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INTEGRATED ANALYSIS OF MANUFACTURING AND SUPERVISORY SYSTEMS

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Abstract

Studies in performance evaluation of automated manufacturing systems have focused on the development of tools that support performance measurement, efficient design, and reconfiguration of manufacturing systems. The use of these tools play a critical role in achieving manufacturing target performances such as average throughput, work in progress and lead time during the design and operation phases of a system. In response, manufacturing systems engineering research in the last decades has developed powerful performance evaluation tools and models that are capable of accurately and efficiently modeling various systems.

Traditionally, many of manufacturing system engineering tools assume that machine reliability parameters, such as (Mean Time to Failure and Mean Time to Repair) are available and precisely known. However, in practical situations these parameters are either estimated from real life data or based on experts' knowledge. In both cases, they are subjected to uncertainty. Indeed, the validity of important system design decisions is dependent on the ability to carry out a significant analysis of the system performance in presence of uncertainty. In addition with digital manufacturing tools becoming increasingly an integral part in the design and operation of manufacturing systems, their design and specification strongly impacts system understanding. Therefore assisting the integrated analysis and design of these tools in relation to manufacturing system configurations is of paramount importance, which motivates this research.

The first part of this work proposes methods for the performance analysis of smaller manufacturing systems using exact analytical methods with uncertain parameters estimates. The impact of performance analysis using real data in contrast with precisely known parameters assumptions is investigated. Performance deviations as high as 15% estimation errors are observed by carrying out the analysis ignoring uncertainty in estimations. Important findings from this analysis are highlighted and the relationships that explain the observed differences are analytically presented.

Emphasizing on the proven advantages of performance analysis on smaller systems with real data the following parts of the work focus on the development of tools that support performance analysis in complex systems. Alternative approximate techniques that are accurate and efficient in measuring the performance of multi-stage manufacturing systems are proposed. Numerical accuracy and applicability of the proposed methods are presented under different conditions. Additionally a new method based on the decomposition of multi-stage manufacturing lines for the estimation of average throughput is proposed. The method is proved to be accurate and computationally efficient to study long lines. It is used to study and understand important system behaviors under uncertainty, providing important insights in system design under practical scenarios.

A gradient based algorithm for the optimal supervisory systems reconfiguration and manufacturing systems reconfiguration is proposed. The method attempts to improve the estimation of the output performance uncertainty by optimally allocating supervisory resources. Exploiting the developed techniques in this work it targets to minimize input uncertainty on the parameters which highly contribute to the output uncertainty. On the other hand it addresses impact of configurations on performance uncertainty by choosing alternative buffer configurations so that target performances can be guaranteed. This allows system designers to evaluate alternative solutions that satisfy a required level robustness for the available resources and knowledge on design parameters.

Based on existing buffer optimization techniques, a new approach for the optimization of manufacturing systems under uncertain parameters is proposed. The approach aims at providing the optimal buffer configuration that guarantees the satisfaction of target performances with a given confidence level. Analysis with the traditional approach that addresses the same problem is observed to provide a guarantee level as low as 43%, which compromises system robustness in achieving target performance. The level of additional information or the necessary buffer configuration required in order to introduce desired level of robustness can be analytically determined using this method. The proposed approach is also used for the analysis of an industrial case featuring a buffered multi-stage manufacturing system.

Finally, based on the result of this study general design and managerial insights are given in the design and operation of manufacturing systems under uncertainty, which is the case in most practical situations. Future research works that extend the work for improvement of analysis techniques and including additional problems in the integrated analysis and design of supervisory and manufacturing areas are identified and suggested.

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Chapter One

1. Introduction

The need for the design, development and operation of manufacturing systems that highly guarantee to achieve target performances is a high priority goal in manufacturing. As manufacturing enters a new era in which enterprises must compete in a global market the importance and challenge of these activities is ever important. Meeting the demanding and dynamic external targets with appropriate design and operation of manufacturing system also plays a decisive role in the success or failure of an enterprise. In order to keep competitive nowadays companies are increasingly interested in assisting the design and operation of advanced manufacturing systems by implementing modern digital manufacturing tools. Technological advances in sensor and information technology enables the acquisition and storage of huge amount of precise data and information about the behavior of the systems for decision making. On the other hand the optimal design, reconfiguration and operation of manufacturing systems is supported by the use of modern and suitable analysis tools, including simulation and analytical methods. Basically, there is the strong link between the technologies which gather system information and the manufacturing systems engineering tools that must be fed with this information to carryout the analysis of manufacturing systems (Gershwin, 1994, 2000). However, in spite of the strong relationship between the two fields they are normally treated independently by researchers and practitioners.

In manufacturing systems there are different phases in which the decision making and the required system information for the decision should be considered together. During the "green field" design phase, the technical efficiencies of the resources/machines that shall compose the manufacturing system are considered as nominal values, provided by the equipment/sensor producers. In the system operational phase, the technical efficiency of the machines can be estimated by using historical data, i.e. the machines' operational records, typically stored in the company production monitoring system database. The high cost associated in changing decisions made during early design phase of manufacturing systems emphasize the need to make these decisions right the first time. These decisions range from the choice of type of manufacturing systems such as dedicated lines, batch or flexible manufacturing systems to specific machine choices and configurations at lower level. In practice, designing the details of manufacturing systems (equipment design and specification, layout, manual and automatic work content, material and information flow, etc.) in a way that is supportive of a firm's business strategy has proven to be a difficult challenge (Cochran et al, 2002). Partly this challenge owes itself to the inherent complexity of manufacturing systems involving many interacting elements. Moreover it can be difficult to understand the impact of detailed, low-level deficiencies and change the performance of a manufacturing system as a whole. Therefore the role of precise information in these activities is immense given the need to design of systems which are inherently complex.

During operational phase of a manufacturing systems traditionally, the reliability of machines is modeled through the characterization of the Mean Time to Failure (MTTF) and the Mean Time to Repair (MTTR) of each failure mode affecting the machine productivity. Normally these parameters are estimated by using historical data on the machines that is collected and stored in the manufacturing monitoring system. Although these estimates are assumed to be the mean of statistical distributions (typically exponential or geometric distributions), their value is considered as known deterministically. However, if they are gathered by using a sample size of 5 instead of 1000 failure observations, the resulting level of confidence on their mean value is clearly different. Depending on the availability of information on the model parameters, the estimates are also subjected to uncertainty. Therefore the subsequent analysis and the reliability of analysis output is dependent on these uncertain estimates, and this should be measured for a valid decision making.

Various frameworks and tools with the goal of assisting decision making at different phases and levels of manufacturing systems design and development have been developed. The fundamental focus of these design and analysis tools is usually targeted at capturing characteristic behaviors that defines most modern manufacturing systems. According to scholars in manufacturing systems engineering (Gershwin 2000), some of the defining elements that are common to most of the manufacturing systems are the following.

- Events that are relevant to manufacturing systems, such as breakdowns, arrival of parts, repairs can be random; consequently manufacturing systems rarely perform as expected.
- Manufacturing systems are complex which are characterized by high interoperability of different resources, functions and objectives, makes the analysis of the impact of local reconfiguration decisions on the system performance a difficult.
- Complexity, multiplicity and uncertainty of variables of different natures and the information that are used to predict and estimate events, process parameters and relationship between interacting subsystems are fundamental.

Many works in the performance analysis of manufacturing systems have been proposed in response to these fundamental requirements. Research in simulation and analytical models has targeted the need to model *complex* and the *stochastic* nature of manufacturing systems. Recent works have perceived the need to address inherently related performance measures of manufacturing systems; such as the *trade-off* between quality and productivity. The importance of obtaining clear, sufficient and precise *information* to carryout valid performance analysis is also a recognized challenge in decision making.

In order to improve productivity of manufacturing systems and minimize errors on final products and process, there is a growing interest in a precise and robust performance analysis of manufacturing systems. The design and analysis of manufacturing systems in terms of choice of machines, decision of how much space to allocate for parts when some of machines are down are important focus of recent research (Gershwin 2002). Strategies to respond and design for the inherent randomness of events that characterize manufacturing systems, failures, repairs, part arrivals, changes to system behavior necessitate the development of many stochastic models and analysis tools. The study of performance measurement in the presence of random events such as machine and quality failures with regard to the choice of processing machines, capacity of material handling equipments and buffers has generated a lot of research interest. Many stochastic analytical and simulation models have been developed to understand and respond to the randomness and the subsequent degraded performance of a manufacturing system.

Performance analysis and decision making under this inherent complexity during design and operational stages of a manufacturing system is a demanding challenge. More importantly, in practice performance analysis has to be carried out with limited information gathered from an operating system or a preexisting knowledge which makes modeling for performance prediction difficult (Gershwin, 1994, 2000). Due to cost reasons it is important to detect incorrect and inefficient behavior in the early stages of the system. For these and additional reasons the design of information systems considering information for the purpose of decision making as opposed to the mere sophistication of keeping of data from the actual manufacturing system is an important element of manufacturing design (MacGregor Smith J. 2005). Traditionally the task of obtaining sufficient data for modeling and analysis in most cases is an area left for software designers, database designers and practitioners. On the other hand most of analytical and simulation models assumed there is enough information to use the appropriate models.

The problem of "sufficient" information is equally relevant even for modern manufacturing systems equipped with state of the art information systems. Quite often, in manufacturing there are critical decisions that don't allow waiting until all the necessary amount of information is collected and obtained for precise estimation of parameters and the subsequent decision making. Real industrial practices require performance analysis and decision to be made with the available limited information on hand. In many cases actual situations require to make decisions under uncertainty.

Embedding uncertainty in the system performance evaluation and design process is of paramount importance for generating system configurations that are robust to input parameter estimation uncertainty. Moreover, it makes it possible to know how the level of uncertainty associated to each input parameter impacts the resulting uncertainty in the output performance measure, and to refine the level of confidence of the input parameters accordingly. For example, if the system is already existing, the sampling plan can be adaptively modified to gather more data about the most critical resources in the system (bottlenecks) and to decrease the monitoring effort for less critical resources, thus providing data management policies that are functional to the achievement of a desired level of confidence in the output system performance estimation.

In spite of the industrial relevance of this problem, in the literature Manufacturing System Engineering approaches, including both simulation and analytical methods never considered this problem. Traditionally, the reliability of machines is modeled through the characterization of the Mean Time to Failure (*MTTF*) and the Mean Time to Repair (*MTTR*) of each failure mode affecting the machine production. Although these are assumed to be the mean of statistical distributions (typically exponential or geometric distributions), their value is considered as known deterministically. However, depending on the resulting level of confidence and knowledge on the estimation of these input parameters they are subjected to uncertainty. Traditionally, the considered performance measures are the average throughput and the average inventory levels of the system. Again, these are considered to be precise estimates, although they are strongly affected by the input parameters' uncertainty. Important issues in using estimated reliability parameters for performance evaluation is discussed in (Denaro et al, 1998) and (Lin et al, 2008). The growing use of online data collection systems for manufacturing systems and the potential of integrating data collection to performance evaluation is also pointed out in literature This further motivates the research.

When performance evaluation has to be carried out using operational data from supervisory systems parameters must be estimated from actual data and this introduces inherent uncertainty in the estimates. This requires performance evaluation techniques that take into consideration this estimation uncertainty introduced in the input parameters. This uncertainty and the complexity of manufacturing systems highly influence the design, management and operation process, by posing serious challenges towards the achievement of their target performance. As a matter of fact, uncertainty analysis and robust system performance measurement are crucial activities for manufacturing competitiveness. Indeed, several important system design decisions are dependent on the ability to carry on a significant analysis of the system performance in presence of uncertainty. Uncertainty in system design/re-design phases may be either generated internally or externally to the system; internal uncertainty is related to imprecise characterization of the events that affect the technical efficiency of the resources in the system, i.e. breakdowns and disturbances; external uncertainty is related to the difficulty in prediction of the system design requirements, mainly due to the market volatility and turbulence. This thesis will focus on the first source of uncertainty, i.e. internal uncertainty.

From a practical point of view, a systematic approach towards uncertainty is an essential step to support both the "green field" design and the re-configuration phases. During the "green field" design phase, the technical efficiencies of the resources/machines that shall compose the manufacturing system are considered as nominal values, provided by the equipment/sensor producers. However, when installed and integrated in the system, these resources typically prove to perform differently from what expected, due to the specific operational conditions and control system settings. Therefore, in order to capture this deviation in the "green field" design phase and to generate a robust system configuration, uncertainty should be associated to the resource efficiency estimates, used as input parameters of the design process. On the contrary, in the system operational phase, the technical efficiency of the machines can be estimated by using historical data, i.e. the machines' operational records, typically stored in the company production monitoring system database. In this case, estimates are subjected to uncertainty due to the specific sampling plan adopted.

The implementation of supervisory control and monitoring systems has a growing importance and role in automated manufacturing systems. One dominant role of implementing these systems is to enable autonomous execution of operations with complex logic and sequence that must be satisfied for the manufacturing system to achieve the desired processing activity. Equally important is their role for the collection of actual data on states and conditions of machines equipments from sensors installed in the manufacturing system. Generally supervisory systems report, display and alert, notify status of machines and equipments and respond automatically to safeguard conditions before equipments enter unsafe states. They record the real time events, states notifications of processing equipments, parts, material handling system from respective sensors installed at different parts of the manufacturing system. These fundamental requirements in manufacturing activities and other additional advantages of employing supervisory control and monitoring systems are increasingly making them an integral part of automated manufacturing systems. Depending on the type of the manufacturing system and the main goals of implementing supervisory control and monitoring systems the alternative solution in terms of specification, design and analysis can be different. Their role as information and data provision for performance analysis is a critical objective and needs a due consideration.

In addition to the choice of manufacturing systems elements, such as processing machines and design of space for storage of semi processed, the performance evaluation and validation process needs to consider supervisory control and monitoring systems. The motivation of involving supervisory and monitoring systems to performance evaluation can be seen from two major perspectives. Firstly, performance evaluation with actual data collected by supervisory systems is crucial in understanding and studying the actual behavior of the manufacturing system (Ioannidis, S. et al, 2004). The collections of signals about events that characterize the manufacturing system from performance evaluation perspective have to be considered in designing the supervisory system (Lafortune S. et al, 2001), (Cao Y. et al, 2005). On the other hand the logical sequence of operations and control rules specified in the design of the supervisory system impacts the manufacturing system behavior, thereby the corresponding performance. The specification of the supervisory systems therefore can determine the type of model and approach to be used for the performance evaluation of the integrated manufacturing system. This strong relationship between supervisory monitoring systems and manufacturing systems on performance analysis requires the system designer to consider the impact of one on the other before arriving on the final decisions of design parameters.

The objective of this research can be viewed from two main perspectives in the integrated analysis of manufacturing and supervisory monitoring tools. The primary objective is to develop performance measurement techniques from operational data for an existing manufacturing system that is controlled by a supervisory monitoring system where real time data collection is performed. This part of the analysis looks how performance evaluation can be performed from actual data, especially when the parameters for performance evaluation are estimated from data and therefore subjected to uncertainty. The ability to carry out performance analysis with uncertainty can be equally applied to the design of manufacturing system during green-field design phase when manufacturing systems parameters are not precisely known. The link between performance evaluation and actual data is rarely considered in manufacturing systems and it is one of the primary goals in this study.

The second goal is to assist the definition of supervision requirements and data gathering needs for improving reliability of the performance measurement and analysis. The proposed analysis is required to provide a feedback on possible uncertainty reduction and improvement of input parameters. This analysis should assist the optimal reduction of uncertainties by adaptively changing resource constrained reconfigurable and adaptive supervisory systems. In practice and the research field of supervisory systems methodologies are developed that guarantee minimal observations for specified requirements. These requirements might arise from different aspects, including observability, controllability and other functional requirements of the system. The analytical nature of the analysis methods proposed in this study enables the measurement of uncertainty as contributions by different input parameters. Unlike statistical methods based on sampling these methods are capable in discriminating between parameters that should be estimated more precisely than others.

The Supervisory Control Theory (SCT) is developed to provide a formal methodology for the automatic synthesis of controllers for Discrete Event Systems (DES). The theory makes a clear definition and distinction between the system to be controlled, called plant and the entity that controls it, called supervisor. Consideration of supervisory systems during the performance evaluation of manufacturing systems is an essential component for various reasons. Increasing observability detail on supervisory system has a monotonically increasing impact on the quality of knowledge of the system parameters and the subsequent performance evaluation. Although it appears that increasing information detail is always desirable for improved analysis, the minimal designs of supervisory systems are preferable from the economical point of view. This has to be studied taking into consideration the functional requirement of the supevisory system from the controllability and observability of events that characterize the given manufacturing system. Significant literature has covered the problem of supervisory control theory based on the framework initially developed by (Ramadge and Wonham 1987) and extended by much research to date. In this research many studies developed and demonstrated basic and fundamental properties of supervisory design solutions. At this link the objective of this research is providing a feedback to supervisory design problems from the resulting performance analysis so that reliability and robustness of analysis is achieved by improved observability. Allocation of sensors regarding the minimal and necessary observability of events can be determined which events should be recorded to perform the desired type of performance evaluation and parameter estimations.

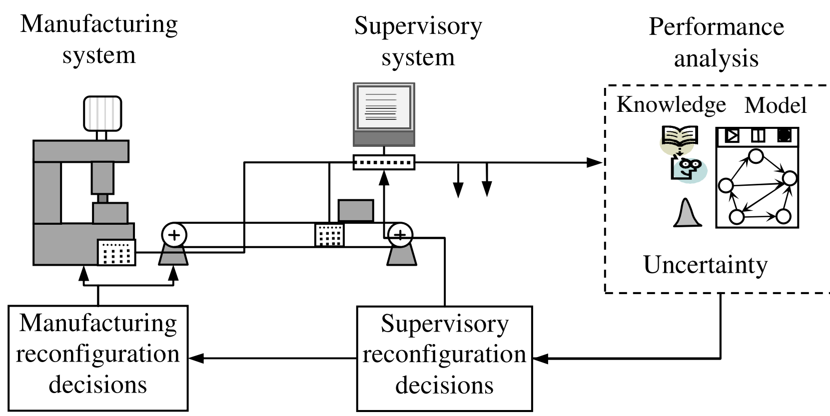


Figure1:1 Integrated data collection and performance evaluation

In this thesis the relationship among manufacturing system performance analysis models and the use of operational data for parameter estimation with uncertainty is investigated. The study is conducted on different manufacturing systems including single machine systems and multi-stage buffered complex manufacturing systems. Different approaches to analyze the performance of manufacturing systems composed of unreliable machines when machine failure and repair parameters are known with uncertainty are proposed. A new method for the analysis of and multi-stage systems with capacitated buffers is developed and used to study the behavior of long lines under uncertainty. Implication of performance analysis under uncertainty and the impacts on decision making during system design/re-design are stressed. The analytical investigation of performance evaluation from actual data and the resulting estimation uncertainty paves the way to the development of a new manufacturing system engineering theory for the robust design of manufacturing systems.

The thesis is organized in the next seven chapters as follows: in chapter 2 literature review covering main contributions and developments on the four areas related to this research are provided. Contributions on (1) analytical performance evaluation models with precise model parameter assumption, developments on analysis tools based on actual data including (2) Bayesian models and (3) fuzzy Markov chains are discussed. Moreover (4) methodologies and considerations in the design and configuration of supervisory systems are highlighted.

Chapter 3 introduces important concepts in the classification, taxonomy and briefly discusses relevant terminologies on typical characteristics and material flow of manufacturing systems. Notations, assumptions and modeling assumptions are discussed that will be used in the subsequent parts of the thesis.

Chapter 4 begins with the discussion on the estimation of input parameter from actual data by using a Bayesian scheme in order to model uncertainty on inputs. Alternative exact and approximate techniques are proposed with detailed procedures for performance evaluation of manufacturing systems with uncertainty parameters. Fundamental differences in conducting performance evaluation with traditional approaches in comparison with proposed techniques with estimated uncertain parameters are presented and analytical proofs are provided. A new method for the performance evaluation of buffered multi-stage serial lines and complex manufacturing systems is presented in detail.

Chapter 5 presents numerical validation for accuracy testing of the methods presented in chapter 4 is provided. Results are reported for each method. Extensive experiments on the comparison between the accuracy and computational efficiency of methods are given. Comparisons on the methods are performed based on exact analytical methods and Monte Carlo simulations on different systems sizes.

Chapter 6 Generalized systems behaviors demonstrated by performance analysis under uncertainty are discussed. Exhibited behaviors are summarized based on systems architectures for two machine single buffer lines and multi-stage lines with accompanying analytical explanations for the observed behaviors. Related practical implications on system configurations and the link between bottle neck resources and estimation uncertainty is explained.

Chapter 7 a gradient algorithm based on the methods developed in chapter 4 is proposed for the reduction input uncertainty and choice of buffer configuration. The first problem deals with how to better allocate sampling and data collecting efforts and resources in order to optimally reduce input uncertainty. Moreover a method considering buffer allocation problem on two machine lines and longer lines based on the original buffer allocation problem is proposed. The impact of uncertain estimates on the buffer capacity decision is demonstrated in comparison with the original buffer allocation method.

Chapter 8 A real case study featuring multi-stage production line with supervisory system for data acquisition is analyzed using proposed method in this thesis. The importance in the application of the proposed framework is highlighted from the analysis results.

In chapter 9 final conclusions and important possible extensions for future research are highlighted.

Chapter Two

2. Literature review

Given the complex challenges during the design and operations of manufacturing systems a significant amount of research effort has been dedicated in the area of manufacturing systems engineering. In the literature, methods that support performance analysis and help to understand the dynamics of manufacturing systems have been developed. Vast research work on manufacturing system analysis addressed manufacturing issues from perspectives which are the most defining characteristics of manufacturing systems. Gershwin in (Gershwin 2002) outlined some of the main challenges that have been the drivers of this research direction in the study of modern manufacturing systems engineering. Out of many challenges he highlighted the most important ones as; *complexity*, *randomness*, *heterogeneity*, *constraints and trade-offs* and *Information*.

Many works in the performance analysis of manufacturing systems have been proposed in response to the above fundamental requirements. Research in simulation and analytical models has targeted the need to model *complex* and the *stochastic* nature of manufacturing systems. Mainly stochastic models for modeling manufacturing systems and performance evaluation aim to capture the behavior of manufacturing systems under unpredictable events such as machine failures and repairs. More recently works have perceived the need to jointly address complexity, multiplicity and uncertainty of variables of different natures and inherent *trade-off* between these variables such as quality and productivity.

The importance in obtaining clear, sufficient and precise *information* to carryout valid performance analysis is also a recognized challenge in decision making. In literature, the stochastic models for performance analysis and the information required to estimate the model parameters are often considered separately. Data driven analysis of complex manufacturing systems is rarely addressed. Model parameter estimation from data such as mean time to failure (MTTF) and mean time to repair (MTTR) and the subsequent performance analysis are considered two independent activities. The impact of one analysis on the other and their mutual interaction in performance analysis of complex systems is not well investigated and understood however important. For instance if the impact of collecting various systems information is a costly activity then observations and data sampling efforts from supervisory systems should be done in an efficient way. Supervisors should be reconfigured in order to facilitate a more reliable performance analysis based on what needs to be estimated more precisely than others. However; the design and reconfiguration of monitoring and supervisory systems in manufacturing systems rarely takes into account the needs of performance analysis.

The main goal of this thesis is to introduce methodologies and techniques that assist an integrated data driven parameter estimation and performance analysis of manufacturing systems. Therefore this chapter discusses relevant areas in the performance analysis of manufacturing systems using information obtained from real manufacturing system. Therefore significant contributions in the literature relevant to these challenges will be discussed here. The review is structured as follows: Firstly, developments and contributions in analytical methods for the performance analysis of manufacturing systems are discussed. Then two main schools of literatures that introduce the use of real operational data and estimation uncertainty in performance analysis of manufacturing systems are highlighted. Developments and issues related to supervisory and data acquisition systems in relation to monitoring of workstations, data collection and acquisition for the performance evaluation for manufacturing systems are discussed.

Performance evaluation of manufacturing systems

In the last few decades research in performance evaluation of manufacturing systems has developed various approaches and techniques that enable the modeling of important performance measures. Alternative solutions are proposed for variety of analysis problems depending on the type and complexity of the manufacturing system. Many modeling tools consider problems such as nature of the parameters stochastic versus deterministic and other characteristics that define the system under consideration. Based on the type of modeling and solution techniques employed in performance analysis of manufacturing systems methodologies can be classified as analytical methods and simulation methods. In this section the review of main contributions made on analytical techniques are discussed, which is the main focus of this research. The proposed techniques in the thesis are generally applicable for performance evaluation by either simulation modeling or analytical models. The choice and effectiveness of either model depends on the nature, size of the system to be modeled and the complexity of the modeling type chosen.

Analytical models

Analytical models in general describe the system using mathematical or symbolic relationships. These relationships are then used to derive a formula or to define an algorithm by which the performance measures of the system can be evaluated. Under conditions where the problem size is complex to be solved by exactly modeled relationship or if the level of complexity is higher to handle with a reasonable computation time further modification can be performed to these relationship. These set of assumptions and approximations from exact analytical models are commonly categorized as approximate analytical techniques. In this report we refer to both types of models as analytical models of performance evaluation.

Although the main target of this thesis is aimed at the analysis of complex manufacturing systems a brief review of queuing systems is presented for the following main purposes. Most advanced works in stochastic analysis of complex manufacturing systems bypass the issue of "adequate knowledge", such as statistical decision on reliability parameters. In the early developments of queuing systems the trend was similar where most of the research assumed parameters such as arrival rates and service rates are known ahead and precisely. Due to their early continual development and ubiquity of applications in many fields they become pioneer to catch the attention of researchers for the statistical treatment of their model parameters estimation. Recent works also well exploited their mathematical simplicity for Bayesian models for uncertainty in developing exact formulas for the quantitative evaluation of performance uncertainty. The same problems in complex manufacturing systems derive the need for the type of research proposed in this thesis. The early development with precisely known parameters assumption and the later developments of data driven parameter estimation are both presented for a good contrast which motivates this research too.

Simple analytical models for performance evaluation such as queuing theory date back as early as in 1909's work of A. K. Erlang. They were used to solve telephone traffic congestion problems. The first mentions of queuing theory appeared in 1951 with well established classifications, notations and theory by D.G. Kendall. (Kendall 1953) published his paper on the queuing notation. An extension of queuing models to network of systems with flexible layouts is researched as the Jackson network model (Jackson 1957), (Jackson 1963) with exponential servers and an exogenous Poisson process. In this work Jackson has shown that the steady state distribution has a product form. (F. Haight 1958) also introduced the concepts of balking and parallel queues. In this paper he investigated the case in which each arrival to a system of two queues joins the shorter queue, or, if they are of equal length, one particular queue using differential-difference equations. (H. White and L.S. Christie 1958) considered server breakdown. They considered the effect on service-time statistics of preempted items re-entering service according to various rules.

In (J. Little 1961) Little proved a formula with dependency of mean number of jobs in systems (and queue) from mean response time (waiting time). (J.F.Ch. Kingman 1962) considered heavy traffic queuing systems with traffic intensity very near but less than unity. In this study algebra of queues and heavy traffic analysis of queuing systems are considered by assuming dependent arrival times and the behavior of related performance measures such as waiting times is investigated. (Jackson 1963) presented queuing networks with arrival process that depend almost arbitrarily upon the number already present, and the mean service rate at each service center depends almost arbitrarily upon the queue length there. He demonstrated how the equilibrium joint probability distribution of queue lengths is obtained for a broad class of jobshop-like "networks of waiting lines,". In (Gordon et al, 1967) the authors studied cyclic queuing systems with restricted queue length. They employed differential-difference equations for the time-dependent stochastic structure to study closed cyclic systems that are considered to be stochastically equivalent to open systems. In

(Mandelbaum et al, 1968) introduced the queuing systems with split and merge structure later referred as “Fork-Join” systems.

In (Buzen J. P. 1973) the convolution algorithm for the computation of normalization constant is proposed. (Basket et al, 1975) introduced a class of interconnected queues named BCMP networks which are a significant extension of Jackson networks by allowing an arbitrary customer routing and service time distribution.(F.P. Kelly 1975) proposed queue networks with multiple type customers and exponential service-time distribution. Each type of customer has a Poisson arrival process and a fixed route through the network and both close and open networks were considered. (Courtois et al, 1977) introduced decomposition for the approximate analysis of queuing networks called a generalized Jackson network. In this network job inter arrival and service times are not required to be exponentially distributed.

In (Reiser et al, 1980) have shown that mean queue sizes, mean waiting tunes, and throughputs in closed multiple-chain queuing networks which have product-form solution can be computed recursively without computing product terms and normalization constants. This work is developed an approximate solution of networks with a very large number of closed chains, and is shown to be asymptotically valid for large chain populations. In (Fdida et al, 1986) queuing systems with a shared common resource where this shared resource is modeled by an allocation queue with a limited number of servers are studied. The authors introduce an approximate technique to evaluate those systems and found the value of the stability condition of those networks.

(Gelenbe E. 1991) introduced new concept of positive and negative customers which can signify work cancellations or customers which don’t need service. (Dai J. G. et al, 1996) considered stability of fluid queuing models with variant rules of FCFS and FCLS. More elaborated and complete reviews of queuing networks for the evaluation of complex manufacturing systems are discussed in (Govil K. et al, 1999), (Papadopoulos and Heavey, 1996) and (Buzzacott and Shantikumar, 1993). (Buzacott and Shanthikumar 1992, 1993), (Hsu et al 1993) and (Bitran et al, 1992) analyzed both performance evaluation models and optimization models for queuing networks. More recent contributions can be found on (B.

Rabata 2009) queuing networks that are useful for modeling and performance evaluation of complex systems such as flow lines and flexible manufacturing systems.

Queuing based performance models have the power and advantages that comes with exact analytical model, such as the shorter evaluation time, and an explicit representation of the dynamic relationship between parameters is preserved. Even though this is of great importance from the performance evaluation perspective particularly to evaluate alternative configuration and reconfigurations they have short comings which initiated the next generation of approximate analytical techniques. Unlike the simplified assumptions that these models consider for the production system parameters, most of real manufacturing systems have specific requirements needed to be modeled. In addition many systems include complex configurations and relatively huge size and network of processing machines and storage spaces which can't be easily evaluated in a reasonable amount of computational time.

The main idea behind the development of approximate analytical methods is to modularly and structurally decompose bigger systems in to smaller building block systems with effective modeling and assumption on the interconnecting parameters that interface these building blocks. Finally the behavior of the whole system is captured by evaluating the building blocks with their interfacing parameters until the assumptions that are used to perform the decomposition are reached. These approaches have given rise to powerful methods that enable to study complex and bigger manufacturing systems and the dynamics of system behavior with satisfactorily accurate approximations. Especially these methods proved to be very effective in bridging the gap between complex simulations required to analyze complex and huge manufacturing systems and the very unrealistic assumptions made in exact analytical methods.

The first results on modern approximate analytical methods appeared in operation research literature in the works of (Gershwin, 1994). However, the process of firm automation and the advances in Information Communication Technology and computers science continued to generate new approaches in this field till now. A first review of early important works done in this area are available in (Koenigsberg, 1959), (M. Buxey et al, 1973) and (Buzacott, 1978). More recent contributions on performance evaluation of serial lines are reviewed in (Dallery and Gershwin, 1992).

The earliest work on modeling a transfer line composed by two machines and one buffer line in are proposed in (Vladzienskii 1953).In (Vladzievskii, 1967), introduced the idea of decomposition to evaluate the performance of a long transfer lines. Since the number of states in which a K stage flow line can explode with the number of machines in the line, he proposed to decompose the whole line into subsystems easy to be studied with the technique previously proposed. Then, the behavior of each subsystem is transmitted to the other subsystems by using opportune decomposition equations. This is the first example of decomposition approach applied to the study of production lines and the first approximate analytical method. Some works had the goal of demonstrating the properties of a production serial line by using the approximate analytical methods. The first numerical analysis of two-machine line with an intermediate buffer is presented in (Okamura and Yamashima, 1977) with important behaviors such as the monotonic function increasing with the buffer capacity. Important characteristics of buffered allocation problem including monotonicity, concavity are studied in (Shantikumar et al, 1989). In (Gershwin and Shick, 1983) the property of conservation of the average throughput in a production line is demonstrated. (Muth, 1979) investigated the property of reversibility of a production line, i.e. inverting the order of the machines in the line, the average production rate remains constant.

Gershwin and Berman (Gershwin et al, 1981) proposed the first effective exact solution for a two-machine line, in which the Markov chain describing the behavior of the system is solved independently on the capacity of the buffer, following a product form solution. Other works improve this method by using the properties of matrixes. In (Gershwin et al, 1983) the first exact solution of a system composed by three machines and two buffers are presented. (Jafari et al, 1987) analyzed flow lines in which, during a stage of production, some imperfect parts had the possibility of being scrapped from the system. Moreover, they extended the analysis to case of two machine lines with general uptime and downtime distributions. (Muth et al, 1987) proposed a method in which repairing personnel was shared by different stations and the repair time depended on the availability of the operator.

A decomposition approach for evaluating performance measures for multistage systems with finite intermediate buffers in which blocking and starvation is considered is presented in (Gershwin, 1987). The approximate decomposition approach is based on system characteristics such as conservation of flow and integrated the solution of the two-machine system already analyzed in (Gershwin et al, 1981). The efficiency, accuracy and flexibility of the modeling methodology made it a pivotal work which guided much of the research in performance analysis of complex systems. The model considered discrete time assumptions, geometrically distributed failure and repair times, unique failure mode and finite buffer capacity. Significant improvements were made to the proposed approximate decomposition algorithm in later works. (Dallery, David and Xie 1988) improved the algorithm by using an iterative technique that with a strong convergence instead of the originally proposed exact solutions.

Additional improvements contributed on the approach's applicability for a wider range of analysis problems with various assumptions. (Gershwin, Matta and Tolio 2002) considered multiple failure modes for each machine, i.e. the possibility that one machine can go down for different reasons and with different probabilities of failure and repair. Moreover, in (Le Bihan and Dallery, 1997) and (Tan and Yeralan, 1997a) new decomposition approaches were proposed. Further research improved the applicability of the proposed approach to real systems with various system architectures, and focused on reducing the approximation error and by generalizing the methodologies assumptions.

Extending the applicability of the decomposition to complex manufacturing architectures, assembly/disassembly systems have been considered (Gershwin and Burman, 2000). Similar architectures are analyzed considering the reconfiguration of resources to increase the system's production rate in (Chiang et al., 2000). Later, systems characterized by non-linear flow of material were analyzed in (Helber, 1999), (Li and Huang, 2005), (Diamantidis and Papadopoulos, 2004) and (Gopalan and Kumar, 1995). Closed loop architectures are studied using the decomposition method in (Gershwin and Werner, 2003) and (Commault et al, 1996). Multiple closed loops are considered in (Levantesi, 2001). Multi-product systems have been recently studied with approximate analytical techniques (Colledani et al. 2005), (Colledani et al., and 2005b). Techniques for the evaluation of generally complex system layouts have been developed in (Li, 2003), where an approach to approximate the production rate for systems with rework loops is considered.

Production control policies for regulating the throughput rate of the system have been also studied with decomposition techniques. Relevant works in this area are the work of Gershwin (Gershwin, 2000) (Gershwin and De Vericourt, 2004) for modeling and evaluating the performance of systems controlled by the Control Point Policy and the work of Bonvik (Bonvik et al. 1997) which review and compare the performance of systems controlled under different policies. Matta in (Matta et al., 2005) analyzed the performance of assembly systems controlled with kanbans with the use of queuing networks. Studies introducing quality control in production systems are developed (Bulgak, 1992), (Cheng et al., 2000) and (Li, 2005) address the problem of studying how different system architectures and quality control policies (Moinzadeh and Tan 2005) impact. (Kim and Gershwin 2005) have shown the importance of integrated analysis of quality and production logistics. In their work they show the trade-off between quality and productivity. (Colledani and Tolio 2006, 2009) extended the decomposition model to serial production lines where machines may experience quality failures. They demonstrated the impact on manufacturing system architecture and inspection allocation and buffer capacity determination for an optimal system yield. (Colledani et al, 2008) proposed Multi-Product Multi-Stage Lines systems that can model flexible manufacturing systems featuring alternative product routes. (Tan and Gershwin 2009) proposed a general methodology using level crossing analysis for solving continuous two machine lines. (Gershwin and Tan 2010) showed that the proposed modeling framework enables the analysis of a wide range of system models, including multiple failure mode lines, identical parallel machine lines, split/merge systems and lines with generally distributed up and down times. (Colledani and Gershwin, 2011) considered multi stage fluid flow systems and proposed a decomposition method for general Markovian complex machines.

In a methodologically different approach to the study of multi-stage lines using decomposition approach another stream of research development is the aggregation method. (Lim and Meerkov 1990) have proposed aggregation method for the analysis of manufacturing lines. The method works by combining the first two machines of the transfer line into a new combined machine and this forward aggregation process is continued until the last machine is reached. Moreover the same approach is used to study different problems that are studied in decomposition methods. In (Kuo, Lim and Meerkov, 1996) proposed a method for the study of bottlenecks and buffer allocation problem. (Li and Meerkov 2009) describe several aggregation approximations of analyzing production systems. Additionally introducing quality and inspection (Meerkov, and Zhang, 2010) have proposed the analysis production systems. However, for the modeling of complex systems such as multiple part-type systems, promising results had been shown in recent decomposition attempts, and thus in this thesis, the Markov modeling approach and decomposition were used as the primary analytical tools.

Analytical models and uncertain parameter estimates

Although the analytical methods for stochastic modeling of manufacturing lines has a long history and has generated of a considerable research, the statistical analysis of model parameters has received comparatively limited attention. Much of the effort is devoted to the probabilistic development of the models and to study the mathematical behavior of the system. The parameters governing the models are for most part assumed to be given. Important modeling building blocks, but relatively simplified systems such as single queues were the first area of investigation of researchers on how to introduce the estimation of parameters required for the evaluation of important performance measures. In the area of statistical analysis of stochastic models the most covered problems in literature are queuing systems. The popularity of these models for various modeling problems and the applicability in many different areas has made them typical target for this analysis. In the coming few section we will see main contribution in this area and recent contributions in the analysis of multistage lines from operational data.

Early mentions on statistical inference of parameters for queuing systems mainly addressed problem of estimating input parameters using frequentist approach. On (Clarke A.B., 1957), Clarke presented a maximum likelihood estimation method for the arrival rates and service rate of M/M/1 queue system. (Basawa et al, 1988) presented in which they demonstrated four alternative ways of data collection and experiments on single server systems. In this work they assumed the service time and the interarrival time densities to be (positive) exponential families. In (L. Schruben, R. Kulkarni 1982) they have shown the estimation of arrival rates and service rates and the resulting discrepancy between the state distribution for the model (estimated parameters) and the state distribution for the actual system (known parameters). They investigated that the mean for the model is infinite even if the estimated traffic intensity is restricted to be strictly less than one. (Zheng et al, 2000) addressed the undesirable properties related to mean estimators and that the expected value of the estimator does not exist and the estimator has infinite mean-squared error and introduced alternative estimators.

Reviews highlighting the significance of statistical analysis of queuing systems which were not covered with in queuing theory or stochastic process models can be found in (N. Bhat and S. Rao 1987). This review raised important questions related to the use of queuing models and the sampling plans that accompany estimation of arrival rates. How long should the system be observed - for a specified length of time or until a specified number of events has occurred? In addition works related to test stationarity, periodicity assumption and the impact of the sampling plan on the stochastic model are discussed. Bayesian works for the inference of parameters for queuing systems presented in (F. McGrath et al, 1987). Their work emphasized on the amount of information conveyed using Bayesian approach for the statistical inference in queues. (C. Armero and M. J. Bayarri 1994a, b) have shown how Bayesian methods are suited to handle the common inferential aims with an emphasis on prediction on M/M/1queue system. In (Armero et al, 1994a) the authors analyzed an exponential single-processor queue, using Gamma prior distributions for the service and arrival rates. They demonstrated the posterior moments of certain performance metrics, such as the steady-state number of customers in a system, do not exist. This problematic issue in prediction of the long-term behavior of the system is addressed in later works in (Armero et al, 1994b). (Insua et al, 1998) have considered statistical analysis of *M*/*G*/1 queuing models with Erlang service time distribution and demonstrated Monte Carlo method for the estimation of interesting performances. A detailed exposition on why Bayesian analysis is good for queues can be found in (Armero e 1999). The popular use of mathematical queuing models for the performance evaluation of production systems (Armero et al, 1998) have conducted a series works on the Bayesian statistical analyses on Markovian bulk arrival queues with a focus on prediction of the usual performance measures of the system in equilibrium. In these papers posterior predictive distribution of the number of customers in the system is obtained through its probability generating function. In (Armero et al, 2000) they have shown the use of Markov Chain Monte Carlo and numerical inversion of these transforms to evaluate the distribution of performance parameters. With an extended work in particular to a production systems (Armero et al, 2003) shown important special features of Bayesian analysis of queuing production systems in comparison with traditional queuing theories. Using conjugate prior they have shown making inference on the posterior density of arrival and service rates. (H. Liyanage and G. Shanthikumar 2005) considered inventory control problem with an ambiguous demand comparing with traditional approach of separating the parameter estimation and the maximization of the expected profit which leads to a suboptimal inventory policy. (Chu et al 2008) demonstrated an integrating parameter estimation and optimization using operational statistics which leads to better solutions compared with the traditional approach. In this paper they also introduced a Bayesian approach for the estimation of input parameter. In (A. Jain et al. 2010) proposed a method for the optimization of single queuing systems with model uncertainty. In this work they have demonstrated the difference of assuming arrival and departure rates as accurate and known parameters against with uncertain model assumption from operational data. (Wazed et al 2009, 2010) identified and provided a review on the different sources of uncertainty in real manufacturing environment. (L. Li et al 2011) proposed an average autoregressive moving average model (ARMA) for a data driven bottleneck detection in multi-stage manufacturing systems. (A. Azizi et al 2012) proposed a Bayesian inference for throughput modeling under uncertainties. They used a Bayesian model utilized prior distributions related to previous information about the uncertainties where likelihood distributions are associated to the observed data with Monte Carlo Markov chain was employed for sampling unknown parameter uncertainties.

Performance analysis with Fuzzy Markov models

An emerging area of research on model parameters uncertainty is Fuzzy Markov models which are capable to deal with uncertain transition rates and probabilities. Fuzzy Markov models are developed to overcome the deterministic assumption by emphasizing on the uncertainty of transition probabilities from real data or insufficient information. Much of the contribution that is made under this research is in the area of mathematical studies and computing systems. Even though currently the use of these models is not widespread in performance analysis of manufacturing systems it is important to briefly discuss their potential for analysis under uncertainty. Early works such as (R.E. Belman and L.A. Zadel 1970) introduced decision making in a fuzzy environment, where constraints and goals can be fuzzy whose range and boundary are not sharply defined. In their work they also investigated the use of these concepts using examples involving multistage decision processes in which the system control is either deterministic or stochastic. They also emphasized the importance of differentiating between randomness and fuzziness. Detailed review on previous main theoretical contribution of fuzzy systems is available in (J. Klir and B. Yuan, 1995). The authors provided summary of works on fuzzy sets, fuzzy logics, fuzzy algebra and applications. More recent developments can be found also in (J. Buckley and E. Eslami, 2002). These reviews summarize the properties of regular, and absorbing, fuzzy Markov chains and show that the basic properties of these classical Markov chains generalize to fuzzy Markov chains. (Dubois et al, 2005) proposed a technique to perform fuzzy interval computation under a condition of local monotony of considered functions, by considering uncertainty as pairs of fuzzy bounds. (D. Kumar et al., 2005) described an application of fuzzy Markov model for the determination of fuzzy state probabilities for generating units including the effect of maintenance scheduling. (T. Binh and D. Khoa, 2006) discussed the application of fuzzy Markov in calculating reliability of power systems. (G. Chongshan 2009) calculated fuzzy availability of a reparable geometric process and fuzzy reliability theory to study a repairable linear. He considered uncertainties in some of the transitions probabilities as modeled by fuzzy numbers.

(D. Kumar et al. 2009) calculated fuzzy reliability and fuzzy availability of the serial processing plant. (I. Uprety and Zaheeruddin, 2009) evaluated the fuzzy reliability of gracefully degradable computing systems. (D. Kumar and Kumar 2010) computed the fuzzy reliability of the stainless steel utensil manufacturing unit for the constant failure and repair rates. (Y. Liu and Huang, 2010) introduced a modified fuzzy multi-state system availability assessment approach to compute the system availability under the fuzzy user demand. (F. Aminifar et al., 2010) proposed reliability modeling of PMU and the Markov process is employed to analyze the proposed model. (D. Kumar and Kumar, 2011) used the concept of fuzzy approach in the evaluation of the reliability of a manufacturing plant. (A. Kumar and S. Lata, 2012) used the fuzzy Kolmogorov's differential equations evaluate the fuzzy reliability of system, the fuzzy Kolmogorov's differential equations are solved analytically for solving n^{th} order fuzzy linear differential equations.

Supervisory systems and manufacturing systems

Supervisory systems are increasingly becoming integral features of modern automated manufacturing systems. The supervisory control of Discrete Event Systems (DES) in accordance with behavioral specification is a new research area which is receiving increasing recognition. Even through supervisory systems are have many diversified functions in manufacturing systems they also play an important role in performance evaluation and analysis. One objective of these systems is to perform data collection and monitor the behavior of individual work stations, work cell behavior and part flow via sensory feedback. Research on supervisory system development and implementation is based on information feedback on the occurrence of events, formal languages and controlled finite state machine concepts and petrinets.

The first important and comprehensive framework on Supervisory Control Theory was developed by (P.J. Ramadge and W. M. Wonham 1987). This work has studied a class of discrete event processes and provided a formal methodology for the automatic synthesis of controllers for Discrete Event Systems (DES). The theory also made a clear definition and distinction between the systems to be controlled, called plant and the entity that controls it, called supervisor.The supervisory theory by Ramadge and Wonham is so far the most comprehensive theory for the control of discrete event systems. It is based on the concept of a supervisor (Ramadge 1983), (Ramadge 1987), i.e., an agent that is capable of disabling the controllable transitions of a DES in response to the traces of events generated. The Supervisory Control Problem (SCP) consists in designing a supervisor which restricts the traces generated by the system within a legal behavior. If the legal behavior is a controllable language (Wonham 1987) a supervisor exists.

The use of Petri nets with inhibitory arcs (PNIA), which are known to have a modeling power equivalent to Turing machines, to describe infinite state systems. Thus, they prove that a PNIA supervisor exists if the system's and specification behaviors are Turing computable languages. However, important properties, such as determining if the behavior of a PNIA is controllable, are undecidable. In (Ramadge 1986) and (Lin F. et al, 1988a, 1988b) a modular approach to the design of supervisors is considered. The case of the infinite state supervisor is discussed by (Sreenivas et al, 1992). The specification language is composed of different specifications, each enforced by a single supervisor. A global control law can be enforced by the conjunction of all the supervisors. In (Ramadge 1989b), (Tadmor 1989) and (Tsitsiklis 1987) different problems of computation and the related issue of computational complexity and modularity are considered. A review of the theory is presented in (Ramadge 1988), (Ramadge 1989b), (Wonham 1988a).

In (Lafortune 1990a) a new control problem is studied: the Supervisory Control Problem with Blocking (SCPB). Here it is assumed that in some cases the solution to the SCP (supremal controllable sublanguage) may be too conservative. A dual concept is defined— the infimal controllable superlanguage — and is used to determine a supervisor that may also permit blocking in order to achieve a larger behavior. In (Lafortune 1990a) Lafortune and Chen introduced two performance measures (in terms of satisficing and blocking), and techniques to improve each of these two conflicting measures. An extension of this work is (Lafortune 1991) where the Supervisory Control Problem with Tolerance (SCPT) is defined. Given a desired and tolerated behavior, the problem is that of designing a controller such that the controlled system never goes beyond the tolerated behavior and achieves as much as possible of the desired behavior. Under very general hypotheses on desired and tolerated behavior, (Lafortune and Lin, 1991) show that a solution to SCPT exists and is unique, but may be blocking. A non blocking solution exists but is not necessarily unique.

In 1991, (Cieslak et al.1991) discuss and solve the Supervisory Control and Observation Problem (SCOP) and the Decentralized Supervisory Control Problem (DSCP). In SCOP the assumption is that a mask is present between the controlled system and supervisor, so that the supervisor cannot observe all the transitions, or cannot distinguish between some of them. In DSCP it is assumed that the control action is enforced by local supervisors that control only subsystems. In (Lin 1988) and (Lin 1990) Lin and Wonham discuss the Decentralized SCOP (DSCOP) where both partial observations and decentralized control are incorporated into the control structure. However, the only mask operator considered in this paper is the language projection operator. In (Brave 1993) Brave and Heymann define stabilization as the ability of a discrete event process to reach a set of target states from an arbitrary initial state and then remain there indefinitely. A slightly different problem that the authors examine is recovery under control failure. In both cases they present design algorithms for controllers that improve the stabilization of processes. In (Ushio 1990) Ushio discusses the conditions under which a finite state supervisor (FSS) may be constructed to solve a SCP. From (Ramadge 1987) it was known that a FSS exists when both system's behavior and specification language are regular. Here the author derives necessary and sufficient conditions for the general case.

Many of the early important works developed in the supervisory control theory are focused on deterministic automata systems, where the transitions between states can be determined or controlled by the supervisor. Studies that are based on original Ramadge-Wonham frame work but which can handle probabilistic transition and control generated a series of new research area. The probabilistic control got more interest from researchers and practitioners due to its power in approximating the behavior of most real systems whose state transition behavior is more represented by probabilistic assumption rather than deterministic.

Lawford and Wonham (1993), a plant under probabilistic control can generate a much larger class of probabilistic languages than deterministic control. The necessary and sufficient conditions for the existence of a supervisor for a class of PDESs are given in

(Lawford and Wonham 1993). The control of different models of stochastic discrete event systems has been investigated in (V.Grag 1992a, 1992a,) and (V. S. Borkar 1991). V. Grag defines probabilistic languages and probabilistic automata over a finite set of events and considered operators under which the set of probabilistic languages (p-languages) is closed. V. Grag in (V. Grag 1997) has extended the use of recursive equations to solved language algebra. He defined the notion of regularity, i.e., finiteness of automata representation of probabilistic languages has been defined. (R. Kumar 2001) provided a condition for the existence of a supervisor and an algorithm to test this existence condition when the probabilistic languages are regular and developed a technique to compute a maximally permissive supervisor online.

Further works on the stochastic systems such as (S. Postma et al, 2004) proved the conditions and provides an algorithm that can be used to compute a solution to the model matching problem when it exists. (Y. Cao and M. Ying 2006) extended important properties such as observability, normality, and co-observability of crisp languages to fuzzy languages. In their analysis they provided the necessary and sufficient condition for the existence of a partially observable fuzzy supervisor. (Chattopadhyay et al, 2007, 2008) demonstrated stochastic discrete-event supervisor is optimal in the sense of element wise maximizing the renormalized language measure vector for the controlled plant behavior and is efficiently computable. A formal proof of the necessity and sufficiency of the conditions and an algorithm for the calculation of the supervisor, if it exists, are presented in (Postma and Lawford 2004), and (Pantelic et al, 2009).

(A. Jayasiri et al, 2010) investigated the decentralized modular supervisory control problem of FDES with partial observation for systems which are composed of concurrently operating, multiple interacting modules with uncertainties in their events and states. (A. Jayasiri et al, 2012) studied modular and hierarchical supervisory control theories of Fuzzy Discrete-Event Systems (FDES). They addressed the horizontal and vertical complexities present in large-scale event-driven systems, which are affected by uncertainties in their event and state representations. Using Probabilistic discrete event systems as a generator of language (V. Pantelic 2012) presented an approximate algorithm to synthesize a probabilistic supervisor that minimizes the distance between generators representing the achievable and required plant behavior.

Role of supervisory systems in performance evaluation of manufacturing systems

Quantitative methods for the performance evaluation of manufacturing systems and design techniques for the supervisory and control systems are traditionally treated as two separate fields of research. Despite the close link and the strong interaction between the two areas both researchers and practicing engineers and system designers treat the study and development of the two aspects independently. The main goal of manufacturing systems design and analysis is to measure and guarantee performance measures such as, average throughput, i.e. the average number of parts produced in a given time, utilization of equipments, lead time, the average duration of time parts spend in the system, the average number of work in progress in the manufacturing system. On the other hand the control of discrete-event systems (DES) is a research area of current vitality, stimulated by the hope of discovering general principles common to a wide range of application domains. Among the latter are manufacturing systems, traffic systems, database management systems, communication protocols, and logistic (service) systems. The contributing specialties are notably control, computer and communication science and engineering, together with industrial engineering and operations research. Regardless of the type of application historic data that are stored in these systems are potential sources of data for performance evaluation of the monitored system.

The importance of system supervision is even more pronounced for understanding and inference of manufacturing systems where the fundamental behavior of these systems is rarely predictable and subjected to high randomness. Some general sources of internal unpredictability, such as excess inventories, long lead times and uncertain delivery dates are caused by randomness and lack of synchronization. There are only two possible solutions: reduce the randomness (due to machine failures, engineering changes, customer orders and so on) and reasons for the lack of synchronization (costly set-up changes, large batch machines and others) or respond to them in a way that limits their disruptive effects. (Gershwin, 1994, 2000). Stochastic performance evaluation models attempt to study and analyze the average and transient behavior of the manufacturing systems caused by this randomness. For this reason most performance evaluation models characterize manufacturing systems as stochastic rather than deterministic. Commonly studied sources of randomness in manufacturing systems are:

1. Arrival times of entities, i.e., parts or raw materials moving throughout the system
2. Processing or assembly times at each workstation or machine for different types of parts
3. Operation times for each workstation or machine without failure or breakdown
4. Time between failures of operational breakdown
5. Repair times for failures or breakdown
6. Set-up times for individual machines and systems

The effectiveness and accuracy of output performance from stochastic models depends on the input parameters associated to the stochastic variable. Examples of such input include the arrival rate of entities to the system, the processing times required at various machines, reliability data (e.g., the pattern of breakdowns of machines). In many practical cases when there are supervisory control systems recording stochastic events related to a manufacturing system these parameters must be estimated from existing data sets. The input data analysis and modeling uncertainty of input parameters of stochastic models and their impact on the performance analysis is less studied problem in performance evaluation models.

More recently there are some literatures available on performance evaluation using operational statistic from actual data, especially for analytical models of stochastic systems. These preliminary works emphasized on how the use of uncertain parameters that are estimated from operational data can greatly impact the performance evaluation output. In some studies they demonstrated how the uncertainty impacts the first moments of evaluated performance and help to estimate higher order moments (H. Liwan et. al, 2004), (LeonYang Chu et. al, 2007), (A. Jain et. al, 2010). These studies highlight the importance of introducing performance evaluation using actual data on smaller systems such as single stage queues, optimization problems. But there are no comprehensive approaches on how to extend solutions to estimated parameters from actual data for the performance evaluation of complex stochastic systems. General evaluation techniques such as approximate analytical approach for the performance evaluation of multistage manufacturing systems are not yet treated from this research perspective.

On the other hand literature and research on supervisory control theory to guarantee the information needs of performance analysis is scantily addressed, except rare qualitative mentions of the problem. The primary focus of supervisory control theory has been so far on insuring safety and other requirements related to the control of operations in the manufacturing systems. Since the introduction of this field in 1982 by P.J. Ramadge and W.M. Wonham there is a huge amount of literature in the study of the synthesis of controlled dynamic invariants by state feedback, and the resolution of such problems in terms of naturally definable control-theoretic concepts and properties, like reachability, controllability and observability. From this perspective the performance evaluation aspect of the supervisory systems get relatively limited attention. Some works that implement stochastic petrinets and automata for the performance evaluation of a realized discrete event systems attempted to evaluate modeled systems with these frameworks. Additionally there are no formal approaches on how to quantify and prioritize observation requirement that come from manufacturing system based on performance analysis that considers real data. On this direction this research attempts to pave one possible way for addressing this problem in a formalized way.

Chapter Three

3. Manufacturing system modeling

This chapter introduces general definitions, attributes and characteristic behaviors in the classification and taxonomy of manufacturing systems. The relevance of the classification system used in this discussion emphasizes on performance modeling and evaluation perspective of manufacturing system. Important terminologies and notations are explained and defined that need to be used in the upcoming chapters of the thesis. The classification is mainly used to underline the category of manufacturing systems that are interesting for this study and gives due consideration on explaining characteristics and parameters that define similar systems which are the main focus of this research.

3.1 Characterization of manufacturing systems

A manufacturing system can be defined as a set of machines, transportation elements, computers, storage buffers, and other items that are used together for manufacturing (Gershwin 2004). These systems can be classified based on different criteria depending on the objective of the classification framework and the intended kind of study. From manufacturing systems engineering point of view the commonly used classification basis are mainly the operational characteristics and the operational flow structures. Classifications of manufacturing systems are comprehensively discussed in (McCarthy, I., 1995). This chapter briefly discusses relevant terminologies and classifications based on typical characteristics and material flow of manufacturing systems. Furthermore important building blocks elements (machines, buffers, material flow) for the modeling and representation of manufacturing systems are introduced.

Discrete manufacturing: The production or assembly of parts and/or finished products that are recognizable as distinct units capable of being identified by serial numbers or other labeling methods-and measurable as numerical quantities rather than by weight or volume. In discrete manufacturing, the manufacturing floor works off orders to build something. Examples include toys, medical equipment, computers and cars.

Process manufacturing: A manufacturing environment often characterized by a batch or continuous transformation of a gas, liquid or powder, low product complexity and manufacturing variations, fixed or dedicated facilities, a flat bill of material and relatively few transactions. The processing of products such as chemicals, gasoline, beverages and food products are typically produced in "batch" quantities rather than discrete units.

The models that are this thesis mainly focus on discrete manufacturing. Depending on the operational characteristics of manufacturing systems, volume and diversity of discrete products in which the manufacturing system is designed for they can be generally be categorized into three main classes as: mass production, batch production and Job shop.

Mass production: Refers to the manufacturing of large quantities of standardized products, using dedicated machines and utilizing assembly line technology. Mass production is typically characterized by some type of automation, as with an assembly line, to achieve high volume, the detailed organization of materials flow, careful control of quality standards. In these systems machines perform operations on incoming parts. In this case, the quantity of products stored in buffers is a real number. Typical applications of this type of systems can be found in food industry, textile production lines, chemical lines and pharmaceutical lines. These are commonly analyzed through the use of discrete and continuous models, which treat the flow of material as a continuous fluid or discrete units. In many cases continuous models can also be used to approximate the behavior of discrete systems.

Batch manufacturing: Refers to a production control method whereby the ranges of products manufactured in a plant are made in batches. Each separate batch consists of a number of the same products/components. In the past, large batches of each product were made to gain efficiencies by reducing the amount of non value adding time spent changing over from one product type to another. However, this type of production results in high inventories and excessive lead times. The Toyota Production System was developed to overcome the limitations imposed by changeovers and allows manufacturers to produce in synch with customer demands at a high level of quality and low cost.

Job Shop manufacturing: Manufacturing systems that produce items that are "one of a kind", for example, manufacturers of automation systems and tooling fall in the job shop category. A distinguishing feature of job shop is that it is capable of processing many different types of jobs, each with its own routing and processing characteristics.

Units and event times in manufacturing systems

Times related to events and phenomena which are common in manufacturing such as sequences in starting and finishing, durations between two events, play important roles for the classification, modeling and the performance evaluation of manufacturing systems. Some important units such as throughput, lead time in performance measurement are linked to statistics of time units. Following are some important units and definitions related to the study and analysis of production discrete flow lines.

Cycle time: the time required for a machine to perform an operation on a product, while working in isolation is named cycle time. It can be deterministic, if it is not varying from one part to the next, concerning a given process. It is stochastic, if it is randomly varying from one part to the next.

Throughput: denotes the number of lots per time-unit that leaves the manufacturing system. At machine level, this denotes the number of lots that leave a machine per time-unit. At factory level it denotes the number of lots that leave the factory per time-unit. The unit of throughput is typically parts/cycle time.

Flow time/Lead-time: denotes the time a part spends in the manufacturing system. At factory level this is the average time from release of the part in the factory until the finished part leaves the factory. At machine level this is the time from entering the machine (or the buffer in front of the machine) until leaving the machine. For modeling purposes flow time is typically measured also in cycle time.

WIP-level: (WIP) work in progress denotes the total number of parts in the manufacturing system, i.e. in the buffers or in the machine. WIP or buffer level is measured in integer in discrete manufacturing.

Utilization: denotes the fraction a machine is not idle. A machine is considered idle if it could start processing a new part. Thus processing time as well as downtime, setup time and preventive maintenance time all contribute to the utilization. Utilization has no dimension. Utilization can never exceed 1.0.

Synchronous manufacturing lines: synchronous systems refer to lines characterized by identical deterministic cycle times for the different machines in a system. It doesn't imply synchronous in the strong sense, where all the machines start and stop processing operations as in the case where all the movement of jobs and parts is coordinated and internal buffers remain constant. In this case if two machines are operational both machines start and stop simultaneously for each machine, while it is possible one machine is operational and the other stays down due to failure or other causes. But all the changes that happen to the system such as repair, failure, completion of processing happen contemporarily.

Asynchronous manufacturing lines: In asynchronous manufacturing lines cycle times may be different among machines and operations do not necessarily start and stop contemporarily for each machine. And on operation completion the part immediately moves to the next work station, as long as there is space for it. In both the asynchronous and synchronous lines the number of jobs in the system may fluctuate (considerably) and buffers are needed to prevent starvation and blocking.

Discrete and Continuous times: in discrete manufacturing each operation requires a fixed time to process a part and the number of products present in buffers, at each time instant, is an integer number. Typical applications of discrete systems can be found in automotive lines, white goods production lines and mechanical components production lines. In continuous production systems machines perform operations on continuously flowing incoming parts. In this case, the quantity of products stored in buffers is a real number. Typical applications of this type of systems can be found in food industry, textile production lines, chemical lines and pharmaceutical lines. This are commonly analyzed through the use of continuous models, which treat the flow of material as a continuous fluid. These continuous models can also Introduction of events that characterize an unreliable manufacturing system

Material flow in manufacturing systems

Starvation and Blocking: Material flow in a manufacturing line can be interrupted for different reasons. One cause can be the failure of the machine itself, but since machines are interconnected in the manufacturing line the failure of other machines in the line can cause other machines to stop due to the starvation and blocking phenomena. A machine is starved if there no part is available for processing from the upstream buffer. A machine is blocked if there is no space to place a completely processed part is in the downstream buffer. Blocking and starvation phenomena are the main causes of interruptions of material flow which propagate through a line. If no buffers are present between machines, a failure of a machine immediately propagates to all the other machines composing the line. The goal of introducing buffers in real production systems is commonly to decouple the behavior of machines and prevent blocking and starvation phenomena from propagating along the line. Once a machine fails, starvation propagates to the downstream machines while blocking propagates to the upstream machines. Therefore, machine M_i is blocked by the failed downstream machine M_j if all the buffers among M_i and M_j are full. On the contrary, machine M_i is starved by the upstream failed machine M_k if all the buffers between M_k and M_i are empty. Capturing the correct dynamics of propagation of blocking and starvation in the system is fundamental for the development of accurate models and methods for the performance analysis of systems.

Basic Elements in a manufacturing line

Basic elements in manufacturing systems modeling such as machines, buffers, material flows and their representation are defined and introduced. The relative arrangement and architecture of these elements defines the layout and configurations of manufacturing systems. A widely used and studied building block models for modular decomposition and performance analysis of the different complex manufacturing architectures are also described. These elements will be used for building and representation of different manufacturing systems architectures that will be discussed here. Their representations will be kept the same for modeling the different types of manufacturing systems that will be discussed throughout this thesis.

Machines: are automated or manual work stations that perform operations on parts that are processed in the manufacturing line. Depending on modeling conveniences and objectives a group of workstations can be considered as single machine. Machines are represented in squares, in cases a machine performs operation on different products they are represented with squares partitioned with horizontal lines, equal to the number of products.

Operational Failures: operational failures are those disturbances which cause the immediate interruption of the manufacturing flow for a machine. Failures which stop the whole production of the system, like energy provision interruptions, are not considered among these types of failures, thus the independence of failures among different machines is considered. In order to restore the machine to the operative conditions, the intervention of an operator is required. Two types of failures are generally observed in real production systems, i.e. Operation Dependent Failures (ODF) and Time Dependent Failures (TDF).

Operation Dependent Failures (ODF): are failures that can happen only if the machine is operational, i.e. not starved nor blocked. This are typically mechanical failures, such that the tool breakage, the errors of sensors while positioning the work piece in the work zone, the lack of material and mechanical jamming.

Time Dependent Failures (TDF): are those failures that can happen even if the machine is starved or blocked, i.e. the failure occurrence do not depend on the machine state. They are typically electronic failures, such that light burn-outs, machine screen problems and machine communication problems.

Buffers: are temporary storage spaces for parts flowing between machines in manufacturing lines. In real manufacturing systems they can be transporting material from one machine to another one, decoupling the behavior of the machines and reducing the effect of the propagation of blocking and starvation phenomena in the line. They can be automatic conveyors, AS/RSs, floor space, etc. Buffers are represented with circles.

Material flow: Material flow represents the direction and routes parts follow in manufacturing systems, which might be derived from the sequence of processing requirements that must be carried out on a given product until it comes out as a finished product from the line. Material flow is represented with arrows connecting machines and buffers and their direction represents the direction of material flow.

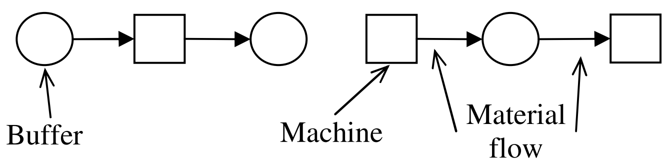


Fig:

Figure 3:1 representations in manufacturing line

3.2 Manufacturing system architectures

In this section a general classification of different manufacturing systems layout configurations is presented. Alternative manufacturing layout choices can be adopted depending on the nature of the product and processing requirements involved. Three widely considered manufacturing layouts are introduced, namely open line, closed line and assembly lines. These lines can be represented and modeled using the basic elements introduced in the previous section and their performance can be analyzed using decomposition approach using two machine single buffer building blocks.

Open line layout (serial manufacturing lines)

These are manufacturing lines composed of workstations and storage areas in which material flows in sequential processing by visiting each work station and storage area in a fixed sequence. Generally such kinds of systems are composed of *K* machines and *K-1* buffers. Material enters into the manufacturing line through the first machine usually represented M_1, crosses a system of *K* machines and *K-1* buffers and finally leaves the system through the last machine M_K. The following manufacturing system producing D12 engine blocks in Scania CV AB is a typical example of open flow line configuration. The line is composed of 22 workstations decoupled by

21 intermediate buffers.

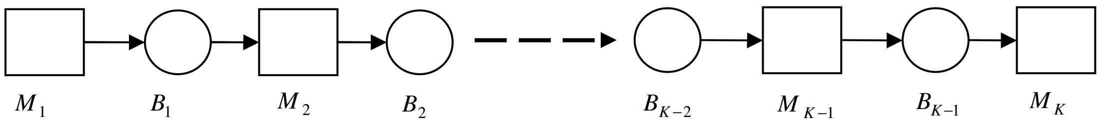


Figure 3:2 Representation of an open flow line with K machines

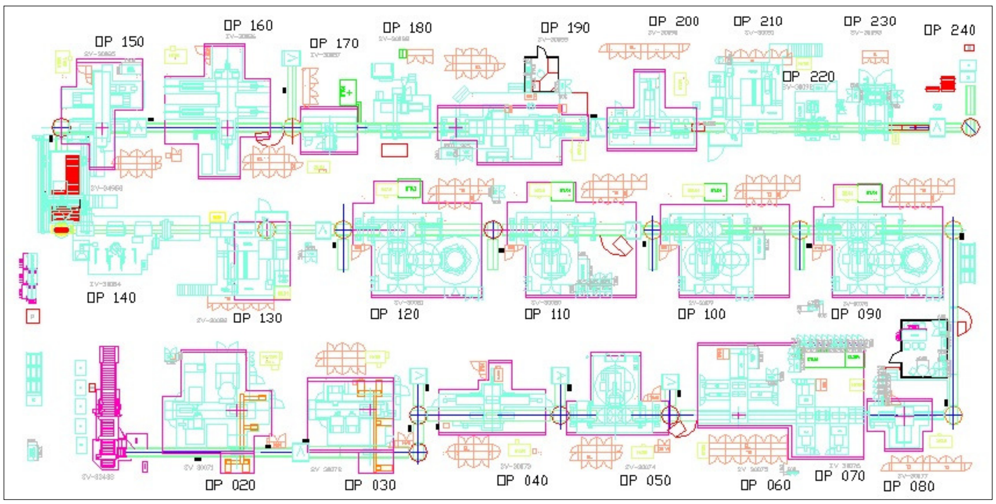


Figure 3:3 A real open flow line producing D12 engine blocks in Scania

Closed loops

Closed loops are lines characterized by a constant number of products circulating in the system. Indeed, a raw part is processed by the first machine M_1 only if a finished product is released by the last machine in the system M_K. Therefore, in closed loop lines, the number of buffer equals the number of machines. A representation of closed loop systems is proposed in Figure 3:4. Given the correlation among the arrival of parts in the system and the delivery of finished products, these type of systems present a particular dynamic behavior concerning the propagation of blocking and starvation phenomena. Indeed, since the number of parts circulating in the system (loop population) is fixed, a failure of a generic machine M_j can cause the blocking phenomenon propagation to involve only a sub-set of the upstream machines which compose the line. The same can be stated for the propagation of the starvation phenomenon. This particular behavior sends this system very complex to analyze, since the blocking and starvation propagation is conditioned to the system state. An example of real closed loop systems is reported in Figure 3:5. In particular, the layout of the system producing printer charger in Olivetti is reported.

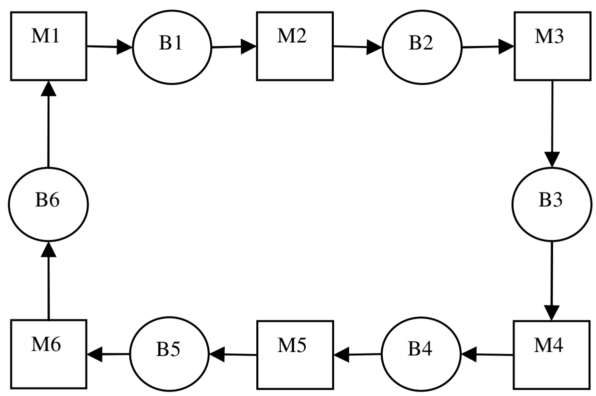


Figure 3:4 Model representation of the loop configuration system

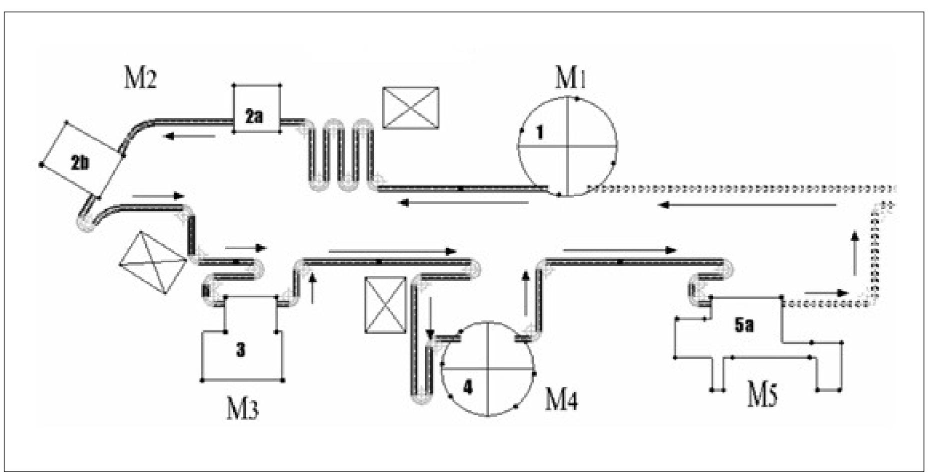


Figure 3:5 A real closed-loop line producing printer chargers in Olivetti

Assembly/disassembly Lines: such layouts are configured such that manufacturing machines can perform assembly (joining operations) and disassembly operations to realize the final product. Machines can take parts from two or more upstream buffers in a join structure to assemble parts. Alternatively machines also can disassemble parts and place in two or more downstream buffers creating a fork structure. Therefore, a more general notation is needed for buffers to indicate which machines are connected through the given buffer. A buffer *B(i,j)* connects the machine M_i to the machine M_j. Unlike the open flow line configuration, an assembly line can have multiple entry ports for the entry of input parts and multiple output ports for the exit of finished parts. Most of complex manufacturing systems can be represented as assembly lines depending on how the join fork structure is. Many real automotive assembly lines and household equipment manufacturing systems adopt similar layout, since the products involve assembly of many subcomponents.

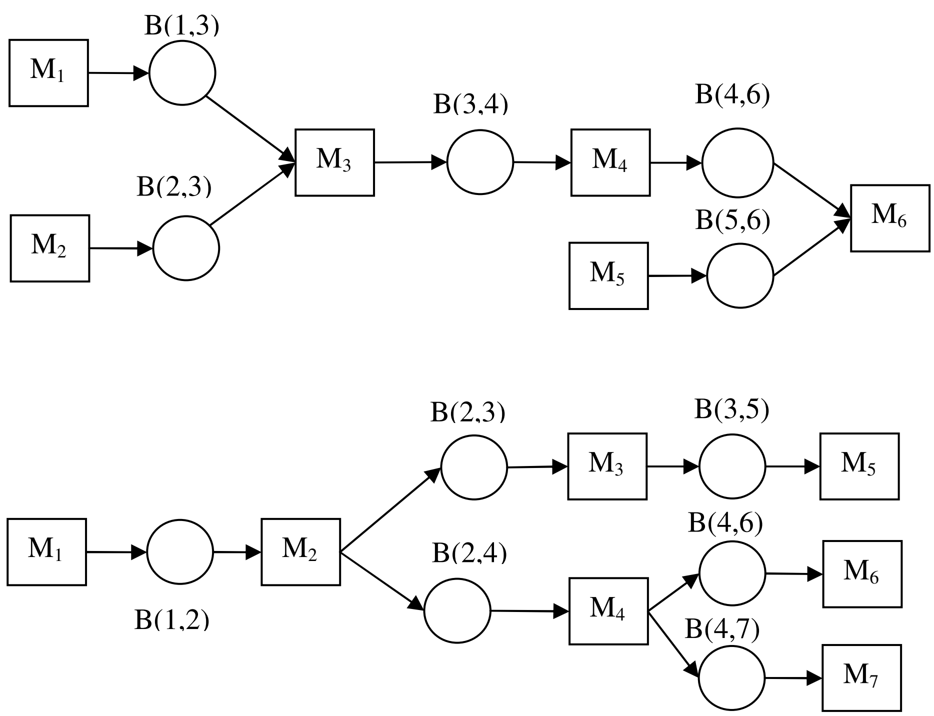


Figure 3:6 Model representation of the assembly/disassembly line configuration system

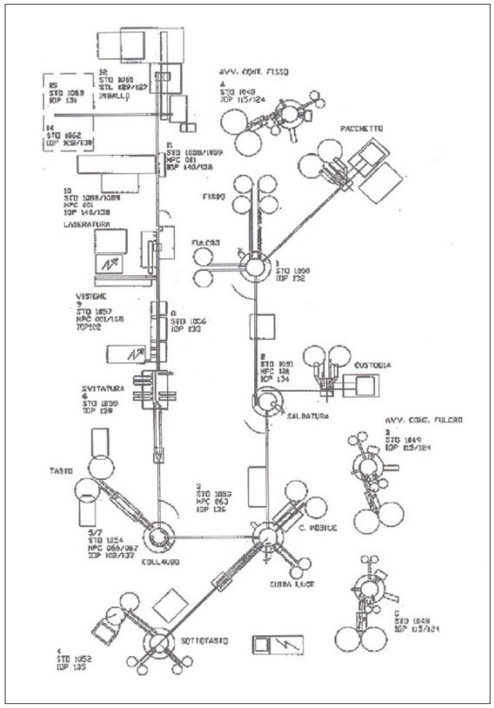


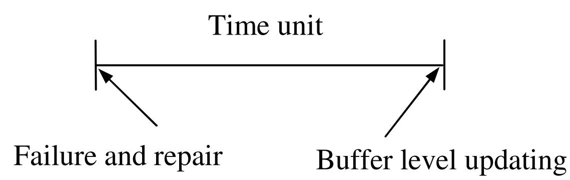
Figure 3:7 Real assembly / disassembly system producing commands in Bticino

3.3 General modeling assumptions

Here the main characteristics of the systems particularly (machines and buffers) that are considered in the coming sections are briefly discussed in the context of the above discussion. Frequently used notations are introduced and unless mentioned all the assumptions adopted for the systems that are analyzed in this thesis are the following.

System architecture:

- In multi-stage cases saturated open layout architectures are assumed
- Discrete flow of parts (discrete production) are assumed discrete times are considered, processing time is scaled to one time unit
- Quality issues are not considered, all produced parts are assumed good
- Blocking before service is considered and times for state transition such as failures and repairs occur at the beginning of time units, when buffers are updated at end of time units.



Conventions for updating state transition and buffer levels

Machines:

- Failures are operational dependent failures (ODF)
- The probability that machine a machine fails in a time unit in the failure mode with a precisely known or unknown failure rate p
- Time to failures, *TTF* are assumed to be geometrically distributed with an unknown failure rate p.
- The Time to repair a machine, *TTR* that is down in a failure mode is assumed to be geometrically distributed with a known or unknown repair rate parameter r.

Buffers:

- Buffers have finite capacity N.
- Transient time is zero.
- Buffers are perfectly reliable.

Chapter Four

4. Proposed method

This section demonstrates the proposed techniques for the performance evaluation of manufacturing lines with uncertain parameter estimates. It begins with an introductory scheme of Bayesian estimation for reliability input parameters, failure and repair probabilities. The alternative techniques introduced here are explained starting with isolated machines for the analysis of isolated efficiency. Mainly expected value of the isolated efficiency E[e] and uncertainty in variance of the isolated efficiency V[e] are evaluated using alternative techniques.

Next the techniques are applied to building block two machine single buffer lines where the estimation of distribution average throughput is investigated. Estimation of the expected value E[TH] and uncertainty V[TH] of the average throughput are evaluated. Finally a method for the analysis of multi-stage long lines is introduced. Although the techniques introduced are generally applicable to continuous time cases, all the analysis in this section is carried out on discrete time systems.

4.1 Inference of Input Parameters

The inference of input parameters begins with the collection of data required for the type of parameters to be estimated. The data can be obtained from an online data base system that record and monitors machine history or any type of log information about failure records. For the purpose of this thesis these parameters are reliability parameters associated to the failure probability and repair probability of individual machines. These data could be randomly taken data or time series data obtained from an operational machine. In some cases there might be prior information from previous estimations or knowledge about specific parameters. On the other hand some parameters have to be estimated for the first time from observation of data.

Essentially, the performance evaluation of manufacturing systems using stochastic models requires estimation of input parameters, particularly when the input parameters have to be estimated from actual operational data. The problem of using actual data for the performance evaluation is rarely addressed and is one of the goals to be discussed in this chapter. Although the inference of parameters is one field of study that needs attention the primary goal remains performance evaluation with inferred uncertain parameters.

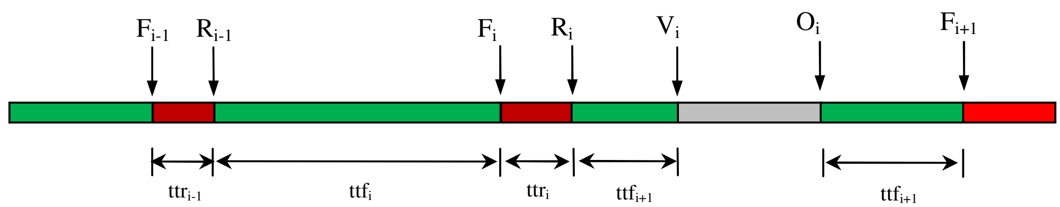


Figure 4:1 Observation of a single machine with time series data

Bayesian Updating scheme

Before the start of making inference on required parameters it might be necessary to perform preprocessing and data cleaning operations depending on the structure of the database, the type of model that is going to be used for analysis and corresponding assumptions in the model. At the end of a given observation period required vectors of observations are collected to make a new inference or update previously made inference on parameters.

Among many available statistical approaches that are used for the estimation and inference of input parameters there are two well known perspectives that commonly used to address these kinds of problems, namely the frequentist approach and the Bayesian approach. In the first few sections the main differences of using each approach is pointed out giving motivations on why the main focus of this work is on the Bayesian approach. Next it will be shown how the Bayesian approach, which is widely addressed in this study, is more appealing from the performance evaluation of manufacturing systems and practical point of view.

An important argument in favor of Bayesian approach over the classical statistical approach is that, Bayesian inference considers all unknowns (both parameters and future observations) as random variables, while in the classical statistical inference the population parameter is assumed to be fixed. The posterior variance on the estimation of parameters is a natural measure of uncertainty on the input parameters. By extension using these inputs in the performance measure enables the measurement of the uncertainty on the measured performance of interest.

In making inference on the unknown parameter that is estimated from an observed data such as failure probability *p* and repair probability *r*, we introduce a general Bayesian updating procedure. The objective of the Bayesian approach being to reach a conclusion about a generic unknown parameter θ from an observation of a stochastic variable X. The distribution of X is not completely known but depends on the value of parameter θ, with a parameter space Θ. Therefore it is possible to write the distribution probability of X as $\pi(x \mid \theta)$, where the notation π is a generic probability density. The stochastic variable X can be assumed as a random sample of $\{Y_1, Y_2, ...Y_n\}$.

$$\pi(x \mid \theta) = \Pi \pi(y_i \mid \theta) \tag{4.1}$$

For instance in the case of collecting failure data from a stochastic time to failure data, TTF, with the goal to make inference on unknown failure probability of a machine *p* the density given to a given random vectors of observations $\{ttf_1, ttf_2, ..., ttf_s\}$ the likelihood can be written as:

$$\pi(ttf \mid p) = \prod \pi(ttf_i \mid p) \tag{4.2}$$

Whether the above vectors of observations are obtained sequentially or once inference of parameter *p* can be made by using the Bayesian updating, by Bayes' theorem.

$$\pi(p \mid TTF) = \frac{\pi(p)\pi(TTF \mid p)}{\int_P \pi(p)\pi(TTF \mid p)dp} \quad p \in P \tag{4.3}$$

After the observation of *ttf* the corresponding marginal likelihood is computed over the parameter space *P*

$$\pi(ttf) = \int_P \pi(p)\pi(TTF \mid p)dp \tag{4.4}$$

Substituting the marginal likelihood in (4.4) the posterior predictive distribution of the failure probability *p* after the observation is

$$\pi(p \mid TTF) = \frac{\pi(p)\pi(TTF \mid p)}{\int_P \pi(p)\pi(TTF \mid p)dp} \tag{4.5}$$

The only remaining information to make the inference on the value of the failure probability conditional to the current observation $\pi(p \mid ttf)$ is the estimation of the prior information about the prior distribution $\pi(p)$. The choice of the prior distribution depends on the objective and nature of information at hand. Some of the most commonly used Bayesian priors fall in one of the three categories namely:

1. Conjugate priors
2. Non-Conjugate priors
3. Non-Informative Priors

Theoretically the choice can be any justified practice in Bayesian approach but in our analysis we limit ourselves to use the conjugate priors, for some of the reasons explained in coming sections.

One of the main advantages of conjugate priors is that they simplify computations, particularly in sequential applications of Bayes' theorem. With these distributions, the integral we need to compute for the posterior has a familiar form. In this particular case of inference on the parameter *p* from a geometrically distributed *TTF* the conjugate prior for the parameter *p* is the two parameter Beta distribution. List of conjugate priors for different continuous and discrete distributions can be chosen accordingly. Table 4:1 shows commonly used discrete and continuous distribution parameters frequently used in performance evaluation of manufacturing systems and their corresponding conjugate priors. Full statistical coverage and list of the conjugate priors are available in statistics books such as (Gelman A. 2003)

Likelihood	Model Parameters	Conjugate prior distribution	Prior Hyperparameters
Bernoulli	p	Beta	α, β
Binomial	p	Beta	α, β
Exponential	λ	Gamma	k, θ
Geometric	p	Beta	α, β
Poisson	λ	Gamma	k, θ
Weibull with known shape	θ	Inverse Gamma	a, b

Table 4:1Conjugate prior of commonly used distributions and hyperparameteres

If the prior distribution of $\pi(p)$ follows a $\sim Beta(\alpha_p^{'}, \beta_p^{'})$ distribution the corresponding density of the prior distribution is written as:

$$\pi(p) = \frac{1}{B(\alpha_p^{'}, \beta_p^{'})} p^{\alpha_p^{'}-1}(1-p)^{\beta_p^{'}-1} \tag{4.6}$$

In order to completely demonstrate how the inference can be made for the whole expression, we will substitute all the above terms with a sample vector of distribution. Suppose the random sample that is collected for the time to failure data composed of s number of sample is available. For the randomly sampled TTF $\{ttf_1, ttf_2, ..., ttf_n\}$ with geometric distribution the likelihood function given by equation (4.2) can be computed

$$\pi(ttf \mid p) = \prod_{i=1}^{n} (1-p)^{ttf_n - 1} p$$

$$= (1-p)^{\left(\sum_{i=1}^{s} ttf_i - n\right)} p^n \tag{4.7}$$

Substituting the likelihood expression in (4.7) and the prior distribution (4.6) gives:

$$\pi(p \mid ttf) = \frac{\frac{1}{B(\alpha,\beta)} p^{\alpha-1}(1-p)^{\beta-1} \times (1-p)^{\left(\sum_{i=1}^{s} ttf_i - n\right)} p^n}{\int_0^1 \frac{1}{B(\alpha,\beta)} p^{\alpha-1}(1-p)^{\beta-1}(1-p)^{\left(\sum_{i=1}^{s} ttf_i - n\right)} p^n dp}$$

$$= \frac{1}{B\left(\alpha+n, \beta+\sum_{i=1}^{n} ttf_i - n\right)} \times p^{\alpha+n-1}(1-p)^{\beta+\sum_{i=1}^{n} ttf_i - n-1} \tag{4.8}$$

$$\sim B\left(\alpha+n, \beta+\sum_{i=1}^{n} ttf_i - n\right)$$

$$\sim Beta(\alpha', \beta')$$

Where $\alpha' = \alpha + n$ and $\beta' = \beta + \sum_{i=1}^{n} ttf_i - n$

As it can be seen in equation (4.8) the use of the prior conjugate as a prior distribution allows to arrive to a numerically simpler solution of the posterior distribution which has the same format as the first one. This has additional advantages for the computations that use this distribution as input parameter for the performance evaluation. If one chooses to use a non-conjugate prior instead of a conjugate prior distribution the final solution one has to compute will be highly complicated in terms of mathematical effort. Further in most cases it might require to use (Markov chain Monte Carlo) MCMC methods as the only approach for deriving the solution of problems involving similar distributions.

As it can be seen from equation (4.8) the estimated failure probability parameter using the Bayesian approach p is a stochastic variable itself, and it is also possible to compute maximum likelihood to find the equivalent of the point estimate using the same data. Additionally in this case we have the information about the natural measure of uncertainty related to the estimation of the parameter.

According to the type of methodology required the above input distribution can be used for the evaluation of the performance. In this case we prefer to use the density function of the distribution which can be given again as:

$$\frac{1}{B(\alpha^{'},\beta^{'})}p^{\alpha-1}(1-p)^{\beta-1}\times(1-p)^{\left(\sum_{i=1}^{s}ttf_{i}-n\right)}p^{n} \tag{4.9}$$

Thus, the maximum likelihood for the posterior distribution can be computed as:

$$\hat{p}=\arg\max\{\pi(p\mid ttf)\}=\frac{\alpha'}{\alpha'+\beta'} = \hat{p}=\frac{s}{s+\sum_{i=1}^{s}ttf_{i}-s}=\frac{1}{mttf} \tag{4.10}$$

The point estimate of the failure probability using maximum likelihood is the same as if it was calculated by using the mean value from the observations of time to failures ttf .

4.2 Analysis of an Isolated Machine

In the first few sections techniques for the performance analysis of the simplest possible systems in production system, i.e., isolate machine are demonstrated. Next fundamental differences and the impact of performance evaluation under uncertainty are investigated in comparison with evaluation using precisely known parameters. The modeling assumptions used for the characterization of machines in the case of individual processing machine are described.

Modeling machine Assumptions

- Failures are operational dependent failures (ODF)
- The probability that machine a machine fails in a time unit in the failure mode with a precisely known or unknown failure rate p
- Time to failures, *TTF* are assumed to be geometrically distributed with an unknown failure rate p.
- The Time to repair a machine, *TTR* that is down in a failure mode is assumed to be geometrically distributed with a known or unknown repair rate parameter r.

Isolated machine model

This section presents how the Markovian model of an isolated single machine with uncertain probabilities of failure and repair can be analyzed. First a univariate case is considered, when there is only one uncertainty parameter. Exact analytical formulas are provided for the output performance distribution for the univariate cases. In the case of multiple uncertainties numerical techniques are proposed for the evaluation of interesting moments and distribution of the output performance. The exact analytical solution and the simplicity of the system is chosen to demonstrate the impact of considering uncertainty in the performance evaluation when compared to the traditional approaches that assume input parameters are precisely known.

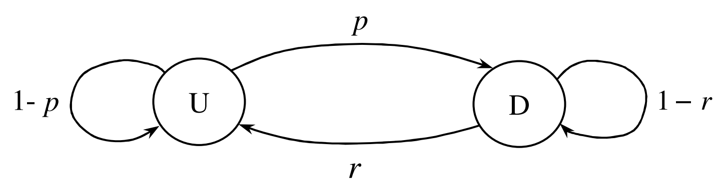


Figure 4:2 Markovian model of single machine with up and down states

An isolated machine under the assumption stated above and its unknown probability of failure p or probability of repair r is estimated as described in section (4.1). The vector of observations of *TTF* and *TTR* are assumed to be geometrically distributed. The estimation for the probability parameter of a geometrically distributed statistics as provided in Table 4:1 is a Beta distribution. Posterior distributions of the unknown failure probability p or the repair probability r which are estimated from geometric *TTF* and *TTR* follow a Beta distribution $\sim Beta(\alpha_p, \beta_p)$ and $\sim Beta(\alpha_r, \beta_r)$ respectively. The densities of these posteriors are given as (4.11) and (4.12).

$$f_P(p) = \frac{1}{B(\alpha_p, \beta_p)} p^{\alpha_p - 1}(1-p)^{\beta_p - 1} \tag{4.11}$$

$$f_R(r) = \frac{1}{B(\alpha_r, \beta_r)} r^{\alpha_r - 1}(1-r)^{\beta_r - 1} \tag{4.12}$$

4.2.1 Exact Analytical Method

Exploiting relative simplicity of the Markovian model of the isolated machine here we derive the exact analytical solutions of the output distribution density of the isolated machine efficiency $f_E(e)$. Using the model presented in Figure 4:2 first the efficiency e for an unknown p and a precisely known r is investigated. The next section follows the same procedure to demonstrate the solution when converse is the case i.e., p is precisely known and r is an unknown parameter that needs be estimated. In all the cases of the isolated efficiency e of a single Markovian machine in Figure 4:2 with a unique failure and precisely known p and r is given as (4.13).

$$e = \frac{r}{r+p} \tag{4.13}$$

Case 1: *Unknown p*

The exact analytical technique aims to solve the density of the average throughput $f_E(e)$ given the posterior density of $f_P(p)$ is expressed as in (4.11). With the precisely known r and using the functional relationship of random variables the density of the isolated efficiency can be expressed as:

$$f_E(e) = \frac{f_P(p)}{\left|\left.\frac{\partial e}{\partial p}\right|_{r=\hat{r}}\right|} \qquad \text{Where } \left|\left.\frac{\partial e}{\partial p}\right|_{r=\hat{r}}\right| = \frac{r}{(p+r)^2}$$

$$f_E(e) = \frac{f_P(p)}{\frac{r}{(p+r)^2}} \tag{4.14}$$

Since the expression $f_E(e)$ is required to be in terms of *e* then all the terms given in *p* should be solved in *e* and substituted giving:

$$p = \frac{r(1-e)}{e} \tag{4.15}$$

Evaluating this expression provides the density of the isolated efficiency of the machine as a function of *e*.

$$f_E(e) = \frac{\left(\frac{r(1-e)}{e}\right)^{\alpha_p - 1}\left(1 - \frac{r(1-e)}{e}\right)^{\beta_p - 1}\left(r + \frac{r(1-e)}{e}\right)^2}{rB(\alpha_p, \beta_p)} \tag{4.16}$$

Once the posterior parameters of the uncertain p, α_p and β_p are known the required moments of the isolated efficiency can be determined from the output density (4.16). In many cases the commonly required moments are the first two moments. If the objective is to determine these two moments i.e., the expected value and variance of the isolated efficiency they can be computed by integrating $f_E(e)$ within the domain of e.

$$E[e] = \int_E e \times f_E(e)de \tag{4.17}$$

$$V[e] = \int_E (e - E[e])^2 f_E(e)de \tag{4.18}$$

Case 2: *Unknown r*

The same analysis can be repeated by assuming p as a precisely known parameter and the repair probability r can be considered an estimate with uncertainty. Following the same approach demonstrated above the corresponding equations in unknown r are evaluated as follows.

$$f_E(e) = \frac{f_R(r)}{\left|\left.\frac{\partial e}{\partial r}\right|_{p=\hat{p}}\right|} \qquad \text{Where} \qquad \left|\left.\frac{\partial e}{\partial r}\right|_{p=\hat{p}}\right| = \frac{p}{(p+r)^2}$$

$$f_E(e) = \frac{f_R(r)}{\frac{p}{(p+r)^2}} \tag{4.19}$$

Since in this case also we need the terms in terms of e and the known parameter p

$$f_E(e) = \frac{\left(\frac{ep}{1-e}\right)^{\alpha_r - 1}\left(1 - \frac{ep}{1-e}\right)^{\beta_r - 1}\left(p + \frac{ep}{1-e}\right)^2}{pBeta(\alpha_r, \beta_r)} \tag{4.20}$$

Similarly the first two moments of an isolated efficiency of the single machine with uncertain repair probability can be computed from the output distribution.

$$E[e] = \int_E e \times f_E(e)de$$

$$V[e] = \int_E (e - E[e])^2 f_E(e) de$$

Direct evaluation of moments (Expected value and Variance) of isolated efficiency

Direct evaluation of the output distribution of the performance measure can be interesting for completeness of the solution. If the density of the output distribution is evaluated exactly then from this distribution the evaluation of higher order moments such as skewness and kurtosis of the distribution can be done precisely. Considering the analytical difficulty involved when considering multivariate cases and dealing with complex performance models, in some cases it can be easier to directly evaluate only the required moments. In addition in most practical cases it suffices to analyze the first two moments i.e., to evaluate the expected value and the uncertainty in variance of the output performance measure. Consequently in many of the future discussions in this thesis the proposed methods focus on the evaluation of these two moments. This widens the range of problems that can be addressed using the techniques including more complex performance models and alternative forms the input uncertain distribution.

Direct evaluation of moments with single uncertainty

Next the exact analytical evaluation that provides the computation of the first and second moments of the isolated efficiency e is given when only one of the parameters i.e., p or r is known with uncertainty.

Single uncertainty (Univariate case)

Provided that the posterior density function of the input is expressed as in (4.11), direct integration techniques can be applied to evaluate the expected value and the uncertainty of the isolated efficiency. Considering the above (case 1) where p is uncertain, the moments of the isolated efficiency e can be directly evaluated without evaluating the density of the output distribution $f_E(e)$.

$$E[e] = \int_P e \times f_P(p)dp$$

$$E[e] = \int_P \frac{r}{r+p} \times f_P(p)dp$$

$$E[e] = \Gamma(\alpha_p + \beta_p)_2 F1(1, \alpha_p, \alpha_p + \beta_p, -\frac{1}{r}) \tag{4.21}$$

Γ is the incomplete Gamma function and ${}_2F1$ is a regularized Hypergeometric function.

Using the expected value obtained above the variance can be evaluated

$$V[e] = \int_P (e - E[e])^2 \times f_P(p)dp$$

$$V[e] = \int_P \left(\frac{r}{r+p} - E[e] \right)^2 \times f_P(p)dp \tag{4.22}$$

Similarly when the repair probability is the parameter with estimation uncertainty the expected value and uncertainty of the isolated efficiency E[e] is evaluated.

$$E[e] = \int_R \frac{r}{r+p} \times f_R(r)dr$$

$$E[e] = \frac{1}{p}(\alpha_r + \beta_r)\Gamma(\alpha_r + \beta_r)_2 F1(1, \alpha_r + 1, \alpha_r + \beta_r + 1, -\frac{1}{p}) \tag{4.23}$$

$$V[e] = \int_R \left(\frac{r}{r+p} - E[e] \right)^2 \times f_R(r)dr \tag{4.24}$$

Multiple uncertainties (Multivariate case)

Continuing the analysis of isolated machine case where both the failure probability *p* and repair probability *r* are unknown the direct evaluation of moments can be used to evaluate the moments of isolated efficiency *e*.

The joint probability density of *p* and *r* can be evaluated from the marginal density of their individual distributions.

$$f_{P,R}(p,r) = f_{R|P}(p,r) \times f_P(p) \tag{4.25}$$

Both failure probability p and repair probability r are assumed independent. Therefore the posterior distribution of p and r is computed from the joint density of the two posterior distributions. Following this the expression (4.25) can be simplified to be the product of the marginal density function of each uncertain parameter.

$$f_{P,R}(p,r) = f_R(r) \times f_P(p) \tag{4.26}$$

Integrating with respect to the two uncertain parameters the expected performance and the corresponding uncertainty in variance can be evaluated.

$$E[e] = \int_P \int_R \frac{r}{r+p} \times f_{P,R}(p,r)drdp$$

$$E[e] = \frac{\Gamma[a]\Gamma[b]\Gamma[d]\left(\frac{\pi Csc[a\pi]\Gamma[a+c]_2F1[1-b,a+c,a+c+d,-1]}{\Gamma[a]\Gamma[b]\Gamma[a+c+d]} + \frac{\Gamma[1+c]pFq[\{1,2-a-b,1+c\},\{2-a,1+c+d\},-1]}{(-1+a)\Gamma[-1+a+b]\Gamma[1+c+d]}\right)}{Beta[a,b]Beta[c,d]} \tag{4.27}$$

$$V[e] = \int_P \int_R \left(\frac{r}{r+p} - E[e]\right)^2 \times f_{P,R}(p,r)drdp \tag{4.28}$$

In most cases where the posterior densities are not in a simplified form, including the Beta distribution the above integrals can be complex to express as a closed analytical expression in terms of the input distributions hyperparameters. Instead numerical integration techniques can be applied to evaluate the required moments of the output distribution.

Analysis of isolated machine with uncertainty

Using the above expressions for the evaluation of the output distribution and its moments we investigate the main differences against using the point estimates by ignoring the uncertainty in estimation. The comparison is primarily carried out only for the expected value of the isolated efficiency E[e] since the uncertainty can’t be measure by using only the point estimates. In order to make an opportune comparison the same data is used for the point estimate of parameters and for making the Bayesian inference. Moreover even if prior subjective knowledge can be integrated into the Bayesian approach it is assumed there is no information available prior to the data set used for this estimation. Therefore equivalently the point estimates can be made from the hyperparameters of the posterior distribution of the input parameters.

Case 1: ***Unknown p***

Considering the data used for the inference of the posterior distribution of p is used also for the point estimate of $\hat{p}$. From the posterior hyperparameters of p which is assumed beta distributed

$$\hat{p} = \arg\max\{\pi(p \mid TTF)\} = E[\pi(p \mid TTF)] = \frac{\alpha_p}{\alpha_p + \beta_p} \quad (4.29)$$

The expected value of the isolated efficiency from the point estimate of the failure probability p and repair probability r is given by the:

$$\hat{e} = \frac{\hat{r}}{\hat{r} + \hat{p}}$$

By numerically evaluating the expression given for the expected value of isolated efficiency E[e] in (4.21) in comparison with the point estimate $\hat{e}$ using the point estimate of $\hat{p}$ the following inequality is always true.

$$\Gamma(\alpha_p + \beta_p)_2 F1(1, \alpha_p, \alpha_p + \beta_p, -\frac{1}{\hat{r}}) \geq \frac{\hat{r}}{\hat{r} + \hat{p}} \quad (4.30)$$

Theorem 1: If the only uncertain parameter is p and the isolated efficiency evaluated using the expected input E[p], $\hat{p}$ considering as deterministic parameter for evaluating $\hat{e}$, then this value is always less than $E[e]$ i.e., $e(E[p]) \leq E[e(p)]$.

The proof of Theorem 1 is given using Jensen inequalities for convex functions in Appendix (A.1). Alternatively the second order derivative test for the isolated efficiency with respect to p can be performed. The same conclusion can be reached by using the condition for the second order derivative test.

$$\frac{\partial^2 e}{\partial p^2} = \frac{2r}{(r+p)^3} > 0 \tag{4.31}$$

Figure 4:3 graphically shows this behavior for the convex isolated efficiency curve as a function of *p* and the corresponding underestimation by using the point estimate $\hat{p}$.

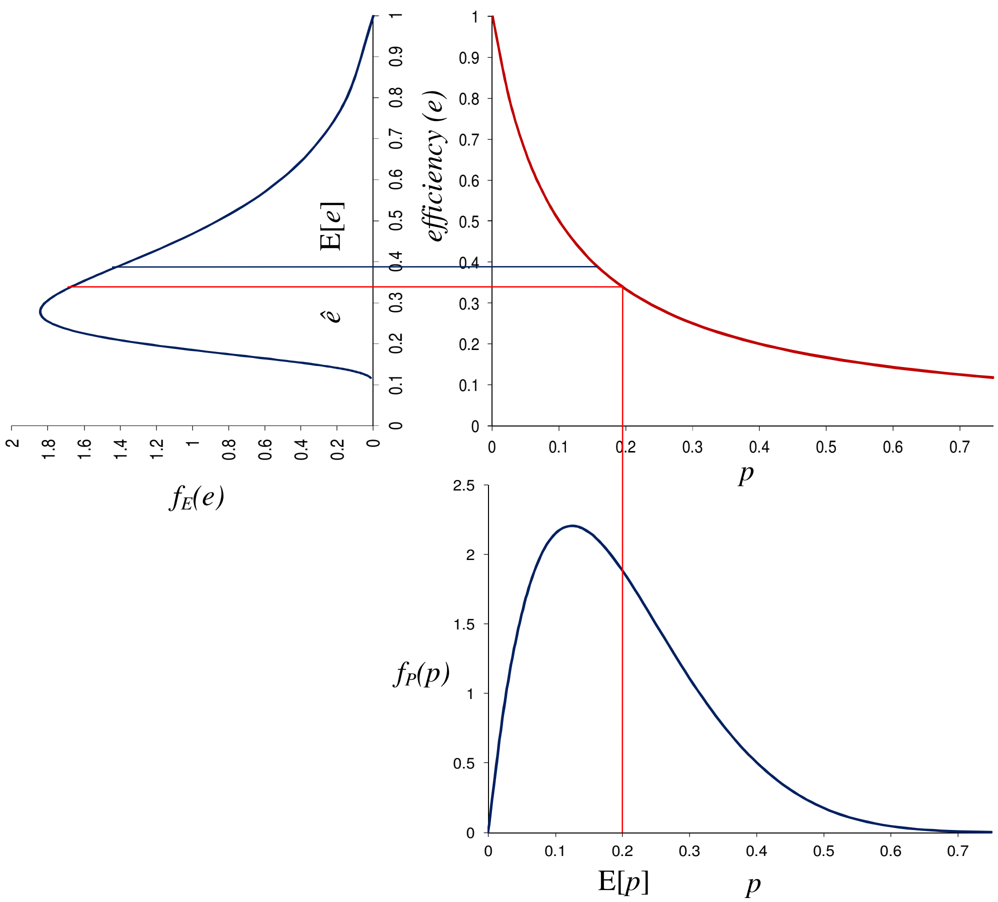


Figure 4:3 Impact of uncertainty of *p* on E[e].

As it is demonstrated in the figure the second order derivative skews the distribution of the average throughput to top of the graph making the expected value to be greater that the value that can be achieve if only the expected value of the uncertain input parameter had been used. Generally the magnitude of the deviation between $\varphi(E[p])$ *and* $E[\varphi(p_i)]$ can be given as a function of the absolute value of the second derivative and the input uncertainty in variance. For demonstrating this behavior the following test data in Table 4:2 is used and the results are reported in Figure 4:4.

p	σ_p	r	(e-E[e])/ E[e]*100%
0.05	0.0001-0.002	0.1	-0.5-7.5

Table 4:2 Data for isolated machine with uncertain p

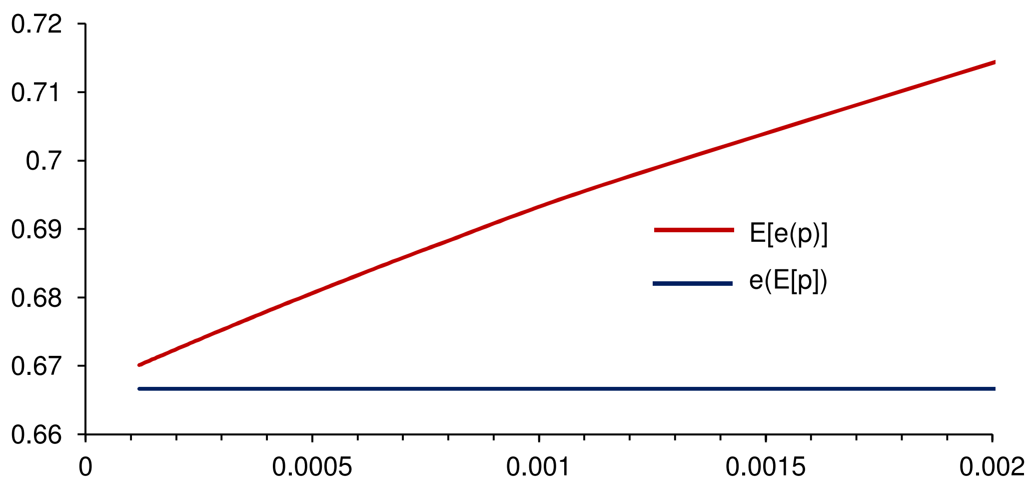


Figure 4:4 Underestimations on E[e] with increasing uncertainty of *p*

Case 2: *Unknown r*

Considering the data used for the inference of the posterior distribution of *r* is used also for the point estimate of $\hat{r}$. From the posterior hyperparameters of r which follows a beta distribution

$$\hat{r} = \arg\max\{\pi(r \mid TTR)\} = E[\pi(r \mid TTR)] = \frac{\alpha_r}{\alpha_r + \beta_r} \tag{4.32}$$

Point estimate of the isolated efficiency from the point estimate of the failure probability *p* and repair probability *r* is given as (4.32).

Numerically evaluating the expression given for the expected value of the isolated efficiency $E[e]$ in (4.23) and comparing the result with the point estimate $\hat{e}$ using the point estimate of $\hat{r}$ by the following inequality is always true.

$$\frac{1}{p}(\alpha_r+\beta_r)\Gamma(\alpha_r+\beta_r)_2F1(1,\alpha_r+1,\alpha_r+\beta_r+1,-\frac{1}{p})\leq\frac{\hat{r}}{\hat{r}+\hat{p}} \tag{4.33}$$

Theorem 2: for an isolated machine if the only uncertain parameter is r and the isolated efficiency evaluated using the expected input E[r], $\hat{r}$ considered as deterministic parameter to evaluate $\hat{e}$, this value is always greater than E[e] i.e., $e(E[r]) \geq E[e(r)]$.

The proof of Theorem 2 is given using Jensen inequalities for concave functions in Appendix (A.2). Performing the second order derivative test of the isolated throughput function with respect to r also leads to the same conclusion.

$$\frac{\partial^2 e}{\partial r^2}=-\frac{2p}{(p+r)^3}<0 \tag{4.34}$$

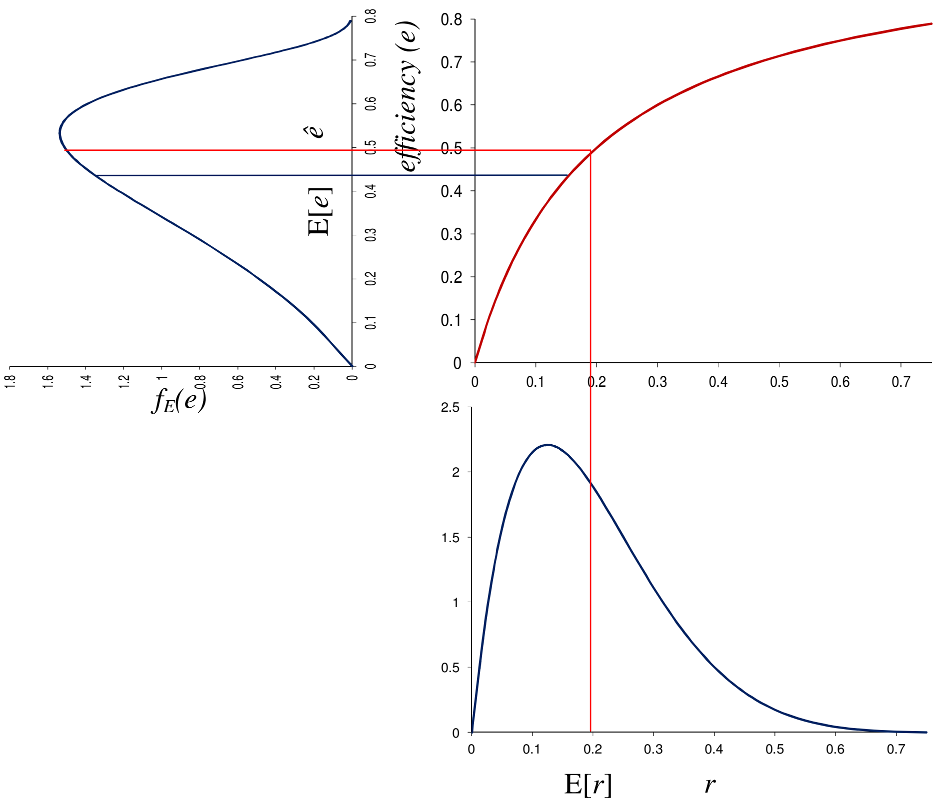


Figure 4:5 Impact of uncertainty on the E[e].

The same procedures used in the case of uncertain p to prove the observed underestimation is always consistence when the uncertainty in the input parameters is not considered can be used. By the converse of the above Jensen theorem that is used for *p* it is possible to show the direction of the inequality changes if the function is concave. It is sufficient to show that the function is strictly concave by using the second derivative test. Therefore it is possible to conclude that the error in the case of carrying out performance evaluation by neglecting the uncertainty in r is a consistent overestimation of the isolated efficiency. For the input data that is given in Table 4:3 the corresponding deviation in the E[e] results are reported in Figure 4:6.

R	σ_r	p	(e-E[e])/ E[e]*100%
0.05	0.0001-0.002	0.05	1.18-22

Table 4:3 Data for isolated machine with uncertain r

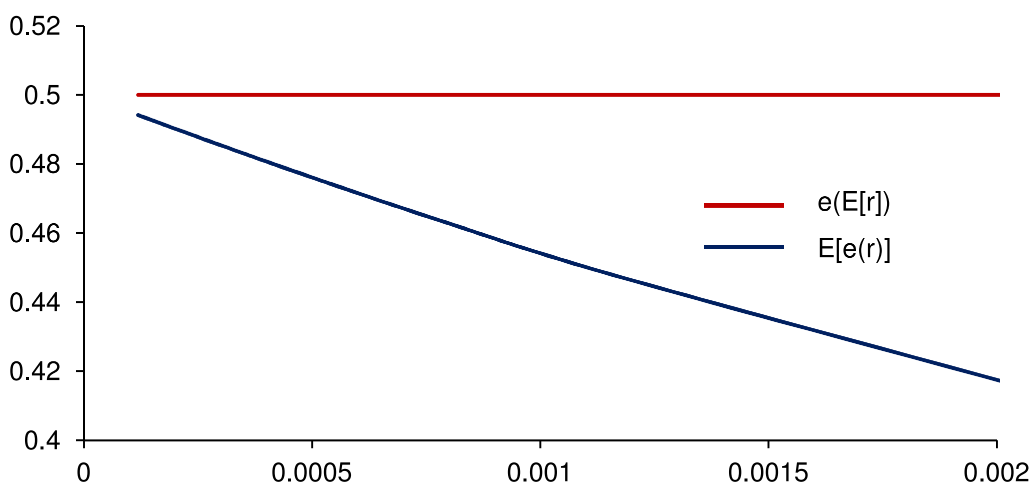


Figure 4:6 Overestimation on E[e] with increasing uncertainty of repair probability

Case 3: uncertain *r* and uncertain *p*

The previous two cases investigated the impact of uncertainty when only one of the parameters is uncertain estimation and other parameter is assumed to be a precisely known value. In this section the impact of uncertainty on the efficiency of an isolated single machine is investigated when both *p* and *r* are unknown. To investigate the behavior of the expected value of the isolated efficiency with multiple uncertain variables one convenient approach is investigation of the concavity or convexity. The Hessian matrix can be used for the analysis of the second order derivative. Considering the Eigen values of the Hessian matrix the behavior of the deviation can be determined.

The Hessian matrix of the isolate efficiency as a function of *p* and *r* is:

$$H(p,r)=\begin{vmatrix} \frac{\partial^2 TH}{\partial p^2} & \frac{\partial^2 TH}{\partial p \partial r} \\ \frac{\partial^2 TH}{\partial r \partial p} & \frac{\partial^2 TH}{\partial r^2} \end{vmatrix}$$

$$H(p,r)=\begin{vmatrix} \frac{2r}{(p+r)^3} & \frac{-p+r}{(p+r)^3} \\ \frac{-p+r}{(p+r)^3} & \frac{2p}{(p+r)^3} \end{vmatrix} \tag{4.35}$$

Evaluating the Eigen values of this matrix gives:

$$\frac{-p+r+\sqrt{2(p^2+r^2)}}{(p+r)^3}$$

$$\frac{-p+r-\sqrt{2(p^2+r^2)}}{(p+r)^3} \tag{4.36}$$

The Eigen values are always opposite in sign the first one is always positive and the second one negative for all sets of the function values. Based on the hessian matrix test this condition confirms that the results are not conclusive. Therefore for the given function a local minimum or maximum in all the domain of *p-r* axis doesn't exist; meaning no local convexity or concavity anywhere on the graph.

Graphically it is also possible to show if the isolated efficiency function is a concave or convex with respect over p-r axis. The conditions for concavity or convexity can be stated as follows. If *f* is a function of many variables, *f* is **concave** if the line segment joining any two points on the graph of *f* is never above the graph; *f* is **convex** if the line segment joining any two points on the graph is never below the graph.

This test can be done by looking at the surface of the isolated efficiency on the *p*-*r* axis and trying to connect any two points on the graph as shown in Figure 4:7.

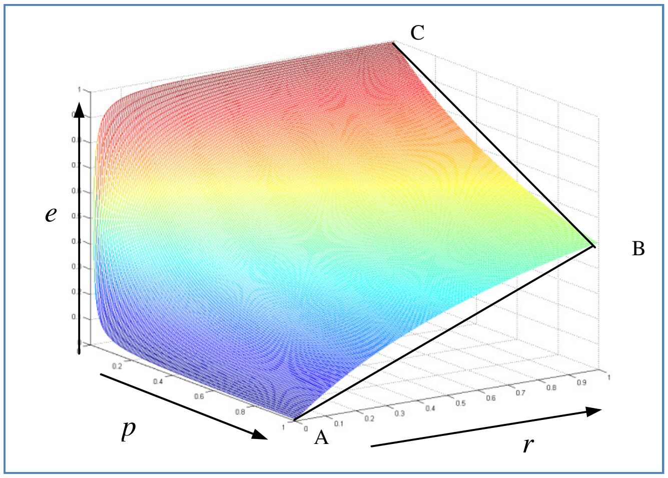


Figure 4:7 Efficiency of an isolated machine as a function of *p* and *r*

If at any point on the surface we try to connect two points by a straight line along the *r* axis, for e.g. (AB) we always fall below the graph and if we try to draw another line along *p* the straight line is always above the graph. Therefore with the above definitions of convexity or concavity, then graphically it can be concluded that the surface is neither concave nor convex.

If there is a region where the graph has a saddle point we can get a locally concave or convex surface. But from the behavior of the isolated efficiency function we know it is strictly concave with *r* and strictly convex along *p*. Consequently this function is neither concave nor convex for anywhere in the given region across *p*-*r* plane.

Estimation errors that might result by ignoring the uncertainty in the parameter estimation can lead to either underestimation or overestimation errors on the expected isolated efficiency. The direction of this estimation errors depend on the particular mean values of the input parameter which determines their corresponding second order partial derivatives and the input uncertainties of the two parameters in variance. Randomly generated test experiments conducted using both uncertain p and r also confirms that the errors in the isolated efficiency could be both underestimation and overestimations. A test run to demonstrate this behavior with 20 sample experiments are reported in Figure 4:8.

In many cases the objective of uncertainty analysis in performance evaluation can be seen from different perspectives, depending on the particular purpose of the performance evaluation. In most of this study we are interested in estimating the expected throughput and the uncertainty in variance of the performance measurement are discussed. The first obvious importance of these approaches is to measure the uncertainty associated to a given performance evaluated with uncertain inputs. Secondly it also assists if there is a deviation in the expected value of a performance measure when uncertainty is introduced in the analysis. This section shows the implication of these two aspects with examples and general proofs how the introduction of uncertainty impacts the performance measurement output compared to the analysis with point estimated not including uncertainty.

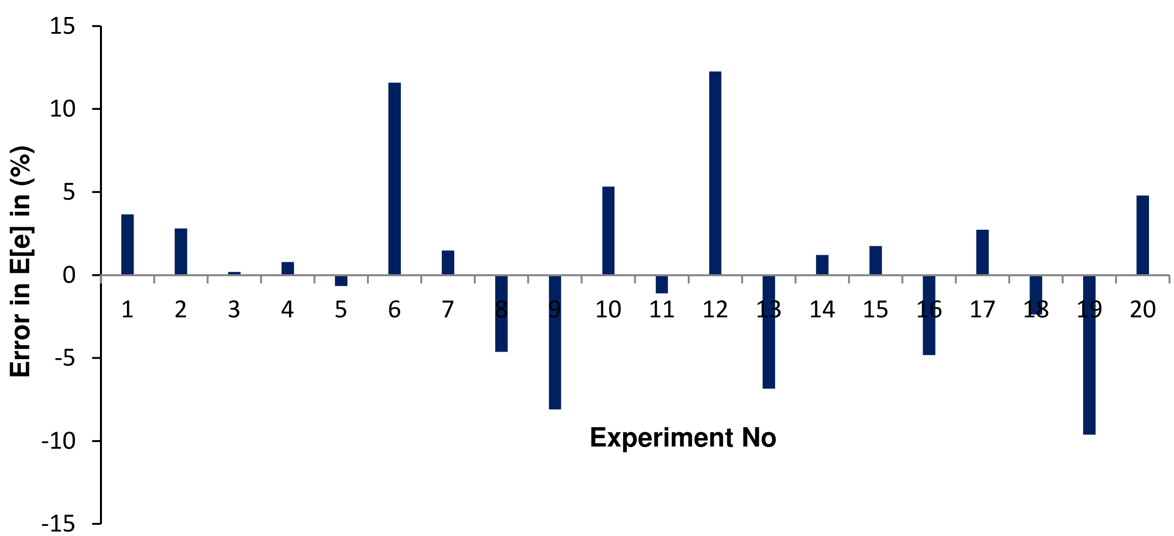


Figure 4:8 Underestimation and overestimation with uncertain p and r

4.2.2 Approximate Methods

The techniques that are proposed in the previous section provide an exact relationship between the unknown input parameter distribution and the output distribution of a performance measure. Generally these methods provide the exact evaluation of the output distribution with respect to the input distributions. Although the techniques introduced in the previous sections are exact and provide precise results they can be limited in application for general performance analysis in practice. Some of the motivations for the need for alternative approximate techniques instead of exact analytical approaches are the following.

Performance evaluation models: Performance functions of relatively simpler models such as an isolated single machine involve simplified function that can be expressed in simple functions. The simplicity of these functions makes them easy to perform inversion, integration, substitution operation with relative less complexity. In practice most of the performance evaluation techniques, including performance models to evaluate simplified systems such as two machine single buffer systems are complex rational functions. Usually the techniques applied for the performance evaluation of complex systems require the use of complex expressions, introduction of multiple uncertain parameters and approximate techniques.

Multiple parameters: Performance analysis of complex manufacturing systems quite often involves the estimation of many parameters. When the number of uncertain parameters grows the evaluation of solution using exact analytical methods requires the application of multiple integrals equal to the number of uncertain parameters. Beyond a limited number of uncertain parameters this often leads to a well known mathematical problem “curse of dimensionality”. Approximate techniques help to avoid this problem and can carryout complex analysis in a reasonable time.

Uncertainty modeling: In addition to the need to involve complex functions and multiple uncertainties into the performance evaluation, the way in which the uncertainty is modeled poses other requirements in the solution technique that can be effectively used for evaluation. For instance in instead of an uncertainty modeling in terms of a probability density function, the available information could be the mean of a parameter and the associated uncertainty in variance.

Considering the above challenges and other advantages of approximate techniques in the coming section alternative approximate techniques for the performance evaluation with uncertainties are introduced. In order to overcome some of these limitations and benefit from other computational and practical advantages approximate techniques for the performance analysis with uncertain parameters are proposed. Alternative approaches are proposed depending on the nature of the problem and requirements in accuracy and computational efficiency some of the methods can be used for solving similar problems.

4.2.2.1 Monte Carlo Method (MC)

Monte Carlo techniques can be used to evaluate the performance distribution of a performance once the posterior distribution is determined. The Monte Caro method uses random sampling from the posterior input distributions. Different kinds of random walk algorithms can be used to sample from the posterior input distribution. By using random points generated from the input posterior then output performance is evaluated at these points. Finally the output distribution or interesting statistics of the output performance can be evaluated.

Many reasons might demand solutions to be obtained only by Monte Carlo techniques. One of this is the "curse of dimensionality" that comes with increasing number of parameters. The curse is that the required computation to solve a problem in many dimensions may grow exponentially with the dimension. For instance if one needs to compute an integral over ten variables by numerical integration in ten dimensional space using twenty points in each coordinate direction, the total number of integration points is $20^{10} \approx 10^{13}$, which is on the edge of what a computer can do in a day for basic iterations. A Monte Carlo computation might reach the reasonable accuracy with only, say, 10^{6} points.

Step 1: For an uncertain parameter p the lower and upper limit p_{min} and p_{max} can be used so that a uniform random number generator can be used for the generation of random samples in the given range. The posterior distribution of the input parameter $F_P(p)$ can be used for the sampling procedure. Considering a particular random number, rnd1 the following figure illustrates how rnd1 selects a particular value x1 from the probability density function p(x). If a large number of such x1 values were selected using a series of random numbers rnd1, the histogram of all the x1 choices, normalized by the integral of the histogram (sum of all histogram bin values times the bin widths), will match the original p(x).

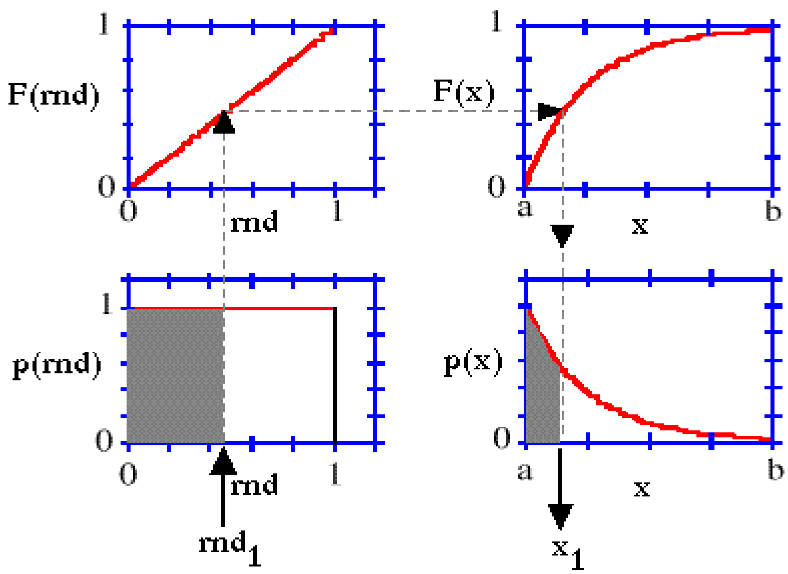


Figure 4:9 Sampling from a single uncertain parameter using Monte Carlo

For density functions there are direct random sample generating functions which can directly carry out this procedure. These functions can be directly used when available. For determined sample size n of p with a generic parameter θ it provides { $p_1, p_2, p_3, ... p_n$ }

Step 2: After the definition of sufficient sample size for the uncertain parameter and vector of random values the output performance is analyzed for each point of these samples. The output distribution $F_E(e)$ of the isolated efficiency can be computed as the corresponding weights of these individual output.

Step 3: Evaluate the required moments of the output.

$$E[e] = \frac{1}{n}\sum_{t=1}^{n} e_t \quad (4.37)$$

$$V[e] = \frac{1}{n-1}\sum_{t=1}^{n}\left(e_t - E[e]\right)^2 \quad (4.38)$$

4.2.2.2 Linearized Probability density function

Problems involving a general performance function *f(x)* or as in the specific case of isolated machine *e(p)* or *e(r)* can be evaluated using this approach. One requirement is that the uncertainty of input parameters should be given as a density function with expression that can be solved by this method. The linearization is performed by using a piece wise functions in different intervals as a linear approximation of the density function. Finally each piece wise evaluation can be used to evaluate the expected value and uncertainty of the performance measure. This method is based on trapezoidal integration techniques and the integral formulas that are used to compute the first two moments of uncertain parameters as indicated in exact numerical integration in section 4.2.1. Besides this kind of approximations are useful in cases where the input posterior density function cannot be directly expressed analytically or the performance function or its inverse might not be continuous. In this particular case similar but a modified approach can be used in a piece wise where the function is invertible and continuous.

The proposed technique first divides the density functions into approximated linear lines. The linearization of the density function using pieces of linear functions that approximate the distribution of the throughput density function and the subsequent integrations required to estimate the expected value and the variance of the performance are explained in the following steps. The steps are shown for the single machine case, even though similarly they can be extended to any performance function with a single uncertainty.

Step 1: Define the density function of the probability of failure p that needs to be estimated using stochastic approach. Furthermore define the number of intervals that are sufficient to achieve the desired accuracy on the throughput function. Specify the maximum and minimum interval of p as, p_{min} and p_{max}, as well the number of intervals n. Use the interval $\frac{p_{max} - p_{min}}{n}$ to calculate the value of the density function at each p_i in terms of the probability density function or the empirical relationship that is defined to approximate the distribution: $f_{p_i}(p_i) = f_p(p) \mid p = p_i$.

Step 2: Calculate the throughput for the maximum and minimum value of the p. This step should provide an interval for the range of possible values of each e_i using the corresponding values of p_{min} and p_{max} in the isolated efficiency function $e(p)$. Determine the domain of the density function of $f_e(e)$ using the range of the throughput function which generally can be written as:

$$e_{min} = e(p_{max}) \text{ or } e_{min} = e(p_n) \text{ and}$$

$$e_{max} = e(p_{min}) \text{ or } e_{max} = e(p_0)$$

The domain of $f_e(e)$ is defined in the interval $e_{min} \le e \le e_{max}$

Step 3: For each value of p_i at an interval $\frac{p_{max} - p_{min}}{n}$ calculate the corresponding throughput values as:

$$e_i = e(p_i); \forall i = 0,1 \ldots n \tag{4.39}$$

Step 4: Calculate the values of the derivatives of the throughput at each point of the failures probabilities that are considered in the above steps. To calculate the derivatives first define an appropriate an infinitesimal interval on the p axis Δp. The calculation of the derivatives at the $e'(e^{-1}(e_i))$ can be computed as the rate of change of Δe versus Δp, we can write this relation as:

$$\left.\frac{\partial e}{\partial p}\right|_{p=p_i} = \frac{e(p_i+\Delta p)-e(p_i)}{\Delta p} \tag{4.40}$$

For each e_i evaluate the above value and form an array of values $D_i; \forall i = 0,1\ldots n$

Step 5: Using the values calculated in step 4 and the density function value at $f_p(p_i)$ calculate the following value as

$$\left.f_e(e)\right|_{e=e_i} = \left|\frac{\left.f_p(p)\right|_{p=p_i}}{\left.\frac{\partial e}{\partial p}\right|_{p=p_i}}\right| = \frac{\left.f_p(p)\right|_{p=p_i}}{D_i} \tag{4.41}$$

Step 6: Develop a piece wise linear equation that connects each of the $\left.f_e(e)\right|_{e=e_i}$ which in intervals define the density function of the efficiency distribution. At this step we have to reorder the $\left.f_e(e)\right|_{e=e_i}$ in increasing order and assign from smallest $f_e(e_0)=\left.f_e(e)\right|_{e=e_n}$ …. $f_e(e_i)=\left.f_e(e)\right|_{e=e_{n-1}}$ and so on in increasing order till $f_e(e_n)=\left.f_e(e)\right|_{e=e_0}$. The same should be applied for the order of e_i i.e. $e_i = e_{n-i}$

For interval i to $i+1$

$$\frac{f_e(e_{i+1})-f_e(e_i)}{e_{i+1}-e_i} = \frac{f_e(e)-f_e(e_i)}{e-e_i}$$

This can be written in the form $f_e(e)_i = Me_i \times e + be_i$

Where

$$Me_i = \frac{f_e(e_{i+1})-f_e(e_i)}{e_{i+1}-e_i} \tag{4.42}$$

$$be_i = f_e(e_i) - e_i \times \frac{f_e(e_{i+1}) - f_e(e_i)}{e_{i+1} - e_i} \tag{4.43}$$

The integration of the functions of these linear lines in their respective domain and their summation yields the expected value and variance of the throughput distribution.

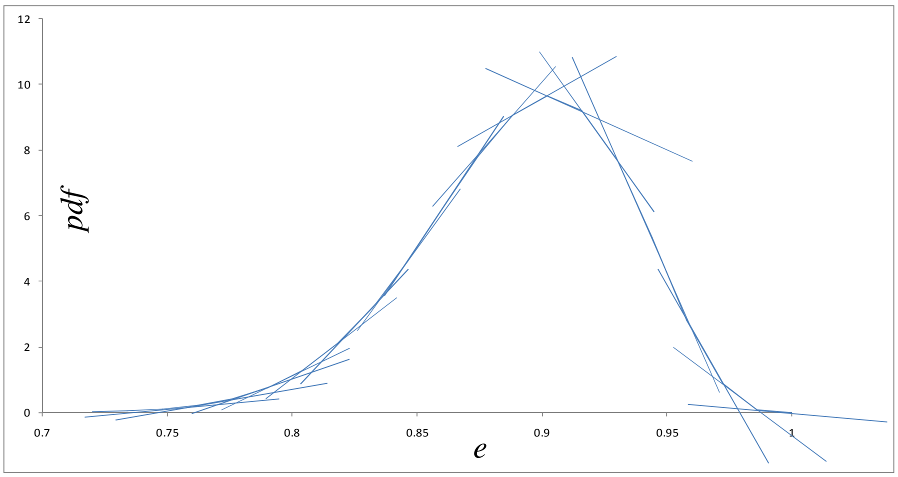


Figure 4:10 Piecewise linear density of an isolated efficiency

$$\mathrm{E}[\mathrm{e}] \approx \int_{e_0}^{e_1} e(Me_0 \times e + be_0)\,de + \ldots \int_{e_1}^{e_2} e(Me_1 \times e + be_1)\,de \;\ldots \int_{e_{n-1}}^{e_n} e(Me_{n-1} \times e + be_{n-1})\,de$$

$$\mathrm{V}[\mathrm{e}] \approx \int_{e_0}^{e_1} (e-\mu)^2 (Me_0 \times e + be_0)\,de + \int_{e_1}^{e_2} (e-\mu)^2 (Me_1 \times e + be_1)\,de + \int_{e_{n-1}}^{e_n} (e-\mu)^2 (Me_{n-1} \times th + be_{n-1})\,de$$

These integrations can be reduced to the following summation terms:

$$\mathrm{E}[\mathrm{e}] \approx \sum_{i=0}^{n-1} \frac{Me_i}{3}({e_{i+1}}^3 - {e_i}^3) + \frac{b_i}{2}({e_{i+1}}^2 - {e_i}^2) \tag{4.44}$$

$$\mathrm{V}[\mathrm{e}] \approx \sum_{i=0}^{n-1} \frac{Me_i}{4}({e_{i+1}}^4 - {e_i}^4) + \frac{be_i - 2\mu Me_i}{3}({e_{i+1}}^3 - {e_i}^3) + \frac{\mu^2 Me_i - 2\mu be_i}{2}({e_{i+1}}^2 - {e_i}^2) + \mu^2 be_i(e_{i+1} - e_i) \tag{4.45}$$

4.2.2.3 Partitioning and discretization

In this section discretization technique of the posterior density function of an estimated uncertain parameter using partition is introduced. The discretized input posterior distribution is then used to evaluate interesting performance measures and related uncertainty of the output performance measure. Generally this method relies on transforming a given density function in to an equivalent probability mass function and use these points as weighted points of evaluation. Particularly this method is important under cases where the possible alternative outcomes are provided as weighted probabilities of occurrence. Additionally it is an easy to use approach to consider multiple uncertain parameters without the need to worry about the mathematical complexity of performance evaluation models and functions.

The discretization technique begins with the definition of a sufficient number of T_u partitions for a predetermined level of accuracy for each uncertain parameter $u = 1,\ldots,U$ The procedure is shown in the following for the uncertain parameters for instance a failure probability p. However, it can be applied in the same way for the repair probabilities r. For simplicity of demonstration the procedure is firstly shown on a single uncertainty and can r be extended to any $u = 1,\ldots,U$ uncertain parameters.

For the single uncertain parameter p the lower and upper limit p_{min} and p_{max} are determined so that the integral area under the density function approximately equals to 1 as that of the original *pdf* $f_P(p)$. The Δp partition width can be determined by using different techniques so the transformation of the density function into a probability mass function can be carried out by preserving all the moments of the original distribution. In general cases particularly in probability density function with low skewness a linear spacing can be used while in the other cases the spacing can be varied depending on the accuracy of linearly approximating a curvature of the density function. These considerations can be formulated in different ways such as a problem of optimizing algorithms to minimize the number of partitions required to satisfy predetermined level of target accuracy on the approximation of the density function.

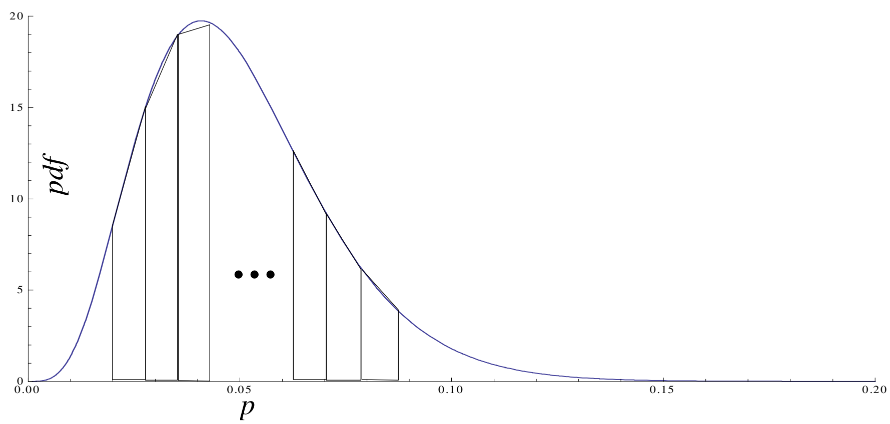


Figure 4:11 Spaced trapezoidal approximations of a density function

For the sake of demonstration in this section we use the equi-spaced intervals for the partition and discretization of a density function. In some of the experiments that will be discussed in the coming sections modified empirical techniques for the determination of spacing is used to achieve a better approximation of the input distribution.

First p_{min} and p_{max} are defined such that

$$\int_{-\infty}^{p_{\min}} f_P(p) + \int_{p_{\max}}^{+\infty} f_P(p) \leq \delta p \tag{4.46}$$

Where δp is the maximum acceptable error in the approximation of the density function

If the number of partition between $p_{\min}$ and $p_{\max}$ is determined to be t then the step width of each partition for p is

$$\Delta p = \frac{p_{\max} - p_{\min}}{T} \tag{4.47}$$

Then the uncertain parameter is partitioned at $T+1$ points so that T number of trapezoids can be formed from the density function. Assuming the vector of these values is x and the first value x_0 corresponds to $p_{\min}$ then the remaining points are assigned as:

$$x_t = p_{\min} + t * \Delta p \quad t = 1, \ldots, T \tag{4.48}$$

Then in the transformed probability mass function the weight of each partition is the area enclosed in that partition which can be evaluated by:

$$w_t = \frac{f_P(x_{t-1}) + f_P(x_t)}{2} \Delta p \tag{4.49}$$

The value representing the center of each partitioned area is well approximated at the centroid of this trapezoidal area and is computed for each partition

$$\hat{p}_{(t)} = x_{t-1} + \frac{\Delta p(2 \times f_P(x_t) + f_P(x_{t-1}))}{3 \times (f_P(x_t) + f_P(x_{t-1}))} \tag{4.50}$$

The above steps transform the density function into probability mass functions with equivalent central points where performance measures can be computed, there by finally determining the distribution and equivalent moment of the performance with the related uncertainty.

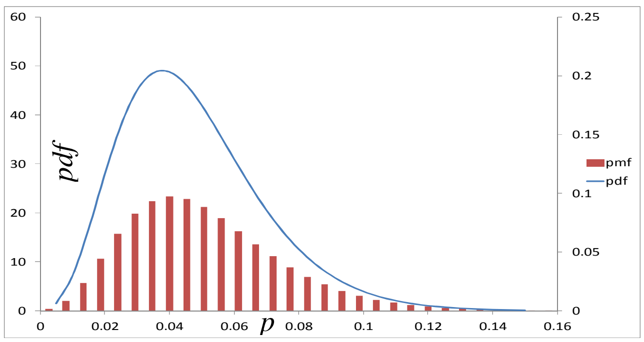


Figure 4:12 A probability density function and equivalent probability mass function

After the discretization of the probability density function interesting performance is evaluated at the centroid of each partition. These performance values with the corresponding weights can be used to reconstruct the distribution of the uncertain performance. The moments of this distribution can be also evaluated from this distribution. For instance in the case of isolated machine efficiency similar results of addition can be performed instead of integration and the expected value and the uncertainty in variance of the isolated efficiency e can be computed as:

$$e_t = e(\hat{p}_t)$$

$$E[e] = \sum_{t=1}^{T} e_t * w_t$$

$$V[e] = \sum_{t=1}^{T} (e_t - E[e])^2 * w_t \tag{4.51}$$

4.2.2.4 Taylor Approximation

The techniques that are discussed in the previous sections make use of the unknown parameters density functions. Instead the Taylor approximation can be used to estimate the moments of the output performance measures given the moments of the input parameters. This method has some main advantages over approximations using density function or distribution functions from computational point of view. By using this method even bigger problems involving complex models can be handled by approximating using their partial derivatives with respect to the input parameters. A further advantage of this method is that it enables the evaluation of the uncertainty when the only available information is about the moments of the input parameter. This allows analysis even with limited information while having fewer details about the behavior of the input distribution.

Moreover it can be also used to analyze output performance moments when the density of the input parameters is available as the required moments can be directly derived from the density function. Particularly this is beneficial when the main objective is to reduce the mathematical complexity of the problem while dealing with complex performance models. It is an indispensable technique especially when the number of uncertain parameters involved in the analysis is considerably high.

Here, we demonstrate simple introduction of the Taylor approximation on an isolated machine parameters for the estimation of the moments of the isolated efficiency. For a reasonable degree of accuracy under many practical cases the second degree approximation is a sufficient, while higher order approximations can be used similarly. Assuming the unknown parameter is p which is estimated from actual *TTF* observations the posterior density of $f_P(p)$ can be estimated as in section 4.1.

From the density function the first two moments or if necessary higher order moments can be estimated. Considering only the first two moments, we denote the input moments, the expected value of failure probability (μ_p) and the second moment the variance as (σ_p^2). Alternatively the input moments can be directly estimated from observations, or an approximate estimate of the moments.

The expected value of the isolated efficiency (*e*), E[e] and its uncertainty i.e., second moment V[e] can be approximated by second order Taylor approximation as:

$$E[e] \approx e(\mu_p) + \frac{1}{2} \left. \frac{\partial^2 e}{\partial p^2} \right|_{p=\mu_p} \sigma_p^2 \tag{4.52}$$

$$V[e] \approx \left(\left. \frac{\partial e}{\partial p} \right|_{p=\mu_p} \right)^2 \sigma_p^2 \tag{4.53}$$

The above two approximations can be easily extended to multiple unknown parameters. Full discussion on multiple uncertainties is provided in two-machine-single-buffer line analysis.

4.3 Analysis of a Two-Machine line

In this section the performance measures of a two-machine line where machines are characterized by a unique failure mode are discussed. Similar to the single machine case assumptions the two machine lines with single failure mode machines are modeled with discrete time. The throughput of the system (*TH*) can be evaluated in closed form as a function of the failure and repair parameters of the upstream machine and downstream machines, p_u, r_u, p_d, r_d and the buffer capacity N. In the analysis of single machine the emphasis was on the introduction of techniques for the performance analysis of simple systems with single uncertainty. Proceeding with similar methods and extending these methods for multiple uncertainty cases is the main goal here. Behavior of the system and the impact of performance evaluation under uncertainty are also investigated.

With the main goal of the coming sections being the development of methods that enable the evaluation of complex systems with multiple uncertainties, the focus will be on the approximate methods that were introduced for the isolated single machine case.

Modeling assumptions

Modeling assumptions that characterize the two machines and the intermediate buffer are described as follows. Additional assumptions related to multi stage lines which govern the relationship between machines and buffers are also mentioned. Some of the approaches in the methodologies can be extended for generic systems which are not addressed in this thesis such as continuous time cases. But the assumptions are narrowed only for the class of systems that are well investigated by the proposed approaches.

Assumptions:

- The time parts spend in each machine is deterministic, known and the two machines have equal and constant service time, and it is scaled to unity.
- Buffers have finite capacity, indicated as N.
- Buffers are perfectly reliable, i.e. they do not fail.

- Transportation takes negligible time compared to processing time, therefore these times are assumed zero.
- When operational, machines start to work a part at the same time, so the time is considered as discrete.
- A machine whose upstream buffer is empty said to be starved and a machine whose downstream buffer is full is called blocked.
- Upstream machine is never starved and downstream machine is never blocked.
- Repairs and failures occur at the beginning of time units, changes in buffer levels take place at the end of the time units.
- Failures are operation dependent (ODF) and the time to failures TTF are assumed to be geometrically distributed
- Time to repair TTR is also assumed to be geometrically distributed.
- When the parameters are estimated with uncertainty the distribution of failure probability p_i and repair probability r_i are estimated as explained in section 4.1 using a Bayesian approach from a vector of observed TTF and TTR respectively.
- Time to failures TTF and time to repairs TTR of each machine are independently distributed.

4.3.1 Two machine line (Gershwin Berman Model)

In the Gershwin Berman model the states and the Markov models of this single buffer two machine system are modeled and introduced. Brief summary of the modeling steps and the functions relevant to the study of uncertainty are revisited. Detailed model description and analysis of the steps to be followed to derive these formulas are available (Gershwin et al, 1981).

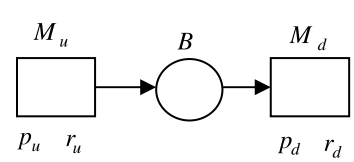


Figure 4:13 Two machine line with single failure mode

Before the demonstration of the method for the analysis of two machine line with uncertainty the Markov model for precisely known model parameters and the main assumptions behind this model are mentioned as follows.

The state of the system is described by three state variables $s = (n, \alpha_1, \alpha_2)$ where:

- n is the buffer level
- $\alpha_i = 1$ if machine M_i is up
- $\alpha_i = 0$ if machine M_i is down

This model analyzes the two machine system by dividing the state space of the system into three distinct categories depending on the buffer size.

- Internal states, the states in which $2 \leq \mathrm{n} \leq \mathrm{N}\text{-}2$
- Lower boundary states: states with $\mathrm{n} \leq 1$
- Upper boundary states: states with $\mathrm{n} \geq \mathrm{N}\text{-}1$

Once the transition probabilities are known, this model which is based on the Markov chain of the above states is solved analytically. Then the solution of the steady state probabilities that characterize the two machine lines are summarized and aggregated in order to recover interesting performance measure of the system.

A reasonable guess on the vectors of internal equation is made in the form

$$\pi(n, \alpha_1, \alpha_2) = CX_2^n Y_{12}^{\alpha_1} Y_{22}^{\alpha_2} = CX^n Y_1^{\alpha_1} Y_2^{\alpha_2}$$

$$\pi(N-1, 0, 0) = CX^{N-1}$$

$$\pi(N-1, 1, 0) = CX^{N-1} Y_1$$

$$\pi(N-1, 1, 1) = \frac{CX^{N-1}}{p_1} \frac{r_1 + r_2 - r_1 r_2 - p_1 r_2}{p_1 + p_2 - p_1 p_2 - p_1 r_2}$$

$$\pi(N, 1, 0) = CX^{N-1} \frac{r_1 + r_2 - r_1 r_2 - p_1 r_2}{p_1 r_2} \tag{4.54}$$

Lower boundary equations

$$\begin{aligned}
\pi(0,0,1) &= CX\,\frac{r_1 + r_2 - r_1 r_2 - r_1 p_2}{r_1 p_2} \\
\pi(1,0,0) &= CX \\
\pi(1,0,1) &= CXY_2 \\
\pi(1,1,1) &= \frac{CX}{p_2}\,\frac{r_1 + r_2 - r_1 r_2 - r_1 p_2}{p_1 + p_2 - p_1 p_2 - r_1 p_2}
\end{aligned} \tag{4.55}$$

Upper boundary equations

$$\begin{aligned}
\pi(N-1,0,0) &= CX^{N-1} \\
\pi(N-1,1,0) &= CX^{N-1}Y_1 \\
\pi(N-1,1,1) &= \frac{CX^{N-1}}{p_1}\,\frac{r_1 + r_2 - r_1 r_2 - p_1 r_2}{p_1 + p_2 - p_1 p_2 - p_1 r_2} \\
\pi(N,1,0) &= CX^{N-1}\,\frac{r_1 + r_2 - r_1 r_2 - p_1 r_2}{p_1 r_2}
\end{aligned} \tag{4.56}$$

By solving the above three sets of equations important performance measures of the two machine line are derived. These are:

Average throughput of the two machine line

$$E = E_1 = \sum_{(n<N),\alpha_1=1} \pi(n,\alpha_1,\alpha_2) = E_2 = \sum_{(n>0),\alpha_2=1} \pi(n,\alpha_1,\alpha_2) \tag{4.57}$$

Average buffer level

$$\bar{n} = \sum_{n=0}^{N} \sum_{\alpha_1=0}^{1} \sum_{\alpha_2=0}^{1} n \cdot \pi(n,\alpha_1,\alpha_2) \tag{4.58}$$

Starvation probability

$$p_s = \pi(0,0,1) \tag{4.59}$$

Blocking Probability

$$p_b = \pi(N,1,0) \tag{4.60}$$

Adding up all the probabilities to get normalization constant for the equations

$$1=\sum_{n=0}^{N}\sum_{\alpha_1=0}^{1}\sum_{\alpha_2=0}^{1}\pi(n,\alpha_1,\alpha_2) \tag{4.61}$$

Finally, substituting the steady state probabilities into the normalization equations the average throughput (*TH*) of a two machine line is evaluated. This function expresses the throughput as a function of failure, repair probabilities and the buffer capacity of the two machine single buffer system.

$$TH = e_d + \frac{e_u - e_d}{1-\frac{m\cdot e_u}{l\cdot e_d}X^{N-2}} \tag{4.62}$$

$$P_s = 1-\frac{E}{e_d} \tag{4.63}$$

$$P_b = 1-\frac{E}{e_u} \tag{4.64}$$

Where

$$l = \frac{r_u + r_d - r_u r_d - r_u p_d}{r_u p_d} \tag{4.65}$$

$$m = \frac{r_u + r_d - r_u r_d - p_u r_d}{p_u r_d} \tag{4.66}$$

$$x = \frac{\dfrac{r_u + r_d - r_u r_d - p_u r_d}{p_u + p_d - p_u p_d - r_u p_d}}{\dfrac{r_u + r_d - r_u r_d - r_u p_d}{p_u + p_d - p_u p_d - p_u r_d}} \tag{4.67}$$

$$e_u = \frac{r_u}{p_u + r_u} \qquad e_d = \frac{r_d}{p_d + r_d} \tag{4.68}$$

The input failure and repair probabilities of, p_u, r_u, p_d and r_d are independent as well as the corresponding posteriors of these distributions are assumed independent. In addition to the discretization techniques with the independency assumption, the output distribution of the performance measures can be approximated using the partial derivatives techniques. In order to approximate using the partial derivatives the first and second moments of each uncertain parameter are used as the input. Then the uncertainties related to each output performance measure can be computed with the expressions given in the above analytical formulas and their partial derivatives with respect to the uncertain parameters. The closed analytical relationship between the input failure parameters and output performance permits the exact evaluation of these partial derivatives in a close form without the need for approximation. For single uncertainty cases the steps demonstrated for the single machine case can be simply followed. The only difference is the evaluation of performance for each precisely known evaluation is carried out using the (Gershwin-Berman) model introduced above. Generally, the upcoming techniques focus on multiple uncertainty cases. In the next sections three alternative discretization techniques and the Taylor approximation for the evaluation of two machine lines are introduced.

4.3.1.1 Discretization with joint distribution

When the problem involves multiple uncertainties, each of the unknown parameter is modeled as a density function as presented in section 4.1 from observed data. Then the input distribution of all the unknown parameters can be considered as the joint distribution of individual marginal distributions. This joint probability distribution is used as an input for the performance evaluation. The discretization techniques used for single uncertainty can be extended for the individual partitioning of the marginal distributions and then the joint probability distribution can be approximated from this discretization.

Considering the distribution of all the U uncertain parameters, both uncertain p and r can be included, in a convenient order to p_u $u=1,...,U$, with corresponding densities $f_{P_u}(p_u)$ $u=1,...,U$. The lower and upper limit $p_{u\min}$ and $p_{u\max}$ are determined depending on the acceptable error for each individual density function and the joint distribution of the parameters as indicated in equation 4.46. Once these thresholds for each parameter are determined then the corresponding step size for each parameter p_u is Δp_u defined as:

$$\Delta p_u = \frac{p_{u\max} - p_{u\min}}{T_u} \tag{4.69}$$

If x_0 corresponds to $p_{u\min}$ and x_{T_u} corresponds to $p_{u\max}$ each partition bound x_{t_u} is obtained as:

$$x_{t_u} = p_{u\min} + t_u * \Delta p_u \qquad t_u = 1,...,T_u \tag{4.70}$$

Each partition weight and the centroid value of the random variable in the considered partition are then approximately computed as follows, for $t_u = 1,...,T_u$:

$$\hat{p}_{(t_u)} = x_{t_u-1} + \frac{\Delta p_u (2 \times f_{P_u}(x_{t_u}) + f_{P_u}(x_{t_u-1}))}{3 \times (f_{P_u}(x_{t_u}) + f_{P_u}(x_{t_u-1}))} \tag{4.71}$$

$$w_{t_u} = \frac{f_{P_u}(x_{t_u-1}) + f_{P_u}(x_{t_u})}{2} \Delta p_u \tag{4.72}$$

Then the joint distribution of the U parameters $u = p_1, p_2..., p_U$ the joint density function is computed

$$f_{P_1,P_2,...,P_U}(p_1, p_2,..., p_U) = f_{P_U|P_2,...,P_U}(p_U \mid p_1,..., p_{U-1}) f_{P_1,P_2,...,P_{U-1}}(p_1, p_2,..., p_{U-1})$$

$$= f_{P_1}(p_1) f_{P_2|P_1}(p_2 \mid p_1) \ldots f_{P_{U-1}|P_1,...,P_{U-2}}(p_{n-1} \mid p_1,..., p_{U-2}) f_{P_U|P_1,...,P_{U-1}}(p_n \mid p_1,..., p_{U-1}) \tag{4.73}$$

In the case of discrete random variables with independence assumption this relation can be reduced to

$$P(P_1 = p_1, P_2 = p_2,..., P_U = p_U) = P(P_1 = p_1)P(P_2 = p_2),..., P(P_U = p_U)$$

For instance for a two dimensional joint distribution with discretized individual density functions, the joint distribution can be shown in a two dimensions with p_1 and p_2 axis as shown in Figure 4:14.

$$f_{P_1,P_2}(p_1, p_2) = f_{P_1}(p_1) f_{P_2}(p_2)$$

$$P(P_1 = p_1, P_2 = p_2) = P(P_1 = p_1)P(P_2 = p_2) \tag{4.74}$$

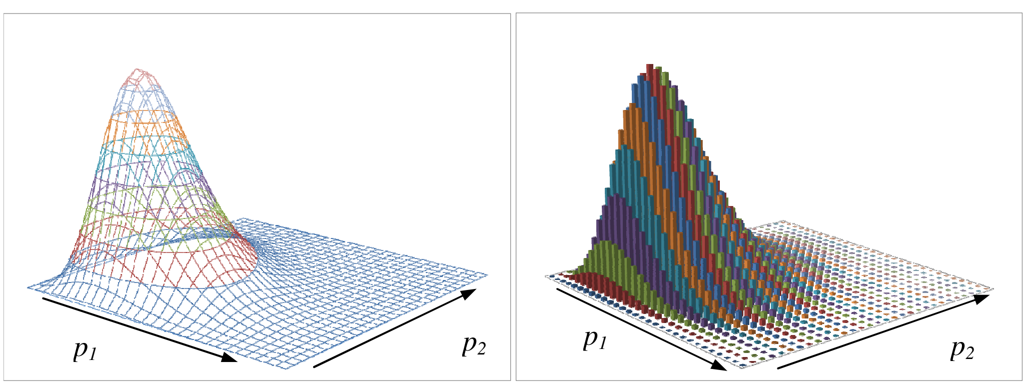


Figure 4:14 Two dimensional joint density function and equivalent mass function

Once the approximate joint distribution is constructed for a general case of U dimension, then experiments with precisely known input are generated. Then number of experiments with precisely known inputs that need to be generated from the combination of all the considered central partition values of U random variables is $\prod_{u=1}^{U} T_u$. The weights that correspond to these experiments are computed with 4.72. These experiments are run to evaluate the required performance measures and the distribution of the performance on a U dimension is reconstructed. For instance the average throughput distribution and the average buffer level can be computed as follows.

$$TH(t_1, t_2, ..., t_U) = TH \mid p_1 = p_{1(t_1)}, p_2 = p_{2(t_2)}, ..., p_U = p_{(t_U)}$$

$$n(t_1, t_2, ..., t_U) = n \mid p_1 = p_{1(t_1)}, p_2 = p_{2(t_2)}, ..., p_U = p_{(t_U)}$$

$$\begin{aligned} w(t_1, t_2, ..., t_U) &= P(P_1 = p_{1(t_1)}, P(P_2 = p_{2(t_2)}), ..., P(P_U = p_{(t_U)}) \\ &= \prod_{u=1}^{U} w_{t_u} \end{aligned} \tag{4.75}$$

By using these weights, the throughput distribution can be easily reconstructed. Moreover, interesting statistics can computed from this distribution, such as the mean and the variance of the average throughput can be estimated.

4.3.2 Approximations for reducing number of experiments

The use of joint distributions for performance evaluation with unknown parameters can be advantageous in terms of the numerical accuracy. Especially when the number of partitions used increases higher accuracy can be achieved. When the number of unknown parameters moderately increases the number of evaluations required to be performed grows exponentially. Two main factors that affect the exponential growth of required computations are the number of unknown parameters U and the partition numbers for each unknown parameter. Therefore the number of precisely known experiments to be evaluated is equal to $\prod_{u=1}^{U} T_u$.

In addition to the large number of experiments needed to be evaluated in some performance models a single point evaluation for the solution of the precisely known parameters can take considerable computational time. Therefore the overall analysis for the required numbers of repeated experiments requires longer time, or in some cases can be impractical. The techniques that are proposed in the coming sections focus on tackling and proposing alternative approach for this problem mathematically known as “curse of dimensionality”.

4.3.2.1 Discretization with one parameter at a time

The joint distributions in multidimensional cases with higher number of parameters can be prohibitive due to the number of experiments that need to be evaluated in order to compute the output distribution. The one parameter at a time discretization is aimed at reducing the number of experiments required when dealing with high number of uncertain parameters. The method relies on the approximate functional relationship between the variance of a function of multiple independent random variables. This relationship assumes that the unknown input parameters are independent. Moreover it assumes that there is low functional covariance between these input variables. Therefore the impact of each unknown parameter can be independently measured and the global impact is evaluated as the superimposition of the individual contributions.

In general the relationship follows if Y is the uncertain output performance measure required to be evaluated and

$$Y = f(X_1, X_2, ..., X_U)$$

If each random input variable has a corresponding variance $\sigma_{x1}^2, \sigma_{x1}^2, ..., \sigma_{xU}^2$

Then the variance of Y can be approximated as

$$V[Y] \approx \left(\frac{\partial Y}{\partial x_1}\right)^2 \sigma_{x1}^2 + \left(\frac{\partial Y}{\partial x_2}\right)^2 \sigma_{x2}^2 +, ..., + \left(\frac{\partial Y}{\partial x_U}\right)^2 \sigma_{xU}^2$$

From the above relationship when each discretized unknown parameter is used to measure the variance on the response Y, effectively we are computing the addend related to that particular variable. As in many statistical methods and partial derivation techniques other parameters are held constant on their maximum likelihood value which is the expected value of each uncertain parameter. The same evaluation is performed for all parameters and the impact of all unknown parameters is approximated as the cumulative effect of all the uncertainties involved in the analysis.

The relation of the approximation on the expected value of Y can be mimicked from second order Taylor approximation. From equation (4.52) the second order Taylor approximation of the expected value of Y can is written as:

$$E[Y] = Y(\mu_{x1}, \mu_{x2}, ..., \mu_{x1,}) + \frac{1\partial^2 Y}{2\partial x_1{}^2}\sigma_{x1}^2 + \frac{1\partial^2 Y}{2\partial x_2{}^2}\sigma_{x2}^2 +, ..., \frac{1\partial^2 Y}{2\partial x_U{}^2}\sigma_{xU}^2$$

Therefore from the above relation the expected value of the variable is the mean expectation plus the individual deviations is a function of the second order derivative and the input variance of the unknown parameters. The individual deviations can be approximated as a function of the variance of the response on *Y* due to that particular input parameter. So these individual deviations can be weighted for the evaluation of the overall deviations.

Extending the assumptions made above and the relative approximations the particular case of two machine line can be analyzed as described in the following steps. The following notations are introduced for performance of *TH* and input unknown parameter p_u. The same notations can be extended for input parameter failure probabilities p_d, r_u, p_d and other output performance measures such as the average buffer level *n*.

- Expected value of TH considering only uncertain p_u: $E[TH]_{(pu)}$
- Expected value of TH considering all parameters at their expected value: $E[TH]_{(0)}$
- Variance of TH considering only uncertain p_u: $V[TH]_{(p_u)}$

Step 1: Using the same procedures explained in section 4.2.2.3 compute the density of each unknown parameter. Considering the distribution of all the *U* uncertain parameters, both uncertain p and r can be included, in a convenient order to p_u $u=1,...,U$, with corresponding densities $f_{P_u}(p_u)$ $u=1,...,U$.

Step 2: Discretize each density $f_{P_u}(p_u)$ $u=1,...,U$ and compute the corresponding individual $\hat{p}_{(t_u)}$ and $w_{(t_u)}$ as shown in 4.2.2.3.

Step 3: Following the convenient order defined in step 1 evaluate the output performance for each point using precisely known inputs. Then from the output distribution compute the expected value $E[TH]_{(pu)}$ and variance $V[TH]_{(p_u)}$.

Step 4: Evaluate the total variance of the performance and the expected value as:

$$V[TH]=\sum_{i=1}^{U}V[TH]_{(p_i)} \tag{4.76}$$

The expected value of the performance measure is approximated based on the relationship of the deviation that results from the use of the expected value of the uncertain parameters and the evaluation by introducing the associated uncertainty. In section 4.2.2.4 it is explained that this deviation is the function of two factors. By using second order Taylor approximation for the difference is a function of second order partial derivative and the input variance of the current uncertain parameter.

Assuming the $\Delta TH_{(p_u)}=E[TH]_{(pu)}-E[TH]_{(0)}$ in the discretization case since there is no direct measurement of the second order derivative, the deviation is approximated as proportional to the function of the variance measured on the performance measure for each uncertain parameter p_u.

Therefore the weighted deviation by each parameter is considered as a percentage of the cumulative uncertainty.

$$W_{p_u}=\frac{V[TH]_{(p_u)}}{V[TH]} \tag{4.77}$$

Then the expected value of the performance is computed as a function of these weights

$$E[TH] = E[TH]_{(0)} + \sum_{u=1}^{U} W_{p_u} \Delta E[TH]_{(p_u)} \tag{4.78}$$

The total number of experiments to be performed in this case is $\sum_{u=1}^{U} T_u$ which grows linearly with the number of uncertain parameter and number of partitions used for discretization.

Comparison of numerical computations required

Assuming the use of constant number partition for each parameter to be $n = T_1 = T_2, ..., T_U$ then the number of two machine line evaluations to be performed as a number of uncertain parameters can be compare in the following table.

	No of required evaluations	
No of uncertainty	Parameter at a time	Joint distribution
1	***n***	***n***
2	***2n***	n^2
3	***3n***	n^3
…	…	…
U	***U*n***	n^U

Table 4:4 Required No experiments in for parameter at a time and joint distribution

From Table 4:4 it can be observed that the joint approach is impractical for multiple uncertainties even with moderately significant numbers. For instance a few parameters as 5 uncertainties has to be considered with each parameter to be discretized in 20 partitions in the case of the joint distribution this requires 3,200,000 evaluations with precisely known parameters, while a one parameter at a time approach needs only 100 evaluations. In the first case even for a single buffer two machine lines requires a computational effort high that could take a considerable time for the overall performance evaluation of the output distribution.

Comparison of numerical accuracy Joint distribution vs one parameter at a time

Two machine lines are used to demonstrate the accuracy difference exhibited by using parameter at a time and the joint distribution of parameters. Detailed accuracy tests and results are reported in the next chapter. Significant differences might arise particularly when the various uncertain parameters strongly interact in a different way in different areas of the hyper-plane function. For instance a comparison two cases where the parameter at a time and joint distribution approach give similar and different results are shown below in Figure 4:15.

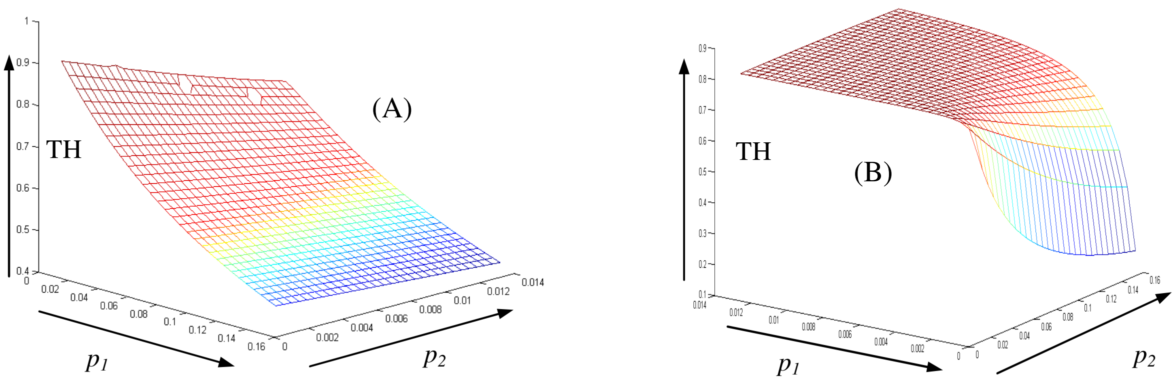


Figure 4:15 Comparison of two profiles where parameter at a time performs well and poor

It can be seen from the above diagrams that in the case of (A) two criss-crossing lines at the central positions of the performance function well represent the trend for each parameter at a time. In the case of (B) by varying one parameter while holding the other at the expected value doesn't capture the functional interaction between the two parameters, especially on the far right of the graph.

4.3.2.2 Two parameters a time approach

Although it is tempting to use one factor at a time approach for higher computational efficiency, it could give a very low accuracy in the particular cases where multiple parameters strongly interact. In order to address these kinds of strong interactions between parameters through the performance function while avoiding the combinatorial growth of experiments with number of uncertain parameter an alternative method is proposed here. This method assumes, if there is strong significant functional interaction as depicted in Figure 4:15 (B), the most significant are the individual variances and the two order interactions. This assumption holds in many cases basically for the following two reasons. One reason is in a particular region usually the factors that influence the response performance are the first or the second order interactions of parameters, as effectively assumed in most statistical experiments too. The second strong support behind this assumption is, even if far points from the expected values have a high order interaction the product of the joint probabilities associated to these extreme points are far small and their product also gets extremely insignificant to influence the overall result of the analysis.

With the above premises the objective of this method is to measure the uncertainty associated to each parameter as in the case of the parameter at a time approach. Then the significant second order interactions will be computed using joint distributions involving two parameters at a time. The basic steps to be followed are described as follows.

1. For each uncertain parameter p_u evaluate the uncertainty by one parameter at a time approach as described in section 4.3.2.1.
   $V[TH]_{(pi)} \approx V[TH \mid P_1 = \hat{p}_1, P_2 = \hat{p}_2, ..., P_U = \hat{p}_U] \ \forall \ \text{u} = 1,...,U, i \neq u$
2. Evaluate the joint distribution of each possible pair of uncertain parameters and the resulting variance in the performance
   $V[TH]_{(pi,p_j)} \approx V[TH \mid P_1 = \hat{p}_1, P_2 = \hat{p}_2, ..., P_U = \hat{p}_U] \ \forall \ \text{u} = 1,...,U, i \neq u, j \neq u$
3. Evaluate the variance due to interaction with (4.79) for every pair of two uncertain parameters by subtracting the results in (1) from two

$$V[TH]I_{(pi,p_j)} = V[TH]_{(p_i,p_j)} - (V[TH]_{(p_i)} + V[TH]_{(p_j)}) \qquad (4.79)$$

These results can be put in a matrix format that summarize the variance due to individual and interaction effects

$$\begin{bmatrix} V[TH]_{(p_1)} & V[TH]_{(p_1,p_2)} & V[TH]_{(p_1,p_3)} & \dots & V[TH]_{(p_1,p_U)} \\ & V[TH]_{(p_2)} & V[TH]_{(p_2,p_3)} & \dots & V[TH]_{(p_2,p_U)} \\ & & V[TH]_{(p3)} & \dots & V[TH]_{(p_3,p_U)} \\ & & & \dots & \dots \\ & & & & V[TH]_{(pU)} \end{bmatrix} \tag{4.80}$$

This matrix can be presented in an alternative form after it is normalized by the variance of the input uncertainties. This format shows the measure of reactivity of the overall system uncertainty with respect to the input uncertainties of individual parameter. The normalizing factor for two pair of uncertain parameters p_i, p_j is the product of the standard deviations.

$$\sigma_{p_i}\sigma_{p_j}$$

Therefore an element a_{ij} in the normalized matrix will be given as

$$a_{ij} = \frac{V[TH]I_{(p_i p_j)}}{\sigma_{p_i}\sigma_{p_j}} \quad \forall i = 1,...U, \forall j = 1,...U \tag{4.81}$$

The resulting matrix can be considered as an index of sensitivity of the overall uncertainty to the input uncertainties and the corresponding interactions.

$$V[TH] = \left[\sigma_{p_1}, \sigma_{p_2}, \sigma_{p_3}, ..., \sigma_{p_U}\right] \times \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1U} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2U} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3U} \\ \dots & \dots & \dots & \dots & \dots \\ a_{U1} & a_{U2} & a_{U3} & \dots & a_{UU} \end{bmatrix} \times \begin{bmatrix} \sigma_{p_1} \\ \sigma_{p_2} \\ \sigma_{p_3} \\ \dots \\ \sigma_{p_U} \end{bmatrix} \tag{4.82}$$

This method gives a compromising solution under conditions where the strong interactions of parameters are expected to occur in the function that relates the input parameters. For systems where the input parameters don't interact the joint considerations could be omitted. For instance in a two machine line, parameters on individual machines separated by significant buffer levels are expected to interact less significantly compared to the parameters of the single machine, such as failure probability and repair probability on the same machine.

The significance of these results is shown in the two machine line analysis the for accuracy test of this method.

In addition this method limits the combinatorial growth in the number of experiments required to be carried out if a pure joint distribution approach were to be used. The number of experiments to be carried out in this case is

$$No_{exp} = \sum_{i=1}^{U} n_i + \frac{\sum_{i=1}^{U} \sum_{j=i+1}^{U} n_i n_j}{2}$$

If the number of partitions for each parameter are the same then the number of experiments is

$$No_{exp} = U \times n + \frac{U(U-1)}{2} n^2$$

	No of required evaluations		
No of uncertain	Parameter at a time	Two Parameters	Joint distribution
1	***n***	***n***	***n***
2	***2n***	n^2	n^2
3	***3n***	$3n+3n^2$	n^3
4	***4n***	$4n+6n^2$	n^4
	…	…	…
U	***U*n***	$U*n+U(U-1)/2*n^2$	n^U

Table 4:5 Comparison of number of evaluations required in the three methods

As it can be seen the number of experiments and the growth with more parameters is considerably lower compared to the joint distribution. The accuracy is also very close to the joint distribution method as will be shown in the numerical accuracy section. Considering the 5 uncertain parameter example mentioned in section 4.3.2.1 the corresponding number of required evaluations in one parameter, two parameters and joint distribution approaches are, 100, 4,050 and 3,200,000 respectively. The latter two almost give practically the same results even in cases where strong interactions are observed emphasizing the advantage of the two factors at a time in terms of computation time while also maintaining a good accuracy in various cases.

4.3.2.3 Approximation of moments using Taylor Expansion

The Second order Taylor approximation can be used for the estimation of the moments in the two machine single buffer line. In the Gershwin Berman model the average throughput can be expressed in terms of the input failure and repair probabilities. This allows the exact analytic evaluation of the partial derivatives with respect to each of these parameters.

For the average throughput approximating the distribution at the expected value of each parameter

$$V[TH] = \left(\frac{\partial TH}{\partial p_u} \bigg| (\mu_{r_u}, \mu_{p_d}, \mu_{r_d}) \right)^2 \times \sigma^2 p_u + \left(\frac{\partial TH}{\partial r_u} \bigg| (\mu_{p_u}, \mu_{p_d}, \mu_{r_d}) \right)^2 \times \sigma^2 r_u + \left(\frac{\partial TH}{\partial p_d} \bigg| (\mu_{p_u}, \mu_{r_u}, \mu_{r_d}) \right)^2 \times \sigma^2 p_d$$
$$+ \left(\frac{\partial TH}{\partial r_d} \bigg| (\mu_{p_u}, \mu_{r_u}, \mu_{r_d}) \right)^2 \times \sigma^2 r_d \qquad (4.83)$$

Similarly for the uncertainty of the probability of starvation

$$V[P_s] = \left(\frac{\partial P_s}{\partial p_u} \bigg| (\mu_{r_u}, \mu_{p_d}, \mu_{r_d}) \right)^2 \times \sigma^2 p_u + \left(\frac{\partial P_s}{\partial r_u} \bigg| (\mu_{p_u}, \mu_{p_d}, \mu_{r_d}) \right)^2 \times \sigma^2 r_u + \left(\frac{\partial P_s}{\partial p_d} \bigg| (\mu_{p_u}, \mu_{r_u}, \mu_{r_d}) \right)^2 \times \sigma^2 p_d$$
$$+ \left(\frac{\partial P_s}{\partial r_d} \bigg| (\mu_{p_u}, \mu_{r_u}, \mu_{r_d}) \right)^2 \times \sigma^2 r_d \qquad (4.84)$$

For the uncertainty in the probability of blocking

$$V[P_b] = \left(\frac{\partial P_b}{\partial p_u} \bigg| (\mu_{r_u}, \mu_{p_d}, \mu_{r_d}) \right)^2 \times \sigma^2 p_u + \left(\frac{\partial P_b}{\partial r_u} \bigg| (\mu_{p_u}, \mu_{p_d}, \mu_{r_d}) \right)^2 \times \sigma^2 r_u + \left(\frac{\partial P_b}{\partial p_d} \bigg| (\mu_{p_u}, \mu_{r_u}, \mu_{r_d}) \right)^2 \times \sigma^2 p_d$$

$$+ \left(\frac{\partial P_b}{\partial r_d} \bigg| (\mu_{p_u}, \mu_{r_u}, \mu_{r_d}) \right)^2 \times \sigma^2 r_d$$

(4.85)

Approximating the expected values of the performance measures using second order Taylor approximation for the same probabilities gives:

For expected average throughput

$$E[TH] = TH(\mu_{p_u}, \mu_{r_u}, \mu_{p_d}, \mu_{r_d}) + \frac{1}{2} \left(\frac{\partial^2 TH}{\partial p_u \partial p_u} (\mu_{r_u}, \mu_{p_d}, \mu_{r_d}) \right) \times \sigma^2 p_u + \left(\frac{\partial^2 TH}{\partial r_u \partial r_u} (\mu_{p_u}, \mu_{p_d}, \mu_{r_d}) \right) \times \sigma^2 r_u$$

$$+ \left(\frac{\partial^2 TH}{\partial p_d \partial p_d} (\mu_{p_u}, \mu_{r_u}, \mu_{r_d}) \right)^2 \times \sigma^2 p_d + \left(\frac{\partial^2 TH}{\partial r_d \partial r_d} (\mu_{p_u}, \mu_{r_u}, \mu_{p_d}) \right) \times \sigma^2 r_d$$

(4.86)

Probability of starvation

$$E[P_s] = P_s(\mu_{p_u}, \mu_{r_u}, \mu_{p_d}, \mu_{r_d}) + \frac{1}{2} \left(\frac{\partial^2 P_s}{\partial p_u \partial p_u} (\mu_{r_u}, \mu_{p_d}, \mu_{r_d}) \right) \times \sigma^2 p_u + \left(\frac{\partial^2 P_s}{\partial r_u \partial r_u} (\mu_{p_u}, \mu_{p_d}, \mu_{r_d}) \right) \times \sigma^2 r_u$$

$$+ \left(\frac{\partial^2 P_s}{\partial p_d \partial p_d} (\mu_{p_u}, \mu_{r_u}, \mu_{r_d}) \right)^2 \times \sigma^2 p_d + \left(\frac{\partial^2 P_s}{\partial r_d \partial r_d} (\mu_{p_u}, \mu_{r_u}, \mu_{p_d}) \right) \times \sigma^2 r_d$$

(4.87)

Probability of blocking

$$E[P_b] = P_b(\mu_{p_u}, \mu_{r_u}, \mu_{p_d}, \mu_{r_d}) + \frac{1}{2} \left(\frac{\partial^2 P_b}{\partial p_u \partial p_u} (\mu_{r_u}, \mu_{p_d}, \mu_{r_d}) \right) \times \sigma^2 p_u + \left(\frac{\partial^2 P_b}{\partial r_u \partial r_u} (\mu_{p_u}, \mu_{p_d}, \mu_{r_d}) \right) \times \sigma^2 r_u$$

$$+ \left(\frac{\partial^2 P_b}{\partial p_d \partial p_d} (\mu_{p_u}, \mu_{r_u}, \mu_{r_d}) \right)^2 \times \sigma^2 p_d + \left(\frac{\partial^2 P_b}{\partial r_d \partial r_d} (\mu_{p_u}, \mu_{r_u}, \mu_{p_d}) \right) \times \sigma^2 r_d$$

(4.88)

The first and second partial derivatives of average performance measures with respect to each reliability parameters can be computed in closed form. For instance the first four partial derivatives for the average throughput are shown in (4.89-4.92)

First order partial derivative with respect to p_u

$$\frac{\partial TH}{\partial p_u}=\frac{e_u}{(p_u+r_u)(-1+z)}+(e_d-e_u)\times\left(\frac{\frac{z(r_u(1+m)+e_u m p_u)}{p_u r_u m}+\frac{1}{x}(N-2)z\left(\frac{r_d+(1-p_d)Y_2}{Y_{2d}Y_1}+\frac{(-1+p_d+r_d)Y_2}{l_n}\right)}{\left(-1+m\right)^2}\right) \tag{4.89}$$

First order derivative with respect to r_u

$$\frac{\partial TH}{\partial r_u}=\frac{\frac{e_u-1}{(p_u+r_u)}}{-1+z}-(e_d-e_u)\times$$
$$\frac{\left(\frac{(-1+p_d+r_d)z}{r_u p_d l}-\frac{(-1+r_d)z}{p_u r_d}+\frac{(2-e_u)z}{r_u}+\frac{(N-2)z}{x}\times\left(\frac{(-1+p_d+r_d)Y_2}{lp_d r_u Y_1}-\frac{(-1+r_d)}{Y_{2d}Y_1}+\frac{Y_{1d}Y_2}{Y_{2d}lr_u}\right)\right)}{\left(-1+z\right)^2} \tag{4.90}$$

First order derivative with respect to p_d

$$\frac{\partial TH}{\partial p_d}=-\frac{{e_d}^2}{r_d}-\frac{{e_d}^2}{r_d\left(-1+z\right)}-(e_d-e_u)\times$$
$$\left(\frac{z}{p_d l}+\frac{z(e_d p_d+r_d)}{p_d r_d}+\frac{\frac{(-2+n)z}{x}\left(\frac{Y_2}{lp_d Y_1}+\frac{Y_{1d}(-1+p_u+r_u)Y_2}{Y_{2d}lp_d r_u}-\frac{(-1+p_u)mp_u r_d}{Y_{2d}lp_d r_u}\right)}{\left(-1+z\right)^2}\right. \tag{4.91}$$

First order derivative with respect to r_d

$$\frac{\partial TH}{\partial r_d} = -\frac{e_d}{(p_d + r_d)} + \frac{e_d}{r_d} - \frac{\frac{e_d}{(p_d + r_d)} - \frac{e_d}{r_d}}{-1+z} + (e_d - e_u) \times$$

$$\left(-\frac{(-1+r_u)z}{r_u l p_d} + \frac{(-1+p_u+r_u)}{p_u r_d m} \frac{-\frac{e_d z}{r_d} + \frac{2 p_d z}{r_d p_d} + \frac{(N-2)z}{x}\left(\frac{Y_2 Y_1 p_u + (1-r_u)Y_2}{l p_d r_u Y_1} + \frac{(-1+p_u+r_u)}{Y_{2d} Y_1} \right)}{\left(-1+z\right)^2} \right)$$

(4.92)

Where

$$z = \frac{m e_u x^{N-2}}{l e_d} \qquad Y_{2d} = p_u + p_d - p_u p_d - r_u p_d \quad l_n = r_u + r_d - r_u r_d - r_u p_d \qquad (4.93)$$

Other partial derivatives can be computed in the same way

4.3.3 Two machine lines with multiple failure modes (TMG model)

The method demonstrated above for two machine lines has a single failure mode. Since the performance has a closed form it is convenient to analytically derive the partial derivatives with respect to uncertain parameters. These derivatives are precise and are useful to study two machines single buffer line systems. For other system such as machines with multiple failure modes performance measures are not evaluated by closed form formulas. The two machine line that is considered in this section is an example of performance analysis where performance evaluation is performed using an approximate method. Therefore the evaluation of the partial derivatives of these systems also requires approximate techniques.

In addition to the generality of the method for performance evaluations carried out by different techniques including approximate methods and results from simulations can be analyzed with this approximate approach. The evaluation of the approximate partial derivatives is carried out by using finite difference on chosen points close to the region where the derivative is required to be estimated. Then delta method is employed for the evaluation of expected value and variance of the performance measures. In the upcoming sections this approach is demonstrated on two machine lines with multiple failure modes which are useful building blocks for the evaluation of various multistage production systems.

The analysis is based on the two machine line analysis model proposed by [Tolio-Matta-Gershwin 2002] (TMG). This model evaluates two machine lines with deterministic processing times with multiple failure modes and finite buffer capacity. Unlike the single failure mode machines these machines can fail in different modes. Each failure mode is characterized by specific time to failure *TTF* and time to repair *TTR*. The summary of the main model assumptions and steps in the evaluation of performance analysis are review below.

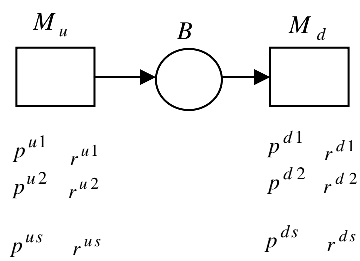


Figure 4:16 Two machine line with multiple failure modes

- The same assumptions for (Gershwin-Berman) model apply for the processing times, buffer capacities and occurrence of failures and repairs.
- In the multiple failure case a machine can be in one failure mode at a time.

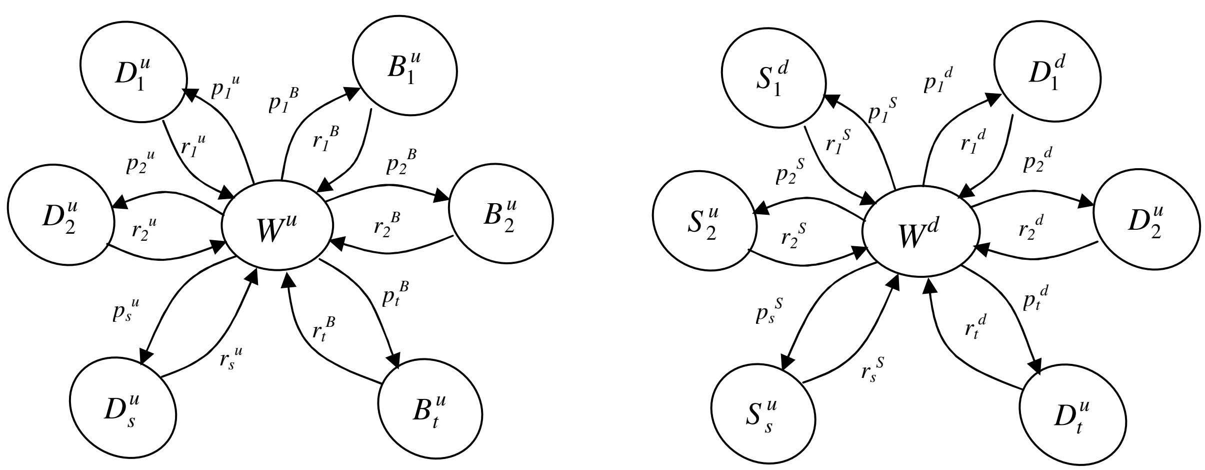


Figure 4:17 Markov chain model for the M_u and M_d with multiple failures

The upstream machine M^u characterized by s failure modes, with parameters p^{ui} and r^{ui}, with $i = 1,...,s$. Similarly the downstream machine M^d has t failure modes with p^{dj} and r^{dj} with $j = 1,...,t$. The general system state is represented by the vector (u_i, d_j, n) where $u_i = 1,...,s+1$ that represents the state of the first machine (u_i = 1then machine is up), $d_j = 1,...,t+1$ represents the state of the second machine ($d_j = 1$ then machine is up) and n is the level of the buffer.

The number of states becomes too many and when the capacity of the buffer *B* becomes large, and this approach can be very cumbersome. Therefore, in (Gershwin et al, 2002) a solution that is independent from the capacity of the buffer is proposed. The aim of the analysis is to evaluate the steady state probability of the generic system state $\pi(u_i, d_j, n)$. Once these probabilities are evaluated, the performance of the system can be calculated.

The approach in the techniques used to solve these steady state probabilities assumes that the steady state probabilities of the internal states must have a product form solution. The guess on the internal state probabilities form is the following.

$$\pi(u_i, d_j, n) = \sum_{m=1}^{R=s+t} C_m X_m^n U_{i,m} D_{j,m} \qquad \text{For } n = 0,...,N; i = 1,...,s; j = 1,....,t \qquad (4.94)$$

Where X_m^n, $U_{i,m}$ and $D_{j,m}$ are calculated as:

$$X_m = \left[1 - P^u + \sum_{i=1}^{s} \frac{p^{u_i} r^{u_i}}{K_m - 1 + r^{u_i}}\right] \frac{1}{K_m}$$

$$U_{i,m} = \frac{p^{u_i}}{K_m - 1 + r^{u_i}}$$

$$D_{j,m} = \frac{p^{d_j}}{\frac{1}{K_m} - 1 + r^{d_j}} \qquad (4.95)$$

and K_m derives from the solution of the following polynomial of degree $R = s + t$

$$f\left(K\right)=\left[1-P^{u}+\sum_{i=1}^{s}\frac{p^{u_i}\ r^{u_i}}{K_m-1+r^{u_i}}\right]\left[1-P^{D}+\sum_{i=1}^{s}\frac{p^{d_j}\ r^{d_j}}{\frac{1}{K}-1+r^{d_j}}\right]-1=0$$

P^u and P^d respectively are the sum of all the upstream and downstream failure probabilities. Constant C_m can be evaluated by solving a linear system formed by the following equations:

$$\frac{P^U}{P^{u_j}}\sum_{m=1}^{R}C_m\ X_m^{N-1}U_{i,m}K_m=\sum_{m=1}^{R}C_mX_m^{N-1}\left[\sum_{k=1}^{s}\sum_{j=1}^{t}U_m\,D_{j,m}r^{u_k}r^{d_j}+\left(1-P^u\right)\sum_{j=1}^{t}D_{j,m}\,r^{d_j}\right]+ \sum_{m=1}^{R}C_m\,X_m^N\,K_m\sum_{j=1}^{t}D_{j,m}\left(1-r^{d_j}\right)\qquad i=1,\ldots,s-1 \tag{4.96}$$

$$\frac{P^D}{P^{d_j}}\sum_{m=1}^{R}C_m\ X_m\ \frac{D_{j,m}}{K_m}=\sum_{m=1}^{R}C_mX_m\quad *\left[\sum_{i=1}^{s}\sum_{k=1}^{t}U_{i,m}D_{k,m}r^{u_i}\ r^{d_k}+\left(1-p^d\right)\sum_{i=1}^{s}U_{i.m}r^{u_i}\right]+ \sum_{m=1}^{R}C_m\sum_{i=1}^{s}\frac{U_{i,m}}{K_m}\left(1-r^{u_i}\right)\qquad j=1,\ldots..,t-1 \tag{4.97}$$

$$\sum_{all\ states}\pi=1 \tag{4.98}$$

This is a system of $s+t-1$ equations in $s+t-1$ unknowns, since one C_m variable is demonstrated to be always equal to zero. By solving the linear system it is possible to calculate C_m unknowns and substituting it and equations all the steady state probabilities can be obtained. For more details on the entire procedure see (Gershwin et al. 2002). The interesting main performance measures of the system can be calculated with the following equations.

Average throughput:

$$TH=TH_1=\sum_{n=0}^{N-1}\left[\pi\left(n,1,1\right)+\sum_{j=1}^{t}\pi\left(n,1,d_j\right)\right]= \\ TH_2=\sum_{n=1}^{N}\left[\pi\left(n,1,1\right)+\sum_{i=1}^{s}\pi\left(n,u_j,1\right)\right] \tag{4.99}$$

Average Buffer Level:

$$\overline{n} = \sum_{n=0}^{N}\sum_{i=1}^{s}\sum_{j=1}^{t} n*\pi\left(n,u_i,d_j\right) + \sum_{n=0}^{N}\sum_{j=1}^{t} n*\pi\left(n,1,d_j\right) + \sum_{n=0}^{N}\sum_{i=1}^{s} n*\pi\left(n,u_i,1\right) + \sum_{n=0}^{N} n*\pi\left(n,1,1\right) \quad (4.100)$$

Probabilities of Blocking, caused by failures of the downstream machine M^d:

$$Pb_j = \pi\left(N,1,d_j\right) = \frac{1-r^{d_j}}{r^{d_j}}\sum_{m=1}^{R} C_m X_m^N D_{j,m} K_m + \frac{P^{d_j}}{P^{u_i} r^{d_j}}\left(1-P^U\right)\sum_{m=1}^{R} C_m X_m^{N-1} U_{i,m} K_m \qquad j=1,\ldots t$$

(4.101)

Total probability of Blocking:

$$pb = \sum_{j=1}^{t}\pi\left(N,1,d_j\right) \quad (4.102)$$

Probabilities of Starvation, caused by failures of the upstream machine M^u:

$$Ps_i = \pi\left(0,u,1\right) = \frac{1-r^{u_j}}{r^{u_j}}\sum_{m=1}^{R} C_m \frac{U_{i,m}}{K_m} + \frac{P^{u_j}}{P^{d_j} r^{u_j}}\left(1-P^D\right)\sum_{m=1}^{R} C_m X_m \frac{D_{j,m}}{K_m} \quad i=1,\ldots.s \quad (4.103)$$

Total probability of Starvation:

$$P^s = \sum_{i=1}^{s}\pi\left(0,u,1\right) \quad (4.104)$$

Based on these formulas derived to evaluate the average performance of the systems the uncertainty in variance of the steady state distributions can be computed using partial derivatives obtained by using finite difference method. For the uncertain transition probabilities p^{ui}, r^{ui}, p^{dj} and r^{dj}, given the corresponding variances as inputs.

4.3.4 Taylor Approximation with approximate partial derivatives

Similar to most of the cases discussed in the previous sections some or all of the input parameters considered might be obtained from uncertain estimations. For the other parameters whose estimate is assumed to be precisely known, only their precise values or the expected values are provided. Instead for the rest of the parameters where their estimation uncertainty has to be considered the first two moments of input parameters are required. For the Taylor approximation evaluation technique, even if the density of the uncertain parameters is available it is sufficient to have the expected value and the corresponding variance from the density function. Alternatively the direct estimation of expected value and associated variance suffices.

Notations for this technique are as follows:

- Precisely known parameters of the upstream machines are denoted simply p^{ui}, r^{ui} while uncertain parameters are characterized by their mean $p^{ui}{}_{(0)}, r^{ui}{}_{(0)}$ and their variances $\sigma^2 p^{ui}, \sigma^2 r^{ui}$.
- Similarly uncertain parameters for the downstream machines with mean $p^{di}{}_{(0)}$, $r^{di}{}_{(0)}$ and variances $\sigma^2 p^{dj}, \sigma^2 r^{ui}$.

Once the input parameters are available then the calculation of the partial derivatives is carried out by using finite difference method. The finite difference computes the first and second order approximate partial derivatives using a second order polynomial approximation. This approximate evaluation needs a finite small difference to evaluate the function at three chosen points close to each other. The choice of the difference size depends on the precision needed on the derivative and the width of the region covered by the distribution needed to be approximated.

Generally the difference can defined depending on a particular criterion. In this case the approximated first and second order derivatives are required to be a weighted derivative in the range of the distribution on parameter axis. An empirical experiment has shown a good approximation is obtained when the difference is defined one standard deviation above and below the expected value of the uncertain parameter.

The difference between the upper and lower limits is defined with the following relation and represented and the following notations are adopted throughout this analysis involving finite difference methods for performance evaluation.

- The expected value given as input for upstream machine parameters and downstream machines; Upstream $p^{ui}{}_{(0)}, r^{ui}{}_{(0)}$ and downstream $p^{di}{}_{(0)}, r^{di}{}_{(0)}$
- The lower threshold limit for the difference of uncertain parameters of upstream machine and downstream machines are defined as;
    - Upstream $p^{ui}{}_{(-)} = p^{ui}{}_{(0)} - \sigma p^{ui}, r^{ui}{}_{(-)} = r^{ui}{}_{(0)} - \sigma r^{ui}$
    - Downstream $p^{di}{}_{(-)} = p^{di}{}_{(0)} - \sigma p^{di}, r^{di}{}_{(-)} = r^{di}{}_{(0)} - \sigma r^{di}$
- The upper threshold limit for the difference of uncertain parameters of upstream machine and downstream machines are defined as;
    - Upstream $p^{ui}{}_{(+)} = p^{ui}{}_{(0)} + \sigma p^{ui}, r^{ui}{}_{(+)} = r^{ui}{}_{(0)} + \sigma r^{ui}$
    - Downstream $p^{di}{}_{(+)} = p^{di}{}_{(0)} + \sigma p^{di}, r^{di}{}_{(+)} = r^{di}{}_{(0)} + \sigma r^{di}$

Therefore each of the uncertain parameter is composed of vectors of three values i.e. the lower threshold value, the mean value and the upper threshold value.

Next the first and the second order partial derivatives can be approximated for each of the input uncertain parameters using the difference formula as;

A centered first order approximation of partial derivative for a generic performance measure $f(p)$ versus parameter p is

$$\frac{\partial f(p)}{\partial p} = \frac{f(p+\Delta p) - f(p-\Delta p)}{2\Delta p} + O(h^2) \qquad (4.105)$$

While the second order centered partial derivative is approximated as:

$$\frac{\partial^2 f(p)}{\partial p^2} = \frac{f(p+\Delta p) - 2f(\hat{p}) + f(p-\Delta p)}{(\Delta p)^2} + O(h^2) \qquad (4.106)$$

Proceeding similarly for the interesting performance measures the partial derivatives of the important performance measures equation (4.99-4.104) with respect to each uncertain parameter can be evaluated. Notations for the evaluated performance measures are also similar to the notations given for the input parameters. The following notational conventions are adopted.

For instance for the probability of blocking:

$$Pb_{j(0)} = Pb_j \mid p^{ui} = p^{ui}{}_{(0)}; r^{ui} = r^{ui}{}_{(0)} \forall \mathrm{i} = 1,..,s;\ p^{dj} = p^{dj}{}_{(0)}, r^{dj} = r^{dj}{}_{(0)} \forall \mathrm{j} = 1,..,t$$

For the upstream failure probabilities

$$Pb_{j(-)}p^{uk} = Pb_j \mid p^{uk} = p^{uk}{}_{(-)}, p^{ui} = p^{ui}{}_{(0)} \forall \mathrm{i} = 1,..,s; i \neq k; r^{ui} = r^{ui}{}_{(0)} \forall \mathrm{i} = 1,..,s;\ p^{dj} = p^{dj}{}_{(0)}, r^{dj} = r^{dj}{}_{(0)} \forall \mathrm{j} = 1,..,t$$
$$Pb_{j(+)}p^{uk} = Pb_j \mid p^{uk} = p^{uk}{}_{(+)}, p^{ui} = p^{ui}{}_{(0)} \forall \mathrm{i} = 1,..,s; i \neq k; r^{ui} = r^{ui}{}_{(0)} \forall \mathrm{i} = 1,..,s;\ p^{dj} = p^{dj}{}_{(0)}, r^{dj} = r^{dj}{}_{(0)} \forall \mathrm{j} = 1,..,t$$

For the upstream repair probabilities

$$Pb_{j(-)}r^{uk} = Pb_j \mid r^{uk} = r^{uk}{}_{(-)}, p^{ui} = p^{ui}{}_{(0)} \forall \mathrm{i} = 1,..,s; r^{ui} = r^{ui}{}_{(0)} \forall \mathrm{i} = 1,..,s; i \neq k;\ p^{dj} = p^{dj}{}_{(0)}, r^{dj} = r^{dj}{}_{(0)} \forall \mathrm{j} = 1,..,t$$
$$Pb_{j(+)}r^{uk} = Pb_j \mid r^{uk} = r^{uk}{}_{(+)}, p^{ui} = p^{ui}{}_{(0)} \forall \mathrm{i} = 1,..,s; r^{ui} = r^{ui}{}_{(0)} \forall \mathrm{i} = 1,..,s; i \neq k;\ p^{dj} = p^{dj}{}_{(0)}, r^{dj} = r^{dj}{}_{(0)} \forall \mathrm{j} = 1,..,t$$

For the downstream failure probabilities

$$Pb_{j(-)}p^{dk} = Pb_j \mid p^{dk} = p^{dk}{}_{(-)}, p^{ui} = p^{ui}{}_{(0)}; r^{ui} = r^{ui}{}_{(0)} \forall \mathrm{i} = 1,..,s;\ p^{dj} = p^{dj}{}_{(0)} \forall \mathrm{j} = 1,..,t;\ j \neq k; r^{dj} = r^{dj}{}_{(0)} \forall \mathrm{j} = 1,..,t;$$
$$Pb_{j(+)}p^{dk} = Pb_j \mid p^{dk} = p^{dk}{}_{(+)}, p^{ui} = p^{ui}{}_{(0)}; r^{ui} = r^{ui}{}_{(0)} \forall \mathrm{i} = 1,..,s;\ p^{dj} = p^{dj}{}_{(0)} \forall \mathrm{j} = 1,..,t;\ j \neq k; r^{dj} = r^{dj}{}_{(0)} \forall \mathrm{j} = 1,..,t;$$

For the downstream repair probabilities

$$Pb_{j(-)}r^{dk} = Pb_j \mid r^{dk} = r^{dk}{}_{(-)}, p^{ui} = p^{ui}{}_{(0)}; r^{ui} = r^{ui}{}_{(0)} \forall \mathrm{i} = 1,..,s;\ p^{dj} = p^{dj}{}_{(0)} \forall \mathrm{j} = 1,..,t; r^{dj} = r^{dj}{}_{(0)} \forall \mathrm{j} = 1,..,t;\ j \neq k;$$
$$Pb_{j(+)}r^{dk} = Pb_j \mid r^{dk} = r^{dk}{}_{(+)}, p^{ui} = p^{ui}{}_{(0)}; r^{ui} = r^{ui}{}_{(0)} \forall \mathrm{i} = 1,..,s;\ p^{dj} = p^{dj}{}_{(0)} \forall \mathrm{j} = 1,..,t; r^{dj} = r^{dj}{}_{(0)} \forall \mathrm{j} = 1,..,t;\ j \neq k;$$

Similar notations can be followed for the remaining performance measures, therefore the following performances are required to be computed.

For probabilities of starvation:

$$Ps_{i(0)}, Ps_{i(-)}p^{uk}, Ps_{i(+)}p^{uk}, Ps_{i(-)}r^{uk}, Ps_{i(+)}r^{uk}, Ps_{i(-)}p^{dk}, Ps_{i(+)}p^{dk}, Ps_{i(-)}r^{dk}, Ps_{i(+)}r^{dk}$$

For the average throughput:

$$TH_{(0)}, TH_{(-)}p^{uk}, TH_{(+)}p^{uk}, TH_{(-)}r^{uk}, TH_{(+)}r^{uk}, TH_{(-)}p^{dk}, TH_{(+)}p^{dk}, TH_{(-)}r^{dk}, TH_{(+)}r^{dk}$$

Using the partial derivative approximations that are presented in equations (4.105-4.106) and the notational conventions adopted above the first and second order derivatives are evaluated for each performance measure.

For instance for the probability of blocking:

$$\frac{\partial Pb_j}{\partial p^{uk}} = \frac{Pb_{j(+)}p^{uk} - Pb_{j(-)}p^{uk}}{p^{uk}{}_{(+)} - p^{uk}{}_{(-)}}, \qquad \frac{\partial Pb_j}{\partial r^{uk}} = \frac{Pb_{j(+)}r^{uk} - Pb_{j(-)}r^{uk}}{r^{uk}{}_{(+)} - r^{uk}{}_{(-)}} \tag{4.107}$$

$$\frac{\partial^2 Pb_j}{\partial p^{uk2}} = \frac{Pb_{j(+)}p^{uk} - 2Pb_{j(0)} + Pb_{j(-)}p^{uk}}{\left(\frac{p^{uk}{}_{(+)} - p^{uk}{}_{(-)}}{2}\right)^2}, \qquad \frac{\partial^2 Pb_j}{\partial r^{uk2}} = \frac{Pb_{j(+)}r^{uk} - 2Pb_{j(0)} + Pb_{j(-)}r^{uk}}{\left(\frac{r^{uk}{}_{(+)} - r^{uk}{}_{(-)}}{2}\right)^2} \tag{4.108}$$

$$\frac{\partial Pb_j}{\partial p^{dk}} = \frac{Pb_{j(+)}p^{dk} - Pb_{j(-)}p^{dk}}{p^{dk}{}_{(+)} - p^{dk}{}_{(-)}}, \qquad \frac{\partial Pb_j}{\partial r^{dk}} = \frac{Pb_{j(+)}r^{dk} - Pb_{j(-)}r^{dk}}{r^{dk}{}_{(+)} - r^{dk}{}_{(-)}} \tag{4.109}$$

$$\frac{\partial^2 Pb_j}{\partial p^{dk2}} = \frac{Pb_{j(+)}p^{dk} - 2Pb_{j(0)} + Pb_{j(-)}p^{dk}}{\left(\frac{p^{dk}{}_{(+)} - p^{dk}{}_{(-)}}{2}\right)^2}, \qquad \frac{\partial^2 Pb_j}{\partial r^{dk2}} = \frac{Pb_{j(+)}r^{dk} - 2Pb_{j(0)} + Pb_{j(-)}r^{dk}}{\left(\frac{r^{dk}{}_{(+)} - r^{dk}{}_{(-)}}{2}\right)^2} \tag{4.110}$$

The first and second partial derivatives for the remaining performance measures similarly:

Probability of starvation

With respect to upstream machine parameters $\frac{\partial Ps_j}{\partial p^{uk}}, \frac{\partial^2 Ps_j}{\partial p^{uk2}}, \frac{\partial Ps_j}{\partial r^{uk}}, \frac{\partial^2 Ps_j}{\partial r^{uk2}}$

With respect to downstream machine parameters $\frac{\partial Ps_j}{\partial p^{uk}}, \frac{\partial^2 Ps_j}{\partial p^{uk2}}, \frac{\partial Ps_j}{\partial r^{uk}}, \frac{\partial^2 Ps_j}{\partial r^{uk2}}$

For the average throughput

With respect to upstream machine parameters $\frac{\partial E}{\partial p^{uk}}, \frac{\partial^2 E}{\partial p^{uk2}}, \frac{\partial E}{\partial r^{uk}}, \frac{\partial^2 E}{\partial r^{uk2}}$

With respect to downstream machine parameters $\frac{\partial E}{\partial p^{uk}}, \frac{\partial^2 E}{\partial p^{uk2}}, \frac{\partial E}{\partial r^{uk}}, \frac{\partial^2 E}{\partial r^{uk2}}$

Now all the parameters required for the evaluation of the expected value and the associated uncertainty of the performance measures are available.

Then the uncertainty of the probability of blocking is approximated as:

$$V[Pb_j] = \sum_{i=1}^{s}\left(\frac{\partial Pb_j}{\partial p^{ui}}\right)^2 \sigma^2 p^{ui} + \sum_{i=1}^{s}\left(\frac{\partial Pb_j}{\partial r^{ui}}\right)^2 \sigma^2 r^{ui} + \sum_{i=1}^{t}\left(\frac{\partial Pb_j}{\partial p^{dj}}\right)^2 \sigma^2 p^{dj} + \sum_{i=1}^{t}\left(\frac{\partial Pb_j}{\partial r^{dj}}\right)^2 \sigma^2 r^{dj} \quad (4.111)$$

$$E[Pb_j]_{p^{ui}} = Pb_{j(0)} + \frac{1}{2}\frac{\partial^2 Pb_j}{\partial p^{ui\,2}}\sigma^2 p^{ui} , \quad E[Pb_j]_{r^{ui}} = Pb_{j(0)} + \frac{1}{2}\frac{\partial^2 Pb_j}{\partial r^{ui\,2}}\sigma^2 r^{ui} \quad (4.112)$$

$$E[Pb_j]_{p^{dj}} = Pb_{j(0)} + \frac{1}{2}\frac{\partial^2 Pb_j}{\partial p^{dj\,2}}\sigma^2 p^{dj} , \quad E[Pb_j]_{r^{dj}} = Pb_{j(0)} + \frac{1}{2}\frac{\partial^2 Pb_j}{\partial r^{dj\,2}}\sigma^2 r^{dj} \quad (4.113)$$

$$\Delta E[Pb_j]_{p^{ui}} = \frac{1}{2}\frac{\partial^2 Pb_j}{\partial p^{ui\,2}}\sigma^2 p^{ui} , \quad \Delta E[Pb_j]_{r^{ui}} = \frac{1}{2}\frac{\partial^2 Pb_j}{\partial r^{ui\,2}}\sigma^2 r^{ui} \quad (4.114)$$

$$\Delta E[Pb_j]_{p^{dj}} = \frac{1}{2}\frac{\partial^2 Pb_j}{\partial p^{dj\,2}}\sigma^2 p^{dj} , \quad \Delta E[Pb_j]_{r^{dj}} = \frac{1}{2}\frac{\partial^2 Pb_j}{\partial r^{dj\,2}}\sigma^2 r^{dj} \quad (4.115)$$

The expected value of the probability of blocking is approximated as:

$$E[Pb_j] = Pb_{j(0)} + \sum_{i=1}^{s}\frac{\Delta E[Pb_j]_{p^{ui}} V[Pb_j]_{p^{ui}}}{V[Pb_j]} + \sum_{i=1}^{s}\frac{\Delta E[Pb_j]_{r^{ui}} V[Pb_j]_{r^{ui}}}{V[Pb_j]} + \sum_{j=1}^{t}\frac{\Delta E[Pb_j]_{p^{dj}} V[Pb_j]_{p^{dj}}}{V[Pb_j]} + \sum_{j=1}^{t}\frac{\Delta E[Pb_j]_{r^{dj}} V[Pb_j]_{r^{dj}}}{V[Pb_j]} \quad (4.116)$$

Uncertainty in variance and expected value of the starvation probability is:

$$V[Ps_i] = \sum_{i=1}^{s}\left(\frac{\partial Ps_i}{\partial p^{ui}}\right)^2 \sigma^2 p^{ui} + \sum_{i=1}^{s}\left(\frac{\partial Ps_i}{\partial r^{ui}}\right)^2 \sigma^2 r^{ui} + \sum_{j=1}^{t}\left(\frac{\partial Ps_i}{\partial p^{dj}}\right)^2 \sigma^2 p^{dj} + \sum_{j=1}^{t}\left(\frac{\partial Ps_i}{\partial r^{dj}}\right)^2 \sigma^2 r^{dj} \quad (4.117)$$

$$E[Ps_i]_{p^{ui}} = Ps_{i(0)} + \frac{1}{2}\frac{\partial^2 Ps_i}{\partial p^{ui\,2}}\sigma^2 p^{ui} , \quad E[Ps_i]_{r^{ui}} = Ps_{i(0)} + \frac{1}{2}\frac{\partial^2 Ps_i}{\partial r^{ui\,2}}\sigma^2 r^{ui} \quad (4.118)$$

$$E[Ps_i]_{p^{dj}} = Ps_{i(0)} + \frac{1}{2}\frac{\partial^2 Ps_i}{\partial p^{dj\,2}}\sigma^2 p^{dj} , \quad [Ps_i]_{r^{dj}} = Ps_{i(0)} + \frac{1}{2}\frac{\partial^2 Ps_i}{\partial r^{dj\,2}}\sigma^2 r^{dj} \quad (4.119)$$

$$\Delta E[Ps_i]_{p^{ui}} = \frac{1}{2}\frac{\partial^2 Ps_i}{\partial p^{ui\ 2}}\sigma^2 p^{ui}, \qquad \Delta E[Ps_i]_{r^{ui}} = \frac{1}{2}\frac{\partial^2 Ps_i}{\partial r^{ui\ 2}}\sigma^2 r^{ui}$$

$$\Delta E[Ps_i]_{p^{dj}} = \frac{1}{2}\frac{\partial^2 Ps_i}{\partial p^{dj\ 2}}\sigma^2 p^{dj}, \qquad \Delta E[Ps_i]_{r^{dj}} = \frac{1}{2}\frac{\partial^2 Ps_i}{\partial r^{dj\ 2}}\sigma^2 r^{dj} \tag{4.120}$$

$$E[Ps_i] = Ps_{i(0)} + \sum_{i=1}^{s}\frac{\Delta E[Ps_i]_{p^{ui}} V[Ps_i]_{p^{ui}}}{V[Ps_i]} + \sum_{i=1}^{s}\frac{\Delta E[Ps_i]_{r^{ui}} V[Ps_i]_{r^{ui}}}{V[Ps_i]} + \sum_{j=1}^{t}\frac{\Delta E[Ps_i]_{p^{dj}} V[Ps_i]_{p^{dj}}}{V[Ps_i]} + \sum_{j=1}^{t}\frac{\Delta E[Ps_i]_{r^{dj}} V[Ps_i]_{r^{dj}}}{V[Ps_i]} \tag{4.121}$$

Uncertainty and expected value for the average throughput are approximated as:

$$V[TH] = \sum_{i=1}^{s}\left(\frac{\partial TH}{\partial p^{ui}}\right)^2 \sigma^2 p^{ui} + \sum_{i=1}^{s}\left(\frac{\partial TH}{\partial r^{ui}}\right)^2 \sigma^2 r^{ui} + \sum_{j=1}^{t}\left(\frac{\partial TH}{\partial p^{dj}}\right)^2 \sigma^2 p^{dj} + \sum_{j=1}^{t}\left(\frac{\partial TH}{\partial r^{dj}}\right)^2 \sigma^2 r^{dj} \tag{4.122}$$

$$E[TH]_{p^{ui}} = TH_{(0)} + \frac{1}{2}\frac{\partial^2 TH}{\partial p^{ui\ 2}}\sigma^2 p^{ui}, \qquad \bar{E}_{r^{ui}} = E_{(0)} + \frac{1}{2}\frac{\partial^2 E}{\partial r^{ui\ 2}}\sigma^2 r^{ui}$$

$$E[TH]_{p^{dj}} = TH_{(0)} + \frac{1}{2}\frac{\partial^2 TH}{\partial p^{ui\ 2}}\sigma^2 p^{ui}, \qquad E[TH]_{r^{dj}} = TH_{(0)} + \frac{1}{2}\frac{\partial^2 TH}{\partial r^{dj\ 2}}\sigma^2 r^{dj} \tag{4.123}$$

$$\Delta E[TH]_{p^{ui}} = \frac{1}{2}\frac{\partial^2 TH}{\partial p^{ui\ 2}}\sigma^2 p^{ui}, \qquad \Delta E[TH]_{r^{ui}} = \frac{1}{2}\frac{\partial^2 TH}{\partial r^{ui\ 2}}\sigma^2 r^{ui}$$

$$\Delta E[TH]_{p^{dj}} = \frac{1}{2}\frac{\partial^2 Ps_i}{\partial p^{dj\ 2}}\sigma^2 p^{dj}, \qquad \Delta E[TH]_{r^{dj}} = \frac{1}{2}\frac{\partial^2 Ps_i}{\partial r^{dj\ 2}}\sigma^2 r^{dj} \tag{4.124}$$

$$E[TH] = TH_{(0)} + \sum_{i=1}^{s}\frac{\Delta E[TH]_{p^{ui}} V[Ps_i]_{p^{ui}}}{V[TH]} + \sum_{i=1}^{s}\frac{\Delta E[TH]_{r^{ui}} V[Ps_i]_{r^{ui}}}{V[TH]} + \sum_{j=1}^{t}\frac{\Delta E[TH]_{p^{dj}} V[Ps_i]_{p^{dj}}}{V[TH]} + \sum_{j=1}^{t}\frac{\Delta E[TH]_{r^{dj}} V[Ps_i]_{r^{dj}}}{V[TH]} \tag{4.125}$$

4.4 Multi-stage lines and uncertainty

This section introduces a technique on how parameters with uncertain estimates can be introduced in the performance evaluation of multistage lines. The previous sections demonstrated the evaluation of parameters related to a building block single buffer two machine lines. Further this approach is extended for the evaluation of multistage lines composed of two machine line building blocks. Previously the performance evaluation with uncertainty of two machine line model is demonstrated for single failure machines. Then the same approach is extended for the case of multiple failure modes machines in two machine building blocks which is introduced in 4.3.4. In the subsequent section these methodologies are adapted for the evaluation of longer lines composed of building blocks with multiple failure modes. A methodology is proposed and implemented with the decomposition method for multistage lines which traditionally are modeled in decomposed to two machine lines.

In the case of multistage lines where the building blocks and concepts applied for modeling an open line manufacturing system are also applied for the proposed method. These assumptions are kept the same with previous works of analytical approximate methods. The main difference of the proposed approach from previous performance models particularly lies on the way the input parameters for these models are considered.

4.4.1 Uncertainty propagation in multistage lines by Decomposition

This section introduces a method for the main steps in the evaluation of performance in multistage lines using uncertain parameters. The method is based on the two machine line decomposition technique proposed by (Tolio-Matta-Gershwin 2002). The multi-stage lines composed of K machines and K-1 buffers is decomposed into K two-machine lines. The performance of these two machine lines with uncertainty is evaluated using the set of decomposition proposed in (TMG) and in combination with the set of equations introduced for uncertain inputs in the previous sections for multiple failure mode two machine lines. The two moments characterizing the parameters in the remote failures are propagated through each two machine line until convergence is achieved for the expected value and the uncertainty in variance of the required performance measures.

The decomposition method studies the performance of K machines by approximating the behavior of the original line to be approximated using K-1 two machine lines.

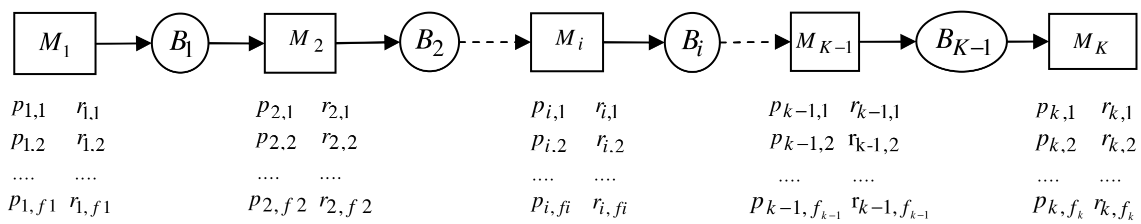


Figure 4:18 Multistage line with multiple failure mode machines

This line is decomposed into K-1 two machine lines so that each upstream pseudo machine mimics the upstream portion of the line and the downstream pseudo machine mimics the failures associate to the downstream machine with reference to the current building block. For instance a multistage line as shown in the Figure 4:18 can be decomposed into two machine lines as shown in figure 4:19.

By referring to the Figure shown in 4:19 to model the line of K machines with K-1 two machine single buffer building blocks we will see how the uncertainties in the local and remote failure probabilities are propagated from one stage into the next one. The failure modes in the line are assigned as local and remote failures in a similar way as shown in Figure 4:19.

In general a two machine line i with upstream pseudo machine $M^u(i)$ and downstream pseudo machine $M^d(i)$ the number of remote failures assigned from upstream portion of the line for the upstream pseudo machine is

$$VF^u(i) = \sum_{j=1}^{i-1} F_j \qquad VF^d(i) = \sum_{j=i+2}^{K} F_j$$

Once parameters related to the two machine building block can be evaluated in this way these expected values and the uncertainties can be used in the DDX algorithm by propagating the parameters from one building block to the next one until convergence in the average throughput and the corresponding variance is reached. For a general two machine line the states of the upstream pseudo machine and the downstream pseudo machines are modeled as shown in the Figure 4:20.

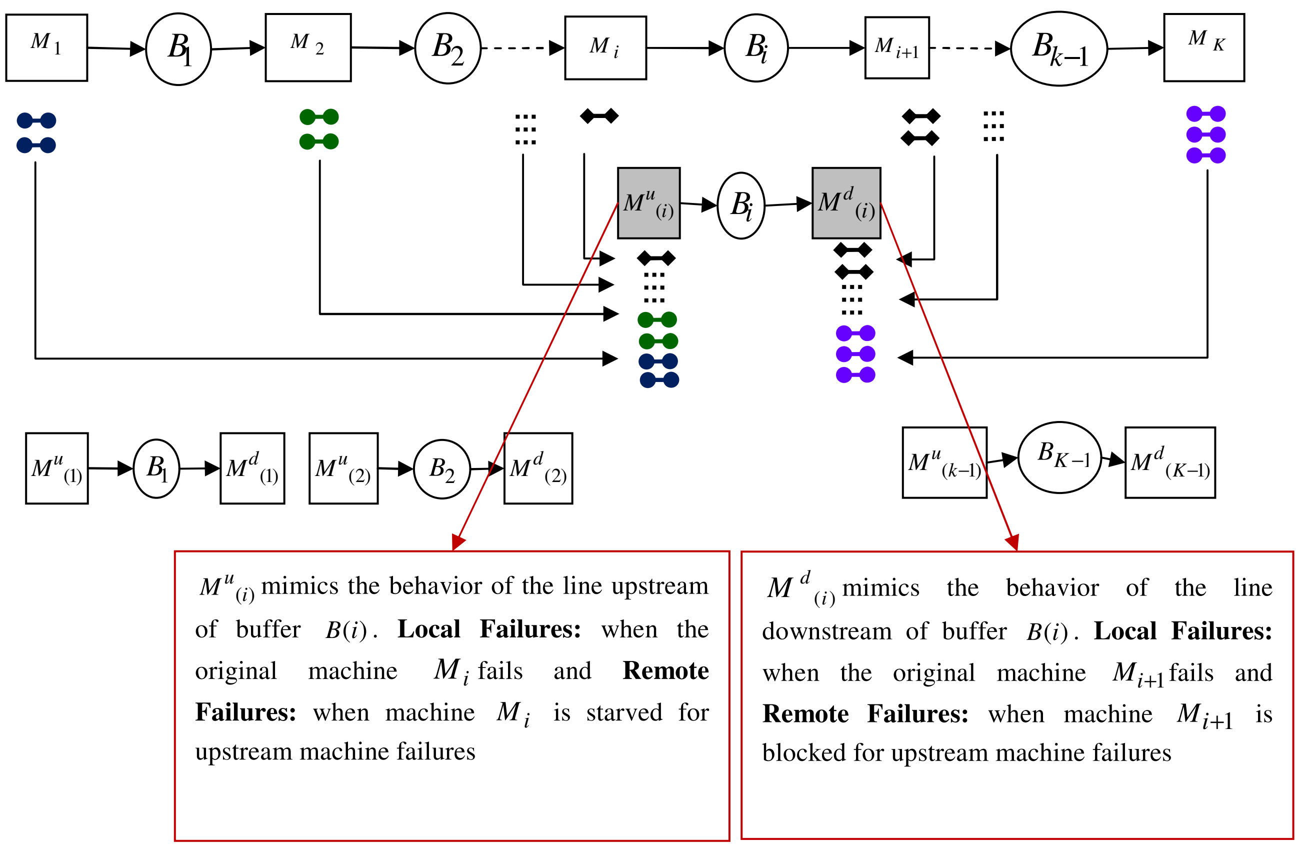


Figure 4:19 Decomposition of multi-stage line into two machine lines

In a similar manner each two machine line building block will be assigned with local failures and remote failures so that they can mimic the average performance of the entire line can be evaluated using DDX algorithm. In the same way the uncertainty related to the estimation of the input parameters for the local failures and uncertainty induced on remote failures by other uncertain parameters are evaluated from one stage to the next one.

Using the approach introduced in section 4.3.4 for the evaluation of two machine single buffer building blocks with multiple uncertain failures modes the decomposition method can be used for the propagation of uncertainty.

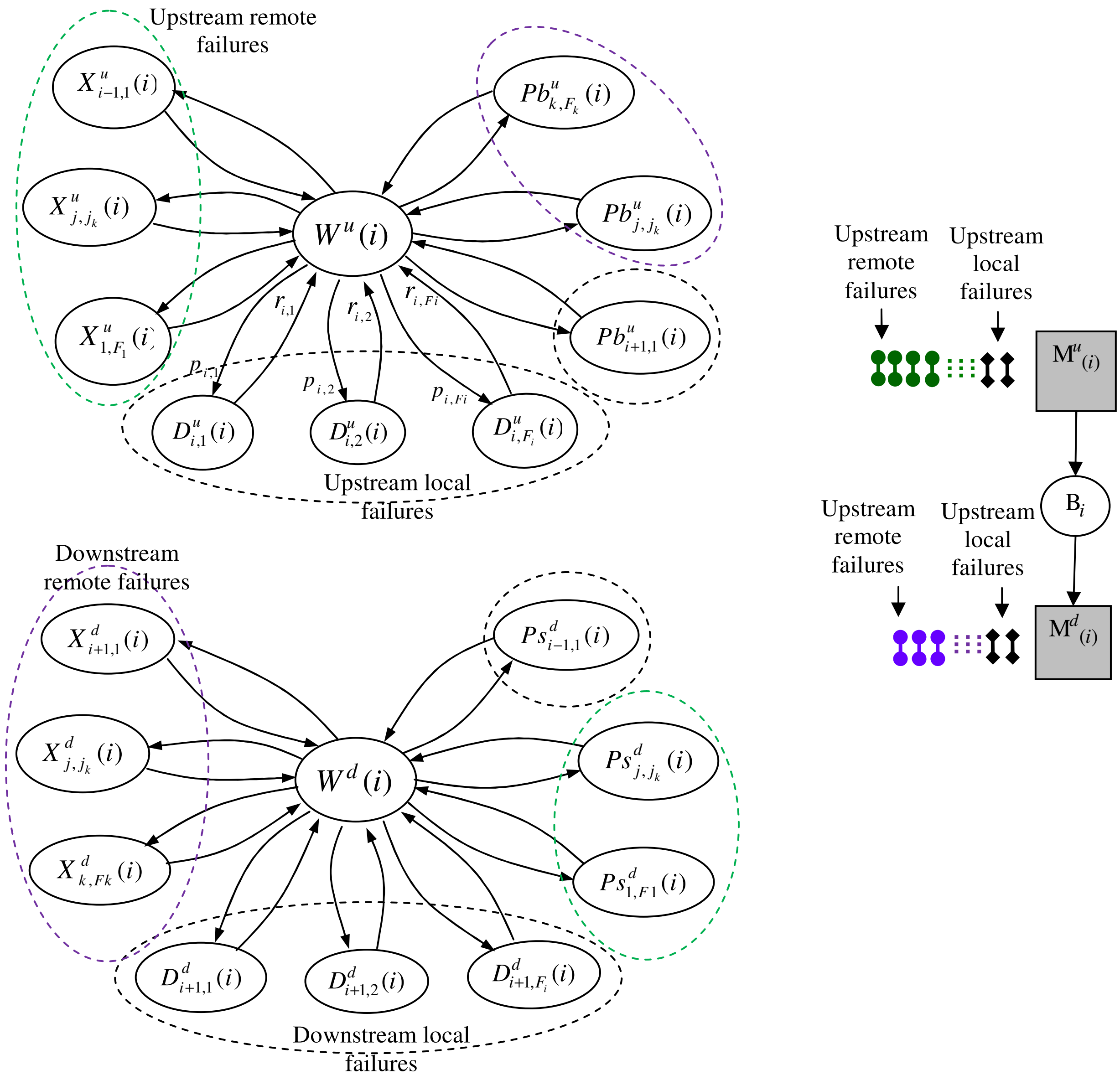


Figure 4:20 Upstream and downstream pseudo machine states

In the decomposition technique evaluating the two machine line model to measure the performance of the multistage line the notations in the previous works are adopted to be as similar as possible with minor modifications for accommodating the introduction of uncertainty in the evaluation. Extending similar notation that are adopted for the demonstration of performance evaluation for two machine line with multiple uncertain failure mode the notations that will be followed for the pseudo machines in the decomposition are the following.

Upstream machine $M^u{}_{(i)}$ parameters $for\ i = 1,...,K-1;$

Local probability of parameters:

- If certain: Failure $p_{j,f}$ and repair probability $r_{j,f}$.
- If uncertain:
  - Expected value, failure $\hat{p}_{j,f} = p_{j,f(0)}$, repair $\hat{r}_{j,f} = \hat{r}_{j,f(0)}$
  - Uncertainty variance: failure $\sigma^2 p_{j,f}$ repair $\sigma^2 r_{j,f}$

Remote probability of parameters:

- If certain: repair probability $r_{j,f}$ all failures in remote are uncertain.
- If uncertain:
  - Expected value, failure $\hat{p}^u_{j,f}(i) = p^u_{j,f}(i)_{(0)}$, repair $\hat{r}_{j,f} = \hat{r}_{j,f(0)}$
  - Uncertainty variance: failure $\sigma^2 p^u_{j,f}(i)$ repair $\sigma^2 r_{j,f}$

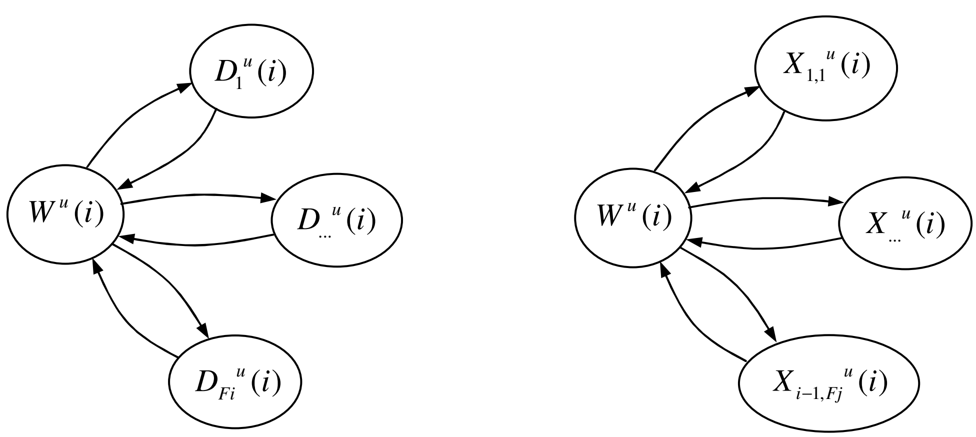


Figure 4:21 Markov model for upstream local and remote failures

For local failures

$$D_f^u(i) = W^u(i) \cdot \frac{p_{i,f}}{r_{i,f}} \quad , \; f = 1,...,F_i;$$

$$V[D_{i,f}^u(i)]_{p_{i,k}} = \left(\frac{\partial D_{i,f}^u(i)}{\partial p_{i,k}}\right)^2 \sigma^2 p_{i,k}; \; \mathrm{V}[D_{i,f}^u(i)]_{r_{i,k}} = \left(\frac{\partial D_{i,f}^u(i)}{\partial r_{i,k}}\right)^2 \sigma^2 r_{i,k} \qquad (4.126)$$

Remote Failures

$$p_{j,f}^u(i) = \frac{X_{j,f}^u(i)}{W^u(i)} \cdot r_{j,f} \quad f = 1,...,F_j \quad j = 1,...,i-1;$$

$$\sigma^2 p_{j,f}^u(i)_{p_{i,k}} = \left(\frac{\partial p_{j,f}^u(i)}{\partial p_{i,k}}\right)^2 \sigma^2 p_{i,k}; \sigma^2 p_{j,f}^u(i)_{r_{i,k}} = \left(\frac{\partial p_{j,f}^u(i)}{\partial r_{i,k}}\right)^2 \sigma^2 r_{i,k} \quad f = 1,...,F_j \quad j = 1,...,i-1; \qquad (4.127)$$

For downstream machine $M^d{}_{(i-1)}$ parameters $for\ i = 2,...,K;$

Local probability of parameters:

- If certain: Failure $p_{j,f}$ and repair probability $r_{j,f}$.
- If uncertain:
  - Expected value, failure $\hat{p}_{j,f} = p_{j,f(0)}$, repair $\hat{r}_{j,f} = \hat{r}_{j,f(0)}$
  - Uncertainty variance: failure $\sigma^2 p_{j,f}$ repair $\sigma^2 r_{j,f}$

Remote probability of parameters:

- If certain: repair probability $r_{j,f}$ all failures in remote are uncertain.
- If uncertain:
  - Expected value, failure $\hat{p}_{j,f}^d(i) = p_{j,f}^d(i)_{(0)}$, repair $\hat{r}_{j,f} = \hat{r}_{j,f(0)}$
  - Uncertainty variance: failure $\sigma^2 p_{j,f}^d(i)$ repair $\sigma^2 r_{j,f}$

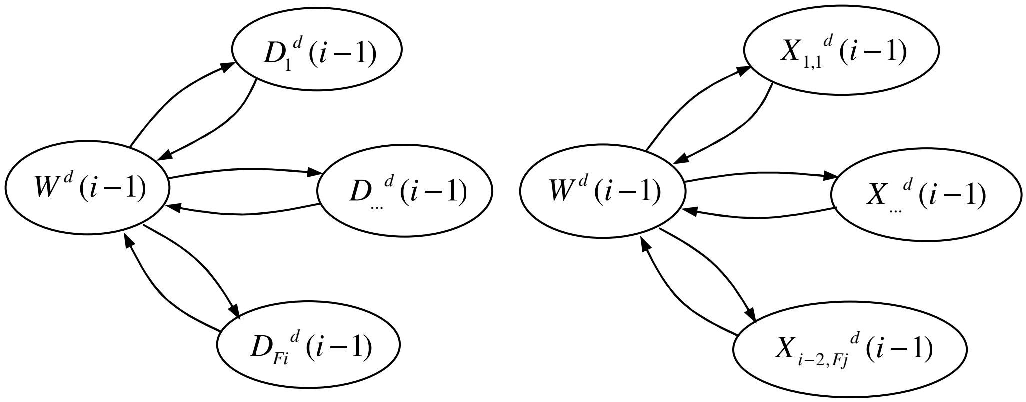


Figure 4:22 Markov model for downstream local and remote failures

Local Failures are evaluated as:

$$D_f^d(i-1) = W^d(i-1) \cdot \frac{p_{i,f}}{r_{i,f}} \quad , \; f = 1,...,F_i;$$

$$V[D_{i,f}^d(i-1)]_{p_{i,k}} = \left(\frac{\partial D_{i,f}^d(i-1)}{\partial p_{i,k}}\right)^2 \sigma^2 p_{i,k}; \; V[D_{i,f}^d(i-1)]_{r_{i,k}} = \left(\frac{\partial D_{i,f}^d(i-1)}{\partial r_{i,k}}\right)^2 \sigma^2 r_{i,k} \qquad (4.128)$$

Remote Failures

$$p_{j,f}^d(i-1) = \frac{X_{j,f}^d(i-1)}{W^d(i-1)} \cdot r_{j,f} \quad f = 1,...,F_j \quad j = 1,...,i-1;$$

$$\sigma^2 p_{j,f}^u(i-1)_{p_{i,k}} = \left(\frac{\partial p_{j,f}^u(i-1)}{\partial p_{i,k}}\right)^2 \sigma^2 p_{i,k}; \sigma^2 p_{j,f}^d(i-1)_{r_{i,k}} = \left(\frac{\partial p_{j,f}^d(i-1)}{\partial r_{i,k}}\right)^2 \sigma^2 r_{i,k} \quad f = 1,...,F_j \quad j = 1,...,i-1;$$

(4.129)

For both the upstream and downstream pseudo machines the sum of the steady state probabilities of the states adds to 1.

For upstream pseudo machines $W^u(i) + \sum_{f=1}^{F_i} D_{i,f}^u(i) + \sum_{j=1}^{i-1}\sum_{f=1}^{F_j} X_{j,f}^u(i) + \sum_{j=i+1}^{K}\sum_{f=1}^{F_j} Pb_{j,f}(i) = 1$ and

For downstream pseudo machines $W^d(i)+\sum_{f=1}^{F_i} D_f^d(i)+\sum_{j=i+1}^{K}\sum_{f=1}^{Fj} X_{j,f}^d(i) \ +\sum_{j=1}^{i-1}\sum_{f=1}^{F_j} Ps_{j,f}(i)=1$ (4.130)

Given the upstream pseudo machine and downstream pseudo machines of a building block and the respective local and remote failure modes the evaluation of the probabilities in Figure 4:21 are updated.

For the upstream machine $M^u{}_{(i)}$ the unknown probabilities and their respective uncertainties are updated

$$p_{j,f}^u(i)=\frac{Ps_{j,f}(i-1)}{E(i)}\cdot r_{j,f}$$

$$\sigma^2 p_{j,f}^u(i)=\left(\frac{\partial p_{j,f}^u(i)}{\partial Ps_{j,f}(i-1)}\right)^2 \sigma^2 Ps_{j,f}(i-1)$$

$$\sigma^2 Ps_{j,f}(i-1)=\left(\frac{\partial Ps_{j,f}(i-1)}{\partial p_{j,f}^u(i-1)}\right)^2 \sigma^2 p_{j,f}^u(i-1) \tag{4.131}$$

For the downstream machine $M^d{}_{(i-1)}$

$$p_{j,f}^d(i-1)=\frac{Pb_{j,f}(i)}{E(i-1)}\cdot r_{j,f}$$

The same analysis as in the case of upstream updating of the uncertainty and expected value gives equation 4.132 for the remote failures of the downstream pseudo machine.

$$\sigma^2 p_{j,f}^d(i-1)_{p_{j,k}}=\left(\frac{\partial p_{j,f}^d(i-1)}{\partial p_{j,f}^d(i)}\right)^2 \sigma^2 p_{j,f}^d(i) \quad f=1,...,F_j \quad j=i+1,...,K-1; \tag{4.132}$$

The performance of the two machine line with multiple failure modes is evaluated using the method proposed by [Tolio-Matta-Gershwin 2002]. The uncertainties related to each probability are computed at two machine level. In the DDX algorithm for the convergence of probabilities the uncertainty variance associated to each parameters is also updated until convergence is reached for the expected value of the probabilities and corresponding variance.

The two machine line model in [TMG] model is as shown below.

4.4.2 Evaluation of partial derivatives of uncertain parameters

The evaluation of the required partial derivatives is performed using the finite difference technique as introduced in the evaluation of the multiple uncertain two machine lines in section 4.3.4. For each uncertain failure probability $p_{j,f}$ define the difference $\Delta p_{j,f}$ and for uncertain repair probability $r_{j,f}$ define the difference $\Delta r_{j,f}$. The difference can be determined depending on a small finite element within the valid domain of the parameter. For all the cases considered in by empirical experiments a better approximation of the first and second order partial derivative approximations are found an when the difference is determined to be results $\sigma p_{j,f}$ or $\sqrt{\sigma^2 p_{j,f}}$.

The following notations are introduced and the empirically chosen approximation difference of two standard deviations as the difference are used for the evaluation of the partial derivatives.

For the uncertain transition probabilities of the upstream machine the following notations are used to refer points used to compute the derivatives. Each uncertain parameter is considered as a vector of three elements, for instance for the upstream failure probability p_i^u, the elements are $p_{i(0)}^u = p_i^u$, the mean point estimate, $p_{i(-)}^u = p_{i(0)}^u - \sigma p_i^u$ lower threshold of the difference, and $p_{i(+)}^u = p_{i(0)}^u + \sigma p_i^u$, and upper threshold for the finite difference are defined.

The same elements will be defined for the rest of the uncertain probabilities to be considered in the analysis.

$p_{i(0)}^u = \hat{p}_i^u$ is the expected value point estimator of failure probability p_i^u

$p_{i(+)}^u = p_{i(0)}^u + \sigma p_i^u$

$p_{i(-)}^u = p_{i(0)}^u - \sigma p_i^u$

$r_{i(0)}^u = \hat{r}_i^u$ is the expected value point estimator of repair probability r_i^u

$r_{i(+)}^u = r_{i(0)}^u + \sigma r_i^u$

$r_{i(-)}^u = r_{i(0)}^u - \sigma r_i^u$

Similarly the uncertain transition probabilities for the downstream machine can be computed at these three points.

Using the notation introduced in for upper and lower values of failure and repair probabilities the interesting parameters, i.e. Average throughput,

$TH_{(0)} = TH \mid p_{i(0)}^u$

$TH_{(+)} p_k^u = TH \mid p_k^u = p_{k(+)}^u, p_i^u = p_{i(0)}^u \forall \text{i} = 1,..,s; i \neq k; r_i^u = r_{i(0)}^u \forall \text{i} = 1,..,s; p_j^d = p_{j(0)}^d, r_j^d = r_{i(0)}^d \forall \text{j} = 1,..,t$

$TH_{(-)} p_k^u = TH \mid p_k^u = p_{k(-)}^u, p_{i(0)}^u \forall i = 1,..,s; i \neq k; r_i^u = r_{i(0)}^u \forall i = 1,..,s; p_j^d = p_{j(0)}^d, r_j^d = r_{i(0)}^d \forall j = 1,..,t$

From these values the first order partial derivative of the average throughput with respect to each uncertain transition probabilities can be evaluated as:

Similarly the remaining performance measures corresponding to these points such as the probability of blocking Pb_j, starvation Ps_i and the average buffer level $\bar{n}$ can be evaluated as:

$Pb_{j(+)} p_k^u = Pb_j \mid p_k^u = p_{k(+)}^u, p_i^u = p_{i(0)}^u \forall \text{ i} = 1,..,s; i \neq k; r_i^u = r_{i(0)}^u \forall \text{ i} = 1,..,s; p_j^d = p_{j(0)}^d, r_j^d = r_{i(0)}^d \forall \text{ j} = 1,..,t$

$Pb_{j(-)} p_k^u = Pb_j \mid p_k^u = p_{k(-)}^u, p_{i(0)}^u \forall \text{ i} = 1,..,s; i \neq k; r_i^u = r_{i(0)}^u \forall \text{ i} = 1,..,s; p_j^d = p_{j(0)}^d, r_j^d = r_{i(0)}^d \forall \text{ j} = 1,..,t$

$Ps_{i(+)} p_k^u = Ps_i \mid p_k^u = p_{k(+)}^u, p_i^u = p_{i(0)}^u \forall \text{i} = 1,...,s; i \neq k; r_i^u = r_{i(0)}^u \forall \text{i} = 1,..,s; p_j^d = p_{j(0)}^d, r_j^d = r_{i(0)}^d \forall \text{j} = 1,..,t$

$Ps_{i(-)} p_k^u = Ps_i \mid p_k^u = p_{k(-)}^u, p_i^u = p_{i(0)}^u \forall \text{i} = 1,..,s; i \neq k; r_i^u = r_{i(0)}^u \forall \text{i} = 1,..,s; p_j^d = p_{j(0)}^d, r_j^d = r_{i(0)}^d \forall \text{j} = 1,..,t$

In the same manner the first order partial derivatives that correspond to these performance measures are computed as:

$$\frac{\partial TH}{\partial p_k^u} = \frac{TH_{(+)} p_k^u - TH_{(-)} p_k^u}{p_{k(+)}^u - p_{k(-)}^u} \tag{4.133}$$

The second order partial derivative is approximated

$$\frac{\partial^2 E}{\partial p_k^u \partial p_k^u} = \frac{E_{(+)} p_k^u - 2E_{(0)} p_k^u + E_{(-)} p_k^u}{\left(p_{k(+)}^u - p_{k(-)}^u \right)^2} \tag{4.134}$$

Similar expression can be written for the computation of the probability of starvation and probability of blocking

$$\frac{\partial Ps_i}{\partial p_i^u} = \frac{Ps_{i(+)} p_k^u - Ps_{i(-)} p_k^u}{p_{i(+)}^u - p_{i(-)}^u} \tag{4.135}$$

$$\frac{\partial^2 Ps_i}{\partial p_i^u \partial p_i^u} = \frac{Ps_{i(+)} p_k^u - 2Ps_{i(0)} p_k^u + Ps_{i(-)} p_k^u}{\left(p_{i(+)}^u - p_{i(-)}^u \right)^2} \tag{4.136}$$

$$\frac{\partial Pb_i}{\partial p_i^u} = \frac{Pb_{i(+)} p_k^u - Pb_{i(-)} p_k^u}{p_{i(+)}^u - p_{i(-)}^u} \tag{4.137}$$

$$\frac{\partial^2 Pb_i}{\partial p_i^u \partial p_i^u} = \frac{Pb_{i(+)} p_k^u - 2Pb_{i(0)} p_k^u + Pb_{i(-)} p_k^u}{\left(p_{i(+)}^u - p_{i(-)}^u \right)^2} \tag{4.138}$$

At a building block level the uncertainty to a given performance measure can be written as:

The expected value of the performance measures is approximated as:

$$E[TH]_{p_i^u} = E_{(0)} + \frac{1}{2} \frac{\partial^2 E}{\partial p_i^u \partial p_i^u} \sigma^2 p_i^u \tag{4.139}$$

$$V[E]_{p_i^u} = \left(\frac{\partial E}{\partial p_i^u} \right)^2 \sigma^2 p_i^u \tag{4.140}$$

$$V[TH]=\sum_{i=1}^{s}\left(\frac{\partial TH}{\partial p_i^u}\right)^2\sigma^2 p_i^u+\sum_{i=1}^{s}\left(\frac{\partial TH}{\partial r_i^u}\right)^2\sigma^2 r_i^u+\sum_{i=1}^{t}\left(\frac{\partial TH}{\partial p_j^d}\right)^2\sigma^2 p_j^d+\sum_{i=1}^{t}\left(\frac{\partial TH}{\partial r_j^d}\right)^2\sigma^2 r_j^d \quad (4.141)$$

Uncertainty in the probability of starvation and blocking can be computed

$$V[Ps_i]=\sum_{i=1}^{s}\left(\frac{\partial Ps_i}{\partial p_i^u}\right)^2\sigma^2 p_i^u+\sum_{i=1}^{s}\left(\frac{\partial Ps_i}{\partial r_i^u}\right)^2\sigma^2 r_i^u+\sum_{i=1}^{t}\left(\frac{\partial Ps_i}{\partial p_j^d}\right)^2\sigma^2 p_j^d+\sum_{i=1}^{t}\left(\frac{\partial Ps_i}{\partial r_j^d}\right)^2\sigma^2 r_j^d \quad (4.142)$$

$$V[Pb_j]=\sum_{i=1}^{s}\left(\frac{\partial Pb_j}{\partial p_i^u}\right)^2\sigma^2 p_i^u+\sum_{i=1}^{s}\left(\frac{\partial Pb_j}{\partial r_i^u}\right)^2\sigma^2 r_i^u+\sum_{i=1}^{t}\left(\frac{\partial Pb_j}{\partial p_j^d}\right)^2\sigma^2 p_j^d+\sum_{i=1}^{t}\left(\frac{\partial Pb_j}{\partial r_j^d}\right)^2\sigma^2 r_j^d \quad (4.143)$$

4.4.3 Evaluation of performance measure with uncertainties

Since remote failures of pseudo-machines mimic starvation or blocking phenomena, due to the propagation of a failure of another machine along the line, repair rates of remote failures are simply equal to those of the original machine that generated the failure. The only unknown parameters in K-1 two-machine lines are remote failure probabilities. To evaluate these probabilities a set of equations is required. These equations are called DECOMPOSITION EQUATIONS. Solving this set of equations it is possible to evaluate unknown failure probabilities in order to mimic with the decomposed line the behavior of the original line.

In the decomposition method the assumption made for the repair probabilities for instance for the upstream machine $M^u(i)$ has a failure (both remote and local) it eventually gets repaired, failure frequency must equal repair frequency for every failure mode. Therefore the expected value and the uncertainty in variance for the local failure probabilities and remote failures can be evaluated as:

$$W^u(i)+\sum_{f=1}^{F_i}D_{i,f}^u(i)+\sum_{j=1}^{i-1}\sum_{f=1}^{F_j}X_{j,f}^u(i)+\sum_{j=i+1}^{K}\sum_{f=1}^{F_j}Pb_{j,f}(i)=1$$

Local Failure modes:

$$D^{u}_{i,f}(i) = W^{u}(i) \cdot \frac{p_{i,f}}{r_{i,f}} \quad , \ f = 1, ..., F_i;$$

$$V[D^{u}_{i,f}(i)]_{p_{i,k}} = \left(\frac{\partial D^{u}_{i,f}(i)}{\partial p_{i,k}} \right)^2 \sigma^2 p_{i,k} \tag{4.144}$$

Remote Failure modes are updated with the expected value and the corresponding uncertainty using a similar approach. For instance the remote failures associated to the upstream machine $M^{u}(i)$ are updated with equations (4.145-4.146)

Therefore substituting the above equations the computation of uncertainty for the updated remote failure probability $p^{u}_{j,f}(i)$ can be simplified as:

$$\sigma^2 p^{u}_{j,f}(i) = \left(\frac{\partial p^{u}_{j,f}(i)}{\partial p^{u}_{j,f}(i-1)} \right)^2 \sigma^2 p^{u}_{j,f}(i-1) \tag{4.145}$$

The expected value and the uncertainty related to the remote failures of the downstream machine $M^{d}(i)$ are updated in the same way.

$$p^{d}_{j,f}(i-1) = \frac{Pb_{j,f}(i)}{TH(i-1)} \cdot r_{j,f} \tag{4.146}$$

The same analysis as in the case of upstream updating of the uncertainty and expected value gives equations (4.147-4.148) for the remote failures of the downstream pseudo machine.

$$\sigma^2 p^{d}_{j,f}(i-1)_{p_{j,k}} = \left(\frac{\partial p^{d}_{j,f}(i-1)}{\partial p^{d}_{j,f}(i)} \right)^2 \sigma^2 p^{d}_{j,f}(i) \quad f = 1, ..., F_j \quad j = i+1, ..., K-1; \tag{4.147}$$

$$E[p^{d}_{j,f}(i-1)]_{p_{j,k}} = p^{d}_{j,f}(i-1)_{(0)} + \frac{1}{2} \left(\frac{\partial^2 p^{d}_{j,f}(i-1)}{\partial p^{d}_{j,f}(i) \partial p^{d}_{j,f}(i)} \right) \sigma^2 p^{d}_{j,f}(i) \tag{4.148}$$

The DDX Algorithm with uncertainty

STEP 1: Initialize parameters for the pseudo machines with the local failures and remote failures. If the parameters are assumed certain: Local failures

$$\mathrm{M}^{\mathrm{u}}(i): p^{u}{}_{j,f_j}(i) = p_{j,f_j}; \sigma^2 p^{u}{}_{j,f_j}(i) = \sigma^2 p_{j,f_j}$$

$$r^{u}{}_{j,f_j}(i) = r^{u}{}_{j,f_j}; \sigma^2 r^{u}{}_{j,f_j}(i) = \sigma^2 r^{u}{}_{j,f_j}$$

Remote failures:

$$\mathrm{M}^{\mathrm{u}}(i): j = 1,...,i-1;$$

$$r^{u}{}_{j,f_j}(i) = r^{u}{}_{j,f_j}; \sigma^2 r^{u}{}_{j,f_j}(i) = \sigma^2 r^{u}{}_{j,f_j}$$

$$p^{u}{}_{j,f_j}(i) = \lambda; \sigma^2 p^{u}{}_{j,f_j}(i) = 0$$

For upstream machines: Local failures

$$\mathrm{M}^{d}(i-1): p^{u}{}_{j,f_j}(i) = p_{j,f_j}; \sigma^2 p^{u}{}_{j,f_j}(i-1) = \sigma^2 p_{j,f_j}$$

Remote failures:

$$\mathrm{M}^{d}(i-1): j = i+1,...,K;$$

$$r^{d}{}_{j,f_j}(i-1) = r_{j,f_j}; \sigma^2 r^{d}{}_{j,f_j}(i-1) = \sigma^2 r_{j,f_j}$$

$$p^{d}{}_{j,f_j}(i-1) = \lambda; \sigma^2 p^{d}{}_{j,f_j}(i-1) = 0$$

STEP 2: Perform phase A and phase B alternately until the Termination Condition is satisfied

PHASE A: For $i=2,...K-1$ calculate $p^{u}_{j,f_j}(i)$ for each uncertain parameter using the following equation and replacing $E(i)$ with $E(i-1)$. Use the two-machine line analytical solution to evaluate the performance of the line.

$$\sigma^2 p^{u}_{j,f}(i)_{p_{j,k}} = \left(\frac{\partial p^{u}_{j,f}(i)_{p_{j,k}}}{\partial p^{u}_{j,f}(i-1)_{p_{j,k}}} \right)^2 \sigma^2 p^{u}_{j,f}(i-1)$$

$$E[p^{u}_{j,f}(i)]_{p_{j,k}} = p^{u}_{j,f}(i)_{(0)} + \frac{1}{2}\left(\frac{\partial^2 p^{u}_{j,f}(i)_{p_{j,k}}}{\partial p^{u}_{j,f}(i-1)\partial p^{u}_{j,f}(i-1)_{p_{j,k}}} \right) \sigma^2 p^{u}_{j,f}(i-1)_{p_{j,k}} \quad f=1,...,F_j \quad j=1,...,i-1;$$

PHASE B: For $i=K-2,...,1$ calculate $p^{d}_{j,f_j}(i-1)$ and for each uncertain parameter using the following equation and replacing $E(i-1)$ with $E(i)$. Use the two-machine line analytical solution to evaluate the performance of the line.

$$\sigma^2 p^{d}_{j,f}(i-1)_{p_{j,k}} = \left(\frac{\partial p^{d}_{j,f}(i-1)}{\partial p^{d}_{j,f}(i)} \right)^2 \sigma^2 p^{d}_{j,f}(i) \quad f=1,...,F_j \quad j=i+1,...,K-1;$$

$$E[p^{d}_{j,f}(i-1)]_{p_{j,k}} = p^{d}_{j,f}(i-1)_{(0)} + \frac{1}{2}\left(\frac{\partial^2 p^{d}_{j,f}(i-1)_{p_{j,k}}}{\partial p^{d}_{j,f}(i)\partial p^{d}_{j,f}(i)_{p_{j,k}}} \right) \sigma^2 p^{d}_{j,f}(i)_{p_{j,k}} \quad f=1,...,F_j \quad j=i+1,...,K;$$

4.4.4 Approximating a linear uncertain isolated machine

Analysis of multi-stage lines with multiple failure mode machines by using the Taylor approximation approach introduced above is fast and numerical results have proved that the accuracy is good. Full numerical results with accuracy and computation times are reported in the next chapter. In some cases where the number of machines in a line is high and each machine features multiple failure modes this might also require considerable time. So an equivalent machine with single failure mode that substitutes the multiple failure mode machines can be a good approximation for providing faster solutions.

The objective of this analysis is to model an isolated machine with multiple failure modes with uncertainty into an equivalent isolated machine with single failure mode with uncertainty. This analysis is important for two reasons. The first advantage is when the uncertainty measurement is performed directly on the average throughput. In this case the equivalent single machine it can be used to an opportune modeling of the uncertainty on the input parameters uncertainty, i.e., on the probability of failure and repair.

The second objective is the analysis of multiple uncertain failure and repair parameters on a single machine can be undesirable for performance evaluation on multistage lines. There are two main drawbacks particularly on using these multiple uncertain parameters in a multistage line evaluation which are on the same machine. The first one is; the interactions of the parameters on a single machine are more significant compared to parameters at two different machines decoupled by a buffer. This interaction can be measured at an isolated machine level and an equivalent linearized uncertainty is computed that can be easily handled with first order Taylor approximation. The second one is related to the required computational effort compared to the same analysis. For instance, if the joint distribution approach for multiple parameters is used so that interaction of parameters through performance function can be analyzed this takes longer computational time. Especially when complex functions involved this time is really high. Finding a linearized equivalent of an isolated machine simplifies the multiple parameters at a single machine level with relative easier to work with the multistage system.

The uncertainty of an isolated efficiency of a machine can be obtained by the different techniques that are discussed in sections 4.2 such as discretization, using multiple integrals or direction estimation on the average throughput. The resulting uncertainty in the throughput for the isolated throughput is denoted as $V[e]$ and can be computed once the equivalent p_{eq} and r_{eq} are calculated with isolated machine equivalency formula. The corresponding equivalent uncertainties can be computed with the following assumptions.

Using the first order Taylor approximation the uncertainty in the isolated efficiency of a single machine is expressed as:

$$V[e] = \left(\frac{\partial e}{\partial p_{eq}}\right)^2 \sigma^2 p_{eq} + \left(\frac{\partial e}{\partial r_{eq}}\right)^2 \sigma^2 r_{eq} \qquad (4.149)$$

This equation equates the overall uncertainty of the isolated machine that is approximated by the first order Taylor series should be equivalent to the total isolated throughput uncertainty obtained from other techniques.

Imposing a second condition is since the approximated uncertain machine is linearized with the first order approximation the deviations due to the second order and higher order derivatives of interacting parameters should add to 0.

From this assumption it can be written that:

$$\frac{1}{2}\frac{\partial^2 e}{\partial {p_{eq}}^2}\sigma^2 p_{eq} + \frac{1}{2}\frac{\partial^2 e}{\partial {r_{eq}}^2}\sigma^2 r_{eq} = 0 \qquad (4.150)$$

Solving equation [1] and [2] and substituting the known parameters of the mean *p*, *r* and the total uncertainty of the isolated machine, with the respective partial derivatives, the equivalent uncertainty in p and r can be computed.

$$\frac{\partial^2 e}{\partial {p_{eq}}^2} = \frac{2 r_{eq}}{\left(p_{eq} + r_{eq}\right)^3}$$

$$\frac{\partial^2 e}{\partial r_{eq}{}^2} = \frac{2r_{eq}}{\left(p_{eq}+r_{eq}\right)^3} - \frac{2}{\left(p_{eq}+r_{eq}\right)^2}$$

$$\left(\frac{\partial e}{\partial p_{eq}}\right)^2 = \frac{r_{eq}{}^2}{\left(p_{eq}+r_{eq}\right)^4}$$

$$\left(\frac{\partial e}{\partial r_{eq}}\right)^2 = \left(\frac{1}{p_{eq}+r_{eq}} - \frac{r_{eq}}{(p_{eq}+r_{eq})^2}\right)^2 \qquad (4.151)$$

Therefore the equivalent uncertainty for p_{eq} is

$$\sigma^2 p_{eq} = \frac{V[e]}{\dfrac{r_{eq}{}^2}{\left(p_{eq}+r_{eq}\right)^4} - \dfrac{\left(\dfrac{1}{p_{eq}+r_{eq}} - \dfrac{r_{eq}}{(p_{eq}+r_{eq})^2}\right)^2 \times \dfrac{2r_{eq}}{\left(p_{eq}+r_{eq}\right)^3}}{\dfrac{2r_{eq}}{\left(p_{eq}+r_{eq}\right)^3} - \dfrac{2}{\left(p_{eq}+r_{eq}\right)^2}}} \qquad (4.152)$$

Therefore the equivalent uncertainty for r_{eq} is:

$$\sigma^2 r_{eq} = \frac{V[e]}{\left(\dfrac{1}{p_{eq}+r_{eq}} - \dfrac{r_{eq}}{(p_{eq}+r_{eq})^2}\right)^2 - \dfrac{\dfrac{r_{eq}{}^2}{\left(p_{eq}+r_{eq}\right)^4} \times \left(\dfrac{2r_{eq}}{\left(p_{eq}+r_{eq}\right)^3} - \dfrac{2}{\left(p_{eq}+r_{eq}\right)^2}\right)}{\dfrac{2r_{eq}}{\left(p_{eq}+r_{eq}\right)^3}}} \qquad (4.153)$$

The new equivalent machine can be substituted with the two new equivalent uncertainties $\sigma^2 p_{eq}$ and $\sigma^2 r_{eq}$ with the expected value p_{eq} and r_{eq} in the multistage line analysis. Numerical tests in the next chapter for the model that uses the multiple failure mode machine against the approximated single failure mode machines show a good accuracy on V[*TH*].

Chapter Five

5. Validation and Accuracy Testing

The first sections of this chapter will demonstrate a summary of numerical experiments conducted to test the numerical accuracy of each technique that is proposed in chapter 4. The rest part of the sections report important summarized behaviors of systems including two machine lines and long multistage systems which are studied by using the proposed techniques. Furthermore qualitative and quantitative explanations on the exhibited behaviors with respect to input parameters and system configurations are provided.

The accuracy of the proposed methods is tested in comparison with exact analytical methods on simpler systems including isolated machines and two machine lines with unique failure modes using fewer uncertain parameters. After a satisfactory precision on the methods that are applicable for simpler systems is achieved some of the techniques are used as a reference to measure the accuracy of other techniques. Some of the methods that have very close precisions and those having a strong mathematical similarity, such as discretization and linearizing are not reported separately considering they provide the same results if applied on the same conditions. Therefore the first few experiments are targeted comparing the accuracy of discretization with exact analytical methods on single machines and two machine lines. A Monte Carlo method with bigger sample sizes to achieve high precision is also used to show the performance in accuracy relative to the proposed methods. The three proposed alternative discretization methods are compared with each other. In later sections, which show the evaluation of longer lines, the Monte Carlo technique and discretization techniques are used as a reference to measure the accuracy of the proposed Taylor approximation method.

Cases for the accuracy testing are generated with an experimental design so that the ranges of parameters characteristic to these systems are chosen to in the range where reasonably designed manufacturing systems are expected to operate. Once the boundary for the system parameter ranges is defined for failure and repair probabilities and intermediate buffers, cases are randomly generated for each experiment with in this boundary. The following are the ranges of parameters that are used in the single machine studies which are discussed briefly. The next few sections show the accuracy of methods by comparison measured for three alternative approaches on isolated machines.

The input parameters are randomly generated within the chosen bounds of ranges for each parameter, while a systematic selection of these values is also done to make the test on a wider range of scenarios. Sets of tests are performed each on 50 test cases for each experimental set. In some instances up to 200 experiments are conducted where increased statistical significance is required. Expected values of operational failure probabilities are varied from 0.0001 to 0.2 and corresponding uncertainty in estimation is varied from 10^{-3} to 10^{-7}. Expected value for repair probabilities are varied from 0.01 to 0.3 for operational failures and the uncertainty in variance is varied from 10^{-3} to 10^{-6}. The inputs are simulated failure and repair data on the Time to Failure (TTF) and Time to Repair (TTR), such that the first two moments of the estimates made from these observations fall in the experimental ranges outlined above.

5.1 Isolated machine Experiments

Three sets of experiments are conducted in this section. In the first two experiments either the failure probability p or the repair probability r is uncertain. The next experiments are cases where both the failure probability and repair probabilities are uncertain. Three alternative techniques are compared with exact analytical solution using integration. The exact integration method is used as a reference for the evaluation of the expected value of the isolated efficiency and the uncertainty in variance. These results are compared with results obtained from MonteCarlo simulation and a discretization of a density function and techniques introduced in section 4.2.2. For the Monte Carlo experiments, data are sampled using bootstrapping, with a bootstrap size of 100000 and 10 replicates to achieve a maximum half width of 0.0002 on the average throughput and 0.000005 on the variance of the average throughput. Even though the number of partitions can be varied depending on the distribution of the uncertain parameters and the desired degree of accuracy in the approximate method, the number of partitions in all experiments is fixed to be 30 for the sake of consistency.

Errors for the Monte Carlo (MC) approach and for the discretization (DT) with single uncertainty are evaluated and reported as: Errors in Monte Carlo,

$$\varepsilon_{MC} = \frac{\theta_{MC} - \theta_{Exact}}{\theta_{Exact}} \times 100\% \tag{5.1}$$

For discretization,

$$\varepsilon_{DT} = \frac{\theta_{DT} - \theta_{Exact}}{\theta_{Exact}} \times 100\% \tag{5.2}$$

are measured for both the average isolated efficiency and uncertainty in variance.

5.1.1 Uncertain p and precisely known r

In these experiments the repair probability r is assumed to be precisely known and the failure probability p is estimated from randomly generated observations of geometrically distributed TTF within as mentioned in section 4.1.

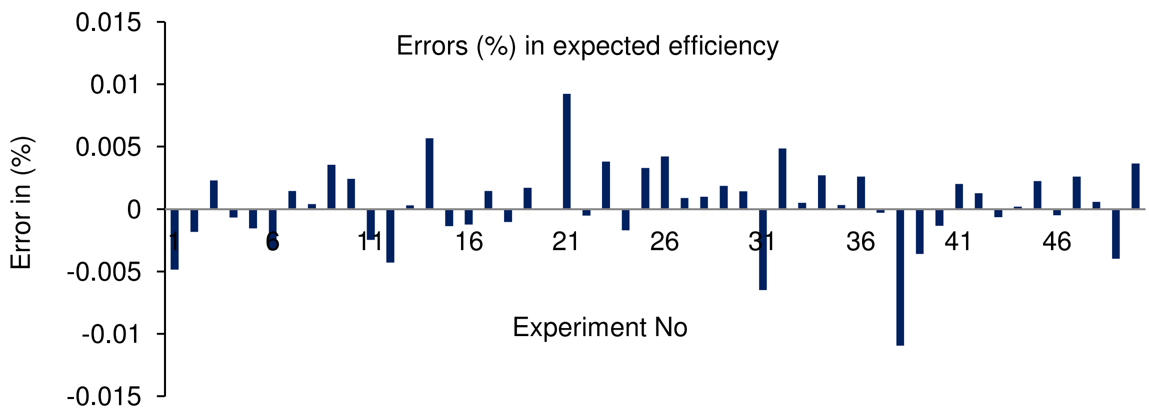


Figure 5:1 Errors on E[e] by Monte Carlo

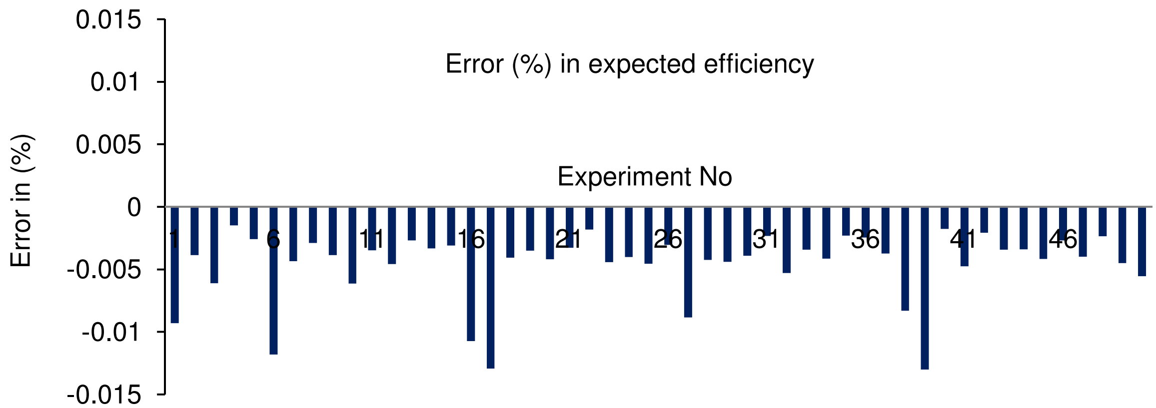


Figure 5:2 Errors on E[e] by discretization

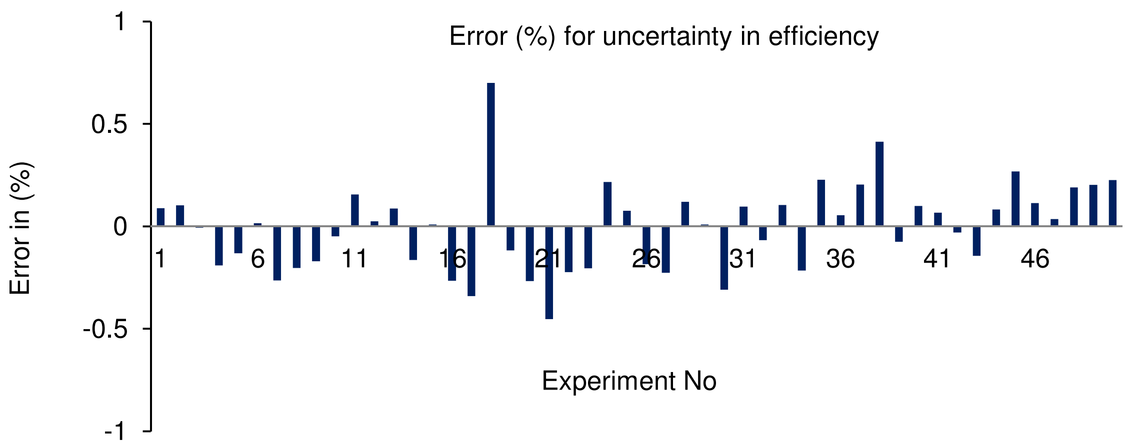


Figure 5:3 Errors on V[e] by Monte Carlo

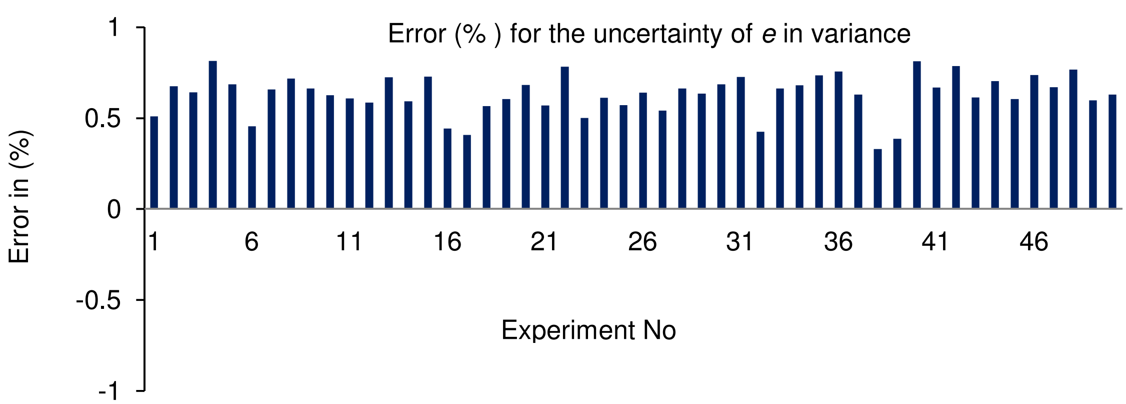


Figure 5:4 Errors on V[e] by discretization

From the above 100 experiments on isolated machines with *p* estimated from randomly generated TTF to evaluate both the expected efficiency and the uncertainty in isolated efficiency the summary is as follows. Using the Monte Carlo experiment with 100,000 runs and 10 replicated, i.e., 10^6 experiments the maximum error on the average efficiency (e) is 0.0092% and 0.70% on the uncertainty in variance. Using the discretization method with 30 runs for each experiment the maximum error in the expected value of efficiency is 0.0130% and 0.81% on the uncertainty in variance of e.

It can be observed that the Monte Carlo errors are centered while in the case of the discretization method the errors are consistent underestimation on the expected value of e. The errors on the uncertainty with discretization are consistent overestimations. This is due to the fact that the discretization technique uses trapezoidal approximation of the partitioned intervals which in most cases lie above the distribution curve, causing the minimal overestimation of the variance. But as it can be seen this deviations are minimal even if consistent, therefore with 30 partitions the level of accuracy is assumed satisfactory.

	Monte Carlo		Discretization	
	E[e]	V[e]	E[e]	V[e]
Average Error (%)	0.00032	-0.00654	-0.00462	0.63
Maximum Error (%)	0.00921	0.70072	0.01299	0.81

Table 5:1 Summary of errors using Monte Carlo and discretization

5.1.2 Uncertain r precisely known p

In these experiments the repair probabilities are estimated from randomly generated observations of geometrically distributed TTR while the failure probabilities are assumed to be known precisely. Similarly to the above experiments the exact analytical method is used as reference in measuring the errors.

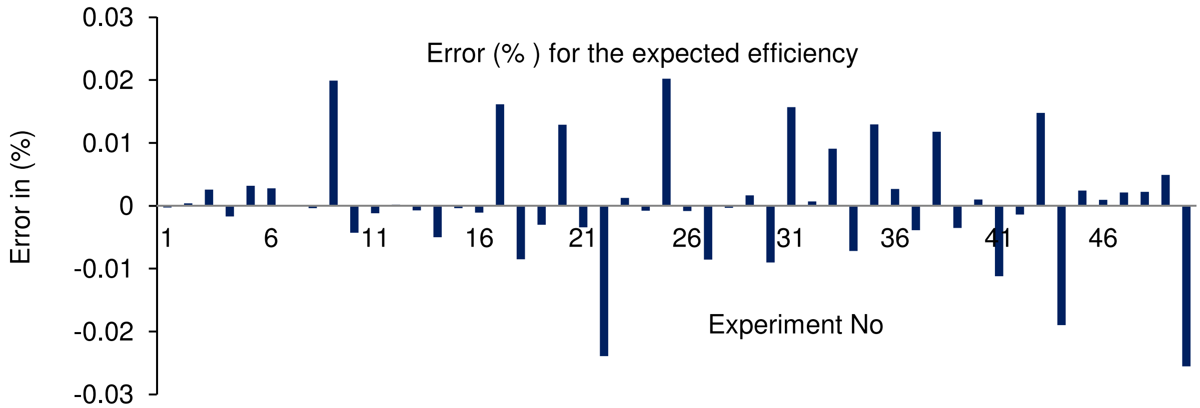


Figure 5:5 Errors on E[e] using Monte Carlo

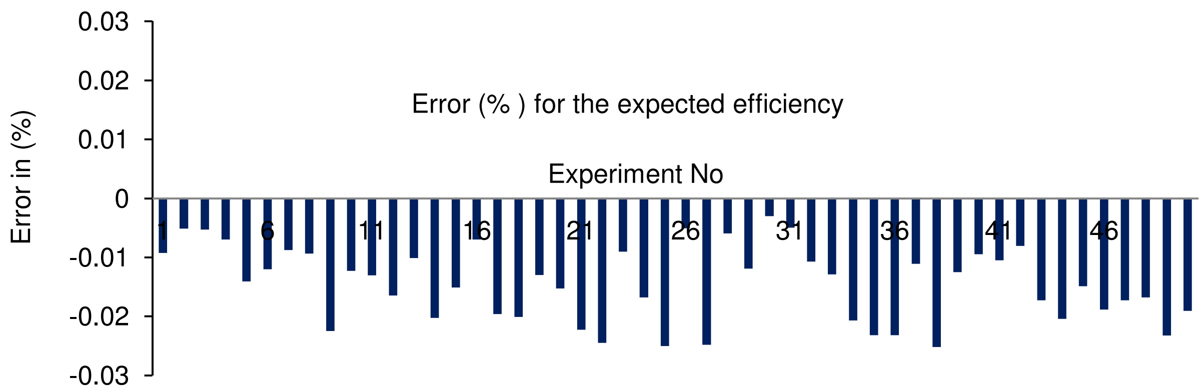


Figure 5:6 Errors on E[e] using Monte Carlo

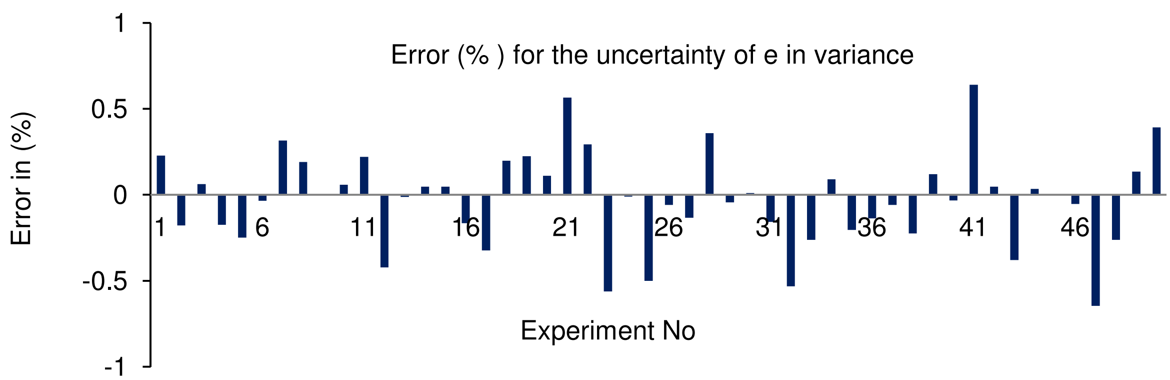


Figure 5:7 Errors on V[e] by Monte Carlo

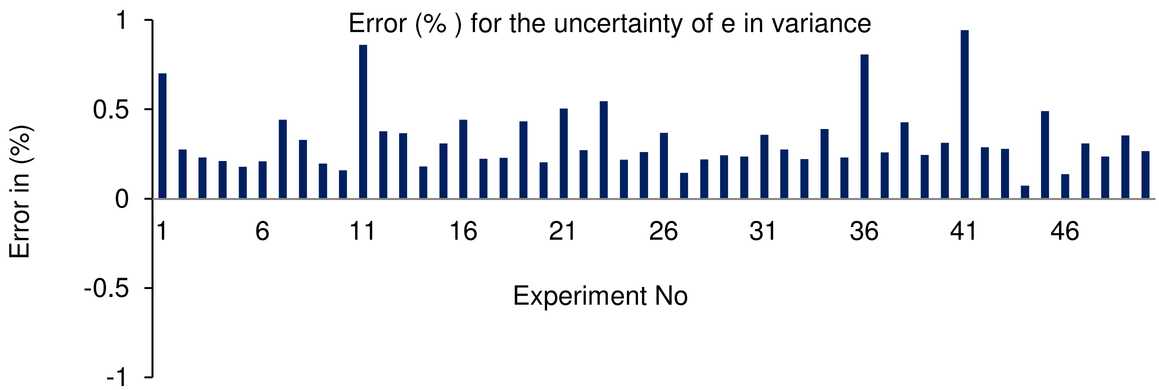


Figure 5:8 Errors on V[e] by Monte Carlo

Additional 50 experiments on isolated machines with *r* estimated from randomly generated TTR to evaluate both the expected efficiency and the uncertainty in isolated efficiency the summary is as follows. The settings for Monte Carlo and discretization technique are kept the same as in the previous experiment. The maximum error on the average efficiency (e) is 0.02556% and 0.65% on the uncertainty in variance. Using the discretization method the maximum error in the expected value of efficiency is 0.0251% and 0.94% on the uncertainty in variance of e.

Similar under estimation and overestimation that are characteristics of the trapezoidal approximation can be observed. The slight increase in the errors compared to the precious case where *p* is uncertain is due to the input variance in the case of *r* is greater than *p*. For an equivalent precision with increasing variance more partitions are required. In general the level of accuracy in both cases is accurate and no errors were observed more than 1% even for the uncertainty in e.

	Monte Carlo		Discretization	
	E[e]	V[e]	E[e]	V[e]
Average Error (%)	0.00033	-0.02882	-0.01448	0.33
Maximum Error (%)	0.02556	0.64502	0.02510	0.94

Table 5:2 Summary of errors using Monte Carlo and discretization

5.1.3 Uncertain p and r

The next experiments measure the accuracy of the discretization and Monte Carlo method under a Bivariate case, when failure probability p and repair probabilities r uncertain estimates. The estimation on *p* and *r* is done on randomly generated observations of geometrically distributed TTF and TTR in the range of the experimental plan explained in section 5.1. Monte Carlo sample size and partitions numbers are kept the same as in the previous experiments.

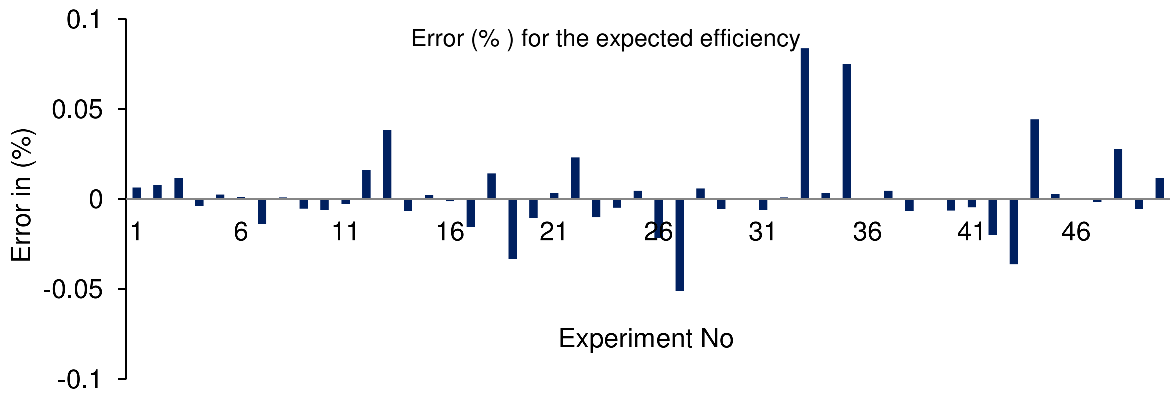


Figure 5:9 Errors on E[e] using Monte Carlo

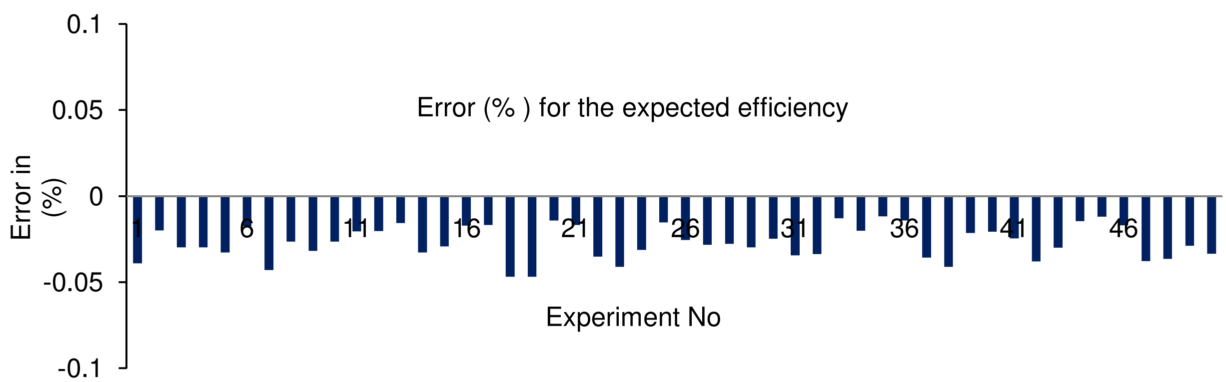


Figure 5:10 Errors on E[e] using discretization

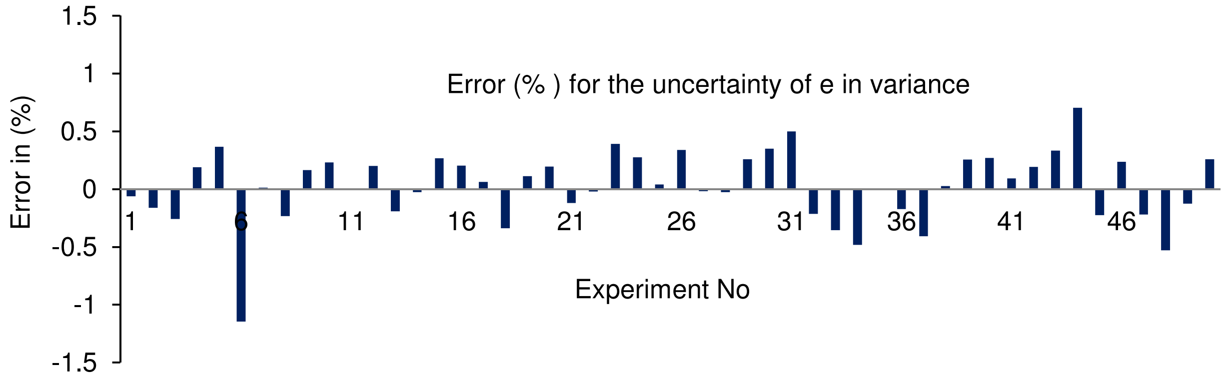


Figure 5:11 Errors on V[e] by Monte Carlo

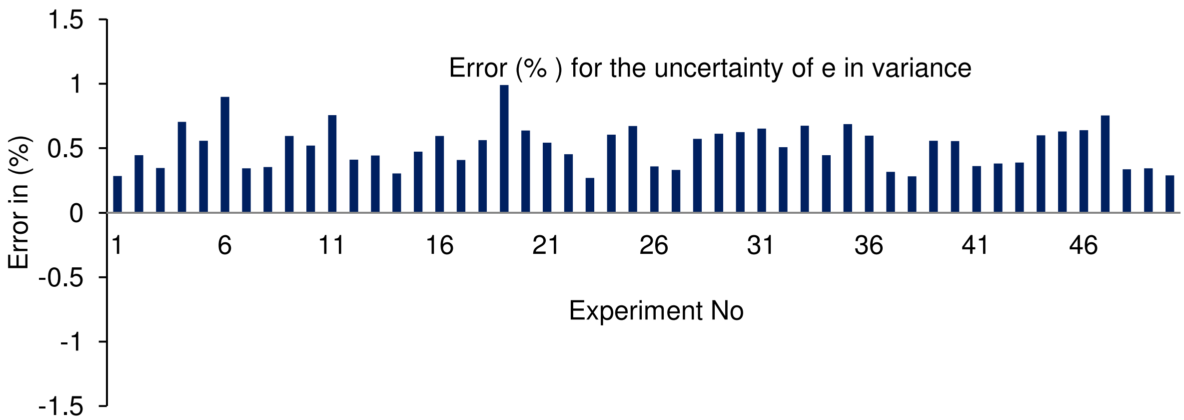


Figure 5:12 Errors on V[e]of by discretization

50 experiments on isolated machines with *p* and *r* estimated from randomly generated TTF and TTR evaluate both the expected efficiency and the uncertainty in isolated efficiency the summary is as follows. The maximum error on the average efficiency (e) using Monte Carlo method is 0.08372% and 1.14% on the uncertainty in variance. Using the discretization method the maximum error in the expected value of efficiency is 0.04689% and 0.990% on the uncertainty in variance of e.

Similar under estimation and overestimation that are characteristics of the trapezoidal approximation can be observed. The slight increase in the errors compared to the precious case where *p* is uncertain is due to the input variance in the case of *r* is greater than *p*. For an equivalent precision with increasing variance more partitions are required. In general the level of accuracy in both cases is accurate and only one case in the Monte Carlo is observed with more than 1% for the uncertainty in e.

	Monte Carlo		Discretization	
	E[e]	V[e]	E[e]	V[e]
Average Error (%)	0.00033	-0.02882	-0.01448	0.33
Maximum Error (%)	0.02556	0.64502	0.02510	0.94

Table 5:3 Summary of errors using Monte Carlo and discretization method

5.2 Two machine lines

Experiments to measure the numerical accuracy of three alternative discretization methods and Taylor approximation technique are also performed using two machine lines. The power of the Monte Carlo method is increased so that it can be used as a reference for the multiple uncertainties, where the application of exact approach is more complex. For the newly adjusted Monte Carlo sampling plan the accuracy is measured so that the Monte Carlo method can be used to measure the accuracy of the three techniques under multivariate cases.

Individual machine parameters of the cases generated for the two machine lines are the same as the ones used in the case of single machine experiments. Two additional constraints besides the individual machine parameters are added for the two machine single buffer line. Firstly, the buffer capacity is varied from 3 to 50. The absolute difference between the isolated efficiency of the upstream and downstream machine is varied from 0 to 0.5, with minimum isolated efficiency of 0.25 and maximum efficiency of 0.99.

In the next experiments involving two machine lines, the main interest is to evaluate multiple uncertainties with two or more uncertain parameters. Only in the case of single uncertainty an exact analytical approach is carried out to measure the power of the Monte Carlo Method. Then Monte Carlo is used as a reference to compare the accuracy of the joint distribution approach which is validated to be sufficiently accurate in the case of isolated machine is used. In this case the size of boot straps is increased to 1,000,000 with 10 replications to increase the power of the Monte Carlo with a certainty of half width 0.000003.

To demonstrate the comparative updated power of the Monte Carlo method the same scale is used in both of the graphs as shown below.

5.2.1 Single uncertainty

5.2.1.1 Uncertain p_u

The Gershwin Berman Model for two machine line single buffer model is used for the evaluation of the average throughput using precise parameters. Only p_u is assumed to be uncertain and the remaining parameters r_u, p_d and r_d are assumed to be precisely known.

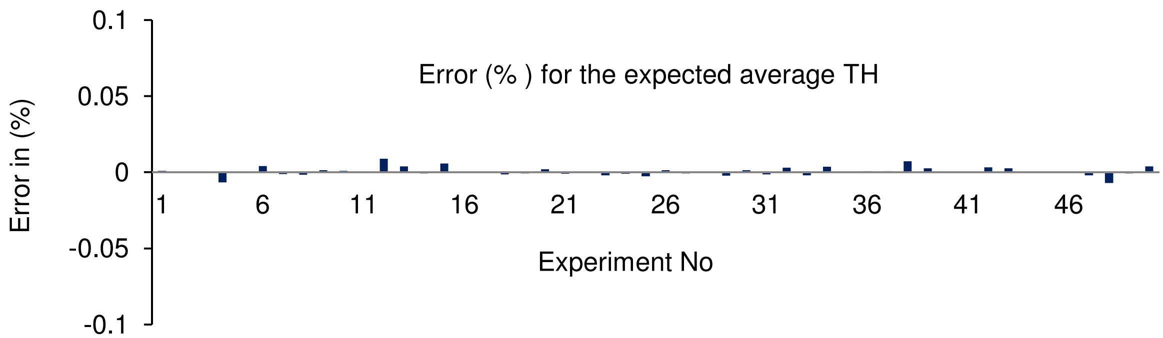


Figure 5:13 Errors on E[TH] by Monte Carlo

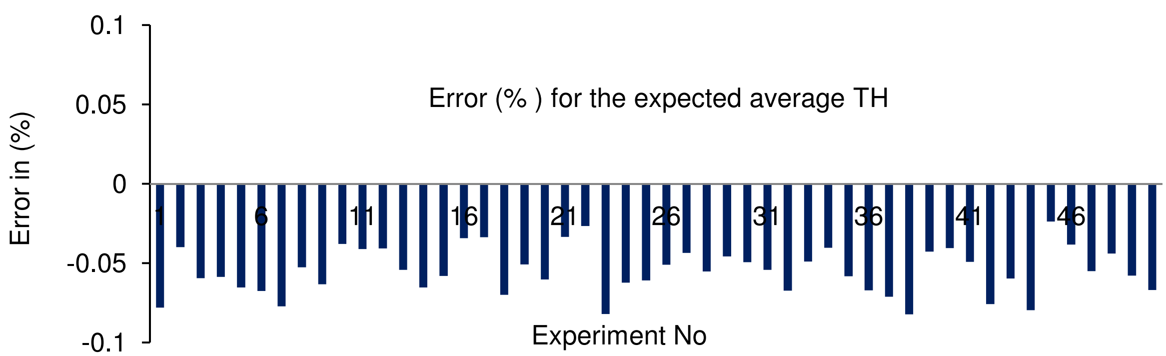


Figure 5:14 Errors on E[TH] by discretization

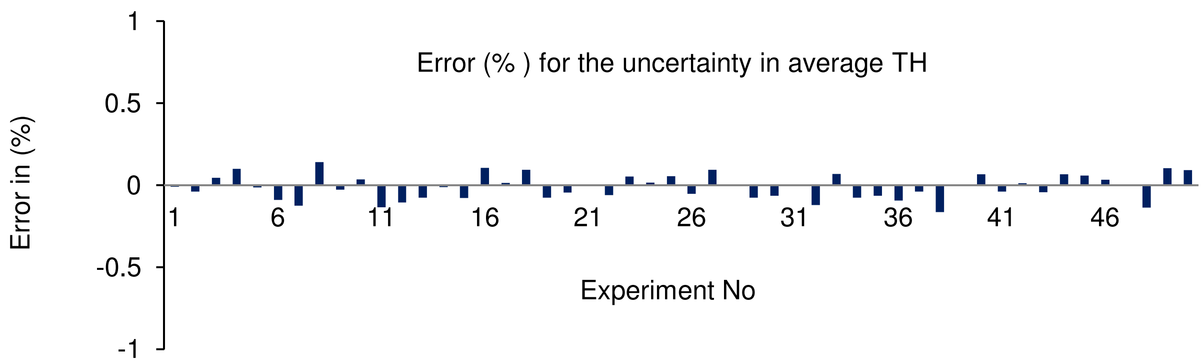


Figure 5:15 Errors in V[TH] by Monte Carlo

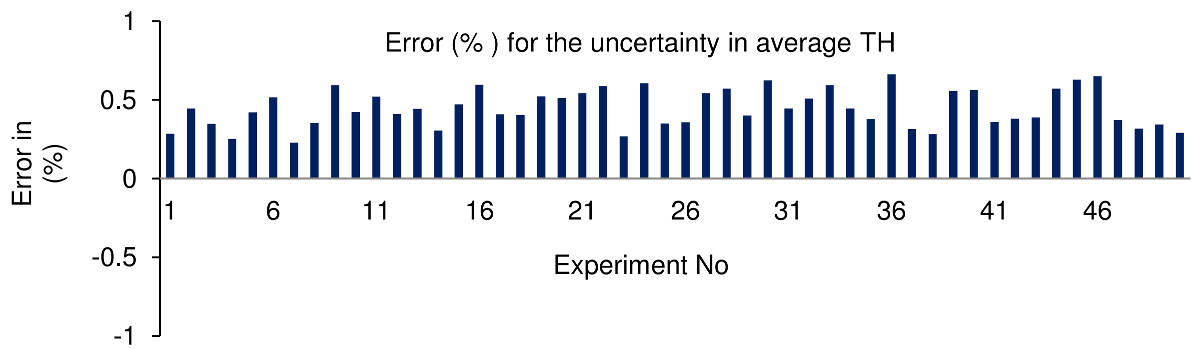


Figure 5:16 Errors in V[TH] by Monte Carlo

The Mote Carlo approach has demonstrated a sufficient power to be used as a reference to measure the errors in the approximate discretization technique used in the upcoming experiments. The maximum error reported for the expected value of the average TH E[TH] is 0.00891% and the error for the uncertainty V[TH] is 0.1652%. A repeated experiment for the isolated machine case with two uncertain parameters, have shown for two uncertainties the maximum E[e] is 0.00942 and maximum error in the uncertainty V[e] is 0.232%. Therefore the new experimental errors are computed with relative to the Monte Carlo results as

$$\varepsilon_{MC} = \frac{\theta_{DT} - \theta_{MC}}{\theta_{MC}} \times 100\% \tag{5.3}$$

5.2.2 Joint distribution discretization

In the next experiments involving multiple uncertainties the joint distribution approach introduced in section 4.3.1.1 is used to evaluate the average throughput (TH) of the two machine lines. The first two experiments show the evaluation errors when there are two uncertain parameters. The errors for these experiments are evaluated as in (5.3). The two uncertainties are shown when in one case they are located on one of the machines. On another case the uncertainties are made to be one on each of the machines. At last four uncertainties are considered when all the two machine line parameters are considered to be uncertain.

5.2.2.1 Uncertain p_u and r_u

For these experiments the uncertainties are located on the same machine. The results are reported only for the upstream machine. The joint distribution approach is performed with the evaluation of 900 individual evaluation with precisely know parameters.

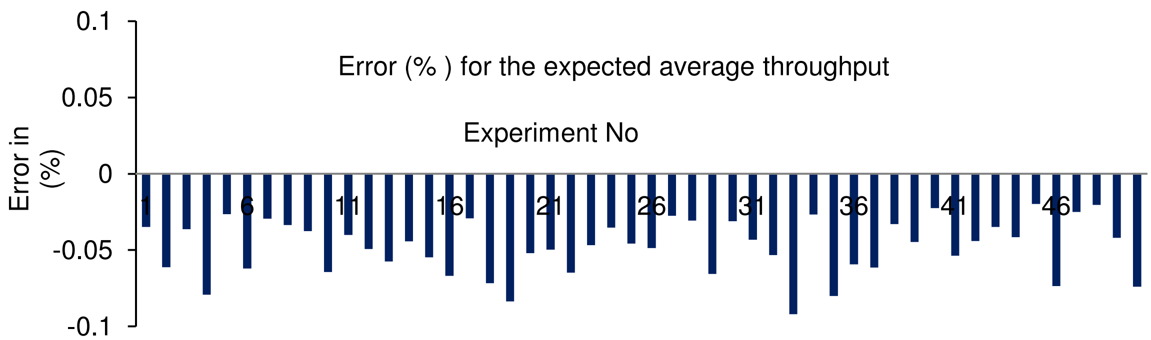


Figure 5:17 Errors on the E[TH] by joint distribution

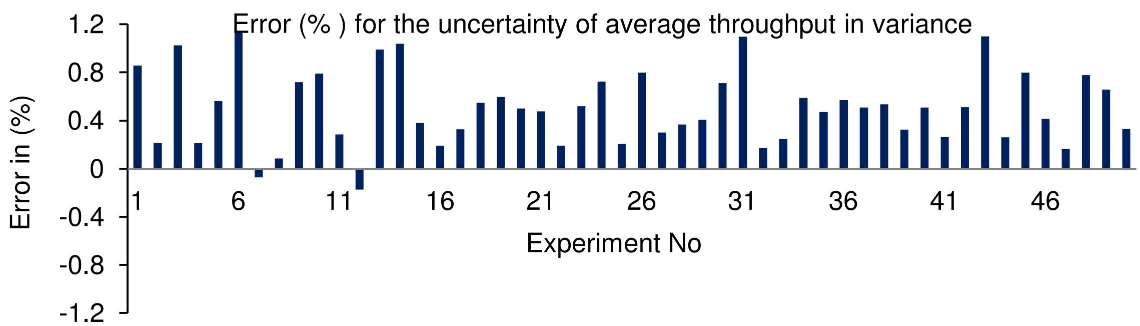


Figure 5:18 Errors on the V[TH] by joint distribution

Summary of the errors for these experiments on the maximum and average errors on the average throughput are the following. The maximum error on E [TH] is 0.0921% and 1.10% on the uncertainty V[TH]. The average error on E [TH] is 0.024% and 0.554% on the uncertainty V[TH]. Errors that are characteristic to the trapezoidal approximation of the partitions, i.e., the overestimation of the variance are also observed here.

5.2.2.2 Uncertain p_u and p_d

The two uncertainties are chosen to be on each of the two machines that compose the two machine lines. One uncertainty is on the failure probability p_u of the upstream machine and the other one is on the failure probability p_d of the downstream machine. With the same experimental settings the following results are reported on the errors of the TH.

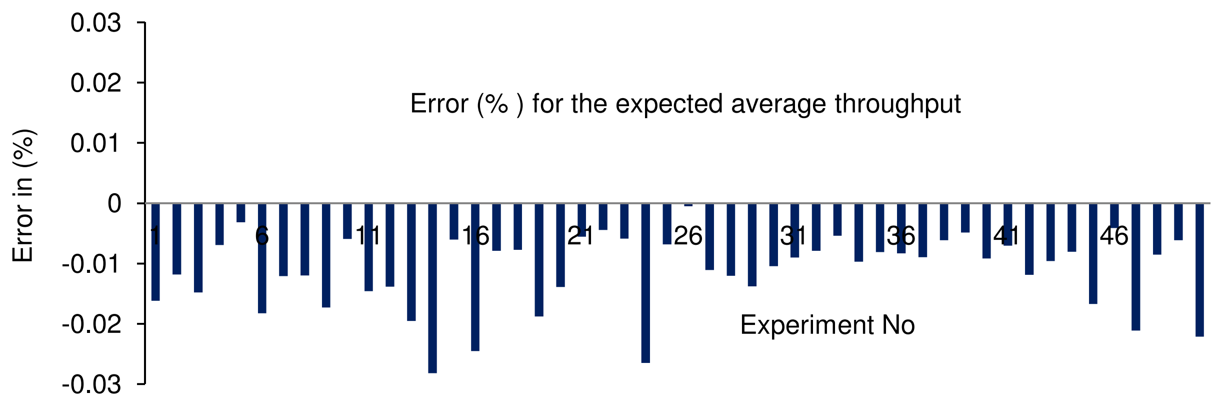


Figure 5:19 Errors on the E[TH] by joint distribution

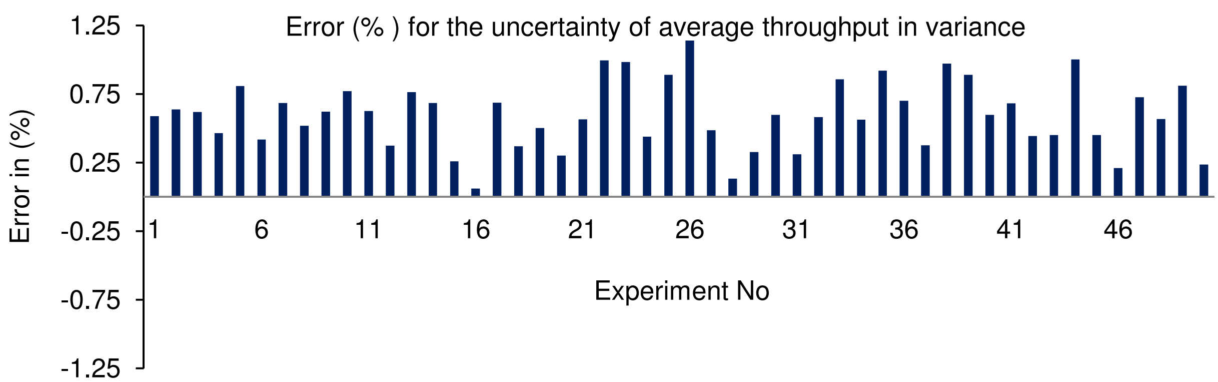


Figure 5:20 Errors on V[TH] by joint distribution

Other combinations on the choice of uncertain parameters are experimented. Since the results are not significantly different from these results only the two cases are reported as a general summary of the performance of the accuracy of the joint distribution approach. The errors for these experiments on the maximum and average errors on the average throughput are the following. The maximum error on E [TH] is 0.02817% and 1.14% on the uncertainty V[TH]. The average error on E [TH] is 0.0087% and 0.361% on the uncertainty V[TH].

5.2.2.3 *Uncertain p_u, r_u, p_d and r_d*

In these experiments all the four parameters of the two machine line are considered to be uncertain. Keeping the same partition number the number of two machine line evaluations for the joint distribution approach is 810,000. Similar to the preceding experiments the errors for the joint distribution approach are evaluated with results obtained from Monte Carlo evaluations.

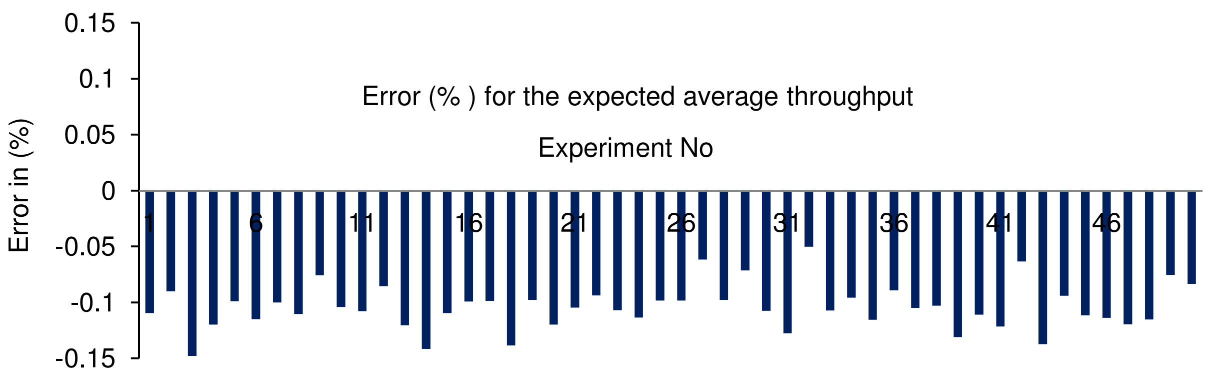


Figure 5:21 Errors on E[TH] by joint distribution

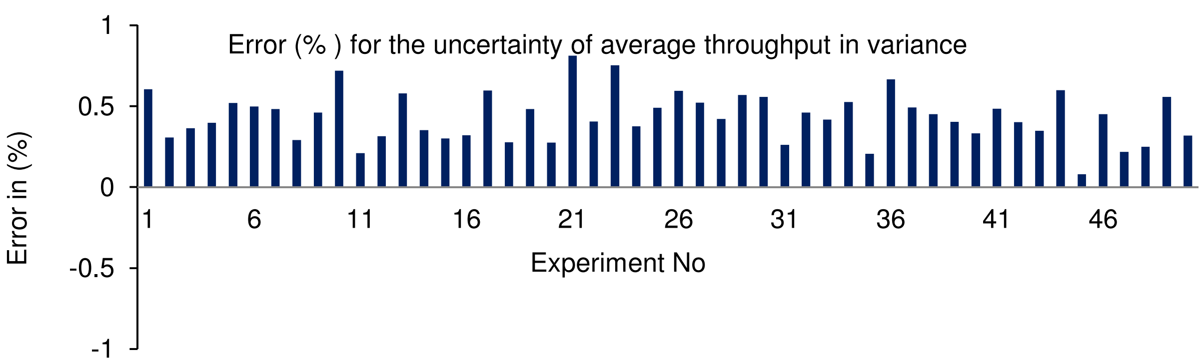


Figure 5:22 Errors on V[TH] by joint distribution

The summary of the errors for all four uncertain parameter experiments on the maximum and average errors on the average throughput are the following. The maximum error on E [TH] is 0.1479% and 0.8131% on the uncertainty V[TH]. The average error on E [TH] is 0.0526% and 0.3319% on the uncertainty V[TH]. Interestingly the errors don't show any particular increase in both the expected value and uncertainty of TH, when more uncertain parameters are considered. In addition since the errors reported are relatively quite small with respect to the objective of evaluating the uncertainty using the methods that will be presented in multistage lines, the joint distribution approach will be used as a reference in multistage lines. A fundamental motivation for using the joint distribution approach is the comparatively lower number of approximation required compared to the Monte Carlo approach, particularly in the evaluation of multistage lines where a single precise evaluation takes a considerable amount of time. So the total evaluation time for a short line such as three machine line with 10^7 evaluations takes an average of 20hrs.

5.2.3 One factor at a time approach two machine line

Same experiments are used to measure the accuracy of the one factor at a time approach. Investigation of this method is interesting, since the method requires few experiments compared to the joint distribution approach. This is particularly important when the number of parameters to be considered grows considerably. The same experiments are used to measure the accuracy. Similarly to the joint approach experiments, first two uncertainties are shown when in one case they are located on one of the machines. On another case the uncertainties are made to be one on each of the machines. Finally four uncertainties are considered when all the two machine line parameters are considered to be uncertain.

5.2.3.1 Uncertain p_u and r_u

In these experiments the uncertainties are on the parameters of the upstream machine. The results are reported only for the upstream machine. The one factor at a time approach requires 60 evaluations with precisely known parameters for an experiment. The same problem using the joint distribution needed 900 individual evaluations with precisely known parameters.

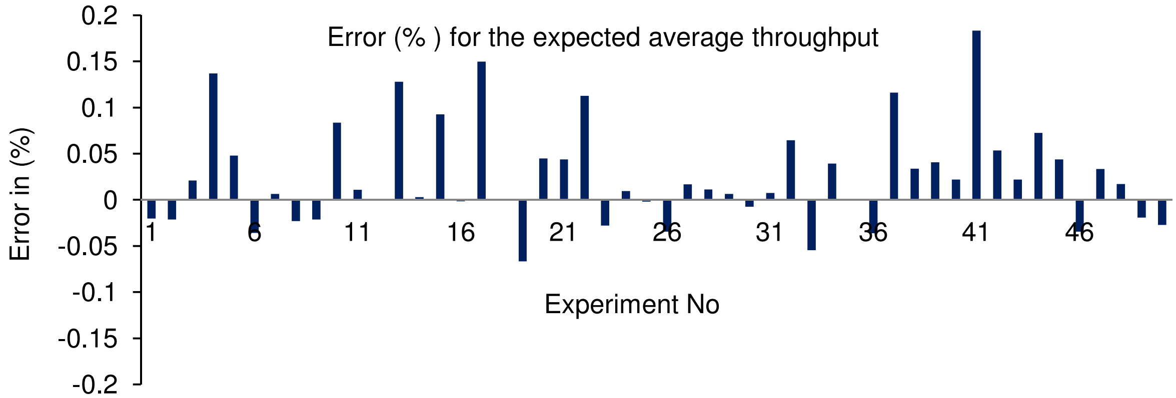


Figure 5:23 Errors on the E[TH] by one factor at a time

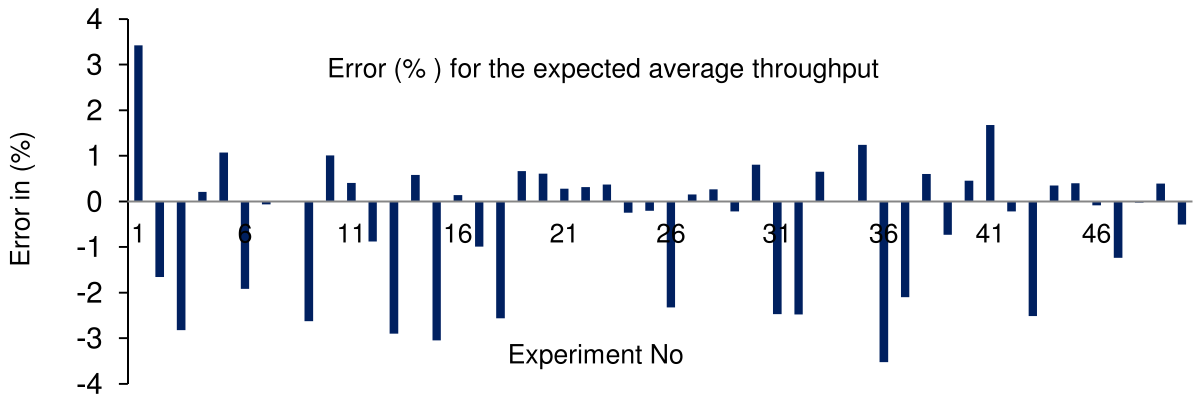


Figure 5:24 Errors on the V[TH] by one factor at a time

Summary of the maximum and average errors for these experiments are the following. The maximum error on E [TH] is 0.1832% and 3.425% on the uncertainty V[TH]. The average error on E [TH] is 0.0549% and -0.8614% on the uncertainty V[TH].

5.2.3.2 Uncertain p_u and p_d

The uncertainties are on these experiments are located on the failure probability, i.e., (p_u and p_d) of each machine comprising the two machine sing buffer line. The results are reported only for the upstream machine. As in case in the above experiment 60 evaluations with precisely known parameters are required for the solution.

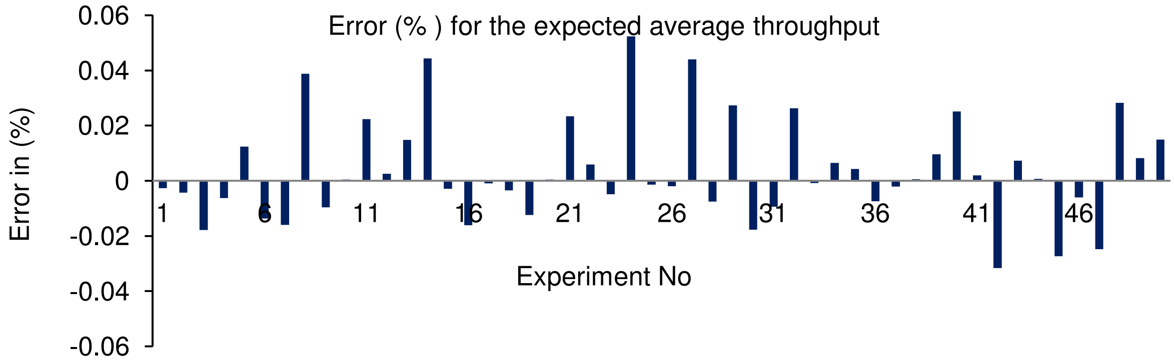


Figure 5:25 Errors on E[TH] by one factor at a time

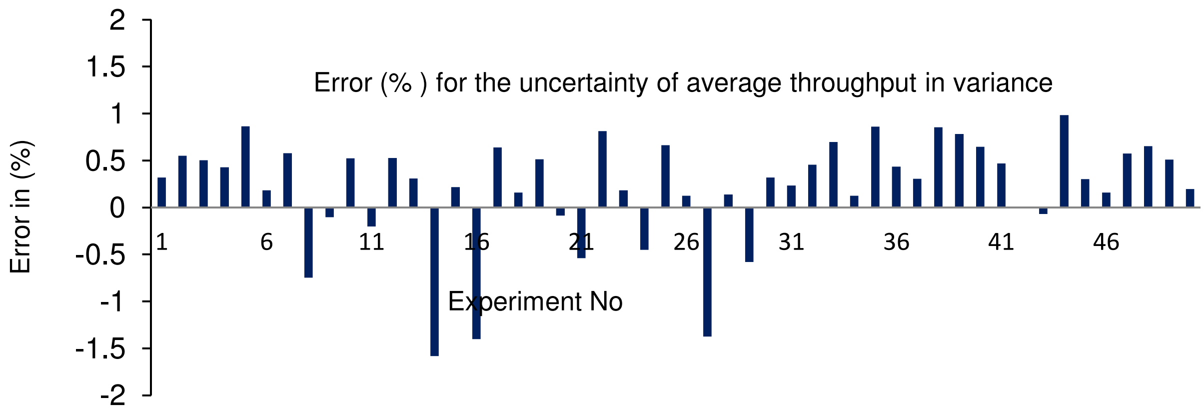


Figure 5:26 Errors on V[TH] by one factor at a time

The errors for these experiments on the maximum and average errors on the average throughput are the following. The maximum error on E [TH] is 0.05232% and 1.58% on the uncertainty V[TH]. The average error on E [TH] is 0.0087% and 0.427% on the uncertainty V[TH].

From the previous two experiments it can be seen that the second experiment has an improved accuracy when the uncertainties are located on two different machines than when they are on the same machine. In general the accuracy of the one factor at a time approach is better when the uncertain parameters are located on two different machines. This is behavior is due to the strong interaction of the two parameters when they are on the same machine, and the one factor at a time approach doesn't consider this interaction. On the other hand when the two parameters are on two machines separated by a buffer this interaction is lower providing a better accuracy for the one factor at a time approach.

5.2.3.3 Uncertain p_u, r_u, p_d and r_d

In these experiments all the parameters of the two machine line are considered to be known with uncertainty. Since there are four uncertain parameters in the evaluation the one factor at a time with 30 partition for each uncertain parameter requires 120 evaluations of precisely known parameters. The same experiments required 810,000 evaluations for the joint distribution approach. This emphasizes, even if for a reduced accuracy this approach highly reduces the computational resources required compared to the joint distribution, particularly when the number of uncertain parameters grow.

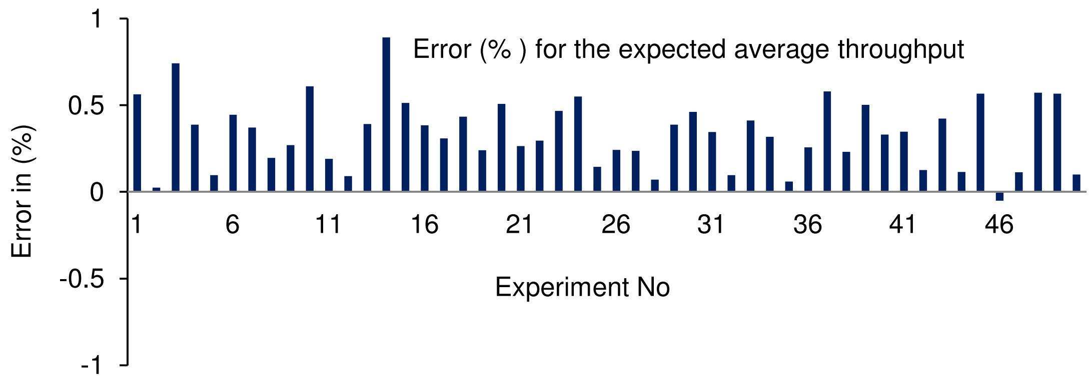


Figure 5:27 Errors on the E[TH] by one factor at a time

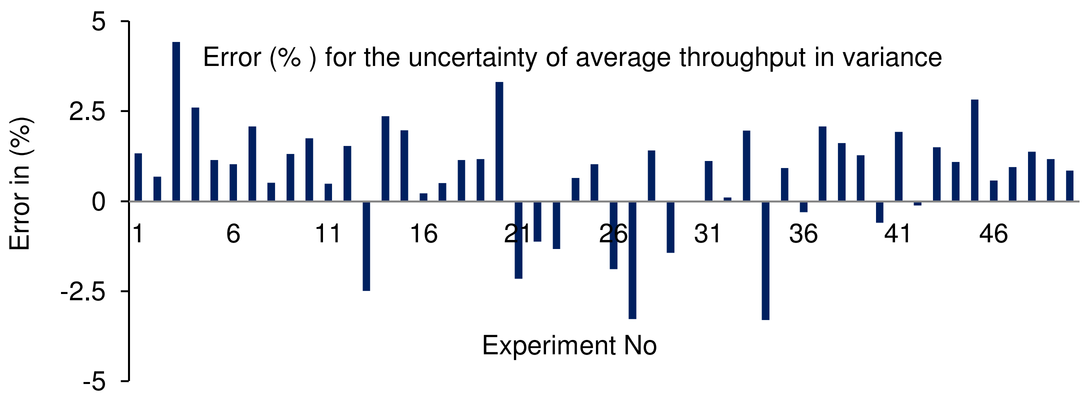


Figure 5:28 Errors on the V[TH] by one factor at a time

Summary of the maximum and average errors for these experiments are the following. The maximum error on E [TH] is 0.8927% and 4.423% on the uncertainty V[TH]. The average error on E[TH] is 0.4426% and 1.239% on the uncertainty V[TH].

5.2.4 Two factors at a time approach in a two machine line

Experiments with three or more uncertain parameters can be solved using this approach. Therefore the experiments with four uncertainties are used to measure the accuracy of the two factors at a time approach. The method requires few experiments compared to the joint distribution approach but more experiments to that of one factor at a time approach. The method can be particularly useful when the numbers of parameters are considerable such as (4 to 10) and at the same time a high accuracy is also required.

The same experiments for four uncertainties are used to measure the accuracy. The numbers of precisely known evaluations required for four uncertainties are 5520. The same problem requires 810000 for joint distribution and 120 for one factor at a time. This method can be used as a compromise between the joint approach and one factor approach when the time required for evaluation is within a reasonable amount of time.

The next experiments demonstrate the accuracy of two parameters at a time parameter at a time approach when all the reliability parameters of the two machine line are considered uncertain.

All parameters uncertain

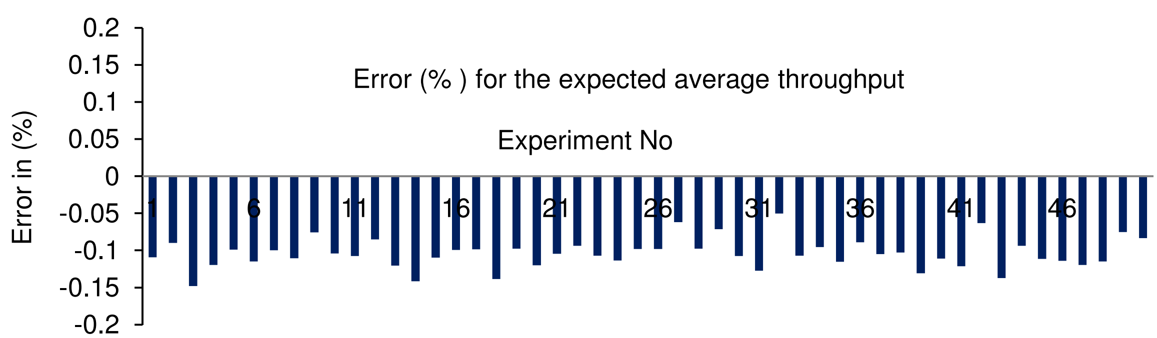


Figure 5:29 Errors on E[TH] by two factors at a time

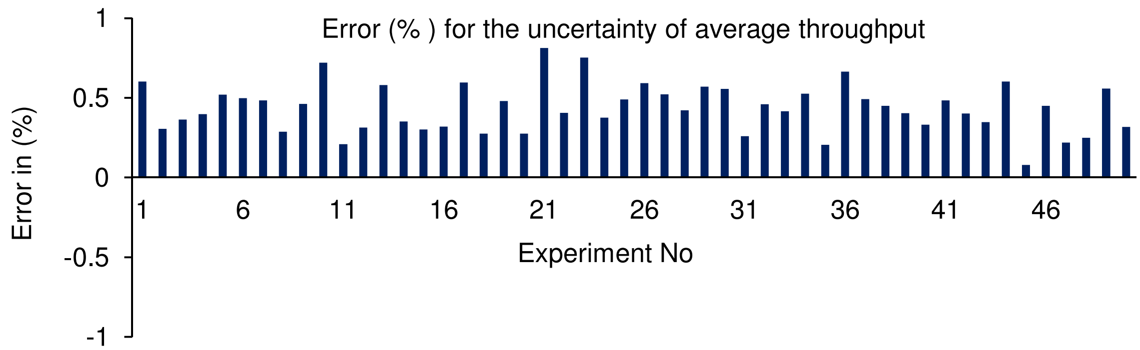


Figure 5:30 Errors on V[TH] by two factors at a time

Summary of the maximum and average errors for these experiments are the following. The maximum error on E [TH] is 0.1484% and 0.8133% on the uncertainty V[TH]. The average error on E [TH] is 0.0573% and 0.3328% on the uncertainty V[TH]. The results and the errors reported are very close to the results obtained from the joint distribution approach. Therefore it is interesting to compare relative difference between these two methods.

The percentage difference between the joint distribution (JD) approach and the two factors at a time experiments (2F) are reported for the expected average throughput and the uncertainty. The differences are computed as:

$$\Delta_{2F} = \frac{\theta_{2F} - \theta_{JD}}{\theta_{JD}} \times 100\%$$

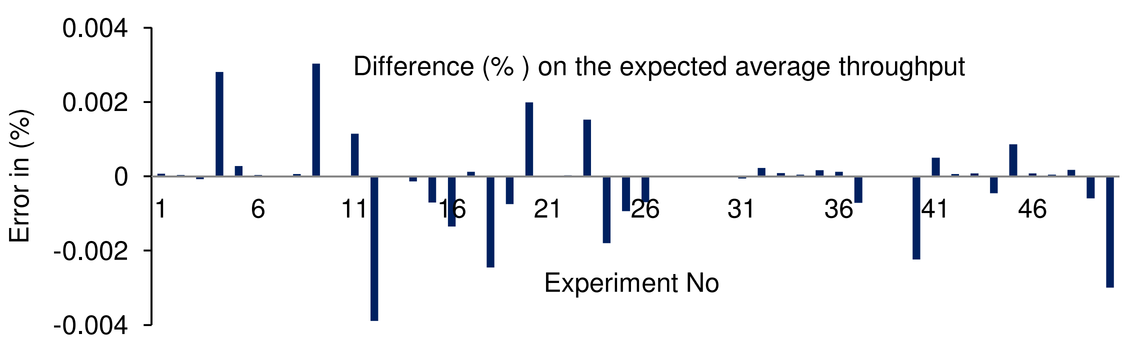


Figure 5:31 Difference on E[TH] by (JD) and (2F)

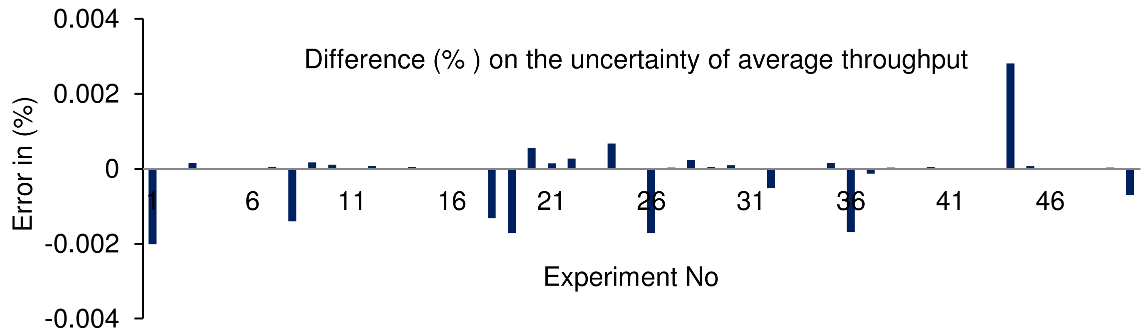


Figure 5:32 Difference on V[TH] by (JD) and (2F)

As it can be seen from the above two figures, the two factors at a time approach almost exactly replicates the results found from the joint distribution approach. The maximum percentage difference on the E[TH] is 0.00389% while 0.00281% for the uncertainty V[TH]. The average difference is 0.000244% for E[TH] and -0.000161% for V[TH]. Few additional experiments conducted with six uncertainties confirm the exact replication behavior of the two methods.

5.2.5 Taylor approximation with approximated derivatives

The following experimental results are the errors reported for the Taylor approximation method for two machine single buffer line with four uncertainties. The number of evaluations required for the single solutions are 3 evaluations per uncertainty. Therefore 12 evaluations are required for four uncertainties. This is comparatively high efficient technique when compared to the number of evaluations required for the other techniques. In the previously discussed techniques the number of evaluations required was, 120 for one factor at a time, 5520 for two factors at a time and 810,000 for the joint distribution.

The errors for this method are shown only for four parameters; a more extended accuracy test of this method will be shown for longer lines. This method is more situated for the analysis of longer lines as it requires fewer evaluations, where the remaining techniques can be difficult because of the long computational time required. The errors are computed with respect to results evaluated from Monte Carlo method.

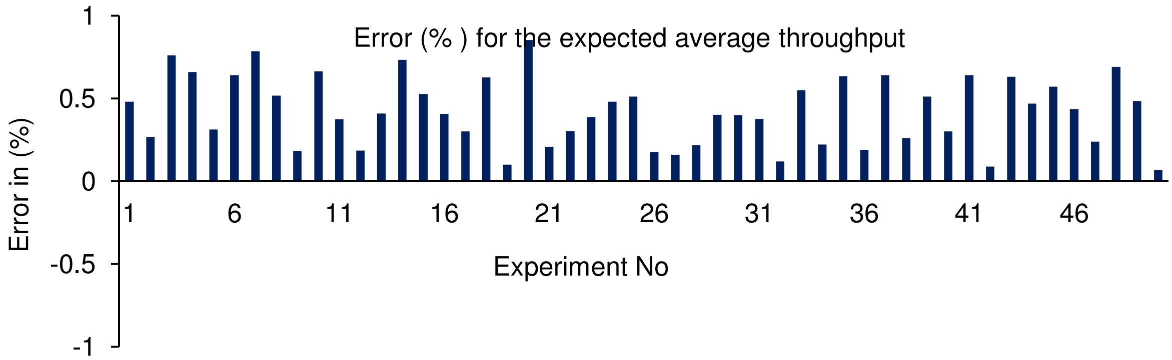


Figure 5:33 Errors on E[TH] by Taylor approximation

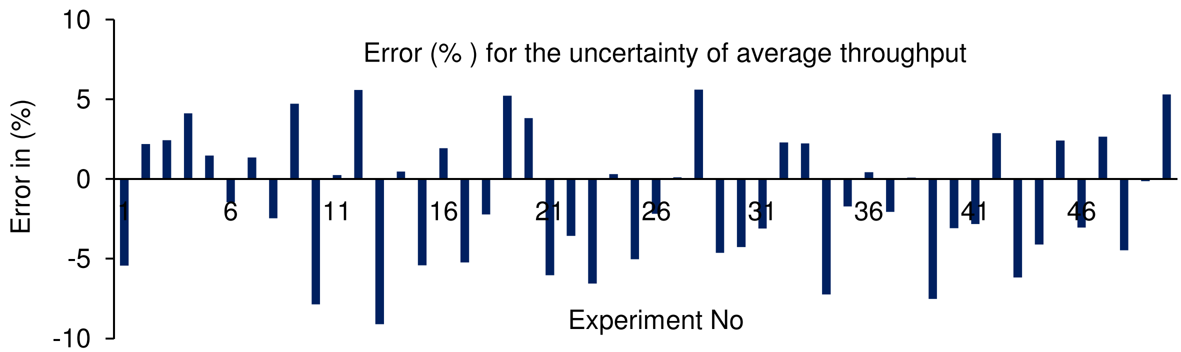


Figure 5:34 Errors on V[TH] by Taylor approximations

The maximum errors observed on the E[TH] is 0.352% while -1.816% for the uncertainty V[TH]. The average errors are on the E[TH] is 0.8522% while -9.098% for the uncertainty V[TH]. The summary for the maximum and average percentage errors on with thresholds TH are reported in Table 5:4.

	Monte Carlo	
	E[e]	V[e]
Average Error (%)	0.352	-1.816
Maximum Error (%)	0.8522	-9.098
Thresholds	(<0.5%) 66%	(<5%) 75%
Thresholds	(<1%) all	(<10%) all

Table 5:4 Summary of errors by Taylor approximation

5.3 Long multistage lines

Accuracy tests are carried out on multi-stage lines from 3 to 10 machine lines. The chosen reference method in the case of longer lines is the two factors at a time approach. The method is selected because for the evaluation of longer a line until a good convergence is achieved requires considerable amount of time making the application of Monte Carlo or joint distribution approach difficult. Moreover the two parameters at a time approach is shown to approximately replicate the results achieved by joint density, with smaller number of runs. For brevity results only for five machine line and ten machine lines with uncertain parameters are reported.

The experimental value ranges are used for generating cases for the accuracy testing of the long lines are the same as the ones used in the two machine lines. Buffer capacities are varied from 3 to 50. The absolute difference between the isolated efficiency of the upstream and downstream machine is varied from 0 to 0.5, with minimum isolated efficiency of 0.25 and maximum efficiency of 0.99.

5.3.1 Five machine line with five uncertainties

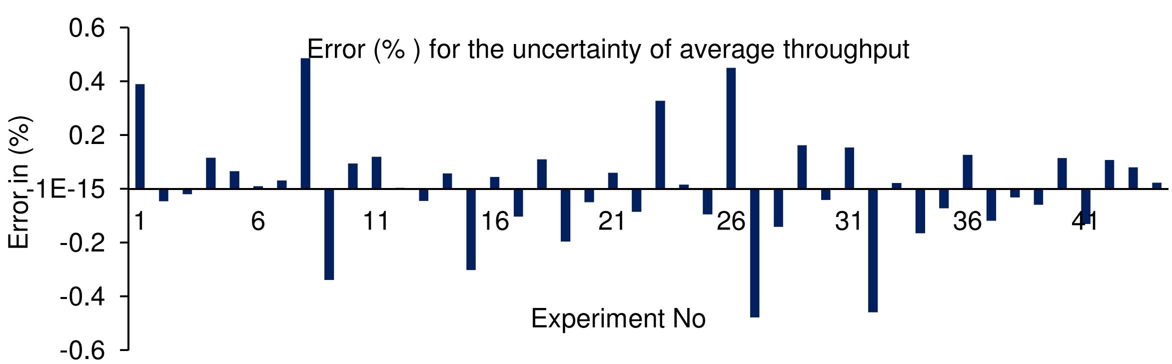


Figure 5:35 Errors on the E[TH] by Taylor approximation

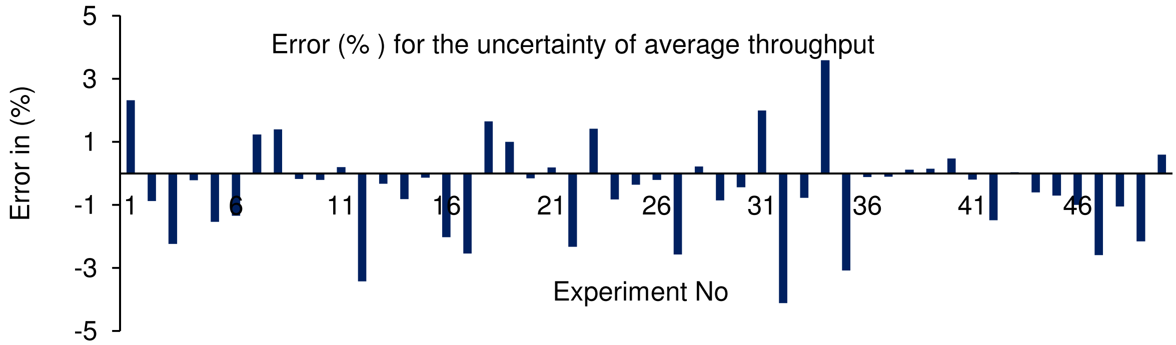


Figure 5:36 Errors on V[TH] by Taylor approximation

5.3.2 Ten machine line with 10 uncertainties

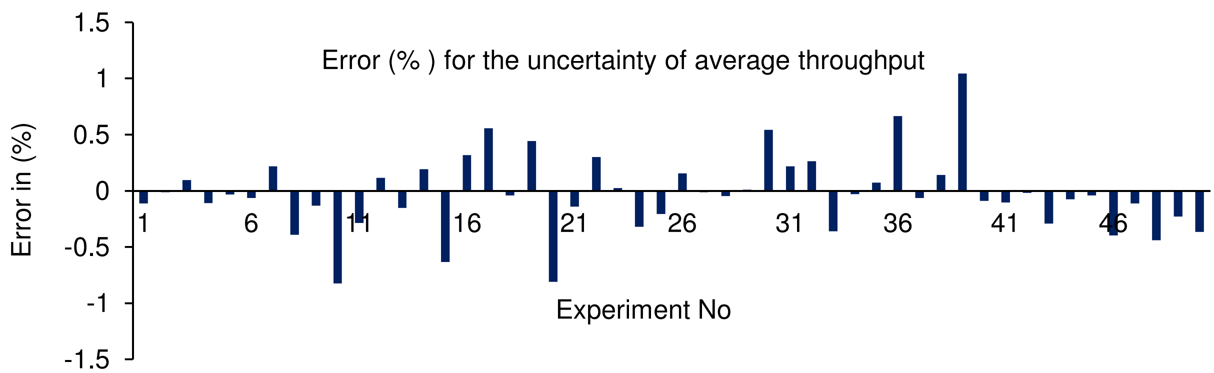


Figure 5:37 Errors on E[TH] by Taylor approximation

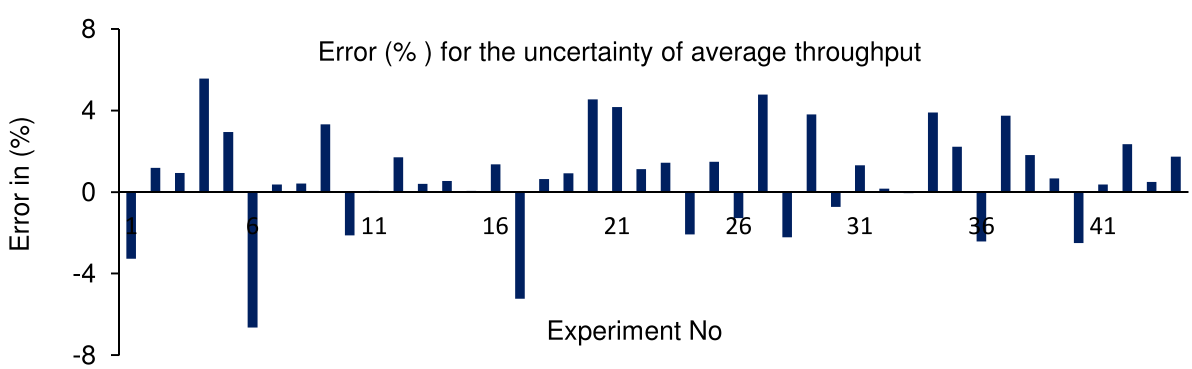


Figure 5:38 Errors on V[TH] by Taylor approximation

5.4 Comparison for Computational efficiency

Sets of experiments are conducted to show the relative efficiency of some of proposed methods. Summary of the run times measured are in seconds and are reported as in the following three tables. The experiments are performed for three machine lines for the proposed methods including 1, 2 and 3 uncertainties for the joint approach, one factor at a time approach (One P) and the Taylor approximation. For the ten machine lines only the one factor at a time and the Taylor approximations are compared as the solutions for the joint distribution approach doesn't yield in a reasonable time.

5.4.1 Three machine line experiments

In this section, 50 experiments of three machine lines and with one, two and three uncertain parameters are carried out to compare the time required using three methods. The average time required for evaluating each problem is reported in seconds. Full results are reported in Appendix (A.3).

	THREE MACHINE LINE							
	1 Uncertain		**2 Uncertain**			**3 Uncertain**		
	One P	**Taylor**	**Joint**	**One P**	**Taylor**	**Joint**	**One P**	**Taylor**
Avg	1.44792	0.10798	17.70454	3.69134	0.14724	1748.193	16.94058	0.42038
Min	0.738	0.06	13.245	1.18	0.049	659.877	6.627	0.186
Max	2.989	0.201	24.321	10.079	0.531	2574.753	26.955	0.652

5:5 Execution time required for evaluation of three machine lines

The average time required for the evaluation of two parameters using one factor a time, joint approach and the Taylor approximation are, 17.7, 3.7 and 0.14seconds respectively while for three uncertainties the times required increase to 1748, 16.9 and 0.42 seconds. This indicates even the time required including for one factor at a time approach can grow quite faster than a linear increase of time. This is attributed to the some farthest points in the distribution values needed to be evaluated needs more time for convergence for each evaluation unlike the central values in the distribution. For the single uncertainty with one factor at a time needed an average evaluation time of 1.44 seconds. This highlights for the analysis of multi-stage systems the Taylor approximation is quite valuable even with lower precision compared to the discretization techniques.

5.4.2 Five machine line experiments

	FIVE MACHINE LINE							
	1 Uncertain		**2 Uncertain**			**3 Uncertain**		
	One P	**Taylor**	**Joint**	**One P**	**Taylor**	**One P**	**Taylor**	**Joint**
Avg	16.08944	0.36522	238.7781	65.4342	0.5231	5094.945	144.2502	1.90828
Min	4.065	0.178	36.635	15.348	0.129	3899.479	35.397	1.17
Max	44.236	0.734	396.34	136.988	0.977	6087.427	249.263	3.322

5:6 Execution times required for evaluation of five machine line

The same conclusion about the comparative time required by each method can be made in the case of three machine experiment. The full result of each experiment is reported in Appendix (A.4).

5.4.3 Ten machine line experiments

TEN MACHINE LINE					
5 Uncertain		**10 Uncertain**		**20 Uncertain**	
One P	**Taylor**	**One P**	**TAYLOR**	**One P**	**Taylor**
518.36	22.557	1945.536	51.962	2591.262	188.7
787.066	14.395	1462.28	77.67	4397.924	149.185
512.368	26.732	1584.397	53.143	3789.9	155.905
735.791	15.203	2006.024	44.769	2600.678	194.575
410.113	24.709	1998.8	57.577	2658.14	147.7575

892.917	21.429	2074.962	47.283	3204.302	137.44
552.242	15.016	2130.644	37.071	5334.076	154.11
532.991	23.689	1466.624	64.186	5071.631	101.745
478.523	18.555	1103.6	47.234	3999.955	164.175
680.833	21.815	1371.97	39.552	3200.816	133.6025

5:7 Execution times required for evaluation of ten machine line

Chapter Six

6. System Behavior

This section discusses the behavior of systems under uncertainty. Behaviors exhibited under single isolated machine cases are already discussed in section 4.2.1 with accompanying exact analytical explanations and proofs. The particular aim of this section is to discuss the behavior of performance evaluation of multi stage production systems under uncertainty. First the behavior of two machine single buffer lines is studied. Generalized behaviors are discussed by grouping peculiar behaviors common to the specific class of systems. Taylor approximation technique is used on the two machine lines to explain behaviors with the objective of relating output distribution to input uncertain parameter distributions. Confirmatory experiments are run using discretization techniques for the accuracy of the observed behaviors.

Later the behavior of multi stage long lines is investigated when reliability parameters are uncertain estimates. Generalized behaviors are given for distinct configurations with additional classifications, such as with respect to the bottle neck resources are provided to explain certain important behaviors.

6.1 Two machine lines

First the impact of individual input parameter distributions on the output the distribution of average throughput TH is discussed. The comparative difference between performance evaluation results obtained by ignoring uncertainty and including it with a single uncertainty is considered. Cases are defined where conclusive results are obtained on the possible overestimation or underestimation of average throughput TH. The next sections investigate the relationship between the uncertainty of the average throughput (TH) with respect to the uncertainty of the input parameters. These general behaviors are discussed for specific classes of systems based on the distribution of the isolated efficiency of the machines and the intermediate buffer.

6.1.1 Expected value of the average TH, E[TH]

Considering each of the individual uncertain reliability parameters possible performance deviations errors that could be committed by ignoring uncertainty are investigated. At the beginning the impact of uncertainty in failure probabilities and the consequence of ignoring these uncertainties in p_u and p_d is discussed. The resulting behaviors are explained using exact Taylor approximation technique and corresponding parameters.

As demonstrated in the cases of isolated machine neglecting input parameter uncertainties from performance evaluation can be lead to either underestimation or overestimation of the average TH. This behavior is also investigated in the two machine line case for the cases where the direction of the performance can be predicted conclusively and also for cases where the deviation can't be generalized. This property can be well explained using the second order partial derivative of the uncertain parameters. The goal of this section is also to see whether the possible errors could be over estimation or under estimation that can be made if uncertainty is ignored.

6.1.1.1 Uncertain p_u and p_d

In this case the two machine single buffer line with single failure mode as given in the (Gershwin-Berman) model is considered. Either the upstream failure probability p_u or the downstream failure probability p_d can be assumed uncertain. Counter examples with second order Taylor approximation can be sufficient to show the behavior of the performance deviation that can be made. In this particular case a two machine single buffer line with the upstream and downstream machine having identical efficiencies are used. In this case the repair probabilities for both machines r_u and r_d are considered to be precisely known values.

Using second order Taylor approximation for the expected average throughput of the two machine line E[TH] can be approximated as:

For the upstream failure probability this can be approximated as:

$$E[TH] = TH(\mu_{p_u}) + \frac{\partial^2 TH}{\partial {p_u}^2}\sigma^2 p_u \tag{6.1}$$

The same expression can be written for the remaining three parameters. We begin with plotting the first and second order derivatives against different levels of intermediate buffer capacities. Input parameters for the experimental output reported in Figure 6:1 are provided in Table 6:1.

The first derivative $\frac{\partial TH}{\partial p_u}$ is given in equation (4.89). The detailed expression is not provided for the second derivative $\frac{\partial^2 TH}{\partial {p_u}^2}$ for sake of brevity.

μ_{pu}	r_u	μ_{pd}	r_d	**N**
0.025	0.1	0.025	0.1	3-50

Table 6:1 Input parameter values for two machine line with uncertain p_u and p_d

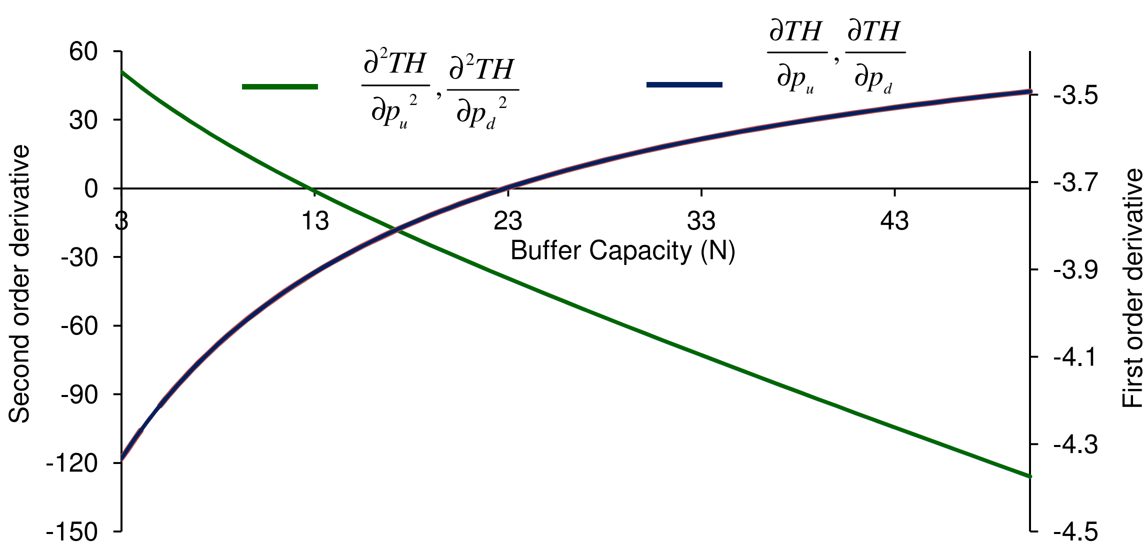


Figure 6:1 First order and second order partial derivatives of TH for a two machine line

In Figure 6:1 the first and second order derivative values for both the p_u and p_d are overlapping since the two machine line is composed of identical isolated machines. As it can be seen the axis for the first order derivatives (right hand axis) the values are always negative. Even if it is not our prime interest here to discuss the behavior of the first order derivative it can be noticed that the values are always negative as the average throughput is a negative function of both the failure probability p_u and p_d.

The interesting parameter in explaining if the consequence of ignoring uncertainties will be underestimation or overestimation is the second order derivative as written in equation 6.1. For the parameters combinations where the second order derivative is positive the deviations by ignoring uncertainty will be less than that of the analysis introducing uncertainty, i.e., underestimation. But observing from the given counter example it can be seen that for the given particular parameter combinations the second order derivatives are greater than 0 for buffer capacity less than 13. Consequently analysis that ignores uncertainty in this region will introduce underestimations. On the other hand for the same case, the second order derivatives with buffer greater than 13 are negative. This introduces an overestimation if the uncertainty of either the p_u or p_d is not included in the analysis.

From the above counter example itself it can be concluded that the possibility of underestimation or overestimation depends on the particular parameter values and buffer configurations. Unless all the parameters are available a conclusive decision on the direction of performance deviation can’t be generalized. To back up this conclusion randomly generated experiments are run with uncertain failure probabilities on the upstream and downstream machine. The repair probabilities for both machines are considered to be known precisely. The resulting comparison between the expected average throughput E[TH] by neglecting the uncertainty and the analysis with uncertainty are reported in Figure 6:2. A maximum underestimation error of -10.7 % is observed; while in some specific configurations more than 15% of performance deviations are found.

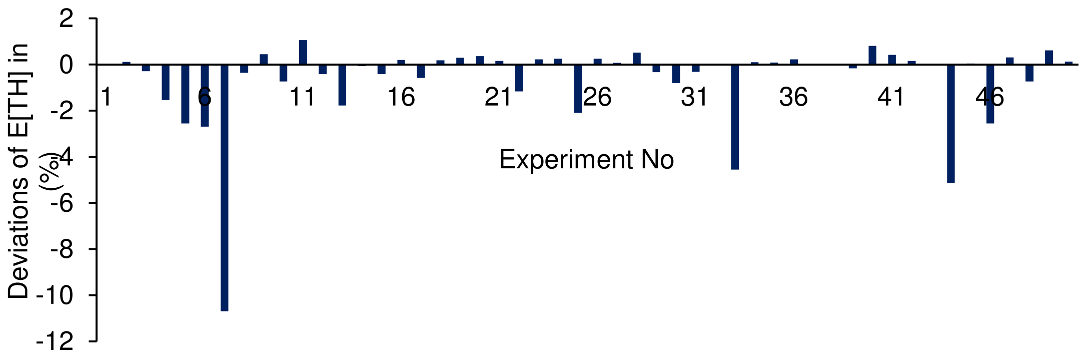


Figure 6:2 Underestimation and overestimations on E[TH] by neglecting uncertainty in p_u and p_d

6.1.1.2 Uncertain r_u and r_d

The same analysis is performed as in the previous case while in this case a two machine line having both machines having identical efficiencies is considered. The repair probabilities for both machines r_u and r_d are considered uncertain estimates. The second order Taylor approximation for the expected average throughput can be written as in the previous case for the E[TH] with respect to r_u and r_d.

For the upstream repair probability this can be approximated as:

$E[TH] = TH(\mu_{r_u}) + \frac{\partial^2 TH}{\partial r_u^{\ 2}} \sigma^2 r_u$ A similar approximation can be applied for the downstream repair probability r_d.

The first derivative $\frac{\partial TH}{\partial r_u}$ is given in equation (4.90), while the expression for second order derivative $\frac{\partial^2 TH}{\partial r_u^{\ 2}}$ is left to avoid complexity. The same counter example is used as in the above case to see the impact of the distribution of average throughput for a two machine line.

p_u	μ_{ru}	p_d	μ_{rd}	N
0.025	0.1	0.025	0.1	3-50

Table 6:2 Reliability input parameters for two machine lines with uncertain r_u and r_d

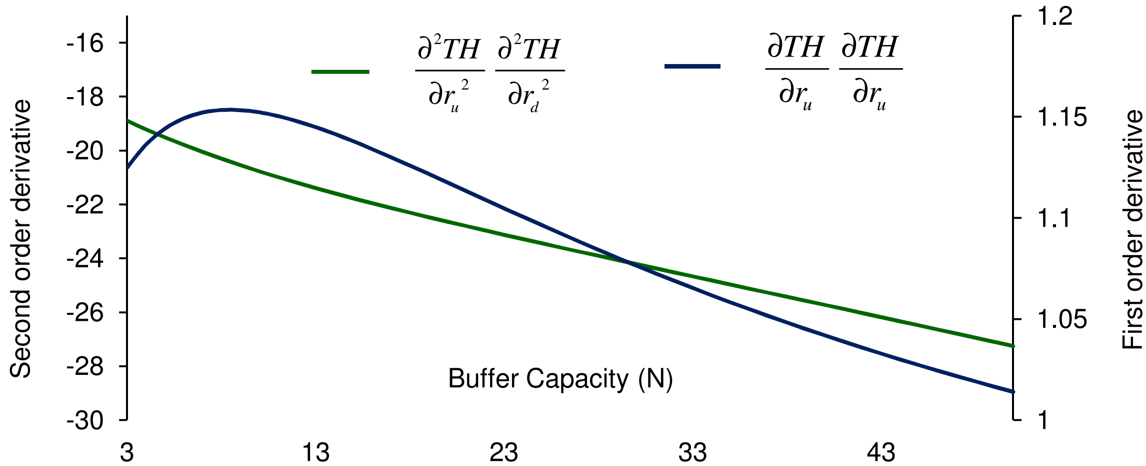


Figure 6:3 First order and second order partial derivatives of TH

The same observation can be made as in the case of uncertain p_u and p_d. From the first order derivatives (right hand axis) it can be see that the average throughput is always a positive function of r_u and r_d. In this specific case the second order derivatives are always negative with respect to r_u and r_d. Although not provided here the numerical evaluation of second order derivatives for all possible parameter values have shown that this quantity is always less than 0 for any given configuration and precisely known p_u and p_d.

Consequently an analysis neglecting uncertainty in repair probabilities while p_u and p_d are precisely known introduces a consistent over estimation of the average throughput E[TH]. Confirmatory experiments from 50 randomly generated two machine lines with uncertain repair probabilities and certain failure probabilities are evaluated. The expected average throughput E[TH] is always less than the value obtained ignoring the uncertainty. Figure 6:4 shows this consistent overestimation in percentage on the E[TH].

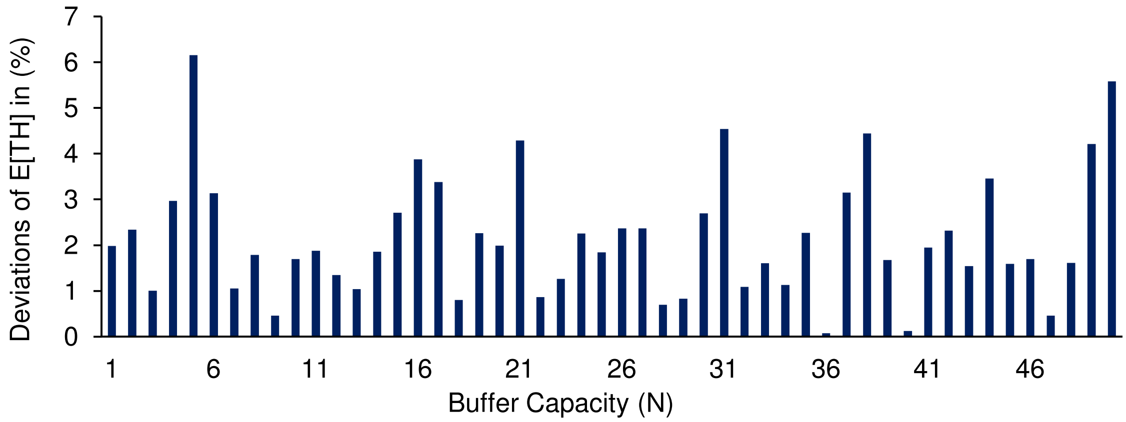


Figure 6:4 Overestimation on a two machine line by excluding the uncertainty in r_u and r_d

If all the parameters in a two machine single buffer line are considered uncertain depending on the specific distributions either underestimation or overestimation can be committed in the expected value of the average throughput.

6.1.2 Uncertainty in variance for the average TH

The previous analysis aims to understand the impact of performance analysis with uncertain parameter estimates particularly on the expected value of the performance measure E[TH]. The objective of the analysis in this section is to investigate the uncertainty of the average throughput measure in variance, V[TH] in a two machine line with respect to parameter settings and buffer capacity. It is of special interest to understand how uncertainty in reliability estimation and the buffer capacity impacts the resulting uncertainty in the performance measure. Such type of analysis helps to understand and how to better address and reduce uncertainty by focusing on optimal reduction of uncertainty from input parameters. On the other hand it also shows the impact of the buffer capacity and how it can be used as a means to reduce uncertainty.

For the sake of making a better summary on the behavior of two machine lines, instead of analyzing specific cases with respect to individual parameter three general behaviors that are characteristic to the two machine line system are discovered. The three general behaviors are based on the distribution of the isolated efficiency of the individual machines that compose the two machine line and the corresponding buffer configuration.

6.1.2.1 Case 1: Highly reliable machine with higher uncertainty

This case is characterized by two machine single buffer lines having the following configurations. The isolated efficiency of the upstream machine and downstream machine are significantly different. The machine with high isolated efficiency has the higher uncertainty on the isolated efficiency and the lower isolated efficiency machine has very low uncertainty on isolated efficiency. Moreover the difference between these uncertainties in the isolated efficiency must be significant. Generally two machine lines that fall in these categories can be expressed as follows.

1. $(E[e_u] \gg E[e_d]) \wedge (Var[e_u] \gg Var[e_d])$
2. $(E[e_u] \ll E[e_d]) \wedge (Var[e_u] \ll Var[e_d])$

In this scenario we are interested in understanding the effect of increasing buffer capacity on the expected value E[TH] and the uncertainty V[TH] of the average throughput. The input parameters used for the analysis are in Table 6:3

The following parameters are used for this test case:

p_u	r_u	p_d	r_d	N
Beta(5,45)	Beta(14,15)	Beta(17800,50000)	Beta(8600,18000)	3-100

Table 6:3 Reliability input parameters for two machine lines with uncertain r_u and r_d

The first two moments of the isolated efficiency of the upstream machine and downstream machines are reported in Table 6:4.

$E[e_u]$	$V[e_u]$	$E[e_d]$	$V[e_d]$
0.8290	0.003794	0.5516	0.0000739

Table 6:4 Expected value and uncertainty of the isolated efficiencies in the two machine line

The results are evaluated from buffer capacity of 3 to 100. As shown in Figure 6:5 the average throughput obviously increases with increasing buffer capacity, while the uncertainty of the average throughput V[TH] goes decreasing close to the value of the uncertainty of the isolated efficiency of the less reliable machine.

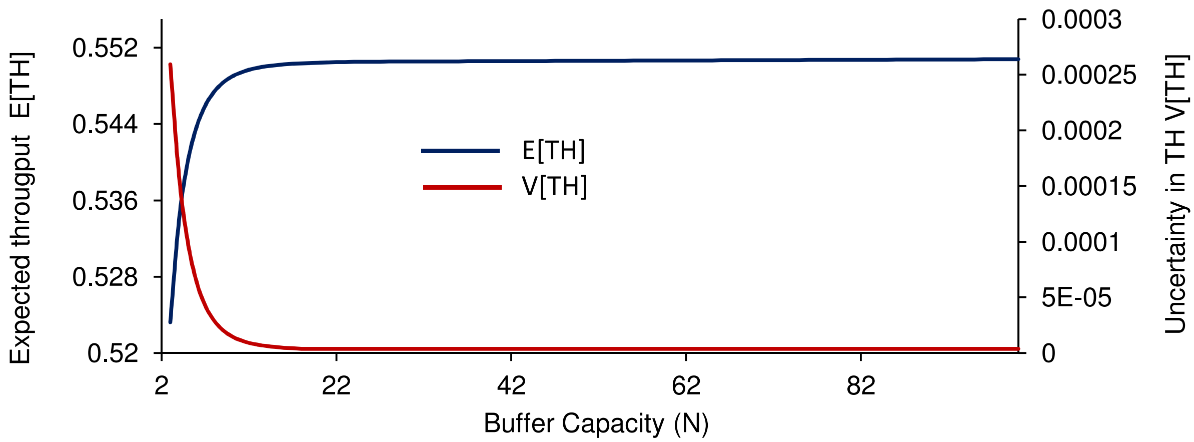


Figure 6:5 Expected value E[TH] and uncertainty V[TH] of the average throughput

It this discussion it is of a particular interest to explain the observed behavior of the uncertainty in the average throughput, V[TH]. To accomplish we will make use of the first order partial derivatives with respect to the uncertain parameters that can be used to approximate V[TH] with a first order Taylor approximation.

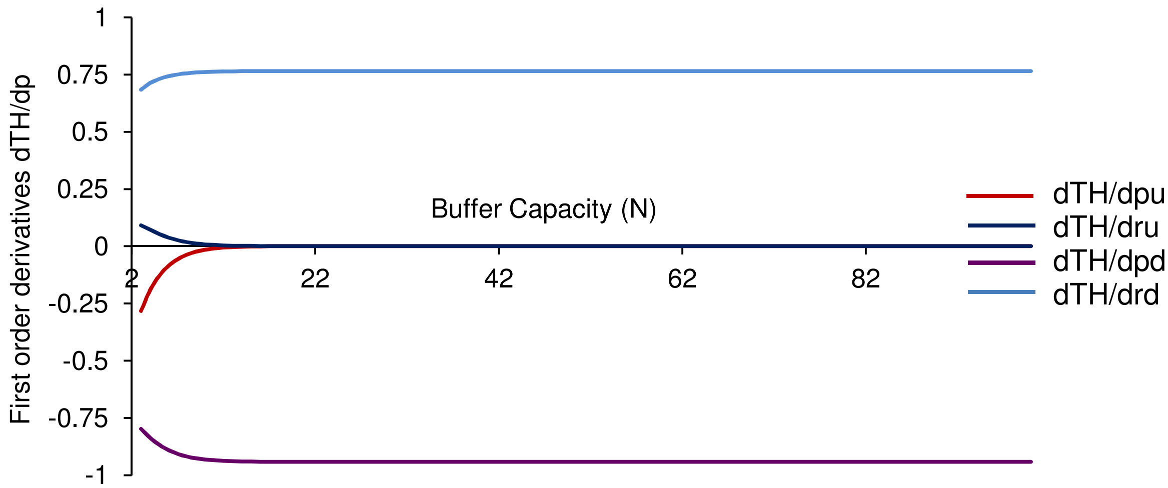


Figure 6:6 First order derivative with respect to uncertain parameters

Following the same example the evaluation of the first derivative provide the specific behavior as shown in Figure 6:6. Generally two machine lines with the above condition will have this kind of configuration for their first order derivatives when plotted against the intermediate buffer capacity. The two lines below the zero line are the derivatives of the failure probabilities, as the throughput is a decreasing function of the failure probability. The bottle neck machine is the one that is farthest from the central line in this case colored in purple. While the failure probability of the higher isolated efficient machine is closer to the zero line. The same can be said about the trend of the derivatives with respect to the repair probability. The farthest line from the center, in this case the light blue is the derivative with respect to the repair probability of the bottleneck machine.

Since the first order approximation of the uncertainty of the average throughput is given as:

$$V[TH] = \left(\frac{\partial TH}{\partial p_u}\right)^2 \times \sigma^2 p_u + \left(\frac{\partial TH}{\partial r_u}\right)^2 \times \sigma^2 r_u + \left(\frac{\partial TH}{\partial p_d}\right)^2 \times \sigma^2 p_d + \left(\frac{\partial TH}{\partial r_d}\right)^2 \times \sigma^2 r_d$$

The square of the derivatives are approximate coefficients of the input uncertainties. Therefore squaring the values in Figure 6:6 gives the results shown in Figure 6:7. The coefficients associated to the bottleneck machine in this case $\left(\frac{\partial TH}{\partial p_d}\right)^2$ and $\left(\frac{\partial TH}{\partial r_d}\right)^2$ will have higher magnitudes and increase with increasing buffer capacity. On the other hand parameters associated with the high isolated efficiency machine *Mu* are close to zero and decrease with decreasing buffer capacity.

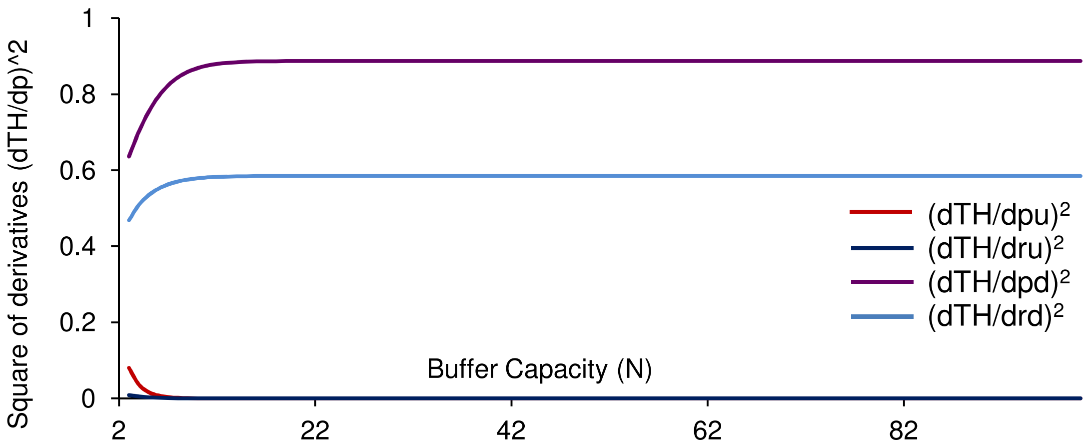


Figure 6:7 Coefficients of the variances of the input parameters (square of first derivatives)

In this case since the machine with higher efficiency have the higher uncertainty and the bottle neck machine has very low uncertainty the higher derivatives will be multiplied with very small coefficients which makes the overall result to be small. It can be seen the purple and light blue lines dropped very low when multiplied by the input uncertainties in Figure 6:7. Comparatively the red and blue lines when multiplied by the input uncertainties have greater influence on the overall behavior of the two machine line. Therefore the overall effect of adding the four lines in Figure 6:8 is a decreasing function of the buffer capacity, which concludes the behavior that is seen in Figure 6:5.

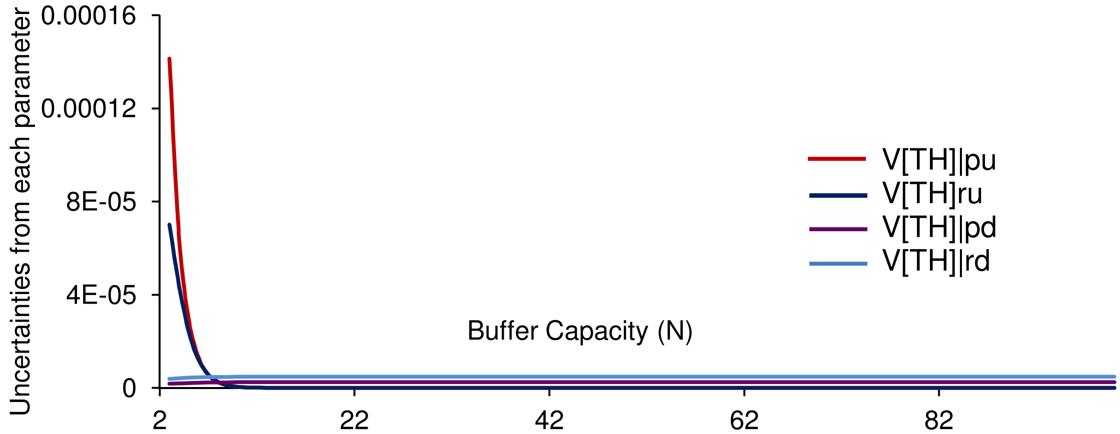


Figure 6:8 Uncertainty from each parameter of a two machine line

6.1.2.2 Case 2: Less reliable machine with higher uncertainty

These cases can be considered as the exact opposite of case 1. They are characterized by two machine single buffer lines having the following configurations. The isolated efficiency of the upstream machine and downstream machine are significantly different. The machine with high isolated efficiency has lower uncertainty of the isolated efficiency. The machine with lower isolated efficiency machine has higher uncertainty of the isolated efficiency. In this case also the difference between these uncertainties in the isolated efficiencies must be significant.

Two machine lines in this category satisfy the following two conditions.

1. $(E[e_u] \gg E[e_d]) \wedge (Var[e_u] \ll Var[e_d])$
2. $(E[e_u] \ll E[e_d]) \wedge (Var[e_u] \gg Var[e_d])$

The same steps as in case 1 are employed for the analysis of this scenario with the objective to understand the effect of increasing buffer capacity and the uncertainty in the average throughput V[TH].

The following parameters are used for this test case:

p_u	r_u	p_d	r_d	N
Beta(4, 1650)	Beta(14,47)	Beta(7,90)	Beta(4,21)	3-100

Table 6:5 Reliability input parameters for two machine lines with uncertain r_u and r_d

$E[e_u]$	$V[e_u]$	$E[e_d]$	$V[e_d]$
0.9880	0.0000404	0.6655	0.01726

Table 6:6 Expected value and uncertainty of the isolated efficiencies in the two machine line

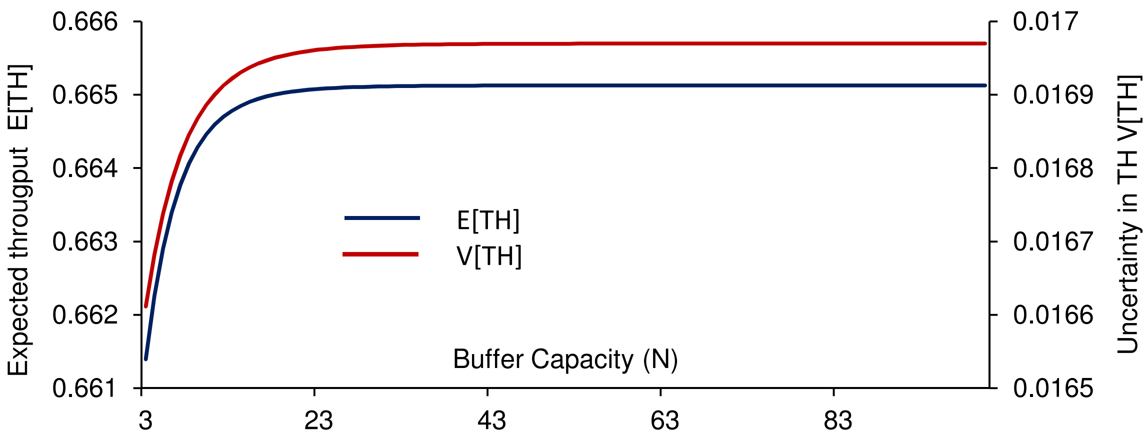


Figure 6:9 Uncertainty V[TH] as a function of buffer capacity

It is already known that the average throughput is always a monotonous function of the buffer capacity. As it can be observed from Figure 6:9 in this case, as the buffer capacity increases the uncertainty of the average throughput V[*TH*] also increases as opposed to the previous case. Extending the same demonstrations used above with Taylor first order approximation, the next two graphs show the derivatives with respect to each parameter of the two machine line.

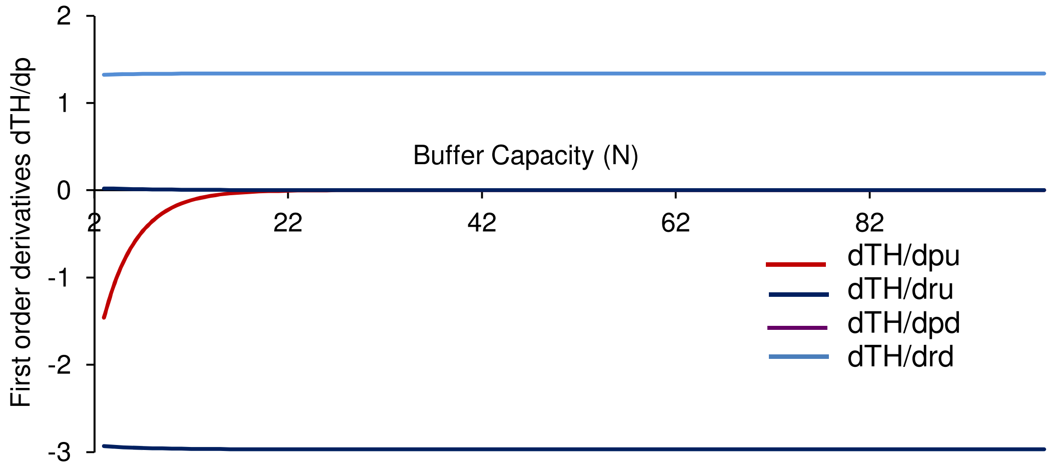


Figure 6:10 First order derivative with respect to uncertain parameters

Using the first order approximation of V[TH] as used in case 1, then the coefficient of each parameters uncertainty will be the square of the values given in Figure 6:10 and the corresponding results are as shown in Figure 6:11.

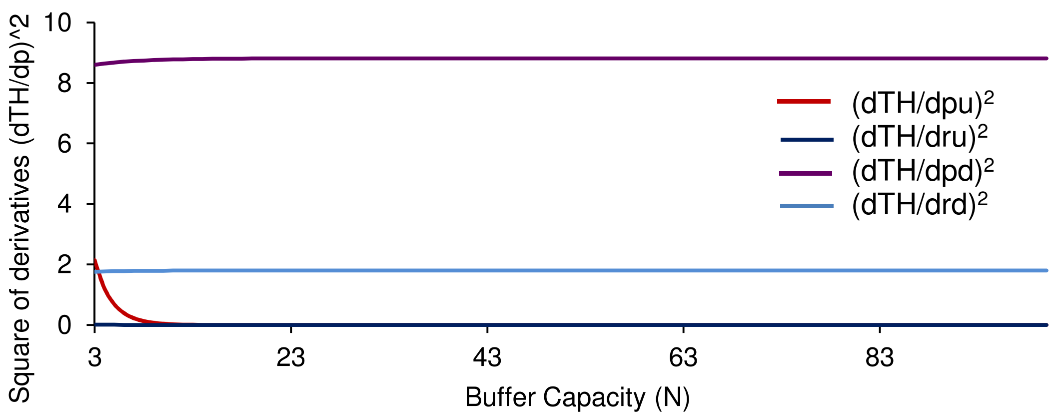


Figure 6:11 Square of the first order derivative with respect to uncertain parameters

In this case since the machine with higher efficiency have the lower uncertainty and the bottle neck machine has very high uncertainty, i.e., the opposite of case 1 then the smaller derivatives will be multiplied with very small coefficients which makes them insignificant in the overall result. The overall effect that results as the uncertainty of the two machine line is the impact of M_d, in which both of them are increasing. The product of the input parameters and the squared derivative is reported Figure 6:12, with the sum superimposed on the right axis.

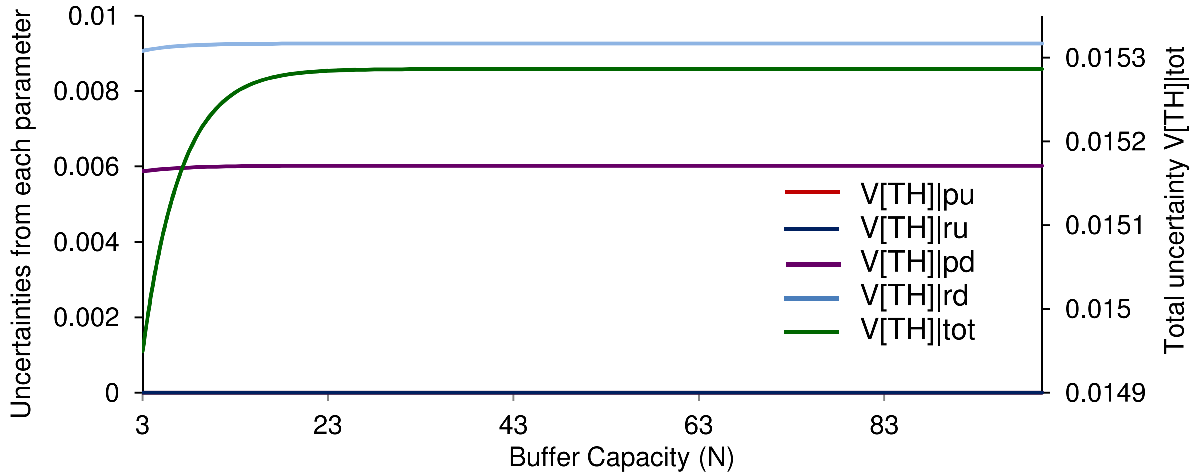


Figure 6:12 Weighted derivatives squared

6.1.2.3 Case 3: intermediate cases with uncertainty

This case includes conditions that the machine with not having significant difference in their isolated efficiency or machines that are not significantly different in the uncertainty of the isolated efficiency. All two machine lines that don't properly fall in the extreme case 1 or case 2 categories would generally have this form of curve for the uncertainty in the TH for increasing buffer capacities. The shape can take different forms depending on the particular values considered for the analysis. But the general configuration is a decreasing uncertainty to some level and the uncertainty starts to rise after some level.

1. $(E[e_u] \approx E[e_d]) \wedge (Var[e_u] \approx Var[e_d])$
2. $(E[e_u] \approx E[e_d]) \wedge (Var[e_u] \approx Var[e_d])$

In this scenario we are interested in understanding the effect of increasing buffer capacity and the response in the average throughput and the uncertainty related to the TH.

The following parameters are used for this test case:

p_u	r_u	p_d	r_d	N
Beta(4,45)	Beta(28,30)	Beta(178,500)	Beta(86,180)	3-100

Table 6:7 Reliability input parameters for two machine lines with uncertain r_u and r_d

$E[e_u]$	$V[e_u]$	$E[e_d]$	$V[e_d]$
0.8290	0.003794	0.5508	0.000746

Table 6:8Expected value and uncertainty of the isolated efficiencies in the two machine line

Two machine single buffer lines with comparable expected isolated efficiencies and uncertainty show widely varied types of curvatures. These curves have a common trend having a decreasing uncertainty and an inflection point and an increase in uncertainty for higher buffer capacities. The drop and the rise can be sharp as seen in Figure 6:13 or a slight drop and slight increase over a wide range of buffer capacities depending on the particular parameters characterizing the two machine line. This unique behavior is also explained with the first order partial derivatives for Taylor approximation.

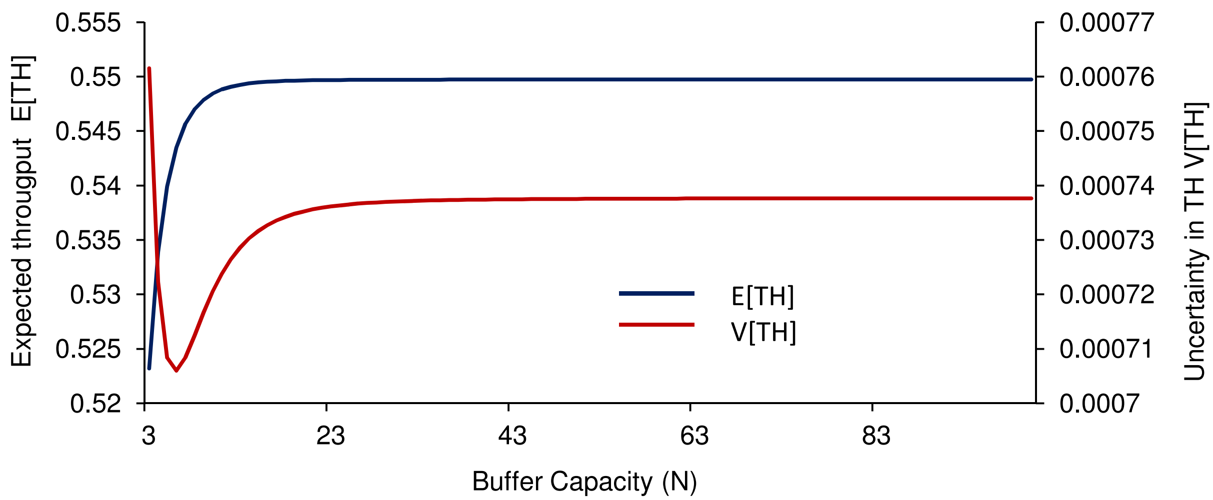


Figure 6:13 Uncertainty V[TH] as a function of buffer capacity

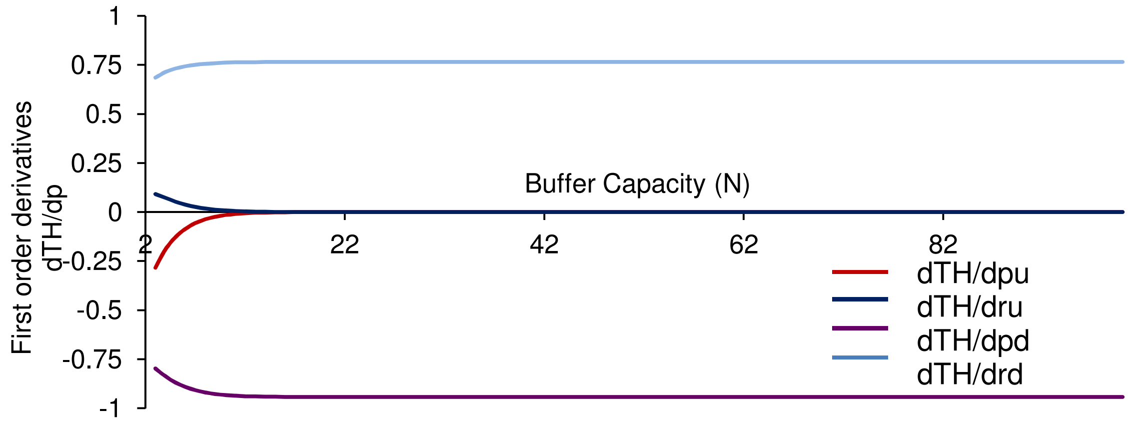


Figure 6:14 First order derivative with respect to uncertain parameters

In this case since the machine with higher efficiency has a moderately higher uncertainty and the bottle neck machine has smaller uncertainty in the isolated efficiency. Therefore as it is in Figure 6:14 the first order derivatives of the downstream machine M_d are the farther away from zero and having the bigger magnitude when square as in Figure 6:15. The coefficients of the input uncertainties, i.e., the squared derivatives of the downstream machine are both increasing while same parameters for the upstream machine M_u they are decreasing. There are two factors that contribute for the overall trend of the uncertainty in the throughput of the two machine line. The first one is the rate of increase of the coefficients of the lower isolated efficiency machine and the decrease of the coefficients of the higher efficiency machine. The second factor is the input uncertainties associated to each uncertain parameter that multiplied these coefficients. In this specific case the input uncertainties for the parameters of the upstream machine are higher. For the first few additional buffer capacities there is a drop for these coefficients, but this value drops to zero after some threshold. The product of input parameters and the coefficients gives a significant drop at the beginning while in the later additional buffers the high efficiency machine doesn't have impact since the coefficients for the input uncertainties goes to zero. In this region the system more influenced by the increase of the derivative coefficients that result from the less efficient machine. The overall contribution of uncertainty from the individual input parameters and the total uncertainty of the two machine line is reported in Figure 6:16, with the sum superimposed on the right axis.

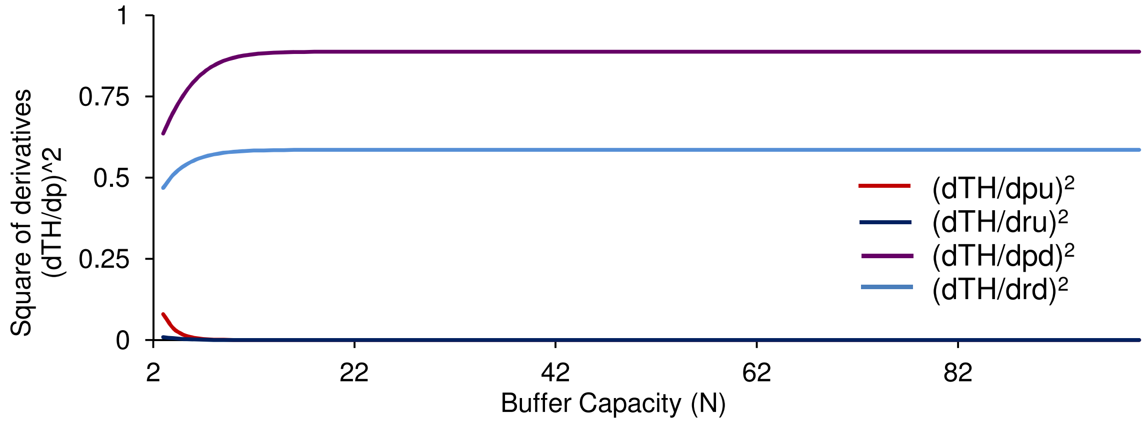


Figure 6:15 Square of the first order derivative with respect to uncertain parameters

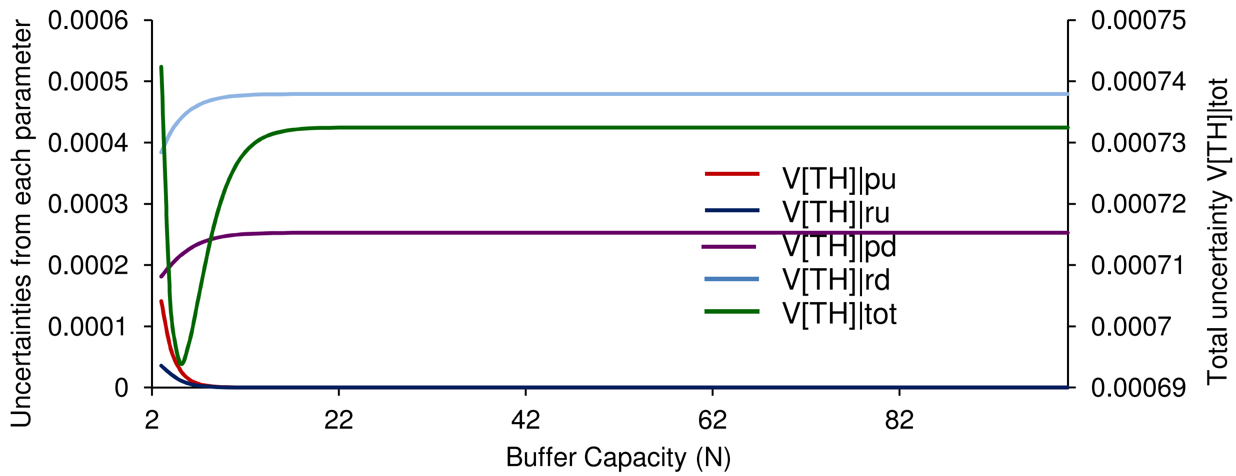


Figure 6:16 Weighted derivatives squared

6.2 Long Multistage Lines

Behavior analysis carried out on two machine single buffer lines with uncertain parameter estimates highlighted the significant role of the bottleneck machine on the overall uncertainty of the system performance measure. Considering the bottleneck machine's importance in the determination of system's uncertainty, it can be a basis for categorizing systems and understand expected the behavior of the systems under alternative buffer configurations. This section aims to generalize the uncertainty in performance measure of long multi-stage lines. The set of systems are classified into three categories with respect to the low efficiency (bottleneck) machine.

6.2.1 Impact of bottle neck on the propagation of uncertainty

Although these behaviors are general for any length of multistage line for simplicity a case with four machines line is used for demonstration. Each of the three machines has equal uncertainty of their isolated efficiencies, with only one of the machine characterized by lower efficiency than the rest of the machines in the lines. As in the case of the two-machine single buffer case the long lines can be treated in three distinct categories on the basis of the bottle neck machine.

Considering the four machine line system with the following parameters we will see the effect of intermediate buffer in three categories of systems. But the behaviors are exhibited by general multistage lines systems depending on the relative position of the uncertainty and the specific reliability parameters.

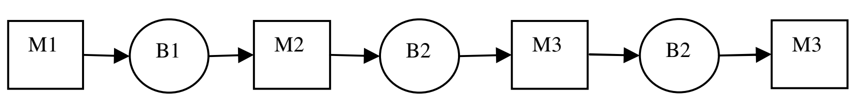


Figure 6:17 Four machine line with uncertain parameter estimates

6.2.1.1 Case 1: Lower uncertainty on the bottle neck machine

This case is characterized by multi-stage lines where the machine with significantly lower isolated efficiency than the rest of the machines has also a significantly lower uncertainty of the isolated efficiency compared to the other machines in the line. The location of the machine can impact the uncertainty on the average throughput of the line. Even through the location of the machine in the line can have an impact the general trends when the buffer capacity increase has the same trend on the uncertainty and expected value of TH. The two machine single buffer line that was discussed in the behavior analysis of the two machine lines can be considered as a specific case of these lines. For demonstrating this behavior a four machine line with the following reliability parameters and buffer capacity range is investigated. The line is equally buffered and the buffer capacity is increased equally in all the buffers for each experiment.

Machine	***Buffer***	***Failure***	***Repair***
M_i	N_i	p_i	r_i
1	5-100	Beta(5,354.85)	0.1
2	5-100	Beta(5,354.85)	0.1
3	5-100	Beta(535.4,35485.4)	0.025
4		Beta(5,354.85)	0.1

Table 6:9 Input parameters considered for case 1

From the above data the third machine M_3 is the machine with minimum isolated efficiency and at the same time the machine with smaller uncertainty. The evaluated expected value and uncertainty of the isolated efficiencies are reported in Table 6:10.

$E[e_3]$	$V[e_3]$	$E[e_{1,2,4}]$	$V[e_{1,2,4}]$
0.6562	0.000010464	0.8796	0.002120

Table 6:10 Expected value and uncertainty of the isolated efficiencies in the four machine line

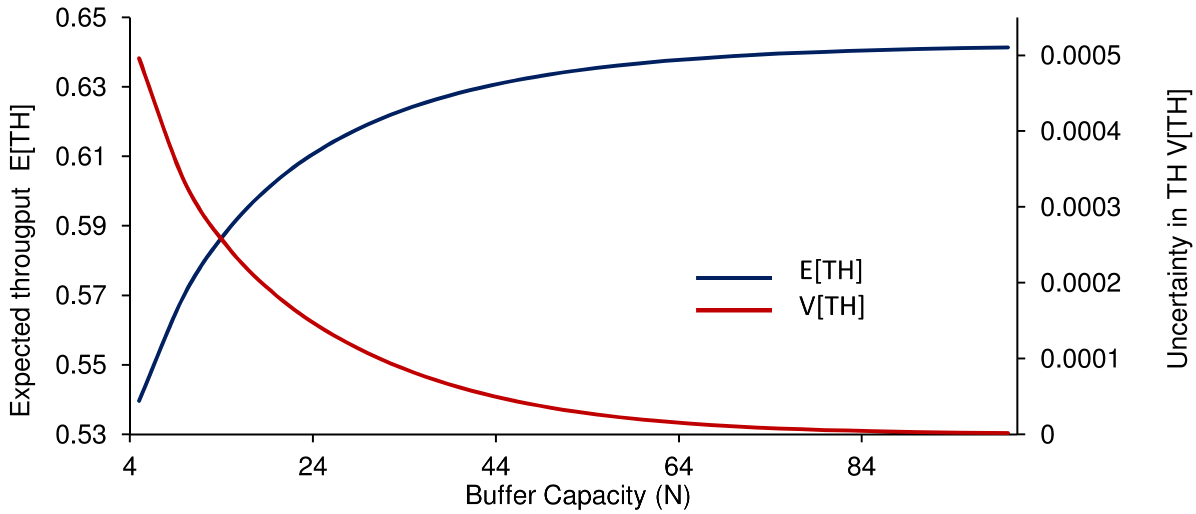


Figure 6:18 Expected value E[TH] and uncertainty V[TH] as a function of N

A similar analysis using the approximated partial derivatives for the approximation of the uncertainty in TH by Taylor expansion can be performed as the two machine lines. As shown in Figure 6:18, the uncertainty of the line decreases with increasing intermediate buffer capacities. This behavior of the line can be explained as follows. For smaller buffer capacities the starvation and blocking with the corresponding uncertainty of even the more efficient machines also have a significant impact resulting higher uncertainty for the whole line. On the other hand when the buffer capacity is increased the propagation of starvation and blocking goes down minimizing the propagation of uncertainty at the same time. The whole behavior of the line becomes dependent on the less efficient machine and the performance also becomes closer to this machine. The expected value and the uncertainty of the throughput become closer to the expected value and uncertainty of the isolated efficiency of the bottle neck machine.

6.2.1.2 Case 2: Higher uncertainty on the bottle neck machine

In short this case can be considered the opposite of case one. The multi-stage line is composed machines with uncertain parameters with one bottleneck machine significantly having a smaller isolated efficiency of the other machines in the line. This line is assumed to be equally buffered and the less efficient machine can be placed anywhere in the line. The bottleneck machine is the one having the higher uncertainty on the isolated efficiency compared to the rest of the machines in the line. The two machine single buffer line that was discussed in the behavior analysis of the two machine lines under case 2 can be considered as a specific case of these lines. In this experiment we use again a four machine line with the following reliability parameters and buffer capacity ranges. The line is equally buffered and the buffer capacity is increased equally in all the buffers for each experiment.

Machine	***Buffer***	***Failure***	***Repair***
M_i	N_i	p_i	r_i
1	5-100	Beta(500,35485.4)	0.1
2	5-100	Beta(500,35485.4)	0.1
3	5-100	Beta(5,354.85)	0.025
4		Beta(500,35485.4)	0.1

Table 6:11 Input parameters considered for case 2

From the above data the third machine M3 is the machine with minimum isolated efficiency and at the same time the machine with smaller uncertainty. The evaluated expected value and uncertainty of the isolated efficiencies are reported in Table 6:12.

$E[e_3]$	$V[e_3]$	$E[e_{1,2,4}]$	$V[e_{1,2,4}]$
0.6562	0.009611	0.8774	0.00022696

Table 6:12 Expected value and uncertainty of the isolated efficiencies in the four machine line

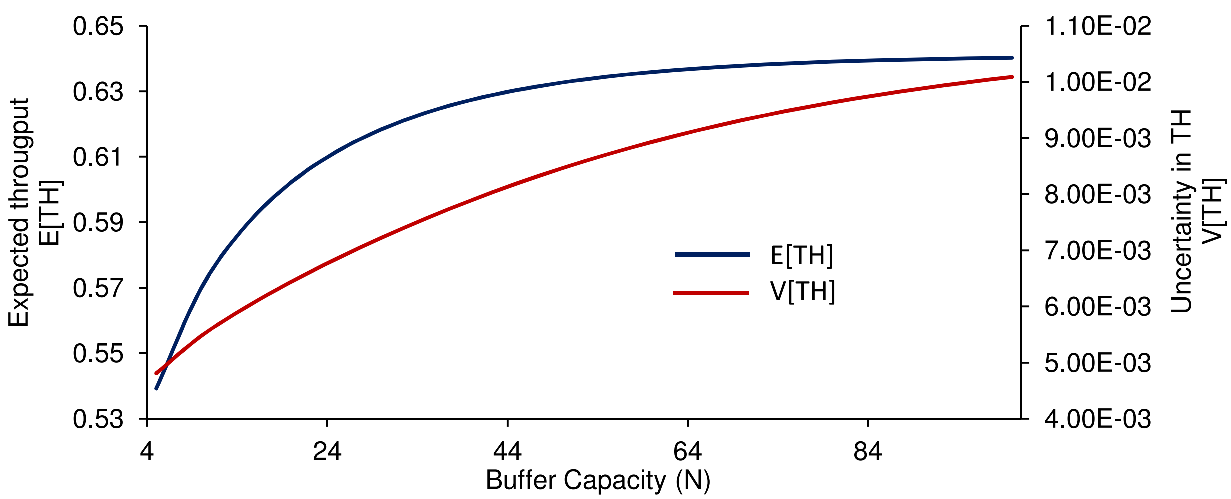


Figure 6:19 Expected value E[TH] and uncertainty V[TH] as a function of N

The same explanation made for case 1 suffices for the behavior seen in Figure 6:19. For smaller buffer capacities the starvation and blocking with the corresponding uncertainty of even the more efficient machines also have a significant impact resulting higher uncertainty for the whole line. Even though the bottleneck machine has higher uncertainty the line is impeded with the other more precisely known machines which decrease the overall uncertainty of the line. When the buffer capacity is increased the propagation of starvation and blocking goes down minimizing the propagation of uncertainty at the same time. In this case also the whole behavior of the line becomes more dependent on the less efficient machine and the performance also becomes closer to this machine. The expected value and the uncertainty of the throughput become closer to the expected value and uncertainty of the isolated efficiency of the bottle neck machine. Looking on Table 6:12 the expected value of the isolated efficiency of the bottleneck machine and the Figure 6:19 for larger buffer capacity indicate this behavior.

6.2.1.3 Case 3: intermediate cases

Most of relatively balanced lines that don't have one bottleneck machine which doesn't have a significantly higher or lower uncertainty from the rest of the machines in the line fall in this category. Therefore machines that don't fall under case 1 or two will show such behavior. This includes also multistage lines composed of machines with identical isolated efficiency with separated by equal buffer capacities. The two machine single buffer line that was discussed in the behavior analysis of the two machine lines under case 3 can be considered as a specific case of these lines. In this experiment we use again a four machine line with the following reliability parameters and buffer capacity ranges. The line is equally buffered and the buffer capacity is increased equally in all the buffers for each experiment.

Machine	***Buffer***	***Failure***	***Repair***
M_i	N_i	p_i	r_i
1	5-100	Beta(5,354.85)	0.1
2	5-100	Beta(5,354.85)	0.1
3	5-100	Beta(5,354.85)	0.025
4		Beta(5,354.85)	0.1

Table 6:13 Input parameters considered for case 3

From the above data the third machine M_3 is the machine with minimum isolated efficiency and at the same time the machine with smaller uncertainty. The evaluated expected value and uncertainty of the isolated efficiencies are reported in Table 6:14.

$E[e_3]$	**$V[e_3]$**	**$E[e_{1,2,4}]$**	**$V[e_{1,2,4}]$**
0.6562	0.009611	0.87959	0.002120

Table 6:14 Expected value and uncertainty of the isolated efficiencies in the four machine line

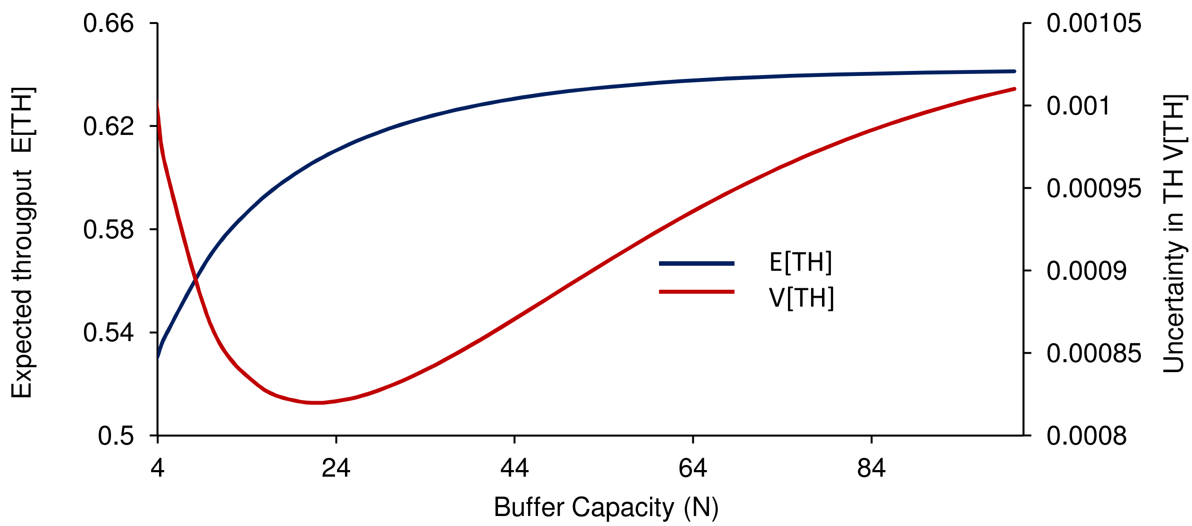


Figure 6:20 Expected value E[TH] and uncertainty V[TH] as a function of N

These curves have a common trend having a decreasing uncertainty and an inflection point and an increase in uncertainty for higher buffer capacities. The drop and the rise can be sharp as seen in Figure 6:20 or a slight drop and slight increase over a wide range of buffer capacities depending on the particular parameters characterizing the multi-stage line.

6.3 Contribution of uncertainty in long lines

In complex multi-stage lines there might be many parameters that must be estimated from real data and known with uncertainty. For an improved certainty on the performance measure it is essential to know where the proportions of the uncertainties arise from. For instance a higher uncertainty in a highly efficient machine might not affect the uncertainty as if the same level of uncertainty is generated by the bottleneck machine. Therefore in addition to evaluating the total uncertainty of a performance measure of the whole system, it is equally important to know how much of the total uncertainty is contributed by each uncertain parameter. The main focus of this section is investigating such analysis on multistage lines particularly equally buffered lines and their behavior under uncertainty.

Generally higher uncertainties in the input parameter increase the uncertainty in the output performance, although the magnitude and the impact of each uncertainty vary depending on the system structure. In this case it is particularly interesting to investigate systems with identical uncertainties on systems having special configurations, such as equally buffered lines. These behaviors in longer lines the following experiments are conducted on longer lines constituting identical machines. The percentage contribution of uncertainty on the performance measure in this case TH by an uncertain parameter *x* is evaluated as:

$$u_x = \frac{V[TH] \mid x}{V[TH]} \times 100\%$$

6.3.1 Multi-stage lines with uncertain identical machines

In order to study longer lines with identical uncertain machines, lines composed of from 3 -15 identical machines with equal buffers are investigated. The general system behavior analysis done is similar even for longer lines. The machines in the line are the same with the following parameters and the results are reported for 3, 5, 10 and 15 machine lines.

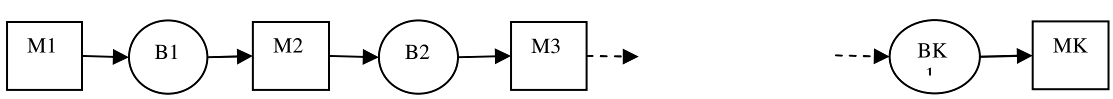


Figure 6:21 Multistage lines with identical machines

	M_i
p_i	Beta(5,355)
r_i	0.05
N_i	5

Table 6:15 Machine parameters for each machine in the line

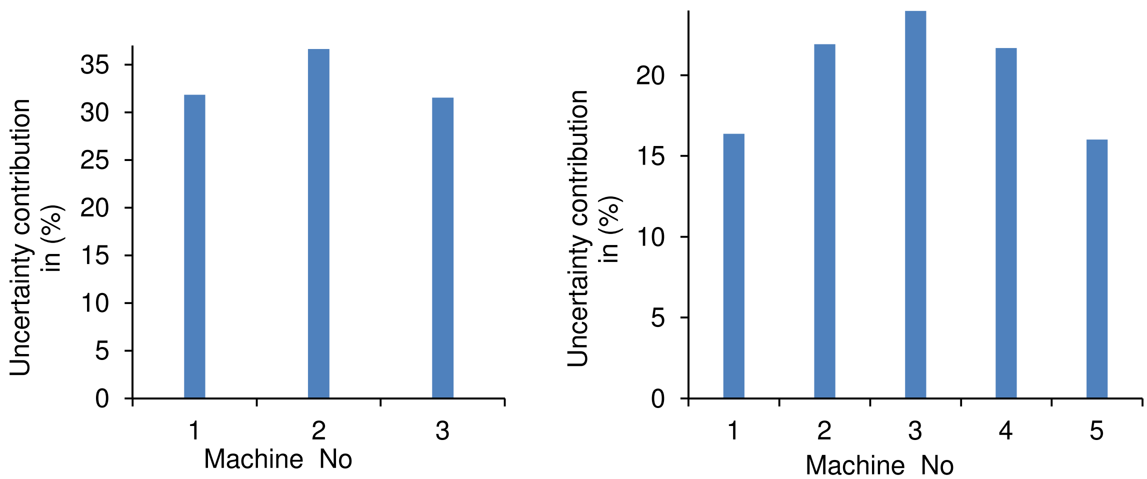


Figure 6:22 Uncertainty contribution in three machine and five machine lines

From Figure 6:22 it can be seen that the contribution o uncertainty from the machine at the middle of the line is bigger than the machine at the entrance and end of the line. This can be verified by computing the partial derivative of the average throughput TH with respect to the individual uncertain parameters. For instance in the case of the three machine line case the relationship of these derivatives can be written as follows. Studies for optimal buffer allocation problem also back up this behavior. A gradient method for the buffer allocation problem shows the highest gradient in a equally buffered identical machines line puts more buffers in the middle of the line.

$$\left(\frac{\partial TH}{\partial p_2}\right)^2 \geq \left(\frac{\partial TH}{\partial p_1}\right)^2, \text{ and } \left(\frac{\partial TH}{\partial p_2}\right)^2 \geq \left(\frac{\partial TH}{\partial p_3}\right)^2$$

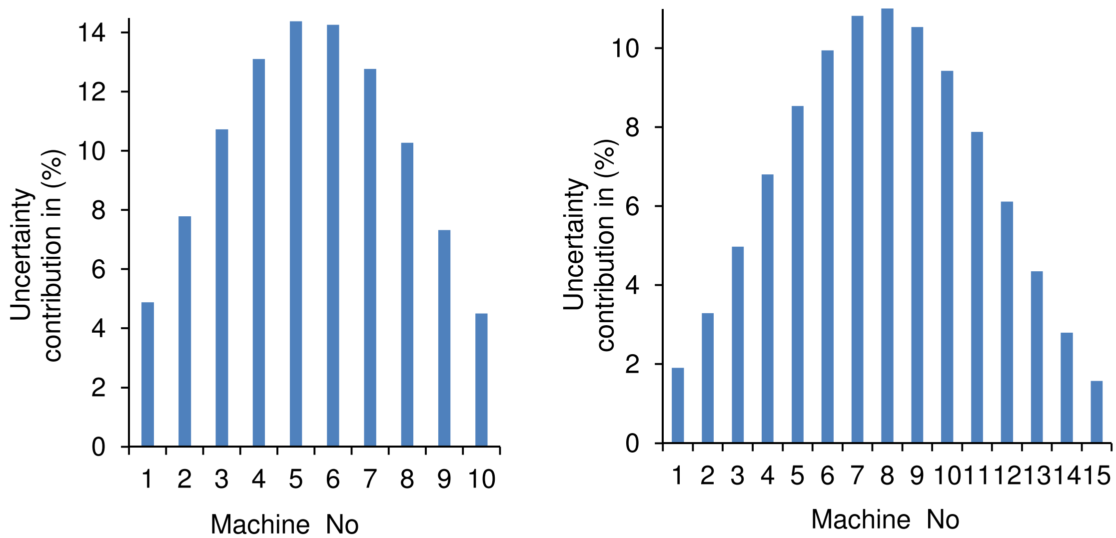


Figure 6:23 Uncertainty contribution in 10 machine and 15 machine lines

Another important observation to make from these graphs is the relative contribution of the lines in the middle and the external machines. If the tree machine line and the five machine lines are compared, the three machine line has an approximated ratio of (1.16:1), while for the five machine line this ratio is (1.5:1). This ratio is even wider for lines with many machines; for the 10 and 15 machine lines that ratio is (3:1) and (5.8:1). Even though this number is not a particular measure for allocating the necessary amount of improvement on the input uncertainties it can be used as a rough approximate of the relative importance of monitoring the particular machine.

6.3.2 Effect of buffering on the contribution of uncertainty

The aim of this experiment is to investigate the impact of more and uniform buffering on the relative contribution of uncertainty. The same machine parameters used in the previous experiment are considered here. Equal buffers with increasing size are used. Each buffer capacity is increased from 5, to 10 and 20 and the resulting relative contributions of uncertainty are reported in Figure 6:24. Results from multi-stage lines with the number of machines 5, 9 and 15 are reported in Figure 6:24-6:26.

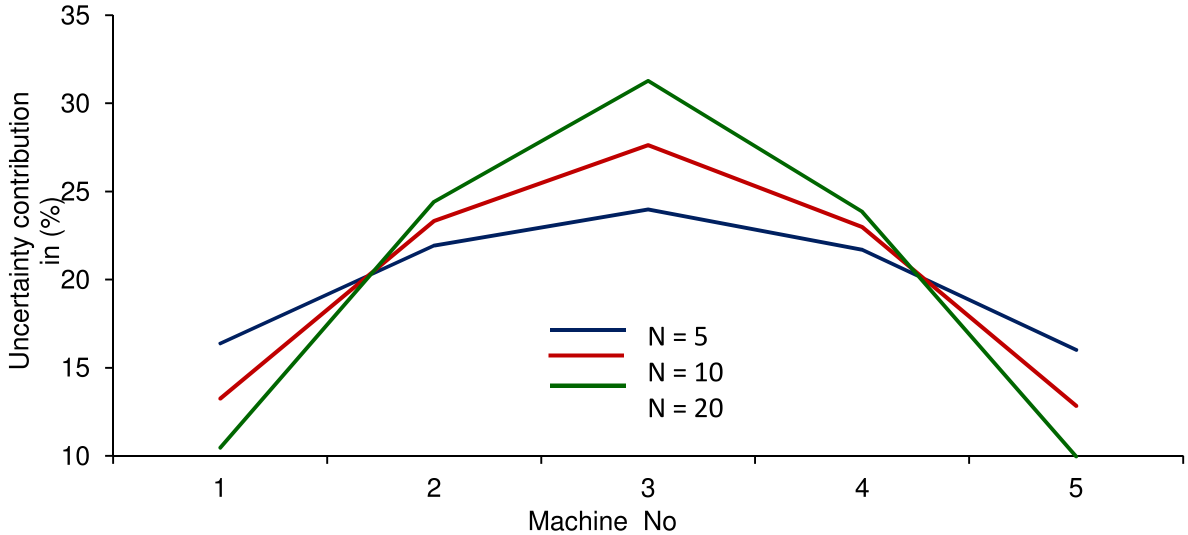


Figure 6:24 Uncertainty contributions in five machine line with three buffer capacities

The results in Figure 6:24 show that when the buffer capacity of the increased with more equal size capacities then the contribution difference between middle machines and external lines even gets more wider. This also how a simple uniform increase of the buffers on the line is not only in less appropriate allocation to bring more impact both on the average throughput but also neglects the possible improvements that can be brought to decrease the corresponding uncertainty in the average throughput.

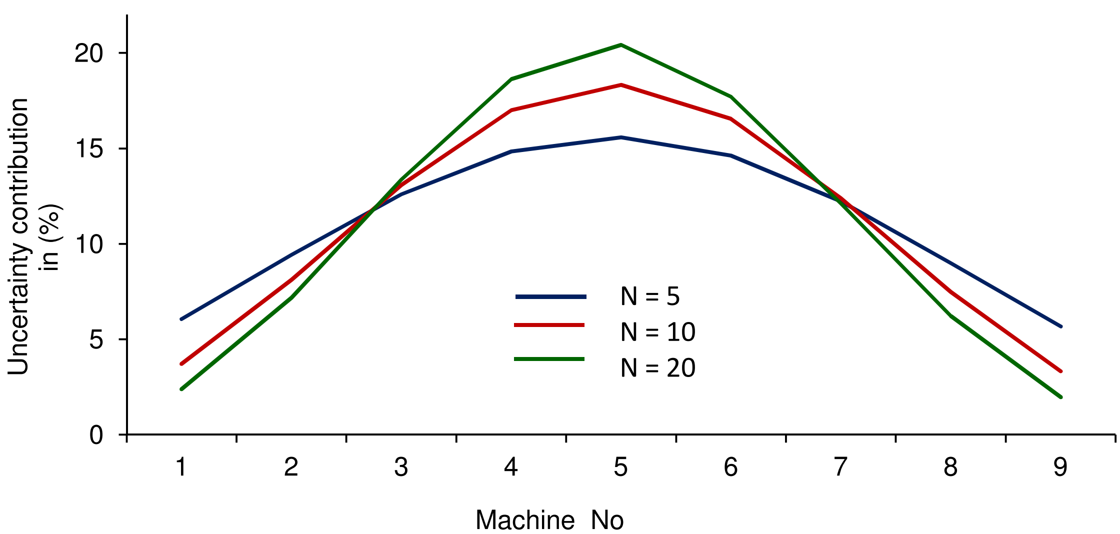


Figure 6:25 Uncertainty contributions in 9 machine line with different N

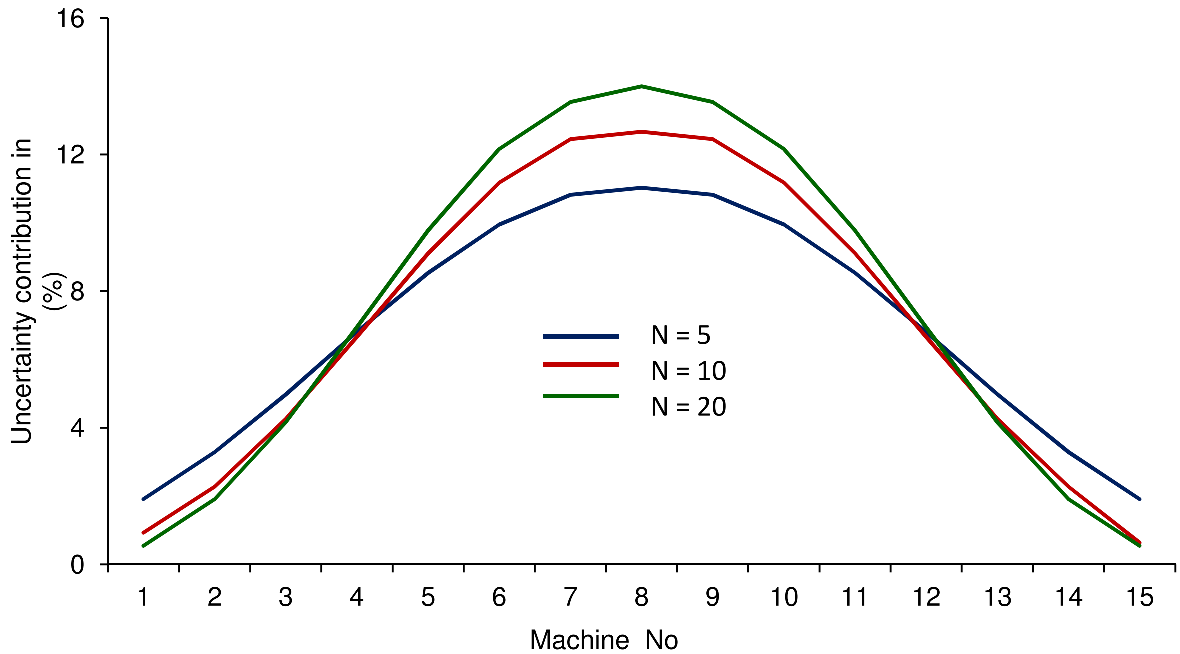


Figure 6:26 Uncertainty contribution in 15 machine line with different N

From the above graphs even when the machine line is longer the difference between the relative contribution of uncertainty between the most inner machines and the outer machines gets bigger as more buffer resources are allocated in a simple uniform manner. For instance for a buffer allocation of 5 buffers between each machine the ratio is (5.68:1) between the inner machines and the outer ones, but when the intermediate buffer capacity is increased to 20 this ratio further grows to (25.5:1). This can be an indicator on appropriate decisions from different perspectives. The first one is if the configuration has to remain the same for other productivity objectives then the improvement action to ensure more certainty on the output performance measure is to reduce input uncertainties on the machines that are located in the middle of the line. Alternatively if the decision lets the system to be reconfigured then a buffer reconfiguration that reduces the uncertainty in the middle of the lines there by decreasing the uncertainty of system performance can be addressed.

6.3.3 Uncertainty and line length

From system design point of view it is also an important aspect to understand how the uncertainty of individual machines that compose a multistage system would impact the overall uncertainty of the system performance. This problem is a particular interest as most production systems are interconnected systems, where simplified statistical assumption, such as independence might result bigger deviations from the real behavior of the line. For instance if one assumes that the presence of buffers between machines makes the uncertainties to be approximately independent, then the expectation of this assumption might be when more machines are introduced to make up a line then the uncertainty of the line would increase. One objective of this analysis is also providing answers to a general expected behavior that result from the combination of uncertain machines. For this goal a sample behavior of longer multistage system composed from identical uncertain machines with increasing line length is demonstrated below. All the machines in the line are identical and reliability parameters of the machines are as provided in Table 6:15. All the intermediate buffer capacities used are 5 for the results reported in Figure 6:27.

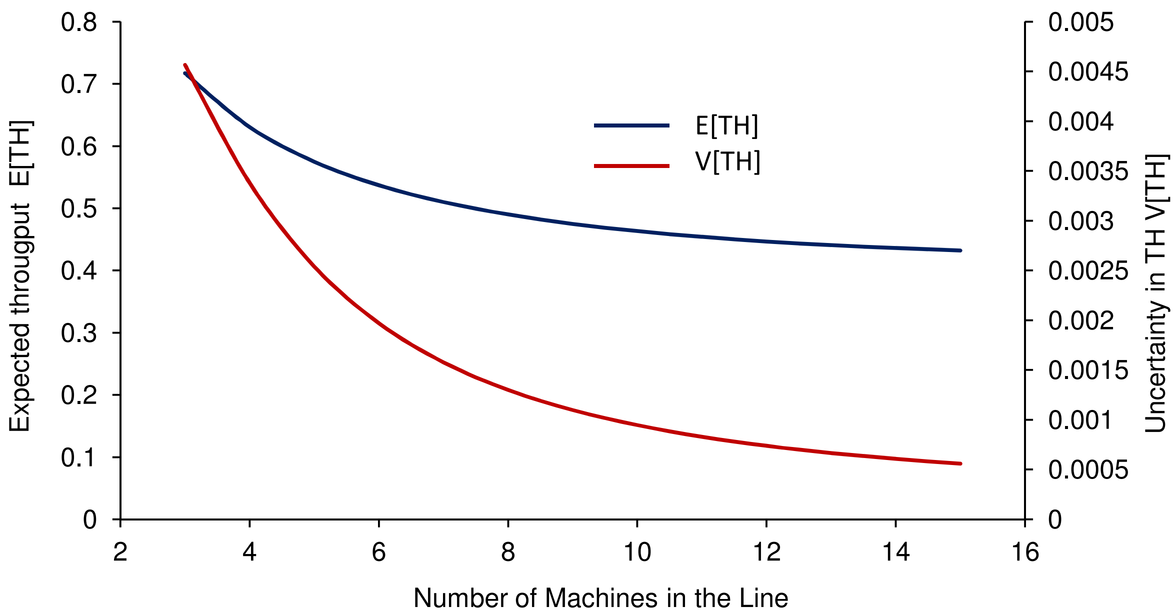


Figure 6:27 Average TH with increasing machine number in a line

On the above Figure 6:27 the analysis is done starting from a three machine line to 15 machine line. The isolated efficiency of each individual machines are identical and E[e] = 0.7881 and V[e] = 0.0052295. It is known from average system performance studies that the average throughput of the line composed of isolated machines goes decreasing as the number of machines increase due to the starvation and blocking phenomena that is introduced with each unreliable machine. This impact is shown in Figure 6:28 for three intermediate buffer capacities. The decrease in the average throughput is even significant with increasing number of machines as smaller buffer capacity means more probability of the line to be blocked and starved due to propagation of the two phenomena.

The uncertainty of the average throughput also decreases with increasing number of machines as in the sample case provided in Figure 6:29. It is important to know that even if the building blocks have an uncertainty on the isolated efficiency 0.00523, the line that is built from these machine has always has smaller uncertainty on the average throughput. For instance the 15 machine line that is made of from these uncertain building blocks has an uncertainty of 0.000558, which is approximately 1/10th of the isolated machines uncertainty.

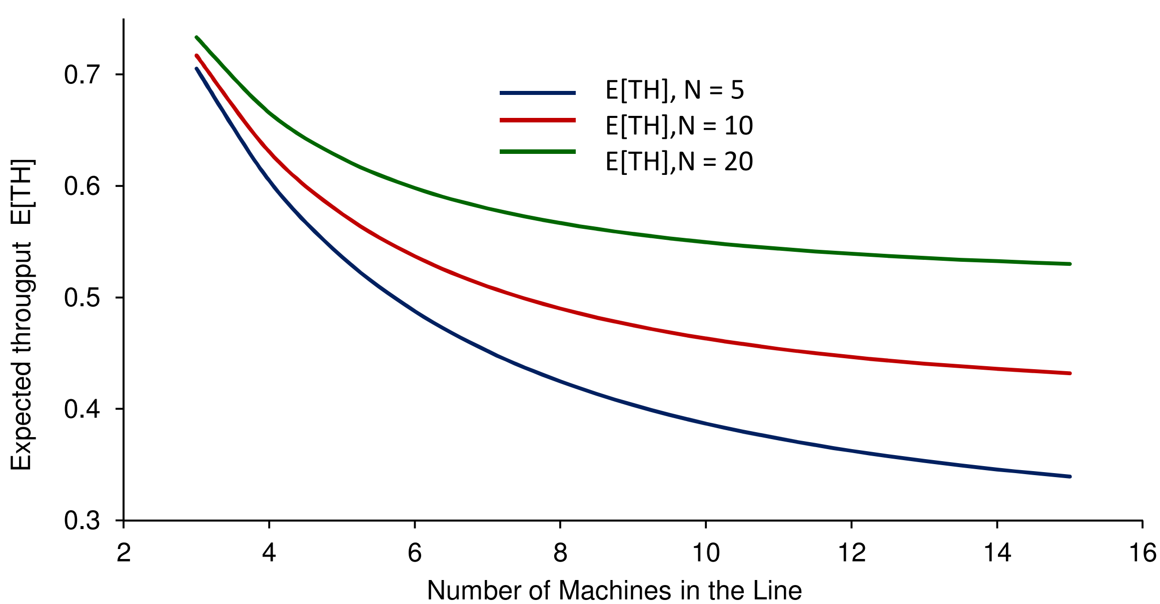


Figure 6:28 E[TH] with increasing line length

Additional experiment investigated the impact of more buffer on the uncertainty of a line is reported in Figure 6:29.

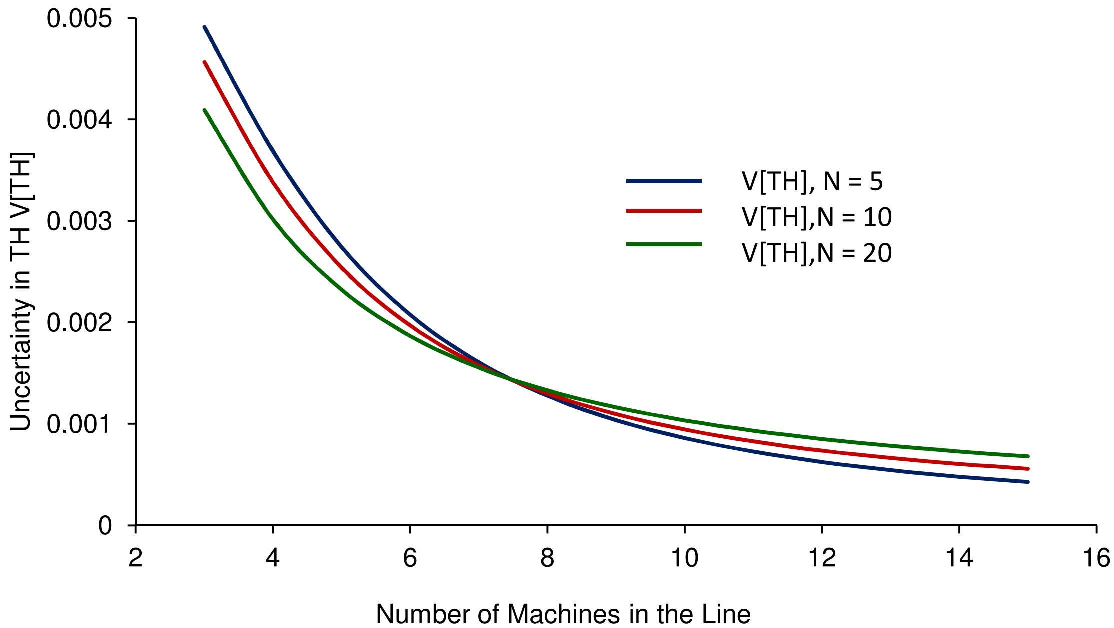


Figure 6:29 Uncertainty of average throughput V[TH] with increasing line length

This analysis addressed the problem of how the system configuration in a two machine line system impacts the uncertainty on the performance measure of the system. Additionally this problem is also revised from in relation to the input parameter uncertainties and bottle neck machine of the two machines system. Understanding how uncertainty in reliability estimation and the buffer capacity impact the resulting uncertainty in the performance measure is important for building blocks so that the overall design impact can be understood.

Similar analysis is extended for the analysis of longer lines considering the critical importance of the bottleneck machine. This analysis highlighted the role of bottleneck machine in determining the overall uncertainty of the line. The impact is also studied in combination with alternative buffer allocations so that decision can be made considering how it influences the overall system behavior.

Finally uncertainty contribution of estimated parameters on the total uncertainty is shown on multistage systems. This analysis investigated the analysis on multistage lines particularly equally buffered lines and their behavior under uncertainty. It discovered important behaviors of the system particularly indicating which machine should be monitored so that a better performance evaluation is achieved. This type of analysis leads to understand and how to better address and reduce uncertainty by focusing on optimal reduction of uncertainty from input parameters and specific system configuration. Similar analysis can provide an essential insight on how to reconfigure systems and how to allocate monitoring effect in a joint manner for a reliable operation of systems guaranteeing target performances.

Chapter Seven

7. System design under uncertainty

In the previous chapters it is introduced how performance analysis can be performed with uncertain parameters. It is also shown how uncertainty in input parameter estimation impacts the output performance measure uncertainty. Furthermore the analysis on multistage lines has shown how a system configuration can impact the performance measure uncertainty. Analysis on two machine line system is also investigated to demonstrate how a particular choice of system configuration also affects the uncertainty in the output performance given the same input parameters. The objective of this chapter is also addressing these problems from the two perspectives on reducing input uncertainty and choice of system configuration. The first section addresses how to better allocate sampling and data collecting efforts and resources in order to optimally reduce input uncertainty. The second views the problem on how to address the uncertainty reduction and design of robust system with configuration decisions.

7.1 Uncertainty reduction by optimal design of sample collection

This section introduces a technique for the optimal design of a sampling plan with the aim of reducing the uncertainty on the output performance measure. The technique shows how to improve the estimation of the output performance by reducing the input uncertainty particularly by aiming the reduction of uncertainty on those parameters which highly contribute to the output uncertainty. The sampling plan is made so that the allocation of resources for the acquisition and storage of new observations is done to bring higher impact on the output uncertainty performance measure. Periodic sampling plan is designed ahead for a period time T and then after actual observation is made the posterior distribution of parameters is updated using the Bayesian approach introduced in previous sections so that higher marginal improvement on the overall uncertainty of the output performance is achieved.

The assumptions made in this case are the behavior of the system parameters remain relatively the same and a planning period of time *T* is determined a priory. The problem can be formulated under constrained resources assumptions, such as the number of sensors to be activated at a time or the maximum number of acquisition modules to be activated in a single unit time is a capacitated resource.

The method uses a gradient approach on the output performance uncertainty model as a function of the parameter uncertainty. The model uses the sensitivity coefficients matrix and the individual marginal reduction of uncertainty in terms of variance for the input uncertain parameters.

$$V[TH] = \left|\sigma p_1, \sigma p_2, \sigma p_3, \ldots.., \sigma p_u\right| \times \begin{vmatrix} a_{11}, a_{12}, a_{13}, \ldots., a_{1u} \\ a_{21}, a_{22}, a_{23}, \ldots, a_{2u} \\ a_{31}, a_{32}, a_{33}, \ldots., a_{33} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a_{u1}, a_{u2}, a_{u3}, \ldots., a_{uu} \end{vmatrix} \times \begin{vmatrix} \sigma p_1 \\ \sigma p_2 \\ \sigma p_3 \\ \ldots. \\ \sigma p_u \end{vmatrix} \quad (7.1)$$

If the systems uncertainty is well approximated by the main factors of the parameters, which are indicated in the diagonal elements of the matrix in equation (7.1) as in the case for most of larger systems with many uncertain parameters the above matrix format can be simplified into a linear model as:

$$V[TH] = a_{11}\sigma^2 p_1 + a_{22}\sigma^2 p_2 + a_{33}\sigma^2 p_3 + \ldots\ldots + a_{uu}\sigma^2 p_u \quad (7.2)$$

Starting with the current uncertainty values of the parameters obtained from the previous observations for each uncertain parameter, we can compute the expected difference of the updated uncertainty of the new uncertainty with the additional observations planned to be made. But the expected number of new observations depends on the mean time to occurrence of the particular event for that parameter.

This mean time to occur (observe) designated *MTTO* can be approximated as:

$$\frac{(MTTR + MTTF)}{E[TH]} \tag{7.3}$$

If the sampling plan is to be made for next observation window of T time units then the vectors of expected number of observation for each parameter X_i can be computed as:

$$E[Xo_i] = \frac{T}{MTTO_i} \tag{7.4}$$

For a Beta distributed parameter with the parameter X_i the new uncertainty of X_i following a Beta distribution can be computed after the inclusion of the new observations is written as:

The variance of an uncertain parameter X_i, which is Beta distributed with parameters $Beta(\alpha_i, \beta_i)$ can be expressed in terms of its' variance as:

$${\sigma_i}^2 = \frac{\alpha_i \times \beta_i}{(\alpha_i + \beta_i)^2 \times (\alpha_i + \beta_i + 1)} \tag{7.5}$$

If it is assumed that the mean of the uncertain parameters to remain relatively the same in the next observation time, with this assumption we can write β_i in terms of the number of events observed α_i and the mean of the parameter μ_i.

$$\beta_i = \frac{(\alpha_i - \mu_i \alpha_i)}{\mu_i} \tag{7.6}$$

Substituting this expression in the above equation gives the variance in terms of the new observations and the parameter mean.

The initial current uncertainty of the parameters in terms of variance is computed as:

$$\sigma^2 p_i c = \frac{\alpha_i \times \frac{(\alpha_i - \mu_i \alpha_i)}{\mu_i}}{\left(\alpha_i + \frac{(\alpha_i - \mu_i \alpha_i)}{\mu_i}\right)^2 \times \left(\alpha_i + \frac{(\alpha_i - \mu_i \alpha_i)}{\mu_i} + 1\right)} \tag{7.7}$$

Furthermore the expected variance after the next set of observations is done is by substituting the current alpha updated with the expected number of observations.

$$\sigma^2 p_i n = \frac{(\alpha_i + E[Xo_i]) \times \frac{(\alpha_i + E[Xo_i] - \mu_i(\alpha_i + E[Xo_i]))}{\mu_i}}{\left((\alpha_i + E[Xo_i]) + \frac{(\alpha_i + E[Xo_i] - \mu_i(\alpha_i + E[Xo_i]))}{\mu_i}\right)^2 \times \left((\alpha_i + E[Xo_i]) + \frac{(\alpha_i + E[Xo_i] - \mu_i(\alpha_i + E[Xo_i]))}{\mu_i} + 1\right)} \quad (7.8)$$

Once we have written the current variance of the uncertain parameter and the expected uncertainty after the new expected observations we can compute the potential reduction of uncertainty on the output performance measure by each of the input parameters as:

$$\Delta\left(\sigma^2 p_i\right) = a_{ii}\left(\sigma^2 p_i c - \sigma^2 p_i n\right) \quad (7.9)$$

The above equation gives the vectors of values that give the expected reduction of uncertainty on the performance measure if the next observations are to be made on the corresponding parameters.

If the capacity of the resources available is limited to *C* then the allocation of these resources is committed so as to bring the largest reduction of uncertainty on *TH* and a gradient approach can be used starting from the vector which has the higher impact.

On the other hand problems formulated as a function of cost can be addressed using a similar approach, the main changes that must be applied in this case is the introduction of unit cost for acquisition of each parameter in the given period and the associated reward on the reduction of uncertainty on the output performance measure. This allows defining stopping criteria when a breakeven is achieved. The break even is reached when the costs of acquiring and collecting more data exceed than the benefits of reduced uncertainty in the performance measure. A simple stopping criterion also can be when an acceptable level of uncertainty is achieved.

Optimal design for uncertain parameters of a manufacturing system

This section introduces a methodology for an optimal design and control of parameters during the design phase of a manufacturing system. The problem of designing manufacturing systems involves decisions on how much uncertainty of the output performance can cost on the long run for an estimated deviation from the target value.

If the unit input costs are quantified for performing additional experiments, cost of underestimation and overestimation of performance then the long run costs can be grouped into the following categories:

- **Cost of uncertainty on the long run:**

Once the distribution of the performance measure is computed using the methods presented in chapter 4 then the corresponding Lower Limit (LL) and Upper Limit (UL) can be defined. Assuming additional information about a lost performance can be quantified as C_U and an idle capacity due to over design defined as C_o then the following costs can be determined.

1. LL and UL for the performance measure with the corresponding level of confidence have an expected cost

   $CU = (UCL - LCL) \times RT \times Cp$ Cost of uncertainty can be computed as the probability of over capacity and under capacity. *RT* is the reconfiguration time to change settings of the manufacturing system.

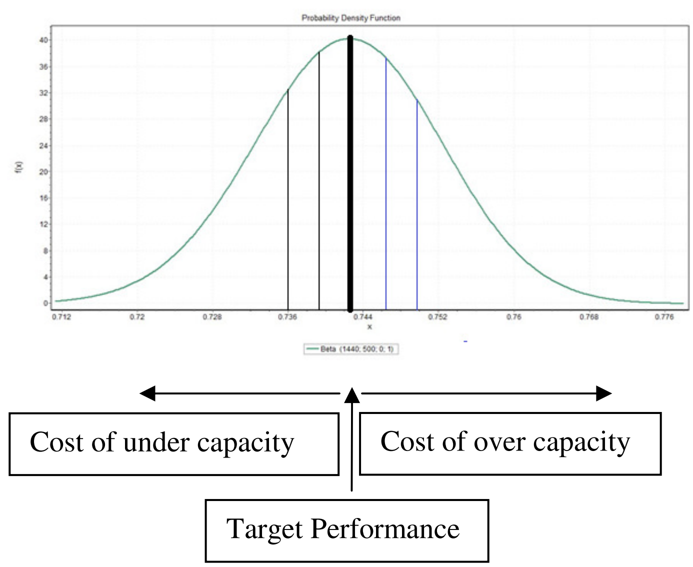


Figure 7:1 Costs related to underestimation and overestimation

If the costs of under capacity and over capacity for a unit time are given as c_u and c_o, then the cost of uncertainty can be computed as:

2. The expected cost of under capacity and can be thought as the number of product units that are not met and lost sales in the case of over capacity it can be thought of as the wasted capacity or number of over produces per unit time and the total cost of uncertainty is computed:

$\sum_{i=1}^{n} \left| TH_i - \widehat{TH} \right| * C_u * p_i * RT$ Cost of underestimation

$\sum_{i=1}^{n} \left| TH_i - \widehat{TH} \right| * C_o * p_i * RT$ Cost of overestimation

$$C_{uncert} = \sum_{i=1}^{n} \left| TH_i - \widehat{TH} \right| * C_u * p_i * RT + \sum_{i=1}^{n} \left| TH_i - \widehat{TH} \right| * C_o * p_i * RT$$

Extra observations and additional experiments reduce the dispersion of the output performance distribution and probability of farthest point gets smaller. Overall cost of uncertainty is expected to drop with the additional experiments as an exponential decay depending on the cost coefficients and the performance function.

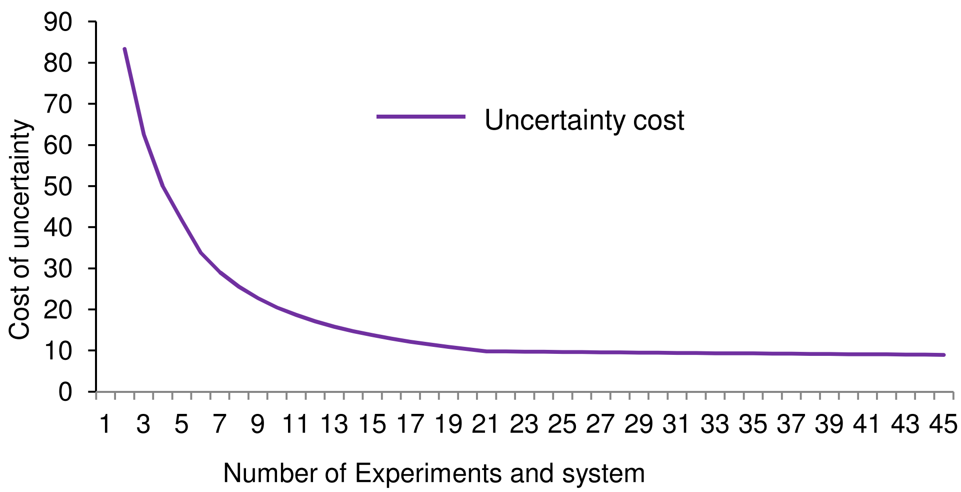


Figure 7:2 Cost of uncertainty with additional observations

Cost of running experiments, collecting data and analysis

Additional data collection/ experimentation and analysis to improve the uncertainty of the output performance incur additional costs that grow with more experiments to be included.

The additional experiments are chosen using a gradient approach on the parameters that introduce the higher improvement on the output performance and the lower cost in terms of obtaining the respective information. Even though these selections are chosen in an optimal way there is additional cost associated to them. The decision of steps is also important to update the sensitivity matrix and where to make the next experiment both on better cost and improved certainty on performance measure.

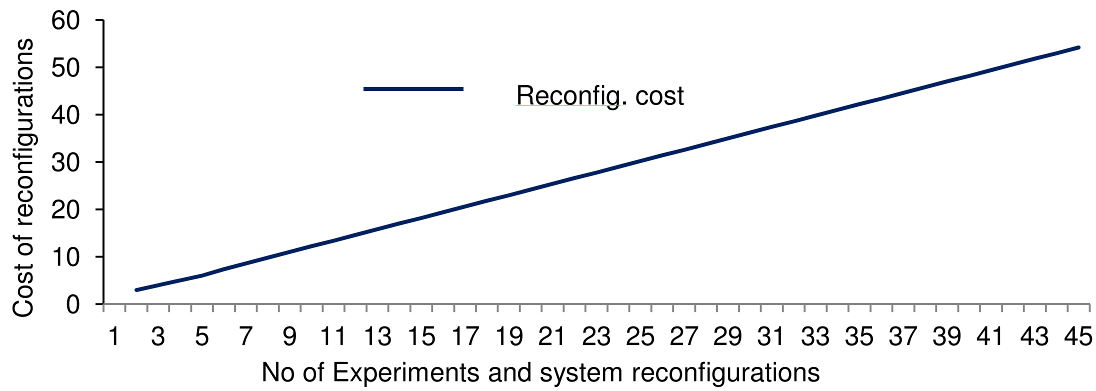


Figure 7:3 Total cost of additional experiment

The number of data collection/experiment can be computed per unit or per batch for all available option of uncertain parameter.

The total cost of the optimal decision is expected to have the following configuration giving an optimal stopping criteria for the decision making process.

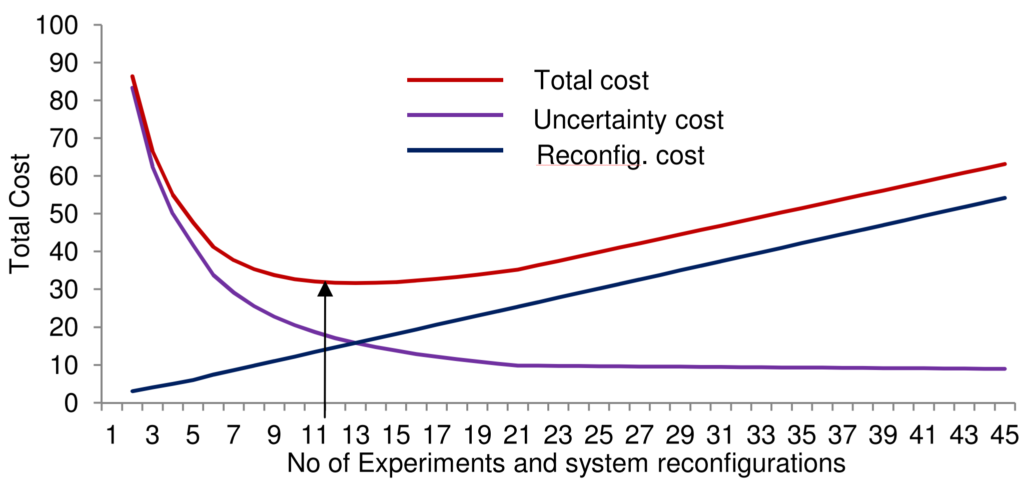


Figure 7:4 Aggregated cost of uncertainty and experiments

The uncertainty in the output performance can be computed as a cost in the long run as a function of the width associated to a given level of confidence interval. On the other hand the costs related to the data collection and analysis can be considered as incremental with respect to the number of data that must be collected in order to achieve the desired level of confidence in the performance measure. The evaluation of the design problem can be done on stages, each stage evaluating the realized improvements and the updated sensitivity matrix. Particularly a starting solution is required to estimate the relationship of parameters and the corresponding uncertainty related to the output performance measure. The structure of the algorithm looks as follows:

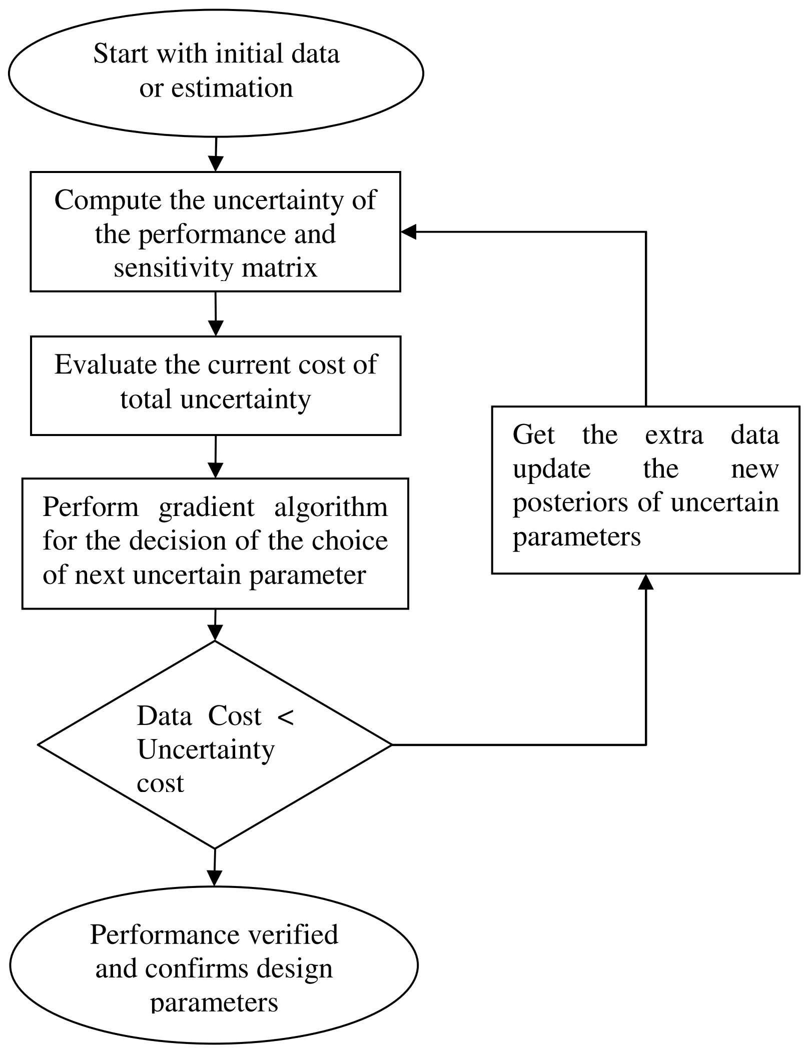


Figure 7:5 Structure of gradient algorithm for reconfiguration decision

7.2 Buffer allocation under uncertainty

In this section, the impact of parameters' uncertainty on the optimal buffer design is investigated. The analysis begins with a two machine line single buffer systems based on the original formulation. The original buffer allocation problem searches for the minimal total buffer capacity that is required to meet a desired target throughput (TH*). However, it is solved in the literature only for precisely known input parameters. The objective of this analysis is to extend this original formulation and study the impact of uncertainty in input parameters on the decision on the subsequent decision on buffer capacity.

7.2.1 Analysis of buffer allocation in a two machine line

Traditionally this problem is formulated to determine an optimal buffer capacity when the parameters are deterministic. It searches the minimal buffer capacity that satisfies a target TH*. This problem can be analytically solved for two machine lines with single failures, as the formula for the average throughput can be determined in a closed form as a function of precisely known p_u, r_u, p_d and r_d.

Using the relation between TH and N in (4.62) for the two machine line the N_{min}, the minimum buffer capacity which satisfies the target average throughput TH* can be determined with (7.10).

$$N_{\min} = \frac{ln\left(2Ln(x)+\dfrac{ed*l-\dfrac{ed(eu-ed)*l}{TH*-ed}}{eu*m}\right)}{ln(x)} \tag{7.10}$$

Where *ln* is the natural logarithm function

When there is uncertainty is introduced in the analysis, the traditional buffer allocation problem can be extended to find the minimal total buffer capacity that is required to meet a desired throughput level TH*, with a specified probability level,(1-α). Analysis including uncertainty in input parameters is important since it introduces a link on how the uncertainty in the input parameter impacts the probability in achieving the target throughput TH*. This would provide robustness to the system design. In this section, we investigate the relation between the solutions to the original formulation and the new formulation with the introduction of uncertainty in a two-machine system with uncertain parameters.

Applying the methods for the performance evaluation under uncertainty in section 4 such as the discretization technique this problem can be solved and the distribution of N* can be reconstructed. This distribution then can be used to determine the level of N that satisfies TH* with a probability of (1-α), i.e., $N_{min} = MIN (N \mid (P (TH \geq TH^*))$.

Therefore when *p* and/or *r* are uncertain the problem can be formulated to determine the minimum buffer capacity (N_{min}) such that N at least guarantees the target throughput TH* with a specified probability (1-α). $N_{min} = MIN(N \mid (P(TH \geq TH^*))$.

Alternatively the problem can be determining the probability (1-α) that can satisfy a target throughput TH* given a buffer capacity N. $(1-\alpha) = P(TH \geq TH^*) \mid N$

To demonstrate the impact of uncertainty in this problem a two machine single buffer line with parameters reported in Table 7:1 is analyzed. The problem is to determine a buffer capacity that will guarantee a target TH* of 0.8 with a probability of (0.95). This problem is solved using both the original formulation and the proposed formulation that includes the uncertainty for the upstream failure probability p_u. The difference between the two analyses is also demonstrated using a graphical interpretation in Figure 7:6.

p_u	r_u	p_d	r_d	TH*
~Beta(5,245)	0.4	0.04	0.4	0.8

Table 7:1 Parameters for buffer allocation in a two machine line

N* with respect to the distribution of p_u is as shown in the Figure 7:6.

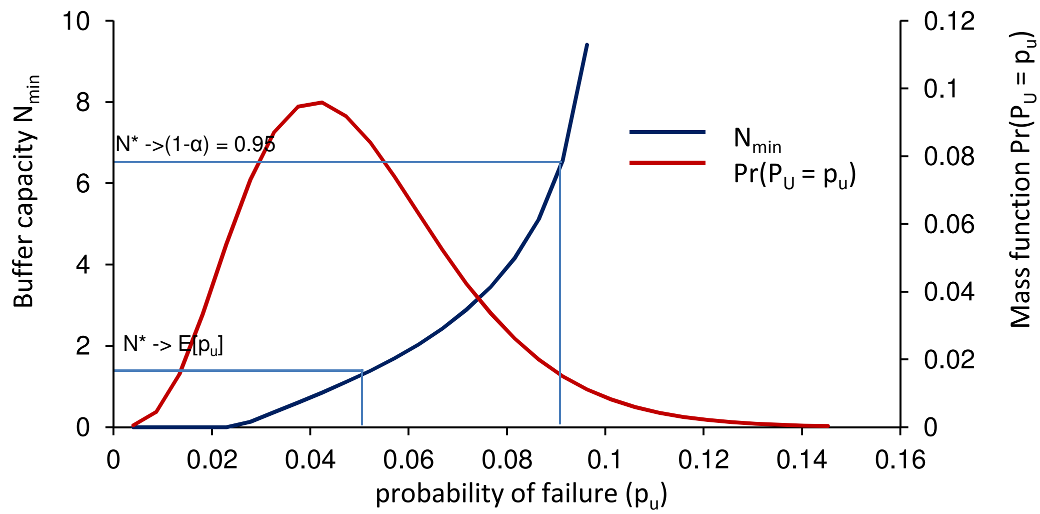


Figure 7:6 Buffer capacity determination with uncertain failure probability

With the original formulation the data necessary for the analysis will be the same as in Table7.1, except p_u in this case is the expected value, i.e., $p_u = 0.05$. Evaluating the minimum buffer capacity N_{min} with results;

$$N_{min} = 1.3$$

The results in the above analysis shows that if only the expected value of the failure probability without the uncertainty is considered then the optimal buffer capacity that satisfies a target TH* of 0.8 is 1.3.

Under the second formulation if the uncertainty in p_u is included and the problem is to achieve the same target TH* of 0.8 with a confidence level of $(1-\alpha) = 0.95$, then the choice of the buffer capacity that satisfies this criteria is;

$$N_{min} = 6.2$$

The buffer capacity decision that ignores the uncertainty in the estimation of p_u i.e., 1.3 can only provide a guarantee of 0.57 that the target TH* can be met. Considering there is a high probability that this decision fails to attain the target TH* of 0.8 it could lead to a highly compromised performance reliability.

Another observation from this particular problem and Figure (7.6) is, for the failure probability distribution $p_u > 0.1$, the target TH* can't be satisfied regardless of how big the buffer capacity choice will be. If this is the case then the N_{min} for this region with $p_u > 0.1$, can be assumed N_{max} a defined level of allowed maximum buffer capacity. For this reason deriving the decision based on the distribution of N* can be less informative when much of the region falls under the infeasible area, where any buffer size can't satisfy the target TH*. This problem instead can be addressed by using the distribution of TH and searching for the N_{min} that satisfies TH* with a probability of (1-α).

Evaluation of (1-α) = P(TH ≥ TH*)| N_{min}

Two cases are provided in the alternative problem formulation to compute the *P(TH ≥ TH*) ∣ N_{min}*, given p_u is uncertain while all the other parameters are considered precisely known. The results provide additional view on how the probability of satisfying a TH* varies with N_{min}. Particularly in the 2nd case, that last row *(inf)* shows how for the TH* even infinite buffer capacity can't guarantee us beyond a certain probability threshold i.e. *P(TH ≥ TH*) ∣ N_{inf}*.

Case 1

P_1	r_1	p_2	r_2
~Beta(5,245)	0.4	0.04	0.4

N	P(TH ≥ 0.8*) ∣N	P(TH ≥ 0.82*) ∣N	P(TH ≥ 0.85*) ∣N	P(TH ≥ 0.9*) ∣N
0	0.741	0.513	0.138	0.000
1	0.992	0.942	0.640	0.001
3	1	1	0.970	0.031
5	1	1	1	0.222
20	1	1	1	0.951
Inf	1	1	1	0.988

Table 7:2 Probability of satisfying TH* in a two machine line case 1

Case 2

P_1	r_1	p_2	r_2
~Beta(5,245)	0.15	0.04	0.25

N	P(TH ≥ 0.8*) ∣N	P(TH ≥ 0.83*) ∣N	P(TH ≥ 0.85*) ∣N	P(TH ≥ 0.86*) ∣N
0	0.074	0.004	0.000	0
5	0.520	0.121	0.003	0
10	0.780	0.339	0.047	0
20	0.910	0.660	0.266	0.010
40	0.953	0.828	0.551	0.182
inf	0.957	0.872	0.778	0.671

Table 7:3 Probability of satisfying TH* in a two machine line case 1

Determination of buffer capacity from the TH distribution

In many practical cases observations are obtained from online data collection system with each estimated parameter subjected to some level of uncertainty. When decisions have to be made based on estimations with limited a high probability of achieving the target TH* can be guaranteed with more buffer capacity. As more observations become available the buffer capacity can be reconfigured depending on the updated new data coming from the new observations. An approximate iterative approach is proposed to solve these problems.

In this formulation a feasible starting value for N_{min} can be obtained such that $N < N^* \mid E[p]$ by using the expected values of the uncertain parameters. The throughput distribution TH of the two machine line the probability $P(TH \geq TH^*)$ can be evaluated using one of the convenient approaches proposed in chapter 4. Then the process continued by incrementing N until the condition $P(TH \geq TH^*) \geq (1-\alpha)$ is satisfied and N_{min} is determined.

Generally the first two moments of TH E[TH] and V[TH] are evaluated directly as in the Taylor approximation method or the distribution of TH can be available if the analysis is done as in the discretization method. In both cases the distribution of TH can be reconstructed from the first two moments. Experiments are conducted on the cumulative distribution using a Beta approximation against the cumulative joint distribution from direct evaluations. The approximations obtained from the two moments have shown the approximation errors are less than 3%. The parameters for the approximated beta distribution can be evaluated by;

$$\alpha_{TH} = \frac{E[TH]^2 - E[TH]^3 - E[TH]V[TH]}{V[TH]} \tag{7.11}$$

$$\beta_{TH} = \frac{E[TH] - 2E[TH]^2 + E[TH]^3 - V[TH] + E[TH]V[TH]}{V[TH]} \tag{7.12}$$

Given the distribution of TH as $Beta\left(\alpha_{TH}, \beta_{TH}\right)$ and its density $f_{TH}(TH)$, can be used to evaluate the probability $P(TH \geq TH^*) = 1 - F_{TH}(TH^*)$.

Equivalently $P(TH \geq TH^*) = 1 - \int_0^{TH^*} f_{TH}(TH)dTH$.

An iterative incremental algorithm is proposed to find the minimum N_{min} following the structure shown in Figure 7:8. If a probability level (1-α) is defined and starting feasible the buffer capacity N is increased with a chosen step size the iteration is continued until $P(TH \geq TH^*) \geq (1-\alpha)$ is satisfied. A general the impact of increasing N is shown to push the distribution of $P(TH \geq TH^*)$ curve further to the right thereby increasing $P(TH \geq TH^*)$ for a specified TH^* as shown in Figure 7:7.

To illustrate how the proposed algorithm works with increasing buffer capacity is applied on a two machine line single buffer line with parameters provided in Table 7:4 are used. The results are reported on Figure 7:7 at four selected points of buffer capacities 3,5,10 and 20.

P_1	r_1	p_2	r_2
~Beta(5,245)	0.15	0.01	0.25

Table 7:4Parameters for buffer allocation in a two machine line

This problem is evaluated for a target TH* = 0.73 and (1-α) = 0.9 and the P(TH≥TH*) for the four buffer capacities shown in Figure 7:7 are;

N	3	5	10	20
P(TH≥TH*)	0.621	0.768	0.875	0.901

Table 7:5 P(TH≥TH*) at four algorithm stages

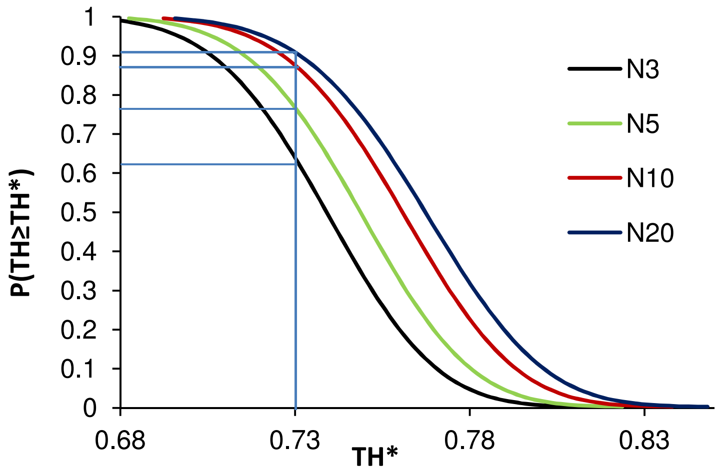


Figure 7:7 Increasing N to achieve TH* in a two machine with (1-α)

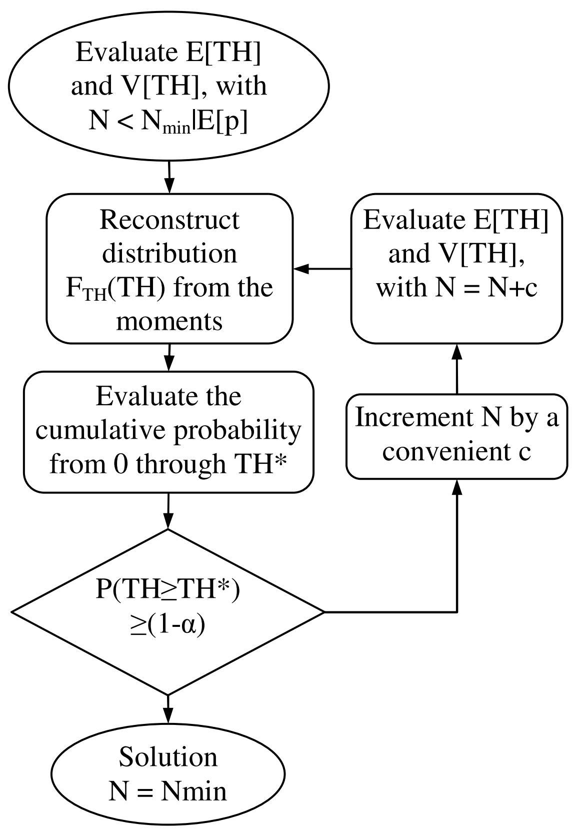


Figure 7:8 Iterative algorithm to evaluate Nmin with uncertain input parameters

In a scenario where $E[TH]$ is assumed to remain the same between successive observations and decreasing $Var[TH]$, the relationship between $P(TH \geq TH^{*})$ and TH^{*} is shown in Figure 7:9 for a fixed N.

E[p_1]	r_1	p_2	r_2	N
0.025	0.21	0.0234	0.23	10

Table 7:6 Input data for the sample buffered two-machine line.

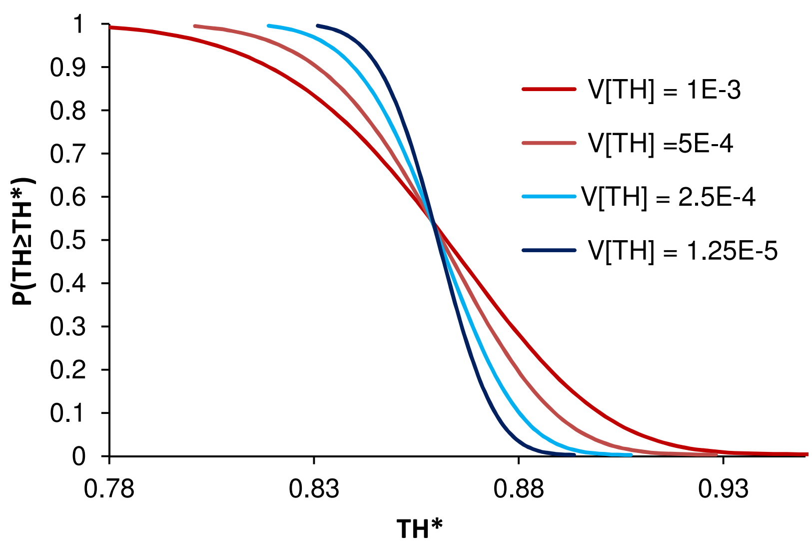


Figure 7:9 P(TH≥ TH*) for different input uncertainties with the same expected value

Replicated experiments are performed on randomly generated reliability data; in this particular case p_u is inferred from generated samples of TTF by considering an observation period of 1600 cycle units as sampling interval. As more data is collected and performance is analyzed similar trends can be seen in Figure 7:12. Same experimental settings are used as in Table 7:6, with sequential observations used to update previous ones.

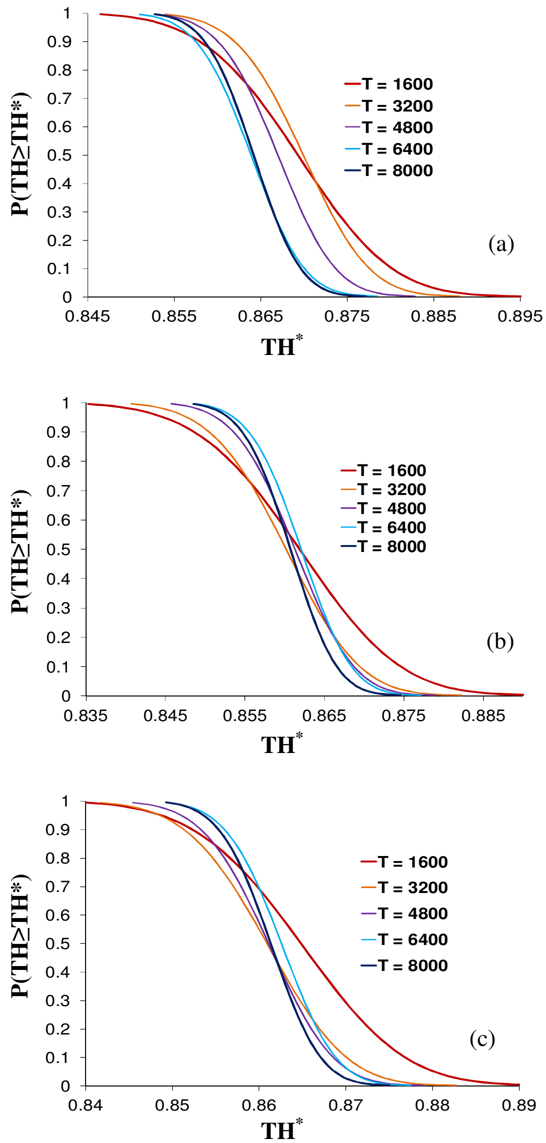


Figure 7:10 (a), (b), (c) results with sequential updating of observation in two machine line

Randomly generated 20 cases are simulated in order to study the effects comparative decisions based on traditional approach and the proposed method. Simulated *TTF* values are used for the estimation of p_u with probability equal to 0.025 until 20 failures observations are available to make an inference on p_u. The first sets of experiments are conducted by using the expected value of the estimated from the observed *TTF*. A second set of experiments are conducted using an uncertain estimate of p_u. In both cases the problem is to determine the minimum level of N^* that is required to achieve the target throughput TH^*

(0.84). A probability level $(1-\alpha)$ = 0.90 is chosen in order to guarantee TH^* for the second set of experiments. Summary of the input parameters for the experiments and results reported in the Table 2 and 3 respectively.

$E[p_u]$	**~p**	**r_u**	**p_d**	**r_d**
0.025	Beta(20,β)	0.21	0.0234	0.23

Table 7:7 Input data for the sample buffered two-machine line.

Exp No	(α+β)	$E[p_u]$	N*| $E[p_u]$	N*|~ p_u
1	834	0.0239	4	9
2	899	0.0222	3	7
3	837	0.0239	4	9
4	803	0.0249	5	10
5	708	0.0284	7	17
6	901	0.0222	3	7
7	704	0.0284	7	17
8	693	0.0289	7	19
9	1069	0.0187	2	5
10	752	0.0266	6	13
11	916	0.0218	3	7
12	999	0.0200	2	6
13	662	0.0302	8	26
14	676	0.0296	8	22
15	761	0.0263	5	12
16	834	0.0240	4	9
17	1003	0.0199	2	5

18	767	0.0261	5	12
19	793	0.0252	5	11
20	896	0.0223	3	7

Table 7:8 Decisions based on the traditional and proposed method

The buffer capacity N^* that guarantees the target TH^* = 0.84 on the long run is 5. From the experimental results the decisions based on the first problem 10 of the decision on the buffer capacity N don't guarantee the desired TH^*.Theoretically if the number of experiments is statistically significant increased there is 43% of achieving the target TH^*. In the problem that considers the uncertainty all decisions on N^* guarantee TH^*. Theoretically since $(1-\alpha)$= 0.90 is chosen there is 90% of probability that TH^* falls inside decisions constructed in this way. In general the effect of increasing N is shown to push the curve further to the right thereby guaranteeing a higher TH^* for the same level of probability $(1-\alpha)$ as shown in Figure 7:11.

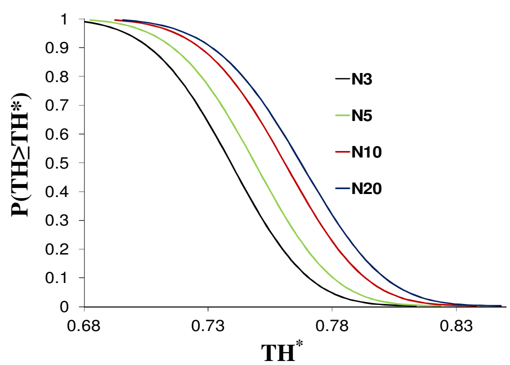


Figure 7:11Probabilities of achieving TH* for different buffer capacities N

7.2.2 Buffer space allocation in long lines

This section demonstrates the buffer allocation problem in long lines considering uncertain parameter estimates. The original buffer allocation problem using the gradient method is used in combination with the performance evaluation using uncertain parameters. In literature the buffer allocation problem is formulated as Primal and Dual problems. In the primal problem the aim is to find the buffer space configuration (N_1,..., N_{K-1}) in a linear multistage line, that minimizes the total buffer space in order to achieve a target production rate requirement, TH_{obj}. The formalization of this problem, by using the formalism of linear programming, is reported in the following equations:

Minimize $N^{Total} = \sum_{i=1}^{K-1} N_i$

Subjected to:

$$P(TH(N_1,...N_{K-1}) \geq TH^*) \geq (1-\alpha)$$

$$N_i \geq N^{MIN} = 3, \qquad i = 1,...,K-1 \tag{7.13}$$

The Dual problem has the goal of finding the maximal production rate of the line subject to a total buffer space constraint. Its formalization is given in the following equation:

Maximize $TH(N_1,...N_{K-1})$

Subjected to: $N^{Total} = \sum_{i=1}^{K-1} N_i$

$$N_i \geq N^{MIN} = 3, \qquad i = 1,...,K-1 \tag{7.14}$$

The following experiments demonstrate the proposed technique on the buffer space allocation problem in long lines. The results are given in comparison with traditional approach that doesn't consider uncertainty in the parameter estimates. The results from the method with the proposed algorithms are compared with those provided by other methods, following the same assumption, such as Gershwin-Goldies method. Four of the six cases of ten machine lines originally studied by Gershwin and Goldies have been analyzed. These ten-machine lines are investigated to show the impact of introducing robustness in the analysis of buffer allocation. The objective is to decide the buffer space allocation that can achieve an objective average throughput, TH* of 0.75. In the analysis considering the uncertainty of input parameters a probability $P(TH \geq TH^*)$ = 0.95 is desired. The analysis for line A and B are conducted using 10,000 cycles of observation time while line C and D are observed for 5,000 cycle time. The failure probabilities of the machines are assumed to be estimated from the corresponding observation times. The resulting mean and the uncertainty in variance for *p* used in the analysis are reported in Table 7:9. Finally results obtained by the proposed method (P) in comparison with the analysis from the buffer allocation using the traditional method (T) are reported in Table 7:10.

Line	A	B	C	D
$E[p_1]$	0.007	0.007	0.007	0.007
$Var[p_1]$	6.93×10^{-7}	6.93×10^{-7}	1.39×10^{-6}	1.39×10^{-6}
r_1	0.095	0.095	0.094	0.095
$E[p_2]$	0.007	0.008	0.008	0.01
$Var[p_2]$	6.93×10^{-7}	7.9×10^{-7}	1.59×10^{-6}	1.98×10^{-6}
r_2	0.095	0.094	0.095	0.09001
$E[p_3]$	0.007	0.006	0.003	0.003
$Var[p_3]$	6.93×10^{-7}	5.93×10^{-7}	5.93×10^{-7}	5.99×10^{-7}
r_3	0.095	0.093	0.045	0.09102
$E[p_4]$	0.007	0.007	0.004	0.005
$Var[p_4]$	6.93×10^{-7}	6.93×10^{-7}	7.93×10^{-7}	1.00×10^{-6}
r_4	0.095	0.094	0.078	0.09903
$E[p_5]$	0.007	0.005	0.006	0.001
$Var[p_5]$	6.93×10^{-7}	4.95×10^{-7}	1.19×10^{-6}	1.96×10^{-7}
r_5	0.095	0.095	0.069	0.09504
$E[p_6]$	0.007	0.006	0.007	0.009
$Var[p_6]$	6.93×10^{-7}	5.94×10^{-7}	1.38×10^{-6}	1.77×10^{-6}
r_6	0.095	0.093	0.094	0.09205
$E[p_7]$	0.007	0.009	0.008	0.009
$Var[p_7]$	6.93×10^{-7}	8.9×10^{-7}	1.59×10^{-6}	1.77×10^{-6}
r_7	0.095	0.095	0.095	0.09706
$E[p_8]$	0.007	0.008	0.003	0.003
$Var[p_8]$	6.93×10^{-7}	7.93×10^{-7}	6.00×10^{-7}	5.98×10^{-7}
r_8	0.095	0.094	0.045	0.09607
$E[p_9]$	0.007	0.007	0.004	0.008
$Var[p_9]$	6.93×10^{-7}	6.93×10^{-7}	7.93×10^{-7}	1.59×10^{-6}
r_9	0.095	0.096	0.078	0.09208

$E[p_{10}]$	0.007	0.008	0.006	0.007
$Var[p_{10}]$	6.93×10^{-7}	7.93×10^{-7}	1.19×10^{-6}	1.38×10^{-6}
r_{10}	0.095	0.095	0.069	0.09409

Table 7:9 Failure and repair parameters for long line experimental case studies

L	Method	Buffer i									**Tot**
		1	2	3	4	5	6	7	8	9	
A	T	5	5	9	10	10	10	9	5	5	**68**
	P	5	6	10	11	11	11	10	6	5	**75**
B	T	5	6	8	9	10	11	10	6	5	**70**
	P	5	7	9	10	11	11	11	8	5	**77**
C	T	5	7	9	12	13	14	11	5	5	**81**
	P	5	9	11	14	16	16	14	7	5	**97**
D	T	5	5	6	7	8	10	7	5	5	**58**
	P	5	7	7	8	8	12	9	6	5	**67**

Table 7:10 Failure and repair parameters for long line experimental cases studies

Comparing the results obtained with the proposed approach it can be seen from the results in Table 7:10 that account for the estimation uncertainty increases the solution of the total buffer capacity. And this difference is bigger for higher uncertainties emphasizing the importance of robustness in design when less information is available.

Line	A	B	C	D
E[TH]	0.7614	0.7610	0.7687	0.7662
Var[TH]	3.97×10^{-5}	3.93×10^{-5}	1.14×10^{-4}	8.38×10^{-5}

Table 7:11 Final statistics of TH that satisfies target TH.

Line	A	B	C	D
TH	0.7500	0.7501	0.7500	0.7501
$P(TH\geq TH^*)$	0.503	0.505	0.500	0.501

Table 7:12 Final TH and level of probability using the traditional approach

The comparative expected values of the using the traditional approach that doesn't consider the uncertainty in estimation and the proposed approach provide different expected average TH. This can be seen from the above results reported in Table 7:11 and 7:12.

Analysis performed in this chapter demonstrated the problem of introducing real data estimated parameter or incomplete knowledge on the evaluation of optimal designs. It emphasizes the importance of introducing level of knowledge about parameters leads to robust designs. In contrast optimal decisions without including this aspect might lead to less reliable and compromised performance of the final system.

The focus on the design phase also takes into account that the decisions that highly impact the operation of a system are done at this phase and costly to change. Therefore the proposed framework that considers the formulation of the problem considering the overestimation and underestimation of performance shows the impact of alternative decisions on the total cost of the system. Taking into individual impact of parameters on the global performance of a system in complex manufacturing system is essential aspect of designing a robust and optimal system.

Chapter Eight

8. Case study: Scania D12 engine production line

The proposed method for the evaluation of multi-stage line chapter 4 is used to assist the performance analysis of a real manufacturing line in Scania Group, producing D12 engines. The problem of performance analysis on this real system is interesting for two main reasons. The manufacturing system implemented a process monitoring and data acquisition system that uses a database system for recording data automatically and semi-automatically from shop floor machines via network. The records from the data collection system can be used for inference making on input parameters and study can be carried out on output uncertainty related to estimations. Besides the line is studied using performance analysis methods that didn't consider the uncertainties in the estimation of reliability parameters of machines. Introducing the estimation uncertainty and studying the systems from this new dimension is the objective of this analysis. Inputs used in this analysis are obtained from authors of a previous study (M. Colledani, 2005), who conducted previous analysis on the manufacturing line. The analyses that are summarized in this study give primary focus on the results obtained with the proposed method in this thesis.

8.1 Company profile

Scania Group is a leading European manufacturer of heavy trucks and buses, as well as industrial and marine diesel engines, with headquarter located in Södertälje, Sweden. It is the world's third largest producer for heavy trucks and the world's third largest producer in the heavy bus segment. Scania is a global company with a sales and service organization in more than 100 countries. Aside from sales and services, Scania offers financial services in many markets. Scania's production units are located in Europe and Latin America.

Currently Scania has approximately 37,500 employees. 16,000 employees work with sales and services in Scania's own subsidiaries worldwide. About 12,400 people work at production units in seven countries and regional product centers in six emerging markets.

8.2 The Processing Line

D12 is a transfer line that produces the 6-cylinder engine-block and belongs to unit Engine Melt and Machining, placed in Södertälje. The line that produces engine block is composed by 23 stations which can be NC machines, light assembly machines, washing machines and quality control stations and 22 inter-operational storage buffers. In total 23 main groups of operations are performed. At the beginning of the line raw parts are automatically unloaded from the pallets and lifted up on the conveyor. A cast block is supplied at beginning of the line and, after all operations take place, the engine block is ready for the assembly line. The cast block enters the line upstream the first station which performs OP 020 and, after all the operations have been visited in sequence, the engine block leaves the system and it is ready for the assembly line. The layout of the system is shown in Figure 3:2.

Therefore, the system can be modeled as a transfer line. Given the fact that the last station has no failures and do not affect the performance of the rest of the line, 22 stations are considered in the analysis. As it can be noticed by the complete layout that was reported in Section 3, three sections can be identified in the line, composing an S-shaped system. In section 1 the work pieces are lifted up on the conveyor and sides, planes and gables are milled. Rough drilling and long-hole drilling are also performed. In section 2 sides, planes and gables are then drilled and threaded. After washing takes place, the automatic assembly of the crankshaft bearing caps is performed. In the last portion of line finishing milling, drilling and facing are performed and camshaft bearings are automatically pressed. After the final washing and drying the cup plugs are assembled. The last two operations are a tightness testing and a final manual control.

From its original design, the line was thought with the current S-shaped layout. The reason for that is the will of highly decoupling the behavior of the three portions of line, in order to allow one section to continue the operation, while another line section is under maintenance, thus preventing stopping the whole line. This concept has been implemented by adding two long buffer conveyors where the two curves take place. Therefore, the need for guarantee the quality of produced products has impacted on the system design, from its conception. One of the objectives of the carried out analysis is to quantify the impact of this choice. Finally, the system does not need major set-up because other types of engine block, with a different number of cylinders are machined by other manufacturing lines.

The 22 machine line is affected by a total of 144 failure modes in total. The number of failure and disruption type for each machine, the intermediate buffer between machines and the processing times in minutes for each coded machine are reported in Table 8:1.

s	Types of disruptions	Downstream Buffer capacity	Cycle times (min)
OP20	6	7	6.40
OP30	6	9	5.85
OP40	6	4	5.62
OP50	6	4	6.08
OP60	6	3	6.08
OP70	6	3	4.58
OP80	5	18	6.27
OP90	6	5	6.40
OP100	6	4	6.08
OP110	6	5	5.77
OP120	6	5	6.20
OP130	5	4	5.35
OP140	8	34	10.00
OP150	5	6	5.32
OP160	6	4	6.22
OP170	5	3	5.25
OP180	7	5	5.85
OP190	7	3	6.20
OP200	6	3	5.42
OP210	5	3	5.80
OP220	5	4	4.13
OP230	6		5.50

Table 8:1 Machines in the processing line and parameters

8.3 Line monitoring and supervisory system

The system is endowed with a production monitoring system. In particular, according with ISO standards each station is equipped with alarm lights representing the machine states. Four different colors can be emitted: white, yellow, green and red. When a machine is processing a part the light is white, when it is blocked or starved the corresponding light is the green and when a failure occurred, red light is used to signal the disruption. Warning state (yellow light) is used for predictive maintenance or before a green state. Also two big electronic tables are hung on the ceiling of the factory shed. One is connected to the machine state lights and represents a scheme of the 23 operations along with the actual state. It is visible to all the operators attending the line. The other shows the produced parts from the beginning of the shift, the respective goal and an estimation of the number of blocks that will be produced up to the end of the shift.

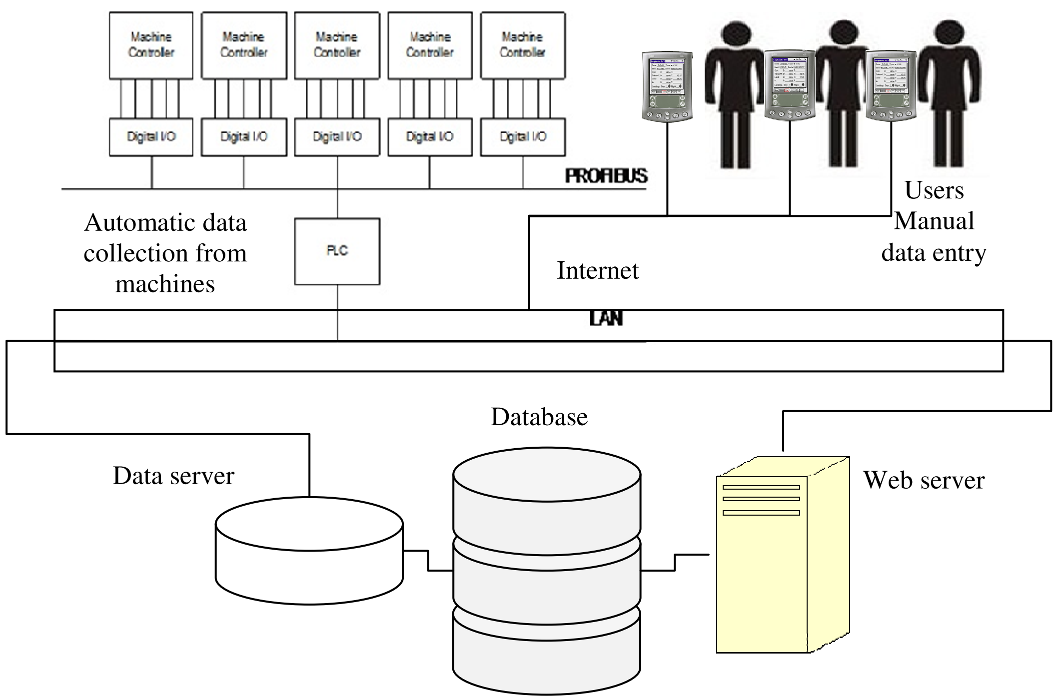


Figure 8:1 Schematic diagram of the data acquisition system

The manufacturing system is also equipped with an automatic data collection system. Data, such as the time between failures and between repairs are logged accurately by the PLC

(Programmable Logic Controller) of each station. The process monitoring system, named PUS (Process Uppföljnings System), is a web-based tool for production monitoring developed by B4Industry Company. A schematic representation of the entire system is given in Figure 8:1.

It contains a database that is used to store production data, collected automatically or semi automatically from the shop floor, and to generate reports via the web system. Data acquisition in PUS can be performed at three different levels of sophistication:

- *Manual data acquisition from operators:* only disruptions are reported but operation states are not logged.
- *Semi-automatic data acquisition:* states data are reported (no classification of failures)
- *Automatic data acquisition:* in addition to state changes, also alarm codes and other parameters are reported.

Since the D12 line is semi-automatically monitored, all stops due to failures are reported automatically as unclassified (disruption ID 973). The operator fills in the type of disturbances (for those lasting more than 3 minutes) while short stops (under 3 minutes) generally remain unclassified.

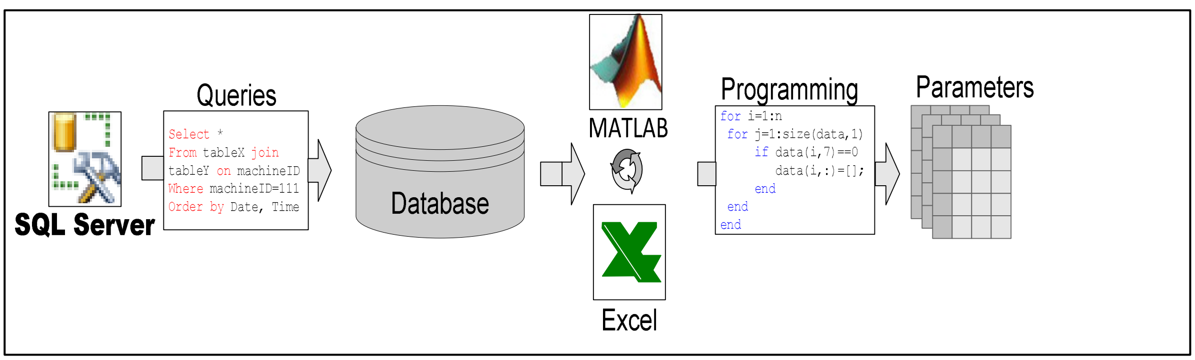


Figure 8:2 Machine parameter derivation procedures

By elaborating the information collected by the production monitoring system, the data concerning the failure and repair probabilities for each machine and concerning each failure mode have been derived, see Figure 8:2. In particular, a set of queries and filters to clean the production data have been designed, in order to obtain reliable information from the semiautomatic monitoring system. This allows eliminating outlier which can be recognized as operator errors. Examples of the introduced filters are the elimination of states lasting 0 seconds or the elimination of the states twice reported. This activity has been carried out in strict collaboration with the production managers and the operators

In order to check the possibility of adopting the geometric distribution to model time to failure and time to repair, the Anderson Darling test have been performed, with positive feedback as regarding time to failures and negative regarding time to repairs. By a further statistical analysis, the fact that the practice adopted while managing semi-automatic data collection is to avoid the registration of short failures (less than 3 minutes) was individuated as the cause for this behavior. Indeed, in case of missing data the distribution is truncated. Following the theory of truncated distribution, time to repair parameters has been recalculated, by assuming geometrical distribution for each type of failure modes.

The first needed activity in order to carry out a performance analysis of the system is to correctly model the production system. According to the system behavior and considering that cycle times of machine could be considered as equal, except for some stations for which a “slow down failure” has been introduced, the production model which matches with the system characteristics has been identified. In particular, a discrete flow line model, with machines characterized by multiple failure modes and deterministic processing times equal to the time unit has been considered.

Therefore, the problems addressed in the next sections deal with:

- The analysis of the production system performance through the proposed approximate analytical method
- Analysis of the methodology in estimating the through comparison with real life production data
- The analysis of the uncertainty contributed by each machine

8.4 Analysis of the system and Results

The reliability data that is collected by the monitoring system is used for the estimation of reliability parameters distribution. Using the Bayesian framework introduced in section 4.1 each of the failure probability and repair probability are estimated. An example of estimated values of the beta distributions hyper parameters are reported in Table 8:2 for the failure probabilities. The same inference is done for each repair probabilities and the inference of parameters and the corresponding uncertainty is carried out for each machine.

Machine ID	Disruption No 1		Disruption No 2		Disruption No 3		Disruption No 4		Disruption No 5		Disruption No 6	
	parameters		parameters		parameters		parameters		parameters		parameters	
	α	β	α	β	α	β	α	β	α	β	α	β
OP20	63	151401	295	178500	126	172105	52.00	170385	17	119137	11	167876
OP30	94	127782	195	152730	615	138379	56.00	166431	32	162271	18	64942
…	…	…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…	…	…
OP230	85	174858	79.00	121569	9	67406	56.00	166555	13	110678	121	131917

Table 8:2 Hyperparameters for input parameters distribution

Using the method introduced in section 4.1 an equivalent approximation for the mean failures using a single failure is performed. The corresponding uncertainty for the isolated machines is also approximated using the equivalent approximation technique that is proposed in section 4.4.4. The joint distribution approach is used for the evaluation of the expected value E[e] and uncertainty in the isolated efficiency V[e] of each machine. The evaluation results from these primary analyses are reported in Figure 8:3.

Machine	E[e]	V[e]
OP20	0.612413	5.63E-06
OP30	0.642701	1.06E-05
OP40	0.712869	3.28E-06
OP50	0.649828	3.11E-06

OP60	0.641053	6.47E-06
OP70	0.804106	2.12E-05
OP80	0.624626	4.3E-06
OP90	0.589251	4.83E-06
OP100	0.62206	9.55E-06
OP110	0.671575	2.91E-05
OP120	0.593876	6.18E-05
OP130	0.669718	0.000263
OP140	0.723564	1.18E-05
OP150	0.739649	1.38E-05
OP160	0.618966	5.65E-06
OP170	0.758323	5.38E-05
OP180	0.688666	2.51E-06
OP190	0.603778	6.33E-06
OP200	0.676284	0.000159
OP210	0.704462	1.93E-06
OP220	0.962625	1.76E-05
OP230	0.720621	5.39E-06

Table 8:3 Equivalent machine with isolated efficiency and uncertainty

The isolated efficiency of each machine is then used for the determination of approximated expected failure probabilities and repair probabilities of each machine. The uncertainty of each parameter is also determined from the evaluated results in Table 8:3. Hyper parameters of the distribution can be constructed from these moments of the uncertain reliability parameters as reported in Table 8:4.

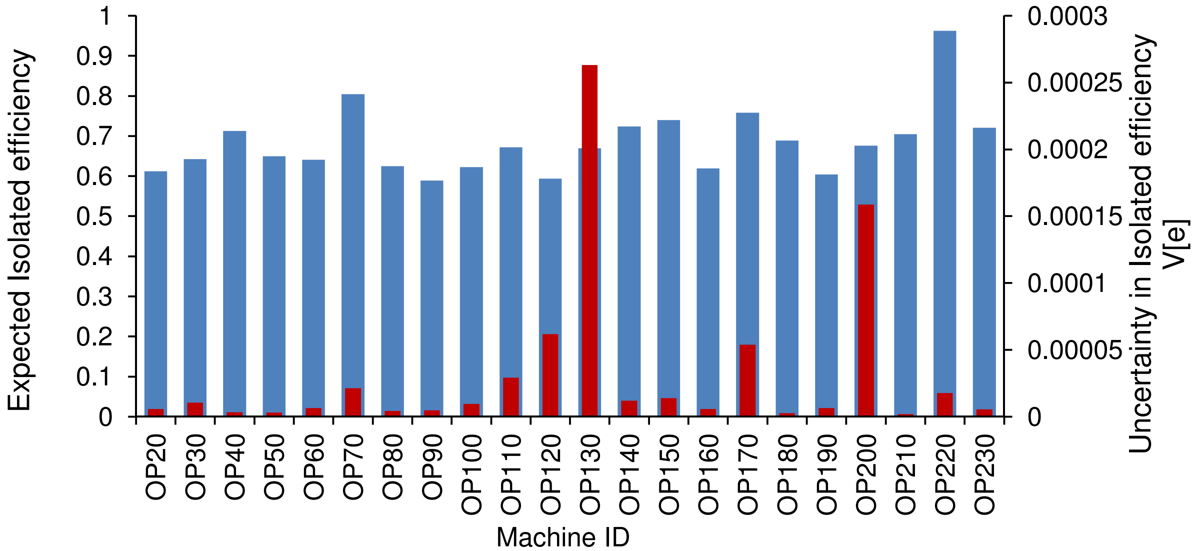


Figure 8:3 Isolated efficiency of machines and the uncertainty of isolated efficiency

Machine ID	p_{eq}	r_{eq}	$\sigma^2 p_{eq}$	$\sigma^2 r_{eq}$	α_p	β_p	α_r	β_r
OP20	0.112988	0.167103	7.41E-07	1.1E-06	15287.29	120012.7	21229.64	105816
OP30	0.09031	0.169379	1.09E-06	2.05E-06	6790.768	68403.22	11629.26	57028.87
OP40	0.074324	0.18846	3.16E-07	8.01E-07	16184.52	201571.2	35978.15	154928.2
OP50	0.097888	0.185544	3.82E-07	7.24E-07	22644.06	208682	38750.53	170098.2
OP60	0.097751	0.17811	7.62E-07	1.39E-06	11313.91	104427.7	18778.67	86654.32
OP70	0.028707	0.129348	6.47E-07	2.92E-06	1236.751	41845.85	4995.205	33623.27
OP80	0.106795	0.181161	5.67E-07	9.62E-07	17972.56	150318.1	27949.34	126329.7
OP90	0.118209	0.174346	6.94E-07	1.02E-06	17753.28	132432.3	24517.32	116107
OP100	0.100306	0.170331	1.11E-06	1.89E-06	8145.712	73062.85	12755.71	62132.17
OP110	0.081171	0.168886	2.7E-06	5.61E-06	2245.395	25417.2	4225.81	20795.92
OP120	0.103106	0.153393	6.8E-06	1.01E-05	1402.422	12199.27	1969.414	10869.61
OP130	0.06036	0.124429	1.33E-05	2.75E-05	256.5653	3994.022	492.8266	3467.866
OP140	0.048339	0.143047	5.81E-07	1.72E-06	3829.89	75399.19	10205.56	61138.72
OP150	0.060688	0.178338	1.06E-06	3.11E-06	3272.105	50644.46	8411.008	38752.34
OP160	0.106694	0.17879	7.35E-07	1.23E-06	13830.08	115793.7	21305.01	97857.15
OP170	0.0547	0.17479	3.72E-06	1.19E-05	760.1764	13136.91	2120.465	10011.03
OP180	0.085314	0.19191	2.79E-07	6.27E-07	23882.58	256054.5	47461.95	199851.6
OP190	0.109818	0.175633	8.39E-07	1.34E-06	12799.15	103749.3	18956.31	88975.23
OP200	0.06357	0.134664	9.18E-06	1.95E-05	412.0522	6069.85	806.6013	5183.112

OP210	0.0613	0.14724	1.19E-07	2.85E-07	29693.58	454705.2	64793.14	375257.6
OP220	0.001253	0.0367	2.63E-08	7.7E-07	59.66628	47549.86	1685.242	44234.54
OP230	0.068768	0.181464	4.65E-07	1.23E-06	9462.797	128142.3	21948.5	99003.71

Table 8:4 Approximated hyper parameters of the 22 machine line

Application of the posterior parameters that are reported in Table 8:4 and using the proposed method is used to evaluate the multi-stage line. Both the expected throughput and the uncertainty are evaluated with final results as reported in Table 8:5. Based on the estimated uncertainty a 95% confidence interval is constructed and the analysis that is performed using real data estimation is found in this interval.

Data estimation	Expected average throughput E[TH]	Uncertainty of throughput V[TH]	95% confidence interval
0.37535	0.36962	1.265×10^{-5}	0.38029-0.36251

Table 8:5 Evaluation results of the 22 machine line

The following analysis is performed to determine the percentage contribution of uncertainty contributed by each individual machine in order to analyze and discriminate the proportion of uncertainty of each factor. The contribution of uncertainty from each parameter is evaluated and the results are reported in Figure 8:4.

It can be seen from this Figure 8:6 only four of the machines i.e., (OP120, OP130, OP160 and OP200) highly contributed to the uncertainty compared to the other machines in the line. Performing a Pareto analysis on the overall contributions of the machines is shown in Figure 8:5. The four machines in the order of their contribution (OP200, OP130, OP160 and OP 120) have contributed for the 90% of total uncertainty.

One obvious advantage of the analysis with the introduction of uncertainty is demonstrated on the possibility of quantifying the resulting uncertainty. Besides it is an informative method on how to improve the reliability of the analysis result since the contribution of each uncertainty can be quantified. The second analysis on the contribution of uncertainty can lead to a targeted effort on data collection and supervision design in order to achieve a more precise estimation. Particularly out of the 22 machines the supervision and estimation effort should be focused on the 4 which contributed for the 90 percent of uncertainty.

Such application even could be more advantageous when applied online. The evaluation here is conducted on a 9 months data, so if the estimation and sampling plan has to be adjusted with a periodic inference much better estimates could be achieve even at earlier observation periods.

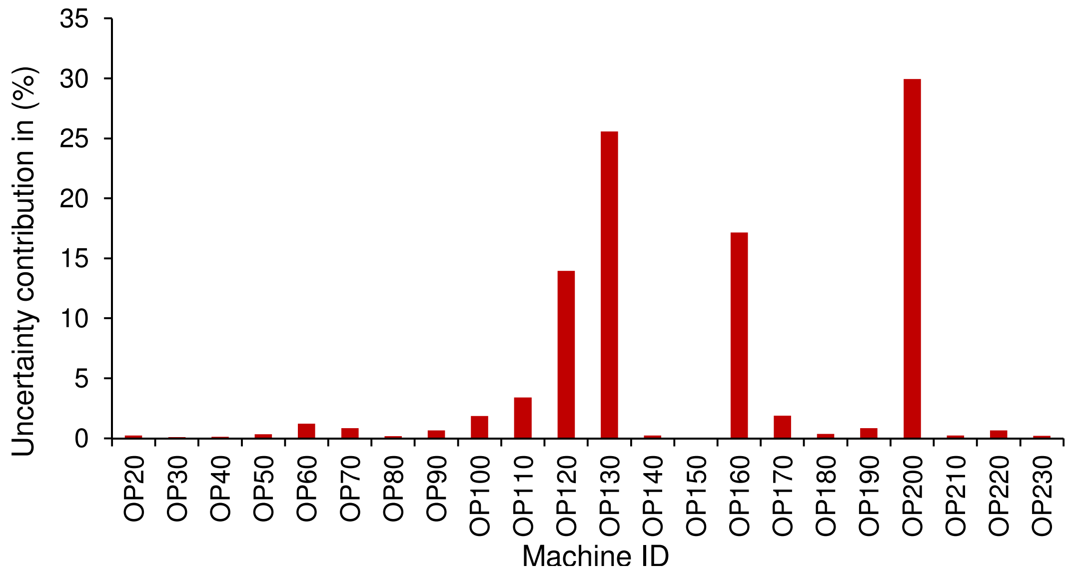


Figure 8:4 Percentage uncertainty contribution by each machine

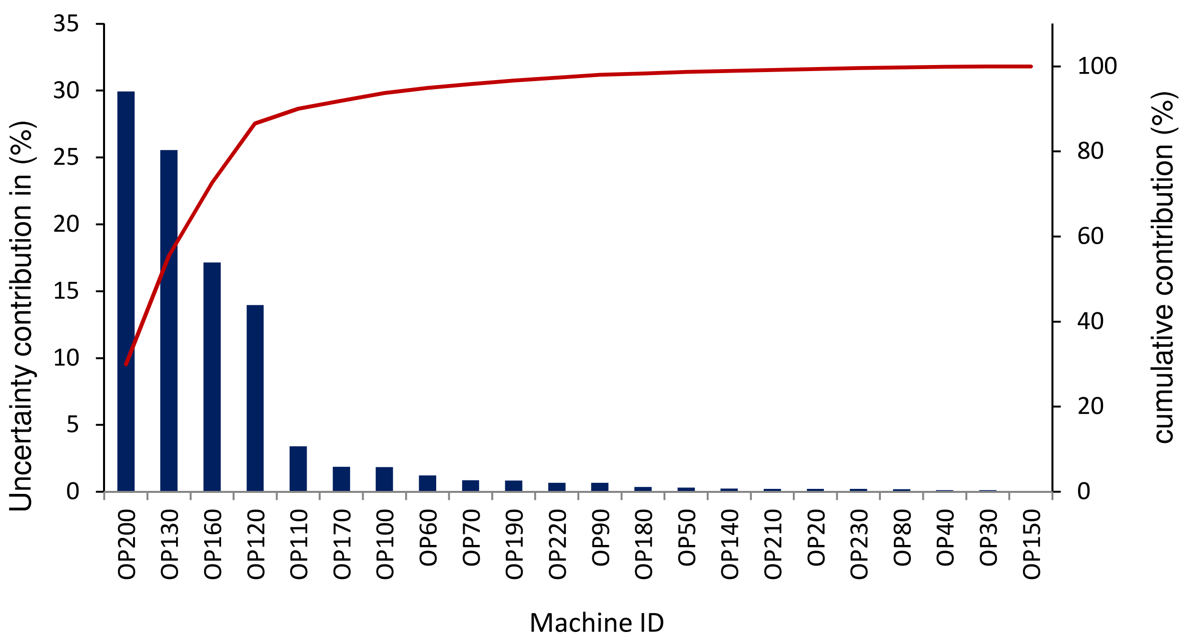


Figure 8:5 Cumulated uncertainties generated by individual machines

Further analysis can be done by analyzing the input uncertainty and the contribution of each parameter at the end of the analysis. For instance keeping the focusing still on the four major contributors of uncertainty, from Figure 8:3 and Figure 8:5 it can be noted that the high input uncertainty of machine OP130, OP200 even though the level of magnification is different for both it is also reflected on the output uncertainty of the V[TH]. For some of the other configurations the relation can also depend on the particular configuration of the machines. For instance an observation on the input parameter for machine no OP160 reveals small input parameters also can be amplified because of the specific buffer configuration and the adjacent machines they are located in.

The analysis of this real case signifies even the important of integrating performance evaluation with parameter estimation. Impact of the joint consideration can be highlighted by the fact that even after 9 months of the effectiveness of system supervision can be determined by few of the machines in the line. Moreover apart from the analysis in this line the online application of this framework can lead to real time intervention on the systems so that the configuration and supervision efforts can be allocated in an effective mode. Additionally, the possibility of introducing integrated analysis of multi-stage systems and their supervisors at early phase could have bigger impact on the understanding of the system with time. The early integration of this framework determines the success of decisions by optimally considering the inherent tradeoff between the time lag required for additional and precise information against making an early and less precise decision.

Conclusions and Future Work

The research reported in this thesis has proposed quantitative methods for the integrated analysis of manufacturing and supervisory systems. The proposed methods in the thesis have the goal of investigating manufacturing performance analysis from real data acquired through supervisory systems. Moreover the quantitative methods that can assist the robust design and reconfiguration of manufacturing and supervisory systems that highly guarantee target performance are provided.

Main results achieved through this work can provide important contributions in performance analysis and design of manufacturing systems that are equally relevant for both practitioners and researchers. The main steps taken, findings of the research and promising future recommendation to better exploit the research direction are briefly discussed as follows.

Primarily, the importance of conducting performance analysis from actual data as opposed to given precisely known model parameters is investigated. The introductory part of the thesis is focused on an in depth analysis of investigating these important differences and providing explanations using quantitative proofs. The next step is dedicated to the development of alternative techniques that can accurately and efficiently analyze the performance of multi-stage manufacturing systems from uncertain data. A new method based on the decomposition of multi-stage manufacturing lines for the estimation of average throughput is also proposed. The method is can be used for the accurate and efficient analysis of complex manufacturing systems from real data obtained through supervisors.

Based on the methods developed in this thesis and existing optimization methods for buffer allocation in multistage manufacturing systems a new method for optimal buffer allocation under uncertain parameters is proposed. The approach aims at providing the optimal buffer configuration that guarantees the satisfaction of target performances with a given confidence level. The level of additional information or the necessary buffer configuration required in order to introduce desired level of robustness can be determined analytically using this method. The proposed approach is also used for the analysis of an industrial case featuring a buffered multi-stage manufacturing system.

Key contributions of the research and their importance of applicability in performance analysis are summarized in the following points:

Model parameter estimation uncertainties and the impact on performance analysis:- Fundamental differences are discovered on performance analysis results when the estimation uncertainties from actual data are ignored. Overestimation and underestimation of performance outputs can be one of the key analysis deviations that can be consistently committed. The level of information on model parameters and amount of data used to make the estimation should be considered in the performance analysis to make valid decisions.

Impact of buffer configuration on output performance uncertainty:- The thesis also investigated the impact of buffer capacity decision on the output performance uncertainty in various system configurations including two machine lines and long lines. The analysis assists on the decision of configurations on performance uncertainty by choosing alternative buffer configurations so that target performances can be guaranteed.

Optimal reconfiguration of supervision and monitoring of parameters:- Based on the developed methods in this thesis, techniques that target to minimize input uncertainty on the parameters which highly contribute to the output uncertainty are applied. The gradient algorithm approach proposed assists the decision on how to better allocate sampling and data collecting efforts and resources in order to optimally reduce input uncertainty. This allows system designers to improve supervision efficiency instead of allocating importance resources equally including parameters that are not critical for the performance.

Optimal buffer allocation under limited knowledge of system parameters:- Existing buffer optimization techniques consider precisely known reliability parameters; in this case a new approach for the optimization of manufacturing systems under uncertain parameters is proposed. The approach enables determining buffer levels that guarantee the achievement of target performances within a specified confidence level. The level of additional information or the necessary buffer configuration required in order to introduce desired level of robustness can be analytically determined using this method.

Future Works:

Provided that this field of integrated analysis is yet unexplored there is a potential development that can benefit the research community and practitioners of manufacturing systems engineering. The main direction of extensions on this research can be outlined as follows:

- *Study and analysis of different manufacturing systems architectures*

The analysis that is carried out in this thesis is conducted on two machine lines and open layout systems. The investigation of alternative configurations and the analysis of architectures such as closed loop lines, assembly/disassembly lines are important to understand how the analyses of these systems behave under uncertainty.

- *Analysis of manufacturing systems with complex and interacting behaviors that exhibit inherent trade-offs*

Previous works have investigated the importance of integrated analysis of interacting manufacturing aspects jointly. One important example is the integrate analysis of productivity and quality performance of a system. Given quality control systems heavily rely on statistical inferences the introduction of uncertainty is an important field of area to study.

- *Resource constrained dynamic optimization problems with robustness for target performance*

Important problem to address with the extension of this research is the study of systems when resources are limited for the design and missing target performance have penalty costs. Unlike the precise assumption formalization for these problems the introduction of uncertainty helps decision making based on level of knowledge on input parameters and uncertain performances. The dynamic nature of the problem assists for online implementation of the techniques for a continuous optimal reconfiguration of resources based on real time feedback from monitoring.

- *Integrated designing supervisory control systems and manufacturing systems with performance validation.*

Although the work that is investigated in this thesis highlights the importance of supervision on the accompanying performance and vice versa, the integration of design methodologies is fundamental. The research filed that is developed formal methods in designing supervisory systems should be taken into account for the sufficiency of enabling desired performance analysis and achieving target performances. Addressing this problem with the feedback adjustment capability of supervisory systems for improved and reconfigurability should be taken in to consideration during the formal specification of both areas.

Appendix

(A:1)

Proof (Theorem 1)

Theorem (Jensen Inequality): let $\varphi(p)$ be a convex function defined on an internal region *I*.

Then $\varphi(E[p]) \le E[\varphi(p)]$

Proof:

Definition: a Let a real function φ is defined on a real interval *I,* then φ is strictly convex on

I iff $\begin{array}{l}\forall p_i, p_j \in I : \forall \lambda \in \mathbb{R} : \lambda > 0 : \\ \varphi(\lambda p_i + (1-\lambda) p_j) < \lambda\varphi(p_i) + (1-\lambda)\varphi(p_j)\end{array}$

Let $p_1, p_2, ..., p_N \in I$ $\lambda_1, \lambda_2, ..., \lambda_N > 0$

Then $\varphi\left(\frac{\sum_{i=1}^{n} \lambda_i p_i}{\sum_{i=1}^{n} \lambda_i}\right) < \frac{\sum_{i=1}^{n} \lambda_i \varphi(p_i)}{\sum_{i=1}^{n} \lambda_i}$

For n = 1statement is trivial, proceeding by induction on n and assuming the theorem holds for some value of n by induction we will show it will hold for n+1

If $y = \frac{\lambda_n p_n + \lambda_{n+1} p_{n+1}}{\lambda_n + \lambda_{n+1}}$

Then

$$\varphi\left(\frac{\sum_{i=1}^{n+1} \lambda_i p_i}{\sum_{i=1}^{n+1} \lambda_i}\right) = \varphi\left(\frac{\sum_{i=1}^{n-1} \lambda_i (p_i) + (\lambda_n + \lambda_{n+1}) y}{\sum_{i=1}^{n-1} \lambda_i + (\lambda_n + \lambda_{n+1})}\right)$$

$< \left(\frac{\sum_{i=1}^{n-1} \lambda_i \varphi(p_i) + (\lambda_n + \lambda_{n+1}) \varphi(y)}{\sum_{i=1}^{n-1} \lambda_i + (\lambda_n + \lambda_{n+1})}\right)$ by induction hypothesis

$< \left(\frac{\sum_{i=1}^{n+1} \lambda_i \varphi(p_i)}{\sum_{i=1}^{n+1} \lambda_i}\right)$ by definition of convexity

$p \in \{p_i : 1,..N\}$ is a random parameter with probabilities $\lambda_1, \lambda_2, ..., \lambda_N > 0$ with $\sum_{i=1}^{N} \lambda_i = 1$ then

$\varphi\left(\sum_{i=1}^{N} \lambda_i p_i\right) < \sum_{i=1}^{N} \lambda_i \varphi(p_i)$

Equivalently $\varphi\left(E[p]\right) < E[\varphi(p_i)]$

By using the second derivative test for $\varphi(p) = \frac{r}{r+p}$ $\varphi^{"}(p) = \frac{2r}{(r+p)^3} > 0 : r > 0, p > 0$ is strictly convex.

Therefore $\varphi\left(E[p]\right) < E[\varphi(p_i)]$ always holds true.

(A:2)

Proof (Theorem 2)

Theorem (Jensen Inequality): let $\varphi(r)$ be a concave function defined on an internal region *I*. Then $\varphi(E[r]) \geq E[\varphi(r)]$

Proof:

Definition: a Let a real function φ is defined on a real interval *I,* then φ is strictly concave on

I iff $\begin{array}{l} \forall r_i, r_j \in I : \forall \lambda \in \mathbb{R} : \lambda > 0 : \\ \varphi(\lambda r_i + (1-\lambda) r_j) > \lambda\varphi(r_i) + (1-\lambda)\varphi(r_j) \end{array}$

Let $r_1, r_2, ..., r_N \in I$ $\lambda_1, \lambda_2, ..., \lambda_N > 0$

Then $\varphi\left(\frac{\sum_{i=1}^{n} \lambda_i r_i}{\sum_{i=1}^{n} \lambda_i}\right) > \frac{\sum_{i=1}^{n} \lambda_i \varphi(r_i)}{\sum_{i=1}^{n} \lambda_i}$

For n = 1statement is trivial, proceeding by induction on n and assuming the theorem holds for some value of n by induction we will show it will hold for n+1

If $y = \frac{\lambda_n r_n + \lambda_{n+1} r_{n+1}}{\lambda_n + \lambda_{n+1}}$

Then

$$\varphi\left(\frac{\sum_{i=1}^{n+1} \lambda_i r_i}{\sum_{i=1}^{n+1} \lambda_i}\right) = \varphi\left(\frac{\sum_{i=1}^{n-1} \lambda_i (r_i) + (\lambda_n + \lambda_{n+1}) y}{\sum_{i=1}^{n-1} \lambda_i + (\lambda_n + \lambda_{n+1})}\right)$$

$> \left(\frac{\sum_{i=1}^{n-1} \lambda_i \varphi(r_i) + (\lambda_n + \lambda_{n+1}) \varphi(y)}{\sum_{i=1}^{n-1} \lambda_i + (\lambda_n + \lambda_{n+1})}\right)$ by induction hypothesis

$> \left(\frac{\sum_{i=1}^{n+1} \lambda_i \varphi(r_i)}{\sum_{i=1}^{n+1} \lambda_i}\right)$ by definition of convexity

$r \in \{r_i : 1,..N\}$ is a random parameter with probabilities $\lambda_1, \lambda_2, ..., \lambda_N > 0$ with $\sum_{i=1}^{N} \lambda_i = 1$ then

$\varphi\left(\sum_{i=1}^{N} \lambda_i r_i\right) > \sum_{i=1}^{N} \lambda_i \varphi(r_i)$

Equivalently $\varphi(E[r]) > E[\varphi(r_i)]$

By using the second derivative test for $\varphi(r) = \dfrac{r}{r+p}$ $\varphi^{''}(r) = -\dfrac{2p}{(p+r)^3} < 0 : r > 0, p > 0$ is strictly concave.

Therefore $\varphi(E[r]) > E[\varphi(r_i)]$ always holds true.

(A:3)

THREE MACHINE LINE							
ONE UNCERTAIN		**TWO UNCERTAIN**			**THREE UNCERTAIN**		
ONEFACT	**TAYLOR**	**JOINT**	**ONEFACT**	**TAYLOR**	**JOINT**	**ONEFACT**	**TAYLOR**
2.989	0.171	20.7	6.811	0.352	2389.735	17.013	0.413
1.258	0.163	15.172	3.799	0.531	1931.153	15.006	0.578
2.298	0.201	14.771	3.074	0.123	1189.713	25.536	0.63
0.768	0.143	15.18	3.296	0.247	1277.343	20.922	0.231
2.167	0.142	19.535	2.026	0.126	2012.715	19.428	0.573
2.31	0.142	16.904	4.34	0.248	868.912	16.287	0.648
0.738	0.111	20.213	1.18	0.143	1457.334	23.778	0.292
1.194	0.166	21.187	8.126	0.286	2574.013	7.836	0.24
0.775	0.094	13.449	4.298	0.129	2179.935	14.556	0.387
1.394	0.088	17.255	1.882	0.253	1554.171	17.88	0.532
1.842	0.162	16.716	2.899	0.124	987.049	11.829	0.245
1.619	0.151	19.797	3.527	0.242	2371.729	16.707	0.432
1.656	0.131	16.372	1.748	0.084	659.877	17.547	0.485
1.289	0.119	18.645	2.22	0.167	1484.959	12.399	0.231
1.206	0.108	15.064	1.905	0.088	1370.564	11.409	0.639
1.065	0.145	24.321	1.496	0.187	752.551	15.618	0.652
1.313	0.147	13.633	2.552	0.119	1524.812	25.443	0.471
1.424	0.079	23.64	1.992	0.231	841.552	23.4	0.538
1.377	0.104	15.198	1.928	0.107	2333.487	6.627	0.416
2.075	0.077	14.443	1.795	0.219	1069.891	16.416	0.281

2.397	0.08	20.968	2.133	0.101	1218.363	26.271	0.595
1.396	0.105	15.386	1.939	0.195	2255.729	17.268	0.544
1.461	0.077	14.149	2.12	0.07	1198.468	23.151	0.339
1.769	0.11	15.012	10.079	0.143	1733.67	9.18	0.421
1.495	0.101	17.377	3.104	0.096	743.913	10.884	0.31
2.1	0.081	23.522	2.600	0.197	2460.998	13.008	0.48
1.262	0.117	14.099	3.665	0.08	2495.219	17.385	0.352
1.361	0.109	19.541	5.806	0.156	2414.856	14.574	0.238
1.094	0.073	21.974	4.031	0.102	2472.02	18.216	0.357
1.355	0.072	13.832	4.527	0.195	1322.376	23.22	0.595
0.965	0.093	13.245	6.751	0.084	2373.082	12.285	0.2
1.333	0.121	18.374	1.795	0.168	847.805	12.129	0.186
1.455	0.103	18.901	2.697	0.087	2458.521	6.795	0.279
1.556	0.075	15.268	1.944	0.176	1581.517	13.2	0.438
1.467	0.112	14.081	5.454	0.051	1535.655	13.398	0.646
1.767	0.065	21.551	5.824	0.102	1729.065	26.496	0.231
0.755	0.168	22.398	4.419	0.07	2226.153	17.883	0.304
1.004	0.086	18.908	4.892	0.136	891.244	22.278	0.454
1.291	0.188	14.306	5.526	0.07	1540.42	7.188	0.403
0.936	0.07	22.954	3.625	0.135	686.855	19.692	0.577
1.471	0.098	18.721	3.584	0.049	2572.525	15.39	0.432
1.283	0.091	23.385	1.422	0.1	2321.048	18.276	0.372
1.489	0.081	21.746	5.482	0.059	2344.712	18.921	0.338
1.322	0.061	17.087	3.828	0.119	2303.174	18.468	0.488
2.442	0.064	13.334	3.306	0.06	2574.753	26.295	0.647
0.94	0.06	15.997	4.617	0.123	1912.529	21.468	0.507
0.861	0.077	17.138	6.079	0.065	1900.841	26.955	0.247
0.988	0.073	17.03	1.202	0.132	2506.559	11.145	0.246
1.009	0.077	14.56	5.987	0.075	1675.859	12.636	0.402
1.615	0.067	18.188	5.235	0.16	2280.233	17.337	0.477

Table A:3 Computation times required for experiments of three machine lines

(A:4)

FIVE MACHINE LINE							
ONE UNCERTAIN		TWO UNCERTAIN			THREE UNCERTAIN		
ONEFACT	TAYLOR	JOINT	ONEFACT	TAYLOR	JOINT	ONEFACT	TAYLOR
13.778	0.511	104.031	35.524	0.941	5040.147	239.583	3.306
20.166	0.5	295.64	120.046	0.474	4114.663	41.767	2.92
7.853	0.599	396.34	90.94	0.191	5252.185	35.397	2.457
6.692	0.734	288.763	133.172	0.47	3899.479	198.297	3.322
4.065	0.404	184.602	55.526	0.897	5236.353	230.95	2.075
19.453	0.395	386.166	67.482	0.272	5339.336	186.627	1.608
7.193	0.378	366.351	45.784	0.834	5279.134	81.017	1.633
11.578	0.348	36.635	15.878	0.686	5125.182	168.914	1.499
8.684	0.402	277.435	18.196	0.499	5344.084	220.3	1.414
6.071	0.325	287.917	43.148	0.378	5284.643	131.892	1.474
6.345	0.366	268.036	136.988	0.232	5282.54	126.402	2.509
7.164	0.443	158.499	135.862	0.552	4875.835	150.76	2.447
12.111	0.207	392.685	98.834	0.634	5068.518	204.157	1.571
9.464	0.299	357.338	15.348	0.828	5101.368	181.277	1.899
18.617	0.248	85.983	112.444	0.289	5043.343	78.008	2.011
9.039	0.257	396.094	97.1	0.719	5074.351	205.287	1.726
33.374	0.273	44.895	71.058	0.725	4998.148	135.983	1.508
7.62	0.344	316.106	112.37	0.69	4927.531	46.926	2.004
9.904	0.309	106.414	65.758	0.48	5234.823	199.051	1.625
5.817	0.324	85.078	38.864	0.593	6087.427	224.286	1.903
9.294	0.391	160.09	29.584	0.246	5293.854	156.94	1.665
5.486	0.478	356.378	38.216	0.765	5290.47	143.955	1.745
9.407	0.414	369.542	61.016	0.894	4849.312	118.97	2.026
39.76	0.286	272.817	58.288	0.452	5078.683	78.868	1.17
16.443	0.393	258.973	55.192	0.812	5134.139	37.983	2.079
7.916	0.404	146.589	34.856	0.479	4943.892	189.803	1.7
8.482	0.445	201.318	28.772	0.977	5398.981	185.937	2.794
6.634	0.352	335.249	77.192	0.259	5257.267	178.505	2.006
44.236	0.286	84.962	116.104	0.545	4805.48	49.052	1.513
9.814	0.347	225.923	30.144	0.546	4895.503	223.953	1.47

28.496	0.317	309.443	18.512	0.681	5055.759	53.407	1.88
14.424	0.409	370.127	73.59	0.221	5096.749	192.389	2.064
7.558	0.243	305.398	42.674	0.129	5135.098	148.422	1.833
6.943	0.292	378.905	52.076	0.516	4957.757	51.23	1.381
7.819	0.275	299.335	90.482	0.163	5201.651	237.858	2.11
6.786	0.253	338.591	24.978	0.505	5000.836	188.544	1.39
20.936	0.417	360.541	87.282	0.38	5303.8	153.23	1.489
29.152	0.178	150.207	41.384	0.791	5042.664	180.92	1.477
43.266	0.304	114.361	29.16	0.451	4824.611	172.806	1.385
24.373	0.438	307.292	118.204	0.218	4850.1	82.28	2.837
33.199	0.431	137.325	63.122	0.552	5251.371	142.759	2.656
6.126	0.294	213.322	103.484	0.777	4990.612	135.747	1.883
9.513	0.311	194.294	47.042	0.462	5260.581	232.819	1.887
37.434	0.322	217.31	61.092	0.193	5011.304	187.269	1.469
29.5	0.322	178.614	126.488	0.152	5128.745	249.263	1.671
31.945	0.303	388.265	48.212	0.412	5256.619	49.928	1.805
9.782	0.249	89.387	65.852	0.426	5398.183	95.866	1.919
27.46	0.317	77.601	57.412	0.51	5259.437	63.518	1.545
43.135	0.433	150.102	25.084	0.901	5099.747	79.132	1.657
4.165	0.691	111.638	55.894	0.356	5064.943	64.278	1.997

Table A:4 Computation times required for experiments of Five machine lines

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