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NETWORK SELECTION GAMES: SIMULATION TOOL DEVELOPMENT AND PERFORMANCE EVALUATION

Supervisor:
Prof. Matteo CESANA

Author:
Camilo PINEDA

737835

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ABSTRACT

The present thesis work is motivated in the recent developments of wireless access technologies, which have given noticeable changes on both operator and end-user side. From the operator side, the wireless access networks have evolved from 1^{st} generation technologies (1G) up to 3^{rd} generation technologies (3G) during the last three decades. Additionally, 4^{th} generation networks (4G) based in advanced radio technologies are already in development phases. The proliferation of wireless access networks yields opportunities to the end-user, however it can also induce a additional problematic due to the complexity of the network to be operated. Therefore, the end-user has the possibility to choose dynamically which access technology (access network) to connect to, in order to obtain the required service. From the moment the decisions from the users occur in a distributed, non-coordinated and generally opportunistic way, the access to wireless networks process dynamics are in a need to be studied. The present thesis work, analyses the competitive access problem by means of game theory.

In particular, non-cooperative game models in which the users tend to maximize their individual utility from the access problem are considered. The thesis work presents the development and implementation of a software instrument capable of simulating game dynamics individualizing equilibria conditions and measuring the transition length to reach such equilibria (convergence time). The second part of the thesis work includes the analysis of different realistic wireless access networks considering different access strategies which are selected by the end-users and different utility function formulations with which the players "play" the game.

SOMMARIO

Il presente lavoro di tesi è motivato dai recenti sviluppi nel campo delle tecnologie di accesso wireless, che hanno portato a notevoli cambiamenti sia lato operatore che lato utente finale. Lato operatore, le reti d'accesso wireless si sono evolute dalla prima generazione (1G) fino alla terza generazione (3G) negli ultimi tre decenni. Inoltre, reti di quarta generazione (4G) basate su tecnologia radio avanzata sono già in fase di sviluppo . La proliferazione di reti di accesso wireless da un lato costituisce un'opportunità per l'utente finale che può, dall'altro introduce problematiche aggiuntive legate alla complessità dell'infrastruttura di rete da gestire. L'utente finale ha quindi la possibilità di scegliere dinamicamente la tecnologia di accesso (rete d'accesso) con cui collegarsi per ottenere i servizi richiesti. Dal momento che le scelte degli utenti avvengono in maniera distribuita, non coordinata e generalmente "opportunistica", nasce il problema di studiare le dinamiche del processo di accesso a reti wireless. Il presente lavoro di tesi analizza il problema dell'accesso competitivo tramite la teoria dei giochi.

In particolare, vengono considerati modelli di gioco non-cooperativo in cui gli utenti mirano a massimizzare la propria utilità percepita a valle del processo di accesso. La tesi presenta lo sviluppo e l'implementazione di uno strumento software in grado di simulare le dinamiche di gioco individuando le condizioni di equilibrio e valutando la lunghezza del transitorio per raggiungere tale equilibrio (tempo di convergenza). La seconda parte del lavoro di tesi include l'analisi di scenari realistici di accesso a reti wireless considerando diverse strategie di accesso scelte dagli utenti e diverse formulazioni della funzione di utilità con cui gli utenti stessi "giocano".

1.

INTRODUCTION

Telecommunications field has experienced a big amount of technological advances in the past decade rendering it one of the most exploited research fields due to the challenging tasks to be addressed. One particular challenging topic is called Network Selection; wireless networks are spread out on metropolitan areas, on schools, on malls, and on particular homes spanning different technologies from 2G going through 3G, Wireless Mesh Networks and the well known IEEE WLANs, at the same pace technology advances are creating more powerful, portable, and multi-service wireless devices, ranging from smart-phones, the family of Apple's devices, as well as laptops and any other particular device which must access an internet backbone through an intermediate wireless station. The increasing demand for portable wireless devices has created an extensive offer of wireless access services by telecom operators making for instance two operators to have partially or totally overlapping coverage areas, whenever a wireless device recognizes more than one available network to connect to, then the Network Selection problem arises.

Network selection agile solutions aim for an automatic and dynamic performance of actions on the network side and user side, the former aims for the automatic distribution of radio resources to stations; for instance a single operator assigning particular channel frequencies to each of its access points in the network in such a way of achieving the lowest overlap and the longest population of covered users, in the latter case, the aim is to develop an automatic selection of the best performing station according to the particular service being requested by the user. The formulated scenario features competition both among users and network operators; indeed two telecom operators might be competing in order to cover the major quantity of users which in turn results in a greater revenue, and users compete for resources: the particular resource that can offer the best service to two common users will yield a competition between these two for achieving a connection to the named resource.

Many literature works model the network selection problem with game theory, more specifically with non-cooperative game theory; the reason behind the use of such particular theory is because network selection yields naturally competition between its players, nevertheless under the competitive framework of network selection certain decisions taken by the players can lead to cooperation. Literature works modeling the problem with non-cooperative game theory either aim to propose new models or algorithms to solve particular instances of the general problem or aim to perform practical analyses of existing algorithms. In the former case, the contribution of the work aims to show and proof existence of game equilibrium and/or how the game converges or not to such equilibria, the latter on the other hands aims toward a merely practical performance evaluation by for instance comparing two or more works made by different authors.

A very well known equilibrium point is the Nash equilibrium, name due to its author John Nash. A Nash Equilibrium is a set of strategies or a set of decisions, say *s*, to be taken by the players such that for a particular player, say *j*, its best decision to be taken is in *s* given that all other players are taking decisions from *s*. In other words if everyone else is playing a Nash equilibrium, then the best thing for everyone to do is play the Nash equilibrium. It is widely known in literature that such a desirable property as Nash equilibrium for games might give arise to different equilibria of different quality, meaning particular Nash equilibria might benefit more a particular user than another, nevertheless once the set of players agree on which Nash equilibrium to play the outcome of the game is a very stable solution. Convergence to equilibrium is often proved by mathematical demonstrations in the available literature, and depending on the game strategies employed (for example: Best Reply dynamics or Air Time metrics) the convergence is guaranteed under particular conditions and if possible the convergence speed is bounded from lowest to fastest achievable rates.

Game strategies span a variety of ways to tackle and manage non-cooperative game models, these strategies endorse the policy of how should the players make their decisions during the game; for instance there are static techniques, such as Best reply and Better reply dynamics and learning techniques such as Fictitious Play and Machine Learning. The first two mentioned techniques do not care about previous decisions made by the players and just base their criteria on actual utilities, Best reply dynamics will yield always the optimal decision meanwhile a Better reply dynamic might not derive the optimal response always; despite the previous fact, literature has proven convergence for Better reply dynamics in a finite number of steps for particular cases of congestion games. The last two mentioned techniques correspond to "active" techniques, in the sense of being aware of past actions taken by all players, at a glance these models are more expensive in computational terms, but can yield interesting results by making each user aware of its opponents decisions through learning to henceforth make Best or Better Reply choices on a much more reduced strategy space when compared to the static counterparts.

The work developed in this thesis work is named *Network Selection Games: Simulation Tool Development and Performance Evaluation*, it is a Network Selection Performance Evaluation Tool, and it was implemented based in non-cooperative game theory algorithms. The algorithms were taken from different literature sources and synthesized together into particular algorithms for solving particular instances of the network selection problem; the language chosen to program the codes was C under *MATLAB* suite.

Network Selection Games: Simulation Tool Development and Performance Evaluation is capable of deploying a network selection scenario by allocating stations (access points) and users on different types of maps; afterwards it simulates the association competition between the users and different stations under different policies. Therefore *Network Selection Games: Simulation Tool Development and Performance Evaluation* contribution is two-fold, firstly it is capable of setting up a customizable Network of wireless devices allocated in customizable topologies and secondly it produces as output the different results derived from the application of non-cooperative game theory to the process of network selection rendering its utility as a performance evaluation tool.

Network Selection Games: Simulation Tool Development and Performance Evaluation is capable of simulating the following scenarios:

Map Topologies:

Customized – Random – Linear Grid – Rectangular Grid

Where grids correspond to the allocation of the access points and not the users

Game Strategies:

Best Response – Better Response – Fictitious Play

Association Policies:

Pure Interference Based- Additive Achievable Rate and Interference Based – Multiplicative Achievable Rate and Interference Based

Type of Outputs:

Elapsed Simulation Time – Number of Iterations Employed – Convergence Probability

A scenario is composed by selecting one option from each of the categories listed above. The game strategies are further divided into subcategories rendering the total available types of simulations to 120. Each of the 120 different scenarios have the outputs presented in the three different types described above with a very few and particular exceptions.

The results obtained through the different simulations range from simple crosschecks to brand new outputs; for instance, the best response algorithm results constitute a baseline result for the better response algorithm results as the latter converges to the results of the former in finite time. It was also found how a learning technique can relax the assumption of allowing the players to have access to a common knowledge base by instead compiling their own knowledge base of its opponent's actions and yet yielding performance results equivalent or similar to the best response algorithms. Other results show how particular geographical distributions of the access points favor a scenario and how for particular ratios between the number of access points and users there exist overshoots or undershoots with respect to the baseline reference scenario.

This thesis works starts with a background review on the network selection problematic in Chapter 2, where access technologies and related work on the matter are reviewed. Later in Chapter 3, a review is made to game theory, focusing on non-cooperative related games, going through important concepts such as Nash equilibria and convergence issues, a special type of non-cooperative games called congestion games are reviewed as well, to finally conclude the Chapter with the different game strategies most commonly applied to non-cooperative game models. Going forward to Chapter 4, a detailed *Network Selection Games: Simulation Tool Development and Performance Evaluation* tool description is found, where all the functionalities, GUI interfaces and capabilities are explored and explained in order to make potential users familiar with the usage of the tool. Rounding up the thesis work a complete Section dedicated to experimental results are reported in Chapter 5, in this Chapter there are simulations for all the association policies described in the paragraphs above, additionally each output result is briefly commented and compared to previous results.

2.

BACKGROUND ON NETWORK SELECTION

During the last decade, the world of wireless communications has experienced a big increase of implemented technologies and services offered to end users, for instance the well known IEEE 802.11 wireless local area networks (WLANs) nowadays are widespread in enterprises, public areas and homes [15], nevertheless alternative technologies such as Wireless Mesh Networks (WMNs), 3G Systems and the so-called in literature coexisting/cooperative wireless access systems: 4G networks [16,17] are enriching the deck of services being offered by network operators. In the future, spectrum agile and radio cognitive devices [11,12,17] will guarantee a user to be always connected and covered by the best service offering station and will allow such users to dynamically adapt in an opportunistic way depending on the quality of the current connectivity opportunities [17].

Cause and effect paradigm cannot be escaped in the network selection context along with all the technology advancements done, several challenging problems arise at different network levels, namely on the network side and on the end-user side, the former posses the problem of resource allocation in principle managing the radio resources when different and possibly competing network operators coexist [17] and the latter accounts the network selection problem, which aims to a dynamic and automatic selection of the "best" available access point to connect to [16].

2.1

TECHNOLOGIES

2.1.1

WLANs

WLANs provide wireless access communication over short distances using radio (e.g IEEE 802.11 protocols) or infrared signals (e.g Bluetooth protocols) instead of traditional network cabling, the communication is offered typically by a station called *access point* which can directly or indirectly connect it's given set of users to the backbone.

Obvious advantages of WLANs over their wired counterparts are the ability to support users mobility over the covering range while still being connected to the network and to the ease of installation.

The commonly widespread WLANs networks over public places and homes are the so called 802.11 based WLANs. 802.11 is a set of standards owned by IEEE that regulate the wireless local area network communications in the 2.4, 3.6 and 5 GHz bands. The most popular standards are the

802.11b and 802.11g both of them operate on the 2.4GHz band, the former achieves maximum data rates of 11Mbps whilst the latter is able to achieve 54 Mbps [4].

Each IEEE 802.11 frequency range has a set of Wi-Fi Wireless LAN channels, each country has its own regulations on the allowable channels, allowable number of users and maximum power ranges within these frequency ranges. Figure 2.1 below shows a graphical representation of the Wi-Fi channels in the 2.4 GHz band:

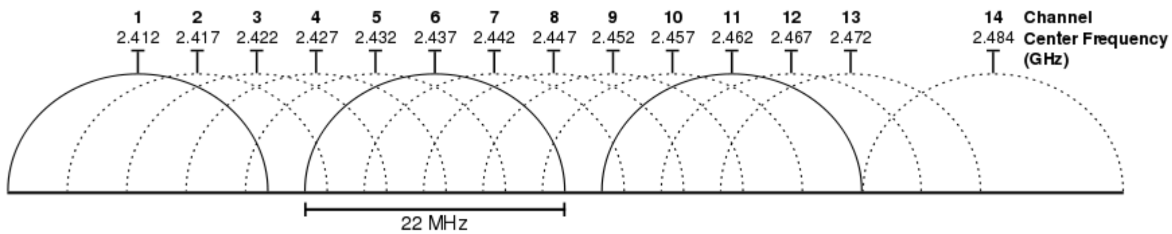


Figure 2.1 Representation of the 2.4GHz band Wi-Fi channels [4]

Security has always been an issue, for instance the WEP security protocol was proven to be easily cracked, and the WPA/WPA PSK security protocols result to be effective only under strong password policies [4, nowadays 802.11b/g security leaks have been amended by 802.11i/n protocols.

For all the scenarios simulated during this work, the considered stations will be access points, using the 802.11b/g protocols and using one and only one frequency amongst the ones shown on figure 2.1, each station will provide connectivity to the internet backbone to their end users. Users are assumed to know what access points are accessible by an apriori passive discovery of them via beacon packets (SSID discovery).

2.1.2

WMNs

Wireless Mesh Networks are rapidly undergoing rapid progress and inspiring numerous applications due to their advantages over other wireless networks, additionally they are considered to be part of the next generation wireless networking [6].

WMNs are dynamically self-organized and self-configured, with the nodes of the network automatically establishing an ad-hoc network maintaining the mesh connectivity.

WMNs are comprised of two types of nodes:

- Mesh routers: Contains besides the typical gateway and bridge functionalities as a conventional wireless router the functionalities to support mesh networking, as for example being able to forward packets via multi-hops the same coverage can be achieved by a Mesh router with less transmission power when compared to a traditional router. Additionally a Mesh router is usually equipped with multiple wireless interfaces built on either the same or different wireless access technologies [6].
- Mesh clients: Typically laptops, cell phones or any other device with wireless connection capabilities.

WMNs connectivity miscellaneous

- Clients can work a mesh routers, providing mesh networking exclusively and not gateway or bridging capabilities offered uniquely by mesh routers. The communication protocols installed on mesh clients to provide mesh networking role as a router is usually lighter than the protocols installed on a router.
- Integration of WMNs with various networks, for instance nodes with wireless NICs can connect to a WMN through wireless mesh routers and customers without a wireless NIC can access a WMN through Ethernet ports.
- Enriching the capabilities of ad-hoc networks by the paragraph described above, and by carrying a low up-front cost, easy network maintenance, robustness, reliable service coverage amongst others [6].

Architecture of WMNs

- **Infrastructure/Backbone WMNs**: This approach provides a backbone for conventional clients and enables integration of WMNs with existing wireless networks, through gateway/bridge functionalities in mesh routers. Figure 2.2 shows a typical infrastructure WMN.
- **Client WMNs**: In this type of architecture, client nodes constitute the actual network to perform routing and configuration functionalities, as well as providing end-user applications to customers, therefore mesh routers are not strictly necessary on this type of networks.
- **Hybrid WMNs**: It is a combination of the formerly described architectures, for instance mesh clients can access the network through mesh routers, or by directly meshing with other mesh clients; the infrastructure provides as well connectivity to Internet, Wi-Fi, sensor networks and others, the routing capabilities on clients provide an improved connectivity and coverage inside WMNs.

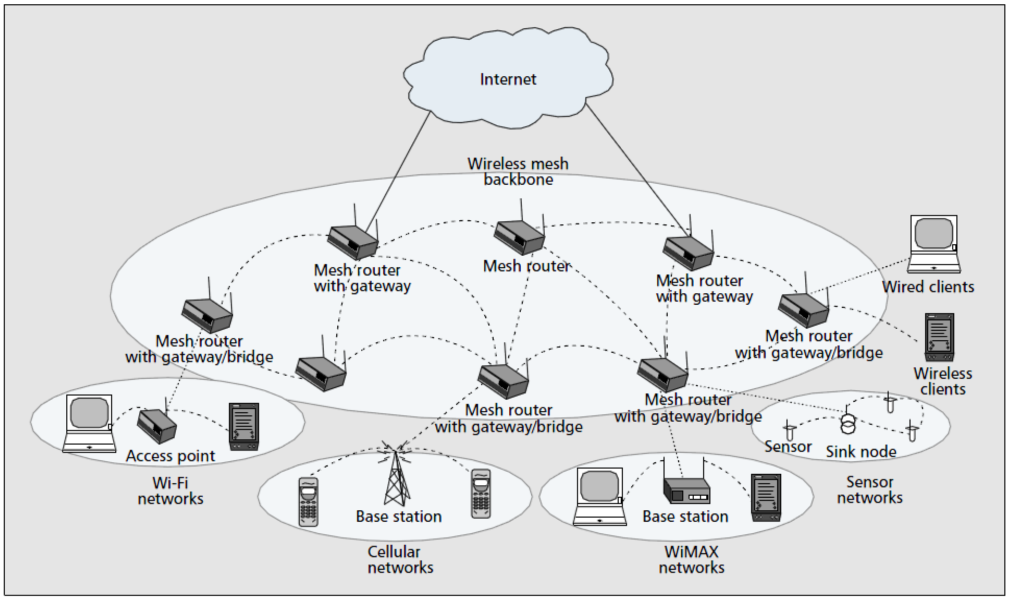


Figure 2.2 Typical Infrastructure WMN architecture [6]

2.1.3

3G systems

3G or 3rd Generation is also known as *International Mobile Telecommunications-2000 (IMT-2000)*, they are a set of standards for mobile phones and mobile telecommunications. Services include amongst others: mobile internet access, video calls and mobile TV. When compared to older generation of standards, 3G fulfills the simultaneous use of speech and data services at peak rates of about 200kbps [7].

3G standards are generally backward compatible with their former 2G networks, however there are revolutionary standards such as the UMTS family of standards that require all new networks and frequency allocations. According to [8] these new 3G systems will trigger an explosion in wireless internet and data applications by delivering high data rates as never seen before, for instance the CDMA2000 set of standards whose latest release called EVDO is capable of offering peak data rates of 14.7Mbps downstream [7,8].

3G / IMT-2000 standards:

- TDMA Single carrier (IMT-SC): also known as *EDGE*, consists of an evolutionary upgrade to GSM/GPRS used worldwide except for Japan and South Korea; EDGE requires no hardware or software changes to be made on the GSM networks, it results four times more efficient as GPRS, and uses nine modulation and coding schemes compared to the four coding schemes used by GPRS. The most advanced EDGE version is called *evolved EDGE* and reaches peak bit rates of 1Mbps and typical rates of about 400Kbps [7].
- CDMA Multi carrier (IMT-MC): also known as CDMA2000, evolutionary upgrade to the so called CDMAone, commonly used in the Americas and Asia. The technology is a set of standards that control the CDMA channel access to send voice and data between mobile

phones and cell sites. The two most known standards are CDMA2000 1X and CDMA2000 1XEV-DO, the former achieves peak data rates of 153Kbps having typical rates around 60-100Kbps, while the latter is able to achieve forward link air interface speeds up to 2.4Mbps, the improvement of the speed rates is due to the use of radio signals along with improved multiplexing techniques which renders EV-DO suitable for broadband IP networks [7,8].

- UMTS, stands for Universal Mobile Telecommunications System, it is a revolutionary set of standards whose deployment requires a whole new set of base stations and frequency allocations. UMTS is currently being used to base the development of an upcoming 4G technology called LTE or 3GPP Long Term Evolution whose aim is to offer downlink rates up to 100Mbps and uplink rates up to 50Mbps while offering simultaneously high throughput, low latency (less than 10ms round trip times) and a plug-and-play feature. The current UMTS technology based devices can reach theoretically rates up to 42Mbps, however most commonly users experience depending on the specific device and technology, rates of 384Kbps for R99 based devices and 7.2Mbps for HSDPA based devices.
- Others, such as FDMA and IP-OFDMA, the latter achieves data rates equivalent to WiMAX systems whose latest update (IEEE 802.16) provides rates up to 40Mbps.

2.1.4

4G systems

4th Generation systems are often confused with evolutions of 3G technologies; wireless access standards as many other technical standards evolve during their lifetime to offer improved performance and capabilities. History began with the analogue 1G systems, whose successor switched from analog networks to digital ones. The definition of 3G systems with respect to the extensive and worldwide deployed 2G systems is either an evolution of the former or a complete revolution.

4G systems involve all IP-packet-switched networks, mobile ultra-broadband (gigabit speed) access and multi-carrier transmission; the so called pre-4G technologies are: 3G Long Term Evolution which was described above and mobile Wi-MAX described above as well. Formal requirements as issued by ITU-R and referred as IMT-Advanced for a cellular system require for instance peak data rates of about 100Mbps for high mobility devices and rates up to 1Gbps for low mobility devices such as local wireless access. Technology migration occurs leaving the formerly used CDMA spread spectrum radio technology into using frequency domain equalization schemes, such as OFDMA.

Deployment dates are around late 2011 early 2012, and they will support IPv6 along with the implementation of almost all the standards from 2G to 4G in order to be a legacy system adopting existing users nowadays for instance [9].

2.2

RELATED WORK

In [10] authors aid the network selection process with a tool they called *Wi-Fi Reports*; their work provides the user with historical information about the different AP performance and application support; the statistical data is collected by means of user reports which in turn results to be a challenging task due to privacy and avoidance of fraudulent reports. The work developed on [10] results to be of great practical interest because nowadays wireless device users expect connectivity wherever they are, Wi-Fi areas covered by wireless access points are dominant each of them offering different services for different subscription prices, therefore a generic user at the state of the art is not able to determine which AP would be best to run his applications before paying for access.

Wi-Fi Reports carried the studies deriving different performance results such as: Basic connectivity, TCP throughput, response time and port blocking, more over the authors performed different tool evaluations, particularly one related to *Network Selection Games: Simulation Tool Development and Performance Evaluation* work, called AP selection performance which is carried in two steps, first the user selects a hotspot to go physically based on a predicted performance of all hotspots nearby, later on the selected hotspot where the users and APs are stationary the user selects the AP that maximizes its performance.

The network selection problem once a user has established a particular AP to connect to can be further divided into a frequency channel selection, authors on [11] propose a Load-Aware Channel (LAC in short) assignment scheme for 802.11 WLANs. The work is motivated by the fact that there is a demand for high throughput wireless internet connectivity which has ultimately ended in the deployment of thousands on WLANs in urban areas resulting in increased interference levels and contention between co-channel APs.

The core of the work is based on the *airtime cost* metric through which the authors discover the most appropriate channel for each AP by measuring uplink and downlink conditions and the number of affiliated users. The work done in a nutshell consists of every AP and user performing a sequential scanning on all the available channels and collect measures aforementioned through the use of LAC, the scanning procedure is divided into 4 steps: *i)* Compute downlink airtime cost, *ii)* Compute uplink airtime cost, *iii)* Decide if the current channel is appropriate and *iv)* Computing the cumulative airtime cost at the next available channel; the set of four steps described previously belong to an iterative process, authors of [11] show through their work that after a set of iterations each AP agrees on the channel with the minimal airtime cost (i.e convergence has been found) yielding the maximum long-term cell throughput.

The framework on which *Network Selection Games: Simulation Tool Development and Performance Evaluation* is developed, the so called Network Selection scheme entails two perspectives, user side and network side, the authors on [12] developed a project called *Simplicity project* which proposes a novel solution on operator oriented network selection schemes. In a broad sense the project aims to simplify the process of using and managing current and future services through personalization, portability and adaptability. Personalization will allow each user to access different services and networks and the automatic selection of services according to specific locations.

Simplicity project architecture encompasses three main components: the simplicity device, terminal broker, and the network broker; the simplicity device (SD for short) will be used to store user profiles, preferences and policies, it also enables automation and exploitation of different network capabilities. The terminal broker (TB for short) enables the interaction between the information stored in the SD and the terminal, allows service adaptation, service discovery and usage. The role of the network broker (NB for short) is to support service advertisement, discovery and adaptation; additional roles of the NB include sharing/allocating available resources and managing network value added functionalities such as differentiation of quality of service, location context awareness among others.

The reason why *Simplicity project* is related with *Network Selection Games: Simulation Tool Development and Performance Evaluation* even though the main focus authors on [12] gave to it was an operator oriented, for which the decisions are taken by the system (e.g GSM systems) and not directly by the user, is because some inputs to the decision process are taken from the SD such as the user profile and preferences. The selection process ultimately drive *Mobile Nodes* (MN for short) to the most suitable access point according to operator policies based on network and application service status. The simulation developed by the authors was done on 802.11 wireless access domain; on the given context the TB perform frequency scanning allowing them to discover surrounding access points by learning their layer 2 identity, then after, they transmit this information to the NB where the selection procedure will be executed. The process results to be transparent to the user, and in the specific case of *Simplicity project* it aimed to achieve load balancing on the network operator side.

Network Selection Games: Simulation Tool Development and Performance Evaluation is steering a performance evaluation for a network selection problem, assuming it has access to a common knowledge base. Such knowledge base includes information about number of interferers and achievable rates between a particular user and access point, therefore, the previously mentioned information must be obtained in some way; the work done by the authors on [13] drives a network selection mechanism that is based on the MAC-layer bandwidth a user would receive after affiliating to certain access point, they call it *potential bandwidth*. The selection process is preceded by passive measurements that do not require a host to be affiliated to an access point just like what authors did on [10] thus allowing an user to evaluate the potential bandwidth of multiple access points in range, additionally the authors have proposed a methodology for the estimation of the potential upstream and downstream bandwidth based on measured delays on the 802.11 beacon frames.

The main challenges on the work done in [13] included: the fact of the implemented algorithm to don't be intrusive in the sense of not introducing noticeable overhead, and the ability to be transparent AP side, not requiring an user to associate and dis-associate to it. The core of the work resides on the measured delays of beacon frames transmitted by the different APs in range, such delays capture the load of a specific AP and the contention inside of a network; the aforementioned delays will provide an estimate for the client's AP downstream, whilst the upstream potential bandwidth relies on frames sent by the user to the AP nevertheless measured with a similar methodology. Authors performed experimental studies under a low noise scenarios which yielded outperforming results when compared to ordinary association policies (based solely on signal strength), however authors state there is challenging work under noisy scenarios and synchronization of time-zero beacon references. All in all the work can clearly aid the network selection process through the implemented methodology on [13].

Related work on channel selection from the access point perspective is analyzed by authors on [14], the motivation starts off by making apparent the increasing number of independently owned IEEE 802.11 WLANs by autonomous users, which results on increased interference, performance degradation and unfairness. The novel proposal of the work consists on the acquisition of a factor called *disruption factor*, along with the proposal of a socially conscious channel selection schemes based on game theoretic learning.

Unfairness on 802.11 collision domains is due to access point location, depending on the geographic location, some access points may experience lower throughput performances compared to others, authors define these as *starved* links or networks; on a channel selection scheme an AP may not always improve its throughput by unilaterally switching to a different channel, therefore the contribution of the work relies on showing that fairness among independent access points can be improved by making the WLANs causing the starvation of other networks, to be able to detect it and try to alleviate it, authors call these networks *socially conscious networks*, because they proactively improve the welfare of disadvantaged networks.

The game theoretic approach authors on [14] used to develop the work, is based on game theoretic learning, arguing to overcome the assumption of common knowledge of the set of players and strategies; particularly during the work authors use two different learning algorithms: Best response learning and Internal Regret minimization learning , the former assimilates the *Best Response* algorithm used on *Network Selection Games: Simulation Tool Development and Performance Evaluation*, similarly the latter assimilates the *Myopic Fictitious Play* developed algorithm in the sense of using a history of periods to base off the decision. A novel contribution on the paper, is the so called disruption factor, which is a measure of the difference between the channel activity when a certain player is participating actively in such channel to when it is not, for example a high value of a disruption factor would indicate that there is more activity when user i is passive compared when it is active, hence player i maybe causing unfair starvation to one or more users of the same channel. Work concludes with performance evaluation compared to two previously existing channel selection schemes, showing an improvement of nearly 30% with respect to achievable throughput and about 17% improvement on fairness.

From the previous studied related work, it has been clearly shown that the way most wireless devices connect to an AP depends on the received signal strength indicator (RSSI for short) procedure which leads to inefficient and unfair usage of radio resources [14,17]; thereafter different metrics have been used such as the airtime cost, for which it can be inferred the achievable throughput a user can obtain from certain AP by making passive measurements on uplink beacon delays. On [17] the author proposes a game theoretic approach which extends results on atomic congestion games and proves convergence to Nash Equilibrium for the implemented model.

The model assumes a generic number of access points and users randomly deployed over a geographical area, the APs periodically broadcast beacon frames which further allow users to identify them for a possible association. An assumption of non-overlapping cells is made, that is adjacent cells in the geographical area do not share the same channel frequency in order to avoid interference. The strategy set a generic user can choose from is the set of APs the user can connect to and the payoff function is the airtime cost of associating with a particular AP. From the previous settlement, the author exposes the idea that the average latency of the network depends on the loads of the APs and it will be minimized if the loads are balanced, however in user association games each user tries to selfishly reduce its own latency not caring for the social welfare, the dynamics of the aforementioned behavior gives rise to a non-cooperative game whose stable outcomes are called Nash Equilibria; the aforementioned type of games belong to a sub-class of games called congestion games on which users compete for a set of resources and the cost of each resource merely depends on the number of users using such resource, more technically the game belongs to the class of atomic congestion games for which a user is completely assigned to a resource (i.e no multi-homing). The core contribution of the work done on [17] is the following theorem: *The airtime metric based user association scheme converges to a Nash Equilibrium solution after a finite number of steps*. The author shows a formal proof of the former theorem and the efficiency of equilibrium when compared to centralized optimum solutions established under different system costs.

Last but not least, *Network Selection Games: Simulation Tool Development and Performance Evaluation* was a thesis proposal made by Prof. Matteo Cesana, co-author of the works in [16,17] which address the problem of network selection [16,17] and resource allocation [16]. The problem *Network Selection Games: Simulation Tool Development and Performance Evaluation* is addressing is the lack of performance evaluation results in literature, therefore based on [16,17] the theoretical background, related topics and works and perspectives were supplied by the aforementioned papers.

On [16] the authors propose a game theoretic approach to tackle the two networking problems, namely the one casted on the user side, and the one casted on the network side, on the former each user tries selfishly to maximize its perceived quality of service, on the latter each network operator will try to maximize the number of associated customers per access point yielding a higher revenue. Through the paper the authors expand the problem in two different scenarios, first, an scenario of solely network selection, where the radio resources are statically assigned, and a joint problem of resource allocation and network selection.

The resource allocation problem is tackled and resolved by resolving to the theorem that states: “Every congestion game with player specific function always admit a pure strategy Nash Equilibrium” from there one, the game theoretic approach is formulated as a mathematical programming formulation (i.e A linear programming formulation) which is used to find and characterize the different Nash Equilibria of the games.

The joint problem is resolved by means of iteratively solving the former problem for different strategy profiles and hence for different assignation of AP frequencies as an example. Authors concluded with numerical results the propositions and theorems stated on the proposed approaches.

3.

BACKGROUND ON GAME THEORY

The reason why this thesis work is called *Network Selection Games: Simulation Tool Development and Performance Evaluation* is because it is indeed using game theory to develop the algorithms that simulate a wireless network (different wireless devices owned by different users and multiple access points spanning different IEEE 802.11 b frequency bands ratios) selection problem by means of modeling the scenario as a game.

The relevance of game theory to the network selection application is almost straightforward, think of an area covered by different wireless access points e.g. a public conference hall in the center of Milan, for the ease of the example consider that the area is covered by a LAN wireless access point which gives free access to the people being inside the conference hall and it is covered as well by a MAN wireless access point which gives free access to all citizens in certain areas of the city, the two named access points are called *strategies* and the wireless devices carried by the citizens will be called *users*. The scenario can be called a game because each user inside the conference hall will have the *choice* to connect to either the organizer wireless access service provided by the LAN AP or by the MAN wireless access service provided by *Comune di Milano* AP, the *payoff* to their choice is the achievable rate or delivered bandwidth, certainly the user that connects to the less crowded and geographically more convenient access point will have most likely the best performance delivered by the strategy selected.

Once the game has started it is not trivial to know the outcome of it without any information [1], for example, if all except one person inside the hall decided not to use their wireless devices then all these users are “helping” the one person utilizing its wireless device, however if two or more users are trying to connect to the same access point they would probably “hurt” each other by making the total deliverable bandwidth lower. *Network Selection Games: Simulation Tool Development and Performance Evaluation* will assume for all the scenarios studied a *non-cooperative game* in which all the users will play the same strategy with the same payoff function under different coverage areas where the access points will be allocated particularly such to study interesting scenarios.

While the game is played and the users interact with each other trying to selfishly maximize their payoff, the sole outcome of the game is to have all users covered under such circumstance that a single player will not deviate from its chosen strategy without making anyone else worse, in other terms the objective of the game is to find a *Nash Equilibrium*.

3.1

Non-cooperative games

The two most basic types of non-cooperative games are called games in *normal* or *extended* form. The reason behind being categorized under the non-cooperative game fashion is because the user preferred set of actions can be in conflict with each other [2], the possible existence of a conflict does not necessarily mean it has to be, hence non-cooperative games can lead to cooperation.

3.1.1

Games in normal form

The games in normal form take place where two or more users will select at the same time one of possible choices the strategy space spans. The strategy space depends on the type of game that is being played, for example and as it was said before on X.1, the strategy space for the games simulated in this work will be access points, however the strategy space can be made up of almost anything imaginable such as: “Cooperate” and “Defect” actions as described on the famous Prisoners Dilemma game [1,2].

Actions under normal form games can be taken specifically (or deterministically) yielding *pure strategy equilibria*, or each action can be taken with some fixed probability yielding in convergence *mixed strategy equilibria*. *Network Selection Games: Simulation Tool Development and Performance Evaluation* deploys across its variety of possible games the two ways users take actions, the decision is deterministic and rational for all games except for the two implemented versions of *Stochastic Fictitious play*.

The way the game develops, its more convenient representation is a *payoff* matrix, whose rows and columns make up for the combination of all possible strategies, rendering each cell of the matrix to be the individual payoff each user receives for their joint actions. When representing a game with its payoff matrix it is guaranteed to find an equilibria as stated by Nash: “*All game matrices have at least one equilibrium strategy, but this strategy might be mixed*”

$User1/User2$	$LAN\ A.P$	$MAN\ A.P$
$LAN\ A.P$	$12Mbps/5Mbps$	$20Mbps/12Mbps$
$MAN\ A.P$	$7Mbps/8Mbps$	$10Mbps/10Mbps$

Figure 3.1 Payoff matrix for a 2-player game

Figure 3.1 is showing the payoff matrix for the hypothetical example of two users inside the conference hall located in Milan; note how the joint actions derive different payoff values, the decisions are made under the *common knowledge* assumption, that is, User1 knows that User2 knows that he knows the utility values User2 will receive from each joint action [2]. Common knowledge is a motivation to further introduce the concepts of *iterated dominance* and *Nash Equilibrium* [1], these concepts are dynamically achieved during the computation of the simulations and will be explained more in detail in the upcoming paragraphs.

Expanding the term *common knowledge* in the *Network Selection Games: Simulation Tool Development and Performance Evaluation* context is reduced to either matrices or cells that contain the number of interferers per access point and the achievable rate each user can obtain from a given access point. For all games except those who use the learning technique *Fictitious Play*, the common knowledge will be a public matrix where the actual payoff each user is receiving is shown, an example is illustrated on figure 3.2

$$\begin{bmatrix} 1 & 1 & 1 & - \\ 2 & 2 & 2 & - \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 3 \end{bmatrix}$$

Figure 3.2 Sequence of actions on a normal form game

Suppose now there are 4 users on the conference hall, 3 of them were already connected to 2 access points, row 1 of figure x.2 matrices represent the LAN AP and row 2 represents the MAN AP, the fourth user decides to turn on his wireless device, and then the device decides to connect to the MAN AP as seen on the rightmost matrix of figure 3.2, in terms of number of interferers the decision was an *efficient* one, efficiency refers to *Pareto Optimal* solutions. These from a social welfare point of view make irrelevant a user complaining about getting more payoff without hurting another user in the process [2]. As seen on the two consecutive actions shown on figure 3.2 the solution presented is not Pareto optimal, since any of 2 users (the two users that were connected before the fourth decided to turn on his wireless device) would complain they could go to the LAN AP and obtain a number of interferers lower than the actual.

The one important problem attached to Pareto optimal solutions is stability, for example, the fact that one user during his last turn decides to play a different strategy because it would yield him a bigger utility not caring about "hurting" all the other users will prevent the system from convergence which is for the specific context of *Network Selection Games: Simulation Tool Development and Performance Evaluation* one of the performance parameters to be studied. The problem of stability was solved by John F. Nash and the concept is studied in the upcoming Section.

Network Selection Games: Simulation Tool Development and Performance Evaluation is not technically capable of simulating normal form games, a piece of code, a program, is a sequential execution of instructions (a batch process) hence without practical manipulation of the code the concurrent actions of users is not achievable. *Network Selection Games: Simulation Tool Development and Performance Evaluation* offers for its *Best Response* and *Better Response* algorithms normal and extended form game simulations, on the former the code is manipulated accordingly to resemble the concurrent execution of actions of a generic number of users.

3.1.2

Games in extended form

In extended form games, users take actions sequentially, therefore their most appropriate representation is a tree where the branches represent the different player actions and the payoff values for the sequence of actions is given at the leafs [2].

A possible extended form version of the payoff matrix of figure 3.1 by adding an additional user is represented in the tree shown on figure 3.3

From the three it can be seen that all leafs except those with payoff (3,3,3) correspond to game equilibrium, and more specifically Nash Equilibria of different quality for each player, however the three player game results to be symmetric giving to each user two outperforming equilibria (1,2,2),(2,1,2) and (2,2,1) respectively for players 1,2 and 3. However it should be noted that the players were distributed on the three in ascending order, giving player 3 always the rational choice to choose its best performing function, however in *Network Selection Games: Simulation Tool Development and Performance Evaluation* to bias-off these behaviors, the order in which users take turns is random for every game simulated.

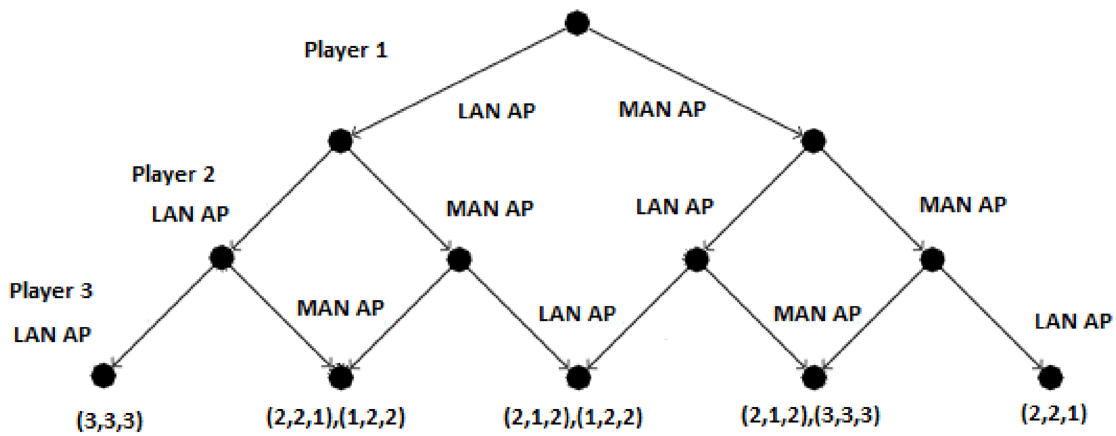


Figure 3.3 Extended form game tree for a three player scenario

The Nash Equilibrium solution concept can be extended from normal form games to extended form games, it will consist of a strategy such that no player can get more utility by playing a different strategy given the fact everyone is playing the Nash equilibrium strategy, however and as noted on the normal game strategy, the Nash equilibrium solution is not guaranteed to be stable under noisy conditions. A stronger solution concept introduced in [2] consists in the so called sub-game perfect equilibrium, considered as a strategy such that for all agents and all sub-games (i.e any sub-tree of the extended game) it holds that a generic user cant gain more utility by playing a strategy different than the sub-game perfect equilibrium one.

3.2

Nash Equilibrium

A quasi formal definition of a Nash equilibrium is a profile of strategies such that each player's strategy is an optimal response to the other player strategies [1]. Resembling the example shown on figure 3.2, it starts off from Nash equilibrium as seen on the leftmost matrix of figure 3.2, however note that before user 4 turns on his wireless device (3 active users) there are multiple Nash equilibria, these are shown on figure 3.4 and 3.5

$$\begin{bmatrix} 1 & 1 & 1 & - \\ 2 & 2 & 2 & - \end{bmatrix}$$

Figure 3.4 Nash equilibrium # 1

$$\begin{bmatrix} 2 & 2 & 2 & - \\ .1 & 1 & 1 & - \end{bmatrix}$$

Figure 3.5 Nash equilibrium # 2

Define a non-cooperative game, $\Gamma = < N, \{S_i, u_i\}_{i \in N} >$ is comprised of a set of agents *N = 1,…, n*, and for each agent $i \in N$, a set of strategies S_i with the interSection of all these strategies making up the global strategy space *S*, an utility function $u_i : S \rightarrow \mathcal{R}$. A joint strategy profile $s \in S$ is referred to as an outcome of the game, where *S* is the set of all possible outcomes, and each agent's utility function specifies the payoff they receive for an outcome by the condition that, if and only if the agent prefers outcome *s* to outcome *s'*, then $u_i(s) > u_{i'}(s)$, in other words, each player's utility function ranks their preferences over outcomes [18].

To formalize completely the definition of a Nash Equilibrium, the one given on [18] is adopted here:

Nash Equilibrium: A joint strategy profile, $s*$, such that no individual agent has an incentive to change to a different strategy is a Nash Equilibrium.

$$u_i(s_i^*, s_{-i}^*) - u_i(s^i, s_{-i}^*) \geq 0 , \forall\, s_i, \forall\, i.$$

Vidal [2] states that a game can have more than one Nash equilibrium and some of these equilibria might be better for one player than another, and this fact is perfectly reflected on the former two figures, note on figure 3.4 how the Nash equilibria benefits the user connected to the LAN, and how it benefits the user connected to the MAN AP on figure 3.5, therefore an argue between the users about which equilibrium to adopt arises, however once the users settle on which one to adopt the solution is very stable; suddenly user # 4 turns on his wireless device, he immediately knows that the current number of interferers are the ones presented on figure 3.4 or figure 3.5, since the previous 3 players played the Nash equilibrium, then the best thing user 4 could do is play its Nash Equilibrium yielding:

- Selecting LAN AP if the former situation was the one presented on figure 3.4
- Selecting MAN AP if the former situation was the one presented on figure 3.5

Any other selection will just give user 4 a worse payoff. Actually, the way *Network Selection Games: Simulation Tool Development and Performance Evaluation* iteratively plays the Nash Equilibrium is choosing the less crowded access point for each user on each iteration, due to the assumption of common knowledge, an *interference matrix* is checked during each turn, if a new access point is found with less interferers than the actual, then the user migrates to it, if not, the user remains on its actual strategy and the game goes on for a fixed number of iterations, a piece of code checks at the end of all iterations if the selected strategies by the users corresponded to a Nash equilibrium, if it is, the elapsed simulation time, number of total iterations (how many times a user selected/changed its strategy) are saved for later processing.

Additional properties of Nash equilibrium

As stated on previous paragraphs, all finite strategic-form games are guaranteed to posses at least one Nash equilibrium; however there are two additional properties that hold only for "almost" all strategic-form games. The author on [1] means by "almost" all the finite strategic-form games, a set of games or namely a set of payoff vectors with the number of players and strategy spaces kept fixed, thereafter these games are considered open and dense in the Euclidean of dimension $I \cdot \prod_{i=1}^{I} S_i$.

Oddness theorem

Stated on 1971 by Wilson [1], states that almost all finite games have a finite and odd number of equilibria.

A game set up properly made on *Network Selection Games: Simulation Tool Development and Performance Evaluation*, can be further checked to see how many equilibria the simulated game had by either doing online debugs, or by checking the output files that are optionally made on the output menu of the tool.

Robustness of Equilibria to Payoff perturbations

A modeler is a role whose job is to specify payoff functions; in reality is highly unlikely that the modeler will have specified payoff functions that are exactly correct [1], the problem of determining whether the Nash predictions of the original game with payoff functions u are sufficiently appropriate Nash predictions of the real game with nearby payoffs $\hat{u}$.
The author on [1] motivates the definition of a forthcoming definition by introducing distances between payoff vectors and between strategy profiles.

Definition: A Nash equilibrium σ of a game u is *essential* or *robust* if for any $\varepsilon > 0$ there exists $\eta > 0$, such that for any $\hat{u}$ such that $D(u, \hat{u}) < \eta$ there exists a Nash equilibrium $\hat{\sigma}$ of game $\hat{u}$ such that $d(\sigma, \hat{\sigma}) < \varepsilon$. A game u is essential if all its equilibrium points are essential.

According to Wu-Jiang's theorem states that the Nash equilibrium of generic strategic-form games are robust to perturbations of the payoffs, and this results of sole interest to simultaneous-move games[2], games such as normal form games or extensive form games having a sub-tree rounded by ellipses.

3.3

Convergence

The mathematical definition of convergence is: The property or manner of reaching a limit, such as a point, line, value or function. Therefore convergence is the most desired property in gaming theory as it makes a game finite and stable to a solution; on the other hand if no convergence exists for a particular game, it is said to be caught in an endless cycle without a stable solution, clearly a non-desirable property for any kind of game. As stated on Section 3.1, on certain particular games there may exist different kinds of equilibria and different rates of convergence to such equilibria if they even exist. The former is addressed by studying the *Price of Anarchy* and *Price of Stability* which are the worst/best case ratio between an optimal solution and a Nash equilibrium respectively [17,19].

In the following, Section 3.3.1 will present results convergence wise for iterative competitive games, Section 3.3.2 will present convergence speed results for singleton congestion games, however before going on with these Sections, the following remarks are taken from the studied literature, found in the "references" Section which point out convergence properties.

- If fictitious play converges to a pure strategy then that strategy must be a Nash Equilibrium (Fundenber & Kreps, 1990) taken from [2].
- Every (unweighted) congestion game possesses a Nash equilibrium in pure strategies, and this equilibrium is found in at most $\binom{n+1}{2}$ steps [3].
- The airtime metric based user association scheme converges to a Nash equilibrium solution after a finite number of steps. [15]
- The finite improvement property ensures that the behavior of agents who play "better responses" in each period of the repeated game converges to a Nash equilibrium in finite time [18].

3.3.1

Speed of convergence to approximate solutions in iterative competitive games

Authors on [19] made an interesting study on the convergence issues on competitive games (i.e non-cooperative games), their study is not focused entirely on studying the obtained Nash Equilibrium but rather on studying the evolution after an initial state how a social function approaches an optimal social value, the former, was modeled by assuming a best response policy on the set of strategies for each user. In a broad sense, the work developed on [19] consists on studying all the paths available in the state transition graph and evaluate the social function at states along these paths.

The state transition graph mentioned above is actually modeling the selfish behavior of each user, and has the following properties:

- Each vertex represents a strategy state: $S = (s_1, s_2, \dots, s_n)$.
- The arcs in the graph correspond to best response moves by players.
- The graph may contain loops.
- A best response path is a directed path in the state graph.

Additional model properties assumed were:

One round path: From an arbitrary ordering of players, a one round path is such that starts from an arbitrary state and contains edges ordered in the same arbitrary way.

k-Covering path: Each player plays at least once on each one of the k-paths.

The contributions of the work done on [19] towards convergence are the following:

Theorem 1: In basic-utility games, the social value of a state at the end of a one-round path is at least $\frac{1}{3}$ of the optimal social value.

Authors stress the above result as a very quick convergence speed, because just after one round of the so-called selfish behavior it is guaranteed to find a social solution with a value at least $\frac{1}{3}$ that of the optimal.

Theorem 2: In basic-utility games, the social value of a state at the end of a one-round path beginning at $T = \phi$ is at least $\frac{1}{2}$ of the optimal social value and this value is tight.

The above theorem matched the fact that any Nash equilibrium in any valid-utility game has a value within a factor of 2 of optimal, and the most interesting result is the fact of having achieved the aforementioned just after one round. It must be reminded that $T = \phi$ stands for an initial strategy state is null.

Theorem 3: In general valid-utility games, the social value of some state on any one-round path is at least $\frac{1}{2n}$ of the optimal social value.

From theorem 2 authors on [19] discovered that there is the possibility of convergence to low quality states in games in which every Nash equilibria is of high quality, the former renders theorem 3 tight as theorem 2.

Convergence to low quality states is called *cyclic equilibria*, it is conducted by a best response path leading onto a cycle whose solutions are extremely bad socially, despite the fact of the existence of socially good Nash equilibria. The presence of low quality cyclic equilibria results disturbing because low price of anarchy values can be achieved yet having states of very poor social quality.

Theorem 4: There are valid-utility games in which every solution on a k-covering path has a social value at most $\frac{1}{n}$ of the optimal solution.

Despite the formalization of theorem 4 on [19], the cyclic equilibria may be left by permutations in the way players make their moves, or take turns.

3.3.2

Best Response dynamics in player-specific singleton congestion games

In the following the convergence time of best response dynamics to pure Nash equilibria in player-specific singleton games is studied. According to the authors on [20] the above dynamics can cycle, therefore the core proposal consists on studying random best response dynamics for player-specific singleton congestion games motivated by the fact that for every of this type of games there exists a polynomially long sequence of best responses leading to a Nash equilibrium, thus the random best response dynamics selecting the next player to play a best response at random terminates with a probability one after a finite number of steps (Milchtaich, 1996).

Experimental results on [20] support the following conjecture:

There exist player-specific singleton congestion games in which the expected number of steps until the random best response dynamics terminates is super-polynomial.

The core contribution made by the authors on [20] towards convergence is two-fold as follows:

- Player-specific congestion games on trees admit a potential function from which it can be derived an upper bound of $O(n^2)$ on the maximum number of best responses until a Nash equilibrium is reached.
- Player-specific games on circles (i.e cyclic games) yield a bound of $O(n^2)$ on the expected number of steps until the dynamics terminate.

Two terms are also introduced to prove the above results, an overload token meaning that a resource is being shared by more than one user, and an underload token indicating that a resource is not being used currently.
For the player-specific congestion games on trees, the following theorem was proposed and proved on [20], considers a tree-game for which the number of resources equals the number of players and that player-specific delays functions can be replaced by common delay functions yielding the game a standard singleton congestion game.

Theorem 2: In every player-specific game on a tree with n nodes, every best response schedule terminates after at most $2n^2$ steps.

The proof of the former, is made by induction on the number of players and by sequentially building n-player specific games with particular properties.

Theorem 4: In every player-specific congestion game on a circle the random best response dynamic terminates after $O(n^2)$ steps in expectation.

Further experimental studies made on [20] with respect to cyclic games are made for the different type of players defined (see Section 3 of [20]). The work concludes with a simulation made on a varying number of users congestion game, which yields a polynomial and almost exponential convergence time as n increases; similar results were obtained in the performance evaluation seen in Section 5, for specific utility functions under the best response strategy the increasing number of simulations tends to be exponential.

3.4

Congestion games

A class of non-cooperative games in which the players share a common set of strategies and for which the particular payoff a player receives for playing certain strategy depends only on the total number of players playing the same strategy are known as *congestion games*; the term congestion[1] involves competition, involves common and possibly conflicting interests, imagine of a limited set of resources available to a particular group of animals, the situation known as ideal free distribution (Maynard Smith, 1982, p.63 taken from [3]) or IFD for short, occurs when each individual settles to the amount of resources needed just for its reproduction and survival, nevertheless even under presence of completely rational agents, there might exist greed amongst one of them no allowing to reach the ideal IFD status.

The aforementioned scenario was modeled by Rosenthal (1973) as a class of games in which each player chooses a particular combination of factors (can be only one factor as well) out of a set of primary factors, the payoff associated to each primary factor is a function of the number of players who included it in their choice, and ultimately the payoff each player will receive is the sum of the payoffs associated with the primary factors associated in his/her choice. For the particular case of the Network Selection problem and *Network Selection Games: Simulation Tool Development and Performance Evaluation* context, the class of games in which the core problem resides is indeed a class of congestion games assuming each player chooses only one primary factor and that the payoff received actually decreases with the number of other players selecting the same primary factor. The relation is direct due to fact of users populating/selecting access points whose obtained payoff results particular to every player depending on the number of associated users to each station and the achievable rate (downlink rate per se.) .

Congestion games have a great practical interest because it was shown by Monderer and Shapley (1991) that this type of games possesses at least one pure-strategy Nash Equilibrium, the former is only valid when the considered game is "weighted", however when the game is "unweighted" a Nash Equilibrium is not always guaranteed to be found, nevertheless under the assumption of a stochastic order of deviators [2] convergence almost always occurs.

The following model is often referred as an *unweighted congestion game* if these conditions hold [3].

- n players share a common set of r strategies.
- The payoff player i th receives for playing the j th strategy is a monotonically decreasing function S_{ij} of the total number n_j playing the j th strategy.
- Denoting the strategy played by the i th player by σ_i, the strategy-tuple $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ is a Nash Equilibrium iff each σ_i is a best reply strategy:
$$S_{i\,\sigma_i}(n_{\sigma_i}) \geq S_{i\,j}(n_j + 1) \text{ for all } i \text{ and } j$$

In the following, two different cases of unweighted congestion games, existence and convergence to equilibrium will be studied.

Symmetric case of Congestion Games

A congestion game is considered symmetric if and only if all players share the same set of payoff functions [3], namely all the players can choose a strategy out of the whole set of existing strategies for the game; under the *Network Selection Games: Simulation Tool Development and Performance Evaluation* scenario, and for simulations that hold the *All in range* assumption, an unweighted congestion symmetric game will be played.

Rosenthal (1973, taken from [3]) defined for this kind of symmetric games and exact potential function:

$$P(\sigma) = \sum_{k=1}^{r} \sum_{m=1}^{n_k} S_k(m)$$

For the previous potential function the following holds:

- When only one player say i th shifts to a new strategy, the j th one, the potential changes by
$$\Delta P = S_j(n_j + 1) - S_{\sigma_i}(n_{\sigma_i}),$$
which results to be equal to what the i th player gains or losses.
- A strategy-tuple where changing one coordinate cannot result in a greater value of P is known as a "local" maximum and corresponds to a pure-strategy Nash Equilibrium.
- The existence of an exact potential function further implies the *finite improvement property* (FIP for short) [3].

In order to understand properties related to the FIP, the following definitions are made:

- Any sequence of strategy-tuples in which each strategy-tuple differs from the previous one in only one coordinate is called *path.*

- If a unique deviator in each step of the sequence strictly increases the payoff he receives is called an *improvement path*. The improvement path is finite.
- The first strategy-tuple of the path is called *initial point.*
- The last strategy-tuple of the path is called *terminal point*.
- Any *maximal* improvement path, an improvement path that cannot be extended is terminated by an equilibrium.

Non-symmetric congestion games do not generally admit an exact potential function, nevertheless, in the special case when $r = 2$ the following theorem holds:

Theorem: Congestion games involving only two strategies posses the finite improvement property.

The proof is done by contradiction on [3], however it can be initially thought as a simple case when $n = 2$ players and $r = 2$ strategies, once in the initial path no player will deviated from the strategy since it will be their one and only strategy as the game is asymmetric. If the game is extended to three players, the same analysis is made, one or more players will only be able to choose one strategy, the rest will enjoy the properties a symmetric congestion game holds as described above.

Games without the Finite Improvement Property

The finite improvement property is equivalent to the existence of a *generalized ordinal potential* for the game under consideration, in other words, it is a real valued function over the set of pure strategy-tuples that strictly increases along any improvement path [3].

Games that do not posses the FIP property in general contain cycles, whose first strategy-tuple results to be the same last strategy-tuple, existence of cycles prevents the admittance of a generalized ordinal potential, however the previous argument does not prevent the existence of pure-strategy Nash equilibrium.

For the games having paths on which on each step the unique deviator shifts to a different strategy which is a best reply against strategies played by other players is called *best reply paths* [3]. When all players deviate from their former strategies when their currently played strategy is not a best reply, then the path is called a *best reply improvement path*, because a unique deviator playing a best reply strategy is guaranteed to have an improvement; FIP implies the *finite best-reply property* (FBRP), however the converse is not true.

Existence of a pure-strategy Nash Equilibrium

Author on [3] demonstrated the following theorem:

Theorem: Every (unweighted) congestion game possesses a Nash Equilibrium in pure strategies.

The previous theorem covers symmetric and asymmetric games, hence backing off the existence of at least one Nash equilibrium for the majority of the scenarios studied on the performance evaluation Section.

The proof is made by induction on the number of players, starting from the trivial $n = 1$, going through n, assuming the theorem holds true for every *(n-1)* player congestion game. In a nutshell the proof made for the previous theorem was constructive, in the sense of finding an equilibrium for a n-player game, by adding one player after another from 1 to n, in at most $\binom{n+1}{2}$ steps.

A more robust theorem is proposed by the same author in [3] where the existence of an equilibrium is found when there are *n* simultaneous players being considered.

Theorem: Given an arbitrary strategy-tuple $\sigma(0)$ in a congestion game Γ, there exists a best-reply improvement path $\sigma(0), \sigma(1), \ldots, \sigma(L)$ such that $\sigma(L)$ is an equilibrium and $L \leq r\binom{n+1}{2}$.

The theorem is again proved by induction, by exploiting the properties of best-reply paths, the author found an upper bound $r\binom{n+1}{2}$ to the length of the shortest best-reply improvement path that connects an arbitrary initial point to an equilibrium.

3.5

Game Strategies

Game theory dynamics involve making decisions (i.e choosing a strategy from the available space), the way the decisions are taken will yield different game outputs; as seen on Section 3.2 a particular way of making decisions can lead to a Nash equilibrium, for instance if all players decide to play a Best Reply strategy, no single player can obtain a greater payoff by deviating from the best reply dynamic onto another strategy.

In the following, three different game strategies will be described, namely, the Best and Better response strategies, along with a learning technique called Fictitious Play. The formerly listed strategies are particularly the same ones implemented on *Network Selection Games: Simulation Tool Development and Performance Evaluation*.

3.5

Best Response Strategy

As it name states, Best reply strategies always involve optimal strategies, under the framework of a non-cooperative game it is assumed that each player selfishly tries to maximize its own payoff conditioned by the choices of its opponents.

Formally defining a Best Response strategy in a broad sense according to [18]:

A *best-response correspondence* $\beta_i(s_{-i})$, is the set of agent *i's* optimal strategies, given the strategy profile of its opponents, $\beta_i(s_{-i}) = argmax_{s_i \in S_i}\{u_i(s_i, s_{-i})\}$. Stable points in such a system are characterized by the set of Nash Equilibria.

Best-reply dynamics and Network Selection Games: Simulation Tool Development and Performance Evaluation

The Best response algorithms developed during the work comply the best-response correspondence described above, by means of a player selecting its optimal strategy given the strategy profile of its opponents; in practical terms, each user during its particular turn is able to know how are the resources being used and make a decision based on this knowledge, in some cases, the optimal strategy can be yielded from an *argmax* or *argmin* function depending on the type of game being considered.

3.6

Better Response Strategy

Better reply dynamics are derived from best reply dynamics by swapping from a deterministic adjustment process onto a stochastic one. The formal definition is taken from [21] and assumes a global objective of converging to a pure strategy Nash equilibrium although not requiring it to be unique, giving greater relevance to the fact of making the system settling to an equilibrium.

Model

Players have status quo actions and these are randomly selected, one at a time, to sample new actions. When a player is selected to sample, she randomly draws one of her available actions and only changes her status quo to the sampled action if it improves her payoff [21] (i.e single player improvement).

The process is called stochastic because the players move from their current actions to a better reply, not necessarily a best reply; even though players move in the direction of their best replies, they can overshoot or undershoot them.

Features

- Better-reply dynamics are consistent with a player not having precise knowledge, or memory, of her own and her opponents payoff functions and past actions.
- Finite games having the weak FIP using better-reply dynamics globally converge to Nash Equilibrium.
-

Definition

Consider a continuous game *g*. Let P_i be a probability measure on the *Borel* subsets of A_i such that for any open interval $I \subset A_i$, $P_i(I) > 0$. At each discrete time period *t* there is a status quo action profile a^t. A single player $i \in N$ is randomly selected, with all players having positive selection probability. Player *i* randomly samples action $x_i \in A_i$ according to the probability measure P_i. If a^t/x_i is a single player improvement over a^t, then it becomes the new status quo $a^{t+1} = a^t/x_i$. If $U_i\left(a^t/x_i\right) \leq U_i(a^t)$ then the status quo does not change, $a^{t+1} = a^t$ [21].

Better-reply dynamics and Network Selection Games: Simulation Tool Development and Performance Evaluation

Network Selection Games: Simulation Tool Development and Performance Evaluation bases its Better Response algorithms on the model and definition based on [21]. For ease of practical uses the way a user samples the strategy space is based in a myopic variable called $\propto$, $0 \leq \propto \leq 1$, $\propto = 0$ being full myopic or blind with respect to the set of strategies meaning a player is not able to draw any strategy and sample it, on the other hand $\propto = 1$ being equivalent to a Best Response algorithm because the player can view the whole set of strategies (i.e the probability for which each strategy can be drawn to be analyzed is equal, whilst on the myopic cases $\propto < 1$ this probability is not equal for all strategies). After the set of strategies is drawn for sampling, each player will perform a best-reply dynamic on the given set, thus deriving a stochastic process as described by [21]

3.7

Fictitious Play Strategy

In the fictitious play process players behave as if they think they are facing a stationary, but unknown, distribution of opponents' strategies [22]. One important issue of the learning processes is the one of determining if the process succeeded in learning, particularly for fictitious play it would correspond to measure how well each player records the frequencies for which each player plays a strategy as if these frequencies were known, or made part of the common knowledge available to all players.

The goal is to determine if the learning technique will converge and to which strategy does converge given that all players playing fictitious play.

Two player Fictitious Play

A player *i* maintains a weight function $k_i : S_i \rightarrow \mathcal{R}^+$. The weight function changes over time as the agent learns. The weight function at time *t* is represented by k_i^t. It maintains the count of how many times each strategy has been played by each player *j*. When at time *t-1* opponent *j* plays strategy s_j^{t-1} then *i* updates its weight function with:

$$k_i^t(s_j) = k_i^{t-1}(s_j) + \begin{cases} 1 \; if \; s_j^{t-1} = s_j \\ 0 \; if \; s_j^{t-1} \neq s_j \end{cases}$$

Using this weight function, agent *i* can assign a probability to *j* playing any of its $s_j \epsilon S_j$ strategies with:

$$Pr_i^t[s_j] = \frac{k_i^t(s_j)}{\sum_{\sim s_j \in S_j} k_i^t(\sim s_j)}$$

From the last equation it is inferred that player *i* determines its best response to a probability distribution over *j's* possible strategies, it can be seen as well how *i* is assuming that *j's* strategy at each time is taken from some fixed but unknown probability distribution.

Asymptotic behavior of Fictitious play

- If *s* is a strict Nash equilibrium and it is played at time *t* then it will be played at all times greater than *t.* (Fudenberg & Kreps, 1990) taken from [2,22]

- If Fictitious play converges to a pure strategy then that strategy must be a Nash equilibrium. (Fudenberg & Kreps, 1990) taken from [2,22]

Since players do not keep track of conditional probabilities but only frequencies about played strategies by other players, they may not recognize the existence of cycles, when adding learning to multi-player systems can easily fall into cycles; a common solution for the former is the use of randomness, players sometimes will take a random action in an effort to exit possible loops and to be able to explore the strategy space [2].

Fictitious play with more than two players

When two or more players are involved it should be decided whether each player should learn individual models of each of the other agents independently or a joint probability distribution over their combined strategies. Individual models assume that each agent operates independently while the joint distributions capture the possibility that the other agents' strategies are correlated. For any interesting system the set of all possible strategy profiles is too large to explore (grows exponentially with the number of players); therefore most learning systems assume that all agents operate independently so they need to maintain only one model per agent [2,22].

Fictitious play and Network Selection Games: Simulation Tool Development and Performance Evaluation

Network Selection Games: Simulation Tool Development and Performance Evaluation has implemented two versions of Fictitious play, namely a deterministic and stochastic version. The former is a simplified version of the formalization made above, on this version, each player will make a Best Response action to all the previously learnt frequencies and weight values that are accounted on properly managed global variables, whilst the latter does its decision assuming a uniform probability distribution of the previous actions of each player, hence the actual player will have a probability distribution over the actions of its opponents rendering the decision to be taken stochastic. In addition to the deterministic and stochastic implemented versions of the Fictitious play algorithm, these were extended to be able to cut off previous history learned by a factor α, rendering the actual user myopic by a controllable percentage .

4.

TOOL DESCRIPTION

Network Selection Games: Simulation Tool Development and Performance Evaluation is a tool whose main objective is to allow its users to set up network selection performance evaluation with ease of use, being capable of simulating various scenarios and presenting results graphically. It can perform automated or step by step simulations. *Network Selection Games: Simulation Tool Development and Performance Evaluation* was developed using *C* language under *MATLAB®* framework, hence the code is not executable without using *MATLAB* engine.

The general features *Network Selection Games: Simulation Tool Development and Performance Evaluation* offers are:

- Various association policy options (insert footnote here, further explaining or making reference to a previous explanation on *association policies*) such as: *Pure Interference Based*, *Additive Interference and Achievable Rate* and *Multiplicative Interference and Achievable Rate Based* policies.

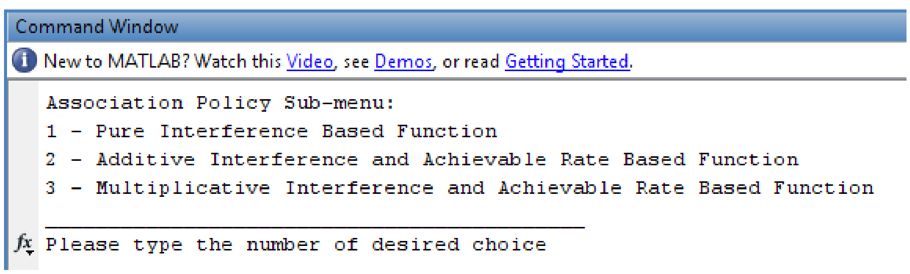


Figure 4.1 Snapshot of the *Network Selection Games: Simulation Tool Development and Performance Evaluation's* Association Policy sub-menu

- Numerous options for game strategy algorithms (insert footnote here, in order to explain more clearly or make reference to previous explanation of *game strategy algorithms*) including: *Best and Better response algorithms*, each one of them divided into sub-categories such as*: Beta-Simultaneous player* and *AP parametric radius coverage analysis* (the latter is only available for *Pure Interference Based* association policies). Learning strategies such as *Fictitious Play algorithms* are included in different flavors.

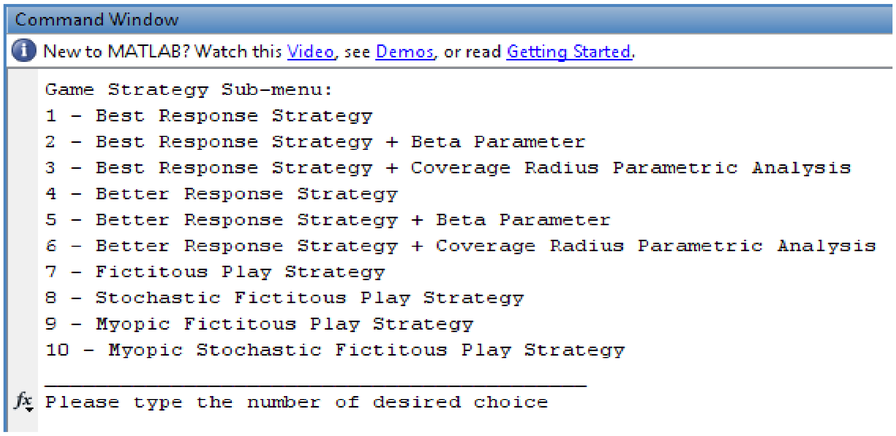


Figure 4.2 Snapshot of the *Network Selection Games: Simulation Tool Development and Performance Evaluation*'s Game Strategy sub-menu

- 4 Different types of game topology characteristics, ranging from randomly deployed APs and Users (*Random Topology)* to fully customized topologies where the user can allocate at will Users and APs into a rectangular map area, between these 2 type of topologies, there are 2 additional semi-customized topologies called *Linear Grid* and *Rectangular Grid* topologies. The following set of figures will show the *Network Selection Games: Simulation Tool Development and Performance Evaluation*'s topologies sub-menu and a preview of the different obtainable map types.

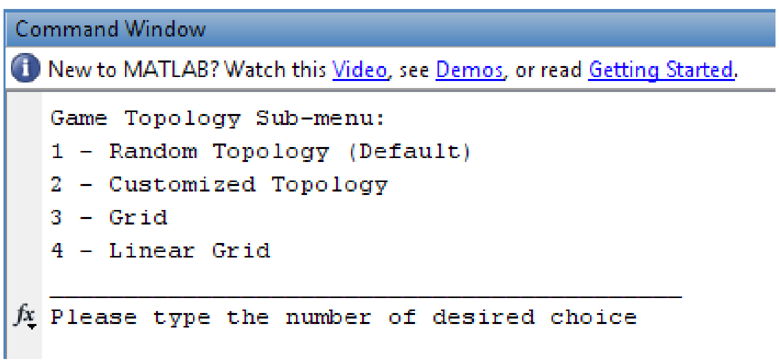


Figure 4.3 Snapshot of the *Network Selection Games: Simulation Tool Development and Performance Evaluation*'s Game Topology sub-menu

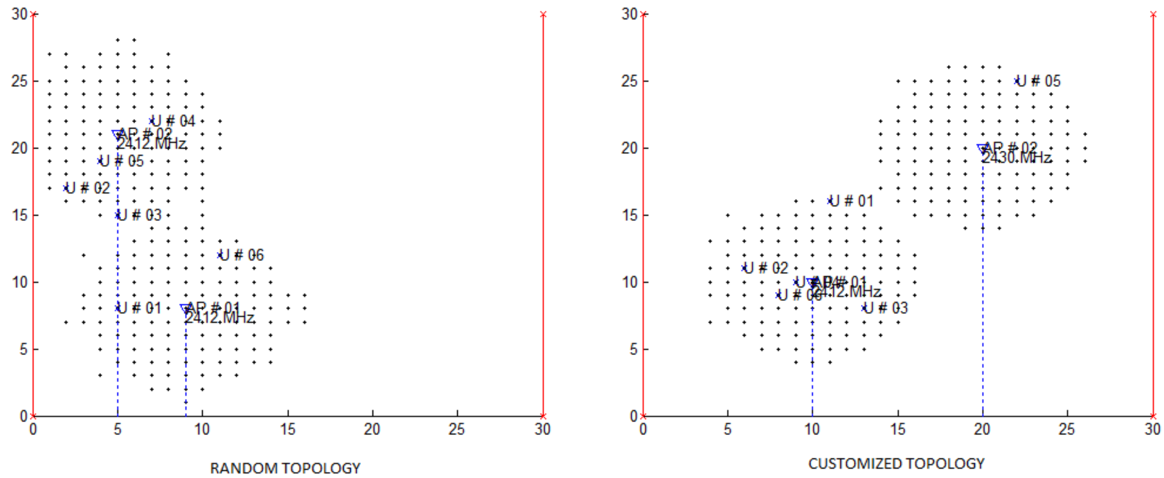


Figure 4.4 Left, random topology; Right, customized topology (AP-1 x=10 y =10, AP-2 x=20 y=20)

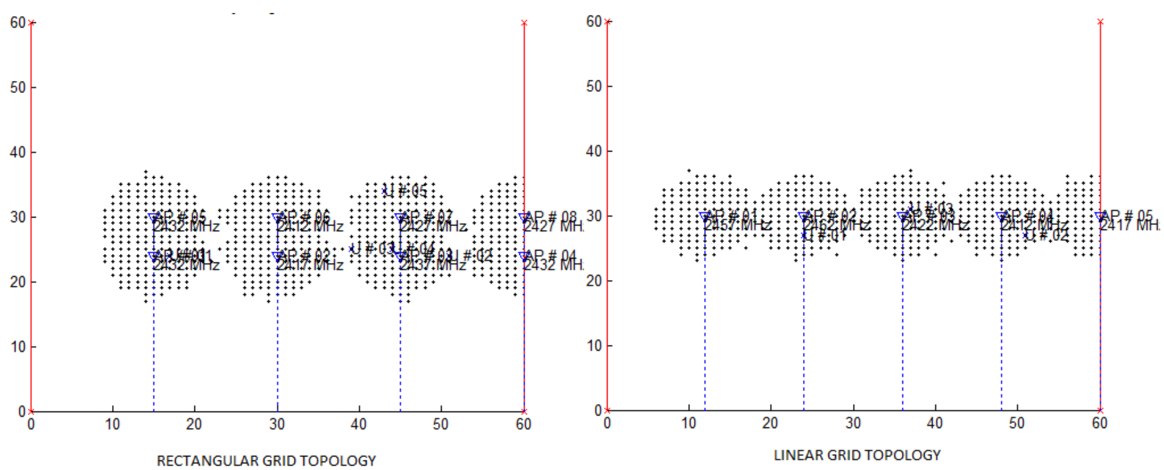


Figure 4.5 Left, Rectangular grid sample; Right, Linear grid sample

- 2 different types of simulation execution mode, single and full, the former executes a single simulation depending on the specific type of association policy, strategy and topology given by the user on the main menu, the latter executes all available game strategy types listed on figure 4.2 for a specific topology and association policy.

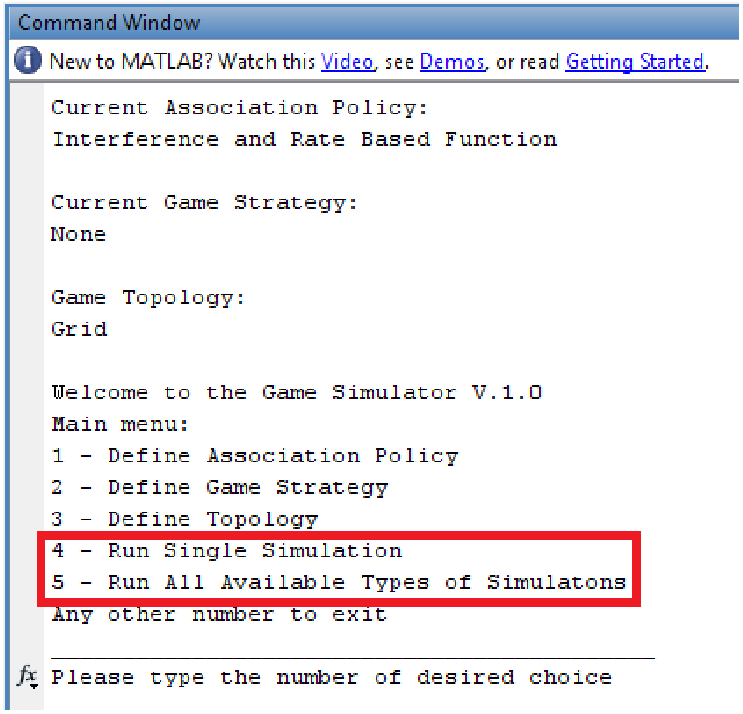


Figure 4.6 Snapshot of *Network Selection Games: Simulation Tool Development and Performance Evaluation's* main menu where the specific execution of simulation is defined

- 2 different types of simulation dynamics mode, either *Fixed Users varying Access point* mode or *Fixed Access Point varying User* mode, both modes allow the user to simulate different scenarios under realistic dynamics. In order to set up *Network Selection Games: Simulation Tool Development and Performance Evaluation* to execute the former listed simulation dynamics, a table with simulation parameters such as the one shown on figure 4.10 needs to be filled in.

The following figure, show how the dynamics work under the 2 different modes.

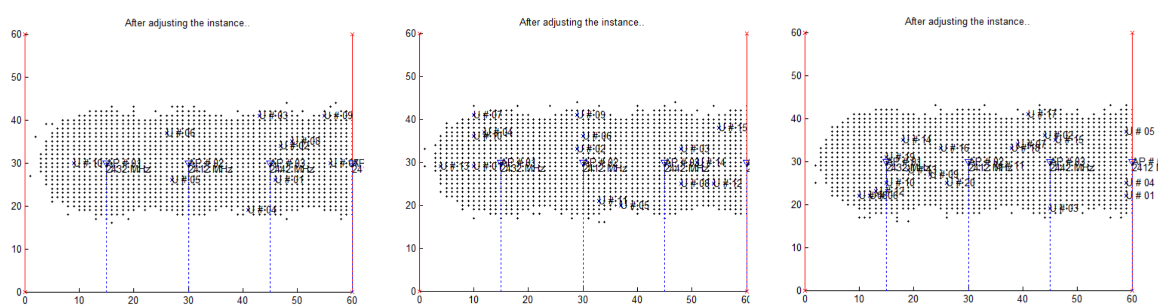


Figure 4.7 Game topology (Linear Grid) showing a fixed number of APs and an increasing number of Users

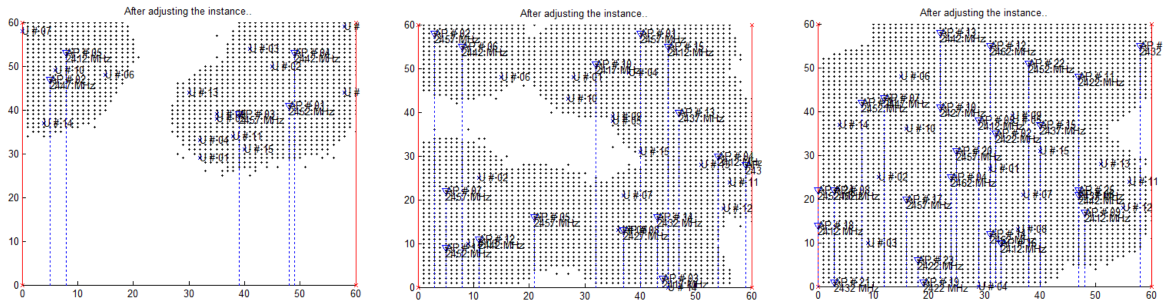


Figure 4.8 Game topology (Random) showing a fixed number of Users and an increasing number of APs

- Single simulation mode, on this mode of operation *Network Selection Games: Simulation Tool Development and Performance Evaluation* will execute only 1 simulation according to the input given by the user, in order to work correctly it needs an association policy, a given game strategy type and finally an existing topology. After these inputs are stated correctly on the *Network Selection Games: Simulation Tool Development and Performance Evaluation's* main menu, the user will be presented with the following message:

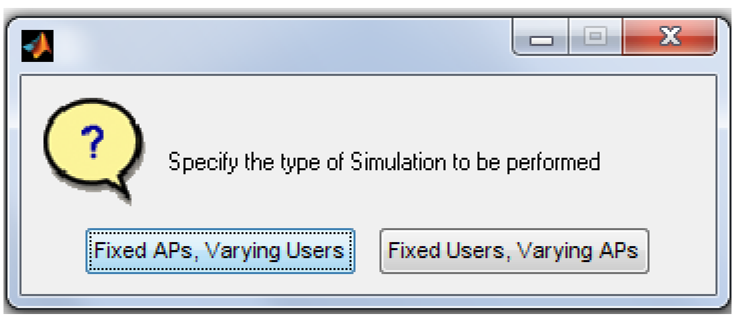


Figure 4.9 Message in which the type of simulation defined

After the user decides on the type of simulation to be performed, it is requested further on to fill a table of simulation parameters, without filling this table adequately the simulation would not be executable.

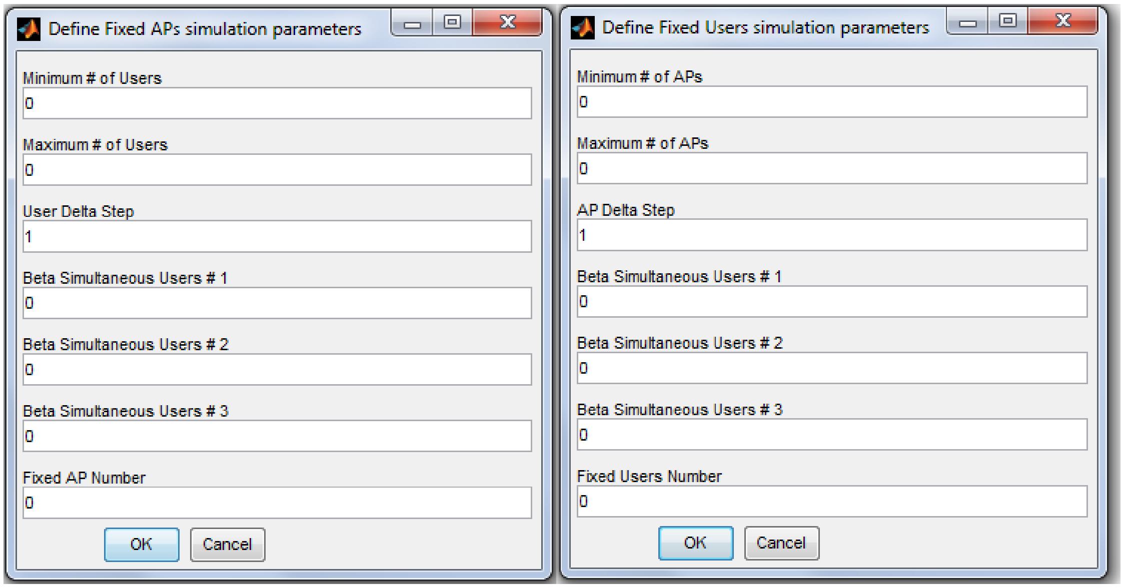


Figure 4.10 Simulation's parameter table

The table shown on the left of figure 4.10 will be presented to the user if Fixed Access Points and varying User simulation mode has been selected, on the other hand the table on the right will be presented if the user has selected the Fixed Users and varying Access Points mode. It is important to note that not all the parameters need to be filled for a specific type of Game Strategy, i.e user is wanting to run a single simulation with random topology, pure interference based association policy and Best Response strategy, hence in this case the fields corresponding to *Beta Simultaneous Users #1-3* won't need to be filled, even though they are filled by mistake by the user, their values will be discarded by *Network Selection Games: Simulation Tool Development and Performance Evaluation*.

Upon clicking "OK" the simulation will commence and depending on the type of simulation chosen different graphics will be presented.

The following set of figures show outputs for different simulation types.

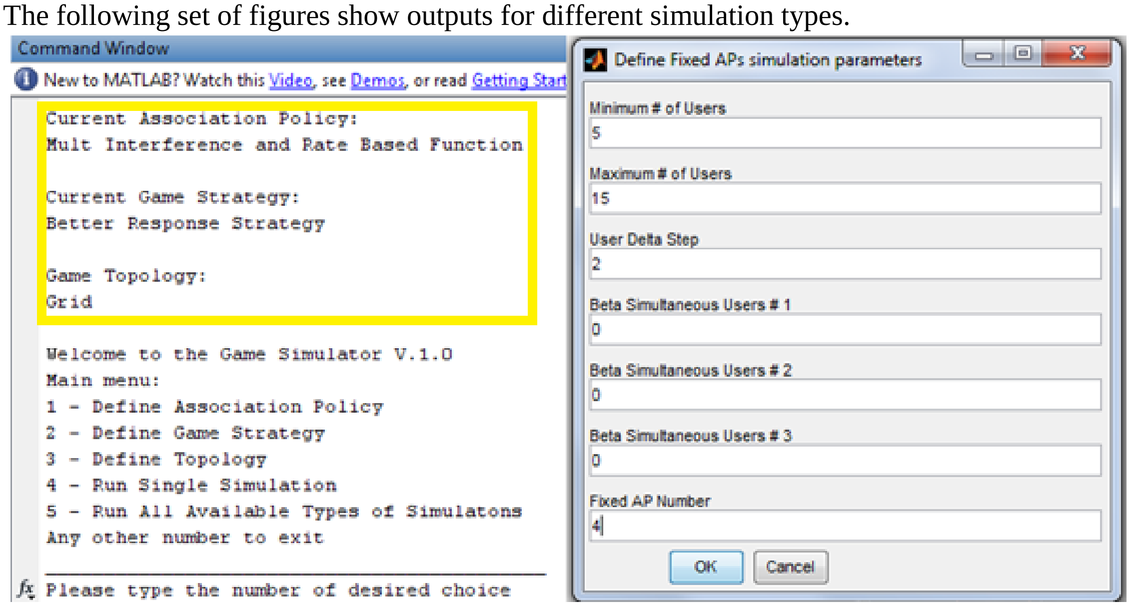


Figure 4.11 Simulation type

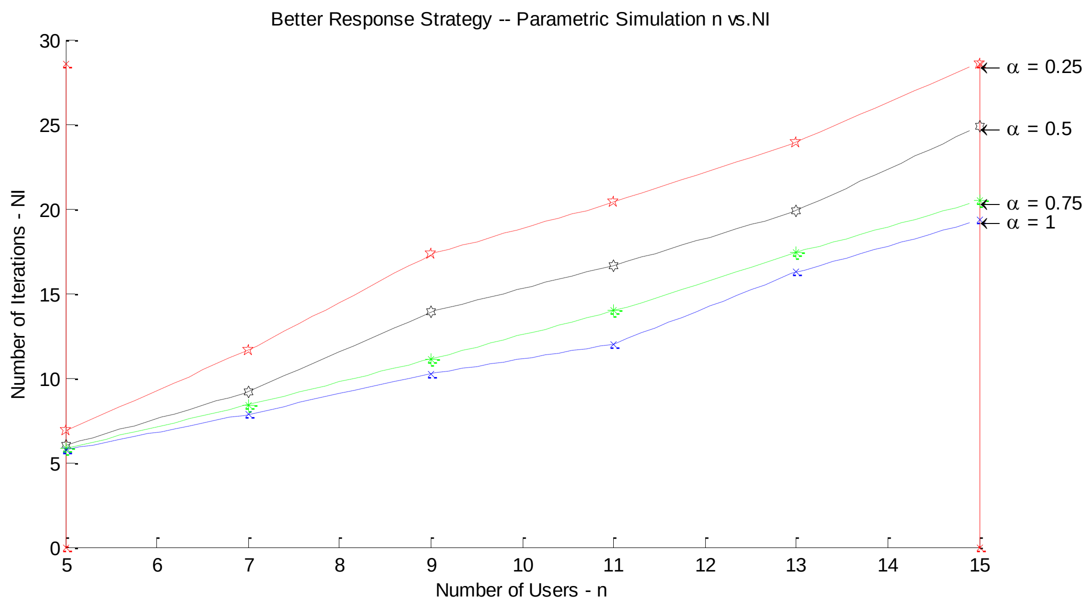
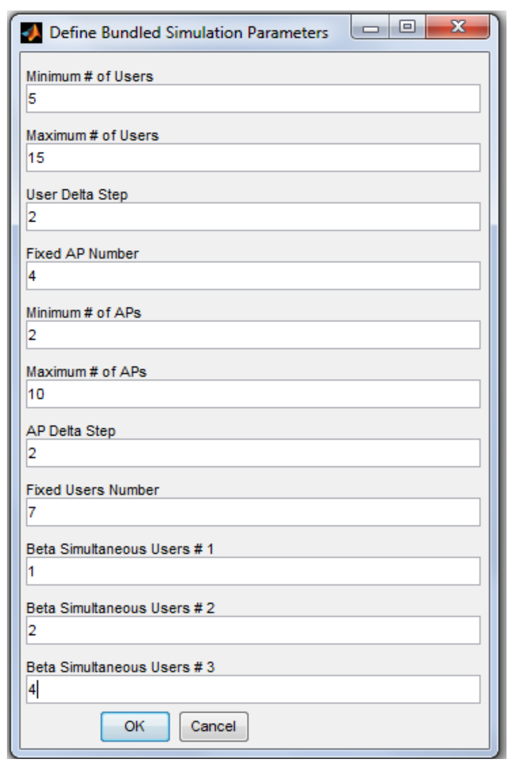


Figure 4.12 Simulation results

- Full simulation mode, also seen in *Network Selection Games: Simulation Tool Development and Performance Evaluation's* menu as *Run All Available Types of Simulations*, is a mode in which the user is previously requested to fill in jointly the parameters shown on both tables of figure 4.10, the joint table is shown in the following figure.


Define Bundled Simulation Parameters

Minimum # of Users
5
Maximum # of Users
15
User Delta Step
2
Fixed AP Number
4
Minimum # of APs
2
Maximum # of APs
10
AP Delta Step
2
Fixed Users Number
7
Beta Simultaneous Users # 1
1
Beta Simultaneous Users # 2
2
Beta Simultaneous Users # 3
4

OK Cancel

Figure 4.13 Bundled Simulation Parameter Table

After clicking OK, the user will be presented as output figures as much as Game strategy profiles exist. By adequately filling the table of the figure shown above, simulations can be easily performed in an automatic way.

- Once *Network Selection Games: Simulation Tool Development and Performance Evaluation* has finished its computation, different figures are shown, figures such as: number of iterations vs. number of users/APs, elapsed simulation time vs. number of users/APs and/or convergence probability vs. number of users/APs, after these figures are plotted the user is presented an output menu shown on the figure below.

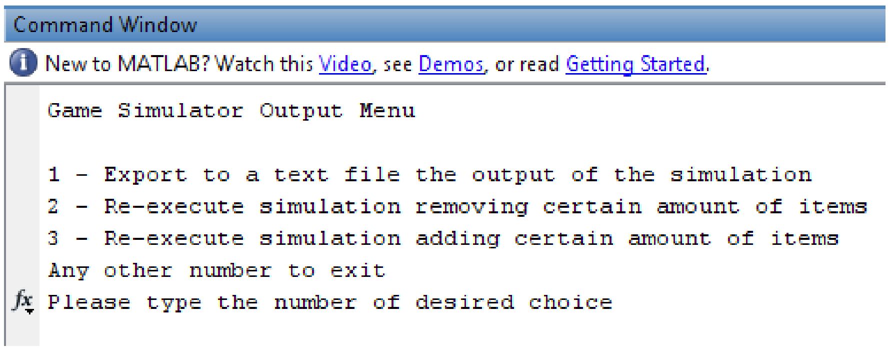


Figure 4.14 Output Menu

The first option as it states, exports in a plain text file all the simulation data, such as users/AP coordinates, map, matrices and vectors that kept track of all the dynamics of the simulated game. This information can be used either, to analyze offline results of a simulation and/or to cross check an expected behavior from a known scenario. *Network Selection Games: Simulation Tool Development and Performance Evaluation* for future works could be able to be extended in order to receive as input similar plain text files such as the one it generates as output with a premade format.

Options 2 and 3 of the output menu shown above, are what is called in the *Network Selection Games: Simulation Tool Development and Performance Evaluation* context as *Perturbation scenarios*, it is a very special feature of the program, making possible to the user add or remove users/APs from a game simulation that has already ended, thus disturbing or adding a perturbation to the previous instance of the game.

Figure 4.15 shows the output results for a pure interference, best response and random topology game, in which there were 3 fixed APs and varying users from 5 to 25.

In the hypothetic case of the user needing to analyze what happens if more users appear given the previous equilibrium states (i.e 5-10-15-20-25 users) or what would happen if suddenly users disappear, the user must input a given disturbance on a certain simulation point, e.g remove 5 users when n = 10, add 2 users when n = 15.

Figure 4.16 shows how the user must input the disturbances and figure 4.17 will show the results adding and removing 2 users from each simulated point.

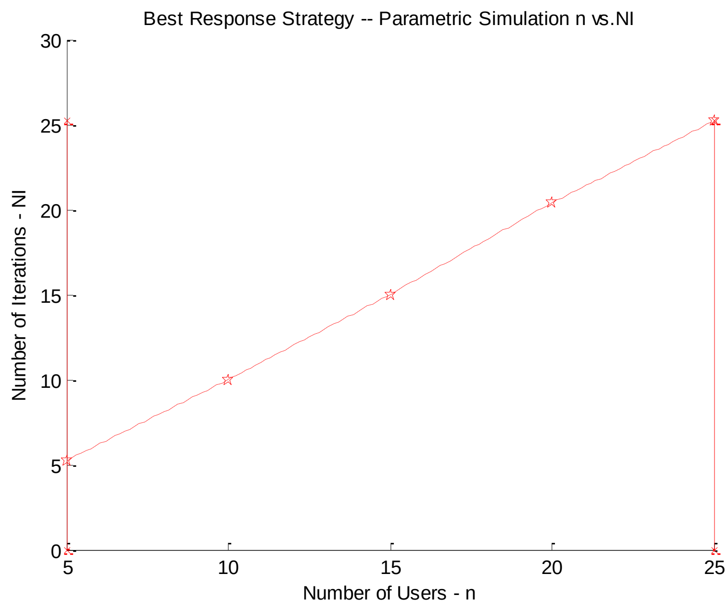


Figure 4.15 Output for n = 5…25 users with m = 3 APs

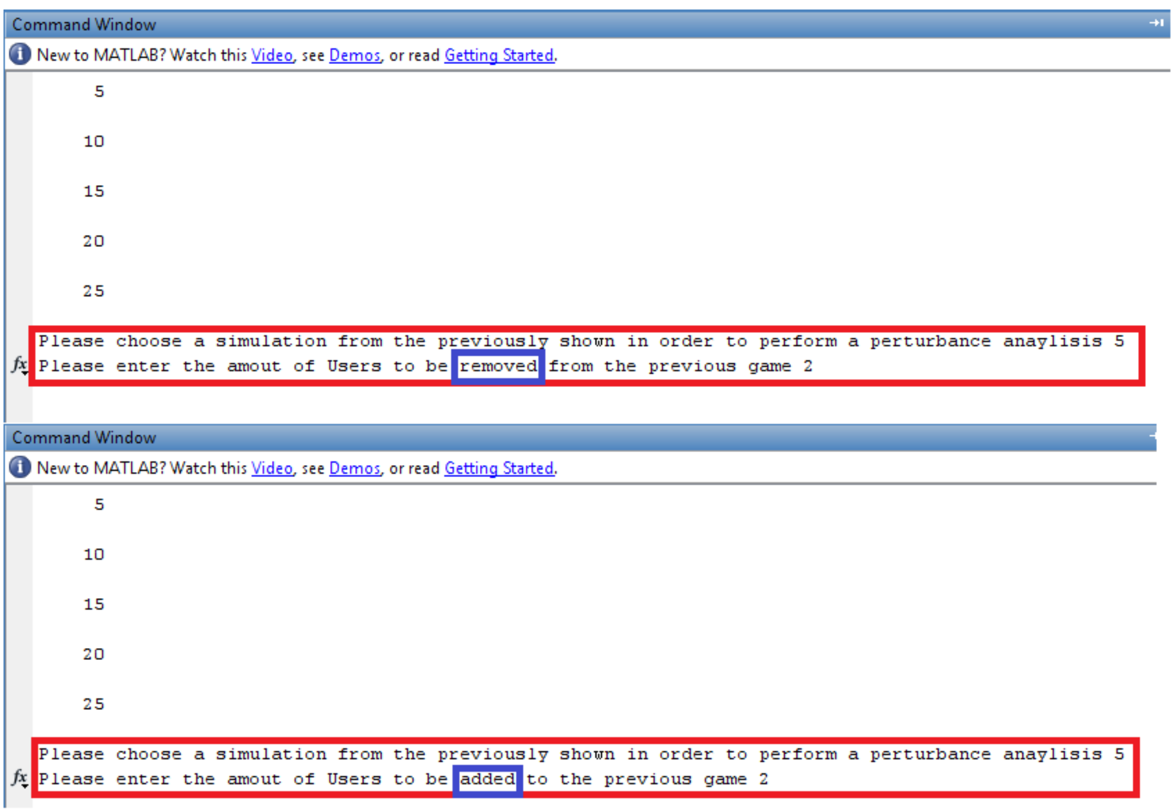


Figure 4.16 Perturbation Scenarios sub-menu, top remove item disturbance, bottom add item disturbance

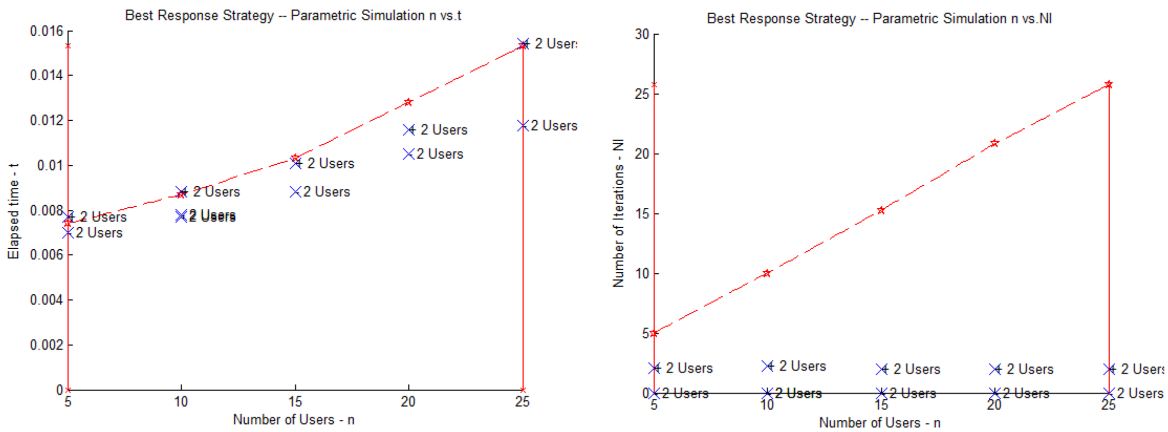


Figure 4.17 Output results for the Perturbation scenarios, left, Elapsed time, right, Number of iterations

NETWORK SELECTION GAMES: SIMULATION TOOL DEVELOPMENT AND PERFORMANCE EVALUATION functionality synthesis

- Three association policy options, Pure Interference Based, Additive Interference and Rate Based, and Multiplicative Interference and Rate based functions.
- Ten game strategy options, ranging from pure strategy types such as Best and Better response algorithms to mixed strategy types such as Fictitious Play algorithms, each one of them presented in different flavors.
- Four types of map topologies, including random and fully customized topologies, linear and rectangular grids of Access Points.
- Two different types of simulation mechanics: run single simulation and run a full bundle of simulations.
- Two types of simulation analysis, Fixed Users and Varying Access Points and Fixed Access Points and Varying Users.
- Two types of disturbance analysis, including adding or removal of Access Points or Users.
- Ability to export simulation data to a plain text file.

5.

PERFORMANCE EVALUATION WITH PURE INTERFERENCE BASED UTILITY FUNCTIONS

General Introduction to Performance Evaluation Sections

Sections 5, 6 and 7 contain the experimental results of *Network Selection Games: Simulation Tool Development and Performance Evaluation*. The results are organized depending on the association policy used, thereafter for each policy different algorithms are compared including the static and learning techniques, along the way additionally, Perturbation scenarios and scenarios in which the coverage area of the access points is parametric are studied.

The results are presented as a set of organized figures, commonly the interesting outputs are those measuring the number of employed iterations and the convergence probability percentage. At the end of each simulation scenario the results will be commented and compared with previously obtained results. Inter-policy results are one of the biggest contributions of this work, these comparisons can be found in some commentaries done after the output figures are presented.

In the following, Section 5 will show the experimental results for the Pure interference based association policy, Section 6 will be showing the results for the Additive interference and achievable rate association policy, and ending the experimental results Chapter, Section 7 showing the Multiplicative interference and achievable rate association policy.

Utility function definition

This type of utility function depends only in the number of interferers, noted as x^i. Formally it would be:

$$c_j\left(i, x^i\right) = x^i$$

These cost functions render the game essentially a single choice asymmetric congestion game, with all users' cost functions being the same [17].

Note: All simulations done will be under the assumption of "All in range" meaning each user is inside the coverage area of at least one access point. The previous holds unless it is differently expressed

5.1

Better Response algorithm Vs. Best Response algorithm

Fixed number of access points, varying number of users static analysis

- Varying number of randomly deployed users, starting from 10 users in steps of 10 users per iteration until 100, giving a total of 10 different games.
- Access point power transmission model modified in order to achieve an *All in Range* scenario.
- Map length $40m^2$

Fixed number of Access Points will be simulated for 5, 7 and 10 APs

<u>Results for 5 APs</u>:

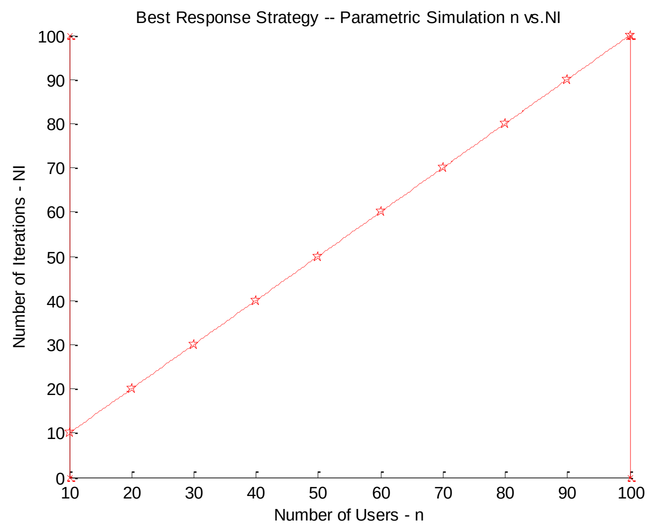


Figure 5.1 Best Response results for 5 APs

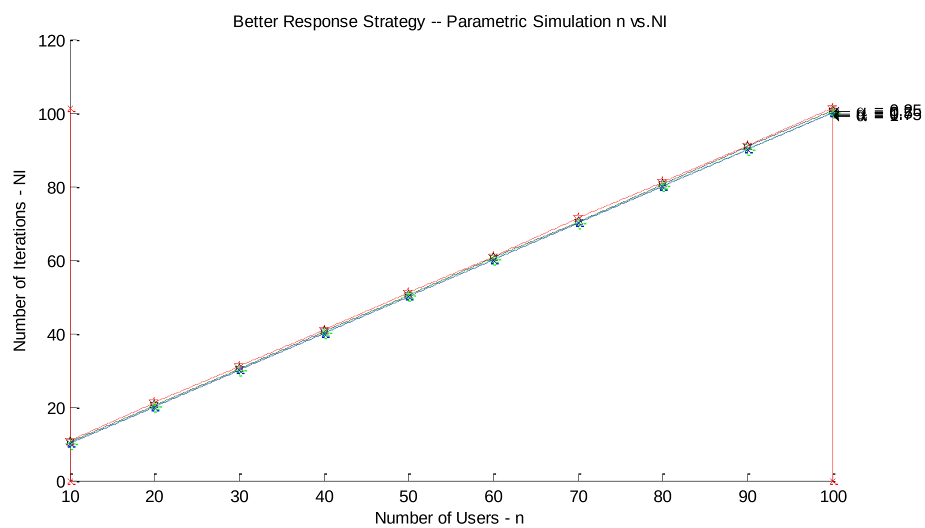


Figure 5.2 Better Response results for 5 APs

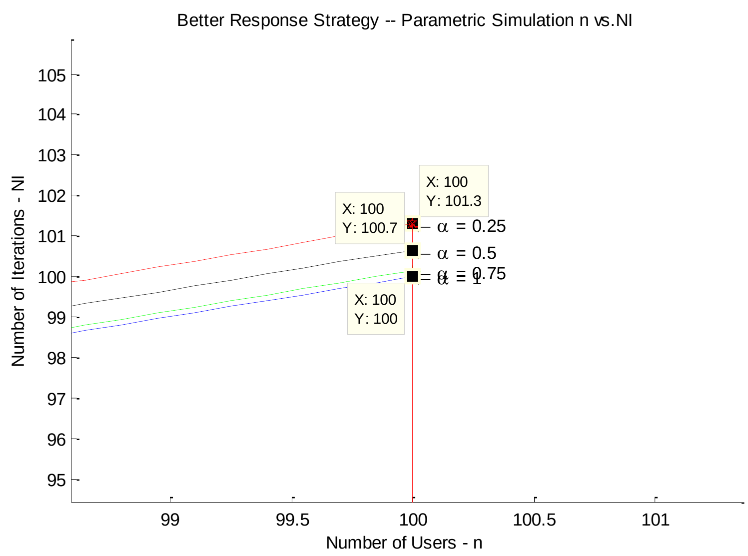


Figure 5.3 Better Response results zoom-in for 5 APs

The behavior in convergence is linear for both types of Game Strategies, result obtained on figure 5.1 it's the base reference for *All in Range* and *Best Response* algorithm game simulations, linear with a slope equal to 1. Similar to this result the 4 different curves obtained in figures 5.2 and 5.3, 5.2 shows the linearity throughout the whole simulated games (i.e 10-20-…100 users @ 5 APs).

Curve $\alpha = 0.25$ corresponds to the ability to select 2 users out of the 5 available ones, in overall it gave 1.3 more iterations compared to its counterpart at $\alpha = 1$ which is precisely the exact *Best Response* algorithm.

<u>Results for 7 APs</u>:

Best Response Strategy results, exactly the same ones obtained in figure 5.1.

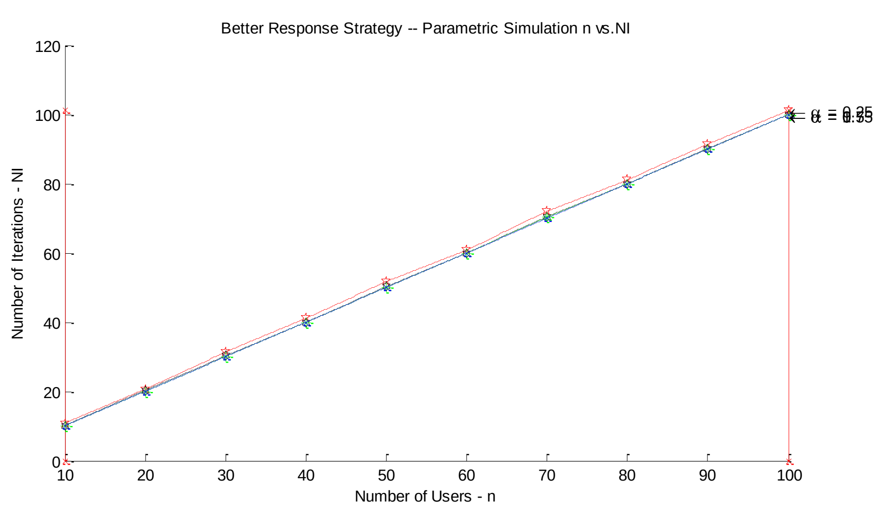


Figure 5.4 Better Response results for 7 APs

As figure 5.4 is showing, for this particular case there is a little discontinuity on the linear behavior for $\alpha = 0.25$ which derives ability to choose 2 access points from the total available ones, a close up to this non linearity is presented on the following figure.

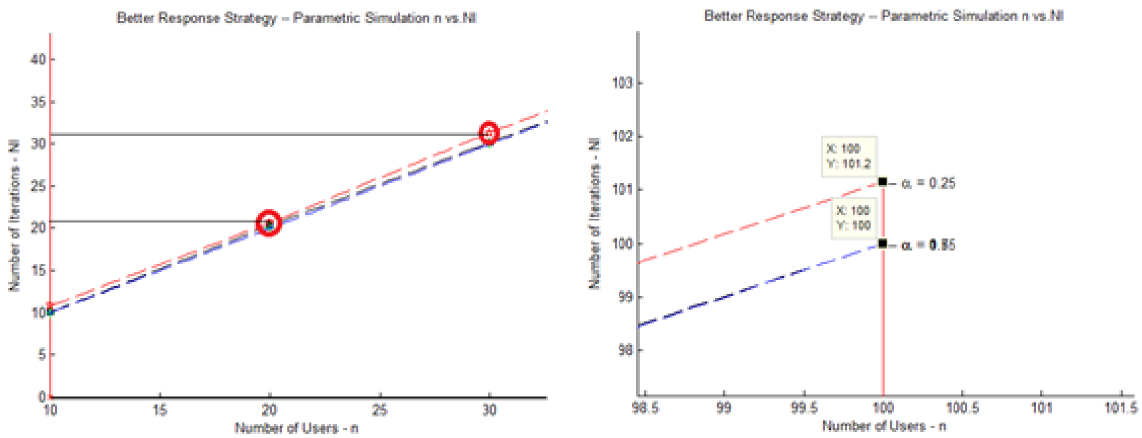


Figure 5.5 Better Response results zoom-in for 7APs

Leftmost graph of figure 5.5 shows where the discontinuance occurred at x ~ 30, which is a simulated point, this behavior is more easily seen because each simulated point is linearly interpolated with its subsequent point.

In the end and as it is shown on the rightmost graph of figure 5.5 at $\alpha = 0.25$ the overall difference of iterations w.r.t to $\alpha = 1$ is only 1.2 iterations, only 0.99% greater. Simulated games with *Better Response* algorithm at $\alpha = 0.5$ (ability to select 4 out of 7 APs), $\alpha = 0.75$ and $\alpha = 1$ are identical iteration wise.

<u>Results for 10 APs</u>:

Best Response Strategy results, exactly the same ones obtained in figure 5.1

Figure 5.6 shows the results obtained allowing the varying number of users to choose between 3 and 10 access points. $\alpha = 0.25$ (3 APs out of 10 available to select) gave the larger number of total iterations, this time 1.8% greater w.r.t $\alpha = 1$, $\alpha = 0.75$ behaving almost identical as if it were a *Best Response* algorithm and $\alpha = 0.5$ giving 0.9% more iterations w.r.t $\alpha = 1$.

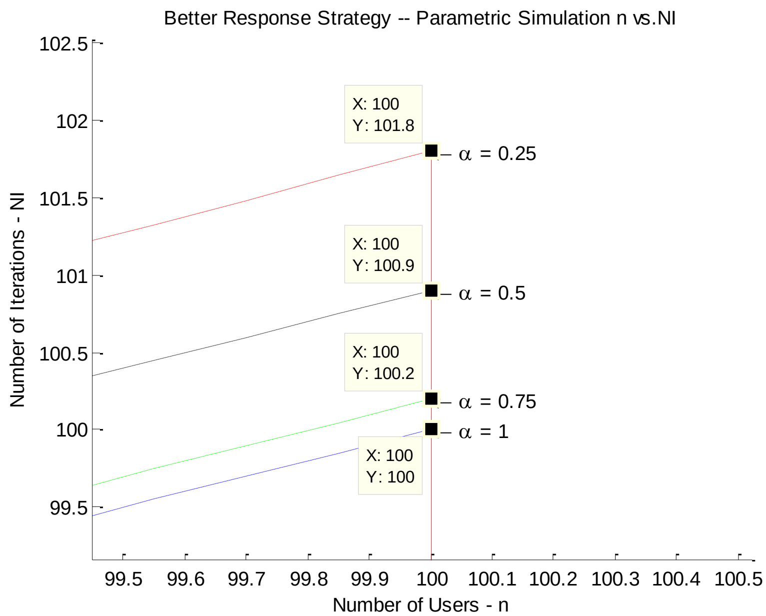


Figure 5.6 Better Response results zoom-in for 10 APs

As concluding remarks for the developed simulations throughout the variation of the number of users keeping fixed the number of access points a linear behavior is obtained for the whole range, allowing the users to choose from only 75% of the total available access points (being *All in Range*) derives identical results as if it was executing a *Best Response* game strategy; moreover for {5-7-10 access points} one can obtain from a 25% of the total available access points a number of iterations just 1.69% higher than the reference (*Best Response* or *Better Response* @ $\alpha = 1$.

Fixed number of users, varying number of access points analysis

- Fixed number of randomly deployed users, varying number of access points starting from 10 APs in steps of 10 APs per iteration until 100, giving a total of 10 different games.
- Access point power transmission model modified in order to achieve an *All in Range* scenario.
- Map length $40m^2$

Fixed number of Access Points will be simulated for 100 and 200 Users

Results for 100 Users:

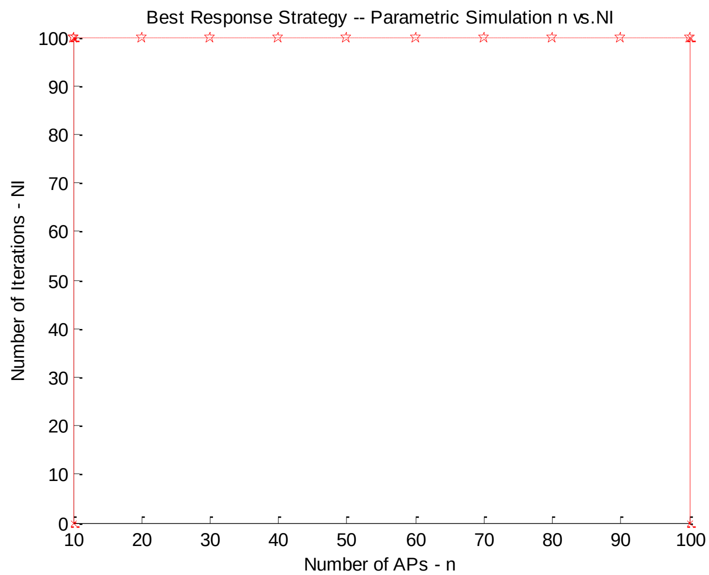


Figure 5.7 Best Response results for 100 users

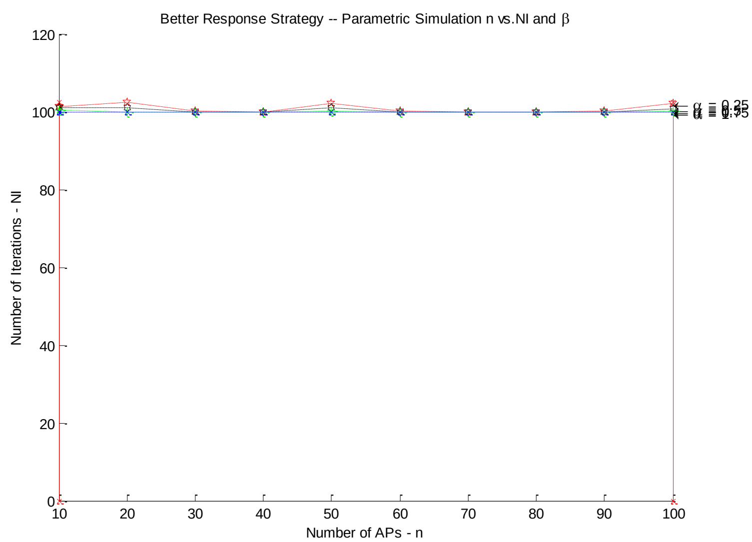


Figure 5.8 Better Response results for 100 users

For the *Best Response* algorithm, constant number of iterations (100) for all simulated games as seen on figure 5.7, similarly figure 5.8 shows constant behavior for majority of simulated games, however on few points this constant behavior is not kept, as shown in a close-up of figure 5.8 by figure 5.9

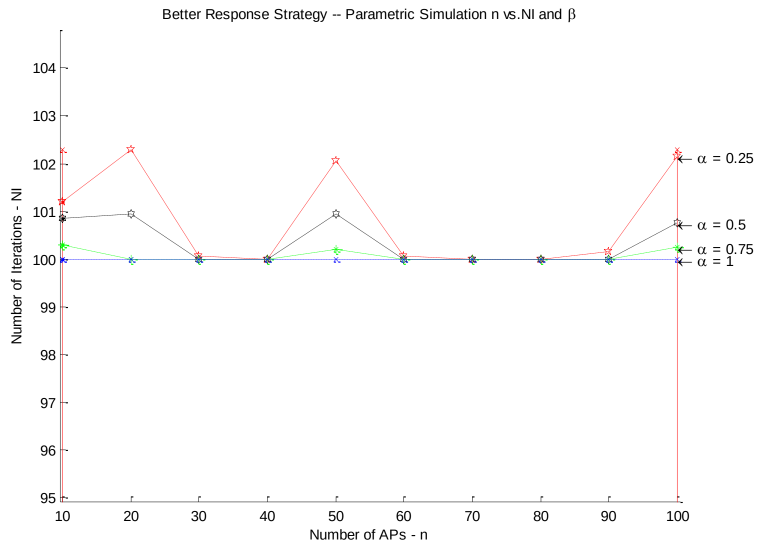


Figure 5.9 Better Response results zoom-in for 100 users

Thanks to the close-up, the points in which each of the different *Better Response* simulations i.e $\alpha = 0.25, \ldots., 1$ differentiate the most are m = 10,20,50 and 100 APs, however as appreciated the maximum difference w.r.t to $\alpha = 1$ is around 2 more iterations, on the other hand $\alpha = 0.75$ is giving an almost identical behavior as it were *Best Response* algorithm, even for the points where the number of AP is considerable w.r.t to the total number of users i.e 100 users for this particular simulation. This gives arise to the conclusion that even when only being able to select up to 25% of the total available AP's one obtains a behavior that is just around 2.5% bigger iteration wise.

<u>Results for 200 Users</u>:

Best Response Strategy results, exactly the same behavior obtained in figure 5.7

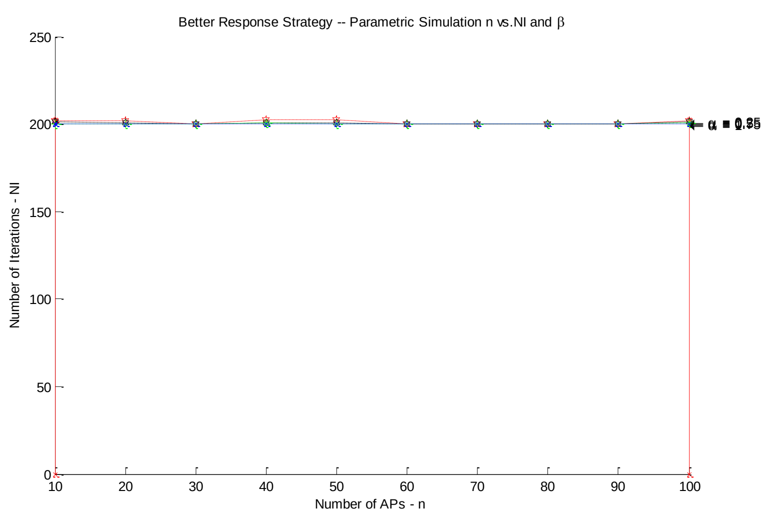


Figure 5.10 Better Response results for 200 users

There is an overall homogeneous constant behavior shown on figure 5.10, similar to its counterpart on figure 5.8.

Close up on figure 5.10 is showing some isolated points which break up the homogenous behavior, but keeping in the worst case i.e $\alpha = 0.25$ an iteration difference margin of up to 2%. Here again from m = 60 and until m = 100 APs the behavior remains constant, for all other points there are greater number of iterations compared to baseline imposed by $\alpha = 1$

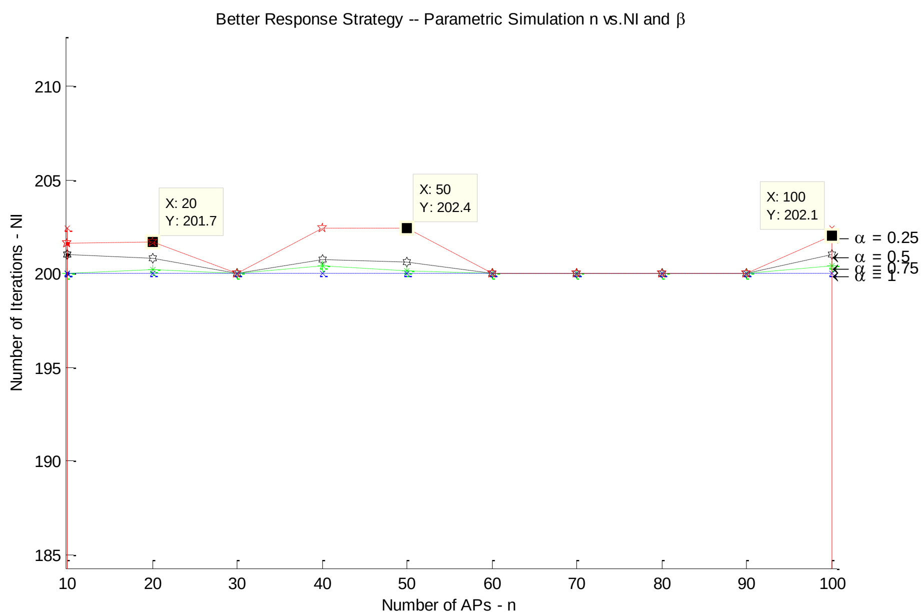


Figure 5.11 Better Response results zoom-in for 200 users

As concluding remarks for fixed number of access point simulations, for both algorithms *Best* and *Better Response* obtained results are close to expected ones under the assumption of *All in Range*, a constant value of number of iterations is expected for *Best Response*, and as α approaches unity *Better Response* algorithm results are closer to the ones of *Best Response*. Difference margins take all values from 0% to 2% for different α simulated values, if 2% is an acceptable margin for the number of iterations, then the more flexible algorithm i.e *Better Response*, would be the adequate choice to use.

5.2

Perturbation Scenarios

Perturbation scenarios consist on adding/deleting users/access points from a given simulation output, such as the ones obtained from figure 5.1 to 5.11. From a *fixed user* game, only users can be added/deleted from the game, and the Perturbation scenario behavior is analyzed and plotted on the same graph previously obtained, on the other hand for the *fixed access points* game, only APs can be added/removed from the previously obtained results in order to analyze further results due to the applied disturbance.

In order to illustrate the idea, a disturbance analysis will be made to all previously shown graphs, since fixed user and fixed access point simulations showed in overall a very similar behavior, a particular case will be chosen (e.g 7 fixed APs, varying users from 10 to 100, and 150 fixed users, APs varying from 10 to 100).

Removing users/access points

<u>Results for 7 APs, varying users from 10 to 100 (User step 10 per iteration)</u>:

Best Response algorithm results

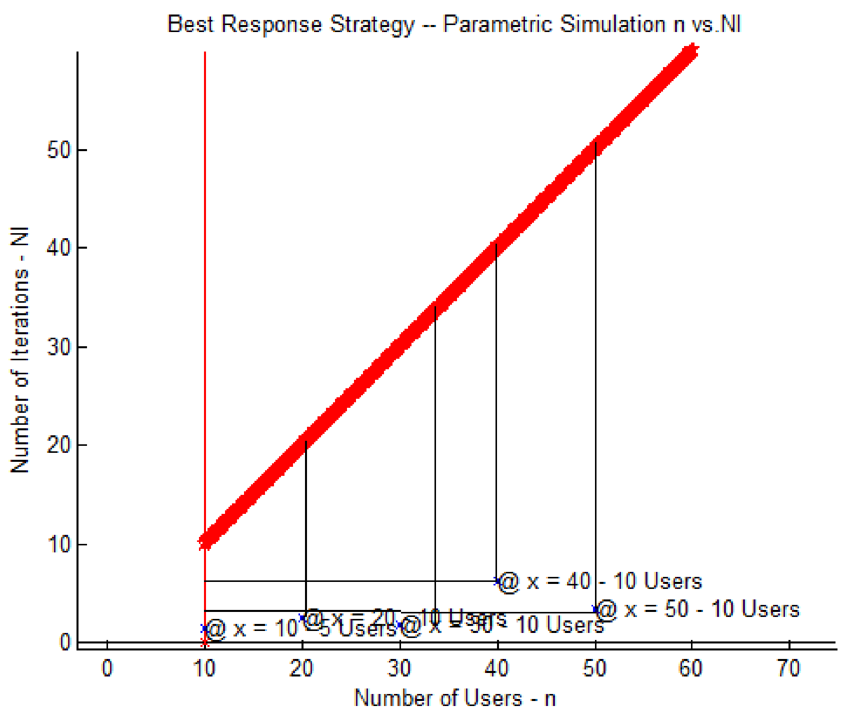


Figure 5.12 Best Response Perturbation scenario results for 7 fixed APs part 1

Figure 5.12 shows Perturbation scenario results for 10-50 number of users using *Best Response* algorithm, whilst figure 5.13 shows the Perturbation scenario results for the remaining 60-100 users, the former graph is showing as it would be expected an additional number of iterations after the disturbance no greater than the current number of access points i.e 7, the maximum number of additional iterations was obtained when n = 40 users and corresponds to almost 7 additional iterations, this behavior might be due to the fact that the 7 removed users, belonged to exactly the same access point, leaving a less crowded AP to the remaining 30 users. All other simulated Perturbation scenarios are showing different averaged results (disturbance scenario is simulated 10 times and averaged) but in all cases less than the one obtained in n = 40 users.

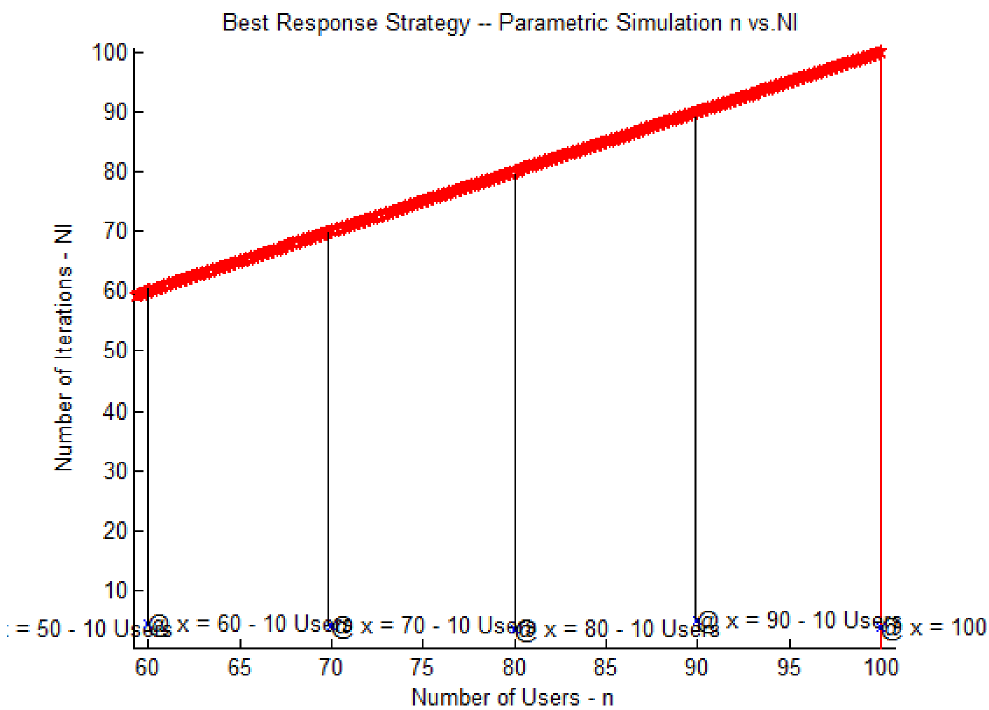


Figure 5.13 Best Response Perturbation scenario results for 7 fixed APs part 2

Figure 5.13 is showing a more homogenous behavior for all points w.r.t to figure 5.12, average is close to 4 additional iterations

It could be concluded from the aforementioned comments, that removing 10 users from any of the simulated points results indifferent to the actual perturbed scenario, e.g whether scenario to be perturbed is n = 20 users or n = 100 users, this is logically valid, since removed users make their previously selected access point less crowded and available to the remaining users despite of the current number of users. From the previous analysis it was also determined that the maximum number of additional iterations in average equals the current number of access points, i.e 7 for this particular simulations, additionally the luckiest case would be whenever the tool removes 10 users all of them belonging to different APs, giving rise to fewer number of additional iterations.

Better Response algorithm results

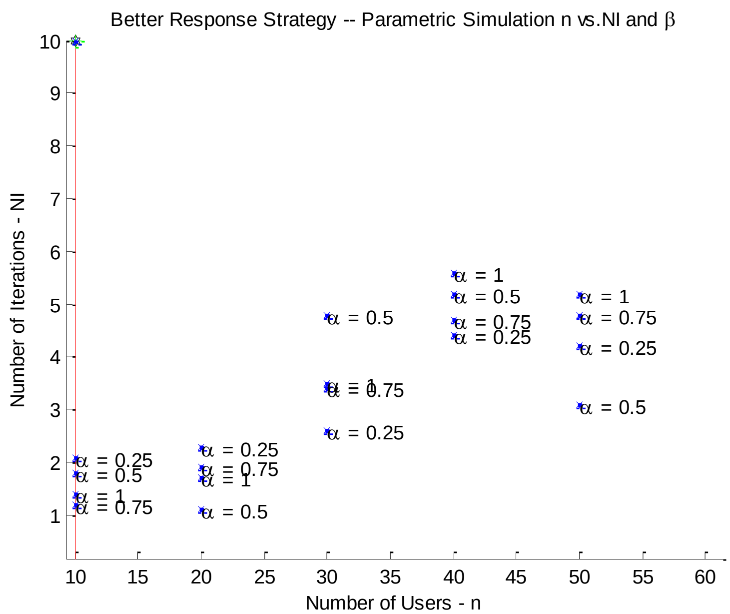


Figure 5.14 Better Response Perturbation scenario results for 7 fixed APs part 1

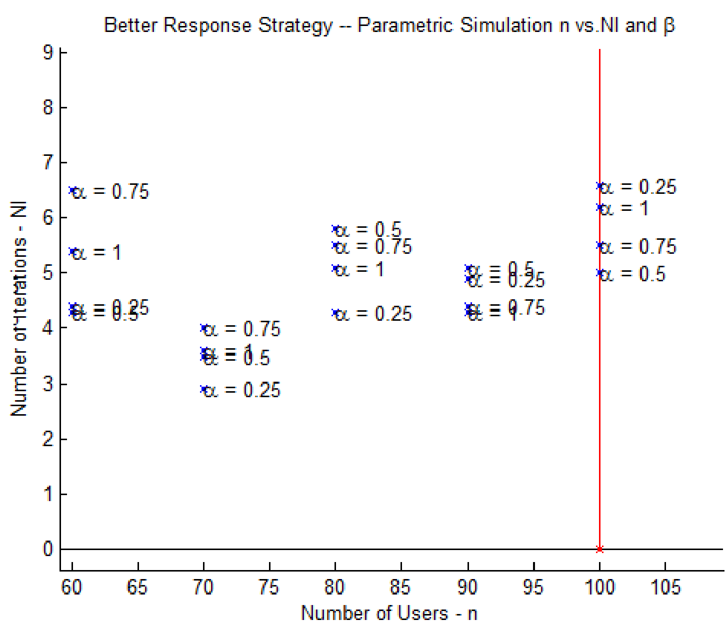


Figure 5.15 Better Response Perturbation scenario results for 7 fixed APs part 2

Figures 5.14 and 5.15 show the results removing 10 users (5 users from n = 10 users) from all simulated points, results at n = 10, 20 results are lower than 2 additional simulations, lower than all other Perturbation scenarios, the lower the users the lower the additional needed iterations to find a new equilibria in average because e.g for n = 10, there were 4 APs with 1 user each and 3 with 2 each, removing 5 users, could lead to in the worst case removing 4 users from 2 APs having 2 users each, giving 2 additional simulations, same analysis is made for n = 20, however here 6 APs have 3 users and only 1 has 2 users, the worst hypothetical case would be to remove 9 users from 3 APs having 3 users each, leading to at most 3 additional iterations, so on and so forth.

Results for 150 fixed users, varying access points from 10 to 100 (AP step 10):

Best Response algorithm results

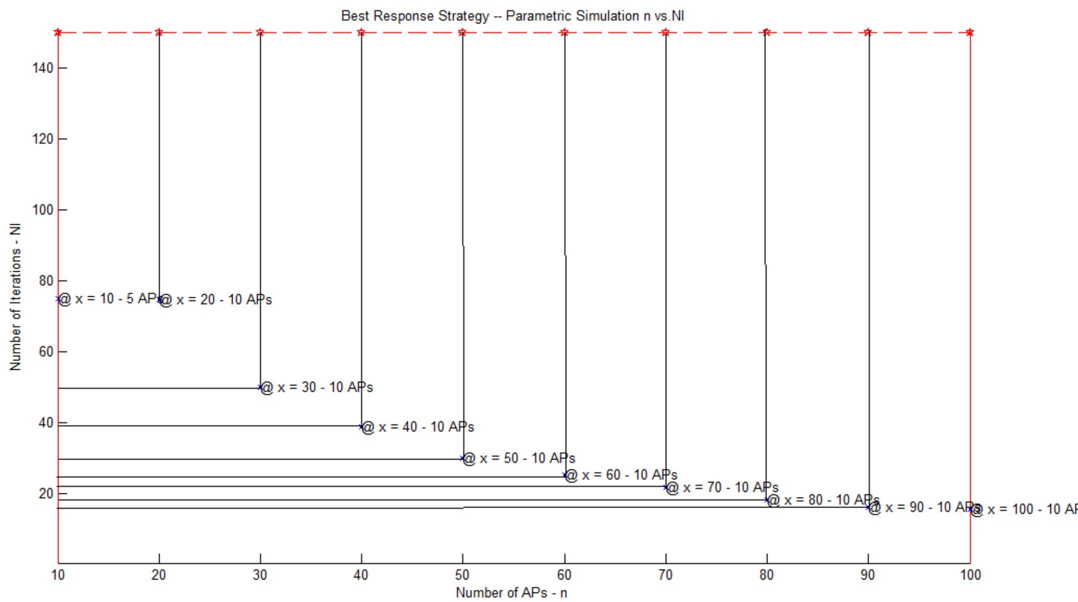


Figure 5.16 Best Response Perturbation scenario results for 150 fixed users

First disturbance scenario studied corresponded to m = 10 APs and removing 5 APs, results obtained are the expected ones, resulting in 75 additional iterations, exactly 75 users were left without selected access point right after 5 APs were removed, exact behavior for m = 30 APs, with 30 APs convergence results assign exactly 5 users to each AP in order to cover the whole 150 users, by removing 10 APs, 10 times 5 users are left without an access point during the disturbance analysis, resulting on what it is plotted on figure 5.14 for m = 30, so on and so forth continuing for increasing number of existing APs, one can notice from figure's 5.16 graph that as m increases impact of removing certain number of APs diminishes because the number of assigned users to each AP is less, hence the number of iterations these users must do in order to encounter a new equilibrium.

Better Response algorithm results

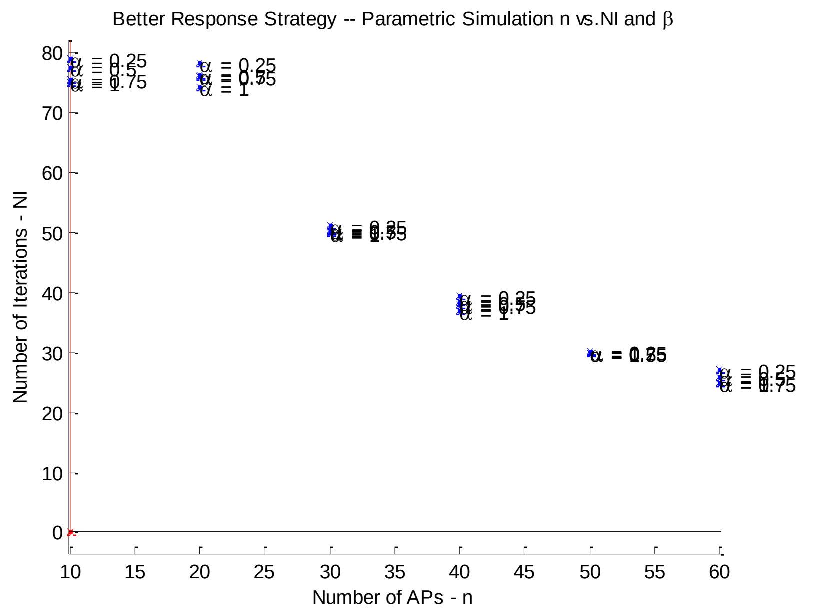


Figure 5.17 Better Response Perturbation scenario results for 150 fixed users part 1

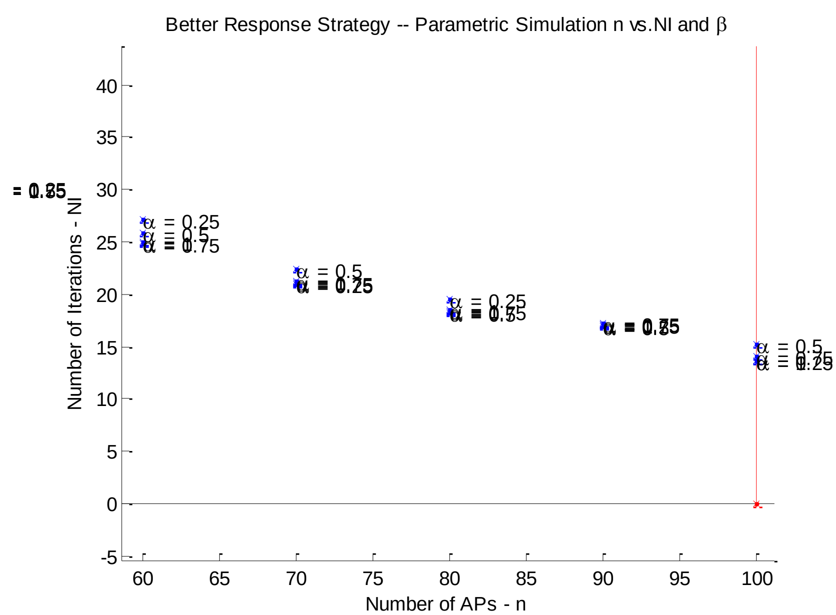


Figure 5.18 Better Response Perturbation scenario results for 150 fixed users part 2

As it is observed on figure 5.16, obtained results for m = 10 and 20 access points, are around 70~80 additional simulated steps, at the same time *Better Response* algorithm found its solutions around the same values, having $\alpha = 1$ for m = 10 exactly allocated at N = 75 additional iterations, for m = 20 results a little bit lower, however close to 75 in average (for m = 10, 5 APs were removed and for m = 20, 10 APs were removed).

From 20 APs until 100 APs a decreasing behavior number of iteration wise is observed, same observed on the *Best Response* algorithm shown on figure 5.16, $\alpha = 1$ *Better Response* algorithm result's can be proved to be the same as the ones obtained from the *Best Response* algorithm* (proof can be done graphically by zooming in or by analyzing the raw data obtained from debug), given the reference, the different α values oscillate over $\alpha = 1$, having a tendency of being $\alpha = 0.25$ the one requiring the greatest number of additional iterations. For the specific cases of m = 10, 20 and 60, margin between $\alpha = 1$ and $\alpha = 0.25$ is around 5 iterations, giving different impact depending on the analyzed disturbance in case.

5.3

Concurrent choice scenarios

In these scenarios, the *Game Simulator* emulates the behavior of users making simultaneous choices. The software allows β players to make simultaneous choices, starting from $\beta = 1$ in β_{step} steps until the maximum number of users is reached. This additional parameter is applicable to *Best Response* and *Better Response* algorithms extending the number of possible output analysis that can be done.

Output graphs include the foreshowed graphs i.e Number of Iterations vs N (users) or M (access points), can include Elapsed simulation time vs N or M and also Convergence Probability vs N or M, convergence probability. The analysis is made because as the β parameter approaches the maximum number of users the algorithms tend to fail in convergence (fact to be proven with the simulations). Output results are the average of 20 different played games, as β increases the number of these games converging decreases.

In order to illustrate the mentioned behaviors, concurrent choice scenarios will be done for the same Perturbation scenarios of Section *5.2*

<u>Results for 7 APs, varying users from 10 to 100 (User step 10 per iteration) and $\beta = 2{,}5$ & 10:</u>

Best Response algorithm results

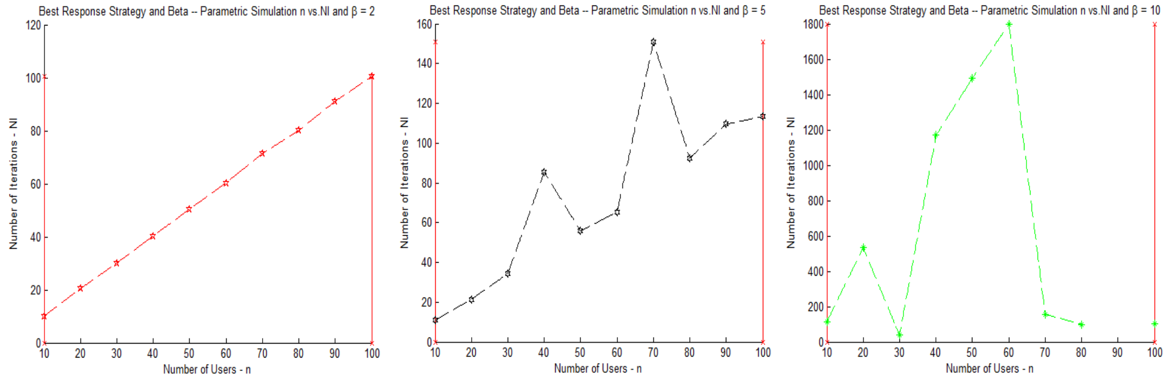


Figure 5.19a Best Response concurrent scenario results for 7 fixed APs

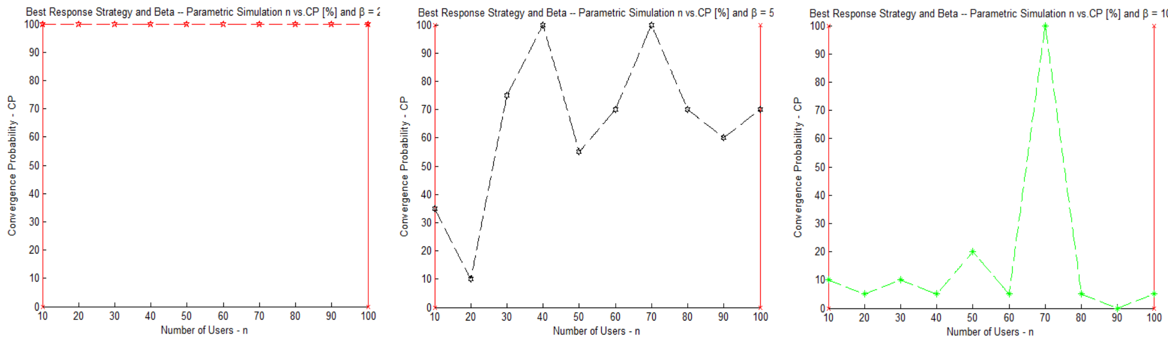


Figure 5.19b Best Response concurrent scenario convergence probability results for 7 fixed APs

Figure 5.19a is showing results number of iteration wise, from leftmost to rightmost corresponds to $\beta = 2$, $\beta = 5$ and $\beta = 10$ simultaneous users respectively. $\beta = 2$ has a monotonic linear behavior similar to the *Best Response* algorithm without simultaneous players, $\beta = 5$ graphic shows an apparent linear behavior like $\beta = 2$, however there are peak values at n = 40 and 70. At $\beta = 10$ there is no uniform behavior for all points, and the number of iterations ranges from few hundreds to more than one thousand iterations.

Convergence probability results shown on figure 5.19b are strictly related to their former parts shown on figure 5.19a, the linear behavior for $\beta = 2$ resulted on a constant convergence probability of 100% for all users, however for $\beta = 5$ the same dynamics do not apply as seen on the “linear” parts of the mid graph of figure 5.19a, for this specific case at $\beta = 5$ the peak values on number of iterations resulted on greater convergence probabilities showing that more effort is required in order to obtain higher %’s of convergence. The particularity of the peaks corresponds to the values in which the ratio of the number of users and the number of APs results on a multiple of the β simultaneous players, hence n= 40 and n = 70 result in n/m = 5.7 and n/m = 10 respectively, the closest multiples to $\beta = 5$ than all other points.

Rightmost graph of figure 5.19b is showing the convergence probability for $\beta = 10$, from these graph it can be seen that the algorithm at n = 60 executed around 1800 user iterations to achieve a convergence probability of around 10%, however for n= 70 convergence probability resulted in average 100% with a number of iterations just close to 200. As it can be seen for n = 90 users there was no convergence, for all other points except for n = 70 the convergence is no greater than 20%.

Better Response algorithm results

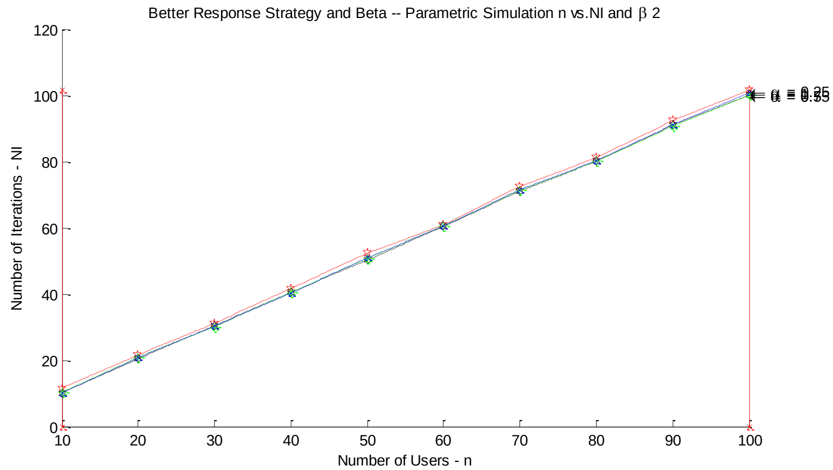


Figure 5.20a Better Response 2 concurrent users scenario results for 7 fixed APs

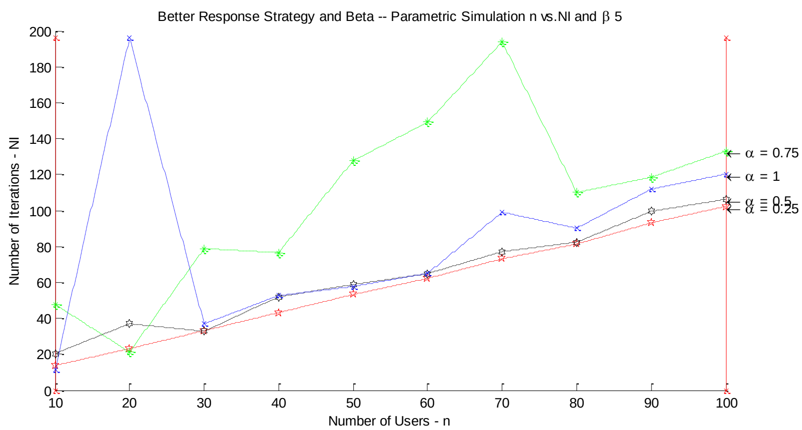


Figure 5.20b Better Response 5 concurrent users scenario results for 7 fixed APs

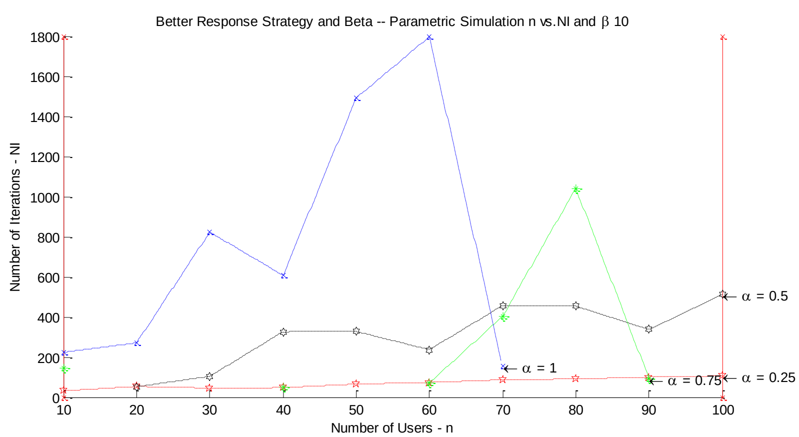


Figure 5.20c Better Response 10 concurrent users scenario results for 7 fixed APs

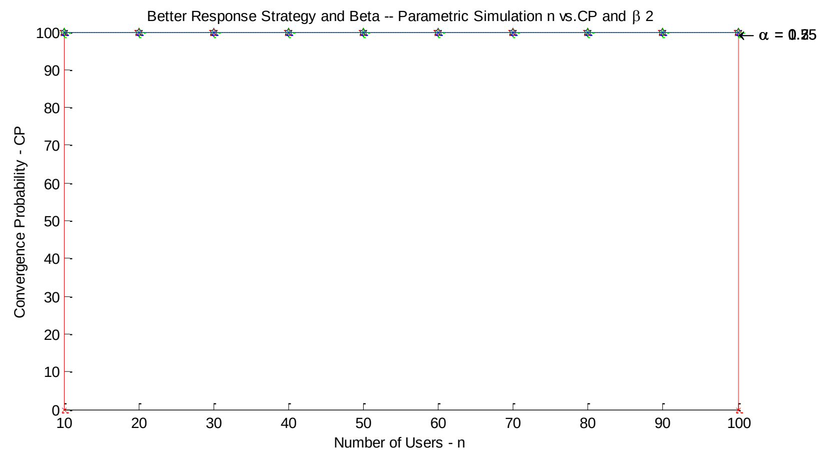


Figure 5.20d Better Response 2 concurrent users scenario convergence probability results for 7 fixed APs

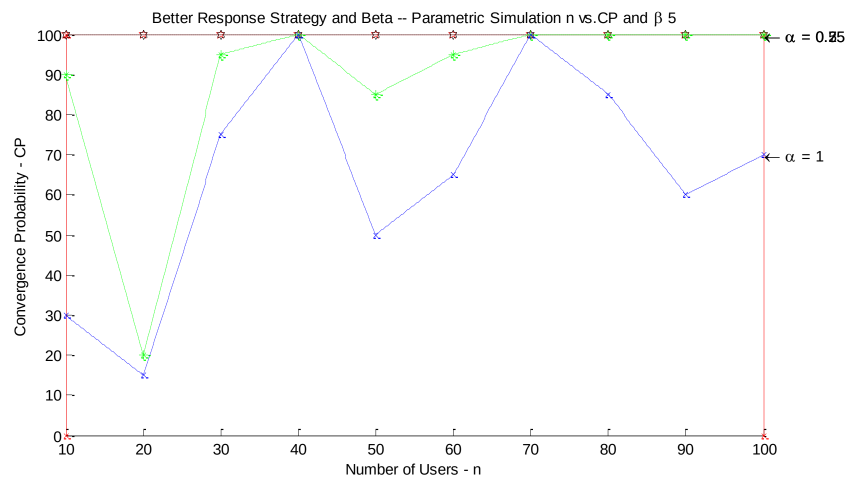


Figure 5.20e Better Response 5 concurrent users scenario convergence probability results for 7 fixed APs

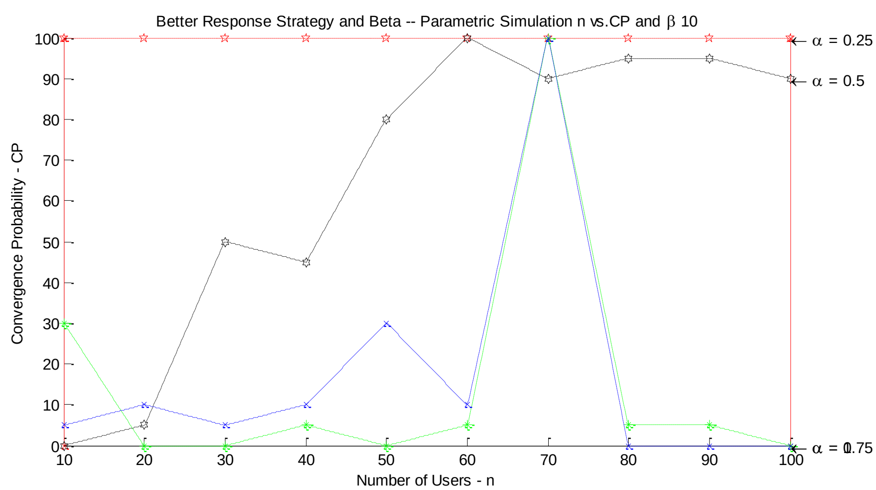


Figure 5.20f Better Response 10 concurrent users scenario convergence probability results for 7 fixed APs

Figures 5.20a through 5.20c are showing the *Better Response* algorithm results for number of iterations, for $\beta = 2$ the behavior for all $\alpha's$ is linear, very similar to the leftmost graph of figure 5.19a, now for $\beta = 5$ the results are different, having $\alpha = 0.25$ and $\alpha = 0.5$ with an apparent linear monotonic behavior unlike the plots for $\alpha = 0.75$ and $\alpha = 1$, $\alpha = 0.75$ gave in overall the biggest number of iterations, however $\alpha = 1$ has a peak value close to the max number of iterations achieved by $\alpha = 0.75$, on the other side for $\beta = 10$, the plot corresponding to $\alpha = 0.75$ outperforms $\alpha = 1$ in an average number of iterations wise.

The reason behind $\alpha = 0.25$ and $\alpha = 0.5$ outperforming $\alpha = 0.75$ and $\alpha = 1$ is because the more players playing simultaneously and the more choices (i.e APs available for them), the bigger the number of iterations along with a lower convergence probability, this proposition is supported by figures 5.20d through 5.20f, on which it can be seen clearly that the convergence probability for $\alpha = 0.25$ and $\alpha = 0.5$ is always greater or equal to that of $\alpha = 0.75$ and $\alpha = 1$. On figure 5.20e for $\beta = 5$ one can see a similar behavior for $\alpha = 0.75$ and $\alpha = 1$ from n = 10 to n = 70, however from n = 70 and on $\alpha = 0.75$ matches $\alpha = 0.25$ and $\alpha = 0.5$ performance achieving a convergence probability of 100%. Finally on figure 5.20f again $\alpha = 0.75$ and $\alpha = 1$ are having the same dynamic behavior convergence probability wise, even with the same peak value at n = 70, in this specific case $\alpha = 0.5$ underperforms $\alpha = 0.25$ for all points except for n = 60.

As concluding remark for the fixed access point simulation it can be concluded that *Better Response* algorithm outperforms *Best Response* algorithms when there are certain β players playing simultaneously, depending on the ratio of the number of users and access points, there would be peak values on both number of iterations and convergence probabilities.

<u>Results for 150 fixed users, varying access points from 10 to 100 (AP step 10) and $\beta = 2,5$ & 10:</u>

Best Response algorithm results

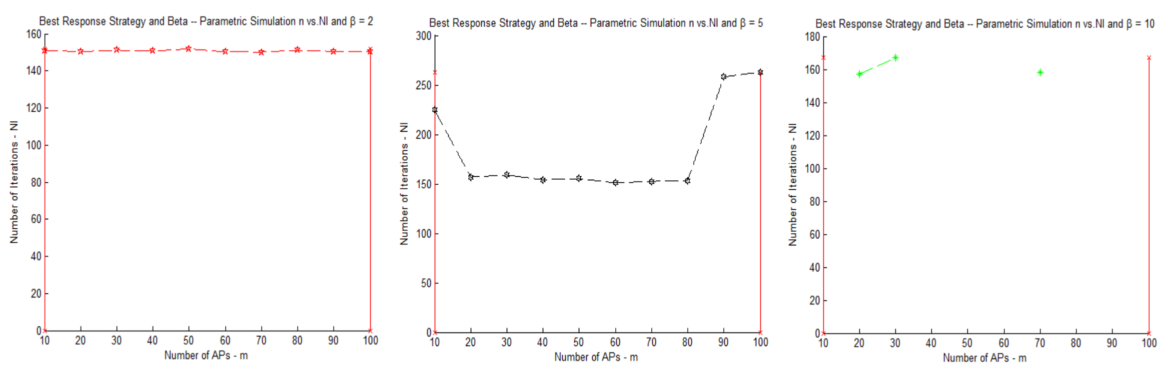


Figure 5.21a Best Response concurrent scenario results for 150 fixed users

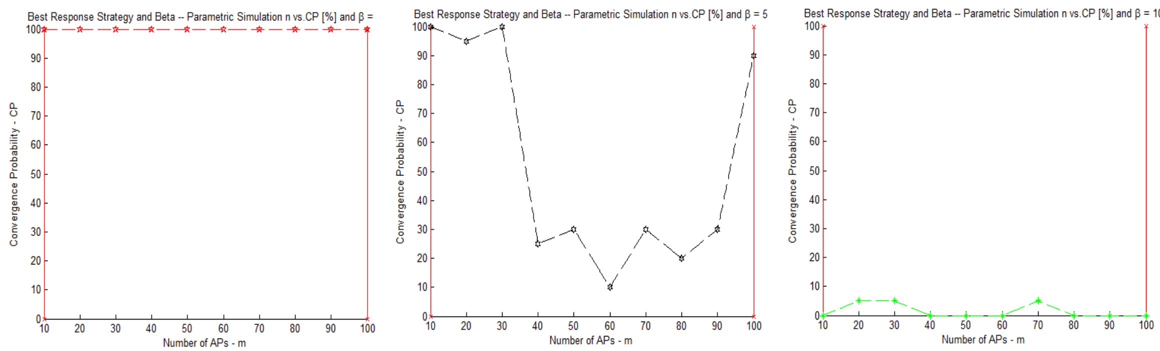


Figure 5.21b Best Response concurrent scenario convergence probability results for 150 fixed users

Figure 5.21a is showing the number of iteration results for the varying number of APs with 150 fixed users, as it can be seen on the leftmost graph of figure 5.21a corresponding to $\beta = 2$ the behavior tends to be constant around 150 iterations, approximate result as it non simultaneous players result obtained on figure 5.7, going right, the center graph of figure 5.21a shows a constant behavior like for $\beta = 2$ except for m = 10, 90 & 100, note that specifically for m = 10 and m = 100 the ratio between users and access points results in a multiple for $\beta = 5$ the correspondence with this behavior will also be seen on the convergence probability center graph shown on figure 5.21b. Lastly on the right of figure 5.21a there are only 3 points that found a convergence probability greater than 0, as seen on the rightmost graph of figure 5.21b, the points correspond to m = 20, 30 & 70

Better Response algorithm results

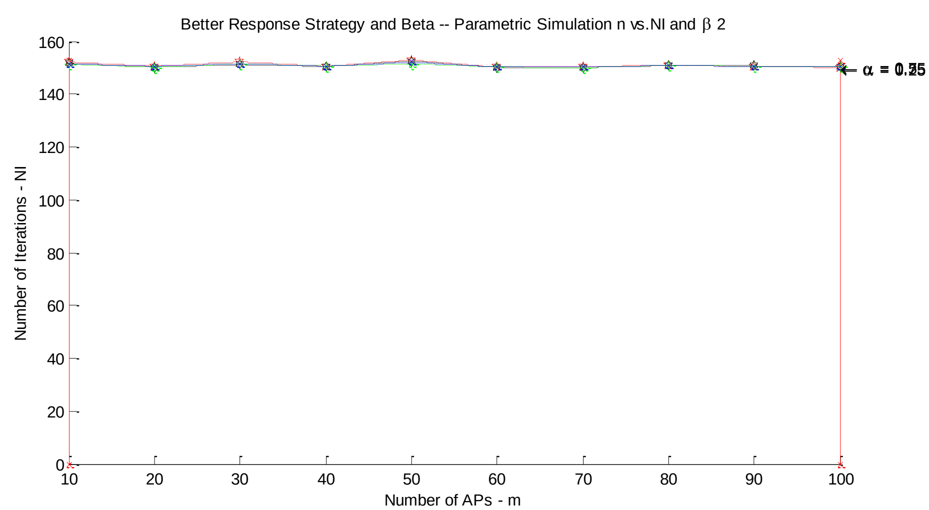


Figure 5.22a Better Response 2 concurrent user scenario results for 150 fixed users

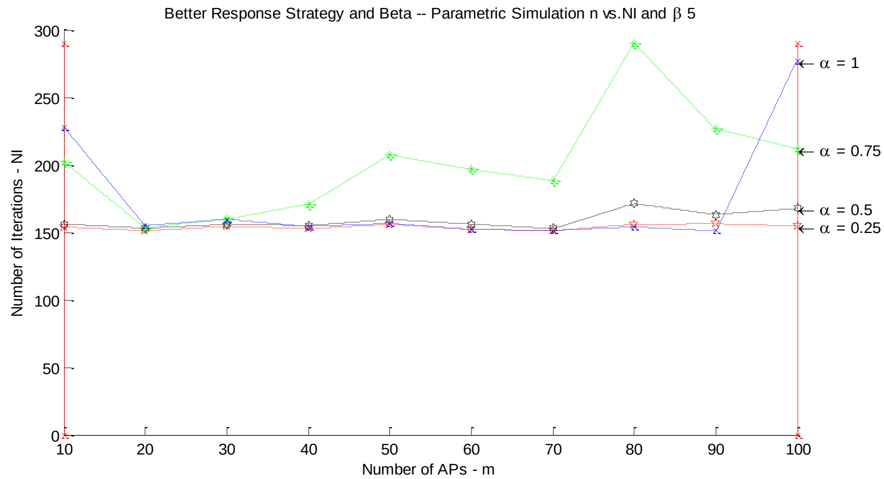


Figure 5.22b Better Response 5 concurrent user scenario results for 150 fixed users

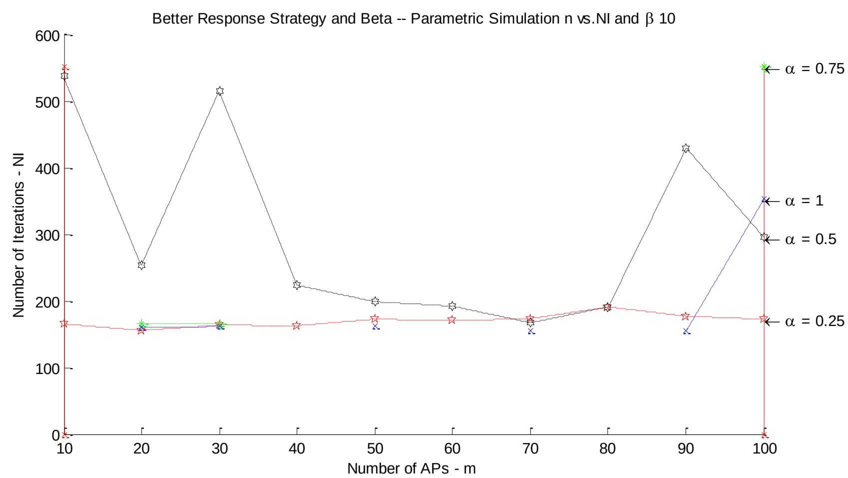


Figure 5.22c Better Response 10 concurrent user scenario results for 150 fixed users

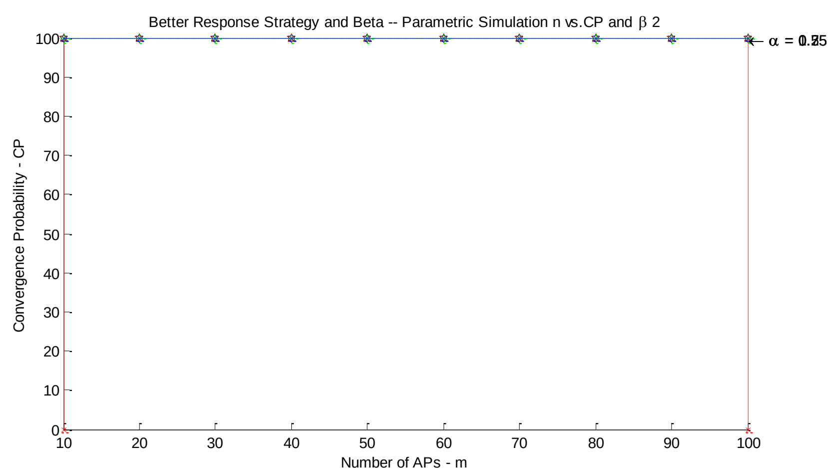


Figure 5.22d Better Response 2 concurrent user scenario convergence probability results for 150 fixed users

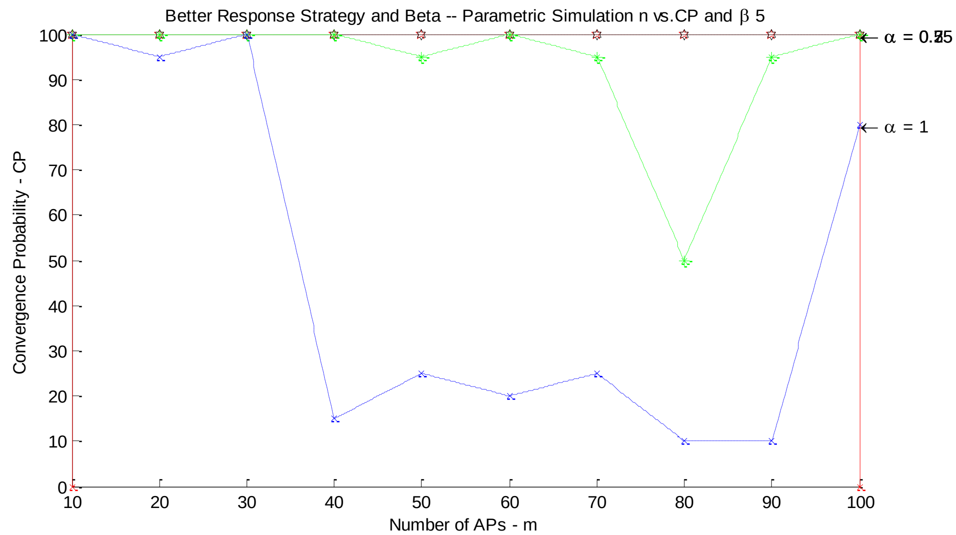


Figure 5.22e Better Response 5 concurrent user scenario convergence probability results for 150 fixed users

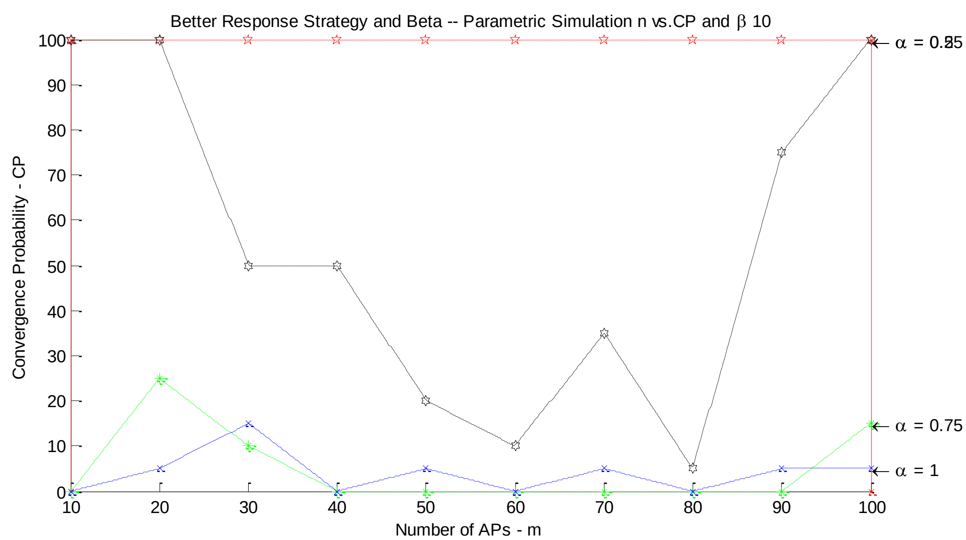


Figure 5.22f Better Response 10 concurrent user scenario convergence probability results for 150 fixed users

The number of iteration results can be observed on figures 5.22a through 5.22c, results for 7.22a do not have much to comment, for all α values at $\beta = 2$ the behavior is constant around 150 iterations, similar behavior a *Best Response* algorithm would have had. Now regarding the behavior on figure 5.22b which corresponds to $\beta = 5$ simultaneous users, it can be seen that $\alpha = 0.25$ and $\alpha = 0.5$ again outperform $\alpha = 0.75$ and $\alpha = 1$ just like it happened on the fixed APs simulation scenario. As seen on figure 5.22b $\alpha = 0.75$ has in average a bigger number of iterations than $\alpha = 1$ however both present a similar peak value at m = 80 and m = 100 respectively.

On the extreme case of β being 10, particular facts are observed, first the fact that the almost equivalent behavior between $\alpha = 0.25$ and $\alpha = 0.5$ is no longer kept, as seen the curve for $\alpha = 0.5$ fluctuates between the average *Best Response* average i.e 150 iterations and high peak values such as the ones observed on m = 10, 30 & 90, secondly, $\alpha = 0.75$ and $\alpha = 1$ exhibit very low convergence probabilities, greater at least than 0% for m = 20-30 and for m = 90-100 as it can be checked on figure 5.22f.

Figure 5.22d shows the convergence probability results for $\beta = 2$, as expected for all α values the convergence probability is 100%, however for the subsequent β values the curves fluctuate greatly, as seen on figure 5.22e where besides $\alpha = 0.25$ and $\alpha = 0.5$, for the other α values it can be seen the cost of outperforming $\alpha = 0.75$ resulted in a lower convergence probability in overall, at the same time figure 5.22f presents different curve oscillations but the behavior can be derived from it number of iteration figure 5.22c.

Concluding remarks for the fixed user and varying number of access points simulation are similar to those stated for fixed number of access points and varying number of users, the lower the α value, the lower the number of total iterations and the higher the convergence probability chance, here also the fact of finding at least 1 Nash equilibria is paid off with a large amount of iterations. For specific values of concurrent users such as $\beta = 5$ and $\beta = 10$ and high values of α such as $\alpha = 0.75$ and $\alpha = 1$ there is no convergence at all except for some values where the ratio between the number of users and access points is close to the specific value of β being evaluated.

5.4

Access point coverage radius parametric analysis

Previously, the analysis made consisted on a fixed access point coverage radius that was modified to cover all users (*All in range*). Being able to analyze the performance analysis of the different games under different AP coverage radius is important because these situations can happen in real life [15,16,17] whenever an AP has a variable transmission power. These type of simulations can be seen as a sort of joint Network Selection problem, in the sense that an AP can increase or decrease its coverage area in order to selfishly maximize the number of users covered, however, the parameterization of the AP coverage area is not done with this policy, but just for practical studies without specifically caring for the AP wealth. The impacts of these disturbances will be studied on the following sub-Sections.

Not all in Range **+ Linear Grid topology**

Results for 50 users and 7 APs:

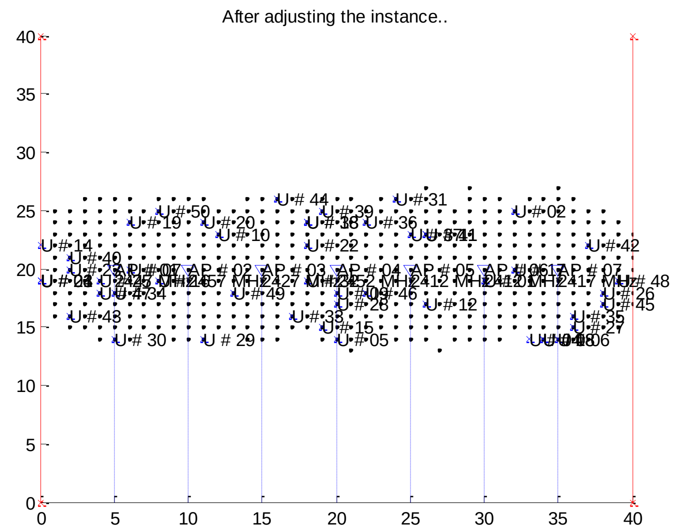


Figure 5.23 Linear grid map with AP Emitted Power = 5dBm

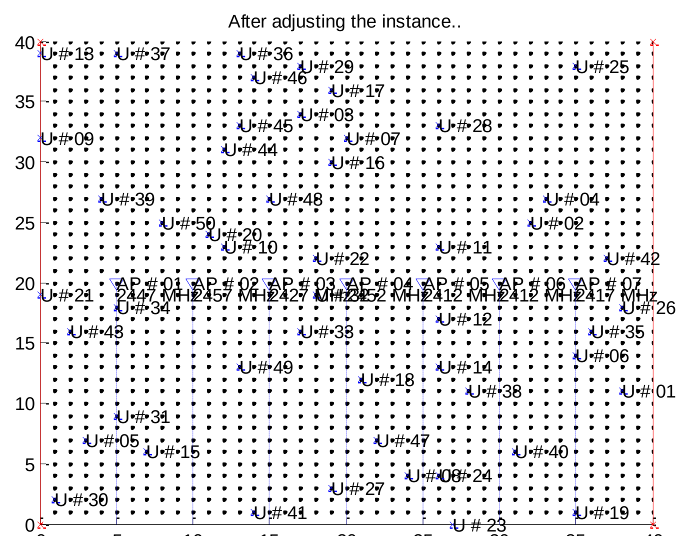


Figure 5.24 Linear grid map with AP Emitted Power = 20dBm

Best Response algorithm results

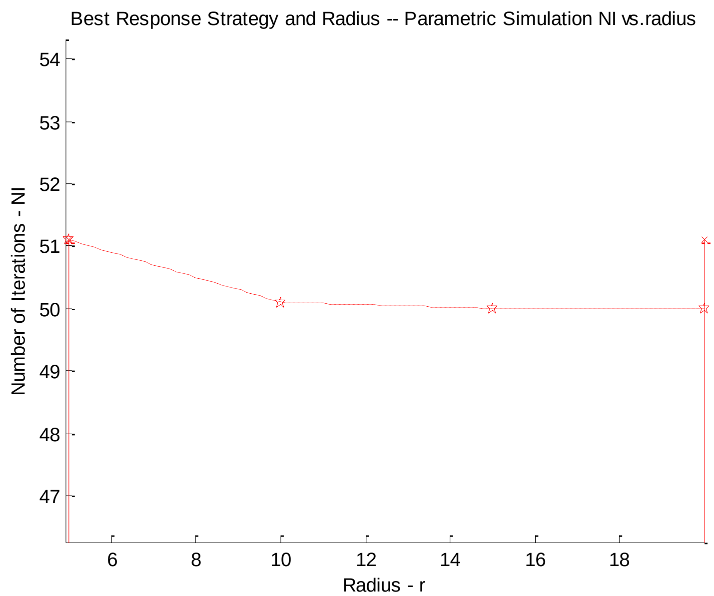


Figure 5.25 Best Response linear grid topology coverage radius results for 50 users and 7 APs

Better Response algorithm results

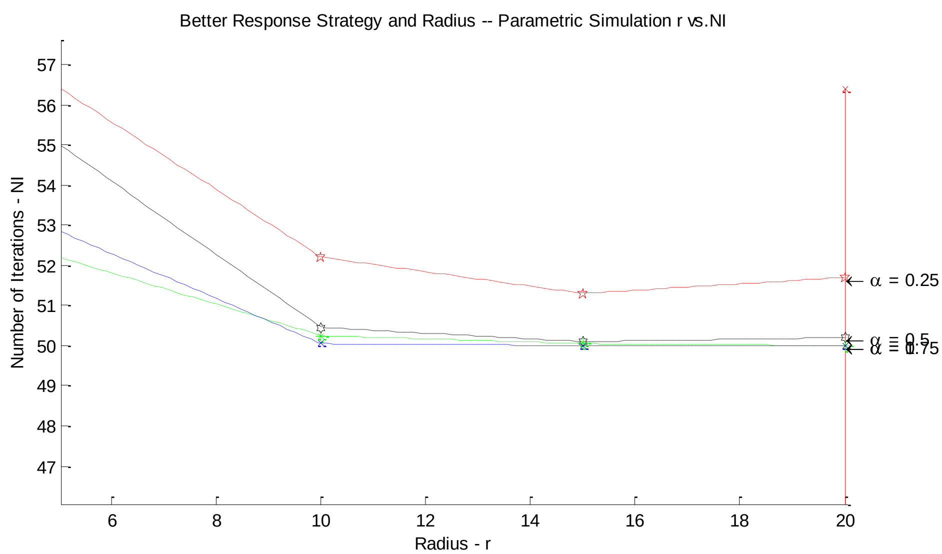


Figure 5.26 Better Response linear topology coverage radius results for 100 users and 7 APs

From figures 5.25 and 5.26 it can be seen that for the linear grid map topology the *all in range* ideal convergence condition is reached faster at around *r* = 10m, no overshoot presented here, however the starting values at *r* = 5m were greater that their couple counterparts. The fast monotonic decrease presented from *r* = 5m to *r* = 10m might be due to the fact that a slight increase in the AP emitted power might suffice to reach or be close to reach the *all in range* condition due to the positioning of the access points, for which in this specific case are close to each other much more compared to a random topology with randomly allocated APs.

<u>Results for 100 users and 7 APs</u>:

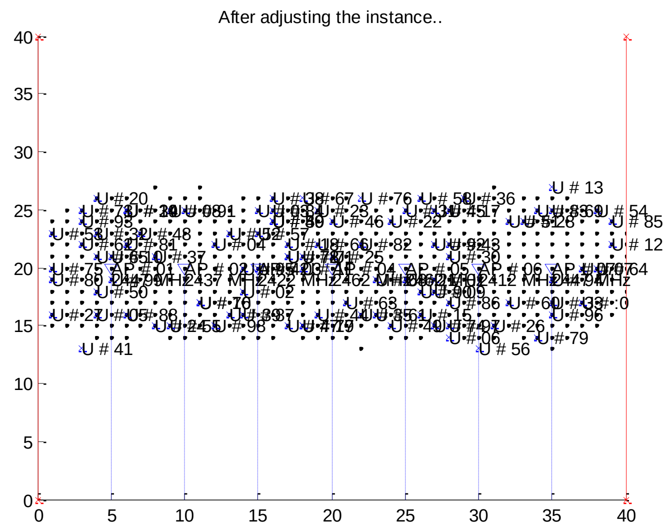


Figure 5.27 Linear grid map with AP Emitted Power = 5dBm

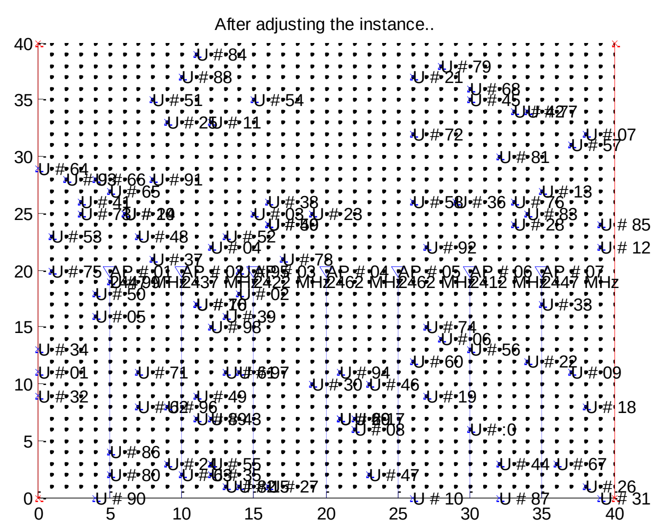


Figure 5.28 Linear grid map with AP Emitted Power = 20dBm

Best Response algorithm results

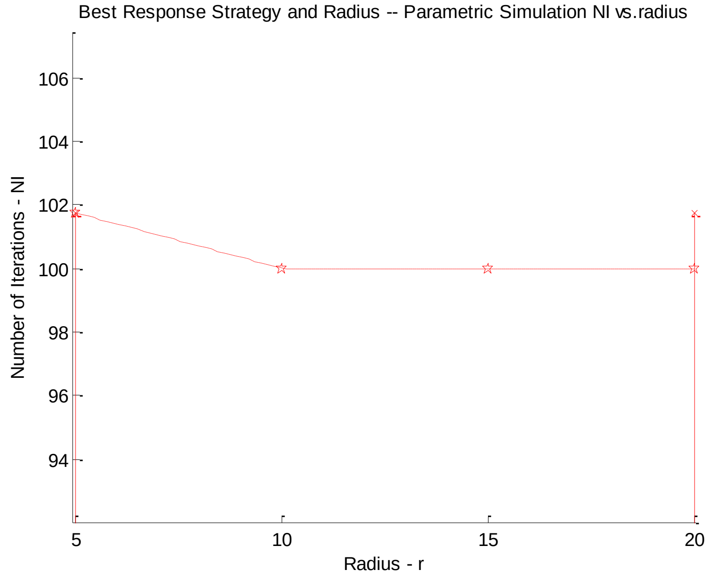


Figure 5.29 Best Response linear grid topology coverage radius results for 100 users and 7 APs

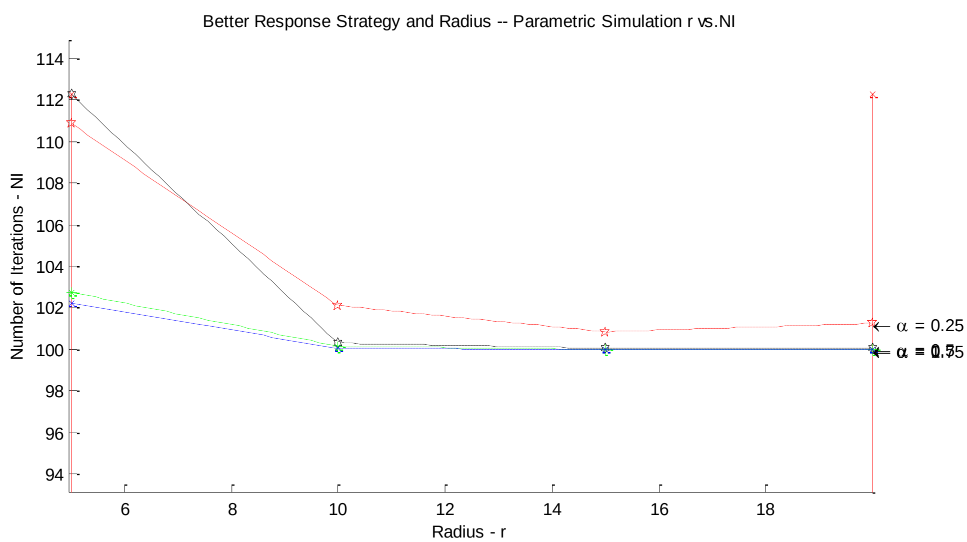


Figure 5.30 Better Response linear grid topology coverage radius results for 100 users and 7 APs

The last couple of figures show not big discrepancies between their former two latest figures even despite the fact of an additional 50 users, same curve dynamics with respect to the increasing value of *r* and same tendency to find the *all in range* convergence faster than the random topology maps.

***Not all in Range* + Rectangular Grid topology**

<u>Results for 50 users and 12 APs</u>:

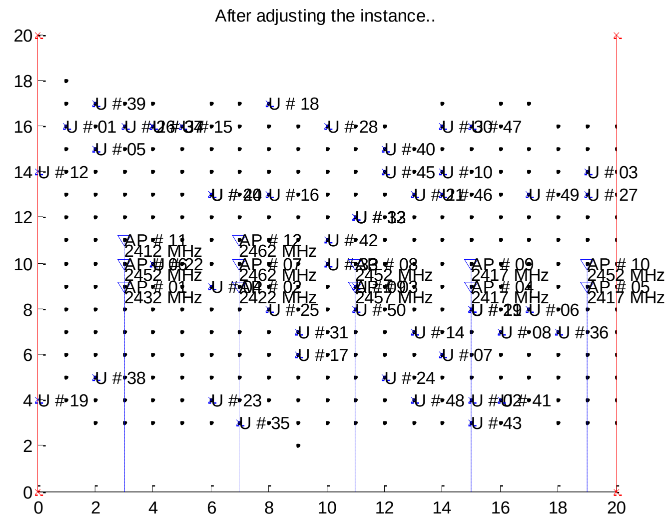


Figure 5.31 Rectangular grid map with AP Emitted Power = 5dBm

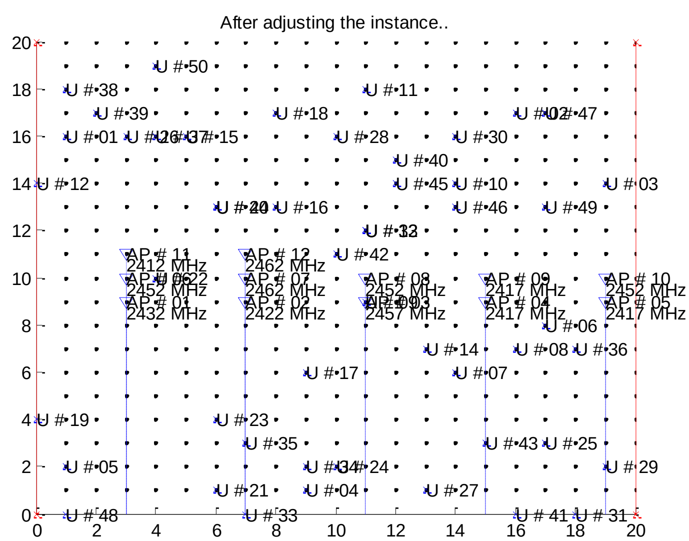


Figure 5.32 Rectangular grid map with AP Emitted Power = 20dBm

Best Response algorithm results

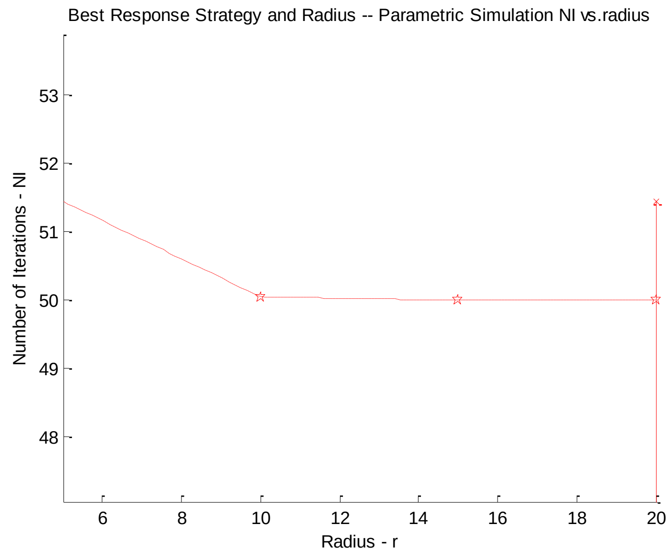


Figure 5.33 Best Response rectangular grid topology coverage radius results for 50 users and 7 APs

Better Response algorithm results

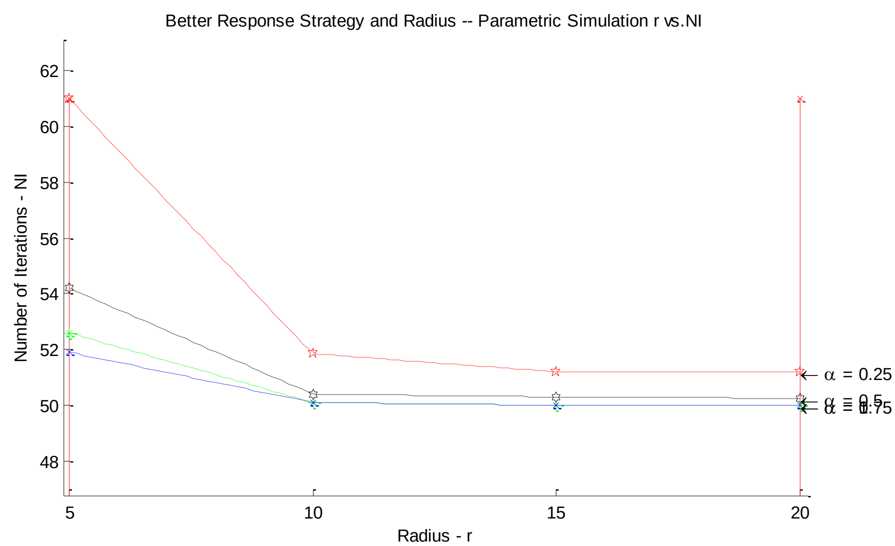


Figure 5.34 Better Response rectangular grid topology coverage radius results for 50 users and 7 APs

Figure 5.33 is not showing anything new when compared with its former result for the previously analyzed topology on figures 5.25, taking into account the strategy space for this simulations is almost the double of the previous scenarios, it could be inferred that even with $\alpha = 0.25$ convergence would have been reached faster, however the overshoot presented at r = 5m is the highest number of iteration value for all 50 user simulated scenarios, possibly making relevant the fact of being able to choose 3 out of 12 APs when their coverage radius is short ends on a bigger effort to find an equilibrium.

<u>Results for 100 users and 12 APs</u>:

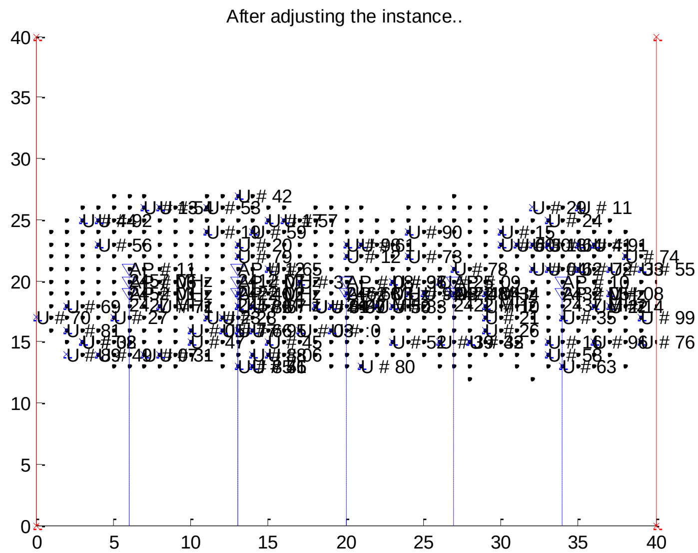


Figure 5.35 Rectangular grid map with AP Emitted Power = 5dBm

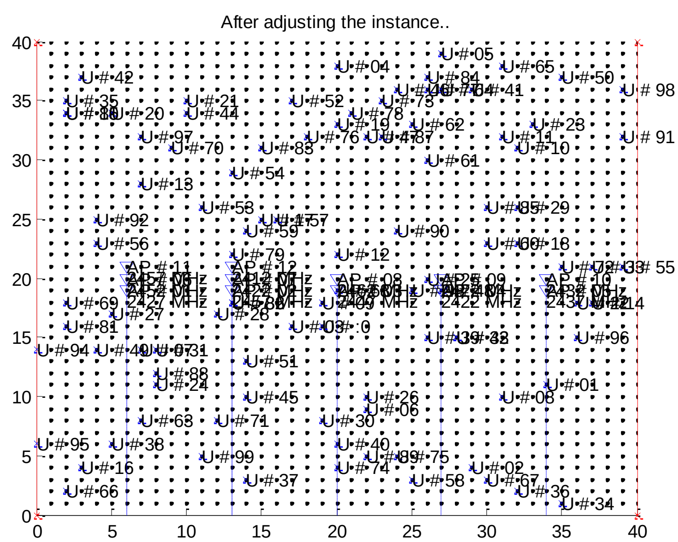


Figure 5.36 Rectangular grid map with AP Emitted Power = 20dBm

Best Response algorithm results

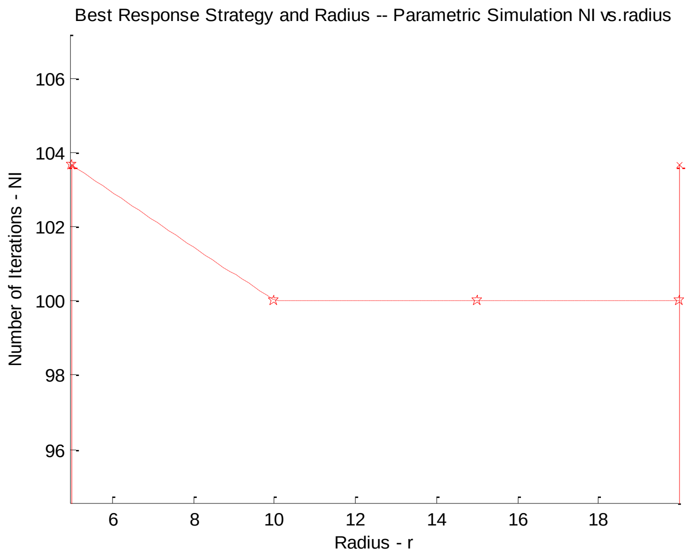


Figure 5.37 Best Response rectangular grid topology coverage radius results for 100 users and 12 APs

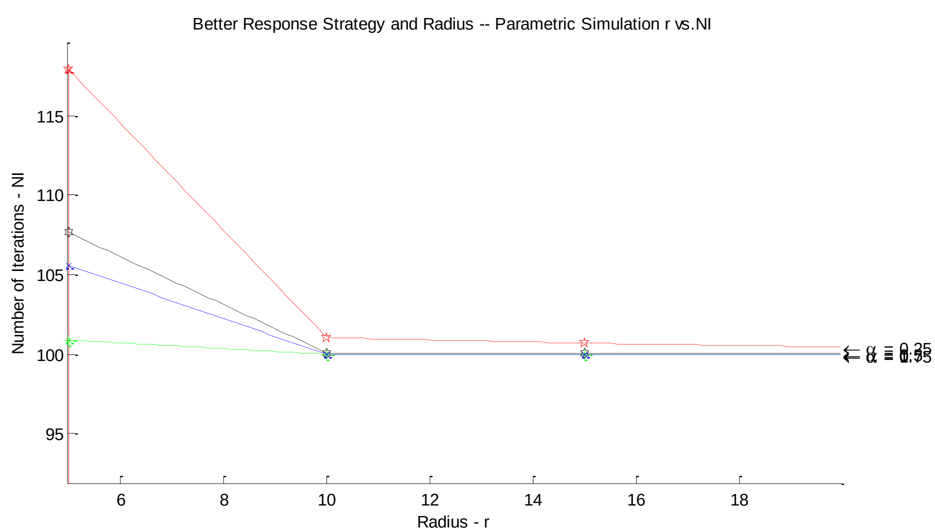


Figure 5.38 Best Response rectangular grid topology coverage radius results for 100 users and 12 APs

Figures 5.37 and 5.38 are showing the number of iteration results vs. the parametric value of AP coverage radius, these couple of graphs are showing the same asymptotic behavior as its former counterpart topology shown on figure 5.30, exposing once again faster convergence at r = 10m; as a relevant fact it is also pointed that the initial value for $\alpha = 0.25$ at r = 5m was the highest one compared to all 100 user simulations.

Concluding remarks for all simulations point out the fact of spread out initial values for r = 5m on all simulations for all α values which monotonically (in almost all cases) decrease afterwards due to a closing up of the *all in range* condition. Additionally the linear grid map topology exhibited the best performing results, having the least number of isolated peaks and having a fast convergence for r = 10m and greater.

5.5

Fictitious Play Algorithm

Myopic Vs. Non-Myopic

Unlike *Best* and *Better Response* algorithms, *Fictitious Play* algorithms base their decisions on the knowledge learnt from previous iterations, rather than from a common knowledge base as the former algorithms did, hence a user playing *Fictitious Play* will have an individual knowledge from all the game that he will use to make the most appropriate decision (for more information on Fictitious Play algorithms go to Section 3.5.3).

Performance analysis will be done over 4 different flavors:

- Deterministic Fictitious Play
- Stochastic Fictitious Play
- Myopic Deterministic Fictitious Play
- Myopic Stochastic Fictitious Play

Fixed number of access points, varying number of users static analysis

- Varying number of randomly deployed users, starting from 5 users in steps of 5 users per iteration until 30, giving a total of 6 different games.
- Access point power transmission model modified in order to achieve an *All in Range* scenario.
- Map length $40m^2$

Fixed number of Access Points will be simulated for 3 and 7 APs

Legend:

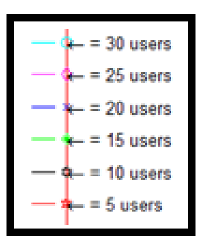


Figure 5.39 User curve legend

Results for 3 APs:

Deterministic FP results

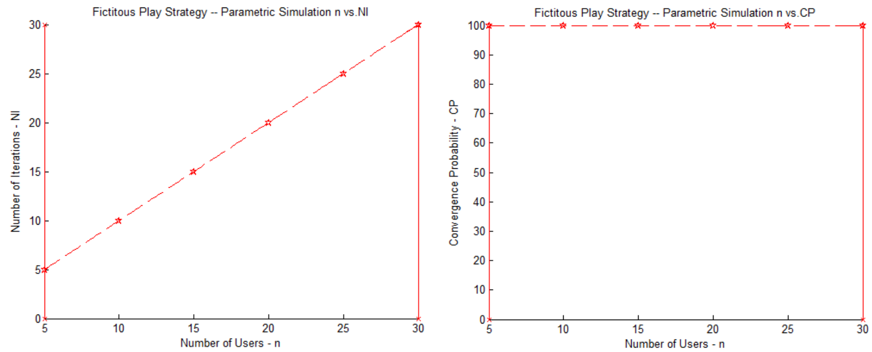


Figure 5.40 Deterministic Fictitious Play results for 3 fixed APs

Stochastic FP results

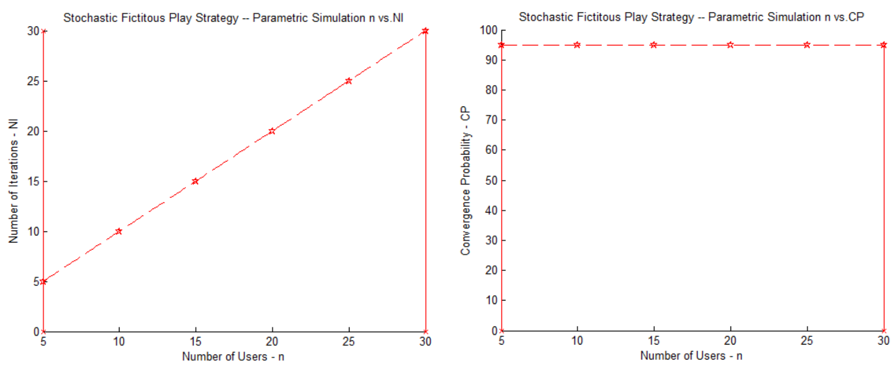


Figure 5.41 Stochastic Fictitious Play results for 3 fixed APs

First result for the learning technique *Fictitious Play* is showing a linear behavior for the deterministic FP, exact behavior as a *Best response* algorithm (line with slope equal to one) as seen on figure 5.40; similarly leftmost graph on figure 5.41 reflects same behavior as its deterministic counterpart, however the same performance number of iteration wise is paid off with a decrease in the average convergence probability equal to 95%. 5% of no convergence accounts for 1 game iteration that did not found a Nash equilibrium, however it should be taken into account that the stochastic decision is taken from the learned history of other players having 3 access points.

Myopic deterministic FP results

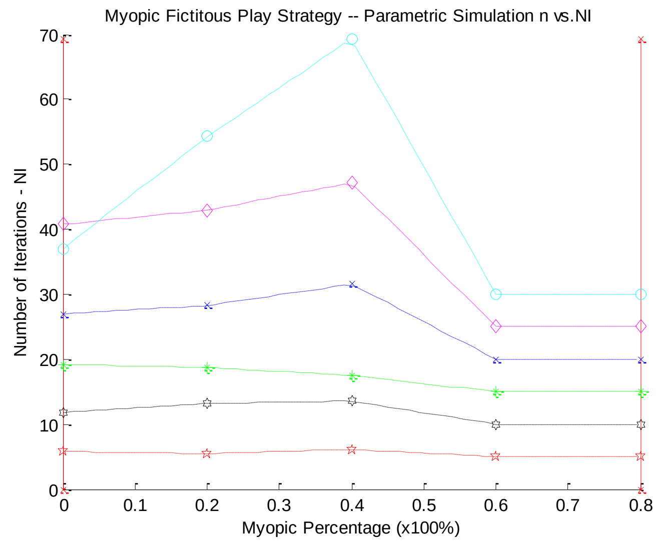


Figure 5.42 Myopic Deterministic Fictitious Play results for 3 fixed APs

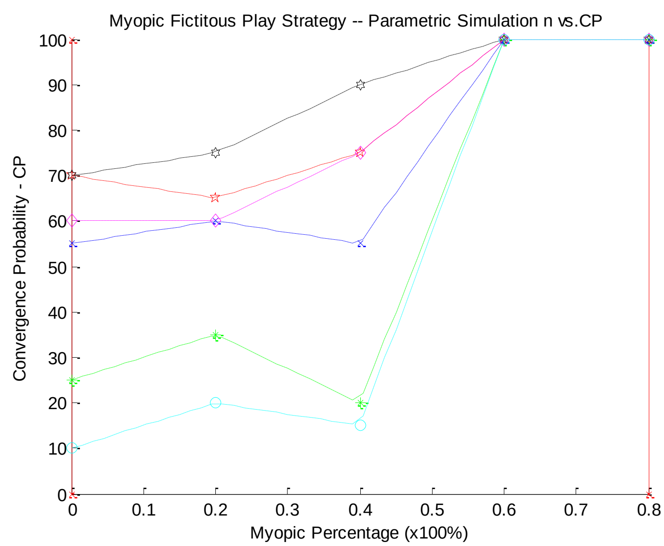


Figure 5.43 Myopic Deterministic Fictitious Play convergence probability results for 3 fixed APs

Figures 5.42 and 5.43 are showing results as they were never showed before, each curve corresponds to each simulated point (5 users, 10 users, … , 30 users) the legend is presented on the figure 5.39, analyzing figure 5.42 from bottom to top, each curve encountered corresponds to the initial iterated value i.e 5 users, from 5 to 15 users an each curve is almost flat (close to 0 slope), from 15 to 30 users a common point presenting an overshoot is observed at m.p = 40%, from m.p = 40% to 60% a negative slope line is seen until the settlement point is found at m.p = 60%. The first three curves from bottom to top seen on figure seem to exhibit immunity with respect to the blind percentage, each one of these three curves is holding an almost constant behavior number of iteration wise paying off a different convergence probability cost, particularly the n = 15 users curve having the second worst behavior just behind n = 30 users. Observe on figure 5.42 another relevant fact, from m.p = 60% to m.p = 80% the difference between each curve is just the delta value of injected users i.e 5 users between each curve, lowest delta value between any pair of curves for all myopic percentage studied.

Myopic stochastic FP results

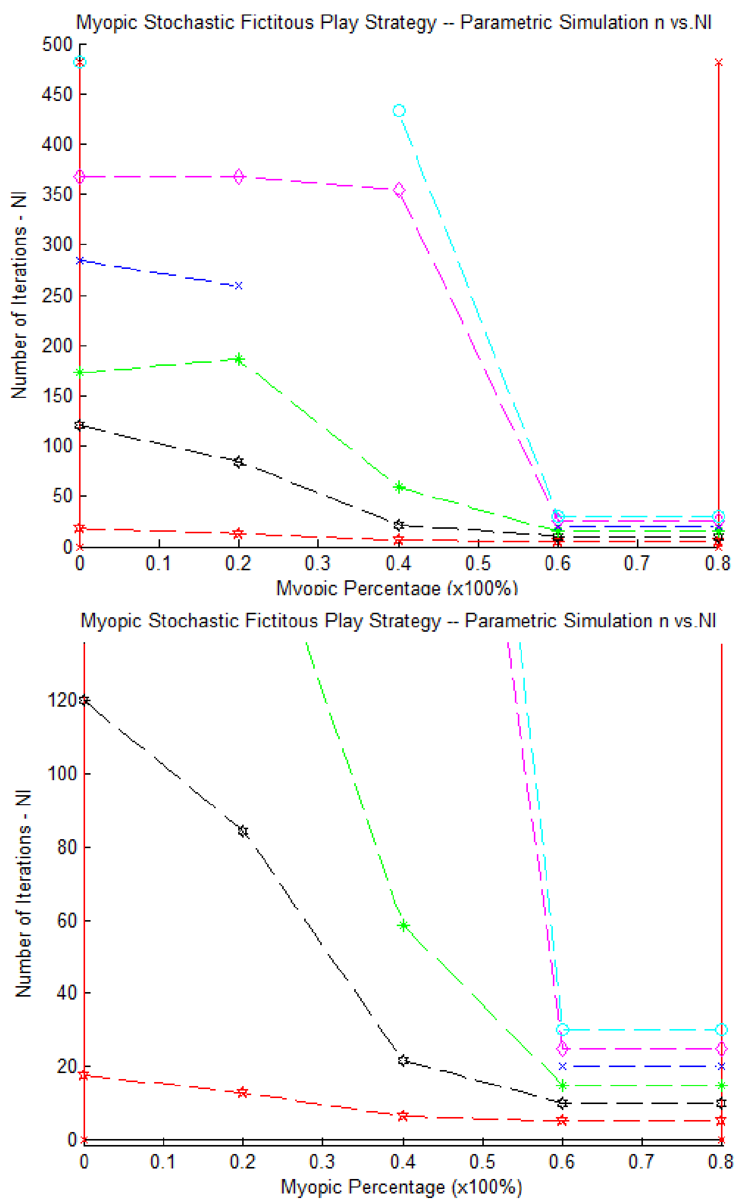


Figure 5.44 Myopic Stochastic Fictitious Play results for 3 fixed APs

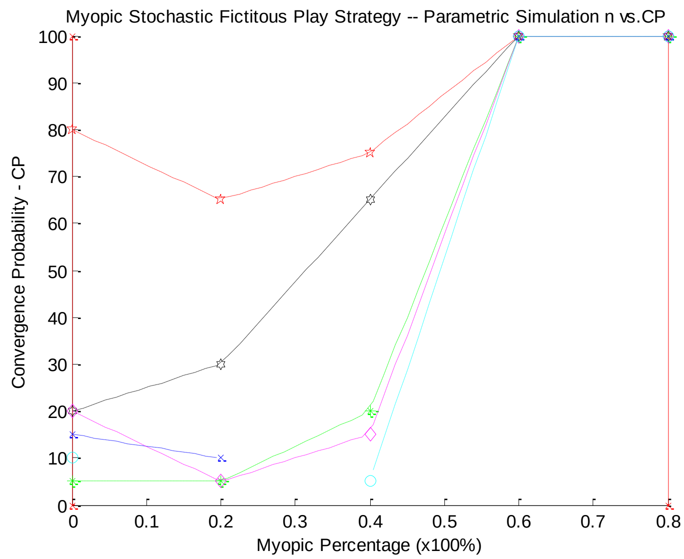


Figure 5.45 Myopic Stochastic Fictitious Play convergence probability results for 3 fixed APs

The fact of choosing from the previous experiences built knowledge base stochastically rather than deterministically is clearly showing relevant differences, in fact comparing figure 5.44 with figure 5.42 it can be observed that the only curve holding a constant behavior for all myopic percentages is n = 5 users, rest of the curves present abysmally big initial values compared to the deterministic version of the same algorithm, and even some curves (cyan and blue) have a convergence probability equal to 0% at particular points. Nevertheless the settlement point is conserved (m.p = 60% to m.p = 80%) and the values presented on this range cross-check the values obtained on the non-myopic version of the algorithm

<u>Results for 7 APs</u>:

Deterministic FP results

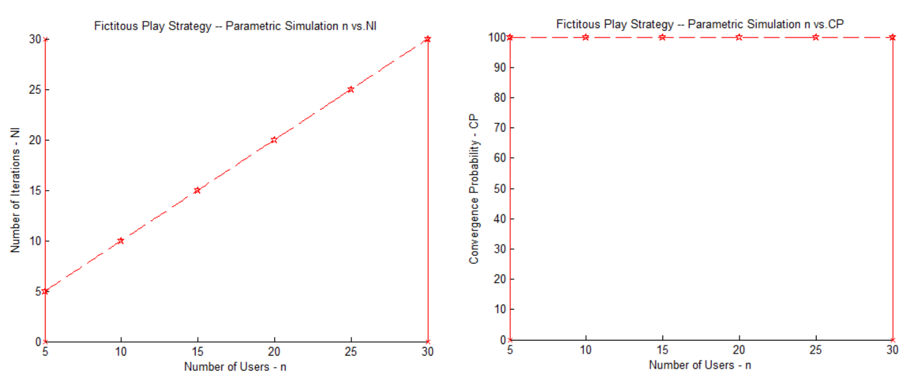


Figure 5.46 Deterministic Fictitious Play results for 7 fixed APs

Stochastic FP results

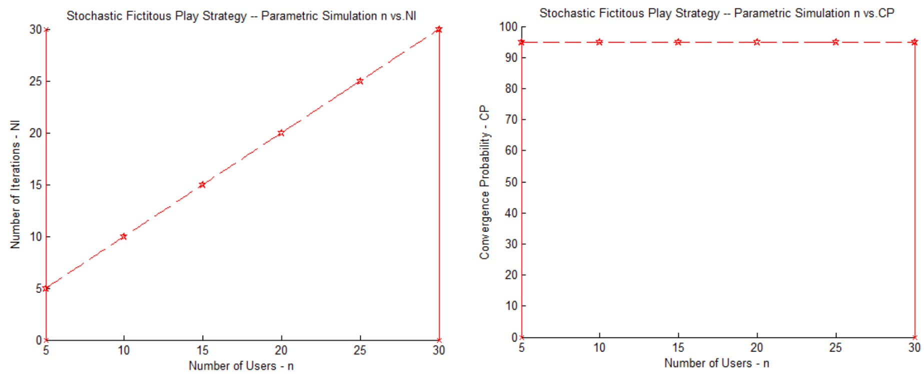


Figure 5.47 Stochastic Fictitious Play results for 7 fixed APs

The addition of 4 access points to the scenario did not alter the outputs shown on figures 5.40 and 5.41 depicted on figures 5.46 and 5.47, the result may point towards a weak or no relation between the amount of strategies and the *Fictitious Play* algorithm.

Myopic deterministic FP results

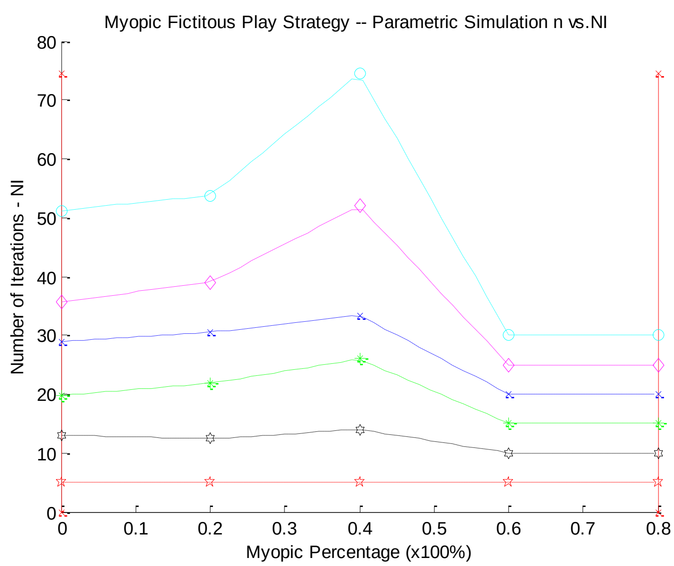


Figure 5.48 Myopic Deterministic Fictitious Play results for 7 fixed APs

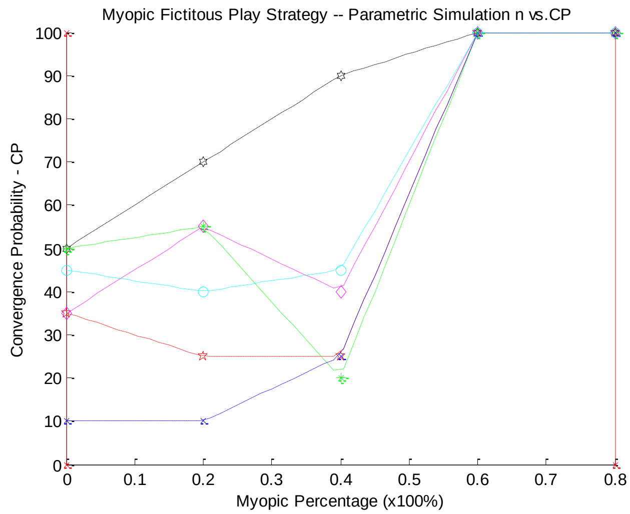


Figure 5.49 Myopic Deterministic Fictitious Play results for 7 fixed APs

Myopic stochastic FP results

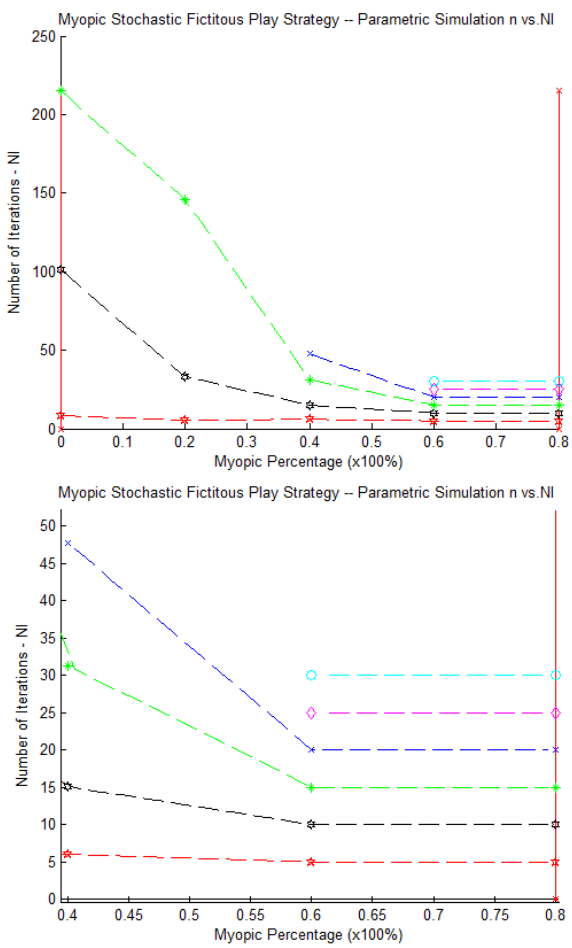


Figure 5.50 Myopic Stochastic Fictitious Play results for 7 fixed APs

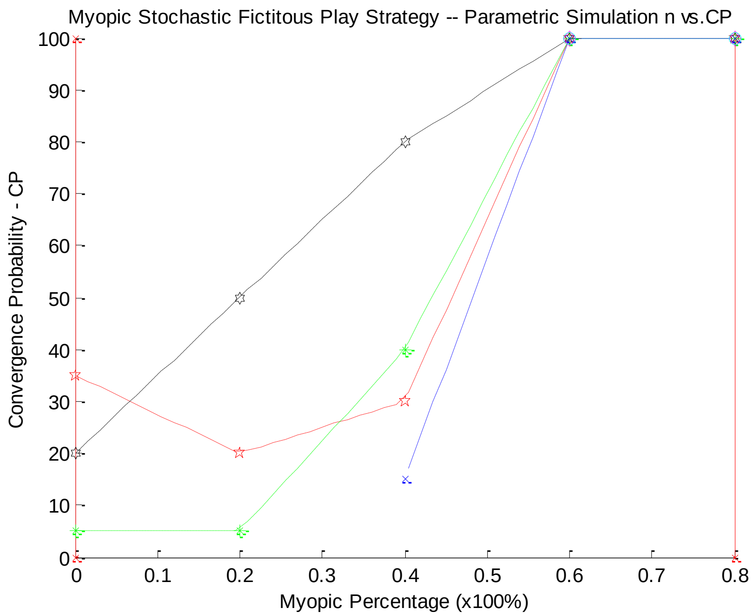


Figure 5.51 Myopic Stochastic Fictitious Play results for 7 fixed APs

Starting off with figure 5.48, appreciate its great similitude with figure 5.42 except for the fact of the starting points for n = 25 and n = 30 users, and also the in-case figure presented a number slightly greater of maximum number of iterations attributed to n = 30 users (cyan curve); bottom graph which corresponds to a close up of figures 5.50 top graph allows to crosscheck the shared behavior of all 4 *Fictitious Play* algorithms, non-myopic algorithms are a special case of the myopic versions of the F.P implementations, hence the crosscheck point must always exist, now the "why" should be answered, non-myopic versions of the F.P algorithm correspond to the deterministic or stochastic *Best Response* to the learnt knowledge based on previous experiences from all the other users, on the other hand the myopic versions of the algorithm have a different update than the non-myopic versions, the way the previous knowledge is stored, is aided by the fact of being myopic with respect previous knowledge because *Fictitious Play* algorithm converges in beliefs to Nash Equilibrium [2], the more recent the knowledge the more refined it will be with respect to *from-scratch* knowledge, hence the particular behaviors observed for high grades of myopic percentage.

Following the last sentences from the previous paragraph, observe from the latest two graphs presented for convergence probability figures 5.49 and 5.51 how the convergence probability improves in an irregular way though for each curve (no direct relation of better performance for a number of users less than to a reference curve), the convergence improvement is not continuous however the increases monotonically from m.p = 40% and on until the settlement point discovered from previous results.

Fixed number of users, varying number of access points static analysis

- Varying number of randomly deployed access points, starting from 1 AP in steps of 1 AP per iteration until 7, giving a total of 7 different games.
- Access point power transmission model modified in order to achieve an *All in Range* scenario.
- Map length $40m^2$

Fixed number of Users will be 30

Deterministic FP results

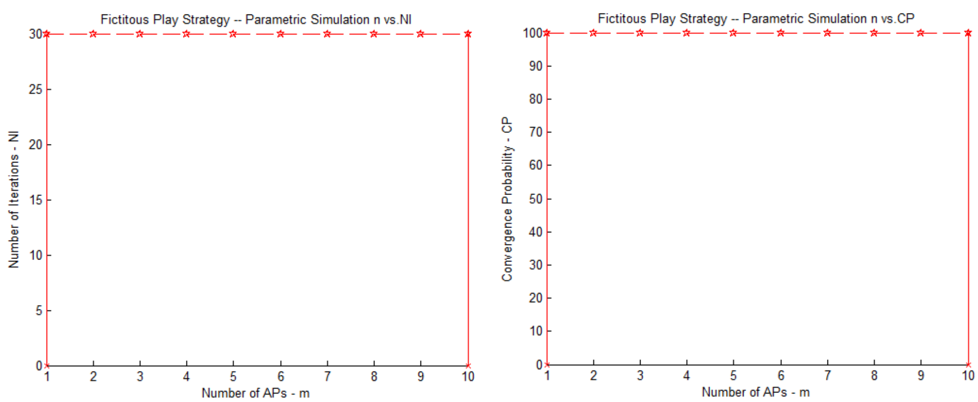


Figure 5.52 Deterministic Fictitious Play results for 30 fixed users

Stochastic FP results

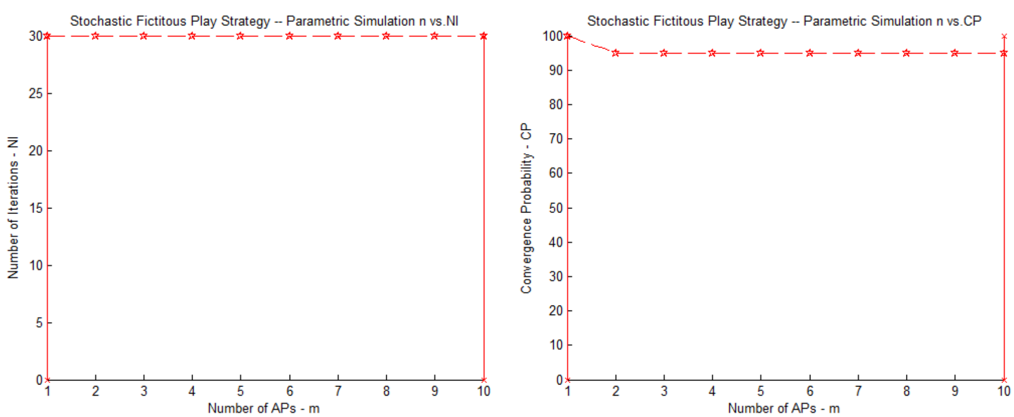


Figure 5.53 Stochastic Fictitious Play results for 30 fixed users

Same behavior under different scenarios is what is showing figures 5.52 and 5.53 when compared to the non-myopic *Fictitious Play* algorithm results presented for a fixed number of access points, the deterministic version of the F.P algorithm output presented on figure 5.52 shows a constant number of iterations (30 iterations) for all simulated points, figure 5.53 similarly holds a constant value of 30 iterations with a payoff of 5% convergence probability with respect to its non-stochastic counterpart.

Myopic deterministic FP results

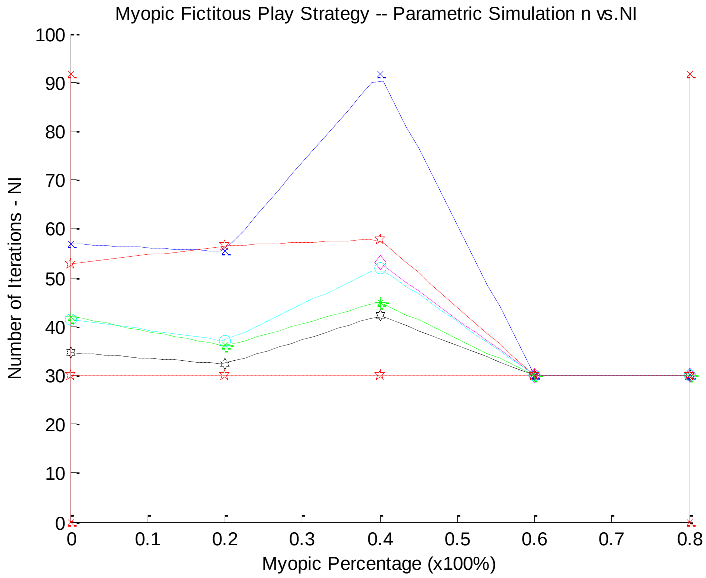


Figure 5.54 Myopic Deterministic Fictitious Play results for 30 fixed users

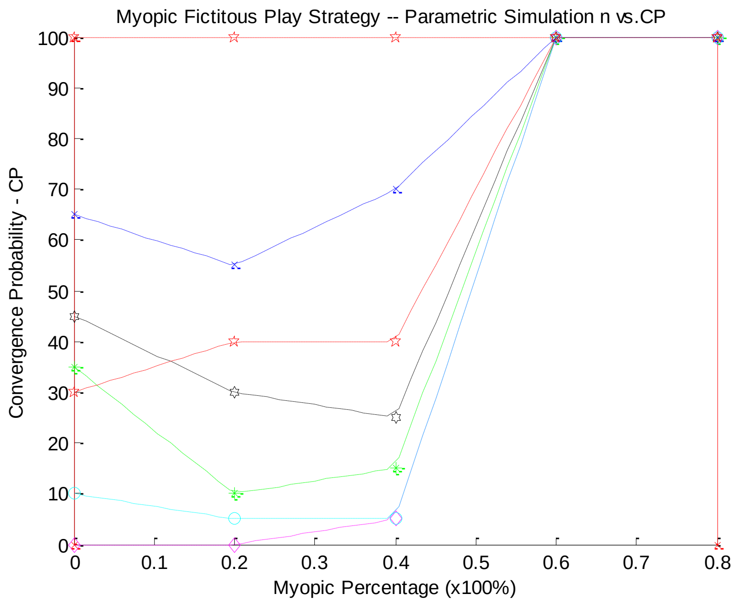


Figure 5.55 Myopic Deterministic Fictitious Play convergence probability results for 30 fixed users

Observe on figure 5.54 how the bottommost red curve is showing an ideal behavior with respect to the blind percentages, the result for this curve results trivial as it is reflecting the output for 30 fixed users with just 1 access point, hence the deterministic and stochastic decisions will derive the exact same results, on the same figure observe how as strategies are added how the myopic percentage makes each curve behave differently presenting a non-linear behavior until the settlement point is reached. The worst performing curve number of iterations wise is not directly the one with most number of strategies as seen on figure 5.54 (resulted to be the blue dashed curve i.e 6 access points) with 300% more iterations than the trivial and settlement value.

Figure 5.55 is presenting an irregular behavior as stated on the fixed access point scenario, however it is observed how the 2 worst performing curves number of iterations wise are outperforming all other curves convergence probability wise (except the trivial case).

Myopic stochastic FP results

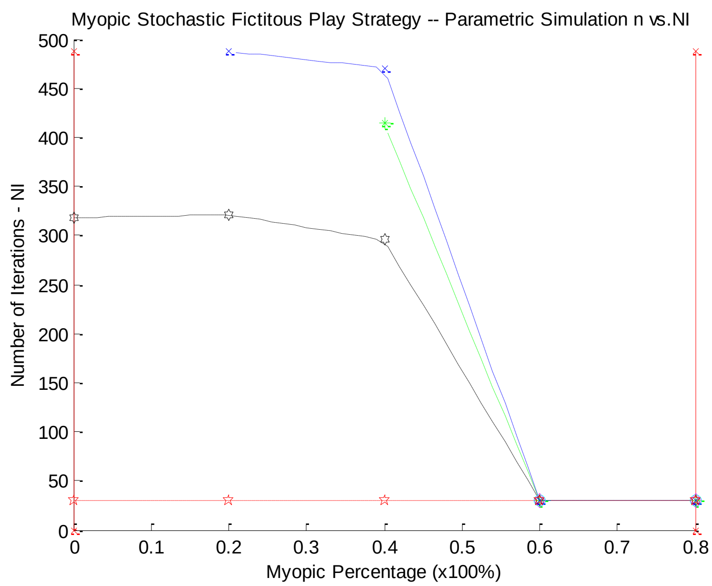


Figure 5.56 Myopic Stochastic Fictitious Play results for 30 fixed users

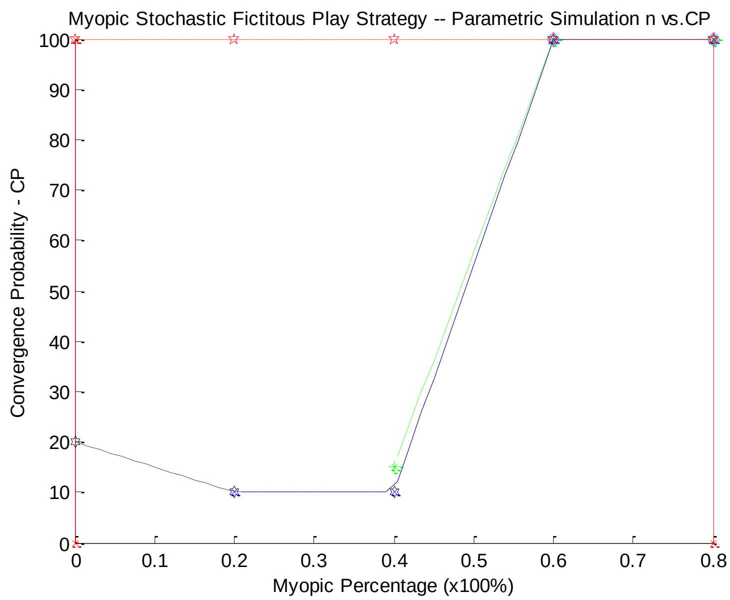


Figure 5.57 Myopic Stochastic Fictitious Play convergence probability results for 30 fixed users

The stochastic version of the studied scenario is showing poor convergence for m = 2, 3 and 4 access points and no convergence at all for the remaining added strategies except for the settlement points m.p = 60% & 80% though it cannot be seen directly on the graph due to the overlapping of the plots on the mentioned range. Behavior observed from the previous results lights up the particular equivalence between the non-myopic versions of the *Fictitious Play* algorithm, on the other hand when dealing with myopic version of the learning technique, it should be known beforehand that being able to choose deterministically from the knowledge base built from previous experiences is clearly a better option than the stochastic version of the latter, however if the myopic percentage can be adapted dynamically for both versions, the latter would be a better option as it would clearly perform more similarly as a real life agent.

6.

PERFORMANCE EVALUATION WITH ADDITIVE INTERFERENCE AND INVERSE RATE UTILITY FUNCTIONS

Utility function definition

The specific case consists of a cost function being a linear combination of the number of interferers and the reverse of the rate perceived by users. It is denoted by T_j^i the reverse of the rate perceived by user *j* to connect to access point A_i.

The user cost functions can be therefore defined as:

$c_j(i, x^i) = \lambda_1 x^i + \lambda_2 T_j^i$, where $\lambda_1, \lambda_2 \in \mathbb{R}_+$ are user independent parameters [17].

Note: All simulations done will be under the assumption of "All in range" meaning each user is inside the coverage area of all existing access points. The previous holds unless it is differently expressed

6.1

BETTER RESPONSE ALGORITHM VS. BEST RESPONSE ALGORITHM

Fixed number of access points, varying number of users static analysis

- Varying number of randomly deployed users, starting from 10 users in steps of 10 users per iteration until 100, giving a total of 10 different games.
- Access point power transmission model modified in order to achieve an *All in Range* scenario.
- Fixed number of Access Points for each game, simulating games for 5 and 10 APs
- Map length $40m^2$

<u>Results for 5 APs</u>:

Best Response algorithm results

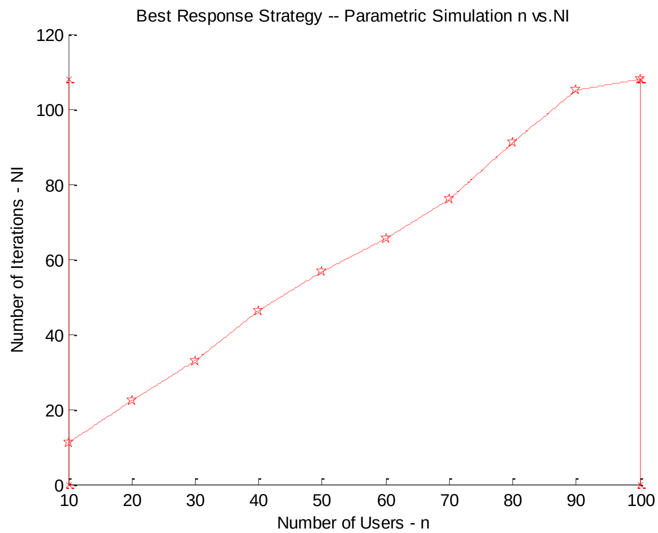


Figure 6.1 Best Response results for 5 APs

Better Response algorithm results

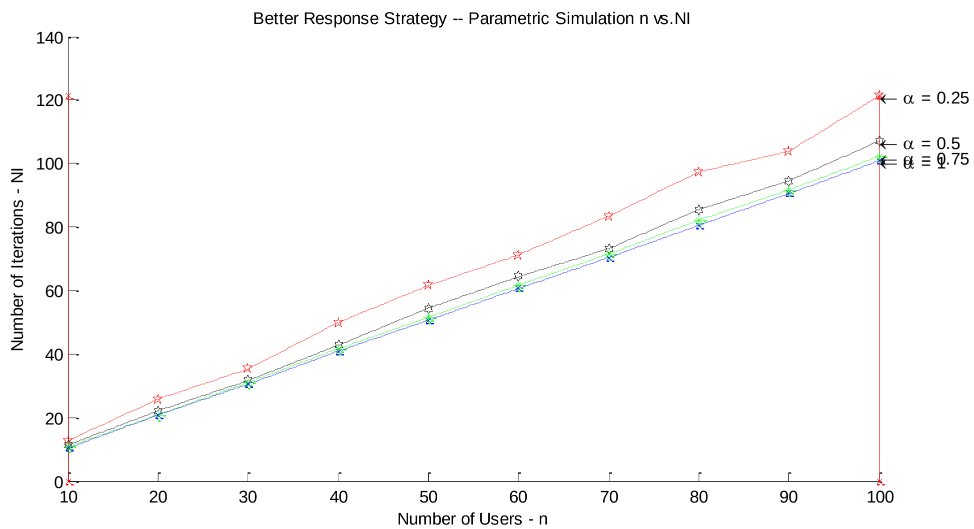


Figure 6.2 Better Response results for 5 APs

Figure 6.1 representing the *Best Algorithm* result shows a tendency to behave linearly with a slope close to 1, however there are fluctuations along the simulation points, in the overall sense it ended up having a total greater number of iterations than its *Pure Interference* based association function which had 100 iterations at n = 100 users. Now regarding the different α curves, the behavior is worse as α decreases from 1 to 0.25, however the curves for $\alpha = 1$ and $\alpha = 0.75$ are almost equivalent.

Results for 10 APs:

Best Response algorithm results

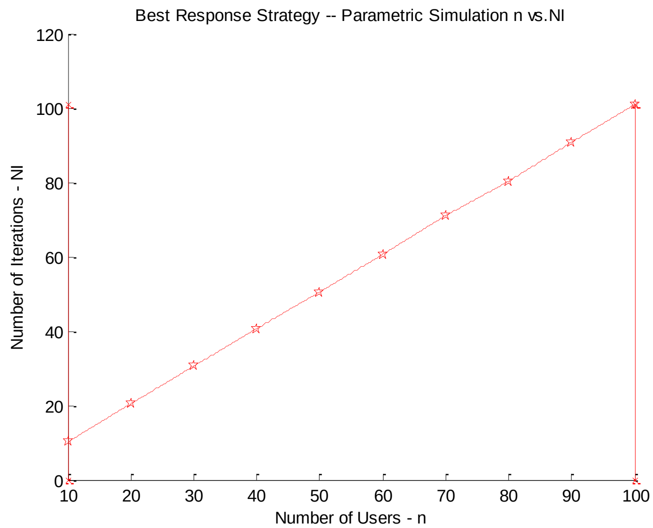


Figure 6.3 Best Response results for 10 APs

Better Response algorithm results

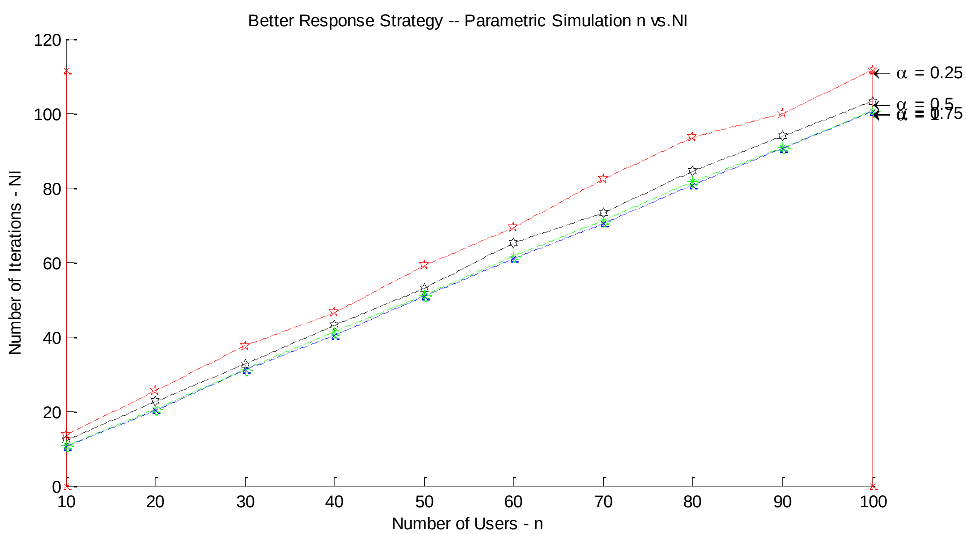


Figure 6.4 Better Response results for 10 APs

When comparing figures 6.3 and 6.4 to its predecessors it can be noticed the lack of noticeable fluctuations in the curves, directly given by the fact of more available strategies for the players, the more strategies available, the higher the probability each user can get a higher achievable rate from an access point that has been randomly allocated closer to him than any other AP. Additionally, with linearity comes along a reduced number of iterations, for ease of comparison take 6.1 and 6.3, 6.3 exhibits the lowest number of iterations in average, as well as a much closer and more linear behavior of the curves $\alpha = 0.75$ and $\alpha = 1$ than any of its other counterparts.

Fixed number of access points, varying number of users static analysis

- Fixed number of randomly deployed users, varying number of access points starting from 10 APs in steps of 10 APs per iteration until 100, giving a total of 10 different games.
- Access point power transmission model modified in order to achieve an *All in Range* scenario.
- Fixed number of Users, simulating games for 100 and 200 users
- Map length $50m^2$

Results for 100 Users:

Best Response algorithm results

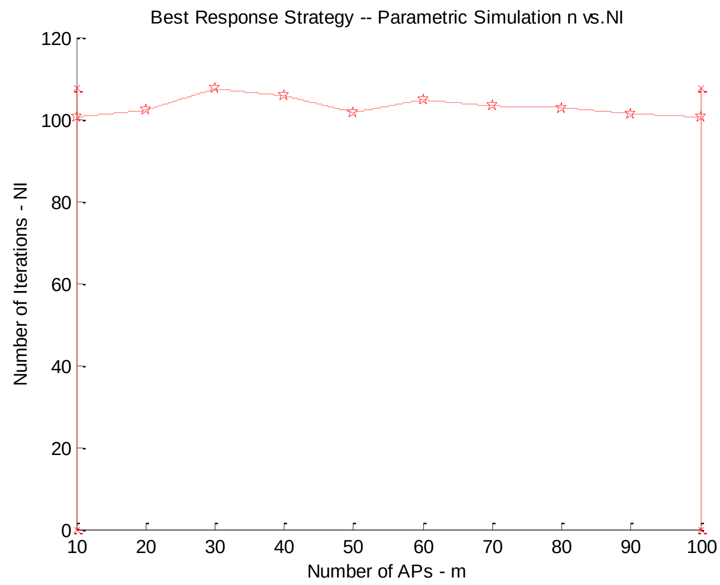


Figure 6.5 Best Response results for 100 users

Better Response algorithm results

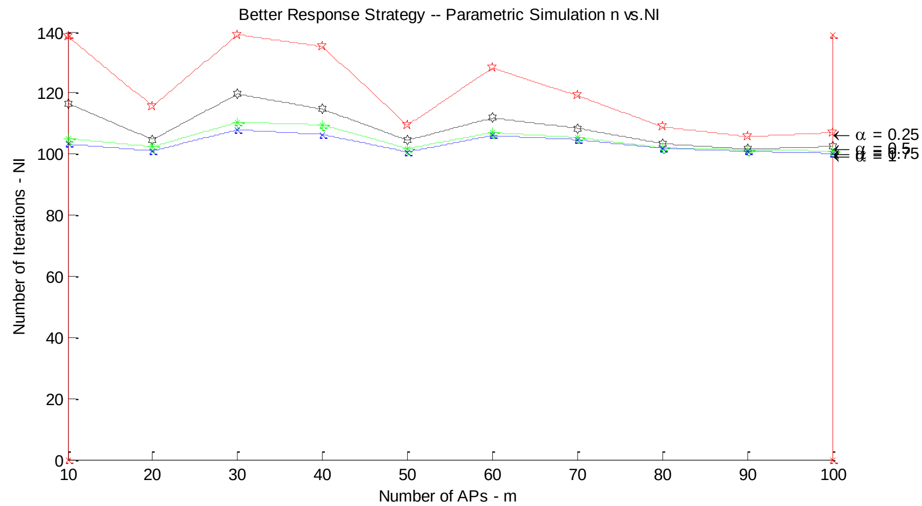


Figure 6.6 Better Response results for 100 users

The constant behavior at the fixed number of users obtained before for *Pure Interference* based association functions is no longer kept here as seen on figure 6.5, fluctuations over the ideal equilibrium value i.e 100 iterations are seen at different points, however a monotonic decrease in the number of iterations is seen from n = 60 and on, same behavior is observed for the *Better Response* algorithm results shown on figure 6.6, big oscillations on $\alpha = 0.25$ curve, summing up to 40% additional iterations at m = 10 and m = 30 APs for 100 users. Apparent equivalent behavior between $\alpha = 0.75$ and $\alpha = 1$ curves is still kept for this scenario, with $\alpha = 0.5$ having a slightly worse performance than the former parameters mentioned.

Results for 200 Users:

Best Response algorithm results

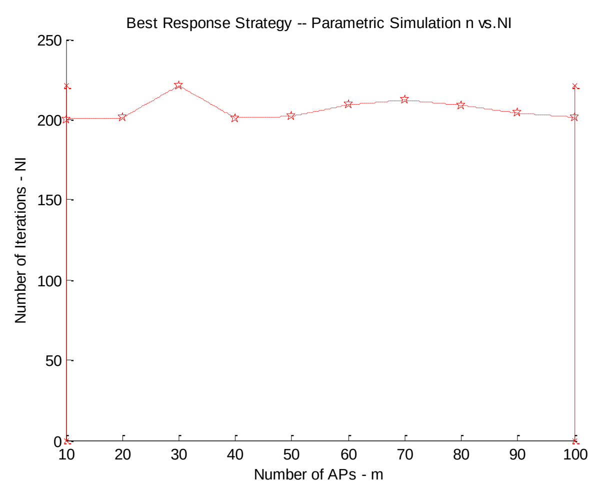


Figure 6.7 Best Response results for 200 users

Better Response algorithm results

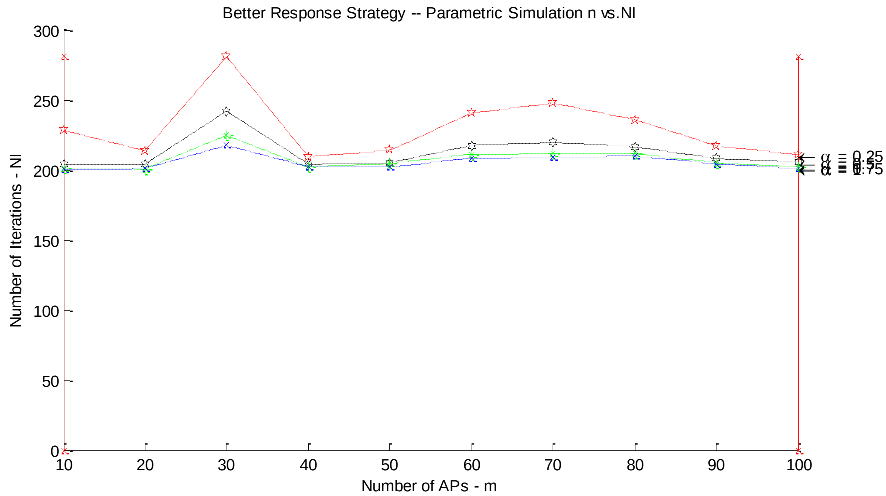


Figure 6.8 Better Response results for 200 users

A relevant fact that is pointed out from figures 6.7 and 6.8 from all the previous results is the fact of all α curves having the least number of oscillations, particularly for $\alpha = 0.25$ through $\alpha = 0.75$ there is only one relevant peak value at m = 30 APs, whilst for $\alpha = 0.25$ there exists additionally a fluctuation between m = 50 and m = 90.

Concluding remarks for the fixed number of users and varying number of access point simulations include the fact of the maximum peak value for all scenarios was of approximately 75% of the base equilibria under "ideal" conditions i.e *pure interference* based association functions and *Best Response* algorithm, for which the results would have been a constant line equal to the number of users for every simulated point.

Finally there is a direct trade-off between the total elapsed simulation time and the number of curve oscillations, for n = 200 fixed users, the elapsed simulation time with a Centrino processor 1.6GHz and 2 GB ram computer the simulation lasted around 1.5 hours while the n = 150 fixed user simulation lasted 55 minutes, however the former exhibits a more constant behavior oscillation wise, nature behind this fact is the presence of more users make their particular payoff function more similar to the other users, since more users are added to the same size map, in the end having a group of users with a base achievable rate equal to a greater number than any of the other previous scenarios (100 and 150 fixed users).

6.2

Perturbation Scenarios

Removing users/access points

The particular scenarios selected to study the Perturbation scenarios were the same as the previous Section (namely the pure interference based association function results Section 5), and the disturbances introduced were done accordingly with respect each study case

Results for 7 APs, varying users from 10 to 100 (User step 10 per iteration):

Best Response algorithm results

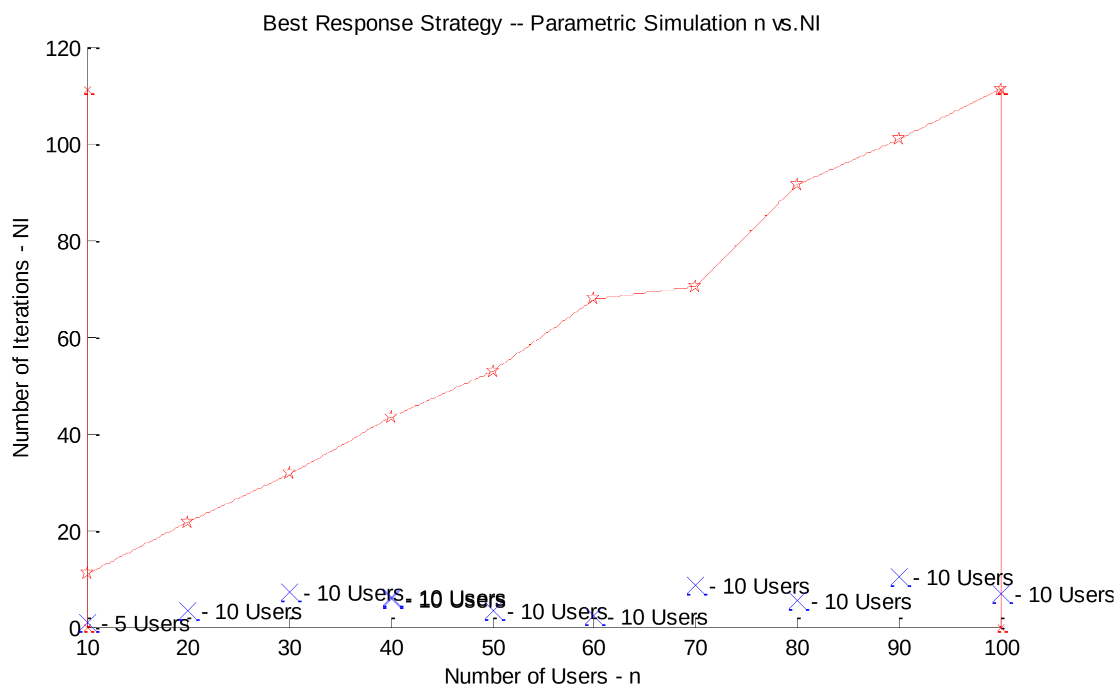


Figure 6.9 Best Response Perturbation scenario results for 7 fixed APs

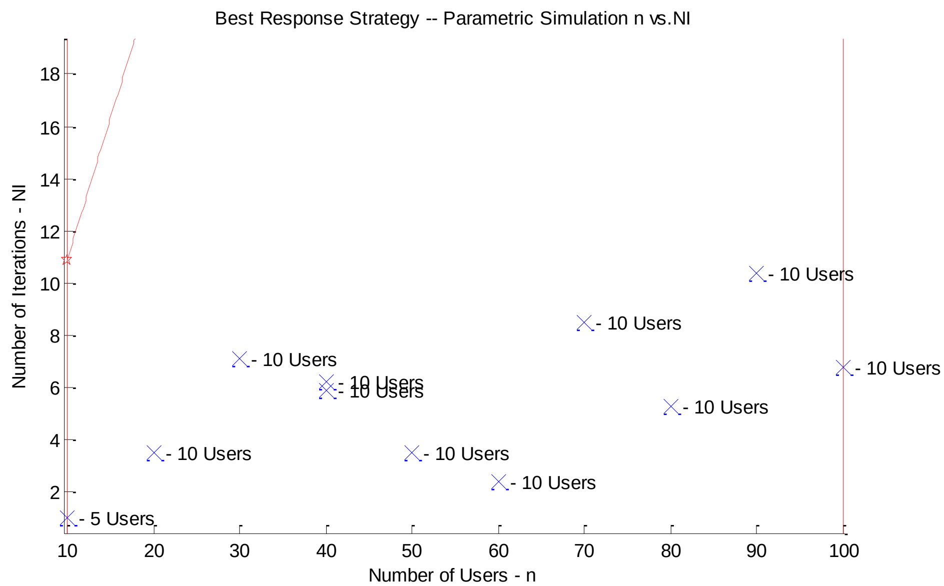


Figure 6.10 Best Response Perturbation scenario zoom-in results for 7 fixed APs

Better Response algorithm results

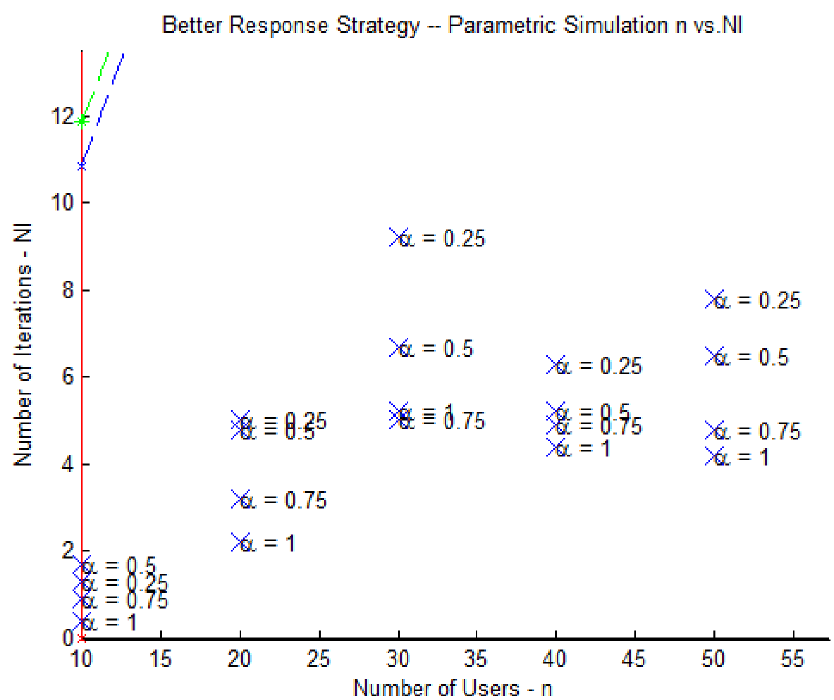


Figure 6.11a Better Response Perturbation scenario results for 7 fixed APs part 1

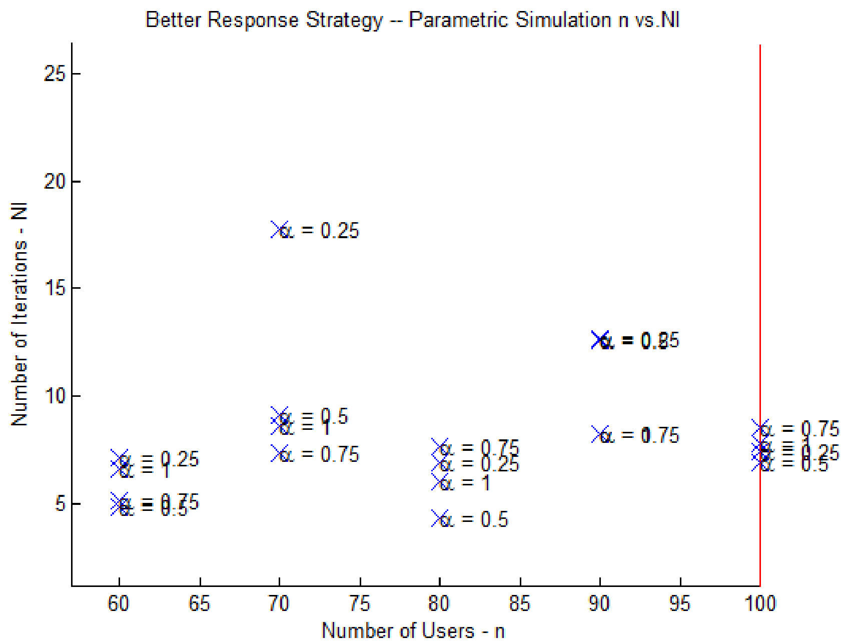


Figure 6.11b Better Response Perturbation scenario results for 7 fixed APs part 2

Figures 6.9 and 6.10 are showing the *Best Response* algorithm results for the fixed AP case, it can be seen in 6.13 and in its close-up figure 6.10 that the number of additional iteration never exceeds the magnitude of the disturbance, in this case 10 users and 5 users for n = 10 users, the values fluctuate in a casual way, however the peak value introduced in the disturbance was found at n = 90 users, where there were needed 10 additional iterations over twenty averaged games.

Figures 6.11a and 6.11b represent the counterpart of the formerly presented figures, it can be easily noted that either from figure 6.11a and 6.11b the peak values are most commonly attributed to $\alpha = 0.25$ with particular large discrepancies presented at n = 30 and n = 70 users, on all other simulated points, the α values tend to be bundled together, nevertheless $\alpha = 0.75$ and $\alpha = 1$ exhibit the best performance for the particular scenario being studied.

<u>Results for 150 fixed users, varying access points from 10 to 100 (AP step 10)</u>:

Best Response algorithm results

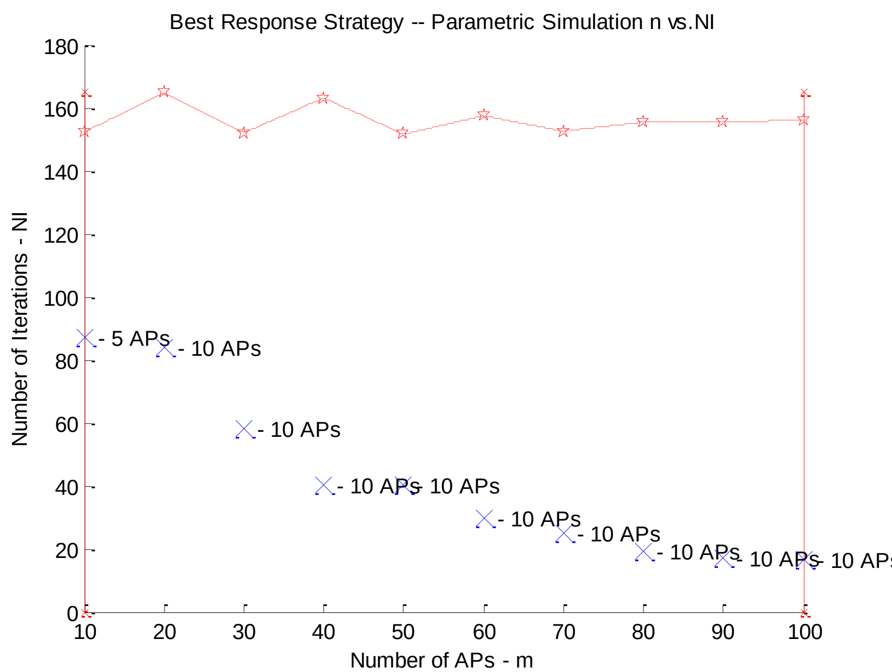


Figure 6.12 Best Response Perturbation scenario results for 150 fixed users

Better Response algorithm results

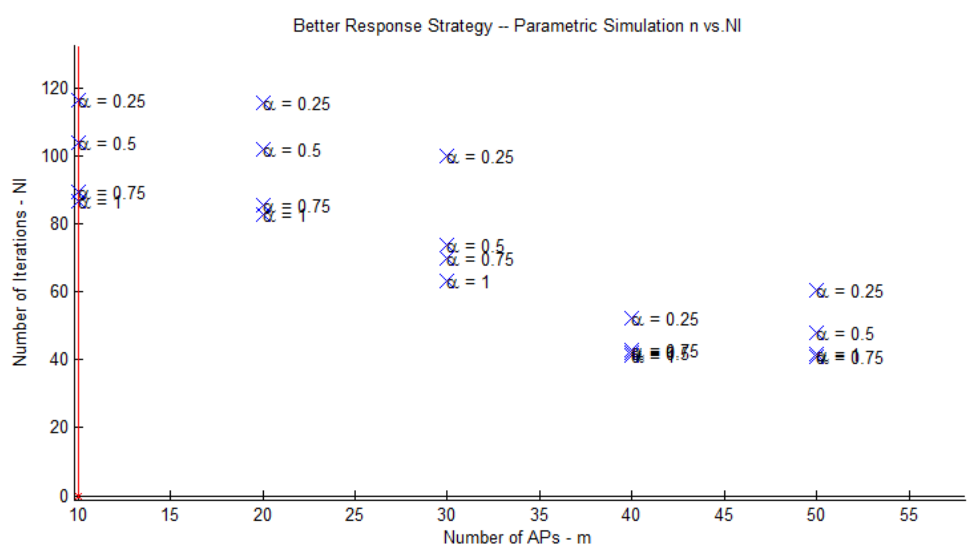


Figure 6.13a Better Response Perturbation scenario results for 150 fixed users part 1

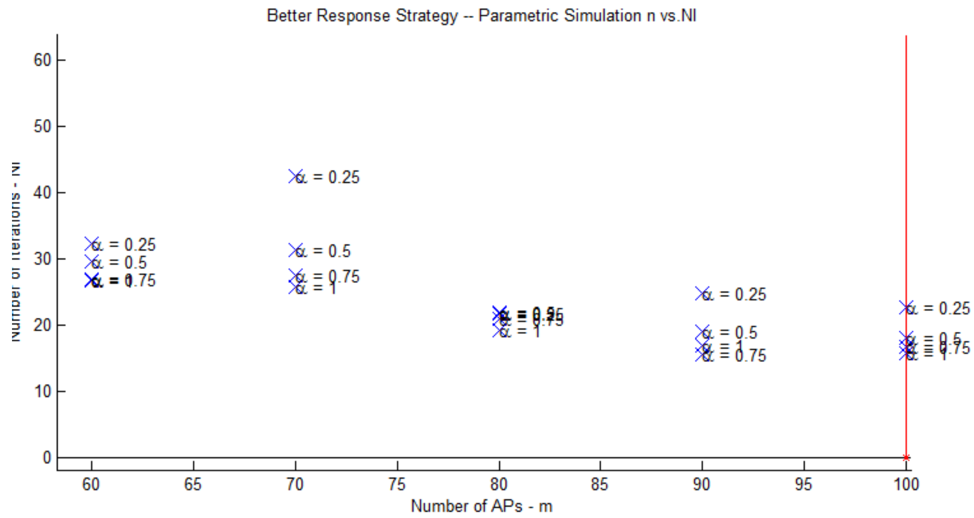


Figure 6.13b Best Response Perturbation scenario results for 150 fixed users part 2

Starting off with figure 6.12 and comparing it directly with the previously obtained results for *Pure Interference* based association functions it can be seen the monotonic decrease in the number of iterations, however the values are greater than the ones presented on the aforementioned Section, i.e at m = 10 APs, if 5 APs were removed, under baseline circumstances the number of additional iterations would have been just 75, however since the association mechanics have changed the game dynamics involving an additive interference and achievable rate functions cause in average a slightly greater number of iterations, it is also appreciated that as m increases, the number of additional iterations decreases, since there are a greater number of available strategies for the same amount of users.

Figures 6.13a and 6.13b are showing the results for *Better Response* algorithm, $\alpha = 1$ values correspond to the plotted points in figure 6.12, above this baseline value, the different α values oscillate, particularly at m = 10 and m = 20 APs the difference on additional iterations by $\alpha = 0.25$ and $\alpha = 0.5$ compared to $\alpha = 0.75$ and $\alpha = 1$ is very noticeable, in facto from m = 10 through m = 50 except for m = 40 APs, the difference can account for up to 40 additional iterations, whilst for the plots on figure 6.13b the maximum difference is presented at m = 70 APs and accounts for around 20 additional iterations, making relevant again the fact of a faster convergence with a larger number of access points despite the fact of being blind of a percentage of the strategies.

6.3

Concurrent choice scenarios

The scenarios to be studied correspond to the same ones studied in Section 5.3.

<u>Results for 7 APs, varying users from 10 to 100 (User step 10 per iteration)</u>:

Best Response algorithm results

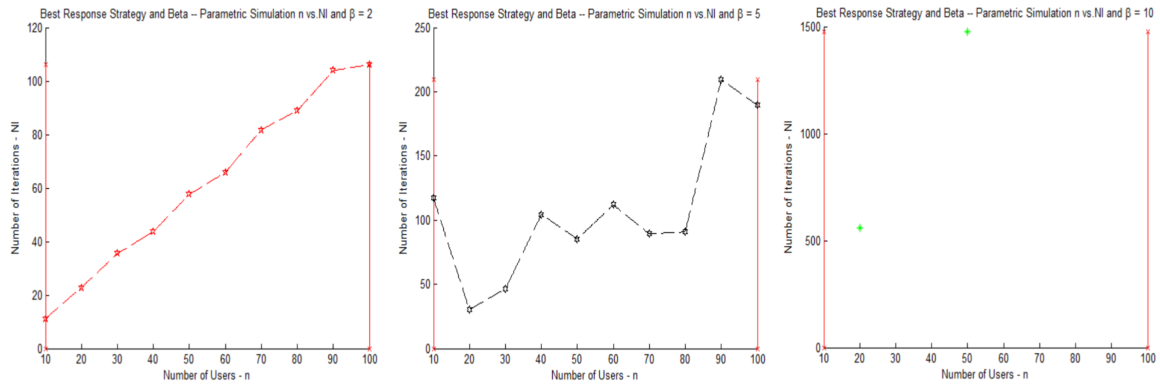


Figure 6.14 Best Response concurrent scenario results for 7 fixed APs

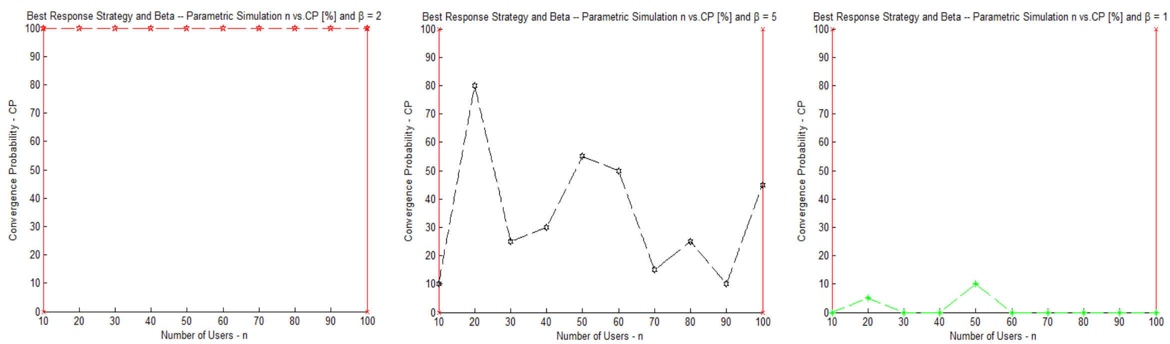


Figure 6.15 Best Response concurrent scenario convergence probability results for 7 fixed APs

Better Response algorithm results

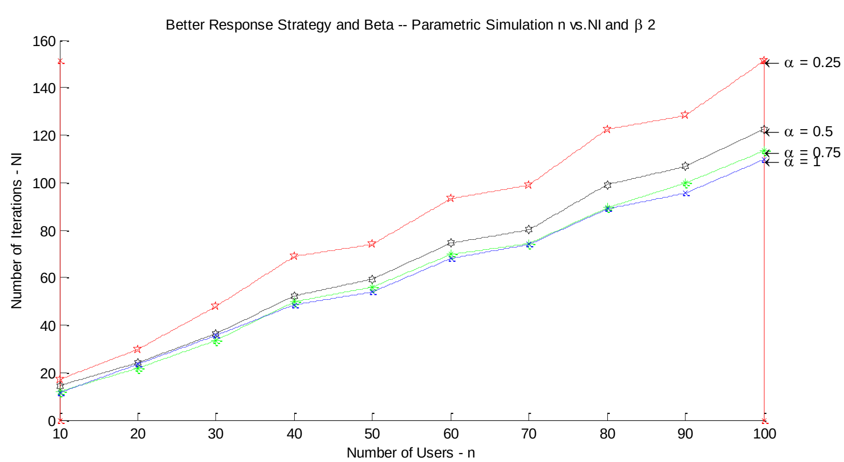


Figure 6.16 Better Response 2 concurrent users scenario results for 7 fixed APs

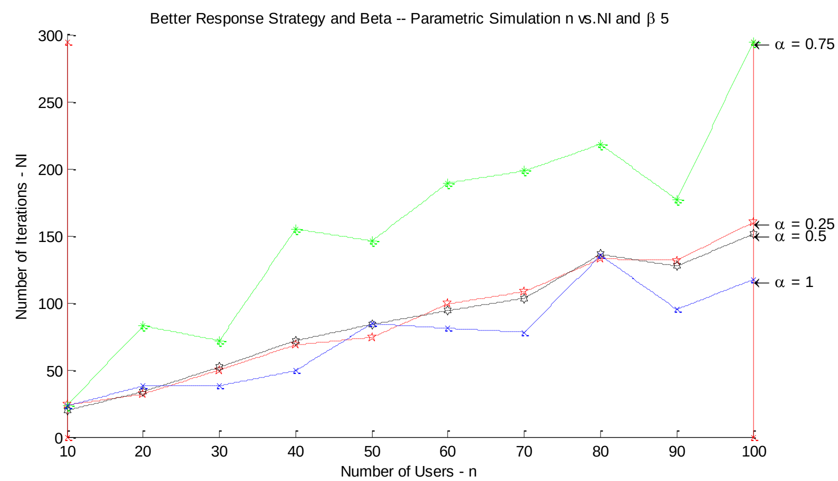


Figure 6.17 Better Response 5 concurrent users scenario results for 7 fixed APs

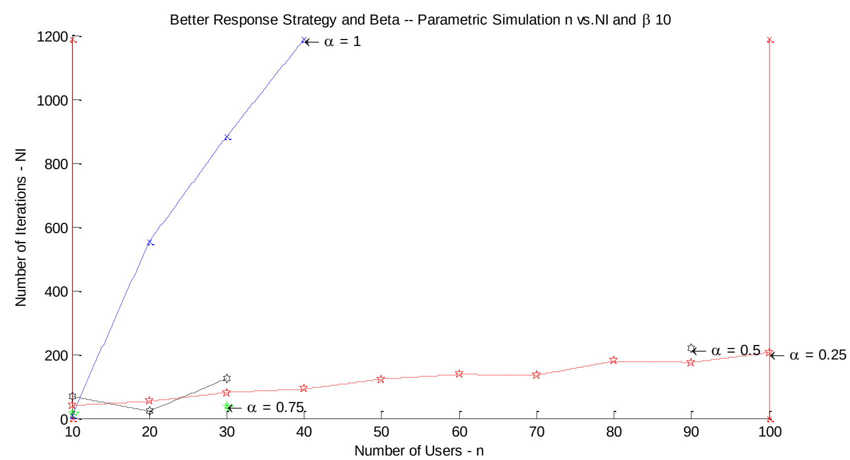


Figure 6.18 Better Response 10 concurrent users scenario results for 7 fixed APs

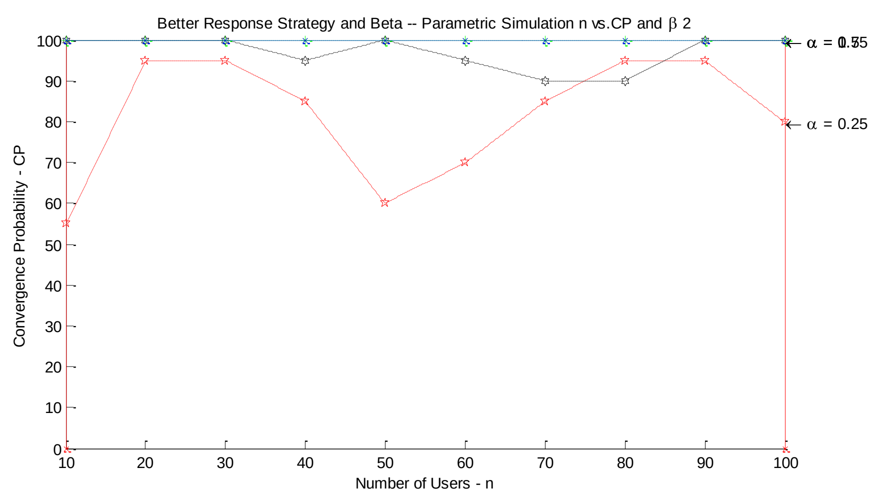


Figure 6.19 Better Response 2 concurrent users scenario convergence probability results for 7 fixed APs

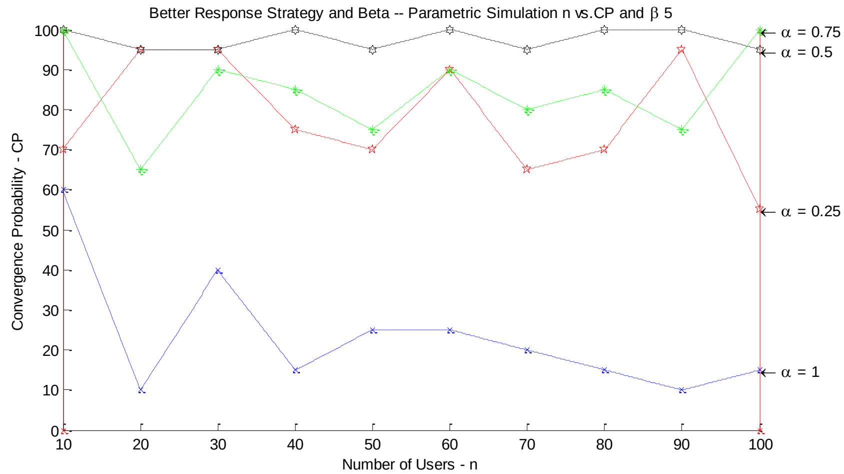


Figure 6.20 Better Response 5 concurrent users scenario convergence probability results for 7 fixed APs

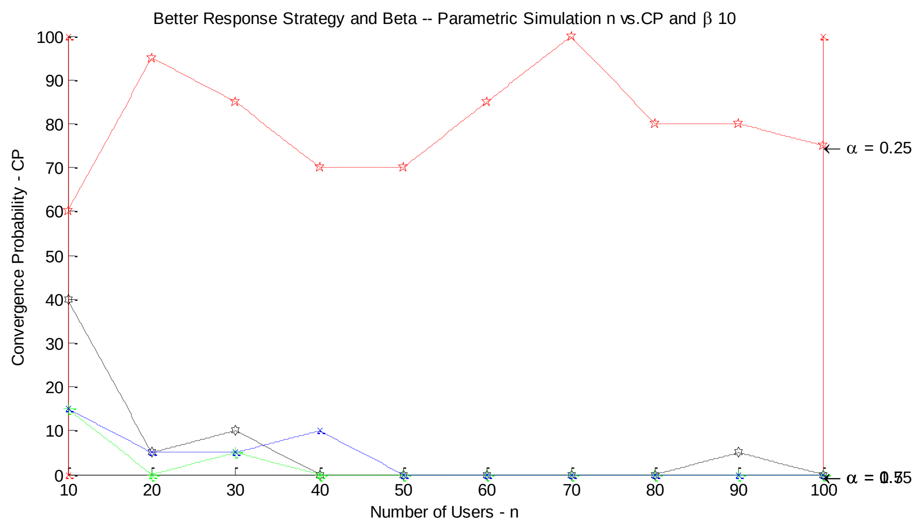


Figure 6.21 Better Response 10 concurrent users scenario convergence probability results for 7 fixed APs

Figures 6.14 and 6.15 are showing some interesting results, going from left to right or equivalently going from $\beta = 2$ to $\beta = 10$ the curves change dramatically, the leftmost graph i.e $\beta = 2$ shows an almost linear behavior having a constant convergence probability of 100%, however with increasing the amount of concurrent players just by three, i.e $\beta = 5$ the linear behavior presented in the center graph of figure 6.14 is lost and becomes a completely irregular curve, having in average a convergence probability less than 50%, finishing with the rightmost graph of both figures i.e $\beta = 10$ which only has a convergence probability greater than 0% on 2 points, n = 20 and n = 50 and for which the number of iterations almost reached the preset software limit.

Better Response algorithm results begin with figure 6.16 presenting the total number of iterations per each game and going throughout until figure 6.18 to show how the algorithm behaves for $\beta = 10$ concurrent users; as curious fact it is observed how $\alpha = 0.25$ is outperformed at $\beta = 2$ by all the other α values, $\beta = 2$ represents a number of concurrent users sufficiently close to the baseline case of 1 concurrent user and so it is natural that for the same amount of strategies available and being blind with respect an increasing percentage of them, the performance is inversely proportional, however going forward to figure 6.17 it can be seen that $\alpha = 0.25$ is no longer outperformed by all other curves, instead, is rapidly approaching $\alpha = 1$ behavior asymptotically, finally on figure 6.18 and for $\beta = 10$ it is noted how $\alpha = 0.25$ curve is the only one presenting a continuous convergence probability greater than 0%, therefore and as it was stated on the *pure interference* based functions being blind under a considerable amount of concurrent players is favorable towards convergence. Figures 6.19 through 6.21 reflect the same statements posed above but in terms of convergence probability.

<u>Results for 150 fixed users, varying access points from 10 to 100 (AP step 10)</u>:

Best Response algorithm results

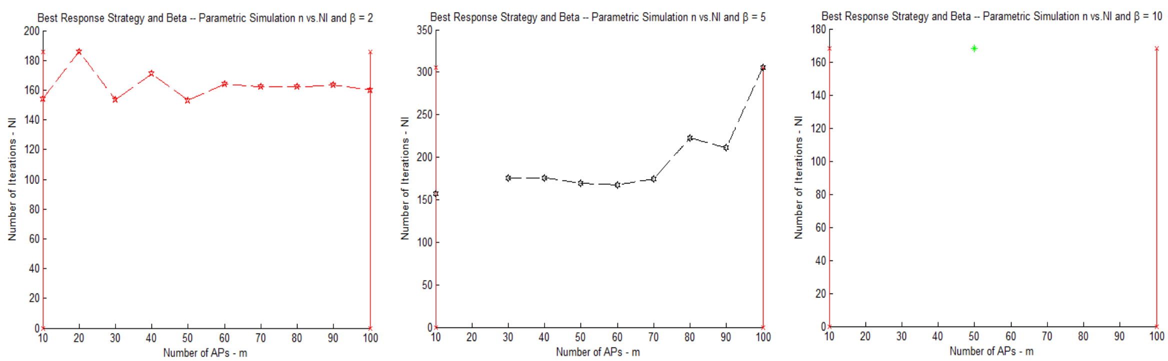


Figure 6.22 Best Response concurrent scenario results for 150 fixed users

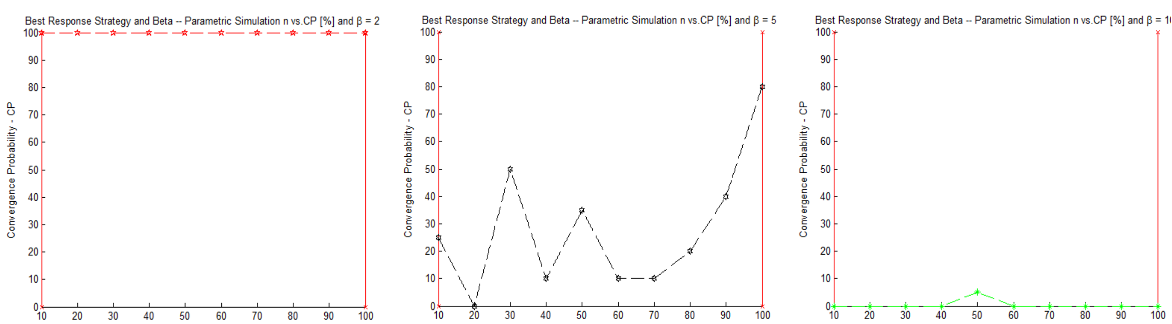


Figure 6.23 Best Response concurrent scenario convergence probability results for 150 fixed users

Better Response algorithm results

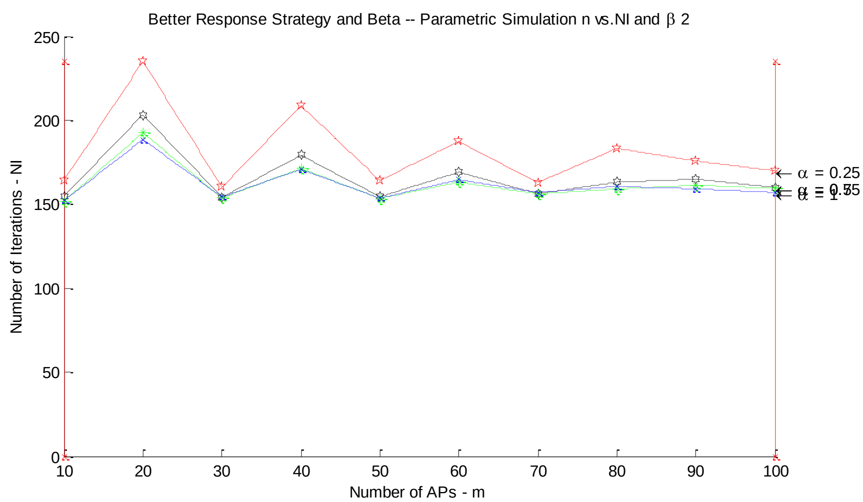


Figure 6.24 Better Response 2 concurrent user scenario results for 150 fixed users

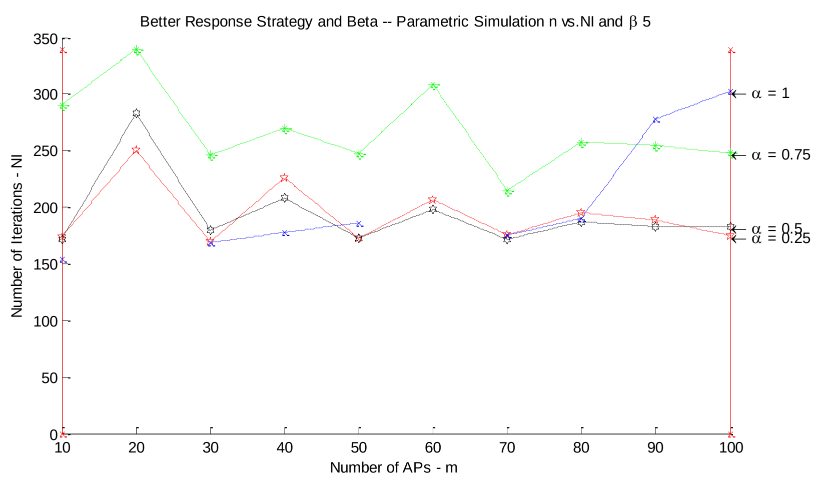


Figure 6.25 Better Response 5 concurrent user scenario results for 150 fixed users

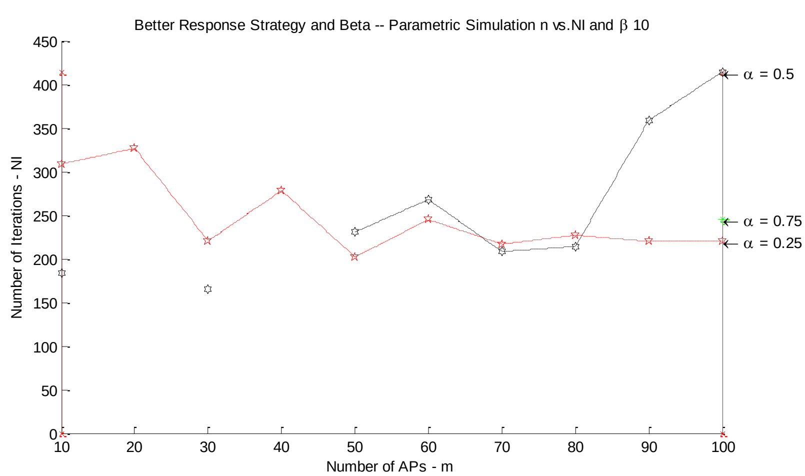


Figure 6.26 Better Response 10 concurrent user scenario results for 150 fixed users

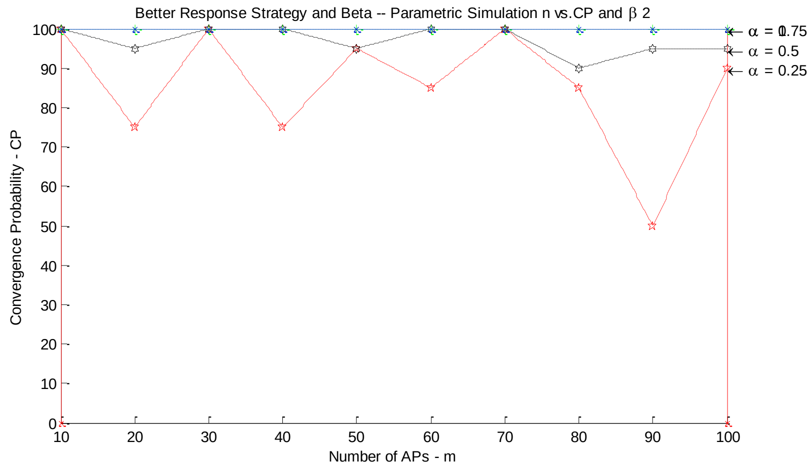


Figure 6.27 Better Response 2 concurrent user scenario convergence probability results for 150 fixed users

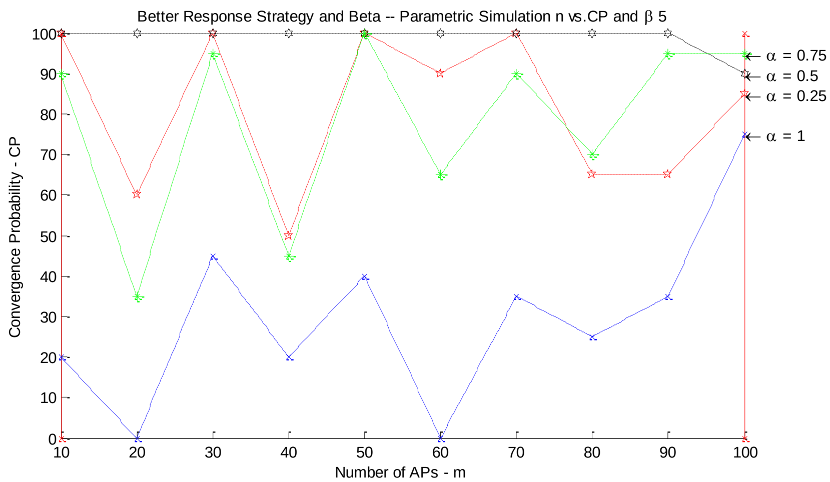


Figure 6.28 Better Response 5 concurrent user scenario convergence probability results for 150 fixed users

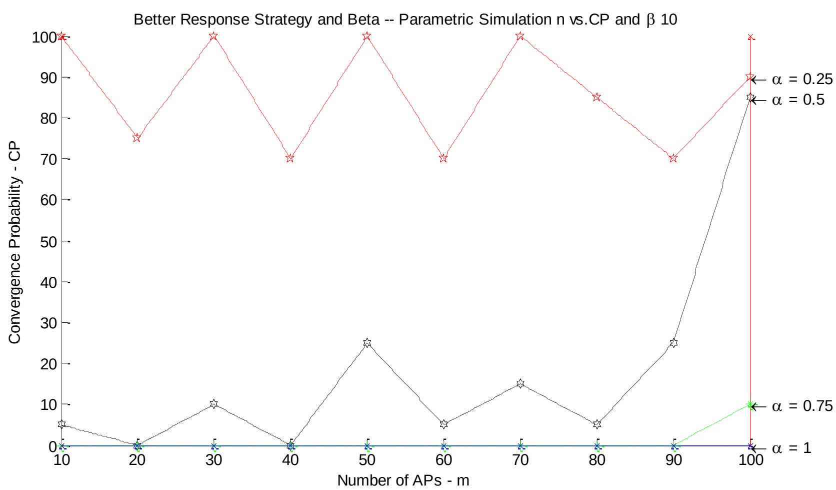


Figure 6.29 Better Response 10 concurrent user scenario convergence probability results for 150 fixed users

Concurrent user analysis begins with figures 6.22 and 6.23 that correspond to *Best Response* algorithm results, leftmost graphs on both figures belong to $\beta = 2$ concurrent users and the number of iterations curve is resembling the output of a compensated 2nd order system, behavior that is familiar to this point of performance analysis, there are peak values, but the decrease asymptotically reaching the ideal case equilibrium i.e 150 iterations for 150 fixed users, the peak values at m = 20 and m = 40 APs didn't not prevent the algorithm to maintain a constant convergence probability fixed to 100%. On the contrary, centermost graphs of figures 6.22 and 6.23 show the resulting curves for $\beta = 5$ and they exhibit a particular behavior, it is observed no convergence between m = 10 and m = 30 APs, further on the number of iterations is maintained close to the baseline reference at a cost of a low convergence probability, however as m increases the number of iterations show a tendency to grow in a quadratic way but incrementing the convergence probability. Finally the rightmost graphs belonging to $\beta = 10$ expose a single convergence point for m = 50 APs with an approximate number of iterations of 170, the convergence probability achieved was around 5% which corresponds to exactly only 1 game out of the 20 considered to average the results.

Better Response algorithm results are not much different than those of its non blind partner presented formerly, as seen on figures 6.24 through 6.29 the blue dashed curves represent the exact same behavior as a *Best Response* algorithm, being blind in terms of game dynamics is not always bad, as seen on figures 6.25, 6.26 and 6.28,6.29 where again as it was stated on the concluding remarks of the previous Section, $\alpha = 0.25$ begins outperformed by all the other $\alpha = 0.25$ values, however as the number of concurrent players increases, the convergence probability for $\alpha = 0.5$, $\alpha = 0.75$ and even for the non blind version of the algorithm $\alpha = 1$ decreases considerably. The direct punishment of the case study of concurrent users is the way the update the common knowledge base of the game, so again, the more concurrent users with the bigger available strategy space, the more likely a bigger amount of total iterations would be needed to reach an equilibrium, on the other side under the same conditions, each user is blinded by certain percentage of the total amount of strategies, the faster is more likely to be found an equilibrium, and this idea is reflected more precisely on figures 6.26 and 6.29 for which $\alpha = 0.25$ is vastly outperforming its counterparts with a limited number of total iterations maintaining a constrained oscillatory behavior on the convergence probability graph.

6.4

Access point coverage radius parametric analysis

Previously, the analysis made consisted on a fixed access point coverage radius that was modified to cover all users (*All in range*). Being able to analyze the performance analysis of the different games under different AP coverage radius is important because these situations can happen in real life [15,16,17] whenever an AP has a variable transmission power. These type of simulations can be seen as a sort of joint Network Selection problem, in the sense that an AP can increase or decrease its coverage area in order to selfishly maximize the number of users covered, however, the parameterization of the AP coverage area is not done with this policy, but just for practical studies without specifically caring for the AP wealth. The impacts of these disturbances will be studied on the following sub-Sections.

***Not all in Range* + Linear Grid topology**

<u>Results for 50 users and 7 APs</u>:

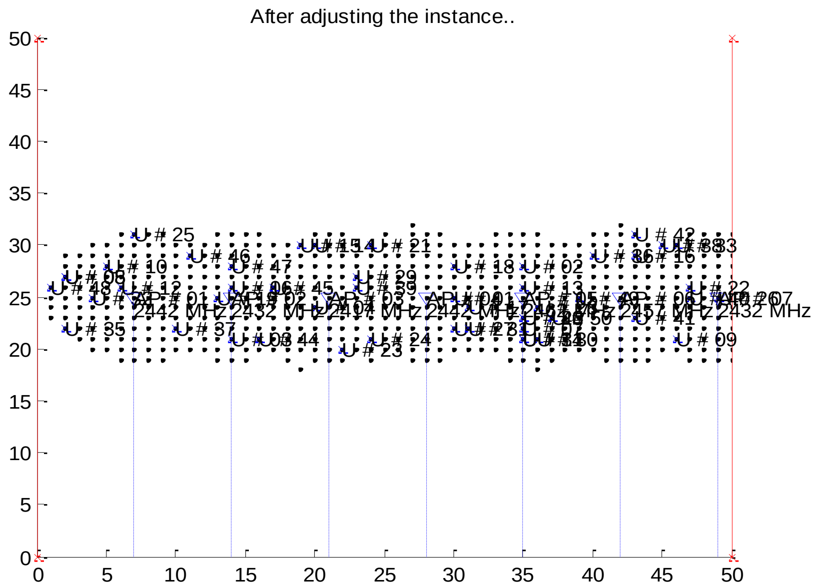


Figure 6.30 Linear grid map with AP Emitted Power = 5dBm

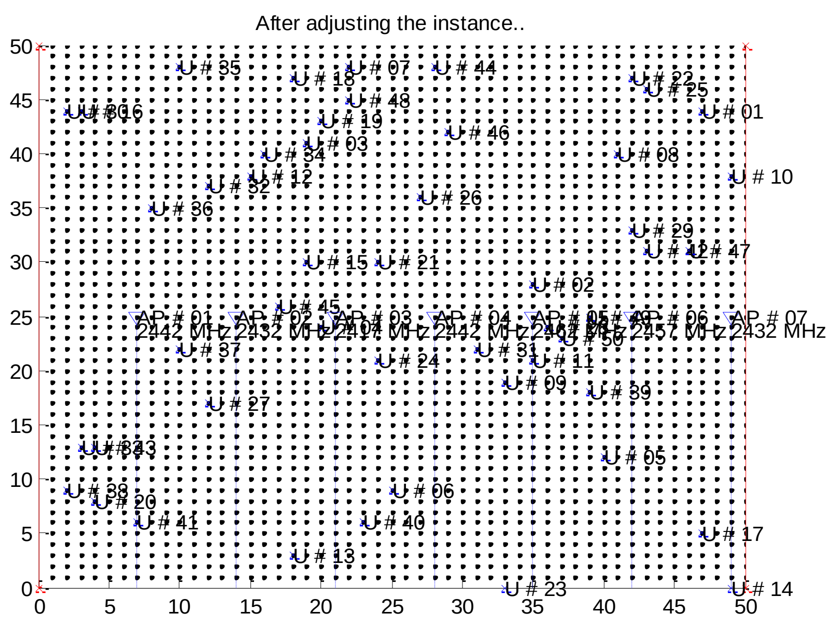


Figure 6.31 Linear grid map with AP Emitted Power = 20dBm

Best Response algorithm results

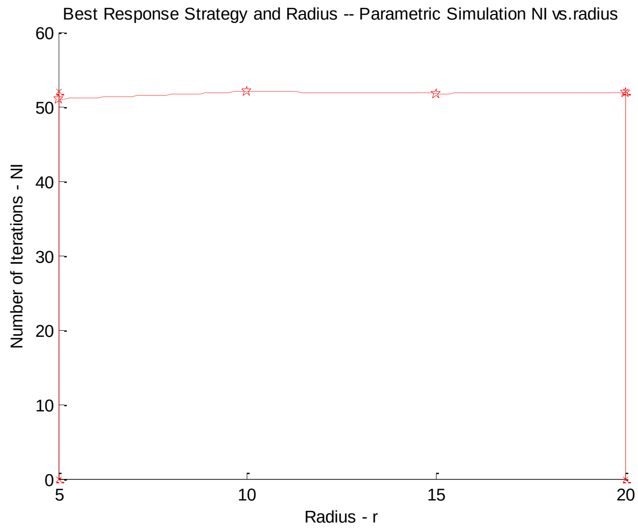


Figure 6.32 Best Response linear grid topology coverage radius results for 50 users and 7 APs

Better Response algorithm results

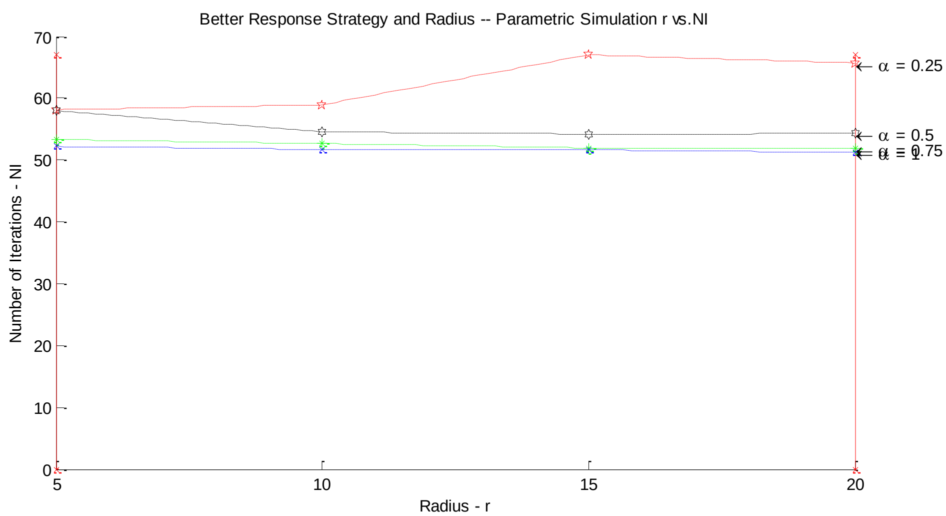


Figure 6.33 Better Response linear grid topology coverage radius results for 50 users and 7 APs

Linear grid topology map is apparently favoring the convergence, having with 50 users and 7 APs an almost ideal case convergence shown in figure 6.32, moreover this aiding is covering $\alpha = 5$ which on the previous studied case, was not as close in asymptotic behavior as the one reflected for n > 10 on figure 6.23, on the contrary $\alpha = 0.25$ still behaves the worse, with a underperformance of around 33% with respect the ideal equilibrium.

<u>Results for 100 users and 7 APs</u>:

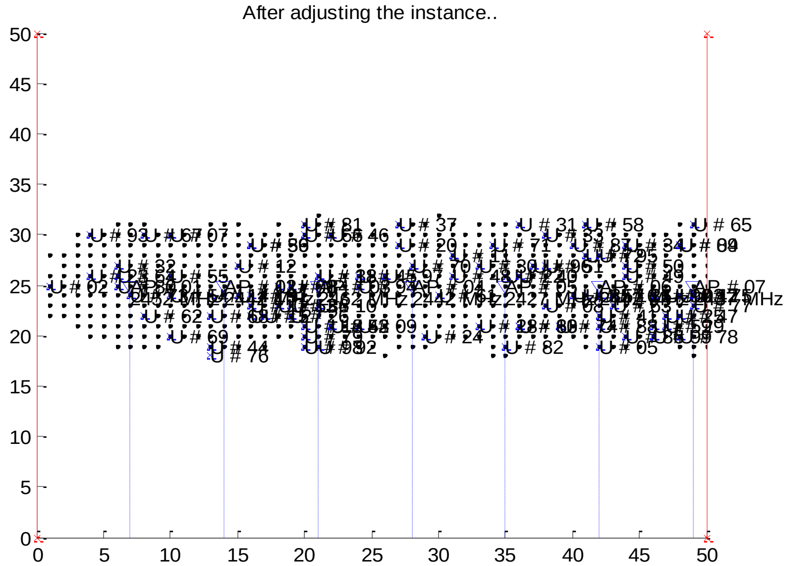


Figure 6.34 Linear grid map with AP Emitted Power = 5dBm

Figure 6.35 Linear grid map with AP Emitted Power = 20dBm

Best Response algorithm results

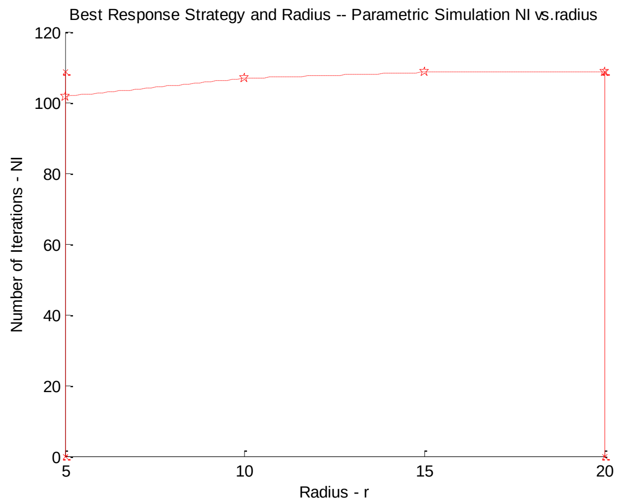


Figure 6.36 Best Response linear grid topology coverage radius results for 100 users and 7 APs

Better Response algorithm results

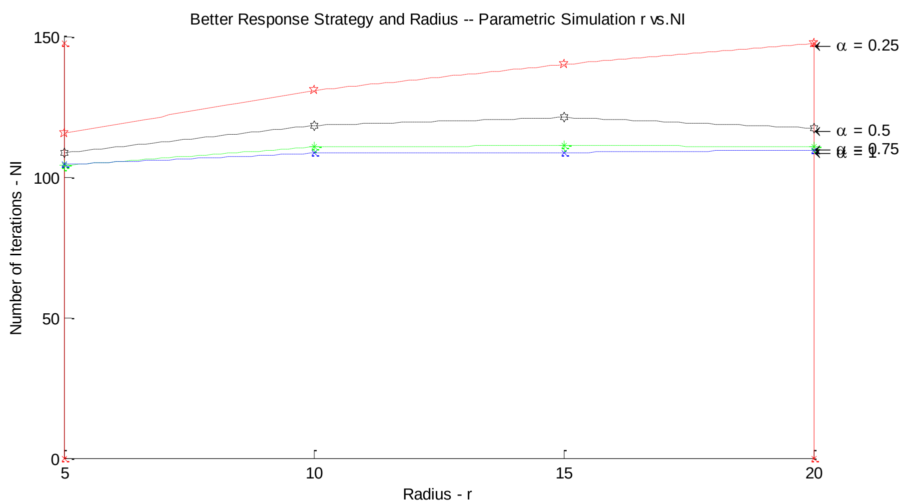


Figure 6.37 Better Response linear grid topology coverage radius results for 100 users and 7 APs

Figure 6.36 is showing a trend to grow slightly as the coverage radio increases, graphically it appears to settle at r = 15, this curve achieved a peak value of around 110 iterations accounting for 10% of the ideal number of iterations to be achieved.

***Not all in Range* + Rectangular Grid topology**

<u>Results for 50 users and 12 APs</u>:

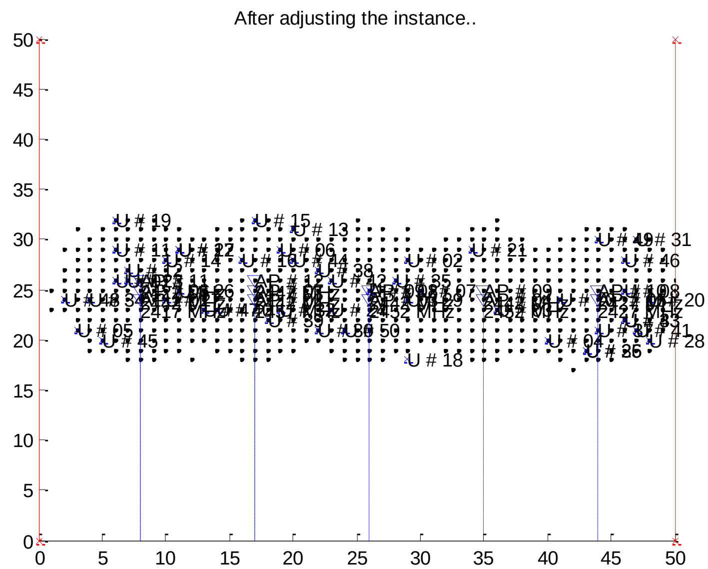


Figure 6.38 Rectangular grid map with AP Emitted Power = 5dBm

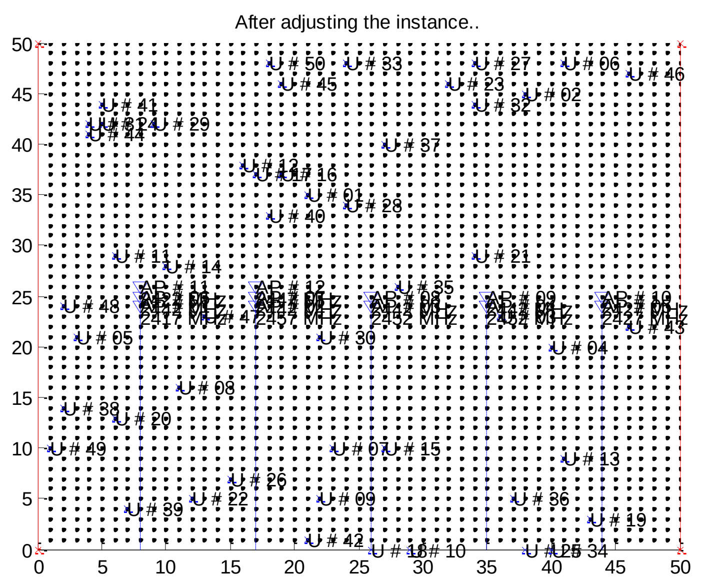


Figure 6.39 Rectangular grid map with AP Emitted Power = 20dBm

Best Response algorithm results

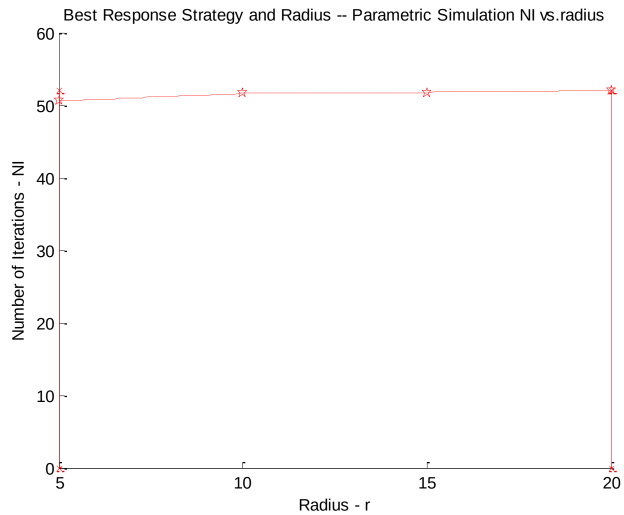


Figure 6.40 Best Response rectangular grid topology coverage radius results for 50 users and 12 APs

Better Response algorithm results

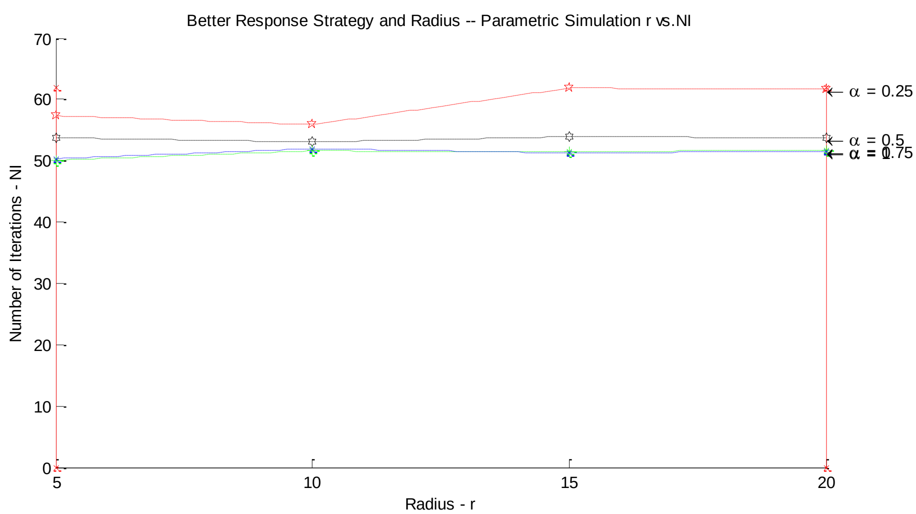


Figure 6.41 Better Response rectangular grid topology coverage radius results for 50 users and 12 APs

Figure 6.40 is equivalent to figure 6.32, hence it can be inferred that the rectangular grid topology under the given circumstances does not favor positively or negatively the dynamic behavior, however it is relevant to point out that the maximum value reached by the worst performing curve i.e $\alpha = 0.25$ is close to 70 iterations whilist in figure 6.40 $\alpha = 0.25$ is slightly greater than 60 representing a difference close to 20%.. With the ability of *Game Simulator* to perform repeated analysis under same topologies, it can be proven that under the rectangular grid topology the blind utility functions tend to be aided by the fact of the AP's being grouped together more closely than a linear topology does.

Results for 100 users and 12 APs:

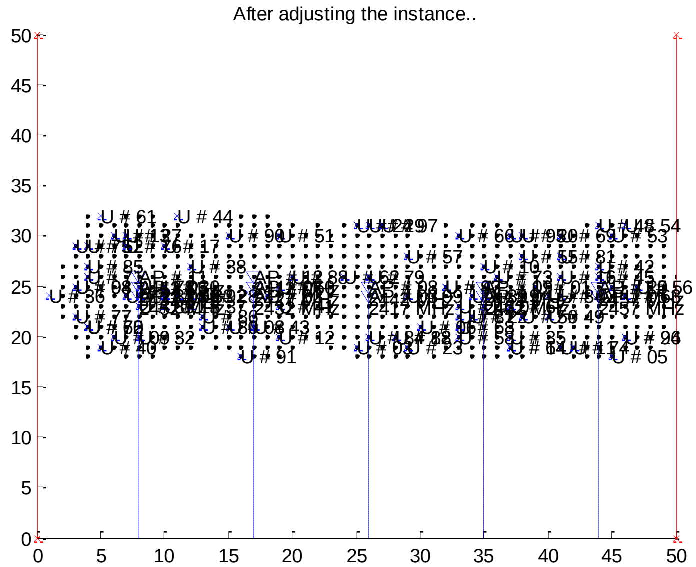


Figure 6.42 Rectangular grid map with AP Emitted Power = 5dBm

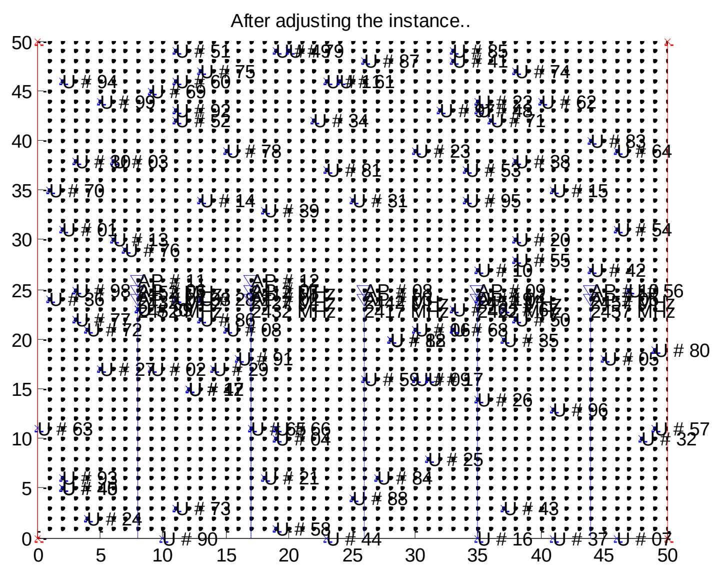


Figure 6.43 Rectangular grid map with AP Emitted Power = 20dBm

Best Response algorithm results

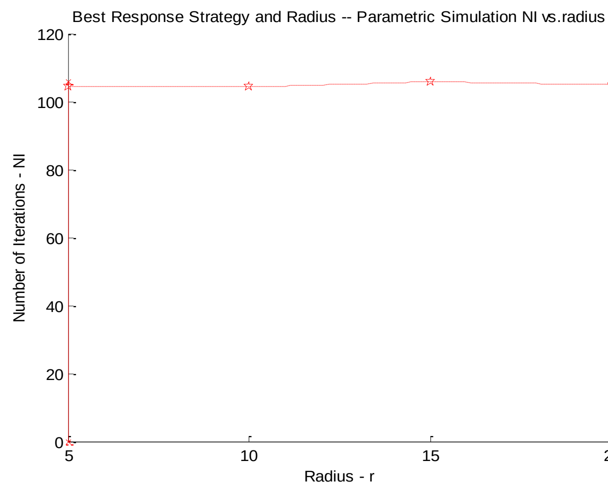


Figure 6.44 Best Response rectangular grid topology coverage radius results for 100 users and 12 APs

Better Response algorithm results

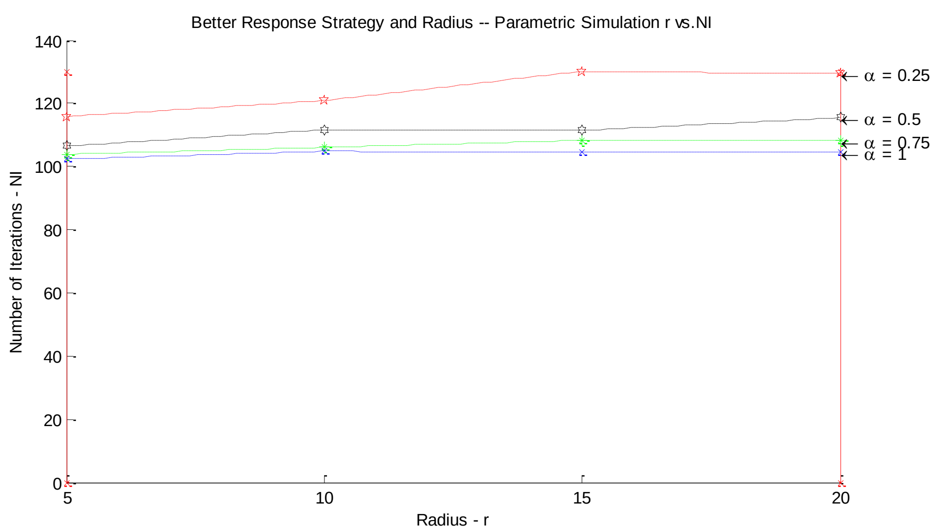


Figure 6.45 Better Response rectangular grid topology coverage radius results for 100 users and 12 APs

Figure 6.44 exhibits an almost constant behavior for the simulated coverage radios, the behavior is kept as well for figure 6.45 curves, where $\alpha = 0.75$ and $\alpha = 0.5$ end up having a discrepancy of 5% and 10% respectively, behavior that is expected to be kept for bigger coverage radios because the *all in range* condition is reached approximately at r = 15.

Under the different topologies studied the growth of the coverage radio has different effects, for example, for the linear and rectangular grid topologies, it won't have such a noticeable effect since *Game Simulator* deploys the APs as the user has determined, but however the users are deployed randomly nevertheless guaranteeing that each user is covered by at least 1 AP, so if this condition is guaranteed, each user will get to know all other access points and have an achievable rate greater than 0 with a much smaller coverage radio than a randomly deployed access point topology would.

6.5

FICTITIOUS PLAY ALGORITHM

MYOPIC VS. NON-MYOPIC

Unlike *Best* and *Better Response* algorithms, *Fictitious Play* algorithms base their decisions on the knowledge learnt from previous iterations, rather than from a common knowledge base as the former algorithms did, hence a user playing *Fictitious Play* will have an individual knowledge from all the game that he will use to make the most appropriate decision (for more information on Fictitious Play algorithms go to Section 3.7).

Performance analysis will be done over 4 different flavors:

- Deterministic Fictitious Play
- Stochastic Fictitious Play
- Myopic Deterministic Fictitious Play
- Myopic Stochastic Fictitious Play

Fixed number of access points, varying number of users static analysis

- Varying number of randomly deployed users, starting from 5 users in steps of 5 users per iteration until 30, giving a total of 6 different games.
- Access point power transmission model modified in order to achieve an *All in Range* scenario.
- Map length $40m^2$

Fixed number of Access Points will be simulated for 3 and 7 APs

Legend:

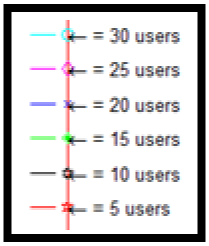


Figure 6.46 User curve legend

Results for 3 APs:

Deterministic FP results

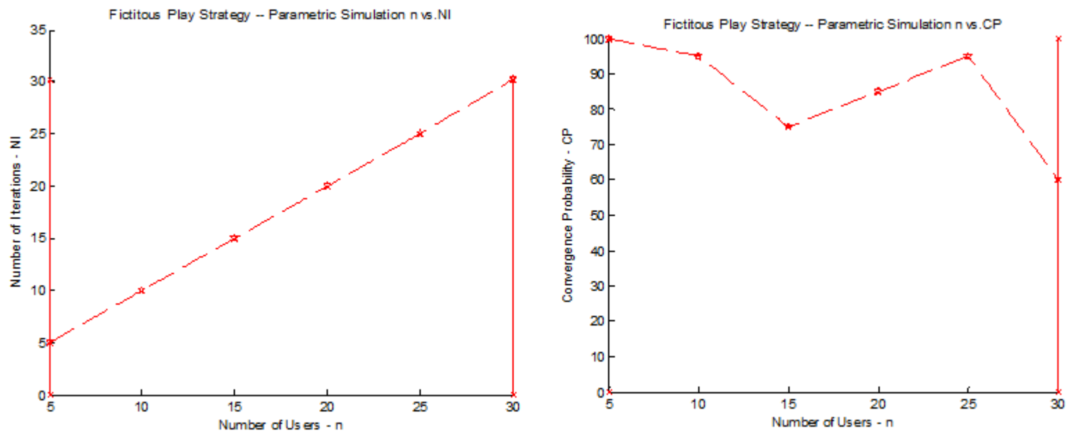


Figure 6.47 Deterministic Fictitious Play results for 3 fixed APs

Stochastic FP results

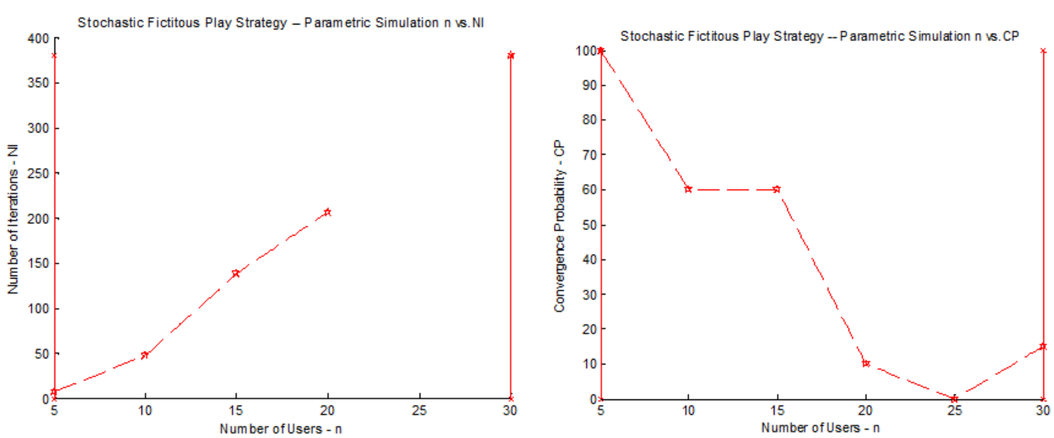


Figure 6.48 Stochastic Fictitious Play results for 3 fixed APs

Differently from what was shown on the *Pure Interference* based function Section 5.5 the non-myopic versions are not showing constant convergence probability behaviors, at figure 6.47 even though the linear line with slope 1 is held, the convergence probability goes as low as 60%, on the other hand figure 6.48 is showing that the stochastic F.P algorithm presented no convergence probability for n = 25 users, its neighbors (n = 20, n = 30) have a convergence probability lower than 20% and the three first simulated points do have a convergence probability greater than 50% but it is paid off by a excessive number of iterations when compared to the deterministic version.

Myopic deterministic FP results

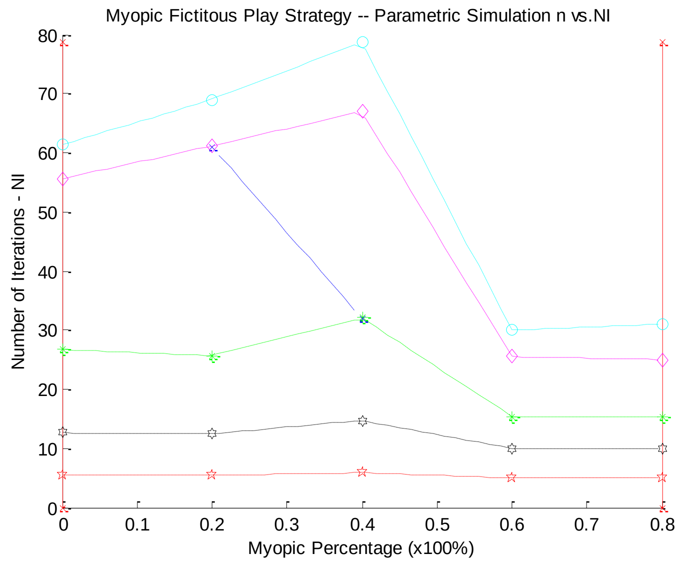


Figure 6.49 Myopic Deterministic Fictitious Play results for 3 fixed APs

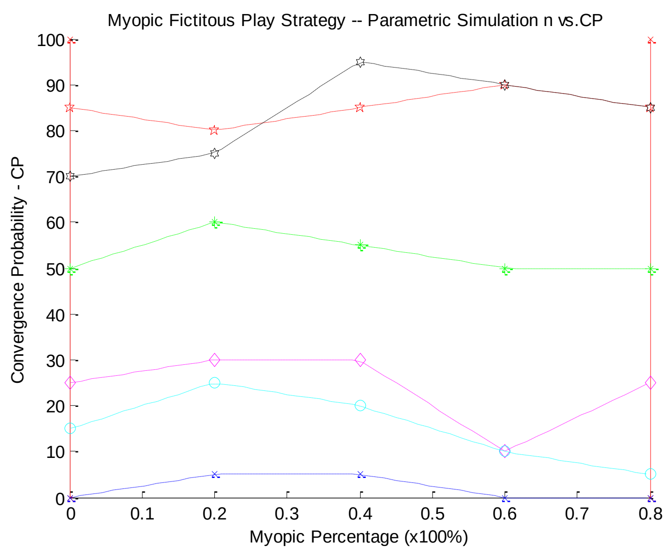


Figure 6.50 Myopic Deterministic Fictitious Play convergence probability results for 3 fixed APs

Myopic stochastic FP results

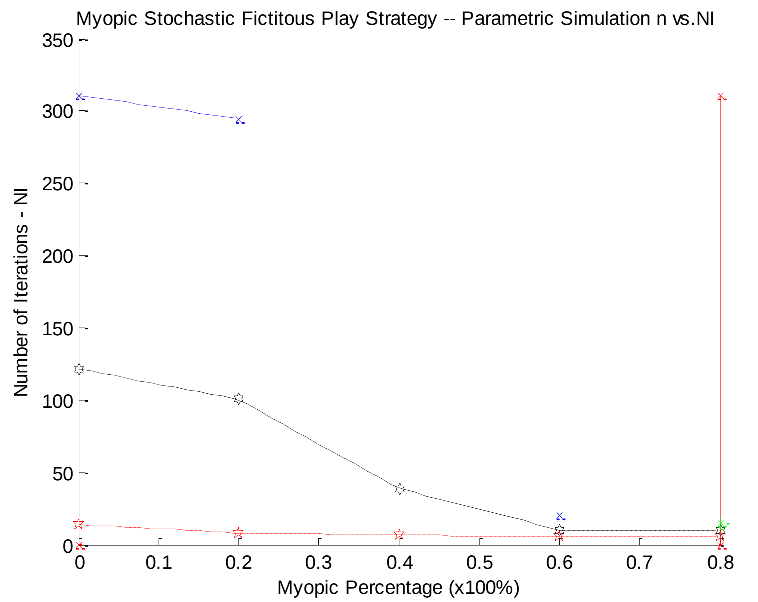


Figure 6.51 Myopic Stochastic Fictitious Play results for 3 fixed APs

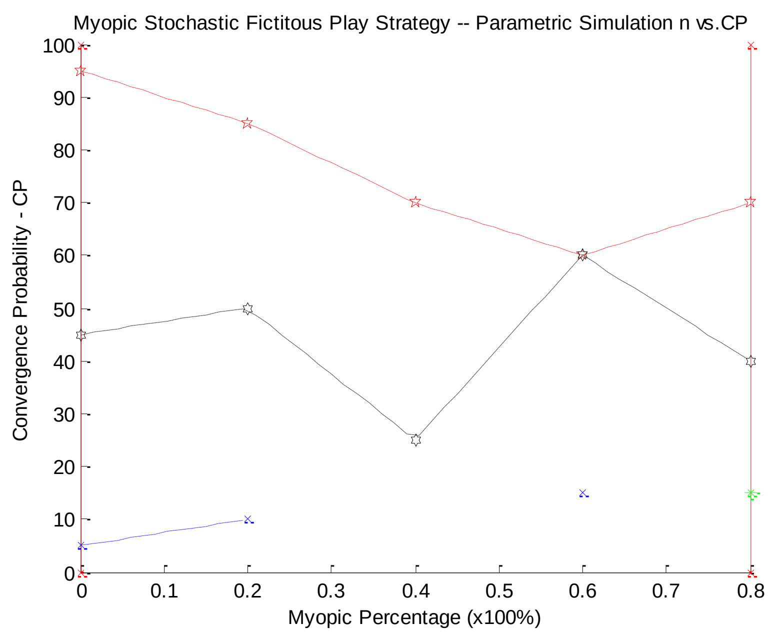


Figure 6.52 Myopic Stochastic Fictitious Play convergence probability results for 3 fixed APs

Figures 6.49 and 6.50 representing the myopic-deterministic outputs are showing similar results as the *Pure Interference* based functions, a settlement point at m.p = 60% for the number of iterations vs. number of users graphs. All curves except for n = 30 users posses convergence probability greater than 0 for all points on the deterministic scenario, additionally a common overshoot at m.p = 40% is observed at figure 6.50 for n = 15, 20 & 25 users, the same set of curves present a convergence probability rounding roughly 50%, 30% and 20% respectively. Unfortunately even though there is a monotonic improvement of the number of iterations when approaching the settlement point, the same rule does not apply for the convergence probability as seen on the irregular behaviors of figures 6.50 and 6.52.

The stochastic version of the scenario with 3 fixed access points is showing a continuous convergence only for n = 5 and 10 users, the rest of the points are having isolated convergence points through the whole range; the results make the deterministic myopic fictitious play much more reliant than the stochastic version.

<u>Results for 7 APs</u>:

Deterministic FP results

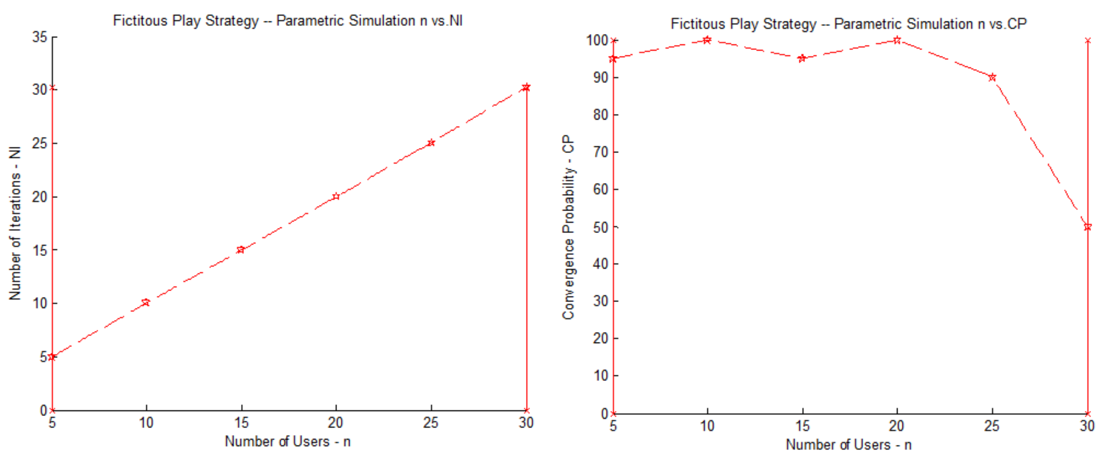


Figure 6.53 Deterministic Fictitious Play results for 7 fixed APs

An improvement convergence probability wise is observed on the rightmost graph of figure 6.53 with respect to 6.47, different of what happened with the *Pure Interference* based functions where there was no difference between the 3 and 7 fixed AP cases. In this case the addition of strategies makes rational for better convergence probability since the new access points offer different achievable rates to each user, and being able to choose the best strategy deterministically leads to an overall improvement of the convergence probability, on the contrary for the stochastic version of the algorithm the improvement does not hold, as observed on the rightmost graph of figure 6.54.

Stochastic FP results

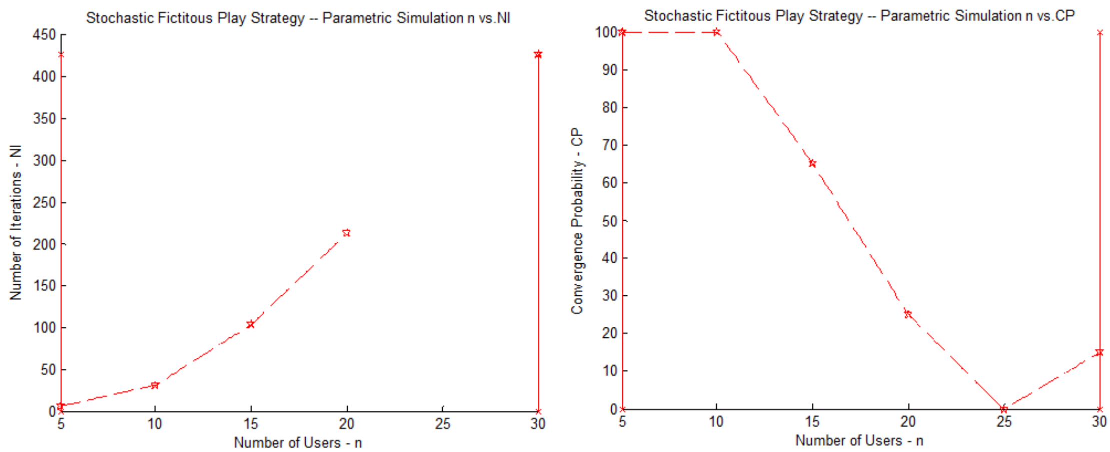


Figure 6.54 Stochastic Fictitious Play results for 7 fixed APs

First of all observe a tendency to have a quadratic behavior on the leftmost graph shown above, add the fact of no convergence at n = 25 users just as like the 3 fixed user scenario, besides the former facts, the convergence probability decreases in a monotonic way from n = 10 to n = 25 highlighting an inverse proportion between number of iterations and convergence probability inside this range. Following what was discussed on the previous paragraph for the stochastic version of the *Fictitious Play* algorithm the improvement of the convergence probability is not direct as the addition of new strategies in fact add possible improvements on the achievable rates and therefore to the global utility function, however now the stochastic decision is made on a bigger set of strategies, given this fact the chances to choose the most relevant strategy in beliefs will indeed exist, but it will require more iterations deriving even quadratic-like behaviors as the one seen on figure 6.54

Myopic deterministic FP results

Unlike the previous results shown by the stochastic F.P algorithm, the myopic deterministic version holds a convergence probability greater than 0% for all myopic percentages for the n = 25 users curve (magenta). The initial values for n = 20 & 25 users are slightly greater than the previous scenario results, however the common overshoot appears yet again at m.p = 40% and the settlement point number of iteration magnitude happens from bottom to top as the number of users increases orderly in steps of 5 users per iteration. It is noted additionally that for this specific scenario the average number of iterations for n = 30 users, outperforms the one for n = 20 and 25 users unlike the case with three fixed access points.

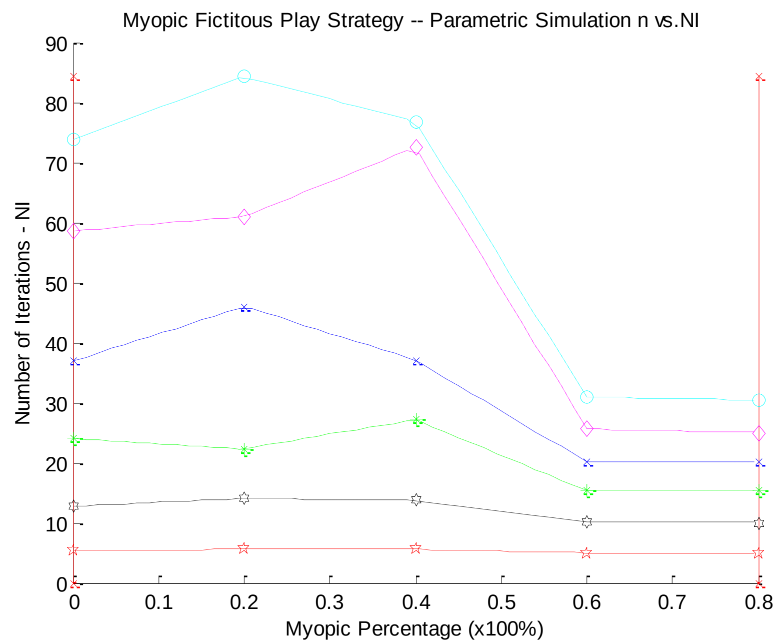


Figure 6.55 Myopic Deterministic Fictitious Play results for 7 fixed APs

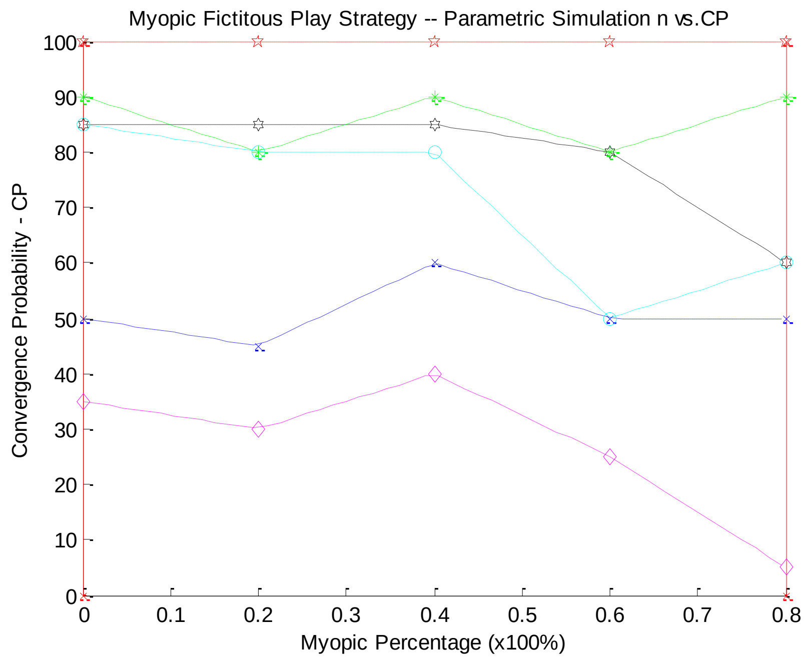


Figure 6.56 Myopic Deterministic Fictitious Play convergence probability results for 7 fixed APs

Myopic stochastic FP results

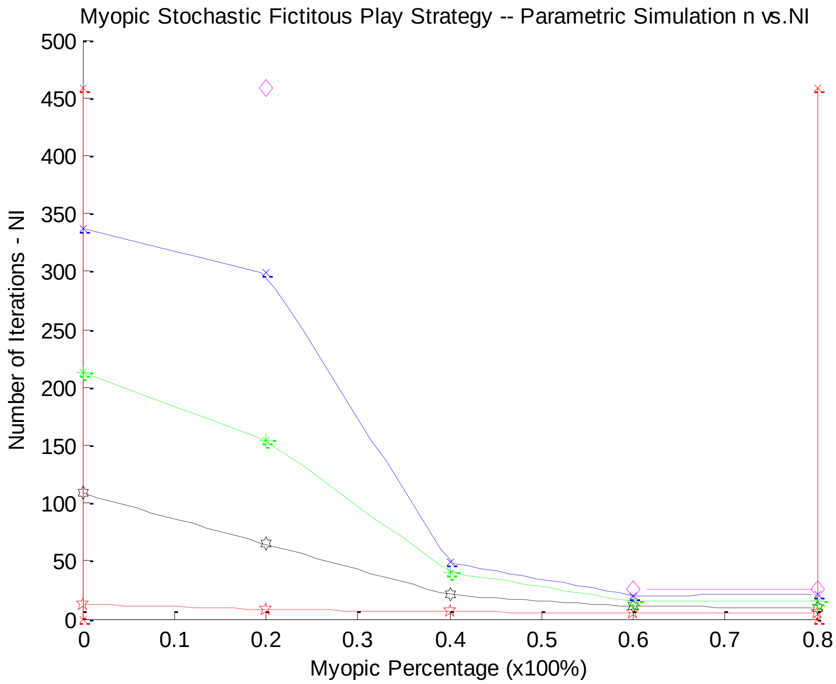


Figure 6.57 Myopic Stochastic Fictitious Play results for 7 fixed APs

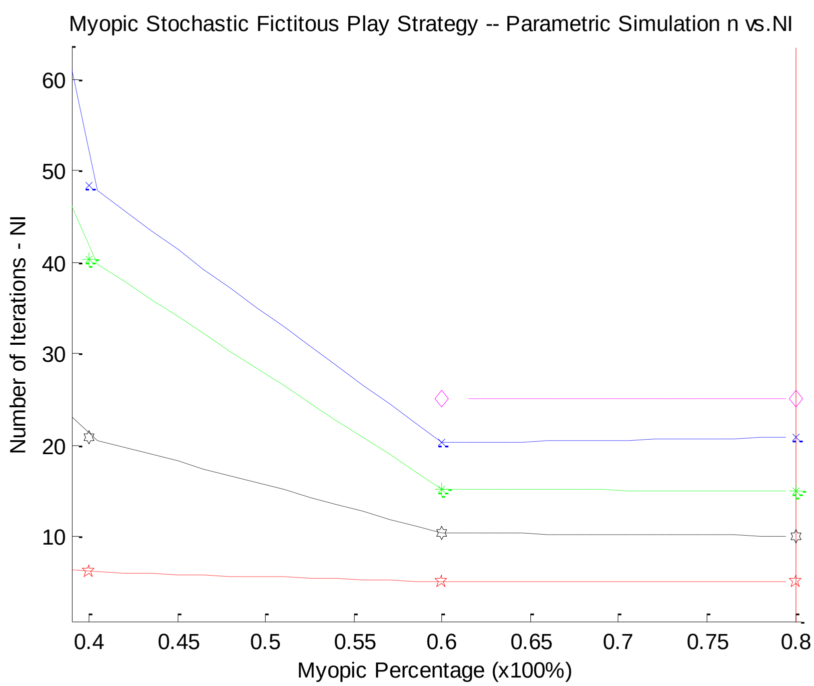


Figure 6.58 Myopic Stochastic Fictitious Play zoom-in results for 7 fixed APs

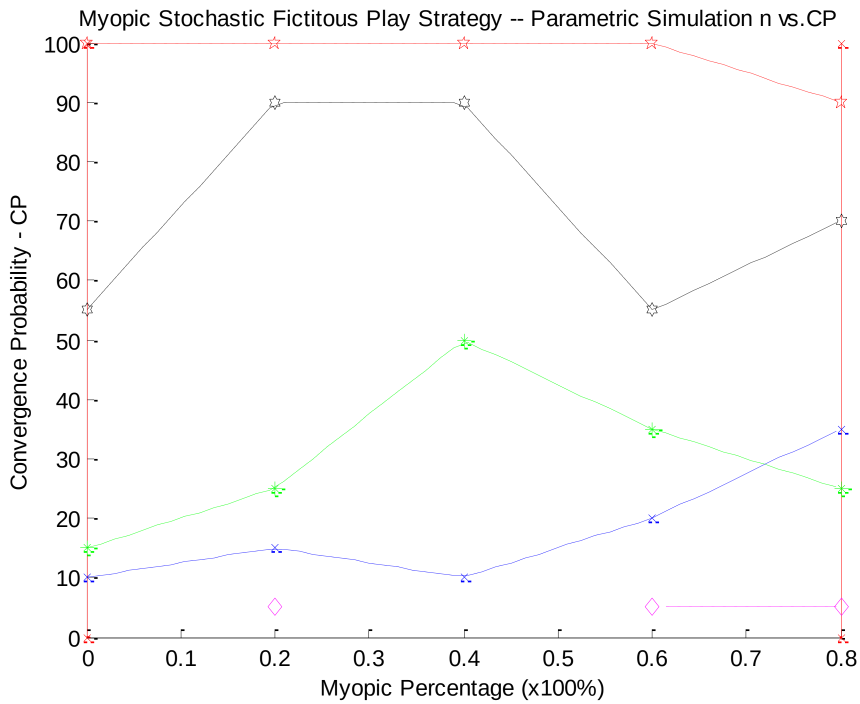


Figure 6.59 Myopic Stochastic Fictitious Play convergence probability results for 7 fixed APs

Figures 6.57 through 6.59 show an overall improvement of the different simulated user points with respect to the myopic percentages when compared with the case having four less strategies; when comparing figure 6.57 with 6.51 it is noted a continuous curve for n = 20 users, and the existence of three convergence points for n = 25 users curve. The number of iterations for all the curves is either constant or having a monotonic decrease from m.p = 0% to m.p = 60% where all the curves settle to a constant value, in direct relation with these dynamics the convergence probability graph is showing a global improvement on the curves as the myopic percentage increases, note though the magenta curve appears it withholds a convergence probability of around 5%

Fixed number of users, varying number of access points static analysis

- Varying number of randomly deployed access points, starting from 1 AP in steps of 1 AP per iteration until 7, giving a total of 7 different games.
- Access point power transmission model modified in order to achieve an *All in Range* scenario.
- Map length $40m^2$

Fixed number of Users will be 30

Deterministic FP results

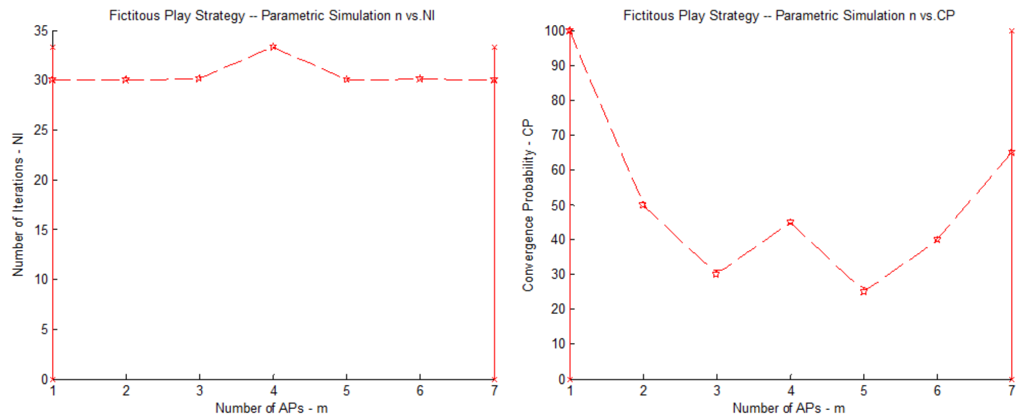


Figure 6.60 Deterministic Fictitious Play results for 30 fixed users

Stochastic FP results

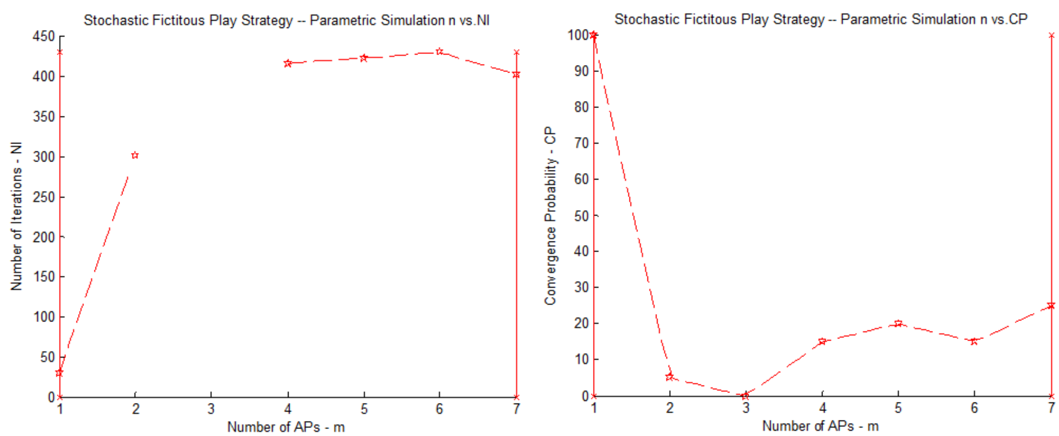


Figure 6.61 Stochastic Fictitious Play results for 30 fixed users

Observing figure 6.60 it is seen that the output almost resembles an ideal number of iteration behavior for the scenario imposed except for the peak value presented at m = 4 APs, even though the convergence probability curve dynamics is nowhere close to the ideal case an interesting fact is seen, a monotonic decrease from m = 1 to m = 3 and a monotonic increase from m =4 to m =7, *Game simulator* can be used to verify if the monotonic increase in the convergence probability is hold for the increasing value of strategies. Opposite of figure 6.60, figure 6.61 is showing a very steep line from m = 1 to m = 2 APs having an intermediate value with 0% convergence probability and then settling up to an excessively high amount of iterations, observe though that after m = 3 APs the convergence probability curve tends to increase its value, and even at m = 6 APs the number of iterations begin to decrease.

The reason behind the behavior on the leftmost graph of figure 6.61 having approximately 300 iterations when 30 users only have to choose between 2 access points results odd at a first glance, however it should be remembered that the decisions here are taken stochastically and having a cumulative dynamics, in the sense that if after 20 iterations, each player has chosen the two available access points the same number of times is the same situation as starting from scratch, hence it is feasible as the number of available strategies increases the tendency for a user to attach in beliefs to a particular access point increases, the former is supported by the fact of the increase of the convergence probability for m > 3 and the number of iterations decrease for m > 6

Myopic deterministic FP results

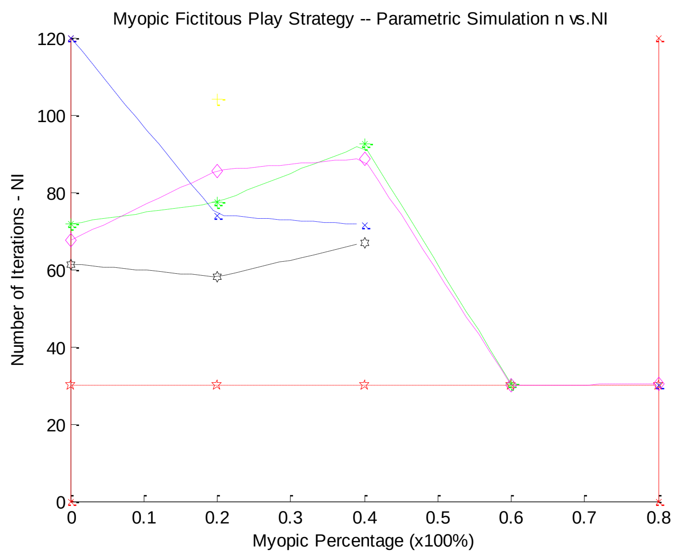


Figure 6.62 Myopic Deterministic Fictitious Play results for 30 fixed users

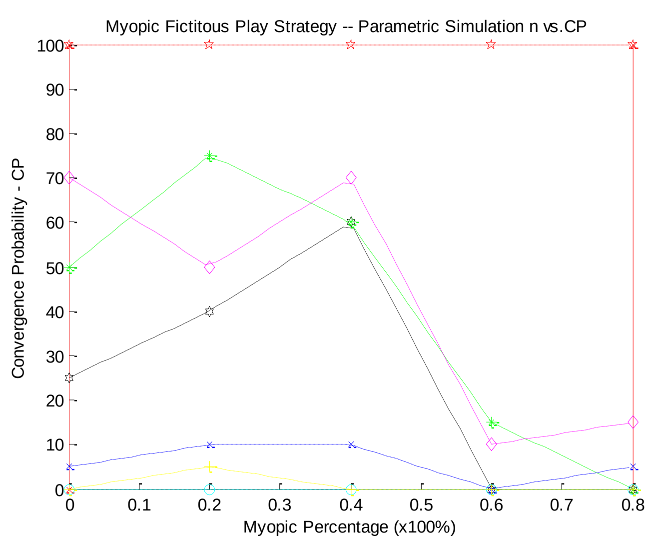


Figure 6.63 Myopic Deterministic Fictitious Play convergence probability results for 30 fixed users

Myopic stochastic FP results

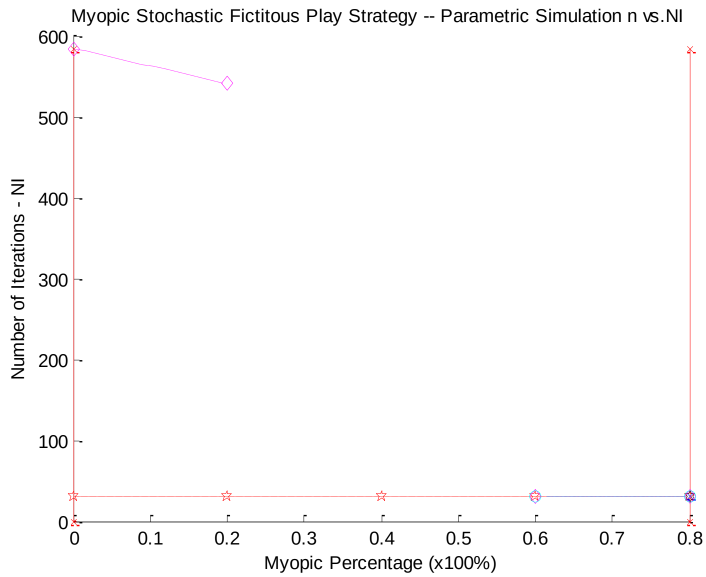


Figure 6.64 Myopic Stochastic Fictitious Play results for 30 fixed users

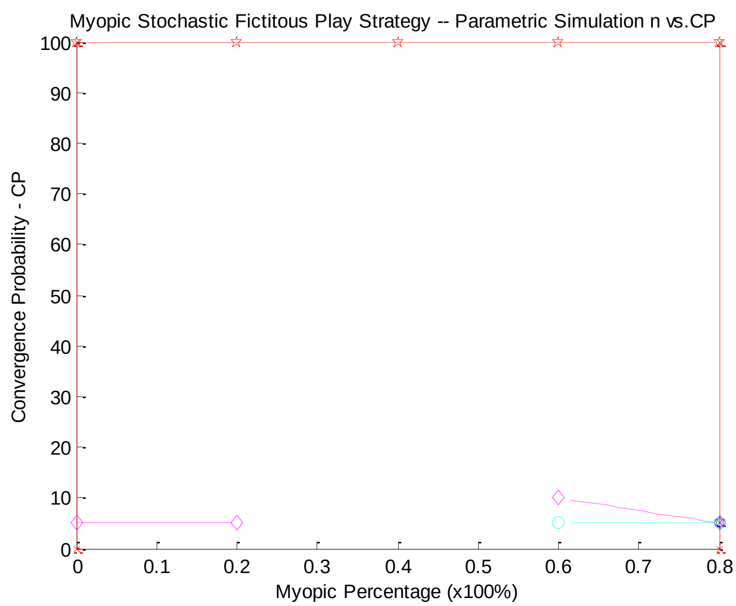


Figure 6.65 Myopic Stochastic Fictitious Play convergence probability results for 30 fixed users

Figure 6.62 with its red curve (1 access point) is showing the ideal behavior for the proposed scenarios, it is observed than the only points where all the other curves meet the ideal number of iteration value is at the settlement point m.p = 60%, before this point m = 6 & 7 APs are showing no relevant convergence probability, whilst the rest of the curves average overall for a convergence probability greater than 30% though after the settlement point, the ideal number of iterations is held with a decreasing payoff convergence probability wise as seen between m.p = 60% and 80% where all curves have a C.P lower than 20%.

Figure 6.65 speaks for itself, continuous convergence for the ideal case having only 1 access point and isolated convergence points for m = 5 and 6 access points, these results can be cross checked with figures 6.51-6.52 and 6.58-6.59 where for 3 and 7 access points respectively there is a consistent absence of the listed curves for n = 30 fixed users (cyan curves).

Being able to choose deterministically from a myopic fictitious play learning technique allows to reach a settlement point however under very low convergence probability rates due to the possibility of being stuck in local equilibria and not on Nash equilibria as desired, on the other hand by choosing stochastically the strategies to play with even when they increase step by step does not show an improvement except for the trivial case with 1 AP.

7.

PERFORMANCE EVALUATION WITH MULTIPLICATIVE INTERFERENCE AND INVERSE RATE UTILITY FUNCTIONS

Utility function definition

This utility functions are given by the multiplication between the number of interferers and the reverse of the rate perceived by users. Formally the users' cost function is defined as:

$$c_j(i, x^i) = T_j^i \cdot x^i$$

The game modeled by such utility function does not admit an exact potential function [17].

Note: All simulations done will be under the assumption of "All in range" meaning each user is inside the coverage area of at least one access point. The previous holds unless it is differently expressed

7.1

BETTER RESPONSE ALGORITHM VS. BEST RESPONSE ALGORITHM*

Fixed number of access points, varying number of users static analysis

- Varying number of randomly deployed users, starting from 10 users in steps of 10 users per iteration until 100, giving a total of 10 different games.
- Access point power transmission model modified in order to achieve an *All in Range* scenario.
- Fixed number of Access Points for each game, simulating games for 5,7 and 10 APs
- Map length $40m^2$

Results for 5 APs:

Best Response algorithm results

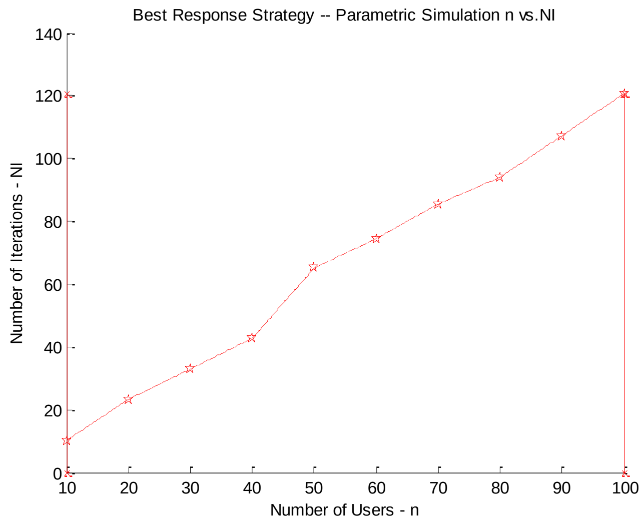


Figure 7.1 Best Response results for 5 APs

Better Response algorithm results

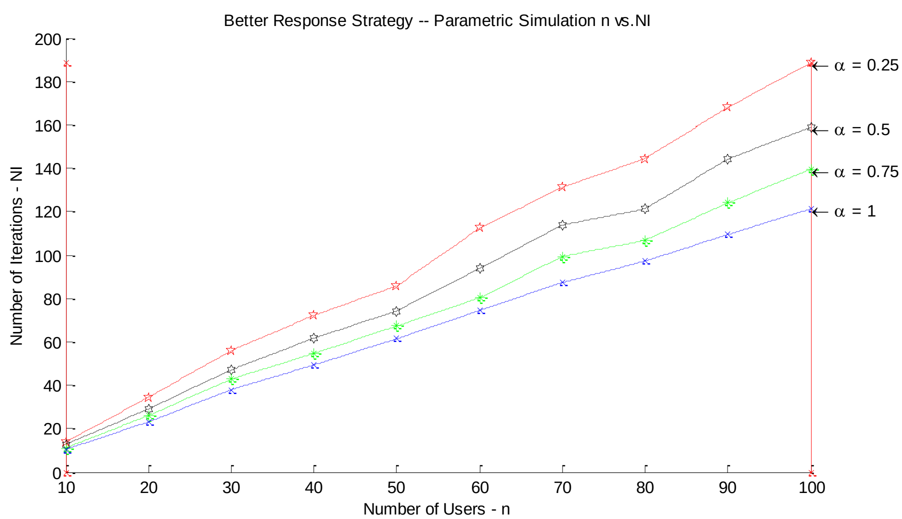


Figure 7.2 Better Response results for 5 APs

For a non-linear utility function it shouldn't be expected a linear behavior number of iterations wise, despite the tendency to this behavior shown by figure 7.1, the slope is clearly greater than 1, which shows a non linear (one by one) increment of the number of iterations with respect to the increasing number of users, the most noticeable change in the linearity is appreciated at n = 50. Figure 7.2 exposes the dynamics for 5 fixed APs, under an increasing number of users, however under different blind percentage of the available strategies, when $\alpha = 0.25$ each user at each turn can only see 2 out of 5 available strategies so it is natural that the number of required iterations is greater in order to find a Nash equilibrium, curves go from worse to best as $\alpha = 0.25$ approaches $\alpha = 1$.

Results for 10 APs:

Best Response algorithm results

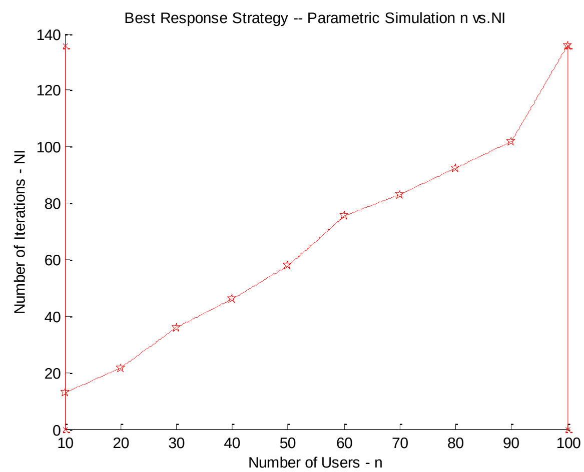


Figure 7.3 Best Response results for 10 APs

Better Response algorithm results

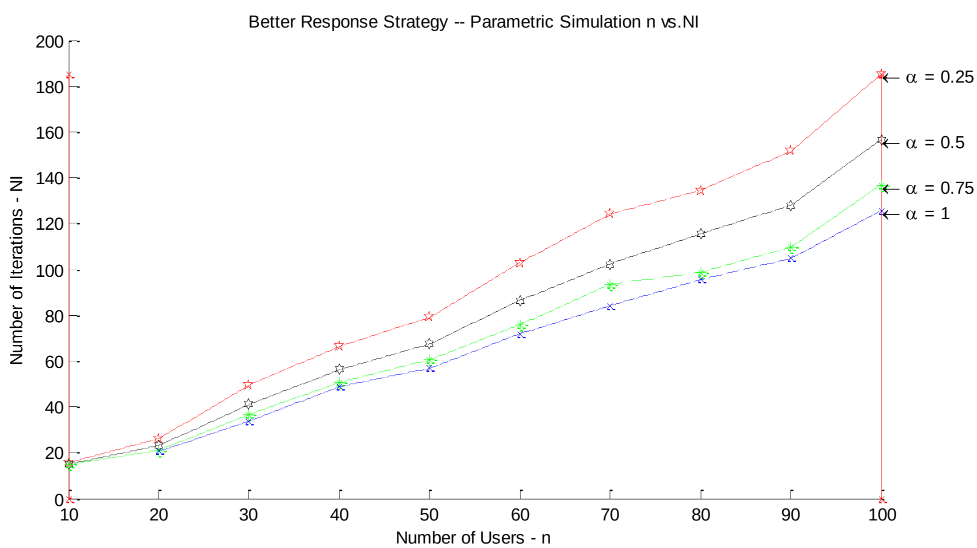


Figure 7.4 Better Response results for 10 APs

Figure 7.3 is having abrupt changes in its slope at n = 50 until n = 60, afterwards it tends to have a linear behavior with a constant slope. Figure 7.4 follows figure's 7.3 dynamics with a set of curves belonging to $\alpha = 0.25$ through $\alpha = 1$, despite the peculiarity of the curve dynamics, it resulted in a lower number of iterations for the worst curve i.e $\alpha = 0.25$ accounting for approximately 190 iterations, most likely the ratio between users and access points for 100 users and 10 fixed APs aided the faster convergence number of iteration wise over twenty averaged games.

Concluding remarks regarding the multiplicative interference and achievable rate functions include the fact of being the curves with most non-linearity's and with the greatest number of total iterations, the former due to the nature of the new non-linear player specific payoff functions.

Fixed number of access points, varying number of users static analysis

- Fixed number of randomly deployed users, varying number of access points starting from 10 APs in steps of 10 APs per iteration until 100, giving a total of 10 different games.
- Access point power transmission model modified in order to achieve an *All in Range* scenario.
- Fixed number of Users, simulating games for 100,150 and 200 users
- Map length $50m^2$

Results for 100 Users:

Best Response algorithm results

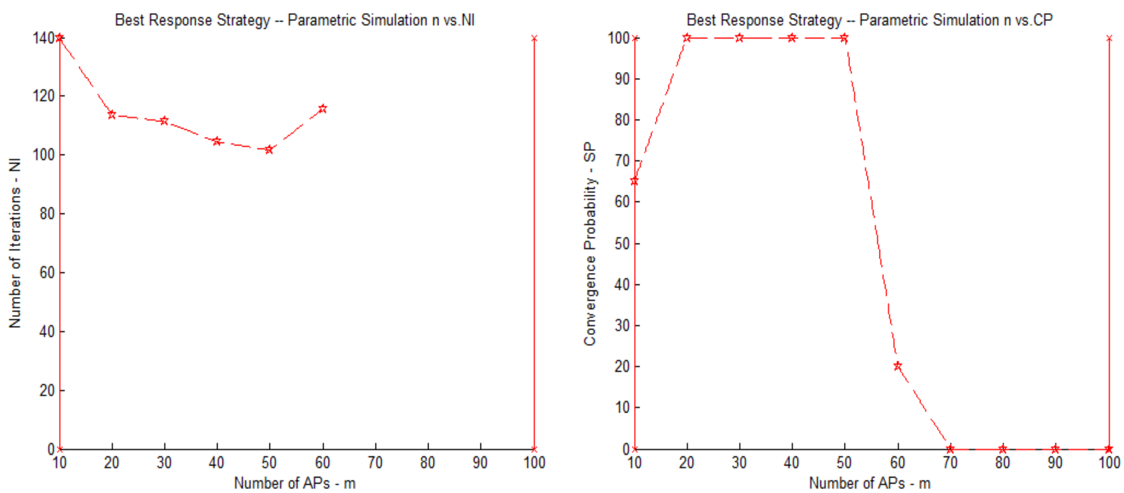


Figure 7.5 Best Response results for 100 users

Better Response algorithm results

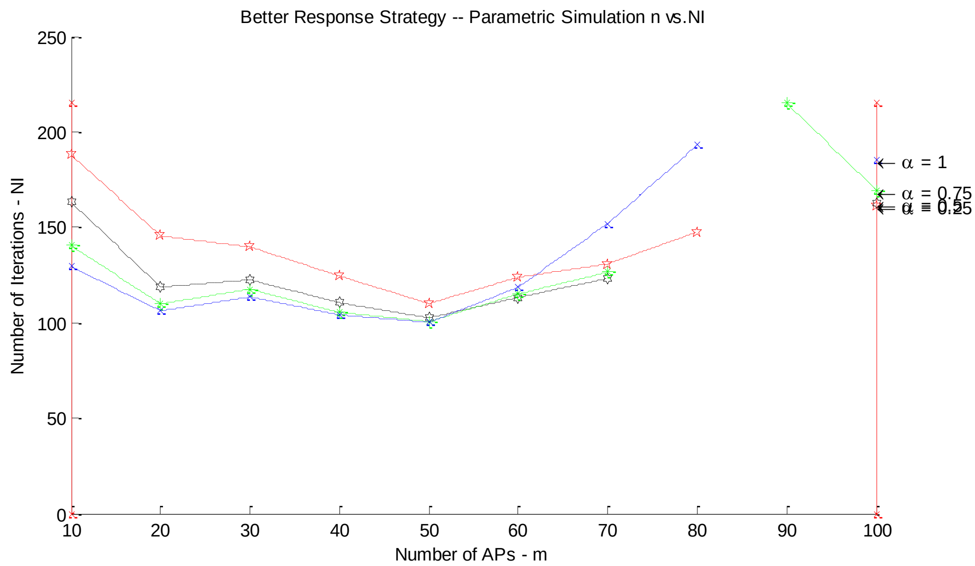


Figure 7.6a Better Response results for 100 users

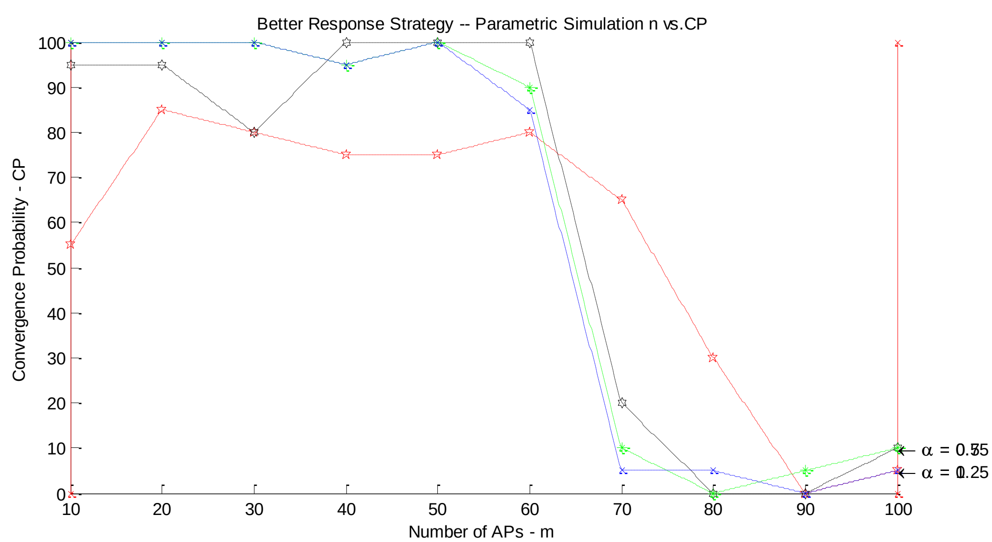


Figure 7.6b Better Response convergence probability results for 100 users

It is relevant to point out before any analysis that for the 1st time on a *Best Response* algorithm simulation, it was needed a convergence probability output, figure 7.5 is showing the results for 100 fixed users and varying APs from 10 to 100, it can be seen from the rightmost graph of figure 7.5 that the output has a convergence probability greater than 0% from m = 10 APs until m = 60 APs, on the points where the convergence probability behaves with a constant value of 100% it is seen an equivalent behavior number of iteration wise, whilst for points with convergence probability lower than 100% the averaged number of iterations required exceeds the constant behavior ones described before. It can be inferred that the lack of convergence is due to an increasing amount of strategy space while the demand for service i.e number of users remains constant, so since the player specific function now depends non-linearly on the number of interferers per AP and the achievable rate, it becomes impossible to find a convergence under the case study given conditions.

Figures 7.6a and 7.6b are showing the results for the *Better Response* algorithm, it can be seen a general behavior similar to the described on the previous paragraph, on the Number of iteration vs. number of APs graphs, the curves for $\alpha = 0.5$ through $\alpha = 1$ behave almost equivalently from m = 10 to m = 50, from m = 50 until m = 80 APs $\alpha = 1$ curve exceeds all of its counterpart values. The green curve corresponding to $\alpha = 0.75$ resulted to be the only one with a convergence probability greater than 0% from m = 90 to m = 100, however paid off with a very low convergence ratio; it is hard to say from the two last graphs wich α curve outperformed the others, from the previous analysis made on pure interference and additive interference and achievable rate functions $\alpha = 0.25$ greatly outperformed all the other α curves, particularly as seen on figure 7.6b, $\alpha = 0.25$ is being outperformed from m = 10 all through m = 70, however, $\alpha = 0.25$ vastly outperforms all other curves from m = 70 to m = 80

<u>Results for 200 Users</u>:

Best Response algorithm results

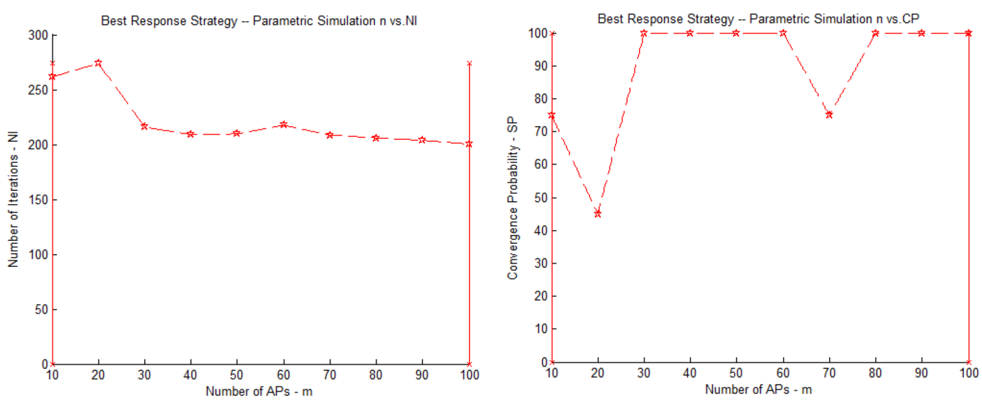


Figure 7.7 Best Response results for 200 users

Better Response algorithm results

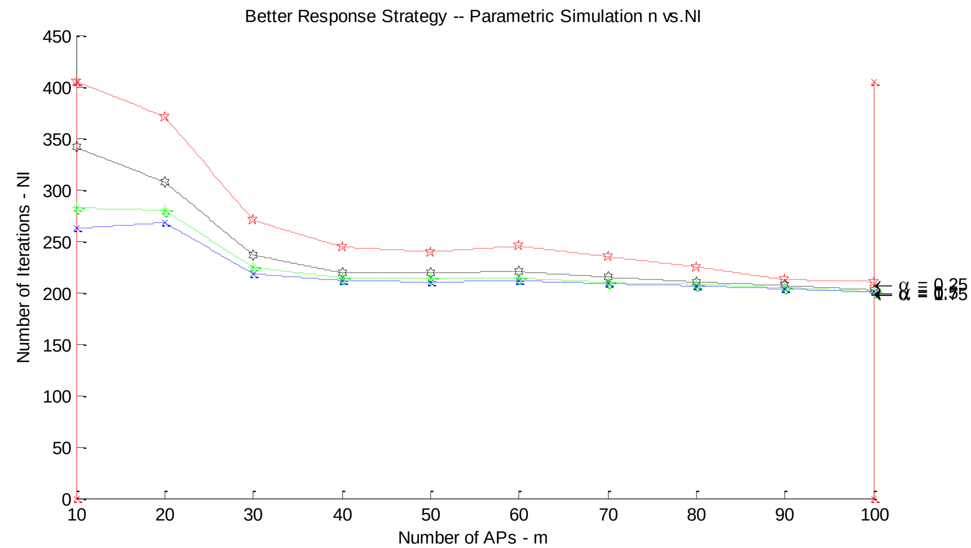


Figure 7.8a Better Response results for 200 users

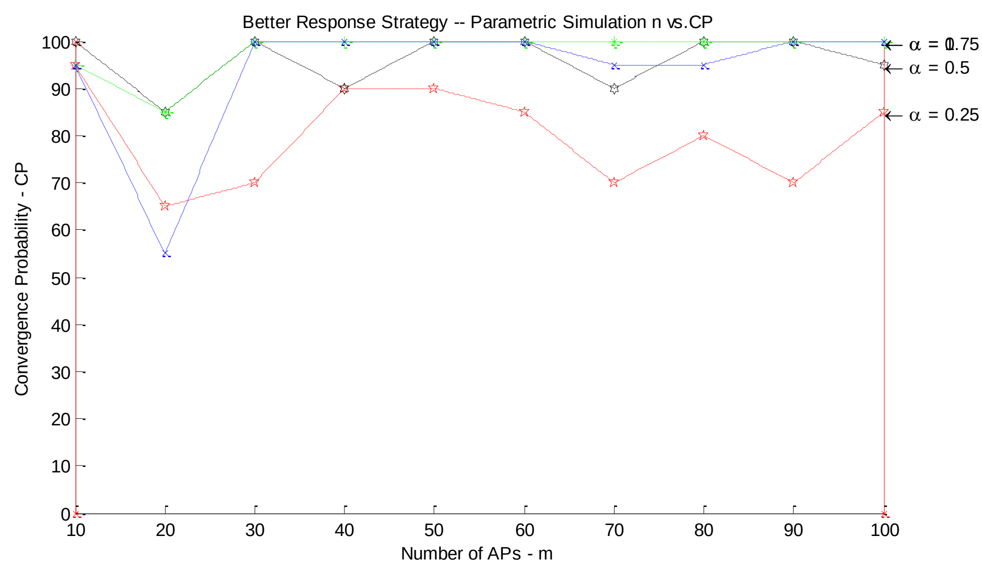


Figure 7.8b Better Response convergence probability results for 100 users

Success! Figure 7.7 is showing on the rightmost graph a convergence probability greater than 0% for all points, again an improvement of 20% with respect to its previous counterpart n = 150 fixed users, analyzing the improvement carries the fact of adding users to the scenario influences the individual payoff function of every users, it is doing so by populating the strategy space with more users, hence by making more decisive the utility function number of interferers wise, in other words the more interferers the more easily a user can choose an AP due to the fact that the distance between each user and AP is unique, it will lead to a unique utility function given by the product of the number of interferers and a equivalent value accounting for the achievable rate, hence, the more users populate the strategy space, the smaller the strategy space left to the other users, ideally leaving the closest APs without interferers.

7.2

Perturbation Scenarios

Removing users/access points

<u>Results for 7 APs, varying users from 10 to 100 (User step 10 per iteration)</u>:

Best Response algorithm results

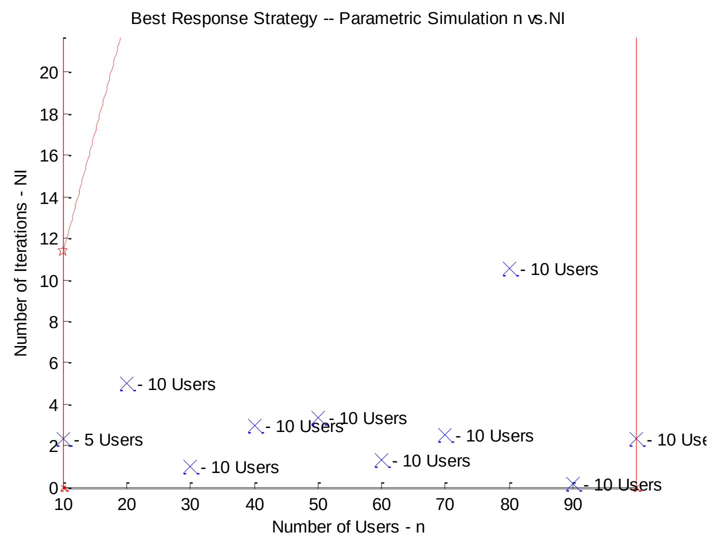


Figure 7.9a Best Response Perturbation scenario results for 7 fixed APs

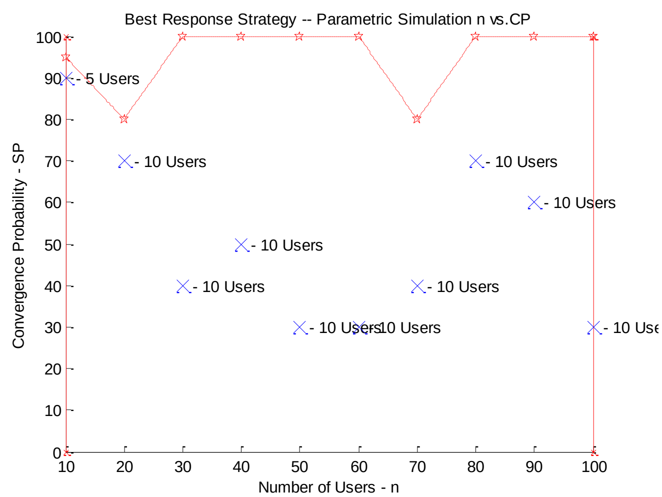


Figure 7.9b Best Response Perturbation scenario convergence probability results for 7 fixed APs

Better Response algorithm results

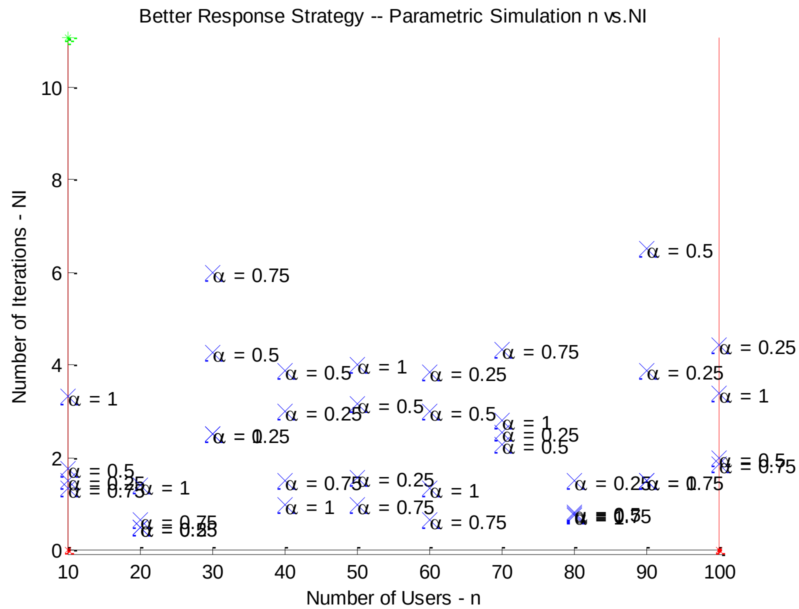


Figure 7.10a Better Response Perturbation scenario results for 7 fixed APs

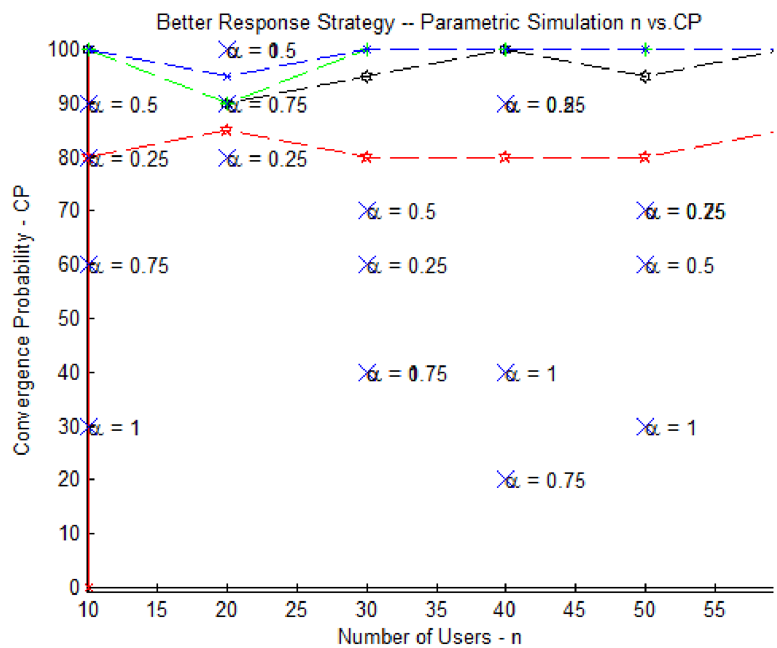


Figure 7.10b Better Response Perturbation scenario convergence probability results for 7 fixed APs part 1

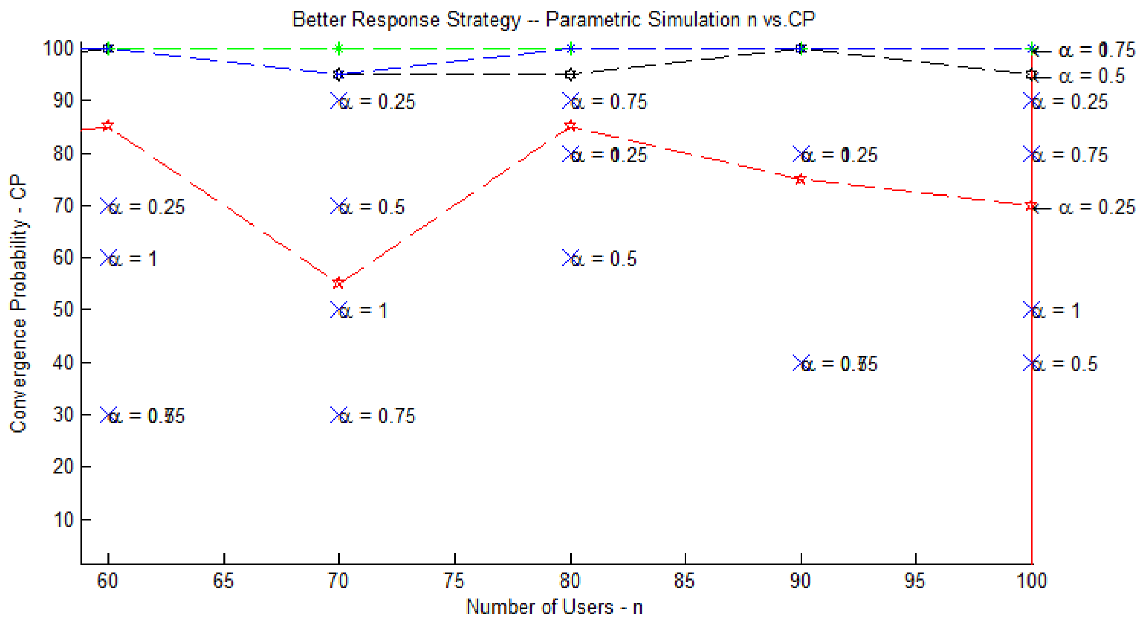


Figure 7.10c Better Response Perturbation scenario convergence probability results for 7 fixed APs part 2

As additional output measure to the multiplicative interference and achievable rate functions exclusively it was added the disturbance analysis to the convergence probability. Figure 7.9a is showing the disturbance accounting for additional simulations done after the disturbance was introduced, for all points, the additional number of iterations are lower or equal to 10, there is no particular pattern in the behavior observed, the points with minimum number of iterations resulted to be n = 30 and n = 60 whilst the point with 10 additional iterations was n = 80 users. Note on figure 7.9b how the convergence probability values for all points oscillate between 80% and 100% however after the disturbance has been produced, the resulting convergence probability is lower, this is due to fact of the individual utility function being non-linear, for example, if 10 users were removed when n = 50, the 40 remaining users might not find a new equilibria in under 30 new iterations, or even if they did, figure 7.9b is showing that for a high percentage of the cases it was not a Nash equilibrium.

Figure 7.10a is showing the *Better Response* algorithm results for the same disturbance scenarios used for *Best Response*, it is seen that for all α values the total number of additional iterations never adds more than 10, and that there is an irregular behavior for all α i.e $\alpha = 0.25, 0.5, 0.75$ & 1 for all simulated points i.e n = 10-100 users in the sense of the non-existance of a α value that always perform better than the other ones. Figure 7.10b and 7.10c are exposing the disturbance results for the convergence probability, $\alpha = 1$ values can be crosschecked with those of figure 7.9b, all the other values exhibit casual behavior, however note that for some particular values, the value of the convergence probability results higher than the original scenario, at figure 7.10b for $\alpha = 0.5$ and $\alpha = 0.75$ at n = 20, for $\alpha = 0.25$ at n = 40, n = 70 and n = 90, the disturbance actually improved the convergence probability of the new scenario, and this is not rare, since it can happen that the removed users, were associated to a certain AP, which with less interferers could yield a higher multiplicative and interference based value to certain users, yet these users being associated to different APs, so since the removal of the users is random, this scenario happens stochastically.

Results for 150 fixed users, varying access points from 10 to 100 (AP step 10):

Best Response algorithm results

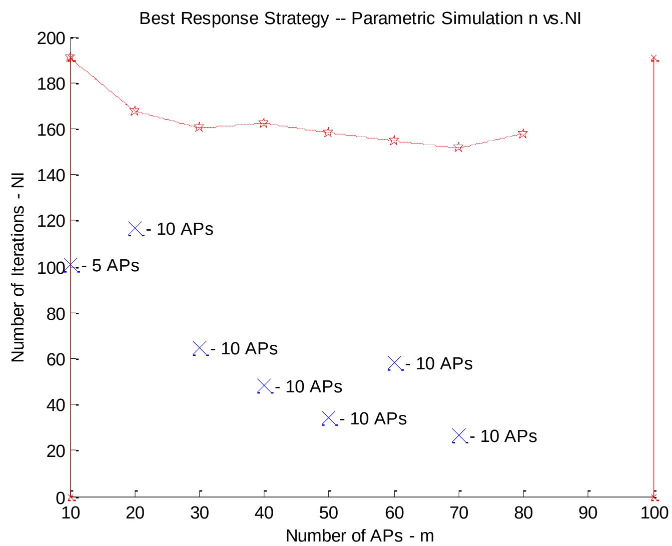


Figure 7.11a Best Response Perturbation scenario results for 150 fixed users

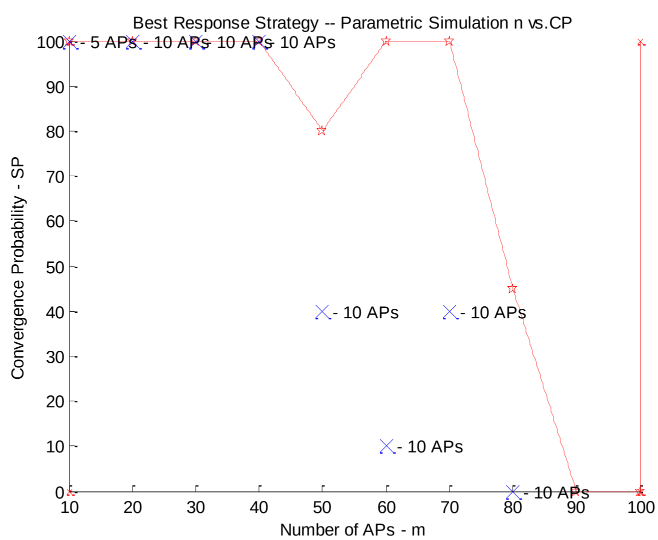


Figure 7.11b Best Response Perturbation scenario convergence probability results for 150 fixed users

Better Response algorithm results

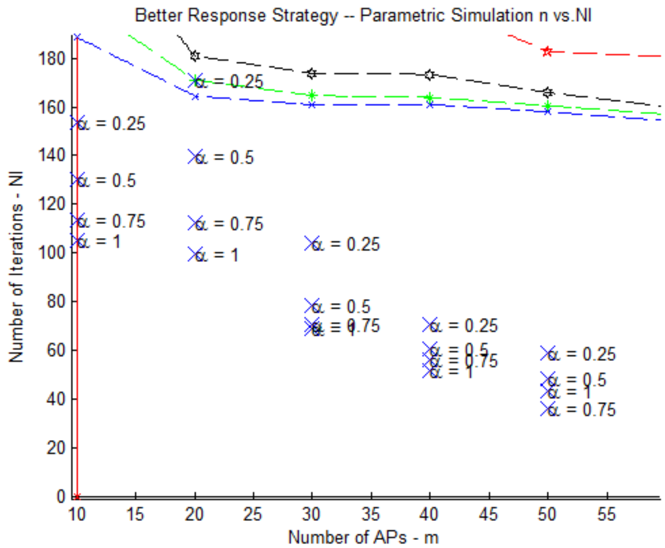


Figure 7.12a Better Response Perturbation scenario results for 150 fixed users part 1

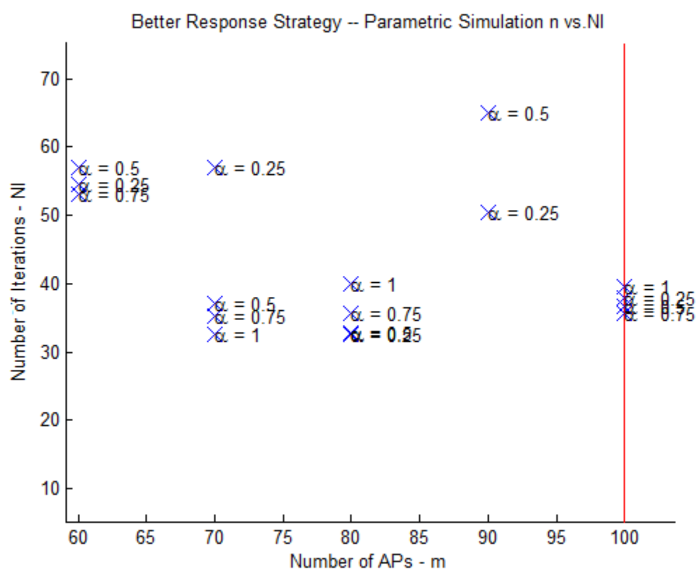


Figure 7.12b Better Response Perturbation scenario results for 150 fixed users part 2

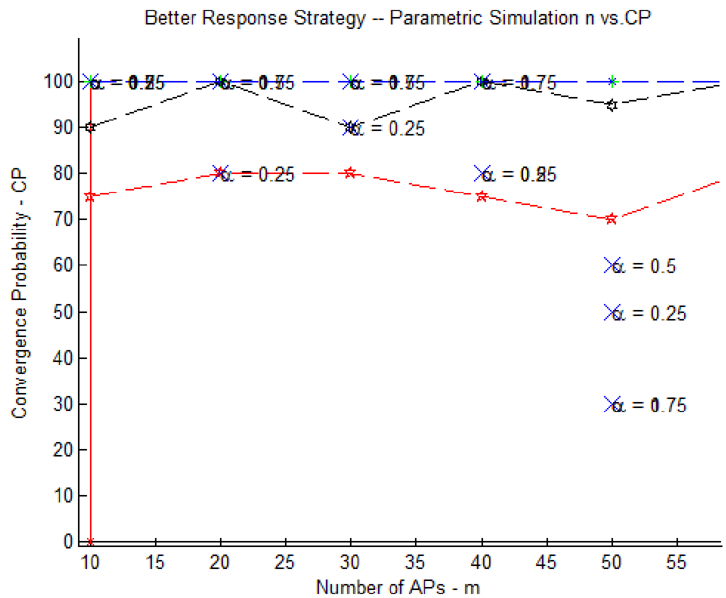


Figure 7.12c Better Response Perturbation scenario convergence probability results for 150 fixed users part 1

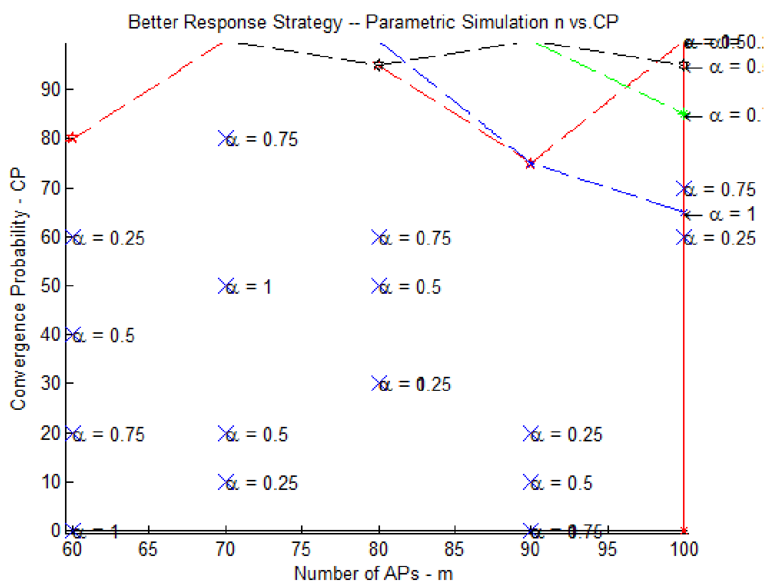


Figure 7.12d Better Response Perturbation scenario convergence probability results for 150 fixed users part 2

The latest six figures are showing in different figures the results for the disturbance analysis done to a fixed users varying number of APs scenario, this particular case of study has shown peculiar behavior on the former Sections, number of iterations wise, it was seen that as the number of APs increases the number of additional iterations required to find an equilibrium decreased because the same amount of users is distributed evenly between the access points that can give them the best utility function, this very same behavior is observed on figures 7.11a at *Best Response* algorithm, and for all points of figure 7.12a and for some points of figure 7.12b, the monotonic decrease for all α values is not kept for instance at n = 60, 70, 80 and 90 APs.

The *Better Response* algorithm whose disturbance application yielded the irregular behavior observed on figure 7.12b, also produces an irregular behavior observed on figures 7.12c and 7.16d, note on 7.16c that from m = 10 to m = 40 APs, all α re-iterations outperform or meet the baseline iteration values (i.e the dashed colored lines), however from m = 40 and on, the re-iterated values never again outperform nor equal their baseline simulation values, even when these were 100% convergence probability, seemingly the irregular behavior starts at m = 50 just one iteration step before the irregular behavior was observed on figure 7.12b; the odd dynamics can be accounted for the fact of being the users more widespread along the increasing number of AP's, the removal of 10 users per iterations results in a little non very noticeable global disturbance, in the sense of updating the local algorithm variables is done in such way that in the case of 1 user removed from 10 different AP's can leave the remaining players stuck in a local equilibria, however in the cases where 10 users are removed from the same AP, can lead to a sensitive global disturbance, motivating the reaming users to look for a new equilibria.

7.3

Concurrent choice scenarios

In these scenarios, the *Game Simulator* emulates the behavior of users making simultaneous choices. The software allows β players to make simultaneous choices, starting from $\beta = 1$ in β_{step} steps until the maximum number of users is reached. This additional parameter is applicable to *Best Response* and *Better Response* algorithms extending the number of possible output analysis that can be done.

Output graphs include the foreshowed graphs i.e Number of Iterations vs N (users) or M (access points), can include Elapsed simulation time vs N or M and also Convergence Probability vs N or M, convergence probability. The analysis is made because as the β parameter approaches the maximum number of users the algorithms tend to fail in convergence (fact to be proven with the simulations). Output results are the average of 20 different played games, as β increases the number of these games converging decreases.

In order to illustrate the mentioned behaviors, concurrent choice scenarios will be done for the same Perturbation scenarios of Section *5.3*

<u>Results for 7 APs, varying users from 10 to 100 (User step 10 per iteration)</u>:

Best Response algorithm results

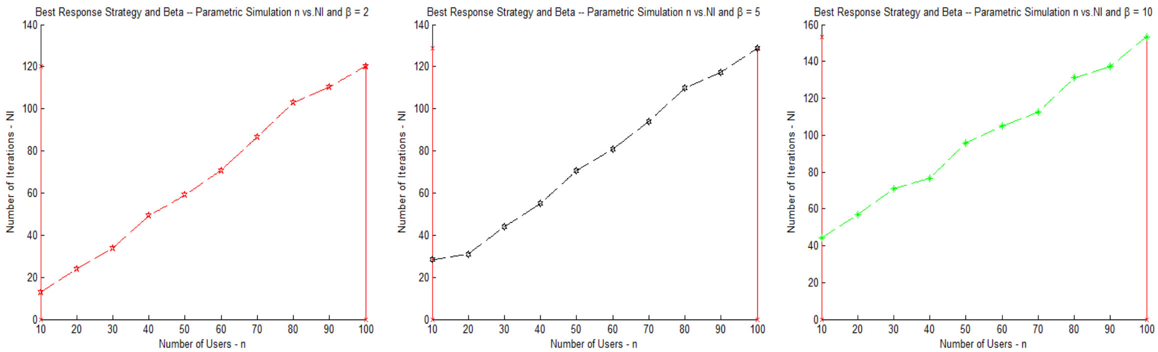


Figure 7.13a Best Response concurrent scenario results for 7 fixed APs

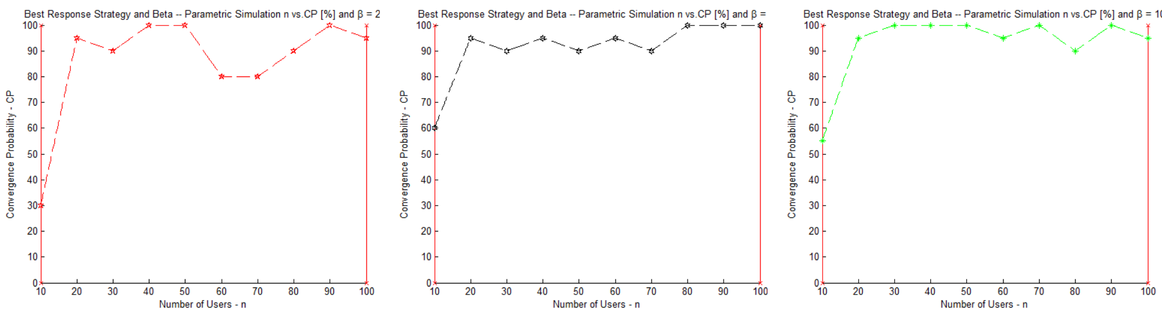


Figure 7.13b Best Response concurrent scenario convergence probability results for 7 fixed APs

Better Response algorithm results

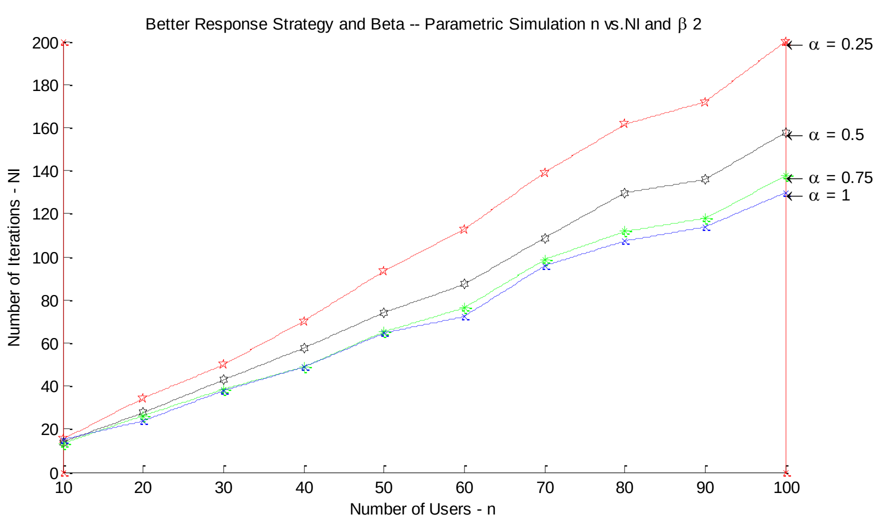


Figure 7.14a Better Response 2 concurrent users scenario results for 7 fixed APs

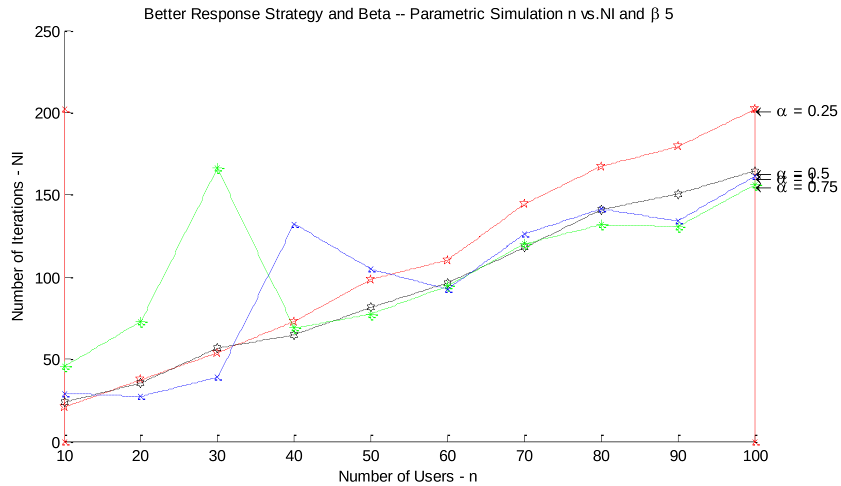


Figure 7.14b Better Response 5 concurrent users scenario results for 7 fixed APs

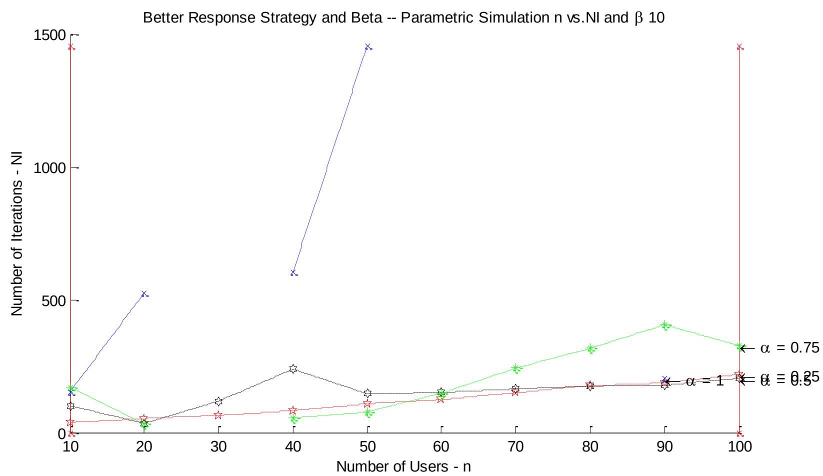


Figure 7.15a Better Response 10 concurrent users scenario results for 7 fixed APs

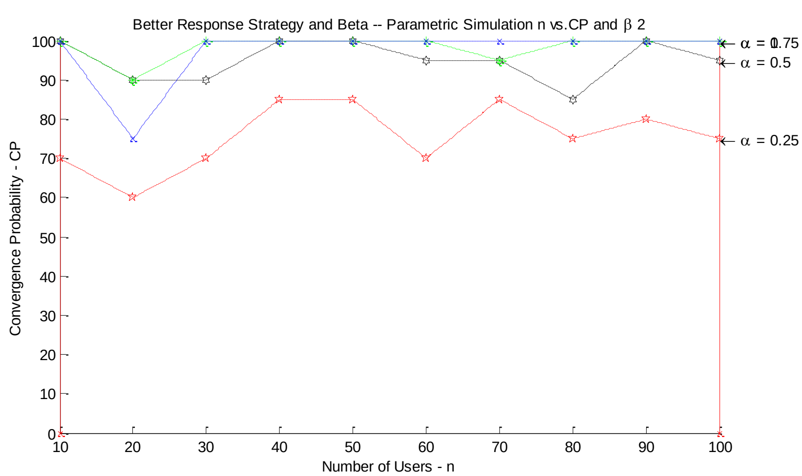


Figure 7.15b Better Response 2 concurrent users scenario convergence probability results for 7 fixed APs

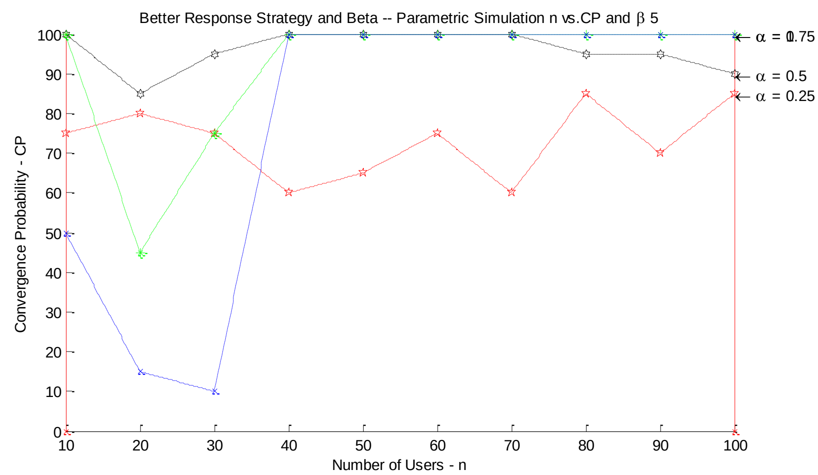


Figure 7.16a Better Response 5 concurrent users scenario convergence probability results for 7 fixed APs

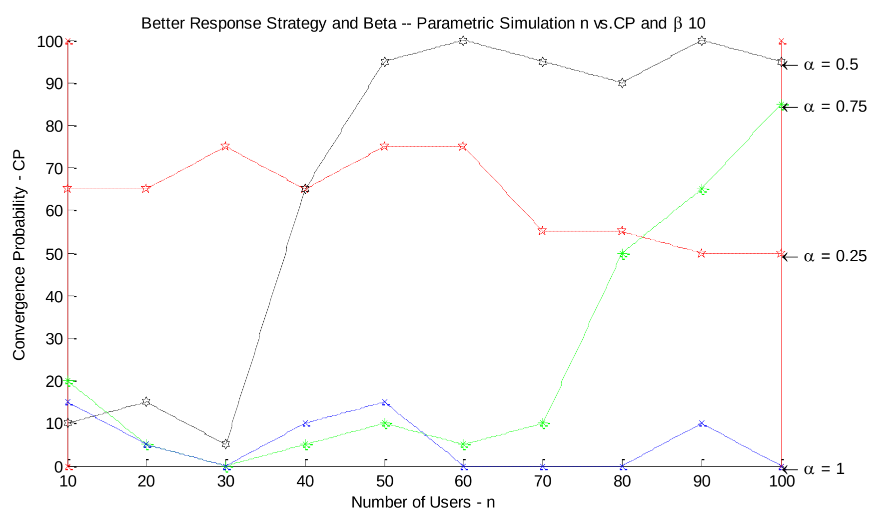


Figure 7.16b Better Response 10 concurrent users scenario convergence probability results for 7 fixed APs

Figures 7.13a and 7.13b are showing an apparent immunity to the change of the β values, the curves are apparently equivalent except for the fact of their starting and finishing points, $\beta = 2$ curve starts at 12 iterations, $\beta = 5$ begins with around 30 iterations and $\beta = 10$ initiates with approximately 40 iterations, then end values are kept accordingly to a constant value slope. Analyzing more in dept the *Better Response* algorithm results, it can be discarded the apparent immunity named on the previous paragraph as $\alpha = 1$ behaves poorly at figures 7.16 and 7.19 compared to any other β curves, however it should be noted that from $\beta = 2$ to $\beta = 5$ the number of points for $\alpha = 0.5$ through $\alpha = 1$ for which the convergence probability is equal to 100% is greater than any other studied case such as pure interference based and additive interference and achievable rate functions, additionally another fact that does not hold here is the fact of $\alpha = 0.25$ outperforming all other α curves as β increases.

Note from figure 7.18 how all α curves have a convergence probability greater than 80%, shows how robust is the multiplicative interference and achievable rate functions with respect being blind of the strategy space and having 5 consecutive players, the robustness over 80% is only kept for $\alpha = 0.5$ and $\alpha = 0.75$ at n = 100 players on figure 7.19, nevertheless still shows a great performance under the wild conditions the scenario is imposing.

Results for 150 fixed users, varying access points from 10 to 100 (AP step 10):

Best Response algorithm results

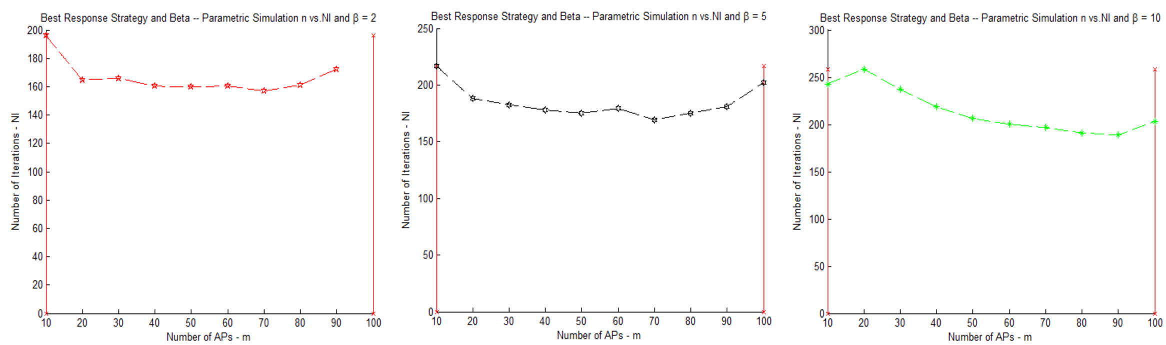


Figure 7.17a Best Response concurrent scenario results for 150 fixed users

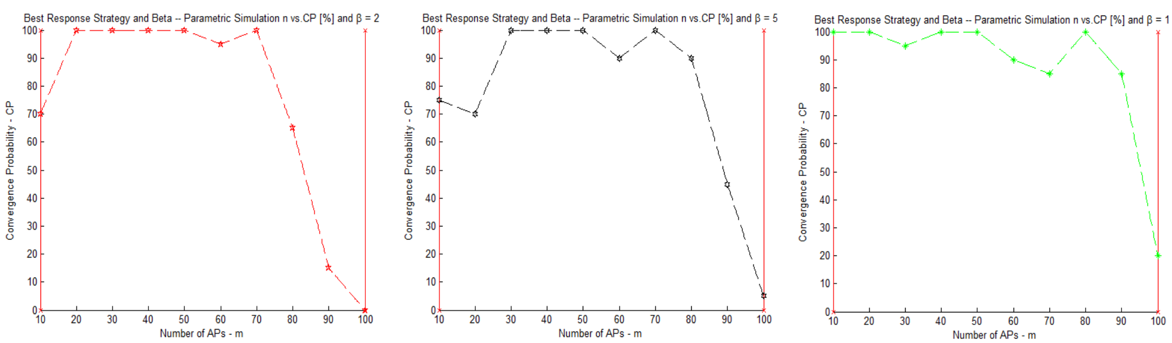


Figure 7.17b Best Response concurrent scenario convergence probability results for 150 fixed users

Better Response algorithm results

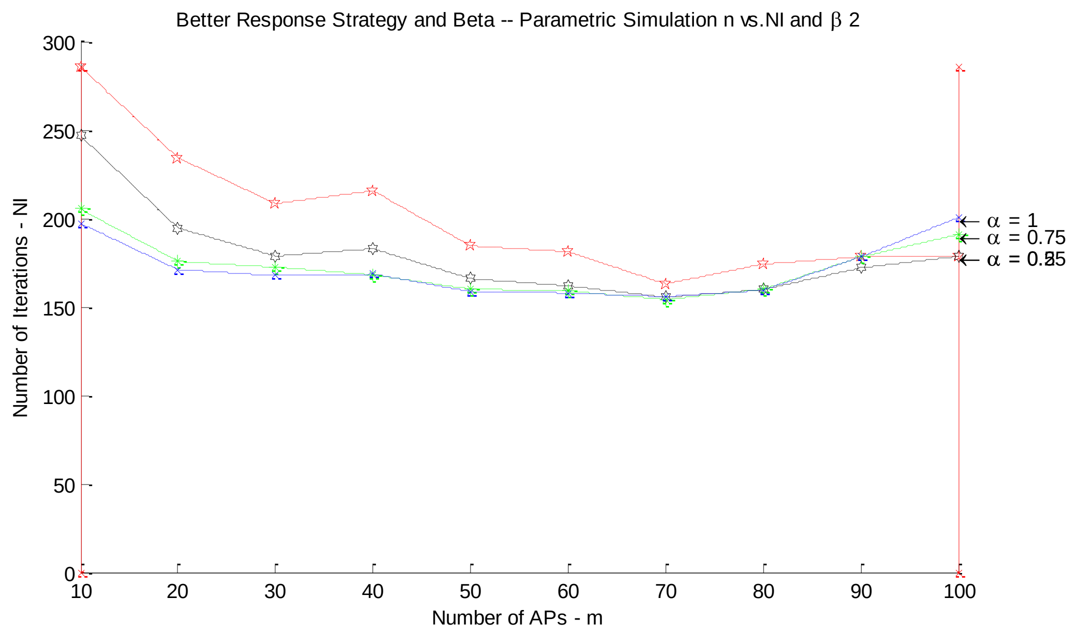


Figure 7.18a Better Response 2 concurrent users scenario results for 150 fixed users

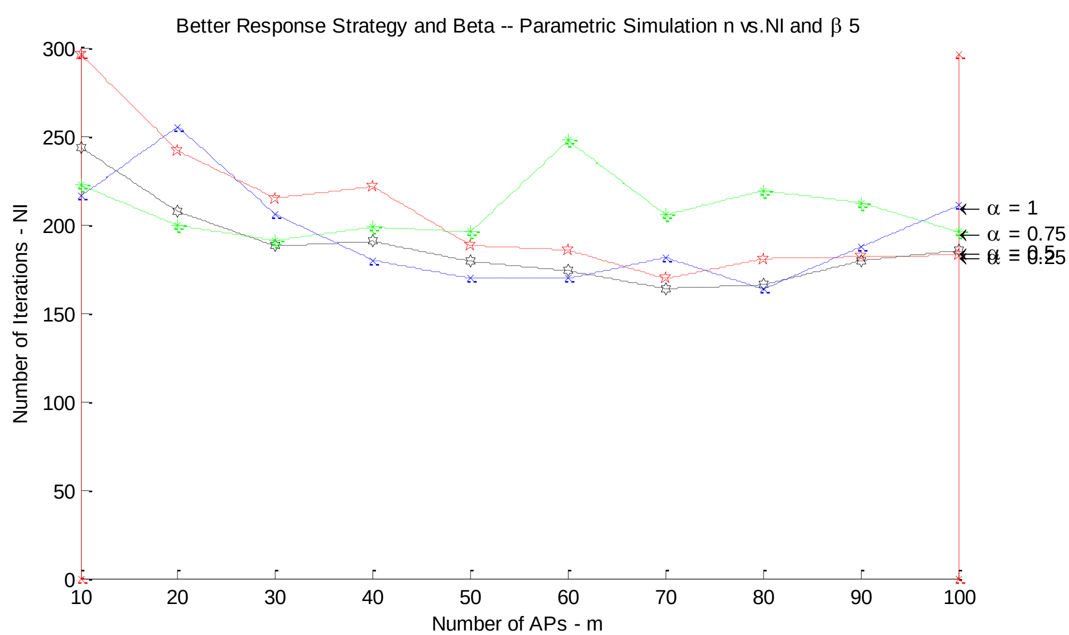


Figure 7.18b Better Response 5 concurrent users scenario results for 150 fixed users

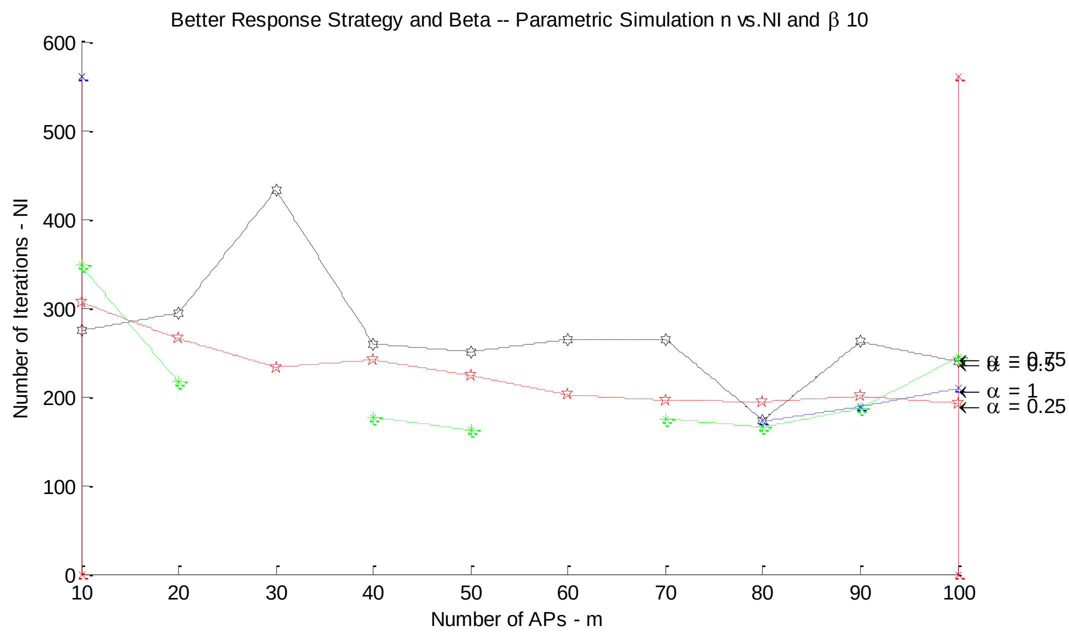


Figure 7.18c Better Response 10 concurrent users scenario results for 150 fixed users

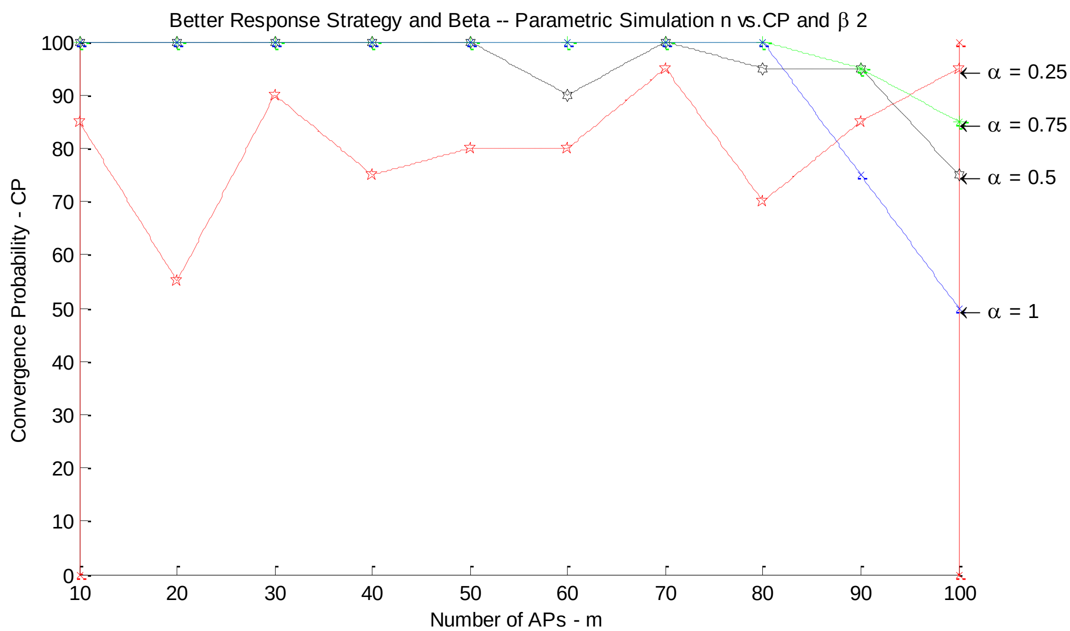


Figure 7.19a Better Response 2 concurrent users scenario convergence probability results for 150 fixed users

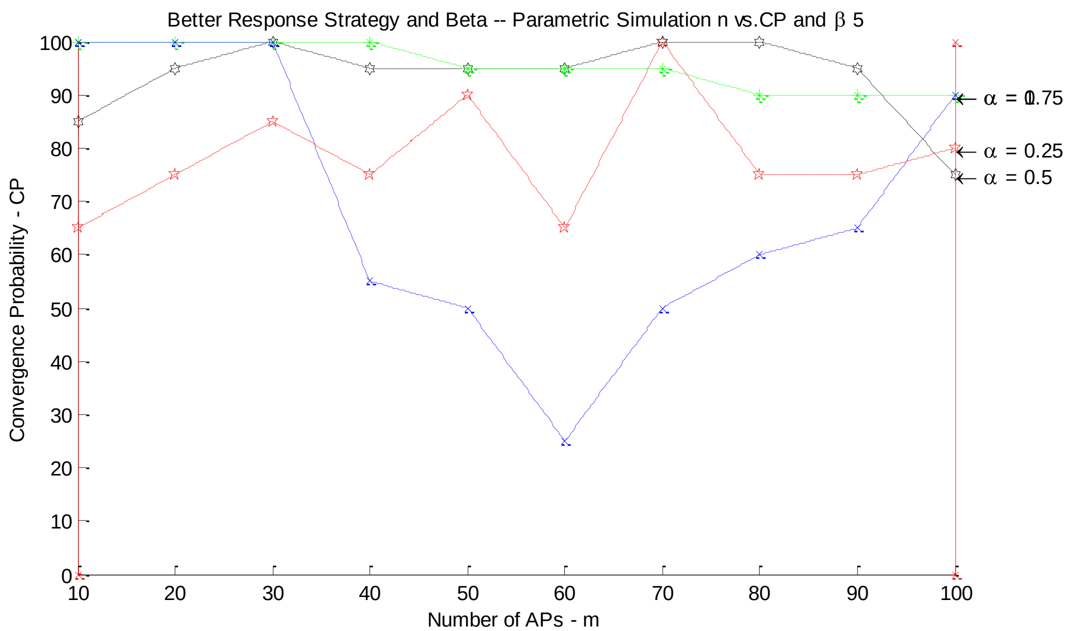


Figure 7.19b Better Response 5 concurrent users scenario convergence probability results for 150 fixed users

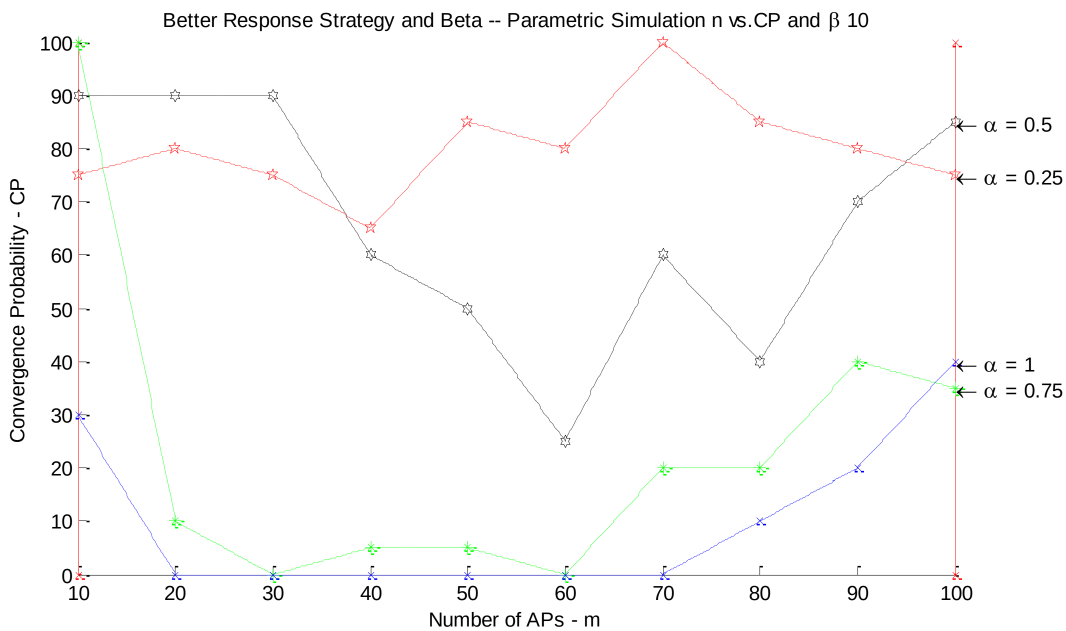


Figure 7.19c Better Response 10 concurrent users scenario convergence probability results for 150 fixed users

7.4

Access point coverage radius parametric analysis

Previously, the analysis made consisted on a fixed access point coverage radius that was modified to cover all users (*All in range*). Being able to analyze the performance analysis of the different games under different AP coverage radius is important because these situations can happen in real life [15,16,17] whenever an AP has a variable transmission power. These type of simulations can be seen as a sort of joint Network Selection problem, in the sense that an AP can increase or decrease its coverage area in order to selfishly maximize the number of users covered, however, the parameterization of the AP coverage area is not done with this policy, but just for practical studies without specifically caring for the AP wealth. The impacts of these disturbances will be studied on the following sub-Sections.

Not all in Range + Linear Grid topology

<u>Results for 50 users and 7 APs</u>:

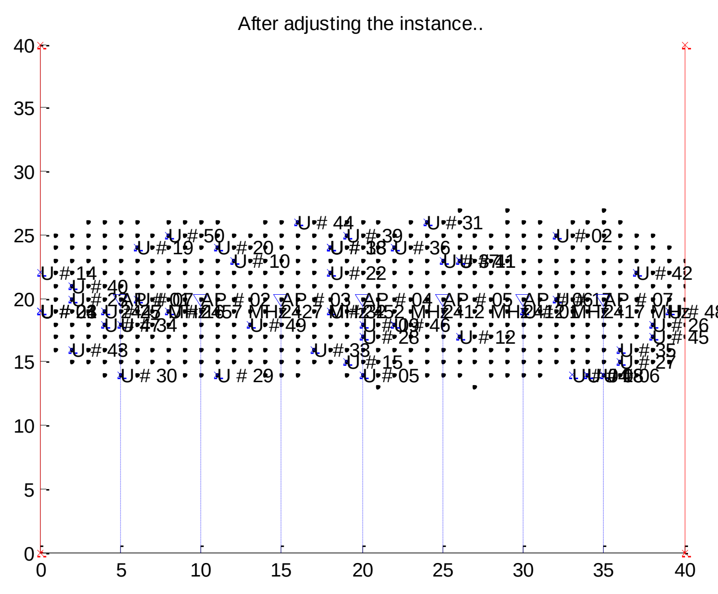


Figure 7.20 Linear grid map with AP Emitted Power = 5dBm

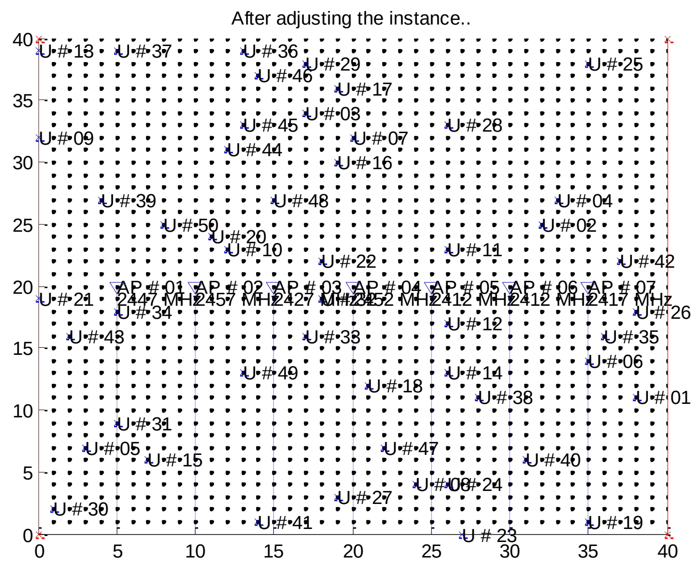


Figure 7.21 Linear grid map with AP Emitted Power = 20dBm

Best Response algorithm results

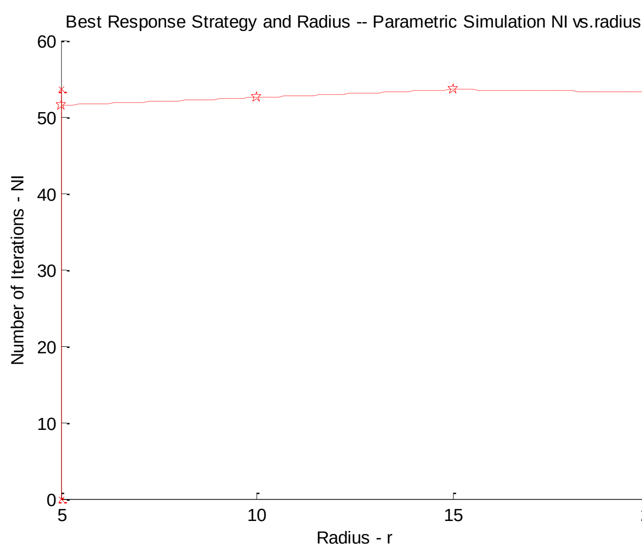


Figure 7.22 Best Response linear grid topology coverage radius results for 50 users and 7 APs

Better Response algorithm results

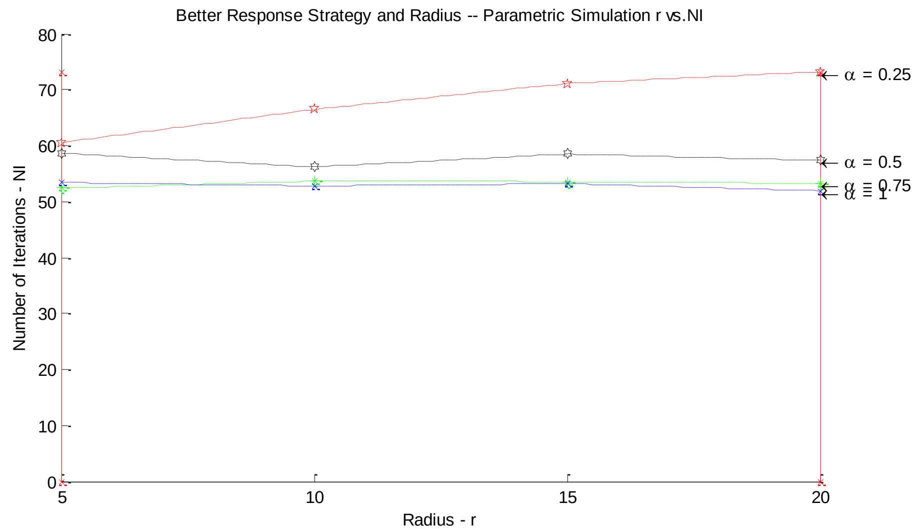


Figure 7.23 Better Response linear grid topology coverage radius results for 50 users and 7 APs

One thing that its noted from figure 7.22 with respect to a random topology scenario is the reduced number of total iterations, for the particular map topology, linear grid aids to find a faster convergence since the APs are more closely allocated yielding very similar utility function values, the maximum value obtained in figure 7.22 was less than 55 iterations compared to 60 iterations obtained with the random topology. Now figure 7.23 is showing the following particular behavior, first the initial point is not common for all α curves, and it should be noted how $\alpha = 0.25$ curve diverges from all other curves, particularly at r = 15, $\alpha = 0.25, 0.5$ & 0.75 curves show a negative value slope while $\alpha = 0.25$ exhibits a positive value one. It can be inferred as conclusion that being able to see 2 APs out of the 7 available (i.e $\alpha = 0.25$) for the linear grid topology while it guarantees convergence the number of iterations increase linearly.

<u>Results for 100 users and 7 APs</u>:

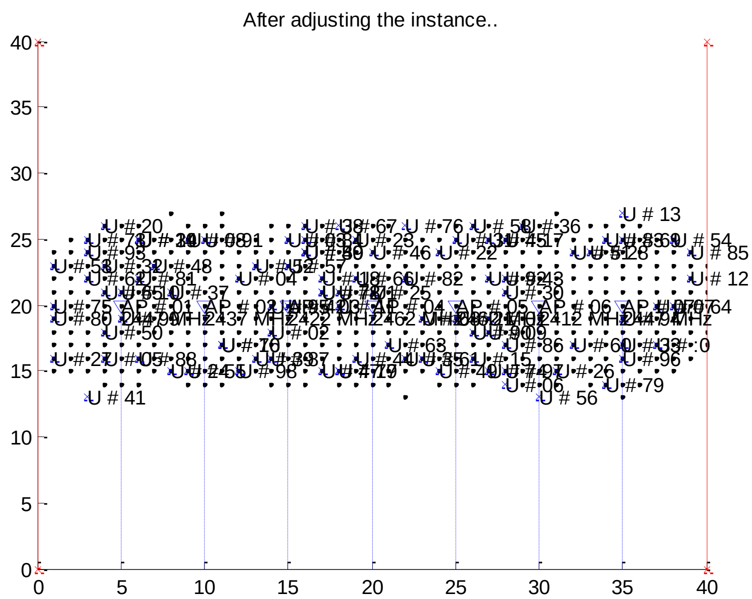


Figure 7.24 Linear grid map with AP Emitted Power = 5dBm

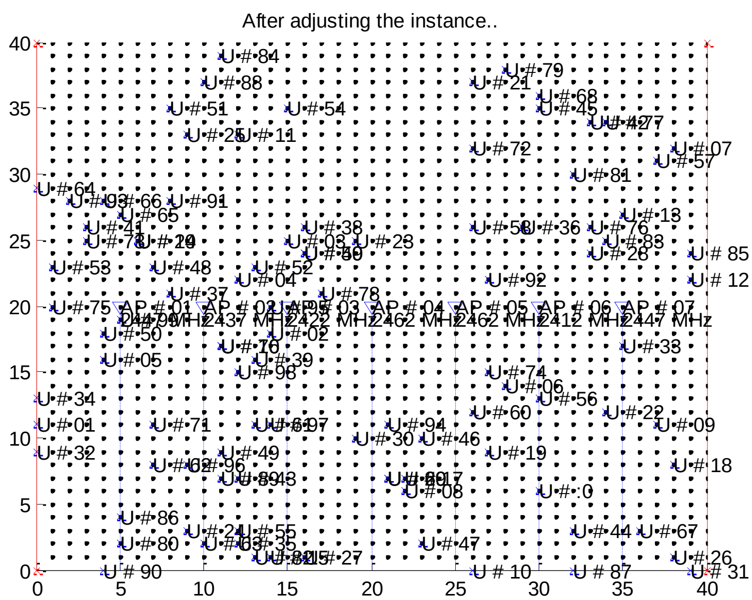


Figure 7.25 Linear grid map with AP Emitted Power = 20dBm

Best Response algorithm results

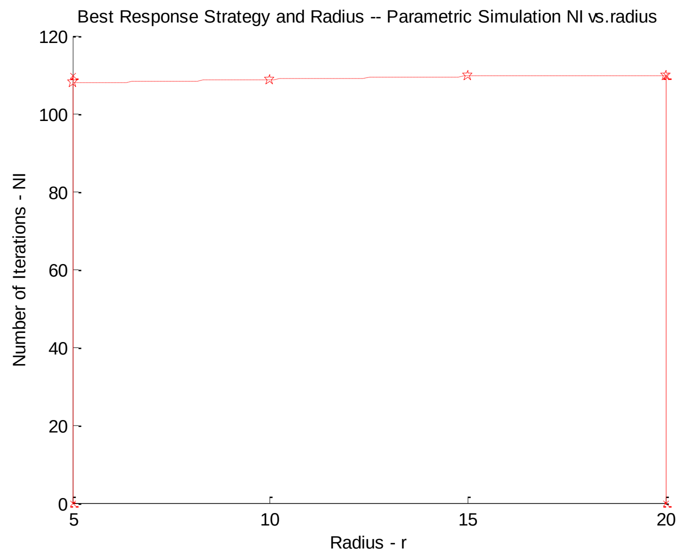


Figure 7.26 Best Response linear grid topology coverage radius results for 100 users and 7 APs

Better Response algorithm results

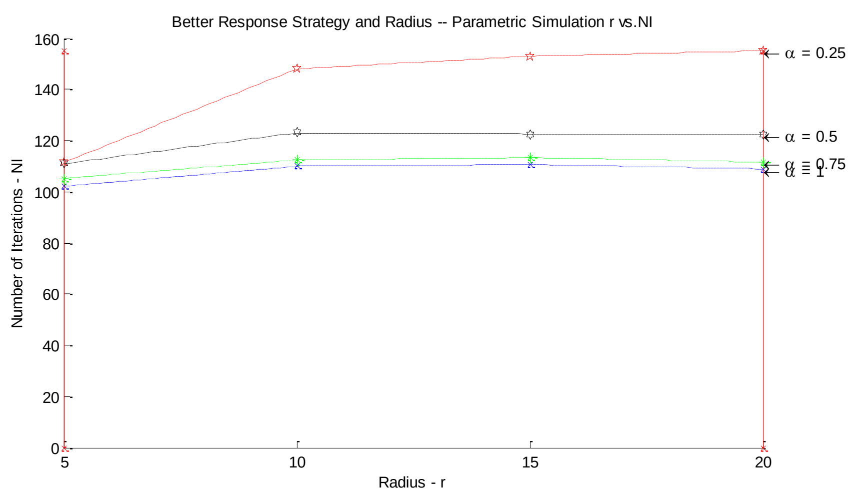


Figure 7.27 Better Response linear grid topology coverage radius results for 50 users and 7 APs

Yet again, the comparison between figures 7.26 and 7.28 is showing how the linear grid topology yields better results than randomly allocated users and APs, note on figure 7.26 how for all α values, the number of iterations is kept constant at around 110 iterations. Never before $\alpha = 0.75$ and $\alpha = 1$ were so equivalent as in figure 7.27, $\alpha = 0.75$ being short 2 APs compared to $\alpha = 1$ behaves in a margin between +5% compared to its non-blind counterpart, a great discovering if allowing users being 2 AP blind would yield computational savings, $\alpha = 0.5$ behavior should not be ignored as it is showing a performance with a +10% margin with respect to the non-blind curve whereas $\alpha = 0.25$ can be discarded as it is showing an underperformance greater than 50%.

***Not all in Range* + Rectangular Grid topology**

<u>Results for 50 users and 12 APs</u>:

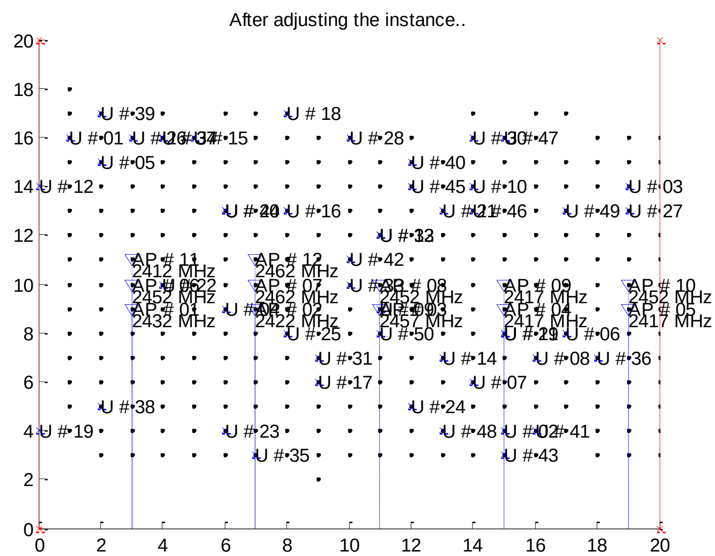


Figure 7.28 Rectangular grid map with AP Emitted Power = 5dBm

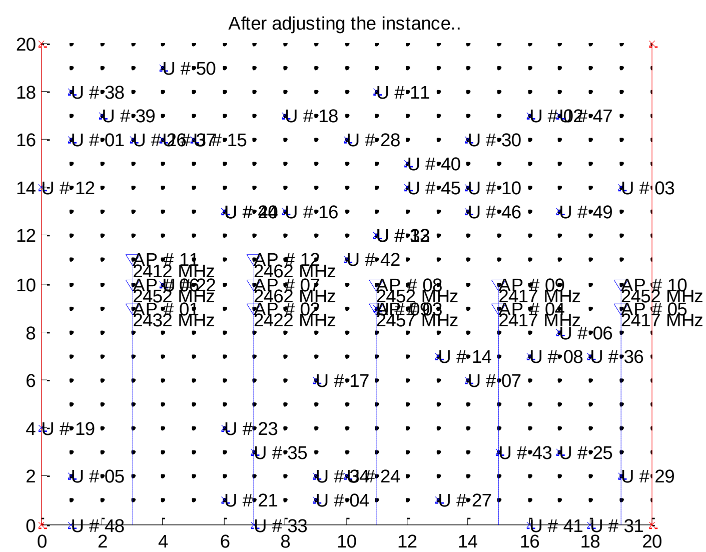


Figure 7.29 Rectangular grid map with AP Emitted Power = 20dBm

Best Response algorithm results

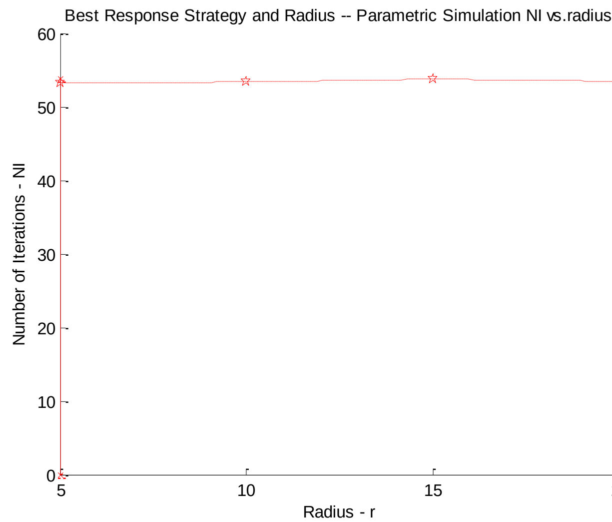


Figure 7.30 Best Response rectangular grid topology coverage radius results for 50 users and 7 APs

Better Response algorithm results

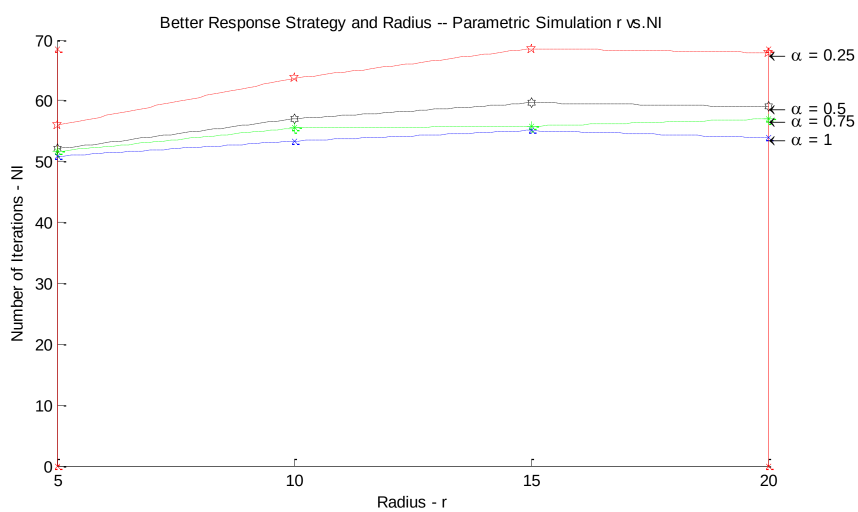


Figure 7.31 Better Response rectangular grid topology coverage radius results for 50 users and 7 APs

Figure 7.30 is showing an initial value (i.e at r = 5) greater than the one presented on figure 7.22, however is the one having the most constant behavior of all two, hence it would be the chosen topology to be deployed if the most important criteria was stability. *Better Response* algorithm results for the coverage radio analysis is showing a continuous increase on the number of iterations from r = 5 until r = 15 where the *All in range* condition is met, thereafter all α curves stabilize or slowly settle down towards the boundary equilibrium value, seen particularly with $\alpha = 0.25$ and $\alpha = 1$ curves, where after r = 15 they exhibit a negative slope value.

Accounting for the total number of iterations behavior, it can be concluded that either the linear grid and rectangular grid topologies had the same number of total iteration behavior, however it should be considered that the rectangular grid topology kept such condition even though the number of strategies was increased by 5, showing a peculiar robust behavior towards the appearance of additional wireless operators for instance on a realistic scenario.

<u>Results for 100 users and 12 APs</u>:

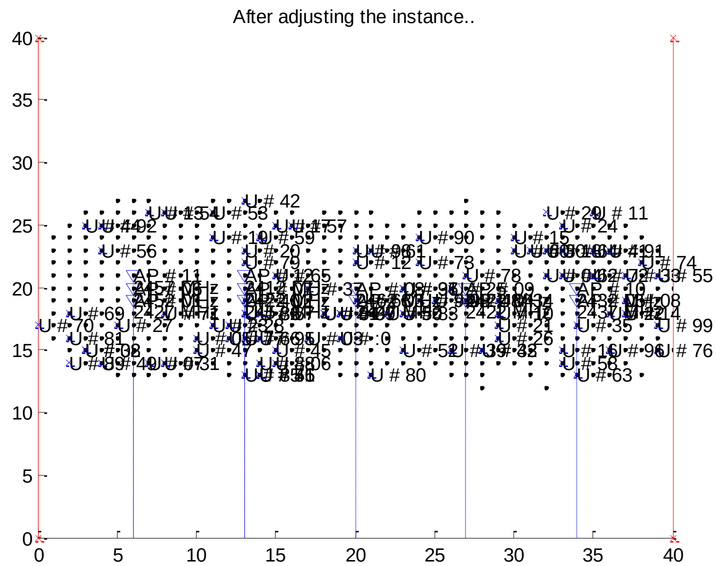


Figure 7.32 Rectangular grid map with AP Emitted Power = 5dBm

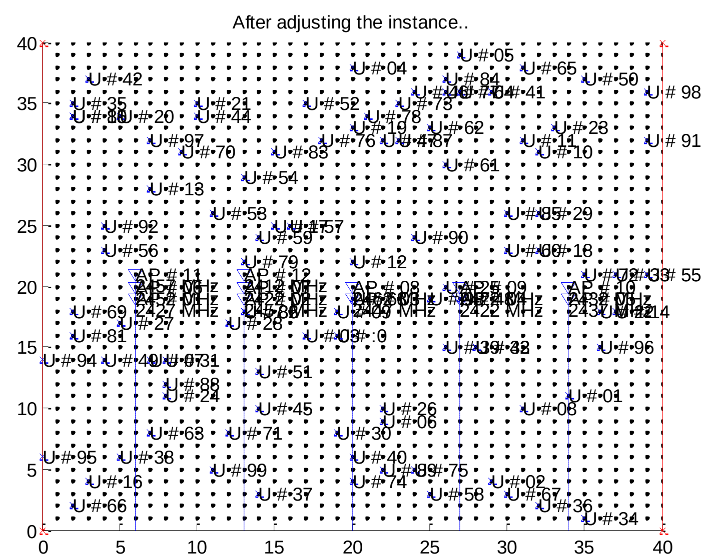


Figure 7.33 Rectangular grid map with AP Emitted Power = 20dBm

Best Response algorithm results

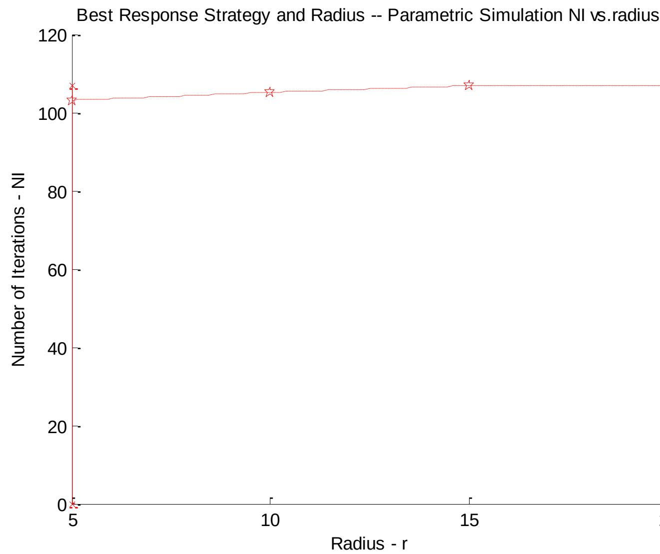


Figure 7.34 Best Response rectangular grid topology coverage radius results for 100 users and 12 APs

Better Response algorithm results

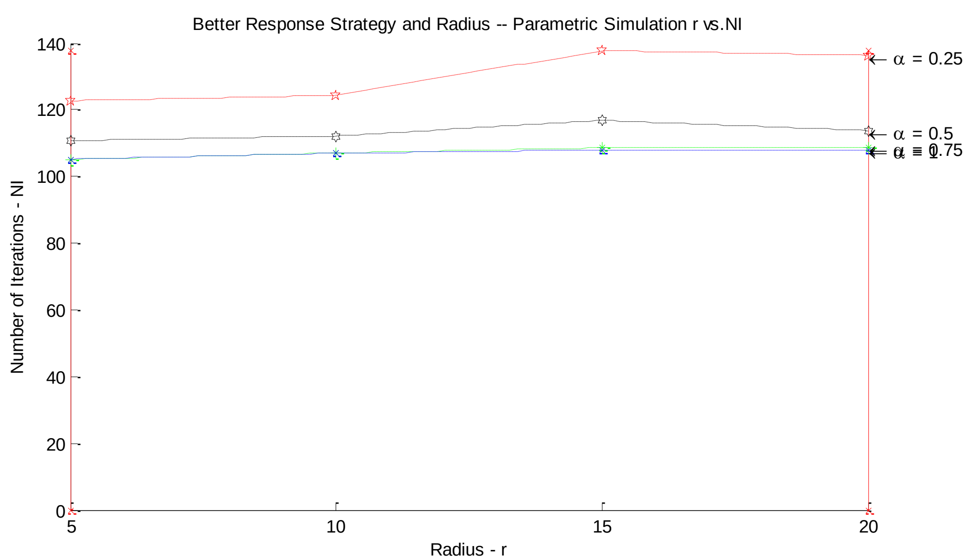


Figure 7.35 Better Response rectangular grid topology coverage radius results for 100 users and 12 APs

The *Best Response* radius analysis results are showing a curve with a slope slightly greater than 0 that settles up at r = 15, and which yielded a maximum number of iterations less than 110, hence with a ratio lower than 10% with respect to the baseline equilibrium (i.e 100 at Best Response, and Pure interference based utility functions), surprisingly figure 7.35 is showing the particularity of having a slope not from r = 5 as it was previously shown but from r = 10 for $\alpha = 0.25$ and $\alpha = 0.5$

curves, the particular behavior can be due to the fact that removing 3 and 6 APs respectively for $\alpha = 0.25$ & 0.5 might have broke up the rectangular original topology yielding something close to a random topology though being in a more constrained space (because rectangular grid APs are deployed close to each other), nevertheless and despite the particular behavior the rectangular grid topology is outperforming the random and linear grid topologies by far with respect to the number of required iterations to find an equilibrium, yielding it the most appropriate topology to be deployed in order to obtain, stable, robust and good performance results.

7.5

FICTITIOUS PLAY ALGORITHM

MYOPIC VS. NON-MYOPIC

Unlike *Best* and *Better Response* algorithms, *Fictitious Play* algorithms base their decisions on the knowledge learnt from previous iterations, rather than from a common knowledge base as the former algorithms did, hence a user playing *Fictitious Play* will have an individual knowledge from all the game that he will use to make the most appropriate decision (for more information on Fictitious Play algorithms go to Section 3.7).

Performance analysis will be done over 4 different flavors:

- Deterministic Fictitious Play
- Stochastic Fictitious Play
- Myopic Deterministic Fictitious Play
- Myopic Stochastic Fictitious Play

Fixed number of access points, varying number of users static analysis

- Varying number of randomly deployed users, starting from 5 users in steps of 5 users per iteration until 30, giving a total of 6 different games.
- Access point power transmission model modified in order to achieve an *All in Range* scenario.
- Map length $40m^2$

Fixed number of Access Points will be simulated for 3 and 7 APs

Legend:

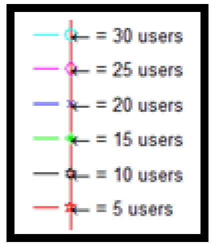


Figure 7.36 User curve legend

<u>Results for 3 APs</u>:

Deterministic FP results

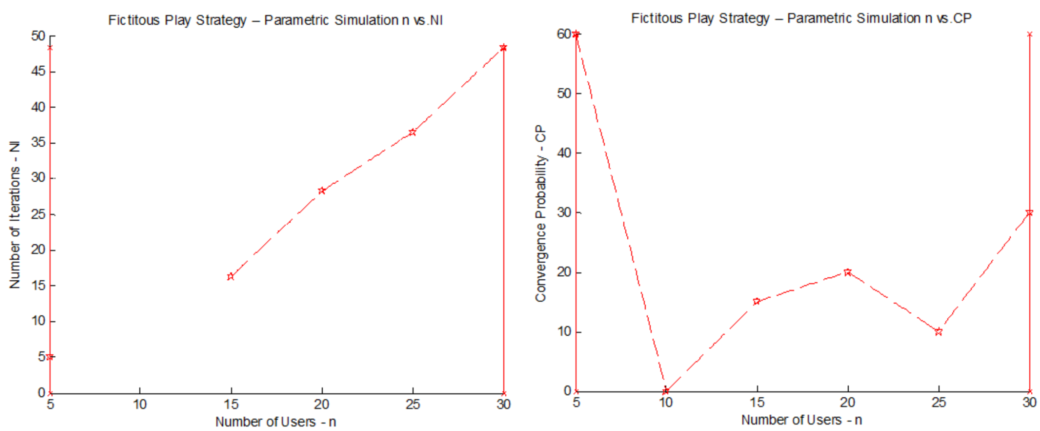


Figure 7.37 Deterministic Fictitious Play results for 3 fixed APs

Stochastic FP results

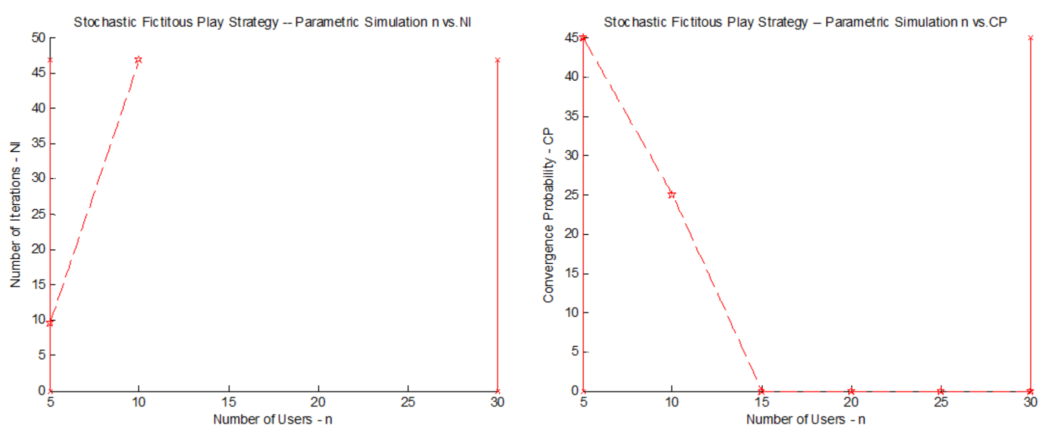


Figure 7.38 Stochastic Fictitious Play results for 3 fixed APs

For the first time, the deterministic *Fictitious Play* strategy results have a simulation point with a convergence probability equal to 0, as seen on figure 7.37 at n = 10, n = 5 and n = 15 show a 1 to 1 correspondence between number of users and iterations however from n = 15 and on, slope variations break up the correspondence. Similarly, the stochastic F.P results has been having a “degradation” from the pure interference based function going through the additive and achievable rate functions to finally end with only 2 points whose convergence probability is 45% and 25% for n = 5 and n = 10 respectively. A non-in-range scenario would show different results that would yield greater convergence probability and lower number of iterations due to the intrinsic nature of the cost function of each user.

Myopic deterministic FP results

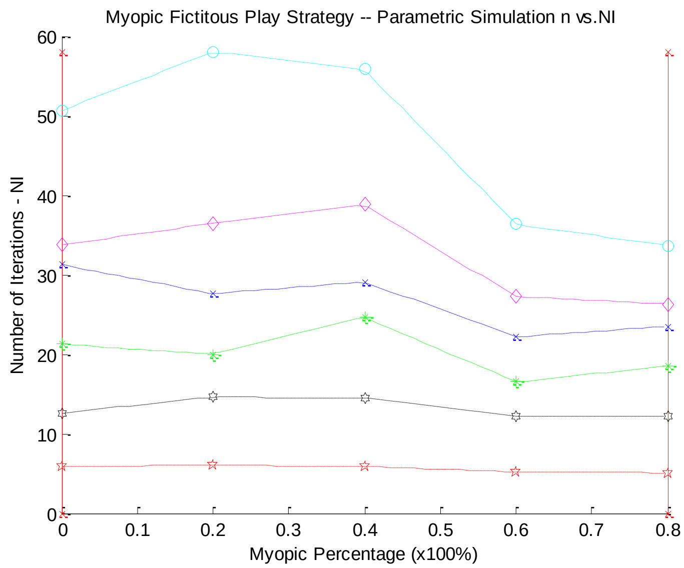


Figure 7.39 Myopic Deterministic Fictitious Play results for 3 fixed APs

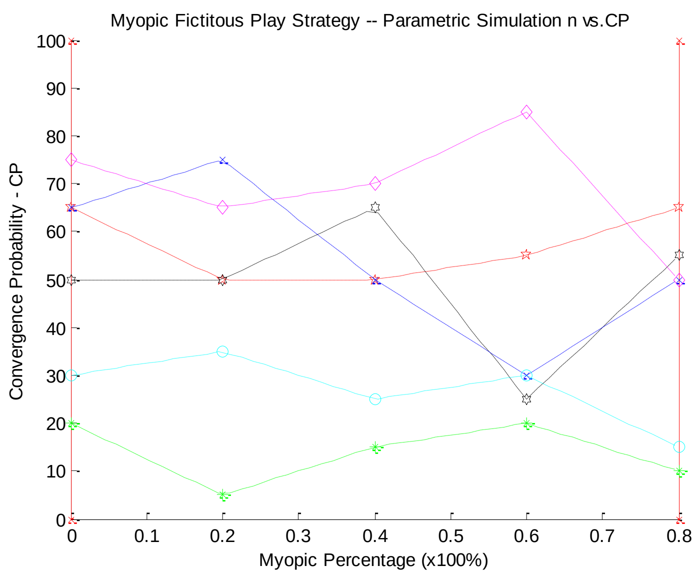


Figure 7.40 Myopic Deterministic Fictitious Play convergence probability results for 3 fixed APs

The settlement point that has been talked before on Sections 5.5 and 6.5 is no longer a complete settlement point for the curves presented on figure 7.39 at m.p = 60%, n = 5 and 10 users have a constant number of iterations from m.p = 60% and on, however the rest of the curves do not, some of them tend to increase afterwards (n = 15 and 20) and others tend to decrease (n = 25 and 30), these behaviors after m.p = 60% cannot be inversely mapped onto figure 7.40 in the sense of a curve increasing the number of iterations increases its convergence probability because figure 7.40 is presenting a casual irregular behavior for each singular curve.

As a side note, it is interesting to note how the n = 10, 20 and 25 user curve outperform n = 5 user curve as seen on figure 7.40 from m.p = 0% to m.p = 40% accounting for a convergence probability greater than 50%, from m.p = 40% and on n = 10 and 20 curves have a degradation behavior whilst n = 25 outperforms all the other simulation curves for the whole myopic range.

Myopic stochastic FP results

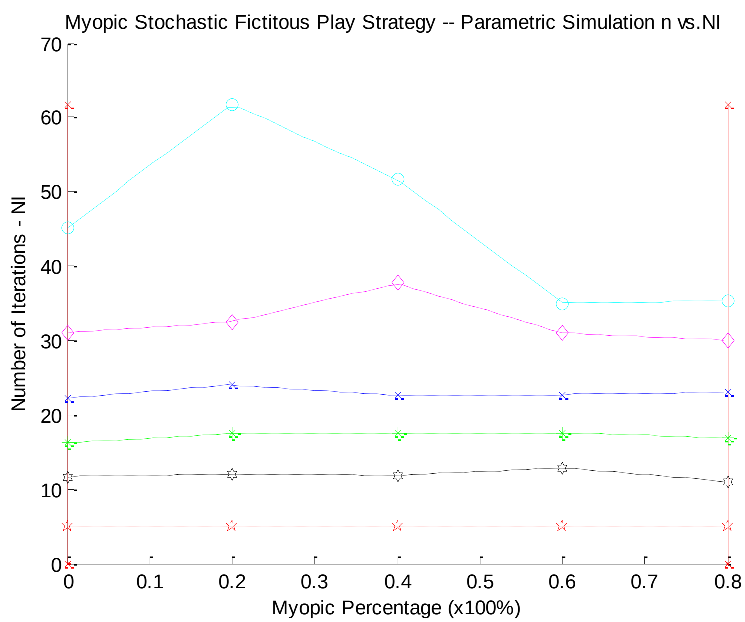


Figure 7.41 Myopic Stochastic Fictitious Play results for 3 fixed APs

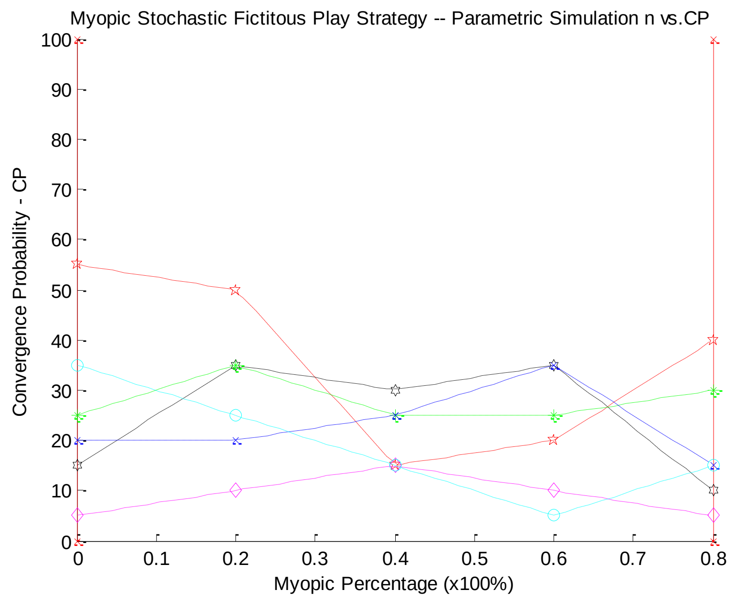


Figure 7.42 Myopic Stochastic Fictitious Play convergence probability results for 3 fixed APs

Great success of the multiplicative and achievable rate based functions under the particular scenario of 3 fixed access points and varying number of users, the comparison is direct with respect to figures 5.44 and 5.45 from Section 5 and figures 6.51 and 6.52 from Section 6, the success accounts for the fact the figures 7.49 and 7.50 shown above have a limited initial value (m.p = 0%) and tend to settle at m.p = 60% in a value close to the starting point which reflects robustness with respect being myopic or not, despite the fact of having an average convergence probability lower than 50% for all curves, it does maintain a continuous convergence probability greater than 0% for all the simulations opposite as the counterparts on previous Sections

<u>Results for 7 APs</u>:

Deterministic FP results

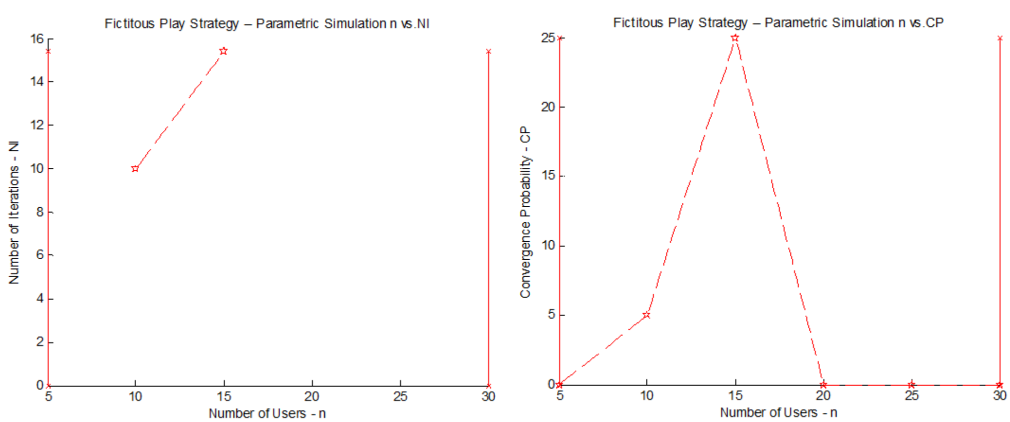


Figure 7.43 Deterministic Fictitious Play results for 7 fixed APs

Stochastic FP results

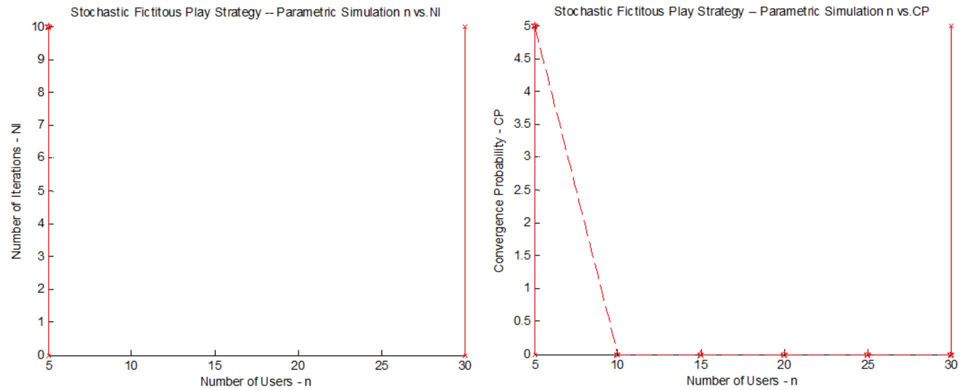


Figure 7.44 Stochastic Fictitious Play results for 7 fixed APs

Complete degradation of the results obtained on the previous Sections, the deterministic version of the learning technique only shows two points who found convergence 5% and 25% respectively for n = 10 and 15 users and a one and only one point having a poor convergence probability of 5% for n = 5 users. The addition of strategies with respect to the fixed 3 access point scenario resulted in a worsening of the performance as reflected by figures 7.43 and 7.44, the particular behavior might be due to different factors such as the fact of the access points given a particular allocation deliver similar achievable rates to each user, and hence the strategy selection ends up in endless loops or in local equilibria, or to the fact of each access point delivering different but very convenient achievable rates to particular users in a way that they decide to maintain selfishly their decisions not caring for the global equilibrium of the system.

Myopic deterministic FP results

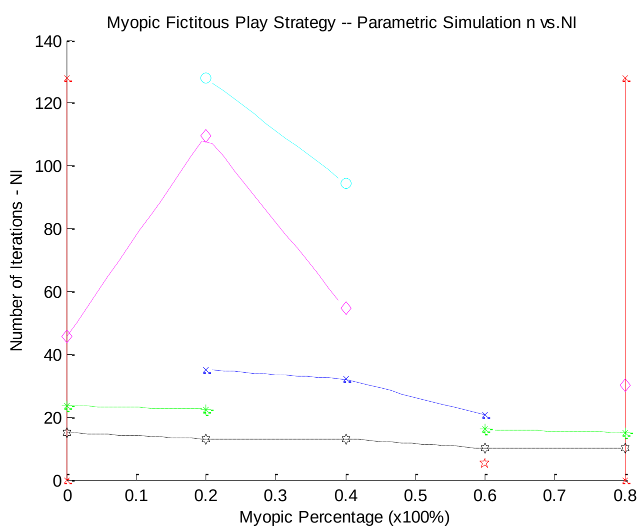


Figure 7.45 Myopic Deterministic Fictitious Play results for 7 fixed APs

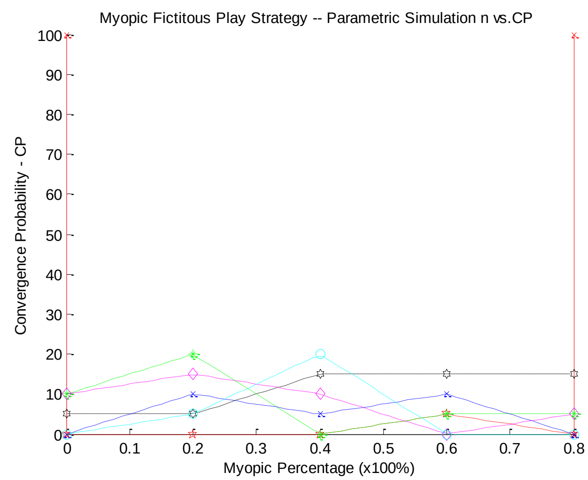


Figure 7.46 Myopic Deterministic Fictitious Play convergence probability results for 7 fixed APs

The addition of 4 access points with respect to the previous scenario whose effects were accounted as a deterioration are mapped directly onto the myopic deterministic *Fictitious play* algorithm results. The comparison is direct between figures 7.39 and 7.40 with figures 7.45 and 7.46, the continual but valid number of iteration curves map onto curves who present isolated and irregular points whose convergence is 0%. Recall from the non-myopic curves how the n = 5, 10 and 15 curves presented somehow a positive convergence probability, happens to be the same curves on figure 7.46 the ones that posses a C.P greater than 0% with the addition of random points from other curves (n = 20, 25 and 30 users). For the deterministic flavors of the implemented *Fictitious Play* algorithm it can be concluded that for a varying number of users under a multiplicative and achievable rate utility function the addition of strategies causes a degradation of the performance.

Myopic stochastic FP results

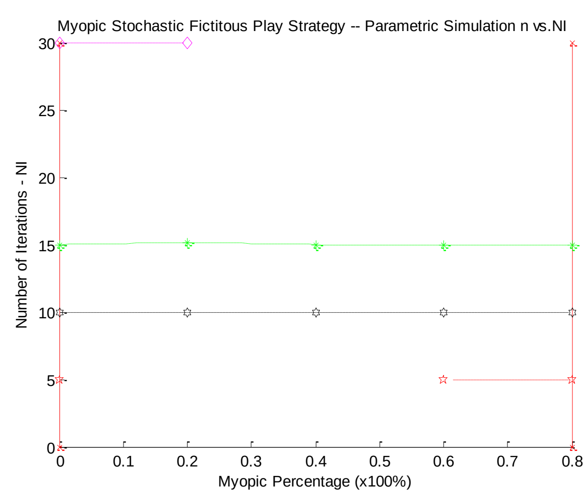


Figure 7.47 Myopic Stochastic Fictitious Play convergence probability results for 7 fixed APs

The stochastic way of choosing strategies from the previously collected knowledge base actually improved the performance number of iterations wise and convergence probability wise for the curves that existed on the non-myopic version of the algorithm and present on the deterministic but myopic F.P algorithm results (i.e curves n = 5, 10 and 15 users, red, black and green curves respectively) as seen at a first glance from figure 7.47 the constant behavior was never that stable for curves n = 10 and 15 users; the improvement on the convergence probability is directly seen by the fact that all the curves on the myopic deterministic F.P output figure 7.46 never are greater than a 20% convergence probability while on figure 7.47 n = 15 users curve is clearly showing a superior C.P

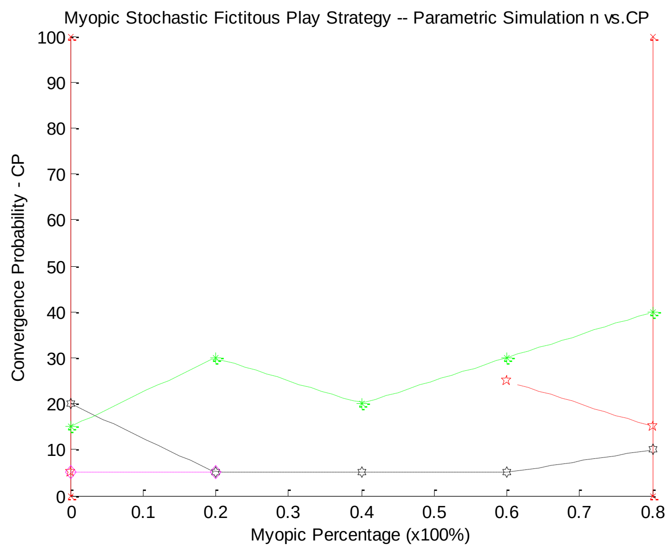


Figure 7.48 Myopic Stochastic Fictitious Play convergence probability results for 7 fixed APs

Fixed number of users and varying number of access points scenario outputs conclude with the fact of the multiplicative and achievable interference based utility functions outperforming all other utility function versions but only under the condition of a limited number of strategies, as access points are eventually added to the space of strategies the multiplicative utility function has a high grade of degradation performance wise when compared to the additive and pure interference based function, both having a greater robustness with respect to the addition of access points to the strategy pool. However the multiplicative version of the utility function under a limited number of access points and varying users also shown a particularity that no other version of payoff function shown and it's the fact of being robust with respect to the myopic percentage as seen on figures 7.41 and 7.47, the previous result is of great interest under practical situations because being able to perform equally well being 0 % myopic to 80% myopic results in a lower computational complexity of the algorithm.

Fixed number of users, varying number of access points static analysis

- Varying number of randomly deployed access points, starting from 1 AP in steps of 1 AP per iteration until 7, giving a total of 7 different games.
- Access point power transmission model modified in order to achieve an *All in Range* scenario.
- Map length $40m^2$

Fixed number of Users will be 30

Deterministic FP results

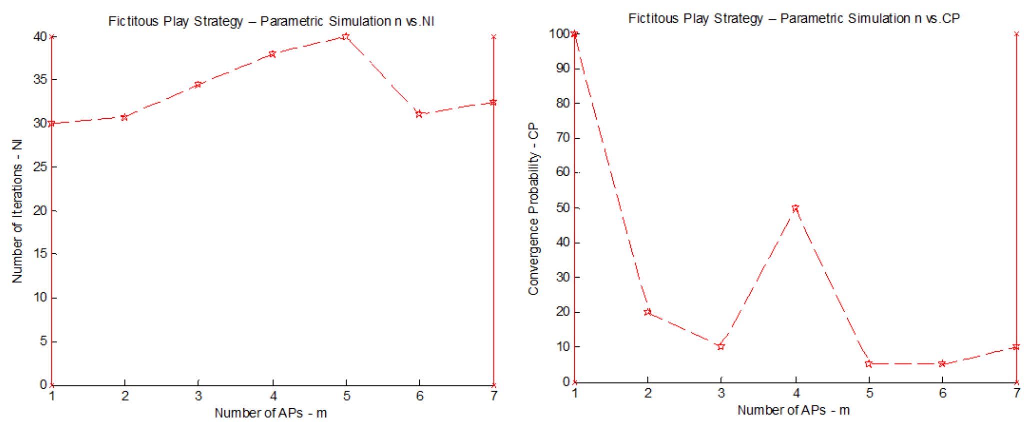


Figure 7.49 Deterministic Fictitious Play results for 30 fixed users

Stochastic FP results

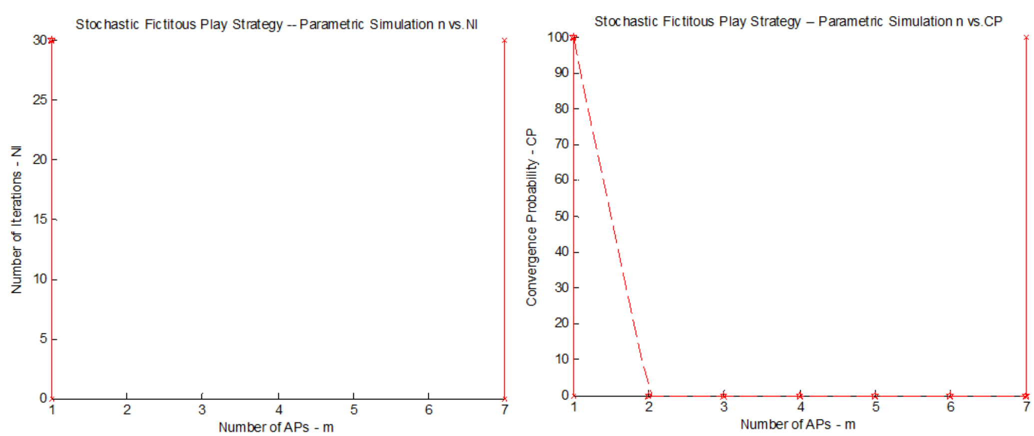


Figure 7.50 Stochastic Fictitious Play results for 30 fixed users

The behavior exposed by figures 7.49 and 7.50 can be crosschecked with the particular points of figures 7.37 and 7.38, observe that for figure 7.37 at n = 30 and having 3 access points results to have a convergence probability greater than 0%, the result for figure 7.49 is still greater than 0% for m = 3APs, and is in fact greater than 0 for all m values, however the number of iterations on the leftmost graph of figure 7.49 are maintained within a 30% ratio as observed with the payoff of having a very low convergence probability.

On figure 7.38 n = 30 users does has a convergence probability equal to 0% as well as it does on figure 7.44 with 7 access points the convergence probability results to be null. The results exposed on figure 7.50 are showing convergence only for the trivial case of 30 fixed users with only one strategy.

Myopic deterministic FP results

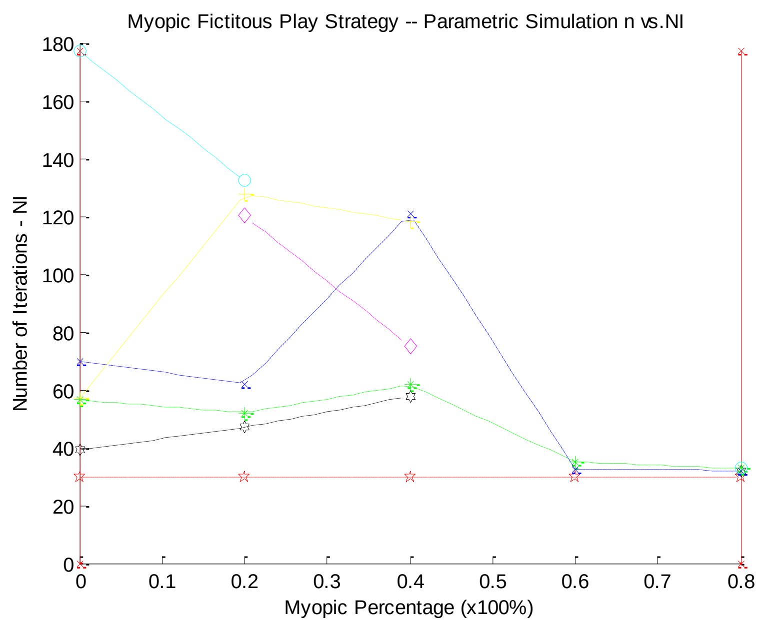


Figure 7.51 Myopic Deterministic Fictitious Play results for 30 fixed users

The non-myopic version of the *Fictitious Play* algorithm which showed on figure 7.49 convergence for all simulated points (m = 1 through m = 7 access points) is showing the same property except for few values on the m = 2, 6 and 7 APs, the baseline reference of the graphs is the red dashed line corresponding to the trivial case. The only curves that have a convergence probability and hence a valid number of iterations for the whole myopic range are m = 1, 3 and 4 APs, however the last 2 curves have an average convergence probability below 20%. Despite the poor performance named before, observe again the robustness of the multiplicative and achievable rate based utility functions with respect the myopic percentage with respect figures 7.62 and 7.71 from Section 6 where at 0% of myopic percentage not all curves had an existing initial point and where figure 7.52 has the larger number of curves finishing at m.p = 80% with a convergence probability greater than 0%

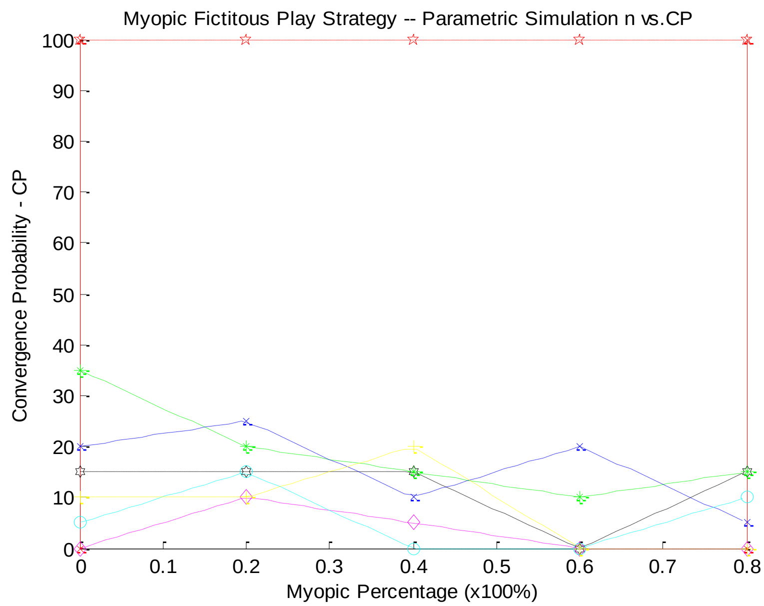


Figure 7.52 Myopic Deterministic Fictitious Play convergence probability results for 30 fixed users

Myopic stochastic FP results

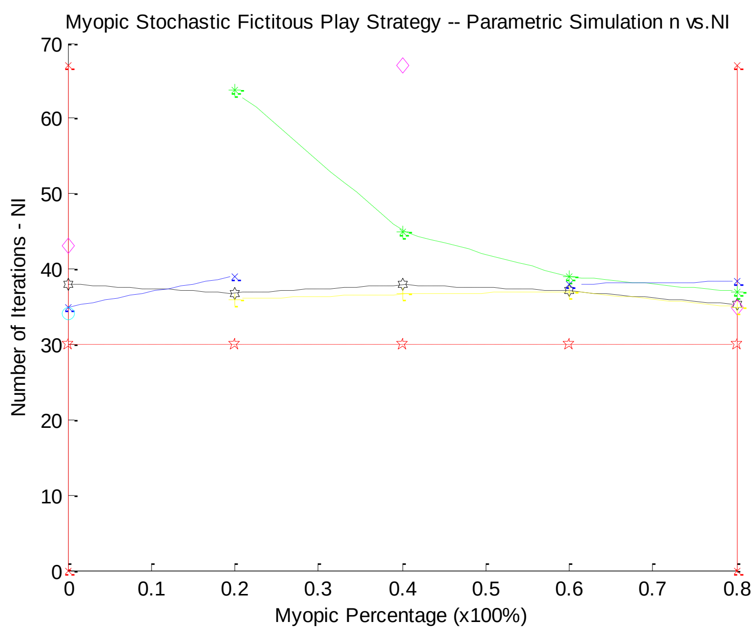


Figure 7.53 Myopic Stochastic Fictitious Play results for 30 fixed users

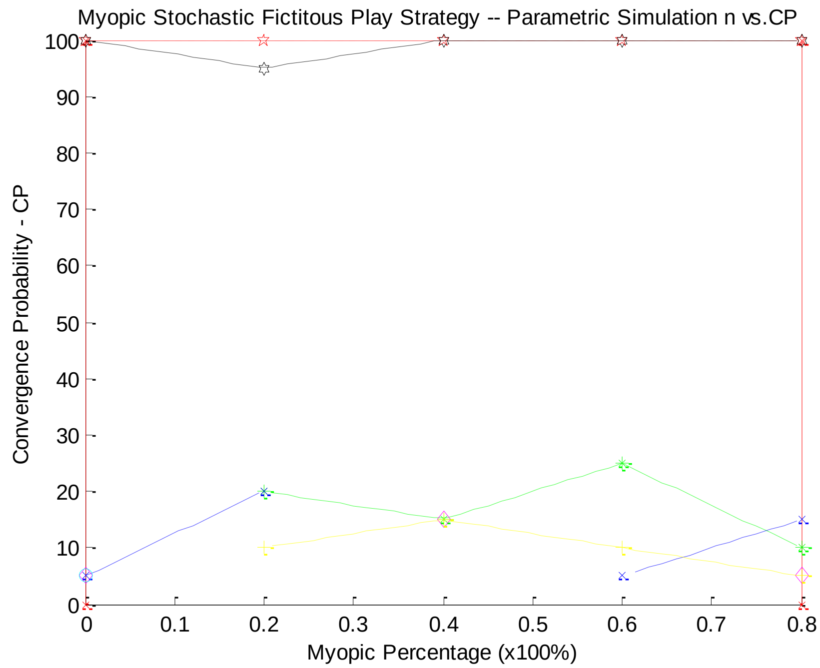


Figure 7.54 Myopic Stochastic Fictitious Play convergence probability results for 30 fixed users

The myopic stochastic outputs are showing interesting results, starting off from figure 7.53 where m = 2 access points curve is having a constant number of iterations throughout the whole myopic range, additionally the curve with the largest number of strategies (i.e the yellow curve = 7 APs) is behaving equivalently as the black dashed curve having 2 access points for the points whose convergence probability is greater than 0% (20%-80% of m-p) with a tendency to improvement after the aforementioned settlement point m.p = 60%.

On figure 7.54 observe how close the convergence probability behavior of m = 2 APs is almost equivalent to the trivial case outperforming the pure interference and additive and rate based functions, additionally for m = 3, 4 and 7 access points throughout their valid points, they have a better performance number of iterations wise and convergence probability wise with respect to the additive and interference based utility functions and outperform exclusively the performance number of iterations wise with respect to pure interference based utility function. Once more the previously shown results exalt the great performance of the multiplicative and achievable rate functions under a myopic fictitious play strategy.

8. CONCLUSIONS

The aim of this thesis was to perform different network selection scenario simulations in order to either cross check well known results or to discover through different experiments possible new results. The previous objective couldn't be achieved without a proper understanding and application of game theory backgrounds into the implemented algorithms. Throughout the process different tunings to the algorithms were needed in order to have sufficiently good performing results when compared to previous literature results. The end product results to be as planned, a two-fold contribution to the network selection literature: it offers an instance generator, by creating a set of users and radio stations, allocated in a customizable way onto a geographical area that can be customized as well, additionally *Network Selection Games: Simulation Tool Development and Performance Evaluation* offers the possibility to perform simulations on user made scenarios under different non-cooperative game modeled algorithms, yielding a very flexible yet good performing tool for simulating network selection problems.

The following are some practical conclusions synthesized from Sections 5, 6 and 7:

Pure interference based association policy

- Better response algorithm is showing a discrepancy for all its blind percentages with respect to Best response algorithm of a maximum of 3%, with a decreasing percentage as the blind portion of the algorithm decreases. When the Better response algorithm is not blind, it matches the performance of the Best response algorithm.
- Perturbation scenarios exposed the fact of best and worst case scenarios on both fixed number of user and fixed number of access point simulations. The number of additional simulations required when *x* users are removed is not greater than x, and the number of additional simulations when *y* access points are removed is in no case greater than the number of users the deleted access points were covering.
- Concurrent player scenarios showed a decreasing performance number of iterations wise and convergence probability wise as the number of concurrent players increases. Additionally the blind versions of the Better Response algorithms tend to outperform the Best response algorithms for a high number of concurrent players.
- In the concurrent player scenarios, there are particular peak values allocated in particular points where the ratio between the number of users and access points results to be a multiple of the number of concurrent players that is being currently simulated.
- There are particular cases of the concurrent players scenario where the performance regarding number of iterations is inversely proportional with respect to the convergence probability, having as reference scenario of a direct proportionality the ideal case of a Best response algorithm output.
- Access point coverage radio analysis, yielded that as the coverage radio increases, the number of iterations decreases linearly or almost linearly tending to a reference value (i.e the number of users playing the game).
- Deterministic and stochastic versions of the myopic Fictitious play algorithms are showing a better performance at high myopic percentages, resembling the fact of an improved

learning as the learning history is filtered and by the fact that F.P algorithms tend to converge to a Nash Equilibrium. On the other hand and exclusively for the stochastic version of the F.P algorithm, low convergence and excessive number of iterations are found due to possible convergence to sub-optimal equilibria.

Additive interference and achievable rate based association policy

- A loss in the linear behavior for the Best response algorithms is found when compared to the Pure interference based policy.
- For the fixed number of users and varying number of strategies scenario, the difference between the blind percentages of the Better response algorithm with respect the number of iterations is coarser when compared to the pure interference based policy.
- Total simulation time increased considerably when compared to the pure interference based policy, particularly, when simulating for fixed number of access points and varying number of users, from 150 fixed users to 200 fixed users the simulation time increased in 200%.
- Noticeable degradation on the convergence probability of the concurrent player scenarios, as beta increases from 2 to 10 players, now the convergence probability approaches much faster 0% when compared to the pure interference policy. The previous result does not resemble an immunity of the player specific payoff with respect a change of the association policy.
- On the radio parametric analyses for random topologies, the behavior found in the former association policy is no longer held, and it makes sense with the fact of being covered by more access points, new conflicts that did not existed with previous users can rise up due to the fact of equivalent additive interference and achievable rate utility functions with new users.
- The previous fact does not hold for linear and rectangular grids, since the access points are allocated by the nature of the topology close to each other, hence, increasing the coverage radio is immune to the effect observed on the paragraph above.
- Stochastic non myopic versions of the F.P algorithms show for the different number of fixed access points non linearity in isolated points up to a tendency to exponential behavior, due to the new nature of the payoff of each user.

Multiplicative interference and achievable rate based association policy

- Best response algorithms showed a convergence probability lower than 100% for all the points, being it the first time happening compared to the two other association policies.
- For the fixed number of users and varying number of access point simulations and inversely when compared to the former association policies, the number of iterations and convergence probability tend to approach the baseline reference values due to the fact of an increasing number of users populating the same geographic area, therefore each user would necessarily associate to the closest access point aiding the association process to all the other users. The behavior as the population grows tends to be welfare aware given the specific association policy studied.
- Convergence probability analyses after disturbances were made and they yielded different results depending on the particular disturbances made as doing so can drive the scenario to a best or worst case disturbance. In other words, certain disturbances made the convergence

probability go to 100%, however on different cases the convergence probability went as low as 20% when the original scenario C.P was greater than 20%. The former fact is aided due to the fact that the removed users or access points have particular coordinates and coverage areas affecting different amount of users.

- A settlement point to baseline reference value that held for the previous association policies at M.P = 60% is no longer held for the multiplicative interference and achievable rate association policy, from this point and on for this policy, the behavior is irregular for each of the particular curves.
- Great outperformance of the multiplicative fashion policy with respect to the former policies, using the myopic deterministic and stochastic versions of the F.P algorithms, having a considerable less number of global iterations and a more constant behavior convergence probability wise for all myopic percentages, however the former holds for a reduced amount of strategies, as the number of strategies increment, the behavior observed drastically deteriorates.

The Network Selection Games: Simulation Tool Development and Performance Evaluation can be easily extended to address other network selection scenarios. Indeed, the algorithms and structure of the whole program are modularly divided so to allow expansions and editing. For instance, the power propagation model used can be changed to a more realistic or even to a more ideal model easily by changing just one line of code. The condition for convergence is also changed easily by modifying one variable value, similarly many other features are easily modified rendering *Network Selection Games: Simulation Tool Development and Performance Evaluation* are very flexible yet robust tool.

Network Selection Games: Simulation Tool Development and Performance Evaluation has demonstrated through different simulation outputs its correctness with respect to previous results obtained in literature, hence, rendering it a correct tool for performing performance evaluation in novel network selection configurations.

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