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OPTIMIZATION OF INSPECTION STATION ALLOCATION IN SERIAL MANUFACTURING LINES

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Abstract

The aim of this thesis is to optimally allocate a limited number of inspection stations in the serial manufacturing systems with the objective of maximizing system effective throughput.

Relation among quality control and production logistics, that is usually neglected in the literature, has been considered in this work. The single machine and entire system has been modeled through proper Markov chains and system performance measures, including total throughput, effective throughput and system yield, are provided by solving the system model.

Based on the solution of the model, a new algorithm is developed to properly allocate and assign inspection stations to different machines.

After automating the algorithm by using Matlab, it is evaluated in different cases and the result shows the high accuracy of the method in finding the optimal solution with substantially lower computational effort. Moreover different experiments are designed to analyze the effect of inspection allocations on system performance. The results of the experiments, confirm the importance of proper allocation and assignment of inspection stations in terms of reaching higher system throughput as well as reducing necessary money investments.

Finally conclusions are given and potential topics that are open for future researches have been suggested.

Table of Contents

Chapter One

1 Introduction

Considering the intensity of ever increasing competitiveness in satisfying market needs, quality of the products along with productivity of production systems, play an important role in design and reconfiguration of manufacturing systems.

In the recent years, valuable technological achievements have facilitated better management of both production and quality control of production systems. Advances in sensors technology, allows to rapid inspection of several quality aspects of the product. On the other hand, many applications are now available to gather information regarding the machine state and production systems, in order to help production managers to take timely and accurate decisions.

However traditionally in manufacturing systems, quality and production control have been considered separately, recently performed research works, as well as industrial practices, confirm that they are mutually related and isolated analysis of them may lead to globally sub-optimal design decisions.

In fact, production system architecture not only affects production logistics of the system but also can greatly influence the performance of quality control system. For instance, dual effect of buffers on quality and productivity performance of the manufacturing systems can be referred. However presence of buffers has positive effect on production rate of a system, they create delay on quality information feedback and therefore on quality problem identification .As a result, a tradeoff among the positive and negative effect of buffers needed to be consider through jointly considering the impact of the buffer design on quality and production logistics.

Preliminary results confirm the high benefits that can be achieved by considering the correlation between quality and productivity at the system level. For instance, it was shown by Prof. Gershwin that for a production line composed of 15 Machines, number of bad parts produced can be reduced by 15% through proper allocation of the same number of inspection stations.

A few research works have been conducted in this area with the aim of developing analytical models to be used in design and reconfiguration of manufacturing systems, jointly considering productivity and quality control. So far these efforts have led to generation of a few mathematical models for performance evaluation of production systems considering quality/productivity issues and the effect of system architecture and quality control system on system productivity.

Due to the novelty of this research area, many aspects of this problem have not been addressed so far. And what has been done is not enough to tackle real problems that companies deal with. However in industrial practices, there is a substantial need to connect these correlated facets.

In this thesis, a mathematical model for a serial machine line has been introduced considering both quality and production logistics, system performance has been evaluated and accordingly new algorithm to optimally allocate and assign a limited number of inspection stations to the machines, with the aim of maximizing the effective throughput of the system is developed.

The thesis is structured as follows: in the Chapter 2 the literature review of jointly analysis of quality and production logistics is reported.

In Chapter 3, problem assumption is described. In chapter 4, single machine is modeled for both locally and remotely monitored cases. System performance measures are introduced and effect of changing inspection location on them is evaluated. Chapter 5 is dedicated to modeling the un-buffered serial line and calculating the performance measures of the system.

In Chapter 6, an iterative algorithm is proposed to find the optimal allocation and assignment inspection stations in order to maximize the effective throughput.

Chapter 7 is dedicated to numerical results. First proposed method is validated through comparing its results to those coming from extensive search. Afterwards a set of experiments are designed and their results are analyzed to better perception of system behavior changing different parameters.

Finally in chapter 8, conclusions and potential future research topics to be explored in the area of research are explained.

Chapter Two

2 Literature Review

Quality control and production logistics are two issues that attracted the great attention of researchers in the area of manufacturing systems and numerous research has been done in these two area , they were traditionally considered as two separate aspects and the link between them is usually neglected.

In the area of production logistics, research works mainly focused on performance measurement of the system such as system throughput, the time parts spend in the system and work in progress. The result has been development of different analytical tools and mathematical models to respond the industrial needs during the design and reconfiguration of manufacturing systems from buffer allocation to repair crew optimization.

In the area of quality control, research has been focused on control charts performance with the aim of developing different kind of control charts with related degree of responsiveness considering the real needs and inspection costs. Different parameters such as Average Run Length, Average Time to Signal and Average Quality Level (AQL) have been considered.

Recently many researchers have perceived that overlooking the relation between these two aspects can lead to take suboptimal decisions. In other words, however considering only one of the aspects can improve the system performance related to that side but it can deteriorate the whole performance of the system that can be affected by both aspects.

Contrary to the importance of integrated analysis of the quality control and production logistics, taking into account the novelty of this area, efficient quantitative tools and mathematical models have still not been developed for this purpose. For this reason, this thesis work aims at focusing on this topic in order to expand the relevant knowledge and providing quantitative methods to support the decision making process of production system designers.

In this chapter, the main contributions in the area of jointly analysis of quality and productivity and the literature of inspection station allocation is explained.

Interaction among quality and productivity introduced to the literature in the recent years. In fact only a few works have focused on jointly considering quality and production logistics on production system performance.

Effect of system design on quality aspects are reported in real cases in automotive sectors. In (Inman et al. 2003) effect of system design on quality performance is studied for several cases from General Motors Corporation

Moreover, [Shingo,1989] analyzed Toyota case, regarding the integration of quality and productivity.

In (Gershwin and Kim 2005), quality failures are considered in system modeling, that means machine can produce both good or bad parts when it is operational. Eventually, system performance as total and effective throughput is evaluated considering the quality control issues.

In (Colledani et al.2005,2008) and (Jang and Gershwin 2007) decomposition technique is applied on multiple product lines.

(Li et al. 2007a,b) addressed the effect of system designe on product quality. Their work is apllied in an automotive paint shop and the results verify the relation amont system design and product quality.

In the literature, mostly cost methods have been used to find optimal allocation of inspection stations. In this area, two approaches have been employed. These approaches can be classified in two groups: exact and approximate methods. Integer programming and dynamic programming techniques fall in the first group. The advantaged of using these method is their ability in providing the optimal solution. Their main restriction is that they are expensive in terms of computational effort. On the other approximate methods, provide the solution close to the optimal one with substantially lower computational effort. This category includes different techniques, among which simulated annealing and genetic algorithms are the most commonly used ones.

In the work of (N.Viswanadham et al.1996) stochastic search algorithm based on genetic algorithm and simulated annealing has been applied. The aim of this algorithm is to find the optimal allocation of inspection station in multistage production lines that minimize the total cost. Total cost has been defined as a function of inspection cost, processing cost, scrapping cost and a penalty cost.

In (S. WHITE 1969) shortest route model has been used to optimally allocate the inspection stations. A base configuration, including potential inspection immediately after each processing station has been considered. Defective parts are divided into repairable and non-repairable, and for each stage, general cost structure is considered for both no inspection and 100% inspection cases. Afterwards, shortest route model is implemented with the aim of finding the configuration that minimize the total cost.

(Eppen and Hurst 1974) used dynamic programming for multistage production systems with imperfect inspection in order to find the tradeoff between inspection, salvage and production costs with the revenue. At each stage parts can be fall in the area of inspected or not inspected and accordingly related cost and revenue is defined for each case.

In some other works, interrelationship of quality features among different stages has been taken into account and it is assumed that quality failures of each stage cannot be evaluated independently and depends on the ones of previous stages. Different production layout have been evaluated based on this assumption.

In (Jin and Tsung 2007), chart allocation is evaluated for multistage serial lines taking into account this interrelationship. Based on this assumption, a strategy is proposed for proper allocation of control charts in a multistage production line aiming at faster detection of out of controls. (Jin,Li,Tsung 2010) studied optimal allocation of control charts for parallel multistage production line that can result in quicker diagnosis of quality problems.

(Taneja and Viswanadham), designed a Genetic Algorithm for allocating inspection stations in a multistage manufacturing systems, for both serial and non-serial cases. They concluded that the problem has high level of consistency with Genetic Algorithm models.

(Bay and Yun, 1996) applied a cost model. Dynamic programming used to solve proposed heuristic method to reach the solution.

In the work of (Emmons and Rabinowitz 2001) three stage decision making process including evaluation of overall inspection capacity, assignment of inspection tasks to inspectors and scheduling of the inspector's tasks in implemented. These decisions are made regarding the trade-off between the cost of inspectors and the loss associated with non-conforming products. A hierarchical heuristic solution procedure is proposed to support these three related decisions. Assembly lines are studied in (Kakade et al., 2004) taking into account inspection accuracy and product yield. Combination of simulated annealing and heuristic solution is used to find the solution.

In (Rau and Chu), inspection allocation for serial production line with two types workstations, workstations of attribute data(WAD) and workstation of variable data(WVD), is studied. the policy regarding to non-conforming parts, is repair, rework or scrap. According to the mentioned assumptions, benefit model is developed and heuristic algorithm is used to solve the model. Results confirm the efficiency of method in terms of computational effort.

Inspection station allocation is an important issue in design and reconfiguration of manufacturing systems. It can highly influence product quality as well as productivity performance of the system. Although the methods developed in the literature only take into account effect of inspection allocation on quality aspects. In this thesis both aspects have been considered in developing a new method to optimally allocate methods In the recent years, inspection station allocation, attracts attention of researchers.

Chapter Three

3 General System Modeling Assumptions

In this chapter, general assumptions and notations that used in this work are explained.

System architecture assumption:

- Serial manufacturing system is considered.
- Continuous time model is considered, due to their consistency with real systems. This means that transition from one state to another can happen at any moment of the time.

Machine assumption:

- Each machine can fail in multiple failure modes.
- Each machine has only one quality failure mode i.e. can only subjects to one type of out of control.
- Machine may only fail in operational mode i.e. not all of the machines are necessarily subjects to out of control.
- Failures are assumed to be operational dependant failures.
- Machines have the same production rate represented by μ.
- When machine M_i operates in control it produces γ_i^w percent of defective items.
- When machine operates out of control it produces γ_i^o percent of defective items.
- When machine M_i is operational, it can be face operational failure with the rate of p_i.
- When machine M_i is failed in operational mode, it can be restart with the ratio of r_i.
- When machine M_i is operating in control, it can be stopped due to the false alarm with the ratio of p_i^{false}.
- When machine M_i is stopped due to the false alarm, it can be restart with the ratio of r_i^{false}.

- When machine M_i goes out of control, out of control can be detected with the ratio of $p_i^{c(i,q)}$.

Inspection assumption:

- Perfectly accurate inspection is assumed.
- Flexible inspection stations are assumed i.e. every inspection station can inspect all product features.

Sampling policy:

- 100% inspection is assumed. That means sample size is one and number of not inspected parts between two samples is zero i.e. $m(C_{i,q})$=1 and $h(C_{i,q})$=0.

Control charts:

- Xbar control charts are assumed to be applied. These control charts are subjects to two types of error. Which are error type I and error type II and are represented by α and β respectively.
- Parameters of control charts are fixed; using available methods in the literature, that only focus on quality control aspects and do not consider the effect of control chart parameters on productivity performances of the system.

Different type of machines in the production system:

Two different types of stations represented in this work. Working stations and inspection stations. Symbols related to each type are as follows:

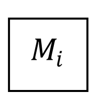

Figure3.1: Working station

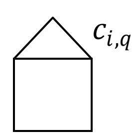

Figure3.2: inspection station

For instance, figure 3.3 represents 3 machines inspected by one inspection station at the end of the line:

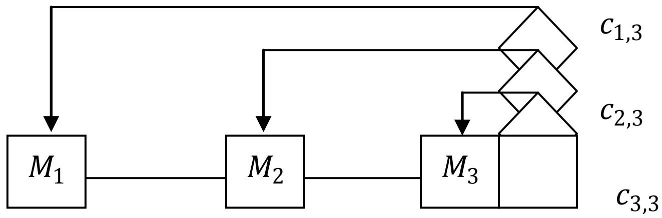

Figure3.3: Three machines inspects by one inspection station

Local vs. remote inspection*:*

In a production line, machines can be monitored locally or remotely. A machine is called locally monitored if parts produced by one machine are inspected just downstream that machine. On the other hand, it is called remotely monitored if inspection takes place downstream of the line. Figure 3.1 represents local and remote inspection.

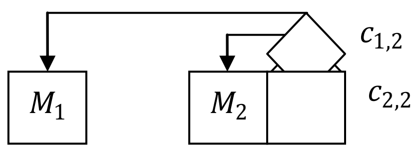

Figure 3.4: Local vs. remote monitoring

Here the second machine is inspected locally and the first one is inspected remotely after the second machine.

Chapter Four

4 Single Machine Model

The simplest production system is a single machine which operates in isolation. A machine is called isolated when it is not affected by presence of other machines or intermediate buffers. This chapter is structured as follows: in section 1, Markov model of locally and remotely monitored machine is explained and solved and related performance measures are calculated. Section 2 is dedicated to analysis of system behavior and effect of shifting the inspection point on system performance measures is evaluated.

Markov model of single machine

In this section, Markov chans of isolated machine is explained for locally and remotely monitored machine. The Markov model is solved and steady state probabilities and system performance measures are determined.

4.1.1 Locally monitored machine

The first case is the locally monitored machine. Since in this case the produced parts are inspected just downstream the machine, the time to identify the out of controls only depends on control chart parameters.

Markov model of the locally monitored machine is represented in figure 4.1.

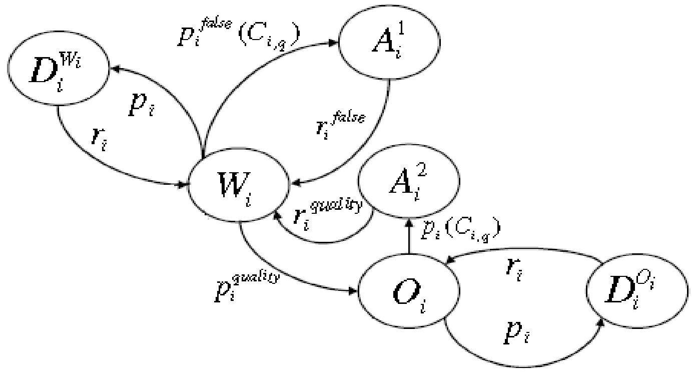

Figure4.1: Markov model of the locally monitored machine

This machine can be found in the following states:

W_i : Machine operates in control

A_i^1 : Machine is stopped due to the of false alarm signal

D_i^w : Machine is stopped from in control state due to operational failure

D_i^o: Machine is stopped from out of control due to operational failure

O_i : Machine operates out of control but out of control is not detected yet

A_i^2 : Out of control state is detected and machine is stopped for maintenance

Corresponding transition rates of the Markov model are as following:

p_i : Transition rate of going to down state due to the operational failure

r_i : Transition rate of repairing the operational failure

p_i^{false} : Transition rate that the machine is stopped due to false signal

r_i^{false} : Transition rate that machine is reset to operational mode after the false alarm

$p_i^{quality}$: Transition rate that machine goes out of control

$r_i^{quality}$: Transition rate that machine has been reset from out of control

$p_i^{c(i,q)}$: Transition rate of identifying the out of control

Some of these transition probabilities are related to machine parameters which are known. The rest are related to control chart parameters and are explained in continue.

Transition rate from the state W_i to the state A_i^1, depends on error type one of control chart and is defined as:

$$\mathrm{ARL}_0 = \frac{1}{\alpha} \tag{4.1}$$

Where ARL_0 (average run length zero) is the number of samples inspected before false alarm happens.

Therefore MTTFA (mean time to false alarm) can be written as:

$$\mathrm{MTTFA} = \mathrm{ARL}_0 \cdot (h + c) = \mathrm{ARL}_0 \tag{4.2}$$

$$\mathrm{MTTFA} = \frac{1}{\alpha} \Longrightarrow p_i^{false} = \alpha \tag{4.3}$$

Transition rate from O_i to A_i^2 depends on reactivity level of control chart which define as error type two or β. Beta is probability of not identifying the shift in one sample.

$$\mathrm{ARL}_1 = \frac{1}{1-\beta} \tag{4.4}$$

Where ARL_1 is the average number of samples to be inspected before finding the out of control? Mean Time to Diagnose (MTTD) can be written as:

$$\mathrm{MTTD} = \mathrm{ARL}_1 \cdot (h + c) = \mathrm{ARL}_1 \tag{4.5}$$

And :

$$p_i^{c(i,q)} = \frac{1}{\text{MTTD}} = \frac{1}{1-\beta} \quad (4.6)$$

Now since all the transition rates are know, it is possible to solve the Markov model of the machine through the following equations:

$$\pi(w_i)\ p_i^{\text{false}} = \pi(A_i^1)\ r_i^{\text{false}} \quad (4.7)$$

$$\pi(w_i)\ p_i = \pi(D)_i^w\ r_i \quad (4.8)$$

$$\pi(o_i)\ p_i^{c(i,q)} = \pi(w_i) p_i^{\text{quality}} \quad (4.9)$$

$$\pi(A_i^2)\ r_i^{\text{quality}} = \pi(o_i)\ p_i^{c(i,q)} \quad (4.10)$$

$$\pi(D_i^o)\ r_i = \pi(o_i)\ p_i \quad (4.11)$$

$$\pi(w_i) + \pi(D)_i^w + \pi(A_i^1) + \pi(o_i) + \pi(D_i^o) + \pi(A_i^2) = 1 \quad (4.12)$$

Solving this set of equations, steady state probabilities of the system is found as follow:

$$\pi(w_i) = \frac{1}{\left(\frac{p_i^{\text{quality}}}{p_i^{c(i,q)}}\right)\left(1+\sum_{f_i=1}^{F_i} p_{i,f_i} + \frac{p_i^{c(i,q)}}{r_i^{\text{quality}}}\right)+1+\sum_{f_i=1}^{F_i} p_{i,f_i} + \frac{p_i^{\text{false}}}{r_i^{\text{false}}}} \quad (4.13)$$

$$\pi(A_i^1) = \pi(w_i)\ \frac{p_i^{\text{false}}}{r_i^{\text{false}}} \quad (4.14)$$

$$\pi(D)_i^w = \pi(w_i)\ \frac{p_i}{r_i} \quad (4.15)$$

$$\pi(o_i) = \pi(w_i)\ \frac{p_i^{\text{quality}}}{p_i^{c(i,q)}} \quad (4.16)$$

$$\pi(A_i^2) = \pi(w_i)\ \frac{p_i^{\text{quality}}}{r_i^{\text{quality}}} \quad (4.17)$$

$$\pi(D_i^o) = \pi(w_i)\ \frac{p_i^{quality}}{p_i^{c(i,q)}} \cdot \frac{p_i}{r_i} \tag{4.18}$$

Performance measures

Using the previously obtained steady state probabilities, performance measures such as total

Throughput, effective throughput and machine yield can be calculated. Total throughput is obtained by adding the probabilities of all the states in which machine are operational, multiplying by production rate of the machine.

$$E_{tot} = \mu[\pi(w_i) + \pi(o_i)] = \frac{\mu(1 + \frac{p_i^{quality}}{p_i^{c(i,q)}})}{\left(\frac{p_i^{quality}}{p_i^{c(i,q)}}\right)\left(1+\sum_{f_i=1}^{F_i} p_{i,f_i} + \frac{p_i^{c(i,q)}}{r_i^{quality}}\right) + 1 + \sum_{f_i=1}^{F_i} p_{i,f_i} + \frac{p_i^{false}}{r_i^{false}}} \tag{4.19}$$

Effective throughput of the system can be written as:

$$E_{eff} = \mu[\pi(w_i)\ (1- \gamma_w) + \pi(o_i)(1- \gamma_o)] = \frac{\mu[1-\gamma_w + \frac{p_i^{quality}}{p_i^{c(i,q)}}(1-\gamma_o)]}{\left(\frac{p_i^{quality}}{p_i^{c(i,q)}}\right)\left(1+\sum_{f_i=1}^{F_i} p_{i,f_i} + \frac{p_i^{c(i,q)}}{r_i^{quality}}\right) + 1 + \sum_{f_i=1}^{F_i} p_{i,f_i} + \frac{p_i^{false}}{r_i^{false}}}$$

(4.20)

Consequently the yield of the system which is the ratio between effective and total throughout is:

$$y_i = \frac{E_{eff}}{E_{tot}} = \frac{p_i^{c(i,q)}\ (1-\gamma_w) + p_i^{quality}\ (1-\gamma_o)}{p_i^{c(i,q)} + p_i^{quality}} \tag{4.21}$$

4.1.2 Remotely monitored machine

Markov model of remotely monitored machine has been presented in figure 4.2.

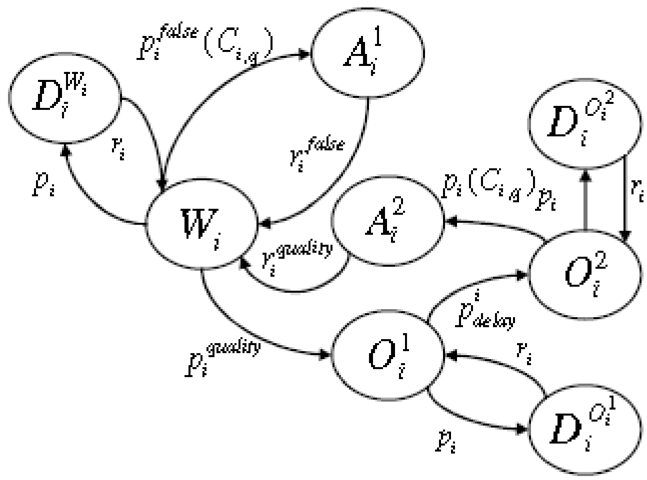

Figure4.2: Markov model of remotely monitored machine

The only difference of this case with the previous one is that a new state o_i^2 must be added to the model.

The rationale of adding this state is that if machine goes to out of control stateo_1, since the parts are not inspected locally, it is not possible to recognize the shift immediately and out of control can only be detected when parts arrive to the inspection station. If the average time parts spend in the system before reaching the inspection point is represented by $LT_{c_{(i,q)}}$, the transition rate of reaching parts to the inspection point can be written as $p_i^{delay} = \frac{1}{LT_{c_{(i,q)}}}$.

Steady state probabilities:

Steady state probabilities can be derived similar to the locally monitored machine:

$$\pi(w_i)\ p_i^{false} = \pi(A_i^1)\ r_i^{false} \tag{4.22}$$

$$\pi(w_i)\ p_i = \pi(D)_i^w\ r_i \tag{4.23}$$

$$\pi(o_i^1)\ p_i^{delay} = \pi(w_i)\ p_i^{quality} \tag{4.24}$$

$$\pi(D_i^{o_i^1})\, r_i = \pi(o_i^1)\ .\ p_i \tag{4.25}$$

$$\pi(o_i^2)\, p_i = \pi(D_i^{o_i^2})\, r_i \tag{4.26}$$

$$\pi(o_i^1)\, p_i^{delay} = \pi(o_i^2)\, p_i^{c(i,q)} \tag{4.27}$$

$$\pi(o_i^2)\, p_i^{c(i,q)} = \pi(A_i^2)\, r_i^{quality} \tag{4.29}$$

$$\pi(w_i) + \pi(A_i^1) + \pi(D)_i^w + \pi(o_i^1) + \pi(D_i^{o_i^1}) + \pi(o_i^2) + \pi(D_i^{o_i^2}) + \pi(A_i^2) = 1 \tag{4.30}$$

Solving this set of equations, steady state probabilities are found:

$$\pi(w_i) = \frac{1}{\frac{p_i^{false}}{r_i^{false}} + \left(1+\frac{p_i^{quality}}{p_i^{delay}}\right)\left(1+\sum_{f_i=1}^{F_i}\frac{p_{i,f_i}}{r_{i,f_i}}\right) + \frac{p_i^{quality}\left(1+\sum_{f_i=1}^{F_i}\frac{p_{i,f_i}}{r_{i,f_i}} + \frac{p_i^{c(i,q)}}{r_i^{quality}}\right)}{p_i^{c(i,q)}}} \tag{4.31}$$

$$\pi(A_i^1) = \pi(w_i)\,\frac{p_i^{false}}{r_i^{false}} \tag{4.32}$$

$$\pi(D)_i^w = \frac{p_i}{r_i}\,\pi(w_i) \tag{4.33}$$

$$\pi(o_i^1) = \frac{p_i^{quality}}{p_i^{delay}}\,\pi(w_i) \tag{4.34}$$

$$\pi(D_i^{o_i^1}) = \pi(w_i)\,\frac{p_i^{quality}}{p_i^{delay}}\ .\ \frac{p_i}{r_i} \tag{4.35}$$

$$\pi(o_i^2) = \pi(w_i)\,\frac{p_i^{quality}}{p_i^{c(i,q)}} \tag{4.36}$$

$$\pi(A_i^2) = \pi(w_i)\,\frac{p_i^{quality}}{r_i^{quality}} \tag{4.37}$$

$$\pi(D_i^{o_i^2}) = \pi(w_i)\ \frac{p_i^{quality}}{p_i^{c(i,q)}} \cdot \frac{p_i}{r_i} \tag{4.38}$$

Performance measures

Similar to the locally monitored machine, performance measures can b obtained for remotely monitored one.

Here probability of finding the machine operational can be attained by adding probability of being in control state to two states that machine operates out of control. Therefore total throughput of the machine is equal to:

$$E_i^{tot} = \mu[\pi(w_i) + \pi(o_i^1) + \pi(o_i^2)] = \frac{\mu\left(1+ \frac{p_i^{quality}}{p_i^{delay}} + \frac{p_i^{quality}}{p_i^{c(i,q)}}\right)}{\frac{p_i^{false}}{r_i^{false}} + \left(1+ \frac{p_i^{quality}}{p_i^{delay}}\right)\left(1+\sum_{f_i=1}^{F_i} \frac{p_{i,f_i}}{r_{i,f_i}}\right) + \frac{p_i^{quality}\left(1+\sum_{f_i=1}^{F_i}\frac{p_{i,f_i}}{r_{i,f_i}} + \frac{p_i^{c(i,q)}}{r_i^{quality}}\right)}{p_i^{c(i,q)}}} \tag{4.39}$$

And system effective throughput is equal to:

$$E_i^{eff} = \frac{\mu\left[(1-\gamma_w) + (1-\gamma_o)\left(\frac{p_i^{quality}}{p_i^{delay}} + \frac{p_i^{quality}}{p_i^{c(i,q)}}\right)\right]}{\frac{p_i^{false}}{r_i^{false}} + \left(1+ \frac{p_i^{quality}}{p_i^{delay}}\right)\left(1+\sum_{f_i=1}^{F_i} \frac{p_{i,f_i}}{r_{i,f_i}}\right) + \frac{p_i^{quality}\left(1+\sum_{f_i=1}^{F_i}\frac{p_{i,f_i}}{r_{i,f_i}} + \frac{p_i^{c(i,q)}}{r_i^{quality}}\right)}{p_i^{c(i,q)}}} \tag{4.40}$$

Finally:

$$y_i = \frac{p_i^{c(i,q)} p_i^{delay}\ (1-\gamma_w) + p_i^{quality}\ (\ p_i^{c(i,q)} + p_i^{delay}\ )\ (1-\gamma_o)}{p_i^{c(i,q)} p_i^{delay} + p_i^{c(i,q)} p_i^{quality} + p_i^{quality}\ \ p_i^{delay}} \tag{4.41}$$

4.2 System behavior

When machine is inspected remotely instead of locally, an additional delay is introduced in identifying the out of control. Contrary to local inspection, the delay of identifying out of control not only depends on control chart parameters, but also depends on the lead time parts spend between inspected machine and inspection point and therefore on p_i^{delay} .

In this section the effect of changing transition rate from state o_1 to state o_2 on performance measures of the system is evaluated.

4.2.1 Effect of changing p_i^{delay} on total throughput of the system

In order to realize the effect of changing p_i^{delay} on total throughput of the system, partial derivative of $\frac{\partial E_i^{tot}}{\partial p_i^{delay}}$ is used.

From solving the Markov model of the remotely monitored machine total throughput of the system is known and the partial derivative can be calculated.

$$\frac{\partial E_i^{tot}}{\partial p_i^{delay}} = \mu \,.\frac{\partial [(\pi(w_i).\left(1+ \frac{p_i^{quality}}{p_i^{delay}} + \frac{p_i^{quality}}{p_i^{c(i,q)}}\right)]}{\partial p_i^{delay}} =$$

$$= \mu \,[\frac{\partial (\pi(w_i))}{\partial p_i^{delay}} (1 + \frac{p_i^{quality}}{p_i^{delay}} + \frac{p_i^{quality}}{p_i^{c(i,q)}}) + \frac{\partial \left(1+ \frac{p_i^{quality}}{p_i^{delay}} + \frac{p_i^{quality}}{p_i^{c(i,q)}}\right)}{\partial p_i^{delay}} \pi(w_i)] \tag{4.42}$$

In order to find the value of p_i^{delay} for which the sign of derivative changes from positive to negative, the derivative is equalized to zero and after simplification following equation is found.

$$\pi(w_i) \, (1+ \frac{p_{i,f_i}}{r_{i,f_i}}) \, (1 + \frac{p_i^{quality}}{p_i^{delay}} + \frac{p_i^{quality}}{p_i^{c(i,q)}}) = 1 \tag{4.43}$$

Substituting $\pi(w_i)$ with the value that is already found, following equation is obtained:

$$\frac{1}{\frac{p_i^{false}}{r_i^{false}}+\left(1+\frac{p_i^{quality}}{p_i^{delay}}\right)\left(1+\sum_{f_i=1}^{F_i}\frac{p_{i,f_i}}{r_{i,f_i}}\right)+\frac{p_i^{quality}\left(1+\sum_{f_i=1}^{F_i}\frac{p_{i,f_i}}{r_{i,f_i}}+\frac{p_i^{c(i,q)}}{r_i^{quality}}\right)}{p_i^{c(i,q)}}}=\frac{1}{\left(1+\sum_{f_i=1}^{F_i}\frac{p_{i,f_i}}{r_{i,f_i}}\right)\left(1+\frac{p_i^{quality}}{p_i^{delay}}+\frac{p_i^{quality}}{p_i^{c(i,q)}}\right)} \quad (4.44)$$

Expanding the denominator of the both sides of equation, it is seen that left side of the equation is always smaller than the right side. In other words, for all values of p_i^{delay} , $\frac{\partial E_i^{tot}}{\partial p_i^{delay}}$ is negative. This means that, increasing p_i^{delay} always results in lower total throughput of the system.

This result is consistent with the common sense. In fact increasing p_i^{delay} leads to faster detection of out of control and therefore more frequent stoppage of machine for repairing out of control. Subsequently transition rate of finding machine in operative mode is diminishing which leads to less total throughput.

Considering a machine with the parameters $p_{quality} = 0.05$, $r_{quality} = 0.6$, $p_{(c,q)} = 0.85$, p_{false} =0.02, r_{false} =0.8,$\gamma_w = 0.04$, $p = 0.04$ *and* $r = 0.8$, figure 4.3, represents total throughput of machine when p_i^{delay} changes in the range of 0.05 to 0.95.

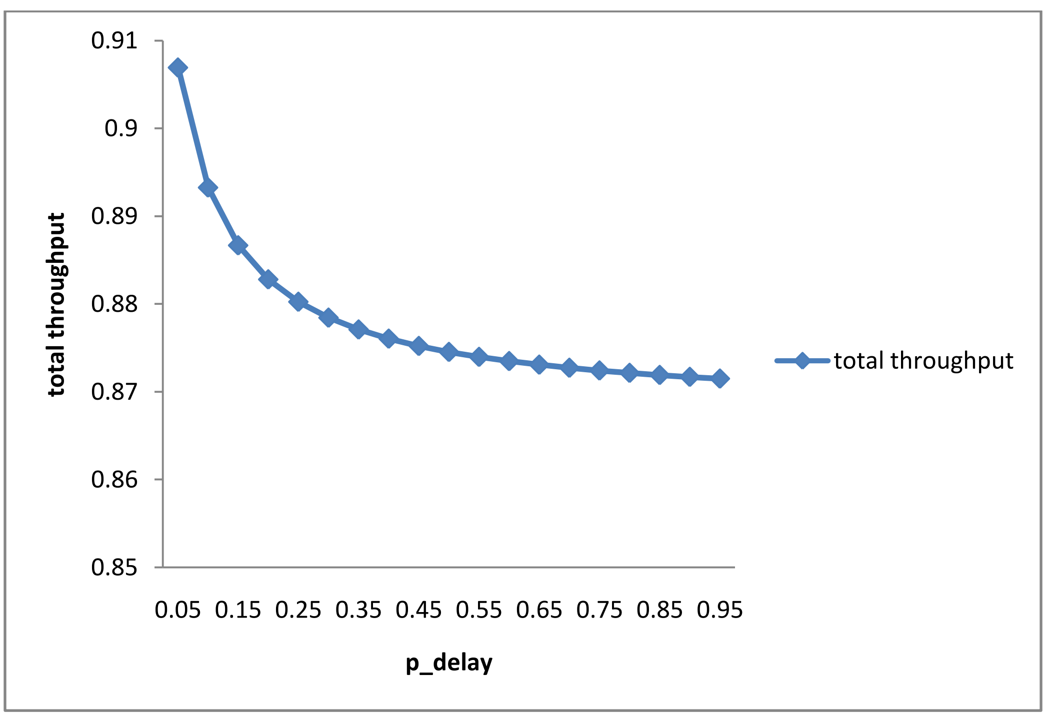

Figure 4.3 Total throughput vs. p_i^{delay}

4.2.2 Effect of changing p_i^{delay} on effective throughput of the system

The same procedure is followed to evaluate the effect of changing p_i^{delay} on effective throughput of the system. Here equalizing the derivative to zero and after some manipulations, following equation is obtained:

$$(1-\gamma_o)\left(\sum_{\mathrm{f_i}=1}^{\mathrm{F_i}}\frac{p_{i,f_i}}{r_{i,f_i}}+\frac{p_i^{quality}}{r_i^{quality}}+\frac{p_i^{false}}{r_i^{false}}+1\right)-(1-\gamma_w)\left(\sum_{\mathrm{f_i}=1}^{\mathrm{F_i}}\frac{p_{i,f_i}}{r_{i,f_i}}+1\right)=0 \qquad (4.45)$$

If this equation holds, $\frac{\partial E_i^{tot}}{\partial p_i^{delay}}$ would be equal to zero. When the left side of equation is greater than zero, $\frac{\partial E_i^{eff}}{\partial p_i^{delay}}$ is positive and otherwise it is negative.

Figure 4.4 represents the effective throughput of the machine that observed in section 4.3.1 when p_i^{delay} and γ_o change from 0.05 to 0.90 and 0.05 to 0.25 respectively, while other parameters kept constant.

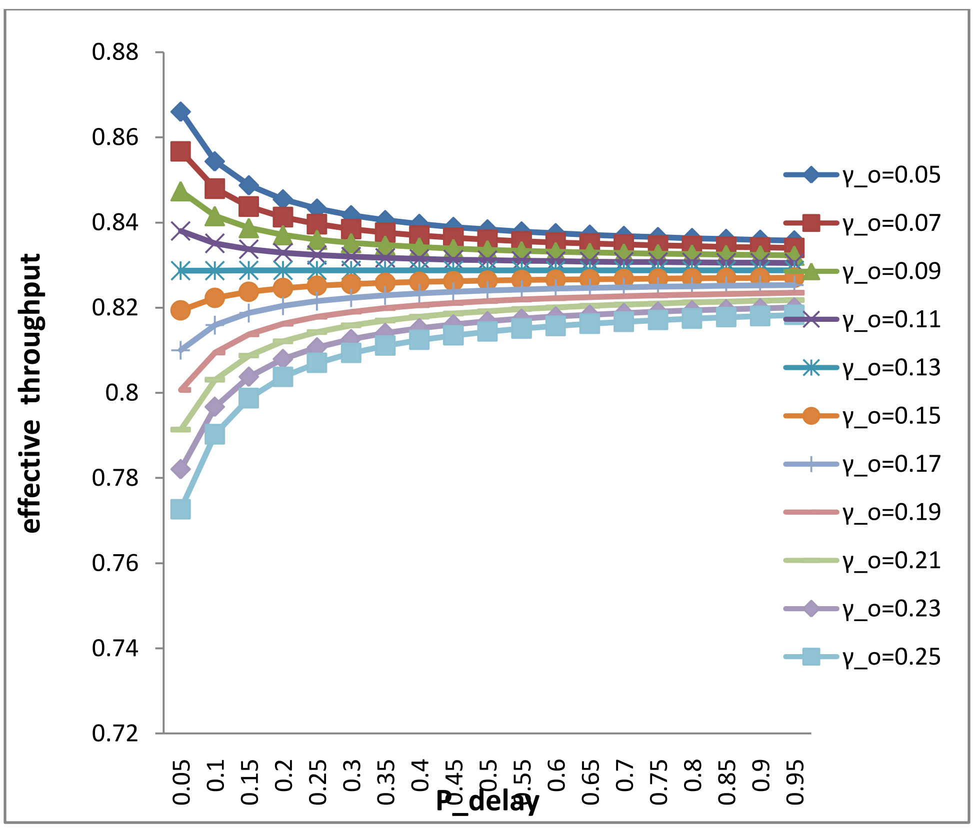

Figure4.4: effective throughput vs. p_i^{delay} for different values of γ_o

According to the results, starting from γ_o=0.05 at the beginning effective throughput decreasing consistently by increasingP_{delay}. However, for γ_o greater or equal to 0.13 increasing P_{delay} results in higher effective throughput. These increasing and decreasing trends are valid for all values ofP_{delay}.

The results are consistent with equation 4.45. For observed remotely monitored machine, solving this equation it is found that for γ_o= 0.1298, derivative of effective throughput is equal to zero for every p_{delay}. From figure 4.4, it is visible that whenγ_o=0.13, effective throughput almost does not change while varyingp_{delay}. For γ_o>0.13 equation is positive that means increasing P_{delay} results in higher effective throughput and for γ_o<0.13 it is negative that means increasing p_{delay} diminishes effective throughput of the system.

In this way it is possible to analyze the behavior of every isolated machine in terms of effect of changing p_{delay} on effective throughput of the machine. It is worth to notice that since equation 4.45 is independent from p_{delay}, effective throughput of the machine, always shows either increasing or decreasing trend while p_{delay} is changing.

4.2.3 Effect of changing p_i^{delay} on the system yield

For a remotely monitored machine, system yield is given by equation 4.41. Therefore $\frac{\partial yield}{\partial p_{delay}}$ is obtained as following:

$$\frac{\partial yield}{\partial p_{delay}} = \frac{\frac{p_i^{quality}}{p_i^{delay\ \wedge 2}}(\gamma_o - \gamma_w)}{\left(1+\frac{p_i^{quality}}{p_i^{delay}}+\frac{p_i^{quality}}{p_i^{c(i,q)}}\right)^{\wedge}2} \tag{4.46}$$

Since defective ratio that machine produces out of control is always greater than the one it produces in control, $\frac{\partial yield}{\partial p_{delay}}$ is always positive. This means increasing p_{delay}, system yield is always increasing.

Figure 4.5, which represents system yield of the machine studied in last two sections, confirms this result.

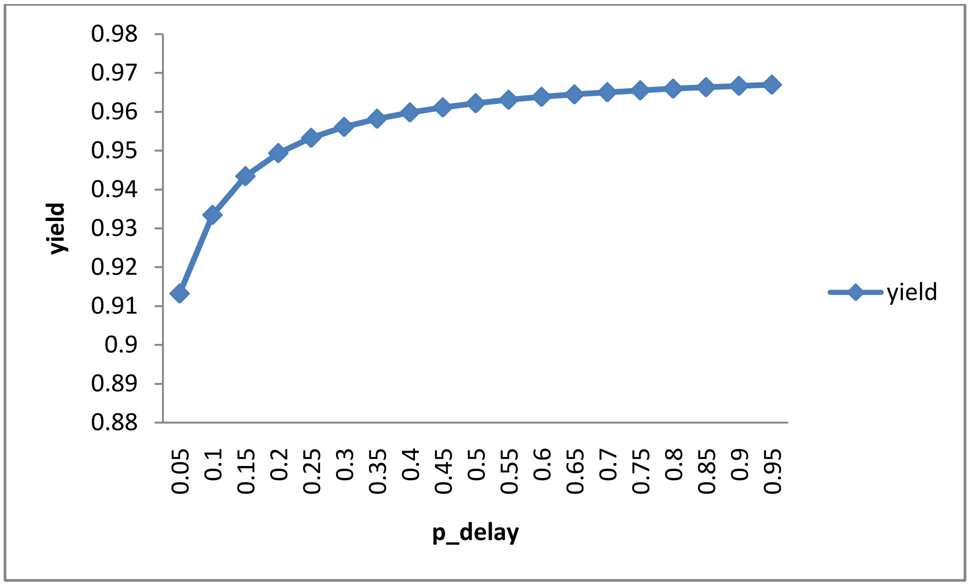

Figure 4.5: system yield vs. p_{delay}

Chapter Five

5 System Model

In this chapter serial un-buffered production line is evaluated. A serial line, is an open multistage production system in which parts transform to final products by passing through different workstations.

System is assumed to be composed of N machine with the same production rate of μ when operational.Since the un-buffer case has been considered, if any machine fails, the whole line stops processing parts immediately. On the other hand, it is worth to notice that a machine in such a system, may stop working for three different reasons: operational failure, out of control diagnosis and false alarm generated by control chart.

Moreover machines are divided into two subsets. The first one includes machines with quality failures and represented by M_Q and the second one includes machines without quality failures and represented by M_{NQ} .

In the following sections, Markov model of a machine that is embedded in a serial un-buffer production line is explained separately for locally and remotely monitored machine and for the machine that is not subjects to out of control. Obtaining the steady state probabilities of the Markov model, it is possible to find performance measure of the whole system. Results is used in chapter 6 to develop an algorithm to find the optimal allocation of limited inspection stations and their assignment to the machines in the serial line with the aim of maximizing effective throughput.

5.1 Markov model of locally monitored machine

The difference between this case with the isolated machine is coming from the fact that performance of each machine is affected by performance of others. As it was mentioned before, when a machine is stopped due to the operational failure, detection of out o control or false alarm generated by control chart, all other machines stop processing immediately.

This phenomenon can be modeled through adding additional states to the Markov model explained in chapter 4 for isolated machine. These states are as follows:

- $A'_{i,j}$: Machine M_i is in this state when it is stopped since machine M_j, has been stopped for repairing the out of control.
- $A''_{i,j}$: Machine is in this state when it is stopped due to the false alarm generated by control chart that monitors machine M_j.
- $D_{i,j}$: Machine is in this state when it is stopped due to operational failure of machine M_j.

All these three states can happen when machine M_i is operational, either in or out of control. Figure 5.1 shows the graphical representation of locally monitored machine.

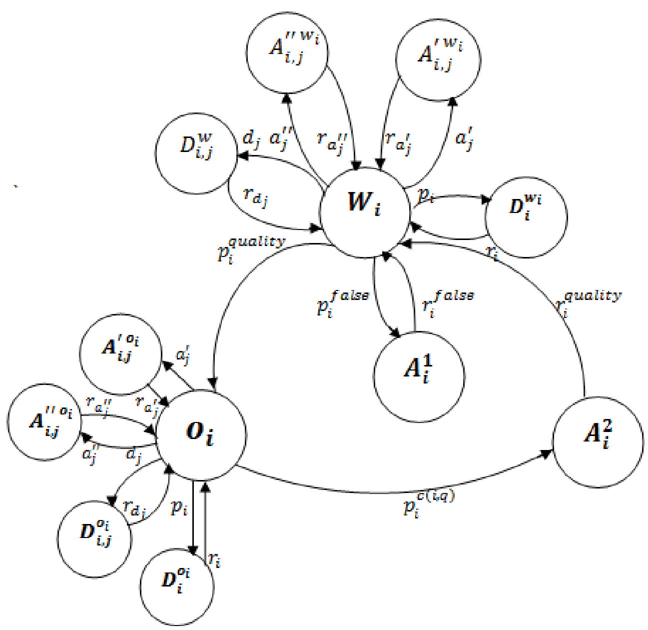

Figure 5.1: Markov model of locally monitored machine

Apart from the transition ratios which already explained in chapter 4 for isolated machine, other transition rates are introduced to describe the additional states. These transition rates are:

- a'_j : the transition rate that machine M_i stopped for repairing out of control of machine M_j
- $r_{a'_j}$: the rate that machine M_i start working when it is stopped in state $A'^{w_i}_{i,j}$
- a''_j : the transition rate that machine M_i stopped due to the false alarm that is generated by

control chart which controls machine M_j

- $r_{a_j''}$: the rate that machine M_i start working when it is stopped in state $A_{i,j}^{''\,w_i}$
- d_j : the transition rate that machine M_i stopped for repairing operational failure of machine M_j
- r_{d_j}: the rate that machine M_i start working when it is stopped in state $D_{i,j}^{w}$

As it is shown in section 5.4, these unknown transition rates only depend on machine M_j parameters. Here equations related to state probabilities are expressed and Markov chain is solved parametrically. Unknown transition rates are calculated in section 5.4

$$\pi(D_i^w)r_i = \pi(w_i)p_i \tag{5.1}$$

$$\pi(A_{i,j}^{'\,w_i})r_{a_j'} = \pi(w_i)a_j' \qquad \forall\, M_j \in M_Q - \{M_i\} \tag{5.2}$$

$$\pi(A_{i,j}^{''\,w_i})r_{a_j''} = \pi(w_i)a_j'' \qquad \forall\, M_j \in M_Q - \{M_i\} \tag{5.3}$$

$$\pi(D_{i,j}^{w})\, r_{d_j} = \pi(w_i)\, d_j \qquad \forall\, M_j - \{M_i\} \tag{5.4}$$

$$\pi(A_i^1)\, r_i^{false} = \pi(w_i)\, p_i^{false} \tag{5.5}$$

$$\pi(D_i^{o_i})r_i = \pi(o_i)p_i \tag{5.6}$$

$$\pi(A_{i,j}^{'\,o_i})r_{a_j'} = \pi(o_i)a_j' \qquad \forall\, M_j \in M_Q - \{M_i\} \tag{5.7}$$

$$\pi(A_{i,j}^{''\,o_i})r_{a_j''} = \pi(o_i)a_j'' \qquad \forall\, M_j \in M_Q - \{M_i\} \tag{5.8}$$

$$\pi(D_{i,j}^{o})r_{d_j} = \pi(o_i)d_j \qquad \forall\, M_j - \{M_i\} \tag{5.9}$$

$$\pi(A_i^2)r_i^{quality} = \pi(o_i)p_i^{c(q,i)} \tag{5.10}$$

$$\pi(w_i)p_i^{quality} = \pi(o_i)p_i^{c(q,i)} \tag{5.11}$$

$$\pi(D_i^w)+\pi(w_i)+\pi(A_{i,j}^{'\,w_i})+\pi(A_{i,j}^{''\,w_i})+\pi(D_{i,j}^{w})+\pi(A_i^1)+\pi(D_i^{o_i})+\pi(o_i)+\pi(A_{i,j}^{'\,o_i})+\pi(A_{i,j}^{''\,o_i})+$$
$$+\pi(D_{i,j}^{o})+\pi(A_i^2)=1 \tag{5.12}$$

Solving this set of equation following transition

$\pi(w_i)=$

$$= \frac{1}{\left(1+\frac{p_i^{quality}}{p_i^{c(q,i)}}\right)\sum_{j=1}^{N-\{M_i\}}\left(\frac{a_j'}{r_{a_j'}}+\frac{a_j''}{r_{a_j''}}+\frac{d_j}{r_{d_j}}\right)+\frac{p_i^{false}}{r_i^{false}}+\frac{p_i^{quality}}{p_i^{c(q,i)}}+\frac{p_i}{r_i}+\frac{p_i p_i^{quality}}{r_i p_i^{c(q,i)}}+\frac{p_i^{quality}}{r_i^{quality}}+1} \tag{5.13}$$

$$\pi(D_i^w) = \pi(w_i)\ \frac{p_i}{r_i} \tag{5.14}$$

$$\pi(A_{i,j}^{'\,w_i}) = \pi(w_i)\ \frac{a_j^{'}}{r_{a_j^{'}}} \tag{5.15}$$

$$\pi(A_{i,j}^{''\,w_i}) = \pi(w_i)\ \frac{a_j^{''}}{r_{a_j^{''}}} \tag{5.16}$$

$$\pi(D_{i,j}^w) = \pi(w_i)\ \frac{d_j}{r_{d_j}} \tag{5.17}$$

$$\pi(A_i^1) = \pi(w_i)\ \frac{p_i^{false}}{r_i^{false}} \tag{5.18}$$

$$\pi(D_i^o) = \pi(w_i)\ \frac{p_i^{quality}}{p_i^{c(i,q)}}\ .\ \frac{p_i}{r_i} \tag{5.19}$$

$$\pi(A_i^2) = \pi(w_i)\ \frac{p_i^{quality}}{r_i^{quality}} \tag{5.20}$$

$$\pi(o_i) = \pi(w_i)\ \frac{p_i^{quality}}{p_i^{c(q,i)}} \tag{5.21}$$

$$\pi(A_{i,j}^{'\,o_i}) = \pi(w_i)\ \frac{a_j^{'}}{r_{a_j^{'}}} . \frac{p_i^{qualit\ y}}{p_i^{c(i,q)}} \tag{5.22}$$

$$\pi(A_{i,j}^{''\,o_i}) = \pi(w_i)\ \frac{a_j^{''}}{r_{a_j^{''}}} . \frac{p_i^{quality}}{p_i^{c(i,q)}} \tag{5.23}$$

$$\pi(D_{i,j}^o) = \pi(w_i)\ \frac{d_j}{r_{d_j}} . \frac{p_i^{quality}}{p_i^{c(i,q)}} \tag{5.24}$$

Performance measures of machine M_i:

$$E_{tot}^{M_i} = \mu[\pi(o_i) + \pi(w_i)] = \mu\pi(w_i)(1 + \frac{p_i^{quality}}{p_i^{c(i,q)}}) = \tag{5.25}$$

$$=\frac{\mu\left(1+\frac{p_i^{quality}}{p_i^{c(i,q)}}\right)}{\left(1+\frac{p_i^{quality}}{p_i^{c(q,i)}}\right)\sum_{j=1}^{N-\{M_i\}}\left(\frac{a_j'}{r_{a_j'}}+\frac{a_j''}{r_{a_j''}}+\frac{d_j}{r_{d_j}}\right)+\frac{p_i^{false}}{r_i^{false}}+\frac{p_i^{quality}}{p_i^{c(q,i)}}+\frac{p_i}{r_i}+\frac{p_i p_i^{quality}}{r_i p_i^{c(q,i)}}+\frac{p_i^{quality}}{r_i^{quality}}+1}$$

$$E_{eff}^{M_i}=\mu[\pi(w_i)(1\text{-}\gamma_w)+\pi(o_i)(1\text{-}\gamma_o)]= \quad (5.26)$$

$$=\frac{\mu\left[1-\gamma_w+\frac{p_i^{quality}}{p_i^{c(i,q)}}(1-\gamma_o)\right]}{\left(1+\frac{p_i^{quality}}{p_i^{c(q,i)}}\right)\sum_{j=1}^{N-\{M_i\}}\left(\frac{a_j'}{r_{a_j'}}+\frac{a_j''}{r_{a_j''}}+\frac{d_j}{r_{d_j}}\right)+\frac{p_i^{false}}{r_i^{false}}+\frac{p_i^{quality}}{p_i^{c(q,i)}}+\frac{p_i}{r_i}+\frac{p_i p_i^{quality}}{r_i p_i^{c(q,i)}}+\frac{p_i^{quality}}{r_i^{quality}}+1}$$

As a ratio between effective and total throughput, machine yield is given by:

$$y_i=\frac{E_{eff}}{E_{tot}}=\frac{p_i^{c(i,q)}(1-\gamma_w)+p_i^{quality}(1-\gamma_o)}{p_i^{c(i,q)}+p_i^{quality}} \quad (5.27)$$

5.2 Markov model of remotely monitored machine

Markov model of remotely monitored machine is similar to the locally monitored one. The difference as it was mentioned in chapter 4 is adding the new state o_2 to the model. As it was mentioned in chapter 4, transition rate p_i^{delay} is equal to $\frac{1}{LT_i}$, where LT_i is the time part processed by machine M_i spend in the system before arriving to the inspection point. Assuming k_i equal to the number of machines between machine M_i and inspection point, LT_i is given by $LT_i=\frac{k_i}{\mu.E_{tot}}$ consequently $p_i^{delay}=\frac{\mu.E_{tot}}{k_i}$.

Markov chain of figure 5.2 represents behavior of remotely monitored machine.

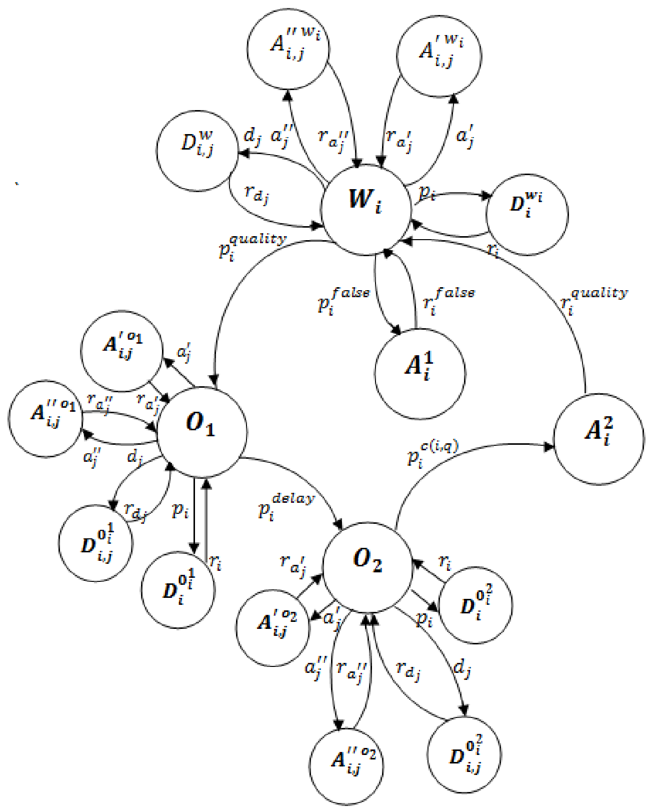

Figure 5.2: Markov chain of remotely monitored machine

Following equations can be obtained subsequently:

$$\pi(D_i^w)r_i = \pi(w_i)p_i \tag{5.28}$$

$$\pi(A_{i,j}^{'\,w_i})r_{a_j'} = \pi(w_i)a_j' \qquad \forall\, M_j \in M_Q - \{M_i\} \tag{5.29}$$

$$\pi(A_{i,j}^{''\,w_i})r_{a_j''} = \pi(w_i)a_j'' \qquad \forall\, M_j \in M_Q - \{M_i\} \tag{5.30}$$

$$\pi(D_{i,j}^w)\, r_{d_j} = \pi(w_i)\, d_j \qquad \forall\, M_j \in M_Q \tag{5.31}$$

$$\pi(A_i^1)\, r_i^{false} = \pi(w_i)\, p_i^{false} \tag{5.32}$$

$$\pi(o_1)p_i^{delay} = \pi(w_i)\, p_i^{quality} \tag{5.33}$$

$$\pi(D_i^{o_i^1})r_i = \pi(o_1)p_i$$

$$\pi(A_{i,j}^{'\,o_i^1})r_{a_j'} = \pi(o_1)a_j' \qquad \forall\, M_j \in M_Q - \{M_i\} \tag{5.34}$$

$$\pi(A_{i,j}^{''\,o_i^1})r_{a_j''} = \pi(o_1)a_j'' \qquad \forall\, M_j \in M_Q - \{M_i\} \tag{5.35}$$

$$\pi(D_{i,j}^{o_i^1})r_{d_j} = \pi(o_1)d_j \qquad \forall\, M_j - \{M_i\} \tag{5.36}$$

$$\pi(o_1)p_i^{delay} = \pi(o_2)p_i^{c(i,q)} \tag{5.37}$$

$$\pi(D_i^{o_i^2})r_i = \pi(o_2)p_i \tag{5.38}$$

$$\pi(A_{i,j}^{'\,o_i^2})r_{a_j'} = \pi(o_2)a_j' \qquad \forall\, M_j \in M_Q - \{M_i\} \tag{5.39}$$

$$\pi(A_{i,j}^{''\,o_i^2})r_{a_j''} = \pi(o_2)a_j'' \qquad \forall\, M_j \in M_Q - \{M_i\} \tag{5.40}$$

$$\pi(D_{i,j}^{o_i^2})r_{d_j} = \pi(o_2)d_j \qquad \forall\, M_j \; - \{M_i\} \tag{5.41}$$

$$\pi(A_i^2)r_i^{quality} = \pi(o_2)p_i^{c(q,i)} \tag{5.42}$$

$$\pi(D_i^w)+\pi(w_i)+\pi(A_{i,j}^{'\,w_i})+\pi(A_{i,j}^{''\,w_i})+\pi(D_{i,j}^w)+\pi(A_i^1)+\pi(D_i^{o_1})+\pi(o_1)+\pi(A_{i,j}^{'\,o_1})+\pi(A_{i,j}^{''\,o_1})+\pi(D_{i,j}^{o_1})+\pi(A_i^2)+$$
$$\pi(D_i^{o_2})+\pi(o_2)+\pi(A_{i,j}^{'\,o_2})+\pi(A_{i,j}^{''\,o_2})+\pi(D_{i,j}^{o_2}) = 1 \tag{5.43}$$

Solving this set of equations, steady state probabilities is obtained:

$$\pi(w_i)= \tag{5.44}$$

$$\frac{1}{\left(1+\frac{p_i^{quality}}{p_i^{c(q,i)}}+\frac{p_i^{quality}}{p_i^{delay}}\right)\sum_{j=1}^{N-\{M_i\}}\left(\frac{a_j'}{r_{a_j'}}+\frac{a_j''}{r_{a_j''}}+\frac{d_j}{r_{d_j}}\right)+\frac{p_i^{false}}{r_i^{false}}+\frac{p_i^{quality}}{p_i^{c(q,i)}}+\frac{p_i}{r_i}+\frac{p_i p_i^{quality}}{r_i p_i^{c(q,i)}}+\frac{p_i^{quality}}{r_i^{quality}}+\frac{p_i^{quality}}{p_i^{delay}}+\frac{p_i p_i^{quality}}{r_i p_i^{delay}}+1}$$

$$\pi(o_1) = \pi(w_i)\,\frac{p_i^{quality}}{p_i^{delay}} \tag{5.45}$$

$$\pi(o_2) = \pi(w_i)\,\frac{p_i^{quality}}{p_i^{c(i,q)}} \tag{5.46}$$

$$\pi(D_i^w) = \pi(w_i)\,\frac{p_i}{r_i} \tag{5.47}$$

$$\pi(A_{i,j}^{'\,w_i}) = \pi(w_i)\,\frac{a_j'}{r_{a_j'}} \tag{5.48}$$

$$\pi(A_{i,j}^{''\,w_i}) = \pi(w_i)\,\frac{a_j''}{r_{a_j''}} \tag{5.49}$$

$$\pi(D_{i,j}^w) = \pi(w_i)\,\frac{d_j}{r_{d_j}} \tag{5.50}$$

$$\pi(A_i^1) = \pi(w_i)\,\frac{p_i^{false}}{r_i^{false}} \tag{5.51}$$

$$\pi(D_i^{o_i^1}) = \pi(w_i)\,\frac{p_i^{quality}}{p_i^{delay}}\cdot\frac{p_i}{r_i} \tag{5.52}$$

$$\pi(A_{i,j}^{'\,o_i^1})=\pi(w_i)\ \frac{a_j^{'}}{r_{a_j^{'}}}.\frac{p_i^{quality}}{p_i^{delay}} \tag{5.53}$$

$$\pi(A_{i,j}^{''\,o_i^1})=\pi(w_i)\ \frac{a_j^{''}}{r_{a_j^{''}}}.\frac{p_i^{quality}}{p_i^{delay}} \tag{5.54}$$

$$\pi(D_{i,j}^{o_i^1})=\pi(w_i)\ \frac{d_j}{r_{d_j}}.\frac{p_i^{quality}}{p_i^{delay}} \tag{5.55}$$

$$\pi(D_i^{o_i^2}) = \pi(w_i)\ \frac{p_i^{quality}}{p_i^{c(q,i)}}\ .\ \frac{p_i}{r_i} \tag{5.56}$$

$$\pi(A_{i,j}^{'\,o_i^2})=\pi(w_i)\ \frac{a_j^{'}}{r_{a_j^{'}}}.\frac{p_i^{quality}}{p_i^{c(q,i)}} \tag{5.57}$$

$$\pi(A_{i,j}^{''\,o_i^2})=\pi(w_i)\ \frac{a_j^{''}}{r_{a_j^{''}}}.\frac{p_i^{quality}}{p_i^{c(q,i)}} \tag{5.58}$$

$$\pi(D_{i,j}^{o_i^2})=\pi(w_i)\ \frac{d_j}{r_{d_j}}.\frac{p_i^{quality}}{p_i^{c(q,i)}} \tag{5.59}$$

$$\pi(A_i^2)=\frac{p_i^{quality}}{r_i^{quality}} \tag{5.60}$$

Performance measures of machine M_i are as follows:

$$E_{tot}^{M_i} = \pi(w_i) + \pi(o_1) + \pi(o_2) = \tag{5.61}$$

$$\frac{\mu[1+\frac{p_i^{quality}}{p_i^{delay}}+\frac{p_i^{quality}}{p_i^{c(i,q)}}]}{\left(1+\frac{p_i^{quality}}{p_i^{c(q,i)}}+\frac{p_i^{quality}}{p_i^{delay}}\right)\sum_{j=1}^{N-\{M_i\}}\left(\frac{a_j^{'}}{r_{a_j^{'}}}+\frac{a_j^{''}}{r_{a_j^{''}}}+\frac{d_j}{r_{d_j}}\right)+\frac{p_i^{false}}{r_i^{false}}+\frac{p_i^{quality}}{p_i^{c(q,i)}}+\frac{p_i}{r_i}+\frac{p_ip_i^{quality}}{r_ip_i^{c(q,i)}}+\frac{p_i^{quality}}{r_i^{quality}}+\frac{p_i^{quality}}{p_i^{delay}}+\frac{p_ip_i^{quality}}{r_ip_i^{delay}}+1}$$

$$E_{eff}^{M_i} = (1-\gamma_w) + (1-\gamma_o)\left(\frac{p_i^{quality}}{p_i^{delay}} + \frac{p_i^{quality}}{p_i^{c(i,q)}}\right) = \tag{5.62}$$

$$\frac{\mu[(1-\gamma_w)+(1-\gamma_o)\left(\frac{p_i^{quality}}{p_i^{delay}}+\frac{p_i^{quality}}{p_i^{c(i,q)}}\right)]}{\left(1+\frac{p_i^{quality}}{p_i^{c(q,i)}}+\frac{p_i^{quality}}{p_i^{delay}}\right)\sum_{j=1}^{N-\{M_i\}}\left(\frac{a_j^{'}}{r_{a_j^{'}}}+\frac{a_j^{''}}{r_{a_j^{''}}}+\frac{d_j}{r_{d_j}}\right)+\frac{p_i^{false}}{r_i^{false}}+\frac{p_i^{quality}}{p_i^{c(q,i)}}+\frac{p_i}{r_i}+\frac{p_ip_i^{quality}}{r_ip_i^{c(q,i)}}+\frac{p_i^{quality}}{r_i^{quality}}+\frac{p_i^{quality}}{p_i^{delay}}+\frac{p_ip_i^{quality}}{r_ip_i^{delay}}+1}$$

$$y_i = \frac{E_{eff}}{E_{tot}} = \frac{p_i^{c(i,q)}p_i^{delay}(1-\gamma_w)+p_i^{quality}\left(p_i^{c(i,q)}+p_i^{delay}\right)(1-\gamma_o)}{p_i^{c(i,q)}p_i^{delay}+p_i^{c(i,q)}p_i^{quality}+p_i^{quality}p_i^{delay}} \tag{5.63}$$

5.3 Markov model of machine without quality failure

The last case is the simplest one and is related to the machine that is not subject to out of control. Therefore such a machine can failed either due to operational failure or failures of other machines.

This kind of machine can be modeled by the Markov chain represented in figure 5.3.

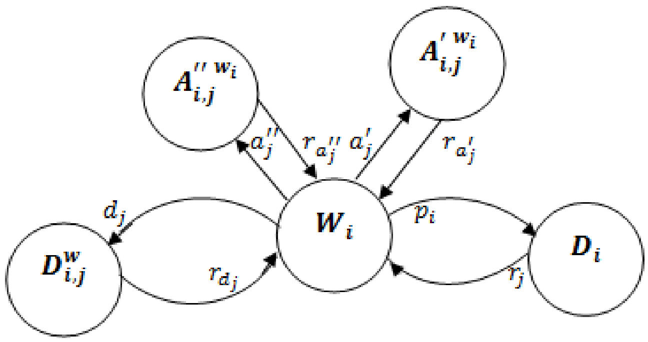

Figure 5.3: Markov chain of machine without quality failure

Corresponding equation are mentioned below:

$$\pi(D_i^w)r_i = \pi(w_i)p_i \tag{5.64}$$

$$\pi(A_{i,j}^{'\,w_i})r_{a_j'} = \pi(w_i)a_j' \qquad \forall\ M_j \in M_Q - \{M_i\} \tag{5.65}$$

$$\pi(A_{i,j}^{''\,w_i})r_{a_j''} = \pi(w_i)a_j'' \qquad \forall\ M_j \in M_Q - \{M_i\} \tag{5.66}$$

$$\pi(D_{i,j}^w)\, r_{d_j} = \pi(w_i)\, d_j \qquad \forall\ M_j\ - \{M_i\} \tag{5.67}$$

Steady states probabilities are equal to:

$$\pi(w_i) = \frac{1}{\sum_{j=1}^{N-\{M_i\}}\left(\frac{a_j'}{r_{a_j'}}+\frac{a_j''}{r_{a_j''}}+\frac{d_j}{r_{d_j}}\right)+\frac{p_i}{r_i}+1} \tag{5.68}$$

$$\pi(D_i^w) = \pi(w_i)\ \frac{p_i}{r_i} \tag{5.69}$$

$$\pi(A_{i,j}^{'\,w_i}) = \pi(w_i)\ \frac{a_j'}{r_{a_j'}} \tag{5.70}$$

$$\pi(A_{i,j}^{''\,w_i}) = \pi(w_i)\ \frac{a_j^{''}}{r_{a_j^{''}}} \tag{5.71}$$

$$\pi(D_{i,j}^{w}) = \pi(w_i)\ \frac{d_j}{r_{d_j}} \tag{5.72}$$

5.4 Transition rates

In this section unknown transition rates $a_j^{'}$, $r_{a_j^{'}}$, $a_j^{''}$, $r_{a_j^{''}}$, d_j and r_{d_j} are determined. Since the result depends on Markov chain of machine M_j i.e. the machine that has stopped other machines, and therefore on type of machine M_j, i.e. locally monitored machine, remotely monitored machine and without quality failure machine, these transition rates are evaluated for each type separately in sections 5.4.1 to 5.4.3.

5.4.1 Transition rates when M_j is locally monitored

Transition rate $a_j^{'}$

To find transition rate $a_j^{'}$ i.e. the rate that machine M_i is stopped due to quality maintenance of locally monitored machineM_j, we should notice that when this transition happens all machines of the line, including machine M_j are working. Probability of finding M_j operational is $\pi(w_j)$ + $\pi(o_j)$. Moreover, machine M_j must be out of control. Therefore it should be in the state o_j. In other words the aim is finding machine M_j in state o_j, knowing that it is operational. Using the Bayes theorem it is possible to find this probability.

Probability of finding machine M_j in state o_j | machine M_j is operational= $\frac{\pi(o_j)}{\pi(o_j) \cup \pi(w_j)}$

From equation 5.11 it is known that:

$$\pi(o_j) = \pi(w_j)\, \frac{p_j^{quality}}{p_j^{c(q,i)}}$$

Substituting $\pi(o_j)$ from this equation:

$$\frac{\pi(o_j)}{\pi(o_j)\cup\pi(w_j)} = \frac{\pi(w_j)\frac{p_j^{quality}}{p_j^{c(q,i)}}}{\pi(w_j)\left(1+\frac{p_j^{quality}}{p_j^{c(q,i)}}\right)} = \frac{\frac{p_j^{quality}}{p_j^{c(q,i)}}}{1+\frac{p_j^{quality}}{p_j^{c(q,i)}}} \tag{5.73}$$

Multiplying numerator and denominator by $p_j^{c(q,i)}$

Probability of finding machine M_j in state o_j | machine is operational=

$$= \frac{p_j^{quality}}{p_j^{quality}+p_j^{c(q,i)}} \tag{5.74}$$

Multiplying this probability by $p_j^{c(q,i)}$i.e. the rate that out of control can be detected, transition rate $a_j^{'}$ is equal to:

$$a_j^{'} = \frac{p_j^{quality}p_j^{c(q,i)}}{p_j^{quality}+p_j^{c(q,i)}} \tag{5.75}$$

Transition rate $\mathbf{r_{a_j^{'}}}$

As soon as machine M_j is repaired, all machines would be operational. Therefore transition rate to the operational state is equal to transition rate of repairing out of control of machineM_j.

$$r_{a_j^{'}} = r_j^{quality} \tag{5.76}$$

Transition rate $\mathbf{a_j^{''}}$

Since transition to $A_{i,j}^{''\,w_i}$, takes place when a false alarm stops machineM_j, based on the same reasoning as what mentioned for calculation of transition rate $a_j^{'}$, this transition can only happens when machine M_j is operational and operates in control.

Probability of finding machine M_j in state w_j | machineM_j is operational=

$$= \frac{\pi(w_j)}{\pi(o_j) \cup \pi(w_j)} \tag{5.77}$$

Substituting $\pi(o_j)$ from equation 5.11

$$\frac{\pi(w_j)}{\pi(o_j) \cup \pi(w_j)} = \frac{\pi(w_j)}{\pi(w_j)\left(1+\frac{p_j^{quality}}{p_j^{c(q,i)}}\right)} = \frac{1}{1+\frac{p_j^{quality}}{p_j^{c(q,i)}}} \tag{5.78}$$

Multiplying numerator and denominator by $p_j^{c(q,i)}$

probability of finding machine M_j in state w_j | machine is operational = $\frac{p_j^{c(q,i)}}{p_j^{quality}+p_j^{c(q,i)}}$

(5.79)

Multiplying this probability by p_j^{false} i.e. the rate that machine M_j stops due to false alarm signal, transition rate a_j'' is equal to:

$$a_j'' = \frac{p_j^{false} p_j^{c(q,i)}}{p_j^{quality}+p_j^{c(q,i)}} \tag{5.80}$$

Transition rate $r_{a_j''}$

Machine M_i is back to operational state when machine M_j is restarted after false alarm with the rate of r_j^{false} . Therefore:

$$r_{a_j''} = r_j^{false} \tag{5.81}$$

Transition rate d_j

Since transition to $D_{i,j}^{w}$ occur when machine M_j faces with an operational failure, transition rate d_j is equal to the operational failure rate of machineM_j .

$$d_j = p_j \tag{5.82}$$

Transition rate r_{d_j}

Similarly transition rate r_{d_j} is equal to operational failure repair rate of machine M_j.

$$r_{d_j} = r_j \tag{5.83}$$

5.4.2 Transition rates when M_j is remotely monitored

Transition rate a_j'

With rationale similar to the one mentioned for locally monitored machine, it is possible say that when transition to the state $A_{i,j}'$ happens, M_j should be out of control and out of control must be detectable. Therefore probability of finding machine M_j in the state 0_j^2 must be identified provided that it is operational.

Probability of finding machine M_j in state 0_j^2 | machine M_j is operational=

$$\frac{\pi(0_j^2)}{\pi(0_j^1)\cup\pi(0_j^2)\cup\pi(w_j)} = \frac{\pi(0_j^2)}{\pi(0_j^1)\cup\pi(0_j^2)\cup\pi(w_j)} \tag{5.84}$$

Substituting $\pi(0_j^1)$ and $\pi(0_j^2)$ from equation 5.45 and 5.46 in equation 5.84

$$\frac{\pi(0_j^2)}{\pi(0_j^1)\cup\pi(0_j^2)\cup\pi(w_j)} = \frac{\pi(w_j)\frac{p_j^{quality}}{p_j^{c(q,i)}}}{\pi(w_j)(1+\frac{p_j^{quality}}{p_j^{c(q,i)}}+\frac{p_j^{quality}}{p_j^{delay}})} = \frac{\frac{p_j^{quality}}{p_j^{c(q,i)}}}{(1+\frac{p_j^{quality}}{p_j^{c(q,i)}}+\frac{p_j^{quali\ ty}}{p_j^{delay}})} \tag{5.85}$$

Multiplying numerator and denominator by $p_j^{c(q,i)}p_j^{delay}$

Probability of finding machine M_j in state 0_j^2 | machine is operational=

$$=\frac{p_j^{delay}\ p_j^{quality}}{(p_j^{c(q,i)}p_j^{delay}+p_j^{quality}\ p_j^{delay}+p_j^{quality}\ p_j^{c(q,i)}} \qquad (5.86)$$

Multiplying this probability by $p_j^{c(q,i)}$i.e. the rate that out of control can be detected, transition rate $a_j^{'}$ is equal to:

$$a_j^{'}=\frac{p_j^{c(q,i)}p_j^{delay}p_j^{quality}}{(p_j^{c(q,i)}p_j^{delay}+p_j^{quality}p_j^{delay}+p_j^{quality}p_j^{c(q,i)}} \qquad (5.87)$$

Transition rate $\mathbf{r_{a_j^{'}}}$

Transition rate of $r_{a_j^{'}}$ is simply equal to transition rate of repairing out of control of machine M_j

$$r_{a_j^{'}}=r_j^{quality} \qquad (5.88)$$

Transition rate $\mathbf{a_j^{''}}$

In order to find $a_j^{''}$, probability of finding machine M_j working in control knowing that it is operational must be determined.

Probability of finding machine M_j in state w_j | machineM_j is operational=

$$\frac{\pi(w_j)}{\pi(0_j^1)\cup\pi(0_j^2)\cup\pi(w_j)}=\frac{\pi(w_j)}{\pi\left(0_j^1\right)+\pi\left(0_j^2\right)+\pi(w_j)} \qquad (5.89)$$

Substituting $\pi\left(0_j^1\right)$ and $\pi\left(0_j^2\right)$ from equation 5.45 and 5.46 in equation 5.89:

$$\frac{\pi(w_j)}{\pi\left(0_j^1\right)+\pi\left(0_j^2\right)+\pi(w_j)}=\frac{\pi(w_j)}{\pi(w_j)(1+\frac{p_j^{quality}}{p_j^{c(q,i)}}+\frac{p_j^{quality}}{p_j^{delay}})}=\frac{1}{(1+\frac{p_j^{quality}}{p_j^{c(q,i)}}+\frac{p_j^{quality}}{p_j^{delay}})} \qquad (5.90)$$

Multiplying numerator and denominator by $p_j^{c(q,i)}p_j^{delay}$, the probability is given by:

Probability of finding machine M_j in state $w_j |$ machine M_j is operational =

$$=\frac{p_j^{c(q,i)} p_j^{delay}}{p_j^{c(q,i)} p_j^{delay} + p_j^{quality}\ p_j^{delay} + p_j^{quality}\ p_j^{c(q,i)}} \tag{5.91}$$

multiplying this probability by p_j^{false} i.e. the rate that machine M_j stops due to false alarm signal, transition rate $a_j^{''}$ is equal to:

$$a_j^{''} = \frac{p_j^{c(q,i)} p_j^{delay}\ p_j^{false}}{p_j^{c(q,i)} p_j^{delay} + p_j^{quality}\ p_j^{delay} + p_j^{quality}\ p_j^{c(q,i)}} \tag{5.92}$$

Transition rate $\mathbf{r}_{\mathbf{a}_j^{''}}$

Transition rate from state $A_{i,j}^{''}$ is equal to the rate that M_j is restarted after the false alarm

$$\boldsymbol{r}_{\boldsymbol{a}_j^{''}} = r_j^{false} \tag{5.93}$$

Transition rate $\mathbf{d_j}$

Transition rate to state $D_{i,j}$ is equivalent to the rate that machine M_j fails in operational mode.

$$d_j = p_j \tag{5.94}$$

Transition rate $\mathbf{r_{d_j}}$

machine M_i leaves the state $D_{i,j}$ when operational failure of machine M_j is repaired. Therefore:

$$r_{d_j} = r_j \tag{5.95}$$

5.4.3 Transition rates when M_j is not subject to out of control

If machine M_j does not have any quality failure, it can stop machine M_i when it fails due to an operational failure. In other words, in this case only transition rates d_j and r_{d_j} exist. They are equal to failure and repair rate of machine M_j respectively.

$$d_j = p_j \tag{5.96}$$

$$r_{d_j} = r_j \tag{5.97}$$

However to make the presentation of general formulation simple, it is possible to assume transition rates $a_j^{'}$ and $a_j^{''}$ equal to zero and $r_{a_j^{'}}$ and $r_{a_j^{''}}$ equal to 1 and the assumption does not affect final result...

5.5 Performance measures of the system

In this part, performance measures for a production line formed by N machine is calculated according to what explained in last sections.

Considering conservation of the flow in a serial manufacturing line, total throughput of every machine when embedded in the system is equal to total throughput of the whole system. Therefore Focusing on specific machine M_i and substituting transition rates determined in section 5.4 (according to type of inspection and machine parameters), $E_{tot}^{M_i}$ and consequently total throughput of the line is found. As a result, total throughput of the line is given by equations 5.98 to 5.100 where M_i is locally monitored, remotely monitored or not subjects to out of control respectively and $a_j^{'}$, $r_{a_j^{'}}$, $a_j^{''}$, $r_{a_j^{''}}$, d_j and r_{d_j} are defined in equations 5.76,5.76,5.80,5.81,5.82 and 5.83 according to inspection and machine type

$$E_{tot}^{line} = \frac{(1+\frac{p_i^{quality}}{p_i^{c(i,q)}})}{\left(1+\frac{p_i^{quality}}{p_i^{c(q,i)}}\right)\sum_{j=1}^{N-\{M_i\}}\left(\frac{a_j^{'}}{r_{a_j^{'}}}+\frac{a_j^{''}}{r_{a_j^{''}}}+\frac{d_j}{r_{d_j}}\right)+\frac{p_i^{false}}{r_i^{false}}+\frac{p_i^{quality}}{p_i^{c(q,i)}}+\frac{p_i}{r_i}+\frac{p_i p_i^{quality}}{r_i p_i^{c(q,i)}}+\frac{p_i^{quality}}{r_i^{quality}}+1} \tag{5.98}$$

$$E_{tot}^{line} = \qquad (5.99)$$

$$\frac{1+\frac{p_i^{quality}}{p_i^{delay}}+\frac{p_i^{quality}}{p_i^{c(i,q)}}}{\left(1+\frac{p_i^{quality}}{p_i^{c(q,i)}}+\frac{p_i^{quality}}{p_i^{delay}}\right)\sum_{j=1}^{N-\{M_i\}}\left(\frac{a_j'}{r_{a_j'}}+\frac{a_j''}{r_{a_j''}}+\frac{d_j}{r_{d_j}}\right)+\frac{p_i^{false}}{r_i^{false}}+\frac{p_i^{quality}}{p_i^{c(q,i)}}+\frac{p_i}{r_i}+\frac{p_i p_i^{quality}}{r_i p_i^{c(q,i)}}+\frac{p_i^{quality}}{r_i^{quality}}+\frac{p_i^{quality}}{p_i^{delay}}+\frac{p_i p_i^{quality}}{r_i p_i^{delay}}+1}$$

$$E_{tot}^{line} = \frac{1}{\sum_{j=1}^{N-\{M_i\}}\left(\frac{a_j'}{r_{a_j'}}+\frac{a_j''}{r_{a_j''}}+\frac{d_j}{r_{d_j}}\right)+\frac{p_i}{r_i}+1} \qquad (5.100)$$

On the other hand, system yield is the product of yield of machines in the line and is given by:

$$y_{tot} = \prod_{i=1}^{N} y_i \qquad (5.101)$$

Finally effective throughput of the system is determined as follow:

$$E_{eff}^{line} = E_{tot}^{line} \,.\, y_{tot} \qquad (5.102)$$

Chapter Six

6 Algorithm of Optimizing Inspection Allocation and Assignment

Here the objective is optimally allocate and assign limited number of inspection stations in a serial un-buffered production line with the aim of maximizing system effective throughput. Therefore the problem can be divided in two correlated parts, allocation and assignment. Allocation problem takes into account the location of inspection stations in the production line while assignment problem is related to assigning inspection stations to different machines.

The system model that explained in chapter 5 has been used in this chapter. Therefore both quality and logistic aspects have been considered to avoid making locally optimized solutions that might highlight one aspect and neglect the other one.

In this chapter, first problem is mathematically formalized, necessary notations are presented and afterwards the algorithm to find the optimal solution is explained.

6.1 Problem formalization

6.1.1 Summary of notations

N: total number of machines

$\boldsymbol{M_Q}$: Subset of machines subjects to out of control

X: is a N× N matrix, in which the element $x_{i,j}$ is equal to 1 if machine M_i is inspected immediately downstream machine M_j and 0 otherwise.

Y: is a 1×N vector, in which $y_{1,j}$ is equal to 1 if for each 1≤ j≤ N, $\sum_{i=1}^{N} x_{i,j} \geq 1$ and 0 otherwise.

$\boldsymbol{\omega}$: Number of available inspection stations

Objective function:

Max F $=\mathrm{E}_{\mathrm{eff}}^{\mathrm{line}}$

s.t. $\sum_{\mathrm{j}=1}^{\mathrm{N}} \mathrm{y}_{1,\mathrm{j}}=\omega$

$\sum_{\mathrm{j}=1}^{\mathrm{N}} \mathrm{x}_{\mathrm{i,j}}=1 \quad \forall \ \mathrm{M}_{\mathrm{i}} \in \mathrm{M}_{\mathrm{Q}}$

The first constraint is related to the number of available inspection stations. If number of allocated stations was greater or less than available one, this constraint would not be satisfied.

The second constraint is to assure that all machines with quality failures are inspected.

6.2 Proposed algorithm

By solving the system in chapter 5, effective throughput of the system is known and used to develop the algorithm.

To initialize the algorithm, all the machines are assumed to be locally inspected. At the first step, starting from the first machine, machines are selected respectively, their local inspection is removed and inspection is shifted to the first available inspection station downstream the machine.

For new configuration total throughput of the line and consequently updated values of p_i^{delay}, called $p_{i_{new}}^{delay}$ is calculated for all the machines:

$$p_{i_{new}}^{delay} = \frac{\mu . E_{tot}}{k_i} \qquad (6.1)$$

Where k_i is the number of machines between machine M_i and the stage that it is inspected and μ is production rate of machines.

Change of p_i^{delay} respect to the last configuration is determined as:

$$\Delta p_i^{delay} = p_{i_{new}}^{delay} - p_{i_{initial}}^{delay} \tag{6.2}$$

Using the Jacobean of effective throughput of system, effect of shifting the inspection point downstream is estimated:

$$\Delta E_{eff}^{line} = \text{jacobian } (E_{eff}^{line})* \Delta p_{delay} \tag{6.3}$$

This step is repeated for all machines and effect of moving inspection point compared to the last configuration is estimated as explained above. Finally the one with the lower negative effect (or higher positive effect) on effective throughput of the system is selected and correlated inspection point is shifted to first inspection station downstream the machine.

In next step, shifting the inspection point of selected machine downstream continues, and effective throughput is evaluated. If any improvement observed in system effective throughput compared to the last configuration, the shift continues. The process is repeated till system effective throughput starts diminishing. At this point moving is stopped and the last configuration substitute the initial one.

During the next step, number of existing inspection stations in the system is updated and constraint related to number of available inspection stations is checked. If the constraint is respected, the algorithm is finished and the last configuration is selected as the optimal one. Otherwise it starts from the first step. These steps repeated iteratively, till the constraint linked to number of available stations is satisfied. Figure 6.1 represents the flowchart of the algorithm.

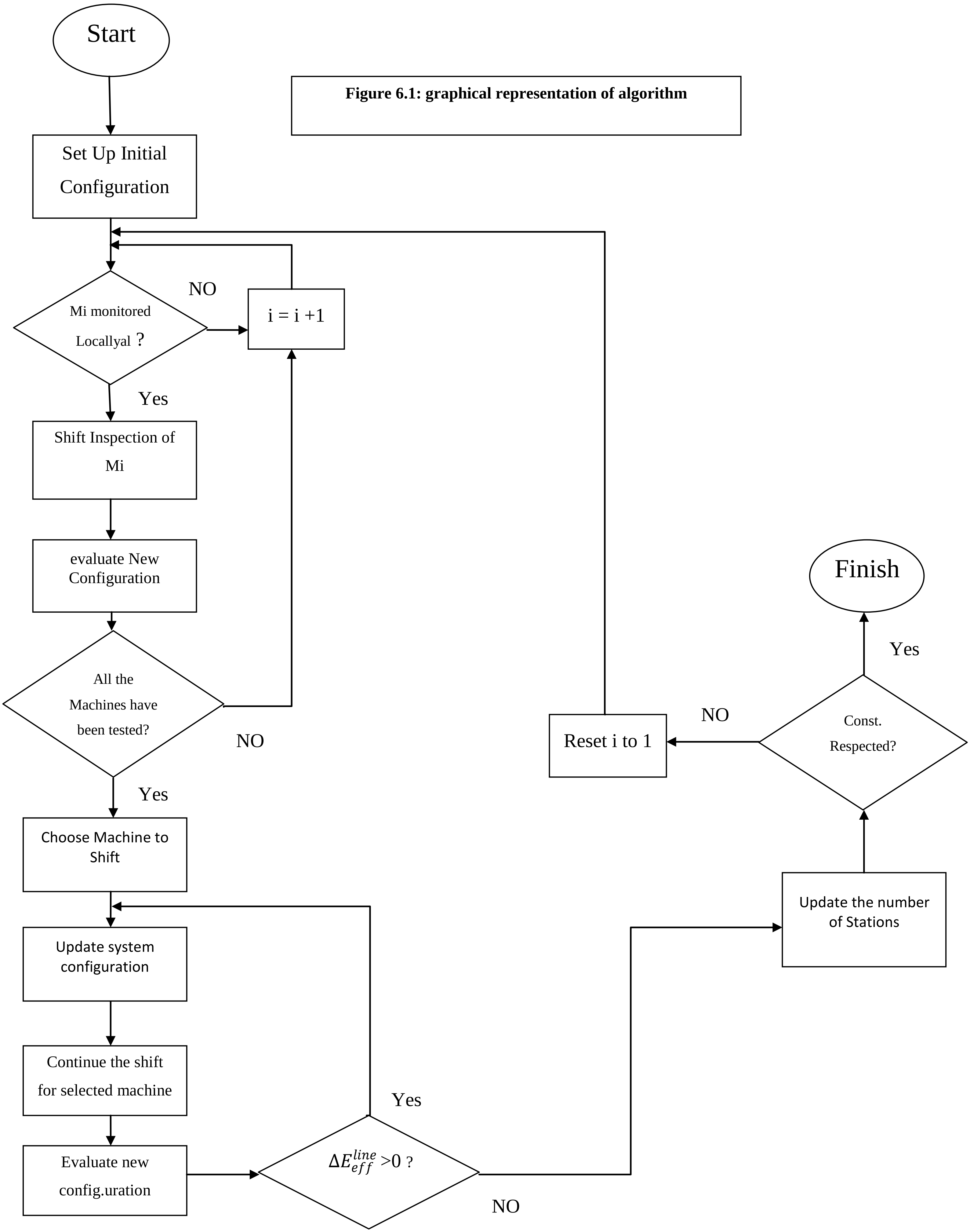

Figure 6.1: graphical representation of algorithm

Chapter Seven

7 Numerical Results

7.1 Accuracy testing

The proposed method has been implemented in Matlab and the results are compared with the results coming from extensive search which solve the system for all possible alternatives to find the optimal solution.

In order to validate the algorithm, it has been applied on 50 different systems, composed of five, six or seven machines where all machines subjects to out of control. In case of seven machine line number of available inspection stations is limited to four while for five and six machine line it is assumed to be three. Data related to the cases is fully reported in the appendix.

For each case, optimal configuration, corresponding effective throughput, required time and number of iterations to reach the solution for extensive search and proposed algorithms have been reported in table7.1. It is worth to notice that in solution vector, the number assigned to each machine is the stage in which it is inspected. For instance if number 3 was assigned to the first machine it means that the first machine is inspected after the third one. Obviously the last machine is always monitored locally.

Table7.1: resuls of proposed method and extensive search

# of case		optimal E_{eff}	optimal allocation	relative error	required time	Difference In time	Percente of the difference	#of evaluated systems
1	complete search	0.3396	2 2 4 4 5 7 7	0	41 min	40:49 min	0.995	2416
	proposed method	0.3396	2 2 4 4 5 7 7		11 s			15

2	complete search	0.3117	1 3 3 5 5 7 7	0.0045	43 min	42:48 min	0.995	2416
	proposed method	0.3103	1 2 5 5 5 7 7		12 s			15
3	complete search	0.2698	7 3 3 5 5 6 7	0	45 min	44:50 min	0.996	2416
	proposed method	0.2698	7 3 3 5 5 6 7		10 s			9
4	complete search	0.3662	2 2 3 5 5 7 7	0	43 min	42:49 min	0.995	2416
	proposed method	0.3662	2 2 3 5 5 7 7		11 s			15
5	complete search	0.3596	7 2 4 4 5 7 7	0	43 min	42:46 min	0.994	2416
	proposed method	0.3596	7 2 4 4 5 7 7		14 s			9
6	complete search	0.286	1 2 7 7 5 7 7	0	45 min	44:46 min	0.995	2416
	proposed method	0.286	1 2 7 7 5 7 7		14 s			15
7	complete search	0.3225	1 2 4 4 7 7 7	0	45 min	44:47 min	0.995	2416
	proposed method	0.3225	1 2 4 4 7 7 7		13 s			15
8	complete search	0.3128	1 2 3 7 7 7 7	0	47 min	46:48 min	0.996	2416
	proposed method	0.3128	1 2 3 7 7 7 7		12 s			15
9	complete search	0.241	7 7 7 4 5 6 7	0	43 min	42:48 min	0.995	2416
	proposed method	0.241	7 7 7 4 5 6 7		12 s			15
10	complete search	0.2448	2 2 4 4 6 6 7	0	42 min	41:47 min	0.995	2416
	proposed method	0.2448	2 2 4 4 6 6 7		13 s			15

11	complete search	0.3784	2 2 3 5 5 7 7		42 min		0.995	2416
	proposed method	0.3784	2 2 3 5 5 7 7	0	11 s	41:49 min		15
12	complete search	0.3556	3 3 3 5 5 6 7		43 min		0.995	2416
	proposed method	0.3556	3 3 3 5 5 6 7	0	14 s	42:46 min		15
13	complete search	0.336	4 4 4 4 5 6 7		46 min		0.996	2416
	proposed method	0.336	4 4 4 4 5 6 7	0	12 s	45:48 min		15
14	complete search	0.3676	7 7 3 5 5 6 7		44 min		0.995	2416
	proposed method	0.3676	7 7 3 5 5 6 7	0	12 s	43:48 min		4
15	complete search	0.3289	7 3 3 7 5 6 7		43 min		0.995	2416
	proposed method	0.3289	7 3 3 7 5 6 7	0	12 s	42:48 min		4
16	complete search	0.3387	7 3 3 7 5 6 7		43 min		0.995	2416
	proposed method	0.3387	7 3 3 7 5 6 7	0	15 s	42:45 min		15
17	complete search	0.2714	1 3 3 5 5 7 7		42 min		0.995	2416
	proposed method	0.2714	1 3 3 5 5 7 7	0	12 s	41:48 min		15
18	complete search	0.3156	1 3 3 5 5 7 7		44 min		0.995	2416
	proposed method	0.3156	1 3 3 5 5 7 7	0	13 s	43:48 min		15
19	complete search	0.2878	1 3 3 5 5 7 7		44 min		0.993	2416
	proposed method	0.2878	1 3 3 5 5 7 7	0	16 s	43:44 min		15

20	complete search	0.3404	2 2 7 5 5 6 7	0	43 min	42:47 min	0.995	2416
	proposed method	0.3404	2 2 7 5 5 6 7		13 s			15
21	complete search	0.3649	1 2 4 4 7 7 7	0	42 min	41:48 min	0.995	2416
	proposed method	0.3649	1 2 4 4 7 7 7		12 s			15
22	complete search	0.3292	1 2 3 7 7 7 7	0	43 min	42:48 min	0.995	2416
	proposed method	0.3292	1 2 3 7 7 7 7		12 s			15
23	complete search	0.298	1 3 3 4 7 7 7	0	43 min	42:46min	0.995	2416
	proposed method	0.298	1 3 3 4 7 7 7		14 s			15
24	complete search	0.3324	3 3 3 4 5 7 7	0.002	45 min	44:47 min	0.995	2416
	proposed method	0.3316	2 2 4 4 5 7 7		13 s			15
25	complete search	0.378	2 2 5 5 5 6	0	3:15 min	3:08 min	0.964	302
	proposed method	0.378	2 2 5 5 5 6		7 s			9
26	complete search	0.3536	2 2 5 5 5 6	0	3:25 min	3:16 min	0.91	302
	proposed method	0.3536	2 2 5 5 5 6		9 s			9
27	complete search	0.3614	2 2 4 4 6 6	0	3:25 min	3:16 min	0.91	302
	proposed method	0.3614	2 2 4 4 6 6		9 s			9
28	complete search	0.3567	2 2 3 6 6 6		3:11 min	3:04 min	0.964	302
	proposed method	0.3567	2 2 3 6 6 6	0	7 s			9

29	complete search	0.2995	2 2 4 4 6 6	0	3.13 min	3:07 min	0.964	302
	proposed method	0.2995	2 2 4 4 6 6		6 s			9
30	complete search	0.3768	2 2 4 4 6 6	0.0085	3.10 min	3:04 min	0.964	302
	proposed method	0.3736	1 3 3 6 6 6		6 s			9
31	complete search	0.3785	4 4 4 4 5 6	0	3.15 min	3:08 min	0.964	302
	proposed method	0.3785	4 4 4 4 5 6		7 s			9
32	complete search	0.2624	2 2 4 4 6 6	0	3.14 min	3:07 min	0.964	302
	proposed method	0.2624	2 2 4 4 6 6		7 s			9
33	complete search	0.3207	1 3 3 6 6 6	0	3.19 min	3:11 min	0.964	302
	proposed method	0.3207	1 3 3 6 6 6		8 s			9
34	complete search	0.3424	6 3 3 5 5 6	0	3.23 min	3:16 min	0.964	302
	proposed method	0.3424	6 3 3 5 5 6		7 s			4
35	complete search	0.2903	2 2 4 4 6 6	0	3.15 min	3:08 min	0.964	302
	proposed method	0.2903	2 2 4 4 6 6		7 s			9
36	complete search	0.3745	2 2 4 4 6 6	0.0072	3.20 min	3:11 min	0.964	302
	proposed method	0.3718	2 2 5 5 5 6		9 s			9
37	complete search	0.3362	3 3 3 5 5 6	0	3.15 min	3:08 min	0.964	302
	proposed method	0.3362	3 3 3 5 5 6		7 s			9

38	complete search	0.3611	6 6 4 4 5 6		3.17 Min	3:10 min	0.964	302
	proposed method	0.3611	6 6 4 4 5 6	0	7 s			3
39	complete search	0.3366	2 2 4 4 5	0	16 s	13 s	0.813	66
	proposed method	0.3366	2 2 4 4 5		3 s			7
40	complete search	0.374	2 2 3 5 5	0	18 s	15 s	0.813	66
	proposed method	0.374	2 2 3 5 5		3 s			7
41	complete search	0.4143	2 2 3 5 5	0	18 s	15 s	0.813	66
	proposed method	0.4143	2 2 3 5 5		3 s			7
42	complete search	0.4004	2 2 3 5 5	0	19 s	15 s	0.813	66
	proposed method	0.4004	2 2 3 5 5		4 s			7
43	complete search	0.4666	5 3 3 4 5	0	18 s	13 s	0.813	66
	proposed method	0.4666	5 3 3 4 5		5 s			3
44	complete search	0.4057	2 2 4 4 5	0	17 s	12 s	0.813	66
	proposed method	0.4057	2 2 4 4 5		5 s			7
45	complete search	0.4049	1 3 3 5 5	0	17 s	14 s	0.813	66
	proposed method	0.4049	1 3 3 5 5		3 s			7
46	complete search	0.5089	5 5 3 4 5	0	16 s	13 s	0.813	66
	proposed method	0.5089	5 5 3 4 5		3 s			3

47	complete search	0.5369	5 5 3 4 5	0	19 s	16 s	0.813	66
	proposed method	0.5369	5 5 3 4		3 s			3
48	complete search	0.3804	3 3 3 4 5	0	19 s	16 s	0.813	66
	proposed method	0.3804	3 3 3 4 5		3 s			7
49	complete search	0.4741	2 2 4 4 5	0	18 s	14 s	0.813	66
	proposed method	0.4741	2 2 4 4 5		4 s			7
50	complete search	0.4658	2 2 4 4 5	0	17 s	14 s	0.813	66
	proposed method	0.4658	2 2 4 4 5		3 s			7

These results confirm the high accuracy and speed of the proposed method. However in 4 out of 50 cases (8%) it does not provide the exact solution but it provides the second best solution which is quit close to the optimal one and the small difference does not affect the system performance. On the other hand, the required time and computational effort is extremely low compared to the extensive search.

As a matter of fact, since increasing the number of machines, both total number of alternatives and therefore number of iterations that is required by extensive search to reach to the optimal solution and complexity of the system is rapidly increasing, performance of the method is quit high for complex systems.

7.2 Robustness of the method

In another experiment to test the reliability of the proposed method to deal with complex systems, a production line composed of 20 machines is selected and number of available inspection station is fixed to 13. Extensive search could not provide the results after three hours and it is stopped. Actual time is estimated to be much longer. However the proposed algorithm reaches to the solution within 13 minutes.

The machine parameters and given solution are represented in table 8.2.

Table7.2. Machine and control chart parameters.

	p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
M1	0.03	0.68	0.05	0.64	0.033	0.173
M2	0.042	0.6	0.05	0.7	0.03	0.16
M3	0.041	0.75	0.03	0.77	0.022	0.222
M4	0.034	0.8	0.01	0.76	0.031	0.281
M5	0.057	0.65	0.06	0.69	0.022	0.192
M6	0.05	0.8	0.06	0.79	0.032	0.432
M7	0.043	0.72	0.01	0.63	0.033	0.403
M8	0.06	0.69	0.07	0.62	0.03	0.12
M9	0.043	0.72	0.03	0.77	0.022	0.302
M10	0.052	0.8	0.01	0.76	0.031	0.301
M11	0.072	0.63	0.06	0.69	0.022	0.242
M12	0.05	0.8	0.02	0.79	0.032	0.312
M13	0.042	0.64	0.05	0.65	0.033	0.283
M14	0.06	0.65	0.04	0.8	0.03	0.25
M15	0.04	0.73	0.033	0.77	0.022	0.282
M16	0.045	0.8	0.022	0.7	0.031	0.311
M17	0.067	0.66	0.064	0.64	0.022	0.362
M18	0.043	0.81	0.03	0.81	0.032	0.362
M19	0.03	0.8	0.042	0.75	0.03	0.25
M20	0.02	0.73	0.06	0.69	0.03	0.28

	$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
C1	1	0	0.05	0.15	0.8	
C2	1	0	0.02	0.15	0.8	
C3	1	0	0.02	0.15	0.8	
C4	1	0	0.03	0.05	0.8	
C5	1	0	0.04	0.6	0.8	
C6	1	0	0.02	0.2	0.8	
C7	1	0	0.06	0.6	0.8	
C8	1	0	0.02	0.05	0.8	
C9	1	0	0.05	0.6	0.8	
C10	1	0	0.02	0.05	0.8	
C11	1	0	0.02	0.6	0.8	
C12	1	0	0.02	0.2	0.8	
C13	1	0	0.06	0.6	0.8	
C14	1	0	0.01	0.05	0.8	
C15	1	0	0.02	0.6	0.8	
C16	1	0	0.02	0.05	0.8	
C17	1	0	0.08	0.6	0.8	
C18	1	0	0.07	0.2	0.8	
C19	1	0	0.03	0.1	0.8	
C20	1	0	0.04	0.1	0.8	

Optimal allocation and association found by the proposed algorithm is as follow.

[2 2 3 5 5 6 7 9 9 11 11 13 13 14 15 17 17 18 20 20]

Optimal allocation is represented in figure 8.1.

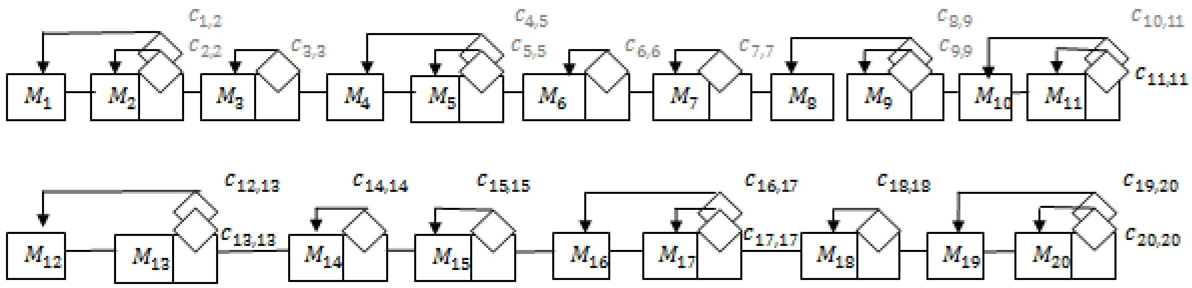

Figure 7.1: representation of optimal allocation

7.3 System behavior analysis

The aim of the last experiment is to test the effect of different allocations on the effective throughput of the system. For this purpose a line including seven machines has been chosen. For this system, number of applied inspection station has been changed from one to seven. Machine parameters and effective throughput of different configurations are presented subsequently.

Table7.3. Machine and control chart parameters.

	p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
M1	0.03	0.5	0.03	0.7	0.01	0.23
M2	0.03	0.6	0.04	0.63	0.03	0.3
M3	0.04	0.58	0.05	0.68	0.02	0.37
M4	0.05	0.56	0.06	0.65	0.02	0.27
M5	0.04	0.53	0.04	0.68	0.02	0.22
M6	0.02	0.57	0.03	0.62	0.01	0.24
M7	0.04	0.55	0.02	0.65	0.02	0.25

	$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
C1	1	0	0.02	0.2	0.7	
C2	1	0	0.02	0.22	0.7	
C3	1	0	0.03	0.31	0.72	
C4	1	0	0.02	0.1	0.76	
C5	1	0	0.03	0.15	0.8	
C6	1	0	0.02	0.17	0.9	
C7	1	0	0.02	0.12	0.7	

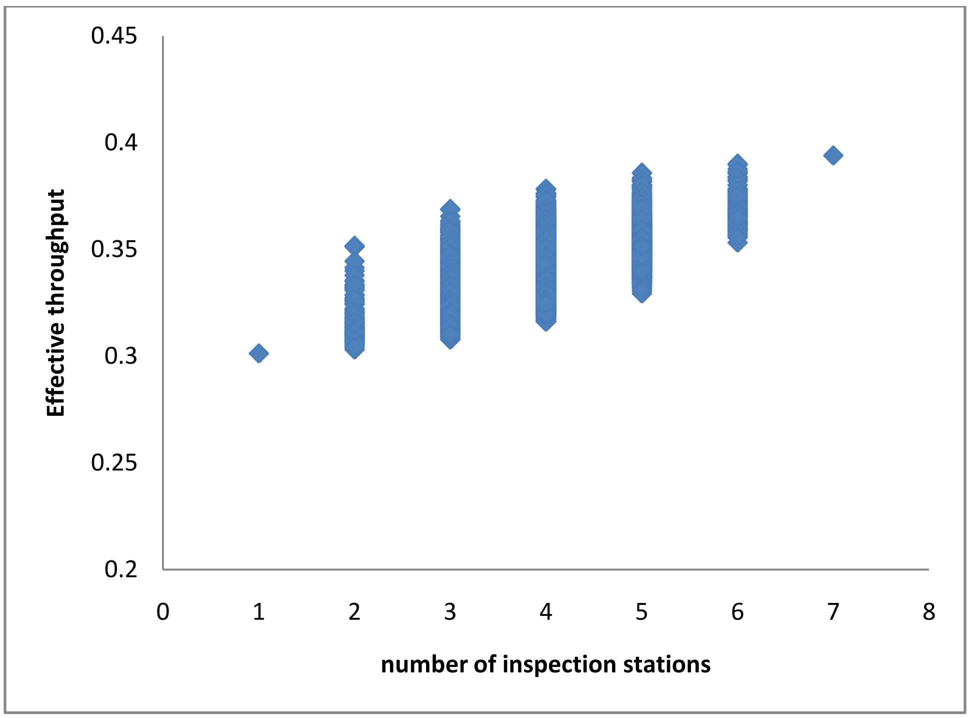

Figure 7.2: Effective throughput vs. number of inspection station

As it is shown in the graph, inspection station allocation and assignment greatly affect the effective throughput of the system. Dispersion between the best and the worst configuration, first increases and then decreases while increasing the number of inspection stations. The maximum happens for the system with 4 inspection stations (+6.2%).

Moreover, as it is visible from figure7.2, since the slope of the optimal curve has diminishing trend while numbers of inspection stations increases, for higher number of inspection stations, adding new inspection stations has minor effect on system throughput compared to lower number of inspection stations. For instance increasing the available inspection stations from one to two raises the effective throughput from 0.3013 to 0.3517 however effect of the adding another station raises system throughput from 0.3517 to 0.3689. Table 7.4 includes the optimal throughput for different number of inspection stations.

Table 7.4: optimal effective throughput vs. number of inspection stations

Number of inspection stations	1	2	3	4	5	6	7
Optimal Effective throughput	0.3013	0.3517	0.3689	0.3784	0.3858	0.39	0.39398

Final consideration is that using lower number of inspection stations which are properly allocated and assigned, may lead to higher effective throughput than using higher number of inspection stations which are poorly allocated. For instance in case of applying three inspection stations, in the optimal configuration, which is [3 3 3 5 5 7 7] system reaches to the effective throughput equal to 0.3689. Although applying 5 inspection stations suboptimaly allocated in the configuration of [1 2 3 7 6 7 7] provides effective throughput of 0.3669, which is slightly lower than previous one. In fact finding the optimal allocation, it is possible to remove two inspection stations and still remain in higher level of system throughput. The consequence is cost optimization only through finding the optimal distribution of inspection stations. This fact verifies the importance of proper allocation and assignment of inspection effort in maximizing the system throughput as well as minimizing the inspection costs.

Chapter Eight

8 Conclusion and Future Research

In this thesis, a new algorithm to optimally allocate the inspection stations in un-buffered serial line jointly considering quality and production logistics aspects is explained. In the first part, isolated machine is evaluated through Markov chain of the locally and remotely monitored cases. Performance measures of system are evaluated and effect of changing inspection location, which considered to be relevant to this work is analyzed. In the second part, a system of machines is considered. Focusing on one machine, effect of other machines on its performance is analyzed and Markov model of the machine is solved to obtain performance measures of the entire system. In the next part, system solution is employed to generate an algorithm aiming at optimally locate and assign limited number of inspection stations in the system. The novel aspects of this work is the followings: Interaction among quality and productivity aspects of the system is considered in the modeling and consequently in developing the algorithm to reach a proper tradeoff among these two aspects of the system. Another novelty is dealing with the problem of location and assignment of inspection stations simultaneously. One regards the location of inspection station in the system whilst the other considers their proper assignment to different machines to reach the best possible equilibrium among quality and productivity aspects.

The algorithm is implemented in Matlab and the results confirm its capability in finding the optimal solution. Appling the software, optimal solution can be found with substantially lower attempt in terms of both required time and computational effort. Obtained benefits expansively increases for large systems.

The results obtained from the software shows the importance of proper allocation and assignment of inspection stations in the production systems in terms of higher system performance and cost reduction.

In fact, results prove the effect of proper inspection allocation on optimizing the money investment in the production systems. It is demonstrated that with lower number of inspection stations that properly allocated it is possible to achieve higher level of effective throughput rather than higher number of inspection stations poorly allocated, which means obtaining higher performance with lower money investment.

Besides, results verify considerable difference between the best and the worst inspection allocation policies; especially in cases that quality problem is critical.

Considering the novelty of the research area, there are several topics that are open for future research. Some of them are mentioned in the following:

Production systems including buffers:

Usually in production systems, buffers are used in order to prevent blocking of upstream machine and starvation of downstream machine. However considering the buffers can increase the complexity of the inspection station allocation, since they are commonly used in production systems; there is a substantial need in the industry to quantitative methods and software applications to properly allocate inspection station in these systems.

Development of cost based allocation methods considering jointly quality and productivity aspect:

In the literature, different cost based methods have been applied to find proper allocation of inspection station. However they have significant constraints. Firstly, they mostly uses methods like genetic algorithm or simulated annealing which are not considered as an efficient optimization approach. Secondly they all have ignored the interaction among productivity and quality aspects. As a result, a proper optimization method that can integrate quality/productivity interaction with cost factors is promising.

Integrating quality, maintenance and production logistics:

Apart from relation among quality and production logistics, maintenance is another relevant aspect in production systems that can be integrated with quality and productivity aspects. Usually in industry, maintenance policy is selected independently from quality information feedback. In fact, generally preventive maintenance are chosen according to the age based rules without considering the quality state of degrading machine. However, as an alternative, quality information feedback coming from downstream inspection machine can be used. Using the latter method, there is a possibility of increasing machine availability either by avoiding critical failures that can stop machine for a long time or by faster maintenance of the machine when it has not been degraded to worse quality state. Moreover it prevents machine from producing more bad parts. The consequence is improving quality and productivity performance of the system.

Another relevant issue is deciding about the frequency of performing preventive maintenance. On one hand, more frequent preventive maintenance means better quality of produced parts. On the other hand it leads to less availability of machine and therefore negative effect on productivity of the system. As a result, it is important to analytically evaluate relation among these three dimensions in design and evaluation of production systems.

Modelling time consuming inspections

Usually inspection time is neglected during the modelling of production systems. However they might be a substantial portion of operating time. This fact can greatly influence the inspection policies as well as allocation of inspection stations. Therefore quantities methods that can evaluate effect of inspection time on quality control policies can play an important role in making more effective decisions that affects performance of the production systems.

9 Appendix

Table1. Data related to algorithm validation

Number of the case		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
1	M1	0.01	0.8	0.02	0.64	0.03	0.34
	M2	0.02	0.7	0.05	0.7	0.04	0.25
	M3	0.045	0.8	0.05	0.65	0.02	0.21
	M4	0.04	0.9	0.04	0.8	0.04	0.33
	M5	0.02	0.78	0.07	0.8	0.01	0.4
	M6	0.04	0.88	0.03	0.83	0.07	0.33
	M7	0.06	0.9	0.05	0.9	0.04	0.43
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.04	0.12	0.8	
	c2	1	0	0.04	0.15	0.7	
	c3	1	0	0.02	0.15	0.85	
	c4	1	0	0.04	0.1	0.5	
	c5	1	0	0.02	0.2	0.75	
	c6	1	0	0.03	0.15	0.6	
	c7	1	0	0.03	0.2	0.8	
2		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.06	0.76	0.06	0.82	0.02	0.4
	M2	0.05	0.7	0.04	0.8	0.03	0.37

	M3	0.045	0.6	0.05	0.7	0.02	0.22
	M4	0.04	0.79	0.04	0.66	0.02	0.3
	M5	0.043	0.88	0.07	0.8	0.03	0.24
	M6	0.05	0.82	0.03	0.83	0.03	0.38
	M7	0.06	0.82	0.05	0.72	0.02	0.4
		m($C_{i,q}$)	*h*($C_{i,q}$)	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.12	0.9	
	c2	1	0	0.05	0.46	0.54	
	c3	1	0	0.02	0.46	0.88	
	c4	1	0	0.06	0.1	0.5	
	c5	1	0	0.02	0.1	0.65	
	c6	1	0	0.03	0.15	0.6	
	c7	1	0	0.03	0.2	0.67	
3		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.07	0.76	0.08	0.7	0.02	0.09
	M2	0.06	0.66	0.067	0.68	0.04	0.37
	M3	0.04	0.8	0.07	0.73	0.04	0.37
	M4	0.04	0.77	0.065	0.7	0.04	0.37
	M5	0.03	0.75	0.068	0.75	0.03	0.35
	M6	0.04	0.73	0.07	0.67	0.03	0.35
	M7	0.06	0.82	0.05	0.83	0.02	0.11
		m($C_{i,q}$)	*h*($C_{i,q}$)	*error*	*error*	*repair probability*	

				type 1	*type 2*	*of false alarm*	
	c1	1	0	0.08	0.15	0.6	
	c2	1	0	0.06	0.15	0.7	
	c3	1	0	0.03	0.15	0.8	
	c4	1	0	0.05	0.15	0.6	
	c5	1	0	0.03	0.15	0.85	
	c6	1	0	0.02	0.15	0.8	
	c7	1	0	0.02	0.15	0.8	
4		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.03	0.8	0.02	0.66	0.02	0.4
	M2	0.02	0.73	0.04	0.8	0.03	0.37
	M3	0.035	0.67	0.05	0.72	0.02	0.38
	M4	0.042	0.82	0.03	0.65	0.02	0.37
	M5	0.033	0.69	0.04	0.83	0.03	0.43
	M6	0.042	0.77	0.032	0.8	0.03	0.5
	M7	0.05	0.8	0.04	0.7	0.02	0.4
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.03	0.15	0.73	
	c2	1	0	0.03	0.15	0.64	
	c3	1	0	0.03	0.15	0.67	
	c4	1	0	0.03	0.15	0.71	
	c5	1	0	0.03	0.15	0.7	

	c6	1	0	0.03	0.15	0.8	
	c7	1	0	0.03	0.15	0.67	
5		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.01	0.65	0.03	0.7	0.02	0.04
	M2	0.02	0.7	0.05	0.8	0.04	0.1
	M3	0.02	0.73	0.01	0.6	0.1	0.15
	M4	0.1	0.68	0.02	0.67	0.08	0.15
	M5	0.03	0.7	0.04	0.74	0.02	0.1
	M6	0.03	0.8	0.02	0.68	0.07	0.12
	M7	0.04	0.75	0.02	0.77	0.08	0.16
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.7	
	c2	1	0	0.02	0.15	0.7	
	c3	1	0	0.02	0.15	0.7	
	c4	1	0	0.02	0.15	0.7	
	c5	1	0	0.02	0.15	0.7	
	c6	1	0	0.02	0.15	0.7	
	c7	1	0	0.02	0.15	0.7	
6		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.04	0.9	0.01	0.9	0.02	0.04
	M2	0.06	0.8	0.07	0.65	0.1	0.3

	M3	0.03	0.65	0.05	0.6	0.03	0.1
	M4	0.02	0.9	0.02	0.8	0.08	0.1
	M5	0.07	0.9	0.06	0.65	0.02	0.2
	M6	0.04	0.6	0.05	0.65	0.07	0.09
	M7	0.01	0.7	0.01	0.9	0.08	0.16
		m($C_{i,q}$)	*h*($C_{i,q}$)	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.1	0.15	0.7	
	c2	1	0	0.02	0.15	0.7	
	c3	1	0	0.1	0.15	0.7	
	c4	1	0	0.02	0.15	0.7	
	c5	1	0	0.1	0.15	0.7	
	c6	1	0	0.02	0.15	0.7	
	c7	1	0	0.02	0.15	0.7	
7		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.06	0.7	0.01	0.67	0.01	0.1
	M2	0.02	0.6	0.02	0.7	0.1	0.3
	M3	0.01	0.65	0.03	0.78	0.06	0.15
	M4	0.04	0.7	0.08	0.65	0.08	0.2
	M5	0.06	0.7	0.02	0.7	0.04	0.08
	M6	0.01	0.65	0.05	0.9	0.07	0.14
	M7	0.01	0.8	0.01	0.7	0.08	0.11
		m($C_{i,q}$)	*h*($C_{i,q}$)	*error*	*error*	*repair probability*	

				type 1	*type 2*	*of false alarm*	
	c1	1	0	0.05	0.4	0.6	
	c2	1	0	0.02	0.2	0.8	
	c3	1	0	0.05	0.15	0.7	
	c4	1	0	0.02	0.15	0.7	
	c5	1	0	0.01	0.15	0.7	
	c6	1	0	0.05	0.15	0.7	
	c7	1	0	0.02	0.15	0.7	
		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.05	0.6	0.01	0.68	0.02	0.08
	M2	0.03	0.78	0.1	0.72	0.04	0.15
8	M3	0.04	0.9	0.05	0.7	0.1	0.2
	M4	0.01	0.56	0.03	0.65	0.08	0.1
	M5	0.04	0.9	0.08	0.7	0.02	0.05
	M6	0.1	0.65	0.01	0.9	0.07	0.1
	M7	0.09	0.9	0.02	0.9	0.08	0.09
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.7	
	c2	1	0	0.02	0.15	0.7	
	c3	1	0	0.02	0.15	0.7	
	c4	1	0	0.02	0.15	0.7	
	c5	1	0	0.02	0.15	0.7	

	c6	1	0	0.02	0.15	0.7	
	c7	1	0	0.02	0.15	0.7	
9		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.01	0.6	0.02	0.6	0.02	0.04
	M2	0.02	0.65	0.04	0.65	0.04	0.07
	M3	0.04	0.7	0.06	0.7	0.1	0.14
	M4	0.06	0.75	0.08	0.75	0.08	0.15
	M5	0.08	0.8	0.1	0.8	0.02	0.12
	M6	0.1	0.85	0.12	0.85	0.07	0.2
	M7	0.12	0.9	0.12	0.9	0.08	0.24
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.65	
	c2	1	0	0.03	0.15	0.7	
	c3	1	0	0.04	0.15	0.7	
	c4	1	0	0.05	0.15	0.72	
	c5	1	0	0.06	0.15	0.76	
	c6	1	0	0.07	0.15	0.8	
	c7	1	0	0.08	0.15	0.9	
10		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.02	0.65	0.04	0.65	0.04	0.07
	M2	0.04	0.7	0.06	0.7	0.1	0.14

	M3	0.06	0.75	0.08	0.75	0.08	0.15
	M4	0.08	0.8	0.1	0.8	0.02	0.12
	M5	0.1	0.85	0.12	0.85	0.07	0.2
	M6	0.12	0.9	0.12	0.9	0.08	0.24
	M7	0.04	0.65	0.05	0.7	0.05	0.15
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.03	0.15	0.7	
	c2	1	0	0.04	0.15	0.7	
	c3	1	0	0.05	0.15	0.72	
	c4	1	0	0.06	0.15	0.76	
	c5	1	0	0.07	0.15	0.8	
	c6	1	0	0.08	0.15	0.9	
	c7	1	0	0.02	0.15	0.7	
11		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.03	0.5	0.03	0.7	0.01	0.23
	M2	0.03	0.6	0.04	0.63	0.03	0.3
	M3	0.04	0.58	0.05	0.68	0.02	0.37
	M4	0.05	0.56	0.06	0.65	0.02	0.27
	M5	0.04	0.53	0.04	0.68	0.02	0.22
	M6	0.02	0.57	0.03	0.62	0.01	0.24
	M7	0.04	0.55	0.02	0.65	0.02	0.25
		$m(C_{i,q})$	$h(C_{i,q})$	*error*	*error*	*repair probability*	

				type 1	*type 2*	*of false alarm*	
	c1	1	0	0.02	0.2	0.7	
	c2	1	0	0.02	0.22	0.7	
	c3	1	0	0.03	0.31	0.72	
	c4	1	0	0.02	0.1	0.76	
	c5	1	0	0.03	0.15	0.8	
	c6	1	0	0.02	0.17	0.9	
	c7	1	0	0.02	0.12	0.7	
		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.02	0.87	0.01	0.8	0.01	0.17
	M2	0.03	0.76	0.02	0.7	0.03	0.23
12	M3	0.04	0.7	0.03	0.68	0.02	0.26
	M4	0.05	0.68	0.045	0.65	0.02	0.3
	M5	0.06	0.65	0.05	0.62	0.02	0.34
	M6	0.07	0.6	0.055	0.6	0.01	0.37
	M7	0.08	0.55	0.06	0.56	0.02	0.42
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.2	0.7	
	c2	1	0	0.02	0.22	0.7	
	c3	1	0	0.03	0.31	0.72	
	c4	1	0	0.02	0.1	0.76	
	c5	1	0	0.03	0.15	0.8	

	c6	1	0	0.02	0.17	0.9	
	c7	1	0	0.02	0.12	0.7	
13		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.08	0.8	0.01	0.53	0.01	0.18
	M2	0.075	0.74	0.02	0.6	0.03	0.23
	M3	0.067	0.7	0.03	0.62	0.02	0.28
	M4	0.058	0.65	0.045	0.68	0.02	0.33
	M5	0.045	0.61	0.05	0.74	0.02	0.37
	M6	0.038	0.6	0.055	0.81	0.01	0.4
	M7	0.02	0.52	0.06	0.88	0.02	0.44
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.05	0.34	0.8	
	c2	1	0	0.045	0.3	0.76	
	c3	1	0	0.04	0.26	0.7	
	c4	1	0	0.035	0.22	0.65	
	c5	1	0	0.03	0.18	0.6	
	c6	1	0	0.025	0.13	0.56	
	c7	1	0	0.02	0.08	0.5	
14		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.04	0.8	0.01	0.5	0.01	0.04
	M2	0.036	0.74	0.02	0.62	0.03	0.05

	M3	0.043	0.76	0.07	0.6	0.02	0.4
	M4	0.042	0.72	0.02	0.65	0.02	0.06
	M5	0.045	0.76	0.09	0.7	0.02	0.48
	M6	0.038	0.77	0.1	0.78	0.01	0.4
	M7	0.041	0.73	0.02	0.843	0.02	0.05
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.05	0.34	0.7	
	c2	1	0	0.045	0.3	0.7	
	c3	1	0	0.02	0.12	0.7	
	c4	1	0	0.035	0.3	0.7	
	c5	1	0	0.02	0.08	0.7	
	c6	1	0	0.02	0.1	0.7	
	c7	1	0	0.04	0.24	0.7	
		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
15	M1	0.032	0.735	0.01	0.562	0.01	0.03
	M2	0.029	0.712	0.06	0.71	0.02	0.45
	M3	0.0387	0.745	0.07	0.71	0.02	0.45
	M4	0.052	0.721	0.02	0.657	0.03	0.06
	M5	0.0531	0.692	0.09	0.71	0.02	0.45
	M6	0.0528	0.721	0.1	0.71	0.02	0.45
	M7	0.041	0.767	0.02	0.843	0.04	0.07
		$m(C_{i,q})$	$h(C_{i,q})$	*error*	*error*	*repair probability*	

				type 1	*type 2*	*of false alarm*	
	c1	1	0	0.035	0.4	0.7	
	c2	1	0	0.02	0.1	0.7	
	c3	1	0	0.02	0.1	0.7	
	c4	1	0	0.04	0.3	0.7	
	c5	1	0	0.02	0.1	0.7	
	c6	1	0	0.02	0.1	0.7	
	c7	1	0	0.03	0.24	0.7	
16		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.03	0.682	0.02	0.562	0.025	0.05
	M2	0.04	0.732	0.05	0.7	0.02	0.37
	M3	0.031	0.811	0.07	0.695	0.02	0.32
	M4	0.05	0.7	0.03	0.72	0.03	0.06
	M5	0.045	0.684	0.08	0.71	0.04	0.35
	M6	0.02	0.754	0.083	0.735	0.04	0.35
	M7	0.07	0.8	0.02	0.8	0.04	0.07
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.035	0.4	0.7	
	c2	1	0	0.02	0.1	0.7	
	c3	1	0	0.02	0.1	0.7	
	c4	1	0	0.04	0.3	0.7	
	c5	1	0	0.02	0.1	0.7	
	c6	1	0	0.02	0.1	0.7	

	c7	1	0	0.03	0.24	0.7	
17		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.06	0.67	0.06	0.63	0.01	0.3
	M2	0.067	0.83	0.05	0.71	0.04	0.23
	M3	0.045	0.76	0.05	0.71	0.05	0.28
	M4	0.0481	0.8	0.05	0.715	0.02	0.29
	M5	0.039	0.74	0.05	0.69	0.03	0.34
	M6	0.061	0.73	0.049	0.7	0.04	0.39
	M7	0.065	0.81	0.06	0.8	0.04	0.2
		m($C_{i,q}$)	*h*($C_{i,q}$)	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.035	0.4	0.7	
	c2	1	0	0.07	0.37	0.7	
	c3	1	0	0.06	0.3	0.7	
	c4	1	0	0.05	0.23	0.7	
	c5	1	0	0.04	0.17	0.7	
	c6	1	0	0.03	0.1	0.7	
	c7	1	0	0.03	0.24	0.7	
18		p_i	r_i	$p_i^{quali\ ty}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.06	0.6	0.05	0.63	0.01	0.3
	M2	0.03	0.8	0.058	0.71	0.05	0.25
	M3	0.04	0.7	0.058	0.71	0.05	0.25

	M4	0.045	0.8	0.03	0.715	0.02	0.34
	M5	0.03	0.7	0.03	0.69	0.02	0.34
	M6	0.05	0.7	0.03	0.7	0.02	0.34
	M7	0.06	0.8	0.06	0.8	0.04	0.2
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.035	0.4	0.7	
	c2	1	0	0.05	0.35	0.7	
	c3	1	0	0.05	0.35	0.7	
	c4	1	0	0.03	0.17	0.7	
	c5	1	0	0.03	0.17	0.7	
	c6	1	0	0.03	0.17	0.7	
	c7	1	0	0.03	0.24	0.7	
19		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.04	0.63	0.06	0.65	0.01	0.3
	M2	0.037	0.76	0.04	0.7	0.05	0.34
	M3	0.05	0.74	0.04	0.72	0.05	0.37
	M4	0.041	0.82	0.041	0.715	0.05	0.34
	M5	0.036	0.77	0.04	0.72	0.05	0.37
	M6	0.051	0.72	0.04	0.72	0.05	0.34
	M7	0.062	0.83	0.055	0.8	0.04	0.2
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	

	c1	1	0	0.035	0.4	0.7	
	c2	1	0	0.05	0.35	0.7	
	c3	1	0	0.05	0.33	0.7	
	c4	1	0	0.02	0.21	0.7	
	c5	1	0	0.02	0.21	0.7	
	c6	1	0	0.02	0.17	0.7	
	c7	1	0	0.03	0.24	0.7	
20		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.02	0.7	0.05	0.8	0.04	0.1
	M2	0.032	0.65	0.037	0.66	0.05	0.3
	M3	0.04	0.7	0.04	0.7	0.02	0.05
	M4	0.03	0.73	0.06	0.57	0.03	0.24
	M5	0.05	0.66	0.055	0.72	0.07	0.32
	M6	0.042	0.65	0.05	0.69	0.02	0.26
	M7	0.051	0.8	0.055	0.8	0.03	0.11
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.8	
	c2	1	0	0.02	0.35	0.8	
	c3	1	0	0.04	0.1	0.8	
	c4	1	0	0.03	0.21	0.8	
	c5	1	0	0.04	0.21	0.8	
	c6	1	0	0.02	0.17	0.8	

	c7	1	0	0.02	0.24	0.8	
		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.041	0.74	0.05	0.8	0.04	0.41
	M2	0.032	0.7	0.05	0.66	0.03	0.41
21	M3	0.042	0.68	0.04	0.7	0.02	0.29
	M4	0.05	0.65	0.04	0.57	0.02	0.35
	M5	0.036	0.76	0.04	0.7	0.02	0.05
	M6	0.031	0.81	0.03	0.65	0.03	0.07
	M7	0.051	0.8	0.055	0.8	0.03	0.11
		m($C_{i,q}$)	*h*($C_{i,q}$)	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.8	
	c2	1	0	0.02	0.35	0.8	
	c3	1	0	0.05	0.4	0.8	
	c4	1	0	0.02	0.21	0.8	
	c5	1	0	0.04	0.15	0.8	
	c6	1	0	0.03	0.25	0.85	
	c7	1	0	0.02	0.24	0.8	
		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
22	M1	0.07	0.6	0.05	0.76	0.04	0.33
	M2	0.03	0.65	0.06	0.65	0.03	0.4
	M3	0.06	0.64	0.03	0.7	0.02	0.29

	M4	0.05	0.6	0.05	0.65	0.04	0.09
	M5	0.02	0.7	0.05	0.8	0.04	0.12
	M6	0.049	0.7	0.04	0.75	0.03	0.11
	M7	0.06	0.8	0.055	0.8	0.03	0.11
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.03	0.2	0.8	
	c2	1	0	0.04	0.32	0.8	
	c3	1	0	0.05	0.22	0.8	
	c4	1	0	0.02	0.2	0.7	
	c5	1	0	0.02	0.15	0.85	
	c6	1	0	0.03	0.1	0.7	
	c7	1	0	0.02	0.24	0.8	
23		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.06	0.8	0.08	0.65	0.02	0.34
	M2	0.05	0.65	0.05	0.65	0.03	0.35
	M3	0.031	0.68	0.06	0.6	0.05	0.37
	M4	0.02	0.6	0.05	0.65	0.04	0.36
	M5	0.031	0.73	0.051	0.72	0.03	0.11
	M6	0.04	0.68	0.045	0.7	0.03	0.15
	M7	0.06	0.8	0.05	0.83	0.02	0.11
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	

	c1	1	0	0.06	0.1	0.65	
	c2	1	0	0.07	0.3	0.7	
	c3	1	0	0.03	0.05	0.7	
	c4	1	0	0.05	0.2	0.6	
	c5	1	0	0.03	0.15	0.85	
	c6	1	0	0.02	0.15	0.7	
	c7	1	0	0.02	0.3	0.8	
		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.05	0.6	0.04	0.8	0.02	0.12
	M2	0.01	0.75	0.07	0.72	0.01	0.14
24	M3	0.1	0.65	0.04	0.7	0.01	0.25
	M4	0.11	0.77	0.08	0.81	0.02	0.22
	M5	0.07	0.65	0.1	0.8	0.01	0.21
	M6	0.06	0.76	0.1	0.92	0.02	0.19
	M7	0.03	0.63	0.03	0.65	0.03	0.17
		m($C_{i,q}$)	*h*($C_{i,q}$)	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.7	
	c2	1	0	0.02	0.15	0.7	
	c3	1	0	0.02	0.15	0.7	
	c4	1	0	0.02	0.15	0.7	
	c5	1	0	0.02	0.15	0.7	
	c6	1	0	0.02	0.15	0.7	

	c7	1	0	0.02	0.15	0.7	
25		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.05	0.65	0.06	0.6	0.04	0.3
	M2	0.07	0.65	0.065	0.58	0.03	0.31
	M3	0.04	0.6	0.035	0.7	0.02	0.25
	M4	0.03	0.75	0.02	0.75	0.02	0.26
	M5	0.08	0.6	0.07	0.65	0.02	0.27
	M6	0.01	0.85	0.02	0.75	0.03	0.28
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.8	
	c2	1	0	0.02	0.15	0.8	
	c3	1	0	0.02	0.15	0.8	
	c4	1	0	0.02	0.15	0.8	
	c5	1	0	0.02	0.15	0.8	
	c6	1	0	0.02	0.15	0.8	
26		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.06	0.63	0.062	0.61	0.04	0.36
	M2	0.07	0.6	0.065	0.59	0.03	0.39
	M3	0.04	0.7	0.036	0.73	0.02	0.32
	M4	0.03	0.75	0.022	0.76	0.03	0.28
	M5	0.08	0.6	0.073	0.67	0.02	0.41

	M6	0.01	0.8	0.021	0.76	0.03	0.28
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.8	
	c2	1	0	0.02	0.15	0.8	
	c3	1	0	0.02	0.15	0.8	
	c4	1	0	0.02	0.15	0.8	
	c5	1	0	0.02	0.15	0.8	
	c6	1	0	0.02	0.15	0.8	
27		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.06	0.68	0.05	0.64	0.035	0.29
	M2	0.07	0.65	0.052	0.6	0.03	0.27
	M3	0.042	0.75	0.03	0.75	0.022	0.33
	M4	0.05	0.8	0.013	0.77	0.031	0.38
	M5	0.07	0.63	0.064	0.67	0.022	0.22
	M6	0.04	0.8	0.02	0.77	0.032	0.4
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.05	0.4	0.7	
	c2	1	0	0.01	0.1	0.7	
	c3	1	0	0.05	0.4	0.7	
	c4	1	0	0.01	0.05	0.7	
	c5	1	0	0.05	0.45	0.7	

	c6	1	0	0.02	0.2	0.7	
28		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.04	0.7	0.05	0.63	0.033	0.29
	M2	0.067	0.67	0.05	0.62	0.03	0.27
	M3	0.04	0.73	0.03	0.77	0.022	0.36
	M4	0.045	0.79	0.01	0.76	0.031	0.38
	M5	0.067	0.62	0.06	0.69	0.022	0.3
	M6	0.044	0.81	0.02	0.79	0.032	0.3
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.07	0.6	0.8	
	c2	1	0	0.01	0.05	0.8	
	c3	1	0	0.07	0.6	0.8	
	c4	1	0	0.01	0.05	0.8	
	c5	1	0	0.07	0.6	0.8	
	c6	1	0	0.02	0.2	0.8	
29		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.03	0.8	0.035	0.68	0.04	0.29
	M2	0.067	0.72	0.03	0.66	0.02	0.27
	M3	0.056	0.76	0.032	0.71	0.03	0.28
	M4	0.07	0.83	0.029	0.7	0.031	0.28
	M5	0.078	0.65	0.33	0.69	0.041	0.29

	M6	0.057	0.84	0.04	0.79	0.05	0.35
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.8	
	c2	1	0	0.01	0.15	0.9	
	c3	1	0	0.05	0.15	0.55	
	c4	1	0	0.03	0.15	0.7	
	c5	1	0	0.04	0.15	0.6	
	c6	1	0	0.02	0.15	0.8	
30		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.035	0.7	0.033	0.68	0.04	0.29
	M2	0.06	0.84	0.028	0.71	0.02	0.27
	M3	0.053	0.73	0.03	0.74	0.03	0.28
	M4	0.072	0.83	0.03	0.72	0.031	0.28
	M5	0.08	0.71	0.035	0.71	0.041	0.29
	M6	0.067	0.69	0.041	0.82	0.05	0.35
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.63	
	c2	1	0	0.02	0.15	0.62	
	c3	1	0	0.031	0.15	0.9	
	c4	1	0	0.025	0.15	0.76	
	c5	1	0	0.03	0.15	0.9	

	c6	1	0	0.02	0.15	0.62	
31		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.02	0.7	0.05	0.8	0.04	0.1
	M2	0.04	0.7	0.04	0.75	0.03	0.13
	M3	0.03	0.75	0.04	0.7	0.02	0.12
	M4	0.03	0.8	0.03	0.65	0.03	0.3
	M5	0.1	0.65	0.06	0.8	0.041	0.32
	M6	0.06	0.67	0.04	0.65	0.06	0.41
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.67	
	c2	1	0	0.03	0.15	0.72	
	c3	1	0	0.04	0.15	0.83	
	c4	1	0	0.03	0.15	0.7	
	c5	1	0	0.03	0.15	0.91	
	c6	1	0	0.02	0.15	0.6	
32		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.1	0.66	0.07	0.74	0.033	0.28
	M2	0.09	0.73	0.11	0.68	0.03	0.41
	M3	0.1	0.69	0.0599	0.63	0.022	0.28
	M4	0.08	0.75	0.07	0.7	0.031	0.29
	M5	0.07	0.8	0.075	0.7	0.022	0.32

	M6	0.11	0.72	0.04	0.66	0.032	0.3
		$m(C_{i,q})$	*$h(C_{i,q})$*	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.1	0.7	
	c2	1	0	0.05	0.2	0.6	
	c3	1	0	0.02	0.1	0.7	
	c4	1	0	0.02	0.05	0.67	
	c5	1	0	0.03	0.2	0.81	
	c6	1	0	0.02	0.2	0.82	
33		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.1	0.6	0.08	0.68	0.03	0.36
	M2	0.03	0.7	0.06	0.72	0.05	0.3
	M3	0.05	0.62	0.04	0.7	0.025	0.28
	M4	0.08	0.65	0.036	0.66	0.03	0.26
	M5	0.04	0.8	0.046	0.73	0.02	0.3
	M6	0.1	0.8	0.04	0.6	0.04	0.25
		$m(C_{i,q})$	*$h(C_{i,q})$*	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.25	0.6	
	c2	1	0	0.02	0.15	0.7	
	c3	1	0	0.02	0.15	0.7	
	c4	1	0	0.02	0.15	0.7	
	c5	1	0	0.02	0.15	0.7	

	c6	1	0	0.02	0.15	0.7	
34		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.05	0.67	0.03	0.75	0.04	0.07
	M2	0.05	0.7	0.05	0.85	0.05	0.11
	M3	0.06	0.85	0.06	0.9	0.07	0.34
	M4	0.04	0.7	0.02	0.8	0.06	0.33
	M5	0.07	0.75	0.14	0.82	0.035	0.2
	M6	0.06	0.82	0.02	0.69	0.06	0.4
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.7	
	c2	1	0	0.02	0.15	0.75	
	c3	1	0	0.02	0.15	0.66	
	c4	1	0	0.02	0.15	0.56	
	c5	1	0	0.02	0.15	0.91	
	c6	1	0	0.02	0.15	0.62	
		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
35	M1	0.09	0.8	0.11	0.65	0.04	0.21
	M2	0.11	0.72	0.04	0.95	0.05	0.2
	M3	0.12	0.86	0.03	0.63	0.07	0.28
	M4	0.06	0.71	0.06	0.67	0.03	0.43
	M5	0.07	0.72	0.04	0.7	0.035	0.06

	M6	0.05	0.8	0.02	0.69	0.06	0.4
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.07	0.3	0.5	
	c2	1	0	0.025	0.3	0.8	
	c3	1	0	0.025	0.3	0.77	
	c4	1	0	0.025	0.3	0.82	
	c5	1	0	0.025	0.1	0.8	
	c6	1	0	0.025	0.15	0.72	
36		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.03	0.75	0.05	0.65	0.035	0.23
	M2	0.032	0.76	0.049	0.65	0.037	0.24
	M3	0.04	0.8	0.047	0.64	0.039	0.22
	M4	0.036	0.74	0.044	0.67	0.036	0.22
	M5	0.039	0.8	0.046	0.65	0.035	0.23
	M6	0.041	0.82	0.044	0.68	0.04	0.24
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.03	0.25	0.7	
	c2	1	0	0.03	0.25	0.7	
	c3	1	0	0.03	0.25	0.7	
	c4	1	0	0.03	0.25	0.7	
	c5	1	0	0.03	0.25	0.7	

	c6	1	0	0.03	0.25	0.7	
37		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.03	0.7	0.05	0.8	0.035	0.23
	M2	0.06	0.65	0.02	0.65	0.05	0.2
	M3	0.11	0.9	0.08	0.6	0.01	0.29
	M4	0.02	0.85	0.01	0.9	0.06	0.3
	M5	0.04	0.8	0.07	0.76	0.07	0.4
	M6	0.08	0.9	0.04	0.73	0.03	0.4
	c1	1	0	0.03	0.2	0.7	
	c2	1	0	0.03	0.2	0.7	
	c3	1	0	0.03	0.2	0.7	
	c4	1	0	0.03	0.2	0.7	
	c5	1	0	0.03	0.2	0.7	
	c6	1	0	0.03	0.2	0.7	
38		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.03	0.66	0.086	0.75	0.03	0.06
	M2	0.02	0.67	0.09	0.65	0.02	0.07
	M3	0.03	0.72	0.11	0.65	0.01	0.29
	M4	0.04	0.71	0.12	0.76	0.03	0.35
	M5	0.04	0.69	0.09	0.7	0.04	0.41
	M6	0.032	0.74	0.011	0.75	0.03	0.41

		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.18	0.75	
	c2	1	0	0.02	0.18	0.75	
	c3	1	0	0.02	0.18	0.75	
	c4	1	0	0.02	0.18	0.75	
	c5	1	0	0.02	0.18	0.75	
	c6	1	0	0.02	0.18	0.75	
		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.05	0.72	0.047	0.65	0.048	0.3
39	M2	0.06	0.75	0.05	0.68	0.05	0.31
	M3	0.05	0.7	0.05	0.67	0.055	0.32
	M4	0.055	0.69	0.06	0.66	0.05	0.31
	M5	0.06	0.72	0.057	0.69	0.055	0.3
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.75	
	c2	1	0	0.02	0.15	0.75	
	c3	1	0	0.02	0.15	0.75	
	c4	1	0	0.02	0.15	0.75	
	c5	1	0	0.02	0.15	0.75	
40		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o

	M1	0.05	0.66	0.02	0.9	0.04	0.22
	M2	0.08	0.9	0.05	0.68	0.02	0.3
	M3	0.01	0.8	0.07	0.7	0.06	0.4
	M4	0.01	0.73	0.08	0.82	0.08	0.19
	M5	0.1	0.65	0.01	0.65	0.07	0.35
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.2	0.8	
	c2	1	0	0.04	0.34	0.7	
	c3	1	0	0.06	0.21	0.65	
	c4	1	0	0.05	0.15	0.9	
	c5	1	0	0.02	0.19	0.75	
41		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.04	0.72	0.02	0.71	0.04	0.36
	M2	0.035	0.74	0.04	0.67	0.02	0.3
	M3	0.041	0.71	0.06	0.69	0.06	0.3
	M4	0.039	0.73	0.08	0.66	0.08	0.28
	M5	0.04	0.75	0.01	0.73	0.07	0.23
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.76	
	c2	1	0	0.02	0.15	0.74	
	c3	1	0	0.02	0.15	0.78	

	c4	1	0	0.02	0.15	0.75	
	c5	1	0	0.02	0.15	0.75	
42		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.03	0.77	0.04	0.76	0.04	0.36
	M2	0.04	0.8	0.07	0.74	0.03	0.3
	M3	0.047	0.79	0.06	0.73	0.05	0.35
	M4	0.03	0.77	0.05	0.77	0.05	0.39
	M5	0.05	0.78	0.01	0.73	0.07	0.43
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.06	0.3	0.76	
	c2	1	0	0.01	0.1	0.74	
	c3	1	0	0.02	0.15	0.78	
	c4	1	0	0.04	0.2	0.75	
	c5	1	0	0.02	0.15	0.75	
43		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.01	0.8	0.02	0.6	0.05	0.08
	M2	0.02	0.7	0.05	0.7	0.03	0.16
	M3	0.02	0.9	0.04	0.65	0.05	0.41
	M4	0.04	0.85	0.03	0.8	0.05	0.41
	M5	0.03	0.75	0.06	0.73	0.07	0.43

		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.8	
	c2	1	0	0.02	0.15	0.7	
	c3	1	0	0.02	0.15	0.85	
	c4	1	0	0.02	0.15	0.75	
	c5	1	0	0.02	0.15	0.7	
44		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.1	0.8	0.02	0.77	0.04	0.3
	M2	0.08	0.65	0.03	0.64	0.01	0.22
	M3	0.03	0.55	0.06	0.71	0.01	0.23
	M4	0.01	0.79	0.11	0.87	0.02	0.35
	M5	0.02	0.9	0.08	0.9	0.04	0.33
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.04	0.2	0.67	
	c2	1	0	0.04	0.2	0.7	
	c3	1	0	0.04	0.2	0.8	
	c4	1	0	0.04	0.2	0.8	
	c5	1	0	0.04	0.2	0.73	
45		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.03	0.6	0.07	0.78	0.005	0.23

	M2	0.03	0.81	0.02	0.56	0.05	0.15
	M3	0.06	0.65	0.078	0.8	0.003	0.34
	M4	0.09	0.7	0.01	0.6	0.07	0.16
	M5	0.1	0.66	0.07	0.8	0.001	0.4
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.05	0.13	0.8	
	c2	1	0	0.01	0.36	0.56	
	c3	1	0	0.06	0.13	0.8	
	c4	1	0	0.01	0.4	0.6	
	c5	1	0	0.05	0.15	0.75	
46		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.04	0.67	0.04	0.8	0.02	0.05
	M2	0.06	0.74	0.02	0.6	0.01	0.2
	M3	0.05	0.66	0.07	0.68	0.005	0.4
	M4	0.02	0.69	0.05	0.66	0.01	0.3
	M5	0.03	0.8	0.04	0.84	0.03	0.3
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.15	0.6	
	c2	1	0	0.02	0.15	0.8	
	c3	1	0	0.02	0.15	0.84	
	c4	1	0	0.02	0.15	0.82	

	c5	1	0	0.02	0.15	0.7	
47		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.03	0.7	0.03	0.7	0.03	0.06
	M2	0.04	0.83	0.04	0.67	0.02	0.05
	M3	0.06	0.73	0.03	0.73	0.006	0.4
	M4	0.03	0.64	0.03	0.77	0.001	0.33
	M5	0.02	0.75	0.02	0.8	0.003	0.3
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.03	0.25	0.56	
	c2	1	0	0.04	0.25	0.86	
	c3	1	0	0.06	0.25	0.8	
	c4	1	0	0.04	0.25	0.81	
	c5	1	0	0.03	0.25	0.72	
48		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.05	0.74	0.08	0.8	0.002	0.16
	M2	0.06	0.8	0.07	0.74	0.002	0.27
	M3	0.03	0.68	0.08	0.7	0.006	0.5
	M4	0.04	0.8	0.11	0.76	0.01	0.4
	M5	0.04	0.9	0.12	0.9	0.004	0.3
		$m(C_{i,q})$	$h(C_{i,q})$	*error*	*error*	*repair probability*	

				type 1	*type 2*	*of false alarm*	
	c1	1	0	0.04	0.2	0.65	
	c2	1	0	0.06	0.2	0.8	
	c3	1	0	0.08	0.2	0.8	
	c4	1	0	0.03	0.2	0.76	
	c5	1	0	0.02	0.2	0.7	
49		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.07	0.67	0.08	0.8	0.04	0.15
	M2	0.03	0.73	0.05	0.8	0.05	0.23
	M3	0.02	0.76	0.02	0.67	0.003	0.33
	M4	0.03	0.78	0.04	0.75	0.003	0.33
	M5	0.04	0.8	0.07	0.8	0.001	0.3
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.03	0.15	0.6	
	c2	1	0	0.03	0.15	0.76	
	c3	1	0	0.03	0.15	0.82	
	c4	1	0	0.03	0.15	0.8	
	c5	1	0	0.03	0.15	0.7	
50		p_i	r_i	$p_i^{quality}$	$r_i^{quality}$	γ_i^w	γ_i^o
	M1	0.035	0.78	0.04	0.71	0.006	0.3
	M2	0.1	0.6	0.035	0.73	0.005	0.29

	M3	0.04	0.75	0.034	0.72	0.004	0.295
	M4	0.1	0.6	0.044	0.69	0.006	0.305
	M5	0.05	0.72	0.05	0.7	0.001	0.36
		$m(C_{i,q})$	$h(C_{i,q})$	*error type 1*	*error type 2*	*repair probability of false alarm*	
	c1	1	0	0.02	0.1	0.75	
	c2	1	0	0.02	0.1	0.75	
	c3	1	0	0.02	0.1	0.75	
	c4	1	0	0.02	0.1	0.75	
	c5	1	0	0.02	0.1	0.75	

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