CHAPTER 2

2.1 DAMAGE INDICES

2.1.1 Introduction

Damage indices are used to estimate the probability of damage within a defined member of a structure. While in new construction a possible damage description is not useful, due to the use of codes, in retrofitting it becomes essential in order to evaluate the right operations to do.
As can be seen in the next paragraphs there are two main ways to quantify damage: using the maximal, usually of deformations, or taking into account the accumulation of damage through hysteretic cycles. In this latter case there are many possibilities: it is possible to use deformation or energy time-history, a combination of them and the concept of low-cycle fatigue.
Usually damage indices are defined locally, that is relatively to each member or plastic hinge of a structure. It is then possible to take a weighted average of these measures in order to evaluate a global damage index for the entire structure. Since in this thesis only local damage is considered, these averages are not described.

2.1.2 Deformation-based indices

These are the simplest indices since they are computed from the maximal values obtained from time-history analysis. But this is also their main advantage because they are not able to take into account the accumulation of plastic deformations due to high number of cycles.

Inter-storey drift
The inter-storey drift is surely the most used damage index because of both its simple computation and the fact that it takes into account also nonstructural damage.

Ductility ratio
The term ductility refers to the ratio between the maximum deformation experienced by the member and its value at yielding. The ductility ratio can be defined in terms of rotation, displacement or curvature. The general formulation can be written as:

$$\mu_x = \frac{x_{max}}{x_{yield}}$$

where x can be rotations, displacements or curvatures.

2.1.3 *Cumulative deformation-based indices*

The description of the accumulation of damage under cyclic loading requires all the time-history data in order to get the deformation-history of the considered member.

Stephens and Yao (1987)
This index is an extension of the displacement ductility made in order to take into account cyclic loading. If inter-storey drifts time-history is considered, as shown in Figure 2.1, this index can be defined as:

$$D_{SY} = \sum \left(\frac{\delta^+}{\delta_f}\right)^{1-br}$$

Where δ_f is the value of the drift at failure for a monotonic load, it is recommended taking it as 10% of the storey height, r is the ratio between the maximum positive and negative drift and b is a constant, recommended value 0.77.

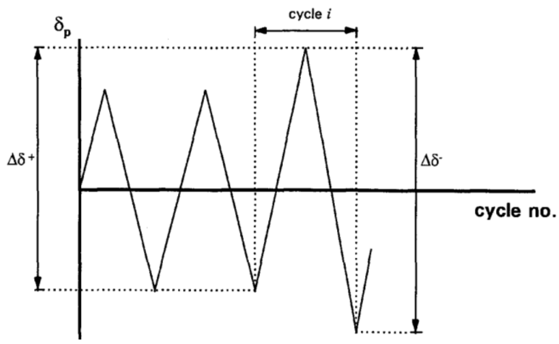


Figure 2. 1

Another way to model the accumulation of damage is through the concept of low-cycle fatigue. In case of civil structures usually large plastic deformations occur under seismic excitation and this model is suitable to describe such behavior.

Jeong and Iwan (1988)
Using Coffin-Manson low, the number of cycles n_f to reach failure at a given ductility μ is given by:

$$n_f \, \mu^s = c$$

The values of *c* and *s* proposed by the author are respectively 6 and 416. The damage index can then be found by the ratio between the number of cycles effectively experienced by the member and the number of cycles at failure:

$$D_{GI} = \frac{n_{eff}}{n_f} = \frac{n_{eff} \, \mu^s}{c}$$

Based on experiments it was showed that the adoption of the Coffin-Manson law is realistic for certain ranges of cycles numbers.

2.1.4 Energy-based indices

As cumulative deformation-based, this class of indices try to consider the effect of the accumulated damage through the hysteretic energy dissipation experienced during plastic behavior. Hereafter the most used of this index is presented.

Cosenza et al. (1993)

This index consists on the ratio between two quantities relative to the hysteretic energy dissipation under a certain seismic excitation and under monotonic load. This quantity is a kind of normalized energy with respect to the yielding condition and it is described with:

$$\mu_e = \frac{E_H}{F_y\, \delta_y} + 1$$

where F_y is the member force at yielding, δ_y its displacement and E_H the dissipated energy. The index is formulated in order to attain a zero value in case of elastic behavior, and a unity value in case of failure, which means in case the hysteretic energy accumulated during the ground motion is equal to the one under monotonic failure. It is written:

$$D_C = \frac{\mu_e - 1}{\mu_{e,mon} - 1}$$

It is to note that cycles in which deformations are small are not taken into account since no dissipation of energy is related with them.

2.1.5 Combined indices

An important group of damage indices takes into account both the energy dissipation under plastic behavior and the maximal deformation experienced by the member. A list with the most used indices is reported. Most of them are also implemented in *RUAUMOKO*.

Park and Ang (1985)

This probably the best known cumulative index and consists simply in a linear combination of deformation and energy indices regulated by factor β:

$$D_{PA} = \frac{\delta_{eff}}{\delta_{u\,mon}} + \beta \frac{E_H}{F_y\, \delta_{u\,mon}}$$

where F_y is the member force at yielding, $\delta_{u\,mon}$ its displacement under monotonic failure and E_H the dissipated energy. Recommended values of β are 0.1-0.5, but it is to note that regressions based on experimental data are not in agreement in the choice of this parameter and therefore there is an undesirable degree of arbitrariness. Firstly adopted for concrete elements, it is possible using this index for steel members as well by changing factor β.

Banion and Veneziano (1982)

Similarly to Cosenza et al, this index is the ratio between the value of one quantity under a seismic excitation and in case of monotonic failure. This time the quantity is composed by both ductility and energy based indices and is given by:

$$\delta_{BV} = \sqrt{\left(\frac{\delta_{eff}}{\delta_y} - 1\right)^2 + \left(a\left(2(\mu_e - 1)\right)^b\right)^2}$$

The authors recommend taking a=1.1 and b=0.38.

The global index thus becomes:

$$D_{BV} = \frac{\delta_{BV}}{\delta_{BV\ mon}}$$

Bracci et al. (1989)

A further development of the combined approach is defined by the concept of damage potential which is the total area between the monotonic load-deformation curve and the fatigue failure envelope. This value represents the maximum damage that an element can accumulate. With cyclic loading in fact the hysteretic loops experience both strength and stiffness degradation, until, at failure, they take a shape similar to the fatigue failure envelope. Figure 2. 2 graphically explain the concept.

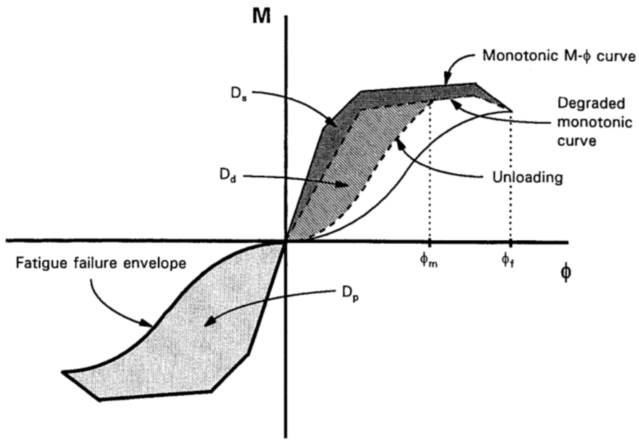


Figure 2. 2

Thus the index can be written as:

$$D_B = \frac{D_{loss}}{D_{pot}}$$

Where D_{loss} is the damage due to the effective degradation experienced by the member while D_{pot} is the maximum potential damage that can be achieved.

Despite of the complex definition of this index, experimental observations showed a spreading on numerical results higher than in the other combined indices (Williams and Sexsmith 1995).

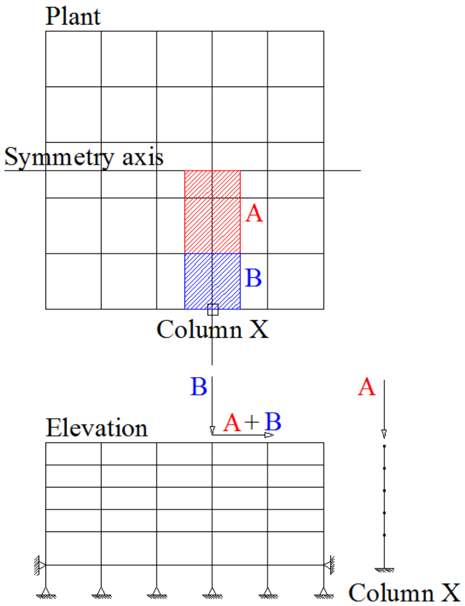
2.2 STRUCTURAL MODELING

RUAUMOKO2D has been chosen in order to model the structure and run inelastic analysis. The input is given using a text file containing the list of all the characteristics of the structure. The main advantages of such a software is the possibility to obtain a visual output and to run it iteratively updating the input file using, like in this work, *MATLAB*. Output is obtained from *.RES* file using a *DYNAPLOT* post-processor. In the next paragraphs the main concepts on the modeling of structure are briefly described.

2.2.1 Mass

In order to run a dynamic analysis is necessary to define the mass of the structure taken into account. Masses are used to compute the inertial effects of an earthquake and don't have influence, in the most part of the existing software, in the static analysis. For the static analysis is necessary to define apart static loads. Mass representation influences the number of degrees of freedom and consequently the computational effort for the analysis. The most used representation is the lumped one where mass is defined as concentrated in one representative node which has three degrees of freedom: two translational and one rotational. Usually in two-dimensional analysis only the translational components are taken into account. The rotational one is used for taking into account torsion effects in three dimensional analysis due to eccentricity between the centre of gravity and the center of rigidity of the same floor. For standard linear analysis the vertical dynamic degrees of freedom are usually not taken into account, except in case of wide bays, horizontal cantilevers or pushing structures.

In case of more detailed analysis where the vertical characterization of the structure is required problems can arise if the considered moment resisting frame is charged also of the weight of other frames which are not designed to withstand horizontal loads. In this case the mass considered for the vertical ground motion is the one relative to the gravity area of influence of the frame, instead for the horizontal loads the pertinent weight comprehend the one of the internal frames. For this reason in each nodal point three different masses, vertical and horizontal translational and rotational, can be set according to the direction in which they act. Finally, if geometrical nonlinearities are considered, also the vertical mass relative to the internal frames must be taken into account. This fact leads the adoption of an auxiliary gravity column endowed of negligible rigidity, whose lateral displacements are constrained to the ones of the main structure. The following figure graphically explain this concept.



For what concern the implementation in RUAUMOKO, the mass of the structure is input in the form of weights and internally converted by the software to mass unit by dividing it by the gravity acceleration, whose unit of measure is specified in the first lines of the model file. The mass is provided by nodal weights and includes both structural and non structural elements. For this reason material density and member weight/unit length are taken as zero. Nodal weights will contribute only to the diagonal terms of the mass matrix while rotational degrees of freedom are taken, in two-dimensional analysis, as zero. Hence a lamped mass matrix is used and the variable IPCONM in the first line of the input data takes the value zero. The diagonal matrix representation concerns elements endowed of weight and considers the contribution of the rotational degrees of freedom equals to the diagonal term of the consistent mass matrix of the member. Finally the consistent matrix representation is usually used for finite element models.

2.2.2 Geometric nonlinearities

Geometric nonlinear effects, also called P-Delta effects, are caused by gravity loading acting on the deformed configuration of the structure. As a consequence supplemental internal forces arise and the structure is pushed even further developing a second order deflection. Theoretically all structures experience this kind of loading, but practically it becomes important in case of slender structures subjected to lateral loads because of the higher magnitude of horizontal displacements.
These large lateral deflections, since they magnify the internal forces demand, cause a loss in the effective lateral stiffness bringing also longer natural periods and lower effective lateral strength.
It is possible to distinguish between P-δ effects, associated with the axial deformation of individual members, and P-Δ effects, associated with the deformations of the whole structure and measured between the ends of each member. This latter effect is more

significant in civil structures subjected to seismic excitation and it is usually related with inter-story drifts.
Then, if large displacements are considered, an iterative procedure must be adopted each time step in order to calculate the exact response of the structure. The coordinates of nodes and the stiffness matrix are updated every time step too and it makes this procedure computationally expansive. For this reason it is suggested to run large displacement analysis only in case the expected inter-story response of the structure exceeds significantly the 1% of the story height or in presence of large axial forces.
In presence of small lateral displacements however the problem can be linearized and the solution does not require iterations if the weight at each column is constant, that is, if the ground motion acts only horizontally. In this case P-Δ effects are considered modifying the stiffness matrix of the structure which affects both the static and dynamic response. As depicted in Figure 2. 3, for two dimensional systems it is possible to write the lateral added forces due to the overturning moment in case of the displacement located in a considered degree of freedom *i* as:

$$\begin{bmatrix} f_i \\ f_{i-1} \end{bmatrix} = \frac{w_i}{h_i} \begin{bmatrix} 1 \\ -1 \end{bmatrix} u_i = \boldsymbol{L}_i\, u_i$$

where w_i, h_i and u_i are respectively the weight, height and displacement of the *i-th* degree of freedom while f_i and f_{i-1} represent the additional lateral forces in the considered floor and in the adjacent one.

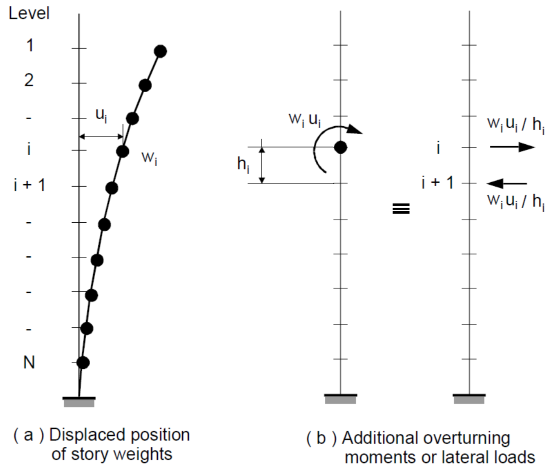


Figure 2. 3

If this procedure is followed for all the storey of the structure it is possible to add these relationships to the equilibrium equation leading:

$$\boldsymbol{K}\,\boldsymbol{u} = \boldsymbol{F} \quad \rightarrow \quad \boldsymbol{K}\,\boldsymbol{u} = \boldsymbol{F} + \boldsymbol{L}\,\boldsymbol{u}$$

The system thus becomes:

$$\widehat{\boldsymbol{K}}\,\boldsymbol{u} = \boldsymbol{F}$$

where:

$$\widehat{\boldsymbol{K}} = \boldsymbol{K} - \boldsymbol{L}$$

Although conceptually identical, the derivation of these equations for more general three dimensional structures is more difficult since rotational inertia and centroids at each floor must be considered.
In the present thesis this simplified approach is chosen since the final design with the added damping system leads to a low level of displacements.

2.2.3 Damping

There are two different source of damping in a structure: the inherent damping which is always present and it is due to complex internal mechanisms and the damping due to particular devices, in this case of study viscous dampers.

2.2.3.1 <u>Inherent damping</u>

The traditional damping model available in most time history program is the Rayleigh damping model. This is model gives a classical damping matrix, that is a matrix that satisfy the ortogonality property:

$$\boldsymbol{C} = \boldsymbol{\Phi}^T \boldsymbol{c}\, \boldsymbol{\Phi}$$

where C is the generalized damping matrix which is diagonal. This fact is very useful in case of modal analysis, where this property is requested in order to uncouple the equations of motions. In case of step-by- step analysis where this requirement is no more necessary, the proportional damping remains a very simple way to get the damping matrix, because it makes use of only matrices already computed during the analysis. It requires only the computation of the two coefficients α and β. It is left to the designer to estimate the two frequencies at which the required amount of damping is specified in order to evaluate the two coefficients..
Before considering the whole model is better to understand separately the meaning of proportionality to mass and stiffness matrices.

$$\boldsymbol{c} = \alpha\, \boldsymbol{m} \qquad \boldsymbol{c} = \beta\, \boldsymbol{k}$$

The most intuitive representation is the one proportional to stiffness, since it appeals intuitive that energy dissipation arises from story deformations. It is possible to relate the modal damping ratio to the coefficient β through the generalized damping of the *n-th* mode:

$$C_n = \beta\, K_n$$

where C_n and K_n are respectively the diagonal terms of the generalized damping and stiffness matrices of a classical damped system. Besides the matrix eigenvalue problem provides:

$$[-\omega_n^2\, \boldsymbol{m}\, \boldsymbol{\Phi}_n + \boldsymbol{k}\, \boldsymbol{\Phi}_n] q_n(t) = \boldsymbol{0}$$

which brings to:

$$\boldsymbol{k}\,\boldsymbol{\Phi}_n = \omega_n^2\,\boldsymbol{m}\,\boldsymbol{\Phi}_n$$

and premultiplying for the *n-th* modal shape vector:

$$\boldsymbol{\Phi}_n^T\boldsymbol{k}\,\boldsymbol{\Phi}_n = \omega_n^2\,\boldsymbol{\Phi}_n^T\boldsymbol{m}\,\boldsymbol{\Phi}_n$$
$$K_n = \omega_n^2\,M_n$$

Substituting this relationship in the first equation follows that:

$$C_n = \beta\,\omega_n^2\,M_n$$

Since the damping ratio of the *n-th* mode is related to the damping coefficient of the same *n-th* mode with:

$$\xi_n = \frac{C_n}{2\,\omega_n\,M_n}$$

it is possible to obtain:

$$\xi_n = \frac{\beta}{2}\,\omega_n$$

It could be noticed that the damping ratio increases linearly with the natural frequency. The coefficient β can be selected to obtain a specific value of damping ratio ξ_d in any one mode, say the *j-th* mode:

$$\beta = \frac{2\ \xi_d}{\omega_j}$$

With β determined, the damping matrix **c** is known and the damping ratio in any other mode is given by:

$$\xi_n = \frac{\beta}{2}\,\omega_n$$

The variations of modal damping ratios with natural frequency is shown in figure 2.4:

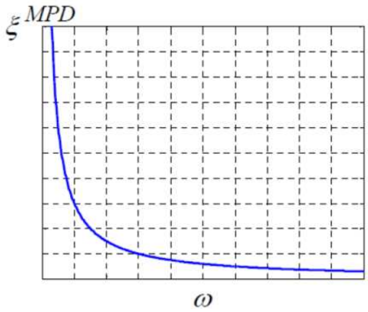


Figure 2. 4 Damping ratio variation with mass proportional damping

A similar procedure can be followed in order to obtain a relationship between the modal damping ratio and the coefficient α in the case of mass proportional damping matrix. Multiplying twice for the *n-th* mode shape vector the equation:

$$\boldsymbol{c} = \alpha\,\boldsymbol{m}$$

brings:

$$\boldsymbol{\Phi}_n^T\,\boldsymbol{c}\,\boldsymbol{\Phi}_n = \alpha\,\boldsymbol{\Phi}_n^T\,\boldsymbol{m}\,\boldsymbol{\Phi}_n$$
$$C_n = \alpha\,M_n$$

Knowing that:

$$\xi_n = \frac{C_n}{2\,\omega_n\,M_n}$$

it is possible to obtain the final relationship:

$$\xi_n = \frac{\alpha}{2\,\omega_n}$$

In this case the dependency from the frequency is inversely proportional as shown in figure 2.5:

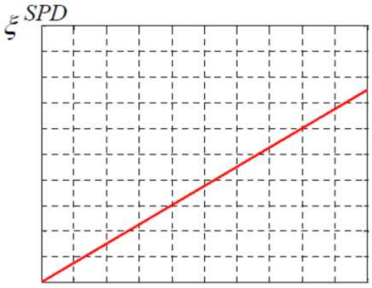


Figure 2. 5 Damping ratio variation with stiffness proportional damping

The variations of modal damping ratios with natural frequencies are not consistent with experimental data that indicate the same damping ratios for several vibration modes of a structure.

As first step toward constructing a classical damping matrix somewhat consistent with experimental data it is possible to combine the two proportionalities:

$$\boldsymbol{c} = \alpha\,\boldsymbol{m} + \beta\,\boldsymbol{k}$$

The damping ratio for the *n-th* mode of such a system is:

$$\xi_n = \frac{\alpha}{2\,\omega_n} + \frac{\beta}{2}\,\omega_n$$

The coefficients α and β can be determined from specified damping ratios ξ_{d1} and ξ_{d2} for the i-th and j-th vibration mode:

$$\frac{1}{2}\begin{bmatrix} \frac{1}{\omega_i} & \omega_i \\ \frac{1}{\omega_j} & \omega_j \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \xi_{d1} \\ \xi_{d2} \end{bmatrix}$$

As can be seen from figure 2.6 the double proportionality ensure a more constant trend of the values of the damping ratios.

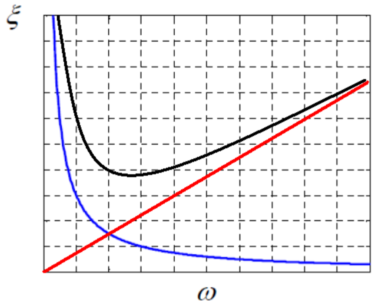


Figure 2. 6 Damping ratio variation with proportional damping model

In applying this procedure to a practical problem the modes *i* and *j* should be chosen to ensure reasonably constant values for the modal damping ratios contributing significantly to the response. Experience suggests specifying Rayleigh damping at the first mode and at a mode number equal to, or a little less than, the number of storey of the frame. On this topic it is also to note that modern dynamic analysis practice usually associates mass to all joints in the frame, including some mass associated with the rotational degrees of freedom. The reason is that the mass representation of the frame has a considerable effect on the response of the members when inelastic behavior takes place. This fact causes a considerable increase of independent dynamic degrees of freedom. With the increase in the fractions of critical damping with increasing frequency typical of the Rayleigh damping model, this brings that although the lower modes of free-vibration may have the order of 5% of critical damping, it is easy to get very large levels of damping in the higher modes. Due to the aim of the present work is the comparison between different methods of optimal design of dampers and not an over-detailed modeling of a structure, as said before, the mass of an entire floor is lumped in a singular joint. In this way a nine degrees of freedom system is obtained, making also simpler the computation of the relative stiffness matrix.

In general the sensitivity of the problem to the amount of damping is rather high. It could be observed in fact that even small amount of critical damping, i.e. 2-5%, reduces significantly the response of the structure.

This is true especially under plastic behavior, where with the formation of plastic hinges the structure looses part of its rigidity and changes its natural frequencies. Work by Crisp (Crisp 1980) in fact, showed that high levels of viscous damping in the high modes of free vibration of a structure has a marked effect on the inelastic response. This fact could be partially compensated using the current tangent stiffness matrix rather than the initial one. Although damping is not supposed to decrease as the structure goes inelastic, the fraction of critical damping tends to remain more constant as the stiffness of the structure reduces. It is to note moreover that, like damping, hysteresis rules and moment-curvature relationships supply a form of energy dissipation. However these hysteretic rules don’t account properly for the energy removed from the system by small cycle oscillations within the structure. And for this reason inherent damping is relevant also for inelastic analysis.

Alternative ways of modeling damping are available, for example the superposition of modal damping matrices and the Caughey damping. This latter representation is particularly suitable to match the required amount of damping at a greater number of modes. The main problem associated with this model is the large powers of the frequencies required to evaluate the coefficients. The algebraic equations in fact are numerically ill conditioned because the coefficients $\omega_n, \omega_n^3, \omega_n^5$ ... can differ by orders of magnitude. In *RUAUMOKO* there are three possible implementation of this model: linear, constant and trilinear damping according to the variation of the fraction of critical damping with frequency (see Figure 2. 7).

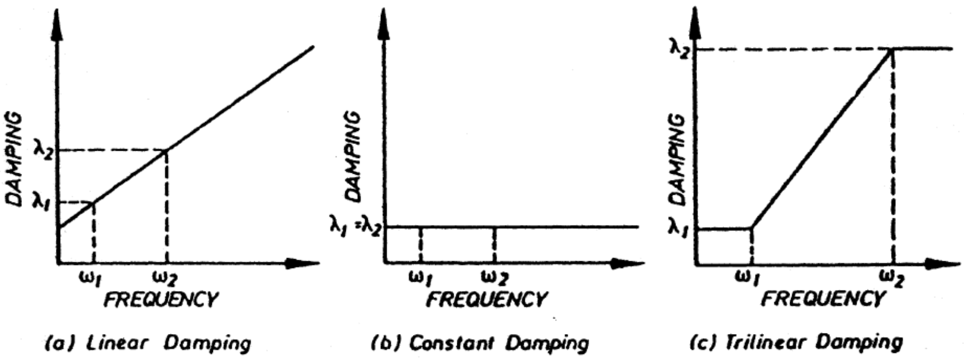


Figure 2. 7

Since the aim of this research is not, as already said, a detailed model of the structural behavior, but the comparison between different optimization schemes, it is found not useful using a too complex damping model. Different damping models can certainly modify the values of the results, but not the general distribution of them. It means that changing damping representation may bring to slightly different values of total added damping, but the distribution of the devices is supposed to be the same.

2.2.4 Stiffness

Usually seismic design is carried on with elastic analysis. In case of retrofitting with use of passive control tools a linear analysis could be accepted if it is designed in such a way not to achieve yielding. Although, in order to provide a better understanding of how these optimal design methods behave, inelastic analysis will also be executed and then compared to the elastic ones.

When elastic analysis is considered the characterization of the structure requires elastic characteristics like the modulus of elasticity of the material, the moment of inertia of sections, their areas…

As known nonlinear analysis involve significantly more effort to be performed and different grades of detailing can be achieved. In general structural member models can be differentiated by the way that plasticity is distributed through the member length. There are two main ways to model it: using distributed or concentrated plasticity. This latter way is the most simple because it concentrates, as the name itself suggests, the inelastic behavior at the two ends of the member. Next paragraphs explain the most used available models.

2.2.4.1 <u>One component model (Giberson plastic hinge)</u>

In the case of the Giberson hinge model rigid plastic hinges are placed at the two extremities where yielding is expected. The part of the member between the two rigid plastic springs remains perfectly elastic while all inelastic deformation is assumed to occur in these springs.

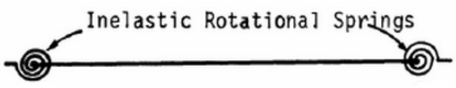


Figure 2. 8 One component model

The stiffness of the hinge is such that the rotation of the hinge together with the rotation associated with the elastic curvature of the beam over the hinge length is the same as the rotation associated with the curvature over the hinge length with the inelastic properties in the hinge zone. When the hinge is in the elastic range its stiffness is infinite.

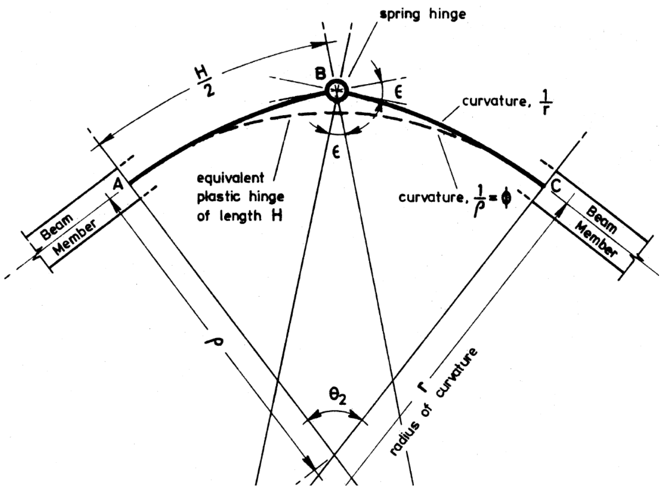


Figure 2. 9 Elastic and plastic rotations

A major advantage of the model is that inelastic member-end deformation depends solely on the moment acting at that end so that any moment-rotation hysteretic model can be assigned to the spring. This fact is also a weakness of the model because the member-end rotation should be dependent on the curvature distribution along the member, hence dependent on moments at both member ends.

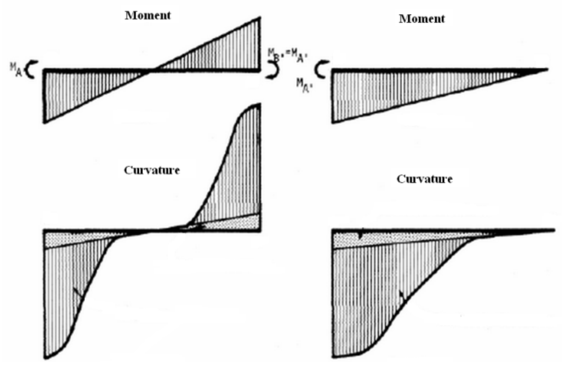


Figure 2. 10 Different moment distribution with relative curvature diagrams

Moreover the stiffness of an inelastic spring is normally defined by assuming an asymmetric moment distribution along a member with the inflection point at midspan. However, once yielding is developed at one member end, the moment at the other end must, moving the inflection point toward the member centre. At the same time, a large concentrated rotation starts near the critical section. For this reason this Giberson model is expected to be reasonably good for relatively low-rise frame structures, where inflection point of a column locates reasonably close to mid-height.
The formulation however, is numerically simple and efficient.

<u>*Other models*</u>

Several more complex models have been proposed in order to give a better understanding on the response of structures under seismic excitation.

For example using multi-component beams proposed by (Clough et al. 1965), that is a member composed by two or more elements in parallel. Each of these elements has a different behavior. In the two component model there are an elasto-plastic element, which represents yielding, in parallel with a completely elastic one which instead represents hardening. The main advantage is that rotations at one end depends on the moments at both ends. On the other hand to give a proper value to the stiffness of this multi component member is necessary to make some assumptions on the moment distribution along the element.

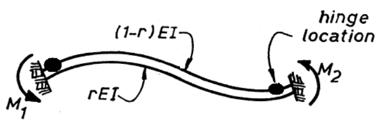


Figure 2. 11 Two component model

Another example is the distributed flexibility model proposed by (Takizawa 1973). The main observation at the base of this model is that where plasticization appears the flexibility increases. Since in under seismic excitation plasticization appears at the ends of the elements, a parabolic distribution of elastic flexibility is adopted. This parabolic shape has an inflection point in the middle length and interpolates the flexibility values at the edges. These latter values depend on a hysteretic model which takes into account the stress history. Despite of the parabolic distribution of flexibility this model may not represent adequately the concentration of deformation at the ends of the element.

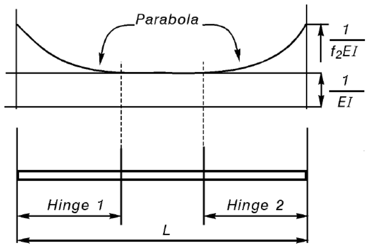


Figure 2. 12 Distributed flexibility model

In the present thesis the most simple one component member is considered since a more detailed description of the structure is not useful.

Hysteretic rules

After the choice of the type of element used to represent the bare frame structure, in this case the two plastic-hinges element, it is necessary to define their behavior in terms of resistance-deformation relationship, which is the so called hysteretic rule. Theoretically, between the several models of existing rules the one which better approximates the real behavior of the beam should be chosen. Real behavior means the behavior observed during experimental tests and includes many phenomena like stiffness and strength degradation, pinching, fracture, local buckling etc. Since for the purposes of this thesis a detailed modeling of the structure is not necessary the simplest hysteretic rule was chosen: the bi-linear model. Moreover it was shown by (Otani 1981) that the structural response for different hysteretic rules is not particularly sensitive to the details of these force-deformation relationships as much as to the basic characteristics of loops, like primary stiffness and fatness.

Bi-linear model was derived from the elastic-perfectly plastic one, assigning it a secondary stiffness after yielding in order to simulate the strain hardening characteristic of steel. Neither stiffness nor strength degradations are taken into account and the resulting model is not realistic.

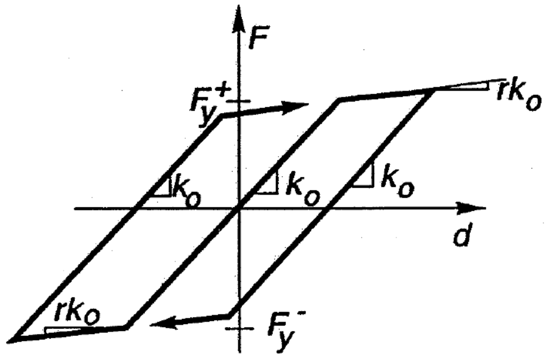


Figure 2. 13 Bilinear hysteretic rule

2.3 PASSIVE CONTROL DEVICES

2.3.1 Introduction

The so called structural control consists on reducing the vibrations induced by horizontal loads without stiffing the structure, or with a limited added stiffness. Vibrations in structures represent the source of both structural and nonstructural damage, therefore their control represents a way to ensure human safety. The applications of structural control concern both new construction and retrofitting. In case of new construction they are usually applied only to high performance buildings like hospitals and bridges. For new standard constructions the costs would be excessive and in general classical anti-seismic design based on ductile response. Retrofitting represents a great field of application for such devices because in many cases classical strengthening is not feasible or too expensive. In order to explain this concept let consider a single degree of freedom structure having a defined columns strength f_i which is not designed for horizontal loads.

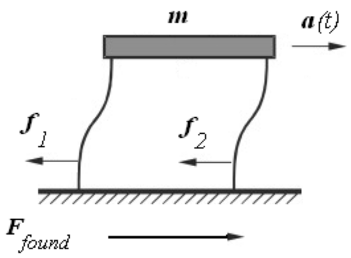


Figure 2. 14 SDOF system and foundations loading

Let consider now a horizontal ground motion which realistically brings the structure under plastic behavior. The maximum force that the foundations will experience is equal to the total resistance of the columns:

$$F_{found} = f_1 + f_2$$

If a strengthening strategy is adopted the force transferred to the foundations will be larger and it could be necessary their reinforcement. As known foundations retrofitting is an expensive task.
Generally increasing the resistance of the structure brings also a higher level of rigidity. As a result the period of the straightened structure is lower than the original one, and the seismic actions due to the same ground motion are stochastically higher. Hence absolute accelerations experienced on the modified structure will be higher. At the time of Northridge earthquake in 1994 there were several hospitals designed to withstand earthquakes mainly without inelastic deformations. In order to ensure elastic behavior a rigid and resistant structure is required. Due to the high ground acceleration of this particular earthquake these kind of buildings experienced such a high absolute acceleration that the nonstructural damage yields to their inoperability, though structural damage was not severe.
Generally the use of structural control avoids, or at least limits, high level of absolute accelerations and foundations reinforcement.
In the last three decades passive control of structures has known a high development and a lot of different methodologies are now available. The most famous of them are tuned mass dampers, base isolators and different types of mechanical devices.
All these technologies were firstly used to control only wind induced vibrations but now their field of application have widen to seismic control.
Passive control differs from active or semi-active one due to the absence of induced forces. In these latter systems in fact there is a constant monitoring of the behavior of the structure and, based on it, forces are applied by an actuating system. As a consequence, an active system requires a significant amount of external power in order to supply the requested internal forces. In case of seismic calamity this external source of power could be unavailable. Moreover these systems are expensive due to the presence of both monitoring devices and actuators which have to provide large amount of energy in small time lapse, and for this reason their applicability is restricted to special cases. Passive control systems are in comparison cheaper and, in general, more reliable.

2.3.2 Types of devices

A possibility to classify passive control devices is by the way they are activated. The activation can be by relative displacements or velocity between the edges of the device or by the motion induced on the structure by the seismic excitation.
A brief description of the most important energy dissipation systems is now presented in order to give a slight comparison between the different devices and to present the main advantages of viscous dampers which have been chosen in the present research.

Tuned mass dampers are not taken into account because represents a particular solution for high rise buildings and their interaction with the structure differs a lot from the other systems.

2.3.2.1 Hysteretic dampers

The first group of devices is displacement-activated and includes metallic and friction dampers. Metallic dampers dissipate energy through their deformations under plastic behavior instead friction dampers dissipation occurs with the friction developed at the interface of two sliding bodies. Both of them exhibit the same hysteretic behavior and therefore are modeled using an elastic-perfectly plastic load-displacement relationship. For this reason the verification must be carried out by inelastic analysis while approximated linearization method is available for pre-design.

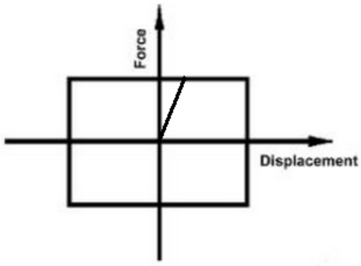


Figure 2. 15 Model of the hysteretic behavior of hysteretic dampers

The main disadvantage of such passive system consists on the added rigidity to the structure which can cause an increasing on the loading accelerations. This effect can be described showing the spectral response of a singular degree of freedom system subjected to a specific ground motion. Figure 2. 16 underlines the increasing of both rigidity and damping resulting in a lower displacement but in a higher acceleration.

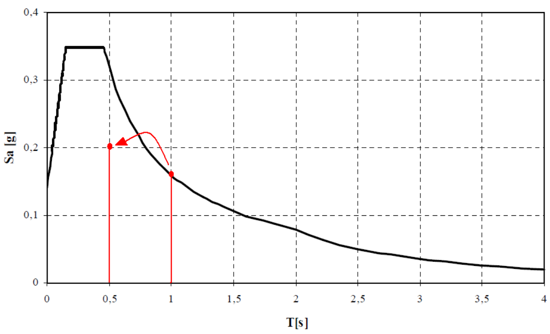


Figure 2. 16

Moreover a careful design of such passive control system is required in order to get a better behavior of the structure. Analyzing a singular degree of freedom system excited by a single wave, it can be shown that the structural response depends on the size of the device and in particular on the ratio between the equivalent static lateral displacement and the displacement that activates the hysteretic damper:

$$\Lambda = \frac{u_{static}}{u_{activ}}$$

For high values of the ratio Λ, that is for low activation displacement and so for undersized dampers, the response of the modified structure is similar to the original one with a slight reduction due to damping effect. On the other hand low values of that ratio make the structure changing its natural frequency and the spectral response shows another peak. It means that dampers are minimally activated and the retrofitted structure behaves like a braced one.

Hence, in order to avoid low efficiency or, even worst, additional stiffness, it is necessary to design these devices for an optimal value of the activation factor Λ. But this factor, and consequently the activation of the devices, depends on the intensity of the excitation. So if the optimal design of the hysteretic system refers to a required ground motion intensity, it is possible that for lower excitation the dampers are not activated and the resulting damage can be higher than the original structure.

In conclusion the design of this kind of passive control system requires high level of precision and accuracy.

2.3.2.2 Viscoelastic dampers

These devices dissipate energy through the shear deformation of a viscoelastic material. This kind of material exhibits both a viscous and an elastic response; therefore dampers are both displacement and velocity dependant and their hysteretic behavior can be modeled using a Kelvin solid with a spring and a dashpot in parallel. The resulting shear stress-deformation constitutive relationship is plotted in Figure 2. 17.

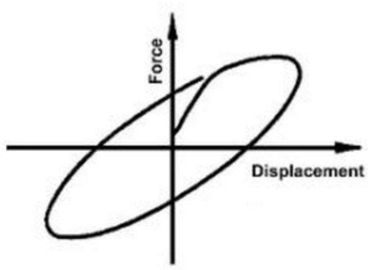


Figure 2. 17 Hysteretic behavior of viscoelastic dampers

As can be seen the constitutive relationship shows the presence of a stiffness component. As for hysteretic dampers the addition of rigidity to the structure is not a positive effect, but on the other hand the design of such devices does not present critical points like the search of an optimal activation ratio.

The representation of the effect of an added viscoelastic system for a singular degree of freedom system is similar to the one of metallic and friction devices and it is shown in Figure 2. 18.

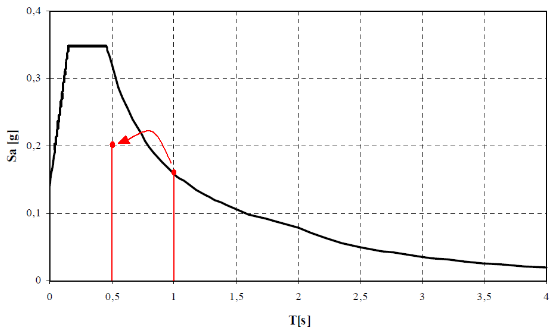


Figure 2. 18

Finally the efficiency of viscoelastic dampers is affected by the environmental condition of exposure and, in case of long ground motions, the high temperature decreases their dissipation capacity. For these reasons viscoelastic dampers are not used in common engineering practice despite of the significant research effort spent on it.

2.3.2.3 Viscous dampers

The energy dissipation occurs with the flow of some viscous fluid through orifices. The flowing is induced by a piston connected to the structure and excited by the ground motion. As better explained in chapter 2.4.3, there is no stiffness component in the response and the device is only velocity-activated. Figure 2. 19 underlines the lack of stiffness in the force-displacement constitutive relationship.

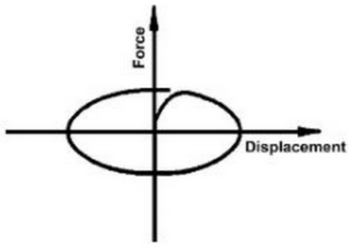


Figure 2. 19 Force-displacement relationship

Due to this characteristic the natural period of the bare frame is the same that the one of the retrofitted building. Therefore viscous damping effect only affects the response of the structure. Figure 2. 20 depicts the change in the response for a single degree of freedom system.

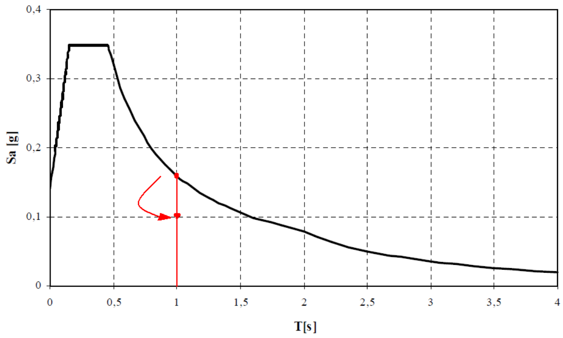


Figure 2. 20

This means that whatever the frequency content of the ground motion, the response will be deterministically lower than the original one. Hence risks related with the correct design of such devices are not so high like for hysteretic dampers.
Moreover, as better explained in chapter 2.4.3, as a result of the velocity dependency the forces due to the damping system are out of phase with the structural forces due to the seismic excitation which are, instead, displacement-dependant.
For these main advantages viscous dampers are now one of the most used passive control system and for this reason these devices have been considered in the present work, rather than the others explained before.

2.4 VISCOUS DAMPERS

2.4.1 Introduction

Firstly used in civil structures to reduce the oscillations due to wind (in the World Trade Center for example), now viscous dampers are commonly used also in seismic applications.
This chapter focuses on the inherent behavior of such devices and on their interaction with the structure in which are installed, giving a brief description on their manufacture.

2.4.2 The devices

Among the variety of energy dissipation devices for passive control of structures it was found that viscous dampers have a series of advantages.
Viscous dampers are composed by a stainless cylinder and piston and are filled with silicon oil. Energy dissipation occurs when silicon oil flows through special orifices, designed in order to provide a specific relation force-displacement. The force is produced

by the differential pressure across the piston head caused by the relative velocity at the edges of the device.
Figure 2. 21 depicts a typical fluid damper and its parts.

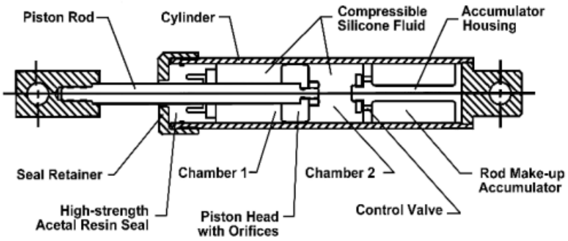


Figure 2. 21 Technical sheme of a viscous damper

Due to the compressibility of silicon oil, a restoring force can arise. Since the absence of added stiffness is one of the advantages of viscous dampers accumulator are used to avoid it. This restoring forces prevention is effective under a certain limit of excitation frequency depending on the geometrical design of the device.
Like other typologies of devices viscous dampers behavior is frequency dependent. This dependency is usually neglected in practical analysis because it is assumed that the device during a ground motion experiences an excitation based on the natural frequency of the structure. Although this assumption is true only for narrow banded systems, and consequently not always realistic for damped structures, the variation of damping coefficient with respect to frequencies is rather slight and so can be neglected.
Differently from other types of energy dissipation devices like for example yielding ones, viscous dampers are able to withstand several earthquakes without the necessity to be substituted.
Moreover the required level of maintenance is low and involves simple inspection of the condition of the device. Usually, visual inspection of the dampers should occur after important seismic events. This inspection consists on looking for eventual leakages or broken parts.

2.4.3 Hysteretic behavior

Fluid viscous dampers operate on the principle of fluid flow through orifices. A stainless steel piston travels through chambers that are filled with silicone oil. The pressure difference between the two chambers cause silicone oil to flow through an orifice in the piston head and seismic energy is transformed into heat, which dissipates into the atmosphere. The force/velocity relationship for this kind of damper can be characterized as following:

$$F = c_d\, \dot{u}^{\alpha}$$

where F is the output force, $\dot{u}$ the relative velocity across the damper, c_d is the damping coefficient and α is a constant exponent. Dampers with α=1 are called linear viscous

dampers in which the damper force is proportional to relative velocity. Devices with α smaller than 1 are the nonlinear viscous dampers used for seismic passive control. Finally, dampers with α larger than 1 give for high velocities really high reactions. These devices are used for so called lock-up applications, for example in bridges, where temperature deformations developed at very low velocity must be enabled, while for high velocity events, rigid behavior must be provided.
In the next paragraphs the description of the first two types of devices is provided.

2.4.3.1 Linear dampers

In order to better understand the behavior of such devices it is useful, as done in the frequency domain analysis, to analyze the response under a simple sinusoidal excitation. Considering the relative displacement at the edges of a pure viscous element in the form:

$$u(t) = U_0 \, sin(\omega t)$$

where U_0 is the maximum displacement amplitude and ω is the circular forcing frequency. The linear damper reaction force is proportional to the relative velocity:

$$F(t) = c_d \, \dot{u}(t)$$

Due to:

$$\dot{u}(t) = \omega \, U_0 \, cos(\omega t)$$

one obtains:

$$F(t) = c_d \, \omega \, U_0 \, cos(\omega t)$$

It can be observed that for a sinusoidal loading the reaction of the device is in counter phase with the displacements. This out of phase response is generally valid also for more complex loading because it can be assumed that when the displacement achieve its maximum value the velocity is equal zero and vice versa. This means that viscous damping systems generate their maximum forces when the structural system is at its minimum displacement, that is, under its minimum solicitation. Hence the forces in the columns due to the action of the damping system do not increase and foundations do not require expensive works of strengthening.

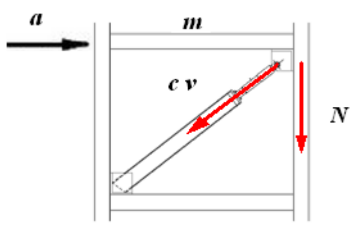


Figure 2. 22 Column forces due to dampers

If now the following basic trigonometric formula:

$$cos(\omega t) = \pm\sqrt{1 - sin^2(\omega t)}$$

is substituted in the previous relationship, one obtains:

$$F(t) = \pm \, c_d \, \omega \, U_0 \, \sqrt{1 - sin^2(\omega t)}$$

$$F(t) = \pm\, c_d\, \omega\, \sqrt{{U_0}^2 - u(t)^2}$$

which can be rewritten as:

$$\frac{F(t)}{c_d\, \omega\, U_0} = \pm \sqrt{1 - \left(\frac{u(t)}{U_0}\right)^2}$$

Hence a relationship between normalized force and displacement is described and can be plotted:

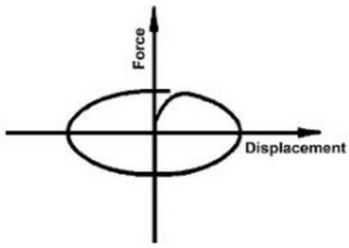


Figure 2. 23 Force-displacement relationship for viscous dampers

Integrating the hysteresis loop it is possible to estimate the energy dissipated by the devices per cycle:

$$E_d = \int_0^{2\pi/\omega} F(t)\, \dot{u}(t)\, dt = c_d\, \omega^2\, {U_0}^2 \int_0^{2\pi/\omega} cos^2(\omega t)\, dt = \pi\, c_d\, \omega\, {U_0}^2$$

As can be seen the energy dissipated per cycle is linearly proportional to damping coefficient and the excitation frequency while it is proportional to the square of the displacement amplitude.

2.4.3.2 Nonlinear dampers

As seen before the relation between force and velocity for a nonlinear damper looks like:

$$F(t) = c_d\, sing\big(\dot{u}(t)\big)\, |\dot{u}(t)|^{\alpha}$$

where the function “sign” and the absolute value are introduced to underline that forces act in the opposite direction of the velocities.

As can be seen from Figure 2. 24, the important advantage of using nonlinear viscous dampers is that forces don’t increase significantly in case of high velocities.

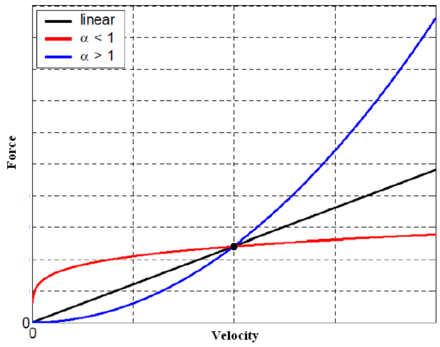


Figure 2. 24 Force-velocity relationship for linear and nonlinear

As done before a sinusoidal excitation is considered:

$$u(t) = U_0 \, sin(\omega t)$$

Substituting its derivative in the nonlinear relationship brings:

$$F(t) = c_d \, sing(cos(\omega t)) \, |\omega \, U_0 \, cos(\omega t)|^{\alpha}$$

Also in the present case the out of phase response behavior of the device can be noted. With the same trigonometric rule:

$$cos(\omega t) = \pm\sqrt{1 - sin^2(\omega t)}$$

one can obtain:

$$\frac{F(t)}{c_d \, (\omega \, U_0)^{\alpha}} = \pm \left(1 - \left(\frac{u(t)}{U_0}\right)^2\right)^{\frac{\alpha}{2}}$$

Hysteresis loops can be plotted for different values of coefficient α. It is clear that in case α=1 linear behavior is obtained. In case α=0 the forces approach a constant value:

$$\frac{F(t)}{c_d \, (\omega \, U_0)^{\alpha}} = \pm \, 1$$

which means that the hysteretic loop becomes a rectangle.

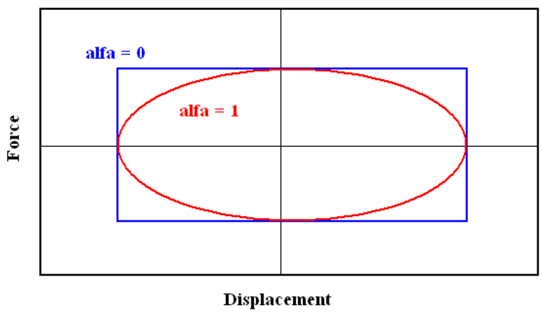


Figure 2. 25 Force-displacement relationship

In order to get a relationship between linear and nonlinear dampers it is useful to calculate the amount of energy dissipated per cycle. As done before, energy is the integral of the force-displacement relationship:

$$E_d = \int_0^{2\pi/\omega} F(t)\,\dot{u}(t)\,dt = 4c_d\,(\omega\,U_0)^{\alpha+1} \int_0^{2\pi/\omega} cos^{\alpha+1}(\omega t)\,dt$$

The solution of this integer require the introduction of the gamma function Γ.
Hence it can be written:

$$E_d = 4c_d\,(\omega\,U_0)^{\alpha+1}\frac{\sqrt{\pi}}{2\omega}\frac{\Gamma(1+\alpha/2)}{\Gamma(3/2+\alpha/2)}$$

2.4.4 Dynamic analysis

Dynamic analysis of systems which incorporate viscous dampers doesn't present particular complexities. Only for multi degrees of freedom systems there is some restriction as explained hereafter.
Considering firstly a single degree of freedom system equipped with an horizontal viscous damper the equation of motion can be written as:

$$m\,\ddot{u}(t) + c\,\dot{u}(t) + k\,u(t) + F_d(t) = -m\,\ddot{u}_g(t)$$

where F_d is the horizontal force provided by the device. As seen before, if the mass and stiffness of the damper can be neglected, this force is equal to:

$$F_d(t) = c_d\,\dot{u}(t)$$

Substituting it in the equation of motion brings:

$$m\,\ddot{u}(t) + (c + c_d)\,\dot{u}(t) + k\,u(t) = -m\,\ddot{u}_g(t)$$

Classical analysis can be performed in order to evaluate the response of the system.
A different approach must be taken with multi degree of freedom system.
If the same approach is carried on with multi degree of freedom systems a similar result is obtained. Considering the equations of motion:

$$\boldsymbol{m}\,\ddot{\boldsymbol{u}}(t) + \boldsymbol{c}\,\dot{\boldsymbol{u}}(t) + \boldsymbol{k}\,\boldsymbol{u}(t) + \boldsymbol{F_d}(t) = -\boldsymbol{m}\,\ddot{u}_g(t)$$

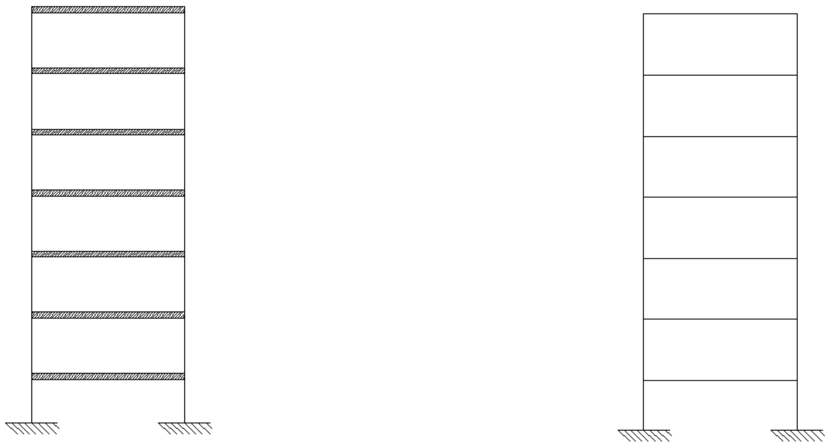
where the force vector is given by:

$$\boldsymbol{F_d}(t) = \boldsymbol{c_d}\,\dot{\boldsymbol{u}}(t)$$

The substitution brings:

$$\boldsymbol{m}\,\ddot{\boldsymbol{u}}(t) + (\boldsymbol{c} + \boldsymbol{c_d})\,\dot{\boldsymbol{u}}(t) + \boldsymbol{k}\,\boldsymbol{u}(t) = -\boldsymbol{m}\,\ddot{u}_g(t)$$

In usual practice damping matrix $\boldsymbol{c}$ is computed in such a way that it has the same ortogonality properties of mass and stiffness matrices, for example in the Rayleigh model. For this reason classical modal analysis can be used in the analysis. In case of added damping system it is not said that the damping matrix in modal coordinate C, obtained from the equivalent damping matrix $(\boldsymbol{c} + \boldsymbol{c_d})$, is diagonal. In this case algorithm for non-classical damping must be adopted. For two-dimensional systems matrix $\boldsymbol{c_d}$ is a tridiagonal one regardless if the structure is shear type or not. As can be seen from the figure below and considering the displacement method, for the bare frame the assumption that floors are taken as rigid body not only in their plane but also transversally brings to a tridiagonal matrix because a singular unit displacement applied at one dynamic degree of freedom has influence only on the neighbor degrees of freedom. Instead for general structures, where transverse rotations are permitted, the unit displacement produces reactions at every floor. Since dampers forces depend on the relative velocity between the edges of the device the chosen model of building doesn't affect the form of the matrix.



$$\begin{bmatrix} c_{d\,1}+c_{d\,2} & -c_{d\,2} & 0 & 0 & 0 & 0 \\ -c_{d\,2} & c_{d\,2}+c_{d\,3} & -c_{d\,3} & 0 & 0 & 0 \\ 0 & -c_{d\,3} & c_{d\,4}+c_{d\,3} & -c_{d\,4} & 0 & 0 \\ 0 & 0 & -c_{d\,4} & c_{d\,4}+c_{d\,5} & -c_{d\,5} & 0 \\ 0 & 0 & 0 & -c_{d\,5} & c_{d\,5}+c_{d\,6} & -c_{d\,6} \\ 0 & 0 & 0 & 0 & -c_{d\,6} & c_{d\,6} \end{bmatrix} \qquad \begin{bmatrix} c_{d\,11} & c_{d\,21} & c_{d\,31} & c_{d\,41} & c_{d\,51} & c_{d\,61} \\ c_{d\,12} & c_{d\,22} & c_{d\,32} & c_{d\,42} & c_{d\,52} & c_{d\,62} \\ c_{d\,13} & c_{d\,23} & c_{d\,33} & c_{d\,43} & c_{d\,53} & c_{d\,63} \\ c_{d\,14} & c_{d\,24} & c_{d\,34} & c_{d\,44} & c_{d\,54} & c_{d\,64} \\ c_{d\,15} & c_{d\,25} & c_{d\,35} & c_{d\,45} & c_{d\,55} & c_{d\,65} \\ c_{d\,16} & c_{d\,26} & c_{d\,36} & c_{d\,46} & c_{d\,56} & c_{d\,66} \end{bmatrix}$$

Figure 2. 26 Added damping matrix for shear type and usual buildings

Always considering two-dimensional systems with only translational dynamic degrees of freedom, $\boldsymbol{c_d}$ can be easily derived from the vector containing dampers coefficients with the inverse of the same matrix $\boldsymbol{T}$ used to compute relative measures from absolute ones. Taking the first degree of freedom as the first floor the inverse of matrix $\boldsymbol{T}$ can be written as:

$$\boldsymbol{T}_{fr} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Since the product of this matrix for its transpose gives the same distribution needed for the tridiagonal damping matrix:

$$(\boldsymbol{T}_{fr})^{\boldsymbol{T}}(\boldsymbol{T}_{fr}) = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

it is possible to allocate the elements of the dampers vector $[c_{d\,1}\, c_{d\,2}\, c_{d\,2}]^T$ in the following way:

$$(\boldsymbol{T}_{fr})^{\boldsymbol{T}}\, \boldsymbol{diag}([c_{d\,1}\, c_{d\,2}\, c_{d\,2}]^T)(\boldsymbol{T}_{fr}) = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{d\,1} & 0 & 0 \\ 0 & c_{d\,2} & 0 \\ 0 & 0 & c_{d\,3} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{d\,1}+c_{d\,2} & -c_{d\,2} & 0 \\ -c_{d\,2} & c_{d\,2}+c_{d\,3} & -c_{d\,3} \\ 0 & -c_{d\,3} & c_{d\,3} \end{bmatrix}$$

The same procedure can be applied to tri-dimensional systems but the computation of the inverse of matrix $\boldsymbol{T}$ is more complex due to the presence of rotational degrees of freedom. Consider one single floor of a 3D structure with dampers allocated in the peripheral frame as in Figure 2. 27. Matrix for plane transformation $\boldsymbol{T_{pl}}$ can be seen as the composition of the vectors whose components $\boldsymbol{T}_j^{(k)}$ are defined as the relative velocity activating damper number *k* caused by a unit velocity at the *j-th* degree of freedom. In the case of the structure shown below these vectors are:

$$\boldsymbol{T}_{pl}^{(1)} = \begin{bmatrix} 0 \\ 1 \\ a \end{bmatrix} \quad \boldsymbol{T}_{pl}^{(2)} = \begin{bmatrix} 0 \\ 1 \\ -a \end{bmatrix} \quad \boldsymbol{T}_{pl}^{(3)} = \begin{bmatrix} 1 \\ 0 \\ -b \end{bmatrix} \quad \boldsymbol{T}_{pl}^{(4)} = \begin{bmatrix} 1 \\ 0 \\ b \end{bmatrix}$$

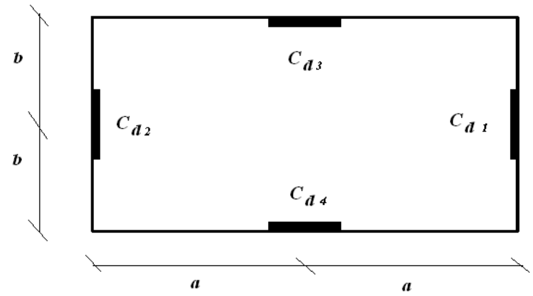


Figure 2. 27 Single story frame

The composition of these vectors gives the transformation matrix for plane systems:

$$\boldsymbol{T}_{pl} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ a & -a & -b & b \end{bmatrix}$$

If now a more general three-dimensional system is considered, the matrix for the transformation from displacements to inter-story drifts is given by the superposition of the two different transformations explained before. Firstly displacements are transformed to inter-story drifts using matrix $\boldsymbol{T}_{fr}$, then with $\boldsymbol{T}_{pl}$ the inter-story drift relative to each damper in the same floor is computed. In order to do that all the devices must be numbered from the first to the last floor. The general transformation to obtain the dampers matrix is:

$$\left(\boldsymbol{T}_{fr}\boldsymbol{T}_{pl}\right)^{\boldsymbol{T}} \boldsymbol{diag}([c_{d\,1}\; c_{d\,2}\; c_{d\,3}\; c_{d\,4}]^T)\left(\boldsymbol{T}_{fr}\boldsymbol{T}_{pl}\right) =$$

$$= \left(\begin{bmatrix} \boldsymbol{T}_{fr} & 0 & 0 \\ 0 & \boldsymbol{T}_{fr} & 0 \\ 0 & 0 & \boldsymbol{T}_{fr} \end{bmatrix} \begin{bmatrix} \boldsymbol{T}_{pl}^{(1)} \\ \boldsymbol{T}_{pl}^{(1)} \\ \boldsymbol{T}_{pl}^{(1)} \end{bmatrix}\right)^T \begin{bmatrix} c_{d\,1} & 0 & 0 & 0 \\ 0 & c_{d\,2} & 0 & 0 \\ 0 & 0 & c_{d\,3} & 0 \\ 0 & 0 & 0 & c_{d\,4} \end{bmatrix} \begin{bmatrix} \boldsymbol{T}_{fr} & 0 & 0 \\ 0 & \boldsymbol{T}_{fr} & 0 \\ 0 & 0 & \boldsymbol{T}_{fr} \end{bmatrix} \begin{bmatrix} \boldsymbol{T}_{pl}^{(1)} \\ \boldsymbol{T}_{pl}^{(1)} \\ \boldsymbol{T}_{pl}^{(1)} \end{bmatrix}$$

where $\boldsymbol{T}_{pl}^{(i)}$ represents the plane transformation matrix for the *i-th* floor.

2.4.4.1 Linearization of nonlinear viscous dampers

For design purposes it is useful to find an approximated relationship which allows to treat nonlinear viscous damping as linear ones. It is possible to relate the two different damping coefficients comparing their energy dissipation per cycle:

$$E_{d\,LIN} = \pi\, c_{d\,LIN}\, \omega\, {U_0}^2$$

$$E_{d\,NONL} = 4c_{d\,NONL}\, (\omega\, U_0)^{\alpha+1} \frac{\sqrt{\pi}}{2\omega} \frac{\Gamma(1+\alpha/2)}{\Gamma(3/2+\alpha/2)}$$

Equating the two energy formulation brings:

$$\frac{c_{d\,NONL}}{c_{d\,LIN}} = (\omega\, U_0)^{1-\alpha}\, \frac{\sqrt{\pi}}{2}\, \frac{\Gamma(1+\alpha/2)}{\Gamma(3/2+\alpha/2)}$$

Typical range of values for α is [0.2;1]. For these values the ratio of the gamma functions is close to unit and the relationship can be rewritten as:

$$\frac{c_{d\,NONL}}{c_{d\,LIN}} \approx (\omega\, U_0)^{1-\alpha}\, \frac{\sqrt{\pi}}{2}$$

Hence results obtained from design using linear dampers can be adapted to use nonlinear ones. The excitation frequency ω can be chosen, for narrow banded systems, as the natural frequency of the structure and U_0 can be taken as the displacement in the dampers corresponding to a desired performance drift level.

2.4.5 Classical design of damping systems

In the present paragraph the traditional pre-design method of viscous dampers, called stiffness proportional, is explained in order to get a comparison between old practice and new optimization methods.

2.4.5.1 Modal damping ratio for damped systems

Although damped multi degrees of freedom systems are generally characterized by non-classical damping, in order to get a measure of the damping ratio achieved with retrofitting, it is useful to neglect off diagonal terms of modal damping matrix and apply modal analysis rules. In this case the damping ratio for each mode can be evaluated knowing that:

$$C_i = \boldsymbol{\Phi}_{(i)}^T \boldsymbol{c}\, \boldsymbol{\Phi}_{(i)}$$

$$M_i = \boldsymbol{\Phi}_{(i)}^T \boldsymbol{m}\, \boldsymbol{\Phi}_{(i)}$$

and substituting it into:

$$\xi_i = \frac{C_i}{2\ \omega_i\ M_i}$$

one obtains:

$$\xi_i = \frac{\boldsymbol{\Phi}_{(i)}^T \boldsymbol{c}\ \boldsymbol{\Phi}_{(i)}}{2\ \omega_i \boldsymbol{\Phi}_{(i)}^T \boldsymbol{m}\ \boldsymbol{\Phi}_{(i)}}$$

This is a quite important relationship because it estimates a reference value for a given amount of added damping. Usually the first vibration mode is taken as representative of the structure.

2.4.5.2 Required modal damping ratio and total added damping

In most of the methods used to design viscous damping systems the value of objective damping needs to be specified. This fact is a limitation in case of optimal design, because it means that the amount of added damping could be more than the necessary and only its distribution is optimally designed.

The first step is the evaluation of the required damping ratio. The simplest way to get a reasonable value for this parameter consists on running time history analysis on the considered structure, increasing the value of the damping ratio used to compose the damping matrix, until the value of some displacements achieves the desired one. In practice a maximum damping ratio of about 40% of critical can be achieved economically with currently available devices. More than one ground motion must be selected in order to obtain credible values. If the design refers to a determined code, ground motions can be derived from the design spectrum relative to the structure. Since damping matrix is commonly composed using proportional models, like Raleigh or Caughey ones, the value of the required damping ratio obtained by this way is realistic if dampers are placed proportionally to stiffness. In this case in fact the damping matrix obtained from the superposition of the inherent damping and of the devices damping is similar to the one obtained from the proportionality models. As dampers are distributed in different ways, typically in case of optimal design, the response of the two systems can be slightly different. However, since the optimally designed system gives the better response, the use of this method to find the requested amount of damping is safety favor.

Once the required damping ratio is found, it is possible to estimate its relative value of damping using energy consumption concepts. Consider firstly a singular degree of freedom system endowed of a viscous damper. The energy dissipated by the damper in one cycle of harmonic excitation is given by:

$$E_d = \int_0^{2\pi/\omega} F(t)\,\dot{u}(t)\,dt = \int_0^{2\pi/\omega} \left(c_d\,\dot{u}(t)\right)\dot{u}(t)\,dt = c_d \int_0^{2\pi/\omega} \dot{u}^2(t)\,dt =$$

$$= c_d \int_0^{2\pi/\omega} \left(U_0\,\omega\cos(\omega t - \Phi)\right)^2 dt = c_d\,\pi\,U_0^2\,\omega = 2\pi\,\xi\,\frac{\omega}{\omega_n} k\,U_0^2$$

where:

$$u(t) = U_0\,sin\,(\omega t - \Phi)$$

The input energy due to seismic action for one cycle is the integral:

$$E_{input} = \int_0^{2\pi/\omega} m\,\ddot{u}_g(t)\,du$$

It can be demonstrated that for steady-state vibrations this latter integral is equal to the dissipated damping energy. In other words the input energy is dissipated by damping. Hence it can be written:

$$E_{input} = 2\pi\,\xi\,\frac{\omega}{\omega_n} k\,U_0^2$$

Knowing that the general form of strain energy is:

$$E_s = \int k\,u\,du = \frac{1}{2} k\,U_0^2$$

Equating the input energy to the dissipated damping energy brings to an equivalent damping ratio:

$$4\pi\,\xi_{eq}\,\frac{\omega}{\omega_n} E_s = E_d$$

The equivalent damping ratio is then:

$$\xi_{eq} = \frac{1}{4\pi}\frac{1}{\frac{\omega}{\omega_n}}\frac{E_d}{E_s}$$

Due to response of narrow-banded systems is characterized by the predominant frequency, the ratio ω/ω_n can be neglected. In this way the resulting damping ratio is no more rate dependant as experimental observations show.

$$\xi_{eq} = \frac{1}{4\pi}\frac{E_d}{E_s}$$

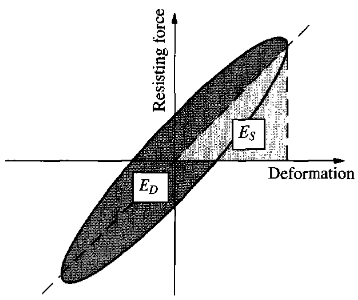


Figure 2. 28 Graphical definition of the energy loss Ed and the strain energy for a cycle of harmonic vibration

As seen in paragraph 2.4.3, the energy dissipated by a single viscous damper is:

$$E_d = \pi \, c_d \, \omega \, {U_0}^2$$

Considering now a multi degree of freedom system endowed of equal viscous dampers, the total energy dissipated is the sum of the amounts dissipated by each device:

$$E_{d\,tot} = \sum E_{d\,i} = \pi \, c_d \, \omega \sum c_{d\,i} \, {\delta_i}^2$$

Note that inter-story drifts are used instead of displacement because dampers response depends on the velocities relative to their edges. Moreover with geometrical amplification a suitable magnification factor must be introduced. A simpler form for this latter equation can be found if displacements are approximated as a straight line, which can be reasonable if the first mode of vibration is the dominant one. In this case in fact the inter-story drift can be described as:

$$\delta_i = \frac{1}{n}$$

where *n* is the number of story of the building.
Then the simplified relationship becomes:

$$E_{d\,tot} = n_d \, \pi \, \omega \, \frac{1}{n^2} \sum c_{d\,i}$$

where n_d is the number of devices.
On the other side the strain energy for a multi degree of freedom system is given by:

$$E_s = \sum_i \frac{1}{2} \, k_i \, \delta_u^2 = \frac{1}{2\,n^2} \sum_i \, k_i$$

Substituting these relationships in the equation of the equivalent damping ratio brings:

$$\xi_{eq} = \frac{1}{4\pi} \frac{E_d}{E_s} = \frac{n_d \, \omega}{2} \frac{\sum c_{d\,i}}{\sum_i \, k_i}$$

which provides the approximated relationship between the total added damping $\sum c_{d\,i}$ and the requested damping ratio ξ_{req}. Formally:

$$\sum c_{d\,i} = \frac{2\,\xi_{eq}}{n_d \, \omega} \sum_i \, k_i$$

2.4.5.3 General consideration on the optimal location of dampers

As for the general seismic design, also the location of damping devices within a structure must follow the criteria of regularity. Their installation in fact causes the presence of additional forces and if their placement is not symmetrical, torsion effects can arise. For this reason dampers must be located in general symmetrically. Furthermore, in order to reduce torsion effects due to accidental eccentricity is better to choose the external frames.
The effects of added damping in irregular structures were studied by many researchers. (Goel 2001) suggested for a single story asymmetric building a disposition of dampers

such that the resulting center of added damping has an equal but symmetrical eccentricity as the rigidity-center with respect to the mass-center. In fact the inertial forces due to the seismic excitation act on the mass-center while, in presence of irregularities, the structural reactions act on the rigidity-center. This difference on the acting lines generates torsion effects. If a damping system must be introduced, it is better to install it in such a way that its reaction forces balance the existing eccentricity.

2.4.5.4 <u>Stiffness proportional method</u>

This method is based on the assumption that since dampers are inserted using bracing systems in between neighbor floors, the damping matrix relative to the added damping is proportional to the stiffness matrix of the structure.
Hence it can be written:

$$\boldsymbol{c_d} = a\,\boldsymbol{k}$$

where a is the proportionality constant.
Using modal analysis the modal damping coefficient in the i-th mode of vibration can be computed as:

$$C_{d\,i} = \boldsymbol{\Phi}_{(i)}^{T}\boldsymbol{c_d}\,\boldsymbol{\Phi}_{(i)} = \boldsymbol{\Phi}_{(i)}^{T}\,a\,\boldsymbol{k}\,\boldsymbol{\Phi}_{(i)} = a\,K_i$$

On the other hand the same modal damping coefficient can be estimate as:

$$C_{d\,i} = 2\,\omega_i\,\xi_i\boldsymbol{\Phi}_{(i)}^{T}\boldsymbol{m}\,\boldsymbol{\Phi}_{(i)} = 2\,\omega_i\,\xi_i\,M_i$$

Knowing that $K_i = {\omega_i}^2 M_i$ and combining the two equations brings:

$$a = \frac{2\xi_i}{\omega_i}$$

Hence the present design process is simple. Once a desired viscous damping ratio is chosen the proportionality constant is computed and so the damping matrix. In case of complex structural systems an approximation of the stiffness matrix, composed only by the lateral stiffness of each floor, can be used.
The idea that the distribution of dampers should be proportional to the lateral stiffness of the structure presents some limit. In case of weak floors at the base of the building for example, the viscous damping added to this floor results less than the one added to the upper levels because of its lower lateral stiffness. Instead it is obvious that weak floors need a higher amount of damping.
In conclusion this method can be used for pre-design in presence of regular structures, but more accurate dynamic analysis must be carried on in order to evaluate eventual deficiencies of the system.

2.4.6 *Geometrical amplification*

The geometrical disposition of viscous dampers between two neighbor floors influences their own efficiency. Consider firstly the two most common ways to install these devices: the diagonal and chevron brace configurations showed in Figure 2. 29.

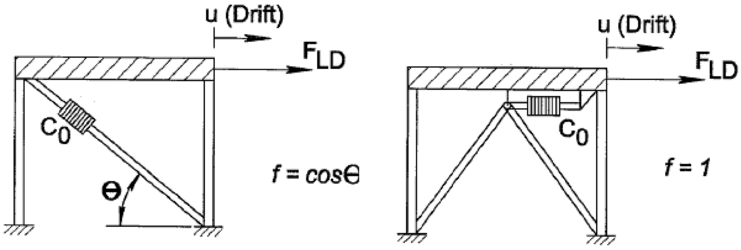


Figure 2. 29 Diagonal and chevron braced configurations

It is possible to note that the horizontal system experiences a displacement between its two edges higher than the diagonal configuration. In fact if the horizontal displacement is denoted as u, then the diagonal one, u_d, can be described using a magnification factor f which depend on the angle of installation of the brace. The relationship can be written as:

$$u_d = f\ u$$

where:

$$f = \cos(\theta)$$

Moreover the derivative with respect to the time suggests:

$$\dot{u}_d = f\ \dot{u}$$

On the other hand the force exerted by the inclined damper on the frame is equal to:

$$F = f\ F_d$$

As seen in paragraph 2.4.3 for linear viscous dampers it can be written:

$$F_d(t) = c_d\ \dot{u}_d(t)$$

Considering now the geometrical disposition, the force due to the device exerted on the neighbor level of the structure is:

$$F(t) = f^2 c_d\ \dot{u}(t)$$

In conclusion a different geometric configuration results in a different values of force and displacement experienced by the damper. In the case of diagonal dampers there is a reduction of these quantities and for this reason there are no advantages. But with other types of disposition is possible to amplify the damper displacement increasing, consequently, the efficiency of the added damping.

The most known configuration is the so called toggle bar which is conceptually similar to the diagonal brace but divided into two parts. The damper is connected with these bracing elements and with the corner of the bay as shown in Figure 2. 30.

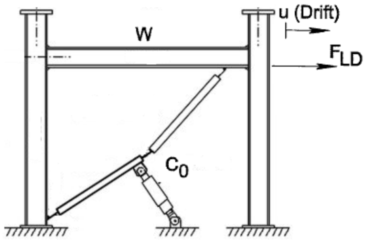


Figure 2. 30 Toggle bar configuration

Another interesting application derived from automobile industry is the so called scissor-jack, shown in Figure 2. 31.

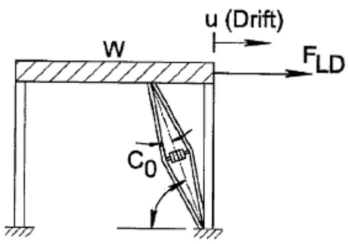


Figure 2. 31 Scissor-jack configuration

For each of these configurations magnification factors can be computed from geometrical characteristics. Geometrical amplification is useful in case of high level of required damping within a storey. In this case however the damper, in front of a higher efficiency proportional to factor f, is going to experience a force increased of f^2 times. The force that the device has to withstand has a large influence on its price. For this reason before deciding the damping configuration is useful to carry on an analysis of the prices, taking into account also the number of installed devices. Especially in retrofitting in fact, the number of dampers to put on a structure has too a large influence due to the installation costs.