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***DESIGN AND CONTROL OF A FIVE BAR
LINKAGE PARALLEL MANIPULATOR WITH
FLEXIBLE ARMS***

A Thesis Submitted in the Partial Fulfillment of the requirements
For degree of Master of Science in Mechanical Engineering
(Study Major: Mechanical Systems Design)

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Abstract

The PRRRP parallel mechanism is made up of five bars, two prismatic joints and three revolute joints, where the two prismatic joints that are attached to the base are actuated. Such a mechanism can be used in high-speed machine tools or in the feeding mechanism in a tuning machine. Considering the promising characteristics of parallel manipulators, and lightweight manipulators, PRRRP parallel manipulators with two lightweight intermediate links are developed, to provide an alternative high-speed pick-and-place positioning mechanism to serial architecture manipulators. Lightweight members are more likely to exhibit structural deflection and vibrate due to the inertial forces from high speed motion, and external forces from actuators. Structural flexibility effects are much more pronounced at high operational speeds and accelerations. Therefore, this thesis presents the kinematic Analysis, a rough work space design and dynamics and vibration control of a PRRRP (2 DOF) parallel manipulator with two flexible links.

The thesis proposes a new method of modeling and simulation of 2 DOF (degree-of-freedom) parallel manipulator with flexible link based on Matlab/SimMechanics, and its direct and inverse kinematics is built up. The controller also designed under the environment of Simulink and SimMechanics to satisfy the performance requirement for making the end-effector track of the reference trajectory. Simulation results and performance of the control system are presented in detail. It indicates that SimMechanics can be used for designing parallel robot with flexible links. In addition the simulation result shows that the controller can control the movement of the robot effectively. The dynamic modeling and control approach presented in this thesis can also be applied to other parallel manipulators with less than six degrees of freedom.

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As the thesis reaches to its final stage, it brings in lot of personal satisfaction & happiness for what I have done intensively for the last six months. It is a great pleasure to present this report, which will reflect the working conditions, problems encountered, solutions obtained and objectives achieved throughout its completion.

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Chapter 1

Introduction

In this introductory chapter, the motivation of this thesis is given and the major research topics relative to this work are reviewed and briefly discussed. The objective, outline, and contributions of the thesis are presented.

1.1 Thesis Motivation

In general, industrial robot manipulators can be classified into two types according to their configurations. The robot manipulator of the first type, as shown in Figure 1.1, is called a serial link manipulator. This type of manipulator consists of successive links, usually hinged at rotary joints which can be actuated in a coordinated fashion to position the end-effector. The other type of robot manipulator, as shown in Figure 1.2, is called a parallel link manipulator (Merlet [1], Tsai [2]). A parallel manipulator consists of two platforms, namely the base platform and the mobile platform. The base platform is fixed in space or attached to the end-effector of another robot. The mobile platform is movable with respect to the base platform. The two platforms are usually linked with six or two linear actuators. Generally speaking, serial link manipulators have the advantage of access to larger workspaces over parallel manipulators. However, parallel link manipulators provide higher strength/weight, stiffness/weight ratios and accuracy than serial manipulators. Moreover, parallel manipulators allow the actuators to be fixed to the base or to be located close to the base of the mechanisms, which minimizes the inertia of the moving parts and which makes it possible to use more powerful actuators. Therefore, since the Stewart platform was proposed to provide rapid multi-axis motion for flight simulators by Stewart [3], parallel robot manipulators have gained growing interest for applications in various manufacturing industries, precision optics (Cash *et al.* [4]), nano manipulation (Chung and Choi [5], Xu and Li [6]), and medical surgery (Xu and Li [7], Taylor and Stoianovici [8]).

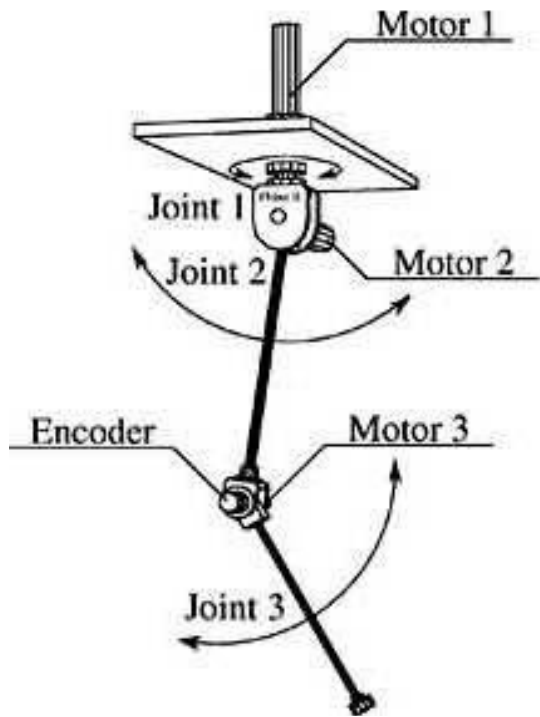


Figure 1.1 A serial manipulator

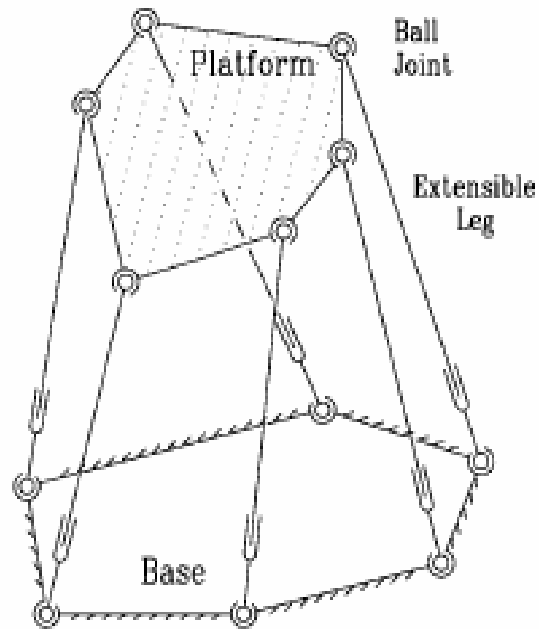


Figure 1.2 A parallel manipulator

On the other hand, traditional industrial robots are built to be massive in order to increase stiffness, as shown in Figure 1.3, and therefore move at speeds much lower than the fundamental natural frequency of the system due to the limitations in the drive motor output torque. The practical solution to this problem is to design and construct light weight manipulators, as shown in Figure 1.4, which are capable of moving swiftly. In contrast to the rigid manipulators, light weight manipulators offer advantages such as higher speed, better energy efficiency, improved mobility and higher payload-to-arm weight ratio. However, at high operational speeds, inertial forces of moving components become quite large, leading to considerable deformation in the light links, generating unwanted vibration phenomena. Such manipulators are called flexible manipulators (Wang and Gao [9], Carlos *et al.* [10]). It is a challenging task to achieve high accuracy end-effector motion for flexible manipulators due to unwanted structural vibrations. Hence, elastic vibrations of light weight links must be considered in the design and control of the manipulators with link flexibility.

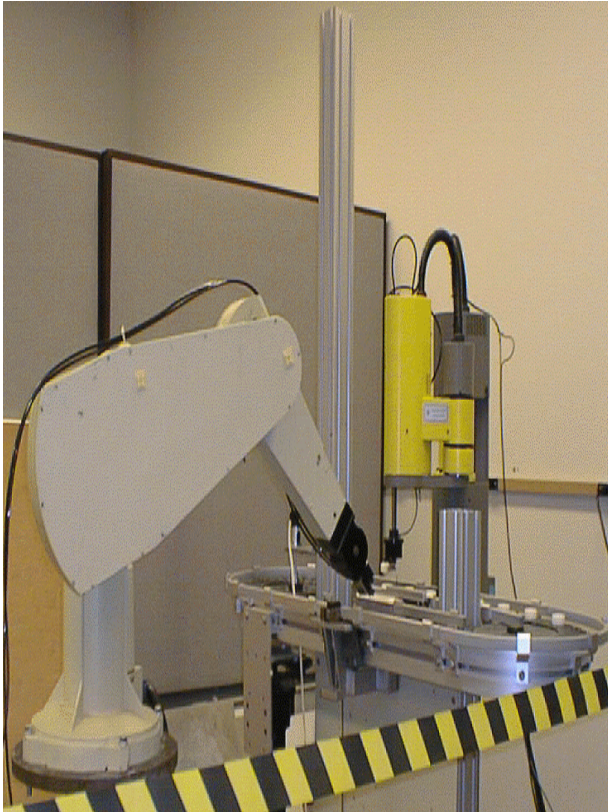


Figure 1.3 A manipulator with rigid links



Figure 1.4 A manipulator with flexible links

In order to Design efficient control for dynamic system of the flexible link Manipulator, the dynamic model of the structure must be built. This is a complex subject and different methods were developed in order to solve it. A classical method for calculating the dynamic models of closed chains is to consider first an equivalent tree-structure, and then to consider the system constraints by the use of Lagrange multipliers or d'Alembert's principle. Other approaches include the use of virtual work, Lagrange formalism, Hamilton's principle, and Newton-Euler equations. This kind of dynamic modeling is quite cumbersome, error prone and time consuming. SimMechanics allows the user to model a physical system while minimizing the rigorous mathematical calculations that are usually required in such a process. By simply inserting a handful of dimensions into specific system blocks, this program is capable of accurately modeling the motion of complex systems.

The process of modeling based on Sim-Mechanics does not need to compute forward dynamics and the derivation of differential equations which is quite cumbersome and error prone. SimMechanics is a new way for robots modeling with its simple, intuitive and accurate characteristic.

In this thesis the dynamic model of the robot is built using the SimMechanics toolbox from Simulink. The toolbox uses the standard Newtonian dynamic of forces and torques in order to solve both direct and inverse problem. The model is built using Simulink blocks; blocks that represents the kinematic elements and the joints of the robot. The blocks from the toolbox allow to model mechanical systems consisting of any number of rigid and flexible bodies, connected by joints representing translational and rotational degrees of freedom.

Considering the promising characteristics of parallel manipulators, and lightweight manipulators, parallel manipulators with light weight links are developed, to provide an alternative high-speed pick-and-place positioning mechanism to serial architecture manipulators. The parallel manipulator presented in this thesis, as shown in Figure 1.5, is categorized as a PRRRP type because it has two symmetric closed-loop chains, each of which consists of a prismatic joint (P), and one revolute joints (R) linked by revolute joint with end-effector. Due to its inherent stiffness and accuracy, and less inertia of moving links, this proposed mechanism can be used in high-speed and high-accuracy robotic applications as a planar positioning and orientation device. Light weight links are used to better meet the demands of high speed and high acceleration placement. However, light weight members are more likely to deflect and vibrate due to the inertial forces and external forces such as those arising from actuators. Structural flexibility effects are much more pronounced at high operational speeds and accelerations. Position feed back control with PID regulator provides a promising solution to the suppression of unwanted deflection.

To address this concern, this thesis aims at the development of dynamic models to investigate the dynamic characteristics of these structural deflections when the flexible parallel robot moves with high speed and acceleration using SimMechanics, Simulink and MATLAB, and the development of PID control methodologies to suppress these unwanted structural deflections.

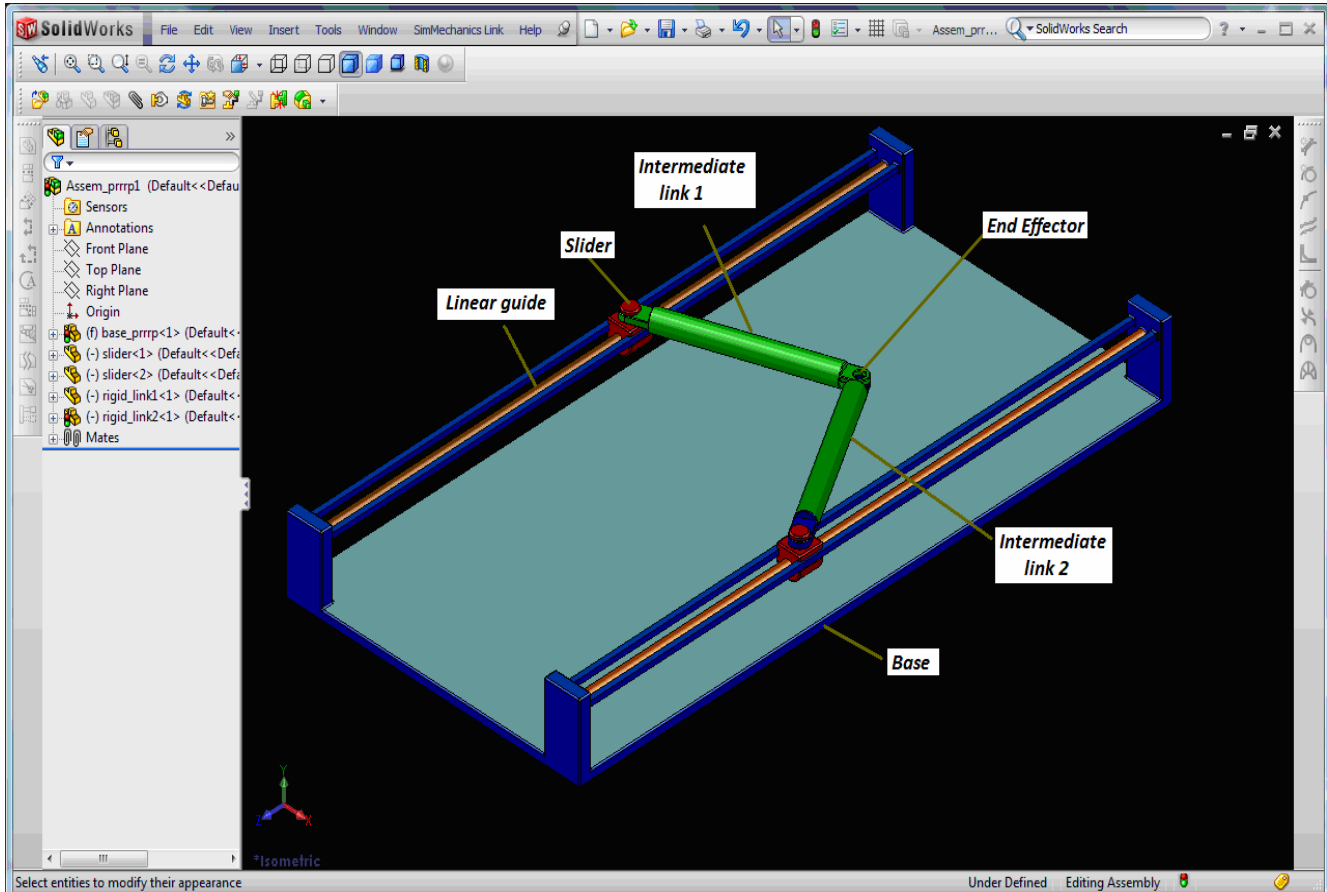


Figure 1.5 Solid work model of PRRRP parallel manipulator

1.2 Literature Review

This thesis involves several research areas, such as parallel manipulators, dynamics of manipulators and mechanisms with link flexibility, dynamics modeling Manipulators in Simmechanics and vibration control of flexible links, etc. This section will review the research literature relative to these research topics.

1.2.1 Parallel Robot Manipulators

In the applications where high load carrying capacity, high-speed, and precise positioning are of paramount importance, it is desirable to have an alternative to conventional serial manipulators. In general, it is expected that manipulators have the end-effector connected to the ground via several

chains having actuation in parallel, and therefore have greater rigidity and superior positioning capability. This makes the parallel manipulators attractive for certain applications and the last two decades have witnessed considerable research interest in this direction. The research efforts mainly involved inverse position kinematics, direct position kinematics, singularities, workspace and dexterity, and dynamics and control, etc. (Dasgupta and Mruthyunjaya [11]). Significant achievements, which have driven research on parallel manipulators from its Infancy into the status of a popular research topic, were reported by Earl and Rooney [12], and Hunt [13]. Work reported by Earl and Rooney [12] presented methods for the analysis and synthesis of the kinematic structures with both serial and parallel mechanisms. Hunt [13] studied the structural kinematics of parallel manipulators on the basis of screw theory. Fichter and McDowell [14] formulated the inverse kinematics equations of individual limbs of parallel manipulators and implemented the method on the Stewart platform. The direct position kinematics problem drew much attention in the research on the Stewart platform during the late 1980s and early 1990s. Lin *et al.* [15] formulated the closed form solutions to the direct position kinematics using the input-output equations of spherical four-bar mechanisms. Nair and Maddocks [16] proposed a decomposition scheme which divides the direct position kinematics into two parts. One part is linear and design-dependant, the other involves solving certain nonlinear design-independent equations. Wang and Chen [17] presented a numerical approach with nonlinear-equation-solving algorithms to obtain the direct position kinematics solution. A neural network solution was developed by Geng and Haynes [18] for the solution of the direct position kinematics of the Stewart platform manipulator. Merlet [19] presented the extensive use of Grassman geometry to enumerate geometric singularity conditions in detail. An investigation was conducted to the force singularity of the Stewart platform in work (Gosselin and Angeles [20]). Gosselin [21] presented the important association of the conditioning of the static transformation with the stiffness of the Stewart platform. Tekeda and Funabashi [22] studied singularity and ill-conditioning in terms of a new transmission index and pressure angle, and pointed out the unusability of the ill-conditioned zones in the workspace. A workspace analysis method of the Stewart platform is developed by Yang and Lee [23] to determine some particular sections of the positional workspace with constant orientation for very specialized structures of the manipulator. Luh *et al.* [24] presented a general formulation for the workspace and dexterity analysis of parallel manipulators in terms of rank-deficiency of the Jacobian of the constraints by incorporating inequality constraints of slack variables. The work reported by Masory and Wang [25] addressed the

problem of determining workspace sections including the constraints of joint angle limits and leg interface. Compared to the vast literature on the kinematics of the parallel manipulators, research reports on the dynamics and control of parallel manipulators are relatively few. Do and Yang [26] presented the inverse dynamics of the Stewart platform using the Newton-Euler approach. Liu *et al.* [27] developed Lagrangian equations of motion with simplified assumptions regarding the geometry and inertia distribution of the parallel manipulator. A dynamic control strategy was proposed in work (Hatip and Ozgoren [28]) for the control for the Stewart platform assumed to be mounted on a ship and used as a motion stabilizer.

To summarize, the research on dynamics and control of parallel manipulators has accomplished much, with significant issues yet to be resolved. With the consideration of link flexibility, the dynamic formulation of parallel manipulators is more complicated and challenging.

1.2.2 Dynamic Modeling of Manipulators and Mechanisms with Link Flexibility

Flexible manipulators and mechanisms have important applications in space exploration, manufacturing automation, construction, mining, hazardous operations, and many other areas, due to their attractive advantages over the rigid-arm counterparts, such as smaller actuators and more maneuverability, higher speeds of operation, greater ratio of payload to manipulator mass, lower mounting strength required and more compact link design, less material and power consumption, etc. However, lightweight members are more likely to deflect and vibrate due to the inertial and external forces. Flexibility effects are much more pronounced at high operational speeds. Therefore, the research interest in the dynamics and control of flexible manipulators and mechanisms has increased significantly in recent decades in order to fully exploit the potential offered by flexible manipulators, and to achieve less vibration and greater accuracy. Significant progresses have been made in many aspects during the past decades, as can be seen from the survey papers (Erdman and Sandor [29], Lowen and Jandrasits [30], Lowen and Chassapis [31], Shabana [32], Dwivedy and Eberhard [33], Book [34], Benosman and Vey [35]). Unfortunately, taking into account the flexibility of the arms, manipulators and mechanisms are highly nonlinear and exhibit coupled dynamics, which make the control problems of such systems difficult. Most of reported works in this field addressed the manipulators and mechanisms with single flexible links, and much fewer experimental works were reported compared with the vast publications of theoretical formulation and numerical simulation.

Compared with the dynamic model of flexible serial manipulators and four-bar mechanisms, research reports on dynamic models for parallel manipulators are rather few in number. Recently, with the consideration of link flexibility, Giovagnoni [36] presented a general approach for the dynamic analysis of flexible closed-chain manipulators using the principle of virtual work. Lee and Geng [37] developed a dynamic model of a flexible Stewart platform using Lagrange equations. Zhou *et al.* [38] established dynamic equations of flexible 3-Parallel-Revolute-joint-and-Spherical-joint (3PRS) manipulator for vibration analysis using the finite element method (FEM). A dynamic model for a 3-PRR planar parallel manipulator with flexible links was developed based on FEM in reported works (Piras *et al.* [39], Wang and Mills [40]) and based on Assume mode method in reported works (Xuping Zhang, Mills, J.K. and Cleghorn, W.L [41]).

Several research topics relevant to dynamic modeling are reviewed as following: the discretization of the continuous elastic deformation, and Dynamic Modeling of Manipulators using SimMechanics

Discretization of Continuous Elastic Deformation

Manipulators and mechanisms with flexible links are inherently continuous dynamic systems with an infinite number of degrees of freedom. Their governing equations of motion are highly nonlinear, coupled, ordinary and partial differential equations, which are normally infeasible to solve. Therefore, the infinite degrees of freedom associated with such distributed parameter systems usually should be approximated by finite-dimensional models based on discretization techniques, such as the finite element method (FEM), the assumed mode method (AMM), the lumped parameter method (LPM), and the transfer matrix methods (TMM). Among them, FEM and AMM are investigated more widely and deeply.

Finite Element Method

The dynamic model of mechanisms and manipulators with link flexibility must generally capture the mass, stiffness and damping characteristics of the links, and external loading. The finite element approach provides an easier and systematic modeling technique for complex mechanical systems and lays the groundwork for a general approach to the modeling of elastic mechanisms and manipulators.

Winfrey [42], Erdman [43], and Iman [44] were among the first investigators to apply the finite element method to elastic mechanism systems. Nath and Ghosh [45] presented a refined application of the finite element method in which the effects of the Coriolis, tangential and normal components of elastic acceleration were considered. In addition, the effects of distributed rigid body as well as elastic axial forces on the transverse vibrations of mechanisms were included. Cleghorn, Fenton and Tabrrok [46] presented a more refined application of FEM in which the need to model the mechanism as a series of instantaneous structures was eliminated. Turic and Midha [47] developed the generalized equations of motion for the dynamic analysis of elastic mechanism systems. Most recently, Piras et al. [39], Wang and Mills [40] the dynamic equations based on FEM to model a planar parallel manipulator with three flexible links were established.

In FEM, all the generalized coordinates are physically meaningful. However, the concepts of natural frequencies are not explicitly exhibited. Furthermore, all of the mathematical models for flexible linkages involve a large number of degrees of freedom, and hence a large number of equations of motion must be solved, which leads to computational inefficiency and hence expensive computational costs. Therefore, its application to the design of controller based model is limited.

Assumed Mode Method

In the assumed mode method, the elastic deflection is described by an infinite number of separable harmonic modes. Since the first few modes are dominant in dynamics, the modes are truncated to a finite number of modal series, in terms of spatial mode eigenfunctions and time-varying mode amplitudes. There are several ways to choose link boundary conditions and mode eigenfunctions. Different kinds of mode shape functions can be obtained from different types of boundary conditions, such as clamped boundary conditions (Book [48]), pinned-pinned boundary

conditions (Asada et al. [49]), and free-free boundary conditions (Baruh and Tadikonda [50]). Hastings and Book [51], and Barbieri et al. [52] have reported that if the beam-to-hub inertia ratio is very small (0.1 or less), the clamped condition yields better results compared with pinned boundary condition.

To obtain accurate dynamic equations of manipulators or mechanisms with link flexibility, another important issue is to determine boundary conditions, and then select the proper sets of modes for problems of elastic beams that undergo large rigid-body displacement. Bellezza et al. [53] presented a mathematical model for a flexible slewing beam with comparison of clamped and pinned boundary conditions at the root end. Low [54] developed experimental investigation of the boundary condition of slewing beams using a high-speed camera system, and experimental results suggested that exact natural frequencies were intermediate between the clamped and pinned cases. Shabana [55] demonstrated that different sets of mode shapes and natural frequencies associated with different sets of boundary conditions can be used to obtain the same solution provided that the coordinate system is properly selected.

Experimental modal analysis (EMA) (Ewins [56]) is a potential way to identify boundary conditions of manipulators and mechanism systems. EMA has been widely used in the experimental identification of structural dynamic characteristics. An EMA formulation was presented for a simply supported plate using piezoelectric actuators and sensors in the reported work (Saunders et al. [57]). Wang et al. discussed the feasibility of modal testing with piezoelectric transducers bonded to a cantilever beam (Wu et al. [58]). Natural frequencies and damping ratios of a parallel robot were measured by Hardage and Wiens [59] using a hammer and accelerometer when the robots is stationary at different configurations. However, experimental identification of flexible manipulators and mechanisms has not been investigated thoroughly due to the coupling effect between rigid body motion and elastic deformation (Midha [60]).

The AMM provides some physical insights, such as the concept of natural frequencies, into the system. However, its generalized coordinates, i.e., the assumed harmonic modes, don't possess any explicit physical meanings. In addition, how to select the appropriate or best mode eigenfunctions for a given flexible mechanism and manipulator system is not a clearly answered problem.

Lumped Parameter Method

There are two kinds of lumped parameter methods which are used to describe the flexible links. One is the lumped-mass method (Sadler and Sandaor [61]) in which each beam is represented by a finite number of equally spaced point masses connected by massless, elastic elements. The lumped mass is called a station, while the elastomer without mass is called field. The flexible mechanisms and manipulators are modeled by combining the stations (distributed joints and lumped masses) and fields (massless elastomers) into a composite system. Ge et al. [62] proposed a new lumped-mass model which is presented by combining both AMM and FEM. The other one of LPM is called the finite segment method which assumes that a flexible body is composed of a number of discrete rigid segments, which are connected by springs and/or dampers. The flexibility is simulated by the springs and dampers. The stiffness and damping coefficients are calculated according to the physical characteristics of the elements. These coefficients are then used in the analysis to simulate the flexibility of the links. Tosunoglu et al. [63] used this model for identification of inaccessible oscillations in n-link flexible robotic systems. Recently, Megahed and Hamza [64] presented a mathematical model for planar flexible link manipulators with rigid tip connections to revolute joints.

Compared with FEM and AMM, no special elements and mode shapes are needed in the lumped parameter method. Therefore, it is not as cumbersome as FEM and AMM. However, LPM is not suitable to the flexible links with complex geometrical shapes.

Transfer Matrix Method

In general, the transient response of flexible link system subjected dynamic loads can be obtained by the assumed modal superposition or the use of finite element models or lumped parameter models. In these methods, however, it is necessary to use a large number of nodes or modes resulting in a need for very large computers for their managements and regulation. In order to overcome these disadvantages, a distinctive alternative to these methods is the use of the transfer matrix method. With the mechanisms and manipulators modeled as an instantaneous structure, transfer matrix techniques can be used to calculate the natural frequencies and corresponding mode shapes. These natural frequencies and mode shapes can then be used to the modal superposition technique to calculate the response of the mechanism.

The transfer matrix method was used widely in structure mechanics and rotor dynamics of linear time invariant system. Dokanish [65] developed finite element-transfer matrix method to solve the problems of plate structure vibration analysis, in which the finite element technique and the transfer matrix technique were combined. Kitis [66] applied the transfer matrix method to investigate the dynamic response of elastic four-bar mechanisms by combining the lumped parameter model of flexible links. Rui and Lu [67] developed the discrete time transfer matrix method for modeling multi-flexible-body systems, in which chain multi-body systems, branched multi-body systems network multi-body systems have been discussed in detail.

1.2.3 Dynamic Modeling of Manipulators using SimMechanics

The process of modeling accurate dynamics of manipulators or mechanisms with link flexibility using SimMechanics does not need to compute forward dynamics and the derivation of differential equations which is quite cumbersome, error prone and time consuming. Sim-Mechanics is a new way for robots modeling with its simple, intuitive and accurate characteristic.

SimMechanics [68] toolbox is a block diagram modeling environment for modeling and simulating mechanical systems which use the standard Newtonian dynamics of forces and torques. The kinematical analyses based on SimMechanics are free from establishing the kinematic model of mechanism. Mechanical systems can be easily represented in a graphical way by connected block diagrams which save time and effort to model. The block set consists of block libraries for bodies, joints, sensors and actuators, constraints and drivers, and force elements. SimMechanics models can be interfaced seamlessly with ordinary Simulink block diagrams. This enables the user to design e.g. the mechanical and the control system in one common environment.

In recent years, many new types of rigid parallel manipulators have been studied and model by using Sim-Mechanics/Simulink. C. C. Ng, S. K. Ong and A. Y. C. Nee [69] analyzed 3-DOF micro Stewart platform using the Sim-Mechanics, Lan Wang etc. [70] analyzed Assistant Robotic Leg using the Sim-Mechanics. Most of the researchers in SimMechanics to simulate their models of mechanism used to analyze dynamics or the workspace of a manipulator [71, 72],

Few researchers also used SimMechanics to design a controller combining the physical model of rigid manipulator [73, 74]. This thesis, introduce the method of modeling, simulation and control for a 2 DOF parallel manipulator with flexible links combining Simulink and SimMechanics.

1.2.4 Vibration Control of Flexible Manipulators and Mechanisms

Concurrent to the work on dynamic modeling as reviewed above, the investigation of vibration control of flexible manipulators and mechanisms has been undertaken by many researchers for several decades. Design or control strategies have been proposed to attenuate the unwanted vibration of the flexible links. For example, links of mechanisms and manipulators were built with composite materials having inherently superior damping characteristics and higher stiffness to weight ratios (Ghazavi et al. [75], Sung and Thompson [76]). The vibration of mechanism was dissipated by introducing additional damping materials (El-Dannah and Farghaly [77], Sisemore et al. [78]). The vibration of mechanisms was attenuated through optimizing the cross-sectional geometrics of the mechanism links (Zhang et al. [79], Cleghorn et al. [80]). These three methods are usually referred to as passive vibration control. Ulbrich and Stein [81] presented a design of an additional electromechanical actuator, which is integrated into a four-bar mechanism at the proximal end of the follower link so that the deflection of the flexible linkages is reduced by controlling the additional actuator. Another alternate strategy advocates reduction of structural vibrations through controlling joint motions or torques based on input shaping (Singhose et al. [82], Shan et al. [83]), singular perturbation techniques (Carlos et al. [10], Siciliano and Book [84]), etc. However, the successful realization of joint motion or torque control schemes may be very difficult, to achieve due to hardware limitations. These limitations include saturation of the motor, signal noise from the sensor, and parameter variations.

1.3 The Objective of the Thesis

The overall objective of this thesis is to investigate the dynamics and vibration control of a planar parallel manipulator with link flexibility.

To establish a dynamic model and investigate the structural vibration characteristics of a PRRRP parallel manipulators with two flexible links using SimMechanics/ Simulink, including the investigation effect of motion laws on the elastic deformations of the flexible model, and the effect of end-effector load on the lateral vibration when the manipulator is moving with high-speed.

To develop effective vibration control strategies to suppress the unwanted vibration of the PRRRP parallel manipulator with flexible links using the traditional PID regulator when the manipulator is moving at high speed and high acceleration.

Many researchers have studied the control of a flexible parallel manipulator. However, most of them have used by driving the differential equations which is quite cumbersome and error prone for modeling the dynamics and vibration control. The objective of this thesis is to develop a control scheme which suppresses the vibration of 2 dof PRRRP flexible parallel manipulator in minimum time. This is approached by using the SimMechanics/ Simulink. The control low result of the proposed research may contribute to the modeling and control of the flexible parallel manipulators with high efficiency and positional accuracy. No previous study of this type is known.

1.4 Thesis Overview

This thesis presents the dynamics and vibration control of a PRRRP parallel manipulator with two flexible links. The details involve establishing Kinematic analysis comprises forward and inverse displacement, a rough workspace study of the manipulator, developing dynamic model for rigid and flexible manipulator using SimMechanics, Conducting numerical simulations, investigation influence of motion laws on the flexible parallel Manipulator trajectory, developing vibration control strategies, designing an effective feed back position controller with traditional PID regulator. The outline of the remainder of the thesis is as follows:

Chapter 2 introduces the solid work model and basic structure of a PRRRP rigid parallel manipulator and presents the kinematic analysis comprises forward and inverse displacement analysis of this parallel manipulator to provide a theoretical foundation for the research work done in this thesis. Based on the results of the kinematic analysis, a rough workspace study of the manipulator is also accomplished. The structure of the said manipulator is especially designed to cover a larger workspace.

Chapter 3 develops the structural dynamic Model for a PRRRP rigid parallel manipulator using the SimMechanics. The mechanical construction is performed with the CAD program Solid Works for PRRRP rigid parallel manipulator and the data is exported to SimMechanics, a simulation tool for mechanical. Using Simulink/SimMechanics and matlab the dynamic behavior of the manipulator is analyzed and tested for circular trajectory with motion Low. Numerical simulation results are given and analyzed.

Chapter 4 will discussed Dynamic modeling of the flexible links of the manipulator using SimMechanics. The effect on end-effector motion for flexible manipulators due to unwanted structural deflection that following the desired trajectory. Analysis on the influence law of motion and simulation results of the flexible Manipulator are given and analyzed.

Chapter 5 presented control of a PRRRP parallel manipulator with two flexible links. The vibration controller is designed based on position feedback control using PID. The simulation of running the robot was based on the Simulink module from MATLAB using SimMechanics. Numerical simulations are performed and the results show that the proposed vibration control strategy is effective.

Finally, the thesis concludes by giving conclusion and recommendation with a discussion of future research considerations.

1.5 Thesis Contributions

The contributions achieved in this thesis include:

- Structural dynamic Model of the proposed PRRRP parallel manipulator with two flexible intermediate links has been developed using SimMechanics. This approach can easily extend to other Parallel Manipulator with flexible links.
- The Numerical simulation of the desired trajectory with motion law provides insight into the design of the joint motion controller.
- The effective position control strategies to suppress the unwanted deflection of the PRRRP parallel manipulator with flexible links using the traditional PID regulator when the manipulator is moving at high speed and high acceleration. The Proposed control strategy has been numerically implemented in Matlab/ Simulink on the Model of a PRRRP parallel manipulator with flexible links using SimMechanics. The developed methods and strategies of dynamic modeling and Control can be extended to other types of parallel manipulators or other multibody dynamic systems with flexible components.

Chapter 2

Manipulator Kinematic Analysis

2.1 Introduction

Manipulator kinematics deals with the study of the manipulator motion as constrained by the geometry of the links. The kinematic analysis is done without regard to the forces or torques that cause or result from the motion. Typically, the study of manipulator kinematics is divided into two parts, inverse kinematics and forward (or direct) kinematics. The inverse kinematics problem involves mapping a known position of the output link of the manipulator to a set of input joint variables that will achieve that position. The forward kinematic problem involves the mapping from a known set of input joint variables to a position of the moving end-effector that results from those given inputs. Generally, as the number of closed kinematic loops in the manipulator increases, the difficulty of solving the forward kinematic relationships increases while the difficulty of solving the inverse kinematic relationships decreases. As an example, the forward kinematics problem for a traditional 6 degree of freedom serial link manipulator is relatively simple, while the inverse kinematic problem is difficult (e.g. Raghavan and Roth, 1993). In contrast, the inverse kinematics of the 6 degree of freedom Stewart platform is relatively simple, but the forward kinematics is difficult (e.g. Zhang and Song, 1994).

The kinematics relation between x and q of these 2 DOF parallel robots can be expressed solving the following equation:

$$\mathbf{f}(\mathbf{x}, \mathbf{q}) = \mathbf{0}$$

For this manipulator, the inverse kinematics problem is solved algebraically and shows that there are two solutions for each leg for the general case manipulator,

2.2 Inverse Kinematics

The objective of the inverse kinematics solution is to define a mapping from the position of the moving end-effector in a Cartesian space to the set of joint position that achieves that position. For this analysis, the position of the moving end-effector is considered known, and is given by the position vector P , which defines the location of P at the center of the revolute joint that connect the two links in the XYZ coordinate frame. The inverse kinematics analysis produces a set of two joint positions for each leg (q_1 and q_2) that define the possible postures for each leg for the given position of the moving end-effector.

The PRRRP 2-DoF parallel mechanism usually consists of two legs, each of which is the PRR (R-revolute joint and P-prismatic joint) chain. The two legs are connected to the end-effector point with a common R joint. The mechanism can position at a point in a plane when the P joint in each of the two legs is actuated by the linear actuator. A PRRRP solid work model mechanism that is actuated horizontally is shown in Figure 2.1.

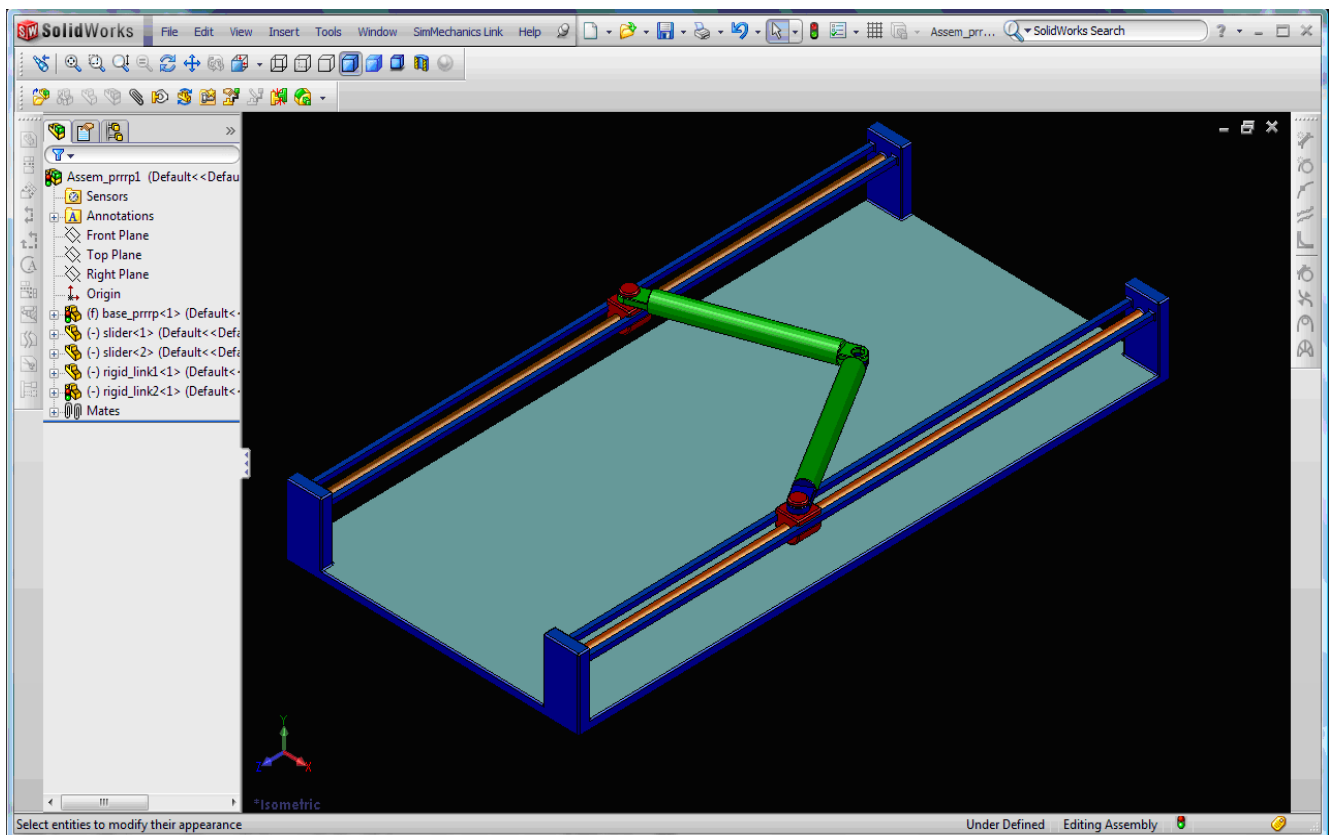


Figure 2.1 Solid work model of PRRRP parallel manipulator

As illustrated in Fig. 2, a reference frame $\mathfrak{R} : \mathbf{O} - XY$ is fixed to the base. Vectors $b_{i\mathfrak{R}} (i=1,2)$ are defined as the position vectors of points B_i in frame \mathfrak{R} . The geometric parameters of the mechanism are $PB_i = R_2$ ($i = 1, 2$), and the distance between two guide ways $2R_1$. The position of point P in the fixed frame \mathfrak{R} is denoted as vector:

$$P_{\mathfrak{R}} = [x, y]^T \quad (2.1)$$

The vectors of $b_{i\mathfrak{R}}$ in the fixed frame \mathfrak{R} can be written as:

$$b_{1\mathfrak{R}} = [-R_1, q_1]^T, \quad b_{2\mathfrak{R}} = [R_1, q_2]^T \quad (2.2)$$

Where q_i are the actuated inputs for the two legs. The inverse kinematics problem of the mechanism can be solved by writing the constraint equation:

$$\|P_{\mathfrak{R}} - b_{i\mathfrak{R}}\| = R_2, \quad i = 1, 2 \quad (2.3)$$

From the geometry of the mechanism, loop closure equations are derived as follows.

$$(x - R_1)^2 + (y - q_1)^2 = R_2^2 \quad (2.4)$$

$$(x + R_1)^2 + (y - q_2)^2 = R_2^2 \quad (2.5)$$

The inverse kinematic problem can then be written as

$$q_1 = y \pm \sqrt{R_2^2 - (x - R_1)^2} \quad (2.6)$$

$$q_2 = y \pm \sqrt{R_2^2 - (x + R_1)^2} \quad (2.7)$$

From which we can see that there are four solutions for the inverse kinematics of the mechanism. The four solutions correspond to four kinds of working modes of the mechanism. Hence, for a given mechanism and for prescribed values of the position of the moving platform, the required actuated inputs can be directly computed from Eqs. (2.6) and (2.7). The working mode shown in Fig. 2 corresponds to the solution in which the signs “ \pm ” in Eqs. (2.6) and (2.7) are both “ $-$ ”.

For the configuration shown in Fig. 2, the inverse solutions of the kinematics are

$$q_1 = y - \sqrt{R_2^2 - (x - R_1)^2} \quad (2.8)$$

$$q_2 = y - \sqrt{R_2^2 - (x + R_1)^2} \quad (2.9)$$

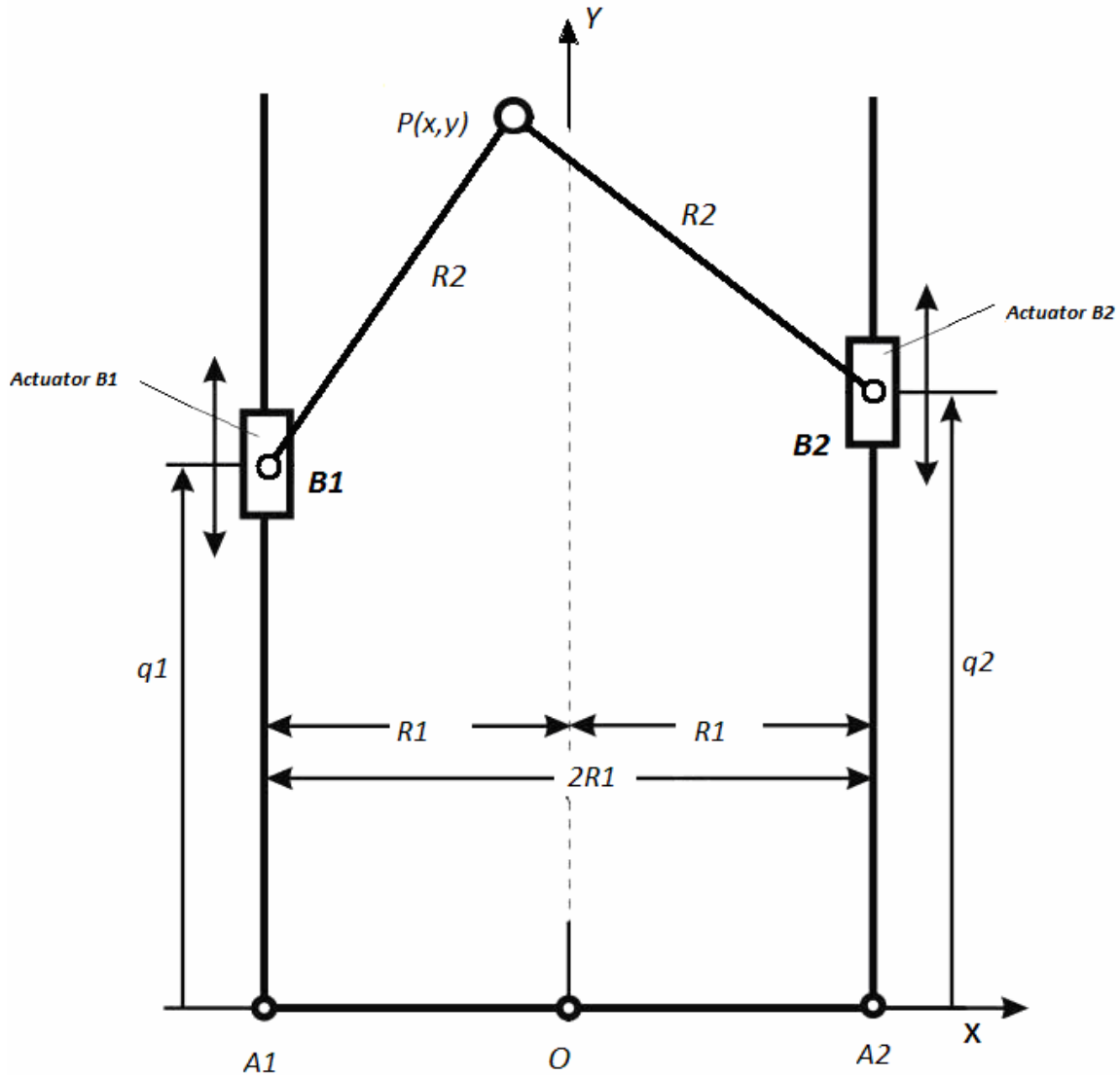


Figure 2.2 General Kinematic Scheme of a PRRRP Parallel Robot

2.3 Forward Kinematics

The objective of the forward kinematics solution is to define a mapping from the known set of the actuated joint position to the unknown position of the moving end-effector. For this manipulator the joint position that are considered known are the position of the sliders q_1 and q_2 . Given the joint position (slider position) derive the forward kinematics equation, expressing a location on the workspace (x, y) , in terms of the slider position (q_1 and q_2). The unknown position of the end-effector is described by the position vector \mathbf{P} , which defines the location of \mathbf{P} at the center of the revolute joint that connects the two links in the XYZ coordinate frame.

From Esq. (2.8) and (2.9), the solutions for the direct kinematics of the manipulator can be expressed as

$$x = ay + b \quad (2.10)$$

Where
$$a = \frac{q_2 - q_1}{2R_2}, \quad b = \frac{q_1^2 - q_2^2}{4R_2} \quad (2.11)$$

and
$$y = \frac{-f \pm \sqrt{f^2 + 4eg}}{2e} \quad (2.12)$$

In which

$$e = a^2 + 1, \quad f = 2a(b - R_2) - 2q_1, \quad g = (b - R)^2 + q_1^2 - R_1^2 \quad (2.13)$$

For the configuration as shown in Figure 2.2, the “ \pm ” of Eq. (2.12) should be only “+”. From the above equations we can see that the direct and inverse kinematics of the manipulator can be described in closed form.

2.4 Jacobian and Singularity Analysis

For parallel manipulators, the Jacobian matrix provides a transformation from the velocity of the end-effector in cartesian space to the actuated joint velocities (e.g. Gosselin and Angeles, 1988; Wang and Gosselin, 1996) as shown in Eq.2.14

$$\dot{\mathbf{q}} = \mathbf{J} \dot{\mathbf{X}} \quad (2.14)$$

Where $\dot{\mathbf{q}}$ is an m -dimensional vector that represents a set of actuated joint rates, $\dot{\mathbf{X}}$ is an n -dimensional output velocity vector of the end-effector, and \mathbf{J} is the $n \times m$ Jacobian matrix. For the manipulator being considered for this thesis, the Jacobian matrix is a square matrix since the two actuated joints map onto the two outputs coordinates of the end-effector. However, it is possible that $m \neq n$. As an example, a redundant manipulator can have more than six actuated joints, while the end-effector will at most have six degrees of freedom, so that $m > n$. For the analysis of the two degree of freedom parallel manipulator, \mathbf{q} is a vector of the actuated joint variables and \mathbf{X} is the position vector of the end effector:

This definition of the Jacobian matrix is slightly different from what is traditionally defined for serial manipulators, where the Jacobian provides a transformation from the joint velocities to the end-effector velocity. This change of the Jacobian definition for parallel manipulators follows naturally from the duality between parallel and serial manipulators (Waldron and Hunt, 1988), and is done as a matter of convenience.

As an extension of this definition, Gosselin and Angeles (1990) presented a two-part Jacobian. It assumes that the relationship between the input coordinates, \mathbf{q} , and the output coordinates, X , can be written in the following form:

$$\mathbf{f}(X, \mathbf{q}) = \mathbf{0} \quad (2.15)$$

Where \mathbf{f} is an n dimensional implicit function of X and \mathbf{q} , $\mathbf{0}$ is an n -dimensional zero vector.

Differentiating Eqn. (2.15) with respect to time, a relationship between the input joint rates and the end-effector output velocity as follows:

$$\mathbf{J}_x \dot{X} = \mathbf{J}_q \dot{\mathbf{q}} \quad (2.16)$$

Where

$$\mathbf{J}_x = \left[\frac{\partial \mathbf{f}}{\partial X} \right] \text{ and } \mathbf{J}_q = \left[\frac{\partial \mathbf{f}}{\partial \mathbf{q}} \right]$$

The Above derivation leads to two separate Jacobian matrices. Hence the overall Jacobian matrix, \mathbf{J} , can be written as

$$\dot{\mathbf{q}} = \mathbf{J} \dot{X} \quad (2.17)$$

Where

$$\mathbf{J} = \mathbf{J}_q^{-1} \mathbf{J}_x$$

And where \mathbf{J}_x is an $n \times n$ Jacobian matrix and \mathbf{J}_q is an $n \times m$ Jacobian matrix. Both \mathbf{J}_x and \mathbf{J}_q are configurations dependent, i.e. $\mathbf{J}_x = \mathbf{J}_x(\mathbf{x}, \mathbf{q})$ and $\mathbf{J}_q = \mathbf{J}_q(\mathbf{x}, \mathbf{q})$. The advantage of the two-part Jacobian is that it allows the identification of various types of singularities.

Independent of which form it takes, the Jacobian provides many useful in-sights to the performance of a manipulator. The Jacobian matrix is often used for trajectory generation purposes since for a given desired end-effector velocity, it's possible to map that velocity back to the joint space. Jacobian analysis is also used to determine the singular positions of a manipulator, since when a manipulator is at a singular position the Jacobian matrix is also singular (Gosselin and Angeles, 1990; Gosselin and Sefrioui, 1992; Wang and Gosselin, 1996). Similarly, the Jacobian is used to describe the workspace boundaries of a manipulator (Oblak and Kohli, 1988). Furthermore, the Jacobian is useful for characterizing the stiffness of a manipulator, which provides some insight to the mechanical advantage the end-effector has relative to the actuated joints (Gosselin, 1990a; Tahmasebi and Tsai, 1995). The condition number of the Jacobian matrix has also been used as a performance index for optimizing manipulator design since it provides a measure of the amplification of the error between the actuated joints and the position of the end-effector (Gosselin and Angeles, 1988, 1989, 1991). A recent novel application of the Jacobian has been the analysis and synthesis of under-actuated force generating mechanisms (Gosselin, 1996).

A manipulator singularity describes a manipulator posture that results in an instantaneous change in the mobility of the manipulator. These are undesirable postures, since the control of the manipulator in these postures becomes problematic or the manipulator is at the limit of the workspace. Accordingly, it's important to understand the conditions that result in a manipulator singularity. The identification of those conditions for this manipulator is the focus of this chapter.

Using the classification scheme presented by Gosselin and Angeles (1990), singularities for parallel manipulators can be categorized into three types. The first type occurs when different branches of the inverse kinematics problem converge. This type of singularity results in a loss of mobility, and occurs at the boundary of a manipulator workspace. The second type of singularity occurs when different branches of the forward kinematics problem converge. This type of singularity results in additional degrees of freedom at the end-effector. So, for a manipulator in this singular posture, the end-effector has one or more degrees of freedom even when the actuated joints are locked. This also means that there are some forces or torques which can be applied to the end-effector that cannot be resisted by the actuators. The third type of singularity occurs when the manipulator is in a posture that produces a singularity of the first type and a singularity of the second type simultaneously.

2.5 Derivation of Jacobian Matrix and Conditioning Indices

In this chapter, the analysis of the two degree of freedom parallel manipulator will be discuss, q is a vector of the actuated joint variables and \mathbf{X} is the position vector of the end effector :

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix}$$

The Jacobian matrices are derived by differentiating the loop closure equation for each leg of the manipulator and then solving the resulting system of equations so that it takes the following form:

$$\mathbf{J}_q \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \mathbf{J}_x \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \quad (2.18)$$

where \dot{x} and \dot{y} are the X and Y components of the velocity of point P on the end-effector in the world coordinate frame see Figure 2, and where \mathbf{J}_x and \mathbf{J}_q are 2×2 Jacobian matrices. The Jacobian matrices are in terms of the manipulator joint position, and are accordingly dependent upon the position and posture of the manipulator. The singularities of the manipulator are then found by examining the conditions that result in singular Jacobian matrices.

$$\mathbf{J}_q = \begin{bmatrix} y - q_1 & 0 \\ 0 & y - q_2 \end{bmatrix} \quad \text{and} \quad \mathbf{J}_x = \begin{bmatrix} x - R_1 & y - q_1 \\ x + R_1 & y - q_2 \end{bmatrix} \quad (2.19)$$

If matrix \mathbf{J}_q is nonsingular, the Jacobian matrix of the manipulator can be obtained as

$$\mathbf{J} = \mathbf{J}_q^{-1} \mathbf{J}_x = \begin{bmatrix} (x - R_1) / \sqrt{R_2^2 - (x - R_1)^2} & 1 \\ (x + R_1) / \sqrt{R_2^2 - (x + R_1)^2} & 1 \end{bmatrix} \quad (2.20)$$

From equation (2.18) and (2.20) one can obtains the speed of the slider:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \mathbf{J} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \quad (2.21)$$

Differentiating above equation with respect to time leads to the acceleration of the sliders:

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \mathbf{J} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \dot{\mathbf{J}} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \mathbf{J} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + AA \quad (2.22)$$

$$\text{Where } AA = \dot{\mathbf{J}} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} -(\dot{x}^2 + \dot{y}^2 + \dot{q}_1^2 - 2\dot{q}_1\dot{y}) / (q_1 - y) \\ -(\dot{x}^2 + \dot{y}^2 + \dot{q}_2^2 - 2\dot{q}_2\dot{y}) / (q_2 - y) \end{bmatrix} \quad (2.23)$$

Let

$$|\mathbf{E}\lambda - \mathbf{J}| = 0 \quad (2.24)$$

In which $\mathbf{E} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$ and there is

$$\lambda^2 + s\lambda + t = 0 \quad (2.25)$$

$$\text{Where } s = \left(\frac{x}{y - q_1} + 1 \right), \quad t = \frac{x - R_1}{y - q_1} - \frac{x + R_1}{y - q_2}$$

Then the singular value of Jacobian matrix \mathbf{J} can be obtained,

$$\lambda = \frac{-s \pm \sqrt{s^2 - 4t}}{2} \quad (2.26)$$

As is well known, the dexterity of a parallel manipulator can be evaluated using the conditioning number (Salisbury and Graig, 1982). The conditioning index (CI) $1/k$ is the reciprocal of the condition number (Gosselin and Angeles, 1991; Liu et al., 2000), and this is written as

$$1/k = \frac{\lambda_2}{\lambda_1} \quad (2.26)$$

Where λ_1 and λ_2 are the maximum and minimum singular values of the Jacobian matrix, which can be obtained from Equation (2.24), of the manipulator, respectively.

The corresponding global conditioning index (GCI) will be

$$\eta = \int_w \frac{1}{k} dW \bigg/ \int_w dW \quad (2.27)$$

Where W is the reachable workspace of the manipulator.

In the conditioning indices, the singular configurations should be avoided. From above analysis, one can see that the manipulator is very simple. And the singularity analysis will also be simple, which can be reached from Jacobian matrices \mathbf{J}_x or \mathbf{J}_q .

Due to existence of two Jacobian matrices, a parallel manipulator is said to be at a singular configuration when either \mathbf{J}_x or \mathbf{J}_q or both are singular. Three different types of Singularities can be identified.

Singularity of First Kind

Singularity of first kind occurs when the determinant of \mathbf{J}_q is equal to zero, namely,

$$\det(\mathbf{J}_q) = \mathbf{0} \quad \text{and} \quad \det(\mathbf{J}_x) \neq \mathbf{0}$$

In the presence of such a singular condition the null space of \mathbf{J}_x is not empty, there exist some nonzero $\dot{\mathbf{q}}$ vectors that result in zero $\dot{\mathbf{X}}$ vectors. Infinitesimal motion of end-effector certain directions cannot be accomplished. Hence the manipulator loses one or more degrees of freedom. From Equation (2.17), there is $y = q_1$ or $y = q_2$, which means that the first or second leg is parallel to x axis. This kind of singularity corresponds to the limit of the workspace.

Singularity of Second Kind

Singularity of second kind occurs when the determinant of \mathbf{J}_x is equal to zero, namely,

$$\det(\mathbf{J}_x) = \mathbf{0} \quad \text{and} \quad \det(\mathbf{J}_q) \neq \mathbf{0}$$

Assuming that in the presence of such a singular condition the null space of \mathbf{J}_x is not empty, there exist some non zero $\dot{\mathbf{X}}$ vectors that result in zero $\dot{\mathbf{q}}$ vectors. That is, the moving platform can possess infinitesimal motion in some directions while all the actuators are completely locked. Hence the moving platform gains one or more degrees of freedom. This is in contradiction with a serial manipulator, which loses one or more degrees of freedom. In other words, at a second kind of singular configuration, the manipulator cannot resist forces or moments in some directions. This kind of singularity occurs, in which $x = R_1$ or $x = -R_1$. Then, if $R_1 \neq 0$, the singularity cannot occur. The physical interpretation of this kind of singularity is that even if all of the input velocities are zero, there are still be instantaneous motion of the end-effector. In this Configuration, the manipulator loses stiffness and becomes uncontrollable. This kind of singularity is located inside the workspace of the manipulator. Such a singularity is very difficult to locate only by analyzing and expanding the equation $\det(\mathbf{J}_x) = \mathbf{0}$. A numerical method is thus a good selection for solving this problem.

Combined Singularities

Combined singularity occurs when the determinants of both \mathbf{J}_x and \mathbf{J}_q are zero. This kind of singularity corresponds to the first and second type of singularity occurring simultaneously. This singularity is both configuration and architecture dependent. In one of these singularities, the two legs are both parallel to the x-axis. The geometric parameter condition for this singularity is $R_1 = R_2$. Additionally, if $R_1 = 0$ the mechanism is in such a singularity when $x = 0$. In this case, the two legs are both parallel to the y-axis. Therefore, in order to make the mechanism be assembled and work freely, there should be $R_1 \leq R_2$. If $R_1 = R_2$, the workspace is only one line section on the y-axis. Parallel singularities are particularly undesirable because they cause the following problems:

- A high increase of forces in joints and links, that may damage the structure,
- A decrease of the mechanism stiffness that can lead to uncontrolled motions of the tool though actuated joints are locked.

2.6 Workspace of the Manipulator

One of the most important issues in the design process of a parallel manipulator is its workspace. For parallel manipulators, this issue may be more critical since Parallel manipulators will sometimes have a rather limited workspace.

The workspace of the planar 2-DoF parallel manipulator is often represented as a region in the plane. The determination of the workspace is simple, and can be obtained geometrically from the inverse kinematics equation. From Equation (2.3), one can obtain Equation (2.4) and (2.5). Which means that if q_i is specified, Equations (2.4) and (2.5) represent two circles centered at $(-R_1, q_1)$ and (R_1, q_2) , respectively. Their radii are R_2 . If $q_i \in [q_{i\min}, q_{i\max}]$, Equations (2.4) and (2.5) represent two enveloping surfaces, each of which is the locus of a circle (the radius is R_2), when the center is rolling on line segments $x = -R_1$ and $x = R_1$ ($q \in [q_{i\min}, q_{i\max}]$), respectively. The intersection of the two enveloping surfaces is the workspace of the manipulator.

Therefore, the reachable workspace of the reference point P is the intersection of the sub-workspaces associated with the two kinematic chains as shown in Figure 2.3.

The task workspace is a part of the reachable workspace. In practical applications, the task workspace is usually defined as a rectangular area in the reachable workspace.

Let the maximum limit of the angles α and β , which are the angles between link B_iP ($i=1, 2$) and the vertical axis, be denoted by α_{\max} and β_{\max} . Let $y_{i,\max}$ and $y_{i,\min}$ represent the maximum and minimum positions of the i -th slider. P reaches point Q_1 when slider B_1 reaches its lower limit and the value of α is the maximum, namely $y_1 = y_{1,\min}$ and $\alpha = \alpha_{\max}$. Similarly, P reaches point Q_4 when $y_2 = y_{2,\min}$ and $\beta = \beta_{\max}$.

A vertical line through Q_1 intersects with the upper bound of the reachable workspace at point Q_2 . Q_3 is directly above Q_4 (see Figure 2.3). The region $Q_1Q_2Q_3Q_4$ then makes up the task workspace, as a rectangle of width b and height h , denoted by W_t .

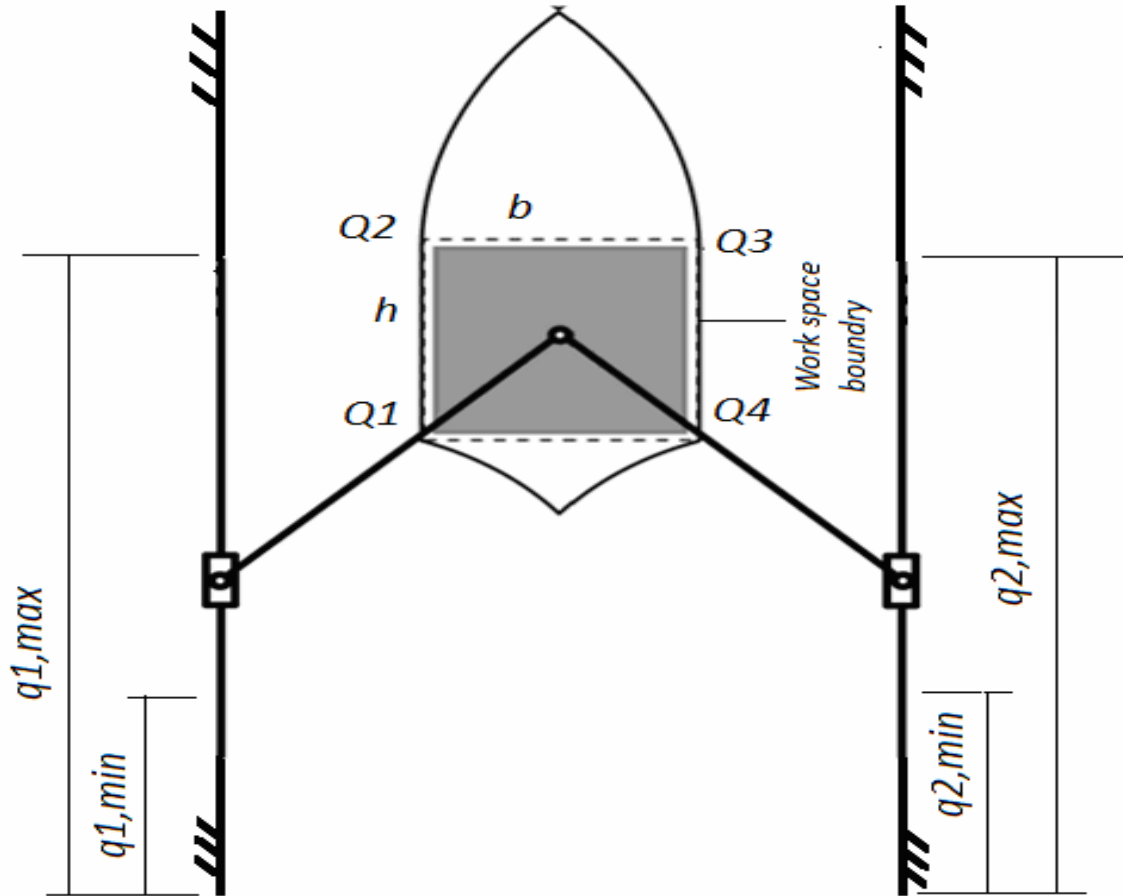


Figure 2.3 General reachable workspace scheme of a PRRRP Parallel Robot

2.7 Determination of Workspace based on the Dimension of a Manipulator

The objective of this section is to determine the maximum area $Q_1Q_2Q_3Q_4$ of the workspace based on the desired dimensions of the manipulator. In this thesis the desired dimensions of the manipulator which is assumed to be given are the length of each intermediate link $R_2 = 300 \text{ mm}$ and the distance between the two sliders $2R_1 = 425 \text{ mm}$. The workspace is usually defined as a rectangular area in the reachable workspace.

To investigate the maximum workspace we can just consider the workspace along the x-axis. It is a line section. Disregarding the input, the maximal region that the mechanism can reach along the x-axis is determined by the singularity, i.e., the first kind of singularity in which either of the two legs B_1P and B_2P is parallel to the x-axis.

As shown in Figure 2.3, if the maximal workspace along the x-axis is denoted as X_w , there is

$$X_w = b = 2(R_2 - R_1) = 2\left(300 - \frac{425}{2}\right) = 175 \text{ mm}$$

On the other hand the advantage of this manipulator is that it gives a total freedom in the choice of the Y direction of the workspace, as shown in Figure 2.3. That means the workspace along the y-axis can be free if the input is enough. Considering the maximum slider translational input $|y_{i,\max} - y_{i,\min}| = 1000 \text{ mm}$.

The maximal workspace along the Y-axis is denoted as Y_w is given by

$$Y_w = h = |y_{i,\max} - y_{i,\min}| - X_w = 1000 \text{ mm} - 175 \text{ mm} = 825 \text{ mm}$$

From this the Maximum workspace of the **PRRRP** parallel robot was found to be

$$W_t = X_w \times Y_w = 175 \times 825 = 144375 \text{ mm}^2$$

The simulation analysis will also be carried out using MATLAB, where a code is written as m-file. Figure 2.4 shows the simulation of maximum work space for PRRRP parallel Manipulator.

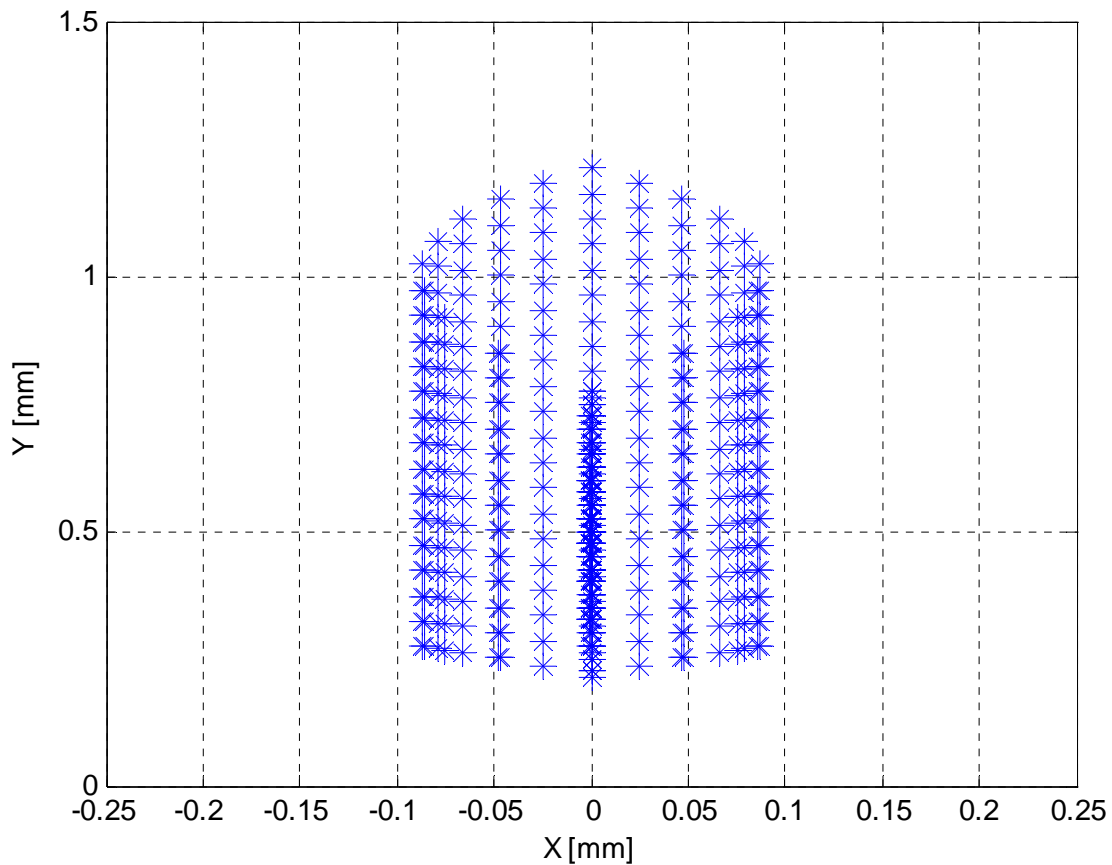


Figure 2.4 The simulation of maximum work space for PRRRP parallel Manipulator

2.8 Summery

Recently, parallel kinematic manipulators with less than 6 DoFs have become more attractive. In this section, the kinematic Analysis of 2-DoF translational manipulator is developed. One advantage of the manipulator is that it can position a rigid body in a 2D plane while maintaining a constant orientation. The inverse and forward kinematics problems are solved. Based on the geometric model of the manipulator, three kinds of singularities are defined. Their physical as well as mathematical meanings are clarified. A rough workspace and conditioning indices are presented in this section. In particular, the maximum work space of the manipulator is investigated. The design result was obtained with respect to a given desired manipulator dimension.

Chapter 3

Dynamic Modeling of a Rigid PRRRP Parallel Manipulator Prototype

3.1 Introduction

In order to simulate the behaviour of the robot, the dynamic model of the structure must be built. This is a complex subject and different methods were developed in order to solve it. A classical method for calculating the dynamic models of closed chains is to consider first an equivalent tree-structure, and then to consider the system constraints by the use of Lagrange multipliers or d'Alembert's principle. Other approaches include the use of virtual work, Lagrange formalism, Hamilton's principle, and Newton-Euler equations.

All the above mentioned dynamic modeling approaches are by explicitly writing out differential equations governing component dynamics which is rigorous mathematical calculations and time consuming that is usually required in such a process.

In this chapter, the dynamic model rigid PRRRP parallel Manipulator is first modeled with the basic connector model forming the basis to the manipulator with flexible link using the SimMechanics toolbox from Simulink. The toolbox uses the standard Newtonian dynamic of forces and torques in order to solve both direct and inverse problem. The model is built using Simulink blocks; blocks that represents the kinematic elements and the joints of the robot. The blocks from the toolbox allow to model mechanical systems consisting of any number of rigid bodies, connected by joints representing translational and rotational degrees of freedom.

The benefit of this approach is that dynamic models of components are much easier to construct using the domain-specific building constructs that Sim-Mechanics provides than by explicitly writing out differential equations governing component dynamics. In addition, the Physical Modeling environment of SimMechanics tool makes the task much easier. Moreover, since SimMechanics allows construction of the mechanical models without the user having to derive the associated equations of motion (time-consuming process), the simulation environment can also be used by researchers from fields other than robotics, even with limited mechanical background.

3.2 *SimMechanics*

SimMechanics allows the user to model a physical system while minimizing the rigorous mathematical calculations that are usually required in such a process. By simply inserting a handful of dimensions into specific system blocks, this program is capable of accurately modeling the motion of complex systems.

SimMechanics is a commercial tool for simulating mechanical system. It is an extension of Matlabs Simulink software. Its key feature is ease of use and integrity to existing Simulink block diagrams. Mechanical systems (linear and nonlinear) can be modeled with Sim-mechanics blocks that can be connected to simulink blocks. CAD models can also be directly translated to function blocks by using separate software. SimMechanics also offers Matlabs powerful mathematical tool, such as integrion and optimization. Furthermore other Matlab extensions, such as the Real-Time Toolkit can be integrated into the development environment. The system also offers automatic C-code.

In order to build a SimMechanics model one have to specify the body inertial properties, degrees of freedom, and constraints, along with coordinate systems attached to each body of the structure. This procedure can be a tough one when it comes to bodies with complex geometric forms, the process can be simplify by using Solid Works.

3.3 *Modeling the Rigid PRRRP Parallel Manipulator in solid work*

The PRRRP rigid planar parallel manipulator developed at the solid work is shown in Figure 3.1. The parallel manipulator presented is categorized a PRRRP type because it has two symmetrical kinematic chains, each of which has one active prismatic (P) joint, followed by two consecutive passive revolute (R) joints. Each active prismatic joint is implemented by a ball screw and linear guide mechanism. The ball screw converts the rotation of motor into the translational motion of the slider along the linear guide. The origin of q_i is selected at the center of each linear guide. The passive revolute joints in sliders connect the slider with the intermediate linkages. The other ends of the intermediate linkages connect each other with the moving end effector by revolute joint, which constrains the motion of the links. The end effector moves in the plane with two degrees of freedom. The two intermediate links connect with the slider and the moving end effector. Each intermediate link has identical geometric parameters. The base is fixed to the ground.

The PRRRP parallel manipulator is made of steel. The material of the intermediate links is mild steel with the density and elastic modulus of $\rho = 7.6 \times 10^3 \text{ Kg} / \text{m}^3$, $E = 2 \times 10^{11} \text{ N} / \text{m}^2$, and the material of brackets (sliders) AS4140 with density and elastic modulus of $\rho = 7.689 \times 10^3 \text{ Kg} / \text{m}^3$, $E = 2 \times 10^{11} \text{ N} / \text{m}^2$, respectively.

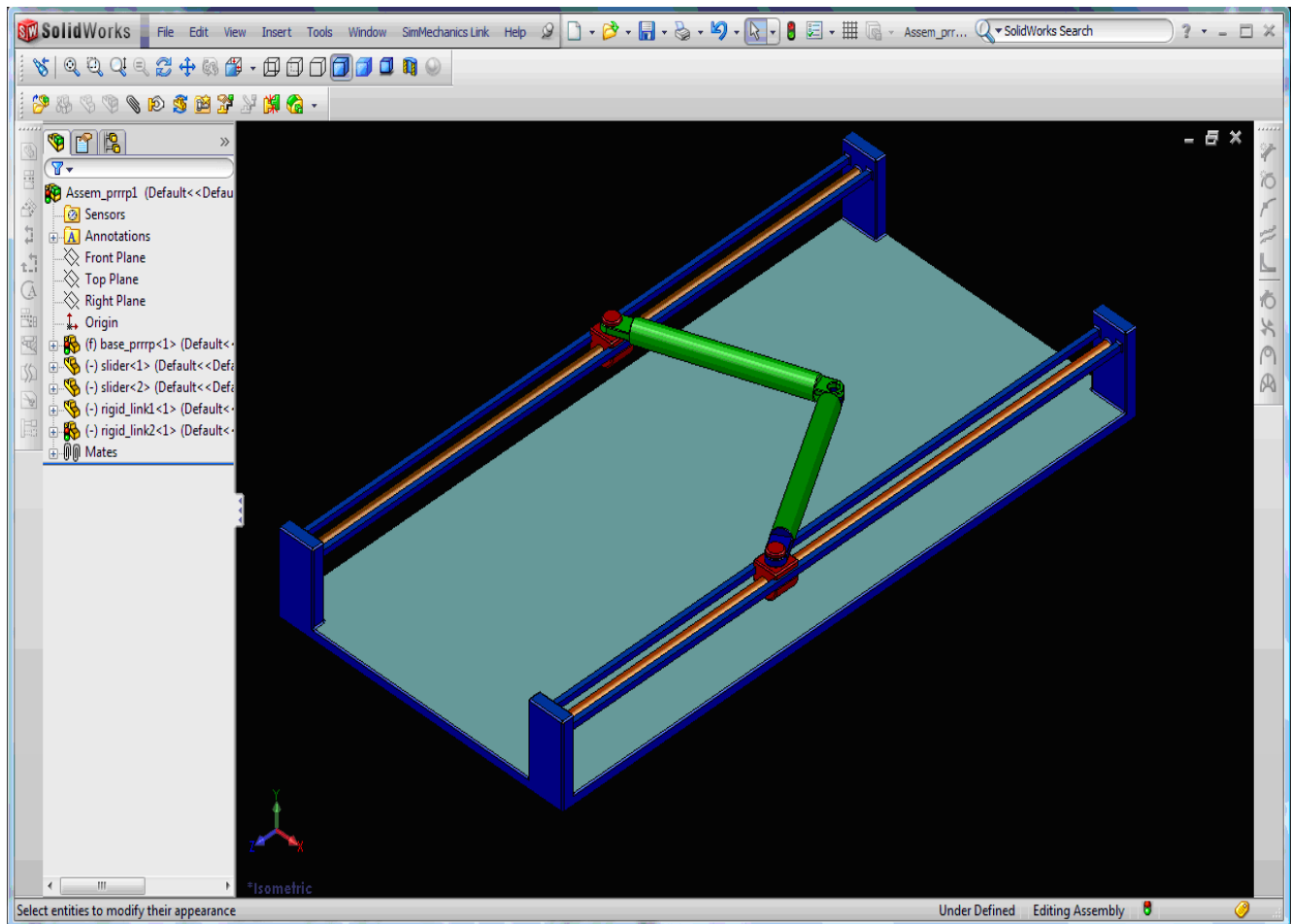


Figure 3.1 Solid work model of rigid PRRRP parallel manipulator

3.4 *Creating SimMechanics Model from Solid Work Model*

After modeling a simplified motion of the Manipulator, CAD translation tool is used to transform geometric CAD assemblies into Simulink block diagram model. The CAD translation tool first exports the assembly model from CAD platform into physical modeling file with xml extension. The physical modeling file is then imported into Simulink, creating a SimMechanics model. Figure 3.2 shows the sequence of CAD to SimMechanics transformation.

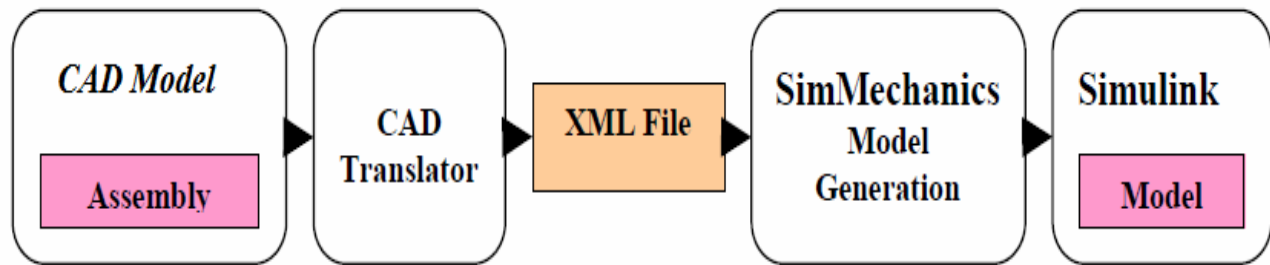


Figure 3.2 CAD to SimMechanics transformation sequence

The imported xml file will be converted to a SimMechanics block model. The model is created using SimMechanics blocks and all the necessary information (mass, inertia, orientation, etc.) is automatically transferred to these blocks. Figure 3.3 shows the automatically-created SimMechanics model of a rigid PRRRP parallel manipulator. The two kinematic chains are defined by body and joint blocks. The inertia properties and the coordinates of the joints for each body were determined automatically when the CAD model was imported in MATLAB. The generated SimMechanics model can be visualized while the simulation is running. Figure 3.4 shows the simplified motion model after it is converted into SimMechanics.

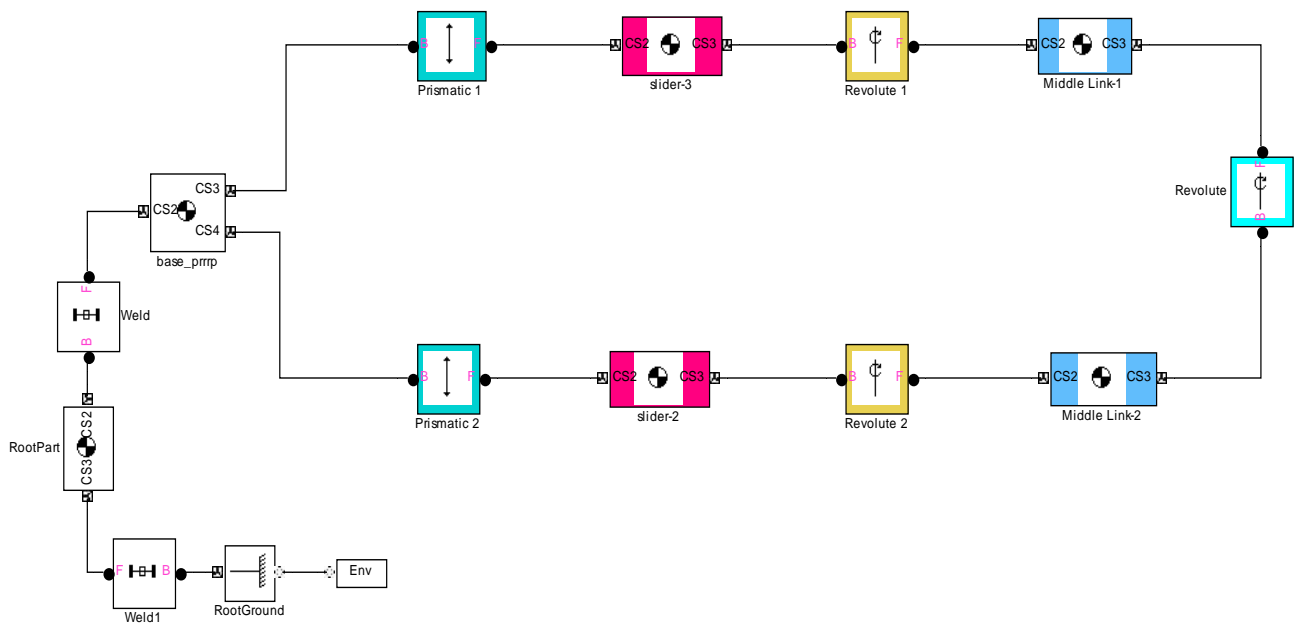


Figure 3.3 SimMechanics model of the PRRRP rigid parallel Manipulator translated from solid work

The SimMechanics blocks that is created as a result of Solid Works to Simmechanics translation are illustrated in the Figure 3.3. The base part, “base-prrrp,” of the manipulator is fixed to the base, which is described as a zero-DOF with respect to the base (ground). The base is then connected to the sliders via a prismatic joint, which is described as one translational DOF in between the base and the slider. The intermediate link is then connected to the slider via a revolute joint, which is described by a rotational motion between the intermediate link and the slider. The two intermediate links is then connected via a revolute joint, “Revolute” (end effector), which is described by a rotational motion between the two intermediate links.

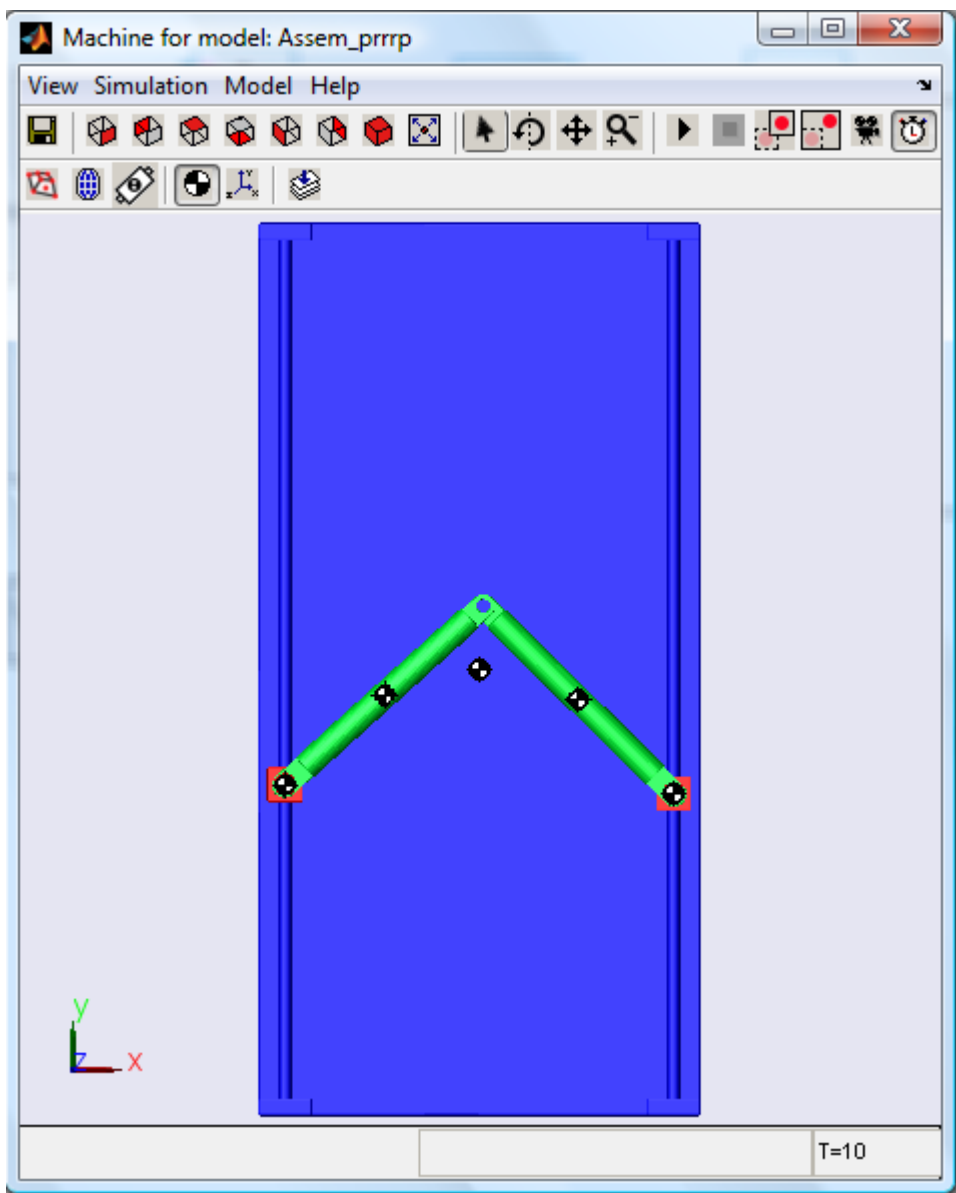


Figure 3.4 Physical model of rigid five-bar linkage PRRRP parallel manipulator in SimMechanics

The connection of the mechanical model of the robot to the rest of the robot model was realized via actuators and sensors. Inputs of the model can be one of the following: the generalized force, the position, speed, or the acceleration of the motor joints. In this model, the inputs chosen were the position, speed, and the acceleration of the two slider joints of the robot from Inverse Kinematics Problem (IKP). As output is the position of the end effector. Figure 3.5 shows SimMechanics model of the PRRRP rigid parallel Manipulator with sensor and actuator

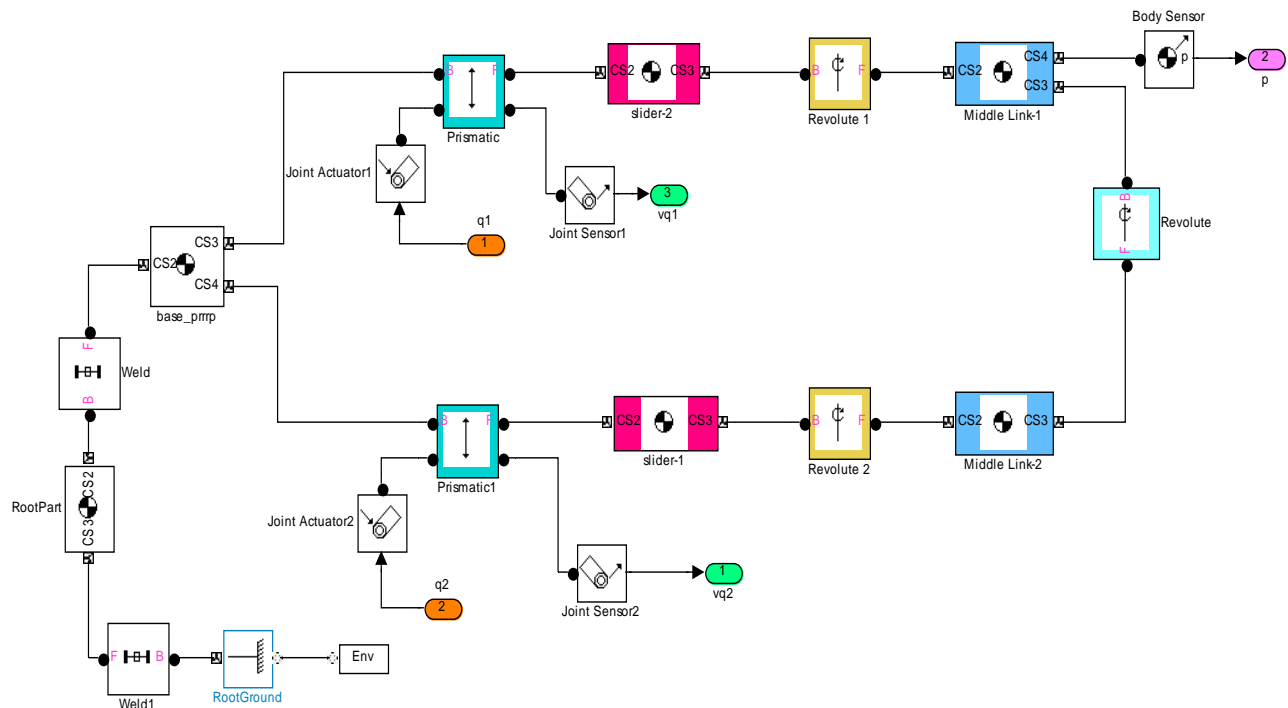


Figure 3.5 SimMechanics model of the PRRRP rigid parallel Manipulator with sensor and actuator

The SimMechanics motion platform (SimPlatform) is incorporated with inverse kinematics model for actuation control. Figure 3.6 shows Simulink Open-loop Model of rigid PRRRP parallel Manipulator without joint motion controls. The equations for Inverse Kinematics Problem (IKP) were implemented via a MATLAB function, where the inputs were the position of the end-effector, while the outputs were the position, velocity and acceleration for each actuator on the slider. The inverse kinematic model controls the actuators to move the sliders relatively to one another. The complete SimPlatform Model allows the motion platform motion cues to be visualized. This also helps to test and validate the performance of inverse kinematics model.

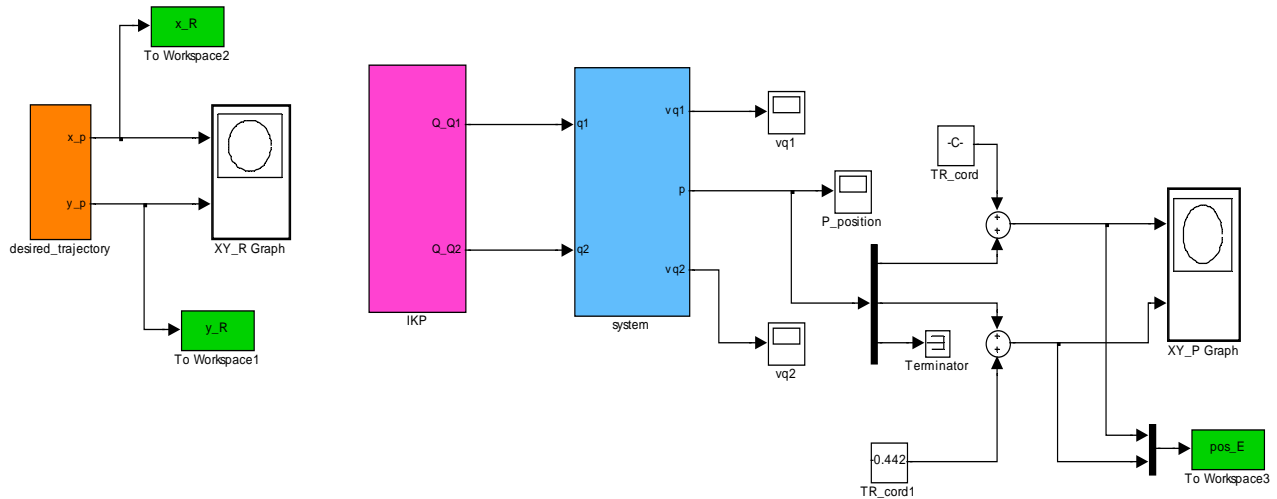


Figure 3.6 Simulink Open-loop model of the rigid PRRRP parallel Robot

3.5 Trajectory planning

For the evaluation of the PRRRP parallel robot model performance, a circular trajectory in 2 dimensional spaces was used, as shown by Figure 3.7 and 3.9. In order to obtain a circle as trajectory it was given a cosine input to the x-axis and sine input to the y-axis using constant acceleration motion law. Radius of the circle trajectory is set to 0.075 m which is inside the work space and its center is $O(0, 0)$.

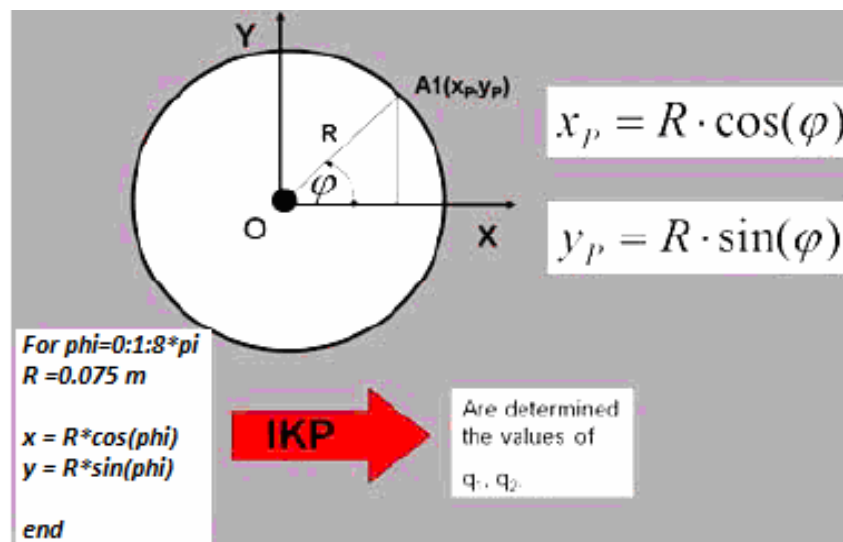


Figure 3.7 2D trajectory generation of circle for the PRRRP parallel robot in MATLAB/Simulink

The constant acceleration motion law represents one of the most common motion law in industrial robotic applications. The end effector moves with maximum angular speed and zero angular acceleration during most of the time interval representing trapezoidal velocity profile. It has an advantage of controlling the angular acceleration at the start and goal positions. The trajectory is composed of three main parts as shown in Figure 8. Figure 8 represents the angular position, angular velocity and angular acceleration for the end effector reference motion law for circular trajectory.

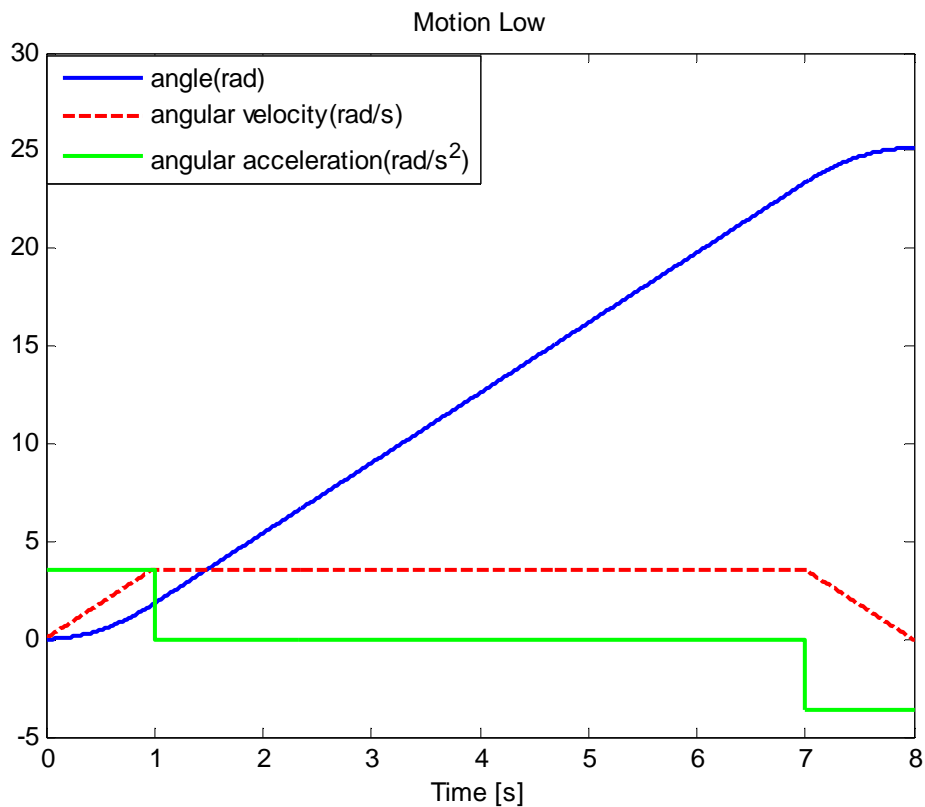


Figure 3.8 Motion low for the reference end effector trajectory

3.6 Simulation results

The main objective of this study is to plot the end effector trajectory by moving the sliders through a prescribed motion from inverse parallel kinematics based on the desired trajectory. The detailed malab function for calculating inverse parallel Kinematics is given in Appendix A. The position and orientation of the end effector can be obtained through the forward Dynamics from the SimMechanics model of the Manipulator using sensor connected on the end effector.

This path is designed to be completed 4 round in 8 seconds when the end-effector reaches the starting point $P_1 (0.075, 0)$ with applying constant acceleration law on change of angle $\varphi(t)$. The actual end-effector trajectory is shown in Figure 3.9.

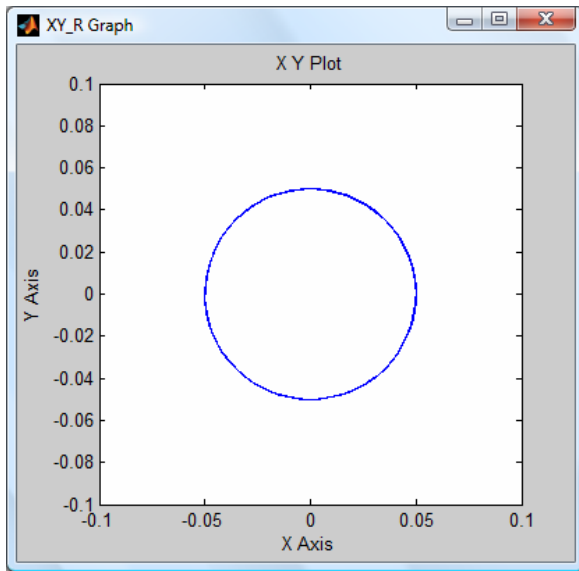


Figure 3.9 Reference End-effector paths for the circular trajectory

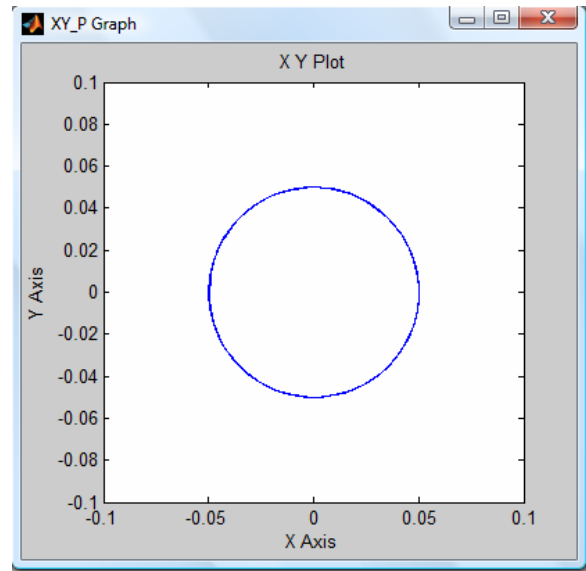


Figure 3.10 Actual 2D plot of trajectory of the rigid PRRRP parallel robot in Simulink

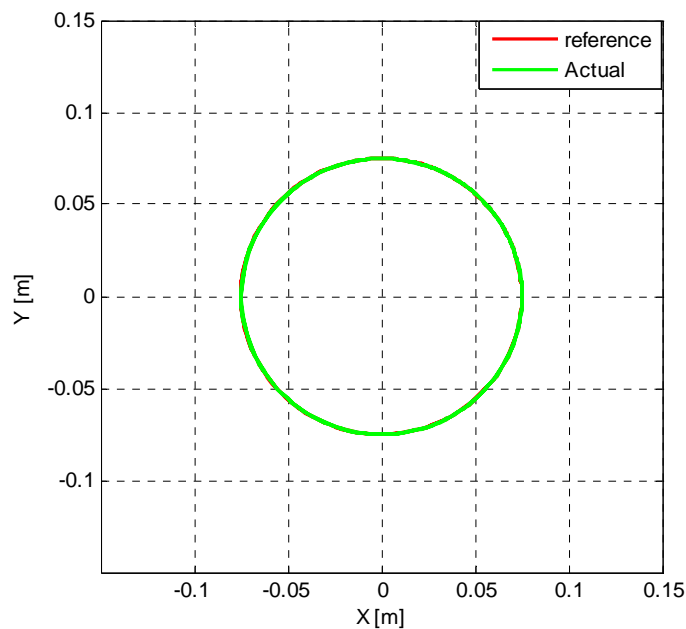


Figure 3.11 Reference and Actual End effector 2D plot of trajectory for the rigid PRRRP parallel robot from MATLAB

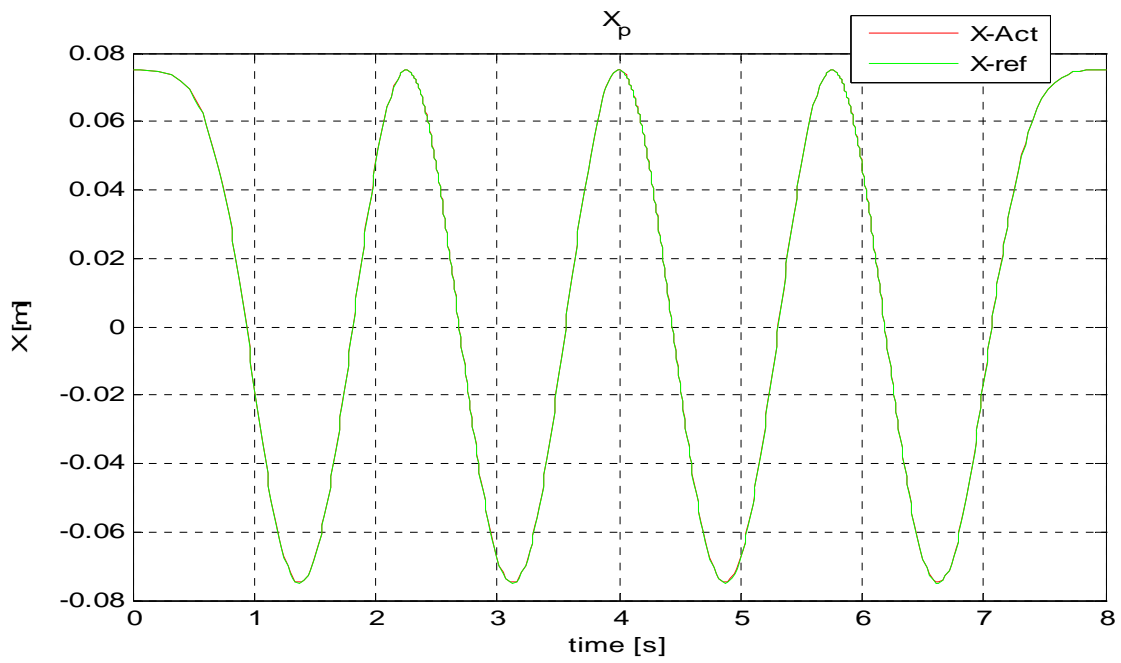


Figure 3.12 Reference and Actual End effector X- trajectory

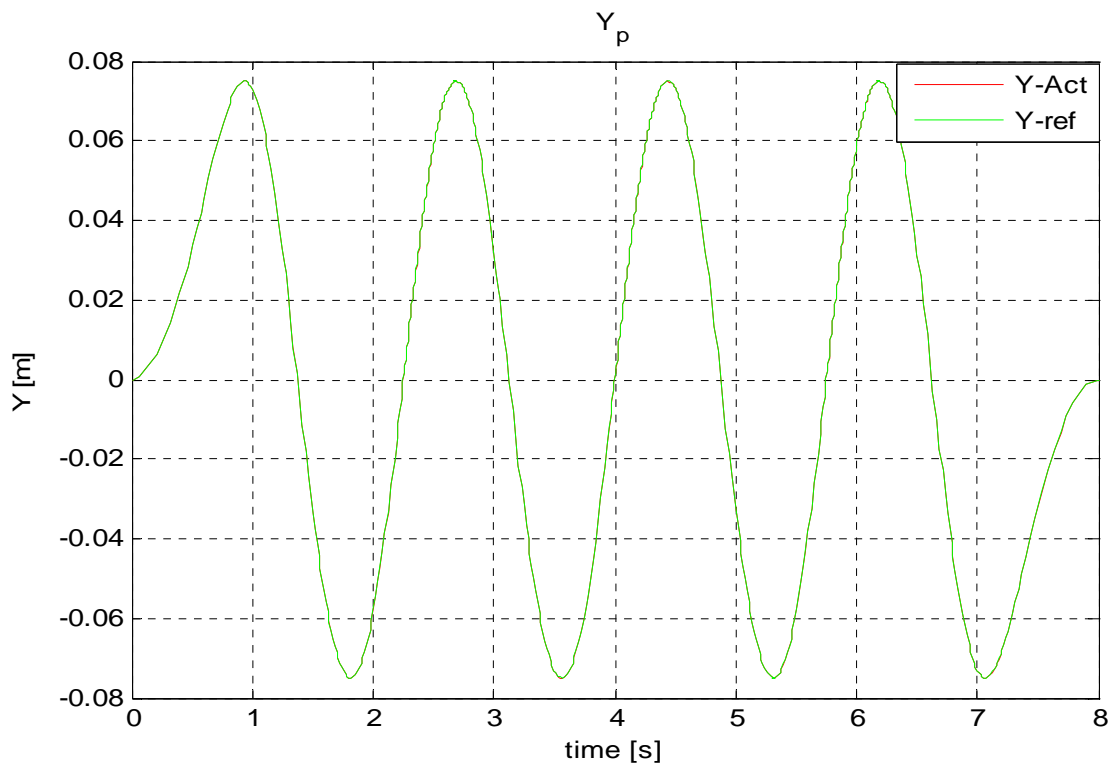


Figure 3.13 Reference and Actual End effector Y- trajectory

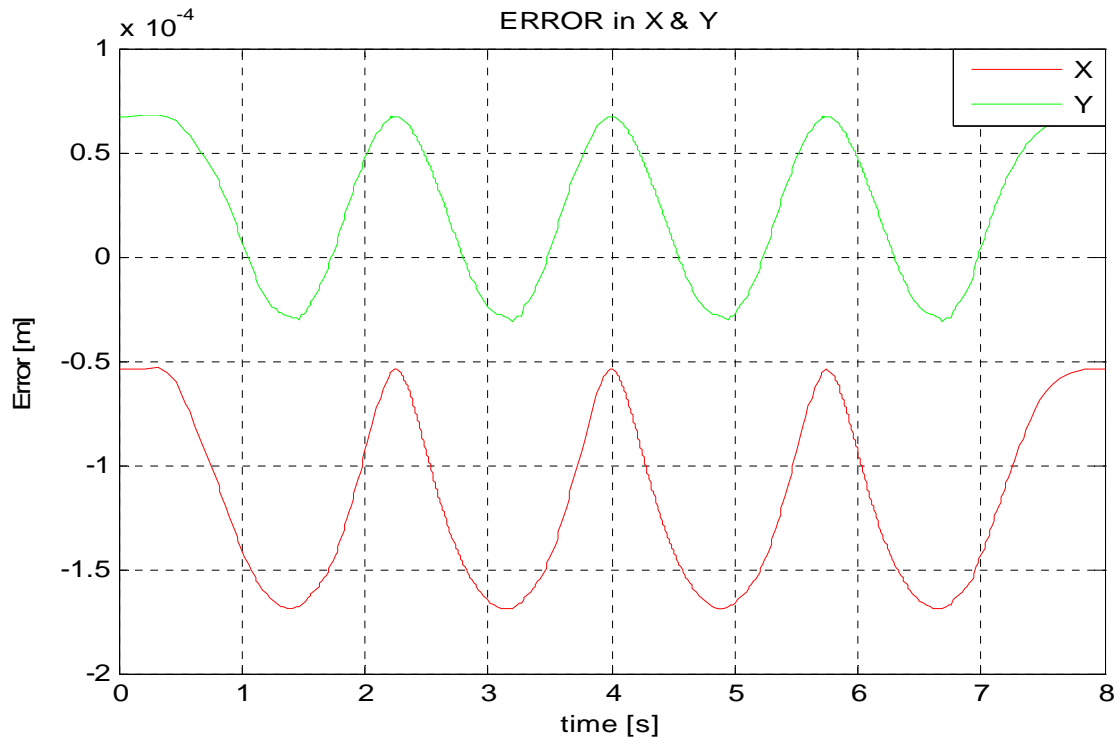


Figure 3.14 Error between Reference and Actual of End effector trajectory for X & Y

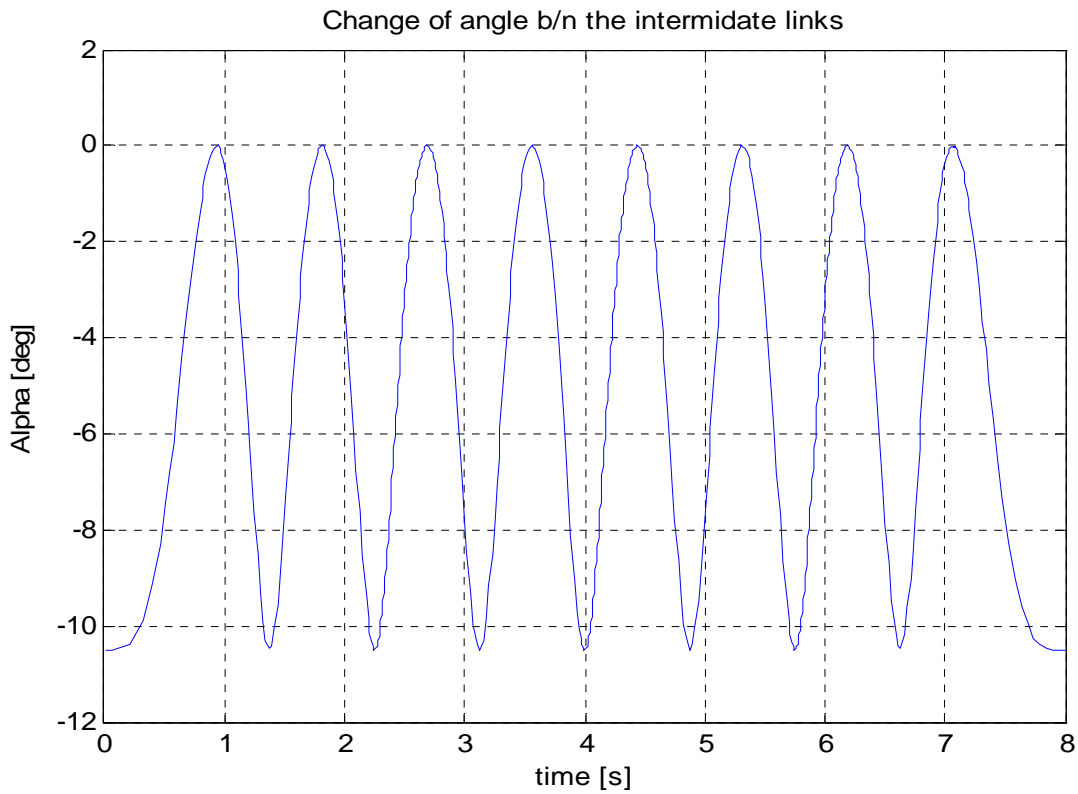


Figure 3.15 change in angle between the two intermediate links

Figure 3.9 to 3.15 present's numerical simulations and results for Open-loop simulations without joint motion controls to investigate the effect of rigid PRRRP parallel manipulator motion for the given circular trajectory. As it can be observed there is no control law over the motion fed into the actuator. Therefore the SimMechanics dynamic model is exactly expresses the rigid parallel manipulator and the result comes out to be a perfect circular path followed by the end-effector.

3.7 Summery

This section presented a new method of modeling and simulation of 2 DOF parallel mechanisms based on Matlab/SimMechanics. The rigid PRRRP parallel Manipulator was first model in solid work then directly translated to function blocks by using separate software in to SimMechanics model. To check the performance requirement for making the end-effector track of the reference trajectory the PRRRP rigid robot system block was connected with inverse parallel kinematics block using actuators of the slider. The simulation of running the robot was based on the Simulink module from MATLAB. MATLAB/Simulink was chosen as a tool that is a widely used for modeling, simulation and testing of dynamical systems. A model in Simulink is represented graphically by means of a number of interconnected blocks. Simulation results are presented in detail. The test results reveal its reliability and accuracy greatly. It indicates that SimMechanics dynamic model of rigid PRRRP parallel Manipulator is exact and accurate.

Chapter 4

Dynamic Modeling of A PRRRP Parallel Manipulator with Flexible Link

4.1 Introduction

Considering the promising characteristics of parallel manipulators and light weight manipulators, a planar parallel manipulator with light weight links is proposed to provide an alternative high-speed pick-and-place positioning mechanism to serial architecture manipulators. Light weight links are used to better meet the demands of high speed and high acceleration placement. However, light weight members are more likely to deflect and vibrate due to the inertial forces and forces from actuators. Structural flexibility effects are much more pronounced at high operational speeds and accelerations of the end-effector. Therefore, we need to develop a dynamic model to investigate the dynamic characteristics of these structural vibrations, with the objective to develop an effective vibration control methodology that suppresses these unwanted structural vibrations.

Investigation on the dynamic modeling of manipulators and mechanisms with flexible links has been studied extensively since the 1970's. Manipulators and mechanisms with flexible links are continuous systems with an infinite number of degrees of freedom and described by nonlinear, coupled, partial differential equations of motion. To design and implement a real-time controller for joint motion and vibration suppression, the dynamic model formulation of flexible linkage manipulators and mechanisms has been carried out based on different discretization approaches of the flexible links. The most common approaches are the finite element method (FEM) (Piras and Cleghorn [45], Wang and Mills [46], Winfrey [47], Erdman *et al.* [48], Imam *et al.* [49], Nath and Ghosh [50], Cleghorn *et al.* [51], Turic and Midha [52]) and the assumed mode method (AMM) (Book [53], Asada *et al.* [54], Baruh and Tadikonda [55], Hustings and Book [56], Barieri and Ozguner [57], Bellezza *et al.* [58], Low and Lau [59], Shabana [60]). Early investigations mainly involved the modeling of flexible serial manipulators and four-bar mechanisms with detailed reviews

in (Erdman and Sandor [35], Lowen and Jandrasits [36], Lowen and Chassapis [37], Shabana [38], Dwivedy and Eberhard [39], Book [40], Benosman and Vey [41]). Compared with the dynamic model of flexible serial manipulators and four-bar mechanisms, research reports on dynamic models for parallel manipulators are rather few in number. Recently, with consideration of link flexibility, Giovagnoni [42] presented a general approach for the dynamic analysis of flexible closed-chain manipulators using the principle of virtual work. Lee and Geng [43] developed a dynamic model of a flexible Stewart platform using Lagrange equations. Zhou *et al.* [44] established dynamic equations of flexible 3-PRS manipulator for vibration analysis using FEM. Based on AMM, Kang and Mills [41] presented a dynamic model of a 3-PRR planar parallel manipulator with flexible links.

All the above mentioned dynamic modeling is by explicitly writing out differential equations governing component dynamics which is rigorous mathematical calculations and time consuming that is usually required in such a process. SimMechanics is used to model link flexibility in this work. The benefit of this approach is that dynamic models of components are much easier to construct using the domain-specific building constructs that Sim-Mechanics provides than by explicitly writing out differential equations governing component dynamics. In addition, the Physical Modeling environment of SimMechanics tool makes the task much easier. Moreover, since SimMechanics allows construction of the mechanical models without the user having to derive the associated equations of motion (time-consuming process), the simulation environment can also be used by researchers from fields other than robotics, even with limited mechanical background.

The flexibility of the manipulator is mainly concentrated in the two intermediate links, as the rest of the structure is mechanically far stiffer. The vibration behavior is excited mainly by the driving forces from motors, the inertial forces, and the reaction due to the payload on the moving platform. It is assumed that the deformations of the intermediate links are small, relative to the length of the link, and hence can be modeled as linear elastic deformations.

4.2 Modeling of the Parallel Manipulator with Flexible Links

Mechanical models often play an important role in control systems simulation. SimMechanics makes it easy to create a representation of a physical mechanism within a Simulink model, but SimMechanics simulates only rigid-body dynamics: none of the parts represented by Body blocks are assumed to change their shape or mass distribution. In real applications, however, there is often a need to model flexible dynamics. The extensibility of MATLAB, Simulink, and SimMechanics makes it possible for users to create their own flexible-body models and libraries.

Different approach can be used for modeling flexible bodies in SimMechanics. The lumped-parameter method, best suited for modeling beam-like geometries, discretizes the flexible body into a series of constituent elements, each of which contains a spring-damper that models the flexibility.

To model the PRRRP parallel manipulator with flexible link in SimMechanics we have to substitute the model of the two rigid intermediate links by the model of two flexible intermediate links. Figure 4.1 shows the Sim-mechanics model of a PRRRP parallel manipulator with flexible links.

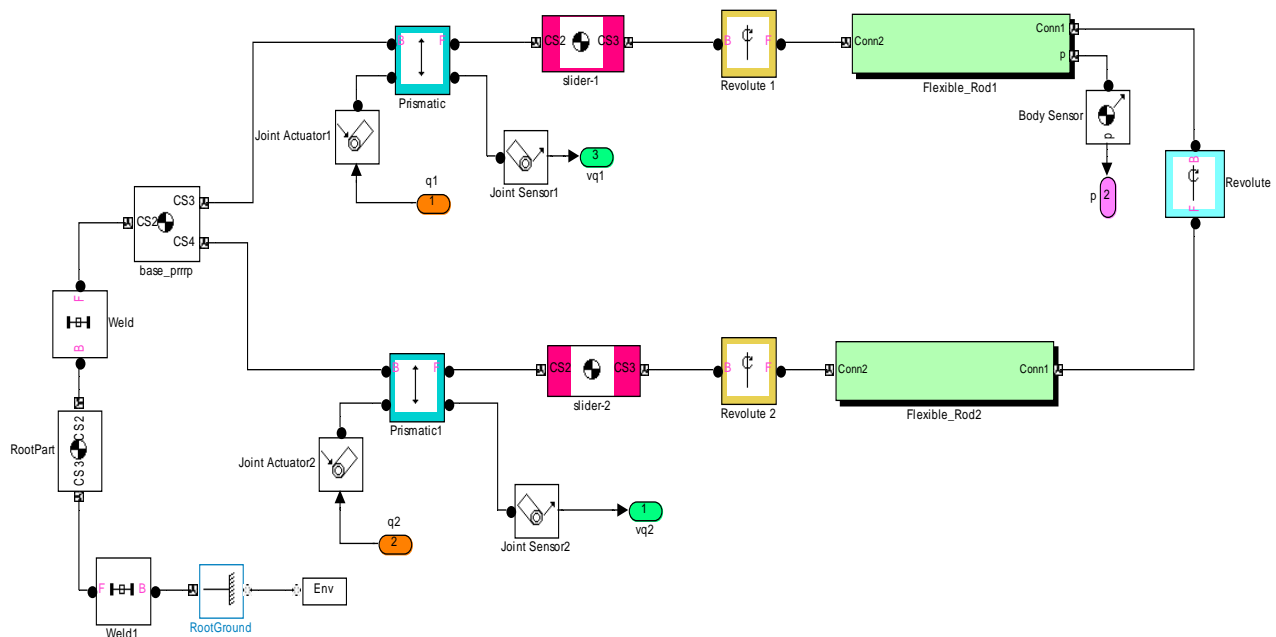


Figure 4.1 SimMechanics model of a PRRRP parallel Manipulator with flexible links

4.3 The Lumped-Parameter Method

For most engineering purposes, a real flexible body is a continuous medium. The lumped-parameter method approximates a flexible body as a set of rigid bodies coupled with springs and dampers and, in SimMechanics, can be implemented by a chain of alternating bodies and joints. The springs and dampers act either on the bodies or the joints. The spring stiffness coefficients and damping coefficients are functions of the material properties and the geometry of the flexible member under consideration. Lumped-parameter methods are best suited to models with linear geometries, such as beams, in which each fundamental flexible element is coupled to two others in a simple chain as shown in Figure 4.2.

The lumped-parameter method discretizes the beam of length L into n identical generalized beam elements (GBEs), each of length $l = L/n$ and mass $m = M/n$. The flexible links are approximated by a chain of rigid elements coupled with springs and dampers. Each GBE is a body-joint-body combination, with the joint primitives chosen to reflect the flexible degrees of freedom being modeled. The material properties determine the spring stiffness and damping that are applied to the joint. The intermediate link, shown in Figure 4.2, consists of adjacent GBEs joined together. The SimMechanics realization of a generalized beam element (GBE) is shown in Figure 4.4.

The entire link is realized as a chain of rigid bodies and joints with springs and dampers either acting on the bodies or the joints. Each beam element is described by a differential equation:

$$I_{zzGBE} \ddot{q}_{GBE} + d_{FD} \dot{q}_{GBE} + K_{FD} q_{GBE} = l_{GBE} f_{GBE} + \tau_{GBE} \quad (4.1)$$

in which d_{FD} denotes the damping coefficient, K_{FD} the spring constant, I_{zzGBE} the inertia of the beam element along the bending axis, l_{GBE} the beam element length, q_{GBE} the bending angle, f_{GBE} the acting bending force and τ_{GBE} the acting bending torque. Assuming that the beam element is only subject to pure bending, the effective spring constant is calculated from the link geometry, material properties and boundary conditions from the theory of flexible bodies:

$$K_{FD} = \frac{E I_{f,zz}}{l_{GBE}} \quad (4.1)$$

in which E denotes the Young modulus of the material and $I_{f,GBE}$ the area moment of inertia. The damping coefficients d_{FD} are identified from experiments on the individual links.

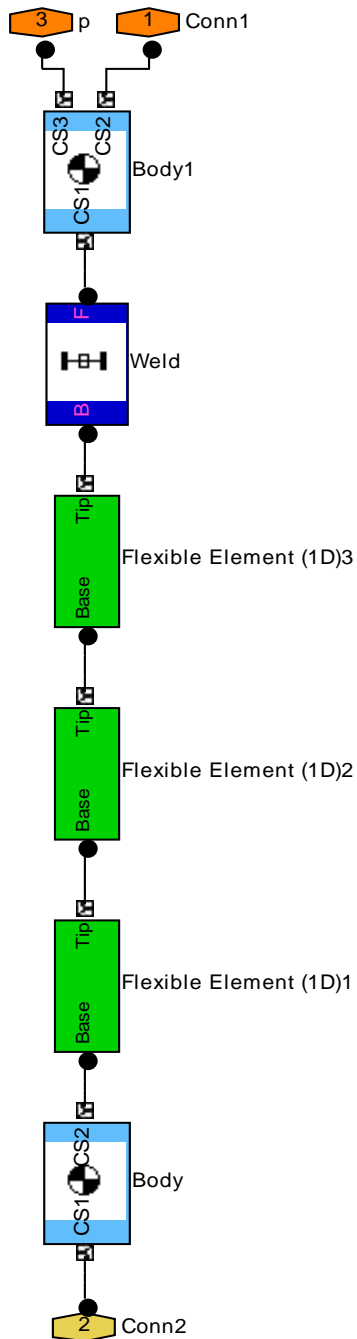


Figure 4.2 SimMechanics model of the Intimidate Flexible link

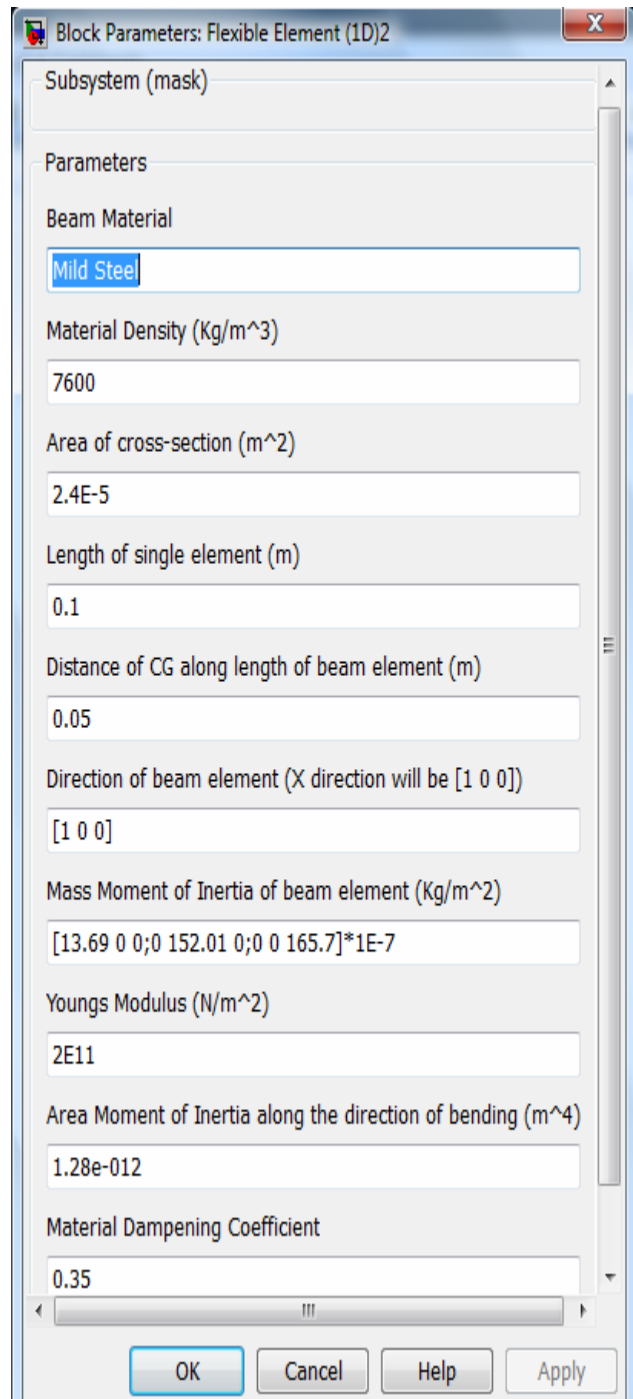


Figure 4.2 Block parameters of flexible element

In SimMechanics, the relative angle q_{GBE} and joint velocity \dot{q}_{GBE} are measured at the joint using a Joint Sensor block. Then K_{FD} the spring constant is multiplied by the angle, and a material damping coefficient d_{FD} is multiplied by the angular velocity. The two resulting moments are added and applied back to the joint using a joint actuator. Figure 4.4 shows the model spring-damper at each joint to encapsulate the flexibility parameters.

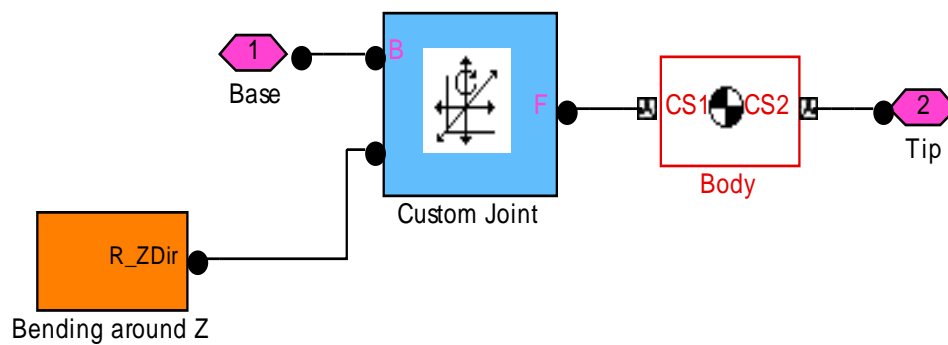


Figure 4.4 Model of Generalized beam element in SimMechanics

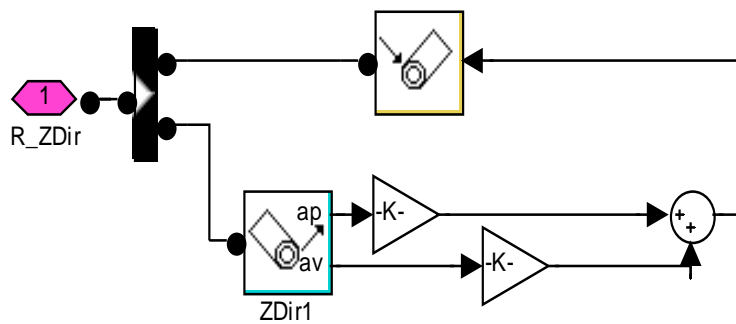


Figure 4.5 SimMechanics Model of Bending around Z direction

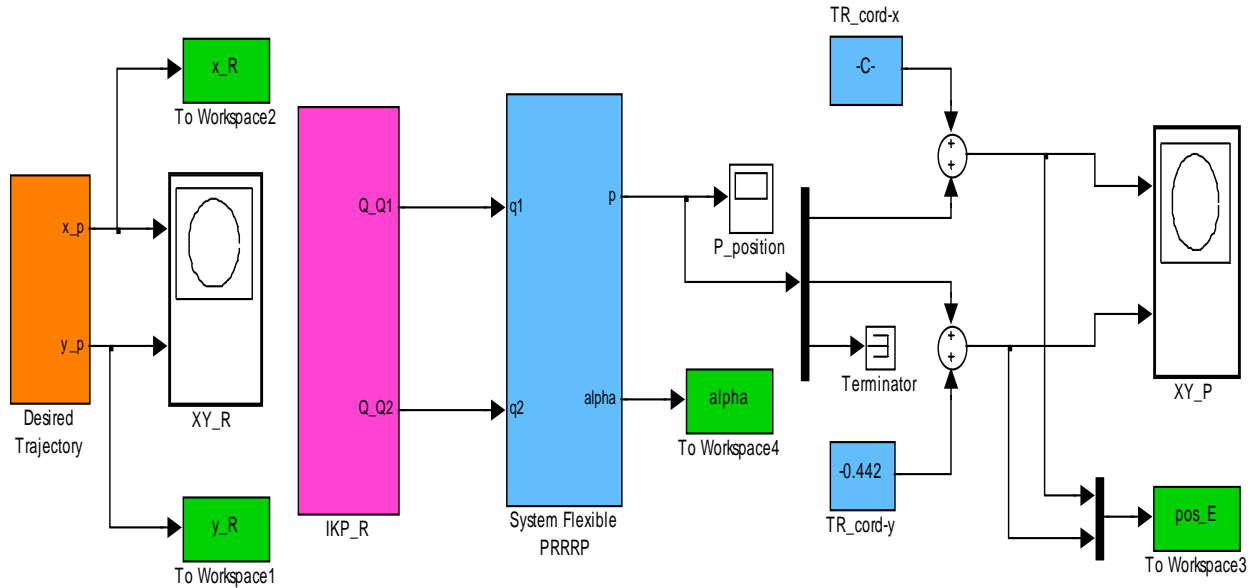


Figure 4.6 Simulink open loop model of the PRRRP parallel Robot with flexible links

4.4 Numerical Simulation and Analysis

Numerical simulations for the PRRRP parallel manipulator with two flexible intermediate links are presented. In these simulations, a circular motion, as shown in Figure 4.9, is assigned as a desired trajectory for the mass center point of the moving end effector with constant radius $R = 0.075m$ the equations for the circle are $X_p = X_0 + R * \cos(\varphi(t)) m$ and $Y_p = Y_0 + R * \sin(\varphi(t)) m$ Here, $\varphi(t)$ is defined as the change in angle of moving end effector along the described circular trajectory which varies from 0 to $8 * \pi$ using constant acceleration motion law and $X_0 = 0$ & $Y_0 = 0$. The intermediate links are modeled as Mild steel with elastic modulus and mass density $E = 2 \times 10^{11} N/m^2$, $\rho = 7.6 \times 10^3 Kg/m^3$, respectively. The two intermediate linkages have identical geometric parameters. The length of each link is 300 mm, cross-section width 30mm, and the height of cross-section 0.8mm. The Block parameters of flexible element Which is input to the model in SimMechanics is shown in Figure 4.5. The mass of the end effector is 0.2 kg: The rigid-body motion of slides and intermediate linkages is derived from the given motion of the moving end effector by solving inverse kinematics of the parallel manipulator which is used as input for the actuator connected with the slider in SimMechanics.

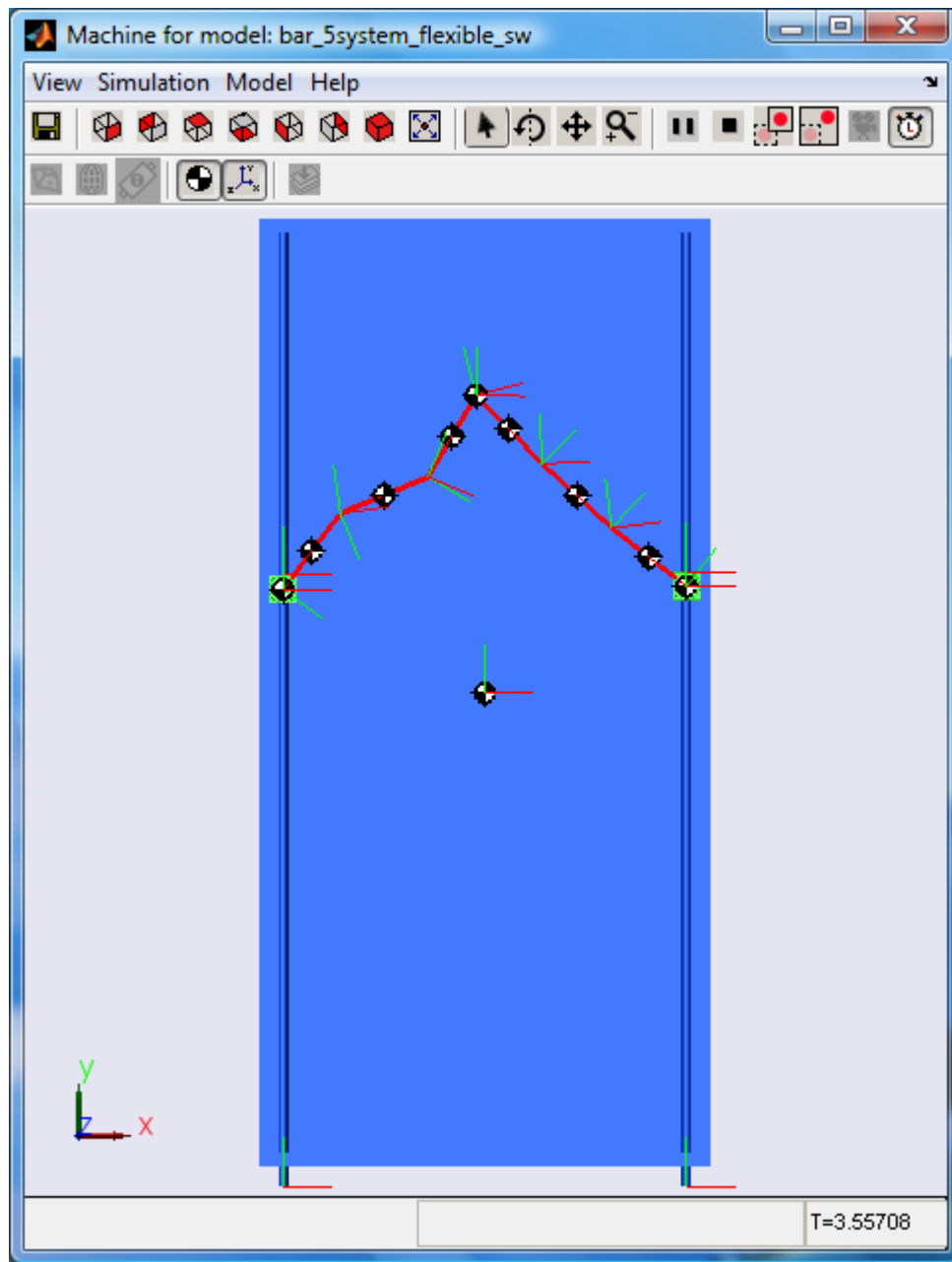


Figure 4.7 Physical model of five-bar linkage PRRRP parallel manipulator
With flexible links in SimMechanics

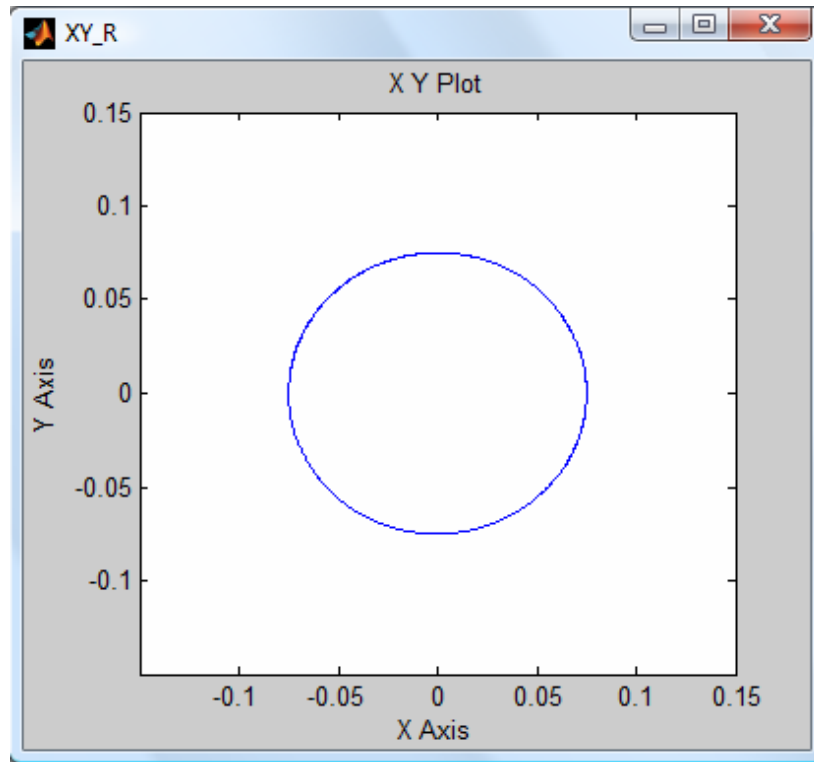


Figure 4.8 Reference End-effector path for the circular trajectory

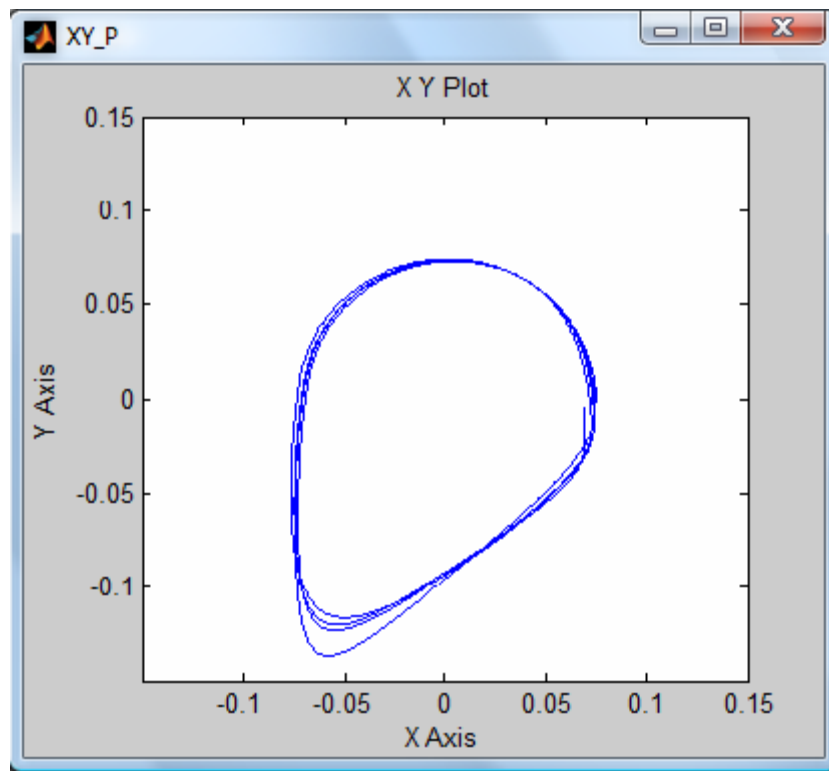


Figure 4.9 Actual 2D plot of trajectory for PRRRP parallel robot with intermediate flexible links from Simulink

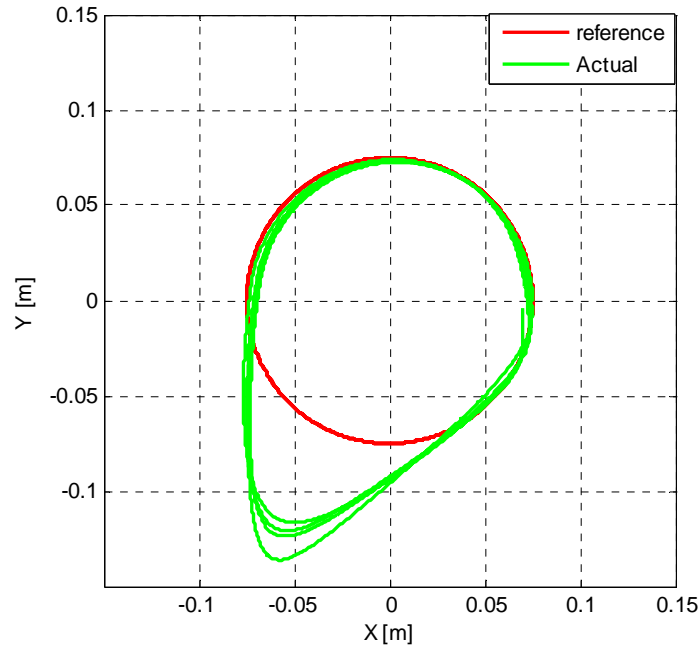


Figure 4.10 Reference and Actual End effector 2D plot of trajectory for PRRRP parallel robot with flexible links from MATLAB

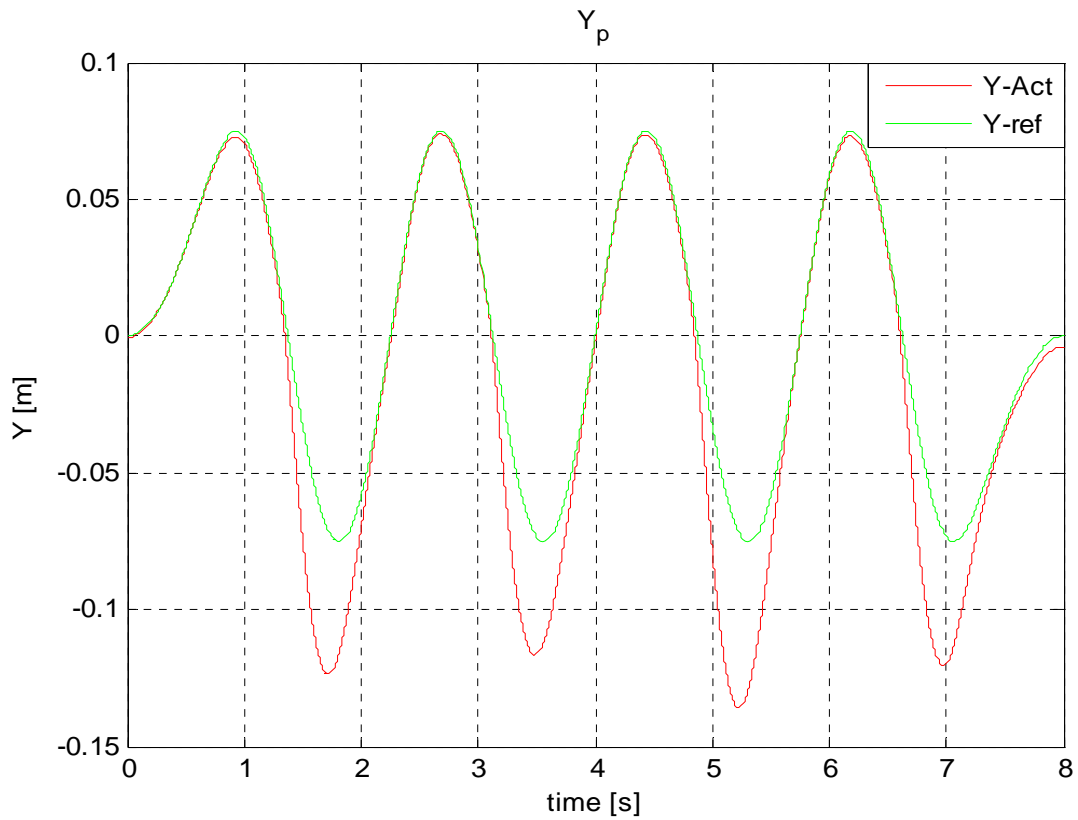


Figure 4.11 Reference and Actual End effector Y- trajectory

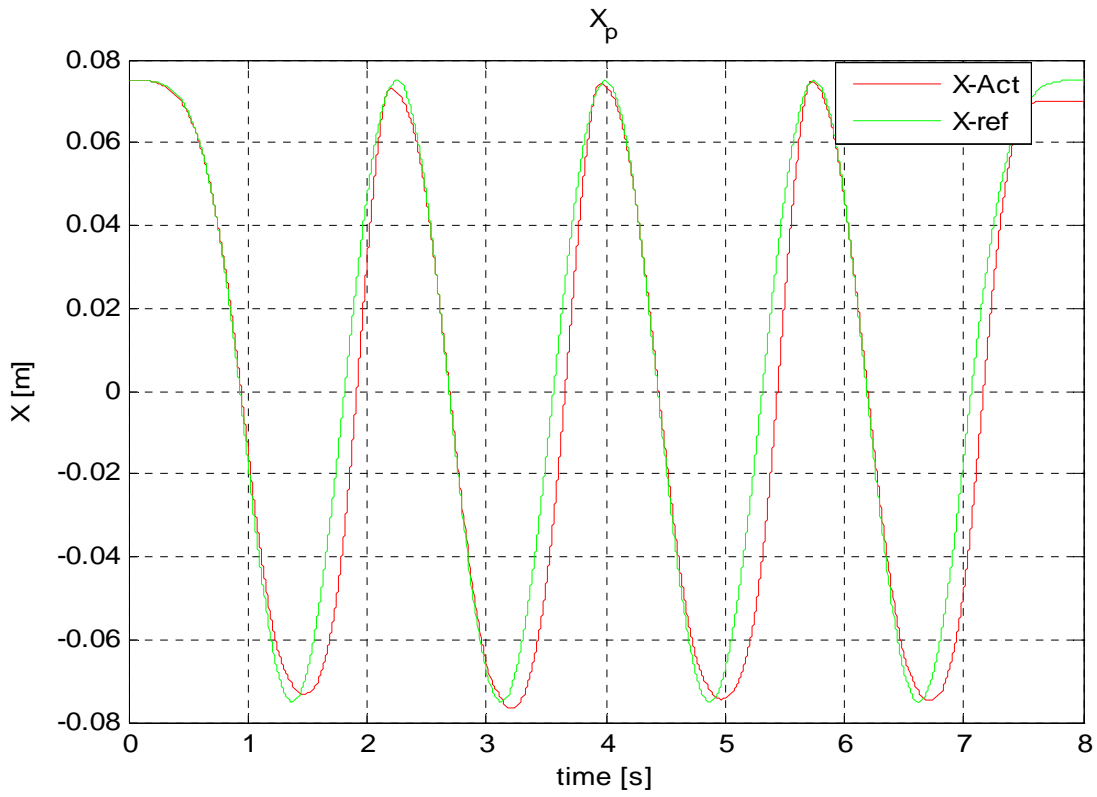


Figure 4.12 Reference and Actual End effector X- trajectory

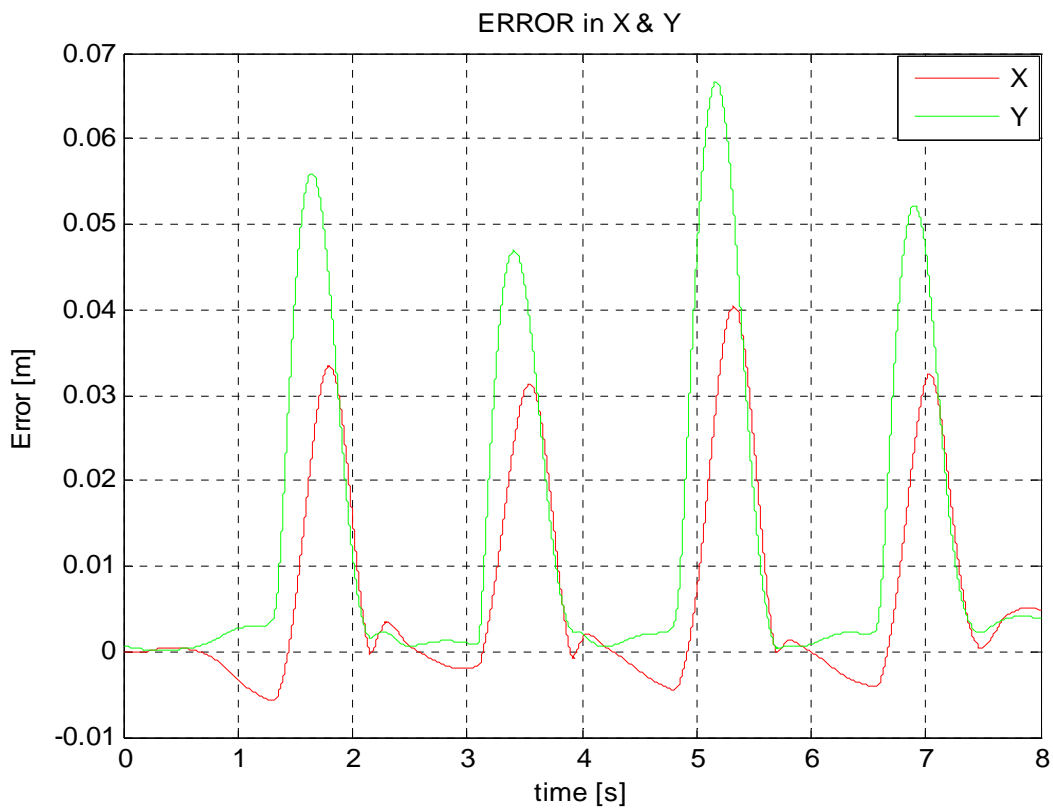


Figure 4.13 Error between Reference and Actual of End effector trajectory for X & Y

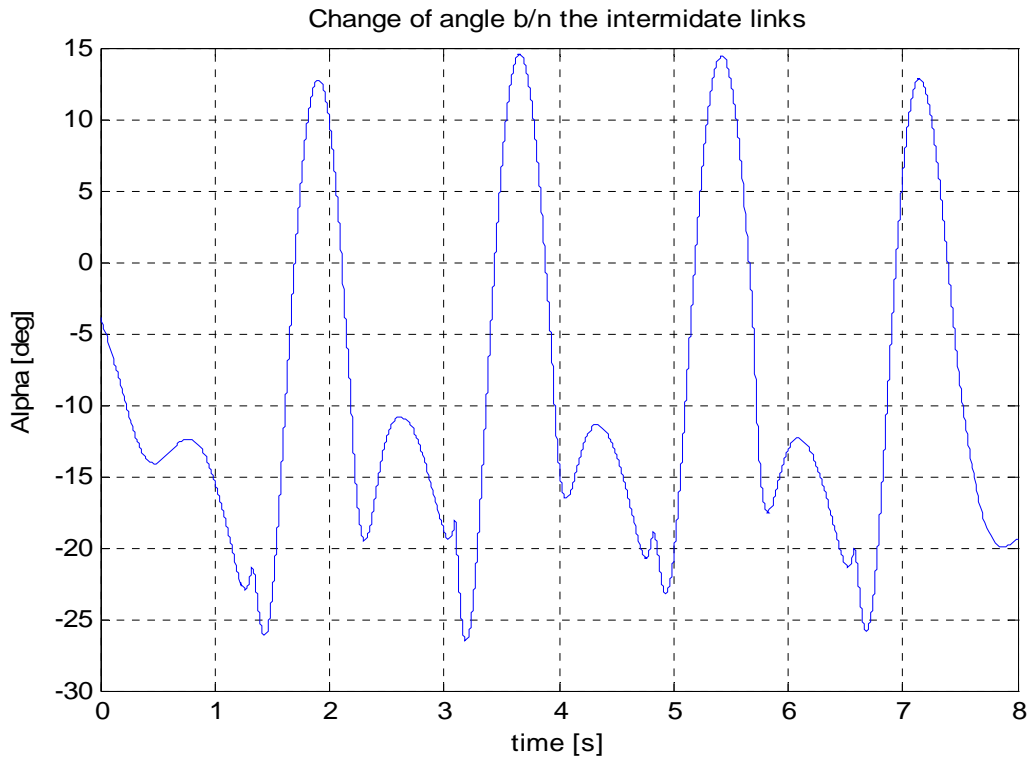


Figure 4.14 Change in angle between the two intermediate links

Figure 4.9 to 3.14 present's numerical simulations and results for Open-loop simulations without joint motion controls to investigate the effect of the PRRRP parallel manipulator with flexible links motion for the given circular trajectory. As it can be observed there is no control law over the motion fed into the actuator. The actual trajectory of the Manipulator is not exact with the reference circular trajectory like the rigid manipulator due to the deflection of the flexible links. The maximum error in x and y is 0.041 m & 0.066 m respectively. The Numerical simulation result also provides insight into the design of the joint motion controller.

4.5 Summery

An approach to dynamic modeling of flexible parallel robots is presented using SimMechanics. The modeling method is applicable for dynamic modeling of spatial parallel robots or complex mechanisms with flexible-body. The model of rigid PRRRP parallel manipulator developed in the pervious section was taken as the base and the two intermediate links are substituted by the model of flexible link using SimMechanics developed by lumped parameter approach.

The validity and accuracy of the model proposed in this section have been tested by Numerical Simulations and Analysis. Therefore the model is useful for dynamic analysis, motion errors suppression, elastic vibration control, optimal design and other subsequent investigations of flexible parallel robots. The flexibility of links has a significant impact on motion errors and elastic vibration of the flexible-links planar parallel robot.

Chapter 5

Design and Implementation of a Control System for a Flexible Planar Manipulator

5.1 Introduction

Concurrent to the work on dynamic modeling as presented in the previous chapters, increasing efforts have been invested to the investigation of vibration control of flexible manipulators and mechanisms. A variety of design or control strategies have been presented to suppress unwanted structural vibration of flexible links (Ghazavi *et al.* [75], Sung and Thompson [76], El-Dannah and Farghaly [77], Sisemore *et al.* [78], Zhang *et al.* [79], Cleghorn *et al.* [80], Ulbrich and Stein [81], Singhose *et al.* [82], Shan *et al.* [83], Siciliano and Book [84]). The control of a manipulator formed by flexible elements bears the study of the robot's structural flexibilities. The control objective is to move the manipulator within a specific trajectory but attenuating the vibrations due to the elasticity of some of its components. The PRRRP parallel robot with flexible arm is controlled by means of traditional PID schemes in position/velocity, considering their kinematics and dynamics: the reference trajectory of the end-effector is established a priori.

5.2 PID Control

The PID controller is a single-input, single-output (SISO) controller that produces a control signal that is a sum of three terms. The first term is proportional (P) to the positioning error, the second term is proportional to the integral (I) of the error, and the third is proportional to the derivative (D) of the error. The PID controller is the most common type of control algorithm used in engineering practice, with one survey in 1991 suggesting that 90% of controllers in Japan were of the PID type (Yamamoto and Hashimoto, 1991). PID controllers are used because the application of the PID controller is relatively straight forward, while providing reasonable results for a wide range of applications.

Since the PID controller is a SISO controller, and the manipulator has two degrees of freedom, two separate PID control loops are used to control the prototype manipulator, with each one controlling the position of slider. The desired positions of the slider are determined by mapping the desired moving end effector trajectory, as given in Cartesian space, into joint space by using the inverse kinematics relationships.

The key to obtaining the best possible performance from a PID controller is the selection of the gains that determine the influence of the three control terms on the control signal. Many gain tuning strategies have been developed. Generally, these tuning strategies are based upon the open-loop system step responses, open-loop frequency responses, optimization techniques, or other analytical approaches. Several of these gain tuning strategies, as well as a general discussion of PID controllers, are presented by Åström and Hägglund (1995).

The gains used to control the prototype manipulator were selected using an optimization technique. This was done on the PRRRP parallel manipulator with flexible links model developed by SimMechanics on Simulink that was used to simulate the open-loop on the previous section for a desired circular trajectory. This simulation was then used with an optimization routine to find a set of gains that minimized an objective function that considered the deflection of the flexible link which produce error in the path of end effector trajectory to follow the desired trajectory. The application of this approach and the results are presented in this section.

5.3 Position Feed back Control with PID Regulator

The control of the robot is implemented using a joint based control scheme. In such a scheme, the end-effector is positioned by finding the difference between the desired quantities and the actual ones expressed in the joint space.

The command of the robot is expressed in Cartesian coordinates of the end-effector. Using the inverse kinematic problem, these coordinates become displacements for the slider. These displacements further become the reference points for the control algorithm. The inputs of the algorithm were the desired position of the slider calculated from inverse kinematic plus the differences between the position of the slider computed (via inverse kinematic problem equations), of the desired and the actual measured from the sensor of end-effector adjusted by tuning the regulator. The control signal was applied on the two Actuators which were actuating on the robot structure separately.

MATLAB/Simulink was chosen as a tool that is a widely used for modeling, simulation and testing of dynamical systems. The model in Simulink is represented graphically by means of a number of interconnected blocks. The Simulink model closed loop feed back position control is shown below in Figure 5.1.

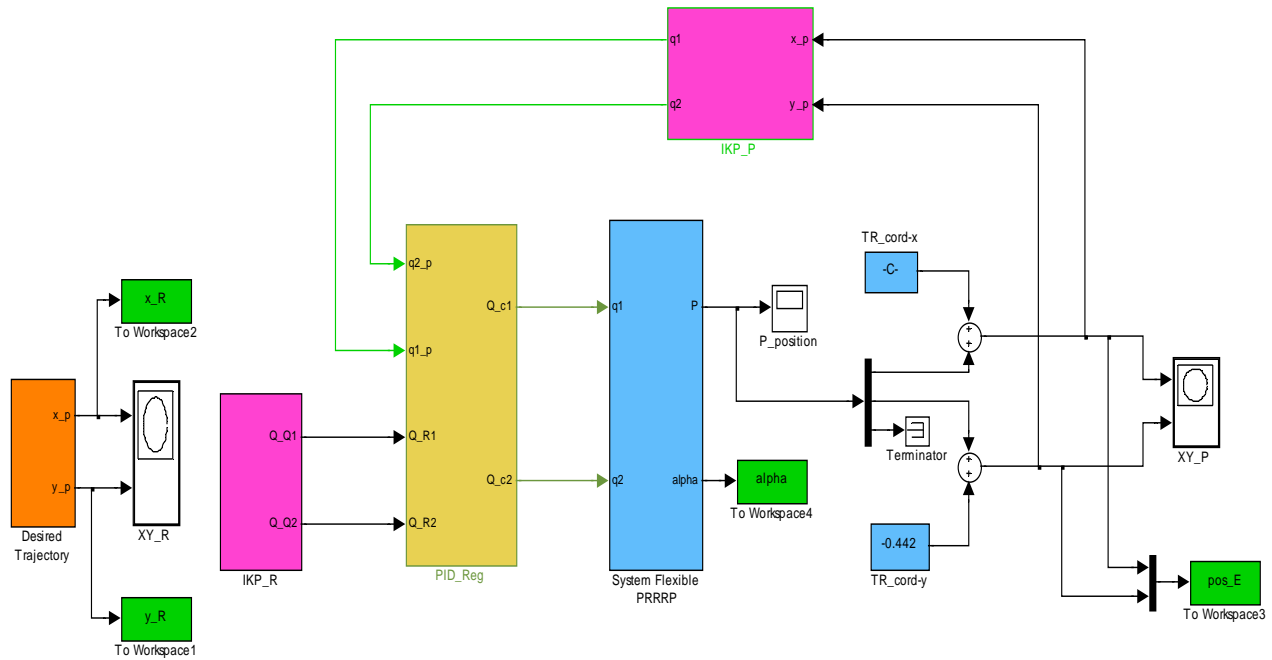


Figure 5.1 Closed-loop Simulink feed back position control Model for PRRRP Parallel Manipulator with flexible links

The two joint motions are controlled separately using a simple proportional – integral – Derivative (PID) feedback controller which is used for two linear actuators. Thus, the driving position applied to the two sliders are given as

$$q(t) = q_{di} + K_p e_i(t) + K_i \int_0^t e_i(t) dt + K_d \frac{de_i(t)}{dt} \quad i = 1,2 \quad (5.1)$$

Where, K_p is proportional gain; K_i is the integral gain coefficient; K_d is the differential coefficient; $e_i(t) = q_{di} - q_{pi}$ is the error and q_{di} and q_{pi} are the desired and feed back values of the i^{th} slider calculated from motion of the end-effector using inverse kinematics. The Simulink Model of PID position control (equation 5.1) is shown in Figure 5.2.

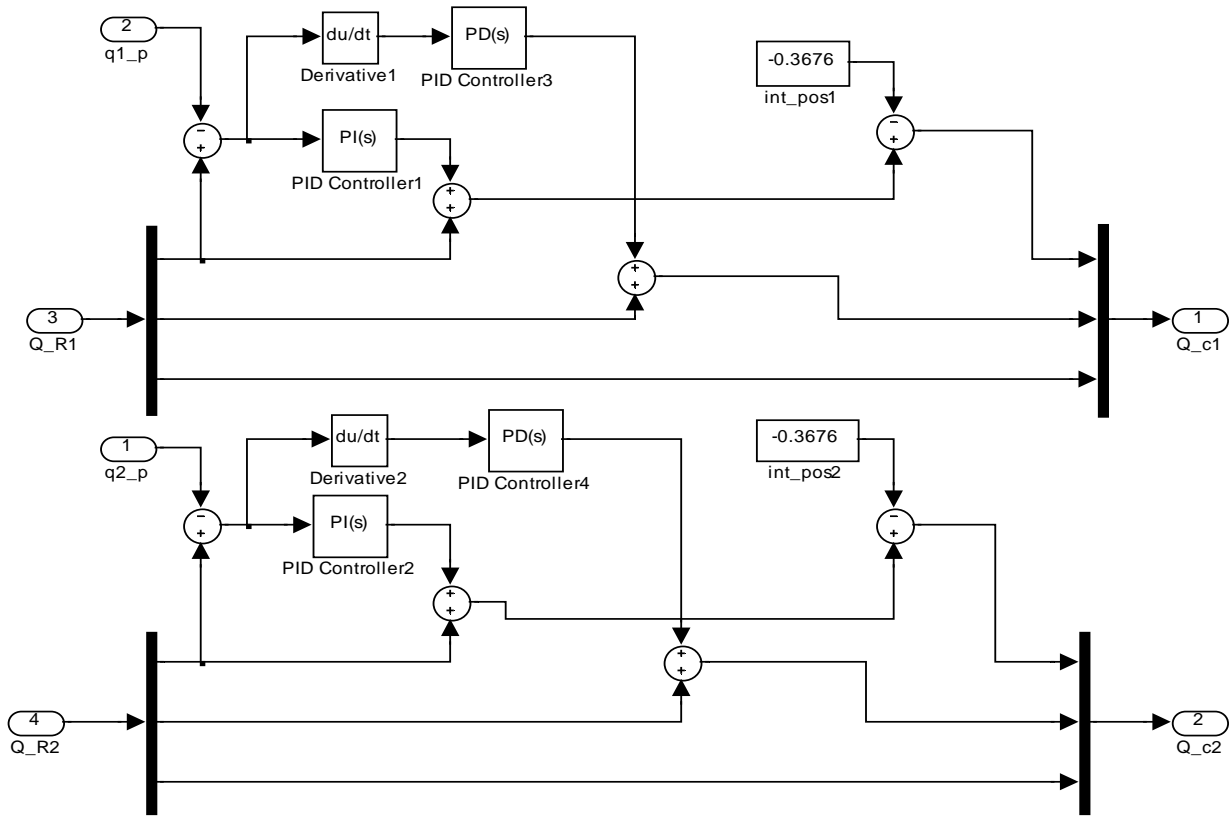


Figure 5.2 Simulink Model of PID Position control

5.4 Numerical Simulation and Results

Numerical simulations for the PRRRP parallel manipulator with two flexible intermediate links using PID position control are presented. In these simulations, a circular motion, as shown in Figure 4.9, is assigned as a desired trajectory for the mass center point of the moving end effector with constant radius $R = 0.075m$ the equations for the circle are $X_p = X_0 + R * \cos(\varphi(t))m$ and $Y_p = Y_0 + R * \sin(\varphi(t))m$ Here, $\varphi(t)$ is defined as the change in angle of moving end effector along the described circular trajectory which varies from 0 to $8 * \pi$ using constant acceleration motion law and $X_0 = 0$ & $Y_0 = 0$ is the center of the circular trajectory. The intermediate links are modeled as Mild steel with elastic modulus and mass density $E = 2 \times 10^{11} N/m^2$, $\rho = 7.6 \times 10^3 Kg/m^3$, respectively. The two intermediate linkages have identical geometric parameters. The length of each link is $300mm$, cross-section width $30mm$, and the height of cross-section $0.8mm$. The Block parameter of flexible element which is the input to the model in

SimMechanics is shown in Figure 4.5. The mass of the end effector is 0.2 kg: The rigid-body motion of slides and intermediate linkages is the output from controller which is used as input for the actuator connected with the slider in SimMechanics.

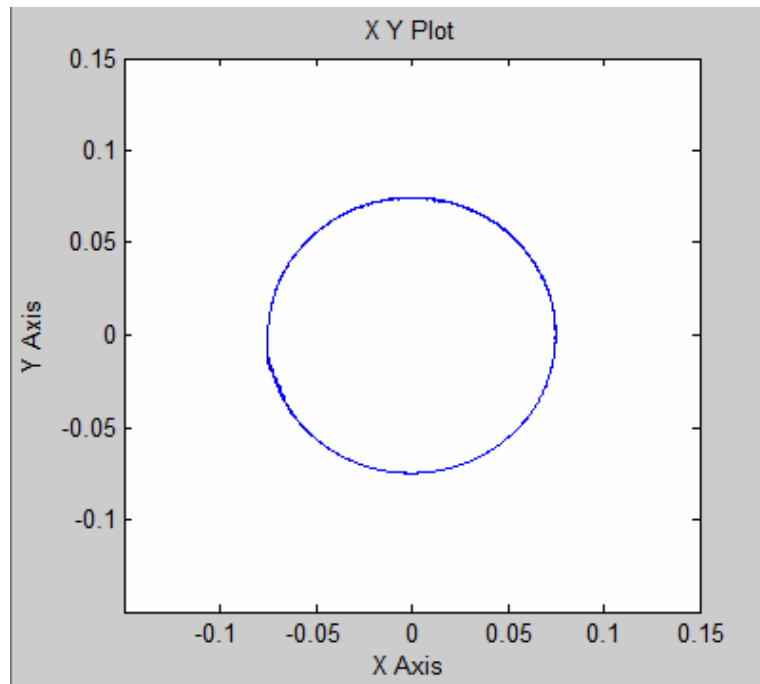


Figure 5.3 Actual End effector 2D plot trajectory using optimum PID regulator from Sumlink

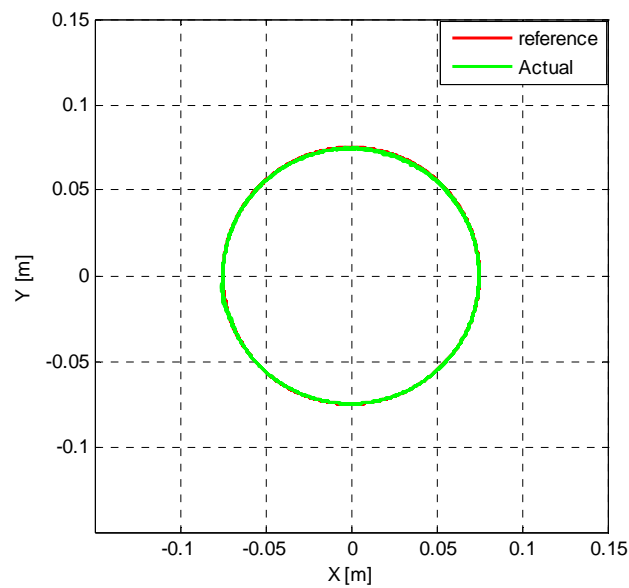


Figure 5.4 Reference and Actual End effector 2D plot of trajectory for PRRRP parallel robot with flexible links from MATLAB using optimum PID regulator

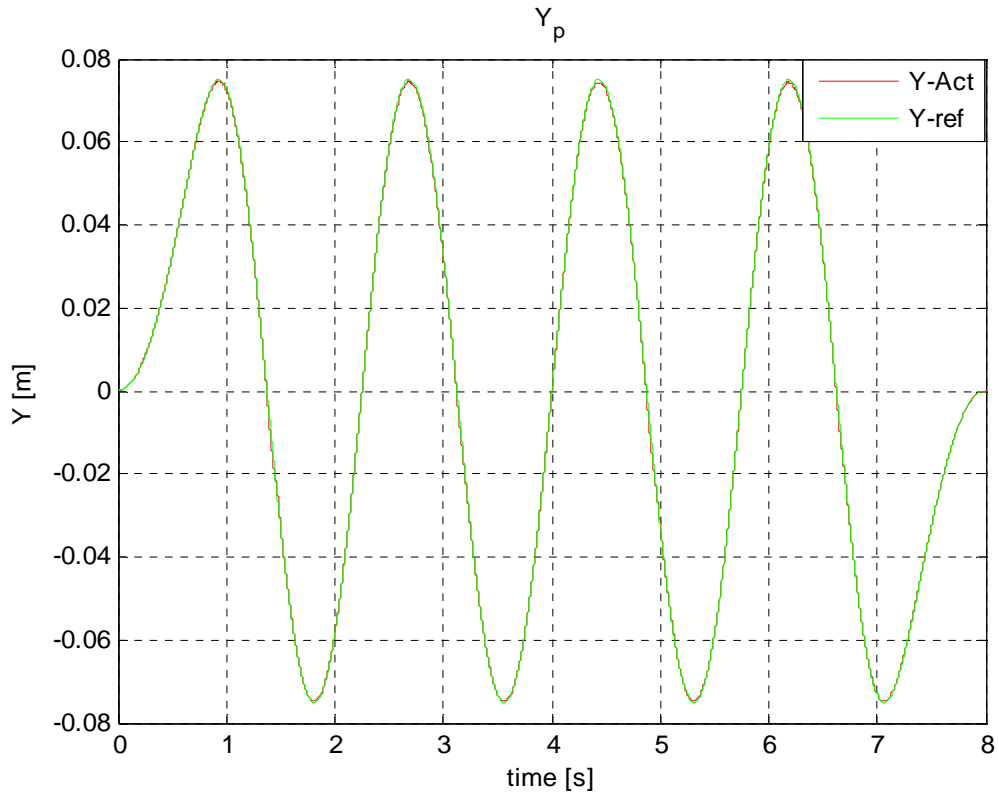


Figure 5.5 Reference and Actual End effector Y- trajectory using optimum PID regulator

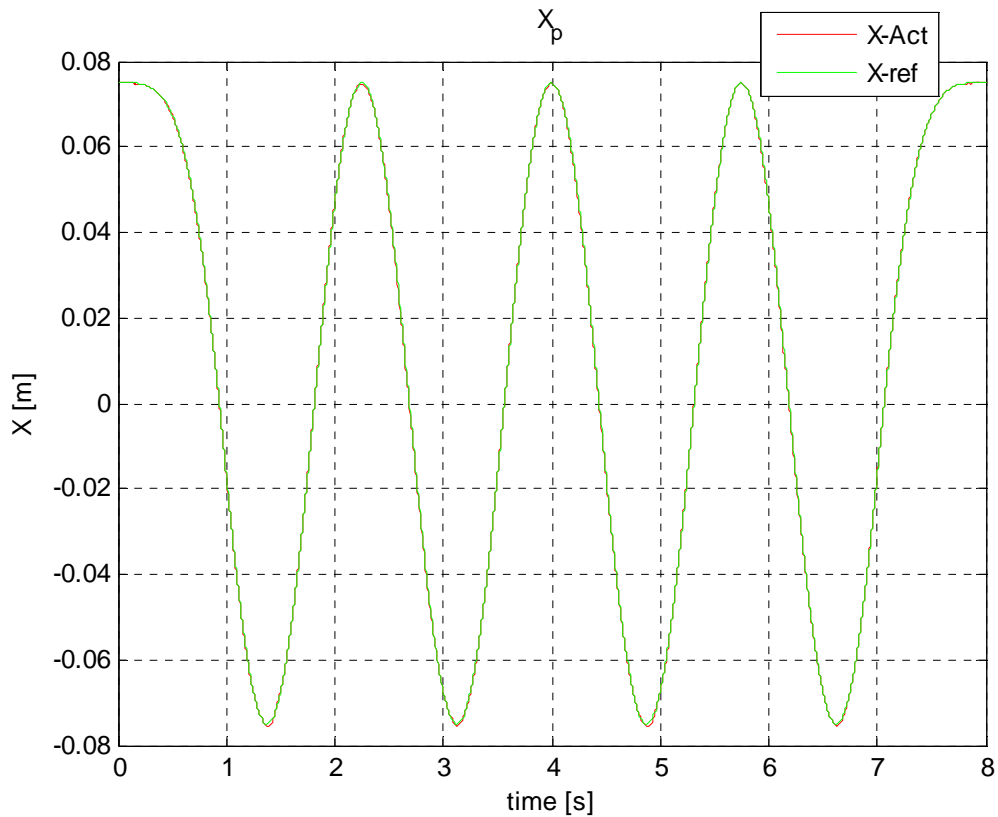


Figure 5.6 Reference and Actual End effector X- trajectory using optimum PID regulator

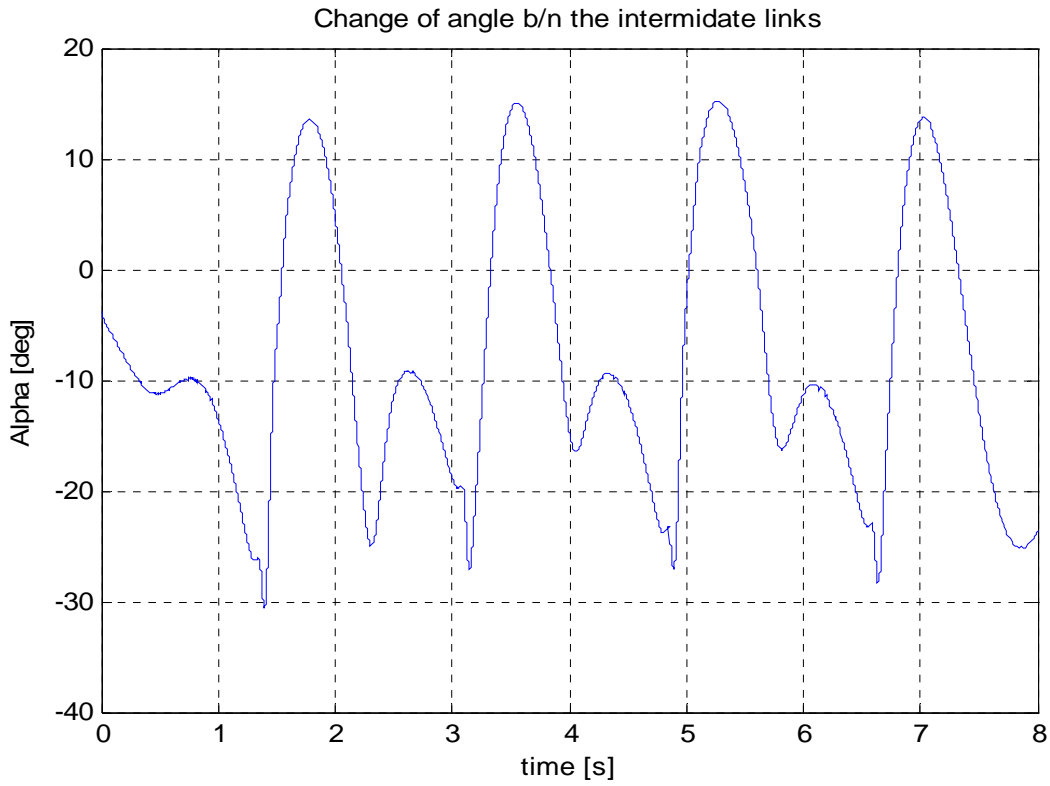


Figure 5.7 Change in angle between the two intermediate links using optimum PID regulator

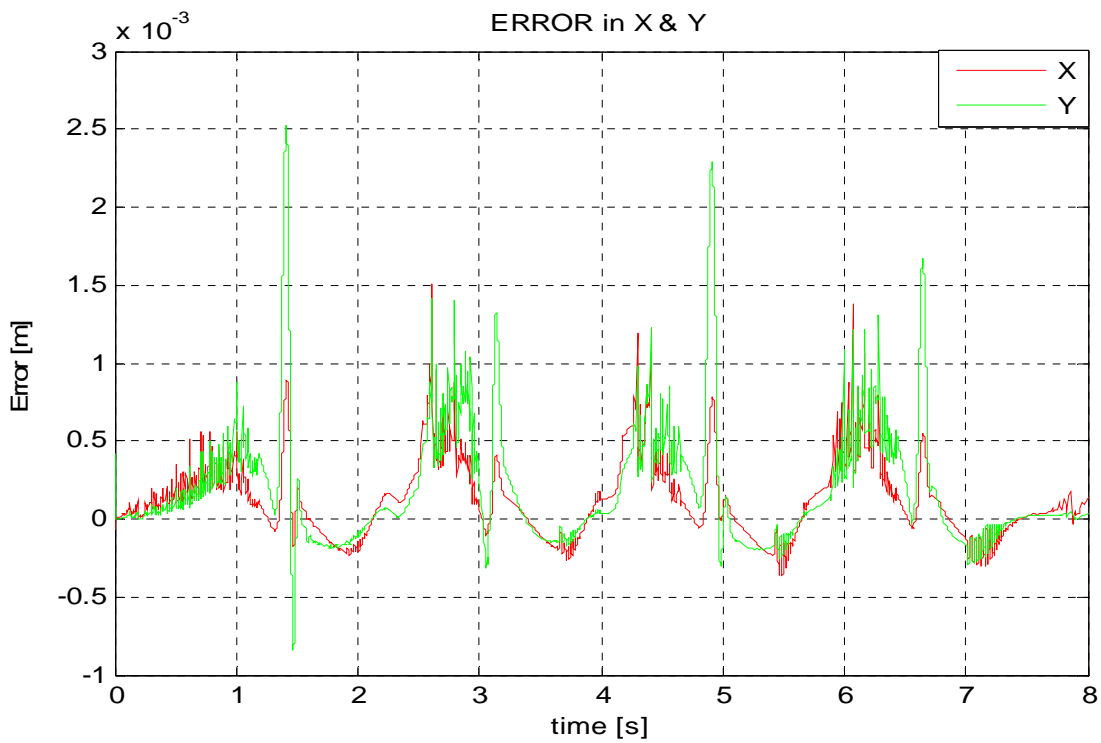


Figure 5.8 Error between Reference and Actual End effector trajectory for X & Y using optimum PID regulator

Figure 5.3 – 5.8 shows the trajectory of the end-Effector with the proposed feed back position control using PID regulator with the best parameters (gains) for minimizing the error $K_p = 0.6$, & $K_i = 500$ and $K_p = 0.6$ & $K_d = -0.03$, which shows that the vibrations of the flexible intermediate links are suppressed due to the PID joint motion controllers. Notice the control gains of the PID joint motion controllers are adjusted using trial and error based on the requirement of the smallest trajectory error and best vibration suppression. The detailed tuning for selecting the best PID gains is given in Appendix B. The maximum error in y is reduced from 0.066 m to 0.0025m and in x from 0.041 to 0.0015 m. Closed-loop simulation results also show that it is feasible to suppress the unwanted vibration through the design of joint motion controllers.

5.5 Summery

This section focuses on the motion control of the two-degree-of-freedom (2-DOF) a PRRRP planar parallel manipulator with flexible links. On the basis of the performance analysis of the kinematic control system, a parameter-tuning method is proposed for regulating the control parameters. To improve the trajectory error, the proportional-integral-derivative controls are introduced to the position-loop controller. The position feed back control is used to control the parallel manipulator. The simulations, that the end effector moves along circular trajectory, show that the position feedback control with PID regulator can reduce the tracking error produced due to the deflection of the flexible link of the manipulator.

Chapter 6

Conclusion and Further Work

6.1 Thesis Summary

This thesis is mainly concerned with dynamic modeling and control of a structural deflection of complex dynamic system, a PRRRP planar parallel manipulator with two flexible links moving at high speed. The methodologies of dynamic modeling of a complex multibody dynamic system have been presented using SimMechanics with the lumped parameter method. The significant dynamic characteristics of the flexible parallel manipulator have been investigated. The position feed control with PID regulator is proposed to counteract structural deflection of the linkage, resulting in precise manipulations of the end effector.

The Proposed control strategy has been numerically implemented in Matlab/ Simulink on the Model of a PRRRP parallel manipulator with flexible links using SimMechanics. The developed methods and strategies of dynamic modeling and Position Control can be extended to other types of parallel manipulators or other multibody dynamic systems with flexible components. In the following, the work done in this thesis is summarized, and some significant contributions of this thesis work are restated:

- To achieve high speed and high acceleration, light weight links are used to reduce inertia forces. However, light weight links are easy to deform and vibrate when the manipulator is moving at high speed. Therefore, link flexibility should be considered in the dynamic model. With the assumption of small and linear elastic deformation, a methodology has been presented based on the lumped parameter method using SimMechanics for the generation of dynamic Model for a PRRRP parallel manipulator with two flexible intermediate links.
- Using Numerical Simulation in Matlab/ Simulink the dynamic behaviour of flexible manipulator has been investigated. The Dynamic characteristics of the flexible parallel manipulator are examined for Circular Trajectory using Motion Low, which provides valuable insights into the design and control of parallel manipulators with flexible intermediate links.

For example, joint motions of parallel manipulators can be optimized and controlled so that the structural deflection of flexible links will counteract by increase the stiffness.

- The numerical simulations have a great contribution in understanding the behavior of the flexible manipulator and to choose the right control method to counteract the unwanted structural deflection the flexible Manipulator.
- The position feedback control is used to control the parallel manipulator. The simulations, that the end effector moves along circular trajectory, show that the position feedback control with PID regulator can reduce the tracking error produced due to the deflection of the flexible link of the manipulator

6.2 Future Work

Dynamical modeling and control of parallel manipulators with multiple flexible links is a challenging task. Many interesting and unsolved problems deserve researchers to invest significant efforts in the future to address, and hence promote the practical application of lightweight parallel manipulators. Reviewing the work done in this thesis, the following research efforts for future work could include:

- In the Modeling and Control developed in this thesis, the dynamics of motors and ball screw mechanisms are neglected. To establish more accurate dynamics Model of the parallel manipulator with link flexibility, motor dynamics and dynamics from ball screw mechanisms should be included, and the effect of Control system dynamics with other component dynamics should be considered and examined as well.
- For the unwanted structural deflection, this thesis only involves the position feed back control with PID regulator. The other strategies could be used for suppression the unwanted vibration, for example, controlling the joint motors with input shapes, vibration suppression with PZT actuators and sensors etc.

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Appendix A: Matlab code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%           Design and control of a five bar linkage           %
%           parallel Manipulator with flexible arms           %
%           Master thesis: by Woldu Zina Gebrehiwot           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clc
clear all
close all

%% INITIALIZATION
global x par

%% INPUT PARAMETERS FOR MANIPULATOR
L = 0.3;           % (m) Length of flexible arm
L2 = 0.425;       % (m) Length of b/n the slider
R = 0.425/2;     % (m)
%% DESIRED TRAJECTORY DATA
rc = 0.075;      % (m) circle radice
xc = 0;   yc = 0; % (m) center of circle [-0.238124 0.20544 -0.211767]
% [-0.096548 -0.193989 -0.183123]

%% INITIAL CONDITIONS
TH0 = 0;         % (m) initial angle
THf = 8*pi;      % (m) final angle
DTH = THf - TH0;

%% SIMULATION PARAMETERS

dt = 0.001;      % Simulation time step
T = 8;           % [s] Period of simulation
t=0:0.001:T;     % Time vector

%% Trajectory information END-EFFECTOR

x=t./T;
Par=[1/4,1/4];

[D,V,A] = Constant_Acceleration_ND(x,par);

TH = D.*DTH;
THD = V.*DTH/T;
THDD = A.*DTH/T^2;
that=[t',TH'];

figure('NumberTitle','off','name','MOTION LAW (constant acceleration)')
subplot(3,1,1),plot(t,TH,'b','linewidth',2),ylabel('TH
[rad]'),title('thata'),grid
subplot(3,1,2),plot(t,THD,'b','linewidth',2),ylabel('THD
[rad/s]'),title('dthata'),grid
subplot(3,1,3),plot(t,THDD,'g','linewidth',2),ylabel('THDD
[rad/s^2]'),title('ddthata'),grid
```



```

% Desired position
xp = xc + rc*cos(TH);
yp = yc + rc*sin(TH);

x_p=[t',xp'];
y_p=[t',yp'];

% Desired velocity
Dxp = -rc*THD.*sin(TH);

Dyp = rc*THD.*cos(TH);

% Desired acceleration
DDxp = -rc*THD.^2.*cos(TH)-rc*THDD.*sin(TH);

DDyp = -rc*THD.^2.*sin(TH)+rc*THDD.*cos(TH);

figure('NumberTitle','off','name','DESIRED END-EFFECTOR MOTION LAW')
subplot(3,2,1),plot(t,xp,'b','linewidth',2),ylabel('x_p
[rad]'),title('x_p'),grid
subplot(3,2,2),plot(t,yp,'b','linewidth',2),ylabel('y_p
[rad]'),title('y_p'),grid
subplot(3,2,3),plot(t,Dxp,'g','linewidth',2),ylabel('Dx_p [rad/s]'),grid
subplot(3,2,4),plot(t,Dyp,'g','linewidth',2),ylabel('Dy_p [rad/s]'),grid
subplot(3,2,5),plot(t,DDxp,'r','linewidth',2),ylabel('DDx_p
[rad/s^2]'),xlabel('t'),grid
subplot(3,2,6),plot(t,DDyp,'r','linewidth',2),ylabel('DDy_p
[rad/s^2]'),xlabel('t'),grid

figure('NumberTitle','off','name','DESIRED END-EFFECTOR TRAJECTORY')
plot(xp,yp,'k','linewidth',2),xlabel('X [m]'),ylabel('Y [m]'),grid
axis equal; axis([-0.3 0.3 -0.3 0.3]); grid;

%% INVERSE KINEMATICS Analysis (SLIDER MOTION)

q10=yc-sqrt(L^2+(xc-R)^2);
q20=yc-sqrt(L^2+(xc+R)^2);
Q0= [q10; q20];
for i=1:length(t)

    TH = D.*DTH;
    THD = V.*DTH/T;
    THDD = A.*DTH/T^2;
    % Desired position
    xp(i) = xc + rc*cos(TH(i));
    yp(i) = yc + rc*sin(TH(i));

% x_p=[t',xp'];
% y_p=[t',yp'];

```

```

% Desired velocity
Dxp(i) = -rc*THD(i)*sin(TH(i));

Dyp(i) = rc*THD(i)*cos(TH(i));

% Desired acceleration
DDxp(i) = -rc*THD(i)^2*cos(TH(i))-rc*THDD(i)*sin(TH(i));

DDyp(i) = -rc*THD(i)^2*sin(TH(i))+rc*THDD(i)*cos(TH(i));

    X(:,i)=[xp(i);yp(i)];
    dX(:,i)=[Dxp(i);Dyp(i)];
    ddX(:,i)=[DDxp(i);DDyp(i)];

    q1(i)=X(2,i)-sqrt(L^2-(X(1,i)-R)^2);
    q2(i)=X(2,i)-sqrt(L^2-(X(1,i)+R)^2);

    Q(:,i) = [q1(i);q2(i)];

    J = jacob(Q(:,i),X(:,i));
    dQ(:,i) = inv(J)*dX(:,i);

    AA = accel(X(:,i),dX(:,i),Q(:,i),dQ(:,i));
    ddQ(:,i) = inv(J)*ddX(:,i)+ AA;

end

figure('NumberTitle','off','name','SLIDER MOTION')
subplot(3,2,1),plot(t,Q(1,:), 'b', 'linewidth', 2), ylabel('q_1
[rad]'), title('Slider 1'), grid
subplot(3,2,2),plot(t,Q(2,:), 'b', 'linewidth', 2), ylabel('q_2
[rad]'), title('Slider 2'), grid
subplot(3,2,3),plot(t,dQ(1,:), 'g', 'linewidth', 2), ylabel('Dq_1 [rad/s]'), grid
subplot(3,2,4),plot(t,dQ(2,:), 'g', 'linewidth', 2), ylabel('Dq_2 [rad/s]'), grid
subplot(3,2,5),plot(t,ddQ(1,:), 'r', 'linewidth', 2), ylabel('DDq_1
[rad/s^2]'), xlabel('t'), grid
subplot(3,2,6),plot(t,ddQ(2,:), 'r', 'linewidth', 2), ylabel('DDq_2
[rad/s^2]'), xlabel('t'), grid

Q_t=Q';
dQ_t=dQ';
ddQ_t=ddQ';

q_1=Q_t(:,1);
q_2=Q_t(:,2);
q_1d=dQ_t(:,1);
q_2d=dQ_t(:,2);
q_1dd=ddQ_t(:,1);
q_2dd=ddQ_t(:,2);

Q_Q1 = [t' q_1 q_1d q_1dd];
Q_Q2 = [t' q_2 q_2d q_2dd];

```

% Analysis with Simulink

```
sim('bar_5system_flexible_sw');
t_p= pos_E.time;
End_px= pos_E.signals.values(:,1);
End_py= pos_E.signals.values(:,2);

figure('NumberTitle','off','name','REFERENCE & ACTUAL END_EFFECTOR TRAJECTORY')
plot(x_R.signals.values,y_R.signals.values,'r','linewidth',2),xlabel('X
[m]'),ylabel('Y [m]'),grid
hold on
plot(End_px,End_py,'g','linewidth',2),xlabel('X [m]'),ylabel('Y [m]'),grid
legend('reference','Actual');
axis equal; axis([-0.15 0.15 -0.15 0.15]); grid;

figure(6)
plot(t_p,End_px,'r');
hold on
plot(x_R.time,x_R.signals.values,'g'),xlabel('time [s]'),ylabel('X [m]'),grid;
title('X_p');
legend('X-Act','X-ref');

figure(7)
plot(t_p,End_py,'r');
hold on
plot(y_R.time,y_R.signals.values,'g'),xlabel('time [s]'),ylabel('Y [m]'),grid;
title('Y_p');
legend('Y-Act','Y-ref');

figure(8)
plot(x_R.time,x_R.signals.values-End_px,'r');
hold on
plot(y_R.time,y_R.signals.values-End_py,'g'),xlabel('time [s]'),ylabel('Error
[m]'),grid;
title('ERROR in X & Y');
legend('X','Y');

figure(9)
plot(alpha.time,alpha.signals.values),xlabel('time [s]'),ylabel('Alpha
[deg]'),grid;
title('Change of angle b/n the intermidate links');
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Work space analysis of parallel planar manipulator %
%                                                    %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

close all
clear all
clc

```

```

%Geometric parameters
L = 0.300; %mm
R = 0.425/2; %mm
d = 1.00; %mm

```

```

q = 0:0.05:1.00;

for i=1:length(q)
    q1=q(i);
    for j=1:length(q)
        q2=q(j);
        a = (q2-q1)/(2*R);
        b = (q1^2-q2^2)/(4*R);
        e = a^2+1;
        f = 2*a*(b-R)-2*q1;
        g = (b-R)^2 + q1^2-L^2;
        y = -(f - sqrt(f^2-4*e*g))/(2*e);
        x=(a*y)+b;
    plot(x,y, '*', 'Markersize',10), grid
    hold on
    xlabel('X [mm]'),ylabel('Y [mm]'),grid
    axis([-0.25 0.25 0 1.5]); grid;

    end

end

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%   Non-dimensional General Constant Acceleration Motion Law   %
%                                                                 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```
function [D,V,A] = Constant_Acceleration_ND(x,par)
```

```

% x = t/tm non-dimensional time
% D = non-dimensional displacement
% V = non-dimensional velocity
% A = non-dimensional acceleration
% par(1) = xa+
% par(2) = xa-

```

```

par=[1/8,1/8];
xap = par(1);
xam = par(2);

```

```

cap = 1/(xap*(1-(xap+xam)/2));
cam = 1/(xam*(1-(xap+xam)/2));

```

```

for k=1:length(x)
if x(k)>=0 & x(k)<=xap
    A(k)=cap;
    V(k)=cap*x(k);
    D(k)=.5*cap*x(k)^2;
end
if x(k)>xap & x(k)<(1-xam)
    A(k)=0;
    V(k)=cap*xap;
    D(k)=cap*xap*(x(k)-xap/2);
end
if x(k)>=(1-xam) & x(k)<=1
    A(k)=-cam;
    V(k)=cap*xap-cam*(x(k)-1+xam);
    D(k)=cap*xap*(x(k)-xap/2)-cam/2*(x(k)-1+xam)^2;
end
end
end

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               Creating Important Matrices                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% the Jacobian matrices

function [J] = jacob(X,Q)

R = 0.425/2;

J= [-(X(1)-R)/(Q(1)-X(2)) 1;
    -(X(1)+R)/(Q(2)-X(2)) 1];

% a Matrices function for Acceleration

function [AA] = accel(Q,dQ,X,dX)

AA =[-(dX(1)^2 + dX(2)^2 + dQ(1)^2 -2*dX(2)*dQ(1))/(Q(1)-X(2));
     -(dX(1)^2 + dX(2)^2 + dQ(2)^2 -2*dX(2)*dQ(2))/(Q(2)-X(2))];

```

Appendix B: Simulation Results with control and with out control
For a PRRRP parallel manipulator with flexible links

With out Control

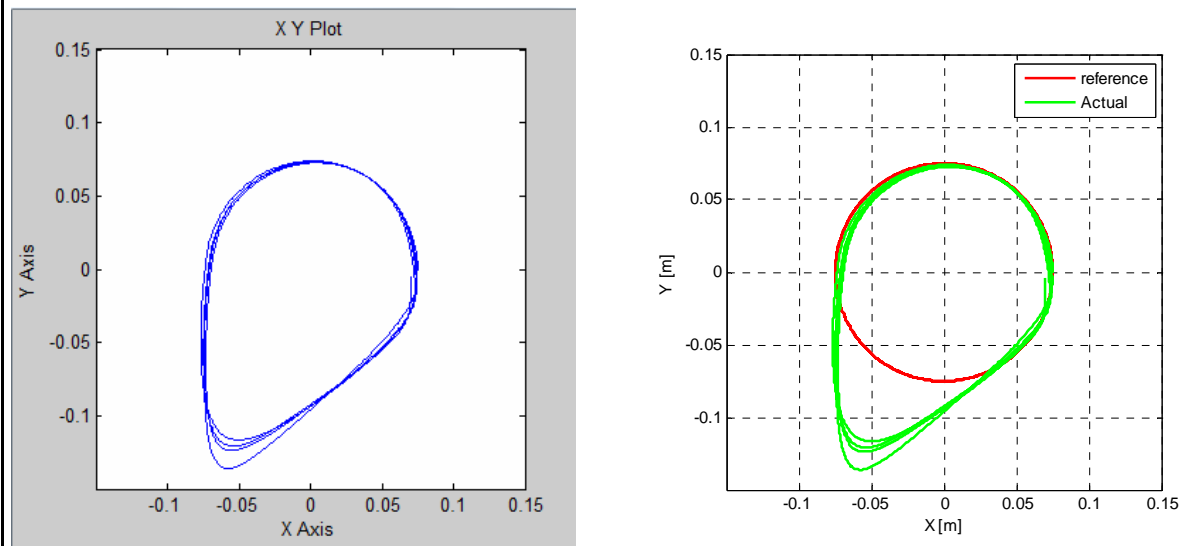


Figure 1 2D Plot from Simulink & matlab with out Control

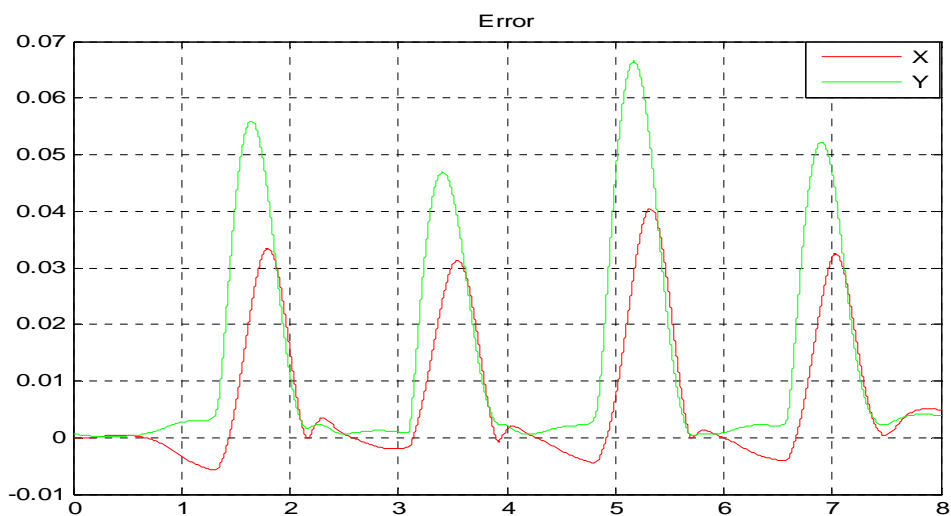


Figure 2 Error for X & Y with out Control : The maximum error in y is = 0.066 m
 And The maximum error in x is = 0.041 m

With Control PID Regulator

For $K_p = 0.2$

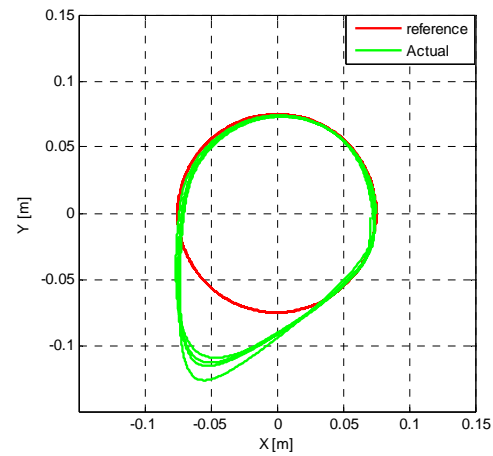
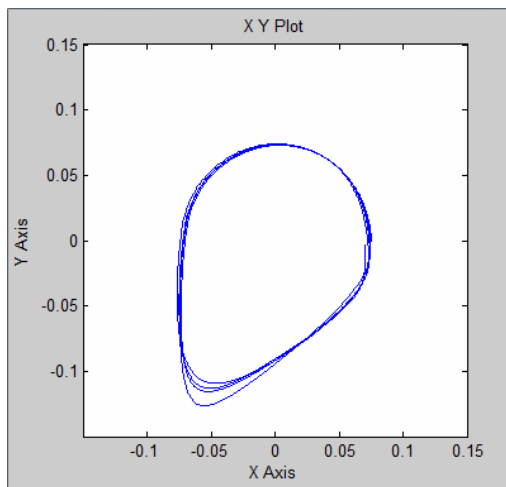


Figure 3 2D Plot from Simulink & matlab for $K_p = 0.2$

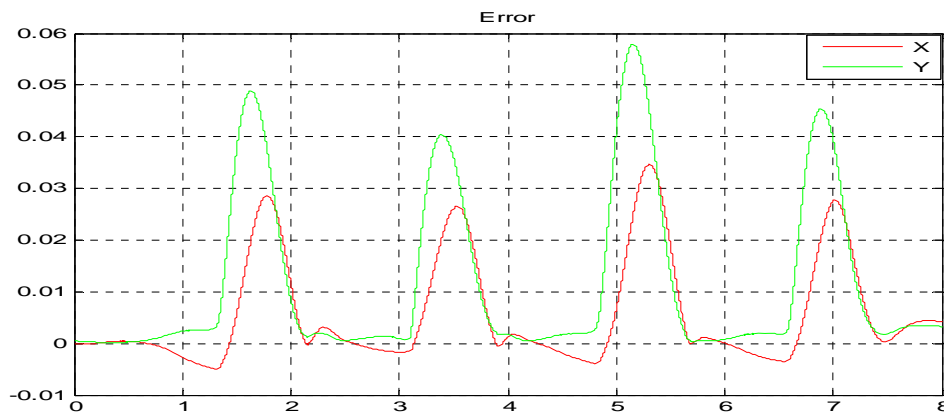


Figure 4 Error for X & Y for $K_p = 0.2$: The maximum error is in $y = 0.058$ m

For $K_p = 0.4$

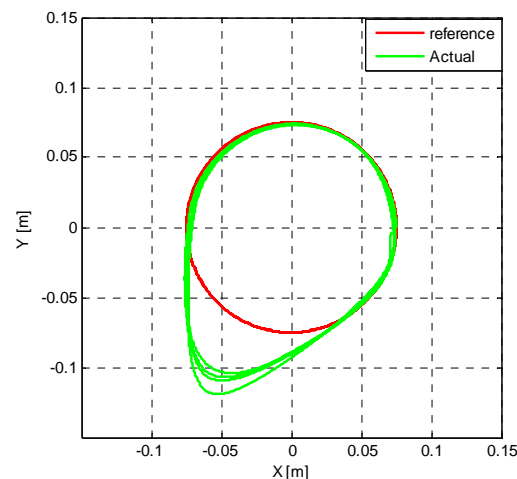
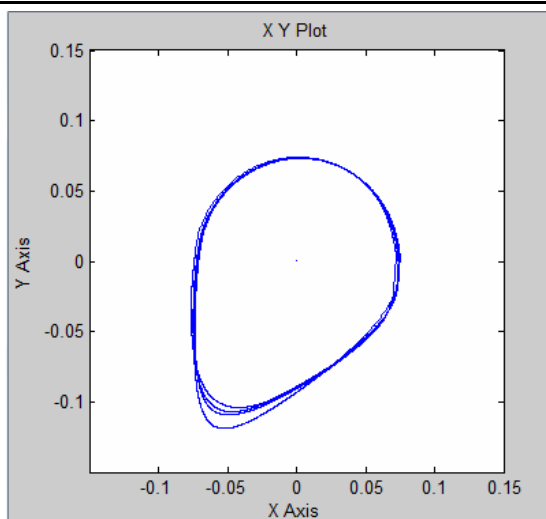


Figure 5 2D Plot from Simulink & matlab for $K_p = 0.4$

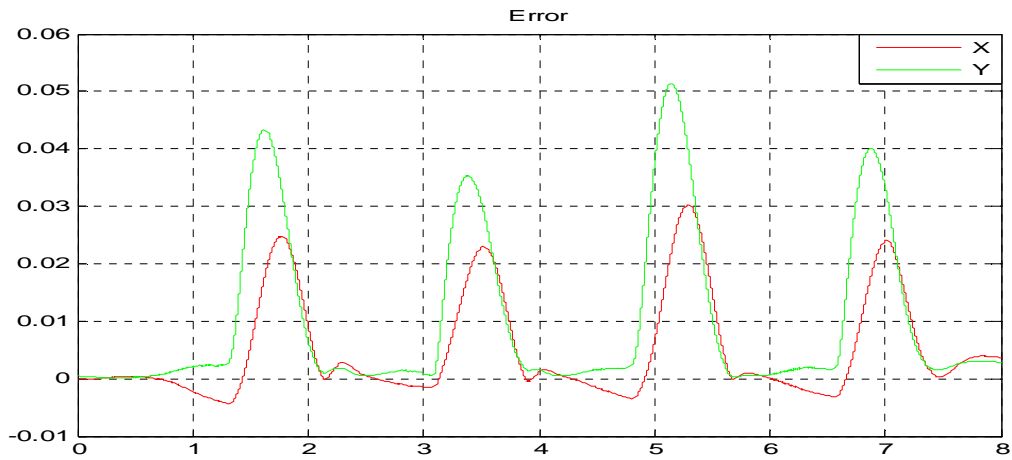


Figure 6 Error for X & Y for $K_p = 0.4$: The maximum error is in $y = 0.051$ m

For $K_p = 0.6$

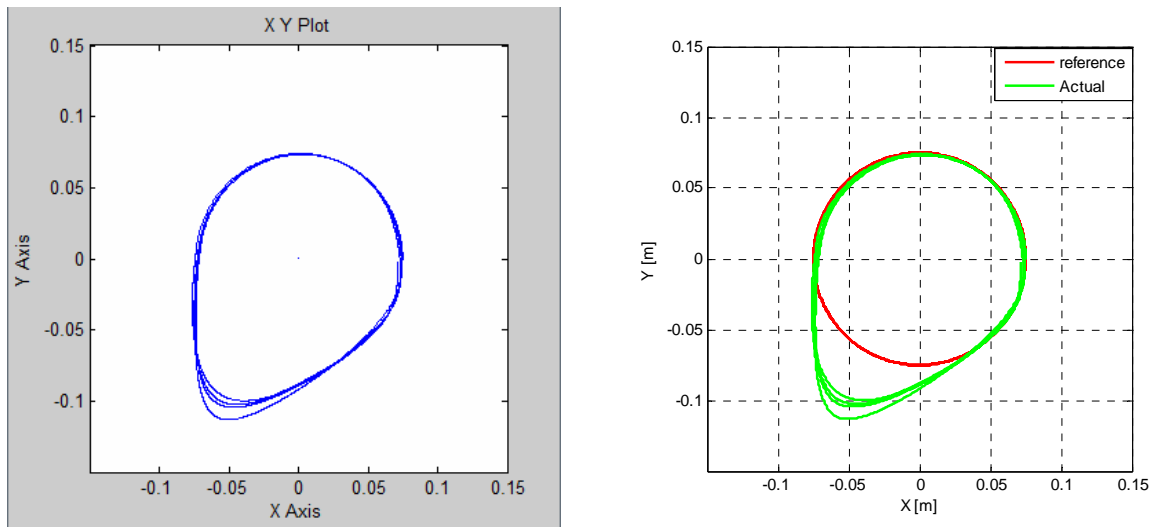


Figure 7 2D Plot from Simulink & matlab for $K_p = 0.6$

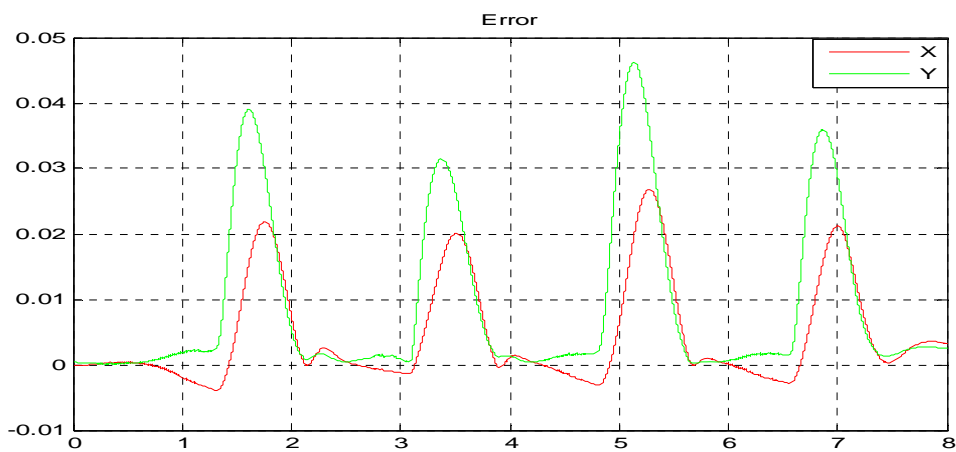


Figure 8 Error for X & Y for $K_p = 0.6$: The maximum error is in $y = 0.046$ m

For $K_p = 0.6$ and $K_i = 20$

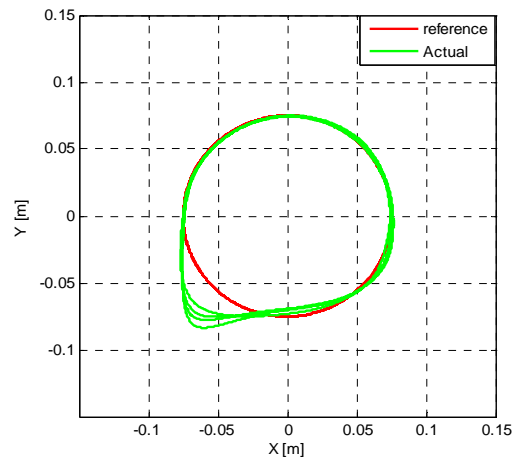
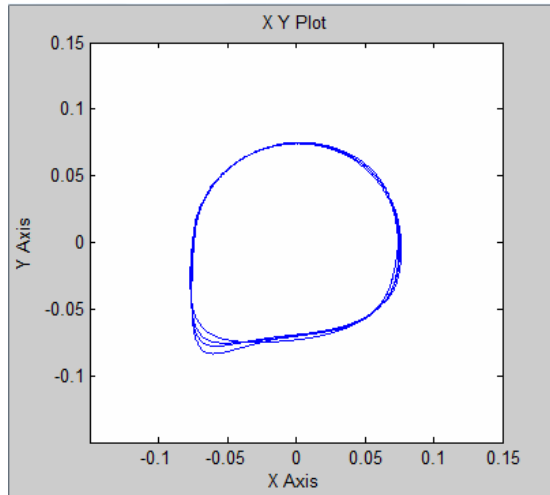


Figure 9 2D Plot from Simulink & matlab for $K_p = 0.6$ & $K_i = 20$

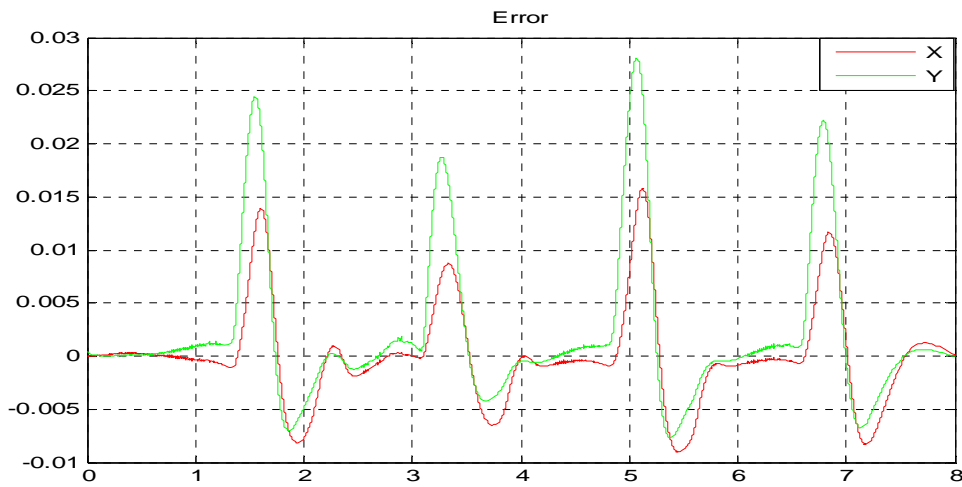


Figure 10 Error for X & Y for $K_p = 0.6$ & $K_i = 20$: The maximum error is in $y = 0.028$ m

For $K_p = 0.6$ and $K_i = 50$

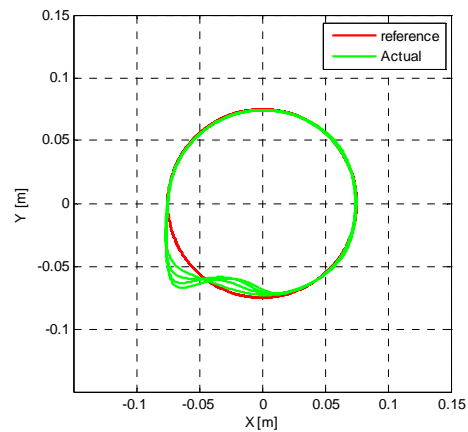
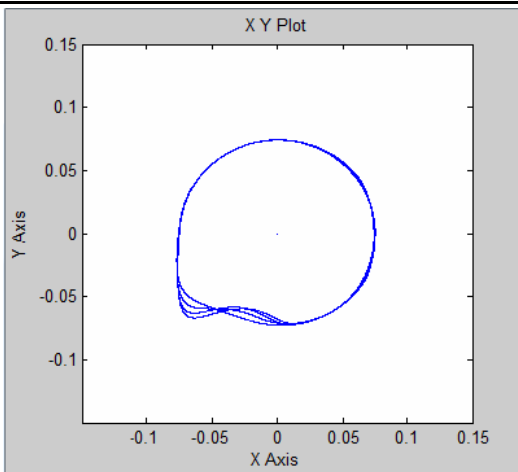


Figure 11 2D Plot from Simulink & matlab for $K_p = 0.6$ & $K_i = 50$

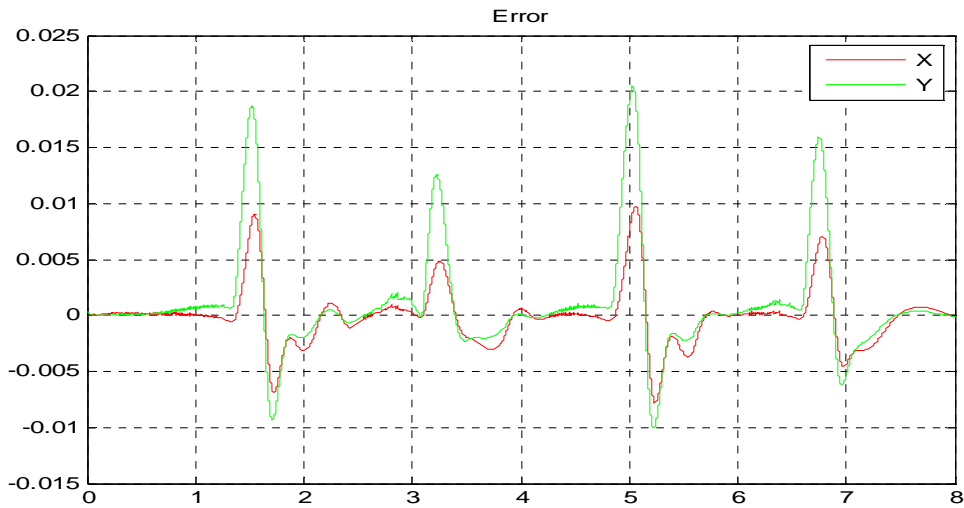


Figure 12 Error for X & Y for $K_p = 0.6$ & $K_i = 50$: The maximum error is in $y = 0.021$ m
 For $K_p = 0.6$ and $K_i = 100$

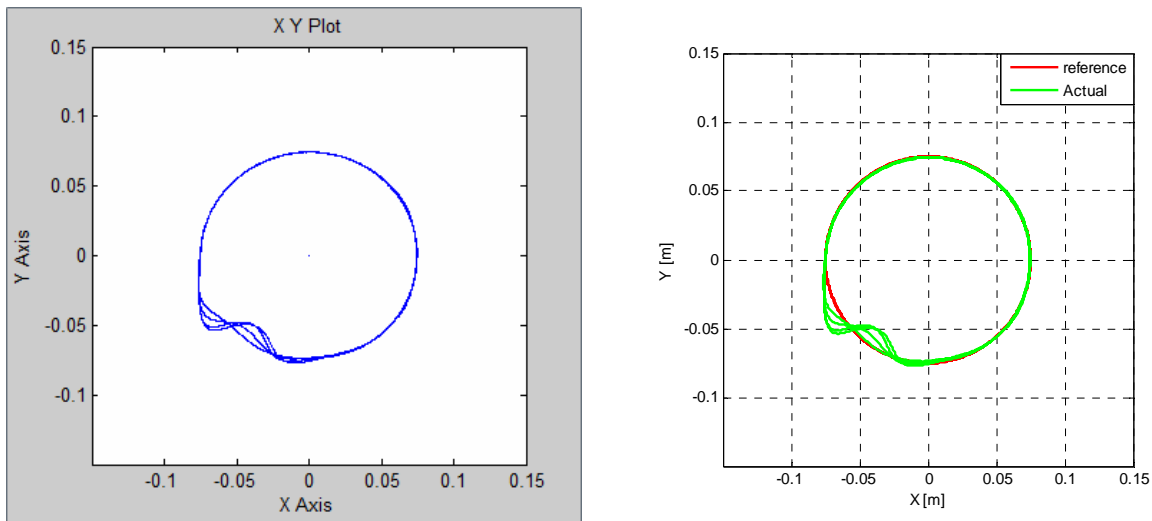


Figure 13 2D Plot from Simulink & matlab for $K_p = 0.6$ & $K_i = 100$

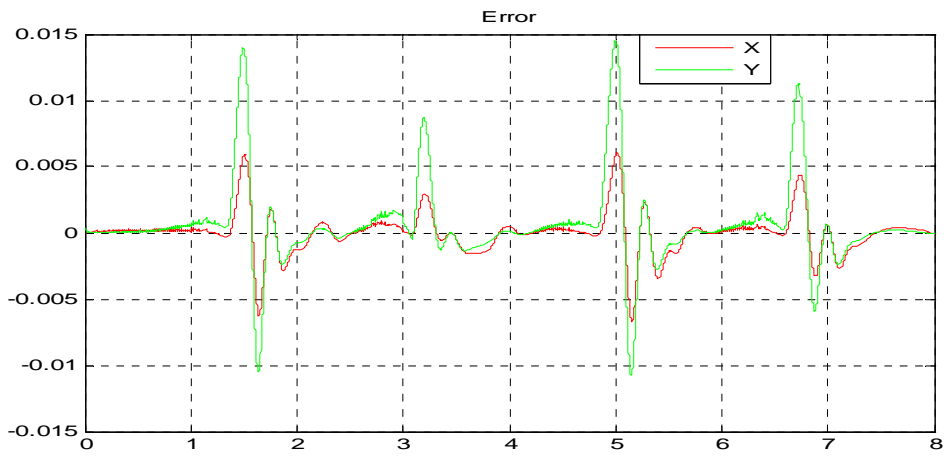


Figure 14 Error for X & Y for $K_p = 0.6$ & $K_i = 100$: The maximum error is in $y = 0.014$ m

For $K_p = 0.6$ and $K_i = 200$

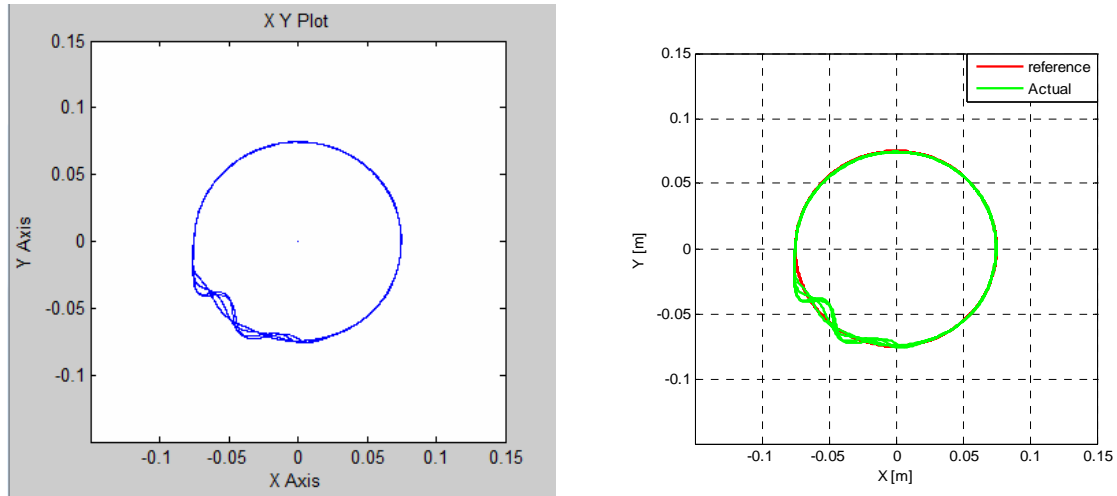


Figure 15 2D Plot from Simulink & matlab for $K_p = 0.6$ & $K_i = 200$

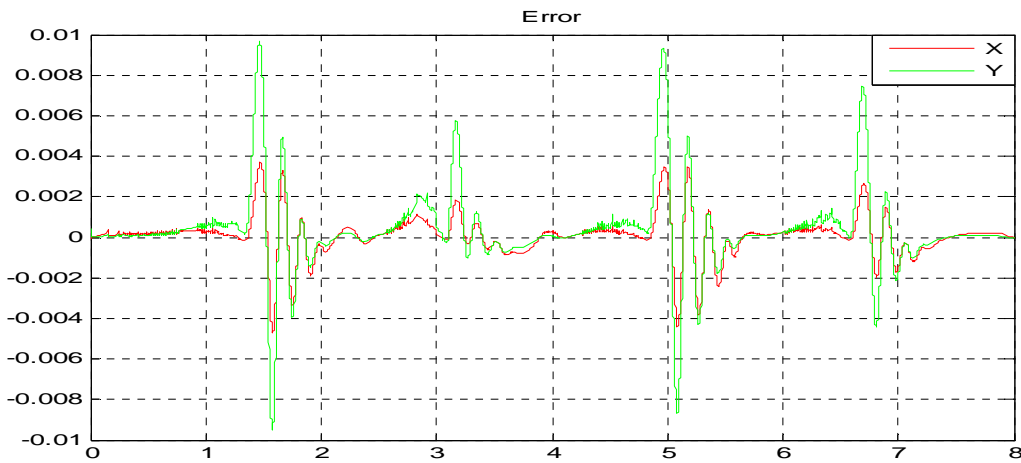


Figure 16 Error for X & Y for $K_p = 0.6$ & $K_i = 200$: The maximum error is in $y = 0.0095$ m

For $K_p = 0.6$ and $K_i = 500$

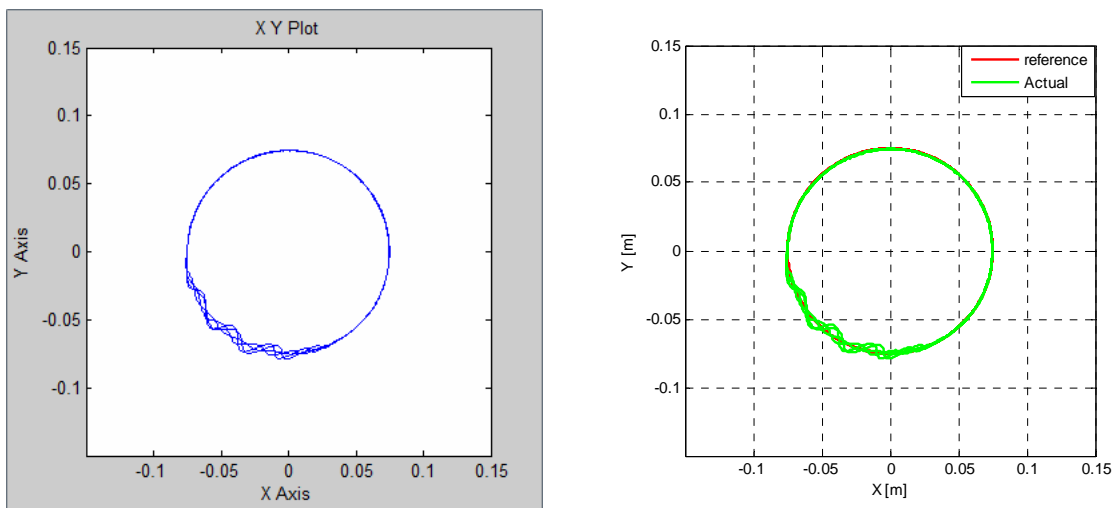


Figure 17 2D Plot from Simulink & matlab for $K_p = 0.6$ & $K_i = 500$

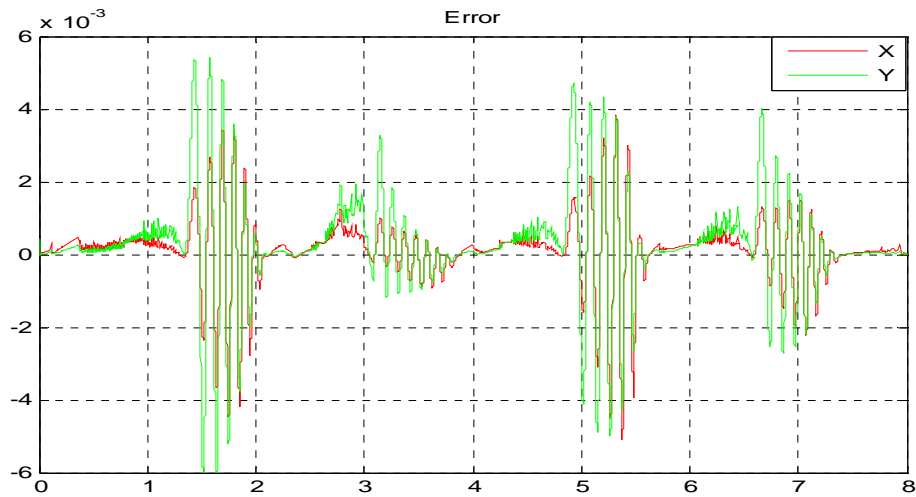


Figure 18 Error for X & Y for $K_p = 0.6$ & $K_i = 500$: The maximum error is in $y = 0.0051$ m
 For $K_p = 0.6$ and $K_i = 800$

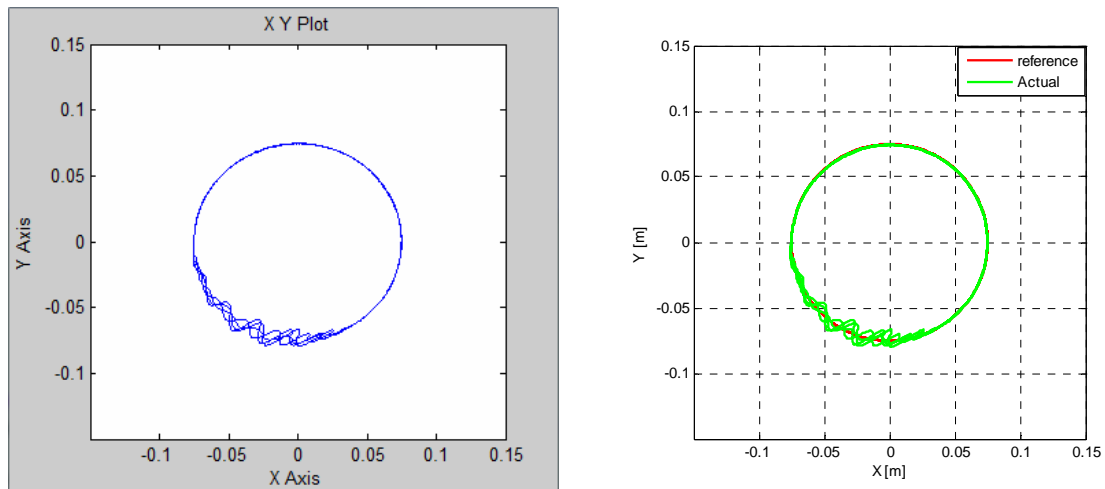


Figure 19 2D Plot from Simulink & matlab for $K_p = 0.6$ & $K_i = 800$

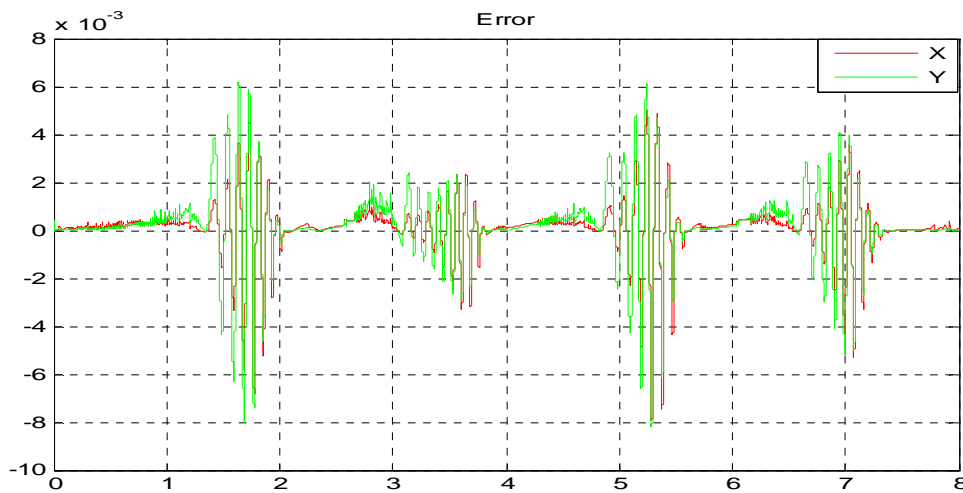


Figure 20 Error for X & Y for $K_p = 0.6$ & $K_i = 800$: The maximum error is in $y = 0.006$ m

For $K_p = 0.6$ and $K_i = 1000$

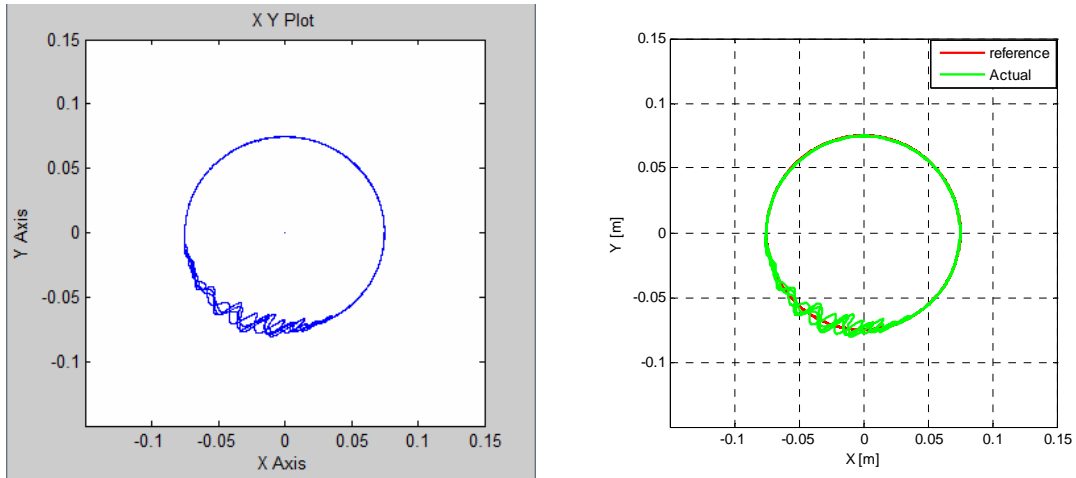


Figure 21 2D Plot from Simulink & matlab for $K_p = 0.6$ & $K_i = 1000$

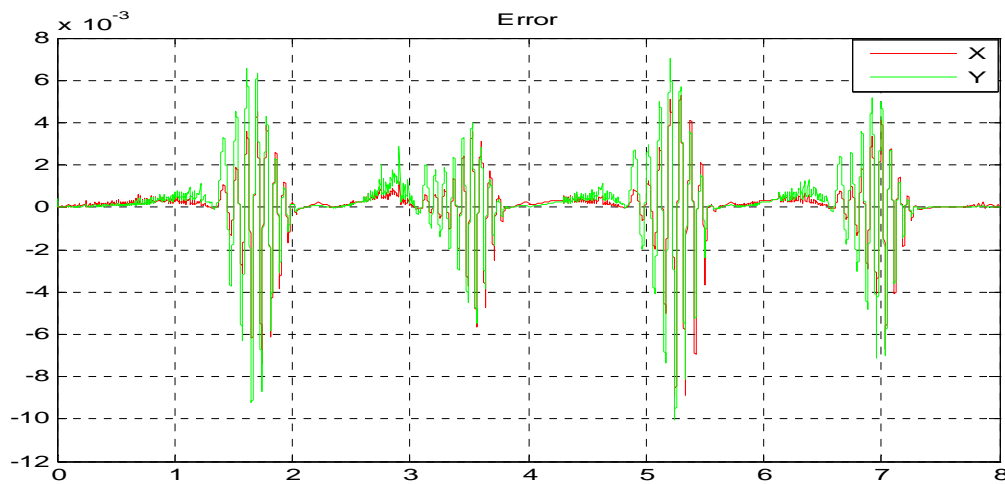


Figure 22 Error for X & Y for $K_p = 0.6$ & $K_i = 1000$: The maximum error is in $y = 0.007$ m

For $K_p = 0.6$ & $K_i = 500$ and $K_p = 0.4$ & $K_d = 0$

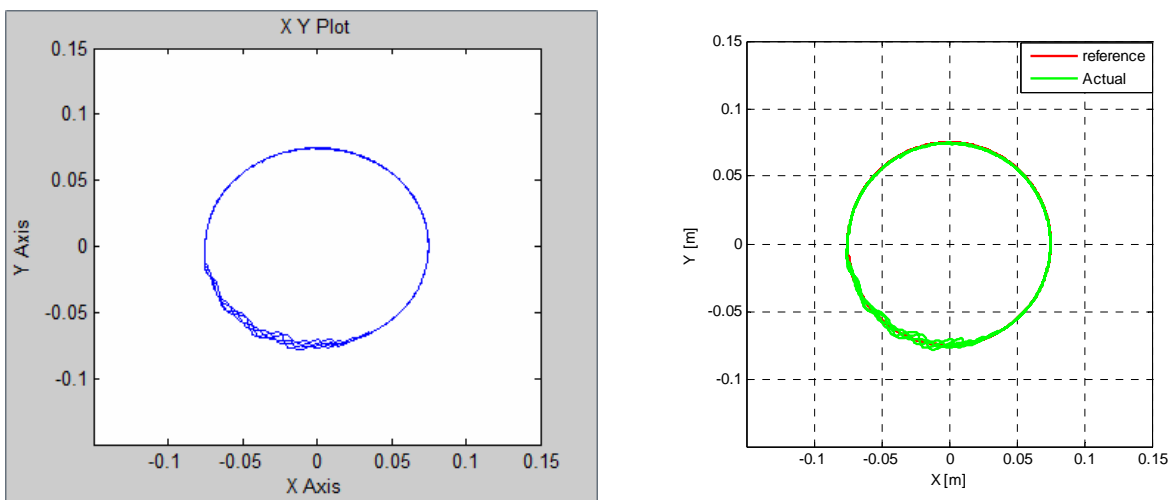


Figure 23 2D Plot from Simulink & matlab for $K_p = 0.6$ & $K_i = 500$ and $K_p = 0.4$ & $K_d = 0$

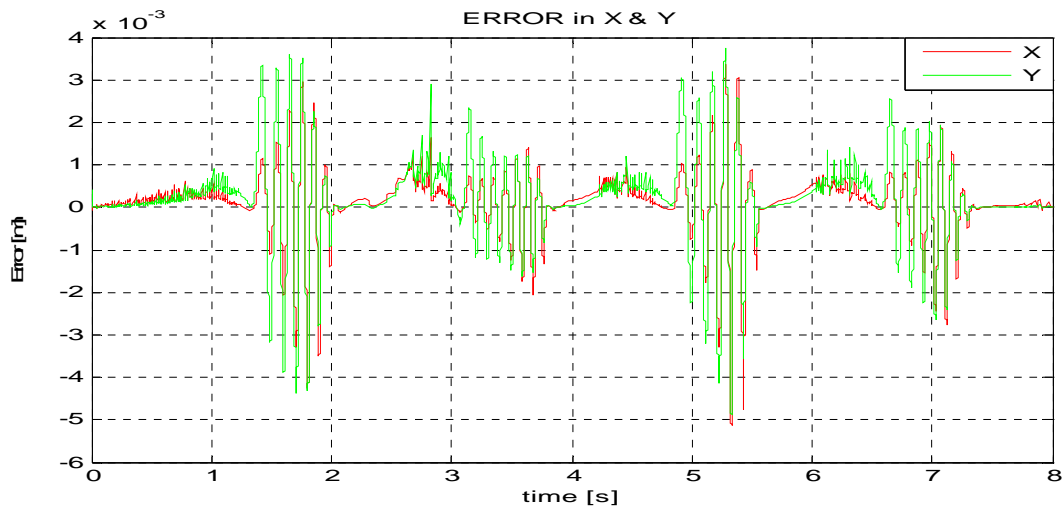


Figure 24 Error for X & Y for $K_p=0.6$ & $K_i=500$ and $K_p=0.6$ & $K_d=0$: Max error in y = 0.0038m
 For $K_p=0.6$ & $K_i=500$ and $K_p=0.6$ & $K_d=0$

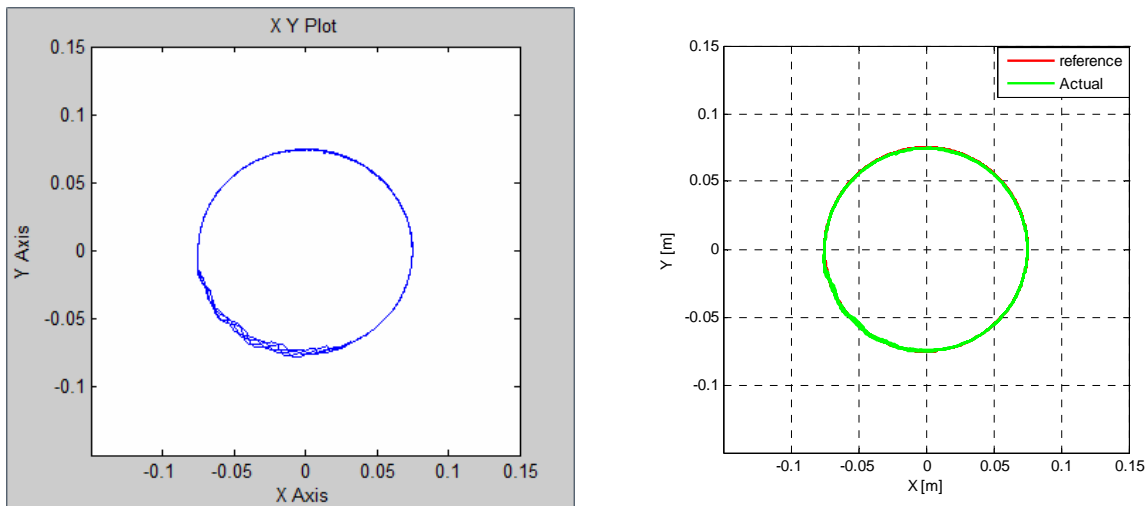


Figure 25 2D Plot from Simulink & matlab for $K_p=0.6$ & $K_i=500$ and $K_p=0.6$ & $K_d=0$

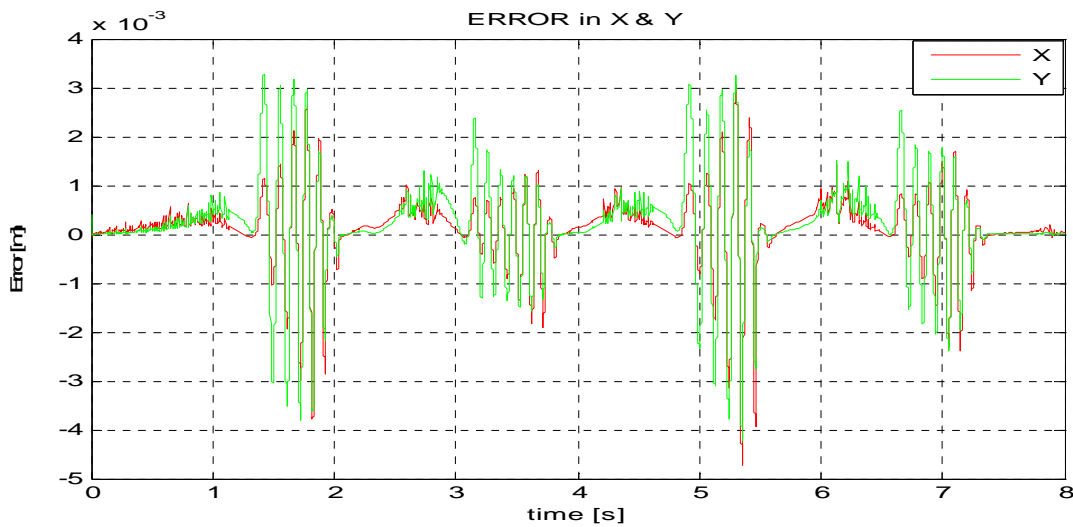


Figure 26 Error for X & Y for $K_p=0.6$ & $K_i=500$ and $K_p=0.6$ & $K_d=0$: Max error in y = 0.0033m

For $K_p = 0.6$ & $K_i = 500$ and $K_p = 0.6$ & $K_d = -0.01$

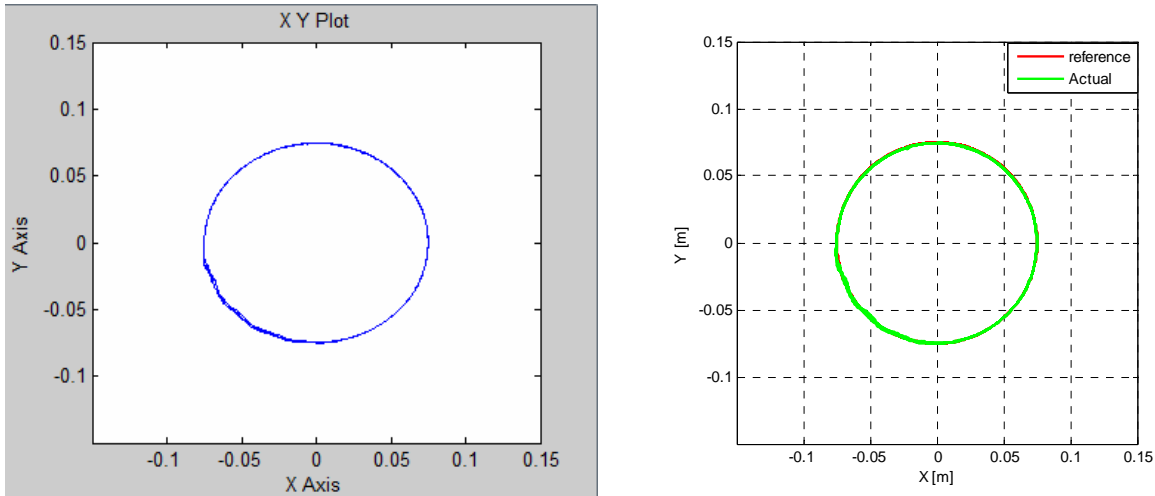


Figure 27 2D Plot from Simulink & matlab for $K_p = 0.6$ & $K_i = 500$ and $K_p = 0.6$ & $K_d = -0.01$

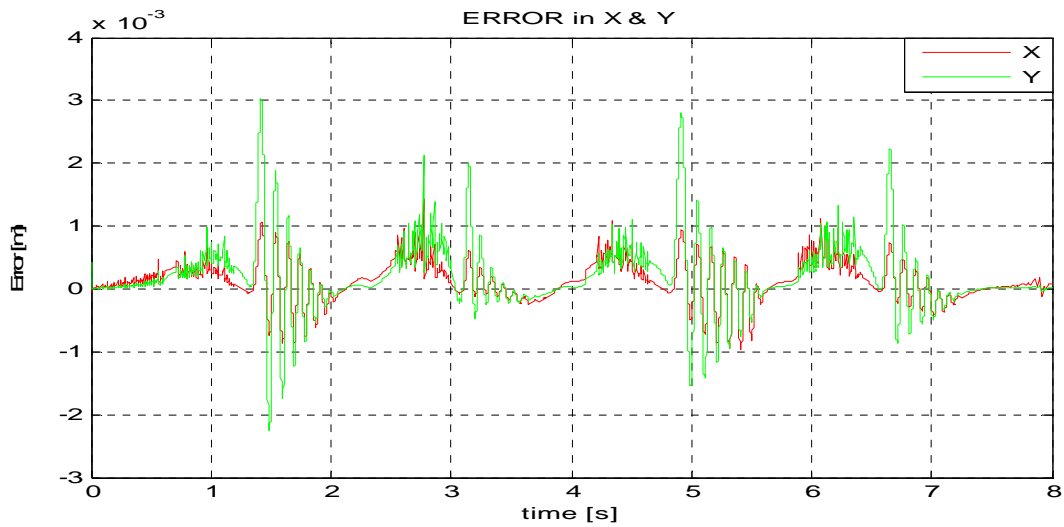


Figure 28 Error for X & Y for $K_p = 0.6$ & $K_i = 500$ and $K_p = 0.6$ & $K_d = -0.01$: Max error in y = 0.003m

For $K_p = 0.6$ & $K_i = 500$ and $K_p = 0.6$ & $K_d = -0.03$

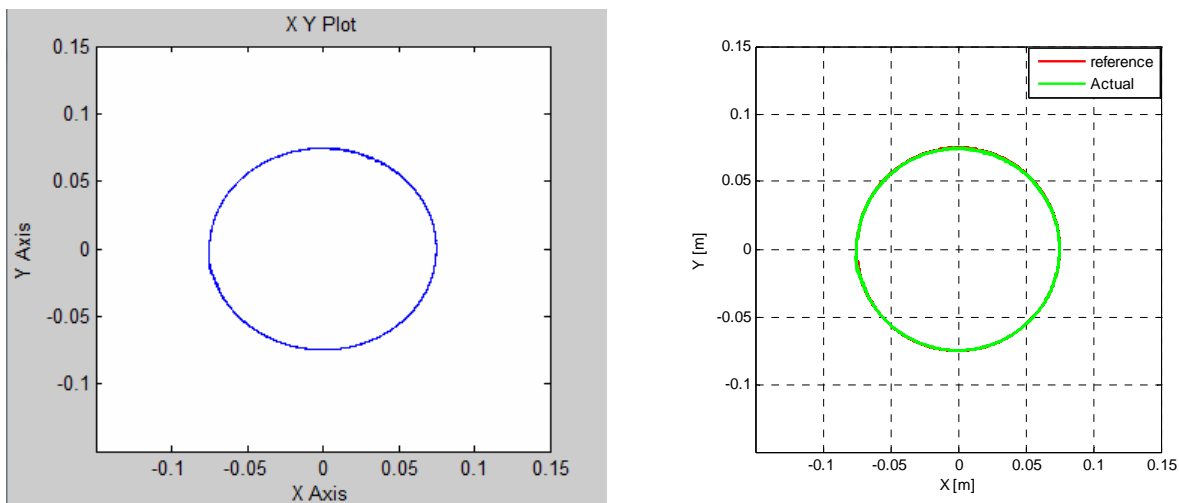


Figure 29 2D Plot from Simulink & matlab for $K_p = 0.6$ & $K_i = 500$ and $K_p = 0.6$ & $K_d = -0.03$

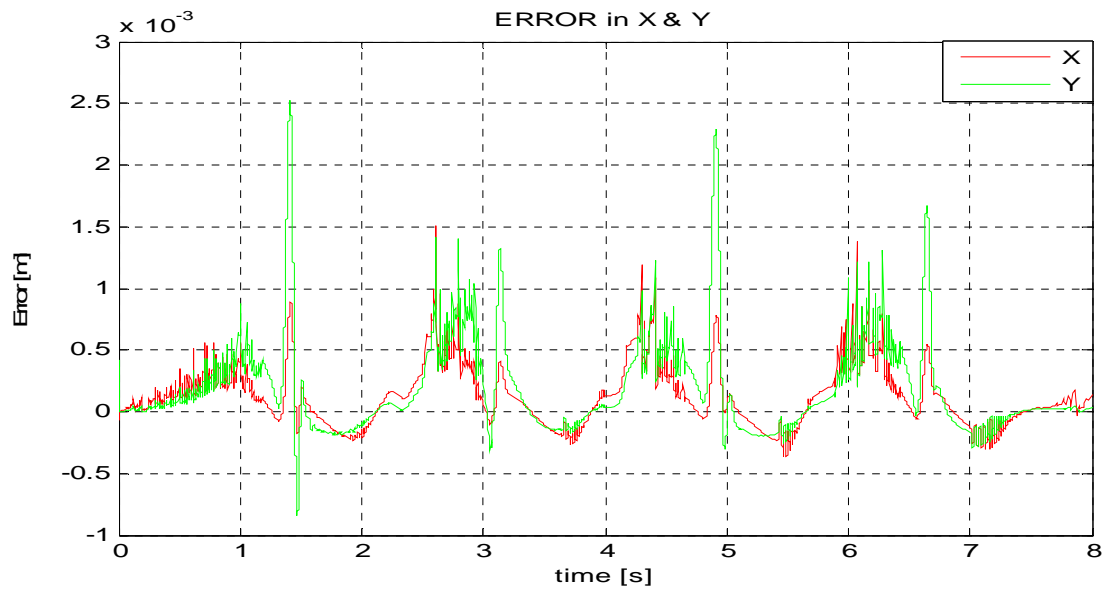


Figure 30 Error for X & Y for $K_p = 0.6$ & $K_i = 500$ and $K_p = 0.6$ & $K_d = -0.03$:
The maximum error is in $y = 0.0025$ m and in $x 0.0015$ m