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Optimal energy management of a building cooling system with thermal storage: modeling and control

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Ad Adele, per la concretezza
A Giuseppe, per la tenacia
A Giulio, per l'intraprendenza

Sommario

La gestione ottima energetica degli edifici è un tema che ha attirato di recente l'attenzione della comunità scientifica internazionale. Questo interesse è motivato in particolare dalla crescente necessità di poter modulare i carichi nella rete elettrica in modo da evitare squilibri tra produzione e consumo di energia, compensando appunto con la flessibilità del carico l'aleatorietà nella produzione sempre più diffusa di energia da fonti alternative, quali l'eolica e la solare. La disponibilità di apparati "smart" per attivare/disattivare in remoto le utenze domestiche e per il rilevamento a distanza dei consumi rende possibile poter concepire soluzioni innovative al problema.

In questo lavoro di tesi si studia il problema del condizionamento di un edificio, utilizzando il set-point di temperatura come variabile di controllo per modulare e traslare nel tempo il carico termico e quindi il corrispondente contributo all'equazione di bilancio energetico di esercizio. Flessibilità aggiuntiva alla modulazione del carico termico, e alla conseguente richiesta di energia elettrica alla rete, viene introdotta considerando un serbatoio termico "attivo", mentre l'edificio contribuisce svolgendo il ruolo di serbatoio termico "passivo", non direttamente manipolabile. Il problema si traduce in un problema di ottimizzazione con vincoli, affetto da incertezza dovuta a disturbi quali il livello di occupazione dell'edificio e la radiazione solare esterna.

Nella prima parte del lavoro si affrontano aspetti modellistici, effettuando in particolare una modellizzazione dettagliata della struttura dell'edificio, che impatta direttamente sulle sue caratteristiche come serbatoio termico. Tecniche di riduzione d'ordine e di approssimazione funzionale vengono utilizzante per ridurre la complessità del modello e rendere le grandezze che poi compaiono nel problema di ottimizzazione vincolata convesse rispetto alle variabili di ottimizzazione. Nel caso in cui si faccia riferimento ai disturbi nominali, il problema di ottimizzazione vincolata risultante è convesso, con un numero finito di vincoli, e semplice da risolvere. Per tenere conto dei disturbi è possibile utilizzare vincoli in probabilità, dove si richiede che il vincolo

sia rispettato per tutte le realizzazioni dei disturbi ad eccezione di un insieme con una certa probabilità prefissata. Il problema chance-constrained risultante non è più convesso, e viene risolto in modo approssimato – con garanzie sulla qualità della soluzione ottenuta – mediante l'approccio a scenario. Da notare che la soluzione a scenario implementa una legge di controllo con compensazione diretta di quella parte dei disturbi che sono misurabili. Le prestazioni delle due leggi di controllo sono state valutate su un esempio numerico. I risultati ottenuti sono promettenti.

Abstract

Optimal energy management of buildings is a research theme that has recently attracted the attention of the international scientific community. The interest in this topic is raised by the need of flexible electrical loads that can be modulated so as to compensate the possible unbalance between energy production and consumption caused mainly by the stochastic behavior of renewable energy sources. The availability of smart appliances and metering systems controlled remotely paves the way for conceiving and developing innovative solutions to the problem.

In this work, we study the optimal energy management of a building cooling system with thermal storage. In the proposed solution, the building temperature set-point is taken as control input, which allows to modulate and shift in time the cooling load during the reference time horizon of interest. The "passive" thermal storage effect of the building structure is considered jointly with the "active" one due to a thermal storage unit introduced to provide additional flexibility to the system. The optimal energy management problem is formulated as a constrained optimization problem affected by uncertainty due to the presence of disturbances acting on the system such as, e.g., the building occupancy and the solar radiation.
Modeling issues are addressed first. In particular a detailed model for the building structure is adopted, which leads to a description of the building thermal characteristics as "passive" storage. We take advantage of model order reduction and functional approximation techniques in order to reduce the size of the problem and provide a convex formulation. In the case of nominal disturbances affecting the system, the constrained optimization problem is convex, subject to a finite number of constraints, and, hence, easy to solve. In order to take into account disturbances, we opt for a probabilistic formulation where constraints are required to be satisfied for all disturbance realizations, except for a set of predefined probability measure. The resulting chance-constrained optimization problem is not convex and it can be approximatively solved (providing guarantees on the quality of the

solution) via a randomized method known as scenario approach. Notably, the scenario-based solution implements a disturbance compensator. Performance of the proposed control strategies is evaluated in a case study, and results appear promising.

Contents

Acknowledgements

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Chapter 1

Introduction

Almost 40% of the US overall electricity consumption can be attributed to buildings, almost a half of this fraction being used by cooling, heating and air conditioning systems (HVAC) [1] in order to guarantee living comfort conditions. In the perspective of the smart grid challenge of integrating renewable energy production and distributed energy generation in the current grid, buildings can be viewed as big consumers that can actively contribute to the electrical energy demand/generation balance. Effective building energy management strategies should be implemented to increase efficiency and eventually track some energy consumption profile, which is possibly modulated during the day in order to avoid peaks in the demand. The introduction of active thermal storage systems can be particularly useful in this respect, since they can shift around in time the electrical energy request from the grid, thus reducing absorbtion peaks, and they also allow cooling systems to work closer to their highest efficiency condition [3], [2], [4]. On the other hand, studies on the "building thermal mass" have shown that it can be exploited as a (passive) thermal storage to delay the building heat release and effectively reduce the overall exchanged heat quantity [5], [6], [7].

In most of the works in the literature on building cooling systems with thermal storage, control at the level of the energy management system involves acting directly on the cooling system, its flows and temperatures. The resulting building temperature behavior is determined based on a detailed model of the system. Comfort conditions are robustly guaranteed against modeling errors and disturbances by means of a secondary controller (usually of the PID type) as shown in Figure 1.1. In [7] it is argued that this configuration leads to unpredictable and hardly quantifiable behaviors in the presence of disturbances. Indeed control design is performed with reference to nominal conditions and involves determining the best operative conditions of every subsystem composing the cooling system, which makes the problem hard to tackle via stochastic optimal control methods in realistic cases where the model size is large.

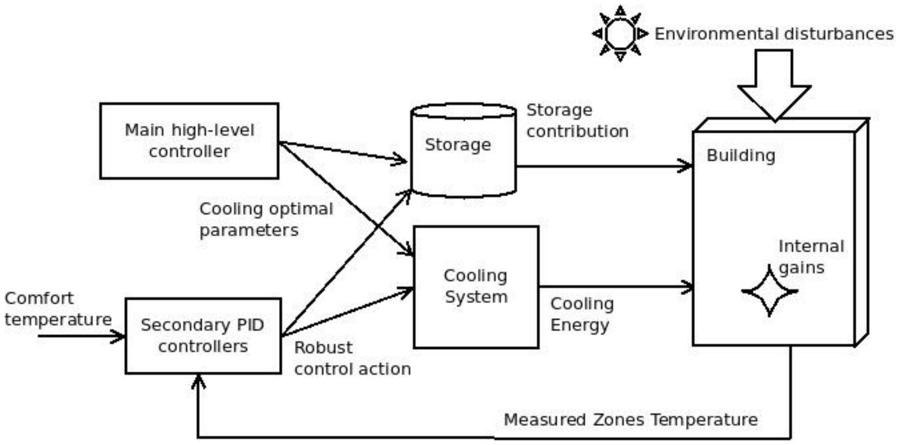

Figure 1.1: Classical energy management system

In this thesis, we propose a different approach to the energy management problem for a building cooling system: we set as control input to be optimized the building temperature set-point and then compute the cooling energy needed for the actual building temperature to track it (Figure 1.2). To this purpose, we resort to a description of the cooling system of "black box" type: we suppose that it is controlled in an optimal way, relegating issues related to the nonlinear characteristics of the system to this sublevel, [8], [9].

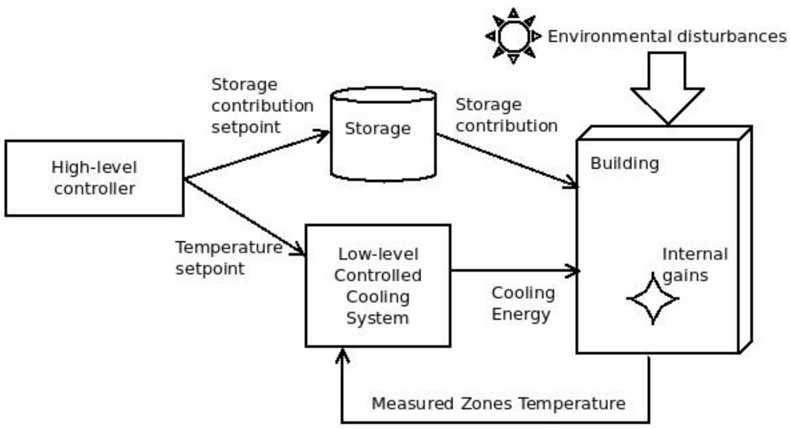

Figure 1.2: Proposed energy management system

The cost function adopted to optimize efficiency or to track some energy consumption profile is defined based on the thermal energy balance within the building, and readily accounts for thermal effects related to the building structure and thermal phenomena related for example to occupancy and radiation through glazed surfaces. The thermal model of the building is derived based on [10] and can make explicit the dependence of the thermal heat exchange as a function of indoor and outdoor temperatures and others environmental conditions. Similarly, a description of the active thermal storage in terms of energy exchange is introduced.

Much effort is spent to derive a convex description of the overall problem, which indeed translates into an easily computable solution if reference is made to the system operating in nominal conditions only (certainty equivalence solution). Results are presented for different variants of the energy management problem, describing the main advantages in using different components and/or strategies as in [11]. Performance degradation is experienced when (non-nominal) disturbances act on the system. A stochastic approach implementing a disturbance compensation mechanism and providing guarantees on constraint satisfaction in probability is hence conceived. The resulting optimization problem is not anymore convex because of the probabilistic constraints, but it can be reduced to a convex one by resorting to a randomized approach known in the literature as “scenario approach”, [12] and [13]. From some preliminary results, the scenario solution appears promising.

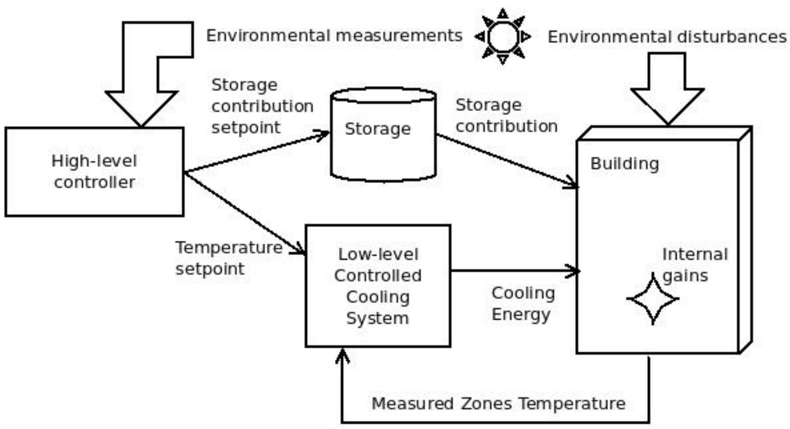

Figure 1.3: Energy management system with disturbance compensation

Chapter 2

Modeling

In this chapter, we address the modeling of the building cooling system with thermal storage to the purpose of optimal energy management. This involves defining the control variables, which include the building temperature set-point. The model consists of a thermal energy balance equation for the building, a description of the thermal storage in terms of energy flows, and a model of the chiller in terms of electrical energy needed to provide the cooling energy for tracking the temperature set-point.

2.1 Temperature set-point as control input

In this work the set-points for the temperatures of the zones in a building or a set of buildings are considered as control inputs. By zone we mean a spatially defined part of a building, composed by one ore more rooms, sharing well stirred and uniform air at the same temperature. Zone temperatures are assumed to track the set-points. This entails that the cooling system (e.g., a chiller, a chiller plant composed of multiple chillers) is effective in promptly and precisely regulating the temperature inside the zones. In order to make this last assumption reasonable, constraints will be added to limit the maximum cooling energy request and this will indirectly affect the set of admissible temperature set-points.

The reason why we make use of temperatures as control variables is that, irrespectively of the method used to solve the optimal management problem, the zone temperature must be comfortable: even if unpredictable events occurs (i.e. a window that is opened) or if errors exist in the model of the building (due to changed destination of use perhaps), the temperature must always lie inside a suitable interval that is defined by constraints. The low level control system is in charge to track the temperature set-point and the cooling power needed is estimated in the optimization process and never directly imposed by the high level controller itself, which ensures robustness in the temperature regulation.

The temperature set-point is specified by setting its values every ten minutes and considering a piecewise linear interpolation. In formulas:

$$T(t) = T_i + (T_f - T_i)\frac{t}{\Delta t}$$

where $\Delta t = 10$ minutes. With this choice we can easily express constraints related to people comfort and provide a closed-form expression for the cost function. More specifically, the temperature derivative has to be suitably bonded in order to avoid too rapid changes in temperature that can be perceived by people as uncomfortable. As for the closed-form for the cost function, we will take advantage of the fact that the time derivative of the temperature is constant and given by:

$$\dot{T}_z = \frac{T_f - T_i}{\Delta t} \tag{2.1}$$

2.2 Thermal energy balance equation

The thermal equation that governs the overall system is:

$$Q_z = Q_w + Q_i + Q_p - Q_c \tag{2.2}$$

This equation represents a power balance that always holds and takes into account different effects, for all of these effects it will be provided a mathematical formulation in the next sections, here they are just briefly introduced:

- Q_z : Represents the thermal power associated to the zone own thermal capacity. The effect considered here is both related to the dynamics of the air contained in rooms and the stuff. We will form now call simply room both air and stuff contained. To increase and decrease the temperature of rooms requires energy, this energy is provided or subtracted by the elements described on the right hand side of the equation;

- Q_w : Thermal power exchanged between zones and walls. This element is considered critical for the purpose of the work since we are exploring a solution such that building thermal inertia is exploited. Most of the environmental factors considered as outdoor temperatures, solar radiation and wind effects act on the system through this term;

- Q_i : Thermal power generated by all the non human sources in zones. This term takes into account the presence of machines, computers, lighting system and everything produces heat and can be considered relevant. Into this term it is also considered the effect of radiation trough windows;
- Q_p : Thermal power generated by humans. Due to the conceptual and technical relevance of this factor (that, like the previous factors, could have been saw as an internal gain) we decide to threat this disjointly from other heat sources;
- Q_c : Cooling power injected into the zone. Positive values means subtracted heat. Constraints ensures that the cooling system can provide as much power as needed in order to climate the rooms, this is actually the only term directly controlled by the cooling system through low level control action;

For the optimization point of view what we are more interested in is to give a formulation that estimates the quantity of cooling energy needed to impose a certain temperature variation when certain environmental condition holds. We are thus more interested in the following reformulation of the problem:

$$Q_c = Q_w + Q_i + Q_p - Q_z \tag{2.3}$$

Moreover we will consider, making use of some assumptions and approximations, the cooling energy request instead of the power request. The energy associated to the sample interval i is intended to be the amount of energy needed to linearly track the temperature of the zone starting from the initial value $T(i)$ (initial temperature condition for sample interval i) to the final condition $T(i+1)$ (final temperature set point for sample interval i) over the sample time Δ_t:

$$\int_t^{t+\Delta_t} Q_c dt = \int_t^{t+\Delta_t} (Q_w + Q_i + Q_p - Q_z) dt$$

from the this equation we introduce the energy notation:

$$E_c = E_w + E_i + E_p - E_z \tag{2.4}$$

We will last derive for each element a finite horizon formulation.

2.2.1 Wall-zone energy contribution

In this section a formulation of the energy interaction in between the building and zones is derived. A suitable procedure to build up a power thermal model for a generic building is provided and then manipulated in order to express the energy quantity over some finite horizon.

We look for low order building models able to accurately predict the future heat demand of the building with the minimum computational effort. The fact that buildings are complex, large, interconnected systems united to the fact that heat exchange is a nonlinear and spatially distributed phenomenon makes the thermal model really challenging to derive. Moreover the existing trade off in between detailed characterization and computational effort makes the problem of providing a suitable control oriented model even harder. Many papers investigates this topic making use of varied methodologies, each with their assumption, advantages and drawbacks.

Review on building modeling

The development of a control-oriented simplified model for buildings plays a basic role when an optimal control policy is considered, moreover a good description of buildings dynamics is mandatory for simulation purposes and first time verifications. For these and more reasons many different modeling techniques has been proposed in literature that, roughly speaking, can be divided in two categories. In the first category we can find models made starting from physical prime equations. This methodology allows the designers to end up with a detailed thermal description of the building without performing any identification, on the other hand the methodology deeply relies on the knowledge of materials and building structure and enforces designers to go deep down in technical details. Usually such models are produced assembling simple subsystems mutually physically interacting, this can usually be done making use of computer tools (Trnsys and EnergyPlus among many) and models are very reliable and detailed. For this reason these models and this way of modeling is more commonly used for simulations purposes. Moreover, despite their reliability, the more detailed such models are the less suitable they will be for control purposes either because of their non explicit formulation or because of their high dimensional and non linear expressions. The second approach is a data driven one: it provides models in an explicit form resorting on identification techniques. A data collection is needed and thus experiments on the building must be performed, this may be difficult especially due to the stochastic nature of the non controllable signals involved). Identification can be made in many different way, from gray to black box, from linear to non linear procedures, until making use of neural networks and genetic algorithms. A short list of the most investigated modeling approaches in thermal building engineering follows:

- Subspace method (4SID): is a statistical based black-box identification method that easily handle a large amount of data, it has been demonstrated [15] that 4SID is suitable even for the identification of a large office building.

- Prediction error method (PEM): it tunes a pre-specified model, usually an autoregressive moving average with external input (ARMAX), so that the one step ahead prediction error is minimized. Despite its simplicity this method is well suited for the identification of linear parametric models able to precisely predict temperature and humidity values of zones over an hourly horizon[16], conversely, buildings inner energy is less well grasped by them.

- Deterministic semi-physical modeling (DSPM): is a gray-box identification technique that makes use of a resistance-capacitance network paradigm to describe the building dynamics. An advantage of this method is to be physical meaningful. Parameters involved are identified as equivalent resistance and capacitance. This approach has been investigated in a lot of papers[17] providing very good results and taking advantage of various identification techniques such as genetic algorithms as well as neural networks.

For the purpose of this work we decided to make use of a modeling technique of the first type. According to the work presented in Kim & Braun[10] we built up a linear model making use of a mono dimensional finite volume discretization of the structure, this lead to an high dimensional linear model that has been successively reduced using proper algorithms. As Kim & Braun already shown in this way it is possible to end up with low order models (magnitude of one tenth or more of the initial one) that are however well representative of thermal dynamics up to frequencies of our interest. Needless to say that whatever model can be well suited for the control issue provided that it is affine in the control variable and that it takes explicitly into consideration disturbances, if this conditions holds, for any dimensionally comparable systems, the control can be implemented expecting similar results in terms of feasibility and performances.

1-D finite volumes model

The model is made assembling simple subsystems, the simplest subsystem composing our building is the wall. As a wall is intended a uniform and geometrically defined portion of the structure that is in contact on both sides and over its whole surface with a defined environment that could either be one conditioned zone or the outside. We consider each wall to be composed by vertical layers (”slices”), each of them with its own width and material composition that is supposed to be uniform within the volume. In this way we discretize each wall in finite volumes having the characteristic that their main thermal time constant is considerably smaller than the entire wall one: the more thin slices are the more the model will capture thermal dynamics to the detriment of the size of the model that grows. The discretization we made is mono dimensional (1-D) and resorts on the assumption that thermal flows are mono directional (i.e. heat flows only perpendicular trough the wall surface) and thus that the temperature is uniform over the slice surface. Notice that 2-D or 3-D finite volume discretization are also possible, they can capture even better the thermal dynamics in the presence of hot spots or in correspondence of wall junctions, windows and forth and son on. However, the more complications introduced by 2-D or 3-D procedures appears unjustified for the overall optimization point of view and for the purpose of this work. Thermal exchange considered is conductive through the wall volumes, convective on both the wall sides and radiative on external surfaces. No radiative exchange is considered on internal surfaces assuming that the inner side of the wall will face other bodies having almost its same temperature (null net radiative thermal exchange). We assume also the external surface to be gray and opaque: diffusivity is equal to absorbivity and equal for every direction, we will consider instead two different absorption coefficients for longwave and shortwave radiation. Next step is to write down the energy balance for each finite volume i (each slice). In order to increase systematic no different formulations are provided for inner or outer volumes, i.e. the following formulation holds for each slice in each wall and case by case some equation components will be null. The balance can be expressed as:

$$\begin{aligned} C_i\dot{T}_i = (k_i^L + h_i^L)T_{i-1} - (k_i^L + k_i^R + h_i^L + h_i^R)T_i + (k_i^R + h_i^R)T_{i+1} + \\ + Qg_i + \alpha_{S,i}Q_i^{SWR} + \alpha_{L,i}Q_i^{LWR} + Q_r(T_i) \end{aligned} \tag{2.5}$$

where:

- T_i = temperature of the node i;
- $T_{i\pm1}$ = temperature of the node that follows or precede i;
- C_i = thermal capacity of the material composing the node, it is expressed as the product of density, specific heat and width of the node;

$$C_i = \rho_i c_i w_i$$

- k_i^L = coefficient of conductive exchange on the left side of the node: it is calculated as the conductivity of the Left surface of the i^{th} node over the distance between the center of the node i and $i-1$;

$$\nexists N_{i-1} \Rightarrow k_i^L = 0$$

- k_i^R = coefficient of conductive exchange on the left side of the node: it is calculated as the conductivity of the Left surface of the i^{th} node over the distance between the center of the node i and $i-1$;

$$\nexists N_{i+1} \Rightarrow k_i^R = 0$$

- k_i^L = convective heat transfer coefficient at the left side of the node i, it is equal to zero if there is no convection acting on the left surface;
- k_i^R = convective heat transfer coefficient at the left side of the node i, it is equal to zero if there is no convection acting on the right surface;
- Qg_i = internal thermal power generation inside node$i$, it can be used for modeling radiant heating systems, however it is always zero in our work;
- $\alpha_{S,i}$ = coefficient that takes into account wall sun exposition of the wall and short wavelength absorption rate, its value can be expressed as the product of coefficients that represents respectively: shortwave absorbivity, shadowing and view factor;

$$\alpha_l = \tilde{\alpha}_s \alpha_w(t) \alpha_v(t)$$

- $\alpha_{L,i}$ = coefficient that takes into account wall sun exposition of the wall and long wavelength absorption rate, its value can be expressed as the product of coefficients that represents respectively: longwave absorbivity, shadowing and view factor;

$$\alpha_l = \tilde{\alpha}_l \alpha_w(t) \alpha_v(t)$$

- Q_i^{SWR} = incoming short wavelength radiation power (measured data);
- Q_i^{LWR} = incoming long wavelength radiation power (measured data);
- $Q_r(T_i)$ radiative emission radiation linearized around the mean emission temperature:

$$Q_r(T_i) = -4\sigma\epsilon_l \bar{T}_i^3 T_i + 3\sigma\epsilon_l \bar{T}_i^4$$

Using this complete definition (2.5) we can write down the equation governing the temperature dynamics of a generic wall n composed by m layers assuming as control variable the zones temperatures and as disturbances the other inputs:

$$\mathbf{C_w^n}\dot{\mathbf{T}}_\mathbf{w}^\mathbf{n} = \mathbf{A_w^n}\mathbf{T_w^n} + \mathbf{B_w^n}\mathbf{T_z} + \mathbf{W_w^n}\mathbf{d} \tag{2.6}$$

This matrix formulation is easily obtainable just considering the balance equation for each volumes composing the wall, according to the order:

$$\text{zone}_a \text{ - volume}_1 \text{ - } \ldots \text{- volume}_m \text{ - zone}_b$$

Notice that inner layers has no convective neither radiative energy exchange, that a zone can be the outdoor and that inner heat generation (useful to model floor heating systems) is not used in our work and thus always zero. The dynamics matrix is expected to be tridiagonal and C_w^n to be diagonal. If for example we define a wall, discretized in m layers, that has on the left side the zone number two ($\text{zone}_a = \text{zone}_2$) and that it is a boundary wall ($\text{zone}_b = \text{outdoor}$), the matrix will be:

$$\mathbf{C_w} = \begin{bmatrix} C_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & C_m \end{bmatrix}_{[m\times m]}$$

$$\mathbf{A_w} = \begin{bmatrix} -(k_1^R + h_1) & k_1^R & 0 & 0 & \cdots & 0 \\ k_2^L & -(k_2^L + k_2^R) & k_2^R & 0 & \cdots & 0 \\ 0 & k_3^L & -(k_3^L + k_3^R) & k_3^R & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & k_m^L & -(k_m^L + h_m) - 4\sigma\alpha_{L,m}\bar{T}_m^3 \end{bmatrix}$$

$$\mathbf{B_w} = \begin{bmatrix} 0 & h_1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & . & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{[m\times nz]}$$

$$\mathbf{W_w} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \\ h_m & \alpha_s & \alpha_l & 3\sigma\epsilon_l\bar{T}_m^4 \end{bmatrix}_{[m\times 4]}$$

The input vector is composed by the nz zone temperatures:

$$\mathbf{T_w^n} = \begin{bmatrix} T_1^n & \cdots & T_m^n \end{bmatrix}^T$$

Disturbances are instead all the inputs related to the environment such as the outdoor temperature, the radiation exchanges with the ground and the sky expressed by the longwave radiation term and the global radiation expressed by the shortwave. All of these factors are stochastic, they represents environmental conditions in which contest the building operates.

$$\mathbf{d} = \begin{bmatrix} T_{out} & Q_{LWR} & Q_{SWR} & 1 \end{bmatrix}^T$$

The last step consists in defining the output transformation: for our control purpose we are interested in tracking the thermal power exchange between walls and zones, this can be directly done with a proper choice of the output transformation of the model. Such a power exchange can be expressed for every wall n and every zone i as:

$$Q_{wn} = S_{wn} h_{wn} (T_{wn} - T_{zi})$$

where S_w is the wall surface and h_w is its associated internal convective exchange coefficient. The overall thermal power released or absorbed by the building is the sum of the contribution of each wall-zone exchange. The resulting output transformation takes thus the linear form:

$$\mathbf{Q_w} = \mathbf{D_w T_w} + \mathbf{E_w T_z}$$

Where D_w and E_w are suitable matrix that links walls and zones according to the shared surface. In our example they will be:

$$\mathbf{D_w} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ sh_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & . & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{[nz \times m]} \qquad \mathbf{E_w} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & -sh_1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & . & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{[nz \times nz]}$$

We can compose now all the n walls forming the building just grouping the matrix calculated for each wall. C and A composes into block diagonal matrix and B, W and D into vectors:

$$\begin{bmatrix} C_w^1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & C_w^n \end{bmatrix} \dot{T}_w = \begin{bmatrix} A_w^1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A_w^n \end{bmatrix} T_w + \begin{bmatrix} B_w^1 \\ \vdots \\ B_w^n \end{bmatrix} T_z + \begin{bmatrix} W_w^1 \\ \vdots \\ W_w^n \end{bmatrix} d$$

$$Q_w = \begin{bmatrix} D_w^1 & \cdots & D_w^n \end{bmatrix} T_w + \sum_{i=1}^{n} E_w^i T_z$$

The system is described into the canonical linear fashion:

$$\begin{cases} \dot{\mathbf{T}}_\mathbf{w} = \mathbf{A}\mathbf{T}_\mathbf{w} + \mathbf{B}\mathbf{T}_\mathbf{z} + \mathbf{W}\mathbf{d} \\ \mathbf{Q}_\mathbf{w} = \mathbf{D}\mathbf{T}_\mathbf{w} + \mathbf{E}\mathbf{T}_\mathbf{z} \end{cases} \tag{2.7}$$

As a result of the modeling process we have a linear MIMO (multi input multi output) system describing the thermal heat power transmitted by the structure to each zone with imposed zone temperatures as control inputs and outdoor temperature and solar radiation acting as disturbances.

Model order reduction

We take advantage of results in state space model reduction methods in order to generate a linear model that can be easily handled when addressing control design. Typically, computational load grows linearly with the model order and super-linearly with the length of the prediction horizon considered. Procedures of model reduction is of great interest in control engineering for the possibility to light systems controllers and easily handle models, nowadays model reduction is performed trough special algorithms appositely made up and on stage of development. Model reduction is called balanced if the full propriety of observability and reachability (controllability) of the original system is preserved. For a given transfer function $W(s)$ of McMillan's order n and chosen the desired order r, the model reduction algorithm based on Hankel's single value decomposition (HSVD) finds out the matrix $\hat{W}(s)$ such that the Hankel norm:

$$||W(s) - \hat{W}(s)||_H$$

is minimized[18] (i.e. the lower order transfer function that best fit the original one in the Hankel's sense.) The algorithm essentially truncates the states that less contributes to the system dynamics evaluating such contribution via the Schmidt's single value decomposition obtained trough the Hankel's operator. Notice that the reformulated states of a reduced system are not the same as the starting states, but they become meaningless. In Kim & Braun [10] it is shown how a physics-based reduced order model can properly work and being representative of the real dynamics, in particular the model reduction factor can be more than ten.

Model discretization

Thanks to the particular shape of the zones temperature evolution (control input) it is better to perform a first-order hold sampling discretization [33].

Consider the continuous-time system described by (2.7), if we assume that the input signal is piecewise affine in between the sampling instants its integration over one sample period gives:

$$x(k+\Delta_t) = e^{A\Delta_t}x(k)+\int_k^{k+\Delta_t} e^{A(k+\Delta_t-s)}B\left[u(k)+\frac{s-k}{\Delta_t}\Big(u(k+\Delta_t)-u(k)\Big)\right]ds$$

Hence

$$\begin{cases} x(k+\Delta_t) = \Theta x(k) + \Gamma u(k) + \frac{1}{\Delta_t}\Gamma_1\Big(u(k+\Delta_t) - u(k)\Big) \\ y(k) = Dx(k) + Eu(k) \end{cases}$$

where

$$\Theta = e^{A\Delta_t}$$

$$\Gamma = \int_0^{\Delta_t} e^{As}dsB$$

$$\Gamma_1 = \int_0^{\Delta_t} e^{As}(\Delta_t - s)dsB$$

Replacing the coordinates by $\xi = x-\gamma_1 u(k+\Delta_t)/h$ and then recalling $\xi = x$ we obtain the standard model:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Wd(k) \\ Q_w(k) = Cx(k) + Du(k) + Vd(k) \end{cases}$$

Notice that the sampled-data system has a direct term even if $E = 0$. First-order hold sampling is particularly useful when approximating continuous transfer functions by sampled systems, because a piecewise affine curve is a good approximation to a continuous function.

In figure (2.1) it is shown a simple test performed on a model with the aim to investigate the effects of order reduction and discretization process. The structure is the one used in next for test and described in subsection (2.6). The structure starts in a thermal equilibrium state and is feed by constant suitable disturbances during the whole time, at time $600s$ the temperature rises about $1K$ in 10 minutes and, as a consequence, the heat quantity released from walls varies and has been calculated. As we can see the first order hold discretization of the reduced model reproduces almost exactly the real behavior, zero order hold discretization is slightly less precise, this is due to the particular linear shape of the input that is taken into account in the first case. Even if the temperature variation would not be linear, but for example exponentially varying, it has been seen that f.o.h. is better representative of the real behavior.

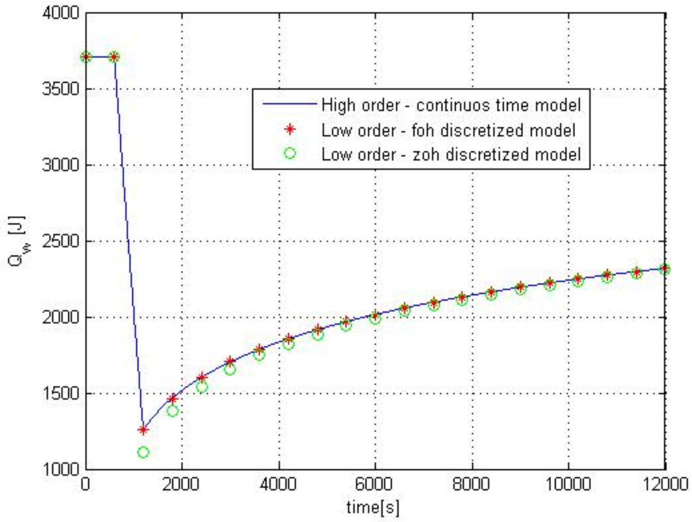

Figure 2.1: Wall response to linear temperature variation

State reconstruction

In order to implement an optimal control policy over a finite horizon we need to get access to a measure of the initial state. Unfortunately the heat exchange between zones and walls is not a measurable quantity and thus the state can not be reconstructed from measures of it. However, recalling equation (2.2) we can

$$Q_z = Q_w + Q_i + Q_p - Q_c$$

notice that the only term that is not quantifiable and actually unpredictable is the one that is related to people. The heating contribute due to people is, needless to say, null where no people is present. Example of these periods can be during the night or soon in the morning, this observation holds also for the inner energy production due to general inactivity of machines and the switch off of lights. In night periods generally, or whenever it exists period where uncertain quantities are negligible, knowing the cooling power provided and the temperature variation of the room is enough to get read of the heat exchanged with the building. From it and through disturbances measurements it is possible to reconstruct the states:

$$x(k) = C^{-1}(Du(k) + Vd(k) - Q_w(k))$$

A more precise states estimation can be derived making use of more complicated tools as the Kalman filter or taking advantage of specific temperature measurements. In a particular case, namely when the system is at a steady state and the inputs are stationary, the state can be calculated also imposing the stationarity condition:

$$\bar{x} = (I - A)^{-1}(B\bar{u} + W\bar{d})$$

This is however an unrealistic condition for the building since it will generally never reach a real equilibrium condition due to non stationarity of involved inputs.

Wall-zone energy exchange

As discussed in section (2.2.1) it is possible to describe thermal interaction in between a building and living zones making use of a multiple input multiple output linear discrete time model:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Wd(k) \\ Q_w(k) = Cx(k) + Du(k) + Vd(k) \end{cases}$$

Inputs are the temperatures of zones and outputs are the thermal power exchanged zone by zone assumed to be positive when heat flows from the structure to the zone. Disturbances are also involved, namely the outdoor temperature and the value of shortwave and longwave radiation incident on the external surfaces. Recall that, due to state order reduction methods involved, states are physically meaningless and in particular they are not representative of temperature values. The difference equation of the system can be iteratively solved explicitly for M samples, here some iterations:

$$Q_w(1) = CAx_0 + CBu_0 + CWd_0 + Du(1) + Vd(1) \tag{2.8}$$

$$\begin{aligned} Q_w(2) = CA^2x_0 + CABu_0 + CAWd_0 + \\ + CBu(1) + CWd(1) + Du(2) + Vd(2) \end{aligned} \tag{2.9}$$

$$\begin{aligned} Q_w(3) = CA^3x_0 + CA^2Bu_0 + CA^2Wd_0 + CABu(1) + \\ + CAWd(1) + CBu(2) + CWd(2) + Du(3) + Vu(3) \end{aligned} \tag{2.10}$$

We can describe the evolution of our system over the time defined by M samples in a matrix compact way simply defining the following vectors of state, initial conditions, inputs and disturbance signals:

$$\mathbf{Q_w} = \begin{bmatrix} \mathbf{Q}_w(0) \\ \mathbf{Q}_w(1) \\ \vdots \\ \mathbf{Q}_w(M) \end{bmatrix} \quad \mathbf{u} = \begin{bmatrix} \mathbf{T_z}(1) \\ \mathbf{T_z}(2) \\ \vdots \\ \mathbf{T_z}(M) \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} \mathbf{d}(1) \\ \mathbf{d}(2) \\ \vdots \\ \mathbf{d}(M) \end{bmatrix}$$

where $\mathbf{T_z}(\tau)$ is a vector containing all the temperature references at time τ of each zone (assumed that for reference temperature is intended the final temperature that as to be reached by the zone at the end of the period):

$$\mathbf{T_z}(\tau) = [T_{zone_1}(\tau), \cdots, T_{zone_n}(\tau)]^T$$

and where $\mathbf{d}(\tau)$ is a vector containing all the disturbances acting on the system at the instant time τ (disturbances are assumed to be piecewise constant over the sample time):

$$\mathbf{d}(\tau) = [T_{out}(\tau), Q_{LWR}(\tau), Q_{SWR}(\tau), 1]^T$$

The vector of the initial condition brings information about the starting values of states, temperatures of each zones and disturbances:

$$\mathbf{\bar{x}_0} = \begin{bmatrix} \mathbf{x_0} & \mathbf{u_0} & \mathbf{d_0} \end{bmatrix}^T$$

Resuming, the system evolution over M samples can be expressed in a compact fashion as an affine system in temperature references and disturbances

$$\mathbf{Q_w} = \mathbf{\bar{F}\bar{x}_0} + \mathbf{\bar{G}u} + \mathbf{\bar{H}d}$$

merely defining matrix:

$$\mathbf{\bar{F}} = \begin{bmatrix} C & D & V \\ CA & CB & CW \\ CA^2 & CAB & CAW \\ \vdots & \vdots & \vdots \\ CA^M & CA^{M-1}B & CA^{M-1}W \end{bmatrix}$$

$$\mathbf{\bar{G}} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ D & 0 & \cdots & 0 & 0 \\ CB & D & \cdots & 0 & 0 \\ CAB & CB & \cdots & 0 & 0 \\ \vdots & \vdots & . & \vdots & \vdots \\ CA^{M-2}B & CA^{M-3}B & \cdots & CB & D \end{bmatrix}$$

$$\bar{\mathbf{H}} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ V & 0 & \cdots & 0 & 0 \\ CW & V & \cdots & 0 & 0 \\ CAW & CW & \cdots & 0 & 0 \\ \vdots & \vdots & . & \vdots & \vdots \\ CA^{M-2}W & CA^{M-3}W & \cdots & CW & V \end{bmatrix}$$

The zone-wall heat exchange must then be integrated over the period Δt in order to obtain the energy value. Hidden dynamics exists because of under sampling of Q_w and, even if their contribution is lost, we can faithfully approximate Q_w to be linear varying within the sampling period and with this assumption directly calculate the exchanged energy using the trapezius method. This is it thanks to the imposed linear variation for temperatures and resorting on the first order hold discretization of the system (figure(2.1)). In figure(2.2) it is depicted the energy error introduced approximating the wall-zone energy exchange making use of the trapezius method for the same example as in (2.2.1), in confirmation of our suppositions we can see that the error is really low and vanishing, less than one percent energy error is introduced for the first sample period. We have to rewrite the system in

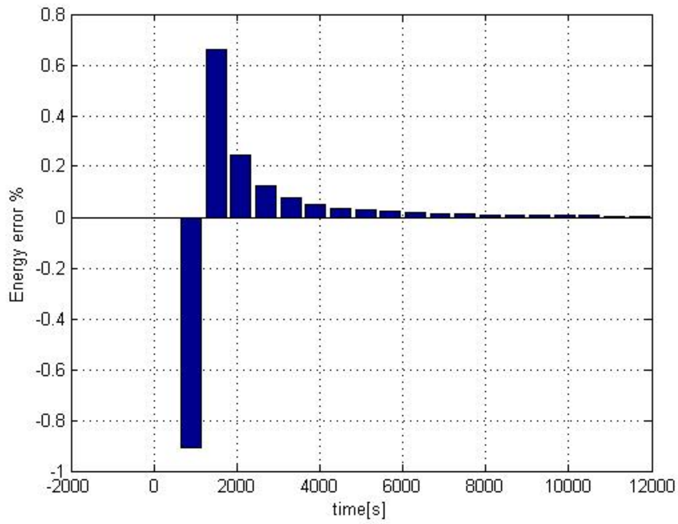

Figure 2.2: Error introduced by trapezius method.

order to explicitly express the energy quantity:

$$\mathbf{E_w}(\tau) = \frac{\Delta t}{2}(\mathbf{Q_w}(\tau) + \mathbf{Q_w}(\tau + 1))$$

making use of the affine vectorial expression:

$$\mathbf{E_w} = \mathbf{F\bar{x}_0} + \mathbf{Gu} + \mathbf{Hd} \tag{2.11}$$

$$\mathbf{F} = \frac{\Delta t}{2}\begin{bmatrix} C+CA & D+CB & V+CW \\ CA+CA^2 & CB+CAB & CW+CAW \\ CA^2+CA^3 & CAB+CA^2B & CAW+CA^2W \\ \vdots & \vdots & \vdots \\ CA^{M-1}+CA^M & CA^{M-2}B+CA^{M-1}B & CA^{M-2}W+CA^{M-1}W \end{bmatrix}$$

$$\mathbf{G} = \frac{\Delta t}{2}\begin{bmatrix} D & 0 & \cdots & 0 \\ D+CB & D & \cdots & 0 \\ CB+CAB & D+CB & \cdots & 0 \\ CAB+CA^2B & CB+CAB & \cdots & 0 \\ \vdots & \vdots & . & \vdots \\ CA^{M-3}B+CA^{M-2}B & CA^{M-4}B+CA^{M-3}B & \cdots & D \end{bmatrix}_{[M\cdot nz\times M\cdot nz]}$$

$$\mathbf{H} = \frac{\Delta t}{2}\begin{bmatrix} V & 0 & \cdots & 0 \\ V+CW & V & \cdots & 0 \\ CW+CAW & V+CW & \cdots & 0 \\ CAW+CA^2W & CW+CAW & \cdots & 0 \\ \vdots & \vdots & . & \vdots \\ CA^{M-3}W+CA^{M-2}W & CA^{M-4}W+CA^{M-3}W & \cdots & V \end{bmatrix}_{[M\cdot nz\times M\cdot nz]}$$

Through formulation (2.11) we express the thermal energy contribution due to heat exchange with the building.

Sample-variant model composition

Matrix in expression (2.11) had been composed considering the system to be time invariant. In a deeper and more detailed description it could be not. For two many reasons: the first one is related to the main radiative temperature of the external surface around which the Plank's equation had been linearized. Simulations showed that this is a relevant parameter in the model. The second is related to the external convective exchange coefficient that is tightly related to the wind strength and in general with the air motion and composition (humidity conditions can be relevant too). A more detailed model could compose the horizon matrix considering mean radiative temperature and convective exchange coefficient nominal profiles describing thus a time variant system. In order to do formulate the equations we can consider some iterations of the state difference equation:

$$X(1) = A_0X_0 + B_0u_0$$

$$X(2) = A_1 A_0 X_0 + A_1 B_0 u_0 + B_1 u(1)$$
$$X(3) = a_2 A_1 A_0 X_0 + A_2 A_1 B_0 + A_2 B_1 u(1) + B_2 u(2)$$

In general the following general expression can be derived:

$$X(n) = \left(\prod_{i=1}^{n-1} A_i\right) X_0 + \sum_{j=0}^{n-1} \left(\prod_{i=j+1}^{n-1} A_i\right) B_j u(j) \tag{2.12}$$

$$Q(n) = C_n \left(\prod_{i=1}^{n-1} A_i\right) X_0 + \sum_{j=0}^{n-1} \left(C_n \prod_{i=j+1}^{n-1} A_i\right) B_j u(j) + D_n u(n) \tag{2.13}$$

According to these relations it is possible to fill newly the matrix **F**, **G**, **H** ending up with a sample-variant model. For the sake of simplicity we will not implement this solution in our work even if it remains a suitable procedure to increase generality and precision of the problem formulation.

Convective heat exchange coefficient

The convective heat exchange of the exterior building surface related to the air flow along it is usually modeled by the convective heat transfer coefficients (CHTCs) whom links the convective heat flux normal to the wall to the difference between the surface temperature of the wall and a reference temperature. As reference the outside T_∞ temperature, correspondent to the measured temperature, is generally assumed. In this way the convective heat flux is expressed by the equation:

$$Q_h = h(T_s - T_o) \tag{2.14}$$

CHTCs for buildings are often correlated to the wind speed at a reference location (i.e. the mean wind speed 10 meters high from ground). Especially the correlations derived by Jürges[19] have been used extensively for building applications: a linear correlation exists at low speeds (also accounting for buoyancy effects) and a power-law correlation holds for forced convection:

$$h = 4.0 W_s + 5.6 \quad W_s < 5m/s$$
$$h = 7.1 W_s^{0.78} \qquad W_s > 5m/s$$

Thereby it is difficult to choose a suitable value for the wind speed since it will be representative of a single and almost arbitrarily chosen point (where data is collected). Nevertheless, a lot of valuable information about CHTCs was obtained from flat-plate experiments, for example the influence of surface roughness on it, and the obtained correlations were for a long time considered sufficient for practical purposes. Many information about convective energy transfer as well as more detailed models can be found in literature[20].

2.2.2 People energy contribution

Occupancy imply heat production and in crowded places this can be a critical factor: think for example to hospitals, offices or wherever many people lives all together. Some of these places are not suitable for application of optimal thermal regulation (optimal thermal control of undergrounds could be unfeasible or even meaningless) despite that, even where occupancy is hardly quantifiable, the thermal energy produced by people has huge importance and it can and should be taken into account. Heat produced by people is often considerable, and if it is, it can even be tapped: for example it has been implemented in a train station in Stockholm a system that makes use thermal energy produced by passers to warm the commercial building aside the station[21], this technology allows 25% electricity savings to heat it. In many places as for example theaters, university classes, canteens, and many others, the occupancy (expressed in number of people and time of occupancy) can be estimated providing useful information: for example being aware of the number of people expected to attend a lecture may be useful allowing considerations about cooling policy (i.e. precooling or energy storage). Another way to make use of occupancy information is that to relax or not comfort constraints: if some rooms in a huge hotel are not booked there may be convenience to erase comfort constraints in order to save.

People heat production is directly related to the body surface area and it is expressed by radiation, convection, and evaporation. Conduction alone is usually insignificant and often taken into account only in combination with convection, a very important form of heat loss. They are both function of body surface area, dry bulb air temperature, and the heat transfer coefficient, which also depends on the ambient air motion (2.2.1). The radiative heat transfer in between two objects is independent of the dry bulb temperature but related to the objects temperature and to the properties of their surfaces only. Water evaporation is a very important means of heat loss too, the latent heat of evaporation of water at normal body temperature is $0.58 kcal/g$, so that with the evaporation of each gram of water from the body surface, the body loses $0.58 kcal$ of heat. This water loss occurs mostly through sweating, but also through water that is breathed out or diffuses through the skin. The rate of evaporation depends on the relative humidity (RH) of the air, and only occurs when $RH < 100\%$[22]. In order to quantify how much heat does the body produces the Basal Metabolic Rate (BMR) was introduced, it is defined as the heat production of a human in a thermoneutral environment (33^oC) at rest mentally and physically more than 12 hours after the last meal. The standard BMR for a 70 kg man is approximately 1.2 W/kg, but it can be altered by changes in active body mass, diet, endocrine levels. Summing up all of these considerations it is clear that too many factors are involved to make a deep estimation, but, for the purpose of our work, it is enough to consider the relationship in between environmental temperature and produced heat. The most used model in that sense express the overall people heat production as the non linear function of temperature and occupancy profile:

$$Q_p(t) = n_p(t)(p_1 T_z(t)^2 + p_2 T_z(t) + p_3)$$

Parameter values are:

$$p_1 = -0.2199$$

$$p_2 = 125.125$$

$$p_3 = -1.7685 \cdot 10^4$$

Since the temperature follows the relation (2.1) and, as the occupancy profile, is linear varying, the expression can be rewritten as:

$$Q_p(t,\tau) = \left(n_p(\tau) + (n_p(\tau+1) - n_p(\tau))\frac{t}{\Delta t}\right)\left(p_1\left(T(\tau) - (T(\tau+1) - T(\tau))\frac{t}{\Delta t}\right)^2 + \right.$$
$$\left. +p_2\left(T(\tau) - (T(\tau+1) - T(\tau))\frac{t}{\Delta t}\right) + p_3\right) \tag{2.15}$$

and thus the thermal energy produced by people in the period can be expressed with the non linear function of temperature and occupancy profile:

$$E_p(\tau) = \int_0^{\Delta t} Q_p(t)dt$$

$$E_p(\tau) = a_i T(\tau)^2 + b_i T(\tau+1)^2 + c_i T(\tau) + d_i T(\tau+1) + eT(\tau)T(\tau+1) + f$$

where:

$$a_i = -\Delta t p_1\left((n(\tau+1) - n(\tau))\frac{1}{12} + \frac{1}{3}n(\tau)\right)$$

$$b_i = -\Delta t p_1\left((n(\tau+1) - n(\tau))\frac{1}{4} + \frac{1}{3}n(\tau)\right)$$

$$c_i = -\Delta t p_2\left((n(\tau+1) - n(\tau))\frac{1}{6} + \frac{1}{2}n(\tau)\right)$$

$$d_i = \Delta t p_2\left((n(\tau+1) - n(\tau))\frac{1}{3} + \frac{1}{2}n(\tau)\right)$$

$$e_i = -\Delta t p_1 \Bigg((n(\tau+1) - n(\tau))\frac{1}{6} + \frac{1}{3} n(\tau) \Bigg)$$
$$f = \Delta t p_3 \Bigg((n(\tau+1) - n(\tau))\frac{1}{6} + n(\tau) \Bigg)$$

We have now to extend this formulation in order to take into considerations many zones. The main solution should be to give an occupancy profile zone by zone. With the aim of ease the computational effort in the stochastic optimization process that we will face we do not make use of a probabilistic description of the occupancy of every single zone in the building, but we introduce many distribution coefficients α_z instead. These coefficients describes the spatial distribution of people inside the building and can even be time-varying: zones that are supposed to be more crowd will have higher coefficient, provided that the sum of them is one. Probabilistic description can be however be formulated for different spaces or areas (each of them may include more zones) that are different by nature as for example commercial and offices areas of the same building, with this description we have high degree of generality in modeling the people heat production.

$$\begin{bmatrix} n_{p,1} \\ n_{p,2} \\ \vdots \\ n_{p,n} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} n_p \quad s.t. \quad \Sigma_{i=1}^{n} \alpha_i = 1$$

Expressing $\bar{n}_p := \alpha \cdot n_p$ and evaluating the coefficients accordingly with the bar notation we can express the problem of estimating the thermal energy produced from people over the time horizon of M sample period as a function of the desired profile temperature in the compact way:

$$P_p^{(M)} = \mathbf{u}^T \mathbf{M}(n_p)\mathbf{u} + [1, \cdots, 1](\mathbf{N}(np)\mathbf{u} + \mathbf{P})$$

where:

$$M = \begin{bmatrix} b_1 + a_2 & \frac{1}{2}e_2 & 0 & & \\ \frac{1}{2}e_2 & b_2 + a_3 & \frac{1}{2}e_3 & \ddots & \\ 0 & \frac{1}{2}e_3 & b_3 + a_4 & & \\ & \ddots & \ddots & \ddots & \frac{1}{2}e_M \\ & & & \frac{1}{2}e_M & b_M \end{bmatrix}$$

$$N = \begin{bmatrix} d_1 & 0 & & 0 \\ c_2 & d_2 & 0 & \\ 0 & \ddots & \ddots & \ddots \\ & & c_M & d_M \end{bmatrix} \quad P = \begin{bmatrix} f_1 + c_1 T_0 + a_1 T_0^2 \\ f_2 \\ \vdots \\ f_M \end{bmatrix}$$

Unfortunately, despite his accuracy, this formulation shows up to be concave, thus not easy to optimize and not suitable for our control purposes. However, having a look at the non linear expression of the internal thermal power production we can observe that its behavior is almost linear in the range we are interested in (figure 2.2.2), we can claim that since during occupancy time the zone temperature is enforced to lie inside the comfort range that is approximately $20 \div 24^oC$. The linearized expression around the average temperature $\bar{T}_z$ is:

$$Q_p \approx n_p(2p_1\bar{T}_zT_z + p_2T_z - p_1\bar{T}_z^2 + p_3) = n_p(\tilde{p}_2T_z + \tilde{p}_3)$$

where

$$\tilde{p}_1 = 0$$

$$\tilde{p}_2 = 2p_1\bar{T}_z + p_2 = -4.6159$$

$$\tilde{p}_3 = p_3 - p_1\bar{T}_z^2 = 1.4515 \cdot 10^3$$

It is possible to reuse the procedure described above just substituting tilde values in it, as a results the **M** matrix will be null and the internal energy produced by people will be described by the linear function of the zone temperatures:

$$\mathbf{E_p} = \mathbf{N}(np)\mathbf{u} + \mathbf{P} \tag{2.16}$$

Notice that the matrix **N** depends on the occupation profile.

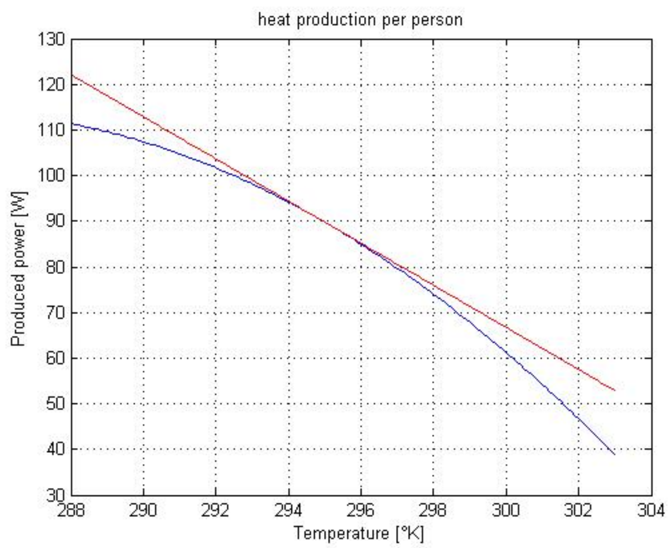

Figure 2.3: Nonlinear and linearized human heat production function

2.2.3 Internal energy contribution

Many other types of heat sources can be present in a building, what we want to model here are just the simplest. For some of them it is possible to quantify their heat contribution thanks to their continuous and uniform availment, others may be instead related to occupancy or business needs: it is the case of machinery and industrial devices. Another relevant internal gain factor, different by nature, is related to the radiation absorbed trough windows. These two phenomena will be addressed in this section.

Light and devices

- Energy emission by electric lamps: a large part energy consumed by lamps is emitted as heat (about 95% for ordinary incandescent lamps and 79% for fluorescent lamps) and the remaining just a little part is emitted as light, which, moreover, when incident on surfaces is converted into heat too. Consequently, almost the total wattage of all lamps in the building when in use, must be added to the Q_i.

- The heat gain due to appliances (machinery, computers, etc.) should also be added to the Q_i. If an electric motor and the machine driven by it are for example both located (and operating) in the zone, a term proportional to the wattage of the motor must be included in order to estimate the percentage of heat energy release into the space. Most of the technological transformations (welding, drilling, milling, ...) produces heat, most of electronic devices produces heat too.

Radiation trough windows

The use of daylight as the primary light source in building is not just assumed to minimize the use of electricity for lighting but mainly for the beneficial effects in terms of workspace productivity and human health [23]. Physically, daylight is just another source of electromagnetic radiation in the visible range and thus tightly related to the Shortwave radiation. Its effect is very consistent so that many strategies had been developed to contain it, the simplest is to make use of window blinds: when they are used regulation is often human made and generally carried out to avert the discomfort produced by direct radiation and over-light, even if effective this technique is unpredictable and not much advisable for the workspace amenity point of view. The development of new glazing materials provides solutions for the uncontrolled influx of solar energy modulating the passage of radiation by intercepting part of it and allowing only a certain component to pass in order to guarantee visual comfort[24].

With this term the radiative thermal fluxes trough windows is thus taken into account. Since we have not access to experimental models of high-tech glazed facades we consider the most of the shortwave radiation incident windows surfaces to be absorbed by rooms, in fact we make use of high transmissivity coefficients for glasses assuming that the most of radiation concentrated on the visible spectra can reach the inside environment and gets absorbed by the air directly or by walls and objects.

$$Q_i(t) = \alpha_s \Sigma_i (A_i \tau_i K_i(t)) Q_{SWL}(t)$$

where some simple but conceptually meaningful parameters are considered:

- α : mean absorptivity of the space (≈ 0.9);
- τ_i : transmissivity of window i (≈ 0.86);
- A_i : area of the i^{th} window;
- K_i : coefficient that takes into account radiation incidence, sun view factor and shading effect for the window. This coefficient varies both over the day and the year;
- Q_{SWL} : shortwave solar radiation value $[W/m^2]$;

Yet data suggest that more than 30% of all energy use in building may nevertheless be attributed to undesirable heat transferred through windows and to artificial lighting. The energy related to this contributions is expressed on future through the vector:

$$\mathbf{E_i} = \begin{bmatrix} E_i(1) & \cdots & E_i(M) \end{bmatrix}^T \tag{2.17}$$

2.2.4 Zone dynamics energy contribution

In the process of heating or cooling a zone we have to consider the amount of energy that is related to the thermal inertia of the air inside the zone and the zone itself. This contribution can be written as:

$$-C_z \frac{T_f - T_i}{\Delta t}$$

where C_z represents the heat capacity of the zone. It is straightforward to rewrite this equation in the compact matrix formulation in order to consider many zones and to express the energy quantity over the whole prediction time (M samples of width Δt):

$$\mathbf{E_z} = \mathbf{Zu} + \mathbf{U} \tag{2.18}$$

where

$$Z = \begin{bmatrix} -C_z & 0 & & \\ C_z & -C_z & 0 & \\ 0 & C_z & -C_z & 0 \\ \ddots & \ddots & \ddots & \ddots \end{bmatrix}_{[M\times M\cdot n]} \qquad U = \begin{bmatrix} C_z T_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{[M\times 1]}$$

Estimating the Heat Capacity of a zone

To find a value of the heat capacity of a room we will need to take two main contributing factors into account:

- Heat capacity of the wet air within the zone: its specific heat depends on many factors, the main one is the humidity value that describes the quantity of steam mixed with dry air. We consider humidity to be treated by the air conditioning system so that its value can be considered stationary at a comfortable level. With this assumptions the steam content is almost constant and the specific heat coefficient of wet air can be assumed to be:

  $$c_{wa} \approx 1870 \frac{J}{KgK}$$

  Another approximation is introduced considering the air density to be constant even if it is temperature dependent in real:

  $$\rho_{wa} \approx 1.2 \frac{Kg}{m^3}$$

  At this point, defining V the volume of the zone, its heat thermal capacity can be calculated as:

  $$C_{z,wa} = \rho_{wa} \cdot c_{wa} \cdot V$$

- Heat capacity of the stuff contained into the zone (belongings, furniture, etc...). Referring to the concept of thermal mass that describes how the mass of buildings provides "inertia" against temperature fluctuations (sometimes known as the thermal flywheel effect) for calculating the contribution of the stuff we approximate the specific heat capacity of all of objects to be of the similar of that of water:

  $$c_s \approx 4200 \frac{J}{KgK}$$

Moreover if we assume that the volume occupied by the stuff is of the order of 1% of V and that the stuff average density is 1000 times bigger than air one we have:

$$C_{z,s} \approx 20 C_{z,wa}$$

We will then assume the following expression for the heat capacity of the zone:

$$C_z \approx 20 \cdot \rho_{wa} \cdot c_{wa} \cdot V \approx 44880 \cdot V$$

2.3 Thermal storage

In this work a technological component that can accumulate, store and release thermal energy is called thermal storage. There are several different kinds and several designs for thermal energy storage (shortly TES), the most common and easiest TES can be seen as tanks containing some thermodynamic fluid at some temperature. Energy is pumped inside in the sense that the contained fluid is cooled down (or heated if we want to store heat), stored through the liquid heat capacity and drawn through energy exchange with some cooling system fluid. Therefore many different implementations exists according to different destinations of use: there may be differences between Heat Thermal Energy Storages and Cold Thermal Energy Storage especially when the fluid is supposed to work in a range of temperatures for which a phase transition happens (the so called Latent TES) or not (Sensible heat TES). Many different fluids can thus be used: from water (near solidification conditions or not) to Phase Change Materials (PCM), each with its own applications, strengths and drawbacks. We simply refer to a generic controlled water Thermal Energy Storage subsystem simply characterizing it through some physical meaningful parameters and his operating modes. The thermal storage operates in just three different modes:

- charging mode;
- storing mode;
- discharging mode;

Parameters that qualifies the storage are instead:

- Maximum energy content S_{max};
- Minimum energy content S_{min};
- Losses coefficient a;

- Maximum discharging rate S_d;
- Maximum charging rate S_c;

The water is extensively used as thermodynamic fluid in thermal storages since it has a good specific heat capacity, it is cheap and easy to find in nature and furthermore it can be used both for cold and heat thermal storages. Water has an intrinsic tendency to stratify due to buoyancy forces that tend to create different layers of water at different temperatures: the coldest layers lay at bottom of the storage while the warmest ones move to the top, due to their different specific weights. Cold water gets in (during the charging phase) or gets out (during the discharging phase) from the bottom of the tank while warm water does it from the top. As a result of this stratification process two different water blocks at almost constant but different temperatures get formed. In between of them a region, called thermocline region, appears where the temperature of water changes steeply. The more the thermocline region is narrow, the better the two blocks are insulated between each other and more it is possible to extract energy from the storage[25]. Mixing among water blocks is promoted by:

- Turbulent flows entering or leaving the storage: designers' effort is intended to avoid them, in some solution heat exchangers are directly plugged inside the tank;
- Inlet temperature variations: the optimal working temperature of the chillers thermovector fluid is however almost constant, there is convenience to maintain it stationary;
- Small temperature difference between the cold and the warm layer: in some solutions the 'non-storing block' is maintained trough exchange at a fixed temperature (i.e. the external environment one) in order to maintain stratification;
- Small height/diameter ratio of the tank: a high ratio improves the stratification but increases also the external surface of the tank and related losses due to heat exchange with the external environment (if the the storage volume is kept fixed);

Thermal storages are becoming widely used in medium size grids thanks to the possibility both to dump energy consumption for thermal regulation and for the possibility to optimize it, moreover using them is in some case the only way to take advantage of renewable as for example thermal solar and geothermic or to easy plug in electrical solar energy via thermal energy transformation. Their importance is even more evident in a smart grid context where electrical energy management and regulation are critical issues. In a cooling scenario for example, the presence of a CTES in a microgrid can yield many advantages:

- Shift the production of cooling energy to electrical consumption off-peak hours;
- Allow chillers to operate at high-efficiency conditions.
- Limit energy peaks power request with benefits both for power production and distribution network systems (i.e. to light grid worse case load conditions);

A thermal storage is usually used in practical according to several strategies:

- If its storing capacity is big enough to supply energy all over the conditioning period during the day (i.e. during the occupation time of a building) it can be defined a working period in which the storage provides all the energy needs and a off period in which the storage is completely charged in the optimal way (i.e. during the night at the maximum efficiency of chillers). The main advantages of this strategy are that electrical energy consumption is completely avoided during the day,that a lower chillers power size is needed and that the scheduling logic is easier. Despite all these characteristics, drawbacks are the needs of storage to be over dimensioned in order to allow this policy to be robust, problematics related to the size of the storage arises since it may be excessive for cost or technical reasons. Moreover store energy means to face energy losses that usually are proportional to the quantity of stored energy itself. Last but not least is the fact that the thermal energy needs are not equally distributed over the year but the system has to be dimensioned over the peak period even if normal working load if far lower from it, this may lead to important system efficiency losses.
- The chillers plant can be forced to work according to an on-off logic being idle or working in its most efficient conditions: using this policy we do not discriminate energy consumption over the day and we make use of storage just as a support, we can hardly predict electrical energy consumption and, moreover, it is very difficult to size both chillers and storage in order to pursue process optimization. Notice that storage does not take part directly and that that no scheduling is generally needed.

- In a third case no predefined working conditions is implemented: the thermal capacity is limited but, nonetheless, the chillers contribution is lowered as much as possible during peak-hours making use of a flexible scheduling policy that interchange operating modes and duty rates. This is the most complicated operating way since it requires a cost estimation but it is also the most suitable for control design. Notice that the previous two policies can be seen as subcases of this one: when respectively the overall thermal needs are less than the storage energy capacity and when cooling needs are really close to the optimal chiller operating conditions.

Modeling

It follows a brief overview on TES modeling:

- The simplest model available is the *fully mixed* where the temperature in the storage is supposed to be homogeneous, this model is described through the differential equation:

$$C_s \frac{dT_s}{dt} = \dot{m} c_p (T_i - T_s) - K_{out}(T_s - T_o)$$

  where C_s denotes the thermal capacity of the storage, T_s the temperature of the fluid, $\dot{m}$ the mass flow and K_{out} the heat exchange coefficient of the external surface with the environment.

- The *fully stratified* model describes the storage so as to be divided in a certain number of different and non-mixing layers, each one of them has different temperature and different heat exchange rate with the external environment but none with adjacent layers of water. If an inlet water temperature variation happens, then the entering block of water occupies the position where its temperature is the closest to adjacent layers;

- The model proposed by *Sharp*[26] is a stratified one. Unlike the fully stratified one, the heat exchanges between adjacent layers are here taken here into account. Even if the turbulence effect is not explicitly considered, increasing the number of layers it is possible to obtain temperature profiles that are similar to the ones that does it.

- *Gahajar*'s model[27] considers a constant inlet water temperature but taking into account the effects of turbulence: this is done varying the diffusivity coefficient depending on storage geometric and thermodynamic characteristics like the fluid flow, Reynolds and Richardson numbers and the shape and position of the inlet. Gahajar's model is one that better predicts the shape of the thermocline in the conditioning context.

To incorporate a detailed model of the thermal storage in our work is by the way unfeasible for two reasons: the first one is because to grasp the inner dynamics of the storage will require a much shorter sample time compared to other relevant dynamics involved as for example occupancy or building heat transportation are. This will inexorably lead to an extremely expensive computational cost. The second motivation is related to major (and unjustified) difficulties introduced for the optimization procedures. We highly resort instead on the separation principle on the same way we do for the chillers description: we consider the thermal storage to be a controlled subsystem where some kind of optimization logic (that makes use of look-up tables perhaps) and low level controllers regulates the system properly, as a result the system may be considered as a black box having as input the energy request, as output the energy drawn and as state the energy content. For our point of view it does not matter *how* the energy is provided (i.e. exchange temperature in between thermovector and thermodynamic fluids, mass flow rate, ...) but just *how much* we can resort on. The easiest model consistent with this description is an autoregressive exogenous model ARX(1,1):

$$Q_s(\tau+1) = aQ_s(\tau) + s(\tau) \tag{2.19}$$

where Q_s is the stored energy, a is a coefficient to introduce losses and s is the inlet or outlet energy according to his sign. The evolution over M sample times of the storage energy content can be expressed in the compact way:

$$\begin{bmatrix} Q_s(1) \\ Q_s(2) \\ \vdots \\ Q_s(M) \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ a & 1 & \cdots & 0 \\ \ddots & \ddots & \ddots & 0 \\ a^{(M-1)} & a^{(M-2)} & \cdots & 1 \end{bmatrix} \cdot \begin{bmatrix} s(0) + aQ_s(0) \\ s(1) \\ \vdots \\ s(M-1) \end{bmatrix}$$

2.4 Chiller

As almost every technology also an HVAC plant has his maximum efficiency in correspondence of a nominal working load, the so called working point. When the system operates in different conditions in respect to the nominal one the performances may decrease quickly introducing as consequence an overconsumption. This effect must be taken into account, otherwise the entire optimization process would be nullified by such factors. The power absorbed from chillers can be expressed according to the Ng-Gordon formulation as:

$$Q_l = \frac{a_1 T_o T_{cw} + a_2(T_o - T_{cw}) + a_4 T_o Q_c}{T_{cw} - a_3 Q_c} - Q_c$$

The Ng-Gordon model[28] is based on both entropy and energy balances and thus incorporates both the first and the second laws of thermodynamics, the performance equation is expressed through physically meaningful parameters: apart from the cooling power, the absorbed power depends either from the stochastic variable T_o (outdoor temperature) and from T_{cw} that is the temperature of the cooling water. The latter can be assumed to be regulated by low level controllers so that it is maintained almost at prescribed fixed value, the first is instead more relevant for our description since it is already a disturbance acting on the building model but we will keep it fixed resorting to its limited importance. To find out an expression for the energy absorbed in the period is however difficult due to non linearities that has to be integrated over the time, to overcome the problem we decided to approximate cooling power to his average value over the period:

$$E_l = \frac{a_1 T_o T_{cw} \Delta_t + a_2(T_o - T_{cw})\Delta_t + a_4 T_o E_c}{T_{cw} - \frac{a_3}{\Delta_t} E_c} - E_c \tag{2.20}$$

With the aim of optimize we need a suitable convex formulation that would be not much computationally heavy, quadratic forms and quartic appears to be the most qualified candidates, in particular we look to the quartic form:

$$E_l = c_1 E_c^4 + c_2 E_c^2 + c_3 \tag{2.21}$$

To choose the best interpolating coefficients the weighted least squares technique had been used trying the best fit in the most critical point like around zero and near the best COP values, coefficients are easily calculated trough normal equations:

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = (T^T T)^{-1} T^T \begin{bmatrix} E_{l,1} \\ \vdots \\ E_{l,n} \end{bmatrix}$$

$$T = \begin{bmatrix} E_{c,1}^4 & E_{c,1}^2 & 1 \\ \vdots & \vdots & \vdots \\ E_{c,n}^4 & E_{c,n}^2 & 1 \end{bmatrix}$$

In figure (2.5) we can see that the quartic approximation fits very well the real function all over his definition range and especially where the COP

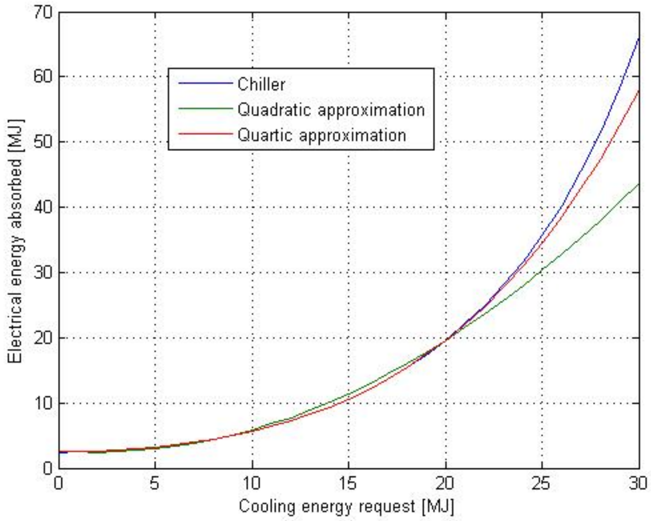

Figure 2.4: Chiller efficiency

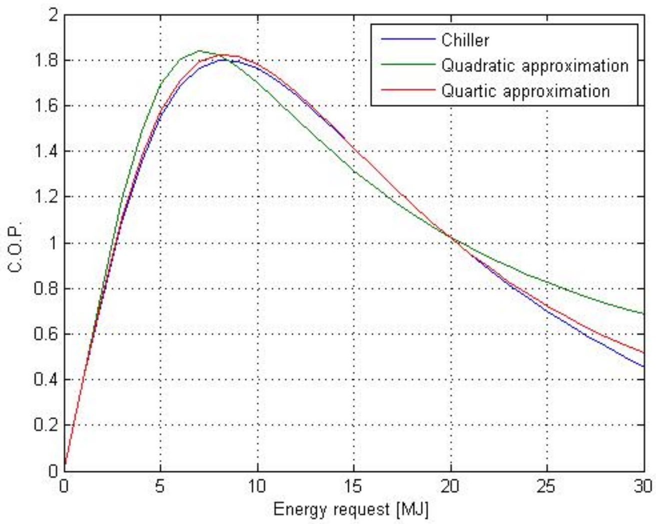

Figure 2.5: Chiller COP

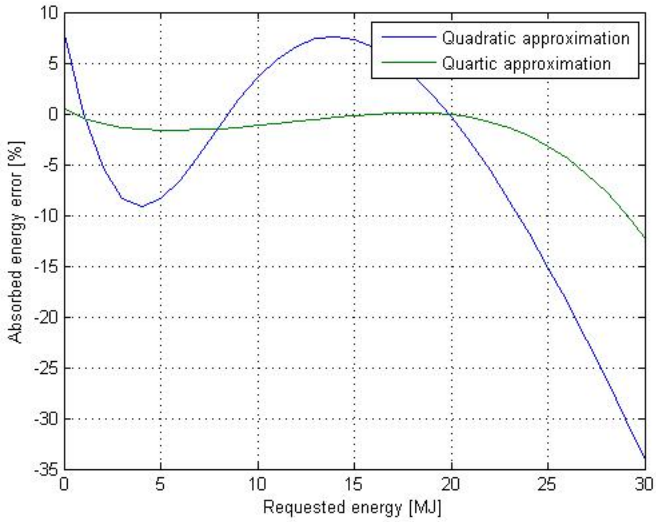

Figure 2.6: Chiller error

assumes his higher values, in figure (2.6) the error percentage is depicted showing again how the curve is well approximated. On figures is also plotted the second order polynomial approximation that despite its simplicity lacks in precision. Last, for big cooling systems chillers bench are normally employed: this choice allowed to increase modularity and performances by the means of optimal cooperation policies obtained through ad hoc algorithms or making use of lookup tables. This kind of optimization can be considered to be at a lower level in respect to ours, that works considering an equivalent description of the optimized chiller bench, namely a single equivalent chiller.

2.5 Disturbances

Disturbances acting on the system are of two different types differentiated according to the way they interacts with zones. Environmental disturbances are outdoor temperature and radiation quantities, they do not influence directly the zones temperatures but indirectly by the means of the building structure. Occupancy and inner energy production, on the contrary, directly act within zones making necessary a prompt compensation reaction from the cooling system.

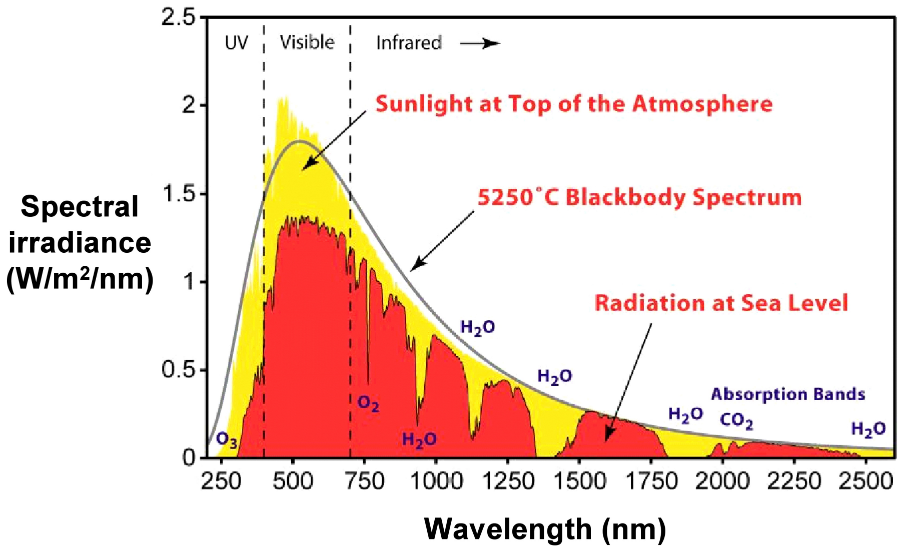

Figure 2.7: Solar spectrum and atmospheric filtering

2.5.1 Solar Radiation

The major Earth's energy source is solar radiation that represents almost all the energy entering the atmosphere, other contributes as for example the one of internal Earth's heat are surely negligible. This radiation is generated by the Sun and propagates trough space as electromagnetic wave at speed of light. It is described by his wavelength λ and frequency f , the relation $\lambda f = c$ holds. Solar radiation has components at every energy level, in fact it covers the whole electromagnetic spectrum from γ to x-ray, from UV to visible and IR, until micro and radio waves, by the way, solar radiation is received on the Earth's surface after undergoing various mechanisms of attenuation, reflection and scattering in the atmosphere. Consequently, two types of radiation affects the Earth's surface: a first one received straight from the sun without change of direction called *beam radiation*, and a second whose direction has been changed by scattering and reflection called *diffuse radiation*. The sum of these two types is known as total or global radiation. Global solar radiation that reaches the ground (due to the atmosphere transparency this part represents almost 50% of the total incident radiation) gets mostly absorbed by oceans, biosphere, lithosphere and cryosphere and just a minimum part gets reflected. According to the first law of thermodynamic radiation that reach ground undergoes heat (the most) and work (the less) transformations. In order for the energy balance to be satisfied (on long time at least), absorbed heat has to be sent back to space as radiative emissions and, due to the enormous difference that exists in between their temperatures, Sun emission frequency bands ($\approx 6000K$) and Earth's ($\approx 255K$) are different. The Sun has his peak value on visible ($\lambda \approx 0.5mm$) as Earth emissions are concentrated in the infrared band (IR, $\lambda \approx 10mm$), most of the outgoing energy is concentrated in between 4 and 60 mm wavelength value and thus are completely IR. These considerations allows us to split radiative contributions into two main categories:

- Short wave radiation or solar radiation ($\lambda < 4mm$);
- Long wave radiation or ground radiation ($\lambda > 4mm$);

To estimate radiative iteration due to neighboring buildings and surrounding environment is very complicate. Therefore, the reflected component from the surrounding ground surface is generally taken for simple calculations or neglected, in our case we will take this effect into consideration through measurements instead.

The way which solar radiation interacts with the Earth determines not just the quantity of absorbed energy but also the way it is distributed into atmosphere, thermal stratification of air, ground temperature, humidity conditions, until to wind formation and a lot of phenomena much important for life. It is glaring that a complete description of factors that condition this interaction is unfeasible, that is also the main reason why solar radiation effect is so difficult to predict and to be taken into account. It follows a brief description of three of these factors, the most important in our work:

- The Zenit angle Z is defined as the angle in between the normal direction to the surface and the Earth-Sun direction, it influence the diffuse-beam shortwave ratio. Roughly speaking from this parameter depends the quantity of atmosphere that the light must cross in order to reach the ground. It depends on the latitude, solar declination and from solar hours. The nature of the effect introduced by the Zenit angle is however astronomical, and thus deterministic and predictable.
- Albedo is the phenomenon that determines the quantity of solar radiation reflected back to space from atmosphere and thus taking not part to the energy balance. The Albedo effect is determined by the thermo-chemical condition of the atmosphere above the interested area, namely: the concentration of steam, aerosols, particles, pollution and many others are that are not easily neither quantifiable nor to be taken into account.

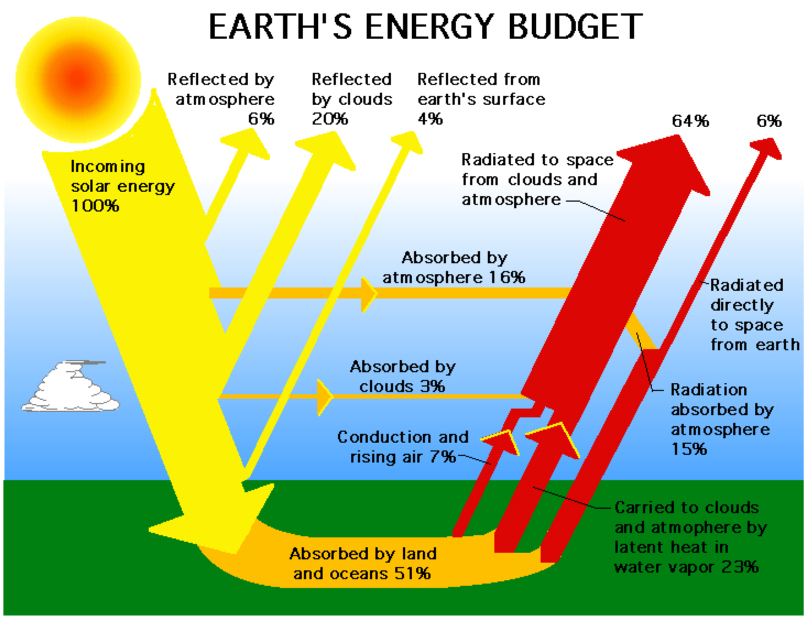

Figure 2.8: Interactions of solar radiations

- Energy reflected (shortwave) and emitted by ground (longwave) is absorbed, re-emitted and reflected back to ground from the sky, the quantity of this energy depends on weather conditions like type and quantity of clouds, the air density and its composition;

Even if the most of variability of solar radiation is determined by astronomical reasons, Albedo and weather conditions generates oscillations that may be considered as noise and that can be really influential on the thermal energy state of buildings.

In our work we will make use of data records of shortwave (beam and diffuse) radiation and longwave radiation.. The reasons why we consider the two contributions separately are both because admissivity coefficients of materials are not the same for different values of the wavelength radiation and because we are interested in the net quantity of energy exchanged as longwave radiation. The impact of solar absorption coefficient of external wall on building energy consumption were investigated in literature[29]. From literature[30] we took the admissivity values $\alpha_l = 0,90$ and $\alpha_s = 0,60$ where the meaning of notation is straightforward. The needs to consider the net value of longwave energy exchange can be explained considering the instantaneous balance equation on the external surface:

$$\alpha_s Q_{SWR} + \alpha_l Q_{LWR} + Q_h + Q_c - \epsilon_l \sigma T_s^4 = 0$$

where Q_h and Q_c are convection and conduction heat fluxes, Q_{SWR} and Q_{LWR} are respectively short and long wave radiation values, σ is the Stefan-Boltzmann constant, T_s is the temperature of the surface. ϵ_l is the emissivity coefficient that, for gray bodies as we assume ours, can be defined to be equal to admissivity. The main advantage in employing measured values is the easy way to consider incoming longwave radiation in the model, usually this quantity is taken into account defining equivalent temperatures for emitting sky and ground. Models exists in literature that defines suitable values for the emission temperature of the sky[31], despite that, they are approximations and calls for other measurements (i.e. humidity, steam partial pressure, ...). Far more difficult is to quantify the emitting temperature of ground where as ground is intended to be not just the grass but miscellaneous like surrounding buildings, trees and so forth and so on.

According to Plank's equation, energy associated to radiative emissions depends to the forth power of the surface temperature, we will make use of its linear approximation around the mean radiative temperature $\bar{T}_s$:

$$\alpha_l \sigma T_s^4 \approx 4\epsilon_l \sigma \bar{T}_s^3 T_s - 3\sigma\epsilon_l \bar{T}_s^4 \tag{2.22}$$

Usually solar radiation incident on the Earth's surface is measured on a horizontal surface, external surfaces of buildings receiving solar radiation are generally tilted (except for the flat roof, which is a horizontal), consequently, it is required to estimate radiation on such surfaces from the data measured on a horizontal surface. Moreover the view factor of Sun changes during year and day long, this has to be taken into account too in a complete model. To do that a coefficient is introduced for each surface:

$$\tilde{\alpha}_l = \alpha_l \alpha_t \alpha_w \tag{2.23}$$

- α_l = admissivity coefficient of the material;
- α_t = coefficient to take into account that the surface is tilted;
- α_w = time dependent coefficient that takes into account the view factor (shading);

The latter coefficients can be estimated making use of computer toolboxes, often used in architecture design and environmental simulations.

Radiation data

In figures (2.9)(2.10) typical data records are presented: they are representative of two scenarios that corresponds respectively sunny and cloudy days. It is available to us the following measures:

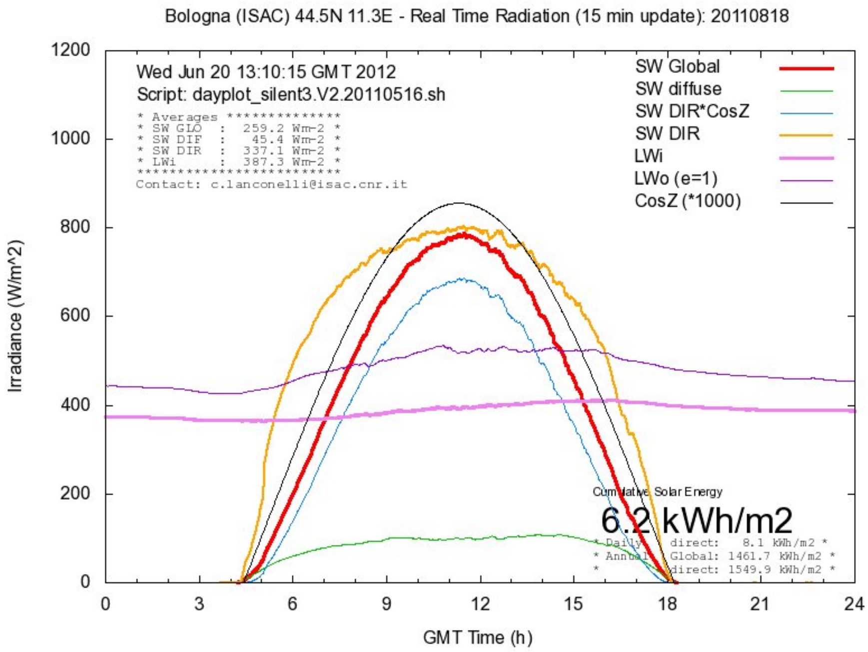

Figure 2.9: Sunny day radiation data

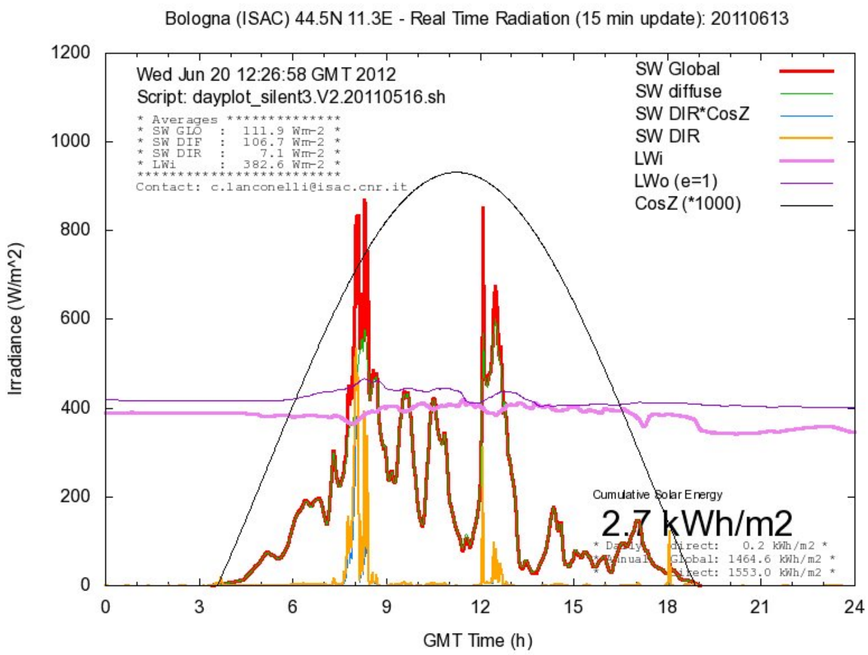

Figure 2.10: Cloudy day radiation data

- SW Global : shortwawe global radiation, the sum of direct ('SW DIR cosz') and diffuse radiation ('SW diffuse');
- LWi : longwave downwelling from Sky;
- LWo : longwave upwelling from Ground;

As we can see the amount of shortwave radiation energy is tightly related to atmospheric conditions, the energy amount is moreover really consistent and rapid changing, in particular direct term generates high gains when the Sun shine, becoming almost null otherwise. Longwave contribution is instead less varying although absolutely important. Notice that after sunset there is no shortwave radiation as well as longwave remain almost constant, this observation could be taken into account when estimation of the state of the wall-zone exchange model is performed.

2.5.2 People occupancy

In this thesis, we model the number of occupants np living the building through a birth-death process with time varying birth (arrivals) and death (departure) rates. Assuming that initially the building is empty, we can define np as follows:

$$np(t) = max(\Delta_{IN}[t_0, t] - \Delta_{OUT}[t_0, t], 0)$$

where $\Delta_{IN}[t_0, t]$ denotes the number of arrivals within $[t_0, t]$ and $\Delta_{OUT}[t_0, t]$ the number of departures within $[t_0, t]$. They are independent Poisson processes with time varying rates $lambda_{IN}(\cdot)$ and $\lambda_{OUT}(\cdot)$, that is:

$$Pr(\Delta_{IN} = k) = \frac{e^{-\int_{t_0}^{t} \lambda_{IN}(\eta)d\eta}(\int_{t_0}^{t} \lambda_{IN}(\eta)d\eta)^k}{k!}$$

$$Pr(\Delta_{OUT} = k) = \frac{e^{-\int_{t_0}^{t} \lambda_{OUT}(\eta)d\eta}(\int_{t_0}^{t} \lambda_{OUT}(\eta)d\eta)^k}{k!}$$

given that

$$E[\Delta_{IN}[t_0, t] - \Delta_{OUT}[t_0, t]] = \int_{t_0}^{t} \lambda_{IN}(\eta)d\eta - \int_{t_0}^{t} \lambda_{OUT}(\eta)d\eta$$

we can define the rates λ_{IN} and λ_{OUT} based on a nominal occupancy profile $\bar{np}(t), t \in [t_0, t_f]$, as follows:

$$\lambda_{IN} = \begin{cases} \bar{np}(t) & \bar{np}(t) > 0 \\ 0 & \bar{np}(t) \leqq 0 \end{cases}$$

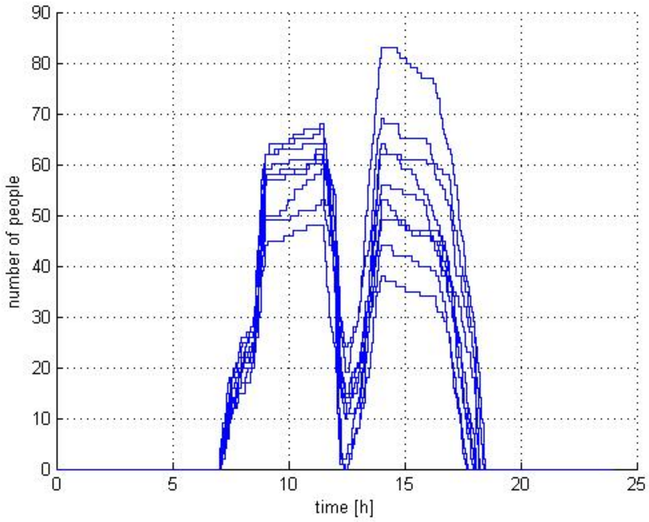

Figure 2.11: Some realizations of occupancy profile

$$\lambda_{OUT} = \begin{cases} \bar{np}(t) & \bar{np}(t) < 0 \\ 0 & \bar{np}(t) \geqq 0 \end{cases}$$

In figure (2.11) some realizations of the occupancy is depicted, the nominal profile is a typical occupancy profile for a public workplace where the most of variability is due to customers affluence during openings hours 9-12, 14-17 (for example a bank or a mail office). At noon, offices are closed and most of employee are supposed to leave the building for lunch.

2.6 Description of a case study

The test building is a simple structure representative of a typical medium-size office building. It is $20m \times 20m$ wide, $9m$ high structure on three floors. Even if it is not an existing building it is however representative of some interesting phenomena:

- Its Sun exposure is different for each facade due to their different orientation, this is relevant in particular for shortwave solar radiation and inner gain trough windows;
- Each facade is equal and comprehensive of $200m^2$ of glazed surface, the rest is walling. External walls are composed by a core layer of concrete, an inside layer made by inner bricks (for wiring and utilities)

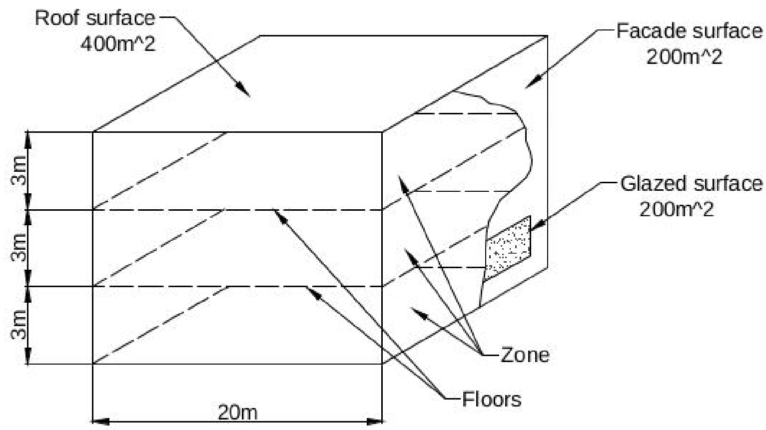

Figure 2.12: Building schema

and an outer layer of insulation panels. The first is characterized by its high thermal capacity, the latter by its high thermal resistance;

- The presence of floors models the thermal inertia of internal walls, they are important since their thermal evolution is completely determined by the movement of zones temperatures;

- A flat roof, due to its large area, is an important heat exchange contributor. Conversely we do not take into account neither the effect of the ground conductive exchange nor the presence of unconditioned zones. They however can be easily pugged in treating them in the same way outdoor temperature is treated;

- The building living space can be considered as a single zone equally regulated or divided in three different zones, one for each floor as in figure(3.14). These two policy will be exploited in two different case study looking for the existence of advantages in dividing the building in more zones;

Physical characteristics of used materials are listed next:

Material	density $\frac{Kg}{m^3}$	specific heat $\frac{J}{Kg}$	thermal conductivity $\frac{W}{mK}$	finite volumes width m
concrete	2400	880	1.6	0.025
insulation	30	2100	0.026	0.01
walling	1200	840	0.36	0.02
substrate	1000	880	0.84	0.02

Wall's composition is different for boundary walls, roof and floors: (boundaries walls and roof are described from inner to outer layers)

Wall type	material	width	material	width	material	width
boundary	brick	$5cm$	concrete	$25cm$	insulation	$8cm$
roof	concrete	$20cm$	insulation	$10cm$	cover	$5cm$
floor	concrete	$20cm$	substrate	$5cm$	/	/

In what concerns view factors they are in first time estimated on the base of their orientation, longwave is divided into upwelling (ground view factor) and downwelling (sky view factor) too:

Wall orientation	Global shortwave	Downwelling longwave	Upwelling longwave
up	0.8	0.8	0
nord	0.3	0.6	0.5
est	0.5	0.6	0.5
sud	0.8	0.6	0.5
west	0.5	0.6	0.5

The full building model ends up to be of order 119 but reducible without lacking in precision to order 22 by the means of HSVD reduction methods (subsection 2.2.1) performing thus a model reduction of magnitude $1/5^{th}$. Other parameters used for the definition of the structure are listed next:

Parameter	Value	Description
σ	$5.67 \cdot 10^{-8}$	Stefan-Boltzmann constant
$\bar{T}$	296 K	Mean radiative temperature
h_{in}	18	Internal convective heat exchange coefficient
h_{out}	15.35	External convective heat exchange coefficient
α_s	0.6	Shortwave admissivity coefficient
α_l	0.9	Longwave admissivity coefficient
α_z	0.8	Mean admissivity of the space
τ_w	0.65	Transmissivity of windows

2.7 Concluding Remarks

We provided models for all the main components of the building cooling problem. Thanks to the assumed characteristic for the control inputs to be linear varying we could end up with an affine formulation for the cooling energy needs through the definition of the thermal energy balance equation. Thermal balance equation is composed by four terms that was examined disjointly: the most interesting contribution is related to the thermal energy exchange with the building, which expression was formulated starting from a finite volumes discretized model of the building structure. Technological components such as the thermal storage and the chiller was introduced, for the latter an important result is represented by the possibility to model it making use of a quartic equation that is convex. To summarize, the main results that should be highlighted are:

- All the proposed models are convex: this propriety is particularly important for optimal control problems issues since convex means solvable in many cases. Convex optimization can be performed efficiently making use of well developed methods and toolboxes, convexity is even more important when stochastic formulations of the problem are introduced;

- All the introduced approximations are reasonable;

- The way the building model is built up is modular and general. It's possible to model the same building with different degrees of detail as well as it's possible to define freely the number and the shape of zones;

- Stochastic factors can enter models directly (i.e. radiation and occupancy) or via the definition of parameters (i.e. external convective exchange coefficients and its relation with wind). To better take into account the latter it was outlined the possibility to make use of a time variant matrix horizon composition;

Chapter 3

Control

In this chapter we address the optimal energy management problem based on the model in Chapter 2. Optimal energy management is formulated as a finite horizon optimal control problem with a suitably defined cost function, subject to certain constraints. Optimization results for the certainty equivalence-based solution are presented and analyzed, discussing possible variants and enlightening the flexibility and modularity of the proposed method. Considerations about the performance degradation of the certainty equivalence-based solution when considering stochastic disturbances naturally lead to a stochastic finite horizon formulation of the control problem, which can be solved via the scenario approach to stochastic optimization.

3.1 Control problem formulation

The aim of the proposed control problem formulation is that to optimize in the future a specific performance criterion choosing (among a suitable set) the best values for control variables. The optimization process makes use of a cooling problem description obtained composing models of interacting subsystems, described in the previous chapter, to calculate future moves in the independent variables. The resulting predicted behavior can be requested to satisfy some constraints. To choose the length of the predicted horizon is a critical point due to the existing trade off in between computational effort and grasped dynamics: zone-wall main thermal constant are of the order of many hours (four or even nine hours depending on insulation degree and composition of walls in general) and thus a sufficiently long optimization horizon is mandatory to exploit the structure thermal mass in the context of a continuously implemented optimization policy. As proposed in [6] we decided to implement a control policy over one day (i.e. temperatures and storage contributions are implemented in open loop over one day), as a difference in our work the optimization horizon is two days long. The main reason for this latter choice is that both to better exploit thermal mass effect and to drop the needs of too restrictive closure constraints for the problem: in the context of a continuously implemented optimization process closure constraints (apart ensuring feasibility for the optimization process) takes the duty to drive the system till suitable end conditions that will be starting conditions for the next day optimization. This problem can be tackled as in [7] making use of an MPC control strategy, however that choice calls for the possibility to precisely observe the building thermal state at every moment and requires much more computational needs to be implemented in practice. The closed loop formulation is obtained through a sort of receding horizon paradigm and actually represents a compromise in between strategies used in [6] and [7]: optimal control is calculated for a two days long horizon but implemented for one day only, until when optimization is performed newly. Notice that assuming the two days long optimization horizon we can better exploit building thermal mass even considering the opportunity of forecasting both weather and disturbances in general, notice also that such a long time prediction can be performed without lacking in resolution thanks to the convex formulation of the problem. Resuming, the resulting control action will be calculated for 48 hours, then implemented for the successive 24 hours and then calculated newly over 48 hours: variables will be in number of 288 temperature set points for each zone plus 288 values for the storage contributions assuming a 10 minutes resolution. The possibility to observe the building thermal state is critical, this can be hopefully better done when major disturbances are less varying and better known as for example during the night when no occupancy and no internal gains conditions hold, to the sake of implementability of the control policy.

3.1.1 Cost function

A novel idea introduced in this work is that to weight differently each sample in the optimization problem according to the time the sample belongs. The motivation of doing that is to investigate the possibility to define preferential periods where energy consumption should be concentrated, let us introduce now the vector $\mathbf{W}$ used to weight each time period and explain some possible choices for it:

1. we just want to sum every period equally in the optimization process: $\mathbf{W}$ will be in this case an unitary vector. In this case the optimization result will be the optimal energy absorption, and weighting factor is

actually not introduced being it equal to the sum operator;

2. the incidence of energy consumption is not the same over the time from the point of view of the dealer and we want to stimulate consumptions in some period and avoid them in others: this is a conceivable policy in a smart grid context with the aim to simplify the task of distribution networks and production sources is some critical periods. **W** will be in this case a vector where weights can be greater or smaller than 1 according to some criteria in order to respectively deject or stimulate consumptions. In this case the results of optimization is not the energy consumption anymore;

3. we want to perform optimization in order to save money: **W** will be in this case a vector containing the price of energy (electricity) differenced sample by sample. In this case the results of optimization is the total cost of the the cooling process, this will also be our choice mainly because we want produce tangible results.

Energy price

Energy markets have been liberalized in the most of countries, from then they are regulated by national and international authorities that seek to discourage volatility of prices and that forbids anti-competitive behavior in order to protect customers from financial speculation and to guarantee the proper work of the electrical network. Electricity price is usually negotiated one day ahead by the major producers, production share is sold and bought by them and its costs is constantly estimated and predicted by many market operators. Although data are available, energy cost is usually not for final users that mostly stipulates fixed contracts for bands. In our work we will calculate the real market energy cost for cooling the building and we will minimize it. The data we use describes the single national purchase cost for end customers in Italy during the year 2012. Electricity price fluctuate over the day and it is different day by day, fluctuations over the day is mainly caused by the so called generation mix effect: in order to produce a particular amount of energy many sources (as for example fossil fuels, hydro or solar energy) can be used and in that sense mixed, all of them has different prime production cost and optimal production rate. In figure(3.1) the average cost profile for the month of June is presented. The price rise starting from six in the morning until ten o'clock, after that it get lower and decrease until two o'clock in the afternoon. Surprisingly the most expensive time for buying energy is late in the evening. We expect chillers to work

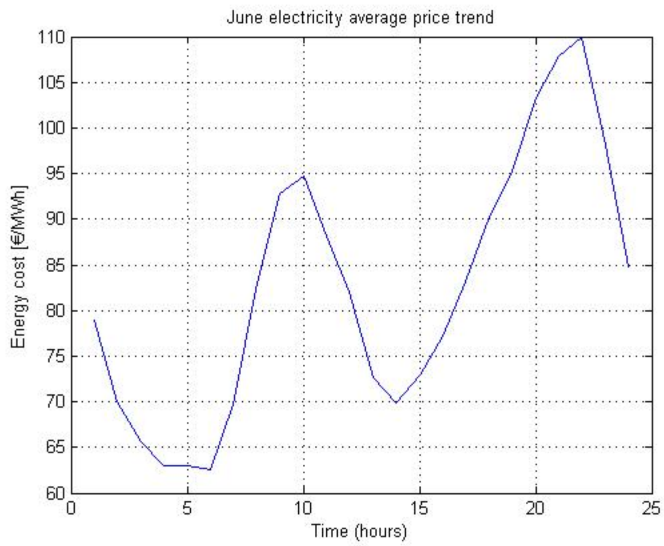

Figure 3.1: Energy price: daily variation

mostly where prices are lower, and to utilize stored energy in correspondence of cost peaks.

Cost function

According to equation (2.4) we can express the cooling energy required by each zone over the horizon M as the sum of different contributes that can be expressed by equations (2.11),(2.16), (2.17) and (2.18). Plugging all these equations together we end up with the affine expression of $\mathbf{u}$ (temperatures):

$$\mathbf{E_c} = \mathbf{Mu} + \mathbf{Q} \tag{3.1}$$

where

$$\mathbf{M} = \mathbf{M(np)} = \mathbf{G} + \mathbf{N(np)} + \mathbf{Z};$$

$$\mathbf{Q} = \mathbf{Q(d)} = \mathbf{H(d)} + \mathbf{Fx_0} + \mathbf{P} + \mathbf{U} + \mathbf{E_i(d)};$$

To express the chiller cooling energy request we must first subtract from the previous equation (3.1) the contribution $\mathbf{s}$ of the storage:

$$\mathbf{E_{ch}} = \mathbf{E_c} - \mathbf{s} = \mathbf{Mu} + \mathbf{Q} - \mathbf{s} \tag{3.2}$$

The last (3.2) represents the amount of energy that the chiller should provide for each sample time. Notice that the contribution of the storage can be so that the chiller duty is lighten ($s > 0$) or increased ($s < 0$) depending on the storage operating mode. This formulation takes also implicitly into account the disturbances realizations. We are now ready to include the relation (2.21) to consider the efficiency of chiller to produce the required amount of cooling energy calculating thus the energy load:

$$\mathbf{E_l} = c_1 \mathbf{E_{ch}}^4 + c_2 \mathbf{E_{ch}}^2 + c_3 \tag{3.3}$$

where products are element-wise. A convex function of an affine expression of the optimization variable is still convex, this propriety is of great importance for the optimization feasibility. Finally, introducing the weighting factor, we can express the cost function as:

$$cost(\mathbf{u}, \mathbf{s}) = \mathbf{W} \cdot \mathbf{E_l} \tag{3.4}$$

Recall that $\mathbf{u}$ and $\mathbf{s}$ are respectively the zones temperature setpoints and the storage contribute for each sample time and that they are the manipulable variables in our optimization problem.

3.1.2 Constraints

Constraints are usually set when some requirements or limitations hold regarding both the system behavior and the inputs. The reason of constraints may be that to prevent models to work in non representative conditions or to pursue desired characteristics of inputs. In our work we will set the following input constraints:

1. $$u_{min} \leq u_n \leq u_{max}, \forall n \in \Xi_1$$

   Zone temperatures are upper and lower bounded in every period in the set Ξ: this constraint allows the definition of a temperature comfort range ($u_{min} \approx 20^oC, u_{max} \approx 24^oC$) in which zone temperatures must lie for a specified periods during the day long (i.e. period of time in which someone lives the building). For $n \notin \Xi_1$ this constraint can either be relaxed or replaced by another one less restrictive defining for example bounding temperatures of 15 and 30 degrees over the rest of the day;

2. $$|u_n - u_{n+1}| \leq \Delta, \forall n \in \Xi_2$$

   for comfort reasons it could be required the maximum temperature derivative to be bounded during the occupancy time (defined by Ξ_2), in our work we set a maximum temperature variation of $0.5K$ in 10 minutes, however we will see that this constraint is never saturated.

This is it because we considered the thermal inertia of zones to be inclusive of the 'stuff' contribution, another way to take into account 'stuff effect' should be to impose this constraint over the whole optimization horizon tuning the maximum temperature variation coherently with a suitable thermal response time of 'stuff'. Drawback of this approach is that thermal heat energy contribution of 'stuff' will not be quantified;

3. $$|s| \leq s_{max}$$

The maximum quantity of thermal energy provided or drawn by the storage is limited. These limitations are merely caused by the thermal exchange process from a technological point of view. In our work we will assume charging and discharging to have the same performances, by the way this is not mandatory. The maximum quantity could be easily made related with the storage energy content simply introducing a proportional relationship with it, despite that we didn't introduce it due to the lack of information about real storages energy behavior;

Energy constraints are instead:

1. $$(\Sigma_{nz}\mathbf{E_c}) - \mathbf{s} = \mathbf{E_{ch}} \leq \mathbf{E_{max}}$$

The overall cooling energy needs must not exceed the maximum cooling capacity of the HVAC. Notice that the thermal storage can help to provide energy if it works in discharging mode or, on the contrary, it can be considered a cooling load if it works in charging mode;

2. $$E_{c,i}(k) \geq 0 \quad \forall \quad i \in (1, \cdots, nz), k \in (1, \cdots, M)$$

The cooling energy required for each zone and for each sample period to track temperatures evolution should not be negative (i.e. HVAC can not heat zones!). This constraint is critical in our work because no heating sources are directly manipulable, the only way the zones can be heated up is taking advantage of the heat produced by people or internally generated or exchanged with the building. This last way is the more intriguing specially because we want to investigate if the complex and slow behavior of building can be exploited;

3. $$S_{min} \leq S(k) \leq S_{max} \quad \forall k \in (1, \cdots, M)$$

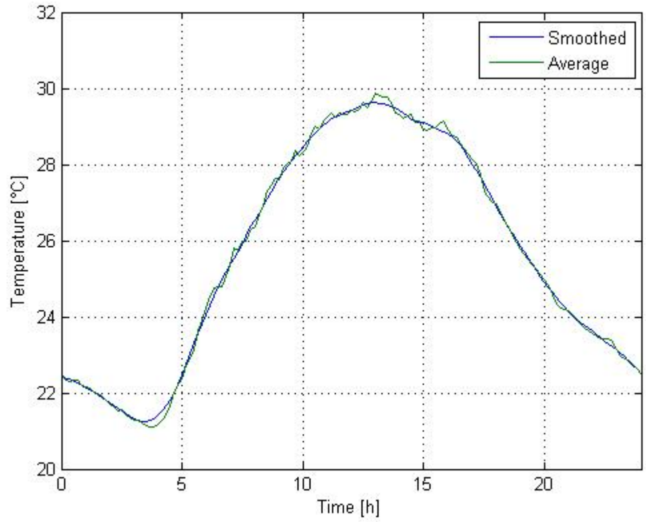

Figure 3.2: Nominal Outdoor Temperature

The storage energy content S is bounded by a maximum and a minimum value, the minimum can either be zero (empty storage) or a nonzero value (technological limitations). In our work we will consider it to be empty, the minimum level condition is however more coherent with the assumption that maximum energy exchange rate is constant in respect to the storage content, this could be not.

3.2 Certainty equivalence-based solution

We aim to investigate the optimal control policy for temperatures evolution and storage contributions when nominal disturbances and occupancy profiles are applied. For nominal disturbances we intend the average values over three months of the data we got: they are plotted in figures (3.2)(3.3)(3.4)(3.5).

According to all the exposed considerations we can now calculate the model predictive control policy numerically solving the convex optimization problem making use of Matlab solvers like CVX or YALMIP. The algorithm

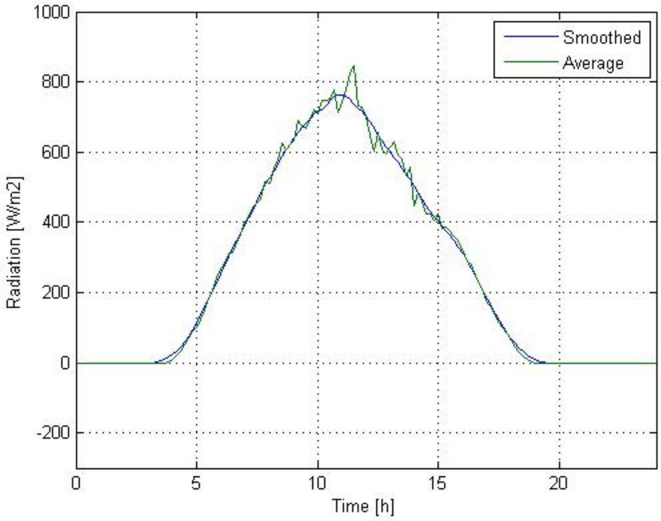

Figure 3.3: Nominal shortwave radiation

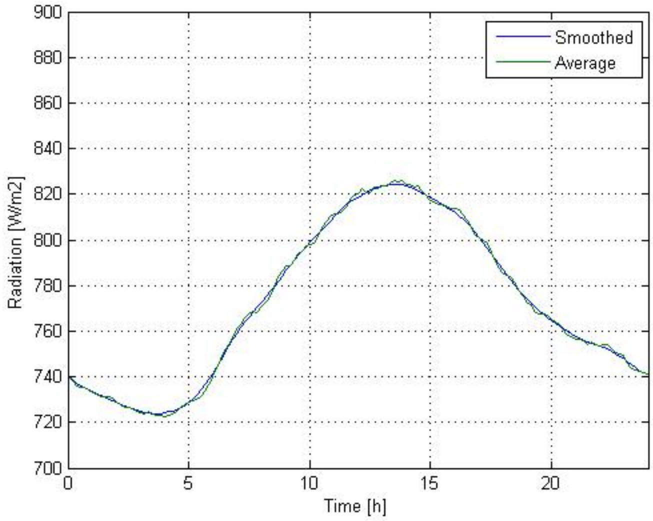

Figure 3.4: Nominal longwave radiation

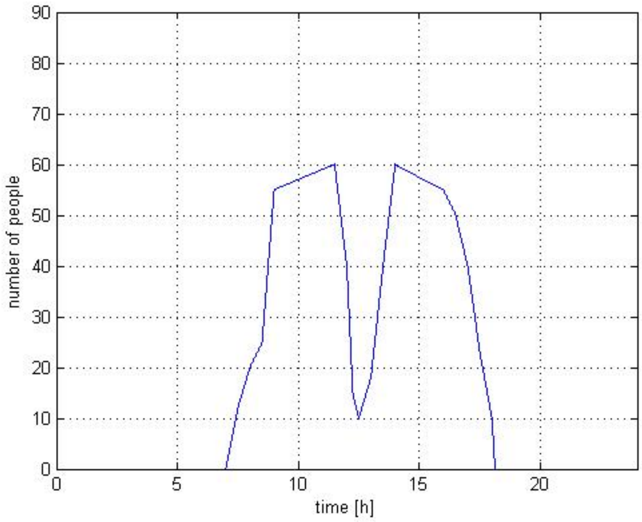

Figure 3.5: Nominal Occupancy profile

for the complete problem (that makes use of the storage) is following:

$$
\begin{array}{ll}
\min\limits_{\mathbf{u},\mathbf{s},S_0} & \mathbf{W} \cdot \mathbf{E_l}(\mathbf{u},\mathbf{s}) \\
\text{subject to:} & u_{min} \leq \mathbf{u} \leq u_{max} \\
 & |\mathbf{s}| \leq s_{max} \\
 & \mathbf{E_l} \leq E_{max} \\
 & \mathbf{E_{c,i}} \geq 0 \\
 & S_{min} \leq \mathbf{S} \leq S_{max}
\end{array}
$$

3.2.1 Performance evaluation in the case study

In this section we compare many control strategies and system configurations with the aim to rate their performances and investigate the way optimal energy management can be pursued. Four different policies are considered:

1. *Fixed* : This trivial policy consists in maintaining the comfort temperature of 24^oC fixed during the whole occupancy time. Outside this period, apart for a short time before occupancy starts were temperature is driven to the comfort one, the chiller operates in idle condition providing no cooling energy to the zones. Notice that, outside the comfort period, temperature evolves freely according to the heat exchanged with the other sources. Storage is unprovided in this case.

This policy, clearly unconvenient, is representative of a real life implemented unoptimized cooling strategy that looks only to maintain comfortable living conditions. It is mainly introduced as a base case policy in order to make comparisons with the others;

2. *Fixed+Storage* : The temperature is regulated as in the previous case, as a difference in this case thermal storage is provided. This control policy is sometimes used in rough real life applications in order to take advantage of the thermal storage, that is fully charged during the night and discharged during the day;

3. *Optimal temperature* : Temperature setpoint is chosen according to proposed control formulation setting $s = 0$ as a constraint so that storage is unused. Optimization is pursued by the means of selecting suitable temperatures to exploit building thermal inertia. Initial temperature and building state are calculated iteratively letting the system evolve over more days and iterating the control strategy as presented;

4. *Optimal temperature + Storage* : The most complete case plugs the thermal storage into optimization. Storage can supply or drawn energy freely during the whole day and its initial and final states are optimization parameters too;

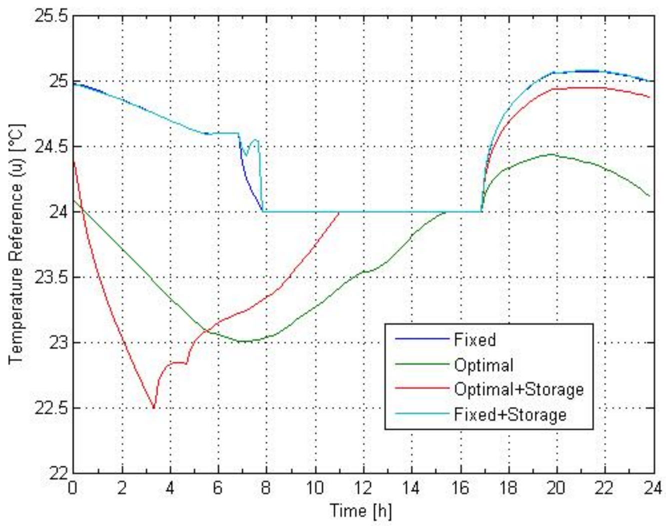

Figure 3.6: Temperature comparison

We introduce first a brief analysis of results and then a numerical evaluation of performances. In what concerns temperature profiles, (figure 3.6) there are no differences (in general) when policies 1 and 2 are applied: temperature rises when comfort period ends until the building gives heat to the zone, during the night (when it is colder outside) the opposite occurs making the zones cooling slowly. In the process of it temperature variations are moderated by the presence of floors (inner walls) that, in some sense, absorbs some of heat released from boundary walls to the zone (for the thermal flywheel effect). When policies 3 and 4 are applied, profiles are qualitatively defined by some phases: at the beginning a 'precooling phase' starts far before the comfort period, here temperature is lowered below 24^oC. In a second phase temperature rises until it stacks to the maximum allowed by the comfort constraint. When the comfort period ends temperature rises as well until precooling starts newly. In what concerns the chiller energy

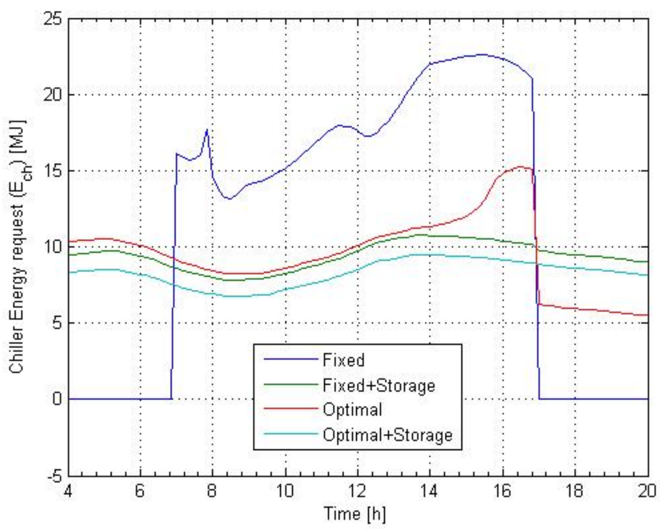

Figure 3.7: Chillers request comparison

request (E_{ch}, figure 3.7) there is a huge difference in between policy 1 and the others, let us we go through details case by case:

1. *Fixed* : The cooling effort is what is needed to climate the zone: from 7 till 8 o'clock the zone is cooled down to the comfort temperature of 24^oC, then all the effort is spent to maintain this temperature condition compensating all the heating factors. Since there is no storage helping, all this energy is directly provided by the chiller. During occupancy both average and peak values are high in respect to the others

(double) and the chiller operates far over its optimal working point;

2. *Fixed+Storage* : The cooling effort is averaged during the day. Occupancy time needs are lowered making use of the storage, during non occupancy time cooling energy is spent to recharge the storage continuously. The amount of stored energy and the charging speed is so that the averaged needs makes the chiller works more closer to its optimal working condition (figure 3.8). A second advantage is that the maximum chiller cooling request is really lower if the storage helps, introducing the possibility to make use of a smaller chiller.

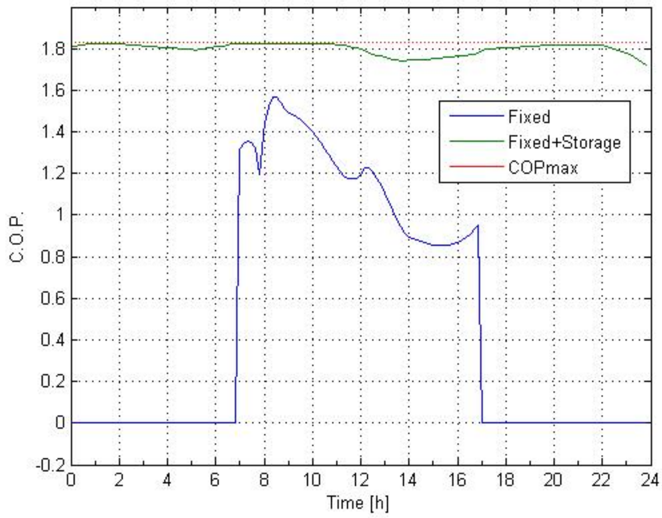

Figure 3.8: Chiller working condition.

3. *Optimal temperature* : We obtain pretty similar results as in the previous case without making use of storage by the means of driving temperature so that exchanged energy with the building is exploited. In figure (3.9) the Wall-Zone energy exchange (E_w) is depicted: (where E_w is below zero it means that the contribution of the building is that to cool down the zone) the advantage of precool the zone is that to lower the internal thermal energy content of the building (cooling it down) in other to let it retain some heat during occupancy. When the temperature rises (from 6 in the morning) this effect is particularly evident being the building elevating its internal temperature according to the new boundary conditions. Due to the long response time to these variations (approximately 10 hours) the structure will never

reach the steady state and for all the occupancy time the effect of precooling will light the building heating contribution. Notice also that precooling starts really early: at 20 in the evening of the day before. What temperature tracking is doing is nothing but making use of the building as a thermal dumper pumping inside it some cooling energy during the precooling phase that will be used later to dump the E_w quantity;

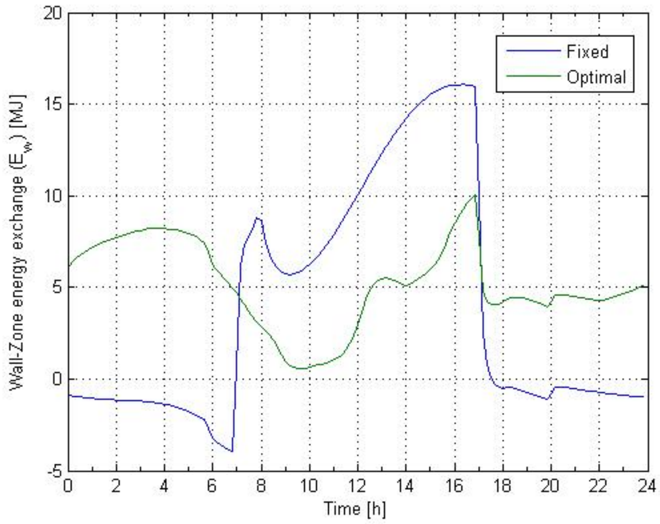

Figure 3.9: Wall-zone enengy exchange comparison.

4. *Optimal temperature+storage* : In this case the situation is even more complicated. The contribution of storage and temperature tracking are interconnected: the storage is used first to boost the precooling (from 0 to 3 a.m.), then the temperature rise makes the opportunity to recharge partially the storage (from 5 to 12 a.m.) that will be used again (from 12 to 17 a.m.) allowing the chiller to work in its best conditions. It is interesting to notice that the 2 degrees wide precooling effort and the following temperature rise allows, during the storage recharging phase, to make the building works to cool down the zone contrasting the inner gains, chiller will work during the whole time at it best COP splitting its contribution in between zone cooling and storage recharging. Another advantage in respect to the case without optimal temperature reference tacking is that to make use of half of the maximum storage capacity allowing a reduction of the storage size.

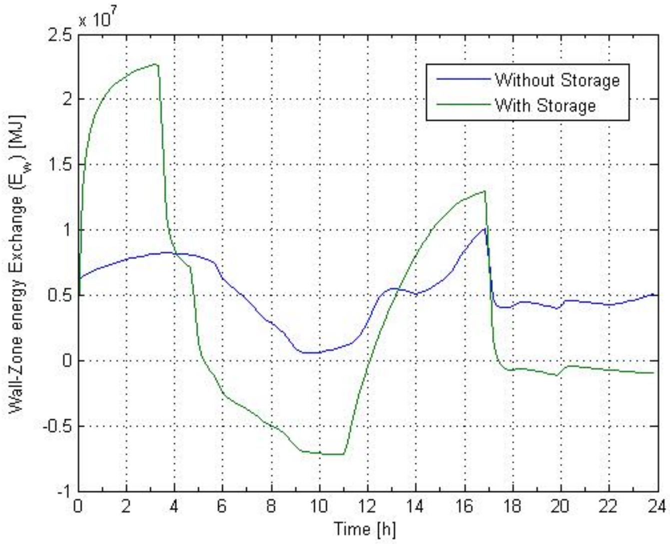

Figure 3.10: Wall-zone energy exchange comparison.

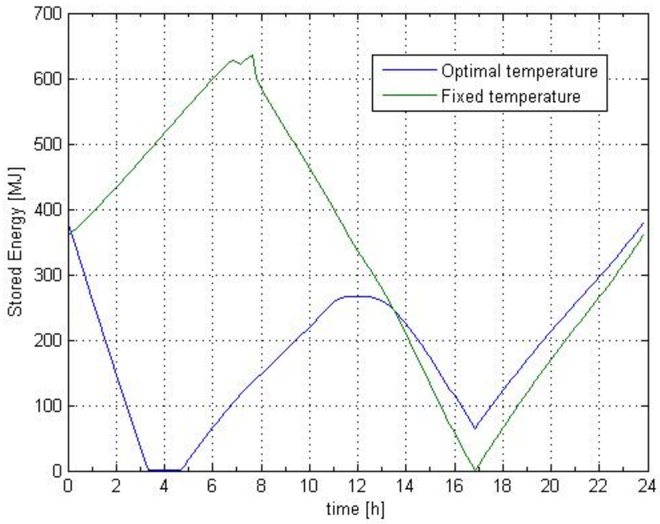

Figure 3.11: Different storage usage.

Some numerical results are presented in the following table:

	F	F+S	O	O+S
€	29.09 (0) (+59%)	16.95 (−37.43%) (0)	16.63 (−38.61%) (−1.89%)	14.44 (−47.1%) (−15.45%)
$E_l[MJ]$	1219	742.3	750.5	694.4
$E_{ch}[MJ]$	1094	1330	1288	1187
$E_c[MJ]$	1094	1087	1288	1076

What we notice first is that Fixed+Storage and Optimal policies are really similar from the costs point of view, that the last policy allows us savings about 15% in respect to the commonly used policy 2 and that policy 1 is clearly disadvantageous. Another relevant difference in between policies 1 and 2 regards the absorbed energy: consumed electricity is almost 39% less if the storage is used even if the cooling energy produced by the chiller is 18% more. The advantage in making the chiller works close to its optimal COP is evident, the more cooling energy request is due to the amount of energy lost by the storage that, in our model, is particularly relevant when the storage is full of charge. As a difference in respect to the fixed policies, the optimal temperature tracking policy alone requires more cooling energy to the zone, this however allows a better distribution for the chiller load over the day that can be translated as a more economical electricity absorption. The energy consumption due to storage losses making use of policy 4 is half than making use of policy 2 ($9,3\%$ vs 18%) showing another advantage in the flexible use of the storage.

Last, it can be seen that costs and energy absorption are not equivalent descriptions of performances: in cases 2 and 3 for example the latter is the cheaper even if the first absorbs less electricity, this is it due to the weighing factor that weights energy consumption differently during the day. The way weighting factor affects the optimization and achievable performances is analyzed next.

Weighting factor

Using optimal temperature tracking and storage configuration we implement now solutions in which energy absorption is differently weighted sample by sample by the means of the definition of a particular shape for the weighting function. Two new different weighting functions are introduced, the first is the simplest possible and simply sums each sample equally over the time. The second one, more interesting, aim to discourage electricity consumption during the period in between 12 and 17 a.m., this is done defining the weighting function to be one all over the time except during the penalized period were it takes values of ten. The electricity consumption profile are depicted in figure (3.12). What we can see is that it is possible to find

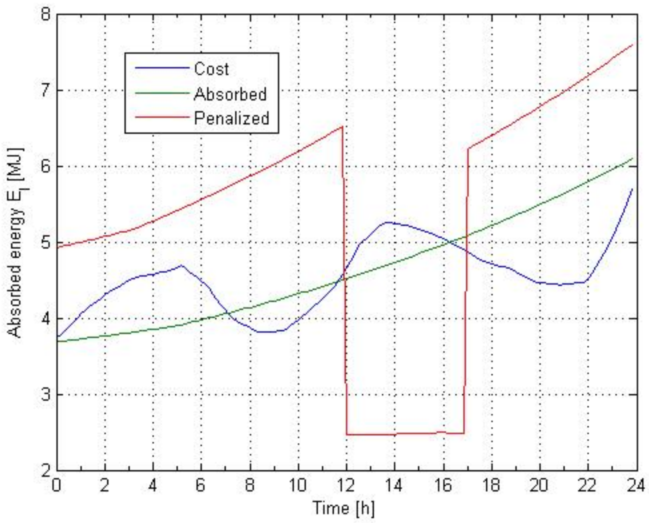

Figure 3.12: Electricity consumption for different weights

out a control strategy so that the energy absorption is the lowest possible during the penalized period, in fact $2.4MJ$ corresponds to the chiller idle condition absorption (i.e. zero cooling energy produced) and is then the less consuming condition achievable without switching off the chiller. The main difference in between cost-weighted and energy-weighted cases consists in some fluctuations whose peaks are in correspondence with the local minima of the energy cost profile (figure 3.1) and that allows to save a little:

	Cooling cost [€]	Absorbed Energy [MJ]
Cost-Weighted	14.96	654.43
Absorption-Weighted	15.57	623.35
Penalized-Weighted	18.10	766.42

With the absorbed energy minimization policy we spend 4% more consuming 4.75% less electricity, the two quantities are by the way strictly related, optimization produces almost the same results with both policies since they both try to make the system works around chiller's optimal COP conditions. In the case of penalized period results are obtained specially taking advantage of the storage: it concentrates its cooling contribution in the penalized period charging in advance so much needed to chill the zone alone, drawbacks on doing that is the less effective precooling action that causes the system consume 17% more electricity and spend 21% more.

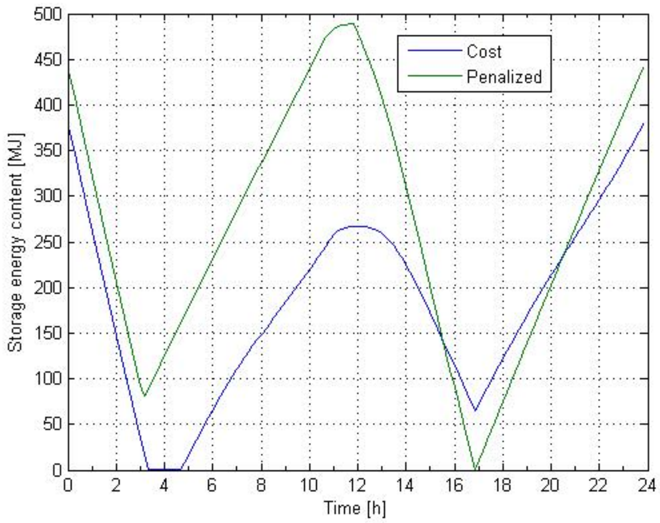

Figure 3.13: Storage usage.

Multizone

For the case in which optimal temperatures tracking alone is implemented we make use in this subsection of a multizone description of the building. The way zones are defined is the most natural one: we divide the building in three living zones, one for each floor as depicted in figure (3.14). In general division criteria can be based on occupancy or destination of use considerations, structural considerations or both of them. For example it could be interesting to investigate the possibility to exploit architectural aesthetic (halls, open spaces, ...) or functional spaces (stair cases, big conference rooms, warehouses, ...) for optimal thermal management purposes. In our configurations the division is made according to the building structure. Some considerations can be made regarding zones: Zone 1, thanks to its position in the building, is little influenced by external disturbances directly and its wall-zone energy exchange is highly related to Zone 2 selected temperatures, we assume this zone to be occupied by the half of the total building occupancy. Zone 2, being in between the others, is highly interconnected with thermal conditions of zones 1 and 3, we assume this zone

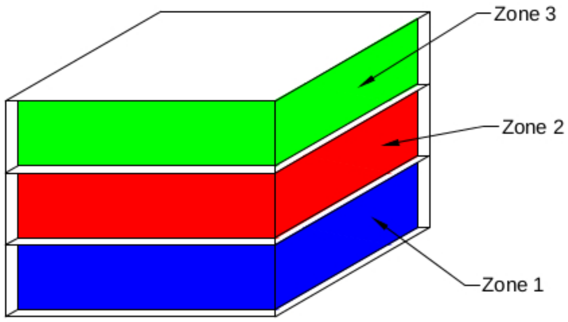

Figure 3.14: Multi zone configuration

occupied by 30% of the total building occupancy. Zone 3, being on the top of the building and including a large roof area, is the most influenced by external conditions, we assume this zone to be occupied by 20% of the total building occupancy.

From the optimization process point of view the same algorithm holds, by the way in this case we have three times more optimization variables defining the temperature optimal profiles for each zone. Results are plotted in figure (3.15). As we can see regime nominal profiles are really different for

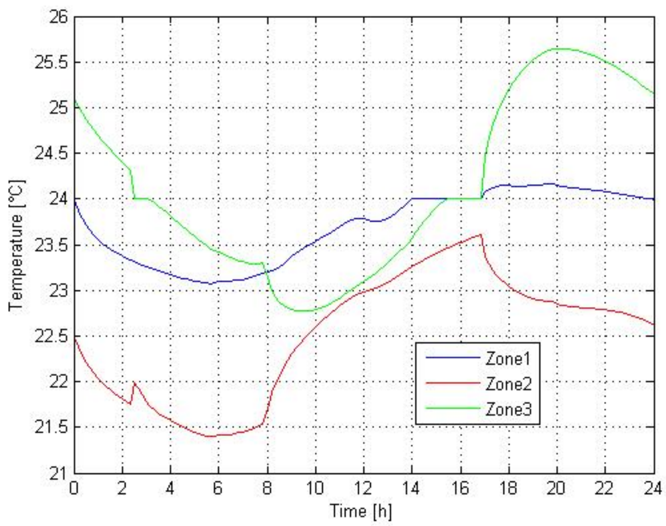

Figure 3.15: Multizone configuration temperatures

the three zones, there is convenience in maintaining zone 2 colder in respect to others as well as in maintaining zone 1 slightly varying. The reason why the middle zone should be strongly cooled seems to be the possibility to exploit it (and zones' boundary walls) as a passive storage in a similar way we saw for walls in the single zone case. It is interesting to have a look at cooling energy injected zone by zone (figure 3.16): as we can see zones are cooled disjointly case by case. During unoccupancy zone 2 is cooled strongly and alone, the existing temperature gradient with the other zones makes them cool down too. During the most of occupancy time zone 2 temperature rise freely absorbing heat from the other zones, only the top zone is cooled. It is also interesting that the bottom zone 1 is almost never directly cooled implementing a sort of passive cooling solution: notice that adding as a constraint cooling energy to be always null for a zone can model the situation in which no cooling system is present. From a numerical point

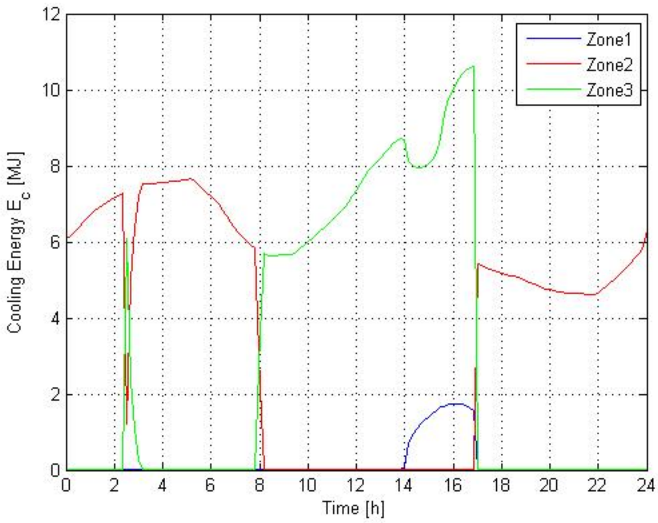

Figure 3.16: Multizone config. cooling effort

of view results are encouraging: we can save more than 20% in respect to the already good result achieved by the single configuration optimal temperature profile just implementing the multizone solution. This result seems to be due to the less needs of cooling energy.

	OSZ	OMZ	
€	16.63	12.79	-21.1%
$E_l[MJ]$	750.5	669.7	-14.5%
$E_c[MJ]$	1288	979.9	-23.9%

This results must of course be deeper studied in particular in what concerns computational feasibility and the trade off it exists in between the number of zones and introduced advantages. Another important aspect that must be investigated is the performance degradation and practical feasibility in real life unknown disturbances scenarios.

Disturbances effect

At this point we want to analyze what happens when real disturbances and occupancy realizations, different from nominals, affect the system. We calculate the optimal control policies making use of realizations of the whole month of July in the case disturbances profiles are known, this is unrealistic in practice but represents the best performances achievable. We then confront performances obtained when nominal control references for temperature is imposed having a look at how much constraints are breached. The constraints violation is very important because their satisfaction implicitly models saturations that are not modeled explicitly even if they exists in practice. In particular imposing the cooling energy to the zone to be nonnegative means that the cooling system cannot heat up the zone, if this constraint is violated the real system cannot track the desired temperature reference making further calculations completely unreliable. We are not interested in quantify what happens when constraints are breached, the cost estimation is the one ideally obtained tracking the nominal temperature reference, and they are, again, not representative at all of the real cooling cost.

Recall: results assumes a particular initial condition that never changes, thermal state of the building and values of initial inputs are representative of a particular condition in which the system is. Optimization is performed starting from such conditions over a two days horizon. Occupancy and environmental factors changes day by day.

Results are presented in the following table where the $E_c \geqq 0$ constraint violation is presented only since the others are never violated.

Day	Optimal Cost	Nominal Cost (%)	Violations %
1	18,16	21,59 (+18,9)	0
2	10,68	12,06 (+13,0)	42
3	10,68	11,41 (+6,8)	42
4	13,21	15,45 (+16,9)	0
5	15,58	17,43 (+11,9)	7
6	9,80	11,23 (+14,7)	63
7	20,02	24,91 (+24,4)	0
8	23,33	29,99 (+28,5)	0
9	27,25	37,54 (+37,8)	0
10	26,16	33,03 (+26,2)	0
11	29,40	40,45 (+37,6)	0
12	24,08	33,02 (+37,1)	0
13	31,25	41,67 (+33,4)	0
14	25,33	34,31 (+35,4)	0
15	30,13	37,27 (+23,7)	0
16	21,02	25,20 (+19,9)	0
17	16,14	17,39 (+7,8)	4
18	16,46	18,86 (+14,6)	0
19	21,79	23,73 (+8,9)	0
20	11,90	12,53 (+5,3)	42
21	11,40	12,10 (+6,1)	0
22	13,69	14,20 (+3,7)	13
23	13,44	14,23 (+5,8)	21
24	11,05	11,80 (+6,8)	42
25	8,93	10,91 (+22,2)	91
26	10,34	10,72 (+3,6)	59
27	12,22	12,52 (+2,4)	37
28	8,90	11,58 (+30,1)	85
29	13,12	13,98 (+6,5)	0
30	11,86	12,44 (+4,9)	42
31	16,14	17,23 (+6,8)	0

As we can see results are not encouraging: constraints are not violated just in few cases, these are realizations of days that show up to be the hottest and with clearest sky. Moreover a sensible mismatch exists in general in between achievable and obtained performances. In many cases constraints violation doesn't correspond to better performances for the nominal case. The reasons for performances violations are mainly :

1. after precooling there is not enough internal gain and wall-zone heat exchange to rise temperature as desired;

2. after occupancy time the heat released from the building is not much as supposed and temperature doesn't evolve as predicted;

There is no problems to track temperatures during cooling phases, constraints violations exist when the amount of heat exchanged to the zone is less than the estimated allowing not temperature to rise as desired, this heat amount is both due to slow building thermal dynamics and fast internal gains: in the unluckiest case environmental conditions heats less the building, occupancy are less than usual and radiation trough windows is weak calling for less cooling effort. To explain performances degradation observe that the main advantages in tracking a particular temperature profile are to deject and in some sense delay the wall-zone heat contribution over the time: in the case when conditions are close to nominal conditions the aim of temperature tracking is to make the chiller works close to its optimal COP working point. When we face particularly hot or cold days (being the chillers respectively under or over dimensioned) this strategy may be not the optimal: selected temperature profile is instead the one that avoids peaks and holes in the chiller duty averaging the cooling load even far from the optimal working point. For example it has been seen that from particular starting conditions and colder days realizations ideal optimal temperature profile may completely drop precooling phase.

Recall: Precooling is intended to be the cooling phase before occupancy period starts. The more width of precooling is, the less Wall-zone heat exchange will be after precooling ends. Precooling lights cooling duty during daylight.

3.3 Scenario-based solution

The presence of disturbances affecting the state evolution is quite common in practice, in our case they are represented by occupancy and weather conditions as outdoor temperature and solar radiation. To account for disturbances we can make use of two different approaches: robust and stochastic. In the robust one a min-max approach is taken where the control cost is optimized against the worst disturbance realization, while guaranteeing constraint satisfaction. Although successful in many cases, the min-max strategy may lead to conservative results, since the disturbance distribution is not accounted for and all disturbance realizations are treated as equally likely. Indeed, it might be the case that low probability disturbance realizations cause a significant deterioration in the cost or even the unfeasibility of some constraint. To overcome these limitations an average cost and probabilistic constraints are typically considered in stochastic problem formulations: in this setup, a violation of the constraint is accepted, although this must happen for few disturbance realizations only, having altogether probability no greater than a chosen threshold. This rules out "bad" situations adversely affecting the robust approach. Moreover, probabilistic constraints are the only way to avoid unfeasibility of state constraints when the disturbance has unbounded support. Unfortunately, probabilistic constraints are in general non-convex and more difficult to treat than usual non probabilistic constraints. The resulting finite-horizon optimization problem with probabilistic constraints belongs, indeed, to the class of the chance-constrained optimization problems,which are known to be hard to solve in general.

The way stochastic realizations that enters our optimization problem can be divided in three types:

1. Structural. It is the case for example of wind and humidity, which variability effects the system within the definition of parameters: humidity level influences both chiller efficiency coefficients and external convective heat exchange. These effects are taken into account respectively by the means of defining time variant performances coefficients and building model;

2. Direct. Is the case for example of solar radiation that affects the system as an exogenous input;

3. Mixed. Is the case for example of outdoor temperature that is an exogenous input as well may condition chiller efficiency;

In this section we tried to explore the possibility to set up and solve the problem in a probabilistic way. In particular we propose and analyze three different configurations for controllers: in the most complete and general formulation (in respect to which others controllers can be viewed as subcases), temperature set points and storage contribution is calculated being aware of the possibility to regulate them on-line, taking advantage of measurements of disturbances (all but occupancy). A sort of (even if not canonical) feedforward compensator is made up in order to do that: it is nothing but a time-variant gain which values are set by the optimization process. Many different configurations were tried, the ones we present here are the more promising and have the aim to show that scenario based solutions can be well suited to tackle the stochastic problem. The temperature profile is made by a nominal profile $\bar{\mathbf{u}}$ calculated once plus a compensator action that sample by sample updates the next setpoint making use of radiation and temperature measures. Shortwave radiation, longwave radiation and outdoor temperature are plugged in all together defining an equivalent disturbance effect:

$$d = \mu_{swr}Q_{swr} + \mu_{lwr}Q_{lwr} + \mu_{To}T_o$$

where coefficients μ are the static gains of disturbances on the building model and weights differently the respective contribution. Doing that we can drop the number of optimization variables grasping as well the environmental effect on building dynamics. Moreover measures used by compensator are the discrete integral (the until-now sum) of the equivalent disturbance: this carries informations about the disturbances effects on the building thermal state over the time and not just punctually, in some sense it implicitly defines how hot is the real day. The storage contribution profile is defined in the same way as temperature but, as a difference, the compensator makes also use of punctual measures of shortwave radiation to better take into account the effect of solar gains trough windows. The probabilistic formulation of the problem is:

$$\begin{array}{ll}
\min\limits_{\bar{\mathbf{u}},\mathbf{C_1},\bar{\mathbf{s}},\mathbf{C_2},\mathbf{C_3},h} & h \\
\text{subject to:} & \mathbf{u} = \bar{\mathbf{u}} + \mathbf{C_1 d} \\
& \mathbf{s} = \bar{\mathbf{s}} + \mathbf{C_2 d} + \mathbf{C_3 Q_{swr}} \\
& \mathrm{Prob}(u_{min} \leq \mathbf{u}(occ.) \leq u_{max}) > 1-\epsilon \\
& \mathrm{Prob}(|\mathbf{s_i}| \leq s_{max}) > 1-\epsilon \\
& \mathrm{Prob}(\mathbf{W} \cdot \mathbf{E_{l,i}}(\mathbf{u}) \leq h) > 1-\epsilon \\
& \mathrm{Prob}(\mathbf{E_{l,i}} \leq E_{max}) > 1-\epsilon \\
& \mathrm{Prob}(\mathbf{E_{c,i}} \geq 0) > 1-\epsilon \\
& \mathrm{Prob}(S_{min} \leq \mathbf{S_i} \leq S_{max}) > 1-\epsilon
\end{array}$$

Where $\mathbf{C_i}$ are diagonal matrix containing compensator gains.

The three control set up we will examine are: the complete one that makes use of storage and compensator, a second configuration in which storage is unprovided and, last, a configuration with neither storage nor compensator that try to find out an 'always feasible profile'. Problem formulation for the two subcases can be derived from the most general simply dropping (or forcing to be zero) unprovided elements.

3.3.1 The scenario approach

As pointed out, introducing probabilistic constraints in the formulation of the constrained control problem leads to a chance-constrained optimization program that is hard to solve since it is generally non-convex except for quite structured cases. Indeed, convexity is central for the real time computation of the finite-horizon policy, since it ensures the well-posedness and tractability of the optimization problem. The scenario approach is an innovative technology that has been introduced to solve complex optimization problems with an infinite number of constraints, a class of problems which often occurs when dealing with uncertainty. This technology relies on random sampling of constraints, and provides a powerful means for solving a variety of design problems in systems and control. We consider general chance-constrained problems:

$$\begin{array}{ll} \min\limits_{\alpha \in R^{n\alpha}} & l(\alpha) \\ \text{subject to:} & \mathrm{Prob}(\eta(\alpha,\omega) \leqslant 0) \geqslant 1-\varepsilon \end{array}$$

α is an $n\alpha$-dimensional optimization variable, whereas ω is the stochastic uncertainty parameter with probability distribution P_ω. The only assumption the scenario approach relies on is the convexity of the cost $l(\alpha)$ and the convexity of $\eta(\alpha,\omega)$ with respect to the optimization variable α only (the dependence on the stochastic parameter ω can be arbitrary). The scenario approach goes as follows. Since we are unable to deal with the wealth of constraints

$$\eta(\alpha,\omega) \leqslant 0, \forall \omega \in \Omega$$

we concentrate attention on just a few of them by extracting a random N instances or 'scenarios' of the uncertain parameter ω according to probability P, only the constraints corresponding to the extracted ω are considered in the scenario approach, in this way Scenario constrained program (SCP) is a standard convex and finite optimization problem which solution can be found at low computational cost via suitable solvers (provided that N is not too large).

$$\begin{aligned} &\min_{\alpha \in R^{n\alpha}} && l(\alpha) \\ &\text{subject to:} && \eta(\alpha, \omega_i) \leqslant 0, \quad i = 1, \cdots, N \end{aligned}$$

Through disregarding all constraints but N of them may appear naive, the scenario approach stands on a very solid mathematical description:

> *Theorem 1* [12]. Select a 'violation parameter' $\epsilon \in (0,1)$ and a 'confidence parameter' $\beta \in (0,1)$, being n the number of optimization variables and N the number of considered scenarios, if:
>
> $$N \geqslant \frac{2}{\epsilon}\left(ln\frac{1}{\beta} + n\right) \tag{3.5}$$
>
> then, with probability no smaller than $1 - \beta$, the optimization result satisfies all constraints in Ω but at most an ϵ-fraction, i.e. $Prob(\omega : \eta(\alpha, \omega_i) \nleqslant 0) \leqslant \epsilon$

For further details see Campi, Garatti, Prandini and the references therein. Theorem 1 says that if N is chosen as indicated, then, the probability of 'bad scenarios' in which constraints are violated is no greater than β. The result holds true irrespective of P, the probability distribution of the noise vector ω, which, hence, can be anything (as disturbances affecting our system are). By making (3.5) explicit with respect to N according to the technique proposed in Alamo et all.[32], it can be shown that the smallest N , say $\bar{N}$, satisfying (3.5) scales as

$$N = O\left(\frac{n + ln\frac{1}{\beta}}{\varepsilon}\right) \tag{3.6}$$

This relationship reveals important features of the computational complexity of the Scenario Algorithm:

- N increases logarithmically with $1/\beta$. Hence, we can enforce a very small value for β (like $\beta = 10^{-5}$ or even $\beta = 10^{-10}$ which guarantee the achievement of $Pr(\omega : \eta(\alpha, \omega_i) \nleqslant 0) \leqslant \epsilon$ beyond any reasonable doubt without affecting N too much.
- According to the parametrization of the control policy the dependence on the horizon length M may pose a hurdle in the applicability of the Scenario Algorithm, in view of the linear dependence of N on n.

This suggests using the alternative parameterizations of the control policy (introduced in subsection 3.3.2) with the aim of reducing the dimensionality of the optimization variable.

A big advantage of making use of scenario approach is that a probabilistic description of stochastic variables is not required whenever we have access of real measurements of involved signals, in particular we do not need to aware of correlations that exists in between of different disturbances sources. In our work the nature of involved stochastic parameters is much complex so that making use of real-happened realizations of them may be straightforward and goes to simplify the problem formulation even from a practical point of view. On the other hand the possibility to make use of forecast possible scenarios (performed by simulations on the weather forecast type) can be used as well.

3.3.2 Performance evaluation in the case study

The aim of this section is to evaluate the possibility to make use of the stochastic control formulation to achieve very good performances in a real life application counteracting the effects of stochastic components. For this purpose we will presents reformulations for the stochastic control problem that take advantage of the scenario approach. We make use of data not particularly refined (i.e. plug scenarios of June for calculations at the end of July) to provide a suitable temperature reference and disturbance feedback parametrization. To ease computational load and in order to increase the probabilistically satisfaction of constraints according to results of scenario approach, we decided to solve the problem defining optimization temperature setpoints every hours instead of every ten minutes and linear interpolating temperatures in between of them (figure 3.17). Unfortunately, the amount of data we can deal with is not such much, so that results are just qualitative. Choosing temperatures set point every hour we get a compromise in between computational feasibility and qualitative description of the problem even if, as we will see, results are not bad performances could reasonably be boosted making use of more optimization variables, more and more precise weather forecasts and so on. Notice that all the calculations are again performed over ten minutes width sampling period so that all the involved dynamics are grasped. It follows a description of the three control configurations and a qualitative description of results, numerical results will be presented and commented altogether at the end.

Feasible temperature profile

Making use of the scenario approach we try first to find out a temperature reference profile so that cost is minimized and constraints never breached. Optimization variables are in this case the 48 temperature setpoints (2 days horizon) $\bar{\mathbf{u}}$ that are linearly interpolated in $\mathbf{u}$. The algorithm is formulated as follows:

$$
\begin{aligned}
\min_{\bar{\mathbf{u}},h} \quad & h \\
\text{subject to:} \quad & u_{min} \leq \mathbf{u}(occ.) \leq u_{max} \\
& \mathbf{W} \cdot \mathbf{E_{l,i}}(\mathbf{u}) \leq h \quad \forall i \\
& \mathbf{E_{l,i}} \leq E_{max} \quad \forall i \\
& \mathbf{E_{c,i}} \geq 0 \quad \forall i
\end{aligned}
$$

where i are 61 different realizations of occupancy and disturbances. The result (figure 3.17) is a conservative profile where temperature rising is discouraged: precooling is evident only in relation to the initial condition (only the first day looking at the two days horizon), temperatures are averaged around the comfort set and it is not encouraged temperature rising after occupancy.

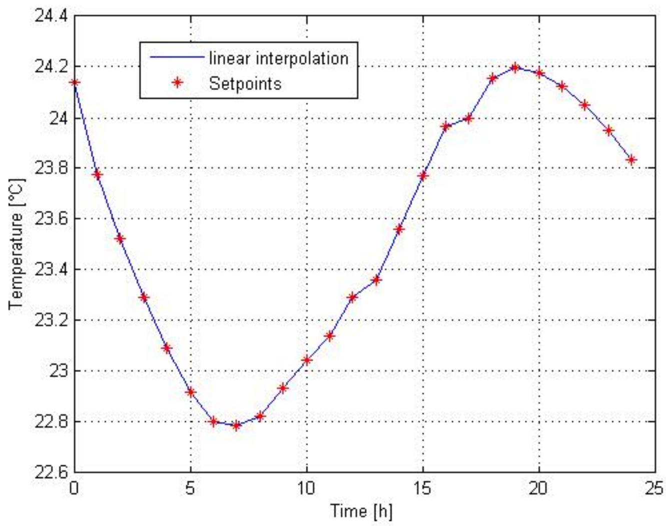

Figure 3.17: Feasible temperature profile

Disturbances compensator

Compensator is a time variant static gain whose values are defined by the optimization processes, the overall optimization variables are thus in number of 48 setpoints + 47 compensator gains; the algorithm is formulated as follows:

$$
\begin{aligned}
\min_{\bar{\mathbf{u}},\mathbf{C},h} \quad & h \\
\text{subject to:} \quad & \mathbf{u_i} = \bar{\mathbf{u}} + \mathbf{C}\mathbf{d_i} \\
& u_{min} \leq \mathbf{u_i}(occ.) \leq u_{max} \forall i \\
& \\
& \mathbf{W} \cdot \mathbf{E_{l,i}}(\mathbf{u}) \leq h \quad \forall i \\
& \mathbf{E_{l,i}} \leq E_{max} \quad \forall i \\
& \mathbf{E_{c,i}} \geq 0 \quad \forall i
\end{aligned}
$$

The resulting profiles for all the realizations used in paragraph 3.2.1 are presented in figure(3.18): as we can see there is a good variability for the temperature profiles and differences arises for every periods in respect to the previous simple policy. Constraints regarding both thermal comfort and positiveness of cooling energy are again never breached. Looking at some

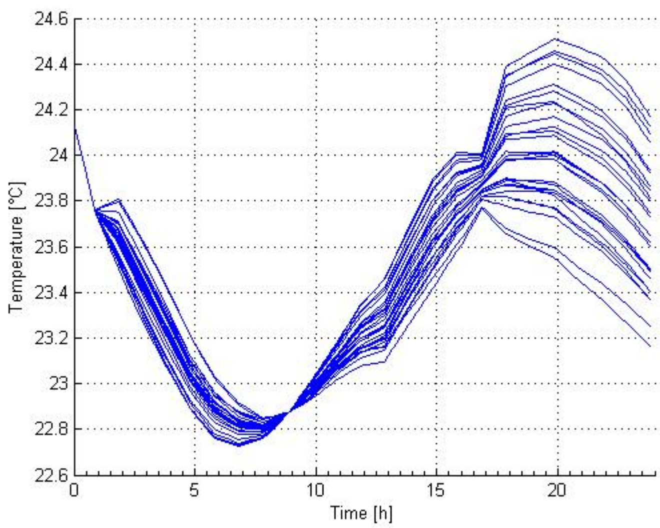

Figure 3.18: Compensator temperature profiles

particular profiles (figure 3.19) we can notice that there is not a general rule according which control evolves: for the first hour all temperatures follows a nominal profile since there is not informations yet about disturbances behavior, then we can see that precooling can either be boosted or slowed. This is true also for temperature rising and especially for the ending phase that could even presents qualitatively different behaviors.

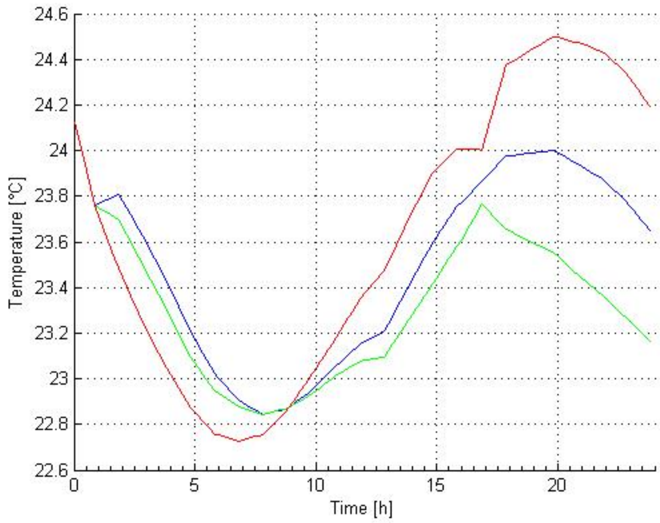

Figure 3.19: Some compensator profiles

Storage

Last the complete formulation that uses the storage. What we were looking for was a control policy that could ensure cooperation in between temperature reference and storage usage on the same way we had seen it works for the nominal case. Storage formulation goes to enrich the compensator policy formulation adding storage contribution setpoints in the same way we did for temperature setpoints. In this way we are considering a one hour picewise constant contribute of the storage to the energy balance, this probably highly limits performances. The reason why we adopted this rough working mode is, once again, computational and due to the scarce amount of data used to ensure probabilistic satisfaction of constraints via scenario approach. Also the storage takes advantage of environmental conditions measures in the same way temperature reference does, as a difference it shown up to be particularly convenient to make use also of the measures of shortwave radiation that affects directly the zone as a window gain. This is done making use of others 47 gains for the compensator and feeding it with the punctual measures of shortwave radiation. Optimization variables are thus in number of 48 temperature setpoints + 47 temperature compensator gains + 47 storage contribution setpoints + 94 storage compensator gains and the problem formulation is:

$$
\begin{array}{ll}
\min\limits_{\mathbf{\bar{u}},\mathbf{C_1},\mathbf{\bar{s}},\mathbf{C_2},\mathbf{C_3},h} & h \\
\text{subject to:} & \mathbf{u_i} = \mathbf{\bar{u}} + \mathbf{C}\mathbf{d_i} \\
& \mathbf{s_i} = \mathbf{\bar{s}} + \mathbf{C_2}\mathbf{d_i} + \mathbf{C_3}\mathbf{Q_{swr,i}} \\
& u_{min} \leq \mathbf{u_i}(occ.) \leq u_{max} \forall i \\
& |\mathbf{s_i}| \leq s_{max} \quad \forall i \\
& \mathbf{W} \cdot \mathbf{E_{l,i}}(\mathbf{u_i}, \mathbf{s_i}) \leq h \quad \forall i \\
& \mathbf{E_{l,i}} \leq E_{max} \quad \forall i \\
& \mathbf{E_{c,i}} \geq 0 \quad \forall i \\
& S_{min} \leq \mathbf{S_i} \leq S_{max} \quad \forall i
\end{array}
$$

Storage content realizations are presented in figure(3.20): strong usages

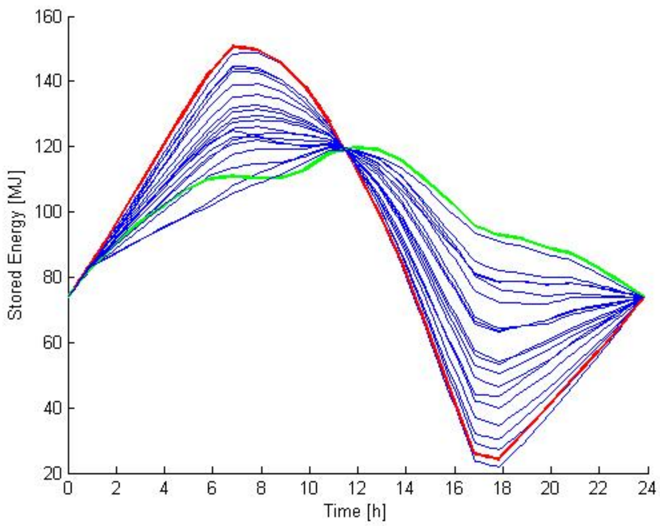

Figure 3.20: Storage level realizations

(red line) or weak ones (green line) are possible, however the way storage is used is different in respect to the nominal case since it is never used to boost the precooling phase. It is interesting to notice that the amount of energy charged is different case by case and that charging and discharging periods are not fixed nor constant. In the red case the storage works charging, discharging and then recharging, this is not in general, in the green case for example the storage just gets charged and then discharged. Moreover in this latter case storage is non participating from 6 to 10.

Results

In this section numerical results are presented in order to qualitatively compare performances. The optimal achievable performances (as in paragraph 3.2.1) are the best performances achievable being aware of the future values of disturbances at the optimization time, this values are introduced as comparison values in order to quantify performances degradation making use of the stochastic formulations previously presented.

- **Feasible temperature reference**: to the detriment of performances, constraints are never breached by the means of selecting a temperature profile suitable for every possible scenarios. Even if this policy may be improved making use of more detailed and suitable weather forecasts it seems to get worse especially when needs are high, the reason of it should be the impossibility to heat up the zone that makes the profile to be prudent in rising temperatures;

- **Disturbances compensator**: almost always the compensator configuration achieve better performances in respect to the Feasible control reference configuration. In particular it does very good in hot days obtaining performances very close to the best achievable at all. This is not when optimal achievable costs are low (index of cold days and low occupancy). As we can see in figure (3.22) the resulting temperature profile making use of the compensator is very close to the optimal achievable one, they differ in particular at the beginning when there's not information yet about disturbances and at the end because compensator is not aware about the effect of occupancy.

- **Storage**: Best performances are achieved making use of the storage. Almost in the half of cases storage usage shows up to be convenient, leading to savings in respect to optimal achievable without storage. In the more expensive days savings are around 30% reaching 37% in the best case. Good results are obtained also for warm days. Nothing can be done in cold days probably because of storage losses that augments consumptions;

Achievable	Feasible		Compens.		Storage	
18,16	24,11	33%	19,62	8,03%	15,72	-13,43%
10,68	12,93	21%	12,84	20,29%	12,39	16,10%
10,68	12,44	16%	12,75	19,41%	12,37	15,84%
13,21	17,44	32%	16,34	23,66%	14,10	6,73%
15,58	19,48	25%	16,97	8,93%	14,15	-9,16%
9,80	12,99	33%	12,56	28,21%	11,63	18,75%
20,02	27,55	38%	21,72	8,50%	16,38	-18,20%
23,33	33,15	42%	24,35	4,40%	17,45	-25,20%
27,25	41,38	52%	28,24	3,64%	18,85	-30,83%
26,16	36,07	38%	26,98	3,15%	18,24	-30,29%
29,40	44,30	51%	30,38	3,35%	19,58	-33,40%
24,08	36,93	53%	25,14	4,38%	17,99	-25,30%
31,25	45,18	45%	31,95	2,25%	19,86	-36,44%
25,33	38,07	50%	26,04	2,79%	17,68	-30,19%
30,13	40,02	33%	30,60	1,54%	18,94	-37,16%
21,02	27,78	32%	22,21	5,66%	16,58	-21,12%
16,14	19,16	19%	17,50	8,43%	14,44	-10,51%
16,46	21,06	28%	17,80	8,13%	14,47	-12,11%
21,79	25,38	16%	22,95	5,35%	16,83	-22,76%
11,90	13,31	12%	13,49	13,42%	12,53	5,33%
11,40	13,72	20%	13,68	20,02%	12,66	11,07%
13,69	15,60	14%	15,43	12,66%	13,63	-0,47%
13,44	15,74	17%	15,13	12,57%	13,25	-1,45%
11,05	13,23	20%	13,28	20,19%	12,36	11,85%
8,93	11,56	29%	11,21	25,57%	11,33	26,92%
10,34	11,73	13%	12,75	23,23%	12,16	17,59%
12,22	13,73	12%	14,07	15,15%	12,90	5,58%
8,90	12,06	35%	11,20	25,82%	11,28	26,78%
13,12	15,69	20%	14,88	13,38%	13,27	1,10%
11,86	13,43	13%	13,52	14,02%	12,62	6,43%
16,14	18,80	17%	17,56	8,81%	14,29	-11,42%

Absorbed energy

Some considerations can be made regarding the absorbed electricity: it is straightforward to derive probabilistic considerations regarding maximum and minimum energy load of the building, in a smart grid context it could be

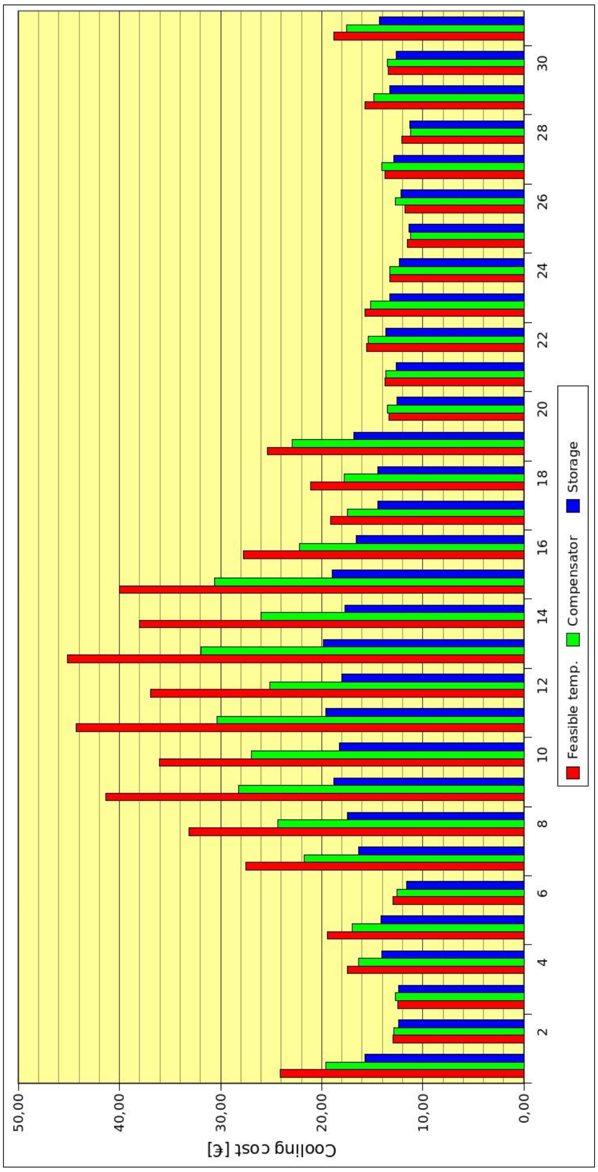

Figure 3.21: Performances compared

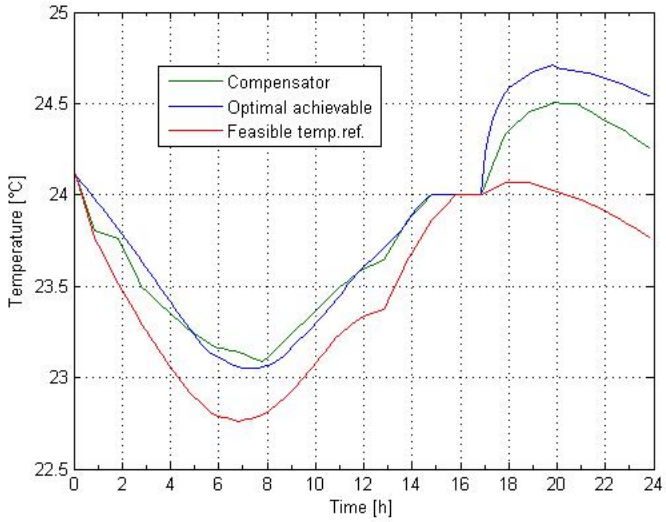

Figure 3.22: Temperature profile comparison

possible to take advantage of this feature for energy scheduling and forecasts. In figure (3.23) we can see the maximum and the minimum electricity needs envelope (they are both different from a single realization). The presence of the absorption peak at 17 o'clock is caused by the switch in the operating mode of the storage and should be discouraged in real applications. Clearly, being aware of such estimations may be useful at a grid level.

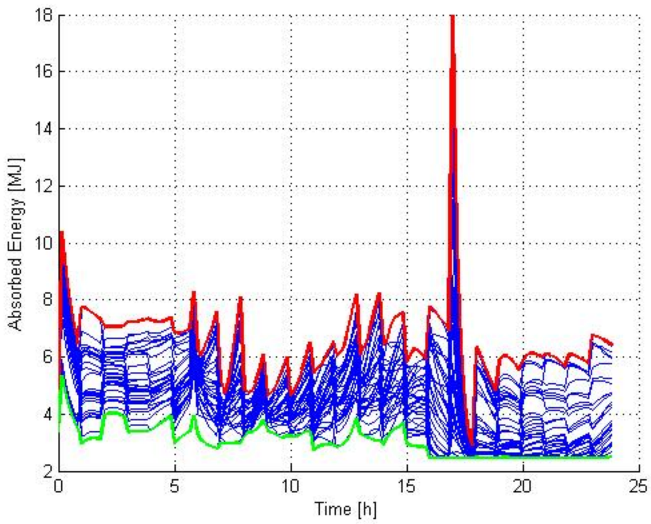

Figure 3.23: Energy needs

Chapter 4

Concluding remarks

We presented a novel approach to face optimal energy management of a building cooling system with thermal storage. A main distinguishing feature of our approach is that we adopt a model based on thermal energy balancing, and use the building temperature set-point as control variables. The optimal energy management problem is formulated as a finite horizon optimal control problem and addressed according to

i) the certainty equivalence-based approach, where stochasticity in the disturbances affecting the system is neglected and reference is made to some nominal operating condition, and

ii) the scenario-based approach, where a finite number N of realizations of the stochastic disturbances is considered in the control problem solution. Theoretical guarantees on the system behavior over all disturbance realizations except for a set of predefined probability can be provided if N is suitably chosen.

Notably, in the scenario-based solution a feedforward disturbance compensator is implemented, which significantly improves the controlled system performance.

We next summarize and highlight the main advantages of the proposed approach to the optimal energy management problem:

- *Considering the cooling system as a black box* allows to simplify the optimization problem since nonlinearities are neglected, and favors a convex formulation of the optimization problem;

- *Adopting a low-order linear model for the building thermal description* allows to grasp the heat dynamics, taking explicitly into account the influence of stochastic factors, while limiting the problem size at the same time;

- *Using temperature set-points as optimization variables* allows to robustly guarantee thermal comfort conditions in case of unpredicted events or modeling errors without making use of a secondary controller of the temperatures. Furthermore, constraints on the temperature evolution can be imposed directly;

- *Using a two days long prediction horizon* allows to avoid the definition of too restrictive constraints at the end of the actual time horizon of interest (1 day);

- *Adopting a stochastic approach to control design* counteracts unfeasibility problems and ensures performance. The design of a feedforward compensator via the scenario approach gives a large degree of flexibility to the system.

Aspects that deserve further investigation include:

- the possibility to preserve convexity of the control problem for a more realistic cooling system that includes chiller benches, pumps, economizers, cooling towers, ...;

- the introduction of a more detailed energy model for the storage system;

- a deeper characterizations of internal gains and zones models;

- the extension to HAVC systems. This will probably call for a problem reformulation that uses hybrid control theory.

Last but not the least, experimental tests should be carried out for assessing and comparatively analyze the performance of the proposed certainty equivalence-based and scenario-based solutions.

List of Figures

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