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Input Data for a State Estimation Algorithm

Tesi di Laurea Magistrale in
Ingegneria Elettrica - Electrical Engineering -

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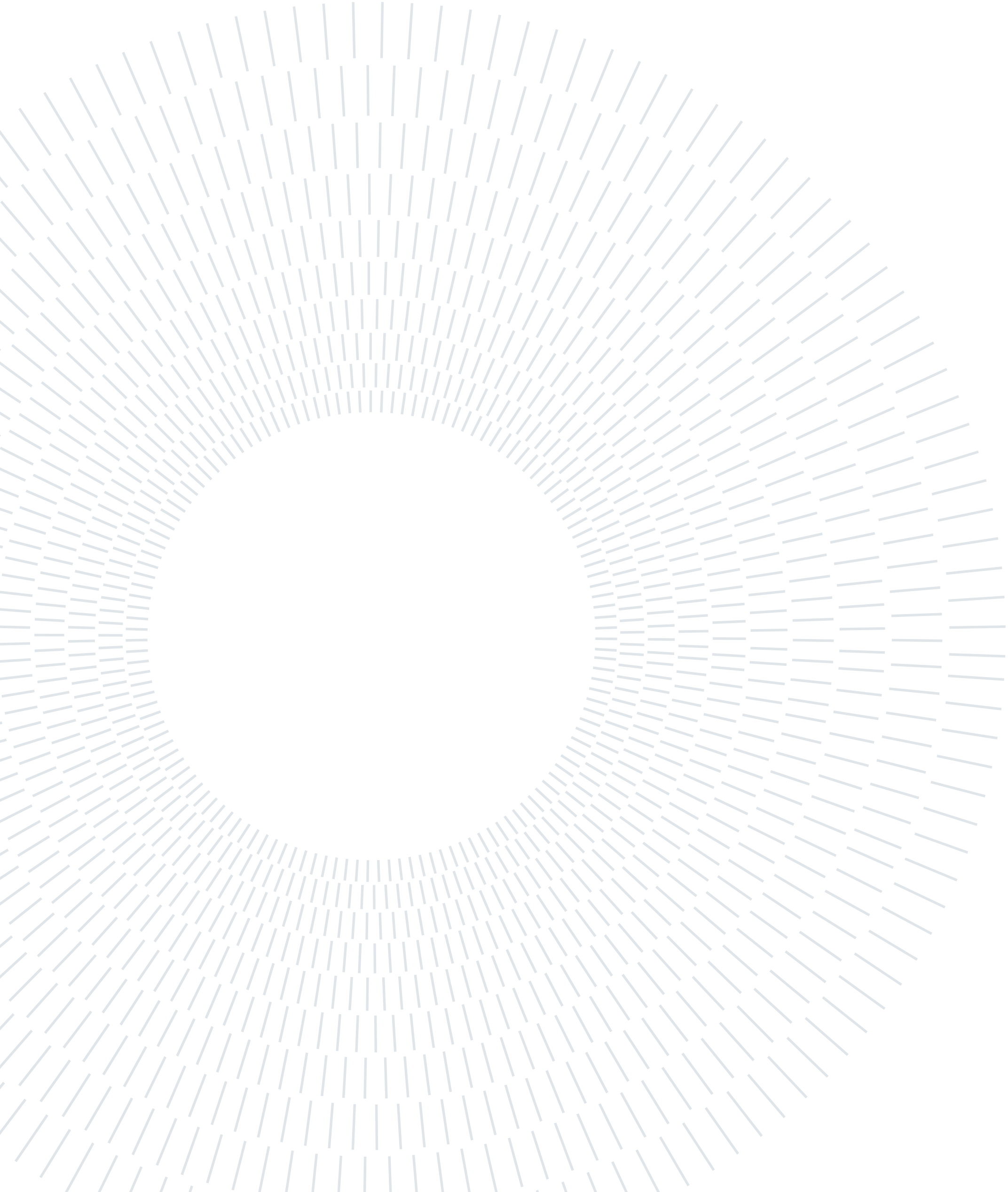

Acknowledge

Today, I just have thanks and good memories of having finally achieved this dream that starts 2 years ago. Thanks God for all your blessings.

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I am also very lucky to have grown in a lovely and united family. My parents, sister, uncles that are like my second parents, cousins that are like my siblings, and grandparents have always been present in every step of my life, and I really hope that they can continue to be by my side for much longer. As it's said, the character is molded at home, and in my house all I always saw was respect, love, and persistence; so, while outside I was preparing myself to become a good professional, at home they were teaching me, in the best possible way, with examples, to be a good person. Thank you Mami y Papi for giving me everything, all you have had and more, and for your support and trust every day of my life. This achievement is all yours.

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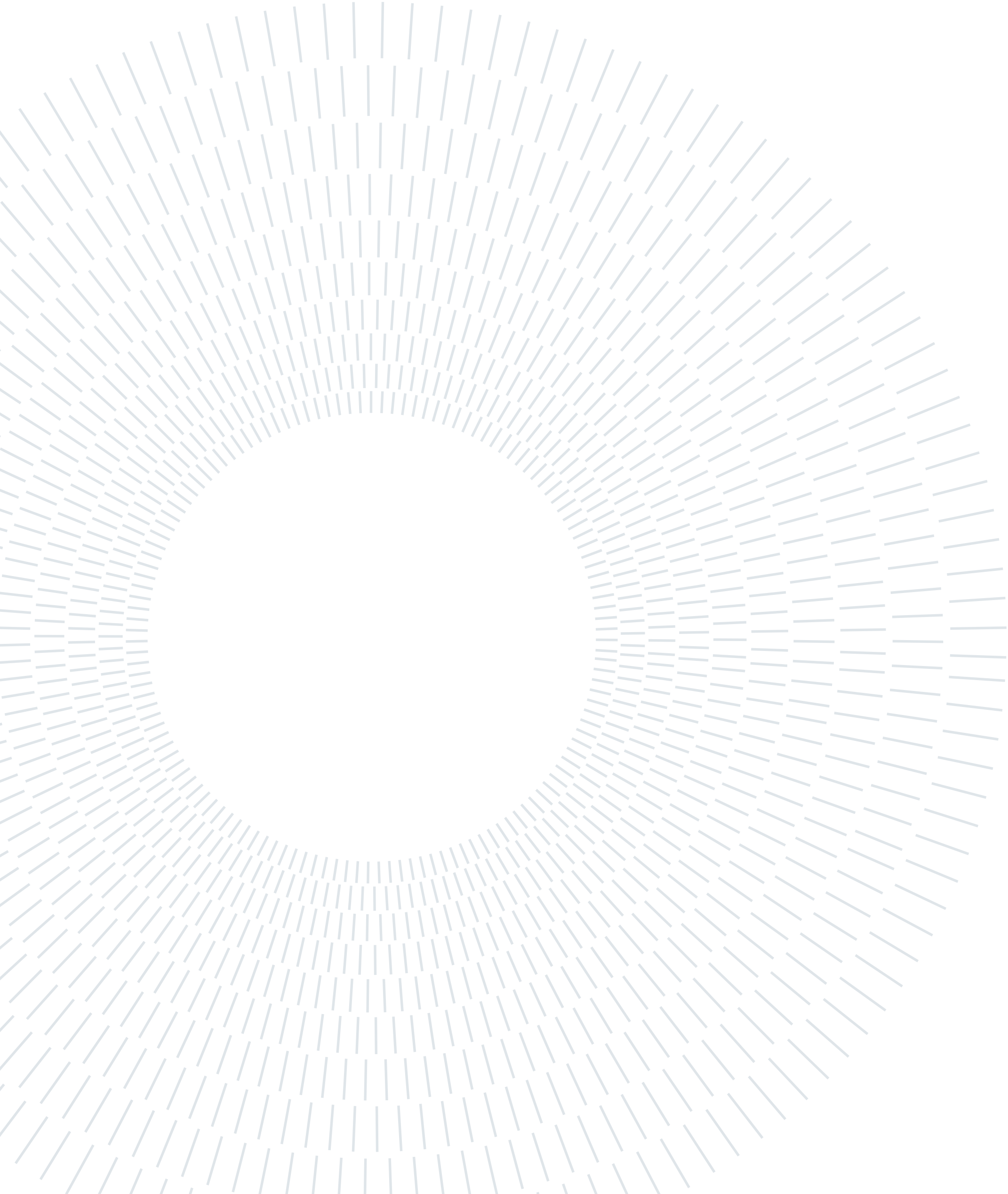
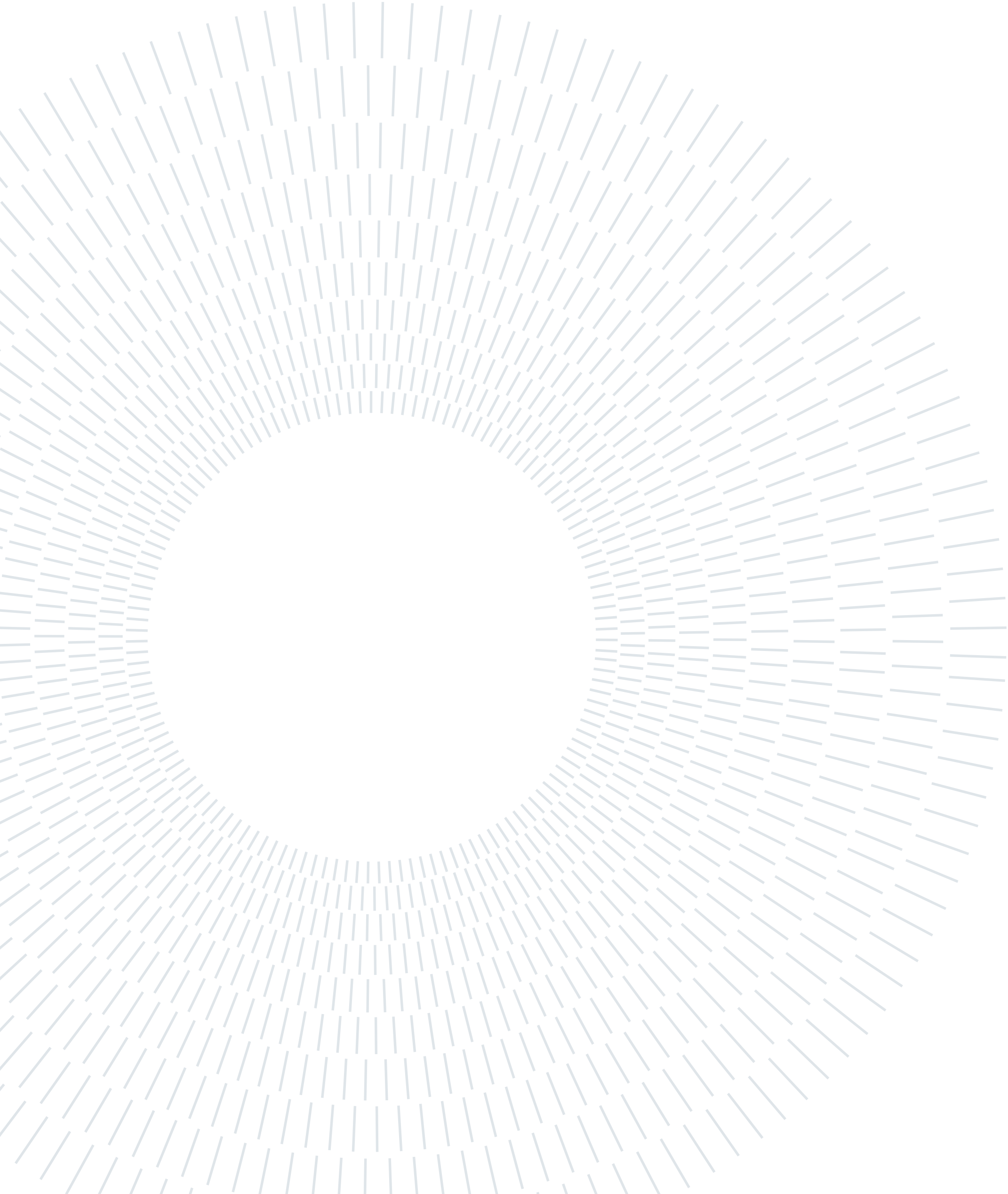


Abstract

Given the "recent" change in the conception of the electrical network is logical to think that the way in which the network has been analyzed and managed has also suffered some alterations; changes that are necessary to keep track of the numerous advances of the grid. One of those changes is, for example, the necessity to make State Estimation analysis (SE) also to the low voltage grid and not only to the transmission network, as it was in the past.

The objective of this project is to prepare the IEEE-906 low-voltage test feeder, to be a suitable input for an algorithm to develop state estimation on a low-voltage distribution network. For a system to be suitable for the estimation process is necessary that the system is observable, meaning that the states can be estimated with the available measured data, That is why, the original information of the system is not enough, and some considerations were done during the development of the project.

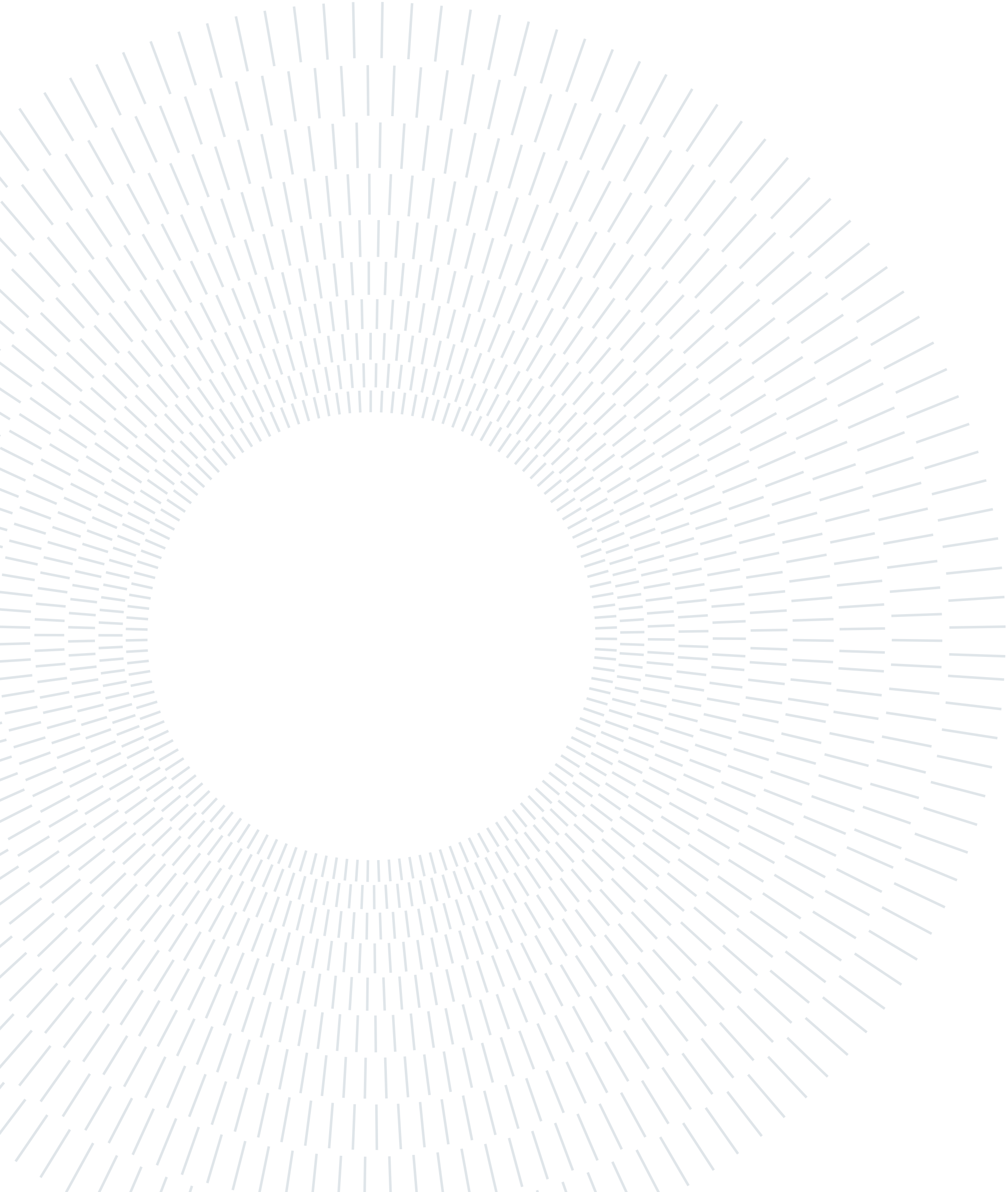
Keywords: Centralized generation, Distributed Generation, State Estimation, Static SE



Abstract in lingua italiana

Considerato il “recente” cambiamento nella concezione della rete elettrica è logico pensare che anche il modo in cui la rete è stata analizzata e gestita abbia subito delle alterazioni; modifiche necessarie per tenere traccia dei numerosi progressi. Uno di questi cambiamenti è, ad esempio, la necessità di effettuare l’analisi di stima dello stato (SE) anche sulla rete di bassa tensione e non solo sulla rete di trasmissione, come avveniva in passato.
L’obiettivo di questo progetto è preparare l’input, utilizzando la rete IEEE-906, per un algoritmo sviluppato per la stima dello stato su una rete di distribuzione a bassa tensione. Affinché un sistema sia adatto al processo di stima è necessario che il sistema sia osservabile, ovvero che gli stati possano essere stimati con i dati misurati disponibili, per questo motivo le informazioni originali del sistema non sono sufficienti e sono state fatte alcune considerazioni durante lo sviluppo del progetto.

Parole chiave: Generazione centralizzata, Generazione distribuita, Stimazione dello Stato, SE statica



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Introduction

Nowadays, the electrical network is not the same as it used to be. Initially, the generation of electrical energy was **centralized**, meaning that the energy was produced in big generation plants (not only in extension but electrically speaking), usually powered by fossil fuels and therefore placed in remote areas, from where the energy was then transmitted to the users. Over time, this panorama has changed. The generation is increasingly becoming **decentralized**.

Decentralized Generation, also known as Distributed Generation (DG), is understood, as the production of energy closer to the consumption place. The possibility of generating energy for one's consumption on the rooftop of a house is all due to the technological advances, that allow everyone to exploit a better kind of electrical energy resources, the so-called Renewable Energy Resources (RES).

With all this being said, is logical to think that the way in which the network has been analyzed and managed has also suffered some alterations; changes that are necessary to keep track of the numerous advances of the network, making it more intelligent every day. One of those changes is, for example, the necessity to make State Estimation analysis (SE) also to the low voltage grid and not only to the transmission network, as it was in the past.

Since SE serves as a real-time network model that is utilized to set up other real-time operation and control functions, it is essential to the real-time operation and control of the electrical network. Nevertheless, given the youth of the DG, the methods for SE in Low Voltage Distribution networks have yet to be researched. This document shows the testing of one proposed algorithm to realize SE, using the IEEE Low Voltage Test Feeder of the European Network (See chapter 2).

The distinctive features of distribution networks have made estimation incredibly challenging. Some of these features are listed below:

- There are unbalanced loads, creating an unbalanced three-phase system.
- The existence of charges dispersed over small distances.
- Feeders with a high R/X ratio.
- Combination of distributed generation (DG) and non-distributed traditional generation on the same grid.
- No measures for feeders (low redundancy).
- Long and short lines connected on the same node.

Therefore, due to these characteristics, it is not possible to directly apply transmission network estimating methodologies to the distribution system.

One could say that most of the research on state estimation in distribution networks started a little over 20 years ago and is now ongoing. Furthermore, the development of technology leading to the emergence of more affordable electronics, and advancements in research to obtain a more accurate estimate and thorough understanding of the system's behavior require searching for models that complement one another.

The attempt to upgrade distribution networks while considering the optimum cost/benefit ratio is one of the primary motivations for researching this topic. In addition, specialized estimators for distribution systems could need to be developed and creative solutions for future state estimators should take into account potential technology advancements.

1 | Theorethical Background and Literature Review

In the section below, you will find the descriptions of useful concepts to further understanding of the study case later described.

1.1. Traditional Power System

A formal definition of a Power System, according to IEEE is "The generation resources and/or transmission facilities operated under common management or supervision to meet load and interchange commitments" [19], which means that the electric power system is nothing more than a network of electrical machines, lines, and electronic devices exploited to generate electricity and supply energy to a group of customers.

The traditional Power System is hierarchically organized into three sectors: Generation, Transmission, and Distribution, as shown in Figure 1.1. Starting on a generation Station, the energy then flows toward the customer, following a unique direction. Generators feed into a High Voltage (HV) Transmission System, to transmit the power over the long distance between the generation station and the customer. On the way to the consumer, there are different Transformation Stations, the first one, known as the Primary substation, steps down the HV into a Medium Voltage level (MV), to enter the distribution network, from where is then distributed to different Secondary Substation, where the MV is transformed into Low Voltage (LV) for immediate use. [2][15]. (See Table 1.1)

Voltage Level	Maximun	Minimum
High Voltage	345 kV	100 kV
Medium Voltage	100 kV	1000 V
Low Voltage	1000 V	100 V

Table 1.1: Voltage levels in the Power System [18]

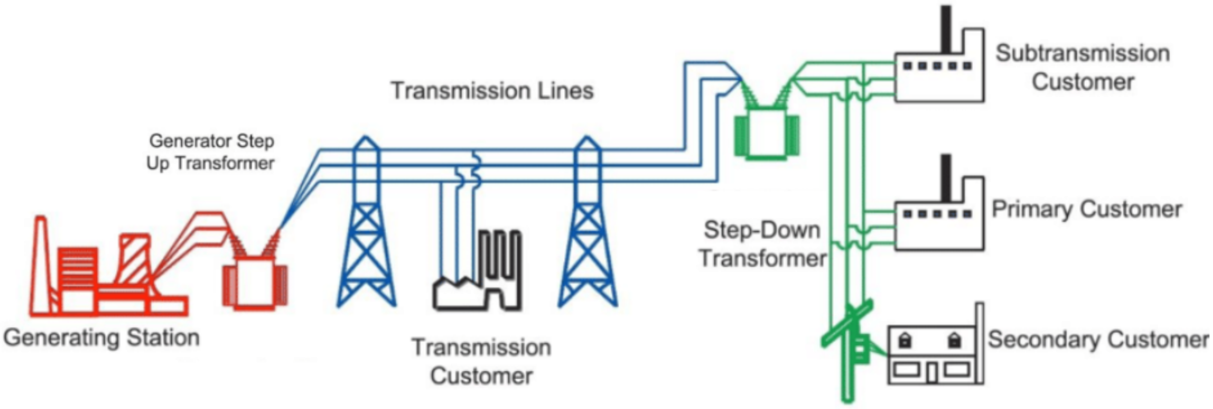


Figure 1.1: Energy flow in a traditional Power System [20]

1.1.1. Generation

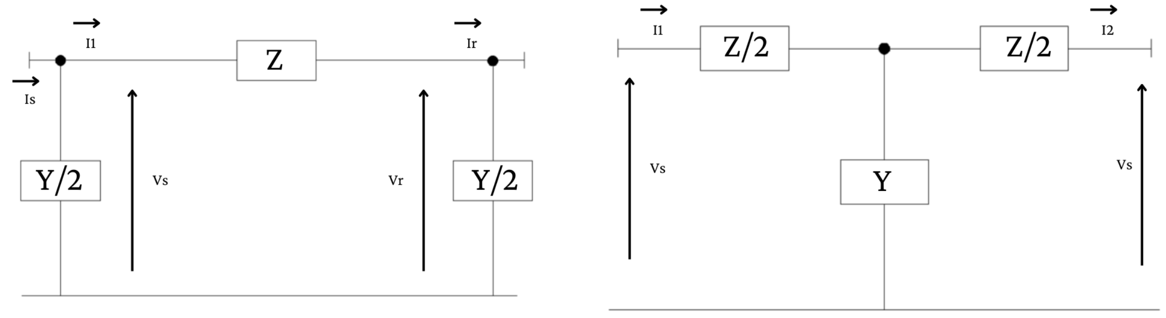
In the Generation Section, of traditional Power Systems, synchronous generators are used to convert mechanical energy into electrical energy. Most of the time, mechanical energy comes from either flowing water or thermal energy, which primarily comes from coal, natural gas, oil, and nuclear fuel. The power produced in these generators is usually generated at LV and then is immediately stepped up by a transformer before going into the Transmission System.[2][15]. Generation is usually modeled as a power injection.

1.1.2. Transmission Network

The electricity is centrally generated in the "Generation Plants". The voltage levels for each section are determined based on the amount of power and the distance it has to cover, therefore, to feed a huge quantity of power over the long distance till the load, an HV network is used, called: Transmission Network. This system interconnects all the generating stations and major load centers in the system and is highly *meshed* to ensure a higher level of reliability.[2][15]

Transmission Lines

Transmission lines are often represented as a two-port model, with the equivalent impedance of the positive sequence equivalent circuit. In this sense, a transmission line with positive sequence impedance $R + jX$ and total line susceptance jB can be modeled by the equivalent circuits shown in Figure 1.2.



(a) Equivalent Π model.

(b) Equivalent T model.

Figure 1.2: Transmission lines for network modeling.

Transformers

Transformers can also be modeled as a two-port system, as shown in Figure 1.3. The equation [20] of this two-port circuit can be computed as follows:

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{1.1}$$

Taking into account that for any transformer with tap ratio a:

$$\begin{cases} i_{primary} = i_{secondary}/a & \text{(1.2a)} \\ v_{primary} = a * v_{secondary} & \text{(1.2b)} \end{cases}$$

The equation 1.1 can be also written:

$$\begin{bmatrix} i_t \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11}/a^2 & y_{12}/a \\ y_{21}/a & y_{22}/a^2 \end{bmatrix} \begin{bmatrix} v_t \\ v_2 \end{bmatrix}$$

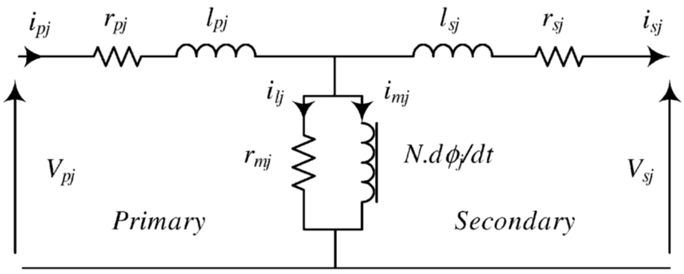


Figure 1.3: Transformer model for network modeling [[15]]

1.1.3. Distribution Network

The distribution represents the final stage of power transfer. It refers to the power supply lines between high-voltage substations and customers[2]. The distribution network, opposite to the Transmission, is generally connected in a radial structure[15], in which we can identify two sections:

- The primary distribution voltage, typically at an MV level, where some industrial loads, with higher voltage requirements are connected.[15]
- The secondary distribution feeders, at the LV level, directly supply residential and commercial loads[15].

1.2. Distributed Generation

As far as it has been said, the electric network was centralized, but in the last years, due to the need for an efficient, reliable, and clean grid, it has been introduced "Distributed Generation", changing the conventional power flow and therefore the traditional method to analyze the complete power system.

In the early history of electricity, DG was the rule rather than the exception, when grids were still based on DC generation. With the technological evolution, the emergence of AC generation allowed to transport power over long distances, and therefore the panorama changed, considering DG as a complement to the centralized generation[27]; but the liberalization and the evolution of the market have enabled DG to participate in energy trading actively. Nevertheless, there are some factors, working as drivers and/or barriers to its integration, such as technical, environmental, and sociopolitical issues [28] making the growth go slow. Further in this section will be deepen this topic.

Let's start by giving an official definition of DG. First, it should be said that there's no agreement on the term, for example, in America it is often used the term "embedded generation", or "dispersed generation" while in Europe and some parts of Asia, the term "decentralized generation", even when everybody refers to the same type of generation [24]. The first definition was given in 2002 referring to a generator with low generating capacity and located close to the load, not connected to the centralized system[10]. Nevertheless, there was no consensus on the limit of this capacity, some said that it meant a generation of up to 30kW, while others said the upper limit could vary from 1MW to 100MW[6]. From another point of view, DG can be also defined based on the technologies and connection type, other than the capacity; then it is also defined as the power source connected directly to the DN on the customer side of the meter. For this project, the definition adopted is "electric power generation connected to the distribution grid serving a customer or providing support to the DN".

In summary, the re-awaken of DG finds an explanation on the following events:

1. Liberalization of the electricity market
2. Sustainability goals.

This summed up to the elevated cost of the transmission and distribution lines for the delivery of electricity, and the impossibility of reaching remote places with the traditional alternative, fasten the research about this topic. However, not all is so good about DG, and some challenges need to be faced before increasing its presence in the network.

1.2.1. Drivers and Barriers

At the beginning of the integration of DG in the grid, the International Energy Agency (IEA) identified 5 major advantages [16][17]:

- Reduced losses of TN and DN.
- Improved grid stability and security.
- Minor environmental impact.

- Increased efficiency.
- Higher integration of RES.

Nowadays, these factors can be grouped into two groups: reliability and power quality, and environmental concerns. In some cases these factors play the role of drivers, reinforcing the integration of DG in the grid, but in some others, they are just a barrier to its growth.

Reliability and Power Quality

With the liberalization of the energy market, the regulatory policies in the EU played an important role. In this sense reliability and power quality, referring to the stability and consistency of the electrical supply, is always a matter of concern for any electricity supplier, and customers are more aware of the value of a reliable service. The IEA, in 2002, recognized the provision of reliable power as the most important future market niche for distributed generation. Distributed generation allows voltage support and power factor corrections because it generally leads to a rise in voltage in the network [27].

One should ask: Will DG improve the efficiency of the electrical network? And even when the pros are considerable, there is not a clear conclusion on whether the widespread of DG would improve energy efficiency or not, because technological advances are still needed.[28] Just to give an example, it is a double-edged sword because the integration of DG also affects the unidirectional structure of the network, for which the protection systems are currently programmed; so, in case of a fault, the protection system can fail, since they are not set for the new bidirectional structure of the network, therefore it should still be analyzed and solved.

Another example is presented on the system losses, generating the electricity at the point of consumption, when analyzing some punctual moments, surely reduces the transmission and distribution losses, but will it be also the case in a real DG dispatch pattern sampled over one year? The situation is not as easy as it could seem. Part of the complexity lies in the fact that the effect of DG on power losses is very dependent on the time and the location. In fact, some studies show that in some areas, initially, the reduction of losses goes from 10 to 30 percent, but while increasing the DG, the percentage starts to decrease as the load moves away from the peak.

Environmental Impact

Environmental concern is another key driver/barrier of DG. Hence, distributed generation, in its majority comes from renewable sources, it can help the environment by reducing the amount of electricity that must be generated at centralized power plants, which minimizes centralized generation's environmental impact. However, due to the dispersion of generating plants, for some kinds of renewable sources, the "footprint" is significantly worse. On the other hand, combustion-based distributed generating methods, can have many of the same negative consequences as bigger fossil-fuel-fired power plants, such as air pollution, affecting the people close to it.[30]

1.3. State Estimation

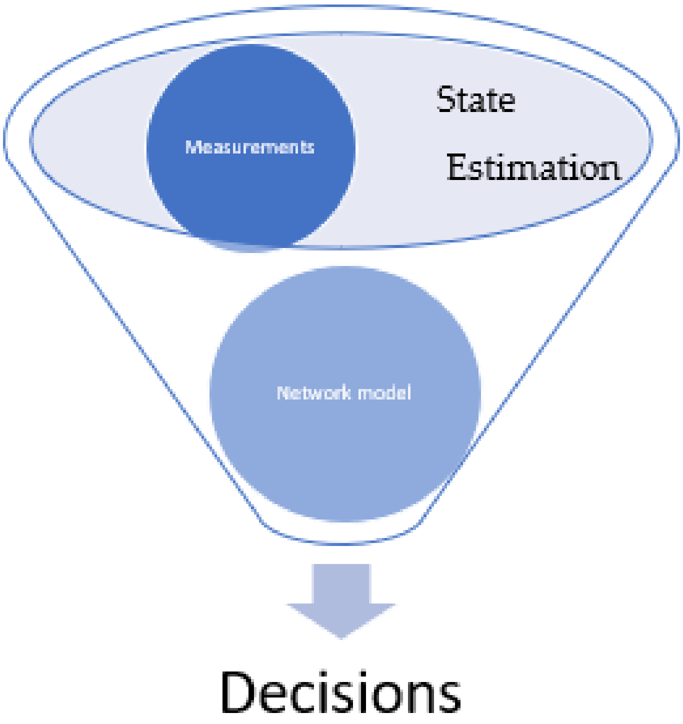


Figure 1.4: Decision making process

For the operation of any electrical power system, control and decision-making processes need real-time data; for this matter, a real-time computer with a model of the network (Central Control System) and a data acquisition system (meters) are used. These control systems can be classified into two types[29]:

- Raw System: The power system data as obtained is considered accurate and adequate for decision-making.
- Estimation System: State Estimation methods are used from the data measured to have a deeper insight into the system and make better decisions about its operation.

Regarding the latter, only the quasi-steady state or static operation of the power system is considered, hence that the process is called: Static-State Estimation.

1.3.1. Static State Estimation

In 1970 F.C. Schweppe first published some concepts and results for state estimation in Power Systems [21]. He defined the *"state"* of a system as the vector of steady state complex (magnitude with phase value) voltages in every bus of a network[13]. With the state being known all other electrical parameters can be easily computed (See equation (1.3)). The problem here is that the power system does not achieve a true steady state, increasing the uncertainty of the methods. Therefore in his models, he proposed just approximated solutions[13].

It should be said that the static state estimation (SSE) results as a combination of 2 fields: **electrical network modeling** and **statistical estimation theory**. For the network model, the electrical characterization of each element in the network (such as generator electrical model, load impedance and power, and lines parameters, as shown in section 1.1) is needed to determine the static structure of the system. Thereby the network is modeled by a complex admittance matrix $\overline{Y}$. In practice, not all the network elements are included in the model, moreover due to uncertainties affecting the network parameters, such as the longitudinal and transversal impedances of transformers and transmission lines, the power system model is also uncertain[7] [9]. On the other hand, the basics of the statistic theory related to estimation are briefly explained in the following sections and can be found, more deeply explained in reference [11]

In this sense, SSE is related to conventional load-flow calculations[29]. Therefore, to estimate the current power flows of the system, it is reasonable first to estimate the state vector, represented as $\vec{x}$; and then use $\overline{Y}$ and Kirchhoff's laws to obtain the desired estimated flows, as follows:

$$P_i = V_i \sum_{j \in \Re} V_j (G_{ij} cos\theta_{ij} + B_{ij} sin\theta_{ij}) \tag{1.3a}$$

$$Q_i = V_i \sum_{j \in \Re} V_j (G_{ij} sin\theta_{ij} - B_{ij} cos\theta_{ij}) \tag{1.3b}$$

$$S_i = P_i + jQ_i = V_i \sum_{j \in \Re} V_j (cos\theta_{ij} + j sin\theta_{ij})(G_{ij} - B_{ij}) \tag{1.3c}$$

See also section 1.3.3 for a better understanding of the use of SE methods in PS.

1.3.2. Measurements

Meters (watt, volt, var, etc.) can be placed on generator buses, transmission lines, and load buses, and the readings can be telemetered in real time to the central control system. However, in reality, real-time measurements are assumed to be available only on generators and some transmission lines, and the readings are not perfect due to the accuracy of the meters. With this in mind, the measurements can be classified as follows[9]:

1. ***Real-time (Raw) measurements:*** These are the measurements obtained from the meters and sent to the control center from the remote station. For example; switches and breakers position, and network topology.
   In this case, let z_{meter} be the set of measurements sent to the central station, z_{ideal}, be the measurement assuming no error, and then η_{meter} the errors resulting from different sources, as communication errors, inaccuracies of the meters, analog/digital conversion, etc. then:
   $$z_{meter} = z_{ideal} + \eta_{meter}$$
   Let also be x the state vector associated with the model and x_{true}, the real but unknown value of the state, the real measurement will be:
   $$z_{meter} = f(x_{true}) + \eta_{meter}$$
   where $f()$ is a function of x determined by the admittance matrix and Kirchhoff's law.

2. ***Pseudo-measurements***: Are values obtained from historical data, the reason why its precision is lower. To represent and model these values, let's consider the real power of a load bus, for example, where the historical consumption is known, and therefore it can be assumed that the real value is P_{nom}, and is entered to the central

stations as z_l, and in the same way as before,

$$z_l = f_l(x_{true}) + \eta_l$$

Let $f_l(x)$ be the function to compute the real power as in (1.3a).
Information similar to P_{nom} should be treated as Pseudo-measurements and depends on the available type and amount of real-time measurements.

3. ***Virtual-measurement***: Evidence that can be inferred from additional background knowledge about the phenomenon under consideration, and thus, does not need to be measured by any physical mean.

1.3.3. Basics and Methods

Summarizing what has been said, the *"State Estimator"* should be capable of the following processes (see also Figure 1.5) [4]

1. Filtering Measurements: A set of rudimentary techniques are carried out to detect and discard measurements that are erroneous or confusing.
2. Modelling the network: Based on the status of the switches and breakers and the physical layout of the substations it is modeled the structure of the electrical network model.
3. Analyze observability: Determine whether the system's state can be obtained using the measurements available.
4. Estimate: Calculates the state that best adapts the measurements captured remotely.
5. Detect the errors: Based on certain statistical properties of the estimation, this function detects the presence of possible errors in the set of measures. If redundancy is adequate, these measures can be identified and eliminated.

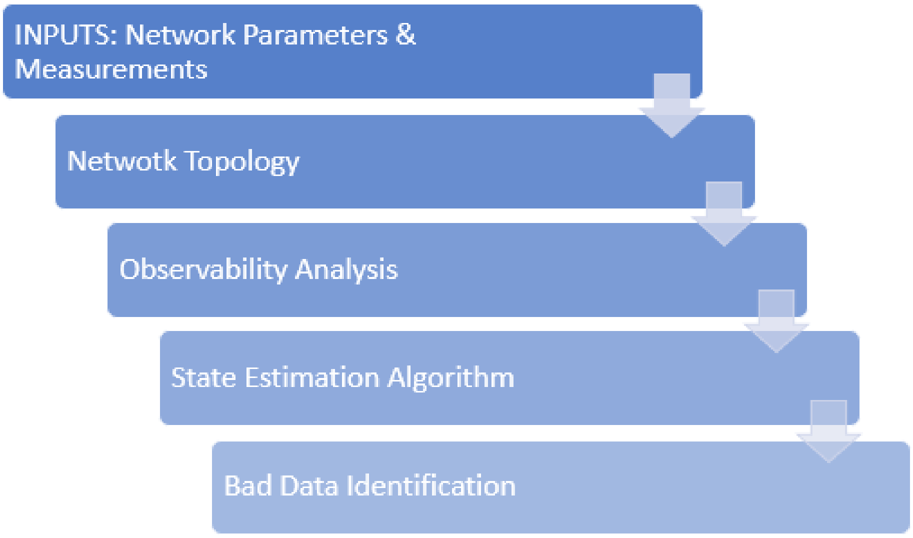


Figure 1.5: Algorithm for state estimation

Saying it in other words, the goal of estimation is converting the "prior knowledge" x^- and the available measurements z, such that $z(x) \sim p(z|x)$, into "posterior knowledge" x^+, whose probability density function can be evaluated at present time (Process known as Filtering or Estimation). The problem of the estimation process is then converting the prior into posterior using the measurements.[8]

$$\pi^+ = \frac{L(x;z) \cdot \pi^-(x)}{p(z)} \tag{1.4}$$

where:
π^+ is the PDF of the posterior or estimate x^+
π^- is the PDF of the prior x^-
$L(x;z)$ is the likelihood function equal to the conditioned probability of z given x

$$L(x;z) = p(z|x) \tag{1.5}$$

In some cases, such as the case of electrical networks, the complexity of the measurement model makes it very difficult to obtain the PDF, therefore the best way to describe the behavior of the meter is to use the mode, mean, and median of the set of measurements $(\tilde{x})$, these values are also called **estimates** of the measurand x, while the function to convert the data into the estimates is called **statistic**.[8]

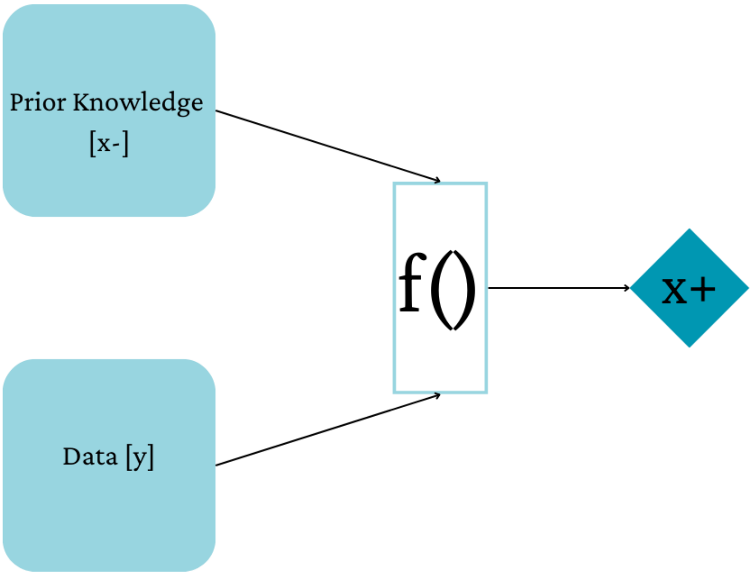


Figure 1.6: Estimate and statistic

Observability Principle

In order to identify the parts of the network where it is possible to calculate the estimated state, the Observability Principle entails examining the set of accessible states. [9].

"If the system cannot be observed, it must be determined which parts of the system can be observed and include pseudo-measurements in the parts that cannot be observed to make the system observable."

Formally, given the measurements of the vector, the estimator tries to find the best feasible solution for the elements of the vector of states. It is important to meet the inequality $m>n$ to arrive at this answer. Therefore, before estimating, the set of measurements needs to be examined to make sure that it includes at least n linearly independent measurements. The **observability test** is the name given to this analysis. The primary goal is to determine whether the system's overall condition can be approximated using the measures that are currently available. In the event that the test is unsuccessful, the analysis keeps going until it finds every observable island, which is every segment of the network whose state can be estimated with the available measurements, separately from every other segment.

In the following sections, the main estimation techniques will be briefly described [9].

Minimun Mean Square Value (MMSE)

A commonly used criterion is minimizing the sum of squares of error. This is a Bayesian approach because it uses the PDF of the prior and the likelihood as well. This method, in other words, uses the mean-square error as the optimality criterion. Generally, the MMSE estimate of one random variable or process x based on other random variables or processes, organized in a vector z such that $\tilde{x} = f(z)$ minimizes the mean of the error(defined as the difference between the actual historical data and the measured or predicted data by the model $x - z$),[8] as follows:

$$\min_{f(y)} E[(x - f(z))^2]$$

The MMSE estimate is $E(x/z)$ in general for any x and z distribution(s).

Minimun Mean Absolute Value

MAE is an error statistic that averages the absolute value of the difference between the actual historical data and the data predicted by the model, its estimate is the median of the posterior PDF. [8]

Maximum A Posteriori

MAP is a Bayesian method for determining the distribution and model parameters that best fit an observed dataset. It consists on solving an optimization problem for estimating the central tendency of the posterior probability. As such, this technique is referred to as “maximum a posteriori estimation". With this technique it’s not calculated the full posterior probability distribution, but just a point estimation such as a moment of the distribution, like the mode, the most common value.[8]

$$\tilde{x}_{MAP} = f_{MAP}(y) = MODE(x^+) = \underset{x}{max} \pi^+(x|y)$$

Maximun Likelyhood

ML is instead a frequentist method, in which the optimisation problem is to maximize the likelihood of observing the data given the model parameters.

With this being explained, it should be added that am an MMSE estimate is not by any means universally superior or inferior to a MAP estimate or an ML estimate. It is just a different estimate with a different goal.[5]

In the case of a power system, the state is defined by the power flows, the injections, the voltages, and the currents. Given the model of the network, these parameters can be expressed as linear or non-linear functions of the state. In this sense, considering $\mathbf{z}$, as a vector composed by m number of measurements, these measurements can be expressed as a function of the n components of the state vector $\mathbf{x}$, as follows:

$$\mathbf{z} = \begin{bmatrix} z(1) \\ z(2) \\ \vdots \\ z(m) \end{bmatrix} = \begin{bmatrix} h_1(x_1, x_2, ...x_n) \\ h_2(x_1, x_2, ...x_n) \\ \vdots \\ h_m(x_1, x_2, ...x_n) \end{bmatrix} + \begin{bmatrix} e(1) \\ e(2) \\ \vdots \\ e(m) \end{bmatrix} = h(\mathbf{x}) + \mathbf{e} \tag{1.6}$$

The peculiarity is that the parameters mentioned above are complex, therefore to work with real numbers, they should be expressed in polar or Cartesian coordinates. Moreover, the conventional state estimator combines these two coordinates, expressing the states in polar coordinates, and the admittance in Cartesian; and that usually the posterior's PDF is not available, making it more difficult to adopt the above methods.[8]

Regarding the measurements, they present some aleatory errors; and when the only quantities available are the measurement errors and the measurand's mean vector and covariance matrix, the maximum entropy principle leads to assume normal PDFs distribution for the prior and the measurement errors. As in every normal distribution, the variance of the measurement shows the interpreter how accurate the measurement is. In the analyzed topic, taking the measurement z as a vector on normal distributed measurements, with mean $h(x)$ and covariance R. According to the *Maximun entropy principle* the prior and the measurement error PDFs normal PDFs.[9]:

$$\pi(x) = \frac{1}{\sqrt{2\pi\Sigma_0}} e^{-\frac{1}{2}(x-x_0)^T\Sigma_0^{-1}(x-x_0)}$$

$$p_e(e) = \frac{1}{\sqrt{2\pi\Sigma_e}} e^{-\frac{1}{2}e^T\Sigma_e^{-1}e}$$

Then, according to 1.5, the likelihood function is expressed by $L(x; z) = p_e(z - h(x))$, that is:

$$L(x; z) = \frac{1}{\sqrt{2\pi\Sigma_e}} e^{-\frac{1}{2}(z-h(x))^T\Sigma_e^{-1}(z-h(x))} \tag{1.7}$$

Since, according to 1.4, the posterior is proportional to the likelihood and the prior, that is

$$p(x|z) \propto L(x;z) \cdot \pi(x) \tag{1.8}$$

$$p(x|z) \propto e^{-\frac{1}{2}(z-h(x))^T\Sigma_e^{-1}(z-h(x))+(x-x_0)^T\Sigma_0^{-1}(x-x_0)} \tag{1.9}$$

Then the MAP estimate is computed by the maximum of the posterior pdf, that will be given by the minimum of the argument of the exponential function in the equation 1.9, let's call it the Loss function J(x;z).[8][9]

$$J(\mathbf{x;z}) = (\mathbf{z} - h(\mathbf{x}))^T\Sigma_e^{-1}(\mathbf{z} - h(\mathbf{x})) + (\mathbf{x} - \mathbf{x}_0)^T\Sigma_0^{-1}(\mathbf{x} - \mathbf{x}_0) \tag{1.10}$$

Let's now study the estimation procedure more often used for Power Systems State Estimation,it is the least square criterion, which is similar to the maximum likelihood method in cases with equal variance and Gaussian error.

Weighted Least Square (WLS)

The Weighted least square estimation, is a technique that minimizes the weighted total of the squares of the measurement residuals [6]. The "Weight" of the measurements will be determined by the accuracy of the device that produces it, and so, the meters with higher accuracy will have a higher weight. This lets us infer that measurements are not perfect, but they have associated errors. This technique requires to have a function to relate the state vector with the measurement vector such that it can be written[8]:

$$\mathbf{z} = h(\mathbf{x}) = A\mathbf{x} \tag{1.11}$$

The above-mentioned error is assumed to follow a Gaussian distribution with zero mean and independent covariance (See [26]). The weight associated with each measurement is related to the covariance of its error, so, it will be:

$$W = \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & 0 & \sigma_m^2 \end{bmatrix} \tag{1.12}$$

The idea then is to minimize the square of the measurement deviation from the initial estimate, to get the best estimate. Then, according to the equation 1.10, the loss function will be:

$$J(\mathbf{x};\mathbf{z}) = (\mathbf{z} - h(\mathbf{x}))^T W^{-1}(\mathbf{z} - h(\mathbf{x}))$$

The WLS estimate is the root of J(**x;z**), as follows:

$$\begin{aligned} g(x) = \frac{\delta}{\delta x} J(\mathbf{x};\mathbf{z}) = \mathbf{0} \\ -H(\mathbf{x})^T W^{-1}(\mathbf{z} - h(\mathbf{x})) = \mathbf{0} \end{aligned}$$

Where $H(x) = \frac{\delta h(x)}{\delta x}$ and $g(x) = g(x^k) + G(x^k)(x - x^k) + \ldots = 0$. The matrix $G(x^k)$ is the gain Matrix, defined as:

$$G(x^k) = \frac{\delta g(x^k)}{\delta x} = H(x^k)^T W^{-1} H(x^k)$$

The final equation for the WLS is given as:

$$G(x^k)\Delta x^{k+1} = H(x^k)^T W^{-1}(\mathbf{z} - h(x^k)) \tag{1.13}$$

Where $\Delta x^{k+1} = x^{k+1} - x^k$.The procedure to solve it is deeply explained in [9].

1.3.4. Errors in SE

The state estimator's reliability and precision are contingent upon the telemeasurements that are received at the control center for further processing. An additional set of errors that come via communications channels are added to the ones related to measurement instruments, whether they are analog or digital. The state calculated from such sets of measurements will be locally insufficient unless these false data are found and identified.[9]

The state estimators have prior filters that rule out obvious inconsistencies in the set of measurements, especially changed signs or values out of range. However, many non-Gaussian errors escape this filtering [14]. The weighted least squares estimator, which in turn is one of the most used as shown in the previous section, uses procedures that are executed after the estimation, based on the residuals of the measurements, to detect and identify erroneous data.

A measurement's error has a null mean, a standard deviation of σ, and follows a normal distribution. The many kinds of analog errors based on their size are[9]:

- Noise: The standard deviation is less than 3σ
- Large errors: The standard deviation is higher than 3σ but less than 20σ. To eliminate this kind of error a **pre-filtering** process is needed. With pre-filtering, errors in measurements are found before the estimation process, identifying them directly from the value of the measurement. For example: Zero voltages, negative intensities, consumption in generating facilities, and generation in consuming facilities are the kinds of data that can be pre-filtered.
- Extreme errors: The standard deviation is higher than 20σ. For them, **Detection/Identification** is needed. Detection can be done through the statistical properties of the error function:
$$J(\tilde{x}) = \sum_{i=1}^{m} \frac{r_i^2}{\sigma_i^2}$$

1.4. SE in Distribution Network

Low-voltage distribution networks (LVDGs) are essential to the idea of the smart grid. The electrification of various industries, including transportation and heating/cooling, as well as the need for a more sustainable grid in the future, are driving up energy demand and pushing modern distribution grids particularly the LVDGs to their operating limits.[23]

However, because of the LVDGs' unbalanced characteristics, a three-phase approach is necessary to accurately account for both the uneven loading of each phase and the uneven coupling effect between the phases. The state variables are related to the available measurements in the formulation of the WLS SE as z=h(x)+e, where, as said before, x is the state vector, e is the Gaussian noise (measurement error) that affects the measurements, and z is a vector containing all available measurements. h(x) is a vector containing non-linear functions that mathematically relate the state variables with the measurements. Within the WLS SE framework, as shown in the section 1.3.3 the state vector is obtained through minimization.

Currently, there is a proposed scheme for DSSE, exploiting the data from the phasor measurement units (PMUs) or the smart meters, that are installed at the MV LV substation for monitoring the load variation inside the LVDG between the execution of the ES, that are done one each minute.[3] This proposal uses the WLS method, where the weight increases the uncertainty of a measurement based on the time that has passed since it was last updated of the devices. Figure 1.7 shows the algorithm followed by this method.

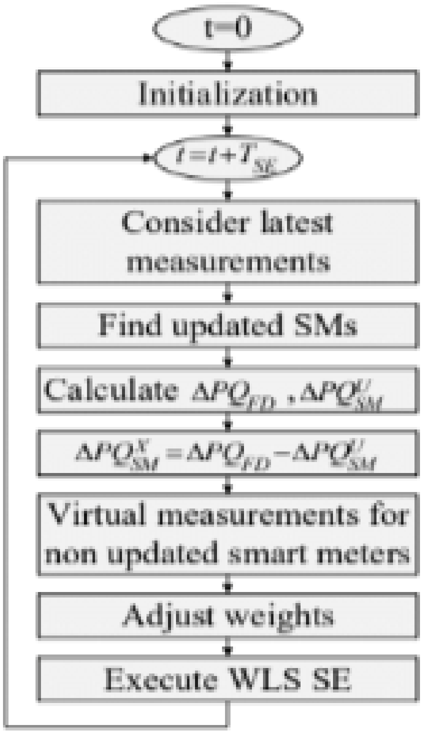


Figure 1.7: Conventional State Estimator

According to the above figure, the first step is *"Initialization"*. In this step, it is assumed that all measurements are available in $t = 0s$, with these measurements the WLS is executed giving as a result the state of the system at that moment. Besides, the measured power is also stored to compute the load variation. The estimated estate is then saved, to be used as virtual measurements. Finished the first iteration, the meters are grouped into: "Updated (U)" and "Not updated (NU)", according to the update time of each device. After one minute ($T_{SE} = 1min$), a new iteration of the WLS SE is made (Step: "Execute WLS SE"), using the latest available measurement of the devices.[3]

The load variation of the feeder on each iteration is calculated as:

$$\Delta P_{LVDG}^{t} = P_{PMU}^{t} - P_{PMU}^{t-T_{SE}} \tag{1.14}$$

In the case that some smart meters have updated their measurements (subset U is not empty), then the load deviation of these smart meters is calculated as,

$$\Delta P_{SM_U}^{t} = \sum_{i \in U} P_{SM_i}^{t} - P_{SM_i}^{t-T_{SE}} \tag{1.15}$$

The load variation related solely to the smart meters that have not been updated at this time can be computed using the load variation as observed by the feeder and the updated smart meters.

$$\Delta P^t_{SM_{NU}} = \Delta P^t_{LDVG} - \Delta P^t_{SM_U} \tag{1.16}$$

The power measurements from smart meters that have not been updated within the last T_{SE} are adjusted according to the calculated $P^t_{SM_{NU}}$ as follows

$$P^t_{SM_{NU,i}} = P^{t-T_{SE}}_{SM_{NU_i}} + \frac{\Delta P^t_{SM_{NU}}}{N_{NU}} \tag{1.17}$$

In the same way as in equations 1.14, 1.15, 1.16 and 1.17 should be computed the reactive power variation on each iteration.

Subsequently, an adjustment of the weight for each measurement in the model, should be executed, due to the small time skewness of the measurements in the transmission system, the measurement weights of the traditional WLS SE are defined by the inverse of the variance that characterizes each measuring, nevertheless, the measurement set that is used in the WLS SE cannot be treated as synchronized, therefore, a method for adjusting the variance of each measured signal is proposed as a means to incorporate this asynchronity in the WLS SE procedure

$$\sigma^i_{new} = \sigma^i_{device}(1 + kT_i)^a$$

where σ^i_{device} is the intrinsic uncertainty of the measuring device associated with the i_{th} measurement, new is the adjusted standard deviation, T, is the time duration since the measurement was last reported and parameters k and a are used as tuning parameters.

1.4.1. Necessity

First, reliability and accuracy in monitoring the functioning state of distribution grids are critical given the growing integration of distributed energy resources (DERs). It is anticipated that LVDGs will replace conventional passive grids with extremely intricate active systems[23]. For example, distribution grids have witnessed a rapid expansion of DG's based on renewable energy, especially the residential-scale PV units in LV grids, which could cause voltage stability problems and affect the security and reliability of grid operation.[23] In order to minimize the negative effects that a high DER penetration rate may have on the LVDG's ability to operate, an advanced distribution management system (DMS) must be implemented. The data is supplied to the control and automation methods for decision-making via the Distribution System State Estimation (DSSE).

In this sense, three voltage control strategies implementing DSSE are being researched.[1] Those are:

- On load Tap Changer Control (OLTC):
- Active Power Management System:
- Reactive Power Management System:

However, these are not the only use cases. Just to mention, the DSSE utilization has been grouped into four groups: Outage management and Power quality, data analysis, renewable and E-Mobility integration, and coordinated control to classify several use cases introduced in different studies, some such as load and generation profiles, topology analysis, short circuit power estimation, fault location on LV and MV, etc.[23]

2 | Parameters Computation

In this project, the goal is to prepare the input data for an algorithm to estimate the state of an LV grid. For that purpose, the IEEE European low voltage test feeder is used. The following sections will give an overall description of the mentioned network and the modifications that have been done to achieve the main goal.

2.1. IEEE European Low Voltage Test Feeder: IEEE 906 LV Network

The first and only model that is now accessible for research and studies at the distribution level in Europe is the IEEE European LV Test feeder.[22] Since an extensive knowledge of the operational condition of the DN is vital for its management and control, this model is frequently utilized for the study and planning of Low Voltage (LV) distribution networks. The European low-voltage test case was developed to meet the following features:

- Phase to phase voltage of 416 V.
- 24h load profile.
- Time-series simulation results over one day and static power flow calculation results at some key moments are provided.

The model is composed of 906 low voltage nodes, radially connected by 905 branches; 55 out of the 906 nodes correspond to load buses. The whole LV network is connected to the MV system by mean of a step-down, MV-LV three-phase transformer. The transformer has a rated power of 0.8 MVA and has a $\Delta - Y$ connection. The resistance and reactance of the windings are 0.4% and 4% respectively.[22]. A diagram of the LV network is shown in Figure 2.1.

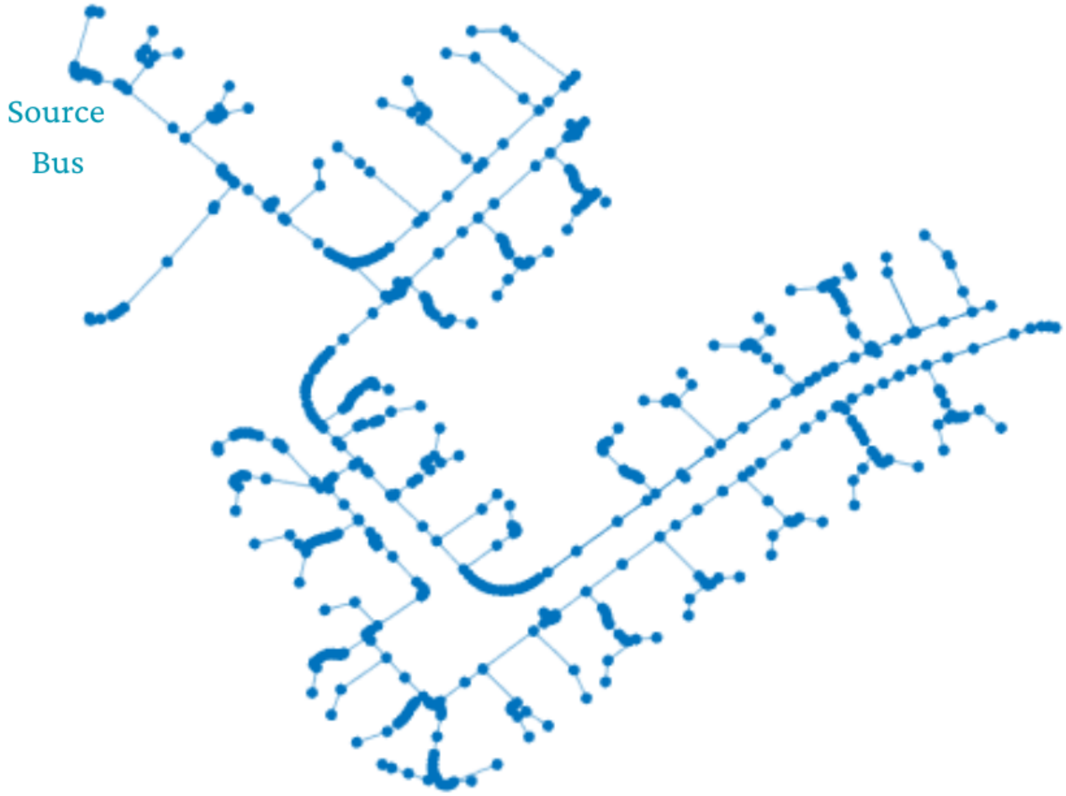


Figure 2.1: IEEE 906 Low Voltage Test Feeder

2.1.1. Modelling

Firstly, the MV system is modeled as a voltage source with impedance, specified by the short circuits currents as follows:

MV System model

Parameter	**Value**
Voltage	11 kV
p.u	1.05
3Φ **Short Circuit**	3000 A
1Φ **Short Circuit**	100 A

Table 2.1: Voltage Source data

As already said, the LV network is connected to the MV System through an MV-LV transformer. The loads, on the other hand, are modeled as PQ nodes, which means, buses with constant active and reactive power. Therefore, to each load is given a base power, the power factor, the phase to which is connected, and the type of connection. Finally, the distribution lines are defined by codes and their length, information available in Appendix C. Each code specifies the zero-sequence resistance and admittance (See Table 2.2); nevertheless, there's no available information about the geometry of each conductor, data that is essential for the computation of the self and mutual inductance, and in this sense, for a more accurate computation of the state.

IEEE 906 conductors

EU Network	R_0	X_0
2c.007	0.00397	0.00010
2c.0225	0.00126	0.00009
2c.16	0.00120	0.00009
35_SAC_XSC	0.00076	0.00009
4c.06	0.00158	0.00009
4c.1	0.00096	0.00008
4c.35	0.00032	0.00008
4c.185	0.00058	0.00008
4c.70	0.00151	0.00008
4c_95_SAC_XC	0.0080	0.00009

Table 2.2: Lines codes data for IEEE 906 Network

2.1.2. Implemented changes

Given that the geometry of the conductors used to model the IEEE-906 network is not available, and this information is needed to compute the inductance for each branch, for this project will be introduced a change: using commercial cables used by Enel, instead of the ones mentioned before, as shown in Table 2.3. The matching of the conductors was based on the per-length unit resistance and admittance, and the structure of the cable (number of phases and neutral).

IEEE 906 Conductor & Enel's cable Matching

EU Network	Enel cable	R_0	X_0
2c.007	2x10	0.00397	0.00010
2c.0225	2x10	0.00126	0.00009
2c.16	2x10	0.00120	0.00009
35_SAC_XSC	3x35+54Al	0.00076	0.00009
4c.06	4x10	0.00158	0.00009
4c.1	4x10	0.00096	0.00008
4c.35	3x150+95Al	0.00032	0.00008
4c.185	3x150+95Al	0.00058	0.00008
4c.70	4x10	0.00151	0.00008
4c_95_SAC_XC	3x35+54Al	0.0080	0.00009

Table 2.3: Matching Enel and IEEE codes

Then, with this modification, we gathered all the required data to compute the needed information, to perform state estimation, such as:

- Topology of the feeders.
- Branches impedances: resistance and self & mutual inductances. (next section)
- Network working conditions.

Impedance Matrix Computation and Reduction

The Figure 2.2 shows all important parameters of a three-phase line, as in the network of study, for the analysis of the line.

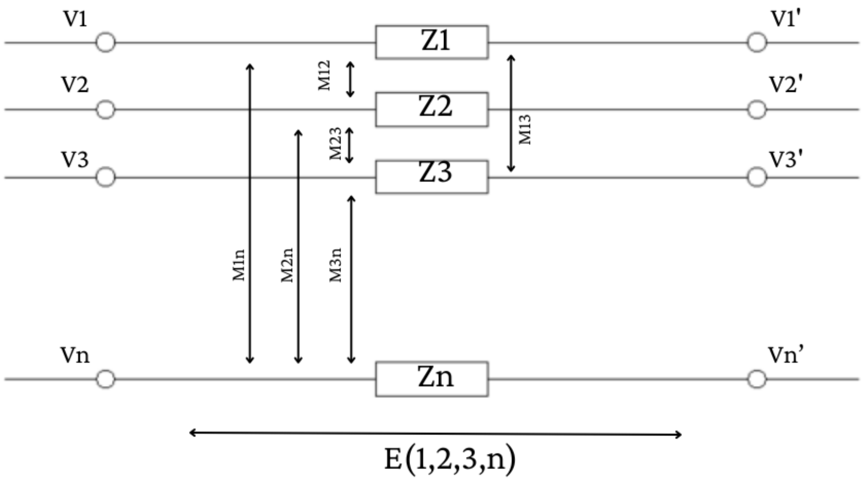


Figure 2.2: Representation of a three phase feeder

From the above scheme, using Kirchhoff's laws it will be obtained:

$$\begin{cases} V_1 - E_1 - V_1^{'} + E_1^{'} = 0 \\ V_2 - E_2 - V_2^{'} + E_2^{'} = 0 \\ V_3 - E_3 - V_3^{'} + E_3^{'} = 0 \end{cases} \tag{2.1}$$

$$\begin{cases} I_1 + I_2 + I_3 + I_n = 0 \\ I_n = -I_1 - I_2 - I_3 \end{cases} \tag{2.2}$$

Expressing Equation 2.2 in matrix form, it results:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_n \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix}}_{C} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$I_{4x1} = C_{4x3} * I_{3x1} \tag{2.3}$$

By the Ohm law, it can be said $E = Z * I$, therefore:

$$\begin{cases} E_1 = R_1 I_1 + jL_1 I_1 + jM_{12} I_2 + jM_{13} I_3 + jM_{1n} I_n \\ E_2 = R_2 I_2 + jL_2 I_2 + jM_{21} I_1 + jM_{23} I_3 + jM_{2n} I_n \\ E_3 = R_3 I_3 + jL_3 I_3 + jM_{31} I_1 + jM_{32} I_2 + jM_{3n} I_n \\ E_n = R_n I_n + jL_n I_n + jM_{n1} I_1 + jM_{n2} I_2 + jM_{n3} I_3 \end{cases}$$

And so it can be written:

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_n \end{bmatrix} = \underbrace{\begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & 0 & R_3 & 0 \\ 0 & 0 & 0 & R_4 \end{bmatrix}}_{R_{4X4}} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_n \end{bmatrix} + j \underbrace{\begin{bmatrix} L_1 & M_{12} & M_{13} & M_{1n} \\ M_{21} & L_2 & M_{23} & M_{2n} \\ M_{31} & M_{32} & L_3 & M_{3n} \\ M_{n1} & M_{n2} & M_{n3} & L_4 \end{bmatrix}}_{L_{4X4}} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_n \end{bmatrix}$$

$$E_{4x1} = \underbrace{R_{4x4} + jL_{4x4}}_{Z_{4x4}} * I_{4x1} \tag{2.4}$$

Now, from Equation 2.1 in general it can be said that $V' = V - (E - E^{'})$, so in this way:

$$\begin{bmatrix} V_1^{'} \\ V_2^{'} \\ V_3^{'} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} - \begin{bmatrix} E_1 - E_n \\ E_2 - E_n \\ E_3 - E_n \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} - \underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}}_{C^T} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_n \end{bmatrix}$$

$$V^{'}_{3x1} = V_{3x1} C^T_{3x4} * E_{4x1} \tag{2.5}$$

Replacing Equation 2.4 into Equation 2.5, it is obtained:

$$V^{'}_{3x1} = V_{3x1} - C^T_{3x4}[Z_{4x4} * I_{4x1}] \tag{2.6}$$

And replacing Equation 2.3 into Equation 2.6, it becomes:

$$V^{'}_{3x1} = V_{3x1} - \underbrace{C^T_{3x4} * Z_{4x4} * C_{4x3}}_{Z_{3x3}} * I_{3x3} \tag{2.7}$$

Now let's work on the portion of Equation 2.7 that was called Z_{3x3}. Since $Z_{4x4} = R_{4x4} + jL_{4x4}$, where $L = \omega X$, it continues:

$$Z_{3x3} = C^T_{3x4} * [R_{4x4} + jL_{4x4}] * C_{4x3}$$

Where

$$R_{3x3} = C^T_{3x4} * R_{4x4} * C_{4x3}$$

$$R_{3x3} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & 0 & R_3 & 0 \\ 0 & 0 & 0 & R_n \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$

$$R_{3x3} = \begin{bmatrix} R_1 + R_n & R_n & R_n \\ R_n & R_2 + R_n & R_n \\ R_n & R_n & R_3 + R_n \end{bmatrix} \tag{2.8}$$

and

$$L_{3x3} = C^T_{3x4} * L_{4x4} * C_{4x3}$$

$$L_{3x3} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} L_1 & L_{12} & L_{13} & L_{1n} \\ L_{21} & L_2 & L_{23} & L_{2n} \\ L_{31} & L_{32} & L_3 & L_{3n} \\ L_{n1} & L_{n2} & L_{n3} & L_n \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$

For the computation of the self and mutual inductance a further analysis should be done, taking into account the geometry of the Enel cables found in [12] and summarized in appendix 2.3, and the following process.

* Self Inductance

To start let's consider we have a solid conductor as in Figure 2.3, porting a current I uniformly distributed, so that $J = \frac{I}{\pi r_m^2}$. Therefore, through an infinitesimal section of radio dr, we will have;

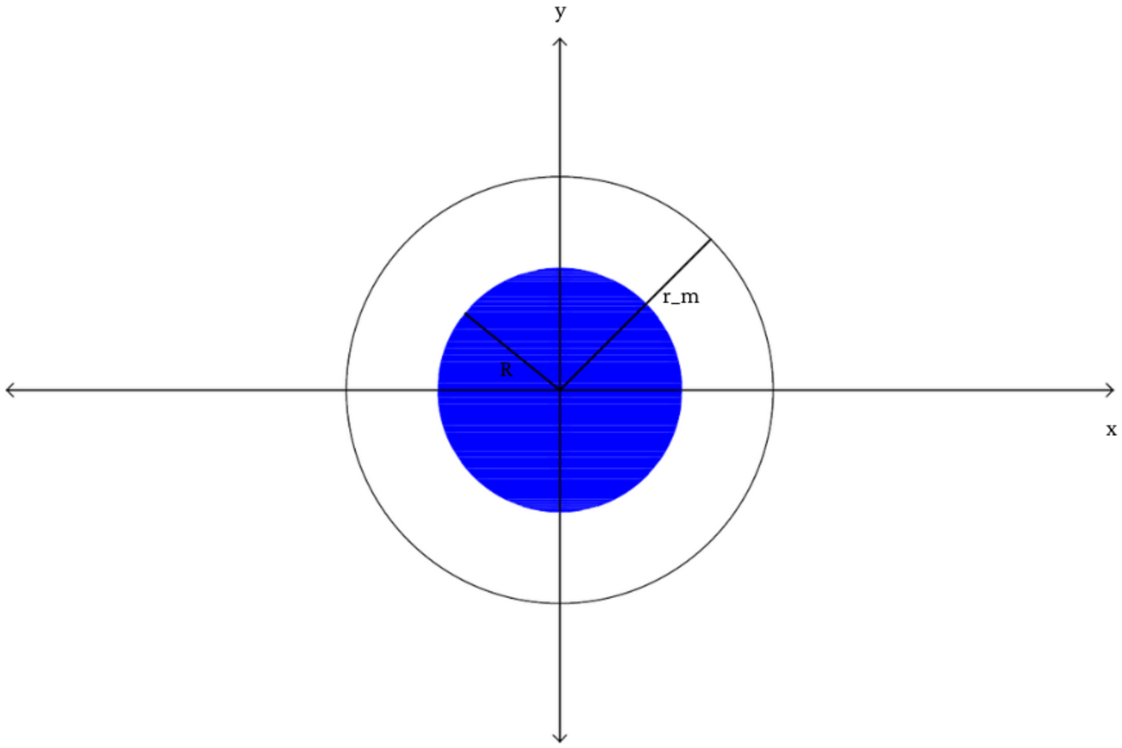


Figure 2.3: Self inductance computation

$$I(r) = J\pi r^2 \Rightarrow I\frac{r^2}{r_m^2}$$

Let's define the function $\alpha(r)$ as

$$\alpha(r) = \begin{cases} \frac{r^2}{r_m^2} & \text{for all } r < r_m \\ 1 & \text{for all } r \geq r_m \end{cases}$$

Therefore, in other terms, $I(r) = \alpha(r)I$. In this same way, knowing that the magnetic field generated by a curent I(r) at a point distance r from it, is:

$$B(r) = \frac{\mu_0}{2\pi r} I(r) = \frac{\mu_0 I}{2\pi r}\alpha(r) \tag{2.9}$$

The magnetic flux and the linked flux in the wire due to the magnetic field produced by the current I(r) will be respectively:

$$\begin{aligned}d\Phi(r) &= B(r)dr\\ d\Phi(r) &= \alpha(r)I\frac{\mu_0}{2\pi r}dr\\ \Rightarrow d\lambda(r) &= \alpha(r)Id\Phi(r)\end{aligned}$$

$$d\lambda(r) = \alpha(r)^2\frac{\mu_0}{2\pi r}Idr \tag{2.10}$$

Now to find the mutual inductance we have to integrate the linked flux (Eq. 2.10) over the space, so till infinity, therefore:

$$L = \lim_{R\to\infty}\int_0^R \frac{d\lambda(r)}{I}$$

where:

$$\begin{aligned}\frac{1}{I}\int_0^R d\lambda(r) &= \left(\frac{\int_0^{r_m} d\lambda(r) + \int_{r_m}^R d\lambda(r)}{I}\right)\\ &= \frac{\mu_0}{2\pi}\left[\int_0^{r_m}\frac{r^3}{r_m^4}dr + \int_{r_m}^R \frac{1}{r}dr\right]\\ &= \frac{\mu_0}{2\pi}\left[\frac{r_m^4}{4} + ln\frac{R_m}{r_m}\right]\end{aligned}$$

Therefore:

$$\begin{aligned}L &= \lim_{R\to\infty}\frac{\mu_0}{2\pi}\left[\frac{r_m^4}{4} + ln\frac{R_m}{r_m}\right]\\ L &= \frac{\mu_0}{2\pi}\left[\frac{r_m^4}{4} + \infty\right]\end{aligned}$$

As noticed, self-inductance is composed of an infinite term, so in theory, this parameter is also infinite, but in practice, it is not and that find an explanation in the following steps, but let's first define the mutual inductance.

* Mutual Inductance

Let's now compute the mutual inductance, we have to proceed the same way as before, considering Figure 2.4.

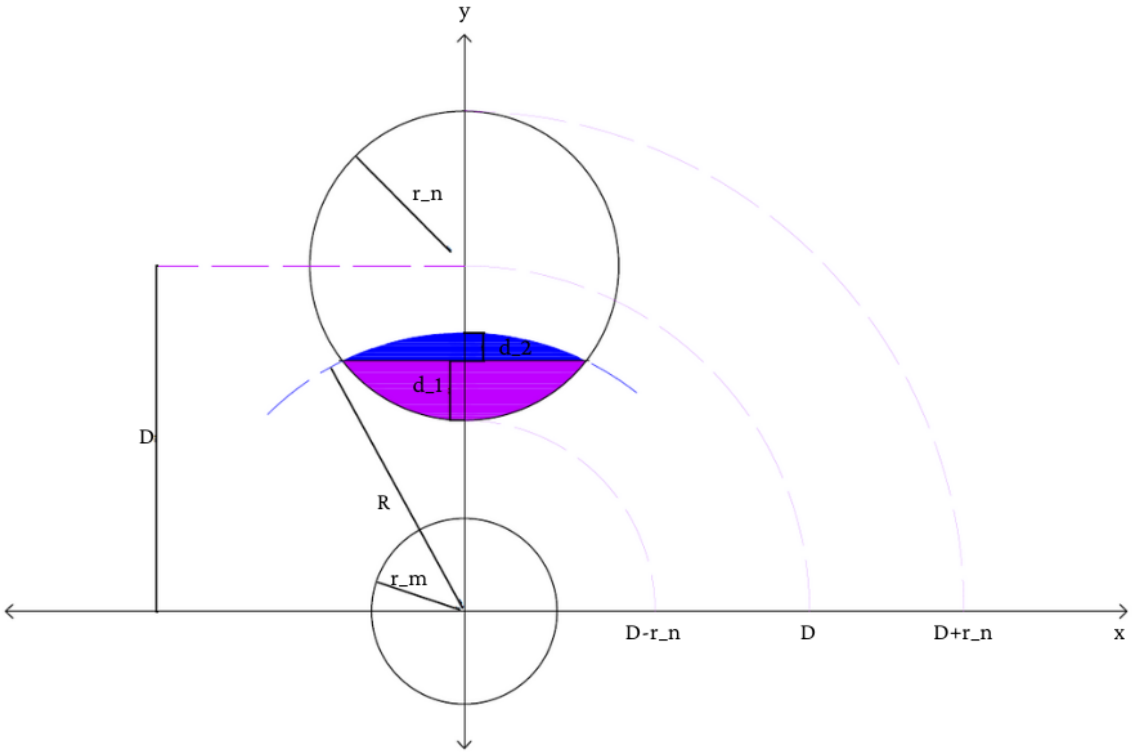


Figure 2.4: Mutual inductance computation

We will define the function $\beta(r)$, similarly to $\alpha(r)$, then

$$\beta(r) = \begin{cases} 0 & \text{for all } r < D - r_n \\ \frac{A_{int}}{\pi r_n^2} & \text{for all } D - r_n < r < D + r_n \\ 1 & \text{for all } r \geq D + r_n \end{cases}$$

Where

$$A_{int} = r^2 cos^{-1}(\frac{d_1}{r}) - d_1\sqrt{r^2 - d_1^2} + r_n^2 cos^{-1}(\frac{d_2}{r_n}) - d_2\sqrt{r_n^2 - d_2^2}$$

So the linked flux will now be:

$$\Rightarrow d\lambda(r) = \beta(r)/d\Phi_m(r)$$

Where $\Phi_m(r)$ is the flux generated by the current in the conductor m, and we have:

$$d\lambda(r) = \beta(r)\alpha(r)\frac{\mu_0}{2\pi r}dr \tag{2.11}$$

As done before, to find the mutual inductance we will have to integrate over the space, so:

$$M = \lim_{R\to\infty}\int_0^R \frac{d\lambda(r)}{I}$$

where:

$$\int_0^R \frac{d\lambda(r)}{I} = \left(\frac{\int_0^{D-r_n} d\lambda(r) + \int_{D-r_n}^{D+r_n} d\lambda(r) + \int_{D+r_n}^{R} d\lambda(r)}{I}\right)$$

Given β the expression before, simplifies as follows:

$$\begin{aligned}\int_0^R \frac{d\lambda(r)}{I} &= \frac{\mu_0}{2\pi}\left[\int_{D-r_n}^{D+r_n}\frac{\beta(r)}{r}dr + \int_{D+r_n}^{R}\frac{1}{r}dr\right]\\ &= \frac{\mu_0}{2\pi}\left[M_{mn} + ln\frac{R_m}{D+r_n}\right]\end{aligned}$$

Therefore:

$$M = \frac{\mu_0}{2\pi}\left[M_{mn} + \infty\right]$$

Where M_{mn} is the $\int_{D-r_n}^{D+r_n}\frac{\beta(r)}{r}dr$, find computationally by the following piece of code:

```
% d - centers distance
% r - radius rn of linked conductor
  function M=mutualL(r,d)
  mu0 = 4e-7*pi;
  rVec = linspace(d-r,d+r,100000);
  b = bet(rVec,r,d);
  M = mu0trapz(rVec,b./rVec)/(2pi);
   function b=bet(r,R,D)
     a = real(acos((r.^2+D^2-R^2)./(2*r*D)));
     b = real(acos((R^2+D^2-r.^2)/(2*R*D)));
     A = b*R^2+a.*r.^2-0.5*r.^2.*sin(2*a)-0.5*R^2*sin(2*b);
     b = A/(pi*R^2);
   end
  end
```

With the above computations, we can write

$$L_{4x4} = L'_{4x4} + L''_{4x4}$$

Where L'_{4x4} is the finite part of the mutual and self inductances, and L''_{4x4} the infinite part, as follows:

$$L'_{4x4} = \frac{\mu_0}{2\pi}\begin{bmatrix} \frac{1}{4} & M_{12} & M_{13} & M_{14} \\ M_{21} & \frac{1}{4} & M_{23} & M_{24} \\ M_{31} & M_{32} & \frac{1}{4} & M_{34} \\ M_{41} & M_{42} & M_{43} & \frac{1}{4} \end{bmatrix}$$

$$L''_{4x4} = \frac{\mu_0}{2\pi}\begin{bmatrix} ln\frac{R}{r_1} & ln\frac{R}{D_{12}+r_1} & ln\frac{R}{D_{13}+r_1} & ln\frac{R}{D_{14}+r_1} \\ ln\frac{R}{D_{21}+r_2} & ln\frac{R}{r_2} & ln\frac{R}{D_{23}+r_2} & ln\frac{R}{D_{24}+r_2} \\ ln\frac{R}{D_{31}+r_3} & ln\frac{R}{D_{32}+r_3} & ln\frac{R}{r_3} & ln\frac{R}{D_{34}+r_3} \\ ln\frac{R}{D_{41}+r_4} & ln\frac{R}{D_{42}+r_4} & ln\frac{R}{D_{43}+r_4} & ln\frac{R}{r_4} \end{bmatrix}$$

Take into account that the index "4" in the above matrices, refers to the *neutro* N in the three-phase feeder; that the distances for $D_{i,j} = D_{j,i}$, and that the R tends to ∞.

Finally, we will have that the inductance reduce matrix L_{3x3} will be:

$$L_{3x3} = C^T_{3x4} * [L'_{4x4} + L''_{4x4}] * C_{4x3} \tag{2.12}$$

$$L_{3x3} = C^T_{3x4}L'_{4x4}C_{4x3} + C^T_{3x4}L''_{4x4}C_{4x3}$$

Resulting:

$$L'_{3x3} = \frac{\mu_0}{2\pi}\begin{bmatrix} \frac{1}{2} - M_{41} - M_{14} & \frac{1}{4} + M_{12} - M_{42} - M_{14} & \frac{1}{4} + M_{13} - M_{43} - M_{14} \\ \frac{1}{4} + M_{21} - M_{41} - M_{24} & \frac{1}{2} - M_{42} - M_{24} & \frac{1}{4} + M_{23} - M_{43} - M_{24} \\ \frac{1}{4} + M_{31} - M_{41} - M_{34} & \frac{1}{4} + M_{32} - M_{42} - M_{34} & \frac{1}{2} - M_{43} - M_{34} \end{bmatrix} \tag{2.13}$$

and

$$L''_{3x3} = \frac{\mu_0}{2\pi} \begin{bmatrix} ln(D_{14}+r_1)+ln(D_{14}+r_4)-ln(r_1)-ln(r_4) & ln(D_{14}+r_1)-ln(D_{12}+r_1)+ln(D_{24}+r_4)-ln(r_4) & ln(D_{14}+r_1)-ln(D_{13}+r_1)+ln(D_{34}+r_4)-ln(r_4) \\ ln(D_{14}+r_4)-ln(D_{12}+r_2)+ln(D_{24}+r_2)-ln(r_4) & ln(D_{24}+r_2)+ln(D_{24}+r_4)-ln(r_2)-ln(r_4) & ln(D-24+r_2)-ln(D-23+r_2)+ln(D_{34}+r_4)-ln(r4) \\ ln(D_{14}+r_4)-ln(D_{13}+r_3)+ln(D_{34}+r_3)-ln(r_4) & ln(D_{24}+r_4)-ln(D23+r_3)+ln(D_{34}+r3)-ln(r_4) & ln(D_{34}+r_3)+ln(D_{34}+r_4)-ln(r_3)-ln(r_4) \end{bmatrix} \tag{2.14}$$

As expected the ∞ term disappears.

2.2. Parameters Computation for Enel Cables

In this section will be summerized the results obtained for each enel cable used and defined in Table 2.3. It will be also shown, as example, the process followed for one type of conductor.

As noticed in the previous section, the mutual and sef inductances are dependent on the geometry of the conductor. In this sense, taking into account the geometry of each enel cable used (See Reference [12]), the distances useful for the computation of the mutual inductance, are shown in Table 2.4

ENEL ID	FROM	TO	DISTANCE	R_{FROM}	R_{TO}
2X10	A	B	5.90	2.03	2.03
	A	C	8.34	5.9	2.03
	A	N	5.90	2.03	2.03
	B	A	5.90	2.03	2.03
	B	C	5.90	2.03	2.03
	B	N	8.34	5.9	2.03
	C	A	8.34	5.9	2.03
	C	B	5.90	2.03	2.03
	C	N	5.90	2.03	2.03
	N	A	5.90	2.03	2.03
	N	B	8.34	5.9	2.03
	N	C	5.90	2.03	2.03

ENEL ID	FROM	TO	DISTANCE	R_{FROM}	R_{TO}
4X10	A	B	7.65	2.03	2.03
	A	C	7.65	2.03	2.03
	A	N	10.82	2.03	2.03
	B	A	7.65	2.03	2.03
	B	C	10.82	2.03	2.03
	B	N	7.65	2.03	2.03
	C	A	7.65	2.03	2.03
	C	B	10.82	2.03	2.03
	C	N	7.65	2.03	2.03
	N	A	10.82	2.03	2.03
	N	B	7.65	2.03	2.03
	N	C	7.65	2.03	2.03
3x35+54Al	A	B	10.70	3.55	3.55
	A	C	10.70	3.55	3.55
	A	N	16.51	3.55	4.73
	B	A	10.70	3.55	3.55
	B	C	15.13	3.55	3.55
	B	N	11.72	3.55	4.73
	C	A	10.70	3.55	3.55
	C	B	15.13	3.55	3.55
	C	N	11.72	3.55	4.73
	N	A	16.51	4.73	3.55
	N	B	11.72	4.73	3.55
	N	C	11.72	4.73	3.55
3x70+54Al	A	B	13.40	4.9	4.9
	A	C	13.40	4.9	4.9
	A	N	18.42	4.9	4.73
	B	A	13.40	4.9	4.9
	B	C	18.95	4.9	4.9
	B	N	13.03	4.9	4.73
	C	A	13.40	4.9	4.9
	C	B	18.95	4.9	4.9
	C	N	13.03	4.9	4.73
	N	A	18.42	4.73	4.9
	N	B	13.03	4.73	4.9
	N	C	13.03	4.73	4.9

ENEL ID	FROM	TO	DISTANCE	R_{FROM}	R_{TO}
3x150+95Al	A	B	21.00	7.1	7.1
	A	C	21.00	7.1	7.1
	A	N	27.15	7.1	5.6
	B	A	21.00	7.1	7.1
	B	C	29.70	7.1	7.1
	B	N	19.28	7.1	5.6
	C	A	21.00	7.1	7.1
	C	B	29.70	7.1	7.1
	C	N	19.28	7.1	5.6
	N	A	27.15	5.6	7.1
	N	B	19.28	5.6	7.1
	N	C	19.28	5.6	7.1

Table 2.4: Distances between phases

With the information shown before, it can be analyzed the case of the Enel cable $4X10$, with a transversal section as shown in Figure 2.5. Further results are found in B

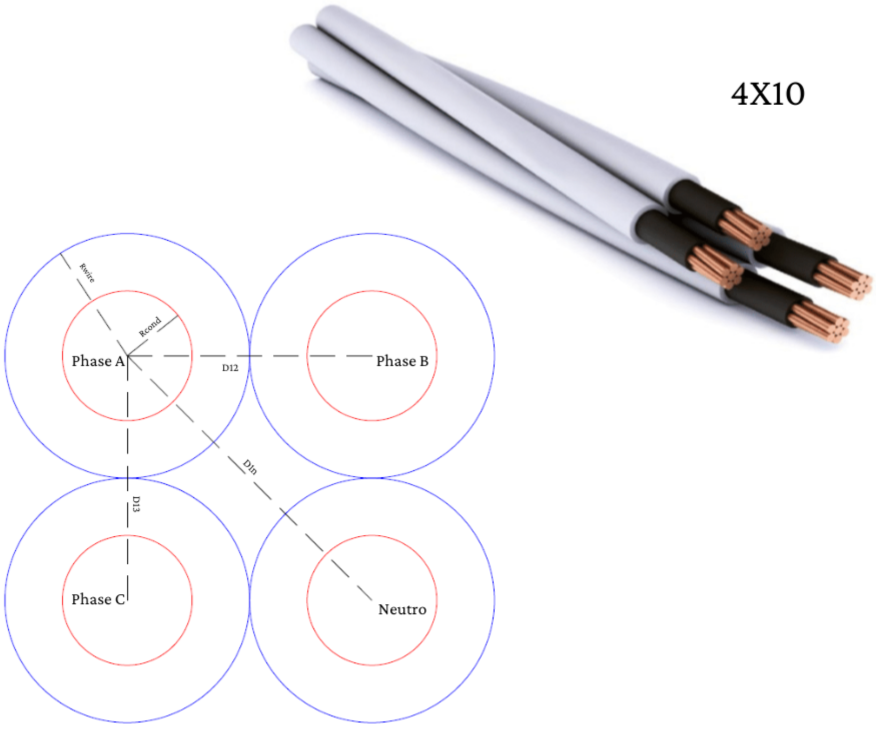


Figure 2.5: Geometry of Enel cable 4X10

With all this, the inductance matrix, computed as the addition $L_{4x4} = L^{'}_{4x4} + L^{''}_{4x4}$, where $L^{'}_{4x4}$ is the matrix of the finite terms, and $L^{''}_{4x4}$ is the matrix of the infinite terms, results:

$$1e^{-5} * \begin{bmatrix} -0.0092 & -0.0454 & -0.0454 & -0.0511 \\ -0.0454 & -0.0092 & -0.0511 & -0.0454 \\ -0.1329 & -0.0511 & -0.0092 & -0.0454 \\ -0.0511 & -0.0454 & -0.0454 & -0.0092 \end{bmatrix}$$

Then, just the reduction has to be done, according to Equation 2.12, as follows:

$$L_{3x3} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} -0.0092 & -0.0454 & -0.0454 & -0.0511 \\ -0.0454 & -0.0092 & -0.0511 & -0.0454 \\ -0.1329 & -0.0511 & -0.0092 & -0.0454 \\ -0.0511 & -0.0454 & -0.0454 & -0.0092 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$

$$L_{3x3} = 1e^{-6} * \begin{bmatrix} 0.8381 & 0.4191 & 0.4191 \\ 0.4191 & 0.7248 & 0.3057 \\ 0.4554 & 0.3057 & 0.7248 \end{bmatrix}$$

3 | Simulations and Results

The data just computed serves to build the impedance matrix for each feeder of the network, information that is useful for the computation and estimation of the state of the system. In this section, the steps to prepare the input data for a state estimator will be described.

As mentioned in 1.3.3 the estimation process starts by filtering the measurements, for that purpose, a dataset of measurements should be available (see Appendix A). In the current study case, we are supposing that the power of the loads is known due to European Network Model. Another available data is the voltage value in the slack bus, understanding the slack bus, as the secondary substation, where the LV network starts, usually provided of voltmeters.

Since for the load, we have a 24-hour profile, for the study case it was selected just an instant of the day (at 7:00 A.M), but the process shown below is valid for the complete profile.

3.1. Load flow for State Estimation

As mentioned in Chapter 1, state estimation is similar to a power flow. Therefore, to gather a suitable dataset to run a state estmation, first it was computed the power flow. The power flow, to use real variables instead of complex ones, each phasor was expressed in polar coordinates, except for the power S that is expressed in Cartesian coordinates, understanding that P is the real part and Q the imaginary part, and therefore $S = P + iQ$. Then, as it will be shown next, with the functions that relate the measurement available with the state, was also considered the topology, implicit in the equations. As for the conventional state estimator, the estimator under study used the functions as in equations 1.3, and the computation was performed using the analytic expression of the Jacobian.

In this case, the state vector was defined as the set of phase voltages (V) at the slack bus, and injection currents (J) in the branches where loads were connected, so the state variable vector in complex format is was defined as:

$$\begin{bmatrix} V_{1A} \\ J_{1A} \\ \vdots \\ J_{nA} \\ V_{1B} \\ J_{1B} \\ \vdots \\ J_{nB} \\ V_{1C} \\ J_{1C} \\ \vdots \\ J_{nC} \end{bmatrix}$$

Below are the results obtained for the bus voltages and the currents flowing through the branches(See Figures 3.3, 3.4 and 3.5). As expected, branches currents are decreasing downward the source bus, and for each phase, on the ramifications, the currents on the branches are zero, if the load is not connected to that phase. In this sense for example, in Figure 3.6, it is shown a comparison plotting of the branches currents, for the three phases, and it is indicate the value of the corrent on branch 33, branch that is connected to the Load1 in phase A, therefore, for phases B and C the current flowing is, as expected 0.

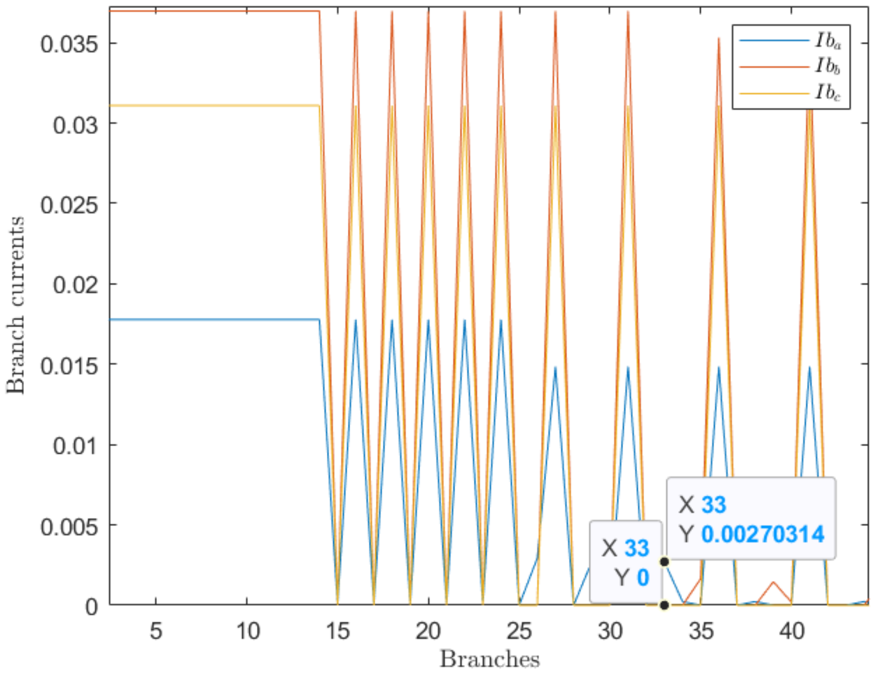


Figure 3.1: Branch currents on the three phases - comparison

The same thing happens with the voltages on every node. For each phase, it is shown a different voltage level, this, because the voltage drop on each branch is proportional to the current flowing in the same. However, this aspect, evidences an imbalance of the system, which can cause damages on the different equipment.

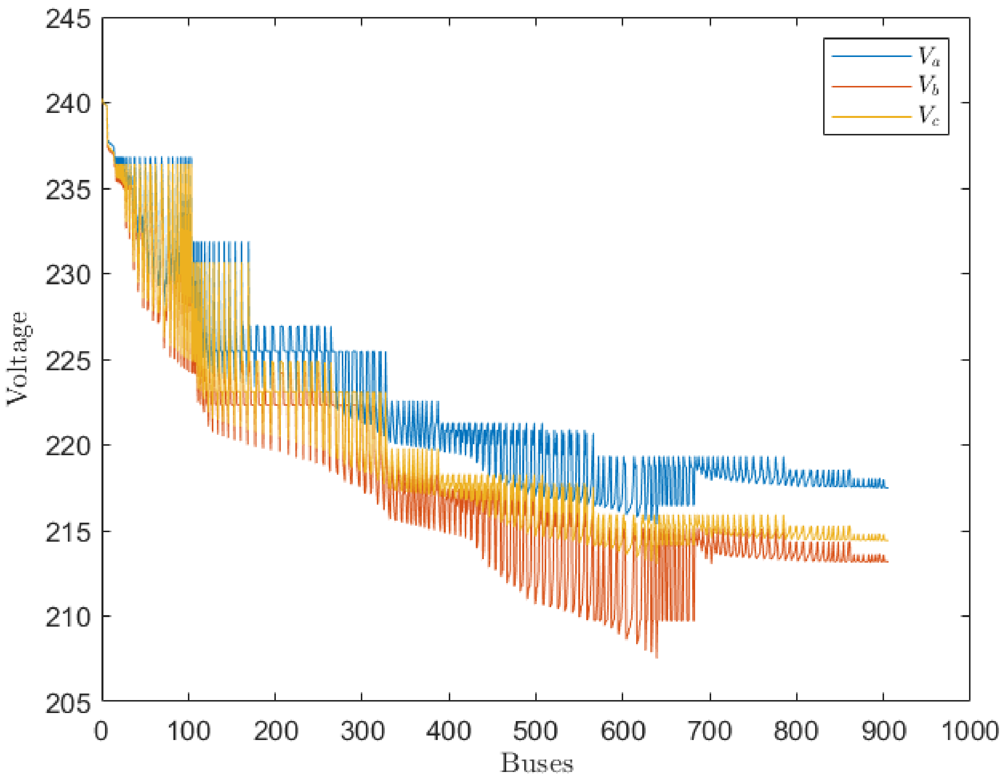
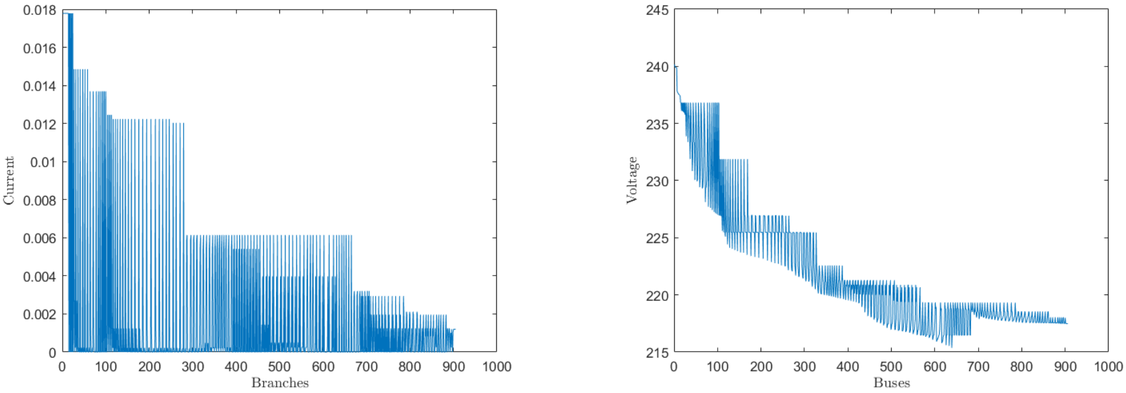


Figure 3.2: Voltages on the three phases-comparison

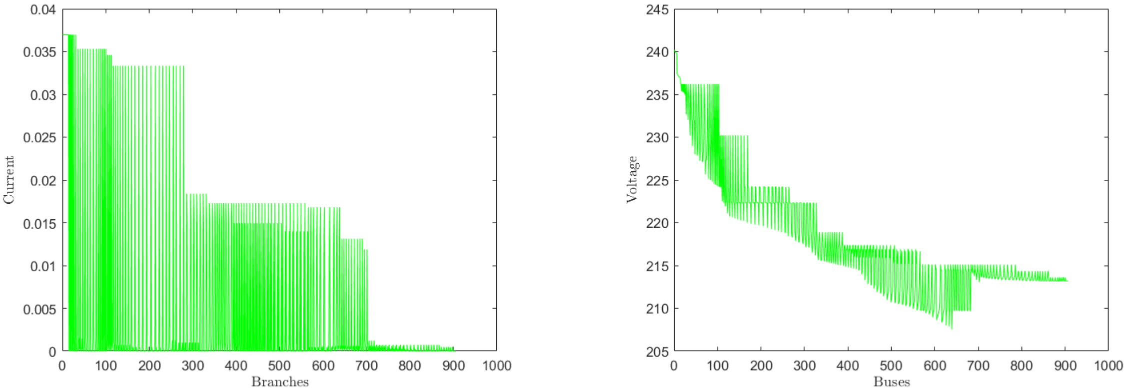
In Figures 3.3, 3.4 and 3.5, there are shown istead the complete profile for branch currents and voltages, for phase A, B and C respectively. It is again evident,as expected, the relation between current and voltage, since, voltages decreases always in the direction of the load, while, currents increase in the direction of the source. It is also evident, how for line with higher branch current flowing, the voltage drop is also higher. In this sence, for example, that the voltage drop in between buses 500 and 600, is higher than the voltage drop at the last buses of the feeder, where branch current are very low.



(a) Branch currents for phase A.

(b) Bus voltages for phase A

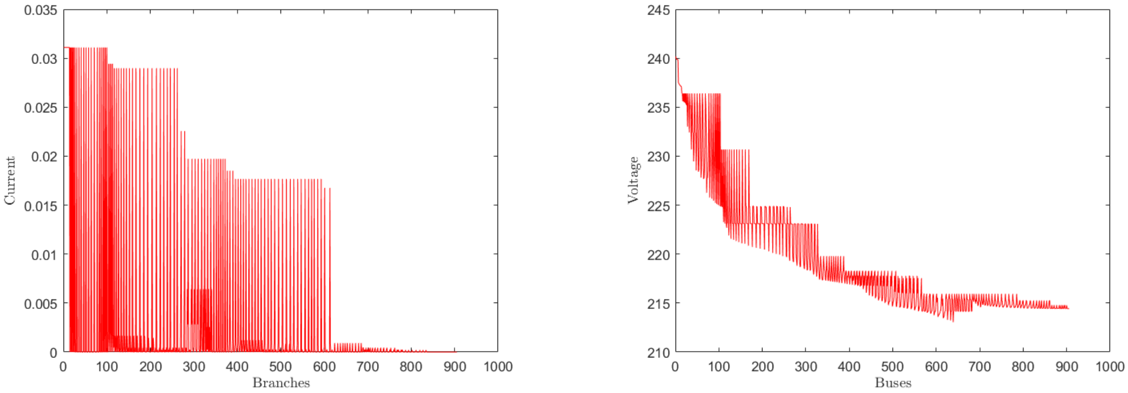
Figure 3.3: Currents and Voltages for phase A.



(a) Branch currents for phase B.

(b) voltages for phase B

Figure 3.4: Currents and Voltages for phase B.



(a) Branch currents for phase C

(b) Bus voltages for phase C

Figure 3.5: Currents and Voltages for phase C.

3.1.1. Comparison with mathematical results

To test the functioning of the algorithm, some mathematical computations were developed, expecting the same result as the shown before. These computations are shown below.

First, it was identified a matrix of currents injected to each node, were the columns correspond to the three phases, and in the rows were the nodes, therefore, it was a 906x3 matrix. Then the apparent power matrix has the following shape (See also A):

$$\begin{bmatrix} 0+j0 & 0+j0 & 0+j0 \\ \vdots & \vdots & \vdots \\ P_{34}+jQ_{34} & 0+j0 & 0+j0 \\ \vdots & \vdots & \vdots \\ 0+j0 & P_{47}+jQ_{47} & 0+j0 \\ \vdots & \vdots & \vdots \\ 0+j0 & 0+j0 & P_{208}+Q_{208} \\ \vdots & \vdots & \vdots \\ P_{906}+jQ_{906} & 0+j0 & 0+j0 \end{bmatrix}_{906x3}$$

Notice that the available measurements were those of the nodes with loads, taking into account the phases in which the loads were connected, for the other nodes it was assumed some "pseudomeasurments". In this sense, it was assumed that the apparent power on nodes with not load connected was 0, for simplicity, meaning that no power was being consumed. Then the three-phase injected current was computed as follows:

$$I_0 = \left(\frac{\bar{S}}{\bar{V}}\right)^* = \left(\frac{P + jQ}{\bar{V}}\right)^*$$

For this, to start iterating, was also assumed that all nodes have the same initial voltage level as the slack, which was $416/\sqrt{3}$V.

Now, we will compute the voltage drop, based on the current on the different branches, having each a load connected.In the studied section of the system, as shown in Figure 3.6, there are two loads present, load 1 and 3 (See Appendix A). The Load 1, connected on the 34^{th} node is only connected to the phase A, as the load 3, connected to the 70^{th} node. Then the power on the loads, at 7:00 A.M are:

$$S_{34} = \begin{bmatrix} 0.6370 + j0.2094 \\ 0.000 + j0.000 \\ 0.000 + j0.000 \end{bmatrix} KVA$$

and

$$S_{70} = \begin{bmatrix} 0.0540 + j0.0177 \\ 0.000 + j0.000 \\ 0.000 + j0.000 \end{bmatrix} KVA$$

As said before, it was initially assumed that on each node where present a voltage level of $416/\sqrt{3}$V, then:

$$V = V_{source} = V_{34} = V_{70}$$

where

$$V = \begin{bmatrix} 240.1777 + j0.0000 \\ -120.08888 - j208.0000 \\ -120.08888 + j208.0000 \end{bmatrix} V$$

Therefore:

$$I_{34} = \begin{bmatrix} 2.6542 - j0.8725 \\ 0.000 - j0.000 \\ 0.000 - j0.000 \end{bmatrix} A$$

And

$$I_{70} = \begin{bmatrix} 0.2291 - j7.5302e - 02 \\ 0.000 - j0.000 \\ 0.000 - j0.000 \end{bmatrix} A$$

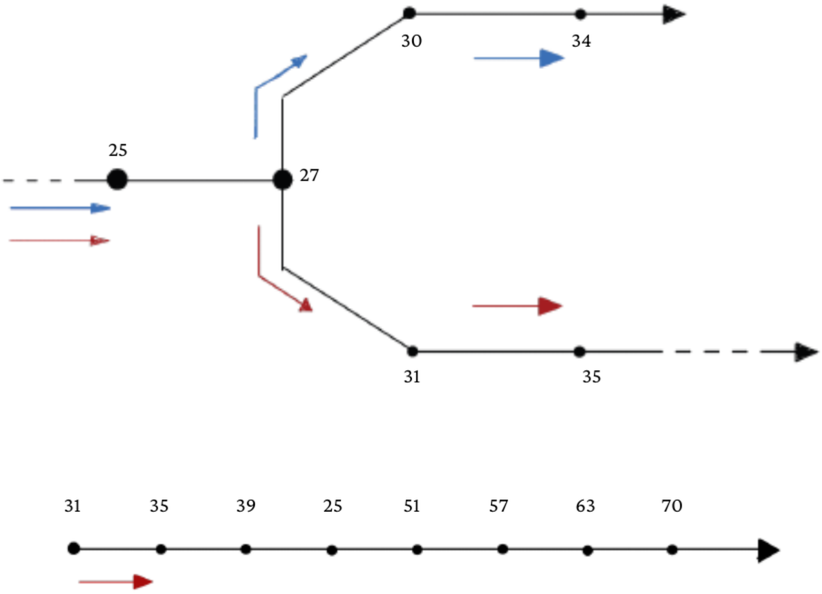


Figure 3.6: System section - Load 34

Then, according to the Figure 3.6, and the currents law of Kirchhoff, the current flowing through the lines in between nodes 27 and 34 will be equal to the current injected to the node34, therefore:

$$Ib_{29} = Ib_{33} = I_{34}$$

Where Ib refers to the branch current and I to the current injected to the node.

With this information, and applying the Kirchhoff's voltages law and the Ohm's law, the voltage drop in the same lines results:

$$\begin{bmatrix} V_{A_{27}} \\ V_{B_{27}} \\ V_{C_{27}} \end{bmatrix} - \begin{bmatrix} V_{A_{34}} \\ V_{B_{34}} \\ V_{C_{34}} \end{bmatrix} = \left(Z_{enel:2x10} * (Length(line_{29}) + Length(line_{33}) * \begin{bmatrix} Ib_{A_{33}} \\ Ib_{B_{33}} \\ Ib_{C_{33}} \end{bmatrix} \right)$$

In the Appendix B, the reduced impedance matrix for each typology of conductor is shown, in the same way, in the Appendix C, the characteristic of each line in the system is shown. With this information, the voltage drop results:

$$\begin{bmatrix} V_{A_{27-34}} \\ V_{B_{27-34}} \\ V_{C_{27-34}} \end{bmatrix} = \begin{bmatrix} 0.0038 + j0.0002 & 0.0019 + j0.0001 & 0.0019 + j0.0001 \\ 0.0019 + j0.0001 & 0.0038 + j0.0003 & 0.0019 + j0.0001 \\ 0.0019 + j0.0001 & 0.0019 + j0.0001 & 0.0038 + j0.0001 \end{bmatrix} *(1627+3725)* \begin{bmatrix} 2.6542 - j0.8725 \\ 0 - j0 \\ 0 - j0 \end{bmatrix}$$

Let's consider just the magnitude, to be able to compare it with the result obtained after running the code, then :

$$\left| \begin{bmatrix} V_{A_{27-34}} \\ V_{B_{27-34}} \\ V_{C_{27-34}} \end{bmatrix} \right| = \begin{bmatrix} 0.0579 \\ 0.0290 \\ 0.0289 \end{bmatrix} V$$

As shown in Figure 3.7, the results coincide, for this first line.

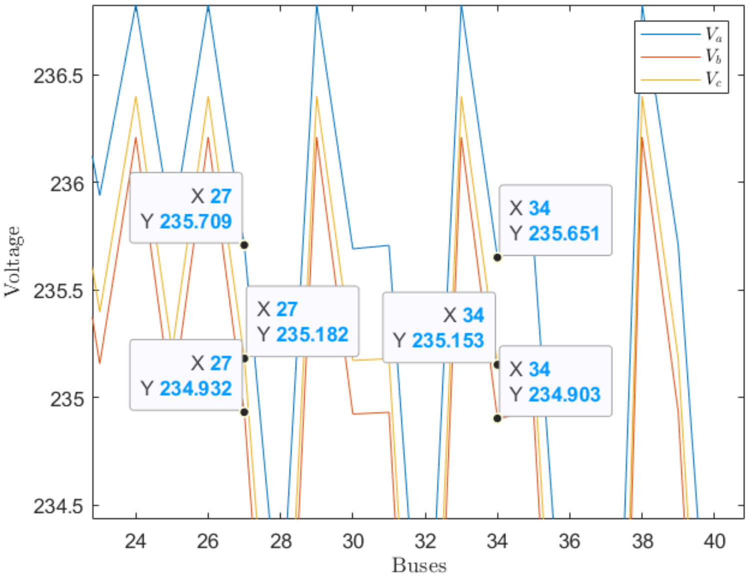


Figure 3.7: Voltages for lines between node 27 and 34

The same process was computed for the lines between nodes 27 and 70, resulting:

$$\left|\begin{bmatrix} V_{A_{27-70}} \\ V_{B_{27-70}} \\ V_{C_{27-70}} \end{bmatrix}\right| = \begin{bmatrix} 0.0062 \\ 0.0031 \\ 0.0031 \end{bmatrix} V$$

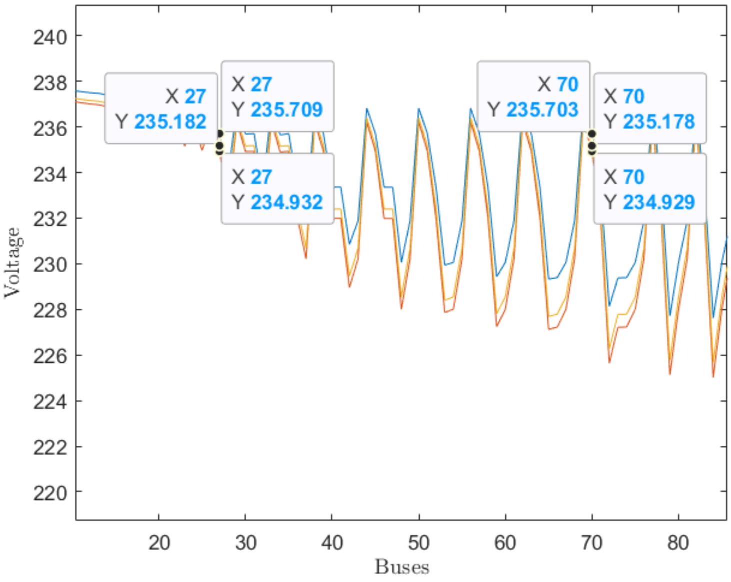


Figure 3.8: Voltages for lines between node 27 and 70

4 | Conclusions

Ever since the State Estimation idea was first developed in the 1970s, it has been an essential component of electrical system management and operation worldwide. As it was mentioned before on this document, State Estimation has been gaining more interest when applied to Distribution Systems. Furthermore, it should be mentioned that traditional distribution networks were passive systems with unidirectional and predictable energy flows, nonetheless, the integration of distributed energy resources is growing at an increasing rate, drawing the interest of multiple academics to this issue.

The objective of this document, in addition to summarizing the evolution of this problem since its interest arose until now, was to prepare a low-voltage network, that was initially developed to be used for a different purpose, but that given its structure, was also usefull to be used as input for an estimation algorithm.

In the first part of the project, were introduced the main methods and features of a conventional state estimator, used for Transmission Networks. Despite the differences between transmission and distribution systems, the same fundamental features of the conventional state estimator were followed and therefore the information of the system to be used needed to be adjusted and completed.

As it was mentioned on this document, the process to make estimation, is similar to run a loadflow, therefore information about the impedances of the lines is essential. This information was not initially available, but taking into account the commercial conductors used in Italy and the available characteristic of the network, the missing parameters, such as resistance and inductance, were computed obtaining a full description of the grid, The result that were obtained, were acceptable since they respect the original characteristic of the network.

The coherence of the system, to be used for this purpose, was used by running the system power flow, and obtaining expected results. In this sense, it was observed that voltages and currents respect all the electrical laws. For example, the relation between current and voltage, was always respected; moreover it was always evident that since the network was passive, and not distributed generation was present, there were not risings on voltages, and instead the voltages decrease always in the direction of the load, while, voltages increases in the direction of the source. the results obtained after running 5 iterations presented a minimum error (less that 1%), compared with the results obtained after mathematical computations; and this is explained by the precision of the methods that were used. Therefore the results were very acceptable.

Moreover, the project's outcomes have created opportunities for further research, since the obtained system is a complete input and this is the very first step to test different methods for estimation, or to obtain a variety of new information of the network, for example it can be thought to realize further development on the algorithm to obtain the thermal state, given the electrical state. It can also be used to propose a different solution according to the developments in algorithms that provide advances and open future lines of research.

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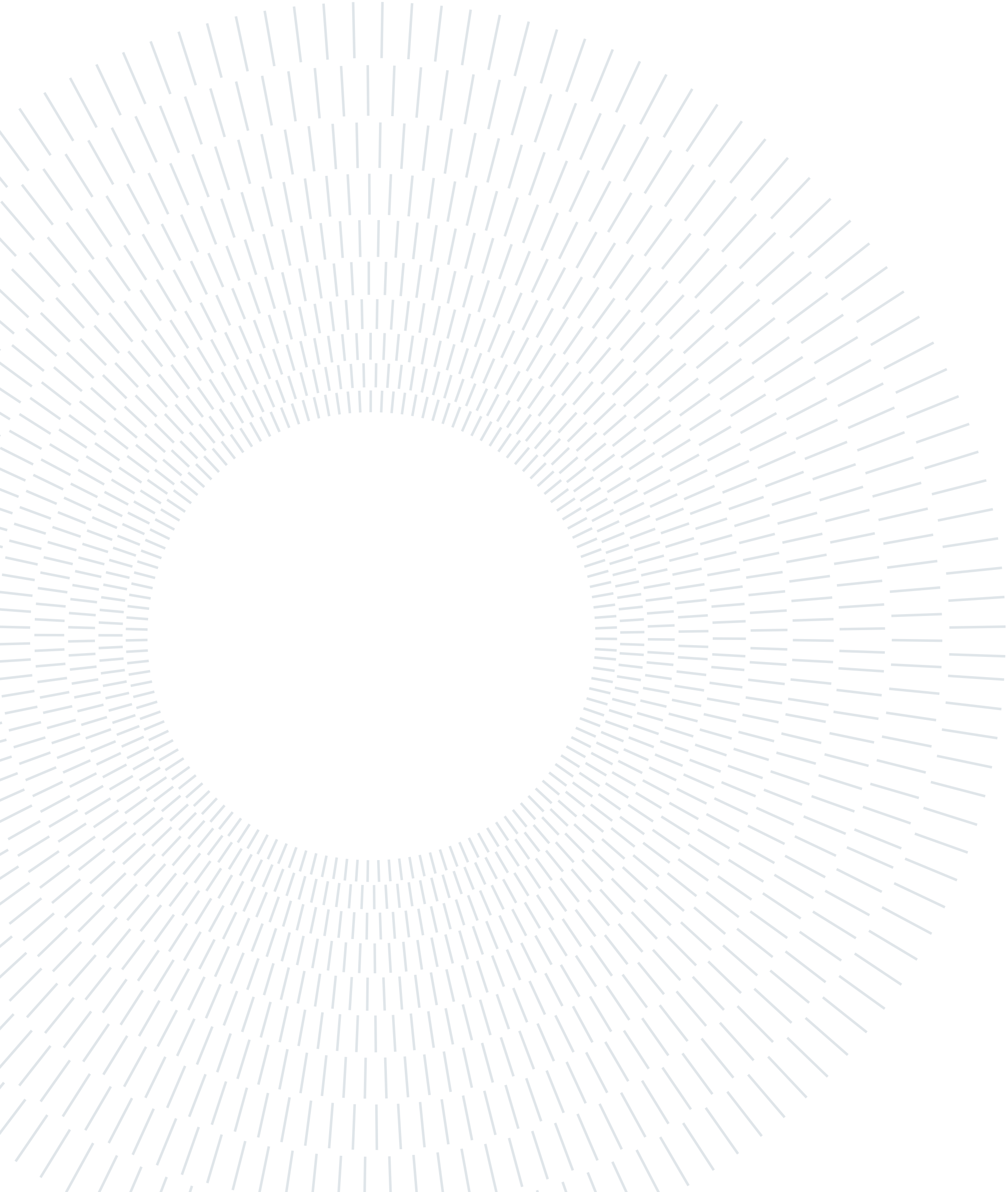


A | Appendix A

ID	Bus	phases	Pa	Pb	Pc	Qa	Qb	Qc
LOAD1	34	A	0.637	NaN	NaN	0.209	NaN	NaN
LOAD2	47	B	NaN	0.044	NaN	NaN	0.014	NaN
LOAD3	70	A	0.054	NaN	NaN	0.018	NaN	NaN
LOAD4	73	A	0.231	NaN	NaN	0.076	NaN	NaN
LOAD5	74	A	0.035	NaN	NaN	0.012	NaN	NaN
LOAD6	83	B	NaN	0.339	NaN	NaN	0.111	NaN
LOAD7	178	B	NaN	0.055	NaN	NaN	0.018	NaN
LOAD8	208	C	NaN	NaN	0.323	NaN	NaN	0.106
LOAD9	225	A	0.281	NaN	NaN	0.092	NaN	NaN
LOAD10	248	B	NaN	0.057	NaN	NaN	0.019	NaN
LOAD11	249	B	NaN	0.049	NaN	NaN	0.016	NaN
LOAD12	264	C	NaN	NaN	0.052	NaN	NaN	0.017
LOAD13	276	B	NaN	0.056	NaN	NaN	0.018	NaN
LOAD14	289	A	0.049	NaN	NaN	0.016	NaN	NaN
LOAD15	314	B	NaN	0.223	NaN	NaN	0.073	NaN
LOAD16	320	C	NaN	NaN	0.052	NaN	NaN	0.017
LOAD17	327	C	NaN	NaN	0.048	NaN	NaN	0.016
LOAD18	337	C	NaN	NaN	0.557	NaN	NaN	0.183
LOAD19	342	C	NaN	NaN	1.406	NaN	NaN	0.462
LOAD20	349	A	0.051	NaN	NaN	0.017	NaN	NaN
LOAD21	387	A	0.045	NaN	NaN	0.015	NaN	NaN
LOAD22	388	A	0.054	NaN	NaN	0.018	NaN	NaN
LOAD23	406	B	NaN	0.236	NaN	NaN	0.078	NaN
LOAD24	458	C	NaN	NaN	0.266	NaN	NaN	0.087
LOAD25	502	A	0.153	NaN	NaN	0.050	NaN	NaN
LOAD26	522	B	NaN	0.208	NaN	NaN	0.068	NaN

ID	Bus	phases	Pa	Pb	Pc	Qa	Qb	Qc
LOAD27	539	C	NaN	NaN	0.155	NaN	NaN	0.051
LOAD28	556	C	NaN	NaN	0.028	NaN	NaN	0.009
LOAD29	562	A	0.054	NaN	NaN	0.018	NaN	NaN
LOAD30	563	A	0.053	NaN	NaN	0.017	NaN	NaN
LOAD31	611	A	0.053	NaN	NaN	0.017	NaN	NaN
LOAD32	614	C	NaN	NaN	3.598	NaN	NaN	1.183
LOAD33	619	C	NaN	NaN	0.057	NaN	NaN	0.019
LOAD34	629	A	0.865	NaN	NaN	0.284	NaN	NaN
LOAD35	639	B	NaN	3.49	NaN	NaN	1.147	NaN
LOAD36	676	B	NaN	0.055	NaN	NaN	0.018	NaN
LOAD37	682	B	NaN	0.045	NaN	NaN	0.015	NaN
LOAD38	688	B	NaN	0.052	NaN	NaN	0.017	NaN
LOAD39	701	C	NaN	NaN	0.048	NaN	NaN	0.016
LOAD40	702	B	NaN	2.534	NaN	NaN	0.833	NaN
LOAD41	755	B	NaN	0.052	NaN	NaN	0.017	NaN
LOAD42	778	C	NaN	NaN	0.054	NaN	NaN	0.018
LOAD43	780	C	NaN	NaN	0.05	NaN	NaN	0.016
LOAD44	785	B	NaN	0.133	NaN	NaN	0.044	NaN
LOAD45	813	B	NaN	0.053	NaN	NaN	0.017	NaN
LOAD46	817	A	0.462	NaN	NaN	0.152	NaN	NaN
LOAD47	835	C	NaN	NaN	0.045	NaN	NaN	0.015
LOAD48	860	A	0.126	NaN	NaN	0.041	NaN	NaN
LOAD49	861	A	0.054	NaN	NaN	0.018	NaN	NaN
LOAD50	886	B	NaN	0.052	NaN	NaN	0.017	NaN
LOAD51	896	A	0.053	NaN	NaN	0.017	NaN	NaN
LOAD52	898	A	0.166	NaN	NaN	0.055	NaN	NaN
LOAD53	899	B	NaN	0.107	NaN	NaN	0.035	NaN
LOAD54	900	A	0.219	NaN	NaN	0.072	NaN	NaN
LOAD55	906	A	0.26	NaN	NaN	0.085	NaN	NaN

Table A.1: Loads data at 7:00 A.M

Further information about load can be found in reference [25]

B | Appendix B

Bellow there are gather the reduced impedance matrix obtained for each conductor type, following the procedure in section 2.1.2

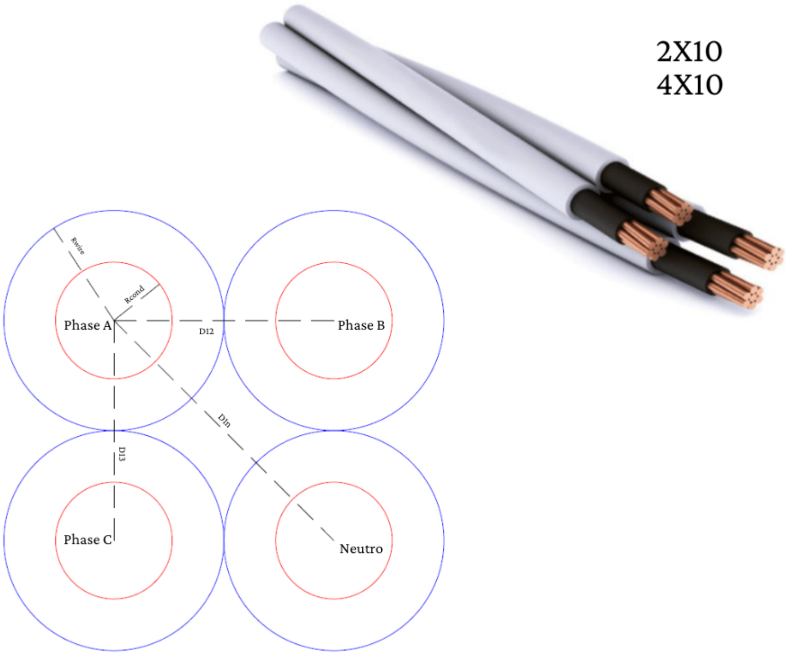


Figure B.1: Geometry of Enel 2x10 and 4x10

B.1. 2x10

$$Z_{enel:2x10} = \begin{bmatrix} 0.0038 + j0.0002 & 0.0019 + j0.0001 & 0.0019 + j0.0001 \\ 0.0019 + j0.0001 & 0.0038 + j0.0003 & 0.0019 + j0.0001 \\ 0.0019 + j0.0001 & 0.0019 + j0.0001 & 0.0038 + j0.0001 \end{bmatrix}$$

B.2. 4x10

$$Z_{enel:4x10} = \begin{bmatrix} 0.0037 + j0.0003 & 0.0018 + j0.0001 & 0.0018 + j0.0001 \\ 0.0018 + j0.0001 & 0.0037 + j0.0002 & 0.0018 + j0.0001 \\ 0.0018 + j0.0001 & 0.0018 + j0.0001 & 0.0037 + j0.0002 \end{bmatrix}$$

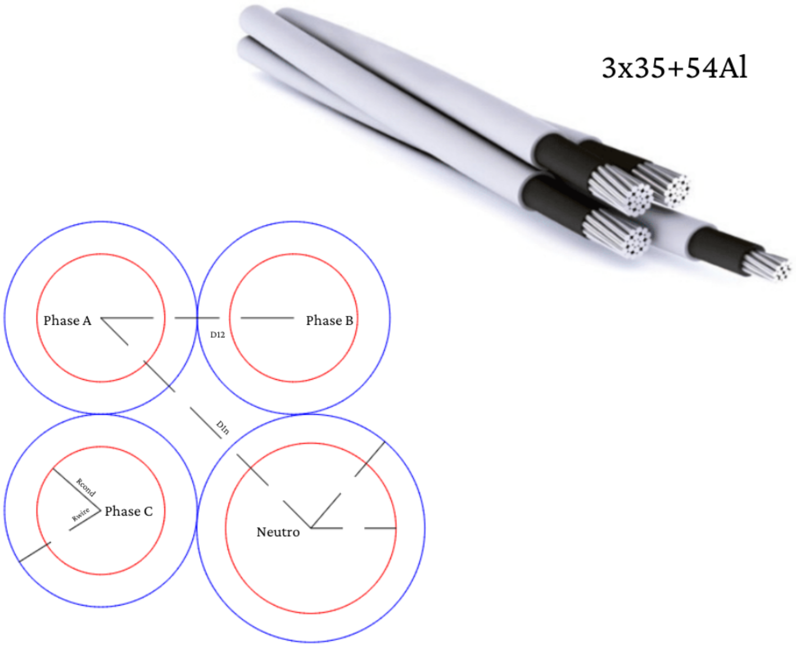


Figure B.2: Geometry of Enel 3x35+54Al

B.3. 3x35+54Al

$$Z_{enel:3x35+54Al} = \begin{bmatrix} 0.0020 + j0.0002 & 0.0012 + j0.0001 & 0.0012 + j0.0001 \\ 0.0012 + j0.0001 & 0.0020 + j0.0002 & 0.0012 + j0.0001 \\ 0.0012 + j0.0001 & 0.0012 + j0.0001 & 0.0020 + j0.0002 \end{bmatrix}$$

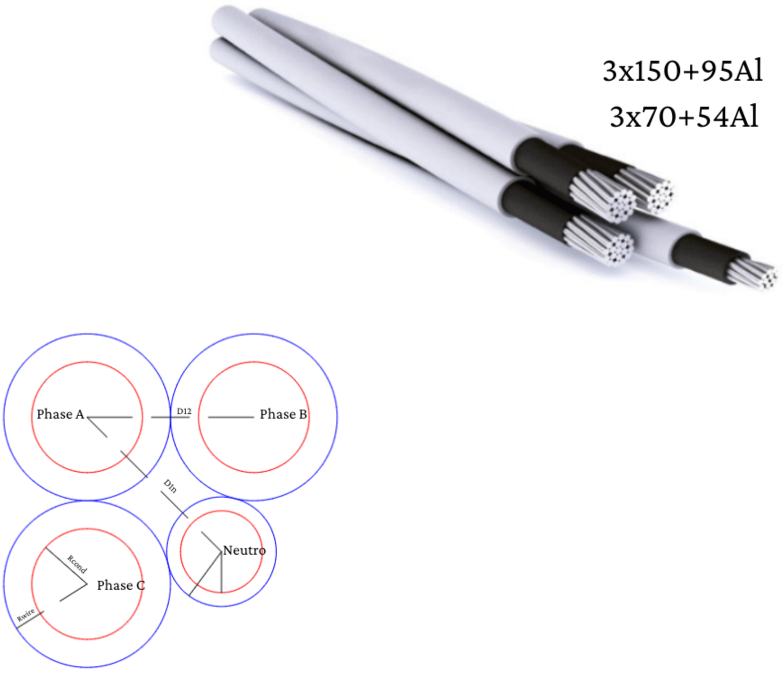


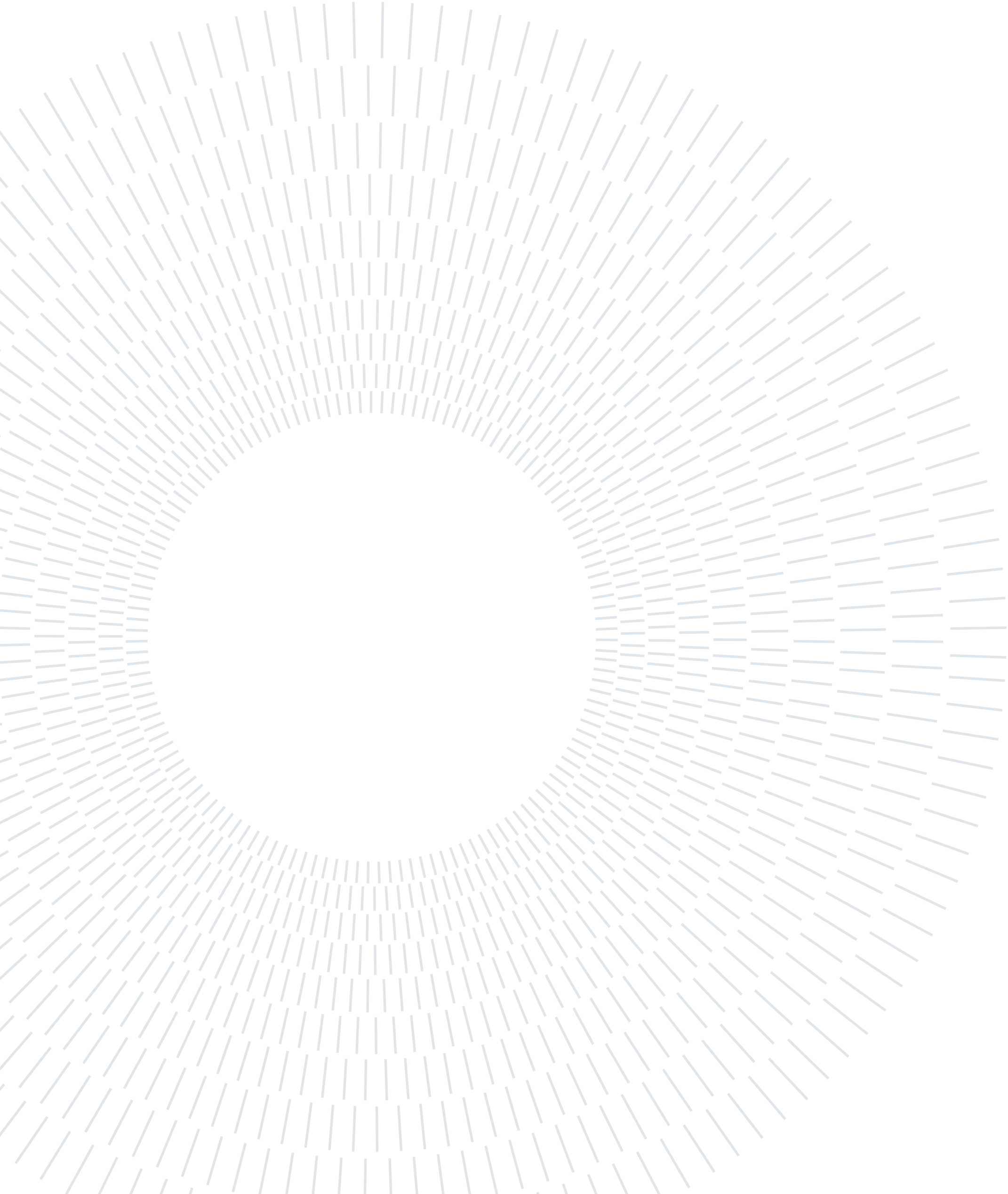
Figure B.3: Geometry of Enel 3x70+54Al and 3x150+95Al

B.4. 3x70+54Al

$$Z_{enel:3x70+54Al} = \begin{bmatrix} 8.6429 + j2.2922 & 4.2429 + j1.1407 & 4.2429 + j1.1407 \\ 4.2429 + j1.1421 & 8.6429 + j1.9605 & 4.2429 + j0.8091 \\ 4.2429 + j1.1407 & 4.2429 + j0.8091 & 8.6429 + j1.9605 \end{bmatrix} * 10^{-4}$$

B.5. 3x150+95Al

$$Z_{enel:-3x150+95Al=} \begin{bmatrix} 8.6429 + j2.2922 & 4.2429 + j1.1407 & 4.2429 + j1.1407 \\ 4.2429 + j1.1421 & 8.6429 + j1.9605 & 4.2429 + j0.8091 \\ 4.2429 + j1.1407 & 4.2429 + j0.8091 & 8.6429 + j1.9605 \end{bmatrix} * 10^{-4}$$



C | Appendix C

ID	From	To	Enel Code	Length
LINE1	1	2	4X10	1.098
LINE2	2	3	4X10	0.115
LINE3	3	4	4X10	0.108
LINE4	4	5	4X10	0.094
LINE5	5	6	4X10	0.148
LINE6	6	7	4X10	10.086
LINE7	7	8	4X10	0.452
LINE8	8	9	4X10	0.376
LINE9	9	10	4X10	0.330
LINE10	10	11	4X10	0.246
LINE11	11	12	4X10	0.231
LINE12	12	13	4X10	0.142
LINE13	13	14	4X10	0.354
LINE14	14	15	4X10	2.866
LINE15	15	16	2X10	0.097
LINE16	15	17	4X10	3.476
LINE17	16	18	2X10	0.044
LINE18	17	19	4X10	0.239
LINE19	18	20	2X10	0.046
LINE20	19	21	4X10	0.269
LINE21	20	22	2X10	0.042
LINE22	21	23	4X10	0.437
LINE23	22	24	2X10	0.051
LINE24	23	25	4X10	0.796
LINE25	24	26	2X10	0.056
LINE26	25	27	2X10	5.905

ID	From	To	Enel Code	Length
LINE27	25	28	4X10	9.969
LINE28	26	29	2X10	0.055
LINE29	27	30	2X10	1.627
LINE30	27	31	2X10	1.585
LINE31	28	32	4X10	2.654
LINE32	29	33	2X10	0.146
LINE33	30	34	2X10	3.725
LINE34	31	35	2X10	0.054
LINE35	32	36	2X10	5.727
LINE36	32	37	4X10	8.024
LINE37	33	38	2X10	0.136
LINE38	35	39	2X10	0.077
LINE39	36	40	2X10	0.400
LINE40	36	41	2X10	1.806
LINE41	37	42	4X10	5.568
LINE42	37	43	3X70+54AL	0.217
LINE43	38	44	2X10	0.034
LINE44	39	45	2X10	0.099
LINE45	40	46	2X10	0.180
LINE46	41	47	2X10	4.346
LINE47	42	48	4X10	4.208
LINE48	43	49	3X70+54AL	0.104
LINE49	44	50	2X10	0.048
LINE50	45	51	2X10	0.241
LINE51	46	52	2X10	0.191
LINE52	48	53	4X10	0.607
LINE53	48	54	4X10	0.639
LINE54	49	55	3X70+54AL	0.116
LINE55	50	56	2X10	0.088
LINE56	51	57	2X10	0.206
LINE57	52	58	2X10	0.182
LINE58	53	59	4X10	2.748
LINE59	54	60	4X10	0.065
LINE60	55	61	3X70+54AL	0.100
LINE61	56	62	2X10	0.078
LINE62	57	63	2X10	0.989
LINE63	58	64	2X10	0.109

ID	From	To	Enel Code	Length
LINE64	59	65	4X10	0.559
LINE65	59	66	2X10	8.258
LINE66	60	67	4X10	0.074
LINE67	61	68	3X70+54AL	0.096
LINE68	62	69	2X10	0.083
LINE69	63	70	2X10	3.496
LINE70	64	71	2X10	0.214
LINE71	65	72	4X10	6.601
LINE72	66	73	2X10	4.037
LINE73	66	74	2X10	4.037
LINE74	67	75	4X10	0.180
LINE75	68	76	3X70+54AL	0.236
LINE76	69	77	2X10	0.389
LINE77	71	78	2X10	1.084
LINE78	72	79	4X10	2.301
LINE79	75	80	4X10	0.104
LINE80	76	81	3X70+54AL	0.747
LINE81	77	82	2X10	0.511
LINE82	78	83	2X10	4.084
LINE83	79	84	4X10	0.502
LINE84	80	85	4X10	0.111
LINE85	81	86	3X70+54AL	1.439
LINE86	82	87	2X10	0.310
LINE87	84	88	4X10	1.018
LINE88	85	89	4X10	0.105
LINE89	86	90	3X70+54AL	0.343
LINE90	87	91	2X10	0.112
LINE91	88	92	4X10	0.960
LINE92	90	93	3X70+54AL	0.065
LINE93	91	94	2X10	0.035
LINE94	92	95	4X10	0.558
LINE95	93	96	3X70+54AL	0.086
LINE96	94	97	2X10	0.037
LINE97	95	98	4X10	0.369
LINE98	96	99	3X70+54AL	0.037
LINE99	97	100	2X10	0.090
LINE100	98	101	4X10	0.509

ID	From	To	Enel Code	Length
LINE101	99	102	3X70+54AL	5.082
LINE102	100	103	2X10	0.286
LINE103	101	104	3X150+95AL	0.288
LINE104	101	105	4X10	0.336
LINE105	102	106	3X70+54AL	0.221
LINE106	104	107	3X150+95AL	0.349
LINE107	105	108	4X10	0.191
LINE108	106	109	3X70+54AL	0.451
LINE109	107	110	3X150+95AL	0.404
LINE110	108	111	4X10	7.518
LINE111	109	112	3X70+54AL	11.977
LINE112	110	113	3X150+95AL	0.617
LINE113	111	114	4X10	0.753
LINE114	112	115	3X70+54AL	10.661
LINE115	113	116	3X150+95AL	0.793
LINE116	114	117	3X150+95AL	1.106
LINE117	114	118	3X35+54AL	3.656
LINE118	115	119	3X70+54AL	0.995
LINE119	116	120	3X150+95AL	0.569
LINE120	117	121	2X10	0.195
LINE121	117	122	3X150+95AL	0.538
LINE122	118	123	3X35+54AL	6.592
LINE123	119	124	3X70+54AL	0.725
LINE124	120	125	3X150+95AL	0.804
LINE125	121	126	2X10	0.169
LINE126	122	127	3X150+95AL	2.529
LINE127	123	128	3X35+54AL	2.965
LINE128	124	129	3X70+54AL	0.625
LINE129	125	130	3X150+95AL	0.870
LINE130	126	131	2X10	0.119
LINE131	127	132	2X10	3.821
LINE132	127	133	3X150+95AL	7.178
LINE133	128	134	3X35+54AL	0.882
LINE134	129	135	3X70+54AL	1.920
LINE135	130	136	3X150+95AL	0.820
LINE136	131	137	2X10	0.184

ID	From	To	Enel Code	Length
LINE137	132	138	2X10	0.329
LINE138	133	139	3X150+95AL	5.340
LINE139	134	140	3X35+54AL	0.996
LINE140	135	141	3X70+54AL	1.413
LINE141	136	142	3X150+95AL	1.349
LINE142	137	143	2X10	0.218
LINE143	138	144	2X10	0.323
LINE144	139	145	3X150+95AL	3.576
LINE145	140	146	3X35+54AL	0.936
LINE146	141	147	3X70+54AL	0.241
LINE147	142	148	3X150+95AL	6.414
LINE148	143	149	2X10	0.194
LINE149	144	150	2X10	0.261
LINE150	145	151	3X150+95AL	3.571
LINE151	145	152	2X10	5.131
LINE152	146	153	3X35+54AL	1.093
LINE153	147	154	3X70+54AL	0.083
LINE154	148	155	3X150+95AL	1.397
LINE155	149	156	2X10	0.219
LINE156	150	157	2X10	0.391
LINE157	151	158	3X150+95AL	9.471
LINE158	152	159	2X10	0.312
LINE159	153	160	3X35+54AL	0.314
LINE160	154	161	3X70+54AL	0.123
LINE161	155	162	2X10	11.742
LINE162	155	163	3X150+95AL	5.284
LINE163	156	164	2X10	0.203
LINE164	157	165	2X10	0.472
LINE165	158	166	3X150+95AL	3.290
LINE166	159	167	2X10	0.285
LINE167	160	168	3X35+54AL	1.786
LINE168	161	169	3X70+54AL	0.250
LINE169	162	170	2X10	2.218
LINE170	163	171	3X150+95AL	7.016
LINE171	164	172	2X10	0.173
LINE172	165	173	2X10	0.356

ID	From	To	Enel Code	Length
LINE173	166	174	3X150+95AL	3.918
LINE174	166	175	2X10	4.205
LINE175	167	176	2X10	0.119
LINE176	168	177	3X35+54AL	1.009
LINE177	170	178	2X10	4.577
LINE178	171	179	3X150+95AL	1.182
LINE179	171	180	2X10	2.422
LINE180	172	181	2X10	0.180
LINE181	173	182	2X10	0.412
LINE182	174	183	3X150+95AL	0.588
LINE183	175	184	2X10	0.454
LINE184	176	185	2X10	0.255
LINE185	177	186	3X35+54AL	0.749
LINE186	179	187	3X150+95AL	4.651
LINE187	180	188	2X10	10.026
LINE188	181	189	2X10	0.154
LINE189	182	190	2X10	0.388
LINE190	183	191	3X150+95AL	2.315
LINE191	183	192	3X150+95AL	0.278
LINE192	184	193	2X10	0.439
LINE193	185	194	2X10	0.651
LINE194	186	195	3X35+54AL	0.673
LINE195	187	196	3X150+95AL	8.304
LINE196	188	197	2X10	1.960
LINE197	188	198	2X10	1.133
LINE198	189	199	2X10	0.191
LINE199	190	200	2X10	0.417
LINE200	191	201	3X150+95AL	0.931
LINE201	192	202	3X150+95AL	0.164
LINE202	193	203	2X10	0.351
LINE203	194	204	2X10	0.382
LINE204	195	205	3X35+54AL	0.836
LINE205	196	206	2X10	3.248
LINE206	196	207	3X150+95AL	2.137
LINE207	197	208	2X10	4.954
LINE208	198	209	2X10	0.160

ID	From	To	Enel Code	Length
LINE209	199	210	2X10	0.448
LINE210	200	211	2X10	0.638
LINE211	202	212	3X150+95AL	0.155
LINE212	203	213	2X10	0.280
LINE213	204	214	2X10	0.267
LINE214	205	215	3X35+54AL	1.543
LINE215	206	216	2X10	10.630
LINE216	207	217	3X150+95AL	3.857
LINE217	209	218	2X10	0.129
LINE218	210	219	2X10	0.672
LINE219	211	220	2X10	1.742
LINE220	212	221	3X150+95AL	0.170
LINE221	213	222	2X10	0.592
LINE222	214	223	2X10	0.514
LINE223	215	224	3X35+54AL	1.346
LINE224	216	225	2X10	4.582
LINE225	217	226	3X150+95AL	1.377
LINE226	218	227	2X10	0.103
LINE227	220	228	2X10	0.149
LINE228	221	229	3X150+95AL	0.233
LINE229	222	230	2X10	0.466
LINE230	223	231	2X10	0.256
LINE231	224	232	3X35+54AL	0.915
LINE232	226	233	2X10	11.895
LINE233	226	234	3X150+95AL	1.127
LINE234	227	235	2X10	0.157
LINE235	228	236	2X10	0.069
LINE236	229	237	3X150+95AL	0.207
LINE237	230	238	2X10	0.323
LINE238	231	239	2X10	0.182
LINE239	232	240	3X35+54AL	0.601
LINE240	233	241	2X10	1.727
LINE241	235	242	2X10	0.185
LINE242	236	243	2X10	0.073
LINE243	237	244	3X150+95AL	0.169
LINE244	238	245	2X10	0.322

ID	From	To	Enel Code	Length
LINE245	239	246	2X10	2.080
LINE246	240	247	3X35+54AL	1.294
LINE247	241	248	2X10	5.302
LINE248	241	249	2X10	5.302
LINE249	242	250	2X10	0.284
LINE250	243	251	2X10	0.070
LINE251	244	252	3X150+95AL	0.133
LINE252	245	253	2X10	0.472
LINE253	246	254	2X10	0.613
LINE254	247	255	2X10	4.562
LINE255	247	256	3X35+54AL	0.857
LINE256	250	257	2X10	1.038
LINE257	251	258	2X10	0.078
LINE258	252	259	3X150+95AL	0.173
LINE259	253	260	2X10	1.774
LINE260	254	261	2X10	0.137
LINE261	255	262	2X10	0.631
LINE262	256	263	3X35+54AL	3.213
LINE263	257	264	2X10	5.050
LINE264	258	265	2X10	0.161
LINE265	259	266	3X150+95AL	0.194
LINE266	260	267	2X10	0.414
LINE267	261	268	2X10	3.458
LINE268	261	269	2X10	0.119
LINE269	262	270	2X10	0.290
LINE270	263	271	2X10	4.255
LINE271	263	272	3X35+54AL	1.079
LINE272	265	273	2X10	0.168
LINE273	266	274	3X150+95AL	0.179
LINE274	267	275	2X10	0.190
LINE275	268	276	2X10	3.427
LINE276	269	277	2X10	0.139
LINE277	270	278	2X10	0.344
LINE278	271	279	2X10	0.309
LINE279	272	280	3X35+54AL	3.912
LINE280	273	281	2X10	0.639

ID	From	To	Enel Code	Length
LINE281	274	282	3X150+95AL	0.605
LINE282	275	283	2X10	0.125
LINE283	277	284	2X10	0.126
LINE284	278	285	2X10	0.531
LINE285	279	286	2X10	0.255
LINE286	280	287	4X10	1.754
LINE287	280	288	3X35+54AL	0.913
LINE288	281	289	2X10	3.390
LINE289	282	290	3X150+95AL	0.810
LINE290	283	291	2X10	0.876
LINE291	283	292	2X10	0.108
LINE292	284	293	2X10	0.127
LINE293	285	294	2X10	0.473
LINE294	286	295	2X10	0.362
LINE295	287	296	4X10	0.807
LINE296	288	297	3X35+54AL	3.827
LINE297	291	298	2X10	0.402
LINE298	292	299	2X10	0.076
LINE299	293	300	2X10	0.137
LINE300	294	301	2X10	0.368
LINE301	295	302	2X10	1.626
LINE302	296	303	4X10	5.275
LINE303	297	304	3X35+54AL	0.845
LINE304	298	305	2X10	0.681
LINE305	299	306	2X10	0.089
LINE306	300	307	2X10	0.794
LINE307	301	308	2X10	0.458
LINE308	302	309	2X10	0.313
LINE309	303	310	4X10	0.394
LINE310	304	311	3X35+54AL	0.281
LINE311	305	312	2X10	1.308
LINE312	306	313	2X10	0.170
LINE313	307	314	2X10	3.360
LINE314	308	315	2X10	0.474
LINE315	309	316	2X10	0.232
LINE316	310	317	2X10	0.649

ID	From	To	Enel Code	Length
LINE317	310	318	4X10	7.216
LINE318	311	319	3X35+54AL	0.146
LINE319	312	320	2X10	3.850
LINE320	313	321	2X10	0.653
LINE321	315	322	2X10	0.559
LINE322	316	323	2X10	0.267
LINE323	317	324	2X10	3.343
LINE324	318	325	4X10	3.463
LINE325	319	326	3X35+54AL	0.125
LINE326	321	327	2X10	2.228
LINE327	322	328	2X10	0.319
LINE328	323	329	2X10	0.265
LINE329	324	330	2X10	1.167
LINE330	325	331	2X10	9.133
LINE331	325	332	4X10	6.673
LINE332	326	333	3X35+54AL	0.143
LINE333	328	334	2X10	1.439
LINE334	329	335	2X10	2.327
LINE335	330	336	2X10	2.540
LINE336	331	337	2X10	3.617
LINE337	332	338	2X10	6.807
LINE338	332	339	4X10	1.213
LINE339	333	340	3X35+54AL	0.112
LINE340	334	341	2X10	0.560
LINE341	335	342	2X10	4.757
LINE342	336	343	2X10	2.082
LINE343	336	344	2X10	0.980
LINE344	338	345	2X10	3.422
LINE345	339	346	4X10	1.039
LINE346	340	347	3X35+54AL	0.094
LINE347	341	348	2X10	0.195
LINE348	343	349	2X10	3.115
LINE349	344	350	2X10	0.136
LINE350	345	351	2X10	0.318
LINE351	346	352	4X10	0.731
LINE352	347	353	3X35+54AL	0.085

ID	From	To	Enel Code	Length
LINE353	348	354	2X10	0.147
LINE354	350	355	2X10	0.148
LINE355	351	356	2X10	0.132
LINE356	352	357	4X10	0.924
LINE357	353	358	3X35+54AL	0.070
LINE358	354	359	2X10	0.107
LINE359	355	360	2X10	0.130
LINE360	356	361	2X10	0.093
LINE361	357	362	4X10	0.579
LINE362	358	363	3X35+54AL	0.082
LINE363	359	364	2X10	0.054
LINE364	360	365	2X10	0.138
LINE365	361	366	2X10	0.095
LINE366	362	367	4X10	0.889
LINE367	363	368	3X35+54AL	0.090
LINE368	364	369	2X10	0.080
LINE369	365	370	2X10	0.197
LINE370	366	371	2X10	0.112
LINE371	367	372	4X10	0.886
LINE372	368	373	3X35+54AL	0.489
LINE373	369	374	2X10	0.134
LINE374	370	375	2X10	0.239
LINE375	371	376	2X10	0.141
LINE376	372	377	4X10	0.994
LINE377	373	378	3X35+54AL	3.685
LINE378	373	379	2X10	1.412
LINE379	374	380	2X10	0.599
LINE380	375	381	2X10	1.483
LINE381	376	382	2X10	0.127
LINE382	377	383	4X10	0.719
LINE383	378	384	3X35+54AL	3.529
LINE384	379	385	2X10	8.667
LINE385	379	386	2X10	1.456
LINE386	380	387	2X10	2.200
LINE387	381	388	2X10	5.073
LINE388	382	389	2X10	0.100

ID	From	To	Enel Code	Length
LINE389	383	390	4X10	0.857
LINE390	384	391	3X35+54AL	0.157
LINE391	385	392	2X10	3.168
LINE392	386	393	2X10	7.834
LINE393	389	394	2X10	0.172
LINE394	390	395	4X10	1.244
LINE395	391	396	2X10	4.060
LINE396	391	397	3X35+54AL	2.879
LINE397	392	398	2X10	0.322
LINE398	393	399	2X10	0.569
LINE399	394	400	2X10	0.600
LINE400	395	401	4X10	0.922
LINE401	396	402	2X10	0.372
LINE402	397	403	3X35+54AL	1.389
LINE403	398	404	2X10	0.225
LINE404	399	405	2X10	0.248
LINE405	400	406	2X10	3.667
LINE406	401	407	4X10	0.890
LINE407	402	408	2X10	0.211
LINE408	403	409	3X35+54AL	4.191
LINE409	403	410	3X35+54AL	0.379
LINE410	404	411	2X10	0.233
LINE411	405	412	2X10	0.167
LINE412	407	413	4X10	0.871
LINE413	408	414	2X10	0.144
LINE414	409	415	3X35+54AL	5.969
LINE415	410	416	3X35+54AL	0.095
LINE416	411	417	2X10	0.979
LINE417	412	418	2X10	0.280
LINE418	413	419	4X10	1.047
LINE419	414	420	2X10	0.234
LINE420	415	421	3X35+54AL	1.249
LINE421	416	422	3X35+54AL	0.067
LINE422	417	423	2X10	0.239
LINE423	418	424	2X10	3.194
LINE424	419	425	4X10	1.754

ID	From	To	Enel Code	Length
LINE425	419	426	4X10	1.754
LINE426	420	427	2X10	0.778
LINE427	421	428	3X35+54AL	0.319
LINE428	422	429	3X35+54AL	0.083
LINE429	423	430	2X10	0.228
LINE430	424	431	2X10	1.133
LINE431	425	432	4X10	5.915
LINE432	425	433	4X10	5.915
LINE433	427	434	2X10	0.866
LINE434	428	435	3X35+54AL	0.167
LINE435	429	436	3X35+54AL	0.071
LINE436	430	437	2X10	0.227
LINE437	431	438	2X10	0.384
LINE438	432	439	4X10	8.366
LINE439	432	440	4X10	8.366
LINE440	434	441	2X10	0.487
LINE441	435	442	3X35+54AL	0.139
LINE442	436	443	3X35+54AL	0.078
LINE443	437	444	2X10	0.260
LINE444	438	445	2X10	0.679
LINE445	439	446	4X10	6.003
LINE446	439	447	4X10	6.003
LINE447	441	448	2X10	0.458
LINE448	442	449	3X35+54AL	0.112
LINE449	443	450	3X35+54AL	0.079
LINE450	444	451	2X10	0.898
LINE451	445	452	2X10	0.594
LINE452	446	453	4X10	1.821
LINE453	446	454	4X10	1.821
LINE454	448	455	2X10	0.437
LINE455	449	456	3X35+54AL	0.172
LINE456	450	457	3X35+54AL	0.054
LINE457	451	458	2X10	4.377
LINE458	452	459	2X10	0.671
LINE459	453	460	4X10	5.391
LINE460	453	461	4X10	5.391

ID	From	To	Enel Code	Length
LINE461	453	462	2X10	3.686
LINE462	455	463	2X10	0.717
LINE463	456	464	3X35+54AL	0.155
LINE464	457	465	3X35+54AL	0.040
LINE465	459	466	2X10	0.634
LINE466	460	467	4X10	5.989
LINE467	460	468	4X10	5.989
LINE468	460	469	4X10	0.883
LINE469	462	470	2X10	0.538
LINE470	463	471	2X10	0.499
LINE471	464	472	3X35+54AL	0.147
LINE472	465	473	3X35+54AL	0.049
LINE473	466	474	2X10	3.201
LINE474	467	475	4X10	2.195
LINE475	467	476	4X10	2.195
LINE476	470	477	2X10	0.294
LINE477	471	478	2X10	0.351
LINE478	472	479	3X35+54AL	0.166
LINE479	473	480	3X35+54AL	0.064
LINE480	474	481	2X10	0.190
LINE481	475	482	4X10	4.643
LINE482	475	483	4X10	4.643
LINE483	475	484	2X10	10.277
LINE484	477	485	2X10	0.199
LINE485	478	486	2X10	0.168
LINE486	479	487	3X35+54AL	0.175
LINE487	480	488	3X35+54AL	0.112
LINE488	481	489	2X10	0.171
LINE489	482	490	4X10	6.655
LINE490	482	491	4X10	6.655
LINE491	484	492	2X10	0.578
LINE492	484	493	2X10	4.073
LINE493	485	494	2X10	0.302
LINE494	486	495	2X10	0.106
LINE495	487	496	3X35+54AL	0.383
LINE496	488	497	3X35+54AL	0.140

ID	From	To	Enel Code	Length
LINE497	489	498	2X10	0.233
LINE498	490	499	4X10	4.061
LINE499	490	500	4X10	4.061
LINE500	492	501	2X10	0.244
LINE501	493	502	2X10	2.606
LINE502	494	503	2X10	0.458
LINE503	495	504	2X10	0.072
LINE504	496	505	3X35+54AL	8.696
LINE505	497	506	3X35+54AL	0.096
LINE506	498	507	2X10	0.306
LINE507	499	508	4X10	0.699
LINE508	499	509	4X10	0.699
LINE509	501	510	2X10	0.144
LINE510	503	511	2X10	0.325
LINE511	504	512	2X10	0.100
LINE512	505	513	2X10	5.565
LINE513	505	514	3X35+54AL	1.337
LINE514	506	515	3X35+54AL	0.085
LINE515	508	516	4X10	1.886
LINE516	508	517	4X10	1.886
LINE517	508	518	2X10	4.732
LINE518	510	519	2X10	0.132
LINE519	511	520	2X10	0.222
LINE520	512	521	2X10	0.076
LINE521	513	522	2X10	4.907
LINE522	514	523	3X35+54AL	0.721
LINE523	515	524	3X35+54AL	0.067
LINE524	516	525	4X10	1.388
LINE525	516	526	4X10	1.388
LINE526	518	527	2X10	2.796
LINE527	519	528	2X10	0.118
LINE528	520	529	2X10	0.176
LINE529	521	530	2X10	0.545
LINE530	523	531	3X35+54AL	0.108
LINE531	524	532	3X35+54AL	0.061
LINE532	525	533	4X10	1.890

ID	From	To	Enel Code	Length
LINE533	525	534	4X10	1.890
LINE534	527	535	2X10	2.215
LINE535	528	536	2X10	0.122
LINE536	529	537	2X10	0.563
LINE537	530	538	2X10	1.923
LINE538	530	539	2X10	5.584
LINE539	531	540	3X35+54AL	0.082
LINE540	532	541	3X35+54AL	0.066
LINE541	533	542	4X10	1.337
LINE542	533	543	4X10	1.337
LINE543	535	544	2X10	0.915
LINE544	536	545	2X10	0.369
LINE545	537	546	2X10	4.553
LINE546	538	547	2X10	2.123
LINE547	540	548	3X35+54AL	0.077
LINE548	541	549	3X35+54AL	0.086
LINE549	542	550	4X10	3.740
LINE550	542	551	4X10	3.740
LINE551	544	552	2X10	0.288
LINE552	544	553	2X10	3.789
LINE553	545	554	2X10	0.371
LINE554	546	555	2X10	0.227
LINE555	547	556	2X10	5.755
LINE556	548	557	3X35+54AL	0.082
LINE557	549	558	3X35+54AL	0.062
LINE558	550	559	4X10	2.708
LINE559	550	560	4X10	2.708
LINE560	552	561	2X10	0.106
LINE561	553	562	2X10	2.866
LINE562	554	563	2X10	3.471
LINE563	555	564	2X10	0.179
LINE564	557	565	3X35+54AL	0.073
LINE565	558	566	3X35+54AL	0.120
LINE566	559	567	4X10	0.601
LINE567	559	568	4X10	0.601
LINE568	559	569	2X10	3.848

ID	From	To	Enel Code	Length
LINE569	561	570	2X10	0.158
LINE570	564	571	2X10	0.202
LINE571	565	572	3X35+54AL	0.074
LINE572	567	573	4X10	0.142
LINE573	567	574	4X10	0.142
LINE574	569	575	2X10	0.486
LINE575	570	576	2X10	0.095
LINE576	571	577	2X10	0.180
LINE577	572	578	3X35+54AL	0.383
LINE578	573	579	4X10	3.856
LINE579	573	580	4X10	3.856
LINE580	573	581	4X10	0.320
LINE581	575	582	2X10	0.359
LINE582	576	583	2X10	0.115
LINE583	577	584	2X10	0.207
LINE584	578	585	2X10	4.732
LINE585	578	586	3X35+54AL	0.826
LINE586	579	587	4X10	2.702
LINE587	579	588	4X10	2.702
LINE588	581	589	4X10	0.116
LINE589	582	590	2X10	0.349
LINE590	583	591	2X10	0.101
LINE591	584	592	2X10	0.212
LINE592	585	593	2X10	0.273
LINE593	586	594	3X35+54AL	3.948
LINE594	587	595	4X10	5.414
LINE595	587	596	4X10	0.642
LINE596	589	597	4X10	0.089
LINE597	590	598	2X10	3.281
LINE598	591	599	2X10	0.659
LINE599	592	600	2X10	0.255
LINE600	593	601	2X10	0.137
LINE601	594	602	2X10	9.167
LINE602	594	603	3X35+54AL	4.642
LINE603	595	604	4X10	4.828
LINE604	595	605	4X10	4.828

ID	From	To	Enel Code	Length
LINE605	595	606	4X10	4.772
LINE606	596	607	2X10	11.664
LINE607	596	608	2X10	11.664
LINE608	597	609	4X10	0.110
LINE609	598	610	2X10	1.527
LINE610	599	611	2X10	4.856
LINE611	600	612	2X10	0.160
LINE612	601	613	2X10	0.115
LINE613	602	614	2X10	4.837
LINE614	603	615	3X35+54AL	4.447
LINE615	604	616	2X10	3.930
LINE616	604	617	4X10	3.109
LINE617	604	618	4X10	3.109
LINE618	607	619	2X10	2.745
LINE619	609	620	4X10	0.064
LINE620	610	621	2X10	0.717
LINE621	612	622	2X10	0.688
LINE622	613	623	2X10	0.094
LINE623	615	624	2X10	1.632
LINE624	615	625	3X35+54AL	1.251
LINE625	616	626	2X10	5.354
LINE626	620	627	4X10	0.078
LINE627	621	628	2X10	0.478
LINE628	622	629	2X10	2.900
LINE629	623	630	2X10	0.147
LINE630	624	631	2X10	0.400
LINE631	625	632	3X35+54AL	0.288
LINE632	626	633	2X10	1.570
LINE633	627	634	4X10	0.065
LINE634	628	635	2X10	0.731
LINE635	630	636	2X10	0.351
LINE636	631	637	2X10	0.275
LINE637	632	638	3X35+54AL	0.143
LINE638	633	639	2X10	5.197
LINE639	634	640	4X10	0.069
LINE640	635	641	2X10	1.427

ID	From	To	Enel Code	Length
LINE641	636	642	2X10	0.243
LINE642	637	643	2X10	0.381
LINE643	638	644	3X35+54AL	0.157
LINE644	640	645	4X10	0.069
LINE645	641	646	2X10	0.395
LINE646	642	647	2X10	0.308
LINE647	643	648	2X10	0.903
LINE648	644	649	3X35+54AL	0.181
LINE649	645	650	4X10	0.094
LINE650	646	651	2X10	0.115
LINE651	647	652	2X10	0.187
LINE652	648	653	2X10	0.687
LINE653	649	654	3X35+54AL	0.183
LINE654	650	655	4X10	0.124
LINE655	651	656	2X10	0.099
LINE656	651	657	2X10	0.528
LINE657	652	658	2X10	0.169
LINE658	653	659	2X10	0.482
LINE659	654	660	3X35+54AL	0.203
LINE660	655	661	4X10	0.365
LINE661	656	662	2X10	0.093
LINE662	657	663	2X10	0.696
LINE663	658	664	2X10	0.285
LINE664	659	665	2X10	0.505
LINE665	660	666	3X35+54AL	0.295
LINE666	661	667	4X10	0.260
LINE667	662	668	2X10	0.096
LINE668	663	669	2X10	2.894
LINE669	664	670	2X10	0.280
LINE670	665	671	2X10	0.415
LINE671	666	672	2X10	0.480
LINE672	666	673	3X35+54AL	0.812
LINE673	667	674	4X10	0.202
LINE674	668	675	2X10	0.693
LINE675	669	676	2X10	1.229
LINE676	670	677	2X10	0.133

ID	From	To	Enel Code	Length
LINE677	671	678	2X10	0.686
LINE678	672	679	2X10	0.288
LINE679	673	680	3X35+54AL	5.139
LINE680	674	681	4X10	0.403
LINE681	675	682	2X10	5.044
LINE682	677	683	2X10	0.146
LINE683	678	684	2X10	2.219
LINE684	679	685	2X10	0.267
LINE685	680	686	3X35+54AL	3.679
LINE686	683	687	2X10	0.140
LINE687	684	688	2X10	5.184
LINE688	685	689	2X10	0.261
LINE689	686	690	2X10	8.145
LINE690	686	691	3X35+54AL	10.525
LINE691	687	692	2X10	0.169
LINE692	689	693	2X10	0.196
LINE693	690	694	2X10	0.762
LINE694	690	695	2X10	2.126
LINE695	691	696	2X10	9.482
LINE696	691	697	3X35+54AL	2.191
LINE697	692	698	2X10	0.269
LINE698	693	699	2X10	0.311
LINE699	694	700	2X10	0.238
LINE700	695	701	2X10	4.331
LINE701	696	702	2X10	5.493
LINE702	697	703	4X10	2.400
LINE703	698	704	2X10	0.736
LINE704	699	705	2X10	0.524
LINE705	700	706	2X10	0.153
LINE706	703	707	4X10	6.331
LINE707	703	708	4X10	0.111
LINE708	704	709	2X10	0.980
LINE709	705	710	2X10	3.928
LINE710	706	711	2X10	0.092
LINE711	707	712	4X10	7.503
LINE712	707	713	2X10	3.945

ID	From	To	Enel Code	Length
LINE713	708	714	4X10	0.105
LINE714	709	715	2X10	0.093
LINE715	710	716	2X10	3.223
LINE716	711	717	2X10	0.093
LINE717	712	718	4X10	7.743
LINE718	713	719	2X10	0.422
LINE719	714	720	4X10	0.097
LINE720	715	721	2X10	0.074
LINE721	716	722	2X10	1.021
LINE722	717	723	2X10	0.083
LINE723	718	724	4X10	3.254
LINE724	718	725	2X10	9.348
LINE725	719	726	2X10	0.161
LINE726	720	727	4X10	0.110
LINE727	721	728	2X10	0.072
LINE728	722	729	2X10	0.929
LINE729	723	730	2X10	0.094
LINE730	724	731	4X10	4.421
LINE731	725	732	2X10	1.254
LINE732	726	733	2X10	0.171
LINE733	727	734	4X10	0.090
LINE734	728	735	2X10	0.077
LINE735	729	736	2X10	0.506
LINE736	730	737	2X10	0.075
LINE737	731	738	4X10	5.176
LINE738	732	739	2X10	0.659
LINE739	733	740	2X10	0.142
LINE740	734	741	4X10	0.138
LINE741	735	742	2X10	0.071
LINE742	736	743	2X10	0.404
LINE743	737	744	2X10	0.090
LINE744	738	745	4X10	4.156
LINE745	739	746	2X10	3.739
LINE746	739	747	2X10	0.122
LINE747	740	748	2X10	0.168
LINE748	741	749	4X10	0.164

ID	From	To	Enel Code	Length
LINE749	742	750	2X10	0.067
LINE750	743	751	2X10	0.345
LINE751	744	752	2X10	0.097
LINE752	745	753	4X10	1.543
LINE753	745	754	2X10	5.923
LINE754	746	755	2X10	3.039
LINE755	747	756	2X10	0.088
LINE756	748	757	2X10	0.163
LINE757	749	758	4X10	0.125
LINE758	750	759	2X10	0.093
LINE759	751	760	2X10	0.405
LINE760	752	761	2X10	0.178
LINE761	753	762	4X10	2.128
LINE762	754	763	2X10	4.951
LINE763	756	764	2X10	0.074
LINE764	757	765	2X10	0.225
LINE765	758	766	4X10	0.122
LINE766	759	767	2X10	0.114
LINE767	760	768	2X10	0.370
LINE768	761	769	2X10	1.329
LINE769	762	770	4X10	5.235
LINE770	763	771	2X10	3.601
LINE771	763	772	2X10	0.130
LINE772	764	773	2X10	0.126
LINE773	765	774	2X10	0.309
LINE774	766	775	4X10	0.079
LINE775	767	776	2X10	0.541
LINE776	768	777	2X10	0.574
LINE777	769	778	2X10	4.072
LINE778	770	779	4X10	4.243
LINE779	771	780	2X10	3.528
LINE780	772	781	2X10	0.125
LINE781	773	782	2X10	0.176
LINE782	774	783	2X10	0.327
LINE783	775	784	4X10	0.097
LINE784	776	785	2X10	4.622

ID	From	To	Enel Code	Length
LINE785	777	786	2X10	1.158
LINE786	779	787	4X10	3.315
LINE787	781	788	2X10	0.134
LINE788	782	789	2X10	0.131
LINE789	783	790	2X10	0.217
LINE790	784	791	4X10	0.092
LINE791	786	792	2X10	0.346
LINE792	786	793	2X10	2.062
LINE793	787	794	4X10	2.996
LINE794	788	795	2X10	0.120
LINE795	789	796	2X10	0.217
LINE796	790	797	2X10	0.221
LINE797	791	798	4X10	0.092
LINE798	792	799	2X10	0.302
LINE799	793	800	2X10	3.300
LINE800	794	801	2X10	0.206
LINE801	794	802	3X35+54AL	2.803
LINE802	795	803	2X10	0.133
LINE803	796	804	2X10	1.503
LINE804	797	805	2X10	0.106
LINE805	798	806	4X10	0.123
LINE806	799	807	2X10	0.249
LINE807	800	808	2X10	2.819
LINE808	801	809	2X10	0.206
LINE809	802	810	4X10	3.075
LINE810	802	811	4X10	3.075
LINE811	803	812	2X10	0.085
LINE812	804	813	2X10	3.320
LINE813	805	814	2X10	0.168
LINE814	806	815	4X10	0.421
LINE815	807	816	2X10	0.194
LINE816	808	817	2X10	1.596
LINE817	809	818	2X10	0.206
LINE818	810	819	4X10	2.578
LINE819	810	820	4X10	2.578
LINE820	812	821	2X10	0.131

ID	From	To	Enel Code	Length
LINE821	814	822	2X10	0.158
LINE822	815	823	4X10	0.487
LINE823	816	824	2X10	0.197
LINE824	818	825	2X10	0.205
LINE825	819	826	4X10	2.772
LINE826	819	827	4X10	2.772
LINE827	821	828	2X10	1.079
LINE828	822	829	2X10	0.226
LINE829	823	830	4X10	0.497
LINE830	824	831	2X10	0.171
LINE831	825	832	2X10	0.206
LINE832	826	833	4X10	2.276
LINE833	826	834	4X10	2.276
LINE834	828	835	2X10	3.720
LINE835	829	836	2X10	2.382
LINE836	831	837	2X10	0.123
LINE837	832	838	2X10	0.206
LINE838	833	839	4X10	2.443
LINE839	833	840	4X10	2.443
LINE840	836	841	2X10	0.953
LINE841	837	842	2X10	0.143
LINE842	838	843	2X10	0.206
LINE843	839	844	2X10	4.809
LINE844	839	845	4X10	3.621
LINE845	839	846	4X10	3.621
LINE846	841	847	2X10	0.484
LINE847	842	848	2X10	0.215
LINE848	843	849	2X10	0.101
LINE849	844	850	2X10	0.704
LINE850	845	851	4X10	4.364
LINE851	845	852	4X10	4.364
LINE852	847	853	2X10	0.107
LINE853	848	854	2X10	0.351
LINE854	849	855	2X10	2.260
LINE855	850	856	2X10	1.982
LINE856	851	857	4X10	6.862

ID	From	To	Enel Code	Length
LINE857	851	858	4X10	6.862
LINE858	853	859	2X10	0.084
LINE859	854	860	2X10	2.820
LINE860	854	861	2X10	7.509
LINE861	855	862	2X10	1.470
LINE862	856	863	2X10	2.328
LINE863	857	864	4X10	2.789
LINE864	857	865	4X10	2.789
LINE865	859	866	2X10	0.090
LINE866	862	867	2X10	1.285
LINE867	863	868	2X10	0.302
LINE868	864	869	4X10	1.737
LINE869	864	870	4X10	1.737
LINE870	866	871	2X10	0.072
LINE871	867	872	2X10	1.348
LINE872	868	873	2X10	0.380
LINE873	868	874	2X10	0.844
LINE874	869	875	4X10	1.200
LINE875	869	876	4X10	1.200
LINE876	871	877	2X10	0.070
LINE877	872	878	2X10	2.302
LINE878	873	879	2X10	0.351
LINE879	874	880	2X10	1.976
LINE880	875	881	4X10	1.011
LINE881	875	882	4X10	1.011
LINE882	877	883	2X10	0.070
LINE883	878	884	2X10	0.565
LINE884	879	885	2X10	0.534
LINE885	880	886	2X10	4.688
LINE886	883	887	2X10	0.069
LINE887	884	888	2X10	0.623
LINE888	884	889	3X35+54AL	3.157
LINE889	885	890	2X10	0.675
LINE890	887	891	2X10	1.237
LINE891	888	892	2X10	1.023
LINE892	889	893	3X35+54AL	2.678

ID	From	To	Enel Code	Length
LINE893	890	894	2X10	2.627
LINE894	891	895	2X10	5.825
LINE895	891	896	2X10	3.259
LINE896	892	897	2X10	0.202
LINE897	893	898	3X35+54AL	4.501
LINE898	894	899	2X10	4.772
LINE899	895	900	2X10	3.868
LINE900	897	901	2X10	0.171
LINE901	901	902	2X10	0.144
LINE902	902	903	2X10	0.187
LINE903	903	904	2X10	0.339
LINE904	904	905	2X10	0.588
LINE905	905	906	2X10	4.815

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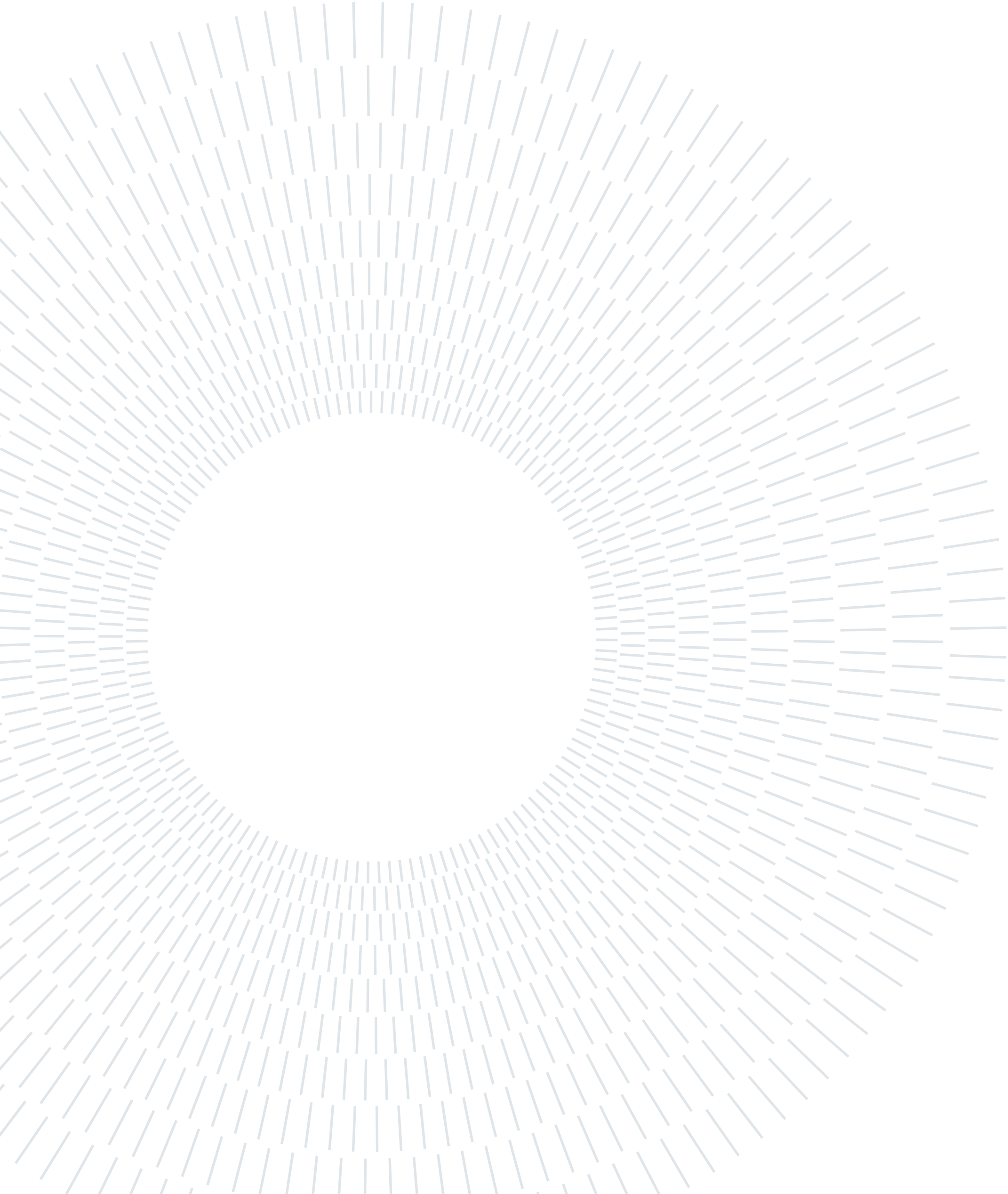

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