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# Distributed Model Predictive Approach for Vehicular Coordination at Intersections

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# Abstract

Autonomous Vehicles are expected to penetrate the consumer market in the near future, potentially resolving many of difficult traffic situations encountered by everyday travellers and increasing safety of travel.In order to maximize their potential cooperation between vehicles offers a way to reach agreements without relying on traffic rules. One of the most critical areas in traffic is the urban intersection which requires efficient coordination strategies.

This Thesis treats the problem of coordination at intersections in a distributed manner, where each vehicle is expected to compute its own trajectory. The proposed algorithms employ branches of optimal control and model predictive control in order to accomplish the safe passage of the vehicles through the intersection space. Two algorithms are presented, one that focuses on avoiding collisions in a distributed manner while the other aims at using the infrastructure for coordination.

In the first algorithm, information is communicated between vehicles, where each vehicle's own MPC solves a constrained optimization problem to achieve safe passage. The second algorithm is composed of two stages, the first stage borrows from reservation based approaches and has an 'Intersection Manager' suggest times of arrival to the critical intersection space for vehicles entering the greater intersection area, while the second stage is the employment of the first algorithm in a smaller constrained space to ensure the ability to keep up with any changes in the environment.

While both algorithms achieve the required safe passage, comparison between the two shows that the second algorithm shows improvements in performance, and robustness. Finally, a new priority scheme defining the crossing order for vehicles is defined to allow for emergency vehicles. This scheme shows the room for improvement in current heuristics that are in use in the literature when distributed approaches are employed to tackle the problem at hand.

**Keywords:** Cooperative Autonomous Vehicles, Intersection Management, Distributed Model Predictive Control,Optimal Control, Constrained Optimization

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# Chapter 1

# Introduction

Autonomous vehicles have received increasing attention in the past years, both in research and industry which has led to an increase in the development of related technologies. Many of the leading automotive and tech-companies are testing highly automated vehicles which they expect to penetrate the market in the foreseeable future. This introduction is expected to have a strong impact on society in general and on mobility trends specifically.
In this thesis, the problem of vehicle coordination at urban intersections will be discussed. This is an important issue since intersections at present are considered both a bottleneck and a safety hazard. Hence, once vehicles are automated, their coordination would offer an alternative to current traffic light technologies as well as rules of passage in a manner that could improve both safety and throughput.

## 1.1 Benefits of Autonomous Vehicles

The switch to AVs is highly motivated by various aspects including safety and efficiency. More than 90% of all traffic related accidents can be traced back to human error [1]. Efficiency on the other hand is more difficult to assess. However, in recent years, urbanization has become a big global trend, where a lot of people are transferring from rural areas to urban ones. In Europe for instance the crossover was witnessed in 2014 and it is projected that about 85% of the population will reside in urban areas by 2050[2].This has led to a noticeable increase in congestion and in the average time an individual spends on the road.Highly autonomous vehicles could thus provide a stimulus towards shared mobility which could ease the strain on the existing infrastructure and offer a resource efficient transportation-as-service alternative to the current situation. Moreover, since AVs are not limited by the decision processes of humans, there is a possibility that they can be operated in a more energetically efficient manner, which could have big ramifications on the traffic system's energy demand[3].

## 1.2 Current Situation

The general public has been enthralled by the prospect of automated vehicles in the last couple of years as technological development has put it within reach.Self driving vehicles have been an attractive feature in the field for long time. In fact, they can be traced as far back as 1930's where an automated traffic system was showcased by General Motors at the world Fair [4].Progress continued slowly in the following decades with the major events including demonstrations of autonomous functionalities on specially designed roads in the 70's [5] and some tests on public roads in the 80's [6]. Research on the topic continued in the 90's when the ability of automated vehicles to travel thousands of kilometers was demonstrated by various research projects [7]. The real boom however, started in the 2000s when vehicles with what could really be described as autonomous capabilities started to emerge. The 2007 Urban challenge is considered a major milestone in this context, where interactions between different automated vehicles was exhibited for the first time [8]. In that period, driver assistance systems such as Adaptive Cruise Control and Lane-keeping Aids were pushed to the market. In the last decade, ADAS (Advanced Driver Assistance Systems) have been evolving rapidly where have seen higher levels of autonomy allowing "hands and feet off" driving at low speeds.Companies such as Tesla, Volvo and Nissan have provided systems offering supervised partial automation at higher speeds outside of the urban environment.

These automakers in addition to service providers such as Waymo,Zenuity,Uber and Lyft are improving on these systems to provide solutions where little to no driver supervision is provided.Such systems are estimated to reach the

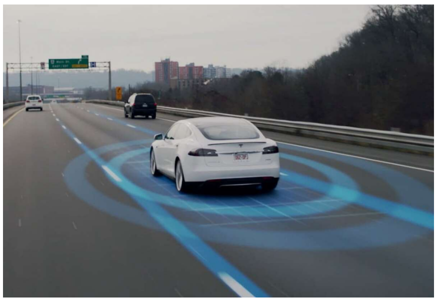

*Figure 1.1: Tesla Vehicle with autopilot*

*Figure 1.2: Waymo (Previously Google car)*

consumers between 2020 and 2030 [9].
In addition, motivated by the increase of availability of wireless communication and potential improvements it offers to the autonomous vehicle's performance due to information sharing, Connected automated vehicles have also been developing at the same time. These systems include cooperative control where better decisions can be made through shared information, in addition to cooperative perception where each vehicle can leverage the sensing abilities of other connected vehicles to make more accurate maps of the surrounding environment [10].
In this domain, vehicle platooning is one of the most popular sectors. Applications of this can be seen where CAVs drive with small inter vehicle distance at highways, or in the cases with lane merging and changing. Platooning can also have an important role in Emergency Vehicle Response, where when a platoon of vehicles encounters emergency vehicles such as Police cars or ambulances, their behavior should be modified to facilitate their passage. Demonstrations of platooning include the European Sartre Project [11] and the Grand Cooperative Driving Challenges in 2010s[12, 13].

A problem that is within the scope of CAVs and which constitutes the major work in this thesis is the issue of intersections in urban areas, where vehicular automation offers a chance at major improvements in all aspects ranging from safety to increasing through-put and reducing congestion.

## 1.3 Objective

The development of Cooperative Autonomous Vehicles aided by the advances in Artificial Intelligence and Control theory opens the door to completely new approaches of management of traffic flow in cities.They offer a solution to many of our current day problems such as the increase of congestion in urban areas, accidents and fatalities. Traffic congestion and automobile accidents are two of the main current day problems where in a study of 85 cities in the United States, it was estimated that the annual time wasted in traffic has increased since 1982 from 16 hours per capita to 46 hours per capita [14].Moreover, even with the incremental advances to vehicle safety through the years, global automobile accidents account for 2.1% of all deaths [15]

Intersections necessitate the usage of a shared space which increases all the risks previously mentioned. This is not to mention problems arising from non-vehicle entities such as pedestrians and cyclists.Intersections thus are the cause of 20% of all traffic related fatalities and to 40% of all injuries in the EU [16].
The hazardous nature of intersections has lead to them being one of the most regulated parts of the traffic network where often, a combination of traffic lights,stop signals and right-of-passage rules apply.This heavy regulation makes them a bottleneck for traffic flow increasing congestion and increasing emissions and energy waste (through idling and varied accelerations and deceleration) [17]. Often the solution quoted for this is the expansion of the infrastructure, or to reshape it in a way to accommodate autonomous vehicles. However, in the US alone there is currently over 300,000 traffic signals which account for more than $82.7 Billion of investments [18], and thus reshaping the network or expanding it would be the expensive solution and cooperative vehicles can offer a better alternative in this case.

Cooperative Autonomous vehicles have the capability to interact with their surroundings and communicate with data clouds. In this context, two main modes of communication can be established, the vehicle to vehicle (V2V) and the vehicle to Infrastructure (V2I). Alot of work has already been done on regulating traffic in a centralized fashion that mainly relies on regulating traffic from the global prespective. While this approach is employed by a lot of researches and has shown its advantages over the existing traffic management systems [17], the decentralized approach which relies on a mixture of V2V and V2I interfaces has been gaining a lot of support recently.This is due to the fact that it would allow researches to account for non-cooperative entities within the intersection space (such as legacy drivers), thus we have decided to tackle the intersection management problem from this angle in order to formalize algorithms that would allow a more efficient sharing of the space.

*Figure 1.3: The problem*

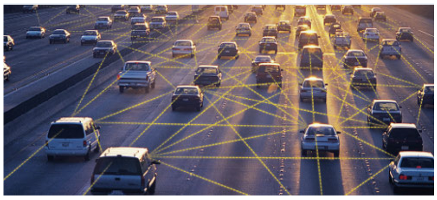

*Figure 1.4: The possible solution*

## 1.4 Structure of the Thesis

The Thesis will be structured as followed

**Chapter 2** offers a brief theoretical summary of Optimization and Relaxation Algorithms, Optimal Control and Model Predictive Control...

**Chapter 3** Presents the main challenges in the intersection management problem, followed by the state of the art and a survey of the various approaches already considered...

**Chapter 4** Introduces the scenarios and models considered with a formulation of the constraints ...

**Chapter 5** Offers a discussion of the results, and a comparison of two implemented approaches ...

**Chapter 6** presents a conclusion and a discussion of possible future works.

# Chapter 2

# Prelimenaries

In this chapter we present a theoretical background on the most important principles that are used throughout this thesis. The main references used are [76–79]

## 2.1 On Optimization

This section gives a general recap of the theory of optimization, constrained and unconstrained. It provides some definitions in addition to discussing a couple of theorems that are integral to the work that has been accomplished.

### 2.1.1 Constrained and Unconstrained Optimization

We consider the general problem of the form :

$$\begin{aligned} \min_{x \in \mathbf{R^n}} \quad & f(x) \\ \text{s.t.} \quad & g_i(x) = 0, \\ & h_i(x) \leq 0 \end{aligned} \tag{2.1}$$

Where x $\in$ $\mathbf{R}$ f: $\mathbf{R}^n \longrightarrow \mathbf{R}$ ; $g_i$: $\mathbf{R}^n \longrightarrow \mathbf{R}$ and $h_i$ : $\mathbf{R}^n \longrightarrow \mathbf{R}$. The solution of the problem (3.1) above is a decision variable x that minimizes the function f while satisfying the constraints $g$ and $h$. This problem can have various characteristics. Special classes that are relevant to this thesis are Quadratic Programs (QPs) and Semi-definite Programs (SDPs).

We will begin our consideration with the unconstrained problem which reduces to:

$$\min_{x \in \mathbf{R^n}} \quad f(x) \tag{2.2}$$

**Local and Global Optima**

A multi-variable scalar function f(x) admits a minimum **x*** if $f(x^*) \leq f(x^* + h)$. In the case that the values of h are small then the minimum is called a **local minimum**. If the inequality holds for all values of h then **x*** is considered a **global minimum**.

**Necessary Conditions for Optimality**

It is said that f(x) admits a stationary point ( maximum, minimum or inflection) at x* if the first partial derivatives of f(x) exist, and are finite and

$$\nabla f(\mathbf{x^*}) = \frac{\partial f}{\partial x}|_{\mathbf{x^*}} = 0$$

In order to determine whether the stationary point is a maximum,minimum or a saddle point it is then enough to check the sufficient condition:

**Sufficient condition theorem**

A Stationary point **x*** is an extremum point if the Hessian matrix evaluated at x*, $\nabla^2 f(x^*)$ is :

- $\nabla^2 f(x^*)$ is positive definite then **x*** is said to be a local minimum
- $\nabla^2 f(x^*)$ is negative definite then **x*** is said to be a local maximum

After defining the Necessary and Sufficient conditions for optimality for the unconstrained case, we can move to the constrained case with equality constraints which becomes of the form:

$$\begin{aligned} \min_{x \in \mathbf{R^n}} \quad & f(x) \\ \text{s.t.} \quad & g_i(x) = 0 \end{aligned} \tag{2.3}$$

For this purpose, it is important to define the Lagrangian duality and the Lagrangian multipliers. In order to move from the above form of the problem to its dual, we need to augment the objective function with the vector containing the equality constraints. Here it is important the the m constraints can be grouped in vector form where $g(x) = [g_1(x)g_2(x)g_3(x)...g_m(x)]^T$. Each of the constraints is then multiplied with a scalar variable $\lambda$ which is called the Lagrange multiplier. Thus the augmented objective function which is now called the Lagrangian which is a function of both the n variables x and the m variables $\lambda$:

$$\mathbf{L}(x) = f(x) + \lambda g(x) = f(x) + \sum_{i=1}^{i=m} \lambda_i g_i(x) \tag{2.4}$$

**Lagrange Multiplier Theorem**

If at **x*** f(x) admits a local minimum satisfying the set of constraints $g(x) = 0$, and if the gradients $\nabla g(x) = [\nabla g_1(x)\nabla g_2(x)\nabla g_3(x)...\nabla g_m(x)]^T$ are linearly independent, then there exists a unique vector, $\lambda^* = [\lambda_1...\lambda_m]$ called the Lagrange multiplier vector such that

- $\nabla f(x^*) + \sum_{i=1}^{i=m} \lambda_i \nabla g_i(x) = 0$
- $g(x^*) = 0$

The problem is thus equivalent to finding the unconstrained minima of the Lagrangian. With the sufficient condition for the pair (x*,λ*) being that the Hessian of the Lagrangian be positive definite.

We finally move to the generalized form of the problem (3.1) with both equality and inequality constraints

$$\begin{aligned} \min_{x \in \mathbf{R^n}} \quad & f(x) \\ \text{s.t.} \quad & g_i(x) = 0 \quad i \in 1....p, \\ & h_j(x) \le 0 \quad \forall j \in 1....p \end{aligned}$$

In order to begin with the solution of this problem, we begin by defining :

1. **Feasible Set**: $\Omega \subset \mathbf{R^n} := \{x \in \mathbf{R^n} | g(x) = 0, \quad h(x) \le 0\}$ the set containing all the variables x that satisfy both the equality and inequality constraints and thus are candidate points to be a solution f the minimization problem.

2. **Active Constraint and Active Set:** the inequality $h_i(x)$ is said to be active at the bound where $l_i(x) = 0$. The active set, $\mathbf{A}(x)$ is the set containing all the active constraints in $\mathbf{h(x)}$

3. **Linear Independence Constraint Qualification (LICQ)**: As mentioned earlier in the definition of the necessary condition on optimality all the gradients of the equality and inequality constraints in the active set are linearly independent

4. A point x* is called a regular point if and only if the LICQ holds at x*.

This enables us to define what is known as **Karush-Kuhn-Tucker Conditions** (KKT): if If x* is a local minimizer to the problem (3.1) and is a regular point, then:

$$\nabla f(x^*) + \sum_{i=1}^{i=m} \lambda_i \nabla g_i(x) + \sum_{j=1}^{j=p} \mu_j \nabla h_j(x) = 0 \tag{2.5}$$

$$g_i(x^*) = 0 \quad \forall i \in \{1....m\} \tag{2.6}$$

$$h_j(x^*) \leq 0 \quad \forall j \in \{1....p\} \tag{2.7}$$

$$\mu_j \geq 0 \quad \forall j \in \{1....p\} \tag{2.8}$$

$$\mu_j h_j(x) \quad \forall j \in \{1....p\} \tag{2.9}$$

In particular, (3.6),(3.7) require that x* is feasible in the primal problem (3.1), and are known as the primal feasibility conditions. Similarly, (3.8) requires that x* is feasible in the dual problem and is known as the dual feasibility condition. The conditions (3.5) and (3.9) are known as the stationarity and complementarity conditions, respectively.Finally, a vector (x*; $\lambda$*; $\mu$*) that satisfies the set of equations above is known as a KKT-point.Here it is important to note that the conditions above are necessary but not sufficient, and thus a KKT-point is not always a stationary point of (3.1).

#### Sufficient Conditions for Optimality

If (x*; $\lambda$*; $\mu$*) is a KKT-point for (3.1) and LICQ holds, x* is a local minimum if:

$$\nu^T \nabla_x^2 \mathbf{L}(x, \lambda, \mu)\nu \geq 0, \tag{2.10}$$

$$\forall \nu \in \left\{ \nu \neq 0 \mid \nabla_x g(x)^T \nu = 0, \nabla_x h_{A^+}(x)^T \nu = 0, \nabla_x h_{A^0}(x)^T \nu l \leq 0 \right\} \tag{2.11}$$

That is, $\mathbf{L}$(x*; $\lambda$*; $\mu$*) must be positive-definite in all the unconstrained directions of x.

#### Importance of Convexity

It is difficult to verify the LICQ without previous knowledge of x*. However, the constraint qualification is always satisfied for problems where all inequality constraints are convex functions and all equality constraints are linear functions and at least one feasible vector $\widetilde{x}$ exists strictly in the feasible region which means that:

$$g_i(x^*) = 0 \quad \forall i \in \{1....m\} \quad h_j(x^*) \leq 0 \quad \forall j \in \{1....p\} \tag{2.12}$$

Moreover, if the conditions above are satisfied, and x* is found to be a local minimum of f(x) and f(x) is a convex function, then x* is also the global minimum of f(x).

### 2.1.2 Quadratic Programs

The optimization problem is classed as a (convex) *quadratic program* (QP) if the objective function is (convex) quadratic, and the constraints are affine [78]. A quadratic Program can thus be written in the form:

$$\begin{aligned} \min_x \quad & \frac{1}{2} x^T P x + q^T x + r \\ \text{s.t.} \quad & A_{eq} x = b_{eq}, \\ & Ax \preccurlyeq b \end{aligned} \tag{2.13}$$

Here, P $\in \mathbb{S}_+^n$, $G \in \mathbf{R}^{mxn}$ and $A \in \mathbf{R}^{pxn}$. This means that we minimize a quadratic function over a polyhedron as can be seen in the figure below .

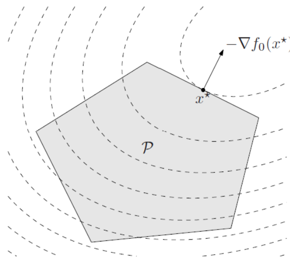


*Figure 2.1: Geometric illustration of a quadratic program showing the feasible set which is the shaded polyhedron and the contour lines of the objective function. The point x* is the optimal point*

Quadratic Programs are considered a special case of a more general class of problems, Quadratically Constrained Quadratic Programs (QCQP) which are of special interest in this thesis. A QCQP is thus formulated as:

$$\begin{aligned} \min_{x} \quad & \frac{1}{2}x^T P_0 x + q_0^T x + r \\ \text{s.t.} \quad & A_{eq}x = b_{eq}, \\ & \frac{1}{2}x^T P_i x + q_i^T x + r \preccurlyeq 0 \quad i = 1....,m \end{aligned} \tag{2.14}$$

The feasible region is no longer a polyhedron but an the intersection of ellipsoids (if $P_i \succcurlyeq 0$. In case the quadratic constraints are non-convex, the QCQP problem becomes NP-Hard, and all known algorithms to solve them have a complexity which grows exponentially with problem dimensions.

### 2.1.3 Semi-Definite Programs

A *semidefinite program* (SDP) minimizes a linear function of a variable $x \in \mathbf{R}^n$ subject to a matrix inequality, its general form is as follows:

$$\begin{aligned} \min_{x} \quad & c^T x \\ \text{s.t.} \quad & F(x) \succcurlyeq 0 \end{aligned} \tag{2.15}$$

where

$$f(x) \triangleq F_0 + \sum_{i=1}^{n} x_i F_i$$

The inequality sign means that F(x) is positive semi-definite and is called a linear matrix inequality LMI[79]. Semi Definite programs are regarded as an extent ion of linear programs where component wise inequalities are replaced by matrix inequalities. SDPs unify a large class of problems such as linear and quadratic programs and is often employed in control engineering, It is worthy to note that while SDPs are considered a richer and more general class of problems than linear programs, they are not harder to solve where they have polynomial worst-case complexity and perform very well in practice.

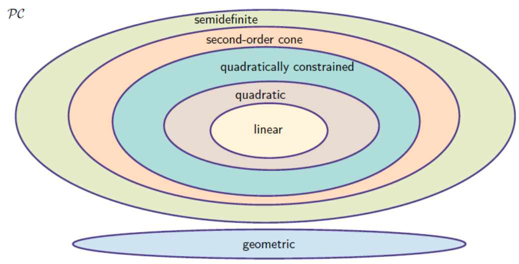


*Figure 2.2: Ven-Diagram of Program Hierarchy, showing the types of programs that can be formulated as a Semi-Definite Program*

### 2.1.4 Semi-Definite Relaxations

In order to solve the non-convex QCQPs defined earlier in the section, some direct relaxations using semi-definite programming are used. This provides a lower bound on the optimal value of them problem which will then be used to find an approximate solution.
starting from the non-convex QCQP:

$$\begin{aligned} \min_{x} \quad & x^T P_0 x + q_0^T x + r \\ \text{s.t.} \quad & x^T P_i x + q_i^T x + r \preccurlyeq 0 \quad i = 1...., m \end{aligned} \tag{2.16}$$

and using $x^T P x = \mathbf{Tr}(P(x^T x))$ it can be re-written as

$$\begin{aligned} \min_{x} \quad & \mathbf{Tr}(P_0(X))_0 x + q_0^T x + r_0 \\ \text{s.t.} \quad & \mathbf{Tr}(P_i(X)) + q_i^T x + r_i \preccurlyeq 0 \quad i = 1...., m, \\ & X = x x^T \end{aligned} \tag{2.17}$$

this can be directly relaxed into a convex problem by replacing the last non-convex equality constraint with a positive semi-definite constraint $X - xx^T \succcurlyeq$

0 which can be formulated as a Schur compliment to obtain:

$$\begin{array}{ll}\min\limits_{x} & \mathbf{Tr}(P_0(X))_0 x + q_0^T x + r_0 \\ \text{s.t.} & \mathbf{Tr}(P_i(X)) + q_i^T x + r_i \preccurlyeq 0 \quad i = 1....,m, \\ & \begin{bmatrix} X & x \\ x^T & 1 \end{bmatrix} \succcurlyeq 0 \end{array} \tag{2.18}$$

This is strictly equivalent to the original problem if the Rank of the Schur matrix is enforced to be 1, however that remains non-convex. The convex relaxation does not enforce this rank constraint and thus only provides a lower bound on the optimal value of the non-convex QCQP. However,since it is formulated as an SDP it is easy to solve. [81]

### Randomization Techniques

Since the relaxation Technique provides a lower bound on the optimal value of the QCQP and does not give any particular clue on how to obtain "good" feasible points, additional work must be done in order to reach these points. The semi-definite relaxation given in (3.18) produces a positive semi-definite matrix ( The Schur Compliment) along with the lower bound on the objective. If this additional output is used, good approximate solutions, with in some cases hard bounds on sub-optimality, can be found.

If x and X are the solutions to the relaxed program, then $X - xx^T$ or the Schur compliment can be considered as the covariance matrix. Therefore, if x is picked as a Gaussian variable with the distribution $\mathcal{N}(x, X - xx^T)$ then by sampling x a sufficient number of times and keeping the best feasible point, the non-convex QCQP will be solved on average over the distribution meaning that the following is solved :

$$\begin{array}{ll}\min\limits_{x} & \mathbf{E}(x^T P_0 x + q_0^T x + r_0) \\ \text{s.t.} & \mathbf{E}(x^T P_i x + q_i^T x + r_i) \preccurlyeq 0 \quad i = 1....,m \end{array} \tag{2.19}$$

This would lead to obtaining a "good" feasible point, if the program does not include an equality constraint and we are able to directly project the random samples onto the feasible set[81].

## 2.2 Optimal Control

Optimal Control is the branch of control Engineering that aims to find a control input for a dynamic system that minimizes a cost functional related to the system state and control histories. The controlled dynamic system can be expressed as a set of ordinary differential :

$$\dot{x}(t) = f(x, u, t) \tag{2.20}$$

where $u(t) : [t_0 t_f] \longrightarrow \mathbb{R}^m$ is the control input and $x(t) : [t_0 t_f] \longrightarrow \mathbb{R}^n$ is the state vector. It is assumed that starting from an initial condition $x_0$ and with a unique control history, only one state time history can be generated.

The optimal control problem formulation is thus :

$$\begin{aligned} \min_{u} \quad & \Phi(x(t_f, t_f) + \int_{t_0}^{t_f} L(x(t), u(t), t)dt \\ \text{s.t.} \quad & \dot{x}(t) = f(x, u, t), \\ & x(t_0) = x_0 \end{aligned} \tag{2.21}$$

In some instances the final time $t_f$ can be an additional parameter to optimize.A famous formulation that is often used is the minimum state and control effort cost function which is put in quadratic form is :

$$x(t_f)^T P(t_f) x(t_f) + \int_{t_0}^{t_f} (x^T Q x + u^T R u + 2x^T N u)dt$$

.

The equations of motion can be added as a constraint, where the augmented cost functional becomes :

$$\tilde{J} = \Phi(x(t_f, t_f) + \int_{t_0}^{t_f} (L(x(t), u(t), t) + \lambda^T(t)[f(x, u, t) - \dot{x}(t)])dt$$

Here the Lagrange multipliers $\lambda(t) : [t_0, t_f] \longrightarrow \mathbb{R}^n$ are also called the costate since they have the same size as the state.

In order to solve the problem at hand it is convenient to start with some definitions, **The Hamiltonian** $H = L(x(t), u(t), t) + \lambda^T(t)[f(x, u, t)]$ allows us to re-write the problem in the following form :

$$\tilde{J} = \Phi(x(t_f, t_f) + \int_{t_0}^{t_f} (H(x(t), u(t), \lambda(t)t)dt - \int_{t_0}^{t_f} \dot{\lambda}(t) x(t)dt \quad + \lambda^T(t_0)x(t_0) - \lambda^T(t_f)x(t_f)$$

A set of first-order necessary conditions can be obtained by nllifying the first variation of the augmented cost functional obtaining:

- $\lambda^*(t_f) = \frac{\partial(\phi(x^*(t_f,t_f)}{\partial x}^T$
- $\dot{\lambda}^*(t) = -\frac{\partial(H(x^*,u^*,\lambda^*,t)}{\partial x}^T$
- $\frac{\partial(H(x^*,u^*,\lambda^*,t)}{\partial u}^T = 0$

these equations should be solved together with the dynamic equation and the initial conditions of the system to find the state vector, the adjoint vector and the control inputs. One criticism of these approaches is that the necessary conditions must apply along a specific trajectory, and thus they obtain a local minimum that does not preclude the existence of other optimizing paths. Here again can be seen the importance of having a convex problem formulation.

## 2.3 Model Predictive Control

Model Predictive Control refers to a large class of control strategies that use a model of the plant in order to obtain a control signal by minimizing an objective function. The current control action is computed on-line rather than using an offline computed control law. Model Predictive control has developed to become the second most used type of control in industry after the classical PID controller due to its ability to handle a wide range of systems and applications while integrating concepts from optimal control, and stochastic control and applying them to both linear and non-linear systems[].

In general, an MPC controller uses at each sampling instant, the plant's current input and output measurements in addition to the current state and the constraints on control action in order to

- calculate (predict), over a finite horizon, a future control sequence that optimizes a cost function while taking into account constraints .
- Use the First control in the sequence as the plant's input

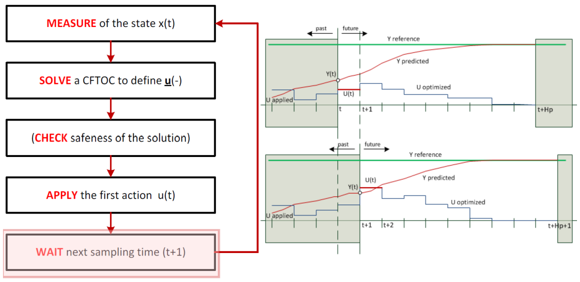


*Figure 2.3: A figure demonstrating the operating principle of an MPC controller*

This process is described in the figure below:

Despite its advantages, it has some drawbacks.The main disadvantages are:

- The computational complexity of obtaining the required signal at every iteration especially when constraints are included which makes it hard to achieve real-time feasibility
- The theoretical complexity in guaranteeing stability and convergence. (For more information refer to []).
- The need for an appropriate and accurate model of the process.

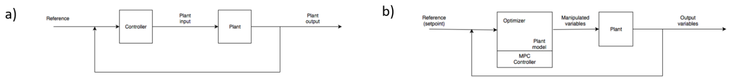


*Figure 2.4: A comparison between the general controller scheme and the MPC scheme*

The elements that go into the design on the MPC controller (aside from the cost function and the prediction model) are [] *the controller sample time, the prediction and control horizons, the constraints and the weights.* The choice of these parameters is important as they affect the controller performance and the computational complexity of the MPC algorithm.

The sampling time $T_s$ determines the rate at which the controller executes the control algorithm. This is of particular importance when disturbances come to play. If the sampling time is too large then the controller can't react to disturbances fast enough. On the other hand as the sampling

time decreases the computational load increases and thus it is important to find a compromise.
The prediction horizon $N_p$ indicates how much the controller looks ahead into the future. The prediction horizon should be selected in order to cover the significant dynamics of the system while not significantly increasing the computational burden on the system to predict states which might be later be affected by unaccounted for disturbances.

The Control Horizon $N_u$ is the number time steps to which the calculated control input is used for the prediction, the inputs for $N_p - N_u$ remains constant. As each control input is a decision variable for the optimizer, the smaller the control horizon the lower the computational cost and less accurate the predictions. For our application the control horizon is taken to be equal to the prediction horizon.

Finally, constraints can be Incorporated on inputs, the rate of change and outputs. These constraints can either be hard or soft constraints. There should be a good balance between constraints on the input and those on the output in order to ensure feasibility. Constraints can be added to achieve the various objectives.

**Implementation**

We will proceed with describing the Linear MPC problem and its formulation as this is the main controller used in our problem The model of the system can be formulated as

$$x_i(k+1) = f_i(x_i(k), u_i(k)) \tag{2.22}$$

which under linear system assumption can be re-written to be

$$x(k+1) = Ax_k + Bu_k \tag{2.23}$$

The MPC problem thus becomes:

$$\underset{u_i}{\text{minimize}} \quad \frac{1}{2}x_N^T P x_N + \sum_{k=0}^{N-1}(\frac{1}{2}x_k^T Q x_k + \frac{1}{2}u_k^T R u_k) \tag{2.24a}$$

subject to

$$x(k+1) = Ax_k + Bu_k, \tag{2.24b}$$

$$y_k = Cx_k, \tag{2.24c}$$

$$y \in \chi \quad , u \in \mathbb{U}_i, , \tag{2.24d}$$

$$y(N) \in Y \subset \chi, , \tag{2.24e}$$

$$x(0) = x_0 \tag{2.24f}$$

Where N is the prediction horizon and the problem becomes that of optimizing the cost function,subject to model dynamics, bounds on states and inputs in addition to bounds on terminal states.
If the states are grouped in $\mathbf{X}$=$[x_1^T,x_2^T,....x_N^T] \in \mathbb{R}^{nN}$ and the control history is likewise grouped in $\mathbf{U}$=$[u_0^T,u_1^T,....u_N^T] \in \mathbb{R}^{mN}$
Being an autonomous system, the states can be obtained knowing the initial state and the control history,

$$x_i = A^i x_o + \sum_{p=0}^{i-1} A^{i-1-i} B u_p \quad i = 1, ...N \tag{2.25}$$

the system equation thus can be written as

$$\mathbf{X} = \bar{T}x_0 + \bar{S}U \tag{2.26}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix} x_0 + \begin{bmatrix} B & 0 & 0 & \dots & 0 \\ AB & B & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 \\ A^{N-1}B & A^{N-2}B & A^{N-3}B & \dots & CB \end{bmatrix} \mathbf{U}$$

Since the constraint is linear, it can be replaced in the cost function in order to reduce the total number of variables to obtain

$$J = x_0^T Y x_0 + U^T H U + 2x_0^T q U \tag{2.27}$$

Similarly, the system outputs can be written as:

$$y_i = CA^i x_o + \sum_{p=0}^{i-1} CA^{i-1-i} B u_p \quad i = 1, ...N \tag{2.28}$$

the system equation thus can be written as

$$\mathbf{Y} = \bar{T}_y x_0 + \bar{S}_y U \tag{2.29}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^N \end{bmatrix} x_0 + \begin{bmatrix} CB & 0 & 0 & \dots & 0 \\ CAB & CB & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 \\ CA^{N-1}B & CA^{N-2}B & CA^{N-3}B & \dots & CB \end{bmatrix} \mathbf{U}$$

This means that the output can be re-written in the following form:

$$\mathbf{Y} = \bar{T}_y x_0 + \bar{S}_y U \leq \begin{bmatrix} y_{max} \\ y_{max} \\ \vdots \\ y_{max} \end{bmatrix} = \mathbf{Y_{max}}$$

which leads to :

$$\bar{S}_y U \leq \mathbf{Y_{max}} - \bar{T}_y x_0$$

similarly,

$$-\bar{S}_y U \leq -\mathbf{Y_{max}} + \bar{T}_y x_0$$

In addition to

$$\mathbf{U} \leq \mathbf{U_{max}} \quad -\mathbf{U} \leq -\mathbf{U_{min}}$$

Leading to

$$\mathbf{G} U \leq \mathbf{W} - S x_0 \tag{2.30}$$

$$\begin{bmatrix} \bar{S}_y \\ -\bar{S}_y \\ I \\ -I \end{bmatrix} \mathbf{U} \leq \begin{bmatrix} \mathbf{Y}_{max} \\ -\mathbf{Y}_{min} \\ \mathbf{U_{max}} \\ \mathbf{U_{min}} \end{bmatrix} + \begin{bmatrix} -\bar{T}_y \\ \bar{T}_y \\ 0 \\ 0 \end{bmatrix} x_0$$

This leads to the optimization problem described previously to be expressed as

$$\underset{U}{\text{minimize}} \quad x_0^T Y x_0 + U^T H U + 2x_0^T q U \tag{2.31a}$$

subject to

$$\mathbf{G}U \leq \mathbf{W} - Sx_0, \tag{2.31b}$$

$$x(0) = x_0 \tag{2.31c}$$

This formulation can be extended for reference tracking which is particularly rel event to our application and will be employed in our problem as will be described in the next chapter.

### Applications in the Automotive Industry

Due to its flexibility, MPC is being implemented in the automotive field, mainly for active safety purposes since it can merge the path planning problem with threat assessment and hazard avoidance. For instance, in [90] used MPC along with the assumption that road lane data is available and that road hazards have been located, and mapped ( which is possible due to the use of various sensors) in order to safely navigate and avoid the obstacle. Another interesting implementation has been used to account for uncertainty in the traffic environment in [91] to simulate possible highway scenarios. Moreover, when it comes to vehicle cooperation, MPC is considered an appropriate method these kind of problems since it allows for the incorporation of anticipated trajectories of conflicting vehicles. To this extent, it has been used quite rigorously in lane change scenarios [92] often coupled with decision systems based on Game theory.
When it comes to the intersection management problem, while initial works had been focused on hybrid systems theory [93] and multi-agent system theory as in [94]. Recent approaches from the control industry formulated the problem in OC terms as mentioned above and the move towards the decentralization of the control is paving the way for MPC to be the main controller used in these scenarios.

# Chapter 3

# Cooperative Autonomous Vehicles

In this chapter, we will begin introduction autonomous and cooperative vehicle technologies. We will then introduce the various considerations that can be taken into account while designing the intersection management control algorithm/strategy and by defining the assumptions taken by the research community. Then,a classification of the implemented approaches will be done based on the methods used. Here, the various components of the system and their interactions will be thoroughly discussed.Finally, a brief survey of the methodologies of the other implemented solutions will be done.

## 3.1 Autonomous Vehicle Technology

An Autonomous car is a classification of vehicles that are able to interact with their surroundings without human input[19].These vehicles can combine a range of technologies in order to perceive the environment(radar,GPS,computer vision). For this purpose, advanced control systems are required in order to identify the appropriate actions in the presence of a dynamic environment. The main functions of an autonomous car can be summed up in the following:

- Perception
- Localization

- Planning
- Vehicle Control
- Systems Management

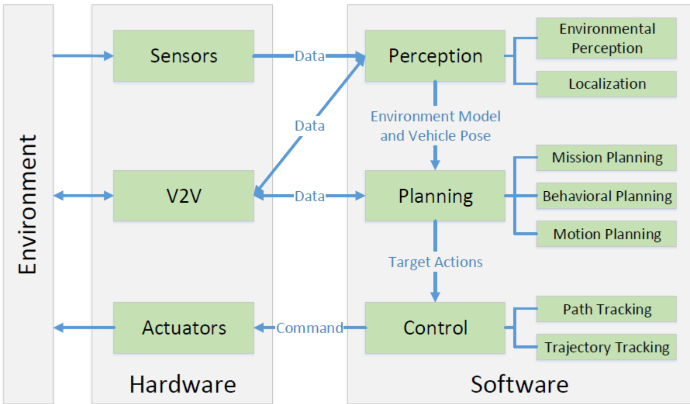


*Figure 3.1: Typical Autonomous Vehicle system overview*

Each of these sectors is challenging in its own respect. This requires a decision-making hierarchy where at the highest level, a route is planned in order to reach the desired location. This could be seen as a problem of finding a minimum-cost path on a road network graph. This is followed by a behavioral layer which decides on the manner the vehicle follows the plotted path.A control system is required to ensure the correct execution of the planned motion satisfying the appropriate constraints.

### 3.1.1 Levels of Autonomy

A classification was published in 2014 by SAE International in order to classify vehicles on six different levels. This ranges from no automation ( requiring the presence of a driver at all times to full autonomy(where the autonomous vehicle operates independently without the intervention of a driver). In 2018, SAE updated the classification in J3016_20806 [20].These levels are reported below as seen in the National Highway Traffic Administration Website [21]

**Level 0:** The human does all the driving

**Level 1:** The driver and the automated system share the control of the

vehicle.The driver should manage directly one of the braking/acceleration and the steering. The suspensions are not managed by the driver. An example of this is Adaptive Cruise Control (ACC), where the driver controls the lateral dynamics while the system controls the speed. Another example can be seen in the Parking Assistance, where the steering is controlled by the system and the speed is controlled by the driver who is available to take over control at any time. A final example is Lane Keeping Assistance.
**Level 2:** An advanced driver assistance system (ADAS) can control both the steering and the breaking/acceleration simultaneously in certain intervals.The human driver is required to be available to take control at any time and to perform the rest of the driving task. As a matter of fact, the driver's hand is required to remain on the driving wheel during SAE2 driving in order to confirm the driver's ability to intervene when required.
**Level 3:** *The quantum Leap:* driver can safely turn his/her attention away from the driving tasks ( they are allowed to text or read). The vehicle thus handles situations requiring immediate response such as emergency braking. The driver must still be available to take control withing a time that is specified by the manufacturer, when called upon by the vehicle. . In Europe the Audi 8 has level 3 up until 60km/h but it is not legal everywhere.
**Level 4:** An Automated Driver Systems (ADS) is able to do all the driving task by itself in addition to monitoring the driving environment(and thus be capable of fulfilling the complete driving task). This kind of driving is enabled only in limited spacial areas (geofenced) or in certain scenarios, like traffic jams.Moreover, the vehicle should be able to safely abort the trip if the driver can't retake control.
**Level 5:** No human intervention is required. This is the vision for the new mobility.

## 3.2 Cooperative Vehicles

Once a certain penetration rate of autonomous vehicles has been exceeded, the opportunity to exploit coordination amongst vehicles arises. Autonomous vehicles are still lacking in terms of their sensing and coordination capabilities as they rely on on-board sensory data and on modeling the behavior of

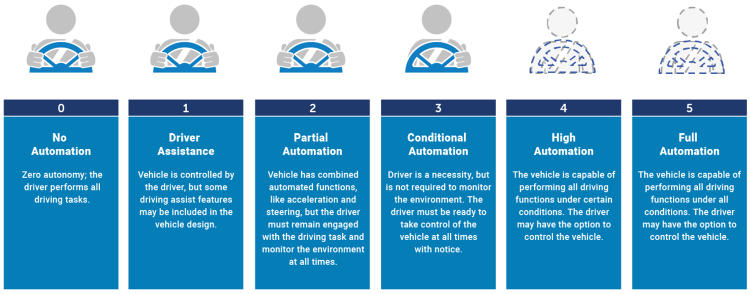


*Figure 3.2: Levels of Autonomy*

other vehicles. A summary of the Urban Grand Challenge [22] mentions that the number of incidents involving autonomous vehicles could be reduced if vehicles were capable of anticipating the actions of other vehicles.The conclusion there was that for autonomous driving to reach its full potential, vehicles should be able to cooperate.

One sector which has fully backed the improvements that cooperation bring is vehicle platooning. This field, relies on Vehicle to Vehicle (V2V) or Vehicle to infrastructure (V2I) communication for the information sharing and joint decision making where all vehicles in the platoon jointly choose safe and efficient control policies [23]

### 3.2.1 Challenges

While cooperation has its benefits, cooperative vehicles face many challenges which play a vital role when developing algorithms for vehicular coordination.

#### Communication

Communication is vital for CAV operation. A vehicle in a cooperative scheme transmits packets of data containing speed,sensor readings and current position to other vehicles through one of the two interfaces mentioned earlier. This communication will be affected by the impairments associated with wireless channels such as the inherent randomness and correlation of the channel, interference due to simultaneous transmissions and limits in communication range. This means that it is necessary to keep the communication load low since it has been predicted that wireless channel congestion will be one of the major challenges related to vehicular networks. The introduction of 5G networks, with assumed reliability of 99% and status updates of 100ms [24] ,is expected to help in many of the previously mentioned challenges.[25]

#### Sensing Challenges

Each vehicle's knowledge of its current location and of its surroundings is accompanied with uncertainty. This knowledge is based on data coming from a range of sensors ( Cameras, Lidar, GNSS, IMUs ...) whose data is corrupted by factors such as noise and clutter. Moreover, and the types of sensors that each vehicle is equipped with vary, the accuracy of each vehicle's understanding of the traffic situation will vary. This would lead to non-coherent uncertainties [26] which could be handled using methods that require that an uncertainty description is communicated among the involved vehicles,thus increasing the demand on the communication system. Even in cases with perfect communication, it is difficult to associate each vehicle's local understanding to other vehicles' understanding of the global problem, this is referred to in the literature as *data association problem* and is considered to be an NP-Hard problem[27].

#### Error Accumulation

This is related to the increase in error when packets are transmitted to other vehicles. In a scenario where a vehicle has noisy or faulty signals, the errors will be accumulated as they are transmitted within the network. A way this can be handled is through comparison between vehicle data and using string stability analysis.

#### Limits on Maximum Scalability

This problem is mainly concerned with the maximum number of vehicles that the network can accommodate.For an algorithm to be good, it has to be scalable, however a limit on scalability should be considered as an important parameter. Since as the limit increases, all the previously mentioned challenges are compounded.

#### Cybersecurity

The security of vehicular networks is an important challenge, since the vehicular network can be seen as a collection of networked sensors and computers connected wirelessly either to the infrastructure (V2I) or to other vehicles (V2V). This network should be safe from online attacks from intruders or hackers to insure the safety of all vehicles in the network.[]

## 3.3 The Intersection Management Problem

Intelligent transportation systems are those which are able to leverage advances in information technology to solve current road traffic problems. As mentioned in the introduction, one of the most regulated areas in traffic is the intersection problem where vehicles must cooperate in order to share a common resource. The inefficiency of the current human based model is expected to become even more apparent as the global number of light vehicles increases. Solving this problem by expanding the infrastructure is considered undesirable and infeasible given the continued urbanization and the associated increase in population density. This brings forward the opportunity of leveraging the types of autonomous vehicles mentioned above where the two tracks are expected to merge forming a new large scale wireless networked control system [28]. In this system, vehicles would be able to drive autonomously while leveraging their communication capabilities to tackle problems such as the traffic intersection.

The problem of vehicular coordination is a popular topic in the research community with most of the work coming after the seminal publications [29, 30], the existing results are fairly recent with the majority of the work being done in the last five years.

### 3.3.1 Initial Considerations

There are many consideration to be taken during the design of coordination algorithms for the intersection problem [31]. To start with, the problem of finding collision free paths for the number of vehicles passing through the intersection is combinatoral [32]. The difficulty is emphasized when there are more than a few vehicles involved in the problem and when the controller is designed in an optimal way. It must also be noted that there is no one correct way to approach the problem of designing the control strategy, as it includes various parameters which can have many combinations leading to different results.These parameters can include the location where the problem is solved, the way the priority order is established, etc... There are hence several plausible approaches that vary, from fully and partly centralized to distributed or decentralized ones.Finally, certain uncertainties can be incorporated in the system in order to account for non-cooperative entities (legacy drivers or cyclists etc.).

The problem of coordinating connected automated vehicles at intersections has been surveyed [33, 34]. The governing contributions disregard non-cooperative entities (legacy vehicles and pedestrians) and are focused on settings where each vehicle is automated. A large part of this work has been performed outside the control community and has relied heavily on tailored heuristics [30, 35, 36]. Nevertheless, coordinating vehicles at an intersection implies fundamentally a constrained optimal control problem (OCP), incorporating collision avoidance constraints,leading to optimized trajectories of the vehicles.

### 3.3.2 Assumptions about Network and Topology

Nearly all the existing literature relies on the fact that vehicles are automated and communication impairments neglected. Moreover, sensing errors and vulnerable road users(pedestrians cyclists) are never considered.This thesis uses the same approach, however at the end it introduces a priority scheme that aims at incorporating non-cooperative vehicles.

## 3.4 Classifications

A simplified way to classify the approaches taken to solve the problem is to divide them into two categories, Centralized and Decentralized. In order to correctly describe these two categories we have to begin by defining the system components.

### 3.4.1 System Components

**The agents: Vehicles**: each vehicle i inside the intersection area, is an agent with a pre-defined trajectory defined as the set of states over the planned time.The pre-defined trajectories do not take into account the other vehicles in the intersection space. An example of the information present to the vehicle can be seen in the figure. More detailed explanation of these parameters will be provided in later chapters.

```
                 ID: 1
         Velocities: [100×1 double]
        PlannedTraj: [100×1 double]
      CurrentLaneID: 1
 CollisionPointsID: [1 3]
   CollsionPointsx: [0 2.6163]
       TargetLaneID: 3
       CurrentState: [3×1 double]
       PlannedEntry: [40 42]
             INSIDE: 0
        Controllers: [1×1 struct]
                 Ta: 0.3000
```

*Figure 3.3: Information Available to Each Vehicle*

**The Intersection Manager** is the entity/cyber agent that is responsible for the intersection. The role of the IM varies along the spectrum between completely centralized approaches and completely decentralized approaches. Where in the former it has the difficult task of finding the collision free-trajectories for the agents, while in the later its only task is relaying information between vehicles in the intersection space. After defining the two

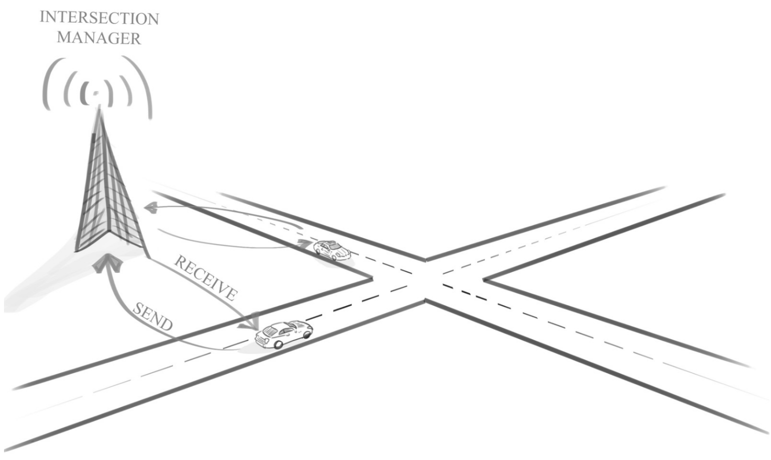


*Figure 3.4: Sketch showing the role of the Intersection Manager*

main players in the intersection scenario, we can proceed with the definition of the classes of algorithms.

### 3.4.2 The two Classifications and the spectrum

**Fully Centralized Algorithms**: are algorithms which rely on the Intersection Manager, in order to calculate the trajectories, and the passing orders for all vehicles in the intersection space, this calculation has to be re-done at every significant event occurrence (vehicle entering or leaving the intersection space).[29, 30, 36–56]
For any initial configuration, arriving at a crossing order involves considering a multitude of feasible orders, an optimal crossing order can thus only be achieved by a structured exploration of the different alternatives. This makes the problem an NP-Hard problem [32] which drives research away from such considerations
**Fully Decentralized Algorithms**: Here, each agent solves its own trajectory planning problem based on the input it receives from other vehicles.
Unlike in the fully centralized approach crossing orders are most often based on heuristics and approximations in order to reach results which are implementable in real systems. The closed loop controller must ensure that the crossing order does not vary upon recalculation[31]

Different levels of centralization occur in literature however, with the role of the intersection manager being more important the more we move towards the fully centralized. And so, several schemes stand in the middle of both ends of the continuum. For example, some approaches use the IM as a mediator to resolve conflicts - i.e, allowing the vehicles to locally compute the actual control commands [29, 30, 36–43].While other formulations obtain a solution through iterative procedures and allowing the computation to take place on-board the vehicles [47–51].

## 3.5 Summary of Implemented Works

Early work tackling the problem focused on centralized reservation-based algorithms [29, 30, 36, 37], whereby a vehicle would initially send a reservation request specifying the time frame it wishes to occupy the intersection

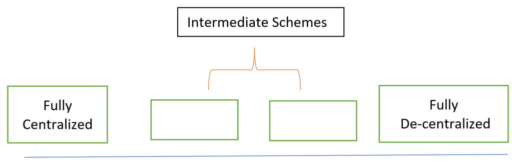


*Figure 3.5: A graphical Representation of the Spectrum Of Approaches*

for. The IM then examines the request and would only grant access if no conflict is detected with vehicles that have already requested times in the intersection. If conflict is detected on the other hand, the IM would demand a modification on the initial request. More recently, decentralized approaches have borrowed on the concept of reservation to find their solution where collision free paths are generated through rule-based interaction protocols.[57–60].

There were several alternative methods proposed to find the equivalent of reservations, mainly through mixed-integer optimization [38, 39], polling-systems theory [40] or scheduling [41–43]. The solution here is computed in two steps - the first being the resolving of potential collisions and the second one being the computation of the vehicle motion profiles. The notion of "extensions of platooning" has also characterized another approach [61, 62], where virtual platoons of existing vehicles on different lanes are created. And it's through a series of delicately selected coordinate transformations that collisions are avoided - however,the virtual platoon features vehicles kept at specified inter-vehicle distances. This approach hence solves the coordination problem featured in standard and decentralized platooning controllers.

With the optimization of the motion profiles of the vehicles at the forefront of the system, there has also been a number of algorithms based on Optimal Control (OC) techniques serving this particular purpose as well. The advantages range from the ability to incorporate various constraints - such as actuator limitations - to the ability to optimize stated performance objectives. These objectives differ from many non OC-based methods that feature performance metrics only used to heuristically derive algorithms. Nonetheless, most OC-based methods rely on heuristics since as mentioned before, finding an optimal crossing order is classified as an NP-Hard problem.Priority schemes will be discussed in more detail in a later section. The approaches taken can be grouped as Sequential/Parallel or Simultaneous.

Starting with Sequential/Parallel schemes, the vehicles at the intersection are ranked according to priority, usually done based on variations of first-come-first-serve (FCFS) policies. After the rankings are set, constraints are set to be matched in order to prevent the collision of higher priority vehicles and smaller Optimal control problems are solved, usually one per vehicle. In schemes that are purely Sequential like [63, 64], also called the "$MPC^0$" alternative in [65] where the control actions of each vehicle is solved according to a decision order which demonstrates in a way the priority of the vehicle. In this case, each vehicle solves its OCP based on the solutions of the OCPs of others (which would already be solved and ready) prior to it in the decision order to prevent collisions.

As for parallel schemes, the OCPs of the vehicles are based on trajectory predictions of vehicles that are of higher priority which are calculated at the previous sampling time. Along these lines, [66] proposes to use conservative estimates, based on predicted trajectories resulting from maximum breaking maneuvers. [65] pursues the "MPC0" solution which is focuses its predictions on constant velocities whereas [67] considers constant acceleration. Another approach used is when the predictions are based on the trajectories of the higher priority vehicles taken from the preceding time instant and is best used for receding horizon implementations ([**?**, 68–70] and the so-called "MPC1" alternative in [65]). This solution becomes close to sequential scheme with delayed information exchange in the case where the order of priorities is the same between two time instants. Sequential and parallel schemes were combined in [71]where the FCFS policy is used to build a crossing time schedule for the vehicles, followed by the parallel solution of the vehicle OCPs to determine state and control trajectories.A Final example which is the starting point of this thesis uses the predicted trajectories of conflicting vehicles in order for each lower priority vehicle to re-plan its trajectory while satisfying a collision avoidance constraint [72].

It is interesting to note that the sequential and parallel schemes differ in many areas such as the objective function considered, the motion models, and the formulation of collision avoidance conditions but are similar in a way that they both are considered "greedy".[73] The actions of the vehicles seldom are in the interest of the intersection as a whole including other vehicles, and instead are focused on limiting the risks on themselves, leaving the responsibility of resolving dangerous maneuvers to lower priority vehicles. The Sequential/Parallel schemes are not ideal in design but can be easily installed in an almost completely decentralized fashion while maintaining low and accurately predictable requirements on both computation and information exchange.

This can't be said about implementations that utilize what is called the simultaneous methods where the solution is found through joint optimization of several vehicles' trajectories. In order to avoid the combinatorial complexity of finding the optimal crossing order heuristics are still employed to determine the crossing order. Examples of these fixed order implementations can be found in [45–53].In other approaches, the coordinator provides a crossing order which is dependent on a first guess, the vehicles then solve their respective OCPs [53, 54].
A few contributions attempt to solve the complete problem optimally by simultaneously optimizing all aspects of the problem. For instance, both [55] and the benchmark discussed in [56] consider mixed integer quadratic programming (MIQP) formulations of the problem, returning both the optimal trajectories and crossing order.Such approaches find globally optimal solutions but do not scale well computationally and thus can't be used for practical purposes.

# Chapter 4

# Modelling of Cooperative Autonomous Driving Scenarios

In this chapter, the modelling of the intersection scenario is tackled, where we will the intersection modelling, the vehicle motion models,in addition to the planning of the initial trajectories, Finally, the collision avoidance constraints and both the vehicle and the Intersection Management Model Predictive Controllers will be presented.

## 4.1 The Intersection and the Intersection Manager

The intersection is a highly regulated area with vehicles following a set of rules and patterns for crossing, in this section we will define the intersection space, the zones inside the intersection space.Finally, we will introduce the Intersection Manager (IM) its various roles throughout the thesis and the different crossing rules that dictate the right of passage for vehicles.

### 4.1.1 Intersection Area Modelling

The main emphasis of this thesis will be on Non-signalized four way intersections where the space considered encompasses the Conflict Area (the central

zone) in addition to a 100 meter radius. We are considering the single lane scenario, but the multiple lane scenario is also feasible. Each lane has a width of 3.7 meters in each direction as per [72].
The intersection is made up of lanes and conflict zones, each properly labelled as can be seen in Figure 4.1.The lane labels allow the intersection manager to be able to place each vehicle in its corresponding location. Conflict Zones signify areas in the intersection in which collision may occur, they are labeled for time suggestion purposes as will be elaborated on later.
Finally the intersection is divided into two 'Zones':

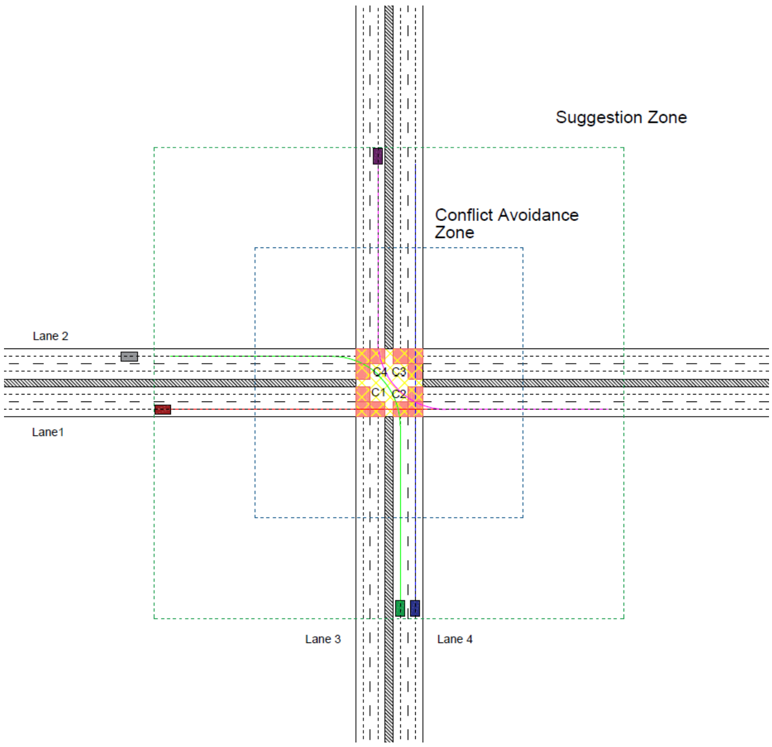


*Figure 4.1: Schematic of modelled intersection space*

- The Suggestion Zone: Upon entry to the intersection area and up to 50m away from the the center of the intersection. When the Intersection Manager is functioning as negotiator, the vehicles receive time suggestions for entering the intersection area while in this zone.

- The Conflict Avoidance Zone: It is the area within 50m of the center of the intersection, the Intersection Manager's role in this zone is to simply relay information (Distances) between vehicles.

### 4.1.2 The Intersection Manager

As mentioned in Chapter 1, the vehicle communication is either be in the form of Vehicle to Vehicle (V2V) or Vehicle to Infrastructure (V2I). In this thesis we will be focusing on the Vehicle to infrastructure mode of communication. The node in the system that represents the infrastructure is called the *'Intersection Manager'* and its role varies on the type of control algorithm implemented.

- For the completely decentralized algorithm, the main role of the intersection manager is to create the priority list which dictates the crossing order in addition to relaying the information received from the various vehicles

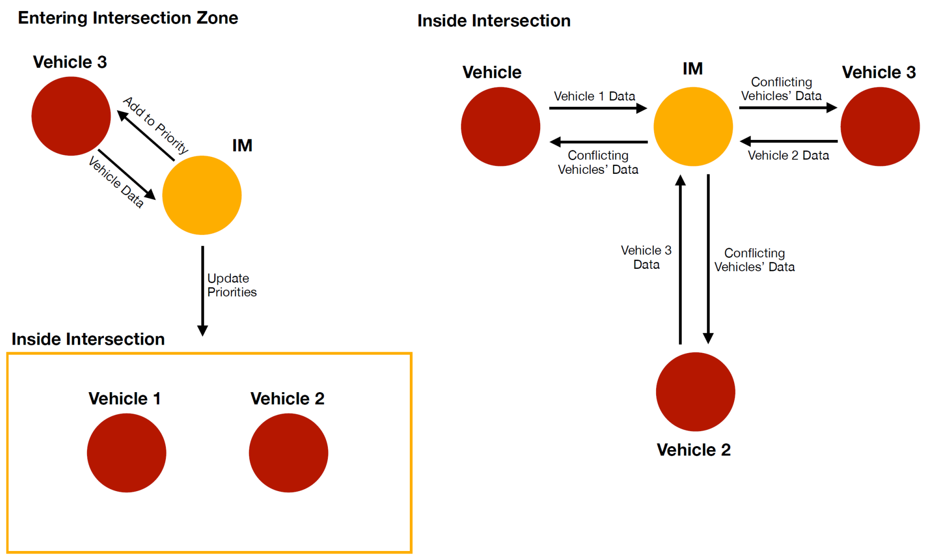


*Figure 4.2: Schematic showing role of Intersection Manager in Decentralized approach*

- In the hybrid algorithm which is influenced by reservation based algorithms, the IM in addition to its previous role has the task of suggesting to the vehicles upon their entry to the intersection space the time of arrival to the conflict zones. It can be seen as a node which receives proposed times of arrival and based on the priority scheme selected for deciding crossing order and the safety time, modifies the request and suggests a new time to the vehicle to ensure safe passage.

The intersection manager thus at all times has as available information the ones shown in the figure 4.3, where each is a data structure which groups vehicles based on:

- Vehicles which will pass through each conflict point
- Vehicles in each intersection zone
- Vehicles on each lane
- All Vehicles in the intersection space

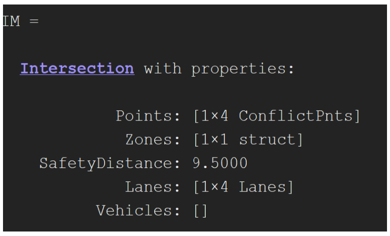


*Figure 4.3: IM stored Information*

### 4.1.3 Priority Schemes

As mentioned earlier, various heuristics are implemented in order to obtain the vehicle priorities while passing the intersection space, below is a summary of the main heuristics implemented in the literature. In this thesis the intersection manager uses variants of the First Come First serve and the Time To React schemes to determine the crossing order.

#### Overpass

This is the benchmark scenario where no coordinating action is taken and is equivalent to the separation of roads. However due to the figures discussed in the introduction, implementing such infrastructure is highly expensive and unlikely.

#### Traffic Light

A fixed-cycle traffic light controller serves as a surrogate for an actual traffic light. It is mainly used in cases with human or non-cooperative autonomous vehicles and forms the second benchmark. Red/Green light alternate every Cycle Time $T_c$. It is assumed however that each vehicle knows the trajectory of the vehicles ahead of it in the lane. This approach can be extended to engulf the scenario with both autonomous and legacy vehicles, where Instead of physical lights, Traffic regulations in regards to right of passage order priority when a legacy driver in cases where a legacy driver is involved (ie. Left turning vehicles concede to straight traveling vehicles, first come first serve at intersection space etc.) An assumption is made in this case that the position of the legacy vehicle is known at all time instances (T) which is a strong assumption. It however becomes a soft one in case the Legacy vehicle is switched to a non-cooperative autonomous vehicle. Such a scenario would be a good gauge for any alternative priority scheme in comparison to the current situation in non-traffic light managed intersections.

#### Parallel/Sequential

A priority ranking of the variables is decided, The solution is then obtained by solving a single optimization problem per vehicle, where conditions are employed to avoid collision with higher priority vehicles. The priority ranking is usually based on a heuristic such as first come first serve which will be discussed later. In sequential schemes,the vehicles compute their solutions in sequence following a decision order which implicitly reflects priority. Parallel schemes on the other hand use data provided by higher priority vehicles at the last iteration in order to produce the current time control input. One main argument against this approach is that it offers a "greedy solution" where no vehicle makes a decision that improves the intersection scenario at its own cost. As a consequence, the effort required to resolve difficult conflict is pushed to the lowest priority vehicle.

#### First Come First Serve/Fixed Order

This is a formulation of the parallel approach mentioned above where each vehicle solves its own MPC problem conceding position to all vehicles already inside the intersection zone. Thus the latest vehicle has to give way to all others already inside the intersection. A clear and coherent representation of the algorithm is present in [75]. Including Traffic light incorporation to include legacy drivers.

### Time To React

This is a variant of the fist come first serve approach mentioned above that assigns a quantity $t_{react}$ to each vehicle which is calculated for each vehicle i as:

$$t^i_{react} = \frac{distance \quad to \quad collision \quad point}{average \quad velocity \quad until \quad collision \quad point} \tag{4.1}$$

For each two conflicting vehicles then, the one with more time to react concedes passage to the one which has more time to react, and thus obtaining a lower position on the priority list. In our thesis, each vehicle then has its own priority list which is a binary array. For a vehicle i in the scenario:

- A priority value of 1 for the other vehicle ID means that vehicle i should concede passage
- A priority value of 0 means that vehicle i does not consider the other vehicle in its decision process

## 4.2 Initial Trajectories and Motion Model

Non-signalized intersections traditionally rely on the interaction between drivers through eye contact for safe passage, due to the lack of controlling facilities. Through inter-vehicular communication, driver interactions are easier and more accurate providing information of vehicles that may be out of the line of sight or blocked. Moreover, an intersection is highly regulated with vehicles generally following set patterns for crossing ( travel routes, regardless of signal existence). Vehicles from different directions with separate trajectories can be seen to follow "predefined routes" as shown in figure 4.1 [33].

This assumption is always used in literature, which mainly focuses on straight trajectories. In this thesis, turning agents will be considered as well. Possible collisions can thus be identified by the intersection manager

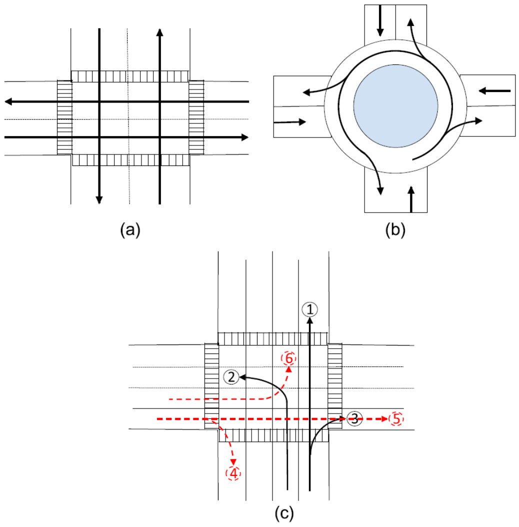


*Figure 4.4: Illustration of possible paths taken while passing an intersection [33]*

through a knowledge of initial and target lanes in addition to expected time of arrival to the intersection. Moreover, due to the critically of the space it is assumed without loss of generality, that risky driving behavior is not present in these scenarios (overtaking, lane changing) and that before and after the intersection area, typical collision avoidance techniques are used (Adaptive Cruise Control), which allows us to limit the number of vehicles to four when simulating the scenario.

### 4.2.1 Initial Pre-planned Trajectories

As mentioned above, the initial trajectories for crossing the intersection can be considered pre-defined in the intersection space. For generating these trajectories, the "*Trajectory and Scenario Generation*" feature of Matlab's Automated driving toolbox [81] was used.

#### Vehicles and Vehicle Trajectories

Each Vehicle has dimensions as reported in the table 4.1.Its pre-planned trajectory is specified through specifying way-points to the MATLAB function trajectory() which generates clothoid based continuous trajectories and allows the specification of velocity, and orientation at each way-point[82].This allows for the generation of a reference velocity profile for trajectory tracking application. An example of the way-points and the generated speed reference can be seen in Figures 4.5 and 4.6

| | |
|---|---|
| Vehicle Length | 4.7 meters |
| Vehicle Width | 1.8 meters |
| Vehicle Height | 1.4 meters |

*Table 4.1: Table of Vehicle Properties*

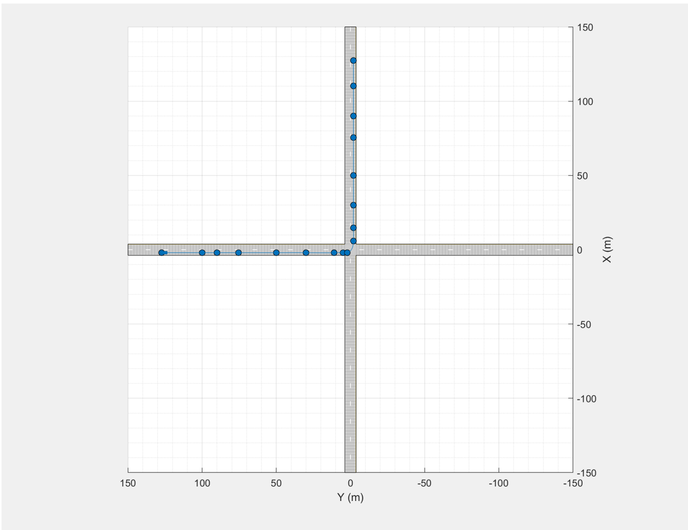


*Figure 4.5: Way-points specified for Left-Turning Vehicle*

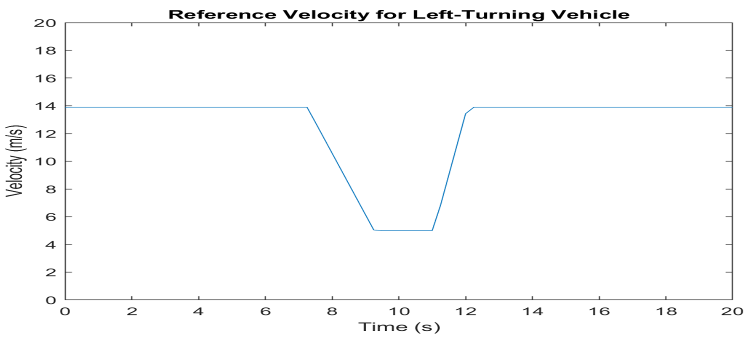


*Figure 4.6: Way-points specified for Left-Turning Vehicle*

### 4.2.2 Motion Models

There are various motion models to describe the motion of ground vehicles, with varying accuracy. The most detailed ones combine longitudinal and lateral motion with power-train models, suspensions and tires which prove to be accurate under various conditions [22, 23]

The intersection problem however, does not require such level of modelling as intersections in general are not placed in roads of high curvature and out of the ordinary driving behavior or overtaking are not common occurrences. Therefore, lateral vehicle dynamics are of less importance in this context and thus in both this thesis, and the majority of other works on the topic, the "Vehicles on Rails" Assumption is employed without loss of generality.

**Assumption 4.1:** (Vehicles on Rails) *Vehicles inside the intersection space move along pre-defined trajectories and do not change lanes. All lateral tracking is handled by a low level controller.*

To this extent, various models are used in the literature. We will begin by recalling them then highlighting the one used in this thesis and motivating our choice.

#### General Model

Based on the assumption stated above, each vehicle's state $x_i(t) : \mathbb{R} \to \mathbb{R}^{n_i}$ in the intersection scenario is then seen as $x_i(t) = [p_i(t), v_i(t), \tilde{x}_i(t)]$ . Here $p_i(t)$ can be considered as the position of the vehicle i's center of mass along its corresponding pre-planned trajectory, $v_i(t)$ is the **scalar** velocity along the path and $\tilde{x_i}(t)$ can be seen as the state which can be seen to describe either the acceleration or the power train dynamics. Therefore, using the control input $u_i(t) : \mathbb{R} \to \mathbb{R}^{m_i}$, the vehicle's motion can be described by a set of ordinary differential equations.

$$\dot{x_i}(t) = f_i(x_i(t), u_i(t)), \tag{4.2}$$

$$h_i(x_i(t), u_i(t)) \leq 0 \tag{4.3}$$

Where both $f_i(t) : \mathbb{R}^{nxm} \to \mathbb{R}^{n_i}$ and $h_i(t) : \mathbb{R}^{nxm} \to \mathbb{R}^{n_i}$ are continuously differentiable functions describing the states and the constraints on the states ( actuation, velocity). Where in this thesis, as in all of the works on the topic the velocities are constrained to be positive (ie. no reverse motion).

### Kinematic Model

A famous model which can often be found in the literature is the double integrator model with input bounds. This only considers the kinematics and models the vehicle through series of integrators where the scalar input $u_i(t)$ enters as the end. such that

$$F_i(x_{i,k}, u_{i,k}, \Delta t) = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} x_{i,k} + \begin{bmatrix} 1/2\Delta t^2 \\ \Delta t \end{bmatrix} u_{i,k}, \qquad a_i^{min} \leq a_i \leq a_i^{max} \quad v_i(t) \geq 0 \quad \forall t \tag{4.4}$$

### Dynamic Models

A minor adjustment allows us to model different vehicle behaviors and thus consider heterogeneous networks by varying the actuation dynamics taking into account the vehicles' dynamic ability to accelerate and decelerate. This is introduced by the following model.

$$\dot{x}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & c_i \\ 0 & -d_i & -a_i \end{bmatrix} x_{i,k} + \begin{bmatrix} 0 \\ 0 \\ b_i \end{bmatrix} u_{i,k}, \tag{4.5}$$

with $a_i, b_i, c_i > 0$ and $d_i \geq 0$ for $i = 1...N$ where N denotes the number of vehicles. The first two state variables can be considered to be the position and the velocity while the third state component can be seen as the actuator state where $a_i$ can be seen to characterize the speed of actuation. The constant $d_i$ in the cases where it is not zero, signifies that the actuator is influenced by the velocity, thus creating an internal feedback loop. It is also assumed that the velocities are the only output variables, where $C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$ Here, the lower the time constant $T_a$, the faster the actuation dynamics.Finally,in order to be utilized in a model predictive control setting, the model is discretized using a Zero-Order Hold technique in order to obtain.

$$\begin{cases} \dot{x}_{i,k+1} = A_{i,k}x_{i,k} + B_{i,k}u_{i,k} \\ y_{i,k} = C_{i,k}x_{i,k} \end{cases} \tag{4.6}$$

By employing this model, distinction between vehicles based on actuation times can be done without delving into power train dynamics which are not important for this application.

### Electric Vehicle Power Train Dynamics

Electric Vehicles are possibly the future and thus vehicle power trains can be considered. For EVs the moving force for the vehicle is a function of the total gear ratio and the torque generated by the electric motor. The equations of motion can thus be written as

$$\dot{p}(t) = v \quad \dot{v}(t) = \frac{1}{m}[F^c(u(t), x(t)) + F^r(u(t), x(t)]$$

and

$$F^c(u(t), x(t)) = G\tau(x(t), u(t));$$

is the propulsion force mentioned above, while G is the gear ratio and tau is the supplied motor torque. While $F^r(u(t), x(t))$ are the restrictive external forces such as friction. However, the power train dynamics can be assumed to be fast enough to be ignored leading to similar system to the dynamic model discussed before[74]

In this thesis we chose to employ the dynamic model, neglecting the effect of velocity on actuation speed. The model of vehicle k's dynamics used is the following:

$$\dot{x}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/T_{a_k} \end{bmatrix} x_{i,k} + \begin{bmatrix} 0 \\ 0 \\ 1/T_{a_k} \end{bmatrix} u_{i,k}, \tag{4.7}$$

with the state vector $x = [p(t), v(t), a(t)]$ where $p(t)$ can be though of as the position along the trajectory of vehicle k,and $v(t), a(t)$ are respectively. the velocity and acceleration along the local trajectory. The constant $T_{a_k}$ is used to summarize each vehicle's drive-train dynamics reflecting each vehicle's unique ability to accelerate and decelerate. The velocity is the system's only output variable and in order to be used in a model predictive framework, the continuous time model above can be discretized using zero-order hold techniques to obtain the Linear time invariant system of form:

$$\begin{cases} x_{i+1} = Ax_i + Bu_i \\ y_i = Cx_i \end{cases}$$

This knowledge is then inputted into an initial trajectory planner which uses the reference velocity generated using the way-points in addition to the system dynamics and initial position to generate the local planned trajectory. Each vehicle then has the information shown in Figure 4.7.Please note that the first center of the first encountered conflict zone is the origin for the local trajectory. This is done for simplicity. Moreover the vehicle would transmit only a subset of the available information.

```
Vehicle with properties:

                          ID: 1
                  Velocities: [100×1 double]
                 PlannedTraj: [100×1 double]
                    Priority: []
               CurrentLaneID: 3
           CollisionPointsID: [1 3]
             CollsionPointsx: [0 2.6163]
  DistancesToConflictPoints: [20×2 double]
                TargetLaneID: 3
                CurrentState: [3×1 double]
                PlannedEntry: [41 43]
                      INSIDE: 4
                 Controllers: [1×1 struct]
                          Ta: 0.3000
        CrossedIntersection: 1
                 ConflictVID: []
                        Type: 0
                  Speedlimit: 13.9000
```

*Figure 4.7: Local Vehicle Information*

- **To the intersection Manager:** The data transmitted to the intersection Manager is: [Current Lane ID, Average Velocity, Collision Points ID, Target Lane ID, Planned Entry]
- **To other Vehicles:** Distances to Common conflict zone as predicted throughout the prediction horizon.

This is done in order to minimize the amount of information shared between vehicles in order to decrease the strain on the network and to minimize the effect of potential cyber attacks ( No vehicle knows the other vehicles' positions).

### Optimal Control Formulation of Initial Trajectory Planning

As mentioned earlier, in order for the vehicle to plan the initial trajectory and obtain information such as the Planned Time of Entry, an optimal control formulation of the reference speed tracking is implemented in order to obtain the trajectory in local coordinates. The problem is stated as the classical Boltza formulation:

$$\underset{u_i}{\text{minimize}} \quad \frac{1}{2}P(x(t_f) - x_{f_{ref}})^2 + \frac{1}{2}\int_{t_0}^{t_f} Q(v(t) - v_{ref}(t))^2 + Ru(t)^2 dt \tag{4.8a}$$

subject to

$$\dot{x} = Ax + Bu, \tag{4.8b}$$

$$u_{min} \leq u(t) \leq u_{max} \tag{4.8c}$$

Where P is the terminal gain matrix, Q is the reference tracking matrix and R is the control gain matrix. A and B are the state matricies mentioned above and the actuation constraints are $u_{min} = -5m/s^2$ and $u_{max} = 5m/s^2$. For a more detailed analysis of the choice of actuation constraints please refer to Appendix A.

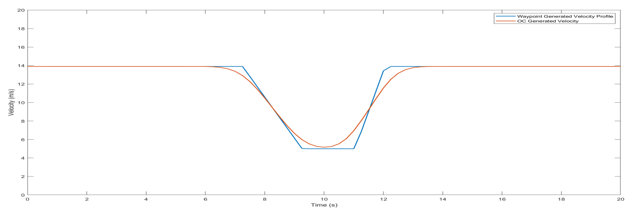

*Figure 4.8: Comparison between Waypoint Generated Velocity and locally Planned Trajectories*

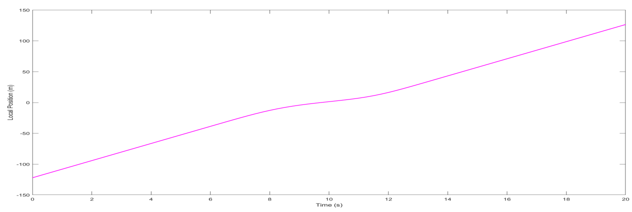

*Figure 4.9: Evolution of Local Trajectory for a Turning Vehicle*

## 4.3 Decentralized Problem Formulation

As previously mentioned, the scenario mainly considers a single four-way intersection with the following simplifying assumptions:

- All vehicles are fully autonomous (disregard of legacy drivers)
- At most one vehicle is approaching from each direction
- All vehicles are equipped with the communication capabilities
- Data that is sent at time k is available to at instant k+1

We then define the set of agents in the intersection space as $\mathcal{A} = \{1, ...., N_A\}$ each having a model as described in the earlier section and equipped with its own trajectory tracking MPC. As a first consideration we need to ensure that every agent $i \in \mathcal{A}$ passes the intersection safely ( by respecting the safety distance set by the intersection manager).To accomplish that, we define the subset $\mathcal{A}_c^i \subset \mathcal{A}$ including all vehicles with whom there might be a chance of conflict.

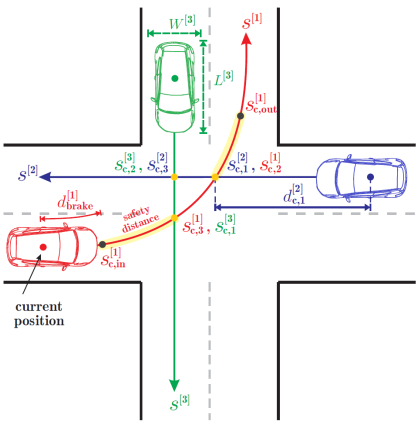


*Figure 4.10: Schematic Describing the scenario [72]*

$$\mathcal{A}_c^i = \{ \, l \mid l \neq i \wedge s^i_{c,l} \neq \infty, \} \tag{4.9}$$

we thus define the value of the coordinate $s^i_{c,l}$ which is the position along the trajectory of the center of the conflict zone between agent l and agent i.In case no such conflict zone exists, we define this quantity as $s^i_{c,l} = \infty$ We then define the distance between any two vehicles [ i , l ] in the intersection area as:

$$d_l = \begin{cases} \mid s^i - s^i_{c,l} \mid + \mid s^l - s^l_{c,i} \mid, & s^l_{c,i}, s^i_{c,l} \neq \infty \\ \infty \quad otherwise \end{cases} \tag{4.10}$$

This is equivalent to stating that the distance between the agents is the sum of their absolute distances to the shared conflict zone or infinite in case there isn't a shared conflict zone. The collision avoidance constraint can then be expressed as:

$$d^i_{l,(k+j)|k} \geq d_{safety,(k+j)|k} \tag{4.11}$$

### 4.3.1 Local Vehicular MPC problem

Each vehicle uses a discrete-time private vehicle model to solve its constrained trajectory tracking MPC problem:

$$\underset{u_i}{\text{minimize}} \quad \sum_{j=0}^{H_p} \delta_i^{-j} Q_i (v^i_{ref,[k+j|k]} - v^i_{[k+j|k]})^2 + S_i \Delta u^{i,2}_{[k+j|k]} + R_i u^{i,2}_{[k+j|k]} \tag{4.12a}$$

subject to

$$x_i(j+1) = f_i(x_i(j), u_i(j)), \tag{4.12b}$$

$$u_i \in \mathbb{U}_i, , \tag{4.12c}$$

$$x_i \in \chi_i, , \tag{4.12d}$$

$$d^i_{l,(k+j)|k} \geq d_{safety,(k+j)|k} \quad \forall l \in \mathcal{A}^i_c, \tag{4.12e}$$

$$\frac{1}{H_p+1} \sum_{j=0}^{j=H_P} v^i_{k+j|k} \geq \bar{v}_{mean} \quad \forall s_i \in [d_{brake}, s_{i,out}] \tag{4.12f}$$

The terms Q,R,and S penalize deviating from the reference, the control effort, and the jump in control effort respectively. The exponential term that accompanies the reference tracking can be seen as an aggressiveness estimator. This term is often used in multi-agent negotiation scenarios where $\delta$ is a number $\in [0, 1]$. The closer $\delta$ is to 1 the more farsighted the agent is where if $\delta$ is close 0 the agent would only want to follow the reference for the short term and deviation from the reference in future instances is penalized less.The subscripts 'i' for the gains signify that vehicles can have different gain values.This is another way in which different vehicles and driver behaviors is simulated in the thesis.

The first constraint is the system dynamics, the second constraints limits the admissible accelerations to $-9m/s^2 \leq u \leq 5m/s^2$.These values of accelerations where chosen to permit us to achieve good braking performance without touching on the limits of stability. For a review of slip dynamics and braking we refer the reader to [89]. The third constraint is on the states, specifically on the velocity where the car is to only drive forwards and to always drive at a velocity lower than the speed limit.The last constraint is for feasibility purposes and it ensures that upon reaching the conflict area the prediction horizon covers the vehicle's crossing of the conflict area. This is to ensure that the safety constraint is satisfied throughout the vehicle's crossing of the conflict area. $\bar{v}_{mean}$ is the mean velocity that ensures the vehicle's crossing within the prediction horizon and is computed by dividing the remaining distance until the exit of the conflict zone by the prediction horizon:

$$\bar{v}_{mean} = \frac{\mid s_i - s_{i,out} \mid}{H_p} \quad \forall s_i \in [d_{brake}, s_{i,out}]$$

- $d_{brake}$ is the distance needed for the vehicle to come to a complete stop before entering the intersection from its reference velocity.

- $c_{i,out}$ marks the exit from the conflict area and it is when the vehicle is at a distance equal to the safety distance away from the last conflict zone center.

**Remark:** Please note that in this thesis, we assume that priority entails right of passage and thus the speed limit is set to 110% of the reference velocity at each time instant. This approach has been sometimes called 'greedy' in the literature when centralized approaches have been employed. However as mentioned in Chapter 2, finding an optimal crossing order centrally allowing for greater vehicle accelerations is a NP hard problem.

### 4.3.2 Convex Relaxation of Collision Avoidance Constraint

The collision Avoidance problem for each vehicle can be stated simply as: *Each vehicle should ensure that it passes within a specified safety distance of vehicles that are of higher priority.* Vehicles thus don't have to worry about vehicles with lower priority, which decreases the computational effort on each vehicle but leads to the solution being sub-optimal in the sense that vehicles don't accelerate above their reference velocities in the intersection zone, but rather only concede passage.This however is a correct portrayal of safe driving as seen from the human driving behavior and is thus a correct modelling of the situation.

The collision avoidance constraint as stated before is:

$$d^i_{l,(k+j)|k} \geq d_{safety,(k+j)|k} \quad \forall l \in \mathcal{A}^i_c$$

by introducing the definition of $d_{l,(k+j)}$ the constraint can be reformulated to become:

$$\mid s^i - s^i_{c,l} \mid + \mid s^l_{(k+j)|k} - s^l_{c,i} \mid \geq d_{safety,(k+j)|k} \quad \forall l \in \mathcal{A}^i_c \tag{4.13}$$

This constraint can be then be separated into a part that is strictly depending on local vehicle information and another that is transmitted to the vehicle.

$$\mid s^i - s^i_{c,l} \mid \geq d_{safety,(k+j)|k} - \mid s^l_{(k+j)|k} - s^l_{c,i} \mid \quad \forall l \in \mathcal{A}^i_c \tag{4.14}$$

For this constraint to be efficiently solved using a numeric solver, there are two approaches according to [83]

1. Replace every absolute value constraint with a mixed-integer linear constraint.

2. Transform every absolute value constraint into a quadratic one.

Each approach has its pros and cons, however in order not to increase the number of optimization variables we chose transform the constraint to a quadratic one. This however imposes a limit to the region in which the constraint can be used. We see then that the constraint becomes:

$$\begin{aligned} (s^i - s^i_{c,l})^2 \geq (d_{safety,(k+j)|k} - d^l_{c,i(k+j)|k})^2 \\ \forall l \in \mathcal{A}^i_c : d_{safety,(k+j)|k} \geq d^l_{c,i(k+j)|k} \end{aligned} \tag{4.15}$$

One shortcoming of this approach is that it is only active when the other vehicle is within the safety distance of the collision point at any point in its prediction horizon. This might lead to complications in a priority scheme such as FCFS where the constraint becomes active too late, this can be seen if a vehicle i enters after j and thus has to concede entry, vehicle i however enters with a higher velocity and reaches the conflict area before vehicle j. The constraint here is activated too late and in some instances collision can't be avoided . In the time to react priority scheme this is not a problem as the vehicle with a higher priority arrives within the activation zone prior to the one that is conceding position.This is addressed and corrected in our time suggestion control algorithm that will be elaborated on later. We thus obtained a non-convex quadratically constrained quadratic program formulation of our MPC problem. We will now perform a semi-definite relaxation to obtain a convex semi-definite program formulation of the same problem.

**Semi-Definite Relaxation**

The reference tracking linear MPC problem with a prediction horizon of N steps, can be reformulated as seen in Chapter 3, to become of the form:

$$\underset{u_i}{\text{minimize}} \quad \frac{1}{2}\mathbf{u}^T(S_yQ_yS_y^T + R)\mathbf{u} + (x_0^T T_y - \mathbf{R})Q_yS_y\mathbf{u} \tag{4.16a}$$

subject to

$$G\mathbf{u} \leq \mathbf{W} + Sx_0, \tag{4.16b}$$

$$d^i_{l,(k+j)|k} \geq d_{safety,(k+j)|k} \quad \forall l \in \mathcal{A}^i_c \tag{4.16c}$$

Where $\mathbf{u}$ is the control action $\mathbf{R}$ is the reference and Y the output (velocity) we have:

$$T_y = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^N \end{bmatrix}$$

$$S_y = \begin{bmatrix} CB & 0 & 0 & 0 & \dots & 0 \\ CAB & CB & 0 & \dots & 0 & \\ \vdots & \vdots & \ddots & \vdots & 0 & \\ CA^{N-1}B & CA^{N-2}B & CA^{N-3}B & \dots & CB & \end{bmatrix}$$

$$G = \begin{bmatrix} S_y \\ -S_y \\ I \\ -I \end{bmatrix}$$

$$W = \begin{bmatrix} Y_{max} \\ -Y_{min} \\ U_{max} \\ -U_{min} \end{bmatrix}$$

Where A,B,and C are the LTI matrices from the state space model.If the avoidance constraint at an instant in the horizon is formulated to have the form:

$$0.5x^T P_k x + q_k^T x + r_k^T r_k \leq 0 \tag{4.17}$$

while the states can be written in terms of the initial time and the control as:

$$x = \bar{T}x_0 + \bar{S}u \tag{4.18}$$

Then the constraint can be reformulated to become

$$\begin{aligned} 0.5(\bar{T}x_0 + \bar{S}u)^T P_k(\bar{T}x_0 + \bar{S}u) + q_k^T(\bar{T}x_0 + \bar{S}u) + r_k^T r_k \leq 0 \\ 0.5u^T \bar{P}_k u + (x_0^T \bar{T}^T P_k + q_k^T)\bar{S}u + 0.5(x_0^T \bar{H}_k x_0) + q_k^T \bar{T}x_0 + r_k^T r_k \leq 0 \\ where \quad \bar{P}_k = \bar{S}^T P_k \bar{S} \quad \bar{H}_k = \bar{T} P_k \bar{T} \end{aligned} \tag{4.19}$$

Moreover, in order to make the problem convex the semi-definite relaxation is applied to obtain:

$$\begin{aligned} 0.5Tr(\bar{P}_k U) + \bar{q}_k^T u + \bar{r}_k^T \bar{r}_k \leq 0 \\ s.t \quad Schur = \begin{bmatrix} U & u' \\ u & 1 \end{bmatrix} \succcurlyeq 0 \end{aligned} \tag{4.20}$$

Where $\bar{r}$ includes all the constant terms (with those relating to the initial condition) and $\bar{T}$,$\bar{S}$ are equivalent to $T_y$,$S_y$ without the elements being multiplied by the output matrix C. This solves the original problem in the case $Rank(Schur) = 1$.However, in the case where this criterion is not satisfied, which means that a sub-optimal or even infeasible solution of the original problem is obtained, We employ a randomization algorithm based in order to obtain a good estimate of the optimal solution.Evaluating the rank numerically is computationally heavy, which is why we consider that the rank-1 condition is satisfied if the second largest singular value $\sigma_2(Schur) \leq \sigma_{2,threshold}$ The problem thus becomes of the form of a semi-definite program:

$$\underset{u_i}{\text{minimize}} \quad \frac{1}{2}Trace(\mathbf{U}(S_y Q_y S_y^T + R)) + (x_0^T T_y - \mathbf{R})Q_y S_y \mathbf{u} \tag{4.21a}$$

subject to

$$G\mathbf{u} \leq \mathbf{W} + Sx_0, \tag{4.21b}$$

$$0.5Tr(\bar{P}_k U) + \bar{q}_k^T u + \bar{r}_k^T \bar{r}_k \leq 0 \quad \forall l \in \mathcal{A}_c^i : d_{safety,(k+j)|k} \geq d_{c,i(k+j)|k}^l, \tag{4.21c}$$

$$\frac{1}{H_p} \sum (T_y x_0 + S_y \mathbf{U}) > v_{mean} \tag{4.21d}$$

**Randomization Algorithm**

We consider the solutions $u^*$ and $U^*$ of the previous relaxed program and define as a covariance the matrix $U^* - u^* u^{*T}$. Then, by drawing random samples from the random variable with a Normal distribution $\tilde{u} \in \mathcal{N}(u^*, U - u^* u^{*T})$ we would be solving the original problem on average over this distribution [72]. In order for the random variable picked to be accepted, it has to also satisfy the various constraints. In the randomization algorithm and to loosen the feasibility set in order to decrease the number of random samples picked by softening the constraints on safety distance and maximum velocity, allocating respective costs for their violation as

$$J_{viol} = k_1 * \sum_{n=1}^{H_p} \epsilon_{ds} + k_2 * \sum_{n=1}^{H_p} \epsilon_v$$

The costs $k_1$ and $k_2$ can be seen as a measure of how much violation over the interval is deemed acceptable. The values used are $k_1 = 10^6, \quad k_2 = 10^3$. Where $\epsilon_{ds}$ is the violation of the safety distance at each sampling instant ( zero if not violated) and $\epsilon v$ is the violation of the maximum velocity (zero if not violated).
The randomization algorithm is then as follows:

**Algorithm 1:** Randomization Algorithm to estimate optimal solution

**Result:** $\tilde{u}_{(.|k)}{}^{*}$, $\tilde{c}_{min}$

$\tilde{u}_{(.|k)}{}^{*} = u_{(.|k)}{}^{*}$ , $\tilde{c}_{min}$ = cost$(x_0, u_{(.|k)}{}^{*})$; $\longleftarrow$ Initialize ;

**for** $n$ = *1 to* $N_{samples}$ **do**

  $\tilde{u}_{(.|k)} = mvnrnd(\mathcal{N}(u^{*}, U - u^{*}u^{*T}))$ $\longleftarrow$ Pick random sample

  **if** *Constraints are satisfied* **then**

    $\tilde{c}$ =cost$(x_0, \tilde{u}_{(.|k)})$ $\longleftarrow$ Calculate sample cost;

    **if** $\tilde{c} \leq \tilde{c}_{min}$ **then**

      $\tilde{c}_{min} = \tilde{c}, \tilde{u}_{(.|k)}{}^{*} = \tilde{u}_{(.|k)}$ $\longleftarrow$ Assign new optimal values;

    **end**

  **end**

**end**

**Function** `cost(`$x_0$, $u_{(.|k)}{}^{*}$`):`

  $\tilde{c} = J(x_0,\ u_{(.|k)}{}^{*}) + J_{viol}(x_0,\ u_{(.|k)}{}^{*})$ $\longleftarrow$ Evaluate cost

  **return** $\tilde{c}$

This control algorithm is the starting point of the thesis, were we will demonstrate its ability to guarantee safe passage for various number of vehicles, considering different scenarios. However, to address its shortcomings we also introduce the hybrid algorithm which makes use of the Intersection Manager in order to suggest times to arriving vehicles.

## 4.4 The Hybrid Algorithm: Time Suggestion

The completely decentralized approach solves the problem at hand under a specific framework and priority scheme.However, we have noticed that there is room of improvement, especially if the Intersection Manager is better utilized, while keeping the framework decentralized. This comes in order to improve the following aspects:

1. The completely decentralized approach is triggered when the higher priority vehicle is within a safety distance from the conflict zone during the prediction horizon. This does not pose a problem when a safety scheme such as the Time to React is employed, since it assures that the higher priority vehicle arrives in the activation space before the lower

priority vehicle. In schemes such as the first come first serve, this is not taken into consideration, and the decentralized approach might be triggered too late that no feasible solution can be found. Having a way that assures the feasibility of the solution is thus needed in such a high risk environment under all priority schemes.

2. Even in the Time to React scheme when a feasible solution of possible, due to the late activation of the safety constraint a high jump in the control action is expected,having a way that would decrease this jump is encouraged for comfort and fuel consumption purposes.

We thus introduce a way to use the intersection manager without increasing the strain on the network. In [], a negotiation algorithm between the vehicles through the intersection manager is implemented in order to arrive at times of arrival to the intersection space that would assure safe passage. This is highly similar to the reservation based algorithms employed early in the literature, it however puts a large strain on the network through requiring the continuous communication between vehicles until a solution is reached. Instead we implement a similar approach while taking advantage of the presence of the collision avoidance algorithm that would be triggered close to the conflict area. The algorithm elaborated below, is triggered whenever a new vehicle enters the intersection space, or more specifically, the 'Time Suggestion Zone'.

**Algorithm 2:** Time Suggestion upon Vehicle Entry Algorithm

**Entering Vehicle**: send entry data;
**Intersection Manager**
Get Conflict Points;
Form conflict set $\mathcal{A}_c$;
Calculate Priorities Request Data from Vehicles in 'Time Suggestion zone';
Calculate time suggestions for vehicles 'Time Suggestion zone';
Broadcast suggested times to vehicles 'Time Suggestion zone';
**Vehicles in 'Time Suggestion Zone'**
Re-plan Trajectories based on local MPCs;

The data sent by the entering vehicle is:

- Current Position/Average Velocity based on priority scheme.
- Lane information ( Current lane, Target Lane).
- Planned time of arrival to conflict zones.

Based on this information, the intersection manager creates the set of vehicles with who there might be conflict, it then calculates the priority list. Based on the priority list and the available information received from the vehicles in the 'Time Suggestion Zone' the Intersection manager then calculates the new suggested times of arrival to the conflict zones and broadcasts this information to the vehicles who re-plan their trajectories based on their local MPCs.

### 4.4.1 Calculation of Suggested Times:

In order to improve the performance and the system and taking into considerations that vehicles enter the intersection zone sequentially, a method that borrows from the spot reservation technique is introduced to suggest to the vehicles the time at which they should enter the intersection space. The Intersection Manager formulates the problem of finding the crossing times as a quadratic program.

$$\underset{t_{ref}}{\text{minimize}} \quad (t_{ref} - t_{sug})^T Q (t_{ref} - t_{sug}) + c^T (t_{ref}) \tag{4.22a}$$

subject to

$$t_{ref,i} + t_{s,i+1} \leq t_{ref,i+1} \tag{4.22b}$$

Here Q weighs the variation between the time presented by the vehicle for the crossing and that suggested by the IM.$t_{ref}$ is the time that will be suggested by the IM to the vehicles while $t_{sug}$ is the already planned time sent by the vehicles to the IM. The constraint makes sure that a safety time is kept between the arrival of two vehicles to the same conflict zone. The safety time is a property of the system and is not based on the vehicle preference. Since the intersection manager plays the role of traffic regulator, it has the freedom of adjusting the safety time depending on the level of traffic where it can be increased in case of light traffic and decreased in case of congestion. The safety time is calculated keeping in mind the safety distance and each vehicles average predicted speed ($\bar{v}^k$) as:

$$t_{s,k} = \frac{d_{safety}}{\bar{v}^k} \tag{4.23}$$

### 4.4.2 Trajectory Re-Planning

The suggested time is then sent to each vehicle which re-plans its trajectory according to the suggestions of the manager through what we call the re-planning Optimal Control problem which can be formulated as:

$$\text{minimize} \quad \frac{1}{2}(x(t_{sug}) - x_c)^T P_i(x(t_{sug}) - x_c) + \frac{1}{2}\int_{t_i}^{t_f} Q\Delta V + R\Delta U \tag{4.24a}$$

subject to

$$u_{min} \leq u \leq u_{max}, \tag{4.24b}$$

$$0 \leq v_i \leq speedlimit \tag{4.24c}$$

The terminal cost introduced to the Planning Algorithm assures that the trajectory is modified to take into account the time suggestion of the intersection manager. The terminal cost weighing matrix is a function of the variation between the previously planned time and the one that is suggested, in this way the more the vehicle is close to collision, the more it will work to follow the suggestion of the intersection manager.Where P is updated at each round of suggestions to be

$$P_i(\kappa) = \epsilon_i |t_{ref,i} - t_{sug,i}| \tag{4.25}$$

The choice of the constant $\epsilon_i$ depends on the priority scheme being followed in addition to the aggressiveness of the vehicle. For example, if the priority scheme employed is First-Come First-Serve, then the value of epsilon is predicted to be larger than that when a Time to React Priority Scheme is employed. This way, the Intersection Manager plays the role of regulating traffic with minimal computation done centrally.

# Chapter 5

# Discussion of Results

In this chapter, we will present and discuss the results of the coordination algorithms discussed in the previous chapters. We will begin by validating the collision avoidance controller for two vehicles, while discussing the effect of the various parameters present in the Vehicle's local MPC. We will then perform an uncertainty analysis in-order to obtain and tune controllers capable of handling different types of uncertainty. We will show the results of the decentralized coordination algorithm on similar and different vehicles in different scenarios while highlighting the room for improvement. Next, we will introduce the hybrid algorithm in two modes, re-planning based on first point of entry and re-planning based on complete trajectory, while comparing these modes to the completely decentralized approach and highlighting the improvements. Finally, We introduce a new priority scheme allowing for the inclusion of an emergency vehicle in the scenario, we then compare the different applied schemes and their application via the designed algorithms while highlighting the advantages of the hybrid control strategy.

## 5.1 Decentralized Coordination

In this section we present and discuss the results obtained applying the completely decentralized approach. Starting from the 2-Vehicle cases ( 4-way Intersection, and Y-Junction Merging), moving to the full 4-Vehicle case.

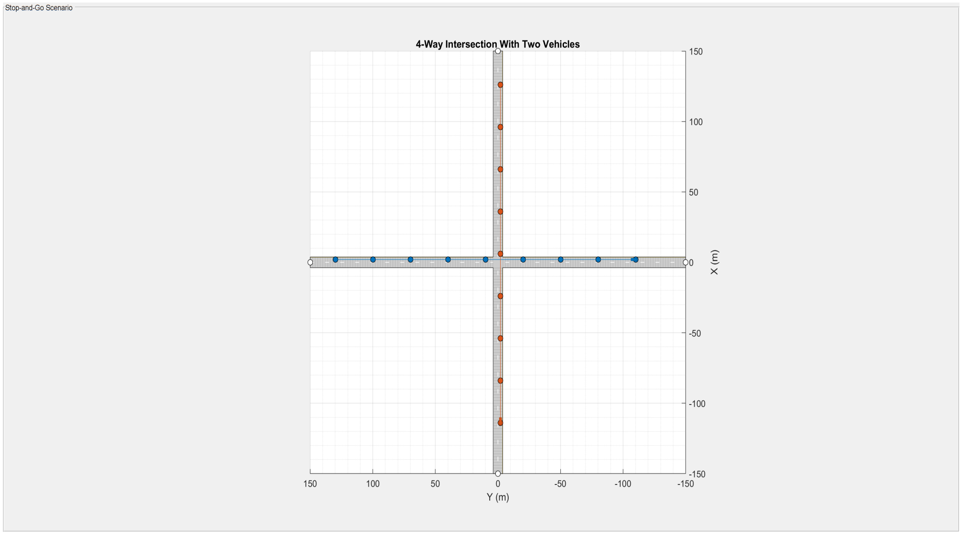


*Figure 5.1: Scenario with planned trajectories*

### 5.1.1 Two-Vehicle Collision Avoidance

We begin our analysis by considering the collision avoidance between two vehicles, we will show case the ability of the controller to avoid collision by simulating two road scenarios.

#### 4-Way Intersection

We will be considering the scenario shown in Fig.5.1 , the various parameters chosen are presented in Table. 5.1. we assume that the planned velocities are constant through-out the vehicle's trajectory for a straight trajectory. The point corresponding to the (0,0) position is the collision point of the two trajectories.

| | |
|---|---|
| Safety Distance | 9.5 m |
| Vehicle 1 Initial Position | (0,-113.95 m) |
| Vehicle 1 Planned Velocity | 13.9 m/s |
| Vehicle 2 Initial Position | (110.15 m, 0); |
| Vehicle 2 Planned Velocity | 13.9 m/s |
| $(Q, R, S, \delta)$ | (1, 1, 0.2, 1) |

*Table 5.1: A table listing the parameters used in the simulated scenario*

We begin our analysis with vehicles sharing the same parameters, in later sections we will discuss different cases.As discussed in the previous chapter, in the decentralized approach the Time to React is used in order to obtain the crossing order. In this scenario :

$$TTR_1 = \frac{113.94}{13.9} = 8.1978 seconds$$

$$TTR_2 = \frac{110.15}{13.9} = 7.9245 seconds$$

The reason why these initial conditions were chosen is to simulate the behavior of two vehicles entering the intersection space within a short time of each other. The velocity Profiles generated after the simulation can be seen below.

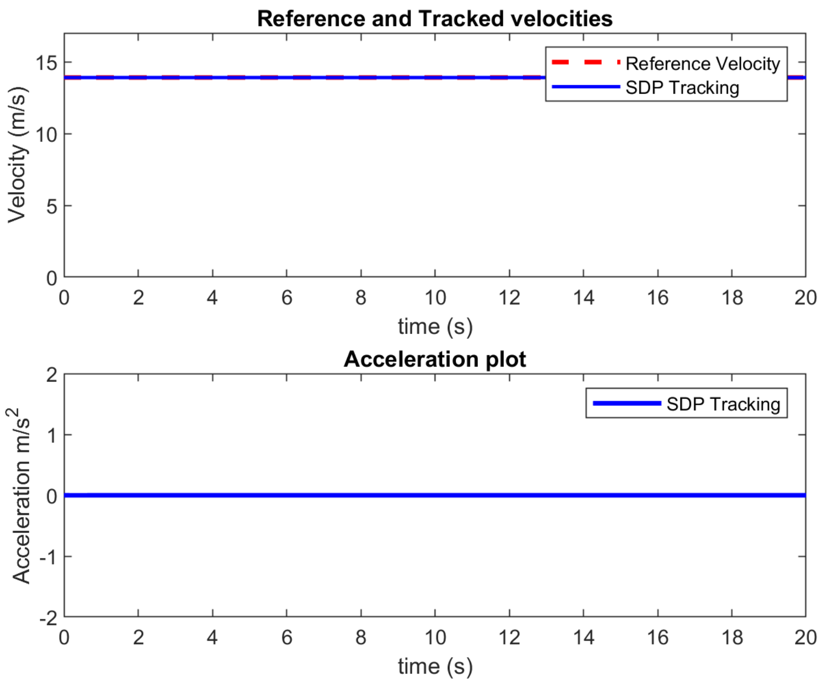


*Figure 5.2: Velocity and Acceleration of Vehicle 2 through the scenario*

The first thing to observe is that the crossing order is respected, where we have that Vehicle 1 gave way to Vehicle 2 as it had the greater time to react. We also notice that the velocity variation wasn't big, and safe passage for two vehicles arriving at very close times to the collision point initially is

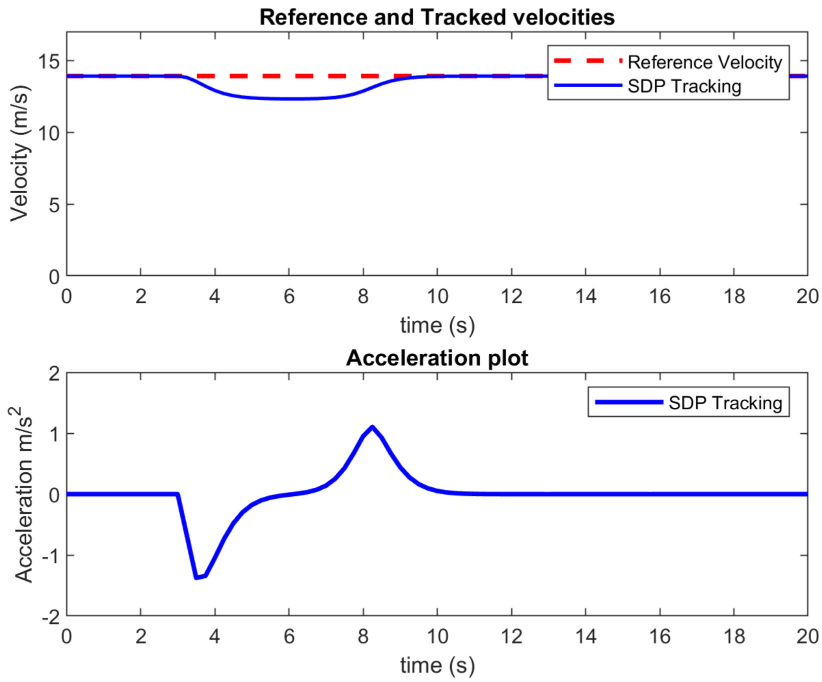


*Figure 5.3: Velocity and Acceleration of Vehicle 1 through the scenario*

guaranteed ( as can be seen by observing Figure 5.4) without stopping, outlining the advantages cooperative vehicles would have over human driving. From the figure of the headways, we observe that the velocity modification

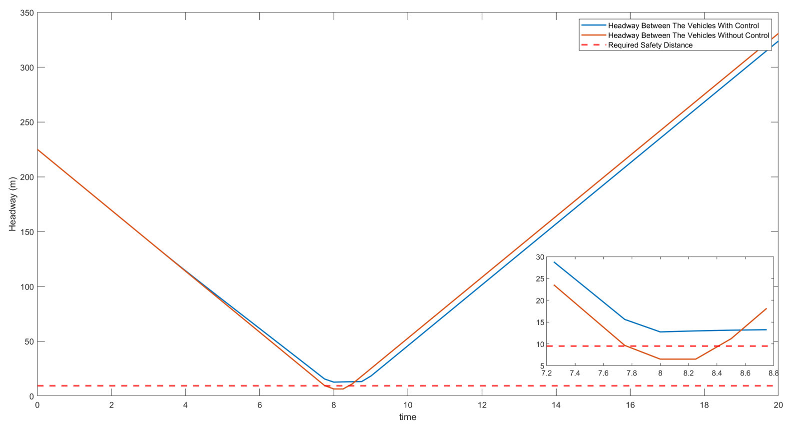


*Figure 5.4: Headway Between the Two Vehicles during the simulation*

was necessary to obtain a headway greater than that required for safety. The next step was to analyze the control signal generated, in order to understand whether actuation is required at a high frequency. From the graph of the acceleration, we notice that there is no sharp jumps, but to get a clearer understanding we perform a wavelet analysis of the control signal From Figure

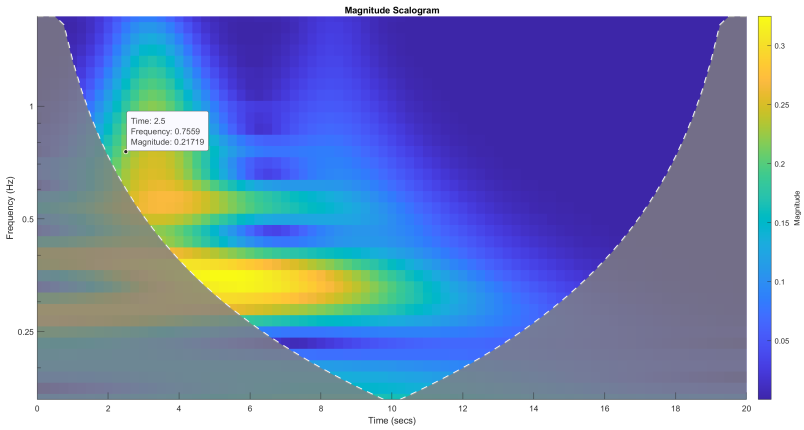


*Figure 5.5: The wavelet transform of the Control Signal of Vehicle 1*

5.5 We can observe that the highest frequency element in the control signal has a frequency in the neighborhood of 1Hz and corresponds to the first time the controller is triggered. We can therefore safely say that the control signal does not contain any high frequency components that might exceed the actuation speed. Moreover, being a real time problem, our sampling frequency is higher than twice the highest reported frequency. Allowing for calculation and execution of control action during the sampling period.

### Y-Junction

We next move to the case of a Y-Junction which is a lane merging scenario. We decided to employ this scenario and divert from the 4-way intersection in order to:

- Show the flexibility off the proposed collision avoidance controller and its applicability to various scenarios
- Show the behavior of a curving vehicle

- Outline that the controller works in different priority schemes as will be shown in what follows

We begin by outlining the scenario which can be seen in the figure below:

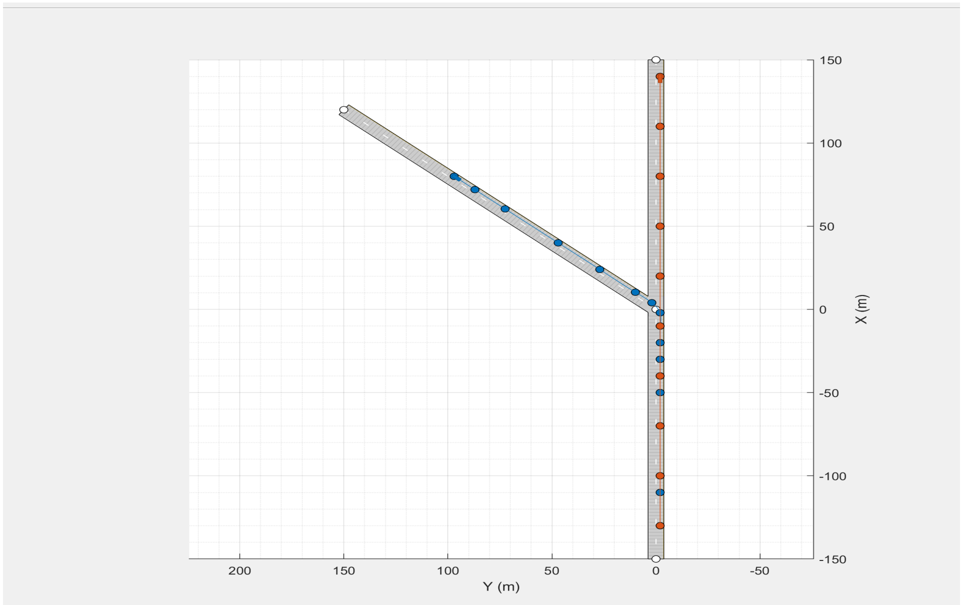


*Figure 5.6: Y-Junction Scenario with Planned Trajectories*

Initial conditions and control parameters are presented in the table 5.2. Unlike in the previous scenario, the planned velocity can't be assumed constant throughout the interval for both vehicles. The velocity profile for the turning vehicle is thus 1 as shown in figure 5.7. As for the priority, and since in

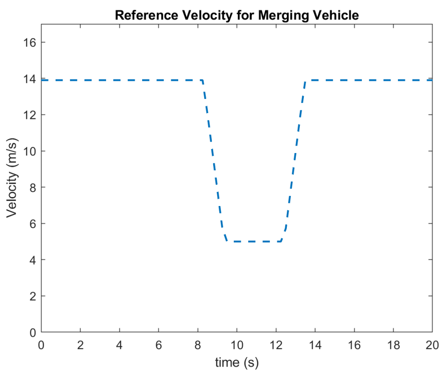


*Figure 5.7: Way-point generated Reference speed for Merging Vehicle*

| | |
|---|---|
| Safety Distance | 9.5 m |
| Vehicle 1 Initial Position | (106 m, 87.2 m) |
| Vehicle 2 Initial Position | (-2 m, 140 m ); |
| Vehicle 2 Planned Velocity | 13.9 m/s |
| $(Q, R, S, \delta)$ | (1, 1, 0.2, 1) |

*Table 5.2: A table listing the parameters used in the simulated scenario*

this section it is defined at the initiation of the scenario and kept constant, we chose to follow the rules of traffic allowing for the vehicle on the main road to pass first. This serves to show the applicability of the controller to different schemes, a topic which will be detailed in later sections. Reported below are the generated velocities and accelerations for different imposed safety distances.

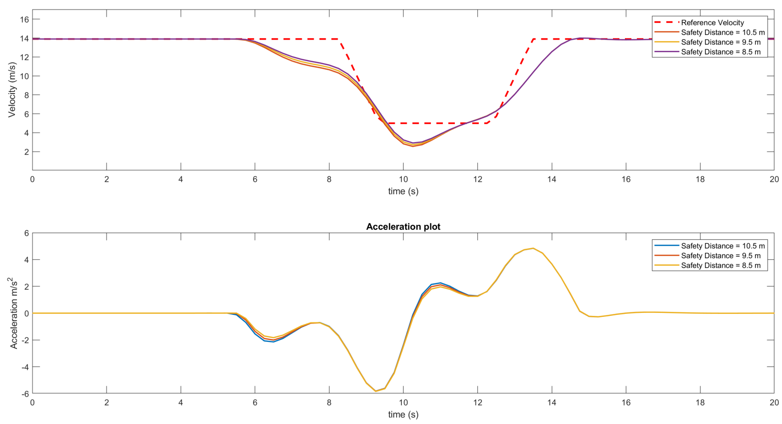


*Figure 5.8: Velocity and Acceleration of Merging Vehicle through the scenario*

From the graphs above, we can observe from the acceleration plots two main sinusoidal components, the one which corresponds to the braking for the turning maneuver while the second smaller and earlier one corresponds to the braking for collision avoidance, Moreover, it can be seen that the level of braking and thus the decrease in velocity is dependent on the safety distance. Moreover, we observe that the vehicle on the main road does not alter its velocity which is in accordance to the mode of functionality of the controller outlined in the previous chapter.

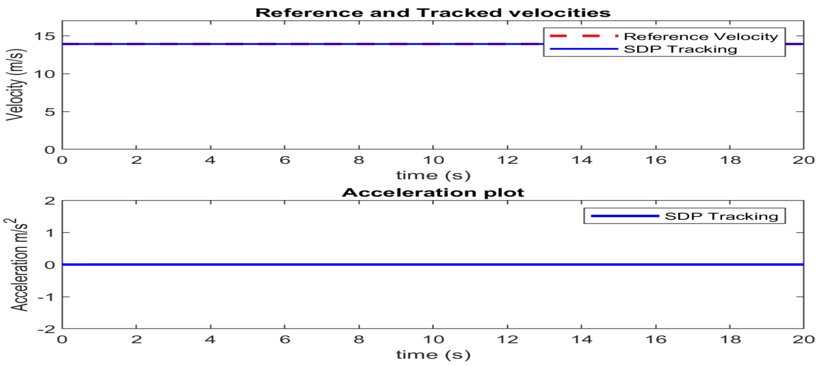


*Figure 5.9: Velocity and Acceleration of on main Road*

The values of the head-ways corresponding to the three different safety distances are reported in the figures below:

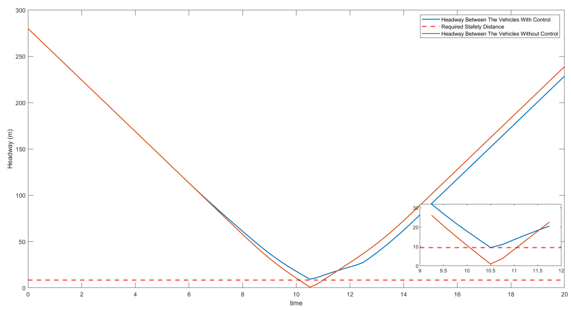


*Figure 5.10: Plot of the Variation of Headway between Vehicles(Safety Distance = 8.5m)*

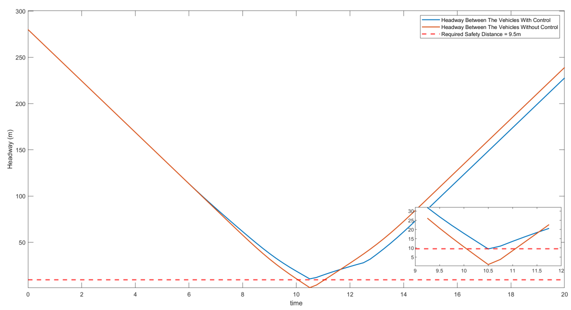


*Figure 5.11: Plot of the Variation of Headway between Vehicles(Safety Distance = 9.5m)*

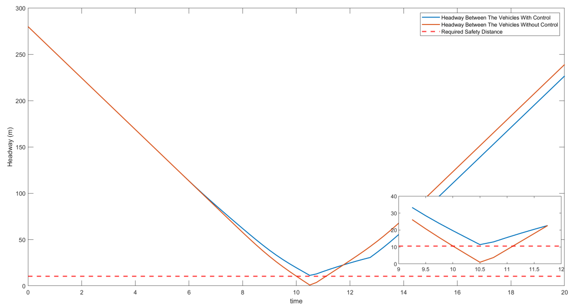


*Figure 5.12: Plot of the Variation of Headway between Vehicles(Safety Distance = 10.5m)*

It can be seen that the safety distance is always kept, and that in all of the reported cases if not for the clash avoidance controller, collision would have occurred as can be seen by the distance between vehicle going below the minimum safety in the non-controlled case.
As mentioned earlier, in the response of the system two main harmonics can be clearly observed, one could further see this by plotting the control signal as a function of time in addition to the wavelet transform of the signal to analyze the response in the frequency domain and make sure that the sampling time is sufficient to capture the required information, and that the frequency of the control signal is within the actuation range. The plots below correspond to the case with a minimum safety distance of 10.5m: As

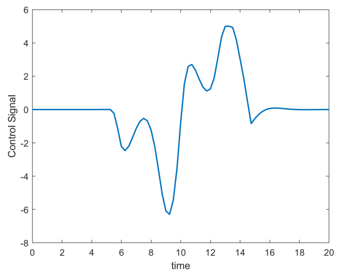


*Figure 5.13: Control signal for safety distance = 10.5m)*

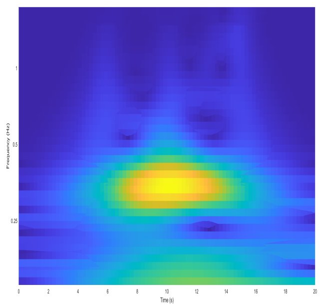

*Figure 5.14: Scalogram of the wavelet plot of the control signal)*

can be seen, we have two main components one from t=6s to t=14s at a frequency of 0.5 Hz. While the second lower frequency component component is reported in the time interval of t= 7s to t=12s with frequency component of around 0.1 Hz It can be noticed that both are below half the sampling frequency and thus the Shannon condition is satisfied for both instances.

### 5.1.2 Four Vehicle Scenarios

In this section we extend the approach of the previous section to the 4 vehicle scenario, where we will simulate the 4-way intersection scenario taking into account all 4 vehicles. In the beginning we will discuss the case of 4 identical vehicles travelling at identical speeds, then we will study the case of different vehicles.

#### Scenario with Four Identical Vehicles

As with the previous section we begin with a description of the scenario, and the initial conditions. However we will stick to analyzing one safety distance, leaving the analysis of different safety distances to the next section. The modelled scenario, along with the vehicle way-points can be seen in Fig 5.16. We have four straight travelling vehicles and one vehicle that is attempting a left turn. In a non-controlled scenario, as will be seen later collision is expected between the three straight-traveling vehicles.

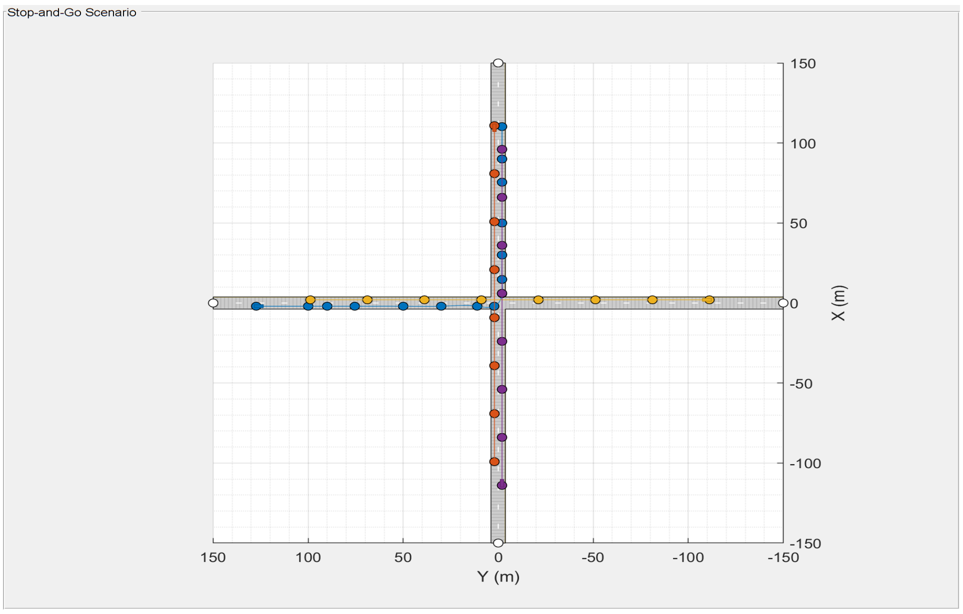


*Figure 5.15: Planned Paths for the 4 vehicles scenario*

In this first modelled 4-way scenario, it was our intention to test the ability of the controller to adapt, the positions and velocities of the previously considered vehicles were kept the same, while we added the other two vehicles in a way so that the vehicle approaching from the top would collide with the vehicle coming from the right if no control was applied, while having the vehicle coming from the left at a distance close enough for conflict to ensue if the other vehicles slow down. In this way it is possible to observe how the controller reacts to other vehicles changing their planned trajectories. Since this was the main emphasis of this scenario, the controller gains and planned velocities were the same for all vehicles. All the significant parameters are reported in the table below:

| | |
|---|---|
| Safety Distance | 9.5 m |
| Vehicle 1 Initial Position | (0,-113.95 m) |
| Vehicle 2 Initial Position | (110.15 m, 0); |
| Vehicle 3 Initial Position | (-3.7 m , 110.85 m); |
| Vehicle 4 Initial Position | (-122.5 m , -3.7 m) |
| Planned Velocity | 13.9 m/s |
| $(Q, R, S, \delta)$ | (1, 1, 0.5, 1) |

*Table 5.3: A table listing the parameters used in the simulated scenario*

The resulting velocity and acceleration profiles are summarized in the following plots, presented in order of entry to the intersection: From the

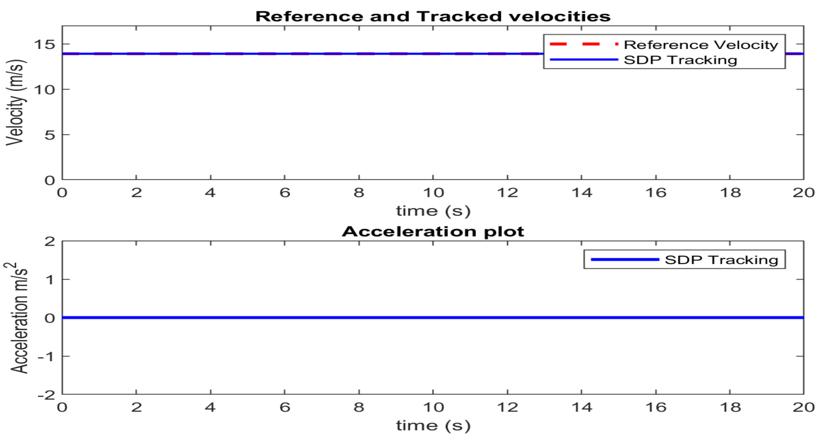


*Figure 5.16: Velocity and Acceleration Vehicle 3 (Top,Red)*

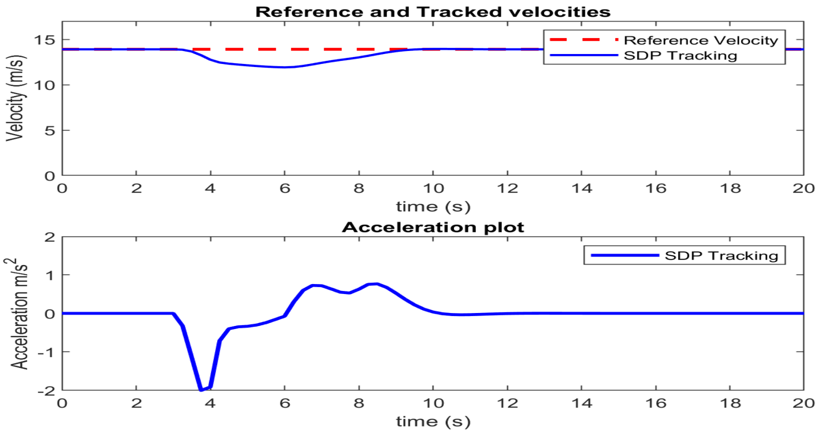


*Figure 5.17: Velocity and Acceleration Vehicle 2 (Right, Yellow)*

velocity and acceleration plots of Vehicle 2, it is clearly seen that in fact, the vehicle was able to react to its counterpart with higher priority where braking initiates at 3.25 seconds from the beginning of the simulation. More interestingly, is the reaction of Vehicle 1 which is reacting to the change in the behavior of vehicle 2. It can be seen that at time 3.5 seconds, the vehicle initiates its braking as in the case of two vehicles, a spike is then observed which is due to the change in the plan of Vehicle 2 (arriving to Vehicle 1 at the consequent time-step). The final result is an increased braking as the priority order is kept constant throughout the scenario, which leads to a velocity decrease greater than that observed in the two vehicle case.

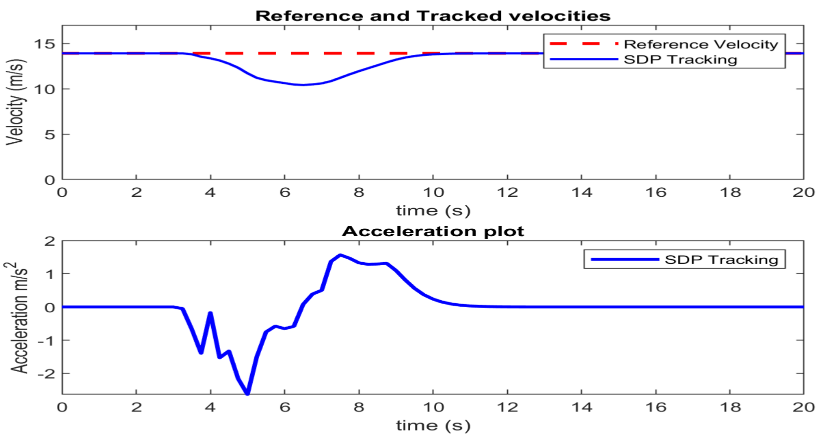


*Figure 5.18: Velocity and Acceleration Vehicle 1 (Down, Purple)*

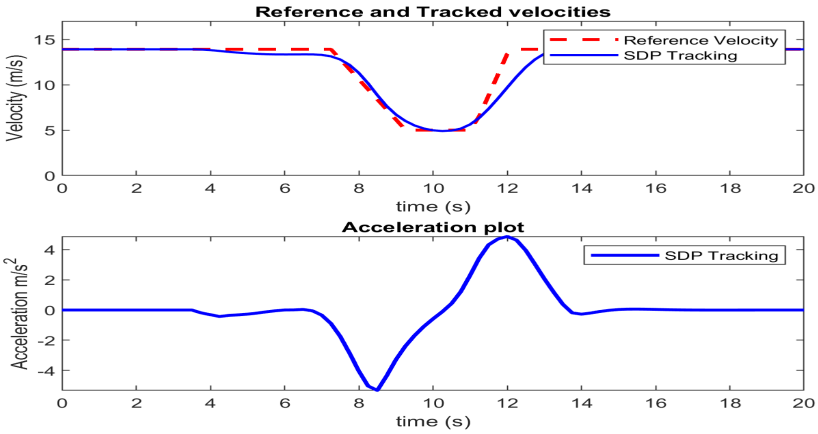


*Figure 5.19: Velocity and Acceleration Vehicle 4 (Left, Blue)*

From the response of the left turning vehicle it can be seen that unplanned braking takes place in the neighborhood of 4 seconds from the start of the simulation as a reaction to vehicle 1 slowing down. The scenario thus proves the ability of the controller to handle the full intersection scenario in addition to showing the adaptive nature of the Model predictive controller and its ability to react to unplanned behaviors which puts it in an advantageous position when compared to traditional reservation based algorithms

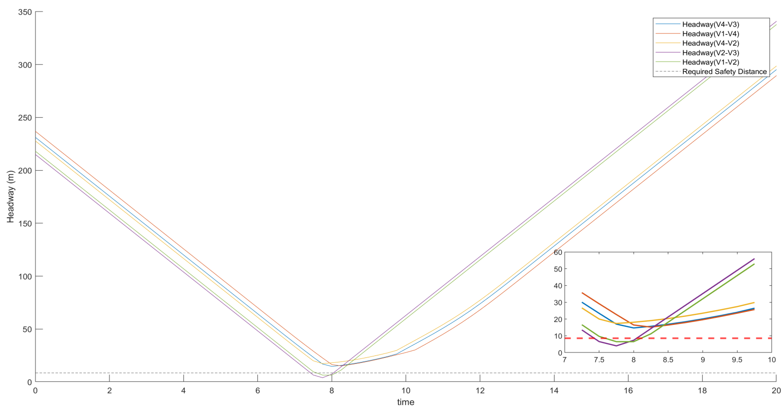


*Figure 5.20: Plot of the Headway between all Vehicles throughout the simulation without Control*

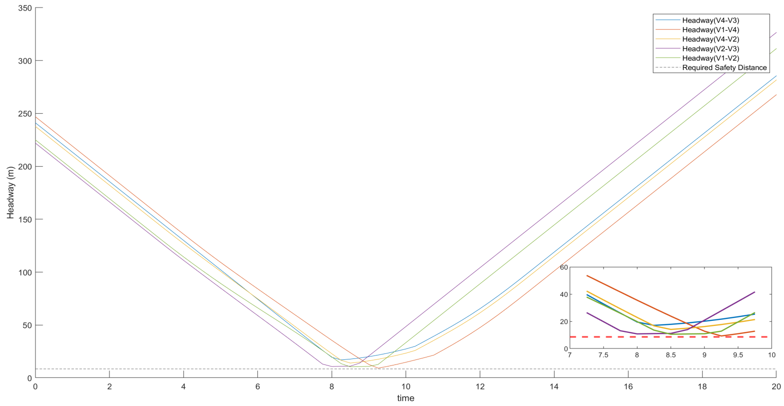


*Figure 5.21: Plot of the Headway between all Vehicles throughout the simulation with Applied Control*

### 5.1.3 Non-Identical Vehicles

The next step is to simulate the scenario with non-identical vehicles in order to establish the generality of the approach. We thus chose to simulate the previous scenario with vehicles having different parameters reported in the Table 5.4:

| | |
|---|---|
| Safety Distance | 9.5 m |
| ($Ta_1$,$Q_1$,$R_1$,$S_1$,$\delta_1$) | (0.1,0.6,1,0.5,1) |
| ($Ta_2$,$Q_2$,$R_2$,$S_2$,$\delta_2$) | (0.5,1,1,0.2,0.7) |
| ($Ta_3$,$Q_3$,$R_3$,$S_3$,$\delta_3$) | (0.3,1,1,1,1) |
| ($Ta_4$,$Q_4$,$R_4$,$S_4$,$\delta_4$) | (0.2,5,1,1,1) |

*Table 5.4: A table listing the controller parameters for the different vehicles used in the simulated scenario*

We will analyze the difference in behavior from the nominal case for each vehicle. Starting from the left turning vehicle 4, where the only parameter which was modified from the nominal scenario is the value of the gain multiplying the variation from the reference velocity. It could thus be seen that the reference profile is more accurately followed at the expense of an increase in the braking done by the vehicle as can be seen in the sharp peak at 8.5 seconds.

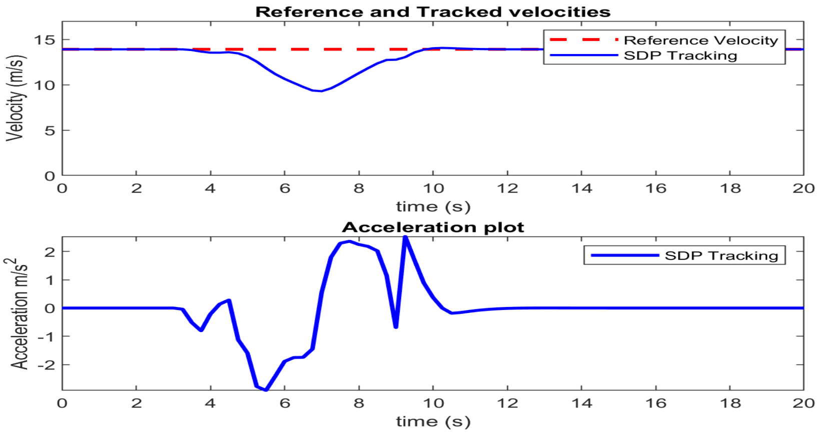


*Figure 5.22: Velocity and Acceleration Vehicle 1*

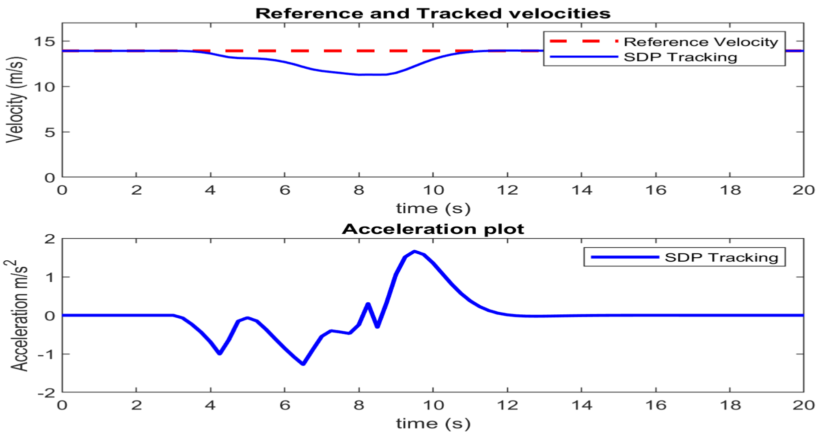


*Figure 5.23: Velocity and Acceleration Vehicle 2*

As the behavior of the vehicles 2 and 1 are linked they will be discussed simultaneously, Looking at vehicle 2's acceleration plot we notice two separate braking instances highlighted by the red boxes, this comes due to the decrease in the parameter $\delta$ coupled with the increase in the actuation time constant $T_a$. The vehicle thus would prefer to brake later than in the nominal case (4 seconds) but due to actuation limitations, it performs a smaller braking maneuver in order to be able to accomplish the avoidance. What is remarkable is that this delayed braking influences the behavior of vehicle 1, which at 4 seconds from the start of the simulation performs braking similar to that observed in the nominal case, however due to the second instance

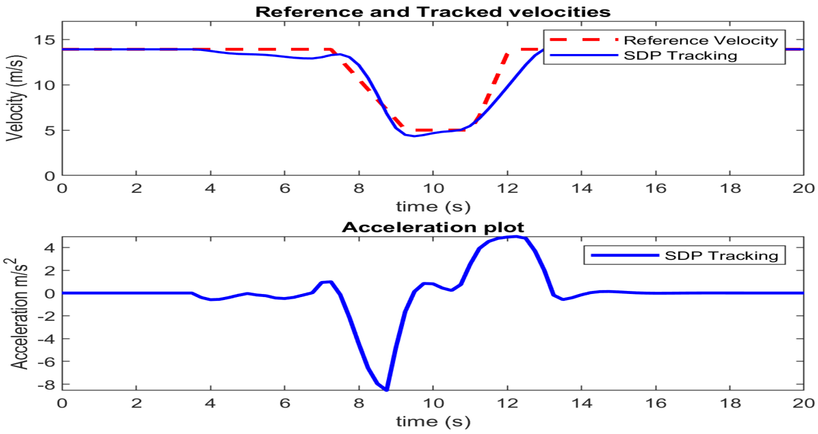


*Figure 5.24: Velocity and Acceleration Vehicle 4*

of braking of vehicle 2, the second braking valley in the acceleration plot of Vehicle one is lower and wider than in the nominal case. From the analyzed behaviors, it can thus be concluded that:

1. The collision avoidance Predictive Controller is capable of handling sudden changes in behavior ( Vehicle 1's reaction to the delayed braking of Vehicle 2)

2. Collision is avoided taking into consideration the lag term in the model

3. Having a single non-cooperative entity decreases the efficiency of the whole system

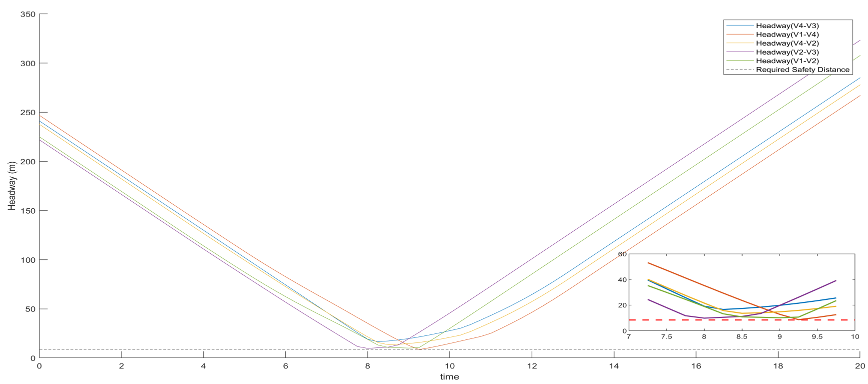


*Figure 5.25: Evolution of Head-ways for the non-Identical Vehicles*

### 5.1.4 Controller Tuning and Uncertainty Analysis

As can be seen from the previous results, the collision avoidance controller is capable of handling various values of the minimum required distance. It is thus of interest to correctly estimate the values of the control parameters that would be able to handle different kinds of uncertainty such as those that arrive from position sensors and mis-modelling. In this section we will begin by outlining the effect of the parameter $\delta$ in the Vehicle's MPC and why it could be of particular interest in this study. We will perform an uncertainty analysis in order to obtain the set of controller parameters suitable for different values of actuation times.

#### Driver Aggressiveness Parameter

In order to account for driver aggressiveness, or vehicle non-cooperativeness which could be a setting in an automated vehicle or a way of accounting for legacy drivers, the weighing term $\delta$ was used in each vehicle's local MPC. The term delta was used as what is commonly referred to in multi-agent systems and negotiation schemes as the Bellman discount factor. In such schemes, it is often used to show that a delay in obtaining a reward results in a discounted utility. In our case however, and as it is used in a minimization framework rather than a maximization one, the value delta weighs future deviations from the reference velocity less than immediate ones, thus resulting in delayed braking behaviour as can be seen in Figure 5.25.
This thus can be used to showcase aggressive driving behavior or to highlight the delayed reaction of the human driver which relies on vision in order to pass the intersection and thus initiates braking at a later time than the cooperative vehicle receiving distance measurements at all time instants.
A rough comparison between traversed trajectories of vehicles having $\delta = 1$ and $\delta = 0.5$ while keeping the other parameters fixed, as shown below. We first construct a cost function penalizing :

- Variation form reference Velocity (Time Costs)
- Jumps in Acceleration (Comfort Cost)
- Maximum absolute values of acceleration (Energy Costs)

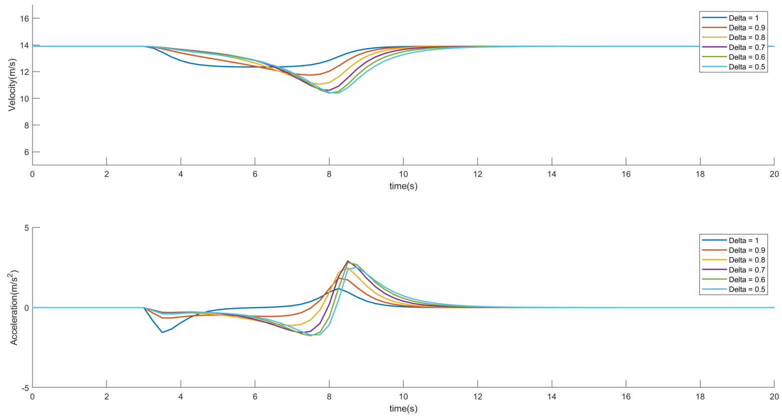


*Figure 5.26: Velocity and Acceleration Variation due to Change in aggressiveness*

to form :

$$J_{\delta_i} = \frac{1}{2}\Delta V_{max}^2 + \frac{1}{2}\Delta a_{max}^2 + \frac{1}{2}a_{max}^2 \tag{5.1}$$

The brake down of the costs is reported in Table 5.5:

| Cost | Cooperative | Non-Cooperative |
|---|---|---|
| Time | 1.2146 | 6.7421 |
| Comfort | 0.3103 | 2.0096 |
| Energy | 1.1186 | 3.3131 |
| Total | 2.6435 | 12.0648 |

*Table 5.5: Breakdown of costs associated with different aggressiveness/cooperativeness levels*

From this simple comparison alone, the advantages of far-sightedness and cooperation is apparent where the costs for passing the intersection in the cooperative ($\delta = 1$) case are almost six times lower than those associated with the ”non-cooperative”/”aggressive” behavior.

**System Performance Analysis**

In this section we try to demonstrate the effect that the power dynamics, summarized as a first order lag element and characterized by the actuation time constant $T_a$ which reflects each vehicle's ability to accelerate and decelerate, have on the response.

Different actuation actuation time constants can be due to different types of engines, or power-train architecture. In the figures below is a graph of the different behaviors vehicles with different actuation speeds have while going through the 2-Vehicles in an Urban four-way intersection discussed in the previous section: It can be seen that the higher the actuation speed, the

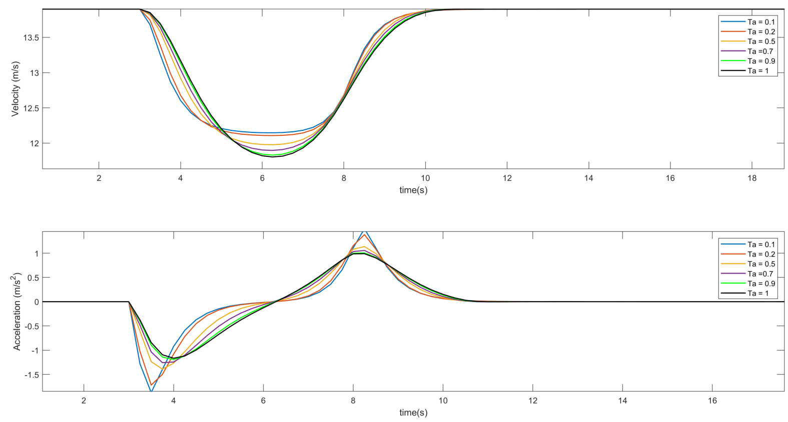


*Figure 5.27: Velocity and Acceleration Variation due to Change in Actuation Speed*

steeper the acceleration, while as the actuation speed decreases the curves become smoother. This smoothness of the response however comes at the expense of greater acceleration values for a longer period of time as can be seen the figure.

### Measurement Uncertainty

Vehicles rely on sensory input from various sensors in order to localize themselves in their environment,however all measurements are subject to a range of uncertainties and a robust control system should be able to safely avoid collision while taking into account a margin of error in the distances with the other vehicle. For this purpose we have simulated the 4-Vehicles in an Urban four-way intersection scenario while taking into account different values of safety distances. This variation in the safety distance serves as a sink for the error in the distances communicated to the vehicle.

**Remark:** A detailed overview of the different kind of sensors, and their respective uncertainties can be found in [87, 88]

### Model Uncertainty

Due to the simplified model of the system, the power train dynamics are summarized in the actuation time constant $T_a$, to add a level of robustness to the controller design, it was thus necessary to simulate the scenarios above for different values of actuation time constants in order to reach a set of controllers that would be applicable to a wide range of vehicles and that could deal with variations in the actuation speeds (either due to the simple modelling of the actuation dynamics or due to errors in the modelling).

### Controller Tuning and Parameter Selection

In order to come up with a robust design, for the criteria mentioned above, it was necessary to simulate the intersection scenario with similar vehicles while varying the following parameters:

- Driving Aggressiveness $\delta \in (0, 1)$
- Penalty on Acceleration Jumps $\mathbf{S} \in (0, 0.5)$
- Safety Distance
- Actuation Time Constant $\mathbf{T_a} \in [0.1, 1]$

We first began our analysis by checking the sets of parameters that would satisfy the safety conditions for the various considered values of the varied parameters. For each value of the minimum distance, In the figures 5.18,5.19 we show for each set of parameter values if the safety distance condition is satisfied (Blue) or not (Red) for safety distances 8.5,9.5 m.
This clearly shows how the slower the actuation of the vehicle, the fewer the number of acceptable controller parameters that are capable of reaching this objective, since for slowly reacting vehicles, highly aggressive driving behaviour coupled with high cost on jumps in acceleration would render the problem infeasible since the collision avoidance constraint is activated in a restricted region of the intersection space that would entail certain limits on actuation in for safety to be achieved. This variation in the number of acceptable parameter values is plotted in the bar graph below for different safety distances.

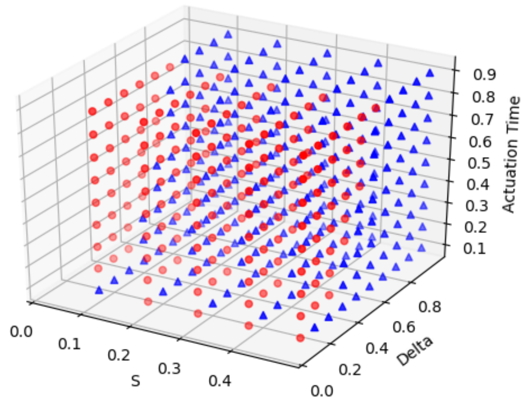


*Figure 5.28: Condition Satisfaction as a function of Parameter values (Safety Distance = 8.5m)*

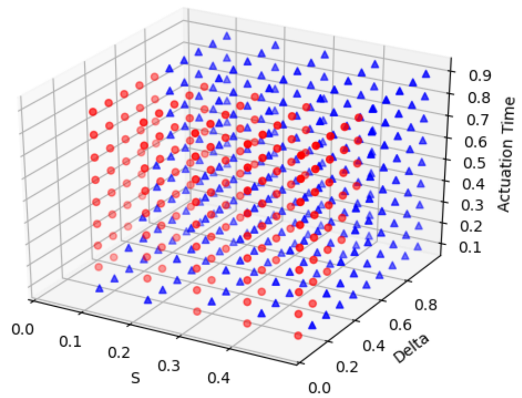


*Figure 5.29: Condition Satisfaction as a function of Parameter values (Safety Distance = 9.5m)*

Being the total number of considered combinations of parameter values for each actuation time 50 for each safety distance two major observations can be made looking at the above bar diagram:

- There exists a set of parameter values that satisfies the safety condition for all values of safety distances and actuation times

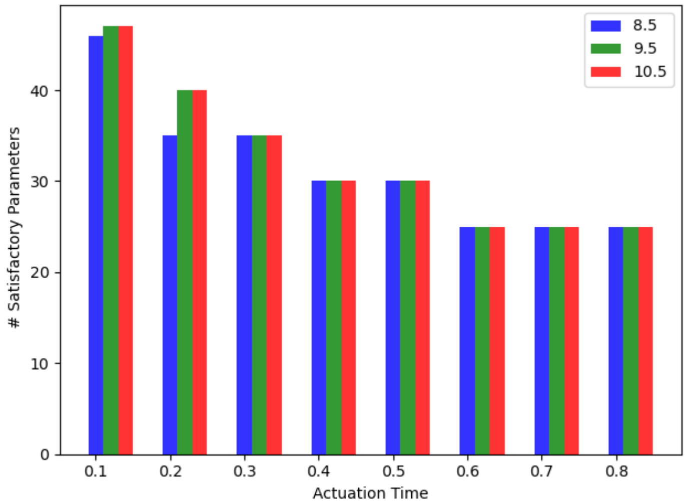


*Figure 5.30: Bar Diagram showing the total number of acceptable Controller Parameters as a function of Actuation Speed for various Safety Distance)*

- As the safety distance increases, the number of parameter values that are applicable increases or stays constant, this again highlights one area of improvement of this control approach. Since with a greater safety distance the controller is triggered earlier and thus has a greater time to satisfy the constraints.

Based on these observations, we decided to first collect all the values of the controller gains that satisfy the safety condition for all values of actuation times and safety distances considered which would be the basis of the robust controller design. We then will perform a comparison of the performances of these controllers based on a min-max cost function in order to come up with a set of parameters that in a way optimize the considered cost function. After the collection of the suitable parameters, and to get a sense of the effect that each of the varied parameters has, the variation of the following Variables was plotted:

1. Minimum Velocity
2. Minimum Acceleration ( Maximum Breaking)
3. Maximum Acceleration
4. Maximum Jerk (Comfort)

Since Actuation time and safety distance are parameters of the system and not of the controller, and since the values of the parameters chosen were satisfactory for all actuation times and safety distances, the values mentioned above are taken as the mean values among all considered actuation times. Plots of the variations for different safety distances are reported below:

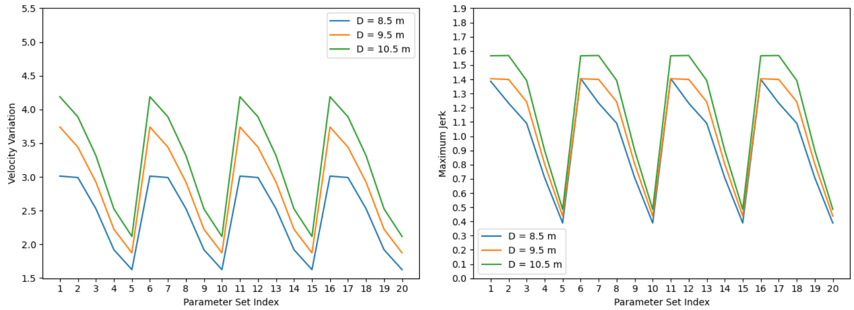


*Figure 5.31: Plots of the maximum variation from reference velocity (right) and the maximum jump in acceleration (left) as a function of chosen controller gains*

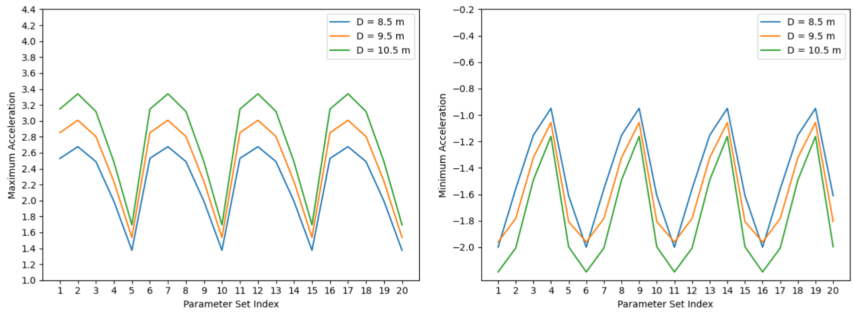


*Figure 5.32: Plots of the maximum values of acceleration (right) and the minimum ones (left) as a function of chosen controller gains*

The plots of the achieved safety margins can be interpreted in two ways, one way is to consider the deviation as a positive which results in an added level of safety or as a disadvantage where it would be optimal to have vehicles achieve the safety distance exactly to have a faster running flow of traffic. Based on either consideration, the cost function was adjusted to arrive at the desired "optimal" parameters.

From the reported figures, it is clear to observe a trend in behavior of the various considered variables, regardless of the considered safety distance, lumping the cost for all considered safety distances is thus a reasonable step.A cost function was then constructed that would enable us to understand the overall performance at each data point :

$$Cost = \pm\alpha\Delta D_{safety_i} + \beta max(\Delta V)_i + \zeta max(a)_i - \eta min(a)_i + \gamma max(|\Delta(a)|)_i \tag{5.2}$$

Where $\Delta D_{safety}$ is the distance from the $D_{safety}$ parameter i, $\Delta V$ is the variation from the reference velocity and a is the acceleration and the constants $\alpha, \beta, \zeta, \eta$ and $\gamma$ are weighing constants. The goal was to obtain a Pareto Optimal Set of Parameters through finding a set that would be the minimum at all Safety Distances From the above plots, what is observed is

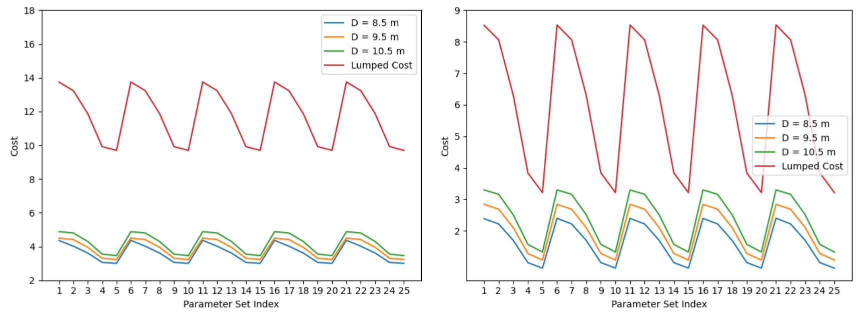


*Figure 5.33: Plots of the lumped costs in both positive and negative effect of safety margin*

that in both cost functions, the cost increases as $\delta$ increases, however when it comes to the cost on the jump in acceleration, what can be observed is that the optimal value is for **S= 0.3** which is in the middle of the considered values. The effect S has on the cost is however not as noticeable as that of the term $\delta$

## 5.2 The Hybrid Algorithm

In this section, the role of the intersection manager first discussed in Chapter 4.4 will be elaborated on, showing the inputs and outputs of the manager in addition to discussing the trajectory re-planning of the entering vehicles and the effect of the terminal cost, which will have a major role in the application of the First Come First Serve Priority Scheme. Then being the main contribution of this report, the advantages of the hybrid algorithm will be highlighted by comparing it to the previously discussed controller.

### 5.2.1 Introduction

In the decentralized approach, the Intersection Manager's role was mainly relaying information among the vehicles, however in the hybrid scenario its role is greatly increased where it has to:

1. Calculate the crossing order based on received information
2. Suggest Arrival time to the various conflict regions for all vehicles
3. Keep track of vehicles in the intersection space ( Possible Conflicts, Target Lanes, Position in the intersection space)

Stage 1: Localization and Forming Property List

When a vehicle then enters the intersection space, It transmits the current lane and the target lane along with the predicted time of arrival to the intersection space from the vehicle's local trajectory planning. Based on this information, the Intersection manager localizes the vehicle in the scenario, it predicts the conflict zones through which it will pass, request information from vehicles in the 'Time Suggestion Zone' and order the information in as seen in Table 5.6:

It is important to note that the time of arrival and Safety Time (calculated

| Vehicle ID | Current Position | Time of Arrival | Safety Time |
|---|---|---|---|
| 1 | -98 | 42 | 3 |
| 3 | -76.42 | 34 | 3 |
| 4 | -70.13 | 33 | 4 |

*Table 5.6: Example of the Input of the Priority List and Time suggestion Algorithm*

based on Equation 4.23) are in discrete time steps rather than actual time. The current position is only requested in case of the First Come First Serve Scenario in order to limit the amount of private information for security purposes. The Vehicle IDs used in this scenario are simple integers without loss of generality, but alpha-numeric strings can also be used.

Stage 2 : Time Suggestion

The input information is thus processed in order to rank the vehicles based on the priority criterion ( First Come First Serve, Time To React ...). The table is reshaped to rank the vehicles in the desired order and the information is sent to the time suggestion optimizer, which based on the current time of arrival sent by the vehicles and the respective safety times calculated the suggested Times of Arrival generating the output Data in the form presented in Table 5.7

| Vehicle ID | Time of Arrival | Safety Time | Suggested Time | Time Difference |
|---|---|---|---|---|
| 4 | 33 | 4 | 33 | 0 |
| 3 | 34 | 3 | 37 | 3 |
| 1 | 42 | 3 | 42 | 0 |

*Table 5.7: Example of the Output of the Priority List and Time suggestion Algorithm*

Stage 3: Trajectory Re-planning

After receiving the time suggestion from the Intersection Manager, each vehicle acts on the information in order to generate a re-planned trajectory based on the Trajectory planning Optimal Control Problem stated in 4.4.2. An example of the re-planned trajectories can be seen in Figure 5.33 It is

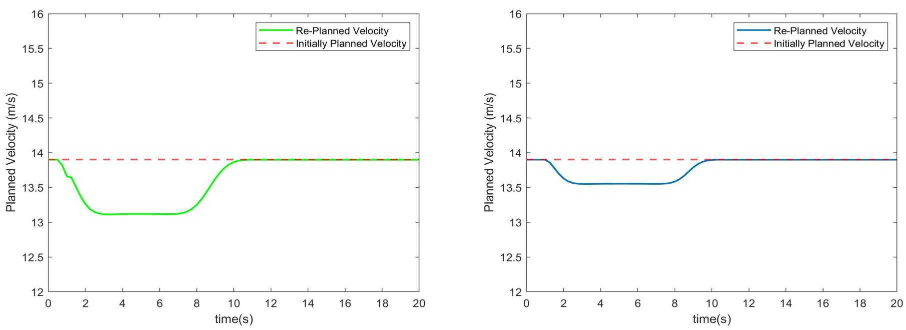


*Figure 5.34: Re-planned Velocities For Vehicles 1 and 2 in the 5.1.2 Identical Vehicle Scenario, P = 0.1*

however important to note that, this variation in velocity is based on each

vehicle's trajectory planner, namely it is directly related to the gain P associated with the variation from the suggested time. This term is left as a free variable which is also influenced by the aggressiveness/cooperativeness of the vehicle.In the first come first serve scenario, and as it required major modification of the planned trajectory, the cost term is put to higher values than in other priority schemes.
It should also be noted that the trajectory re-planning does not guarantee the collision-free passage of the vehicles through the intersection as the constraint is a soft one and there is no continuous feedback with the intersection manager to guarantee convergence to the suggested time. The role of the Intersection manager is thus to help the vehicle's adjust in order facilitate the avoidance of collisions in a more efficient manner as will be shown in what follows.

### 5.2.2 Two Vehicle Scenario Comparison

We begin our analysis of the Hybrid control algorithm by analysing the improvements it has in the two vehicle scenario, by comparing it to the already analyzed behaviors. We, and for avoiding repetition will only compare the behavior of the Y-Junction scenario since more insight will be obtained for the urban intersection scenario in the four vehicle case.
Since the same scenario is analysed, we refer the reader to Figure 5.6 and table 5.2 for a description of the scenario. The resulting behavior of the merging vehicle is reported in Figure 5.35.
It is clear to see the advantage of the (minimal) early braking action, where the minimum velocity achieved in greater than that in the scenario without re-planning. The greatest improvements however, are seen in the acceleration plots where early breaking of 0.2637 $\frac{m}{s^2}$ magnitude results in a smoother response having peaks of lower magnitudes. The 'smoothness of motion' is perhaps better observed by looking at the wavelet plots of the response. The difference from the nominal case is apparent where the amplitude of the frequency components is decreased and the range of frequencies due to the collision avoidance braking is decreased from around 0.5Hz to 0.3Hz. It is important to note that the frequency of the applied control action is not only important for the practical purposes but also for comfort purposes.

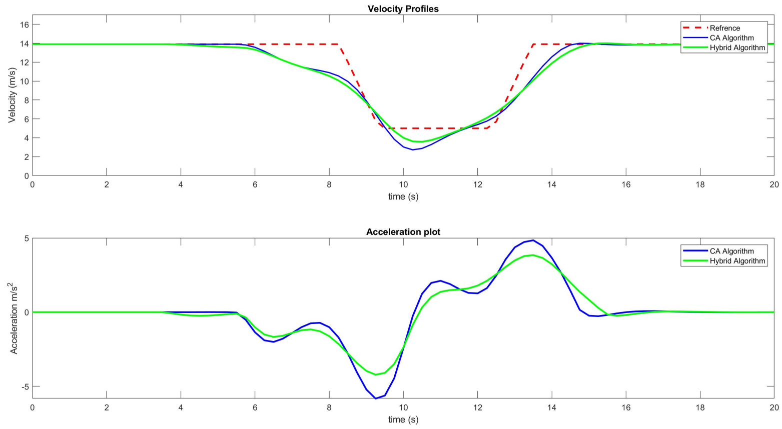


*Figure 5.35: Plots showing achieved Velocity and Acceleration Profiles for both algorithms*

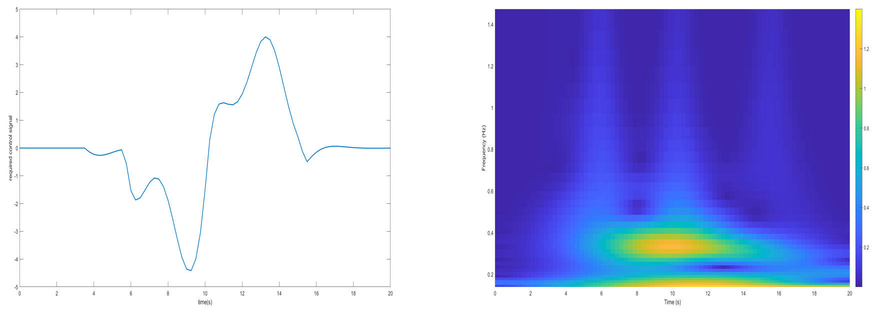

*Figure 5.36: Plots of the Control Signal and corresponding wavelet Transform Spectrogram*

### 5.2.3 Four Vehicle Comparison

It has been mentioned in earlier chapters that the previously discussed algorithm exhibits some weaknesses and shows areas for improvements. We thus in what follows will begin by showing the performance improvement of the hybrid algorithm over the Collision Avoidance one, by simulating the same scenario in the 'Four Way Urban Intersection Identical Vehicles' Scenario. We will then simulate a different scenario in both the 'Time to Collision' and 'First Come First Serve' priority schemes comparing the outputs of the controllers and comparing the efficiencies of the priority schemes.

**Performance Comparison**

In the development of the hybrid algorithm, we had to consider various possible approaches when expanding the problem from the two vehicle case to the four vehicle case. In order not to overly strain the network, we investigated the following possible methods:

- Re-plan trajectory based on the first entry point: In this way only the first entry point is considered in the time suggestion algorithm of the intersection manager.

- Re-plan trajectory based on the first occupied point of entry: In contrast with the previous approach, the time suggestion algorithm takes into consideration the first conflict point that will be passed by other vehicles in the 'Time Suggestion Zone'.

- Re-plan based on all conflict areas: In this algorithm the time suggestion loops over all possible conflict areas and suggests times to all affected vehicles in the 'Time Suggestion Zone'.

**Remark:** It should be noted that time suggestion occurs only to vehicles within the 'Time Suggestion' Zone by design, as we don't want the two controllers to work at the same time which might lead to unpredicted errors. The sizes of the zones were selected on the basis of the considered safety distance.

We begin our comparisons by plotting the velocities and accelerations for all vehicles in the considered scenario ( We refer the reader to Figure 5.15 and Table 5.3 for scenario topology and Initial conditions).

We begin by discussing the response of vehicle 4, which upon entry has only vehicle 1 (Coming from the top) in the 'Suggestion Zone'. We notice that its response is thus very similar to the case without the time suggestion application when the 'Entry Point' method is used, This also leads to a similar response from vehicle 2.

This motivated us to explore the possibility of using the 'First Occupied Point of Entry' For time suggestion. This allows vehicle 4 to consider Vehicle 1 which is not at its entry point, the modified response can be seen plotted in pink in Figure 5.37. Due to a small braking action upon entry, the required braking for avoiding collision is reduced, moreover this translates to lower effort required from Vehicle 2 as can be seen in figure 5.38.

The 'smoothest' and least demanding responses however, were observed in the Re-plan All' Approach where all possible conflict points are considered by the Intersection manager for suggesting the times, this leads to very small and minimal braking from the vehicles, as can be seen by looking at the cyan plots in the before mentioned figures. As for vehicle 3, minimal

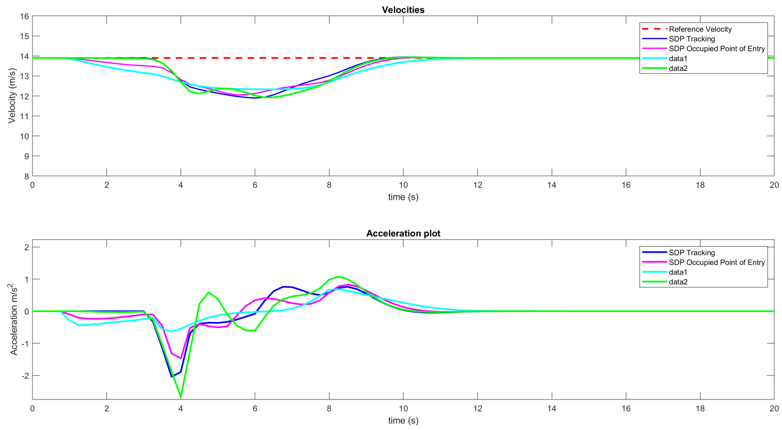


*Figure 5.37: Plot comparing responses for Vehicle 4 (Right, Yellow)*

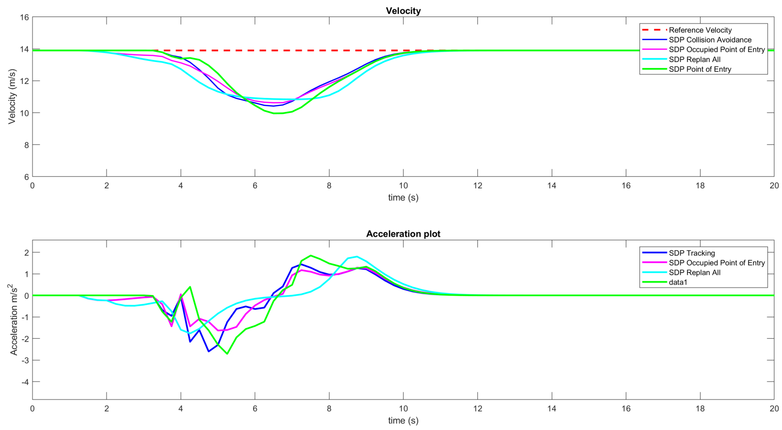


*Figure 5.38: Plot comparing responses for Vehicle 2 (Down, Purple)*

differences between the responses can be observed, mainly due to the fact that little deviation from the reference trajectory is required for the safe passage through the intersection space.

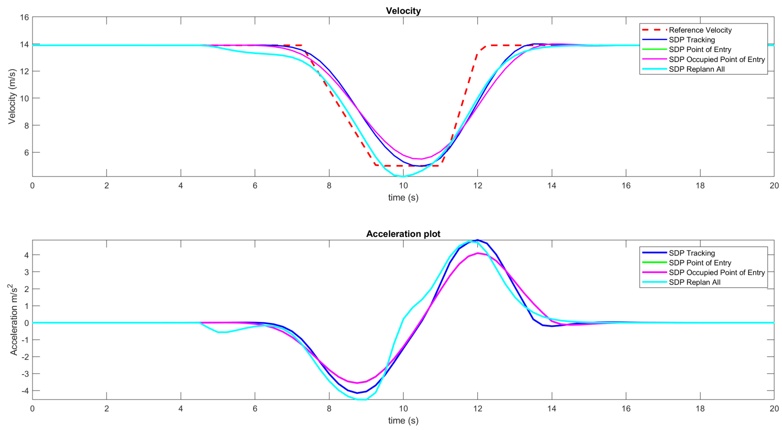


*Figure 5.39: Plot comparing responses for Vehicle 3 (Right, Blue)*

Analysing the time response of the vehicles is however not sufficient, we thus produce the response in the time frequency domain using the wavelet transform. In the figures below, only the responses of the 'Occupied Point of Entry' and the 'Re-plan All' approaches are reproduced, as they are the responses with most promise.

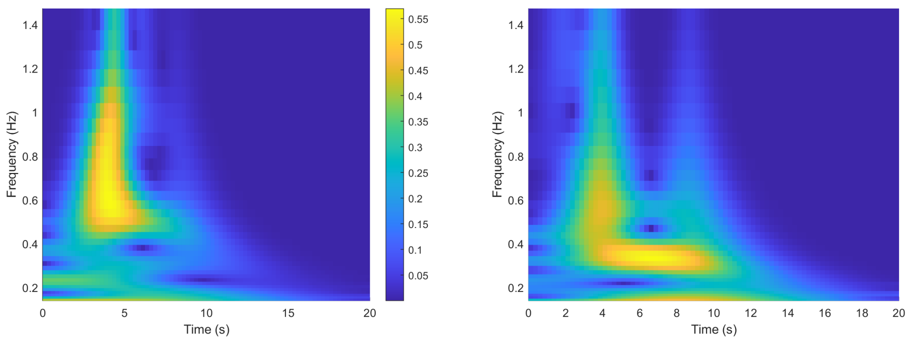


*Figure 5.40: Spectrogram of the wavelet transform for the Occupied Entry Point (Right) and Replan All (left) for Vehicle 4*

Looking at the time-frequency plots for vehicle 3, for the Occupied Point of entry method it is seen that the dominant frequency band during the braking maneuver is between 0.6 and 1.2 Hz, for the 'Re-plan all' (RA) method however, the band is 0.4 Hz to 0.8 Hz which can be translated to a softer motion profile. As the 'Occupied Point of Entry' (OPE) method shows very similar response to the collision avoidance response (with shorter peaks in the acceleration graph) the improvements (RA) has over (OPE) can be extended to the other approaches discussed earlier.

The most significant improvement can be observed in Figure 5.41, where

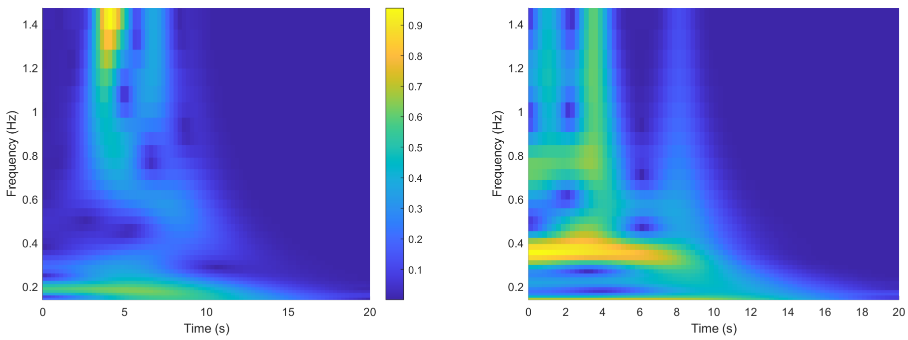


*Figure 5.41: Spectrogram of the wavelet transform for the Occupied Entry Point (Right) and Re-plan All (left) for Vehicle 2*

the most dominant frequency changes from 1.2Hz to 0.4 Hz, this occurs due to spreading the needed braking over a long period which leads to avoiding excessive late braking.

The results both in the time and frequency domains show the performance related improvements that the Hybrid algorithm has over the Collision Avoidance once. It is however also of interest to be able to achieve the desired safe passage regardless of the considered priority scheme, which is not always the case for the collision avoidance controller.

The last performance check was to check the range of actuation time constants $T_a$ for which the controller was capable of achieving the desired outcome. The simulation was carried out for the same set of parameters studied in Section 5.1.4 and the outcomes are shown in Figure 5.43. It can be seen that the set of parameter values that achieve the desired outcome is greater than in the previously considered case for the entire range of $T_a$ considered. In fact the simulations ended in success for all the considered values, proving the Hybrid approach to me more robust.

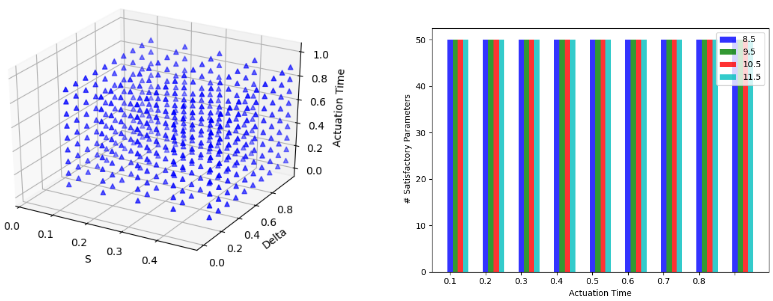


*Figure 5.42: Plots showing the improved ability of the Re-plan All Hybrid method at achieving safety for all $T_a$*

### First Come First Serve Priority Scheme

In this section, we aim to show that on its own, the collision avoidance controller is not capable of satisfying the First come First Serve Priority Scheme.This is important since human legacy drivers in urban 4-way intersections often rely on a passage rule that is similar to the First Come First Serve Priority Scheme, thus for any integration between cooperative vehicles and legacy driving, it should be possible to enforce the crossing order on the basis of the human drivers.

Our scenario is simulated using the normal 4-way urban intersection setting, however the initial conditions of the vehicles have changed from the previously modelled scenarios and are summarized in the following table :

From the table we can see that the order of entry is [2,3,1,4]. The

| | | | |
|---|---|---|---|
| Safety Distance | 9.5 m | | |
| Vehicle 1 Initial Position | (0, - 113.95m) | Planned Velocity | 15 m/s |
| Vehicle 2 Initial Position | (110.15m, 0) | Planned Velocity | 13.9 m/s |
| Vehicle 3 Initial Position | (3.7m, 110.85m) | Planned Velocity | 12.5 m/s |
| Vehicle 4 Initial Position | (122.5m, 3.7m) | Planned Velocity | 13.9 m/s to 5m/s |

*Table 5.8: Table showing the initial conditions for the considered Scenario*

priority ranking based on the Time to React is however [1,3,2,4]. It is thus important to see if the controllers are able to force the crossing or-

der, knowing that vehicle 2 arrives within a safety distance of the conflict area at a time when vehicle 3 is already inside the intersection zone.
<u>Collision Avoidance Controller Results:</u>

We can see that Vehicle 2 doesn't not deviate from its reference velocity,

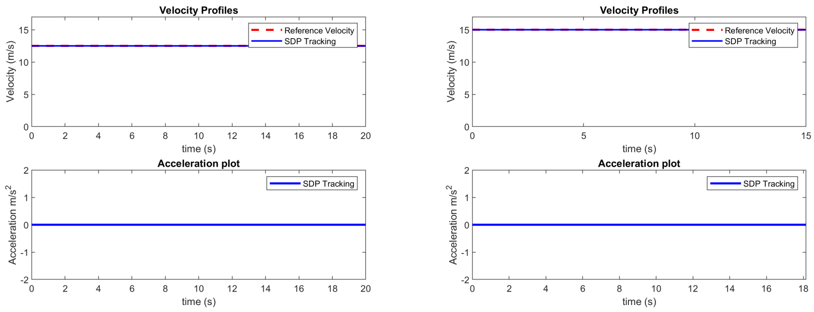


*Figure 5.43: Time Response of Vehicles 2 and 3*

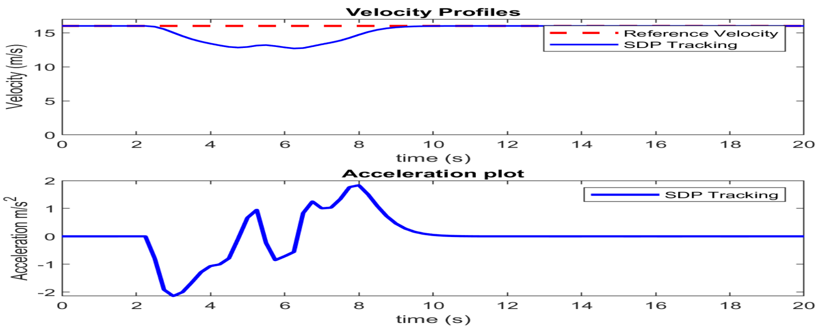


*Figure 5.44: Time Response of Vehicle 1*

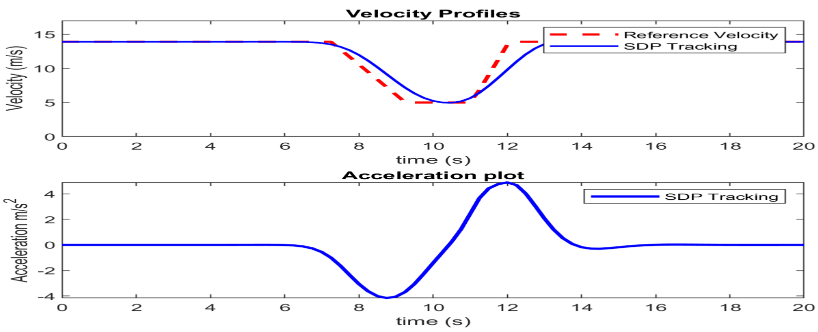


*Figure 5.45: Time Response of Vehicle 4*

which means that it does not concede its position to vehicle 3, since vehicle 3 arrives within a safety distance of the conflict region at a very late time. On the other hand, Vehicle 1 concedes entry to Vehicle 2 as can be seen from the time response (Fig. 5.44). Safe Passage however was achieved as the safety distances was kept as can be seen from Figure 5.46. No emergency braking was done since the controller for Vehicle 2 did not predict any violation of the safety distance which could not be avoided throughout the vehicle's motion.

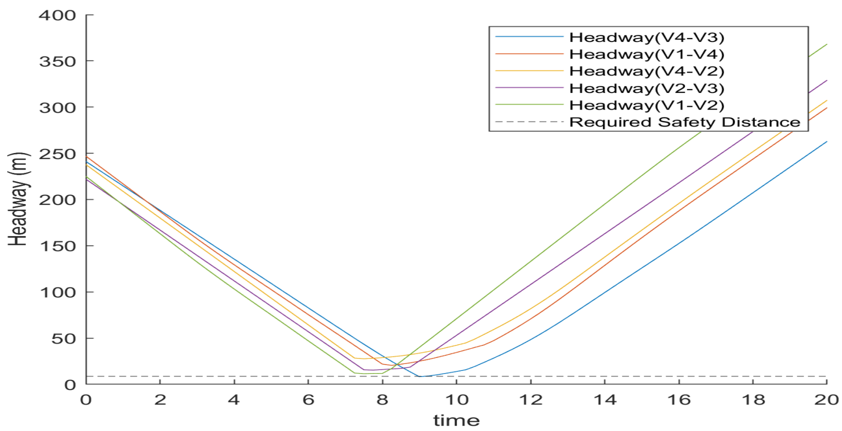


*Figure 5.46: Achieved Head-ways with FCFS Priority Scheme,CA controller*

Hybrid Control Algorithm Results

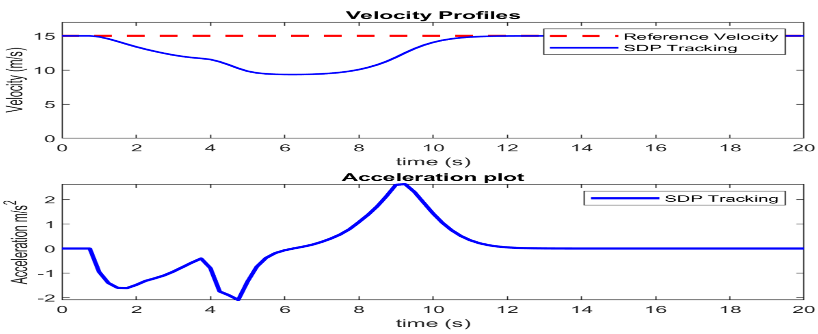


*Figure 5.47: Time Response of Vehicle 2*

What can be seen from the results,besides the smoothness of the response

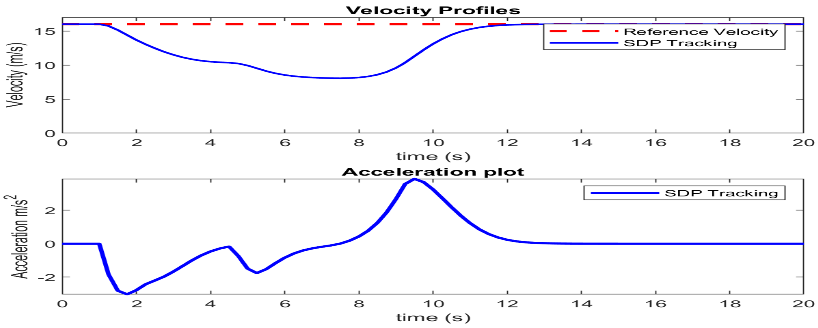


*Figure 5.48: Time Response of Vehicle 1*

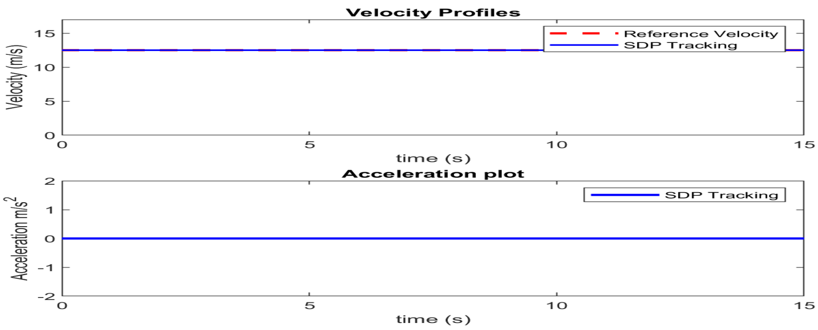


*Figure 5.49: Time Response of Vehicle 3*

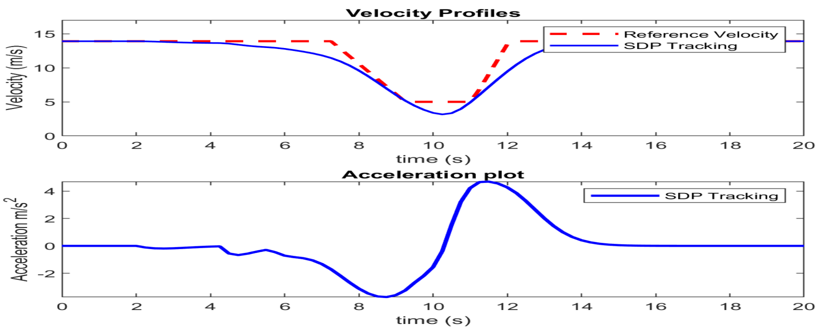


*Figure 5.50: Time Response of Vehicle 4*

is that the crossing order follows that of the Priority scheme, where significant deceleration is observed in Vehicles 1 and 2. Vehicle 2 thus brakes allowing Vehicle 3 to pass while Vehicle 1 concedes to both vehicles. It is also noteworthy that the suggestion does not on its own guarantee the safe passage, where the collision avoidance controller is triggered at t = 5 seconds for vehicle 2 while it is also triggered at t =5.25 for vehicle 1. We can finally observe that safe passage was achieved satisfying the minimum safety distance constraint (Fig. 5.51).

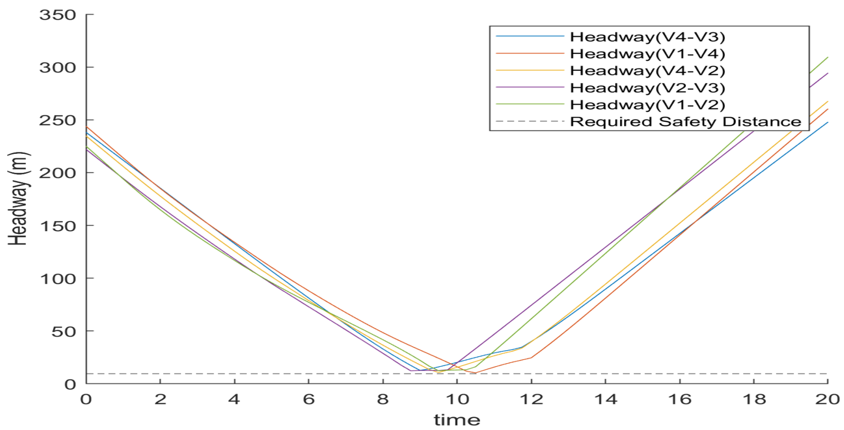


*Figure 5.51: Achieved Head-ways with FCFS Priority Scheme,Hybrid Control Algorithm*

**Conclusion**

From the preceding analysis, it can be clearly observed that the Hybrid Algorithm provides improvements in various areas over the Collision avoidance controller alone.

1. It allows for early adjustment of behavior which leads to a lower deviation from target velocity and a smoother response improving comfort.

2. It improves performance for the various considered actuation time constants $T_a$ and head-ways increasing the robustness of the system.

3. It provides regulation for passing order when needed

## 5.3 Comparison of Priority Schemes

In previous sections the priority schemes employed were not analysed, and due to the fact that passing order can be enforced by the Hybrid algorithm, it is of interest to compare the results coming from the various priority schemes.In this section we will begin by introducing a variant of the Time to

Collision scheme which allows for the consideration of an emergency vehicle, we will then compare the three schemes (First Come First Serve, Time to Collision, TTCEM) based on the scenario employed in the preceding section . We will only be using the Hybrid Algorithm since it proved its advantage over the Collision Avoidance controller alone.

### 5.3.1 Time To Collision - Emergency Vehicle

In a cooperative vehicle scenario, it should be considered the cases with emergency vehicles, they in the setting should take priority over the rest. This motivated a variation of the Time to collide priority scheme. Recall that the safety distance was defined in (4.23) as:

$$t_{s,k} = \frac{d_{safety}}{\bar{v}^k}$$

We thus use this definition to define the region in space in which cars should concede to the emergency vehicle. The reasoning is that if another vehicle is inside the intersection space and is potentially in conflict with the emergency vehicle, it should be the one to concede passage. The new Time to React for the emergency vehicle is calculated as:

$$TTR_{Em} = TTR_0 - t_{s,em} \tag{5.3}$$

In this sense all vehicles who plan on arriving within a time period of the safety time (i.e. vehicle which might lead to the emergency vehicle slowing down to concede entry) would have to prioritize the emergency vehicle which leads to minimizing the emergency vehicle's need to decrease its own velocity. This is done through adding the type parameter to the vehicle class which is sent in the packet to the Intersection Manager, if the type is detected to be 'Emergency', (5.3) is used to calculate the time to React and update the priority lists.

We simulated the same scenario discussed in Table 5.8, while specifying the type of Vehicle 2 to be an emergency vehicle. In this way, Vehicle 3 should concede to Vehicle 2 (the opposite is true for the normal TTR case).

We notice that again the priority was followed and the control signals were not excessive while keeping the minimum required safety distance(Fig 5.54).

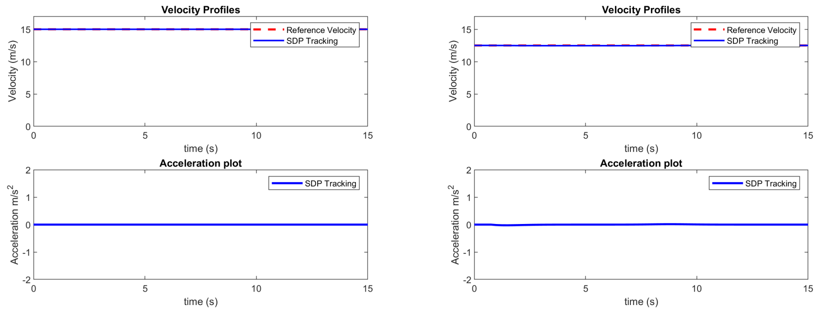


*Figure 5.52: Time Response of Vehicles 2 and 3 and employing the EMTTR priority scheme*

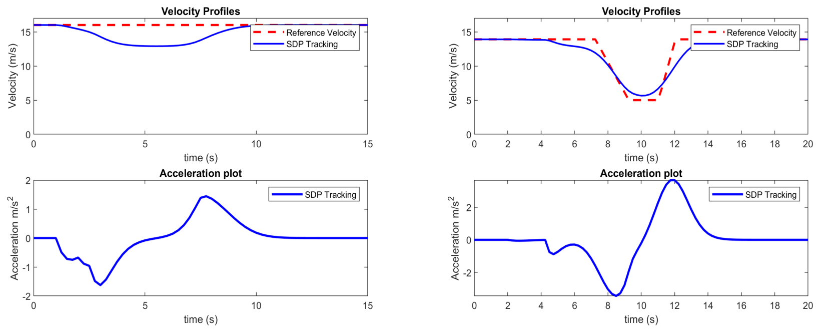


*Figure 5.53: Time Response of Vehicles 1 and 4 employing the EMTTR priority scheme*

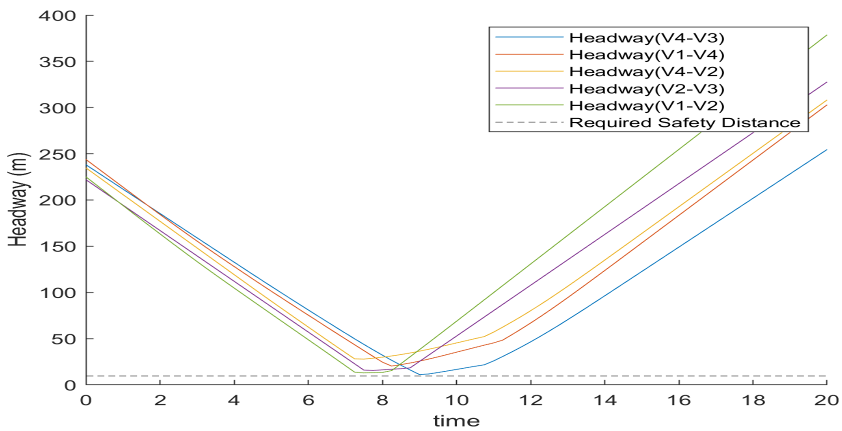


*Figure 5.54: Achieved Head-ways with EMTTR Priority Scheme,Hybrid Control Algorithm*

### 5.3.2 Priority Scheme Comparisons

In this section, we will compare the outputs of the main priority schemes in this thesis, for the last discussed scenario. To do so, we first report the time response of the Vehicles when the Time To React Scheme is employed We

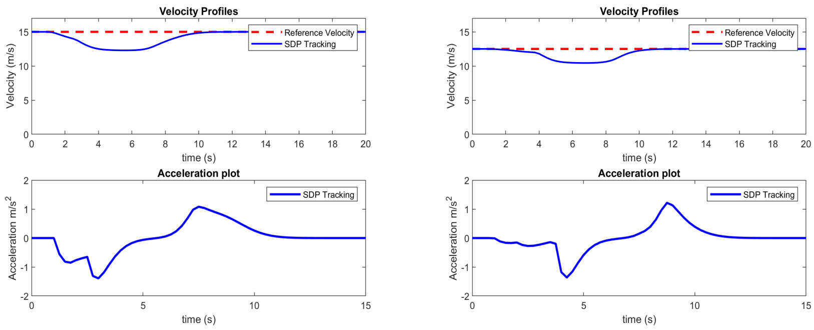


*Figure 5.55: Time Response of Vehicles 2 and 3 employing the TTR priority scheme*

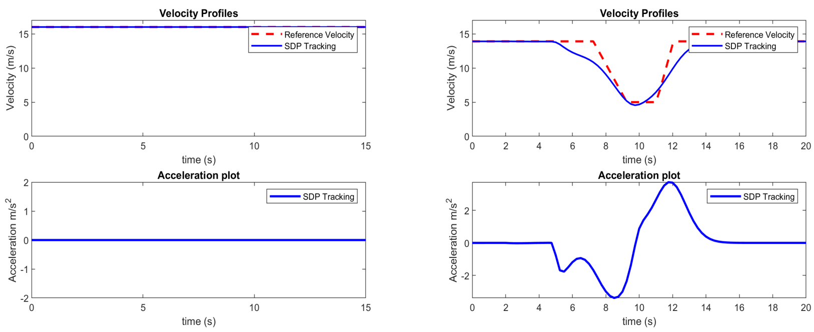


*Figure 5.56: Time Response of Vehicles 2 and 3 employing the TTR priority scheme*

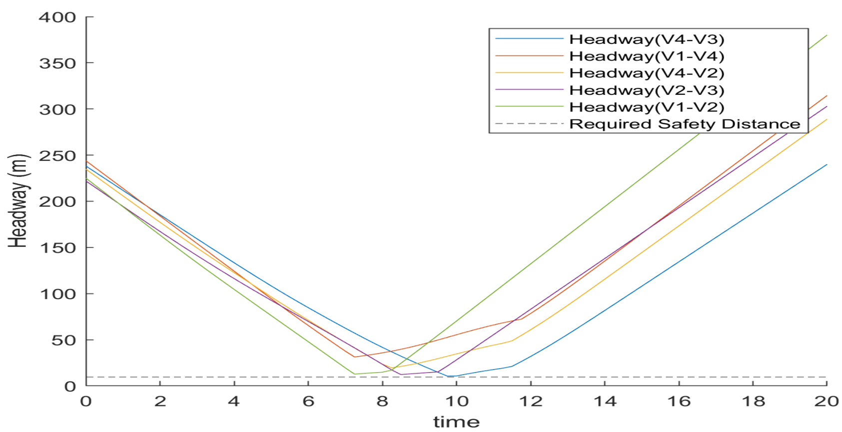


*Figure 5.57: Achieved Head-ways with TTR Priority Scheme,Hybrid Control Algorithm*

can see that the crossing order again changed with respect to the previously

considered schemes. We thus compared the outputs to see which of the approaches is more efficient. We thus employed two cost functions to help us quantify the results.
The first cost function is a minmax function of the form:

$$\sum_{i=1}^{4} max(v_i - v_{ref_i}) + \sum_{i=1}^{4} maxa_i + \sum_{i=1}^{4} |mina_i| + \sum_{i=1}^{4}\sum_{j=1}^{4} \min Headway_{ij} - D_{safety} \tag{5.4}$$

which is a collective cost over all the vehicles taking into consideration how much the vehicles are deviating from the reference velocity, the maximum acceleration s and deceleration in addition to the deviation from the safety distance. The costs for the three schemes are reported in Table 5.9

| Scheme | Cost |
|---|---|
| TTR | 35.347 |
| TTREM | 40.8479 |
| FCFS | 50.0689 |

*Table 5.9: Costs associated with the various schemes following the minmax Cost formulation*

It could thus be seen that the TTR scheme entails a much lower cost than the FCFS, which motivated its use in the cooperative vehicle scenarios. We also used the cost term which is often used in the literature to evaluate the efficiency of the applied control, and is a measure of how much each vehicle is giving up in terms of velocity :

$$M = \sum_{i=1}^{4} \mu_i = \sum_{i=1}^{4} \sum_{t=0}^{t_f} 1 - \frac{v_{actual}}{v_r ef} \tag{5.5}$$

It is interesting to see that the emergency variant of the TTR performed

| Scheme | M |
|---|---|
| TTR | 16.9764 |
| TTREM | 6.8964 |
| FCFS | 31.47 |

*Table 5.10: Caption*

the best in this metric. This shows one disadvantage of the distributed approaches, which is the reliance on a heuristic for the calculation of the crossing order. This shows that possible modification for the widely used TTR approach which takes into consideration the number of vehicles that would react in response to the variation in the behavior of each vehicle would possibly lead to an improvement in performance.

# Chapter 6

# Conclusions and Future Works

## 6.1 Conclusion

This thesis has studied a distributed algorithm for the coordination of connected and autonomous vehicles at intersections. The problem is formulated based on an individual vehicle basis, where each vehicle is computing its own trajectory in order to guarantee the collective safe passage.
The first proposed algorithm, based on a completely decentralized approach performed well in scenarios including 2 or and 4 vehicles. This showed the ability of the discussed algorithm to be generalized for problems such as highway lane merging. Some room of improvement was present when it came to enforcing crossing order (which is desirable in the non-fully cooperative scenarios).
Being a collective problem, we therefore added a level of centralization through the second discussed algorithm which allows a higher level of coordination between vehicles prior to activation of a collision avoidance controller. This showed improvements over the completely decentralized approach, in performance and robustness while also allowing to enforce a strict crossing order.

## 6.2 Future Works

Throughout the development and discussion of our results, we mentioned and often addressed some of the main issues that are still open in this area of research, we now will summarize them highlighting areas of improvement and possible future works:

**Robustness to perception and localization Uncertainties:**

In this thesis, we performed a simple robustness analysis that takes into consideration, measurement uncertainties and model uncertainties. We found out the second proposed algorithm satisfied the safety condition for a range of model uncertainties along and a range of safety distances ( this range was assumed to account for measurement uncertainties).
The distance to the intersection, usually found by combining maps with an absolute positioning system (GNSS) is highly sensitive to disturbances, therefore more in depth analysis and construction of control algorithms that are provably robust to positional uncertainties should be constructed in order to ensure safety. It is important to note that such algorithms are an open question in literature.

**Inclusion of Legacy Drivers and non-cooperative entities:**

This thesis along with the majority of the literature focus on completely autonomous and cooperative driving scenarios. While autonomous vehicles are expected to penetrate the market in the next decade, reaching a full autonomous setting will still take more time. It is therefore of practical importance to extend the fully cooperative algorithms to include non-cooperative entities.
In this thesis we included the aggressiveness term to account for driver aggressiveness and possible non-cooperativeness can also be expanded in the second algorithm by showing no compliance to the suggested times. Our algorithms can be expanded to include legacy drivers if correct prediction models of driver behaviors and aggressiveness can be used to generate predicted velocities for legacy drivers. The current literature in this aspect [84, 84, 86] can predict the direction of travelling (straight, left-turning, right-turning). Accurate velocity predictions would require an aggressiveness estimation

based on historic data along from previous and current trips and is a much more involved task.
Including human drivers also means that priority schemes that follow human traffic laws are important for seamless integration, for this task, our second algorithm was capable of correctly enforcing the order of passage based on a first come first serve priority scheme which is very similar to current traffic laws. We also proposed a priority scheme which takes into consideration emergency vehicles giving them precedence over the rest of vehicles in the scenario. This law was correctly enforced by our second algorithm, a similar scheme giving non-cooperative entities priority can be employed when the number of automated vehicles on the roads reaches high percentages.

**Multiple intersections:**

Current works in the literature along with this thesis focus on a 4-way urban intersection setting, in real life scenarios the traffic grid is constructed of many intersections, and other traffic components. A coordination algorithm that would be able to take into consideration, the traffic flow maximizing throughput of the whole traffic network would lead to much more efficient roads. This however would increase the complexity of the optimization problem and might be infeasible for real-time applications.Swarm Intelligence and Formation Control methods can be used for these purposes if fully cooperative scenarios are considered.

**Improved Heuristic Priority Schemes:**

In this thesis, we presented a priority scheme that takes into consideration emergency vehicles, in the analysis of a scenario it was shown that the output of that scheme out-performed currently used heuristics. This shows one area of weakness for distributed approaches, where the crossing order due to heuristics does not always yield an optimal result. Supervised or Reinforcement learning techniques can be used in this area in order to come up with optimal crossing orders based on certain inputs (number and types of vehicles, velocities, traffic congestion at target lanes) that would lead to improvements on current results.

### Cyber Security

Communication of private information between vehicles remains important for the feasibility of a coordinated scenario, this information can be corrupted or used for malicious purposes, for this reasons, further consideration should be taken to consider different kinds of attacks.

We thus see, looking at the possible areas of improvement, why the topic of this thesis has been popular in the last couple of years, on a personal level we aim to in the future validate the simulation results experimentally, while expanding the network to include more than 4 vehicles and multiple intersections. The most pressing area of improvement is involving legacy drivers and considering robustness to measurement uncertainties which should be the next steps in any future work.

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