3 RICCATI

3.1 Introduction

The method presented hereafter was first proposed by (Gluck and Reinhorn 1996) and is based on optimal control theory used for active control of structures. In active control there are force-generating devices which are able to process information from all observable sensors and then introduce forces in the structure to reduce unwanted vibrations. In the case of passive devices the forces induced in the structure depends on the displacement and/or velocity at the extremities of the devices, dictated by the building motion. This difference results in the presence of off-diagonal terms in the control force equation, which means there is interaction between non neighbor stores. Such an interaction can't be provided by passive devices, which are installed between contiguous floors. For this reason some methodologies are necessary in order to approximate the exact solution. A brief comparison of such methodologies will be then presented.

3.2 Mathematical formulation

3.2.1 Equations of motion

For a structure braced by general devices the equation of motion can be written as:

$$\boldsymbol{m}\,\ddot{\boldsymbol{u}}(t) + \boldsymbol{c}\,\dot{\boldsymbol{u}}(t) + \boldsymbol{k}\,\boldsymbol{u}(t) = \boldsymbol{e}\,\boldsymbol{f}(t) + \boldsymbol{d}\,\boldsymbol{x}(t)$$

where matrices **m**, **c** and **k** characterize mass, damping and stiffness of the structure at the different degrees of freedom while $\boldsymbol{x}(t)$ is the vector of control forces located in accordance to matrix $\boldsymbol{d}$ and $\boldsymbol{f}(t)$ is the vector of excitations forces located in accordance to matrix $\boldsymbol{e}$.

A second degree differential equation can always be compacted to a named state space formulation, that is, to a system of first order differential equations. If the variable displacement is substituted by:

$$\boldsymbol{z}(t) = \begin{Bmatrix} \boldsymbol{u}(t) \\ \dot{\boldsymbol{u}}(t) \end{Bmatrix}$$

then is possible to write the equation of motion in the following way:

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{A}\,\boldsymbol{z}(t) + \boldsymbol{B}\,\boldsymbol{x}(t) + \boldsymbol{H}\,\boldsymbol{f}(t)$$

where:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{m}^{-1}\boldsymbol{k} & -\boldsymbol{m}^{-1}\boldsymbol{c} \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} \boldsymbol{0} \\ -\boldsymbol{m}^{-1}\boldsymbol{d} \end{bmatrix} \quad \boldsymbol{H} = \begin{bmatrix} \boldsymbol{0} \\ -\boldsymbol{m}^{-1}\boldsymbol{e} \end{bmatrix}$$

Assuming that the control forces are of linear form, that is:

$$\boldsymbol{x}(t) = \boldsymbol{G}\,\boldsymbol{z}(t) = \boldsymbol{G}_u\boldsymbol{u}(t) + \boldsymbol{G}_{\dot{u}}\dot{\boldsymbol{u}}(t)$$

the equation of motion reduces to:

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{A_c}\,\boldsymbol{z}(t) + \boldsymbol{H}\,\boldsymbol{f}(t)$$

where the matrix of the controlled system $\boldsymbol{A_c}$ is:

$$\boldsymbol{A_c} = \boldsymbol{A} + \boldsymbol{BG}$$

3.2.2 Optimal function

The aim of optimal control is to minimize the displacements of the system during the time interval of the earthquake. Being the displacement a vector, it is possible to make the following scalar function:

$$J = \int \boldsymbol{z}^t\, \boldsymbol{z}\, dt$$

However in general a physical system contains different types of variables with different units of measure (in this case variables of space and velocity). For this reason it is necessary to introduce a matrix $\boldsymbol{Q}$ to bring the same dimension to all of the terms of the state vector:

$$J = \int \boldsymbol{z}^t\, \boldsymbol{Q}\, \boldsymbol{z}\, dt$$

Minimizing this function would bring to a $\boldsymbol{G}$ that is over-proportionate, because it is independent from the control forces, that is from the work done in order to maintain the specifics. I t is possible to update the object function in this way:

$$J = \int [\boldsymbol{z}^t \boldsymbol{Q}\, \boldsymbol{z} + \boldsymbol{x}^t \boldsymbol{R}\, \boldsymbol{x}]\, dt$$

This approach is named quadratic control because *J* is a quadratic function of the state and the control vectors **z** and **x**. Matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ are weighting matrices of the factors of optimization.
The gain matrix $\boldsymbol{G}$ is obtained from the minimization of the objective function *J*:

$$\boldsymbol{G} = -\frac{1}{2}\boldsymbol{R}^{-\mathbf{1}}\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{P}$$

Where $\boldsymbol{P}$ is the solution of the Riccati equation:

$$\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{P} + \boldsymbol{PA} - 1/2\, \boldsymbol{PBR}^{-\mathbf{1}}\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{P} + 2\boldsymbol{Q} = \mathbf{0}$$

3.2.3 Relative values

First it has to be noted that $\boldsymbol{G}$ relates the absolute values of displacements and forces. Nevertheless the design of the viscous devices has to be done with respect to the relative displacement between neighbor floors and forces acting between the extremities of the devices.
The change between relative quantities $\boldsymbol{\delta}(t)$ and absolute ones $\boldsymbol{u}(t)$ is made using the following simple transformation:

$$\boldsymbol{u}(t) = \boldsymbol{T}\, \boldsymbol{\delta}(t)$$

where $\boldsymbol{T}$ depends on the order of the degrees of freedom that has been used. Assuming that the first degree of freedom is the one at the bottom of the building matrix $\boldsymbol{T}$ is written as:

$$\boldsymbol{T} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

The inverse transformation can be done simply using the inverse of the matrix $\boldsymbol{T}$:

$$\boldsymbol{inv}(\boldsymbol{T}) = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Substituting the previous equation in:

$$\boldsymbol{x}(t) = \boldsymbol{G}\,\boldsymbol{z}(t)$$

Results:

$$\boldsymbol{x}_{rel}(t) = \boldsymbol{T}^{-1}\boldsymbol{G}\,\boldsymbol{T}\,\boldsymbol{\delta}(t)$$

The new gain matrix which relates relative measures is then:

$$\boldsymbol{G}_{rel} = \boldsymbol{T}^{-1}\boldsymbol{G}\,\boldsymbol{T}$$

3.2.4 Approximation

Moreover, it has to been noted that $\boldsymbol{G}_{rel}$ is a full matrix. But since the devices for the vibration control are going to be installed only between neighbor floors a diagonal matrix is needed:

$$\boldsymbol{x}_{rel}(t) = \begin{bmatrix} c_{\mathbf{1}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{c_2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots \end{bmatrix} \boldsymbol{\delta}(t)$$

In order to use passive control the gain matrix must be approximated. This approximation has to be done in order to give the most similar results, that is, with least squares method:

$$min\colon \int_0^T \sum_j \left[g_{kj}\dot{\delta}_j - c_k\dot{\delta}_j\right]^2 dt$$

$$\frac{d}{d\dot{\delta}_k}\left\{\int_0^T \sum_j \left[g_{kj}\dot{\delta}_j - c_k\dot{\delta}_j\right]^2 dt\right\} = 0$$

and leads to:

$$c_k = \int_0^T \sum_j g_{kj}\dot{\delta}_j\, dt \Big/ \int_0^T \dot{\delta}_j dt$$

The preceding coefficient can be determined using different grades of simplification as outlined next.

Response spectrum approach

Assume that the velocity can be obtained from a modal spectrum approach using the square root of sum of square (SRSS) superposition:

$$\dot{\delta}_{ji} = \left[\sum_i \left(\Phi_{ji} P_i S_{vi}\right)^2\right]^{\frac{1}{2}}$$

where i represents the number of the mode, j the degree of freedom, $S_{vi}$the spectral velocity of *j-th* mode, Φ_{ji} the mass normalized mode shapes and P_i the modal participation factor defined as:

$$P_i = \sum_j m_j \Phi_{ji}$$

The equivalent damping factor becomes:

$$c_k = \frac{\sum_j g_{kj} \left[\sum_i \left(\Phi_{ji} P_i S_{vi}\right)^2\right]^{\frac{1}{2}}}{\left[\sum_i (\Phi_{ki} P_i S_{vi})^2\right]^{\frac{1}{2}}}$$

Single Mode approach

The previous formulation takes into account the influence of all the different modes and degree of freedom of the system. But in application involving building structures in earthquakes, most often only one mode of vibration is relevant. If only one mode *m* is taken the previous equation becomes:

$$c_k = \frac{\sum_j g_{kj} \Phi_{jm}}{\Phi_{km}}$$

Truncation approach

Finally, if only one gain factor is considered, the one that corresponds of the same degree of freedom, the formulation of the control factor is given by:

$$c_k = g_{kk}$$

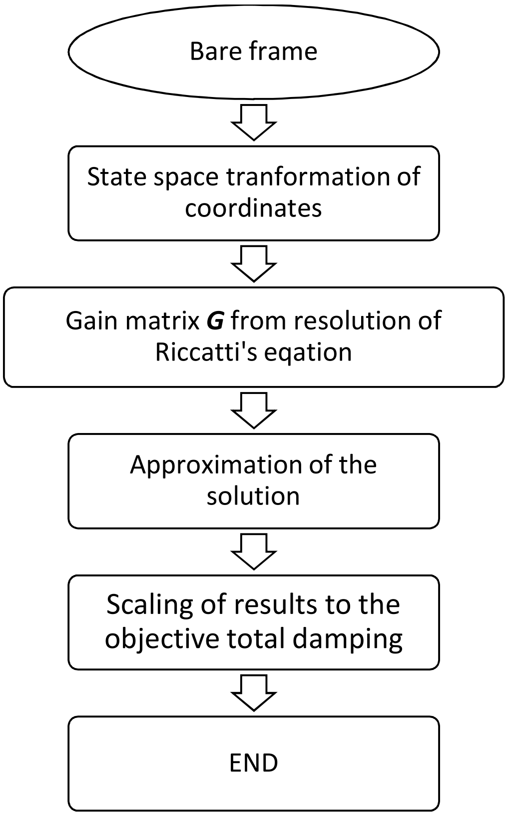
It has to be noted that with the simplification coming from the modal spectrum approach the coefficient are no longer dependent on the history of the event, but only from the characteristics of the structure.

The procedure here exposed gives a damper distribution for a specific value of the parameter *p*. Since the total damping depends linearly on this parameter, the value of the resulting added damping can be scaled to the value of the objective damping, which is decided by the designer on the basis of what explained in chapter 2.4.5.2. The mathematical formula of the scaling follows

$$c_{k\,fin} = \frac{c_k}{\sum_k c_k} C_{obj}$$

Where c_k and $c_{k\,fin}$ are respectively the damper sizes at the *k-th* degree of freedom while C_{obj} is the amount of total added damping.

Hereafter is presented a flowchart that summarizes the design process.



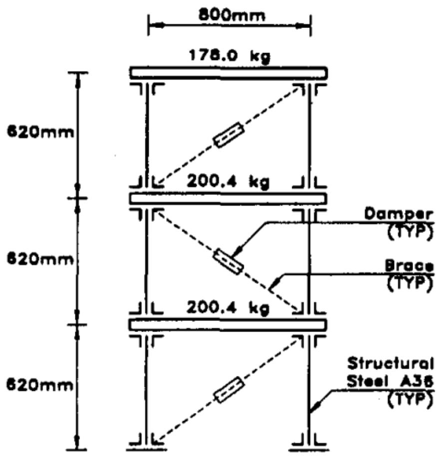
3.3 Example: 3-story shear frame

Mass, stiffness and damping matrices of the 3-story building used by (Guck et al. 1996) follow:

$$\boldsymbol{m} = \begin{bmatrix} 200.4 & 0 & 0 \\ 0 & 200.4 & 0 \\ 0 & 0 & 178.0 \end{bmatrix} kg$$

$$\boldsymbol{k} = \begin{bmatrix} 238{,}932 & -119{,}466 & 0 \\ -119{,}466 & 238{,}932 & -119{,}466 \\ 0 & -119{,}466 & 119{,}466 \end{bmatrix} N/m$$

$$\boldsymbol{c} = \begin{bmatrix} 264.99 & -78.09 & -16.08 \\ -78.09 & 246.89 & -92.15 \\ -16.08 & -92.15 & 162.02 \end{bmatrix} Ns/m$$



Matrices of the state space notation follow:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1192.3 & 596.14 & 0 & -1.322 & 0.390 & 0.080 \\ 596.14 & -1192.3 & 596.14 & 0.390 & -1.232 & 0.460 \\ 0 & 671.06 & -671.16 & 0.090 & 0.518 & -0.910 \end{bmatrix}$$

$$\boldsymbol{B} = \boldsymbol{D} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.005 & 0 & 0 \\ 0 & 0.005 & 0 \\ 0 & 0 & 0.0056 \end{bmatrix}$$

Riccati matrix for p=6 is:

$$\boldsymbol{P} = \begin{bmatrix} 357.49 & -160.73 & -10.656 & 0.0132 & 0.0247 & -0.009 \\ -160.73 & 345.98 & -170.42 & -0.0163 & 0.0136 & 0.0371 \\ -10.656 & -170.42 & 184.73 & 0.0081 & -0.0184 & -0.0148 \\ 0.0132 & -0.0163 & 0.0081 & 0.3121 & 0.0247 & 0.0060 \\ 0.0247 & 0.0136 & -0.0184 & 0.0247 & 0.3193 & 0.0298 \\ -0.009 & 0.0371 & -0.0148 & 0.0060 & 0.0298 & 0.3018 \end{bmatrix}$$

$$\boldsymbol{G}_{\dot{u}} = \begin{bmatrix} 778.79 & 61.66 & 14.99 \\ 61.66 & 796.56 & 74.36 \\ 16.88 & 83.72 & 847.85 \end{bmatrix}$$

$$\boldsymbol{G}_{rel} = \begin{bmatrix} 2736.48 & 1879.15 & 937.21 \\ 1881.04 & 1802.50 & 922.21 \\ 984.45 & 931.57 & 847.85 \end{bmatrix}$$

$$\boldsymbol{G}_{rel} = \begin{bmatrix} 2736.48 & 1879.15 & 937.21 \\ 1881.04 & 1802.50 & 922.21 \\ 984.45 & 931.57 & 847.85 \end{bmatrix}$$

Hereafter the results for the single mode approach:

$$\boldsymbol{c}_d = \begin{bmatrix} 4933.3 & 0 & 0 \\ 0 & 4667.4 & 0 \\ 0 & 0 & 4605.8 \end{bmatrix}$$

3.4 Observations

The LQR approach is first optimal design procedure proposed for seismic retrofitting of structures using viscous dampers.

The Linear quadratic regulator (LQR) solution provides a simple way to design the distribution of dampers. The procedure is analytical and no iterations are needed.

On the other hand this method has several weak points that make modern methods more adequate.
The main weakness of this methodology is the quadratic form of the objective function:

$$J = \int [\boldsymbol{z}^t \boldsymbol{Q}\, \boldsymbol{z} + \boldsymbol{x}^t \boldsymbol{R}\, \boldsymbol{x}]\, dt$$

As can be seen, this is a smeared measure both in time and along the floors. The objective function should take into account the peak values of performance indices and not their integral along the time. The same is for the distribution in the space of this function. Considering the sum of the drifts at all different story leads to the loss of the information relative to where peak drifts occur. In general seismic engineering requires more attentions for details and peak measures.
This concentration of damping is necessary in order to efficiently reduce the structural response. Since additional damping is required only where local damage exceeds allowable values a spread distribution of it results in a lower dynamic performance. Moreover in presence of irregular structures the concentration of damping in discontinuities zones is essential in order to prevent local damage mechanism. Hence in general the amount of damping required in order to achieve a certain performance is higher than other method.
A further factor of simplification is the model of seismic excitation which is considered as a white noise process. This representation is not the source of the smeared distribution of dampers as demonstrated by (Levy and Lavan 2009) and, in case of narrow-banded systems, gives a good approximation.
The distribution of damping resulting from the design is the same for every value of the parameter *p*. Its magnitude, however, changes with the value of *p*. Hence the solution obtained with a certain value of *p* can be scaled to obtain the objective total added damping. This means that the result concerns the distribution of damping and not the total amount of it which is decided by the designer. This seems to be an advantage of the method as no iterations are required. Nonetheless, in terms of performance, experience shows that the optimal distribution of damping strongly depends on the magnitude of total damping.
The value of the objective total added damping can be evaluated with the methods explained in chapter 2.4.5.2 or in alternative time-history analyses can be run using the real dampers configuration for different values of the total amount of damping. This latter option better reproduce the real response of the structure.
Finally, as other algorithms later explained, the stiffness matrix is required in order to carry out the analysis. The computation of this matrix, especially in case of three dimensional structures, is not always available in commercial software and its manual extraction, through the force method for example, is quite demanding.
Note that the LQR method can only consider linear elastic behavior.

3.5 MATLAB code

```
% INPUT 3story frame
% Stiffness matrix [kN/m]
k=1000*[238.932 -119.466 0;-119.466 238.932 -119.466;0 -119.466
119.466]
% Mass matrix [tons]
m=diag([ 200.4 200.4 178 ])
% Hinerent damping matrix [kNs/m]
c=[264.99 -78.09 -16.08;-78.09 246.89 -92.15;-16.08 -92.15 162.02]
% Number of degrees of freedom
gdl=max(size(m));
% Matrix for the transformation in inter-story quantities
T=tril(ones(gdl))
% State Space notation
D=eye(gdl);
E=eye(gdl);
invm=inv(m);
A=[zeros(gdl) eye(gdl); -invm*k -invm*c];
B=[zeros(gdl);invm*D];
H=[zeros(gdl);invm*E];
% parameter p for the weight of the optimization function
p=6;
% Optimization function matrices
Q=eye(2*gdl);
R=10^(-p)*eye(gdl);
% Riccati's solution
[G,P] = LQR(A,B,Q,R);
% Gain matrix
G1=G(1:gdl,gdl+1:2*gdl)
% Inter-story gain matrix
G1d=T'*G1*T;
% Modal analysis
[S,w2]=eig(k,m);
S=inv(T)*S;
w=sqrt(w2);
Periodi=2*pi*inv(w);
Tmax=norm(T,inf);
% TRUNCATION APPROACH
dc0=diag(G1d)'
% SINGLE MODE APPROACH
dc1=zeros(1,gdl);
for jj=1:gdl
    dc1(jj)=G1d(jj,:)*S(:,1)/S(jj,1);
end
figure
barh(1:gdl,dc1)
title('Dampers');
xlabel('Damping kNs/m')
ylabel('Floor number')
```

4 ANALYSIS-REDESIGN METHODS

These kinds of analyses consist of two different phases. Firstly the analysis is carried on and then, based on its results, the initial design is changed. This procedure goes on until all the constrains are satisfied.
Hereafter the algorithm developed by (Levy and Lavan 2005) is first described. This method is based on time-history analysis and it takes into account both linear and nonlinear behavior, so that it can be applied also for irregular structures.
Than an implementation of the previous method developed always by (Lavan Levy 2009) is explained. This procedure attempts to simplify the time domain analysis replacing them with Lyapunov equations and good agreement in the results is shown for linear analysis.

TIME HISTORY ANALYSIS-REDESIGN

4.1 Introduction

This method is named fully stressed due to an analogy with classical design of trusses, whereby the weight is optimized for a given allowable stress. The optimal solution is achieved iteratively. In the design of viscous dampers presented hereafter, the size of these devices will be minimized attending certain performances. With this method optimal design is achieved iteratively using a two step algorithm in each iteration cycle. In the first one an analysis is performed for a given preliminary design, whereas in the second step the design is changed using a recurrence relationship. The process ends when the analysis shows the achievement of the requested performances.

4.2 Mathematical formulation

4.2.1 Equations of motion

The equations for a two-dimensional linear dynamic system are given by:

$$\boldsymbol{m}\,\ddot{\boldsymbol{u}}(t) + \boldsymbol{c}\,\dot{\boldsymbol{u}}(t) + \boldsymbol{k}\,\boldsymbol{u}(t) = -\boldsymbol{m}\,\mathbf{1}\,\ddot{u}_g(t)$$

where matrices $\boldsymbol{m}$, $\boldsymbol{c}$ and $\boldsymbol{k}$ characterize mass, damping and stiffness of the structure at the different degrees of freedom. $\boldsymbol{u}$ denotes the horizontal floor displacement vector and $\ddot{u}_g(t)$ represents the ground motion which acts at all the considered degrees of freedom as shown by the unity vector **1**. The damping matrix $\boldsymbol{c}$ is composed of two contributions: the original structural damping $\boldsymbol{c_0}$ and the one of the viscous devices $\boldsymbol{c_d}$

$$\boldsymbol{c} = \boldsymbol{c_0} + \boldsymbol{c_d}$$

The inter-story drifts $\boldsymbol{\delta}(t)$ are related to the floor displacements with the following relationship:

$$\boldsymbol{u}(t) = \boldsymbol{T}\,\boldsymbol{\delta}(t)$$

where, if the first degree of freedom corresponds to the first floor:

$$\boldsymbol{T} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

The inverse relation depends on the inverse matrix of $\boldsymbol{T}$ which takes the form of:

$$\boldsymbol{inv}(\boldsymbol{T}) = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

The response in terms of inter-story drifts can thus be computed with the classical methods of structural dynamic explained in Annex A.

4.2.2 Performance index

Regular structures can be modeled using more common linear methods. Despite during a ground motion the bare frame can achieve inelastic range it can be reasonably assumed that, for this kind of structures, the vibration shape does not change significantly. Moreover, once an added damping system is installed, the retrofitted structure does not undergo large plastic deformations. So if linear behavior is requested and consequently represents both a constrain of the problem and the final response of the retrofitted building, linear analysis tools can be used to perform the time-history response.

In this linear case inter-story drift becomes an important damage index because it takes into account not only nonstructural damage but also it gives a good description of the structural one. This can be assumed to be valid in case of low plastic behavior.

In conclusion, the maximal inter-story drift is chosen as the local performance index for regular buildings.

4.2.3 Optimization problem formulation

The optimization problem can be formulated as:

minimize: $J = c_d^T \mathbf{1}$

subject to: $max_i\left(max_t(|\delta_i(t)|)\right) < \delta_{all,i}$

where $\delta_i(t)$ *satisfy the equations of motion:* $\boldsymbol{x}(t) = \boldsymbol{T}\,\boldsymbol{\delta}(t)$

4.2.4 Recurrence relationship

Experience with rigorous optimization methods such as the cutting planes method (Lavan Levy 2005) has shown two interesting aspects. Firstly the optimal design c_d attains zero value where the local performance index is less than the allowable while in case the local performance index is equal to the allowable dampers must be placed. In other words viscous devices are added only where damage is presents while where the damage is less than the acceptable value there is no need of dampers. Based on this observation, a recurrence mathematical relationship can be generated in order to either increase the added damping if damage is higher than the constrain value or decrease it in case the damage is less than the constrain value for the considered floor. This can be done, as suggested by (Levy and Lavan 2005), multiplying the damping coefficient of each floor by the ratio between the real performance index and the allowable one. As a result damper size increases in case of high damage and it decreases in the opposite case. The relationship can be written as:

$$c_{di}^{(k+1)} = c_{di}^{(k)}(|\delta_i(t)|/\delta_{all,i})^{\frac{1}{q}}$$

where $c_{di}^{(k+1)}$and $c_{di}^{(k)}$are the values of the damping vector at the *i-th* degree of freedom and at the *k-th+1* and *k-th* iteration, q is a convergence parameter and $PI_i^{(k)}$is the *i-th* component of the performance index at the *k-th* iteration.

The choice of q affects the efficiency of the method. In fact for larger values of this constant the method is more stable, that is, the method is more likely to converge, although the convergence is slower. Values of 0.5 for linear analysis and 2 for nonlinear ones are suggested by the authors.

The second interesting aspect shown by rigorous optimization criteria is the monotonic convergence to the solution. It means that the configuration achieved in the present step is surely better than the previous one if large initial added damping was chosen. Hence it is possible to end the iterations whenever the designer wants. This could be useful in case the software used for the time history analysis can't be used iteratively, which is the case of the most part of the commercial programs. As depicted in Figure 4-1 the convergence of the objective function is rather fast and can bring in five-ten iterations to reasonably values.

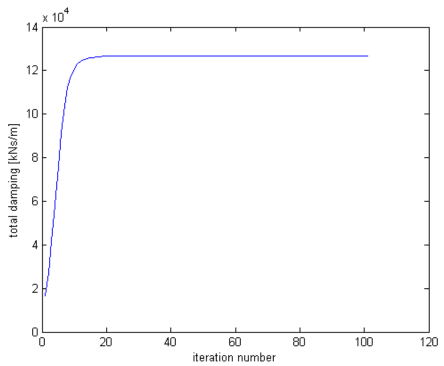


Figure 4-1 Convergence of the objective function

4.2.5 *Design methodology*

Since time-history analysis is needed to obtain the maximum values of the performance indices the first step on the optimization procedure is the choice of the input ground motion. Since seismic excitation is a stochastic function while a particular ground motion is a deterministic one, several accelerograms must be considered in order to give to the resulting configuration a general validity. Hence an ensemble of ground motions must be derived from design spectrum and then analyzed.

On this point it is important to underline that the final configuration of added damping is not given by the superposition of the configurations resulting from each time-history analysis. The final configuration is achieved instead superposing at each iteration the performance indices. In the first case in fact the interaction of damping on the performance index at a different floor is neglected resulting in a higher value of final added damping. In other words, if two specific ground motions, with a different frequency content, bring to two different inter-story drift configurations and the performance indices are superposed, at the next iteration the dampers added by the first ground motion influence the response also for the second ground motion. Instead if the superposition is at the end of the iterative process this reciprocal influence does not exist.

For the most part of software which can’t be run iteratively from an independent platform it is not possible this kind of direct superposition because of the great number of requested analysis, hence a more intelligent procedure must be followed. It is possible, for a given ground motion ensemble, to find the so called active ground motion, that is the most important one, by the computation of displacements of a singular degree of freedom system having the same period of the examined structure for all the different excitations and choosing the higher value. This should be done for different values of damping ratio because its change may not have the same effect on all the different responses. In Figure 4-2 the results for the LA 10% in 50 years ensemble for the nine-story building analyzed in chapter 7.1 are shown.

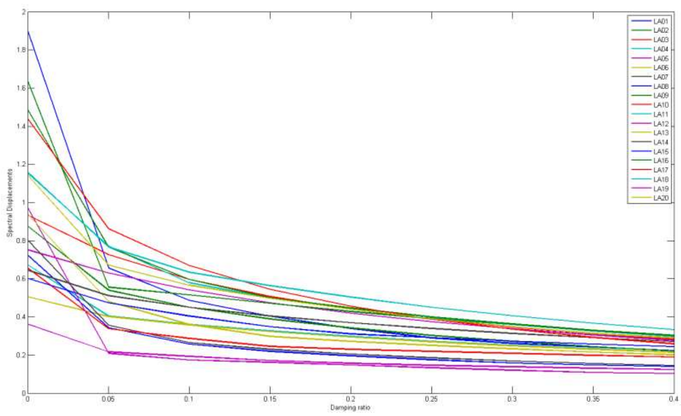


Figure 4-2 Spectral displacements vs damping ratio for the LA 10% in 50 years for T=2.16 s

Once the active ground motion is selected its optimal damper configuration is found. Then the remaining records are applied to the current design. If one of the responses

overpasses the allowable limit then the superposition of the performance indices of the two considered excitations at each iteration must be adopted.
It has been seen that with large initial values of added damping the convergence is faster. For this reason an initial values of the damping ratio due to viscous dampers will be chosen. The simplest way to do it is adopting a uniform distributed damping ratio related to the first mode. In a multi degree of freedom system damping ratio and coefficients of the damping matrix are linked with the well known following relationship:

$$\xi_i = \frac{C_i}{2\,\omega_i\,M_i}$$

that is:

$$C_i = 2\,\omega_i\,M_i\,\xi_i$$

where $M_i$and $C_i$are the terms of generalized mass and damping matrices $\boldsymbol{M}$ and $\boldsymbol{C}$ which can be obtained from:

$$C_i = \boldsymbol{\Phi}_{(i)}^T \boldsymbol{c}\,\boldsymbol{\Phi}_{(i)}$$

$$M_i = \boldsymbol{\Phi}_{(i)}^T \boldsymbol{m}\,\boldsymbol{\Phi}_{(i)}$$

Where $\boldsymbol{\Phi}_{(i)}$ is the *i-th* mode shape. Substituting these last relationships and considering only the first mode we obtain:

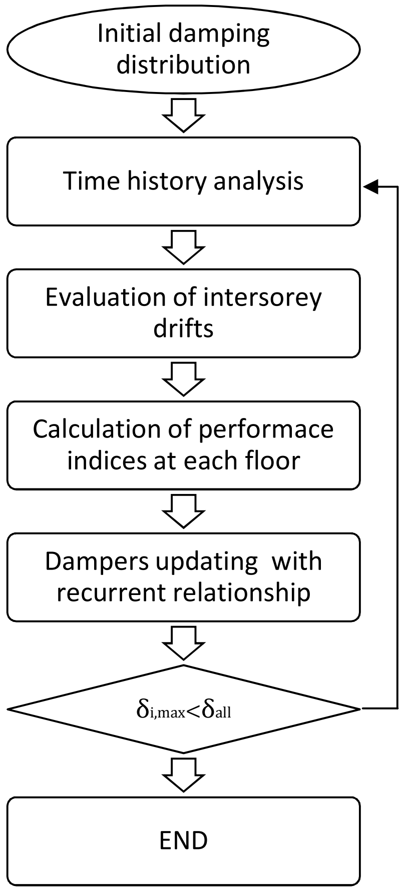
$$c_d^{(1)} = 2\,\xi_{d1}\,\omega_1\,\frac{\boldsymbol{\Phi}_1^T \boldsymbol{M} \boldsymbol{\Phi}_1}{\boldsymbol{\Phi}_1^T \boldsymbol{\Phi}_1}$$

After the choice of the loading and the initial values of damping the analysis/redesign procedure can be carried on. The process will end when the constrain error takes value lower than a predetermined tolerance:

$$max_i(PI_i) - 1 < tollerance$$

that is, when the drift is smaller than the allowable value.

The solution algorithm is summarized in the following flowchart.



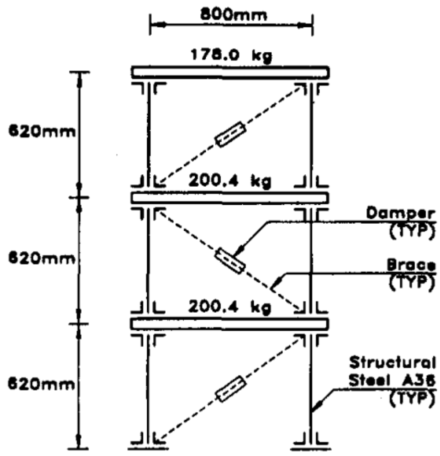
4.3 Example: 3-story shear frame

Mass, stiffness and damping matrices of the 3-story building used by (Guck et al. 1996) follow:

$$\boldsymbol{m} = \begin{bmatrix} 200.4 & 0 & 0 \\ 0 & 200.4 & 0 \\ 0 & 0 & 178.0 \end{bmatrix} kg$$

$$\boldsymbol{k} = \begin{bmatrix} 238{,}932 & -119{,}466 & 0 \\ -119{,}466 & 238{,}932 & -119{,}466 \\ 0 & -119{,}466 & 119{,}466 \end{bmatrix} N/m$$

$$\boldsymbol{c} = \begin{bmatrix} 264.99 & -78.09 & -16.08 \\ -78.09 & 246.89 & -92.15 \\ -16.08 & -92.15 & 162.02 \end{bmatrix} Ns/m$$



LA02 ground motion is taken in account.
Due to the linearity of the problem a convergence factor q=2 is chosen.
The value of the initial damping, using a damping ratio of ξ_{d1}=15%, is equal to 633.6 Ns/m and the initial added damping matrix is:

$$\boldsymbol{c_d} = \begin{bmatrix} 1267.1 & -633.6 & 0 \\ -633.6 & 1267.1 & -633.6 \\ 0 & -633.6 & 633.6 \end{bmatrix} Ns/m$$

The first three iterations are here considered:

$$\boldsymbol{\delta}^{(1)} = \begin{bmatrix} 0.0732 \\ 0.0571 \\ 0.0295 \end{bmatrix} m \qquad \boldsymbol{c}_{d\,i}^{(1)} = \begin{bmatrix} 989.8 \\ 873.9 \\ 627.8 \end{bmatrix} Ns/m$$

$$\boldsymbol{\delta}^{(2)} = \begin{bmatrix} 0.0693 \\ 0.0540 \\ 0.0279 \end{bmatrix} m \qquad \boldsymbol{c}_{d\,i}^{(2)} = \begin{bmatrix} 1504.2 \\ 1172.2 \\ 605.4 \end{bmatrix} Ns/m$$

$$\boldsymbol{\delta}^{(2)} = \begin{bmatrix} 0.0644 \\ 0.0502 \\ 0.0260 \end{bmatrix} m \qquad \boldsymbol{c}_{d\,i}^{(2)} = \begin{bmatrix} 2204.1 \\ 1516.5 \\ 563.5 \end{bmatrix} Ns/m$$

The final values obtained after one hundred iterations are resumed:

$$\boldsymbol{\delta}^{(fin)} = \begin{bmatrix} 0.0300 \\ 0.0300 \\ 0.0161 \end{bmatrix} m \qquad \boldsymbol{c}_{d\,i}^{(fin)} = \begin{bmatrix} 9765.7 \\ 2777.7 \\ 0.0 \end{bmatrix} Ns/m$$

4.4 Observations

Based on the results obtained from formal optimization procedures (Lavan and Levy 2005 and 2006), this method represents a simplification suitable for engineering practice. In fact the solution is achieved iteratively using simple time-history analyses.

Furthermore, a value for the total added damping to the structure is not needed because the solution converges to the exact amount of damping necessary to satisfy the constraints of the problem.

This fact makes this algorithm suitable for the so called performance based design, which is a design that concerns not only life safety, but also a prescribed level of damage throughout the structure. This level of damage can be computed quantitatively, as explained in chapter 2.1, by damage indices which thus can be used, once the allowable value is decided, to mathematically describe the recurrence relationship.

With respect to the other methods presented in this thesis although, this methodology requires time-history analyses which are computationally expensive, especially if used iteratively. On the other hand these analyses can be run with commercial software and consequently there is no need to derive the stiffness matrix, which represents one of the points of more concern for other methods. Furthermore commercial software packages usually allow inelastic analysis and consequently plastic behavior of structures can be taken into account for example in presence of irregular building. This represents another limit for most of the optimization methods here presented.

One point to note regarding the fully stressed design is the use of several input ground motion due to the fact that seismic excitation is in the form of accelerograms which is a deterministic description of a stochastic event. On this point it is interesting to see how two ground motions with different frequency content can bring to really different design. In Figure 4-3 the results obtained for the nine-story building for LA10 and LA07 ground accelerations are shown. As can be seen, although the value of total added damping is rather similar, its distribution along the floors is really different.

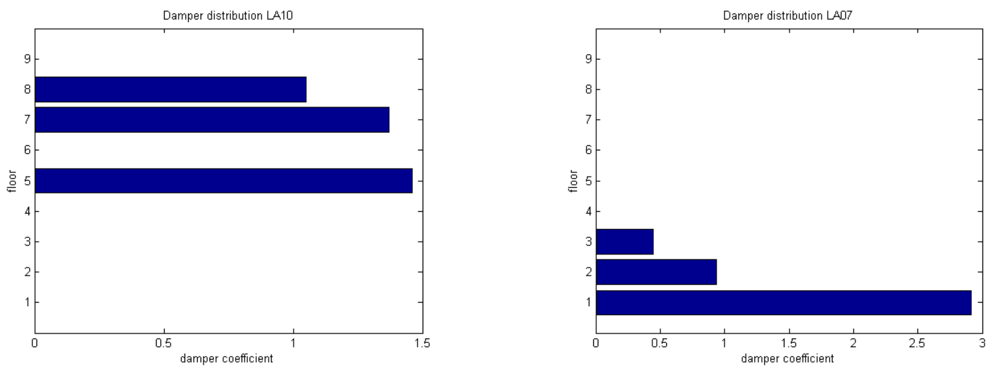


Figure 4-3 Damping distribution for LA10 and LA07

Form the spectral pseudo-acceleration depicted in Figure 4-4 it can be argued that LA07 affects more the second mode (period of 0.812 s) while LA10 the third one (period of 0.47 s).

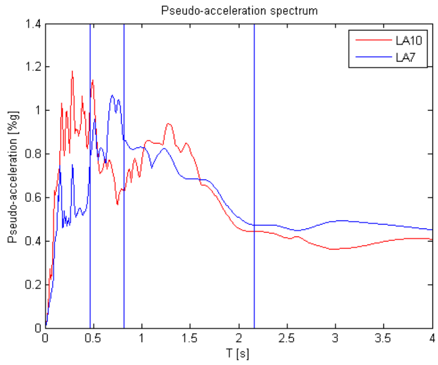


Figure 4-4 Spectral pseudo-acceleration for LA10 and LA07

Also if it has been observed that for high levels of damping results trend to achieve similar distributions, it is evident how the choice of seismic inputs affects the results.
In order to avoid these effects an ensemble of ground motions must be considered, as explained in paragraph 4.2.5. As a result a set of active accelerograms must be analyzed at the same time in order to superpose at each iteration the values of the performance indices. If more than two active ground motions have to be run the procedure becomes computationally difficult if commercial software is used. In this case however it is possible to adopt a superposition at the level of results instead of at each step, knowing that the final configuration is not the optimal solution but a safety sure approximation of it.
The objective function to minimize in the optimization problem is the sum of the sizes of the viscous dampers, then the minimization of the number of devices, which has great influence on costs in case of retrofitting, is not directly taken into account.
Although if in the final configuration smaller values of damping coefficients are obtained for some particular location, then it can be assumed that they have low influence on the response. Hence a second analysis-redesign can be carried on without considering dampers at those locations, resulting in a lower number of devices to install. Finally the costs of the two or more solutions must be compared.

4.5 MATLAB code

```
% INPUT 3story frame
% Stiffness matrix [kN/m]
k=1000*[238.932 -119.466 0;-119.466 238.932 -119.466;0 -119.466
119.466]
% Mass matrix [tons]
m=diag([ 200.4 200.4 178 ])
% Hinerent damping matrix [kNs/m]
c0=[264.99 -78.09 -16.08;-78.09 246.89 -92.15;-16.08 -92.15 162.02]
```

```
% Initial conditions
IC=zeros(3);
% Number of degrees of freedom
gdl=max(size(m));
% Allowable inter-story drift
DriftsAllow=[0.03 0.03 0.03];
% Ground motion
nomefile='LA2.txt';
Dt=0.02;
fact=1/9.806;
% Modal Analysis
[S,w2]=eig(k,m);
w=sqrt(w2);
% Matrix for the transformation in inter-story quantities
T=eye(gdl)-diag(ones(1,gdl-1),-1)
% Initial dampers values
csi1=0.15;
cdi=2*csi1*w(1,1)*(S(:,1)'*m*S(:,1))/(S(:,1)'*S(:,1))
cdi=cdi*ones(1,gdl);
cd=T'*diag(cdi)*T% cd=matrice(cdi*ones(1,gdl)')
% Pramiters for the iterations control
q=2;            % convergence parameter
toll=0.0001;    % tollerance
maxRapporto=1000;
while maxRapporto-1>toll
    c=c0+cd;
    [u]=Newmark(Dt,nomefile,fact,m,c,k,IC);
    % Maximum inter-story drifts
    d=T*u;
    dmax=max(abs(d'));
    % Changing of dampers coefficients
    for jj=1:gdl
        cdi(jj)=cdi(jj)*(dmax(jj)/DriftsAllow(jj))^(1/q);
    end
    cd=T'*diag(cdi)*T;
    % Maximal value to control the end of the cycle
    for jjj=1:gdl
        rapporto(jjj)=dmax(jjj)/DriftsAllow(jjj);
    end
    maxRapporto=max(rapporto);
end
figure
barh(1:gdl,cdi)
title('Dampers');
xlabel('Damping kNs/m')
ylabel('Floor number')
```

LYAPUNOV'S SOLUTION ANALYSIS-REDESIGN

4.6 Introduction

The present methodology, proposed by (Lavan Levy 2009), consists on a Lyapunov-based analysis/redesign approach similar to the one used in (Levy Lavan 2005). The main difference is the use of control theory tools, i.e. Lyapunov equations, instead of classical time history analysis. The optimal solution minimizes the total added damping while the mean squared drifts are constrained to allowable values under a white noise excitation.
Like in general all problems belonging to classical *fully stressed design* approach, the solution is achieved iteratively using a two-step algorithm for each iteration. In the first step an analysis is performed while in the second one the design is changed using a recurrence relationship, which dictates the *fully stressedness*.

4.7 Mathematical formulation

4.7.1 Equations of motion

The equations of motion for an N-storey building model with added viscous dampers can be formulated as:

$$\boldsymbol{m}\,\ddot{\boldsymbol{u}}(t) + \boldsymbol{c}\,\dot{\boldsymbol{u}}(t) + \boldsymbol{k}\,\boldsymbol{u}(t) = -\boldsymbol{m}\,\underline{1}\,\ddot{u}_g(t)$$

where $\boldsymbol{m}$ and $\boldsymbol{k}$ are the mass and stiffness matrix respectively and $\boldsymbol{u}$ denotes the horizontal floor displacement vector. The damping matrix $\boldsymbol{c}$ is composed of two contributions: the original structural damping $\boldsymbol{c_0}$ and the one of the viscous devices $\boldsymbol{c_d}$

$$\boldsymbol{c} = \boldsymbol{c_0} + \boldsymbol{c_d}$$

The inter-story drifts $\boldsymbol{\delta}(t)$ are related to the floor displacements with the following relationship:

$$\boldsymbol{u}(t) = \boldsymbol{T}\,\boldsymbol{\delta}(t)$$

where, if the first degree of freedom corresponds to the first floor:

$$\boldsymbol{T} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

The inverse relation will depends on the inverse matrix of T which takes the form of:

$$\boldsymbol{T^{-1}} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

The second order differential equation system can be rewritten, as explained in annex A, in the form of a first order of equations:

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{A}\,\boldsymbol{z}(t) + \boldsymbol{H}\,\ddot{u}_g(t)$$

where:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{m}^{-1}\boldsymbol{k} & -\boldsymbol{m}^{-1}\boldsymbol{c} \end{bmatrix} \qquad \boldsymbol{H} = \begin{bmatrix} \boldsymbol{0} \\ -\boldsymbol{1} \end{bmatrix}$$

and the new variable $\boldsymbol{z}(t)$ id defined as:

$$\boldsymbol{z}(t) = \begin{bmatrix} \boldsymbol{u}(t) \\ \dot{\boldsymbol{u}}(t) \end{bmatrix}$$

Inter-story drifts can be evaluated using:

$$\boldsymbol{\delta}(t) = \boldsymbol{D}\,\boldsymbol{z}(t)$$

where:

$$\boldsymbol{D} = [\boldsymbol{T}^{-1} \quad \boldsymbol{0}]$$

4.7.2 Lyapunov equations

The mean square response of a linear system, as explained in annex A, can be derived using the solution of Lyapunov's equation:

$$\boldsymbol{A}^T\boldsymbol{Q} + \boldsymbol{Q}\boldsymbol{A} + \boldsymbol{B}\boldsymbol{W}\boldsymbol{B}^{\boldsymbol{T}} = \boldsymbol{0}$$

This latter relationship is named Lyapunov's equation and its solution gives the response variance of the system $\boldsymbol{Q}_z = E(\boldsymbol{z} \cdot \boldsymbol{z}^{\boldsymbol{T}})$. In order to evaluate the response in terms of inter-story drifts $\boldsymbol{Q}_\delta = E(\boldsymbol{\delta} \cdot \boldsymbol{\delta}^{\boldsymbol{T}})$ the following transformation can be applied:

$$\boldsymbol{Q}_\delta = \boldsymbol{D}\boldsymbol{Q}_z\boldsymbol{D}^{\boldsymbol{T}}$$

The values on the diagonal of matrix $\boldsymbol{Q}_\delta$ represent the mean squared of the inter-story drifts. These values are taken as control values since they control both the achievement of the objective function and the updating of the damping matrix, through the performance index described hereafter.

4.7.3 Performance index

The performance index at the *i-th* degree of freedom is taken as the ratio between the mean squares value of inter-story drift at the present iteration and the allowable one:

$$PI_i = max_i(Q_{\delta i}/Q_{\delta\, all,i})$$

The allowable value is chosen taking into account the inelastic characteristics of the building and the performances that must be achieved.

4.7.4 Optimization problem formulation

The optimization problem is thus formulated as:

Minimize: $J = \boldsymbol{c}_d^T\,\boldsymbol{1}$

Subject to:

$$max_i(Q_{\delta i}/Q_{\delta\, all,i}) < 1$$

where $Q_\delta$satisfy the Lyapunov equation:

$$\boldsymbol{AQ} + \boldsymbol{QA^T} + \boldsymbol{HWH^T} = \boldsymbol{0}$$
$$\boldsymbol{Q}_\delta = \boldsymbol{DQ}_z\boldsymbol{D^T}$$

4.7.5 Recurrence relationship

The updating of the damping matrix is carried out through a recurrent relationship similar to the one used in time-history analysis-redesign:

$$c_{di}^{(k+1)} = c_{di}^{(k)}(PI_i^{(k)})^{\frac{1}{q}}$$

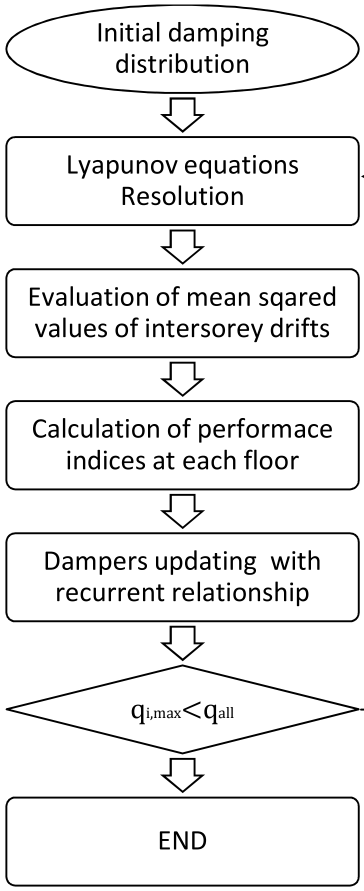
where $c_{di}^{(k+1)}$and $c_{di}^{(k)}$are the values of the damping vector at the *i-th* degree of freedom and at the *k-th+1* and *k-th* iteration, q is a convergence parameter and $PI_i^{(k)}$is the *i-th* component of the performance index at the *k-th* iteration.

4.7.6 Solution algorithm

This method consist on an iterative process which makes use of Lyapunov formulation to find control values, instead of standard dynamic analysis as in (Levy Lavan 2005). The control values are represented by the mean squares of the inter-story drift.
As usually the most part of analysis/redesign algorithms it needs starting values for unknown added damping. In order to do that an uniform distribution of damping is selected. The value of the single damper is obtained considering the first mode shape and a feasible damping ratio, using modal analysis tools:

$$c_d^{(1)} = 2\,\xi_{d1}\,\omega_1\,\frac{\boldsymbol{\Phi}_1^T\boldsymbol{M}\boldsymbol{\Phi}_1}{\boldsymbol{\Phi}_1^T\boldsymbol{\Phi}_1}$$

The iterative process contemplates a first stochastic analysis using provisional values of viscous damping and following Lyapunov formulation to obtain the mean squared values of drifts. Secondly, using these values, the updating of added damping is carried out making use of the recurrent relationship explained before.
As seen in *fully stressed design*, the damping is added on the locations in which the control quantity exceeds the allowable value for that floor. This process goes on until the maximum value along the structure of the control quantity becomes less or at least equal to the allowable one, chosen by the designer.
The solution algorithm is summarized in the following flowchart.



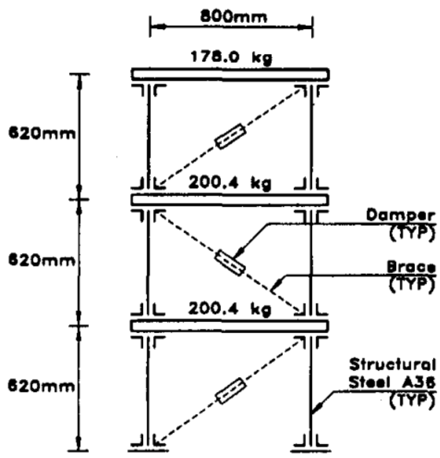
4.8 Example: 3-story shear frame

Mass, stiffness and damping matrices of the 3-story building used by (Guck et al. 1996) follow:

$$\boldsymbol{m} = \begin{bmatrix} 200.4 & 0 & 0 \\ 0 & 200.4 & 0 \\ 0 & 0 & 178.0 \end{bmatrix} kg$$

$$\boldsymbol{k} = \begin{bmatrix} 238{,}932 & -119{,}466 & 0 \\ -119{,}466 & 238{,}932 & -119{,}466 \\ 0 & -119{,}466 & 119{,}466 \end{bmatrix} N/m$$

$$\boldsymbol{c} = \begin{bmatrix} 264.99 & -78.09 & -16.08 \\ -78.09 & 246.89 & -92.15 \\ -16.08 & -92.15 & 162.02 \end{bmatrix} Ns/m$$



The value of the initial damping, using a damping ratio of ξ_{d1}=15%, is equal to 633.6 Ns/m and the initial added damping matrix is:

$$\boldsymbol{c_d} = \begin{bmatrix} 1267.1 & -633.6 & 0 \\ -633.6 & 1267.1 & -633.6 \\ 0 & -633.6 & 633.6 \end{bmatrix} Ns/m$$

An allowable mean square value of inter-story drift of $Q_{\delta\, all,i} = 0.012589^2$ is chosen in order to get similar results to the example of the time history design.
Matrices of the state space notation follow:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1192.3 & 596.14 & 0 & -37.58 & 14.23 & 0.080 \\ 596.14 & -1192.3 & 596.14 & 14.23 & -19.06 & 0.445 \\ 0 & 671.06 & -671.16 & 0.090 & 5.007 & -5.399 \end{bmatrix}$$

$$\boldsymbol{H} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

The first three iterations are here considered:

$$diag\left(\boldsymbol{Q}_\delta^{(1)}\right) = 10^{-3} \begin{bmatrix} 1.1258 \\ 0.6951 \\ 0.2002 \end{bmatrix} \qquad \boldsymbol{c}_{d\,i}^{(1)} = \begin{bmatrix} 4500.4 \\ 2778.9 \\ 800.4 \end{bmatrix} Ns/m$$

$$diag\left(\boldsymbol{Q}_\delta^{(2)}\right) = 10^{-3} \begin{bmatrix} 0.2963 \\ 0.1887 \\ 0.0594 \end{bmatrix} \qquad \boldsymbol{c}_{d\,i}^{(2)} = \begin{bmatrix} 8413.9 \\ 3308.1 \\ 300.0 \end{bmatrix} Ns/m$$

$$diag\left(\boldsymbol{Q}_\delta^{(3)}\right) = 10^{-3} \begin{bmatrix} 0.1783 \\ 0.1413 \\ 0.0514 \end{bmatrix} \qquad \boldsymbol{c}_{d\,i}^{(2)} = \begin{bmatrix} 9464.4 \\ 2949.9 \\ 97.4 \end{bmatrix} Ns/m$$

The final values obtained after eighty iterations are resumed:

$$diag\left(\boldsymbol{Q}_\delta^{(fin)}\right) = 10^{-3} \begin{bmatrix} 0.1585 \\ 0.1585 \\ 0.0583 \end{bmatrix} \qquad \boldsymbol{c}_{d\,i}^{(fin)} = \begin{bmatrix} 10240.0 \\ 2296.0 \\ 0.0 \end{bmatrix} Ns/m$$

4.9 Observations

The main advantage of this method is the low computational cost due to the use of efficient control design tools in alternative to expensive dynamic time-history analyses. The optimization process itself is really simple once the stiffness matrix is computed. This latter passage represents probably the most concerning point in usual engineers practice because of the difficulties related to the computation of the dynamic stiffness matrix. Commercial software packages do not offer the possibility to extract it and applying the forces or displacements methods for each floor is not feasible for a high number of degrees of freedom.

On the other hand it is no more necessary to analyze a great number of ground motions, because the excitation is modeled as white noise process. The method could also make use of filtered white noise input if use is made of an appropriate filter. Moreover as a consequence of the adoption of a stochastic description of the seismic input, the response is no more dependent on the choice of the set of ground motions and hence engineers are not required to choose and scale ground motions to represent the seismic hazard. Choosing and scaling ground motions requires some expertise most practicing engineers do not have.

Also if the white noise model can be substituted with a more realistic one, it does not have many influence on the final results. Usual structures in fact have a narrow banded frequency response which means that the most significant part of their transfer function is concentrated near to the first natural frequency. As a result the values of the seismic input that have more influence are the values in correspondence to this first natural period which is usually located in the low range of the spectrum. Since in this range seismic excitations are characterized by a broad band process the approximation of white noise model gives good results. Nonetheless, the method could consider a filtered white noise, if desired.

Although the main disadvantage of this method is the lack of a realistic performance index. In fact the values of the mean square displacements obtained from the mathematical solution of Lyapunov's equations are not in agreement with the real structural response also if they provide a meaningful performance index. Hence a performance based design is not possible and the value of total added damping must be decided a priori. An alternative procedure consists of evaluating the maximal response of the bare frame structure excited by a set of ground motions and, on the base of these results, decide of what percentage the response must be reduced. Then this percentage reduction is adopted for the performance indices of the Lyapunov-based algorithm and the obtained configuration is verified with the same ensemble of records.

Finally as other methods also this Lyapunov-based analysis-redesign does not consider inelastic behavior and for this reason it can't be used in case of irregular buildings.

4.10 MATLAB code

```
% INPUT 3story frame
% Stiffness matrix [kN/m]
k=1000*[238.932 -119.466 0;-119.466 238.932 -119.466;0 -119.466
119.466];
% Mass matrix [tons]
m=diag([ 200.4 200.4 178 ]);
% Hinerent damping matrix [kNs/m]
c0=[264.99 -78.09 -16.08;-78.09 246.89 -92.15;-16.08 -92.15 162.02];
% Initial conditions
IC=zeros(3);
% Number of degrees of freedom
gdl=max(size(m));
% Allowable values of mean square drift
dallow=0.012589*ones(1,3);
Pallow=dallow.^2;
% Modal analysis
[S,w2]=eig(k,m);
w=sqrt(w2);
% State space notation
invm=diag(1./diag(m));
H=[zeros(gdl,1); -ones(gdl,1)];
T=eye(gdl)-diag(ones(1,gdl-1),-1);
D=[T zeros(gdl)];
% Initial dampers values
csi1=0.15;
cdi=2*csi1*w(1,1)*(S(:,1)'*m*S(:,1))/(S(:,1)'*S(:,1));
cdi=cdi*ones(1,gdl);
cd=T'*diag(cdi)*T;
maxRapporto=1000;
while maxRapporto-1>0
    c=c0+cd;
    % State space notation matrices
    A=[zeros(gdl) eye(gdl); -invm*k -invm*c];
    Q=lyap(A,H*H');
    P=D*Q*D';
    % Mean squared inter-story drifts
    Pi=diag(P)';
    % Updating of dampers
    for jj=1:gdl
        cdi(jj)=cdi(jj)*(Pi(jj)/Pallow(jj));
    end
    cd=T'*diag(cdi)*T;
    % Maximum value of performance index
    for jjj=1:gdl
        rapporto(jjj)=Pi(jjj)/Pallow(jjj);
    end
    maxRapporto=max(rapporto);
end
sum(cdi')
```

5 SEQUENTIAL SEARCH ALGORITHMS

The term *sequential search* underlines how the procedures exposed hereafter achieve the final design adding sequentially certain amounts of viscous damping in the locations that are supposed to be optimal.
The first method, developed by (Zhang and Soong 1992), provides a frequency domain analysis with a stochastic description of the input and of the response.
The second method was developed by (Garcia 2001) and it is known as *simplified sequential search algorithm* because it provides a more ordinary time domain analysis.

Original Sequential Search Algorithm

5.1 Introduction

The present method, developed by (Zhang Soong 1992), is based on the concept that additional damping is added only where a preselected damage index is maximized. For this reason this procedure is named *sequential search algorithm*. Between several existing damage indices, inter-storey drift has been chosen, because of its simplicity and the fact that it takes in account also nonstructural damage. Due to the stochastic nature of earthquakes a non deterministic analysis is carried out. The values that are obtained don't represent therefore the deterministic value of drifts, but are measure of the mean squared response of the structure. In order to obtain such quantities is necessary to carry out a frequency domain analysis, where the seismic input is more properly described from a statistical viewpoint than in time history accelerograms.
Despite these non deterministic measures allow a more proper description of the problem, they don't offer a practical mean to stop the iterative process and to understand if the level of damping achieved is or not enough. Therefore it is presented a deterministic way to calculate the target value of added damping which represents the end of the iterative procedure.

5.2 Mathematical formulation

5.2.1 <u>Equations of motion</u>

Consider an N-storey building model with added viscous dampers. The system can be described by the following differential equation:

$$\boldsymbol{m}\,\ddot{\boldsymbol{u}}(t) + \boldsymbol{c}\,\dot{\boldsymbol{u}}(t) + \boldsymbol{k}\,\boldsymbol{u}(t) = -\boldsymbol{m}\,\underline{1}\,\ddot{u}_g(t)$$

where $\boldsymbol{m}$, $\boldsymbol{c}$ and $\boldsymbol{k}$ are the mass damping and stiffness matrix respectively, $\boldsymbol{u}$ denotes the horizontal floor displacement vector, $\ddot{u}_g(t)$ represents the ground motion and the unit vector indicates that all dynamic degrees of freedom are excited. The damping matrix $\boldsymbol{c}$ is composed of two contributions: the original structural damping $\boldsymbol{c_0}$ and the one of the viscous devices $\boldsymbol{c_d}$

$$\boldsymbol{c} = \boldsymbol{c_0} + \boldsymbol{c_d}$$

The basic concepts of frequency domain analysis are now briefly summarized. For a more detailed description see Annex A.3.

Let $\boldsymbol{U}(\omega)$ and $\ddot{U}_g(\omega)$ denote the Fourier transforms of $\boldsymbol{u}(t)$ and $\ddot{u}_g(t)$ respectively, and ω the generic circular frequency of a sinusoidal excitation. The equations of motion in the frequency domain become:

$$(-\omega^2 \boldsymbol{m} + i\,\omega\,\boldsymbol{c} + \boldsymbol{k})\,\boldsymbol{U}(\omega) = -\boldsymbol{m}\,\underline{1}\,\ddot{U}_g(\omega)$$

The transfer function matrix of steady-state harmonic response, $\boldsymbol{H}(\omega)$, is given by the ratio between response displacement and seismic acceleration:

$$\boldsymbol{H}(\omega) = \frac{\boldsymbol{U}(\omega)}{\ddot{U}_g(\omega)} = (-\omega^2 \boldsymbol{m} + i\,\omega\,\boldsymbol{c} + \boldsymbol{k})^{-1}(-\boldsymbol{m})$$

However the choice of the location of dampers is made considering inter-story drift instead of displacements. The relation between these two quantities can be written in matrix formulation:

$$\boldsymbol{\delta}(\omega) = \boldsymbol{T^{-1}}\,\boldsymbol{U}(\omega)$$

If first story is assumed to be at the bottom, then the inverse transform matrix $\boldsymbol{T^{-1}}$ is:

$$\boldsymbol{T^{-1}} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Consequently the transfer function matrix relative to inter-story drifts is given by:

$$\boldsymbol{H_\delta}(\omega) = \boldsymbol{T^{-1}}\boldsymbol{H}(\omega)$$

If the external excitation is modeled as a stationary random process characterized by its power spectral density (PSD), then the PSD of the response of the structural system is given by:

$$S_U(\omega) = |\boldsymbol{H}(\omega)|^2 S_{\ddot{U}_g}(\omega)$$

where $S_{\ddot{U}_g}(\omega)$ is the power spectral density of the ground acceleration $\ddot{u}_g(t)$. The squared absolute value of the transfer function is defined by:

$$|\boldsymbol{H}(\omega)|^2 = \boldsymbol{H}(-i\omega)\,\boldsymbol{H}(i\omega)$$

and takes the form:

$$|\boldsymbol{H}(\omega)|^2 = ((\boldsymbol{k} - \omega^2\boldsymbol{m})^{\boldsymbol{2}} + \omega^2\,\boldsymbol{c}^2)^{-1}(-\boldsymbol{m}^2)$$

It is to point out that the squared absolute value of the transfer function doesn't depend no more from complex quantities.

The power spectral density of the earthquake ground motion has been modeled here with a Kanai-Tajimi formulation given by:

$$S_{\ddot{U}_g}(\omega) = \frac{1 + 4\xi_g^2 \left(\frac{\omega}{\omega_g}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_g}\right)^2\right]^2 + 4\xi_g^2 \left(\frac{\omega}{\omega_g}\right)^2} S^2$$

where S^2 is a measure of the intensity of the ground motion and ξ_g and ω_g are parameters that depend on the local geological characteristics of the site. This spectrum is obtained by passing a white noise process, S^2, through a second order linear single degree of freedom system whose parameters depend on approximations of real ground motions.

Using this characterization, we can now find the mean square response of inter-story drifts σ_i^2 of the structure at every floor i :

$$\sigma_i^2 = \int_{-\infty}^{+\infty} |\boldsymbol{H}_{\delta i}(\omega)|^2 \, S_{\ddot{U}_g}(\omega) \, d\omega$$

This last value is taken as optimal placement location index since dampers are placed between neighbor floors.

5.2.2 Design methodology

Before starting the design an objective damping value have to be estimated in order to be able to stop the procedure once the aim is achieved. As explained in chapter 2.4.5.2 the estimation of this value is done considering a singular degree of freedom system with the period equal to the first one of the examined structure. Damping ratio is increased until the maximum displacement resulted from time-history analysis takes the desired value. Hence a set of ground motions must be considered. Finally the objective damping ratio can be related to the viscous added damping with:

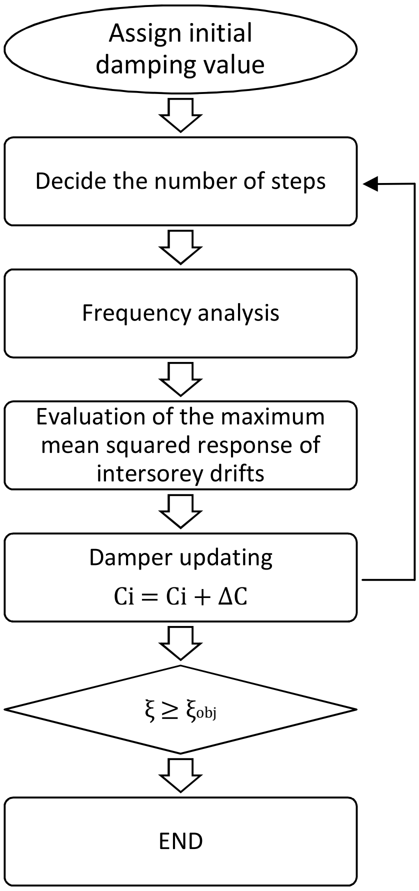
$$c_{tot} = \frac{\xi_{obj} \, T \, \sum_i K_i}{\pi \, f}$$

where ξ_{obj} is the objective dumping ratio, T is the first period of the building, $\sum_i K_i$ is the sum of the lateral rigidity of all floors and f is the factor which considers geometrical amplification.

As seen before the stochastic input is modeled with a Kanai-Tajimi power spectral density. This function can be obtained by fitting the Fourier transforms of design ground motions. For the detailed explanation see Annex A.4.6.

The design methodology is an iterative process which consists of two different steps. Firstly a frequency domain analysis is required in order to find the mean squared values of inter-story drifts, as seen before. These values are taken as indices of the optimal location of dampers. Then in the second phase an increment of damping is provided at the floor that shows the highest index. This procedure is repeated until the required value of damping is achieved.

The solution algorithm is summarized in the following flowchart.



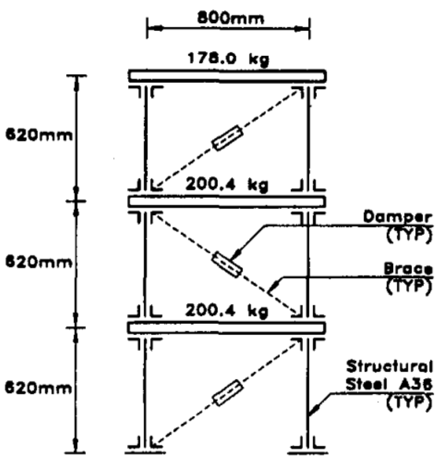
5.3 Example: 3-story shear frame

Mass, stiffness and damping matrices of the 3-story building used by (Guck et al. 1996) follow:

$$\boldsymbol{m} = \begin{bmatrix} 200.4 & 0 & 0 \\ 0 & 200.4 & 0 \\ 0 & 0 & 178.0 \end{bmatrix} kg$$

$$\boldsymbol{k} = \begin{bmatrix} 238{,}932 & -119{,}466 & 0 \\ -119{,}466 & 238{,}932 & -119{,}466 \\ 0 & -119{,}466 & 119{,}466 \end{bmatrix} N/m$$

$$\boldsymbol{c} = \begin{bmatrix} 264.99 & -78.09 & -16.08 \\ -78.09 & 246.89 & -92.15 \\ -16.08 & -92.15 & 162.02 \end{bmatrix} Ns/m$$



The process starts without any initial viscous damper. The value of the total added damping is 12513 *kN s/m* while 20 steps are considered, bringing to an increment of damping of 625.6 *kNs/m* each iteration.
The first three steps are here considered:

$$\boldsymbol{\sigma}^{2^{(1)}} = \begin{bmatrix} 0.0098 \\ 0.0061 \\ 0.0018 \end{bmatrix} \qquad \boldsymbol{c}_{d\,i}^{(1)} = \begin{bmatrix} 625.65 \\ 0.0 \\ 0.0 \end{bmatrix} Ns/m$$

$$\boldsymbol{\sigma}^{2^{(2)}} = \begin{bmatrix} 0.0054 \\ 0.0033 \\ 0.0010 \end{bmatrix} \qquad \boldsymbol{c}_{d\,i}^{(2)} = \begin{bmatrix} 1251.3 \\ 0.0 \\ 0.0 \end{bmatrix} Ns/m$$

$$\boldsymbol{\sigma}^{2^{(3)}} = \begin{bmatrix} 0.0037 \\ 0.0023 \\ 0.0007 \end{bmatrix} \qquad \boldsymbol{c}_{d\,i}^{(3)} = \begin{bmatrix} 1876.9 \\ 0.0 \\ 0.0 \end{bmatrix} Ns/m$$

The final values obtained after eighty iterations are resumed:

$$\boldsymbol{\sigma}^{2^{(final)}} = 10^{-3} \begin{bmatrix} 0.5765 \\ 0.5694 \\ 0.1855 \end{bmatrix} \qquad \boldsymbol{c}_{d\,i}^{(fin)} = \begin{bmatrix} 10636 \\ 1877 \\ 0.0 \end{bmatrix} Ns/m$$

5.4 Observations

Using frequency domain analysis this method attempts to avoid the dependency on the input ground motions providing a more general result.
Although, the amount of total added damping to locate along the structure is evaluated from a set of time-history analyses as described in chapter 2.4.5.2.
Hence also if the final distribution of dampers along the structure is not affected by particularities of the ground motion responses, the total value of this distribution is based on a deterministic approach.
As in other methods the evaluation of the damping to add to the structure is the main weakness because the value of the target damping ratio is estimated considering a proportional damping system time-history responses and the equivalent damping is computed assuming an approximated shape of deformation (see chapter 2.4.5.2). This drawback can be partially avoided taking advantage of the sequential nature of the algorithm. Since the solution is achieved incrementing damper sizes at the optimal location each step, the performance of the structure can be evaluated when certain levels of added damping are reached. Despite there is anyway dependency on deterministic

excitations, this approach enables to consider the effects of the real distribution of damping and not the distribution relative to the Rayleigh model, resulting in a more efficient estimation of the required total damping.
The modeling of the seismic excitation by power spectral density aims to a stochastic description of the seismic input. Although, it was found the parameters of this function does not influence the dampers placement. In fact modeling the power spectral density as a constant white noise, the results change slightly. The comparison between a white noise excitation and the Kanai-Tajimi results for the three story building examined in the example is given below:

Floor	White noise	Kanai-Tajimi
1	10010	10636
2	2503	1877
3	0.0	0.0

Damping coefficients [kNs/m] at different floors

In case of the nine-story building taken in exam in chapter 7.1 the difference is even less significant. This fact is due to the similar shape of the different components of the square transfer vector, which is composed by the sum of the rows of the transfer matrix. The peaks in fact are all located at the same natural frequency of the system and the first peak relative to the first mode of vibration is the dominant. Whatever input power spectral density is chosen it has only a scaling effect on the integral to find mean square inter-story drifts.
As other methods the design of the damped configuration requires the computation of the dynamic stiffness matrix, which is a complicated procedure.
Finally inelastic behavior is not taken into account. This is not significant in case of regular building while could represent a limit in presence of irregular ones.

5.5 MATLAB code

```
MAIN PROGRAM
% INPUT 3story
% Structure
% Mass matrix
m=diag([ 200.4 200.4 178 ])
% Stiffness matrix
k=1000*[238.932 -119.466 0;-119.466 238.932 -119.466;0 -119.466
119.466]
% Inherent damping matrix
c0=[264.99 -78.09 -16.08;-78.09 246.89 -92.15;-16.08 -92.15 162.02]
% Pramiters of Kanai-Tajimi function
Kanaj=[1.4 , 11 ,  61.46];
% Number of degree of freedom
gdl=max(size(m));
% Matrix for the transformation in inter-story quantities
T=eye(gdl)-diag(ones(1,gdl-1),-1);
```

```
% Damping values
nd=20;      % Number of damping increment
W=12513;    % Total added damping
ci=W/nd;    % single increment of damping
% Initializing dampers
cdi=zeros(gdl,1);
cd=T'*diag(cdi)*T;
c=c0+cd;
for l=1:nd
    % Frequency analyisi
    [sigmaout]=MeanSquareValues(m,k,c,Kanaj);
    % Maximal value of mean square drift
    [Max,colonna]=max(sigmaout);
    % Updating damping matrix
    cdi(colonna)=cdi(colonna)+ci;
    cd=T'*diag(cdi)*T;
    c=c0+cd;
end
% Plot of dampers configuration
figure
barh(1:gdl,cdi)
title(['Dampers number: ',num2str(nd)])
%____________________________________________
```

SUBRUTINE MeanSquareValues

```
function [sigma]=MeanSquareValues(m,k,c,Kanaj)

% Function MeanSquareValues estimates the mean square values of inter-
story
% drifts of a given structure and for a given Kanai-Tajimi spectrum
% Input variables:
% m,k,c are the mass, stiffness and damping matrices
% Kanaj is a matrix containing the Kanai-Tajimi parameters
% Internal variables:
% H2 frequency transfer matrix
% Sin Kanai-Tajimi power spectral density
% Sout response power spectral density

% Number of degrees of freedom
gdl=max(size(m));
% Modal Analysis
[S,w2]=eig(k,m);
w=sqrt(w2);
% Pramiters of Kanai-Tajimi function
csig=Kanaj(1);
omegag=Kanaj(2);
Swhite=Kanaj(3);
% Matrix for the transformation in inter-story quantities
T=eye(gdl)-diag(ones(1,gdl-1),-1);
% State space notation
invm=eye(gdl)*diag(1./diag(m));
A=[zeros(gdl) eye(gdl); -invm*k  -invm*c];
B=[zeros(gdl,1);-ones(gdl,1)];
C=[eye(gdl)  zeros(gdl)];
II=eye(2*gdl,2*gdl);
```

```
% Highest frequency
mmm=round(norm(w,inf));
% Initializing matrices
H2=zeros(gdl,100*mmm);
Sin=zeros(1,100*mmm);
Sout=zeros(gdl,100*mmm);
% For cycle on the frequency range
for indice=1:mmm*100
    omega=0.01*indice;
    % Frequency transfer matrix
    HH=C*inv(1i*omega*II-A)*B;
    HH=T*HH;
    Hconiu=conj(HH);
    H2(:,indice)=HH.*Hconiu;
    % Kanai-Tajimi function
    Sin(indice)=(1+4*csig^2*(omega/omegag)^2)/((1-
(omega/omegag)^2)^2+4*csig^2*(omega/omegag)^2)*Swhite;
    % Output power spectral density
    Sout(:,indice)=H2(:,indice)*Sin(indice);
end

% Integral of the power spectral density
for n=1:gdl
        sigma(n)=trapz(0.01:0.01:mmm,Sout(n,:));
end
end
```

Simplified Sequential Search Algorithm

5.6 Introduction

A simplification of the sequential search algorithm presented in (Zhang and Soong 1992) has been developed by (Garcia 2001) and is now taken in exam. The original sequential search algorithm requires frequency domain analyses which are not appropriate for usual engineers practice. For this reason the previous method has been modified, loosing, although, some of his advantages, such as the generality of the solution. The essential idea behind the SSA is that dampers are placed sequentially where their effect is maximized, that is where the mean squared value of inter-story drift is maximized. The simplified approach changes the optimal location index from this latter statistical quantity to values which derive from usual time history analysis such as inter-story drifts or velocities.

The analysis can be carried on deciding an objective value of the total added damping and running the iterative process until this value is achieved, as made in (Zhang and Soong 1992) and (Takewaki 2010). The author although decided, in order to offer a further simplification, to divide the total amount of damping in a defined number of dampers. In this way the damper sizes, usually different at every storey of the building, are equal or multiple of the standard one, decided by the designer. Although interesting from a practical point of view, in order to carry on a comparison between other methods, this last possibility will not be considered.

5.7 Mathematical formulation

5.7.1 Equations of motion

The equations for a linear dynamic viscously damped system are given by:

$$\boldsymbol{m}\ddot{\boldsymbol{u}}(t) + \boldsymbol{c}\,\dot{\boldsymbol{u}}(t) + \boldsymbol{k}\,\boldsymbol{u}(t) = -\boldsymbol{m}\,\underline{1}\,\ddot{u}_g(t)$$

where matrices $\boldsymbol{m}$, $\boldsymbol{c}$ and $\boldsymbol{k}$ characterize mass, damping and stiffness of the structure at the different degrees of freedom. $\boldsymbol{u}$ denotes the horizontal floor displacement vector. The damping matrix $\boldsymbol{c}$ is composed of two contributions: the original structural damping $\boldsymbol{c_0}$ and the one of the viscous devices $\boldsymbol{c_d}$

$$\boldsymbol{c} = \boldsymbol{c_0} + \boldsymbol{c_d}$$

The inter-story drifts $\boldsymbol{\delta}(t)$ are related to the floor displacements with the following relationship:

$$\boldsymbol{u}(t) = \boldsymbol{T}\,\boldsymbol{\delta}(t)$$

where, if the first degree of freedom corresponds to the first floor:

$$\boldsymbol{T} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

The inverse relation will depends on the inverse matrix of T which takes the form of:

$$\boldsymbol{inv}(\boldsymbol{T}) = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

5.7.2 Performance index

The basic idea of this simplified sequential search algorithm is to place added damping where their effects are maximized. The effect takes into account by the author of the paper although is not the inter-story drift, as made in the majority of other researches, but the dissipation of energy. Due to viscous damper energy dissipation depends on the velocity at the extremity of the device, dampers are placed where the inter-story velocity is maximized. Thus this value becomes the performance index:

$$PI = max_i\left(max_t\left(\left|\dot{\delta}_i(t)\right|\right)\right)$$

where i refers to the different floors and t represents the duration of the time history analysis.

5.7.3 *Solution algorithm*

Before starting the design an objective damping value have to be estimated in order to be able to stop the procedure once the aim is achieved. As explained in chapter 2.4.5.2 the estimation of this value is done considering a singular degree of freedom system with the period equal to the first one of the examined structure. Damping ratio is increased until the maximum displacement resulted from time-history analysis takes the desired value. Hence a set of ground motions must be considered. Finally the objective damping ratio can be related to the viscous added damping with:

$$c_{tot} = \frac{\xi_{obj}\ T\ \sum_i K_i}{\pi\ f}$$

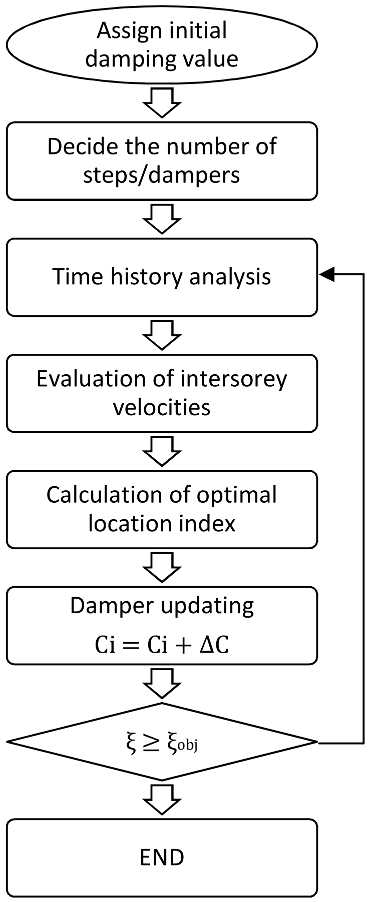
where ξ_{obj} is the objective dumping ratio, T is the first period of the building, $\sum_i K_i$ is the sum of the lateral rigidity of all floors and f is the factor which considers geometrical amplification.

The authors suggested to divide the value of the total added damping in a discrete number n_d of equal dampers in order to obtain a more realistic final configuration. Hence at each iteration one damper of size $c_d = c_{tot}/n_d$ is placed.

The simplified sequential search algorithm is based on time-history analysis, hence a set of ground motions must be chosen for example deriving it from design spectrum.

Then for each accelerogram the optimal configuration is achieved. As already seen in the previous method the single iteration is divided in two steps. In the first one an analysis is carried on to obtain the values of the optimal location indices. In the second part damping is added where the index takes the maximum value. The procedure is repeated until the objective value of added damping is achieved. The final configuration is the envelope of the distribution evaluated for each ground motion.

Hereafter a flowchart of the solution algorithm is presented.



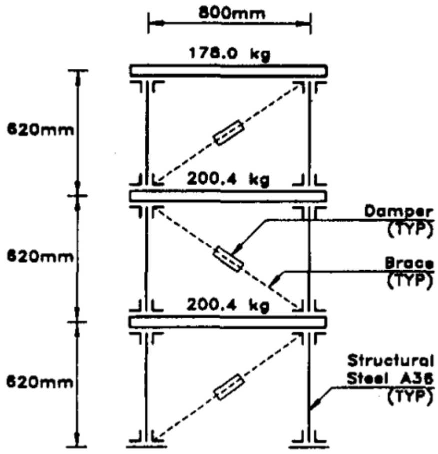
5.8 Example: 3-story shear frame

Mass, stiffness and damping matrices of the 3-story building used by (Guck et al. 1996) follow:

$$\boldsymbol{m} = \begin{bmatrix} 200.4 & 0 & 0 \\ 0 & 200.4 & 0 \\ 0 & 0 & 178.0 \end{bmatrix} kg$$

$$\boldsymbol{k} = \begin{bmatrix} 238{,}932 & -119{,}466 & 0 \\ -119{,}466 & 238{,}932 & -119{,}466 \\ 0 & -119{,}466 & 119{,}466 \end{bmatrix} N/m$$

$$\boldsymbol{c} = \begin{bmatrix} 264.99 & -78.09 & -16.08 \\ -78.09 & 246.89 & -92.15 \\ -16.08 & -92.15 & 162.02 \end{bmatrix} Ns/m$$



LA02 ground motion is taken into account, no initial added damping and a total of ten steps are considered.
The first three iterations are here considered:

$$\dot{\boldsymbol{\delta}}^{(1)} = \begin{bmatrix} 1.0871 \\ 0.8437 \\ 0.5112 \end{bmatrix} m/s \qquad \boldsymbol{c}_{d\,i}^{(1)} = \begin{bmatrix} 1253.9 \\ 0 \\ 0 \end{bmatrix} Ns/m$$

$$\dot{\boldsymbol{\delta}}^{(2)} = \begin{bmatrix} 0.7290 \\ 0.7058 \\ 0.4541 \end{bmatrix} m/s \qquad \boldsymbol{c}_{d\,i}^{(2)} = \begin{bmatrix} 2507.9 \\ 0 \\ 0 \end{bmatrix} Ns/m$$

$$\dot{\boldsymbol{\delta}}^{(2)} = \begin{bmatrix} 0.6239 \\ 0.6274 \\ 0.4164 \end{bmatrix} m/s \qquad \boldsymbol{c}_{d\,i}^{(2)} = \begin{bmatrix} 2507.9 \\ 1253.9 \\ 0 \end{bmatrix} Ns/m$$

The final values obtained are resumed:

$$\dot{\boldsymbol{\delta}}^{(fin)} = \begin{bmatrix} 0.3146 \\ 0.3104 \\ 0.2855 \end{bmatrix} m/s \qquad \boldsymbol{c}_{d\,i}^{(fin)} = \begin{bmatrix} 7523.6 \\ 5015.8 \\ 0.0 \end{bmatrix} Ns/m$$

5.9 Observations

The proposed method represents a simple and efficient alternative to other more complex solutions. Its implementation is intuitive and can be carried out with common commercial software if a discrete number of dampers is chosen. In fact it is only necessary the computation of the peak values of inter-story velocities to place each damper and if their number is not high the procedure can be controlled manually.

This algorithm is based on time-history analysis which highly simplifies the frequency domain analysis used in the first version. On the other hand a set of ground motions must be chosen in order to obtain results endowed of general validity. As seen in the case of analysis-redesign time-history the results for different records can be really different as shown in Figure 5-1 for the case of LA06 and LA08.

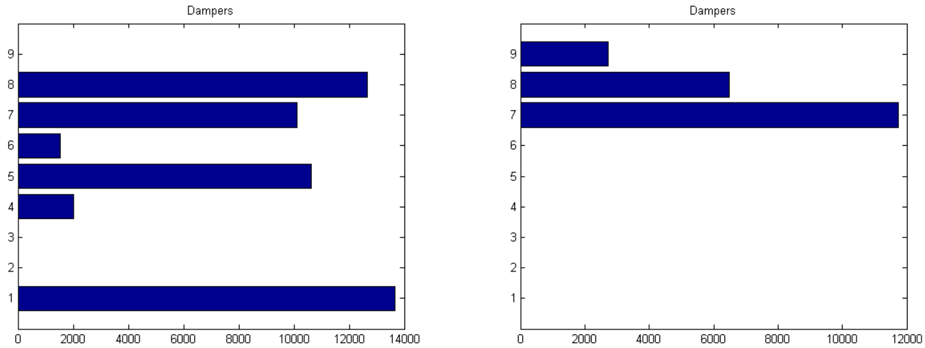


Figure 5-1 Dampers configuration for LA06 and LA08

As a result of the adoption of a set of accelerograms, the computational effort due to the number of analyses to carry out can easily increase and the manual control of the procedure is no more feasible.

A point of great concern of this method is the way to superpose the effects of the different records. The simplest way is to superpose the final configuration achieved in each one of the analyzed cases. Although in this manner the effects of each damper at the other floors is neglected and the obtained solution is no more optimal. In fact the envelope of the different distributions characterized by the same amount of damping results in configuration with a higher level of total added damping

The other way, as seen in analysis-redesign, is to superpose the performance indices obtained from all the different records. In this case the time-history analyses are performed for all ground motions and then the maximum value of the performance index is chosen.

As seen before the criteria to identify the optimal location of dampers consists on finding the place where the viscous device is able to exploit its capabilities, that is the dissipation of energy, in the best way. As a result the parameter taken into account is the inter-story velocity (viscous dampers are velocity-dependent devices). This concept of optimal placement is quite singular between all the procedures here proposed. It is more common in fact the criteria based on the effects that the device produces on the response of the structure. Hence damping is added in correspondence to the place where the structure suffers more damage that usually is where the inter-story drift is maximized. The difference on the results between these two criteria is evident in case of low level of damping, that is for seismic excitations that excite higher modes, as shown in Figure 5-2 for the case of the LA07 record.

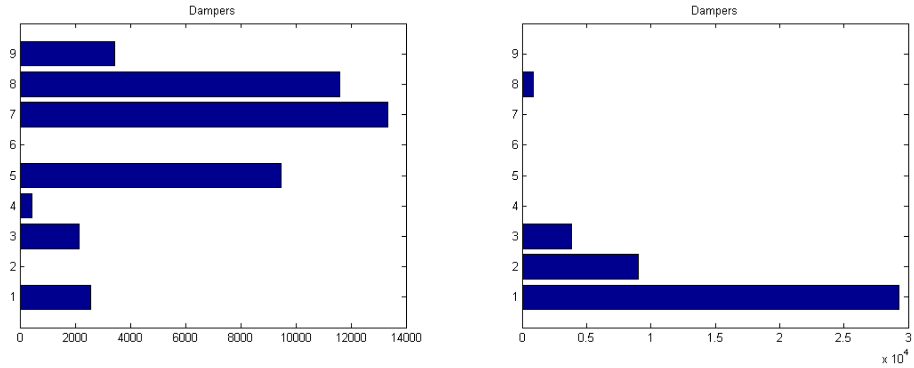


Figure 5-2 Configurations for LA07 using a) original velocity criteria b) drift criteria

For the ground motions that influence mainly the first mode of vibration, and consequently require a higher level of damping, the differences trends to expire because inter-story velocities achieve larger values in correspondence of significant inter-story drifts. Considering for example LA03 accelerogram, a total amount of damping equals to four times the one requested for the LA07 record results from design by analysis-redesign time-history. The final distributions in case inter-story velocities or drifts are taken into account are quite similar as can be seen in Figure 5-3.

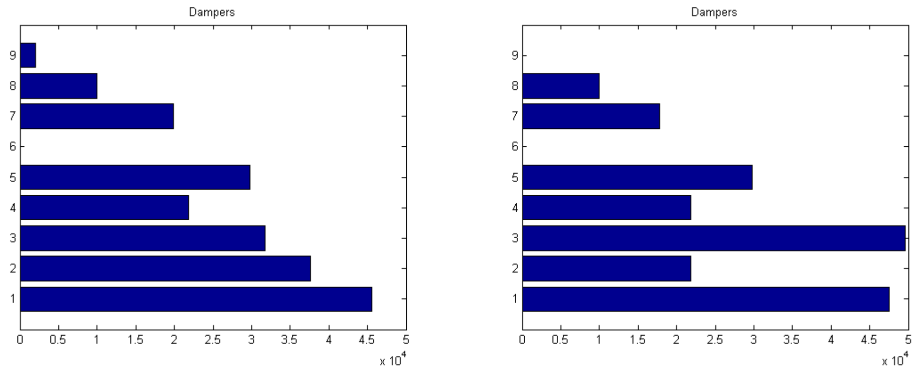


Figure 5-3 Configurations for LA03 using a) original velocity criteria b) drift criteria

As explained in chapter 7.4 the results obtained using inter-story drift criteria are very similar to the one obtained using time-history analysis-redesign optimization method, due to the similar performance index used in the two cases.

This similarity in the final configuration is obtained if the total added damping is divided in a number of parts enough high to be comparable to a continuous analysis. In fact a low number of devices, characterized thus by a larger size, does not fit well the optimal solution. Depicted in Figure 5-4 for the case of the nine-story structure analyzed in chapter 7.1, the results of the method for different divisions of the total added damping under the excitations of LA7 and LA3. The objective damping is divided in different parts and a time-history analysis is carried out using the final configuration of dampers. In figure 5.4 the maximum and the mean value of inter-story drift are shown.

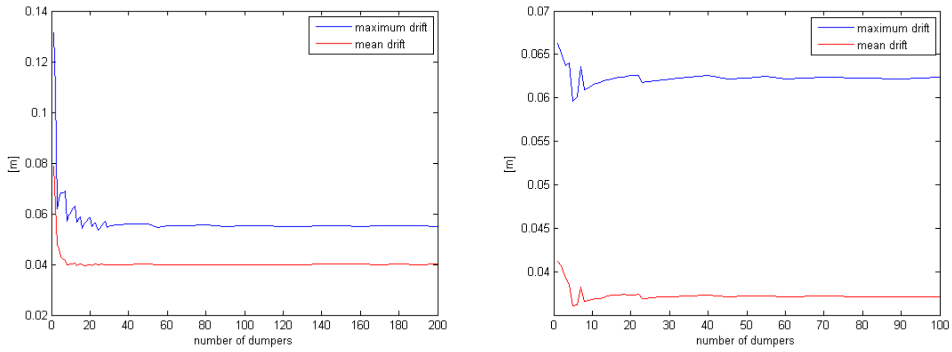


Figure 5-4 Influence of number of dampers for a) LA07 b) LA03

As can be seen the influence of the number of steps is different for the two cases. In case of low intensity ground motion and consequently low level of damping, such as LA7 record, the influence expires after about twenty dampers. Instead for high levels of excitation the influence is lower. Moreover the solution achieved in the continuous case is not the optimal one as can be observed by the minimum in correspondence to the five number of dampers.

5.10 MATLAB code

```
% INPUT 3story frame
% Stiffness matrix [kN/m]
k=1000*[238.932 -119.466 0;-119.466 238.932 -119.466;0 -119.466
119.466]
% Mass matrix [tons]
m=diag([ 200.4 200.4 178 ])
% Hinerent damping matrix [kNs/m]
c0=[264.99 -78.09 -16.08;-78.09 246.89 -92.15;-16.08 -92.15 162.02]
% Initial conditions
IC=zeros(3);
% Number of degrees of freedom
gdl=max(size(m));
% Allowable inter-story drift
DriftsAllow=[0.03 0.03 0.03];
% Ground motion
nomefile='LA2.txt';
Dt=0.02;
fact=1/9.806;
% Modal Analysis
[S,w2]=eig(k,m);
w=sqrt(w2);
Periodo=2*pi/w(1,1)
% Matrix for the transformation in inter-story quantities
T=eye(gdl)-diag(ones(1,gdl-1),-1)
% Initial dampers values
cdi=zeros(1,gdl);
cd=zeros(gdl);
c=c0;
```

```
% Total added Damping Value
TotalDamping=12539.4;
nd=10; % Increments number
ci=TotalDamping/nd; % Single increment
for l=1:nd
    % Time-history analysis
    [u,u1]=Newmark(Dt,nomefile,fact,m,c,k,IC);
    % Inter-story maximal velocities
    d=T*u1;
    Max=max(abs(d'))
    for jjj=1:gdl
        rapporto(jjj)=Max(jjj)/DriftsAllow(jjj);
    end
    [Max,Place2]=max(Max);%max(rapporto);
    colonna=Place2;
    % Updating of damping matrix c
    cdi(colonna)=cdi(colonna)+ci
    cd=T'*diag(cdi)*T;
    c=cd+c0;
end
figure
barh(1:gdl,cdi)
title('Dampers');
xlabel('Damping kNs/m')
ylabel('Floor number')
```

6 MINIMUM TRANSFER FUNCTION

6.1 Introduction

The present methodology, developed by (Takewaki Yamamoto and Fujita 2010), takes into account the inter-story drifts transfer function evaluated at the undamped natural frequency of the structural system. The first version of this method considered as objective function the sum along the height of the building of amplitudes of the transfer function (Takewaki 1997). A similar approach doesn't describe properly structural damage that, especially in irregular buildings, can be concentrated. An appropriate measure to describe structural safety is the maximum value of inter-story drifts and it was adopted in the last version of this method. This latter version is now presented.

6.2 Mathematical formulation

6.2.1 Equations of motion

Consider a N-storey building model with added viscous dampers. The equation of motion is:

$$\boldsymbol{m}\,\ddot{\boldsymbol{u}}(t) + \boldsymbol{c}\,\dot{\boldsymbol{u}}(t) + \boldsymbol{k}\,\boldsymbol{u}(t) = -\boldsymbol{m}\,\underline{1}\,\ddot{u}_g(t)$$

where $\boldsymbol{m}$ and $\boldsymbol{k}$ are the mass and stiffness matrix respectively and $\boldsymbol{u}$ denote the horizontal floor displacement vector. The damping matrix $\boldsymbol{c}$ is composed of two contributions: the original structural damping $\boldsymbol{c_0}$ and the one of the viscous devices $\boldsymbol{c_d}$

$$\boldsymbol{c} = \boldsymbol{c_0} + \boldsymbol{c_d}$$

Let $\boldsymbol{U}(\omega)$ and $\ddot{U}_g(\omega)$ denote the Fourier transforms of $\boldsymbol{u}(t)$ and $\ddot{u}_g(t)$ respectively, and ω the generic circular frequency of a sinusoidal excitation. The equation of motion in the frequency domain becomes:

$$(-\omega^2\boldsymbol{m} + i\,\omega\,\boldsymbol{c} + \boldsymbol{k})\,\boldsymbol{U}(\omega) = -\boldsymbol{m}\,\underline{1}\,\ddot{U}_g(\omega)$$

That can be modified to:

$$\boldsymbol{A}(\omega)\,\boldsymbol{U}(\omega) = \boldsymbol{B}\,\ddot{U}_g(\omega)$$

where

$$\boldsymbol{A}(\omega) = -\omega^2\boldsymbol{m} + i\,\omega\,\boldsymbol{c} + \boldsymbol{k}$$

$$\boldsymbol{B} = -\boldsymbol{m}\,\underline{1}$$

The inter-story drifts $\boldsymbol{\delta}(\omega)$ are related to the floor displacements with the following relationship:

$$\boldsymbol{U}(\omega) = \boldsymbol{T}\,\boldsymbol{\delta}(\omega)$$

where, if the first degree of freedom corresponds to the first floor:

$$\boldsymbol{T} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

The inverse relation will depends on the inverse matrix of T which takes the form of:

$$\boldsymbol{inv(T)} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

The inter-story drifts transfer function can now be defined by the ratio of the drift vector to the ground acceleration:

$$\widehat{\boldsymbol{\delta}}(\omega) = \boldsymbol{\delta}(\omega)/\ddot{U}_g(\omega)$$

$$\boldsymbol{H_\delta}(\omega) = \boldsymbol{\delta}(\omega)/\ddot{U}_g(\omega)$$

which, with the previous equations, takes the form:

$$\widehat{\boldsymbol{\delta}}(\omega) = \boldsymbol{T}\,\boldsymbol{A^{-1}}(\omega)\,\boldsymbol{B}$$

The amplitude of this transfer function is meaningful since the mean squares of the response can be evaluated by multiplying the power spectral density function of the ground acceleration on the squared transfer function itself and integrating that in the frequency domain. The shape of this function for a certain degree of freedom is normally composed by the peaks of each mode of vibration. The higher peak is the first, which corresponds to the first mode. In this lower range of frequency usually also ground motions show their highest values. It means that the most significant contribution of the response is due to the first mode, in which earthquakes principal frequency range is resonant to the fundamental natural frequency of the structure. For this reason the values of the inter-story drift transfer function at the natural frequency will be considered as representative quantities of the response of the building:

$$\widehat{\boldsymbol{\delta}}(\omega_1) = \boldsymbol{T}\,\boldsymbol{A^{-1}}(\omega_1)\,\boldsymbol{B}$$

6.2.2 Optimality criteria

The problem of optimal dampers placement consist on finding the optimal distribution of a given value of viscous damping capacity *W* so as to minimize the magnitude of inter-story-drift transfer function at the natural frequency of the system. The problem can be stated as:

$$\text{Minimize} \qquad J = max_{i,\omega_1}\left|\hat{\delta}_i(\omega_1)\right|$$

$$\text{Subject to} \qquad \textstyle\sum_i c_{d,i} = \overline{W}$$

The Lagrangian *L* for the optimal design problem can be defined as:

$$L(\boldsymbol{c}_d, \lambda) = J + \lambda\left(\sum\nolimits_i c_{d,i} - \overline{W}\right)$$

where λ is the Lagrange multiplier. The optimality criteria can be derived from stationary conditions of *L* with respect to λ and $\boldsymbol{c}_d$:

$$f_{,i} + \lambda = 0 \qquad \text{for i=1,2…N}$$

$$\sum_i c_{d,i} - \overline{W} = 0$$

The symbol $(\ )_{,i}$ denotes the partial differentiation with respect to $c_{d,i}$.

In order to define the present objective function it is necessary to proceed in two steps. First it is to find the degree of freedom associated to the maximum absolute value of the inter-story drift of the transfer function. Then, in correspondence of this story, the maximum value of first-order sensitivities should be searched, or, in other words, it is to find where the placement of a damping device has more influence on that inter-story drift.

The derivation of these quantities is now discussed. Differentiation of frequency domain form of equation of motion with respect to $c_{d,i}$ provides:

$$\boldsymbol{A}_i\widehat{\boldsymbol{U}} + \boldsymbol{A}\widehat{\boldsymbol{U}}_{,i} = 0$$

$\boldsymbol{A}_i$ can be expressed, depending on if the derivation is made respectively the last floor or one of the others, as:

$$\boldsymbol{A}_{,i\neq Ngdl} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \boldsymbol{A}_{,i=Ngdl} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The first-order sensitivities of displacements can be derived as:

$$\widehat{\boldsymbol{U}}_{,i} = -\boldsymbol{A}^{-1}\boldsymbol{A}_i\widehat{\boldsymbol{U}}$$

Due to $\boldsymbol{\delta} = \boldsymbol{TU}$**,** it is possible to obtain also the first-order drifts:

$$\widehat{\boldsymbol{\delta}}_{,i} = -\boldsymbol{TA}^{-1}\boldsymbol{A}_i\boldsymbol{T}^{-1}\widehat{\boldsymbol{\delta}}$$

Because of the frequency domain analysis, drifts and displacements are complex numbers composed by a real and an imaginary part:

$$\widehat{\boldsymbol{\delta}}_i = Re[\widehat{\boldsymbol{\delta}}_i] + i\ Im[\widehat{\boldsymbol{\delta}}_i]$$

The absolute value of drifts is defined by:

$$|\widehat{\boldsymbol{\delta}}_i| = \sqrt{\left(Re[\widehat{\boldsymbol{\delta}}_i]\right)^2 + \left(Im[\widehat{\boldsymbol{\delta}}_i]\right)^2}$$

The first order sensitivities of absolute values of drifts can be expressed as:

$$|\widehat{\boldsymbol{\delta}}_i|_{,j} = \frac{1}{|\widehat{\boldsymbol{\delta}}_i|}\left\{Re[\widehat{\boldsymbol{\delta}}_i]\left(Re[\widehat{\boldsymbol{\delta}}_i]\right)_{,j} + Im[\widehat{\boldsymbol{\delta}}_i]\left(Im[\widehat{\boldsymbol{\delta}}_i]\right)_{,j}\right\}$$

where $\left(Re[\widehat{\boldsymbol{\delta}}_i]\right)_{,j}$and $\left(Im[\widehat{\boldsymbol{\delta}}_i]\right)_{,j}$ are calculated from previous equation.

6.2.3 Solution algorithm

The procedure aims to arrange in the optimal way a given amount of damping capacity.

As explained in chapter 2.4.5.2 the estimation of this value is done considering a singular degree of freedom system with the period equal to the first one of the examined structure. Damping ratio is increased until the maximum displacement resulted from time-history analysis takes the desired value. Hence a set of ground motions must be considered. Finally the objective damping ratio can be related to the viscous added damping with:

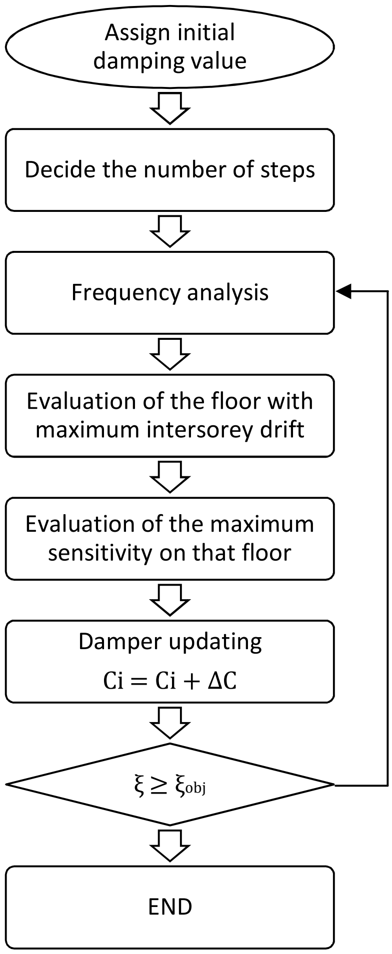
$$c_{tot} = \frac{\xi_{obj}\, T\, \sum_i K_i}{\pi\, f}$$

where ξ_{obj} is the objective dumping ratio, T is the first period of the building, $\sum_i K_i$ is the sum of the lateral rigidity of all floors and f is the factor which considers geometrical amplification.

Naturally a change in the disposition of damping along the structure modifies the values of displacements and drifts. For this reason an iterative procedure must be adopted and the total damper capacity W is increased gradually. Defined N as the number of steps in which the final value of damper capacity is achieved, the increment of damping in each steps is:

$$\Delta c = \frac{W}{N}$$

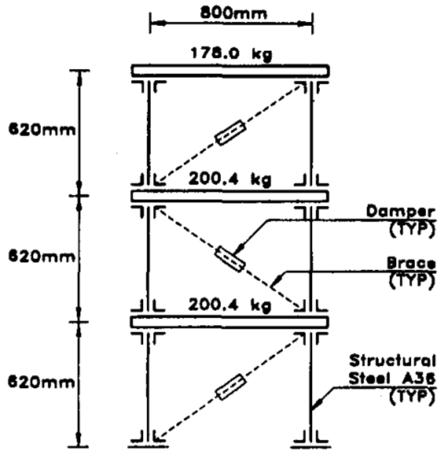
The algorithm starts finding the natural frequency of the structure in order to calculate the peak values of inter-story drift. Then it is necessary to identify the storey which exhibits the maximum amplitude of those peak values. The location of the added damping corresponds to the one in which the first-order sensitivity of drift at the predetermined floor is maximized. As damping increase the performance indices assume similar values and for this reason added damping must be spread between them. The authors proposed a formal way which requires the computation of the second order sensitivities. In order to avoid this expansive calculation the added damping is simply divided in equal parts and distributed in the floors that show similar first order sensitivities. It is to underline that this procedure is not possible in the first version of the algorithm (Takewaki 1997) because in that case an initial amount of damping was modified in order to bring the optimal configuration. Hence damping in a specific location could be added or removed. In this last version instead the initial damping equals zero, and the optimal configuration is achieved adding damping where the performance indices are maximized.



After the placement of the damper increment the damping matrix is updated and the procedure is repeated until the achievement of the prescribed amount of damper capacity

6.3 Example: 3-story shear frame

Mass, stiffness and damping matrices of the 3-story building used by (Guck et al. 1996) follow:



$$\boldsymbol{m} = \begin{bmatrix} 200.4 & 0 & 0 \\ 0 & 200.4 & 0 \\ 0 & 0 & 178.0 \end{bmatrix} kg$$

$$\boldsymbol{k} = \begin{bmatrix} 238{,}932 & -119{,}466 & 0 \\ -119{,}466 & 238{,}932 & -119{,}466 \\ 0 & -119{,}466 & 119{,}466 \end{bmatrix} N/m$$

$$\boldsymbol{c} = \begin{bmatrix} 264.99 & -78.09 & -16.08 \\ -78.09 & 246.89 & -92.15 \\ -16.08 & -92.15 & 162.02 \end{bmatrix} Ns/m$$

There is no initial added damping.
A total added damping of $W = 12512.9\,kNs/m$ and a steps number of $N = 100$ are considered.
The derivatives of matrix $\boldsymbol{A}$ follow:

$$\boldsymbol{A}_1 = \begin{bmatrix} 11.21\,i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \boldsymbol{A}_2 = \begin{bmatrix} 11.21\,i & -11.21\,i & 0 \\ -11.21\,i & 11.21\,i & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{A}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 11.21\,i & -11.21\,i \\ 0 & -11.21\,i & 11.21\,i \end{bmatrix}$$

The first three steps are here considered:

$$\left|\widehat{\boldsymbol{\delta}}_i\right|^{(1)} = \begin{bmatrix} 0.1110 \\ 0.0876 \\ 0.0457 \end{bmatrix} \quad \left|\widehat{\boldsymbol{\delta}}_i\right|^{(2)} = \begin{bmatrix} 0.0954 \\ 0.0753 \\ 0.0393 \end{bmatrix} \quad \left|\widehat{\boldsymbol{\delta}}_i\right|^{(3)} = \begin{bmatrix} 0.0836 \\ 0.0660 \\ 0.0345 \end{bmatrix}$$

For each maximum absolute value of inter-story drift the first order sensitivities are given:

$$\left|\widehat{\boldsymbol{\delta}}_i\right|_{,j}^{(1)} = 10^{-3} \begin{bmatrix} 0.1453 \\ 0.0903 \\ 0.0246 \end{bmatrix} \quad \left|\widehat{\boldsymbol{\delta}}_i\right|_{,j}^{(2)} = 10^{-3} \begin{bmatrix} 0.1073 \\ 0.0667 \\ 0.0182 \end{bmatrix} \quad \left|\widehat{\boldsymbol{\delta}}_i\right|_{,j}^{(3)} = 10^{-3} \begin{bmatrix} 0.0824 \\ 0.0513 \\ 0.0139 \end{bmatrix}$$

Then the updated dampers vectors are:

$$\boldsymbol{c}_{d\,i}^{(1)} = \begin{bmatrix} 125.1 \\ 0.0 \\ 0.0 \end{bmatrix} Ns/m \quad \boldsymbol{c}_{d\,i}^{(2)} = \begin{bmatrix} 250.3 \\ 0.0 \\ 0.0 \end{bmatrix} Ns/m \quad \boldsymbol{c}_{d\,i}^{(3)} = \begin{bmatrix} 375.4 \\ 0.0 \\ 0.0 \end{bmatrix} Ns/m$$

The final values obtained are resumed:

$$\left|\hat{\boldsymbol{\delta}}_i\right|^{(fin)} = \begin{bmatrix} 0.0068 \\ 0.0068 \\ 0.0037 \end{bmatrix} \; \left|\hat{\boldsymbol{\delta}}_i\right|_{,j}^{(fin)} = 10^{-6} \begin{bmatrix} 0.5899 \\ 0.4086 \\ 0.1073 \end{bmatrix} \; \boldsymbol{c}_{d\,i}^{(fin)} = \begin{bmatrix} 9509.8 \\ 3003.1 \\ 0.0 \end{bmatrix} Ns/m$$

6.4 Observations

Since the analysis is carried out in the frequency domain, it avoids the dependency from the particularities of seismic excitation, present instead in all the methods that use time domain time history and, consequently, particular ground motions. This independency gives to the obtained results a more general validity against phenomena like earthquakes characterized by a wide variability. On the other hand, as the specific conditions at the site are not considered, the efficiency of the damping attained is lesser. Moreover, the input considered by the methodology is actually a harmonic excitation with a frequency equal to the fundamental frequency of the structure.

Note also that it is not immediate to determine a proper value for the total amount of damping *W*. As seen in the descriptions of the other methods its evaluation is possible a priori, as suggested in chapter 2.4.5.2, or during the course of the optimization process. In this latter case damping is added until the performance of the building is acceptable.

Finally, as other methods the design of the damped configuration requires the computation of the dynamic stiffness matrix and cannot account for nonlinear response.

6.5 MATLAB code

```
% INPUT 3story frame
% Stiffness matrix [kN/m]
k=1000*[238.932 -119.466 0;-119.466 238.932 -119.466;0 -119.466
119.466]
% Mass matrix [tons]
m=diag([ 200.4 200.4 178 ])
% Hinerent damping matrix [kNs/m]
c0=[264.99 -78.09 -16.08;-78.09 246.89 -92.15;-16.08 -92.15 162.02]
% Initial conditions
IC=zeros(3);
% Number of degrees of freedom
gdl=max(size(m));
% Matrix for the transformation in inter-story quantities
```

```
T=eye(gdl)-diag(ones(1,gdl-1),-1)
invT=tril(ones(gdl,gdl));
% Total added damping
W=12512.9;
N=100;  % number of increments
Dc=W/N; % single increment of damping
% Initial dampers values
cdi=zeros(gdl,1);
cd=T'*diag(cdi)*T;
c=cd+c0;
% Modal Analysis
[S,w2]=eig(k,m);
w=sqrt(w2);
omega1=w(1,1)
periodo1=2*pi/omega1
% Main cycle on the step number N
for q=1:N
    % Transfer matrix
    A=k+1i*omega1*c-omega1^2*m;
    invA=inv(A);
    % Derivative of the transfer matrix
    for nn=1:gdl
        if nn==1
            Aderivata(nn,nn,nn)=1;
        else
            Aderivata(nn,nn,nn)=1;
            Aderivata(nn-1,nn-1,nn)=Aderivata(nn,nn,nn);
            Aderivata(nn,nn-1,nn)=-1;
            Aderivata(nn-1,nn,nn)=Aderivata(nn,nn-1,nn);
        end
    end
    Aderivata=Aderivata*omega1*i;
    % Inter-story drifts
    delta=-T*invA*m*ones(gdl,1)
    deltaABS=abs(delta);
    % First order sensitivity of inter-story drifts
    for nn=1:gdl
        delta1derivato(:,nn)=-T*invA*Aderivata(:,:,nn)*invT*delta;
    end
    % First order sensitivity of absolute values of inter-story drifts
    for nn=1:gdl
        for nnn=1:gdl

deltaABS1(nn,nnn)=(real(delta(nn))*real(delta1derivato(nn,nnn))+...
            imag(delta(nn))*imag(delta1derivato(nn,nnn)))/deltaABS(nn);
        end
    end
    deltaABS1=abs(deltaABS1);
    % Maximum inter-story absolute value
    [mass1,piano]=max(deltaABS');
    % Maximum first order sensibility absolute value
    [mass2,dove]=max(deltaABS1(piano,:));
    % Counting how many values are similar(2% of difference)
    posizioni=[];
    cont=1;
    for bb=1:gdl
        if (mass2-deltaABS1(piano,bb))/mass2<0.02
```

```
            posizioni(cont)=bb;
            cont=cont+1
        end
    end
    % Number of maximum values
    fine=size(posizioni,2)
    % Updating damping matrix
    if fine>1
        for tt=1:fine
            colonna=posizioni(tt);
            cdi(colonna)=cdi(colonna)+Dc/fine;
        end
    else
        colonna=dove;
        cdi(colonna)=cdi(colonna)+Dc;
    end
    cd=T'*diag(cdi)*T;
    c=c0+cd;
end
figure
barh(1:gdl,cdi)
title('Dampers');
xlabel('Damping kNs/m')
ylabel('Floor number')
```