POLITECNICO DI MILANO
Dipartimento di elettrotecnica

Corso di Laurea Specialistica in Ingegneria Elettrica

# On board systems modeling and power flow analysis for Light Rail Vehicles design

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## FIGURES INDEX

## TABLES INDEX

# ABSTRACT IN LINGUA ITALIANA

Lo sviluppo di modelli e metodi di calcolo capaci di rappresentare fedelmente il funzionamento dei sistemi elettrici ausiliari di veicoli ferroviari e di fornire predizioni dell'evoluzione dinamica degli stessi ai fini del dimensionamento delle parti componenti e della verifica del buon funzionamento degli impianti è un'esigenza sentita in maniera sempre più crescente dai produttori di materiale rotabile, a causa dell'acquisto di un ruolo sempre più importante delle apparecchiature ausiliarie.

Il presente lavoro di tesi si propone di descrivere nel dettaglio l'attività che il candidato ha svolto nell'ambito di un progetto di collaborazione tra Politecnico di Milano ed Alstom Transport, per la realizzazione di uno studio volto a sviluppare un modello di funzionamento e diverse metodologie di calcolo in grado di fornire agli ingegneri della suddetta compagnia un valido supporto nell'attività di studio e di progetto, con particolare riguardo al dimensionamento dei convertitori statici di potenza, delle batterie elettrochimiche e dei cavi di alimentazione dei carichi ausiliari.

I modelli proposti in questo lavoro sono stati successivamente implementati su di una piattaforma software, in maniera da rendere reiterabili e meglio fruibili i risultati del presente studio.

# ABSTRACT IN ENGLISH LANGUAGE

The development of models and calculation methods that are able to accurately represent the auxiliary electrical systems' functioning of rail vehicles and to provide predictions of the dynamical evolution of them, in order to size the components and to verify the proper operation of the systems, is a need always more felt by the rolling stock manufacturers, due to the increasing in importance of the role of auxiliary equipments.

The present work aims to describe in detail the activities that the candidate has done during his internship in France, inside a collaborative project between Politecnico di Milano and Alstom Transport. This work pursued the realization of a study aimed at the development of an analytical model and several calculation methodologies that can provide to the engineers of this company a valuable support in study and design stages, with particular regard to the sizing of static power converters, electrochemical batteries and wiring supplying of the auxiliary loads.

The models proposed in this work have been subsequently implemented on a software platform, so as to make the results of this study more usable and repeatable.

# ESTRATTO IN LINGUA ITALIANA

Il presente lavoro si propone di fornire una spiegazione quanto più efficace del lavoro svolto e degli obiettivi raggiunti dal candidato durante il periodo di stage svolto per Alstom Transport, nel sito di Valenciennes Petite-Forêt (Francia). Il progetto di stage, nasce dalla necessità concreta di sviluppo di un modello di calcolo, in grado di fungere da strumento versatile per il dimensionamento dei sistemi ausiliari a bordo di veicoli ferroviari, riferendosi in particolar modo ai veicoli LRV.

L'attività inoltre, si propone di realizzare infine un'applicazione software in grado di integrare le soluzioni proposte, così da costituire un valido supporto ingegneristico alla realizzazione dei progetti Alstom Transport.

Le necessità di calcolo che si chiede di soddisfare sono la capacità di effettuare valutazioni statiche e dinamiche del funzionamento (flussi di potenza, cadute di tensione, perdite...) dei sistemi ausiliari di distribuzione di un veicolo di trasporto ferroviario, nonché l'abilità nel dimensionamento ottimo dei cavi, dei convertitori statici e delle batterie di bordo.

I veicoli oggetti dello studio afferiscono al gruppo del trasporto elettrico urbano e sub-urbano; i sistemi, alla distribuzione BT di classificazione ferroviaria (24-110 V c.c.). Per sistemi di distribuzione ausiliaria si intendono quei sistemi che esulano dall'alimentazione dei motori di trazione, dedicati quindi al servizio di tutte le altre apparecchiature elettriche di bordo.

L'attività da cui nasce questo lavoro può essere suddivisa in tre parti principali; una fase di studio, una di modellizzazione ed un'ultima di implementazione software.

## 1 – STUDIO TEORICO

L'iniziale fase di studio ha previsto la valutazione e la comparazione dei diversi procedimenti che fino ad allora venivano usati per il dimensionamento dei suddetti sistemi, al fine di valutarne i pro, i limiti ed i miglioramenti necessari. Questa prima fase è stata inoltre dedicata ad un primo approccio con il veicolo su cui lo sviluppo è stato maggiormente concentrato, il tram Citadis. In particolare sono state analizzate le peculiarità del *casus studii*, i dispositivi di alimentazione, le diverse architetture della distribuzione, la presenza, ridondanza ed i sistemi di comando delle apparecchiature ausiliarie equipaggiate.

È seguita una fase di ricerca sui metodi e modelli attualmente disponibili in letteratura, al fine di arrivare alla creazione di un modello di analisi che sia rappresentativo, nei limiti di utilizzazione caratteristici di un'applicazione ferroviaria urbana, del comportamento dei sistemi elettrici ausiliari. Questa prima fase è stata affrontata nell'ottica di giungere ad una soluzione che potesse essere facilmente adattata ad un'applicazione di tipo software.

## 2 – MODELLIZZAZIONE

La fase di modellizzazione ha avuto lo scopo di realizzare un modello analitico dell'intero sistema ausiliario, che tenesse conto delle esigenze di calcolo espresse nel primo periodo dello stage dai colleghi ingegneri, nonché futuri fruitori del mio lavoro, in Alstom.

La fase di modellizzazione può essere suddivisa in tre parti:

- Modellizzazione dei carichi ausiliari.
- Modellizzazione delle batterie elettrochimiche (Ni-Cd, Ni-MH,...).
- Modellizzazione degli impianti di distribuzione.

I modelli, realizzati nell'ottica di una loro integrazione in un modello di calcolo numerico, permettono di creare un bilancio di potenza evolvente nel tempo, funzione dei carichi intermittenti e della condizione di funzionamento del veicolo (nominale, degradata, di soccorso, di emergenza) e della configurazione della rete, di stimare la potenza transitante nelle diverse parti del veicolo per il dimensionamento dei cavi di alimentazione e di tutte le altre parti del sistema di distribuzione, dal punto di alimentazione al carico.

La strada intrapresa è stata orientata alla creazione dei modelli mancanti, al perfezionamento di quelli in uso, ed alla definizione rigorosa e normalizzazione dei metodi di calcolo ad essi associati, relativamente alle diverse piattaforme di trazione elettrica interessate (Tram, Tram-Treni, Metropolitane).

Le metodologie sviluppate per rispondere alle espresse esigenze di calcolo, hanno riguardato principalmente:

- La definizione dei coefficienti parametrici di intermittenza, dei carichi non permanenti.
- Il calcolo della potenza di dimensionamento dei convertitori statici, funzione dei coefficienti di intermittenza dei carichi.
- Il calcolo del profilo di scarica di una batteria elettrochimica (caratteristica V/t), al fin di definirne le caratteristiche di dimensionamento della stessa, in rapporto agli scenari di funzionamento dinamico più critici.

Si è inoltre provveduto alla scelta di un metodo di calcolo numerico, adeguato in termini di facilità di utilizzo e semplicità di implementazione, in grado di valutare in tutti i punti di interesse il buon funzionamento del sistema (in termini di correnti, cadute di tensione, perdite di potenza). A tal fine, il metodo di Gauss, in quanto algebrico, è stato ritenuto il più indicato.

## 3 – IMPLEMENTAZIONE SOFTWARE

La creazione di uno strumento software in grado di replicare il calcolo, ha permesso una più agevole fruizione dei risultati ottenuti nei presenti studi. La versatilità, unita alla semplicità di utilizzo, alla potenza e accuratezza di calcolo, hanno caratterizzato il design di questo Tool Software, un prodotto pensato per sostituire un buon numero di software e modelli spesso viziati di obsolescenza ma attualmente utilizzati come supporto alla progettazione, alle offerte di gara, agli studi di fattibilità e di ricerca.

Il tool software, sviluppato in linguaggio Visual Basic, su base Microsoft Excel è stato provvisto di una interfaccia grafica e di tutti gli accorgimenti necessari a renderlo uno strumento semplice, versatile ed affidabile.

Come prima e contestuale applicazione, gli studi effettuati sono stati utilizzati per l’ottimizzazione degli schemi di principio della nuova generazione di tram Citadis X02NG, uno dei principali progetti di R&D all’interno della sede Alstom di Valenciennes. In particolare, il software permetterà di valutare la qualità del nuovo sistema di distribuzione sul precedente e della bontà della ripartizione delle apparecchiature di bordo.

# INTRODUCTION

The following work has been developed during a six months internship in the Alstom Transport site in Valenciennes Petite Foret (France). The partnership between ALSTOM and Politecnico di Milano, began thanks to Prof. Morris Brenna and Ing. Nicola Frigerio, gives to me the chance to have a brief introduction to the complex world of job in the field of electrical transports, on which the following work has been developed.

The internship starts from a request of the company for a young engineer interested in having an experience about the study of the DC auxiliary systems on Light Rail Vehicles (LRV). My answer came from my deep interest in the electrical traction, mainly because the contemporary working of mechanical systems, electric engines, signal electronics, power electronics, automatic control devices and at least three different voltage levels on a vehicle often rolling in urban environment, represents for me the meeting point among all the knowledge that I developed during this five years of studies at the Politecnico di Milano. In addition, my desire to live for a long period of time in a foreign country in order to meet another culture, to grow with another point of view and to learn another language (the French language) did the rest.

The internship's topic was a Research & Development project on the modeling of the distribution system, the loads and the development of calculation procedures for the sizing of the supplying devices and the network cabling. The consequential goal of this project was the development of a software able to perform electrical calculation, implementing the knowledge and the procedures already known in Alstom or just developed. Moreover, this work have to be considered as a basis for the future development of a complete railway distribution network calculation software.

The need of such kind of calculation has firstly to be searched in the development of a new LV auxiliary distribution system, supplied at a very low voltage rate (24 V), enough low to require a detailed and accurate voltage drops calculation. In general, the lack of unified methods and models required the improvement and formalization of several design procedures, most of which are fundamental in project stage and R&D works. In addition, some of these calculation procedures, capable to improve the design efficiency, accuracy and speed was to develop from the beginning. To reach a suitable and well-shared goal, since all these kinds of calculation normally have to be repeated several times (usually in a strict amount of time), the software implementation was mandatory.

# HISTORY OF ELECTRIC RAILWAYS

Here there is a brief historical introduction to the world of electric rail transportations. Starting from a general overview on the historical development of the railway's technology, the attention is focused on the introduction of the electric traction and the explosion of urban and sub-urban light rail vehicle (LRV) systems. The social and technological aspects that permitted the revolution of transports are here analyzed, in order to introduce and clarify how and why the LRV systems evolve nowadays.

The history of railways, as the use of rails to carry wagons for transportation, starts probably around the early XVI century. These systems was originally exploited in coal and iron mines, for the transportation of the minerals outside the galleries. these ancient systems was really primitive: the first documented application, at the beginning of the industrial revolution, was a short line developed in Newcastle by Mr. Beaumont, in 1602. This line, used to transport the coal from the mine to the Tyne river, was made by wooden carts rolling on wooden rails. The mechanical power to carry the carts was directly provided by horses. Although this first experiment will be quickly overcome, its importance have to be searched in the introduction of a new philosophy in the transportation field (the use of rails), which advantages became the key factor for its development. In fact Mr. Beaumont named its device "Tramway", which etymology probably comes from the German term "Trame" that means wooden beam or tie. So "Tramway" was a way made by beams.

## FROM THE STEAM ENGINE TO THE FIRST LOCOMOTIVES

The first industrial revolution saw the imposition of the steam engine. All the industrial fields had the possibility to emancipate themselves from the animal power and to apply new mechanic machines to their production cycles. In that period, the European society was quickly growing up: the progress of technical sciences and medical knowledge produced a rising in the population and the growth of the industry led a lot of people to leave the country for the largest cities. In this scenario a new need was always more felt: a distributed and efficient transport system.

Prior to the development the modern railways most of the overland transport had consisted of horse powered vehicles. The transport by horse presented several intrinsic matters, maily due to the fact that the horses cannot work for long time and they are exposed to all the problems of the living creatures (feeding, illness…). In addition, its limited power set severe bounds to its use on large scale.

In the second half of the XVIII century, the steam engine started to cover a prominent role in the industry; the capability in providing high rate of mechanical power signed its success. James Watt, a Scottish engineer, was the responsible for the improvements to the steam engine of Thomas Newcomen, mainly used to pump water out of mines. James Watt developed the reciprocating engine, a device capable to powering a wheel. Nevertheless the steam engines was still very heavy stationary machines and even if some experiment for carts transport through rope taken place, we have to wait until the technology permitted to build smaller steam boilers, adapted to be implemented on a running application. Watt started to investigate the use of high pressure steam acting directly upon a piston. His studies rapidly lead him to the patent of the first steam locomotive in 1784.

In the XIX century, Europe and United States saw the born of a large number of prototypes. The iron rails gauge were normalized in most countries and the first passengers and goods transport services started. The evolution of the railways started with the creation of short lines connecting close cities, proceeding afterwards with the interconnection among them. This first models of steam locomotives usually had a four wheels bogie with a mechanical transmission made by piston rod. The engineers started to build more powerful locomotives, capable to run at considerably high speed (for that time). The adhesion problem so starts to be studied, producing the first classification in the railway systems. Since the transmission was mechanic type, they adapted the reduction rate to different needs: larger wheels for high speed passenger transport and smaller ones for goods transport, in which the higher weight required more power and a better adhesion’s control.

The first classification in the railway systems come up: the difference in the services required became a difference in the locomotive's technology. This set the baseline for a consequential sub-classification that will start the era of the urban light railway transport, now called LRV.

## DEMAND FOR MOBILITY IN THE XIX CENTURY SOCIETY

As already stated, in the XVIII and XIX century the European cities was rapidly growing up because of the born of the new industrial middle class. This development marked the end of a subsistence economy for the new market economy, where always larger industrial centers called new workers from the country. These centers, mainly located near the larger cities, starts to modify the characteristics of them; the urban center, from a place exclusively dedicated to the luxury lives of the nobility became the place where the new industrial middle class organized its markets, while the suburbs saw the raise up of new residential zones and industrial enclaves. The evolution of the medicine strongly decrease the children dead rate and, in the meanwhile, the new socio-economic conditions permits a strong urbanization of large parts of population. In this context, transportation problems became more and more stringent, due to the fact that:

1. Goods and primary materials needed to be transported from the production centers to the selling/shipping centers and vice versa.
2. Always more persons needed to easily and quickly move from one point of the city to another and among different cities.

The new society so started to ask for high speed/capacity transport's technologies, answering to a demand that the old ones wasn't able to satisfy anymore, both for the large scale industrial trades and for persons mobility. The philosophy behind them was the same: private companies tried the economic advantages in offering to everyone a pay service to satisfy transport requests. The introduction of the railways systems in the industrial activities shown a big advantage: the use of rail instead of streets significantly increase the comfort and the safety of the travel and the better stability offered by iron rails permits to perform the vehicles at higher speed.

In parallel with the development of the steam engine's technology, some engineers started to study its application on an urban track. The implementation in the urban environment stated several limits: the first steam locomotives was still too large and heavy. These and other technical impediments temporarily prevented the use of steam locomotives in the cities. The first application of urban railway was so horse powered, with one or two light two-axle carts, normally towed by two horses. Nevertheless, the animal traction presented several limits to ensure a regular service on a large railways network: the exploitable horse life was very short and their service was limited to a little amount of hours per day. Moreover horses was often killed by illness and their feeding and recovering required many attention and work.

Figure 1 – A horse-powered tramway.

With the technological evolution of the steam engines many cities started to use the more powerful steam locomotives, whereas it was possible, but their safety in the urban environment and the production of pollutant smokes still constituted problems. The revolution in the urban railway transports really started only with the development of the first electric locomotives.

After several years of studies and tests, in 1881 Werner Von Siemens presents the world's first tram vehicle in Lichterfelde, near Berlin. It was supplied by the rails at a continue voltage rate of 150 V, with a power of about 7,5 kW. It was able to travel at a maximum speed of 40 km/h carrying on up to 20 passengers. After this first successful experiment, all the Europe starts to electrify all its tramway lines, supplied by a continue voltage of 500-600V through an overhead electric line. The current collector (introduced 8 years later by Siemens-Halske) was a long rod with a little wheel at the bottom, called "bow trolley".

Figure 2 – The first Siemens's electrical tramway in Berlin (1881).

On the other side, on the large scale transports, the new electric technology had to fight against steam locomotives, which technology was already deeply investigated and exploited. In addition, the more efficient diesel locomotives (thermal engines with internal combustion) started to be produced. Nevertheless, the electric revolution was started and, starting from the end of the XIX century, due to their technical advantages, alternative ad continue system will be studied and realized, covering a role always more important in the transport field, both for passengers and for goods. From the technical point of view we can state that the electric propulsion became prominent for three main reasons:

1. The higher specific power (per weight and per volume) offered in respect of engines supplied by fossil fuels (from coal for steam locomotives to oil products for diesel locomotives), very important in countries with high slope passages.
2. The lack of pollutant gases, more and more important with the growth of sensibility in respect of environmental problems and whereas their disposal constitutes a serious problem (i.e. metro tunnels).
3. The higher energetic efficiency and robustness of electric engines.

On the other side, some problems prevent an overcome of the steam (and afterwards diesel) technology:

2. The need of a distribution system working at its own voltage level, constituted by electrical lines and transformation sub-stations (SSE), which increase the global cost of the railway's infrastructure.
3. The difficulties met in realize efficient and reliable train control systems, very easy for fuel engines through a simple valve control.

Nowadays, with the development of high performance power electronic, the last point has been completely overwhelmed. Nevertheless, diesel locomotives are still performing and they are employed whereas this system is economically convenient, like in long low density lines, especially for goods transportation. To choose what kind of railway system is economically convenient we can refer to a concept chart in which fixed and variable costs are parameterized on the traffic density; the breakeven point marks the theoretical passage from one technology to another (excluding all the other analysis).

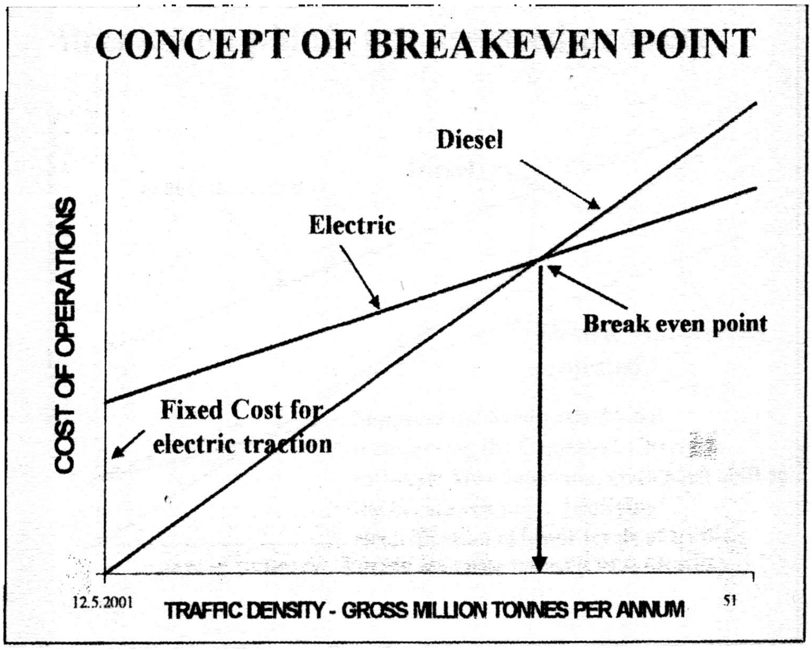


Figure 3 – Economic comparison between electric and diesel railway technologies.

## URBAN RAILWAYS IN THE MODERN AGE

In the urban scenario, the electric technology had no rivals. The advantages offered in terms of comfort, safety, noise and pollution have strongly supported the development of the first urban transport services. The railways systems (tram, train and metro) grew up all over the world, often built and performed from the public administration, both for the high costs and for the strategic and economic role covered by these new transport mediums. Nevertheless after more than fifty years of growing, the after-WW2 saw the superimposition of a new concept: the outback of the consumption's economy gave the property of the urban roads to private car's circulation; the public transport was limited to the underground and most of the European cities stopped the development of a tramway network, evolving with the city.

This wicked choice will be understood only several years later, when traffic jam started to paralyze the cities and pollutant gases became a serious problem. Public administrations recognized that the complexity of the mobility problem cannot be left to the uncontrolled private initiative. In addition, they saw how the development of the urban ground strongly influences the development of a transport network and vice versa: the administration of the ground and the building cannot be done regardless for the administration and the exercise of the common transports. In the end, starting from the 70's, most of the cities where the mobility already became a problem came back to the tramway's technology.

The resurgence of the tramway set down in a new social scenario, characterize by the presence of the new metropolis and the growth of an high number of little and medium cities, deserving of public transport. The big leading companies in the railways field (Alstom, Bombardier, Siemens, ABB…) started so to develop new tramway lines and vehicles suitable, cheap and adaptable as possible to every city characteristics and national standards.

## THE MODERN TRAMWAYS

The new economical logic impose in the last decades new product lines. The philosophy behind imposes several criteria to design and realize every products. In the meantime, the railways products became more and more complex, with the implementation of new devices able to provide for passengers' comfort and safety, starting to define the modern auxiliary system, complementary to the traction one.

In the electric transport field, the product is so complex that only a little number of companies are able to afford such kind of effort. The high cost of the investments and the need of a complex structure made by specialists and technicians permitted to only few groups to grow, going to satisfy all the world demand for electrical railway systems. On the other hand, the very high price and transport's laws permits only to a restrict number of clients to afford such kind of charge, most of which are public administrations. These clients are usually very specialized in evaluating the market's products and allowed to claim for a very personalized product. Due to the high specialization of the work behind electrical railways and the high cost of them, few multi-nationality companies developed different kind of product that can be adapt to the multitude of markets. In fact, each country normally had an independent history of its railways, following the development of its own national standards. Even if in the last century has been made a strong effort to reach the interoperability of the railway systems, quite each country differs from the other for:

- The genre of supplying voltage (AC/DC)
- The characteristics of the voltage, in term of level (from 500 V up to 25 kV), frequency ($16^2/_3$ Hz, 25 Hz, 50 Hz…)
- The track gauge
- The track characteristics, in terms of maximum slope, minimum curve's ray…
- The signaling system
-

## LIGHT RAILWAY VEHICLES (LRV)

With the resurgence of the tramway, a new classification in the railways has been outlined, the LRV. Despite of the intrinsic meaning of a classification process, very different definition of LRV are actually present worldwide and is not so easy to define which kind of rail vehicles belong to this group. Even its most characteristic features are shared from the other rail systems.

In the spectrum of rail modes, while high-speed rail, intercity passenger rail and heavy goods convoys have quite clear boundaries, the light railway has blurred edges, considerably overlapping some of the other modes. The acronym LRV means Light Railway Vehicle and it includes all the short range, low capacity and low speed passenger transport vehicles. The term "light" is normally referred to the load instead of the train's physical weight. In addition, the investments in infrastructure are usually lighter than would be for heavy rail systems.

In order to keep a clear idea of LRV, in this document I will refer to the most generally accepted definition of LRV in Alstom Transport.

This group is so composed by:

- Tramway
- Metro
- Tram-train

The tram-train, the last included, is an hybrid vehicle similar to a tram but able in performing both on urban track and on sub-urban railway lines. It’s so able to work with several voltage rate , since normally urban and sub-urban railways work, for technical and economical reasons, to different voltage levels. Tram-train has been included in the LRV group because, due to its use, it presents all the peculiar characteristics of these vehicles.

The main characterizing features of LRV vehicles are:

1. Distributed traction, that constitutes a technical advantage in respect of sliding/slipping, noise and vibrations control, cars’ swinging, mechanical efforts, braking efficiency, rolling stability. In addition, the lack of locomotives at the train’s edge (there are only the driver cabs) permits to gain additional space  for passengers.
2. Partial or complete low floor, which guarantees, with a gap between doors and rail level of about 30 cm, an increased accessibility to the train.
3. Safety devices, like magnetic brakes, that are mandatory for each vehicle performing in urban environment, due to the fact that persons or other vehicles normally can cross the rails (this device is complementary to mechanic and dynamic brakes).
4. Shorter and lighter trains, in respect of heavy lines (from one up to 8 cars).
5. Frequent stops, which hardly influences the traction/braking system.

## TARGET

The target of the present work is to describe, summarize and explain the achievements reached through my internship in Alstom Transport. This internship, focused on the development of several calculation procedures and their implementation on a software base, started with my acknowledgment about the company, its products and their characteristics. After a period of study of the current design methods, my efforts concentrated on the development of a model capable in representing the electrical behavior of the auxiliary systems, in order to provide for an effective aid in the design process. The development of these models and methods is the main topic of this work. After that, a significant part has been dedicated to the implementation of models and methods on informatics support, which strongly influenced the development of the whole work.

# CITADIS: A MODERN TRAMWAY

My experience in Alstom Transport has been focused on the development of the calculation procedures capable to offer a concrete and quick aid to all the Alstom's electrical engineers working on the electrical distribution systems. Although it was required to reach a result exploitable to all the traction platform (from tramway to high speed trains), my work has been necessarily based on a single platform, the one on which my work would be more appreciated.

The platform on which I conduced my studies and my work is the Citadis X02NG tramway. The name is composed as a code: the letter NG are for New Generation, while X02 contains information about the length of the tram. It is proposed in three lengths versions:

- 40 meters long for X=4
- 30 meters long for X=3
- 20 meters long for X=2

## AUXILIARY SYSTEM

The auxiliary system is the collection of all the electrical devices not directly involved in traction/braking as a whole. It comprises the supplying units, the distribution system and the auxiliary equipments. Such kind of equipments are mainly used to:

- Provide for safety requirements (magnetic brakes, fire detection system…)
- Improve passengers' comfort (air cooling and conditioning, lights, information devices…)
- Provide for functional requirements (doors opening/closing, signaling and train control…)

## DISTRIBUTION SYSTEM

As already said the topic of my work has been focused on train's distribution systems. A distribution system is the set of circuits and devices which common role is to supply all the auxiliary equipments plugged to the distribution system's terminals. The distribution systems are intended wholly located on board, so the overhead line or the supplying rail are not included. The auxiliary distribution systems excludes all the traction circuits, which are completely independent from the other.

In every urban railway distribution system there are three normalized voltage levels:

1. High Voltage: 600 V; 750 V; 1500 V; 3000 V dc – 15000; 25000 V ac. (EN 50163 – Ch.4)
2. Medium Voltage: 400 V; 220 V ac.
3. Low Voltage (battery voltage): 24 - 110 V dc. . (EN 50155 – Ch.5)

Hereafter there is a short description of the main parts which compose an auxiliary distribution system in a modern rail vehicle. Afterwards, more attention will be pay to the study case peculiar characteristics, the Citadis.

The key components of a distribution system are:

- High Voltage Current Collector, which ensures external power supply from catenary line or third rail.
- High Speed Circuit Breaker.
- Static Converter (CVS), which transforms and converts High Voltage into Medium and Low Voltage. CVS outputs are:

    1. AC output 400 V, 50 Hz - driver's cab HVAC and fan motors supply.
    2. AC output 360-480 V, 35-65 Hz, V/f constant rate - passengers' cabs HVAC supply.
    3. AC output 220 V, 50 Hz - sockets supply.
    4. DC output 24 V – 110 V - equipments supply.

- Battery, branch connected to CVS's DC output. Battery is charged from CVS DC output and it works only when:
    1. The power required by equipments is higher than maximum power limit of CVS (battery's buffer mode functioning).
    2. CVS's DC output is out of service, because of LV side failure.
    3. HV Supply System is not available because of CVS breakdown, collector system failure, line disconnection or special functioning mode (autonomous traction mode).
    4. Train Preparation/Depreparation time (Pantograph rising, CVS starting/CVS ventilation after EoD).

- Distribution Circuits that carry on current from CVS to equipments. There are several MV and LV circuits, divided by functions (normal functioning, train preparation/de-preparation, emergency, rescuing…), so as each circuit can be well dimensioned on its typical functioning cycle.

  Distribution is often a radial system, sometimes with petal (ring) shape branches. The primary line is often on the vehicle roof, spaced by distribution boxes. These primary boxes can contain bus bars, switches, electronic devices and circuit breakers. Electrically under this first level, we can find several distribution sub-levels, distribution boxes and directly plugged equipments.

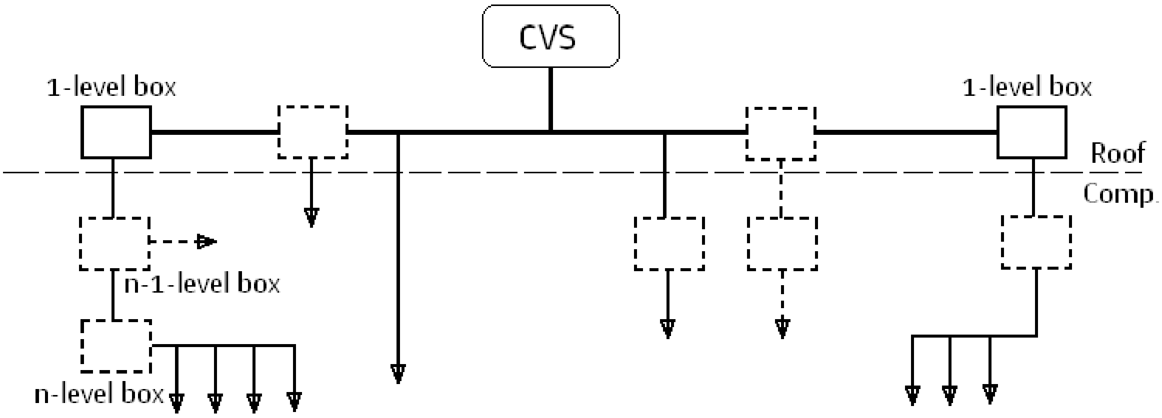
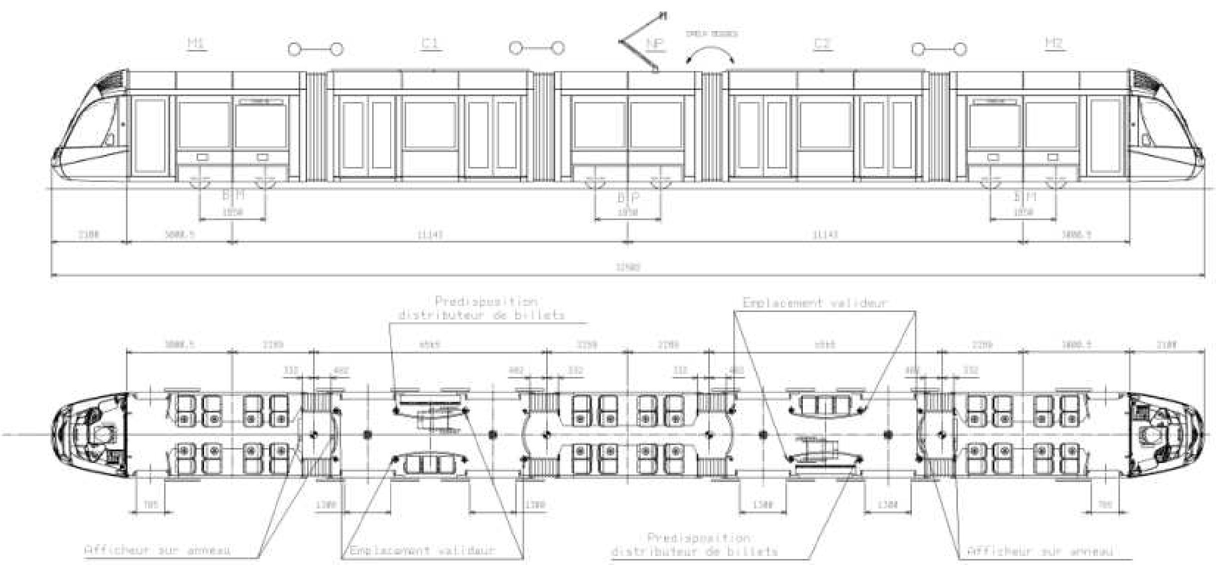


Figure 4 – Principle scheme of auxiliary distribution system.

- Equipments performing safety, comfort, functional and emergency requirements. They are mainly made of electronics, motors and resistive loads.

## CITADIS X02NG ARCHITECTURE

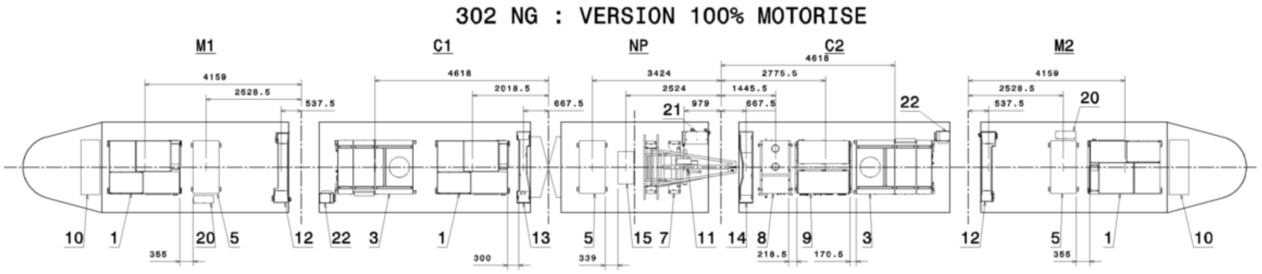
Citadis X02NG is the new tramway product made by Alstom Transport. It has been developed starting from the old version Citadis, giving it some improvements about cars, electronics and distribution systems. In addition, some new optional packs (developed in the last years) have been implemented.



Citadis X02NG Tramway presents several peculiar characteristics:

- It is a modular vehicle made by several cars that can be composed in order to meet customer’s requirements (plug-and-play logic). It is sold all over the world, adapting itself to very different environmental and service conditions.
- It’s a 100% low-floor vehicle able to operate with 20 meters minimum curve radius.
- It has several optional packs that can be integrated on the vehicle, modifying architecture and power consumptions.
- It’s cost-oriented, it means with a strict orientation to reduce fix and variable costs, and weight-oriented, in order to reduce the gap with the main competitors and to join the 10 tons per axle target imposed by the German market.
- Because of safety reasons, LV level is 24 V. This very low level makes distribution system critical in respect of voltage drops’ limits.
- The new electrical architecture has been improved to achieve the 0 V voltage drops goal.
- In all Citadis X02NG versions there is no redundancy on CVS and battery (only in 40 meters version a smaller ventilation CVS is added).

- It has a predisposition for the new Ecopack and APS (ground rail supply) systems implementation.



| | | | |
|---|---|---|---|
| 1 | Traction unit | 12 | End box |
| 2 | Chopper box | 13 | End box |
| 3 | Air conditioning | 14 | End box |
| 4 | Super capacitor box | 15 | High speed circuit breaker Box |
| 5 | Rheostat | 16 | NA |
| 6 | NA | 17 | NA |
| 7 | Pantograph | 18 | NA |
| 8 | Battery Box | 19 | Auxilliary ACE Box |
| 9 | ACE Box | 20 | Cable box |
| 10 | Cab air conditioning | 21 | Cable box |
| 11 | Lightning arrester | 22 | Cable box |

### Citadis HV architecture

The HV system is composed by:

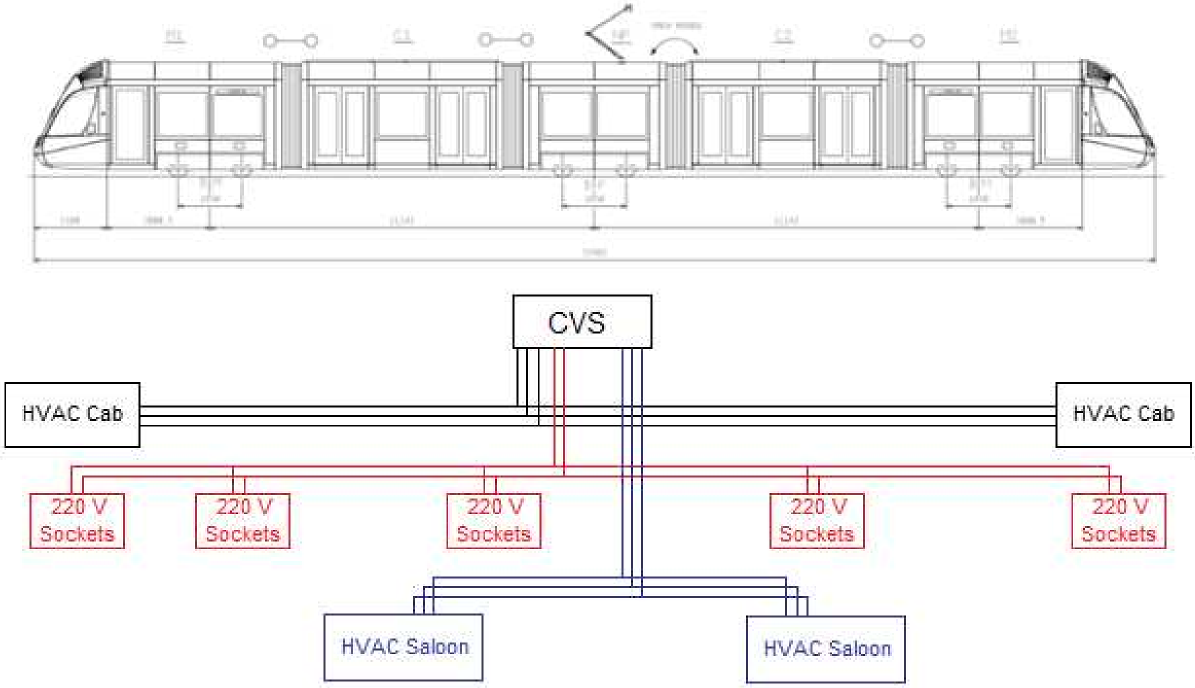
- The pantograph
- The HV box, containing the HSCB
- The HV side of the auxiliary converter
- Inverters and traction motors

### Citadis MV architecture

In all the Citadis range, the MV distribution is used exclusively for the air-conditioning and ventilation (HVAC) and the sockets distributed in the cars. There are two kinds of CVS's MV output:

1. 400 V fixed frequency output for the cabs' HVAC.
2. 360-480 V, 45-60 Hz, V/f=8 constant rate - passengers' compartments HVAC.

The use of variable voltage rate/frequency is used to obtain the compartment's air control through the CVS regulation. A synoptic scheme follow hereafter:



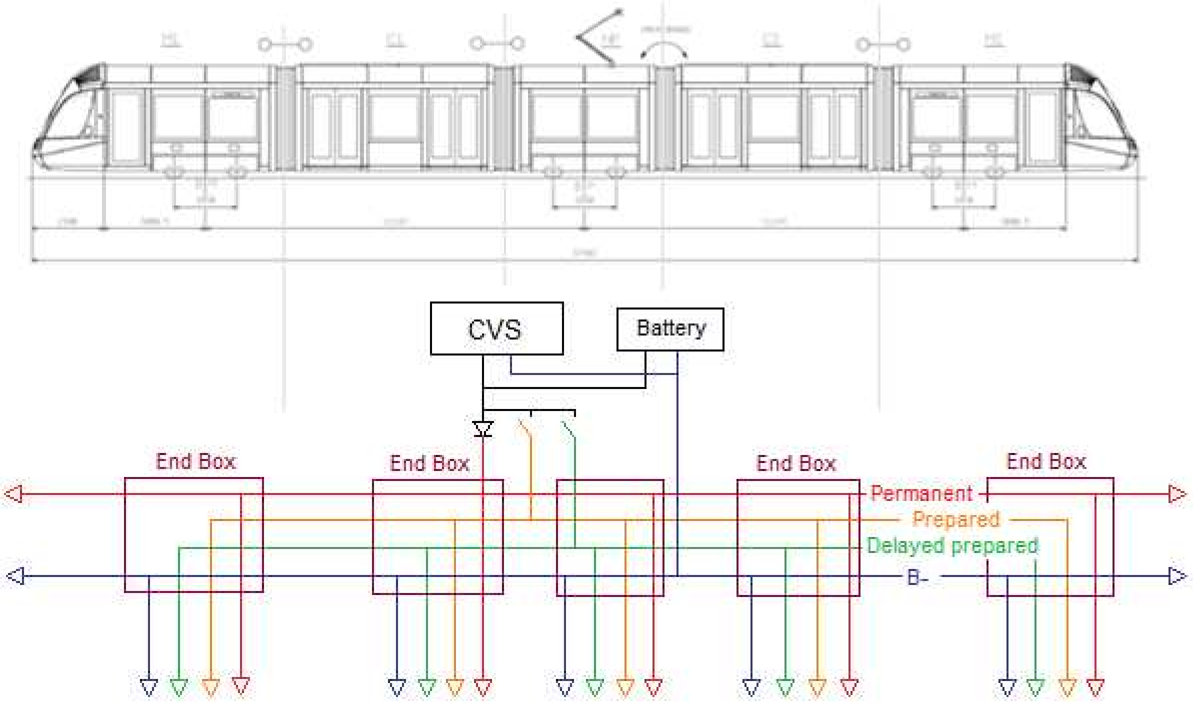
**Citadis LV architecture**

LV equipments are usually supplied by several 24 V circuits, in order to achieve functional differentiation. The LV network can be so split into 4 sub-networks:

- **Permanent LV**: This network is used to supply equipments that must remain available when the train is not prepared. It is always supplied by the battery until it's flat or from the CVS when it's active. This network is also used to supply equipments that must remain available when the train is in Towing/Pushing operation; it is separated from the CVS/Battery couple by a diode which avoids supplying the other low voltage networks when the train is rescued by another one. Only safety and essential equipments/apparatus must be connected to it.
- **Prepared LV**: This network is used to supply equipments that are necessary when the train is in service. This network is powered from the CVS/Battery couple only when the preparation contactor (represented in green colour) is closed (train prepared). As soon as the train is deprepared, this network is not energised anymore. Most of low voltage

equipments are connected to it. This network is the one that can hold the highest current among all the networks.

- **Time-delayed prepared LV**: This network is used to supply equipments that are necessary when the train is in service and cannot be abruptly de-energized without being previously advised. This is why this network remains supplied several minutes (30-60) after train de-preparation. Rack UMC, Wi-Fi modem, switches Ethernet, SAE box, radiotelephone and ticket marker central unit. In case of Towing/Pushing operation, this network cannot be supplied by the rescuing train.
- **B-**: This network represent the wired return of the current. It is common to all the other LV networks. This achievement has been introduced to contain and control the voltage drops, moving them from the car body. As the permanent network, it have to be plugged when the train is rescued.



As already stated, for several reasons, the X02NG LV distribution network has been completely redesigned.

The new low voltage architecture has been developed to allow the deletion of low voltage boxes on the roof, in order to leave enough space for the EcoPack equipments - chopper box and super capacitors boxes - that permits the braking energy saving and to perform parts of the track with the lack of the primary HV supply. This led on designing a new based on end-boxes situated on the roof of each car to realise a local electrical distribution for each car.

The battery and CVS locations have been moved for a better electrical balance (closer to the CVS), allowing reducing distribution cable sections.
Another criterion was to reduce the voltage drops in line due to the cable lengths and sections, and main supplying voltage, since all the 24 V equipments must be supplied within a correct voltage range included between 16,8V and 30V (refer to standard EN 50155).

The last novelty has been done with the introduction of the current cabling B-, so as the current return is wired and not through the car body anymore.

The DC low voltage network is so important since it is responsible for all the equipments on board supplying (except for heating and ventilation), which cover all the functional, security and safety roles.

The following table shows the functional sub-systems supplied by the LV distribution network:

| # | Acronym | Description |
|---|---|---|
| 19 | ECOPACK | ECOPACK SYSTEM |
| 20 | APS | GROUND SUPPLY SYSTEM |
| 21 | MVS | AUXILIARY POWER SUPPLY SYSTEM |
| 22 | DRV | PREAPARATION/DEPREPARATION SYSTEM |
| 23 | SFT | SAFETY CONTROL SYSTEM |
| 24 | DSF | DRIVE SAFETY SYSTEM |
| 25 | BRK | FRICTION BRAKING SYSTEM |
| 26 | EBR | ELECTROMAGNETIC BRAKING SYSTEM |
| 27 | ESG | EXTERNAL SIGNAL SYSTEM |
| 28 | LIG | LIGHT SYSTEM |
| 30 | DRS | DOORS SYSTEM |
| 31 | EAS | EXTERNAL ACCESS MONITORING |
| 32 | TMR | TACHOMETER AND EVENT RECORDER SYSTEM |
| 33 | PAS | PASSENGERS INFORMATION SYSTEM |
| 34 | OAS | OPERATING AID SYSTEM |

| | | |
|---|---|---|
| 35 | CLM | CLIMATIC CONFORT SYSTEM |
| 36 | WWD | WIPER AND REMOVE ICE SYSTEM |
| 37 | SAN | SANDING SYSTEM |
| 38 | TPS | REMOTE NEEDLE AND CROSSROADS' PRORITY SYSTEM |
| 39 | WFL | WHEEL FLANGE LUBRIFICATION |
| 40 | CAL | PASS. SCREENS AND INFORMATIONS SYSTEM |
| 41 | TMS | PRINTER/MARKER TICKET SYSTEM |
| 42 | RTS | RADIO FM SYSTEM |
| 43 | VSS | INTERNAL VIDEO MONITORING SYSTEM |
| 44 | CRP | PRIORITY CROSSROADS AERIAL SYSTEM |
| 45 | PCS | PASSENGERS COUNT SYSTEM |
| 51 | ATP | AUTOMATIC TRAIN PROTECTION SYSTEM |
| 56 | FSD | FIRE DETECTION SYSTEM |
| 99 | TCN | TRAIN ELECTRONIC INFRASTUCTURES |

Some of these sub-systems are optional (Ecopack or APS). Nevertheless this table shows that the all the rolling support systems, monitoring systems and safety systems (especially electromagnetic brakes) are LV supplied.

A model doesn’t exhaust its meaning in what
it is but in what it represents.
O.Heaviside

# CHAPTER 1 – EQUIPMENTS MODELIZATION

## ON MODELING

The mathematical modeling aim is to describe in mathematical terms different aspects of the natural behaviors and their dynamics. The mathematical modeling, with the theoretical analysis and the empirical experimentation constitutes the base of the scientific method. The mathematical modeling in the last century known a huge success, mainly due to the fact that:

1. It enables to translate every kind of model (e.g. electrical models) in the language of mathematic, so as every algorithm became solvable with a numerical method (explicit solutions are allowed only for a little part of models), by always more powerful calculators.
2. The mathematical language permits to achieve models by abstraction process and it can be apply to every knowledge field.
3. It also allow to apply all the mathematical tools that have been developed, so algorithms that reach for optimal solutions with the maximum calculation efficiency.
4. In general, it provide for a growth in the knowledge.

In the last decades the technology and the mathematical modeling grew together because the technological progress and the industrial world is completely dependent from the prediction that this kind of modeling gives, in respect of any kind of dynamic, system and study case.

Mathematical models offer new possibilities to master the growing complexity of industrial technologies, exploring new solutions quickly, allowing to speed up the cycles innovation.
Since the early 60's, numerical analysis, as the discipline that allows the resolution mathematical equations (algebraic, functional, differential and integral) using algorithms has played a leading role in solving problems associated with mathematical models derived from engineering and applied sciences. In the wake of this success, new disciplines have

appreciate the use of modeling mathematics, such as communication, bioengineering, financial engineering. The extraordinary complexity of these applications has spurred mathematicians to reconsider their approach to putting the issue to center stage as such, and looking models and algorithms to find solutions. This paradigm shift led to the rising of scientific computing, whose aim is to build better algorithms for efficient and accurate simulation and optimization of problems of real interest. The ultimate goal of scientific computing is to create versatile and reliable models, accurate within limits dictated by the specific class of problem, which can be verified on a large and significant variety of test cases, analog or experimental, for which we can have reference solutions. In addition to universal concepts that a mathematical model to reproduce, such as the conservation of mass and energy of a fluid, the moment of inertia of a structure, etc.. for a successful numerical simulation is necessary to establish what level of detail it makes sense to associate to the different components of a model and what kind of simplifications can be done to facilitate its integration with different models. Models that simulate reality very complex should also take into account the uncertainty resulting from inadequate availability of data that characterize the model. Models that enjoy these properties will be used to predict natural processes, biological, environmental, to better understand the physics of complex phenomena and contribute to designing innovative products and technologies.

## LOAD MODELING

In order to analyze an electrical system (representing its behavior using an analytical model), since the global behavior is the resulting of the interactions among the different parts that compose it, the modeling of each part of the system is mandatory. The different models, developed taking into account the integration with the others, shall be gathered to form the whole system model.

Load modeling is the activity that permits to define the load answer when perturbations apply.

Due to the huge variety of perturbations that can change the load’s behavior, several models can be developed to study the same load. The set of mathematical relationships, linking any input vector (of perturbations) to any other output vector (of variables) is called mathematical model.

These relationships can be defined either on deterministic base or on stochastic base (e.g. aggregate loads), linking different inter-acting systems (electrical, mechanical, thermal…), in a way depending on the nature of the load, the type of study required, the variables taken into account or neglected.

In transmission/distribution systems, load modeling covers a prominent role in most of the more common analysis, such as steady state, transient stability, voltage stability, small-signal stability, planning studies.

The load modeling activity, gathered with the models of supplying devices and distribution system, permits to give a complete mathematical representation of an electrical system. The whole model achieved in this way, enable the engineering in making all the analytical simulations needed, providing for all the behavioral predictions used for the projects and research activities.

### Steady-state studies

Steady-state studies evaluate the system variables in an equilibrium point.

Most currently used calculation models don't represent the voltage sensitivity of the load, since all the loads are represented as constant active/reactive power draws. This approach can be used in transmission and distribution systems as a baseline for planning studies or to evaluate the steady-state functioning point after a contingency, when voltage regulation devices has brought all the voltages close to the nominal values.

### Transient stability studies

This kind of study provides information about the capability of an AC system to maintain the synchronism of the generators after perturbations apply (usually as load step). The dynamic response of generators and motors is approached through differential equation, while for non-motor loads the used model is still algebraic, sometimes sensitive to voltage magnitude and frequency changes.

### Voltage stability studies

Voltage stability also includes all the non-linear time-variant control actions able to modify the network state vector due to transient recovery or rescheduling actions. Since these kinds of perturbations can have significant effects on loads, detailed models are very important to represent such kind of response.

To correctly estimate the current's flows in an electrical network, a good knowledge in the loads adsorbed currents is mandatory.

Although their effects are often negligible, a sizing calculation (whatever solved) implies certain hypothesis and simplifying approximations that have to be discussed. These kinds of hypothesis can be often linked with the stability of the voltage across the load terminals or the power adsorbed rate and in general they ensure that the load functioning conditions remain inside the nominal bounds.

Nevertheless, these kinds of representations are still an area of great uncertainty and many studies have shown that load representation can have a significant impact on analysis results, also in steady-state evaluations [6].

Until few years ago, a common philosophy was, in absence of accurate data on load characteristics, to use always the most pessimistic representation, in order to provide a safety margin. Although still used, this approach presents two problems: the use of expensive overestimations and the fact that is not always possible to define the worst representation that can be exploited in all the system under every functioning condition.

From the mathematical point of view, considering a state variable (as voltages in a conservative network) as independent from the system's evolution means to reduce the order of the system, so the accuracy of the calculation.

This simplification can be done wherever the analysis is conduct around a nominal functioning mode point (of the state-vector) or when the variables have independent control and, in general, wherever the questioned variable has not a strong effect on the system behavior during the considered period.

In this way, we can consider as constant the active power generated by a turbine in a load flow calculation or the supplying voltage at a domestic circuit terminals, due to presence of voltage regulators and/or contractual constraints.

A railway vehicle’s electrical system cannot be considered as a domestic or industrial system.

In fact, in railways field, all the equipment and devices are specially developed to perform under the heaviest condition, due to the fact that the exploiting is in outside environment, the whole system is able to move so as technical requirements are more stringent.

Furthermore, when passengers’ transportation is involved (especially in LRV vehicles) a large number of safety requirements have to be taken into account from the manufacturer.

One of the most important safety exploiting condition is the battery functioning; in case of primary supply loss, the onboard batteries shall be able at least to provide current for the equipments useful to prevent panic or danger situations (i.e. lighting, air-ventilation, fire-protection systems…) and to avoid damages to that equipments which suffer unexpected switch off or voltage holes (i.e. computers and other electronic devices).

Since these equipments have to correctly work under supplying condition that can widely vary (from LVPS output to battery minimum voltage), the previous hypothesis falls and the nominal values cannot be enough accurate under all the conditions.

A wrong choice in load modeling can lead to additional costs due to oversizing and dangerous situation in undersizing.

This chapter has been developed in order to improve calculations method with more reliable and accurate procedures, to reduce the presence of empirical security factors and so, overestimations. It is divided in several sections; the first one will show the used model in Alstom calculations and the need of improvements, the second one will describe some of the modeling method present in the literature, the third one present the chosen model and the last part show how to implement and characterize it.

# USED MODEL AND NEED OF IMPROVEMENTS

Actually, in steady-state calculation made in Alstom, each equipment is modeled as a constant power load. This kind of modeling is very simple to implement, rather than all the others, since only one data is needed to characterize it, the nominal power rate, taken at nominal voltage rate. Nevertheless, it ensures the validity of the power value only in the nominal functioning point and, maybe, in its neighborhoods.

The validity of the model in the range of use depends from the width of this range and from the intrinsic nature of the concerned load.

International specifications EN 50155, explain that railway equipments' manufacturers and suppliers shall provide, in their specification document, for three different power or current rates, measured at 3 different voltage levels, representative of the functioning range for a railway application. Such equipments, shall be able to operate satisfactorily for all the values defined within the range below (measured at the output terminals of the equipment):

- Minimum voltage 0,7 Un
- Nominal voltage Un
- Rated voltage 1,15 Un
- Maximum voltage 1,25 Un

Considering the normalized nominal low voltage values:

| Un | 0,7 Un | 1, Un | 1,15 Un | 1,25 Un |
|---|---|---|---|---|
| **24 V** | 16,8 | 24 | 27,6 | 30 |
| **48 V** | 33,6 | 48 | 55,2 | 60 |
| **72 V** | 50,4 | 72 | 82,8 | 90 |
| **96 V** | 67,2 | 96 | 110,4 | 120 |
| **110 V** | 77 | 110 | 126,5 | 137,5 |

Table 1 - LV normalized rates.

The voltages between 0,7 Un and 1,25 Un form the range inside which low voltage equipments are able to function continuously. In addition the norm specify that if the voltage falls between 0,6 Un and 1,4 Un not exceeding 0,1 s, the load shall not present deviation of function (§ 5.1.1.1).

To make consideration about the importance of considering different models we will use an example, from Knorr Magnetic Shoes, one of the most important safety equipments.

The manufacturer provides for a chart in which is reported the attractive force on the rails, depending from current rate and so from voltage applied.

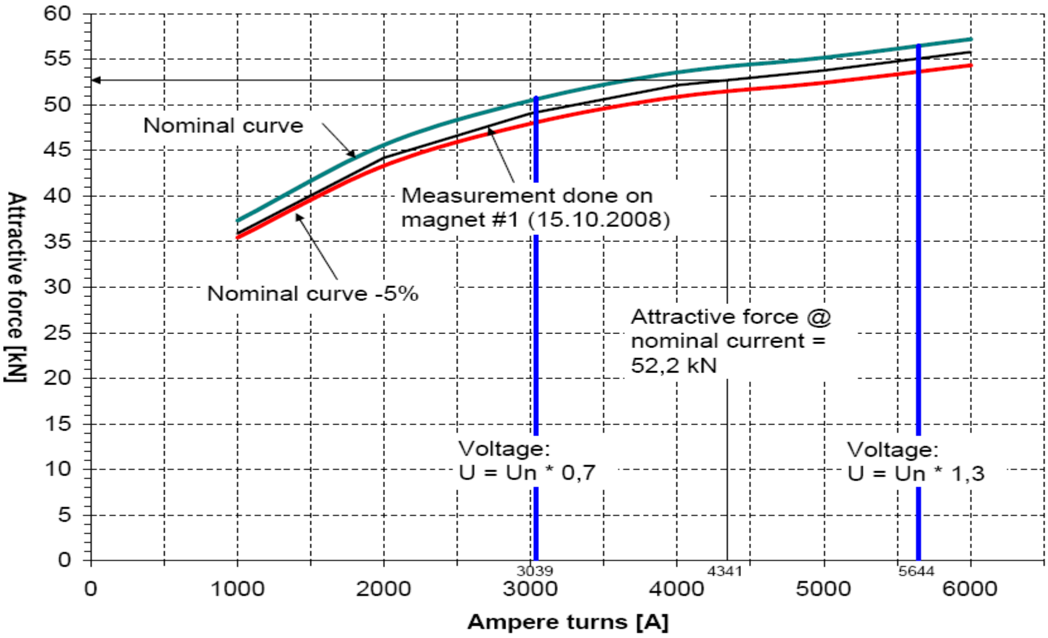


Figure 5 - Knorr magnetic shoes, effort/current caracteristic curve.

The black (measured) curve shows that as the voltage drop, the current drop and so the force. This is in contrast with what should happen using a constant power model: to keep constant the product between current and voltage, when the voltage decrease (i.e. due to battery discharge or voltage drops) the current should increase, showing a different attractive effort. Even if the braking force is not so affected by the voltage variation, it is for the current adsorbed, which strongly influence the supplying wire sizing.

Since such kind of overestimation could lead to an underestimation in security or project parameters, is easy to understand the importance of correct modeling.

# LOAD MODELING LITERATURE

A load model is a mathematical representation of the relationship between the power (or current) adsorbed by equipment and its state variables. Load's state variables are usually voltage and frequency.

Load modeling literature is enough reach; as already said, the right model is strongly related with the conditions under which the load is used and so the model is simulated. These conditions include the observed interval, the type of supplying condition and perturbation, the presence of simplifying constraints.

The first model classification can be done between static and dynamic models.

Static model are the simplest ones. They usually relate the power with the voltage through a simple algebraic equation. A static load model is used to model those components that present negligible dynamic behavior (like lights) or to approximate dynamic loads (i.e. motors), when their dynamics are not interesting (for example in steady-state analysis).

Dynamic model usually require to be explicated through differential equations, often using a state system. These kind of models are used whereas a dynamic analysis is required, for instance in transient (short-term) evaluations. Dynamic models, once simplified and adapted to steady-state conditions, leads to the relative static models.

## STATIC MODELS

Using static models imply the ability of the system to reach a steady-state condition. A steady-state condition is a system functioning point in which all the variables root mean square values (RMS) are standing at constant level. From a strictly mathematical point of view a steady-state condition cannot be never achieved: for instance, in a distribution network this is due to the fact that network topology change naturally with time and because of some kind transients can presents very long time constants. Nevertheless, to use such kind of simplified models, we have to find that conditions under which it is possible.

The analysis concerning the sizing calculation of static converter and batteries (so power balance calculation and battery discharge profile calculation) have different nature. The first ones, intrinsically static, consist of evaluating during different functioning scenarios the functioning system variables vector, to evaluate the steady-state flows used in define converter’s thermal sizing current. The second ones, concerning a discharge profile evaluations in time, are dynamic calculations. Nevertheless, some approximation can be provided to let us use the same load models for both calculations.

Battery discharge profile calculations are evaluating in time just to take into account the fact that the voltage on battery terminals gradually drops as the discharging rate grows, so as the integral of the battery current in time raise.

So if we can consider the current overshoot energetically negligible (so with a negligible product current for time duration in respect of the battery capacity [Ah]) or if we imagine that the current transient are strongly limited due to the battery capacity inertia, we are enabled in ignore this behaviour and in approaching the battery discharge calculation as a series of steady-state calculations.

## CONSTANT POWER LOAD MODEL

This is the load model already used. The power rate is always the same, whatever voltage value is applied. Considering two different functioning points:

$$P_0 = V_0 \cdot I_0$$

$$P_1 = V_1 \cdot I_1 = P_0$$

In case of AC loads, is sufficient to add the equation $Q_1 = Q_0$ to the previous one.

$P_0$ and $Q_0$ are the values of active and reactive power in the nominal functioning point ($V_0, I_0$) or, in general, the values associated to any already known functioning point.

## CONSTANT CURRENT LOAD MODEL

In a constant current model the power vary directly with the voltage magnitude. This kind of model is often used in those equipments that present a controlled current supplying device (most of electronics). For DC:

$$I_0 = P_0 / V_0 \qquad I_1 = P_1 / V_1 \qquad I_0 = I_1$$

$$P_1 = P_0 \cdot (V_1 / V_0)$$

While for AC systems:

$$P_1 = P_0 \cdot \frac{V_1 \cdot \cos\varphi_1}{V_0 \cdot \cos\varphi_0} \cong P_0 \cdot \frac{V_1}{V_0}$$

$$Q_1 = Q_0 \cdot \frac{V_1 \cdot \sin\varphi_1}{V_0 \cdot \sin\varphi_0} \cong Q_0 \cdot \frac{V_1}{V_0}$$

Considering the power factor quite constant for little voltage variations.

## CONSTANT IMPEDENCE LOAD MODEL

In a constant current model the power vary with the square of the voltage magnitude. Ideal linear and time-invariant impedance can be so represented:

$$Z_0 = V_0^2 / P_0 \qquad Z_1 = V_1^2 / P_1 \qquad Z_0 = Z_1$$

$$P_1 = P_0 \cdot (V_1 / V_0)^2$$

In the same way, for AC systems:

$$R_0 = V_0^2 / P_0 \qquad R_1 = V_1^2 / P_1 \qquad R_0 = R_1$$

$$X_0 = V_0^2 / Q_0 \qquad X_1 = V_1^2 / Q_1 \qquad X_0 = X_1$$

$$P_1 = P_0 \cdot (V_1 / V_0)^2$$

$$Q_1 = Q_0 \cdot (V_1 / V_0)^2$$

## POLYNOMIAL (ZIP) LOAD MODEL

This model is a weighted combination, through a polynomial equation, of the other three static models (const Z, const I, const P). It can be expressed as:

$$P_1 = P_0 \cdot \left[ a \cdot \left( \frac{V_1}{V_0} \right)^2 + b \cdot \frac{V_1}{V_0} + c \right]$$

$$Q_1 = Q_0 \cdot \left[ d \cdot \left( \frac{V_1}{V_0} \right)^2 + e \cdot \frac{V_1}{V_0} + f \right]$$

The weight coefficients *a*, *b*, *c*, *d*, *e*, *f* and the power factor of the load (for AC loads) are the parameters of the equation.

The only one constraint for these two sets of parameter is:

$a + b + c = 1$

$d + e + f = 1$

From another point of view:

1. *a* and *d* can be seen as the constant impedance fraction
2. *b* and *e* can be seen as the constant current fraction
3. *c* and *f* can be seen as the constant power fraction

## EXPONENTIAL LOAD MODEL

Exponential load model is another way to express the relationship between power and voltage, with the same accuracy of the ZIP load model:

$$P_1 = P_0 \cdot a \cdot \left(\frac{V_1}{V_0}\right)^{np}$$

$$Q_1 = Q_0 \cdot b \cdot \left(\frac{V_1}{V_0}\right)^{nq}$$

Other exponents can be added, so this kind of model is useful in representing the aggregate effect of different kind of loads. Exponential greater than 2 and less then 0 can be used to model special types of load. In a simpler form $a$ and $b$ can be neglected making this model is very simple in empirical characterization:

$$P_1 = P_0 \cdot \left(\frac{V_1}{V_0}\right)^{np} = S_0 \cdot \cos\varphi \cdot \left(\frac{V_1}{V_0}\right)^{np}$$

$$Q_1 = Q_0 \cdot \left(\frac{V_1}{V_0}\right)^{nq} = S_0 \cdot \sqrt{1-\cos^2\varphi} \cdot \left(\frac{V_1}{V_0}\right)^{nq}$$

This form, which is the most present in the scientific literature, can be characterized only with two functioning points:

$$np = \frac{\log\left(\frac{P_1}{P_0}\right)}{\log\left(\frac{V_1}{V_0}\right)} \qquad nq = \frac{\log\left(\frac{Q_1}{Q_0}\right)}{\log\left(\frac{V_1}{V_0}\right)}$$

Although the meaning of *np* and *nq* is not really intuitive is easy to find in the specialized and technical literature several sets of empirical data useful for different kinds of domestic and industrial loads and common load patterns (commercial, industrial and residential) [1], [4], [5].

## FREQUENCY DEPENDENT LOAD MODEL

For some equipment the dependence of the power from the frequency have to be explicated.
This behaviour is usually taken into account by multiplying either a polynomial or an exponential load model by a factor of the following form:

$$1 + a_f \cdot (f - f_0)$$

Where $f$ is the supplying frequency and $f_0$ is the nominal frequency.
$a_f$ is the frequency sensitivity parameter of the load model.

This kind of model is required in modelling static frequency dependent load and induction motors, during rotor generator oscillation analysis or whereas they are supplied by a variable frequency supplying device (like PWM inverter).

# MODEL CHOICE

To characterize ZIP and exponential model we need three measurements in three different functioning points. Both models give a parabolic approximation of the P/V load characteristic chart.

Nevertheless, because of the presence of only two parameters instead of three, the use of the exponential model is more useful and to prefer wherever one or both of the following conditions are satisfied:

1- The use of empirical exponential values (e.g. using aggregate loads).
2- The functioning point moves close to the nominal functioning point.

On the other side, with a large functioning range and with three measured power values at disposal, parabolic interpolation gives more precise results.

The following chart has been traced on a double door motors' opening consumption. Manufacturer measurements gave us three functioning points:

| Absolute Values | | Relative Values | | |
|---|---|---|---|---|
| Voltage | Power | Voltage | Power | Funct. Point |
| 16,8 | 66 | 0,7 | 0,66 | (P2,V2) |
| 24 | 100 | 1 | 1 | (P0,V0) |
| 30 | 145 | 1,25 | 1,45 | (P1,V1) |

Table 2 - Door's opening power data.

Using all the three points to characterize the ZIP load model we obtain:

$$P(V) = P_0 \cdot \left[ 1{,}2121 \cdot \left( \frac{V}{V_0} \right)^2 - 0{,}9272 \cdot \left( \frac{V}{V_0} \right) + 0{,}7151 \right] \text{ [W]}$$

While using two points (points (P0,V0) and (P1,V1)) to characterize the exponential model we obtain:

$$P(V) = P_0 \cdot \left( \frac{V}{V_0} \right)^{1{,}6651} \text{ [W]}$$

Their graphical representations are implemented in the following chart:

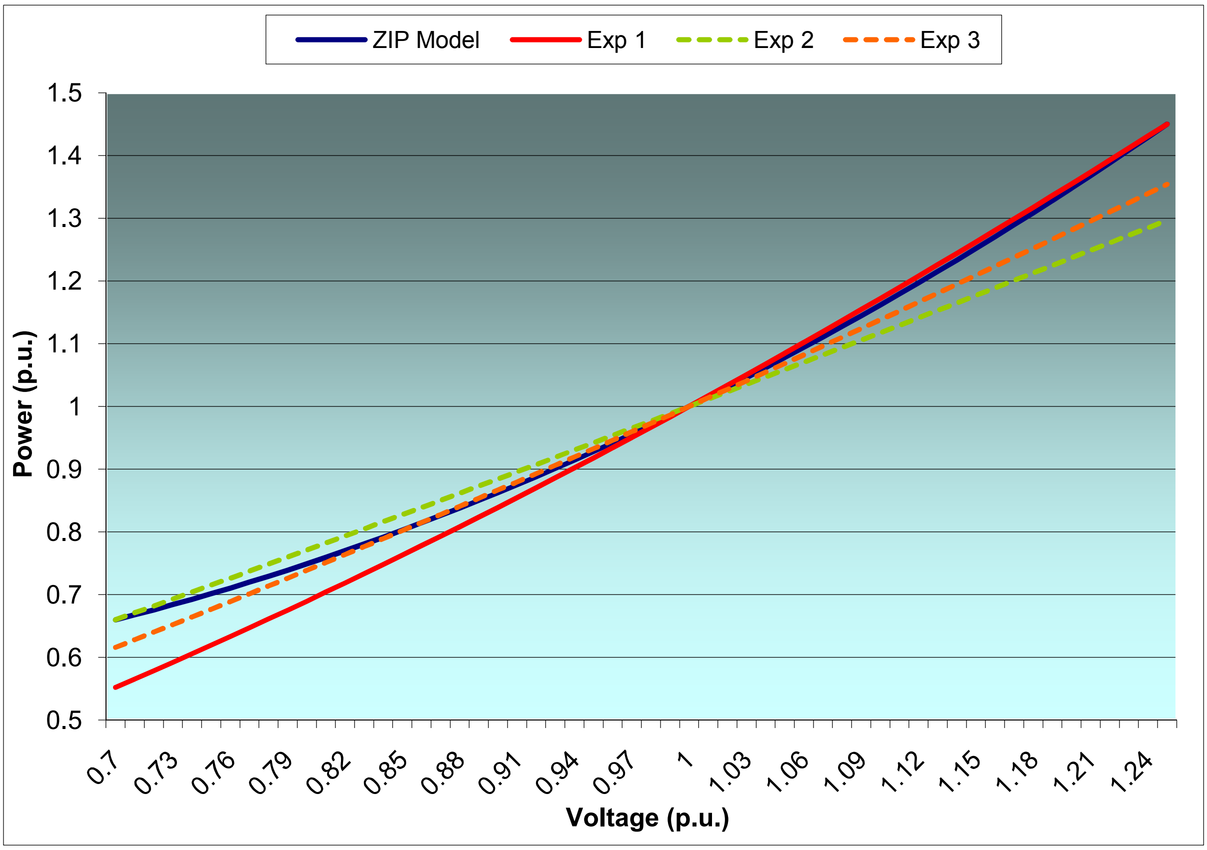


Figure 6 – Comparison between ZIP model and exponential models.

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While the parabolic interpolation of the ZIP model passes through all the three functioning points, the exponential model, parameterized only on two functioning points leads to a 17% power underestimation in the third point, (P2,V2).

In the same way, the dashed curves Exp2 (np=1,1649695) and Exp3 (np=1,3574576), traced evaluating the exponential parameter using respectively (P0,V0), (P2,V2) and (P1,V1), (P2,V2) show high errors, when they are evaluated in such a wide voltage range.

We agree that this kind of error cannot be tolerated, especially in a calculation model that mainly bases its accuracy on the correct estimations in power flows.

Moreover the ZIP model is to prefer whether and user-friendly application is aspect: The ZIP parameters have clear physical meaning since this model allow a lighted passage from the universally valid ZIP model to the three particular models constant Z, constant I and constant P.

## PARAMETRIZATION

Because of for all equipments consummations must be done test at the same three different voltage (0,7 ; 1 ; 1,25) we can approximate the characteristic like a parabolic curve, since through three point pass one and only one parabola. This approximation is useful to determine the parameters that link power or current to voltage applied. This calculation gives a higher error as high is the global exponent *b*:

$$P = \alpha \cdot v^2 + \beta \cdot v + \gamma = P_0 \cdot v^{np}$$

With $\alpha = P_0 \cdot a$, $\beta = P_0 \cdot b$, $\gamma = P_0 \cdot c$, $v = \frac{V}{V_0}$

Since coefficients are determined from experimental data, for physical reasons *b* will be rarely greater than 2, in a real load. We can so consider it a good approximation.

For each load we can write:

$P_1 = \alpha \cdot {v_1}^2 + \beta \cdot v_1 + \gamma$ $\qquad$ $I_1 = \phi \cdot v_1 + \varphi + \varepsilon / v_1$

$P_2 = \alpha \cdot {v_2}^2 + \beta \cdot v_2 + \gamma$ or $\qquad$ $I_2 = \phi \cdot v_2 + \varphi + \varepsilon / v_2$

$P_3 = \alpha \cdot {v_3}^2 + \beta \cdot v_3 + \gamma \qquad I_3 = \phi \cdot v_3 + \varphi + \varepsilon / v_3$

In matrix form:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} {v_1}^2 & v_1 & 1 \\ {v_2}^2 & v_2 & 1 \\ {v_3}^2 & v_3 & 1 \end{bmatrix} \times \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0{,}49 & 0{,}7 & 1 \\ 1 & 1 & 1 \\ 1{,}5625 & 1{,}25 & 1 \end{bmatrix} \times \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

Now we can find for each equipment the coefficients $\alpha$, $\beta$ and $\gamma$ with a simple matrix inversion:

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 6{,}0606 & -13{,}3333 & 7{,}2727 \\ -13{,}6363 & 26 & -12{,}3636 \\ 7{,}5757 & -11{,}6666 & 5{,}0909 \end{bmatrix} \times \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

In this way is possible to determine a voltage-dependent model for any equipment. A similar process can be applied in event of different set of voltage (e.g. 0,7 ; 1 ; 1,15).

## ZIP LOADS

In order to characterize equipments' static behavior through the ZIP model, we always need three static functioning points. Nevertheless, (although the norm EN 50155, § 5.1.1.1) most of times is not possible to find these data nor to perform measurements.
Most of times, for each equipment, manufacturers provide for a power or a current value, often given at nominal voltage.

Nevertheless I found consumption's data about several kinds of loads, from which I chose some of the most important and representative, on the basis of several criteria:

1. Maximum power
2. Number of equipments on board
3. Capability in representing a larger group of equipments
4. Absorption during, intermittence factor[1]
5. Functioning during different rolling modes

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[1] An equipment showing frequently starts and stops adsorbed current that widely fluctuate and is more able to interfere with the supplying voltage.

6. Operation in conjunction with other loads
7. Operation with other equipments' load shedding
8. Operation with temporary or permanent inhibition of other loads

Starting from their description and modeling, is possible to drive for analogy or similitude the better model choice for quite any kind of equipment. The use of the lower case power $p = P/P_0$, allow to determine load parameters that can be used (inside the validation limits) regardless for the rated power of the equipment. In the end, the methods hereafter used to find the models, can suggest the way to reach all the other models.

### Doors

Doors are very critical equipments, firstly due to their number, power consumption (about 100 W per door) and a power adsorption strongly dependent on voltage; in addiction, their correct functioning become fundamental in degraded mode, like HV loss mode, where, after the train stand stopped along track with passengers inside, batteries must ensure energy and correct voltage supply to enable all the doors to open, just when voltage on batteries is near to sharply drop.

| | **Current @ voltage [A]** | | | **Peak power** | | **Power @ voltage [W]** | | | **R_eq @ voltage[Ω]** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Name** | **16,8** | **24** | **30** | **Value** | **during(ms)** | **16,8** | **24** | **30** | **16,8** | **24** | **30** |
| Door opening | 3,93 | 4,17 | 4,83 | 440 W | max 500 | 66 | 100 | 145 | 0,234 | 0,174 | 0,161 |
| Door closing | 1,90 | 1,54 | 1,37 | 440 W | max 500 | 32 | 37 | 41 | 0,113 | 0,064 | 0,046 |
| Door perm. | 0,8 | 0,72 | 0,64 | | | 13,44 | 17,28 | 19,2 | 0,048 | 0,030 | 0,021 |

**Table 3 - Door's opening data.**

### 1 – Door's opening

From the table above, we can obtain:

$$\alpha = 121,212161$$
$$\beta = -92,727244$$
$$\gamma = 71,5151211$$
$$P(v) = 121,212161 \cdot v^2 + -92,727244 \cdot v + 71,5151211$$
$$p(v) = 1,212122 \cdot v^2 + -0,927272 \cdot v + 0,7151512$$

With $p = \frac{P}{P_0}$, $P_0 = 100W$.

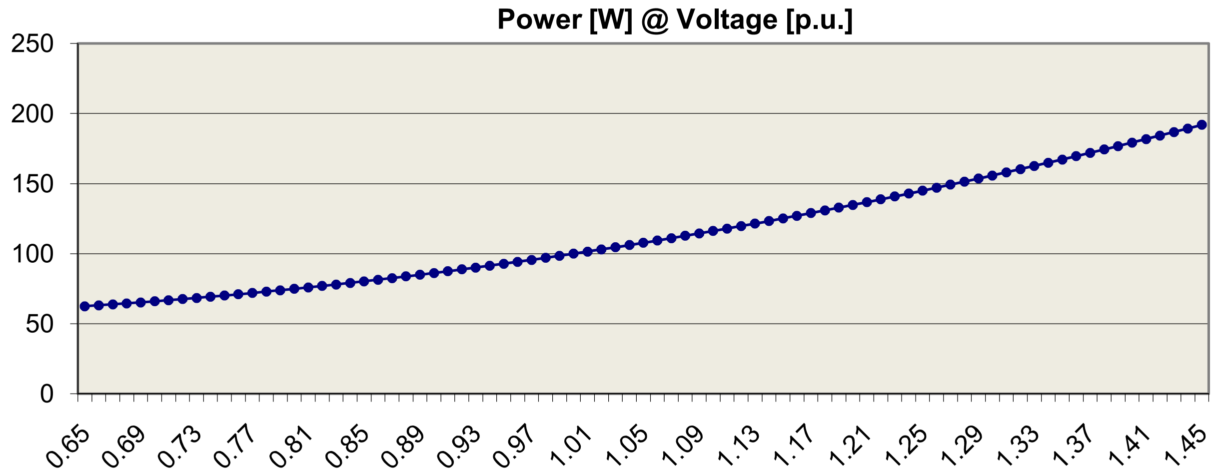

Figure 7 – Door's opening static characteristic.

During the opening operation the door shows a behaviour that cannot be conducted to the other elementary models.

## 2 – Door's closing

From the table above, we can obtain:

$$\alpha = -1,2121065$$
$$\beta = 18,727276$$
$$\gamma = 19,4848373$$
$$P(v) = -1,2121065 \cdot v^2 + 18,727276 \cdot v + 19,4848373$$
$$p(v) = -0,012121065 \cdot v^2 + 0,18727276 \cdot v + 0,194848373$$

With $p = \frac{P}{P_0}$, $P_0 = 100W$.

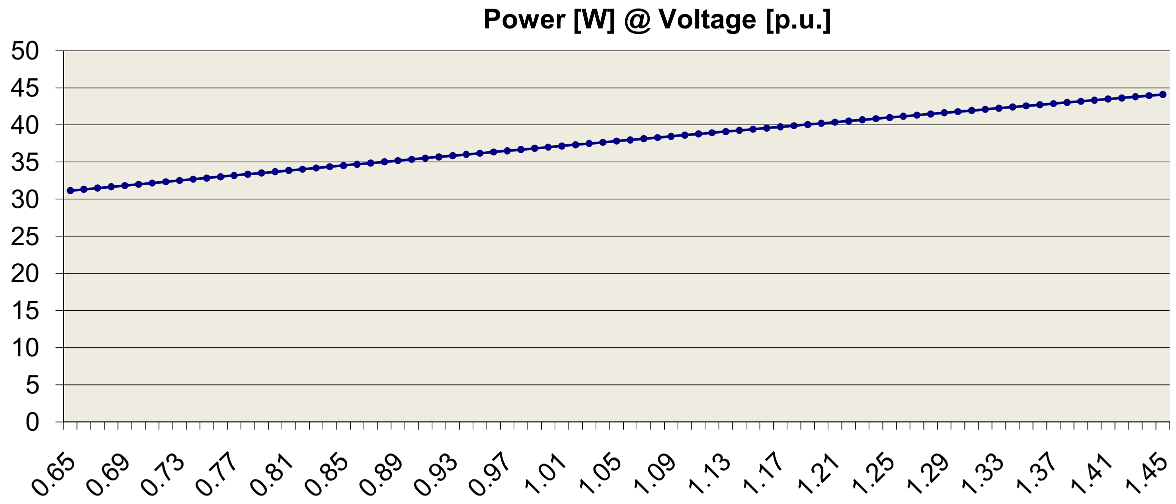

Figure 8 - Door's closing static characteristic.

During the closing operation the door shows an hybrid behaviour composed for half part from constant power and half part from constant current.

## 3 – Door's opening & closing

To take into account only one model for closing and opening, we will do an average of absolute values α, β and γ. The hypothesis under which we do this average is the very little time between opening and closing, so referring to the Power Balance Calculation, we can consider both like a single operation with the same voltage applied. Nevertheless, for dynamic evaluations like the Battery Discharge Profile, we can use two different models for opening and closing.

$$\alpha = (121{,}212161 - 1{,}2121065)/2 = 60$$
$$\beta = (-92{,}727244 + 18{,}727276)/2 = -37$$
$$\gamma = (71{,}5151211 + 19{,}4848373)/2 = 45{,}5$$
$$P(v) = 60 \cdot v^2 - 37 \cdot v + 45{,}5$$
$$p(v) = 0{,}875912 \cdot v^2 \text{ - } 0{,}54015 \cdot v + 0{,}664234$$

With $p = \dfrac{P}{P_0}$, $P_0 = 100W$ .

## Emergency brakes

The main characteristic that contributes in defining this equipment as an important load is inside its name. Emergency or electro-magnetic brakes play a fundamental role in emergency operation, like during emergency or safety braking. The use of this equipment imply an huge current adsorbed, and used in safety braking, it works contemporary with HSCB opening (on HV line and/or SC, SNCF specifications), that lead to the HV loss mode (with all the load shedding provided and with battery supply).

| Characteristic | Value |
|---|---|
| Nominal voltage | 24V |
| Nominal current | 44 A |
| Resistance at 20°C | 0,54 ohm ±5% |
| Inductance | 0,11H |
| Attractive effort | 54 kN ±5% |

Figure 9 - TTNG, KNORR magnetic shoes

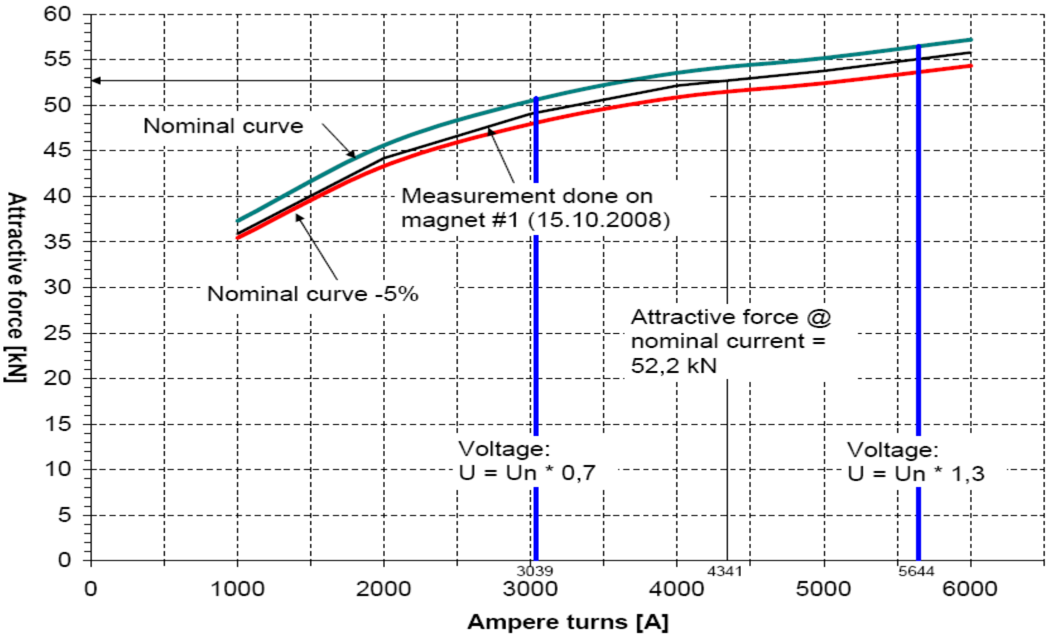

Figure 10 – Magnetic shoes static characteristic.

As shown in the figure above, current grow from 3030 A at 0,7 Un to 5644 A at 1,3 Un (for 10 shoes).

The curve also shows the dependence of the attractive force from the supplying voltage so that to have an emergency braking within length limit, we must have a supplying voltage within predefined limits. Stopping distance grow up proportional with voltage drop.

| I | V | P | Z |
|---|---|---|---|
| 30,39 | 16,8 | 510,552 | **0,552813** |
| 43,41 | 24 | 1041,84 | **0,552868** |
| 56,44 | 30 | 1693,2 | **0,531538** |

Table 4 – Emergency brakes functioning points.

As shown in the table, emergency brakes can be well modelled as a constant impedance load. Magnetic brakes, in a 24 V LV system are considered normally supplied from the CVS at 28 V and from the battery at 21 V.

### Traction control unit electronic

TCU (AGATE electronic) can be modeled as constant power equipment due to his supply control system. This equipment is very important for train control and braking control so it must be protected against incorrect supplying. In addition, due to its supplying device, its behavior can be associate to every current-controlled electronic device.

|  | **Current @ voltage** |  |  | **Peak current** |  | **Power @ voltage** |  |  | **R_eq @ voltage** |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Name** | **16,8** | **24** | **30** | **Value** | **t(ms)** | **16,8** | **24** | **30** | **16,8** | **24** | **30** |
| AGATE control (AA3x) | 3,570 | 2,455 | 2,000 | 65,00 | 0,001 | **59,976** | **58,92** | **60** | 0,213 | 0,102 | 0,067 |

Table 5 - Traction control unit data.

The table above shows that this kind of electronic can be modelled as a constant power equipment.

## Relays

Relays, according to their design features, present constant impedance load behaviour.

Despite they often require a little power consumption, this kind of equipment is present in any train in a huge number.

| | Current @ voltage | | | Peak current | | Power @ voltage | | | R_eq @ voltage | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Name** | **16,8** | **24** | **30** | **Value** | **t(ms)** | **16,8** | **24** | **30** | **16,8** | **24** | **30** |
| Input contactor (KLx) | 0,958 | 1,352 | 1,690 | 2,07 | - | 16,094 | 32,448 | 50,7 | **17,54** | **17,75** | **17,751** |
| Precharge contactor (KPREx) | 0,546 | 0,771 | 0,964 | 1,18 | - | 9,173 | 18,504 | 28,92 | **30,769** | **31,13** | **31,120** |
| Power Module fans control contactor (K1x) | 0,120 | 0,170 | 0,210 | 0,26 | - | 2,011 | 4,08 | 6,3 | **140,35** | **141,18** | **142,86** |
| HSCB TRAIN opening - control contactor (K2x) | 0,120 | 0,170 | 0,210 | 0,26 | - | 2,011 | 4,08 | 6,3 | **140,35** | **141,18** | **142,86** |
| HSCB SC closing - control contactor (KCDJx) | 0,120 | 0,170 | 0,210 | 0,26 | - | 2,011 | 4,08 | 6,3 | **140,35** | **141,18** | **142,86** |
| HSCB SC keeping - control contactor (KMDJx) | 0,120 | 0,170 | 0,210 | 0,26 | - | 2,011 | 4,08 | 6,3 | **140,35** | **141,18** | **142,86** |
| HSCB SC loop opening - control contactor (KQC) | 0,137 | 0,196 | 0,245 | 0,33 | - | 2,302 | 4,704 | 7,35 | **122,63** | **122,45** | **122,49** |

Table 6 - Relays data.

## Lights

Incandescent lamp can be modelled as constant impedance load. Another question is for gas/LEDs lamp, when they are driven from an inverter ballast configuration: in this case they can be modeled as constant current loads.

## Other equipments

For most of the auxiliary equipments is possible to approximate the static behavior using one of the elementary models; constant P, constant I, constant Z. On the basis of datasheets, transactions [1-5] and international specifications [6] has been made a list of Citadis equipments with their reference load model. For the other equipments, technical datasheets or measurements are required.

| Constant power loads | Constant current loads | Constant impedance loads |
|---|---|---|
| TBCU | Gas/LED lamps | DJ |
| Switching box | Sanding compressor | TBCU - Relays |
| Battery charger | | Cars' relays |
| Charger | | CVS |
| BCU | | Super capacitor box |
| Platine | | Pantograph |
| Photo-electric door's cell | | Magnetic shoes |
| Information screens | | Driver's feet air dryer |
| Lateral information panels | | Wheel flange greasing valve |
| Internal information panels | | Incandescent lamps |
| Front information panels | | |
| Radio FM | | |
| Public address/intercom | | |
| Ticket marker/distributor | | |

Table 7 – List of Z, I, P equipments.

To take care that for some of these equipments, the static behavior is driven by the supplying control device; in absence of which the load's behavior can radically change.

## CABLES

All the wires in the distribution network are taken into account like constant impedance equipments, considering only the longitudinal resistance as characterizing parameter. This model is justified by the very low DC voltage of the system (24 V). Despite the presence of large section cables (up to 240 mm2), since the short-term transients are considered non-significant in respect of the LVPS sizing, there is no need to take into account the conservative behavior of the cables.

# CONCLUSIONS

The choice of static model enables only to represent a static simulation that in the reality doesn't exist. In fact, static simulations are evaluations of variables during stable functioning, regardless any kind of dynamics. Even if we would be able to reach a long-term steady-state condition, this kind of simulation implies the ability of the system in reaching this steady-state point.
That's why, to represent such dynamics like transient stability, this approach not enough. During this kind of study, a static model could give a realistic result while a more accurate dynamic model could give a failure and in the reality the real network would collapse [2].

So the use of static models in converter and battery sizing calculations is subordinate to the check, by the manufacturers, of the right functioning of their devices and to the evaluation, through complete dynamic models (also able to simulate the interaction between different systems, like electrical and mechanical ones), of the most critical transients and functioning conditions.

Once ensured the supplying devices stability under all the concerned conditions, we are able to use only static load models, since in energetic calculations, like power balance and battery sizing, the energetic content of the transient periods can be always neglected.

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[4] K. Morrison, H. Hamadani, L. Wang, “Load Modeling for Voltage Stability Studies” Power Systems Conference and Exposition, 2006. PSCE '06. 2006 IEEE PES, pp 564 - 568.

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[6] IEEE Standard 1476-2000.

# CHAPTER 2 - BATTERY MODELING

## INTRODUCTION

Electrochemical rechargeable batteries are of great importance in many electrical systems because they can store chemical energy that can be converted in and send whenever and wherever is needed, in electrical form.

Especially in Light Railway Systems, this kind of batteries covers an important role, both for safety and for functioning. In case of loss of the main low voltage supplying system (LVPS), train's batteries shall be able to supply, with their stored energy, al the equipments covering functioning and safety roles - during time and modes specified in national and international standards -, ensuring a supplying voltage level kept inside the good and safety edges of any equipment.

Moreover, batteries can be used during normal functioning like in "buffer mode", that consists in to connect battery output terminal directly to the Static Converter low voltage output and so on to distribution network. This kind of architecture forces to set the Converter output voltage to a level compatible with the battery charging limits, keeping this one always charged. Due to this operational mode, during switching operations or transients, once reached Converter output current limits, battery naturally provide for the missing current.

While in the past Lead-acid battery was the leader technology in the railway systems (and not only), nowadays they are no more used in passenger transportation - also due to the intrinsic danger in having sulfuric acid as electrolyte- , preferring new and safer technologies.

Today, the main used technologies in the railways are Nickel-based and Lithium-based.

Since all the Alstom's LRV vehicles carry on Nickel-Cadmium batteries, special attention will be paid to the development of a mathematical model able to provide for discharge predictions of this kind of batteries.

A battery discharge profile calculation is a runtime evaluation of the battery output voltage. This kind of simulation is very important in order to ensure energy and right voltage value to all the supplied equipments.

Battery sizing is similar to cable sizing; after the first energetic sizing (thermal current), a second check level is required to ensure minimum voltage threshold  respect (voltage drop), without which battery size must be increased. Battery discharge profile calculations are so used to check that, during specified runtime conditions, all the equipments are supplied with a voltage capable to avoid functioning failures.

This chapter, starting from an overview on battery's physic, technologies and actual solutions, provides for a battery discharge model suitable to perform the Battery Discharge Profile Calculations on Visual Basic platform.

## ELECTROCHEMICAL BATTERIES

Despite differences among technologies, all the electrochemical batteries present similar intrinsic behaviors that is useful to summarize.

An electrochemical cell is normally defined through its electric parameters: nominal voltage and charge. While the terminal's voltage is defined by the chemistry of the electrochemical reaction, the storable charge depends from its geometry.

One or more elementary parts, the electrochemical cell, form an electrochemical battery. Several cells can be linked through series/branches connections in order to achieve the battery desired voltage and charge.

An electrochemical cell, as the name suggest, shows iteration between two different physical systems, chemical and electromagnetic. Regardless the historical and epistemological implications that this kind of statement carries on, we are interested in analyze this iteration in order to find an equivalent model able to represent two coupled physical systems as only one.

The chemical behavior of a cell can be described by a red-ox equation. Two semi-reactions of reduction and oxidation generate a differential of potential, as the electromotive force that sustains the reaction. To have a useful electrochemical cell, is sufficient to separate the semi-reactions and convoy the electronic flow (between the two sub-systems, anode and cathode) into an electric circuit.

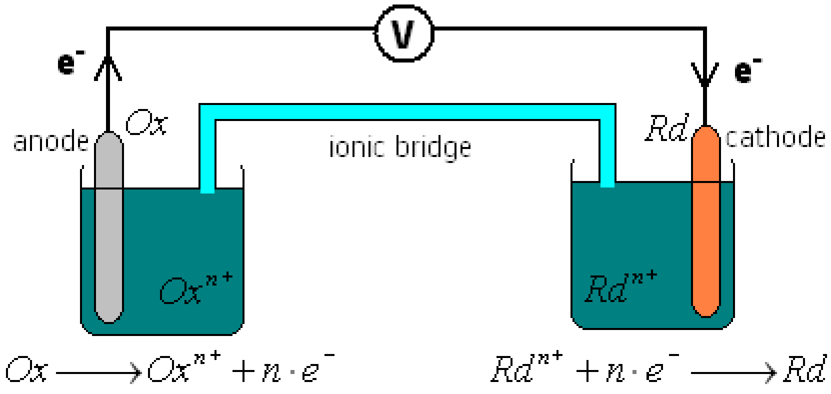


Figure 11 - Electrochemical cell's principle scheme.

Oxidation semi-reaction: $Ox \longrightarrow Ox^{n+} + n \cdot e^-$

Reduction semi-reaction: $Rd^{n+} + n \cdot e^- \longrightarrow Rd$

Total red-ox reaction: $Ox + Rd^{n+} \longrightarrow Ox^{n+} + Rd$

The Nernst Equation, directly derived from the thermodynamic potential definition (Gibbs Potential), permits to calculate, in each state of thermodynamic equilibrium, the potential generated by an electrochemical cell. In a general formulation, for each electrode:

$$E = E_0 + \frac{RT}{nF} \cdot \ln \frac{\prod a^{vox}{}_{i,ox}}{\prod a^{vrd}{}_{i,rd}}$$

Where:

- $R$ is the gas constant (8,3144 J/(K mol)),
- $T$ is the absolute temperature (K),
- $F$ is the Faraday constant (98485 C/mol),
- $n$ is the number of electrons transferred in the semi-reaction,
- $a$ are the activities of the reduced and oxidized reagents (for a reduction, right and left side of the reaction) and
- $v$ are them stoichiometric coefficients.

This formula provides the voltage generated from each semi-reaction, referring to the standard semi-reaction of hydrogen reduction $2H^+ + 2e^- \rightarrow H_2(g)$, which gives $E = 0$ V (under standard measurement conditions)[2].

For aqueous solutions with low reagents concentrations, activities can be expressed as concentrations:

$$E = E_0 + \frac{RT}{nF} \cdot \ln \frac{\prod [ox]_i^{vox}}{\prod [rd]_i^{vrd}}$$

For oxidation semi-reaction:

$$E = E_0 + \frac{RT}{nF} \cdot \ln \frac{[Ox]^{n+}}{[Ox]}$$

While for reduction semi-reaction:

$$E = E_0 + \frac{RT}{nF} \cdot \ln \frac{[Rd]^{n+}}{[Rd]}$$

So, for the global reaction, taking into account a unitary concentration for solid electrodes, we obtain the cell output voltage:

---

[2] Standard measurement conditions are defined as: T=25°C; pressure of gaseous reagents 1 atm; reagents in concentration 1M (1 mol/l); metallic electrodes.

$$\Delta E = \Delta E_0 + \frac{RT}{nF} \cdot \ln \frac{[Rd]^{n+}}{[Ox]^{n+}}$$

This equation is so able to link, in steady-state condition, the external electrical variable Voltage (in Volt) with the internal chemical variables Concentration (of all the reagents acting in the reaction).

## BATTERIES FOR LRV TRACTION

During the last two centuries, a lot of different battery technologies have been developed, basing on different electrodes and electrolytes, so red-ox reactions.

In battery sizing, the first choice is on technology: since in most of application weight is a very important parameter (aircraft, railways, mobile-phones, satellites…) energy and power, depending on service requirements are always referred to unit of weight.

Nevertheless, for most stationary applications, is possible to ignore the following graph, choosing the cheapest technology:

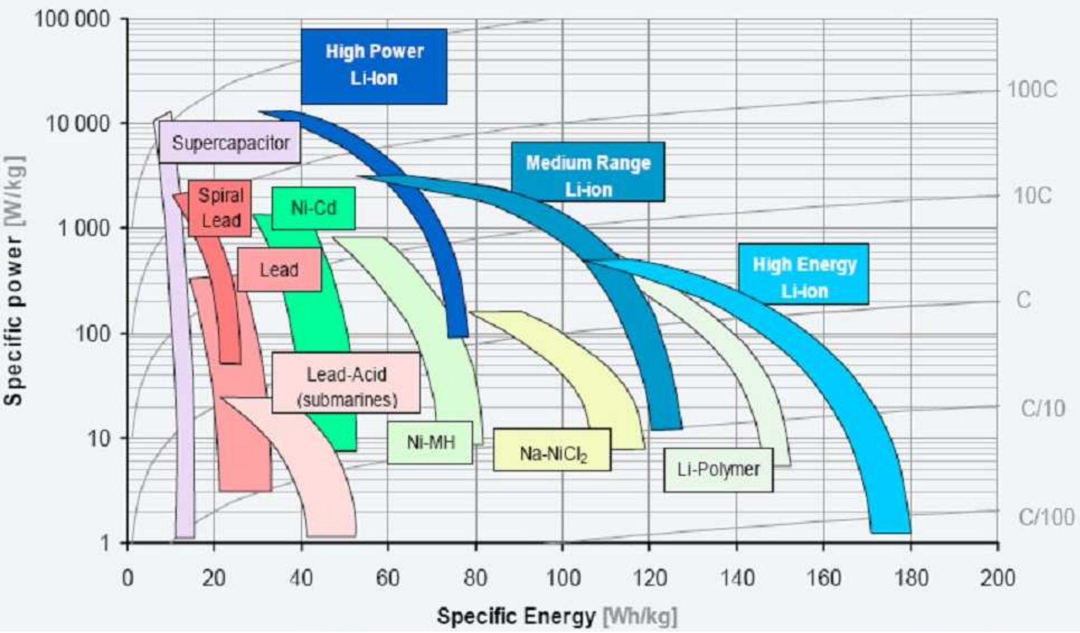


Figure 12 - Comparison among different cell's technologies.

Regardless of weight, most application often have to meet other requirements; Railways is a field in which persons are involved, so safety requirements apply since various dangers are linked to electrochemical batteries (explosion, burning, loss of acid or toxic electrolyte…).

Actually, on LRV vehicles, most used cells are based on Nickel (even though its toxicity):

- Nickel-Cadmium Cell (Ni-Cd)
- Nichel-Metal Hydride Cell (Ni-MH)

Nevertheless, other appreciated technologies are Lithium-ion and Polymer Lithium-ion ones.

Nickel-based cells are very suitable for LRV application. Their main features are:

1. Very low internal series resistance: a low internal resistance ensures low internal voltage drops, so as battery output voltage vary as less as possible, varying the applied load. They're so suitable to supply equipments with high insertion current.
2. High robustness: They require less maintenance and can be used on a non-stationary application. In addition, electrolyte doesn't take part in the chemical reaction, so there's no need to fill periodically the electrolyte tank. In this way the battery can be mounted regardless for the short-period maintenance.
3. Very long life: they can be charged and discharged more time in respect of the other technologies (LRV vehicles usually refer to 30 years exploitable life).

4. High self-discharge rate: Ni-Cd cells presents self discharge rate from 15% per month, up to 20% per month and Ni-MH cells of about 30% per month while for Pb-Acid cell this value is around 5%.
5. Higher Specific power and energy: in respect of Lead-acid technologies
6. High cost: Materials and production cycle are more expensive than for other technologies.
7. Memory effect: This technology suffer the so called “memory effect”, a loss in net storable energy if the battery is charged until it is not completely discharged.

Ni-Cd Reaction

For the anodic electrode:

$$2 \cdot Ni(OH)_2 + 2 \cdot OH^- \xrightarrow{CHARGE} 2 \cdot NiOOH + 2H_2O + 2 \cdot e^-$$

$$E_{OA} = 0{,}38V$$

While for the cathodic one:

$$Cd(OH)_2 + 2 \cdot e^- \xrightarrow{CHARGE} Cd(s) + 2 \cdot OH^-$$

$$E_{OC} = -0{,}91V$$

The total redox reaction is:

$$2 \cdot Ni(OH)_2 + Cd(OH)_2 \xrightarrow{CHARGE} 2 \cdot NiOOH + 2H_2O + Cd(s)$$

Giving a theoretical potential of $E_0 = E_{OA} - E_{OC} = 1{,}29V$

# BATTERY VARIABLES AND PARAMETERS

Although the Nernst equation is able to represent the cell's voltage at each equilibrium point, it is not so useful for our purposes. The use of non-linear equations expressed through internal chemical variables constitutes a serious problem for the implementation and the integration of the model. We need so to begin a modeling process that takes into account the physical phenomena and the dynamics that we are interested in representing. In addition, this process shall lead to a battery model expressed through the external electrical variables, so that it could be implemented on an electrical network's model.

This work shall start with the study of the main variables and parameters affecting the battery's behavior, according to its application and its use on a LRV vehicle.

In a Ni-Cd battery, the main variables are the ones represented in the most manufacturers' discharge charts: Voltage and State of Charge. We can consider the battery voltage as a state variable, since the conservative chemical behavior acts like a capacitor's electrical field. According to the network model developed, the most suitable battery model will be the one that will provide for the actual battery voltage, depending on the environmental and functional variables and parameters.

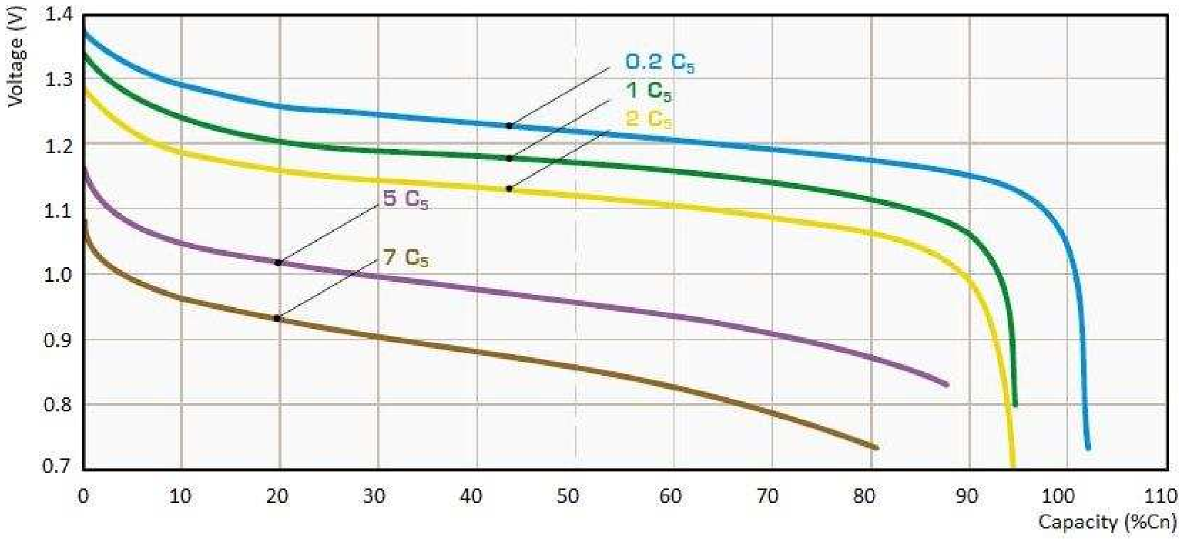


Figure 13 – SAFT's MSX Ni-Cd battery, Voltage/State of Charge plot.

Battery’s manufacturers, in their technical brochures, usually represent several discharge curves measured at different constant discharge’s current rate. These levels are referred to multiple of C5 (Ah), the capacity declared on 5 hours constant current discharge.  The reference test current, in Ampére, is obtained dividing the reference capacity by one hour (EN 61434). Starting from the full-charge conditions, on the Y axle there is the cell voltage while on the X axle the used capacity is reported. This chart shows the cell internal resistance non-linearity and time-variance (with the SOC). Instead of a V/SOC plot, some manufacturers sometimes provide for V/time plots, like the following one:

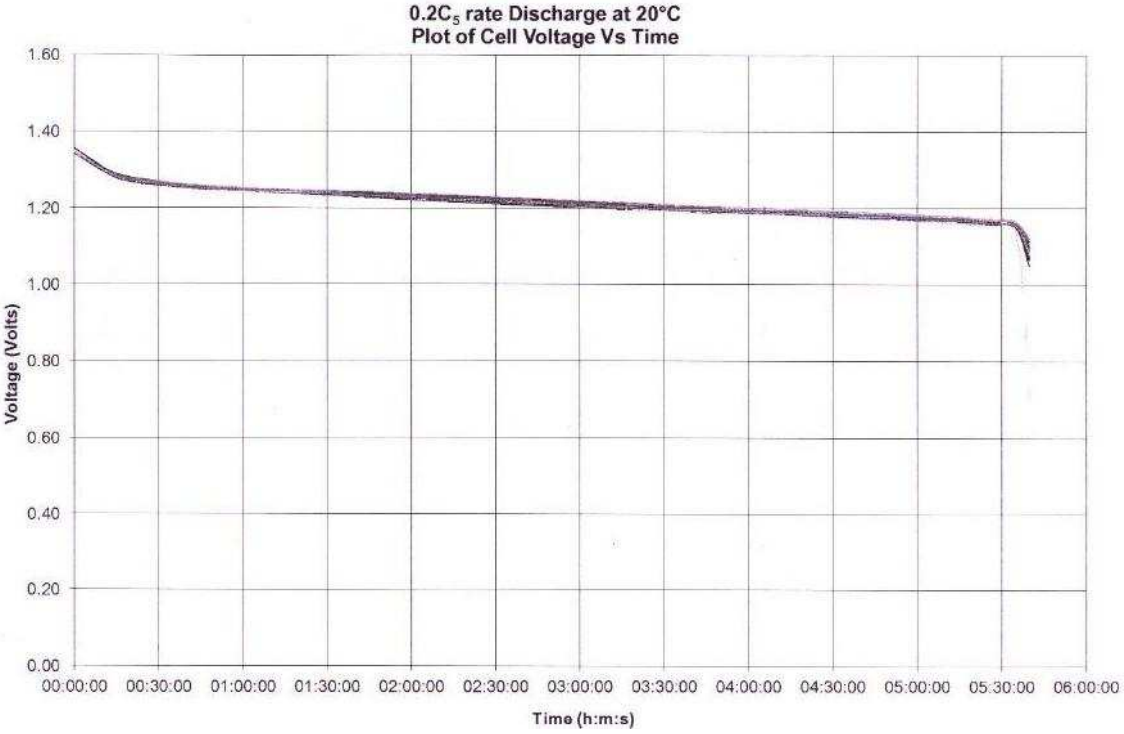


Figure 14 – HBL battery discharge test (Voltage/Time). Model KRH 130 Ah.

Since V/SOC plots are drawn at constant discharge current rate, figure 13 and figure 14 are equivalent.

Under steady-state conditions output voltage mainly depends on current, state of charge, aging and temperature.

### Current dependence

Voltage vary with current mainly because of the fact that a conductive medium, when crossed by a current flow, show a voltage drop proportional to the capability of the medium to oppose to this flow: the resistivity.

In cables, electrical resistivity is a parameter that can be considered constant with the current rate, at least inside the normal functioning range of a cable (with high overheating due to over currents or environmental conditions, resistivity can be modeled as linearly dependent from the temperature).

In electrochemical batteries, resistivity property can be seen as the sum of two different contributions: electrical resistivity and electro-chemical resistivity. The integral of this property on the cell geometry lead to the equivalent parameter resistance. The sum of electrical and electro-chemical resistances makes the so-called cell's internal resistance.

While the electrical resistance can be considered as constant, electro-chemical resistance doesn't refer to a simple electronic flow but to a more complex phenomenon, which makes this kind of resistance non-linear (and time-variant).

First of all, while in a solid metal the current conduction is carried only by electrons, in a electrolytic solution, charge flow (the current) is maintained by a ionic flow, energized by an electric field. Positive and negative ions have, due to their bigger size, lower mobility in respect of electrons so as, for the same charge concentration, they present higher resistivity.

The second effect is more linked to the structure of an electrochemical cell. It is stated that cell's functioning is mainly based on the interaction between two different systems: the electrical one and the chemical one. This interaction occurs in a little layer between them, made by the electrode surface and a little electrolyte film. In fact, here it is where the redox semi-reactions act, linking through a chemical reaction an electronic flow and an ionic flow.

It's intuitive to understand that such kind of interface can strongly limit the system charge transport capability.

The redox semi reactions work with an exchange of material between anode or cathode and their relative electrolytic solutions. Each elementary reaction needs place to permit to molecules to reach the right orientation, needed to break the old chemical links and to form the new ones. In addition, to promote this situation, the reagent concentration in the layer closer to the electrode surface has to be high, or, in another way, the formed product shall be able to quickly diffuse in the rest of the electrolytic solution.

For these reasons the electrochemical resistance cannot be considered constant with the current density $J\left[A/mm^2\right]$, but it presents two saturation zones (anodic and cathodic) where it is higher.

Giving a look on a manufacturer's constant current discharge curve is simple to identify a third order system, with two non-linear parts and a quite constant slope in the central part. This curve directly comes from V/J curve, since $C\% \propto \int i \cdot dt \propto i(t) \propto J(t)$, the current density $(A/mm^2)$.

State of charge dependence

Decreasing in voltage due to decreasing in state of charge presents the same characteristic shown for the current dependence.

Decreasing in state of charge (SOC), first means a decrease in the reagents concentration at disposal of the reaction. The redox, working in debt of reagent, is not able to provide for the theoretical potential anymore. Nernst equation shows in detail this kind of behavior; the external equivalent effect is a decreasing of the open circuit voltage of the Thevenin equivalent generator, or, in the same way, the increase of the output equivalent resistance.

Temperature

Temperature strongly affects the mobility of ions in the electrolyte solution. Higher temperature means lower viscosity of the solution and higher thermal excitation of molecules so higher speed in diffusion and transport processes.

The following plot shows, starting from the same point, the variation in the energy capability of a Ni-Cd battery (given by the integration of the curves):

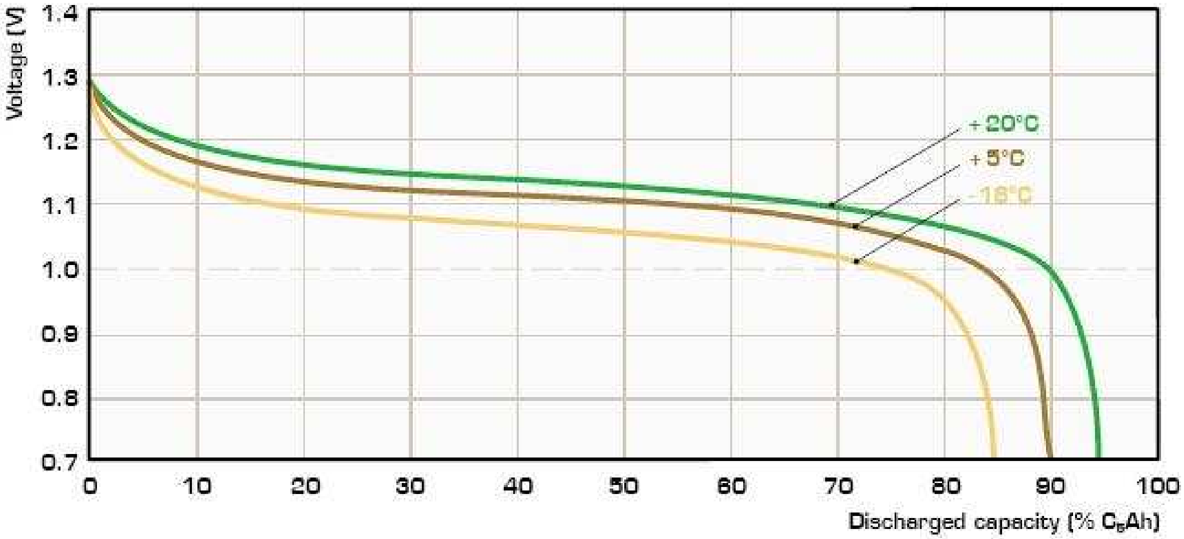


Figure 15 – Temperature dependance in discharge characteristic.

Another effect of the temperature variation is the presence in all the traction batteries of a charging control system that automatically reduces the charging voltage when the temperature in the battery case is too high (over-temperatures due to over-currents can lead to cell damages). Therefore, for high environment temperature the full charging voltage will be lower.

## Aging

The age of the battery influence its storage capability and the internal resistance.

Assuming that the battery is discharged from an equal charge state to the same end of discharge voltage, the usable capacity declines with cycle number.

With aging, the available reagents' quantity decrease, mainly due to the fact that:

- Electrolyte degrades, e.g. forming inert composites.
- Electrolyte could take part in the reaction, so its tank has to be refilled periodically.

While about the internal resistance:

- Electrodes' surfaces decrease due oxidation/reduction cycles.

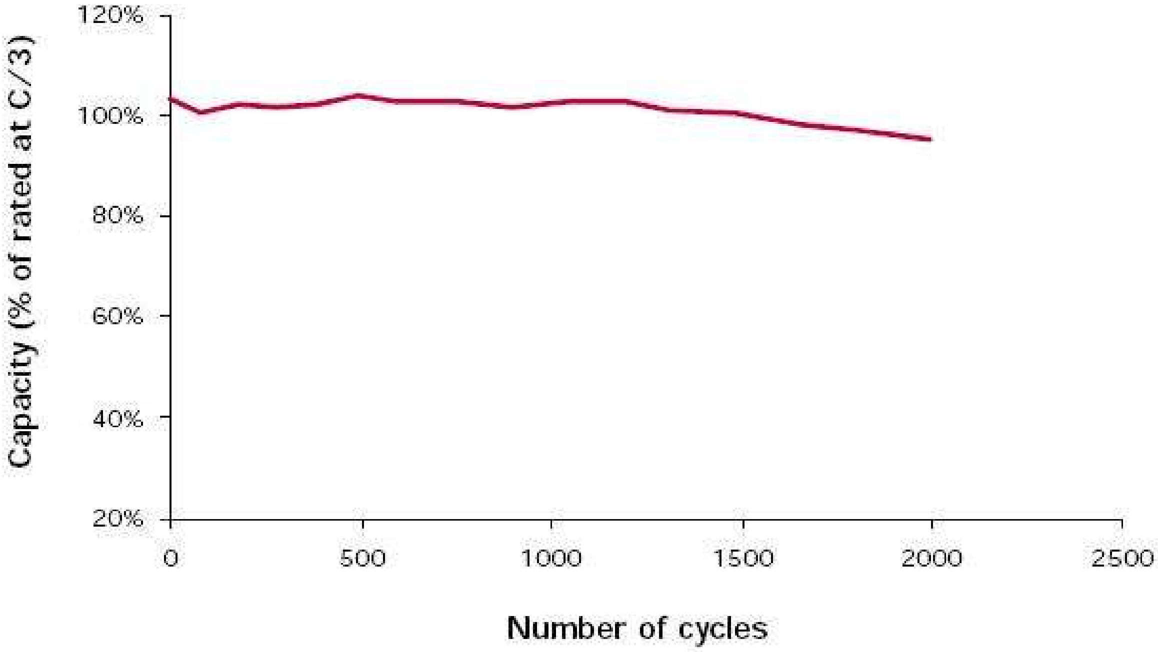


Figure 16 - Aging dependance of storable capacity.

Under dynamic conditions other effects have to be taken into account: the main one considers that saturation occurs when a current step is applied. In this event, battery shows an additional resistance due to the fact that is not possible to reach the new thermodynamic equilibrium point in a null time. From the electrical point of view it's the equivalent presence of a big capacitor, able, thanks to its stored energy, to give continuity to its state variable, the voltage applied to its terminals.

However, due to the short duration of these transients, the light influence that they've got on the rest of the network (especially on the farther loads) and on the battery state of charge, we can consider them as a negligible phenomena, representing so the battery dynamic as a collection of steady-state functioning points. Moreover, we can state that not considering this phenomena constitutes a safety gap, since it should give an higher battery voltage like, for instance, during an emergency braking. It can be so considered a licit approximation.

# BATTERY DISCHARGE PROFILE CALCULATION

Battery Discharge Profile Calculations are used to size the auxiliary batteries on board.

It is a dynamical study. Battery Discharge Profile Calculations consists in to analyze the more critical battery's uses in order to choose, once the LV voltage level is defined, the storable energy, in Ampere-hour [Ah].

International norms (EN 61434, EN 60623) and Alstom specifications define the most critical battery uses. ALSTOM Transport currently use the following test scenarios:

- 30 minutes CVS breakdown.
- 1 hour HV loss.
- 48 hours vehicle parked in depot with red lights switched on.

Battery's behavior analyses, regarding test scenarios, have to ensure respect of equipments' supplying voltage bounds (to ensure no equipments' failures) and the cell's voltage limit (to prevent damage from deep discharge).

The battery sizing is normally dealt from the battery manufacturers. It consists in making several simulation with different growing battery sizes, until all the technical requirements are completely met. Battery suppliers need from the client a timing list of the most severe power consumptions (the scenario on which the sizing will be done), environmental conditions and other specifications. Starting from these data the manufacturer apply its discharge algorithm, finally providing for a discharge chart, like this one:

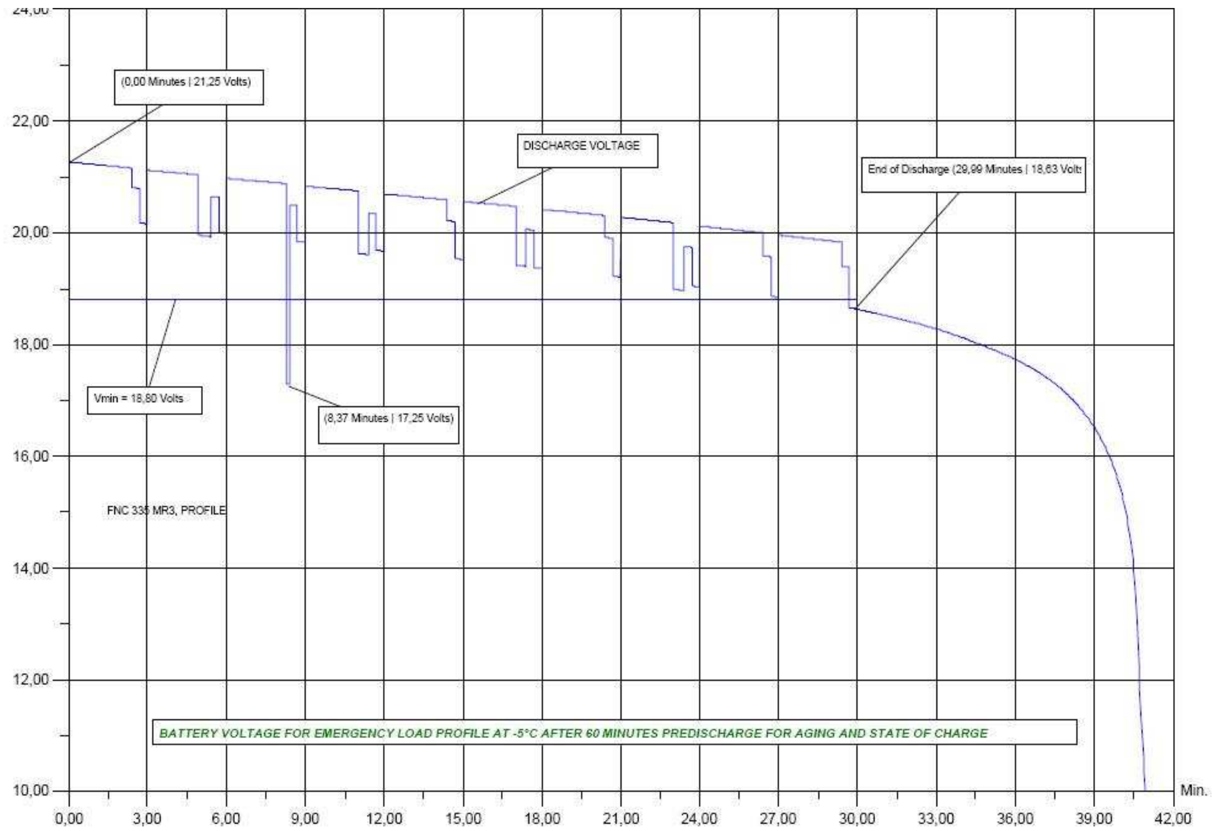


Figure 17 – Oppecke's 30 minutes battery discharge profile calculation for Citadis Rouen.

However, this process requires sometimes a lot of time, from the notice for tender to the final battery size. In addition, the separation between the distribution system architecture (from Alstom) and the supplying device (from battery manufacturer) doesn't permit to have a global and complete overview on the whole system's functioning. Some behaviors like the voltage dependence of the loads aren't taken into account at all, while is not possible to estimate the real voltage drops on the equipments, so as the manufacturers use to apply an average voltage drop in the evaluation of the minimum battery voltage. For instance, for 24 V systems:

16,8 V (minimum eqp voltage) + 2V (average voltage drop) = 18,8 V (minimum batt. voltage)

The discharge simulations provided with the algorithm that will be introduced in this chapter don't want to substitute to the manufacturers' simulations. The manufacturer experience and know-how is a fundamental prerequisite for a correct battery sizing.

This method wants to be an useful tool capable to provide for a first battery size on which better define the price of one of the most expensive parts of the vehicle’s electrical system (with the converter). In addition, it can be used to:

- make comparisons with the manufacturer results,
- study the behavior of new equipments or sub-systems supplied by the battery,
- better evaluate the real voltage drops on the equipments and their real consumptions,
- constitute a base for a future improvement of the battery calculation inside Alstom.

This model, that will necessarily be less complete and complex in respect of the manufacturers' ones, will be develop taking into account all the requirements of implementation on Visual Basic platform and user friendship, in respect of all the target users.

# BATTERY MODELING

The purpose of this modeling study is to develop the more suitable battery model able to be implemented on excel base and to be used on different batteries from different persons. This means that the model shall be simple and at the same time easy to be characterized and parameterized from anyone, at least only with the aid of a basic technical datasheet.

## ACTUAL SOLUTIONS

We have seen, in steady-state condition, as Nernst equation is able to evaluate the external electrical potential, function of the internal chemical concentration of the reagents.

Nevertheless, it is only capable to estimate the theoretical potential during electrochemical equilibrium condition, so as it’s not useful in dynamic evaluations. Moreover, internal variables as concentration are not suitable in modeling the external behavior of a circuit-integrated battery.

For these reasons, all over the world, a huge variety of battery models have been developed. The main features that they present are the ability in calculating V-I performances and in predicting runtime, so they are oriented to battery sizing. Since they are based on the same electrochemical behavior, almost all the models can be adapted to all the technologies.

These models can be divided into several groups.

Electrochemical models

These models are mainly used to optimize physical aspects in batteries design. They take into account each aspect of the thermodynamic behavior inside cells and, starting from internal chemical variables (achieved from the chemical reaction), lead to the external variables of voltage and current.

In Electrochemical models (starting from Nernst equation), the internal impedance is more often composed by three terms, achieved from the infinite Taylor series representation (here shown up to the first order terms):

$$\Delta i = \left( \frac{\delta i}{\delta E} \right) \cdot \Delta E + \left( \frac{\delta i}{\delta C_O} \right) \cdot \Delta C_O + \left( \frac{\delta i}{\delta C_R} \right) \cdot \Delta C_R + \ldots$$

The first term, called charge-transfer resistance ($\delta \cdot E / \delta \cdot i$), takes into account the electrical behavior of the cell response, while the other two terms, called impedance of mass transfer or semi-finite Warburg impedance ($\delta \cdot E / \delta \cdot C_{O,R}$) take into account the effect of the concentration rates, in anodic and cathodic zones [2].

Nevertheless, these kinds of equation are not so useful in Discharge Profile Evaluations, because they are excessively detailed and they use partial differential and non-linear equations like Maxwell and Nernst ones (i.e. differential equation are needed to represent in diffusion and convection in electrolyte), which implementation, whether is possible, takes a lot of time.

In addition, these kinds of models, resulting from a linearization close to an equilibrium point (Taylor series), are able to characterize battery behavior only for little perturbations.

### Mathematical models

These kinds of models are generally based on empirical equation or stochastic approaches, mainly to predict runtime, capacity or efficiency. They are often too abstract and not enough accurate to be applied to a critical system like traction, since they provide results with 5%-20% error. For example, the maximum error of Peukert's law can rise up to 100% for time-variant load [1].

### Electrical models

There are a wide variety of circuital models, which are characterized by measurements. Most of them represent battery dynamics with good accuracy but, at the same time, they are difficult in characterizing.

### Thevenin-based models

Thevenin-based models refer to Thevenin equivalent circuit, composed by a constant voltage generator with series impedance. This impedance can widely vary in composition, from a simple resistance to an n-order sub-circuit (where n is the number of conservative elements, so state variables).

The simplest one is just a Thevenin circuit; a constant voltage generator is in series with a constant resistance, which provides for different output voltages depending on load current. This model is really too much simple and it cannot be used for static calculations neither for dynamic ones.

## Impedance-based models

Impedance-based models use the method of electrochemical spectroscopy to obtain an equivalent series impedance capable to replicate the frequency response of the cell. [2]

Measurement modelling explains the experimental impedances in terms of mathematical functions, in order to obtain good fit between the calculated and experimental impedances. In the latter case, the parameters obtained do not necessarily have a clear physical-chemical significance.

The ideal process begins from the global impedance that, once measured in the frequency domain and identified number and nature of the parameters, is reproduced combining the elementary elements R, L and C [2]. This representative process also allow to achieve the same frequency response with different electrical circuits.

These kinds of models are useful to reproduce dynamical battery behaviour for a wide range of frequencies: since each perturbation is decomposable in Fourier series, they ensure the ability to manage runtime and transients.

However, this kind of model is difficult to characterize due to complexity in spectral analysis. Is so possible to find impedance equivalent circuit, showing only the most important dynamical characteristics.

## First order Impedance model

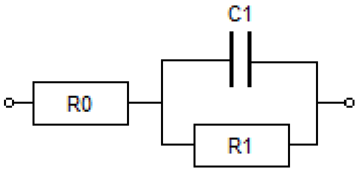


Figure 18 – First order impedance model.

This kind of model is composed of one series resistor and a capacitor parallel connected with another resistor. While the series resistor represents the metallic part of the internal resistance, the first order parallel dipole simulate the short-term dynamical behavior that opposites to instantaneous changes in voltage when a step load is applied.

This model need a series voltage generator (usually controlled by current or SOC) to be completed. Sometimes a complex generator circuit is connected, to take into account several parameters that influence the state of charge:

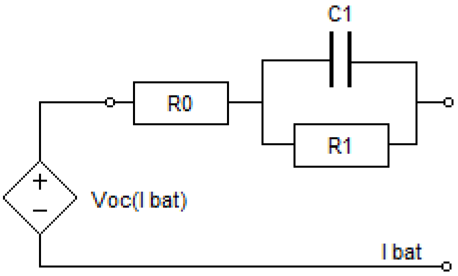


Figure 19 – First order impedance model with current-dependent voltage generator.

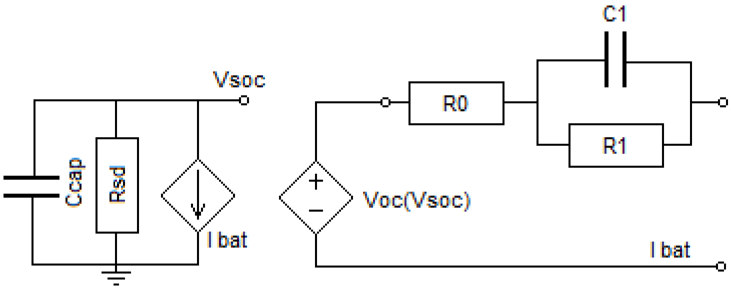


Figure 20– First order impedance model with voltage-dependent voltage generator. The supplying circuit on the left takes into account the self-discharge rate (Rsd), the battery stored capacity (Ccap) and the battery current (I bat), while the circuit on the right represents the voltage/current characteristic.

## Second order Impedance model

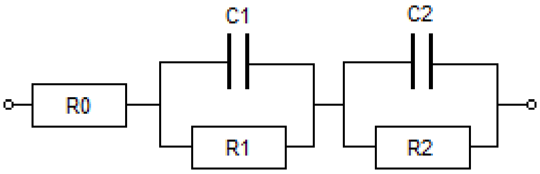


Figure 21 - Second order impedance model.

As it is for the first order model, there is a series resistance representing the metallic electric resistance of the cell. Two RC dipoles are then series connected to represent the short-term and long-term responses to the load step. On the basis of several experimental curves, this kind of model is considered the best compromise between accuracy and complexity, as shown in [1], in representing the most common charge and discharge dynamics.

As already done for the first order model, this model have to be completed with a voltage supply.

Nth order model – Voigt model

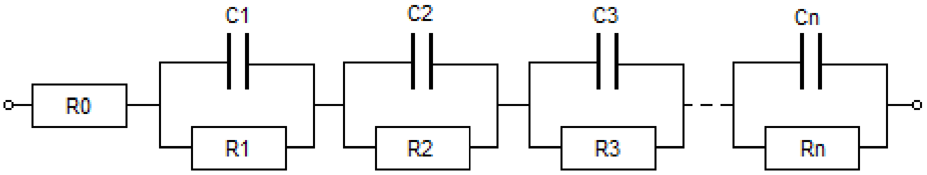


Figure 22 – Nth order impedance model.

Through an extension process, is possible to add to the second order model a desired number of RC elements, accorded on the frequency responses that we are interested in representing. Of course, this process increase the complexity of the model.

As already done for the other models, this model have to be completed with a voltage supply.

## PROPOSED BATTERY MODEL

Through the use of circuital equivalent or the implementation of differential equation is possible to represent electrochemical battery dynamics in a lot of ways. Nevertheless, they present always a constraint: the use of non-algebraic equations. Since the need to implement the model on the network algebraic equations' system, we have to develop a battery model able to represent battery dynamics through algebraic relationships.

The battery model has to be chosen basing on three different requirements:

1) The ability in representing battery discharge dynamics.
2) The possibility of its implementation on Visual Basic platform.
3) The ease in parameterize it.

The model adopted is achieved adapting a battery analytical model developed for Hybrid Electric Vehicles [4]. It has been developed to be used in dynamic simulation software, using only the state of charge (SOC) as state variable.

The battery is modelled using a simple controlled voltage source in series with a constant resistance:

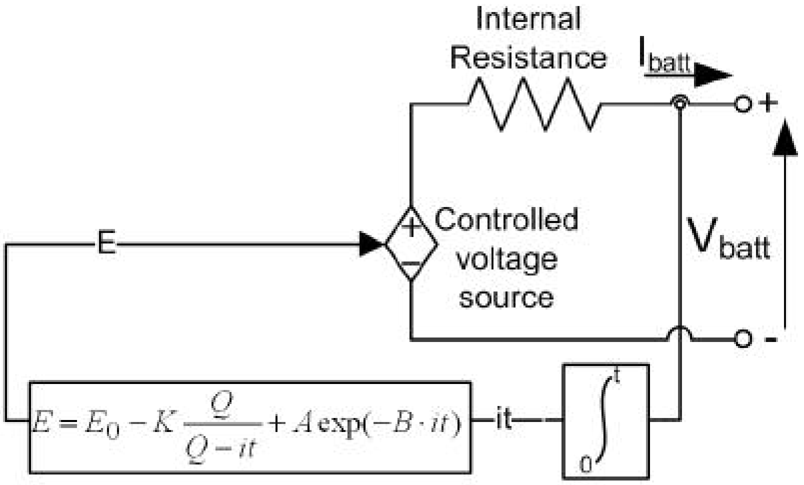


Figure 23 – Proposed battery model scheme.

The generator voltage is controlled through a non-linear equation based on the actual state of charge, retrieved from the time integration of the battery current.

The control is analytically described by the following equation:

$$E = E_0 + A \cdot e^{-B \cdot \int i \cdot dt} - K \cdot \frac{Q}{Q - \int i \cdot dt}$$

$$V_{batt} = E - R \cdot i$$

Where:

- $E$ is the open circuit output voltage [V]
- $E_0$ is the battery full charged voltage [V]
- $K$ is the polarization voltage [V]
- $Q$ is the battery capacity [Ah]
- $\int i \cdot dt$ is the used charge [Ah]
- $A$ is the exponential zone amplitude [V]
- $B$ is the inverse of the exponential zone time constant [1/Ah]
- $V_{batt}$ is the battery output voltage [V]
- $R$ is the battery internal resistance [Ω]
- $i$ is the battery output current [A]

The model is able to fit the manufacturer discharge characteristics , through the superimposition of three main terms:

- $E_0$: this term is the constant full charged voltage.
- $A \cdot e^{-B \cdot \int i \cdot dt}$: this term reproduce the exponential discharge zone, which behavior represents the first saturation zone.
- $K \cdot \frac{Q}{Q - \int i \cdot dt}$: the last term show the voltage drop due to the discharge of the battery.

## PARAMETERIZATION

The main problem in the use of battery models is the ease in find data or do measurements useful to characterize the analytical model. That is due to the fact that battery manufacturers take care to their know-how, giving to their clients a short list of data, most of which not useful, from our point of view.

Nevertheless in every product brochure a discharge plot is always present. It is usually a plot of the voltage/SOC characteristic or voltage/time characteristic. The main feature of this model is the ability in deducing all the battery parameters from them.

### Internal resistance

Internal resistance is very important in order to adequately represent the voltage drop caused by a current variation in the battery. The internal resistance is generally specified in technical datasheets. Nevertheless, due to the very low values that especially characterize Ni-Cd cells and the wide variance in the measurements, it could be useful to achieve it from the efficiency formula:

$$\eta = 1 - \frac{R \cdot I_n^2}{V_n \cdot I_n} \qquad I_n = \frac{Q_n \cdot 0{,}2}{1h}$$

$$\eta = 1 - \frac{0{,}2 \cdot R \cdot Q_n}{V_n}$$

More than 30 empirical test based on several nominal capacities have established an average efficiency able to fit the manufacturers' curves.

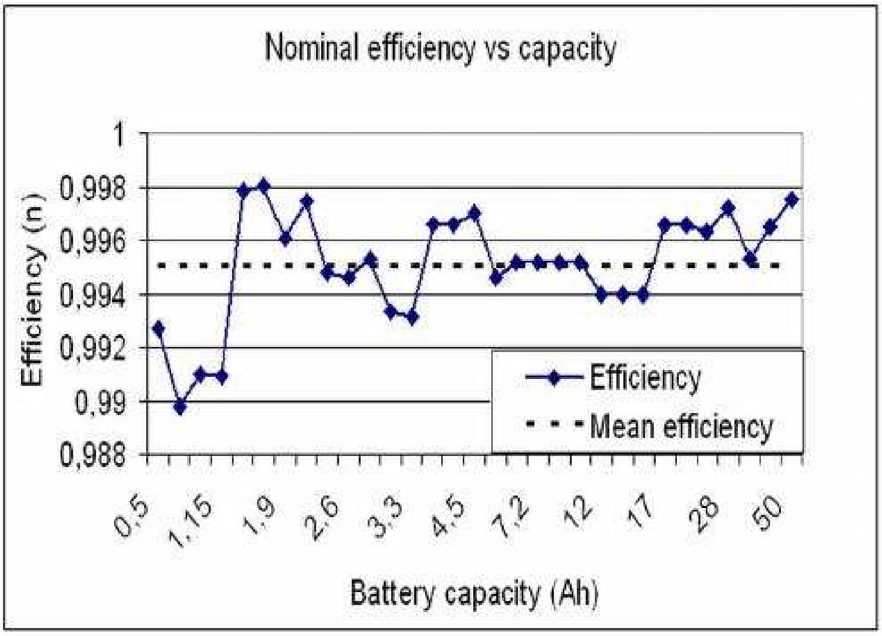


Figure 24 – Efficiency at nominal current / Battery capacity (Ah).

The here above figure shows that the 99,5% of average efficiency covers a wide range of batteries. The internal resistance can be so calculated as:

$$R = V_n \cdot \frac{1-\eta}{0{,}2 \cdot Q_n}$$

This method can be used whereas no more information about the internal resistance is available.

In order to characterize the rest of the model, we need only three points, taken from the 0,2 C discharge curve:

- The fully charged voltage
- The end of the exponential zone, voltage and charge
- The end of the nominal zone, voltage and charge

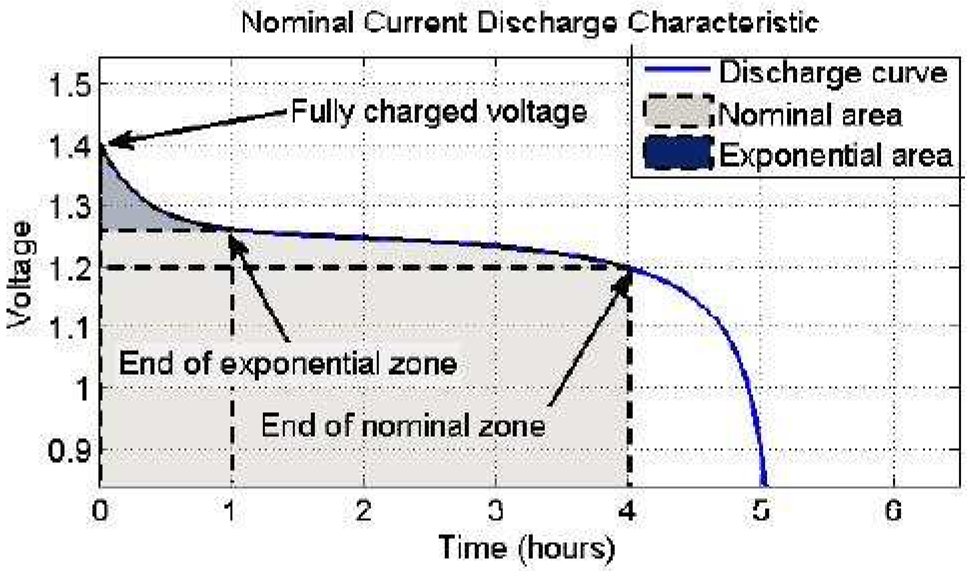


Figure 25 - Nominal battery discharge characteristic (at 0,2C), used to find the three functioning points.

Exponential part

$A \cdot e^{-B \cdot \int i \cdot dt}$ is calculated with the first two points, in this way:

A is the voltage drop during the exponential zone (V).

$$A = E_{Full} - E_{Exp}$$

B is the time constant of the exponential zone (Ah). Considering the exponential transient exhausted after 3B:

$$B = \frac{3}{Q_{Exp}}$$

Other parameters

The other parameters are calculated using the discharge characteristic equation,

$$E = E_0 + A \cdot e^{-B \cdot \int i \cdot dt} - K \cdot \frac{Q}{Q - \int i \cdot dt}.$$

$$K = \frac{(E_{Full} - E_{Nom} + A(\exp(-B \cdot Q_{Nom}) - 1)) \cdot (Q - Q_{Nom})}{Q_{Nom}}$$

Then, the constant voltage $E_0$ is deduced from the fully charged voltage:

$$E_0 = E_{Full} + K + Ri - A$$

This kind f approach is very general and can be applied to every kind of electrochemical cells. Of course, the parameters are approximate and depend on the precision of the points obtained from the discharge curve.

## CHAPTER REFERENCES

[1] Min Chen, Gabriel A. Rincon-Mora, “Accurate Electrical Battery Model Capable of Predicting Runtime and *I-V* Performance”, IEEE Transaction on Energy Conversion, vol.21, no.2, June 2006, pp. 504-511

[2] A. Lasia, Electrochemical Impedance Spectroscopy and Its Applications, Modern Aspects of Electrochemistry, B. E. Conway, J. Bockris, and R.E. White, Edts., Kluwer Academic/Plenum Publishers, New York, 1999, Vol. 32, p. 143-248.

[3] Mathworks, “Implement Generic Battery Model”, User Guide to Simulink Toolbox, http://www.mathworks.com/help/toolbox/physmod/powersys/ref/battery.html

[4] Olivier Tremblay, Louis-A. Dessaint, Abdel-Illah Dekkiche, “A Generic Battery Model for the Dynamic Simulation of Hybrid Electric Vehicles”, IEEE Transaction.

# CHAPTER 3 - GAUSS ITERATIVE METHOD

Gauss iterative method is an iterative method that enables to solve non-linear equation systems.

Given a non-linear equation, for simplicity in only one variable x:

$$f(x) = 0$$

To search its solutions with the Gauss iterative method we have to write the equation in the Canonical Form:

$$x = F(x)$$

After that, we have to choose a starting point $x^{(0)}$ to calculate, at each iteration $k$, $x^{(k)}$ as a function of the previous iteration $x^{(k-1)}$, so as:

$$x^{(k)} = F(x^{(k-1)})$$

We have now to repeat the iterations until the difference between the previous and the actual falls under a defined threshold; the given value will be our result.

The writing of the equation in the Canonical Form is always possible, often in more than one way, and the starting iterative point is a completely free choice. Nevertheless, we will see that the convergence of the method, the speed of convergence and the physical meaning of the results obtained depend strongly on these arbitrary choices.

For example we will take a simple expression:

$$f(x) = x^2 - 5x + 4 = 0$$

which solutions are 1 and 4, of course.

$$x = F(x) = \frac{1}{5}x^2 + \frac{4}{5}$$

With a simple Excel implementation we can see the differences, varying the starting point:

| **Starting point** | **3** | **2** | **5** |
|---|---|---|---|
| **Iter 1** | 2,6 | 1,6 | 5,8 |
| **Iter 2** | 2,152 | 1,312 | 7,528 |
| **Iter 3** | 1,726221 | 1,144269 | 12,13416 |
| **Iter 4** | 1,395968 | 1,06187 | 30,24755 |
| **Iter 5** | 1,189745 | 1,025514 | 183,7829 |
| **Iter 6** | 1,083099 | 1,010336 | 6756,03 |
| **Iter 7** | 1,034621 | 1,004156 | 9128788 |
| **Iter 8** | 1,014088 | 1,001666 | 1,67E+13 |
| **Iter 9** | 1,005675 | 1,000667 | 5,56E+25 |
| **Iter 10** | 1,002276 | 1,000267 | 6,17E+50 |
| **Iter 11** | 1,000912 | 1,000107 | 7,6E+100 |
| **Iter 12** | 1,000365 | 1,000043 | 1,2E+201 |
| **Iter 13** | 1,000146 | 1,000017 | #NUM! |
| **Iter 14** | 1,000058 | 1,000007 | #NUM! |
| **Iter 15** | 1,000023 | 1,000003 | #NUM! |
| **Iter 16** | 1,000009 | 1,000001 | #NUM! |
| **Iter 17** | 1,000004 | 1 | #NUM! |
| **Iter 18** | 1,000001 | 1 | #NUM! |
| **Iter 19** | 1,000001 | 1 | #NUM! |
| **Iter 20** | 1 | 1 | #NUM! |

Table 8 - Comparison in convergence starting from different starting points, for a 1st order model. Example 1.

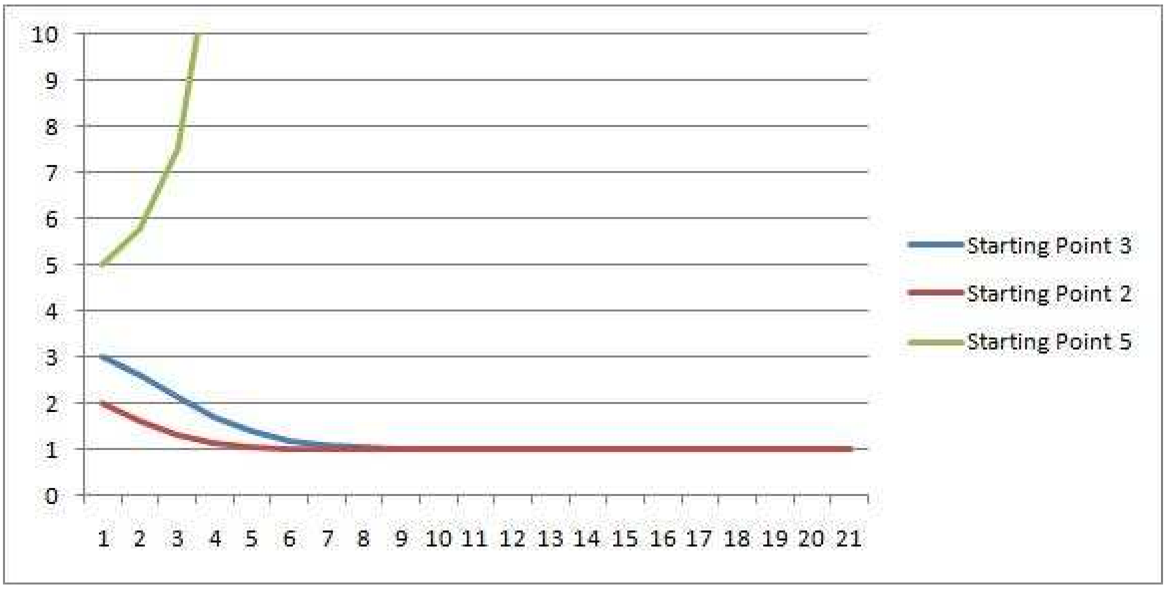


Figure 26 - Comparison in convergence starting from different starting points, for a 1st order model. Example 1.

We can see that starting from a closer point, the solution is reached quicker, while if we choose a too far point, the method diverges. We can see also that the chosen Canonical Form lead only to the solution x=1. For example, using a the Canonical form:

$$x = F(x) = \frac{5x-4}{x}$$

The Excel simulation, with the same starting values, gives the following results:

| **Starting point** | **3** | **2** | **5** |
|---|---|---|---|
| **Iter 1** | 3,666667 | 3 | 4,2 |
| **Iter 2** | 3,909091 | 3,666667 | 4,047619 |
| **Iter 3** | 3,976744 | 3,909091 | 4,011765 |
| **Iter 4** | 3,994152 | 3,976744 | 4,002933 |
| **Iter 5** | 3,998536 | 3,994152 | 4,000733 |
| **Iter 6** | 3,999634 | 3,998536 | 4,000183 |
| **Iter 7** | 3,999908 | 3,999634 | 4,000046 |
| **Iter 8** | 3,999977 | 3,999908 | 4,000011 |
| **Iter 9** | 3,999994 | 3,999977 | 4,000003 |
| **Iter 10** | 3,999999 | 3,999994 | 4,000001 |
| **Iter 11** | 4 | 3,999999 | 4 |
| **Iter 12** | 4 | 4 | 4 |
| **Iter 13** | 4 | 4 | 4 |
| **Iter 14** | 4 | 4 | 4 |
| **Iter 15** | 4 | 4 | 4 |
| **Iter 16** | 4 | 4 | 4 |
| **Iter 17** | 4 | 4 | 4 |
| **Iter 18** | 4 | 4 | 4 |
| **Iter 19** | 4 | 4 | 4 |
| **Iter 20** | 4 | 4 | 4 |

Table 9 - Comparison in convergence starting from different starting points, for a 1st order model. Example 2.

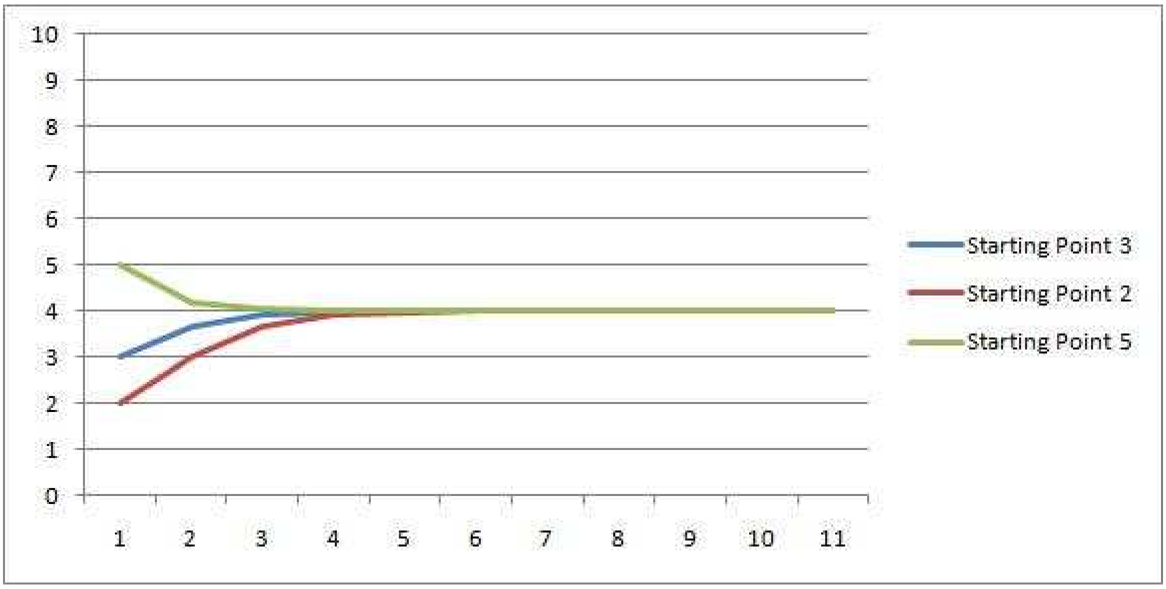


Figure 27 – Comparison in convergence starting from different starting points, for a 1st order model. Example 2.

As we can see, the solutions lead always to 4. Achieving convergence is faster than the case before.

The Gauss Iterative Method is applicable also if x is a vector formed from the variables x1, x2, x3,…, xn. In this situation we have to solve a system of *n* non-linear equations.

A second order example is showed above:

$$f_1(x_1, x_2) = 2x_1 + x_1 x_2 - 1 = 0$$

$$f_2(x_1, x_2) = 2x_2 - x_1 x_2 + 1 = 0$$

Which solution is $x_1$=1, $x_2$=-1.

$$x_1 = F_1(x_1, x_2) = 0{,}5 \cdot (1 - x_1 x_2)$$

$$x_2 = F_2(x_1, x_2) = 0{,}5 \cdot (x_1 x_2 - 1)$$

Again, we will proceed choosing a starting point ${x_1}^{(0)}, {x_2}^{(0)}$ and performing iterations:

$$ {x_1}^{(k)} = F_1({x_1}^{(k-1)}, {x_2}^{(k-1)}) $$

$$ {x_2}^{(k)} = F_2({x_1}^{(k-1)}, {x_2}^{(k-1)}) $$

| | X1 | X2 | | X1 | X2 | | X1 | X2 | | X1 | X2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Start** | 0 | 0 | | | | **Start** | 0,8 | -0,8 | | | |
| **Iter 1** | 0,5 | -0,5 | **Iter 21** | 0,923096 | -0,9231 | **Iter 1** | 0,82 | -0,82 | **Iter 21** | 0,937903 | -0,9379 |
| **Iter 2** | 0,625 | -0,625 | **Iter 22** | 0,926054 | -0,92605 | **Iter 2** | 0,8362 | -0,8362 | **Iter 22** | 0,939831 | -0,93983 |
| **Iter 3** | 0,695313 | -0,69531 | **Iter 23** | 0,928788 | -0,92879 | **Iter 3** | 0,849615 | -0,84962 | **Iter 23** | 0,941641 | -0,94164 |
| **Iter 4** | 0,74173 | -0,74173 | **Iter 24** | 0,931323 | -0,93132 | **Iter 4** | 0,860923 | -0,86092 | **Iter 24** | 0,943344 | -0,94334 |
| **Iter 5** | 0,775082 | -0,77508 | **Iter 25** | 0,933681 | -0,93368 | **Iter 5** | 0,870594 | -0,87059 | **Iter 25** | 0,944949 | -0,94495 |
| **Iter 6** | 0,800376 | -0,80038 | **Iter 26** | 0,935881 | -0,93588 | **Iter 6** | 0,878967 | -0,87897 | **Iter 26** | 0,946464 | -0,94646 |
| **Iter 7** | 0,820301 | -0,8203 | **Iter 27** | 0,937936 | -0,93794 | **Iter 7** | 0,886292 | -0,88629 | **Iter 27** | 0,947897 | -0,9479 |
| **Iter 8** | 0,836447 | -0,83645 | **Iter 28** | 0,939862 | -0,93986 | **Iter 8** | 0,892756 | -0,89276 | **Iter 28** | 0,949254 | -0,94925 |
| **Iter 9** | 0,849821 | -0,84982 | **Iter 29** | 0,94167 | -0,94167 | **Iter 9** | 0,898507 | -0,89851 | **Iter 29** | 0,950542 | -0,95054 |
| **Iter 10** | 0,861098 | -0,8611 | **Iter 30** | 0,943372 | -0,94337 | **Iter 10** | 0,903657 | -0,90366 | **Iter 30** | 0,951765 | -0,95177 |
| **Iter 11** | 0,870745 | -0,87075 | **Iter 31** | 0,944975 | -0,94497 | **Iter 11** | 0,908298 | -0,9083 | **Iter 31** | 0,952928 | -0,95293 |
| **Iter 12** | 0,879098 | -0,8791 | **Iter 32** | 0,946489 | -0,94649 | **Iter 12** | 0,912503 | -0,9125 | **Iter 32** | 0,954036 | -0,95404 |
| **Iter 13** | 0,886407 | -0,88641 | **Iter 33** | 0,947921 | -0,94792 | **Iter 13** | 0,916331 | -0,91633 | **Iter 33** | 0,955093 | -0,95509 |
| **Iter 14** | 0,892859 | -0,89286 | **Iter 34** | 0,949277 | -0,94928 | **Iter 14** | 0,919831 | -0,91983 | **Iter 34** | 0,956101 | -0,9561 |
| **Iter 15** | 0,898598 | -0,8986 | **Iter 35** | 0,950563 | -0,95056 | **Iter 15** | 0,923045 | -0,92304 | **Iter 35** | 0,957064 | -0,95706 |
| **Iter 16** | 0,90374 | -0,90374 | **Iter 36** | 0,951785 | -0,95179 | **Iter 16** | 0,926006 | -0,92601 | **Iter 36** | 0,957986 | -0,95799 |
| **Iter 17** | 0,908373 | -0,90837 | **Iter 37** | 0,952947 | -0,95295 | **Iter 17** | 0,928743 | -0,92874 | **Iter 37** | 0,958869 | -0,95887 |
| **Iter 18** | 0,91257 | -0,91257 | **Iter 38** | 0,954054 | -0,95405 | **Iter 18** | 0,931282 | -0,93128 | **Iter 38** | 0,959715 | -0,95971 |
| **Iter 19** | 0,916392 | -0,91639 | **Iter 39** | 0,95511 | -0,95511 | **Iter 19** | 0,933643 | -0,93364 | **Iter 39** | 0,960526 | -0,96053 |
| **Iter 20** | 0,919887 | -0,91989 | **Iter 40** | 0,956117 | -0,95612 | **Iter 20** | 0,935845 | -0,93584 | **Iter 40** | 0,961305 | -0,96131 |

Table 10 - Comparison in convergence from different starting points for a 2nd order system.

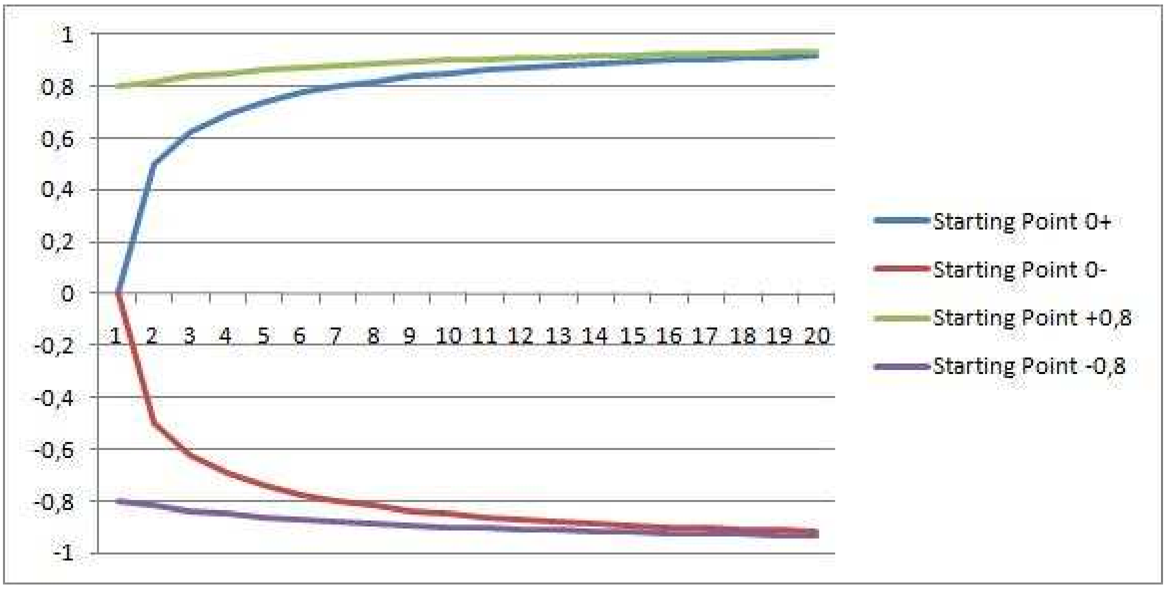


Figure 28 - Comparison in convergence from different starting points for a 2nd order system.

As the system order grow up, the convergence speed falls down, even choosing a closer starting point.

An improvement in speed can be obtained using the actual determinations of the variables, whereas it is possible (depending on calculation order):

$$x_1^{(k)} = F_1(x_1^{(k-1)}, x_2^{(k-1)}, \ldots, x_n^{(k-1)})$$

$$x_2^{(k)} = F_2(x_1^{(k)}, x_2^{(k-1)}, \ldots, x_n^{(k-1)})$$

……………………………………………………

$$x_n^{(k)} = F_n(x_1^{(k)}, x_2^{(k)}, \ldots, x_n^{(k-1)})$$

This solution, proposed by Glimn and Stagg is called Gauss-Seidel iterative method.

On our example:

| | X1 | X2 | | X1 | X2 | | X1 | X2 | | X1 | X2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Start** | 0 | 0 | | | | **Start** | 0,8 | -0,8 | | | |
| **Iter 1** | 0,5 | -0,5 | **Iter 21** | 0,937777 | -0,93908 | **Iter 1** | 0,82 | -0,828 | **Iter 21** | 0,948431 | -0,94932 |
| **Iter 2** | 0,625 | -0,65625 | **Iter 22** | 0,940322 | -0,94152 | **Iter 2** | 0,83948 | -0,84754 | **Iter 22** | 0,950183 | -0,95102 |
| **Iter 3** | 0,705078 | -0,73135 | **Iter 23** | 0,942665 | -0,94377 | **Iter 3** | 0,855748 | -0,86264 | **Iter 23** | 0,95182 | -0,9526 |
| **Iter 4** | 0,757831 | -0,77712 | **Iter 24** | 0,944828 | -0,94585 | **Iter 4** | 0,869102 | -0,87486 | **Iter 24** | 0,95335 | -0,95408 |
| **Iter 5** | 0,794463 | -0,8087 | **Iter 25** | 0,946832 | -0,94778 | **Iter 5** | 0,880173 | -0,88501 | **Iter 25** | 0,954786 | -0,95547 |
| **Iter 6** | 0,82124 | -0,83207 | **Iter 26** | 0,948694 | -0,94958 | **Iter 6** | 0,889483 | -0,8936 | **Iter 26** | 0,956135 | -0,95678 |
| **Iter 7** | 0,841663 | -0,85016 | **Iter 27** | 0,950429 | -0,95125 | **Iter 7** | 0,897422 | -0,90097 | **Iter 27** | 0,957405 | -0,95801 |
| **Iter 8** | 0,857774 | -0,86462 | **Iter 28** | 0,952049 | -0,95282 | **Iter 8** | 0,904275 | -0,90736 | **Iter 28** | 0,958603 | -0,95918 |
| **Iter 9** | 0,870826 | -0,87647 | **Iter 29** | 0,953566 | -0,95429 | **Iter 9** | 0,910252 | -0,91296 | **Iter 29** | 0,959735 | -0,96028 |
| **Iter 10** | 0,881625 | -0,88636 | **Iter 30** | 0,954988 | -0,95567 | **Iter 10** | 0,915514 | -0,91792 | **Iter 30** | 0,960806 | -0,96132 |
| **Iter 11** | 0,890718 | -0,89475 | **Iter 31** | 0,956325 | -0,95696 | **Iter 11** | 0,920182 | -0,92232 | **Iter 31** | 0,961822 | -0,96231 |
| **Iter 12** | 0,898484 | -0,90196 | **Iter 32** | 0,957585 | -0,95819 | **Iter 12** | 0,924354 | -0,92628 | **Iter 32** | 0,962785 | -0,96325 |
| **Iter 13** | 0,905197 | -0,90823 | **Iter 33** | 0,958773 | -0,95934 | **Iter 13** | 0,928104 | -0,92984 | **Iter 33** | 0,963701 | -0,96414 |
| **Iter 14** | 0,911061 | -0,91372 | **Iter 34** | 0,959895 | -0,96043 | **Iter 14** | 0,931494 | -0,93307 | **Iter 34** | 0,964572 | -0,96499 |
| **Iter 15** | 0,91623 | -0,91859 | **Iter 35** | 0,960958 | -0,96147 | **Iter 15** | 0,934575 | -0,93601 | **Iter 35** | 0,965402 | -0,9658 |
| **Iter 16** | 0,92082 | -0,92293 | **Iter 36** | 0,961965 | -0,96245 | **Iter 16** | 0,937387 | -0,9387 | **Iter 36** | 0,966194 | -0,96658 |
| **Iter 17** | 0,924925 | -0,92682 | **Iter 37** | 0,962921 | -0,96338 | **Iter 17** | 0,939964 | -0,94117 | **Iter 37** | 0,96695 | -0,96732 |
| **Iter 18** | 0,92862 | -0,93033 | **Iter 38** | 0,96383 | -0,96427 | **Iter 18** | 0,942334 | -0,94345 | **Iter 38** | 0,967673 | -0,96802 |
| **Iter 19** | 0,931962 | -0,93352 | **Iter 39** | 0,964696 | -0,96511 | **Iter 19** | 0,944523 | -0,94556 | **Iter 39** | 0,968365 | -0,9687 |
| **Iter 20** | 0,935001 | -0,93642 | **Iter 40** | 0,96552 | -0,96592 | **Iter 20** | 0,946549 | -0,94751 | **Iter 40** | 0,969027 | -0,96935 |

Table 11 - Comparison in convergence from different starting points for a 2nd order system, using the Gauss-Seidel method.

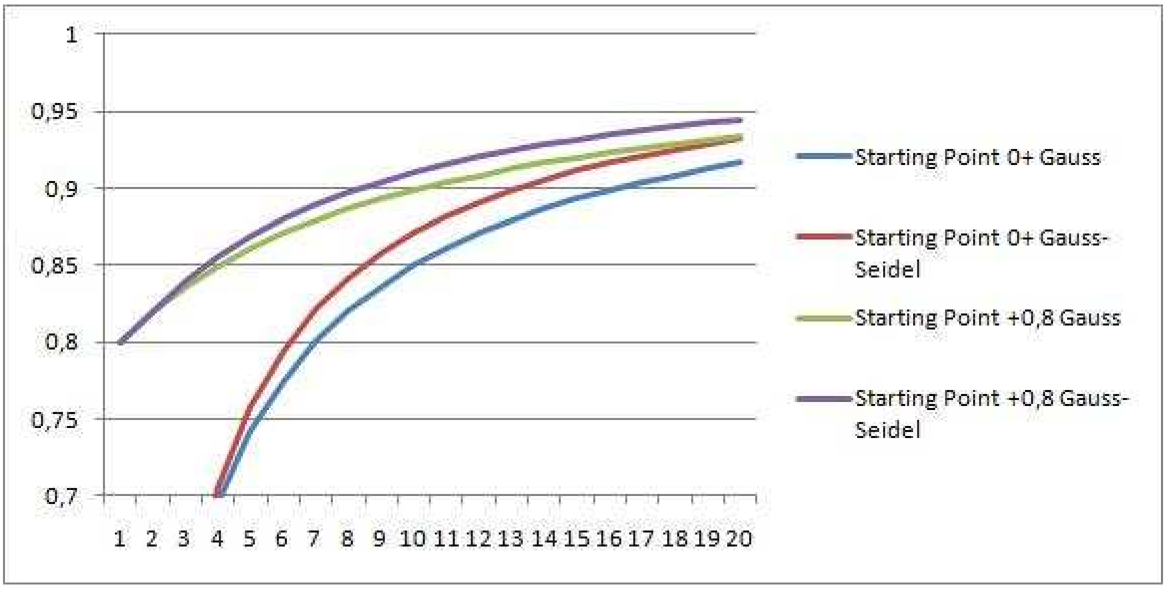


Figure 29 – Comparison between the positive solution of the 2nd order system between Gauss Method and Gauss-Seidel Method.

Another improvement in the convergence speed is using an Acceleration Factor (α>1); at k iteration we will not start from the point $x_i^{(k)}$, but from:

$$x_i^{(k-1)} + \alpha \cdot (x_i^{(k)} - x_i^{(k-1)})$$

The choice of the better α varies with case and for certain values it can lead to divergence in the iterative process, otherwise stable (in the last example for α=3).

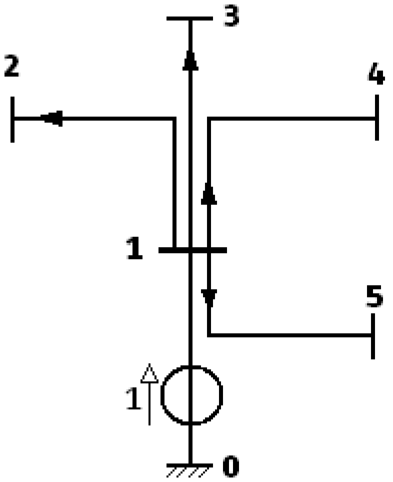
# GAUSS ITERATIVE METHOD ON LOAD FLOW

Gauss Iterative Method can be profitably allows evaluating system loss of energy (for economical or thermal evaluation) and the overload margin inside stability bounds [2]. used in Load Flow calculations. A Load Flow Calculation is a static calculation that permits to calculate all the currents and voltages in the network. This is essential not only in order to design the different system so that these can withstand the stresses they are exposed to during steady state operation without any risk for damages. Furthermore, it

Starting from a generic network, AC supplied, based on *n* nodes and *m* branches, we can write the Kirchhoff Current Law in a suitable form:

:
$$\bar{I}_p = \sum_q \overline{Y}_{pq} \cdot \overline{E}_q = \overline{Y}_{pp} \cdot \overline{E}_p + \sum_{q \neq p} \overline{Y}_{pq} \cdot \overline{E}_q$$

Where *p* and *q* are two generic nodes. Transversal elements' effects ($G_{po}$ and $B_{po}$) are considered negligible. The generic $\overline{Y}_{pq}$ element is equal to the current injection in the node *p* ($\bar{I}_p$) when the voltage applied on the node *q* is equal to 1 and all the other voltage sources are short-circuited to the ground node. This operative definition enables to find graphically all the values of the $\left[\overline{Y}\right]$ matrix. This is a five nodes example network:



$$\overline{Y}_{1p} = \overline{Y}_{p1} = -\frac{1}{z_{1p}}$$

$$\overline{Y}_{11} = \sum_p \frac{1}{z_{1p}} \qquad \left[\overline{Y}\right] = \begin{bmatrix} \overline{Y}_{11} & \overline{Y}_{12} & \overline{Y}_{13} & \overline{Y}_{1n} \\ \overline{Y}_{21} & \overline{Y}_{22} & \overline{Y}_{23} & \overline{Y}_{2n} \\ \overline{Y}_{31} & \overline{Y}_{32} & \overline{Y}_{33} & \overline{Y}_{3n} \\ \overline{Y}_{n1} & \overline{Y}_{n2} & \overline{Y}_{n3} & \overline{Y}_{n1} \end{bmatrix}$$

$$p = 2{,}3{,}4{,}5\,.$$

To show the load flow method we write:

$$\underline{A}_p = P_p - jQ_p = \underline{E}_p \cdot \bar{I}_p = \underline{E}_p \cdot \sum_q (\bar{Y}_{pq} \cdot \bar{E}_q) = \underline{E}_p \cdot \left[ \bar{Y}_{pp} \cdot \bar{E}_p + \sum_{q \neq p} (\bar{Y}_{pq} \cdot \bar{E}_q) \right]$$

$$\bar{E}_p = \frac{1}{\bar{Y}_{pp}} \cdot \left[ \frac{\underline{A}_p}{\underline{E}_p} - \sum_{q \neq p} \bar{Y}_{pq} \cdot \bar{E}_q \right] = \frac{1}{\bar{Y}_{pp}} \cdot \left[ \frac{P_p - jQ_p}{\underline{E}_p} - \sum_{q \neq p} \bar{Y}_{pq} \cdot \bar{E}_q \right]$$

The last equation is the one expressed in the Canonical Form; the load flow calculation provide to determine all the voltages across all nodes to enable the determination of all the currents, voltage drops and power losses across branches. All the network variable are so determined.

From the same power equation we can find:

$$P_p = E_p \cdot \sum_{q \neq p} Y_{pq} \cdot E_q \cdot \cos(\delta_p - \delta_q - \theta_{pq})$$

$$Q_p = E_p \cdot \sum_{q \neq p} Y_{pq} \cdot E_q \cdot \sin(\delta_p - \delta_q - \theta_{pq})$$

With $\bar{Y}_{pq} = Y_{pq} \cdot e^{j\theta_{pq}}$ , $\bar{E}_p = E_p \cdot e^{j\delta_p}$ and $\bar{E}_q = E_q \cdot e^{j\delta_q}$

Usually in a network are defined three types of nodes; Load nodes, Generator nodes, Balance nodes. Often there is only balance node, which is a node capable to be represented like an infinite-power node. Every kind of node has different data and different variables.

| NODE | DATA | VARIABLES |
|---|---|---|
| Load node | P, Q | E, δ |
| Generator node | P, E | Q, δ |
| Balance node | E, δ | P, Q |

**Table12 – Data and variables for different kinds of node, in AC load flow.**

The iterative method is applied as:

LOAD NODE

$$\overline{E}_p{}^{k} = \frac{1}{\overline{\overline{Y}}_{pp}} \cdot \left[ \frac{\underline{A}_p}{\underline{E}_p{}^{(k-1)}} - \sum_{q \neq p} \overline{Y}_{pq} \cdot \overline{E}_q{}^{(k-1)} \right] = \frac{1}{\overline{\overline{Y}}_{pp}} \cdot \left[ \frac{P_p - jQ_p}{\underline{E}_p{}^{(k-1)}} - \sum_{q \neq p} \overline{Y}_{pq} \cdot \overline{E}_q{}^{(k-1)} \right] \qquad [1]$$

GENERATOR NODE

$$\overline{E}_p{}^{k} = \frac{1}{\overline{\overline{Y}}_{pp}} \cdot \left[ \frac{\underline{A}_p{}^{(k-1)}}{\underline{E}_p} - \sum_{q \neq p} \overline{Y}_{pq} \cdot \overline{E}_q{}^{(k-1)} \right] = \frac{1}{\overline{\overline{Y}}_{pp}} \cdot \left[ \frac{P_p - jQ_p{}^{(k-1)}}{\underline{E}_p} - \sum_{q \neq p} \overline{Y}_{pq} \cdot \overline{E}_q{}^{(k-1)} \right]^{*}$$

$$Q_p{}^{k} = E_p \cdot \sum_{q \neq p} Y_{pq} \cdot E_q{}^{(k-1)} \cdot \sin(\delta_p{}^{(k-1)} - \delta_q{}^{(k-1)} - \theta_{pq})$$

**In generator nodes, only phase values resulting from calculation have to be taken into account*

BALANCE NODE

$$P_p^{\ k} = E_p \cdot \sum Y_{pq} \cdot E_q^{\ (k-1)} \cdot \cos(\delta_p - \delta_q^{\ (k-1)} - \theta_{pq})$$

$$Q_p^{\ k} = E_p \cdot \sum Y_{pq} \cdot E_q^{\ (k-1)} \cdot \sin(\delta_p - \delta_q^{\ (k-1)} - \theta_{pq})$$

The Gauss-Seidel method improvement can be applied, of course. The implementation order can be freely chosen, starting from voltages, without significant variations in the convergence speed. The iteration will continue until all the variables (inside the same iteration k) will show a difference value that falls under a predefined threshold, i.e. when the norm of the state variables $\Delta x^k = x^{k+1} - x^k$ falls under a certain threshold $\varepsilon$ . Then the steady state of the whole system will be completely known.

The iteration starting point is made by all the iterative variables starting values. To increase the process speed these starting value should be as close as possible with the expected value.

For example, in *p.u.*, if the balance node has a voltage magnitude value of 1.1 times the reference voltage, the starting values of load nodes could be set equal to 1, because we aspect little voltage drops in a well sized system. In the same way, if the reference voltage phase value is set at zero degree, we could set at the same value all the other voltage phases, because for transmission stability reasons, two nodes require very little phase delay.

## GAUSS ITERATIVE METHOD ON LV, DC POWER FLOW

Gauss-Seidel iterative method is often applied to very high voltage grid networks. Its use is driven by the presence of more than one supplying node; so the current flows can be only calculated through determining all the voltages, in terms of phase and magnitude, in all nodes. In fact, each voltage depends from all the other voltage, as shown in equation [1].

A radial system can be usually solved with the basic laws and methods of the Circuit Theory, like Kirchoff’s Postulates and Boucherot Method.

Nevertheless, even in a radial network, this simple method fails when the system looses its linearity, for instance when the load's power is modelled with a voltage dependent model.

In this event, a numerical method shall be applied.

As a general concept, Power Flow can be defined as a simplified Load Flow. Power Flow Calculations can be used to solve a decoupled active load flow problem, in which only $P$ and $\delta$ variables are taken into account. In another way, further simplification could be done to apply Load Flow methods to DC power systems.

Starting from the Gauss iterative method we can write:

$$\overline{Y}_{pq} = G_{pq} + jB_{pq} = G_{pq}$$

$$\overline{A}_p = P_p + jQ_p = P_p$$

$$\delta_p = \delta_q = 0$$

$$\overline{E}_q = E_q \cdot e^{j\delta_q} = E_q\,,\ \ \overline{E}_p = \underline{E}_p = E_p$$

Without phase reference need, system has determined solution without a balance node anymore.

| NODE | DATA | VARIABLES |
|---|---|---|
| Load node | P(E) | E |
| Generator node | E | P |

Table 13 – Data and variables for different kinds of node, in DC power flow.

So as:

LOAD NODE

$$P_p^{\ k} = f(E_p^{(k-1)})$$

$$E_p^{\ k} = \frac{1}{G_{pp}} \cdot \left[ \frac{P_p^k}{E_p^{\ (k-1)}} - \sum_{q \neq p} G_{pq} \cdot E_q^{\ (k-1)} \right]$$

GENERATOR NODE

$$E_p^{\ k} = E$$

$$P_p^{\ k} = E \cdot \sum_{q \neq p} G_{pq} \cdot E_q^{\ (k-1)}$$

Each node voltage depends from the strength of the same node and from the other nodes' voltages, weighted by the mutual conductance factors.

The iterative method and all the other variations and hypothesis will be the same.

GAUSS POWER FLOW IN LRV DISTRIBUTION SYSTEM

As already said, numerical methods in solving distribution problems are mandatory whether the superimposition principle cannot be applied. Even if inside several LRV vehicles the distribution system can be easily solved with algebraic methods, the need of generality leads us to choose a method that can be used with an high accuracy in all the railways distribution systems, also in the ones not covered by the LRV group and their future evolutions. Moreover, the use of certain kind of models in load and battery modeling (due to accuracy needs) force the whole distribution system to a strong non-linearity.

The main reasons in adopting a numerical calculation method so are:

- The possible presence of two or more supplying devices with the contemporary presence of gridded network.

- The presence of electrochemical batteries sized for emergency conditions or to size on the network consumptions anyway.
- The use of a voltage dependent model for each equipment, so as each node's voltage depends from itself and from all the other voltages.
- The need of a general model as suitable as possible for the future technologies.

Another important feature is the capability in iterating all the calculation procedures in an easy and quick way, implementing them on Visual Basic platform.

After a study of Visual Basic abilities in calculations, its logics and structures, I saw that the only one method that can be implemented with a calculation time accorded to the calculation needs shouldn't call differential equations.

Inside the large group of the numerical methods, the only one responding to these requirements and on which my knowledge was enough strong was the Gauss method.

## CHAPTER REFERENCES

[1] Notes from the course "Sistemi Elettrici per l'Energia", kept by Professor Andrea Silvestri at "Politecnico di Milano" University.

[2] Modeling and Analysis of Electric Power Systems, Göran Andersson, EEH - Power Systems Laboratory, ETH Zürich, March 2003

# CHAPTER 4 – POWER BALANCE CALCULATION

## ON SIZING POWER

A sizing calculation consists of identifying and numerically defining the parameters that permit to exploit the device's nominal performances (with enough reliability rate) for the entire project life.

From the electrical point of view (i.e. ignoring the mechanical sizing) the sizing calculation means the determination of the sizing power

.The sizing power takes into account the two main electrical factors that concur to the life duration of the whole device: the voltage (V), to correctly size the device insulation and the current (A), to size the device from the thermal point of view, in order to avoid dangerous over-heating during the normal functioning cycle.

While for DC devices it can be merely achieved as the product between DC current and voltage (usually multiplied by a security factor)[3], for an AC device, sizing power is defined as the product of current and voltage root mean square values, with equivalent meaning.

In fact, current's root mean square value is defined as the constant current value that gives the same Joule loss of the alternative real current, evaluating them during the same time period $T$ :

$$R \cdot I_{RMS}^2 \cdot T = R \cdot \int_0^T i(t)^2 \cdot dt$$

$$I_{RMS} = \sqrt{\frac{1}{T} \cdot \int_0^T i(t)^2 \cdot dt}$$

$$A_d = V_{RMS} \cdot I_{RMS}$$

---

[3] More often, for DC devices like converters, the DC sizing power is calculated as the product between the open circuit voltage and the rated current, with equivalent thermal meaning [1]

In a DC time-invariant device, root mean square value coincides with the DC value, while in an alternative sinusoidal (AC) device (like an induction motor), root mean square value is equal to the maximum value divided by the square root of two ( $\sqrt{2}$ ).

Nevertheless, in static converter devices and in all the time-variant DC systems, hybrid behavior may occur. It is so possible to have a constant variable and an alternative variable. This is the case on which the Power Balance Calculation has been developed.

Power Balance Calculation is the static converter's sizing calculation. Like every other equipment or device, it's a sizing calculation made in order to find the sizing power $A_d$ .

In static converter devices, the valve's gate control system is extremely rapid. We can so state that under whatever load condition, the output DC voltage will be the same set value. On the other side the output current, inside its functioning limits, is free to evolve, providing the power asked from the load (deterministic or random) evolution.

The power balance calculation is an useful way to provide the dimensioning power of a DC supplying device presenting a large number of load variation during its functioning cycle, especially when this kind of time evolution is not well known.

## RAILWAY’S STATIC CONVERTER

In the railway systems, a static converter device (CVS) is normally located close to the current collector and it is used to modify the characteristics of the input power that have to be distributed all along the vehicle. Its main parts are:

- An input filter, consisting of a choke and a capacitor, act to remove disturbances from the catenary voltage to the converter and vice versa.
- An active voltage limiter handles the inductive energy stored in the input-filter choke by firing a thyristor when positive ringing at the diode exceeds a maximum value. If the input voltage exceeds the maximum value, the converter is cut off. Low voltage also shuts the inverter down.
- An optional step-up converted used to rise the input voltage in case of 600V catenary voltage.
- A diode that protects the converter in case of reversed input polarity and blocks the energy stored in the input filter capacitor in case the catenary is being grounded.
- An IGBT inverter working according to the PWM principle converts the input voltage to a sinusoidal three-phase AC voltage (3 x 400 V /50 Hz / 60Hz) to supply the AC consumers. An AC filter smoothes the voltage signals of each of the three phases.
- A regulating circuit that limits the current and keeps the output voltage constant independently of load and fluctuations of the input voltage.
- A DC power module that converts the intermediate voltage to a DC voltage which supplies the onboard electric consumers and charges the vehicle battery. An IGBT high-frequency transformer in H-bridge arrangement chops the constant DC intermediate voltage into a pulse-width modulated AC voltage at a frequency of approximated 20 kHz. High-frequency switching allows the use of a particularly compact and light-weight ferrite transformer. This provides galvanic isolation and transforms the intermediate voltage to the required level.
- A regulating circuit that limits the current and keeps the output voltage constant independently of the load. Thus even a short circuit on the output cannot damage the DC/DC output module.

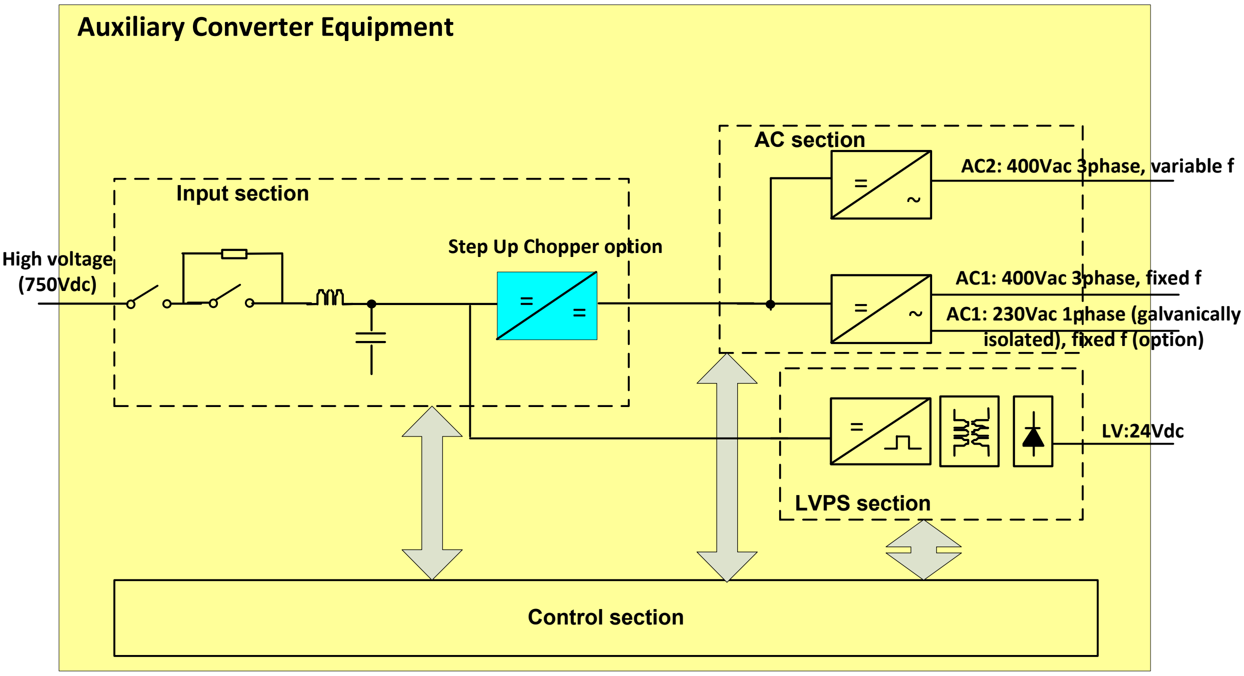


Figura 30 – Functional scheme of a Citadis static coverter.

The power balance calculation is not the real sizing process of the CVS; its scope is to define on the load's consumptions basis, the total thermal required power. Since in the Citadis range, the MV output exclusively supply a little number of ventilation and cooling units (HVAC), this kind of calculation is focused on the low voltage DC side of the CVS.

As it will be for the battery sizing, the customer (Alstom) demands to the manufacturer for the CVS's sizing, providing for a set of technical requirements. The result of the power balance calculation is one of them. Once the specification document has been done, the manufacturer have to provide to the sizing of all the electrical components inside the CVS.

In another way (in the most common uses), the power balance calculation is used to check if the equipment configuration used for the current project requires a power inside the CVS limits.

In a railway vehicle the network electrically above the static converter output cannot be considered as a time-invariant equivalent load; a lot of intermittent equipments like compressors' motors, ventilation and air-conditioning, lights, battery charger, screens are switched on and off periodically (by control system) or randomly, depending on train functioning conditions. So the adsorbed current rate will vary in time, on a huge number of levels.

To take into account this kind of variance in the integral sum, we have to know the current shape during the entire considered interval. In another way we should have at disposal all the equipments functioning time plot, which sum gives the converter current plot. Nevertheless is often difficult to find this information due to equipments' random functioning, especially during train tender stage (most equipments are not yet defined)[4].

During this kind of stage is not possible to define a functioning profile on which to define the thermal current useful to determine the sizing power, because of doing such kind of study for a time period enough large to gives the minimum required accuracy (at least one a one way track) should require a lot of time. In addition most of times the whole track and the functioning logic of the train control system are not well defined until the executive project and some of them can be achieved only after the track tests.

For these reasons, railways' engineers prefer to use some useful tools called 'Intermittence Coefficients'. These numerical terms are defined in different ways and are useful to provide and equivalent time representation of the intermittent use of each equipment. The mathematical combination can be used in Power Balance Calculations instead of the real functioning plots.

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[4] In addition, many of these intermittent equipments are controlled through a Train Control Monitoring System (TCMS), which controls the train state variables and commands equipments functioning when needed.

# INTERMITTENCE COEFFICIENT

In this way equipment's power is divided in two parts: permanent power and intermittent power. Permanent power is the power rate used with continuity during the normal functioning of the concerned equipment. An intermittence coefficient (included between 0 and 1) is associated to the intermittent part.

Intermittence coefficients take into account the intermittent use of the loads' intermittent power on a functioning cycle. They have so the same meaning of the duty cycle.

An intermittence coefficient $K$ is defined for any equipment, as the ratio between the functioning time (function of reference period) and the reference period. Considering ideal switching[5]:

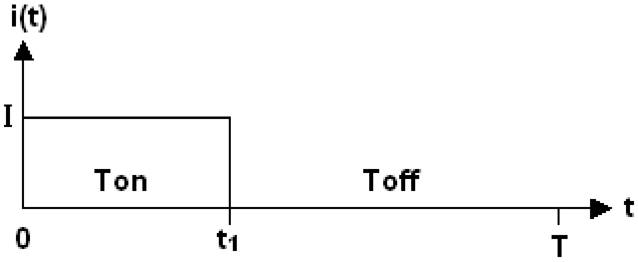


Figura 31 – Example of load's functioning cycle.

$$T_{on} + T_{off} = T$$

$$I_{RMS} = \sqrt{\frac{1}{T} \cdot \int_0^T i(t)^2 \cdot dt} = \sqrt{\frac{1}{T} \cdot \sum_i I_i^2 \cdot \Delta t_i} = \sqrt{\frac{1}{T} \cdot \left[ I^2 \cdot T_{on} + 0 \cdot T_{off} \right]} = \sqrt{I^2 \cdot \frac{T_{on}}{T}} = \sqrt{I^2 \cdot K}$$

Since intermittence coefficients are defined at equipment level, we need to arrange them in a form useful to represent the entire intermittence functioning at converter level. To combine all

---

[5] Ideal switch commutates current from o and $I$ and vice versa in null time. Another implicit hypothesis is the presence of only one current level; if switch apply on more than one level, equipment can be divided into sub-equipment to reach this condition, like is already done with the permanent part of power.

these coefficients, they have to be homogeneous; first of all, we have to define them on a common reference period.

Since the reference period $T$ is the same for all the equipments, it has to be enough wide to catch all the intermittent functioning, so at least equal to $Max(T_{on\_i} + T_{off\_i})$.

Taking a large reference period some equipment intermittence is represented more then one time.

If the equipment functioning is periodic, its intermittence coefficient can be defined on the minimum period with the same meaning (see fig. 32).

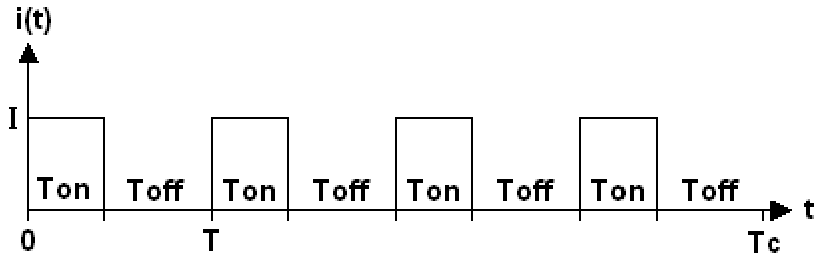


Figure 32 - Example of load's functioning cycle on larger period.

$$I_{RMS} = \sqrt{\frac{1}{Tc} \cdot \int_0^{Tc} i(t)^2 \cdot dt} = \sqrt{\frac{1}{Tc} \cdot \sum_i I_i^2 \cdot \Delta t_i} = \sqrt{\frac{1}{Tc} \cdot \left[4 \cdot I^2 \cdot T_{on}\right]} = \sqrt{4 \cdot I^2 \cdot \frac{T_{on}}{4T}} = \sqrt{I^2 \cdot K}$$

Else, if the functioning is random or it cannot be considered periodic, is better to take into account a period very large as reference time, so as a large number of non periodic functioning can be seen as "averagely periodic" (see fig. 33).

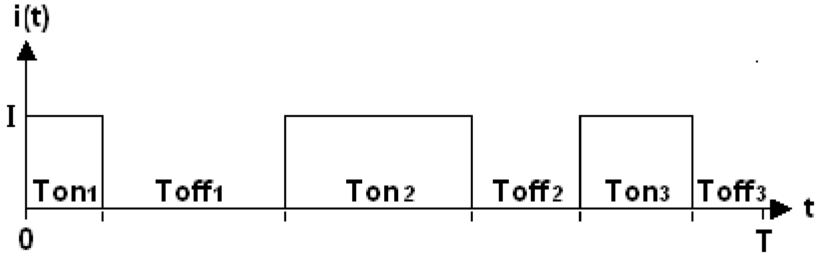


Figure 33 - Example of non-periodic load’s functioning cycle.

$$I_{RMS} = \sqrt{\frac{1}{Tc} \cdot \int_0^T i(t)^2 \cdot dt} = \sqrt{\frac{1}{Tc} \cdot \sum_i I_i^2 \cdot \Delta t_i} = \sqrt{\frac{1}{Tc} \cdot \left[ I^2 \cdot T_{on1} + I^2 \cdot T_{on2} + I^2 \cdot T_{on3} \right]} =$$

$$= \sqrt{I^2 \cdot \frac{T_{on}}{Tc}} = \sqrt{I^2 \cdot \frac{T_{on\_avg}}{3 \cdot T_{avg}}} = \sqrt{I^2 \cdot K}$$

$$T_{on} = T_{on1} + T_{on2} + T_{on3}$$

$$T_{on\_avg} = \left( T_{on1} + T_{on2} + T_{on3} \right) / 3$$

Average switch-on period approximation is allowed especially since the reference period is wide. In urban railway systems, for example, shall be wrong to take into account the period between two stations as reference period, because several auxiliary equipments’ functioning (like braking compressor’s motor) strongly depends from the traction system functioning.

That’s why a single way track time is normally used as reference time (more than 3000 seconds); above 20-30 stations, differences in inter-station tracks can be neglected, leading to use of average values for $T_{on\_i}$ parameters. In this way the total “on” period can be expressed as # $\cdot T_{on\_avg}$, where # is the number of functioning during the considered reference period.

# FROM EQUIPMENTS TO CVS LEVEL

As like is easy to define coefficients for single equipment's intermittence power it is difficult for the converter. Converter current shape is made by the linear combination of all the equipments current shapes so as its intermittence coefficients, those are made of the linear combination of equipments' intermittence coefficients.

Here below is reported a simple example with two equipments:

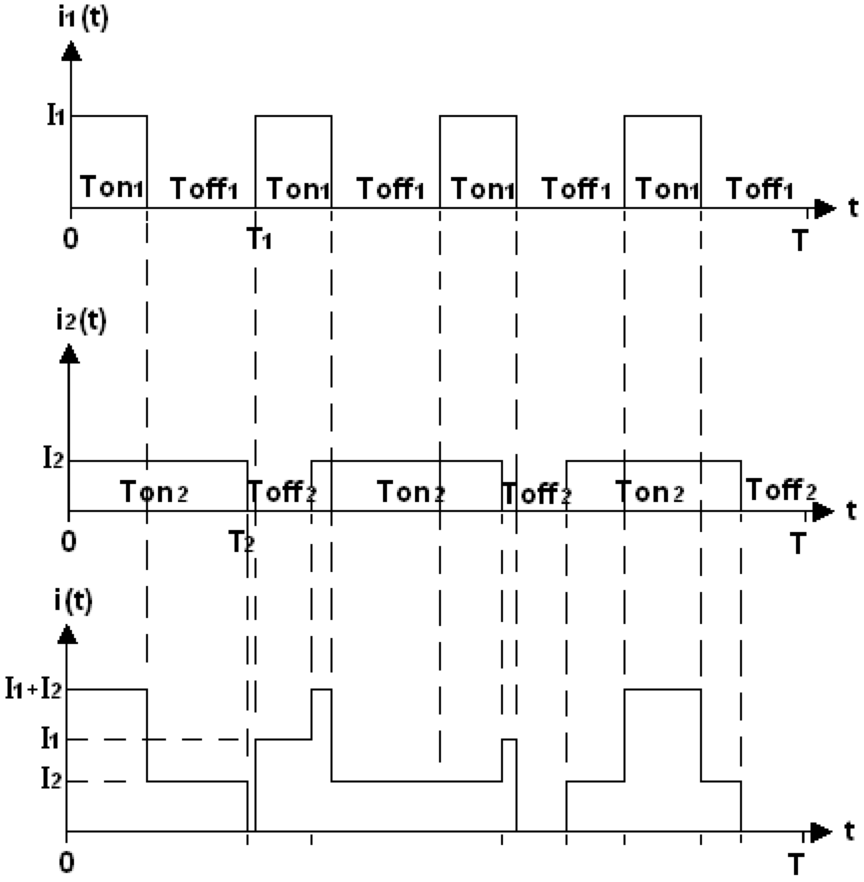


Figure 34 – Superimposition of two different load's cycles.

For a single equipments is:

$$I_{RMS1} = \sqrt{\frac{1}{T}\cdot\int_0^T i_1(t)^2\cdot dt} = \sqrt{\frac{1}{T}\cdot\left[4\cdot I_1^{\ 2}\cdot T_{on1} + 0\cdot T_{off1}\right]} = \sqrt{I_1^{\ 2}\cdot\frac{4\cdot T_{on1}}{T}} = \sqrt{I_1^{\ 2}\cdot K_1}$$

$$I_{RMS2} = \sqrt{\frac{1}{T} \cdot \int_0^T i_2(t)^2 \cdot dt} = \sqrt{\frac{1}{T} \cdot \left[4 \cdot I_2^{\ 2} \cdot T_{on2} + 0 \cdot T_{off2}\right]} = \sqrt{I_2^{\ 2} \cdot \frac{4 \cdot T_{on2}}{T}} = \sqrt{I_2^{\ 2} \cdot K_2}$$

While for total (converter) shape is:

$$I_{RMS} = \sqrt{\frac{1}{T} \cdot \int_0^T i(t)^2 \cdot dt} = \sqrt{\frac{1}{T} \cdot \left[I_2^{\ 2} \cdot T_{onA} + I_1^{\ 2} \cdot T_{onB} + (I_1 + I_2)^2 \cdot T_{onC}\right]} =$$
$$= \sqrt{I_2^{\ 2} \cdot \frac{T_{onA}}{T} + I_1^{\ 2} \cdot \frac{T_{onB}}{T} + (I_1 + I_2)^2 \cdot \frac{T_{onC}}{T}} = \sqrt{I_2^{\ 2} \cdot K_A + I_1^{\ 2} \cdot K_B + (I_1 + I_2)^2 \cdot K_C}$$

It’s easy to see that to find $K_A, K_B$ and $K_C$ we need to know $i(t)$ total shape, so all the $i_n(t)$ equipment shapes.

First of all is better to arrange $K_A, K_B$ and $K_C$ in a condensed form:

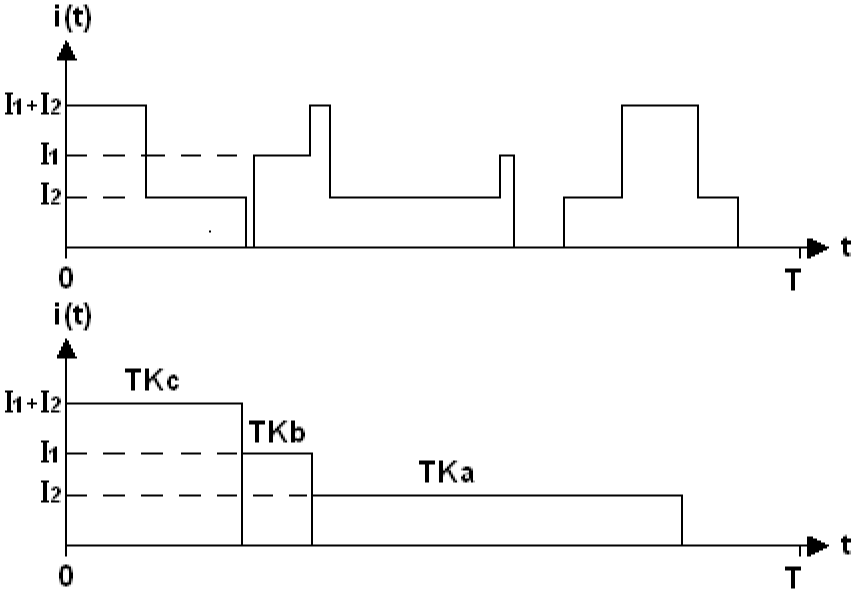


Figure 35 – First arrangement of CVS's current shape.

Now, we need to express the condensed graph in an approximated way that considers only the equipment’s intermittence coefficients. Without loss of generality we can state that $K_1 < K_2$:

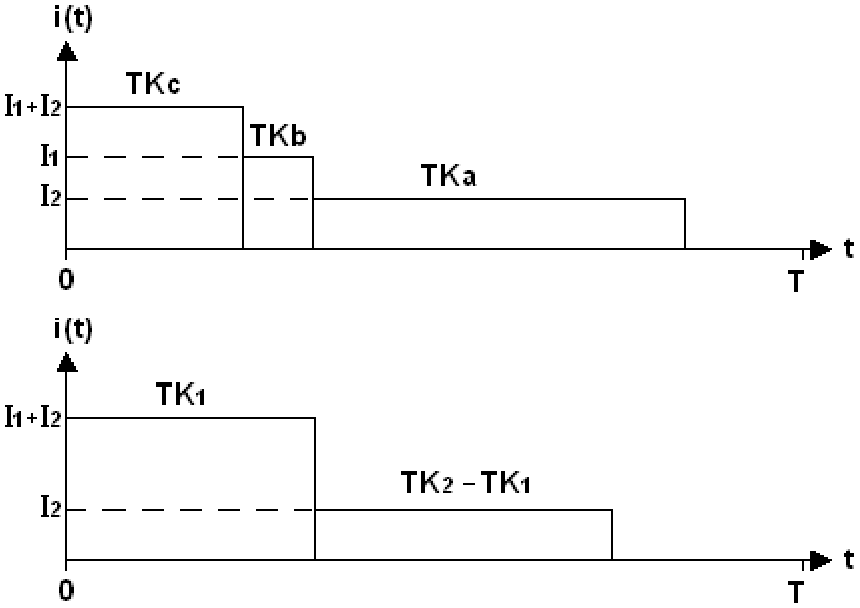


Figure 36 - Second arrangement of CVS's current shape.

This graph says that the current shape 1 is completely superimposed to current 2 in time, so its correlation factor is 1, which is a security approximation.

Nevertheless, to use this kind of arrangement rather than the real total current profile in power balance calculations, we have to check that from the thermal point of view the equivalent model provide for an equivalent or greater sizing current. To check this statement the following inequality has to be satisfied:

$$\sqrt{(I_1+I_2)^2 \cdot K_1 + I_2{}^2 \cdot (K_2 - K_1)} \geq \sqrt{I_2{}^2 \cdot K_A + I_1{}^2 \cdot K_B + (I_1+I_2)^2 \cdot K_C}$$

As $K_A, K_B$ and $K_C$ are defined, we can state that:

$$K_B + K_C = K_1$$

$$K_A + K_C = K_2$$

$$(I_1 + I_2)^2 \cdot K_1 + I_2{}^2 \cdot (K_2 - K_1) \geq I_2{}^2 \cdot K_A + I_1{}^2 \cdot K_B + (I_1 + I_2)^2 \cdot K_C$$

$$I_1{}^2 \cdot K_1 + I_2{}^2 \cdot K_1 + 2 \cdot I_1 \cdot I_2 \cdot K_1 + I_2{}^2 \cdot K_2 - I_2{}^2 \cdot K_1 \geq I_2{}^2 \cdot K_A + I_1{}^2 \cdot K_B + I_1{}^2 \cdot K_C + I_2{}^2 \cdot K_C + 2 \cdot I_1 \cdot I_2 \cdot K_C$$

Using the definitions of $K_1$ and $K_2$:

$$I_1{}^2 \cdot K_1 + 2 \cdot I_1 \cdot I_2 \cdot K_1 + I_2{}^2 \cdot K_2 \geq I_2{}^2 \cdot K_2 + I_1{}^2 \cdot K_1 + 2 \cdot I_1 \cdot I_2 \cdot K_C$$

$$2 \cdot I_1 \cdot I_2 \cdot K_1 \geq 2 \cdot I_1 \cdot I_2 \cdot K_C$$

$$K_1 \geq K_C$$

That is always verified. The equality condition is verified whenever $K_B = 0$, so there is no superimposition between the two loads.

In this way, each functioning time period can be defined as a linear combination of equipments' intermittence coefficients, disengaging $I_{RMS}$ from its intrinsic stochastic dependence.

# POWER BALANCE CALCULATION

## DEFINING SCENARIOS

Once ensured the energetic equivalence between the real current wave and the rearranged one, we have to define the last one in an useful way, taking into account only the permanent and intermittent equipments used in the normal exploiting condition. Normal condition is normally considered the most stringent one (i.e. rather than train preparation/de-preparation, in standstill condition), so its thermal cycle is used for thermal sizing. The emergency conditions, that don't involve the use of the static supplying device, are not taken into account.

Equipments are divided into the so-called “scenarios”; each scenario is made from a collection of equipments that are able to work during the same time. As already said, we will suppose that they will work contemporary, to take into account the worst service condition.

Scenarios are defined on functional classification; each functional performance exploited by the train defines a group of equipments that cooperate together, even without complete time superimposition. Inside this group, the smaller intermittence coefficient will be the scenario’s intermittence coefficient. Scenario’s current will be the sum of all the involved equipments’ currents.

The sum of the intermittence periods of the scenarios in which an equipments is involved can’t be more than the equipment’s intermittence coefficient.

As a consequence, if the smaller coefficient belongs to equipment already considered in another scenario, scenario’s intermittent factor will appear as the subtraction between equipment intermittence coefficients and the other scenario coefficient.

Permanent equipments are considered as global intermittent equipment with $K_{PP} = 1$ and it’s involved in all the scenarios, of course.

Finally:

$$I_{RMS} = \sqrt{\frac{1}{T} \cdot \int_0^T i(t)^2 \cdot dt} = \sqrt{\sum_j I^2{}_{SC\_j} \cdot K_{SC\_j}}$$

## POWER BALANCE ON CITADIS

Power balance calculation performed for Citadis tramway is a bit different from the general method. Instead of on current, equipments and scenarios are defined through the power. Of course, under the hypothesis of constant voltage (independent from time and current) the meaning shall be the same:

$$I_{RMS}^2 = \frac{1}{T} \cdot \int_0^T i(t)^2 \cdot dt = \sum_j I^2{}_i \cdot K_i \cong \sum_j I^2{}_{SC\_j} \cdot K_{SC\_j}$$

$$V_{RMS}^2 \cdot I_{RMS}^2 = V_{RMS}^2 \cdot \sum_j I^2{}_{SC\_j} \cdot K_{SC\_j}$$

$$P_{RMS}^2 = \sum_j V_{RMS}^2 \cdot I^2{}_{SC\_j} \cdot K_{SC\_j} = \sum_j P^2{}_{SC\_j} \cdot K_{SC\_j}$$

## SIZING POWER

Although this method is really simple to apply, it presents an intrinsic error, due to the hypothesis of constant voltage. In fact, such a hypothesis means that all the equipments are connected to a common bus bar that provides the same voltage for each load. Of course, this kind of hypothesis ignores the presence of a distribution network between CVS's output and equipments.

Taking into account the network mainly means taking into account its power losses and voltage drops.

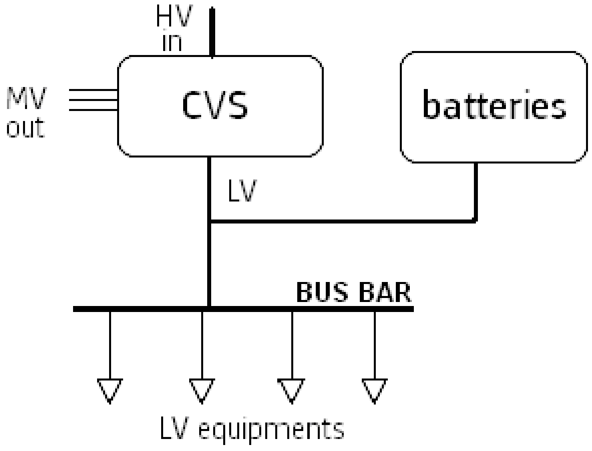


Figure 37 - Functional scheme of low voltage power supply system.

To take into account the effect of power losses (on total sizing power) and voltage drops (on equipments' powers) a security factor is introduced, so:

$$P_d = P_{RMS} \cdot F = P_{RMS} \cdot \frac{V_{RMS}}{V_{Eqp}}$$

Since equipments powers are defined at nominal voltage rate, the root mean square voltage, will be the nominal one:

$$P_d = P_{RMS} \cdot F_n = P_{RMS} \cdot \frac{V_{RMS}}{V_{Eqp}} = P_{RMS} \cdot \frac{V_n}{V_{Eqp}} = P_{RMS} \cdot \frac{V_n}{V_n - \Delta V_{avg}}$$

$V_{Eqp}$ is the equipments supplying voltage and it's calculated like nominal voltage minus the average voltage drop $\Delta V_{avg}$, set as an experienced limit.

## METHOD IMPROVEMENTS

The Power Balance Calculation, as done in Alstom has been just explained.

The main critical to this method is the complete lack of a network structure between supplying devices and equipments, so the lack of any kind of dependence (in term of power) that a real network implies. In fact, taking into account a real network means to take into account 2 main effects:

1. The heat loss in distribution cables, collectors and diodes due to Joule effect.
2. The modifying in equipments' adsorbed powers due to variable voltage drops, whereas a voltage dependent model is used in load modelling.

As already shown, the first effect's error in power estimation is taken into account through a security-oversizing coefficient. That is due to the fact that, even if the network structure is available and well defined, is always faster to perform calculations that doesn't take it into account, so as they don't need the cabling input.

Nevertheless, following accuracy and cost-oriented philosophy, is not possible to accept this kind of over-estimation in probably the most expensive device of the distribution system.

To avoid the use of this kind of semi-empirical security factor, using a more precise and severe method, we should refer to currents instead of powers, calculating the consumptions just at the CVS's output. In this way, the power balance dependence from the network architecture could be shed whereas its global effect (CVS output current measurement) is at disposal.

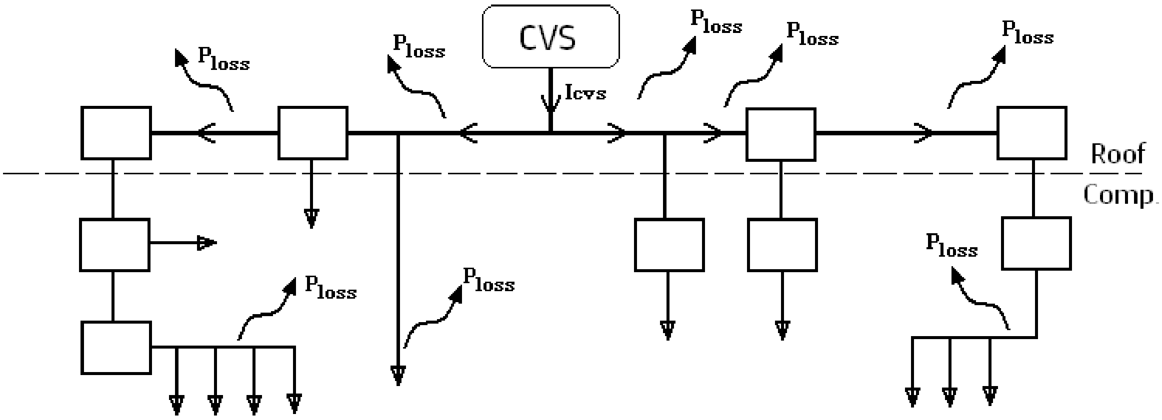


**Figure 38 -** As shown in the figure, is possible to avoid estimation of branches' power losses knowing CVS's current rate.

After that, for each scenario, the square of CVS current shall be multiplied for the relative scenario's intermittence coefficient. So on, the total square root is multiplied for the CVS output voltage, finding the real CVS sizing power.

# POWER BALANCE CALCULATION IMPLEMENTATION

To take into account how the CVS current evolve with equipments adsorbed power we could have to use a numerical method. In a sizing process, a numerical model is required whereas:

1. Distribution network is gridded.
2. A “top-down” approach is used, rather than a “bottom-up” one (e.g. Boucherot).
3. Voltage source cannot be considered as infinite power node (like for batteries).

In power balance calculations, even if radial network and infinite power supply conditions are met, the top-down approach (from supplying node voltage to equipments) and the use of voltage dependent load models, force the use of numerical methods.

Due to ability in estimating all the electrical variables characterizing the whole electrical system behaviour and due to the ease of its implementation, Gauss iterative method has been chosen to perform all kinds of calculation (static and dynamic).

Power balance calculation is a static estimation, of course. The iterative process flow chart is reported below:

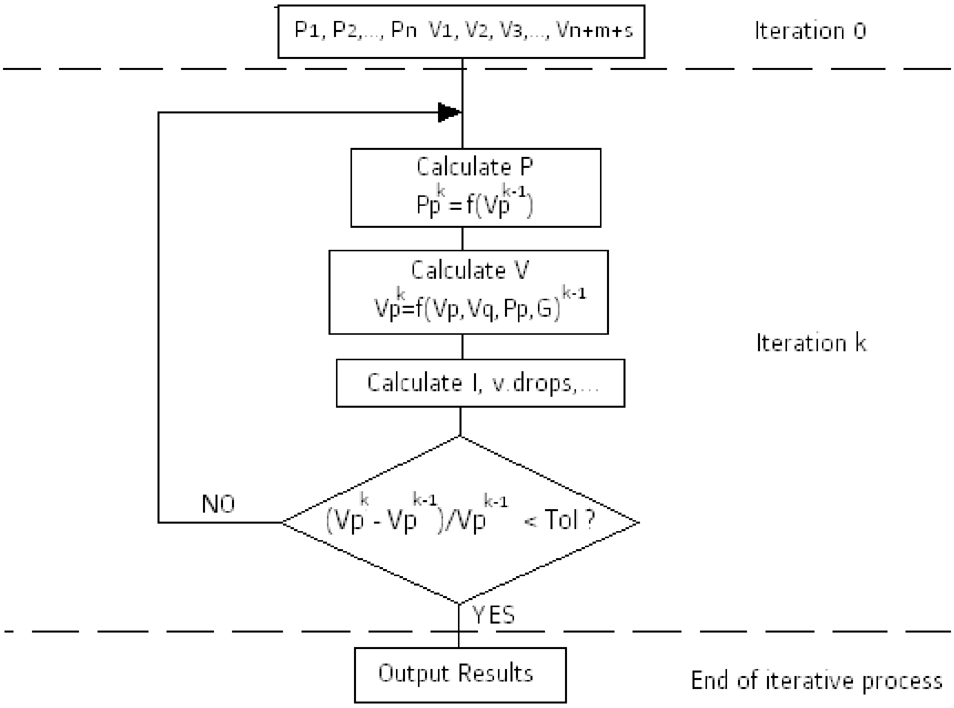


Figure 39 - Gauss method flow chart.

It begins with the iterative variable first value settings, at iteration 0. Once entered in the iterative cycle, the system is free to evolve to the mathematical solution of the equations' system.

In order have been calculated:

1. Nodal adsorbed powers
2. Nodal supplying voltages
3. All the other nodal/branch variables required (current draws, voltage drops, branch currents, power losses…)

The convergence condition, so the exit cycle condition, has to be satisfied contemporary in all the nodes and it has been applied to *p.u.* voltages, which are the most sensitive variables in the network.

The method just shown shall be applied to all defined scenarios; the scenario CVS's currents so found will be used in the thermal current calculation (RMS value):

$$I_{RMS}^2 = \frac{1}{T} \cdot \int_0^T i(t)^2 \cdot dt = \sum_j I^2{}_i \cdot K_i \cong \sum_j I^2{}_{SC\_j} \cdot K_{SC\_j}$$

$$P_d = \sqrt{\sum_j V_{CVS}^2 \cdot I_{SC\_j}^2 \cdot K_{SC\_j}}$$

## DEFINING SCENARIOS

To define Scenarios' characteristics we need to do further equipments classification, a functional one. It's based on grouping equipments witch simultaneously work to achieve a result or to reply to a command (from driver or automatic drive) or another interference. I will call them Functional Equipment Group (FEG), so as, to each Scenario is associated a FEG.

For example at the Emergency Braking command will follow the magnetic brakes supplying, the sanding compressors and ejectors supply, the red lights lightening and all the other implied subsystems supply. To achieve the Emergency Braking consumption, we also shall add the permanent loads consumption.

Here below is reported an example list of Functional Equipments Group (FEG).

| **FEG** | Involved Equipments |
|---|---|
| Emergency brake | PP+P1+P2+P4+P5 |
| Security brake | PP+P1+P2+P4+P5 |
| Station announcement | PP+P1+P6 |
| Tickets marking | PP+P7 |
| Opening/Closing doors | PP+P1+P8 |
| Electric Cloche | PP+P9 |
| Maximum service brake | PP+P1+P4+P5 |
| Service and stop brake | PP+P1+P5 |
| Station standstill | PP+P1 |
| Wheel flange greasing | PP+P10 |
| Casual functioning | PP+P11+P13+P14+P15 |
| Windscreen and lateral windows defrosting | PP+P16+P17 |
| Driver comfort setting | PP+P18+P19+P1 |
| APS mode commutation | PP+P20+P21 |
| Permanent Functions | Puissance Permanente |

| |
|---|
| PP: Permanent Power |
| P1: Stop Lights |
| P2: Magnetic Brakes |
| P4: Sanding |
| P5: Hydraulic Brakes (BM+BP) |
| P6: UMC (carte+DPAI) |
| P7: Ticket Marker |
| P8: Doors |
| P9: Cloche |
| P10: Wheel Flange Grease |
| P11: Windscreen Defrost Motor |
| P13: SAE |
| P14: Remote Needle |
| P15: Crossroads Priority Aerial |
| P16: Windscreen Defrost |
| P17: Lateral Windows Defrosting |
| P18: Seat Motor |
| P19: Security Driver's Presence Pedal |
| P20: Pantographe Motor |
| P21: APS Shoes Motor |

Table 14 - Scenarios/functional equipments group list.

# CHAPTER REFERENCES

[1] Notes from the course “Complementi di elettronica di potenza”, kept by Professor Antonino Di Gerlando at “Politecnico di Milano” University.

[2] IEEE Standard 1476-2000.

# CHAPTER 5 – SOFWARE IMPLEMENTATION

## LEADING PRINCIPLES

All the models and the calculation processes so far explained have been developed in order to be implemented on informatics basis. The software implementation has been provided during the modeling stage, in order to build an analytical model accorded to several constraints. The main constraints can be synthesize so on:

1. The use of Microsoft based or freeware software platform, to allow all the Alstom engineers to use the calculation software tool (hereafter tool).
2. The use of graphical input/output.
3. The possibility to be used during tender, project, research, feasibility stage.
4. The possibility to simplify the input stage, proving for equipments and devices' databases.

In addition, during the preliminary stage, several criteria has been defined to lead the software development. These general software criteria comes from all the requirements expressed by the TCMS team during several meetings. In fact, as final users, the TCMS team, during our continuous team work, gave to me all the suggestions and the main issues to complete my work with a software tool capable to exploit all the models developed, performing the required calculations. The main criteria are so summarized:

- <u>Calculation accuracy</u>: This tool performs all the calculations and all the evaluation required, with the maximum computing accuracy, according to the other purposes. Accuracy in the system mathematical model is strongly linked with the accuracy in the computational model; in fact a really accurate mathematical model, used out of its validation limits, can lead to completely wrong results (without physical meaning). Furthermore, these wrong results shouldn't be out of a reasonable range, so as the error couldn't be detected. To avoid validity mistakes, all the calculations should provide a list of the hypothesis under which they are performed.
- <u>Calculation speed</u>: This tool, for its purposes, is designed to be used up to a real time use. In this way, it must show very high process speed, to show very little time between the first input and the last output. First, this requirement leads to the achievement of a high

calculation speed. Calculation speed depends mainly on system's model choice, system size, simulation type, accuracy reached, and analysis duration. But a high speed opposes to high calculation accuracy, when a numerical calculation occurs. This antagonism leads to compromise choices. Because of calculation requirements, calculations should be performed with a model numerically solvable. In this event, the order of the system shall be limited, eventually dividing the calculation into sub-calculations. Integration step and convergence threshold should vary according to computation accuracy and Stage Requirements. An algebraic model will be always preferred to a differential model. The computational speed must be followed from an input time as short as possible. The input sheets have to be easy understandable and non-repetitive. In addition, they should show only what is useful to the user calculation's needs. This requirement is useful in output evaluation speed too.

- Error immunity: The human factor covers a leading role in every computer-based simulation. In each calculation, a more complicate model presents higher accuracy but at the same time more and more complicate user inputs are required. The input stage is very critical in respect of error propagation, especially when the input number grows up. Is not so difficult to have critical data error that leads to plausible but wrong results, without user noticing. The tool must be as much as possible unaffected from errors that can derive from lack of data, incorrect data, input errors. The way to reduce error probability is totally preventive and it consists in to follow and to suggest user during input period and during outputs overview. In addition, it is very important to provide for any equipment a list of data, because of the often-low reliability of part of user's data. Finally, for each calculation type a model as simple as possible should be provide, according to accuracy requirements.

- Modularity, especially in input stage, is useful to reduce process time. Since most of equipments are present in a vehicle more than one time, an input structure (circuits, cars, sub-systems...) should be replicable as a whole.
- Flexibility: The tool must present several degrees of flexibility in order to enable a more general use, starting from a development on a particular one (Citadis tramway):

    1. Flexibility in respect of platforms (tram, metro, tram-train…), because it should be useful for the most possible large part of Alstom transport engineers.
    2. Flexibility in respect of test scenarios, because the users should be able to perform the larger possible group of studies and test scenarios that can occur nowadays and in the future.
    3. Flexibility in respect of extensibility, because with time improvements and modifications should be done, in order to adapt the tool to new requirements, specifications, technologies and platforms.

We have to remember that for each platform the same auxiliary load can be supplyed at different voltage and regime; we have to take into account possible different behaviors of the same physical device.

- Continuity: This tool is developed on a continuity principle. The continuity principle implies the use, whereas is possible, of predefined structures and Alstom's know-how. This principle is useful to link the old users' tools to the new one, to mitigate the changing effects and to make this software tool easier understandable. Continuity is met about methods, because nowadays most of engineers often use the same theoretical method for each kind of calculation, so that the tool will be easily used by a larger group of persons. If there will be strong needs to improve a method, this improvements must be simple to understand and well explained. Continuity is met about software, to enable all the persons in the company to run the program. Continuity is met about classification, for example in circuit, functional groups, scenarios, if there is no strong reasons to change them.
- User friendship: Most of the proposed requirements pertain to a general user friendship principle. This principle ensured the minimum effort of the user in understanding and performing the software tool, achieving the better result.

Taking into account all the preliminary requirements, I decided to use Microsoft Visual Basic as software platform.

Visual Basic, as the name suggests, is a visual-based programming language. In fact, besides the possibility to use the classic logical structures (common to all the main programming languages), it is able to interface the code with Excel sheets and the so-called "Userforms", that are graphical windows equipped with buttons, data lists, textboxes, etcetera.

The tool has been structured with a step-by-step logic on several Excel sheets, using only buttons to pass from a sheet to another, driving the user in the calculation process that he wants to afford.

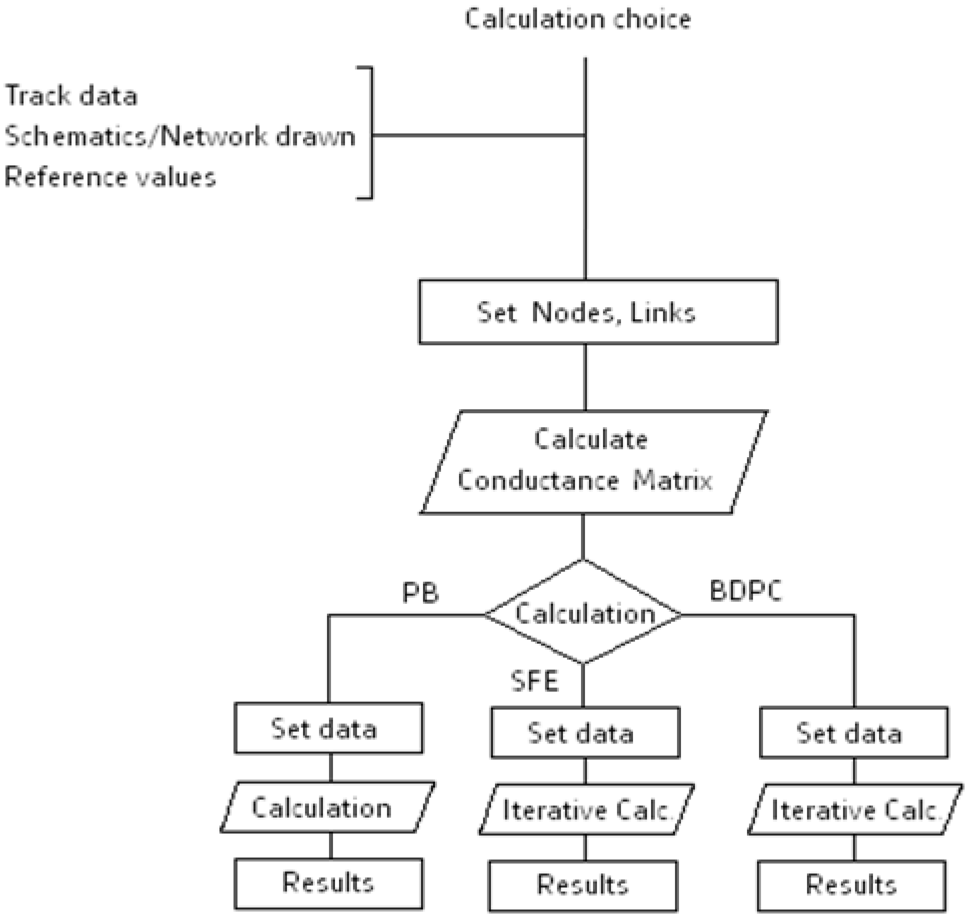
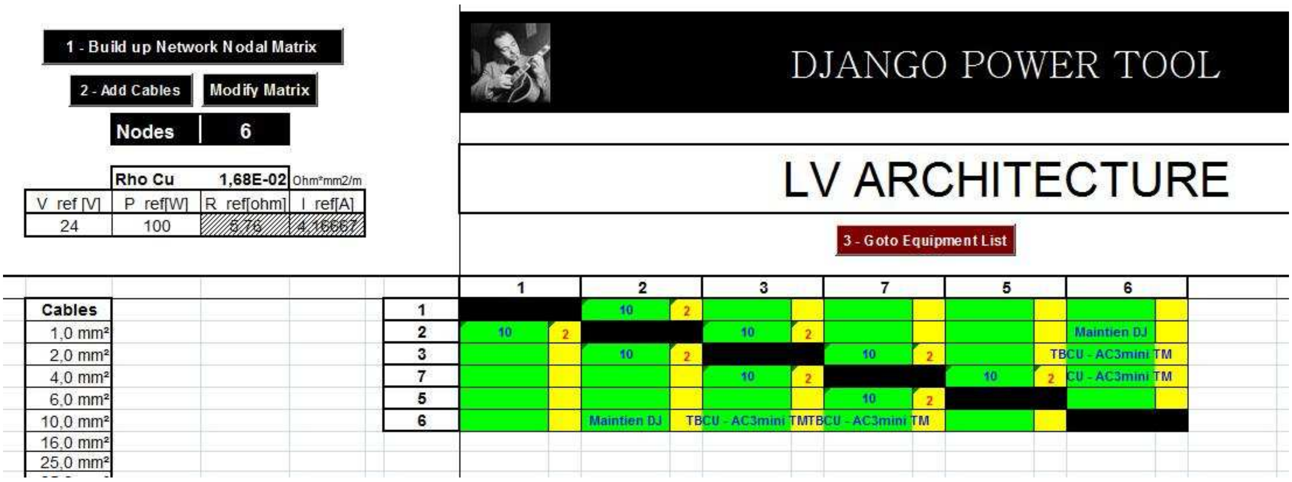


Figure 40 – Tool's structure flow chart.

## NETWORK INPUT

The first step, common to all the calculations choices, is the definition of the studied network. The network is represented as a collection of nodes, among which there are wires, diodes or equipments. Is possible to define the network starting from an electrical scheme or a simple draw, just enumerating the interested nodes. The number of nodes defines the order of the system and the size of the matrix that represents the network.



| Nodes | 6 |
|---|---|

| | Rho Cu | 1,68E-02 | Ohm*mm2/m |
|---|---|---|---|
| V ref [V] | P ref[W] | R ref[ohm] | I ref[A] |
| 24 | 100 | 5,76 | 4,16667 |

| Cables |
|---|
| 1,0 mm² |
| 2,0 mm² |
| 4,0 mm² |
| 6,0 mm² |
| 10,0 mm² |
| 16,0 mm² |
| 25,0 mm² |

| | 1 | | 2 | | 3 | | 7 | | 5 | | 6 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | 10 | 2 | | | | | | | | |
| 2 | 10 | 2 | | | 10 | 2 | | | | | Maintien DJ | |
| 3 | | | 10 | 2 | | | 10 | 2 | | TBCU - AC3mini TM | | |
| 7 | | | | | 10 | 2 | | | 10 | 2 | CU - AC3mini TM | |
| 5 | | | | | | | 10 | 2 | | | | |
| 6 | | | Maintien DJ | TBCU - AC3mini TM | | TBCU - AC3mini TM | | | | | | |

Figure 41 - Input matrix.

An userform has been provided to fill the n<sup>th</sup> network matrix:

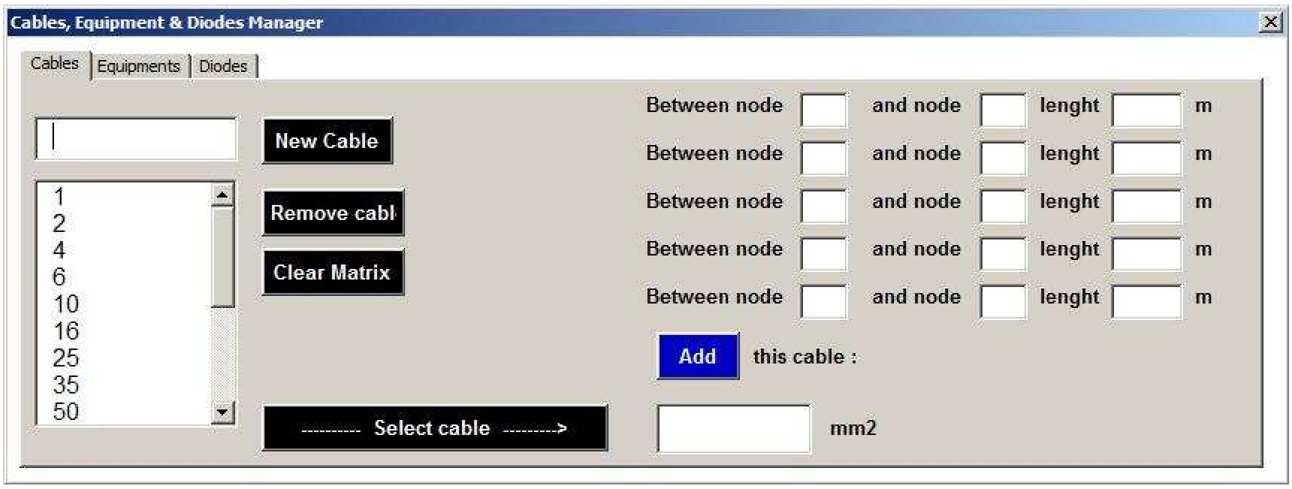


Figure 42 - Network userform.

For cables and equipments a data list is provided, while for diodes the user have to choose the diode's orientation (anode node – cathode node) and the rated current.

The supplying devices are kept out of this matrix while is not for the equipments: in this way, at the expense of computational speed, is possible to represent also the networks with the cabled return of the current (instead of car body plugging).

## CONDUCTANCE MATRIX

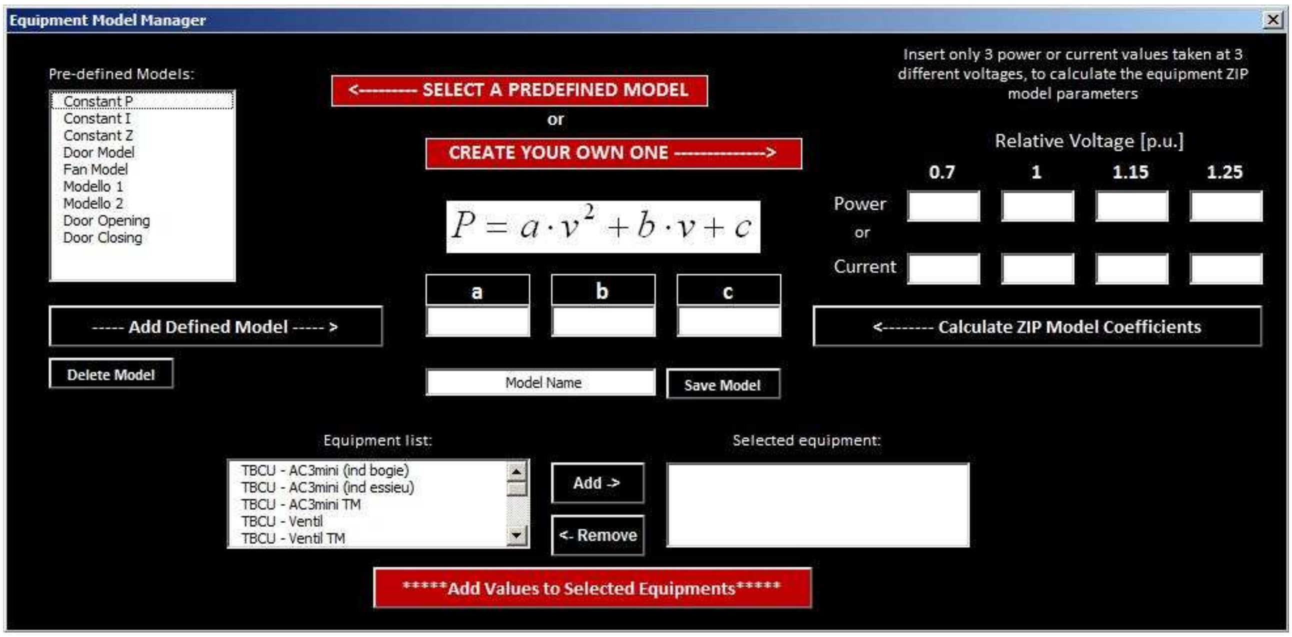
Pressing the button "Go to equipment list", the tool calculate through the inspection process the conductance matrix, with all the self and mutual terms required from the Gauss method.

In the equipment list the user defines all the characteristic of the equipments that he needs:

| | Equipments List on DC network | Nominal Power [W] | | Load Model Coefficients | | | Remarks |
|---|---|---|---|---|---|---|---|
| | | Continuative | Intermittent | a | b | c | |
| | **10 - TBCU** | | | | | | |
| | TBCU - AC3mini (ind bogie) | | | 3,703704 | -4,62965 | 1,925905 | |
| | TBCU - AC3mini (ind essieu) | | | | | | |
| | TBCU - AC3mini TM | 300,00 | | 0 | 1 | 0 | |
| | TBCU - Ventil | | | | | | |
| | TBCU - Ventil TM | | | | | | |
| X | Maintien DJ | | 400,00 | 0 | 1 | 0 | |
| | TBCU - Relayage (ind bogie) | | | | | | |
| | TBCU - Relayage (ind essieu) | | | | | | |
| | TBCU - Relayage TM | | | | | | |
| | Relayage motrice | | | | | | |
| | Relayage CC | | | | | | |
| | Relayage secours | | | | | | |
| | **19 - ECOPACK** | | | | | | |
| | Coffre Hacheur | | | | | | |
| | Ventilation coffre hacheur | | | | | | |
| | CSC30 | | | | | | |
| | CSC40 | | | | | | |
| | Ventilation CSC30 (idem CSC40) | | | | | | |

Figure 43 - Equipments sheet.

For each equipment is possible to define the continuative power and/or the intermittence power. In addition, three columns are dedicated to the ZIP model coefficients. To calculate them, the "calculate coefficients" button leads to the following userform:



Pre-defined Models:

Constant P
Constant I
Constant Z
Door Model
Fan Model
Modello 1
Modello 2
Door Opening
Door Closing

$$P = a \cdot v^2 + b \cdot v + c$$

Insert only 3 power or current values taken at 3 different voltages, to calculate the equipment ZIP model parameters

Equipment list:

TBCU - AC3mini (ind bogie)
TBCU - AC3mini (ind essieu)
TBCU - AC3mini TM
TBCU - Ventil
TBCU - Ventil TM

Selected equipment:

Figure 44 - Equipments model userform.

## CALCULATION CHOICE

After the network and the used equipments have been characterized, we have to choose what kind of calculation we want to perform. The choice have to be made among three possibilities:

1. Static flow calculation
2. Power balance calculation
3. Battery discharge profile calculation

For all the three calculations other inputs are required; first of all, an userform is dedicated to choice of the number and the location of the LV supplying devices (one or more than one). After that, on the left of each calculation sheet, there is a little table in which is possible to specify some calculation parameters:

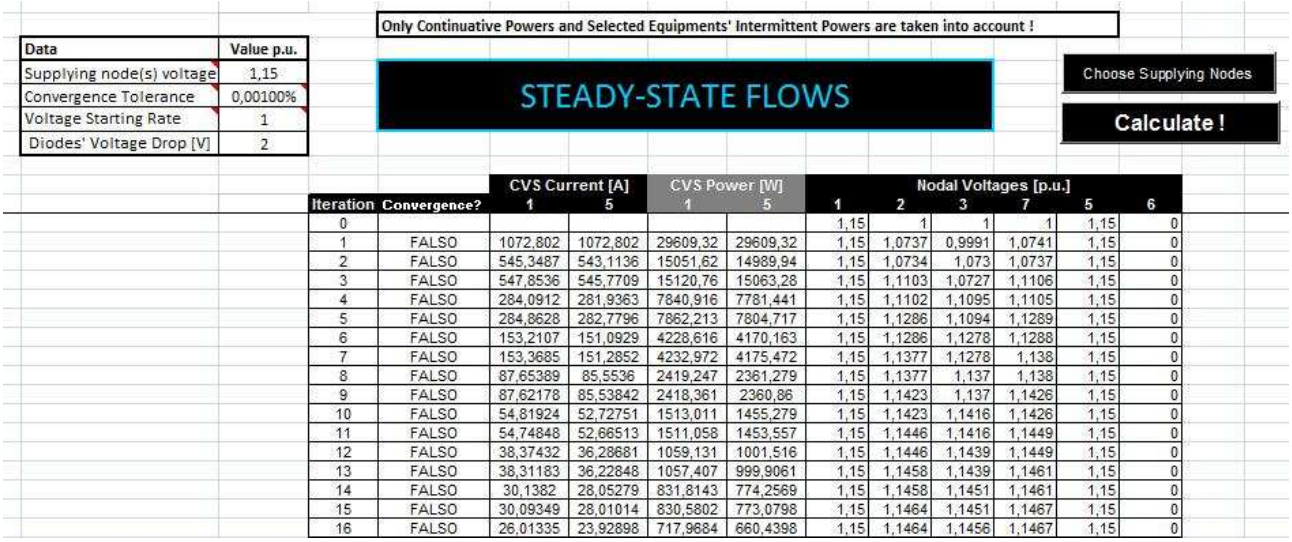
- Supplying node(s) voltage rate (the same for all the supplying devices)[6]
- Convergence tolerance
- Starting voltage rate (for all the other nodes)
- Diodes' conducing voltage drop.
- Time step (only for battery discharge profile calculation)

### STATIC POWER FLOW

The first calculation performs a power flow calculation. The iterative process go ahead until the convergence of all the nodes' voltages. The calculation output is the voltage rate of all the nodes and power and current draws for the supplying node(s).

---

[6] If there is more than one CVS, they always work synchronously on the same network, or, at least, they are decoupled on two sub-networks.



Only Continuative Powers and Selected Equipments' Intermittent Powers are taken into account !

| Data | Value p.u. |
|---|---|
| Supplying node(s) voltage | 1,15 |
| Convergence Tolerance | 0,00100% |
| Voltage Starting Rate | 1 |
| Diodes' Voltage Drop [V] | 2 |

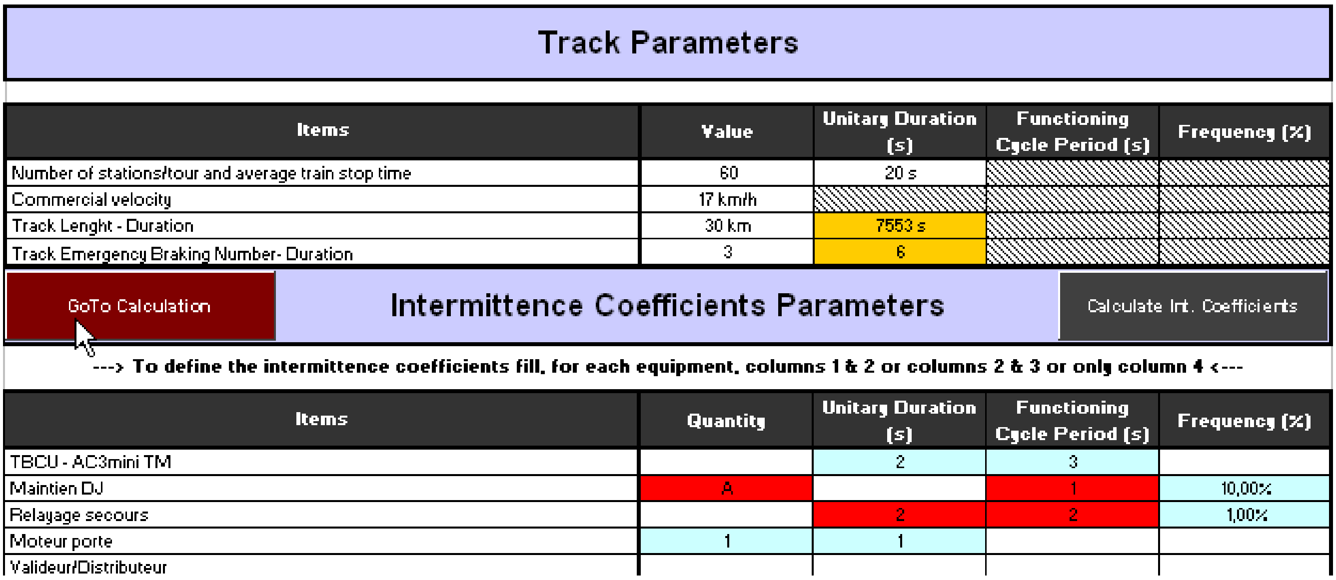
| | | CVS Current [A] | | CVS Power [W] | | Nodal Voltages [p.u.] | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Iteration | Convergence? | 1 | 5 | 1 | 5 | 1 | 2 | 3 | 7 | 5 | 6 |
| 0 | | | | | | 1,15 | 1 | 1 | 1 | 1,15 | 0 |
| 1 | FALSO | 1072,802 | 1072,802 | 29609,32 | 29609,32 | 1,15 | 1,0737 | 0,9991 | 1,0741 | 1,15 | 0 |
| 2 | FALSO | 545,3487 | 543,1136 | 15051,62 | 14989,94 | 1,15 | 1,0734 | 1,073 | 1,0737 | 1,15 | 0 |
| 3 | FALSO | 547,8536 | 545,7709 | 15120,76 | 15063,28 | 1,15 | 1,1103 | 1,0727 | 1,1106 | 1,15 | 0 |
| 4 | FALSO | 284,0912 | 281,9363 | 7840,916 | 7781,441 | 1,15 | 1,1102 | 1,1095 | 1,1105 | 1,15 | 0 |
| 5 | FALSO | 284,8628 | 282,7796 | 7862,213 | 7804,717 | 1,15 | 1,1286 | 1,1094 | 1,1289 | 1,15 | 0 |
| 6 | FALSO | 153,2107 | 151,0929 | 4228,616 | 4170,163 | 1,15 | 1,1286 | 1,1278 | 1,1288 | 1,15 | 0 |
| 7 | FALSO | 153,3685 | 151,2852 | 4232,972 | 4175,472 | 1,15 | 1,1377 | 1,1278 | 1,138 | 1,15 | 0 |
| 8 | FALSO | 87,65389 | 85,5536 | 2419,247 | 2361,279 | 1,15 | 1,1377 | 1,137 | 1,138 | 1,15 | 0 |
| 9 | FALSO | 87,62178 | 85,53842 | 2418,361 | 2360,86 | 1,15 | 1,1423 | 1,137 | 1,1426 | 1,15 | 0 |
| 10 | FALSO | 54,81924 | 52,72751 | 1513,011 | 1455,279 | 1,15 | 1,1423 | 1,1416 | 1,1426 | 1,15 | 0 |
| 11 | FALSO | 54,74848 | 52,66513 | 1511,058 | 1453,557 | 1,15 | 1,1446 | 1,1416 | 1,1449 | 1,15 | 0 |
| 12 | FALSO | 38,37432 | 36,28681 | 1059,131 | 1001,516 | 1,15 | 1,1446 | 1,1439 | 1,1449 | 1,15 | 0 |
| 13 | FALSO | 38,31183 | 36,22848 | 1057,407 | 999,9061 | 1,15 | 1,1458 | 1,1439 | 1,1461 | 1,15 | 0 |
| 14 | FALSO | 30,1382 | 28,05279 | 831,8143 | 774,2569 | 1,15 | 1,1458 | 1,1451 | 1,1461 | 1,15 | 0 |
| 15 | FALSO | 30,09349 | 28,01014 | 830,5802 | 773,0798 | 1,15 | 1,1464 | 1,1451 | 1,1467 | 1,15 | 0 |
| 16 | FALSO | 26,01335 | 23,92898 | 717,9684 | 660,4398 | 1,15 | 1,1464 | 1,1456 | 1,1467 | 1,15 | 0 |

Figure 45 - Steady state flow sheet.

## POWER BALANCE

The second calculation, the power balance, iterates the first calculation on each defined scenario. The PB scenarios are defined through another excel sheet. First, the user have to define the intermittence factor for each load on which an intermittence power has been associated. The track parameters are used to define a common reference basis on which to refer equipments' duty cycles. The intermittence coefficients can be defined in here ways:

1. Directly, as a percentage of the total track time (empirical or estimated values).
2. Defining "functioning cycle period" and "unitary duration", so equipment's duty cycle.
3. Defining "unitary duration" and "quantity", so how much times the equipment works in the track period and how many time each time.



Track Parameters

| Items | Value | Unitary Duration (s) | Functioning Cycle Period (s) | Frequency (%) |
|---|---|---|---|---|
| Number of stations/tour and average train stop time | 60 | 20 s | ▨ | ▨ |
| Commercial velocity | 17 km/h | ▨ | ▨ | ▨ |
| Track Lenght - Duration | 30 km | 7553 s | ▨ | ▨ |
| Track Emergency Braking Number- Duration | 3 | 6 | ▨ | ▨ |

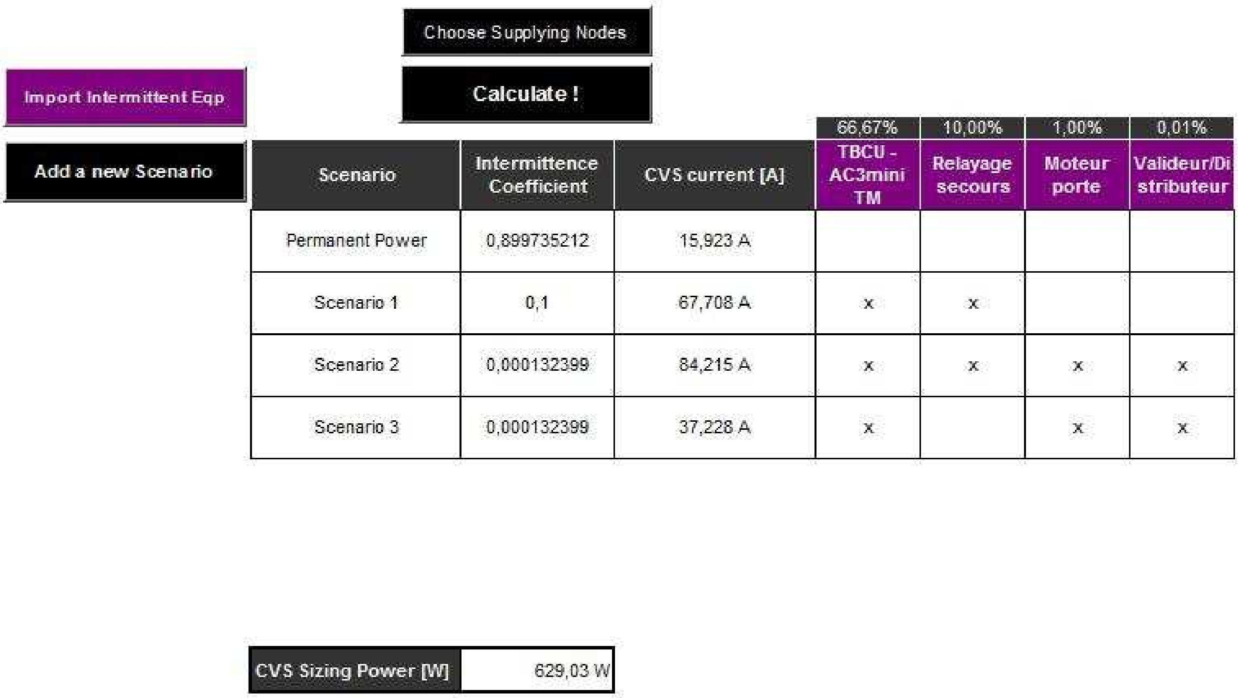
Intermittence Coefficients Parameters

---> To define the intermittence coefficients fill, for each equipment, columns 1 & 2 or columns 2 & 3 or only column 4 <---

| Items | Quantity | Unitary Duration (s) | Functioning Cycle Period (s) | Frequency (%) |
|---|---|---|---|---|
| TBCU - AC3mini TM | | 2 | 3 | |
| Maintien DJ | A | | 1 | 10,00% |
| Relayage secours | | 2 | 2 | 1,00% |
| Moteur porte | 1 | 1 | | |
| Valideur/Distributeur | | | | |

Figure 46 - Parameters sheets.

Once all the intermittent equipments are fully characterized, "go to calculation" button leads to the power balance calculation sheet, in which the user have to define the test scenarios.



| | | | 66,67% | 10,00% | 1,00% | 0,01% |
|---|---|---|---|---|---|---|
| Scenario | Intermittence Coefficient | CVS current [A] | TBCU - AC3mini TM | Relayage secours | Moteur porte | Valideur/Distributeur |
| Permanent Power | 0,899735212 | 15,923 A | | | | |
| Scenario 1 | 0,1 | 67,708 A | x | x | | |
| Scenario 2 | 0,000132399 | 84,215 A | x | x | x | x |
| Scenario 3 | 0,000132399 | 37,228 A | x | | x | x |

| CVS Sizing Power [W] | 629,03 W |
|---|---|

Figure 47 - Power balance sheet.

The "calculation" button lunches one power flow for each scenario. The scenarios' intermittence coefficients are rearranged in order to avoid the superimposition of intermittent equipments already taken into account. Every time, only the permanent power and the selected intermittent equipments are included in the power flow. The result is the CVS (one or more) current provided for each scenario. Afterwards, the quadratic sum leads to the CVS's sizing power.

## BATTERY DISCHARGE CALCULATION

To calculate a battery discharge profile, we need to define this load time profile, so how the intermittent equipments' time scheduling is made and which equipments are shed during the discharge of the battery, in order to define, for each second, the network topology on which the gauss method apply. To reach this goal, a "discharge scenario" sheet has been provided:

Pressing the "go to calculation" button, the user goes to the battery discharge calculation sheet. After calculation parameters and type of discharge calculation (30 min. CVS breakdown; 1 hour HV loss; 48 hours parking) have been chosen, the "calculate" button launches the same power flow calculation, repeating it at each second, regarding at the current topology (defined in "discharge scenario" sheet ). The battery discharge calculation works like a camera, making at each time step (one second or more), a photograph of the circuit static behavior, once the calculation has reached the convergence. The result is a collection of static functioning points that can be represented on a output chart.

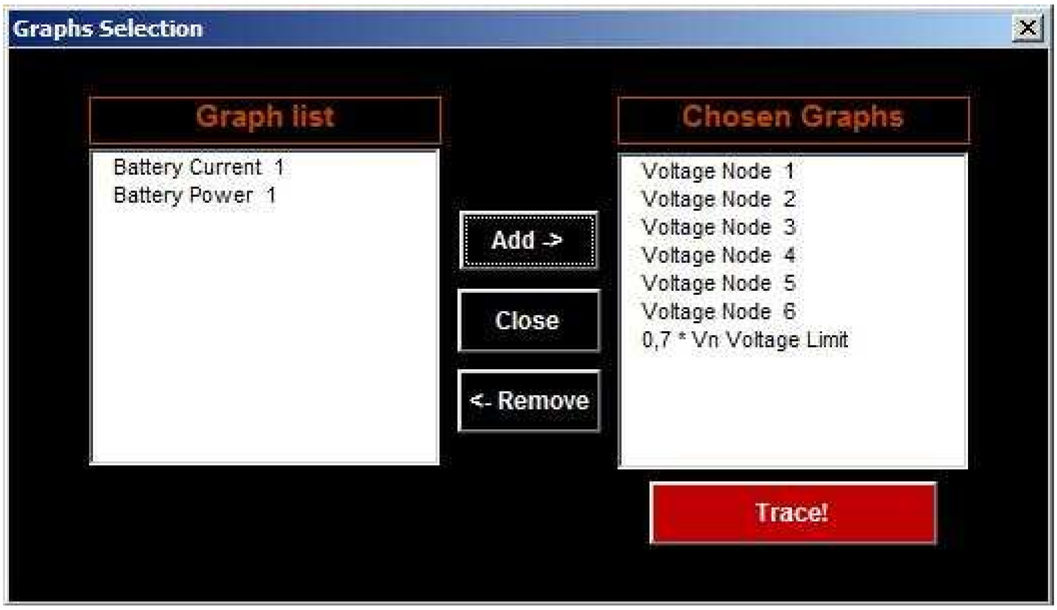


Figure 48 - Graph selection userform.

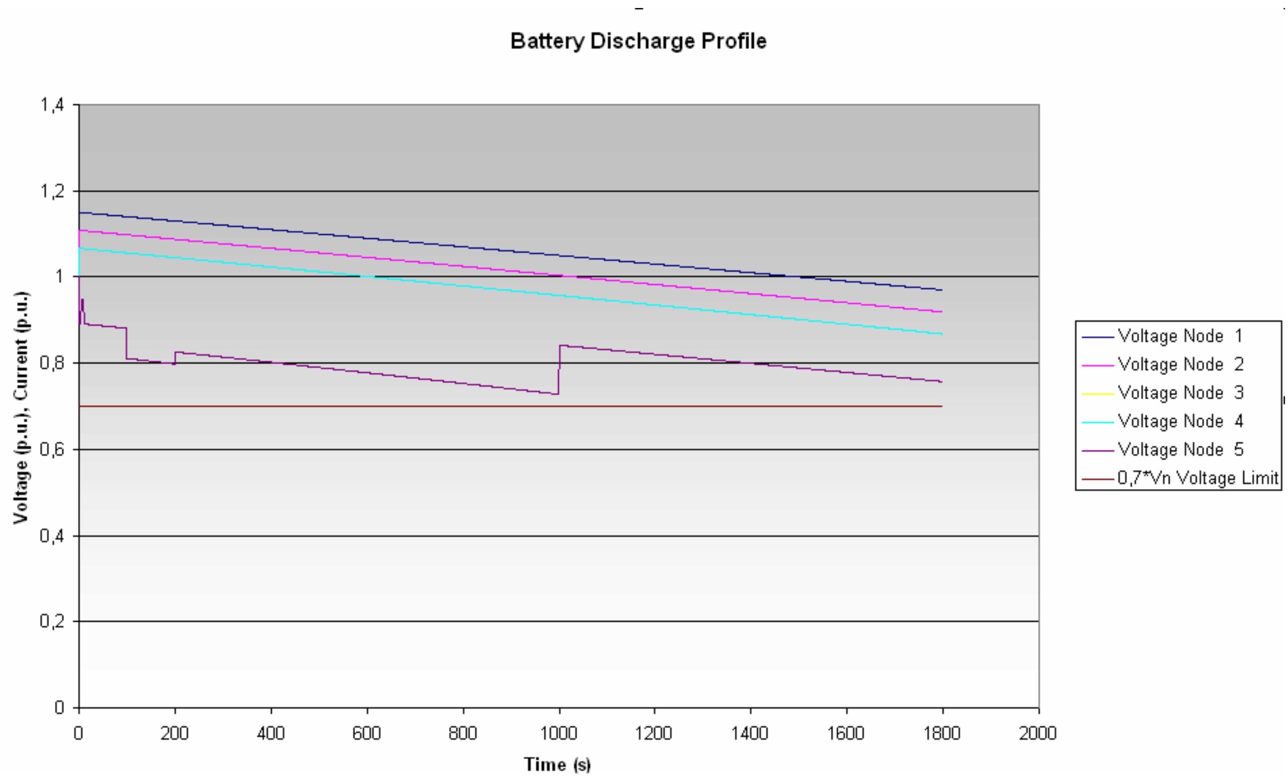


Figure 49 – Example of battery discharge profile output.

# CONCLUSIONS

These six months internship, gave me the possibility to explore a little part of huge work that stay behind a rolling stock vehicle. The TCMS and traction team of Alstom Transport VPF, that constantly work to provide always more efficient, comfortable and safety trains, during this period shared with me their knowledge, giving me a wide overview on what is a real rolling stock and what you have to do to reach a performing product, starting from a requirement or an idea.

Moreover, I appreciated what is a team work and what kinds of processes have to be followed to administrate the synergy of a work group.

This thesis, with the tool software, constitutes the final product of my internship.

The software tool, due to the huge possibility offered, has been developed in order to be modified and extended, following the technical evolution and the users' requirements.

In particular, some efforts can be done to:

- Add equipment models list, providing for measurements.
- Improve the battery model.
- Permit the sub-network study (cutting the network).
- Allow the automatic sizing of the wires (implementing the standard EN 50343).
- Increase calculation speed.
- Provide for economical estimations.
- Extend all the structures to the MV.

I want to say thanks for these experience to Prof. Morris Brenna, Ing. Nicola Fringerio, Ing. Jean-Luc Terrier, Ing. Pascal Vivegnis, Ing. Nicolas Shoemaecker, Ing. Nicolas Henry, Ing. Mathieu Gloriant, Ing. Jourgen Wax, Ing. Vincent Vanhede, Ing. Agathe Haussman, to the friends and the city of Valenciennes, to Mr. Stefane Paul and Mrs. Chaterine Pendolino and to all the other persons that offered me their support during this period.

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[C2] Modeling and Analysis of Electric Power Systems, Göran Andersson, EEH - Power Systems Laboratory, ETH Zürich, March 2003

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[D2] IEEE Standard 1476-2000.