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Modeling the euro exchange rate using the GARCH framework

An application of GARCH models in a dynamic currency hedging strategy

RELATORE:

PROF. ROCCO MOSCONI

LAUREANDO: JEAN-BAPTISTE BONNAUD

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Abstract

This paper considers the Generalized Autoregressive Conditional Heteroscedastic approach to model the Eurodollar spot and futures exchange rate's volatilities using daily and weekly observations over the period of 19th June 2007 to 18th March 2013. In this paper, I estimate different symmetric and asymmetric bivariate models that capture most of the common stylized facts about exchange rates such as volatility clustering and leverage effect. I apply them in different currency futures hedging strategies which minimize the foreign exchange risk in USD. I observed the performance of the different models: dynamic strategies outperform the static strategies, the bivariate GARCH model with a diagonal VECH covariance was the most effective hedging model and the introduction of a TARCH term doesn't increase the effectiveness. I could also observe a structural problem: the estimation based on daily data underestimates the optimal hedge ratio and causes poor hedging performances.

Keywords Exchange rate volatility, multivariate GARCH models, Optimal Hedge ratio, Futures contracts, Hedging effectiveness

Riassunto

Questo documento considera l'approccio eteroschedastico condizionale autoregressiva generalizzata per modellare la volatilità del tasso di cambio spot e futuro del eurodollar utilizzando osservazioni giornaliere e settimanali nel periodo del 19 giugno 2007 al 18 marzo 2013. In questo lavoro, ho stimato diversi modelli bivariate simmetrici e asimmetrici che catturano la maggior parte dei fatti stilizzati comuni sui tassi di cambio. Li applico nelle diverse strategie di copertura che riducano il rischio di cambio in USD. Ho osservato le prestazioni dei diversi modelli: le strategie dinamiche superano le strategie statiche, il modello GARCH bivariato con una covarianza di tipo diagonale VECH è stato il modello il più efficace e l'introduzione di un termine TARCH non aumenta l'efficacia. Ho potuto osservare anche un problema strutturale: la stima basata su dati giornalieri sottovaluta il rapporto di copertura ottimale e provoca povere efficacia di copertura.

Parole chiave Tasso di cambio, Modelli multivariati GARCH, Tasso di copertura ottimo, Contratti Futuri, Efficacia di copertura

Introduction

Since the collapse of the Bretton Woods agreement of fixed exchange rate in 1972, the movements and fluctuations of the currency exchange rates have become a central subject of academic research in macroeconomics as well as in empirical finance. Black (1976) and Mandelbrot (1963) were among the first to work on the volatility of stock prices. Later, all kind of asset prices such as foreign exchange rates became the subject of research in an attempt to model their behavior. It has become a particular topic of interest in a period of financial and economic crisis of the euro area where the euro exchange rate stability is questionable due to the raise of possible default for some European sovereign debt owners. The euro instability could affect international trade, competitiveness of economies, local inflation as well as security valuation, investment decisions and risk management. Modeling a currency exchange rate is therefore an important subject of research in econometrics. Among the numerous econometric models studied in the literature, I chose to focus on Bollerslev's Generalized Autoregressive Conditional Heteroscedastic model (GARCH framework). According to Engle (1982) and Bollerslev (1986), time series' models are more reliable for capturing the volatility in financial time series as these models are specifically designed for volatility modeling and capture some characteristics of financial time series. Brooks (2008) and Campbell (1996) describe in details in their books how they particularly suit for capturing the characteristics of exchange rates' volatility.

Besides,they are the volatility models the most commonly used by traders and structuring teams. Indeed, modeling exchange rates volatilities is essential in the determination of the hedging strategy for companies operating within different markets with different currencies. They are indeed exposed to the exchange rate risk which makes their foreign cash flows more uncertain and need to implement currency hedging strategies. Döhring (2008) reports to the E.U. commission how multinational companies operating in Europe try to manage the euro exchange rate volatility through financial or operational hedging. Its report illustrates the problem of currency hedging and the purpose of modeling and forecasting the exchange rate appreciation or depreciation. Financial hedging strategies can involve different financial instruments and requires the modeling of a joint distribution and the determination of an optimal hedge ratio. For this purpose, the literature proposes different approaches about the nature of the co-distribution, the model to employ, the purpose of the hedging and its performance or the estimation method to employ. Most of the dynamic models will involve a multivariate GARCH framework for modeling the covariance matrix of the distribution. For example, Kroner and Sultan (1995) proposed a conditional constant correlation model for foreign exchange spot and futures prices and studied static and dynamic optimal hedge ratios minimizing the variance of the hedged portfolio. Some other models, easier to compute were proposed as well (see Jonhson in 1960). What remains is the complexity in the determination of an optimal hedging strategy due to the number of approaches and parameters to consider.

My study focused on the case of a short term financial hedging strategy involving a simple currency derivative for which modeling volatility will be possible: three months futures contracts. I shortly describe in the two first parts the characteristics of foreign exchange rates from a macroeconomic point of view and the characteristics of financial time series. Then, I present the GARCH framework used to describe the distributions of exchange rates (spot and futures). Finally, I explain the different currency hedging strategies and apply some of them on the sample with different data frequency. The final objective is to see the benefits of the different hedging models and illustrate some structural aspect of the data that should be considered in an estimation process. My study should focus on the spot and future EUR/USD exchange rates: it is indeed a floating rate and the most traded asset on the FOREX market, giving it a good sensitivity to macro announcements and a stable level. The Historical Data of the spot exchange rates used in this study is provided by the Statistic Data Warehouse of the European Central Bank (http://sdw.ecb.europa.eu) and the Historical Data of daily quotations for the EUR/USD futures contracts is provided by the Wall Street Journal (http://wsj.com) whose source is Thomson Reuters. I use the daily average bilateral exchange rates for the spot rate and the settlement prices of 3-months futures contracts. The data covers from the 19th of June 2007 to the 18th of March 2013, period where the financial crisis has spread. The sample description and the different tests on individual series is performed with the software GRETL used in courses of Applied Econometrics at Politecnico di Milano but the estimations of the univariate and bivariate models used in our hedging strategy study are done with the software EVIEWS 6.

1. About Foreign Exchange Rates

For more details, see https://en.wikipedia.org/wiki/Exchange_rate and *International Economics – Theory and Policy* – 9th Edition - by Paul Krugman (2012).

1.1. Definition

The foreign-exchange rate is the value of one country's currency in terms of another currency and is determined in the foreign exchange market, which is open to a wide range of different types of buyers and sellers and where currency trading is continuous (24 hours a day except weekends). The spot exchange rate refers to the current exchange rate and the forward or future exchange rate refers to an exchange rate that is quoted and traded today but for delivery and payment on a specific future date. Market convention gives most currencies in terms of EUR, GBP or USD and other common currencies with a 4 decimal quotation. I indicate the main currency codes:

- USD: US Dollar
- EUR: Euro
- JPY: Japanese Yen
- GBP: British Pound
- CHF: Swiss Franc
- CAD: Canadian Dollar
- AUD: Australian Dollar

For example, on the 8th of June 2013 at 3.43pm (Paris time), 1 Euro was worth and therefore quoted in average on the Foreign Exchange Market 1,3219 US Dollar (data provided by reuters.com). The quotation is given as 1,3219 EUR/USD.

1.2. Exchange rate regime

Every country manages the value of its currency and determines the exchange rate regime that will be applied: free-floating, fixed or hybrid.

If a currency is free-floating, its exchange rate is allowed to vary and is determined by the market forces of supply and demand. A movable or adjustable peg system is a system of fixed exchange rates with a provision for the devaluation of a currency. For example, between 1994 and 2005, the Chinese Yuan (RMB) was pegged to the United States dollar at 8.2768 RMB/USD. This system allows a government to keep their currency within a narrow range. As a result currencies become over-valued or under-valued, causing trade deficits or surpluses. Indeed, a weak exchange rate will make exportation cheaper for foreign markets and importations more expensive for the domestic market helping therefore domestic production and causing a trade surplus (exportation becomes superior to importations). That's why it is interesting for an exportation based economy such as China to maintain an undervalued exchange rate.

1.3. Exchange rate fluctuation: a macroeconomic overview

A market-based exchange rate will change according to the force of supply and demand of the two component currencies. A currency will tend to become more valuable whenever demand for it is greater than the available supply (a depreciation does not mean people no longer want the currency, it just means they prefer holding their wealth in some other form or currency). Increased demand for a currency can be due to either an increased transaction demand for money or an increased speculative demand for money. The transaction demand is highly correlated to a country's level of business activity, gross domestic product (GDP), and employment levels. The more people that are unemployed, the less the public as a whole will spend on goods and services. Central banks typically have little difficulty adjusting the available money supply to accommodate changes in the demand for money due to business transactions. Speculative demand is much harder for central banks to accommodate, which they influence by adjusting interest rates. A speculator may buy a currency if the return (that is the interest rate) is high enough. In general, the higher a country's interest rates, the greater will be the demand for that currency.

1.4. Balance of payment model Versus Asset Market model

The Balance of payment model holds that a foreign exchange rate must be at its equilibrium level which is the rate giving a stable current account balance : a nation with a trade deficit will experience reduction in its foreign exchange reserves, which ultimately depreciates the value of its currency. The cheaper currency renders the nation's goods more affordable in the global market place while making imports more expensive. After an intermediate period, imports are forced down and exports rise, thus stabilizing the trade balance and the currency towards equilibrium. The balance of payments model focuses largely on trade of goods and services, ignoring the increasing role of global capital flows. In other words, money is not only chasing goods and services, but to a larger extent, financial assets such as stocks and bonds. Their flows go into the capital account item of the balance of payments, thus balancing the deficit in the current account.

The increase in capital flows has given rise to the asset market model: economic variables such as economic growth, inflation and productivity are no longer the only drivers of currency movements and the proportion of foreign exchange transactions generated from trading of financial assets became significant relatively to the extent of currency transactions generated from trading in goods and services. The asset market approach views currencies as asset prices, traded in an efficient financial market. Like the stock exchange, money can be made on the foreign exchange market by investors and speculators buying and selling at the right times. Currencies can be traded at spot and forward rates as defined before but we observe as well a multitude of currency based products traded on different markets or over the counter (swaps, options, futures, …).

1.5. Foreign Exchange Market (FOREX)

The foreign exchange market assists international trade and investment by enabling currency conversion. It also supports direct speculation in the value of currencies, and the carry trade, speculation based on the interest rate differential between two currencies. In a typical foreign exchange transaction, a party purchases some quantity of one currency by paying some quantity of another currency. The foreign exchange market is unique because of its huge trading volume representing the largest asset class in the world and leading to high liquidity, its geographical dispersion, its continuous operations, the variety of factors that affect exchange rates, the low margins of relative profit compared with other markets of fixed income, the use of leverage to enhance profit and loss margins. As such, it has been referred to as the market closest to the ideal of perfect competition.

We present some of the characteristics of this market given by a survey of April 2010 from the Bank for International Settlements:

- 24 hour market, from Sunday 5pm EST through Friday 4pm EST; trading begins in the Asia-Pacific region followed by Middle East, Europe, and America
- An average daily turnover estimated at 3.98 trillion USD, a growth of approximately 20% over the 3.21 trillion USD daily turnover as of April 2007. It's more than 12 times the daily turnover of global equity markets (about 320 billion USD) and an annual turnover more than 10 times the world GBP (about 58 trillion USD). The US & UK markets account for over 50% of daily turnover (37% for UK and 18% for US) and the US dollar is involved in over 80% of all foreign exchange transactions.

The 3.98 trillion USD break-down per product is as follows:

- 1.490 trillion USD in Spot transactions (one third of daily turnover)
- 475 billion USD in Forward Contracts
- 1.765 trillion USD in Foreign Exchange Rates Swaps
- 43 billion USD in Currency Swaps
- 207 billion USD in Options and other products

In 2013, the daily turnover of the FOREX market was estimated over 5 trillion USD. We indicate as well the average daily turnover by currency and currency pairs:

Currency	Share	Currency pairs	Share
USD	84,9%	EUR/USD	28%
EUR	39,1%	USD/JPY	14%
JPY	19,0%	GBP/USD	9%
GBP	12,9%	AUD/USD	6%
AUD	7,6%	USD/CAD	5%
CHF	6,4%	USD/CHF	4%
CAD	5,3%	EUR/JPY	3%
HKD	2,4%	EUR/GBP	3%
SEK	2,2%	EUR/CHF	2%
Other	20,6%	Other	26%
Total	200%	Total	100%

The Total of the currency share should be 200% because two currencies are involved in every transaction.

I chose to study the EUR/USD exchange rate, the most traded currency pair over all.

1.6. Exchange rate volatility: definition and measurement

Exchange rate volatility is a measure of the fluctuation in an exchange rate. It is also known as a measure of risk in asset pricing, risk management,… and is taken into account for a variety of economic decisions. It can be measured on an hourly, daily, weekly, monthly or annual basis. Based on the assumption that changes in an exchange rate follow a normal distribution for example, volatility provides an idea of how much the exchange rate can change within a given period. Volatility of an exchange rate, just like that of other financial assets, is usually calculated from the standard deviation of movements of exchange rates. Clearly, it is unobservable and its measure is a matter of serious contention.

Two measures of volatility are commonly employed in financial calculations: historical and implied volatility. Historical volatility is calculated from the past values of an exchange rate. Given a series of past daily exchange rates, we can calculate the standard deviation of the daily price changes and then the annual volatility of the exchange rate. Historical volatility provides a good assessment of possible future changes when the financial markets and economies have not gone through structural changes. Statistically, it is often measured as the sample standard deviation:

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(r_i - \mu)^2}$$

Where r_i is the returns on day *t* and μ is the average return over the n-days period.

Implied volatility is a forward looking measure of volatility and is calculated from the market participants' estimates of what is likely to happen in the future. More precisely, implied volatility is estimated from the quoted price of a currency option when the values of all other determinants of the price of an option are known. The basis of this calculation is the Black Scholes option pricing model, according to which the price of an option is determined by the following parameters: the current price of the asset (the spot exchange rate), the strike price at which the option can be exercised, the remaining time for the maturity of the option, the risk free interest rate, and the volatility of the asset. For more details on option pricing theory, you can see *Options, Futures and Other Derivatives* by Hull (2005). Knowing the current price of the option and all the others parameters values we can therefore determine backwards the volatility implied by the option price. It is a good start for traders to observe the volatility implied by the market.

Exchange rate volatility, like the volatility of any other financial asset, changes in response to information. Currency traders are sensitive to information that might influence the value of one currency in terms of another. The most important information is that about the macroeconomics performance of the economies behind the currencies. Changes in the levels of uncertainty about the future of either economy will cause traders to become restless and less willing to hold a particular currency. Uncertainty about the future is the most important reason for the change in the volatility of a currency. Central banks can also influence the volatility of their currencies. While it is commonly believed that central banks can influence the value of their currency at most in the short run, they can certainly cause a change in the volatility.

We will discuss further about the GARCH models used to investigate volatility characteristics. They have two distinct specifications: the conditional mean and the conditional variance.

1.7. About Monetary Policy

To understand fully the mechanism behind the exchange rate determination I present shortly the role of Monetary Policy. It is the process by which the monetary authority of a country such as Central banks controls the supply of money, the availability of money and the cost of money (rate of interests); it differs from fiscal policy, which refers to taxation, government spending, and associated borrowing. The purpose of controlling the money supply is often to promote economic growth and stability, relatively stable prices, and low unemployment. Each Central Bank has therefore a type of monetary policy with a given long term objective. For example, the European Central Bank has an Inflation Targeting policy with an objective of a close to 2% inflation whereas the Federal Reserve has a Mixed Policy with a long term objective of a controlled inflation and low unemployment.
We observe then two kind of monetary policy:

- An expansionary policy which increases the total supply of money in the economy in order to reduce unemployment and stimulate economic growth by lowering interest rates in the hope that easy credit will help businesses into expanding.
- A contractionary policy which reduces or expands more slowly than usual the money supply and intends to slow inflation in order to avoid the resulting deterioration of asset values.

Numerous Keynesian Macroeconomics models (see the Dornbush Model proposed by Dornbush in 1976) describe the dynamic of exchange rates and how a shock such as a change in monetary aggregates or interest rates would impact on the exchange rate level on the short term (phenomenon called "overshooting" of the exchange rate) and how it could lead on the long term to for example an increased inflation and a new stable level of the exchange rate close to the previous one.
To achieve their objectives the central banks can use a variety of tools such as interest rates or reserve requirement. For example, to achieve a contraction of money supply, the central bank can increase interest rates, reduce the monetary base, and increase reserve

requirements. The primary tool of monetary policy is open market operations: the management of the quantity of money in circulation through the buying and selling of various financial instruments, such as treasury bills, company bonds, or foreign currencies. All of these purchases or sales result in more or less base currency entering or leaving market circulation. In US style central banking, liquidity is furnished to the economy primarily through the purchase of Treasury bonds by the Federal Reserve System. The Eurosystem uses a different method. There are about 1500 eligible banks which may bid for short term repo contracts of two weeks to three months duration (a repo contract being the abbreviation for repurchase agreement which is the sale of securities together with an agreement for the seller to buy back the securities at a later date). The banks in effect borrow cash and must pay it back; the short durations allow interest rates to be adjusted continually. When the repo notes come due the participating banks bid again. An increase in the quantity of notes offered at auction allows an increase in liquidity in the economy. A decrease has the contrary effect. In facts, an increase in deposits in member banks, carried as a liability by the central bank, means that more money has been put into the economy. The other primary means of conducting monetary policy include: Discount window lending (lender of last resort), Fractional deposit lending (changes in the reserve requirement), Moral suasion (cajoling certain players to achieve specified outcomes), Open Mouth Operations (talking monetary policy with the market).
It is also important for policymakers to make credible announcements: private agents must believe that these announcements will reflect actual future policy.

Some papers such as Engle and Ng (1991) studied the effects of the differential impact of the different monetary measures used by Central Banks on the exchange rates level and volatility: they introduced Dummies in a simple GARCH framework for changes in interest rates, reserves requirement … or public announcements. My study will not focus on this matter.

For more details on the role and tools of the European Central Bank, you can read *La Banca Centrale Europea: la politica monetaria nell'area dell'euro*, by Pifferi and Porta (2003).

2. Stylized facts of Financial Time Series

As explained in the introduction, this study involves modeling the EUR/USD exchange rates level and volatility and we will therefore be working on time series. Modeling financial time series is complex, not only because of the wide variety of series (stock prices, interest rates, exchange rates, …), the influence of the observation's frequency (second, minute, hour, day etc.) or the availability of very large sample but because of the existence of statistics regularities (stylized facts) common to a large number of financial series and difficult to artificially reproduce from stochastic models.

Since the early work of Mandelbrot (1963), researchers have documented empirical regularities regarding these series. Due to a large body of empirical evidence, many of the regularities can be considered as stylized facts.

We introduce first some notations: let p_t be the asset price at time t and $\varepsilon_t = \log(p_t / p_{t-1})$ the logarithm of the asset return.

The series (ε_t) is often close to the one describing relative price changes:

$$r_t = \frac{p_t - p_{t-1}}{p_{t-1}} \text{ or } \varepsilon_t = \log(1 + r_t).$$

These two series have the advantage of being without unit, which facilitates comparisons between different assets. The following properties have been extensively discussed in the financial literature concerning daily series of stock prices though it can be observed for other series. Charpentier (2002) distinguished seven main properties.

2.1. Non Stationary Process

The stochastic process of p_t are usually not stationary at the second order whereas the processes built on asset return or price changes are. The price trajectories are actually close to that of a random walk without constant term.

Let's remind the definition of a strong and weak stationary process (or a process stationary at the second order). Let $(x_t, t \epsilon Z)$ be a stochastic time process:

Definition of a stationary process

The process (x_t) is said to be strictly stationary if $\forall$ the n values $t_1 < t_2 < \cdots < t_n$ such as, $\forall$ i $\in [\![1, n]\!]$, $\forall$h $\in$ Z, $t_i \in$ Z and $t_i + h \in$ Z, the series $(x_{t_1+h}, x_{t_2+h}, \dots, x_{t_n+h})$ follow the same probability law as the series ($x_{t_1}, x_{t_2}, \dots, x_{t_n}$).

Definition of a stationary process at the second order

The process (x_t) is said to be stationary at the second order if the following conditions are satisfied:

(i) $\forall t \in$ Z, E$({x_t}^2) < \infty$

(ii) $\forall t \in$ Z, E(x_t) = *m* , independent of *t*

(iii) $\forall$(*t, h*) $\in$ Z², *cov*(x_t, x_{t+h}) = E[$(x_{t+h}$ -- *m*)(x_t − *m*)] = γ(*h*), independent of *t*

Especially for high-frequency data like exchange rates, volatility is highly persistent and there exists evidence of near unit root behavior of the conditional variance process (Longmore and Robinson, 2004). This means the characteristic equation could admit a unit root and the time series model could be non-stationary and therefore non stable.

2.2. Auto-correlations of squared price changes

We see that the series (ε_t) have very low autocorrelation, making it close to a white noise. In contrast, the series of squares $({\varepsilon_t}^2)$ or absolute values $(|\varepsilon_t|)$ are often highly auto-correlated.

These two properties are not incompatible, but show that the white noise is not independent. The non-existence of auto-correlations is a direct consequence of an efficient market. I won't detail any further the theory of market efficiency and the test of the Efficient Market Hypothesis. Under this hypothesis, the price p_t incorporates all the pertinent information.

2.3. Non normality: Existence of fat tails

When one considers the sample distribution of returns or price changes, or the logarithm of the price changes, it is generally perceived that they do not correspond to a normal distribution. Conventional tests of normality tend to clearly reject the hypothesis of a normal distribution. Specifically, the density probability function of these series have fat tails and spike in zero: they are called leptokurtic distributions.

A measure of this effect is obtained from the coefficient of kurtosis: E $\left[\left(\frac{X-E(X)}{\sigma_X}\right)^4\right]$ with V(X) = σ^2 the variance of the distribution. This coefficient can be calculated with sample variance and average and is asymptotically equal to 3 for a normal distribution and is much higher for these series. This observation is also referred to as excess kurtosis.

2.4. Volatility Clustering

Large values of $|\varepsilon_t|$, or price swings, tend to be followed by large values, and small by small. We see sub-periods of high agitation prices (we say that the market is more volatile), followed by sub-periods much calmer (called low volatility). As these sub-periods are recurrent but not periodic, the series of returns is not incompatible with a stationary process, in particular homoscedastic process (with constant marginal variance).

However, since a high value of $\varepsilon_{t-1}{}^2$ appears to increase the probability of observing a high value for $\varepsilon_t{}^2$, the variance of ε_t conditional on its past values do not seem constant. This phenomenon called conditional heteroscedasticity is not incompatible with a stationary process.

2.5. Leverage effects

In financial markets, it is a stylized fact that a downward movement (depreciation) is always followed by a higher volatility. This characteristic exhibited by percentage changes in financial data is termed leverage effects. According to past studies in this field, price movements are negatively correlated with volatility. Volatility is higher after negative shocks than after positive shocks of the same magnitude. This feature was first suggested for stock returns by Black (1976).

2.6. Asymmetry of Gain-Loss

The distribution of prices is usually asymmetric: there are stronger moves of depreciation than appreciation.

A measure of this effect is obtained from the coefficient of Skewness: $E\left[\left(\frac{X-E(X)}{\sigma_X}\right)^3\right]$ with $V(X) = \sigma^2$ the variance of the distribution. This coefficient can be calculated with sample variance and average and is asymptotically equal to 0 for a symmetric distribution.

2.7. Seasonality

Regular events like holidays and weekends have effects on exchange rate volatility. Studies indicate that volatility of exchange rates returns or percentage changes is lower during weekends and holidays than during the trading week. Many studies attribute this phenomenon to the accumulative effects of information during weekends and holidays.

3. The GARCH Framework

The ARCH model was first applied in modeling the currency exchange rate by Hsieh in 1988. He showed that a generalized ARCH model could explain a large part of the nonlinearities for the five exchange rates of his study. Since then, applications of the GARCH family models have increased tremendously (see Danielsson, 1994; Christie, 1982; Brooks and Burke, 1998; Longmore and Robinson, 2004; Hafner and Herwartz, 2006). In many of the applications, it was found that a very high-order ARCH model is required to model the changing variance.

The alternative and more flexible lag structure is the Generalised ARCH introduced by Bollerslev (1986). He is the one who indicated that the squared returns of not only exchange rates but all speculative price series exhibit autocorrelation and volatility clustering. It is also proven that small lag such as GARCH(1,1) is sufficient to model the variance changing over long sample periods (French, 1987; Choo, 1999; Brooks and Burke, 2003). Some other models based on the simple GARCH one has been proposed as well. Engle (1987) proposed the GARCH in the mean model (GARCH-M) to formulate the conditional mean as function of the conditional variance, standard deviation or logarithm of the variance and a constant; the conditional variance following a classic GARCH model. This model can be seen as a natural extension due to the suggestion of the financial theory that an increase in variance will result in a higher expected return.

Even though the GARCH model can effectively remove the excess kurtosis and capture the volatility clustering in returns, it cannot capture the skewness of financial time series. A few modifications to the GARCH model have been proposed to take into account this phenomenon. Different alternatives were proposed: the Exponential GARCH or EGARCH model introduced by Nelson (1991) or the Threshold ARCH (TARCH) applied by Zakoïan (1994).

Choo (1999), Longmore and Robinson (2004) or Engle (1993) among numerous authors studied the performance of these GARCH models in forecasting the stock market and other asset prices volatility and concluded the symmetric GARCH-M (1,1), asymmetric EGARCH (1,1) and TARCH (1,1) models were effective in modeling returns volatility. I will therefore present in a first time the symmetric GARCH model, more specifically the GARCH(1,1) and the GARCH-M(1,1), before presenting the asymmetric models EGARCH(1,1) and TARCH(1,1). These models can be estimated using the Maximum Likelihood method.

For more details on the theory and estimation methodology of the GARCH framework with financial time series, you can see *Econometric Analysis* by Greene (2011), *The Econometrics of Financial Markets* by Campbell (1996) or *Introductory Econometrics for Finance* by Brooks (2006).

3.1. Testing for Autocorrelation and Heteroscedasticity

One of the most important issues before applying the GARCH methodology is to first examine the residuals of the returns series of exchange rate for evidence of autocorrelation and heteroscedastocity in the residuals. To test it, the Lagrange Multiplier (LM) test proposed by Engle (1982) can be applied. In the study, I use the software GRETL which provides the LM statistic to test for ARCH effects once a first regression is done on the mean equation of the model via the Ordinary Least Squares method.

In summary, the test procedure is performed by first obtaining the residuals e_t^2 from the ordinary least squares regression of the conditional mean equation which might be an autoregressive (AR) process, moving average (MA) process or a combination of AR and MA process (ARMA).

After obtaining the residuals e_t the next step is to do the regression of the squared residuals on a constant and q lags in the following equations:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \cdots + \alpha_q e_{t-q}^2 + v_t$$

The null hypothesis there is no ARCH effect up o order q can be formulated as:

$$H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_q = 0$$

Against the alternative $H_1: \alpha_i > 0$ for at least one $i = 1, 2, \ldots, q$

The test statistic for the joint significance of the q-lagged squared residuals is the number of observations times the R^2 from the regression. It is evaluated against the $\chi^2(q)$ distribution.

3.2. The Generalized ARCH Model

The GARCH model used in this study has only three parameters that allows for an infinite number of squared errors to influence the current conditional variance (volatility). The conditional variance determined through GARCH model is a weighted average of past squared residuals. However, the weights decline gradually but they never reach zero. Essentially, the GARCH model allows the conditional variance to be dependent upon previous own lags. The general framework of this model, GARCH (*p, q*), is expressed by allowing the current conditional variance to depend on the first *p* past conditional variances as well as the *q* past squared innovations:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

where *p* is the number of lagged σ^2 terms and *q* is the number of lagged ε^2 terms.

In this study, the following simple specifications GARCH (1, 1) are used:

Mean equation $\quad r_t = \mu + \varepsilon_t$

Variance equation $\quad \sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

where $\omega > 0$, $\alpha_1 > 0$ and $\beta_1 > 0$

r_t is the return of the asset at time *t*

μ is the average returns

ε_t is the residual returns defined as $\varepsilon_t = \sigma_t z_t$ where z_t is the standardized residual returns (i.e. *iid* random variable with zero mean and variance 1) and σ_t^2 is the conditional variance.

The constraints $\omega > 0$, $\alpha_1 > 0$ and $\beta_1 > 0$ are needed to ensure σ_t^2 is strictly positive. In this model, the mean equation is written as a function of constant with an error term. Since σ_t^2 is the one period ahead forecast variance based on past information, it is called the conditional variance. The conditional variance equation specified as a function of three terms: the constant term, ω; the news about volatility from the previous period, measured as the lag of the square residuals from the mean equation, ε_{t-1}^2 ; and the last period forecast variance, σ_{t-1}^2.

The conditional variance equation models the time varying nature of volatility of the residuals generated from the mean equation. This specification is often interpreted in a financial context, where a trader predicts this period's variance by forming a weighted average of a long term average (the constant), the forecast variance from last period and information about volatility observed in the previous period. If the asset return was unexpectedly large in either the upward or the downward direction, then the trader will increase the estimate of the variance for the next period.

3.3. The GARCH in Mean model

The GARCH-M model introduced by Engle (1987) allows the conditional mean to depend on its own conditional variance. The following GARCH-M (1,1) specifications are used:

<u>Mean equation</u> $\quad r_t = \mu + \delta\, {\sigma_t}^2 + \varepsilon_t$

<u>Variance equation</u> $\quad \sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

where ω > 0, α_1 > 0 and β_1 > 0

r_t is the return of the asset at time *t*

μ is the average returns

ε_t is the residual returns defined as ε_t = $\sigma_t z_t$ where z_t is the standardized residual returns (i.e. *iid* random variable with zero mean and variance 1) and σ_t^2 is the conditional variance.

3.4. The Exponential GARCH model

The GARCH model is not the best model to explain the leverage effects or the asymmetry observed in financial time series. Indeed, the conditional variance is a function only of the magnitudes of the past values and not their sign. The effects of a shock on the volatility are asymmetric or in other words the effect of good news, a positive lagged residual, may be different from the effects of the bad ones, a negative lagged residual. The development and the presentation of EGARCH model done by Nelson (1991) takes into account such an asymmetric response to a shock and ensures the variance is always positive. In the general form, the conditional variance is written as:

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{q} \alpha_i \left\{ \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{p} \beta_j \ln(\sigma_{t-j}^2)$$

The EGARCH model is asymmetric because the level $\frac{\varepsilon_{t-i}}{\sigma_{t-i}}$ is included with coefficient γ_i and since this coefficient is typically negative, positive returns shocks generate less volatility than negative shocks.

The following EGARCH (1, 1) characteristics are used:

<u>Mean equation</u> $r_t = \mu + \varepsilon_t$

<u>Variance equation</u> $\ln(\sigma_t^2) = \omega + \alpha_1 \left\{ \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \ln(\sigma_{t-1}^2)$

3.5. The Threshold ARCH model

The TARCH model used by Zakoïan (1994) proposes the independence for the asymmetric effect of shocks. The conditional variance is written as:

$$\sigma_t^2 = \omega + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{k=1}^{r} \gamma_k \, \varepsilon_{t-k}^2 \, I_{t-k}^-$$

Where $I_{t-k}^- = \begin{cases} 1 \; if \; \varepsilon_{t-k} < 0 \\ 0 \; otherwise \end{cases}$

In the study I will use the following TARCH (1,1):

<u>Mean equation</u> $r_t = \mu + \varepsilon_t$

<u>Variance equation</u> $\sigma_t^2 = \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 \, \varepsilon_{t-1}^2 \, I_{t-1}^-$

Where $I_{t-1}^- = \begin{cases} 1 \; if \; \varepsilon_{t-1} < 0 \\ 0 \; otherwise \end{cases}$

In the TARCH model, good news ε_t > 0 and bad news ε_t < 0 have different effects on the conditional variance. When $\gamma_k \neq 0$, it can be concluded the news impact is asymmetric and there is presence of leverage effects. The difference between the TARCH and EGARCH is that TARCH assumes leverage effect as quadratic and the EGARCH assumes leverage effect as exponential.

4. A Currency Hedging Strategy

Commercial transactions in foreign currencies are subject to exchange rate risk: changes in foreign exchange rates vary continuously on the FOREX market and can affect a company's cash flow. If the exchange rate exposure has disappeared in the euro area, it is still present for trades with the rest of the world. Depending on the currency, you can record losses in your profit margin if the transaction is made in the currency of the foreign company with which you conduct your business operations. On the contrary, if the transaction is carried out in your currency is your counterparty is exposed to currency risk.

The exchange rate at the time of settlement will be indeed different from the exchange rate at the billing time. During this period, the exchange rate's fluctuations may significantly affect the final cash flow converted into your national currency. Furthermore, a change in exchange rate of one currency to another can affect the competitiveness of a company's products: the prices offered to a foreign buyer can become more or less expensive for foreign buyers when the currency is appreciated or depreciated. For all these reasons, the choice of the currency plays a very important role. Given this currency exposure, you can either do nothing and run the risk of losses or protect yourself against the risk of exchange rates movements with the use of hedging strategies. A currency hedge will be an investment position intended to offset potential losses or gains that may happen for an investment in a foreign currency.

Döhring (2008) describes in a report for the EU commission how multinational companies operating in Europe try to manage the euro exchange rate volatility with financial or operational hedging strategies. A structural hedging also called operational or natural hedge is a long term strategy where a company tries to match its cash flows of revenue and cost in the same local currency. For example, an exporter to the United States faces a risk of changes in the value of the U.S. dollar and chooses to open a production facility in that market to match its expected sales revenue to its cost structure. The structural hedging takes time but is proven to be more effective on the long term. Some other natural hedges are:

- Currency loan: the company receives the amount of the deal immediately and suffers no change in the price. It will pay back later with the cash provided by the sales.

- The use of insurance policies: it allows the hedging of the exchange rate exposure through an insurance premium. If the company suffers a loss of exchange, it will be compensated, and if the company recorded a foreign exchange gain, it must be repaid to the insurer.

A financial hedging will use financial instruments such as stocks, exchange traded funds, swaps, options and many types of over-the-counter and derivatives products. Here is a set of hedging instruments used for hedging purpose:

- Forward exchange contract for currencies: once the company has signed a sales agreement with its counterpart abroad, it informs the bank the elements of the contract (billing currency, amount, …) and settle a forward agreement on the exchange rate and amount of currencies it will exchange on a given date. Thus, the currency risk is fully hedged because the company knows the amount it will receive at maturity, with the forward rate that is set with the bank.
- Currency future contracts: they are similar to forward contracts but are traded on regulated markets giving them some advantages I will explain later.
- Money Market Operations for currencies
- Forward Exchange Contract for interest
- Money Market Operations for interest
- Future contracts for interest
- Currency option: it offers more flexibility. A currency option is a right to buy or sell an amount of currency, which means that whoever owns the option has the right to enforce it or not. The company can hedge against exchange loss but can also benefit from a gain in the spot foreign exchange rate.

My study will focus on a short term financial hedging strategy involving three months futures contracts. I will first present the product and its use in currency hedging before presenting the methodology followed for the hedging strategy. I will focus in particular on the determination of the optimal amount of three months futures contracts to hold in a portfolio. I will use different data frequencies for the estimation process and for the observation of the hedged portfolio returns.

4.1. Futures contracts

For more details on this topic, you can see *Options, Futures and Other Derivatives* by Hull (2005). The following definition of a futures contract is actually extracted from this book.

"A futures contract, like a forward contract, is an agreement between two parties to buy or sell an asset at a certain time in the future for a certain price. Unlike forward contracts, futures are normally traded on an exchange which specifies certain standardized features of the contract. It also provides a mechanism which gives the two parties a guarantee that the contract will be honored. The largest exchanges on which futures are traded are the Chicago Board of Trade and the Chicago Mercantile Exchange. A very wide range of commodities and financial assets such as stock indices or currencies form the underlying assets of the contracts. The contract usually specifies the amount of the underlying asset, the delivery month or week of the month, how the futures price is to be quoted and possibly the limits on the amount by which the price can move in one day. Futures prices are regularly reported in the financial press and are determined on the floor of the exchange like other financial products by the force of supply and demand from investors. "

A currency future is therefore a futures contract to exchange one currency for another at a specified date in the future at a price (exchange rate) that is fixed on the purchase date. This rate fluctuates every day on the market where the contract is quoted. For most contracts, one of the currencies is the US dollar. For example, a contract will involve the sales of 100000 EUR for 134250 USD, i.e. a price of 1,3425EUR/USD. To enter into a futures contract a trader needs to pay a deposit called an initial margin then his position will be tracked on a daily basis and whenever his account makes a loss for the day, the trader would receive a margin call requiring him to pay up the losses. Further details on issues such as margin requirements, daily settlement procedures, delivery procedures, bid-ask spreads, and the role of the exchange clearinghouse are exposed in *Options, Futures and Other Derivatives* by Hull (2005). Most contracts have physical delivery, so for those held at the end of the last trading day, actual payments are made in each currency. However, most contracts are closed out before that. These contracts, mostly traded electronically, are traded on different markets: the International Monetary Market, the Euronext Liffe, Tokyo Financial Exchange, Intercontinental Exchange. The existence of futures markets is mainly due to the need and difficulty in finding counterparty for forward contracts.

The futures contracts used in this study are EUR/USD contracts traded on the Chicago Mercantile Exchange in the United States. The contracts' amount is 125000EUR, and the delivery months are March, June, September and December of the current year or the next year. The delivery occurs on the second day of the third week of the month. The quotation is expressed in term of exchange rate. For information, the Chicago Mercantile Exchange represented in 2009 a volume of more than 750 000 futures contracts a day and an average daily notional value close to 100 billion USD.

4.2. Hedging using Futures

The following definition of a futures hedging is extracted from *Options, Futures and Other Derivatives* by Hull (2005).

"Investors use these futures contracts to hedge against foreign exchange risk. A company that knows it is due to sell an asset such as currencies at a particular time in the future can hedge by taking a short futures position (sales of futures contracts). It is known as short hedge. If the price of the asset goes down, the company does not fare well on the sale of the asset but makes a gain on the short futures position. If the price goes up, the company gains from the sales of the asset but make a loss on the futures position. Similarly, a company that knows it is due to buy an asset in the future can hedge by taking a long futures position (the company buys futures). This is known as long hedge. For example, an investor based in the United States will receive 1,000,000EUR on December 1. The current exchange rate implied by the futures is 1.2347USD/EUR. He can lock in this exchange rate by selling 1,000,000EUR worth of futures contracts expiring on December 1. That way, he is guaranteed an exchange rate of 1.2347USD/EUR regardless of exchange rate fluctuations in the meantime. It is important to recognize that hedging does not necessarily improve the overall financial outcome. In fact, we can expect the outcome to be worse. The futures hedge exists to reduce risk by making the outcome more certain but has a cost. Besides, there is a number of reasons why hedging using futures works less than perfectly in practice: the asset whose price is to be hedged may not be exactly the same as the underlying of the contract; the hedger may be uncertain as to the exact date when the asset will be bought or sold; the hedge may require the futures contract to be closed out well before its expiration date. "

We can easily summarize the pro and cons of hedging with futures:

- (Pro) liquid and central market: with many market participants it is easy to buy or sell a futures contract.
- (Pro) position easily closed out: a trader who has taken a position in futures can easily make an opposite transaction and close his position.
- (Pro) leverage: a trader takes a large position with only a small initial deposit.
- (Pro) convergence: spot and futures prices tend to converge at expiration of the contract. Besides, their prices' fluctuations are highly correlated and are easy to model.
- (Cons) legal obligation: if hedging is done with futures for a project whose cash flows are not yet estimated or won't be estimated in a correct way the futures position becomes a speculative position.
- (Cons) standardized features make perfect hedging difficult: it is close to impossible to find the amount or maturity matching with a project's cash flows.
- (Cons) Daily margin calls can cause significant cash flow burdens for traders.
- (Cons) Futures hedging don't allow the trader to benefit from a gain in the spot transaction.

The disadvantages of futures contract can actually be solved by hedging with currency options, more flexible but because of the numerous futures hedging models proposed by the literature and the easy computability of these models, I choose to focus on the futures hedging strategies. I will therefore assume there aren't any of these problems or any transaction costs and that the only available hedging instrument will be futures contracts. I will therefore be able to focus the study on the determination of the optimal ratio of futures needed to hedge our position.

4.3. Optimal hedge ratio

The following definition of an optimal hedge ratio is extracted from *Options, Futures and Other Derivatives* by Hull (2005).

"The hedge ratio is the ratio of the size of the position taken in futures contracts to the size of the exposure. Up to now we have always assumed a hedge ratio of 1. We now show that, if the objective of the hedger is to minimize risk, a hedge ratio of 1 is not necessarily optimal."

We can understand that different approaches are available for the determination of an optimal hedge ratio. For example, an approach could be to minimize the variance of the hedged portfolio. This approach proposed by Johnson (1960) is called the minimum variance (MV) hedge ratio. The following definition is based on this approach and uses some other implied assumptions: the hedge ratio is constant through time and we observe a constant correlation independent of the time and the hedge ratio.

"We define:

ΔS *: Change in spot price, S, during a period of time equal to the life of the hedge (gain or loss)*
ΔF *: Change in futures price, F, during a period of time equal to the life of the hedge (gain or loss)*
σ_S *: Standard deviation of* ΔS
σ_F *: Standard deviation of* ΔF
ρ *: Coefficient of correlation between* ΔS *and* ΔF
h : Hedge ratio

The portfolio of the hedger is composed of a position in cash that we named S and the futures position that we named F. When the hedger is long in a currency and short in futures, the change in the value of the hedger's position during the life of the hedge is therefore $\Delta S - h.\Delta F$*. It is also called the payoff of the hedge. For a long hedge, the payoff will be* $h.\Delta F - \Delta S$ *.*

In either case, the variance ϑ *of the change in payoff is given by*

$$\vartheta = \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F$$

So that $\frac{\partial\vartheta}{\partial h} = 2h\sigma_F^2 - 2\rho\sigma_S\sigma_F$ *. Setting it equal to zero and noting that* $\frac{\partial^2\vartheta}{\partial h^2}$ *is positive, we see that the value of h that minimizes the variance is*

$$h = \rho\frac{\sigma_S}{\sigma_F}$$

If ρ =1 and $\sigma_S = \sigma_F$ *then h = 1 but for example if ρ =1 and* $\sigma_S = 0.5\ \sigma_F$ *then h = 0.5."*

This definition shows us that the hedge ratio is not always equal to 1 (approach called naïve hedge) and that hedging a currency in practice doesn't need the existence of a future on this particular currency but a future on an asset whose price is correlated to the price of this currency. This result illustrates the purpose of cross correlation between the different markets or financial instruments involved in the hedging strategy: it explains why a loss in some position can be offset by a gain in some other in order to reduce the variance of the returns. Because of the nature of the individual series, the multivariate GARCH models for the estimation of the covariance matrix of the joint distribution of spot and futures rates seem to be a natural framework.

Other approaches different from the MV hedge ratio are studied in the literature: Cecchetti, Cumby and Figlewski (1988) incorporated the expected returns in the determination of the optimal hedge ratio. They tried to find the optimal dynamic futures hedge for 20-year Treasury bonds by maximizing the expected returns and minimizing the variance of the returns. It is called the Mean-Variance optimal hedge ratio and is more consistent with the mean variance framework proposed in section 3 for modeling the exchange rates' distribution. This approach is also used by Kroner and Sultan in their paper from 1990 and 1993 but under their assumption (the futures prices follow a martingale process i.e. the expected delivery price is the purchase price) they actually use a simplified expression of the hedge ratio which is the expression of the MV hedge ratio. The literature proposes as well the determination of an optimal hedge ratio based on the maximization of a utility function or an expected utility function of the investor. This approach has the inconvenient of the determination of a utility function. Some more recent papers proposed stochastic approaches: hedge ratios based on the generalized semi-variance or lower partial moments.

In addition to the different definition of an optimal hedge ratio, the literature differs in term of the static or dynamic nature of the hedge ratio. Conventional approach to hedging will assume the hedge ratio is constant over time but it is clear that the distributions of spot and futures prices are changing through time, and estimating a constant hedge ratio may not be appropriate. The position to hedge evolves and might need a dynamic approach. In practice, a trader will anyway manage his position on a daily basis and a company will do it on a weekly or monthly basis. Another matter needs to be considered: the data frequency we shoud use for the estimation of the hedge ratios.

Finally, many different techniques are used to estimate the static and dynamic hedge ratios from the simple Ordinary Least Squares (OLS) to the Conditional Heteroscedastic (GARCH and ARCH), the random coefficient method, the cointegration method or the cointegration-heteroscedastic method (Kroner and Sultan in 1993). A review of the different futures hedge ratios is proposed by Chen, Lee and Shrestha (2003). The models they proposed performed with varying degrees of effectiveness and the different papers available propose two different conclusions: a time invariant model performs better than a dynamic model against the opposite conclusion.

In my study, I make the assumption that the objective of the hedger (i.e. the company) is to minimize the variance of his returns which is seen as the widely used MV approach. Then, I use different estimation methods for the constant and dynamic hedge ratios and observe the effectiveness of the hedge by constructing the hedge portfolio implied by the hedge ratios, observing its returns distribution and measuring the percentage of reduction in the variance of the spot price changes (Geppert 1995).

I will also use different data frequencies (daily, weekly and monthly) for the estimation and the observation of the portfolio returns. This way, I can see if there is any relationship between the hedge ratio, data frequency and effectiveness of the hedge.

4.4. The Conventional Hedging Model

This model is constructed assuming the investor is looking for a minimization of his returns' variance and that the only hedging instrument available is the futures contract. This model also considers the hedge ratio is static (i.e. it remains the same over time).

Let's F_0 and F_1 be the purchase price and settlement price of a futures contract, and S_0 and S_1 be the price of the spot at the time the future was purchased and the price at the time the future was settled. Then, $x = (S_1 - S_0) + h(F_1 - F_0)$ denotes the random return for holding one unit of the spot and h units of the futures. A perfect hedge would be the one for which this return remains null (mean and variance null). The variance of the portfolio return is therefore:

$$\sigma_x^2 = \sigma_S^2 + h^2\sigma_F^2 - 2h\sigma_{S,F}$$

By minimizing the variance with the first order condition with respect to h we can find the optimal number of futures contracts in the investor's portfolio:

$$h = -\frac{\sigma_{S,F}}{{\sigma_F}^2}$$

The sign is just the indication of opposite position in spot and futures. This solution called the optimal MV constant hedge ratio gives the optimal proportion of the spot position which is hedged. Then, if the distribution of spots and futures is not changing through time, it becomes possible to extend this model to a multi-period framework with a given variance for each period and to find the optimal sequence of hedge ratios $\{h_1, h_2, \dots; h_t\}$ for each period. Therefore, under these assumptions the optimal hedge could be calculated as the ordinary least squares estimator from a time series regression of changes in spot prices on changes in futures prices:

$$\Delta S_t = \alpha + \beta\Delta F_t + \varepsilon_t$$

where the estimate of the MV hedge ratio is given by (-β).

In my study, the periods used for the regression are days, weeks and months. The OLS technique is quite robust and simple to use but some assumptions need to be satisfied. In particular, the error term cannot be heteroscedastic. The simplicity of the model brought several applications in the literature: Hill and Schneeweis (1982) used the model to show that foreign exchange futures can provide a good hedge for risk in foreign exchange markets. However, this model presents some disadvantages: it uses unconditional moments and therefore doesn't account that the distributions (and therefore the variances and covariance) of spots and futures are changing through time as new information is incorporated to the prices.

4.5. The Dynamic Hedging Model

I consider here some alternatives allowing the variance and covariance to vary through time. It involves a bivariate GARCH framework for the joint distribution of spot and futures rates allowing therefore a time varying hedge ratio. The purpose of these models is to estimate the covariance matrix in order to give an estimation of the hedge ratio. Kroner and Sultan

(1990 and 1993) used this approach with a conditional constant correlation and a mean variance optimal hedge ratio under the assumption that the futures prices follow a martingale process. A review of the different multivariate models is proposed by Ding and Engle (2001). I considered here again the investor is looking for minimizing the variance of the hedged portfolio (MV approach). I present then shortly the different characteristics of the models I applied in my study.

Let's ΔS_t and ΔF_t be the changes in the prices of the spot and the futures between time $t'$and t, and $h_{t'}$ the purchases of futures at time t'. Then the payoff at time *t,* for purchasing one unit of the spot and $h_{t'}$units of the future at some time t' in the past, is:

$$x_t = \Delta S_t + h_{t'} \Delta F_t \qquad t < t'$$

The investor chooses his optimal one-period holdings of futures at each time t by minimizing his variance, the index *t* reminds that the variance is calculated conditional on all information available at time *t*. We have therefore a risk which is now measured by conditional, not unconditional, variances.

Following the minimization at the first order respect to the hedge ratio, we find the optimal MV hedge ratio at time *t*:

$$h_t = -\frac{COV_t(\Delta S_{t+1},\ \Delta F_{t+1})}{VAR_t(\Delta F_{t+1})}$$

This is similar to the conventional hedge ratio except that conditional moments replace unconditional moments. A direct consequence is that the optimal number of futures held will change through time because conditional moments are changing as new information is integrated to the prices. Kroner and Sultan (1993) called this model the conditional model since it's based on conditional moments.

In order to estimate the hedge ratio we need to specify the joint conditional distribution of the changes in prices. Because we need a stationary process, we choose the first log difference of spot price and the futures price $(\Delta s_t, \Delta f_t)$ with the condition of the first and second moments depending on time. I consider the bivariate GARCH framework:

$$\begin{cases} \Delta s_t = \beta_S + \varepsilon_{S,t} \\ \Delta f_t = \beta_F + \varepsilon_{F,t} \end{cases} \text{ with } \begin{pmatrix} \varepsilon_{S,t} \\ \varepsilon_{F,t} \end{pmatrix} |\varphi_{t-1} \sim N(0, \Omega_t) \text{ and } \Omega_t = \begin{bmatrix} \sigma_{S,t}^2 & \sigma_{SF,t} \\ \sigma_{SF,t} & \sigma_{F,t}^2 \end{bmatrix}$$

Where φ_{t-1} is the information available at time t and Ω_t is the covariance matrix.

The different models estimation will then use different assumptions on the form of the covariance matrix but they will all be estimated with the maximum likelihood method and the estimation of the optimal hedge ratio will then be given by:

$$\widehat{h_t} = -\frac{\widehat{\sigma_{SF,t}}}{\widehat{\sigma^2_{F,t}}}$$

A simple computation of the hedge ratio based on the expression of the covariance matrix expression will therefore give us the time varying hedge ratio.

We understand here a new matter need to be considered: what is the form of this covariance matrix and how do we estimate it? The literature proposes again a multitude of different models among which I noticed some popular ones: the general VECH model, the Conditional Constant Correlation and the BEKK model. I expose shortly these models.

4.5.1. The Diagonal VECH model

Under the assumption of a diagonal VECH model, the conditional covariance matrix of the dependent variables can follow a flexible dynamic structure. The VECH model is the most general model for the covariance matrix and was proposed by Bollersev, Engle and Wooldrige in 1988. It has the inconvenient to involve the determination of a prohibitive number of parameters (21 parameters just for my bivariate system). Another inconvenient is the estimated covariance matrix is not guaranteed to be positive. Bollerslev et al. (1988) proposed then the diagonal VECH model to reduce the number of parameters to estimate. The conditional covariance depends then only on its own past and on past shocks covariance (we only consider the case with 1 lag for the ARCH and GARCH terms):

$$\sigma_{SF,t} = \omega_{SF} + \alpha_{SF}\varepsilon_{S,t-1}\varepsilon_{F,t-1} + \beta_{SF}\sigma_{SF,t-1}$$

The covariance matrix is therefore given by:

$$\Omega_t = \begin{bmatrix} \sigma^2_{S,t} & \sigma_{SF,t} \\ \sigma_{SF,t} & \sigma^2_{F,t} \end{bmatrix} = \omega + \alpha\varepsilon_{t-1}\varepsilon'_{t-1} + \beta\Omega_{t-1}$$

Where $\varepsilon_t = \begin{bmatrix} \varepsilon_{S,t} \\ \varepsilon_{F,t} \end{bmatrix}$, $\omega = \begin{bmatrix} \omega_S & \omega_{SF} \\ \omega_{SF} & \omega_F \end{bmatrix}$, $\alpha = \begin{bmatrix} \alpha_S & \alpha_{SF} \\ \alpha_{SF} & \alpha_F \end{bmatrix}$ and $\beta = \begin{bmatrix} \beta_S & \beta_{SF} \\ \beta_{SF} & \beta_F \end{bmatrix}$

This simplification let us 9 parameters to estimate. Ding and Engle (2001) proposed then different models based on the diagonal VECH with restrictions on these parameters allowing the covariance matrix to be positive semi-definite. EVIEWS 6 proposes to estimate the diagonal VECH and allows some of restrictions on the different matrix of parameters: I can impose a scalar, diagonal, rank 1 or full rank matrix for each of the parameters matrix. I chose the simple model proposed by Ding and Engle (2001) for which I impose α and β to be scalars. It allows in a simple way the variance and covariance to vary through time.

4.5.2. The Conditional Constant Correlation model (CCC)

Another model of covariance is the conditional constant correlation (CCC) proposed by Bollerslev (1992). Under such assumption we have the following covariance matrix:

$$\Omega_t = \begin{bmatrix} \sigma_{S,t} & 0 \\ 0 & \sigma_{F,t} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} \sigma_{S,t} & 0 \\ 0 & \sigma_{F,t} \end{bmatrix}$$

where $\sigma_{S,t}^2$ and $\sigma_{F,t}^2$ follow a GARCH process. I consider only the GARCH (1,1) process in my study. It is the model used by Kroner and Sultan (1990 and 1993). This model has the inconvenient to consider a correlation constant through time which is sometimes unrealistic. Tse (1998) proposed then a simple test based on the Lagrange Multiplier (LM) test to determine if that hypothesis is sustainable. He found in his study that correlation could be considered constant for spot-futures and foreign exchange data but not for national stock market returns.

4.5.3. The Diagonal BEKK model

The last model proposed in my study is the diagonal BEKK based on the BEKK model first proposed by Baba, Engle, Kraft and Kroner (1991) and republished by Engle and Kroner (1995). This model imposes restrictions over parameters and includes all positive definite diagonal VECH models. It solves the problem of the positive definiteness.

The covariance matrix is given then by:

$$\Omega_t = \begin{bmatrix} \sigma_{S,t}^2 & \sigma_{SF,t} \\ \sigma_{SF,t} & \sigma_{F,t}^2 \end{bmatrix} = \omega + \alpha\varepsilon_{t-1}\varepsilon_{t-1}'\alpha + \beta\Omega_{t-1}\beta$$

In EVIEWS 6, a simplified diagonal BEKK model is proposed for which α and β are diagonal. The covariance follows then the same form of equations as for the diagonal VECH and the positive definitiveness is guaranteed.

5. The Data: Sample Description and Empirical Results

Exchange rates used in this study are daily bilateral spot and futures rates from the 19th of June 2007, first trading date of the three months contract of September 2007, to the 18th of March 2013, last trading date of the three months contract of March 2013. I remind the data is provided respectively by the Statistic Data Warehouse of the European Central Bank for the spot rates and by The Wall Street Journal for the future rates. The study is limited to EUR/USD exchange rates. The spot rates are the average daily rates obtained from the ECB reference rate at 2.15p.m. (based on Bloomberg's Data of the day). The futures rates are the settlement prices published by the Wall Street Journal whose source is Thomson Reuters for EUR/USD contracts traded on the Chicago Mercantile Exchange in the United States. Spot and Futures are quoted with 4 decimals and are expressed in terms of USD per EUR.

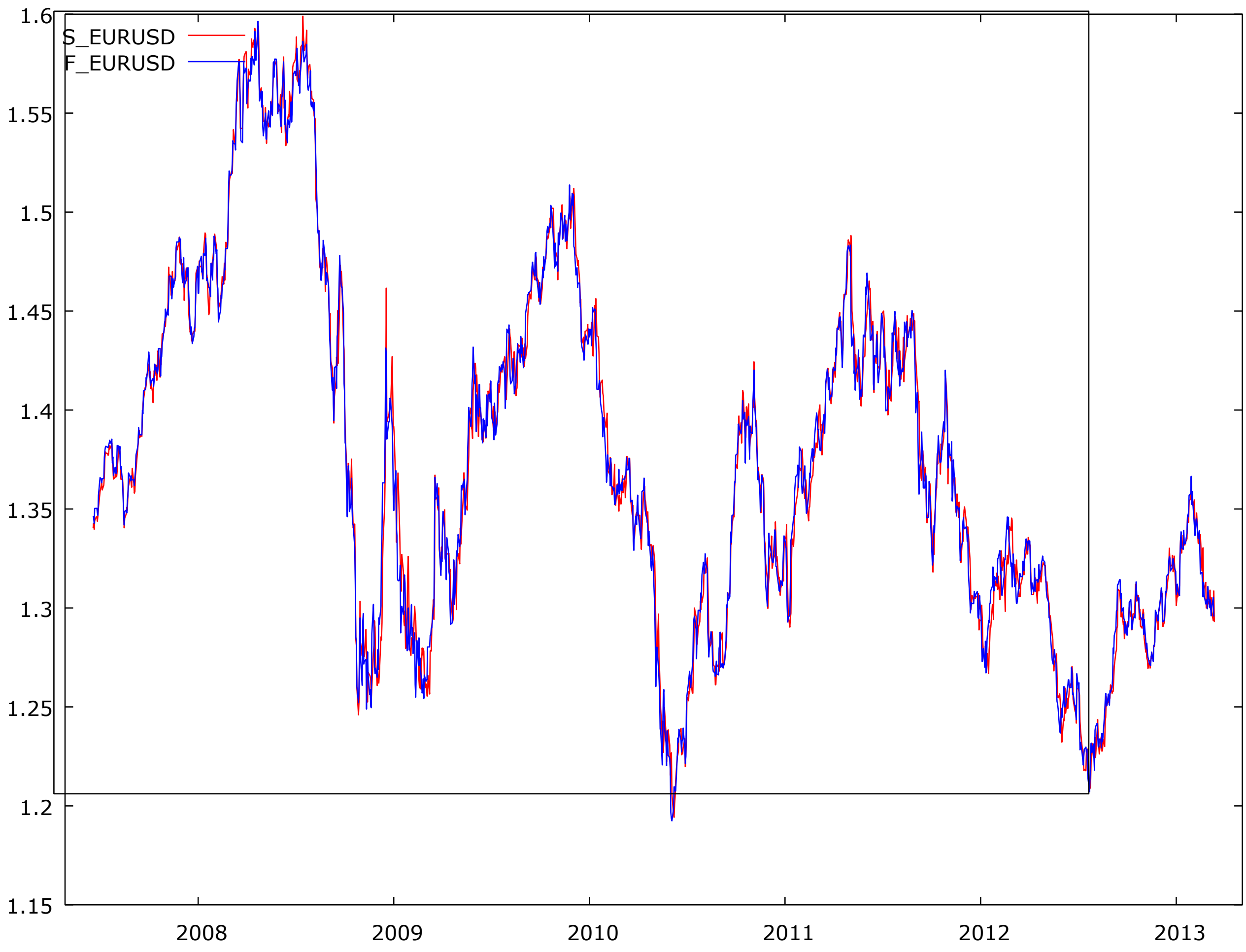
The study is made with the help of two different software: GRETL for the sample description and the different tests on the individual series; EVIEWS 6 for the estimation of the univariate and bivariate GARCH models. GRETL is used and provided during the course of Applied Econometrics at Politecnico di Milano.

5.1. Sample Description

The Data set was constructed the following way: I collected first the daily spot for the EUR/USD; then, I collected daily settlement prices during the trading days for every three months futures contracts from 2007 to 2013. The trading days for spot and futures are the same since the data are provided for every day except weekends and holydays. When the future rate was missing for an unknown reason while the spot rate was provided, we kept the future rate of the previous trading date. I will not try to describe the seasonality effects on these series. I chose to focus only on the asymmetry, excess kurtosis and clustering effect. For the futures data, I proceed by collecting the sequential 60 to 64 trading days of the shortest term contract (a three months contract) until delivery before keeping the observations of the next three months contract. An example to illustrate the process: I collected the 64 daily quotations of the March2008 EUR/USD futures contract, from the 18th of December 2007 (first trading day after delivery of the previous contract) to the 17th of March 2008 (last trading day of the current contract); I then started collecting the daily settlement prices for the next contract (June2008 EUR/USD) from the first trading day after delivery of the previous contract to the last trading day of the current contract.

This filter process gives us a sequence of 1496 daily settlement prices for future rates and 1496 average daily spot rates for the EUR/USD, for a total of 23 different three month futures contracts. I proceed then with the collect of weekly and monthly data by taking the average values of spot and futures rates for every weeks and month of the period of study. We obtain then 69 monthly spot and futures rates and 301 weekly spot and futures rates.

A first plot of the daily spot and futures EUR/USD rates is provided as well as the summary statistics:



Tab 5.1.Daily Spot and Three Month Futures EUR/USD exchange rates

Variable	Mean	Median	Minimum	Maximum
S_EURUSD	1.37530	1.36410	1.19420	1.59900
F_EURUSD	1.37501	1.36510	1.19240	1.59640
Variable	Std. Dev.	C.V.	Skewness	Ex. kurtosis
S_EURUSD	0.0887528	0.0645334	0.436533	-0.428354
F_EURUSD	0.0878860	0.0639166	0.424231	-0.443976

Tab 5.2. Summary Statistics, using the daily observations 2007/06/19 - 2013/03/18

We can observe a stable level with a really small downward trend but without any structural changes. We consider therefore no linear trend in the spot or future rate over the period of study. The futures prices following the spot prices closely we can expect a high correlation factor in the joint distribution of returns. A first series of unit roots test is performed on the logarithms of the spot and futures prices using the Dickey Fuller test provided by GRETL with four lags and a constant:

```
sample size 1496
unit-root null hypothesis: a = 1

test with constant
model: (1-L)y = b0 + (a-1)*y(-1) + e
1st-order autocorrelation coeff. for e: 0.012
estimated value of (a - 1): -0.00586748
test statistic: tau_c(1) = -2.05384
p-value 0.2639
```

Tab 5.3. Dickey-Fuller test for the logarithm of the daily Spot EURUSD rate

```
sample size 1496
unit-root null hypothesis: a = 1

   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + e
   1st-order autocorrelation coeff. for e: 0.037
   estimated value of (a - 1): -0.00582157
   test statistic: tau_c(1) = -2.04594
   p-value 0.2672
```

Tab 5.4. Dickey-Fuller test for the logarithm of the daily Future EURUSD rate

Based on the results, the number of lags chosen is sufficient (regarding the small autocorrelation coefficients) and I can't reject the hypothesis of a unit root in the logarithm of the prices at the usual level of 5 or 10% of significance. These are therefore non-stationary processes. Similar results are obtained with weekly and monthly data.
As explained in the section about Financial Time series characteristics, I then need to work with the first log differences of the spot and the futures prices: $\Delta s_t = \log\left(\frac{S_t}{S_{t-1}}\right)$ and $\Delta f_t = \log\left(\frac{F_t}{F_{t-1}}\right)$.

I proceed with the same tests and observe this time the log differences follow a stationary process for all the different data frequencies:

```
sample size 1495
unit-root null hypothesis: a = 1

   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + e
   1st-order autocorrelation coeff. for e: 0.000
   estimated value of (a - 1): -0.99053
   test statistic: tau_c(1) = -38.2387
   p-value 5.692e-017
```

Tab 5.5. Dickey-Fuller test for the first log difference of the daily Spot EURUSD rate

```
sample size 1495
unit-root null hypothesis: a = 1

   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + e
   1st-order autocorrelation coeff. for e: 0.001
   estimated value of (a - 1): -0.966147
   test statistic: tau_c(1) = -37.3527
   p-value 1.144e-019
```

Tab 5.6. Dickey-Fuller test for the first log difference of the daily Future EURUSD rate

```
sample size 69
unit-root null hypothesis: a = 1

   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
   1st-order autocorrelation coeff. for e: 0.014
   lagged differences: F(2, 62) = 1.646 [0.2012]
   estimated value of (a - 1): -0.910616
   test statistic: tau_c(1) = -3.97255
   asymptotic p-value 0.001567
```

Tab 5.7. Dickey-Fuller test for the first log difference of the monthly Spot EURUSD rate

```
sample size 69
unit-root null hypothesis: a = 1

   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + e
   1st-order autocorrelation coeff. for e: 0.024
   estimated value of (a - 1): -0.778508
   test statistic: tau_c(1) = -6.47691
   p-value 8.141e-007
```

Tab 5.8. Dickey-Fuller test for the first log difference of the monthly Futures EURUSD rate

```
sample size 301
unit-root null hypothesis: a = 1

   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + e
   1st-order autocorrelation coeff. for e: -0.004
   estimated value of (a - 1): -0.74477
   test statistic: tau_c(1) = -13.2941
   p-value 1.231e-024
```

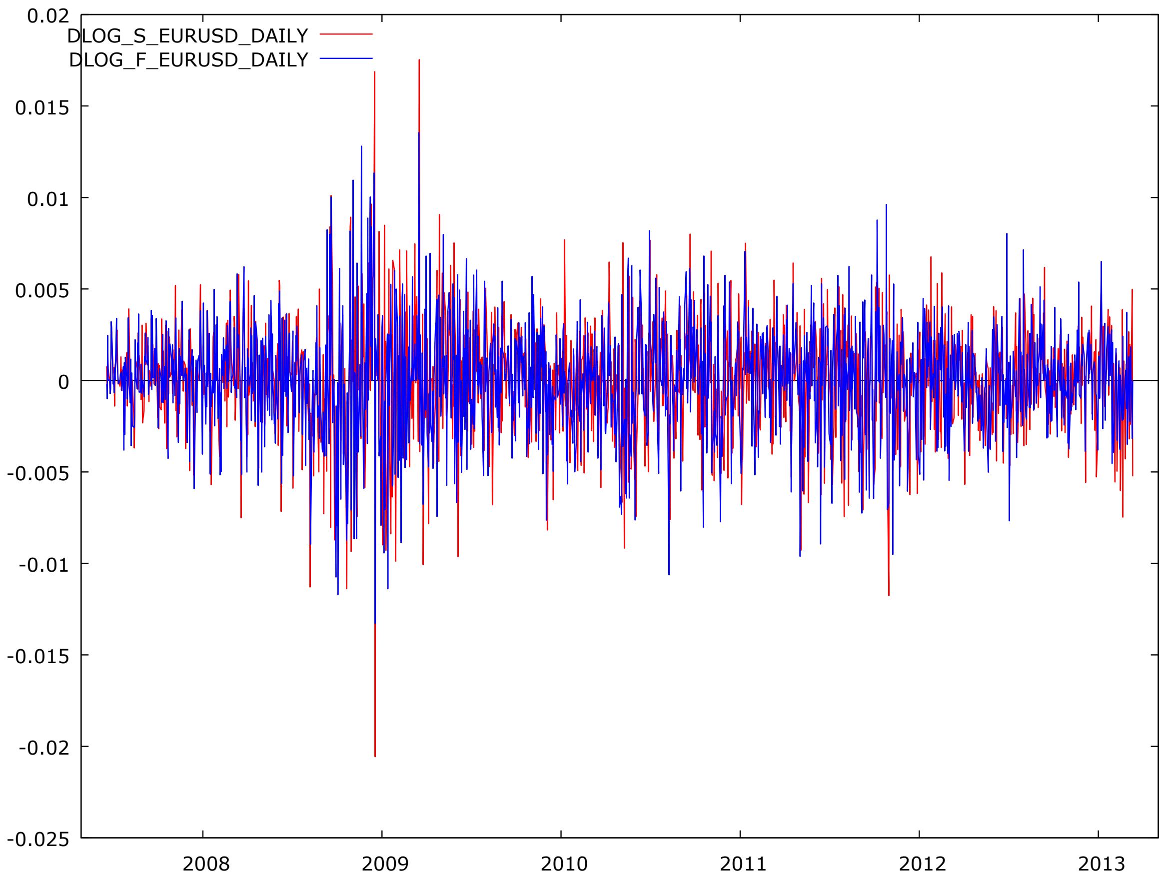
Tab 5.9. Dickey-Fuller test for the first log difference of the weekly Spot EURUSD rate

```
sample size 301
unit-root null hypothesis: a = 1

   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + e
   1st-order autocorrelation coeff. for e: 0.005
   estimated value of (a - 1): -0.69076
   test statistic: tau_c(1) = -12.5392
   p-value 4.651e-023
```

Tab 5.10. Dickey-Fuller test for the first log difference of the weekly Futures EURUSD rate

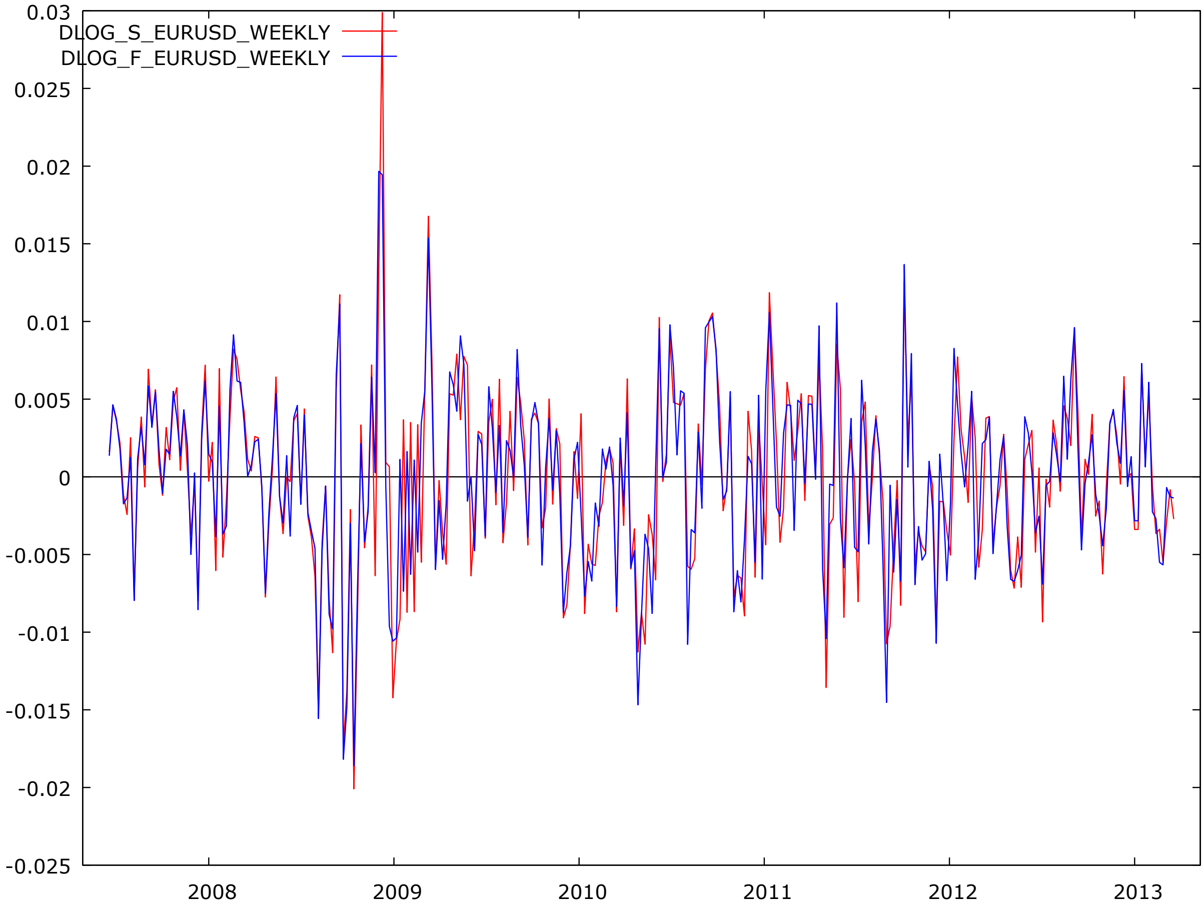
We can observe the unconditional distributions of Δs_t and Δf_t noted DLOG_S_EURUSD_"Frequency" and DLOG_F_EURUSD_"Frequency" and provide the summary statistics. I can already notice graphically that the futures movements follow the spot movements for the weekly and monthly data:



Tab 5.11. First log difference of the daily Spot and Futures EURUSD prices

Variable	Mean	Median	Minimum	Maximum
DLOG_S_EURUSD_DAIILY	-2.40680e-005	0.000136837	-0.0473544	0.0403771
DLOG_F_EURUSD_DAILY	-2.55541e-005	0.000000	-0.0305682	0.0311839
Variable	Std. Dev.	C.V.	Skewness	Ex. kurtosis
DLOG_S_EURUSD_DAIILY	0.00706512	293.548	-0.186343	3.14246
DLOG_F_EURUSD_DAILY	0.00697277	272.863	-0.0960317	1.52958

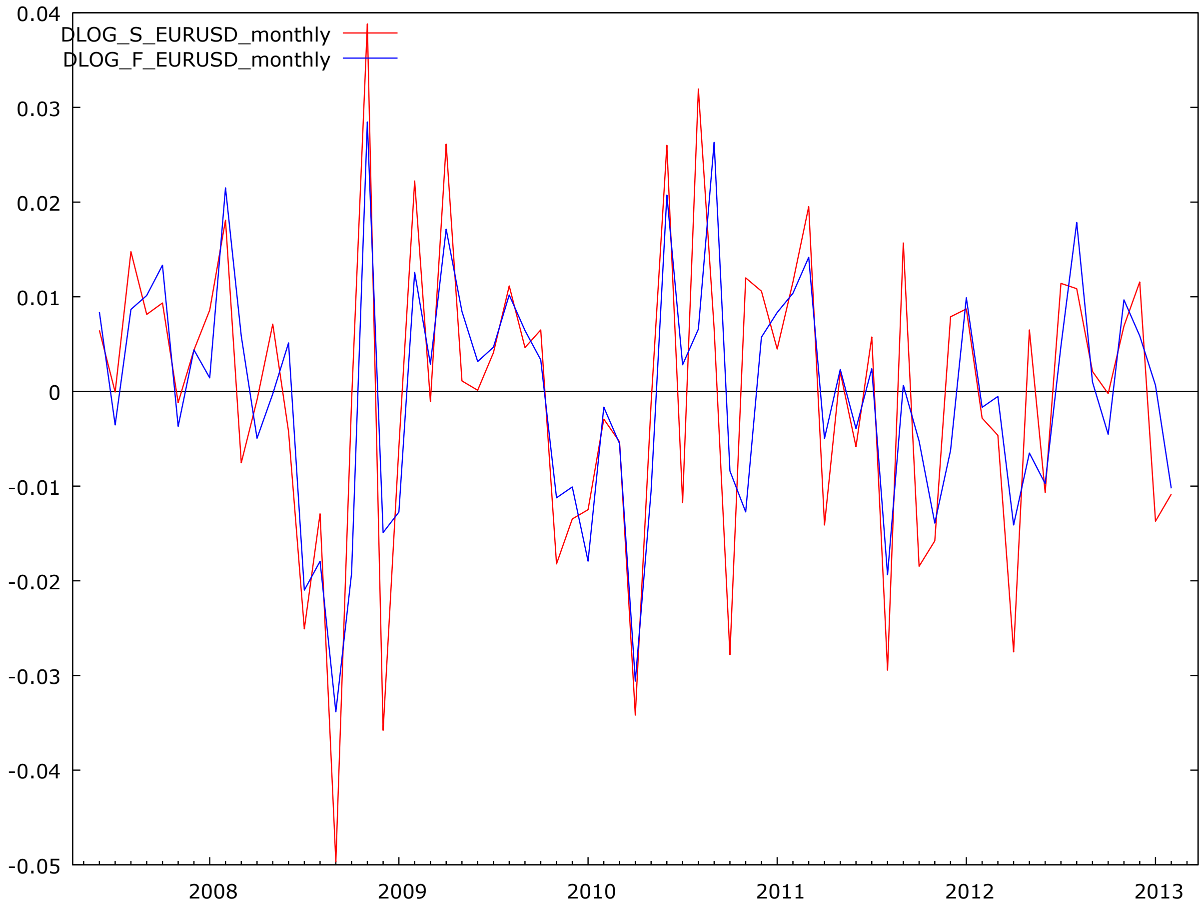
Tab 5.12. Summary Statistics, using the daily observations 2007/06/19 - 2013/03/18



Tab 5.13. First log difference of the weekly Spot and Futures EURUSD prices

Variable	Mean	Median	Minimum	Maximum
DLOG_S_EURUSD_WEEKLY	-6.38035e-005	-5.59813e-006	-0.0200856	0.0298877
DLOG_F_EURUSD_WEEKLY	-6.09499e-005	0.000316682	-0.0185899	0.0196629
Variable	Std. Dev.	C.V.	Skewness	Ex. kurtosis
DLOG_S_EURUSD_WEEKLY	0.00582143	91.2400	0.130093	2.13894
DLOG_F_EURUSD_WEEKLY	0.00560525	91.9648	-0.0649090	1.03258
Variable	5% Perc.	95% Perc.	IQ range	Missing obs.
DLOG_S_EURUSD_WEEKLY	-0.00916185	0.00788459	0.00768063	0
DLOG_F_EURUSD_WEEKLY	-0.00885479	0.00913432	0.00725955	0

Tab 5.14. Summary Statistics, using the weekly observations 2007/06/19 - 2013/03/18



Tab 5.13. First log difference of the monthly Spot and Futures EURUSD prices

Variable	Mean	Median	Minimum	Maximum
DLOG_S_EURUSD_monthly	-0.000335000	0.00110758	-0.0496786	0.0388196
DLOG_F_EURUSD_monthly	-0.000224151	0.000989333	-0.0338319	0.0284620
Variable	Std. Dev.	C.V.	Skewness	Ex. kurtosis
DLOG_S_EURUSD_monthly	0.0160591	47.9375	-0.480466	0.751745
DLOG_F_EURUSD_monthly	0.0123516	55.1041	-0.226084	0.196719
Variable	5% Perc.	95% Perc.	IQ range	Missing obs.
DLOG_S_EURUSD_monthly	-0.0318133	0.0260604	0.0197944	0
DLOG_F_EURUSD_monthly	-0.0201764	0.0211196	0.0174264	0

Tab 5.14. Summary Statistics, using the monthly observations 2007/06/19 - 2013/03/18

Variable	Jarque-Bera test
DLOG_S_EURUSD_monthly	4.27947, with p-value 0.117686
DLOG_F_EURUSD_monthly	0.69907, with p-value 0.705016

Tab 5.15. Test for Normality for the first log differences of monthly Spot and Future rates

Variable	Jarque-Bera test
DLOG_S_EURUSD_WEEKLY	58.2277, with p-value 2.26992e-136
DLOG_F_EURUSD_WEEKLY	13.5837, with p-value 0.00112291

Tab 5.16. Test for Normality for the first log differences of weekly Spot and Future rates

Variable	Jarque-Bera test
DLOG_S_EURUSD_DAIILY	624.205, with p-value 2.85543e-136
DLOG_F_EURUSD_DAILY	148.135, with p-value 6.80647e-033

Tab 5.17. Test for Normality for the first log differences of daily Spot and Future rates

The same way, I don't observe any linear trend but what looks like a random walk around a constant level for the daily and weekly data. They are non-normal, as evidenced by high skewness for the daily and weekly spot rates, high excess kurtosis for the daily and weekly spot and future rates and highly significant Bera-Jarque statistics. The excess kurtosis will be reduced with a GARCH framework and the asymmetry implied by the high skewness can be managed by asymmetric models. The monthly data doesn't show any of the characteristics of the financial time series and the hypothesis of non-normality can't be rejected. It could be explained by the low number of observations (69).

A plot of the Correlogram giving the autocorrelation and partial autocorrelation functions for the first log differences of spot and futures rates for the different data frequencies is provided in Annex 1. It doesn't seem to show any specific order of autocorrelation for the daily and monthly observations since we can't observe any specific pattern or significance in the autocorrelations but we can observe a significant first order correlation for the weekly data.

But the test provided by GRETL for ARCH effects in the residuals from the mean equation regression via the OLS method confirmed the presence of autocorrelation and heteroscedasticity: I obtained LM statistic well above the $\chi^2(10)$ critical value of 18,307 (for a 5% level of confidence) for the daily data.

```
Null hypothesis: no ARCH effect is present
Test statistic: LM = 172.612
with p-value = P(Chi-square(10) > 172.612) = 7.98013e-032
```

Tab 5.18. Test for ARCH(10) in residuals of an OLS regression on the first log difference of daily Spot rates

```
Null hypothesis: no ARCH effect is present
Test statistic: LM = 120.878
with p-value = P(Chi-square(10) > 120.878) = 3.35696e-021
```

Tab 5.19 Test for ARCH(10) in residuals of an OLS regression on the first log difference of daily Future rates

The similar tests are performed for weekly data and monthly data with 2 lags: I obtained LM statistic well above the $\chi^2(2)$ critical value of 5,991 (at a 5% level of confidence) for the weekly data but no evidence of autocorrelation in the error term for the monthly data.

```
Null hypothesis: no autocorrelation
Test statistic: LMF = 10.3937
with p-value = P(F(2,298) > 10.3937) = 4.33045e-005
```

Tab 5.20. Test for ARCH(2) in residuals of an OLS regression on the first log difference of weekly Spot rates

```
Null hypothesis: no autocorrelation
Test statistic: LMF = 15.8056
with p-value = P(F(2,298) > 15.8056) = 2.99178e-007
```

Tab 5.21 Test for ARCH(2) in residuals of an OLS regression on the first log difference of weekly Future rates

Null hypothesis: no autocorrelation Test statistic: LMF = 0.310909 with p-value = P(F(2,66) > 0.310909) = 0.733848

Tab 5.22. Test for ARCH(2) in residuals of an OLS regression on the first log difference of monthly Spot rates

Null hypothesis: no autocorrelation Test statistic: LMF = 1.94298 with p-value = P(F(2,66) > 1.94298) = 0.151385

Tab 5.23 Test for ARCH(2) in residuals of an OLS regression on the first log difference of monthly Future rates

In conclusion, unlike the monthly data, the series of test performed on the weekly and daily data set shows that GARCH models seem appropriate for modeling the daily and weekly EUR/USD spot and future rates distributions and a bivariate GARCH framework seem therefore natural for modeling the joint distribution. I will now use the software EVIEWS 6 for the estimation of the univariate and bivariate models.

5.2. The GARCH framework: Empirical Results

In this section, I quickly estimate the different models proposed in the literature on the individual daily and weekly series and try to determine which ones perform the best. The following univariate models estimation are performed with the Maximum Likelihood estimation method. I provide the full results of the estimation in the Annex.

Based on the Akaike Info Criterion, I notice the models which performed the best for the individual series are:

- For the daily data: the GARCH-M (1,1) performed the best for modeling the first log difference of the spot rates and the GARCH (1,1) performed the best for the futures rates.
- For the weekly data: the TARCH (1,1) performed the best for modeling both the first log difference of the spot and futures rates.

5.2.1. The daily data: GARCH-M (1, 1) and GARCH (1,1)

The complete estimation is provided in Annex 2 and 8. I have the following results for the first log difference of the spot rates:

$$\Delta s_t = \underset{(0{,}000147)}{0{,}000292} - \underset{(17{,}81152)}{32{,}31904}\, \sigma_{s,t}^2 + \varepsilon_{S,t}$$

$$\sigma_{s,t}^2 = \underset{(0{,}0000000234)}{0{,}0000000807} + \underset{(0{,}006091)}{0{,}035079}\, \varepsilon_{s,t-1}^2 + \underset{(0{,}006966)}{0{,}956635}\, \sigma_{s,t-1}^2$$

The log likelihood for the model is 6643,905 and the Akaike info criterion (AIC) is -8,875541.

And for the first log difference of the futures rates:

$$\Delta f_t = \underset{(0{,}0000669)}{0{,}0000472} + \varepsilon_{f,t}$$

$$\sigma_{f,t}^2 = \underset{(0{,}0000000231)}{0{,}0000000698} + \underset{(0{,}006146)}{0{,}033016}\, \varepsilon_{f,t-1}^2 + \underset{(0{,}006825)}{0{,}959340}\, \sigma_{f,t-1}^2$$

The log likelihood for the model is 6650,967 and the Akaike info criterion (AIC) is -8,886319.

5.2.2. The weekly data: TARCH (1,1)

The complete estimation is provided in Annex 7. We have the following results for the first log difference of the spot rates:

$$\Delta s_t = \underset{(0{,}000315)}{0{,}0000945} + \varepsilon_{s,t}$$

$$\sigma_{s,t}^2 = \underset{(0{,}000000781)}{0{,}00000141} + \underset{(0{,}055863)}{0{,}893005}\, \sigma_{s,t-1}^2 + \underset{(0{,}041816)}{0{,}003462}\, \varepsilon_{s,t-1}^2 + \underset{(0{,}045386)}{0{,}113940}\, \varepsilon_{s,t-1}^2\, I_{s,t-1}^-$$

Where $I_{s,t-1}^- = \begin{cases} 1 \; if \; \varepsilon_{s,t-1} < 0 \\ 0 \; otherwise \end{cases}$

The log likelihood for the model is 1144,168 and the Akaike info criterion (AIC) is -7,569226.

And for the first log difference of the futures rates:

$$\Delta f_t = \underset{(0{,}000307)}{0{,}000133} + \varepsilon_{f,t}$$

$$\sigma_{f,t}^2 = \underset{(0{,}000000689)}{0{,}00000133} + \underset{(0{,}050386)}{0{,}904052}\, \sigma_{f,t-1}^2 - \underset{(0{,}038817)}{0{,}011850}\, \varepsilon_{f,t-1}^2 + \underset{(0{,}049545)}{0{,}121800}\, \varepsilon_{f,t-1}^2\, I_{f,t-1}^-$$

Where $I^{-}_{f,t-1} = \begin{cases} 1\ if\ \varepsilon_{f,t-1} < 0 \\ 0\ otherwise \end{cases}$

The log likelihood for the model is 1152,272 and the Akaike info criterion (AIC) is -7,623072.

5.2.3. Conclusion

The results confirmed the clustering effect: highly significant coefficient on the GARCH terms of the variance equations. It also confirmed the variance has negative effects on the spot returns. I notice the asymmetry for the weekly data with a higher coefficient for the downwards movement than the upwards movements of the residuals. These results promote therefore the use of bivariate GARCH models for the estimation of the optimal hedge ratio and the introduction of a TARCH term in the case of the weekly data.

5.3. The Optimal Hedge Ratio: Empirical Results

The bivariate models of our study are estimated in EVIEWS with the Autoregressive method.

5.3.1. The conventional hedging model

Here we consider the case of a constant variance and hedge ratio (no GARCH and ARCH terms in the variance equation) for the spot and future rates distributions. We can therefore apply this model for the daily, weekly and monthly data. The optimal hedge is calculated as the ordinary least squares estimator from a time series regression of changes in spot prices on changes in futures prices. The complete estimation is provided in Annex 10. I obtained the following results (standard error in parentheses):

Daily data:
$$\Delta s_t = -0{,}00000666 + 0{,}342062\ \Delta f_t + \varepsilon_t$$
(0,0000747) (0,024675)

Weekly data:
$$\Delta s_t = -0{,}00000887 + 0{,}901367\ \Delta f_t + \varepsilon_t$$
(0,000167) (0,029836)

Monthly data:
$$\Delta s_t = -0{,}0000984 + 1{,}055477\ \Delta f_t + \varepsilon_t$$
(0,001137) (0,092751)

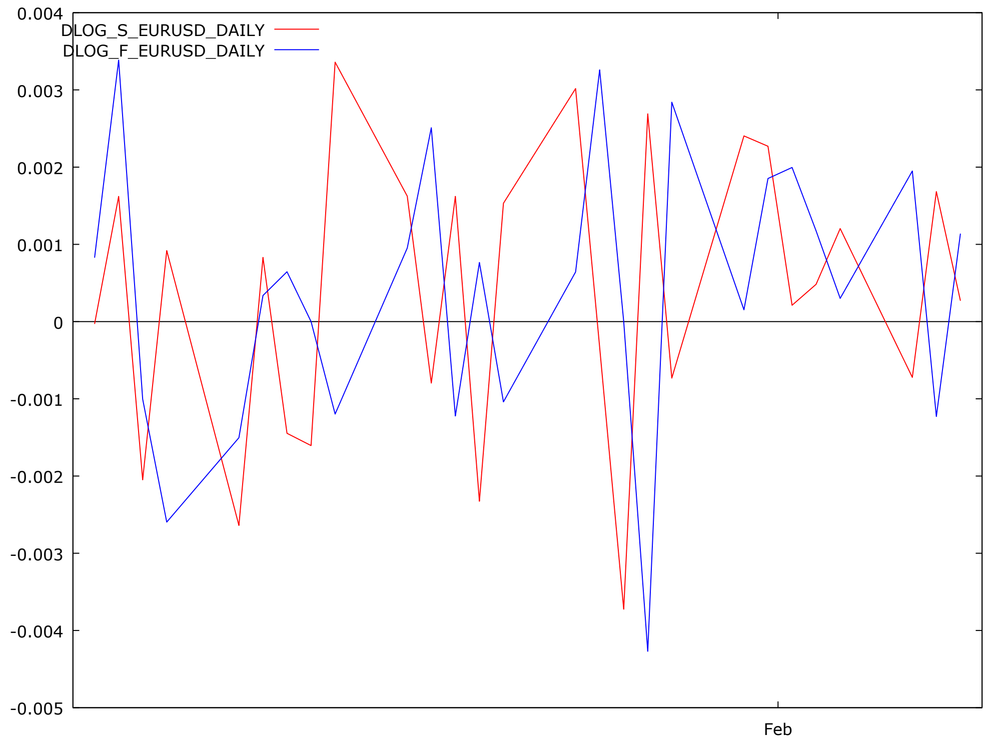
The implied hedge ratios would be:

$$\widehat{h_{DAILY}} = -0{,}342062$$

$$\widehat{h_{WEEKLY}} = -0{,}901367$$

$$\widehat{h_{MONTHLY}} = -1{,}055477$$

If the variances and covariance are constant through time then this set of ratios would be the optimal risk-minimizing hedge ratio based on daily, weekly and monthly data. I can notice right away the differences in the value of the hedge ratio depending on the frequency of the data I used. It seems the longer is this frequency, the bigger is the hedge ratio. This result suggests a structural feature of the daily spot and future prices: it seems the daily spot and futures prices have a low correlation unlike weekly and monthly prices. Indeed, by taking a closer look at the daily data and the weekly, I can clearly see that daily spot and futures prices are not perfectly correlated unlike weekly spot and futures prices. I took random sub-samples of 5 weeks of data and clearly observed that daily futures prices movements follow daily spot prices movements but with a time lag of one or two days. Here is a print of the feature:



Tab 5.24. First log differences of spot and futures prices over a random period of 5 weeks

This result actually shows that the futures market needs time to incorporate the information in its prices and therefore suggests that hedging using daily observations for estimating the optimal hedge ratio might not be appropriate because the correlation between spot and futures prices will be underestimated. The time lag should be taken into consideration. The application of the dynamic hedge ratios models using daily observations for the estimation process and the measure of the hedging effectiveness will confirm this result.

Anyway, point was clear from the sample description: variances and covariance have a time varying nature for the individual daily and weekly time series, implying that the hedge ratio should also be continually changing.

5.3.2. The dynamic hedging model

5.3.2.1. The bivariate GARCH (1,1) and restricted diagonal VECH covariance model

The model is estimated using the maximum likelihood and the results are in Annex 11. I indicate here the expression for the covariance matrix I obtained and used for the computation of the optimal hedge ratio:

For the daily data:

$$\sigma_{S,t}^2 = 0{,}0000000789 + 0{,}032277\, \varepsilon_{S,t-1}^2 + 0{,}959855\sigma_{S,t-1}^2$$

$$\sigma_{F,t}^2 = 0{,}0000000751 + 0{,}032277\, \varepsilon_{F,t-1}^2 + 0{,}959855\, \sigma_{F,t-1}^2$$

$$\sigma_{SF,t} = 0{,}0000000242 + 0{,}032277\, \varepsilon_{S,t-1}\varepsilon_{F,t-1} + 0{,}959855\, \sigma_{SF,t-1}$$

For the weekly data:

$$\sigma_{S,t}^2 = 0{,}00000127 + 0{,}103248\, \varepsilon_{S,t-1}^2 + 0{,}873055\, \sigma_{S,t-1}^2$$

$$\sigma_{F,t}^2 = 0{,}00000118 + 0{,}103248\, \varepsilon_{F,t-1}^2 + 0{,}873055\, \sigma_{F,t-1}^2$$

$$\sigma_{SF,t} = 0{,}00000113 + 0{,}103248\, \varepsilon_{S,t-1}\varepsilon_{F,t-1} + 0{,}873055\, \sigma_{SF,t-1}$$

5.3.2.2. The bivariate GARCH (1,1) and the CCC covariance model

The model is estimated using the maximum likelihood and the results are in Annex 12. I found the following expression for the covariance matrix:

For the daily data:

$$\sigma_{s,t}^2 = 0{,}0000000659 + 0{,}030559\, \varepsilon_{s,t-1}^2 + 0{,}962852\, \sigma_{s,t-1}^2$$

$$\sigma_{f,t}^2 = 0{,}0000000604 + 0{,}029602\, \varepsilon_{f,t-1}^2 + 0{,}963870\, \sigma_{f,t-1}^2$$

And $\hat{\rho} = 0{,}330101$ with standard error 0,021881. We remark again the unusual low correlation between the spot and futures rates movements.

For the weekly data:

$$\sigma_{s,t}^2 = -0{,}0000000381 + 0{,}095103\, \varepsilon_{s,t-1}^2 + 0{,}916823\, \sigma_{s,t-1}^2$$

$$\sigma_{f,t}^2 = 0{,}0000000828 + 0{,}074988\, \varepsilon_{f,t-1}^2 + 0{,}930272\, \sigma_{f,t-1}^2$$

And $\hat{\rho} = 0{,}900656$ with standard error 0,010628. We remark here the high correlation between the spot and futures rates movements which is more consistent with the description of the individual series.

5.3.2.3. The bivariate GARCH(1,1) and the diagonal BEKK covariance model

The model is estimated using the maximum likelihood and the results are in Annex 13. I found the following expression for the covariance matrix:

For the daily data:

$$\sigma_{S,t}^2 = 0{,}0000000764 + 0{,}03171106\varepsilon_{S,t-1}^2 + 0{,}960744991\sigma_{S,t-1}^2$$

$$\sigma_{F,t}^2 = 0{,}0000000785 + 0{,}032907774\varepsilon_{F,t-1}^2 + 0{,}958199347\sigma_{F,t-1}^2$$

$$\sigma_{SF,t} = 0{,}0000000244 + 0{,}032303876\varepsilon_{S,t-1}\varepsilon_{F,t-1} + 0{,}959771676\sigma_{SF,t-1}$$

For the weekly data:

$$\sigma_{S,t}^2 = 0{,}00000308 + 0{,}133636307\varepsilon_{S,t-1}^2 + 0{,}770281809\sigma_{S,t-1}^2$$

$$\sigma_{F,t}^2 = 0{,}00000205 + 0{,}065909779\varepsilon_{F,t-1}^2 + 0{,}8985371\sigma_{F,t-1}^2$$

$$\sigma_{SF,t} = 0{,}00000116 + 0{,}093850623\varepsilon_{S,t-1}\varepsilon_{F,t-1} + 0{,}831941602\sigma_{SF,t-1}$$

5.3.2.4. The bivariate TARCH(1,1)

This model adds a TARCH term in the variance and covariance expressions to solve the problem of asymmetry. Based on the results for the individual series in section 5.2 I use this model only with the weekly data. The model is estimated using the maximum likelihood and the results are in Annex 14, 15 and 16. The following results are obtained for the covariance matrix:

For the CCC model:

$$\sigma_{s,t}^2 = 0{,}000000113 + 0{,}920517\,\sigma_{s,t-1}^2 + 0{,}041346\,\varepsilon_{s,t-1}^2 + 0{,}083742\,\varepsilon_{s,t-1}^2\,I_{s,t-1}^-$$

$$\sigma_{f,t}^2 = 0{,}000000250 + 0{,}935657\,\sigma_{f,t-1}^2 + 0{,}016056\,\varepsilon_{f,t-1}^2 + 0{,}0088920\,\varepsilon_{f,t-1}^2\,I_{f,t-1}^-$$

Where $I_{s,t-1}^- = \begin{cases} 1\ if\ \varepsilon_{s,t-1} < 0 \\ 0\ otherwise \end{cases}$ and $I_{f,t-1}^- = \begin{cases} 1\ if\ \varepsilon_{f,t-1} < 0 \\ 0\ otherwise \end{cases}$

And $\hat{\rho} = 0{,}899351$ with standard error 0,011395.

For the diagonal VECH model with the scalar restriction on parameters:

$$\sigma_{s,t}^2 = 0{,}000000112 + 0{,}891915\,\sigma_{s,t-1}^2 + 0{,}028029\,\varepsilon_{s,t-1}^2 + 0{,}113506\,\varepsilon_{s,t-1}^2\,I_{s,t-1}^-$$

$$\sigma_{f,t}^2 = 0{,}00000107 + 0{,}891915\,\sigma_{f,t-1}^2 + 0{,}028029\,\varepsilon_{f,t-1}^2 + 0{,}113506\,\varepsilon_{f,t-1}^2\,I_{f,t-1}^-$$

$$\sigma_{SF,t} = 0{,}00000101 + 0{,}891915\,\sigma_{SF,t-1} + 0{,}028029\varepsilon_{S,t-1}\varepsilon_{F,t-1} + 0{,}113506\,\varepsilon_{S,t-1}\varepsilon_{F,t-1}\ I_{s,t-1}^- I_{f,t-1}^-$$

Where $I_{s,t-1}^- = \begin{cases} 1\ if\ \varepsilon_{s,t-1} < 0 \\ 0\ otherwise \end{cases}$ and $I_{f,t-1}^- = \begin{cases} 1\ if\ \varepsilon_{f,t-1} < 0 \\ 0\ otherwise \end{cases}$

For the diagonal BEKK model:

$$\sigma_{s,t}^2 = 0{,}00000155 + 0{,}89913065\sigma_{s,t-1}^2 + 0{,}0024993\,\varepsilon_{s,t-1}^2 + 0{,}08914703\,\varepsilon_{s,t-1}^2\,I_{s,t-1}^-$$

$$\sigma_{f,t}^2 = 0{,}00000132 + 0{,}912989805\sigma_{f,t-1}^2 + 0{,}0058494\,\varepsilon_{f,t-1}^2 + 0{,}06883539\varepsilon_{f,t-1}^2\,I_{f,t-1}^-$$

$$\sigma_{SF,t} = 0{,}00000148 + 0{,}906033728\sigma_{SF,t-1} - 0{,}0038235\,\varepsilon_{S,t-1}\varepsilon_{F,t-1} + 0{,}07833562\varepsilon_{S,t-1}\varepsilon_{F,t-1}\ I_{s,t-1}^- I_{f,t-1}^-$$

Where $I_{s,t-1}^- = \begin{cases} 1\ if\ \varepsilon_{s,t-1} < 0 \\ 0\ otherwise \end{cases}$ and $I_{f,t-1}^- = \begin{cases} 1\ if\ \varepsilon_{f,t-1} < 0 \\ 0\ otherwise \end{cases}$

5.3.2.5. *The time varying hedge ratio*

I compute the within sample hedge ratio from the results of the conditional models: I first compute the residuals, then the expression of variances and covariance and finally calculate within sample the time varying hedge ratio from the following expression.

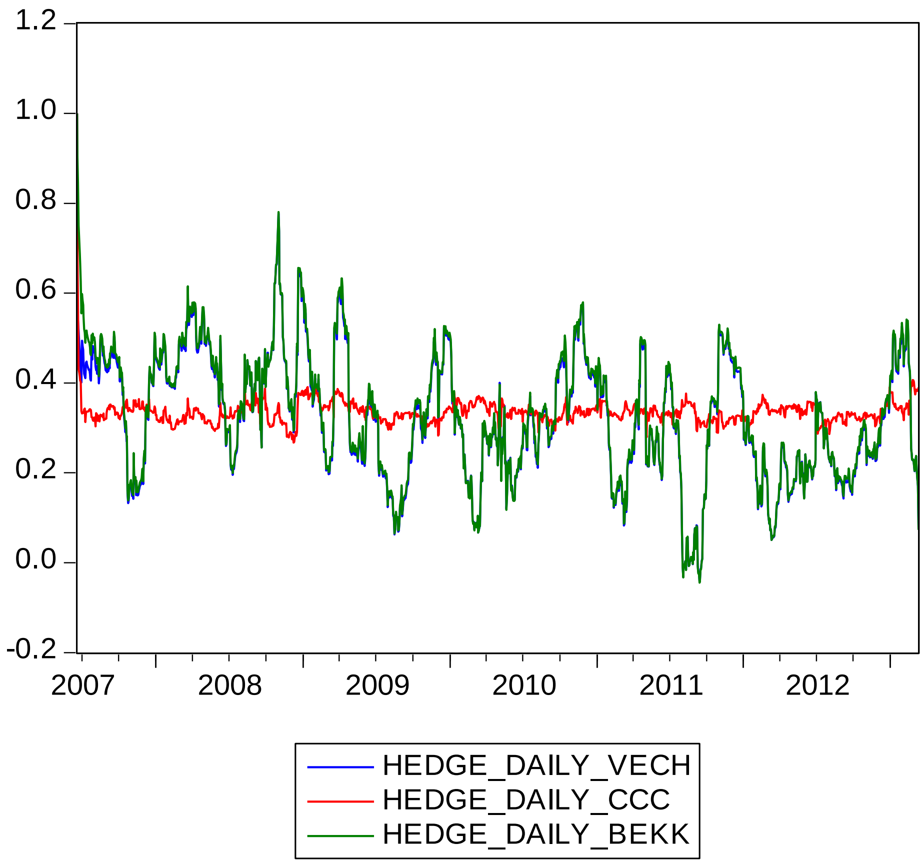
$$\widehat{h_t} = -\frac{\widehat{\sigma_{SF,t}}}{\widehat{\sigma^2_{F,t}}}$$

I took a positive hedge ratio to be more convenient, the result needs to be multiplied by (-1) for indicating the opposite position taken in the futures. The expression of the variances and covariance are estimated from daily and weekly data. I obtained the following results for the daily hedge ratios:

	HEDGE_DAILY_VECH	HEDGE_DAILY_CCC	HEDGE_DAILY_BEKK
Mean	0.323192	0.335304	0.330585
Median	0.321690	0.333626	0.325854
Maximum	1.000000	1.000000	1.000000
Minimum	-0.042380	0.267157	-0.044729
Std. Dev.	0.138781	0.029378	0.144043

Tab 5.25. The daily GARCH hedge ratios statistics

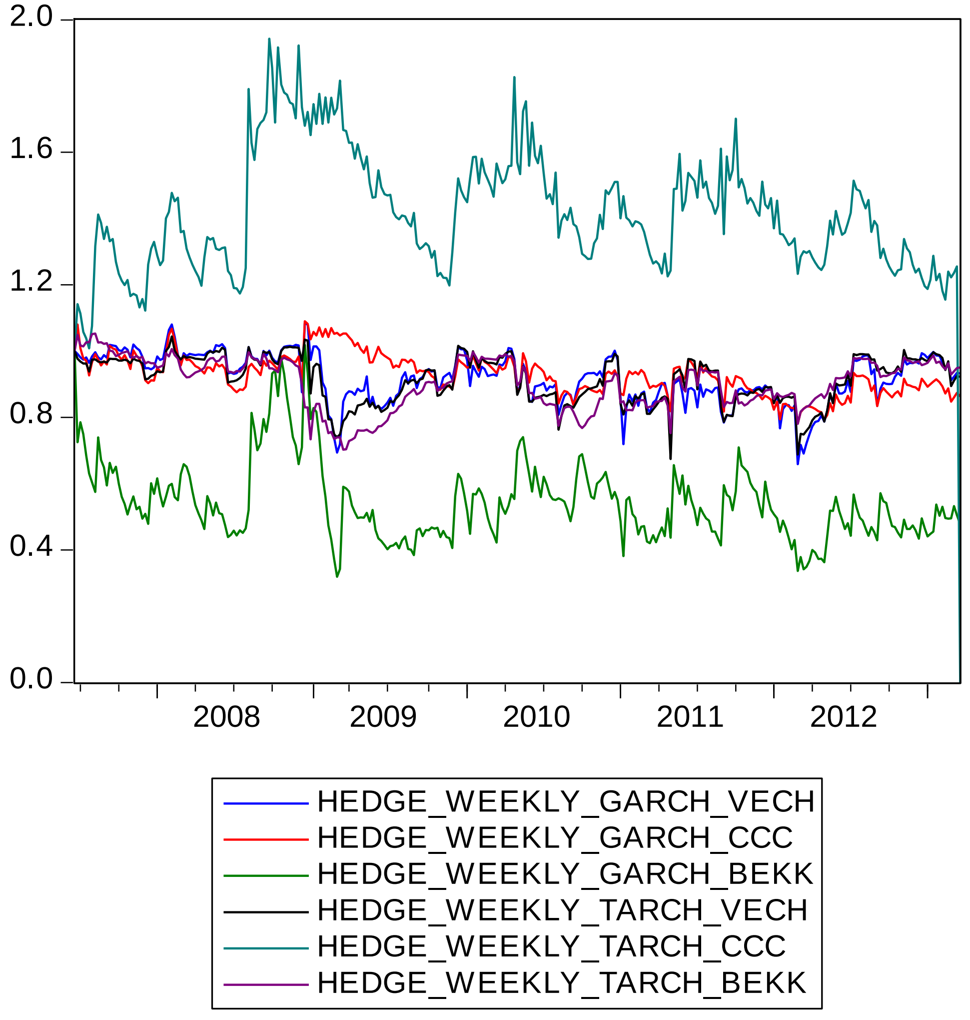
Despite the time varying nature of the hedge, we can observe the same problem as for the static hedge. Estimations based on daily observations give a really low hedge ratio and clearly doesn't take in consideration the time lag existing between the spot and futures market. It underestimates the correlation between spot and futures prices and gives therefore an underestimated hedge ratio. The measure of the effectiveness of the hedge should confirm this observation. We can still clearly see a difference between the models allowing the correlation to vary through time and the conditional constant correlation model. The diagonal VECH and BEKK models give similar results for the hedge ratios which vary widely through time unlike the CCC hedge ratio.



Tab 5.26. Daily time varying hedge ratios from 06-19-2007 to 03-18-2013 for the bivariate GARCH models

In the next pages, I give the results for the hedge ratios estimated from weekly data. We can see different group of results for the level and variance of the hedging ratio: we have similar results for all the models except for the GARCH_BEKK and the TARCH_CCC models which gave significant different levels of hedge and a higher variance in the hedge ratio. The level of hedging, close to 1 for the main group of models is consistent with the high correlation I observed between the spot and futures prices.

Both daily and weekly models show clearly the hedge ratio is not constant through time.



Tab 5.27. Weekly time varying hedge ratios from 06-19-2007 to 03-18-2013 for the bivariate GARCH and TARCH models

	HEDGE_WEEKLY_GARCH_VECH	HEDGE_WEEKLY_GARCH_CCC	HEDGE_WEEKLY_GARCH_BEKK	HEDGE_WEEKLY_TARCH_VECH	HEDGE_WEEKLY_TARCH_CCC	HEDGE_WEEKLY_TARCH_BEKK
Mean	0.918207	0.929151	0.544791	0.914909	1.411344	0.903957
Median	0.924843	0.932670	0.524989	0.930430	1.395024	0.921961
Maximum	1.082921	1.090554	1.019252	1.043922	1.943310	1.053432
Minimum	0.658798	0.781582	0.319020	0.673815	0.000000	0.702367
Std. Dev.	0.074329	0.060364	0.120402	0.070704	0.193896	0.076278

Tab 5.28. The weekly GARCH and TARCH hedge ratios statistics

5.4. The effectiveness of the different models

This section focuses on the effectiveness of the different hedging models from an investor point of view. Two approaches are proposed: I first observe the distribution of the hedged portfolio returns knowing that a perfect hedge would have a zero mean and zero variance; then, I measure the hedge effectiveness as the percentage of reduction in the variance of the spot price changes. The last measure of effectiveness was proposed by Geppert (1995) for his study on stock and futures prices. The computation requires to build the hedged portfolio implied by the hedge ratios and to calculate the variance of the hedged returns over a given hedge horizon, i.e. we evaluate $VAR(\Delta S_t + h_t \Delta F_t)$ over the hedge horizon where ΔS_t and ΔF_t are the prices changes. Finally, the degree of effectiveness can be expressed as follows:

$$\frac{VAR(\Delta S_t) - VAR(\Delta S_t + h_t \Delta F_t)}{VAR(\Delta S_t)}$$

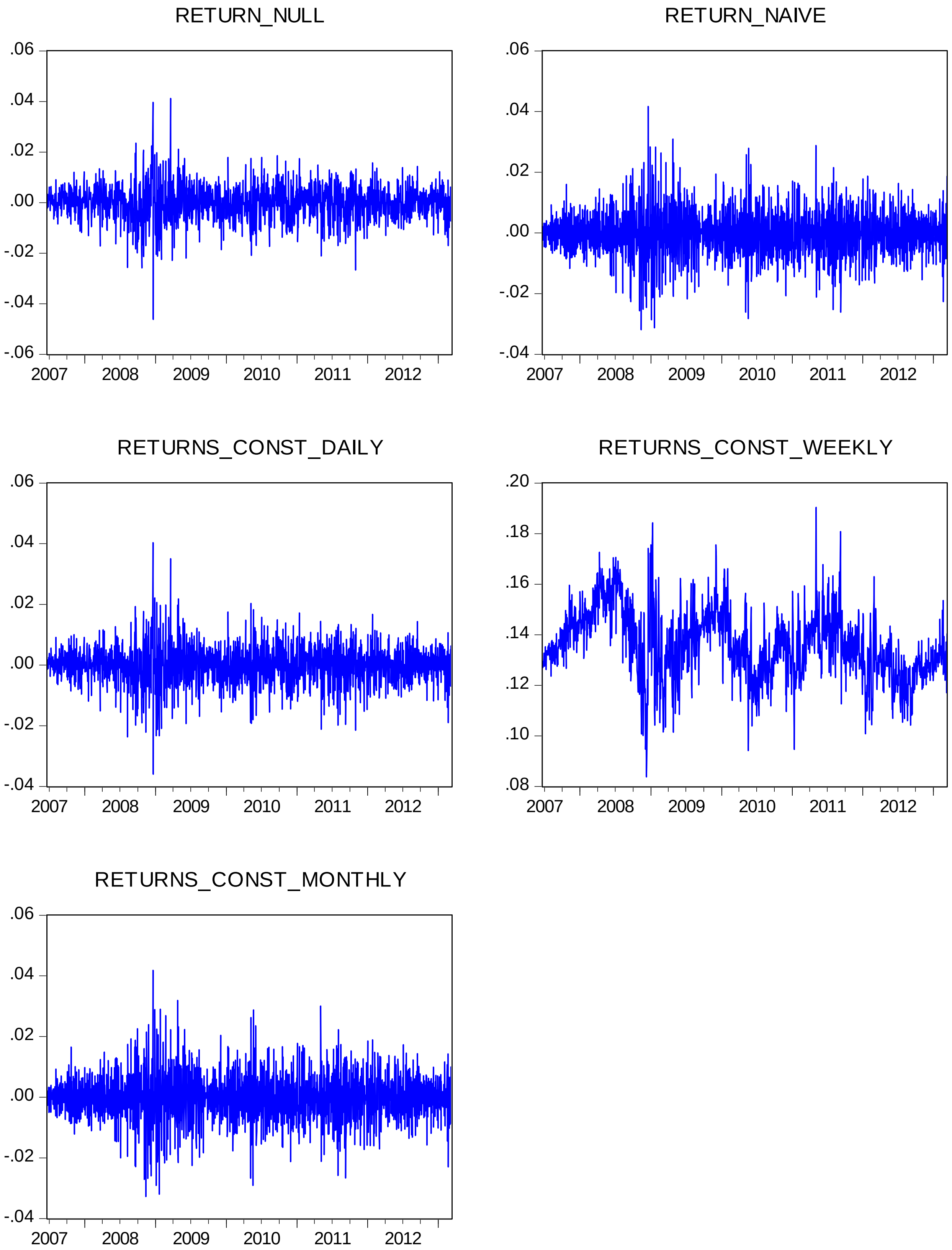
The higher it is, the more effective is the hedge over the given period.

5.4.1. The distribution of returns

I begin with the observation of the return distribution for the daily and weekly returns with different hedge ratios: a null hedge ratio ($h_t = 0$), a naïve hedge ratio ($h_t = -1$), the different constant hedge ratios and the different dynamic hedge ratios. I use dynamic daily hedge ratios for the daily returns and dynamic weekly hedge ratios for the weekly returns:

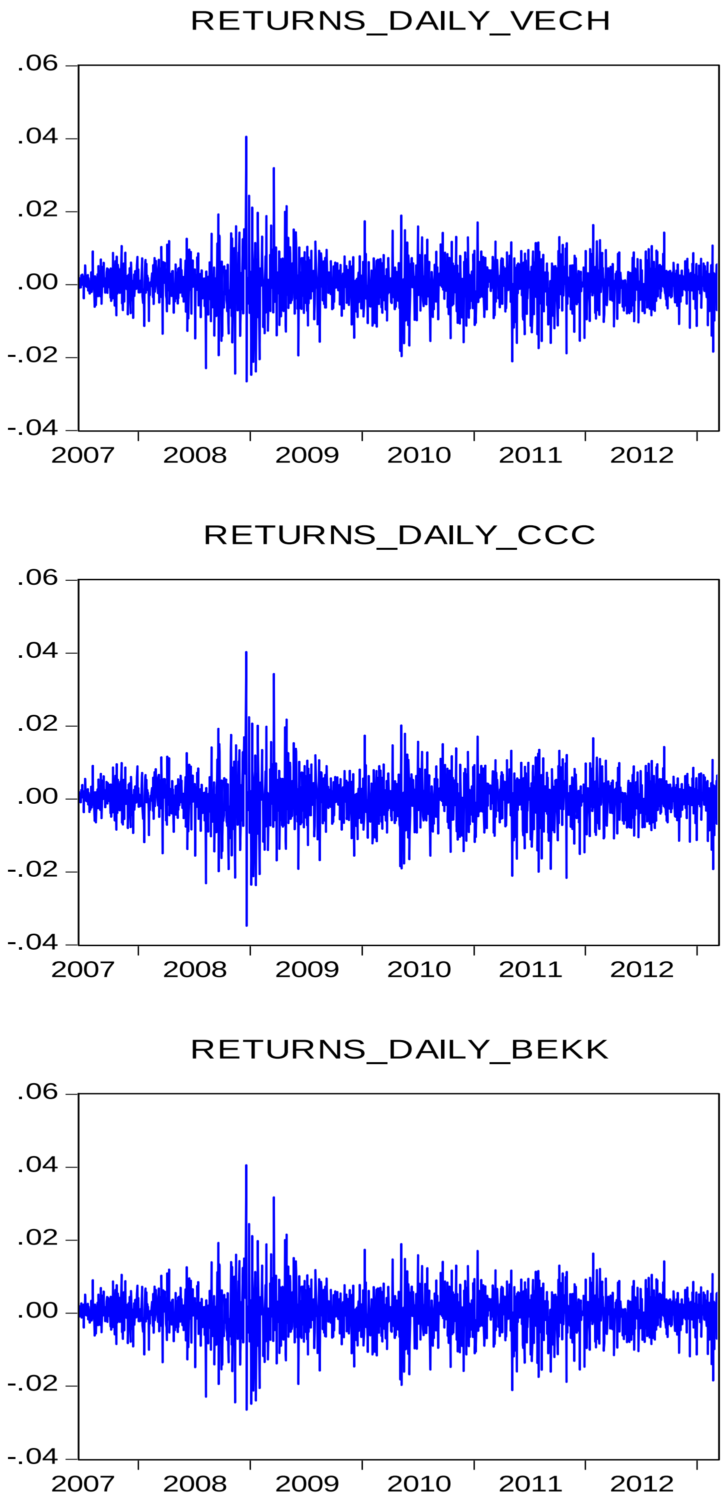
	NULL HEDGE	NAIVE HEDGE	CONST_DAILY HEDGE	CONST_WEEKLY HEDGE	CONST_MONTHLY HEDGE
Mean	8.89E-06	1.02E-05	9.32E-06	0.135926	1.02E-05
Median	0.000144	6.92E-05	0.000263	0.135348	3.85E-05
Maximum	0.041203	0.041715	0.040336	0.190372	0.041831
Minimum	-0.046251	-0.031917	-0.035953	0.083720	-0.032743
Std. Dev.	0.007056	0.008075	0.006643	0.013866	0.008301

Tab 5.29. Summary statistics for the daily returns with constant hedge ratios



Tab 5.30. Daily returns of the hedged portfolio with the constant hedge ratios

I give now the distribution of the daily returns for the dynamic daily hedge ratios:



<u>Tab 5.31. Daily returns of the hedged portfolio with the dynamic hedge ratios</u>

	DAILY_VECH HEDGE	DAILY_CCC HEDGE	DAILY_BEKK HEDGE
Mean	4.80E-05	1.64E-05	4.22E-05
Median	0.000181	0.000274	0.000192
Maximum	0.040588	0.040339	0.040603
Minimum	-0.026657	-0.034878	-0.026490
Std. Dev.	0.006414	0.006632	0.006413

Tab 5.32. Summary statistics for the daily returns with dynamic hedge ratios

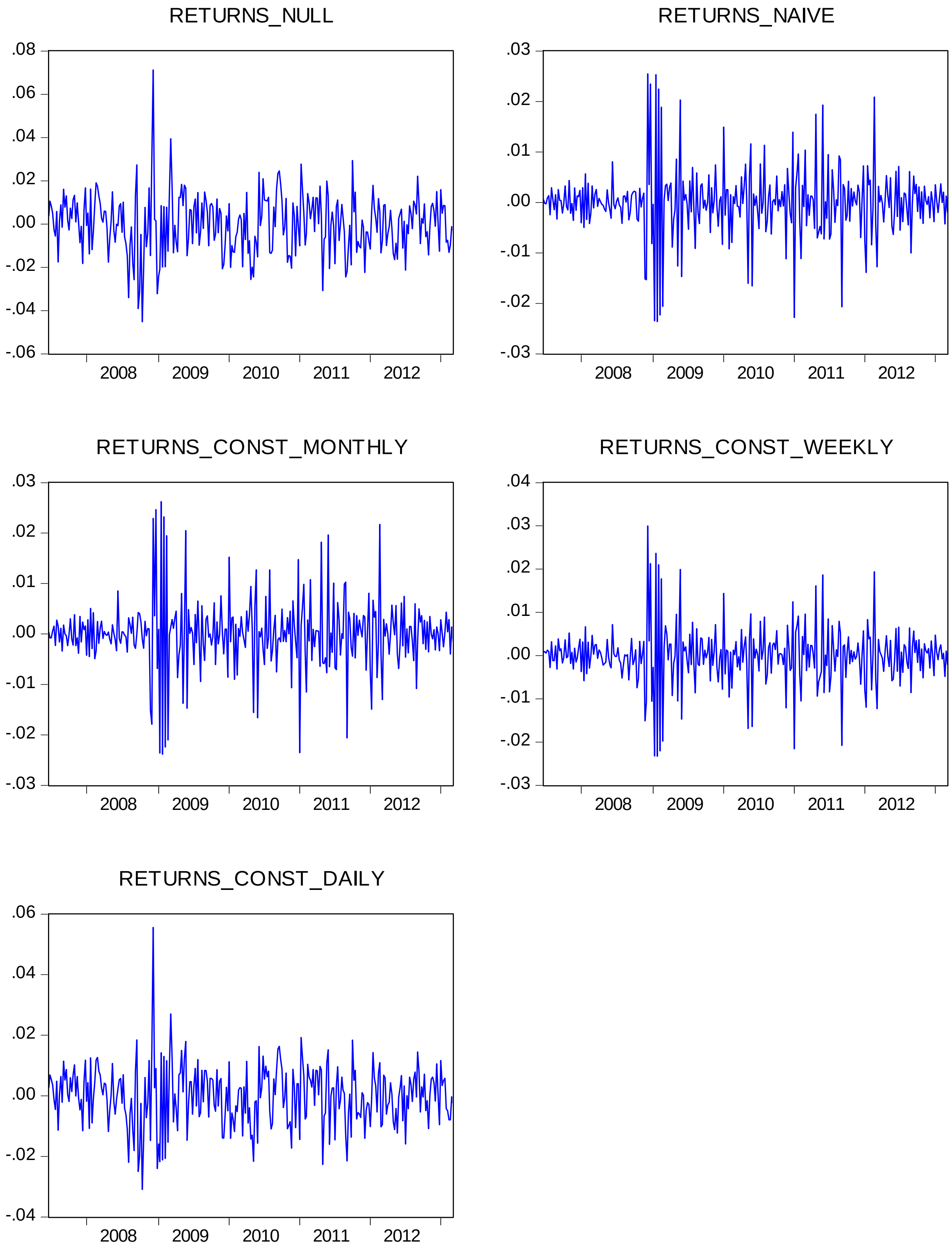
We can't notice any real effectiveness in the hedging strategies using a constant hedge ratio for the daily returns. In facts, the hedging with the constant ratio estimated with daily observations decreased a bit the variance of the portfolio and the hedging using a constant hedge ratio estimated from weekly data actually made it worse. For the dynamic hedging strategies, we observe better results: they all decreased the variance of the portfolio. We clearly see here it is difficult to evaluate the effectiveness of the hedging strategy by simply observing the distribution of returns.

Finally, we observe the weekly returns with the different constant hedge ratios and the dynamic weekly hedge ratios:

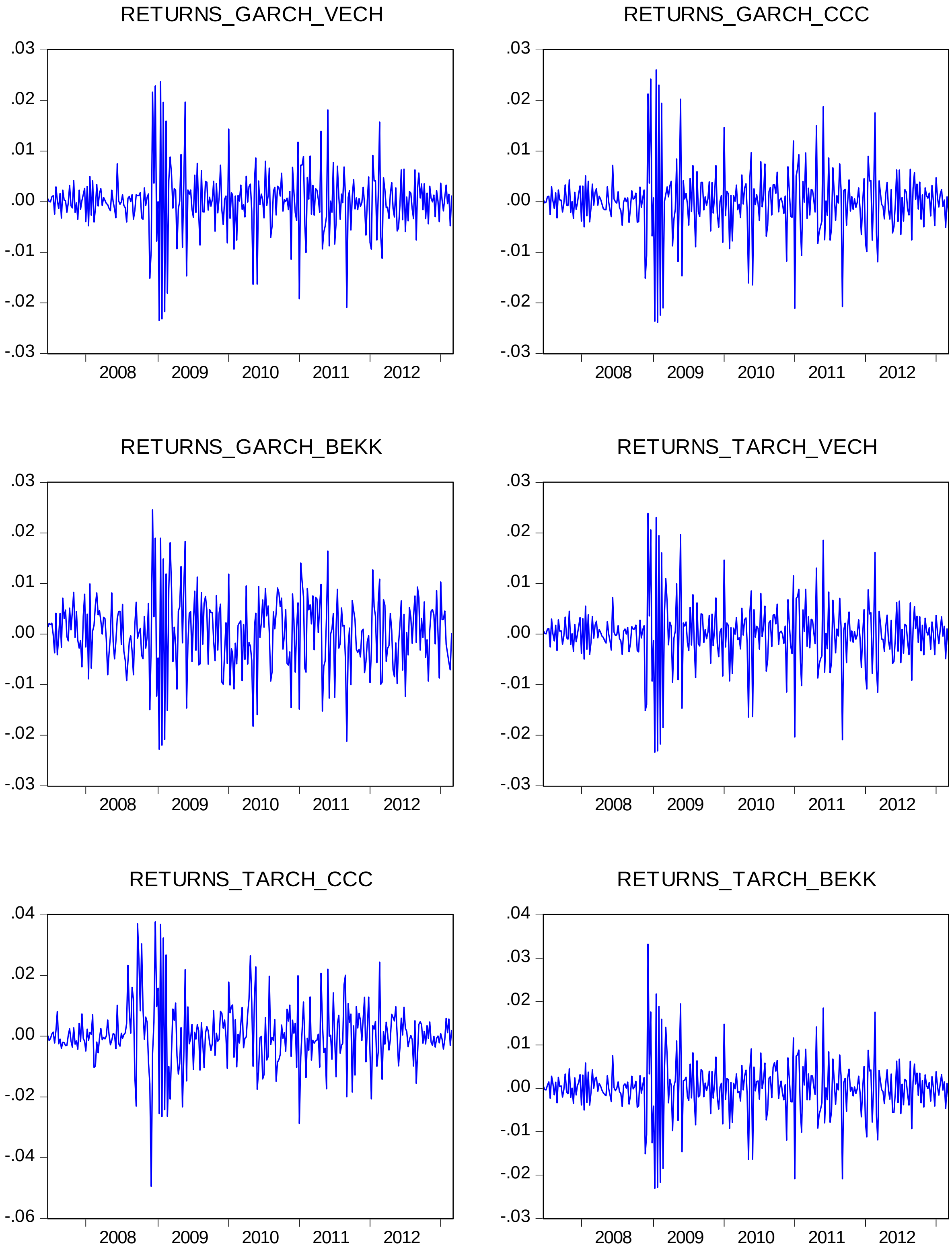
	NULL HEDGE	NAIVE HEDGE	CONST_DAILY HEDGE	CONST_WEEKLY HEDGE	CONST_MONTHLY HEDGE
Mean	-1.94E-05	1.67E-05	1.87E-05	1.31E-05	-7.07E-06
Median	0.000371	0.000203	9.99E-05	0.000296	0.000315
Maximum	0.071242	0.025453	0.026241	0.029969	0.055580
Minimum	-0.045196	-0.023629	-0.023837	-0.023259	-0.030863
Std. Dev.	0.013471	0.006794	0.006960	0.006681	0.009876

Tab 5.33. Summary statistics for the weekly returns with constant hedge ratios

The results shows the hedging strategies are actually effective in reducing the variance of our portfolio. The best performance is obtained with the constant hedge ratio estimated from weekly data.



Tab 5.34. Weekly returns of the hedged portfolio with the constant hedge ratios



Tab 5.35. Weekly returns of the hedged portfolio with the dynamic hedge ratios

	VECH HEDGE	CCC HEDGE	BEKK HEDGE	TARCH_VECH HEDGE	TARCH_CCC HEDGE	TARCH_BEKK HEDGE
Mean	3.85E-05	-2.67E-05	0.000239	2.17E-05	0.000428	0.000119
Median	0.000263	0.000142	0.000606	0.000401	-9.45E-05	0.000282
Maximum	0.023703	0.026033	0.024571	0.023892	0.037740	0.033230
Minimum	-0.023481	-0.023878	-0.022808	-0.023366	-0.049453	-0.023064
Std. Dev.	0.006304	0.006536	0.007200	0.006389	0.010519	0.006573

Tab 5.36. Summary statistics for the weekly returns with dynamic hedge ratios

We can see that all the dynamic models helped reducing the variance of the portfolio and the introduction of a TARCH term in the covariance expression for the asymmetry isn't really improving the results except for the diagonal BEKK model. Here, the best strategy is the GARCH model with diagonal VECH covariance. In term of minimizing the variance of the portfolio, it performs better than the constant hedge model.

5.4.2. The variance of spot prices

I apply now the effectiveness measure proposed by Geppert (1995) who defined it as the percentage of reduction in the variance of the spot price changes. This measure is easier to use as it only involves the calculation of a percentage and is more flexible since it allows us to calculate the variance of the prices changes for different hedging horizons. I used the same way different hedge ratios: a null hedge ratio ($h_t = 0$), a naïve hedge ratio ($h_t = -1$), the different constant hedge ratios and the different dynamic hedge ratios. I compute this coefficient for the daily returns and the weekly returns. I use the dynamic hedge ratios estimated from daily data for the daily returns and the dynamic hedge ratios estimated from weekly data for the weekly returns.

I first consider the full period of study as the hedging horizon (69 months). The following results are obtained for the null hedge, naïve hedge and the different constant and dynamic hedge ratios:

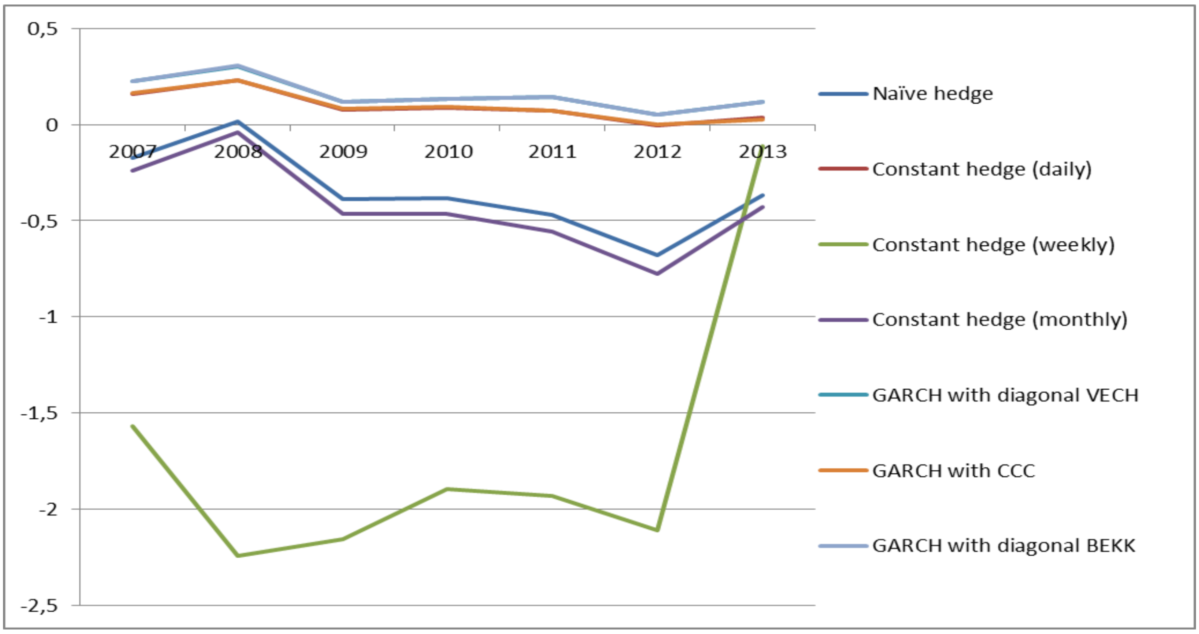
	Daily returns	Weekly returns
Null hedge	0	0
Naïve hedge	-0,3096	0,7456
Constant hedge (daily)	0,1138	0,4625
Constant hedge (weekly)	-2,8612	0,7540
Constant hedge (monthly)	-0,3840	0,7331
GARCH with diagonal VECH	0,1738	0,7810
GARCH with CCC	0,1167	0,7646
GARCH with diagonal BEKK	0,1742	0,7143
TARCH with diagonal VECH	[redacted]	0,7751
TARCH with CCC		0,3902
TARCH with diagonal BEKKK		0,7619

Tab 5.37. Effectiveness of the hedging strategies for a 69 months hedging horizon

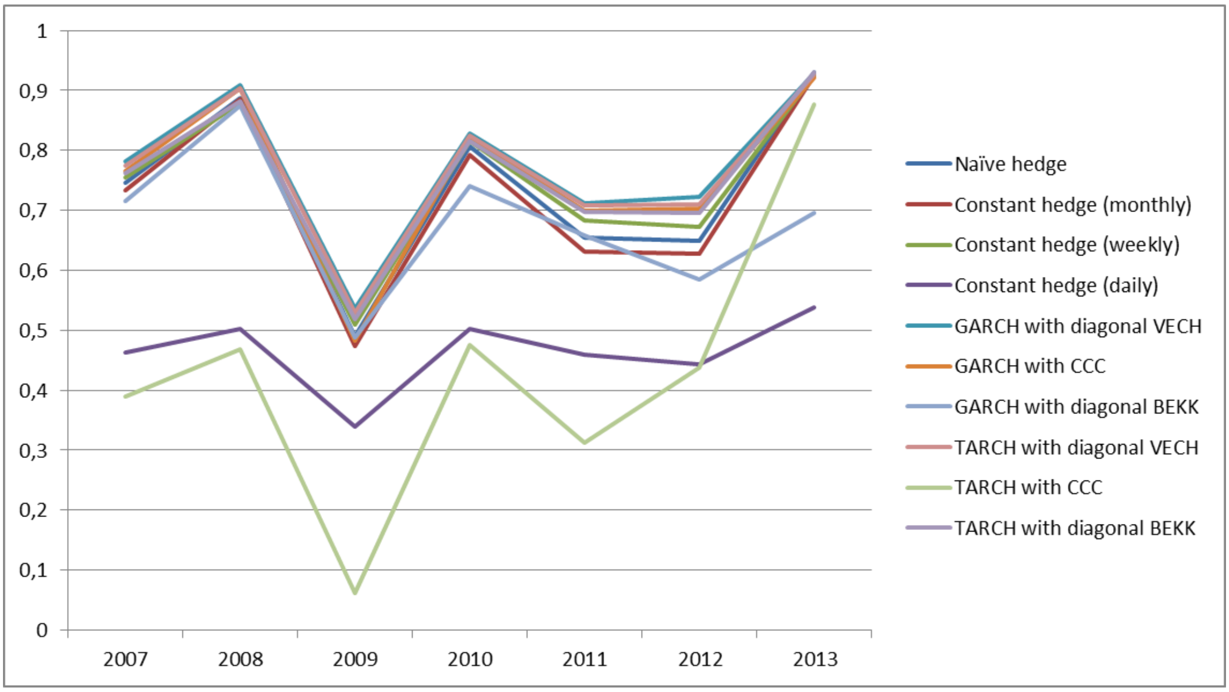
We can make a first series of observations:

- A hedging strategy is necessary to decrease the variance of the spot price (by definition of the coefficient).
- Hedging strategies are clearly more effective overall when observing the weekly returns and we observed poor hedging performance for the daily returns, some strategies actually increased the spot prices variance.
- The time varying hedge ratios strategies clearly outperform the constant hedge ratios strategies with both daily and weekly returns.
- The constant hedge ratio estimated from daily data doesn't perform well for the weekly returns confirming the observation in section 5.3.1: we need to use weekly data for the estimation of a constant hedge ratio.
- The diagonal BEKK and VECH models which allow the correlation to vary through time are the most effective for both series of returns.
- The introduction of the TARCH term in the covariance doesn't improve the effectiveness of the hedge except for the BEKK model

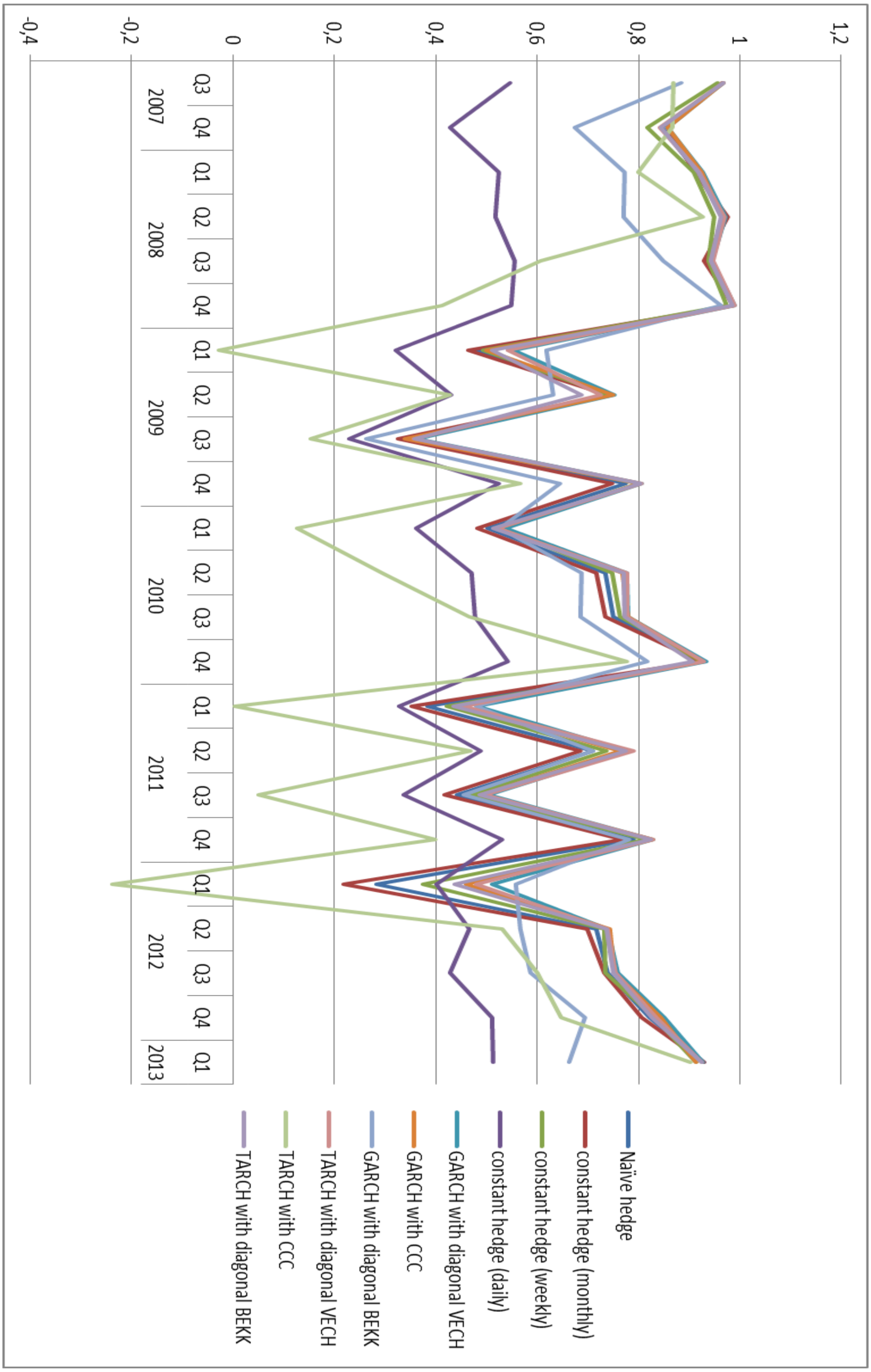
The following results are obtained for a hedging horizon of three months (the life time of the futures contract) and one year. I didn't indicate the results for the null hedge knowing it will always be ineffective by definition. The full results are provided in Annex17.



Tab 5.38. Effectiveness of the hedging strategies for daily returns with a 12 months hedging horizon



Tab 5.39. Effectiveness of the hedging strategies for weekly returns with a 12 months hedging horizon



Tab 5.40. Effectiveness of the hedging strategies for weekly returns with a 3 months hedging horizon

I don’t provide the results for the daily returns with a three months hedging horizon since I obtained poor performances like for the 12 and 69 months horizons. A first conclusion would therefore be that we can’t use daily returns to measure the effectiveness of the hedge and that the estimation of a hedge ratio from daily data leads to an underestimated hedge ratio which brings poor hedging performance. The time lag between the spot and futures market seems to enable the estimation of an effective optimal hedge ratio from daily data. The performance for the 12 months hedging horizon clearly shows the poor performance of all the strategies in 2009 indicating the pick of the financial crisis and instability of the different markets. As observed before, the introduction of the TARCH term doesn’t improve the hedging performance. Finally, we can see the dynamic approach improves a bit the hedging effectiveness and that the flexibility of the VECH and BEKK models allowing the correlation to vary through time gives better results.

I have to remind the assumption of the model that the futures contract is the only available hedging instrument is violated in practice. The model should be therefore extended to multiple hedging instruments. I remind finally the results above are computed within sample, the constant and dynamic optimal hedge ratios are not forecasted. An analysis of the out of sample hedging performance could be done by working on a sub-sample to estimate the model and forecast the optimal hedge for the next period. This process being iterative we can obtain a series of forecast optimal hedge. We would probably obtain then similar results for the variance of our portfolios as suggested by the literature.

Conclusion

It is an acknowledged fact that Multinational companies are exposed to foreign exchange risk and that they need to use effective operational or financial hedging strategies. My study proposed to examine a short term financial hedging strategy involving the EUR/USD exchange rate and a three months currency futures contract. I focused especially on the determination of the optimal hedge ratio that minimizes the variance of the hedged portfolio's returns. For this purpose, I use different approaches. One involved the modeling of the time varying joint distribution of spot and futures prices. I therefore presented and applied the different multivariate GARCH models proposed by the literature with daily, weekly and monthly data and measured the effectiveness of the different hedging models with daily and weekly returns. The results are clear: a dynamic hedge ratio is more effective than a static one. Among the different dynamic models I applied, the bivariate GARCH model with a diagonal VECH covariance allowing a time varying correlation between spot and futures prices is the most effective hedging model. This study also showed that despite the observable asymmetry of individual series for the weekly data, the hedging effectiveness is not improved with the introduction of a TARCH term in the covariance model. Concerning the data frequency, I noticed that the estimation of the different models based on daily data caused ineffective hedging strategies: the hedge ratio is indeed underestimated since I could see an unusual correlation of spot and futures prices (about 0,3). A quick observation of the data showed a structural problem: we observe a time lag between the daily spot prices and the daily futures prices. Finally, the results showed we need to use weekly returns to measure the effectiveness of the hedging strategy. I remind that multiple hedging instruments are actually available for companies and that the study could therefore be extended to incorporate them in the hedging strategy. A way to do it could be to consider a multitude of portfolios, each involving a different hedging instrument and then estimate their optimum weights in the final portfolio.

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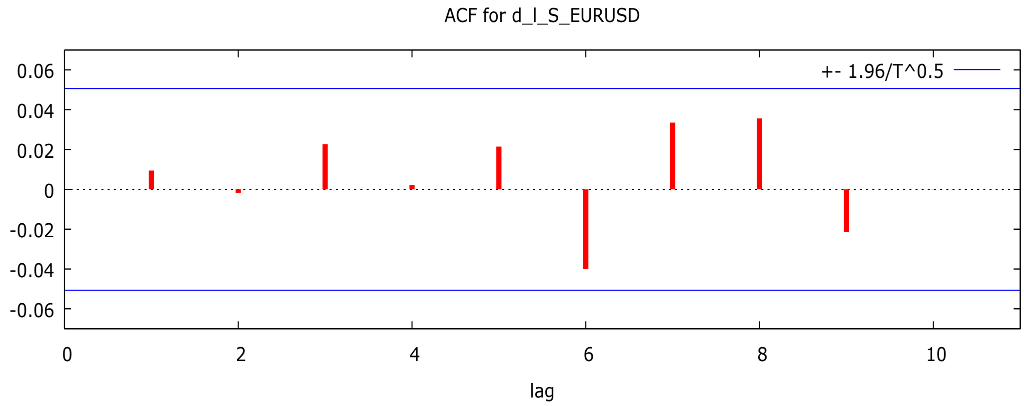
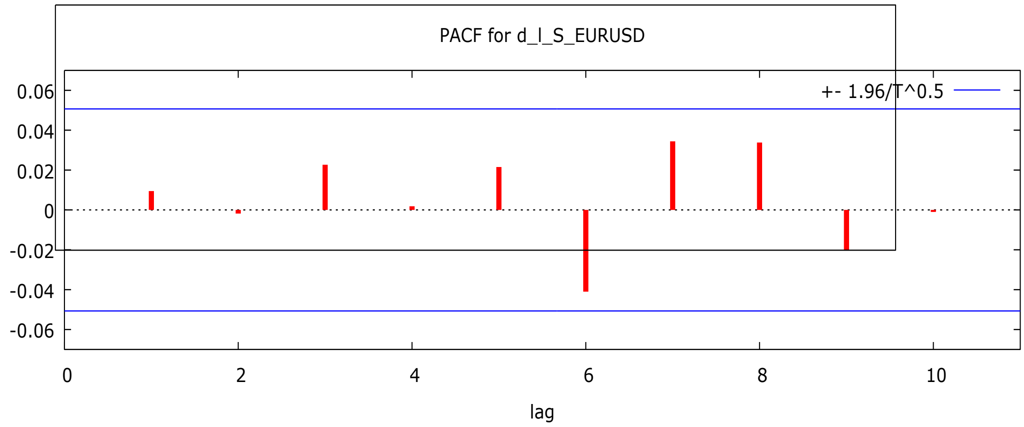
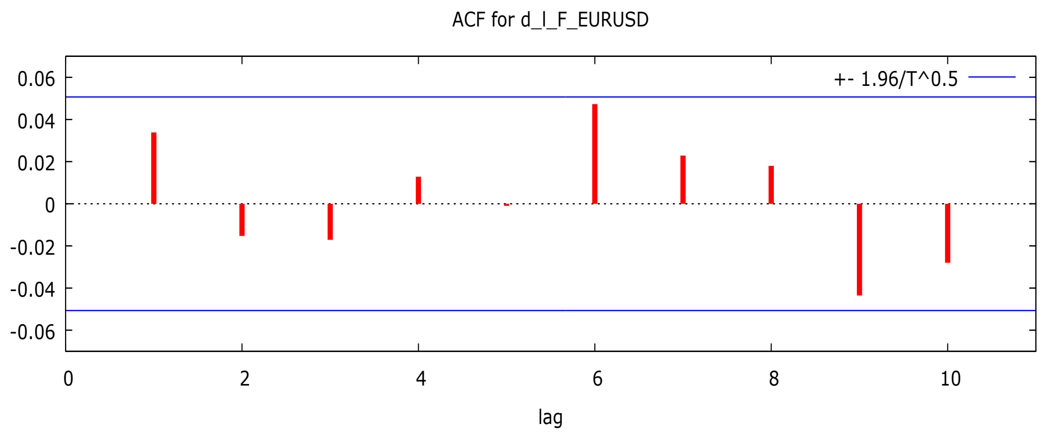
Annex 1. Correlogram of the log difference of spot and futures rates






ACF for d_l_F_EURUSD
+- 1.96/T^0.5
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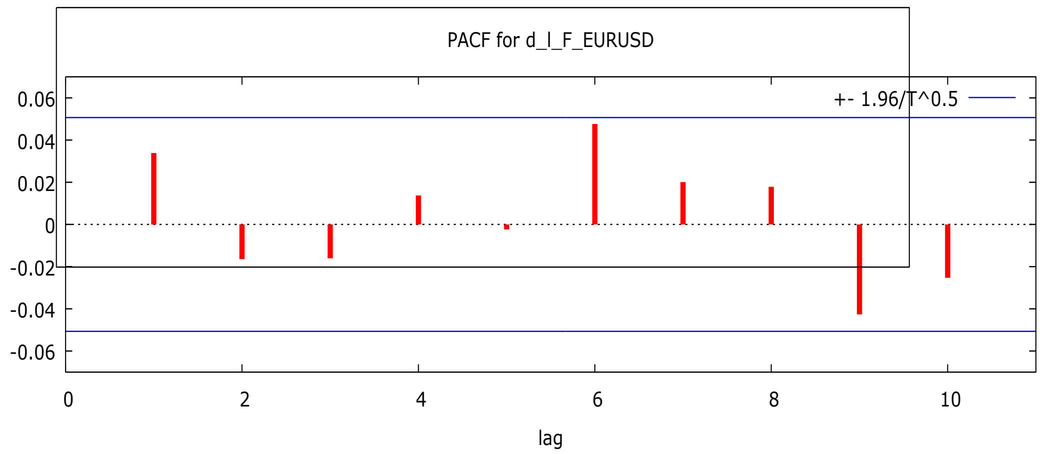
PACF for d_l_F_EURUSD
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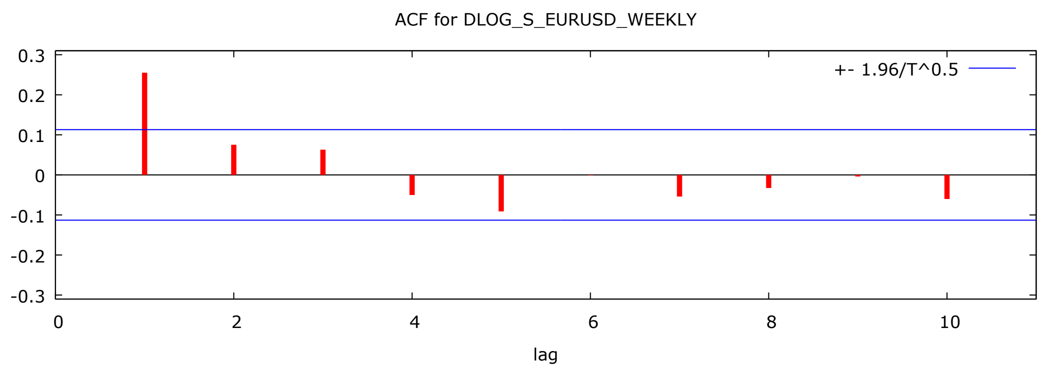
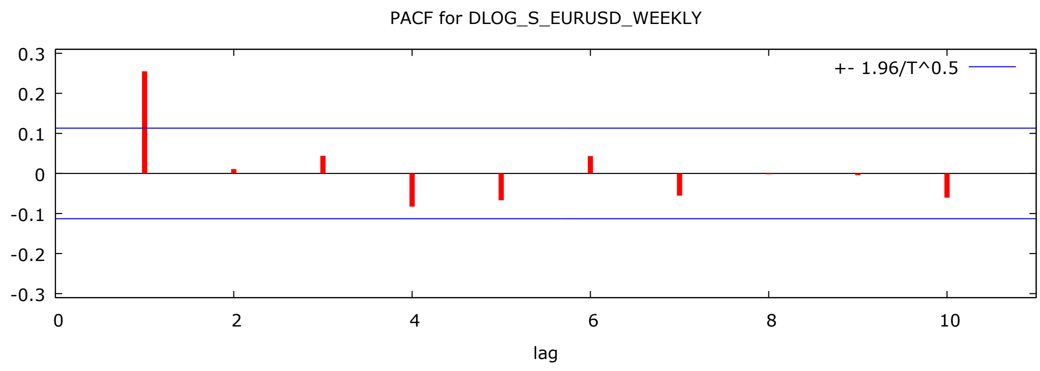
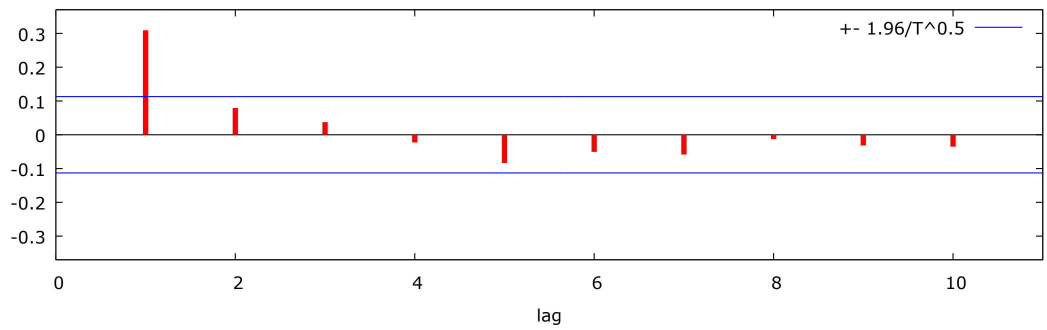
10
lag

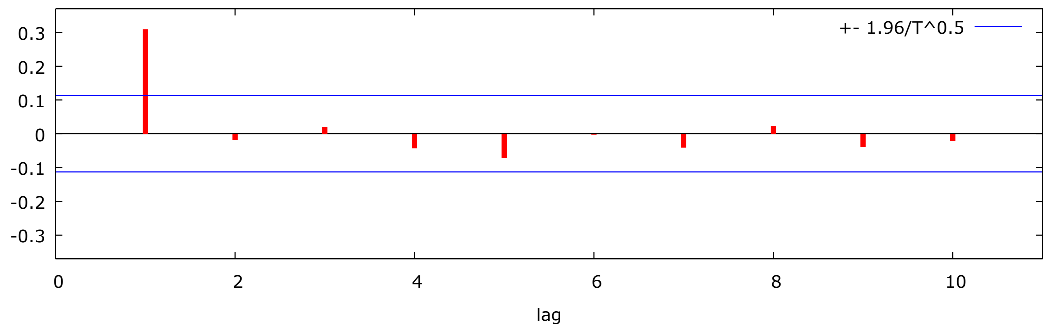

ACF for DLOG_S_EURUSD_WEEKLY
+- 1.96/T^0.5
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
2
4
6
8
10
lag


PACF for DLOG_S_EURUSD_WEEKLY
+- 1.96/T^0.5
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
2
4
6
8
10
lag

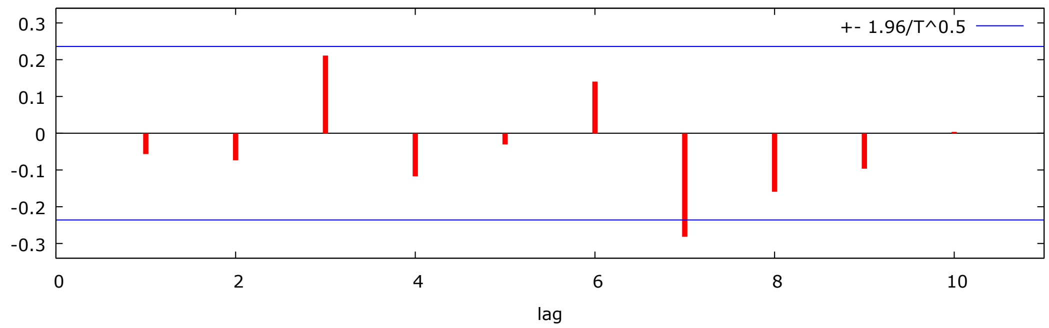
ACF for DLOG_F_EURUSD_WEEKLY



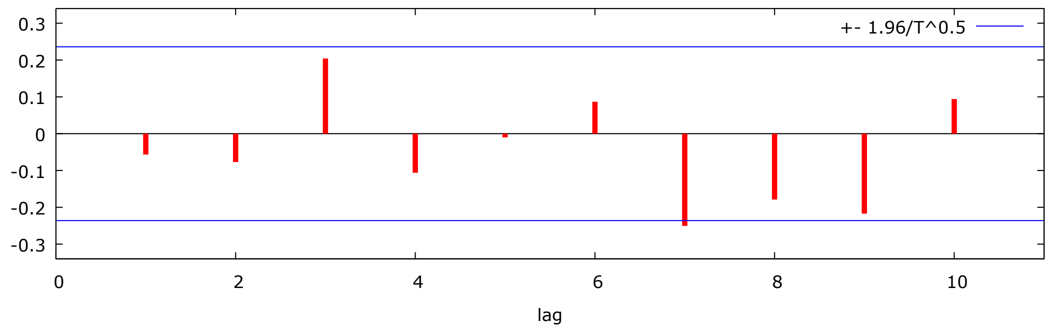
PACF for DLOG_F_EURUSD_WEEKLY

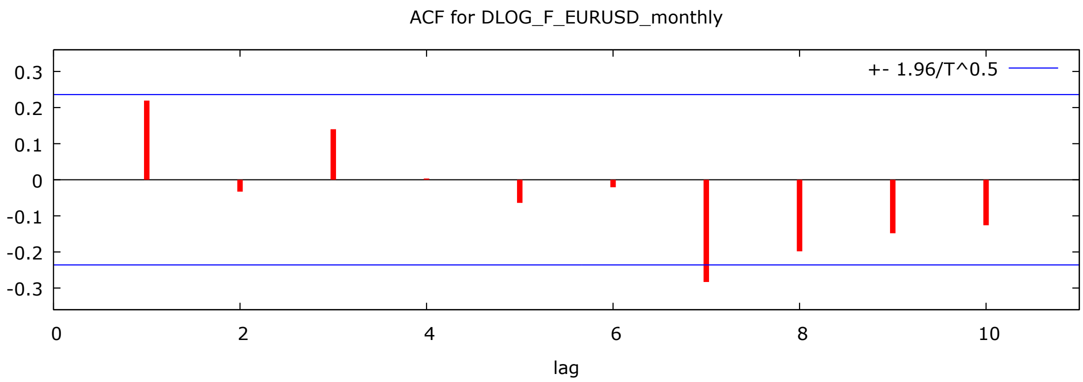


ACF for DLOG_S_EURUSD_monthly



PACF for DLOG_S_EURUSD_monthly




ACF for DLOG_F_EURUSD_monthly
+- 1.96/T^0.5
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
2
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6
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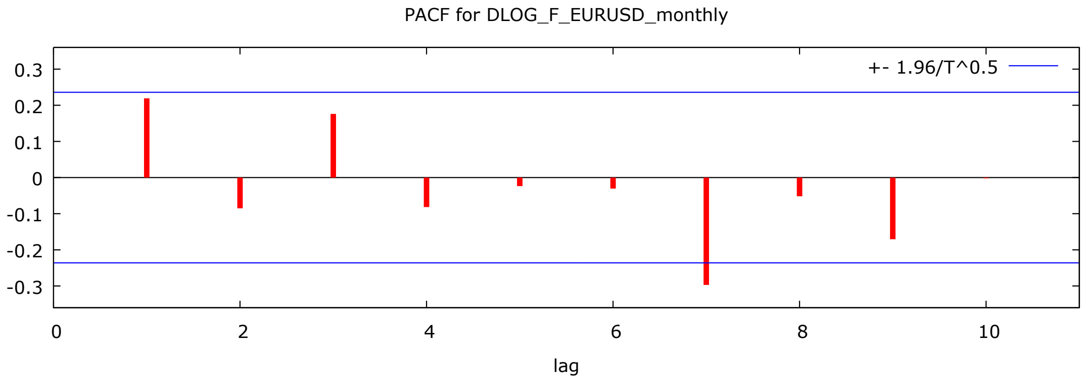
lag


PACF for DLOG_F_EURUSD_monthly
+- 1.96/T^0.5
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
2
4
6
8
10
lag

Annex 2. GARCH(1,1) for the daily spot and futures rates

Dependent Variable:
DLOG_S_EURUSD_DAILY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/29/13 Time: 22:16
Sample (adjusted): 6/19/2007 3/18/2013
Included observations: 1496 after adjustments
Convergence achieved after 10 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	5.66E-05	6.92E-05	0.818388	0.4131
		Variance Equation		
C	7.75E-08	2.34E-08	3.317385	0.0009
RESID(-1)^2	0.033731	0.005734	5.882893	0.0000
GARCH(-1)	0.958327	0.006603	145.1426	0.0000

R-squared	-0.000478	Mean dependent var	-1.05E-05
Adjusted R-squared	-0.002490	S.D. dependent var	0.003068
S.E. of regression	0.003072	Akaike info criterion	-8.874509
Sum squared resid	0.014082	Schwarz criterion	-8.860310
Log likelihood	6642.133	Hannan-Quinn criter.	-8.869219
Durbin-Watson stat	1.978162		

Dependent Variable: DLOG_F_EURUSD_DAILY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/29/13 Time: 22:16
Sample (adjusted): 6/19/2007 3/18/2013
Included observations: 1496 after adjustments
Convergence achieved after 10 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	4.72E-05	6.69E-05	0.704891	0.4809
		Variance Equation		
C	6.98E-08	2.31E-08	3.029280	0.0025
RESID(-1)^2	0.033016	0.006146	5.371632	0.0000
GARCH(-1)	0.959340	0.006825	140.5689	0.0000

R-squared	-0.000371	Mean dependent var	-1.11E-05
Adjusted R-squared	-0.002382	S.D. dependent var	0.003028
S.E. of regression	0.003032	Akaike info criterion	-8.886319
Sum squared resid	0.013715	Schwarz criterion	-8.872120
Log likelihood	6650.967	Hannan-Quinn criter.	-8.881029
Durbin-Watson stat	1.931575		

Annex 3. GARCH(1,1) for the weekly spot and futures rates

Dependent Variable: DLOG_S_EURUSD_WEEKLY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/12/13 Time: 02:07
Sample: 6/20/2007 3/20/2013
Included observations: 301
Convergence achieved after 12 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000190	0.000314	0.606823	0.5440
		Variance Equation		
C	1.65E-06	1.08E-06	1.526689	0.1268
RESID(-1)^2	0.100236	0.043250	2.317614	0.0205
GARCH(-1)	0.850417	0.065784	12.92740	0.0000

R-squared	-0.001912	Mean dependent var	-6.38E-05
Adjusted R-squared	-0.012033	S.D. dependent var	0.005821
S.E. of regression	0.005856	Akaike info criterion	-7.558046
Sum squared resid	0.010186	Schwarz criterion	-7.508782
Log likelihood	1141.486	Hannan-Quinn criter.	-7.538332
Durbin-Watson stat	1.486094		

Dependent Variable: DLOG_F_EURUSD_WEEKLY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/12/13 Time: 02:10
Sample: 6/20/2007 3/20/2013
Included observations: 301
Convergence achieved after 9 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000234	0.000318	0.736640	0.4613
		Variance Equation		
C	1.70E-06	9.99E-07	1.703732	0.0884
RESID(-1)^2	0.088238	0.038711	2.279403	0.0226
GARCH(-1)	0.856763	0.059501	14.39912	0.0000

R-squared	-0.002780	Mean dependent var	-6.09E-05
Adjusted R-squared	-0.012909	S.D. dependent var	0.005605
S.E. of regression	0.005641	Akaike info criterion	-7.609759
Sum squared resid	0.009452	Schwarz criterion	-7.560495
Log likelihood	1149.269	Hannan-Quinn criter.	-7.590046
Durbin-Watson stat	1.377411		

Annex 4. EGARCH(1,1) for the daily spot and futures rates

Dependent Variable: DLOG_S_EURUSD_DAILY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/29/13 Time: 22:11
Sample (adjusted): 6/19/2007 3/18/2013
Included observations: 1496 after adjustments
Convergence achieved after 14 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(4)
*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	6.10E-05	7.02E-05	0.868747	0.3850
Variance Equation				
C(2)	-0.200449	0.044767	-4.477576	0.0000
C(3)	0.089521	0.013646	6.560090	0.0000
C(4)	0.000678	0.007743	0.087551	0.9302
C(5)	0.988626	0.003408	290.0891	0.0000

R-squared	-0.000542	Mean dependent var	-1.05E-05
Adjusted R-squared	-0.003226	S.D. dependent var	0.003068
S.E. of regression	0.003073	Akaike info criterion	-8.868172
Sum squared resid	0.014083	Schwarz criterion	-8.850423
Log likelihood	6638.393	Hannan-Quinn criter.	-8.861559
Durbin-Watson stat	1.978036		

Dependent Variable: DLOG_F_EURUSD_DAILY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/29/13 Time: 22:13
Sample (adjusted): 6/19/2007 3/18/2013
Included observations: 1496 after adjustments
Convergence achieved after 14 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(4)
*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	4.65E-05	6.69E-05	0.694693	0.4872
Variance Equation				
C(2)	-0.163911	0.032390	-5.060499	0.0000
C(3)	0.067295	0.013290	5.063726	0.0000
C(4)	-0.012356	0.007694	-1.605908	0.1083
C(5)	0.990365	0.002459	402.7388	0.0000

R-squared	-0.000362	Mean dependent var	-1.11E-05
Adjusted R-squared	-0.003045	S.D. dependent var	0.003028
S.E. of regression	0.003033	Akaike info criterion	-8.879875
Sum squared resid	0.013714	Schwarz criterion	-8.862126
Log likelihood	6647.147	Hannan-Quinn criter.	-8.873262
Durbin-Watson stat	1.931592		

Annex 5. EGARCH(1,1) for the weekly spot and futures rates

Dependent Variable: DLOG_S_EURUSD_WEEKLY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/12/13 Time: 02:13
Sample: 6/20/2007 3/20/2013
Included observations: 301
Convergence achieved after 24 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
C(4)
*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000119	0.000310	0.384910	0.7003
Variance Equation				
C(2)	-0.628863	0.275083	-2.286087	0.0222
C(3)	0.153105	0.082614	1.853259	0.0638
C(4)	-0.067347	0.033703	-1.998254	0.0457
C(5)	0.951455	0.022961	41.43855	0.0000

R-squared	-0.000993	Mean dependent var	-6.38E-05
Adjusted R-squared	-0.014520	S.D. dependent var	0.005821
S.E. of regression	0.005864	Akaike info criterion	-7.554281
Sum squared resid	0.010177	Schwarz criterion	-7.492701
Log likelihood	1141.919	Hannan-Quinn criter.	-7.529639
Durbin-Watson stat	1.487459		

Dependent Variable: DLOG_F_EURUSD_WEEKLY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/12/13 Time: 02:16
Sample: 6/20/2007 3/20/2013
Included observations: 301
Convergence achieved after 17 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
C(4)
*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000154	0.000297	0.520350	0.6028
Variance Equation				
C(2)	-0.624224	0.263635	-2.367762	0.0179
C(3)	0.127911	0.069608	1.837583	0.0661
C(4)	-0.075376	0.036418	-2.069718	0.0385
C(5)	0.950114	0.023153	41.03601	0.0000

R-squared	-0.001482	Mean dependent var	-6.09E-05
Adjusted R-squared	-0.015016	S.D. dependent var	0.005605
S.E. of regression	0.005647	Akaike info criterion	-7.611734
Sum squared resid	0.009440	Schwarz criterion	-7.550154
Log likelihood	1150.566	Hannan-Quinn criter.	-7.587092
Durbin-Watson stat	1.379197		

Annex 6. TARCH(1,1) for the daily spot and futures rates

Dependent Variable: DLOGS
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/29/13 Time: 22:07
Sample (adjusted): 6/19/2007 3/18/2013
Included observations: 1496 after adjustments
Convergence achieved after 14 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0) +
C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	6.09E-05	7.07E-05	0.862232	0.3886
		Variance Equation		
C	7.90E-08	2.45E-08	3.222624	0.0013
RESID(-1)^2	0.036514	0.007280	5.015358	0.0000
RESID(-1)^2*(RESID(-1)<0)	-0.003691	0.009400	-0.392678	0.6946
GARCH(-1)	0.957373	0.006791	140.9675	0.0000

R-squared	-0.000542	Mean dependent var	-1.05E-05
Adjusted R-squared	-0.003226	S.D. dependent var	0.003068
S.E. of regression	0.003073	Akaike info criterion	-8.873235
Sum squared resid	0.014083	Schwarz criterion	-8.855486
Log likelihood	6642.180	Hannan-Quinn criter.	-8.866622
Durbin-Watson stat	1.978036		

Dependent Variable: DLOGF
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/29/13 Time: 22:09
Sample (adjusted): 6/19/2007 3/18/2013
Included observations: 1496 after adjustments
Convergence achieved after 11 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0) +
C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	3.58E-05	6.92E-05	0.516492	0.6055
		Variance Equation		
C	6.83E-08	2.13E-08	3.200763	0.0014
RESID(-1)^2	0.023534	0.008245	2.854543	0.0043
RESID(-1)^2*(RESID(-1)<0)	0.013222	0.010410	1.270111	0.2040
GARCH(-1)	0.961997	0.006638	144.9175	0.0000

R-squared	-0.000240	Mean dependent var	-1.11E-05
Adjusted R-squared	-0.002923	S.D. dependent var	0.003028
S.E. of regression	0.003033	Akaike info criterion	-8.885821
Sum squared resid	0.013713	Schwarz criterion	-8.868072
Log likelihood	6651.594	Hannan-Quinn criter.	-8.879208
Durbin-Watson stat	1.931828		

Annex 7. TARCH(1,1) for the weekly spot and futures rates

Dependent Variable: DLOG_S_EURUSD_WEEKLY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/12/13 Time: 02:22
Sample: 6/20/2007 3/20/2013
Included observations: 301
Convergence achieved after 16 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0) +
C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	9.46E-05	0.000315	0.300473	0.7638
Variance Equation				
C	1.41E-06	7.81E-07	1.799201	0.0720
RESID(-1)^2	0.003462	0.041816	0.082792	0.9340
RESID(-1)^2*(RESID(-1)<0)	0.113940	0.045386	2.510448	0.0121
GARCH(-1)	0.893005	0.055863	15.98570	0.0000

R-squared	-0.000743	Mean dependent var	-6.38E-05
Adjusted R-squared	-0.014266	S.D. dependent var	0.005821
S.E. of regression	0.005863	Akaike info criterion	-7.569226
Sum squared resid	0.010174	Schwarz criterion	-7.507646
Log likelihood	1144.168	Hannan-Quinn criter.	-7.544584
Durbin-Watson stat	1.487830		

Dependent Variable: DLOG_F_EURUSD_WEEKLY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/12/13 Time: 02:24
Sample: 6/20/2007 3/20/2013
Included observations: 301
Convergence achieved after 15 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0) +
C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000133	0.000307	0.433359	0.6648
Variance Equation				
C	1.33E-06	6.89E-07	1.930865	0.0535
RESID(-1)^2	-0.011850	0.038817	-0.305270	0.7602
RESID(-1)^2*(RESID(-1)<0)	0.121800	0.049545	2.458387	0.0140
GARCH(-1)	0.904059	0.050386	17.94259	0.0000

R-squared	-0.001204	Mean dependent var	-6.09E-05
Adjusted R-squared	-0.014734	S.D. dependent var	0.005605
S.E. of regression	0.005646	Akaike info criterion	-7.623072
Sum squared resid	0.009437	Schwarz criterion	-7.561492
Log likelihood	1152.272	Hannan-Quinn criter.	-7.598430
Durbin-Watson stat	1.379580		

Annex 8. GARCH-M(1,1) for the daily spot and futures rates

Dependent Variable:
DLOG_S_EURUSD_DAILY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/29/13 Time: 19:38
Sample (adjusted): 1 1496
Included observations: 1496 after adjustments
Convergence achieved after 18 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
GARCH	-32.31904	17.81152	-1.814501	0.0696
C	0.000292	0.000147	1.992402	0.0463
		Variance Equation		
C	8.07E-08	2.43E-08	3.326061	0.0009
RESID(-1)^2	0.035079	0.006091	5.759524	0.0000
GARCH(-1)	0.956635	0.006966	137.3227	0.0000

R-squared	0.002315	Mean dependent var	-1.05E-05
Adjusted R-squared	-0.000361	S.D. dependent var	0.003068
S.E. of regression	0.003069	Akaike info criterion	-8.875541
Sum squared resid	0.014042	Schwarz criterion	-8.857792
Log likelihood	6643.905	Hannan-Quinn criter.	-8.868928
F-statistic	0.865088	Durbin-Watson stat	1.983390
Prob(F-statistic)	0.484192		

Dependent Variable: DLOG_F_EURUSD_DAILY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/29/13 Time: 19:39
Sample (adjusted): 1 1496
Included observations: 1496 after adjustments
Convergence achieved after 17 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
GARCH	-22.60476	18.02173	-1.254305	0.2097
C	0.000211	0.000146	1.444662	0.1486
		Variance Equation		
C	7.20E-08	2.34E-08	3.084755	0.0020
RESID(-1)^2	0.033817	0.006322	5.348716	0.0000
GARCH(-1)	0.958301	0.006981	137.2714	0.0000

R-squared	0.000321	Mean dependent var	-1.11E-05
Adjusted R-squared	-0.002361	S.D. dependent var	0.003028
S.E. of regression	0.003032	Akaike info criterion	-8.885942
Sum squared resid	0.013705	Schwarz criterion	-8.868192
Log likelihood	6651.684	Hannan-Quinn criter.	-8.879329
F-statistic	0.119839	Durbin-Watson stat	1.933286
Prob(F-statistic)	0.975456		

Annex 9. GARCH-M(1,1) for the weekly spot and futures rates

Dependent Variable: DLOG_S_EURUSD_WEEKLY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/12/13 Time: 02:29
Sample: 6/20/2007 3/20/2013
Included observations: 301
Convergence achieved after 21 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
GARCH	-18.99962	24.76473	-0.767205	0.4430
C	0.000760	0.000774	0.981897	0.3262
		Variance Equation		
C	1.83E-06	1.19E-06	1.542885	0.1229
RESID(-1)^2	0.104412	0.044649	2.338491	0.0194
GARCH(-1)	0.841017	0.070707	11.89437	0.0000

R-squared	0.008417	Mean dependent var	-6.38E-05
Adjusted R-squared	-0.004983	S.D. dependent var	0.005821
S.E. of regression	0.005836	Akaike info criterion	-7.553245
Sum squared resid	0.010081	Schwarz criterion	-7.491665
Log likelihood	1141.763	Hannan-Quinn criter.	-7.528603
F-statistic	0.628140	Durbin-Watson stat	1.497862
Prob(F-statistic)	0.642776		

Dependent Variable: DLOG_F_EURUSD_WEEKLY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/12/13 Time: 02:31
Sample: 6/20/2007 3/20/2013
Included observations: 301
Convergence achieved after 26 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
GARCH	-41.26577	29.70371	-1.389246	0.1648
C	0.001409	0.000893	1.576801	0.1148
		Variance Equation		
C	2.09E-06	1.08E-06	1.927462	0.0539
RESID(-1)^2	0.095594	0.043323	2.206513	0.0273
GARCH(-1)	0.837026	0.064431	12.99110	0.0000

R-squared	0.016914	Mean dependent var	-6.09E-05
Adjusted R-squared	0.003629	S.D. dependent var	0.005605
S.E. of regression	0.005595	Akaike info criterion	-7.608947
Sum squared resid	0.009266	Schwarz criterion	-7.547367
Log likelihood	1150.146	Hannan-Quinn criter.	-7.584305
F-statistic	1.273175	Durbin-Watson stat	1.406067
Prob(F-statistic)	0.280553		

Annex 10. The conventional hedging model: OLS with daily, weekly and monthly data

Dependent Variable: DLOG_S_EURUSD_DAILY
Method: Least Squares
Date: 07/12/13 Time: 02:37
Sample: 6/19/2007 3/12/2013
Included observations: 1496

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DLOG_F_EURUSD_DAILY	0.342069	0.024675	13.86281	0.0000
C	-6.66E-06	7.47E-05	-0.089110	0.9290

R-squared	0.113972	Mean dependent var	-1.05E-05
Adjusted R-squared	0.113379	S.D. dependent var	0.003068
S.E. of regression	0.002889	Akaike info criterion	-8.854361
Sum squared resid	0.012471	Schwarz criterion	-8.847262
Log likelihood	6625.062	Hannan-Quinn criter.	-8.851716
F-statistic	192.1776	Durbin-Watson stat	2.242505
Prob(F-statistic)	0.000000		

Dependent Variable: DLOG_S_EURUSD_WEEKLY
Method: Least Squares
Date: 07/12/13 Time: 02:33
Sample: 6/20/2007 3/20/2013
Included observations: 301

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DLOG_F_EURUSD_WEEKLY	0.901367	0.029836	30.21093	0.0000
C	-8.87E-06	0.000167	-0.053095	0.9577

R-squared	0.753239	Mean dependent var	-6.38E-05
Adjusted R-squared	0.752414	S.D. dependent var	0.005821
S.E. of regression	0.002897	Akaike info criterion	-8.843917
Sum squared resid	0.002509	Schwarz criterion	-8.819285
Log likelihood	1333.009	Hannan-Quinn criter.	-8.834060
F-statistic	912.7002	Durbin-Watson stat	2.852626
Prob(F-statistic)	0.000000		

Dependent Variable: DLOG_S_EURUSD_MONTHLY
Method: Least Squares
Date: 07/12/13 Time: 02:58
Sample: 2007M06 2013M02
Included observations: 69

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DLOG_F_EURUSD_MONTHLY	1.055477	0.092751	11.37970	0.0000
C	-9.84E-05	0.001137	-0.086519	0.9313

R-squared	0.659029	Mean dependent var	-0.000335
Adjusted R-squared	0.653940	S.D. dependent var	0.016059
S.E. of regression	0.009447	Akaike info criterion	-6.457670
Sum squared resid	0.005980	Schwarz criterion	-6.392913
Log likelihood	224.7896	Hannan-Quinn criter.	-6.431979
F-statistic	129.4975	Durbin-Watson stat	2.761424
Prob(F-statistic)	0.000000		

Annex 11. Bivariate GARCH(1,1) with a restricted diagonal VECH covariance

System: DAILY DATA
Estimation Method: ARCH Maximum Likelihood (Marquardt)
Covariance specification: Diagonal VECH
Date: 07/12/13 Time: 02:55
Sample: 6/19/2007 3/12/2013
Included observations: 1496
Total system (balanced) observations 2992
Presample covariance: backcast (parameter =0.7)
Convergence achieved after 12 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	5.78E-05	6.68E-05	0.865252	0.3869
C(2)	3.44E-05	6.65E-05	0.517968	0.6045
	Variance Equation Coefficients			
C(3)	9.24E-08	2.21E-08	4.187666	0.0000
C(4)	1.40E-07	4.28E-08	3.266056	0.0011
C(5)	9.36E-08	2.66E-08	3.516188	0.0004
C(6)	0.032698	0.005484	5.961909	0.0000
C(7)	0.042058	0.006983	6.022496	0.0000
C(8)	0.034088	0.006351	5.367487	0.0000
C(9)	0.957130	0.006476	147.7988	0.0000
C(10)	0.896544	0.021989	40.77145	0.0000
C(11)	0.955062	0.007296	130.9067	0.0000

Log likelihood	13392.54	Schwarz criterion	-17.85071
Avg. log likelihood	4.476117	Hannan-Quinn criter.	-17.87521
Akaike info criterion	-17.88976		

Equation: DLOG_S_EURUSD_DAILY = C(1)

R-squared	-0.000495	Mean dependent var	-1.05E-05
Adjusted R-squared	-0.000495	S.D. dependent var	0.003068
S.E. of regression	0.003069	Sum squared resid	0.014082
Durbin-Watson stat	1.978129		

Equation: DLOG_F_EURUSD_DAILY = C(2)

R-squared	-0.000226	Mean dependent var	-1.11E-05
Adjusted R-squared	-0.000226	S.D. dependent var	0.003028
S.E. of regression	0.003029	Sum squared resid	0.013713
Durbin-Watson stat	1.931854		

Covariance specification: Diagonal VECH
GARCH = M + A1.*RESID(-1)*RESID(-1)' + B1.*GARCH(-1)
M is an indefinite matrix
A1 is an indefinite matrix
B1 is an indefinite matrix

Tranformed Variance Coefficients

	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	9.24E-08	2.21E-08	4.187666	0.0000

M(1,2)	1.40E-07	4.28E-08	3.266056	0.0011
M(2,2)	9.36E-08	2.66E-08	3.516188	0.0004
A1(1,1)	0.032698	0.005484	5.961909	0.0000
A1(1,2)	0.042058	0.006983	6.022496	0.0000
A1(2,2)	0.034088	0.006351	5.367487	0.0000
B1(1,1)	0.957130	0.006476	147.7988	0.0000
B1(1,2)	0.896544	0.021989	40.77145	0.0000
B1(2,2)	0.955062	0.007296	130.9067	0.0000

System: WEEKLY DATA
Estimation Method: ARCH Maximum Likelihood (Marquardt)
Covariance specification: Diagonal VECH
Date: 07/12/13 Time: 17:14
Sample: 6/20/2007 3/20/2013
Included observations: 301
Total system (balanced) observations 602
Presample covariance: backcast (parameter =0.7)
Convergence achieved after 21 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.000187	0.000290	0.643764	0.5197
C(2)	0.000187	0.000289	0.646388	0.5180
	Variance Equation Coefficients			
C(3)	1.27E-06	4.81E-07	2.630020	0.0085
C(4)	1.13E-06	4.17E-07	2.703542	0.0069
C(5)	1.18E-06	4.00E-07	2.954133	0.0031
C(6)	0.103248	0.016312	6.329730	0.0000
C(7)	0.873055	0.018574	47.00503	0.0000

Log likelihood	2531.017	Schwarz criterion	-16.68466
Avg. log likelihood	4.204347	Hannan-Quinn criter.	-16.73638
Akaike info criterion	-16.77087		

Equation: DLOG_S_EURUSD_WEEKLY = C(1)

R-squared	-0.001862	Mean dependent var	-6.38E-05
Adjusted R-squared	-0.001862	S.D. dependent var	0.005821
S.E. of regression	0.005827	Sum squared resid	0.010186
Durbin-Watson stat	1.486168		

Equation: DLOG_F_EURUSD_WEEKLY = C(2)

R-squared	-0.001962	Mean dependent var	-6.09E-05
Adjusted R-squared	-0.001962	S.D. dependent var	0.005605
S.E. of regression	0.005611	Sum squared resid	0.009444
Durbin-Watson stat	1.378537		

Covariance specification: Diagonal VECH
GARCH = M + A1.*RESID(-1)*RESID(-1)' + B1.*GARCH(-1)
M is an indefinite matrix
A1 is a scalar
B1 is a scalar

Tranformed Variance Coefficients

	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	1.27E-06	4.81E-07	2.630020	0.0085
M(1,2)	1.13E-06	4.17E-07	2.703542	0.0069
M(2,2)	1.18E-06	4.00E-07	2.954133	0.0031
A1	0.103248	0.016312	6.329730	0.0000
B1	0.873055	0.018574	47.00503	0.0000

Annex 12. Bivariate GARCH(1,1) with a CCC covariance

System: DAILY DATA
Estimation Method: ARCH Maximum Likelihood (Marquardt)
Covariance specification: Constant Conditional Correlation
Date: 06/29/13 Time: 22:18
Sample: 6/19/2007 3/18/2013
Included observations: 1496
Total system (balanced) observations 2992
Presample covariance: backcast (parameter =0.7)
Convergence achieved after 8 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	7.05E-05	6.87E-05	1.026169	0.3048
C(2)	5.75E-05	6.69E-05	0.859415	0.3901
	Variance Equation Coefficients			
C(3)	6.59E-08	2.04E-08	3.229996	0.0012
C(4)	0.030559	0.005234	5.838660	0.0000
C(5)	0.962852	0.005911	162.8945	0.0000
C(6)	6.04E-08	2.14E-08	2.826405	0.0047
C(7)	0.029602	0.005624	5.263527	0.0000
C(8)	0.963870	0.006265	153.8592	0.0000
C(9)	0.330101	0.021881	15.08602	0.0000

Log likelihood	13378.76	Schwarz criterion	-17.84206
Avg. log likelihood	4.471511	Hannan-Quinn criter.	-17.86211
Akaike info criterion	-17.87401		

Equation: DLOGS = C(1)

R-squared	-0.000697	Mean dependent var	-1.05E-05
Adjusted R-squared	-0.000697	S.D. dependent var	0.003068
S.E. of regression	0.003069	Sum squared resid	0.014085
Durbin-Watson stat	1.977729		

Equation: DLOGF = C(2)

R-squared	-0.000513	Mean dependent var	-1.11E-05
Adjusted R-squared	-0.000513	S.D. dependent var	0.003028
S.E. of regression	0.003029	Sum squared resid	0.013717
Durbin-Watson stat	1.931300		

Covariance specification: Constant Conditional Correlation
GARCH(i) = M(i) + A1(i)*RESID(i)(-1)^2 + B1(i)*GARCH(i)(-1)
COV(i,j) = R(i,j)*@SQRT(GARCH(i)*GARCH(j))

Tranformed Variance Coefficients

	Coefficient	Std. Error	z-Statistic	Prob.
M(1)	6.59E-08	2.04E-08	3.229996	0.0012
A1(1)	0.030559	0.005234	5.838660	0.0000
B1(1)	0.962852	0.005911	162.8945	0.0000
M(2)	6.04E-08	2.14E-08	2.826405	0.0047
A1(2)	0.029602	0.005624	5.263527	0.0000
B1(2)	0.963870	0.006265	153.8592	0.0000
R(1,2)	0.330101	0.021881	15.08602	0.0000

System: WEEKLY DATA
Estimation Method: ARCH Maximum Likelihood (Marquardt)
Covariance specification: Constant Conditional Correlation
Date: 07/12/13 Time: 02:45
Sample: 6/20/2007 3/20/2013
Included observations: 301
Total system (balanced) observations 602
Presample covariance: backcast (parameter =0.7)
Convergence achieved after 20 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.000349	0.000258	1.352659	0.1762
C(2)	0.000369	0.000249	1.482988	0.1381
	Variance Equation Coefficients			
C(3)	-3.81E-08	3.72E-07	-0.102474	0.9184
C(4)	0.095103	0.023084	4.119807	0.0000
C(5)	0.916823	0.020559	44.59370	0.0000
C(6)	8.28E-08	2.85E-07	0.290644	0.7713
C(7)	0.074988	0.019579	3.829933	0.0001
C(8)	0.930272	0.017276	53.84801	0.0000
C(9)	0.900656	0.010628	84.74657	0.0000

Log likelihood	2530.493	Schwarz criterion	-16.64326
Avg. log likelihood	4.203476	Hannan-Quinn criter.	-16.70975
Akaike info criterion	-16.75410		

Equation: DLOG_S_EURUSD_WEEKLY = C(1)

R-squared	-0.005036	Mean dependent var	-6.38E-05
Adjusted R-squared	-0.005036	S.D. dependent var	0.005821
S.E. of regression	0.005836	Sum squared resid	0.010218
Durbin-Watson stat	1.481474		

Equation: DLOG_F_EURUSD_WEEKLY = C(2)

R-squared	-0.005906	Mean dependent var	-6.09E-05
Adjusted R-squared	-0.005906	S.D. dependent var	0.005605
S.E. of regression	0.005622	Sum squared resid	0.009481
Durbin-Watson stat	1.373131		

Covariance specification: Constant Conditional Correlation
GARCH(i) = M(i) + A1(i)*RESID(i)(-1)^2 + B1(i)*GARCH(i)(-1)
COV(i,j) = R(i,j)*@SQRT(GARCH(i)*GARCH(j))

Tranformed Variance Coefficients

	Coefficient	Std. Error	z-Statistic	Prob.
M(1)	-3.81E-08	3.72E-07	-0.102474	0.9184
A1(1)	0.095103	0.023084	4.119807	0.0000
B1(1)	0.916823	0.020559	44.59370	0.0000
M(2)	8.28E-08	2.85E-07	0.290644	0.7713
A1(2)	0.074988	0.019579	3.829933	0.0001
B1(2)	0.930272	0.017276	53.84801	0.0000
R(1,2)	0.900656	0.010628	84.74657	0.0000

Annex 13. Bivariate GARCH(1,1) with a diagonal BEKK covariance

System: DAILY DATA
Estimation Method: ARCH Maximum Likelihood (Marquardt)
Covariance specification: BEKK
Date: 07/12/13 Time: 02:56
Sample: 6/19/2007 3/12/2013
Included observations: 1496
Total system (balanced) observations 2992
Presample covariance: backcast (parameter =0.7)
Convergence achieved after 10 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	5.55E-05	6.75E-05	0.821246	0.4115
C(2)	3.05E-05	6.69E-05	0.455514	0.6487
Variance Equation Coefficients				
C(3)	7.64E-08	2.05E-08	3.730832	0.0002
C(4)	2.44E-08	8.58E-09	2.838527	0.0045
C(5)	7.85E-08	2.46E-08	3.191526	0.0014
C(6)	0.178076	0.014234	12.51060	0.0000
C(7)	0.181405	0.016128	11.24814	0.0000
C(8)	0.980176	0.002985	328.3777	0.0000
C(9)	0.979183	0.003378	289.8927	0.0000

Log likelihood	13381.37	Schwarz criterion	-17.84555
Avg. log likelihood	4.472382	Hannan-Quinn criter.	-17.86559
Akaike info criterion	-17.87749		

Equation: DLOG_S_EURUSD_DAILY = C(1)

R-squared	-0.000462	Mean dependent var	-1.05E-05
Adjusted R-squared	-0.000462	S.D. dependent var	0.003068
S.E. of regression	0.003069	Sum squared resid	0.014082
Durbin-Watson stat	1.978194		

Equation: DLOG_F_EURUSD_DAILY = C(2)

R-squared	-0.000189	Mean dependent var	-1.11E-05
Adjusted R-squared	-0.000189	S.D. dependent var	0.003028
S.E. of regression	0.003029	Sum squared resid	0.013712
Durbin-Watson stat	1.931926		

Covariance specification: BEKK
GARCH = M + A1*RESID(-1)*RESID(-1)'*A1 + B1*GARCH(-1)*B1
M is an indefinite matrix
A1 is diagonal matrix
B1 is diagonal matrix

Tranformed Variance Coefficients

	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	7.64E-08	2.05E-08	3.730832	0.0002
M(1,2)	2.44E-08	8.58E-09	2.838527	0.0045
M(2,2)	7.85E-08	2.46E-08	3.191526	0.0014
A1(1,1)	0.178076	0.014234	12.51060	0.0000
A1(2,2)	0.181405	0.016128	11.24814	0.0000

B1(1,1)	0.980176	0.002985	328.3777	0.0000
B1(2,2)	0.979183	0.003378	289.8927	0.0000

System: WEEKLY DATA
Estimation Method: ARCH Maximum Likelihood (Marquardt)
Covariance specification: BEKK
Date: 07/12/13 Time: 02:54
Sample: 6/20/2007 3/20/2013
Included observations: 301
Total system (balanced) observations 602
Presample covariance: backcast (parameter =0.7)
Convergence achieved after 36 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.000219	0.000270	0.810711	0.4175
C(2)	0.000198	0.000252	0.785019	0.4324
Variance Equation Coefficients				
C(3)	3.08E-06	9.05E-07	3.399679	0.0007
C(4)	2.05E-06	5.50E-07	3.725958	0.0002
C(5)	1.16E-06	3.21E-07	3.607646	0.0003
C(6)	0.365563	0.027562	13.26320	0.0000
C(7)	0.256729	0.026116	9.830503	0.0000
C(8)	0.877657	0.018225	48.15772	0.0000
C(9)	0.947912	0.006871	137.9600	0.0000

Log likelihood	2538.272	Schwarz criterion	-16.69495
Avg. log likelihood	4.216399	Hannan-Quinn criter.	-16.76144
Akaike info criterion	-16.80580		

Equation: DLOG_S_EURUSD_WEEKLY = C(1)

R-squared	-0.002370	Mean dependent var	-6.38E-05
Adjusted R-squared	-0.002370	S.D. dependent var	0.005821
S.E. of regression	0.005828	Sum squared resid	0.010191
Durbin-Watson stat	1.485415		

Equation: DLOG_F_EURUSD_WEEKLY = C(2)

R-squared	-0.002145	Mean dependent var	-6.09E-05
Adjusted R-squared	-0.002145	S.D. dependent var	0.005605
S.E. of regression	0.005611	Sum squared resid	0.009446
Durbin-Watson stat	1.378285		

Covariance specification: BEKK
GARCH = M + A1*RESID(-1)*RESID(-1)'*A1 + B1*GARCH(-1)*B1
M is an indefinite matrix
A1 is diagonal matrix
B1 is diagonal matrix

Tranformed Variance Coefficients				
	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	3.08E-06	9.05E-07	3.399679	0.0007
M(1,2)	2.05E-06	5.50E-07	3.725958	0.0002

M(2,2)	1.16E-06	3.21E-07	3.607646	0.0003
A1(1,1)	0.365563	0.027562	13.26320	0.0000
A1(2,2)	0.256729	0.026116	9.830503	0.0000
B1(1,1)	0.877657	0.018225	48.15772	0.0000
B1(2,2)	0.947912	0.006871	137.9600	0.0000

Annex 14. Bivariate TARCH(1,1) with a restricted diagonal VECH covariance

System: WEEKLY DATA
Estimation Method: ARCH Maximum Likelihood (Marquardt)
Covariance specification: Diagonal VECH
Date: 07/12/13 Time: 17:17
Sample: 6/20/2007 3/20/2013
Included observations: 301
Total system (balanced) observations 602
Presample covariance: backcast (parameter =0.7)
Convergence achieved after 21 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	5.02E-05	0.000281	0.178565	0.8583
C(2)	2.86E-05	0.000288	0.099349	0.9209
Variance Equation Coefficients				
C(3)	1.12E-06	3.60E-07	3.102951	0.0019
C(4)	1.01E-06	3.16E-07	3.195549	0.0014
C(5)	1.07E-06	3.16E-07	3.377083	0.0007
C(6)	0.028029	0.015487	1.809908	0.0703
C(7)	0.113506	0.026027	4.361082	0.0000
C(8)	0.891915	0.015117	58.99919	0.0000

Log likelihood	2536.506	Schwarz criterion	-16.70218
Avg. log likelihood	4.213466	Hannan-Quinn criter.	-16.76128
Akaike info criterion	-16.80071		

Equation: DLOG_S_EURUSD_WEEKLY = C(1)

R-squared	-0.000385	Mean dependent var	-6.38E-05
Adjusted R-squared	-0.000385	S.D. dependent var	0.005821
S.E. of regression	0.005823	Sum squared resid	0.010171
Durbin-Watson stat	1.488363		

Equation: DLOG_F_EURUSD_WEEKLY = C(2)

R-squared	-0.000256	Mean dependent var	-6.09E-05
Adjusted R-squared	-0.000256	S.D. dependent var	0.005605
S.E. of regression	0.005606	Sum squared resid	0.009428
Durbin-Watson stat	1.380888		

Covariance specification: Diagonal VECH
GARCH = M + A1.*RESID(-1)*RESID(-1)' + D1.*(RESID(-1)*(RESID(-1)<0))
(RESID(-1)(RESID(-1)<0))'D1.*(RESID(-1)*(RESID(-1)<0))*(RESID(
-1)*(RESID(-1)<0))' + B1.*GARCH(-1)
M is an indefinite matrix
A1 is a scalar
D1 is a scalar
B1 is a scalar

Tranformed Variance Coefficients

	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	1.12E-06	3.60E-07	3.102951	0.0019
M(1,2)	1.01E-06	3.16E-07	3.195549	0.0014
M(2,2)	1.07E-06	3.16E-07	3.377083	0.0007
A1	0.028029	0.015487	1.809908	0.0703
D1	0.113506	0.026027	4.361082	0.0000
B1	0.891915	0.015117	58.99919	0.0000

Annex 15. Bivariate TARCH(1,1) with a CCC covariance

System: WEEKLY DATA
Estimation Method: ARCH Maximum Likelihood (Marquardt)
Covariance specification: Diagonal VECH
Date: 07/12/13 Time: 02:48
Sample: 6/20/2007 3/20/2013
Included observations: 301
Total system (balanced) observations 602
Presample covariance: backcast (parameter =0.7)
Convergence achieved after 150 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.000178	0.000289	0.615005	0.5386
C(2)	0.000152	0.000281	0.540654	0.5887
Variance Equation Coefficients				
C(3)	1.23E-06	3.57E-07	3.456632	0.0005
C(4)	1.17E-06	2.90E-07	4.039521	0.0001
C(5)	1.07E-06	2.82E-07	3.810531	0.0001
C(6)	-0.022932	0.018443	-1.243367	0.2137
C(7)	-0.030361	0.015691	-1.935004	0.0530
C(8)	-0.018822	0.017639	-1.067107	0.2859
C(9)	0.301991	0.042680	7.075676	0.0000
C(10)	0.271534	0.046520	5.836919	0.0000
C(11)	0.934751	0.016514	56.60218	0.0000
C(12)	0.943367	0.013450	70.13797	0.0000
C(13)	0.944369	0.015342	61.55454	0.0000

Log likelihood	2554.322	Schwarz criterion	-16.72575
Avg. log likelihood	4.243060	Hannan-Quinn criter.	-16.82179
Akaike info criterion	-16.88586		

Equation: DLOG_S_EURUSD_WEEKLY = C(1)

R-squared	-0.001731	Mean dependent var	-6.38E-05
Adjusted R-squared	-0.001731	S.D. dependent var	0.005821
S.E. of regression	0.005826	Sum squared resid	0.010184
Durbin-Watson stat	1.486362		

Equation: DLOG_F_EURUSD_WEEKLY = C(2)

R-squared	-0.001450	Mean dependent var	-6.09E-05
Adjusted R-squared	-0.001450	S.D. dependent var	0.005605
S.E. of regression	0.005609	Sum squared resid	0.009439
Durbin-Watson stat	1.379242		

Covariance specification: Diagonal VECH
GARCH = M + A1.*RESID(-1)*RESID(-1)' + D1.*(RESID(-1)*(RESID(-1)<0))
(RESID(-1)(RESID(-1)<0))'D1.*(RESID(-1)*(RESID(-1)<0))*(RESID(
-1)*(RESID(-1)<0))' + B1.*GARCH(-1)
M is an indefinite matrix
A1 is an indefinite matrix
D1 is a rank one matrix
B1 is an indefinite matrix

Tranformed Variance Coefficients				
	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	1.23E-06	3.57E-07	3.456632	0.0005
M(1,2)	1.17E-06	2.90E-07	4.039521	0.0001
M(2,2)	1.07E-06	2.82E-07	3.810531	0.0001
A1(1,1)	-0.022932	0.018443	-1.243367	0.2137
A1(1,2)	-0.030361	0.015691	-1.935004	0.0530
A1(2,2)	-0.018822	0.017639	-1.067107	0.2859
D1(1,1)	0.091199	0.025778	3.537838	0.0004
D1(1,2)	0.082001	0.022792	3.597723	0.0003
D1(2,2)	0.073731	0.025264	2.918460	0.0035
B1(1,1)	0.934751	0.016514	56.60218	0.0000
B1(1,2)	0.943367	0.013450	70.13797	0.0000
B1(2,2)	0.944369	0.015342	61.55454	0.0000

Annex 16. Bivariate TARCH(1,1) with a diagonal BEKK covariance

System: WEEKLY DATA
Estimation Method: ARCH Maximum Likelihood (Marquardt)
Covariance specification: BEKK
Date: 07/12/13 Time: 15:17
Sample: 6/20/2007 3/20/2013
Included observations: 301
Total system (balanced) observations 602
Presample covariance: backcast (parameter =0.7)
Convergence achieved after 140 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.000158	0.000291	0.544266	0.5863
C(2)	0.000122	0.000287	0.424936	0.6709
Variance Equation Coefficients				
C(3)	1.55E-06	4.99E-07	3.108674	0.0019
C(4)	1.48E-06	3.64E-07	4.078950	0.0000
C(5)	1.32E-06	3.51E-07	3.770146	0.0002
C(6)	-0.049993	0.089597	-0.557977	0.5769
C(7)	0.076482	0.081691	0.936228	0.3492
C(8)	-0.298575	0.045873	-6.508686	0.0000
C(9)	-0.262365	0.043879	-5.979238	0.0000
C(10)	0.948225	0.012817	73.98426	0.0000
C(11)	0.955505	0.009876	96.75016	0.0000

Log likelihood	2553.054	Schwarz criterion	-16.75525
Avg. log likelihood	4.240953	Hannan-Quinn criter.	-16.83651
Akaike info criterion	-16.89072		

Equation: DLOG_S_EURUSD_WEEKLY = C(1)

R-squared	-0.001460	Mean dependent var	-6.38E-05
Adjusted R-squared	-0.001460	S.D. dependent var	0.005821
S.E. of regression	0.005826	Sum squared resid	0.010182
Durbin-Watson stat	1.486765		

Equation: DLOG_F_EURUSD_WEEKLY = C(2)

R-squared	-0.001068	Mean dependent var	-6.09E-05
Adjusted R-squared	-0.001068	S.D. dependent var	0.005605
S.E. of regression	0.005608	Sum squared resid	0.009436
Durbin-Watson stat	1.379768		

Covariance specification: BEKK
GARCH = M + A1*RESID(-1)*RESID(-1)'*A1 + D1.*(RESID(-1)*(RESID(
-1)<0))*(RESID(-1)*(RESID(-1)<0))'D1*(RESID(-1)*(RESID(-1)<0))
(RESID(-1)(RESID(-1)<0))'*D1 + B1*GARCH(-1)*B1
M is an indefinite matrix
A1 is diagonal matrix
D1 is diagonal matrix
B1 is diagonal matrix

Tranformed Variance Coefficients

	Coefficient	Std. Error	z-Statistic	Prob.

M(1,1)	1.55E-06	4.99E-07	3.108674	0.0019
M(1,2)	1.48E-06	3.64E-07	4.078950	0.0000
M(2,2)	1.32E-06	3.51E-07	3.770146	0.0002
A1(1,1)	-0.049993	0.089597	-0.557977	0.5769
A1(2,2)	0.076482	0.081691	0.936228	0.3492
D1(1,1)	-0.298575	0.045873	-6.508686	0.0000
D1(2,2)	-0.262365	0.043879	-5.979238	0.0000
B1(1,1)	0.948225	0.012817	73.98426	0.0000
B1(2,2)	0.955505	0.009876	96.75016	0.0000

Annex 17. Measure of Hedging Effectiveness

	2007	2008	2009	2010	2011	2012	2013
Naïve hedge	-0,1727	0,01607	-0,3878	-0,3855	-0,471	-0,6806	-0,3656
Constant hedge (daily)	0,16075	0,22756	0,0773	0,08837	0,07209	-0,0055	0,03698
Constant hedge (weekly)	-1,5673	-2,2443	-2,1544	-1,8951	-1,9309	-2,1086	-0,1128
Constant hedge (monthly)	-0,2395	-0,0408	-0,4639	-0,4642	-0,5578	-0,7775	-0,4281
GARCH with diagonal VECH	0,22319	0,30337	0,1193	0,13358	0,14424	0,05059	0,11733
GARCH with CCC	0,16192	0,23056	0,08264	0,09009	0,0731	0,00176	0,02509
GARCH with diagonal BEKK	0,22262	0,30419	0,11921	0,13439	0,14437	0,05058	0,11834

Daily returns with a 12 months hedging horizon

	2007	2008	2009	2010	2011	2012	2013
Naïve hedge	0,74559	0,8876	0,48999	0,80632	0,65533	0,6491	0,93073
Constant hedge (monthly)	0,73293	0,88533	0,47273	0,79192	0,63068	0,62743	0,92511
Constant hedge (weekly)	0,75408	0,87825	0,50915	0,81659	0,68331	0,67264	0,92585
Constant hedge (daily)	0,46269	0,50157	0,33845	0,50312	0,45863	0,4437	0,53841
GARCH with diagonal VECH	0,7811	0,90949	0,53571	0,82879	0,71185	0,7224	0,92943
GARCH with CCC	0,76472	0,90364	0,48266	0,82369	0,69987	0,70223	0,92184
GARCH with diagonal BEKK	0,71491	0,87487	0,48779	0,73985	0,65832	0,58427	0,69586
TARCH with diagonal VECH	0,77517	0,90281	0,52826	0,82538	0,70857	0,71009	0,92599
TARCH with CCC	0,38929	0,46859	0,06139	0,4757	0,31257	0,43721	0,87632
TARCH with diagonal BEKK	0,76194	0,88233	0,51923	0,81499	0,69825	0,69499	0,93101

Weekly returns with a 12 month hedging horizon

		Naïve hedge	constant hedge (monthly)	constant hedge (weekly)	constant hedge (daily)	GARCH with diagonal VECH	GARCH with CCC	GARCH with diagonal BEKK	TARCH with diagonal VECH	TARCH with CCC	TARCH with diagonal BEKK
2007	Q3	0,9675	0,9650	0,9573	0,5466	0,9680	0,9669	0,8856	0,9669	0,8691	0,9687
	Q4	0,8469	0,8578	0,8183	0,4287	0,8572	0,8572	0,6737	0,8434	0,8676	0,8451
2008	Q1	0,9167	0,9124	0,9100	0,5255	0,9268	0,9242	0,7733	0,9215	0,7989	0,9193
	Q2	0,9708	0,9760	0,9489	0,5191	0,9716	0,9657	0,7715	0,9698	0,9278	0,9623
	Q3	0,9386	0,9297	0,9385	0,5553	0,9456	0,9429	0,8481	0,9470	0,6084	0,9440
	Q4	0,9878	0,9874	0,9742	0,5501	0,9902	0,9876	0,9638	0,9900	0,4125	0,9866
2009	Q1	0,4792	0,4649	0,4941	0,3212	0,5532	0,5000	0,6198	0,5430	-0,0283	0,5104
	Q2	0,7449	0,7405	0,7409	0,4306	0,7529	0,7511	0,6312	0,7256	0,4294	0,6886
	Q3	0,3362	0,3249	0,3484	0,2300	0,3639	0,3401	0,2636	0,3581	0,1532	0,3596
	Q4	0,7744	0,7495	0,8011	0,5259	0,8072	0,8026	0,6461	0,8058	0,5673	0,8055
2010	Q1	0,5036	0,4825	0,5285	0,3609	0,5383	0,5224	0,5273	0,5229	0,1262	0,5147
	Q2	0,7345	0,7180	0,7488	0,4709	0,7771	0,7689	0,6892	0,7780	0,2953	0,7699
	Q3	0,7515	0,7358	0,7646	0,4778	0,7791	0,7790	0,6860	0,7773	0,4677	0,7726
	Q4	0,9288	0,9220	0,9257	0,5418	0,9344	0,9284	0,8169	0,9289	0,7776	0,9082
2011	Q1	0,3831	0,3532	0,4231	0,3279	0,4820	0,4464	0,4799	0,4555	0,0048	0,4350
	Q2	0,7106	0,6858	0,7380	0,4898	0,7791	0,7649	0,7095	0,7902	0,4700	0,7749
	Q3	0,4413	0,4176	0,4709	0,3361	0,5054	0,4887	0,4547	0,4987	0,0513	0,4862
	Q4	0,7915	0,7676	0,8163	0,5312	0,8294	0,8291	0,7823	0,8286	0,3987	0,8235
2012	Q1	0,2828	0,2193	0,3752	0,4013	0,5119	0,4609	0,5577	0,4804	-0,2385	0,4376
	Q2	0,7168	0,6988	0,7338	0,4672	0,7385	0,7430	0,5663	0,7403	0,5319	0,7384
	Q3	0,7385	0,7334	0,7356	0,4298	0,7589	0,7532	0,5878	0,7560	0,6003	0,7499
	Q4	0,8215	0,8070	0,8316	0,5118	0,8525	0,8446	0,6957	0,8354	0,6477	0,8277
2013	Q1	0,9295	0,9302	0,9151	0,5135	0,9281	0,9133	0,6643	0,9245	0,9025	0,9261

Weekly returns with a 3 month hedging horizon