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Generation of Human Walking Paths Based On Inverse Optimal Control Approach

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(**ABSTRACT**)

The purpose of this thesis is to present the approach method to generate human path which based on inverse optimal control problem. The main aim is to simulate human locomotion and build optimal control models that can be used to control robot motion. To determine the optimization criterion by solving inverse optimal control problem for a given dynamic process and an observed solution, we here use a dual-level approach and estimate parameters to guarantee a match between experimental measurement and optimal control problem. We apply this approach to both simulated and experimental data to obtain a simple model of human walking trajectories. The performance of the approached methods and generated path were tested in MALTAB simulation and V-REP simulation, which include the reference paths generation, parameters estimation and approached paths certification. The approach methods were based on Least Square and Fréchet Distance.

Keywords: Inverse Optimal Control, Path Following, Human Locomotion

(ABSTRACT)

L'obiettivo di questo lavoro è di presentare un metodo, basato sulla soluzione di un problema di controllo ottimo, per la generazione di percorsi di cammino il più possibile simili a quelli pianificati da un essere umano. Lo scopo del lavoro riguarda sia la simulazione del movimento di un essere umano che la pianificazione di percorsi per robot umanoidi il più possibile simili a quelli di un essere umano.

Il metodo studiato parte dall'ipotesi che gli esseri umani pianifichino il loro moto ottimizzando un funzionale di costo. Ciò può essere facilmente tradotto in un problema di controllo ottimo basato su un modello dinamico semplificato e su una cifra di merito di cui si suppone nota la struttura e incogniti i parametri. Il problema affrontato riguarda quindi l'implementazione di un algoritmo per la stima di tali parametri, a partire da un insieme di traiettorie di moto ottenute sperimentalmente.
Una volta stimati i parametri della funzione di costo è stato risolto il problema di controllo ottimo, utilizzando MATLAB, in modo da rigenerare al calcolatore le traiettorie e confrontarle con quelle sperimentali. Tale confronto è stato eseguito utilizzando sia la distanza euclidea che la distanza di Fréchet.

E' stato infine utilizzato l'ambiente di simulazione V-REP per mostrare la differenza tra la camminata registrata sperimentalmente e quella ricreata al calcolatore.

Keywords: problema di controllo ottimo inverso, inseguimento di percorso, analisi del movimento umano

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Chapter 1

Introduction

The simulation of human motion and the performance evaluation of human in virtual environments is becoming increasingly important in computer animation, mechanical engineering, medical, military and space exploration applications. The pioneers have developed several methods to generate locomotion of human beings [1-4]. Goal-directed locomotion in humans has mainly been investigated with respect to how different sensory inputs are dynamically integrated. Visual, vestibular, and proprioceptive inputs were analyzed during both normal and blindfolded locomotion in order to study how humans could continuously control their trajectories [5]. However the estimation of the human intention tout court is obviously impossible, in the case of walking of a human being it is possible to try to predict her/his trajectory in a Goal-directed motion, based on the known model of motion or instead, on a model of the way human beings plans their path [6].

The general hypothesis is that locomotion of animals and humans is optimal which the experts have done a lot of research of [7]-[12]. According to this assumption, if we have some specific optimization criterion as an observed solution to such dynamic process, we can easily find an approach to the real trajectories. As before the optimization criterion is unknown in such condition, but the new trend leads us to solve this optimal control problem based on an inverse optimal control problem. Recently, it has been observed that for predefined paths, an inverse relationship between the path geometry (curvature profile) and body kinematics (walking speed) exists [13],[22]. From the biomechanics or the neuroscience point of view, by given an end position and orientation, human will select a very specific path out of numerical possibilities. There are different perspectives from which human and humanoid locomotion can be investigated—the biomechanics or the neuroscience point of view. Most researchers in biomechanics study locomotion on joint level along a given straight or bent overall path on the floor to be followed. The study of the selection and optimal generation of this overall path has however been widely neglected in humanoid robotics and also in biomechanics so far. If humans are asked to walk towards a given end position and orientation in an empty space with no obstacles, they will select a very specific path, out of an infinity number of possibilities. This choice is not so much influenced by biomechanical properties but rather by neuroscience aspects. In the attempt to control humanoids in a biologically inspired manner, it would be desirable to understand and imitate that behavior of human. In the problem of optimal control we are asked to find input and state trajectories that minimize a given cost function. In the problem of inverse optimal control, we are asked to find a cost function with respect to which observed input and state trajectories are optimal [14]. These methods are used to predict optimal movements by searching the control law according to some performance criterion.

As these method based on numerical experiments also, however, inverse optimal control is the problem of computing a cost function that would have resulted in an observed sequence of decisions. The standard formulation of this problem assumes that decisions are optimal and tries to minimize the difference between what was observed and what would have been observed given a candidate cost function [15]. We assume instead that decisions are only approximately optimal and try to minimize the extent to which observed decisions violate first-order necessary conditions for optimality. For a discrete-time optimal control system with a cost function that is a linear combination of known basis functions, this formulation leads to an efficient method of solution as an unconstrained least-squares problem. We apply this approach to both simulated and experimental data to obtain a simple model of human walking trajectories. This model might subsequently be used either for control of a humanoid robot or for predicting human motion when moving a robot through crowded areas. We see the understanding of the optimality principles of human locomotion as one of the keys to generate biologically inspired locomotion on autonomous robots. If the human optimization criterion of locomotion can correctly be formulated in mathematical terms, it is straightforward to mimic this optimization approach on a humanoid robot.

The method presented in this thesis is inspired from [16]. This new formulation of inverse optimal control assumes that the observations are perfect, while the system is considered to be only approximately optimal. This change in assumption allows us to define residual functions based on the Karush Kuhn-Tucker (KKT) necessary conditions for optimality [17], [18]. The inverse optimal control problem then simplifies to minimizing these residual functions in order to recover the parameters that govern the cost function. As a result, the inverse optimal control problem reduces to a simple Least Squares minimization or Fréchet Distance minimization, which can be solved very efficiently. In [17], the authors restrict their attention to convex optimization problems, so in this thesis, we apply a similar approach to solve an optimal control problem of that. Our approach can be extended to a wide range of discrete-time nonlinear problems, with the assumption that the unknown parameter vector needs to enter the cost function linearly. This technique of approximating a cost function using linear combinations of basic functions is common to most inverse optimal control methods.

The outline of this thesis is as following: In Chapter 2 you can find the basic methodology of the inverse optimal control problem which will cover the path following formulation, the unicycle time model and the basic background of two approach methods. In Chapter 3 we will discuss how the approach methods implemented in simulation environment which is following the outline discussed in Chapter 2, it will also describe the representation of the problem and the processing of solving problem and how we solve it. The simulation results of each simulation and the comparison of reference paths and approached paths will be presented in Chapter 4 and we will also analysis and discuss these results to give an overall conclusion then implant these methods. Finally, in Chapter 5, we will conclude the main achievement, prime result and re-call the thesis topic. Finally give some possible improvement in the future.

Chapter 2

Methodology

2.1 Problem statement

As the Goal-directed locomotion in humans, we still try to model this problem based on such goals and orientations of trajectories. The frame of this problem is two-dimension in space which is fixed on ground floor, whose axes are redefined as $\varphi_i = (x_i, y_i, \theta_i)$, the random point on such trajectory, which stand for North, East and rotation angle. Assuming that human starts at orientation point $\varphi_0 = (x_0, y_0, \theta_0)$ and ends at $\varphi_T = (x_T, y_T, \theta_T)$, which means as the experiments those have done by the researchers are the fundamental data used to rebuild the criterion. Considering $v(t)$

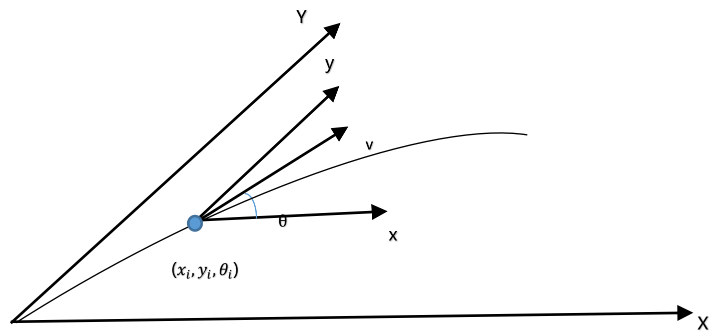


Figure 2.1 Problem Frame

as the forward speed of human and $\omega(t)$ as the angular speed of human, these two we called inputs as $u(t)$ =($v(t)$,$\omega(t)$). These all is presented in Figure 2.1.

To determine the formulation of an optimal control problem, in our case is the cost function of human locomotion, which is according to the experiments we have done, in each time, human will always choose the lowest potential energy cost path for himself. Where the unicycle time kinematic model can be described as following,

$$\begin{cases} \dot{x} = v \cdot \cos\theta \\ \dot{y} = v \cdot \sin\theta \\ \dot{\theta} = \omega \end{cases} \tag{2.1}$$

Former survey proof that locomotion trajectories are the optimal solutions of a dynamic extension of a simple unicycle control model. The validation method consists in comparing the optimal trajectories of the system with the trajectories of the data basis. The locomotion trajectories minimize the time derivative of the curvature, and the locomotion trajectories are well approximated by clothoid arcs. However, the number of concatenated arcs of switching points are under study [15], this is a very good approach algorithm right now. But in our case the optimal control problem is replaced by the corresponding first order optimality conditions which become constraints of the parameter estimation problem.

The problem of determining the “best” of cost function J thus is transformed into the problem of determining the best weight factor. In a combined objective function the relative size of the weight factors is crucial since they determine the influence of the respective term on the overall sum: the larger the weight, the more the corresponding term is punished and therefore is likely to be reduced in the overall context. The inverse optimal control problem reduces to a simple least-squares minimization, which can be solved very efficiently. We here introduce J as the energy cost during the whole time in continuous as our objective function in formula 2.2

$$J = \frac{1}{2}\int_0^T c_1 \cdot v^2(t) + \omega^2(t) dt \tag{2.2}$$

In order to apply optimal control techniques on this model, a discrete version is adopted. In the following we consider the Explicit Euler discretization method. We here re-present the dynamic model as,

$$\begin{cases} x(k+1) = x(k) + \tau \cdot v(k) \cdot \cos(\theta(k)) \\ \mathrm{y}(k+1) = y(k) + \tau \cdot v(k) \cdot \sin(\theta\,(k)) \\ \theta(k+1) = \theta(k) + \tau \cdot \omega(k) \end{cases} \tag{2.3}$$

where τ is the sampling time and k is the discrete-time index.

The inverse optimal control problem becomes,

$$\begin{aligned} &\min_{u(k),\varphi(k)} \frac{1}{2}\tau \sum_{k=0}^{N-1} (c_1 \cdot v^2(k) + \omega^2(k)) \\ \text{s.t.} \quad & \varphi(0) - \varphi_0 = 0 \\ & \varphi(N-1) - \varphi_T = 0 \\ & x(k+1) - x(k) + \tau \cdot v(k) \cdot \cos\big(\theta(k)\big) = 0 \\ & \mathrm{y}(k+1) - y(k) + \tau \cdot v(k) \cdot \sin(\theta\,(k)) = 0 \\ & \theta(k+1) - \theta(k) + \tau \cdot \omega(k) = 0 \\ & \forall k = 0, \cdots, N-1 \end{aligned} \tag{2.4}$$

As the dynamic model and initial and final conditions are given, here we are interested in two different cases: the optimal solution $u^*(k)$, $\varphi^*(k)$ is known when weight parameters c_1 fixed and the optimal duration T^* is known. Only some components of the optimal states and controls are known at k discrete points. The inverse optimal control problem now consists in determining the exact objective function $\varphi^*(\cdot)$ that produces the best fit to the measurements in the least squares sense.

For a given c_1, assuming that $\chi^* = [u^*(k)^T \ \ \varphi^*(k)^T]^T$ is a local minimum of the problem and is regular, the cost function $f(\chi; c_1) \in \mathbb{R}$ and the set of constraints $g(\chi) \in \mathbb{R}^{\mathrm{m}}$, there exist unique Lagrange multiplier vectors $\lambda^* \in \mathbb{R}^{\mathrm{m}}$ so that,

$$\begin{cases} \nabla_x f(\chi^*; c_i) + \sum_{j=1}^{m} \lambda_j{}^{*T} \nabla_x g_j(\chi^*) = 0 \\ g(\chi^*) = 0 \end{cases} \tag{2.5}$$

assuming that $f(\cdot)$ and $g(\cdot)$ are continuously differentiable functions which known as the KKT necessary and sufficient conditions for quality constraint optimization problems: the first one is the stationary condition and the second equation ensures primal feasibility, so that the KKT conditions for the Lagrangian of the problem can be described as

$$\nabla_{(x,\lambda)} \Lambda(\chi, \lambda, c_1) = \nabla_{(x,\lambda)} \left(f(\chi; c_1) + \sum_{j=1}^{m} \lambda_j{}^T g_j(\chi) = 0 \right) \tag{2.6}$$

The inverse optimal control problem can be solved by minimizing the residual function

$$\min_{\lambda,c} \frac{1}{2} \left\| \nabla_{(x,\lambda)} \Lambda(\chi, \lambda, c_1) \right\|^2 = \min_{\lambda,c} \frac{1}{2} \| J^*(z) \|^2 \tag{2.7}$$

where $z = [c_1 \ \lambda]^T$ and J^* is the identification criterion where we can implant approach methods. So this problem becomes a convex unconstrained least-squares optimization or a discrete distance minimizing problem which are easier to solve than the initial constrained optimization one.

As the consideration of the geometry of the walking path only, we here can implant a space method instead of the complete trajectory as a function of time, where we have an assumption that along the path human only walk forward without back velocity. So we need to rewrite the dynamic model with natural coordinate s as the independent variable. By this method we will reduce our input control variance to angular speed only and check the optimization problem again. As well we can find the relationship between the distance and the forward speed as $s = \int_0^t v(\tau)d\tau$ which inverted defined as $t = t(s)$. So the time variance can be represented as a function of s where the derivative of path in time $\varphi(t) = (x(t), y(t), \theta(t)$ will be as,

$$\frac{\mathrm{d}\varphi}{\mathrm{dt}} = \frac{\mathrm{d}\varphi}{\mathrm{ds}} \cdot \frac{\mathrm{d}s}{\mathrm{dt}}$$

Without the time variance but only considering the natural coordinate, the unicycle model in (2.1) can be rewritten as following,

$$\begin{cases} x'(s) = \cos(\theta(s)) \\ y'(s) = \sin(\theta(s)) \\ \theta'(s) = \omega(s) \end{cases} \tag{2.8}$$

And (2.3) can be,

$$\begin{cases} x(k+1) = x(k) + \sigma(k) \cdot \cos(\theta(k)) \\ \mathrm{y}(k+1) = y(k) + \sigma(k) \cdot \sin(\theta\,(k)) \\ \theta(k+1) = \theta(k) + \sigma(k) \cdot \omega(k) \end{cases} \tag{2.9}$$

Where $\sigma(k) = s(k) - s(k-1)$ and k is the space index. So that here we only considering the angular speed as the unique input. After analysis of such model we give another weight parameter to angular speed, so formula (2.2) can be,

$$J = \frac{1}{2}\int_0^T c_1 \cdot v^2(t) + c_2\omega^2(t)dt$$

The general idea of how to solve the inverse optimal control problem is divided into two levels. The upper level is minimizing the cost function of the statement from experimental data by Least Square or Fréchet Distance method to estimate such weight factors c_i. The lower level is to solve the optimal control problem with such specified weight factors c_i and direct boundary conditions as the orientations, goals and speed.

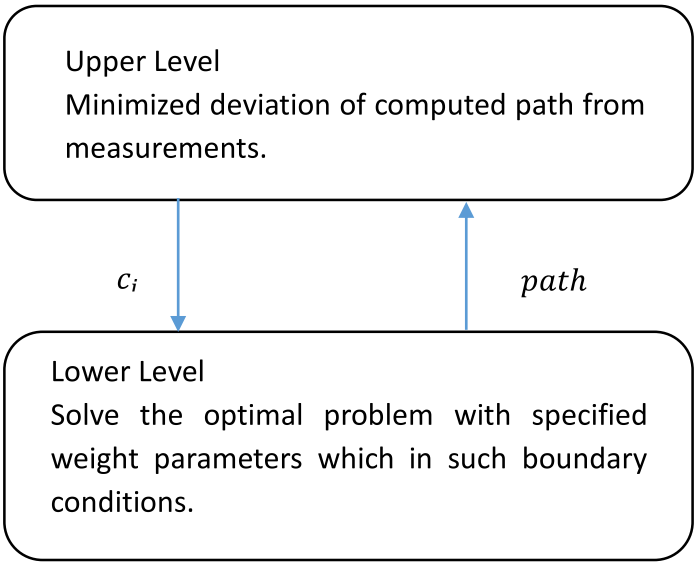


Figure 2.2 Structure of solution

According to method we used, we also need to define the differential criterion J^* which stands for the differential value or the distance between the generated paths and the reference ones. This criterion is a function of the weight parameters whenever the weight values minimized this criterion then return such values as the estimated weight value from the Upper Level, so here J^* will be described as $J^*(c_i)$.

First, we should consider the Lower level how to solve the optimal problem, before that we do have a clarification that the locomotion of human being is optimal and then we can generally plot this graph of how human being intended their paths. Based on the survey that human's locomotion only related to the speed of motion and energy consumed, we can generate such statement from the goal directed motion problem as an optimal problem and the criterion in such problem. So here we do not discuss how to generate such energy cost function from normal method, because we do not know it now. It can be approached through a lot of numerical methods [20]. Since our approach method is based on inverse optimal control, we now consider the criterion only depends on the human walking speed and rotation speed with such specified weight values. The Lower level solution is not the first calculated part, it is only a calculation level after we estimate the weight value for such criterion.

As here if we save this path as our reference trajectory for it is same as experimental one, by using inverse optimal method, we only need to identify the weight factors c_i. If our algorithm estimates the correct values of c_i, then we can find the proper objective function. After compare these data with approached ones we can examine the approached method and then by given experimental date, the approach method would work as well as with the simulated ones, this will save the experimental data sheet and time. It won't deal with numerical experiments to estimate different weight values for such certain criterion, only boundary conditions needed.

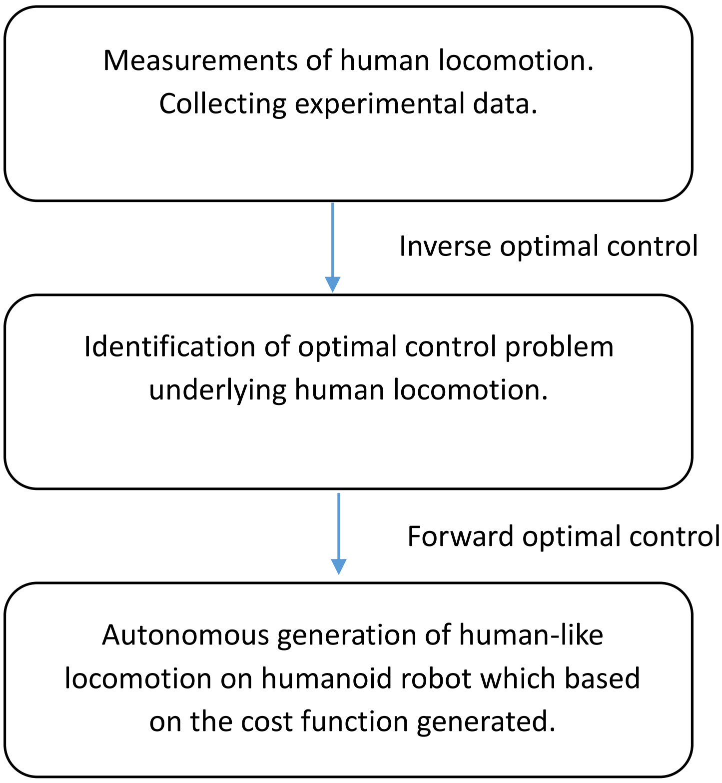


Figure 2.3 Completed Construction

For in our case we need to discuss with single weight parameter and double weight parameter, to be compared with these two approach method and find the most suitable numerical parameters for the cost function which is much more approached to the realistic ones. Other more discussion will be presented in the next chapter and also in the chapter of results. After we build such Lower level, the next step is to discuss the approach methods. As there are a lot of minimizing and approach algorithms, we here choose Least Squares approach and Fréchet Distance approach for our inverse optimal control problem and estimate weight parameters suitable for the optimal control problem identified criterion.

Our final goal is to control the human-like locomotion humanoid robot, in this thesis we only use the human walking model in V-rep software to simulate the humanoid robot and check these approached paths with the experimental ones. The V-rep allows users to setup the human walking model with nature coordinates and supply various control scheme and also with user interface modification. This process will looks like the forward optimal control part where we solve the optimal control problem and implant the time variant control values to mimic human locomotion. So even if we change the simulated human walking model to a real humanoid robot, the results will be the same. How to build the environment and how to simulate such human walking model will be introduced in Chapter 3.

2.2 Approach methods

The basic idea to apply such approach methods based on the differential value between the experimental data and the generated path data, which is satisfied minimum value of the criterion with specified weight parameters. To estimate such weight parameters, we here introduce two minimize approach methods, one is based on Least Squares approach and the other is Fréchet Distance approach. For these two method we both defined an identification criterion J^*, which is the differential value and a function of weight parameters.

The identification criterion is different from the cost function in optimal control problem. The basic idea of how to build such criterion is based on the minimum value point of view. How to define such minimum value is different in two approach method. However the same thing is to compare the approached paths with the reference one, which gives us the idea of both in nature coordinates and curve distance.

The processing of these two approach methods are the same. Here we firstly use random values of weight parameters in optimal problem criterion and generate a reference path which based on the optimal problem solution and then set a serial of weight values in a random range. It is like a fitting experiment where we give such serial values of weight parameters into the optimal problem criterion function each time and generate a path contrast to such parameters, then compare such path to the reference path which generated at beginning to specify which weight values are minimized the identification criterion then estimate such weight values. If we instead the first generated reference path of the realistic experimental data, this method will also work and estimate the weight values which are satisfied the optimal problem solution and return the certain criterion for the optimal problem that will be the general criterion for goal-directed human locomotion in such specified goal and orientation. The flow chart is described in figure 2.4.

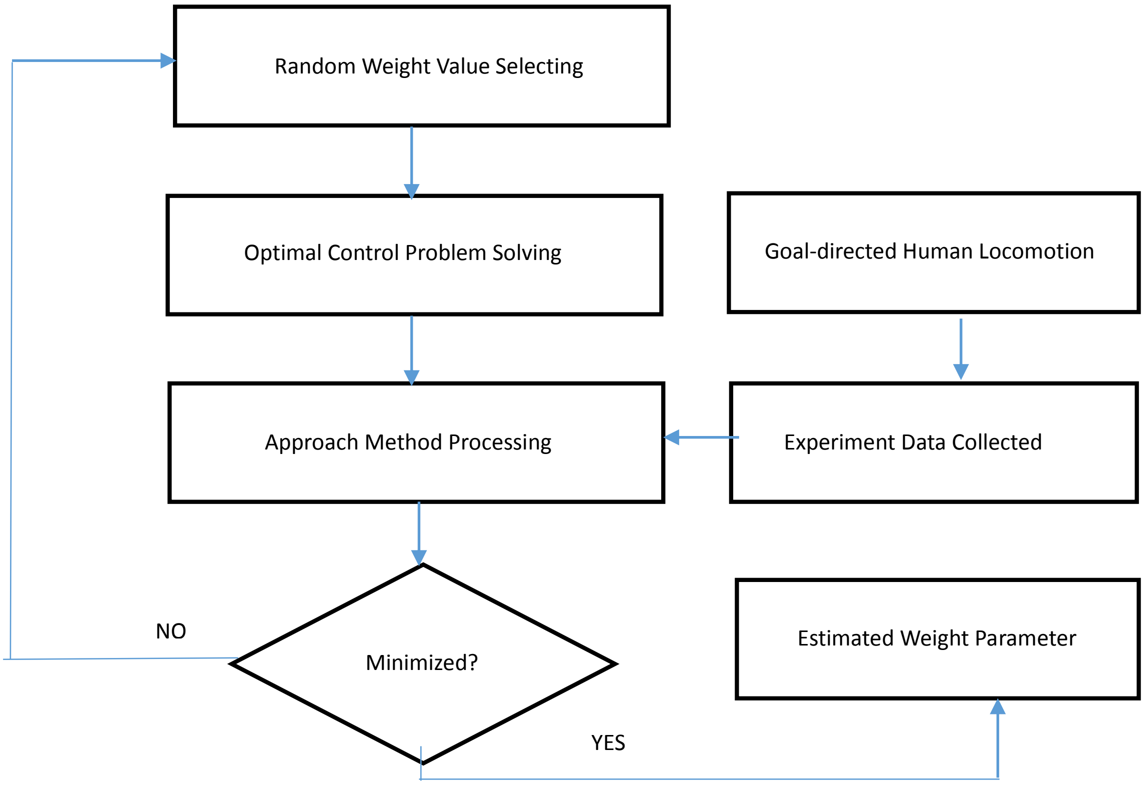


Figure 2.4 Flow Chart of processing

After we gained such weight parameter, return them the optimal control problem and rebuild the cost function of human walking, then solve the forward optimal control problem to find the mostly human-like path generated. By inputting such paths and control parameters to V-rep software we can easily check the walking model based on our approach methods. If this simulation is also well done, we can implant such paths to the real humanoid robots and the processing program will be also embedded in such robot to control it walking like human beings which is our final goal.

2.3.1 Least Squares method.

Here we introduce the Least Squares method to be one of our approach methods. The basic idea is to minimizing the sum of each generated point compared with the ones on the reference path which is time variant. A mathematical procedure for finding the best-fitting curve to a given set of points by minimizing the sum of the squares of the offsets or the residuals of the points from the curve. The sum of the squares of the offsets is used instead of the offset absolute values because this allows the residuals to be treated as a continuous differentiable quantity. The linear least squares fitting technique is the simplest and most commonly applied form of linear regression and provides a solution to the problem of finding the best fitting straight line through a set of points. However in our case the weight value is not fixed which we should run around a specified range and in each time and use Least Squares approach to estimate the weight value to gain the minimum of the identification criterion. At each fixed time point we can generate an approach point based on the weight parameter in the Lower level which we defined as (x_i^*, y_i^*) . So the deviations which here we called identification criterion J^* in Least Squares approach case is described as following in our case.

$$\min_{c} J^*\left(c_j\right) = \left\| \begin{matrix} x^* - x \\ y^* - y \end{matrix} \right\|_2^2 = \frac{1}{2}\sum_{i=0}^{M} {r_i}^2 \tag{2.10}$$

For the identification criterion J^* is a function of c_j which is also the sum of squared residuals, we express the relationship between each weight value and the criterion and then we can find the minimum value of J^* contrasted to a certain c_j which can be estimated. The squared residual which we defined as

$$r_i = f(x, y) - g(x^*, y^*; c_j) \tag{2.11}$$

where $f(x, y)$ is the reference path and $g(x^*, y^*; c_j)$ is the approached path with different c_j. To solve such Least Squares problem, we can gain that:

$$\frac{\partial J^*}{\partial c_j} = 2\sum_i r_i \cdot \frac{\partial r_i}{\partial c_j} = 0, j = 1, \cdots, m \tag{2.12}$$

$$r_i = f(x^*, y^*) - g(x, y, c_j)$$

$$-2\sum_i r_i \cdot \frac{\partial g}{\partial c_j} = 0$$

$$f(x, y) = g(x^*, y^*; c_j) \tag{2.13}$$

$$g(x^*, y^*; c_j) = \sum_{j=1}^{m} c_j \Phi(x^*, y^*)$$

$$\hat{c} = (\Phi^{\mathrm{T}} \cdot \Phi)^{-1} \cdot \Phi^{\mathrm{T}} \cdot f(x, y)$$

From this equation we can apparently find the minimum point of J^* and the weight value of $\hat{c}$ where c is unique, so that we can return such weight value c back to the criterion of optimal control problem and then solve the forward control problem in such goal-directed orientation.

However, at sometimes, the minimum is not apparent which looks like a bottom of the curve and the curve cannot even converge. Because some weight values can be fitted to that reference path and some of which are near the minimum which cannot be observed and discriminated in such situation. But in fact the minimum is unique and by here we also ignore the effect of the turning angle. So at contra pose we here introduce another approach method to examine whether the minimum point is unique or not.

2.3.2 Fréchet distance approach.

The Fréchet distance is a measure of similarity between two curves, in our case is the distance between the reference path which based on the experience and the path which generated from different weight parameters. The Fréchet distance of these two curves is the minimal length of any leash necessary for the dog and the handler to move from the starting points of the two curves to their respective endpoints. The Fréchet distance and its variants have been widely used in many applications such as dynamic time-warping, speech recognition, signature, verification, and matching of time series in databases.

Formally, the Fréchet distance is defined as follows. A parameterized curve in R^d can be represented as a continuous function: $f : [0,1] \to \mathbb{R}^d$. A monotone reparametrization α is a continuous non-decreasing function: $\alpha : [0,1] \to [0,1]$ with $\alpha(0)$=0 and $\alpha(1)$=1. Given two curves $f, g: [0,1] \to \mathbb{R}^d$, their Fréchet distance, $\delta_F(f,g)$, is defined as

$$\delta_F(f,g) \coloneqq \inf_{\alpha,\beta} \max_{t\in[0,1]} \mathrm{d}(f(\alpha(t)), g(\beta(t))) \tag{2.14}$$

where d(x, y) denotes the Euclidean distance between points x and y, and α and β range over all monotone reparametrizations.

And the Discrete Fréchet distance is defined as following: A simpler variant of the Fréchet distance for two polygonal curves $\pi = \langle p_1, p_2, \cdots, p_n \rangle$ and $\sigma = \langle q_1, q_2, \cdots, q_n \rangle$ is the discrete Fréchet distance, denoted by $\delta_D(\pi, \sigma)$. The discrete Fréchet distance is defined as the minimal leash necessary at these discrete moments. To formally define the discrete Fréchet distance, we first consider a discrete analog of (α, β), i.e., the correspondences between continuous reparametrizations. In particular, an order-preserving complete correspondence between π and σ is a set $M \subseteq \{(p,q) \mid p \in \pi,\ q \in \sigma\}$ of pairs of vertices which is one case order-preserving: if $(p_i, q_i) \in M$, then no $(p_s, q_t) \in M$ for $s < i$ and $t > j$, nor $s > i$ and $t < j$; and the other case complete: for any $p \in \pi$ (respectively $q \in \sigma$) there exists some pair involving p (respectively, q) in M. The discrete Fréchet distance between π and σ, $\delta_D(\pi,\sigma)$, is then

$$\delta_D(f,g) \coloneqq \min_{M} \max_{(p,q)\in M} \mathrm{d}(p.q) \tag{2.15}$$

where M range over all order-preserving complete correspondences between π and σ. It is well known that discrete and continuous versions of the Fréchet distance relate to each other as follows:

$$\delta_F(f,g) \leq \delta_D(f,g) \leq \delta_F(f,g) + \max\{l_1, l_2\} \tag{2.16}$$

where l_1 and l_2 are the lengths of the longest edges in π and σ, respectively. This suggests using δ_D to approximate δ_F. So the identification criterion becomes,

$$\delta_F(f,g;c_i) = J_F{}^*(c_i) \tag{2.17}$$

Based on the discrete Fréchet distance we can find a minimum of this value which is also a function of weight parameter in such case. So the identification criterion becomes the Fréchet distance that is much similar to the one in Least Squares approach. But why we introduce Fréchet distance is because in some conditions, especially with a very large weight value, the convergence of the Least Squares approach is not so much apparent. There is a flat area on the minimum curve which will return weight parameters incorrectly. In fact the minimum point should be unique, however in Least Squares approach since the error during calculation and toolbox we used will cause such problem, which is main reason we introduce Fréchet Distance approach to verify the minimum point we gain in Least Square approach.

Chapter 3

Parameter Processing and Estimation

In this chapter we will discuss how to use our simulator and how to estimate our parameters. The general idea is based on the experimental paths which are collected through dataset of human walking experiments. The simulator is program which can simulate the human walking trajectories with our cost function and can be implanted with our approach methods to estimate weight parameters which can be return to solve the optimal control problem. The approach methods and basic idea of parameters estimation have been introduced in the chapter before. By following such methods we can check whether the simulator works or not. We will also introduce to basic idea of how to achieve the approach paths in such environment.

3.1 Path Generation

The generation of the path is divided into two parts, one is how to implant experimental protocol which we can collect realistic case data from. The other method is to simulate such paths with MATLAB environment and compare them with the experimental ones. If we can simulate these experimental data in programming, to implant our approach methods will be much easier. We can only use the simulated trajectories to verify these approached paths and then be checked with the experimental paths which will save a lot of time and experimental data.

3.1.1 Experimental Path Setup

As mentioned before our based reference paths are all from experiments, so here we describe the experimental setup which used to collect a dataset of human walking trajectories. There are about one thousand paths were recorded which using a 6 cameras motion capture system (SMART system by BTS S.p.A.). Each subject was equipped with 3 light reflective markers, two located on the hips, anterior superior iliac spine (asis), and one located on the sacrum which we can see from Figure 3.1.

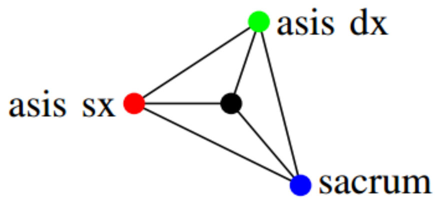


Figure 3.1 Maker positions and barycenter

The experimental protocol was inspired to the one adopted in [5]. More specifically, we restrict the study to the “natural” forward locomotion, excluding goals located behind the starting position and goals requiring side-walk steps. Goals are defined both in position and orientation, and in order to cover at best the accessibility region, the space for the experiments, a 4m×6m rectangle corresponding to the calibrated volume, was sampled with 144 points defined by 12 positions on a 2D grid and 12 orientations each. The final orientation varied from 0 to 2π in intervals of π/6 at each final position which is described in Figure 3.2. The starting position was always the same.

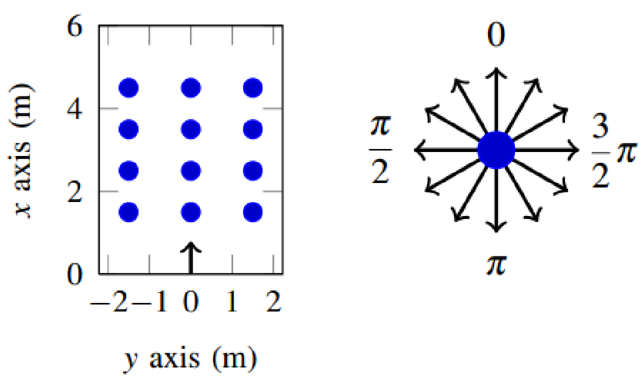


Figure 3.2 Final porch positions (left) and orientations (right)

Locomotion trajectories of seven normal healthy people (both males and females), who volunteered for participation in the experiments, were recorded. Their ages, heights, and weights ranged from 24 to 50 years, from 1.60 to 1.85m, and from 50 to 90kg, respectively. Each subject performed all the 144 trajectories. Subjects walked from the same initial configuration to a randomly selected final configuration. The target consisted of a porch that could be rotated around a fixed position in order to show the desired final orientation which is shown in Figure 3.3.

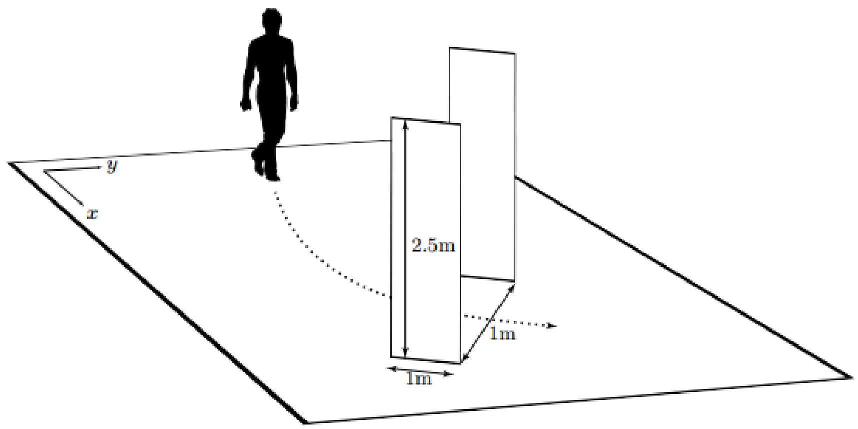


Figure 3.3 Example of the experiment

The subjects were instructed to freely cross over this porch without any spatial constraint relative to the path they might take. Further, they were allowed to choose their natural walking speed to perform the task.

A pre-processing phase on the paths collected by the optoelectronic system was required in order to remove the outliers, fill in the missing data and smooth the curves, and the path of each marker was interpolated with a smoothing spline. Then, considering the triangle that the three markers form, the path of a unique “virtual” marker representing the human walking path was computed as the barycenter of the triangle.

3.1.2 Reference Path Simulation

Since the experimental trajectories cannot be easily used in MATLAB environment so that we introduced a simulated method to generate reference paths which can be compared with the paths which generated with estimated parameters. The simulated trajectories are based on specified parameters and which are also the solution of the optimal control problem. So the basic idea of such simulation is to solve the optimal control problem with specified parameter. If such parameter are given, the solution of the optimal control problem is fixed and the solutions can be restored in a database. To solve this optimal control problem, here we used the optimal tool box Acado for Matlab which developed by David Ariens at al. This tool box can be easily used in Matlab environment. By given certain criterion parameter value, the solution of the optimal control problem will be present as a set of data which contrast to the generated path. The change of the weight parameters can be inputted outside of each optimal problem solution to deal with numerical calculation and returned values, which will be used in the Upper Level and return such specified weight values to the Lower Level to fix the criterion.

As for easily describing such level, we give a simple example of single weight value express, If we consider the optimal control problem as following which is with single weight c, and the boundary conditions are fixed. Recall the optimal control problem which can be presented as following in continuous:

$$
\begin{aligned}
&\min_{v(t),\omega(t)} J = \tfrac{1}{2}\int_0^T c \cdot v^2(t) + \omega^2(t)dt \\
&\text{s.t} \qquad \mathrm{x}(0) = 0, \mathrm{y}(0) = 0 \\
&\qquad\quad \mathrm{x}(\mathrm{T}) = 2.25, \mathrm{y}(\mathrm{T}) = 2.5 \\
&\qquad\quad \theta(0) = \frac{\pi}{2}, \theta(\mathrm{T}) = \frac{\pi}{4}
\end{aligned} \tag{3.1}
$$

$$0 < v < 1.37$$
$$-\pi < \theta < \pi$$

Where $v(t), \omega(t)$ are the two control variables, The differential states are (x, y, θ) , which are the characters of each point on generated path.

From some surveys concluded that human will walk at an average speed around 1.37 meters per second, so in our assumption the input speed value will be in a range from 0 to 1.37 which considered the starting speed and final boundary. The rotation angle should be also limited that in realistic that human will only turn from left to right or inverse, so that the limited rotation angle would be from -180° to 180°. These would be the boundary of the input values in the optimal problem solving.

It is obliviously that if we have given a value to the certain weight factor, this problem becomes a known criterion optimal control problem and the solution can be easily found. That is the normal method which have done by pioneers of goal-oriented locomotion researches. So that our conditions now are, first optimal solution is known which is based on collected experimental data, and second the initial and final boundaries are given for such experiment. So to complete the objective function we still need to determine the best value of weight factor c. With different c we can suspect that each time we can solve the optimal control problem by giving such fix weight factor. If with certain c, a generated trajectory is completely approached to the experimental one, we have found the correct objective function for such conditions. Here we assume c equals to 3.7 and the generated trajectory is as the one in Figure 3.4. According to such figure we can easily simulate the human locomotion and check the walking speed and rotation speed which are based on goal-directed and discuss why human being will choose such path and walking manner.

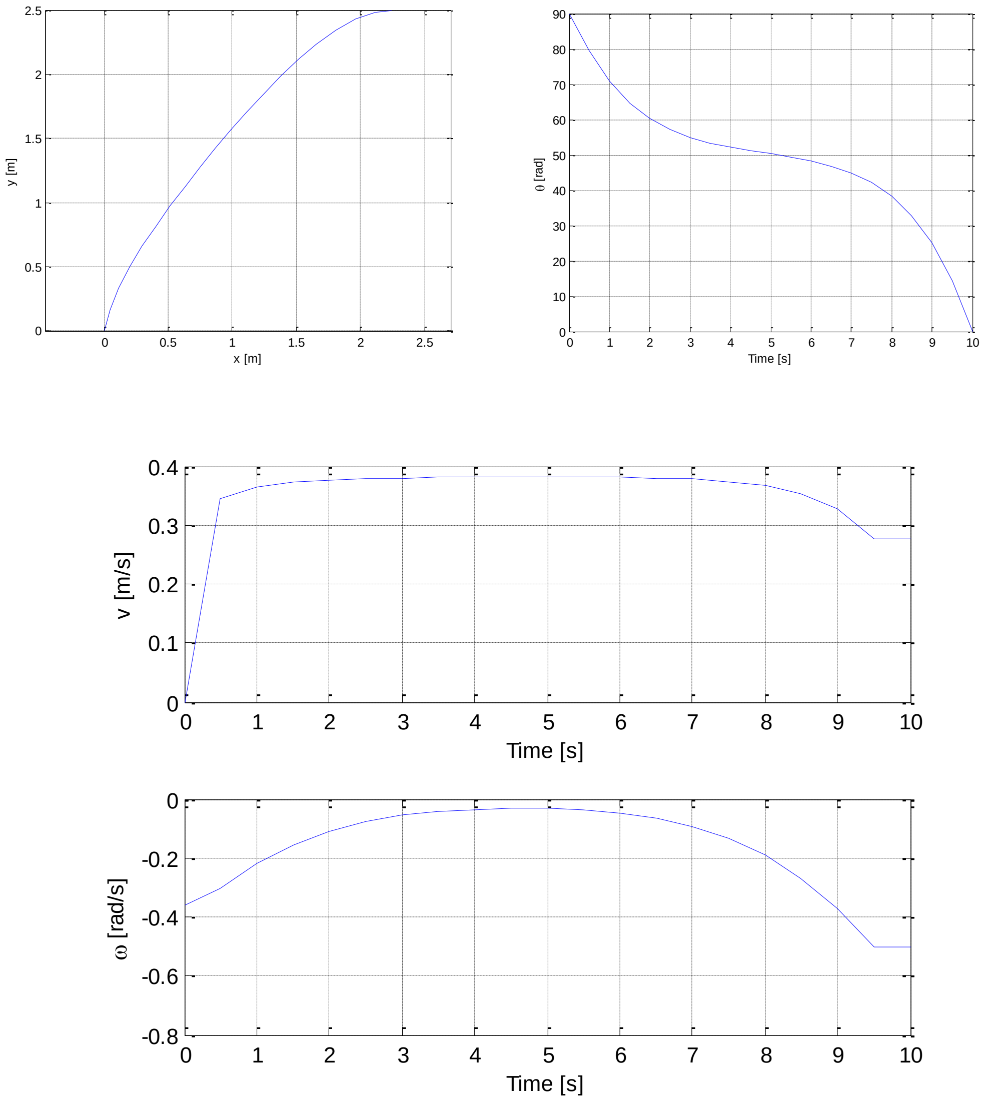


Figure 3.4. Generated data with single parameter

However we should also discuss the model is correct or not. To demonstrate such assumption we can change the orientations and goals to identify such model is suitable for the real case, also the dual parameters model. So here we changed some boundary conditions in the optimal control problem to check our simulation results which can be seen from figure 3.5 and figure 3.6. These are different orientation paths and the generated path which compared with the experimental data based on dual parameters model. By seen from these generated reference paths and compared with the experimental dataset, it can be easily demonstrated this reference model can be used to implant approach method and define the energy cost function of human walking which based on the two controllable input parameter, the walking speed and the rotation speed.

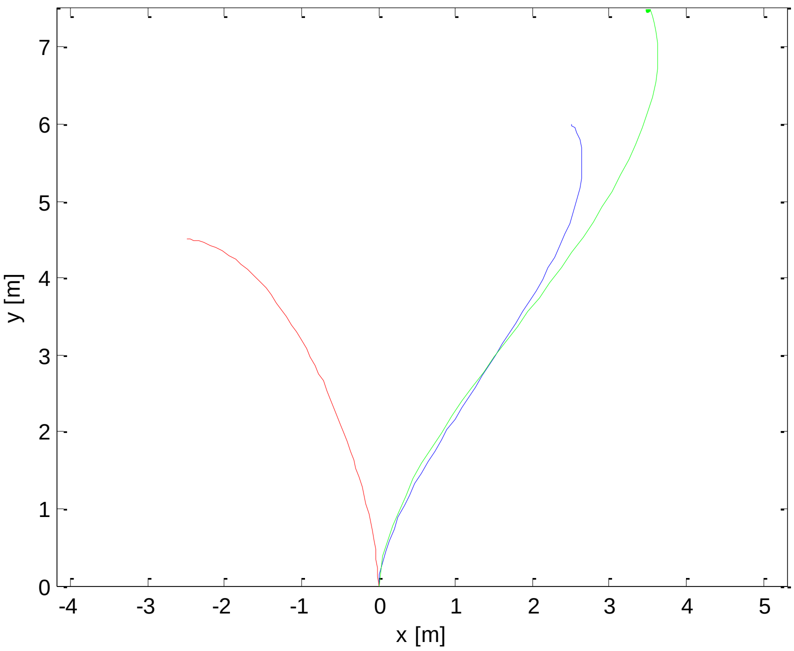


Figure 3.5 Simulation of Different goals

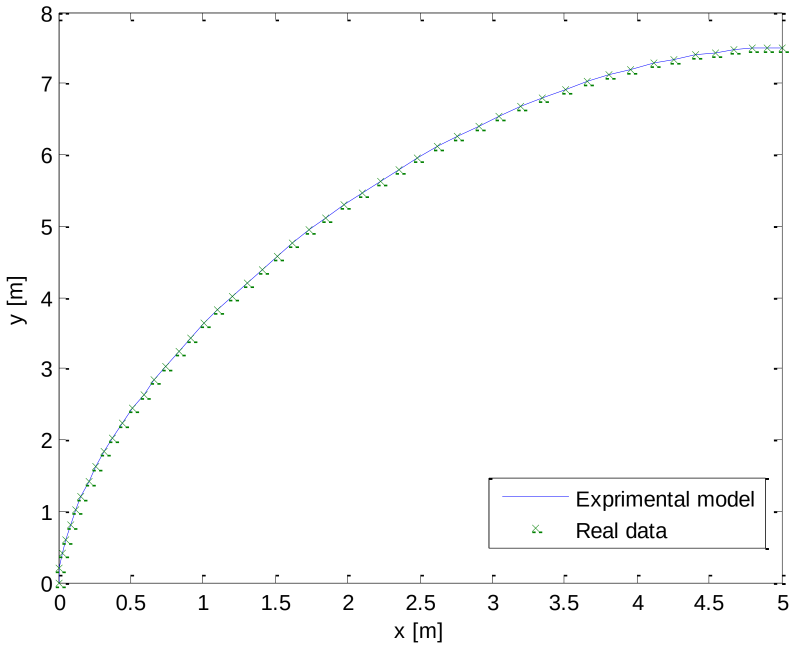


Figure 3.6 Simulation path

So the next step is estimate such weight value to build the real case of cost function and return such values to verify the approach methods.

3.2 Parameter Estimation

Although we have obtain the reference paths and demonstrated the model is near the real case, even if we have known such specified weight parameter, however, in real case the parameter is still unknown. According to the method we described in Chapter 2, to generate an approached trajectory should be based on the parameter estimation and then return such parameter into the optimal control problem to gain the solution. So here we should check the approach methods which are suitable or not. To check and demonstrate these method we should first generate numeral paths with random weight value and compare such paths with the reference one and estimate the parameter which is fitted the minimum criterion.

3.2.1 Single Weight Parameter Estimation

3.2.1.1 Single Weight Parameter with Least Square Approach

To test the first approach method Least Square we should find the relationship between the minimum criterion and the estimate value. Recall the minimum criterion and re-define it as following:

$$\min_{c} J^* (c_i) = \left\| \begin{matrix} x^* - x \\ y^* - y \\ \theta^* - \theta \end{matrix} \right\|_2 = \frac{1}{2} \sum_{i=0}^{M} \sqrt{(x_i^* - x_i)^2 + (y_i^* - y_i)^2 + (\theta_i^* - \theta_i)^2} \qquad (3.2)$$

The ideally result would be only one minimum value of J^* estimated and since the estimated c_i is unknown at beginning so that we should try a range of random value of c_i to find which one is fitted the minimum value of J^*.

The basic approach tool is Linear Least Squares function in MATLAB, by given the reference path, we can build a path corelate to the weigthing parameter c_i and by comparing each point betwenn the reference one and the generated one to calculate the difference in natrual coordinates. As we have already built the simulate to generate reference path, the same tool can be used to generate the appoached paths too. Based on the method we metioned in last chapter, the minimun point of the indentify criterion becomes the nearest generated path with specified c_i. So our first test is to recurve all possible c_i value to generate a serial of paths and compare these paths with the reference one and find the minimum value of J^* then return the c_i which one cause the minimum value.

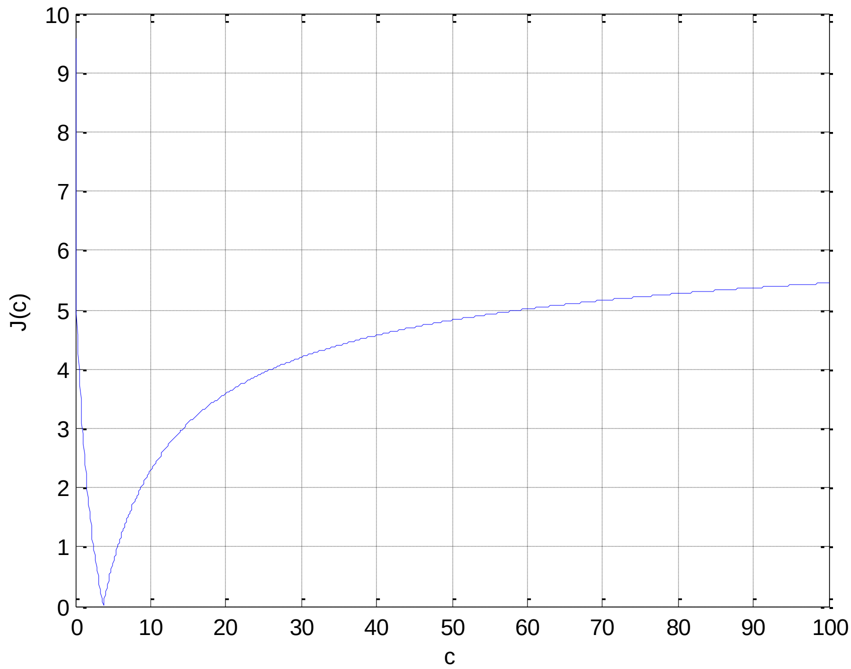


Figure 3.7 Relationship between weight value and criterion

Follow such test method we fixed the range as 0 to 100 for c_i, and we can see the simulated relationship between J^* and c_i in figure 3.7. Apparently the minimum point of J^* can be easily found, in this example is $\mathrm{c} = 3.7$. After this, we should return the value of c_i back to the optimal control problem to solve it and find the estimated path. Compared to the experimental path we generated before, the new path which based on the estimated parameter value c_i fits the experimental path very well. When we return such estimated parameter to the cost function and re-generate the approached path, by comparing to the reference one we can easily see that this kind of approach However the experimental path is based on a very small value of c, where the approximated range of c_i is known in such case, but in fact we do neither know the range and the value of the parameter. So our new problem is if c is large or in a very large range, what kind of relationship will be appeared in such approach method and how can solve this problem?

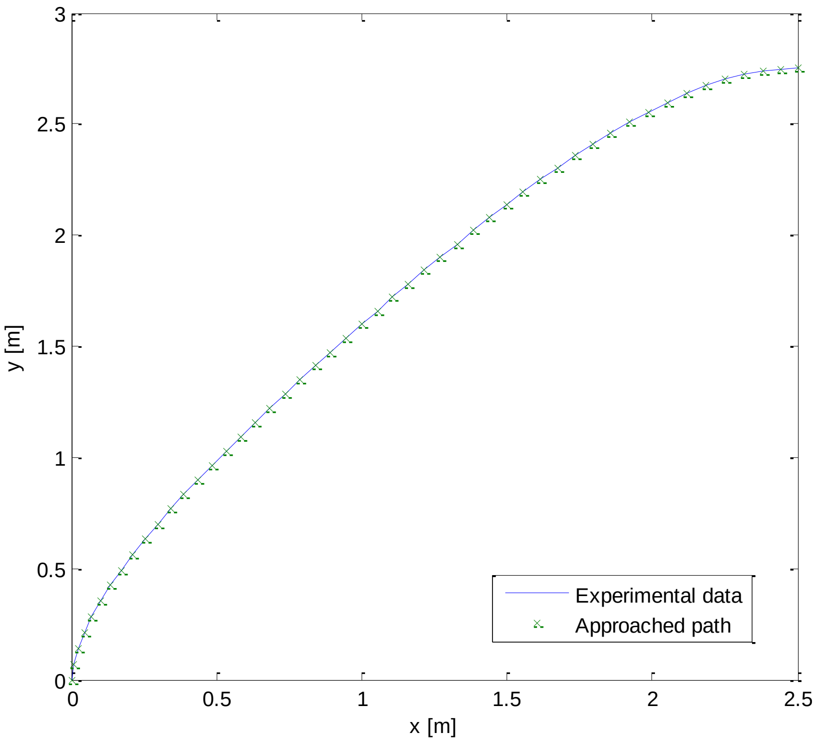


Figure 3.8 Simulation result of approached path

So we changed the experimental path which based on a very large c, the relationship between J^* and c_i becomes as in figure 3.9.

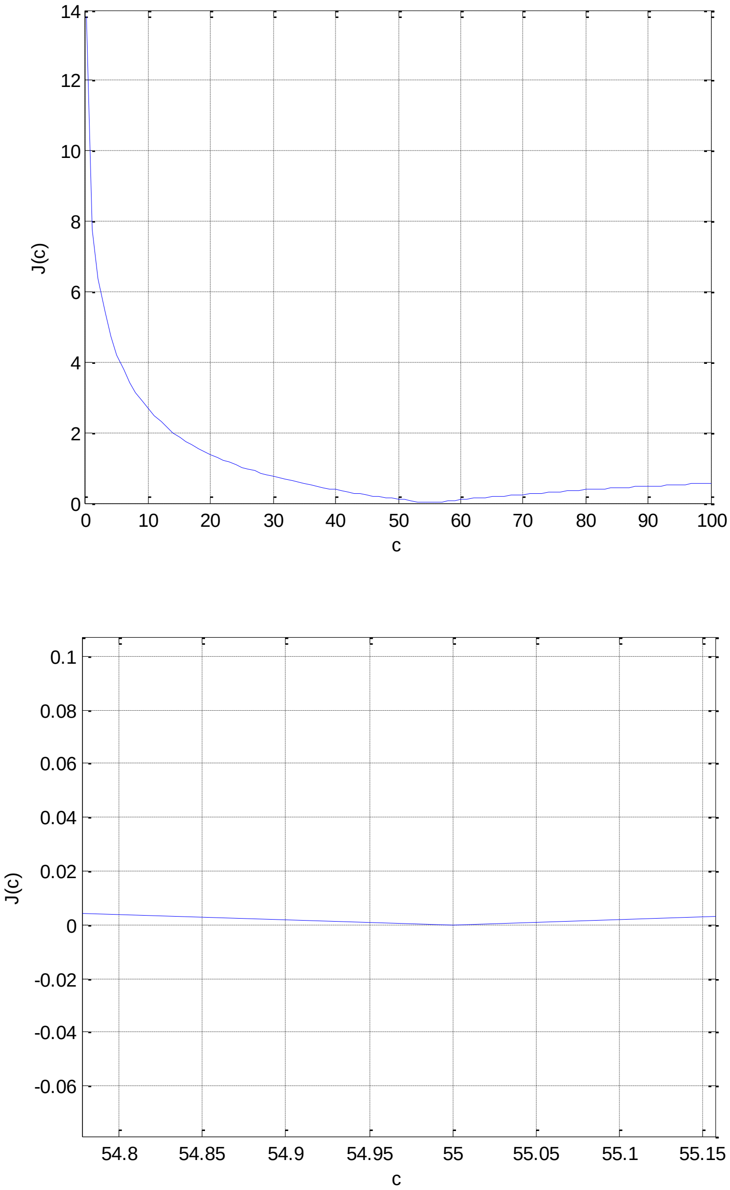


Figure 3.9 Relationship between weight value and criterion when c_i=55

Apparently, this Least Squares algorithm works, we can find the minimum point of J^*, but not as well when c is small.

After we finished the finding minimum part we should estimate such weight value out and return it to the optimal control solution part. So we store all the possible value of J^* in a column, by finding the minimum element of the column return its row value to the column of c_i. As we followed ascending sequence of c_i where the returned row value points to a c_i value that is the one we need to estimate out. By giving such value to the weight parameter in the energy cost function in optimal control problem and solving it, the solution which we described as $\varphi_c(k) = (x_c(k), y_c(k), \theta_c(k))$ and this solution is our approached path.

However, after several generation of such approached path and analysis of the relationship between generated path and weight value, the curve described the criterion, we found that there is always a flat area near the minimum point so that the simulation result cannot tell which one is minimum and the estimated c is not correct as before and this difference causes the approached path has a little distance with the experimental path which we can see from figure 3.10.

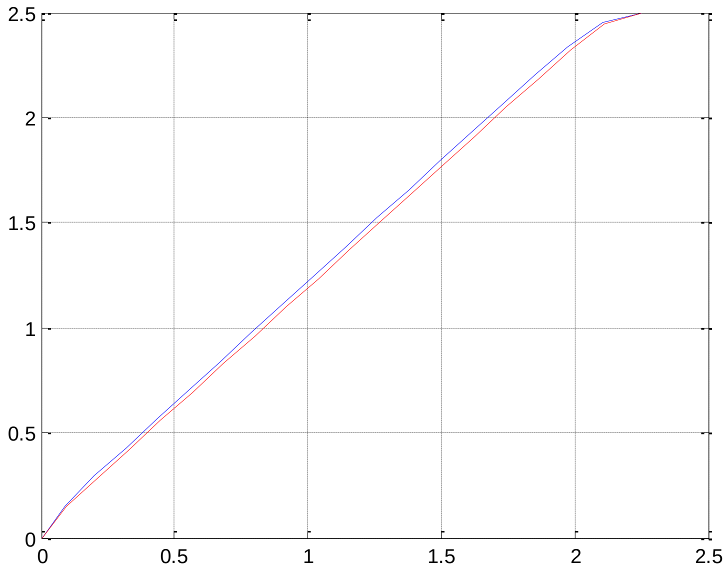


Figure 3.10 Generated path with large weight value

To avoid such difference in this kind approach method there are two attempts, first one is only use this method in small value range of c, this kind of attempt is a little tricky which will lose a large serial numbers of c but works perfect. The second attempt is to change the algorithm to Fréchet Approach which we will discuss in the following.

3.2.1.2 Single Weight Parameter with Fréchet Approach

For the Least Squares method does not work well when c is too large, we try another approach method to estimate c as Fréchet Distance method. This method only calculates the distance between two paths and find the closet estimated path to the reference one.

The basic idea of generation of the path is the same as Least Square method, by giving a range value of c and simulate each weighted cost function to gain a serial of paths and find the closest path to the reference one. It is defined as the minimum cord-length sufficient to join a point traveling forward along the reference path P and one traveling forward along generated path Q, although the rate of travel for either point may not necessarily be uniform.

The Fréchet distance is not in general computable for any given continuous P and Q. However, the discrete Fréchet Distance, also called the coupling measure, where we define cm as a metric that acts on the endpoints of curves represented as polygonal chains. The magnitude of the coupling measure is bounded by Fréchet Distance plus the length of the longest segment in either P or Q, and approaches Fréchet Distance in the limit of sampling P and Q

Calculates the discrete Fréchet distance between curves P and Q in program is expressed as following,

$$[cm, csq] = \mathrm{DiscreteFréchetDist}(P, Q) \tag{3.3}$$

where P and Q are two sets of points that define polygonal curves with rows of vertices or data points and columns of dimensionality. The points along the curves are taken to be in the order as they appear in the experimental data and the generated data which mentioned before.

Returned in cm is the discrete Fréchet distance, also known as the coupling measure, which is zero when P equals Q and grows positively as the curves become more dissimilar. The secondary output csq is the coupling sequence, which is the sequence of steps along each curve that must be followed to achieve the minimum coupling distance cm. The output is returned in the form of a matrix with column 1 being the index of each point in P and column 2 being the index of each point in Q, although the coupling sequence is not unique in general.

By using such returned discrete Fréchet distance we easily find the relationship between the weight parameter and the minimum value of such distance. When we achieved the minimum point which means the generated path is the closet one to the reference one, we can estimate the parameter value contrast to that minimum point. So we stored all the possible value of weight parameter in $c(i)$ and all the returned discrete Fréchet distance in $\mathrm{cm}(i)$, where $c(i)$ and $\mathrm{cm}(i)$ have a linear relationship between each other. If there is a minimum value in $\mathrm{cm}(i)$ which we noted as $\mathrm{cm}(K)$ and the corresponding $c(K)$ will be the weight value which caused such minimum value of $\mathrm{cm}(K)$. When we return such $c(K)$ to the optimal control problem and obtain the solution, we gained an approached path $\varphi_c(k) = (x_c(k), y_c(k), \theta_c(k))$ in such approached method.

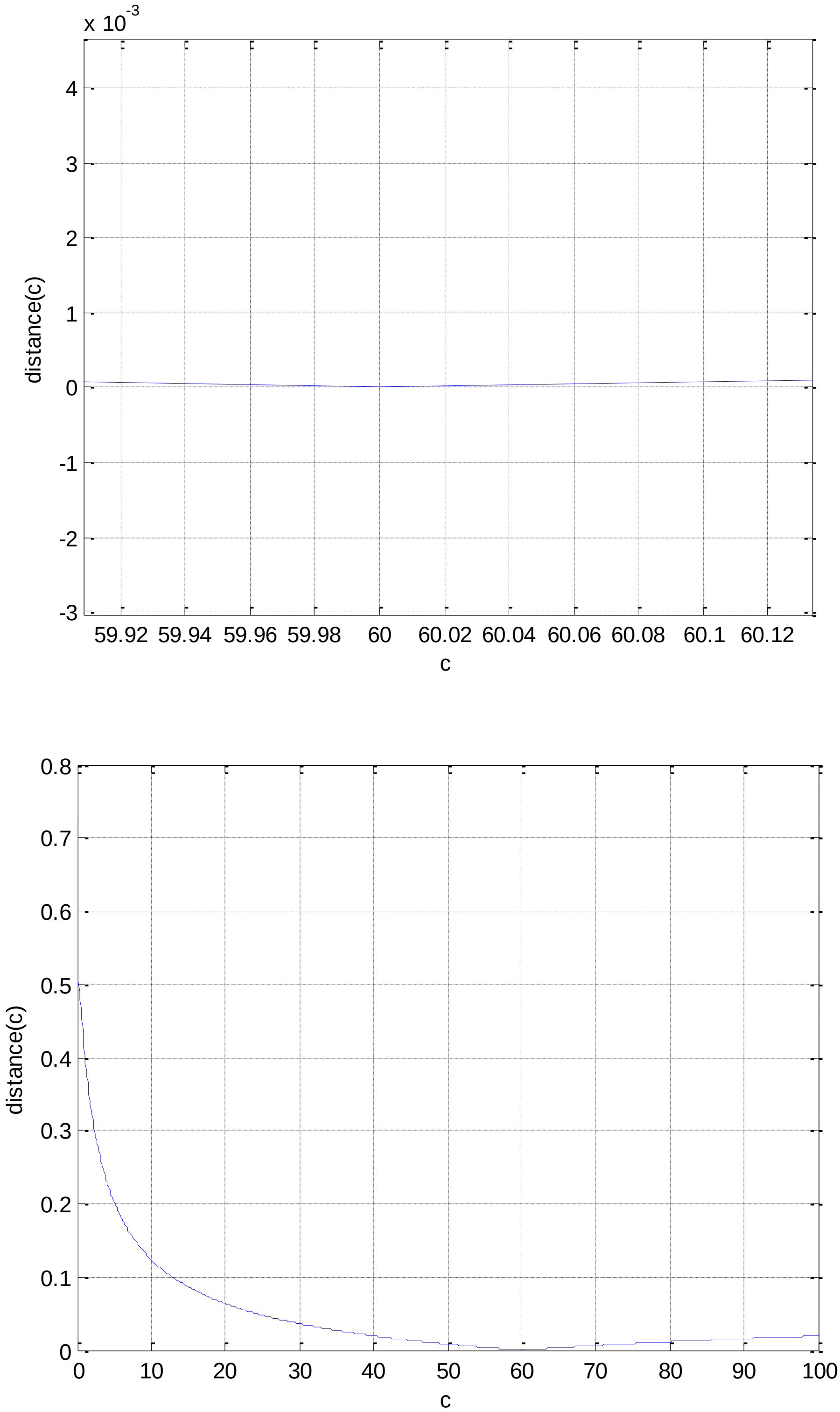


Figure 3.11 Large weight value relationship with criterion

In figure 3.11 we can see that at the same condition where the weight parameter is very large, there still have a gathered points near the minimum point. But this approach algorithm can be much more fitted to the reference paths even the path is strange which we can check from the figure 3.12. In this figure, the simulated reference path is generated by random value of weight value and we do not know such value. So we toured all the possible weight value from 0 to 100 to find a possible minimum point of criterion cm and apparently the value is found and perfect approached even though the generated path is strange which was caused by large weight parameter.

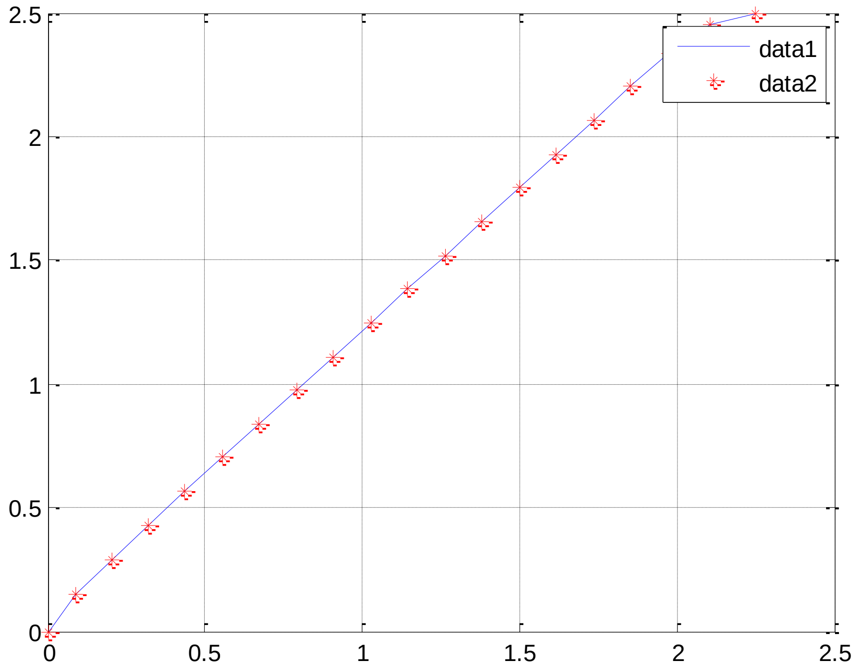


Figure 3.12 Approached path with large weight

Although the reference model generates strange paths, the approach methods are demonstrated work in such conditions. So the next improvement will be the cost function itself. Since at the beginning, we set the cost function with only one parameter which only effect the forwarding speed, we here try to effect the angular speed also. This cost function model we call it dual weight parameters model.

3.2.2 Dual Weight Parameters Estimation

Since the single weight parameter model will cause a 'straight' curve shape which is not good compared to the realistic case when c is large. So how to modify and remove it and present more preciously become our new goal. The basic idea is to modify the cost function. However rebuild a new cost function which is also based on velocity and rotation through energy minimized theory can be much similar to the model what we have done. So that we are trying to introduce two weight parameters which gives the rotation speed a weight value also so that the weight of forward speed and rotation speed become balanced. We changed our optimal control problem as following,

$$\min_{v(t),\omega(t)} J = \frac{1}{2}\int_0^T c_1 \cdot v^2(t) + c_2 \cdot \omega^2(t)dt \tag{3.4}$$

By here the weight parameter c_1 is as the same as the single parameter model which only effect the forward speed and c_2 is the new introduced parameter.

The figure 3.14 here is generated based on same goal and orientation in experiments. The much more 'straight' line is the single parameter, and the much more curve line is the double parameters. We can easily see from that double parameter will avoid the strange linear part of the generated path where double parameter will be more like the realistic one.

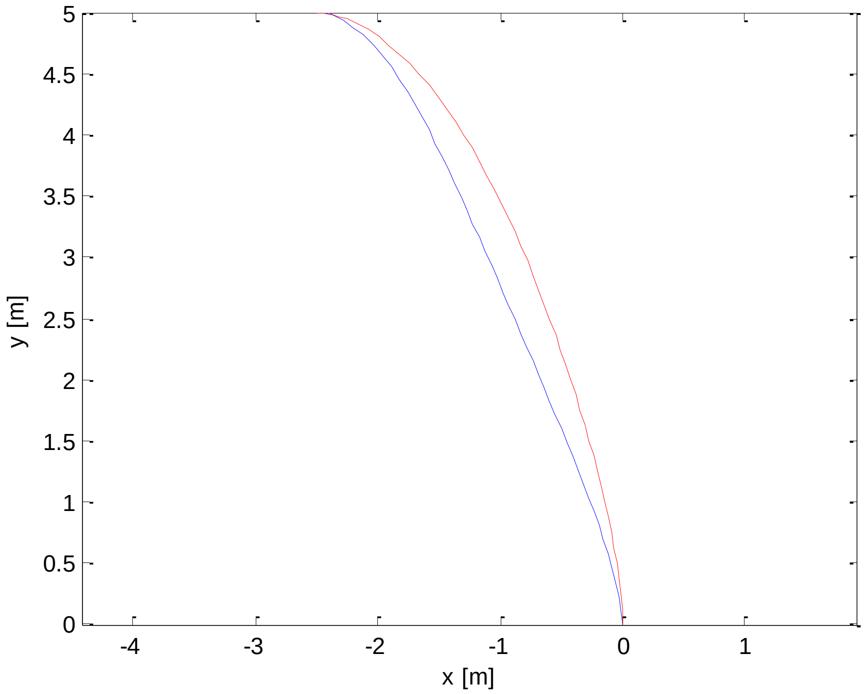


Figure 3.13 Compare between single and dual

To implant such cost function with our approach methods, the basic idea is to fix one parameter's value and go through all the possible value of the other parameter then change the first parameter's value to re-tour all the possible value of the other parameter which is like a recursive method.

Although this kind of cost function will cause a long time to estimate two parameters which mostly cost a lot of times that is much longer than the previously one but the results are better than single parameter which seems more realistic. So that we should try again these two approach methods in such situation to check this assumption works or not. For the first step is still the single parameter estimation which we have introduced before and the second step of the Least Square approach method is the same as the Fréchet distance approach, so we here only discuss one of them for example which is Fréchet distance approach.

In dual parameters estimation, we re-define the returned discrete Fréchet distances as in matrix $\mathrm{cm}(i,j)$, where corresponding to $c_1(i)$ and $c_2(j)$ which are the two weigh parameter. If there is a minimum value in $\mathrm{cm}(i,j)$ which we noted as $\mathrm{cm}(r,s)$ and the corresponding $c_1(r)$ and $c_2(s)$ will be the weight values which caused such minimum value of $\mathrm{cm}(r,s)$. When we return such $c_1(r)$ and $c_2(s)$ to the optimal control problem and obtain the solution, we gained an approached path $\varphi_{c_i}(k) = (x_{c_i}(k), y_{c_i}(k), \theta_{c_i}(k))$ in such approached method.

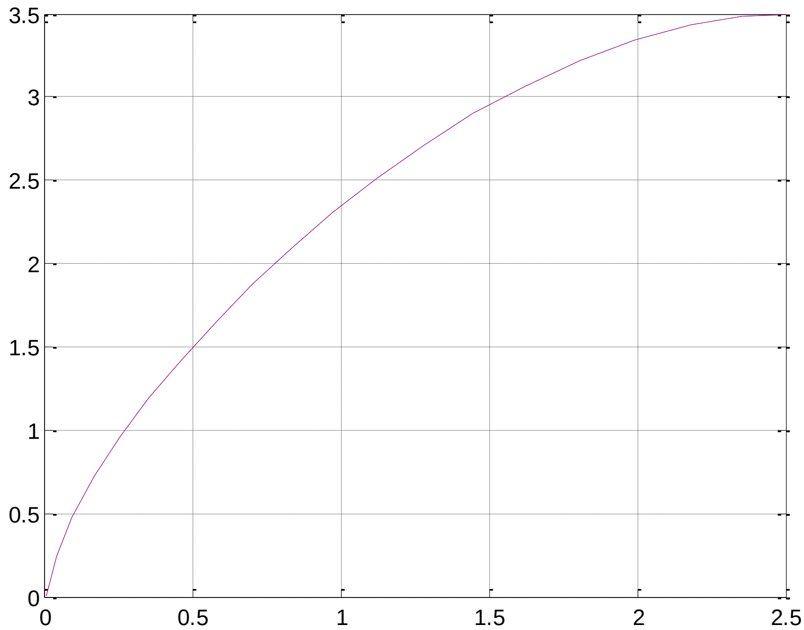


Figure 3.14 Dual parameters with FD

In figure 3.14 which is based on dual parameters model and Fréchet distance approach, we can find the approached path which is in dashed line is extremely close to the reference one. This dual parameters model and the method of Fréchet distance approach works very well.

We can also check the relationship between the parameters and the identification criterion in figure 3.15 where we follow the trend of the values and find each minimum point of the criterion in such figures. When the two minimum points in each figure join together where the unique minimum point for two relationship has found and these two values of parameters are the ones which should be estimated and returned to the optimal control problem.

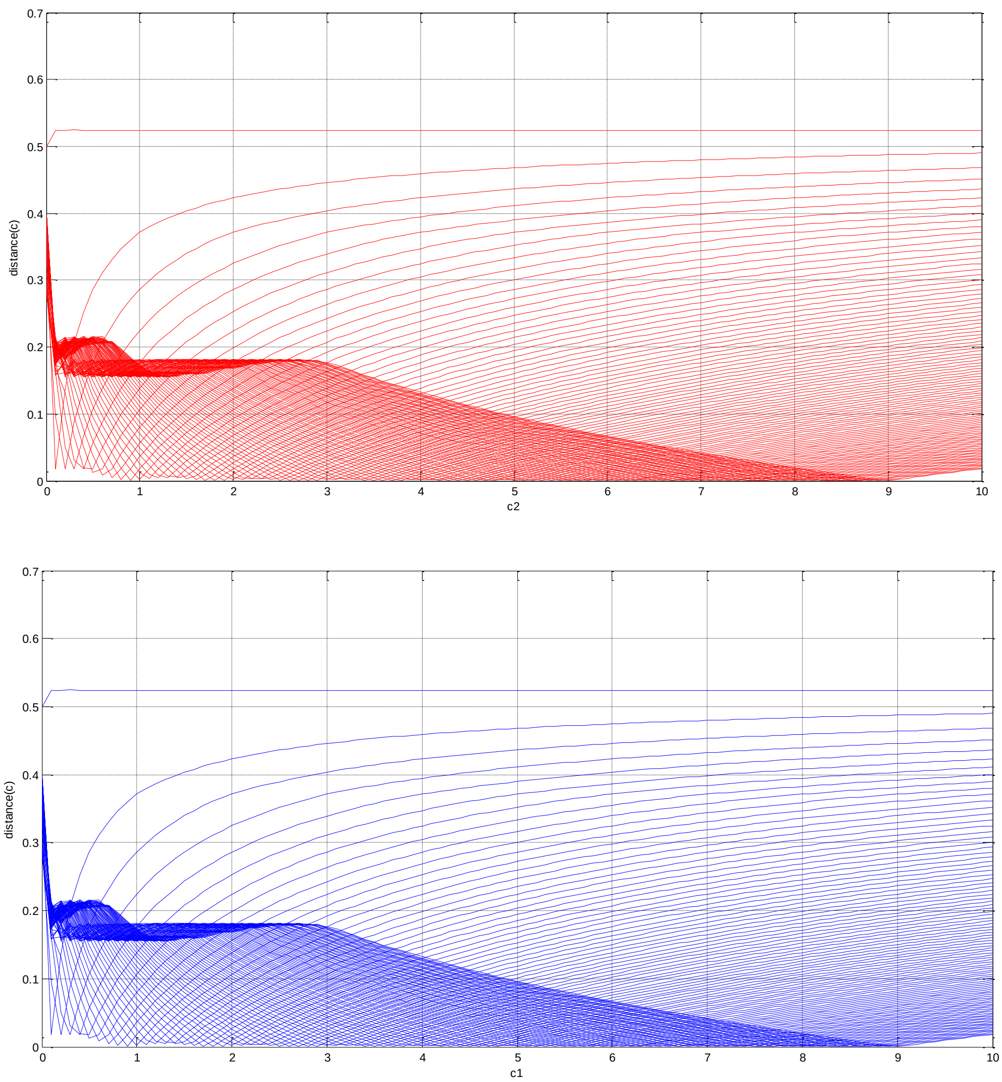


Figure 3.15 Full relationship between weight parameters and criterion

After the estimation of all the parameters, the next step is to control humanoid robots. For we have solve the optimal control problem with fixed parameters and fixed orientations and goals, the two control valuable walking speed and angular speed can be obtain with solving such problem. If we transfer these data to the control interface of the humanoid robots, the robot will follow the path as expected, further more by giving such approach method, the robot can calculate its own path through such steps. That would be our expect result.

3.3 V-REP Environment

V-REP's path planning module allows handling path planning tasks in 3D-space, and in 2D-space for dummy models. In V-REP environment, a path has a position and orientation component (or channel), and can additionally also have a component that describes a velocity profile. A path is defined by control points that describe the path as a succession of linked segments. However the human model in V-REP is not dynamic and path planning is just based on the algorithm what they have. But the simulation can be implanted based on imported data files, which is called remote API. But the path following model will ignore the effect of obstacles when only bonded with imported data. For such reasons we here only use the V-REP software to demonstrate our final approached paths and compare them to the algorithm in V-REP and experimental ones.

As we have obtained the approached path $\varphi_{c_i}(k) = (x_{c_i}(k), y_{c_i}(k), \theta_{c_i}(k))$ and the control values $u(t) =(v(t),\omega(t))$, we store such path and control parameters in a data file path.csv. Such file can be implant into V-rep software and automatically related to the path which the dummy will follow by. The control values will control the dummy at each point on the path and simulate how human being walking. . The goal directed model which is the original model in V-rep, but the algorithm of how to generate the path is not good enough.

The algorithm in V-rep is based on the minimum distance between two points on the path and the path is divided into several parts. By calculating each part of the path points, it gains the minimum curve which re-construct the following path. This kind of approach is simple but efficiency which is similar to some goal-directed approach method others done before. So we choose this algorithm as a contrast example to the ones which we generated.

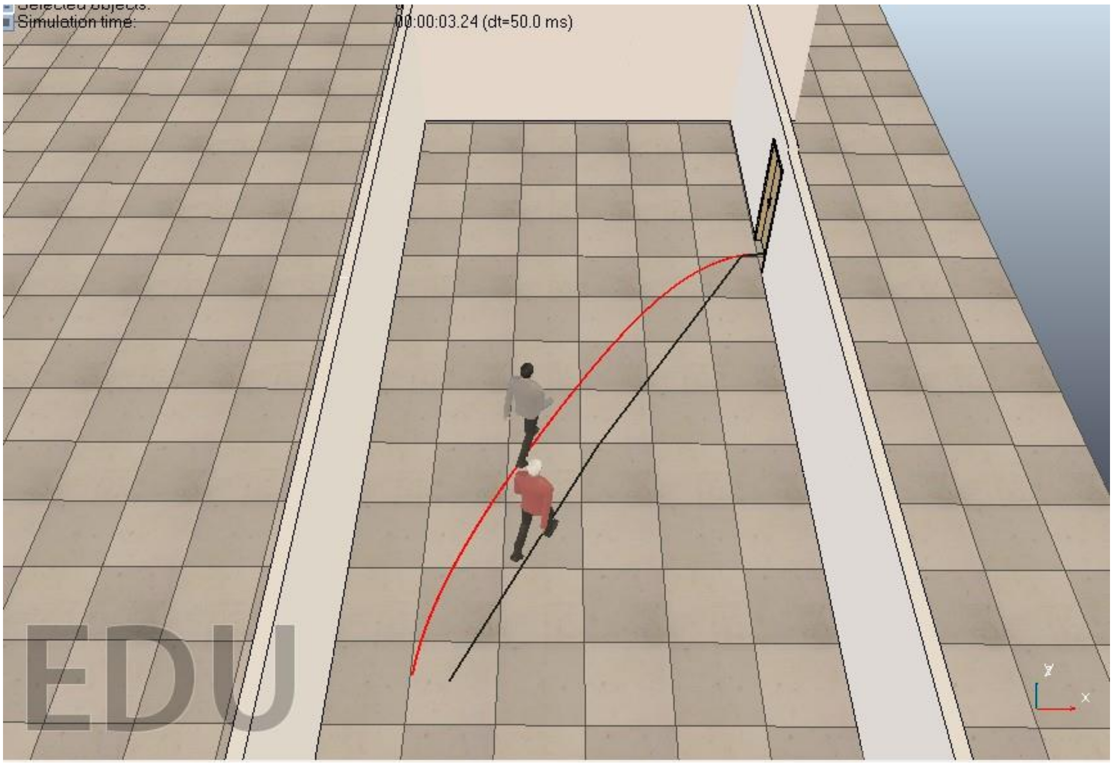


Figure 3.16 V-rep presentation of approached path

In figure 3.16, the path following dummy who follows the approached path generated from MATLAB simulated data. We can also use this software to compare the approached paths and the experimental ones.

Chapter 4

Results and Analysis

In this chapter we will analysis the simulation results we have obtain through these two approach methods and verify them with the experimental data. For the simulation results are based on the methodology and simulation method mentioned before, we here only compare the generated paths with reference ones and experimental ones.

4.1 Generation of Reference Paths

To identify the cost function of our inverse optimal control problem, we here settle down four goal points and four orientations to test these two cost function model. The four points and identity are given in table 4.1. These four points are randomly selected and we do not know the exact weight values which can help us to solve the optimal control problem. When we finished the experimental part we only collect the discrete time data points from which we can simply draw an experimental path and the control values also. After this we now can simulate such paths with random weight values and compare these paths with the experimental ones and then set them to be the reference paths. Then we can implant our approach methods to approach such reference paths and verify these methods works or not.

Table 4.1 Experimental Points

POINTS	A	B	C	D
X	-3	2	1	-5.25
Y	4.5	2	-3	-1
THETA	$\pi/2$	$7\pi/4$	$2\pi/3$	$7\pi/6$
ORIGIN	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

They can also be presented in figure 4.1 as example.

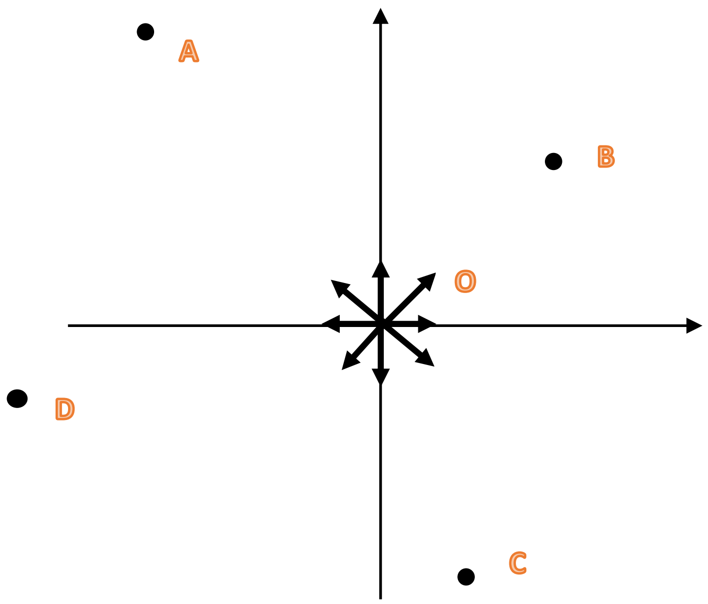
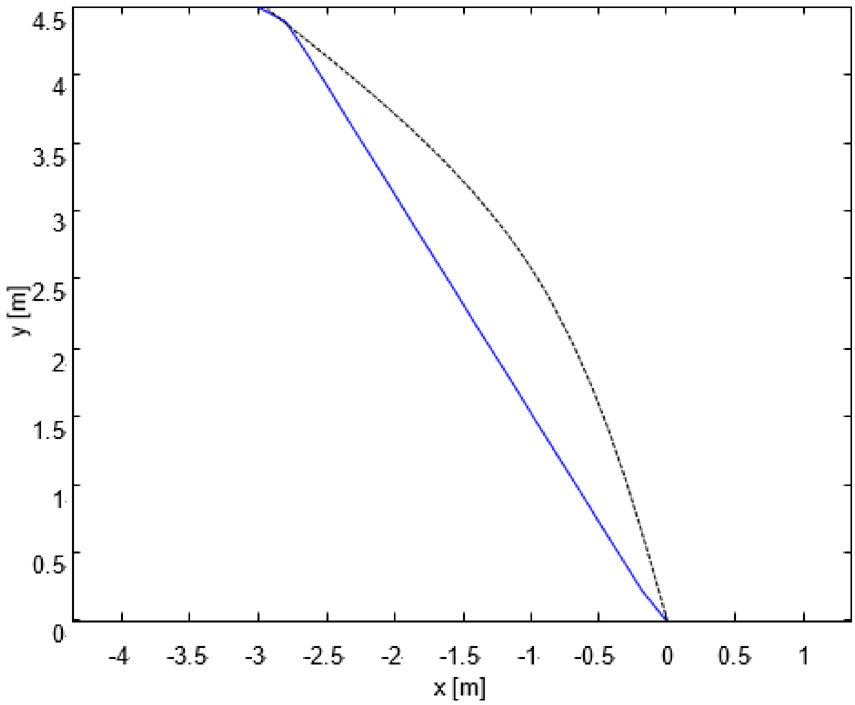
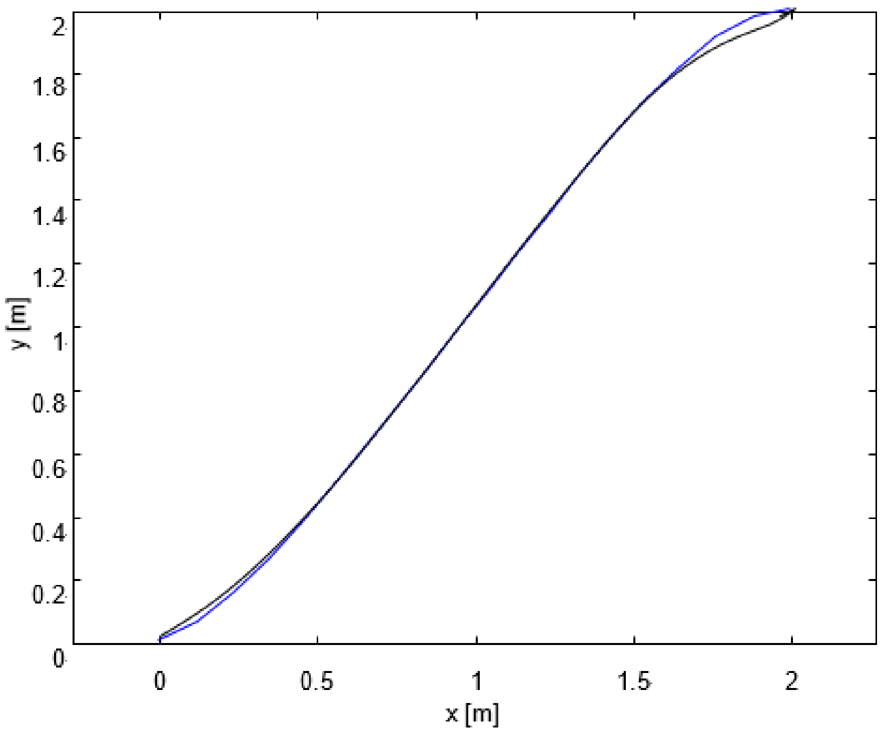


Figure 4.1 Positions of experimental points

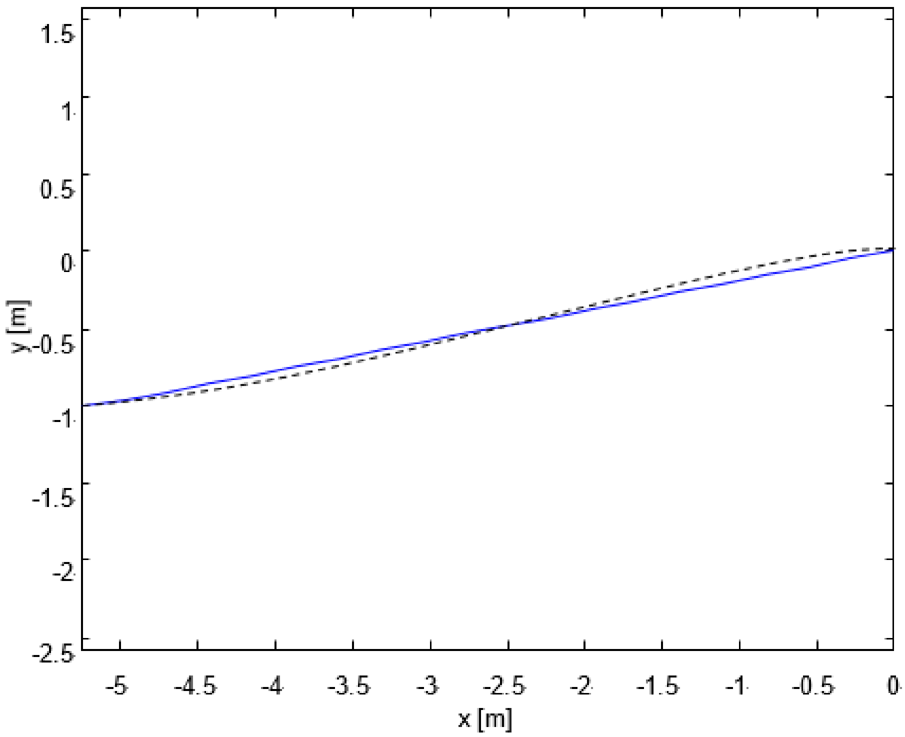
As these points are fixed, we can collect these data from the experiments and the simulation of the reference can be done and collected in MATLAB environment. Then we can compare these with each other to verify our cost function and our simulation methods. So we collected the experimental data of these points and the results are shown in figure 4.2 and figure 4.3 where the dashed lines are the experimental data. The generated paths is not fixed with weight parameters so that there are some difference between the simulation results and the experimental data. Furthermore, we here only present the time method results which has some orientation problem, for the time method based on the time variance not the natural coordinates, as the time differential will affect the control valuable forward speed and rotation speed which two we defined can be negative. That would be also simulated and compare with the experimental data. For choosing other reference points and orientations, the analysis of space method will be discussed in next part.



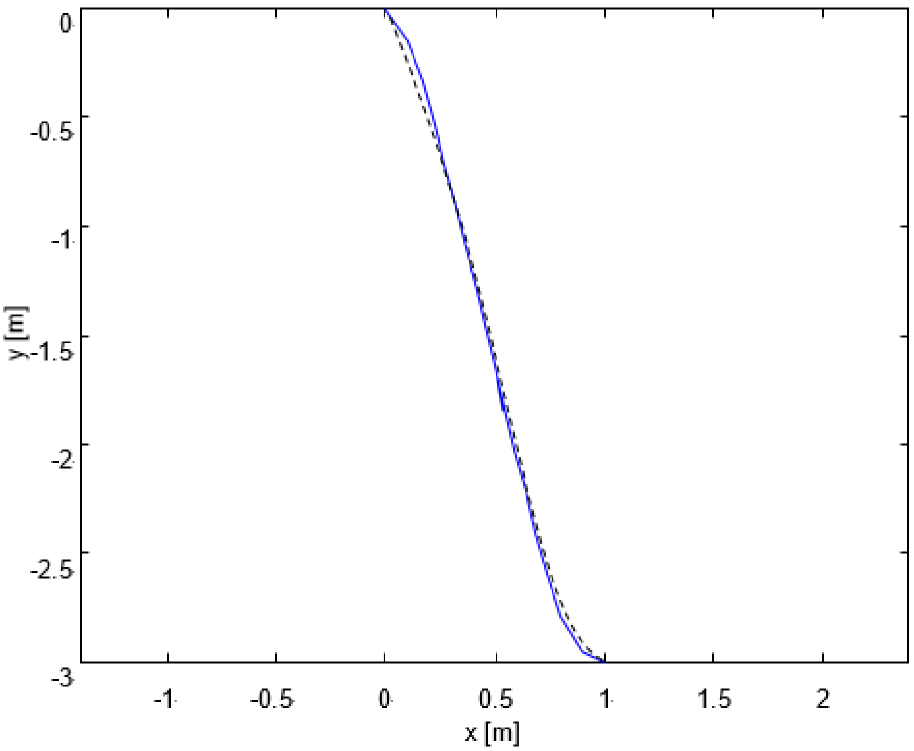
Point A



Point B



Point D



Point C

Figure 4.2 Single parameter model compared with experimental data

In figure 4.3 are the results of reference paths with single parameter model. Apparently the results are not smooth enough for human being would not walk straightly like this. However, in real forward speed which along the path should be only positive. That is the reason why in these simulated paths human will turn first then start to walk but not as turning as walking. We can check the forward speed and angular speed in figure 4.4 compared with the experimental data in discrete time index and there is problem we can find from this.

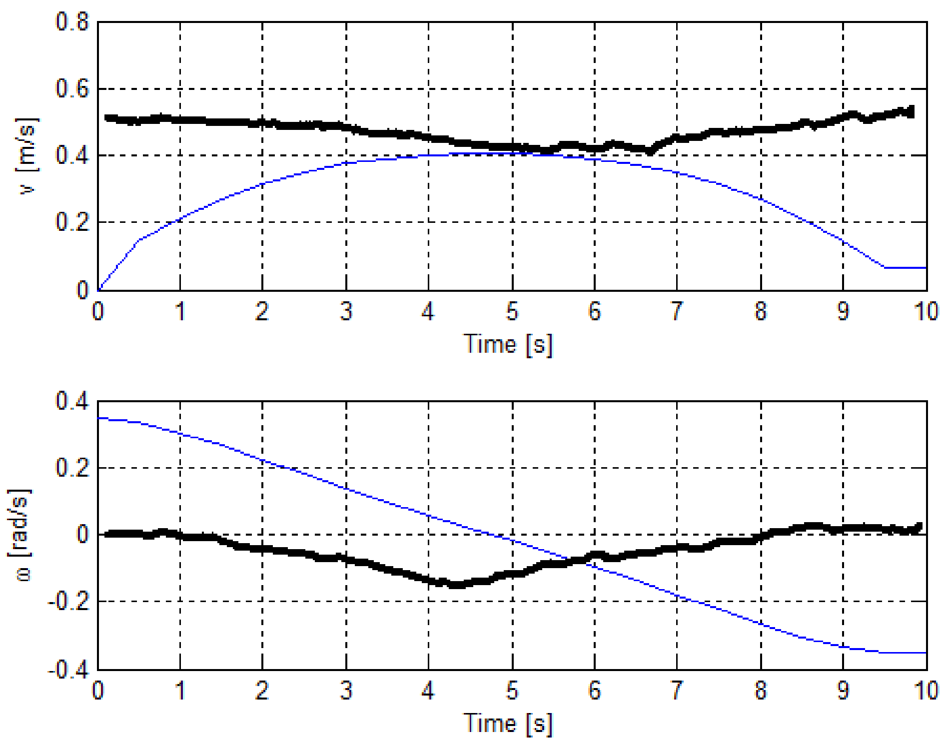
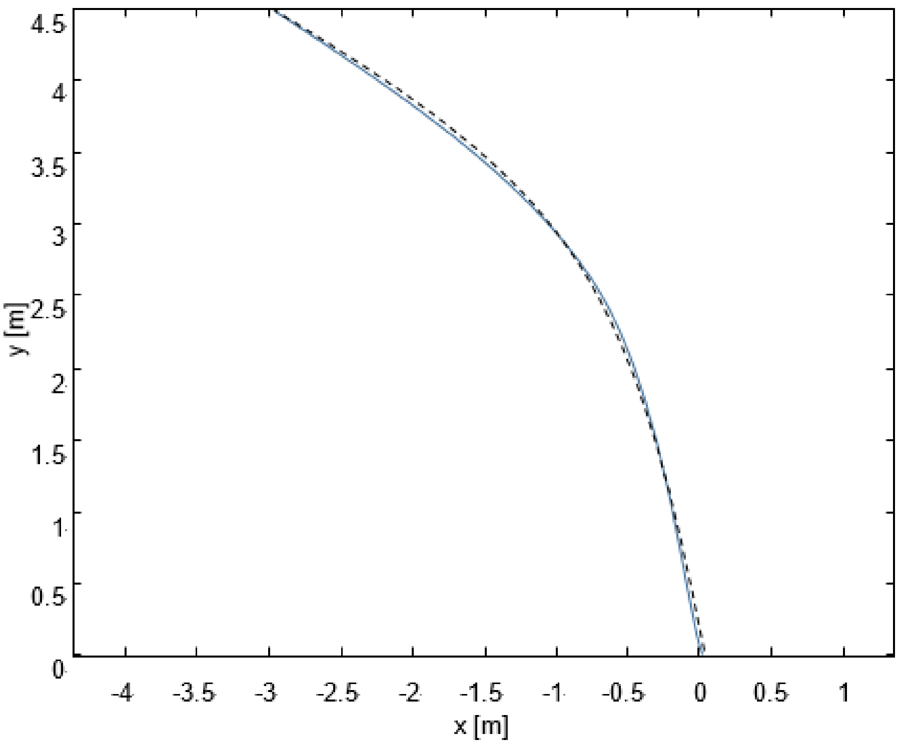


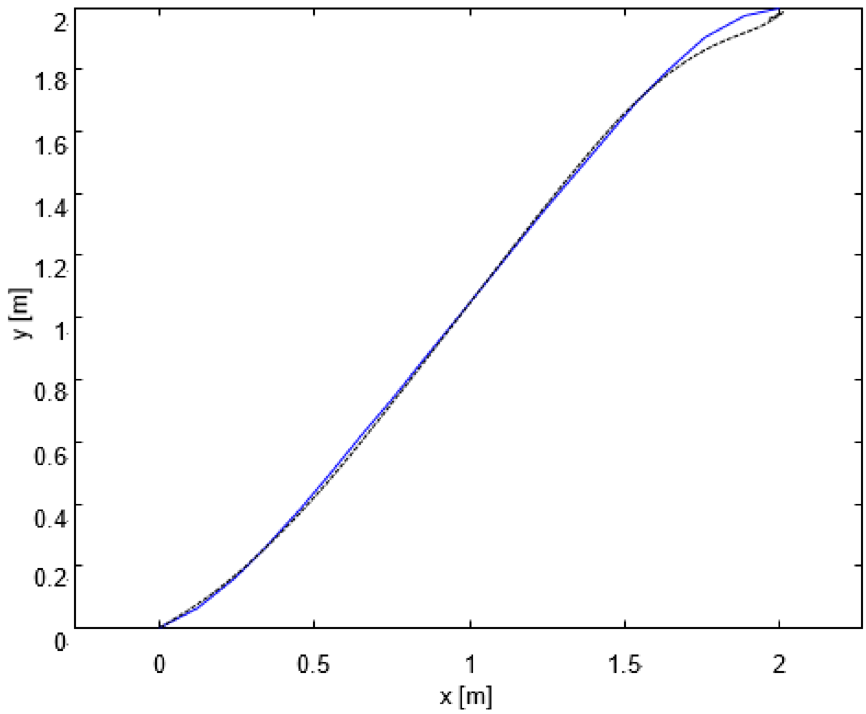
Figure 4.3 Generated control value compared with experimental data

In Figure 4.5 the bold lines are from the collect experimental data and the thin line is our simulation results in single parameter model. Apparently the simulation of the control valuables are so different from the realistic ones although the approached path is so close to the real one. There are two reasons cause this problem in single parameter model, one is the velocity which we set in the optimal control problem has a boundary from 0 to 1.36 which means we allow the simulated human can walk at a very small velocity but not always constant. And we ignored the effect of the starting acceleration of human at the orientation which we only set the starting velocity as zero. However in this thesis we only consider the approached methods as well but not the control values in different path points. The other reason is we only concerned about the discrete time variance not the natural space coordinates of how human walks following such path.

Compared with the generated path in figure 4.4 which are the results of reference paths with dual parameter model, we can see clearly that the two parameters model is much smoother and more like human being's walking paths for these generated paths will consider the effect of the angular speed as more important especially at the starting point. If the orientation is different the discrete time model will only turn at origin then walk but the space model will as turning as walking. We can also check the difference between the discrete time and space model in figure 4.6.



Point A



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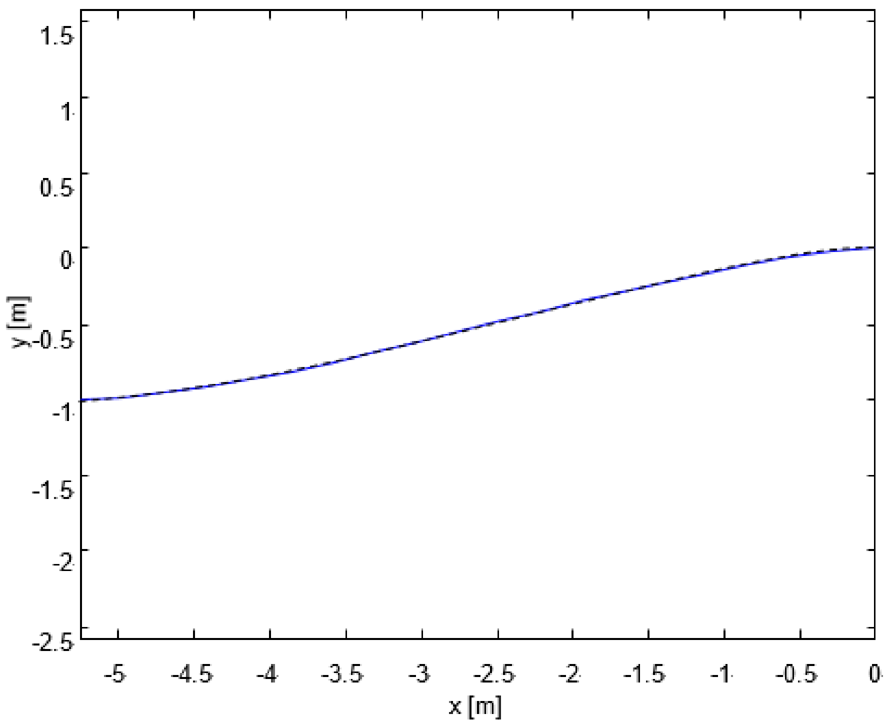
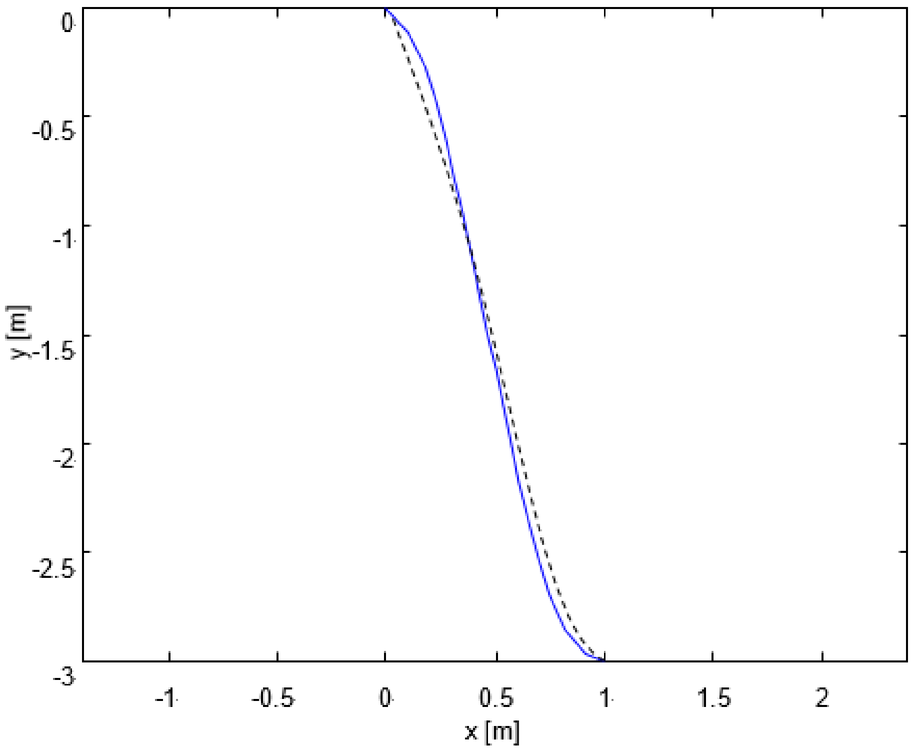


Point D



Point C

Figure 4.4 Dual parameters model compared with experimental data

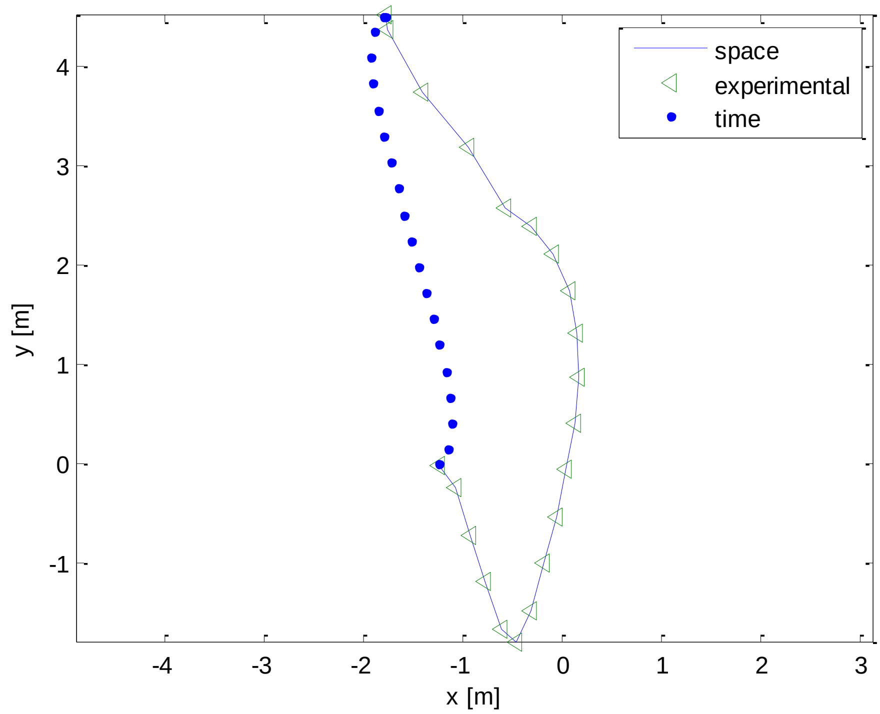


Figure 4.5 Generated reference path compared with experimental data

4.2 Least Square approach

With Least Square approach, we first need to test different value of weight parameters to find the relationship with the criterion. The value of weight values are in table 4.2 and 4.3 which are the random selected weight values for test. As we have two models of the energy cost function, so we should test these two separately.

Table 4.2 Random value of C_1

Parameter	Value
$C_1(1)$	0.3
$C_1 2)$	4.8
$C_{1(}3)$	45
$C_1(4)$	98

Table 4.3 Random value of C_2

Parameter	Value
$C_2(1)$	2.2
$C_2 2)$	3.5
$C_{2(}3)$	58
$C_2(4)$	89

After we settled these random parameter values, we can go back to solve the optimal control problem with these values and compare the simulation results of the approach method with these reference paths. Before that we now analysis the relationship between the criterion and parameters first. With the single parameter model we only consider the c_1 as the only parameter and check the relationship with Least Square criterion based on B point character.

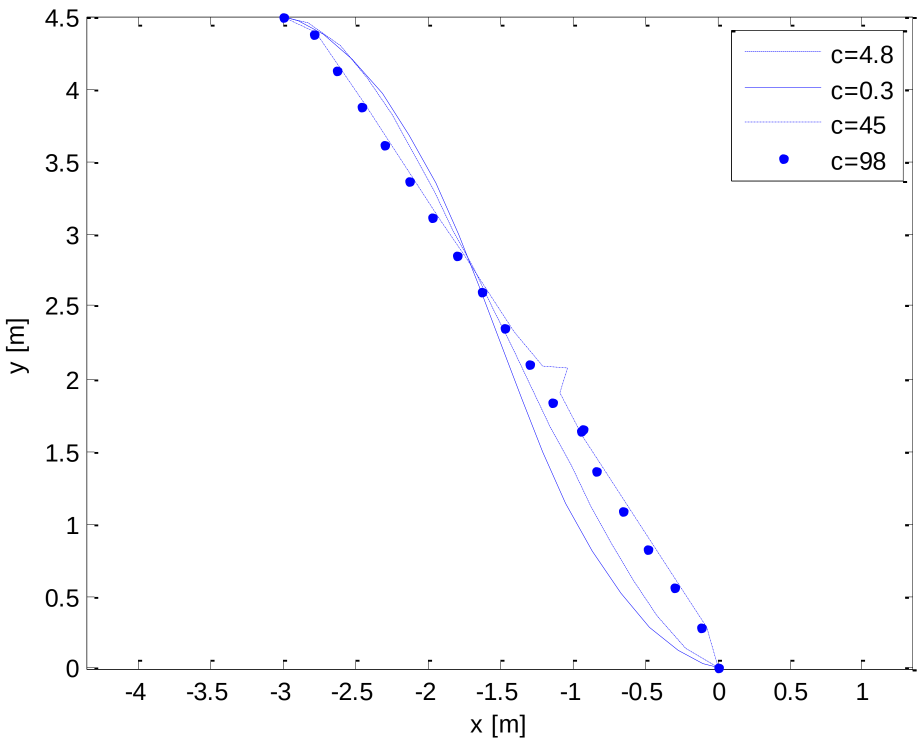


Figure 4.6 different weight parameter at A point

From figure 4.6 we can apparently obtain that the different value of weight parameter will affect the shape of the generated path at same goal and orientation, which means as the larger of the weight value, the generated path will be straighter from the overall point of view. This can be a problem but not an advantage for in real the human beings walking not that straightly in verse has some little curve which caused by the angle turning on specified points. For instance as c equals to 98 which is large in such case, the simulated path shows us that human turned so fast at beginning and then walk straightly to a point near goal then turn again. He cannot walk as adjusting the angle as forwarding. So the single parameter model which only affect the forward speed gives the forward speed more weight along such path which means we give a very large effect only on forward speed and ignore the effect of the angular speed. That is the reason why we reconsider the effect of the angular speed and give another weight parameter to it.

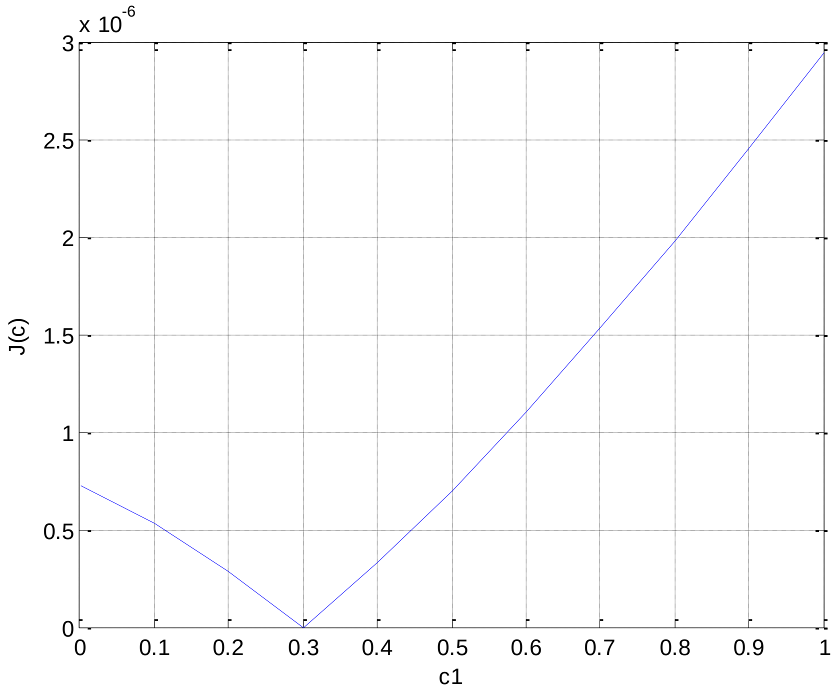


Figure 4.7(a) c_1=0.3

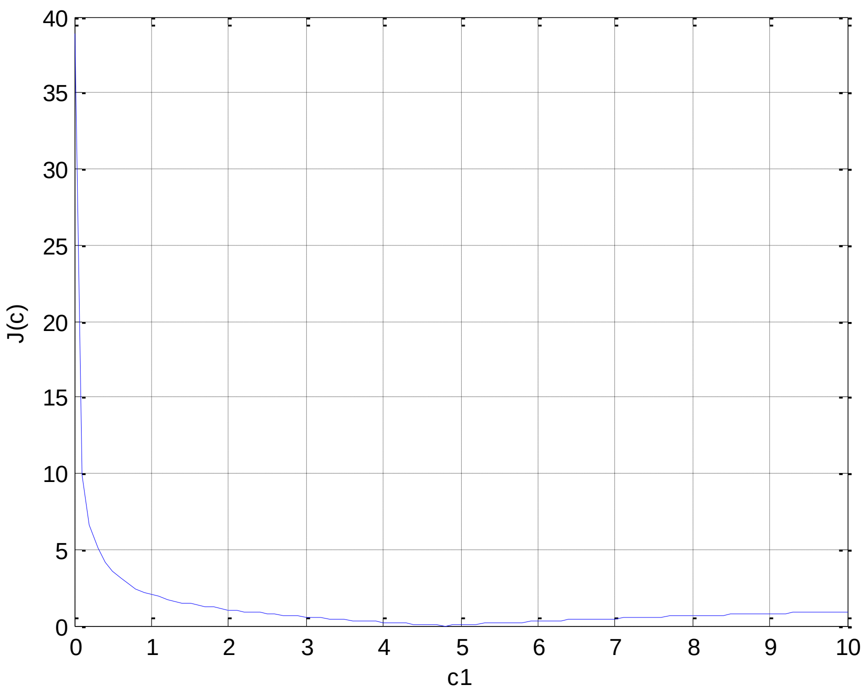


Figure 4.7(b) c_1=4.8

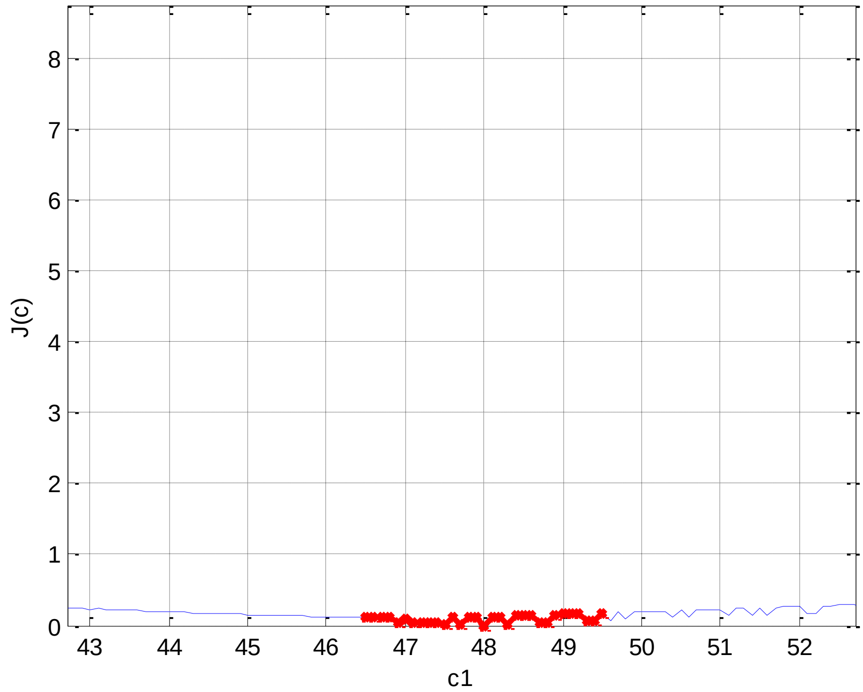


Figure 4.7(c) c_1=48

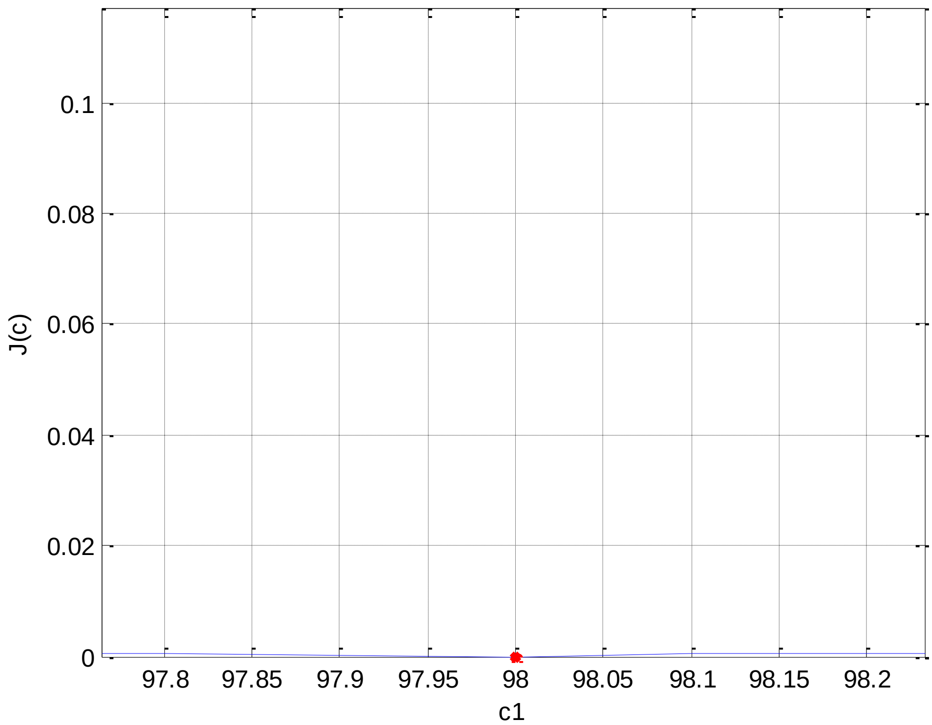
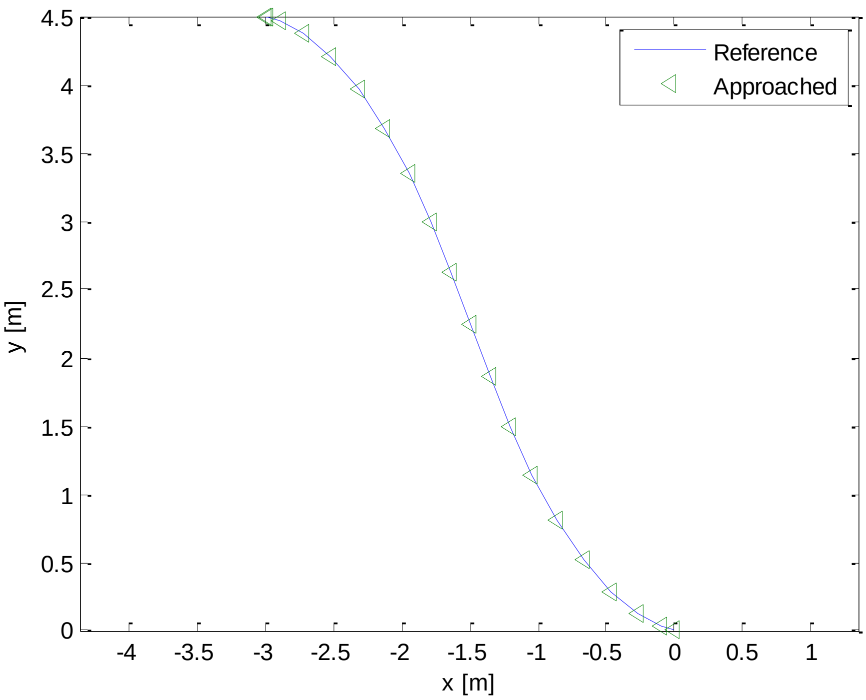


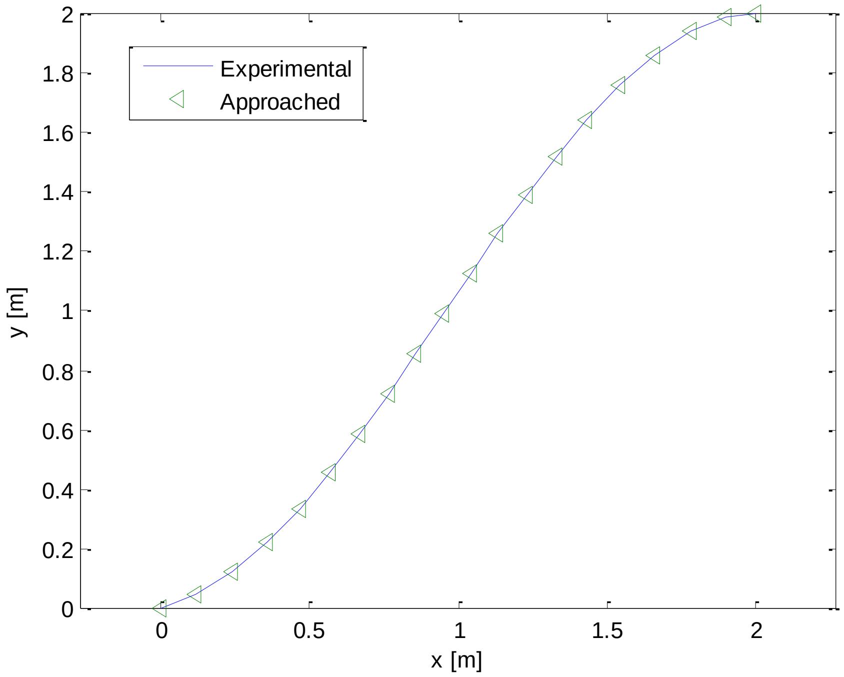
Figure 4.7(d) c_1=98

Figure 4.7 Single parameters and criterion in LSQ

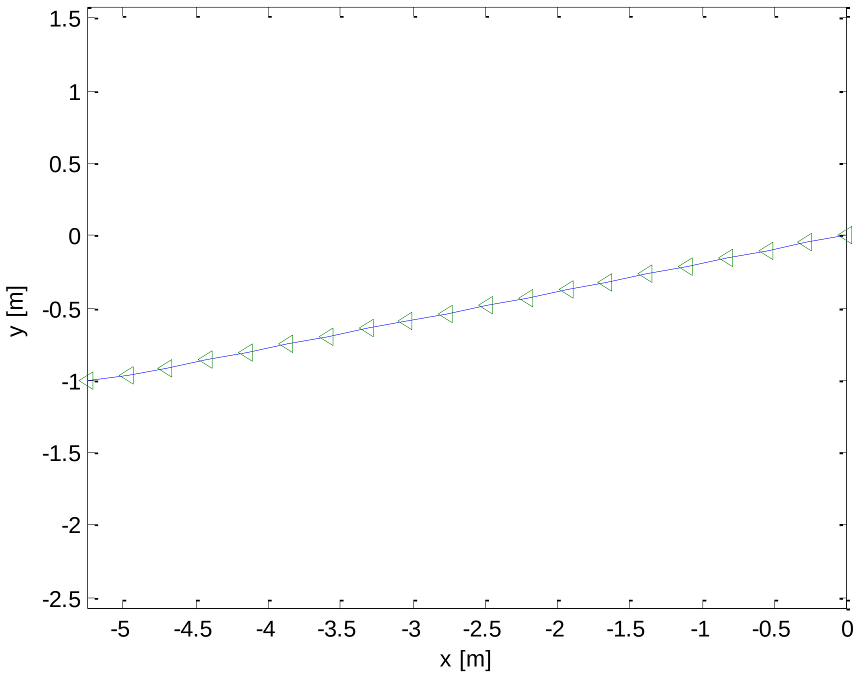
From figure 4.7, the relationship between the weight parameter's value and the criterion in Least Square approach, the minimum point of the curve can be easily found from such figure when weight value is small then 5. However when the weight value is large, we can also find the minimum value of the criterion but which one is not so much obvious in such situation. The reason that cause such problem is when the weight value is large, the generated path is become more flat than before, in another word, the weight of the forward speed is so large that the effect of the angular speed becomes smaller and smaller. So the path following becomes straight forward going motion in such condition. That is also the reason why we try to arise the effect of the angular speed. In such assumption, as the curves no longer convex, the minimum point will be lost and we only gain a similar approached path to the reference one but not exactly the closest one. However, even such problem exists, even the path no longer convex, we still found the approached path in fact.



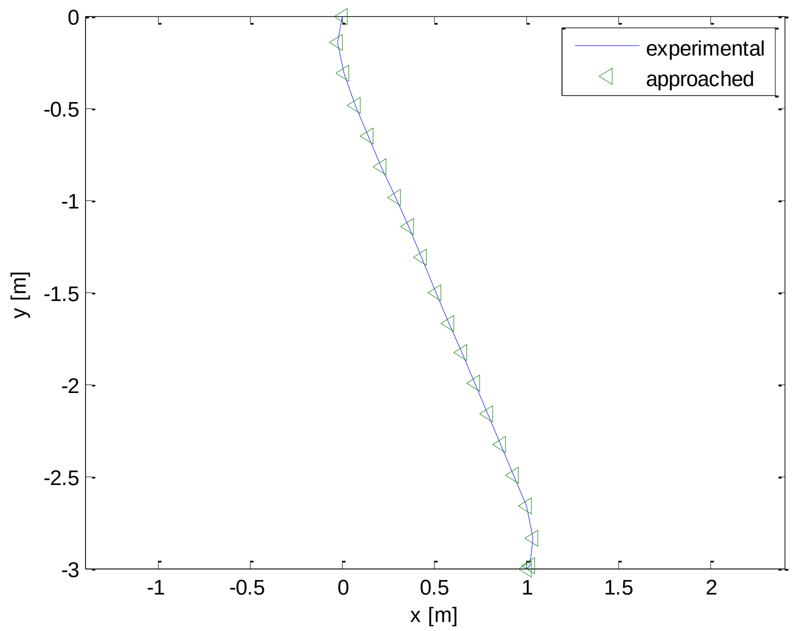
Point A



Point B



Point D



Point C

Figure 4.8 Single parameter in LSQ

According to such approach method we can see the obtained results in Figure 4.8. The approach method works very well with comparing to the reference paths and experimental paths. From this point of view, the Least Square in single parameter model can approach to such reference paths.

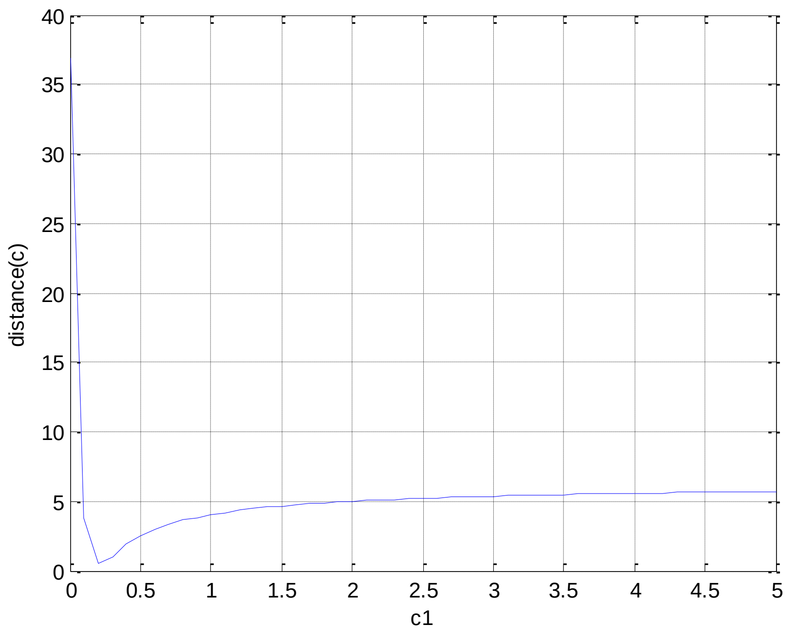


Figure 4.9 (a) c_1=0.3

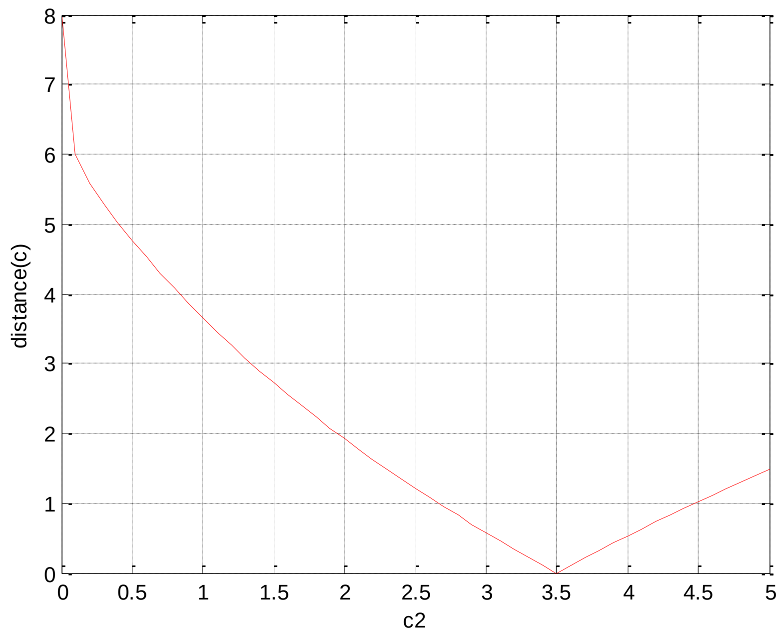


Figure 4.9(a) c_2=3.5

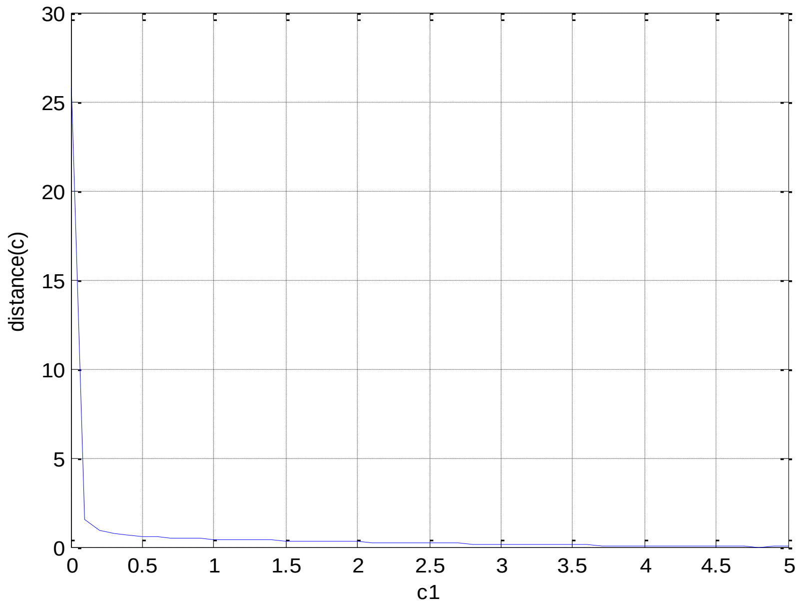


Figure 4.9 (b) c_1=4.8

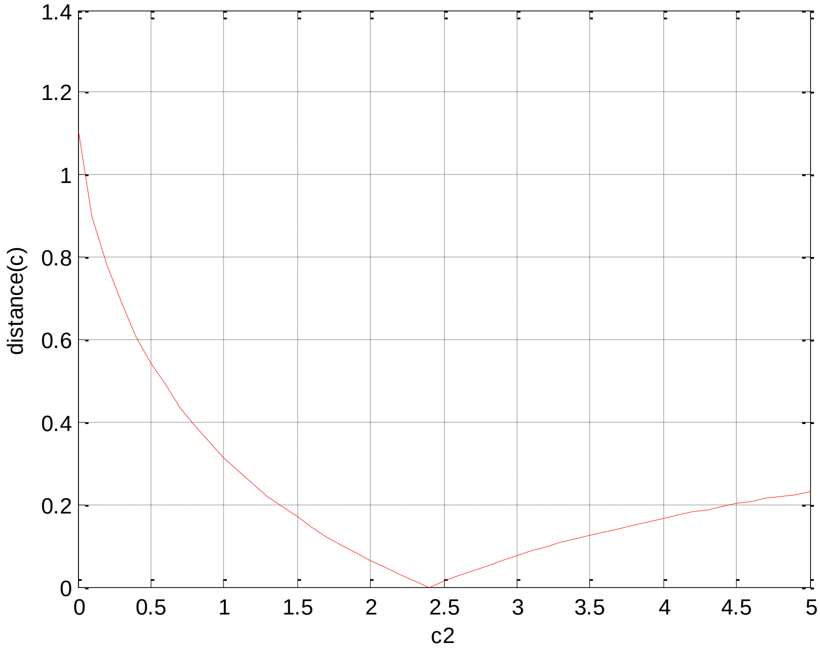
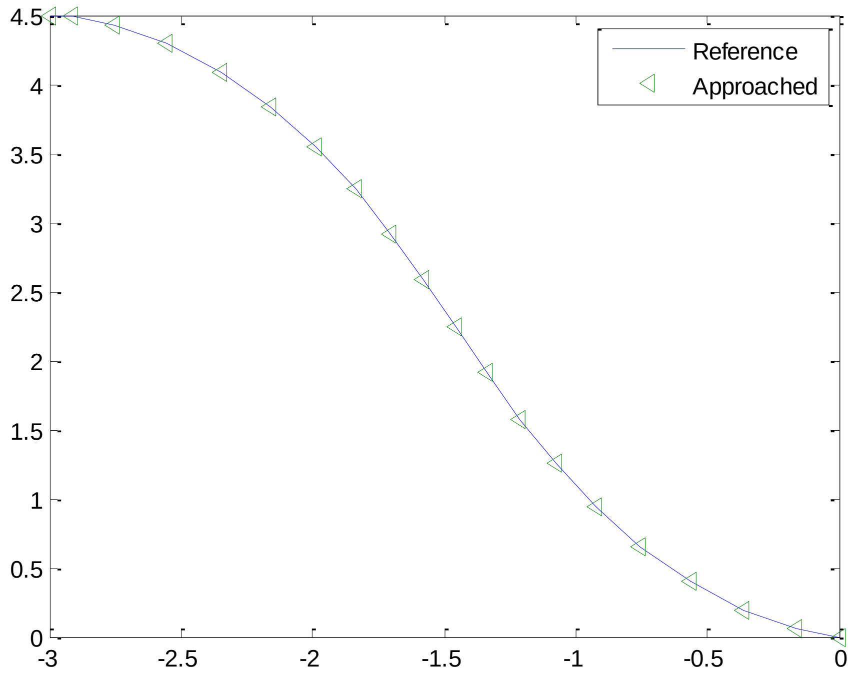


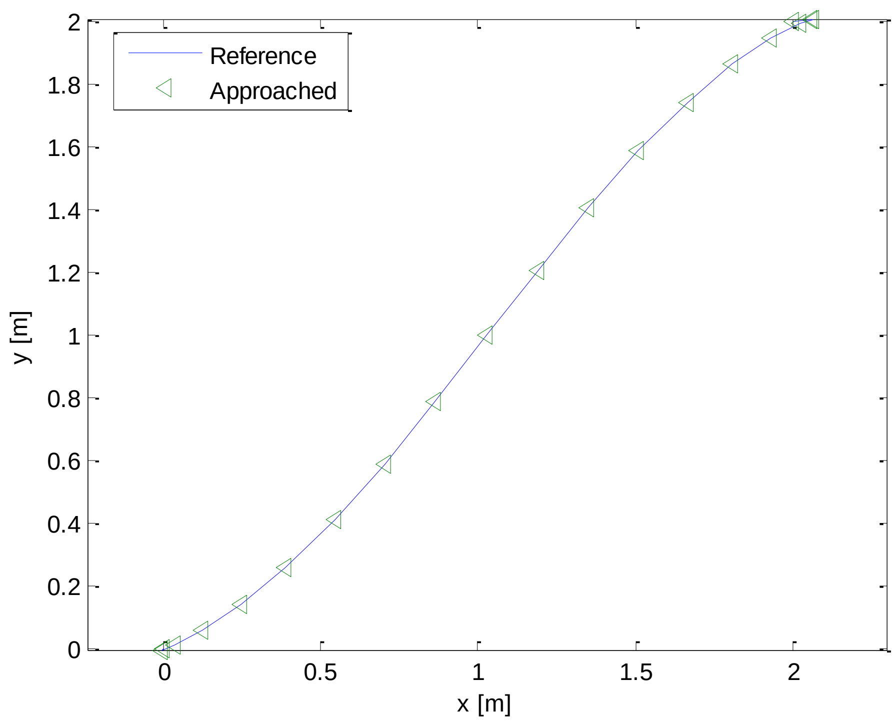
Figure 4.9 (b) c_2=2.2

Figure 4.9 Dual parameters and criterion in LSQ

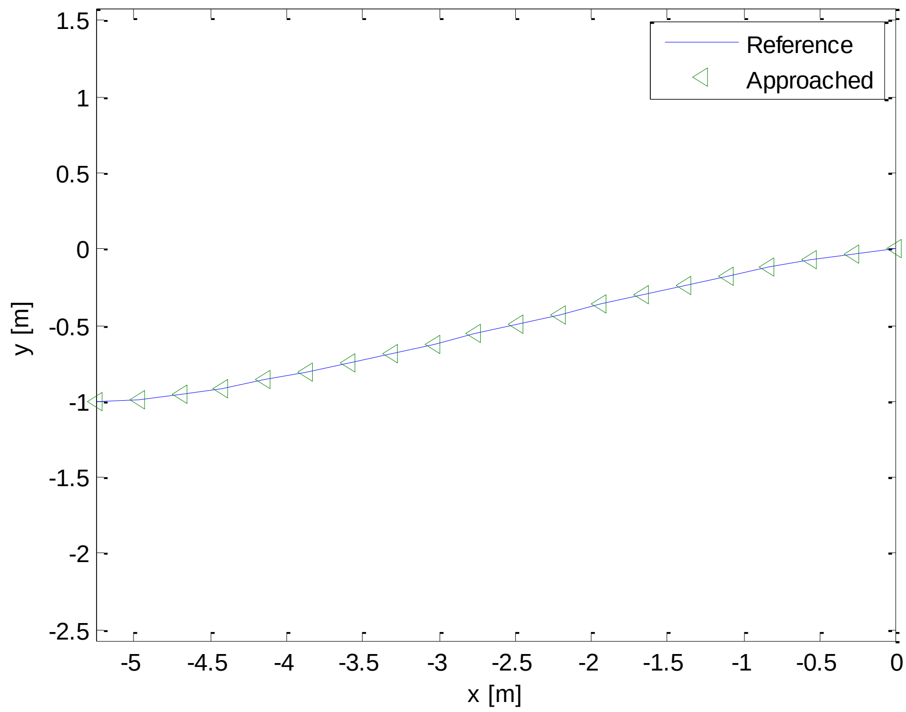
Figure 4.9 presented the relationship between dual parameters and criterion in Least Square approach. From the added difference of each point on reference paths and the generated paths, there is a minimum value of the criterion that both weight value suit for such minimum and in our case the minimum value is zero. The difference is described as diff (row, column) and the unique minimum point is diff(p^{th}, q^{th}) which means the p^{th} value of c_1 and the q^{th} value of c_2 cause the minimum value of diff (·), so that we return such values of parameter and solve the optimal control problem and find the approached paths which are present in figure 4.10. From this figure we can obtain that the Least Square method can approach the reference paths very well in such dual parameters model.



Point A



Point B



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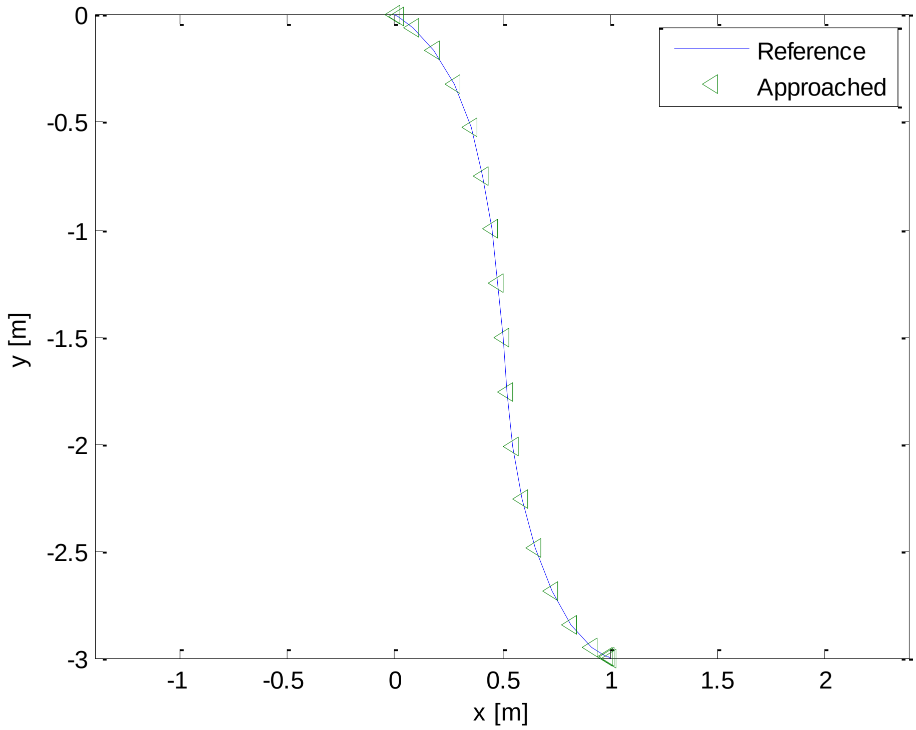


Point C

Figure 4.10 Dual parameter LSQ

4.3 Fréchet approach

The result analysis of Fréchet distance approach is the same as the Least Square approach method. We can check the relationship between single weight parameter and distance criterion for figure 4.11. In such figure we can conclude the difference between Least Square method and Fréchet distance approach. When the weight value is small the minimum value point of the distance can be obliviously found. However at the same condition even when the weight value is smaller than 10, the Least Square criterion becomes unclear and without such program it is hard to tell which value is the minimum which only happen when weight is very large in Fréchet distance.

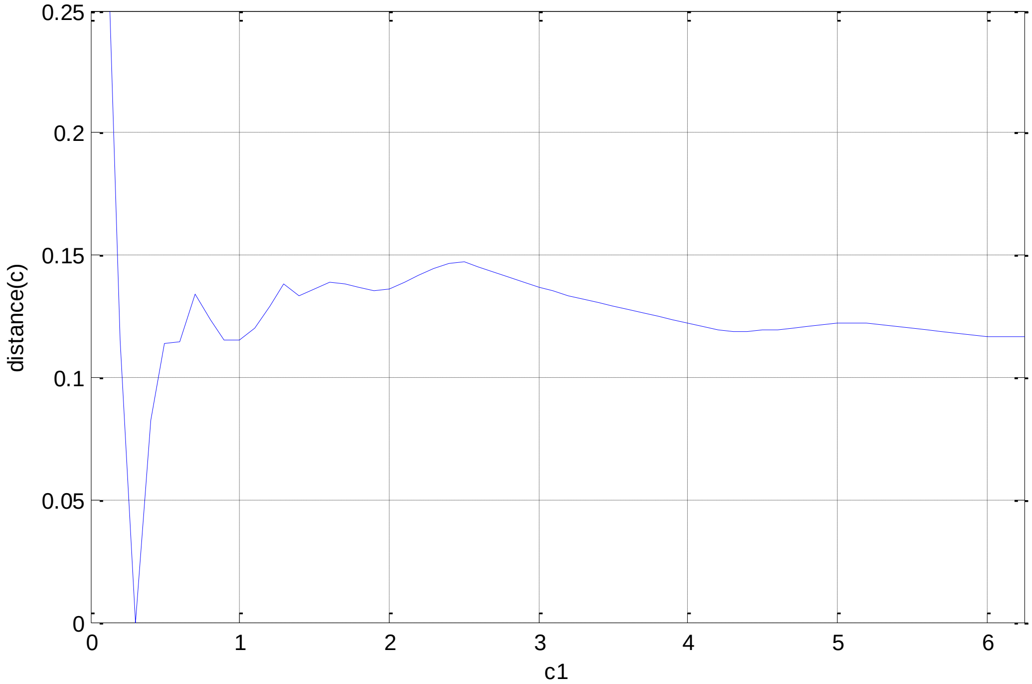


Figure 4.11 (a) c_1=0.3

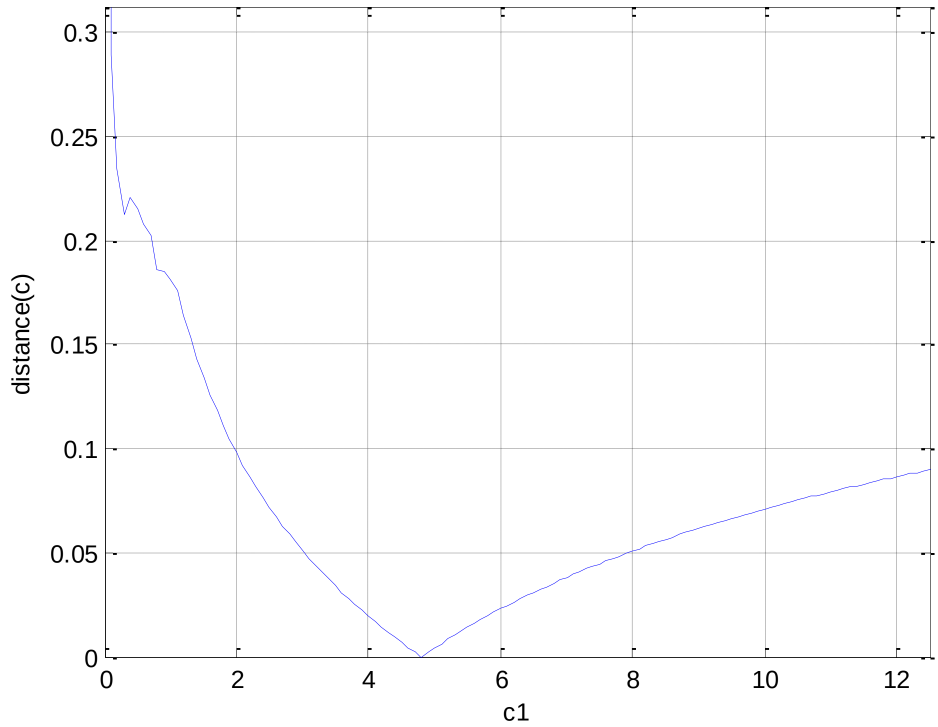


Figure 4.11 (b) c_1=4.8

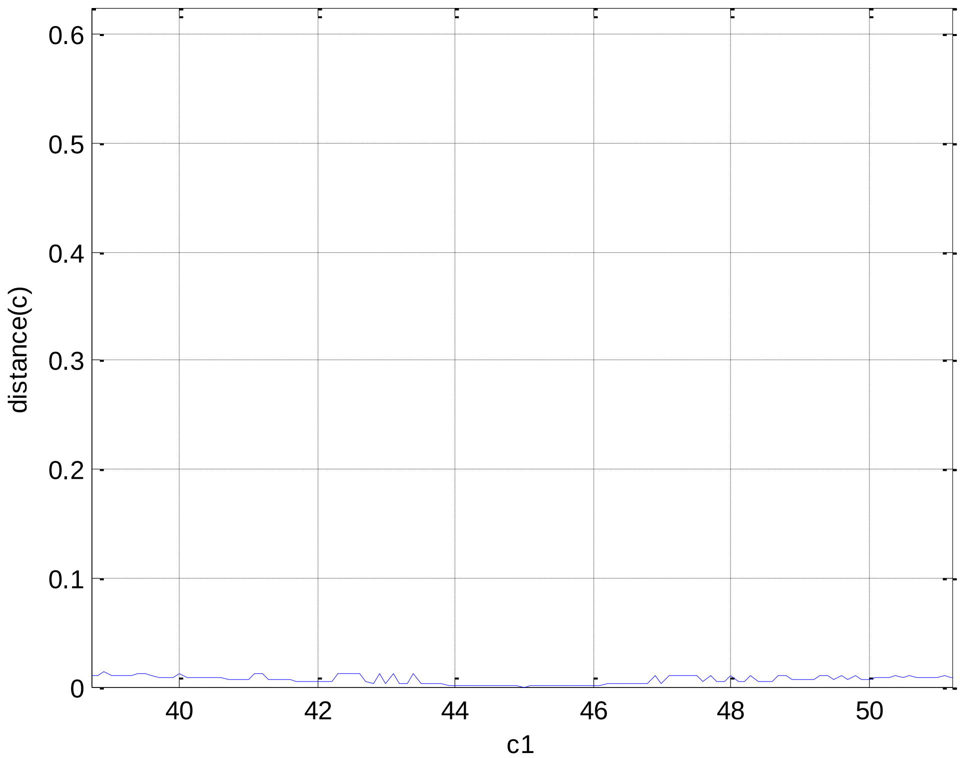


Figure 4.11 (c) c_1=45

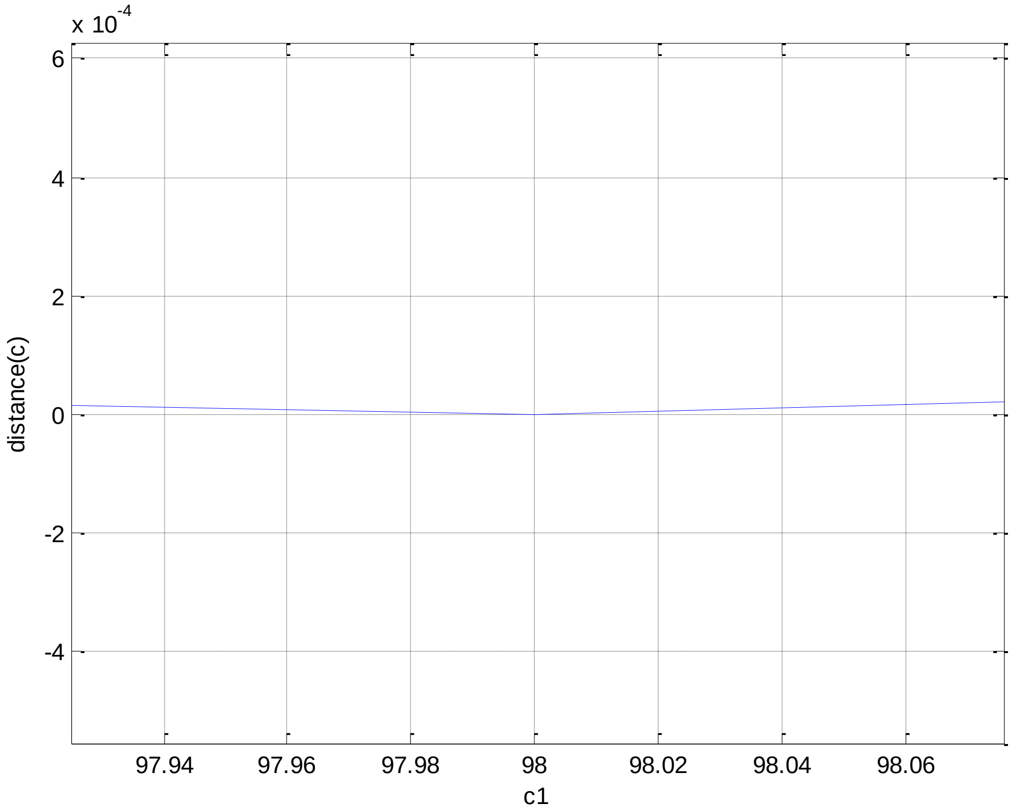


Figure 4.11 (d) c_1=98

Figure 4.11 Single parameter and criterion FD

We can also check these results in Figure 4.12 which is the relationship between two weight value and the distance. Even when weight values are very large the minimum points of the Fréchet distance are still obvious which ones performed better than the Least Square approach criterion. But the results are the same, Fréchet distance method approached the reference paths and experimental paths very well which we can read from Figure 4.13 and 4.14. They are the results of single parameter model approach and dual parameter model approach. Based on such figures, the performance of Fréchet distance approach is good each for path generation.

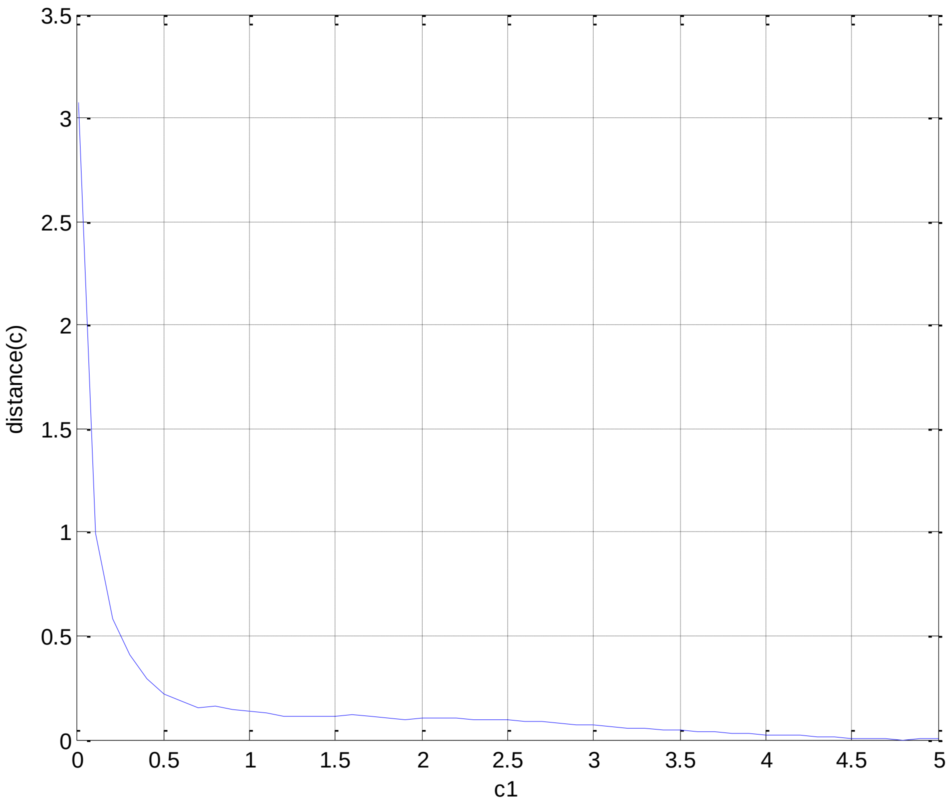


Figure 4.12 (a) c_1=4.8

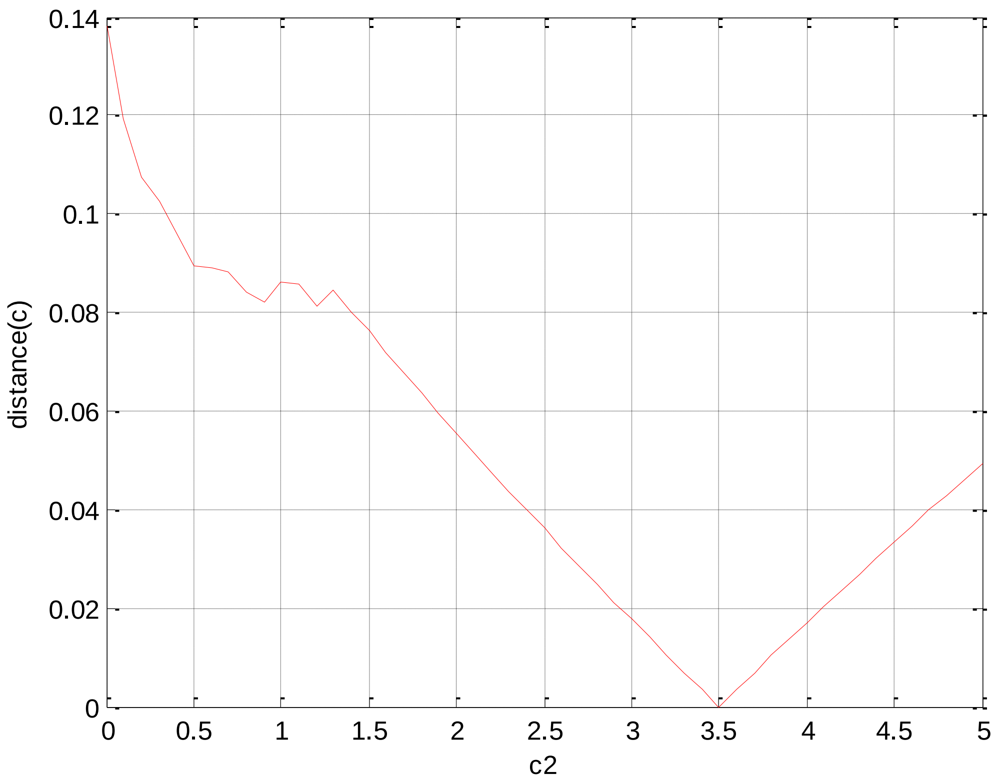


Figure 4.12 (a) c_2=3.5

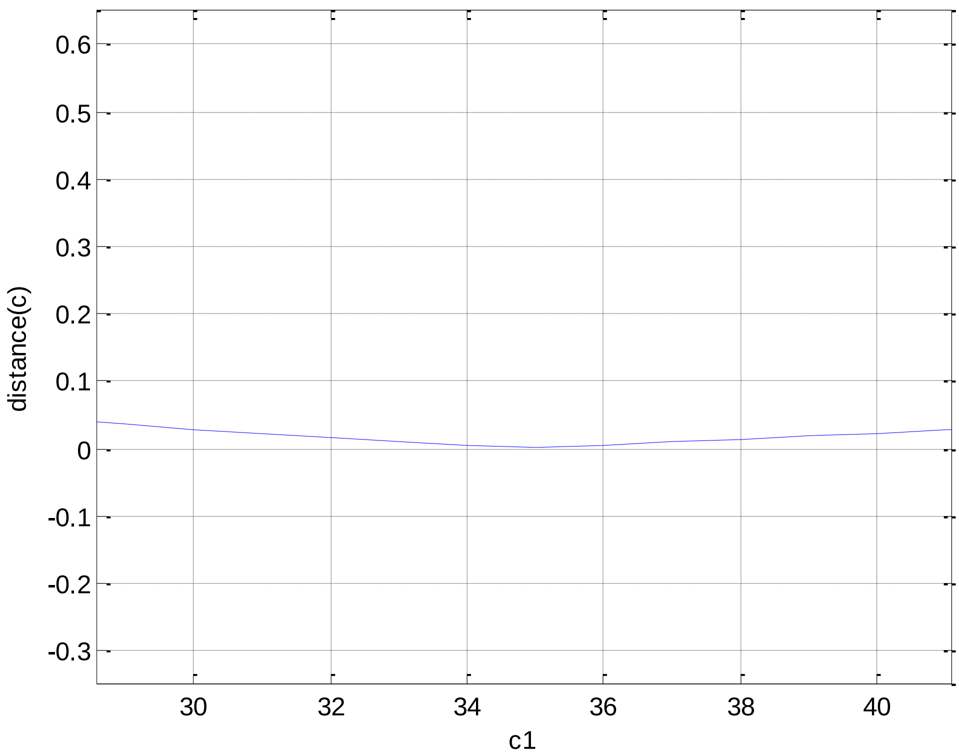


Figure 4.12 (b) c_1=35

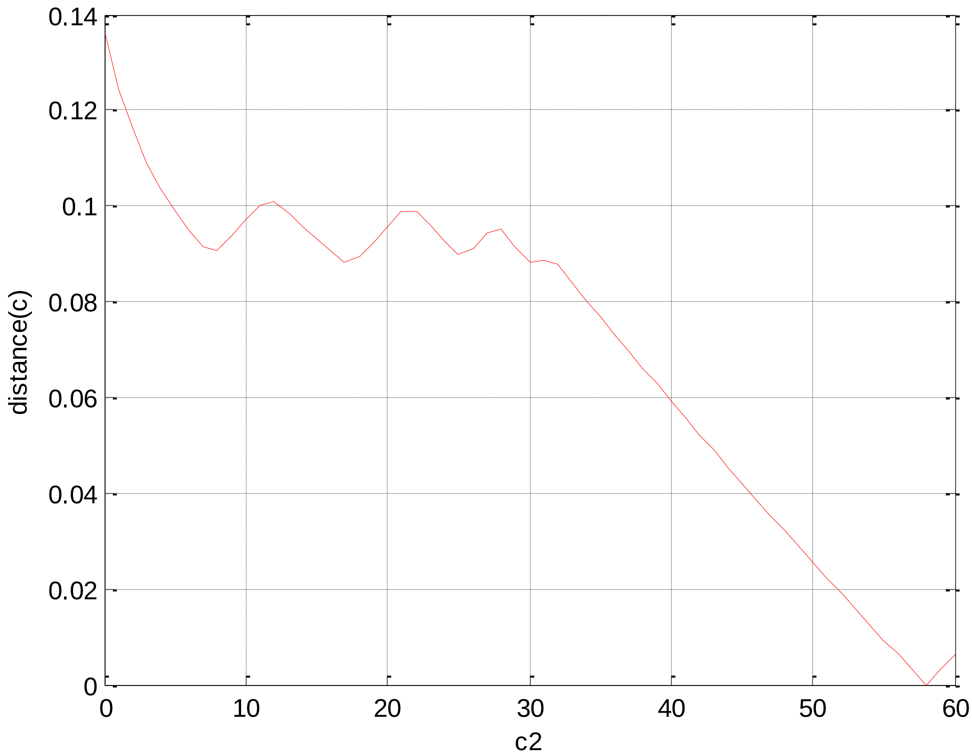
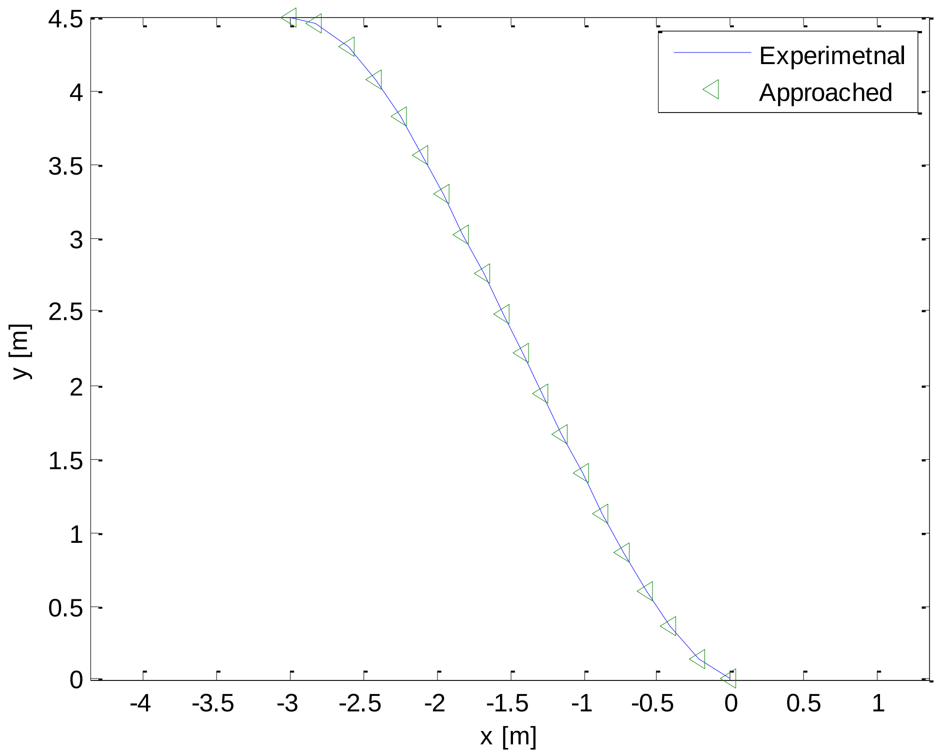


Figure 4.12 (a) c_2=58

Figure 4.12 Dual parameter and criterion FD



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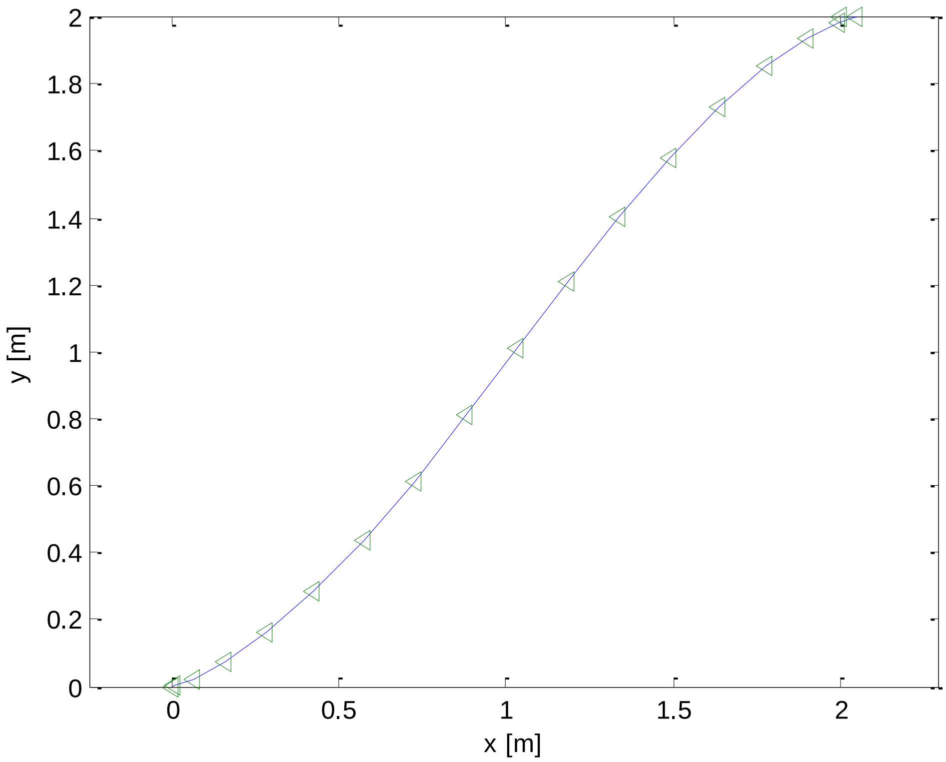
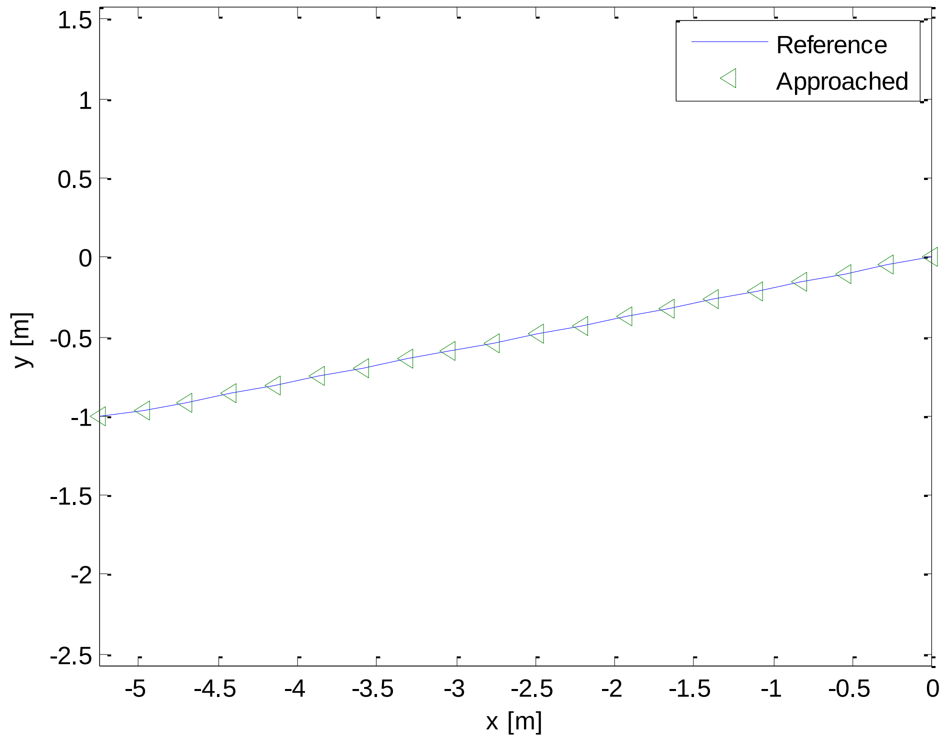
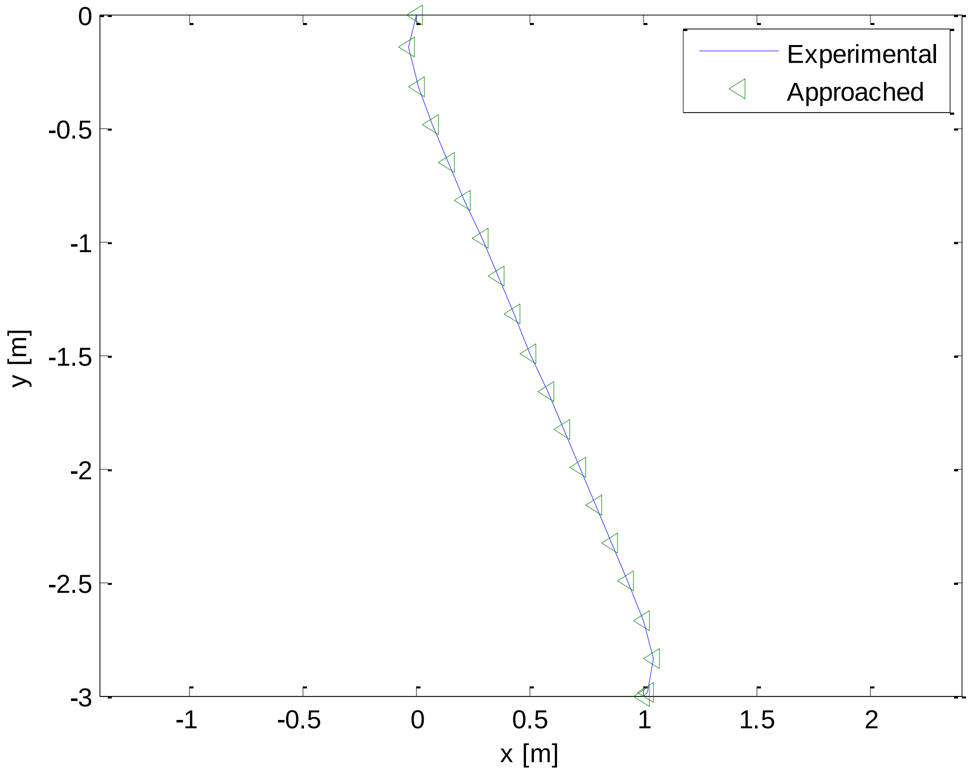


Point B

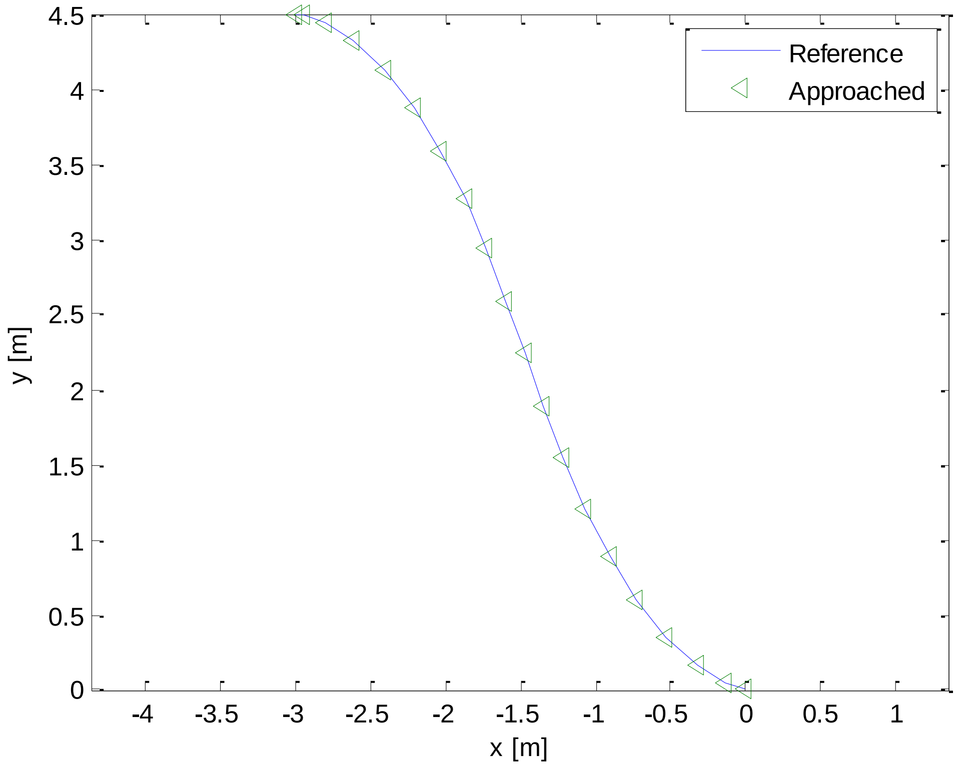


Point D

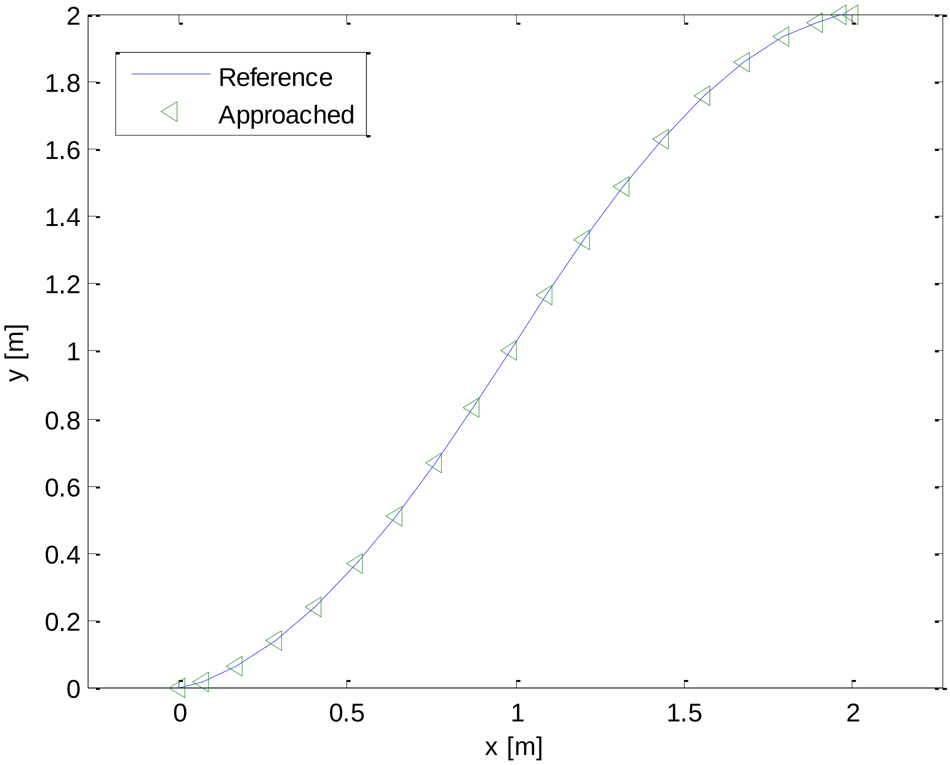


Point C

Figure 4.13 Single parameter approach in FD



Point A



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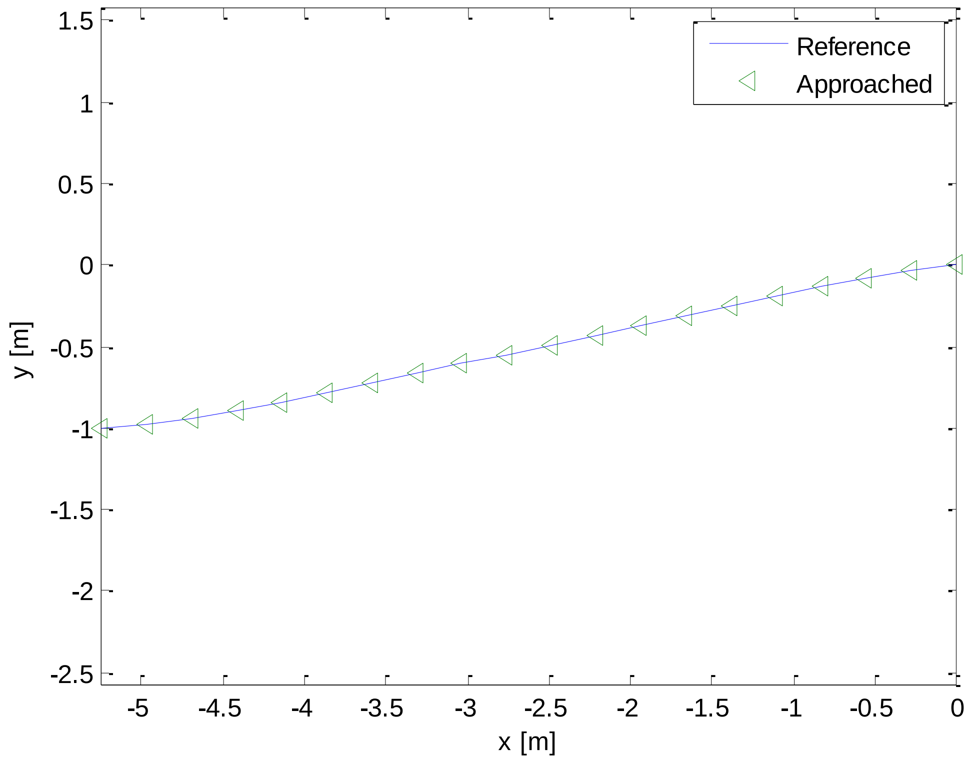
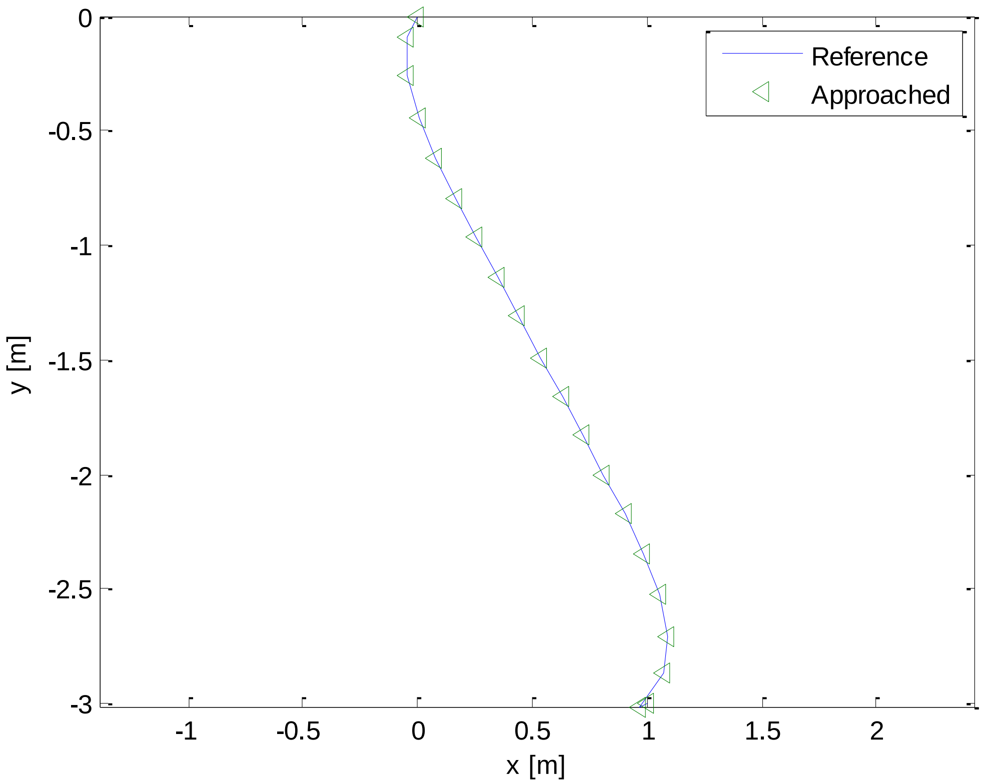


Point D



Point C

Figure 4.14 Dual parameter approach in FD

4.4 V-rep simulation

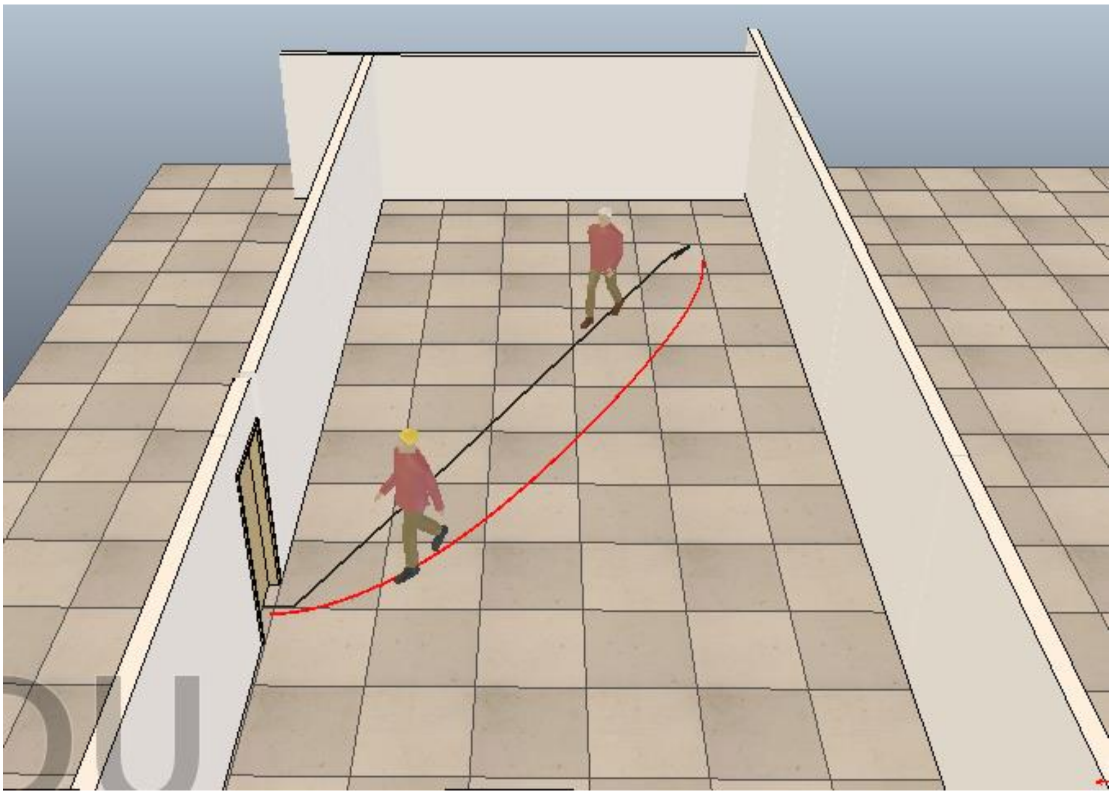


Figure 4.15 V-rep simulation Result

The simulation result of V-rep software is a movie file which present the processing of human locomotion which based on following the approached path obtained from the simulation in MATLAB. By comparing with the original approach method in the software, we can see the difference between these two algorithms. The former one which based on minimization of simple two-point-distance is not so realistic and the approached path we obtained can be more smooth and convex.

4.5 Summary of the results

It is apparent that the Least Square and the Fréchet distance methods can approach the reference paths very well. It is difficult, however, to draw direct and definitive comparisons, and as such, the conclusions are based on the specific problem formulations. After this the inverse optimal control problem can be solved and the results is apparent, that we can generate the approached paths close to the human walking paths according to such methods.

However, in some aspect, we can find the Fréchet distance method can be better than Least Square and the inverse optimal problem is solved effectively when the generated path is not convex. So based on this point of view, the assumption mention in Chapter 1 can be re-write that this kind of approach method which based on inverse optimal control could also work on non-convex path generation.

Additionally, the discrete time model of problem is not good enough for a lot of problems, especially when at the assumption of human forward walking speed is always along the path. That is why we need to improve the cost function into a space one.

There are still some dissatisfactory problem with such approach methods. For the inverse optimal control will cost a lot time during calculation which means this kind of method is not so much efficient compared to other method like only solve forward optimal control problem, because it is a method which solved the forward optimal control problem with random parameters and find the most proper one to the ideal case, so the accuracy is very good but not so much efficiency.

Chapter 5

Conclusions and Future Work

In this thesis, we have presented an inverse optimal control approach which allows us to identify optimization criterions of processes such as human locomotion from measured data. A flexible numerical technique has been described which allows the solution of a large class of problems. By using such method we can obtain the ideal model to solve the optimal control problem in goal-directed locomotion. So we gain the conclusion through all experimental results and simulation results that using inverse optimal control, it was possible to establish a simple and unique optimal control model that seems to represent a good approximation of the collected data.

Current efforts are focused on implementing the previously described Least Square approach method and Fréchet distance approach method. Maybe more minimization approach methods can be applied to such path following problem. However, the inverse optimal control method would be the optimal solving method for such human path generation problem which will give us a new numerical algorithm direction to handle similar problems.

The results in Chapter 4 presented within are in no way intended to be general, and are highly dependent on the system being analyzed. They are even more highly dependent on the control designer, and other choices of outputs to penalize or penalty weight formulations may prove to produce better results for one or several of the controllers or methods presented within which would be my future work to do.

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