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SOCIALLY RESPONSIBLE INVESTING: A MEAN VARIANCE ANALYSIS

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O caráter de uma pessoa é forjado ao longo dos anos,
o que sou e conquistei devo aos meus pais,
agredeço por me apoiarem e sempre acreditarem em mim.

Abstract

Socially responsible investing has been growing over the past few years more quickly than ever; only in United States of America professionally managed funds which are aligned to socially responsible investing strategies represented \$3.07 trillion in the beginning of 2010.

This study focuses on in analyzing the behavior of the Markowitz Mean Variance efficiency frontier due to changes in the criteria intensity level characterizing the funds composing the efficient portfolios. In order to overcome the shortcomings of the traditional Markowitz's theory, Ledoit-Wolf estimators were utilized in conjunction with resampling procedures.

The results obtained in this study show that during the financial crisis of 2008 to 2010, portfolios composed by funds with higher ”ethicalness” level were more successful than those characterized by lower levels. Over the long run, from 2001 to 2010, portfolios with higher ”ethicalness” presented higher maximum Sharpe-ratio than those with lower levels, but the difference between these two groups decreases as the time span under consideration increases.

Sommario

Investimenti socialmente responsabile hanno cresciuto rapidamente negli anni passati, solo negli Stati Uniti i fondi gestiti per professionisti che seguono le strategie socialmente responsabile erano \$3.07 trilioni nell´inizio di 2010.

Questo studio tiene come focus analizzare il comportamento della Markowitz Mean Variance frontiera efficiente quando i livelli d´intensità dei criteri sociali, governance e ambientale che compongono i fondi dei portfolio efficienti cambiano. I problemi dalla teoria tradizionale di Markowitz sono combatti con l´uso degli estimatori di Ledoit-Wolf e procedimenti di ricampionamento.

I risultati ottenuti mostrano che durante la crisi finanziaria dei 2008 al 2010, i portfolio composti di fondi più etiche hanno avuto migliori risultati da quelli in cui i livelli etici sono più bassi. In periodo più lungo, dal 2001 al 2010, i portfolio in cui i livelli sociali e ambientali sono più alti hanno presentato valori per il massimo Sharpe-ratio più alto da quelli con bassi livelli, ma questa differenza tra questi due gruppi non rimangano cosi, ma diminuisce col tempo più lungo.

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Chapter 1

Introduction

Socially responsible investment constitutes a new trend that has gained momentum over the past few years. The shift in perspective pushed by this trend has been a mark in the financial community, before investors only sought in maximizing financial returns but now this is not enough anymore. Now investors seeking socially responsible investment opportunities seek more than only the financial dimension, new dimensions arose under this trend, now these investors are looking for financial returns aligned with outcomes in the social, environmental and governance spheres.

New terms arose in order to fit and satisfy newly discovered needs, now companies do not only seek financial returns, but blended returns. Corporations in order to keep up with the new needs and demands from society, governments and financial markets, actively engage and promote CSR (corporate social responsibility).

What was seen as an obstacle, a barrier to business and shareholder wealth, now is faced as opportunity by organizations. SRI assets have been growing at a fast pace throughout the world, especially in developed countries. In US only, in the beginning of 2010, professionally managed assets aligned with SRI strategies were \$3.07 trillion. This value represents a growth of 380% compared to 1995[1].

SRI aligned investments represent an important asset class, not only to the financial community but also to the society. This search for blended value represents a great opportunity to improve and integrate marginalized individuals, fairer business relations and sustainable use of natural resources.

Over history, using knowledge, human kind has been able to develop new technologies. This study makes use of several of these aiming in developing knowledge intended to support the development of such important asset class.

Several studies aimed in analyzing if a trade-off exists when engaging in SRI, whether in return terms, risk terms or risk-adjusted terms. Unfortunately little consensus exists regarding these matters.

This study aims to analyze what are the effects over the investor's portfolio when the ethicalness level of this investment shifts. Markowitz showed that when analyzing risk and return of a portfolio, these are not a mere linear relation, but how these assets swing over time with each other has a big impact over this relationship. Analyzing assets separately may not be optimal, as the covariance between assets is an important dimension in an investment decision. The modern portfolio theory shows that optimal portfolios can be reached when these assets relationships are taken into consideration.

Several studies aim in analyzing separately SRI strategy aligned assets in order to draw conclusions. This studies do not take into account how these assets correlate with each other, when these relationships are considered the overall analysis

[1] Data retrieved from the ”Report on Socially Responsible Investing Trends in the United States”, Social Investment Forum Foundation.

may present a completely new and different set of alternatives and behavior which could not be seen when assets were considered to be independent.

The Modern Portfolio Theory constitutes a mark in the modern financial history; this study uses the Markowitz theory as its foundation. In order to obtain robust results this study makes use of several tools and methodologies widely accepted by the financial community. The Ledoit-Wolf estimator was utilized to cope with instability and estimation errors; Markowitz's single index model is one of the components which shrinkage estimator uses as anchor to impose some structure. The resampling technique widely known and utilized by the practitioners and academics were applied in this study in order to take into account different scenarios and the Mean Variance efficient frontier variance.

The results obtained in this study constitutes an important step for socially responsible investing, as true SRI strategy aligned portfolios showed improved risk/performance when compared to other investments alternatives characterized by lower ”ethicalness” level.

Chapter 2

Literature Review

2.1 Socially Responsible Investment

Social investing is ”that investing which seeks to produce both financial and social/environmental value and returns”, Emerson and Bonini (2003). The search for not only financial, but other types of returns - social, environmental - is what is called the blended return.

The shift in investor perspective has brought new opportunities. In the past, financial returns were all that matters but now they are not enough. Social investing covers a huge area of investments possibilities and alternatives, Emerson and Bonini (2003) provides a separation of social investing in two main groups:

- Socially Responsible Investing (SRI);
- Community and Double Bottom Line Investing.

The main difference between these two groups is the degree in which these investments must reach market returns. The same author also highlights the fact that most of the studies related to social investing are developed in silos, without cross-communication and information sharing, and in order to fully develop and support social investment many points still need to be improved, as its actual condition is mostly characterized by lack of efficiency, low diversity of investment instruments, inefficient information flow, no clear investment framework and lack of performance metrics.

Some authors developed frameworks aimed in explaining and modeling the investor behavior towards SRI via the utility obtained by engaging in this kind of investment. In Beal et al. (2005) work, the investor's utility function in investing socially is analyzed by linking the utility received from the investment with psychic returns, investor's ethicalness level and make use of tools developed by happiness researchers.

The main actors in the social investment field according to Nicholls and Pharoah (2008) are finance professionals, third sector entrepreneurs and government policy makers. They also highlight the fact that the demand side - the social enterprises - is mainly characterized by three key features: sociality, innovation and market orientation.

Corporations engage in socially responsible behavior in order to attract investors, improve their image and create blended value, what is known as corporate social responsibility (CSR). In Petersen and Vredenburg (2009), the authors highlight some shared ideas among the financial community regarding SRI, among those it is argued that CSR was considered a type of insurance, the title socially responsible provides some opportunities and economic value was created due to better management, enhanced competitiveness, better government and community relations.

In Godfrey et al. (2009) and Bird et al. (2007) the links between CSR activities and returns are discussed.

Several studies tried to analyze whether or not there is a trade-off between financial performance and non-financial dimensions. Most empirical studies seem to point that no trade-off seems to exist; others show that the trade-off exist or SRI stocks may present enhanced returns. Hence little agreement seems to exist on this matter.

2.2 Portfolio Construction

Portfolio is a set of securities put together with a certain purpose. Several are the techniques and methods created to address this issue, building a portfolio. Different positions exist regarding risk and return when analyzing portfolios:

- Targeting a certain return that matches the investor risk profile;
- Mimic a certain index, trying to be as close as possible of its return and risk profile;
- Beat benchmark index.

The positions just mentioned are also relative, this is to say that these positions may or may not exist, depending on your beliefs and knowledge. For instance, a money manager that believes in the strong form of the Efficient Markets Theory, will never try to beat the market. But on the other hand, an investor who believes in the weak form of the Efficient Markets Theory believes that it is possible to beat the market and will try do to so.

2.2.1 Modern Portfolio Theory

The Modern Portfolio Theory was developed by Markowitz (1959). It constitutes the base for the allocation structure of risky assets.

The theory states that departing from certain information regarding the assets - return and risk - it is possible to reach an optimized set of portfolio which represents the best opportunities for investors to hold the basket of assets defined by these optimized portfolios.

It is important to notice that risk has several meanings, interpretations and impacts - how a certain outcome is perceived - risks will be further analyzed in the next chapters, but it is important to define that risk is how a certain attribute, characteristic deviates from a certain positioning measure. For instance, as a positioning measure we can use mean - μ - and for a risk measure we could use the standard deviation - σ .

According to the Modern Portfolio Theory, departing from estimates of average return and variance of several risky assets, it is possible to define optimized portfolios.

Markowitz states that using the mean, variance and the relationship between the risky assets it is possible to define a group of efficient portfolio - which is called the efficient frontier. This group is characterized by several portfolios which differ from each other by the return and risk trade-off, as the risk increases the expected return also does. An interesting characteristic of this group is that considering a certain risk level σ_0 ,the portfolio that belongs to this group and has σ_0 as its volatility will have the highest expected return - μ_0 - among all feasible portfolio options. This particular fact can be seen on the Figure 2.1.

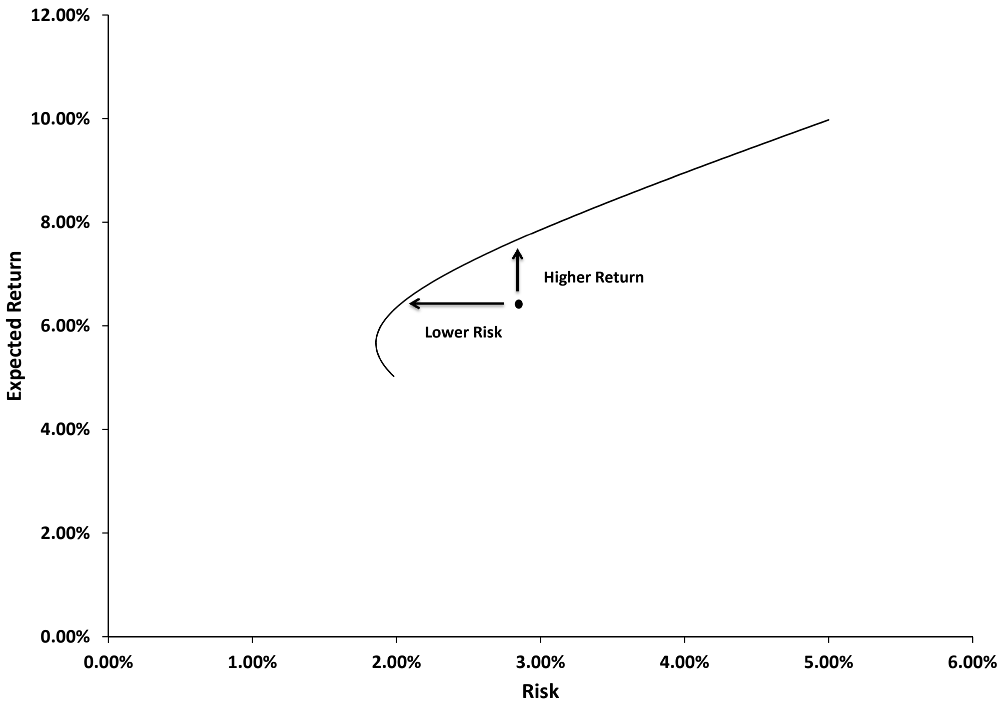

Figure 2.1: Example of an efficient frontier

On Figure 2.1 the portfolio highlighted does not belong to the efficient frontier, for this reason there is one portfolio which is located on the efficient frontier that has the same return - μ_0 - but with a lower risk - σ_1 ($\mu_0 = \mu_1$ with $\sigma_1 < \sigma_0$) . The same rationale can be applied for the risk, for the same risk of this portfolio - σ_0 - there is one portfolio on the efficient frontier that has a higher expected return - μ_2 ($\sigma_0 = \sigma_2$ with $\mu_2 > \mu_0$).

The Mean - Variance theory assumes that mean, variance and covariance are known factors. Money managers, when carrying out their analysis and methods, do not have available the true mean, variance and covariance of their population. This fact is very important when analyzing Markowitz theory and in fact is one of the most criticized points.

As the true values of the population μ and Σ (the covariance matrix of the assets) are unknown, when using the Mean Variance theory, it is necessary to use estimates of them. This fact can generate serious problems, the model stability is closely related to the Σ estimation, which represents an increasingly complicated matter as the number of assets under consideration increases. This matter is discussed by Kyj et al. (2009) when exposing the problems related to the covariance matrix , as the smaller eigenvalues shrinks towards 0 the matrix condition number goes to infinity which can be fought by imposing structure to the estimators.

The condition number of the matrix Σ plays an important role in defining the efficient frontier stability. When we have an ill-conditioned matrix, any small variance on its components will result in big changes on the portfolio set that composes the efficient frontier. Some methods were developed to minimize the problems caused by ill-conditioned matrices by generating well-conditioned estimates of the true Σ:

- Shrinkage;
- Single-factor models.

The problem concerning the amount of assets under consideration and the time span necessary of the return data to be covered in order to avoid the problem regarding the stability of the model was pointed out by Grinold and Kahn (1999).

2.2.2 Efficient Frontier

The portfolios located over the efficient frontier are said to be Mean-Variance efficient. According to Michaud and Michaud (2008), they must obey the following criteria:

- "A portfolio P* is MV efficient if it has least risk for a given level of portfolio expected return;
- The MV efficiency criterion is equivalent to maximizing expected portfolio return for a given level of portfolio risk;
- A portfolio P* is MV efficient if it has the maximum expected return for a given level of portfolio risk."

Due to the fact that Mean-Variance efficient portfolios obey these criterions, they present very interesting characteristics, explaining why so much attention has been focused on them.

As mentioned earlier, the main requisites that Markowitz theory uses when defining and generating the efficient frontier are the portfolio μ and σ. These are represented by the following equations:

$$\mu_P = \sum_{i=1}^{N} \mu_i \omega_i \tag{2.1}$$

$$\sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_i \sigma_j \omega_i \omega_j \rho_{ij} \tag{2.2}$$

Where:

- μ_i is the expected return of asset i;
- ω_i is the percentage of asset i on portfolio P;
- σ_i is the standard deviation of returns of asset i;
- ρ_{ij} is the correlation between assets i and j;

- N is the number of assets under consideration.

The equations 2.1 and 2.2 can be represented by:

$$\mu_P = \omega^T \mu \tag{2.3}$$

$$\sigma_P^2 = \omega^T \Sigma \mu \tag{2.4}$$

Where:

- ω is the vector of portfolio weights;
- ω^T is the transpose of the vector ω ;
- μ is the vector of expected returns of the assets under consideration;
- Σ is the covariance matrix regarding the assets under consideration.

In order to exemplify the implications of equations 2.3 and 2.4, consider the following situation:

- 2 distinct assets - A and B;
- Asset A

  μ_A = 10 % and σ_A = 5 %

- Asset B

  μ_B = 5 % and σ_B = 2 %

- 4 Scenarios

  1 - $\rho_{AB} = 1$

  2 - $\rho_{AB} = 0$

  3 - $\rho_{AB} = -0.5$

  4 - $\rho_{AB} = -1$

The situation just described can be seen at the figure 2.2 :

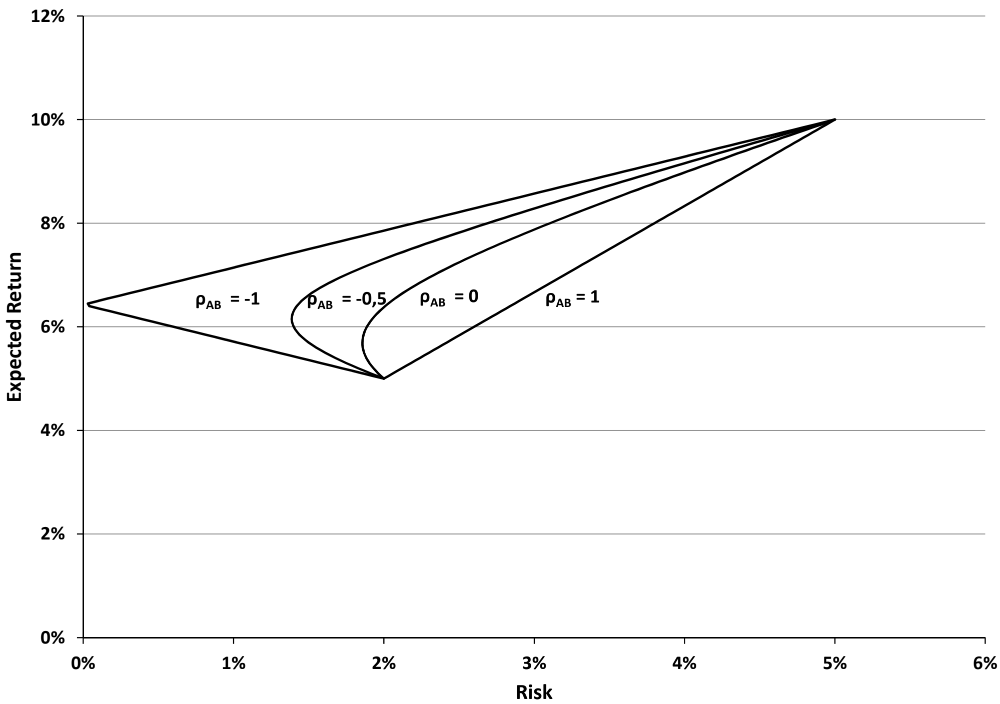

Figure 2.2: Example of an efficient frontier for a portfolio of two assets under different scenarios

As expected, the expected return of the portfolio is a linear combination of the expected return of each asset that composes it. On the other hand, the portfolio variability is not a simple linear combination of the variability of each portfolio asset - except on the case where ρ_{AB} is equal to one - this particular fact shows that by utilizing different assets to compose a portfolio, the investor will be better off than just investing in a single asset. As the ρ_{AB} moves from 1 to -1, in this 2 assets case, an interesting transition occurs on the efficient frontier, at the same return it is possible to pick assets A and B in such proportion that the overall portfolio will have a lower risk. An extreme case happens when ρ_{AB} is equal to minus one, it is possible to create a riskless portfolio which have an expected return of 6.45% , the composition of such portfolio is approximately 29% of asset A and 71% of asset B. This can be done because our two assets portfolio is composed by securities which are perfectly negatively correlated.

2.2.3 The Mean-Variance Efficiency Problem

Markowitz defined in his work what efficient portfolios are, according to him an efficient portfolio is characterized by:

$$Min_{\omega} \quad \omega^T \, \Sigma \, \omega \tag{2.5}$$

Subject to

- $\omega^T \, \mu = \mu_*$
- $\omega^T \, \mathbf{1} = 1$

Where μ_* is the target return.

When an investor is defining his portfolio, he may wish to impose restrictions that he may consider necessary in order to increase the investment value of the portfolio. Other reasons can also be mentioned that justify the inclusion of further constraints.

For instance, the existence of a portfolio which overweights stock A and have very high levels of stock B shorted may be a possible outcome from the model, but in the real world such portfolio may not be possible to be created. In this case, constraints regarding the ω value are used to avoid such situations. Also unbounded portfolios may present very high instability, small changes at the inputs may cause big variations on the portfolio composition, in such cases the amount of trades necessary to rebalance the portfolio may be far too high, which would compromise the portfolio viability. In order to cope with trading costs, some changes on the efficient frontier criteria are deemed necessary, one example of these can be seen at Fernando (2000) work.

The Mean-Variance problem can be grouped in two groups:

- Unbounded Mean-Variance problem;
- Constrained Mean-Variance problem.

2.2.4 Unbounded Mean-Variance Efficiency problem

Unbounded portfolio optimization has been used for a long time by academic and practitioner research, as stated by Michaud and Michaud (2008).

The unbounded framework allows the existence of defined analytical solution, the scenario described by equation 2.5 can be solved by the following equations:

$$\omega = \frac{B\Sigma^{-1}\mathbf{1} - A\Sigma^{-1}\mu + \mu_*(C\Sigma^{-1}\mu - A\Sigma^{-1}\mathbf{1})}{D} \tag{2.6}$$

$$A = \mu^T \Sigma^{-1} \mathbf{1} \tag{2.7}$$

$$B = \mu^T \Sigma^{-1} \mu \tag{2.8}$$

$$C = \mathbf{1}^T \Sigma^{-1} \mathbf{1} \tag{2.9}$$

$$D = BC - A^2 \tag{2.10}$$

The introduction of a constraint blocking the existence of short selling, for instance, causes the efficient frontier to change.

Another interesting application of unbounded framework where no-short-selling constraint exists is to decide the optimal portfolio that should be held by a rational investor. Rational investors seek to maximize their utility function, Markowitz defined the expected utility function by an investor who decides to hold only one fund i:

$$\text{Expected Utility} = \mu_i - \lambda_k\ \sigma_i^2 \tag{2.11}$$

The utility of investor k depends on the expected return of his investment on fund i and it is reduced by the risk of this investment - σ_i^2 - corrected by a risk aversion measure - λ_k. Markowitz's Mean-Variance is not used to measure *ex post* performance because it does not represent a single and universal measure, "each investor must evaluate performance by using a measure designed for his or her degree of risk aversion", Sharpe (1998). In order to achieve such purpose, other measure is widely used, such measure is known as the Sharpe ratio. According to Sharpe (1998):

> "... the fund with the greatest Sharpe ratio is the best for any investor regardless of the investor's degree of risk aversion. In this sense, the measure is universal."

The Sharpe ratio is a measure of excess returns per unit of risk, due to this it can be applied to evaluate the performance of an investment and allow the comparison among different investments. The objective of an investor is to maximize the Sharpe ratio when deciding between two investments alternatives.

An interesting fact for the unbounded framework is that any portfolio that belongs to the efficient frontier is composed by a linear combination of the maximum Sharpe ratio portfolio and the minimum variance portfolio. Specifically on the unbound Mean-Variance efficiency case, any portfolio of the efficient frontier is a linear combination of any other two different Mean-Variance efficient portfolios.

2.2.5 Constrained Mean-Variance Efficiency problem

Despite the fact that unbounded Mean-Variance optimization has at its disposal a well known set of analytical solutions, it does not receive acceptance on the professional money management world. Mostly because the portfolios created under this framework are too volatile, unfeasible under real investing situation and are said to be error optimized.

Normally, professional money managers deal with portfolio construction rules that impose constraints to the portfolio assembly, as: risk exposition limitation, short position constraints, leverage limits, increasing the investment value of portfolio generated. These are few facts that justify why ”Linear constrained, not unbounded, MV optimization is typically the framework of choice for asset management in practice”, Michaud and Michaud (2008).

Under this scenario where linear equality and inequalities are introduced in the problem modeling, defined analytical solutions are not available. In order to solve the Mean-Variance efficiency problem in this case, computational methods are applied.

Common constraints can be related to budget (the amount of value invested cannot result more than the amount of resources available) and shorting level (if no shorting is allowed then it is equivalent to use a non-negative constraint on the portfolio weights vector).

The use of constraints changes the efficient frontier. The Sharpe ratio of the portfolios that belong to the efficient frontier also change. The maximum Sharpe ratio for the constrained frontier is lower than the one that we had when no constraints were utilized. This happens because constraints reduce in-sample risk and/or return of the portfolios of the efficient frontier. A logical question to do is ”Why should I add constraints ?”. Such contradiction can be explained when errors are taken into account, as the true value of return and risk are unknown, when the efficient portfolios are being calculated, estimation of those parameters are used and these present error on their calculation. Unbounded Mean-Variance optimization has the tendency to overweigh or underweigh some securities.

According to Michaud and Michaud (2008), the overweighted securities are those which present large estimated returns, negative correlations and small variance, the underweighted ones present small estimated returns, positive correlations and large variance. These securities are the ones most likely to have higher estimation errors. This is what is known as the error maximization effect which results in heavily underweighted and overweighted asset allocation. In order to counterbalance this effect, constraints are utilized. Constrained portfolios are proved in Frost and Savarino (1986) to be improved on average compared to unconstrained Mean-Variance efficient portfolios. The Mean-Variance efficiency problem can be formulated in several different ways as shown by Fernando (2000).

2.3 Mean-Variance Efficiency Critics

Markowitz's Mean-Variance theory is a very known mark on the finance theory field, a huge amount of work and research were based on his theory. Due to that the prestige that his theory enjoys is notable. Due to this, many people simply ignored its shortcomings and limitations. This fact may have led, in some cases, to negative results which induced a behavior described by discredit, disbelief among part of the financial community.

Over time, practitioners and academics have raised several objections regarding Markowitz's Mean-Variance theory as the most suitable framework when the objective being targeted is portfolio optimality. In Michaud and Michaud (2008) four main categories are highlighted based on their similarities regarding this fact:

1. Investor Utility;
2. Normal Distribution;
3. Multi-period Framework;

4. Asset-Liability Financial Planning.

Even though these categories cover some weak points of Markowitz's theory, the main drawback of it is not covered. According to Michaud and Michaud (2008), the most important limitations that must be highlighted regarding Markowitz's theory are instability and ambiguity.

Chaotic systems are characterized by their unpredictability regarding the output when small disturbances are added into the input. Mean-Variance optimization may behave as a chaotic investment tool, as small changes in the input parameters may result in substantially different efficient portfolios. The model sensitivity to the input parameters is a very well known deficiency from the Mean-Variance efficiency optimization theory, which has been addressed by many authors throughout time (Kim and Boyd (2008), Black and Litterman (1992), Best and Grauer (1991), Britten-Jones (1999), Jorion (1986), Michaud and Michaud (2008)).

2.3.1 Unbounded Mean-Variance Efficiency limitations

Unbounded Mean-Variance efficiency framework does not enjoy a good reputation among practitioners or researchers. In Michaud and Michaud (2008), the studies of J.D. Jobson and Bob Korkie caused a serious impact on the unbounded Mean-Variance optimized portfolios reputation, according to their studies, biases in the Mean-Variance optimized portfolios can be significant, through Monte Carlo resampling they showed that the Mean-Variance efficiency frontier had an average maximum Sharpe ratio of 0,08, while the true Sharpe ratio for the data was 0,32 and the Sharpe ratio of an equally weighted portfolio was 0,27.

The investment value of a Mean-Variance optimized portfolio is strictly related to the input quality, information and methodology. Jobson and Korkie results showed the importance of meaningful investment related constraints in order to increase the performance and investment value of Mean-Variance optimized portfolios.

2.4 Mean-Variance Efficiency Alternatives

Despite the fact that many authors have criticized Markowitz's Mean-Variance theory, it still remains the base, the foundations of portfolio analysis. Thus it can be said that it is a robust theory. Nonetheless, over time several other methodologies have been proposed as alternative to the Mean Variance efficiency proposed by Markowitz. As any model that tries to represent, recreate reality, some simplifications are made. It is the user responsibility to define which trade offs are being considered and at what cost. When different models are under consideration, advantages and disadvantages of each model will arise when these are being compared against each other.

Michaud and Michaud (2008) proposed five broad categories to host these alternatives. According to the same author, these alternatives do not address the main basic limitation of the Mean Variance efficiency theory. The five categories are:

- Alternative risk measures;
- Utility function optimization;
- Multi period objectives;
- Monte Carlo financial planning;

- Linear programming.

2.4.1 Alternative risk measures

The traditional Mean Variance theory uses standard deviation or the variance as a measure of risk, but several other indicators can be used to represent risk as:

- Semi variance;
- Mean absolute deviation;
- Range measures;

In Michaud and Michaud (2008), the author highlights the fact that ”alternative risk measures are often more difficult to estimate accurately. Analysts must weigh the trade-off between estimation error and a more conceptually appealing measure of risk.”

As in any analysis, some caution must be taken whether or not a certain measure makes sense in a specific context, the trade-offs involved when using a certain measure and what are the costs involved to estimate it.

2.5 Mean Variance Methods

Efficient portfolios according to the modern portfolio theory are those located over the mean variance efficient frontier. Markowitz proposed an approach to obtain this set of portfolios through the use of the mean, variance and covariance of returns obtained from historical data. Such task becomes burdensome as the number of securities increases, for instance, in the case of a portfolio composed by n securities we have:

- n estimates of expected returns,
- n estimates of variance,
- (n^2 - n) / 2 estimates of covariances.

Hence for a portfolio composed of 50 stocks, 1325 estimates will be necessary. As the number of estimates necessary does not vary linearly but exponentially, when the number of stocks considered is for instance 3000, more than 4,5 million estimates will be required.

A common point of criticism against the traditional Mean Variance approach is regarding the covariance matrix instability, which can result in very volatile portfolios and with little investment meaning.

2.5.1 MCDM approach

A multi-attribute utility optimization model was proposed by Ehrgott et al. (2004). In this work five sub-objectives - 12-month performance, 3-year performance, annual dividend, Standard and Poors Star Ranking and volatility - are targeted by the optimization procedure. The motivation of the proposed method is due to the lack of flexibility of the traditional Mean Variance efficiency optimization where only the maximization of expected returns at a specific risk level is considered, not taking into account other investor's objectives and preferences.

2.5.2 Single Index Model

The Mean Variance efficiency theory proposed by Markowitz constitutes the base of many others portfolio related methods and theories. When Markowitz developed it, he also acknowledged some limitations regarding the difficulties related to the estimation of the several input parameters, to deal with it he proposed a single index model. His objective was to explain the correlation among securities returns, the underlying assumption was that such correlation was dependent on an index which represents a general property of the market.

2.5.3 Capital Asset Pricing Model

The Capital Asset Pricing model, CAPM, was developed by William Sharpe in Sharpe (1964). The CAPM is a single index model in which the return of asset i is due to its relationship with a risk index representing the overall market movements, the market risk or systematic risk. In Markowitz and Fabozzi (2002), the term market risk is described as "By market risk it is meant the risk associated with holding a portfolio consisting of all assets, called the market portfolio".

An important point that needs to be highlighted is the fact that even though the CAPM and the Market Model seem very similar at first glance, they are different and this fact has created some confusion, Markowitz highlighted this fact in Markowitz (1984).

2.5.4 Multi-Factor Risk Models

Asset pricing models are not restricted to single-index models; several models proposed in the literature consider more than one risk factor. The CAPM considers only market risk as the risk index, the arbitrage pricing theory - APT - goes at the opposite direction. According to the APT, the asset's expected return is influenced by several risk factors and such relationship is characterized by linear function. What the APT does not specify is how many of these factors there are and what they are.

Several multi-factor risk models were proposed by academics and practitioners. These can be grouped in three main types:

- Statistical factor models;
- Macroeconomic factor models;
- Fundamental factor models.

2.5.5 Resampled Mean Variance Optimization

The Mean Variance efficient frontier is directly linked to the input parameters used in the optimization, hence variations on the input will result in variation on the output - the mean variance efficient frontier. Unless one knows the true value of the inputs and exactly how they behave, these inputs will present variations and there is some uncertainty surrounding the expected values.

Resampled Mean Variance optimization takes into account this input uncertainty through a set of Monte Carlo simulation. Simulated data regarding the inputs are fed into the traditional Mean Variance optimization procedure, a simulated Mean Variance efficient frontier is obtained, and this frontier is recorded. This process is repeated several times (this process can be seen at figure 2.3). The final Mean Variance efficient frontier is composed by an average of assets allocation obtained previously.

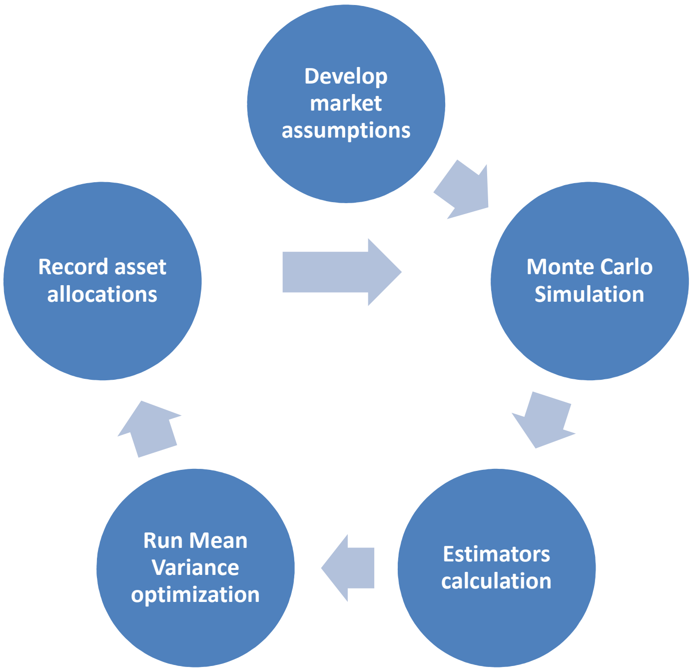

Figure 2.3: Mean Variance efficient frontier calculation through resampling procedure.

The resampling process can be carried out through two types of Monte Carlo simulation:

- Non-parametric simulation;
- Parametric simulation.

2.5.6 Stein Estimators

Bayesian statistics assumes a prior. A prior can be interpreted as an aimed guess or an assumption that imposes an external constraint into the set of potential solutions. Stein estimators belong to the Bayesian estimation procedures groups, but differ a little from the general body of Bayesian procedures, as Stein estimators impose structure while Bayesian are designed to be used in a wide range of contexts.

Several Stein estimators have been developed so far intended to be used on Mean Variance optimization context. Among these are the following:

- James-Stein;
- Frost-Savarino;
- Ledoit;
- Stein.

James-Stein Estimator

The James-Stein estimator (James and Stein. (1961)) is an estimator for asset means. It assumes the existence of a global sample mean, the sample mean of each asset under analysis will shrink or not towards the global mean according to the assets variance. The more volatile is the asset, the more its mean will shrink toward the sample mean.

Frost-Savarino Estimator

The Frost-Savarino estimator (Frost and Savarino (1986)) is intended to obtain mean and covariances using the efficiency of an equally weighted portfolio as its prior.

Ledoit Estimator

The Ledoit estimator (Ledoit and Wolf (2004a)) is intended to deal with the covariance matrix. This estimator shrinks the covariance matrix towards the model prior, which is the CAPM used in this case for risk estimation.

2.5.7 The Bays Return

Equity management relies on the team expertise in forecasting market outlooks, assets future behavior and risk-adjusted excess returns. This data can be gathered from numerous sources within and outside the management team, thus it is important to consolidate such external point of view into historically estimated data in order to add value to the latter. Bayesian procedures do so through the assumption of a prior.

2.6 Diversification

The idea behind creating portfolios is to reach a certain expected return for a risk level that is inferior to the one that exists by holding a single security which is characterized by the expected return under consideration. It is the same rationale as the insurance principle, in the case of independent risk sources, insurance companies will achieve risk reduction by writing policies which insures against independent sources of risk.

Risk reduction through diversification presents its limits; one can reduce his risk levels as low as possible until a certain point. The amount of risk that can be achieved through diversification is called diversifiable risk, firm-specific risk, unique risk or nonsystematic risk. The remaining portion of the risk that cannot be removed through diversification is called market risk, systematic risk or nondiversifiable risk.

Market risk is due to macroeconomic factors, these can be inflation, business cycle, interests rates, exchange rates, GDP growth, unemployment rates and price indices. In Statman (1987), the author develops a study using the NYSE stocks to check the effect of portfolio diversification according to the number of random selected stocks to compose an equally weighted portfolio, the results of such study are shown at figure 2.4, where the expected standard deviation of annual returns as the number of stocks varies are shown. The portfolio standard deviation decreases as the number of stocks composing it increases, showing the effect of diversification. As the number of stocks increases, the marginal standard deviation reduction diminishes, until a certain point where the reduction stops and no more standard deviation reduction through diversification can be achieved; this is the point where the systematic risk is the only one remaining.

2.7 SRI Implications

The decision whether or not to formulate strategies and develop portfolios based on socially responsible criteria compliance generates impacts on the methodology, procedures and final outputs of the process. Thus when an organization decides to do so, it is important that the desired changes are implemented and emphasized through all the processes. The impacts on the final output need also to be evaluated, what may seems a good idea due to trends, if not perfectly understood may result in undesirable outcomes.

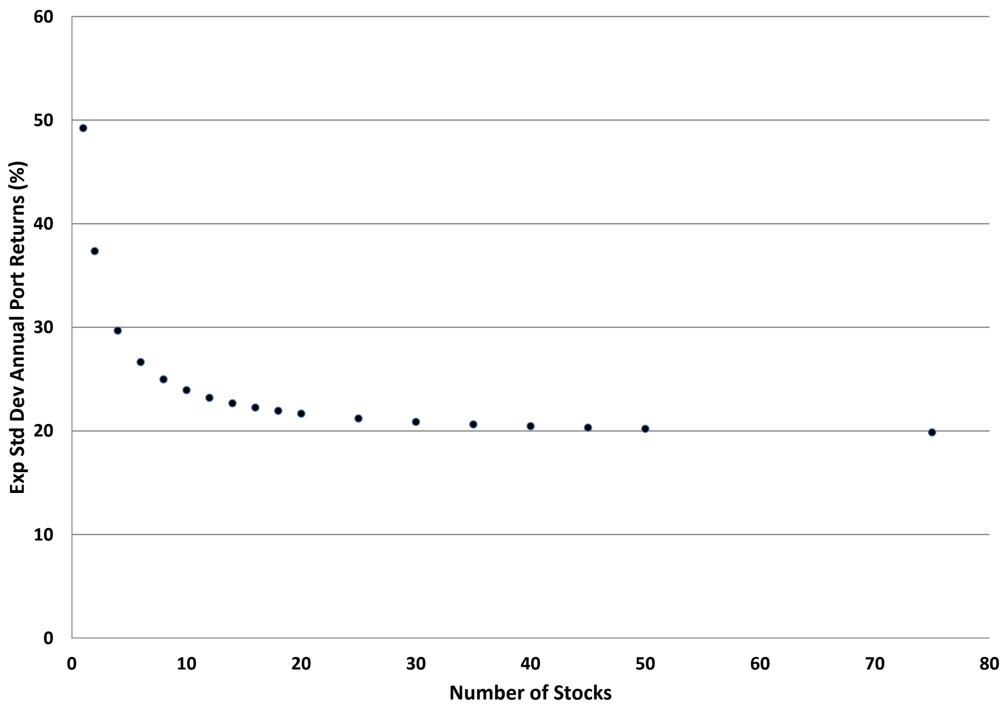

Figure 2.4: Expected standard deviation of annual portfolio returns according to the number of stocks composing the portfolio

Several studies were carried out through time in order to analyze and measure the impacts of socially responsible investments, whether or not they present higher or lower returns, risks or constitute a new class of assets with its own properties and behavior.

2.7.1 SRI and Diversification

Creating a portfolio of securities requires a set of information regarding the securities under consideration, but not only this information affects the final output, the range of securities available are a critical point that a money manager must take into account. The broader is the range of securities available, the broader will be the possibilities of portfolios to be created.

Inserting limitations, constraints on the range of securities at the disposal of the money manager may result in counterproductive portfolio possibilities, shrinking the Mean Variance efficient frontier. Several studies addressed the return differences of socially responsible portfolios against SRI compliant, but few addressed the risk-adjusted performance of socially responsible funds. Rudd (1981), Grossman and Sharpe (1986) and Diltz (1995) argue that when a portfolio is subjected to constraints, its performance is affected negatively. They also assert that socially responsible investment compliance introduces certain biases in the portfolio resulting in reduction of the long-run performance. Not only this, but also socially screened presents higher extra market covariation in returns.

In Bello (2005), through the study of non-systematic variance of portfolio returns of socially responsible funds and non-SRI compliant, the author defends that no significant difference between these founds were found regarding the characteristics of assets, degree of portfolio diversification or long-run investment performance. While Rudd (1981) defends the existence of higher levels of extra market covariation in the case of socially responsible funds, Bello (2005) goes at the opposite direction, the residual variance of socially responsible funds and non-SRI compliant funds do not present statistical significant difference.

2.7.2 SRI and Mean Variance efficiency frontier

According to Markowitz's theory, an investor that decides to pursue a goal different from the maximization of expected portfolio return for a given risk level, will not end up with a portfolio that belongs to the efficient frontier, thus a non-mean variance efficient portfolio. An investor pursuing socially responsible objectives adds constraints to the portfolio in order to satisfy his needs. Constraints generate shrinkage of the mean variance efficient frontier. Hence one could expect that socially responsible portfolios will present lower returns for the same risk level as a non-SRI compliant portfolio. According to Geczy et al. (2005), the insertion of constraints in order to satisfy socially responsible criteria can affect harmfully diversification.

In Drut (2010), a study of the efficient frontier shape subjected to socially responsible constraints shows that the efficient frontier may change according to the socially responsible rating behavior in comparison to the expected return along the traditional efficient frontier and the constraint strength. In this study, the socially responsible constraint is represented by a grade, a rate assigned to each security composing the portfolio, and the portfolio grade is composed by the weighted sum of the individual securities grades. Hence the constraint strength is defined by the investor as "ethicalness" level that the portfolio must comply.

The effects on the efficient frontier due to the need to meet socially responsible criteria was also studied by Galema et al. (2009). In this work, when the entire efficient frontier is being considered, if there are no-short-sale constraints, investor will be worse off (the result stands for the social, environment, product and sin dimensions, in the case of corporate governance the result does not hold). The investor's loss is not due to foregone returns, but due to foregone risk reduction opportunities. In the case where short sales were not allowed, investors will not be worse off when engaging in socially responsible investing.

The effects of the use of socially responsible screening over the Mean Variance efficient frontier are very important from the investor point of view, as it shows what are the expected results and behavior of the investment due to engaging in socially responsible investment compliance. In Herzel et al. (2011), the three main areas of socially responsible investment - environmental, social and governance - are used to study the effects of screenings criteria that fit with these areas over the Mean Variance efficient frontier. In this study, the Fama and French model was adopted to generate the Mean Variance efficient portfolios based on three risk levels - minimum variance efficient portfolio, medium variance efficient portfolio and maximum variance efficient portfolio - an indicator named "price of sustainability" was used to measure the cost of engaging in socially responsible investing. This indicator measures the loss of Sharpe Ratio after screening. Their results show that engaging in socially responsible investment that fits the social area presents the highest sustainability price, followed by governance and environmental. A spanning test showed that investment opportunities by diversification through the inclusion of non-SRI compliant securities only holds when short selling is allowed. Market capitalization of the investment universe considered is heavily impacted by social screening.

Chapter 3

Data

Nowadays socially responsible funds are widespread throughout the world; this work will be based on European funds. In order to have a full set of reliable socially responsible funds list, the Eurosif data was used.

The European Sustainable Investment Forum - Eurosif - is a ”pan-European network and think-tank whose mission is to Develop Sustainability through European Financial Markets. Current Member Affiliates of Eurosif include institutional investors, financial service providers, academic institutes, research associations, trade unions and NGO’s. The association is a not-for-profit entity that represents assets totaling over 1 trillion through its affiliate membership”, Eurosif (2011).

Data regarding the existence of social, environmental and governance criteria used by the Eurosif listed funds was obtained at Eurosif-Avanzi database. Monthly returns of these funds were provided by Morningstar.

3.1 Dataset

The initial dataset was composed by 531 funds. From this initial dataset, funds which presented missing data regarding social, environmental and governance criteria, funds with short-selling positions and non-European funds were removed. The resulting dataset is composed by 226 funds. In this 226 funds group, in order to create a homogeneous dataset where the fund size does not create disturbs, outlier analysis was carried out. The tool used was the box plot using the Net asset Value data. The result can be seen at figure 3.1.

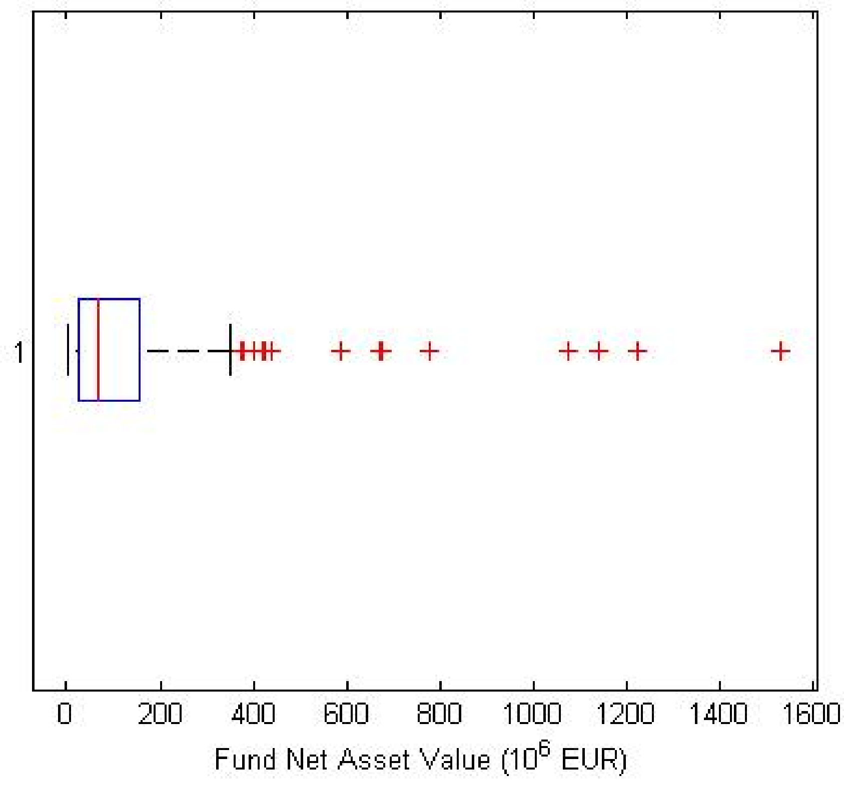

Figure 3.1: Net Asset Value outlier analysis - Box Plot

The resulting database is composed by 213 funds. The latter dataset was the one used to conduct this research, and from now on will be referred to only as dataset. Table 3.1 provides a fund size summary of the dataset composition.

Table 3.1: Funds Net Asset Value Summary

Fund Net Asset Value (10^6 EUR)	
Average	93,04
Median	60,48 t
Mode	70,82
Std Error	91,41
Min	2,17
Max	349,93

3.1.1 Dataset Overview

In order to provide an overview of the data used in this study, this section focus on extracting and providing a few descriptive statistics from the data. Information regarding the funds size can be seen at figure 3.2. More than 60% of the funds of the dataset has net asset value up to 100 million Euros.

Figure 3.3 provides information regarding the fund domicile distribution of the dataset.

The criteria used by funds to screen and choose which assets to invest vary according to the fund objectives and constraints. A variety of such criteria was used in this study, in total 24 different criteria were considered; the list of these can be seen at table 3.2.

In the dataset, all funds make use of at least one criterion. The percentage of funds which apply at least one criterion of the four groups can be seen at table 3.3. In our sample, the criteria least used is the environmental one, followed by corporate governance and controversial business involvement. The criteria which

Table 3.2: Controversial Business Involvement, Social, Environmental and Governance criteria used by the funds that compose the dataset.

Environmental Criteria	Excessive negative impact
	Harmful products/services
	Beneficial products/services
	Environmental protection
Social Criteria	Human rights violations
	Labor rights violations
	Oppressive regimes
	Human rights protection
	Community development
	Quality of life
Corporate Governance Criteria	Corporate governance
	Customer relations
	Employee relations
Controversial Business Involvement	Firearms
	Weapons/military
	Nuclear power
	Tobacco
	Gambling
	Adult entertainment
	Alcohol
	Animal testing
	Factory farming
	Furs
	Gmo

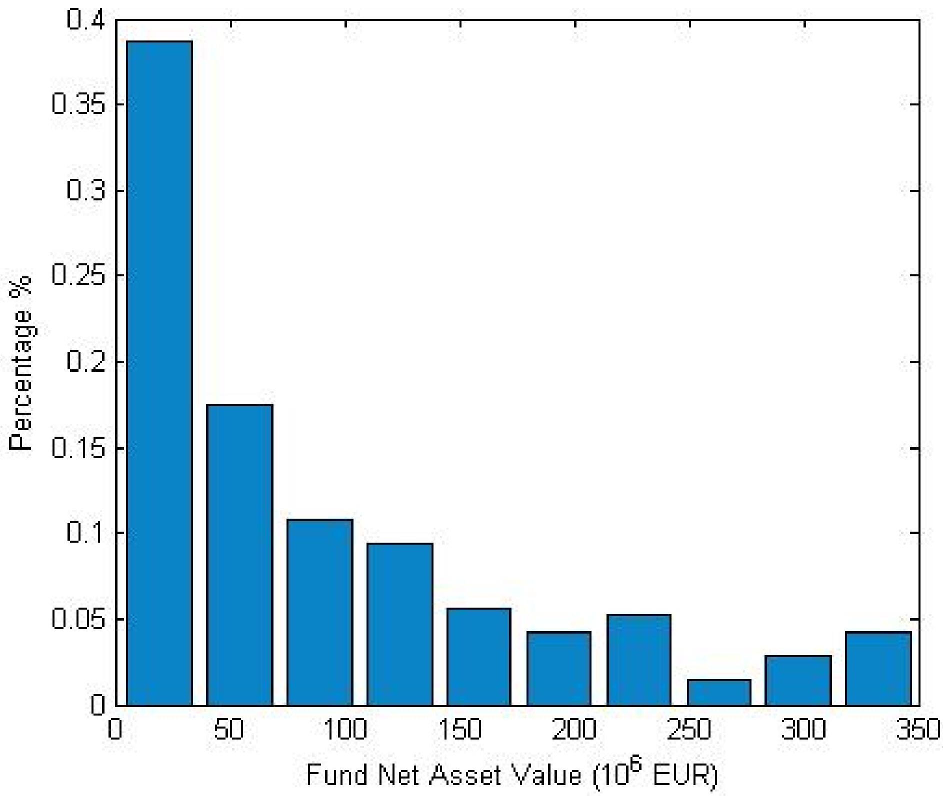

Figure 3.2: Fund sizes distribution.

belong to the social group are the mostly used ones.

Table 3.4 shows that the controversial business involvement are the criteria that, when used, is the most used one, mainly due to higher number of criteria belonging to this group. Social criteria are the second one in the rank. On average, the number of criteria utilized by the funds, independently of its criteria intensity level, in the dataset is 10,12.

The criteria intensity - number of any criterion belonging to the four distinct groups mentioned earlier used by a fund - is a measure of major interest in this study. Figure 3.4 shows the distribution of the criteria intensity in the dataset

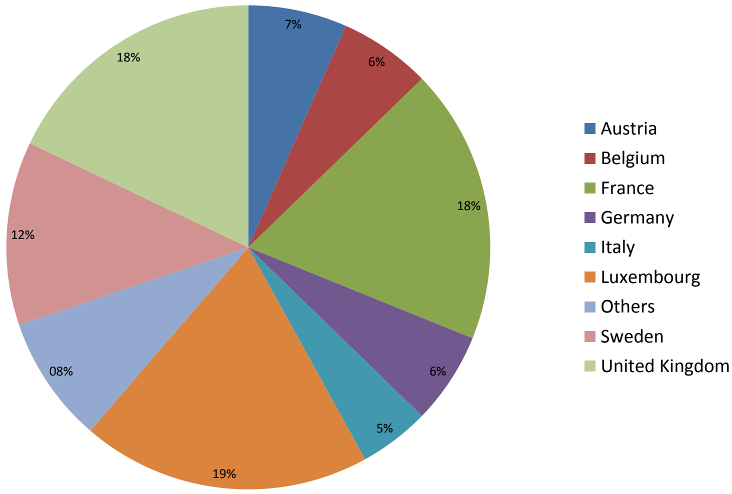

Figure 3.3: Fund domicile distribution

Table 3.3: Percentage of funds that make use of at least one criterion of the following groups.

Controversial Business Involvement	81,1%
Corporate Governance Criteria	69,3%
Environmental Criteria	55,2%
Social Criteria	83,0%

and the accumulative distribution function. The adoption of a certain quantity of criteria belonging to any of the considered groups is widespread, which indicates that there is no consensus regarding an optimal amount of criteria to be adopted.

The marginal increase in the criteria intensity level as the number of criteria changes is very stable, which suggests a linear relationship between criteria intensity and the accumulative distribution function of the number of funds at that specific level. A linear regression with a R^2 of 0.986 can be seen at figure 3.5. This means that each set of funds characterized by a certain criteria intensity level

Table 3.4: Criteria distribution in the dataset.

Criteria / Parameters	Mean	Std Deviation
Controversial Business Involvement	5,04	3,22
Corporate Governance Criteria	1,43	1,14
Environmental Criteria	1,14	1,24
Social Criteria	2,52	1,80
Any Criterion	10,12	5,50

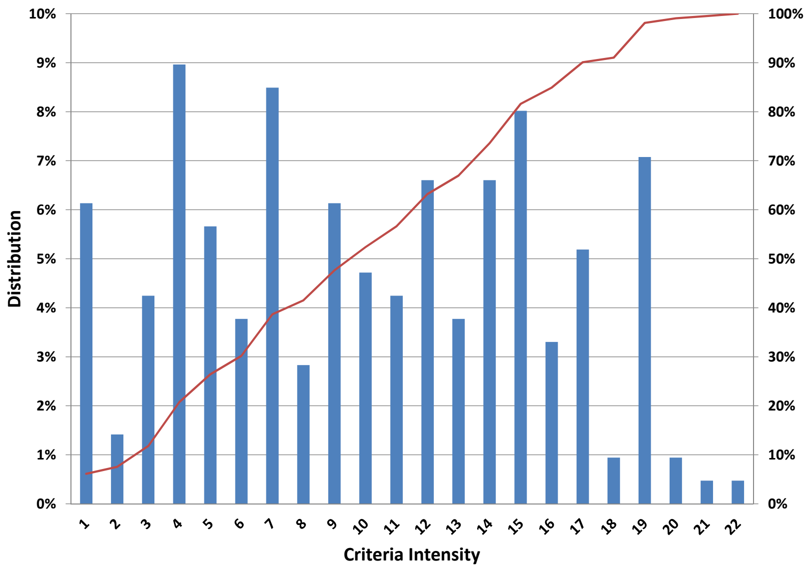

Figure 3.4: Criteria intensity distribution

will not differ substantially from another group, thus propitiating the creation of homogeneous groups regarding the group size.

Regarding risk and return of the dataset, tables A.1, A.2 and A.3 (Appendix A) provides a summary of annualized risk and return using monthly return data from 2008 to 2010, 2006 to 2010 and 2001 to 2010 respectively, by group (groups differ by the criteria intensity level of the funds composing them, further information about the groups structure utilized in this study can be seen on chapter 4).

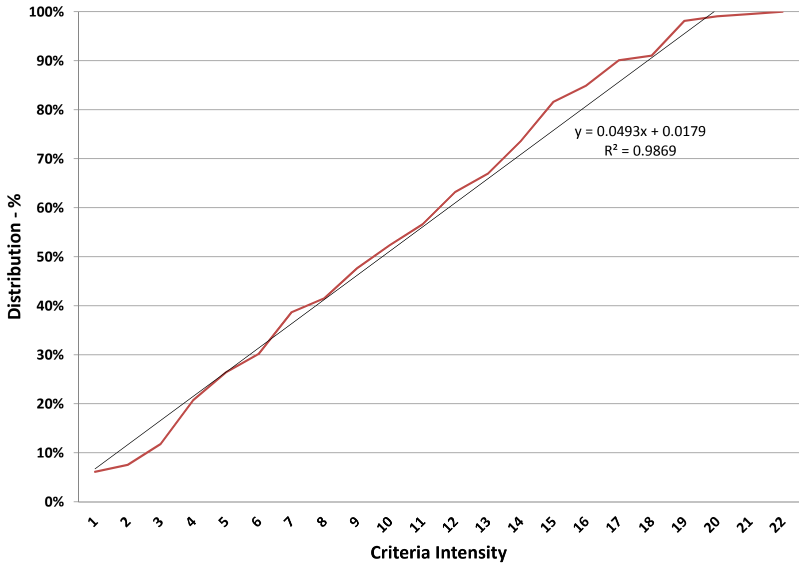

Figure 3.5: Regression of the accumulative distribution function of the number of funds at a certain criteria intensity level.

Chapter 4

Methodology

4.1 Motivation

Socially responsible investing has gained huge momentum in the past few years, the amount to resources related to such investment class has been growing at very fast pace. Analyzing such asset group constitutes a matter of major interest and importance in modern society, as it is an investment class that not only brings financial returns, but also general society improvement. This blended return constitutes a new form of developing our society in such way to bring and improve the life of those who inhabit the 'peripheral' areas, push and stimulate business to act in a sustainable way and do not act recklessly.

But when one decides to invest socially, does it represent changes on its capabilities to be financially rewarded or the risk level that he will face in order to achieve those returns? Several studies in the literature traced the relationship between return and SRI, or risk and SRI. But so far the results can be considered inconclusive, as there is no defined and unique answer for those points.

This study tries to analyze how engaging in socially responsible investments will shift the efficient frontier. Through the analysis of the Mean Variance efficient frontier behavior by changing the number of socially responsible criteria being considered changes - the criteria intensity level.

4.2 Method

The analysis of the Mean Variance efficient frontier based on Markowitz's framework is greatly affected by the inputs quality, as mentioned by several authors. In order to avoid what Michaud and Michaud (2008) calls as ”error maximization”, other methodology rather than the one proposed initially by Markowitz was used in this study.

As this study focuses on past performance data of socially responsible funds, no forecasting model is deemed necessary. The Markowitz Mean Variance efficiency framework using the mean and covariance matrix obtained from the sample would be a good model to analyze the effects of the criteria intensity over the Mean Variance efficiency frontier if it was reasonable to assume that the extreme positions generated by such methodology were adopted by funds, which is not true as no fund knows the true value of securities expected returns and volatility. As more diversified portfolios are more likely to be adopted by funds, a method which does not generate such extreme positions is more adequate.

Balancing the estimator error with a biased structural model is the key to achieve a good estimator. This line of thought was proposed in Ledoit and Wolf (2003b), the author highlights the trade-off between a very structured estimator but biased and an unbiased estimator but with very high estimation error, there must be an optimal point where these two distinct estimators are put together in order to minimize bias and estimation error.

Ledoit and Wolf proposed a method to estimate the covariance matrix balancing the trade-offs between estimation error and bias. Their estimator is a Stein estimator, as it shrinks one value towards another optimally. Their shrinkage estimator for the covariance matrix uses the covariance matrix obtained from the sample data and the covariance matrix obtained through the single-index model to compose a new and improved estimator.

The Ledoit-Wolf model is good alternative as it does not require the development of complex and burdensome multi-factor models, has investment sense (shrinking the sample covariance matrix towards a widely known and accepted structural model), presents good performance compared to other alternatives (Ledoit and Wolf (2003b)), provides robust estimator of the covariance matrix and in Michaud and Michaud (2008) the author defends the superiority of Stein estimators as historical information can be merged with structural models resulting in a superior estimator.

Optimization methods normally assume the existence of a very high level of confidence over the model inputs, which results the optimal solution for that specific set of data. But the optimal solution for that set does not mean that it represents the true optimal point. Thus uncertainty regarding the inputs is not considered.

As the inputs change, the Mean Variance efficiency frontier will also shift. In order to take into account this uncertainty of the Mean Variance efficiency frontier, a Monte Carlo simulation was used to support the development of a robust analysis.

The simulation process is the following: from the sample data, the mean, variance and covariance of returns are estimated; these estimation are fed into a Monte Carlo simulation where 1000 sets of 60 month data are estimated; each set is used to generate 100 equally spaced Mean Variance efficient portfolios; the final set of Mean Variance efficient portfolios are the average of the 1000 portfolios generated (for each of the 100 equally spaced portfolios).

The data retrieval and analysis was separated by groups, in total six groups are utilized in this study. The criteria to create the groups was the criteria intensity level (CIL), the groups and their CIL are the following:

- Group 1 - $1 \leq CIL \leq 3$;
- Group 2 - $4 \leq CIL \leq 6$;
- Group 3 - $7 \leq CIL \leq 9$;
- Group 4 - $10 \leq CIL \leq 12$;
- Group 5 - $13 \leq CIL \leq 15$;
- Group 6 - $16 \leq CIL$;

The Sharpe-ratio is an adequate measure to compare and pick best performing portfolios when the investor decision compress mutually exclusive alternatives, as stated in Bodie et al. (2003). Using the CAPM perspective, the market portfolio is the optimal investment option when combined with risk-free asset as it will deliver a superior performance when compared to other portfolios of the Mean Variance efficient frontier and it is the tangent portfolio of the capital market line.

A characteristic of such portfolio is that it is the maximum Sharpe-ratio portfolio among all portfolios composing the Mean Variance efficient frontier.

The maximum Sharpe-ratio[1] portfolio of each criteria intensity level will be compared to each other in order to provide information regarding the effects of different CIL over the Mean Variance efficient portfolios. The maximum Sharpe-ratio data was obtained from the several simulation rounds, at each round the maximum Sharpe-ratio for each set of efficient portfolios was recorded to be analyzed in the analysis phase.

An overview of the framework utilized in this study can be seen on figure 4.1. From the sample data, information regarding mean monthly returns, standard deviation and covariance are gathered. The sample covariance matrix is fed into Ledoit-Wolf estimator which shrinks it towards the covariance matrix calculated from the single index model. The next step is to use this data as input into the Monte Carlo simulation, each simulation will provide monthly returns data for the desired time length. These simulated returns are utilized as inputs to obtain the shrinkage estimator correspondent to this new dataset. The next step is to feed the optimization model with the newly obtained estimators, obtaining 100 equally spaced Mean Variance efficient portfolios. The portfolio allocation is calculated for each portfolio of each group frontier, also the maximum Sharpe-ratio data is recorded for each CIL. This process repeats for 1000 times.

[1]The Sharpe-ratio was calculated using the EURIBOR obtained at the European Banks Federation web page.

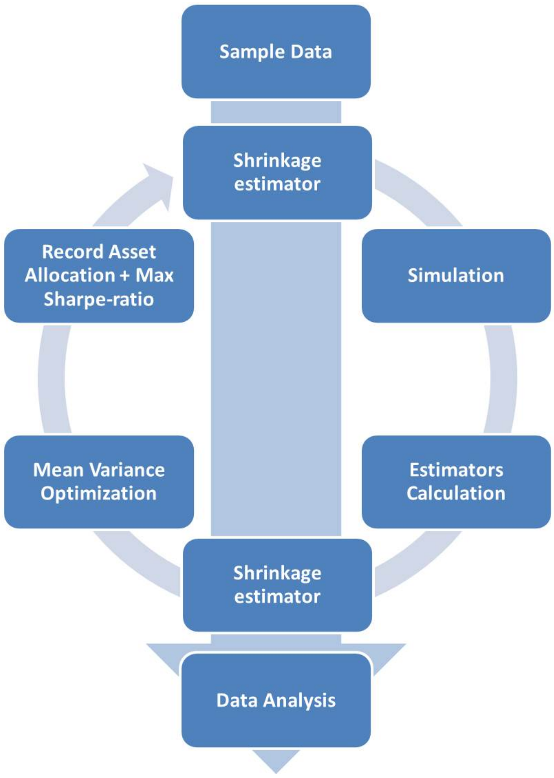

Figure 4.1: Overview of the framework utilized in this study.

4.3 Mathematical Model

The optimization model used to generate the Mean Variance efficient portfolios is:

$$Min_w \quad \sigma_P^2 = \omega^T \Sigma \mu \tag{4.1}$$

Subject to:

$$\mu_P = \omega^T \mu = \mu^* \tag{4.2}$$

Where:

- ω is the vector of portfolio weights;
- ω^T is the transpose of the vector ω ;
- μ is the vector of expected returns of the assets under consideration;
- Σ is the covariance matrix regarding the assets under consideration.

This study used the Ledoit-Wolf shrinkage model, proposed in Ledoit and Wolf (2003b) to obtain the covariance matrix. The mathematical formulation is as follows:

Sample Data

$$\mathbf{m} = \frac{1}{T}\mathbf{X}\,\mathbf{1} \tag{4.3}$$

$$\mathbf{S} = \frac{1}{T}\mathbf{X}(\mathbf{I} - \frac{1}{T}\mathbf{1}\,\mathbf{1'})\mathbf{X'} \tag{4.4}$$

Where:

- **X** is a NxT matrix (N securities, T returns);
- **m** is the mean vector of the sample;
- **S** is the covariance matrix;
- **1** is a vector of ones;
- **I** is an identity matrix;

Single-Index Model

$$x_{it} = \alpha_i + \beta_i x_{0t} + \varepsilon_{it} \tag{4.5}$$

$$\Phi = \sigma_{00}^2 \beta\beta' + \Delta \tag{4.6}$$

Where:

- x_{0t} is the market return on time t;
- Φ is the covariance matrix of the single-index model;
- σ_{00} is the market returns variance;
- Δ is the diagonal matrix containing Var(ε_{it}).

In order to estimate this model, securities' returns are regressed to market returns. The single-index model estimator for the covariance matrix results in:

$$\mathbf{F} = s_{00}^2 \, \mathbf{b} \, \mathbf{b'} + \mathbf{D} \tag{4.7}$$

Where:

- s_{00}^2 is the sample variance of market returns;
- **b** is the vector of slopes estimates;
- **D** is the matrix of residual variance estimates.

Shrinkage Estimator

$$\hat{\mathbf{S}} = \frac{\kappa}{T}\mathbf{F} + (1 - \frac{\kappa}{T})\mathbf{S} \tag{4.8}$$

$$\kappa = \frac{\pi - \rho}{\gamma} \tag{4.9}$$

$$\pi = \sum_{i=1}^{N}\sum_{j=1}^{N}\pi_{ij} \tag{4.10}$$

$$\rho = \sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{ij} \tag{4.11}$$

$$\gamma = \sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_{ij} \tag{4.12}$$

Where:

- $\hat{\mathbf{S}}$ is the covariance matrix shrinkage estimator;
- π_{ij} = Asymptotic Var $[\sqrt{T}s_{ij}]$;
- ρ_{ij} = Asymptotic Cov $[\sqrt{T}f_{ij}, \sqrt{T}s_{ij}]$;
- γ_{ij} = $(\phi_{ij} - \sigma_{ij})^2$.

Consistent estimators of π_{ij}, ρ_{ij} and γ_{ij} are the following:

π_{ij} consistent estimator is:

$$p_{ij} = \frac{1}{T}\sum_{t=1}^{T}\{(x_{it} - m_i)(x_{jt} - m_j) - s_{ij}\}^2 \tag{4.13}$$

ρ_{ij} consistent estimator is:

If $i = j$ then:

$$r_{ii} = p_{ii} \tag{4.14}$$

If $i \neq j$ then:

$$r_{ij} = \frac{1}{T}\sum_{t=1}^{T} r_{ijt} \tag{4.15}$$

Where:

$$\begin{aligned} r_{ijt} = &\frac{s_{j0}s_{00}(x_{it}-m_i)+s_{i0}s_{00}(x_{jt}-m_j)-s_{i0}s_{j0}(x_{0t}-m_0)}{s_{00}^2}. \\ &.(x_{0t}-m_0)(x_{it}-m_i)(x_{jt}-m_j)-f_{ij}s_{ij} \end{aligned} \tag{4.16}$$

Where:

- m_0 is the sample mean of market returns;
- s_{i0} is the sample covariance of market returns and assets returns.

γ_{ij} consistent estimator is:

$$c_{ij} = (f_{ij} - s_{ij})^2 \tag{4.17}$$

Chapter 5

Results

This chapter presents the results obtained in this study. Initially the results obtained from data spanning from 2008 to 2010 will be presented, followed by the results regarding 2006 to 2010 and 2001 to 2010.

5.1 Criteria Intensity Effects over the MV Frontier - Period 2008/2010

Using data spanning from January 2008 to December 2010, Mean Variance efficient portfolios were created according to their criteria intensity level (CIL). Each set represents socially responsible oriented funds grouped together in such way to maximize the portfolio return for a given risk level. One way to interpret the position of the Mean Variance efficient frontier using socially responsible aligned funds as assets is assuming the same point of view of a fund of funds. A fund which, instead of investing directly in securities or bonds directly, will invest in funds. Another point of view which can be used to interpret the data is, if a group of assets is superior compared to other group (is characterized by higher return and lower risks) it will have a superior Mean Variance efficiency frontier.

The Mean Variance efficient frontiers obtained on the first round of analysis can be seen at figure 5.1.

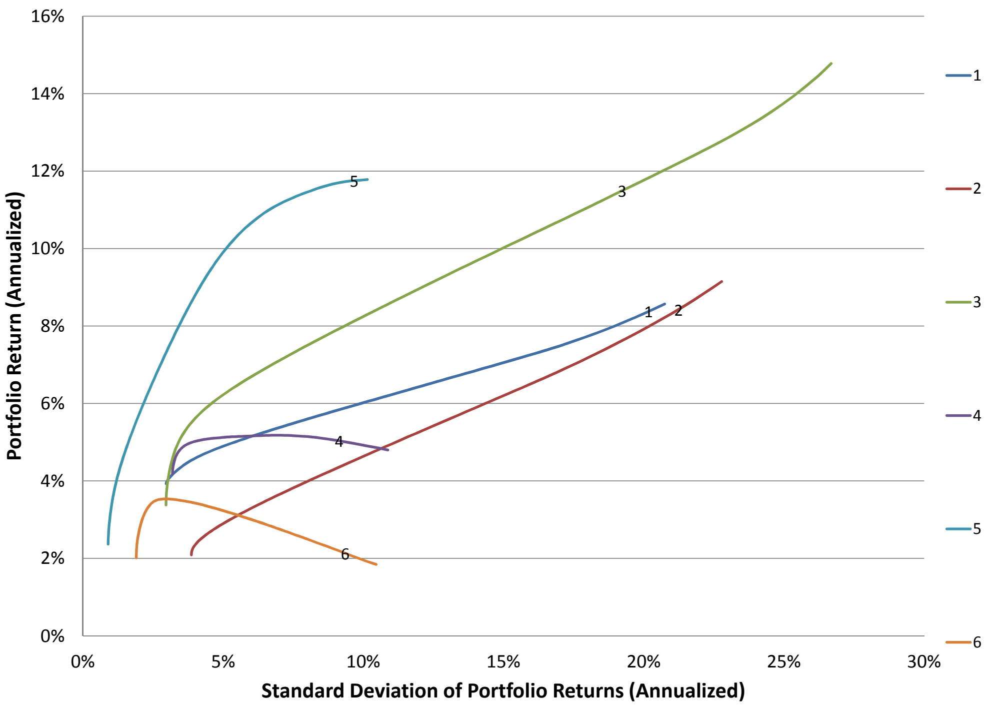

Figure 5.1: Mean Variance efficient frontier according to the groups criteria intensity level.

At figure 5.1, a transition pattern can be seen as the CIL shifts. The group 1 frontier dominates the group 2, which is dominated by group 3. Group 4 does not show any improvement level compared to 3, but group 5 has a clear dominance over the others, representing a possible maximum point, which is followed by a decrease of investment utility, represented by group 6 frontier. Analyzing the Mean Variance efficient frontiers just by plotting and comparing them does not represent a very easy task nor the observations can be taken as final. As the frontiers span over different areas, it is not possible to fully assess the dominance of one CIL over the others, this issue was addressed by using the maximum Sharpe-ratio data.

Analyzing the portfolio composition maps, a good diversification level seems to exist at the Mean Variance efficient portfolios, thus showing that the frontier behavior is not only due to very high positions in very few assets. This diversification occurs because the Mean Variance efficient frontier variance is taken into account, as it is the result of several simulation scenarios.

Analyzing the risk vs. return profile of the Mean Variance efficiency curve, almost all curves support the common assumption about it, higher the risk translates into higher expected returns. However is not always true, as shown by the frontiers 4 and 6. This may happen due to lack of investment alternatives during a crisis period, as the number of portfolio composition constraints is high, you may exclude a considerable part of the market from your investment possibilities.

All Mean Variance Efficient frontier and portfolio composition maps utilized in this case can be seen at Appendix B.

5.1.1 Sharpe-Ratio Analysis

In order to further analyze the relationship between the Mean Variance efficient frontier and the criteria intensity level, Sharpe-ratios were calculated for each of the 100 Mean Variance efficient frontier portfolios at each simulation round for each group. At each round, the maximum Sharpe-ratio was recorded for each group. A summary[1] of the maximum Sharpe-rations obtained for the data span-

[1] Box-plot was utilized to identify and remove outliers.

ning from 2008 to 2010 can be found at Appendix B.

When defining the best investment among a set of options, as long as these options are exclusive, the maximum Sharpe-ratio represents a good indicator. The best investment will be the one with the maximum Sharpe-ratio. An one-way ANOVA analysis[2] was carried out to point whether or not the groups are characterized by different maximum Sharpe-ratio average. The test can be seen on Appendix B.

As the null hypothesis can be refuted, a t-test[3] was carried out in order to analyze the maximum Sharpe-ratio average by group separately, Appendix B presents the test, which can be seen graphically at figure 5.2.

All maximum Sharpe-ratios are different but groups 4 and 6. Analyzing the Mean Variance efficient frontier and figure 5.2, a pattern seems to exist, portfolios with very low CIL - group 1 - perform better than groups with some criteria, this may be due to a loss of diversification capability and higher management costs incurred due to the adoption of screening criteria. As the CIL increases, the investor will be better-off, the benefits may be due to better management, governance, practices and superior performance at this CIL range. As the CIL increases too much, group 6, there is a loss of performance probably due to loss of assets availability as much of the market cap are not eligible due to the high number of constraints.

Figure 5.2 shows that portfolios characterized by higher CIL are better off than those with lower levels (portfolios with CIL ranging from 7 to 24 performed better than thos with CIL ranging from 1 to 6).

[2]At 95% significance level.
[3]At 95% significance level.

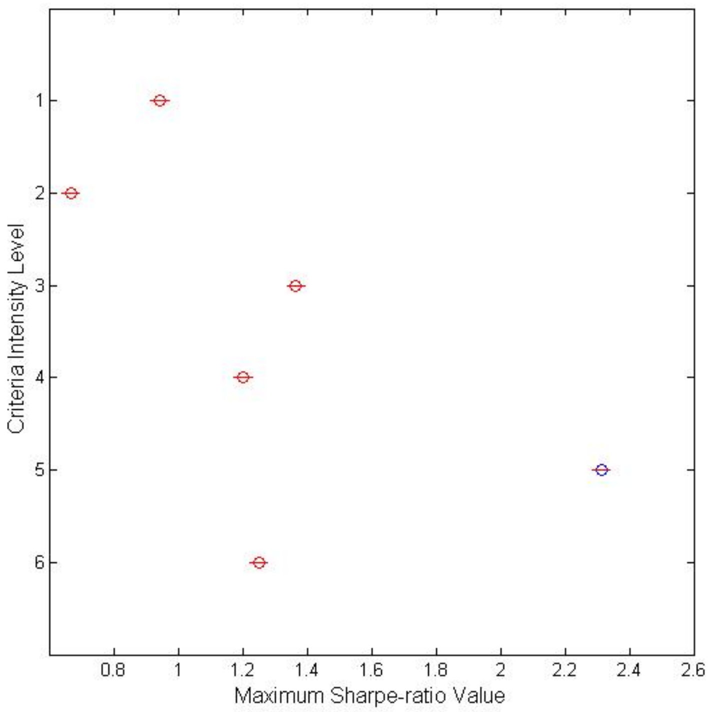

Figure 5.2: Maximum Sharpe-ratio confidence interval by group.

5.2 Criteria Intensity Effects over the MV Frontier - Period 2006/2010

Using data spanning from January 2006 to December 2010 the same structure utilized on 5.1 was repeated in this step.

The Mean Variance efficient frontier by group can be seen on figure 5.3.

Figure 5.3 indicates a very similar pattern as the one obtained from data spanning from 2008 to 2010. But now the frontiers are shifting, the differences regarding shape and position seem to be smoothened when compared to the frontiers presented on figure 5.1. This transition seems to be pushing the groups with

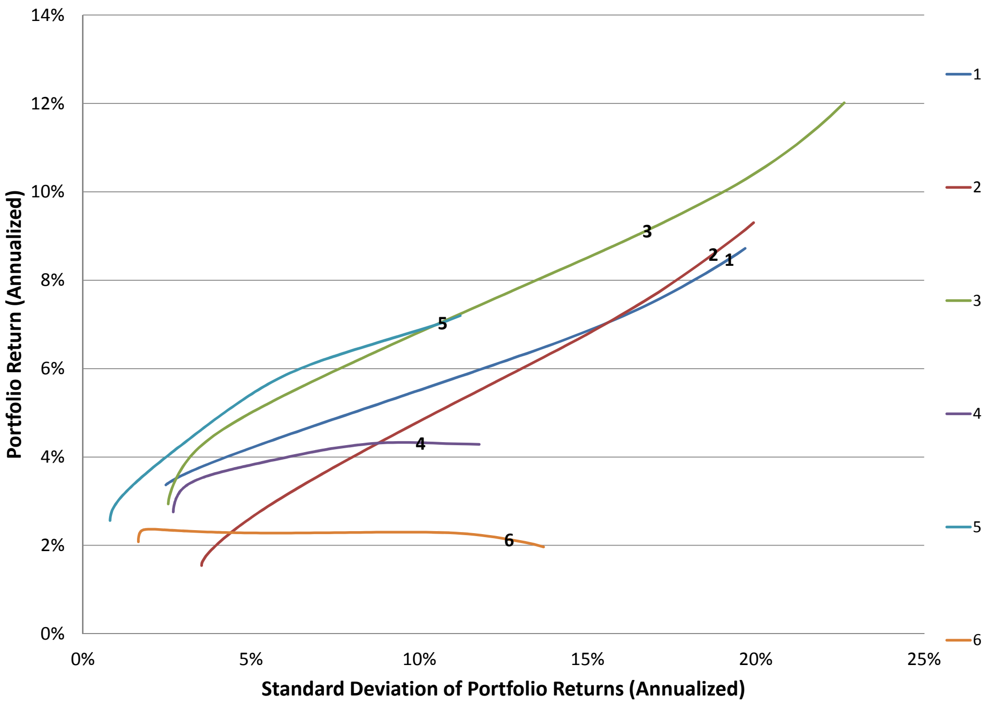

Figure 5.3: Mean Variance efficient frontier by group.

higher CIL - groups 4, 5 and 6 - towards a region dominated by less Mean Variance efficient portfolios compared to the low CIL groups.

The portfolio composition maps show a good level of diversification among the groups, indicating that the behavior presented is not the result of very few assets, thus the frontiers obtained are due to a group characteristics rather than an exception. As the frontiers span over different areas, analyzing solely the frontiers does not indicate which CIL is preferable, it only provides a brief idea regarding the behavior of risk and return relationship as the CIL changes.

Mean Variance efficient frontiers and portfolio composition of all groups studied can be seen on Appendix C.

5.2.1 Sharpe-ratio Analysis

The maximum Sharpe-ratio summary obtained for 2006 to 2010 by group can be seen on Appendix C.

A one-way ANOVA test was carried out (data can be seen on Appendix C), as the null hypothesis was refuted[4], a t-test was conducted to compare the maximum Sharpe-ratio average of each group against each other (data can be seen on Appendix C). Figure 5.4 shows a similar graphical representation of this test.

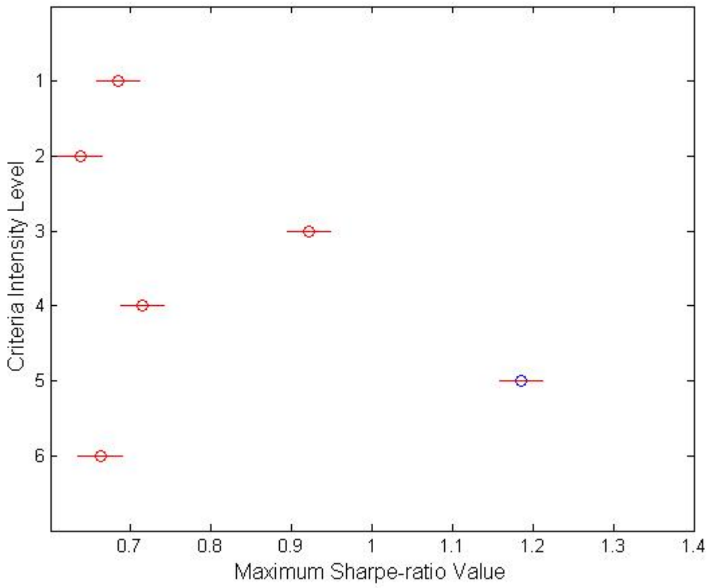

Figure 5.4: Maximum Sharpe-ratio confidence interval by group.

Groups 5 followed by group 3 are the groups which presented the best results over this time period. As observed before when analyzing the frontiers, there is a convergence. The distance between the maximum value and minimum value of the Sharpe-ratio reduced in 67.8%. Most of all the remaining groups are not

[4]At 95% significance level.

statistically significant[5] different from each other.

The behavior observed on figure 5.4 can be considered very confusing at a first glance; during the crisis period portfolios characterized by higher CIL performed better than those with lower levels but now, during 2006 to 2010 this behavior does not apply to all groups tested. This may have occurred due to the existence of two very distinct moments covered in this time frame, a bullish one and a crisis one. During bullish periods, the extra constraints levels incurred due to the adoption of higher CIL ended up impacting the performance level. This result agrees with the literature, as many researches proposed that SRI may represent a type of insurance which explains the improved performance of high CIL during the crisis period.

5.3 Criteria Intensity Effects over the MV Frontier - Period 2001/2010

Using data spanning from January 2001 to December 2010 the same structure utilized on 5.1 was repeated in this part.

The Mean Variance efficient frontier by group can be seen on figure 5.5.

Figure 5.5 shows the behavior of the Mean Variance efficient frontier over a 10 years period of time. The effects of the most recent financial crisis - 2008 to 2010 - which were most evident on figure 5.1, are now smoothened, reflecting the behavior of the Mean Variance efficiency frontier over a longer term period as the CIL changes.

The groups characterized by high CIL (groups 4, 5 and 6) dominate the lower

[5] At 95% significance level.

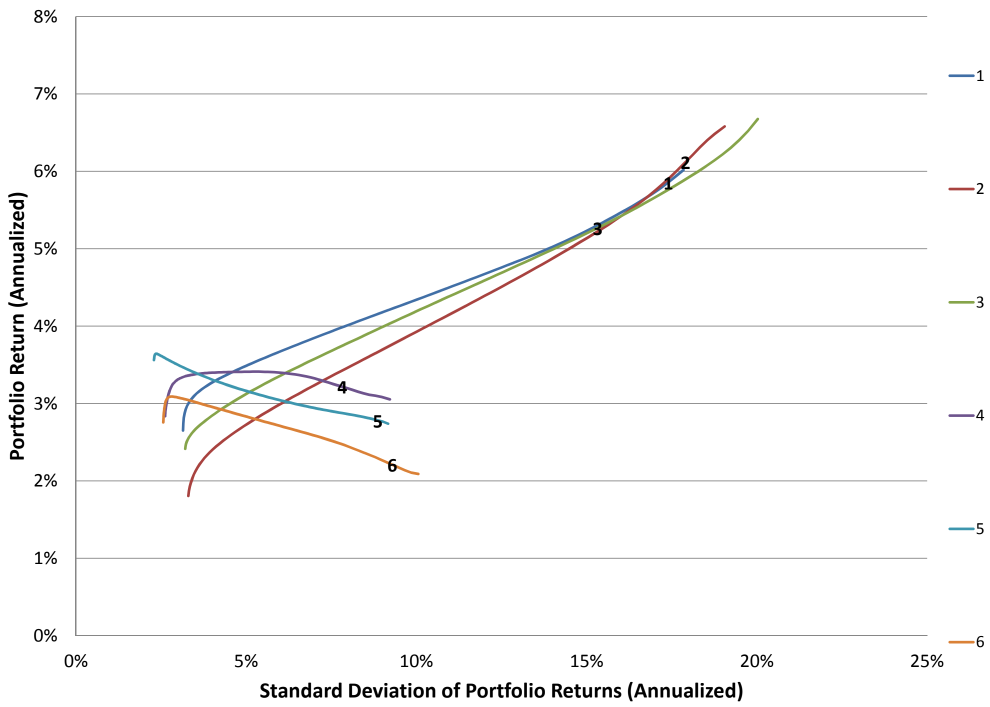

Figure 5.5: Mean Variance efficient frontier by group.

risk area ($0 < \sigma < 5\%$). The area with higher risk levels ($\sigma > 5\%$), are dominated by the low CIL groups (groups 1, 2 and 3). This pattern suggests that when the number of criteria utilized to compose the portfolio increases, less successful the portfolio will be as the portfolio volatility increases.

Groups 4, 5 and 6 (high CIL) present an inversion point, in this case, riskier portfolios does not result in higher returns. This behavior does not appear on groups 1, 2 and 3 (low CIL)

In the lower CIL groups (1,2 and 3), the frontier dominance shifts, thus no clear pattern can be observed. Mean Variance efficient frontiers and portfolio composition of all groups studied in this part can be seen on Appendix D.

As the Mean Variance efficiency frontiers span over different areas of the return vs. risk graph, it is not possible to make a clear and quantifiable comparison by only analyzing the curves, the next section covers this issue.

5.3.1 Sharpe-ratio Analysis

The maximum Sharpe-ratio summary obtained for 2001 to 2010 by group can be seen on Appendix D.

A one-way ANOVA test was carried out (data can be seen on Appendix D), as the null hypothesis was refuted[6]. A t-test was conducted to compare the maximum Sharpe-ratio average of each group against each other (data can be seen on Appendix D). Figure 5.6 shows a similar graphical representation of this test.

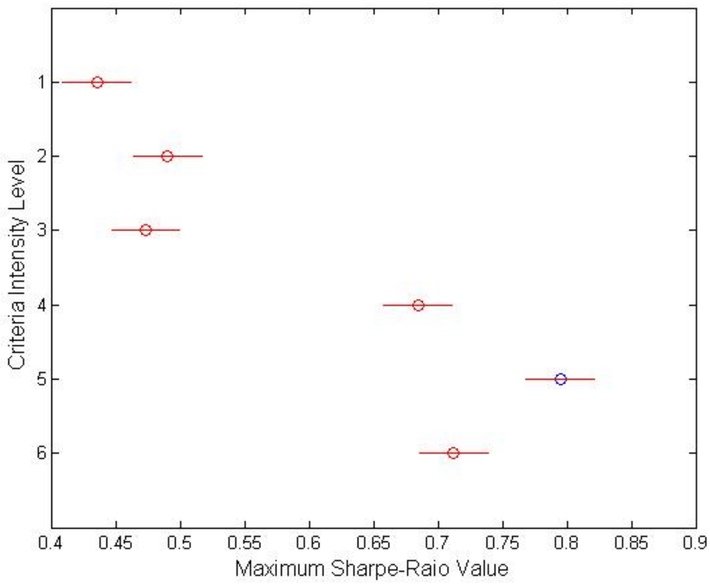

Figure 5.6: Maximum Sharpe-ratio confidence interval by group.

Analyzing figure 5.6, two main distinct groups appear. The low CIL group

[6]At 95% significance level.

(groups 1,2 and 3) and high CIL group (groups 4, 5 and 6). The low CIL group has an average maximum Sharpe-ratio value of 0,47, while the high CIL group has 0,73. These results suggest that in the long term, portfolios with a higher CIL will be better off.

Analyzing the range between the highest value of the maximum Sharpe-ratio and the lowest one for each time period, it decreases as the time span under consideration increases. From 2008/2010 to 2006/2010 the range decreases in 67,8% and from 2006/2010 to 2001/2010 the range decreases in 32%. Which points out that over the long run the maximum Sharpe-ratio value of the different groups converges, reducing the gap between high and low CIL.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

Socially responsible investment has been growing over the past years at a very fast pace and has been attracting a lot of attention from several actors. Governments are dedicating more attention to SRI and creating measures to stimulate and develop this sort of investments; the financial community has been creating and developing new products to satisfy the demand for socially responsible investments.

Being able to deliver market financial returns aligned with social and environmental value (triple bottom line) is a differential that has been attracting investors' attention. The search for blended value is a major shift of perspective, not only financial returns are deemed necessary but how these are achieved and what are the social and environmental repercussions.

This study utilized very solid and well accepted theories to obtain its results. The modern portfolio theory proposed by Markowitz was utilized to generate and obtain efficient Mean Variance portfolios. To address the instability and estimation errors, Ledoit-Wolf estimator was used together with simulation procedures. Together, this methodology provides a robust and consistent approach. The simulation procedure propitiates an opportunity to take into account several possible scenarios, thus addressing the frontier variance and instability issues. The approach utilized to develop this study differs from several other studies as the focus is not on the single asset level, analyzing separately and ignoring the relationship between assets; but face these assets as joint components that can be assembled together obtaining completely different solutions as now their relationship is a component of the analysis.

In this study, the behavior of socially responsible investments was analyzed in terms of the Mean Variance efficiency frontier and the investment "ethicalness" level (criteria intensity level). Three time frames were used in order to capture the behavior of SRI over short and long term; also another point which deserves special attention was the results obtained for the period between 2008 and 2010 which corresponds to the most recent financial crisis.

During the most recent financial crisis (2008 to 2010), among socially responsible aligned funds, those with higher SRI criteria are the ones which performed better (group 3 to 6). Lower CIL groups performed clearly poorly compared to higher CIL groups. During this time period, the optimal CIL was between 13 to 15, portfolios characterized by very high levels (over than 15 criteria) presented a loss of Sharpe-ratio when compared to the others high CIL groups, but still performed better than the lower CIL.

The results obtained for the period 2006 to 2010 show a transition to the situation observed for the period spanning from 2001 to 2010. The long term behavior of portfolios composed of socially responsible aligned funds shows very distinctive and clear patterns. The low criteria intensity portfolios (groups 1, 2 and 3) performed poorly compared to those characterized by higher criteria intensity levels, but this performance difference appears to fade away as the time frame under analysis increases. Another clear pattern is related to the Mean Variance efficiency frontiers shape, the ones with low criteria intensity levels agree with the behavior that higher risks results in higher returns, on the other hand the frontiers with high criteria intensity level goes into the opposite direction, their behavior suffer an inversion at a certain risk level.

During crisis period, portfolios characterized by higher CIL performed distinctively better than their low CIL counterparts, but this gap closes during bullish periods.

Fortunately the results of this study are favorable towards socially responsible investing, companies looking for business opportunities may try to disguise themselves as SRI aligned and put into practice the so called ”Green Washing” as they advertise and offer investment alternatives which do not present SRI as its foundation (for instance just adding very few screens in order to lure investors). These companies should be aware that this practice may not propitiate the desired results.

Portfolios characterized by higher criteria intensity levels showed improved performance compared to their lower level counterparts. This performance difference widens during slowdown periods, thus agreeing with several studies that showed correlation between SRI aligned strategy and an insurance effect due to SRI. Over long time periods, the increased performance of higher CIL levels is very distinct from the lower CIL levels. During bullish periods the gap between low and high CIL closes. Thus a practical result for an investor looking for investment alternatives or for a money manager willing to initiate a SRI strategy aligned fund, the utilization of higher criteria intensity levels showed improved performance, specially for the portfolios composed by funds characterized by a criteria intensity level ranging from 13 to 15.

There are still a lot to be done in order to develop and bring other socially related vehicles into mainstream as community investing, micro-finance and social enterprises. The purpose of this study, as many others, is to characterize and analyze the behavior of this new class of investment that brings with it a shift of perspective. The results obtained here join many others studies which presented favorable outcomes for socially responsible investments.

6.2 Future Work

The literature about socially responsible investing is vast and still growing. This study adds important and bright perspectives for this investment class.

In order to obtain clearer results regarding the behavior of the Mean Variance efficiency frontier and the investment attractiveness as the ”ethicalness” level of the assets under consideration changes, there are still few points that could be addressed. Larger database with higher availability of funds and longer time span would provide an important dataset to be analyzed and compared with the results obtained in this study. Other portfolio creation methods should be tested and used jointly with other analysis techniques in order to test if the results obtained are coherent.

Appendix A

Appendix

This appendix contains further information regarding the dataset utilized in this study.

Table A.1: Summary of annualized risk and return using monthly data from 2008 to 2010.

Group	Avg Annualized Return	Avg Annualized Risk
1	-1,6%	17,0%
2	-1,0%	20,2%
3	-0,9%	19,1%
4	-1,9%	17,2%
5	0,0%	14,9%
6	-2,3%	16,2%

Table A.2: Summary of annualized risk and return using monthly data from 2006 to 2010

Group	Avg Annualized Return	Avg Annualized Risk
1	1,8%	14,0%
2	2,5%	17,0%
3	1,7%	16,4%
4	0,8%	14,3%
5	2,1%	12,1%
6	1,0%	14,0%

Table A.3: Summary of annualized risk and return using monthly data from 2001 to 2010

Group	Avg Annualized Return	Avg Annualized Risk
1	1,4%	14,1%
2	1,6%	17,7%
3	0,3%	16,7%
4	-0,2%	14,7%
5	0,7%	13,3%
6	0,3%	13,3%

Appendix B

Appendix

This appendix contains all the results obtained for section 5.1 (refers to the analysis comprehending the time frame from 2008 to 2010).

Mean Variance efficient frontiers

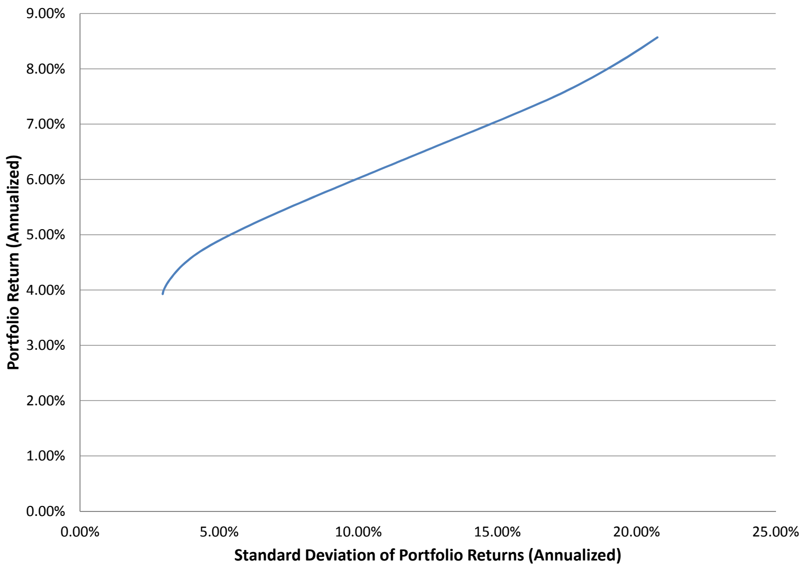

Figure B.1: Mean Variance efficient frontier for assets belonging to the group 1.

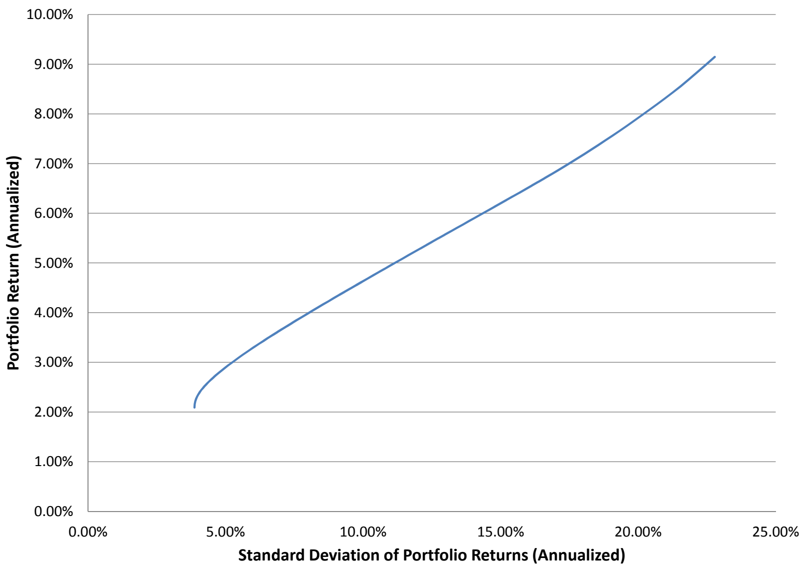

Figure B.2: Mean Variance efficient frontier for assets belonging to the group 2.

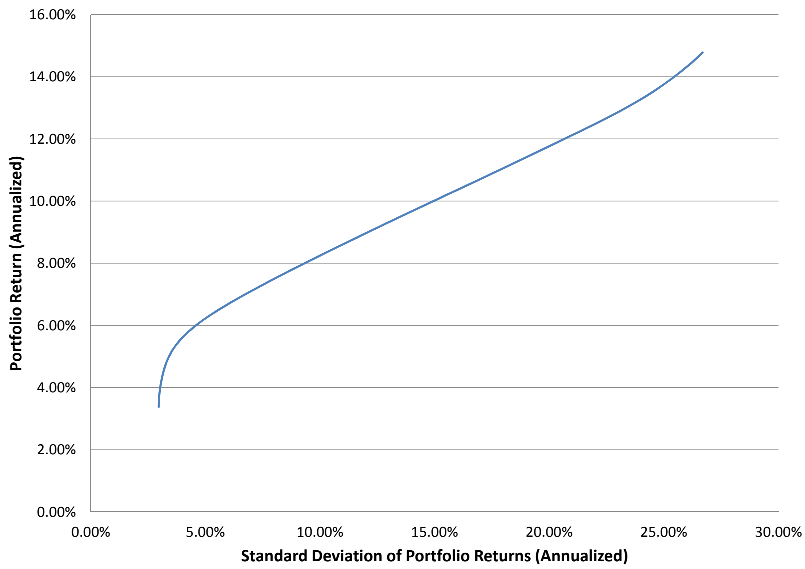

Figure B.3: Mean Variance efficient frontier for assets belonging to the group 3.

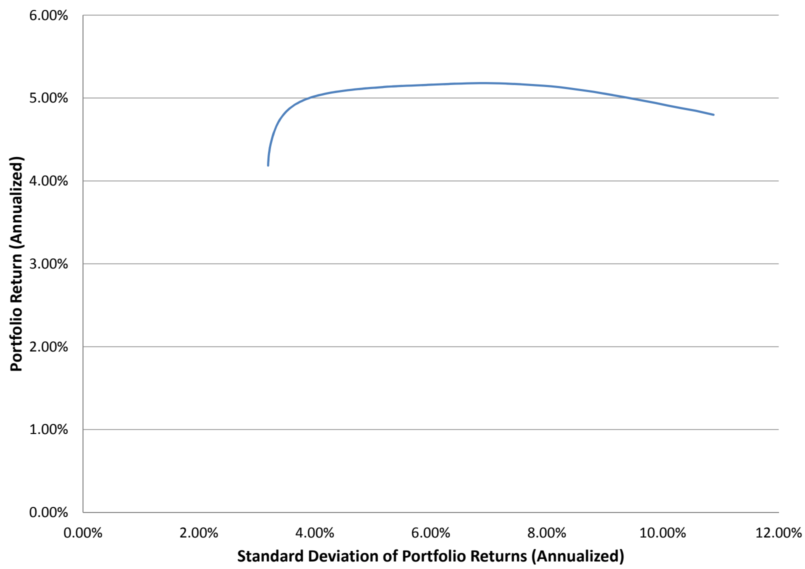

Figure B.4: Mean Variance efficient frontier for assets belonging to the group 4.

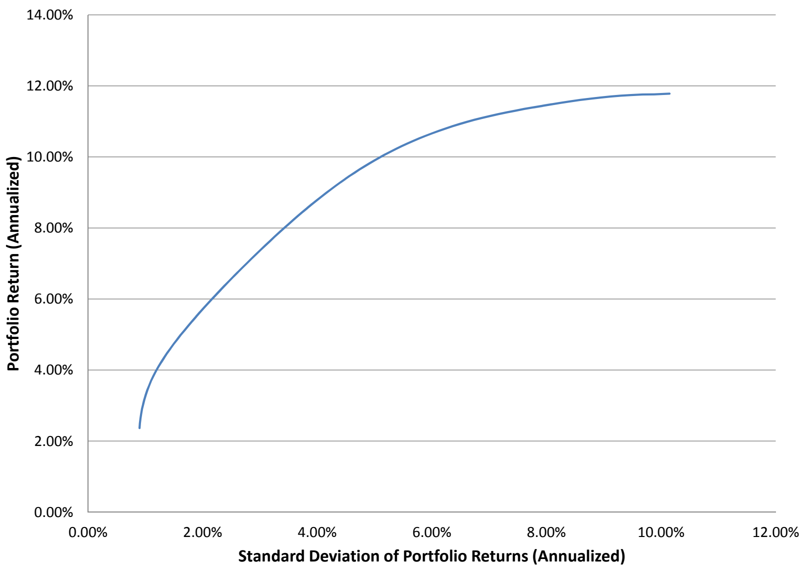

Figure B.5: Mean Variance efficient frontier for assets belonging to the group 5.

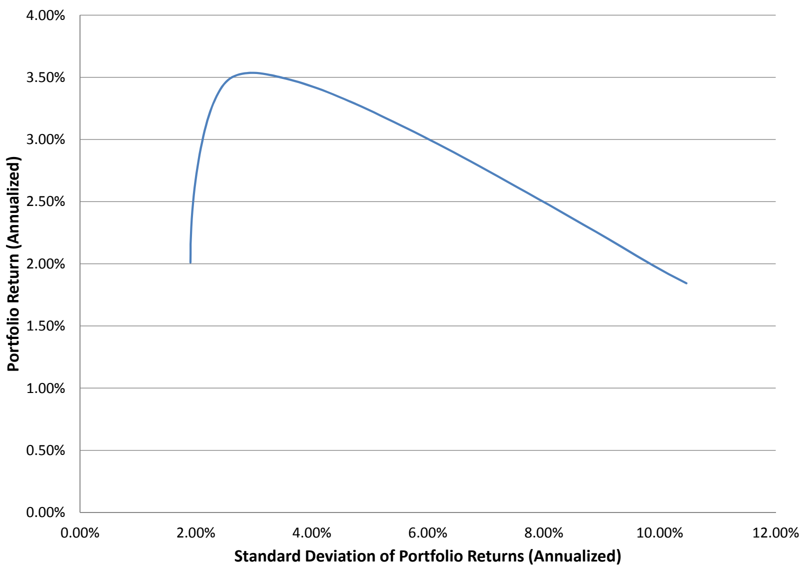

Figure B.6: Mean Variance efficient frontier for assets belonging to the group 6.

Portfolio Composition Maps

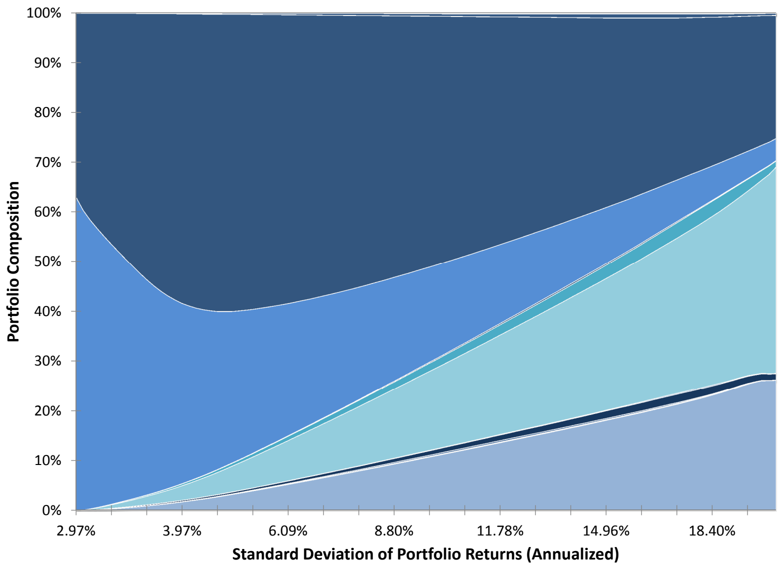

Figure B.7: Portfolio composition map for assets belonging to the group 1.

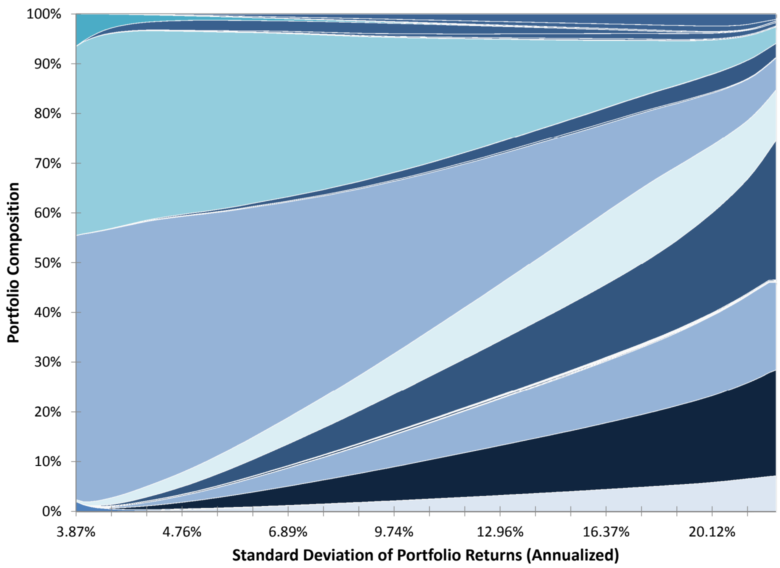

Figure B.8: Portfolio composition map for assets belonging to the group 2.

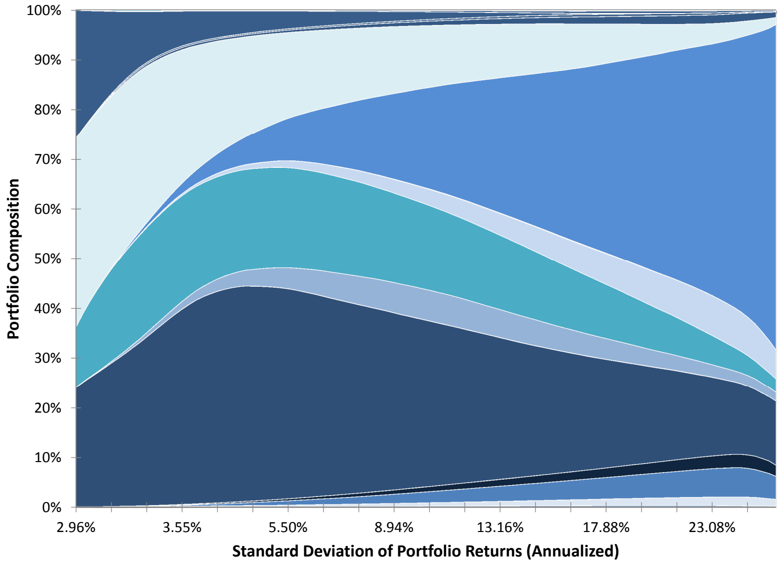

Figure B.9: Portfolio composition map for assets belonging to the group 3.

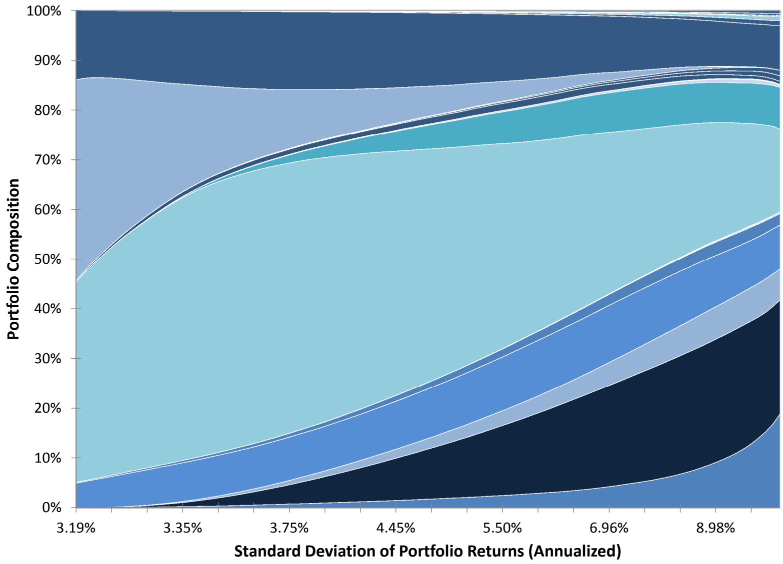

Figure B.10: Portfolio composition map for assets belonging to the group 4.

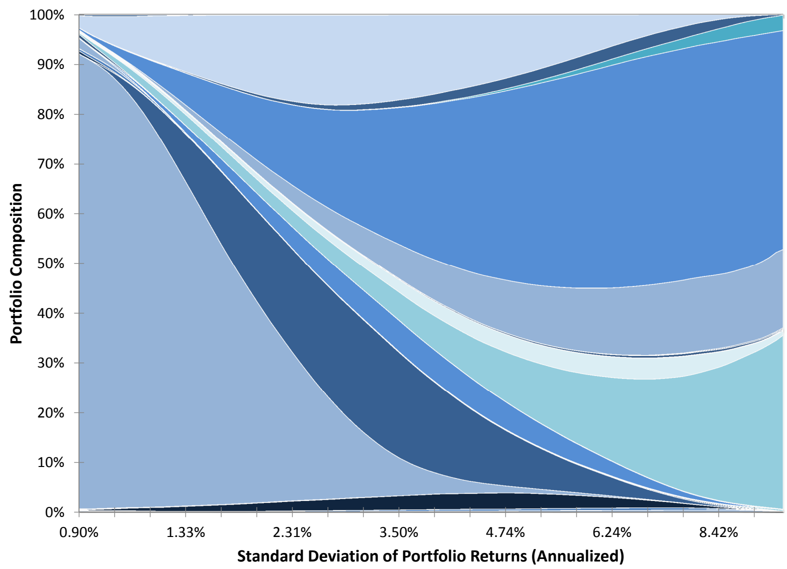

Figure B.11: Portfolio composition map for assets belonging to the group 5.

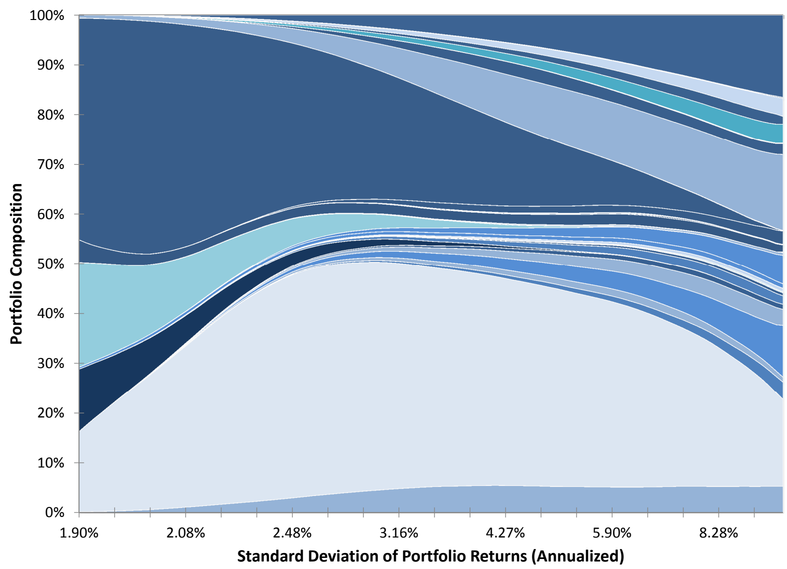

Figure B.12: Portfolio composition map for assets belonging to the group 6.

Maximum Sharpe-ratio Summary

Table B.1: Maximum Sharpe-ratio Summary by groups for 2008 to 2010

Group	Mean	Std Dev	Min	Max
1	0,94	0,41	-0,14	2,06
2	0,66	0,43	-0,46	1,89
3	1,36	0,46	0,15	2,64
4	1,20	0,46	0,01	2,38
5	2,31	0,48	1,00	3,64
6	1,25	0,44	0,04	2,45

Tests

Table B.2: One-way ANOVA at 95% confidence level for the maximum Sharpe-ratio average by groups for 2008 to 2010

One-way ANOVA					
Source	**SS**	**df**	**MS**	**F**	**Prob>F**
Groups	1565,439	5	313,0879	1578,926	0
Error	1177,456	5938	0,198292		
Total	2742,895	5943			

Table B.3: Mean and confidence interval at 95% confidence level for GroupA - GroupB mean.

Group A	Group B	Lower Bound	Mean	Upper Bound
1	2	0,220	0,277	0,334
1	3	-0,481	-0,424	-0,367
1	4	-0,317	-0,260	-0,203
1	5	-1,431	-1,374	-1,317
1	6	-0,365	-0,308	-0,250
2	3	-0,758	-0,701	-0,644
2	4	-0,594	-0,537	-0,480
2	5	-1,708	-1,651	-1,594
2	6	-0,642	-0,585	-0,528
3	4	0,107	0,164	0,221
3	5	-1,007	-0,950	-0,893
3	6	0,059	0,116	0,173
4	5	-1,171	-1,114	-1,057
4	6	-0,105	-0,048	0,009
5	6	1,009	1,066	1,123

Appendix C

Appendix

This appendix contains all the results obtained for section 5.2 (refers to the analysis comprehending the time frame from 2006 to 2010).

Mean Variance efficient frontiers

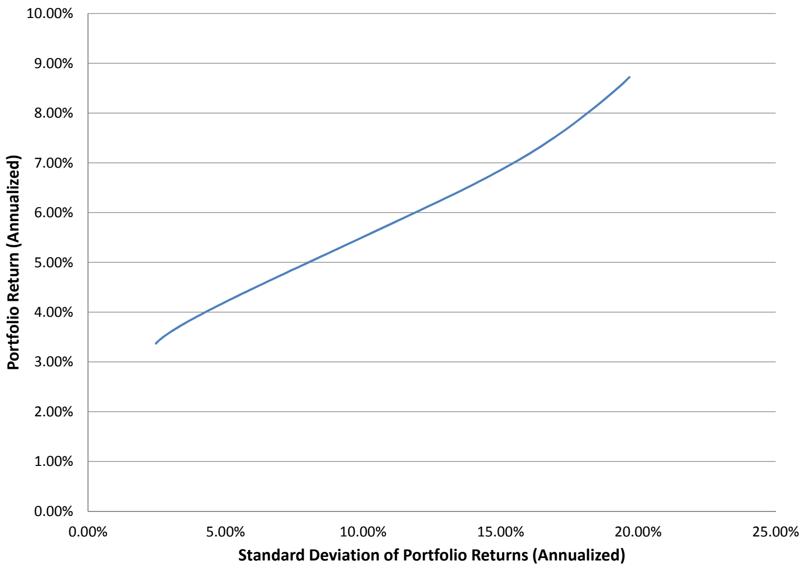

Figure C.1: Mean Variance efficient frontier for assets belonging to the group 1.

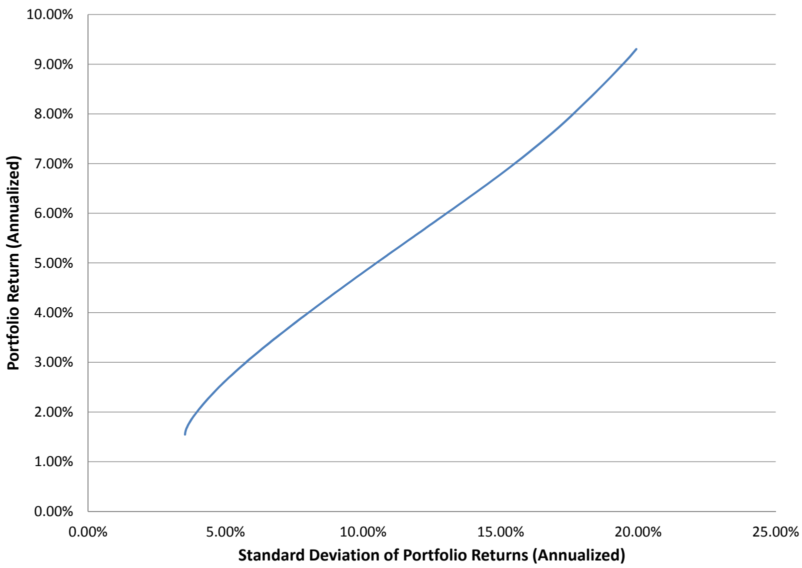

Figure C.2: Mean Variance efficient frontier for assets belonging to the group 2.

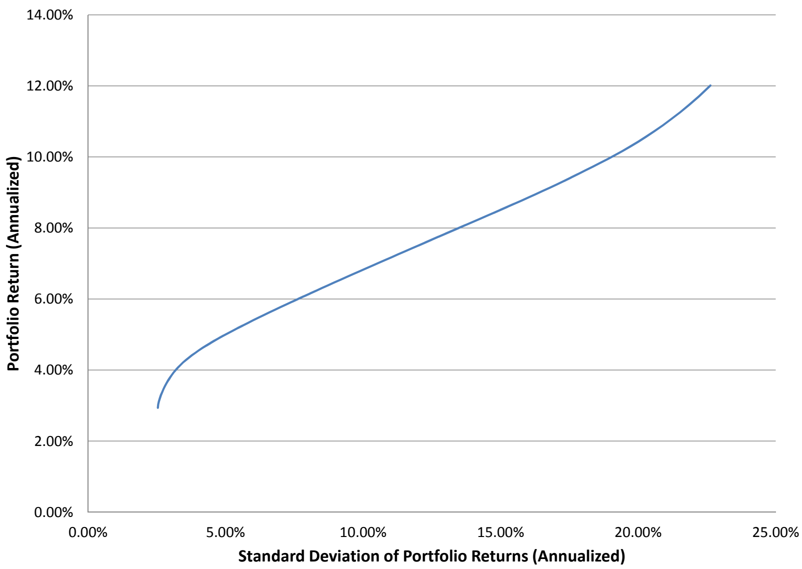

Figure C.3: Mean Variance efficient frontier for assets belonging to the group 3.

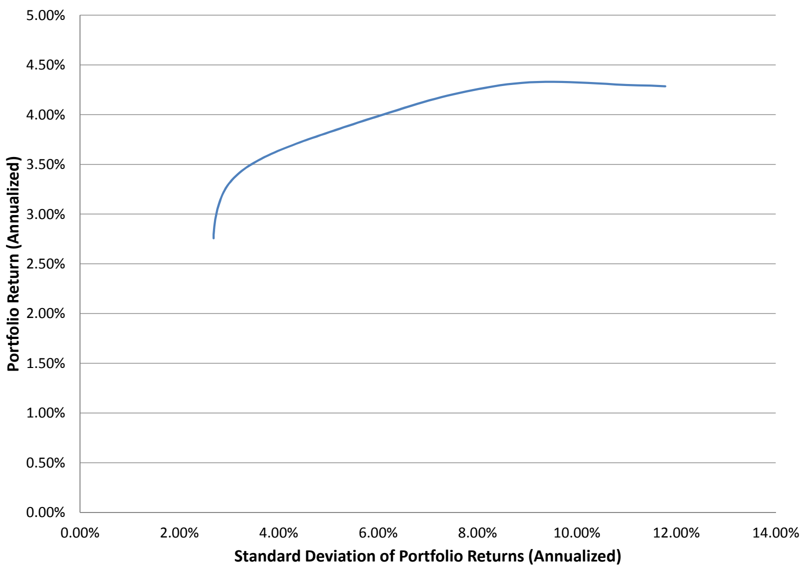

Figure C.4: Mean Variance efficient frontier for assets belonging to the group 4.

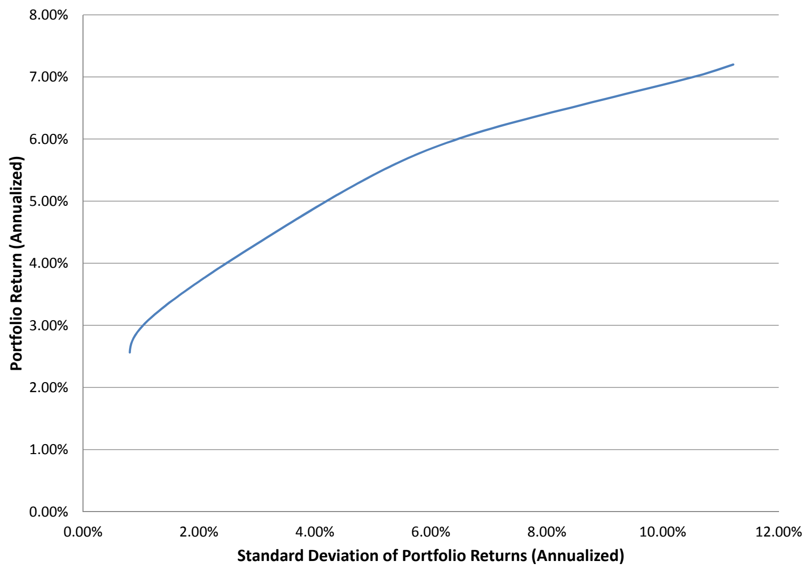

Figure C.5: Mean Variance efficient frontier for assets belonging to the group 5.

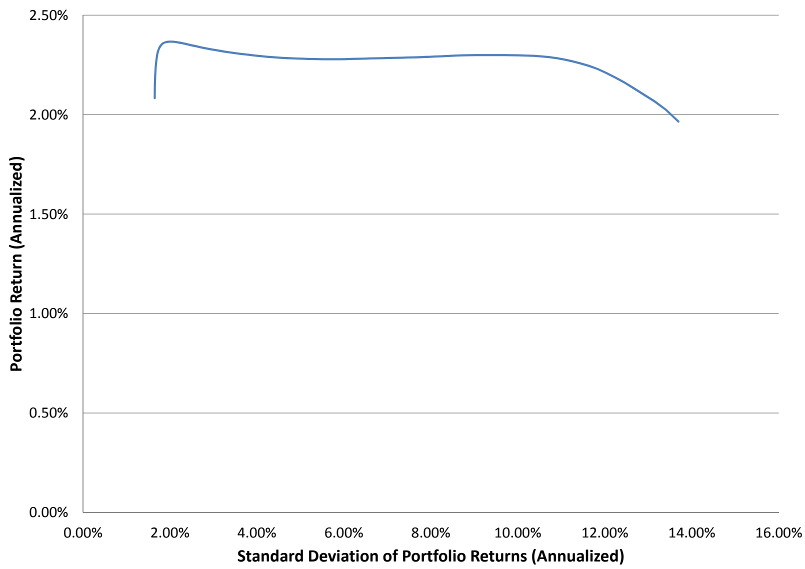

Figure C.6: Mean Variance efficient frontier for assets belonging to the group 6.

Portfolio Composition Maps

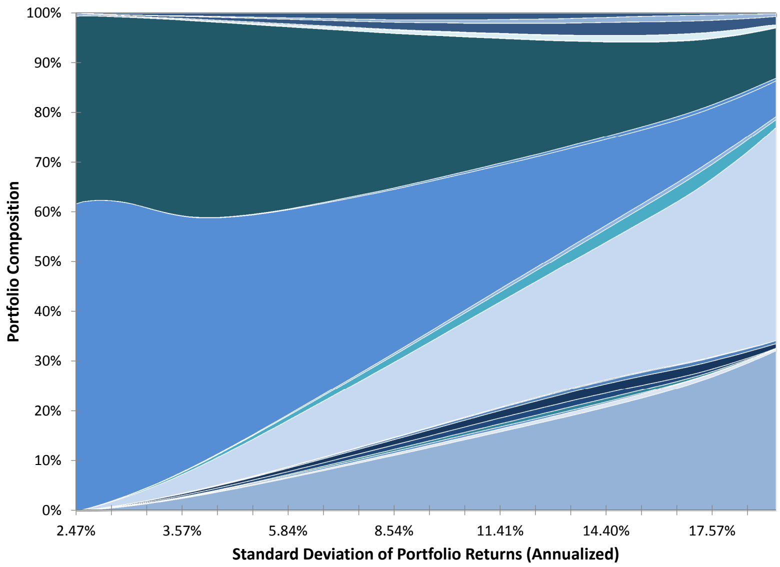

Figure C.7: Portfolio composition map for assets belonging to the group 1.

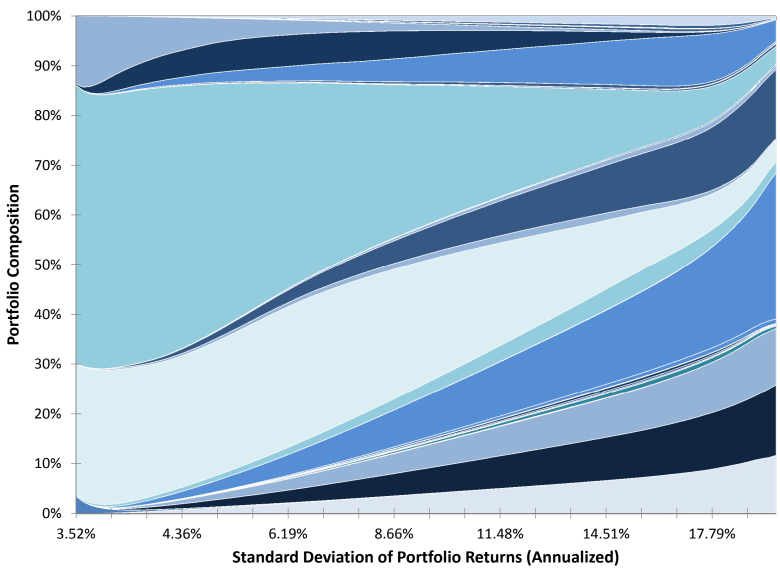

Figure C.8: Portfolio composition map for assets belonging to the group 2.

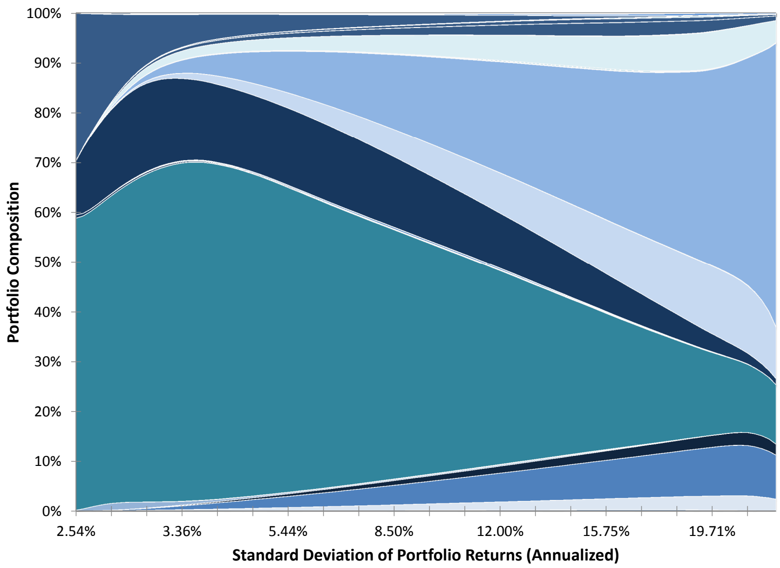

Figure C.9: Portfolio composition map for assets belonging to the group 3.

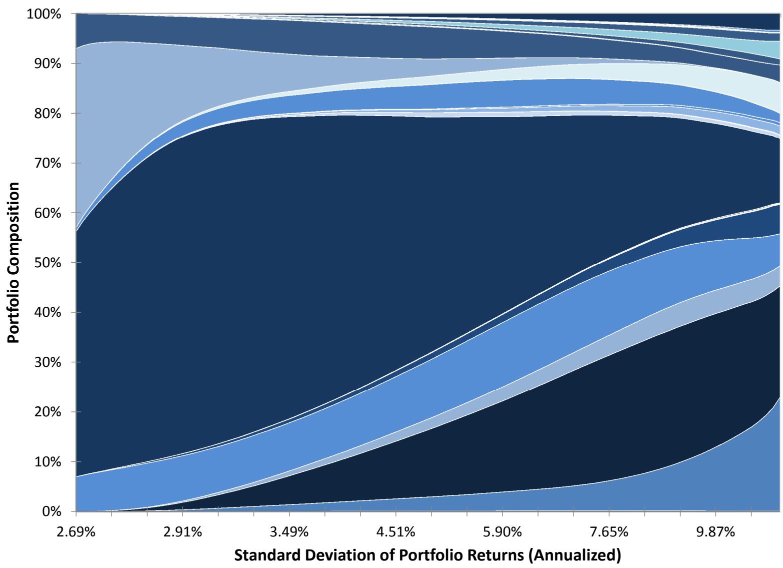

Figure C.10: Portfolio composition map for assets belonging to the group 4.

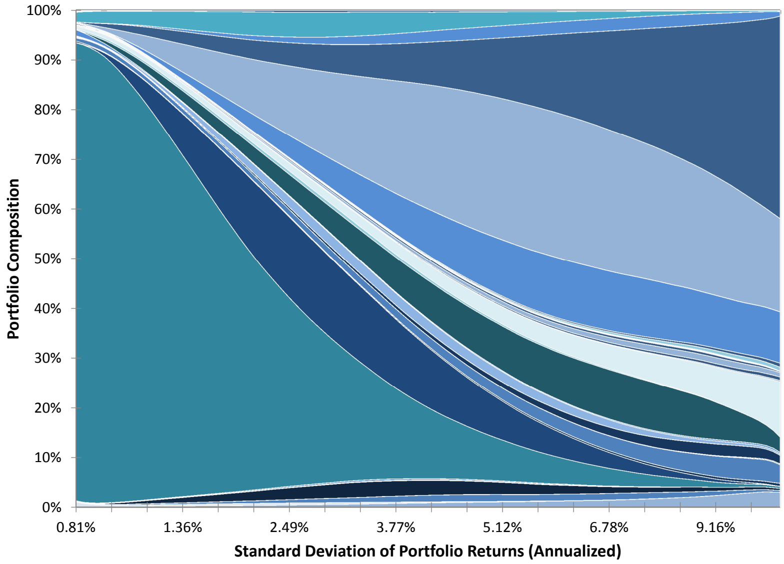

Figure C.11: Portfolio composition map for assets belonging to the group 5.

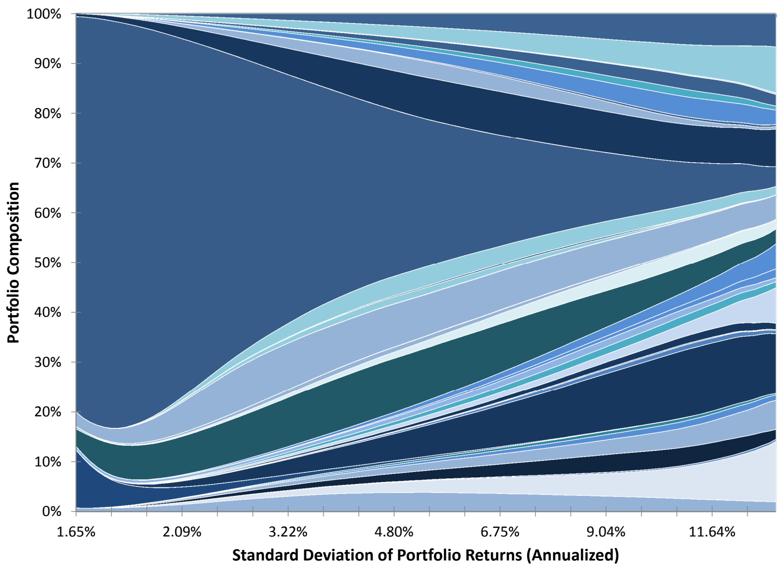

Figure C.12: Portfolio composition map for assets belonging to the group 6.

Maximum Sharpe-ratio Summary

Table C.1: Maximum Sharpe-ratio Summary by groups for 2006 to 2010

Group	Mean	Std Dev	Min	Max
1	0,68	0,41	-0,45	1,81
2	0,64	0,44	-0,59	1,91
3	0,92	0,43	-0,06	2,09
4	0,72	0,42	-0,46	1,85
5	1,19	0,43	0,00	2,39
6	0,66	0,38	-0,37	1,74

Tests

Table C.2: One-way ANOVA at 95% confidence level for the maximum Sharpe-ratio average by groups for 2006 to 2010

One-way ANOVA					
Source	**SS**	**df**	**MS**	**F**	**Prob>F**
Groups	227,431	5	45,48620165	258,9028616	5,2E-251
Error	1041,305	5927	0,175688292		
Total	1268,736	5932			

Table C.3: Mean and confidence interval at 95% confidence level for GroupA - GroupB mean.

Group A	Group B	Lower Bound	Mean	Upper Bound
1	2	-0,006	0,048	0,101
1	3	-0,291	-0,237	-0,183
1	4	-0,084	-0,031	0,023
1	5	-0,555	-0,501	-0,447
1	6	-0,031	0,022	0,076
2	3	-0,338	-0,284	-0,231
2	4	-0,132	-0,078	-0,025
2	5	-0,602	-0,548	-0,495
2	6	-0,079	-0,025	0,029
3	4	0,153	0,206	0,260
3	5	-0,318	-0,264	-0,210
3	6	0,205	0,259	0,313
4	5	-0,524	-0,470	-0,417
4	6	-0,001	0,053	0,107
5	6	0,469	0,523	0,577

Appendix D

Appendix

This appendix contains all the results obtained for section 5.3 (refers to the analysis comprehending the time frame from 2001 to 2010).

Mean Variance efficient frontiers

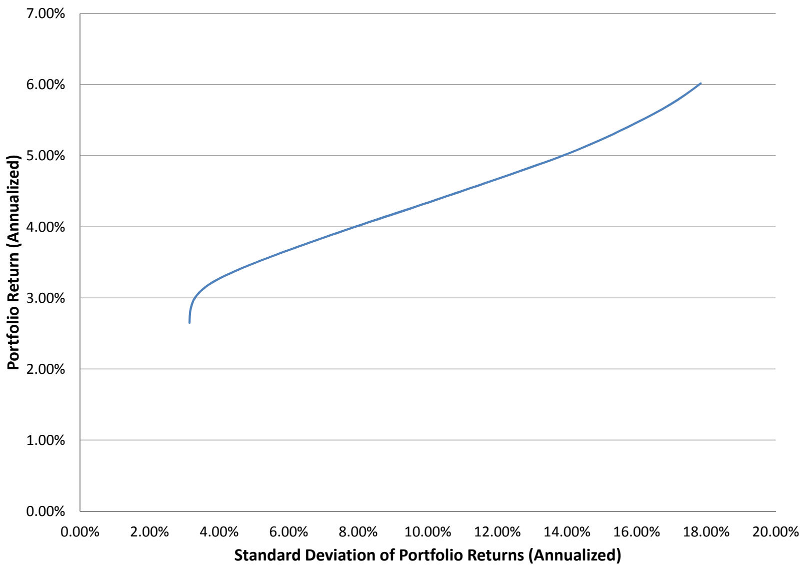

Figure D.1: Mean Variance efficient frontier for assets belonging to the group 1.

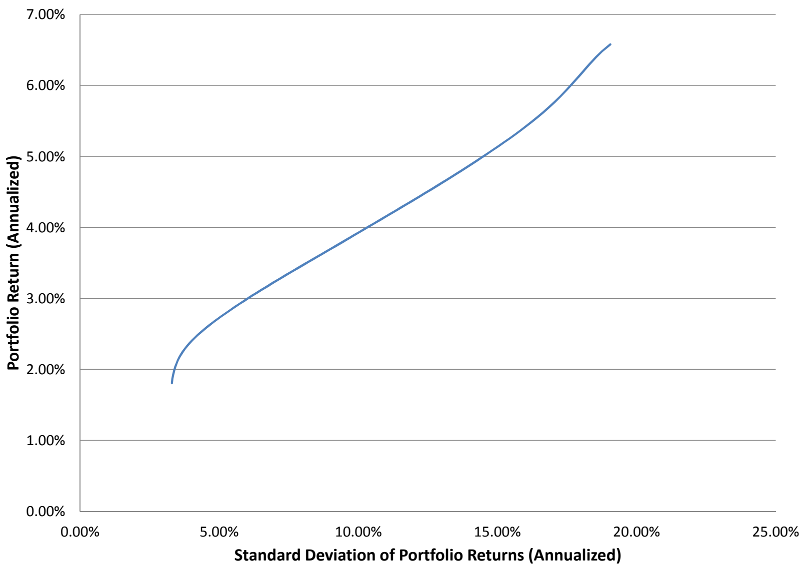

Figure D.2: Mean Variance efficient frontier for assets belonging to the group 2.

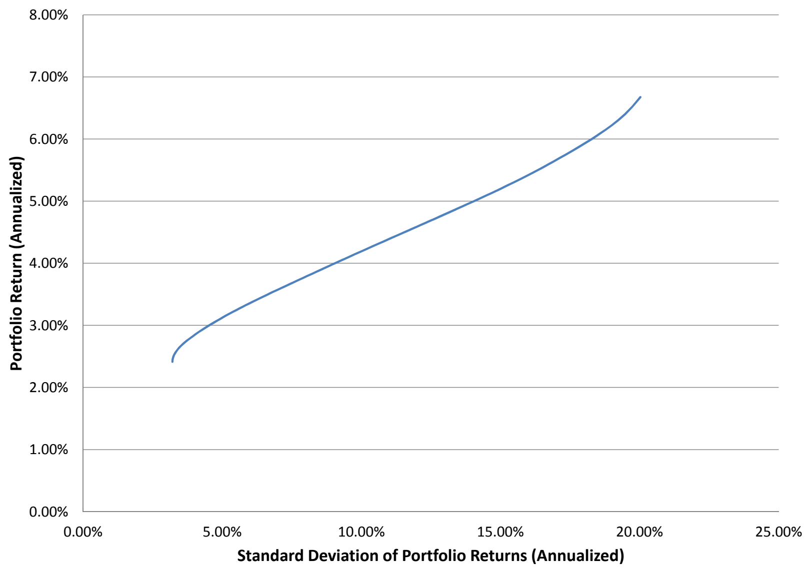

Figure D.3: Mean Variance efficient frontier for assets belonging to the group 3.

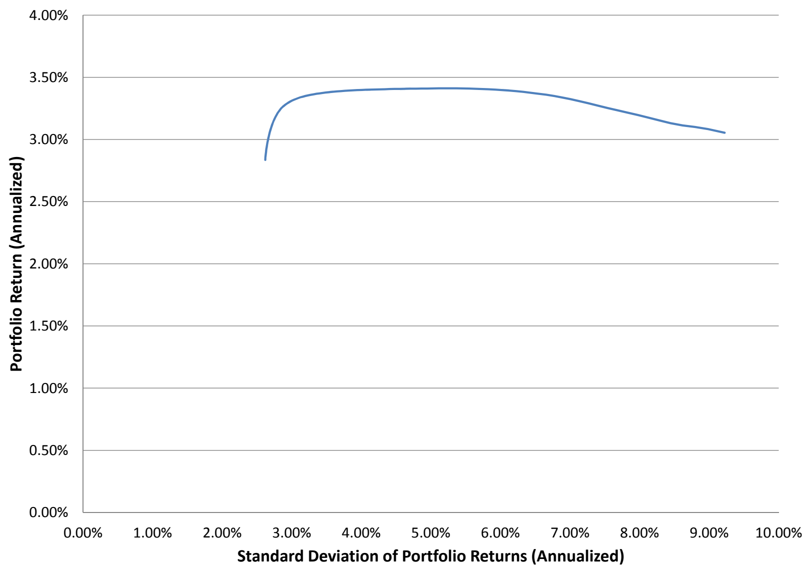

Figure D.4: Mean Variance efficient frontier for assets belonging to the group 4.

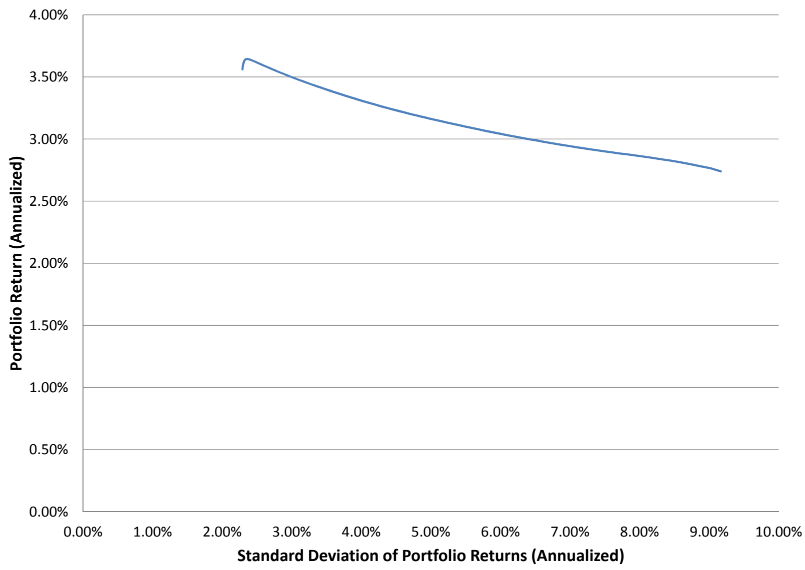

Figure D.5: Mean Variance efficient frontier for assets belonging to the group 5.

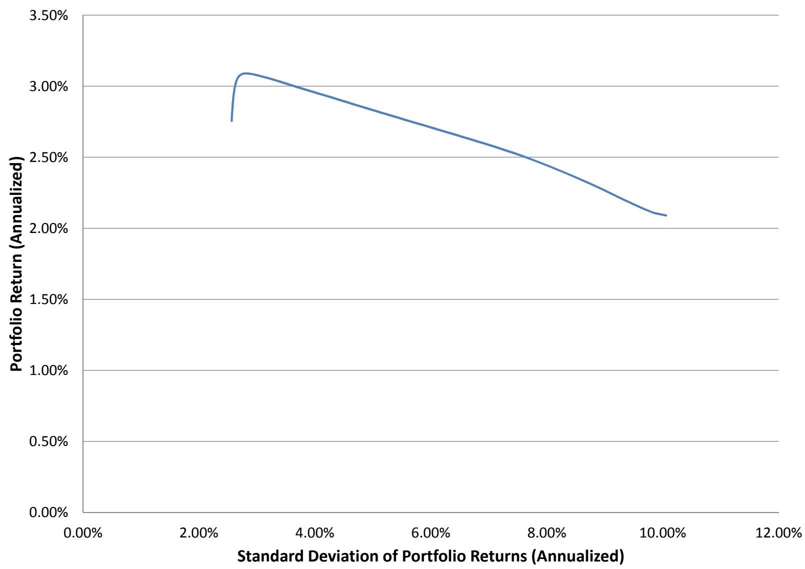

Figure D.6: Mean Variance efficient frontier for assets belonging to the group 6.

Portfolio Composition Maps

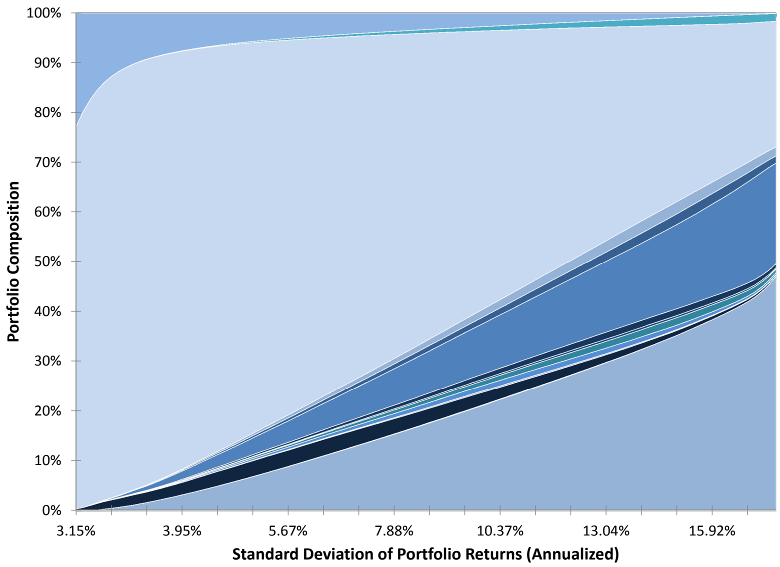

Figure D.7: Portfolio composition map for assets belonging to the group 1.

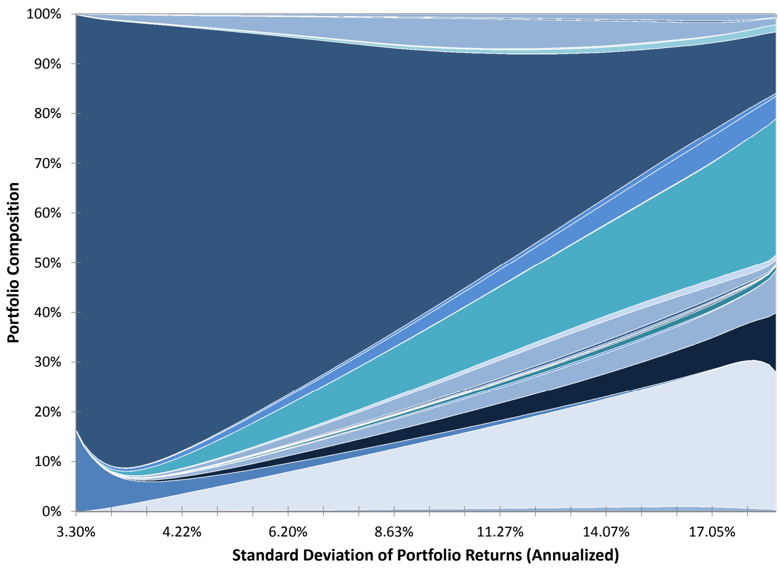

Figure D.8: Portfolio composition map for assets belonging to the group 2.

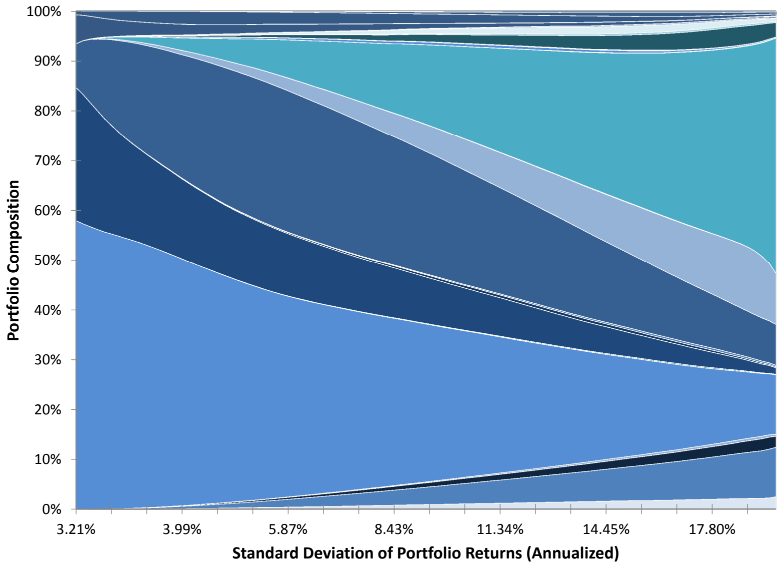

Figure D.9: Portfolio composition map for assets belonging to the group 3.

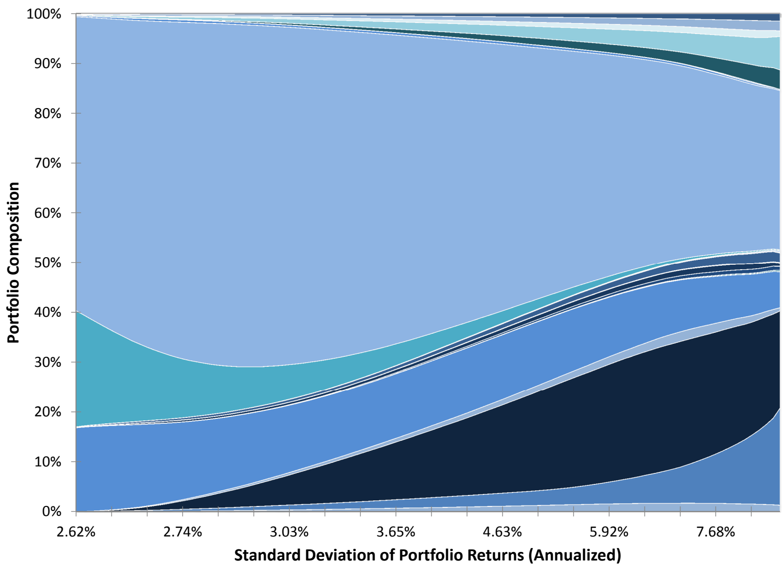

Figure D.10: Portfolio composition map for assets belonging to the group 4.

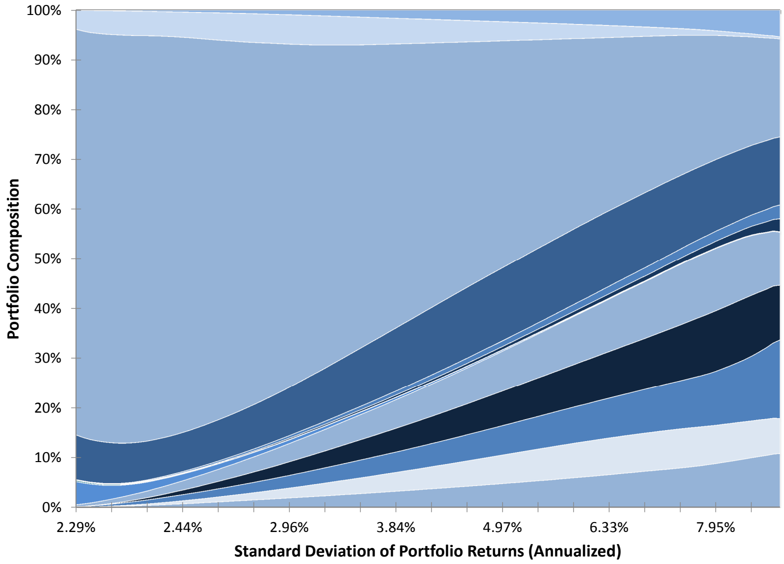

Figure D.11: Portfolio composition map for assets belonging to the group 5.

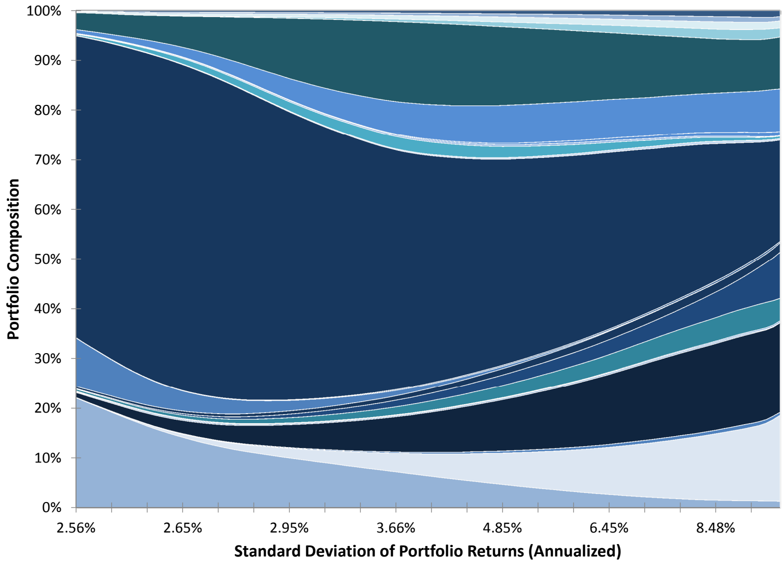

Figure D.12: Portfolio composition map for assets belonging to the group 6.

Maximum Sharpe-ratio Summary

Table D.1: Maximum Sharpe-ratio Summary by groups for 2001 to 2010

Group	Mean	Std Dev	Min	Max
1	0,43	0,42	-0,71	1,57
2	0,49	0,42	-0,65	1,64
3	0,47	0,43	-0,63	1,60
4	0,68	0,41	-0,43	1,81
5	0,79	0,41	-0,33	1,90
6	0,71	0,39	-0,36	1,77

Tests

Table D.2: One-way ANOVA at 95% confidence level for the maximum Sharpe-ratio average by groups for 2001 to 2010

One-way ANOVA					
Source	**SS**	**df**	**MS**	**F**	**Prob>F**
Groups	112,467848	5	22,49356953	130,9860727	4,7E-132
Error	1019,87415	5939	0,171724895		
Total	1132,342	5944			

Table D.3: Mean and confidence interval at 95% confidence level for GroupA - GroupB mean.

Group A	**Group B**	**Lower Bound**	**Mean**	**Upper Bound**
1	2	-0,108	-0,055	-0,002
1	3	-0,091	-0,038	0,015
1	4	-0,303	-0,250	-0,197
1	5	-0,413	-0,360	-0,307
1	6	-0,330	-0,277	-0,224
2	3	-0,036	0,017	0,070
2	4	-0,248	-0,195	-0,142
2	5	-0,359	-0,305	-0,252
2	6	-0,276	-0,223	-0,169
3	4	-0,265	-0,212	-0,159
3	5	-0,375	-0,322	-0,269
3	6	-0,292	-0,239	-0,186
4	5	-0,163	-0,111	-0,058
4	6	-0,081	-0,028	0,025
5	6	0,030	0,083	0,136

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