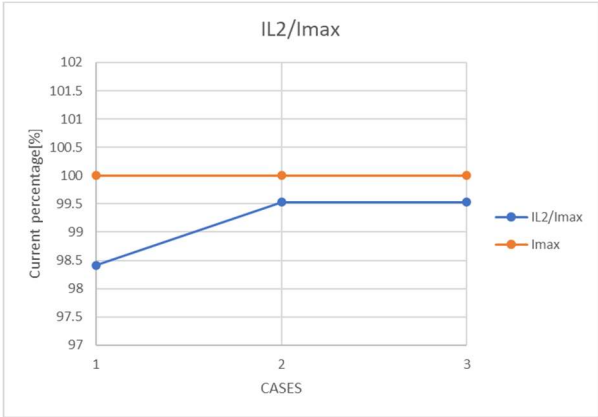


Figure 2-60: $i_{l,2}$ CASE-1-2-3

As shown by Figure 2-60 in CASE 1 i_{l2} is very close to the limit but does not exceed it, and therefore no flexibility is activated. In CASE 2 and CASE 3, instead, i_{l2} is brought back to a value very close to 100%, confirming that GAMS correctly minimizes the amount of flexibility to be activated. It is worth noting that even a slight perturbation in CASE 1 would require the activation of flexibility, which further emphasizes the importance of having flexibility available.

Comparing the $i_{l2} = i_{l2}^0 + \Delta i_2^{\xi, max}$ with the branch current of branch 2-3 obtained by performing AC power flows i_{l2}^{ACPF} in Table 2-12:

	i_{l2} [A]	i_{l2}^{ACPF} [A]	$\Delta IL2$ [%]
CASE 1	115.14	115.16	0.0163
CASE 2	116.45	116.47	0.0177
CASE 3	116.45	116.46	0.0122

Table 2-12: 1 i_{l2} and i_{l2}^{ACPF} after flexibility activation- CASE 3

$\Delta IL2$ is computed as the relative error between i_{l2}^{ACPF} and i_{l2} related to i_{l2}^{ACPF} (most accurate).

From the order of magnitude of the errors, it is clear that the proposed RO model provided an accurate estimation of the current, despite the inclusion of the uncertainty term. Nevertheless, both in the results of the proposed RO model and in AC power flow the current values, while very close to 117 A, show a deviation of about 0.5 A from the maximum limit set at 117 A. This discrepancy is due to the linearization of the constraint itself rather than to the introduction of the uncertainty term $\Delta i_2^{\xi, max}$: the piecewise approximation (32 sectors in the plane) indeed represents the actual current flowing through branch 2, but, although fairly precise, it inevitably introduces a limited approximation error.

By construction, the intersecting-plane approximation here used yields an inscribed polygon; the feasible set is contained within the true circular bound and is therefore conservative, systematically underestimating the admissible current. This method

may eliminate some truly feasible points, but it guarantees feasibility with respect to the original constraint for any point it admits.

To illustrate this aspect, three elements are represented within the same plot. The first is a circle with a radius of 117, corresponding to the value imposed as maximum branch current magnitude for the line 2 ($I_{max} = 117 A$). The second is the piecewise linearization of the constraint obtained through the 32 planes, depicted as 32 segments inscribed within the circle ($I_{max,lin}$). Finally, the third element corresponds to the coordinates of the real and imaginary components of the current computed by GAMS (I_{GAMS}), reported for instance in CASE 3, whose length is, by definition, the vector's magnitude.

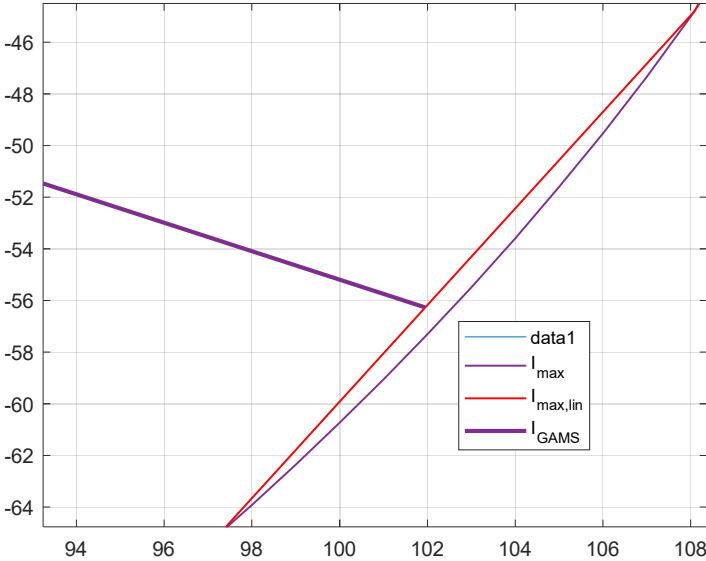


Figure 2-61: Current linear approximation check-CASE 3

In Figure 2-61 it can be observed that, although the current vector reaches the linearized current limit, there remains a gap between this limit and the surrounding circular arc representing the actual constraint. This gap explains the error of approximately 0.5 A, which originates from the linearization process. Such an issue can be easily addressed either by adopting a different type of linearization or by employing the same approach while increasing the number of planes in the subsection of the circle where

the current is expected to lie. Since this type of error stems from the chosen approximation, it is correctable, unlike the case in which the error would have arisen from the linearization of the uncertainty term. In that latter case, the possibilities of intervention would be much more limited.

Conclusions

The aim of this thesis was to extend the deterministic model presented in [29] by incorporating the uncertainty associated with the power set points of flexible resources. To this end, a robust optimization approach with a box-type uncertainty set was adopted, whereby each value within the interval $\xi \in [-1,1]$ is treated as equally probable. Such a formulation naturally drives the analysis towards worst-case scenarios, thereby ensuring a high level of operational security, albeit sometimes at the expense of more conservative and economically less efficient solutions.

This intrinsic conservatism represents both a strength and a limitation: on the one hand, it guarantees full protection of the distribution network against even low-probability disturbances; on the other hand, it may lead to overly cautious estimates of the flexibility required. A promising direction for further research lies in the introduction of a *budget of uncertainty*, which allows for a more nuanced trade-off between robustness and cost. By explicitly linking the degree of risk acceptance to the level of security to be guaranteed, such an approach enables the operator to tolerate minor network congestions in exchange for reduced activation of costly flexibility resources.

From a practical perspective, this would mean, for example, accepting slight overloads, such as an overcurrent of 102% on a given line, if they do not compromise system safety, while clearly rejecting excessive violations, such as an overcurrent of 120%, which would no longer be compatible with operational standards. Overall, the analysis carried out demonstrates how the robust approach, despite its conservative nature, constitutes a solid foundation for future modelling efforts aimed at achieving a more balanced integration of security and economic efficiency.

In the model developed in this thesis, each generator is associated with an uncertainty within the interval $(-1,1)$ and each perturbation is treated as independent and potentially realizable in its worst-case form. In other words, it is implicitly assumed that

extreme perturbations may occur simultaneously for all generators, without any statistical relationship among them. While this assumption simplifies the formulation, it can also prove excessively conservative, as it neglects the possible correlation that may exist among the outputs of geographically close distributed resources.

In particular, in the case of photovoltaic panels, it is reasonable to expect that the production of two units located in different but nearby points of the network may be affected by common environmental factors, such as solar irradiance, cloud coverage, or local temperature. Under such shared conditions, the deviations of the set points are not fully independent but may exhibit a positive (or, more rarely, negative) correlation, which should be duly considered in the definition of the uncertainty set.

Thus, a possible extension of the model would therefore be to replace or complement the box-type uncertainty with sets that account for the observed statistical correlations. The difference can be visualized by comparing the box uncertainty, represented by a square that treats the variables as independent, with an ellipsoidal set elongated along the diagonal, which instead reflects the presence of correlations. In the first case, extreme and rather unrealistic scenarios are considered, where all generators may simultaneously reach their maximum or minimum values without any reciprocal relationship; in the second, by contrast, uncertainty is constrained along statistically consistent directions, in line with the fact that, for instance, nearby photovoltaic panels tend to vary their production in a similar way under the same environmental conditions. Among the approaches that can implement this concept are ellipsoidal sets based on the covariance matrix, which jointly constrain the deviations of uncertain variables, or the more general copula-based models, capable of capturing nonlinear dependencies. The integration of such techniques would allow for the construction of more realistic uncertainty scenarios, thereby reducing the level of conservatism without compromising the robustness of the model.

In conclusion, the proposed model, based on a box-type uncertainty, constitutes the starting point of this analysis. It has made it possible to highlight both the strengths and the limitations of the robust approach, while laying the groundwork for further methodological developments, such as the introduction of uncertainty budgets and the modeling of statistical correlations among distributed resources. These extensions hold significant interest, as they promise to refine the trade-off between security and economic efficiency producing uncertainty models that are more realistic and better aligned with the behavior of real-world distribution networks.

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