



**HUMAN RESOURCE PLANNING MODELS FOR HOME HEALTH CARE
SERVICES: ASSIGNMENT AND ROUTING PROBLEMS**

by
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To my parents

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Abstract

The care givers' assignment and routing problems are relevant issues for Home Health Care (HHC) service providers. The first problem consists of deciding which care givers will provide services to which patients, whereas the second aims at determining the visiting sequences of care givers. From a modelling perspective, these problems can be solved with either a two-stage approach or a simultaneous approach. Although the currently most known simultaneous approach yields more accurate results by solving the assignment and routing problems at the same time, its resolution remains computationally difficult and not viable for large scale applications.

In this thesis, we focus on the two-stage approach that sequentially solves an assignment and a routing problem in order to compare its performances to those of the simultaneous approach. Hence, several variants of mathematical models are developed by taking into account: (1) the skill compatibilities between patients and operators; (2) single or multiple planning periods; (3) imposed or released operator capacity restrictions. An important point regarding the two stage approach concerns the estimation of care givers' travel times that are required to solve the assignment problem. For this purpose, we propose an empirical data-driven method that is based on the Kernel Regression technique to estimate travel times. Such a method uses care givers' historical travel times that integrate several realistic factors such as cared patients' clinical conditions and locations or care givers' personal preferences to estimate the time necessary for visiting a set of patients located in the HHC service area.

Numerical studies based on realistic problem instances are used to analyze the performances of the proposed data-driven travel time estimation method and the two-stage approach. Results obtained show that both the newly developed travel time estimation method and the two-stage models are promising approaches for the HHC human resource planning process.

Keywords: Home health care; human resource planning; assignment; routing; skill management; travel time estimation; kernel regression

Résumé

L'affectation des patients aux soignants et le séquençement des visites à effectuer par les soignants sont deux problématiques intéressantes observées dans les établissements de soins décentralisés tels que les établissements d'HAD (Hospitalisation à Domicile), de SSIAD (Soins et services infirmiers à Domicile) ou de MAD (Maintien à Domicile). Le premier problème consiste en effet à décider quels soignants fourniront quels services (visites) à quels patients, tandis que le second vise à déterminer la séquence de visites de chaque soignant. Du point de vue de la modélisation, ces deux problèmes peuvent être résolus par une approche séquentielle qui comprend deux étapes ou une approche simultanée. Bien que les résultats de l'approche simultanée soient plus précis en raison de la résolution des problèmes d'affectation et de routage en même temps, son application semble être peu adaptée à des situations réelles, souvent de grande échelle. Dans cette thèse, nous nous concentrons sur l'approche en deux étapes qui considère successivement le problème d'affectation (assignment) et de séquençement (routing) afin de comparer ses performances à celles obtenues par l'approche simultanée. Ainsi, plusieurs variantes de modèles mathématiques sont développés en tenant compte de : (1) la compatibilité de compétences entre les patients et les opérateurs, (2) périodes de planification uniques ou multiples, (3) contraintes au niveau des capacités disponibles des soignants. Le verrou scientifique au niveau de l'approche en deux étapes concerne essentiellement l'estimation de la durée des déplacements des soignants, estimations qui sont nécessaires pour résoudre le problème d'affectation. À cette fin, nous proposons une méthode utilisant des données empiriques basée sur la technique de régression de Kernel (Kernel Regression Technique) permettant d'estimer les durées de déplacement. Cette méthode utilise des données historiques sur les durées de déplacement qui intègrent plusieurs facteurs réalistes concernant les conditions cliniques des patients et les conditions géographiques, ou encore les préférences personnelles des soignants afin d'estimer la durée nécessaire pour visiter un ensemble de patients situés dans la zone de service donnée. Des études numériques basées sur des données réelles en provenance d'un établissement d'HAD Italien sont réalisées pour analyser les performances de la méthode d'estimation proposée. Les résultats obtenus montrent que cette nouvelle méthode d'estimation ainsi que l'approche en deux étapes sont des approches prometteuses pour traiter des problématiques de planification de ressources humaines dans les établissements d' HAD, SSIAD ou MAD.

Mots-clés : Hospitalisation à domicile ; planification de ressources humaines ; affectation ; routage ; gestion des compétences ; estimation durées de déplacement ; régression de kernel

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GENERAL INTRODUCTION

With the ever increasing costs of operations, the service industry is faced with the tough challenge of offering better service quality while keeping costs as low as possible. This issue is even more important for *mobile services* that involve the traveling of service operators among customer sites and eventually, the realization of on-site activities. Indeed, home delivery, appliance (elevator, technical equipment, etc.) installation and repair services are typical examples of such services that include the transportation of goods and personnel (competencies) spending some time at customers' places. Hence, with the increase in energy costs and various constraints coming from customers or operators (e.g. different service offerings that require different skills, customer time windows or service preferences, lunch break constraints, etc.), to be able to tackle the human resource planning process for service operations becomes even more challenging.

Home Health Care (HHC) is an example of such mobile services that has known a fast growth recently in the health care sector, representing an alternative to the conventional hospitalization in developed countries [39]. As such, HHC providers deliver medical, paramedical and social services to patients in their homes. The development of the HHC concept can be attributed to demographic changes related to population aging, social changes in families, more people having chronic diseases, improved medical technologies, new drugs and governmental pressures to contain health care costs ([2,19]). Since many resources are involved in the HHC service delivery, including operators (e.g., nurses, physicians, physiotherapists, social workers, psychologists, home support workers, etc.), the human resource planning process is of particular interest and consists of several decisions such as: resource dimensioning (see [16]), partitioning of a territory into districts (see [9]), allocation of resources to districts (see [23]), assignment of operators to patients (or to visits) and routing (see [7]). In this research, we focus on the last two levels of planning that are the assignment and the routing problems. Although the importance of these short term processes, most of HHC providers still lack of methodologies and tools to improve the performance of these processes.

The assignment problem refers to the decision of which operators will take care of which patients, whereas the routing problem specifies the sequence in which the patients are visited. To determine each operator's route, the assignment lists of operators and, thus, the travel times between the assigned patients should be known. Traditionally, these two problems are solved simultaneously by using a single model. In this manuscript, we refer to this integrated model as the "simultaneous approach" which, in terms of modelling, corresponds to the Vehicle Routing Problem (VRP) that exists in the current HHC literature. An alternative approach is to solve these two problems sequentially: first, the assignment problem is solved, and then, the assignment results and travel times between patients serve as inputs to define the routing that each operator will perform. In this case, the individual operator route is often obtained by solving a Traveling Salesman Problem (TSP). This approach is often used in practice and in this manuscript, we refer to this sequential approach model as the "two-stage procedure".

As will be detailed in the coming chapters, the current HHC literature mostly focuses on the simultaneous decision of assigning patients to operators and defining their routes. Because both decisions are made at the same time, the simultaneous approach is known as theoretically the best alternative to solve such problems. One main drawback of this approach is that it requires solving an NP-Hard problem. In particular, in practice, there are several features that would have an impact on the assignment and routing decisions such as the geographical locations of patients, the care profile of patients, the availability of the person that provides help to the operator and the geographical aspects of the territory where the HHC provider is operating. Because of these features, in practice, minimizing total travel time may (as in the most of the existing works in HHC literature) not be the only criterion that is required to be achieved. However, modeling and using these features in the simultaneous approach would not be computationally tractable since each feature would be formulated and integrated as a new decision variable or a new constraint in the formulation of the model. An alternative method to capture such features can be obtained by using the available historical data which would provide information regarding the choices made in previous tours accomplished by operators. Thus, these features can be integrated into the models with the use of the data-driven approach which will allow to make future assignment and routing decisions based on the operator's past behaviors. However, it is still complicated to incorporate the data-driven technique into the simultaneous approach modeling framework.

In the two-stage approach, since the routing optimization can be considered independently

and exact travel times between patients are unavailable when the assignment problem is solved, an estimation of operator travel times is required to solve the assignment problem. Travel times can be estimated through different approaches. In this research we first use a basic approach based on Average Values (AV) which indicates the average traveling time to reach a patient from all other patients. Although this approach is intuitive, more accurate travel time estimation methods might be necessary to obtain results that would more closely approximate the results of the simultaneous approach. Thus, we propose to use the operator specific estimates via two different techniques. As the first one, we use an extended version of the AV technique, Operator Specific Average Value (OSAV), where each average value is calculated only with the assigned patients of a specific operator and repeated for each operator independently. The second operator specific estimate is proposed with the use of the previously discussed data-driven approach which might enable the capture the accomplished route choices and permitting travel time estimations based on the past behaviors.

Finally, we also developed different models to be able to consider several realistic situations where different operator and patient skills are incorporated hierarchically (simultaneously) or independently for single (i.e. a day) and multiple planning periods (i.e. multiple days). Since in real practice patients usually have different care requirements and operators have various qualification (skills), alternative skill management ways for the skill compatibility between patients and operators are crucial. Thus, in addition to the case where all skill levels are managed independently, we also consider the case where over skilled operators are able to care patients with lower skill requirements (i.e. hierarchical skill management). Such variety of models enable us to consider as much different cases as possible that can be applicable to differently structured health care providers from different regions and countries.

Contributions

This work has several contributions that can be described as follows:

The first one is to present the existing HHC literature that is available for the assignment and routing problems. To do this, we first present a detailed framework in Chapter 1 which is used to classify the important features (i.e. travel time, service time, planning period etc.) of the assignment and routing problems based on organization, geography and patient related aspects. We then, classify the existing works according to this framework in Chapter 2, section I.2 and

II.2 respectively. To our knowledge, such a recent literature review does not exist.

The second contribution is to propose a methodology to decompose the assignment and routing problems into two stages by considering several criteria such as operators' workload balancing, continuity of care, multiple operator and patient skills, multiple planning periods and travel time reduction. Details related to the models are provided in Chapter 2. The two-stage approach provides a significant contribution to the HHC literature since it enables to take into account more realistic situations than the currently available simultaneous approach and also allows to tackle complex instances characterized by larger number of patients and operators.

The third contribution, which is presented in Chapter 3, is to provide alternative travel time estimation methods for the assignment problem of the two-stage approach. Among the presented estimation methods, data-driven technique is the most crucial one for HHC services because it uses the travel times observed from previous periods to estimate the time for visiting a set of patients located in specific geographical locations.

The last contribution of this thesis is to analyze the performance of the proposed two-stage models in comparison to the simultaneous approach. To this end, several numerical experiments are conducted in Chapter 5.

All these goals have already been or will be presented in journals or conference publications as follows:

- The presented literature review in Chapter 2 is published in the proceedings of the 37th Conference on Operational Research Applied to Health Services (ORAHS 2011) with the title "Human Resource Scheduling and Routing Problems in Home Health Care Context: A Literature Review".
- The two-stage approach model (presented in Chapter 2) with the average travel time estimation technique (provided in Chapter 3) is published in the proceedings of the 8th annual IEEE International Conference on Automation Science and Engineering (CASE 2012) with the title " Operator Assignment and Routing Problems in Home Health Care Services".
- A new data-driven travel time estimation method (provided in Chapter 3) for the assignment problem of the two-stage approach is developed and published in the proceedings of the 1st International Conference on Health Care Systems Engineering (HCSE 2013) with the title "A Two-Stage Approach for Solving Assignment and Routing Problems in Home

Health Care Services”.

- An extension of the work published in the proceedings of HCSE 2013 has been submitted to European Journal of Operational Research for publication with the title ”The Assignment Problem in Home Health Care: A Data-Driven Method to Estimate Travel Times of Operators”.
- Models considering skill management alternatives that are presented in Chapter 2 for the two-stage and simultaneous approaches are being compared in the ongoing work and will be submitted to the Journal of Production and Operations Management.

Outline

This structure of this manuscript is presented on Figure 1 below:

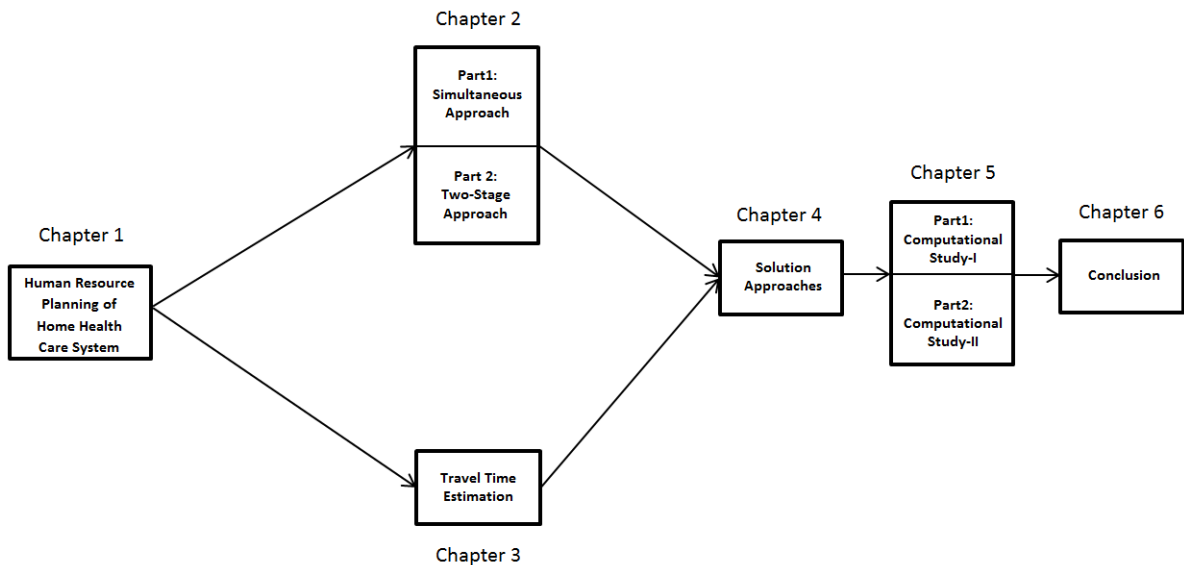


Figure 1: The structure of the Manuscript

In **Chapter 1**, we focus on the human resource planning process related to HHC systems. After a general overview of the human resource planning activity, due to high expected old-age dependency ratio by 2050¹, we provide a comparative description of HHC operations planning for France and Italy, by emphasizing difficulties, advantages, drawbacks and differences etc. In the second part of this Chapter, we present the hierarchial steps of human resource planning process

¹<http://www.pewglobal.org/2014/01/30/attitudes-about-aging-a-global-perspective>

including the assignment and routing problems that are analyzed in further details afterwards. As such, a framework that classifies HHC attributes related to organization, geography and the patient is proposed to classify the existing work in the HHC literature.

Chapter 2 consists of two main parts. **Part I** presents the simultaneous approach where we first provide a comprehensive literature review based on the framework presented in Chapter 1.4. Then, we present the mathematical programming models including assumptions considered and different modeling alternatives. **Part II** describes the two-stage approach that we propose. We start with the first-stage, the assignment problem, of the presented approach and then we provide details for the second-stage where details related to the routing problem are given. For the first-stage, we present the literature related to the stand-alone assignment problem applied to HHC services and then we provide the existing and newly developed mathematical programming models with the related assumptions and alternatives. Finally, the routing models that are used to solve the second stage of the two-stage process are presented in details.

Chapter 3 focuses on the travel time estimation methods to be able to use the two-stage process. We first explicitly present the details of the different alternative methods based on the average value and data-driven approaches. Since data-driven approach is one of the main focus of this thesis, the chapter ends with a convergence and accuracy analysis of this estimation alternative.

Chapter 4 presents the solution techniques that are used throughout this research. We present two different approaches that are adopted to solve the previously detailed mathematical models where the first one is based on a commercial CPLEX solver and the second one is based on a heuristic approach. Since, the choice of the solution algorithm depends on the used travel time estimation method, implementation details for each travel time function are also provided throughout this chapter.

In **Chapter 5**, several experiments are executed with the instances generated from real data and then associated numerical analysis is presented under two main parts. The first part (see Part III) presents the results with hierarchical skill models and the second part (see Part IV) provide results for the independent skill models. According to the analysis based presented in these parts, it is observed that the two-stage provides as good solutions as the simultaneous approach does based on the objective functions. It is also seen that the presented data-driven approach performs better than other travel time estimation methods. Thus, this makes the two-

stage approach with the data-driven travel time estimation technique as a promising method for future works.

Finally in **Chapter 6**, conclusions, limits of this research are discussed. Some future perspectives are also provided.

Chapter 1

HUMAN RESOURCE PLANNING FOR HOME HEALTH CARE SYSTEM

1.1 INTRODUCTION

HHC services emerge as an increasingly promising alternative for providing health and social services to patients at their homes. Many factors drive the need and demand for HHC such as the demographic trends, changes in the epidemiological landscape of disease, the increased focus on user-centered services, the availability of new support technologies and the pressing need to reconfigure health systems to improve responsiveness, continuity, efficiency and equity. HHC aims at satisfying people's health and social needs at their home by providing appropriate and high-quality health and social services within a balanced and affordable continuum of care.

HHC is considered and serviced differently around the countries across Europe. The differentiation occurs because health services are usually regulated within the framework of a national health system (i.e. Greece, Italy, the United Kingdom) or a national social insurance system (i.e. Austria, France, Germany and the Netherlands), while the social welfare systems usually administered by regional or local governments.

The proportion of older people in the general population is increasing in many European countries and is predicted to rise further in the coming decades. In this thesis, we consider 2 European countries, France and Italy, where this case is observed for the last 10 year¹ (See Figure 1.1). In particular, the ratio of care-dependent people in these counties are expected to increase steadily even more than many other European Countries (i.e. 25 Countries) for the coming three decades as well² (See Figure 1.2).

¹<http://www.who.int/>

²http://epp.eurostat.ec.europa.eu/portal/page/portal/population/data/main_tables

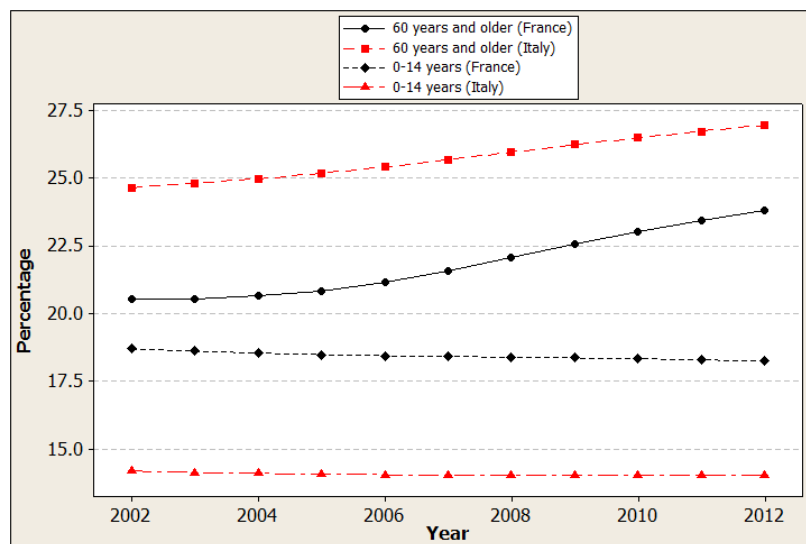


Figure 1.1: Old vs Young Population Change between 2002 and 2012 for France and Italy

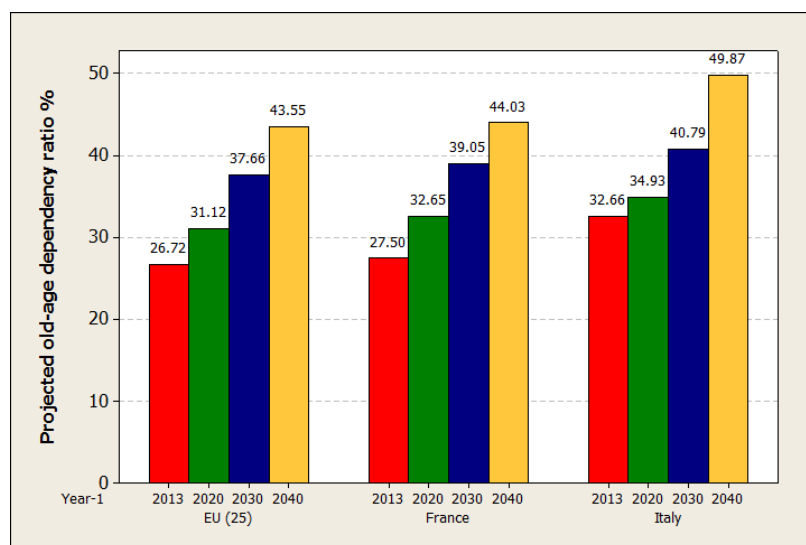


Figure 1.2: Expected Old Person Dependency Ratio in France, Italy and Europe

In the following part, we present the health care system of these countries to better analyze the differences and similarities of the provided HHC service. In particular, we provide details for 2 real health care providers from France and Italy as well. Lastly, we mention and position the human resource planning process in the HHC services.

1.2 HOME HEALTH CARE SYSTEM IN FRANCE AND ITALY

HHC services are organized under different structures in France and Italy. Generally, in France, the organization is managed by social insurance and local government or by the municipality. On the other hand, in Italy central or regional government takes care the organization issue of the HHC services. In France, the service delivery is provided to different age categories (i.e. child care or elderly people etc). On the contrary, in Italy although several age categories are considered as well, the main focus is on the elderly people. One other difference in the HHC system of these countries is the pricing condition. In France, big percent of the HHC service cost (i.e. around %80) is supported by the health insurance and the remaining part is paid by the patient. In Italy, HHC services are delivered free of charge and supported directly by the national health system and by the local health units. Beside the differences, the admission conditions for HHC services are mainly similar in both service structures. In both countries, the patient is moved from hospital to home care with the decision of the doctor in charge. In particular, after the decision of the doctor, family members of the patient should also agree with the transfer from hospital to the home environment. Here below details for 2 real service providers are presented.

1. *CHU Grenoble, France:*

It is founded in 1969 as the first HHC provider of the province. It is dedicated to home support for adults, pediatric patients with relatively severe disease and requiring hospital care, and maternity patients. The service is provided in a geographical area ranging up to about 40 km from the hospital that the operators mainly work for. They have capacity of 80 people, divided into three areas: 58 adults, 14 seats maternity and 8 pediatric patients and these patients are served by the team consists of 48 people including doctors, nurses, social workers, coordinator etc.

2. *MOSAICO Milan, Italy:*

MOSAIC is a company of SEGESTA group that provides health and social care services at home. Since 1999 it works in partnership with local health authorities (ASL and hospitals) in the provinces of Milan, Monza and Pavia. MOSAICO provides free service to the patients mainly with age of 65 and over (i.e. around % 87 of the total capacity) without considering their economic conditions. The service is provided with different category of operators including doctors, nurses, physiotherapists, coordinator etc.

As it can be recognized, these providers have different management strategies where the one from France is a part of the hospital and resources are shared between the hospital and home services. On the other hand, the Italian provider serves as an autonomous center and receives patients from several hospitals. Although it is also possible to see this management structure in France as well, the reverse case is not available. In particular, although both provide service to different age categories, the Italian provider has higher percentage of caring older patients than the French provider. Thus, we can conclude that different countries have different considerations and management strategies for the HHC services.

Even there are differences in the HHC systems, human resource planning process is always in the center of attention almost in all countries. Although we can also observe differences on the human resource planning strategies, the decision making processes is quite general. Hence, in the following section, we present details of the general decision making process in details.

1.3 DECISION MAKING PROCESS ON THE HUMAN RESOURCE PLANNING

There are several issues that should be considered in the decision making process of the human resource planning of the HHC services, such as the resource dimensioning, partitioning of a territory into districts, allocation of resources to districts, assignments of operators to patients or the visits and the operator routing.

These issues can be classified as long, medium or short term decisions. Among them, partitioning of a territory into districts can be considered as a long term decision whereas assignment and routing processes of operators can be considered as medium and/or short term decisions.

Although the focus of this thesis is on resource assignment and routing processes, it is also interesting to be aware of processes that take place before the assignment and routing processes. Figure 1.3A presents all the procedure explicitly. The first step is the resource dimensioning issue. Here, the number of operators are determined to meet the predetermined care demand with the minimum cost and the adequate service quality. The second step is partitioning of a territory into districts. This consists of grouping small geographic areas into larger clusters, which are named as districts, according to relevant criteria where each district is under the responsibility of a multidisciplinary team. Once districts are determined, resources are assigned to districts and then to patients equitably. After that the successive steps are the assignment and routing

processes.

We have discussed the human resource planning procedure where each step has been considered independently. As shown in the Figure 1.3B, 1.3C, and 1.3D, it is also possible to see that some of the processes can be carried out simultaneously.

Since the main focus of this thesis is based on the assignment and routing problems, we assume that the previous decisions are already held. Thus, in this research we compare models for the cases presented in Figure 1.3A and Figure 1.3C. Among these cases, the one that is presented in Figure 1.3C is the widely studied one in the HHC literature where the assignment and routing problems decisions are held simultaneously. On the other hand, the case shown in Figure 1.3A, which is sequentially solving the assignment and routing problems, has not been considered in this literature yet. Hence, with this thesis we develop different models and tools for this case and compare the performances with respect to the widely considered simultaneous case.

In the following part, we present the fundamental elements of the assignment and routing problems that are required for the development of the decision making tool.

1.4 FUNDAMENTAL ELEMENTS OF THE ASSIGNMENT AND ROUTING PROBLEMS

The modelling of the assignment and routing problems encountered in HHC services depends on several characteristics. In this part we present a framework to classify these characteristics based on organization, geography and patient related aspects. Figure 1.4 provides a general view for this framework where the details of the presented elements are discussed later in this section.

Such a framework enables us to see the impact of different HHC characteristics on the planning process of the HHC services. As such, elements in the two way and three way relation (intersection) can be seen as the more essential ones to be able to plan the service as in real practice. For instance, elements of the three way relation (i.e. Therapeutic Project (ThP) of patients, operators and travel time etc.) are the core of the HHC service planning issue and they are required to plan the service independent from the provider characteristics. Beside, difference parts in the framework (i.e. organizational objective or home environment etc.) can be considered as HHC specific characteristics that is more dependent on the HHC provider, the service region or the country etc.

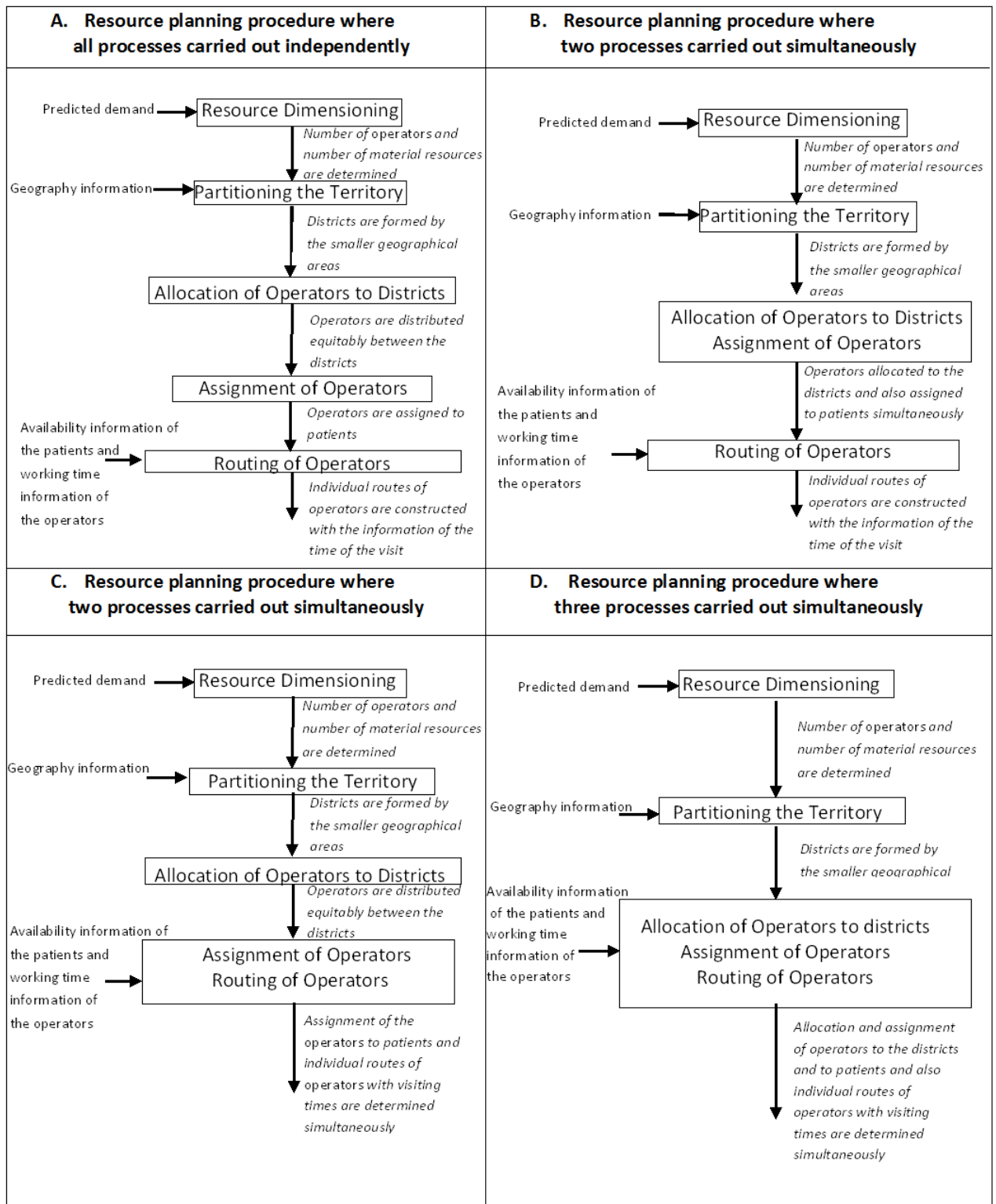


Figure 1.3: Alternatives for the Decision Making Process on the Human Resource Planning

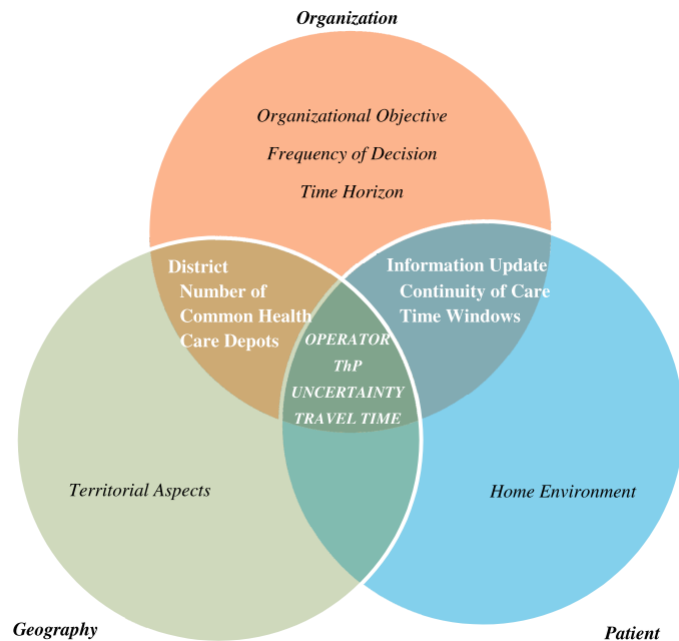


Figure 1.4: Human Resource Planning Elements

In the following part all the fundamental elements (characteristics) of the HHC services are defined and explained based on the organizational, geographical or patient related aspects in details.

1.4.1 Organizational Aspects

Here we present the main aspects that are related to organization of human resources which usually depends on the structure of the HHC provider.

1. Objective:

1.1 *Single Criterion:*

The assignment and routing objectives are set according to the structure and needs of the health care provider. These objectives can be set according to three assignment criteria: service quality, cost and use of resources. The most important objective for the use of resources criterion is usually required for assigning patients to operators with the balancing aspects (e.g., balancing the operator utilizations, balancing the number

of patients per operator etc.). An objective related to the cost criterion can be required for the both assignment and routing decisions. For the routing, it can be the reduction of travel times traversed by all of the operators or reduction of penalty costs of visiting other districts. For the assignment case, cost criteria can be the reduction of the cost of hiring external operator or the reduction of overtime costs etc. Moreover, the objective for the service quality aspect can be imposed by considering the continuity of care issue especially for the assignment decision.

1.2 *Trade-off Function:*

Depending on the service structure, it is also possible to consider trade-off between the use of resources and cost criteria (i.e., trade-off between the balancing values and total travel times of operators) while assigning patients to operators and obtaining visiting sequences of operators.

2. **Time Horizon:**

Planning horizon in HHC context is a time period, during which, health care provider will plan the service. In the planning horizon, decision is based on three aspects: elementary assignment and/or routing period, assignment and routing horizon and information update interval.

2.1 *Assignment and routing horizon (T):*

Planning process in the HHC is based on the available information (e.g., patient demand). To make decisions, planners should consider the maximum length of the available information (for how many periods the data is available). Thus, the assignment and routing horizon is the period where the overall assignment and routing decisions are supposed to be planned based on the length of the available information.

2.2 *Information update interval (U):*

The health care provider should also decide how to update the assignment plan. Information update is usually based on the information related to patients and operators (i.e., patient demand, operator availability, etc.). Since new patients are entering to the system, the conditions of previously admitted patients and operator availibilities are subject to change, the information update interval is important to respond the needs of patients at the right time. Different alternative cases are present and can be clas-

sified under two main groups: *decision with a fixed time frequency* or *decision with a condition*.

In the fixed time frequency case, independent from the specific patient or operator information, the information can be updated at the beginning of each period (e.g., day, week or month). Alternatively, in the decision with a condition case, information can be updated by a given condition. This condition can be updating with the arrival of each new patient (when a patient enters to the system, assignment process is held) or with the arrival of certain number of new patients (group of patients).

Using one of these alternatives depends on the organizational structure of the health care provider. Each provider may decide to use either alternative depending on their structure. If the provider chooses to use the second alternative, they also need to decide the condition of the decision. As indicated before, condition can be repeating decisions for each single newly admitted patient or a batch of patients. The single patient case can be used to increase the quality of the service. Serving patients as they arrive to the system will help to increase their satisfaction levels because they do not wait in the system and as soon as they enter, they are served with an operator. Decreasing waiting times is one of the important aspects to serve with a higher quality. On the other hand, assigning when the predetermined number of patients are obtained can be more efficient. This case is similar to the inventory control problem. With this alternative health care provider is also able to have more control over the capacity of the HHC system. In addition, health care provider can assign nurses to the waiting patients to do pre-assisting and this could be useful to avoid the quality problems due to waiting times to obtain the predetermined number of arrivals.

2.3 *Elementary assignment (A) and/or routing (R) period (P):*

Previous parts provided details for the planning horizon based on the whole decision period and the plans update interval. In addition to these information, it is also important to emphasize the elementary period where this responds the question of when to make or repeat each assignment and routing decisions. Usually, the whole decision period is relatively larger than this elementary decision period. This period(s) can be considered as sub-periods of the whole decision period where the assignment and routing decisions are held. For example, HHC provider may decide to do the assignments

and routings for 3 months and each assignment and routing decision can be repeated weekly. In particular, there can be also differences between the lengths of the elementary periods for the assignment and routing decisions. In such case, usually assignment periods are held for longer periods than the routing decisions. For example, within each week, routing decisions can be held for each day whereas assignment decision is held once for the whole week.

Figure 1.5 is used to show the planning horizon with the three important aspects.

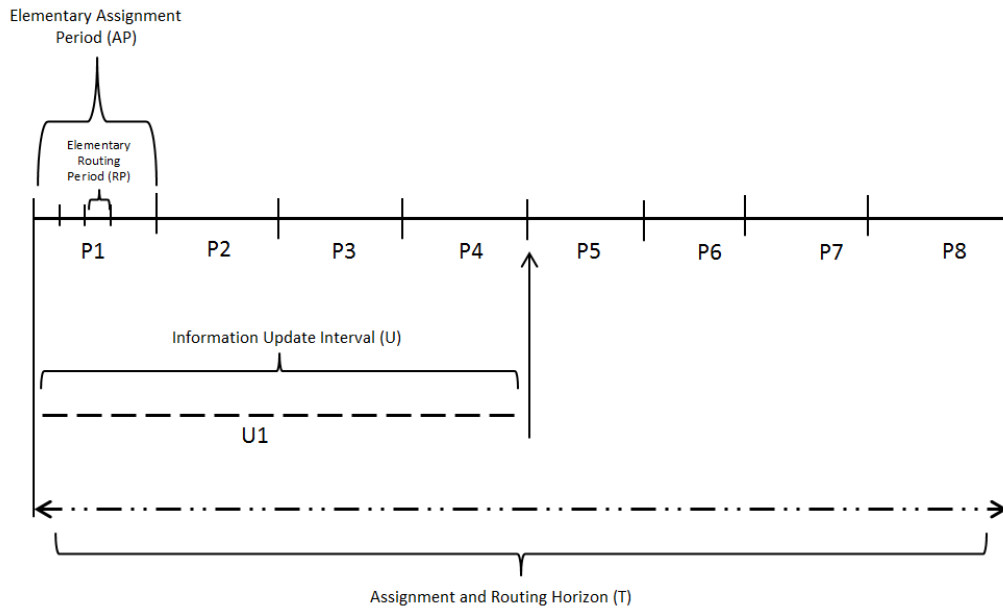


Figure 1.5: Human Resource Planning Horizon

3. Operator Characteristics:

3.1 Operators:

There are different categories of operators that are able to handle the specific requirements of patients such as nurses, physicians, physiotherapists, psychologists, social assistants, home assistants, etc. Each type of operators have different skill to handle different requirements of patients. Assignment and routing decisions are usually made for *single operator* type or if it is held for more than one operator types, it is generally assumed that assignment and routing decision are made for each type of operators

independently. In another case, if it is decided to use more than one operator type simultaneously, this can be considered as the *team* assignment and routing decisions.

3.2 *Operator Skills:*

Operators from each type have a main skill and may also have some additional skills to serve the different needs of the patients. The main skill of the operator is the one that is best suited to care a particular patient from a specific category (full knowledge of patient characteristics, etc.). With the additional skills an operator is also able to handle patients from different categories in addition to the category of his/her main skill.

For example, if a provider is serving for both palliative and non-palliative patients, hierarchical skilled operators can take care of both palliative patients (main skill) and non-palliative patients (additional skill). On the other hand, if the service is provided to only one type of patients, operators will be called *identical skilled operators*. In this case, there is not a distinction between main and additional skills.

3.3 *Operator Availability:*

Operator availability is the period, over a time frame, where the operator is able to serve the patients. Operator availability is usually considered according to the working contract and reliability. *Availability* is based on the contract type of the operator. The operator can be *full time*, *half time* or *external employee* of the provider. Full time employment is the general case where the health care provider is just accepting new patients according to its available *capacity*. Some health care providers may want to work in shifts (with half time employees) and instead of using an operator during whole day, they may want to split daily workload among different operators and ask operators work half a day (i.e., according to specific *operator time windows*). Although planning working days in shifts can be more complex to organize, asking less work during a day might be useful to increase operators motivation levels. Lastly, due to some special care types or insufficient number of operators, the health care provider may also need to use employees from external resources. Depending on the structure of the health care provider, they can use one, two or all of these different contract types to response the needs of the patients. Reliability of the operator is another important aspect related to the availability of the operator. Reliability is the ability

of an operator to perform and maintain its functions in routine circumstances. Due to personal reasons or illnesses, one or more operators may not be available during some time periods. Thus, considering the unavailability might be crucial to make better decisions. Other important concepts related to the contract and reliability aspects are the utilization rates, overtime hours and efficiency of operators. Utilization rate is the ratio between the actual workload and the operator's capacity. This rate can be used to measure the availability of the operator. In addition, it is also possible for operators to work beyond their capacities according to the need of the HHC provider (overtime) (i.e., soft operator time window). In such case, HHC provider usually needs to pay additional costs for each overtime hour exceeding the regular capacity of an operator. In particular, efficiency is defined as the ratio of time that the operator is available to the total time it is required.

Figure 1.6 is used to show the differences between operator capacity, operator availability and the overtime period.

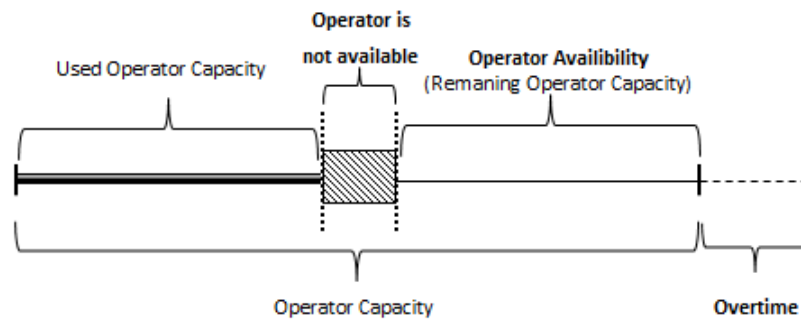


Figure 1.6: Operator Availability

3.4 Service Time:

The service time is the duration in which the operator provides the required care service to the patient. In real practice, since the service is provided with different operators to different patients, the service time can be *variable*. There are several factors that may cause this variability. The main factor is the operator-patient match which is based on the Thp of the patient. For example, different operator skills require different care durations according to the associated complexities. Similarly, if an operator has an

additional skill, the intervention times of using the additional skill might be longer in comparison to his/her main skills. Thus, this results in different operation times. In particular, in some cases where all patients have identical requirements from identical skilled operators, service time can also be *identical* and fixed (i.e., 45 minutes per visit).

1.4.2 Geographical Aspects

Here are the main aspects that are related to territory where the HHC provider is providing the service on.

1. District:

Districts are clusters where operators and patients grouped according to relevant criteria such as territory, skill and compatibility condition of the patient. There are two main ways of considering districts: *single district* or *multi districts*.

1.1 Single District:

The organizational structure has a important impact on the provider's districting scheme. The single district case is the simpler alternative that the provider does not split the geography into smaller clusters. However, to fully exploit special skills, operators can be preferred to be controlled among multiple districts. In such case, the health care provider is managing more than one district to serve its patients. Since there are more than one district, the provider may decide to serve each district independently as an autonomous decision center.

1.2 Multiple District:

Alternatively, the provider can also decide to serve districts in a integrated way. In this way, operators are also able to serve other districts that are not their primary one with a penalty cost. In another case, districts can also be formulated as they are intersecting. This provides more flexibility to the provider and they can allocate operators to more than one district without any additional cost.

2. Health Care Center:

2.1 Single Health Care Center:

In general, operators start and finish their daily activities in a common health care center.

2.2 *Multiple Health Care Center:*

If the service area is big and not easily accessible because of territorial aspects (i.e., urban, non-urban area), there can be more than one common health care centers and operators start and finish their service in one of these centers. Another alternative location can be the houses of the operators. In such a case, operators may only need to visit the common center on the beginning of the planning period and in the rest of the time they can serve their patients starting from their own houses.

1.4.3 Patient Related Aspects

Here are the aspects that characterizes the main profiles of patients that are associated to the HHC provider.

1. Patient:

1.1 *Classification:*

Patients are usually classified into different categories depending on the type (required operator type and skill) and intensity (volume) of the service requested (e.g., palliative and non palliative). Patients are also classified according to their physical presence in the system such as currently in service or planned future arrivals. With the admission to the system, the health care provider performs the Therapeutic Project (ThP). Once a ThP is defined, a category, namely care profile (CP), is assigned to the patient based on the pathology, requested operator type, required number of visits and home environment (either the people at home eligible to do services like cleaning or not).

1.2 *Demand:*

Patient demand is the service requirement of each patient in the given time period. There are different cases related to the patient demand as considering only the demand of the single period or multiple periods. It is also possible to update the demand information according to the condition of the patient. More detailed information was given in the time horizon part of this section.

1.3 *Availability:*

Patient availability is usually identified by the physical presence or *time window* of the patient. As described before according to ThP of the patient, a CP is assigned to the patient and this is usually revised periodically. According to this information, sometimes patients need to be served at hospital instead of their home. In such case, the patient is removed from the HHC system for some periods and he/she is not anymore available (physically not present) until the next arrival. In particular, time windows (i.e., hard time windows) restrict the times at which a patient is available to receive a service. Other than these time slots, patients are considered as unavailable and it is not possible to serve them. On the other hand, the patient may also ask to be visited on the preferred time slots due to some personal reasons. This case is commonly considered as soft time windows and these time windows can be violated with a certain penalty.

2. **Continuity of Care:**

The continuity of care aspect can be grouped as full, partial or no continuity of care. The provider use one of these groups according to the patient availability, operator availability and also the length of assignment horizon. The full continuity of care is pursued by several HHC providers to assign a patient to only one operator who is responsible for the care during his/her stay in the HHC service. Since loss of information between operators is avoided and the patient does not need to develop new relations with new operators, the full continuity of care is considered as a crucial indicator of the service quality. The partial continuity of care is also important where a patient requires more than one type of care. If one of the care types is more frequent (more than fifty percent of total care required), then a reference operator can also be assigned (like in the full continuity of care case) but some other operators are also needed to provide other required care services. In the no continuity of care case the provider does not need to respect the operator-patient assignments from previous periods. Each new assignment period starts with new assignment process where all available operators can be assigned to all patients according to their skills and requirements.

3. **Uncertainty:**

In real practice, it is usually not possible to know all the necessary information at the beginning of the planning horizon such as patient demand, patient availability, operator availability, travel time or service time. Since all these information are usually uncertain,

to have more realistic assignment and routing solutions considering the uncertainty of one or more than one of these elements might be significant.

4. **Travel Time:**

Travel time is the duration that the operator spends on the way between each patients and the common health care center (depot).

4.1 *Real Travel Time:*

If the assignment and routing decisions are held at the same time by the simultaneous approach, real travel times (i.e. euclidian distance) between each patient is available.

4.2 *Estimated Travel Time:*

The time needed to travel from one patient to the other depends on the sequence of visits defined for each operator. In the two stage problem, the optimal visit sequence is obtained by solving a sequencing problem based on patients assigned to a given operator. That is why, for the stand-alone assignment problem since the visiting sequence of patients are not yet obtained (i.e. the sequencing problem is not solved), relevant travel time estimations are necessary. This modeling aspect and different alternatives of the travel time estimation will be discussed on following chapters.

Until now, we have identified and detailed characteristics for the assignment and routing problems of the HHC services. Here below, Table 1.1 summarizes these characteristics in the light of the framework that we have developed and with respect to the available HHC literature. This table will be used in the following chapter (in Section I.2 and Section II.2) for presenting and comparing the available literature.

Note that this is a subjective analysis where each attribute is considered in one of the three attribute families according to our choice. Other considerations like considering operator under patient attribute family can also be a alternative representation.

In the following chapter, we present the assumptions, literature reviews and models for the assignment and routing problems of the HHC services.

<p>1 Organization</p> <p>1.1 Objective</p> <p>1.1.1 Single Criterion</p> <p>1.1.1.1 <i>Travel Time Minimization</i></p> <p>1.1.1.2 <i>Penalty Cost Minimization</i></p> <p>1.1.1.3 <i>Workload Balancing</i></p> <p>1.1.2 Trade-off Function</p> <p>1.2 Time Horizon</p> <p>1.2.1 Single Period</p> <p>1.2.2 Multiple Period</p> <p>1.3 Operator Characteristics</p> <p>1.3.1 <i>Single Operator</i></p> <p>1.3.2 <i>Multiple Operator</i></p> <p>1.3.3 <i>Identically Skilled Operators</i></p> <p>1.3.4 <i>Different Skilled Operators</i></p> <p>1.3.5 <i>Operator Availability</i></p> <p>1.3.6 <i>Operator Capacity</i></p> <p>1.3.7 <i>Full Time Contract</i></p> <p>1.3.8 <i>Half Time Contract</i></p> <p>1.3.9 <i>External Operator</i></p> <p>1.3.10 <i>Service Time</i></p> <p>1.3.10.1 <i>Identical</i></p> <p>1.3.10.2 <i>Variable</i></p> <p>1.3.11 <i>Time Windows</i></p> <p>2 Geography</p> <p>2.1 Number of District</p> <p>2.1.1 Single District</p> <p>2.1.2 Multiple Districts</p> <p>2.2 Number of Common Health Care Center</p> <p>2.2.1 Single Health Care Center</p> <p>2.2.2 Multiple Health Care Centers</p> <p>3 Patient</p> <p>3.1 Time Window Constraint on Visits</p> <p>3.2 Continuity of Care</p> <p>3.3 Uncertainty</p> <p>3.4 Travel Time</p> <p>3.4.1 <i>Real Travel Time</i></p> <p>3.4.2 <i>Estimated Travel Time</i></p> <p>3.4.3 <i>Travel Time is Not Explicitly Considered</i></p>

Table 1.1: Assignment and Routing Decision Attributes in HHC Services

Chapter 2

ASSIGNMENT AND ROUTING PROBLEMS

2.1 INTRODUCTION

As presented in Section 1.4, once the patient is admitted to the HHC service, according to his/her therapeutic project, the resource assignment and routing problems are solved to plan the visiting activities of operators. The assignment problem decides which operator will provide care for which patients and the operator routing problem specifies the sequence in which the patients assigned are visited. The planner tries to provide patients with convenient service according to their specific needs such as planning visit to the patient within appropriate time interval according to the availability of the person who provides help to the operator. He/she also tries to minimize operational costs in terms of distances traveled by operators such as planning the visiting sequence of an operator according to the geographical locations of the assigned patients (i.e. visiting the closely located patients one another). Lastly, the planner also tries to satisfy eventual operator preferences such as avoiding the planning of specific patient visits (i.e. located in the city center) in rush hours.

To specify each operator's route, the assignment lists of operators as well as the travel times between the assigned patients should be known. Most of existing work in the literature solve these two problems simultaneously where the assignment and routing decisions are held at the same time (i.e. VRP). Generally, this problem is formulated in a single model using data such as required visiting frequencies of patients (i.e., number of times that the patient should be visited in a given week), durations of patient visits, operators' capacities, operators' skills, and Euclidean distances that separate patients, which are deduced from their geographical locations (given by Euclidean distances).

Because both decisions occur at the same time, the simultaneous approach is known as

theoretically the best alternative to solve such problems. However, in practice, other features than the geographical locations of patients (i.e., euclidian distances) would have an impact on the assignment of patients and the routes that each operator would use. Examples of such features can stem from features related to patient care requirements (i.e., their care profiles) or the geographical aspects of the territory the HHC provider is operating. For instance, the visit of a patient requiring a blood test would most probably be done early in the morning, although it could be optimal to visit him/her at the end of the day if only a travel time minimization criterion is applied. Furthermore, because of physical constraints over the territory or implicit operator personnel preferences, some sequences of visits would never be realized in practice (although possible theoretically). Other features such as information regarding the availability, for a given day, of patient family members that help operator is another feature that would drive the operator to modify the planned sequence of visits. Because of these features, in practice, the HHC planner would assign a patient to a different operator than the one to whom she/he would be assigned when only the geographical criterion based on average or euclidian traveling values is used. In other words, minimizing total travel time may not be the only criterion that is wanted to be achieved.

Modeling and integrating such features to the simultaneous approach would not be computationally tractable since one would have to formulate each feature as a new decision variable or a new constraint and integrate it to the formulation of the model. Thus, we propose a new approach where assignment and routing problems are solved sequentially with the two-stage procedure: first, the assignment problem is solved, and then, assignment results serve as inputs to define the route that each operator performs. In this case, the individual operator route is often obtained by solving a Traveling Salesman Problem (TSP) model.

In the first stage, the assignment problem needs to be solved to obtain the assignment list of each operator based on patients' care requirements (i.e., required visiting frequency, service time and operator skill) and operator availabilities. Because the routing optimization is considered independently and exact travel times between patients are unavailable when the assignment problem is solved, an estimation of the travel time necessary to reach each patient is also required to solve the assignment. To this end, in Chapter 3 we present different travel estimation methods in details. Even if the two-stage approach is an approximation of the simultaneous approach, it has several advantages that makes it worth to use. First, it enables to take into account

the impact of several factors and operator behaviors observed in practice while HHC planners determine operator assignment lists and routes. Thus, this makes it a more realistic planning approach than the existing ones (i.e., simultaneous models). Second, most of time in practice, while operator assignment lists are defined over long periods (i.e., weekly or monthly planning horizon), operators' routes might be required for shorter periods (i.e., daily routes). In such cases, the two stage approach would enable to reach an increased planning flexibility since it would permit to work over different horizons when solving the assignment and routing problems. Third, depending on patients' needs, some adjustments of the scheduled plans (of the assignments and/or routes) may be necessary in practice. With the simultaneous approach, it could be difficult to make these adjustments directly because both the assignment and routing decisions have to be determined at the same time such that adjusting only one of them might be impossible. Although the simultaneous approach can be expressed over several periods, this expression results in complex formulations that would require demanding solution procedures and computational times.

Figure 2.1 represents the general framework for the simultaneous and two-stage approaches where input parameters (i.e. travel times, service times etc.) and output decisions (i.e. assignment lists and/or visiting sequences of operators) for each approach are explicitly identified.

We divide this chapter into two main parts: first, the simultaneous approach (see Part I), and then, the two-stage approach (see Part II) are presented. In Part I, we first present the existing literature for the simultaneous approach. Since this approach is used for the benchmark analysis (i.e. to be able to compare the models of two-stage approach), we refer to the existing models from the literature. We also develop non existing variant models by applying minor modifications on these models. On the other hand, in Part II, models for the two-stage approach are presented. In this part, in addition to the existing literature, newly developed mathematical models for the first stage problem and modified mathematical formulations (i.e. with respect to the existing routing literature) for the second stage problem with alternative considerations are presented. In the following section all the associated assumptions for both approaches are identified explicitly.

2.2 ASSUMPTIONS FOR THE ASSIGNMENT AND ROUTING PROBLEMS

In this thesis, we present several variants of the assignment and routing problems for both the simultaneous and two-stage approaches. For simplicity, all models are presented in the

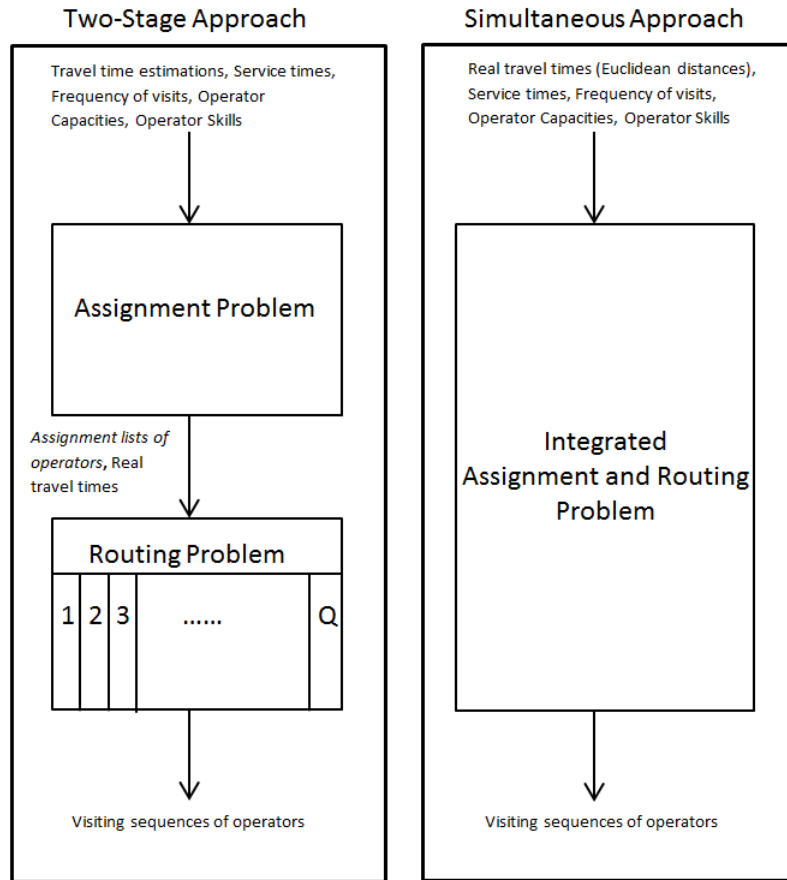


Figure 2.1: Two-Stage Approach vs. Simultaneous Approach

most general form which is composed of multiple planning periods (i.e. more than one day), hierarchical operator skills (i.e. operators with higher skills is able to handle the patients from lower skill level) and daily operator capacities (i.e. the maximum amount of time that the operator works according to his (her) contract). In particular, simpler variant cases are also considered with single planning period and/or relaxed operator capacities.

Assumptions related to the most general cases (i.e. most frequent and complicated case from practical and modeling perspectives) are described below. These assumptions are valid for both the simultaneous and two-stage approaches.

Planning Problem Characteristics

- Although districting is a priori step before handling the assignment and routing decision as presented in Chapter 1.3, in our work we assume that the districts have been defined earlier and our models are developed for one of the previously defined districts.

- We consider a planning period W (i.e. see for Chapter 1.4 for more details), usually a week (i.e. Elementary assignment period is a week and elementary routing period is a day, see Figure 1.5 in Chapter 1) for the models with *multiple planning periods* .

Patient Characteristics

- Models are defined on a complete directed network $G = (N, A)$ with n nodes, where each node j corresponds to a patient (with $j = 1, \dots, |N|$). We assume to have an extra node (node 0), which is used to denote the common health care center (i.e. basis of the operators) where each operator starts and comes back for each daily tour.
- A set K of \bar{k} levels of skill is assumed for patients, where skill \bar{k} corresponds to the highest skill (i.e. main skill of the operator) and skill 1 to the lowest level. All the available skill levels lower than \bar{k} is also considered as the additional skill of operators.
- Each patient is assumed to have a *care plan* r_j indicating weekly total service requests from one or more skill levels to be operated according to his/her therapeutic project. In other words, for each level of skill k , the care plan associated with patient j specifies the number (frequency) of visits required by patient j in the planning period W relatively to that skill. Thus, each care plan r_j is composed of one or more skill requests and each is denoted by r_{jk} with $k \in K$, representing the number of visits of skill k required by j in the planning period.
- It is assumed that the patient requests are operated according to a set P of a priori given (i.e. input parameters) patterns and they are used to identify all of the possible visiting combinations. For example, if a patient requires three visits of a given skill in the planning period, they can be operated according one of the pre-determined patterns Monday-Wednesday-Friday or Monday-Tuesday-Thursday etc. Formally, for each pattern $p \in P$ we define $p(d) = 0$ if no service is offered at day d , while it is $p(d) = k$ if a visit of skill k is operated according to pattern p on day d . Each patient is visited once in the day.
- Each patient j is assumed to have a deterministic demand λ_j (expressed in time), which denotes the total amount of care volume (in terms of service and travel time) the patient requires in the planning period. The demand of patient j is assumed to be calculated as follows:

$$\lambda_j = \sum_{k=1}^{\bar{k}} r_{jk}(\bar{\tau}_j + sv_j) \quad (2.1)$$

where sv_j is the service time that an operator spends at a patient location during a visit. It is considered as a standard value and without loss of generality, is assumed to have the same value for all patients. $\bar{\tau}_j$ is the estimated travel time (or can be replaced with t_{ij} as Euclidean distances) to reach the patient from any other patient or from the common health care center.

- Each patient receives at most 1 visit per day in total.
- Patient visits do not have precise time windows to be respected.
- We do not consider synchronous visits (i.e. only one operator is simultaneously required to visit a patient).

Operator Characteristics

- We consider a single category of operators (nurses or doctors).
- Each operator t , $t \in \Omega = \{1, \dots, O\}$, is assumed to have a deterministic capacity a_t , which corresponds to the maximum amount of daily time that he/she works according to his (her) contract (see Figure 1.6 in Chapter 1).
- The set of available operators on day d , for each $d \in W$ is denoted by O_d .
- HHC operators usually have a main skill and also some additional skills to serve the different needs of the patients. The main skill of the operator is the one that is best suited to care a particular patient from a specific category (full knowledge of patient characteristics, etc.). With the additional skills an operator is also able to handle patients from different categories in addition to the category of his/her main skill. In this work, we assume a hierarchical structure of skill levels where an operator with skill k is able to handle all the requests characterized by a skill level up to k and this is denoted with s_t . For instance, if two skill levels are assumed with skill 1 as ordinary (basic) request (non-palliative) and skill 2 as intensive request (palliative). Then, operator t with skill level 2 (i.e. characterized by $s_t \geq 1$) is able to handle patients from both skill 1 and skill 2 levels. In this thesis, we also

assume independent operators skills for some models. If this is the case, operators are only allowed to handle patients belonging to his/her main skill level.

- Each patient can be assigned to only one operator in the set of existing operators who is responsible for the care during his/her stay in the HHC service (i.e. Continuity of care is ensured).
- Each visit requires only one operator.

In this thesis, in the light of these assumptions the assignment and routing problems for HHC problems are addressed with three types of decision, the *care plan scheduling*, *operator assignment* and *routing decisions*, for both simultaneous and two-stage approaches. The care plan scheduling consist in assigning a pattern from P to each patient j to be able to schedule the requests of a patient j (i.e expressed by r_j) during the planning horizon. This is a crucial decision in the case of several planning days (i.e. multiple planning periods). On the other hand, the operator assignment decision corresponds to assigning operators to patients for each day where requests of patients have been scheduled. Lastly, the routing is the decision of computing the tour of each operator for each scheduled day.

In addressing these decisions, the skill constraints (i.e. the compatibility between the skills associated with patient requests and the skills of operators) and the daily workload constraints for the operators are taken into account as well as the *continuity of care* consideration. Models are solved under three types of objective functions that are: workload balancing, travel time minimization and workload balancing/travel time trade-off. Workload balancing refers to the case where the utilization rates of operators (defined as the ratio between the actual workload of the operator and his (her) capacity) are to be balanced. The second objective function, cost minimization, is used to minimize the total traveling times of operators. Lastly, the trade-off objective function tries to balance the trade off that exists between these two functions (i.e., workload balancing and travel time minimization). The selection of the objective function depends on the preference of the HHC service provider as presented in Chapter 1.4.

As a simple case, it is possible to consider the models with single planning period (i.e. a day) and without operator capacities. Such cases are applicable to real practice as well especially for small providers (i.e. few operators and small set of patients) or providers working with external operators. As such, the visiting plan is created one day before the service with the permission

of overtime (i.e. no restriction on operator capacities). Overtime is usually allowed when the operators are paid based on the visiting frequency and not the working time (i.e 8 hours per day). The single planning period can be easily obtained by assuming $W = 1$, $r_j = 1, \forall j \in N \setminus \{0\}$ and $P = 1$.

In the following parts, first we present the details of the considered models from literature for the simultaneous approach and then, we provide models that are developed for the two-stage approach.

Part I

THE SIMULTANEOUS APPROACH

I.1 INTRODUCTION

As described before, the simultaneous approach (VRP) is used to decide the assignment and routing decisions at the same time. It is important to note that, since VRP is a well known and widely studied approach in the literature, in this work, we do not intend to have a significant contribution to the VRP literature. Rather, we try to use this problem as the benchmark to be able to analyze the performance of the two-stage approach.

In Section I.2, we first present the literature review for the simultaneous approach of HHC services. Then, in Section I.3 we provide the general mathematical modeling formulation for this approach which is developed in a recent work of Cappanera and Scutella [17] where assumptions such as hierarchical operator skills, and multiple planning periods are considered. We also formulate other variants of this model by considering independent operator skills and/or single planning period.

The main contribution of this part is the presentation of the existing model from the literature (see [17]) and formulation of other variants for benchmark analysis with the models that are developed for the two-stage approach.

I.2 LITERATURE REVIEW FOR THE SIMULTANEOUS APPROACH

Almost all the existing assignment and routing models in the HHC literature are devoted to the simultaneous approach. Hence, in this section, we provide a general overview for existing papers on this topic (see also [26, 49]).

This literature review is provided according to the chronological order of the existing works. Table I.1 provides the overview of the existing works according to the framework provided in Section 1.4 (see Table 1.1).

Begur *et al.* [3] propose a Spatial Decision Support System (SDSS) that contains a special module for the daily scheduling of operators' activities. This module assigns simultaneously care providers to visits and generates the sequence in which the visits would be carried out. It is based on a heuristic approach that combines a set of procedures for building and improving the daily routes of care providers. The objective of this heuristic is to minimize the total travelling time while respecting constraints related to the route construction, care providers time windows, and skills requirements.

In the work of Cheng and Rich [21], a daily scheduling problem is developed as a multi-depot VRP with time windows (VRPTW) and the compability information. The problem is formulated as a mixed integer linear program. The objective is to minimize the total cost associated with the amount of overtime hours of full-time nurses and the amount of hours assigned to part-time nurses. Meanwhile this objective is obtained with respect to visiting each patient exactly once, assigning each nurse at least one patient, starting and ending at his/her home, taking a lunch break within the given time interval and respecting the maximum nurse shift length constraints. The problem is solved by a two-phase heuristic: the first phase falls into the parallel tour-building procedure category and the second phase attempts to make an improvement on tours identified in the first phase.

Eveborn *et al.* [27] develop a decision support system for the local authorities in Sweden, called Laps Care. In this system, they formulate the scheduling problem as a VRPTW with the set partitioning model and then solved by a repeated matching algorithm. The objective is to minimize a total cost related to the travel time, scheduled hours, preferences, etc., while respecting the following constraints: time windows for visits, operators' skill requirements, and accomplishment of each visit by one operator.

Bertels and Fahle [10] propose a weekly plan by using the VRPTW which combines linear programming, constraint programming, and heuristics in order to assign operators to visits and sort visits assigned to each operator optimally. The objective is to minimize the total transportation cost while maximizing the satisfaction level of patients and operators with respect to a variety of soft constraints. These soft constraints include affinities between the patients and care providers, preferences for certain visits and soft visits' and care providers' time windows. Besides, there are also some hard constraints that must be satisfied: skill requirements, work time limitations, time window constraints for visits, and the assignment of each visit exactly once.

Thomsen [47] addresses the daily scheduling problem as a VRPTW and shared visits (visits by two operators). The objective of this model is to minimize the total travelling cost, the number of unshared (visit is carried out by a non-reference operator) and unlocked visits and the number of shared (visit is carried out by two non-reference operators) and unlocked visits. The constraints of the model are as follows: respecting the visits' and operators' time windows, assignment of at least one visit to each operator and starting, ending a shared visit at the same

time. The model is solved by using an insertion heuristic and Tabu search technique.

Akjiratikarl *et al.* [1] generate daily schedules by using the VRPTW. Since this problem is a combinatorial optimization problem, they develop a heuristic based approach to solve it. They develop the Particle Swarm Optimization Problem (PSO) and also incorporate the Local Improvement Procedure (LIP) into the PSO solution approach to improve their solutions. Finally, they combine their approach with the Earliest Start Time Priority with Minimum Distance Assignment technique to generate the initial solutions. Within this framework, they focus on the determination of routes for each operator while minimizing the total distance travelled with respect to visits' and operators' time windows and assignment of each visit to only one operator.

Ben Bachouch *et al.* [5] develop the VRP with time windows as a mixed linear programming model with the objective of minimizing the total distance travelled by the operators. The model is subject to visits' and operators' time windows, nurses' meal breaks, care continuity and the restriction on the nurses' maximum distance travel limit constraints.

Elbenani *et al.* [25] develop a model for determining routes for operators that incorporates constraints of the VRP with the medical and continuity of care constraints. Here, each patient is assigned to a region with respect to his/her home address. Similarly, each nurse is also assigned to a region but there can be more than one nurse in a specific region. They allow a nurse to visit a different region with a certain penalty. In this model, they add blood sample related constraints as a medical constraint and they consider the objective function as minimizing the total travelling cost of operators. As a final step, they solve this problem with a meta-heuristic approach based on Tabu search.

Ben Bachouch *et al.* [6] address the daily drug delivery problem in the French home care structure as a VRP with time windows. The objective of the model is to minimize the total distance travelled. In this model they assign carriers to specific regions so that each tour is realized by the same carrier. In addition, they develop four different strategies as follows: starting deliveries when a specified number of deliveries is received, starting deliveries if a specified distance is reached regarding to the planned deliveries, starting deliveries on a fixed number of deliveries per carrier, and starting deliveries on fixed hours. They compare results for each strategy in order to identify which one is the most efficient to solve the drug delivery problem in the HHC context.

Chahed *et al.* [20] couple the production and distribution of anti-cancer drugs within the

context of the chemotherapy at home. They present six models based on three main criteria: time windows, objective function and distribution of drugs. The objective is either to minimize the delivery cost or maximize the number of visited patients.

Bredstörms and Rönnqvist [12] develop a mathematical model that incorporates synchronization and precedence constraints between visits. The proposed model is based on the traditional VRP with the additional synchronization and precedence constraints. They use a heuristic approach based on the local branching heuristic to solve their model. In their previous study (see Bredstörms and Rönnqvist [11]), they developed a branch-and-price algorithm to solve the same model without including the precedence constraints.

Kergosien *et al.* [33]) formulate the routing problem of the HHC operators as a Multiple Traveling Salesman Problem (MTSP) with time windows. The objective of the proposed model is to minimize the total travelling cost while respecting visits' and operators' time windows constraints, the assignment of each service to one operator constraints, synchronized (some visits require more than one operator) and disjunctive (some operators cannot work together) services constraints, continuity of care and the assignment of all operator constraints.

Trautsmawieser *et al.* [48] develop a model for the daily planning of the HHC services. The main aim of their work is securing the HHC services in times of natural disasters. They develop the daily scheduling model as a VRP with state-dependent breaks. The objective of the model is minimizing the sum of travel times and waiting times, and also the dissatisfaction levels of the patients and health care operators. The proposed model is first solved for small data with a state of art solver. Then, they also solve the real life-sized data with a neighborhood search based heuristic.

Recently, Rasmussen *et al.* [45] address the daily scheduling problem as a multi-depot VRPTW and connections between visits. They use a multi-criteria objective rather than only minimizing the total distance travelled. The proposed formulation is very similar to the one that is developed by Bredstörms and Rönnqvist [11] but here they allow also a visit to be uncovered (visit is not carried out). Thus, the proposed multi-criteria objective includes the minimization of uncovered visits, the maximization of operator-visit preference and the minimization of the total distance travelling costs. In particular, in the objective function they assign a higher priority to the uncovered visit criterion than the other criteria. Finally, the constraints of this model include: each visit can be covered exactly once or left as uncovered, operators can only handle allowed

visits, visits' and operators' time windows and precedence relations of visits.

Nickel *et al.* [44] address the weekly scheduling problem as the combination of the VRPTW and the nurse rostering problem. The objective of the proposed model is minimizing the weighted sum of the patient-nurse loyalty (continuity of care), unscheduled tasks, the overtime costs, and the traveling distance. They solve the proposed models using different meta-heuristics combined with methods from constraint programming which allows a very flexible treatment of realistic constraints.

Lastly Cappanera and Scutella [17] propose an extended VRP models to formulate the weekly planning of HHC services by considering skill compatibility between patients and operators. They develop a hierarchical skill management concept where they allow the over skilled operators to serve patients with lower skill requirements. The developed model aim at balancing the operator workload via two objectives with respect to continuity of care and operator skill constraints. The first considered objective is to maximize the minimum operator utilization factor whereas the alternative one is to minimize the maximum operator utilization factor. Finally, they analyze the computational results on a set of real instances.

Note that, the literature on the simultaneous assignment and routing problems is also recently reviewed by Hulshof *et al.* [32] that proposes a taxonomic review on planning-related decisions in health care services, including HHC.

This literature review is provided to present the details of the works that has been provided for the HHC VRP literature. Table 1.1 presents more specific details of these works as well.

In the following sections, we present the mathematical models for the simultaneous approach that will be used as the benchmark models for the two-stage models in Chapter 5.

I.3 MATHEMATICAL MODELS

In this section, we present the most general simultaneous model that is considered in this thesis. The term most general corresponds to the case where hierarchical operator skills, multiple planning periods and operator capacities are assumed. Then, we also formulate the models where independent operator skills (i.e. operators assigned only according to their main skill) are considered. Several alternative models are presented since in practice there are several different structured health care providers according to their budget, patient and operator profile, region, country etc. For example, some providers only provide unique service (i.e. blood sample testing)

No	Feature	Begur <i>et al.</i> [3]	Cheng and Rich [21]	Berteles and Fahle [10]	Eveborn <i>et al.</i> [27]	Thomsen [47]	Akjritrakarn <i>et al.</i> [1]	Bredstorm and Romqvist [11]	Bredstorm and Romqvist [12]	Elibanani <i>et al.</i> [25]	Ben Bachouch <i>et al.</i> [5]	Ben Bachouch <i>et al.</i> [6]	Chahed <i>et al.</i> [20]	Kargosien <i>et al.</i> [33]	Trautsammwieser <i>et al.</i> [48]	Rasmussen <i>et al.</i> [45]	Nickel <i>et al.</i> [44]	Cappanera and Scutella [17]
1.1.1.1	Travel Time Minimization Obj.	✓		✓	✓					✓	✓	✓	✓	✓				
1.1.1.2	Penalty Cost Minimization Obj.		✓															
1.1.1.3	Workload Balancing Obj.																	
1.1.2	Trade-off Obj.					✓	✓	✓	✓						✓	✓	✓	✓
1.2.1	Single Time Horizon		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
1.2.2	Multiple Time Horizon	✓																
1.3.1	Single Op.		✓	✓	✓	✓	✓	✓	✓									
1.3.2	Multiple Op.									✓								
1.3.3	Identically Skilled Op.					✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
1.3.4	Different Skilled Op.	✓	✓	✓	✓													
1.3.5	Operator Availability		✓	✓	✓	✓	✓	✓	✓									
1.3.6	Operator Capacity	✓	✓	✓	✓	✓	✓	✓	✓									
1.3.7	Full Time Op. Contract	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
1.3.8	Half Time Op. Contract	✓	✓															
1.3.9	External Operator																	
1.3.10.1	Identical Service Time																	
1.3.10.2	Variable Service Time	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
1.3.11	Operator Time Window	✓	✓	✓	✓	✓	✓	✓	✓									
2.1.1	Single District	✓	✓	✓	✓	✓	✓	✓	✓									
2.1.2	Multiple District				✓					✓								
2.2.1	Single Health Care Center	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
2.2.2	Multiple Health Care Center		✓															
3.1	Patient Time Window		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
3.2	Continuity of Care									✓	✓	✓	✓	✓	✓	✓	✓	✓
3.3	Uncertainty																	
3.4.1	Real Travel Time	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
3.4.2	Estimated Travel Time																	
3.4.3	Travel Time is Not Explicitly Considered																	

Table I.1: Classification of Existing Models Related to the Simultaneous Approach in HHC services

to their patients thus, a model with independent operator skill for multiple planning periods is required to be able to formulate such case. On the other, some providers serve patients with regular (non-palliative patients) or intensive (palliative patients) service requirements. Hence, such case can be formulated with a model considering the hierarchical skills either for multiple or single planning period.

In order to formulate the HHC models in a more formal way, let us recall or define the following definitions:

W the planning period

K the set of skill levels

\bar{k} the highest skill

r_{jk} the number of visits required by patient j in W from skill k , $j \in N(j \neq 0), k \in K$

sv_j the service time that an operator spends at the location of patient j

t_{ij} the traveling time from patient i to j , $(i, j) \in A$

a_t the daily capacity of operator t , $t \in O$

O_d the set of available operators on day d , for each $d \in W$

D_{td} the daily workload of operator t , $t \in O_d, d \in W$

The following variables are used to model the care plan scheduling, the assignment and the routing decisions:

$$z_{jp} = \begin{cases} 1 & \text{if pattern } p \text{ is assigned to patient } j \\ 0 & \text{otherwise} \end{cases} \quad j \in N \setminus \{0\}, p \in P$$

$$u_{tj} = \begin{cases} 1 & \text{if operator } t \text{ is assigned to patient } j \\ 0 & \text{otherwise} \end{cases} \quad t \in O, j \in N \setminus \{0\}$$

$$x_{ij}^{td} = \begin{cases} 1 & \text{if operator } t \text{ travels along } (i, j) \text{ on day } d \\ 0 & \text{otherwise} \end{cases} \quad (i, j) \in A, i \neq j, d \in W, t \in O_d$$

and

y_{ij}^d auxiliary flow variable which represents the number of the patients visited after patient i by the operator moving along (i, j) on day d , $(i, j) \in A$, $d \in W$.

Using these variables and definitions, in the following section we present the simultaneous hierarchical skill model (see [17]). Then, variant of this model with independent skills is formulated. For both cases, variant models including multiple and single planning periods with and without capacity constraints are also presented.

I.3.1 Hierarchical Skill, Multiple Planning Periods Model

Remind that the hierarchical skill corresponds to the case where an operator with skill k is able to handle all the requests characterized by a skill level up to k . In particular, multiple planning periods are defined as $W \geq 2$.

In this case, it is assumed that the previously mentioned three decisions are considered simultaneously for all patient types (i.e., with all required skill levels) and for all skill levels of operators via a joint approach. In other words, a single simultaneous problem is formulated involving all operators (i.e. from each skill level), to handle all the patient requirements from each skill level with the use of the hierarchical skill consideration. This refers to the case where more qualified operator, e.g. with skill level $\bar{k} = 2$, is assumed to serve patients that belong to either skill $k = 1$ and/or skill $k = 2$ levels. This hierarchical skill consideration for the simultaneous approach with the workload balancing objective is given by the following model:

$$\min h \quad (2.2)$$

$$\sum_{i \in N} \sum_{t \in O} x_{ij}^{td} \leq \sum_{p: p(d) \geq 1} z_{jp} \quad \forall j \in N \setminus \{0\}, \forall d \in W \quad (2.3)$$

$$\sum_{i \in N} \sum_{t \in s_t \geq k} x_{ij}^{td} \geq \sum_{p: p(d) = k} z_{jp} \quad \forall j \in N \setminus \{0\}, \forall d \in W, \forall k \in K \quad (2.4)$$

$$\sum_{p \in P} z_{jp} = 1 \quad \forall j \in N \setminus \{0\} \quad (2.5)$$

$$\sum_{t \in O} u_{tj} = 1 \quad \forall j \in N \setminus \{0\} \quad (2.6)$$

$$x_{ij}^{td} \leq u_{tj} \quad \forall (i, j) \in A, \forall j \in N \setminus \{0\}, \forall d \in W, \forall t \in O_d \quad (2.7)$$

$$u_{tj} \leq \sum_{i \in N} \sum_{d \in W} x_{ij}^{td} \quad \forall j \in N \setminus \{0\}, \forall t \in O \quad (2.8)$$

$$D_{td} = \sum_{(i, j) \in A} (t_{ij} + sv_j) \cdot x_{ij}^{td} \leq a_t \quad \forall d \in W, \forall t \in O_d \quad (2.9)$$

$$\sum_{i \in N} x_{ij}^{td} = \sum_{i \in N} x_{ji}^{td} \quad \forall j \in N \setminus \{0\}, \forall d \in W, \forall t \in O_d \quad (2.10)$$

$$\sum_{j \in N} y_{0j}^d = \sum_{j \in N \setminus \{0\}} \sum_{p: p(d) \geq 1} z_{jp} \quad \forall d \in W \quad (2.11)$$

$$\sum_{i \in N} y_{ij}^d - \sum_{i \in N} y_{ji}^d = \sum_{p: p(d) \geq 1} z_{jp} \quad \forall j \in N \setminus \{0\}, \forall d \in W \quad (2.12)$$

$$y_{ij}^d \leq n \sum_{t \in O_d} x_{ij}^{td} \quad \forall (i, j) \in A, \forall d \in W \quad (2.13)$$

$$\frac{\sum_{d \in W} D_{td}}{|W| \cdot a_t} \leq h \quad \forall t \in O \quad (2.14)$$

Constraint (2.3) states that (exactly) one operator per day can visit patient j only if a visit has been scheduled on that day for node j . Constraint (2.4) guarantees that, on day d , exactly one operator, of adequate skill, must visit patient j if a service of that skill has been scheduled for j on day d . In particular, the least skilled operators can perform only visits of skill 1 (case $k = 1$), whereas the most skilled operators can perform all types of visits (case $k = \bar{k}$). Constraint (2.5) ensures that each patient is assigned exactly to one pattern. Constraint (2.6) ensures that exactly one operator is assigned to each patient during the planning period. This is included in order to guarantee the continuity of care. Constraint (2.7) guarantees that an operator can visit

a patient only if he/she has been assigned to that patient (links between routing and assignment variables). Furthermore, constraint (2.8) forces variables u_{tj} to zero if operator t never visits patient j during the planning period. Constraint (2.9) ensures that the workload of each operator in each day, expressed as the sum of the service times and the traveling times, does not exceed the corresponding daily capacity. Constraint (2.10) is the classical flow conservation constraint on the routing variables. Constraint (2.11) and (2.12) are the flow conservation constraints on the auxiliary y variables, which are introduced to avoid sub-tours in solutions. They also guarantee the link between scheduling decisions and auxiliary flow variables. Finally, constraints (2.13) link together routing variables and auxiliary flow variables.

Note that a pattern variable z_{jp} can have a value other than zero only if:

$$|\{d : p(d) = k\}| = r_{jk} \quad \forall k \in K. \quad (2.15)$$

Therefore, in the preprocessing step $z_{jp} = 0$ if anyone of the \bar{k} constraints (2.15) is not satisfied. In addition, in the preprocessing step $x_{ij}^{td} = 0$ if patients i and j have only requests of skill at least k during the planning horizon, and t is an operator of skill less than k (i.e. $s_t \leq k$).

The objective function defines a workload balancing between operators over the planning period by minimizing the maximum operator utilization rate.

An alternative objective function considers the cost minimization where total travel time of operators are considered like in the classical VRP model. The model with the cost minimization can be formulated as follows:

$$\min \quad v \quad (2.16)$$

$$\text{Constraints} \quad (2.3) - (2.13) \quad (2.17)$$

$$\sum_{d \in W} \sum_{(i,j) \in A} t_{ij} \cdot x_{ij}^{td} = v \quad \forall t \in O, \quad (2.18)$$

where the auxiliary variable v defines the total traveling time of operators in the planning period.

The last alternative objective functions is defined as the trade-off function that balances the trade-off that exists between workload balancing and total travel time minimization. This case is defined as follows:

$$\min \quad h + \gamma \cdot v \tag{2.19}$$

$$\text{Constraints} \quad (2.3) - (2.14) \tag{2.20}$$

$$\text{Constraint} \quad (2.18) \tag{2.21}$$

where h is the auxiliary variable that is used to estimate the maximum utilization rate of the operators, v is used to calculate the total travel time and γ is a penalty parameter between 0 and 1.

I.3.2 Variants of the Hierarchical Skill Multiple Planning Period Model

Hierarchical Skill, Single Planning Period Model

Until now, we present the model for the hierarchical skill case where multiple planning periods and operator capacities are considered. Here we present the modifications required to obtain the model for a single planning period.

This case can be easily obtained by assuming $W = 1$, $r_{jk} = 1, \forall j \in N \setminus \{0\}$ and $O_d \equiv O$.

Since here we consider the single planning period, and we can only provide a single visit per day, each patient is considered according to the skill requirement associated with that day. For instance, if there are two skill levels, then patients are assumed to require the service either from skill level 1 or 2 on the given day.

Hierarchical Skill without Operator Capacity Restrictions Model

As discussed before, in practice sometimes operators are paid based on the number of visit that they provide instead of their working duration. Thus, in such a case it is assumed that, operators can handle an excess load beyond their capacities, i.e. operators' overtime is allowed, with or without explicitly considering associated overtime costs.

This case can be obtained by modifying the constraints (2.9) as follows

$$D_{td} = \sum_{(i,j) \in A} (t_{ij} + sv_j) \cdot x_{ij}^{td} \quad \forall d \in W, \forall t \in O_d \tag{2.22}$$

Note that, this case is applicable to both multiple and single planning period models. Since

there could be more feasibility problems due to single day restriction in the single planning period model, it would be useful to eliminate the operator capacities (see 2.22) to avoid feasibility problem.

Rather than considering all skill levels in a single model, each skill level can be managed independently. Thus, for each skill level independently considered models (i.e. each model has different patients and operators) has to be solved. This alternative case is presented in the following section.

I.3.3 Independent Skill, Multiple Planning Periods Model

In this case, one model is formulated for each skill level and all of these models are solved independently. In other words, mathematical models for each skill level $k \in K$ is required to be formulated and solved in a separate way.

To be able to present such models, first we need to modify some of previously given definitions and variables.

Different than the hierarchical skill case, we need to consider a subset of patients and operators according to the highest required skill level. Thus, a subgraph of the previously defined directed network $G = (N, A)$ is introduced as $G_k = (N_k, A_k)$ where N_k is used to denote the subset of patients with the highest required skill k . Similarly, operators that have the highest skill k are grouped under the subset $O_k \subseteq O$, with $k \in K$. For any day $d \in W$, the available set of operators with the highest skill k are represented with the subset $O_{dk} \subseteq O_k$.

Since subsets of patients and operators are considered for each skill level, we have to modify the decision variables which were given at the beginning of this section.

$$z_{jp} = \begin{cases} 1 & \text{if pattern } p \text{ is assigned to patient } j \\ 0 & \text{otherwise} \end{cases} \quad j \in N_k \setminus \{0\}, p \in P$$

$$u_{tj} = \begin{cases} 1 & \text{if operator } t \text{ is assigned to patient } j \\ 0 & \text{otherwise} \end{cases} \quad t \in O_k, j \in N_k \setminus \{0\}$$

$$x_{ij}^{td} = \begin{cases} 1 & \text{if operator } t \text{ travels along } (i, j) \text{ on day } d \\ 0 & \text{otherwise} \end{cases} \quad (i, j) \in A_k, i \neq j, d \in W, t \in O_{dk}$$

and

y_{ij}^d auxiliary flow variable which represents the number of the patients visited after patient i by the operator moving along (i, j) on day d , $(i, j) \in A_k$, $d \in W$.

The general model that is given for the hierarchical skill case (see Section I.3.1) is modified for the independent skill case as follows:

$$\min h_k \tag{2.23}$$

$$\sum_{i \in N_k} \sum_{t \in O_k} x_{ij}^{td} = \sum_{p:p(d) \geq 1} z_{jp} \quad \forall j \in N_k \setminus \{0\}, \forall d \in W \tag{2.24}$$

$$\sum_{p \in P} z_{jp} = 1 \quad \forall j \in N_k \setminus \{0\} \tag{2.25}$$

$$\sum_{t \in O_k} u_{tj} = 1 \quad \forall j \in N_k \setminus \{0\} \tag{2.26}$$

$$x_{ij}^{td} \leq u_{tj} \quad \forall (i, j) \in A_k, \forall j \in N_k \setminus \{0\}, \forall d \in W, \forall t \in O_{dk} \tag{2.27}$$

$$u_{tj} \leq \sum_{i \in N_k} \sum_{d \in W} x_{ij}^{td} \quad \forall j \in N_k \setminus \{0\}, \forall t \in O_k \tag{2.28}$$

$$D_{td} = \sum_{(i,j) \in A_k} (t_{ij} + sv_j) \cdot x_{ij}^{td} \leq a_t \quad \forall d \in W, \forall t \in O_{dk} \tag{2.29}$$

$$\sum_{i \in N_k} x_{ij}^{td} = \sum_{i \in N_k} x_{ji}^{td} \quad \forall j \in N_k \setminus \{0\}, \forall d \in W, \forall t \in O_{dk} \tag{2.30}$$

$$\sum_{j \in N_k} y_{0j}^d = \sum_{j \in N_k \setminus \{0\}} \sum_{p:p(d) \geq 1} z_{jp} \quad \forall d \in W \tag{2.31}$$

$$\sum_{i \in N_k} y_{ij}^d - \sum_{i \in N_k} y_{ji}^d = \sum_{p:p(d) \geq 1} z_{jp} \quad \forall j \in N_k \setminus \{0\}, \forall d \in W \tag{2.32}$$

$$y_{ij}^d \leq n \sum_{t \in O_{dk}} x_{ij}^{td} \quad \forall (i, j) \in A_k, \forall d \in W \tag{2.33}$$

$$\frac{\sum_{d \in W} D_{td}}{|W| \cdot a_t} \leq h_k \quad \forall t \in O_k \tag{2.34}$$

In this model, mainly set definitions associated with the constraints are modified according to skill compatibility in comparison with the model presented for the hierarchical skill case.

Since each single skill level is considered independently, Constraints (2.3) and (2.4) of the hierarchical skill case are replaced with Constraints (2.24) that specify that exactly one operator per day can visit patient j only if a visit has been scheduled on that day. Moreover, the auxiliary variable h that is used in the hierarchical skill case is also modified as h_k to be able to represent

each skill level k with a independent model.

The cases where alternative objective functions are presented also need to be modified with the new subset definitions as follows:

The model with the travel time minimization objective:

$$\min v_k \tag{2.35}$$

$$\text{Constraints (2.24) – (2.33)} \tag{2.36}$$

$$\sum_{d \in W} \sum_{(i,j) \in A_k} t_{ij} \cdot x_{ij}^{td} = v_k \quad \forall t \in O_k, \tag{2.37}$$

where the auxiliary variable v_k defines the total traveling time of operators in the planning period associated with skill k .

The model with the trade-off objective function:

$$\min h_k + \gamma \cdot v_k \tag{2.38}$$

$$\text{Constraints (2.24) – (2.34)} \tag{2.39}$$

$$\text{Constraint (2.37)} \tag{2.40}$$

where h_k is the auxiliary variable that is used to estimate the maximum utilization rate of the operators for the skill level k , v_k is used to calculate the total travel time with the skill set k and γ is a penalty parameter between 0 and 1.

I.3.4 Variants of the Independent Skill Multiple Planning Period Model

The *single planning period* model for the independent skill case can be directly obtained as presented for the hierarchical skill case by imposing new versions of the patient and operator set definitions, $j \in N_k \setminus \{0\}$ and $O_{dk} \equiv O_k$, and keeping $W = 1$ and $r_{jk} = 1$.

Similarly, models *without operator capacity restrictions* can be obtained by deleting the daily capacity term a_t from constraints (2.29).

Note that if we consider the independent skill case (i.e. for any skill level k) for a single planning period and without capacity restrictions then, this model turns out to be the well-known Multiple Traveling Salesman Problem (MTSP) that has been extensively studied in the VRP domain (for more details please refer to [4]).

Part II

THE TWO-STAGE APPROACH

II.1 INTRODUCTION

In this part, we present an alternative approach for solving the assignment and routing problems of HHC services. As presented before, in this procedure first the assignment problem, and then, the routing problem are solved independently. Note that, to the best our of knowledge there is no work related to the two-stage approach for the assignment and routing problems in the HHC literature. Thus, with this part, we contribute to the related HHC literature by providing a new two-stage approach concerning several variants of the assignment and routing problems.

Several new models for the two-stage approach for the hierarchical and independent skill cases are developed for single or multiple planning periods. Since the visiting sequences for operators are not known in the assignment phase of these models, an estimation $\bar{\tau}_j$ for the travel times is required to be able solve the assignment and routing problems with the two-stage approach. To this end, different travel times estimation methods are considered and presented in Chapter 3.

Remind that in the multiple planing period case (i.e. $W \geq 2$), in addition to the Assignment (A) and Routing (R) decision, we also need to consider the Care Plan (CP) decision (i.e. pattern assignment). In the two-stage approach, this decision can be incorporated via three alternative scenarios. The first one (i.e. Model II) is solving the assignment model for the whole planning period W by considering all patient service requirements but without deciding the care plan schedules of the patients (i.e. no pattern assignment). Then, with the assignment decisions from the first stage, routing models with care plan decision is required to be solved for each operator. The second alternative (i.e. Model III) is simultaneously deciding the assignment and care plan decision and then solving several routing models for each day and for each operator. The last scenario (i.e. Model IV) is the combination of these two previous alternatives where care plan decision is incorporated into both stages of the two-stage approach. To present complete schema, we provide Figure II.1 with all alternative cases for models with multiple planning periods as well as the model with the single planning period (i.e. Model I).

All these models are important to analyze several scenarios encountered in the two-stage approach especially for the multiple planning periods. For example, Model II and Model III are useful to analyze when to make the care plan decision to be able to obtain solutions close to the ones obtained from the simultaneous approach. Particularly, Model IV can be considered as the combination of the Model II and III where the care plan decision that is obtained in the assignment stage is modified in the routing stage. This model (Model IV) might be useful to

	Single Planning Period W=1	Multiple Planning Periods W≥2		
MODEL	I	II	III	IV
PATIENT ASSIGNMENT (A)	MODEL I A	MODEL II A		MODEL IV A
CARE PLAN (PATTERN ASSIGNMENT) (CP)	NO CARE PLAN DECISION		MODEL III A + CP	MODEL III A + CP ↑ IN BOTH STAGES CP ↓ R + CP
ROUTING (R)	MODEL I R	MODEL II R+CP	MODEL III R	MODEL IV R

Figure II.1: Alternative Models for the Two-Stage Approach

analyze the effect of the travel time estimation on the solutions of the two-stage approach. This effect can be investigated by comparing the solutions of the Model IV with the simultaneous approach.

In Section II.2, we start with the presentation of the literature review on the assignment problem of the two-stage process. Note that here we do not present any literature related to the second stage of the two-stage approach since the current literature is either devoted to the stand-alone assignment problem or the simultaneous assignment and routing problems (presented in Section II.2). Then, we provide the mathematical modeling formulations for the assignment problem for both hierarchical and independent skill cases with the single (i.e. Model I) and multiple planning period (i.e. Model III, III and IV) assumptions. As the next step, we also present the models for the routing problem which are formulated as the TSP models with and without care plan decisions.

II.2 LITERATURE REVIEW ON THE ASSIGNMENT MODELS

As in the previous literature review presented in Section I.2, the overview of the existing works is also provided for the stand-alone assignment problem in Table II.1.

The assignment problem in the HHC literature has been rarely studied as a stand-alone problem (i.e. without considering the routing problem). Hertz and Lahrichi [30] propose two different mixed integer programming models for assigning operators to patients. The objective of both models is to balance the nurses' workloads by minimizing a weighted sum of the visit load (based on the weight of each visit), the case load (due to the number of patients assigned) and the travel load (related to the distances traveled) while respecting constraints related to maximum acceptable loads and continuity of care. The travel load is calculated on the basis of the average distance of the patient location from the district where operator works. Since the estimate does not consider sequencing, it should be accurate for small districts. Borsani *et al.* [15] propose assignment and scheduling models where the output of the assignment model is incorporated as the input to the scheduling model. In this work, the assignment process is held to ensure workload balance among operators while respecting continuity of care, qualification requirements and geographical coherence constraints. Travel times are constant and independent from the sequence.

An extended modeling framework related to the assignment problems of the HHC services was developed by Lanzarone *et al.* [36], where the authors provide different assignment models to balance the operators' workloads by considering several peculiarities of HHC services, such as the operators' skills, the geographical locations of patients and operators, and the stochastic patient requests. Travel times are modeled as in [15]. The same problem with stochastic demand is then tackled in the works of Lanzarone and Matta [35, 37] who propose simple policies to assign patients to operators instead of mathematical programming. Carello and Lanzarone [18] develop a cardinality-constrained robust assignment model where their aim is to exploit the potentialities of a mathematical programming formulation and to evaluate the capability of such model in reducing the costs related to nurses' overtimes. Also in this case, the travel time for reaching homes is the same for all the patients and operators. Lastly, Koeleman *et al.* [34] represent the HHC system as a Markov chain and they develop admittance policies for patients with the use of a trunk reservation heuristic to control the system by considering a general visiting time containing a travel load that does not consider routes.

No	Feature	Hertz and Lahrichi [30]	Borsani <i>et al.</i> [15]	Lanzarone <i>et al.</i> [36]	Lanzarone and Matta [35]	Koelman <i>et al.</i> [34]	Carello and Lanzarone [18]	Lanzarone and Matta [37]
1.1.1.1	Travel Time Minimization Obj.				✓			✓
1.1.1.2	Penalty Cost Minimization Obj.							
1.1.1.3	Workload Balancing Obj.		✓			✓	✓	
1.1.2	Trade-off Obj.	✓		✓				
1.2.1	Single Time Horizon							✓
1.2.2	Multiple Time Horizon	✓	✓	✓	✓	✓	✓	✓
1.3.1	Single Op.	✓	✓	✓	✓	✓	✓	✓
1.3.2	Multiple Op.			✓				
1.3.3	Identically Skilled Op.							
1.3.4	Different Skilled Op.	✓	✓	✓	✓	✓	✓	✓
1.3.5	Operator Availability	✓	✓		✓	✓	✓	✓
1.3.6	Operator Capacity	✓	✓	✓	✓	✓	✓	✓
1.3.7	Full Time Op. Contract	✓	✓	✓	✓	✓	✓	✓
1.3.8	Half Time Op. Contract		✓					
1.3.9	External Operator							
1.3.10.1	Identical Service Time		✓					
1.3.10.2	Variable Service Time	✓		✓	✓	✓	✓	✓
1.3.11	Operator Time Window		✓					
2.1.1	Single District		✓		✓	✓		✓
2.1.2	Multiple District	✓		✓			✓	
2.2.1	Single Health Care Center		✓		✓	✓		✓
2.2.2	Multiple Health Care Center	✓		✓			✓	
3.1	Patient Time Window							
3.2	Continuity of Care	✓	✓	✓	✓	✓	✓	✓
3.3	Uncertainty			✓	✓	✓	✓	✓
3.4.1	Real Travel Time							
3.4.2	Estimated Travel Time							
3.4.3	Travel Time is Not Explicitly Considered	✓		✓	✓	✓	✓	✓

Table II.1: Classification of Existing Models Related to the First Stage of the Two-Stage Approach in HHC services

A common characteristics of the assignment related papers is the need of balancing the workload among operators. Indeed, this problem is extremely important to have equal working conditions in the same organization. However, these papers does not consider the hierarchical skills and they do not focus on the travel time estimation methods. These are two important issues that has to be considered to be able to better reflect the real cases. To this end, in the following sections, we present new mathematical models with hierarchical skills that we have developed for the assignment problem of the two-stage approach. Then, in Chapter 3, we provide different travel time estimation methods.

II.3 PROPOSED MATHEMATICAL MODELS FOR THE ASSIGNMENT PROBLEM

In this section, we present the models that are developed for the first stage of the two-stage approach. As shown in Figure II.1, single and multiple planning period cases are considered for the assignment problem. In particular, for the multiple planning period case, two main models are developed where in the first one the assignment decision is held with the care plan decision (i.e. Model III and IV) whereas in the second one only the assignment decision is considered (i.e. Model I and II) and the care plan decision is left for the routing stage. Furthermore, all these models are investigated under two skill cases, hierarchical and independent skills, as in the simultaneous approach.

In the literature of the assignment models for HHC services, there are works that considers different operator skills however, there is no work that focus on the management of the skills independently or hierarchically. In particular, considering the care plan decision within the assignment problem is also new to the literature.

In this section, models are presented mainly based on definitions, notations and variables given for the simultaneous approach. The only new variable is defined as follows:

$$\mu_j^{td} = \begin{cases} 1 & \text{if operator } t \text{ is assigned to patient } j \text{ on day } d \\ 0 & \text{otherwise} \end{cases} \quad j \in N \setminus \{0\}, d \in W, t \in O$$

Note that this notation is provided only for the hierarchical skill case. In the independent skill case, the given set definitions are modified as $j \in N_k \setminus \{0\}$ for patients and $t \in O_k$ for operators.

II.3.1 Hierarchical Skill Case

We first start by presenting the models with multiple planning periods (see Model II (A), Model III (A+CP) and Model IV (A+CP)) in Figure II.1). As detailed before, two alternative modeling structures are proposed where the first one includes simultaneous assignment and care plan decisions (Model III (AC+P) and Model IV (AC+P)) and the second one is only based on the assignment decision (Model II (A)). The main difference between these models lies on the care plan decision which is included in the assignment stage of Model III and IV and not considered in the assignment stage of Model II.

1. Model III (A+CP) and Model IV (A+CP) with Hierarchical Skills

Model III (A+CP) and Model IV (A+CP) is represented with the same assignment model where care plan decision is also made. The aim of the model is to assign patients to operators with the daily scheduling information (e.g. Patient 3 (with 2 visit requirements) is assigned to the Operator 1 and the visits are scheduled on Monday and Thursday).

The formulation of this model is given as follows:

$$\min h \tag{2.41}$$

$$\sum_{t \in s_t \geq k} \mu_j^{td} = \sum_{p: p(d) \geq 1} z_{jp} \quad \forall j \in N_k, \forall d \in W, \forall k \in K \tag{2.42}$$

$$\sum_{p \in P} z_{jp} = 1 \quad \forall j \in N \tag{2.43}$$

$$\sum_{t \in s_t \geq k} u_{tj} = 1 \quad \forall j \in N_k, \forall k \in K \tag{2.44}$$

$$\mu_j^{td} \leq u_{tj} \quad \forall j \in N, \forall d \in W, \forall t \in O \tag{2.45}$$

$$D_{td} = \sum_{j \in N} (\bar{\tau}_j + sv_j) \cdot \mu_j^{td} \leq a_t \quad \forall d \in W, \forall t \in O \tag{2.46}$$

$$\frac{\sum_{d \in W} D_{td}}{|W| \cdot a_t} \leq h \quad \forall t \in O \tag{2.47}$$

$$u_{tj} \in \{0, 1\} \quad \forall j \in N_k, \forall t \in s_t \geq k, \forall k \in K \tag{2.48}$$

$$\mu_j^{td} \in \{0, 1\} \quad \forall j \in N, \forall d \in W, \forall t \in O \tag{2.49}$$

$$z_{jp} \in \{0, 1\} \quad \forall j \in N, \forall p \in P \tag{2.50}$$

Equation (2.42) states that (exactly) one operator per day with the appropriate skill can visit patient j only if a visit has been scheduled on that day for node j . Equation (2.44) ensures that exactly one operator is assigned to each patient during the planning horizon. Equation (2.45) guarantees that an operator can visit a patient only if he has been assigned to that patient. Equation (2.46) ensures that the workload of each operator in each day, expressed as the sum of the service times and the traveling times, is not exceeding the operator capacity. Inequality (2.47) expresses the maximum utilization rate h , which is minimized in the objective function. As presented in the simultaneous approach, other objective functions can also be considered.

2. Model II (A) with Hierarchical Skills

Remind that in the previous models (i.e. Model III (A+CP) and Model IV (A+CP)), daily scheduling decision for the whole planning period (i.e. care plan decision) is also held within the assignment problem. However, in practice it is also possible to make assignment

decision for the whole planning period without considering daily care plan decision.

Hence, the formulation presented for Model III (A+CP) and Model IV (A+CP) can be modified as follows:

$$\min h \tag{2.51}$$

$$\sum_{t \in s_t \geq k} u_{tj} = 1 \quad \forall j \in N_k, \forall k \in K \tag{2.52}$$

$$D_t = \sum_{j \in N} \lambda_j \cdot \mu_j^{td} \leq |W| \cdot a_t \quad \forall t \in O \tag{2.53}$$

$$\frac{D_t}{|W| \cdot a_t} \leq h \quad \forall t \in O \tag{2.54}$$

$$u_{tj} \in \{0, 1\} \quad \forall j \in N_k, \forall t \in s_t \geq k, \forall k \in K \tag{2.55}$$

In comparison with the formulation given for Model III (A+CP) and Model IV (A+CP), here care plan decision constraints and variables are removed. In particular instead of considering daily patient service request, we consider the service request of the whole planning period λ_j and corresponding operator workload D_t by constraints (2.53).

For Models II (A), Model III (A+CP) and Model IV (A+CP) the formulated objective function is the balancing function that minimizes the maximum operator utilization rate. As in the simultaneous approach, cost minimization objective (Equation (2.16)) and trade-off objective (Equation (2.19)) functions can also be considered in the same manner.

3. Variants

3.1 Hierarchical Skills Case with Single Planning Period (Model I (A))

Single planning period case is represented by Model I (A) in Figure II.1. This model can be formulated by the same set of constraints (2.51)-(2.55) that are presented for Model II (A). The only difference lies on the value of the patient demand λ_j which represents a single period ($W = 1$) in Model I (A).

consideration where in Model I (A) it should only include a single period value ($W = 1$).

3.2 Hierarchical Skills Case without Operator Capacity Restrictions

The capacity restrictions of operators can be relaxed by omitting the right-hand-side terms $\leq a_t$ from equation (2.46) and $\leq |W| \cdot a_t$ from equation (2.52).

All models presented with the hierarchical skill case are also applicable to the independent skill case. The following part presents the details of Model I (A), Model II (A), Model III (A+CP) and Model IV (A+CP) for the independent skill case.

II.3.2 Independent Skill Case

As discussed in Section I.3.3, \bar{k} (i.e. the number of available skill level) models are required to be able to manage each skill level independently. Set definitions, notations and variables presented in Section I.3.3 are also valid for this case. The set definition of the assignment variable μ_j^{td} that is presented in the previous section is modified according to independent skill compatibility and this modification is presented in the model formulation.

Here below, we first present the developed model with the multiple planning periods (Model II (A), Model III (A+CP) and Model IV (A+CP)), and then, the model with the single planning period (Model I) respectively.

1. Model III (A+CP) and Model IV (A+CP) with Independent Skills

Remind that Model III (A+CP) and Model IV (A+CP) represents the case where the assignment decision is held simultaneously with the care plan decision for multiple periods. The corresponding formulation with independent skills (i.e. for each skill level k) and the balancing objective are shown as follows:

$$\min h_k \tag{2.56}$$

$$\sum_{t \in O_k} \mu_j^{td} = \sum_{p: p(d) \geq 1} z_{jp} \quad \forall j \in N_k, \forall d \in W \tag{2.57}$$

$$\sum_{p \in P} z_{jp} = 1 \quad \forall j \in N_k \tag{2.58}$$

$$\sum_{t \in O_k} u_{tj} = 1 \quad \forall j \in N_k \tag{2.59}$$

$$\mu_j^{td} \leq u_{tj} \quad \forall j \in N_k, \forall d \in W, \forall t \in O_k \tag{2.60}$$

$$D_{td} = \sum_{j \in N_k} (\bar{\tau}_j + sv_j) \cdot \mu_j^{td} \leq a_t \quad \forall d \in W, \forall t \in O_k \tag{2.61}$$

$$\frac{\sum_{d \in W} D_{td}}{|W| \cdot a_t} \leq h_k \quad \forall t \in O_k \tag{2.62}$$

$$u_{tj} \in \{0, 1\} \quad j \in N_k, t \in O_k \tag{2.63}$$

$$\mu_j^{td} \in \{0, 1\} \quad \forall j \in N_k, \forall d \in W, \forall t \in O_k \tag{2.64}$$

$$z_{jp} \in \{0, 1\} \quad \forall j \in N_k, \forall p \in P \tag{2.65}$$

2. Model II (A) with Independent Skills

The model formulation that is developed with only the assignment decision for multiple planning periods is almost the same as the Model II (A) that is presented in the hierarchical skill case of the previous section (Section II.3.1). The difference lies on the set definitions of the constraints where all of the considered patients for each skill level is considered with subset N_k (instead of N) and all the available operators with the appropriate skills are considered with the subset O_k (instead of O).

3. Variants

3.1 Independent Skills Case with Single Planning Period (Model I (A))

This model is developed with the same constraint family of Model II (A) with independent skills. The only difference is on the patient demand λ_j calculation (i.e. it is calculated for $W = 1$) as described in the hierarchical skill case.

Note that all of previously presented alternative objective functions are applicable to the models of the independent skill case as well.

3.2 Independent Skills Case without Operator Capacity Restrictions

The operator capacity relation can also be obtained as it is presented in the hierarchical skill case of the two-stage approach.

Till now we have only presented the models for the first stage of the two-stage approach where we obtain either the assignment or the assignment and care plan decisions. In the following section, we present the details of the second-stage models where the visiting sequences of the operators are obtained.

II.4 MATHEMATICAL MODELS FOR THE ROUTING PROBLEM

At this level of the two-stage approach, TSP models are used to create each operator's route with the information obtained from the assignment step for the given planning period.

Different than the existing routing models in the HHC services, here the stand alone routing problem with the care plan decision is formulated. In particular, travel time balancing among operators with the already assigned patients is also new to this literature.

As presented in Figure II.1, similar to the assignment decision, the routing decision can be held with (Model II (R+CP) and Model IV (R+CP)) or without the care plan decision (Model I (R) and Model III (R)). In this thesis, these cases are formulated with two TSP formulations (with care plan decision (R+CP) or without care plan decision (R)) independent from the skill compatibility issue. The first mathematical formulations is used to consider Model II (R+CP) and Model IV (R+CP) where the routing decision is held with the care plan decision. The other formulation is considered for the stand-alone routing decision to be able to solve Model I (R) and Model III (R).

Since the operator to patient assignment has already been decided according to the appropriate skill configuration in the first stage of the problem, this issue is not incorporated into the routing models.

In this part, we need to reconsider the previously given definitions, notation and variables for the simultaneous approach and present some new ones. The new definitions are given as follows:

- The subset of patients that are assigned to operator t is denoted with subset N_t
- The feasible subset of patterns associated with operator t is presented by P_t where it must be created in a way that $p \in P_t$ implies $p(d) = 0$ if $t \notin O_d$
- The pattern assignment variable z_{jp} is modified according to assigned patients and feasible pattern subset of the operator t as follows:

$$z_{jp} = \begin{cases} 1 & \text{if pattern } p \text{ is assigned to patient } j \\ 0 & \text{otherwise} \end{cases} \quad j \in N_t \setminus \{0\}, p \in P_t$$

1. TSP Formulation for Model II (R+CP) and Model IV (R+CP)

The following formulation presents the model for the routing and care plan decisions of operators with multiple planning periods.

$$\min \sum_{d \in W: t \in O_d} \sum_{(i,j) \in A} t_{ij} \cdot x_{ij}^{td} \quad (2.66)$$

$$\sum_{i \in N} x_{ij}^{td} = \sum_{p: p(d) \geq 1} z_{jp} \quad \forall j \in N_t, \forall d \in W : t \in O_d \quad (2.67)$$

$$\sum_{p \in P_t} z_{jp} = 1 \quad \forall j \in N_t \quad (2.68)$$

$$D_{td} = \sum_{(i,j) \in A} (\bar{\tau}_j + sv_j) \cdot x_{ij}^{td} \leq a_t \quad \forall d \in W : t \in O_d \quad (2.69)$$

$$\sum_{i \in N} x_{ij}^{td} = \sum_{i \in N} x_{ji}^{td} \quad \forall j \in N \setminus \{0\}, \forall d \in W : t \in O_d \quad (2.70)$$

$$\sum_{j \in N} y_{0j}^d = \sum_{j \in N_t} \sum_{p: p(d) \geq 1} z_{jp} \quad \forall d \in W : t \in O_d \quad (2.71)$$

$$\sum_{i \in N} y_{ij}^d - \sum_{i \in N} y_{ji}^d = \sum_{p: p(d) \geq 1} z_{jp} \quad \forall j \in N_t, \forall d \in W : t \in O_d \quad (2.72)$$

$$y_{ij}^d \leq |N_t| x_{ij}^{td} \quad \forall (i,j) \in A, \forall d \in W : t \in O_d \quad (2.73)$$

$$y_{ij}^d \geq 0 \quad \forall (i,j) \in A, \forall d \in W \quad (2.74)$$

$$x_{ij}^{td} \in \{0, 1\} \quad \forall (i,j) \in A, \forall d \in W : t \in O_d \quad (2.75)$$

$$z_{jp} \in \{0, 1\} \quad \forall j \in N_t \setminus \{0\}, \forall p \in P_t \quad (2.76)$$

Constraint (2.67) states that operator t must visit patient $j \in N_t$, on day d , if a visit has been scheduled on that day for node j . Constraint (2.68) ensures that each patient is assigned exactly to a pattern. Constraint (2.69) imposes that the workload of t in each day where t is available, expressed as the sum of the service times and the traveling times, does not exceed the duration of a workday for operator t . Constraint (2.70) is the classical flow conservation constraints on the routing variables. Constraints (2.71) and (2.72) are the flow conservation constraints on the auxiliary y variables, which are introduced to avoid subtours in the model solutions. They also guarantee the correct linking between scheduling decisions and auxiliary flow variables. Finally, constraints (2.73) link together routing variables and auxiliary flow variables.

The objective function (2.66) is trying to minimize the overall traveling cost of operators where the set of assigned patients has already been given as input information. In other words, with the presented model several TSP models are simultaneously solved to be able to minimize the utilization rates of operators and obtain a balanced utilization rate among them. Such a balancing objective is selected to be able to solve both stages of the two-stage problem with the same type of function.

The other alternative objective can be directly minimizing the traveling cost (i.e. travel cost minimization) of operators without considering any balancing among them. Such a case can be obtained by solving the presented TSP model for each operator independently with the following objective function:

$$\min \sum_{d \in W} \sum_{(i,j) \in A_t} t_{ij} \cdot x_{ij}^{td}, \quad (2.77)$$

where A_t represents the (i, j) couples for only the assigned patient of operator t . Since each operator is considered independently, the set of all patient N should be replaced with the subset of assigned patient N_t in all necessary constraints. In particular, the term related to operators $t \in O_d$ should also be removed from all necessary constraints because of the same reason.

Further observe that, for each $j \in N_t$, a pattern variable z_{jp} can assume a value other than zero only if:

$$|\{d : p(d) = k\}| = r_{jk} \quad \forall k \in K. \quad (2.78)$$

Therefore, in the preprocessing phase $z_{jp} = 0$ if anyone of the \bar{k} constraints (2.78) is not satisfied.

2. TSP Formulation for Model I (R) and Model III (R)

If only the routing decision is required from the second stage then , several basic TSP (i.e. without care plan decision) models have to be solved to obtain the visiting sequences of operators.

The corresponding TSP model for Model I (R) can be simply obtained by considering $W = 1$ and providing each t value (for only cost minimization objective) as input parameter in the previous model. The formulation for Model III (R) can be easily represented by using each d and t (for only cost minimization objective) values as input information in the previous model as well. Actually in both cases models are formulated for a single period and single operator thus, the formulations that are used for these models are equivalent. Although the TSP formulation for Model I (R) and Model III (R) are equivalent, the number of times that this model has to be solved is different. For example, if we consider the travel cost minimization objective, for Model I (R) $|O|$ and for Model III (R) $\sum_{d \in W} |O_d|$ independent TSP models has to be solved.

As an alternative the formulation proposed by Miller *et al.* [41] can also be adopted to solve the Model I (R) and Model III (R).

2.3 CONCLUSION

In this chapter, we demonstrate the newly developed two-stage models to cope with the computational and modeling complexities encountered in the models of the simultaneous approach. We start by demonstrating the existing simultaneous models from the literature. Then, we also show some variant simultaneous models that are also developed for benchmark analysis. After presenting these models, we provide newly developed two-stage models as the main contribution of this chapter. These models reflect several realistic cases by considering multiple planning periods and skill compatibility between operators and patients etc.

Although several models are developed for the two-stage approach, travel time estimations methods are required to be able to solve the assignment problem of this approach. Hence, in the following chapter (Chapter 3), we provide different estimation methods. Performances of these estimators and correspondingly the two-stage approach are further analyzed in Chapter 5.

Chapter 3

TRAVEL TIME ESTIMATION METHODS

3.1 INTRODUCTION

In the two-stage approach, the routing optimization is considered independently from the assignment decision thus, exact travel times (Euclidean distances) among patients are not available when the assignment problem is solved. Hence, to be able to solve the assignment problem, an estimation of travel times is required. In this chapter, we present three alternative travel time estimation methods based on two techniques: operator specific and operator independent estimations. The difference between these two techniques is based on how the patients are considered. In the operator independent case, the estimation is done based on the data relative to all patients whereas in the operator specific case, it is done based on the subset of patients specifically assigned to a given operator.

Section 3.2 presents a basic approach for the operator independent estimation technique based on Average Values (AV). Then, Section 3.3 provides the operator specific estimates via two different approaches where the first one is the Operator Specific Average Value (OSAV) approach and the second one is the data-driven approach based on Kernel Regression (KR) technique.

3.2 OPERATOR INDEPENDENT ESTIMATION TECHNIQUE

In this part, we present a first approach based on Average Values (AV) which does not depend on operator related information (i.e. assigned patients).

In the AV approach, the estimation of the travel time required to visit a particular patient is calculated as the weighted average travel time to reach him/her from all other patients. In such

cases, the weights can be assumed to be proportional to the weight of the visits required by each patient. Thus, the following estimator $\bar{\tau}_j$ is used:

$$\bar{\tau}_j = \frac{\sum_{i=1}^N r_j t_{ij}}{\sum_{j=1}^N r_j} \quad (3.1)$$

where t_{ij} denotes the real time (Euclidean distances) separating patient i from j (expressed in time unit), $(i, j) \in A, i \neq j$, and r_j is the weight (i.e. frequency of required visits) related to patient j .

Because an average value is used to calculate the time to reach a patient, higher travel times might be observed compared to the optimal travel times that would be obtained with the simultaneous approach. This difference occurs because the AV approach assumes a uniform probability of passing by patient j and does not integrate operator specific information (i.e. assigned patients). Thus, better estimation methods are needed to solve the assignment problem independently. To this end, in the following section we propose operator specific estimates.

3.3 OPERATOR SPECIFIC ESTIMATION TECHNIQUE

Instead of focusing on all patients for estimating travel times, it could be more relevant to calculate the travel time estimation relative to each operator based on patients assigned to him/her. Here below two operator specific estimation techniques are presented where the first one is the variant of the AV approach and the second is a data-driven learning procedure.

3.3.1 Operator Specific Average Value (OSAV) Approach

In this case, the previously defined AV value approach is used to calculate the estimated travel time for each operator based on patients assigned to him/her. To apply the AV approach, the previously defined patient related estimate value $\bar{\tau}_j$ is modified as an operator specific value $\bar{\tau}_t$ and the estimate for the given operator t is calculated with equation (3.1) with the set of assigned patients N_t .

3.3.2 Operator Specific Data-Driven Approach

Due to the distribution-free property of non-parametric methods and the asymptotic convergence of some estimators, we use a data-driven non-parametric method to estimate travel times from real data observations with the KR technique.

KR is a non-parametric regression technique that does not require a predetermined form, as the predictor is built with the information derived from existing data [52]. KR exploits correlations that exists among observations by assuming a radial basis function explaining the data. In our context, since HHC patients have unique characteristics depending on their features (i.e., geographical location, care profile, etc.), KR seems to be proper to estimate the travel time to visit a set of patients. Indeed, such data-driven approach is important for HHC services since historical observations would enable to capture what really happened in the system in terms of executed planning decisions. For instance, for some reason, if a patient has been visited in the first order of the visiting sequence for a certain period of time, then it is likely to observe similar behaviors for the following periods. Thus, the KR technique would enable to capture this situation when estimating travel times by assigning a certain weight to that patient for that specific sequence based on the information coming from historical observations. Hence, travel times can be estimated in a more realistic way via the use of KR.

To our knowledge, although such data-driven approaches and the KR technique have been used for problems such as inventory control, call center staffing and dynamic assortment optimization [51], they have not been applied to the HHC setting yet.

There are some advantages to use this technique in HHC services. First, this method uses past data to infer the travel time related to a set of patients with specific attributes. Since the method needs several samples to build its estimators, HHC service fits quite well because it consists of a periodic and repetitive service. Thus, a particular patient can be observed several times in the past observations and the Kernel estimator gains significance by time. Another advantage is related to districting, which is a priori step involved in the HHC planning problem before the assignment is tackled. The districting process consists of partitioning a territory into smaller areas [8]. The use of Kernel or other regression techniques for larger areas would require a more important volume of historical data not available in practice. Hence, the proposed method is an efficient way to estimate travel times for smaller regions without requiring a lot of historical information.

The travel time of a HHC operator may be affected by several patient related features (attributes) such as the care profiles of patients (i.e., pathology, type and intensity of care), temporal constraints (i.e., availability of the family member that is present for help) and geographical locations of patients. In this work, we basically focus on features that are related to the geographical locations of patients, but the mathematical expressions proposed for estimating travel time is general enough to consider other kinds of patient attributes.

We use historical information to bring the routing consideration into the assignment problem by estimating operators' travel times based on patients' geographical locations, depending on the related operator's past behavior. Indeed, there are several factors related to patients' geographical locations that would have an impact on the travel time of a operators. Examples of such factors are related to daily traffic conditions (i.e., dense or calm), personal preferences of operators and/or difficulties related to the access to patients' homes.

To illustrate this, the following section provides an example observed in a real case to emphasize the importance of using the data-driven approach in HHC services.

Real Case Example

This example presents a real operator tour which has to visit 7 patients (identified as A-F) in a particular day. The operator considers several geographical and physical aspects while planning his/her visits. For example, due to high traffic density, he/she chooses to visit patient A at the end of his/her working day. Similarly, due to the absence of elevator at patient D, he/she chooses to visit patient D at the beginning of his working day since he/she feels more energetic. Thus, according to such personal preferences, he/she executes a route that may be non optimal from a total travel time minimization perspective.

For the given case, if he/she wanted to obtain his/her route as the optimal one, according to the travel distance (time) minimization, he/she would need to travel 16.2 *km* with the following sequence Center-A-B-C-D-E-F-Center, see Figure 3.1 (i.e., real patient locations are used to obtain the total travel distance). However, since he/she considers other features (i.e., high traffic density or personal preferences), the observed executed tour length turns out to be 20 *km* with the sequence of Center-D-C-E-F-B-A-Center, see Figure 3.1 (i.e., provided by the real case). Indeed, in practice, planners not only take into account the criteria regarding travel distances but also other geographical and physical features while planning visits. On way to assess the

effect of geographical locations or other features on the corresponding operator’s route is to consider his/her past behavior based on a data-driven technique (i.e., Kernel Regression).

In the following section, we present the mathematical derivation and implementation details of the KR technique.

Mathematical Derivation and Implementation Details

The KR technique estimates the expectation of the outcome array Y (i.e., an operator’s total travel time) conditional on the random variable array \mathbf{X} (i.e., patients’ geographical locations), $E(Y|\mathbf{X})$. In our work, the outcome array Y is denoted with its component, Y_p , that is used to express the total travel time of an operator to reach the assigned patients on the given period p (with $p = 1, \dots, m$). Similarly, the geographical location array, X , is also denoted with its component, \mathbf{X}_p , to represent the geographical locations of patients in each period p . The main reason for using KR is that doing so imposes few restrictions on the functional relationship between the covariate array \mathbf{X} and the outcome array Y . This relationship can be formulated as follows:

$$Y = \tau(\mathbf{X}) + \varepsilon \tag{3.2}$$

where τ is an unknown function, and ε is the error term, which is independent and identically distributed with $[0, \sigma^2(X)]$. In our work, we replace τ with τ_k to be able to present the estimation of total travel time function related to each operator k .

We consider the case of Multivariate Kernel Regression method because our response variable Y depends on a vector of exogenous variables \mathbf{X} . Thus, we aim to estimate the following conditional expectation:

$$E(Y|\mathbf{X}) = E(Y|x_1, \dots, x_d) = \tau(\mathbf{X}), \tag{3.3}$$

where $\mathbf{X} = (x_1, \dots, x_d)^T$, d is the dimension of the covariate \mathbf{X} .

In our case, d corresponds to the number of patient locations that an operator t visited in one of the periods p , over all of the historical periods m .

To estimate the unknown function, we use the Nadaraya-Watson estimator [52]:

$$\hat{\tau}(x) = \frac{\sum_{p=1}^m K\left(\frac{\mathbf{X}_p - \mathbf{x}}{\mathbf{h}}\right) Y_p}{\sum_{p=1}^m K\left(\frac{\mathbf{X}_p - \mathbf{x}}{\mathbf{h}}\right)}, \quad (3.4)$$

where $K(\cdot)$ is a d dimensional kernel function, \mathbf{h} is the bandwidth array and the point \mathbf{x} corresponds to the newly assigned patients according to which total travel time estimation should be calculated. In the Nadaraya-Watson approach, the function τ is estimated with a locally weighted average by using the kernel as a weighting function. The selection of the bandwidth value is relevant, as it affects the predictor's smoothness. Several methods are available in the literature to select an optimal value for \mathbf{h} ([14]).

The kernel function, $K(\mathbf{Z}_p)$, is chosen as the widely applied Gaussian Kernel,

$$K(\mathbf{Z}_p) = \frac{1}{\sqrt{2\pi}} e^{-\mathbf{Z}_p^2}, \quad (3.5)$$

where $\mathbf{Z}_p = \frac{\mathbf{X}_p - \mathbf{x}}{\mathbf{h}}$.

As a summary, in this work, \mathbf{X}_p and Y_p values are used to estimate the total travel time function, $\hat{\tau}_t$, with the use of newly assigned patients, \mathbf{x} . Hence, in the KR approach, the main objective is to estimate the total travel time of an operator k , $\hat{\tau}_t$, with the given set of newly assigned patients, \mathbf{x} , and his/her history on the previously realized tours, \mathbf{X} and \mathbf{Y} . Thus, $\hat{\tau}_t$ can be estimated based on operators' past behavior. Figure 3.2 presents the implementation procedure of the KR technique.

As can be seen from Figure 3.2, the Kernel procedure starts with the data gathering or data generation step. Within this step, all required information to calculate the estimation function should be provided (see Step 1 in the figure). The required information includes the geographical locations of patients, \mathbf{X}_p , and the corresponding total travel times, Y_p , for each period p . Bandwidth (h) values should also be calculated or estimated according to the given historical data. With the provided historical information, the next step is to estimate the travel time according to the newly admitted patients (\mathbf{x}) (see Step 2.1 in the figure). Then, the next step is the deviation calculation for each period p (Z_p) between the locations of the newly admitted patients, \mathbf{x} , and historical patients, \mathbf{X}_p (see Step 2.2 in the figure) which is directly used in Step 2.3. Step 2.3 is the weight calculation (F_p) step of each historical period p that is used to identify which past periods will have more impact on the estimation according to the newly admitted patients. The last step is the incorporation of the calculated weights, F_p , to the Equation 3.4

and the calculation of the travel time estimate of an operator (see Step 2.4 in the figure).

Note that, in this thesis, we randomly generate the historical data that is composed of locations of patients. With this information visiting sequences and corresponding total travel times of operators are optimally calculated by using the TSP model and no other geography related information is incorporated. If real historical data is available, the following KR procedure can be easily applied to incorporate other available attributes (i.e., other geographical information, care profile, temporal constraint etc.) as well.

Convergence and Accuracy

In this part we provide some information on the convergence properties of the multivariate kernel estimate. Then, we present the accuracy analysis with numerical experiments.

Here we list some of the important aspects of the convergence properties as follows (for more information see [43, 46]):

- In higher dimensions d the observations are sparsely distributed even for large sample sizes where the estimator has lower performance.
- Speed of convergence decreases dramatically for higher dimensions.
- Convergence rate of the nonparametric estimator behave like $m^{4/5}$ if τ is assumed to have an integrable second derivative.
- In d dimensions the risk behaves like $m^{-4/(4+d)}$
- To maintain the given degree of accuracy with lower dimensions, the sample size must increase exponentially with d .
- In higher dimensions, the distribution tails is more important.
- Beyond three dimensions ($d \geq 3$), the number of observations required for reliable estimation is very large.
- In practice $d \geq 10$ requires very high sample size.

To test the accuracy of the proposed KR technique, we use a small instance generated from real data. In the experiment, we randomly generate five patients in a geographical area where

these locations are obtained from real data provided by an Italian HHC. A single operator is considered for this analysis.

We first conduct an experiment to calculate the predictor for new patients with equation (3.4) using the historical (observed) total travel times. Each historical total travel time represents one observation (of m observations) based on which the predictor is developed. We use different sizes of historical data ($m = 5, 100, 500, 750, 2500, 5000, 7500, 15000, \dots \text{days}$) to study the behavior of the predictor as the number of observations increases. For each data set, we calculate $\hat{\tau}$ based on m observations. Then, the predictor is used to estimate the travel times for 100 new data sets (with each data set composed of 5 new patients) randomly generated *out-of-sample*. After obtaining the estimated travel time using the KR approach, the TSP model was used to calculate the optimal travel time to visit the new patients according to the travel time minimization criterion. Optimal travel times are used as benchmarks to determine the estimator's accuracy.

The error percentage between the estimated values, $T(KR)$, and the optimal TSP values, $T(TSP)$, are given in Table 3.1. The term $T(\cdot)$ indicates the total travel time value obtained by the KR approach or the TSP. As the number of historical observations increases, the predictor provides more accurate estimates. Because of the large amount of historical data ($m=10,000,000$ days) and the associated computational complexity, results presented in Table 3.1 correspond to the case of five patients and a single operator.

To obtain the bandwidth array, \mathbf{h} , we follow the optimal bandwidth technique suggested by Bowman and Azzalini [14]:

$$\mathbf{h}_p = \sigma_p \left(\frac{4}{(d+2)r} \right)^{1/d+4}, \quad p = 1, \dots, d. \quad (3.6)$$

Here r indicates the number of points to be used for the regression analysis and σ_p is the standard deviation of the p^{th} variate.

With Table 3.1, we can conclude that with more historical data, we can better observe the real operator behavior and obtain assignment lists that are similar to the ones that operators usually have.

Number Days (m)	T(KR)	T(TSP)	% Error
5	19.04	23.20	17.94
100	19.19	23.20	17.28
500	19.20	23.20	17.27
750	19.21	23.20	17.19
2500	19.22	23.20	17.18
5000	19.26	23.20	17.01
7500	19.32	23.20	16.75
15000	19.38	23.20	16.48
100000	19.77	23.20	14.81
1000000	20.24	23.20	12.76
10000000	20.72	23.20	10.69

Table 3.1: Error between the estimated and optimal travel times (as average over 100 samples)

3.4 CONCLUSION

In this chapter we present three alternative travel time estimation methods to be able to solve the assignment problem of the two-stage approach. Among these approaches, we emphasize the importance of the data-driven approach in HHC services via a real case example. We also present the mathematical derivation, implementation details and accuracy analysis of the selected non-parametric regression technique, KR.

The presented KR technique can be extended by using analytical approximations. In the current setting, an estimation is built by averaging the contribution of all the observations by assigning higher weight to the closer observations than the farther ones. On the other hand, hybrid method that uses KR technique with analytical methods can further improve the estimate. With this method, Kernel can be built in order to give more importance to the historical data when the observation is close to the point that is trying to be estimated. On the contrary, in the points far from observation more importance can be given to the response from analytical methods.

In the following chapter, we provide details for how to consider the travel time estimation functions in the assignment problem of the two-stage approach.

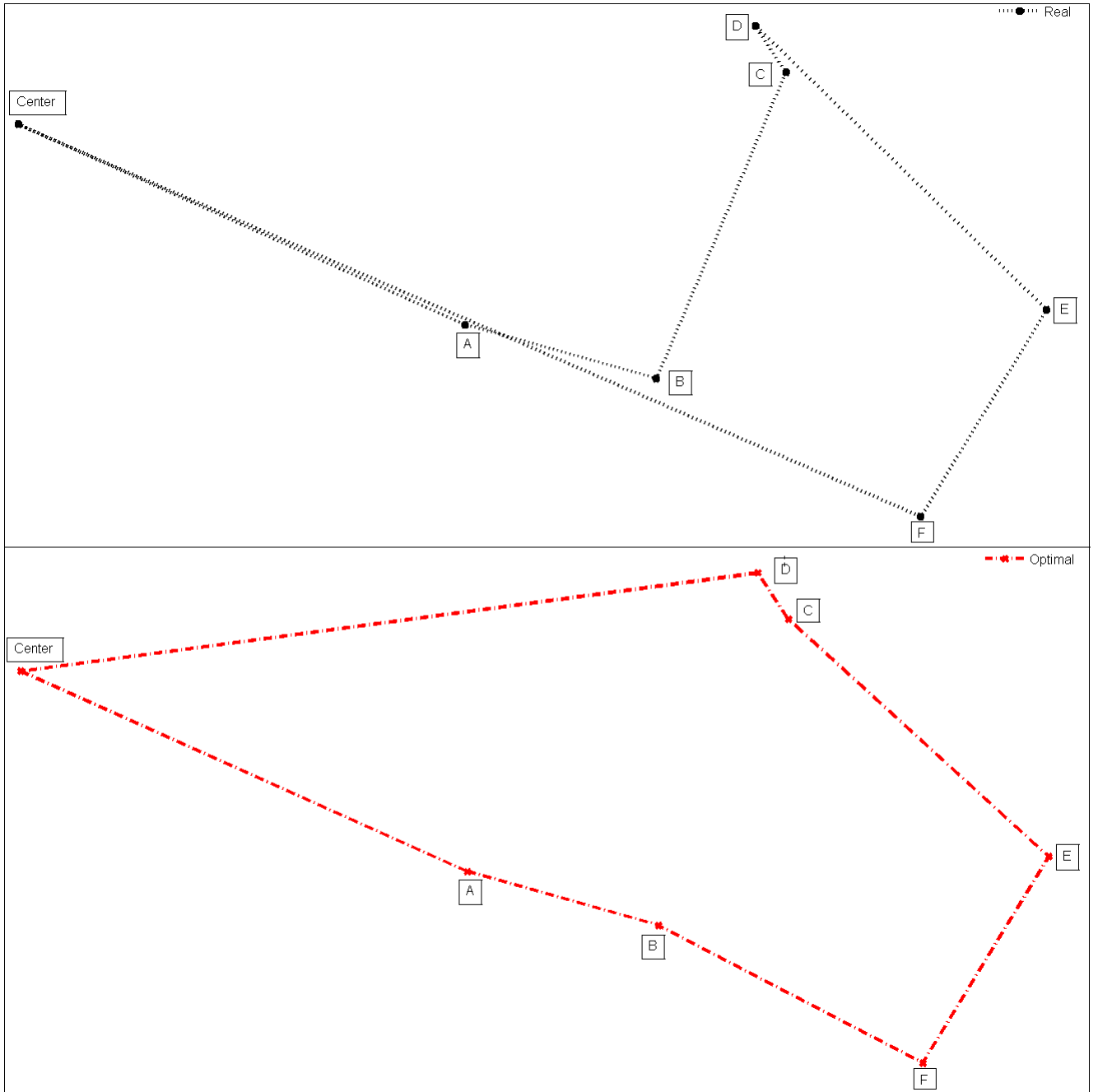


Figure 3.1: Optimal vs Realized (executed) Operator Tour

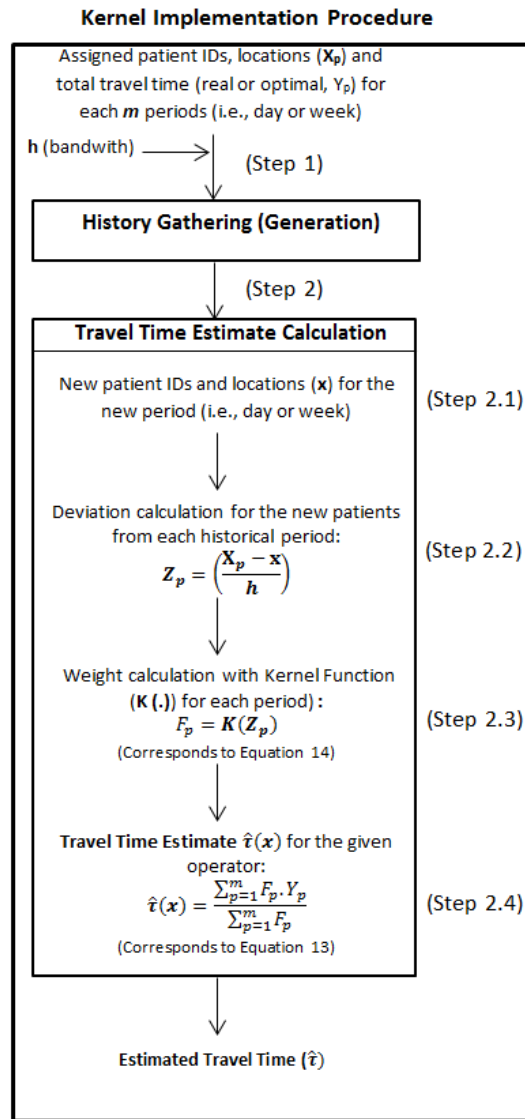


Figure 3.2: Procedure for Implementing Kernel Technique

Chapter 4

SOLUTION APPROACHES

4.1 INTRODUCTION

In this chapter, we use two solution methods based on the standard CPLEX solver and Genetic Algorithm (GA) to solve models presented in Chapter 2. The aim of this chapter is to present the details of the considered solution methods especially the GA. The chapter is organized by first presenting a general procedure for the GA and then, providing details for the developed GAs. Finally, information related to embedding the travel time functions are given.

Although it is possible to use CPLEX to solve both the two-stage and simultaneous approaches, in some cases GA is preferred because of three main reasons. The first one is the computational complexity encountered in the simultaneous approach, especially for large instances. The second reason is that KR and OSAV functions are fitted to calculate directly the total travel time of an operator (not on each patient separately) and considering this in a GA method is much more easier rather than in a mathematical programming model. The last reason is the non-linear property of the Kernel function.

Note that, in Chapter 5 results of the numerical experiment are provided and these results are grouped in two parts (i.e. in Part III and Part IV). The first part is used to analyze the performance of different travel time estimation methods on the two-stage approach whereas the second part is based on the different skill management (i.e. hierarchical or independent skill management) alternatives. Different models are considered in each part and they are solved with either CPLEX solver or GA in the first part and only CPLEX solver is used in the second part. Table 4.1 summarizes the considered solutions methods for each part and case explicitly.

To sum up, as presented in Table 4.1 while a GA is used for the simultaneous approach in Part III of Chapter 5 and the two-stage approach using the KR and OSAV functions (i.e. in

Part	III			IV	
Approach	Two-Stage		Simultaneous	Two-Stage	Simultaneous
Estimation Technique	AV	OSAV	KR	Euclidean	Euclidean
Solution Method	CPLEX	GA	GA	GA	CPLEX

Table 4.1: Solution Methods for Different Models

Part III), the standard CPLEX solver is used for the two-stage approach using the AV technique in both Part III and IV of Chapter 5. Lastly, all the simultaneous approach models of Part IV are also solved by using the CPLEX solver. All the details are presented explicitly in Chapter 5. Note that, for the two-stage approach both stages are also solved with same solution method (i.e. either CPLEX or GA).

In the following sections, we first present the GAs that are implemented and then, provide details about how different travel times functions are incorporated into the two-stage approach.

4.2 GENETIC ALGORITHMS

Genetic Algorithm (GA) is a metaheuristic (local search technique) that mimics the evolution process in order to solve the combinatorial optimization problems. It was first developed by Holland [31] and followed by several others ([28]). GA is an adaptive search procedure applied to a set of solutions and uses the properties from population genetics (i.e., crossover and mutation) to guide the search. At each iteration GA discards some solutions (the poor ones) and generates new ones based on superior members of the current set of solutions. The evaluations of the solutions (e.g., poor or good) are based on a problem specific function that is named as *fitness function*. The general representation of the GA is presented in Algorithm 1 below.

Algorithm 1: Genetic Algorithm

1. Generate an initial population
 2. Evaluate the chromosomes with the fitness function
 3. Perform selection operation with tournament system
 4. Perform crossover and mutation and check feasibility
 5. Repeat steps 2, 3 and 4 until the stopping (i.e maximum number of iterations is satisfied) criterion is met
-

In this research, we use a variant of existing GA approaches and for simplicity reasons,

we refer to it as GA as well. Different than the classical published GA works that deal with the assignment and routing problems ([42]), here we produce children from a single parent so crossover operator transforms a single chromosome. Due to such a property, the GA procedure turns out to be a random local search method where new solutions are derived from a current solution by transformation, but without any neighborhood exploration. Since management of populations solutions is similar to the classical GA, our approach can be considered as a variant of the GA.

Since GA is applied for both the two-stage and simultaneous approaches, there exists differences between each GA. The following parts provide details on the first GA that is used to solve the simultaneous approach (see Section 4.2.1), the second one is adopted for the assignment problem of the two-stage approach (see Section 4.2.2), the last one is used to solve the routing problem of the two stage approach (see Section 4.2.3).

Note that the presented GAs are only applied to the models with independent skill, multiple planning periods and capacity constraints of operators. The models with hierarchical skills are solved using the two-stage approach by only considering the AV technique. Thus, the corresponding solutions are obtained with the CPLEX solver and no GA is developed neither for this two-stage approach nor for the benchmark approach.

4.2.1 Simultaneous Approach

In this section we present each phase of the GA such as the encoding, fitness function evaluation, population selection procedure, crossover and mutation operations, and the feasibility. Finally, we also provide a performance analysis of the developed GA.

Encoding (Representation)

Each solution in GA is represented as a chromosome where in each chromosome, the visiting sequences of each operator is identified with *ones*. In particular, for each chromosome, patients and associated patterns are identified with numbers between 2 and $N+1$ for patients, 2 and $P+1$ for patterns. Figure 4.1 represents the chromosome for the visiting sequence with 3 operators with 10 patients and 4 patterns (i.e. each gene is divided into two parts representing the id of the patient and id of the assigned pattern (**|patient|pattern|**)).

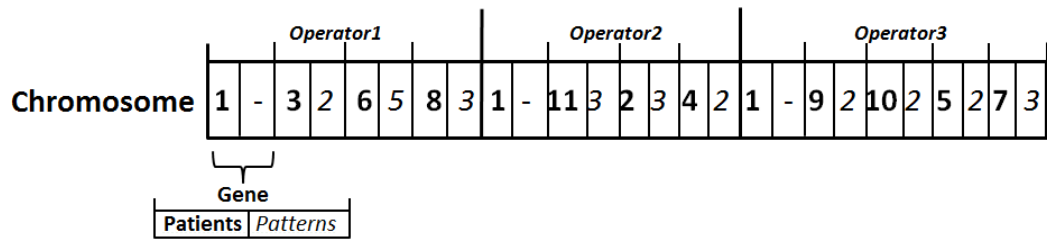


Figure 4.1: Chromosome representation for simultaneous approach with 3 operators, 10 patients and 4 patterns

Fitness Function

The fitness function is the objective function of the simultaneous approach and is selected as one of the equations (Equation (2.16) or (2.19)) that are presented in Chapter I.3.1.

Note that, since subset of patients are visited according to the associated patterns in each day, the visiting sequences of operators are built with the sequence presented in the chromosome with the available patients for the given day. For example, for Operator 1, according to the patterns of patients only patient 6 and patient 8 are available on Monday. Thus, according to the chromosome presented in Figure 4.1, the operator first visits patient 6 and then patient 8 and the associated fitness function for this day is calculated according to this visiting sequence.

Population Selection

Population selection process involves choosing the chromosomes that would serve as parents for the next population generation. In this thesis, tournament system ([40]) is used in which q chromosomes are randomly selected from the population. Then, the chromosome with the minimum fitness function value is selected among these q individuals to be used as the parent one. This process is performed several times to populate the next generation.

Crossover and Mutations Operations

After selecting the parents, within each parent chromosome single point (gene) crossover is held to generate the new solution (chromosome). First, this is obtained for patients by randomly choosing two crossover genes and simply changing the places of these genes. Since there are operator identifiers in each chromosome (i.e. 1), selected crossover points should be different than these identifier points. If one of the randomly selected points turns out as an identifier point then

the selection process is repeated until a patient point is found. This operation corresponds to either changing the operator-patient match or keeping the match same and changing the visiting sequence of the selected patients for an operator. For example, in Figure 4.1, if crossover is held for the 2nd and 6th patients (corresponds to 3rd and 7th genes of the chromosome) then in the new chromosome, patient 2 will be visited by Operator 1 and patient 6 will be visited by Operator 2. As the next step, this crossover operation is also repeated for the patterns by randomly choosing two points from the second part of each gene (i.e. first part of the gene is not replaced thus, only associated patterns of patients are changed) without considering the gene that corresponds to the operator identifier.

With the mutation operation two new solutions are generated by using a parent chromosome. The first one is obtained by moving a gene randomly to another position in the chromosome (slide the gene). For example, moving the second gene (i.e. patient 3 with pattern 2) to the last position in Figure 4.1 where both the selected gene and the new position is chosen randomly. In the new chromosome patient 3 is assigned to Operator 3 and will be visited according to the associated pattern. The second mutation operation is the flipping where the order of the selected genes are flipped. For example, 3rd and 8th genes are randomly chosen from the chromosome of Figure 4.1 and all genes in between are flipped. The new chromosome is presented in Figure 4.2. Here the only concern is again the selected gene. Any selected gene should not contain the operator identifier 1.

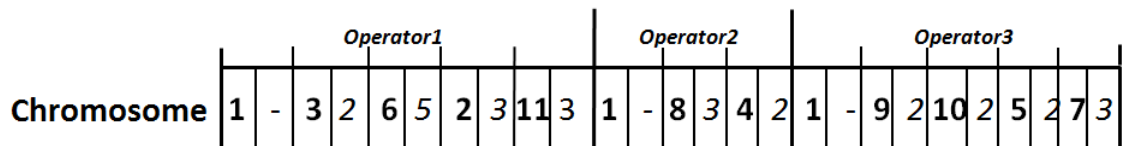


Figure 4.2: Chromosome representation after flipping the genes between 3th and 8th genes

Feasibility

Since each operator should have at least one assigned patient and each patient can only be assigned to a single operator, for each new chromosome generated by the crossover and mutation operations, the feasibility should be checked. The structure of the population matrix (see chromosome representation) always ensures the constraint where each patient can only be assigned once. Thus, no extra effort is required for this constraint. Concerning the pattern feasibility,

after the crossover operation if the corresponding pattern does not satisfy the required visiting frequency of the patient, the crossover operation is repeated until a feasible match between the patient and the pattern is seen. The next feasibility issue is to check the daily operator capacity constraint for each chromosome. If daily workload of any operator exceeds his/her daily capacity then this chromosome is discharged from the population matrix. Lastly, after any crossover and mutation operation, it is also possible to observe an operator that has no assigned patient. For such a case, this chromosome is also discharged from the population matrix. For the last two cases, new chromosomes are generated instead of the discharged ones and this process is repeated until the feasibility conditions are satisfied.

Performance

To be able to test the performance of the presented GA, we use the Unified Hybrid Generic Search (UHGS) method presented in the paper of Vidal *et al.* and solve the same problem instances with both GA and UHGS. Due to complexity problem of the simultaneous approach and limitations of UHGS method, we use a simplified model and consider a single planning period (i.e. a day). Thus, we assume single visit for each patient and also a single pattern for all patients. The tested model is solved with the travel time minimization objective and without considering operator capacities (i.e. MTSP formulation is considered).

Table 4.2.1 shows the objective function values minimized by the implemented GA and the method used as benchmark [50]. All results obtained with two groups of instances generated from real data which are also used in Chapter 5 for the numerical experiments. The first group (Group A.1) contains 56 patients and 7 operators whereas the second one (Group A.2) has 150 patients and 15 operators (see Chapter 5 and Part for more details of these instances).

It is observed from the Table that the maximum error for the instance group A is less than %2.0 and for the instance group B is %16.0. Thus, we can conclude that the implemented GA is performing good enough for the case with 56 patients and 7 operators. When we increase the size of the instance, we observe higher error values. Since the solution algorithm is not the primary goal of this research, we assume that the actual differences are expected and acceptable.

Instance	UHGS	GA	Error %
A.1.1	37.03	37.32 ± 0.22	0.80
A.1.2	44.05	44.63 ± 0.39	1.32
A.1.3	37.41	37.47 ± 0.05	0.15
A.1.4	36.92	37.28 ± 0.36	0.95
A.2.1	53.84	62.41 ± 3.38	15.91
A.2.2	54.36	60.74 ± 2.74	11.73
A.2.3	54.11	62.50 ± 2.39	15.50
A.2.4	54.30	60.80 ± 1.64	11.99

Table 4.2: Performance Analysis of the GA implemented for the simultaneous approach

4.2.2 Two-Stage Approach: Assignment Problem

Similar to the previous GA, here we also present the encoding structure, fitness function evaluation, population selection, crossover and mutation operations and feasibility check process of this GA.

Encoding (Representation)

Here, each solution to the genetic algorithm is represented as a chromosome with the size of number of patients and the associated patterns ($2N$) and each chromosome contains the information for the operator-patient match (i.e. first part of each gene) and the patient-pattern match (i.e. second part of each gene). Figure 4.3 represents the chromosome for 3 operators, 10 patients and 5 patterns.

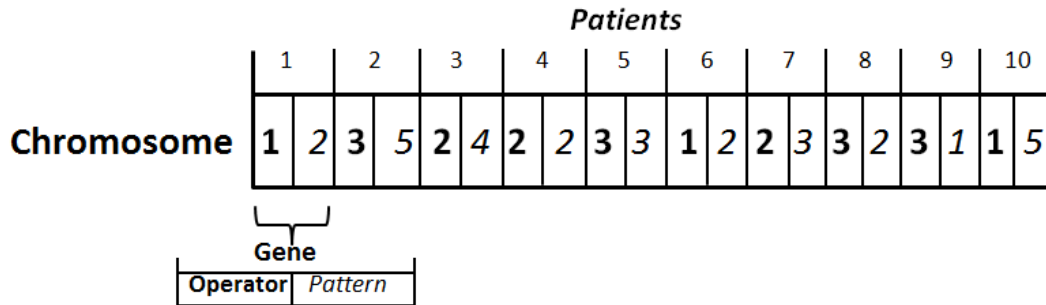


Figure 4.3: Chromosome representation for the assignment problem with KR approach

Fitness Function

The fitness function can be one of the objective functions presented in Chapter II.3 and it is calculated for each day with the available patients (i.e. according to the assigned pattern). We use Equation 2.19 as the fitness function.

Population Selection

The population selection is the same as the previously described GA.

Crossover and Mutation Operation

Crossover and mutation operations are also same as the previous GA. The only difference is, here we do not need to consider the operator identifiers thus, these operation can be held for each gene.

Feasibility

Since the population matrix is generated according to the constraint where each patient can only be assigned to single operator, feasibility is always ensured through out the whole procedure. In particular, feasibilities regarding to the patient-pattern matches and operator capacities can be ensured as presented for the previous GA.

4.2.3 Two-Stage Approach: Routing Problem

The encoding structure, fitness function evaluation, population selection, crossover and mutation operations, feasibility and performance analysis of the GA is provided as follows:

Encoding (Representation)

Since the routing problem deals with the visiting sequences of a single operator, different than the previous GAs here the chromosome represents the visiting sequence of the corresponding operator. (see Figure 4.4 for the visiting sequence of the operator with 10 patients and associated patterns).

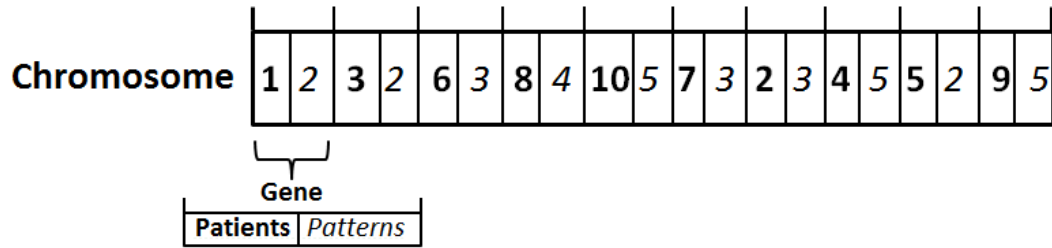


Figure 4.4: Chromosome representation for TSP with 10 patients

Fitness Function

The fitness function is the objective function of the routing model that is given with the Equation (2.77) which is trying to minimize the total travel time of the operator. Note that, the objective function of the routing problem where balancing between operators are ensured is only solved with CPLEX solver thus, in this part no details are given for this case.

Population Selection, Crossover, Mutation

The population selection, crossover, mutation operations are the same as the previously described GAs.

Feasibility

Since the population matrix is generated according to the constraint where each patient can be visited only once, this feasibility issue is always ensured through out the whole procedure. The other feasibility problems related to patient-pattern assignments and operator capacities are solved with the same procedure presented for the previous GAs.

Performance

This part provides details about the performance of the implemented GA for the second stage of the two-stage approach. Solutions of the GA are compared with the optimal solutions that are executed by the ILOG CPLEX solver. Table 4.3 reports the objective function values minimized by the implemented GA and the CPLEX solver. All of the provided results are obtained with instances of 15 patients generated from real data. As in the simultaneous approach, here we also use the model with the single planning period with the same assumptions presented in the GA

for the simultaneous approach (i.e. basic TSP model is considered). It is possible to observe that the GA for the TSP always provided the optimal solution.

Instance	CPLEX	GA	Error %
1	333.94	333.94	0.0
2	327.66	327.66	0.0
3	341.43	341.43	0.0
4	321.21	321.21	0.0

Table 4.3: Performance Analysis of the GA implemented for the routing problem of the two-stage approach

The following section provides details about how the travel time estimation functions that are presented in Chapter 3 can be implemented into the two-stage approach.

4.3 HOW TO CALL THE TRAVEL TIME FUNCTION

Regarding the AV approach, since the average travel times are calculated over all patients before solving the assignment problem, the incorporation of calculated estimations can be accomplished by using these values as patient related input parameters. Then putting them into the relevant equation (e.g. Equations (2.46)) provides the total travel time estimation of each operator.

With respect to KR and OSAV estimations, because the functions are fitted to calculate directly the total travel time of an operator, the incorporation of these estimations into the assignment problem is more difficult than the AV approach. As mentioned before, GA is adopted to be able to cope with such complexities. Indeed, it is not difficult to embed these functions into this heuristic approach since in each iteration of the GA, the assignment list of each operator is known. Thus, direct computation of total travel times for each operator can be completed using either the generated KR (each operator has its own KR function based on his learning) or OSAV functions. Then, the algorithm can proceed for the next step where the fitness value is obtained.

4.4 CONCLUSION

In this chapter, we present the solution methods used in order to analyze the performance of the models proposed for two-stage approach (cf. Part II of Chapter 2). Note that, in this thesis, our objective is to assess the performance of the simultaneous and two-stage approaches (i.e. in

terms of objective function and total travel time of operators). In other words, the performances of the presented GAs are not the main interest since we are not trying to contribute to the VRP literature, rather we try to develop new decision tools for the assignment and routing problems of the HHC literature. Hence, in the following chapter, we use these GAs to solve and present the results of the two-stage approach using KR and OSAV techniques. On the other hand, the commercial CPLEX solver is used to obtain the solutions of the models with AV approach.

In addition to the developed GAs, another proper approach to solve the two-stage models with KR and OSAV functions can be the enumeration of all possible assignment combinations for all operators and the estimation of the related travel times using these functions. These two steps can be completed off-line, i.e., before solving the assignment problem and the corresponding values can be evaluated with the use of an exact method like Column Generation (CG). However, in this thesis, we only implement the GA as a solution method for the two-stage approach with KR and OSAV.

Chapter 5

COMPUTATIONAL STUDY

5.1 INTRODUCTION

In this chapter, we present results of the numerical experiments that analyze the performance of models proposed for the two-stage approach with respect to the simultaneous approach.

Results are grouped in two parts where **Part III** assesses the performance of different travel time estimation methods on the two-stage approach and **Part IV** assesses the impact of alternatives in terms of operator skill management. In each part different models and different solution techniques are used to analyze the performance of some specific cases considered. More specifically, the first part mainly focuses on the performance of the travel time estimation methods. Thus, different travel time estimation methods are used for different variants of the two-stage model. Since the nature of operator skills is not the focus, we assume that operators have identical skills. The focus of the second part is the skill management issue. Both two-stage and simultaneous approach models are tested with independently and hierarchically managed operator skills by considering the AV technique as the travel time estimation method (see Table 5.1 for the summary).

Part	III	IV
Focus	Performance of Travel Time Estimation Methods	Skill Management Alternatives
Models	I,III,IV,V	I,II,III,IV,V
Travel Time Estimation Method	AV,OSAV,KR and Euclidean	AV and Euclidean
Operator Skills	Single and Identical	2 Skill Levels and Not Identical
Planning Period	Single and Multiple	Multiple
Solution Approach	GA and CPLEX	CPLEX

Table 5.1: Summary for the Experimentations

Each part starts by presenting the design of experiments carried and then, provides information related to the experimental settings used for the instances as well as solution methods. Finally, several numerical results are analyzed in details.

Note that all instances used in this work are obtained from real data provided by an Italian HHC provider for which patient locations, patient demands and standard service times for the past four years are available.

Part III

ASSESSING THE PERFORMANCE OF TRAVEL TIME ESTIMATION
METHODS USED IN THE TWO-STAGE PLANNING APPROACH

III.1 INTRODUCTION

The aim of this part is to analyze the performance of each travel time estimation method used in the two-stage approach (i.e. the AV, OSAV and KR methods) in comparison to models developed for the simultaneous approach.

All alternative modeling scenarios (excluding Model II) that are proposed for the two-stage approach (cf. Figure II.1 in Chapter 2) are tested on several instances. Numerical studies are developed for various settings such as single or multiple planning periods, with or without operator capacity restrictions.

Section III.2, provides details on the design of experiments . Then, in Section III.3, we present parameters related to the instances and solution methods considered. Lastly, in Section IV.4, numerical results are provided with a detailed analysis.

Note that in addition to modeling scenarios presented for the two-stage approach (cf. Figure II.1) the model referring to the simultaneous approach is called Model V in the rest of this thesis.

III.2 DESIGN OF EXPERIMENTS

Two groups of experiments are considered: Group A experiments that are based on a single planning period and Group B that considers multiple planning periods. In Group A, Model I (two-stage model) and Model V (simultaneous model) are used to test the performances of different travel time estimation methods (i.e. AV, OSAV and KR) based on the simplest model setting that assumes a single day of planning and released operator capacities.

Group B experiments focus more on the performance of the two-stage approach using the KR technique. In order to do this, in addition to the KR technique, we also consider a second travel time estimation technique (either AV or OSAV) that yields the best performance in the experiments carried in Group A. Different than Group A experiment, the considered modeling assumptions are multiple planning periods assumption and restricted operator capacities. This enables to further analyze the performances of the KR technique and the two-stage approach under more complex conditions.

In both experiment groups, we assume that all operators are identical in terms of skill qualifications and daily working capacities. Furthermore, all models are solved to balance the trade-off between operators' workload balancing and operators' total travel times (i.e. Equation 2.19). In

particular, the second-stage of the two stage approach is solved with the travel time minimization objective.

III.3 EXPERIMENTAL SETTINGS

In this section, we provide details regarding parameters used in the instances and solution methods considered.

As mentioned before, the real data used in our analysis includes information related to patient locations, demands and service times but it does not provide the visiting sequences of patients for each operator. Therefore, to be able to use the historical data in our calculations, we calculate optimally the travel time necessary for reaching all patients assigned to a given operator by solving a Travel Salesman Problem (TSP). In the experiments this travel time is then considered as the historical value which the Kernel estimator is built on.

III.3.1 Parameters Regarding the Instances

The parameters and assumptions related to the experiment groups A and B are as follows:

Group A:

- A total of 8 instances grouped in two sets are used in Group A experiments.
 1. The first set of instances consists of 4 medium-size instances. Those are A.1.1, A.1.2, A.1.3, A.1.4 that consider 56 patients and 7 operators.
 2. The second set of instances consists of large size instances. Those are A.2.1, A.2.2, A.2.3, A.2.4 that consider 150 patients and 15 operators.
- The first instance in each set (i.e. A.1.1 and A.2.1) is directly generated from real data, with 56 or 150 patients distributed across 7 cities (see Figure III.1 for an example with 56 patients). By using the same patient information from these instances, 3 additional instances are generated for each set (i.e. A.1.2, A.1.3, A.1.4 for the first set and A.2.2, A.2.3, A.2.4 for the second set) such that in each instance the cities that the patients belong to are re-sampled according to the probabilities and the city locations reported on Table III.1.

City	$x_{\text{city},1}$	$x_{\text{city},2}$	Probability
1	49.21	42.32	0.1544
2	51.63	40.01	0.1611
3	48.16	36.92	0.1544
4	46.14	38.02	0.2819
5	41.90	39.05	0.0604
6	42.60	38.79	0.1007
7	44.36	39.75	0.0872

Table III.1: Location and probabilities of cities

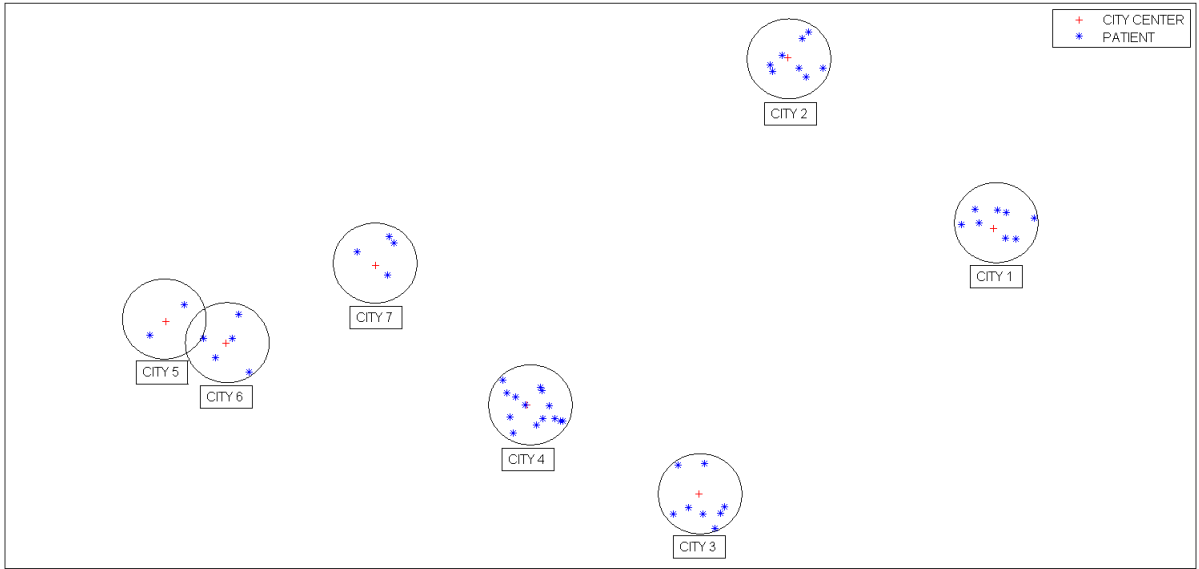


Figure III.1: Example for the map of cities and patient locations

- The planning horizon is assumed to be a single day (i.e. single planning period).
- Each patient requires a single visit.
- The standard service time, sv_j , that is required to visit a patient is set to 45 minutes for all patients which is directly set by the health care provider.
- KR learning is obtained based on $m=100$ days (i.e. approximately 6 months) and $m=1000$ days (i.e. approximately 4 years) of history with only geographical locations used as the patient attributes. Historical data is used for 6 months or 4 years because using information less than 6 months is not enough to build a knowledge on and operators usually work for a provider at least 6 months. In particular, 4 years is usually the average time period that an operator works for the same health care provider.

- The trade-off parameter γ is assumed to be 1/100 for medium-sized and 1/750 for large-sized instances. In addition to these values, in Section III.4.2.2, we also provide a sensitivity analysis to show that for several other γ values the solutions are consistent according to the nature of the trade-off function (i.e. increasing γ results in higher total travel time values and lower balance between operators).

Group B:

- A total of 10 instances are considered in Group B. All instances are medium-size instances that use 44 to 56 patients and 4 operators as reported in Table III.2.
- Different than Group A instances, all instance are directly generated from real data.
- Operators are allowed to work 400, 350, 300 and 300 minutes per day respectively and these values are directly obtained from the data given by the health care provider.
- The planning horizon consists of 6 days (i.e. multiple planning periods).
- Each patient requires 1 to 4 visits in total during the planning period of 6 days. The total number of visits required among all patients is provided in column 3 of Table III.2.
- Remind that a pattern is a priori given schedule for a given set of visits (i.e. in total 2 visits on Tuesday and Friday etc.) and several patterns are usually defined to satisfy the requirements of all patients. In this work, patterns for care plan decision are generated with the flow based pattern policy given in the work of Cappanera and Scutella [17] for each instance.
- The standard service time, sv_j , that is required to visit a patient is set to 45 minutes for all patients.
- KR learning is obtained based on $m=100$ and $m=1000$ days of history with only geographical locations qs in the Group A instances.
- Several values between 0 and ∞ are assumed for the trade-off parameter γ to be able to analyze the effect of workload balancing on the total travel times and vice versa. All these values are specified explicitly in Section III.4.2. In particular, in Section III.4.2.4, a sensitivity analysis is also done for different γ values as we do for the Group A instances.

Instance	Number of Patients	Number of Visits	Number of Patterns	Number of Days	Number of Operators
B.1.1	51	69	8	6	4
B.1.2	51	76	5	6	4
B.1.3	48	67	5	6	4
B.1.4	50	70	5	6	4
B.1.5	44	69	3	6	4
B.1.6	53	72	7	6	4
B.1.7	52	74	6	6	4
B.1.8	52	75	6	6	4
B.1.9	56	80	5	6	4
B.1.10	49	62	4	6	4

Table III.2: Parameter values used for Group B experiments

III.3.2 Parameters Regarding to the Solution Methods Used

In this work, we use two solution methods based on a commercial CPLEX solver and GA. The CPLEX solver is only used for the two-stage approach with AV technique whereas all other models for the two-stage and simultaneous approaches are solved by the developed GAs (cf. Table 4.1 in Chapter 4). Details related to these solutions methods are as follow:

CPLEX Solver:

- Models are coded in Python 2.7.2 programming language.
- CPLEX 12.3 is used for solving the two-stage model with the AV technique.

GA Configuration:

- Algorithms are coded in Matlab R2013b.
- The population size is selected as 100.
- For the iteration number, two different values are used as 1000 and 5000.
- Algorithms are terminated when the maximum iteration number is reached.
- To keep the computational effort of experiments manageable, we conduct 5 replications for each experiment.

Note that, all numerical experiments are performed on a computer with Intel Core i7 2.2 GHz CPU, and 8 GB of RAM.

III.4 NUMERICAL RESULTS

In this section, the proposed two-stage approach is compared to the simultaneous approach using Group A and B instances. Section III.4.1 presents the performance indicators used to analyze results. Then, in Section III.4.2 numerical results for the simple setting with Model I and Model V are provided (i.e single period and no restriction on operator capacity) by using the instances from Group A. The performances of KR, AV and OSAV methods in the two-stage approach are evaluated. As the next step, we extend the numerical analysis to the cases where multiple planning periods and operator capacities are considered. Results are developed for Group B.1 instances. Hence, Model III, Model IV and Model V are used to analyze the performance of the two-stage approach with KR technique and the operator specific travel time estimation method OSAV. Since we show that the OSAV method outperforms the AV method, the AV technique is not included in further analysis. Lastly, sensitivity analysis is also carried out for both Group A and B instances.

III.4.1 Performance Indicators

Three indicators are used to assess the performance of each model analyzed.

The total travel time of all operators obtained in the two-stage approach with the AV, OSAV and KR methods are represented by $T(AV)$, $T(OSAV)$ and $T(KR)$, respectively, and the workload balance value between the maximally and minimally utilized operators are represented by $B(AV)$, $B(OSAV)$ and $B(KR)$, respectively. Similarly, the total travel time in the simultaneous approach is denoted by $T(VRP)$, and the balancing value is denoted by $B(VRP)$. Because the models are solved to balance the trade-off between operators' workload and total travel times, the corresponding value is denoted as $Obj.$, which equals to $h(.) + \gamma T(.)$ where $h(.)$ corresponds to the maximum operator utilization level.

$T(AV)$, $T(OSAV)$ and $T(KR)$ values are obtained by solving several (as the number of operators) independent TSP models (i.e. the objective function is travel time minimization), with the outputs obtained from the assignment stage and summing up the results obtained from each TSP model across the seven operators. $T(VRP)$ values are directly calculated from the corresponding simultaneous model as the sum of each operator's route time.

Because we use a genetic algorithm for both the two-stage approach with the KR and OSAV techniques as well as the simultaneous approach, $T(.)$ and $B(.)$ values are obtained as the average

values from 5 replications of the algorithms, and corresponding *Obj.* values are calculated based on a %95 confidence interval. Moreover, the results of the two-stage approach using the AV technique are obtained with the ILOG Cplex solver for both stages thus, we do not provide a confidence interval for the two-stage approach using the AV technique.

III.4.2 Results

Section III.4.2.1 presents the results associated with Group A instances that compare Model I to Model V where AV, OSAV and KR methods are used as travel time estimators in the first stage of Model I. Then, Section III.4.2.3 provides other results with Group B instances for Model III, Model IV and Model V. In this case, only OSAV and KR methods are used as travel time estimator within the Model III and Model IV. In Section III.4.2.2 and Section III.4.2.4, results for the sensitivity analysis are presented as well.

III.4.2.1 Results Pertaining to Group A Instances

In this section, we analyze the performance of the KR technique in comparison to AV and OSAV techniques. Then, we conduct a sensitivity analysis based on different historical data (i.e. m) values.

Results for Group A.1 Instances:

As Table III.3 indicates, there are two types of results related to the two-stage approach. One of them is referred as the Non Uniformized two-stage KR approach (cf. columns 2,3,4) and the other one is called the Uniformized (Unif.) two-stage KR approach (cf. columns 5,6,7). For the second case, we consider the uniformization of the input data used in the KR function. For example, a patient can occupy the first position (i.e., the rank in the input matrix) in a planning horizon and can be considered in the third position in the following horizon. Thus, if we do not uniformize the input data, the KR function might not recognize that these patients refer to the same patient and may spend unnecessary time providing a better estimate by building a structure across all the dimensions (as the number of patients). To avoid this computation, we use a simple ordering technique for the input data that sorts patients according to their geographical locations (based on X and Y coordinates). This technique calculates an Order Value (OV) for each patient, $OV = X + aY$ (a is a positive integer), and generates the input data by sorting patients in descending order of their OV values.

Results reported for the $T(\cdot)$ values obtained with with both No Unif. and Unif. Two-Stage approaches are generally close to the ones with the simultaneous approach. The differences between the No Uniformized two-stage approach and the simultaneous approach are approximately %13.6 (i.e., average difference for values of the 4 instances used for the total travel time $T(\cdot)$).

1	2	3	4	5	6	7	8	9	10
Instance	No Unif. Two-Stage KR (Model I)			Unif. Two-Stage KR (Model I)			Simultaneous (Model V)		
	$T(KR)$	$B(KR)$	Obj_{KR}	$T(KR)$	$B(KR)$	Obj_{KR}	$T(VRP)$	$B(VRP)$	Obj_{VRP}
A.1.1	92.22	0.003	1.706 ± 0.1157	79.51	0.016	1.578 ± 0.0011	78.08	0.019	1.562 ± 0.1395
A.1.2	98.64	0.002	1.769 ± 0.0970	85.64	0.002	1.640 ± 0.0144	87.63	0.063	1.683 ± 0.1630
A.1.3	96.27	0.003	1.745 ± 0.1202	83.47	0.011	1.619 ± 0.0011	83.55	0.063	1.642 ± 0.0702
A.1.4	87.55	0.003	1.659 ± 0.0491	79.84	0.016	1.582 ± 0.0312	80.53	0.032	1.594 ± 0.0549

Table III.3: Results for Group A.1, $\gamma = 1/100$, $m=100$

The uniformization generates in general the smallest $T(KR)$ values. We observe lower differences between the $T(KR)$ values of Uniformised Two-Stage and simultaneous approaches. By applying the uniformization, we are able to decrease the difference of $T(\cdot)$ values between these approaches to %1.2 on average (cf. column 5 and 8). Even if we consider the maximum error of %2 observed as the result of using the GA to solve the simultaneous approach with the medium-size instances (see Chapter 4.2.1), the differences still seems clearly to be acceptable. Thus, we can conclude that by ordering the input data, we improve results for the two-stage approach and obtain almost the same solutions than the simultaneous approach. In the rest of this study, all results presented for the two-stage approach are obtained with the uniformization technique.

Results for Group A.2 Instances:

Results obtained for Group A.2 instances are presented in Table III.4.

1	2	3	4	5	6	7
Instance	Unif. Two-Stage KR (Model I)			Simultaneous (Model V)		
	$T(KR)$	$B(KR)$	Obj_{KR}	$T(VRP)$	$B(VRP)$	Obj_{VRP}
A.2.1	236.82	0.015	1.291 ± 0.0156	226.24	0.024	1.285 ± 0.0123
A.2.2	233.85	0.016	1.287 ± 0.0348	232.69	0.021	1.293 ± 0.0169
A.2.3	233.25	0.022	1.288 ± 0.0317	231.47	0.019	1.289 ± 0.0284
A.2.4	226.61	0.019	1.276 ± 0.0379	230.15	0.021	1.290 ± 0.0196

Table III.4: Results with Group A.2, $\gamma = 1/750$, $m=100$

The difference between the total travel times of the two-stage approach and the simultaneous approach remains acceptable (i.e. an average difference of %1.5, a maximum difference of %4.7 among all four instances considered)

approximately %1.5 (see columns 2 and 5 of Table III.4). Additionally, the solution times for both approaches are comparable. However, when we consider the maximum error of %16 as the result of using the GA to solve the simultaneous approach with large instances (see Chapter 4.2.1), one can conclude that the solutions of the two-stage approach with larger instances does not provide as good results as it does for the medium-size instances. One solution to address this issue can be to increase the number of historical data used for the KR approach. Thus, in the sensitivity analysis part (Section III.4.2.2), we consider these instances with a larger number of historical data and present the corresponding results.

Comparison of the KR Estimator with Other Estimators:

To be able to analyze the performance of the proposed KR estimator, we compare results of the two-stage method using the KR technique to two other methods, AV and OSAV. The analysis is done for Group A.1 and A.2 instances for two different γ values.

1	2	3	4	5	6	7	8	9	10
Instance	Two-Stage AV			Two-Stage OSAV			Two-Stage KR		
	T(AV)	B(AV)	Obj _{AV}	T(OSAV)	B(OSAV)	Obj _{OSAV}	T(KR)	B(KR)	Obj _{KR}
A.1.1	147.02	0.004	2.456	80.79	0.021	1.598 ± 0.0663	79.51	0.016	1.578 ± 0.0011
A.1.2	157.63	0.006	2.562	90.08	0.017	1.687 ± 0.0705	85.64	0.002	1.640 ± 0.0144
A.1.3	141.33	0.003	2.402	83.95	0.015	1.622 ± 0.0598	83.47	0.011	1.619 ± 0.0011
A.1.4	141.90	0.017	2.401	82.95	0.017	1.613 ± 0.1135	79.84	0.016	1.582 ± 0.0312

Table III.5: Model I results for Group A.1 and $\gamma = 1/100$ with different travel time estimators

1	2	3	4	5	6	7	8	9	10
Instance	Two-Stage AV			Two-Stage OSAV			Two-Stage KR		
	T(AV)	B(AV)	Obj _{AV}	T(OSAV)	B(OSAV)	Obj _{OSAV}	T(KR)	B(KR)	Obj _{KR}
A.2.1	336.90	0.013	1.427	221.68	0.019	1.274 ± 0.0117	236.82	0.015	1.291 ± 0.0156
A.2.2	323.04	0.016	1.472	233.48	0.021	1.290 ± 0.0460	233.85	0.016	1.287 ± 0.0348
A.2.3	322.43	0.015	1.419	234.03	0.021	1.292 ± 0.0126	233.25	0.022	1.288 ± 0.0317
A.2.4	308.63	0.017	1.398	230.13	0.019	1.283 ± 0.0104	226.61	0.019	1.276 ± 0.0379

Table III.6: Model I results for Group A.2 and $\gamma = 1/750$ with different travel time estimators

As Table III.5 and Table III.6 present, operator specific travel time estimation methods outperform the AV for both medium-size and large-size instances. From Table III.5 it is observed that, the average gap on the T(.) values between the AV and OSAV methods is %74 and the maximum gap is %82. On the other hand, the average and maximum gaps between AV and KR techniques are %79 and %85 respectively. Moreover, when large-size instances are considered the average and maximum gaps between the AV and OSAV methods are observed as %41 and %52

respectively. Similarly, the average and maximum gaps between AV and KR techniques turn out to be %39 and %42 respectively. In particular, it can also be observed that when m is set to 100, the OSAV method performs close to the KR technique. Thus, in the sensitivity analysis part below, we present some results with an increased number of the historical data.

III.4.2.2 Sensitivity Analysis of Group A Instance

This section analyzes the sensitivity of size of the historical data m and the trade-off parameter γ .

Results obtained from increasing the number of historical data (for $m = 100$ and $m = 1000$) with 56 patients are presented in Table III.7 and with 150 patients are presented in Table III.8.

Instance	m	Two-Stage KR			Simultaneous		
		T(KR)	B(KR)	Obj _{KR}	T(VRP)	B(VRP)	Obj _{VRP}
A.1.1	100	79.51	0.016	1.578 ± 0.0011	78.08	0.019	1.562 ± 0.1395
	1000	76.75	0.010	1.551 ± 0.0432			
A.1.2	100	85.64	0.002	1.640 ± 0.0144	87.63	0.063	1.683 ± 0.1630
	1000	82.94	0.000	1.613 ± 0.0051			
A.1.3	100	83.47	0.011	1.619 ± 0.0011	83.55	0.063	1.642 ± 0.0702
	1000	79.99	0.002	1.584 ± 0.0111			
A.1.4	100	79.84	0.016	1.582 ± 0.0312	80.53	0.032	1.594 ± 0.0549
	1000	76.41	0.009	1.548 ± 0.0041			

Table III.7: Results with 56 patients and $\gamma = 1/100$ with increasing the number of history from 100 to 1000

Instance	m	Two-Stage KR			Simultaneous		
		T(KR)	B(KR)	Obj _{KR}	T(VRP)	B(VRP)	Obj _{VRP}
A.2.1	100	236.82	0.015	1.291 ± 0.0156	226.24	0.024	1.285 ± 0.0123
	1000	188.09	0.022	1.223 ± 0.0035			
A.2.2	100	233.85	0.016	1.287 ± 0.0348	232.69	0.024	1.293 ± 0.0169
	1000	181.11	0.022	1.213 ± 0.0129			
A.2.3	100	233.25	0.022	1.288 ± 0.0317	231.47	0.019	1.289 ± 0.0284
	1000	182.83	0.021	1.216 ± 0.0196			
A.2.4	100	226.61	0.019	1.276 ± 0.0379	230.15	0.021	1.290 ± 0.0196
	1000	184.59	0.020	1.218 ± 0.0146			

Table III.8: Results with 150 patients and $\gamma = 1/750$ with increasing the number of history from 100 to 1000

As the number of historical data increases, the performance of the two-stage approach with KR technique significantly increases.

When the $T(\cdot)$ values of KR solutions from Tables III.7 and III.8 are compared with solutions of the OSAV method from Table III.5 and Table III.6, one can remark that when the number of historical days is increased to 1000, the KR technique starts to perform much more better than the OSAV approach.

In particular, in the previous analysis, we observe that for large-size instances with $m=100$, the results of KR approach do not seem to be as good as the ones of the medium-sized instances. However, when we compare the solutions of the large-sized instances of the KR approach with $m=1000$ (cf. Table III.8) with the solutions of the simultaneous approach (cf. Table III.4) by also considering the GA error of %16, we can conclude that with an enough number of historical points, the two-stage approach with KR method presents similar solutions in comparison to the simultaneous approach even for large instances.

For simplicity, all results presented so far are obtained by using two trade-off values, $\gamma = 1/100$ (i.e. Group A.1 instances) and $\gamma = 1/750$ (i.e. Group A.2 instances). Two different γ values are chosen for each group since number of patients and operators are different as well. To be able to show that the two-stage approach using KR estimation (i.e. OSAV performs similar as well, see the sensitivity analysis in Section III.4.2.4) is also consistent with other penalty values, we plot Figure III.2 using one of the medium-size instances. With this figure, we show the trade-off between the workload balancing and total travel times of operators for decreasing values of the trade-off term; results refer to the assignment phase of the two-stage process. As expected, it is evident that when we decrease the trade-off value, the effect of the total travel time decreases while better workload balancing is ensured. Thus, the KR predictor seems to correctly guide the assignment problem in different situations.

In the following part, we extend our analysis with Group B instances to analyze the performance of the two-stage approach with KR under more complex conditions.

III.4.2.3 Results with Group B Instances

We consider a single set of 10 instances to analyze the models with multiple planning periods and operator capacities. Instances consist of 44 to 56 patients and 4 operators as presented on Table III.2.

We start our analysis with a single instance (i.e. Instance B.1.1) and we consider 6 trade-off values, γ , between zero and infinity. Then, for simplicity, we restrict our computations with two

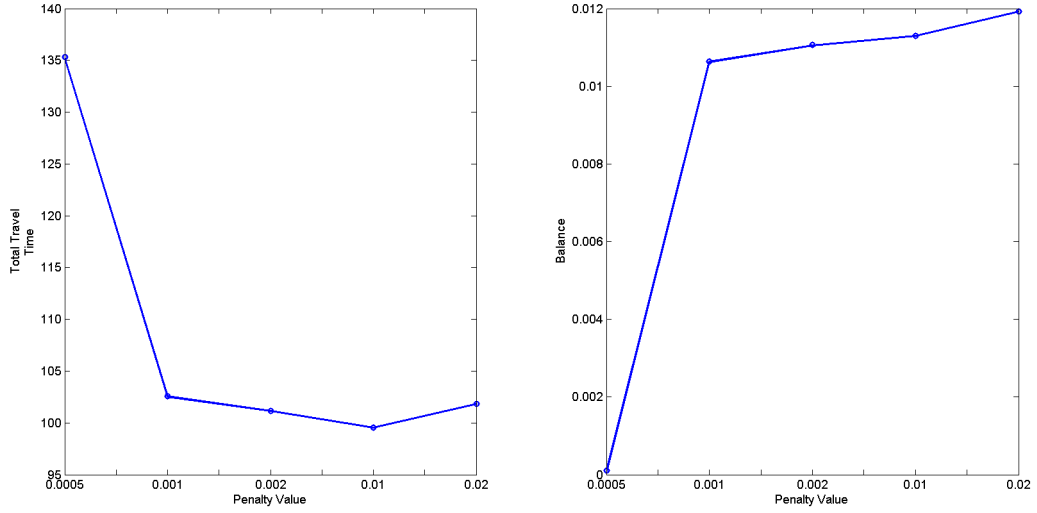


Figure III.2: The trade-off between the workload balancing and the total travel time minimization for the assignment phase of the two-stage model

γ values zero and infinity for the rest of the instances. All the analysis regarding to the two-stage approach are held with KR and OSAV methods by using Model III and Model IV.

Table III.9 present results for Model III, Model IV for the two-stage approach and Model V for the simultaneous approach with different γ values for instance B.1.1. Results obtained from Model III and Model IV show how solutions are effected when care plan decision is held within different stages of the two-stage approach (cf. Chapter 2). We also analyze the performance of the two-stage models (i.e Model III and Model IV) with respect to the simultaneous models (i.e. Model V). Lastly, the performance of KR estimator is compared with the OSAV estimator.

1	2	3	4	5	6	7	8	9	10
Instance B.1.1	Two-Stage OSAV (Model III)			Two-Stage KR (Model III)			Simultaneous (Model V)		
γ	T(OSAV)	B(OSAV)	Obj_{OSAV}	T(KR)	B(KR)	Obj_{KR}	T(VRP)	B(VRP)	Obj_{VRP}
0	283.09	0.011	0.430 ± 0.0097	276.36	0.010	0.422 ± 0.0036	334.95	0.001	0.424 ± 0.0017
0.00002	276.62	0.007	0.434 ± 0.0096	241.31	0.009	0.425 ± 0.0039	327.31	0.001	0.425 ± 0.0331
0.0004	250.14	0.016	0.529 ± 0.0149	242.52	0.013	0.516 ± 0.0106	278.38	0.008	0.483 ± 0.0271
0.008	195.82	0.061	1.996 ± 0.1133	222.77	0.032	2.220 ± 0.1251	260.87	0.033	2.469 ± 0.0912
0.16667	191.46	0.148	32.393 ± 2.7358	209.67	0.151	35.401 ± 2.0926	260.84	0.048	43.721 ± 1.6180
∞	198.00	0.143	198.000 ± 18.2610	210.61	0.173	210.609 ± 18.9264	259.70	0.103	259.699 ± 6.2491
Instance B.1.1	Two-Stage OSAV (Model IV)			Two-Stage KR (Model IV)			Simultaneous (Model V)		
γ	T(OSAV)	B(OSAV)	Obj_{OSAV}	T(KR)	B(KR)	Obj_{KR}	T(VRP)	B(VRP)	Obj_{VRP}
0	234.50	0.018	0.423 ± 0.0086	259.35	0.014	0.422 ± 0.0056	334.95	0.001	0.424 ± 0.0017
0.00002	236.99	0.016	0.429 ± 0.0054	231.97	0.014	0.423 ± 0.0043	327.31	0.001	0.425 ± 0.0331
0.0004	238.82	0.018	0.523 ± 0.0102	240.60	0.018	0.517 ± 0.0107	278.38	0.008	0.483 ± 0.0271
0.008	195.21	0.06	1.996 ± 0.0844	220.82	0.033	2.205 ± 0.0243	260.87	0.033	2.469 ± 0.0912
0.16667	186.37	0.148	31.543 ± 2.3112	205.08	0.147	34.632 ± 1.2790	260.84	0.048	43.721 ± 1.6180
∞	186.32	0.142	186.314 ± 13.042	204.45	0.189	204.448 ± 22.3561	259.70	0.103	259.699 ± 6.2491

Table III.9: Results for Instance B.1.1 with different γ values and $m=100$ for Models III, IV and V

As it can be seen from Table III.9, if the care plan decision is considered in both stages

(i.e. Model IV) where it is first decided in the first stage of the two-stage approach and then adjusted in the second stage, the total travel time of operators start to decrease for all γ values (see columns 2 and 5) . Moreover, we also observe that both two-stage models (Model III and Model IV) with either KR or OSAV methods perform close to the models of the simultaneous approach in terms of total travel time (see columns 2, 5 and 8). In some cases KR method performs better than OSAV and in some others vice versa. Thus, to be able to comment more, we present the KR approach with more historical data (i.e. $m=1000$) in Section III.4.2.4 below. Lastly, as expected, increasing the trade-off value γ results in lower total travel times but more unbalanced operator workloads. Since for all γ values the two-stage approach is performing good enough, we continue to analyze other instances with only two γ values, 0 and ∞ . Table III.10 and Table III.11 present results with these trade-off values for all instances of Group B.

1	2	3	4	5	6	7	8	9	10	11
Instance	γ	Two-Stage OSAV (Model III)			Two-Stage KR (Model III)			Simultaneous (Model V)		
		T(OSAV)	B(OSAV)	Obj _{OSAV}	T(KR)	B(KR)	Obj _{KR}	T(VRP)	B(VRP)	Obj _{VRP}
B.1.1	0	283.09	0.011	0.430 \pm 0.0097	276.36	0.010	0.422 \pm 0.0036	334.95	0.001	0.424 \pm 0.0017
	∞	198.00	0.143	198.000 \pm 18.2610	210.61	0.173	210.609 \pm 18.9264	259.70	0.103	259.699 \pm 6.2491
B.1.2	0	244.18	0.017	0.459 \pm 0.0019	228.90	0.014	0.458 \pm 0.0027	308.74	0.007	0.464 \pm 0.0026
	∞	178.08	0.162	178.081 \pm 15.9055	181.17	0.334	181.174 \pm 8.5506	228.97	0.079	228.972 \pm 8.6700
B.1.3	0	205.90	0.010	0.4016 \pm 0.0019	205.53	0.011	0.403 \pm 0.0039	266.37	0.003	0.406 \pm 0.0017
	∞	161.61	0.127	161.611 \pm 18.5159	159.08	0.335	159.076 \pm 11.3078	209.58	0.119	209.580 \pm 6.7713
B.1.4	0	237.22	0.020	0.428 \pm 0.0020	234.52	0.016	0.425 \pm 0.0034	307.39	0.010	0.432 \pm 0.0055
	∞	181.32	0.261	181.323 \pm 9.5673	177.33	0.338	177.328 \pm 3.6413	238.12	0.174	238.123 \pm 7.9943
B.1.5	0	217.86	0.012	0.361 \pm 0.0044	194.94	0.009	0.432 \pm 0.0033	285.18	0.000	0.363 \pm 0.0018
	∞	122.03	0.195	122.027 \pm 10.5859	146.13	0.196	146.126 \pm 12.0489	204.94	0.084	204.939 \pm 4.6616
B.1.6	0	213.99	0.017	0.435 \pm 0.0040	211.47	0.010	0.432 \pm 0.0032	272.53	0.005	0.436 \pm 0.0029
	∞	185.43	0.131	185.430 \pm 15.2338	188.70	0.146	188.700 \pm 5.9041	232.71	0.095	232.714 \pm 7.1368
B.1.7	0	243.46	0.067	0.455 \pm 0.02098	242.04	0.016	0.451 \pm 0.0015	323.86	0.008	0.455 \pm 0.0084
	∞	198.00	0.144	197.997 \pm 15.6440	195.59	0.181	195.591 \pm 9.3370	256.49	0.059	256.487 \pm 8.8888
B.1.8	0	247.40	0.083	0.489 \pm 0.0524	239.39	0.018	0.456 \pm 0.0025	353.15	0.011	0.465 \pm 0.0029
	∞	203.44	0.160	203.4422 \pm 10.4303	211.59	0.231	211.590 \pm 7.2127	268.99	0.104	268.991 \pm 14.8023
B.1.9	0	249.57	0.023	0.529 \pm 0.0431	268.97	0.020	0.4844 \pm 0.0018	348.16	0.008	0.497 \pm 0.0091
	∞	238.74	0.241	238.7447 \pm 14.788	236.33	0.179	236.331 \pm 25.3273	296.00	0.102	295.999 \pm 12.4094
B.1.10	0	206.62	0.023	0.383 \pm 0.0023	217.65	0.015	0.378 \pm 0.0035	262.13	0.005	0.379 \pm 0.0013
	∞	163.85	0.220	163.8519 \pm 19.0895	171.48	0.289	171.478 \pm 6.7729	218.30	0.198	218.303 \pm 6.7673

Table III.10: Results for Instance Group B with $m=100$ for Models III and V

The other nine instances provided in Table III.10 and Table III.11 confirm our previous observations. Results obtained with Model IV for any instance and trade-off value provides lower total travel times than all other models. Figure III.3 is another visual representation of pertaining results to Table III.10 and Table III.11.

Figure III.3 confirms that Model IV seems to be the most successful model in terms of total travel time. Furthermore, all alternative models of the two-stage approach are performing good enough when compared to the simultaneous approach. As in the analysis of the Instance B.1.1, we can not conclude which travel time estimator is performing better than the other. Thus,

1	2	3			4			5			6			7			8			9			10			11		
Instance	γ	Two-Stage OSAV (Model IV)						Two-Stage KR (Model IV)						Simultaneous (Model V)														
		T(OSAV)		B(OSAV)		Obj _{OSAV}		T(KR)		B(KR)		Obj _{KR}		T(VRP)		B(VRP)		Obj _{VRP}										
B.1.1	0	234.50	0.018	0.423 ± 0.0086		259.35	0.014	0.422 ± 0.0056		334.95	0.001	0.424 ± 0.0017																
	∞	186.32	0.142	186.314 ± 13.042		204.45	0.189	204.448 ± 22.3561		259.70	0.103	259.699 ± 6.2491																
B.1.2	0	225.60	0.013	0.456 ± 0.0020		226.09	0.016	0.458 ± 0.0015		308.74	0.007	0.464 ± 0.0026																
	∞	167.45	0.163	167.452 ± 10.3921		179.74	0.333	179.742 ± 12.3954		228.97	0.079	228.972 ± 8.6700																
B.1.3	0	186.45	0.013	0.402 ± 0.0032		201.74	0.014	0.405 ± 0.0049		266.37	0.003	0.406 ± 0.0017																
	∞	156.37	0.130	156.374 ± 15.9795		152.00	0.334	151.999 ± 10.7377		209.58	0.119	209.580 ± 6.7713																
B.1.4	0	221.69	0.023	0.426 ± 0.0011		227.25	0.019	0.425 ± 0.0042		307.39	0.010	0.432 ± 0.0055																
	∞	180.17	0.244	180.174 ± 6.8892		175.86	0.333	175.859 ± 8.6199		238.12	0.174	238.123 ± 7.9943																
B.1.5	0	204.49	0.012	0.361 ± 0.0067		181.10	0.006	0.354 ± 0.0019		285.18	0.000	0.363 ± 0.0018																
	∞	119.93	0.211	119.928 ± 15.9631		144.25	0.204	144.253 ± 9.8666		204.94	0.084	204.939 ± 4.6616																
B.1.6	0	173.51	0.016	0.429 ± 0.0032		208.54	0.012	0.432 ± 0.0032		272.53	0.005	0.436 ± 0.0029																
	∞	160.11	0.107	160.113 ± 12.5401		184.70	0.102	184.6977 ± 2.8198		232.71	0.095	232.714 ± 7.1368																
B.1.7	0	227.08	0.098	0.492 ± 0.0439		229.67	0.022	0.452 ± 0.0025		323.86	0.008	0.455 ± 0.0084																
	∞	189.38	0.145	189.383 ± 7.1874		188.79	0.180	188.794 ± 19.0097		256.49	0.059	256.487 ± 8.88884																
B.1.8	0	237.53	0.082	0.488 ± 0.0587		229.85	0.023	0.457 ± 0.0039		353.15	0.011	0.465 ± 0.0029																
	∞	186.21	0.105	186.210 ± 20.2977		210.60	0.228	210.600 ± 11.1603		268.99	0.104	268.991 ± 14.8023																
B.1.9	0	231.90	0.023	0.562 ± 0.0955		255.62	0.021	0.483 ± 0.0038		348.16	0.008	0.497 ± 0.0091																
	∞	226.96	0.135	226.9577 ± 12.7062		209.99	0.119	209.990 ± 24.6529		296.00	0.102	295.999 ± 12.4094																
B.1.10	0	205.51	0.017	0.378 ± 0.0147		215.54	0.011	0.378 ± 0.0027		262.13	0.005	0.379 ± 0.0013																
	∞	162.00	0.246	161.996 ± 4.1591		167.12	0.290	167.119 ± 14.8358		218.30	0.198	218.303 ± 6.7673																

Table III.11: Results for Instance Group B with $m=100$ for Model IV and V

here below we conduct a sensitivity analysis to investigate the effect of increasing the size of the historical data on KR in comparison to OSAV. We variate values of γ and GA iterations.

III.4.2.4 Sensitivity Analysis of Group B Instance

We conduct a sensitivity analysis based on different γ , m and GA iteration values.

Figure III.4 presents the comparison between two travel time estimates KR and OSAV. This figure is generated by solving instances with a single γ (i.e. selected as ∞) value for Model III and Model IV. This case is specifically chosen since setting γ to ∞ is the most unfavorable case for the KR method.

In this figure, in addition to the previous analysis based on 100 historical data, we also incorporate the case with 1000 historical data for the KR method. This case is held for two main reasons. The first one is to show how KR method performs when the size of the historical data increases even for the most unfavorable case and the second one is to be able to comment on the comparison between KR and OSAV. Remind that in the previous analysis, we were not able to provide a general conclusion for these methods.

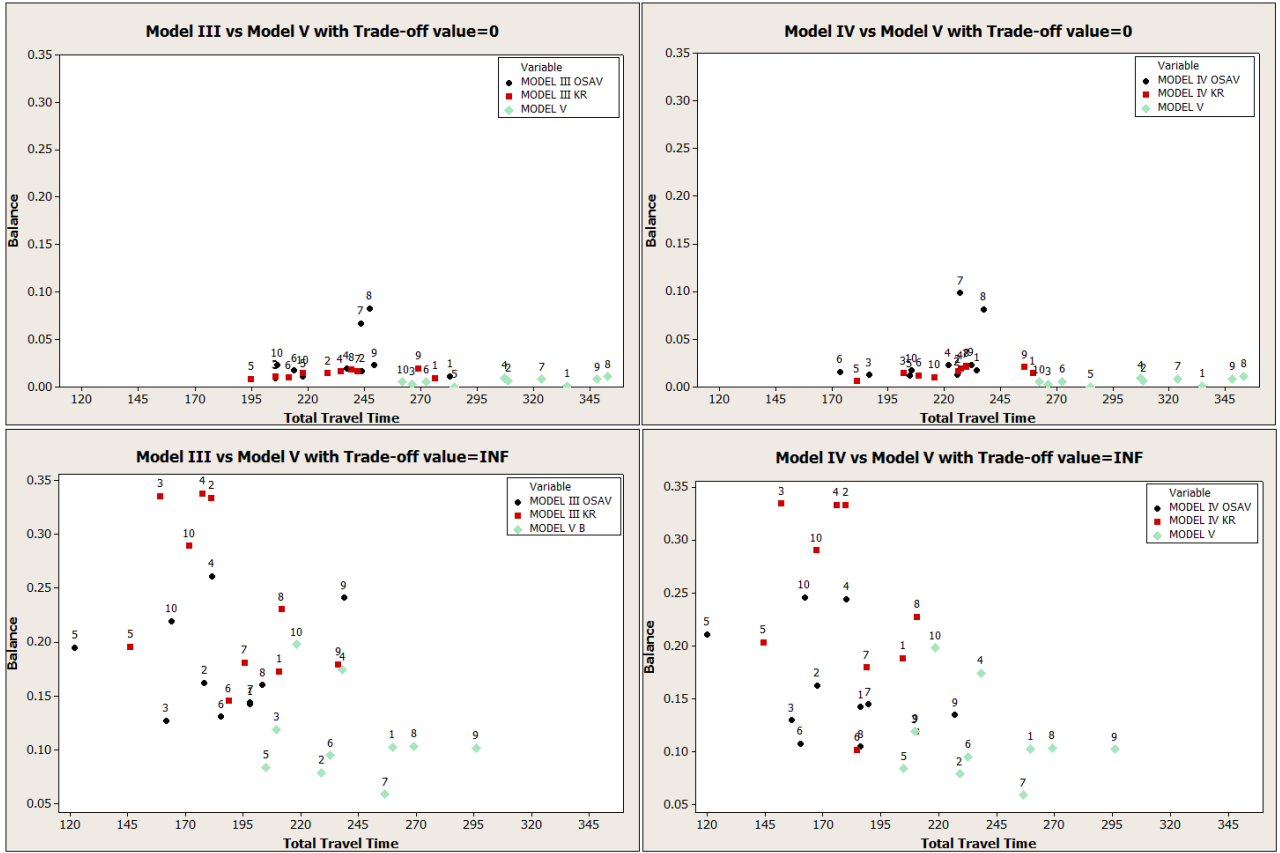


Figure III.3: Results with Instance Group B with $m=100$ for Model III, IV and V

It can be observed from Figure III.4 that increasing the number of historical data results in lower total travel times with the KR technique for all instances (see blue and red points). In particular, with more historical data, KR method performs better than the OSAV technique almost for all instances with both Model III and Model IV (see blue and black points). The only exception where OSAV performs better in terms of total travel times is observed on the experiment of Model IV with the Instance B.1.8. Thus, to be able to observe lower travel times for this instance as well, more historical data is required.

As the next analysis, we analyze the sensitivity of the model to γ by considering several values. Thus, we plot Figure III.5 with the instance B.1.1 and we show the trade-off between the workload balancing and total travel times of operators for the increasing values of the γ (i.e. penalty value) for the first stage of the two-stage process.

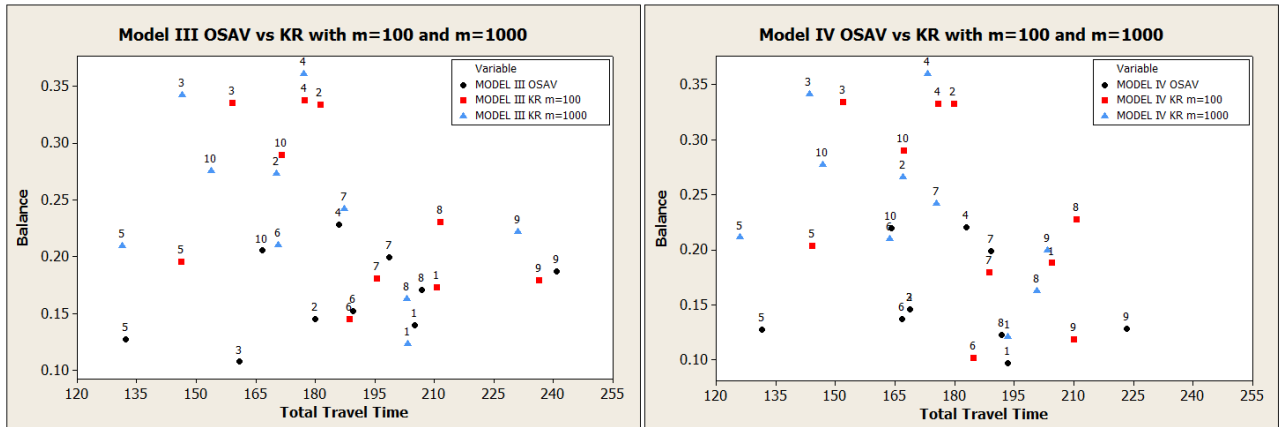


Figure III.4: Sensitivity Analysis with Instance Group B for $m=100$ and $m=1000$

As expected, from Figure III.5 it is evident that when we decrease the γ value, the effect of the total travel time decreases and better workload balancing is ensured for both travel time estimation methods. In particular, since the KR method provides more stable and better outputs than the OSAV technique, this makes KR approach more preferable method for the estimation of travel times.

Last sensitivity analysis is done to analyze the performance of the models when different iteration number is chosen for the GA. Figure III.6 presents the result for Model III, Model IV and Model V where KR method is used for the two-stage models. The analysis is done for instance B.1.1 with 6 different γ where GAs are run for 1000 (as presented in Table III.9) and 5000 iterations. From Figure III.6 we can conclude that the quality of the solutions starts to increase when GAs are lunched with more iterations. Moreover, with both iterations setting for the GAs, the two-stage approach with KR performs good in terms of total travel time of operators for both Model III and Model IV in comparison to the simultaneous approach.

III.5 CONCLUSION

In this part, we analyze the impact of the travel time estimation methods especially the KR technique in a two-stage planning approach. We conduct several experiments with different modeling assumptions such as single or multiple planning periods, existing or relaxed operator capacities restrictions. All assumptions are tested with two-stage models, Model I, Model III and Model IV and compared to the simultaneous approach models, Model V.

As the result of all these experiments and analysis, we see that the two-stage approach with KR method provides similar results to those of the simultaneous approach. Particularly, adjusting the care plan decision, which is initially held in the assignment stage of the two-stage approach, in the routing stage provides lower total travel times (i.e. results with Model IV). Another important observation is based on the size of the historical data. As the size of the historical data increases (i.e. $m = 1000$), the total travel time values start to decrease as well. Hence, we can conclude that the KR technique used in Model IV with a realistic number of historical data seems to be a promising tool for approximately solving the HHC VRP (i.e $m = 1000$ can be considered as a realistic case since an operator usually works 250 days per year).

The experiments in this part have some limitations. For example, operator qualifications has not been considered while analyzing the travel time estimation methods. Thus, one important extension will be to include operator skills to analyze more realistic situations. Another limitation is only focusing on the geographical locations of patients which actually might not be the only criteria for defining operators' visits. In some cases, operators might also need to consider other significant HHC specific features while planning their visits, such as patients' care profiles (i.e., corresponding pathology), special service requests (i.e., requests for clinical tests) and temporal constraints (i.e., requests for visits at specific times). Using these features is also important for capturing real operator behaviors and estimating more accurate travel times. Lastly, an exact solution approach based on the Column Generation method can be implemented to better analyze the performance of the travel time estimators and the two-stage approach.

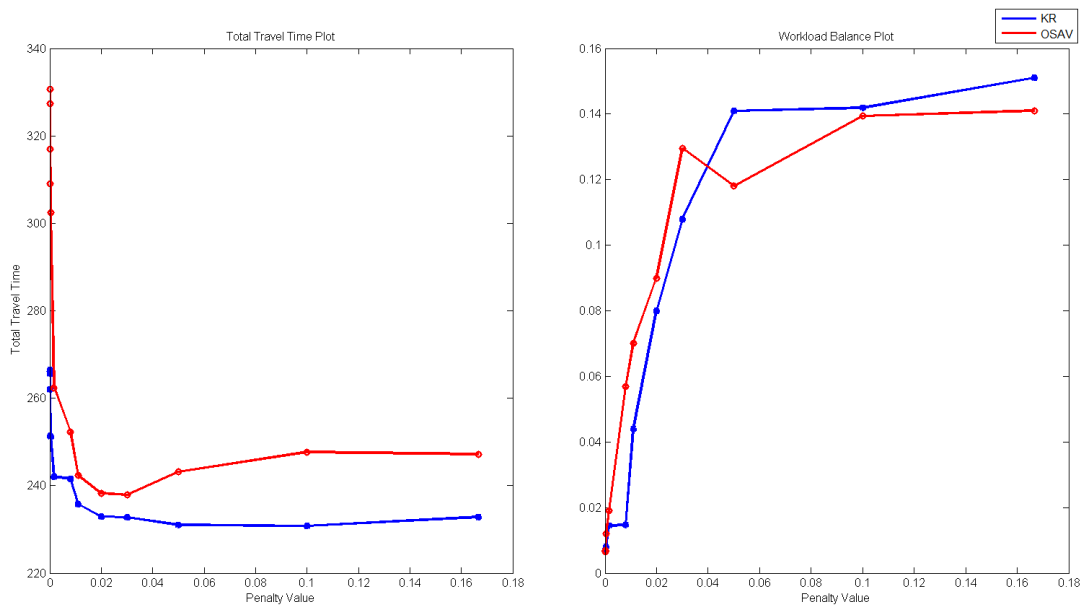


Figure III.5: The Trade-Off Analysis for the Assignment Phase Of the Two-Stage Model with the KR and OSAV methods with $m=100$

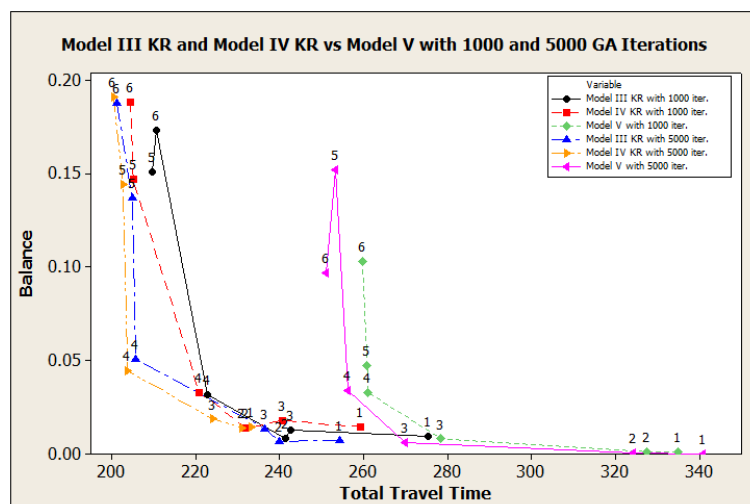


Figure III.6: Sensitivity Analysis on the Iteration Number of the GA with Instance B.1.1 and $m=100$

Part IV

ASSESSING THE IMPACT OF THE SKILL MANAGEMENT

IV.1 INTRODUCTION

In this part, we present the results of the two-stage models considering different patient requirements and different operators skills. The issue of different requirements/skills is relevant from a practical perspective since patients would often require various services that can be grouped as regular or specific/intensive requirements. These services are usually provided by operators having different abilities. For instance, patients with regular requirements would be served by operators having basic skills whereas patients having intensive requirements should be cared by more qualified operators. Hence, the health care provider has to manage skill compatibility issues to ensure that the required skill is deployed for a given type of patient requirement. Note that, generally operators that are able to serve patients with intensive requirements are capable to serve regular patients as well. Thus, the HHC providers can either manage operator skills independently (i.e. without allowing an overskilled operator performing care activities that require a lower level of skill) or they can use more qualified operators for basic services as well (i.e. hierarchical skill management). Hence, the goal of this section is to analyze the impact of the alternative skill management ways (i.e. independent or hierarchical skill management) on the two-stage approach and compare the impact with the ones of the simultaneous planning approaches.

The alternative modeling variants of the two-stage approach presented in Figure II.1 (cf. Chapter II) for the multiple planning periods are tested on several instances. Daily operator capacity restrictions are considered in all experiments.

In Section IV.2, details of the experiments are presented. Then, in Section IV.3, parameter settings for the instances and solution methods are identified. Finally, numerical results and the corresponding analysis are presented in Section IV.4.

IV.2 DESIGN OF EXPERIMENTS

Experiments are done based on a single instance group (Group C) generated from real data. We consider three sets of instances depending on the size of instances, (i.e. one medium-sized, Group C.1, and two large-sized sets, Group C.2 and C.3). These instances are used for both the two-stage and simultaneous approach models with different operator and patient skills, multiple planning periods and operator capacity restriction assumptions.

Several two-stage modeling scenarios are considered (i.e Model II, Model III and Model IV) and compared to the simultaneous models (i.e. Model V). With Group C.1 instances, Model II, Model III and Model IV are used for the two-stage approach with independent and hierarchical skills. Then, as the result of the analysis made on these models, the best model in terms of solution quality is identified (i.e. Model IV) and used for further analysis with Group C.2 and C.3 instances.

The two-stage models are solved by only considering the AV approach as the travel time estimation method for the first stage of the two-stage approach. In particular, two different objective functions are used to solve each model where the first one is the balancing objective (i.e. minimizing the maximum operator utilization factor, MinMax) and the second one is the cost minimization objective (i.e. minimizing total travel time of all operators, MinCost). The formulation of these objectives in the simultaneous approach is straightforward. On the other hand, the formulation of these two objectives for the two-stage approach needs more attention. Thus, when we refer to the case of balancing objective (MinMax), we try to minimize the maximum operator utilization factor in the both objective functions. However, when we consider the travel time minimization objective (MinCost), we use balancing objective in the first stage and travel time minimization in the second stage. Since the assignment problem (i.e. first stage problem) is already solved to balance the workloads of operators, we try to analyze if it is worth to consider by possibly increasing the tour lengths of some operators and balance the workloads in the second stage problem as well (with MinMax objective).

In the following section, we present the selected parameter settings of the considered instances and solution methods in more details.

IV.3 EXPERIMENTAL SETTINGS

In this section, we present the details of the instance and solution method parameter settings respectively.

IV.3.1 Parameters Regarding the Instances

Group C:

- 2 skill levels are assumed for patients.

1. Skill 1 (i.e. patients having regular requirements) and Skill 2 patients (i.e. patients having with intensive requirements)
- 2 skill levels are assumed for operators.
 1. Skill 1 (i.e. ability to care patients with regular requirements) and Skill 2 operators (i.e. ability to care patients with regular and intensive requirements)
 - A total of 20 instances are grouped in three sets and used in Group C experiments, see Table IV.1 for all the details.
 1. The first set of instances consists of 10 medium-size instances (i.e. Group C.1). Those are identified between C.1.1 and C.1.10 that consider 60 patients and 4 operators.
 - (a) Group C.1: 44 to 56 skill 1, 4 to 16 skill 2 patients and 2 skill 1, 2 skill 2 operators.
 2. The second and third set of instances consists in total of 10 large-size instances (i.e. Group C.2 and Group C.3). The second set is identified between C.2.1 and C.2.5 for Group C.2 that consider 100 patients and 7 operators. The third set of the instances are identified between C.3.1 and C.3.5 for Group C.3 that consider 200 patients and 10 operators.
 - (a) Group C.2: 80 to 88 skill 1, 12 to 20 skill 2 patients and 4 skill 1, 3 skill 2 operators.
 - (b) Group C.3: 159 to 174 skill 1, 26 to 41 skill 2 patients and 6 skill 1, 4 skill 2 operators.
 - Daily operator capacities for each instance are as follows:
 1. Group C.1: 400 and 350 minutes respectively for skill 1 operators and 300 minutes for each skill 2 operator.
 2. Group C.2: 400, 350, 300 and 300 minutes respectively for skill 1 operators and 300 minutes for each skill 2 operator.
 3. Group C.3: 500, 500, 400, 400, 350, 350 minutes respectively for skill 1 operators and 300 minutes for each skill 2 operator.
 - All the instances in each set is directly generated from read data, with 60, 100 and 200 patients and distributed across 7 cities as it is in Part III.

- The planning horizon consists of 6 days (i.e. multiple planning periods).
- Each patient requires 1 to 4 visits in total during the planning period of 6 days. The total number of visits required among all patients is provided in column 3 of Table IV.1.
- Patterns for care plan decision are generated with the flow based pattern policy given in the work of Cappanera and Scutella [17] for each instance.
- The standard service time, sv_j , that is required to visit a patient is set to 45 minutes for all patients.
- Distances between patients are considered as integer values (i.e. values are rounded to nearest integer) to lessen the solution complexity.

Instance	Number of Skill1 Patients	Number of Skill1 Patient2	Number of Skill1 Visits	Number of Skill2 Visits	Number of Patterns	Number of Days	Number of Skill1 Operators	Number of Skill2 Operators
C.1.1	51	9	69	28	9	6	2	2
C.1.2	51	9	76	28	8	6	2	2
C.1.3	48	12	67	38	8	6	2	2
C.1.4	50	10	70	28	8	6	2	2
C.1.5	44	16	69	46	7	6	2	2
C.1.6	53	7	72	23	10	6	2	2
C.1.7	52	8	74	21	9	6	2	2
C.1.8	52	8	75	26	8	6	2	2
C.1.9	56	4	80	7	7	6	2	2
C.1.10	49	11	62	36	7	6	2	2
C.2.1	84	16	115	46	8	6	4	3
C.2.2	82	18	122	48	9	6	4	3
C.2.3	86	14	124	39	9	6	4	3
C.2.4	88	12	123	39	10	6	4	3
C.2.5	80	20	111	66	10	6	4	3
C.3.1	168	32	199	104	16	6	6	4
C.3.2	165	35	253	96	16	6	6	4
C.3.3	171	29	246	92	18	6	6	4
C.3.4	174	26	253	71	12	6	6	4
C.3.5	159	41	226	132	16	6	6	4

Table IV.1: Instance parameters for the experiment Group C

IV.3.2 Solution Method Parameters

In this part solutions are obtained with the commercial CPLEX solver. Here we present the details related to these solutions methods as follow:

CPLEX Solver:

- The models are coded in Python 2.7.2 programming language.
- CPLEX 12.3 is used for solving the all the models.

- CPU time limit is set as 12 hours.
- Memory limit is set equal to 1 Gb.
- In the tables that are presented in numerical result section, a \star is used to denote an instance for which the memory limit is exceeded and a \diamond is used to show that the presented results are reported with the upper CPU time limit.

Note that, all these numerical experiments are performed on a computer with Intel Core i.7 2.2 GHz CPU, and 8 GB of RAM.

IV.4 NUMERICAL RESULTS

The proposed two-stage models with hierarchical and independent skills are tested and compared to the simultaneous approach. We start by presenting the performance indicators used to analyze the results. Then, numerical results with Model II, Model III, Model IV and Model V are provided. With the provided results, we try to analyze the the impact of different skill management cases on the two-stage and simultaneous approaches. In particular, by using different modeling scenarios for the two-stage approach, we try to investigate the best model to consider the assignment, care plan and routing decisions via two-stage approach. Hence, we start solving all the indicated models with the medium-sized instances and then extend our analysis with large-sized instances by only considering the best two-stage model among the alternatives. For all the set of the instances models of the simultaneous approach are solved and compared with the ones of the two-stage approach as well.

IV.4.1 Performance Indicators

Four indicators are used to assess the performance of each model analyzed. Total travel time of all operators obtained in the two-stage and simultaneous approaches is shown by T , and the workload balance value between the maximally and minimally utilized operators is shown by B . The models are solved with either balancing (i.e. MinMax) or cost minimization (i.e. MinCost) objectives thus, the corresponding objective function value is denoted as Obj and is either set equal to h or T value where h corresponds to the maximum operator utilization level. Lastly, $NOvSk$ is used for the hierarchial skill case. It is used to denote the total number of times that

operators with skill level 2 are used for lower skilled patients (i.e. with patients belonging to skill level 1) for the models with hierarchical skill consideration.

The T values for the two-stage models are obtained by solving several independent TSP models either with balancing or travel time minimization objectives with the outputs obtained from the assignment stage and summing the results of each TSP model across the all operators. On the other hand, T values for the simultaneous models are directly calculated from the corresponding model as the sum of each operator's route time.

IV.4.2 Results

We first present the results for the medium-sized instances (i.e. Group C.1) to compare Model II, Model III and Model IV of the two-stage approach with the models of the simultaneous approach. Then we also provide results with the best performing two-stage model (according to the analysis based on Group C.1 instances) and corresponding simultaneous model for the large-sized instances (i.e. Group C.2 and Group C.3). All the instances are solved with the independent and hierarchical skill cases by considering both the balancing and cost minimization objectives.

IV.4.2.1 Results with the Instance Group C

We consider three subset of instances for Group C to analyze the models with 2 skill levels, multiple planning periods and operator capacity restrictions.

Results for Group C.1 Instances:

Table IV.2 and Table IV.3 provide results for Model III, Model IV and Model V with the hierarchical operator skills. Regarding the solutions of the independent skill case where an operator can perform only the visits requiring exactly his/her skill, all of the instances turn out to be infeasible. One possible cause of this infeasibility might be the insufficient number of patterns. However, when the skills are managed hierarchically, we are able to obtain feasible solutions for the two-stage and simultaneous models with most of the instances even with the same pattern sets. The only exception is observed for Model II of the two-stage approach. Results of this model also turns out to be in feasible with all of the instances except the Instance C.1.9. The reason for these infeasibilities are also possibly the number of used pattern.

Although we observe infeasibilities in some cases due to limited number of pattern, we con-

tinue our analysis with the hierarchical skill case since most of the models with hierarchical skills are successful to provide feasibility. Here below we analyze solutions in details.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Objective	Instance	Two-Stage Model III				Two-Stage Model IV				Simultaneous Model V			
		T	B	NOvSk	Obj	T	B	NOvSk	Obj	T	B	NOvSk	Obj
MinMax	C.1.1	728	0.053	18	0.656	666	0.065	18	0.651	n.a. [◊]	n.a.	n.a.	n.a.
MinMax	C.1.2	700	0.167	24	0.753	716	0.162	24	0.751	668 [◊]	0.125	22	0.721
MinMax	C.1.3	751	0.249	17	0.798	762	0.239	17	0.798	705 [◊]	0.201	14	0.763
MinMax	C.1.4	659	0.084	18	0.666	679	0.072	18	0.666	613 [◊]	0.061	16	0.649
MinMax	C.1.5	784	0.177	7	0.777	788	0.164	7	0.762	768*	0.159	7	0.765
MinMax	C.1.6	n.a.	n.a.	n.a.	n.a.	626	0.047	20	0.624	800 [◊]	0.193	26	0.735
MinMax	C.1.7	677	0.062	22	0.642	667	0.059	22	0.637	n.a. [◊]	n.a.	n.a.	n.a.
MinMax	C.1.8	688	0.109	23	0.706	641	0.116	23	0.705	802 [◊]	0.274	26	0.833
MinMax	C.1.9	644	0.029	31	0.580	613	0.017	31	0.567	776 [◊]	0.149	36	0.662
MinMax	C.1.10	754	0.388	18	0.793	736	0.382	18	0.789	731*	0.298	18	0.796

Table IV.2: Results for Hierarchical Skills with Instance Group C.1 and Balancing Objective

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Objective	Instance	Two-Stage Model III				Two-Stage Model IV				Simultaneous Model V			
		T	B	NOvSk	Obj	T	B	NOvSk	Obj	T	B	NOvSk	Obj
MinCost	C.1.1	626	0.086	18	626	595	0.086	18	595	736 [◊]	0.180	20	736
MinCost	C.1.2	669	0.172	24	669	644	0.174	24	644	698 [◊]	0.279	27	698
MinCost	C.1.3	685	0.255	17	685	663	0.265	17	633	877 [◊]	0.343	17	877
MinCost	C.1.4	599	0.087	18	599	588	0.089	18	588	724*	0.267	22	724
MinCost	C.1.5	713	0.187	7	713	667*	0.182	7	677	766 [◊]	0.296	8	766
MinCost	C.1.6	n.a.	n.a.	n.a.	n.a.	582*	0.044	20	582	932 [◊]	0.282	26	932
MinCost	C.1.7	614	0.063	22	614	588	0.063	22	588	756 [◊]	0.144	24	756
MinCost	C.1.8	634	0.114	23	634	615	0.119	23	615	963 [◊]	0.289	28	963
MinCost	C.1.9	608	0.038	31	608	584	0.025	31	584	812 [◊]	0.187	35	812
MinCost	C.1.10	670	0.389	18	670	630	0.391	18	630	731 [◊]	0.538	21	731

Table IV.3: Results for Hierarchical Skills with Instance Group C.1 and Cost Minimization Objective

With Table IV.2 and Table IV.3, we present results of the hierarchical skill case with the MinMax and MinCost objectives respectively. As it can be seen from these tables both models of the two-stage approach performs better the simultaneous approach in terms of total travel times. The differences between Model III and Model V and between Model IV and Model V are approximately %7.9 and %11.5 (i.e. average of the 10 instances) respectively with the MinMax objective. In particular these differences increases to %16.8 for the first case and %22.1 for the second case when the MinCost objective is considered. Thus, two-stage models seems to outperformed the simultaneous approach.

It is also interesting to compare 2 two-stage models to be able to analyze the best modeling

scenario for the assignment, care plan and routing decisions. When we consider the total travel times for these models, Model IV seems to be performing better than Model III and the differences between them turns out to be %3.6 on average for the MinMax objective and %4.4 on average for the MinCost objective. Hence, considering care plan decision in both stages of the two-stage approach seems to provide lower total travel times as expected.

Another analysis based on the results of Table IV.2 and Table IV.3 can be made on the choice of the objective function. As it can be observed from the percentage differences provided above, solutions of the MinCost objective provides better solutions than the MinMax objective. Thus, we can conclude that using balancing objectives in each stage of the two-stage approach is not advantageous because it increases unnecessarily the route lengths of some operators when it is also considered in the second stage of the problem. Since the assignment decision is already held with the balancing criterion, obtaining routes with the travel time minimization criteria seems to be a better alternative.

In addition to the analysis based on the total travel times, with Figure IV.1 we provide analysis based on two different objective function values (i.e Maximum utilization level and total travel times) and with Figure IV.2 we also provide analysis for the trade-off between the total travel time and workload balancing levels of operators (i.e. difference between the maximum and minimum utilized operators).

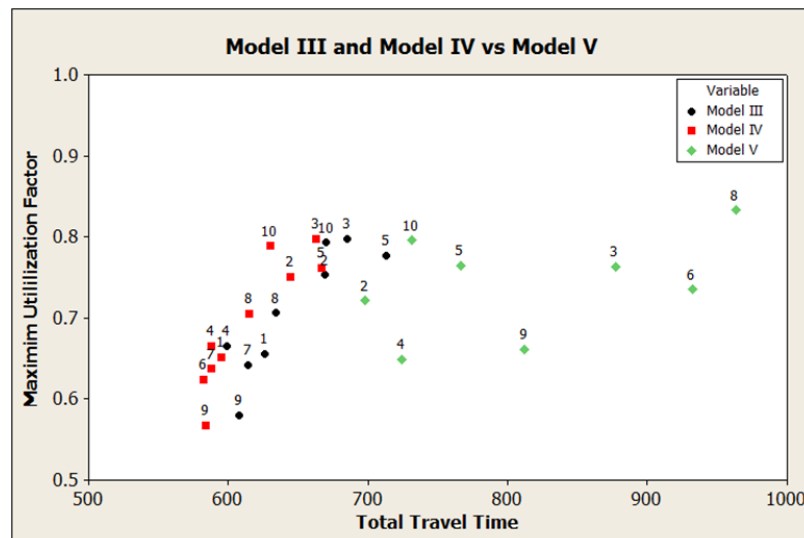


Figure IV.1: Comparison Between the MinMax and MinCost Objectives with Hierarchical Skills

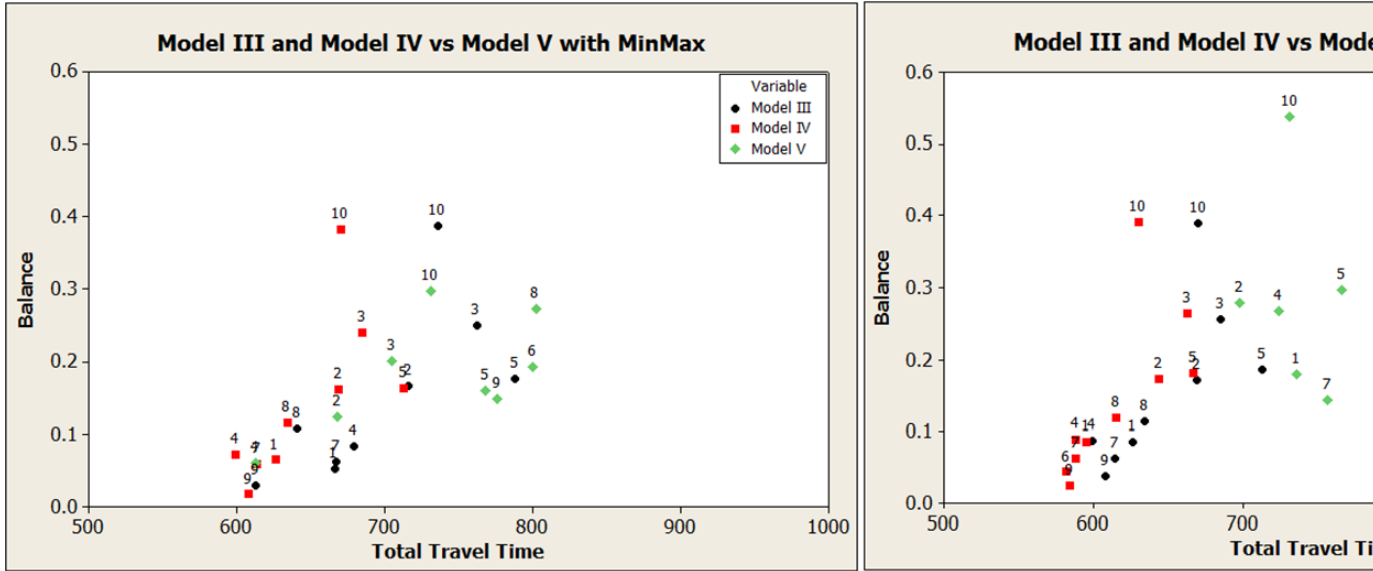


Figure IV.2: Trade-off Analysis for MinMax and MinCost Objectives with Hierarchical Skills with Medium-Sized Instances

Similar to the previous analysis, from Figure IV.1 it can be observed that two-stage models and especially Model IV provides better solutions (i.e. more balanced workloads with lower total travel time) by considering both objective function values (i.e. *Obj* values). In particular, from Figure IV.2 we can see that MinCost objective provides more stable and better solutions than the MinMax objective.

Hence, from all these analysis we can conclude that if we manage different skills in a hierarchical way, we are able to obtain feasible and interesting results. In particular, we also observe that making the assignment, care plan and routing decisions via two stage approach provides better solutions (i.e. in terms of balancing and total travel time) than the ones of the simultaneous approach especially with the MinCost objective. Particularly, the best alternative is observed when the first stage care plan decision is modified in the second stage of the problem (i.e. Model IV).

In the following part, we extend the analysis with large-size instances by considering the best performing two-stage model (i.e. Model IV) and the simultaneous model.

Results for Group C.2 and C.3 Instances:

Here we present results with larger instances. Similar to the previous analysis of the medium-size instances, we can not obtain any feasible solution for the independent skill case. In addition

to this, when the size of the instances is increased, we also can not obtain any feasible solution with the models of the simultaneous approach even with the hierarchial skill case. On the other hand, considered model of the two-stage approach is able to provide feasible solutions with almost all instances.

In Table IV.4 and Table IV.5 results of Model IV with MinMax and MinCost objectives are reported.

1	2	3	4	5	6
Objective	Instance	Two-Stage Model IV			
		T	B	NOvSk	Obj
MinMax	C.2.1	1203	0.088	23	0.672
MinMax	C.2.2	1232	0.179	30	0.761
MinMax	C.2.3	1268	0.108	32	0.691
MinMax	C.2.4	1115	0.114	31	0.676
MinMax	C.2.5	1266	0.378	21	0.837
MinMax	C.3.1	2279	0.133	0	0.757
MinMax	C.3.2	2335	0.023	17	0.818
MinMax	C.3.3	2512	0.040	17	0.812
MinMax	C.3.4*	2118	0.024	34	0.762
MinMax	C.3.5 [◊]	n.a.	n.a.	n.a.	n.a.

Table IV.4: Results for Hierarchical Skills with Instance Group C.2, Group C.3 and Balancing Objective

1	2	3	4	5	6
Objective	Instance	Two-Stage Model IV			
		T	B	NOvSk	Obj
MinCost	C.2.1*	1098	0.094	23	0.672
MinCost	C.2.2	1114	0.191	30	0.759
MinCost	C.2.3	1114	0.112	32	0.688
MinCost	C.2.4	997	0.114	31	0.676
MinCost	C.2.5	1092	0.382	21	0.833
MinCost	C.3.1*	2309	0.129	0	0.761
MinCost	C.3.2*	1906	0.039	17	0.816
MinCost	C.3.3*	2432	0.062	17	0.827
MinCost	C.3.4 [◊]	1841	0.033	34	0.754
MinCost	C.3.5 [◊]	n.a.	n.a.	n.a.	n.a.

Table IV.5: Results for Hierarchical Skills with Instance Group C.2, Group C.3 and Cost Minimization Objective

Similar to the analysis of the instance Group C.1, here we also observe improvements on the

total travel times when the second stage objective is considered as the travel time minimization instead of balancing. We observe approximately %12.3 and %10.5 of reduction on the total travel times by considering the MinCost objective for Group C.2 and Group C.3 instances respectively. With Figure IV.3 we further analyze the trade-off between the total travel time and workload balancing levels of operators separately for two instance groups with both MinMax and MinCost objectives.

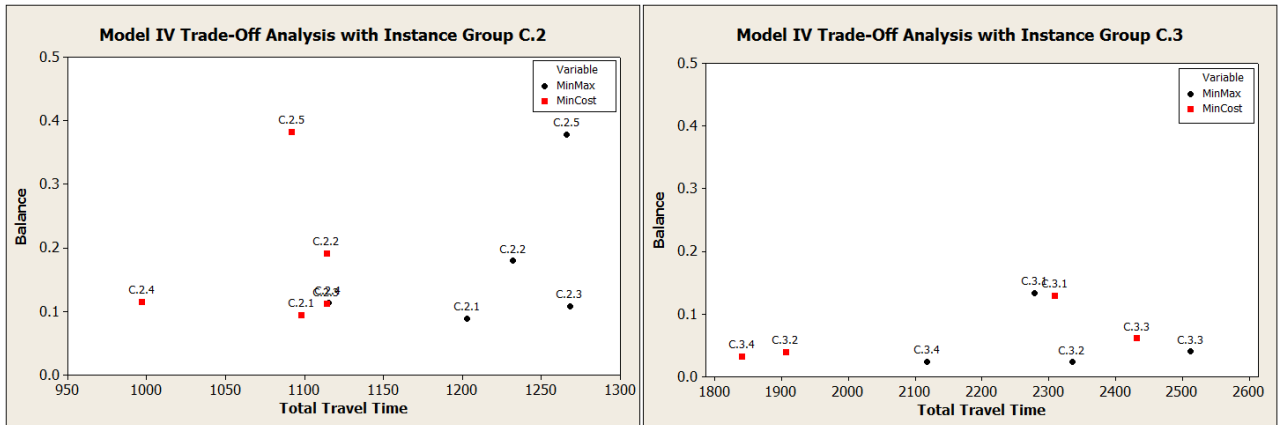


Figure IV.3: Trade-off Analysis for MinMax and MinCost Objectives with Hierarchical Skills with Large-Sized Instances

It can be seen from the figure that even the analysis is considered based on the trade-off between the workload balancing and total travel times, the MinCost objective provides better results with almost all instances. Since in most of the instances the balance values for both objectives are close to each other, we can derive this conclusion by mainly focusing on the total travel time scale.

IV.5 CONCLUSION

In this part, we try to analyze the impact of the different skill management alternatives on the two-stage and simultaneous approaches. We conduct several experiments for the models with the multiple planning period and operator capacity restriction assumptions for the independent and hierarchical skill cases.

As the result of all these experiments and analysis, we observe that when the skills are

managed independently, we could not obtain any feasible solutions. On the other hand, hierarchically considering the skills helps us to obtain feasible solutions with both the two-stage and simultaneous approaches.

We also see that the solutions of the two-stage models outperformed the solutions of the simultaneous models. Especially, when the size of the instances are increased the simultaneous approach does not make possible providing feasible solutions whereas the two-stage approach does. Moreover, for the two-stage models considering the assignment problem with the balancing objective and routing problem with the cost minimization objective provides the best solutions. Additionally, we also compare three modeling scenarios for deciding the assignment, care plan and routing decisions for the two-stage models. This comparison shows us that considering the care plan decision in both stages seems to be the best alternative modeling case.

One limitation regarding to the experiments of this part can be considered as the number of patterns used to solve the models. Due to limited number of patterns, we are not able to observe feasible solutions for the independent skill case. Thus, it will be interesting to repeat all these experiments with the increased number of patterns and compare the corresponding solutions with ones of the hierarchical skill case. Another extension can be done by considering data-driven travel time estimation method (i.e. KR approach) instead of using the AV approach. In the previous part (see Part III), we show that the performance of the KR approach outperforms the AV approach. Hence, all the reported solutions of this part can be improved by considering the KR technique.

Chapter 6

CONCLUSION AND PERSPECTIVES

6.1 CONCLUSION

In this study, we develop new methods and models to support the human resource planning process of the HHC services. We particularly focus on the assignment and routing problems of the human resource planning process. To this end, we first consider an alternative and more convenient method than the widely used simultaneous approach. Hence, we focus on the two-stage approach that is an approximate approach for sequentially solving the assignment and routing problems rather than solving them at the same time. We propose several two-stage models with various realistic assumptions such as skill compatibility between operators and patients, multiple planning periods and operator capacity restrictions. Then, we analyze the performance of these models in comparison to the simultaneous approach models. In particular, since the assignment decision of the two-stage approach is typically made without knowing the visiting sequence, we also need to consider good travel time estimates for being used in the assignment problem. Thus, we propose a Kernel Regression method to estimate travel times using the historical routing information of the operators. Then, we analyze the performance of the proposed estimator and show the improvements achieved in comparison with the AV and OSAV methods via two-stage approach.

As a result of a comprehensive computational study, we observe that the solutions of the two-stage approach are comparable with the simultaneous approach and this process seems to be a promising tool for approximately solving the HHC VRP. It is observed that the use of the KR technique is promising as well in practical HHC organizations where the number of patients and operators can be significant and the assignment and routing problems have different time scales, e.g. operators' assignment lists are gathered weekly, and routes are obtained daily. In particular,

among different skill management alternatives, the hierarchical skill case turns out to be a better alternative than the independent skill case with feasible solution for almost all instances. Lastly, among several modeling alternatives for the two-stage approach (See Figure II.1 in Chapter 2 Part II), it is seen that the Model IV, where the care plan decision is held in both stages, turns out to be the best alternative with lower total travel times and equitably distributed workload balancing levels.

As presented in the previous chapters, there are some limitation of this research and so several research paths to follow. For example, while estimating the travel times of operators with the KR method, we only consider the geographical locations of patients. However, to capture real operator behavior, we also need to consider other features such as patients' care profiles, special service requests and temporal constraints. Hence, as a future work more accurate travel time estimates can be obtained by using these features. In particular, KR estimates can be further improved by using analytical approximations. In the current setting, KR uses a spatial correlation function (generally radial basis functions) to build a predictor in the unobserved domain space. However, the knowledge from analytical methods about some structural properties of the function can help to build the estimator, particularly when the point we want to evaluate is far from unobserved points and spatial correlation is not likely to hold. Another limitation of this research is based on the solution approach. Since our main goal in this research is towards modeling approaches, we do not intent to propose efficient solution algorithms and we use basic solution methods based on GA and CPLEX solver. However, to better analyze the performance of two-stage approach (i.e. with any travel time estimation method) with respect to the simultaneous approach, an efficient exact solution method (i.e. Column Generation) could be useful and provide more transparent analysis. Such approach could also enable to avoid the randomness occur due to use of GA or it could decrease the time of finding feasible solutions for the complex cases (i.e. solving the simultaneous approach with the instance Group C.3) when the CPLEX solver is used. Lastly, in this research we assume equal costs for different skill levels while using the hierarchial skill case. Although using this case is more efficient, it could increase the total resource cost incurred when we assume different costs for different skill levels. Thus, a further analysis with different cost policies should be investigated as a future work.

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