POLITECNICO DI MILANO

Faculty of systems engineering

Masters of Science in Management, Economics,
and Industrial Engineering

Optimization of the Physical Cash Logistics in Bank Monte dei Paschi di Siena

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Abstract

Banca Monte dei Paschi di Siena (MPS), the 3rd largest Italian bank, puts a lot of efforts to improve the effectiveness and efficiency of its operations. In this thesis possible way to improve the performance of the **Physical Cash Logistics** process is described.

The bank has very wide cash Distribution Network with four types of distribution channels. This work is focused only on the Logistics process optimization **in Branches**. A branch consists of Cash Desk and one or several ATMs.

Currently, **Bank** MPS **doesn't use rational inventory management policy**. So, **the objective** of the work was **to find**, adopt and test the **methodology of Inventory management**, which will allow Banca Monte dei Paschi di Siena to manage its Physical cash Inventories in a structured way, **minimizing the logistics costs** and **assuring the pre-defined service level** for customers.

The methodology, described in this thesis, achieves the predetermined goals, **by forecasting the Demand** and **planning the Inventories** for **ATM** and **Cash Desk**.

For forecasting the demand in ATMs and Cash Desks, three forecasting approaches were compared. At the end, the decision was made in favor of **Time series approaches.** Specifically, **Winter's model** was chosen as it provides the best forecasting accuracy.

For the **ATM** Inventory Planning the **Periodic Review model** was used. The model was adjusted to overcome the problem of ATM replenishments during week-ends. While for the **Cash Desk** Inventory Planning the **Reorder point model** was chosen. The model works under the assumption that there is more cash demand than supply in a Cash Desk, i.e. that customers withdraw more cash from the branch (from both ATMs and Cash Desk), than bring it there.

The whole optimization process was realized in the Mathcad **program**. Having imported the input data into the program from the bank's IT system, it **automatically forecasts the demand**, **calculates Inventories planning parameters** and **assesses the expected benefits.** The program was applied to the data collected from the branches in Prato.

The results showed, that implementation of proposed methodology allows to **decrease the costs** of physical cash logistics **on 22%**; to **define the service level** in ATMs and Cash Desks; to **decrease the number of transportations** from external vaults to branches **on 47%**.

The **limitations** of the proposed methodology and the ways of its **further development** were presented.

Abstract

Il Monte dei Paschi di Siena ad oggi è la terza banca più grande d'Italia, mirando ad un continuo miglioramento in termini di efficacia ed efficienza delle proprie attività. In questa tesi di laurea verranno descritte le possibili modalità di miglioramento del **processo logistico dei contanti materiali**.

La banca possiede una rete di distribuzione dei contanti molto ampia, disponendo di quattro diversi tipi di canali di distribuzione. Questo studio si focalizza integralmente sull'ottimizzazione del processo logistico **nelle filiali**. In ogni filiale è presente una cassa, ed uno o più bancomat.

Attualmente **la banca non implementa una politica di gestione della giacenza razionale**. **L'oggetto** di studio è stato quindi quello di **trovare**, adottare e testare **una metodologia di gestione della giacenza** che permetterà al Monte dei Paschi di Siena di gestire le sue scorte di contanti materiali in modo strutturato, **minimizzando i costi logistici** e **assicurando un predeterminato livello di servizio** per i clienti.

La metodologia descritta in questa tesi di laurea raggiunge gli obiettivi di cui sopra **mediante la previsione della domanda** e **la pianificazione delle giacenze per il bancomat** e **per la cassa**.

Nello specifico, sono state comparate tre diverse modalità di previsione della domanda di contanti per bancomat e cassa. Alla fine, i modelli a **Serie storiche** sono risultati essere la soluzione migliore. In particolare si è deciso di utilizzare il **modello di Winter** per la sua maggiore accuratezza previsionale.

Per quanto riguarda la pianificazione della giacenza nei **bancomat** è stato applicato un **modello di Revisione periodica** "aggiustato" per potere superare il problema di rifornimento durante i giorni festivi. Per la pianificazione della giacenza di **cassa**, invece, è stato scelto il **modello del Punto di riordino**, assumendo che la domanda di contanti sia maggiore della sua offerta: i clienti prelevano più denaro di quanto ne versano.

L'intero processo di ottimizzazione è stato realizzato in Mathcad. Una volta importati i dati dal sistema informativo della banca, **il programma stima automaticamente la domanda**, **i parametri progettati di giacenza** e **valuta i profitti previsti**. I dati utilizzati sono stati raccolti dalle filiali di Prato.

I risultati mostrano che l'implementazione della metodologia proposta permette di: abbassare del 22% i costi della logistica dei contanti materiali; definire il livello di servizio di bancomat e casse; ridurre del 47% il numero di viaggi per rifornimento dai depositi esterni alle filiali.

In fine sono stati presentati i **limiti della metodologia** proposta e i **potenziali metodi di sviluppo futuro**.

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Abbreviations

MPS – Monte dei Paschi di Siena

ATM – Automated Teller Machine

MSE – Mean Squared Error

MPE – Mean Percentage Error

SDE – Standard Deviation Error

MAPE – Mean Absolute Percentage Error

TS – Tracking Signal

TC – Total Costs

ICC – Inventory Carrying Costs

PC – Procurement Costs

AICC – Annual Inventory Carrying Costs

APC – Annual Procurement Costs

EOQ – Economic Order Quantity

SS – Safety Stock

ROP – Reorder Point model

LT – Lead Time

ELT – Expected Lead Time

ED – Expected Demand

AIL – Average Inventory Level

AT – Availability Target

phc – Inventory Carrying Costs as a percentage of item value

upc – Unitary Procurement Costs

0. Summary

0.1. Problem

Banca Monte dei Paschi di Siena, the 3rd largest Italian bank, in 2007 and 2008 made two big acquisitions. These acquisitions damaged the economic state of the bank. Unfavorable conditions, having come together with the crisis, didn't facilitate the recovering of losses occurred because of big acquisitions. For that reason now the bank works on the **improving of the effectiveness and efficiency of** its **operations** even more than usually.

One of the areas which MPS decided to review and to find a way for possible improvements was the **Logistics of Physical Cash**.

Banca Monte dei Paschi di Siena has very wide cash Distribution Network. The bank owns four types of distribution channels: Branches, External ATM, Home serviced customers, External "cash in" safes. The current **work is focused only on** the Logistics process optimization in **Branches**. It was done for the following reasons:

- According to the bank's estimation, it incurs the biggest Logistics Costs in Branches,
- Optimization of the Physical Cash Logistics process for External ATMs is already in process,
- MPS can only partly influence on the Logistics flow of its cash in External Vaults,
- It is difficult to retrieve data necessary for the Home Serviced customers and External "Cash in" safes Physical Cash Logistics optimization.

Currently, **Bank** MPS **doesn't use rational inventory management policy**. Stocks are managed in an intuitive way. As a result, sometimes people in a branch:

- Use an ATM as a place for keeping stocks;
- Short-term demand forecasting for ATMs;
- Replenish and then immediately withdraw almost the same amount of money from a Cash Desk;
- Keep stock level in a Cash Desk much higher than maximum authorized level.

0.2. Objectives

The objective of the work was **to propose the methodology**, which will allow Banca Monte dei Paschi di Siena **to optimize the Physical cash Inventories management**.

The methodology must:

- Minimize logistics costs for a branch,
- Assuring the pre-defined service level for customers

By:

- Forecasting the Demand and
- Planning the Inventories

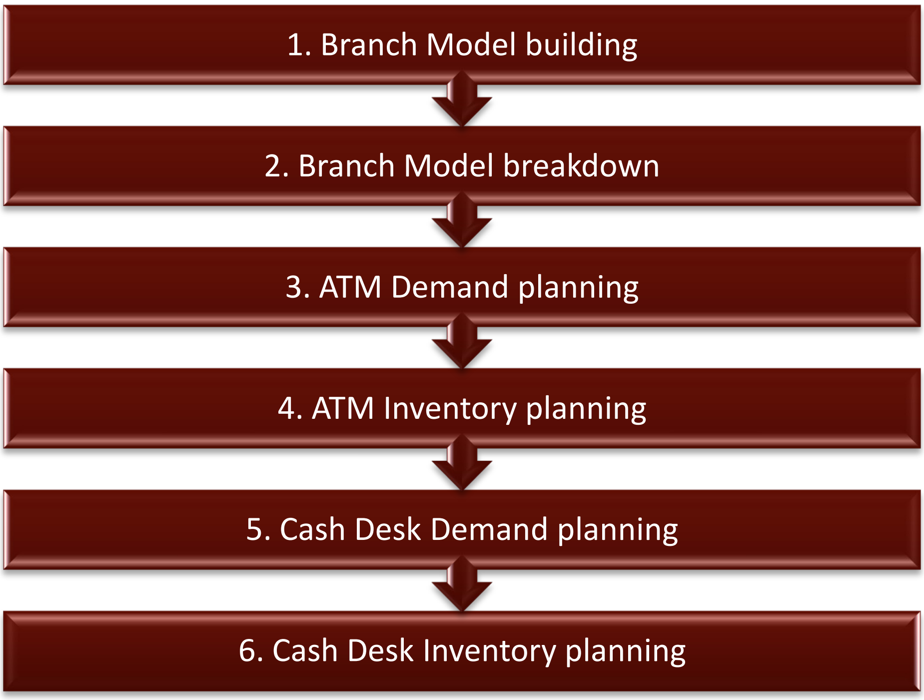
For:

1) ATM
2) Cash Desk

The proposed methodology has to be applied to the data obtained from the 11 branches in Prato (the bank uses branches in Prato as a testing ground for proving new approaches). The expected costs savings and other possible benefits must be evaluated. Moreover, limitations and fields of possible further development of the methodology have to be identified.

0.3. Methodology

The **optimization of Physical cash Inventories management process** in branches has the following steps:

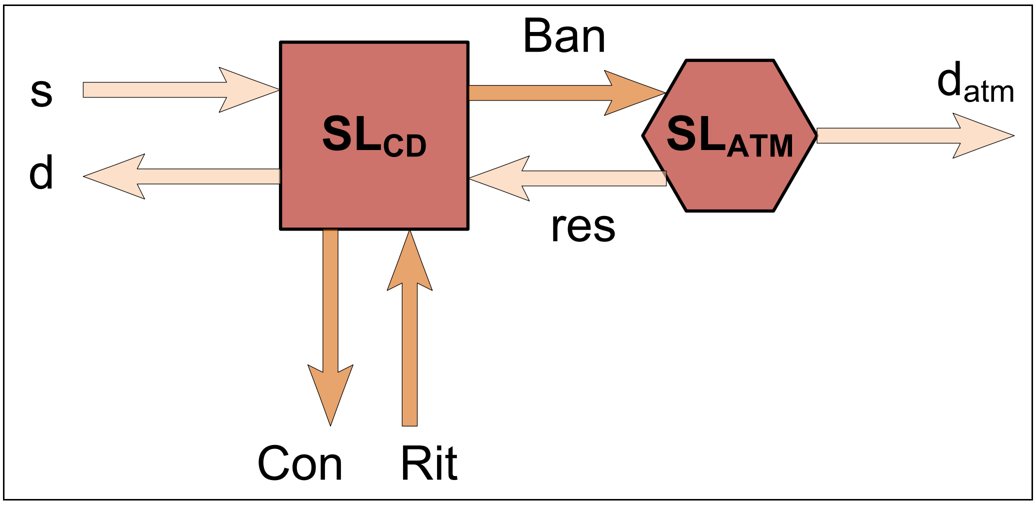


The whole optimization process was realized in the Mathcad **program**. Having imported the input data into the program from the bank's IT system, it **automatically forecasts the demand** and **calculates Inventories planning parameters**, which minimize logistics costs for an ATM and a Cash Desk, assuring the pre-defined service level and assessing the expected benefits.

0.3.1. Branch model building

For the branch Inventory management optimization, a model of a branch, shown on the figure below, was build. The model consists of a Cash Desk and an ATM. It has three types of parameters: external, controlled, decisional.

Model of a branch



Legend:

External variables:

- **d** – physical cash flow from Cash Desk to Customers
- **s** – physical cash flow from Customers to Cash Desk
- $\mathbf{d_{atm}}$ – physical cash flow from ATM to Customers
- **res** – physical cash left in ATM at the moment it has been opened for replenishment

Controlled variables:

- $\mathbf{SL_{CD}}$ – stock level in Cash Desk
- $\mathbf{SL_{ATM}}$ – stock level in ATM

Decisional variables:

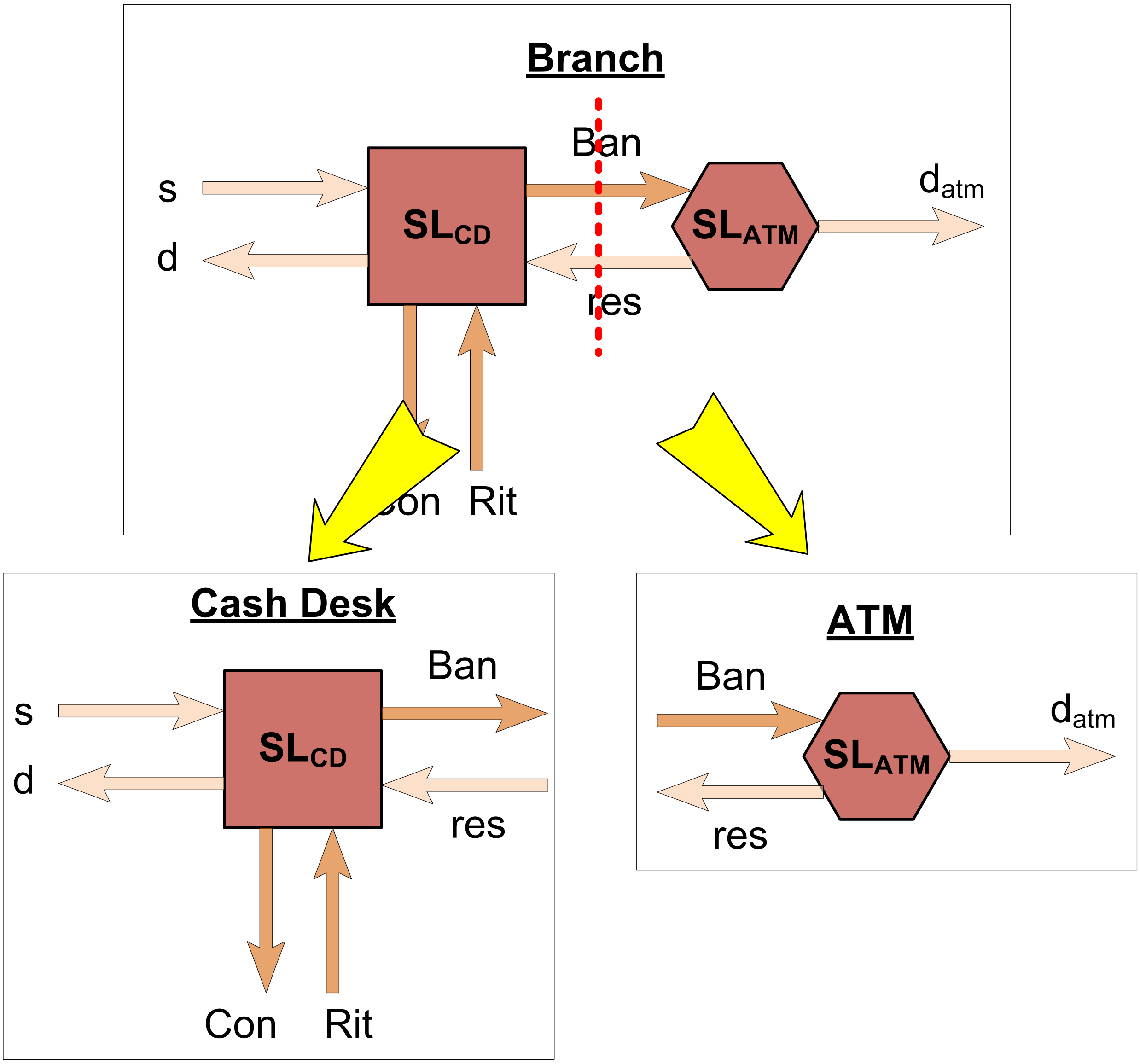
- **Ban** – physical cash flow to replenish ATM from Cash Desk
- **Con** – physical cash flow to replenish Cash Desk from External Vault
- **Rit** – physical cash flow from Cash Desk to External Vault

In order to optimize the Inventory Management process in a Branch, first of all, the external variables must be forecasted, then, the values of decisional variables, allowing reaching the objectives of the optimization process, must be identified.

0.3.2. Branch Model breakdown

Decisional variables, optimizing the inventory, cannot be found directly from the model, as it is a bit comprehensive. To solve this problem the model was broken down into two parts, as it is shown on figure below.

Branch model breakdown



First of all, the future values of External variables (**d_{atm}, res, d/s**) were forecasted.

Then, using the model for ATM, decisional variable **Ban** (physical cash flow to replenish ATM from Cash Desk) was identified. Using the model for Cash Desk, decisional variables **Con** (physical cash flow to replenish Cash Desk from External Vault) and **Rit** (physical cash flow from Cash Desk to External Vault) were found.

Finally, controlled variables **SL_{CD}** and **SL_{ATM}** were used in order to assess the benefits, which the methodology provides.

0.3.3. ATM. Demand planning

Objective: forecast the future demand for cash in ATM from the bank's customers.

Choosing forecasting Methodology

Choosing the best approach for cash demand forecasting, three following methodologies were considered:

1) "Time series" approaches
2) Causal approaches
3) Qualitative approaches

Having compared Pros and Cons of the approaches mentioned above and having adopted them to our specific case, **the choice was done in favor of <u>Time series approaches</u>.**

Choosing the Time Series model

The following Time series models were considered:

1. <u>Exponential smoothing models:</u>
   1) Simple exponential smoothing (Brown's model);
   2) Exponential smoothing with trend adjustment;
   3) Exponential smoothing with trend and seasonality (Winter's model);
   4) Simple adaptive exponential smoothing;
   5) Exponential smoothing with damped trend.
2. <u>Autoregressive models:</u>
   1) Moving average models;
   2) Autoregressive moving average models;
   3) Autoregressive integrated moving average models.

"The greater effort, required to develop and identify autoregressive models, is not always rewarded by an improvement in prediction accuracy over simpler methods." [(Vercellis, 2009), p.211]

The Winter's model provided the best results in terms of accuracy out of all exponential smoothing models having been considered.

Based on the reasons explained above, **Winter's model** (exponential smoothing with trend and seasonality) was taken as a model for the ATM cash demand forecasting.

Forecasting with Winter's model

Forecasting with Winter's model process consists of the following steps:

1. Data Importing
2. Data Transformation
3. Data Validation
4. Data Analysis
5. Initialization of the Time Series
6. Adaptation of the Time Series
7. Accuracy measurement
8. Demand forecasting

0.3.4. ATM. Inventory planning

Objective: define the Inventory management policy and parameters, which will minimize Logistics Costs for an ATM, assuring the pre-defined service level for customers.

Choice of the Inventory management policy and parameters

For the **ATM** Inventory Planning the **Periodic Review model** was chosen. The main reason of such choice is that Periodic Review model doesn't require the continuous control of the inventory level, which cannot be implemented for ATMs, because there are no operators in branches on week-ends, while there is still demand, as customers withdraw money from ATMs 7 days a week.

The parameters of Inventory management were found according to the Periodic Review model.

Adjustment of the replenishment plan (for week-ends)

The Periodic review model could not be applied in the pure way to all the branches, because sometimes operators in a branch will have to replenish the stock of an ATM in the week-end, which is not possible.

To overcome the problem with the ATM replenishments during week-ends 4 types of solution (**4 policies**) were considered. Then, the "ideal" replenishment plan (with replenishments during week-end) was converted into 4 real (without replenishments during week-end).

The best plan, providing the **lowest annual total costs,** was chosen.

0.3.5. Cash Desk. Demand planning

Objective: forecast the future demand for cash in Cash Desk from the bank's customers.

Winter's model (exponential smoothing with trend and seasonality) was chosen as forecasting model for Cash Desk demand planning for the same reasons as in Demand planning for ATMs.

Forecasting the future demand process for a Cash Desk consists of the same steps as the one for an ATM.

0.3.6. Cash Desk. Inventory planning

Objective: define the Inventory management policy and parameters, which will minimize Logistics Costs for a Cash Desk, assuring the pre-defined service level for customers.

Choice of the Inventory management policy and parameters

For the **Cash Desk** Inventory Planning the **Reorder point model** was chosen, as, on average, it ensures lower inventory level than in the Periodic review model. A branch manager can control the stock level of Cash Desk continuously 5 days a week, and the stock level is not changed during the week-end, as branches are closed.

Constraint:

Reorder Point model works only if there is more cash demand than supply in a Cash Desk ($\Sigma \boldsymbol{D}_{\Sigma} > 0$), i.e. if customers withdraw more cash from the branch (from both ATMs and Cash Desk), than bring it there.

The parameters of Inventory management were found according to the Reorder Point model.

0.4. Results

The methodology of Inventory management, allowing Banca Monte dei Paschi di Siena to optimize the Physical cash Inventories management, was proposed.

The Mathcad program, automatically calculating the Inventory management plan for a branch (both for ATMs and Cash Desk), according to the proposed methodology, was developed. The program was applied to define the Inventory management plans and expected benefits for the branches in Prato. Moreover, limitations and fields of possible further development of the methodology were identified.

The results showed, that implementation of proposed methodology allows:

- **decrease the costs** of physical cash logistics **on 22%**;
- **define the service level** in ATMs and Cash Desks;
- **decrease the number of** cash **transportations** from an external vault to a branch **on 47%.**

0.4.1. Costs Savings

The estimation of expected Costs Savings for all branches of MPS, obtained by testing the methodology on branches in Prato, is presented in the table below.

Estimation of expected Costs Savings for all branches of MPS

Branch	Δ	Δ%
Prato AG.1	5 338 €	34%
Prato AG.2	Supply>Demand[1]	Supply>Demand
Prato AG.3	2 962 €	19%
Prato AG.4	360 €	3%
Prato AG.5	3 089 €	24%
Prato AG.6	Supply>Demand	Supply>Demand
Prato AG.7	6 044 €	29%
Prato AG.8	1 718 €	14%
Prato AG.9	3 530 €	30%
Prato	Supply>Demand	Supply>Demand
Prato AG.11	Supply>Demand	Supply>Demand
Average Saving per branch:	**€3 292**	**22%**
Total Savings (2 600 branches)	**€8,6 mln**	**22%**

0.4.2. Defined Service Level

The proposed approach for Inventory management assures the predefined Service level.

For ATMs

For all calculations, the **Service Level** for ATMs $\boldsymbol{k_{atm} = 2,33}$ was used. Such service level will ensure the 99% probability to avoid stock out.

For Cash Desk

For all calculations, the **Service Level** for Cash Desks $\boldsymbol{k_{cd} = 1,75}$ was used. Such service level will ensure the 96% probability to avoid stock out.

We couldn't estimate the current Service Level in ATMs and Cash Desks. However, using the Mathcad program, which was developed for Inventory Management optimization, MPS can easily calculate the economic impact of the Service Level variation and choose the best values of $\boldsymbol{k_{atm}}$ and $\boldsymbol{k_{cd}}$.

[1] The proposed way of Cash Desk Inventory Planning works only if there is more Demand than Supply in a Cash Desk.

0.4.3. Improvements in environmental and social performance

The proposed methodology reduces the number of transportations from External Vault to a branch (see table below). Decreasing the number of transportation will lead to the reducing of the CO_2 emission and decreasing of the number of car accidents in the streets. All this will improve the environmental and social performance of the bank.

Estimation of expected reduction in the number of transportations from MPS branches to External Vaults

Branch	As Is	To Be	Δ	Δ%
Prato AG.1	152[2]	74	78	51%
Prato AG.2	Supply>Demand[3]	Supply>Demand	Supply>Demand	Supply>Demand
Prato AG.3	146	68	78	53%
Prato AG.4	106	66	40	38%
Prato AG.5	126	72	54	43%
Prato AG.6	Supply>Demand	Supply>Demand	Supply>Demand	Supply>Demand
Prato AG.7	154	100	54	35%
Prato AG.8	100	54	46	46%
Prato AG.9	84	32	52	62%
Prato	Supply>Demand	Supply>Demand	Supply>Demand	Supply>Demand
Prato AG.11	Supply>Demand	Supply>Demand	Supply>Demand	Supply>Demand
Average Reduction per branch:			**57**	**47%**
Total Reduction (for 2 600 branches):			**148 200**	**47%**

0.4.4. Limitations and Further development

During the tests of the proposed methodology on different branches, some limitations of the described approach and several ways for its further development were identified.

Limitations

- The Cash Desk Inventory Planning methodology works only under the assumption that during the year there is more cash Demand than Supply in the Cash Desk. In other words, customers withdraw more cash from the branch, than bring it there.
- Forecasting model for Cash Desks is not very flexible. For some branches it doesn't provide forecast with sufficient accuracy. This can be explained by the small amount of available past data for Cash Desks.

[2] Unit of measure: [number of transportations/year]

[3] The proposed way of Cash Desk Inventory Planning works only if there is more Demand than Supply in a Cash Desk.

Further development

- To include into the Cash Desk Inventory Planning the case, when there is more Supply than Demand.
- To check the flexibility of forecasting model for Cash Desk, when the data for past 2 years will be available.

The methodology presented in this thesis will be tested in branches in Prato. If the practical results will correspond to the theoretical estimations, the methodology will be implemented in other branches of Monte dei Paschi di Siena.

MONTE DEI PASCHI DI SIENA
BANCA DAL 1472

1. Introduction

1.1. About the Montepaschi Group

"Banca Monte dei Paschi di Siena (MPS), founded in 1472, is considered to be the oldest bank in the world. Today's parent company of Italy's third largest banking group, the bank holds significant market shares in all areas of business.

The Montepaschi Group is present all over Italy and in the major international financial centres, with operations ranging from traditional banking activities to Private Banking (mutual funds, wealth management, pension funds, and life insurance policies) and Corporate Banking (project finance, merchant banking, and financial advisory), with a special vocation for household accounts and small and medium enterprises. With some 33,000 employees and over 3,000 branches, the Montepaschi Group offers its services to more than six million customers (Banca Monte dei Paschi di Siena)."

1.2. Context

Just before the economic crisis MPS made two big acquisitions:

"2007. Acquisition of 55% of Biverbanca - Cassa di risparmio di Biella e Vercelli (700 employees and 105 branches in 7 provinces in Piedmont, Lombardy, Lazio and the Aosta Valley). **2008.** Acquisition of Banca Antonveneta, company founded by a process of mergers and aggregation of banks with strong territorial roots. (Banca Monte dei Paschi di Siena)"

These acquisitions damaged the economic state of the bank. Unfavorable conditions, having come together with the crisis, didn't facilitate the recovering of losses occurred by the reason of big acquisitions. For that reason now the bank works on the improving of the effectiveness and efficiency of its operations even more than usually.

1.3. Optimization of the Physical Cash Logistics

One of the areas which MPS decided to review and to find a way for possible effectiveness and efficiency improvements was the Logistics of Physical Cash.

The project of Physical Cash Logistics Optimization was developed under the supervision and information support of Guido Giorgetti, who is Head of the Group Logistical Support in the Area of Costs and Logistics[4].

[4] Responsabile Servizio Economato di Gruppo, Area Costi e Logistica

2. Description of the Physical Cash Distribution Network.

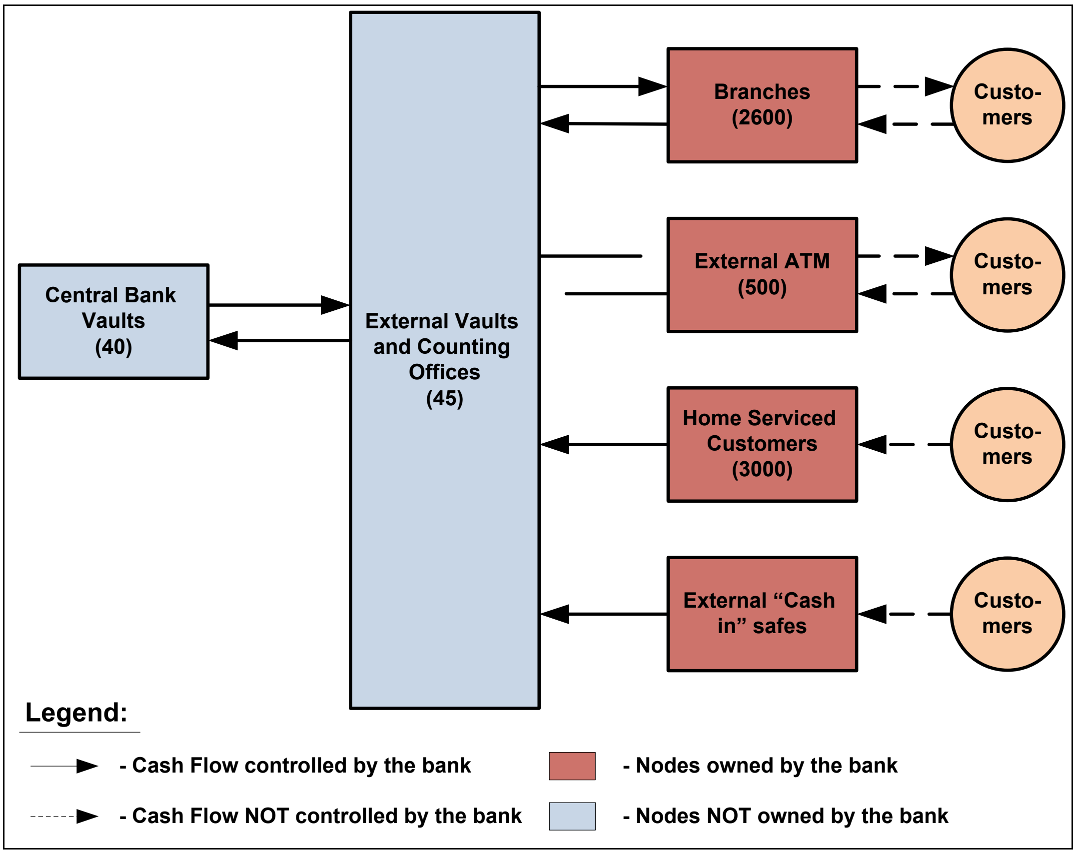
Banca Monte dei Paschi di Siena has very wide cash Distribution Network (see Figure 1). The bank owns four types of distribution channels:

- Branches
- External ATM
- Home serviced customers
- External “cash in” safes

External Vaults and **Central Bank Vaults** are owned and controlled NOT by the bank.

All transportations of cash are outsourced. The bank pays fix rate for one shipment. The rate depends only on type of being connected entities, but not on the distance between two nods, on the amount of money to be transported or on something else.

Figure 1. Physical Cash Distribution Network



1. **Branches**

Branch is a classic retail entity where a bank offers a range of services to its customers. The services can be offered face-to-face in a Cash Desk or automatically by means of ATM.

Banca Monte dei Paschi di Siena has about 2 600 branches.

2. **External ATM**

External ATM is an ATM standing alone, beyond bank's branch. MPS has about 500 External ATM.

3. **Home serviced customers**

Home serviced customer is a company which has an account in MPS. Such company can be a store, for example. Home service customer accumulates cash during its operation. Then it contacts with MPS to withdraw the cash from its cash desk or safe and to put the money on company's account in MPS.

MPS has about 3 000 Home serviced customers.

4. **External "cash in" safes**

External "cash in" safes are safes standing alone, beyond the bank's branches. One can put the cash into special box and then to put the box into the "cash in" safe. In the box there must be a paper with the information about client's account, so that the bank could put the money on it.

3. Inventory Management in Branches. AS IS situation.

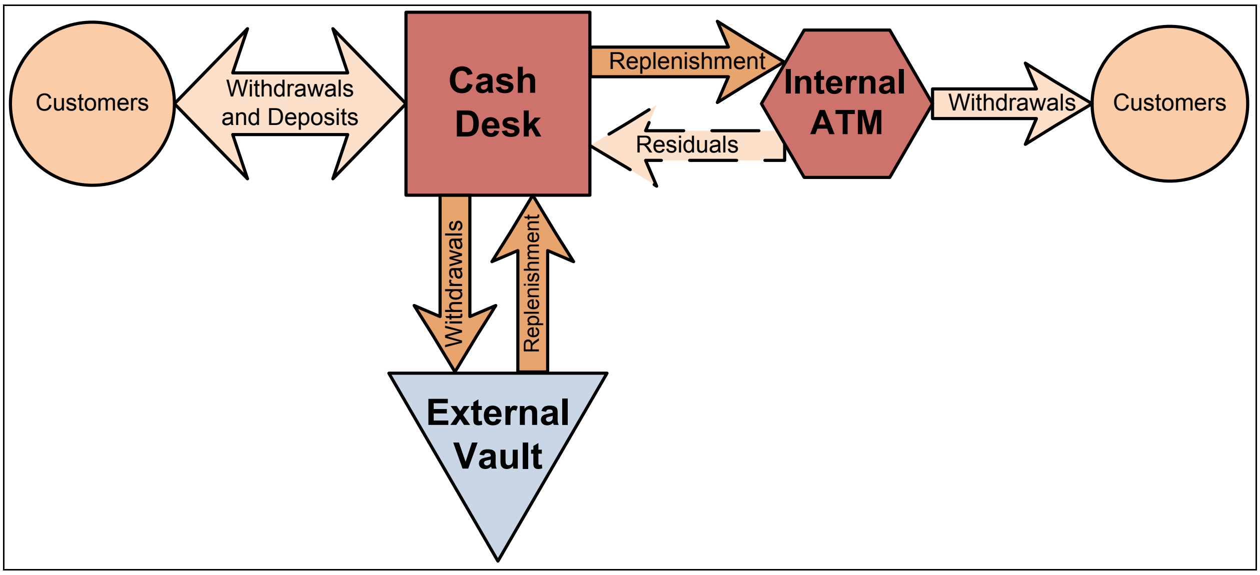
The current **work is focused only on** the Logistics process optimization in **Branches**. It was done for the following reasons:

- According to the bank's estimation, it incurs the biggest Logistics Costs in Branches,
- Optimization of the Physical Cash Logistics process for External ATMs is already in process,
- MPS can only partly influence on the Logistics flow of its cash in External Vaults,
- It is difficult to retrieve data necessary for the Home Serviced customers and External "Cash in" safes Physical Cash Logistics optimization.

3.1. Description

Banca Monte dei Paschi di Siena has about 2 600 branches. A branch consists of Cash Desk and one or several internal ATM (see Figure 2). Branches work 5 days a week, from Monday to Friday.

Figure 2. Principal scheme of a branch with one Internal ATM



Through **Cash Desk** the bank provides face-to-face services. Among such services are the withdrawals of cash from client's account and depositing money on it.

The manager of branch decides when and how much to withdraw cash from the branch to External Vault, if it has accumulated too much of. The branch manager also decides when and by how much to replenish the stock of cash in the branch, if its stock draws to the zero. He can continuously control the stock level in the Cash Desk.

The physical cash transportation to/from External Vault from/to branch is outsourced. The lead time of delivery is one day, provided that the order was put at the first part of the day, otherwise it takes two days. MPS pays the fix rate for one shipment for all branches, for all External Vaults.

The cost of a shipment is 31,1+20% (Imposta sul Valore Aggiunto) = 37,32€.

Internal ATMs (or just ATMs further) are ATMs standing on the branch territory. They allow customers to withdraw cash from their accounts 24 hours a day, 7 days a week.

The stock of Internal ATMs is replenished by operators working in the branch. It can be done only 5 days a week, when the branch is opened. To replenish an ATM two operators are needed. On the average it takes 30 minutes to replenish an ATM. The replenishment is implemented only from the Cash Desk, not from the External Vault directly. Operators put the predefined amount of money from a Cash Desk to an ATM, withdraw the residuals – the money remained in an ATM – and put it back to the Cash Desk.

3.2. Problems identified

Currently, Bank MPS doesn't use rational inventory management policy. Stocks are managed in an intuitive way. As a result, sometimes people in a branch:

1) Use an ATM as a place for keeping stocks;
2) Short-term demand forecasting for ATMs;
3) Replenish and then immediately withdraw almost the same amount of money from a Cash Desk;
4) Keep stock level in a Cash Desk much higher than maximum authorized level.

Using ATM as a place for keeping stocks, when there is too much cash in Cash Desk (see Figure 3). Sometimes people in branch transfer the money from Cash Desk to ATM only because the level of stock in Cash Desk has sharply increased and became too high. Even though there may be enough cash in ATM. Such actions do not decrease the overall stock level in a branch, but operators spend their working time on the secondary activities, thus incurring "procurement" costs. Moreover, for both Cash Desk and ATM it is not defined which stock level is considered to be high and which one is to be low.

Short-term demand forecasting for ATMs. The bank has forecasting system which forecasts the future demand from an ATM in a short term and signalizes operators in branch when the ATM will be out of stocks. The amount of money to replenish the ATM's stock is decided by operators in branch and it is always intuitive. Such approach allows avoiding the stock out in week-end (in practice it happens not always), but it doesn't minimize the yearly total costs (Inventory Carrying + Replenishment Costs).

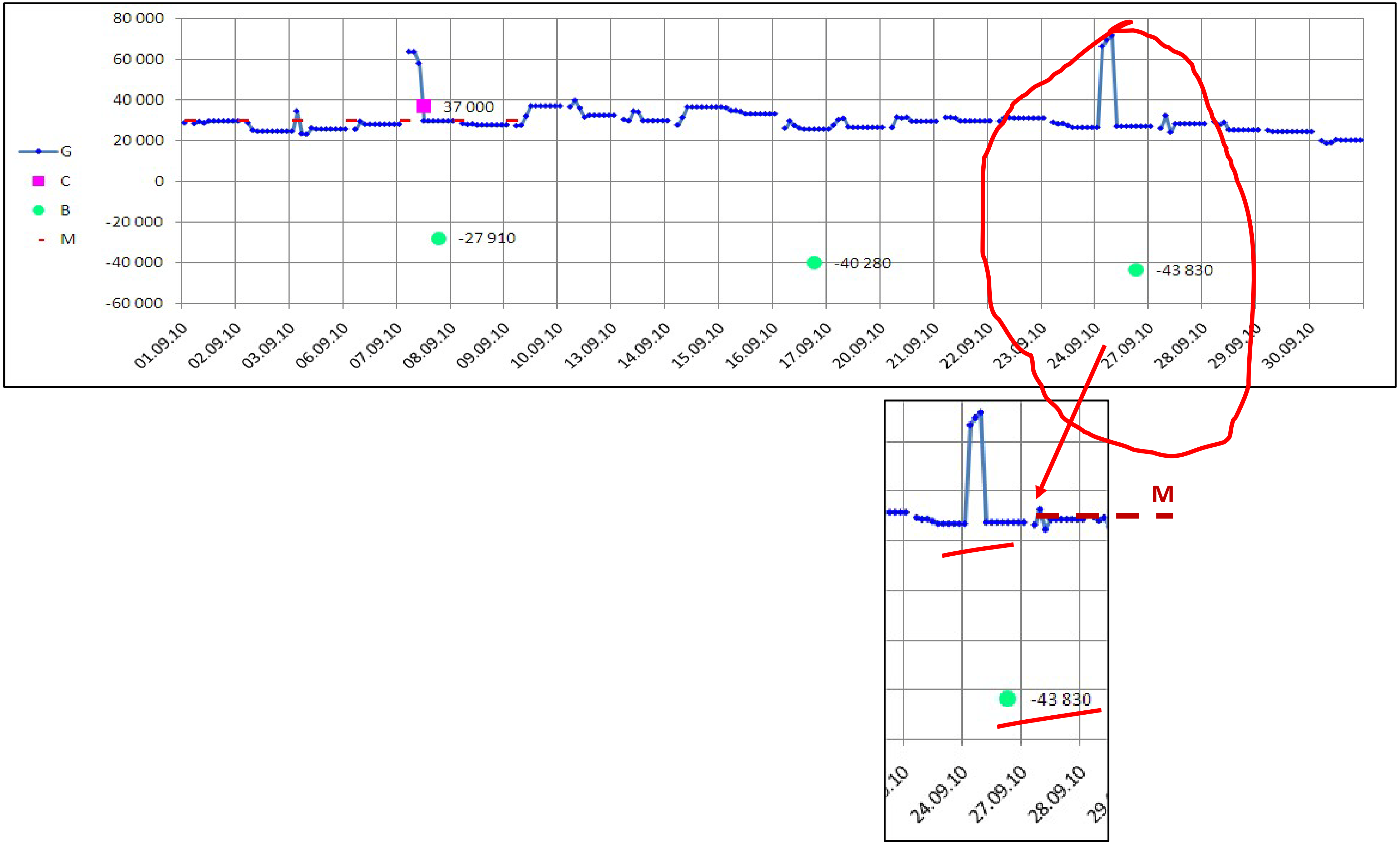
Replenishment and then immediate withdrawal of almost the same amount of money from a Cash Desk **(**see Figure 4**)** lead to the unfounded procurement costs, not decreasing the actual stock level in the branch.

Keep stock level much higher than Maximum authorized level (see Figure 5**)**, which was once defined for each branch in order to push branches' managers not to keep big stocks. The problem is that this maximum level in practice is considered like kind of minimum. Moreover, now it is hard to recall which logic was behind setting one or another value of the Maximum authorized level.

IT system allows collecting all necessary data for the applying of rational inventory management approach in branches.

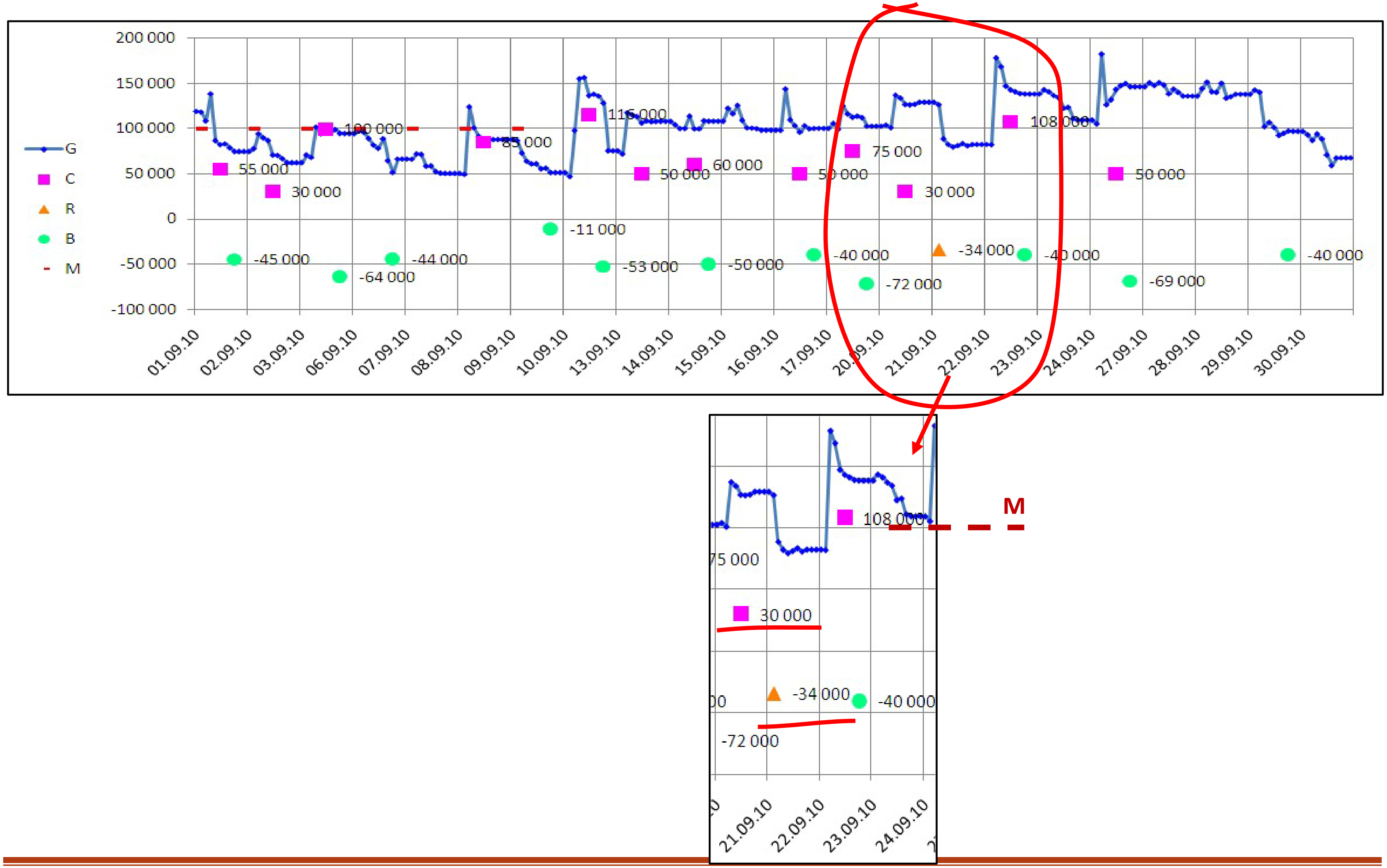
LEGENDA: B=BANCOMAT (VOCE 1003.02.003); T=TRANSFERT (VOCE 1003.02.005); R=RITIRO; C=CONSEGNA; G=GIACENZA DELLA CASSA (VOCE 1.01);M=GIACENZA

Figure 3. Using ATM as place for keeping stocks. Carmignano, 5072



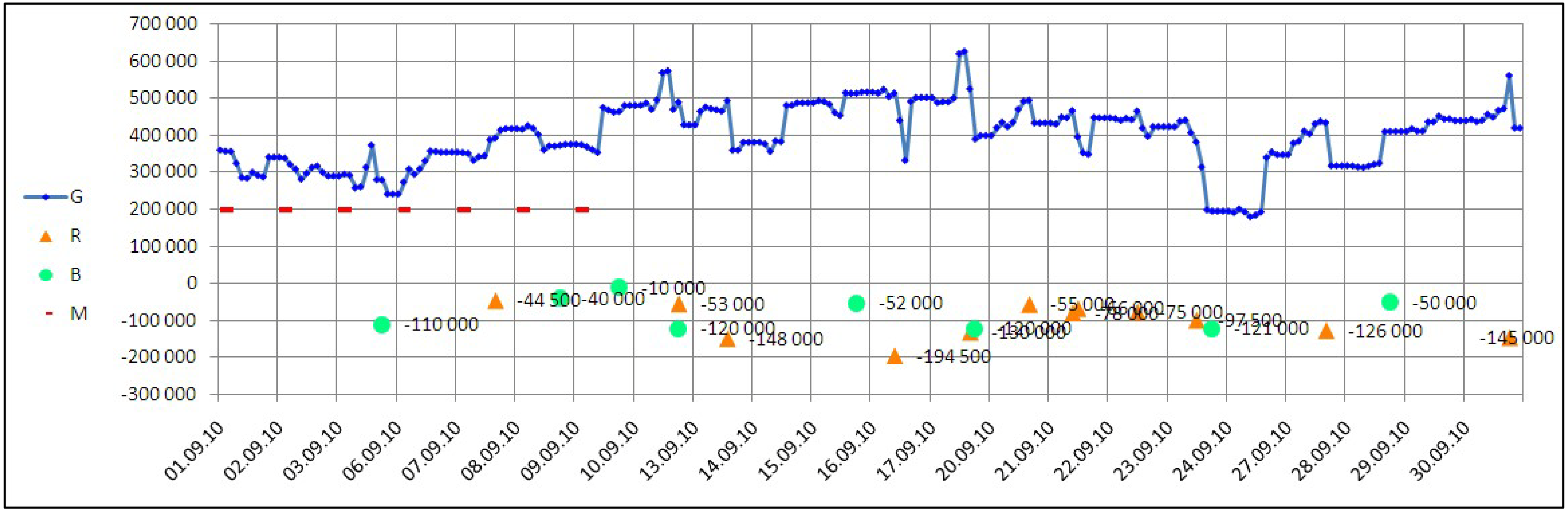
LEGENDA: B=BANCOMAT (VOCE 1003.02.003); T=TRANSFERT (VOCE 1003.02.005); R=RITIRO; C=CONSEGNA; G=GIACENZA DELLA CASSA (VOCE 1.01);M=GIACENZA

Figure 4. Replenishment and immediate withdrawal of almost the same amount of money. Casellina, 5072



LEGENDA: B=BANCOMAT (VOCE 1003.02.003); T=TRANSFERT (VOCE 1003.02.005); R=RITIRO; C=CONSEGNA; G=GIACENZA DELLA CASSA (VOCE 1.01);M=GIACENZA

Figure 5. The stock level is always much higher than maximum authorized level (red dashed line). Empoly, 5072



4. Proposed methodology for Inventory Management optimization in Branches

4.1. Objectives

Having analyzed the problems, described above, the objectives of Inventory Management optimization were formulated, and the methodology for their achievement was suggested.

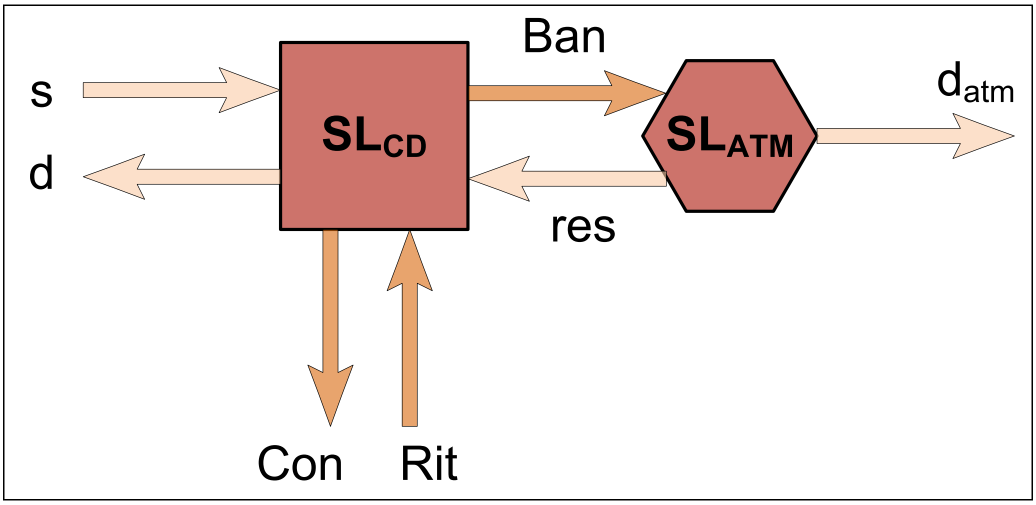
Objectives:

- Minimize logistics costs for a branch,
- Assuring the pre-defined service level for customers.

4.2. Model of a branch

On the basis of a branch description the model of a branch, consisting of a Cash Desk and one ATM was build (see Figure 6).

Figure 6. Model of a branch



Legend:

External variables:

- **d** – physical cash flow from Cash Desk to Customers
- **s** – physical cash flow from Customers to Cash Desk
- **d_{atm}** – physical cash flow from ATM to Customers
- **res** – physical cash left in ATM at the moment it has been opened for replenishment

Controlled variables:

- **SL_{CD}** – stock level in Cash Desk
- **SL_{ATM}** – stock level in ATM

Decisional variables:

- **Ban** – physical cash flow to replenish ATM from Cash Desk
- **Con** – physical cash flow to replenish Cash Desk from External Vault
- **Rit** – physical cash flow from Cash Desk to External Vault

The objectives, formulated above, can be rephrased in this way:

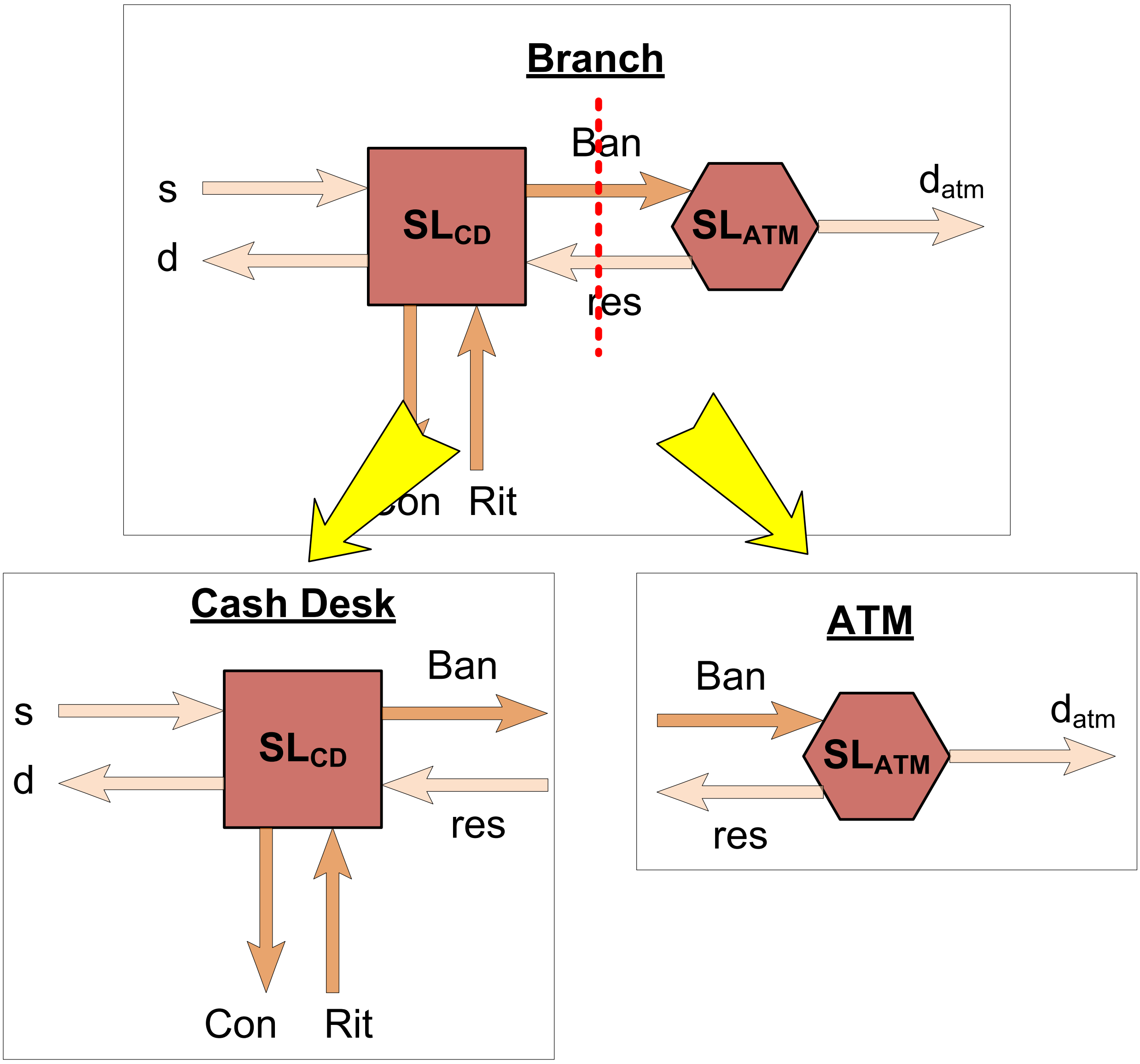
- Define the decisional variables (Ban, Con and Rit), which will
- Minimize logistics costs for a branch,
- Assuring the pre-defined service level for customers

To define a decisional variable means to define when and how much cash to put/withdraw to/from the stock.

4.3. Model breakdown

Decisional variables, optimizing the inventory, cannot be found directly from the model, presented on the Figure 6, as it is a bit comprehensive. To simplify it we can break it down into two parts, as it is shown on Figure 7.

Figure 7. Branch model breakdown



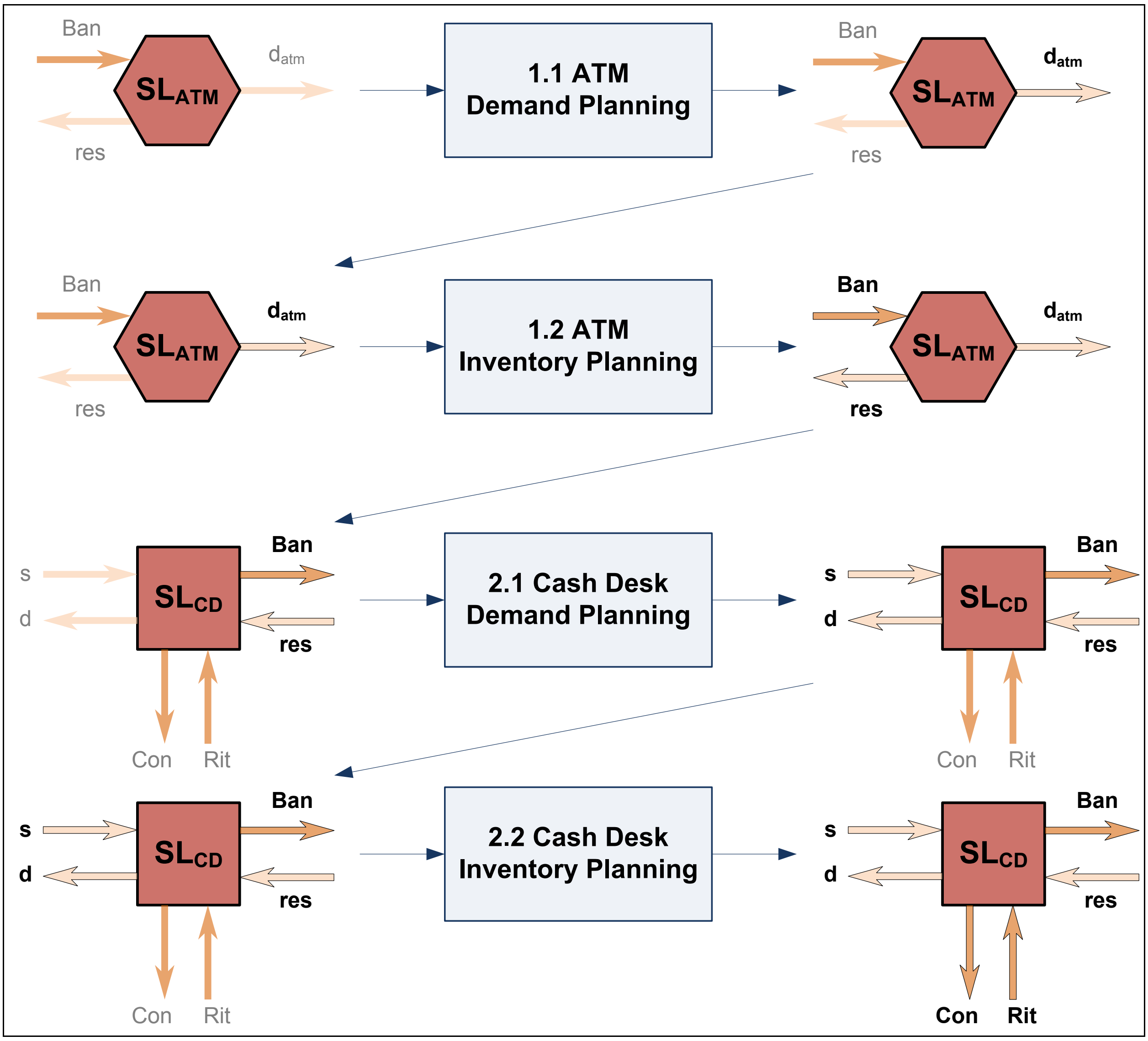
Starting with the model for ATM, we can identify decisional variable **Ban** (physical cash flow to replenish ATM from Cash Desk). Then, using the model for Cash Desk, it is possible to identify decisional variables **Con** (physical cash flow to replenish Cash Desk from External Vault) and **Rit** (physical cash flow from Cash Desk to External Vault).

Before the identification of Decisional variables, the future values of External variables (**d_{atm}, res, d/s**) must be forecasted.

4.4. Branch Inventory Management optimization process

The steps of the Inventory Management optimization process for a branch are shown on the Figure 8.

Figure 8. Inventory Management optimization process



Summing up everything written above, it can be said that Branch Inventory Management optimization has the following objectives and process:

Objective:

- Minimize logistics costs for a branch,
- Assuring the pre-defined service level for customers

By:

- Forecasting the Demand and
- Planning the Inventories

For:

1) ATM
2) Cash Desk

The whole optimization process is realized in the Mathcad **program**. Having imported the input data into the program, it **automatically forecasts the demand** and **calculates Inventories planning parameters**, which minimize logistics costs for ATM and Cash Desk, assuring the pre-defined service level and assessing the expected benefits.

5. ATM. Demand planning

ATM Demand Planning is the first step of Branch Inventory Management optimization Figure 9.

Figure 9. ATM Demand Planning as a part of Branch Inventory Management optimization

<u>Objective:</u>

Forecast the future demand for cash [euro] in ATM from the bank's customers.

<u>Input:</u>

- Time series of the cash withdrawals from ATMs in the past.

<u>Output:</u>

- Daily demand forecast for next year;
- Measure of the plan accuracy.

<u>Process settings:</u>

Time bucket: 1 day

Forecasting horizon: 1 year

Frequency: 1 month

5.1. Choosing forecasting Methodology

Choosing the best approach for cash demand forecasting, three following methodologies were considered:

1) “Time series” approaches
2) Causal approaches
3) Qualitative approaches

The Pros and Cons of each methodology are adduced in the Table 1, Table 2, Table 3. (Perego & Mangiaracina, Demand Forecasting, "Logistics Management" course, lecture 14, 2010)

5.1.1. “Time series” approaches

Table 1. PROs and CONs of “Time Series” approaches

PROs	CONs
Relatively simple	Limited consideration of the external factors
Based on historical data, which is available	
Easy to automate	Long set up periods might be required to select and set the models
If preceded by the analysis of the time series give a good level of understanding of the demand behavior (trend and seasonality)	
Easy to update the models	

We have the historical data about the daily cash withdrawals from all ATMs of MPS starting from December 2009. The fact that Time series can be automated and easily updated is important, as the bank owns almost 3’000 branches and manual implementation of forecasting technique is almost impossible. During interview with a manager from MPS Costs and Logistics department, it got clear that most likely there will be weekly and yearly seasonality in the ATM cash withdrawals.

Even though Time series has limited consideration of the external factors, it is not the main problem, as there are many external factors influencing ATM cash withdrawals. It is almost impossible to take into account all external factors. Moreover for ATMs in different areas, such factors can be completely different.

Having applied Pros and Cons of “Time series” approaches to our specific case, it seems that Time series fit well to the problem being considered.

5.1.2. Causal approaches

Table 2. PROs and CONs of Causal approaches

PROs	CONs
They take into account the external factors that influence or explain the demand (price, weather conditions, etc.).	They require heavy data analysis
Higher "intelligence" on the factors explaining the demand	A functional relationship has to be built and validated
Possibility to integrate/correct the forecasts based on time series	It's necessary to be able to forecast the casual factors better than the demand (dependent variable)

It is difficult to mark out specific factors mainly influencing on the ATM cash withdrawals. Moreover for ATMs in different areas, such factors can be completely different.

Causal approaches seem to be not the best ones for our case.

5.1.3. Qualitative approaches

Table 3. PROs and CONs of Qualitative approaches

PROs	CONs
They can take into account all the factors that have never happened and can influence the demand	Limited capability of quantitative analysis and difficulty in managing "lots of numbers"
	High costs in terms of "manual" analysis of data and facts + meetings, or external costs (surveys, etc.)
	Risk of fallacious correlations to support the thesis, excess of confidence regarding the conclusion, excess of conformity among the group members

Even though Qualitative approaches can take into account all the factors that have never happened and can influence the demand, the "manual" analysis is quite costly. And keeping in mind that we deal with almost 3'000 branches, it is almost impossible to apply Qualitative approaches to our case.

Having compared Pros and Cons of the approaches mentioned above and having adopted them to our specific case, **the choice was done in favor of <u>Time series approaches</u>.**

5.2. Choosing the Time Series model

The following Time series models were considered:

1. Exponential smoothing models:
    1) Simple exponential smoothing (Brown's model);
    2) Exponential smoothing with trend adjustment;
    3) Exponential smoothing with trend and seasonality (Winter's model);
    4) Simple adaptive exponential smoothing;
    5) Exponential smoothing with damped trend.
2. Autoregressive models:
    1) Moving average models;
    2) Autoregressive moving average models;
    3) Autoregressive integrated moving average models.

I. At the first stage, the choice was made between two groups of models:

1. Exponential smoothing models;
2. Autoregressive models.

"Autoregressive models are more flexible and general than exponential smoothing models. However, empirical analyses conducted on a large number of benchmark time series, arising in different domains and characterized by different profiles, have shown that the greater effort required to develop and identify autoregressive models is not always rewarded by an improvement in prediction accuracy over simpler methods…" [(Vercellis, 2009), p.211]

Thus, the choice fell on the Exponential smoothing models as it is easier to implement and in practice gives sufficient results in terms of accuracy.

1. **Exponential smoothing models;**
2. Autoregressive models.

II. At the second stage, the choice was made between different models of Exponential smoothing:

1) Simple exponential smoothing (Brown's model) (A);
2) Exponential smoothing with trend adjustment (A+T);
3) Exponential smoothing with trend and seasonality (Winter's model) (A+T+S);
4) Simple adaptive exponential smoothing (adaptive);
5) Exponential smoothing with damped trend.

First of all, we made an assumption that there is no damped trend in data analyzed and got rid of the model #5 (Exponential smoothing with damped trend). Later this assumption was proved by the visual analysis of data.

Choosing between four models (#1-4), each of them was implemented for an ATM in the branch in Prato.

The results provided by Simple exponential smoothing (Brown's model), Exponential smoothing with trend adjustment and Simple adaptive exponential smoothing models are worse than ones provided by Exponential smoothing with trend and seasonality (Winter's model) in terms of all indicators mentioned in the Table 4.

Table 4. Exponential smoothing models performance

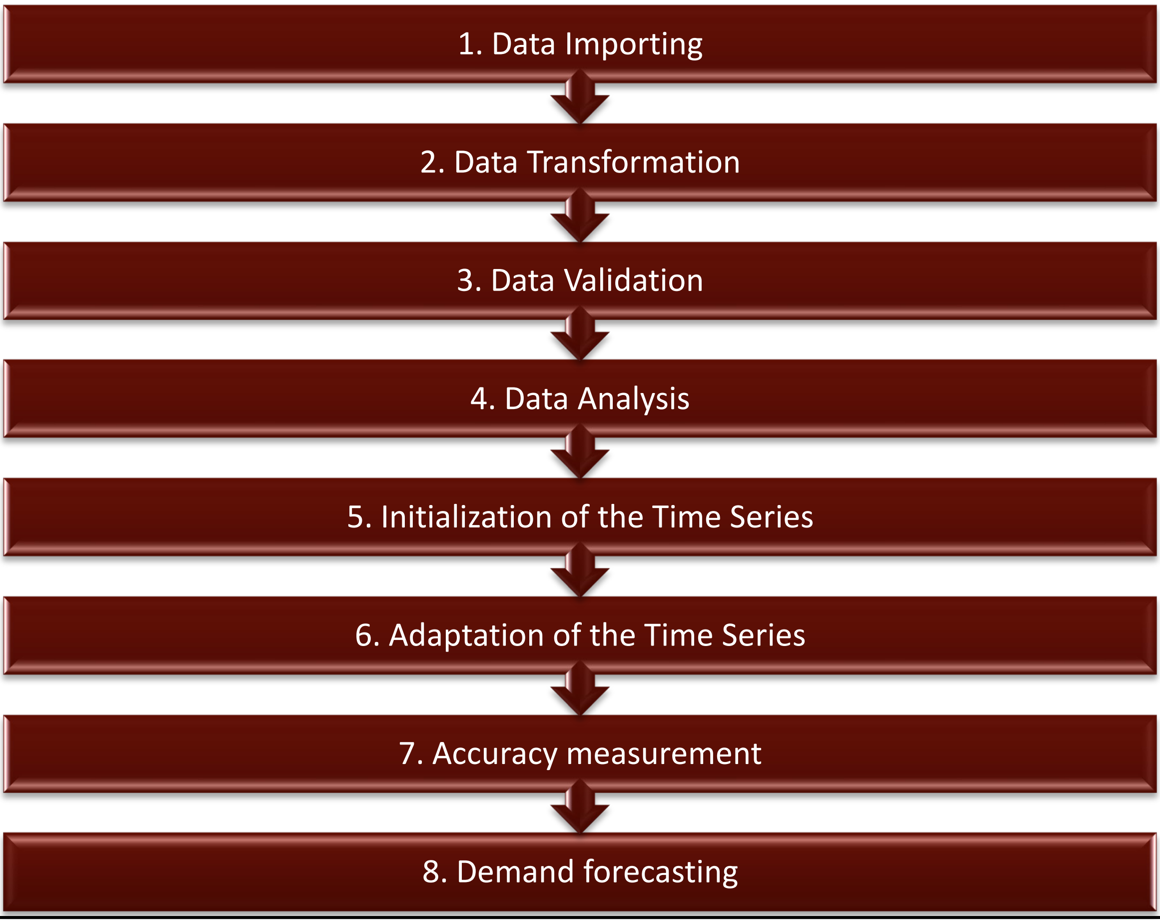
Indicator\Model	A+T+S	A+T	A	adaptive
MPE[5]	-10,6 %	-18 %	-18,1 %	-18,1 %
SDE	2 524	3 061	3 060	3 060
MAPE	29,8 %	42,9 %	42,9 %	42,9 %
TS	0,09	0,14	0,13	0,13

Based on the reasons explained above, **Winter's model** (exponential smoothing with trend and seasonality) was taken as a model for the ATM cash demand forecasting.

[5] The description and calculation of the coefficients is given in the paragraph "Accuracy measurement"

6. Forecasting with Winter's model

The process is described on the example of its application for Inventory Management optimization for the branch **Prato AG.8, 2397, area 5072**. This branch was chosen, because MPS uses the branches in Prato as a testing ground for proving new approaches and methodologies. Branch Prato AG.8 was to be tested first.



6.1. Data Importing

As an input for the forecasting process we use the data about the branch, obtained from the bank's IT system. The structure of the input data is presented in the Table 5.

Table 5. Input data structure

Date	SL_Op	SL_Cl	Con	Rit	Ban	Withdrawals
1 January 2010						2560
2 January 2010						7620
…	…	…	…	…	…	…
1 September 2010	107095,96	92521,25	61000		-30000	7 540
…	…	…	…	…	…	…
29 April 2011	76602,02	85190,84				4630
30 April 2011						14700

LEGEND:

SL_Op – Stock Level in the Cash Desk at the opening of the Branch.

SL_Cl – Stock Level in the Cash Desk at the closing of the Branch.

Con – physical cash replenished Cash Desk from External Vault.

Rit – physical cash withdrawn from Cash Desk to External Vault.

Ban – physical cash replenished ATM from Cash Desk.

Withdrawals - physical cash withdrawn from ATM by customers.

There are records for the period of **16 months** (from 1 January, 2010 to 30 April 2011) in the Input data. Fields SL_Op, SL_Cl, Con, Rit, Ban have records only starting from 1 September 2010

The original dataset was divided into two subsets: for Initialization and for Adaptation) in the way shown in the Table 6.

Table 6. Subdivision of the original Data

<table>
<tr><th>years</th><th colspan="12">2010</th><th colspan="4">2011</th></tr>
<tr><th>months</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th><th>11</th><th>12</th><th>1</th><th>2</th><th>3</th><th>4</th></tr>
<tr><td>ATM</td><td colspan="12">Initialization (12 mon.)</td><td colspan="4">Adaptation (4 m.)</td></tr>
<tr><td rowspan="2">Cash Desk</td><td rowspan="2" colspan="8">no records</td><td colspan="8">Initialization (8 mon.)</td></tr>
<tr><td colspan="8">Adaptation (8 mon.)</td></tr>
</table>

Such subdividing of the original dataset is based on the following:

ATM: 12 months is taken for Initialization to neutralize possible yearly seasonality, the remained 4 months – for the Adaptation.

Cash Desk: having tried different decompositions, the best results were obtained with using the whole available data for both Initialization and Adoption.

6.2. Data Transformation.

Data transformation consists in extracting the demand from the Original data.

The extraction the demand from the Original data starts with the assumption that the daily demand is equal to daily withdrawals of cash from ATMs. So, as an Input of Forecasting model, we can use the **Withdrawals** field of the Original data.

6.3. Data Validation (Substitution)

6.3.1. Zero values

Preliminary visual analysis of data showed that for some dates the registered withdrawals are equal to 0. The fact that such zero values appear only during weekends or holidays means that the possible reason of the phenomenon is the stock-out of cash in ATM (In weekends Internal ATMs cannot be replenished). However, the fact that there are no withdrawals doesn't mean that there is no demand. So, the zero values must be substituted.

To substitute zero values, the average withdrawals for that day of the week were used.

For example, if a zero value appeared in the dataset corresponds to Sunday. To substitute it the average withdrawals of all Sundays in the Original dataset is used.

6.3.2. Outliers

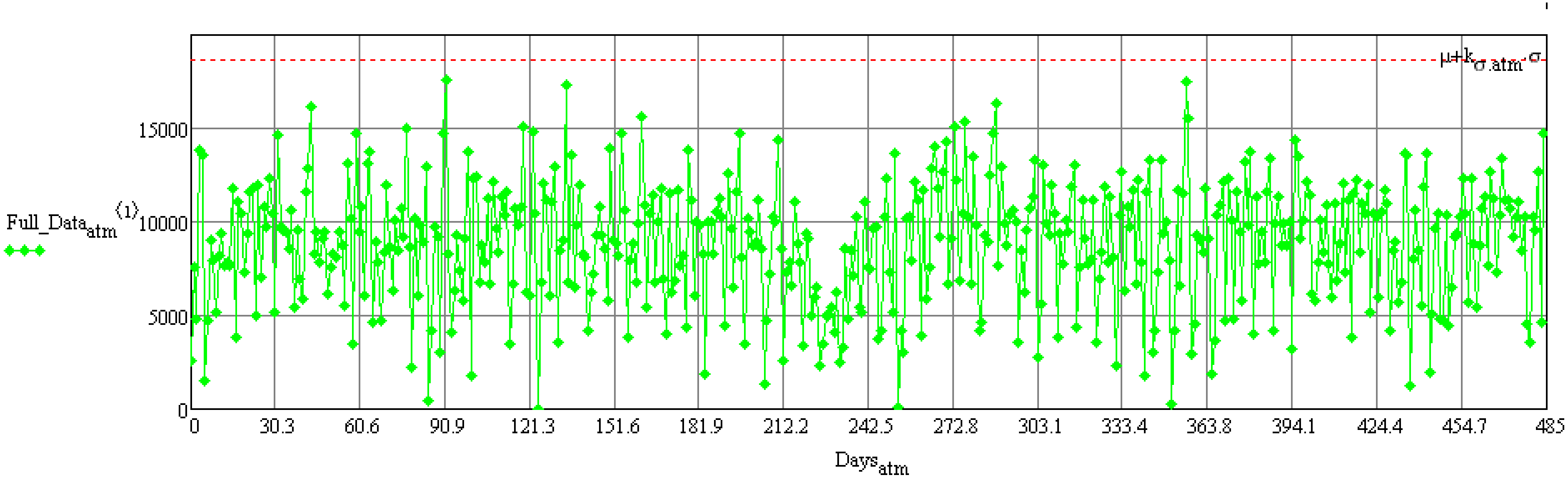
Outliers are abnormally high or low input values, which can distort the forecasting model. To identify them, **3σ approach** was used.

At the beginning, the mean value **μ** and the standard deviation **σ** were calculated. Then, the $(\mu-3\sigma;\mu+3\sigma)$ range was build. All values following beyond of that range are tagged as outliers.

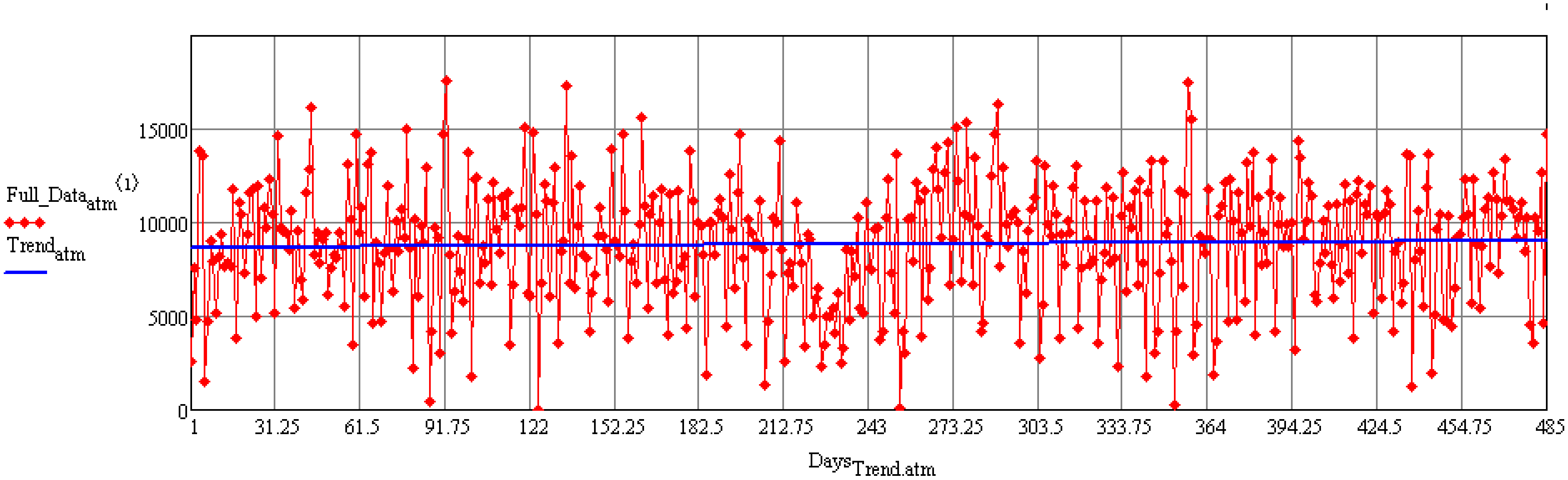
To substitute outliers, the average withdrawals for that day of the week were used, as for the zero values substitution.

The example of outliers identification for ATM in the branch Prato AG.8 is shown on the Graph 1. *All the values fell inside the $(\mu-3\sigma;\mu+3\sigma)$ range, thus there are no outliers in that specific case.*

Graph 1. Outliers identification. (ATM in branch Prato AG.8)



Graph 2. Trend identification. (ATM in branch Prato AG.8)



6.4. Data Analysis

For the Times Series analysis the full dataset $\boldsymbol{D_{Full}}$ with the observations covering 16 months was used.

6.4.1. Trend

Trend identification

In order to identify Trend component we applied **Linear Regression** to the full dataset $\boldsymbol{D_{Full}}$.

For the demand from ATM in the branch Prato AG.8 the slope of Linear Regression is equal to 0.72 (see Graph 2*).*

Trend elimination

Having identified the Trend component we subtracted it from the original dataset and obtained stationary Time series.

$\boxed{D_t^{S,R} = D_t^{T,S,R} - (b + k * (t-1))}$ - stationary Time Series (without Trend component),

$D_t^{T,S,R}$ - past demand with Trend, Seasonality and Random components;
k- slope of a linear Trend obtained before;
$t = 1, 2.. N_{Full}$ – daily indexes;
N_{Full} - is the number of days in the data subset for Initialization $\boldsymbol{D_{Full}}$.

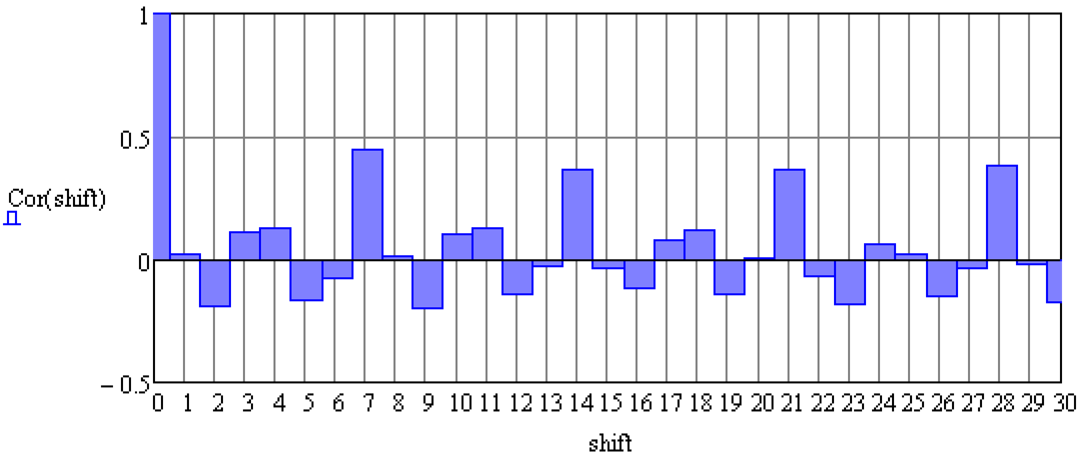
6.4.2. Seasonality

Identification of the Seasonality period

Autocorrelation coefficients were used to identify the seasonality period. A coefficient is calculated as a correlation between two data subsets shifted on k-periods far from each other, where k=1, 2..30. It was done for the stationary Time Series $D_t^{S,R}$ obtained in the previous paragraph.

The autocorrelation coefficients of ATM data from the branch Prato AG.8 is shown on the Graph 3.

Graph 3. The autocorrelation coefficients (ATM in branch Prato AG.8)



We assume that the Time series has the seasonality period equal to the shift that provides the highest absolute value of autocorrelation coefficient.

Seasonality period for the demand from ATM in the branch Prato AG.8 is equal to 7 days. There is weekly seasonality.

6.5. Initialization of the Time Series model

In order to define initial values for Time series, the data subset for Initialization $\boldsymbol{D_{Initial}}$ was used.

6.5.1. Initial Trend

Initial Trend for an ATM is calculated as a **slope *k*** of regression line $\boldsymbol{regl(t) = b + k * t}$**,** which best fits the data subset for Initialization $\boldsymbol{D_{Initial}}$.

For the demand from ATM in the branch Prato AG.8 the Initial Trend T_0 = -0.14.

6.5.2. Initial Average

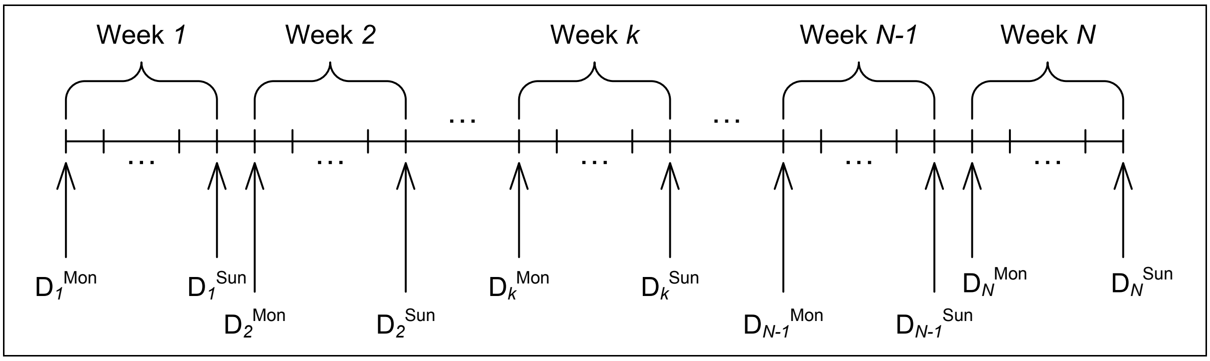
Initial Average for an ATM is calculated as a value of regression line for the last day of Initialization data subset. $\boldsymbol{A_0 = regl(N_{Initial}) = b + k * N_{Initial}}$, where $\boldsymbol{N_{Initial}}$ - is the number of days in $\boldsymbol{D_{Initial}}$.

For the demand from ATM in the branch Prato AG.8 the Initial Average A_0 = 8733.

6.5.3. Initial Seasonality

As it was found out before, the seasonality period for an ATM is equal to 7 (weekly seasonality). Figure 10 helps to explain the procedure of calculation of Initial seasonality indexes.

Figure 10. Calculation of Initial seasonality indexes



First of all, we divided the Initialization subset $\boldsymbol{D_{Initial}}$ on ***N*** parts (periods). The size of each part is equal to ***L*** – period of seasonality (in the considered example L=7).

Then, the Initial seasonality index, for example, for Monday $\boldsymbol{S_0^{Mon}}$ were calculated using the following formulas:

$$S^{Mon} = \frac{\sum_{i=1}^{N} S_i^{Mon}}{N}, \qquad S_i^{Mon} = \frac{D_i^{Mon}}{\overline{A_i}}, \qquad \overline{A_i} = \frac{D_i^{Mon} + \ldots + D_i^{Sun}}{7}$$

By analogy, the seasonality indexes for all days of the week were calculated (*branch Prato AG.8*):

$S^{Mon} = 1.13$,
$S^{Tue} = 1.07$,
$S^{Wed} = 0.9$,
$S^{Thu} = 1.08$,
$S^{Fri} = 1.28$ – the highest coefficient. Probably customers withdraw more money before week-ends,
$S^{Sat} = 1.01$,
$S^{Sun} = 0.52$ – the lowest coefficient. Partly it can be explained by the fact that customers maybe do not pass next to the ATM on Sundays. And partly by the fact that the demand is assumed to be equal to recorded withdrawals, and there maybe stock outs on Sundays, as there is no one who can replenish it during week-end.

At the end of the Initialization process Initial Trend T_0, Initial Average A_0 and *L* (the seasonality period) initial seasonality indexes $S_1^0 \ldots S_L^0$ were calculated.

6.6. Adaptation of the Time Series model

The aim of adaptation is to define the values of smoothing coefficients α^*, β^*, γ^*, which will minimize the Mean Squared Error (MSE).

$$\mathrm{MSE}(\alpha^*, \beta^*, \gamma^*) = \min\{\mathrm{MSE}\}$$

Mean Squared Error function is defined as follows:

$$\mathrm{MSE}(\alpha, \beta, \gamma) = \frac{\sum_{t=1}^{N_{Adapt}} (D_t^{Adapt} - F(\alpha, \beta, \gamma)_t)^2}{N_{Adapt}}$$

$\boldsymbol{D_{Adapt}}$ - is the data subset for Adaptation,

N_{Adapt} - is the number of days in that data subset,

$t = 1$ for the first day in $\boldsymbol{D_{Adapt}}$ and $t = N_{Adapt}$ for the last one.

$\mathbf{F}(\alpha, \beta, \gamma)$ **-** is the array of forecasted values of demand, which can be calculated, using the following formulas:

$$F(\alpha, \beta, \gamma)_{t+1} = (A_t + T_t) \cdot S_{t-L+1} \text{ , where}$$

Average: $A_t = \alpha \cdot \frac{D_t}{S_{t-L}} + (1 - \alpha) \cdot (A_{t-1} + T_{t-1})$

Trend: $T_t = \gamma \cdot (A_t + A_{t-1}) + (1 - \gamma) \cdot T_{t-1}$

Seasonality: $S_t = \beta \cdot \frac{D_t}{A_t} + (1 - \beta) \cdot S_{t-L}$

The Initial values A_0, T_0, $S_1^0 \ldots S_L^0$ (array, containing *L* elements, where *L* is the seasonality period) for these recursive formulas are calculated in the paragraph "6.5 Initialization of the Time Series model".

$D_t = D_t^{Adapt}$ for $t = 1..N_{Adapt}$

Having defined the $\mathrm{MSE}(\alpha, \beta, \gamma)$ function, the smoothing coefficients α^*, β^*, γ^*, minimizing that function, are obtained by varying α, β and γ from *0* to *1* with a discrete *0,1*.

For the ATM in the branch Prato AG.8:

$$\alpha^* = 0$$

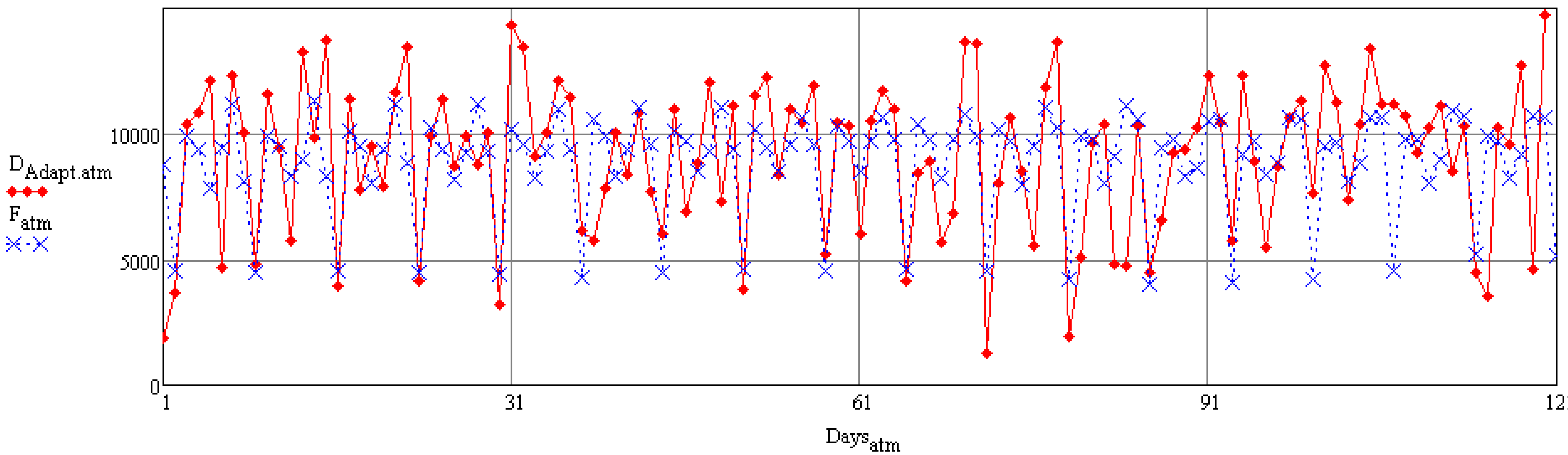
$$\beta^* = 0.1$$

$$\gamma^* = 0$$

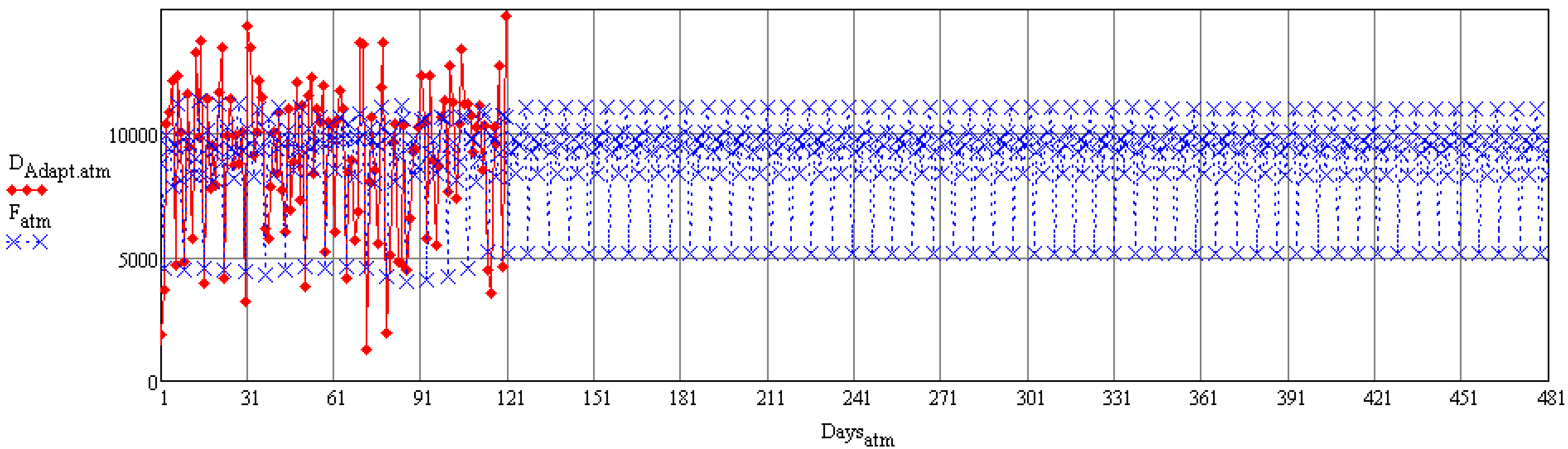
$$\min\{\mathrm{MSE}\} = \mathrm{MSE}(0{,}0.1{,}0) = 6.37 \cdot 10^6$$

The real value of the Demand and the Forecast for the Adaptation period (for the ATM in branch Prato AG.8) are shown on the Graph 4.

Graph 4. The real value of the Demand and the Forecast for the Adaptation period (ATM in branch Prato AG.8)



Graph 5. The real value of the Demand for the Adaptation period and the Forecast for the Adaptation period and the next year (ATM in branch Prato AG.8)



6.7. Accuracy measurement

Accuracy of the model was measured comparing the actual demand in the data subset for adaptation $\boldsymbol{D}_{Adapt}$ with the forecast for the adaptation period, which includes N_{Adapt} days. Three types of indicators were calculated:

1) Distortion
2) Dispersion
3) Tracking Signal

6.7.1. Distortion

Mean Percentage Error (MPE):

$$MPE = \frac{\sum_{t=1}^{\mathrm{N_{Adapt}}} \frac{D_t^{\boldsymbol{Adapt}} - F_t}{D_t^{\boldsymbol{Adapt}}}}{\mathrm{N_{Adapt}}} \times 100\%$$

$MPE = -10{,}6\ \%$ *- for the ATM in branch Prato AG.8*

6.7.2. Dispersion

Standard Deviation Error (SDE):

$$SDE = \sqrt{\frac{\sum_{t=1}^{\mathrm{N_{Adapt}}} (D_t^{\boldsymbol{Adapt}} - F_t)^2}{\mathrm{N_{Adapt}}}}$$

$SDE = 2\ 524$ *- for the ATM in branch Prato AG.8*

Mean Absolute Percentage Error (MAPE):

$$MAPE = \frac{\sum_{t=1}^{\mathrm{N_{Adapt}}} \left| \frac{D_t^{\boldsymbol{Adapt}} - F_t}{D_t^{\boldsymbol{Adapt}}} \right|}{\mathrm{N_{Adapt}}} \times 100\%$$

$MAPE = 29{,}8\%$ *- for the ATM in branch Prato AG.8*

6.7.3. Tracking Signal

Tracking Signal (TS):

$$TS = \frac{\sum_{t=1}^{\mathrm{N_{Adapt}}} D_t^{\boldsymbol{Adapt}} - F_t}{\sum_{t=1}^{\mathrm{N_{Adapt}}} \left| D_t^{\boldsymbol{Adapt}} - F_t \right|}$$

$TS = 0{,}09$ *- for the ATM in branch Prato AG.8*

6.8. Demand forecasting

6.8.1. Forecast for next year

The forecast for the next year is done using the formulas described before.

$$F(\alpha^*, \beta^*, \gamma^*)_{t+1} = (A_t + T_t) \cdot S_{t-L+1} \text{ , where}$$

Average: $A_t = \alpha^* \cdot \frac{D_t}{S_{t-L}} + (1 - \alpha^*) \cdot (A_{t-1} + T_{t-1})$

Trend: $T_t = \gamma^* \cdot (A_t + A_{t-1}) + (1 - \gamma^*) \cdot T_{t-1}$

Seasonality: $S_t = \beta^* \cdot \frac{D_t}{A_t} + (1 - \beta^*) \cdot S_{t-L}$

The values of smoothing coefficients α^*, β^*, γ^*, which minimize the Mean Squared Error (MSE) were obtained in the paragraph "6.6 Adaptation of the Time Series model".

The Initial values A_0, T_0, $S_1^0 \ldots S_L^0$ (array, containing *L* elements, where *L* is the seasonality period) for these recursive formulas are calculated in the paragraph "6.5 Initialization of the Time Series model".

$D_t = D_t^{Adapt}$ for $t = 1..N_{Adapt}$

$D_t = F_t$ for $t = N_{Adapt} + 1..N_{Adapt} + 365$

The real value of the Demand for the Adaptation period and the Forecast for the Adaptation period and the next year (for the ATM in branch Prato AG.8) are shown on the Graph 5.

6.8.2. Yearly seasonality adjustment

Visual analysis of past data showed us that along with the weekly seasonality in cash demand, there is yearly seasonality, as well. For example, in cities like Milan the demand in summer, especially in August, is much lower than average demand during the year. While in the towns on the seaside the situation is the opposite. For that reason the forecast for the next year can be adjusted with yearly seasonality coefficients.

In order to obtain yearly seasonality coefficients for ATMs the data subset for Initialization $\boldsymbol{D_{Initial}}$ was used, as it contains the records during the whole year 2010. The coefficients were calculated using the following formulas:

$$S_k = \frac{\sum_{i=1}^{30} D_{(k-1)\cdot 30+i}}{\overline{A}}, k = 1..12; \qquad \overline{A} = \frac{\sum_{i=1}^{360} D_i}{360}$$

For the ATM in branch Prato AG.8 the following results were obtained:

$S_{year.atm}^{T}$ =

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.98	1.05	0.97	1.08	1	1.06	0.98	0.75	0.98	1.14	1.04	0.97

- The lowest yearly seasonality coefficient corresponds to August, when customers demand only 75% of average amount.

- The highest yearly seasonality coefficient corresponds to October, when the demand is 114% of the average value.

The adjusted forecast is shown on the Graph 6.

6.8.3. Accuracy measurement

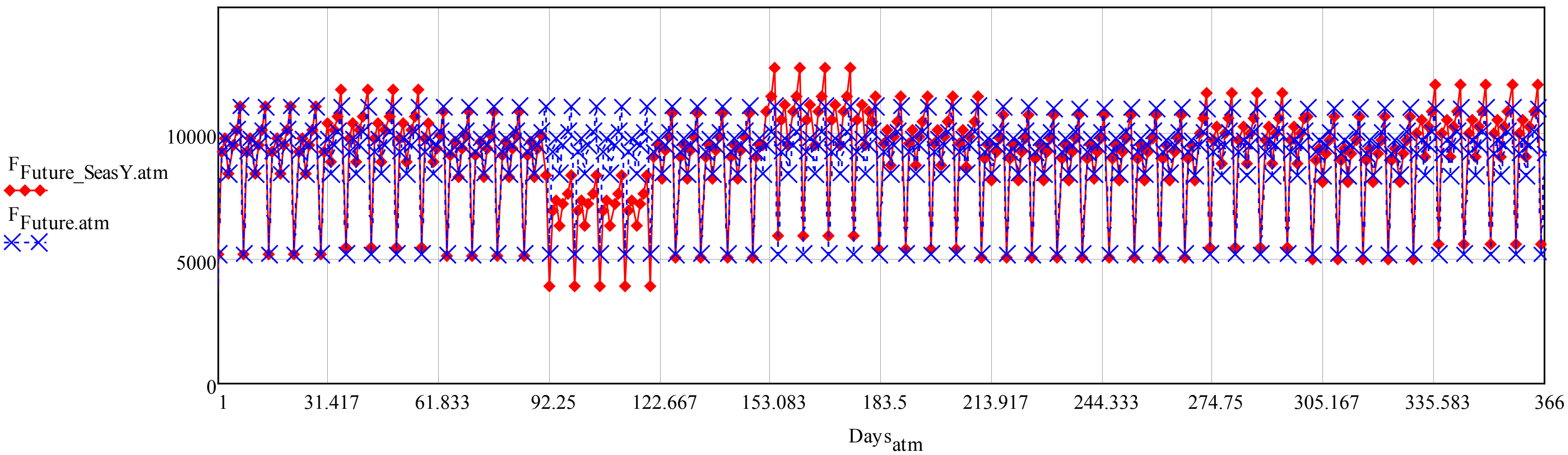
Having done the yearly seasonality adjustment, we measured the accuracy again and compared it with the one that was obtained in the paragraph "6.7 Accuracy measurement" for a forecast without yearly seasonality adjustment. Then, forecast corresponding to the best accuracy was chosen (see Table 7).

Table 7. Choosing the best forecast (ATM in the branch Prato AG. 8)

Accuracy Measure		Yearly seasonality adjustment	
Name	Formula	NO	YES
MPE	$MPE = \frac{\sum_{t=1}^{\mathrm{N_{Adapt}}} \frac{D_t^{Adapt} - F_t}{D_t^{Adapt}}}{\mathrm{N_{Adapt}}} \times 100\%$	-10,6 %	**-12,2 %**
SDE	$SDE = \sqrt{\frac{\sum_{t=1}^{\mathrm{N_{Adapt}}} (D_t^{Adapt} - F_t)^2}{\mathrm{N_{Adapt}}}}$	2 524	**2 501**
MAPE	$MAPE = \frac{\sum_{t=1}^{\mathrm{N_{Adapt}}} \left\lvert \frac{D_t^{Adapt} - F_t}{D_t^{Adapt}} \right\rvert}{\mathrm{N_{Adapt}}} \times 100\%$	29,8 %	**29,5 %**
TS	$TS = \frac{\sum_{t=1}^{\mathrm{N_{Adapt}}} D_t^{Adapt} - F_t}{\sum_{t=1}^{\mathrm{N_{Adapt}}} \lvert D_t^{Adapt} - F_t \rvert}$	0,09	**0,01**

The forecast with yearly seasonality adjustment **was chosen**, as it provides lower SDE. Tests on different branches showed that such criterion in choosing the better forecast provides lower MAPE and better tracks the signal, as well.

Graph 6. The Forecast for the next year with yearly seasonality adjustment (ATM in branch Prato AG.8)



6.9. Output

The output of ATM Demand Planning process (see Figure 11):

- the daily forecast of the cash demand from ATM for the next year $\mathbf{d}_{atm}$;
- estimated forecast accuracy SDE_{atm}.

It will be the Input for the next step, ATM Inventory Planning.

Figure 11. ATM Demand Planning as a part of Branch Inventory Management optimization

7. ATM. Inventory planning

The second step of Branch Inventory Management optimization process is ATM Inventory Planning (see Figure 12).

Figure 12. ATM Inventory Planning as a part of Branch Inventory Management optimization

Objective:

- To define the Inventory management policy and parameters, which will
- minimize Logistics Costs for an ATM,
- assuring the pre-defined service level for customers.

Input:

- Daily forecast of cash demand from ATM for next year $\boldsymbol{d_{atm}}$;
- Estimated forecast accuracy SDE_{atm};
- Service target;
- Replenishment lead times;
- Logistics Costs structure.

Output:

- Inventory management policy and parameters,
- Replenishment plan for the next year **Ban**.
- Expected residuals in the ATM at the moment of replenishment **res**.

7.1. Choice of the Inventory management policy

Two main models for Inventory planning were considered (see Table 8):

- Reorder Point
- Periodic Review

In <u>Reorder Point model</u>, the amount of money to replenish is fixed, but the time of replenishment varies. The replenishment occurs only when Stock Level reaches pre-calculated level (Reorder Point).

In <u>Periodic Review model</u>, the interval between 2 replenishments is fixed, but the amount of money to replenish varies. Every T days the stock must be replenished to achieve Availability Target.

Table 8. Inventory planning models comparison

	Reorder point	Periodic review
+	On average, the Inventory level is lower than in the Periodic review model	The Inventory level control is easy to be done (only when an order has to be placed)
−	The Inventory level control has to be continuous and as a consequence expensive	On average, the Inventory level is higher than in the Reorder point model

(Perego & Mangiaracina, Pull Inventory Control, "Logistics Management" course, lecture 16, 2010)

For the **ATM** Inventory Planning the **Periodic Review model** was chosen. The main reason of such choice is that Periodic Review model doesn't require the continuous control of the inventory level, which cannot be implemented for ATMs, because there are no operators in branches on week-ends, while there is still demand, as customers withdraw money from ATMs 7 days a week.

For the **Cash Desk** Inventory Planning the **Reorder point model** was chosen, as, on average, it ensures lower inventory level than in the Periodic review model. A branch manager can control the stock level of Cash Desk continuously 5 days a week, and the stock level is not changed during the week-end, as branches are closed.

7.2. Periodic Review model. Choice of the Inventory management parameters

7.2.1. Logistics costs

Total Costs structure

During the meeting with MPS managers we agreed on that Annual logistics Total Costs (TC) consist of two components:

- Annual Inventory Carrying Costs (ICC)
- Annual Procurement Costs (PC)

$$TC(T) = ICC(T) + PC(T)$$

$\boldsymbol{ICC(T) = AIL(T) \cdot V \cdot phc}$ $[€/year]$

T $[year]$ – Order interval,

AIL $[€]$ – Average Inventory Level per year,

$\boldsymbol{V \equiv 1}$ $[-]$ – Unitary item Value. Equal to 1 in our case, as in the stock we keep the money,

phc $[\%/year]$ – Inventory Carrying Costs as a percentage of item value.

$\boldsymbol{PC(T) = \frac{D_y}{Q(T)} \cdot upc}$ $[€/year]$

D_y $[€/year]$ – Annual Demand,

$Q(T)$ $[€]$ - Order Quantity (depends on the order interval *T*),

upc $[€]$ – unitary procurement costs.

phc estimation

$\boldsymbol{phc} = \boldsymbol{rc} + \boldsymbol{imc}\ [\%/year]$

$rc\ [\%/year]$ – robbery costs as a percentage of money keeping in stock,

$imc\ [\%/year]$ - immobilization costs as a percentage of money keeping in stock.

Robbery costs (rc) occur when a Cash Desk and/or ATM are exposed to the robbery. According to the statistics provided by the bank's managers, MPS every working day somewhere in Italy one of their Cash Desk or ATM are exposed to robbery.

$rc = \frac{270[days/year]}{2800+3600} \cdot 75\% = 3{,}16\ [\%/year]$, where 2800 and 3600 are the number of branches and ATMs respectively, 75% is the average percentage of money stolen from the stock in case of robbery.

Immobilization costs (imc) occur because of the fact that cash "frozen" in the stock cannot be invested.

$imc = 1\ [\%/year]$ – is the number given by the MPS managers.

$\boldsymbol{phc} = rc + imc = 3{,}16 + 1 = \mathbf{4{,}16}\ [\%/\boldsymbol{year}]$ - this value is valid for both ATMs and Cash Desks.

upc estimation. ATM

The replenishment of money in internal ATMs is performed by operators, working in a branch. It requires two operators to replenish the money and it takes about 30 minutes on average. So, this activity requires $1\ \text{operator} \times \text{hour}$. So, $\boldsymbol{upc_{atm}}$ was calculated as a salary paid for an operator per hour, provided that $60\ 000[€/(year \cdot operator)]$ - is the gross yearly salary of an operator in the branch.

$$\boldsymbol{upc_{atm}} = \frac{60\ 000[€/(year \cdot operator)]}{365 \cdot \frac{5}{7} \cdot 8[hours/year] \cdot \frac{2[replenishments/hour]}{2[operators]}} = \mathbf{28{,}77}[\frac{€}{\boldsymbol{replenishment}}]$$

7.2.2. Order interval

Order interval T, minimizing the Total Costs is calculated with the following formula:

$$\boldsymbol{T = \sqrt{\frac{2 \cdot upc_{atm}}{phc \cdot V \cdot D_y^{atm}}} \times 365[days]}$$

Annual demand from an ATM D_y^{atm} is obtained by summing of all the elements of $\mathbf{d_{atm}}$ array (forecast of cash demand from ATM for next year).

For the ATM in the branch Prato AG.8:
$D_y^{atm} = €3{,}29\ mln.$ *per year. And order interval:*

$$T = \sqrt{\frac{2 \cdot 28{,}77\ [€]}{4{,}16 \left[\frac{\%}{year}\right] \cdot 1 \cdot 3{,}29 \cdot 10^6 \left[\frac{€}{year}\right]}} \times 365[days] = 7{,}49[days]$$

Rounding up this number to the closest lower integer, we obtain that the order interval for the ATM in the branch Prato AG.8 $T = 7\ [days]$.

Expected Demand during the interval $[\boldsymbol{t_1}, \boldsymbol{t_2}]$:

$$\boldsymbol{ED^{atm}[t_1, t_2] = \sum_{i=t_1}^{t_2} d_i^{atm}}$$

d_{atm} - is the daily forecast of cash demand from ATM for next year.
t_1- is the day of the stock review;
$t_2 = t_1 + T - 1 + LT$

T – is the interval between two closest reviews;

LT – replenishment lead time.

7.2.3. Safety stock and Availability Target

$$\boldsymbol{SS^{atm} = k \cdot \sigma_{D,LT+T}^{atm}}$$

$\boldsymbol{k = 2,33}$ - is the **service level** defined as the Probability to avoid Stock out during the Order interval and replenishment Lead time. Such service level will ensure the 99% probability to avoid stock out.

$\sigma_{D,LT+T}^{atm}$ –is the standard deviation of the demand during T+LT (order interval plus replenishment lead time):

$$\sigma_{D,LT+T}^{atm} = \sqrt{(ELT^{atm} + T) \cdot {SDE_{atm}}^2 + (\sigma_{LT}^{atm})^2 \cdot {ED^{atm}}^2}$$

$\boldsymbol{ELT^{atm}}$ =**0** – is the expected lead time of an ATM stock replenishment.
It requires 30 minutes, on average, to replenish an ATM, while the order interval is several days. So we can neglect the expected lead time.

$\boldsymbol{\sigma_{LT}^{atm} = 0}$ – is the standard deviation of the lead time of an ATM stock replenishment.
The time to replenish an ATM lies in the range [20,50] minutes, which is negligible when we operate with order interval of several days.

SDE_{atm} - is the standard deviation of error, the accuracy measure of the demand in an ATM forecast,
T – is the order interval,
ED^{atm} – is the forecasted demand from an ATM.

As ELT^{atm} =0 and $\sigma_{LT}^{atm} = 0$, we can rewrite the formula of $\sigma_{D,LT+T}^{atm}$ as follows:

$$\boldsymbol{\sigma_{D,LT+T}^{atm} = \sqrt{T \cdot {SDE_{atm}}^2}}$$

For the ATM in the branch Prato AG.8:
$SS^{atm} = 2{,}33 \cdot \sqrt{7 \cdot (2\,501)^2} = 15\,420$ [€]

Availability Target:

$$\boldsymbol{AT[t_1, t_2] = ED^{atm}[t_1, t_2] + SS^{atm}}$$

t_1- is the day of the stock review;
$t_2 = t_1 + T - 1 + LT$
T – is the interval between two closest reviews;
LT=0 – replenishment lead time.

7.3. Adjustment of the replenishment plan (for week-ends)

The Periodic review model cannot be applied in the pure way to all the branches. For the ATM in Prato AG 8 it works in the pure way, because the order interval is 7 days (exactly 1 week). Operators in the branch can replenish the stock of ATM, let say, each Monday. But what if the order interval is equal to 5 days? In this case, having replenished the stock of an ATM on Monday, next replenishment must be done on Saturday. But there are no operators in the branch on week-ends to replenish the stock, while there is the demand from ATM 7 days a week. Thus, there will be stock-out during the week-end. It means that the Periodic review model must be somehow adjusted.

7.3.1. 4 replenishment policies "for week-ends"

To overcome the problem with the ATM replenishments during week-ends 4 types of solution (4 policies) were considered. Let us assume that the **order interval is 4 days** and on **Tuesday** we must replenish the stock. The amount of money to put must replenish current stock up to the $AT[Tue, Fri]$ and must be enough until Saturday, but on Saturday the replenishment cannot be done.

<u>There are 2 possible solutions how to overcome the problem of replenishment, which is planned for Saturday:</u>

1) **To put** on Tuesday the amount of **money** up to the $AT[Tue, Thu]$, that will be enough for 3 days (**until Friday**), and on Friday to make "normal" replenishment for the next 4 days. In this way we **shift** the replenishment from week-end to Friday **backward "←".**

2) **To put** on Tuesday the amount of **money** up to the $AT[Tue, Sun]$, that will be enough for 6 days (**until next Monday**), and on Monday to make "normal" replenishment for 4 days. In this way we **shift** the replenishment from week-end to Monday **forward "→".**

The same 2 solutions we can use if the replenishment is planned to be done on Sunday. Totally, there are 4 replenishment policies "for week-ends", schematically described in the Table 9.

Table 9. 4 replenishment policies "for week-ends"

Day of the week	Policy - 1	Policy - 2	Policy - 3	Policy - 4
Saturday	←	←	→	→
Sunday	←	→	←	→

Then, the "ideal" replenishment plan was converted into 4 real.

For the ATM in the branch Prato AG.8, the order interval is 7 days. Thus 4 replenishment plans are equal. The 4 different replenishment plans for Prato AG.8 for May 2011 are shown in the Table 10*.*

Table 10. The 4 different replenishment plans for May 2011 (ATM in the branch Prato AG.8)

Replen =

Day of the month	Day of the week	Policy-1	Policy-2	Policy-3	Policy-4
	0	1	2	3	4
0	0	0	0	0	0
1	7	0	0	0	0
2	1	78666	78666	78666	78666
3	2	0	0	0	0
4	3	0	0	0	0
5	4	0	0	0	0
6	5	0	0	0	0
7	6	0	0	0	0
8	7	0	0	0	0
9	1	78659	78659	78659	78659
10	2	0	0	0	0
11	3	0	0	0	0
12	4	0	0	0	0
13	5	0	0	0	0
14	6	0	0	0	0
15	7	0	0	0	0
16	1	78652	78652	78652	78652
17	2	0	0	0	0
18	3	0	0	0	0
19	4	0	0	0	0
20	5	0	0	0	0
21	6	0	0	0	0
22	7	0	0	0	0
23	1	78645	78645	78645	78645
24	2	0	0	0	0
25	3	0	0	0	0
26	4	0	0	0	0
27	5	0	0	0	0
28	6	0	0	0	0
29	7	0	0	0	0
30	1	81791	81791	81791	81791
31	2	0	0	0	...

For the ATM in the branch Prato AG.3 order interval is 4 days. The 4 different replenishment plans for Prato AG.3 for May 2011 are shown in the Table 11.

Table 11. The 4 different replenishment plans for May 2011 (ATM in the branch Prato AG.3)

Replen =

Day of the month	Day of the week	Policy-1	Policy-2	Policy-3	Policy-4
	0	1	2	3	4
0	0	0	0	0	0
1	7	0	0	0	0
2	1	117635	117635	117635	117635
3	2	0	0	0	0
4	3	0	0	0	0
5	4	0	0	0	0
6	5	109547	109547	109547	109547
7	6	0	0	0	0
8	7	0	0	0	0
9	1	0	0	0	0
10	2	93857	93857	156960	156960
11	3	0	0	0	0
12	4	0	0	0	0
13	5	109717	109717	0	0
14	6	0	0	0	0
15	7	0	0	0	0
16	1	0	0	118006	118006
17	2	93996	93996	0	0
18	3	0	0	0	0
19	4	0	0	0	0
20	5	109887	109887	109887	109887
21	6	0	0	0	0
22	7	0	0	0	0
23	1	0	0	0	0
24	2	94135	94135	157483	157483
25	3	0	0	0	0
26	4	0	0	0	0
27	5	110056	110056	0	0
28	6	0	0	0	0
29	7	0	0	0	0
30	1	0	0	125668	125668
31	2	101565	101565	0	...

7.3.2. Choosing the vest policy

Having obtained 4 different possible replenishment plans for the next year, we must choose one to implement. As all out of 4 designed plans do not decrease the pre-defined service level, **the best plan** is the plan which provides the **lowest annual total costs**:

$$TC(pl)_{atm} = ICC(pl)_{atm} + PC(pl)_{atm}\ [€/year]$$

Procurement Costs:

$$PC(pl)_{atm} = N(pl)_{pr} \cdot upc_{atm}\ [€/year]$$

$N(pl)_{pr}[\frac{replenishments}{year}]$ – number of replenishments per year. It depends on the type of policy pl;

$upc_{atm} = 28{,}77[\frac{€}{replenishment}]$ – unitary procurement costs for ATM.

Inventory Carrying Costs:

$$ICC(pl)_{atm} = AIL(pl)_{atm} \cdot V \cdot phc\ [€/year]$$

$AIL(pl)_{atm}\ [€]$ – expected average inventory level in ATM. It depends on the type of policy pl;

$V \equiv 1[-]$ - unitary item Value. Equal to 1 in our case, as in the stock we keep the money;

$phc = 4{,}16\ [\%/year]$ – inventory carrying costs as a percentage of item value.

$$AIL(pl)_{atm} = \frac{\sum_{t=1}^{365} SL(pl)_t}{365}$$

Stock Level at the day ***t*** depending on type of policy:

$$SL(pl)_t = \begin{cases} SL(pl)_{t-1} - D_t^{atm}, if\ Replen(pl)_t = 0 \\ Replen(pl)_t - D_t^{atm}, otherwise \end{cases}$$

D^{atm} - is the forecasted demand for the next year;

$Replen(pl)$ – is the replenishment plan for the policy pl (see 7.3.1 4 replenishment policies “for week-ends”).

The best replenishment policy corresponds to the minimum annual Total Costs:

$$\boldsymbol{TC(pl_{best})_{atm}} = \mathbf{min}\,\{\boldsymbol{TC(pl)_{atm}}\} \Rightarrow \boldsymbol{pl_{best}}$$

For the ATM in the branch Prato AG.8 the expected profile of Stock Level for the best replenishment policy is shown on the Graph 7*.*

Expected residuals:

$$res_t = \begin{cases} SL(pl_{best})_{t-1}, if\ Replen(pl_{best})_t \neq 0 \\ 0, otherwise \end{cases}$$

7.4. Output

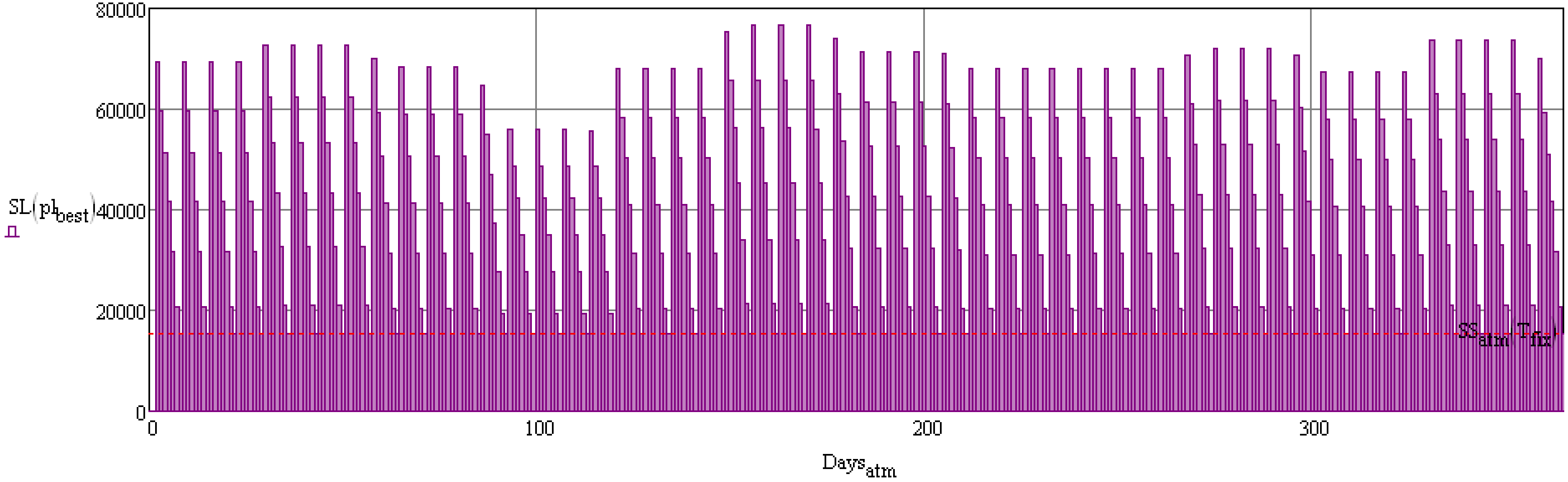
The output of ATM Inventory Planning process (see Figure 13) is the

- Replenishment plan for the next year $\boldsymbol{Ban} = \boldsymbol{Replen}(pl_{best})$;
- Expected residuals in the ATM at the moment of replenishment $\boldsymbol{res}$.

It will be the Input for the next step, Cash Desk Demand Planning.

Figure 13. ATM Inventory Planning as a part of Branch Inventory Management optimization

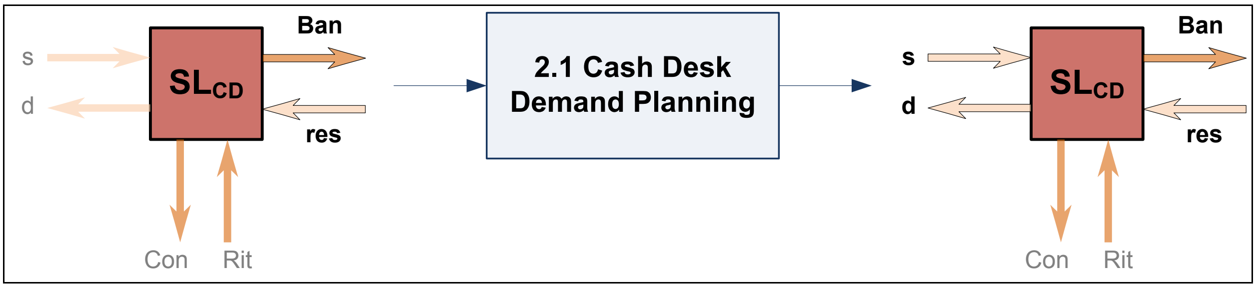
Graph 7. Profile of expected Stock Level for the best replenishment policy. ATM in the branch Prato AG.8



8. Cash Desk. Demand planning

Cash Desk Demand Planning is the third step of Branch Inventory Management optimization (see Figure 14).

Figure 14. Cash Desk Demand Planning as a part of Branch Inventory Management optimization



Objective:

- Forecast the future demand for cash [euro] in Cash Desk from the bank's customers.

Input:

- Time series with past data about:
  - stock level in Cash Desk **SL_{CD},**
  - physical cash flow to replenish Cash Desk from External Vault **Con,**
  - physical cash flow from Cash Desk to External Vault **Rit**,
- Time series with data about:
  - defined replenishment plan to replenish ATM from Cash Desk for the next year **Ban,**
  - forecast for the next year about physical cash left in ATM at the moment it has been opened for replenishment **res.**

Output:

- Daily total demand/supply forecast **d/s** for next year for Cash Desk from both customers and ATM.
- Measure of the plan accuracy.

Process settings:

Time bucket: 1 day

Forecasting horizon: 1 year

Frequency: 1 month

Winter's model (exponential smoothing with trend and seasonality) was chosen as forecasting model for Cash Desk demand planning for the same reasons as in the Chapter "5 ATM. Demand planning".

8.1. Data Importing

Demand forecasting for Cash Desk uses exactly the same Original data as for the ATM demand forecasting. For forecasting process the data about the branch, obtained from the bank's IT system, is used. The structure of the input data is presented in the Table 12.

Table 12. Input data structure

Date	SL_Op	SL_Cl	Con	Rit	Ban	Withdrawals
1 January 2010						2560
2 January 2010						7620
…	…	…	…	…	…	…
1 September 2010	107095,96	92521,25	61000		-30000	7 540
…	…	…	…	…	…	…
29 April 2011	76602,02	85190,84				4630
30 April 2011						14700

LEGEND:

SL_Op – Stock Level in the Cash Desk at the opening of the Branch.

SL_Cl – Stock Level in the Cash Desk at the closing of the Branch.

Con – physical cash replenished Cash Desk from External Vault.

Rit – physical cash withdrawn from Cash Desk to External Vault.

Ban – physical cash replenished ATM from Cash Desk.

Withdrawals - physical cash withdrawn from ATM by customers.

There are records for the period of **16 months** (from 1 January 2010 to 30 April 2011) in the Input data. Fields SL_Op, SL_Cl, Con, Rit, Ban have records only starting from 1 September 2010.

The original dataset was divided into two subsets: for Initialization and for Adaptation) in the way shown in the Table 13.

Table 13. Subdivision of the original Data

<table>
<tr><th>years</th><th colspan="12">2010</th><th colspan="4">2011</th></tr>
<tr><th>months</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th><th>11</th><th>12</th><th>1</th><th>2</th><th>3</th><th>4</th></tr>
<tr><td>ATM</td><td colspan="12">Initialization (12 mon.)</td><td colspan="4">Adaptation (4 m.)</td></tr>
<tr><td rowspan="2">Cash Desk</td><td colspan="8" rowspan="2">no records</td><td colspan="8">Initialization (8 mon.)</td></tr>
<tr><td colspan="8">Adaptation (8 mon.)</td></tr>
</table>

Such subdividing of the original dataset is based on the following:

ATM: 12 months is taken for Initialization to neutralize possible yearly seasonality, the remained 4 months – for the Adaptation.

Cash Desk: having tried different decompositions, the best results were obtained with using the whole available data for both Initialization and Adoption.

8.2. Data Transformation.

Data transformation for Cash Desk consists in:

- cutting the data without records about Cash Desk (from 1 January 2010 to 1 September 2010),
- week-ends elimination,
- demand/supply extraction from the Original data.

As the Original data, gathered from the bank IT system, doesn't contain any records concerning Cash Desk from 1 January 2010 to 1 September 2010, these first 8 months are cut.

Cash Desk is closed during the week-end, so there are no records, concerning Cash Desk on Saturdays and Sundays. So, they are eliminated as well.

In order to extract from the Original data demand (supply, if negative) the following formula was used:

$$D = \mathrm{SL_{Op}} - \mathrm{SL_{Cl}} + \mathrm{Con} + \mathrm{Rit} + \mathrm{Ban}^6$$

8.3. Data Validation (Substitution)

8.3.1. Zero values

Preliminary visual analysis of data showed that for some dates the records are missing. Maybe branch was closed for some days because of holidays. However, the fact that there are no records doesn't mean that there is no demand. So, the zero values must be substituted.

To substitute zero values, the average demand in Cash Desk for the whole period was taken.

8.3.2. Outliers

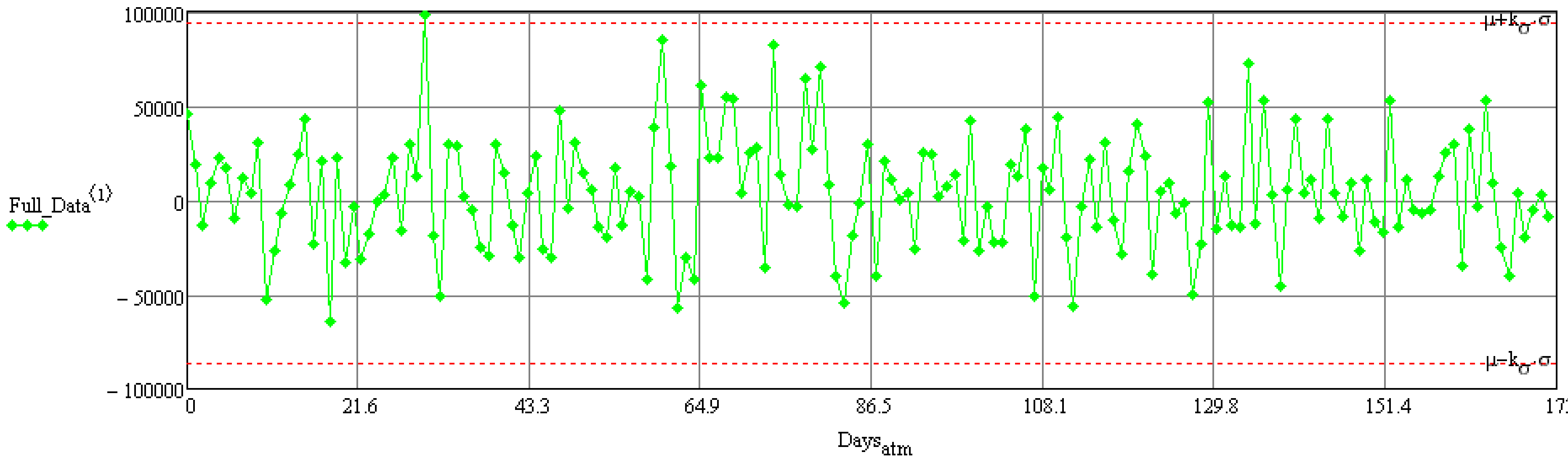
In order to identify outliers in the demand in Cash Desk the same **3σ approach** was used, as for the demand in ATM.

At the beginning, the mean value **μ** and the standard deviation **σ** were calculated. Then, the (μ-3σ;μ+3σ) range was build. All values following beyond of that range are tagged as outliers. To substitute outliers, the average for the whole period was taken, like as for the zero values substitution.

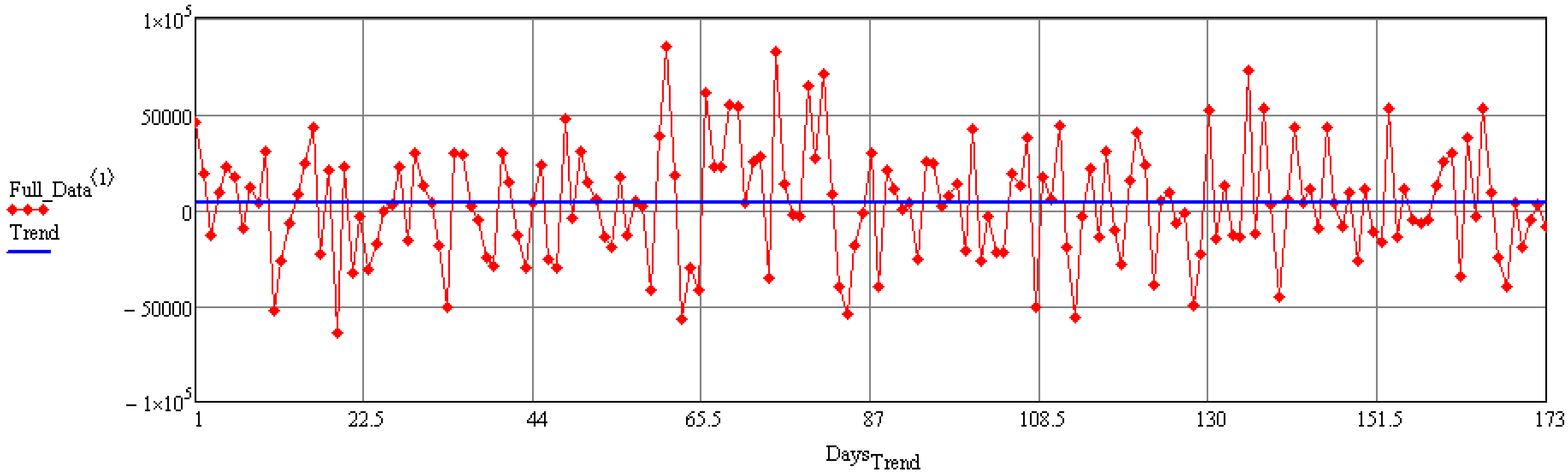
The example of outliers identification for Cash Desk in the branch Prato AG.8 is shown on the Graph 8 *. One value falls out of the (μ-3σ;μ+3σ) range and will be substituted.*

[6] Rit and Ban are recorded with minuses in the Original dataset

Graph 8. Outliers identification. (Cash Desk in branch Prato AG.8)



Graph 9. Trend identification. (Cash Desk in branch Prato AG.8)



8.4. Data Analysis

For the Times Series analysis for Cash Desk the full dataset $\boldsymbol{D_{Full}}$ (first 8 months without records are not included (see "8.1 Data Importing")) with the observations covering 8 months was used.

8.4.1. Trend

Trend identification

In order to identify Trend component we applied **Linear Regression** to the full dataset $\boldsymbol{D_{Full}}$.

For the demand from Cash Desk in the branch Prato AG.8 the slope of Linear Regression is equal to 0.1 (see Graph 9*).*

Trend elimination

Having identified the Trend component we subtracted it from the original dataset and obtained stationary Time series.

$D_t^{S,R} = D_t^{T,S,R} - (b + k * (t-1))$ - is the stationary Time Series (without Trend component), where

$D_t^{T,S,R}$ – is the past demand with Trend, Seasonality and Random components;
k- slope of a linear Trend obtained before;
$t = 1, 2..N_{Full}$ – daily indexes,
N_{Full} - is the number of days in the data subset for Initialization $\boldsymbol{D_{Full}}$.

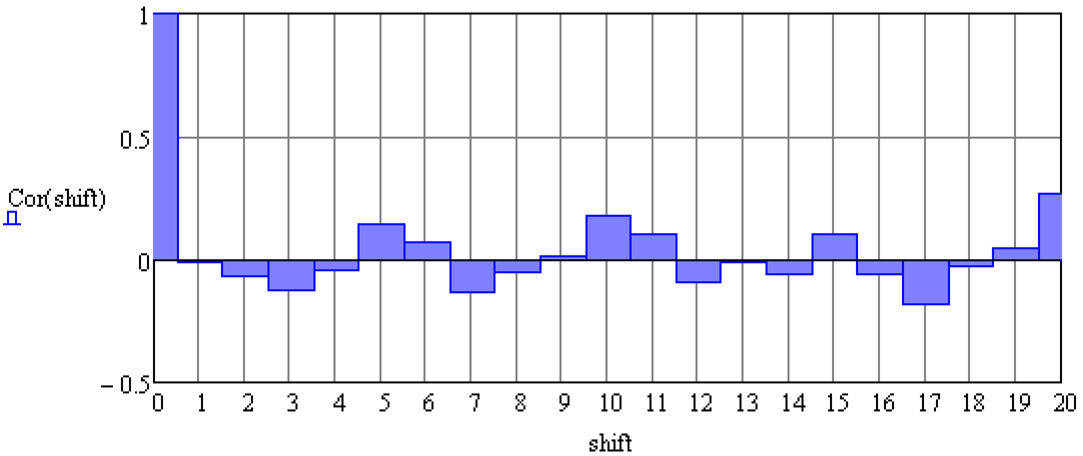
8.4.2. Seasonality

Identification of the Seasonality period

Autocorrelation coefficients were used to identify the seasonality period. A coefficient is calculated as a correlation between two data subsets shifted on k-periods far from each other, where k=1, 2..20. It was done for the stationary Time Series $D_t^{S,R}$ obtained in the previous paragraph.

The autocorrelation coefficients of Cash Desk data from the branch Prato AG.8 is shown on the Graph 10*.*

Graph 10. The autocorrelation coefficients (Cash Desk in branch Prato AG.8)



We assume that the Time series has the seasonality period equal to the shift that provides the highest absolute value of autocorrelation coefficient. However, it is better to check the seasonality periods multiple of 5 and choose the one, which provides the lowest SDE.

Seasonality period for the demand from Cash Desk in the branch Prato AG.8 is equal to 5 days.

The highest absolute value of autocorrelation coefficient *corresponds to the seasonality period equal to 20. However, the deeper analysis showed that Seasonality period equal to 5 assures lower SDE.*

8.5. Initialization of the Time Series model

For the Cash Desk, in order to define initial values for Time series, the full dataset $\boldsymbol{D_{Full}}$ (first 8 months without records are not included (see "8.1 Data Importing")) with the observations covering 8 months was used (see Table 13).

8.5.1. Initial Trend

Initial Trend for a Cash Desk is calculated as a **slope** $\boldsymbol{k}$ of regression line $\boldsymbol{regl(t) = b + k * t}$**,** which best fits the full dataset $\boldsymbol{D_{Full}}$.

For the demand from the Cash Desk in the branch Prato AG.8 the Initial Trend T_0 = 0,1.

8.5.2. Initial Average

Initial Average for a Cash Desk is calculated as a value of regression line for the first day of the full dataset $\boldsymbol{D_{Full}}$, i.e. as an interception point of regression line $\boldsymbol{regl(t) = b + k * t}$ with Y axis.

$$\boldsymbol{A_0 = regl(0) = b}$$

For the demand from the Cash Desk in the branch Prato AG.8 the Initial Average A_0 = 3241.

8.5.3. Initial Seasonality

Initial seasonality indexes for demand from Cash Desk are calculated in the same way as indexes for ATM in paragraph "Initial Seasonality".

As it was found out before, the seasonality period for the Cash Desk in the branch Prato AG.8 is equal to 5 days, thus 5 seasonality indexes were calculated:

$$S_{L\_1y}^{T} = (-5.5 \quad 6.4 \quad 1.3 \quad 5.4 \quad -2.5)$$

At the end of the Initialization process Initial Trend T_0, Initial Average A_0 and *L* (the seasonality period) initial seasonality indexes $S_1^0 \ldots S_L^0$ were calculated.

8.6. Adaptation of the Time Series model

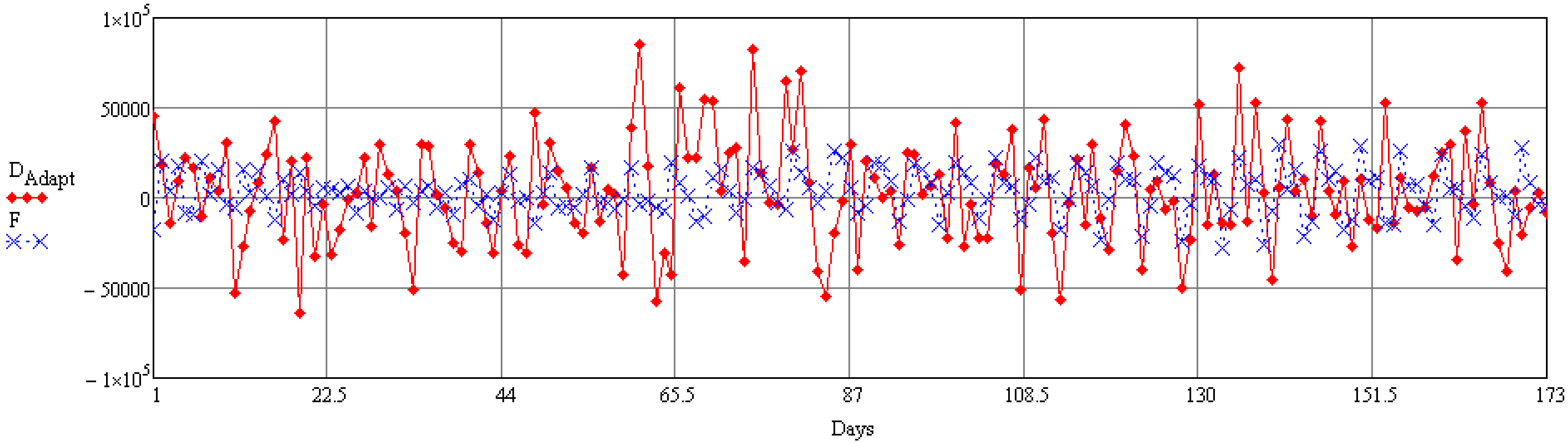
The adaptation process for a Cash Desk is implemented by the analogy with the adaptation process for an ATM, described in the paragraph "6.6 Adaptation of the Time Series model".

For the Cash Desk in the branch Prato AG.8:

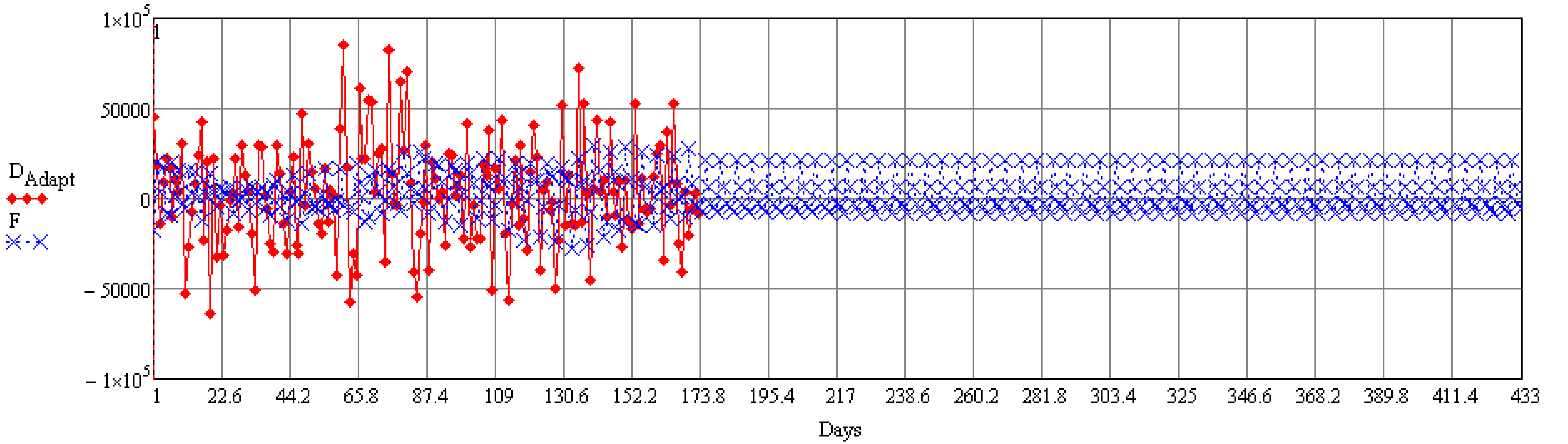
$$\alpha^{*} = 0$$
$$\beta^{*} = 0.14$$
$$\gamma^{*} = 0$$
$$\min\{\text{MSE}\} = \text{MSE}(0,0.14,0) = 8.39 \times 10^{8}$$

The real value of the Demand and the Forecast for the Adaptation period (for the Cash Desk in branch Prato AG.8) are shown on the Graph 11*.*

Graph 11. The real value of the Demand and the Forecast for the Adaptation period (Cash Desk in branch Prato AG.8)



Graph 12. The real value of the Demand for the Adaptation period and the Forecast for the Adaptation period and the next year (Cash Desk in branch Prato AG.8)



8.7. Accuracy measurement

Accuracy of the model was measured comparing the actual demand in the data subset for adaptation $\boldsymbol{D}_{Adapt}$ with the forecast for the adaptation period, which includes N_{Adapt} days. Three types of indicators were calculated:

1) Distortion
2) Dispersion
3) Tracking Signal

8.7.1. Distortion

Mean Percentage Error (MPE):

$$MPE = \frac{\sum_{t=1}^{\mathrm{N_{Adapt}}} \frac{D_t^{\boldsymbol{Adapt}} - F_t}{D_t^{\boldsymbol{Adapt}}}}{\mathrm{N_{Adapt}}} \times 100\%$$

$MPE = 46\ \%$ *- for the Cash Desk in branch Prato AG.8*

8.7.2. Dispersion

Standard Deviation Error (SDE):

$$SDE = \sqrt{\frac{\sum_{t=1}^{\mathrm{N_{Adapt}}} (D_t^{\boldsymbol{Adapt}} - F_t)^2}{\mathrm{N_{Adapt}}}}$$

$SDE = 28\ 969$ *- for the Cash Desk in branch Prato AG.8*

Mean Absolute Percentage Error (MAPE):

$$MAPE = \frac{\sum_{t=1}^{\mathrm{N_{Adapt}}} \left| \frac{D_t^{\boldsymbol{Adapt}} - F_t}{D_t^{\boldsymbol{Adapt}}} \right|}{\mathrm{N_{Adapt}}} \times 100\%$$

$MAPE = 204\%$ *- for the Cash Desk in branch Prato AG.8*

8.7.3. Tracking Signal

Tracking Signal (TS):

$$TS = \frac{\sum_{t=1}^{\mathrm{N_{Adapt}}} D_t^{\boldsymbol{Adapt}} - F_t}{\sum_{t=1}^{\mathrm{N_{Adapt}}} \left| D_t^{\boldsymbol{Adapt}} - F_t \right|}$$

$TS = 0$ *- for the Cash Desk in branch Prato AG.8*

8.8. Forecasting the demand by customers

8.8.1. Forecast for next year

The forecast for the next year for a Cash Desk is implemented by the analogy with the same process for an ATM, which was described in the paragraph "6.8.1 Forecast for next year". In case of Cash Desk the size of the array with the forecast is not equal to 365, as for an ATM, but it is equal to the number of working days (Monday,…, Friday) in the next year.

The real value of the Demand for the Adaptation period and the Forecast for the Adaptation period and the next year (for the Cash Desk in branch Prato AG.8) are shown on the Graph 12.

8.8.2. Yearly seasonality adjustment

As we have data only for the last 8 months we calculated yearly seasonality indexes only for these 8 months. For four remained months we assign the "1" values.

$S_{year}^T =$

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.62	-0.52	0.7	4.43	0.17	0.39	1.45	0.76	1	1	1	1

Later, after the data for the last year is collected, all yearly seasonality indexes can be recalculated.

Having done the yearly seasonality adjustment, we measured the accuracy again and compared it with the one that was obtained in the paragraph "8.7 Accuracy measurement" for a forecast without yearly seasonality adjustment. Then, forecast corresponding to the best accuracy was chosen (See Table 14).

Table 14. Choosing the best forecast (Cash Desk in the branch Prato AG. 8)

Accuracy Measure		Yearly seasonality adjustment	
Name	Formula	NO	YES
MPE	$MPE = \frac{\sum_{t=1}^{N_{Adapt}} \frac{D_t^{Adapt} - F_t}{D_t^{Adapt}}}{N_{Adapt}} \times 100\%$	**94 %**	65 %
SDE	$SDE = \sqrt{\frac{\sum_{t=1}^{N_{Adapt}} (D_t^{Adapt} - F_t)^2}{N_{Adapt}}}$	**30 260**	46 419
MAPE	$MAPE = \frac{\sum_{t=1}^{N_{Adapt}} \left\lvert \frac{D_t^{Adapt} - F_t}{D_t^{Adapt}} \right\rvert}{N_{Adapt}} \times 100\%$	**151 %**	176 %
TS	$TS = \frac{\sum_{t=1}^{N_{Adapt}} D_t^{Adapt} - F_t}{\sum_{t=1}^{N_{Adapt}} \left\lvert D_t^{Adapt} - F_t \right\rvert}$	**-0,002**	-0,083

The forecast without yearly seasonality adjustment **was chosen**, as it provides lower SDE. Tests on different branches showed that such criterion in choosing the better forecast provides lower MAPE and better tracks the signal, as well.

8.9. Total demand/supply forecast

8.9.1. Expected demand by ATM

$$\boldsymbol{F_{by\ atm} = Ban - res}$$

$\boldsymbol{Ban}$ **-** is the replenishment plan to replenish ATM from Cash Desk for the next year;
$\boldsymbol{res}$ **-** is the forecast for the next year about physical cash left in ATM at the moment it has been opened for replenishment.

8.9.2. Total demand from the Cash Desk

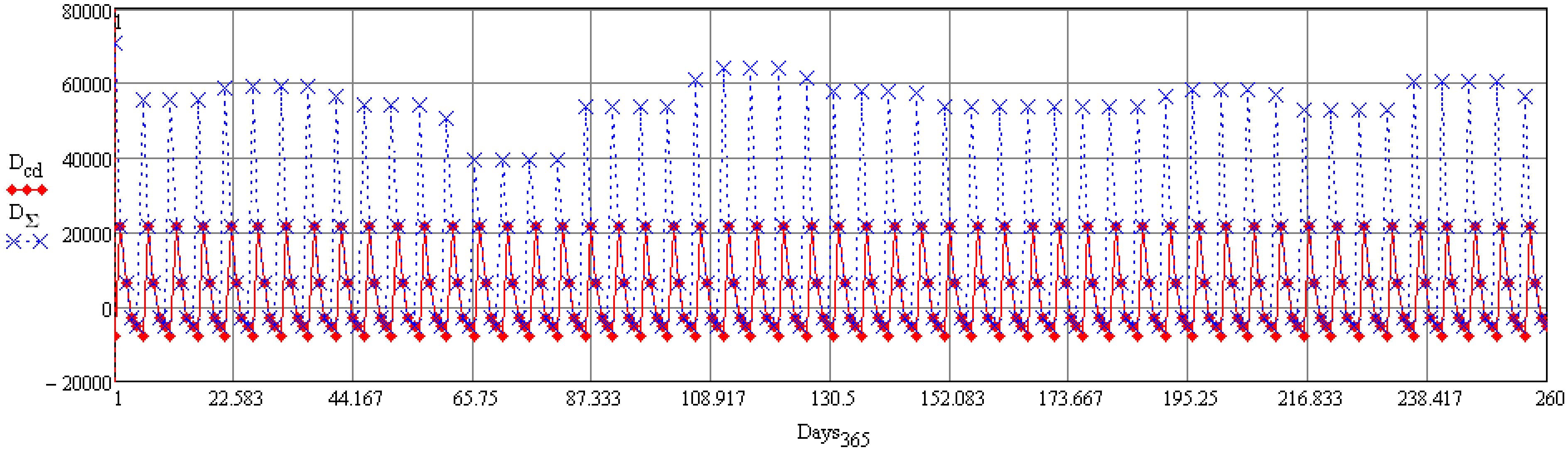
Before the calculation of Total demand $\boldsymbol{D_\Sigma}$, we must eliminate week-ends from the array of expected demand by ATM $\boldsymbol{F_{by\ atm}}$.

$$\boldsymbol{D_\Sigma = F_{by\ cust} + F_{by\ atm}}$$

$\boldsymbol{F_{by\ cust}}$ – is forecasted demand from Cash Desk by customers (see “8.8 Forecasting the demand by customers”).
$\boldsymbol{F_{by\ atm}}$ – is the forecasted demand from Cash Desk by ATM, i.e. the amount of money operators must add to an ATM (see “8.9.1 Expected demand by ATM”).

Total demand D_Σ *for the Cash Desk in branch Prato AG.8 is shown on the* Graph 13.

Graph 13. Total demand for the Cash Desk in branch Prato AG.8



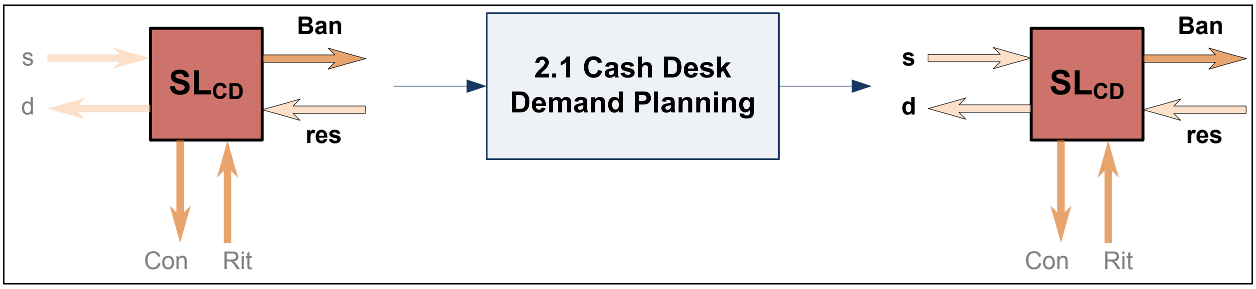
8.10. Output

The output of Cash Desk Demand Planning process (see Figure 15):

- daily total demand/supply forecast for the next year for Cash Desk from both customers and ATM $\boldsymbol{D}_{\Sigma}$;
- estimated forecast accuracy SDE_{cd}.

It will be the Input for the next step, Cash Desk Inventory Planning.

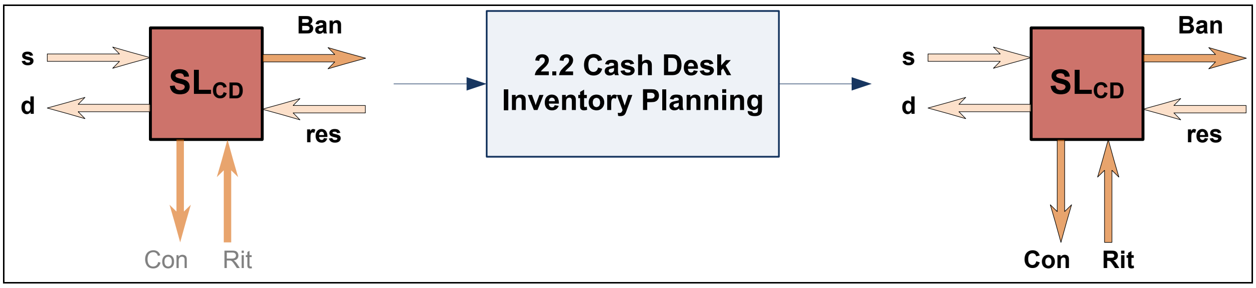
Figure 15. Cash Desk Demand Planning as a part of Branch Inventory Management optimization



9. Cash Desk. Inventory planning

The last step of Branch Inventory Management optimization process is Cash Desk Inventory Planning (see Figure 16).

Figure 16. Cash Desk Inventory Planning as a part of Branch Inventory Management optimization



Objective:

- To define the Inventory management policy and parameters, which will
- minimize Logistics Costs for a Cash Desk,
- assuring the pre-defined service level for customers.

Input:

- Daily forecast total demand/supply forecast for the next year for Cash Desk from both customers and ATM $\boldsymbol{D}_{\Sigma}$,
- Estimated forecasts accuracies SDE_{atm} and SDE_{cd};
- Service target,
- Replenishment lead times,
- Logistics Costs structure.

Output:

- Inventory management policy and parameters,
- Replenishment plan for the next year **Con/Rit**.

9.1. Choice of the Inventory management policy

Two main models for Inventory planning were considered:

- Reorder Point
- Periodic Review

For reasons, described in paragraph "7.1 Choice of the Inventory management policy", for the **Cash Desk** Inventory Planning the **Reorder point model** was chosen.

On average, it ensures lower inventory level than in the Periodic review model stock level, and a branch manager can control the stock level continuously.

Constraint:

Reorder Point model works only if there is more cash demand than supply in a Cash Desk ($\Sigma \boldsymbol{D}_{\boldsymbol{\Sigma}} > 0$), i.e. if customers withdraw more cash from the branch (from both ATMs and Cash Desk), than bring it there.

9.2. Reorder Point model. Choice of the Inventory management parameters

9.2.1. Logistics costs

Total Costs structure

During the meeting with MPS managers we agreed on that Annual logistics Total Costs (TC) consist of two components:

- Annual Inventory Carrying Costs (ICC)
- Annual Procurement Costs (PC)

$$TC(Q) = ICC(Q) + PC(Q)$$

$\boldsymbol{ICC}(\boldsymbol{Q}) = \boldsymbol{AIL}(\boldsymbol{Q}) \cdot \boldsymbol{V} \cdot \boldsymbol{phc}\ [€/year]$

$Q\ [€]$ – Order quantity,

$AIL\ [€]$ – Average Inventory Level per year,

$\boldsymbol{V} \equiv \boldsymbol{1}\ [-]$ – Unitary item Value. Equal to 1 in our case, as in the stock we keep the money,

$phc\ [\%/year]$ – Inventory Carrying Costs as a percentage of item value.

$$\boldsymbol{PC(Q)} = \frac{\boldsymbol{D_y}}{\boldsymbol{Q}} \cdot \boldsymbol{upc}\ [€/year]$$

D_y $[€/year]$ – Annual Demand,

upc $[€]$ – unitary procurement costs.

phc estimation

$$\boldsymbol{phc = 4,16}\ [\%/\boldsymbol{year}]$$

The value of phc is valid for both ATMs and Cash Desks. The description of its estimation is given in the paragraph “7.2.1 Logistics costs”.

upc estimation. Cash Desk

MPS outsourced all the deliveries of Cash to/from the External Vaults. The bank pays the fix rate for one shipment for all branches, for all External Vaults. The cost of a shipment is 31,1€ plus 20% of IVA.

$$\boldsymbol{upc_{cd}} = 31{,}1 + 20\% = \boldsymbol{37,32}\ [€/\boldsymbol{delivery}]$$

9.2.2. Economic Order Quantity

Economic Order Quantity (EOQ), minimizing the Total Costs, is calculated with the following formula:

$$\boldsymbol{EOQ} = \sqrt{\frac{\boldsymbol{2 \cdot upc_{cd} \cdot D_y}}{\boldsymbol{phc \cdot V}}}\ [€]$$

Annual demand from a Cash Desk D_y is obtained by summing of all the elements of $\boldsymbol{D_\Sigma}$, array (Daily forecast total demand/supply forecast for the next year for Cash Desk from both customers and ATM).

D_y must be bigger than 0, otherwise we cannot use the Reorder Point model.

For the Cash Desk in the branch Prato AG.8:

$D_y = €3{,}91\ mln.$ *per year;*

$$EOQ = \sqrt{\frac{2 \cdot 37{,}32\ [€] \cdot 4{,}11 \cdot 10^6 \left[\frac{€}{year}\right]}{4{,}16 \left[\frac{\%}{year}\right] \cdot 1}} = 83\ 695 [€]$$

9.2.3. Safety stock

$$\boldsymbol{SS = k \cdot \sigma_{D,LT}}$$

$\boldsymbol{k = 1,75}$ - is the **service level** defined as the Probability to avoid Stock out during the Order interval and replenishment Lead time. Such service level will ensure the 96% probability to avoid stock out.

$\sigma_{D,LT}$ –is the standard deviation of the demand during lead time (LT):

$$\sigma_{D,LT} = \sqrt{ELT \cdot SDE^2 + \sigma_{LT}{}^2 \cdot ED(1)^2}$$

$\boldsymbol{ELT} = \mathbf{1}$ – is the expected lead time of a Cash Desk stock replenishment.
Branch manager has to check the Stock level at the first part of the day so as to put an order and receive cash the next day (see "3.1 Description").

$\boldsymbol{\sigma_{LT} = 0}$ – is the standard deviation of the lead time of a Cash Desk stock replenishment.

SDE - is the standard deviation of error, the measure of forecast accuracy for the demand in Cash Desk (by both customers and ATMs),
$ED(1)$ – is the expected daily demand from Cash Desk.

As $\sigma_{LT} = 0$, we can rewrite the formula of $\sigma_{D,LT}$ as follows:

$$\boldsymbol{\sigma_{D,LT} = \sqrt{ELT \cdot SDE^2}}$$

Standard deviation of error (SDE):

$$SDE = \sqrt{SDE_{cd}{}^2 + SDE_{atm}{}^2}$$

SDE_{cd} – is the standard deviation of error of the forecasted demand by customers ($\boldsymbol{F_{by\ cust}}$);
SDE_{atm} – is the standard deviation of error of the forecasted demand by ATM ($\boldsymbol{F_{by\ atm}}$).

For the Cash Desk in the branch Prato AG.8:

$SDE = \sqrt{28\ 969^2 + 2\ 501^2} = 29\ 077[€]$

For the Cash Desk in the branch Prato AG.8:

$SS = 1{,}75 \cdot \sqrt{1 \cdot 29\ 077^2} = 50\ 884\ [€]$

9.2.4. Reorder Point

$$ROP_t = ED_t^{LT} + SS$$

$ED_t^{LT} = \mathbf{D}_t^{\Sigma}$ – is the expected demand during lead time for an order, which was put at day t. As the lead time is 1 day, ED_t^{LT} is equal to the forecasted total demand/supply forecast for the next year for Cash Desk from both customers and ATM $\mathbf{D}_t^{\Sigma}$.

9.2.5. Average Inventory Level in a Cash Desk

All the calculations, presented above, were made under the assumption that a week consists out of 5 days only (without week-ends). In order to calculate expected Average Inventory Level for a Cash Desk, we must also consider the stock being carried during week-ends.

If the Stock Level at the day t reaches the ROP_t, a branch manager has to order the EOQ from External Vault to the branch.

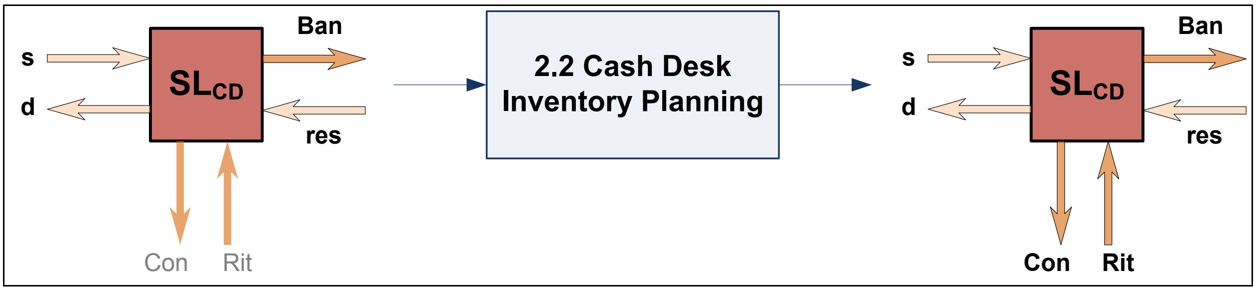
Profile of expected Stock Level and Reorder Points *for the Cash Desk in the branch Prato AG.8 is shown on the* Graph 14.

9.3. Output

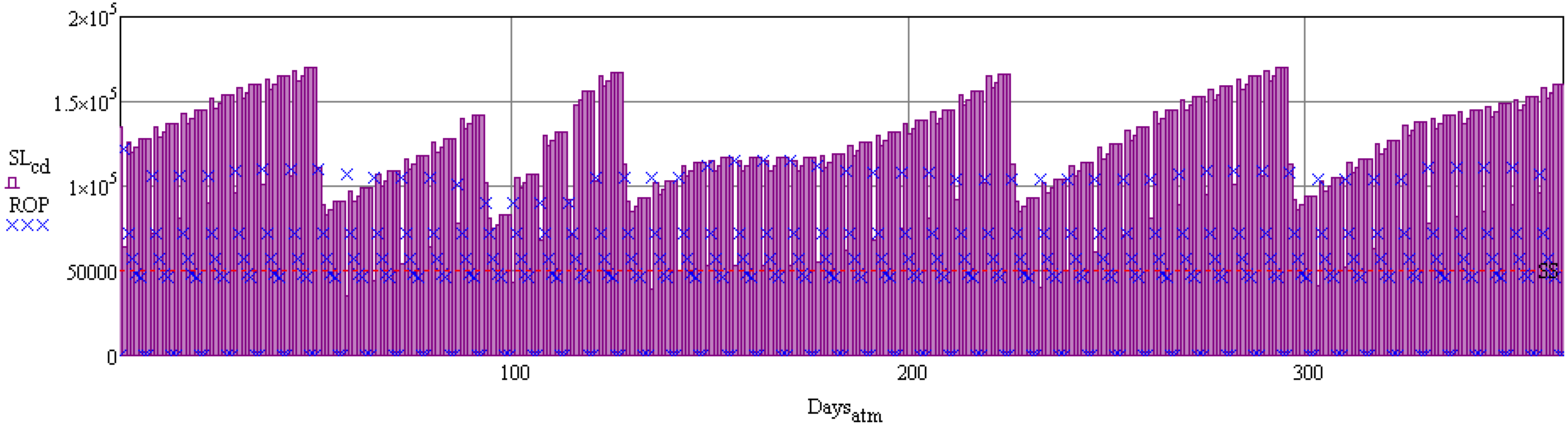
The output of the Cash Desk Inventory Planning process (see Figure 17) is the

- Replenishment plan for Cash Desk for the next year Con (if the Total Demand > 0).

Figure 17. Cash Desk Inventory Planning as a part of Branch Inventory Management optimization



Graph 14. Profile of expected Stock Level and Reorder Points. Cash Desk in the branch Prato AG.8



10. Estimation of expected benefits

10.1. Costs Savings

10.1.1. Costs TO BE for the Forecast

In Costs To Be for the Forecast calculation the forecasted demand is used as an input for Inventory planning.

For the branch Prato AG.8:

$$ICC_{CD} = AIL_{CD} \cdot V \cdot phc = 121\,256 \cdot 1 \cdot 0{,}0416 = 5\,049\ [€]$$
$$ICC_{ATM} = AIL_{ATM} \cdot V \cdot phc = 41\,153 \cdot 1 \cdot 0{,}0416 = 1\,714\ [€]$$
$$PC_{CD} = N_{CD}^{pr} \cdot upc_{CD} = 48 \cdot 37{,}32 = 1\,791\ [€]$$
$$PC_{ATM} = N_{ATM}^{pr} \cdot upc_{ATM} = 52 \cdot 28{,}77 = 1\,496\ [€]$$

10.1.2. Costs TO BE for the Past Demand

In Costs To Be for the Past Demand calculation the data about past demand are used as an input for Inventory planning. For Inventory planning we used the same methodology as it was described in the current work, but instead of Forecasted Demand we used Expected Demand in the Past and standard deviation of Past demand instead of SDE.

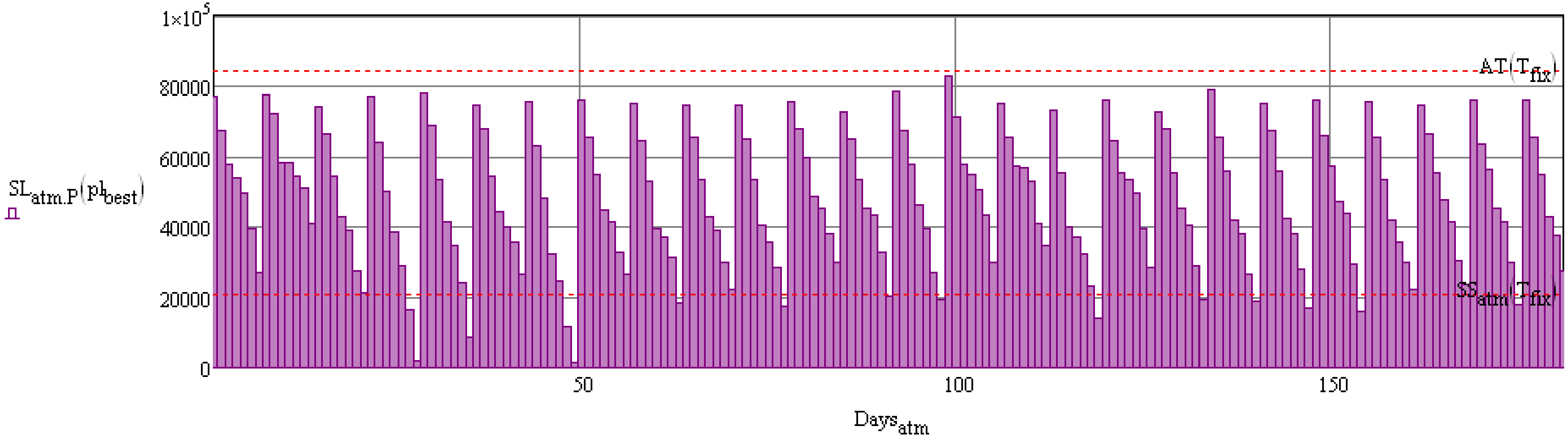
Unfortunately, the bank started to record the data, from which it is possible extract the information about demand from Cash Desk, only in September 2010. So, to calculate AIL and number of procurements, the records starting from the September 1st 2010 till March 1st 2011 (half a year) were used. The AIL were calculated using the data from that half a year. Number of procurements was obtained by counting the number of procurements during that half a year and by its multiplication by two.

For the branch Prato AG.8:

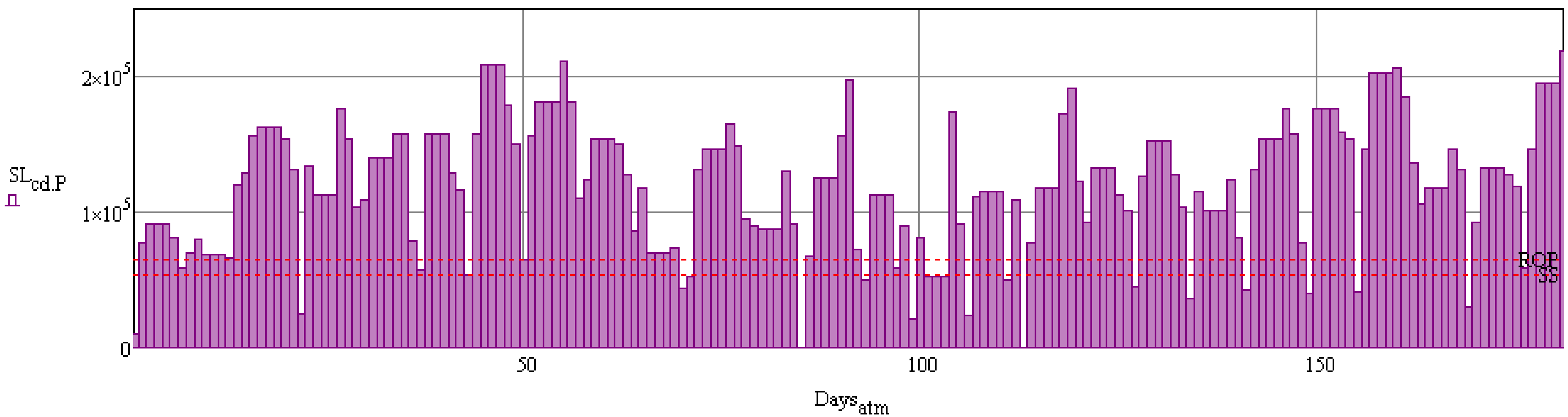
$$ICC_{CD} = AIL_{CD} \cdot V \cdot phc = 119\,548 \cdot 1 \cdot 0{,}0416 = 4\,978\ [€]$$
$$ICC_{ATM} = AIL_{ATM} \cdot V \cdot phc = 47\,784 \cdot 1 \cdot 0{,}0416 = 1\,990\ [€]$$
$$PC_{CD} = N_{CD}^{pr} \cdot upc_{CD} = 54 \cdot 37{,}32 = 2\,015\ [€]$$
$$PC_{ATM} = N_{ATM}^{pr} \cdot upc_{ATM} = 52 \cdot 28{,}77 = 1\,496\ [€]$$

The profiles of Stock Level for branch Prato AG.8 are shown on the Graph 15 *and on the* Graph 16.

Graph 15. Profile of expected Stock Level for Past Demand. ATM in the branch Prato AG.8



Graph 16. Profile of expected Stock Level for Past Demand. Cash Desk in the branch Prato AG.8



10.1.3. Costs AS IS

In order to estimate Costs As Is the past data were used. Unfortunately, the bank started to record the data, from which it is possible extract the information about Stock level, only in September 2010. So, to calculate AIL and number of procurements, the records starting from the September 1st 2010 till March 1st 2011 (half a year) were used. The AIL were calculated using the data from that half a year. Number of procurements was obtained by counting the number of procurements during that half a year and by its multiplication by two.

For the branch Prato AG.8:

$$ICC_{CD} = AIL_{CD} \cdot V \cdot phc = 89\,830 \cdot 1 \cdot 0{,}0416 = 3\,741\ [€]$$

$$ICC_{ATM} = AIL_{ATM} \cdot V \cdot phc = 26\,414 \cdot 1 \cdot 0{,}0416 = 1\,100\ [€]$$

$$PC_{CD} = N_{CD}^{pr} \cdot upc_{CD} = 100 \cdot 37{,}32 = 3\,732\ [€]$$

$$PC_{ATM} = N_{ATM}^{pr} \cdot upc_{ATM} = 126 \cdot 28{,}77 = 3\,625\ [€]$$

10.1.4. Costs Savings

The expected Costs Savings for branch Prato AG.8 are presented in the Table 15.

Table 15. Expected Costs Savings for branch Prato AG.8

	Cost type	AS IS	TO BE (Past D.)	Δ	Δ%
ATM	ICC	1 100	1 990	-890	-81%
	PC	3 625	1 496	2 129	59%
	Total Costs	**4 725**	**3 486**	**1 239**	**26%**
Cash Desk	ICC	3 741	4 978	-1 237	-33%
	PC	3 732	2 015	1 717	46%
	Total Costs	**7 473**	**6 993**	**479**	**6%**
Branch	**Total Costs**	**12 197 €**	**10 479 €**	**€1 718**	**14%**

The estimation of expected Costs Savings for all branches of MPS, obtained by testing the methodology on branches in Prato, is presented in the Table 16.

Table 16. Estimation of expected Costs Savings for all branches of MPS

Branch	Δ	Δ%
Prato AG.1	5 338 €	34%
Prato AG.2	Supply>Demand[7]	Supply>Demand
Prato AG.3	2 962 €	19%
Prato AG.4	360 €	3%
Prato AG.5	3 089 €	24%
Prato AG.6	Supply>Demand	Supply>Demand
Prato AG.7	6 044 €	29%
Prato AG.8	1 718 €	14%
Prato AG.9	3 530 €	30%
Prato	Supply>Demand	Supply>Demand
Prato AG.11	Supply>Demand	Supply>Demand
Average Saving per branch:	**€3 292**	**22%**
Total Savings	**€8,6 mln**	**22%**

The expected savings for all branches per year were calculated by the multiplying the average savings per branch by the number of branches: $3\,292\left[\frac{€}{branch}\right] \times 2\,600\,[branches] = €8{,}6\,mln$

For comparison, the following items from the Income Statement 2010 of MPS (Banca Monte dei Paschi di Siena, 2010) are given:

Transportation Costs: €40 mln/year

Costs of robbery: €9 mln/year

[7] The proposed way of Cash Desk Inventory Planning works only if there is more Demand than Supply in a Cash Desk.

10.2. Defined Service Level

The proposed approach for Inventory management assures the predefined Service level.

For ATMs

For the calculations presented above, the **Service Level** for ATMs $\boldsymbol{k_{atm} = 2,33}$ was used. Such service level will ensure the 99% probability to avoid stock out.

For Cash Desk

For the calculations presented above, the **Service Level** for Cash Desks $\boldsymbol{k_{cd} = 1,75}$ was used. Such service level will ensure the 96% probability to avoid stock out.

We couldn't estimate the current Service Level in ATMs and Cash Desks. However, using the Mathcad program, which was developed for Inventory Management optimization, MPS can easily calculate the economic impact of the Service Level variation and choose the best values of $\boldsymbol{k_{atm}}$ and $\boldsymbol{k_{cd}}$.

10.3. Improvements in environmental and social performance

Banca Monte dei Paschi di Siena pays a lot of attention to the Environmental and Social Responsibility. The director of Procurement, Logistics, HSE and Security Management, Luca Guzzabocca put a lot of efforts to increase the environmental and social performance of MPS. As a result, the bank obtained several awards in 2010. The most important ones are the following (Guzzabocca, 2011):

Procurement Leaders Awards 2010, 26 May 2010

5 Short List Finalist for "CSR Initiatives in supply management". The only Italian company (all the categories) and Bank (short listed) on 50 Initiatives.

CIPS Supply Management Awards 2010, 8 Sept 2010

6 Short List Finalist for "Best contribution to CSR". The only Italian company (all the categories) and Bank (short listed) on 222 Initiatives.

ProcureCon2010 Awards -4 Nov 2010

7 Short list Finalist for "Sustainability and Green Sourcing". The only Italian company (all the categories) and Bank (short listed).

The proposed methodology can decrease the number of transportations from External Vault to a branch (see Table 17). Decreasing the number of transportation will lead to the reducing of the CO_2 emission and decreasing of the number of car accidents in the streets. All this will improve the environmental and social performance of the bank.

Table 17. Estimation of expected reduction in the number of transportations from MPS branches to External Vaults

Branch	As Is	To Be	Δ	Δ%
Prato AG.1	152[8]	74	78	51%
Prato AG.2	Supply>Demand[9]	Supply>Demand	Supply>Demand	Supply>Demand
Prato AG.3	146	68	78	53%
Prato AG.4	106	66	40	38%
Prato AG.5	126	72	54	43%
Prato AG.6	Supply>Demand	Supply>Demand	Supply>Demand	Supply>Demand
Prato AG.7	154	100	54	35%
Prato AG.8	100	54	46	46%
Prato AG.9	84	32	52	62%
Prato	Supply>Demand	Supply>Demand	Supply>Demand	Supply>Demand
Prato AG.11	Supply>Demand	Supply>Demand	Supply>Demand	Supply>Demand
Average Reduction:			**57**	**47%**
Total Reduction:			**148 200**	**47%**

The expected reduction of the number of transportations from External Vault to branches per year was calculated by the multiplying the average reduction per branch by the number of branches: $57\left[\frac{1}{branch}\right] \times 2\,600\,[branches] = 148\,200.$

The proposed methodology **reduces the number of transportations** from External Vault to a branch **on 47%.**

[8] Unit of measure: [number of transportations/year]

[9] The proposed way of Cash Desk Inventory Planning works only if there is more Demand than Supply in a Cash Desk.

Conclusions

The methodology of Inventory management, allowing Banca Monte dei Paschi di Siena managing its Physical cash Inventories in a structured way, was proposed.

The Mathcad program, automatically calculating the Inventory management plan for a branch (both for ATMs and Cash Desk), according to the proposed methodology, was developed. The program was applied to define the Inventory management plans and expected benefits for the branches in Prato.

The results showed, that implementation of proposed methodology allows:

- decrease the costs of physical cash logistics on 22%;
- define the service level in ATMs and Cash Desks;
- decrease the number of cash transportations from an external vault to a branch on 47%.

Limitations

- The Cash Desk Inventory Planning part of the methodology works only under the assumption that during the year there is more cash Demand than Supply in the Cash Desk. In other words, customers withdraw more cash from the branch, than bring it there.
- Forecasting model for Cash Desks is not very flexible. For some branches it doesn't provide forecast with sufficient accuracy. This can be explained by the small amount of available past data for Cash Desks.

Further development

- To include into the Cash Desk Inventory Planning the case, when there is more Supply than Demand.
- To check the flexibility of forecasting model for Cash Desk, when the data for past 2 years will be available.

The methodology presented in this thesis will be tested in branches in Prato. If the practical results will correspond to the theoretical estimations, the methodology will be implemented in other branches of Monte dei Paschi di Siena.

Bibliography

Banca Monte dei Paschi di Siena. (s.d.). *About the Group*. Tratto il giorno 2011 da http://english.mps.it/GMPS/

Banca Monte dei Paschi di Siena. (2010). *Annual Report.*

Banca Monte dei Paschi di Siena. (s.d.). *Five centuries of history*. Tratto da http://english.mps.it/GMPS/Storia/

Guzzabocca, L. (2011). Sustainable Procurement at Gruppo Montepaschi: delivering Efficiency, delivering Value.

Perego, A., & Mangiaracina, R. (2010). *Demand Forecasting, "Logistics Management" course, lecture 14.* Politecnico di Milano.

Perego, A., & Mangiaracina, R. (2010). *Pull Inventory Control, "Logistics Management" course, lecture 16.* Politecnico di Milano.

Vercellis, C. (2009). Business Intelligence: Data Mining and Optimization for Decision Making. A John Wiley and Sons, Ltd. Publications.

1/7 ATM. Demand Planning

1. Data Importing

$Full\_Data_{Original} :=$

	0	1	2	3	4	5	6
		G.ap	G.ch	C	R	B	ATM Withdrawals
0	'01/01/2010"	0	0	0	0	0	2560
1	'01/02/2010"	0	0	0	0	0	7620
2	'01/03/2010"	0	0	0	0	0	4820
3	'01/04/2010"	0	0	0	0	0	13800
4	'01/05/2010"	0	0	0	0	0	13600
5	'01/06/2010"	0	0	0	0	0	1550
6	'01/07/2010"	0	0	0	0	0	4740
7	'01/08/2010"	0	0	0	0	0	8990
8	'01/09/2010"	0	0	0	0	0	7990
9	'01/10/2010"	0	0	0	0	0	5140
10	'01/11/2010"	0	0	0	0	0	...

$N_{Full} := rows(Full\_Data_{Original}) = 485$

Adding weekly indexes to the original data

```
Week_index := | for i ∈ 0 .. 7 − 1
              |    Week_index_i ← i + 1
              | Week_index
```

```
Week_index_per_full_data := | k ← −1
                            | for i ∈ 0 .. N_Full − 1
                            |    | k ← k + 1
                            |    | Week_index_per_full_data_i ← Week_index_k
                            |    | k ← −1  if  k = 7 − 1
                            | Week_index_per_full_data
```

Indexes shfting

$period := 7$ - period = 7 (1 week)

Full_Data.Original (Jan2010_April2011) starts with Janaury 1st, Friday.

$fd := 5$ - First day of the week of data Past data

```
Index_week := | for i ∈ fd .. N_Full
              |    (QW)_(i−fd) ← (Week_index_per_full_data)_(i−1)
              | for i ∈ 1 .. fd − 1
              |    (QW)_(N_Full−fd+i) ← (Week_index_per_full_data)_(mod(N_Full, period)+i−1)
              | QW
```

Check the last day of the week of the Original data:

$Index\_week_{N_{Full}-1} = 6$ - April 2011, Satturday

$Full\_Data_{Original} := augment(Full\_Data_{Original}, Index\_week)$

	G.ap	G.ch	C	R	B	ATM Withdra.	Day of week

$Full\_Data_{Original} =$

	0	1	2	3	4	5	6	7
0	"01/01/2010"	0	0	0	0	0	2560	5
1	"01/02/2010"	0	0	0	0	0	7620	6
2	"01/03/2010"	0	0	0	0	0	4820	7
3	"01/04/2010"	0	0	0	0	0	13800	1
4	"01/05/2010"	0	0	0	0	0	13600	2
5	"01/06/2010"	0	0	0	0	0	1550	3
6	"01/07/2010"	0	0	0	0	0	4740	4
7	"01/08/2010"	0	0	0	0	0	8990	5
8	"01/09/2010"	0	0	0	0	0	7990	6
9	"01/10/2010"	0	0	0	0	0	5140	7
10	"01/11/2010"	0	0	0	0	0	8170	...

2. Data Transformation. Extracting demand from ATM from the original data

$Dates := submatrix(Full\_Data_{Original}, 0, N_{Full} - 1, 0, 0)$

$F\_D_{atm} := submatrix(Full\_Data_{Original}, 0, N_{Full} - 1, 6, 7)$

$Full\_Data_{atm} := augment(Dates, F\_D_{atm})$

$Full\_Data_{atm} =$

	0	1	2
0	"01/01/2010"	2560	5
1	"01/02/2010"	7620	6
2	"01/03/2010"	4820	7
3	"01/04/2010"	13800	1
4	"01/05/2010"	13600	2
5	"01/06/2010"	1550	3
6	"01/07/2010"	4740	4
7	"01/08/2010"	8990	5
8	"01/09/2010"	7990	6
9	"01/10/2010"	5140	7
10	"01/11/2010"	8170	...

$N_{Full.atm} := rows(Full\_Data_{atm}) = 485$

3. Data Substituition

A) Zero values identification and substituition

To substituite zero values, the average withdrawls for that day of the week were used.

Calculating 7 averages per every day of the week

```
aver_atm :=  for i ∈ 0 .. 7
                 sum_i ← 0
                 num_i ← 0
             for i ∈ 0 .. rows(Full_Data_atm) − 1
                 k ← (Full_Data_atm^⟨2⟩)_i
                 sum_k ← sum_k + [(Full_Data_atm)^⟨1⟩]_i
                 num_k ← num_k + 1
                 aver_k ← sum_k / num_k
             Average ← augment(sum, num, aver)
             aver
```

Zero values substituition

```
Full_Data_atm :=  for i ∈ 0 .. rows(Full_Data_atm) − 1
                      k ← (Full_Data_atm^⟨2⟩)_i
                      average ← 0
                      if (Full_Data_atm^⟨1⟩)_i = 0
                          k ← (Full_Data_atm^⟨2⟩)_i
                          (Full_Data_atm^⟨1⟩)_i ← aver_atm_k
                  Full_Data_atm
```

Creating the arrays with Natural numbers

$$Days_{atm} := \left| \begin{array}{l} Days_{atm_0} \leftarrow 0 \\ \text{for } i \in 1..N_{Full.atm} + 365 \\ \quad Days_{atm_i} \leftarrow i \\ Days_{atm} \end{array} \right.$$

$$Days_{Full.atm} := \text{submatrix}(Days_{atm}, 0, N_{Full.atm}, 0, 0)$$

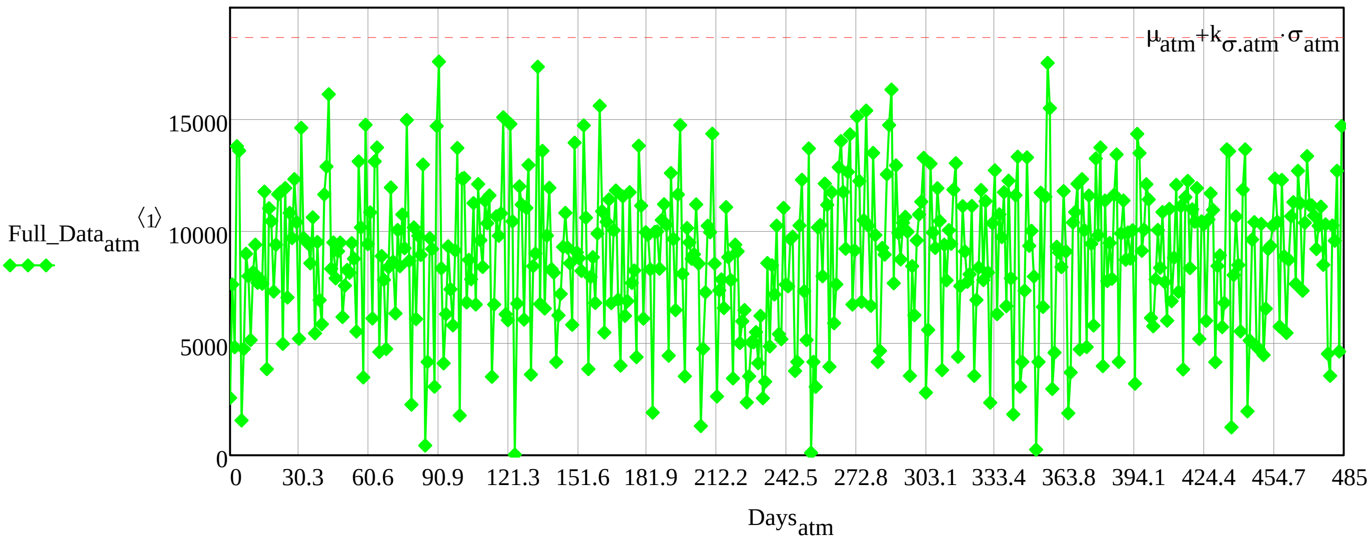
$$Days_{365} := \text{submatrix}(Days_{atm}, 0, 365, 0, 0)$$

B) Outliers identification and substituition

Outliers identification

$$\mu_{atm} := \frac{\sum_{i=0}^{N_{Full}-1} \left(Full\_Data_{atm}^{\langle 1 \rangle}\right)_i}{N_{Full}} = 8838 \qquad \sigma_{atm} := \sqrt{\frac{\sum_{i=0}^{N_{Full}-1} \left[\left(Full\_Data_{atm}^{\langle 1 \rangle}\right)_i - \mu_{atm}\right]^2}{N_{Full}}} = 3292$$

$$k_{\sigma.atm} := 3$$



Outliers substituition

$$Full\_Data_{atm} := \left| \begin{array}{l} \text{for } i \in 0..\text{rows}(Full\_Data_{atm}) - 1 \\ \quad \left| \begin{array}{l} k \leftarrow \left(Full\_Data_{atm}^{\langle 2 \rangle}\right)_i \\ average \leftarrow 0 \\ \text{if } \left[\left(Full\_Data_{atm}^{\langle 1 \rangle}\right)_i > \mu_{atm} + k_{\sigma.atm} \cdot \sigma_{atm}\right] \vee \left[\left(Full\_Data_{atm}^{\langle 1 \rangle}\right)_i < \mu_{atm} - k_{\sigma.atm} \cdot \sigma_{atm}\right] \\ \quad \left| \begin{array}{l} k \leftarrow \left(Full\_Data_{atm}^{\langle 2 \rangle}\right)_i \\ \left(Full\_Data_{atm}^{\langle 1 \rangle}\right)_i \leftarrow aver_{atm_k} \end{array} \right. \end{array} \right. \\ Full\_Data_{atm} \end{array} \right.$$

4. Analysis

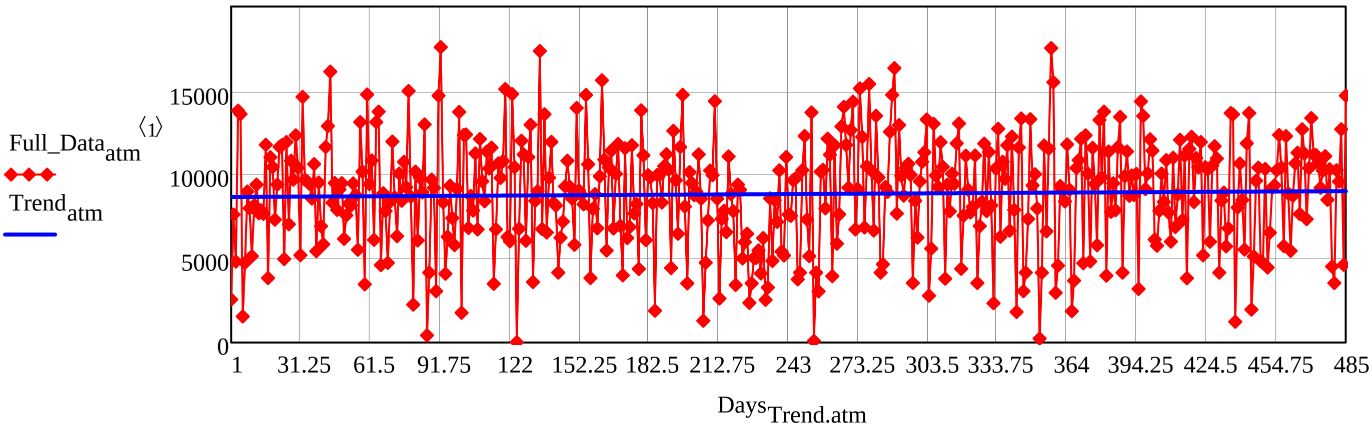
Identification of the Trend

$Days_{Trend.atm} := submatrix(Days_{Full.atm}, 1, N_{Full.atm}, 0, 0)$

$B_{atm} := intercept\left(Days_{Trend.atm}, Full\_Data_{atm}^{\langle 1 \rangle}\right) = 8661.81$

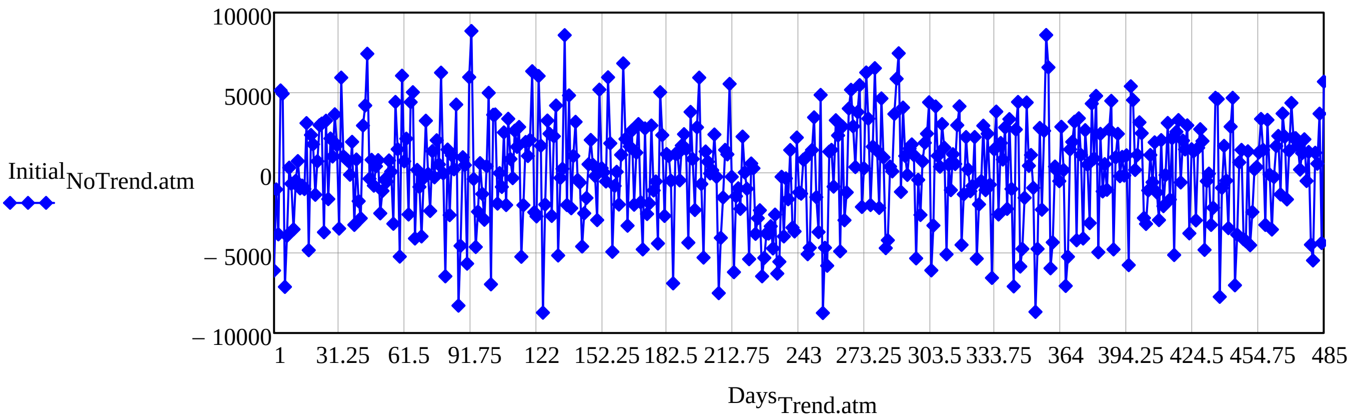
$K_{atm} := slope\left(Days_{Trend.atm}, Full\_Data_{atm}^{\langle 1 \rangle}\right) = 0.72$

$Trend_{atm} := B_{atm} + K_{atm} \cdot Days_{Trend.atm}$



Trend removing

$$Initial_{NoTrend.atm} := \begin{array}{|l} \text{for } i \in 0 .. N_{Full.atm} - 1 \\ \quad Initial_{NoTrend.atm_i} \leftarrow \left(Full\_Data_{atm}^{\langle 1 \rangle}\right)_i - (B_{atm} + K_{atm} \cdot i) \\ Initial_{NoTrend.atm} \end{array}$$



Seasonality period identification

$N_{shift.atm} := 30$

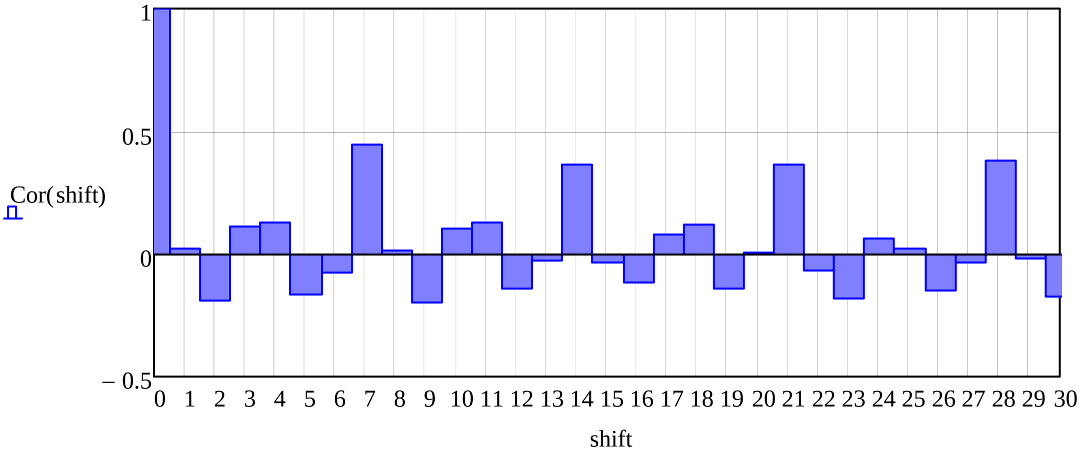
$corr(Initial_{NoTrend.atm}, Initial_{NoTrend.atm}) = 1$

$VX := submatrix(Initial_{NoTrend.atm}, N_{shift.atm}, N_{Full.atm} - 1, 0, 0)$

$shift := 0 .. N_{shift.atm}$

$VY(shift) := submatrix(Initial_{NoTrend.atm}, N_{shift.atm} - shift, N_{Full.atm} - 1 - shift, 0, 0)$

$Cor(shift) := corr(VX, VY(shift))$



$$r_{k.atm} := \left| \begin{array}{l} r_{k_0} \leftarrow 0 \\ \text{for } i \in 1 .. N_{shift.atm} \\ \quad r_{k_i} \leftarrow |Cor(i)| \\ r_k \end{array} \right.$$

$r_{k.max} := \max(r_{k.atm}) = 0.44$

$$L_{atm} := \left| \begin{array}{l} \text{for } i \in 1 .. N_{shift.atm} \\ \quad L \leftarrow i \text{ if } r_{k.atm_i} = r_{k.max} \\ L \end{array} \right.$$

$L_{atm} = 7$ - Seasonality period

Adding L-indexes

$$L\_index_{atm} := \left| \begin{array}{l} \text{for } i \in 0 .. L_{atm} - 1 \\ \quad L\_index_{atm_i} \leftarrow i + 1 \\ L\_index_{atm} \end{array} \right.$$

$$Full\_Data_{atm} := \left| \begin{array}{l} k \leftarrow -1 \\ \text{for } i \in 0 .. N_{Full.atm} - 1 \\ \quad \left| \begin{array}{l} k \leftarrow k + 1 \\ L\_index_{atm_i} \leftarrow L\_index_{atm_k} \\ k \leftarrow -1 \text{ if } k = L_{atm} - 1 \end{array} \right. \\ Full\_Data_{atm} \leftarrow \text{augment}(Full\_Data_{atm}, L\_index_{atm}) \\ Full\_Data_{atm} \end{array} \right.$$

5. Initialization

Substracting data subset for Initialization

Set the size of the data subset for Initialization:

$N_{Initial.atm} := 365$ - 12 months (Jan2010-Dec2010)

$Initial_{atm} := submatrix(Full\_Data_{atm}, 0, N_{Initial.atm} - 1, 0, 2)$

$Initial_{atm} =$

	0	1	2
0	"01/01/2010"	2560	5
1	"01/02/2010"	7620	6
2	"01/03/2010"	4820	7
3	"01/04/2010"	13800	1
4	"01/05/2010"	13600	2
5	"01/06/2010"	1550	3
6	"01/07/2010"	4740	4
7	"01/08/2010"	8990	5
8	"01/09/2010"	7990	6
9	"01/10/2010"	5140	7
10	"01/11/2010"	8170	...

1) Initial Trend. ATM

$Days_{In.atm} := submatrix(Days_{atm}, 1, N_{Initial.atm}, 0, 0)$

$K_{Initial.atm} := slope\left(Days_{In.atm}, Initial_{atm}^{\langle 1 \rangle}\right)$

$T_{atm_0} := K_{Initial.atm} = -0.14$

2) Initial Average. ATM

$B_{Initial.atm} := intercept\left(Days_{In.atm}, Initial_{atm}^{\langle 1 \rangle}\right) = 8783.75$

$A_{atm_0} := B_{Initial.atm} + K_{Initial.atm} \cdot N_{Initial.atm} = 8733$

3) Initial Seasonality coefficients

$Initial_{atm} := submatrix(Full\_Data_{atm}, 0, N_{Initial.atm} - 1, 0, 3)$

```
S_L_ly.atm := | for j ∈ 1 .. floor(N_Initial.atm / L_atm)
              |   sum_j ← 0
              | for j ∈ 1 .. L_atm
              |   summa_j ← 0
              | k ← 1
              | for i ∈ 0 .. floor(N_Initial.atm / L_atm)·L_atm − 1
              |   | sum_k ← sum_k + (Initial_atm^<1>)_i
              |   | if mod(i + 1, L_atm) = 0
              |   |   | aver_k ← sum_k / L_atm
              |   |   | k ← k + 1
              | for k ∈ 1 .. floor(N_Initial.atm / L_atm)
              |   for l ∈ 1 .. L_atm
              |     S_l,k ← (Initial_atm^<1>)_(l+(k−1)·L_atm−1) / aver_k
              | for l ∈ 1 .. L_atm
              |   | for k ∈ 1 .. floor(N_Initial.atm / L_atm)
              |   |   summa_l ← summa_l + S_l,k
              |   | S_L_ly_l ← summa_l / floor(N_Initial.atm / L_atm)
              | S_L_ly
```

6. Adaptation

Substracting data subset for Adaptation

Set the size of the data subset for Initialization:

$N_{Initial.atm} = 365$ - 12 months (Jan2010-Dec2010)

Set the size of the data subset for Adaptation for ATM:

$N_{Adapt.atm} := 120$ - 4 months (Jan2011-April2011)

$Adapt_{atm} := submatrix(Full\_Data_{atm}, N_{Initial.atm}, N_{Initial.atm} + N_{Adapt.atm} - 1, 0, 3)$

$Adapt_{atm} =$

	0	1	2	3
0	"01/01/2011"	1870	6	2
1	"01/02/2011"	3690	7	3
2	"01/03/2011"	10390	1	4
3	"01/04/2011"	10860	2	5
4	"01/05/2011"	12120	3	6
5	"01/06/2011"	4720	4	7
6	"01/07/2011"	12330	5	1
7	"01/08/2011"	10050	6	2
8	"01/09/2011"	4830	7	3
9	"01/10/2011"	11590	1	4
10	"01/11/2011"	9440	2	...

Adjusting the Seasonality Indexes for the Adaptation subset

$fd_{atm} := Adapt_{atm_{0,3}} = 2$ - first L-index of the 1st day of the array of data for Adaptation

$$
\begin{aligned}
S_{0\_adapt.atm} := \;& S_{0_0} \leftarrow 0 \\
& \text{for } i \in fd_{atm} .. L_{atm} \\
& \quad S_{0(i+1-fd_{atm})} \leftarrow S_{L\_ly.atm_i} \\
& \text{for } i \in 1 .. fd_{atm} - 1 \\
& \quad S_{0(L_{atm}+1-fd_{atm}+i)} \leftarrow S_{L\_ly.atm_i} \\
& S_0
\end{aligned}
$$

2 arrays musts be shifted on: $fd_{atm} - 1 = 1$

$S_{0\_adapt.atm}^{T} = (0 \quad 1.01 \quad 0.52 \quad 1.13 \quad 1.07 \quad 0.9 \quad 1.08 \quad 1.28)$

$S_{L\_ly.atm}^{T} = (0 \quad 1.28 \quad 1.01 \quad 0.52 \quad 1.13 \quad 1.07 \quad 0.9 \quad 1.08)$

MSE(α,β,γ) function

$S_{0.atm} := S_{0\_adapt.atm}$

$D_{Adapt.atm_0} := 0$

$D_{Adapt.atm} := stack\left(D_{Adapt.atm_0}, Adapt_{atm}^{\langle 1 \rangle}\right)$

$$
\begin{aligned}
MSE_{atm}(\alpha,\beta,\gamma) := \;& F \leftarrow 0 \\
& F \leftarrow stack\left[F, \left(A_{atm_0} + T_{atm_0}\right)\cdot S_{0.atm_1}\right] \\
& MSE_{atm} \leftarrow 0 \\
& S \leftarrow 0 \\
& \text{for } t \in 1 .. N_{Adapt.atm} \\
& \quad \text{if } t < L_{atm} \\
& \quad\quad A_{atm} \leftarrow stack\left[A_{atm}, \alpha\cdot\frac{D_{Adapt.atm_t}}{S_{0.atm_t}} + (1-\alpha)\cdot\left(A_{atm_{t-1}} + T_{atm_{t-1}}\right)\right] \\
& \quad\quad T_{atm} \leftarrow stack\left[T_{atm}, \gamma\cdot\left(A_{atm_t} - A_{atm_{t-1}}\right) + (1-\gamma)\cdot T_{atm_{t-1}}\right] \\
& \quad\quad S \leftarrow stack\left[S, \beta\cdot\frac{D_{Adapt.atm_t}}{A_{atm_t}} + (1-\beta)\cdot S_{0.atm_t}\right] \\
& \quad\quad F \leftarrow stack\left[F, \left(A_{atm_t} + T_{atm_t}\right)\cdot S_{0.atm_{t+1}}\right] \\
& \quad \text{if } t = L_{atm} \\
& \quad\quad A_{atm} \leftarrow stack\left[A_{atm}, \alpha\cdot\frac{D_{Adapt.atm_{L_{atm}}}}{S_{0.atm_{L_{atm}}}} + (1-\alpha)\cdot\left(A_{atm_{L_{atm}-1}} + T_{atm_{L_{atm}-1}}\right)\right] \\
& \quad\quad T_{atm} \leftarrow stack\left[T_{atm}, \gamma\cdot\left(A_{atm_{L_{atm}}} - A_{atm_{L_{atm}-1}}\right) + (1-\gamma)\cdot T_{atm_{L_{atm}-1}}\right] \\
& \quad\quad S \leftarrow stack\left[S, \beta\cdot\frac{D_{Adapt.atm_{L_{atm}}}}{A_{atm_{L_{atm}}}} + (1-\beta)\cdot S_{0.atm_{L_{atm}}}\right] \\
& \quad\quad F \leftarrow stack\left[F, \left(A_{atm_{L_{atm}}} + T_{atm_{L_{atm}}}\right)\cdot S_1\right] \\
& \quad \text{if } t > L_{atm} \\
& \quad\quad A_{atm} \leftarrow stack\left[A_{atm}, \alpha\cdot\frac{D_{Adapt.atm_t}}{S_{t-L_{atm}}} + (1-\alpha)\cdot\left(A_{atm_{t-1}} + T_{atm_{t-1}}\right)\right] \\
& \quad\quad T_{atm} \leftarrow stack\left[T_{atm}, \gamma\cdot\left(A_{atm_t} - A_{atm_{t-1}}\right) + (1-\gamma)\cdot T_{atm_{t-1}}\right] \\
& \quad\quad S \leftarrow stack\left[S, \beta\cdot\frac{D_{Adapt.atm_t}}{A_{atm_t}} + (1-\beta)\cdot S_{t-L_{atm}}\right] \\
& \quad\quad F \leftarrow stack\left[F, \left(A_{atm_t} + T_{atm_t}\right)\cdot S_{t-L_{atm}+1}\right] \\
& MSE_{atm} \leftarrow \frac{\sum_{r=1}^{N_{Adapt.atm}} \left(F_r - D_{Adapt.atm_r}\right)^2}{N_{Adapt.atm}}
\end{aligned}
$$

<u>Finding the best α,β,γ providing the minimum MSE</u>

```
AD_atm := | min ← 10^100
          | for α ∈ 0, 0.1 .. 1
          |   for β ∈ 0, 0.1 .. 1
          |     for γ ∈ 0, 0.1 .. 1
          |       if MSE_atm(α, β, γ) < min
          |          | min ← MSE_atm(α, β, γ)
          |          | α_min ← α
          |          | β_min ← β
          |          | γ_min ← γ
          | AD_atm ← stack(min, α_min, β_min, γ_min)
```

$$AD_{atm} = \begin{pmatrix} 6.37 \times 10^{6} \\ 0 \\ 0.1 \\ 0 \end{pmatrix}$$

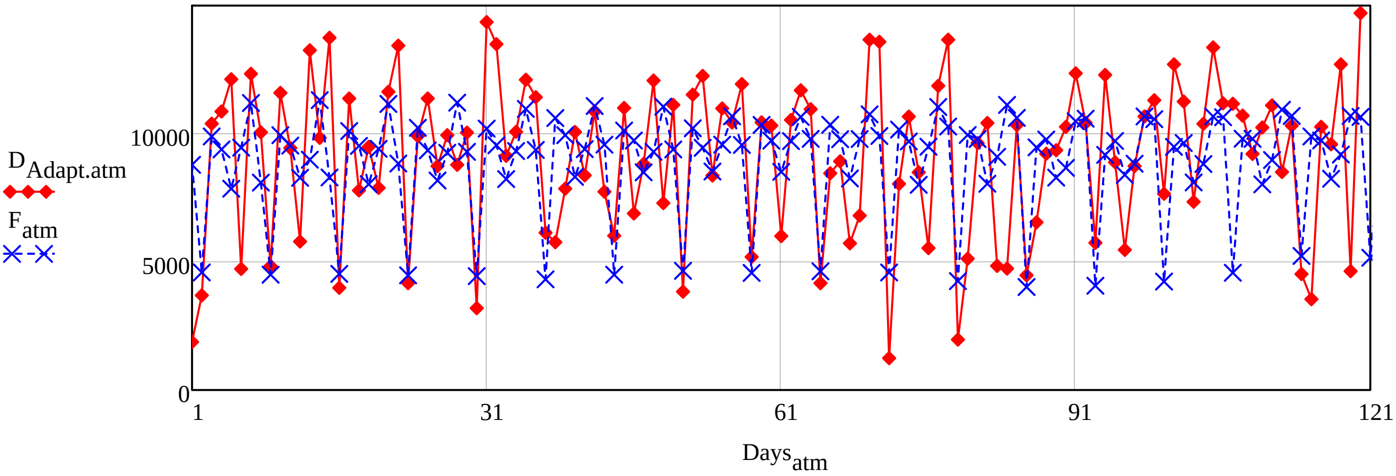
$\alpha_{atm} := AD_{atm_1}$ $\beta_{atm} := AD_{atm_2}$ $\gamma_{atm} := AD_{atm_3}$

$\alpha_{atm} = 0$ $\beta_{atm} = 0.1$ $\gamma_{atm} = 0$

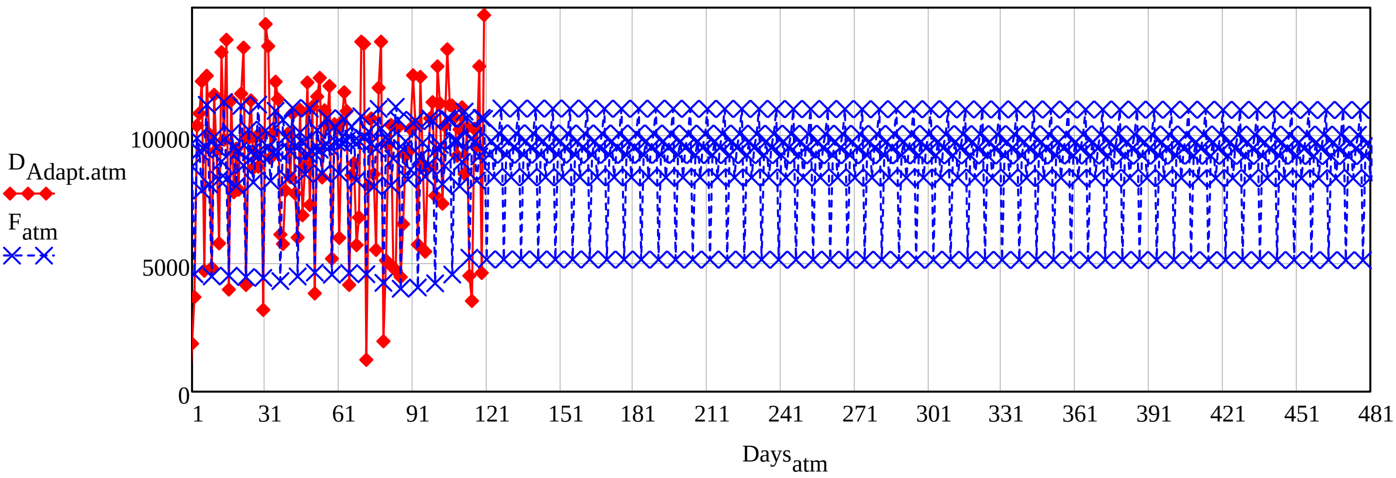
8. Forecasting

$$
\begin{aligned}
F_{atm} :=\ & F \leftarrow 0 \\
& F \leftarrow \text{stack}\left[F, \left(A_{atm_0} + T_{atm_0}\right)\cdot S_{0.atm_1}\right] \\
& S \leftarrow 0 \\
& \text{for } t \in 1 .. N_{Adapt.atm} + 365 \\
& \quad \text{if } t < L_{atm} \\
& \qquad A_{atm} \leftarrow \text{stack}\left[A_{atm}, \alpha_{atm}\cdot\frac{D_{Adapt.atm_t}}{S_{0.atm_t}} + \left(1-\alpha_{atm}\right)\cdot\left(A_{atm_{t-1}} + T_{atm_{t-1}}\right)\right] \\
& \qquad T_{atm} \leftarrow \text{stack}\left[T_{atm}, \gamma_{atm}\cdot\left(A_{atm_t} - A_{atm_{t-1}}\right) + \left(1-\gamma_{atm}\right)\cdot T_{atm_{t-1}}\right] \\
& \qquad S \leftarrow \text{stack}\left[S, \beta_{atm}\cdot\frac{D_{Adapt.atm_t}}{A_{atm_t}} + \left(1-\beta_{atm}\right)\cdot S_{0.atm_t}\right] \\
& \qquad F \leftarrow \text{stack}\left[F, \left(A_{atm_t} + T_{atm_t}\right)\cdot S_{0.atm_{t+1}}\right] \\
& \quad \text{if } t = L_{atm} \\
& \qquad A_{atm} \leftarrow \text{stack}\left[A_{atm}, \alpha_{atm}\cdot\frac{D_{Adapt.atm_{L_{atm}}}}{S_{0.atm_{L_{atm}}}} + \left(1-\alpha_{atm}\right)\cdot\left(A_{atm_{L_{atm}-1}} + T_{atm_{L_{atm}-1}}\right)\right] \\
& \qquad T_{atm} \leftarrow \text{stack}\left[T_{atm}, \gamma_{atm}\cdot\left(A_{atm_{L_{atm}}} - A_{atm_{L_{atm}-1}}\right) + \left(1-\gamma_{atm}\right)\cdot T_{atm_{L_{atm}-1}}\right] \\
& \qquad S \leftarrow \text{stack}\left[S, \beta_{atm}\cdot\frac{D_{Adapt.atm_{L_{atm}}}}{A_{atm_{L_{atm}}}} + \left(1-\beta_{atm}\right)\cdot S_{0.atm_{L_{atm}}}\right] \\
& \qquad F \leftarrow \text{stack}\left[F, \left(A_{atm_{L_{atm}}} + T_{atm_{L_{atm}}}\right)\cdot S_1\right] \\
& \quad \text{if } \left(t > L_{atm}\right) \wedge \left(t \le N_{Adapt.atm}\right) \\
& \qquad A_{atm} \leftarrow \text{stack}\left[A_{atm}, \alpha_{atm}\cdot\frac{D_{Adapt.atm_t}}{S_{t-L_{atm}}} + \left(1-\alpha_{atm}\right)\cdot\left(A_{atm_{t-1}} + T_{atm_{t-1}}\right)\right] \\
& \qquad T_{atm} \leftarrow \text{stack}\left[T_{atm}, \gamma_{atm}\cdot\left(A_{atm_t} - A_{atm_{t-1}}\right) + \left(1-\gamma_{atm}\right)\cdot T_{atm_{t-1}}\right] \\
& \qquad S \leftarrow \text{stack}\left[S, \beta_{atm}\cdot\frac{D_{Adapt.atm_t}}{A_{atm_t}} + \left(1-\beta_{atm}\right)\cdot S_{t-L_{atm}}\right] \\
& \qquad F \leftarrow \text{stack}\left[F, \left(A_{atm_t} + T_{atm_t}\right)\cdot S_{t-L_{atm}+1}\right] \\
& \quad \text{if } t > N_{Adapt.atm} \\
& \qquad A_{atm} \leftarrow \text{stack}\left[A_{atm}, \alpha_{atm}\cdot\frac{F_t}{S_{t-L_{atm}}} + \left(1-\alpha_{atm}\right)\cdot\left(A_{atm_{t-1}} + T_{atm_{t-1}}\right)\right] \\
& \qquad T_{atm} \leftarrow \text{stack}\left[T_{atm}, \gamma_{atm}\cdot\left(A_{atm_t} - A_{atm_{t-1}}\right) + \left(1-\gamma_{atm}\right)\cdot T_{atm_{t-1}}\right] \\
& \qquad S \leftarrow \text{stack}\left[S, \beta_{atm}\cdot\frac{F_t}{A_{atm_t}} + \left(1-\beta_{atm}\right)\cdot S_{t-L_{atm}}\right] \\
& \qquad F \leftarrow \text{stack}\left[F, \left(A_{atm_t} + T_{atm_t}\right)\cdot S_{t-L_{atm}+1}\right] \\
& F
\end{aligned}
$$

Past Data



Past Data+Forecast



7.1 Accuracy measurement before Year seasonality adjustment

1) Distortion

$$MPE_{atm.1} := \frac{\sum_{i=1}^{N_{Adapt.atm}} \frac{D_{Adapt.atm_i} - F_{atm_i}}{D_{Adapt.atm_i}}}{N_{Adapt.atm}} \cdot 100 = -10.6$$

- Mean Percentage Error

2) Dispersion

$$SDE_{atm.1} := \sqrt{\frac{\sum_{i=1}^{N_{Adapt.atm}} \left(D_{Adapt.atm_i} - F_{atm_i}\right)^2}{N_{Adapt.atm}}} = 2524$$

- Standard Deviation Error

$$MAPE_{atm.1} := \frac{\sum_{i=1}^{N_{Adapt.atm}} \left| \frac{\left(D_{Adapt.atm_i} - F_{atm_i}\right)}{D_{Adapt.atm_i}} \right|}{N_{Adapt.atm}} \cdot 100 = 29.8$$

- Mean Absolute Percentage Error

3) Tracking Signal

$$TS_{atm.1} := \frac{\sum_{i=1}^{N_{Adapt.atm}} \left(D_{Adapt.atm_i} - F_{atm_i}\right)}{\sum_{i=1}^{N_{Adapt.atm}} \left|D_{Adapt.atm_i} - F_{atm_i}\right|} = 0.09$$

- Tracking Signal

- if it is close it zero, it means that there is almost no predominant positive or negative errors.

Yearly seasonality

$$
\begin{aligned}
S_{year.atm} := \; & sum_0 \leftarrow 0 \\
& summa \leftarrow 0 \\
& S_{year.atm_0} \leftarrow 0 \\
& \text{for } k \in 1..12 \\
& \quad sum_k \leftarrow 0 \\
& \text{for } k \in 1..12 \\
& \quad \text{for } i \in 0..29 \\
& \quad\quad sum_k \leftarrow sum_k + \left(Full\_Data_{atm}^{\langle 1 \rangle}\right)_{(k-1)\cdot 30+i} \\
& \text{for } k \in 1..12 \\
& \quad summa \leftarrow summa + sum_k \\
& average \leftarrow \frac{summa}{12} \\
& \text{for } k \in 1..12 \\
& \quad S_{year.atm} \leftarrow stack\left(S_{year.atm}, \frac{sum_k}{average}\right)
\end{aligned}
$$

$S_{year.atm}$

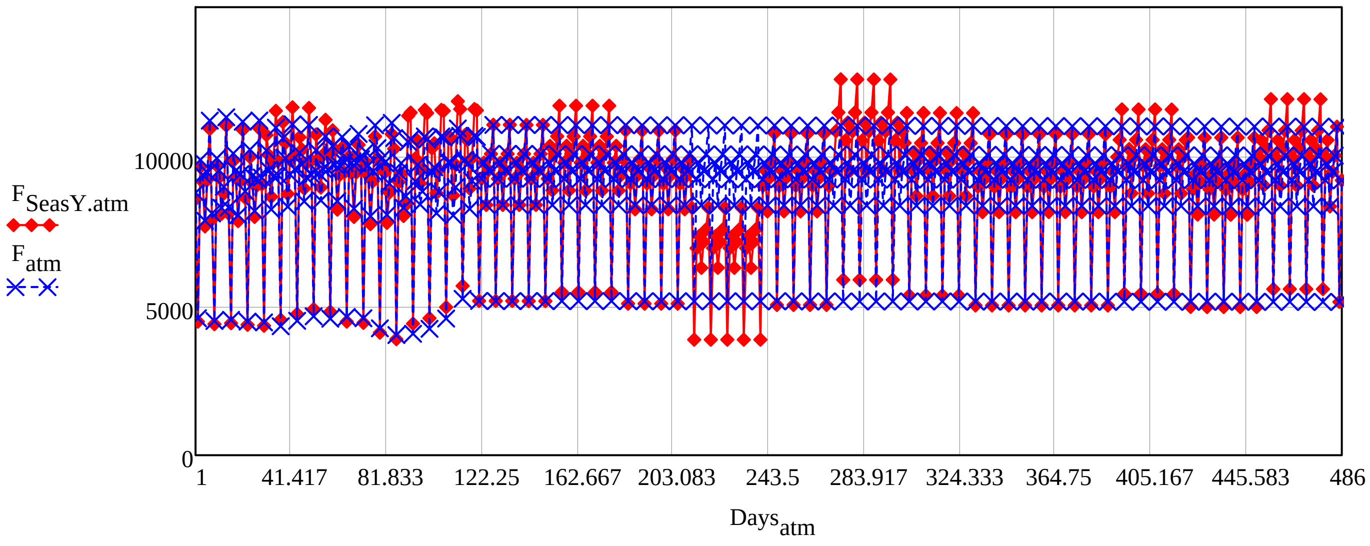
Adopting of yearly seasonality indexes for the future forecast

First month of the array of data for Adaptation:

$fd_{atm} := 1$ - January 2011

$$S_{atm} := \left| \begin{array}{l} S \leftarrow 0 \\ \text{for } i \in fd_{atm} .. 12 \\ \quad S_{(i+1-fd_{atm})} \leftarrow S_{year.atm_i} \\ \text{for } i \in 1 .. fd_{atm} - 1 \quad \text{if } fd_{atm} \neq 1 \\ \quad S_{(12+1-fd_{atm}+i)} \leftarrow S_{year.atm_i} \\ S \end{array} \right.$$

$$F_{SeasY.atm} := \left| \begin{array}{l} F_{SeasY.atm_0} \leftarrow 0 \\ k \leftarrow 1 \\ j \leftarrow 0 \\ \text{for } i \in 1 .. N_{Adapt.atm} + 365 + 1 \\ \quad \left| \begin{array}{l} F_{SeasY.atm_i} \leftarrow F_{atm_i} \cdot S_{atm_k} \\ j \leftarrow j + 1 \\ \text{if } j = 30 \\ \quad \left| \begin{array}{l} j \leftarrow 0 \\ k \leftarrow k + 1 \end{array} \right. \\ k \leftarrow 1 \quad \text{if } k = 13 \end{array} \right. \\ F_{SeasY.atm} \end{array} \right.$$



7.2 Accuracy measurement after Year seasonality adjustment

1) Distortion

$$MPE_{atm.2} := \frac{\displaystyle\sum_{i=1}^{N_{Adapt.atm}} \frac{D_{Adapt.atm_i} - F_{SeasY.atm_i}}{D_{Adapt.atm_i}}}{N_{Adapt.atm}} \cdot 100 = -12.2$$

- Mean Percentage Error

2) Dispersion

$$SDE_{atm.2} := \sqrt{\frac{\displaystyle\sum_{i=1}^{N_{Adapt.atm}} \left(D_{Adapt.atm_i} - F_{SeasY.atm_i}\right)^2}{N_{Adapt.atm}}} = 2501$$

- Standard Deviation Error

$$MAPE_{atm.2} := \frac{\displaystyle\sum_{i=1}^{N_{Adapt.atm}} \left|\frac{\left(D_{Adapt.atm_i} - F_{SeasY.atm_i}\right)}{D_{Adapt.atm_i}}\right|}{N_{Adapt.atm}} \cdot 100 = 29.5$$

- Mean Absolute Percentage Error

3) Tracking Signal

$$TS_{atm.2} := \frac{\displaystyle\sum_{i=1}^{N_{Adapt.atm}} \left(D_{Adapt.atm_i} - F_{SeasY.atm_i}\right)}{\displaystyle\sum_{i=1}^{N_{Adapt.atm}} \left|D_{Adapt.atm_i} - F_{SeasY.atm_i}\right|} = 0.01$$

- Tracking Signal

- if it is close it zero, it means that there is almost no predominant positive or negative errors.

Choosing the best forecast

Before YearSeas adjust:	After YearSeas adjust:
$MPE_{atm.1} = -10.6$	$MPE_{atm.2} = -12.2$
$SDE_{atm.1} = 2524$	$SDE_{atm.2} = 2501$
$MAPE_{atm.1} = 29.8$	$MAPE_{atm.2} = 29.5$
$TS_{atm.1} = 0.09$	$TS_{atm.2} = 0.01$

$$SDE_{atm} := \min\left(SDE_{atm.1}, SDE_{atm.2}\right) = 2501$$

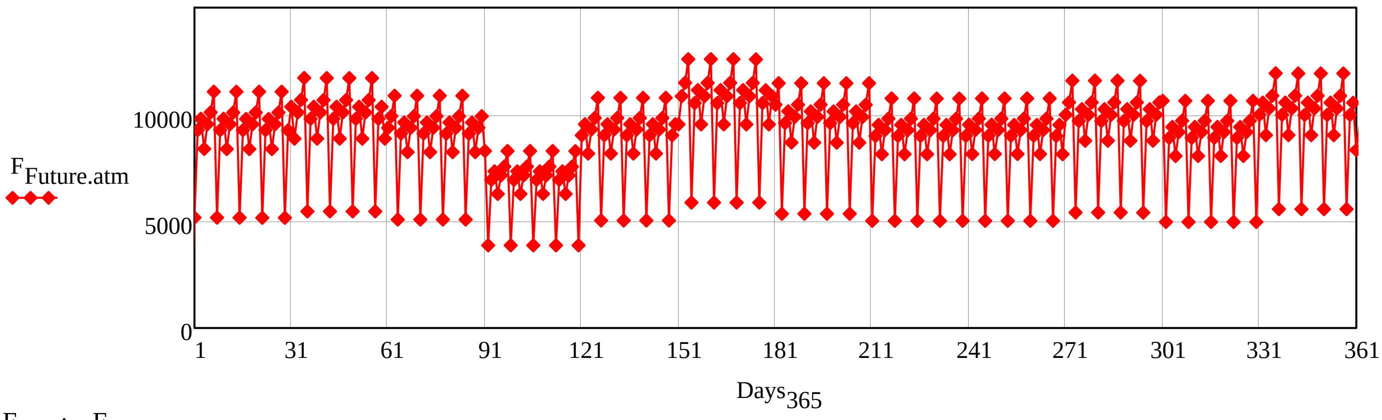
$$D_{atm} := \left| \begin{array}{l} D_{atm} \leftarrow F_{atm} \text{ if } SDE_{atm} = SDE_{atm.1} \\ D_{atm} \leftarrow F_{SeasY.atm} \text{ if } SDE_{atm} = SDE_{atm.2} \\ D_{atm} \end{array} \right.$$

Extracting the Forecast only for Future

Forecast for May2011-April2012

$F_{Future.atm} := submatrix(D_{atm}, N_{Adapt.atm}, N_{Adapt.atm} + 365 + 1, 0, 0)$

$F_{Future.atm_0} := 0$



$F_{atm} := F_{Future.atm}$

==

2/7 ATM. Inventory Planning

Periodic Review Model

1. Choice of the Inventory management parameters

1) Logistics Costs structure

$GrossSalary := 60000$ [euro] - gross salary of operator per year

$upc_{atm} := \frac{GrossSalary}{365 \cdot \frac{5}{7} \cdot 8 \cdot \frac{2}{2}} = 28.77$ [euro] -unitary procurement cost (cost of one replenishment)

$rr := \frac{270}{2800 + 3600} \cdot 100 = 4.22$ [%] - risk of robbery

On the average, in case of robbery, 75% of stock is stalen

$rc := rr \cdot 0.75 = 3.16$ [%/year] - robbery cost as a percentage of money keeping in stock

$imc := 1$ [%/year] - immobilization cost as a percentage of money keeping in stock

$phc := rc + imc = 4.16$ [%/year] - Inventory Carrying Costs as a percentage of money keeping in stock

$phc := \frac{phc}{100}$

2) Order interval and Cycle Stock

$V := 1$ [-] - unitary item value. Unitless value equal to 1, as we measure the Demand in euros.

$D_{y.atm} := \sum_{i=1}^{365} F_{atm_i} = 3.28 \times 10^6$ [euro] - annual demand

$T_y := \sqrt{\frac{2 \cdot upc_{atm}}{phc \cdot V \cdot D_{y.atm}}} = 0.02$ [year] - Order Interval

$T_d := T_y \cdot 365 = 7.49$ [days] - Order Interval

$\boxed{T_{fix} := floor(T_d) = 7}$

$ED_{atm}(t_1, t_2) := \sum_{i=t_1}^{t_2} F_{atm_i}$ [euro] - Excpected demand in the interval [t1;t2]

3) Safety Stock

$k_{atm} := 2.33$ - assures Service Level 99% k=1.75 (96%) k=2.33 (99%)

$SDE_{atm} = 2501$

$ELT_{atm} := 0 \quad \sigma_{LT.atm} := 0$

$$\sigma_{D\_LT.atm}(T) := \sqrt{(ELT_{atm} + T)\cdot SDE_{atm}^{2} + \sigma_{LT.atm}^{2}\cdot ED_{atm}^{2}}$$

$$\sigma_{D\_LT.atm}(T) := \sqrt{T\cdot SDE_{atm}^{2}}$$

$SS_{atm}(T) := k_{atm}\cdot\sigma_{D\_LT.atm}(T) \qquad SS_{atm}(T_{fix}) = 15420$ [euro] - safety stock

4) Availability Target

$AT(t_1, t_2) := ED_{atm}(t_1, t_2) + SS_{atm}(T_{fix})$ - Availability Target

Adding indexes of the days of a week for data Forecasted

Week_index_per_year := submatrix(Week_index_per_full_data, 0, 364, 0, 0)

Week_index_per_year := stack(0, Week_index_per_year)

First day of the week of data forecasted:

$fd := \left(Full\_Data_{Original}^{\langle 7\rangle}\right)_{N_{Full}-7} = 7$ May 1st, 2011 is Sunday

Index_week$_y$:= - for 365 days

```
QW_0 ← 0
for i ∈ fd .. 365
    (QW)_(i+1−fd) ← (Week_index_per_year)_i
for i ∈ 1 .. fd − 1
    (QW)_(365−fd+i) ← (Week_index_per_year)_i
QW_365 ← (Week_index_per_year)_fd
QW
```

2. Adjustment of the replenishment plan (for week-ends)

1) Replenishment plan, depending on a type of policy

```
Putin(pl) := | Putin_0 ← 0
             | k ← Index_week_(y_1)
             | j ← 1  if  1 ≤ mod(k,7) ≤ 5
             | j ← 3  if  mod(k,7) = 6
             | j ← 2  if  mod(k,7) = 0
             | if  pl = 1
             |   | sh_6 ← −1
             |   | sh_7 ← −2
             | if  pl = 2
             |   | sh_6 ← −1
             |   | sh_7 ← 1
             | if  pl = 3
             |   | sh_6 ← 2
             |   | sh_7 ← −2
             | if  pl = 4
             |   | sh_6 ← 2
             |   | sh_7 ← 1
             | while  (j ≤ 365)
             |   | k ← Index_week_(y_j) + T_fix
             |   | if  mod(k,7) = 6
             |   |   | Putin_j ← AT(j, j + T_fix − 1 + sh_6)
             |   |   | j ← j + T_fix + sh_6
             |   | if  mod(k,7) = 0
             |   |   | Putin_j ← AT(j, j + T_fix − 1 + sh_7)
             |   |   | j ← j + T_fix + sh_7
             |   | if  1 ≤ mod(k,7) ≤ 5
             |   |   | Putin_j ← AT(j, j + T_fix − 1)
             |   |   | j ← j + T_fix
             |   | Putin_365 ← 0  if  j < 365
             | Putin
```

$Putin(pl) := submatrix(Putin(pl), 0, 365, 0, 0)$

$Replen := augment\left(Index\_week_y, Putin(1), Putin(2), Putin(3), Putin(4)\right)$

Replen =

	Day of the month / Day of the week	Policy-1	Policy-2	Policy-3	Policy-4
	0	1	2	3	4
0	0	0	0	0	0
1	7	0	0	0	0
2	1	78666	78666	78666	78666
3	2	0	0	0	0
4	3	0	0	0	0
5	4	0	0	0	0
6	5	0	0	0	0
7	6	0	0	0	0
8	7	0	0	0	0
9	1	78659	78659	78659	78659
10	2	0	0	0	0
11	3	0	0	0	0
12	4	0	0	0	0
13	5	0	0	0	0
14	6	0	0	0	0
15	7	0	0	0	0
16	1	78652	78652	78652	78652
17	2	0	0	0	...

2) Nuber of replenishments, depending on type of policy

$$N_{pr}(pl) := \left| \begin{array}{l} N_{pr} \leftarrow 0 \\ \text{for } i \in 0..365 \\ \quad N_{pr} \leftarrow N_{pr} + 1 \text{ if } (Putin(pl))_i \neq 0 \\ N_{pr} \end{array} \right.$$

$j := 0..3$

$N_{pr_j} := N_{pr}(j + 1)$

$$N_{pr} = \begin{pmatrix} 52 \\ 52 \\ 52 \\ 52 \end{pmatrix}$$

3) Expected Avarage Annual Inventory Level, depending on type of policy

$$SL(pl) := \begin{array}{|l} SL_0 \leftarrow 0 \\ N_{Stock.Out} \leftarrow 0 \\ \text{for } i \in 1..365 \\ \quad \begin{array}{|l} SL_i \leftarrow SL_{i-1} - F_{atm_i} \;\; \text{if } (Putin(pl))_i = 0 \\ SL_i \leftarrow (Putin(pl))_i - F_{atm_i} \;\; \text{otherwise} \\ \text{if } SL_i < 0 \\ \quad \begin{array}{|l} SL_i \leftarrow 0 \\ N_{Stock.Out} \leftarrow N_{Stock.Out} + 1 \end{array} \end{array} \\ SL_0 \leftarrow N_{Stock.Out} \\ SL \end{array}$$

$M := augment(SL(1), SL(2), SL(3), SL(4), Index\_week_y)$

$j := 0..3$

$$AIL_{atm_j} := \frac{\sum_{i=1}^{365} \left(M^{\langle j \rangle}\right)_i}{365} = ... \qquad AIL_{atm} = \begin{pmatrix} 41153 \\ 41153 \\ 41153 \\ 41153 \end{pmatrix}$$

4) Total Costs for different policies

$$ICC_{atm} := AIL_{atm} \cdot V \cdot phc = \begin{pmatrix} 1714 \\ 1714 \\ 1714 \\ 1714 \end{pmatrix} \qquad PC_{atm} := N_{pr} \cdot upc_{atm} = \begin{pmatrix} 1496 \\ 1496 \\ 1496 \\ 1496 \end{pmatrix} \qquad TC_{atm} := ICC_{atm} + PC_{atm} = \begin{pmatrix} 3210 \\ 3210 \\ 3210 \\ 3210 \end{pmatrix}$$

5) Choosing the best policy

$$Pol := \begin{array}{|l} min \leftarrow 10^{100} \\ \text{for } j \in 0..3 \\ \quad \text{if } TC_{atm_j} < min \\ \quad\quad \begin{array}{|l} min \leftarrow TC_{atm_j} \\ j_{min} \leftarrow j \end{array} \\ Pol \leftarrow stack(min, j_{min}) \end{array}$$

$TC_{atm.min} := Pol_0 = 3210$

$pl_{best} := Pol_1 + 1 = 1$ - Best policy

$AIL_{atm.to.be} := AIL_{atm\left(pl_{best}-1\right)} = 41153$ [euro] - average inventory level

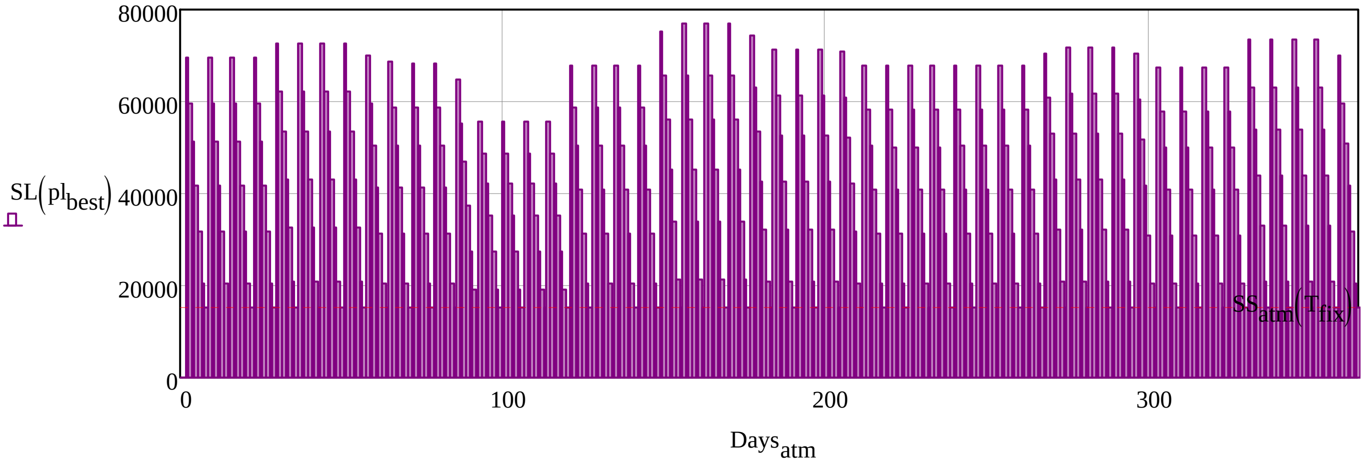
$N_{pr.atm.to.be} := N_{pr\left(pl_{best}-1\right)} = 52$

ATM. Costs To Be

$ICC_{atm.to.be} := AIL_{atm\left(pl_{best}-1\right)} \cdot V \cdot phc = 1714$ $PC_{atm.to.be} := N_{pr\left(pl_{best}-1\right)} \cdot upc_{atm} = 1496$

$TC_{atm.to.be} := ICC_{atm.to.be} + PC_{atm.to.be} = 3210$

Expected Stock Level profile for the best policy



6) Calculating of the Residuals for the best policy

$$Residuals := \left| \begin{array}{l} R_0 \leftarrow 0 \\ \text{for } i \in 1..365 \\ \quad R_i \leftarrow SL(pl_{best})_{i-1} \text{ if } (Putin(pl_{best}))_i \neq 0 \\ R_{365} \leftarrow 0 \text{ if } (Putin(pl_{best}))_{365} = 0 \\ R \end{array} \right.$$

$Putin(pl_{best}) =$

	0
0	0
1	0
2	78666
3	0
4	0
5	0
6	0
7	0
8	0
9	78659
10	...

$Residuals =$

	0
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	15420
10	...

$N_{Stock.Out} := SL(pl_{best})_0 = 1$ - number of stockouts

The maximum capacity of ATM is 140 000 euro

Expexted demand by ATM calculation

Demand by ATM

$$Demand_{ATM} := Putin(pl_{best}) - Residuals$$

Week-ends elimination

$$Demand_{ATM} := augment(Demand_{ATM}, Index\_week_y)$$

$$Demand_{ATM.adjust} := \left| \begin{array}{l} \text{for } j \in 0..1 \\ \quad D_{0,j} \leftarrow 0 \\ \text{for } i \in 1..365 \\ \quad D \leftarrow stack\left[D, \left(Demand_{ATM}{}^T\right)^{\langle i \rangle T}\right] \text{ if } 1 \le \left(Demand_{ATM}{}^{\langle 1 \rangle}\right)_i \le 5 \\ D \end{array} \right.$$

$N_{forecast.branch} := rows(Demand_{ATM.adjust}) - 1 = 260$

==

3/7 Cash Desk. Demand Planning

1. Data Importing

2. Data Transformation. Extracting demand from Cash Desk from the original data

Cuting data without records

The difference between #of days of past data for ATM and for Branch:

$N_{dif} := 243$ - days from [January to August2010]

$$Full\_Data_3 := submatrix(Full\_Data_{Original}, N_{dif}, N_{Full} - 1, 0, 5)$$
$$QW := submatrix(Full\_Data_{Original}, N_{dif}, N_{Full} - 1, 7, 7)$$
$$Full\_Data_3 := augment(Full\_Data_3, QW)$$

$Full\_Data_3 =$

	0	1	2	3	4	5	6
0	"09/01/2010"	107096	92521	61000	0	-30000	3
1	"09/02/2010"	92521	74217	0	0	0	4
2	"09/03/2010"	74217	132915	45000	0	0	5
3	"09/04/2010"	0	0	0	0	0	6
4	"09/05/2010"	0	0	0	0	0	7
5	"09/06/2010"	132915	93891	0	0	-30000	1
6	"09/07/2010"	93891	71413	0	0	0	2
7	"09/08/2010"	71413	89856	35000	0	0	3
8	"09/09/2010"	89856	80605	0	0	-19000	4
9	"09/10/2010"	80605	99334	30000	0	0	5
10	"09/11/2010"	0	0	0	0	0	...

$$N_{Full\_Data.3} := rows(Full\_Data_3) = 242$$

Week-ends elimination

As Cash Desk doesn't work during the weekend, the rows,corresponding to the week-end observations must be eliminated

$$Full\_Data_2 := \left| \begin{array}{l} \text{for } j \in 0..6 \\ \quad Full\_Data_{2_{0,j}} \leftarrow 0 \\ \text{for } i \in 0..N_{Full\_Data.3} - 1 \\ \quad Full\_Data_2 \leftarrow stack\left[Full\_Data_2, \left(Full\_Data_3{}^T\right)^{\langle i \rangle T}\right] \text{ if } 1 \le \left(Full\_Data_3{}^{\langle 6 \rangle}\right)_i \le 5 \\ Full\_Data_2 \end{array} \right.$$

$Full\_Data_2 =$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	"09/01/2010"	$1.07 \cdot 10^5$	92521.25	61000	0	-30000	3
2	"09/02/2010"	92521.25	74217.27	0	0	0	4
3	"09/03/2010"	74217.27	$1.33 \cdot 10^5$	45000	0	0	5
4	"09/06/2010"	$1.33 \cdot 10^5$	93891.2	0	0	-30000	1
5	"09/07/2010"	93891.2	71413.06	0	0	0	...

$N_{Full\_Data.2} := rows(Full\_Data_2) = 174$

Demand (Supply, if negative) extraction

$Demand := Full\_Data_2^{\langle 1 \rangle} - Full\_Data_2^{\langle 2 \rangle} + Full\_Data_2^{\langle 3 \rangle} + Full\_Data_2^{\langle 4 \rangle} + Full\_Data_2^{\langle 5 \rangle}$

$Full\_Data_1 := augment\left(Full\_Data_2^{\langle 0 \rangle}, Demand, Full\_Data_2^{\langle 6 \rangle}\right)$

$Full\_Data_1 =$

	0	1	2
0	0	0	0
1	"09/01/2010"	45574.71	3
2	"09/02/2010"	18303.98	4
3	"09/03/2010"	-13698.05	5
4	"09/06/2010"	9024.12	1
5	"09/07/2010"	22478.14	...

$N_{Full\_Data.1} := rows(Full\_Data_1) = 174$

3. Data Substituition

A) Zero values identification and substituition

To substituite zero values, the average Demand/Supply for the considered period was taken.

$$aver := \frac{\sum_{i=1}^{N_{Full\_Data.1}-1} \left(Full\_Data_1^{\langle 1 \rangle}\right)_i}{N_{Full\_Data.1} - 1} = 3689$$

$$Full\_Data := \left| \begin{array}{l} \text{for } i \in 0 .. N_{Full\_Data.1} - 1 \\ \quad \left(Full\_Data_1^{\langle 1 \rangle}\right)_i \leftarrow aver \ \text{ if } \left(Full\_Data_1^{\langle 1 \rangle}\right)_i = 0 \\ Full\_Data_1 \end{array} \right.$$

$Full\_Data =$

	0	1	2
0	"09/01/2010"	45575	3
1	"09/02/2010"	18304	4
2	"09/03/2010"	-13698	5
3	"09/06/2010"	9024	1
4	"09/07/2010"	22478	2
5	"09/08/2010"	16557	3
6	"09/09/2010"	-9749	4
7	"09/10/2010"	11271	5
8	"09/13/2010"	3411	1
9	"09/14/2010"	30361	2
10	"09/15/2010"	-52452	...

$N_{Full} := rows(Full\_Data) = 173$

Creating the arrays with Natural numbers for Cah Desk

$$Days := \left| \begin{array}{l} Days_0 \leftarrow 0 \\ \text{for } i \in 1 .. N_{Full} + 365 \\ \quad Days_i \leftarrow i \\ Days \end{array} \right.$$

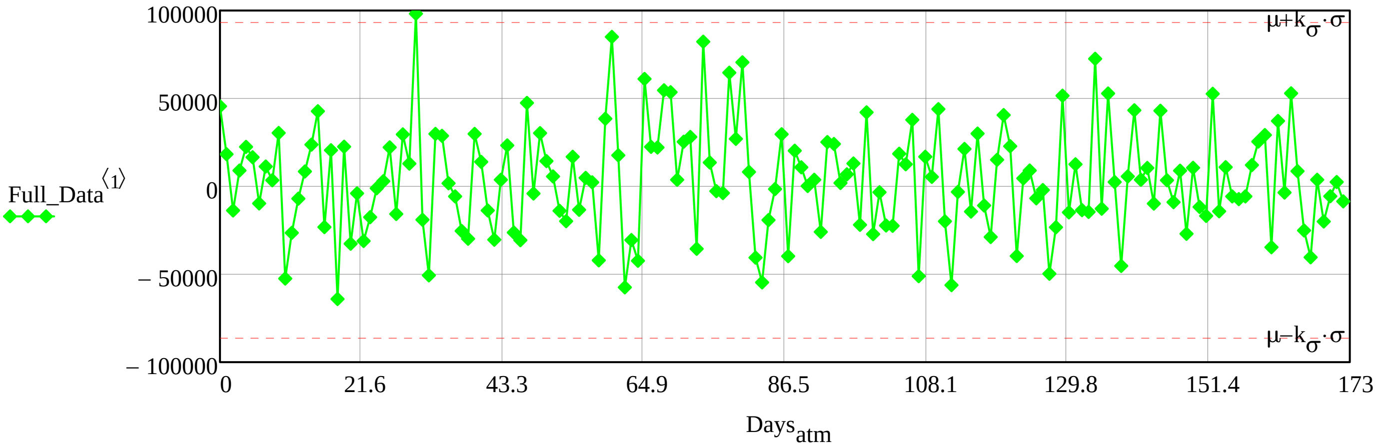
$$Days_{Full} := submatrix(Days, 0, N_{Full}, 0, 0)$$
$$Days_{365} := submatrix(Days, 0, 365, 0, 0)$$

B) Outliers identification and substituition

Outliers identification

$$\mu := \frac{\sum_{i=0}^{N_{Full}-1} \left(Full\_Data^{\langle 1 \rangle}\right)_i}{N_{Full}} = 3796 \qquad \sigma := \sqrt{\frac{\sum_{i=0}^{N_{Full}-1} \left[\left(Full\_Data^{\langle 1 \rangle}\right)_i - \mu\right]^2}{N_{Full}}} = 30014$$

$$k_\sigma := 3$$



Outliers substituition

$$Full\_Data := \left| \begin{array}{l} \text{for } i \in 0..N_{Full}-1 \\ \quad \left(Full\_Data^{\langle 1 \rangle}\right)_i \leftarrow aver \text{ if } \left[\left(Full\_Data^{\langle 1 \rangle}\right)_i > \mu + k_\sigma \cdot \sigma\right] \vee \left[\left(Full\_Data^{\langle 1 \rangle}\right)_i < \mu - k_\sigma \cdot \sigma\right] \\ Full\_Data \end{array} \right.$$

Full_Data =

	0	1	2
0	"09/01/2010"	45575	3
1	"09/02/2010"	18304	4
2	"09/03/2010"	-13698	5
3	"09/06/2010"	9024	1
4	"09/07/2010"	22478	2
5	"09/08/2010"	16557	3
6	"09/09/2010"	-9749	4
7	"09/10/2010"	11271	...

- Data with substituited outliers

4. Analysis

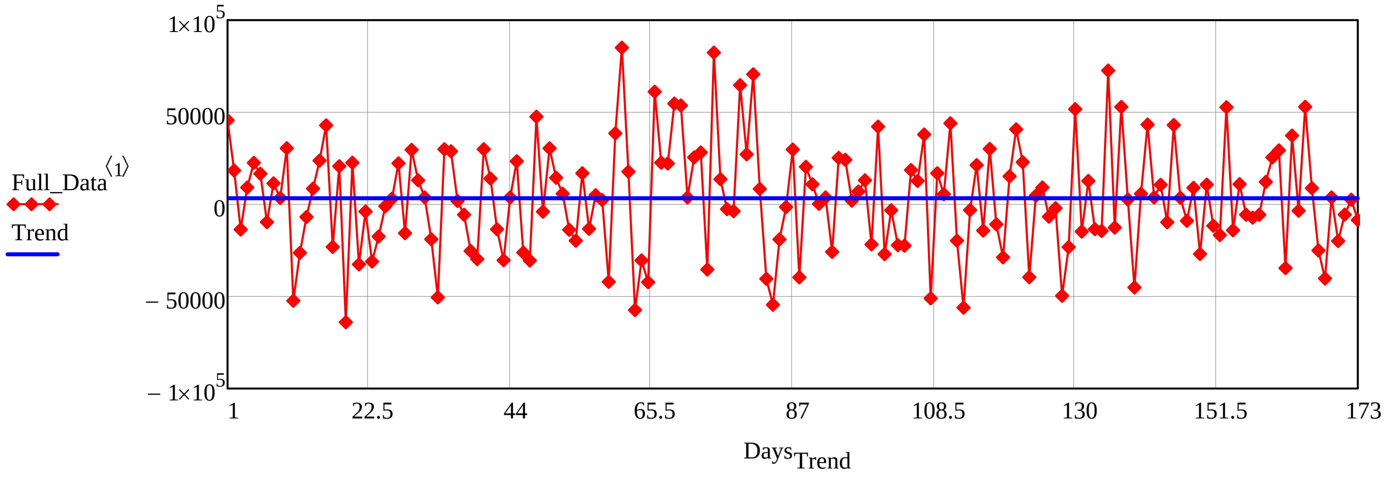
Identification of the Trend

$$Days_{Trend} := submatrix(Days_{Full}, 1, N_{Full}, 0, 0)$$

$$B := intercept\left(Days_{Trend}, Full\_Data^{\langle 1 \rangle}\right) = 3241.37$$

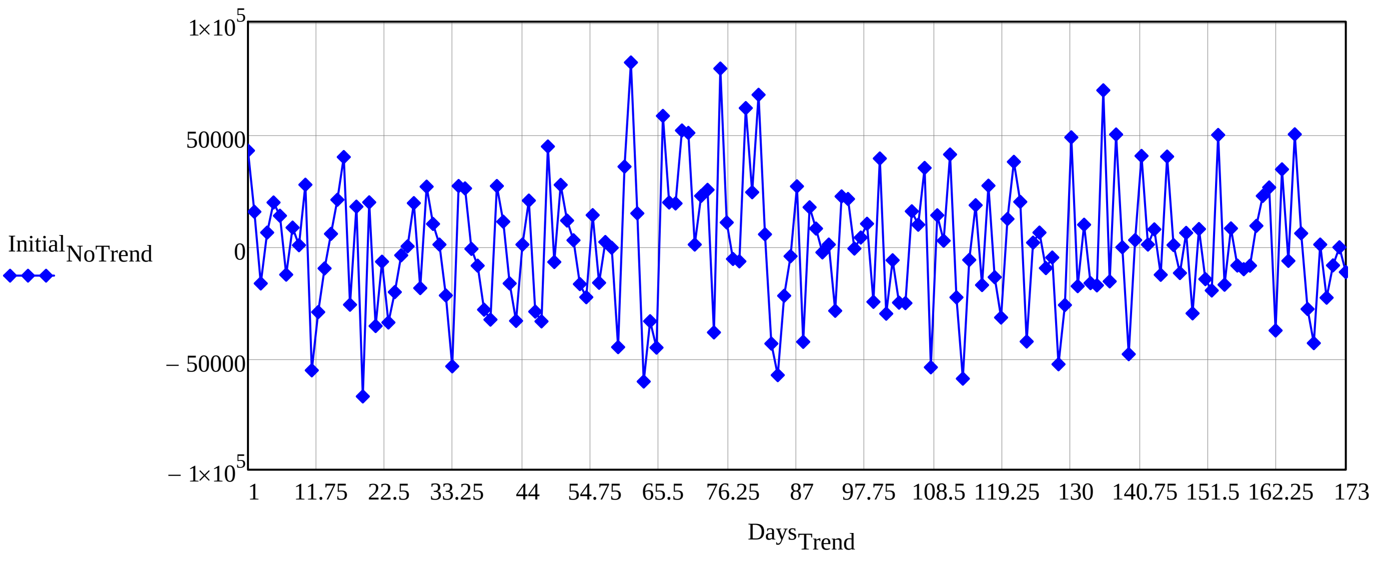
$$K := slope\left(Days_{Trend}, Full\_Data^{\langle 1 \rangle}\right) = 0.1$$

$$Trend := B + K \cdot Days_{Trend}$$



Trend removing

$$Initial_{NoTrend} := \left| \begin{array}{l} \text{for } i \in 0 .. N_{Full} - 1 \\ \quad Initial_{NoTrend_i} \leftarrow \left(Full\_Data^{\langle 1 \rangle}\right)_i - (B + K \cdot i) \\ Initial_{NoTrend} \end{array} \right.$$



Seasonality period identification

$N_{shift} := 30$

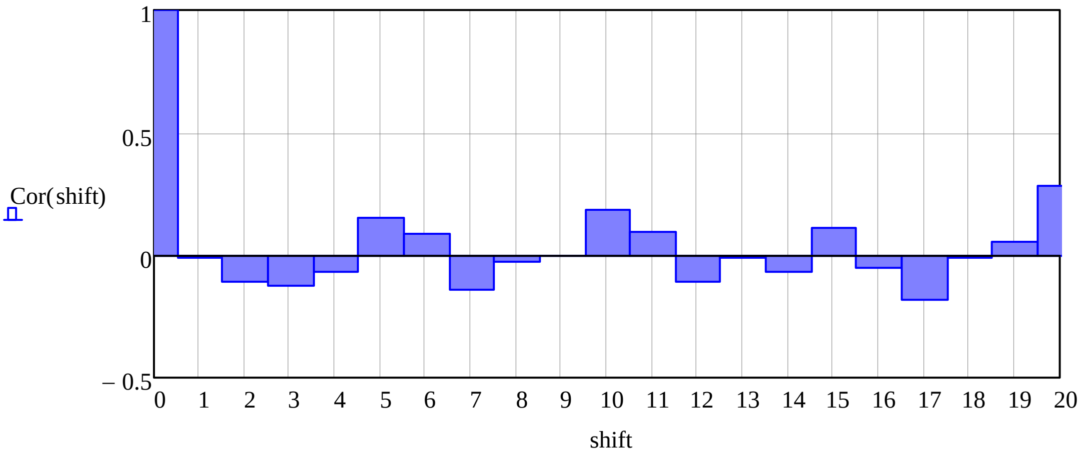
$corr(Initial_{NoTrend}, Initial_{NoTrend}) = 1$

$VX := submatrix(Initial_{NoTrend}, N_{shift}, N_{Full} - 1, 0, 0)$

$shift := 0 .. N_{shift}$

$VY(shift) := submatrix(Initial_{NoTrend}, N_{shift} - shift, N_{Full} - 1 - shift, 0, 0)$

$Cor(shift) := corr(VX, VY(shift))$



$r_k := \begin{array}{|l} r_{k_0} \leftarrow 0 \\ \text{for } i \in 1 .. N_{shift} \\ \quad r_{k_i} \leftarrow |Cor(i)| \\ r_k \end{array}$

$r_{k.max} := max(r_k) = 0.28$

$L := \begin{array}{|l} \text{for } i \in 1 .. N_{shift} \\ \quad L \leftarrow i \text{ if } r_{k_i} = r_{k.max} \\ L \end{array}$

$L = 20$ - shift, providing the highest autocorrelation coefficient

$L := 5$ - Seasonality period, providing better SDE

Adding L-indexes

$L\_index := \begin{array}{|l} \text{for } i \in 0 .. L - 1 \\ \quad L\_index_i \leftarrow i + 1 \\ L\_index \end{array}$

$Full\_Data := \begin{array}{|l} k \leftarrow -1 \\ \text{for } i \in 0 .. N_{Full} - 1 \\ \quad \begin{array}{|l} k \leftarrow k + 1 \\ L\_index_i \leftarrow L\_index_k \\ k \leftarrow -1 \text{ if } k = L - 1 \end{array} \\ Full\_Data \leftarrow augment(Full\_Data, L\_index) \\ Full\_Data \end{array}$

5. Initialization

Substracting data subset for Initialization

Set the size of the data subset for Initialization for Cash Desk:

$N_{Initial} := N_{Full} = 173$ - days without week-ends, 8 months (Sept2010-April2010)

$Initial := submatrix(Full\_Data, 0, N_{Initial} - 1, 0, 2)$

Initial =

	0	1	2
0	"09/01/2010"	45574.71	3
1	"09/02/2010"	18303.98	4
2	"09/03/2010"	-13698.05	5
3	"09/06/2010"	9024.12	1
4	"09/07/2010"	22478.14	2
5	"09/08/2010"	16557	3
6	"09/09/2010"	-9749.02	4
7	"09/10/2010"	11271.44	5
8	"09/13/2010"	3410.66	1
9	"09/14/2010"	30361.46	2
10	"09/15/2010"	-52452.22	3
11	"09/16/2010"	-26437.55	4
12	"09/17/2010"	-6963.59	5
13	"09/20/2010"	8441.2	1
14	"09/21/2010"	23682.45	2
15	"09/22/2010"	42769.01	...

1) Initial Trend for Cash Desk

$T_0 := K = 0.1$

2) Initial Average for Cash Desk

$A_0 := B = 3241$

3) Initial Seasonality coefficients for Cash Desk

$\text{Initial} := \text{submatrix}\left(\text{Full\_Data}, 0, N_{\text{Initial}} - 1, 0, 3\right)$

$$
\begin{aligned}
S_{L\_ly} := \;& \text{for } j \in 1 .. \text{floor}\left(\frac{N_{\text{Initial}}}{L}\right) \\
& \quad \text{sum}_j \leftarrow 0 \\
& \text{for } j \in 1 .. L \\
& \quad \text{summa}_j \leftarrow 0 \\
& k \leftarrow 1 \\
& \text{for } i \in 0 .. \text{floor}\left(\frac{N_{\text{Initial}}}{L}\right) \cdot L - 1 \\
& \quad \text{sum}_k \leftarrow \text{sum}_k + \left(\text{Initial}^{\langle 1 \rangle}\right)_i \\
& \quad \text{if } \text{mod}(i + 1, L) = 0 \\
& \quad\quad \text{aver}_k \leftarrow \frac{\text{sum}_k}{L} \\
& \quad\quad k \leftarrow k + 1 \\
& \text{for } k \in 1 .. \text{floor}\left(\frac{N_{\text{Initial}}}{L}\right) \\
& \quad \text{for } l \in 1 .. L \\
& \quad\quad S_{l,k} \leftarrow \frac{\left(\text{Initial}^{\langle 1 \rangle}\right)_{l+(k-1)\cdot L-1}}{\text{aver}_k} \\
& \text{for } l \in 1 .. L \\
& \quad \text{for } k \in 1 .. \text{floor}\left(\frac{N_{\text{Initial}}}{L}\right) \\
& \quad\quad \text{summa}_l \leftarrow \text{summa}_l + S_{l,k} \\
& \quad S_{L\_ly_l} \leftarrow \frac{\text{summa}_l}{\text{floor}\left(\frac{N_{\text{Initial}}}{L}\right)} \\
& S_{L\_ly}
\end{aligned}
$$

$S_{L\_ly}{}^{T} = (0 \;\; -5.5 \;\; 6.4 \;\; 1.3 \;\; 5.4 \;\; -2.5)$

6. Adaptation. Cash Desk

The size of the data subset for Adaptation for Cash Desk:

$N_{Initial} = 173$ - days without week-ends, 8 months (Sept2010-April2010)

Set the size of the data subset for Adaptaion for Cash Desk:

$N_{Adapt} := N_{Full} = 173$ - days without week-ends, 8 months (Sept2010-April2010)

$Adapt := submatrix(Full\_Data, N_{Full} - N_{Adapt}, N_{Full} - 1, 0, 3)$

Adapt =

	0	1	2	3
0	"09/01/2010"	45574.71	3	1
1	"09/02/2010"	18303.98	4	2
2	"09/03/2010"	-13698.05	5	3
3	"09/06/2010"	9024.12	1	4
4	"09/07/2010"	22478.14	2	5
5	"09/08/2010"	16557	3	1
6	"09/09/2010"	-9749.02	4	2
7	"09/10/2010"	11271.44	5	3
8	"09/13/2010"	3410.66	1	4
9	"09/14/2010"	30361.46	2	5
10	"09/15/2010"	-52452.22	3	...

Adjusting the Seasonality Indexes for the Adaptation subset

$fd := Adapt_{0,3} = 1$ - First day of the array of data for Adaptation

$$S_{0\_adapt} := \begin{array}{|l} S_{0_0} \leftarrow 0 \\ \text{for } i \in fd..L \\ \quad S_{0_{(i+1-fd)}} \leftarrow S_{L\_ly_i} \\ \text{for } i \in 1..fd-1 \quad \text{if } fd \neq 1 \\ \quad S_{0_{(L+1-fd+i)}} \leftarrow S_{L\_ly_i} \\ S_0 \end{array}$$

2 arrays musts be shifted on: $fd - 1 = 0$

$$S_{0\_adapt} = \begin{pmatrix} 0 \\ -5.5 \\ 6.37 \\ 1.27 \\ 5.35 \\ -2.49 \end{pmatrix} \qquad S_{L\_ly} = \begin{pmatrix} 0 \\ -5.5 \\ 6.37 \\ 1.27 \\ 5.35 \\ -2.49 \end{pmatrix}$$

MSE(α,β,γ) function

$S_0 := S_{0\_adapt} \quad D_{Adapt_0} := 0$

$D_{Adapt} := stack\left(D_{Adapt_0}, Adapt^{\langle 1 \rangle}\right)$

$$
\begin{aligned}
\mathrm{MSE}(\alpha,\beta,\gamma) :=\ & F \leftarrow 0\\
& F \leftarrow \mathrm{stack}\left[F,\left(A_0 + T_0\right)\cdot S0_1\right]\\
& d \leftarrow 1\\
& \mathrm{MSE} \leftarrow 0\\
& S \leftarrow 0\\
& \text{for } t \in 1 .. N_{Adapt}\\
& \quad \text{if } t < L\\
& \quad\quad A \leftarrow \mathrm{stack}\left[A,\alpha\cdot\frac{D_{Adapt_t}}{S0_t} + (1-\alpha)\cdot\left(A_{t-1} + T_{t-1}\right)\right]\\
& \quad\quad T \leftarrow \mathrm{stack}\left[T,\gamma\cdot\left(A_t - A_{t-1}\right) + (1-\gamma)\cdot T_{t-1}\right]\\
& \quad\quad S \leftarrow \mathrm{stack}\left[S,\beta\cdot\frac{D_{Adapt_t}}{A_t} + (1-\beta)\cdot S0_t\right]\\
& \quad\quad F \leftarrow \mathrm{stack}\left[F,\left(A_t + T_t\right)\cdot S0_{t+1}\right]\\
& \quad \text{if } t = L\\
& \quad\quad A \leftarrow \mathrm{stack}\left[A,\alpha\cdot\frac{D_{Adapt_L}}{S0_L} + (1-\alpha)\cdot\left(A_{L-1} + T_{L-1}\right)\right]\\
& \quad\quad T \leftarrow \mathrm{stack}\left[T,\gamma\cdot\left(A_L - A_{L-1}\right) + (1-\gamma)\cdot T_{L-1}\right]\\
& \quad\quad S \leftarrow \mathrm{stack}\left[S,\beta\cdot\frac{D_{Adapt_L}}{A_L} + (1-\beta)\cdot S0_L\right]\\
& \quad\quad F \leftarrow \mathrm{stack}\left[F,\left(A_L + T_L\right)\cdot S_1\right]\\
& \quad \text{if } t > L\\
& \quad\quad A \leftarrow \mathrm{stack}\left[A,\alpha\cdot\frac{D_{Adapt_t}}{S_{t-L}} + (1-\alpha)\cdot\left(A_{t-1} + T_{t-1}\right)\right]\\
& \quad\quad T \leftarrow \mathrm{stack}\left[T,\gamma\cdot\left(A_t - A_{t-1}\right) + (1-\gamma)\cdot T_{t-1}\right]\\
& \quad\quad S \leftarrow \mathrm{stack}\left[S,\beta\cdot\frac{D_{Adapt_t}}{A_t} + (1-\beta)\cdot S_{t-L}\right]\\
& \quad\quad F \leftarrow \mathrm{stack}\left[F,\left(A_t + T_t\right)\cdot S_{t-L+1}\right]\\
& \mathrm{MSE} \leftarrow \frac{\sum_{r=1}^{N_{Adapt}}\left(F_r - D_{Adapt_r}\right)^2}{N_{Adapt}}
\end{aligned}
$$

<u>Finding the best α,β,γ providing the minimum MSE</u>

```
AD := | min ← 10^100
      | for α ∈ 0, 0.1 .. 1
      |   for β ∈ 0, 0.1 .. 1
      |     for γ ∈ 0, 0.1 .. 1
      |       if MSE(α, β, γ) < min
      |         | min ← MSE(α, β, γ)
      |         | α_min ← α
      |         | β_min ← β
      |         | γ_min ← γ
      | AD ← stack(min, α_min, β_min, γ_min)
```

$$AD = \begin{pmatrix} 8.45 \times 10^{8} \\ 0 \\ 0.1 \\ 0 \end{pmatrix}$$

$\alpha := AD_1$ $\beta := AD_2$ $\gamma := AD_3$

$\alpha = 0$ $\beta = 0.1$ $\gamma = 0$

$\beta := 0.14$

$MSE(\alpha, \beta, \gamma) = 8.39 \times 10^{8}$

8. Forecasting

```
F := | F ← 0
     | F ← stack[F, (A_0 + T_0)·S0_1]
     | for t ∈ 1 .. N_Adapt + N_forecast.branch
     |   | if t < L
     |   |   | A ← stack[A, α·(DAdapt_t / S0_t) + (1 − α)·(A_(t−1) + T_(t−1))]
     |   |   | T ← stack[T, γ·(A_t − A_(t−1)) + (1 − γ)·T_(t−1)]
     |   |   | S ← stack[S, β·(DAdapt_t / A_t) + (1 − β)·S0_t]
     |   |   | F ← stack[F, (A_t + T_t)·S0_(t+1)]
     |   | if t = L
     |   |   | A ← stack[A, α·(DAdapt_L / S0_L) + (1 − α)·(A_(L−1) + T_(L−1))]
     |   |   | T ← stack[T, γ·(A_L − A_(L−1)) + (1 − γ)·T_(L−1)]
     |   |   | S ← stack[S, β·(DAdapt_L / A_L) + (1 − β)·S0_L]
     |   |   | F ← stack[F, (A_L + T_L)·S_1]
     |   | if (t > L) ∧ (t ≤ N_Adapt)
     |   |   | A ← stack[A, α·(DAdapt_t / S_(t−L)) + (1 − α)·(A_(t−1) + T_(t−1))]
     |   |   | T ← stack[T, γ·(A_t − A_(t−1)) + (1 − γ)·T_(t−1)]
```

=

	0
0	0
1	-17828.73
2	20633.81
3	4129.94
4	17347.47
5	-8073.69
6	-8953.59
7	20310.68
8	1634.26
9	16184.63
10	-3797.01
11	-5382.91
12	16104.73
13	2983.92
14	14398.43
15	...

$$S \leftarrow \text{stack}\left[S, \beta \cdot \frac{D_{Adapt_t}}{A_t} + (1-\beta) \cdot S_{t-L}\right]$$

$$F \leftarrow \text{stack}\left[F, \left(A_t + T_t\right) \cdot S_{t-L+1}\right]$$

if $t > N_{Adapt}$

$$A \leftarrow \text{stack}\left[A, \alpha \cdot \frac{F_t}{S_{t-L}} + (1-\alpha) \cdot \left(A_{t-1} + T_{t-1}\right)\right]$$

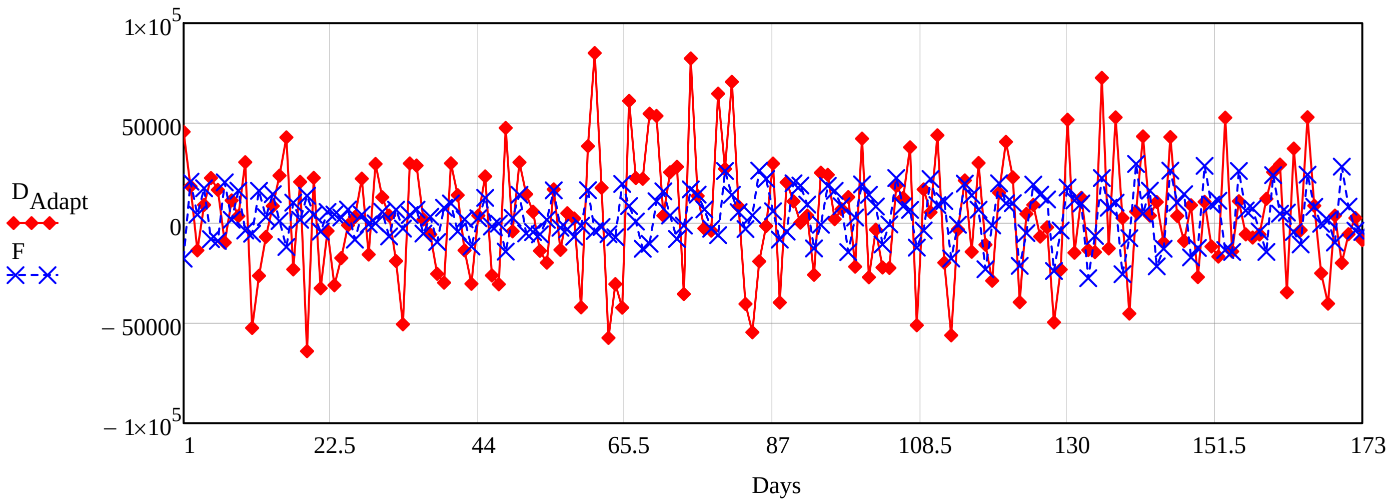
$$T \leftarrow \text{stack}\left[T, \gamma \cdot \left(A_t - A_{t-1}\right) + (1-\gamma) \cdot T_{t-1}\right]$$

$$S \leftarrow \text{stack}\left[S, \beta \cdot \frac{F_t}{A_t} + (1-\beta) \cdot S_{t-L}\right]$$

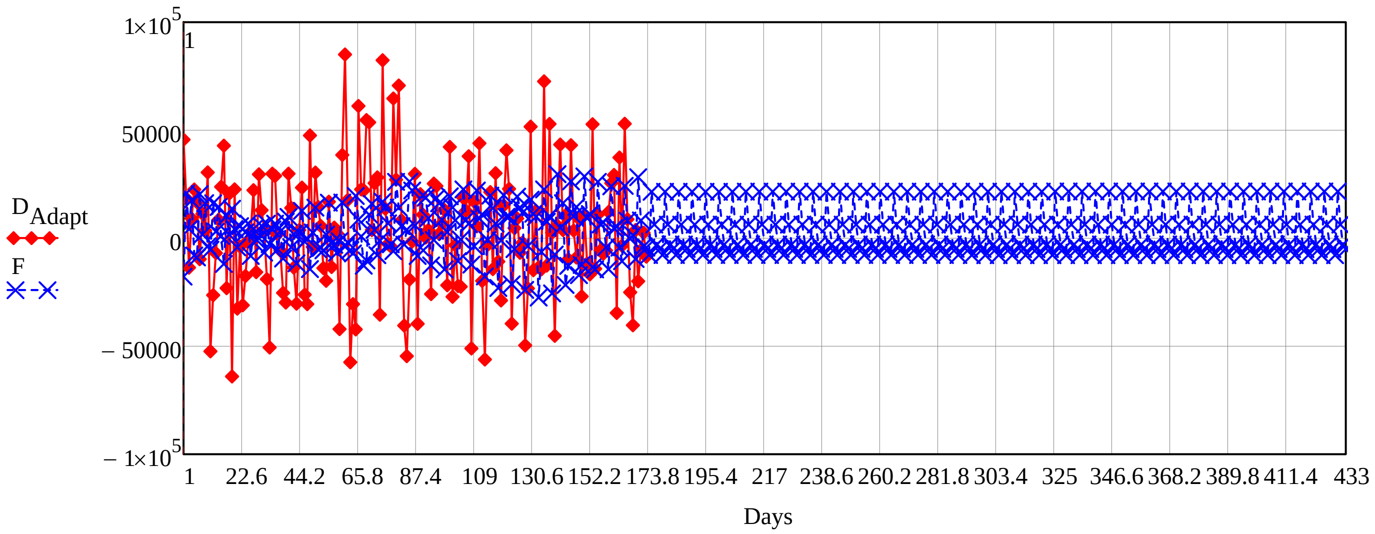
$$F \leftarrow \text{stack}\left[F, \left(A_t + T_t\right) \cdot S_{t-L+1}\right]$$

F

Past Data



Past Data+Forecast



7. Accuracy measurement

1) Distortion

$$MPE := \frac{\sum_{i=1}^{N_{Adapt}} \frac{D_{Adapt_i} - F_i}{D_{Adapt_i}}}{N_{Adapt}} \cdot 100 = 46.1$$

- Mean Percentage Error

2) Dispersion

$$SDE := \sqrt{\frac{\sum_{i=1}^{N_{Adapt}} \left(D_{Adapt_i} - F_i\right)^2}{N_{Adapt}}} = 28969$$

- Standard Deviation Error

$$SDE_{cd} := SDE$$

$$MAPE := \frac{\sum_{i=1}^{N_{Adapt}} \left| \frac{D_{Adapt_i} - F_i}{D_{Adapt_i}} \right|}{N_{Adapt}} \cdot 100 = 203.8$$

- Mean Absolute Percentage Error

3) Tracking Signal

$$TS := \frac{\sum_{i=1}^{N_{Adapt}} \left(D_{Adapt_i} - F_i\right)}{\sum_{i=1}^{N_{Adapt}} \left|D_{Adapt_i} - F_i\right|} = -0.008$$

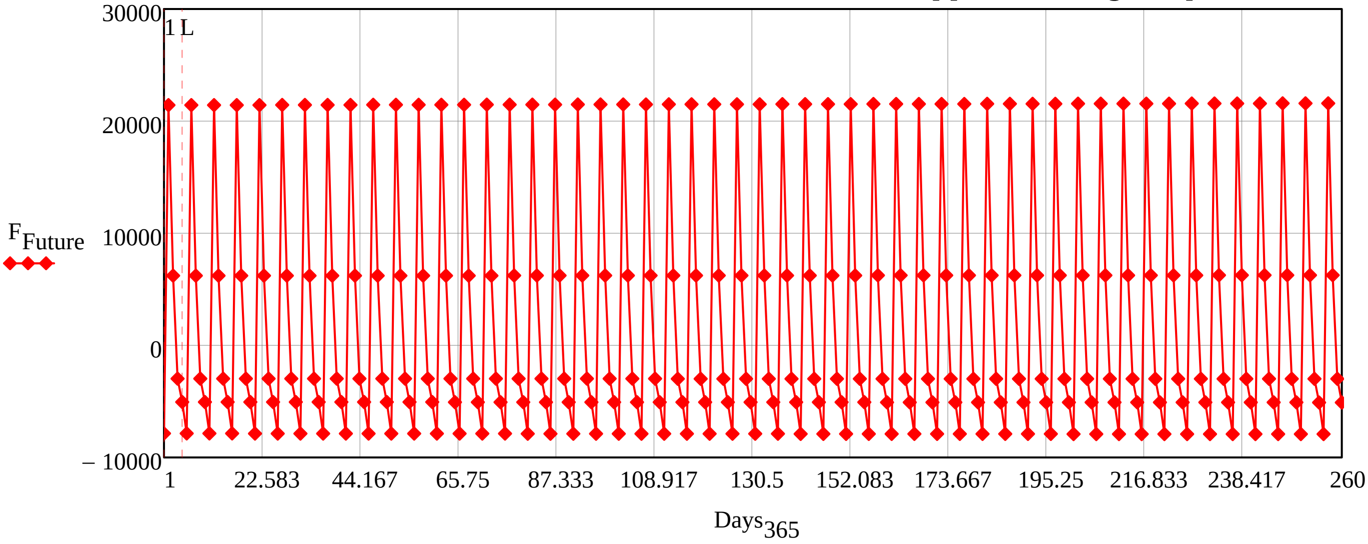
- Tracking Signal

If it is close it zero, it means that there is almost no predominant positive or negative errors.

Extracting the Forecast only for Future **Forecast for May2011-April2012**

$$F_{Future} := \mathrm{submatrix}\left(F, N_{Full}, N_{Full} + N_{forecast.branch}, 0, 0\right)$$

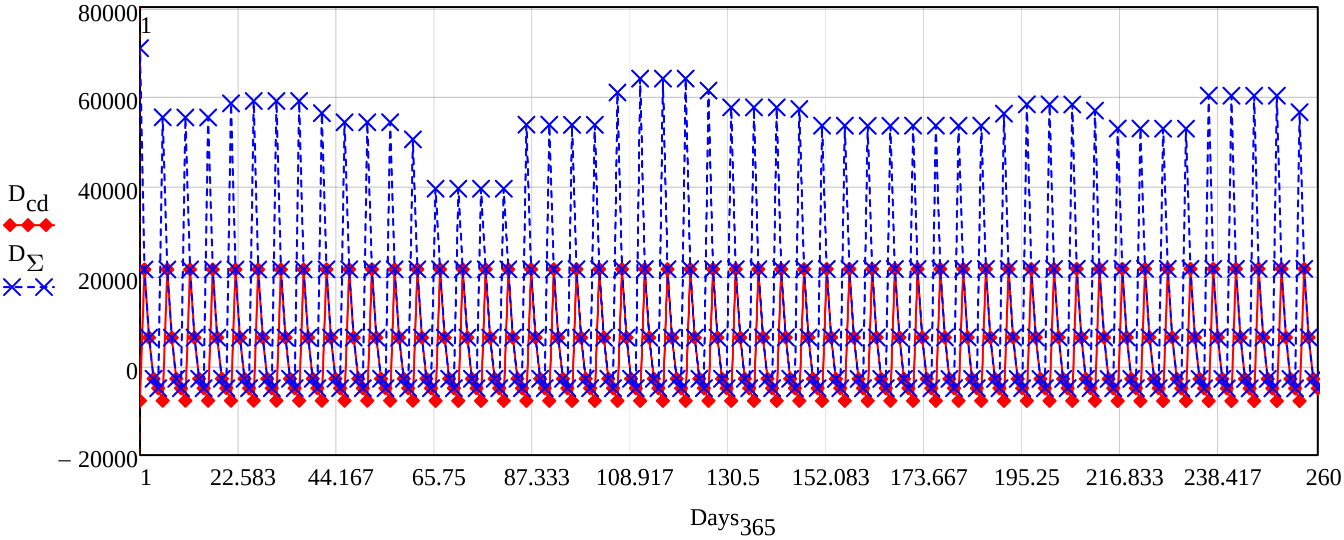
$$F_{Future_0} := 0$$



Total demand in the Cash Desk

$D_{cd} := F_{Future}$ $\qquad$ $D_{atm} := Demand_{ATM.adjust}^{\langle 0 \rangle}$

$D_{\Sigma} := D_{cd} + D_{atm}$



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4/7 Cash Desk. Inventory Planning

Reorder Point Model

$V := 1$ [-] - unitary item value. Unitless value equal to 1, as we measure the Demand in euros.

$upc := 31.1 \cdot 1.2 = 37.32$ [euro] -unitary procurement cost (20% IVA)

$rr := \frac{270}{2800 + 3600} \cdot 100 = 4.22$ [%] - risk of robbery

On the average, in case of robbery, 75% of stock is stalen

$rc := rr \cdot 0.75 = 3.16$ [%] - robbery cost as a percentage of money keeping in stock

$imc := 1$ [%] - immobilization cost as a percentage of money keeping in stock

$phc := rc + imc = 4.16$ [%/year] - Inventory Carrying Costs as a percentage of money keeping in stock

$phc := \frac{phc}{100}$

$D_y := \sum_{i=1}^{N_{forecast.branch}} D_{\Sigma_i} = 3.91 \times 10^6$ [euro] - annual demand

$EOQ := \sqrt{\frac{2 \cdot upc \cdot D_y}{phc \cdot V}} = 83695$ [euro] - Economic Order Quantity

Safety Stock

$k := 1.75$ - assures Service Level 96%

$ELT := 1 \quad \sigma_{LT} := 0 \quad SDE_{cd} = 28969 \quad SDE_{atm} = 2501$

$SDE := \sqrt{SDE_{cd}^2 + SDE_{atm}^2} \quad SDE = 29077$

$SS := k \cdot \sqrt{ELT \cdot SDE^2} = 50884$

Reorder Point

$ED_{LT} := D_{\Sigma}$

$ROP := ED_{LT} + SS \qquad ROP_0 := 0$

$ROP^T =$

	0	1	2	3	4	5	6	7	8	9	10	11
0	0	121690	72307	57092	47915	45829	106261	72310	57093	47915	45828	...

Calculating expected Stock Level in Branch

Week-ends adding

$M := \text{augment}\left(D_{\Sigma}, ROP, Demand_{ATM.adjust}^{\langle 1 \rangle}\right)$ - Expected Demand in Branch with week-ends

```
M7days :=  | for  j ∈ 0 .. 2
           |   QW[0,j] ← 0
           | D ← QW
           | D ← stack(D,QW)  if [(Demand_ATM^<1>)_1 = 6] ∨ [(Demand_ATM^<1>)_1 = 7]
           | D ← stack(D,QW)  if (Demand_ATM^<1>)_2 = 7
           | for  i ∈ 1 .. N_forecast.branch
           |   | D ← stack[D, ((M^T)^<i>)^T]  if (M^<2>)_i ≠ 5
           |   | otherwise
           |   |   | D ← stack[D, ((M^T)^<i>)^T]
           |   |   | D ← stack(D,QW)
           |   |   | D ← stack(D,QW)
           | D
```

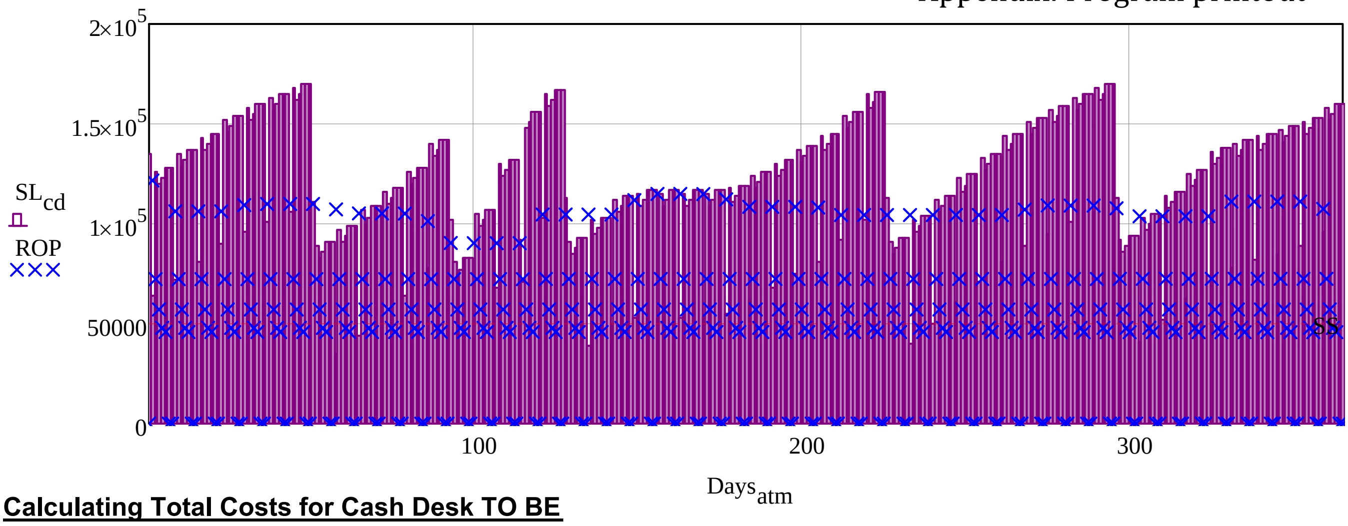
$M_{7days} := \text{submatrix}\left(M_{7days}, 0, 365, 0, 2\right)$

$D_{\Sigma} := M_{7days}^{\langle 0 \rangle}$ $\qquad ROP := M_{7days}^{\langle 1 \rangle}$

```
SL_cd := | ordered ← 0
         | N_pr ← 1
         | SL_0 ← 0
         | for i ∈ 0 .. 365
         |   ORD_i ← 0
         | SL_1 ← EOQ + SS − DΣ_1  if EOQ + SS − DΣ_1 ≥ 0
         | SL_1 ← 0  otherwise
         | if SL_1 ≤ ROP_1
         |   | ORD_2 ← EOQ
         |   | N_pr ← N_pr + 1
         | for i ∈ 2 .. 365
         |   | ordered ← ordered − 1
         |   | SL_i ← SL_(i−1) − DΣ_i + ORD_i  if SL_(i−1) − DΣ_i + ORD_i ≥ 0
         |   | SL_i ← 0  otherwise
         |   | if (SL_i ≤ ROP_i) ∧ (i ≤ 365 − 2) ∧ (ordered ≤ 0)
         |   |   | ORD_(i+1) ← EOQ  if (M7days^⟨2⟩)_(i+1) ≠ 0
         |   |   | otherwise
         |   |   |   | ORD_(i+2) ← EOQ  if (M7days^⟨2⟩)_(i+2) ≠ 0
         |   |   |   | ORD_(i+3) ← EOQ  otherwise
         |   |   | ordered ← 1
         |   |   | N_pr ← N_pr + 1
         | SL_0 ← N_pr
         | SL
```

$N_{pr.cd.to.be} := SL_{cd_0} = 48$

$$AIL_{cd.to.be} := \frac{\sum_{i=1}^{365} SL_{cd_i}}{365} = 121256$$ - AIL, considering week-ends and with Lead Time=1 day



Calculating Total Costs for Cash Desk TO BE

$ICC_{cd.to.be} := AIL_{cd.to.be} \cdot V \cdot phc = 5049$

$PC_{cd.to.be} := N_{pr.cd.to.be} \cdot upc = 1791$

$TC_{cd.to.be} := ICC_{cd.to.be} + PC_{cd.to.be} = 6841$

==

5/7 AS IS Costs

$Full\_Data_{Original} =$

		G.ap	G.ch	C	R	B	ATM Withdrawals	Day of week
	0	1	2	3	4	5	6	7
0	'01/01/2010"	0	0	0	0	0	2560	5
1	'01/02/2010"	0	0	0	0	0	7620	6
2	'01/03/2010"	0	0	0	0	0	4820	7
3	'01/04/2010"	0	0	0	0	0	13800	1
4	'01/05/2010"	0	0	0	0	0	13600	2
5	'01/06/2010"	0	0	0	0	0	1550	3
6	'01/07/2010"	0	0	0	0	0	4740	4
7	'01/08/2010"	0	0	0	0	0	8990	5
8	'01/09/2010"	0	0	0	0	0	7990	6
9	'01/10/2010"	0	0	0	0	0	5140	7
10	'01/11/2010"	0	0	0	0	0	8170	1
11	'01/12/2010"	0	0	0	0	0	9400	2
12	'01/13/2010"	0	0	0	0	0	7700	...

# of days from the Initial day of Original data to the September 1st, 2010:

$N_{Sept10} := 243$ - 8 months (Jan-Aug2010)

# of days from the Initial day of Original data to the March 1st, 2010:

$N_{March11} := 424$ - 6 months (Jan2010-Feb2011)

$N_{March11} - N_{Sept10} = 181$ - # of days from September 1st, 2010 to February 31st, 2011 (half a year)

$N_{Full} := rows(Full\_Data_{Original}) = 485$

Adding Stock Level at week-ends to Cash Desk

$$
\begin{aligned}
F\_D := \; & \text{for } i \in N_{Sept10} .. N_{Full} - 1 \\
& \quad \text{if } \left(Full\_Data_{Original}^{\langle 1 \rangle}\right)_i = 0 \\
& \quad\quad \left(Full\_Data_{Original}^{\langle 1 \rangle}\right)_i \leftarrow \left(Full\_Data_{Original}^{\langle 1 \rangle}\right)_{i-1} \\
& \quad\quad \left(Full\_Data_{Original}^{\langle 2 \rangle}\right)_i \leftarrow \left(Full\_Data_{Original}^{\langle 2 \rangle}\right)_{i-1} \\
& Full\_Data_{Original}
\end{aligned}
$$

$$F\_D^{\langle 6 \rangle} := Full\_Data_{atm}^{\langle 1 \rangle}$$

Number of procurements in the Cash Desk during last year

We calculate the number of procurements during for half a year (from September 1st, 2010 to February 31st, 2011). Multiplying it by 2, we obtain the number of procurements per year.

$$N_{pr.cd.as.is} := \left| \begin{array}{l} N \leftarrow 0 \\ \text{for } i \in N_{Sept10} .. N_{March11} \\ \quad \left| \begin{array}{l} N \leftarrow N + 1 \text{ if } \left(F\_D^{\langle 3 \rangle}\right)_i \neq 0 \\ N \leftarrow N + 1 \text{ if } \left(F\_D^{\langle 4 \rangle}\right)_i \neq 0 \end{array} \right. \\ N \cdot 2 \end{array} \right. \qquad N_{pr.cd.as.is} = 100$$

AIL in the Cash Desk during last year

$$AIL_{cd.as.is} := \frac{\sum_{i = N_{Sept10}}^{N_{March11}-1} \left(F\_D^{\langle 2 \rangle}\right)_i}{N_{March11} - N_{Sept10}} = 89830$$

Number of procurements in the ATM during last year

$$N_{pr.atm.as.is} := \left| \begin{array}{l} N \leftarrow 0 \\ \text{for } i \in N_{Sept10} .. N_{March11} \\ \quad N \leftarrow N + 1 \text{ if } \left(F\_D^{\langle 5 \rangle}\right)_i \neq 0 \\ N \cdot 2 \end{array} \right. \qquad N_{pr.atm.as.is} = 126$$

AIL in the ATM during last year

$$SL_{atm} := \left| \begin{array}{l} SL_0 \leftarrow 0 \\ j \leftarrow 1 \\ \text{for } i \in N_{Sept10} .. N_{March11} \\ \quad \left| \begin{array}{l} QW \leftarrow SL_{j-1} - \left(F\_D^{\langle 6 \rangle}\right)_i - \left(F\_D^{\langle 5 \rangle}\right)_i \\ QW \leftarrow 0 \text{ if } QW \leq 0 \\ SL \leftarrow \text{stack}(SL, QW) \\ j \leftarrow j + 1 \end{array} \right. \\ SL \end{array} \right.$$

$$AIL_{atm.as.is} := \frac{\sum_{i = 1}^{\left(N_{March11}-N_{Sept10}\right)+1} SL_{atm_i}}{\left(N_{March11} - N_{Sept10}\right) + 1} = 26414$$

TOTAL SAVINGS for AS IS vs. TO BE for Future Deamd

ATM

As Is

$ICC_{atm.as.is} := AIL_{atm.as.is} \cdot V \cdot phc = 1100$

$PC_{atm.as.is} := N_{pr.atm.as.is} \cdot upc_{atm} = 3625$

$TC_{atm.as.is} := ICC_{atm.as.is} + PC_{atm.as.is} = 4725$

To Be

$ICC_{atm.to.be} := AIL_{atm.to.be} \cdot V \cdot phc = 1714$

$PC_{atm.to.be} := N_{pr.atm.to.be} \cdot upc_{atm} = 1496$

$TC_{atm.to.be} := ICC_{atm.to.be} + PC_{atm.to.be} = 3210$

Cash Desk

As Is

$ICC_{cd.as.is} := AIL_{cd.as.is} \cdot V \cdot phc = 3741$

$PC_{cd.as.is} := N_{pr.cd.as.is} \cdot upc = 3732$

$TC_{cd.as.is} := ICC_{cd.as.is} + PC_{cd.as.is} = 7473$

To Be

$ICC_{cd.to.be} := AIL_{cd.to.be} \cdot V \cdot phc = 5049$

$PC_{cd.to.be} := N_{pr.cd.to.be} \cdot upc = 1791$

$TC_{cd.to.be} := ICC_{cd.to.be} + PC_{cd.to.be} = 6841$

$Save_{atm.icc} := ICC_{atm.as.is} - ICC_{atm.to.be} = -613.76$

$Save_{atm.pc} := PC_{atm.as.is} - PC_{atm.to.be} = 2128.77$

$Save_{atm} := TC_{atm.as.is} - TC_{atm.to.be} = 1515$

$Save_{cd.icc} := ICC_{cd.as.is} - ICC_{cd.to.be} = -1308.59$

$Save_{cd.pc} := PC_{cd.as.is} - PC_{cd.to.be} = 1940.64$

$Save_{cd} := TC_{cd.as.is} - TC_{cd.to.be} = 632$

$$Save_{atm.icc.\%} := \frac{Save_{atm.icc}}{ICC_{atm.as.is}} \cdot 100 = -55.8$$

$$Save_{atm.pc.\%} := \frac{Save_{atm.pc}}{PC_{atm.as.is}} \cdot 100 = 58.73$$

$$Save_{atm.\%} := \frac{Save_{atm}}{TC_{atm.as.is}} \cdot 100 = 32.07$$

$$Save_{cd.icc.\%} := \frac{Save_{cd.icc}}{ICC_{cd.as.is}} \cdot 100 = -34.98$$

$$Save_{cd.pc.\%} := \frac{Save_{cd.pc}}{PC_{cd.as.is}} \cdot 100 = 52$$

$$Save_{cd.\%} := \frac{Save_{cd}}{TC_{cd.as.is}} \cdot 100 = 8.46$$

$TC_{as.is} := TC_{atm.as.is} + TC_{cd.as.is} = 12197.11$

$TC_{to.be} := TC_{atm.to.be} + TC_{cd.to.be} = 10050.05$

$T\_SAVINGS := TC_{as.is} - TC_{to.be} = 2147$

$$TotSave\% := \frac{T\_SAVINGS}{TC_{as.is}} \cdot 100 = 18$$

TOTAL SAVINGS for AS IS vs. TO BE for Future Deamd

	As Is	To Be	Δ	Δ [%]
ATM	$ICC_{atm.as.is} = 1100$ $PC_{atm.as.is} = 3625$ $TC_{atm.as.is} = 4725$	$ICC_{atm.to.be} = 1714$ $PC_{atm.to.be} = 1496$ $TC_{atm.to.be} = 3210$	$Save_{atm.icc} = -614$ $Save_{atm.pc} = 2129$ $Save_{atm} = 1515$	$Save_{atm.icc.\%} = -56$ $Save_{atm.pc.\%} = 59$ $Save_{atm.\%} = 32$
Cash Desk	$ICC_{cd.as.is} = 3741$ $PC_{cd.as.is} = 3732$ $TC_{cd.as.is} = 7473$	$ICC_{cd.to.be} = 5049$ $PC_{cd.to.be} = 1791$ $TC_{cd.to.be} = 6841$	$Save_{cd.icc} = -1309$ $Save_{cd.pc} = 1941$ $Save_{cd} = 632$	$Save_{cd.icc.\%} = -35$ $Save_{cd.pc.\%} = 52$ $Save_{cd.\%} = 8$
Branch	$TC_{as.is} = 12197$	$TC_{to.be} = 10050$	T_SAVINGS = 2147	TotSave% = 18

===

6/7 ATM TO BE Costs for Past Demand

Past Demand preparation

Costs are calculated From September 1st 2010 until February 28th 2011

$Full\_Data_{atm.Past} := submatrix(Full\_Data_{atm}, 243, 423, 1, 1)$

$Full\_Data_{atm.Past} := stack(0, Full\_Data_{atm.Past})$

$D_{atm.P} := Full\_Data_{atm.Past}$

Periodic Review Model

1. Choice of the Inventory management parameters

1) Logistics Costs structure

$GrossSalary := 60000$ [euro] - gross salary of operator per year

$upc_{atm} := \frac{GrossSalary}{365 \cdot \frac{5}{7} \cdot 8 \cdot \frac{2}{2}} = 28.77$ [euro] -unitary procurement cost (cost of one replenishment)

$rr := \frac{270}{2800 + 3600} \cdot 100 = 4.22$ [%] - risk of robbery

On the average, in case of robbery, 75% of stock is stalen

$rc := rr \cdot 0.75 = ▮$ [%/year] - robbery cost as a percentage of money keeping in stock

$imc := 1$ [%/year] - immobilization cost as a percentage of money keeping in stock

$phc := rc + imc = 4.16$ [%/year] - Inventory Carrying Costs as a percentage of money keeping in stock

$phc := \frac{phc}{100}$

2) Order interval and Cycle Stock

$V := 1$ [-] - unitary item value. Unitless value equal to 1, as we measure the Demand in euros.

$$D_{y.atm} := 2 \cdot \sum_{i=1}^{N_{March11} - N_{Sept10}} D_{atm.P_i} = 3.3 \times 10^6$$

$$T_y := \sqrt{\frac{2 \cdot upc_{atm}}{phc \cdot V \cdot D_{y.atm}}} = 0.02$$ [year] - Order Interval

$$T_d := T_y \cdot (365) = 7.47$$ [days] - Order Interval

$$\boxed{T_{fix} := \text{floor}(T_d) = 7}$$

$$ED_{atm}(d) := \frac{\sum_{i=1}^{N_{March11} - N_{Sept10}} D_{atm.P_i}}{N_{March11} - N_{Sept10}} \cdot d \qquad ED_{atm}(1) = 9113$$ [euro] - Excpected demand per day

$$ACS_{atm}(T) := \frac{ED_{atm}(T)}{2} \qquad ACS_{atm}(T_{fix}) = 31895$$ [euro] - average value of the Cycle Stock

3) Safety Stock

$k_{atm} := 2.33$ - assures Service Level 99% k=2.33 (99%) k=1.75 (96%)

$$\sigma_{D.atm} := \sqrt{\frac{\sum_{i=1}^{N_{March11} - N_{Sept10}} \left(D_{atm.P_i} - ED_{atm}(1)\right)^2}{N_{March11} - N_{Sept10}}} = 3367$$

$ELT_{atm} := 0$ $\sigma_{LT.atm} := 0$

$$\sigma_{D\_LT.atm}(T) := \sqrt{\left(ELT_{atm} + T\right)\cdot\sigma_{D.atm}{}^{2} + \sigma_{LT.atm}{}^{2}\cdot ED_{atm}(1)^{2}}$$

$$SS_{atm}(T) := k_{atm}\cdot\sigma_{D\_LT.atm}(T)$$

$SS_{atm}(T_{fix}) = 20759$ [euro] - safety stock

4) Average Inventory Level and Availability Target

$AIL_{atm}(T) := SS_{atm}(T) + ACS_{atm}(T)$ $AIL_{atm}(T_{fix}) = 52653$ [euro] - average inventory level

$AT(T) := ED_{atm}(T + ELT_{atm}) + SS_{atm}(T)$ $AT(T_{fix}) = 84548$ [euro] - availability terget

Adding indexes of the days of a week for Past data

First day of the week of data forecasted:

$fd := \left(Full\_Data_{Original}{}^{\langle 7\rangle}\right)_{N_{Full}-7-(N_{March11}-N_{Sept10})-61} = 3$ Sept 1st, 2010 is Wednesday

```
Index_week_y := | QW_0 ← 0                                                  - for 181 days
                | for  i ∈ fd .. 181
                |   (QW)_(i+1−fd) ← (Week_index_per_year)_i
                | for  i ∈ 1 .. fd
                |   (QW)_(181−fd+i) ← (Week_index_per_year)_(i+mod(181,7)−1)
                | QW
```

2. Adjustment of the replenishment plan (for week-ends)

1) Replenishment plan, depending on a type of policy

```
Putin(pl) := | Putin_0 ← 0
             | k ← Index_week_y_1
             | j ← 1  if  1 ≤ mod(k,7) ≤ 5
             | j ← 3  if  mod(k,7) = 6
             | j ← 2  if  mod(k,7) = 0
             | if  pl = 1
             |   | sh_6 ← −1
             |   | sh_7 ← −2
             | if  pl = 2
             |   | sh_6 ← −1
             |   | sh_7 ← 1
             | if  pl = 3
             |   | sh_6 ← 2
             |   | sh_7 ← −2
             | if  pl = 4
             |   | sh_6 ← 2
             |   | sh_7 ← 1
             | while  (j ≤ N_March11 − N_Sept10)
```

$k \leftarrow Index\_week_{y_j} + T_{fix}$

if $mod(k,7) = 6$

$Putin_j \leftarrow AT(T_{fix} + sh_6)$

$j \leftarrow j + T_{fix} + sh_6$

if $mod(k,7) = 0$

$Putin_j \leftarrow AT(T_{fix} + sh_7)$

$j \leftarrow j + T_{fix} + sh_7$

if $1 \le mod(k,7) \le 5$

$Putin_j \leftarrow AT(T_{fix})$

$j \leftarrow j + T_{fix}$

$Putin_{365} \leftarrow 0$ if $j < N_{March11} - N_{Sept10}$

Putin

$Putin(pl) := submatrix(Putin(pl), 0, N_{March11} - N_{Sept10}, 0, 0)$

$Replen := augment(Index\_week_y, Putin(1), Putin(2), Putin(3), Putin(4))$

Replen =

	Day of the month	Day of the week	Policy-1	Policy-2	Policy-3	Policy-4
		0	1	2	3	4
	0	0	0	0	0	0
	1	3	84548	84548	84548	84548
	2	4	0	0	0	0
	3	5	0	0	0	0
	4	6	0	0	0	0
	5	7	0	0	0	0
	6	1	0	0	0	0
	7	2	0	0	0	0
	8	3	84548	84548	84548	84548
	9	4	0	0	0	0
	10	5	0	0	0	0
	11	6	0	0	0	0
	12	7	0	0	0	0
	13	1	0	0	0	0
	14	2	0	0	0	0
	15	3	84548	84548	84548	...

2) Nuber of replenishments, depending on type of policy

$$N_{pr}(pl) := \begin{array}{|l} N_{pr} \leftarrow 0 \\ \text{for } i \in 0 .. N_{March11} - N_{Sept10} \\ \quad N_{pr} \leftarrow N_{pr} + 1 \text{ if } (Putin(pl))_i \neq 0 \\ 2 \cdot N_{pr} \end{array}$$

$j := 0 .. 3$

$N_{pr_j} := N_{pr}(j+1)$ $\qquad N_{pr} = \begin{pmatrix} 52 \\ 52 \\ 52 \\ 52 \end{pmatrix}$

3) Expected Avarage Annual Inventory Level, depending on type of policy

$$SL_{atm.P}(pl) := \begin{array}{|l} SL_0 \leftarrow 0 \\ N_{Stock.Out} \leftarrow 0 \\ \text{for } i \in 1 .. N_{March11} - N_{Sept10} \\ \quad \begin{array}{|l} SL_i \leftarrow SL_{i-1} - D_{atm.P_i} \text{ if } (Putin(pl))_i = 0 \\ SL_i \leftarrow (Putin(pl))_i - D_{atm.P_i} \text{ otherwise} \\ \text{if } SL_i < 0 \\ \quad \begin{array}{|l} SL_i \leftarrow 0 \\ N_{Stock.Out} \leftarrow N_{Stock.Out} + 1 \end{array} \end{array} \\ SL_0 \leftarrow N_{Stock.Out} \\ SL \end{array}$$

$M := \text{augment}\left(SL_{atm.P}(1), SL_{atm.P}(2), SL_{atm.P}(3), SL_{atm.P}(4), Index\_week_y\right)$

$j := 0 .. 3$

$$AIL_{atm_j} := \frac{\sum_{i=1}^{N_{March11} - N_{Sept10}} \left(M^{\langle j \rangle}\right)_i}{N_{March11} - N_{Sept10}} \qquad AIL_{atm} = \begin{pmatrix} 47784 \\ 47784 \\ 47784 \\ 47784 \end{pmatrix}$$

4) Total Costs for different policies

$$ICC_{atm} := AIL_{atm} \cdot V \cdot phc = \begin{pmatrix} 1990 \\ 1990 \\ 1990 \\ 1990 \end{pmatrix} \qquad N_{pr} = \begin{pmatrix} 52 \\ 52 \\ 52 \\ 52 \end{pmatrix}$$

$$PC_{atm} := N_{pr} \cdot upc_{atm} = \begin{pmatrix} 1496 \\ 1496 \\ 1496 \\ 1496 \end{pmatrix} \qquad TC_{atm} := ICC_{atm} + PC_{atm} = \begin{pmatrix} 3486 \\ 3486 \\ 3486 \\ 3486 \end{pmatrix}$$

5) Choosing the best policy

$$Pol := \begin{array}{|l} min \leftarrow 10^{100} \\ \text{for } j \in 0..3 \\ \quad \text{if } TC_{atm_j} < min \\ \qquad min \leftarrow TC_{atm_j} \\ \qquad j_{min} \leftarrow j \\ Pol \leftarrow stack(min, j_{min}) \end{array}$$

$$Pol = \begin{pmatrix} 3486 \\ 0 \end{pmatrix}$$

$TC_{atm.min} := Pol_0 = 3486$

$pl_{best} := Pol_1 + 1 = 1$ - Best policy

$AIL_{atm.to.be.P} := AIL_{atm}(pl_{best}-1) = 47784$

$N_{pr.atm.to.be.P} := N_{pr}(pl_{best}-1) = 52$

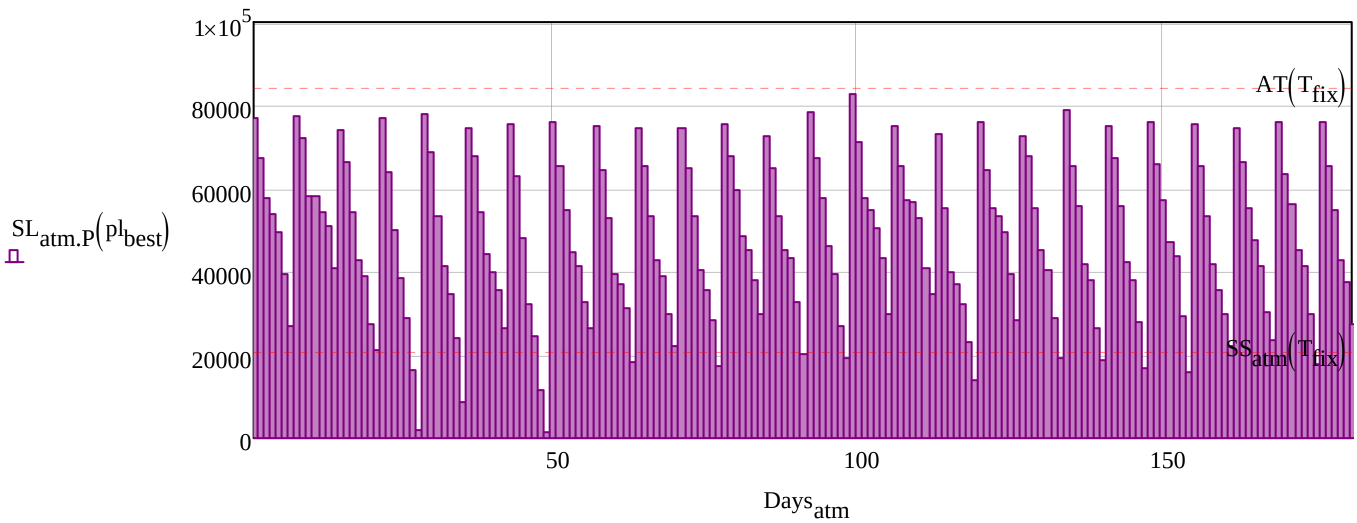
ATM. Costs To Be

$ICC_{atm.to.be.P} := AIL_{atm.to.be.P} \cdot V \cdot phc = 1990$

$PC_{atm.to.be.P} := N_{pr.atm.to.be.P} \cdot upc_{atm} = 1496$

$TC_{atm.to.be.P} := ICC_{atm.to.be.P} + PC_{atm.to.be.P} = 3486$

Expected Stock Level profile for the best policy



6) Calculating of the Residuals for the best policy

$$Residuals := \begin{array}{|l} R_0 \leftarrow 0 \\ \text{for } i \in 1..N_{March11} - N_{Sept10} \\ \quad R_i \leftarrow SL_{atm.P}(pl_{best})_{i-1} \text{ if } (Putin(pl_{best}))_i \neq 0 \\ R_{N_{March11}-N_{Sept10}} \leftarrow 0 \text{ if } (Putin(pl_{best}))_{N_{March11}-N_{Sept10}} = 0 \\ R \end{array}$$

$N_{Stock.Out} := SL_{atm.P}(pl_{best})_0 = 0$ - number of stockouts

$Putin(pl_{best}) =$

	0
0	0
1	84548
2	0
3	0
4	0
5	0
6	0
7	0
8	84548
9	0
10	...

$Residuals =$

	0
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	27103
9	0
10	...

The maximum capacity of ATM is 140 000 euro

Expexted demand by ATM calculation

Demand by ATM

$Demand_{ATM} := Putin(pl_{best}) - Residuals$

Week-ends elimination

$Demand_{ATM} := augment(Demand_{ATM}, Index\_week_y)$

$$Demand_{ATM.adjust} := \left| \begin{array}{l} \text{for } j \in 0..1 \\ \quad Demand_{0,j} \leftarrow 0 \\ \text{for } i \in 1..N_{March11} - N_{Sept10} \\ \quad Demand \leftarrow stack\left[Demand, \left(Demand_{ATM}{}^{T}\right)^{\langle i \rangle T}\right] \text{ if } 1 \le \left(Demand_{ATM}{}^{\langle 1 \rangle}\right)_i \le 5 \\ Demand \end{array} \right.$$

$Demand_{ATM.adjust} =$

	0	1
0	0	0
1	84547.97	3
2	0	4
3	0	5
4	0	1
5	0	2
6	57445.36	3
7	0	4
8	0	5
9	0	1
10	0	...

===

7/7 Cash Desk TO BE Costs for Past Demand

Costs are calculated From September 1st 2010 until February 28th 2011, without weekends

$D_{atm.P} := Demand_{ATM.adjust}^{\langle 0 \rangle}$ Sept 1st, 2010 is Wed

$N_{Sept\_MarchNW} := rows(D_{atm.P}) - 1 = 129$

$QW := (0\ \ 0\ \ 0\ \ 0)$

$FD_{cd.by.cust.past} := stack(QW, Full\_Data)$ - Past Demand by customers from Cash Desk

$D_{cd.P} := submatrix(FD_{cd.by.cust.past}, 0, N_{Sept\_MarchNW}, 1, 1)$

$D_{\Sigma.P} := D_{cd.P} + D_{atm.P}$

Calculating Stock Level in Branch for Past Demand

Week-ends adding

```
D_cd.7days := | for j ∈ 0..3
              |    QW_0,j ← 0
              | D ← QW
              | for i ∈ 1..173
              |    | D ← stack[D, (FD_cd.by.cust.past^T)^<i>^T]  if (Full_Data^<2>)_i−1 ≠ 5
              |    | otherwise
              |    |    | D ← stack[D, (FD_cd.by.cust.past^T)^<i>^T]
              |    |    | D ← stack(D, QW)
              |    |    | D ← stack(D, QW)
              | D
```

$D_{cd.7days} := submatrix(D_{cd.7days}, 0, N_{March11} - N_{Sept10}, 1, 1)$

Reorder Point Model

$V := 1$ [-] - unitary item value. Unitless value equal to 1, as we measure the Demand in euros.

$upc := 31.1 \cdot 1.2 = 37.32$ [euro] -unitary procurement cost (20% IVA)

$rr := \frac{270}{2800 + 3600} \cdot 100 = 4.22$ [%] - risk of robbery

On the average, in case of robbery, 75% of stock is stalen

$rc := rr \cdot 0.75 = 3.16$ [%] - robbery cost as a percentage of money keeping in stock

$imc := 1$ [%] - immobilization cost as a percentage of money keeping in stock

$phc := rc + imc = 4.16$ [%/year] - Inventory Carrying Costs as a percentage of money keeping in stock

$phc := \frac{phc}{100}$

$D_y := 2 \cdot \sum_{i=1}^{N_{Sept\_MarchNW}} D_{\Sigma.P_i} = 4.06 \times 10^6$ $EOQ := \sqrt{\frac{2 \cdot upc \cdot D_y}{phc \cdot V}} = 85283$ [euro] - Economic Order Quantity

Safety Stock

$k := 1.75$ - assures Service Level 96%

$ELT := 1$

$$ED(d) := \frac{\sum_{i=1}^{N_{Sept\_MarchNW}} D_{\Sigma.P_i}}{N_{March11} - N_{Sept10}} \cdot d$$

$ED(1) = 11209$ [euro] - Excpected demand per day

Demand by customers:

$$\mu_{cd.P} := \frac{\sum_{i=1}^{N_{Sept\_MarchNW}} D_{cd.P_i}}{N_{Sept\_MarchNW}} = 2730$$

$$\sigma_{D.cd.P} := \sqrt{\frac{\sum_{i=1}^{N_{Sept\_MarchNW}} \left(D_{cd.P_i} - \mu_{cd.P}\right)^2}{N_{Sept\_MarchNW}}} = 29928$$

Demand by ATM:

$$\mu_{atm} := \frac{2 \cdot \sum_{i=1}^{N_{Sept\_MarchNW}} D_{atm.P_i}}{N_{pr.atm.to.be.P}} = 64485$$

$$\sigma_{D.atm.P} := \left| \begin{array}{l} QW \leftarrow 0 \\ \text{for } i \in 1 .. N_{Sept\_MarchNW} \\ \quad QW \leftarrow QW + \left(D_{atm.P_i} - \mu_{atm}\right)^2 \text{ if } D_{atm.P_i} \neq 0 \\ \sigma_{D.atm} \leftarrow \sqrt{\frac{2 \cdot QW}{N_{pr.atm.to.be.P}}} \\ \sigma_{D.atm} \end{array} \right.$$

$\sigma_{D.atm.P} = 9490$

Total Demand:

$$\sigma_{D.P} := \sqrt{\sigma_{D.cd.P}^2 + \sigma_{D.atm.P}^2} = 31397$$

$ELT := 1$

$$\sigma_{D\_LT} := \sqrt{ELT \cdot \sigma_{D.P}^2 + \sigma_{LT}^2 \cdot ED(1)^2} = 31397$$

$SS := k \cdot \sigma_{D\_LT} = 54945$

$ROP := ED(ELT) + SS = 66154$

Calculating expected Stock Level in Branch

Week-ends adding

$D_{\Sigma} := \text{augment}\left(D_{\Sigma.P}, Demand_{ATM.adjust}^{\langle 1\rangle}\right)$ - Expected Demand in Branch with week-ends

$$
\begin{aligned}
D_{\Sigma.7days} := \;& \text{for } j \in 0..1 \\
& \quad QW_{0,j} \leftarrow 0 \\
& D \leftarrow QW \\
& D \leftarrow \text{stack}(D, QW) \text{ if } \left[\left(Demand_{ATM}^{\langle 1\rangle}\right)_1 = 6\right] \vee \left[\left(Demand_{ATM}^{\langle 1\rangle}\right)_1 = 7\right] \\
& D \leftarrow \text{stack}(D, QW) \text{ if } \left(Demand_{ATM}^{\langle 1\rangle}\right)_2 = 7 \\
& \text{for } i \in 1..N_{Sept\_MarchNW} \\
& \quad D \leftarrow \text{stack}\left[D, \left(D_{\Sigma}^{T}\right)^{\langle i\rangle T}\right] \text{ if } \left(D_{\Sigma}^{\langle 1\rangle}\right)_i \neq 5 \\
& \quad \text{otherwise} \\
& \quad\quad D \leftarrow \text{stack}\left[D, \left(D_{\Sigma}^{T}\right)^{\langle i\rangle T}\right] \\
& \quad\quad D \leftarrow \text{stack}(D, QW) \\
& \quad\quad D \leftarrow \text{stack}(D, QW) \\
& D
\end{aligned}
$$

$D_{\Sigma.7days} := \text{submatrix}\left(D_{\Sigma.7days}, 0, N_{March11} - N_{Sept10}, 0, 1\right)$

$D_{\Sigma.7days} =$

	0	1
0	0	0
1	130123	3
2	18304	4
3	-13698	5
4	0	0
5	0	0
6	9024	1
7	22478	2
8	74002	3
9	-9749	4
10	11271	...

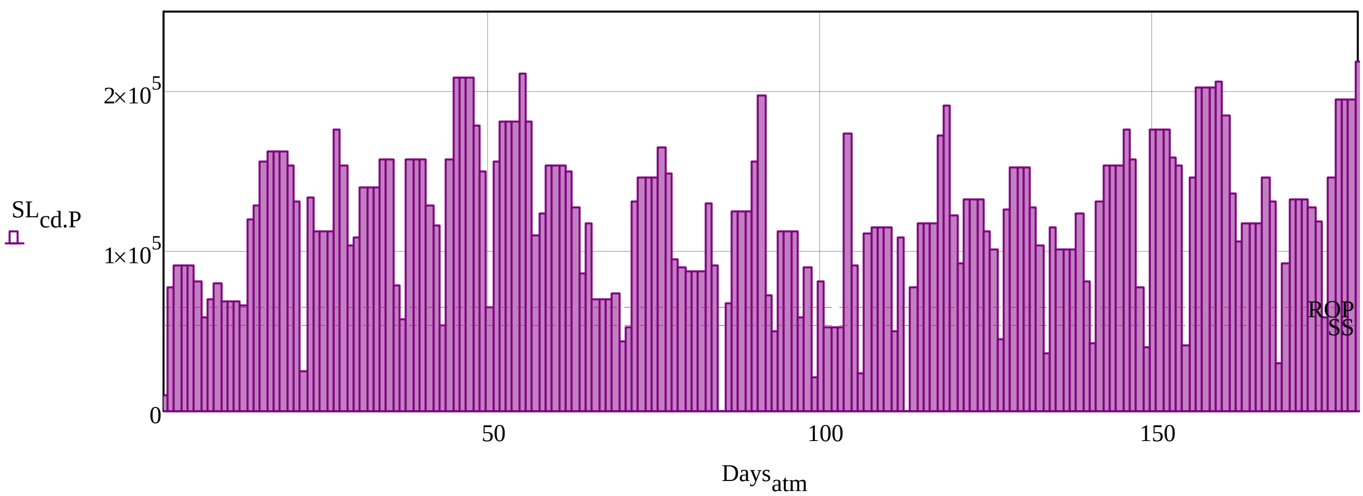
$D_{\Sigma} := D_{\Sigma.7days}$

$$
\begin{aligned}
SL_{cd.P} := \;& ordered \leftarrow 0 \\
& N_{pr} \leftarrow 1 \\
& SL_0 \leftarrow 0 \\
& \text{for } i \in 0..N_{March11} - N_{Sept10} \\
& \quad ORD_i \leftarrow 0 \\
& SL_1 \leftarrow EOQ + SS - \left(D_{\Sigma}^{\langle 0\rangle}\right)_1 \text{ if } EOQ + SS - \left(D_{\Sigma}^{\langle 0\rangle}\right)_1 \geq 0 \\
& SL_1 \leftarrow 0 \text{ otherwise}
\end{aligned}
$$

$$
\begin{array}{l}
\text{if } SL_1 \le ROP \\
\quad \left| \begin{array}{l} ORD_2 \leftarrow EOQ \\ N_{pr} \leftarrow N_{pr} + 1 \end{array} \right. \\
\text{for } i \in 2 .. N_{March11} - N_{Sept10} \\
\quad \left| \begin{array}{l}
ordered \leftarrow ordered - 1 \\
SL_i \leftarrow SL_{i-1} - \left(D_\Sigma^{\langle 0 \rangle}\right)_i + ORD_i \ \text{ if } SL_{i-1} - \left(D_\Sigma^{\langle 0 \rangle}\right)_i + ORD_i \ge 0 \\
SL_i \leftarrow 0 \ \text{ otherwise} \\
\text{if } \left(SL_i \le ROP\right) \wedge \left(i \le N_{March11} - N_{Sept10} - 2\right) \wedge (ordered \le 0) \\
\quad \left| \begin{array}{l}
ORD_{i+1} \leftarrow EOQ \ \text{ if } \left(D_\Sigma^{\langle 1 \rangle}\right)_{i+1} \ne 0 \\
\text{otherwise} \\
\quad \left| \begin{array}{l} ORD_{i+2} \leftarrow EOQ \ \text{ if } \left(D_\Sigma^{\langle 1 \rangle}\right)_{i+2} \ne 0 \\ ORD_{i+3} \leftarrow EOQ \ \text{ otherwise} \end{array} \right. \\
ordered \leftarrow 1 \\
N_{pr} \leftarrow N_{pr} + 1
\end{array} \right.
\end{array} \right. \\
SL_0 \leftarrow N_{pr} \\
SL
\end{array}
$$

$$N_{pr.cd.to.be.P} := 2 \cdot SL_{cd.P_0} = 54$$

$$AIL_{cd.to.be.P} := \frac{\displaystyle\sum_{i=1}^{N_{March11}-N_{Sept10}} SL_{cd.P_i}}{N_{March11} - N_{Sept10}} = 119548$$ - AIL



Calculating Total Costs for Cash Desk TO BE

$$ICC_{cd.to.be.P} := AIL_{cd.to.be.P} \cdot V \cdot phc = 4978$$

$$PC_{cd.to.be.P} := N_{pr.cd.to.be.P} \cdot upc = 2015$$

$$TC_{cd.to.be.P} := ICC_{cd.to.be.P} + PC_{cd.to.be.P} = 6993$$

TOTAL SAVINGS (for Past Demand)

$Save_{atm.icc.P} := ICC_{atm.as.is} - ICC_{atm.to.be.P} = -890$

$Save_{atm.pc.P} := PC_{atm.as.is} - PC_{atm.to.be.P} = 2129$

$Save_{atm.P} := TC_{atm.as.is} - TC_{atm.to.be.P} = 1239$

$Save_{cd.icc.P} := ICC_{cd.as.is} - ICC_{cd.to.be.P} = -1237$

$Save_{cd.pc.P} := PC_{cd.as.is} - PC_{cd.to.be.P} = 1717$

$Save_{cd.P} := TC_{cd.as.is} - TC_{cd.to.be.P} = 479$

$$Save_{atm.icc.\%.P} := \frac{Save_{atm.icc.P}}{ICC_{atm.as.is}} \cdot 100 = -81$$

$$Save_{atm.pc.\%.P} := \frac{Save_{atm.pc.P}}{PC_{atm.as.is}} \cdot 100 = 59$$

$$Save_{atm.\%.P} := \frac{Save_{atm.P}}{TC_{atm.as.is}} \cdot 100 = 26.22$$

$$Save_{cd.icc.\%.P} := \frac{Save_{cd.icc.P}}{ICC_{cd.as.is}} \cdot 100 = -33$$

$$Save_{cd.pc.\%.P} := \frac{Save_{cd.pc.P}}{PC_{cd.as.is}} \cdot 100 = 46$$

$$Save_{cd.\%.P} := \frac{Save_{cd.P}}{TC_{cd.as.is}} \cdot 100 = 6$$

$TC_{as.is} := TC_{atm.as.is} + TC_{cd.as.is} = 12197$

$TC_{to.be.P} := TC_{atm.to.be.P} + TC_{cd.to.be.P} = 10479$

$TOTAL\_SAVINGS_P := TC_{as.is} - TC_{to.be.P} = 1718$

$$TotSave\%_P := \frac{TOTAL\_SAVINGS_P}{TC_{as.is}} \cdot 100 = 14$$

TOTAL SAVINGS for AS IS vs. TO BE for Future Deamd

	As Is	To Be	Δ	Δ [%]
ATM	$ICC_{atm.as.is} = 1100$	$ICC_{atm.to.be} = 1714$	$Save_{atm.icc} = -614$	$Save_{atm.icc.\%} = -56$
	$PC_{atm.as.is} = 3625$	$PC_{atm.to.be} = 1496$	$Save_{atm.pc} = 2129$	$Save_{atm.pc.\%} = 59$
	$TC_{atm.as.is} = 4725$	$TC_{atm.to.be} = 3210$	$Save_{atm} = 1515$	$Save_{atm.\%} = 32$
Cash Desk	$ICC_{cd.as.is} = 3741$	$ICC_{cd.to.be} = 5049$	$Save_{cd.icc} = -1309$	$Save_{cd.icc.\%} = -35$
	$PC_{cd.as.is} = 3732$	$PC_{cd.to.be} = 1791$	$Save_{cd.pc} = 1941$	$Save_{cd.pc.\%} = 52$
	$TC_{cd.as.is} = 7473$	$TC_{cd.to.be} = 6841$	$Save_{cd} = 632$	$Save_{cd.\%} = 8$
Branch	$TC_{as.is} = 12197$	$TC_{to.be} = 10050$	$T\_SAVINGS = 2147$	$TotSave\% = 18$

TOTAL SAVINGS for AS IS vs. TO BE for Past Deamd

	As Is	To Be (Past Demand)	Δ (Past Demand)	Δ [%] (Past Demand)
ATM	$ICC_{atm.as.is} = 1100$	$ICC_{atm.to.be.P} = 1990$	$Save_{atm.icc.P} = -890$	$Save_{atm.icc.\%.P} = -81$
	$PC_{atm.as.is} = 3625$	$PC_{atm.to.be.P} = 1496$	$Save_{atm.pc.P} = 2129$	$Save_{atm.pc.\%.P} = 59$
	$TC_{atm.as.is} = 4725$	$TC_{atm.to.be.P} = 3486$	$Save_{atm.P} = 1239$	$Save_{atm.\%.P} = 26$
Cash Desk	$ICC_{cd.as.is} = 3741$	$ICC_{cd.to.be.P} = 4978$	$Save_{cd.icc.P} = -1237$	$Save_{cd.icc.\%.P} = -33$
	$PC_{cd.as.is} = 3732$	$PC_{cd.to.be.P} = 2015$	$Save_{cd.pc.P} = 1717$	$Save_{cd.pc.\%.P} = 46$
	$TC_{cd.as.is} = 7473$	$TC_{cd.to.be.P} = 6993$	$Save_{cd.P} = 479$	$Save_{cd.\%.P} = 6$
Branch	$TC_{as.is} = 12197$	$TC_{to.be.P} = 10479$	$TOTAL\_SAVINGS_P = 1718$	$TotSave\%_P = 14$

Prato_AG_1:	$TOTAL\_SAVINGS_p = 5338$	$TotSave\%_p = 34$	$TS_1 := 5338$	$TS\%_1 := 34$
Prato_AG_2:	SUPPLY>DEMAND		$TS_2 := 0$	$TS\%_2 := 0$
Prato_AG_3:	$TOTAL\_SAVINGS_p = 2962$	$TotSave\%_p = 19$	$TS_3 := 2962$	$TS\%_3 := 19$
Prato_AG_4:	$TOTAL\_SAVINGS_p = 360$	$TotSave\%_p = 3$	$TS_4 := 360$	$TS\%_4 := 3$
Prato_AG_5:	$TOTAL\_SAVINGS_p = 3089$	$TotSave\%_p = 24$	$TS_5 := 3089$	$TS\%_5 := 24$
Prato_AG_6:	SUPPLY>DEMAND		$TS_6 := 0$	$TS\%_6 := 0$
Prato_AG_7:	$TOTAL\_SAVINGS_p = 6044$	$TotSave\%_p = 29$	$TS_7 := 6044$	$TS\%_7 := 29$
Prato_AG_8:	$TOTAL\_SAVINGS_p = 1718$	$TotSave\%_p = 14$	$TS_8 := 1718$	$TS\%_8 := 14$
Prato_AG_9:	$TOTAL\_SAVINGS_p = 3530$	$TotSave\%_p = 30$	$TS_9 := 3530$	$TS\%_9 := 30$
Prato:	SUPPLY>DEMAND		$TS_{10} := 0$	$TS\%_{10} := 0$
Prato_AG_11:	SUPPLY>DEMAND		$TS_{11} := 0$	$TS\%_{11} := 0$

$$\text{AverageSaving} := \frac{\sum_{i=1}^{11} TS_i}{7} = 3292$$

- average Saving per branch

$$\text{AverageSaving\%} := \frac{\sum_{i=1}^{11} TS\%_i}{7} = 22$$

- average Percantage Saving per branch