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The E-bike Charger Location Problem

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Abstract

The urgency of facing climate change has had a positive impact on the electric vehicle market, which, combined with the pandemic situation of the last few years, is leading customers to move on a new way of sustainable mobility, choosing short- range electric vehicles, such as scooters and e-bikes. This trend importantly impacts in holiday resorts where e-bikes are becoming one of the most requested activities, suited to a large spectrum of tourists, promoting a new way of appreciating the natural landscapes. In this scenario the range of the battery mounted on e-bikes could be a limit to these activities, especially in resorts characterized by steep climbs and long trails. This thesis proposes two main optimization models to address the problem of locating charger facilities on the network of trails of a resort. A first formulation models the scenario where a set of predefined itineraries is devoted to the specific cultural journey, and hence a location problem is derived to cover the charge demand on the network. The second formulation models the scenario where trails are not given, but are part of the decisions process. For this case, we exploit an existing model in the literature, the so called Tourist Trip Design Problem, which we generalize to accumulate charging stations locations decisions. The derived models are applied to three randomly generated instances. Finally, we validate the programs on a real test case scenario, evaluating their performances and deriving some conclusions.

Keywords: E-bike; Charging Stations; Location Problem; Routing Location Problem; Tourist Trip Design Problem.

Sommario

L'urgenza di affrontare il cambiamento climatico ha avuto un impatto positivo sul mercato dei veicoli elettrici, che, combinato con la situazione pandemica degli ultimi anni, sta portando i clienti a muoversi su un nuovo tipo di mobilità sostenibile, scegliendo veicoli elettrici a corta percorrenza come monopattini e biciclette elettriche. Questa tendenza si riflette soprattutto sulle località di villeggiatura in cui l'e-bike sta diventando l'attività più gradita ai turisti di ogni tipo, promuovendo un modo di apprezzare i siti piu gettonati della regione, apprezzando la natura dei paesaggi e le attività proposte. In questo scenario l'autonomia della batteria montata sulle e-bike potrebbe essere un limite a questa attività, soprattutto nelle località dove le salite ripide e i lunghi sentieri limitano i chilometri raggiungibili. Questa tesi propone due principali modelli di ottimizzazione che sono in grado di trovare correttamente una soluzione al problema della posizione dei caricabatterie. Una prima formulazione fornisce una soluzione a uno scenario in cui un insieme di itinerari predefiniti è dedicato a specifici viaggi culturali e quindi un problema di localizzazione viene derivato per coprire la domanda di ricarica sulla rete. La seconda formulazione modella lo scenario in cui gli itinerari non sono dati, ma fanno parte del processo decisionale. Per questo caso, sfruttiamo un modello esistente in letteratura, il cosiddetto Tourist Trip Design Problem, che generalizziamo per accumulare le decisioni di localizzazione delle stazioni di ricarica. I modelli derivati sono applicati a tre istanze generate casualmente. Infine, convalidiamo i programmi su uno scenario reale, valutando le loro prestazioni e derivando alcune conclusioni.

Parole chiave: E-bike; Stazioni di Ricarica; Problema di Localizzazione; Problema di Localizzazione e Instradamento; Problema di Progetto di Reti Ciclo-Turistiche.

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Introduction

The need to address climate change is one of the most challenging and important problems of our century. In a 2018 report from United Nations [1], thousands of scientists and government reviewers agreed that to avoid the worst climate impacts and maintain livable climate conditions we must limit the increase of global temperature within 1.5°C. However, based on current climate plans, global warming is projected to reach 2.7°C by the end of the century. The pressure to make real actions is resulting in agreements to guide this progress, such as the Sustainable Development Goals, the UN Framework Convention on Climate Change and the Paris Agreement. What emerges form these agreements is that meaningful climate mitigation solutions exist to reduce greenhouse gas emissions, which would in turn slow down climate change and prevent natural disasters.[2].

Hence it is of fundamental importance to switch energy systems from fossil fuels to renewable energy. This directly reflects on the private transportation vehicle market, where the emissions of road vehicles, despite the continuous technological improvements, is still one of the largest contributing factor to air pollution in cities, often outside of the air quality standards provided by the World Health Organizations [3]. Governments and businesses are hence transitioning to green solutions, spurred on by regulations aimed at discouraging costumers and companies to invest in combustion engine vehicles, promoting instead, initiatives aimed at speeding up the transition to electric vehicles. This combination of facilitation-regulations and the quick growing of electrical vehicle market, is leading an electrical transition also in the low range vehicle market, where e-scooter and e-bicycles are quickly becoming among the most trendy personal urban vehicles. This new way of transportation is one of the best solutions to fight the rise of greenhouse gas emissions, it avoids traffic congestion and promote public transport in and outside of large metropolitan areas. The global market of two wheeler electric vehicles is expected to grow at a compound annual grow rate (CAGR) of 29.4 % for the end of 2028 [4], and a bigger movement is expected in the e-bike/scooter rental market that is quickly growing in the Asia Pacific regions, following the rapidly emerging economies like China and India. Moreover, the Covid-19 pandemic has significantly spurred on this market, bringing an overall positive impact in the selling volumes. With authorities worldwide discouraging people to take crowded public transportation systems wherever possible, people are getting used to their bikes for short-distance commuting.

This trend is reflected also in the touristic locations where e-bicycle is spreading not only as a mean of transportation but also as common ludic activity: enjoying a trail with pedal assist is attractive not only for family but also for sporty users who do not want to compromise between long distance and speed even when the climbs become harder. Resort and touristic administrations feel the pressure to develop cycling networks to face the needs of this market, able to lead the economies of this locations thanks to targeted investments and strategic decisions. A wider picture is presented in an article commissioned by the the European Parliament's Committee on Transport and Tourism in 2009 and conducted by the Institute of Transport and Tourism at the University of Central Lancashire (UK) and the Center for Sustainable Transport and Tourism, at Breda University, in the Netherlands (ITT, 2009). In this study it is stressed how developing a cycle tourism networks, emphasizes the economic impact on the local economies, increasing job opportunities.

Setting up such a kind of project is quite challenging: quantitative and qualitative tools aimed at evaluating the cost and the expected potential benefits of this investment are of fundamental importance for local administrations where the budget and the resources are quite limited. The main challenge stems from the development of the electrical infrastructure aimed at supporting the e-bike along the trails. Although the European Union directive No 168/2013 [5] for the definition of e-bikes, limits the maximum continuum rated power to less then 250 W, the market present models much more powerful. It follows that in a network open to each type of tourist, it is possible to find different types of electrical bicycles, which can require a higher presence of charger facilities. Hence, in such type of networks it is mandatory to have a reliable infrastructure of e-charger in order to face the needs of all types of users.

Among all types of resorts, mountain regions are the ones that most require a rapid development of these infrastructures. Indeed, e-bikes are quickly becoming the most popular activities in these sites: driven by electric motors, tourists have the opportunity to experience this new type of mobility for sports, spending time with the family and enjoying the beautiful natural landscapes. Furthermore, e-bikes make it possible to cover long distances and helping face the most difficult climbs even for less trained users. Consequently, these means of transportation use much of their capacity, requiring a frequent and strategic positioning of the charging stations. In addition, the paths and mule tracks characterizing the landscapes, lead the cyclists to arrive at very remote locations in the network. This translates into a demand for chargers in places far from the power grid, such as huts and mountain passes, where the implementation of a charging station would lead to a great expenditure of money and resources. Hence, the problem of find a strategic and optimal positioning of the columns is of extreme importance in these tourist resorts, requiring appropriate tools and methodologies.

In mathematical optimization, the family of Location Problems comprehend several studies studying the positioning of charging stations for electric vehicles. However, the proposed models are designed for large cities and long distance vehicles such as cars and buses. Consequently, if the formulations treated work well in urban contexts where charging stations capacity is generally treated as a coverage radius, in a mountain environment this is not possible, both for the distances and differences in height between possible charging points, and for the vehicles treated, very distant from the electric bicycles.

This thesis aims at proposing a new formulation that, modeling the mountain trails as a directed graph, exploits the territory conformation to find the best charging stations positions. This type of model is completely new in the literature, as it combines two field of studies such as the problem of charger location with the Tourist Trip Design, leading to competitive solutions which combine the constraints of battery capacity with the maximization of itineraries attractiveness.

In particular we propose two main types of models that solve the problem in different contexts. The first one can be defined as a Location Problem, i.e. with a formulation able to find the charging stations positions in a selected set of nodes. The goal is to minimize the implementation costs by finding the minimum number of chargers and ensuring in this way a full coverage of the routes that the cyclist can travel. This model is in fact implemented in a scenario where the feasible paths have been previously defined by local authorities or particular territorial conformations that oblige the user to cover a certain set of arcs to reach a desired destination. The second model is more detailed and flexible, allowing to find the optimal chargers positions without knowing in advance the routes taken by the cyclists. This formulation takes the name of Location Routing Problem because it exploits a routing problem to define the paths of the network and the position of the chargers. Such a strategy is applied by exploiting the potential of the Tourist Trip Design Problem that allows to design an offer of routes able to maximize the cyclists pleasure, exploiting the territory characteristics.

The thesis has the following structure, after the literature review , where the main contributions in similar problems are summarized, in Chapter 1 we propose a Location Problem that aims at finding the best locations of charger facilities given a set of predefined itineraries followed by the cyclists. In Chapter 2 we exploit the methodology of the Tourist

Trip Design Problem to propose a Location Routing Problem which locates the charging stations in a scenario where the set of predefined itineraries is missing. Then, in Chapter 3, these two models are tested on randomly generated networks to study their performance on various types of graphs. Chapter 4 is devoted to analyze the results on a real network, implementing the models on the Asiago Plateau, which represents a paradise for mountain bikers. Finally, Chapter 5 brings some conclusions on the work developed in this thesis, highlighting possible future developments in such a promising field.

Literature Review

Analyzing how many charging stations are needed on a specific topological area and which is their best position is a quite complex aspect for administration executives. It follows that a proper analysis and evaluation of the possible infrastructure is mandatory to ensure an e-charger network able to face the needs of the costumers and ensure a certain level of reliability. The majority of the researchers agree that different demands need different charger locations approaches [6], hence an appropriate model able to describe all the feature and needs of the network-costumer relationship must be created ad-hoc for the instance under analysis.

In the literature the problem of the location of charging stations for Electric Vehicles is extensively studied. Based on the facility location theory, many studies [7],[8],[9],[10], have analyzed EV refueling features to create an optimization problem able to size and properly locate a charging facilities infrastructure. The literature is therefore rich of articles that needs to be classified in different families. An appropriate classification is proposed in [11] where the different models are organized based on the hypothesis done on refueling demand, where the main groups are flow based, arc-based, and node-based models.

Flow-based models are the most studied one in the literature and focus on the assumption that the refueling capacity is associated to a traffic flow over the charger facilities. This method originates from the problem of maximum covering and basically it aims to find the best locations for p facilities in order to capture as much traffic flow as possible. The ancestor of this family of models is the flow capturing location model (FLRM), proposed in [12].

The main drawback of this method is that it does not take into account the trip distance, assuming that all the traffic flows on a path can be covered by a single refueling capacity. To overcome this problem Kuby and Lim [13] proposed a flow refueling location model (FLRM), where the combination of different refueling stations on a path ensures a sufficient flow coverage. Building on this work, different researchers proposed models where the costumer is not constrained to follow a predefined itinerary, but is allowed to deviate from the original path to charge the vehicle, for example the so called deviation refueling location model (DFLRM) [14] and the multi path refueling location model (MPRLM) [15].

In addition to the flow-based models, several other formulations have been proposed to solve the EV charging location problem. For example, a different approach was developed by He at al. [16] where a method called network equilibrium is applied to find the optimal location of n charging facilities.

Following this strand of problems a greatly developed branch over the years, is the facility location on networks, where demand arises continuously along the edges. This field of studies was first introduced by Revelle, Toregas, and Falkson [17]. Here, the model aims at serving population along street networks, where a set of nodes and edges should be covered by a minimum number of facilities within a maximum service distance.

These models tend to solve the problem of chargers positioning in urban contexts where roads form a dense and complex network, allowing flexible itineraries between two points of the city. This characteristic, combined with the urban electric grid, allows to choose the charger position analyzing crowed traffic areas and main roads. From a recharging perspective, these models totally differ from touristic resorts, where the conformation of the territory and the isolated electric grid, force tourists to follow specific trails to achieve possible charging points. As a consequence, although the literature presents a wide variety of formulations and models, the location problem applied to touristic resorts is missing.

Another stream of works in the location of charging stations, focuses on the idea of combining the vehicle routing with the optimal location problem, going under the popular classification of Location Routing Problem (LRP). For example, the work by Yang and Sun [18] models and solves a location-routing problem for EV battery swapping stations (BSS). Such a problem aims at combining the strategic covering problem (such as the location of chargers in a network), with the routing design, finding a set of itineraries that minimizes the total BSS costs and the EVs shipping cost.

This family of problems descends from the study of Boventer [19] and evolved in the modern LRP over time. In [20] we can find an extensive review of the classic LRP problem, that can be defined as a "deterministic, static, discrete, single-echelon, single-objective problem" , and aims at "opening a set of facilities, visiting every costumer by a capacited vehicle starting at one of the open facilities" [21].

Combining two NP-hard problems leads to quite challenging solving methodologies, both for the limitation of commercial solvers such as Cplex and GUROBI, and both for the advanced solutions techniques. Hence many researchers concentrate their studies in finding the best methodology to solve this kind of models, proposing heuristic and exact methods. To this end, Berger et al. [22] proposed a model that aims at solving by a branch-and-price methodology the standard LRP with uncapacitated facilities and vehicles, enriched by a bound on the total length of each route. [23] present an Elementary Shortest Path Problem with Resource Constraints (ESPPRC), solved by a heuristic and three exact branch-and-price approach. Baldacci et al. [24] decomposes the problem in a finite set of multi capacited depot and then solved the LRP with an algorithm composed by various bounding procedures.

Despite these notable studies and all the LRP variants and extension, resumed in the article by Drexl and Schneider [20], there is a lack of knowledge for what concerns the LRP applications to a context of charging stations for tourist destinations. Indeed, as the classic Location Problem, this methodology it has never been applied to find the optimal charger location on holiday resorts. However, the growing market of electric bicycles, urgently asks to the municipalities for the implementation of an appropriate charging infrastructure. Therefore this methodology could be implemented to find a solution to the e-bike charging location problem, providing different benefits such as maximize the network competitiveness and minimize the implementation costs.

Speaking of touristic destinations, the touristic trip design problem (TTDP) propose a technique that can be exploit by the LRP on cyclist resorts. The TTDP aims at maximizing the total reward obtained in a path by passing over different Points of Interest (POI) [25],[26],[27],[28],[29],[30]. However, this type of problems explored also case studies where different objectives are optimizes: in [31] the goal is to maximize the reward with respect to heterogeneous mode of transportation, while minimizing the time spent on the network and the weight of the used arcs. In [32] instead, the object is to minimize the time spent on each route.

Following the e-bike market direction, this type of problems could be the right solution for the design and implementation of a touristic trails design enriched with an e-charger facilities infrastructure, optimally located along the routes. Indeed, the capacity of routing the possible path on a touristic e-bike infrastructure gives a fundamental advantage in the strategic planning of the e-charger locations, contributing significantly to the entire attractiveness of the territory. Reliable charging facilities and a dedicated trails network results in a minimization of the implementation investments and a maximization of the economic upturn. Indeed, the TTDP plays a fundamental role in improving the planning of tourist travel experience, increasing the economic benefits and improving the competitiveness of the entire infrastructure as proven by many researchers in this field (

[33],[34],[35],[36],[37]). This is also proven by the fact that this family of problems is quite recent and has growing importance following the direction of the e-bikes market, making his first appearance in 2007 [38].

Close to this family of problems it is important to recall a work proposed in [39] where a multi commodity orienteering problem with network design (MOP-ND) is formulated. This paper generalizes the orienteering problem, developed in [40], where the origin and destination nodes collapsed to a single depot on the path. Moreover, it allows path more complicated than a simple cycle and uses a concave function to estimate the reward for multiple passages over the trails. Based on a previous work [41] where this attractiveness function is introduced, this paper gives a more realistic implementation to the tourist trip design problem.

Therefore, it is evident that although the literature presents a wide variety of models, to the best of our knowledge, no one has ever applied the methodologies listed above for the localization of chargers in tourist contexts. This thesis aims to cover this lack developing two promising models, respectively classified as a Location Problem and a Location Routing Problem for E-bike charging facilities on a touristic network. In particular the formulations are modeled as Mixed Integer Linear Programs (MILP) applied to direct graphs $G(N, A)$, faithful representation of the grid of trails, muleteer and roads present in cyclist resorts. On this wave, following the work analyzed in [39], the second model proposed in Chapter 2, exploits the attractiveness function to create a Location Routing Problem combining the Tourist Trip Design Problem with the optimal location of charging facilities along the network. Moreover, the complexity of this formulation results in an exponential number of constraints that requires the implementation of a Branch & Cut to solve the problem in a reasonable computational time.

1 | E-bike Charger Location Problem

1.1. Problem Description

Cycle tourism represents one of the largest considerably income for many resorts in the world. In the last years this market has growth, spurred on by the anti-climate change trend and the presence of new types of bicycle that attracts every kind of users. Some of the locations where it is possible to see this growing trend are the major touristic destinations, where several organized activities offer the possibility to relax, do sports and discover the food and culture of the territories for families and tourist of every kind. Among them, mountain resorts are the most popular for cycle tourism, where a large network of trails and muleteers are the perfect combination for Mountain Bikes and excursionists. Starting from strategic points in the network, such as bus stations, information hubs or small villages, the trails take different directions, passing trough attractive viewpoints and picnic-areas, to continue then their journey to others trails and villages.

This combination of trails can lead users to achieve distances up to hundreds of kilometers. Although these distances are normal for trained cyclist, thanks to the comfort offered by the e-bike, also less trained tourists can achieve this span by exploiting the e-bike electric motor. It follows that to face the needs of this new costumers, with the object of keeping the flow of tourism as much as possible on the network, a supporting charger infrastructure is mandatory to ensure a reliable charge spare capacity.

The cost related to the implementation of this charger facilities is quite high, making it impossible to place chargers frequently along the whole trails. Hence, especially in large networks, is fundamental to properly identify the demand of the e-bikes and consecutively install the charger facilities in the best sites in the network.

Another aspect that must be taken into account is the fact that in many touristic resorts, the paths can be devoted to specific cultural journey where the right sequence of the activities along the trails is fundamental to have a full immersion experience and appreciate the cultural message of the territory. Moreover, in mountain resorts the paths are constrained to follow the conformation of the territory, leading to have few intersection along the network, especially for mountain bikes, that usually have dedicated paths and cannot approach all the trails devoted to excursionist users. It follows that the network is described by a sequence of predefined paths connecting different sites in the resort.

This chapter aims at proposing and solving a formulation for this problem that fits these scenarios, finding the best possible location for the charger implementation along the network.

1.2. Methodologies

Choosing the best locations on a cycle tourist network for chargers facilities is a task that can be achieved with different types of models and techniques. As we will see in the next chapter the Location Routing Problem with Touristic Trip Design implementation, is the most promising technique that can be adopted over this kind of problems. Indeed, choosing to optimize the map of trails leads to have more degrees of freedom in the optimization routine that is able to create an infrastructure conformation where both the charger locations and the trails contribute to an optimal result in terms of attractiveness of the network and implementation cost.

Anyway, as mentioned above, redefining the map of trails could not be the right solutions for resorts where a proper offer of paths is already defined on the network.

Analyzing the literature, this problem can be seen as a facility location problem for EV. Works like [42] or [43], propose a way to solve the problems taking into account complicated factors such as people driving behaviors or the amount of charging request.

It follows that the formulations proposed in the literature are mainly devoted to cars and urban areas where the roads grid is quite different from a touristic resort, then models that take into account e-bicycle for this kind of problem are not treated, leading to have a lack of knowledge in this promising field. Moreover, the majority of the location problems assumes the implementation of the charger facilities possible along the whole network. Instead the formulation proposed here is based on a network, where the paths followed by the e-bicycles are constrained by the conformation of the territory or by cultural objectives and the location of the charger facilities is limited to a closed set of nodes.

1.3. Model

As written above the following model has the objective of providing a solution to a location problem applied to a particular network infrastructure devoted to cycle tourists. The described territory made of trails, common roads and muleteers can be modeled as directed graph $G = (N, A)$, where N represents the set of nodes corresponding to sites in the resort where an intersection between trails is present and or an infrastructure, rest area is located in that position. Some of these nodes are listed in a subgroup $N' \subset N$ due to the presence of a connection to the electric grid infrastructure and hence a faster and cheapest implementation of the charger facilities. The set A contains the arcs (i, j) of the graph associated to a real connection between the nodes along the network. An energy consumption e_{ij} characterizes the weight of the arc (i, j) that must be carefully calculated taking into account the cyclist behavior and the type of trail. Taking into account that some trail can be traveled in both the direction, e_{ij} is not necessarily equal to e_{ji}, in case of a steep hill for example, e_{ij} can be large and e_{ji} could be 0. Moreover, in case of trails dedicated to uphill or downhill, we may have $(i, j) \in A$ and $(j, i) \notin A$. Finally, at each node $i \in N'$ a cost c_i for installing a charging station is associated.

For the e-bikes a maximum batery capacity E is defined, moreover two important assumption are made:

- The e-bikes are supposed to have a full charged battery when starting the trails. This assumption is feasible because users are likely to charge their own e-bike before a long trail. This statement is also valid when e-bicycles are provided by rental facilities at the beginning of the trails, because the provider ensures a fully charged battery.
- At each charging session the maximum capacity is reached. Thus partial charges are not contemplated.

Let P denote the set of path in the network, each path $p \in P$ is described by a sequence of arcs a_{ij} connecting a starting node O_p to a destination node D_p, hence for each path a subset $A^p \subset A$ is defined.

It is important to underline that with respect to the location problem for classic EV, here it is not necessary to analyze the volume of cyclists along each trail to properly size the charger facilitates. Considering as model objective the minimization of the total number of chargers along the network, the proposed solution choose a set of strategic points, possibly common to more than one trail, where cyclists coming from different directions are able to rest and charge the bicycle. Hence, knowing the possible paths and the maximum turnout of tourists who visit the whole network every day we have a complete knowledge of the flow of charging that must be covered. It follows that each charger facilities should be sized according to the maximum turnout of tourists, and guarantee an infrastructure able to face the needs of the whole network.

To summarize, the model proposed here defines a location problem able to cover the flow of charge of each trail, locating major chargers facilities along the network; resulting then in a minimization of the total number of facilities to be installed and hence minor implementation costs for the entire infrastructure.

1.3.1. Notations

For sake of clearness, here the proper notation used in the model is listed:

- $G(N, A)$ directed graph, N set of nodes, A set of arcs;
- e_{ij} energy for crossing arc $(i, j) \in A$;
- E maximum capacity of a full charged battery;
- c_i cost for installing a charger in node d_i;
- v_i binary number equal to 1 if in node i is a possible to install a charger station, 0 otherwise, hence $v_i = 1 \leftrightarrow i \in N'$;
- P set containing all paths p;
- O_p Origin of path p;
- D_p Destination of path p;
- A^p subset of A containing all the arcs (i, j) of the path p.

1.3.2. Model Formulation

The model can be formulated as a Mixed Integer Linear Problem (MILP) where the used decision variables are:

- b_i^p Continuous variables associated to path $p \in P$, equivalent to the remaining battery at node i;
- y_i Binary variables, equal to 1 if the location problem chooses to put a charger in node i, 0 otherwise.

The MILP model is formulated as follow:

$$\min_{b\,y} \quad \sum_{i \in N} y_i c_i \tag{1.1}$$

s.t.

$$b_{O_p} = E \qquad \forall\, p \in P \tag{1.2}$$

$$b_j^p \le b_i^p - e_{ij} + E y_i v_i \qquad \forall\, (i,j) \in A^p, \forall\, p \in P \tag{1.3}$$

$$b_j^p \le E - e_{ij} \qquad \forall\, (i,j) \in A^p, \forall\, p \in P \tag{1.4}$$

$$b_j^p \ge (E - e_{ij}) y_i v_i \qquad \forall\, (i,j) \in A^p, \forall\, p \in P \tag{1.5}$$

$$b_j^p \ge b_i^p - e_{ij} \qquad \forall\, (i,j) \in A^p, \forall\, p \in P \tag{1.6}$$

$$b_i^p \in R^+ \qquad \forall\, i \in N, \forall\, p \in P \tag{1.7}$$

$$y_i \in \{0,1\} \qquad \forall\, i \in N \tag{1.8}$$

The objective function (1.1) minimizes the total cost associated to the charging station implementation in each selected sites. The first constraint (1.2) ensures that e-bikes has a maximum charged battery at the beginning of each path. The constraints from Eq. (1.3) to Eq. (1.6) defines the remaining battery at each nodes i for each path p, hence the behavior of the variable b_i^p. In particular Eqs. (1.4) and (1.5) ensure that at the presence of a charging station at node i, hence $y_i = 1$, the remaining capacity at the next node j is equal to the total energy E subtracted by the energy e_{ij} along the arc a_{ij}. Instead, if there is no a charging station at node i, Eqs. (1.3) and (1.6) ensure that variable b_j^p is equal to the remaining charging capacity at node i, hence b_i^p minus the energy e_{ij} to reach node j along the arc a_{ij}.

Finally, Eq.s (1.7) and Eq. (1.8) bind respectively variable b_i^p to the domain of continuous numbers and y_i to be a binary variable.

This formulation guarantees that the remaining charging capacity at each node is always greater or equal to the energy required to go to the following node along the trail, with the possibility of recharging the e-bike when it runs out of charge.

1.3.3. Model Implementation

The model was implemented and solved by the Python-MIP package. This package provides tools to formulate different types of Mixed-Integer Linear programming problems (MIPs), from the default installation it includes a large offer of solver such as the COIN-OR Linear Programming Solver (CLP), the COIN-OR Branch-and-Cut solver (CBC), and it supports the state-of-the-art GUROBI MIP solver, a strong mathematical optimization solver that provides different configuration tools and settings [44].

In this work the GUROBI solver was used, because is the most powerful one and it guarantees the fastest and most precise solutions. However, several commercial packages can be used, even if generally less performing.

Here a simple example of the results provided by the Location routing model exposed above is presented. The network under analysis it is a fictional map corresponding to a touristic resort infrastructure modeled as a direct graph $G(N, A)$ composed by 11 nodes and 17 arcs.

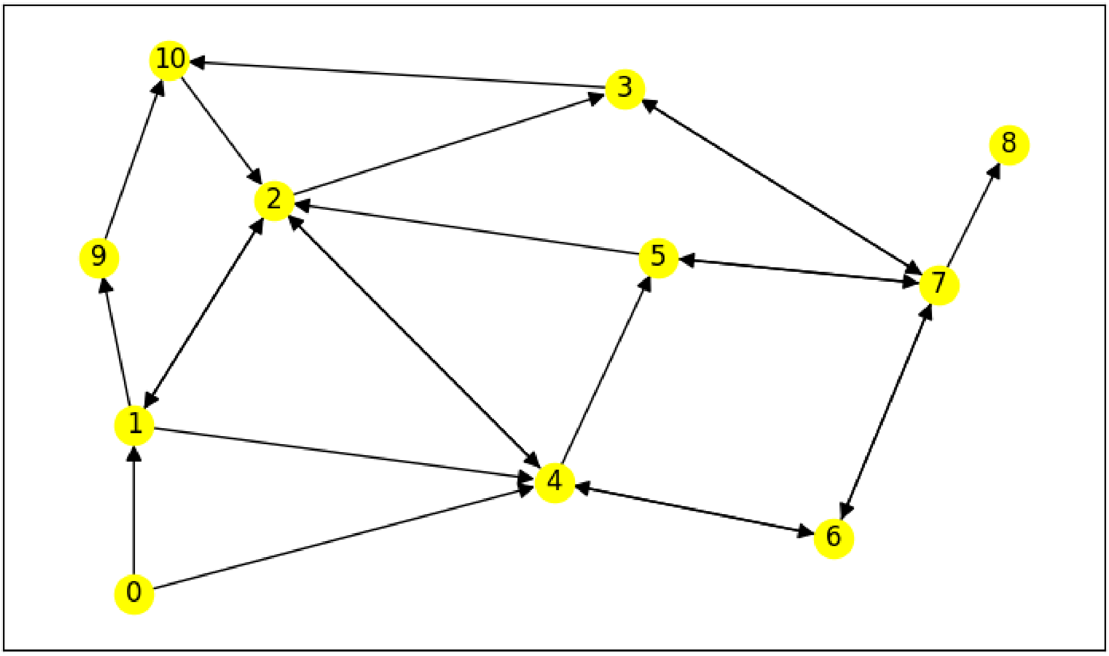

Figure 1.1: Network example

Given a set of five trails, a random energy consumption e_{ij} for each arc and a charger implementation cost c_i for each node $i \in N'$, a simplistic result follows in figure 1.2.

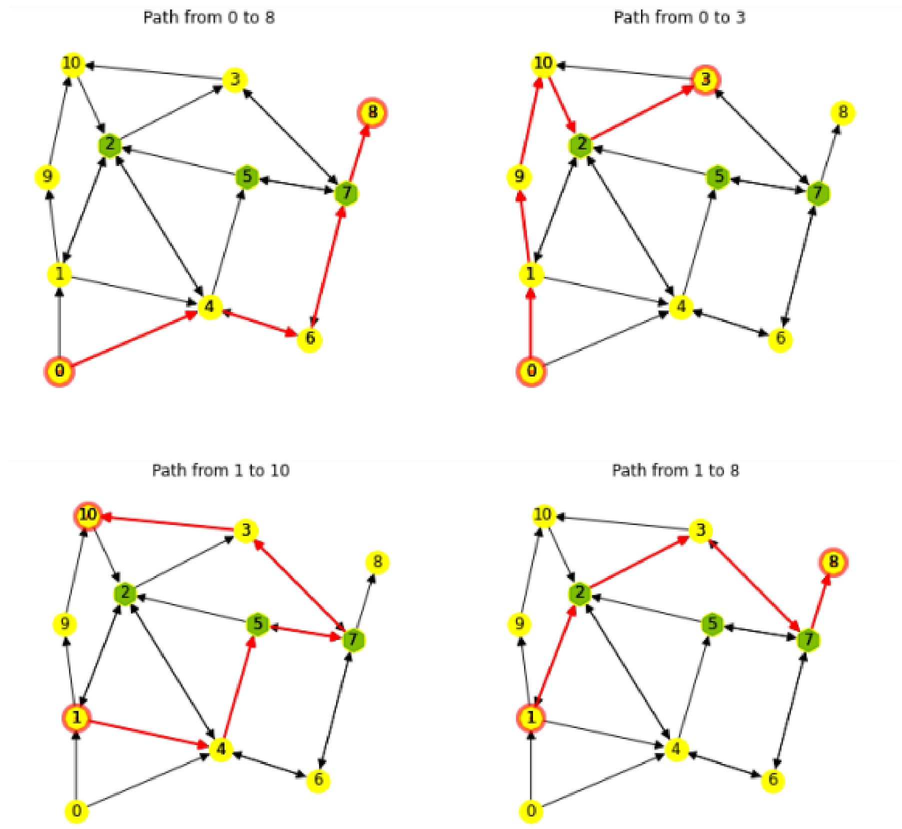

Figure 1.2: E-charger Location Problem Solution

For each path a proper scenario is shown and the trails followed by the cyclists is highlighted in red. The node chosen to implement the charger facilities are the one highlighted in green and it is important to underline that the infrastructure is always the same for each scenario, hence the chosen charger locations are common to all the five scenarios. As can be seen the implemented facilities are less than the traveled paths, this is possible because the program selects nodes shared by more trails. A more precise analysis of the results is done in Chapter 3, where the model is applied to a set of different randomly generated graphs.

2 | E-bike Charger Location Routing Problem

2.1. Problem Description

The assumption of having a predefined network of trails as stated in the first Chapter 1, remains true for resort where paths are devoted to cultural journey and the right sequence of activities is fundamental to appreciate the cultural message of the territory. However, the majority of the resorts present an offers of trails, mule tracks and roads that are not devoted to specific cultural paths but create a huge network of connection between villages and point of interest along the territory, with the presence of few specific paths that connect origin-destination pairs simply following the shortest way.

As extensively dealt in the introduction to this thesis, to follow the growing trend of e-bicycle users, the infrastructure managers of this territories must properly consider the implementation of charger facilities along the trails, considering different factors and information to optimally allocate the spare budget of these resorts. However, the absence of predefined tracks lead to have a lack of information for what concern the location of the flow of cyclists along the network. Hence the first model cannot be used in this scenario but a proper formulation must be adopted.

In this chapter we first analyze how this problem is dealt in the literature, to then explain the Tourist Trip Design methodology used to derive a Routing Model explain in section 2.3.1. Subsequently we exploit this formulation to propose a Location Routing Model capable to find a solution for the e-bike charger location problem.

2.2. Methodologies

In case of extra urban roads and tracks such as in a mountain resorts, to minimize the cost of installation of the chargers along the network is fundamental to know the most common tracks by the cyclists and hence understanding which are the best possible sites for the location of the chargers facilities. This issue can be solved with information about paths defined by a sequence of arcs connecting different pairs of origin-destination nodes, as the scenario analyzed in Chapter 1. Without this information, understanding where is located the flow of charge that must be covered along the network, lead to make the e-charger location problem quite challenging.

In the literature papers such as [45], defines a methodology to optimally locate charging stations using a trip origin/destination matrix, coupled with information about the elevation of the territory to enable a specific characteristic of the energy needs. In this work a routing problem on the Lyon Metropolitan Area to compute the various trips is solved, providing with a complete knowledge of the flow of charge required along the network. Hence, the literature presents studies adopting a location routing problem (LRP) to find a solution for the optimal chargers locations problem. However these models mainly analyzed large cities scenarios and they concentrate on classic EV such as private long range vehicle.

This chapter aims at defining a formulation modeled on a touristic resort scenario for e-bicycle users. To do so, the methodologies introduced by the Tourist Trip Design Problem perfectly fit with the goal of finding the trips along a cyclists network, maximizing the economic benefits and improving the competitiveness of the entire infrastructure. By using information such as the conformation of the territory, the activities along the tracks and the kind of users that every day visit the resort it is possible to find optimal paths for different class of users, maximizing the attractiveness of the whole network.

Matching the potentiality of this type of problems with the formulation provided in the Chapter 1, the model here exposed proposes a solution to the Location Routing Problem offering a strategic design of the paths that aims at maximizing the competitiveness of the network and minimize the total implementation cost of the charger stations.

2.3. Tourist Trip Design Problem

By using territory and cultural information, these kinds of models propose different techniques to design an optimal offer of tracks that aims at maximizing the competitiveness and attractiveness of the whole tourist resort. In general the objective, of these models is to maximize the total score associated with the visit to Points of Interest (POI) along the network. Schilde [46] for example, proposed a model where the goal was to find routes that maximize the total score associated to each POI category; Abbaspour [31], instead formulated a model to maximize the score considering multi-modal transports where the weights of the arcs and the travelling time were used.

Here the goal is to design the most attractive itineraries from a pool of origin-destination pairs in order to maximize the competitiveness of the network for different kind of user. To do so, we based our routing problem on a model proposed in [39]. This work proposes a routing problem which design an offers of trails on the Trebon region in the South Bohemia province of Czech Republic, with the goal to exploit the old network of muleteers, common tracks and roads, in order to create an offers of cyclist paths, respecting the budget and maximizing the competitiveness of whole the network. To do so, an attractiveness function is proposed, based on information about tourist behaviors and strategic known sites such as lakes and viewpoint along the network. This method defined a concave attractiveness function inversely proportional to the number of visit of the same arcs or nodes along the paths. Moreover, it assumes different values with respect to the kind of tourist approaching the network. In this way a proper offers of trails dedicated to different kinds of cyclists is defined, ensuring a maximization of the attractiveness of the whole territory. The result is a cyclist network that emphasize the economic impact of the tourist in that region, spurring the local businesses and increasing the job opportunities.

Here we exploit the power of this method utilizing a simpler version of the attractiveness function proposed in [39], keeping the different rewards with respect to the kind of user under analysis, leading to a more realistic routing problem, able to design an set of tracks that reflects the tourist's needs. This is done because the model proposed in [39] is able to collect the attractiveness reward over more traversal without keeping track of the correct arc sequence, hence it is not compliant with the routing problem required by the Location Routing Model. It follows that a simpler formulation is required to match the two methodology.

2.3.1. Routing Model

The routing problem proposed here is a simpler version of the problem, where decisions on recharging infrastructure are disregarded, that will be then integrated in the Location Routing Problem exposed in the next sections of this Chapter 2, where the decisions on the location of charging station are integrated in section 2.4.

Once the grid of trails, muleteer and roads present on the resort territory is modeled as a direct graph $G(N, A)$, starting from a pool of origin-destination pairs previously defined, the routing model here proposed aims at finding the best possible paths. To do so a modified version of the attractiveness function described above is used to properly collect different rewards over the nodes and arcs of the trip, reflecting the preferences of the users. In particular we suggest three different category of cycle tourist, namely the classic, the gastronomic and the sporty tourists. However different kind of profiles can be considered without loss of generality. The classic tourist is attracted by observation decks, waterfalls, villages and lakes, more likely choosing paths on scenic landscapes, without strong slopes where a physical preparation is required. The gastronomic profile privileges rest area like picnic parks and restaurants, but also mountain chalets and vineyards. Moreover, like the classic tourist, tries to avoid difficult trails. Contrary, the sporty cyclist is more oriented to difficult trails, where the effort required by both the e-bicycle and the user is higher, with the goal of riding more kilometers or achieving the highest peaks of the mountain resort. It is important to underline that the majority of the trails is accessible to each class of tourist, hence it is possible that in order to achieve a viewpoint or a chalet also the classic or the gastronomic tourist goes trough difficult slopes, with a bigger effort required by the e-bike.

Hence the objecting function is a maximization of the total reward obtained by the trip for each users for each origin-destination pair. It follows that some constrains must be defined in order to keep the paths designed by the model in a certain range of time and kilometers. In particular a constraint is defined on the maximum time available for each class of cyclist. The total time required by a path for a certain origin destination pair, is then calculated summing the time spent on each arc, corresponding, as in chapter 1, to a trail, muleteer or roads between two nodes along the network. Here an important assumption is taken: considering that each user is provided with an electric bicycle and high velocity are strongly not recommended for the presence, on the same network, of different kind of users such as children and elders, an average velocity of $25km/h$ up-going and $35km/h$ down-going is stated. This is valid for each class of cycle tourists and it rules the timing on the network.

For simplicity, in this first version of the problem, multiple visits of an arc or node along the same path are not allowed. As explained in the introduction to this section 2.3, this allows a model flexible enough to be integrated with the Location Routing Problem exposed in the next section 2.4.

Notations:

The terms described above and the terminology used in the model are here summarized:

- $G(N, A)$ directed graph, N set of nodes, A set of arcs;
- t_{ij} time for crossing arc a_{ij};
- T^u maximum time for user u;
- P set containing all the paths p defined by O-rigin and D-estination;
- O_p Origin of path p;
- D_p Destination of path p;
- N^p any node subset containing both O-rigin, D-estination of a certain path p;
- U set containing each user type u;
- a_{ij}^u is the attractiveness on the arc (i, j) for user u;
- d_i^u is the attractiveness on node i for user u.

Model Formulation

The decision variables used in the model are the following:

- x_{ij}^{up} Binary variables associated to user type u for $p \in P$, equal to 1 if the routing problem chooses the arc $a_{i,j}$, 0 otherwise;
- γ_i^{up} Binary variables associated to user type u for $p \in P$, equal to 1 if the routing problem chooses the node i, 0 otherwise.

The problem can be solved by a Boolean optimization program with the following mathematical formulation:

$$\max_{x\ \gamma} \quad \sum_{p \in P} \sum_{u \in U} \Big(\sum_{(i,j) \in A} a^u_{ij} x^{up}_{ij} + \sum_{i \in N} d^u_i \gamma^{up}_i \Big) \tag{2.1}$$

s.t.

$$\sum_{(j,i) \in BS(i)} x^{up}_{ji} - \sum_{(i,j) \in FS(i)} x^{up}_{ij} = b_i \qquad \forall i \in N, \forall u \in U, \forall p \in P \tag{2.2}$$

$$\sum_{(i,j) \in A} t_{ij} x^{up}_{ij} \leq T^u \qquad \forall u \in U, \forall p \in P \tag{2.3}$$

$$\sum_{(i,j) \in FS(i)} x^{up}_{ij} = \gamma^{up}_i \qquad \forall i \in N, i \neq D_p, \forall u \in U, \forall p \in P \tag{2.4}$$

$$\sum_{(j,i) \in BS(d)} x^{up}_{jD_p} = \gamma^{up}_{D_p} \qquad \forall u \in U, \forall p \in P \tag{2.5}$$

$$\gamma^{up}_v \leq \sum_{(i,j) \in A: i \notin N^p, j \in N^p} x^{up}_{ij} \qquad \forall v \notin N^p, \forall N^p \subset N, \forall u \in U, \forall p \in P \tag{2.6}$$

$$x^{up}_{i,j} \in \{0,1\} \qquad \forall (i,j) \in A, \forall u \in U, \forall p \in P \tag{2.7}$$

$$\gamma^{up}_i \in \{0,1\} \qquad \forall i \in N, \forall u \in U, \forall p \in P \tag{2.8}$$

The objective function (2.1) maximize the total attractiveness collected by each user at each edge and node of the network, for the chosen itinerary. The object function is also the only function that ties together the decisions involving the different users. The first constraint (2.2) ensures a flow balance between the origin and destination nodes of the path p for each users class. In particular $BS(i)$ and $FS(i)$ are respectively the Backward and Forward Star of node i and b_i is equal to -1 if i is the origin node, $+1$ if i is the destination node and 0 elsewhere. Eq.s (2.3) impose a constraint on the maximum travelling time for each path p. The maximum time T is different for each class of users, respecting the physical preparation of each cyclist. Eq.s (2.4) bounds together x^{up}_{ij} and γ^{up}_i, moreover combined with eq. (2.5) they ensure a correct population of variable γ^{up}_i. For each user flow, the connectivity of the path proposed by the program is enforced by eq.s (2.6), indeed this constraints cut all the possible sub-tour disconnected from the main track. However, they create an exponential number of constraints which ask for a positive user flow going into subset N^p when a node $v \notin N^p$ is selected as part of the itinerary for user u from O_p to D_p. Finally, constraints (2.7 -2.8) bind the decision variables to be binary, in this way we ensure the single visit of the nodes and arcs of the network.

Model Implementation:

As for the model proposed in Chapter 1, the program was implemented using Python-MIP. Here the routing model is applied as example to the same network provided in the first chapter. The difference is that the data associated to the nodes and arcs are enriched with randomly generated attractiveness values, corresponding to the user's preferences related to the point of interest of the territory.

Figure 2.1 shows the discrete graph interpretation of the territory. The nodes number (0-1-5-8) are connected to the urban grid, with the presence of bus stations and possible e-bike rental locations. Then the attraction of territory are highlighted by symbols such as the winery, the wood, some pic-nic area, a castle and the mountains.

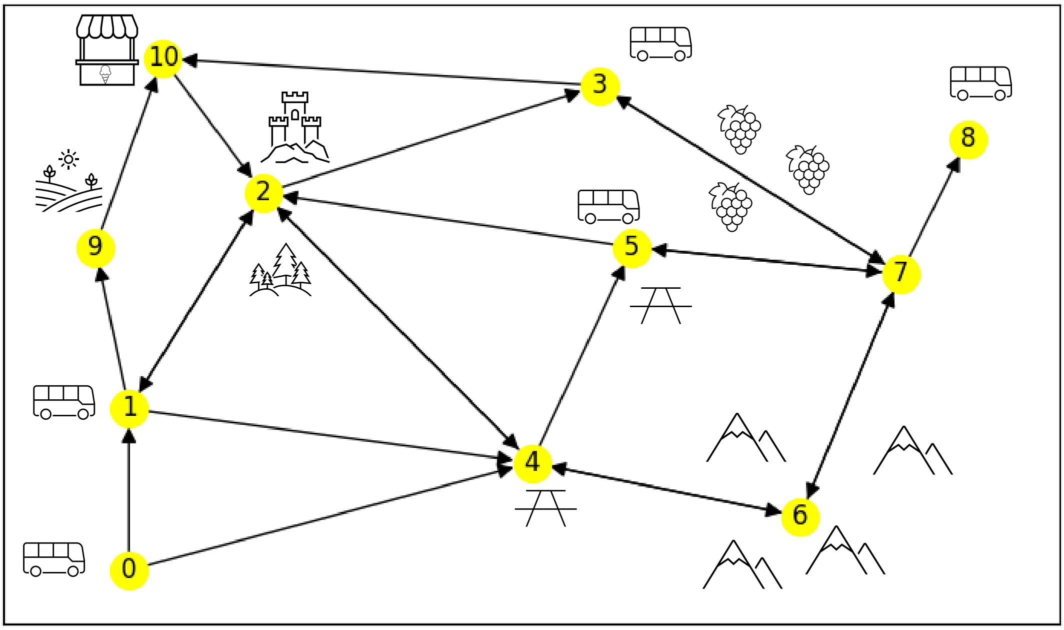

Figure 2.1: Network with Attractive Sites

Assuming that the bus stations are the possible entrances to the resort, this point are used to create a pool of Origin/Destination pairs. It follows that, applied to this scenario, the routing problem provides with a set of trails maximizing the reward of the cyclists in terms of attractiveness. Hence, the program propose for each origin-destination pair different tracks dedicated to the preferences of the class of users under analysis. The solutions is shown in Figure 2.2:

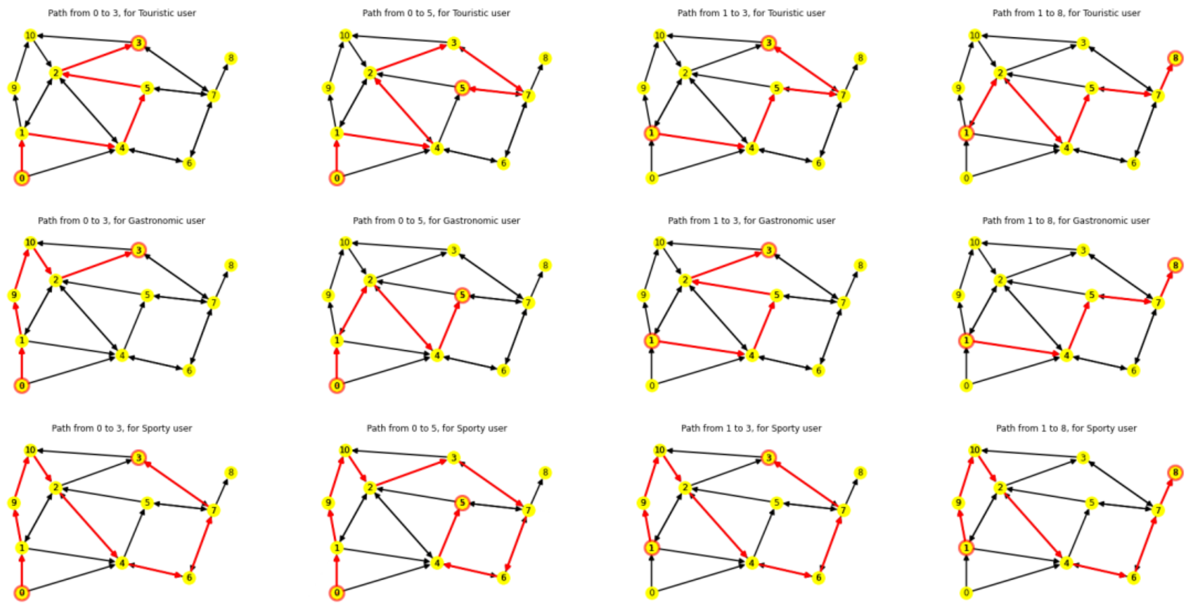

Figure 2.2: Network example

2.4. Location Routing Problem

Based on the routing problem presented in the previous section, we now integrate the optimization of the charging stations location. The resulting problem is a variant of the location Routing Problem.

The classic Location Routing Problem (LRP) aims at finding the optimal location of some facilities and the routes of the vehicles to serve the customers demand under facilities and vehicles constraints. With the growing of the electric vehicle market, this methodology was exploited by many reseachers to find the best locations for charger facilities, in order to cover the energy demand asked by EV present on the network. As previously explained, the majority of this models analyzed large cities scenario, taking into account classic electric vehicles such as cars and bus.

In this work we propose a location routing problem that aims at finding the best chargers locations for e-bike users, matching the formulation stated by the Tourist Trip Design Problem with the Location Problem expose in Chapter 1. This is possible by exploiting the routing variables populated by the Tourist Trip Design model to activate the constraints of the charger location model, ensuring a routing solution that match the energy constrained of the electric bicycle. Indeed, bounding the routing problem to satisfy the never empty battery requirement, ensure the implementation of a certain number of charger facilities along the tracks. Then a proper formulation of the object function consist in the maximizing the attractiveness reward collected by the routing problem, while minimizing the total implementation costs of the charger facilities. It follows that the derived solution exploits the attraction points of the network to design a set of tracks with possible common sites that satisfy the energy needs, minimize the charger's investments and maximizes the competitiveness of whole resort.

2.4.1. Notations

Here the notation utilized in the past models and properly adapted for the following LRP model is summarized:

- $G(N, A)$ directed graph, N set of nodes, A set of arcs;
- P set containing all the paths p defined by O-rigin and D-estination;
- O_p Origin of the path p;
- D_p Destination of the path p;

- U set containing each user type u;
- e_{ij}^u energy for crossing arc (i, j) for user u;
- t_{ij}^u time for crossing arc (i, j) for user u;
- T^u maximum time for user u;
- E maximum capacity of a full charged e-bike battery;
- c_i cost for installing a charger in node i;
- v_i binary number equal to 1 if it is possible to install a charger in node i;
- N^p any node subset containing both O-rigin, D-estination of a certain path p;
- a_{ij}^u is the attractiveness on the arc (i, j) for user u;
- d_i^u is the attractiveness on node i for user u;
- W is the weight to properly scale in the object function the total implementation cost with respect to the attractiveness reward.

2.4.2. Model Formulation

The decision variables used in the model are the following:

LOCATION:

- b_i^{up} Continuous variables associated to user u for $p \in P$, equal to the remaining battery at node i;
- y_i Binary variables, equal to 1 if the location problem chooses to put a charger in node i, 0 otherwise.

ROUTING:

- x_{ij}^{up} Binary variables associated to user type u for $p \in P$, equal to 1 if the routing problem chooses the arc $a_{i,j}$, 0 otherwise;
- γ_i^{up} Binary variables associated to user type u for $p \in P$, equal to 1 if the routing problem chooses the node i, 0 otherwise.

The **MILP** is:

$$\max_{b\ y\ x\ \gamma} \quad \sum_{p\in P}\sum_{u\in U}\Big(\sum_{(i,j)\in A} a^u_{ij}x^{up}_{ij} + \sum_{i\in N} d^u_i\gamma^{up}_i\Big) - W\sum_{i\in N} y_i c_i \tag{2.9}$$

s.t.

$$\sum_{(j,i)\in BS(i)} x^{up}_{ji} - \sum_{(i,j)\in FS(i)} x^{up}_{ij} = b_i \qquad \forall\, i\in N, \forall\, u\in U, \forall\, p\in P. \tag{2.10}$$

$$\sum_{(i,j)\in A} t^u_{ij}x^{up}_{ij} \le T^u \qquad \forall\, u\in U, \forall\, p\in P \tag{2.11}$$

$$\sum_{(i,j)\in FS(i)} x^{up}_{ij} = \gamma^{up}_i \qquad \forall\, i\in N \leftrightarrow i\neq O_p, \forall\, u\in U, \forall\, p\in P \tag{2.12}$$

$$\sum_{(j,i)\in BS(O_)} x^{up}_{jO_p} = \gamma^{up}_{O_p} \qquad \forall\, u\in U, \forall\, p\in P \tag{2.13}$$

$$\gamma^{up}_v \le \sum_{(i,j)\in A: i\notin N^p, j\in N^p} x^{up}_{ij} \qquad \forall\, v\notin N^p, \forall\, N^p\subset N, \forall\, u\in U, \forall\, p\in P \tag{2.14}$$

$$b^{up}_o = E \qquad o\in p, \forall\, u\in U, \forall\, p\in P \tag{2.15}$$

$$b^{up}_j \le b^{up}_i - e_{ij}x^{up}_{ij} + E y_i v_i + (1 - x^{up}_{ij})E \qquad \forall\,(i,j)\in A, \forall\, u\in U, \forall\, p\in P \tag{2.16}$$

$$b^{up}_j \le E - e_{ij}x^{up}_{ij} \qquad \forall\,(i,j)\in A, \forall\, u\in U, \forall\, p\in P \tag{2.17}$$

$$b^{up}_j \ge (E - e_{ij})y_i v_i - (1 - x^{up}_{ij})E \qquad \forall\,(i,j)\in A, \forall\, u\in U, \forall\, p\in P \tag{2.18}$$

$$b^{up}_j \ge b^u_i - e_{ij} - (1 - x^{up}_{ij})E \qquad \forall\,(i,j)\in A, \forall\, u\in U, \forall\, p\in P \tag{2.19}$$

$$x^{up}_{i,j} \in \{0,1\} \qquad \forall\,(i,j)\in A, \forall\, u\in U, \forall\, p\in P \tag{2.20}$$

$$\gamma^{up}_i \in \{0,1\} \qquad \forall\, i\in N, \forall\, u\in U, \forall\, p\in P \tag{2.21}$$

$$b^{up}_i \in R^+ \qquad \forall\, i\in N, \forall\, u\in U, \forall\, p\in P \tag{2.22}$$

$$y_i \in \{0,1\} \qquad \forall\, i\in N \tag{2.23}$$

The objective function (2.9) is the maximization of a value that corresponds to a trade-off between the total attractiveness collected by each user on each path and the total costs related to the e-charger implementations. it is important to underline that in order to minimize the total implementation costs, the corresponding sum is preceded by a negative sign, in this way maximizing the function we are also minimizing these costs. Moreover, considering that the attractiveness data associated to each arc and node are of the orders of units, the total implementation costs are scaled by a weight W. This value must be properly tune in accordance with the objective of the resort. In particular a lower weight correspond to an offers of trails closer to the best preferences of the users and potentially an higher implementation cost marginally due to the higher number of chargers facilities. Instead if the main task is to minimize the implementation costs without losing the possibility to have an attractive offers of paths, a smaller value of W should be selected.

From eq.s (2.10) to (2.14) the constraints bound and properly populate the routing variables x_{ij}^{up} and γ_i^{up} for each user u and path p as in the routing problem previously exposed. Then this variables are used in the next constraints (2.15-2.19) appertaining to the covering problem of the first chapter. The routing decision variables gives hence a perfect knowledge of which tracks are followed by the cycle tourist, defining visited arcs and nodes. In this way these variables can be used to activate the covering constraints and hence properly populate the positional variable y_i, locating the charger facilities in order to cover the energy demand along the paths. Finally, eq.s (2.20-2.23) bound the decision variables to their proper domain.

2.4.3. Model Implementation

As the previous models, the Location Routing MILP formulation was implemented in Python using the Python-MIP package. Here a simple example showing the capacity of the program is shown. The scenario under analysis is the simple fictional network used also in the previous examples, formulated as a Graph $G(N, A)$ with 11 nodes and 23 arcs. Merging than the data used for the routing problem with the one used for the location problem, such as the energy requirements, we end up with the following solution:

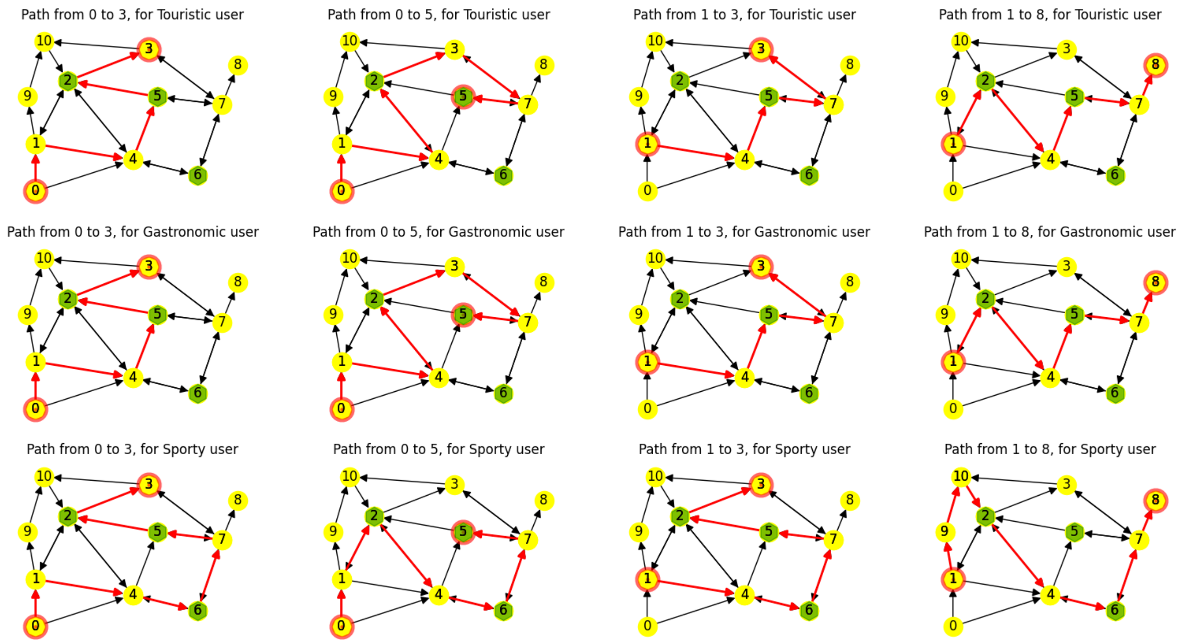

Figure 2.3: LRP solution example

The solution reflects the aim of the algorithm, proposing different itineraries for each class of user, maximizing the attractiveness of the network and minimizing the number of charger stations.

Branch and Cut

The strong formulation of the Location Routing Program exposed above include an exponential number of constraints that could limit the performance of the algorithm when applied to large graph. Model of this kind can be directly solved by the Python-MIP solver only for small instances, indeed adding all this constraints at once is usually not practical, asking a strong computational effort that can lead to a not convergence of the algorithm.

This kind of problems can hence be solved using the cutting planes method where a relaxation of the model is solved and only the violated constraints are inserted. The most famous relaxation method is the Linear Problem Relaxation that aims at removing the integrality constraints of the real formulation obtaining an easier solvable Linear Problem (LP). Then the relaxed model is optimized and if an integral solution is found the original problem is solved. If the solution violates one of the relaxed constraints, the method generates a cutting plane, hence a linear constraint that excludes the LP solution but does not consider any integer points. Finally, the cut is added to the LP and the routine is repeated. Hence the cutting plane method generates linear constraints which aim at closing the region of the possible solutions getting closer to the so called convex hull, defined as the region containing all the feasible integral points of the original model.

Computing the convex hull may become very difficult with a large number of decision variables. In the LRP proposed above, the computational effort grows significantly when applied to large graphs where an important number of paths and users are defined. Better results can be obtained with the Branch & Cut algorithm, a method of combinatorial optimization, in which cut generation is combined with branching. Exploiting tree-based search method, so called Branch & Bound, the algorithm explores the space of possible variable's values and combining the method of cutting planes improves the bounds found via the LP relaxation. Finally, as the classic Branch & Bound, it prunes the branches of the tree finding the optimal integer solution of the problem without exploring all the possible leafs of the search tree.

As explained in the documentation of Python-Mip [47], the package allows to implement this method using a BC algorithm implemented in the solver engine using callbacks. Cut generation callbacks (CGC) are called at each node of the search tree where a fractional solution is found. Cuts are generated in the callback and returned to the MIP solver engine which adds these cuts to the Cut Pool. The most active cuts are merged with the cuts generated by the solver builtin cut generators and then the most active ones are included to the relaxed model.

This methodology could be useful also to produce lazy constraints. Lazy constraints are dynamically generated, just as cutting planes, with the difference that lazy constraints are also applied to integer solutions. They should be used when the initial formulation is incomplete.

In our model, this approach could be useful to avoid the sub-tours elimination constraints Eq. (2.14) from the initial formulation. Indeed, these constraints are stated for every subset of nodes, leading to have an exponential number of constraints proportional to the number of nodes of the Graph under analysis. Moreover, these constraints cannot be neglected because are fundamental to avoid disconnected sub-tours created by the routing problem. Hence, using the lazy constraint generator, MIP allows to solve an incomplete initial formulation, adding the sub-tour generation constraints on demand.

In this scenario is fundamental to solve the Separation Problem, an optimization routine that leads to know when and which are the missing violated sub-tour elimination constraints, activating the relatives lazy constraints.

The separation problem and hence the lazy constraints generation is solved eat each leaf of the tree. Analyzing the results of the incomplete problem formulation denied from the sub-tour elimination constraints, for each origin-destination pair and user Figure 2.3, an appropriate logic is executed.

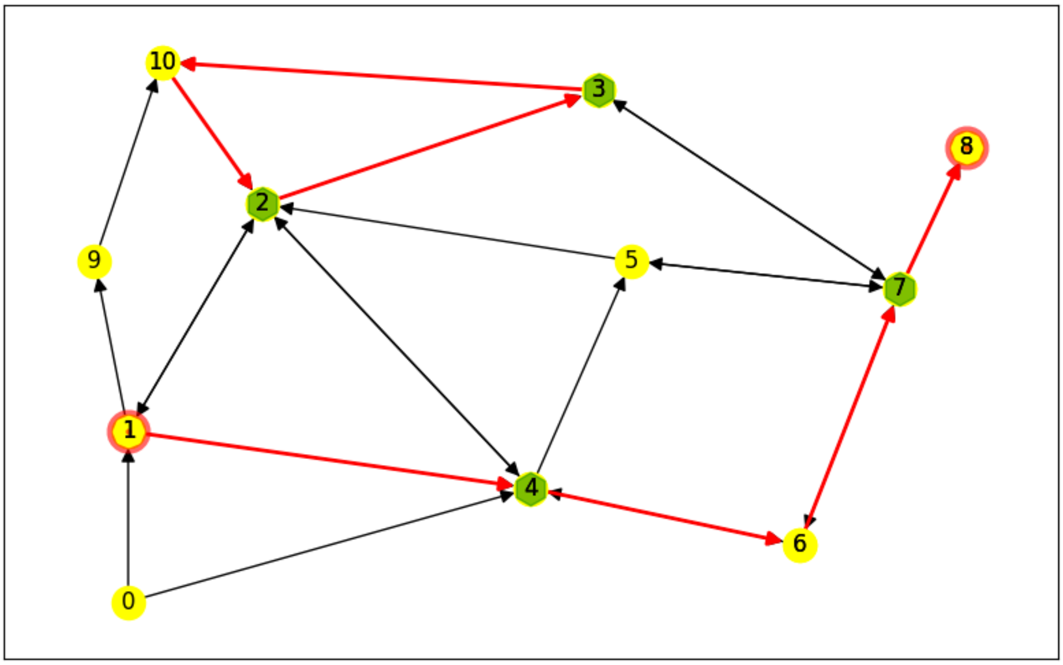

Figure 2.4: Path from 1 to 8 for Sporty user

After generating the graph composed by arcs active in the unfeasible solution for each user and path, the code collects the value γ_d^{up} associated to the destination node d. Then for each node v, different from the destination one and with a value γ_v^{up} greater than 0, the Minimum Cut Problem is executed. This method makes a partition of the graph in two subset (NS, S) that are disjointed for a certain minimum quantity, corresponding to the value of the minimum arc that connects these two subsets. The subsets (NS, S) contain respectively the node v and the destination node d. It follows that if these two subset are isolated a sub-tour characterize the solution.

For example Figure 2.2 if the selected node appertain to the subset composed by nodes $(2-3-10)$, the two subsets returned by the Minimum Cut Problem are isolated and a lazy constraint must be activate to eliminate the disconnected sub-tour. To automatically identify if the subsets are isolated a simply inequality is checked:

$$value \leq \gamma_v^{up} - \epsilon \tag{2.24}$$

Where *value* corresponds to the value of the minimum cut hence the arc connecting the two subset. It follows that if this value is smaller than γ_v^{up}, it means that v belongs to a disconnected sub-tour. Hence a lazy constraint of this form 2.25 is added to the program:

$$\gamma_v^{up} \leq \sum_{(i,j)\in A: i\in NS, j\in S} x_{ij}^{up} \tag{2.25}$$

This procedure is done for each path and user of the scenario under analysis, ensuring the elimination of disconnected sub-tours, moving the number of sub-tour elimination constraint from 2^N to N. However, these procedure allows us to cut only sub-tours of dimension greater than two, hence simple constraints of the form 2.26 are added to the initial formulation. In this way we eliminate also sub-tours of dimension two.

$$x_{ij}^{up} + x_{ji}^{up} \leq 1 \qquad \forall a_{ij} \in A, \forall u \in U, \forall p \in P \tag{2.26}$$

2.5. Model Extension with Multiple Level

The LRP formulation obtained above allows us to find the best charger facilities locations, disposed along the network to cover the energy demand of the cycle tourists travelling on a strategically designed offer of trails. However following the Tourist Trip Design methodology, the maximization of the attractiveness reward obtained from the paths traveled by the cyclist should allow the cyclists to visit multiple times an arcs or nodes along the same path. Indeed, because the cycle tourists ride for pleasure, it could happen that riding for more than one time the same track increase the satisfaction of the user even though it is not a new experience.

It follows that the LRP model exposed above limits the possibility to obtain a higher reward from a routing problem with multiple visit as exposed in [39]. The formulation proposed in this thesis adds to the classic routing problem a binary variable ξ_{ij}^{k} associated to each one of the k^{th} traversal of arc a_{ij}. In this way the model allows to keep track of the number of times that the cyclist rides along the same arc. The same method is applied to the nodes of the graph with a proper variable. Then the proposed attractiveness function returns a specific reward with respect to the number of traversal of the same tracks. In particular after the second visit the total reward can still be greater than the one obtained at first visit but, less than the double: riding the same tracks several times can still be enjoyable after many traversal although the absolute pleasure may, reasonably, decrease.

However, the mathematical formulation exposed in this model cannot be used here because although if it is able to keep track of the number of traversal, it does not allow to know the right sequence of tracks from origin to destination. Moreover, the LRP formulation here proposed already presents many variables and constraints, hence adding other terms could lead to have a not easy solvable problem.

Exploiting the Branch & Cut method applied above, the solution we propose suitable modifies the data of the problem in order to allow multiple visits of the same arc or node. Duplicating the nodes of the graph in two levels we can associate the first level to the first visit of the network and the second one to the revisit. In general it is possible to use this techniques to create more than two levels, allowing the cyclists to do more traversals. However, after the third ride along the same track the pleasure of the tourist starts to decrease. Hence here we propose only two levels leading to have a maximum number of three traversal of the same arc and only two of the same nodes. hence the third visit of the same node is not allowed.

The graph is now composed by two levels, a first one corresponding to the real network of trails and sites along the territory infrastructure, and a second one that is a straight copy of the real network, with the same conformation of arcs and nodes. Between these two levels, a pair of arcs are stated for each connected nodes in the original network, one for rise and one for fall from the actual level. Considering the fictional network used in the previous examples, the multiple level graph takes the following shape:

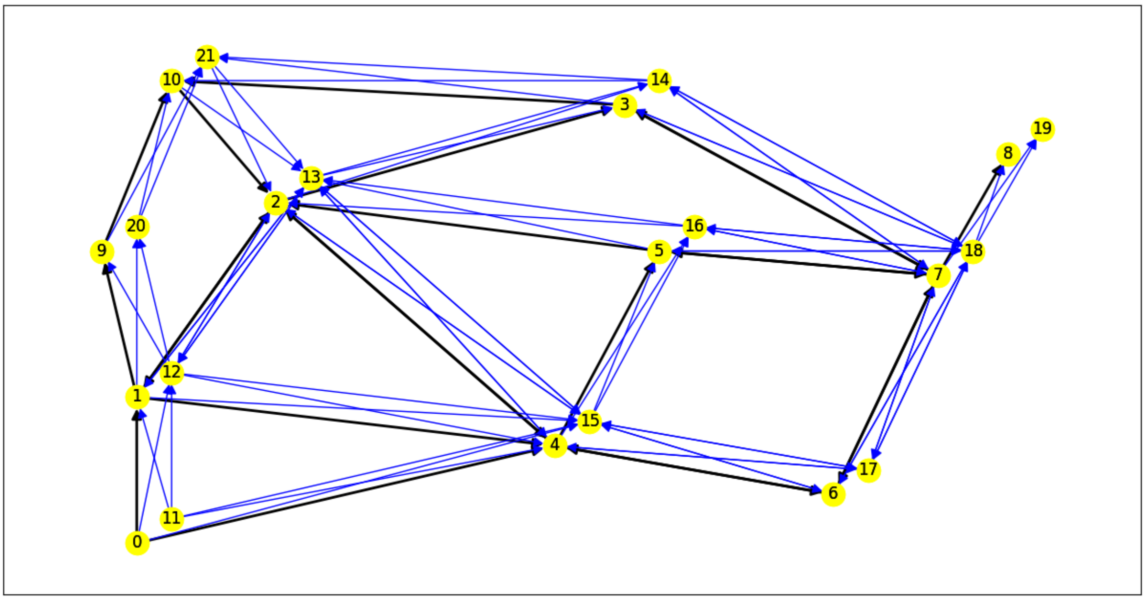

Figure 2.5: Fictional Multi-level Network Example

As can be seen, the two levels are defined by a shift in the nodes enumeration ($n0$), equal to the cardinally of the original nodes set $N0$. It follows that the obtained graph has now two levels with a double number of nodes and four times the arcs of the original grid. The data used for the original LRP model, such as the energy consumption, the time and the installation costs associated to each candidate sites, are the same for both the levels and for the arcs moving between them. The only difference is the attractiveness function that associates different values on each level. In particular considering that the pleasure at the second and third traversal should be smaller than the one obtained at the first one, the attractiveness value associated to each arc of the upper level are half with respect to the original one; the same is done for the arc that rise between the first and the second level, leaving to the original attractiveness value the fallen arcs. In this way the routing problem is allowed to obtain a higher reward from designing paths on the first level and only in case of high attractiveness it may choose to route on the second level.

However, the object function maximizes the total attractiveness obtained from the combination of all the itineraries. It follows that the routing problem can decide to route on the second level of some nodes for then going to the first one, obtaining a final reward higher then the one passing through first level of the previous arc in the sequence. This is caused by an attractiveness value of some arcs that is smaller of the half of other ones. Hence the following constraints are defined in order to keep the routing result on the first level and only in case of an already visited trail going to the second one.

$$x_{i,j} + x_{j,i} = x_{i+n0,j+n0} + x_{i,j+n0} \qquad \forall a_{i,j} \in Asl1 \subset A \tag{2.27}$$

$$x_{i,j} = x_{i+n0,j+n0} + x_{i,j+n0} \qquad \forall a_{i,j} \in Asl2 \subset A \tag{2.28}$$

Where $Asl1$ contains all the arcs $a_{i,j}$ of A of the first level that has also a returning arc $a_{j,i}$ and $Asl2$ the ones that are of the first level but do not have a returning arc.

To remain consistent with the e-charger implementation, a constraint on the binary decision variable y_i, corresponding to the installation of the charger on node i is stated:

$$y_i = y_{i+n0} \qquad \forall i \in N0 \tag{2.29}$$

In this way we ensure that if a node is chosen to hold a charger facility, the charger is present on both the level. Moreover, in the object function, the term associated to the total implementation cost is calculated only for the nodes of the first level:

$$\max_{b\ y\ x\ \gamma} \quad \sum_{p \in P} \sum_{u \in U} \Big(\sum_{(i,j) \in A} a_{ij}^{u} x_{ij}^{up} + \sum_{i \in N} d_i^u \gamma_i^{up} \Big) - W \sum_{i \in N0} y_i c_i \tag{2.30}$$

The final formulation is identical to the one stated at paragraph (2.4.2), with the object function here proposed (2.30) and the added constraints (2.27-2.28-2.29).

3 | Results on Artificial Data

This chapter aims at apply the location model, the routing location model and the version with multiple levels on a set of randomly generated instances, which mimics reals networks, in order to evaluate the performance of the various algorithms and the possible limitations.

3.1. Data Generation

The generation of the data-set used for the simulations is a crucial aspect of this chapter, including a directed graph $G(N, A)$, main representation of the resort under analysis, a proper energy consumption and timing associated to the arcs of the graph, the subset of nodes chosen to possibly implement the charger facilities and the associated costs. Finally, the values of attractiveness of the entire network. To properly generate all this data more than 3000 lines of code were written in Python, exploiting different Python package and script which are here summarized.

3.1.1. Graph Generation

Since e-bikes are spreading typically in mountain resorts, the directed graph $G(N, A)$ here generated, represents the network of trails, common roads and muleteers typical of these locations. To simulate the energy consumption required by travelling on these infrastructure, the graph generation must associate to the nodes of the network an altitude and a proper slope to the arcs connecting them.

To do so, we adapted an open source Python package called Random_Planar_Graphs [48]. The algorithm proposed here, allows for creating a random 2-D planar graph with different specifications such as the number of nodes, arcs, the dimension of the network and the minimal arc length. With the right choice of options, the obtained graph perfectly models the 2-D map of trails, common roads and muleteer present on a mountain resort, which, combined with the strategic sites along the network, completes the graph $G(N, A)$ needed by the program.

Anyway the package does not allow for creating a 3-D graph which is, fundamental to obtain altitude which would be used to get the data necessary for the energy consumption and timing calculation. Hence Python program was coded for this specific problem: after picking from the nodes set N of the 2-D graph, three random points A, B, C are associated to random altitudes between 2500 and 1500 meters, creating a landscape with three upside-down cones, representing the mountain peaks in the resort. Then the following formula is used to associate an altitude h to the remaining nodes:

$$h_i = Max(200, \Delta h_{A,i}, \Delta h_{B,i}, \Delta h_{C,i}) \qquad \forall i \in N \setminus \{A, B, C\} \tag{3.1}$$

$$\Delta h_{A,i} = h_A - slope * distance(A, i)) \tag{3.2a}$$

$$\Delta h_{B,i} = h_B - slope * distance(B, i)) \tag{3.2b}$$

$$\Delta h_{C,i} = h_C - slope * distance(C, i)) \tag{3.2c}$$

The *slope* is generated from a random routine between two limit values $(3\%, 9\%)$ and the *distance* correspond to the one computed for the 2-D planar graph. These assignments ensure that each point of the network has an altitude proportional to the selected slope and the original distances with respect to those of the main peaks. It follows that the created network follows the conformation of the territory, leading to have trails with a slope smaller than 12%. This value is perfect for our purpose because the bicycle trails usually have slopes smaller than 12%, for different limitations due to the achievable human power, friction and the center of mass of the system [49].

The final 3-D discrete graph is close to the representation of a real mountain area, with a vast offer of trails, common roads and muleteers that connect strategic sites, possibly provided with a connection to the electric grid, and hence feasible implementation points for the charger facilities. To properly test the models on different scenarios the script is used to generate three instances with a growing number of arcs and nodes. The three networks are shown with the selected options in Figure 3.1, 3.2, 3.3.

Network Width	25km
Network Length	25km
Network Height	2500m
Number of Nodes	15
Number of Arcs	25
Minimum Node Distance	5km

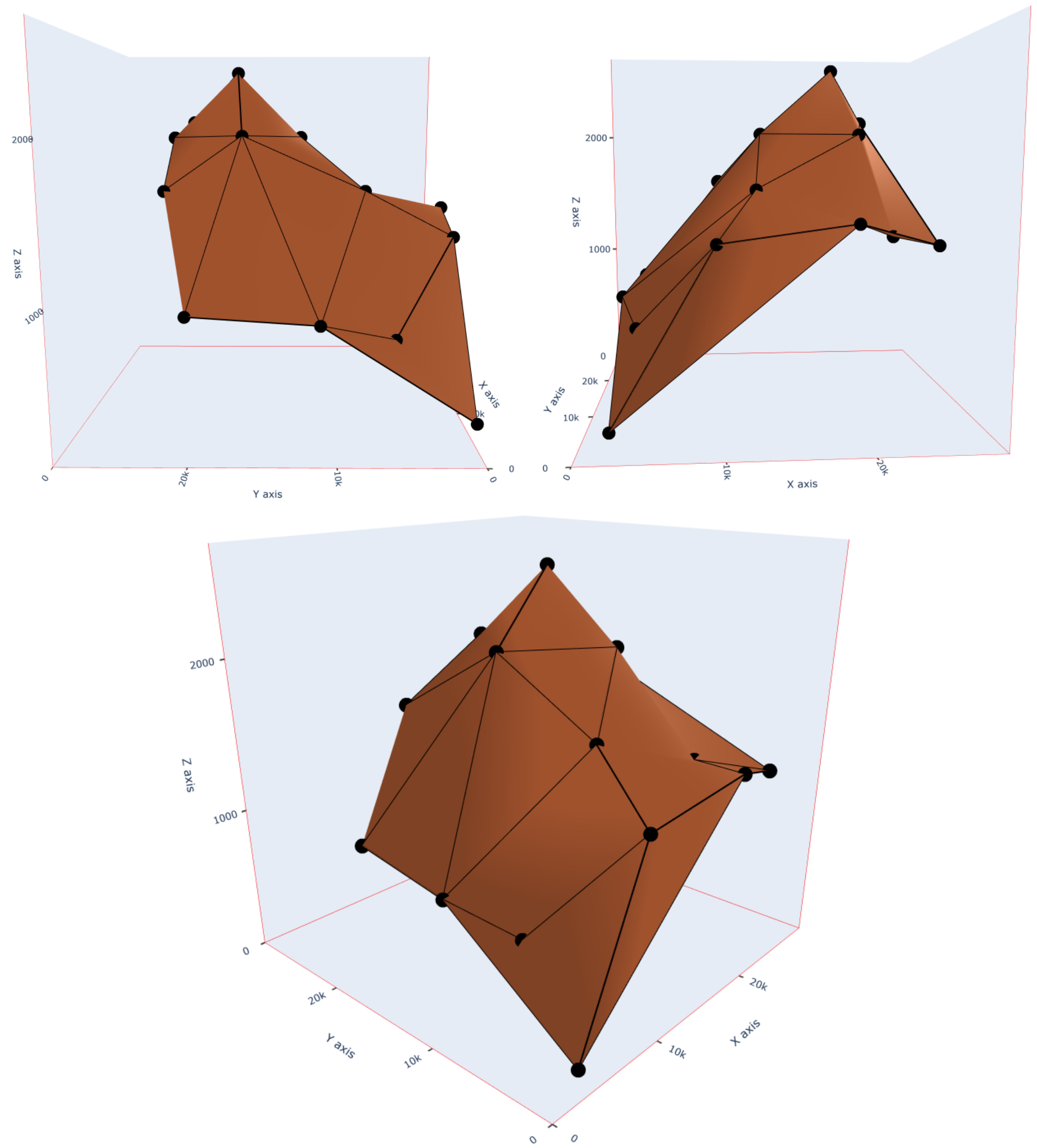

Figure 3.1: 3-D Small Network Example

Network Width	50km
Network Length	50km
Network Height	2500m
Number of Nodes	20
Number of Arcs	40
Minimum Node Distance	5km

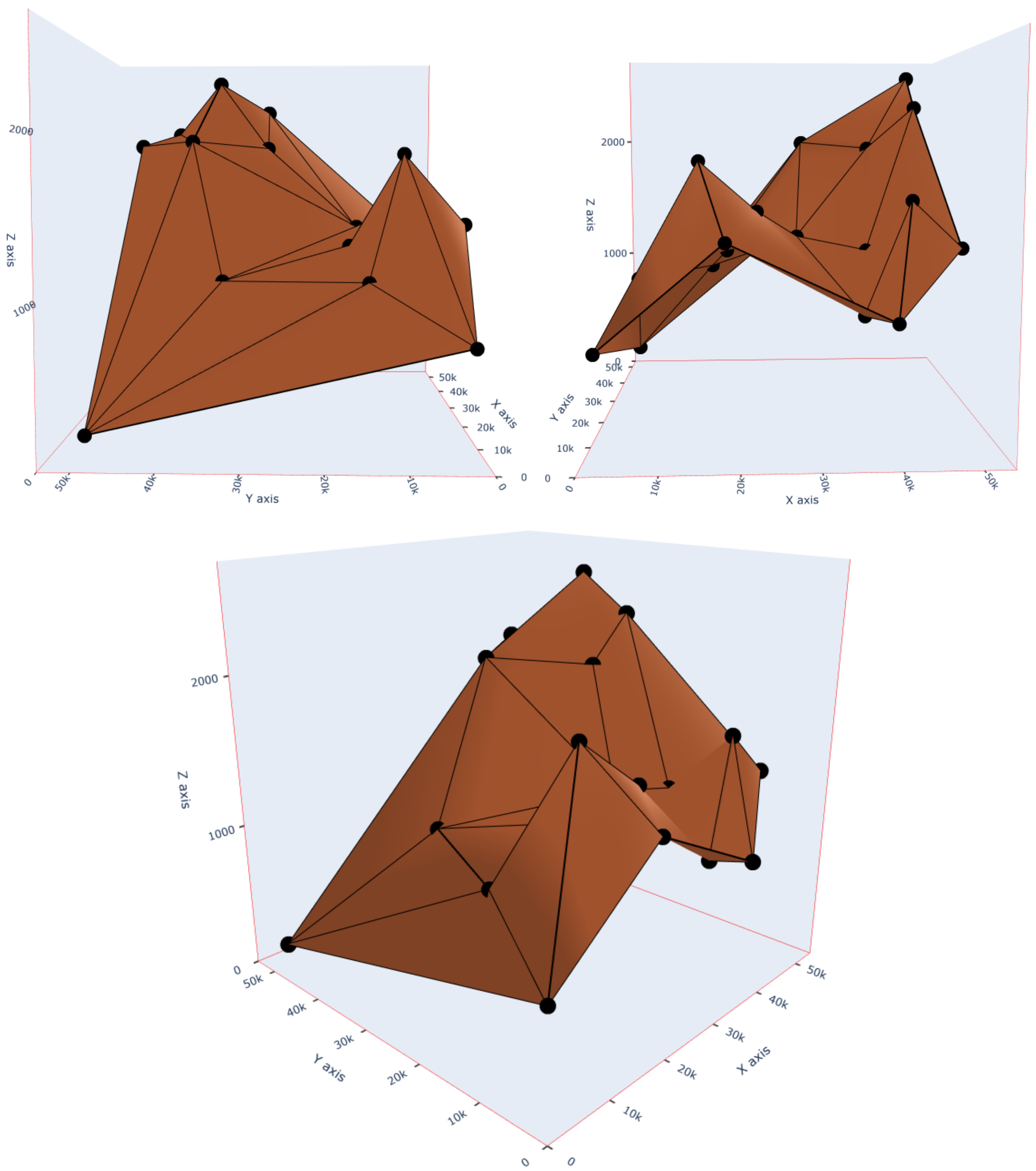

Figure 3.2: 3-D Medium Network Example

Network Width	75km
Network Length	75km
Network Height	2500m
Number of Nodes	50
Number of Arcs	130
Minimum Node Distance	5km

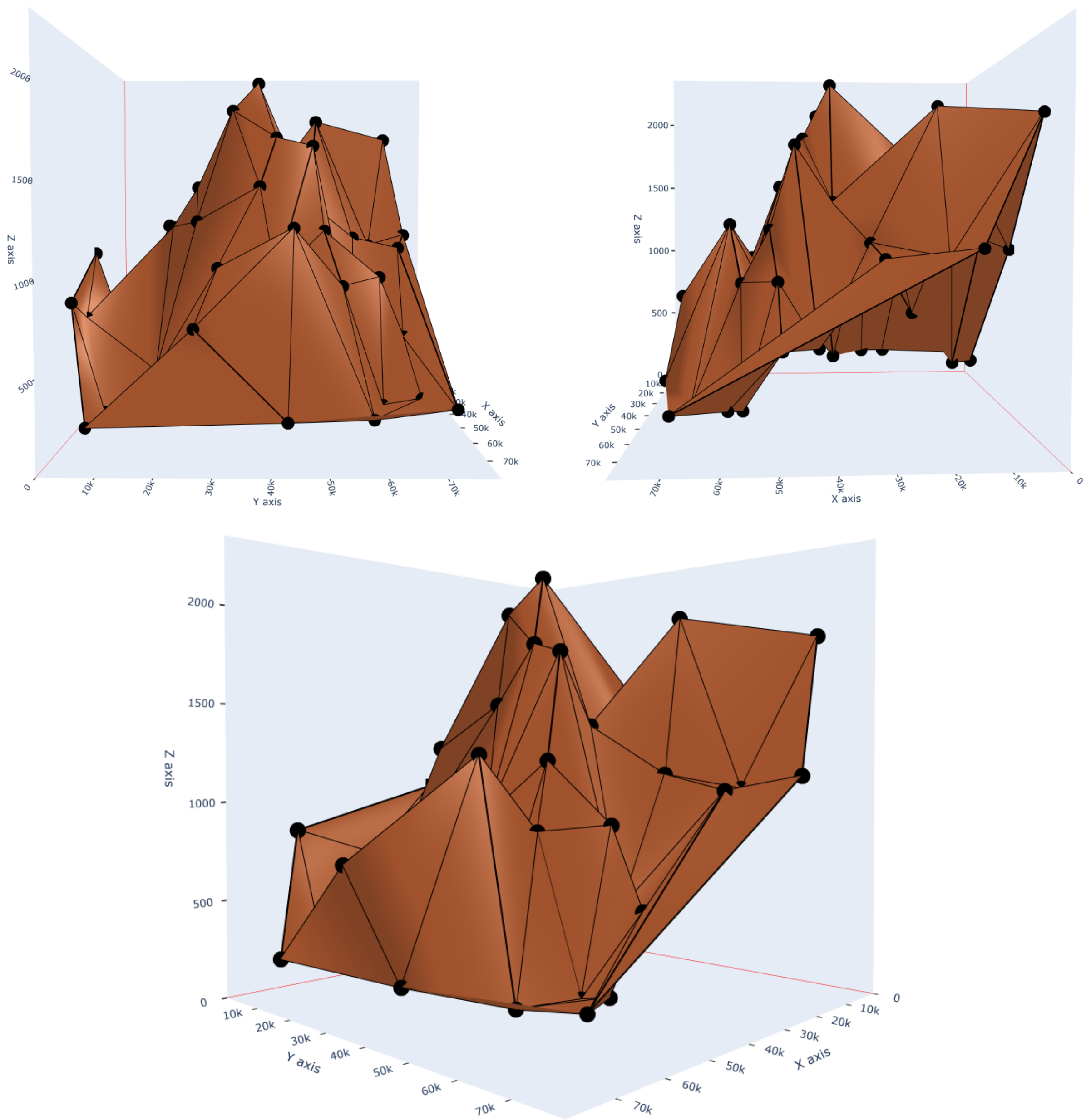

Figure 3.3: 3-D Large Network Example

It is interesting to observe how the generated graphs assume a shape closer to a mountain as the number of nodes and arcs increase. This is probably due to the fact that the constraint related to the slopes, the dimensions and the altitude of the network brings the program to create a graph with more details, capable of describing in this way the typical shape of plateaus, valleys and peaks, typical of a mountain area.

3.1.2. E-bike Model

To apply the models developed in this thesis, it is fundamental to know the e-bike battery consumption along the trails, a proper characterization of this data is indeed mandatory to end up with a realistic solution that reflects the behaviour of the battery. However, understanding the energy required by the e-bike is a challenging task that must consider heterogeneous factor such as the e-bicycle dynamics, the terrain type, the weight and physical preparation of the cyclist under analysis. In the literature it is possible to find various papers that formulate an accurate model to the e-bike energy consumption.

These studies proposed detailed mathematical formulations far from the scope of this thesis. Moreover, modelling the real behavior of this system is almost impossible because of all the influencing factors to which the e-bike is subjected. Among them there is the surface of the road and the mechanical condition of the e-bike that are very variable and subjective.

The optimization models proposed here, are formulated with the purpose to find a reliable implementation of the charging facilities in order to ensure the possibility to charge the e-bike for 3 different classes of tourists. It follows that for each class, the final infrastructure should ensure a fully charged battery also for the cyclist that exploits the maximum power of the electric motor mounted on the bicycle. Hence a simple model considering the worst case scenario for each class is enough to describe the energy requirements of each trail.

First of all, a consideration on the e-bike must be stated: According to the European classification standard, an e-bike can provide a maximum rated power of 250 W, which decreases over the time until the maximum speed of $25km/h$ is reached. Moreover, the e-bike is defined as a Pedelec Drive, hence the motor is activated by the cyclist's pedalling effort and it is cut immediately if the cyclist stops pedaling. [5]

Based on this limitation and on the assumption that, reasonably, the velocity on the cyclist network is limited to values lower than $25km/h$ uphill and $35km/h$ downhill, some mathematical equations can be stated to model the e-bike energy consumption with this specification.

It follows that when the e-bike goes uphill, the power required by the electric motor P_{e-bike} is equal to the total power P_{Tot} necessary to keep a constant speed of $25km/h$ minus the power generated by the cyclist P_{user}. In particular for a bicycle the total power P_{Tot} is given by the sum of three different components: the power to win the drag forces P_{drag}, the friction $P_{friction}$ and the power to go uphill P_{hill}, defined with a formula (3.6) proposed by Lim [50].

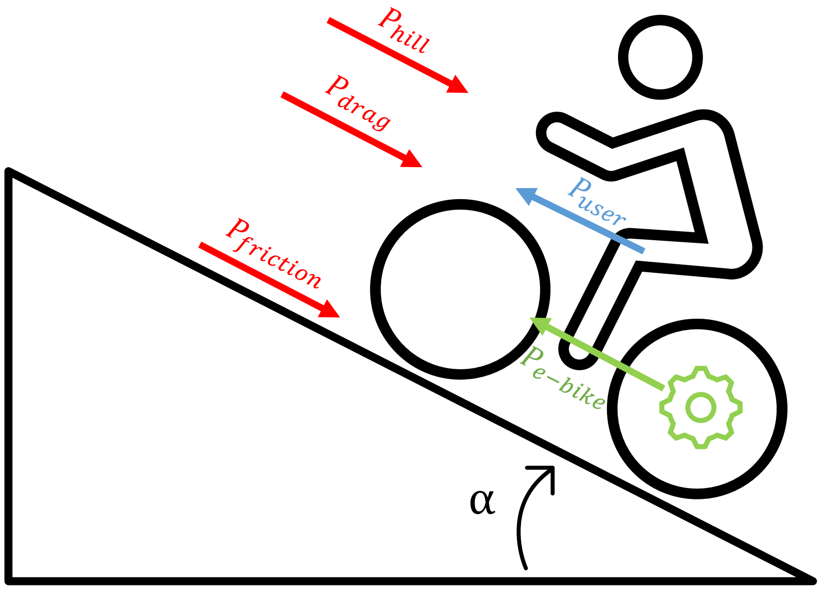

Figure 3.4: Power Balance E-bike

The following equations describe this relationship:

$$P_{Tot} = P_{drag} + P_{friction} + P_{hill} \tag{3.3}$$

$$P_{drag} = \frac{(C_d * A * \rho)}{2} * v^3 \tag{3.4}$$

$$P_{friction} = g * M * R_c * v \tag{3.5}$$

$$P_{hill} = g * M * v * slope \tag{3.6}$$

Finally, it is important to underline that, in order to properly describe the energy consumption of the e-bike with respect to the class of user under analysis, P_{user} assumes 3 different values. The power generated by the cyclist mainly depend on its physical preparation. To have an idea, a Tour de France athlete is able to generate 200/300 Watt for almost four hours; instead an amateur athlete does not exceed values of 120 Watt. Hence we will assume 3 different values of P_{user} characterizing the user class. In particular the tourist will produce 80 watt per hour, the gastronomic 60 watt per hour and the sporty one reaches to 100 watt per hour. Hence the final equations that define the ideal motor power is the following:

$$P_{e-bike} = P_{Tot} - P_{user} \tag{3.7}$$

$$P_{real} = \frac{P_{e-bike}}{\eta} \tag{3.8}$$

It is important to underline that the power requirement P_{e-bike} resulting from these relationships is divided by a factor η that reflects the efficiency of the power provided by the motor on a real track. In fact the real power P_{real} will be larger than the ideal one, due to various external factors such as rocks, roots and holes that contributes to a greater power consumption.

Hence knowing the power that the e-bike should provide to the wheel it is possible to calculate the energy spent on the arc as $[Wh]$. To do so the real power required P_{real} is multiplied by the time $t_{arc_{ij}}$ needed to go through the arc (i, j), calculated as the track length divided by the average velocity of $25km/h$. This relationship is stated by the following formulas:

$$e_{arc_{ij}} = P_{e-bike} * t_{arc_{ij}} \tag{3.9}$$

$$t_{arc_{ij}} = \frac{L_{arc_{ij}}}{v} \tag{3.10}$$

Finally, note that in case of a downhill track the energy required by the battery on that arc $e_{arc_{ij}}$ is set to 0 due to the acquired potential energy that allows one to descend the mountain without using any power from the e-bike.

Below a table defining all the parameters used above is presented:

Symbol	Parameter	Unit	Comment
M	Mass	kg	E-bike and cyclist
g	Gravity acceleration	m/s^2	Equal to $9.81m/s^2$
v	Speed	km/h	$25km/h$ uphill and $35km/h$ downhill
C_d	Drag coefficient	$-$	Equal to 1 for a cyclist
A	Frontal Area	m^2	$0.6m^2$ for upright cyclist
R_c	Rolling friction coefficient	$-$	0.0125 for mtb
$slope$	Slope %	$-$	$\frac{\Delta h_{ij}}{Larc_{ij}} * 100$
ρ	Density of air	kg/m^3	equal to 0.4
η	External factors	$-$	equal to 0.8

Table 3.1: E-bike Model Parameters

3.1.3. Attractiveness Values

For the location routing problem the attractiveness values associated to the arcs and nodes of the network must be generated. To do so the data set is constructed by a random routine that choose values between 0 and 3, with different indications with respect to the type of tourist under analysis. In particular for the Touristic and Sporty classes, the values of attractiveness are bigger on mountain peaks and smaller on lower nodes, moreover the arcs associated with a large energy consumption has an higher attractiveness for sporty cyclists. Then all the remaining attractiveness values are randomly generated within the range specified above.

3.1.4. Charger Cost and Implementable Sites

In the creation of this realistic instances, the implementable sites, described by the binary variables v_i are assumed to be all the nodes of the graph and the charger costs are stated to a symbolic cost of 100 units. This is done to have an easy readable solution of the models without bounding the program to choose the implementation sites only in a certain subset of nodes. This simplification is then removed in the real test case scenario proposed in Chapter 4, where the nodes are associated to specific sites on the network and hence fundamental information are retrieved to populate variables v_i and the implementation costs.

3.2. Computational Results

The model was coded in Python and solved on a two-core machine with an Intel i7 processor and 2.50GHz of clock. The used solver was GUROBI, one of the fastest and most powerful solver available on the market. Here the results of the different problems over the 3 instances exposed above are shown and analyzed.

3.2.1. Location Model

To test the Location Model exposed in Chapter 1, the program was fed with different data-sets associated to the 3 graphs exposed above. Moreover, a set of 6 routes composed by a sequence of nodes between selected origin-destination pairs were randomly generated. The following images show the different instances with the best locations for the charger facilities, highlighted on the 2-D graph by green nodes. Moreover, the red arcs show up the itineraries followed by the cyclists, between an origin and destination nodes always circled in red. For sake of conciseness only three out of six routes are here reported:

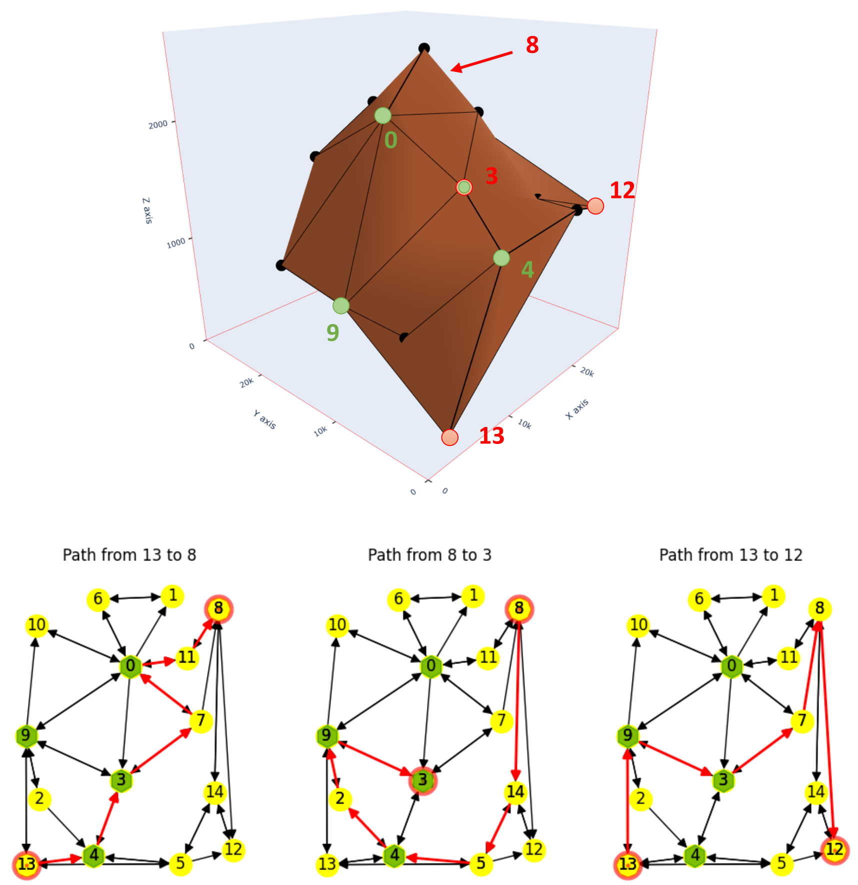

Figure 3.5: Location Problem Solution: Small Network Scenario

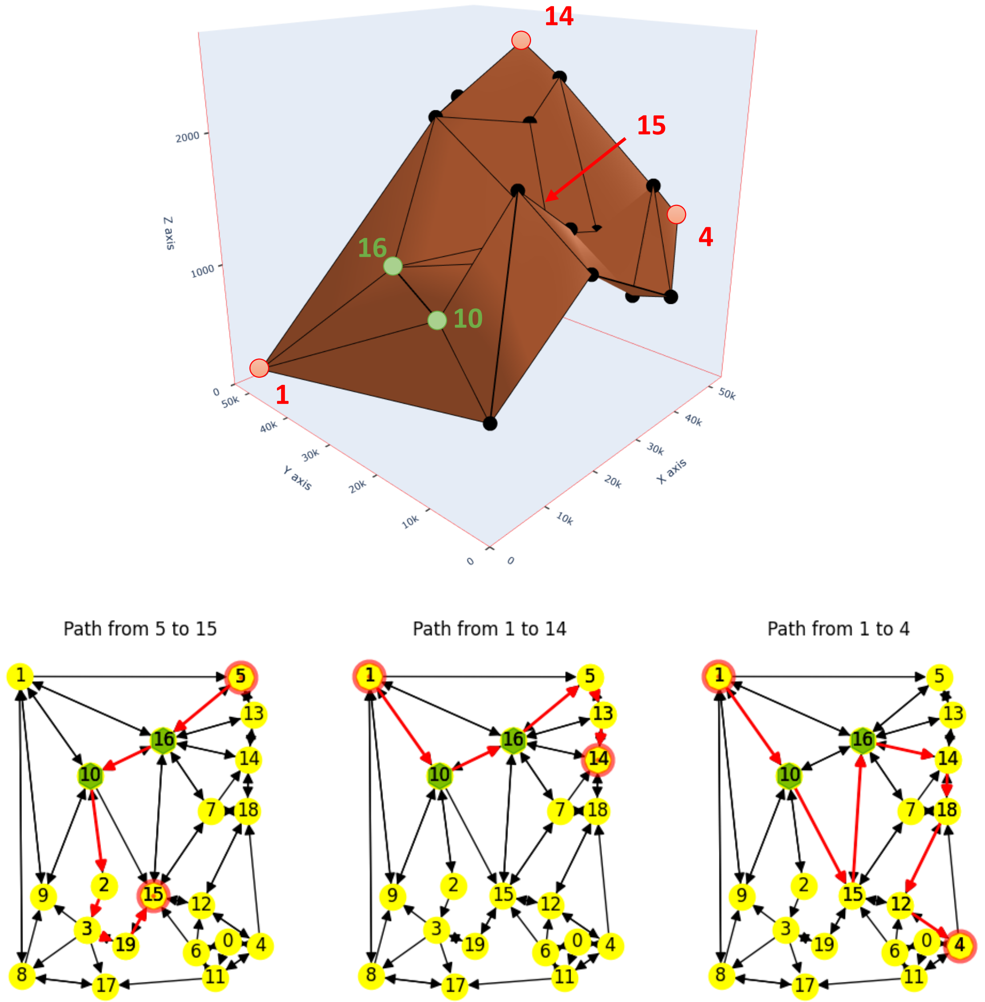

Figure 3.6: Location Problem Solution: Medium Network Scenario

The first two instances show that the model works properly, indeed although if the routes are quite long and hence they require at least one charger to complete the trip, the total number of facilities allocated on the network is less than the number of routes. This is possible because the routes pass through some common nodes, hence the program decides to put the chargers on these sites, minimizing the total implementation costs. Moreover, it is possible to observe that the number of chargers along the network is not directly proportional to the size of the area. The reason is that the routes are randomly generated on the graph and are not designed to minimize the charger implementation costs. Hence the result is strongly influenced by the itinerary design, leading for example in a larger number of chargers on the small instance (3.2) with respect to the medium one (3.3), where the paths share more nodes.

Finally, the model was tested on the biggest network (3.4), here the result of two out of six routes is shown:

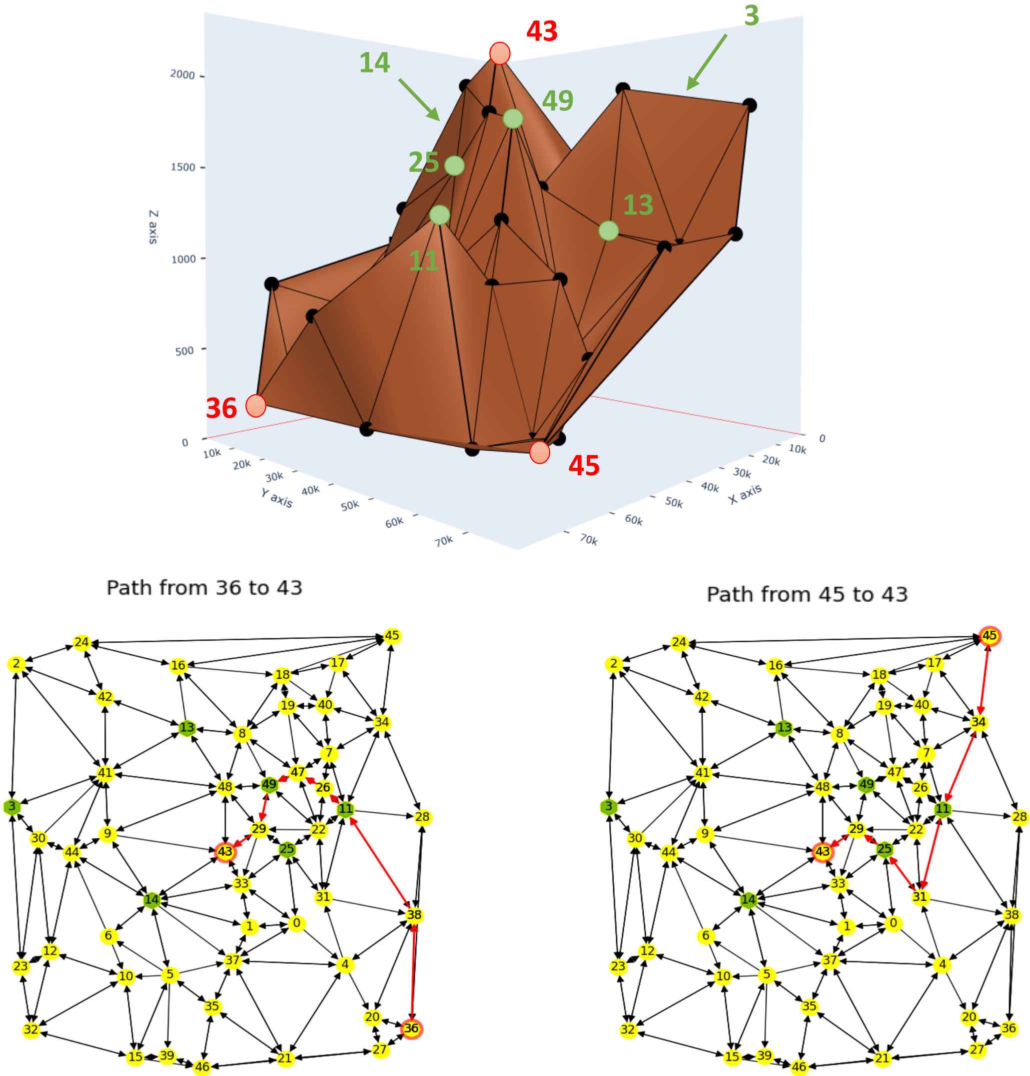

Figure 3.7: Location Problem Solution: Large Network Scenario

Although the dimension of the graph results in an optimization model with 350 variables and 186 constraints, the simple build in GUROBI Branch and Cut routine solves the problem at the root node in only 0.01 s . Here it is clear that the dimension of the graph leads to have some itineraries that are totally isolated from the others, consequently requiring isolated chargers.

3.2.2. Location Routing Model

Here the results of the LRP are shown, for the sake of conciseness only the third path out of six is reported, in this way we are able to analyze the difference with respect to the class of cyclist under analysis. However, it is important to underline that the presence of three kind of cyclist and six different origin-destination pairs mean that we must solve 18 different instances for each network.

The pool of origin-destination pairs is defined by taking four border points, representing the resort entrances at the boundaries of the network, and three characteristic points, respectively characterized as the lowest and the highest at the center of the graph and the most prominent peak of the entire resort. Then from the combination of these points, a random routine picks six couples that define the final origin-destination pairs. The maximum time on the network T_u is reasonably set to two hours and a half for the Touristic and Gastronomic cyclist, and four hours for the Sporty one.

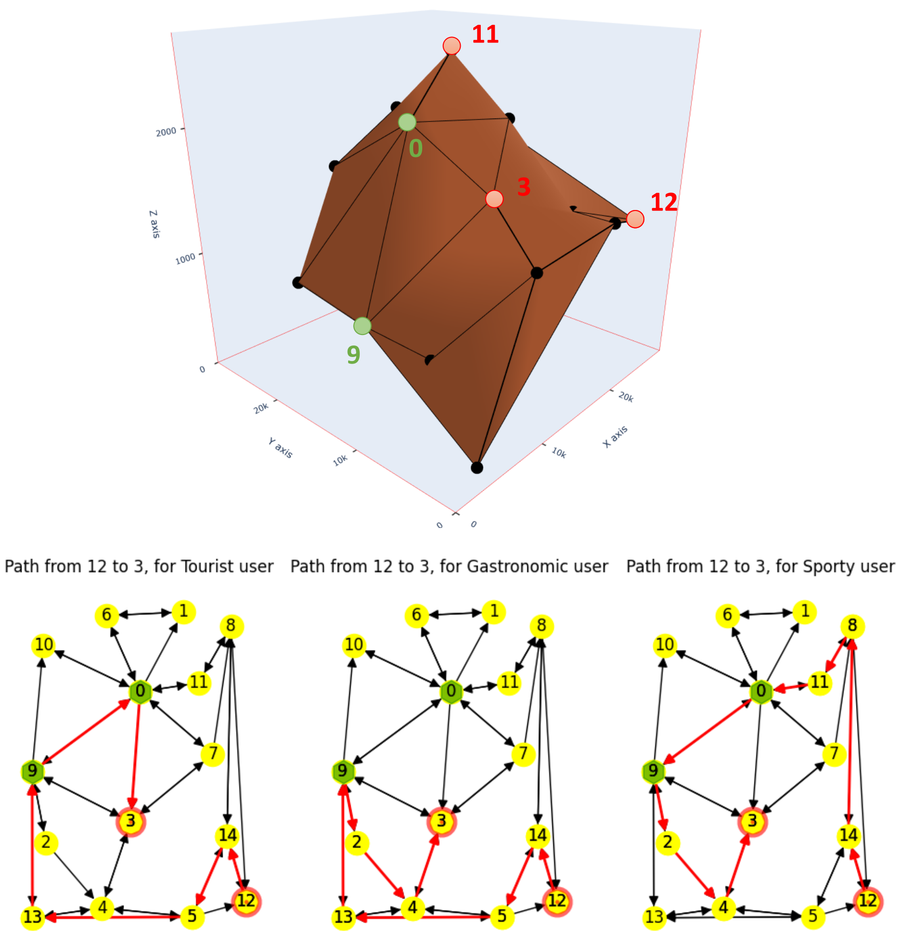

Figure 3.8: Location Routing Problem Solution: Small Network Scenario

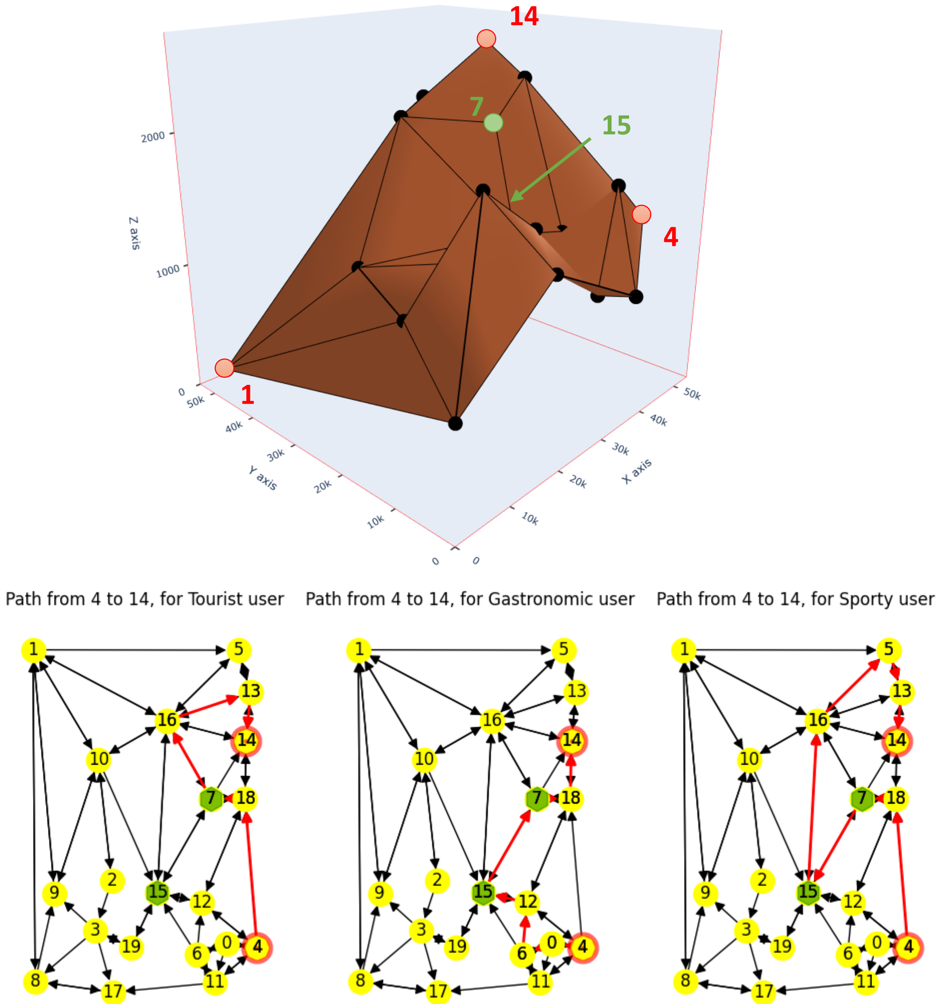

Figure 3.9: Location Routing Problem Solution: Medium Network Scenario

From the small and medium instance it is clear that the routed path is different for the three cyclists, maximizing the attractiveness reward gained along the trip. For example the sporty class has a longer trail, traveling on steeper slopes and gaining an important delta of elevation. However, the limit on the multiple visit is evident on the small instance where the sporty user visits almost all the network without repeating the hardest trails, where he could gain an higher reward. Finally, as it can be seen from the small instance, the number of chargers is lower with respect to the Location Model. This is possible for the strategic design of the routes, which shares the charger facilities minimizing the implementation costs.

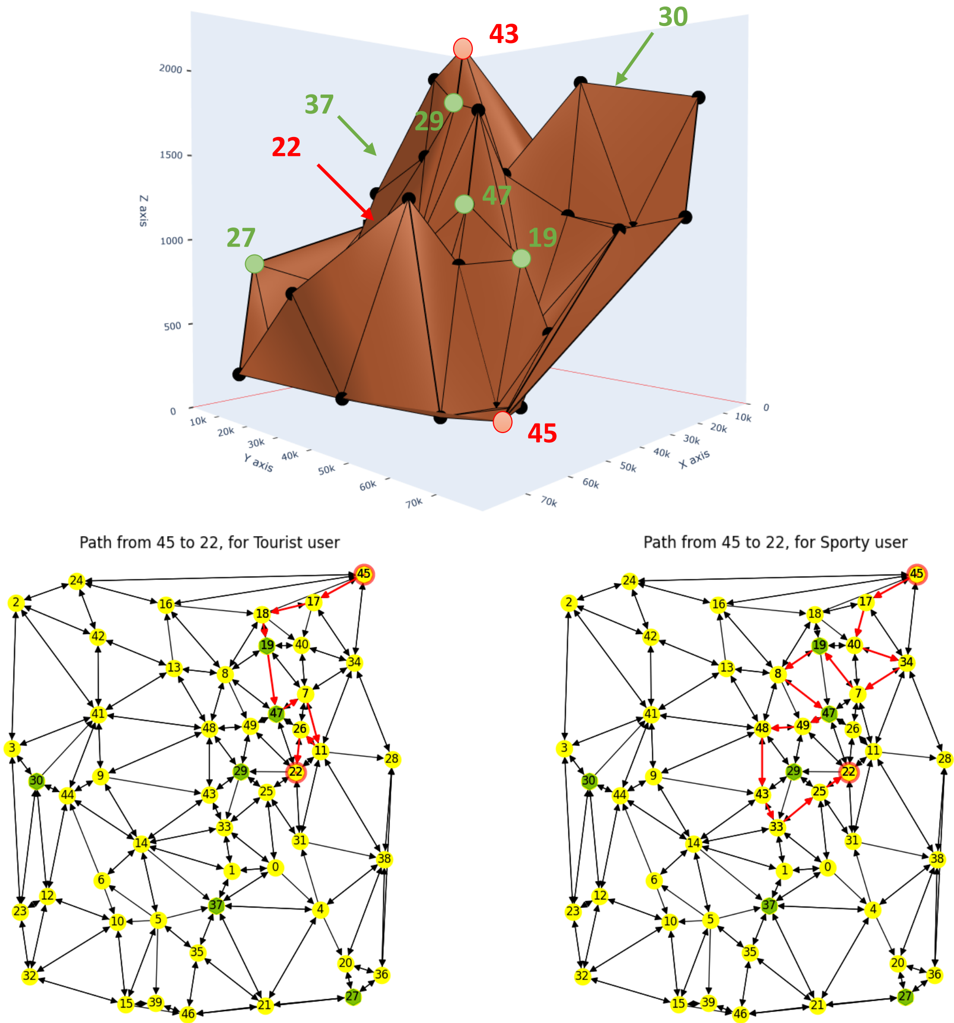

Figure 3.10: Location Routing Problem Solution: Large Network Scenario

On the largest instance, the dimensions of the network require more chargers along the routes; this can be limited by increasing the weight on the implementation costs inside the object function W. This way the designed paths visit more shared sites, leading to have a smaller number of facilities. The computational effort significantly increases with the size of the graph: if, in the medium instance, the number of variables and constraints remain respectively at 1946 and 6552, in the largest graph the variables grows to 6098 and the problem presents 22644 constraints. However, the Branch and cut is still efficient, speeding up the optimization process with the generation of more than 1000 cutting planes and exploring 190'000 nodes.

3.2.3. Location Routing Model with Multiple Level Extension

In this section the output of the model with Multiple Level Extension is shown. The three instances grow in complexity for the presence of the second level, leading to double the nodes and quadruple the arcs of the original instances. Even so, the solver is able to find a solution in a reasonable amount of time for all the three instances. The following pictures show the results:

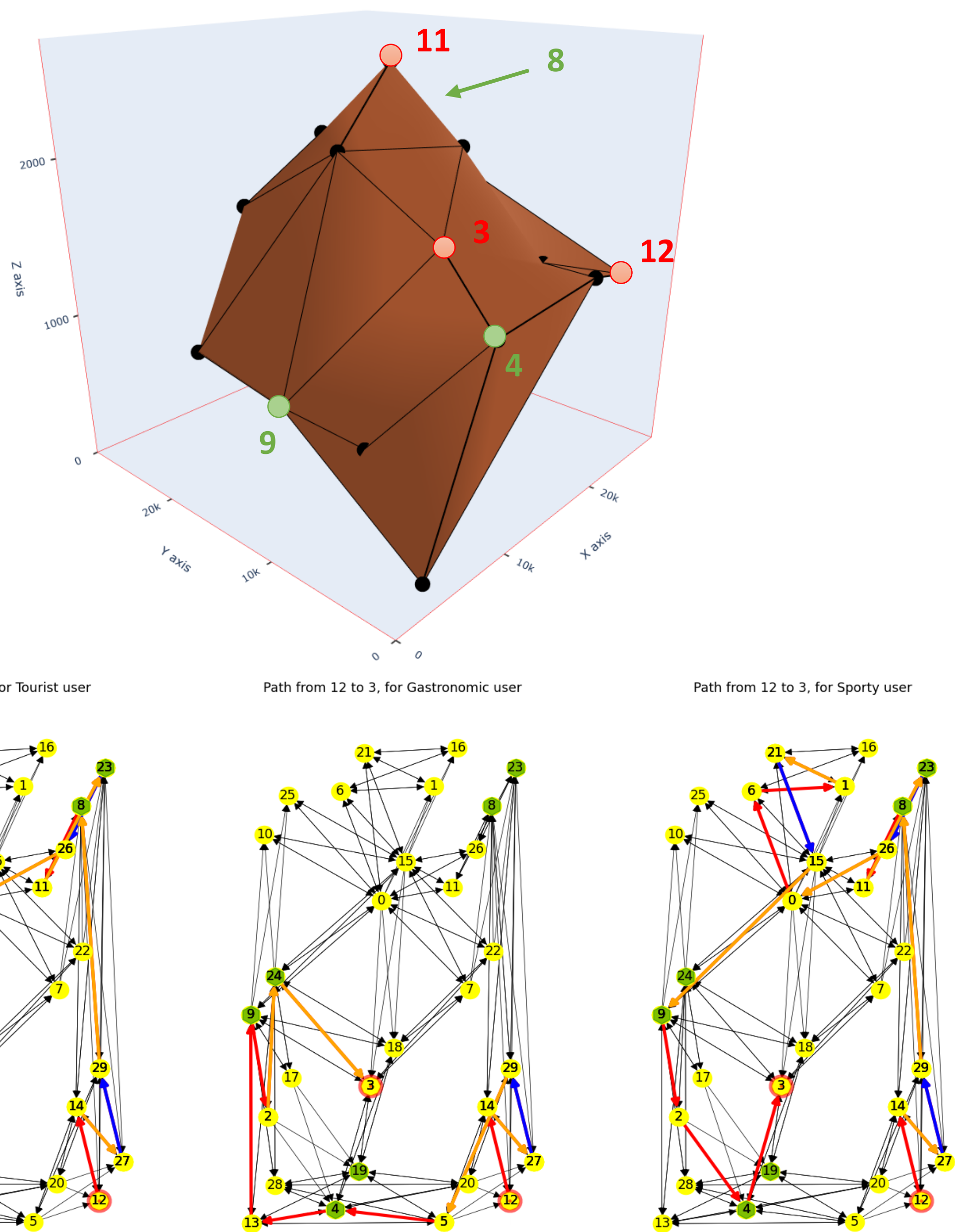

Figure 3.11: LRP-ML Solution: Small Network scenario

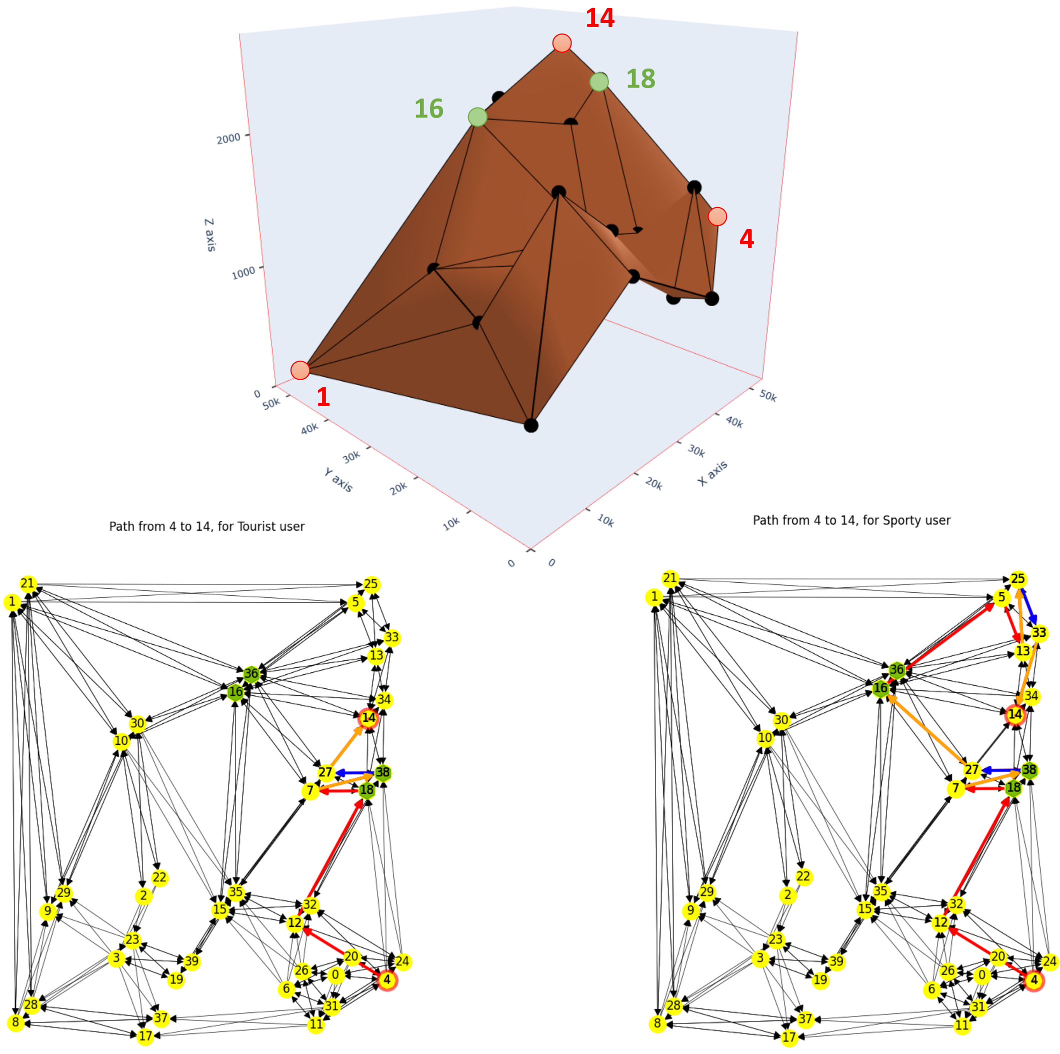

Figure 3.12: LRP-ML Solution: Medium Network scenario

To make the comprehension clearer, an appropriate color scale highlights the selected arcs. In particular the red arcs appertain to the original graph, hence to the first level, the arcs colored in orange instead show the transition between the two levels; finally the blue ones highlight the movement on the second level.

As can be seen from these two examples, the model allows the cyclists to pass trough the same arc multiple times after visiting the first level. This happens when the attractiveness reward of the region is enough to justify more than one traversal. For example, in the tracks proposed on the medium instance (3.12), the Tourist user moves multiple times between nodes 18 and 7. However, this causes longer itineraries that can results in a higher demand of charging stations along the network. Indeed, in the small instance (3.11), the sporty user discharge frequently his battery leading to higher number of charging stations along the itinerary.

Analyzing the performance of the program we can conclude that this version of the LRP model leads to an important number of variables that slows down the resolution process, requiring a large computational efforts by the machine. In particular the multiple level method applied to the large instance (3.3) results in 20692 variables and 91166 constraints. Applying the solver we can see in the progress line a feasible solution after 255 seconds with a gap of 32% with respect to the upper bound. Then it is possible to observe a decreasing progress in the gap between the lower and upper bound, achieving the optimal solution after 484 seconds.

Here the 2-D map of the full graph for the found partial on the Sporty user solution is shown:

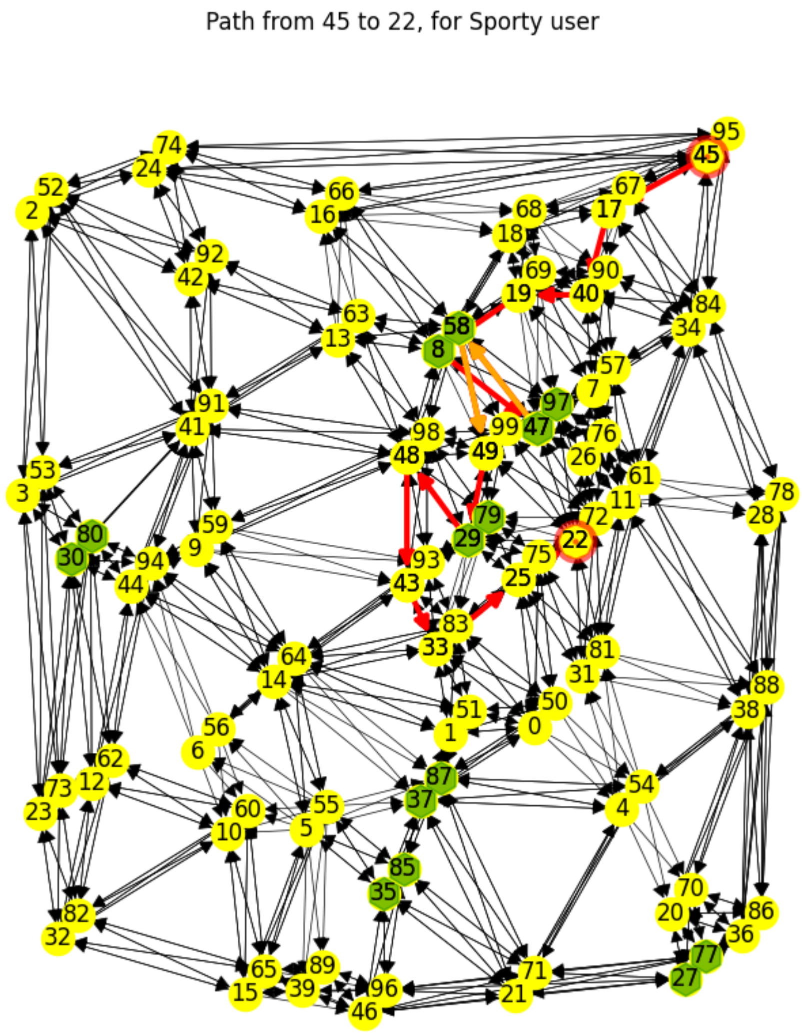

Figure 3.13: LRP-ML Solution: Large Network scenario

Highlighting the nord - est region it is possible to observe the path between 45 and 22:

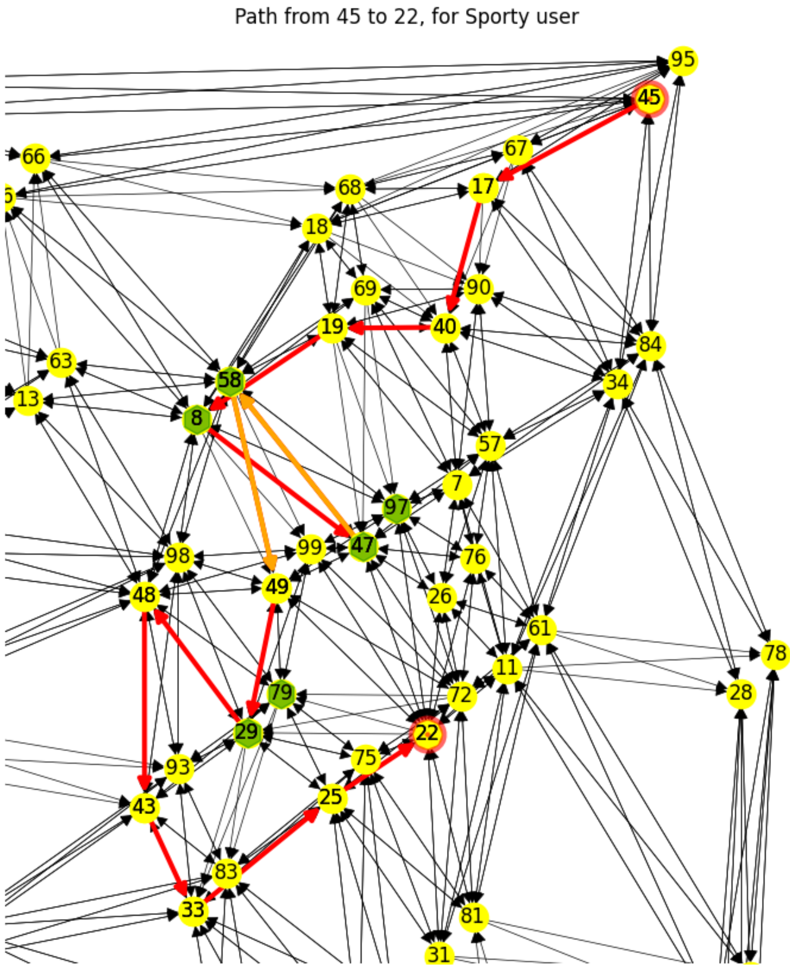

Figure 3.14: LRP-ML Solution: Large Network scenario

4 | Case study:

In this chapter the charger location problem is applied to a real mountain network. We analyzed the Asiago Sette Comuni Plateau, located in Trentino Alto Adige, Italy. This territory is a true paradise for cycling lovers and MTB excursionists, because it offers a myriad of itineraries and routes, with various degrees of difficulty, ideal for bike traveling.

If until a few years ago discovering the Plateau by mountain bike was an experience that only trained cyclists could enjoy, because of the long and difficult routes and the hard climbs, lately, thanks to the advent of E-bikes, even those who do not have a particular sports and technical preparation can travel the mountain itineraries on a bicycle, managing to reach the most beautiful destinations and views. The resort present also different rental facilities that push this growing market, providing e-bikes directly at the beginning of the trails, because of their strategic positions on the different municipalities of the territory.

4.1. Data Generation

Although not all the trails are achievable with an e-bike due to *portage* passage, combining the different paths, muleteers and roads it is possible to construct a large graph connecting all the seven major municipality of the territory: Asiago, Enego, Foza, Gallio, Lusiana-Conco, Roana and Rotzo. Hence, in the proposed test case, a pool of the major trails were chosen to construct a 3D graph $G(N, A)$ on which applying the models developed in this thesis. In particular the chosen trails are: Passo Vezzena, Il Sentiero 800, Monte Cengio e Kaberlaba, Ortigara, Monte a Fior la Città di Roccia, Marcesina - Anepoz, Monte Lisser, Monte Verena, I 3 Monti della Battaglia, Rubbio - Col D'Astiago, Le Incisioni Rupestri della Val D'assa, Il vecchio Trenino di Asiago and Cima della Caldiera.

The obtained planar graph $G(N, A)$ hs 28 nodes and 45 arcs, that cover all the Plateau and includes also the municipality of Bassano del Grappa. Here a graphic representation is shown:

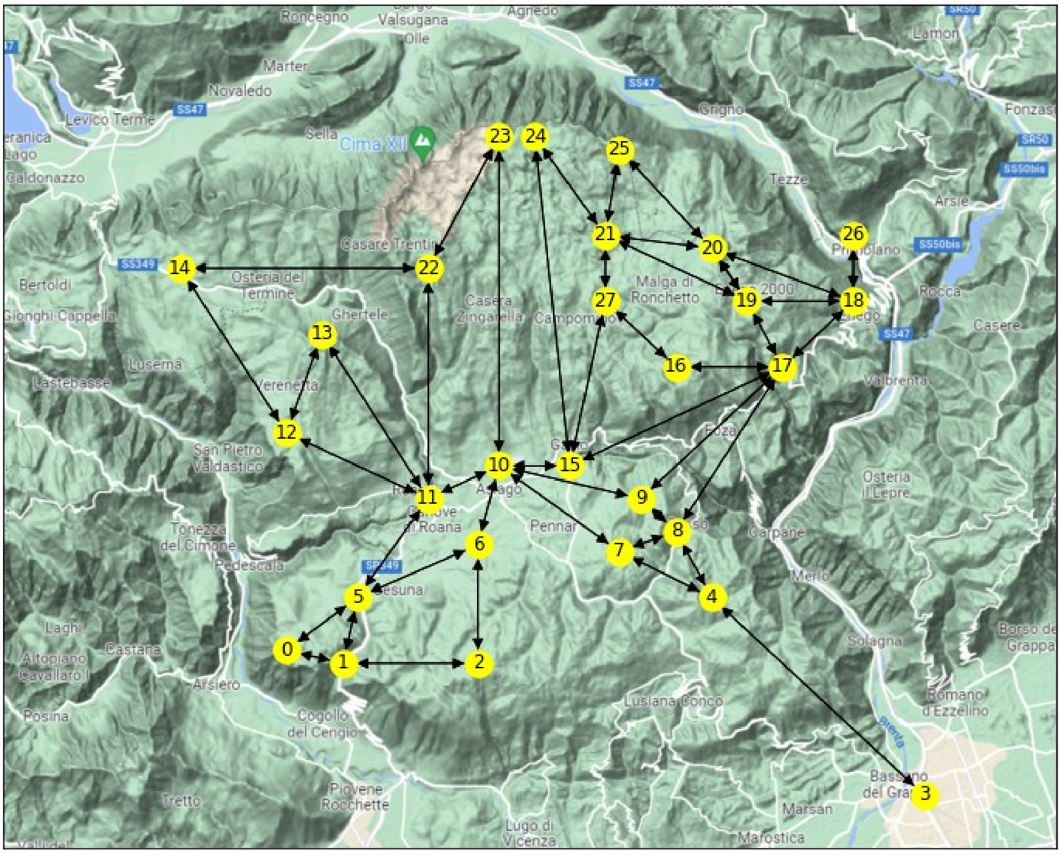

Figure 4.1: Asiago Trails Graph

The nodes are associated to points of interest along the territory such as mountain peaks, villages and farm holidays, connected by a grid of trails synthesized by the arcs. It follows that the nodes are associated to specific altitudes and a proper length characterize the trails. This data was retrieved from topographic maps of the territory [51]. Then the data related to the energy requirements were computed exploiting the Bike Model presented in Chapter 3.

The attractiveness reward was then specifically stated with respect to the preferences of the different classes of cyclists. For example the attractiveness of a farm-restaurant is bigger for the gastronomic class and smaller for the sporty one who, instead, earns a higher reward by passing through a mountain peak or a hard climb.

As stated for Chapter 3 these values are limited in a range between a lower bound of zero and an upper bound equal to three. However, if in the simulation on artificial data the values were stated with an almost random routine, here the values are specifically stated considering the main characteristic of the territory.

The following table lists the nodes with the associated sites, the relative possibility to install a charger and the implementation costs:

Node	Location	Installing point	Implementation Costs
0	Monte Cengio	No	–
1	Sp 349	Yes	100
2	Cima del Porco	No	–
3	Bassano del Grappa	Yes	100
4	Malga Col dei Remi	Yes	200
5	Tresche' Conca	Yes	100
6	Roncalto - Mela	Yes	100
7	Cima Echar	No	–
8	Sasso	Yes	100
9	Monte Valbella	No	–
10	Asiago	Yes	100
11	Roana	Yes	100
12	Malga Erio	Yes	200
13	Monte Verena	No	–
14	Passo Vezzena	Yes	200
15	Gallio	Yes	100
16	Monte Fior	No	–
17	Stoner	Yes	100
18	Enego	Yes	100
19	Monte Lisser	No	–
20	Passo della Forcellona	No	–
21	Malghe Mandrielle e Buson	Yes	200
22	Monumenti	Yes	200
23	Monte Ortigara	No	–
24	Cima della Caldera	No	–
25	Anepoz	No	–
26	Primolano	Yes	100
27	Rifugio Campo Muletto	Yes	200

Table 4.1: Node Locations and Data

The data related to the charger implementation such as the cost c_i and the possibility to install the charger on the nodes i, ruled by the binary variable v_i, were formulated taking into account two important assumption. First of all the possibility to implement a charger facilities is possible only if there is a connection to the electric grid or a farmhouse, that can provide the required electricity thanks to autonomous power supplies such as solar panels. Then the implementation costs of a single charger were set to a symbolic value of 100 units for easy accessible sites such as provincial roads, municipalities or small villages. The cost doubles for farmhouses, where the isolated positions and the cost of electricity lead to higher implementation costs.

4.2. Results

Here the results achieved by the 3 models on the Asiago Plateau are shown.

4.2.1. Location Problem

For the Location Problem different trails were combined to create a set of 6 paths on which the problem is executed. The following table shows the 6 paths and the related trails:

Sequence of Nodes	**Used Trails**
10 – 15 – 27 – 16 – 17 – 9 – 8 – 7 – 4 – 3	Il Sentiero 800 - Monte Fior
3 – 4 – 7 – 8 – 9 – 17 – 16 – 27 – 15 – 10	Il Sentiero 800 - Monte Fior
26 – 18 – 17 – 19 – 20 – 25 – 21 – 24 – 15 – 10	Marcesina Anepoz - Monte Lisser
10 – 23 – 22 – 14 – 12 – 13 – 11	Passo Vezzena - Ortigara
11 – 5 – 0 – 1 – 2 – 6 – 10 – 15	Monte Cengio
14 – 12 – 13 – 11 – 5 – 0 – 1 – 2 – 6 – 10	Passo Vezzena- Il vecchio Trenino

Table 4.2: Asiago Predefined Paths

As done for the fictional test case of the location problem the data related to the energy consumption are calculated considering the power requirement for a classic tourist user. Analyzing the result in Figure 4.2 we can see that, following the itineraries proposed, the e-bike riders need to charge the battery in order to finish the paths. Hence the resort should install five chargers to ensure a reliable infrastructure. Moreover, it is interesting to observe how only a charger is located close to a farm-restaurant, asking for a double cost of installation.

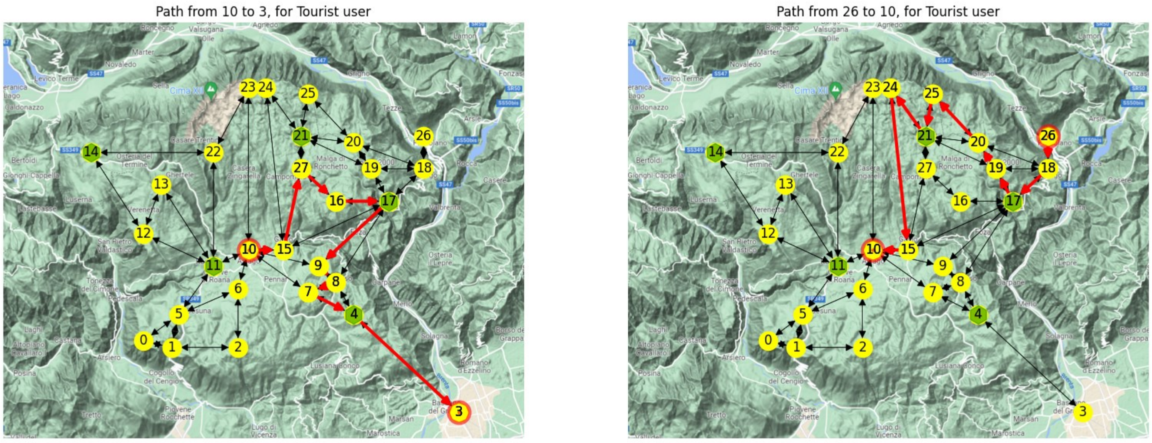

Figure 4.2: Asiago Location Problem Solution

4.2.2. Location Routing Problem

For the scenario with predefined tracks a set of six origin-destination pairs were chosen among the main entrance to the network, represented by municipalities and provincial roads. This way the model is executed over six different instances for the three classes of cyclists, resulting in a total number of 18 scenarios. Here for brevity only the itinerary that brings from Primolano to Asiago is shown, for the tourist and sporty cyclist:

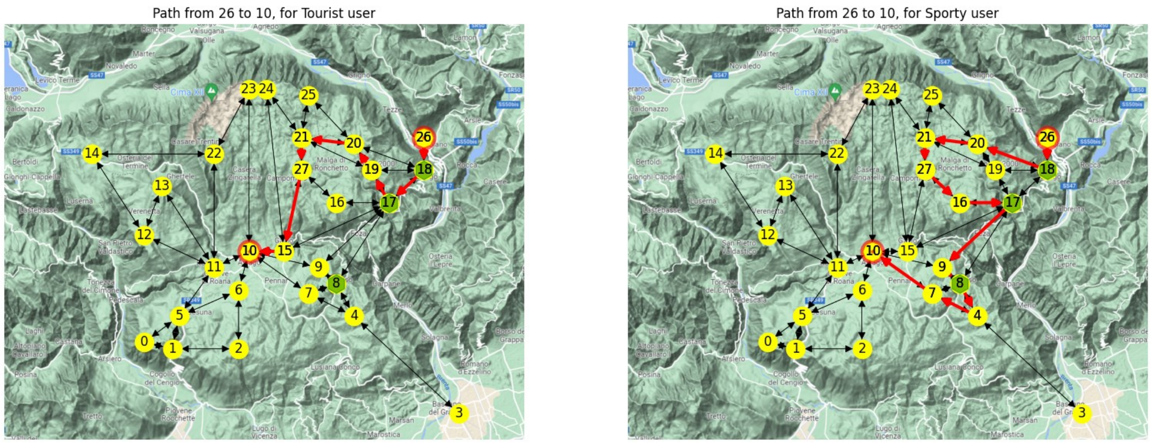

Figure 4.3: Asiago Location Routing Problem Solution

The graphical result shows how the sporty user passes through different peaks of the network, asking for more charging stations and designing a longer path.

4.2.3. Location Routing Problem with Multiple Level Extension

Finally, the LRP with Multiple Level Extension is applied to the Asiago Plateau keeping the same set of origin - destination pairs used for the classic LRP model. The following figure shows the graphic results of the chosen tracks between the municipalities of Primolano and Asiago for the Sporty user:

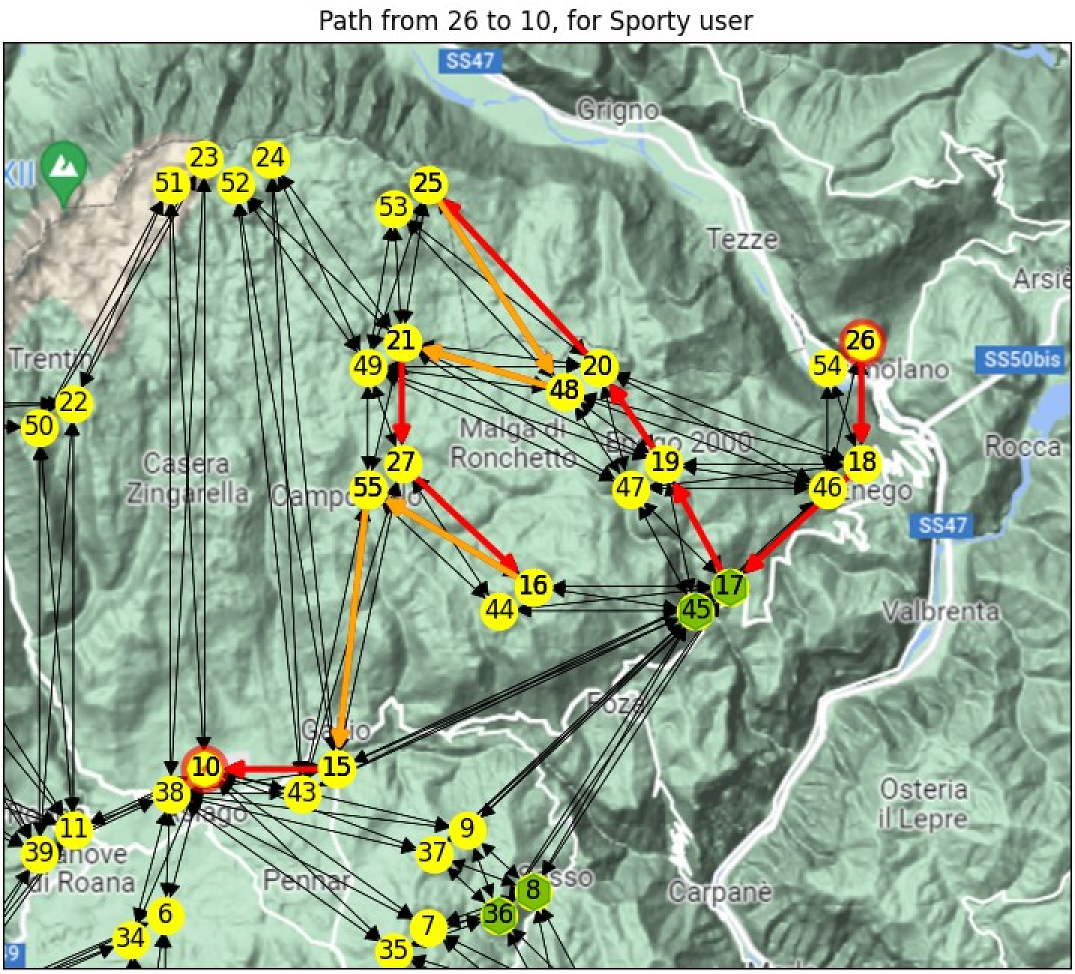

Figure 4.4: Asiago LRP-ML Solution

The map shows clearly that the solution brings the sporty user to pass through the main peaks of the region, such as Passo della Forcellona (20), Anepoz (25) and Monte Fior (16). There are only two charger facilities for all the resort and they are placed at Stoner (17) and Sasso (8), where the implementation cost are lower due to a direct connection to the electric grid. Moreover, these locations are shared by more itineraries that are almost constrained to pass through this nodes, moving between Bassano del Grappa and Asiago or Enego and Asiago.

The model has 8408 variables and 35344 constraints that are used by the algorithm to find the first feasible solution in 105 seconds. The optimization routine is then able to improve again, moving from a gap of 35% with a lower bound of 320.5, to the final solution after 228 seconds, with a lower bound of 379.5 and a gap of 14.4%. The process terminates after exploring 1657 nodes at 550 seconds, generating more 500 cuts.

5 | Conclusions and Future Developments

The results shows that the developed models are perfectly capable to find a solution to the Charger Location Problem for E-bikes, providing flexible formulations that reflects the needs of the interested resorts. The best results can be obtained by the Location Routing Model with Multiple Level Extension which yields an offer of trails that limits the number of charging stations required on the network. Indeed, it results in a smaller number of installed facilities that can be further decreased playing with the weight of the object function (2.1). Moreover the possibility to visit multiple times the same node and arc leads to a more realistic scenario that exploits the charging locations to formulate a competitive and reliable infrastructure. Instead the output of the Location Problem strongly depends on the set of predefined itineraries, statistically leading to higher implementation costs. The performance of the Branch & Cut are clearly visible in the LRP-ML, when it is applied to huge instances as the one shown in Figure 3.13, allowing the program to find a solution in a reasonable amount of time. Although the program is meant to be applied in an offline context, this is a datum that shows the competitiveness of the algorithm.

This field of studies will increase in the next years, following the e-bicycles growing market and in general the wave of low range vehicles mobility. Further implementation, in a touristic context, should consider possible partial charges taking into consideration the time spent on each charging point.

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List of Symbols

Variable	Description	Domain
$\boldsymbol{P}$	Set containing paths p	–
$\boldsymbol{O_p}$	Origin of path p	–
$\boldsymbol{D_p}$	Destination of path p	–
$\boldsymbol{U}$	Set containing users u	–
$\boldsymbol{y_i}$	LP and LRP charger implementation decision variable	$\{0, 1\}$
$\boldsymbol{b_i^p}$	LP remaining battery decision variable	R^+
$\boldsymbol{b_i^{up}}$	LRP remaining battery decision variable	R^+
$\boldsymbol{x_{ij}^{up}}$	LRP arc routing decision variable	$\{0, 1\}$
$\boldsymbol{\gamma_i^{up}}$	LRP node routing decision variable	$\{01\}$
$\boldsymbol{e_{ij}}$	LP energy required on arc (i, j)	R^+
$\boldsymbol{e_{ij}^u}$	LRP energy required on arc (i, j) for user u	R^+
$\boldsymbol{E}$	Maximum capacity of an e-bike battery	R^+
$\boldsymbol{t_{ij}^u}$	Time required to travel arc (i, j)	R^+
$\boldsymbol{T^u}$	Maximum travel time for user u	R^+
$\boldsymbol{v_i}$	Possibility to install a charger on node i	$\{0, 1\}$
$\boldsymbol{c_i}$	Cost of installing a charger in node i	R^+
$\boldsymbol{a_{ij}^u}$	Attractiveness for user u on arc (i, j)	R^+
$\boldsymbol{d_i^u}$	Attractiveness for user u on node i	R^+

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