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Infrastructure Sharing Problem in future generation networks: a MILP based analysis

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Abstract

The ever increasing use of mobile-connected devices has caused a dramatic increase in data traffic. By 2018 there will be nearly 1.4 of such devices per capita, while global mobile data traffic is expected to increase 11-fold with respect to 2013. Mobile Network Operators (MNOs) can deal with such demand only by investing in new and costly infrastructure. On the one hand, they have to face high capital and operational costs to run their network. On the other hand, the current earning releases of leading MNOs show a decoupled traffic growth from their revenues. However, if MNOs decide to share their network infrastructure, they will also share its costs, which in turn improves the return on investment, giving them an incentive to upgrade.

While network sharing is a commercial reality for 3G mobile networks, there is only limited research regarding 4G.

In this thesis we propose a Mixed Integer Linear Programming (MILP) model to address the problem of sharing the Radio Access Network (RAN) infrastructure of Heterogeneous Networks (HetNet) among MNOs that want to deploy small cell LTE base stations in dense urban areas. The proposed model binds technical issues related to the radio communication at the access interface (area coverage, transmission rate, user density and quality observed by users) with economic issues (deployment costs and revenues), allowing us to investigate the decision of a MNO on whether to share infrastructure with other MNOs when either the quality offered to users or the return on investment is prioritized.

The quality observed by users is obtained by simulating the deployment of picocell base stations in the considered areas. Our problem formulation allows to solve to optimality real-size instances in negligible time. Numerical results are obtained both for shared investment in a single area and simultaneously over multiple areas. It is concluded that independently of the operator's objective, that is, maximizing user quality or the return on investment, it is almost always in its best interest to share the infrastructure with other operators.

Keywords: Mobile network sharing; RAN sharing; LTE; HetNet; Small cells; Mathematical Programming; Network economics.

Abstract – Italian

L'ampio utilizzo di dispositivi mobili ha causato un significativo aumento del traffico dati nelle reti cellulari. Entro il 2018 ci saranno 1.4 dispositivi mobili pro capite, mentre il traffico dati dovrebbe aumentare di 11 volte rispetto al 2013. Tale domanda può essere sostenuta solo se gli operatori di rete mobile investono in infrastrutture nuove e costose. Da un lato gli operatori devono affrontare elevati costi iniziali per costruire l'infrastruttura e costi operativi per gestire la propria rete. Dall'altro, gli attuali ritorni in guadagno degli operatori leader del settore mostrano che la crescita del traffico e quella dei ricavi sono disaccoppiate. Tuttavia, se gli operatori decidono di condividere le infrastrutture di rete, condividono anche i costi, cosa che a sua volta migliora il ritorno sugli investimenti, dando loro un incentivo per migliorare la rete.

Mentre la rete condivisa è una realtà commerciale per le reti mobili 3G, per quanto riguarda 4G, ci sono solo pochi studi nell'ambito della ricerca scientifica.

In questo lavoro di tesi si propone un modello matematico di Programmazione Lineare Mista Intera (MILP) per affrontare il problema della condivisione dell'infrastruttura di rete di accesso per reti eterogenee (HetNet) tra operatori che intendono attivare picocelle LTE in aree urbane densamente popolate. Il modello proposto mette in relazione questioni tecniche relative alla comunicazione radio a livello di interfaccia di accesso (copertura, velocità di trasmissione, densità degli utenti e qualità percepita dagli utenti) con questioni economiche (costi di implementazione e ricavi), e permette di indagare la decisione di un operatore di condividere o meno l'infrastruttura di rete con altri operatori sia quando l'operatore dà priorità alla qualità offerta agli utenti, che quando il suo interesse principale è il ritorno sugli investimenti.

La qualità percepita dagli utenti è calcolata simulando l'attivazione delle picocelle nelle aree considerate. La nostra formulazione del problema permette di ottenere la soluzione ottima per istanze di dimensioni reali in tempo trascurabile. I risultati numerici sono ottenuti sia considerando un'unica area che simultaneamente più aree in cui gli operatori possono cooperare. Essi mostrano che indipendentemente dall'obbiettivo dell'operatore, ovvero massimizzare la qualità percepita dall'utente oppure il ritorno sugli investimenti, è quasi sempre nel suo interesse condividere la rete con altri operatori.

Parole chiave: Condivisione della rete mobile; Condivisione della rete di accesso; LTE; HetNet; Picocelle; Programmazione matematica; Economia di rete.

Executive summary

During the last two decades mobile telephony has reinvented itself more than once in terms of offered services and the rates at which they are delivered to the end user. Such services extend the basic voice and text messaging of the first generation mobile networks not only to Internet access but also a variety of applications that continue to be developed for user handsets only.

To keep up with the traffic growth and provide higher rates, mobile operators need to upgrade their network technology. Technology upgrade results in greenfield investment, as an already existing infrastructure has to be replaced with a new and expensive one. Consequently, mobile operators have to face high upfront and operational costs for running their network. Moreover, the vicious circle of technology upgrade and traffic increase in mobile networks does not payoff well for mobile operators. Leader operators earning releases show a decoupled growth of traffic demand from their revenues.

One way to improve the return on investment from deploying new technologies such as (Long Term Evolution) LTE or its successor LTE-Advanced, is for mobile operators to jointly invest in the new network infrastructure. By sharing the infrastructure, mobile operators also share its capital and operational costs, which positively affects their return on investment and, as a result, gives them an incentive to upgrade their networks.

Network sharing has been largely addressed in scientific literature. As far as Third Generation (3G) mobile networks are concerned, there is not only a significant contribution from scientific research but also commercial solutions provided by top vendors. On the contrary, for 4G mobile networks, which are currently under deployment phase, scientific literature related to network sharing is quite limited.

Consequently, the focus of this thesis will be on LTE radio access network (RAN) infrastructure sharing, being the latter the most viable level of sharing among operators. Furthermore, we have addressed the deployment of small cell base stations as RAN technology, which are a key enabler of Heterogeneous Networks (HetNet) for providing high capacity to dense urban areas.

We have approached the infrastructure sharing problem by means of mathematical modeling. The decision of an operator on whether to invest by itself or

collaborate with either a subset or all the other coexisting operators over a dense urban area, can be modeled as a Mixed integer linear programming (MILP), which allows us not only to model the operators decision to join a coalition, but also to flexibly build a complex model that combines technical issues related to the radio communication at the LTE access interface (area coverage, transmission rate, user density and quality observed by users) with economic issues (deployment costs and revenues).

We have proposed revenue functions in terms of the quality observed by the user, deployment costs as function of the shared investment (number of activated base stations) and user rate for each coalition as function of LTE nominal rate and the coalition load (number of users). Different objective functions have been investigated. They focus either on the user's main indicator (perceived quality) or the operator's main indicator (return on investment). Furthermore, we have defined objectives that can only be imposed by a regulatory entity in order to provide fairness in the outcome with respect to all operators. Due to the social nature of the proposed objectives, that is, the joint maximization of operators' user rate (return on investment), we decided to capture the operators' selfish behavior by introducing expected lower bounds on the return on investment. If such lower bounds are not satisfied, operators do not invest at all.

At a second step, we altered the proposed model in order to address the problem of simultaneously sharing infrastructure among operators over multiple areas. Five dense areas, that approximately coincide with five hotspots of the city of Milan, have been used to test the multiple area model. Several instances have been tested in order to perform a sensitive analysis of the operators's decision on whether to invest and with whom to collaborate with respect to its expectations from the return on investment and the user willingness to pay for the new service. As far as operators' customer base is concerned, we have accounted for both a uniform and a non-uniform distribution of users in the area(s) among operators.

Testing the model for the several generated instances (different cases and scenarios) allows to analyze its behavior under different objectives with respect to the two main parameters: expectations on the return on investment and how much users are willing to pay for the new service. The results show how it is almost always in the operator's best interest to collaborate with either a subset or all the operators independently of the objective function. However, instances characterized by operators with high expectations from their investment and users willing to pay very little for the new service result unaffordable for any objective. Differences between the two types of objectives (user rate/return on investment) are observed only when users are willing to pay little, otherwise all objectives tend to have the same behavior. Such differences are slightly more emphasized for the non-uniform user distribution even when users are willing to pay more. It is also concluded that the shared investment is reasonably intermediated by a regulatory entity when all operators have the same market share. However, when operators have significantly different market shares, the objectives imposed by the regulatory entity disadvantage the bigger operator with respect to the smaller operators, making the shared investment less attractive for the former.

Thesis layout

This thesis is organized as follows.

The first chapter provides a review of the scientific literature regarding network sharing. It also explains the rationale behind our choice of the level of infrastructure sharing and of small cell technology as an essential part of the thesis framework, concluding with the research objective and methodology.

Chapter 2 gives a general description of the considered problem for the single area investment scenario and its extension for the multiple area scenario. It also states the assumptions that have been made and defines the main components (revenues, costs, user rate etc.) of our problem.

In Chapter 3, we provide the complete mathematical models for both scenarios, starting with their non-linear formulation and then going through the necessary steps to achieve their linerization.

The fourth chapter describes how we set the parameters for the different considered cases and scenarios in order to generate the instances. It also explains, at a reasonable level of detail, the simulations that have been carried out to obtain user rate for each coalition when small cell base stations are deployed over the areas under study. Results of the tested instances are therefore summarized in tabular and graphical fashion for both scenarios and cases.

Conclusions that have been drawn from the results of Chapter 4 and recommendations for future related research conclude this work.

Chapter 1

Introduction

1.1 Background

In the last two decades we have witnessed several cellular network technology migrations, starting with the introduction of the 3rd Generation (3G) after the turn of the century on top of the existing 2G (also notorious as GSM). As from 2010, standards of 4G technology emerge, aiming to provide higher rates compared to 3G under an all IP-based communication. Its first candidate, *i.e.*, Long Term Evolution (LTE) failed to meet the technical requirements imposed by the 3GPP consortium, while LTE-Advanced formally satisfies them. While LTE-Advanced is still in deployment phase, by 2020 it is believed that the 5G will be introduced.

Such rapid evolution of generations (*i.e.*, approximately one generation per decade) has made it possible for the Over The Top (OTT) providers to enrich the set of applications offered to the end users, such as HD TV, cloud computing, video conferencing etc. Consequently, there is a persistent traffic growth that comes hand in hand with the technology migration. It is, however, the job of the underlying network operator and not of the OTT one to deal with both the former and the latter. Unfortunately, network roll-out is highly expensive and becomes affordable only if Mobile Network Operators (MNOs) start charging their customers accordingly.

Therefore, migrating to new generations, that is, investing in additional expensive infrastructure may result in marginal profits or even unprofitable for MNOs due to a separate growth of traffic from their return on investment (ROI) [3].

Consequently, the conventional business models and the complete vertical control that a MNO has on its network (managing physical infrastructure, users and providing voice and data services) has been challenged for quite a while now.

Network sharing among MNOs can be seen as the first step in disintegrating such centralized and inflexible approach. Some primitive forms of network sharing are as old as the mobile networks, *e.g.*, collocation of 2G base station transceivers (BTS), or 2G BTSs with 3G ones in shared premises. As we will see shortly, this is the simplest way to perform sharing and nowadays it is practically taken for granted.

In literature there is not a unique classification of sharing scenarios. Nevertheless, the common ground is that multiple levels of sharing are defined; in other words, there are joint-ventures among MNOs at different depths of the network architecture, with distinct business contracts and, as a result, significantly different implementation complexity and inter-relatedness. The work in [4] provides a top level categorization of sharing scenarios based on two major degrees of freedom: network infrastructure and radio spectrum as the operator's essential resources to deliver mobile services. The resulting possible scenarios are the following:

- Full sharing, *i.e.*, common infrastructure and spectrum
- Spectrum sharing, *i.e.*, inter-operator spectrum sharing, separate infrastructure
- Network sharing, *i.e.*, common infrastructure, individual radio spectrum

For the network sharing scenario, we refer to the classification in [1]. Meddour *et al.* ([1]) have defined three broad categories for network sharing taking into account its complexity which ranges from sharing of towers to complete network sharing: (i) *passive* sharing, (ii) *active* sharing and (iii) *roaming-based* sharing. These levels of sharing are graphically represented in Figure 1.1. A short summary of the suggested classification in [1] is given in the following paragraph.

According to the work in [1], sharing is *passive* when spaces such as premises, sites and masts are shared among operators. *Active* sharing is a more involved type that applies to critical components of the operators network, such as antennas, base stations[1] (BTS/Node B), node controllers[2] (BSC/RNC), backhaul and

[1]BTS and Node B are the acronyms for base station respectively in 2G and 3G networks.
[2]The node controller acronym is BSC for 2G networks but RNC for 3G.

backbone transmission and partly to the core network components[3] (MSC/SGSN). *Roaming-based* sharing allows an operator to use the coverage provided by the infrastructure of another operator in areas that cannot be reached by its own infrastructure. The extreme case of sharing is the one of a Mobile Virtual Network Operator (MVNO), which in [1] has been defined as follows:

> "The MVNOs operate by reselling wholesale minutes that they have purchased from an existing infrastructure owner (a MNO). Most MVNOs have their own core network (including a billing and identification system) and only require access to the mobile operators radio access network."

MVNOs are therefore new actors in the telecom market whose subscribers will always roam in the network of an incumbent MNO.

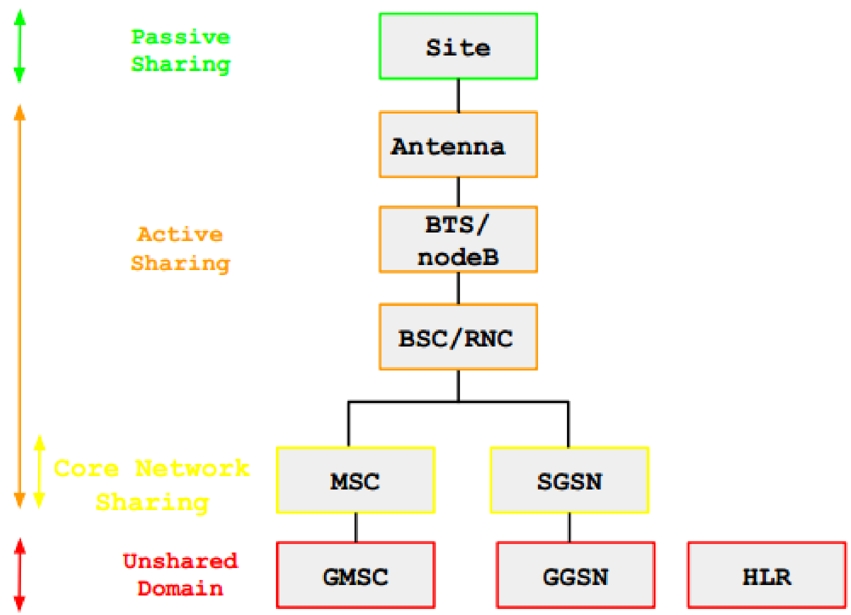

Figure 1.1: Different levels of infrastructure sharing [1].

[3]MSC and SGSN are the switching centers respectively for voice and data.

Network infrastructure sharing became popular with the migration from 2G to 3G. By lowering the initial capital expenditures and operational costs during the network lifetime, it accelerated the roll-out of 3G networks and reduced the new services time to market [5]. As a result, in addition to scientific research, there are well-established commercial solutions for 3G mobile networks, that is, devices compliant with the different levels of sharing are provided in the market by leader manufacturers. The report in [6] presents four levels of sharing: *common shared network*, *geographically split network*, *shared Universal Terrestrial Radio Access Network (UTRAN)* and *site sharing*. The first type implies a complete sharing of the UTRAN (BTS and RNC) and a partial sharing of the core network (MSC, Visitor Location Register (VLR) and SGSN). Clearly, core network components that are responsible for subscriber data, services, billing and interconnections with other networks are kept separate in such a way that operators can still tailor their own services and manage interconnection rates (represented as Unshared Domain in Figure 1.1). The *geographically split* solution allows operators to improve coverage in poorly/uncovered areas by means of national roaming in the network of another operator. Through virtualization of the physically shared Wideband Code Division Multiple Access (WCDMA) radio network, *shared UTRAN* provides the operators with logically separate radio networks. Lastly, under the *site sharing* solution, premises equipment, transmission to the the RNC and antennas can be shared.

Apart from its convenience and appeal, network sharing makes MNOs face certain trade-offs. The study on drivers and barriers for network sharing in [7] is summarized in the two following paragraphs .

According to Berkers *et al.* ([7]) one of the main drivers for infrastructure sharing is cost-effectiveness. The amount of cost reduction depends heavily on the level of sharing. The deeper the level of sharing, the more network elements are shared among the operators; so will the initial deployment costs (Capital Expenditure (CAPEX)) and those of maintenance for the investment lifetime (Operational Expenditure (OPEX)). In particular for spectrum sharing, which is a scarce resource, pooling of resources results in higher utilization and therefore higher spectral efficiency. Spreading the investment risk among collaborating operators is another important driver, given that technology migration or greenfield investment has very large costs and consequently very high risks. Joint-ventures among operators also speed up the process of obtaining a license or acquiring sites. Lastly, sharing has an important social contribution in reducing both the environmental impact of ICT and the digital divide phenomena, respectively by installing less base stations (BSs) to cover an area and by improving coverage in areas that are financially unattractive to MNOs.

On the other hand, infrastructure sharing comes at the cost of high inter-relatedness and as a result, high exposure of one's financial situation to collaborating operators and loss of the ability to differentiate services. In other words, it causes loss of competitive advantage. Moreover, the cost reduction from the shared investment has to be compared with the additional costs introduced by the interoperability issues as it is not trivial to find the appropriate combination of operators.

Figure 1.2 shows in a simplified way the tradeoff between the financial benefits of RAN sharing and loss of competitive advantage as investigated in [2].

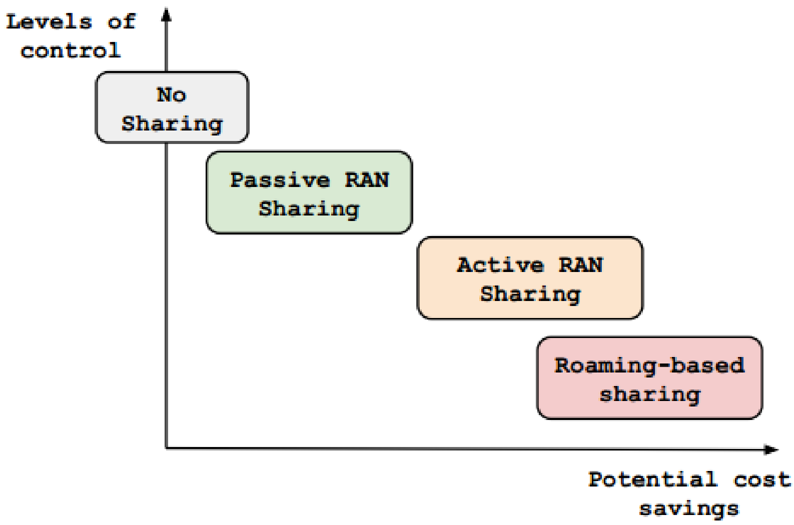

Figure 1.2: Loss of control *vs.* cost reduction for different levels of RAN sharing [2].

1.2 Literature review

In this section we provide a considerate literature review regarding network sharing.

It is important to notice that the concept of shared infrastructure does not apply exclusively to mobile networks. As its main driver is cost reduction, it results attractive also to other networks with expensive infrastructure like Next Generation Access (NGA) networks. The work in [8] investigates the appropriate conditions that make shared access viable for Fiber To The Premise (FTTP).

In this section we restrict the analysis of previous work to mobile networks, which are the focus of this thesis.

In broad terms, there are two major tracks of related work: (i) techno-economic literature on network sharing and (ii) models and algorithms for the management and allocation of shared network resources.

The first track includes mostly qualitative and quantitative study of different sharing scenarios and models for estimating capital and operational expenditures ([1], [2]). Particular attention is dedicated to the identification of drivers and barriers to network sharing ([7]) and potential new organizations of the mobile network value chain for sharing to be viable ([9], [10], [2]).

Meddour *et al.* ([1]) suggest a classification of sharing scenarios and give an estimation of savings for certain uses cases. Their work also assesses technical constraints, suggests guidelines for MNOs involved in the sharing process and emphasizes the need for subsidization and assistance from regulatory entities. Similarly, according to the work in [9], the role of regulatory entities is crucial to avoid the decline of market competition. Moreover, Beckman *et al.* ([9]) use the general product life cycle to give insight on the role that network sharing has in "disaggregating the mobile networks value chain" and, as a result, in facilitating the emergence and development of 3G technology. In [2] capital and operational expenditures for different levels of sharing are modeled, and outsourcing is suggested as the solution to the challenges posed by network sharing. The work in [10] proposes a benchmark-based model that provides high-quality cost estimates for alternative delivery options of the MNO processes such as "regionalization", "centralization" and "outsourcing".

The second track assumes that sharing is indeed feasible but new algorithms for managing and distributing shared resources are needed. At a general level, in [11] it is suggested that radio resource management is handled by the service provider or an inter-connection provider to preserve competition and reduce exposure. The authors in [12] introduce Network without Borders (NwoB) as the pool of virtualized wireless resources with a shared Radio Resource Management

(RRM) unit, for which they provide an efficient coverage model (virtualization as a network sharing enabler has been addressed in [13]). Johansson ([14]) provides an algorithm that fairly allocates the shared radio resources among multiple operators.

An important subset of literature that classifies in the latter track comes from the recent but growing use of game theory in resource allocation problems ([15], [5], [16]). Malanchini *et al.* ([15]) resort to non-cooperative game theory to model both the network selection by the user when multiple heterogeneous wireless access networks are available and the resource allocation among competing operators. In [5] cooperative game theory is used to address the resource allocation problem in a shared network as a two step problem: resource sharing among operators; and resource bargaining among users and MVNOs of each operator. The work in [16] considers not only sharing among MNOs but also among operators of different wireless access technologies. Khan *et al.* ([16]) formulate the allocation of bandwidth within the operator's network and distribution of excess bandwidth among operators as cooperative bargaining games.

In most cases, the above references refer to 3G networks for which sharing is currently a commercial reality. As far as 4G technology is concerned, given that it is still under deployment phase, there are significantly less works that address the network sharing problem. The work in [17] and more extensively the one in [18] use a non-cooperative game to model the strategic decision of a MNO regarding sharing of its LTE infrastructure in a non-monopolistic telecom market. Another example of 4G infrastructure sharing is given in [19] which considers sharing LTE access network femtocells with other access technologies such as Wi-Fi.

1.3 Research framework

1.3.1 Sharing level and RAN technology

As it can be inferred from the previous section, infrastructure sharing in cellular networks is not exactly a recent concept by itself if we refer to passive sharing done with the purpose of migrating from 2G to 3G macrocell technology. However, it is new to the small cell technology, a key enabler for next-generation cellular networks including 5G.

Small cells coexist with the conventional macrocells. They are the latest trend for improving user throughput, network capacity and consequently operator's revenues. Different terms have been used in literature for the new low cost, low power and low range BSs, such as femtocells, picocells, metrocells and microcells depending on their range and therefore on their application scenarios [20].

In this work we focus on picocells as representative of small cells technology for the following reasons.

Picocells are mainly deployed in areas of dense phone/mobile-connected device usage: stadiums, train stations, offices, city centers etc. A picocell BS is significantly smaller compared to the traditional BS; its coverage radius is limited to few hundreds of meters compared to few kilometers of macrocell one. From an economic perspective, there is still some common ground between the two, since the costs for acquiring and maintaining a picocell BS reside on the operator. However, the work in [21] shows how small cells are financially more beneficial for the operators.

On the other hand, Home eNBs[4] (HeNB) or alternatively femtocells[5] have significant techno-economic differences with picocells. As it can easily be deduced from their name, HeNBs are used to improve indoor coverage and serve a limited number of users. Indoor users are usually disfavored by the additional attenuation that buildings' walls introduce. Therefore, from a technical point of view HeNBs have a different emitted power and propagation channel model. As far as HeNBs use cases are concerned, there are two main scenarios: Open Subscriber group (OSG) and Closed Subscriber Group (CSG). For the former any user can access the service provided by the femtocell whereas for the latter, there is only a restricted group of users who are given access. However, hybrid solutions have been proposed in [22]. Moreover, since the costs of HeNBs for CSG[6] rely on the user side [23], infrastructure sharing at RAN level cannot be justified. Nevertheless, there can be other types of sharing applicable to femtocell technology, *e.g.*, at back-hauling level instead of RAN level. The latter is quite critical at the moment and it would result in a sharing scenario that involves non-cellular wireless technologies, such as Wi-Fi.

With reference to [4], sharing of LTE microcell antennas allows the operators to provide their services in hotspot areas such as stadiums, airports, stations etc.

[4]eNB/eNodeB are the 4G acronyms for base station.

[5]HeNB is the term used in the LTE specifications for femtocells.

[6]For OSG, costs will be on the operator.

Even though antenna sharing is classified as active sharing, it is also considered to be an extension of site sharing and therefore has minimal improvement on the reduction of costs with respect to completely sharing the BS [1]. Therefore, we consider sharing picocell BSs which in LTE terminology are known as eNodeB, and they integrate the functionalities of both a conventional BS and a radio controller with respect to previous generations. In addition, infrastructure sharing can be naturally combined with spectrum sharing, once the new releases of LTE can support such thing [24].

In this thesis we address RAN level of sharing, which means that the operators will have to perform their own back-hauling and maintain separate core networks. Referring to Section 1.1, all levels of sharing are theoretically possible. However, the deeper the level of sharing, the more complex and less attractive it becomes for operators to cooperate. Therefore the choice made in this work on the level of sharing captures a more practical scenario for MNOs. In other words, it is the level with the most balanced trade-off between cost reduction and loss of control of the operator's network (Figure 1.2).

Thus, only picocell BSs deployment and infrastructure sharing at RAN level will be the scope of this thesis.

1.3.2 Objective and research methodology

Keeping in mind the choice on technology and level of sharing stated in the previous subsection, we now introduce the approach used in this work and its objective.

In essence, the problem we are addressing is to determine whether it is affordable for operators to upgrade their technology (invest in picocell LTE BSs) and if so, whether it is more profitable to invest by themselves or collaborate with others. We formulate the problem as a Mixed Integer Linear Programming (MILP) model, which was coded in AMPL [25] and solved by the commercial MILP solver CPLEX [26].

As described in the previous section, Offergelt ([18]) has modeled the decision of a MNO regarding sharing of its LTE RAN by means of a non-coperative game. Our research has a more social approach, in the sense that the problem we consider has only one objective function which is a function of each operator's payoff. In other words, there is only one decision-maker instead of many. We account for the fact that each operator should be a decision-maker by introducing constraints that reflect the operators' expectations from their payoffs. Moreover, by introducing appropriate objective functions, we investigate the shared investment in the presence of a regulatory entity. The latter intermediates the sharing process in order to provide fairness among the involved MNOs.

Unlike previous works, we do not refer to the breakdown of different infrastructure cost components to calculate payoffs for each coalition. Instead, we use a simulation to approximately obtain LTE user rate for each coalition allowing us to express the technical and economic terms as function of these rates. In other words, our approach aims at modeling at a reasonable level of detail the use of radio resources for communication between users and network, rather than being a top level cost analysis.

To the best of our knowledge, this is the first work that models the relation between technical issues related to the radio communication at the access interface (area coverage, transmission rate, user density and quality observed by users) with economic issues (deployment costs and revenues) in mobile network infrastructure sharing. Moreover, we focus on the new and critical Heterogeneous Networks (HetNet) scenario where a large number of small cells must be installed by operators in dense urban areas where both costs and quality offered to users play an important role in the decision if sharing network infrastructure with other operators.

Since at the moment small cell technology does not allow to easily enlarge the bandwidth used by each base station, we consider a fixed bandwidth per base station (10 MHz) both in sharing and non-sharing scenarios. This means that spectrum sharing is basically not explicitly included in the model. However, in the last part of the thesis we discuss how it is possible to extend the model to consider possible spectrum sharing policies.

This research aims to determine how the MNO's decision on whether to share its network infrastructure with other MNOs is affected by parameters that reflect the operators and users standpoint in the investment both when the former's objective is to maximize their return on investment and when the quality offered to their users is to be maximized. The same is investigated also in the presence of a regulatory entity.

Chapter 2

Problem description

In Chapter 1 we provide the rationale behind the choice of small cell technology and RAN level of sharing. These choices are the first two assumptions of our definition of the RAN Infrastructure Sharing Problem (ISP). A general description of its key components (technical and economic) and the remaining assumptions are given in the following two sections.

Section 2.1 introduces the first application scenario of the problem, that is, the Single Area Infrastructure Sharing Problem referred to as SISP throughout this work. Section 2.2 describes how we extend the SISP definition for a second application scenario that we denote by Multiple Area Infrastructure Sharing Problem (MISP).

2.1 Single Area scenario

The SISP addresses the problem of sharing small cell LTE infrastructure among MNOs that plan to simultaneously invest in such technology in a dense urban area. Investment over a dense urban area instead of a rural one has been taken into account in order to be in line with the current picocell deployment scenarios introduced in Section 1.3 of Chapter 1.

Beside the assumptions regarding the technology (small cell LTE) and level of sharing (RAN), we also assume that each MNO comes with an already deployed pre-LTE[1] macrocell infrastructure over the considered area. As a result, this

[1]We do not specify the exact technology previously deployed by the operator. It is simply assumed to be pre-LTE, *e.g.*, 3G, HSPA+ etc.

problem does not classify into a coverage problem, as basic coverage and pre-4G data services are already provided by the MNO. However, there is a financial incentive to invest in additional infrastructure over areas of dense mobile-connected device usage as a large number of users are willing to pay for an improved service. On the contrary, rural areas are usually over-capacitated as long as coverage is guaranteed. Consequently, they are not attractive for picocell deployment.

It is important to notice that since the considered MNOs are not new players in the market, each of them has its own customer base inherited from the already deployed pre-LTE technology. The users share of each operator is represented as a fraction of the overall number of users that already exploit the pre-LTE network services of the considered area. We furthermore assume that each user will keep his/her old operator but will potentially subscribe to a different data plan, given that installing picocell BSs will lead to higher throughput[2]. Therefore, users will not have any subscription costs, but they will have to pay for an improved service.

Given a set of coexisting MNOs, that are planning to deploy picocell BSs over the considered area, we want to examine if and when operators decide to invest by themselves or join a coalition under different objective functions and configuration of parameters.

The SISP considers all possible coalitions that can be created for the given set of MNOs. If an operator invests by himself, we refer to that coalition as a singleton, whereas to the one made up by all operators as the big coalition. When affordable, each operator will either invest by itself or join a coalition that is made up by a subset or all the other operators. In addition, we also account for the particular case when operators do not invest because it is not affordable or, in other words, it is impossible to meet their financial targets. As it will be observed latter on, such financial targets allow us to model the realistic selfish behavior of the operators. The problem therefore is to determine which coalitions are created and how many BSs are activated per coalition. We assume that a maximum number of BSs can be installed over the area. We consider two cases. In the first, the maximum number limits the number of BSs installed by all the selected coalitions. In the second, we divide the available set of BSs (*i.e.*, the maximum allowed) among the selected coalitions according to their number of users in order to be fair to all operators.

[2]In this work throughout is used interchangeably with user rate.

We now introduce the techno-economic components and related parameters for the SISP, which allow us to define the objective functions in the last subsection. The key components are the following:

- User rate per operator
- Revenues per operator
- Costs per operator
- Operator's expectations on the return on investment

2.1.1 User rate

User rate per operator is the average LTE rate perceived by any user of an operator. Therefore it is the same for all the users of an operator. This rate is calculated by scaling down the LTE nominal rate[3] with a load factor.

The LTE nominal rate corresponds to the maximum rate that a user can experience for a given level of Signal to Noise Ratio (SNR), that is, when the entire set of LTE resource blocks[4], obtained by the bandwidth that the operator has purchased, are allocated to that user. For a fixed bandwidth, a higher value of SNR translates into higher LTE spectral efficiency[5]. As a result, a user with a better SNR, perceives a higher rate. The LTE nominal rate is also a function of the number of BSs activated in the area, as this number affects the value of SNR. As the number of BSs increases, on the average, the user is closer to his/her serving BS but also closer to the non-serving ones that cause interference. A small increase of the number of BSs contributes positively on the SNR. However, as the BSs deployment becomes dense, a further increase on the number of BSs has a negative impact in the SNR. As a result, the user rate saturates for a certain number of BSs in the area. Such behavior is investigated by means of simulation (explained in detail in Chapter 4). The SISP is therefore explored for a limited number of BSs for which rate saturation is reached. This limit can also be justified as the maximum number of BSs that a regulatory entity allows to be installed over the given area.

[3]The LTE nominal rate in this work is calculated for 10 MHz of bandwidth.

[4]A resource block is the atomic resource unit in LTE.

[5]LTE rate can be increased either by purchasing larger bandwidth, improving spectral efficiency or a combination of both.

It is important to notice that when an operator is part of a coalition, since any of its users can be assigned to any of the BSs activated for that coalition, both nominal and user rate for any operator in the coalition will result the same. This is why we define the coalition rate as the average LTE rate perceived by any user belonging to the members of the coalition. In other words, operators that collaborate offer the same average rate to their users.

We propose a load factor in order to derive the average coalition rate (and as a consequence the user rate per operator) from the nominal one. The load factor is a function of the number of activated BSs for the coalition, of the number of users of the coalition and of a parameter introduced to capture, on the average, the percentage of time during which the user experiences nominal rate (he/she obtains all LTE resource blocks).

As the number of BSs per coalition increases, on the average, there are less users per BS, and as a result, more capacity (user rate) for each of them. Therefore, the load factor is lower for a larger number of BSs, whereas the coalition rate is higher. However, it saturates for a certain threshold on the number of BSs due to the fact that a further increase in the density of BSs per area starts degrading the SNR instead of improving it. The limit on the number of BSs allows us to go up to the saturation point. Given the number of activated BSs per coalition, the coalition rate is smaller for a larger number of users since more users share the given capacity of the single BS.

The activity factor captures, on the average, the percentage of time during which the user obtains all LTE resource blocks. The higher the activity factor, the smaller the coalition rate as more users are requesting more resources simultaneously.

Considering the elements that compose the load factor, it can be derived that the load factor depends also on the coalition. In addition, in this work, the nominal rate depends also on the coalition, as we use the same load factor in the simulation from which we obtain the nominal rate instead of using a generic value. Such load factor allows to correctly reduce the interference generated by the non-serving BSs when calculating the SNR[6]. Under a generic load factor, *i.e.*, a typical constant value used in simulations, the nominal rate would be the same for any coalition. This is firstly due to the fact that we do not consider

[6]This is also explained in detail in Chapter 4.

spectrum sharing. As a result, each BS operates with the same fixed bandwidth, independently of the coalition. Secondly, the nominal rate, being the maximum LTE rate for a given SNR, is only a function of the number of installed BSs and not of any other coalition-dependent parameter (*e.g.*, number of users).

2.1.2 Revenues

It is sensible to think that the higher the rate provided by the operator, the larger its gain from the investment. Thus, revenues per operator, that is, the operator's earnings from its investment, are defined as a linear function of the user rate per operator. Moreover, they will also be proportional to the number of users per operator. However, to fully define revenues we need to introduce a parameter that captures the user's willingness to pay for an improved service. In other words, this parameter shows how much a user is willing to pay every month for an additional 1 Mbps of rate, or equivalently, the monthly price of 1 Mbps of service. In addition, since we are dealing with greenfield investment, the investment lifetime has to be taken into account when calculating not only revenues but also costs and expected return on investment. Thus, revenues per operator for the investment lifetime are calculated as the product of all the elements above.

2.1.3 Costs

We define costs of an operator as a linear function of the number of installed BSs in the coalition it belongs to. Each operator is accounted for a fraction of the BSs costs of the coalition it decides to join. In other words, costs for deploying and maintaining the infrastructure of a coalition are divided among its member operators according to their number of users. In case of a singleton the operator will be accounted for all the costs of the installed infrastructure.

There are both capital and operational expenditure terms that contribute in the overall costs of the infrastructure, that for simplicity we refer to as BSs costs. Referring to the costs breakdown in [1], the CAPEX component accounts mainly for site and BS acquisition, whereas OPEX include hardware and software maintenance, land renting and electricity. Clearly there can be other terms, but we have to address only those that concern RAN level of sharing. The complex cost of a single BS is calculated using as reference pricing model the one in [23].

Equations (2.1), show how the complex cost of a single BS (g) is derived. g_{capex} is the fixed CAPEX component, whereas g_{opex}^{year} is the annual OPEX component expressed as a fixed percentage (ξ) of the initial CAPEX component. The price of a single BS (g) for the investment lifetime D is the sum of the fixed initial CAPEX with the accumulated OPEX from the investment lifetime.

$$\begin{aligned} g_{opex}^{year} &= \xi g_{capex} \\ g_{opex} &= D g_{opex}^{year} \\ g &= g_{capex} + g_{opex} \end{aligned} \tag{2.1}$$

2.1.4 Expected return on investment

The return on investment is by definition the difference between the operators revenues and costs. For the considered investment lifetime, the operator's financial target on the investment is modeled as expectations on a minimum return on investment. Such expectations allow us to capture the role of each operator as decision-maker in the joint investment, despite the social nature of our formulation (*i.e*, having a single objective which is a function of each operator's payoff). We therefore define another key parameter, that is, the monthly return from a single user, to account for the contribution that each user has in the expected minimum return on investment. This parameter is kept the same for all the operators, but the overall expected return on investment depends on the operator since their customer base can vary. The consistent lower bound on the return on investment has to be normalized by the operator's clients share and the investment lifetime, similarly to the approach used for calculating the operator's revenues.

2.1.5 Objective functions

In principle, what we are trying to achieve is figure out the answer to one question: "When do MNO collaborate?"

The answer to this will be explored under two families of objective functions: *user-oriented* and *operator-oriented* while varying the techno-economic parameters that capture the user and the operator standpoint in the investment. We introduce two such families in order to explore the outcome either when the user's or operator's best interest has been prioritized. In other words, we investigate the operator's decision both when its objective is the maximize the return on investment and when it aims at maximizing the quality offered to its users. Nevertheless, as pointed out in Chapter 1, a non game theoretic approach has been used in this work. Therefore, both families of objective functions will jointly consider the operators' payoffs (user rate/return on investment).

The first type of *user-oriented* objective function maximizes the sum of user rate over all the operators, whereas the *operator-oriented* one maximizes the sum over all the operators of their return on investment for the considered investment lifetime. In order to investigate the case when a regulatory entity intermediates the shared investment, for each family, we introduce an objective that maximizes the minimum user rate (return on investment) over all operators. In other words, the role of the regulatory entity is to guarantee fairness among the involved operators by accounting for the operator with worst served users (in the case of *user-oriented* objective) or for the operator with the smallest return on investment in the market (in the case of *operator-oriented* objective). For both families, we introduce a third type of objective that combines the previous two, that is, it maximizes the sum of the total and the minimum user rate (return on investment). Such objective aims at avoiding solutions that disadvantage certain operators with respect to others, that is, it provides more balanced solutions at the cost of being less optimal (with respect to the total user rate/return on investment) compared to corresponding solutions provided by the objective that simply maximizes the sum of operators' payoffs.

In any case, under such objective functions, the sum of all payoffs is maximized. The selfish behavior of each operator is captured by introducing the constraint on the minimum expected return on investment.

2.2 Multiple Area scenario

Since it is quite probable that operators want to invest in more than one area at a time, we extend the SISP for a multiple area investment scenario (MSIP). We consider a set of dense urban areas, which are not necessarily adjacent, but could be part of the same large city or distributed over different cities.

Each area is characterized by its size and number of users. We keep the distribution of users among operators the same for all the areas. Furthermore, we assume that operators select the same coalition for all the areas, considering that it becomes difficult for an operator to simultaneously manage different collaboration contracts for different areas. On the other hand, since each area is characterized by its own size and number of users, the number of activated BSs for each coalition will be different over different areas. The problem therefore consists in determining the coalition selected by each operator and the number of activated BSs for each coalition in each area. Once the number of BSs per coalition becomes a function of the area, so will the nominal, coalition and user rate according to the definitions given in the previous section.

These changes affect the definitions of revenues and costs. As revenues depend on the operator's user rate and costs on the number of activated BSs by the coalition joined by the operator, they will both be different for each area. Clearly, the two families of objective functions defined above need to be slightly modified as well.

The first type of both families of objective functions that focuses on maximizing the total user rate (*user-oriented*) and the total return on investment (*operator-oriented*) has to be calculated over all the areas and operators. The objective function that maximizes the minimum user rate is calculated over all areas and operators, since we want to account for the worst served user. This is reasonable given that such objective function is imposed by a regulatory entity. However, the *operator-oriented* objective function that maximizes the minimum return on investment over all operators is calculated over the global return per operator and not over the return from each area. This is due to the fact that operators are more interested in the global earnings from their investment, rather than how well they are doing financially in individual areas.

On the same note, the expected return on investment applies to the overall return on investment instead of separately to each area.

Chapter 3

Mathematical model

This chapter is dedicated to the proposed mathematical models for both the SISP and MISP. Although the formulation results in a non-linear model, we derive a MILP formulation by applying suitable linearization and approximation. The number of variables and constraints of our formulation grows exponentially with the size of the problem, that is, with the number of coexisting MNOs in a certain area. This is due to the fact that, for a given set of operators, we consider all possible coalitions that can be created.

Section 3.1 describes in detail the mathematical model of the SISP and the necessary steps to obtain its linearization. In Section 3.2, we introduce the necessary modifications to the SISP model in order to obtain the MISP model.

3.1 SISP mathematical model

3.1.1 Sets and parameters notation

We denote by $\mathcal{O}$ the set of n coexisting MNOs over a given area where they want to invest in LTE picocell technology. $\mathcal{S}$ is the set of all possible coalitions that can be created. The number of such coalitions, that is, the cardinality of $\mathcal{S}$ denoted by m, is equal to $2^{|\mathcal{S}|} - 1$. The members of each coalition s from set $\mathcal{S}$ are the elements of the corresponding set $\mathcal{C}_s$. Parameter σ_i gives the share of users of operator $i \in \mathcal{O}$ out of the total N users that populate the area. The SISP consists in determining the subset of coalitions selected by the MNOs and the number of BSs activated for each. We denote by U_{max} the maximum number of BSs that can be activated in total over the given area.

The following parameters reflect the techno-economic aspects for both scenarios.

MNOs consider investment in additional infrastructure (*i.e.*, picocell LTE with respect to the existing macrocell pre-LTE) only if their users are willing to pay for an improved service. User's willingness to pay for 1 Mbps of LTE rate on a monthly basis, or, in other words, the monthly price of 1 Mbps, is denoted by δ. Moreover, when investing, MNOs set certain financial targets, such as a minimum return on investment for the investment lifetime. The investment lifetime is denoted by D. The minimum return on investment is modeled by introducing a parameter γ which represents the minimum monthly contribution that each user should provide. The normalized BS cost over the investment lifetime D was calculated in Chapter 2, by means of Equations (2.1). In our model it is used as a single parameter denoted by g. The last parameter that we introduce is the user activity factor (η), that is, the average percentage of time during which a user obtains maximum LTE rate.

3.1.2 Definition of variables

The decision of an operator to join coalition s or not is captured by the binary variables x_{is}, one for each operator and coalition. When operator i does not belong to coalition s ($i \notin \mathcal{C}_s$), the corresponding variable x_{is} is forced to zero. Otherwise, it is equal to zero in case operator i selects any other coalition but s or when all operators in s do not invest at all (when not affordable). Binary variables y_s are introduced to keep track of the selected coalitions. If not all the operators in s select coalition s or there is no investment, y_s equals zero. Otherwise, coalition s has been selected by all its members (operators that belong to the set $\mathcal{C}_s$). If coalition[1] s is created, a certain number of BSs will be activated for that coalition. Such number can only be a positive integer, reflected by the non-negative integer variables u_s, one for each coalition. If coalition s has not been selected or there is no investment, the corresponding variable u_s equals zero.

Three types of LTE user rate were defined in Chapter 2: nominal user rate, coalition user rate and user rate per operator. Since each of them is a function of the number of activated BSs per coalition, we need to define one family of

[1]If not explicitly stated a coalition can be also a singleton.

non-negative continuous variables for each: ρ_s^{nom}, ρ_s and q_i, respectively.

Referring to Section 2.1 of Chapter 2, revenues per operator are a function of its user rate, whereas costs are a function of the number of activated BSs in the coalition joined by the operator. As a result, we introduce two families of non-negative continuous variables (one per operator): r_i and c_i. Again if no investment is made by operator i, both r_i and c_i equal zero.

Additional parameters and variables are needed to perform the linearization of the initially non-linear model. They will be introduced in Subsection 3.1.4 together with the linear model for the SISP.

3.1.3 SISP non-linear model

The set of constraints and variable domains are the following:

$$\sum_{s \in \mathcal{S}: i \in \mathcal{C}_s} x_{is} \leq 1, \quad \forall i \in \mathcal{O} \tag{3.1}$$

$$x_{is} = 0, \quad \forall s \in \mathcal{S}, \forall i \in \mathcal{O} : i \notin \mathcal{C}_s \tag{3.2}$$

$$x_{is} = y_s, \quad \forall s \in \mathcal{S}, \forall i \in \mathcal{O} : i \in \mathcal{C}_s \tag{3.3}$$

$$u_s \leq U_{max} y_s, \quad \forall s \in \mathcal{S} \tag{3.4}$$

$$\sum_{s \in \mathcal{S}} u_s \leq U_{max} \tag{3.5a}$$

$$u_s \leq \sum_{i \in \mathcal{C}_s} \sigma_i U_{max}, \quad \forall s \in \mathcal{S} \tag{3.5b}$$

$$\rho_s = \rho_s^{nom} (1 - \eta)^{\frac{\sum_{i \in \mathcal{C}_s} \sigma_i N}{u_s}}, \quad \forall s \in \mathcal{S} \tag{3.6}$$

$$q_i = \sum_{s \in \mathcal{S}} \rho_s x_{is}, \quad \forall i \in \mathcal{O} \tag{3.7}$$

$$r_i = \delta D \sigma_i N q_i, \qquad \forall i \in \mathcal{O} \tag{3.8}$$

$$c_i = \sum_{s \in \mathcal{S}} g \frac{\sigma_i}{\sum_{j \in \mathcal{C}_s} \sigma_j} u_s x_{is}, \qquad \forall i \in \mathcal{O} \tag{3.9}$$

$$r_i - c_i \geq \gamma D \sigma_i N \sum_{s \in \mathcal{S}} x_{is}, \qquad \forall i \in \mathcal{O} \tag{3.10}$$

$$x_{is} \in \{0,1\}, \quad \forall i \in \mathcal{O}, \, \forall s \in \mathcal{S} \tag{3.11}$$

$$y_s \in \{0,1\}, \; u_s \in Z_N^+, \; \rho_s \geq 0, \; \rho_s^{nom} \geq 0, \quad \forall s \in \mathcal{S}$$

$$q_i \geq 0, \; r_i \geq 0, \; c_i \geq 0, \quad \forall i \in \mathcal{O}$$

The family of Constraints (3.1) guarantees that each operator joins only a feasible coalition, that is, any coalition in which it results a member. However, they are relaxed and not an equality in order account for the case when it is impossible to satisfy the operators' expectations on the return on investment for certain combinations of δ and γ. In such scenarios an all zeros solution is obtained. Equations (3.2) do not allow inappropriate assignments, that is, an operator cannot join a coalition to which it does not belong. Constraints (3.3) make sure that a coalition exists only if all of its members select it, in other words, all members agree to collaborate. Equations (3.4) guarantee consistency between the coalitions that are created and the number of BSs installed in each of them. In case y_s equals zero, that is, coalition s has not been created, the number of BSs for that coalition has to be forced to zero, otherwise it is upperbounded by U_{max}.

Constraints (3.5a) and (3.5b) are used in alternative, and they both make sure that the overall number of BSs installed in the area does not exceed U_{max}. However, while Constraint (3.5a) simply limits the number of BSs installed by all the coalitions to be at most U_{max}, Constraints (3.5b) limit the number of BSs for each coalition to be proportional to the number of users that make up that coalition, guaranteeing a fair distribution of the available BSs among operators when the regulatory entity imposes the objective function that maximizes the minimum return on investment in the case of non-uniformly distributed users among operators (see Chapter 4).

Equations (3.6) define the coalition rate ρ_s as function of the nominal rate ρ_s^{nom}, scaled down by the factor $(1-\eta)^{\frac{\sum_{i\in\mathcal{C}_s}\sigma_i N}{u_s}}$. Parameter η, referred to as the user activity factor, is the probability that a user requires the maximum achievable downlink rate from his/her serving BS. Being u_s the number of BSs activated by coalition s and $\sum_{i\in\mathcal{C}_s}\sigma_i N$ the total number of users that belong to this coalition, on the average, the ratio $\frac{\sum_{i\in\mathcal{C}_s}\sigma_i N}{u_s}$ equals to the number of users served by one BS. Therefore the load of the coalition s, which we denote by l_s, being equal to the probability that there is at least one user that demands the maximum LTE downlink rate from its serving BS is calculated as $1-(1-\eta)^{\frac{\sum_{i\in\mathcal{C}_s}\sigma_i N}{u_s}}$. However, the higher this probability, that is, the higher l_s, the smaller the rate perceived by users of coalition s (ρ_s). As a result, the nominal rate is scaled down by $1-l_s$ (*i.e.*, by the factor $(1-\eta)^{\frac{\sum_{i\in\mathcal{C}_s}\sigma_i N}{u_s}}$).

Equations (3.7) define the user rate per operator (q_i) as equal to the coalition user rate, referring to the coalition joined by the operator.

Equations (3.8) define revenues per operator (r_i) as a linear function of the operator's user rate (q_i), while the proportionality constant is the product of the monthly price of 1 Mbps (parameter δ), the investment lifetime D and the number of users of the operator, namely the product of the operators share σ_i and the total number of users in the area N.

Equations (3.9) define the costs for each operator (c_i) as a linear function of the number of BSs installed in the coalition selected by the operator. The rationale behind this definition is that the overall BSs costs of a coalition are divided among its members according to their number of users.

Constraints (3.10) model the minimum return on investment that the operators expect in the investment lifetime period D. This lower bound is obtained as the product of the monthly return on investment expected from the single user (parameter γ), the investment lifetime D and the operator's number of users $\sigma_i N$ to make it consistent with the definitions of costs and revenues. The right handside of Constraints (3.10) is also multiplied by the term $\sum_{s\in\mathcal{S}} x_{is}$ which deactivates the constraint in case operator i does not invest at all (*i.e.*, $\sum_{s\in\mathcal{S}} x_{is}$ equals zero). This is done in order to avoid an unsatisfiable lower bound when both revenues and costs of operator i equal zero (no investment and therefore no gain).

Finally variable domains (3.11) complete the model.

For the same set of constraints and variables, we define six different objective functions as follows.

$$\max \sum_{i \in \mathcal{O}} q_i \tag{3.12}$$

$$\max \min_{i \in \mathcal{O}} q_i \tag{3.13}$$

$$\max \left(\min_{i \in \mathcal{O}} q_i + \sum_{i \in \mathcal{O}} q_i \right) \tag{3.14}$$

$$\max \sum_{i \in \mathcal{O}} (r_i - c_i) \tag{3.15}$$

$$\max \min_{i \in \mathcal{O}} (r_i - c_i) \tag{3.16}$$

$$\max \left(\sum_{i \in \mathcal{O}} (r_i - c_i) + \min_{i \in \mathcal{O}} (r_i - c_i) \right) \tag{3.17}$$

Objectives (3.12), (3.13), (3.14) are referred to as *user-oriented* objective functions since they focus on the quality observed by the user (user rate), being the latter the main indicator of the user's level of satisfaction. On the other hand, Objectives (3.15), (3.16), (3.17) focus on the operators' return on investment. As a result, we refer to them as *operator-oriented* objective functions.

Objective (3.12) maximizes the sum of operators' user rate, whereas (3.13) maximizes the smallest user rate among all operators. As a result, Objective (3.13) introduces some fairness in the solution provide by our model. Even though it is not the goal of any operator to improve the user rate of another operator while degrading its own, such objective function can be imposed by a regulatory entity. That is why we explore the outcome of the formulation under such objective function. Objective (3.14) maximizes both the total user rate over all operators and the minimum one. Such objective is introduced to avoid solutions that advantage a subset of operators by disadvantaging the others. Even though the optimal solution provided by of Objective (3.12) may result in a total user rate higher than the optimum of (3.14), the solution provided by (3.14) is expected to be fair to all the operators.

The same reasoning lies behind the other three *operator-oriented* objective functions. The only difference is that such objective functions focus on the most important indicator for the operators, that is, the return on investment. Objective (3.15) maximizes the sum of the return on investment of all operators, (3.16) maximizes the minimum return on investment over all operators, whereas Objective (3.17) maximizes the sum of both factors.

3.1.4 SISP linear model

As mentioned before, there are non-linear objective functions and constraints in our formulation. We linearize them so that the SISP can be formulated as a MILP.

The right hand side of Equations (3.7) and (3.9), involves the product between two variables: $\rho_s x_{is}$ in (3.7) and $u_s x_{is}$ in (3.9). Since for both terms the product is between a binary and a non-binary variable, their linerization is carried out in the same fashion. We introduce non-negative, continuous auxiliary variables z_{is} and the set of Equations (3.18), (3.19), (3.20) which force z_{is} to always be equal to $\rho_s x_{is}$. It is important to notice that such linearization is feasible only because one of the variables in the product is binary. Finally Equations (3.21) redefine q_i in terms of z_{is}.

$$z_{is} \leq R_s^{max} x_{is}, \quad \forall i \in \mathcal{O}, \, \forall s \in \mathcal{S} \tag{3.18}$$

$$z_{is} \leq \rho_s, \quad \forall i \in \mathcal{O}, \, s \in \mathcal{S} \tag{3.19}$$

$$z_{is} \geq \rho_s - R_s^{max}(1 - x_{is}), \quad \forall i \in \mathcal{O}, \, \forall s \in \mathcal{S} \tag{3.20}$$

$$q_i = \sum_{s \in \mathcal{S}} z_{is}, \quad \forall i \in \mathcal{O} \tag{3.21}$$

Similarly, w_{is} are another family of non-negative, continuous auxiliary variables which we introduce to linearize $u_s x_{is}$. Equations (3.22), (3.23), (3.24) guarantee that w_{is} equal $u_s x_{is}$, whereas (3.25) redefine c_i in terms of w_{is}.

$$w_{is} \leq U_{max} x_{is}, \quad \forall i \in \mathcal{O}, \, \forall s \in \mathcal{S} \tag{3.22}$$

$$w_{is} \leq u_s, \quad \forall i \in \mathcal{O}, \, s \in \mathcal{S} \tag{3.23}$$

$$w_{is} \geq u_s - U_{max}(1 - x_{is}), \quad \forall i \in \mathcal{O}, \, \forall s \in \mathcal{S} \tag{3.24}$$

$$c_i = \sum_{s \in \mathcal{S}} g \frac{\sigma_i}{\sum_{j \in \mathcal{C}_s} \sigma_j} w_{is}, \quad \forall i \in \mathcal{O} \tag{3.25}$$

Equations (3.6) involve two non-linear terms: the nominal rate ρ_s^{nom} and the load factor. From (3.6), it can be easily seen why the load factor is not linear (variable u_s appears in the exponent). As explained in Section 2.1 of Chapter 2, the nominal rate is obtained through simulation. An example of the behavior of ρ_s^{nom} and ρ_s is given in Figure 3.1.

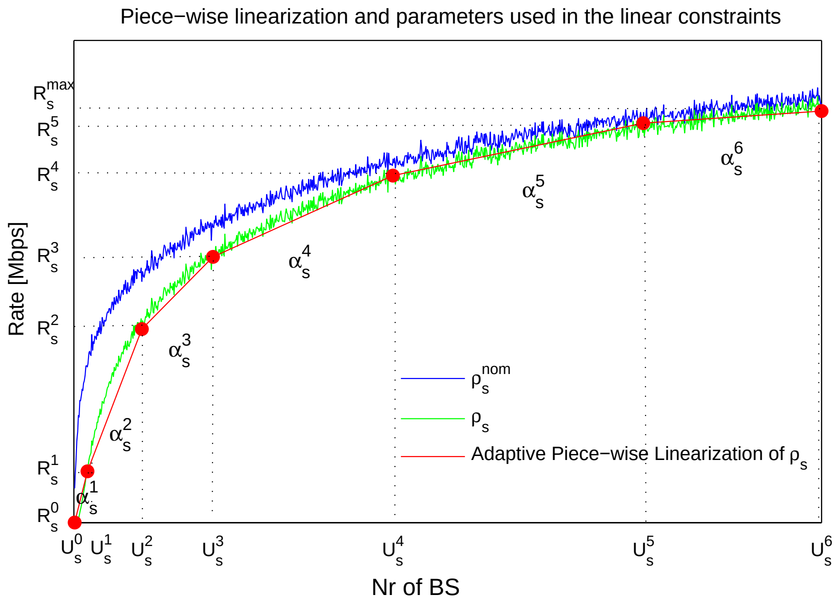

Figure 3.1: Simulated nominal rate, coalition rate and adaptive piece-wise linearization.

It can easily be observed that the non-linear behavior of the simulated nominal ρ_s^{nom} and coalition rate ρ_s with respect to the number of BSs. We approximate the coalition rate ρ_s with a piece-wise linear function which is computed by means of an algorithm that performs adaptive piece-wise linearization. Even though such algorithm has to be applied to every coalition rate curve, we consider the same number of linear pieces for all the coalitions so that the same family of linear constraints can linearize each of the Equations (3.6).

In order to perform such linearization, we need to introduce some additional parameters.

We denote by α_s^k the gradient of the k^{th} linear piece of the user rate of coalition s (ρ_s). $[U_s^{k-1}, U_s^k]$ is the range of number of BSs for which α_s^k serves as the gradient of the k^{th} linear piece provided by the adaptive piece-wise linearization algorithm. R_s^k is the user rate of coalition s, respectively when U_s^k BSs have been activated in coalition s. U_s^0 are equal to 1, whereas R_s^{max} is the user rate of coalition s

obtained by activating U_{max} BSs.

Equations (3.26) show how the value of parameters R_s^k has been calculated.

$$\begin{aligned}
R_s^0 &= \rho_s(1), \quad \forall s \in \mathcal{S} \\
R_s^1 &= R_s^0 + \alpha_s^1(U_s^1 - U_s^0), \quad \forall s \in \mathcal{S} \\
R_s^2 &= R_s^1 + \alpha_s^2(U_s^2 - U_s^1), \quad \forall s \in \mathcal{S} \\
R_s^3 &= R_s^2 + \alpha_s^3(U_s^3 - U_s^2), \quad \forall s \in \mathcal{S} \\
R_s^4 &= R_s^3 + \alpha_s^3(U_s^4 - U_s^3), \quad \forall s \in \mathcal{S} \\
R_s^5 &= R_s^4 + \alpha_s^3(U_s^5 - U_s^4), \quad \forall s \in \mathcal{S}
\end{aligned} \tag{3.26}$$

Finally we introduce six families of constraints that model the piece-wise linearization (Equations (3.27), (3.28), (3.29), (3.30), (3.31) and (3.32)). It is possible to use such set of linear constraints given that the user rate has concave convexity (Figure 3.1). Equations (3.33) set to zero the coalition rate for the coalitions that are not created, otherwise if the coalition exists, they behave as an upperbound on the coalition rate. These constraints will be tight only if the big coalition is created and the entire set of U_{max} BSs is installed, due to Constraints (3.5). Parameter M is a large positive constant that deactivates all the constraints for the coalitions that are not created, that is, for y_s equal to zero, allowing an appropriate behavior of the formulation of the model.

$$\rho_s \le R_s^0 + \alpha_s^1(u_s - U_s^0) + M(1 - y_s), \quad \forall s \in \mathcal{S} \tag{3.27}$$

$$\rho_s \le R_s^1 + \alpha_s^2(u_s - U_s^1) + M(1 - y_s), \quad \forall s \in \mathcal{S} \tag{3.28}$$

$$\rho_s \le R_s^2 + \alpha_s^3(u_s - U_s^2) + M(1 - y_s), \quad \forall s \in \mathcal{S} \tag{3.29}$$

$$\rho_s \le R_s^3 + \alpha_s^4(u_s - U_s^3) + M(1 - y_s), \quad \forall s \in \mathcal{S} \tag{3.30}$$

$$\rho_s \le R_s^4 + \alpha_s^5(u_s - U_s^4) + M(1 - y_s), \quad \forall s \in \mathcal{S} \tag{3.31}$$

$$\rho_s \le R_s^5 + \alpha_s^6(u_s - U_s^5) + M(1 - y_s), \quad \forall s \in \mathcal{S} \tag{3.32}$$

$$\rho_s \le R_s^{max} y_s, \quad \forall s \in \mathcal{S} \tag{3.33}$$

Moreover, Objectives (3.13), (3.14), (3.16) and (3.17) are non-linear since they include *max min* terms. We need two more auxiliary variables, P_{min} and Q_{min}

and Constraints (3.34), (3.35) to linearize the objective functions.

$$P_{min} \leq r_i - c_i, \quad \forall i \in \mathcal{O} \tag{3.34}$$

$$Q_{min} \leq q_i, \quad \forall i \in \mathcal{O} \tag{3.35}$$

The linearized versions of Objectives (3.13), (3.14), (3.16) and (3.17) are respectively (3.36), (3.37), (3.38) and (3.39).

$$\max \; Q_{min} \tag{3.36}$$

$$\max \left(Q_{min} + \sum_{i \in \mathcal{O}} q_i \right) \tag{3.37}$$

$$\max \; P_{min} \tag{3.38}$$

$$\max \left(\sum_{i \in \mathcal{O}} (r_i - c_i) + P_{min} \right) \tag{3.39}$$

Tables 3.1 and 3.2 summarize the sets, parameters and variables of the SISP model.

$\mathcal{O}$	Set of operators, $\vert\mathcal{O}\vert = n$
$\mathcal{S}$	Set of all subsets of $\mathcal{O}$, $\vert\mathcal{S}\vert = m$
$\mathcal{C}_s$	Set of member operators of coalition $s \in \mathcal{S}$
N	Total number of users in the given area
σ_i	Share of users out of total N for operator $i \in \mathcal{O}$
U_{max}	Max number of BSs allowed in the considered area
δ	User's willingness to pay for 1 Mbps every month [€/month×Mbps]
γ	Expected monthly return from one user [€/month×user]
g	Cost of a single BS normalized for the investment lifetime [€]
η	User activity factor
D	Investment lifetime [months]
α_s^k	Rate gradient for linear piece $k \in 1..6$, coalition $s \in \mathcal{S}$
U_s^k	Number of BSs for which the rate gradient changes from α_s^k to α_s^{k+1}
R_s^k	User rate obtained from installing U_s^k BSs for coalition s [Mbps]
R_{max}^s	Max user rate for coalition $s \in \mathcal{S}$ obtained by U_{max} BSs [Mbps]

Table 3.1: Sets and parameters of the SISP model

x_{is}	1 if operator $i \in \mathcal{O}$ joins coalition $s \in \mathcal{S}$, 0 otherwise
y_s	1 if coalition $s \in \mathcal{S}$ is created, 0 otherwise
u_s	Number of BSs activated for coalition $s \in \mathcal{S}$
ρ_s^{nom}	Nominal user rate of coalition $s \in \mathcal{S}$
ρ_s	User rate of coalition $s \in \mathcal{S}$
q_i	User rate of operator $i \in \mathcal{O}$
c_i	Costs of operator $i \in \mathcal{O}$
r_i	Revenues of operator $i \in \mathcal{O}$
Q_{min}	Min user rate calculated over all the operators $i \in \mathcal{O}$
P_{min}	Min return on investment calculated over all the operators $i \in \mathcal{O}$
z_{is}	Auxiliary variable used to linearize $\rho_s x_{is}$ for $i \in \mathcal{O},\ s \in \mathcal{S}$
w_{is}	Auxiliary variable used to linearize $u_s x_{is}$ for $i \in \mathcal{O},\ s \in \mathcal{S}$

Table 3.2: Variables of the SISP model

We conclude this section with the complete linear model for the SISP.

$$\sum_{s \in \mathcal{S}: i \in \mathcal{C}_s} x_{is} \leq 1, \quad \forall i \in \mathcal{O} \tag{3.40}$$

$$x_{is} = 0, \quad \forall s \in \mathcal{S},\ \forall i \in \mathcal{O} : i \notin \mathcal{C}_s \tag{3.41}$$

$$x_{is} = y_s, \quad \forall s \in \mathcal{S},\ \forall i \in \mathcal{O} : i \in \mathcal{C}_s \tag{3.42}$$

$$u_s \leq U_{max} y_s, \quad \forall s \in \mathcal{S} \tag{3.43}$$

$$\sum_{s \in \mathcal{S}} u_s \leq U_{max} \tag{3.44a}$$

$$u_s \leq \sum_{i \in \mathcal{C}_s} \sigma_i U_{max}, \quad \forall s \in \mathcal{S} \tag{3.44b}$$

$$\rho_s \leq R_s^0 + \alpha_s^1 (u_s - U_s^0) + M(1 - y_s), \quad \forall s \in \mathcal{S} \tag{3.45}$$

$$\rho_s \leq R_s^1 + \alpha_s^2 (u_s - U_s^1) + M(1 - y_s), \quad \forall s \in \mathcal{S} \tag{3.46}$$

$$\rho_s \leq R_s^2 + \alpha_s^3 (u_s - U_s^2) + M(1 - y_s), \quad \forall s \in \mathcal{S} \tag{3.47}$$

$$\rho_s \leq R_s^3 + \alpha_s^4 (u_s - U_s^3) + M(1 - y_s), \quad \forall s \in \mathcal{S} \tag{3.48}$$

$$\rho_s \leq R_s^4 + \alpha_s^5 (u_s - U_s^4) + M(1 - y_s), \quad \forall s \in \mathcal{S} \tag{3.49}$$

$$\rho_s \leq R_s^5 + \alpha_s^6 (u_s - U_s^5) + M(1 - y_s), \quad \forall s \in \mathcal{S} \tag{3.50}$$

$$\rho_s \leq R_s^{max} y_s, \quad \forall s \in \mathcal{S} \tag{3.51}$$

$$z_{is} \le R_s^{max} x_{is}, \quad \forall i \in \mathcal{O}, \forall s \in \mathcal{S} \tag{3.52}$$

$$z_{is} \le \rho_s, \quad \forall i \in \mathcal{O}, \forall s \in \mathcal{S} \tag{3.53}$$

$$z_{is} \ge \rho_s - R_s^{max}(1 - x_{is}), \quad \forall i \in \mathcal{O}, \forall s \in \mathcal{S} \tag{3.54}$$

$$q_i = \sum_{s \in \mathcal{S}} z_{is}, \quad \forall i \in \mathcal{O} \tag{3.55}$$

$$r_i = \delta D \sigma_i N q_i, \quad \forall i \in \mathcal{O} \tag{3.56}$$

$$w_{is} \le U_{max} x_{is}, \quad \forall i \in \mathcal{O}, \forall s \in \mathcal{S} \tag{3.57}$$

$$w_{is} \le u_s, \quad \forall i \in \mathcal{O}, \forall s \in \mathcal{S} \tag{3.58}$$

$$w_{is} \ge u_s - U_{max}(1 - x_{is}), \quad \forall i \in \mathcal{O}, \forall s \in \mathcal{S} \tag{3.59}$$

$$c_i = \sum_{s \in \mathcal{S}} g \frac{\sigma_i}{\sum_{j \in \mathcal{C}_s} \sigma_j} w_{is}, \quad \forall i \in \mathcal{O} \tag{3.60}$$

$$r_i - c_i \ge \gamma D \sigma_i N \sum_{s \in \mathcal{S}} x_{is}, \quad \forall i \in \mathcal{O} \tag{3.61}$$

$$P_{min} \le r_i - c_i, \quad \forall i \in \mathcal{O} \tag{3.62}$$

$$Q_{min} \le q_i, \quad \forall i \in \mathcal{O} \tag{3.63}$$

$$x_{is} \in \{0, 1\}, \ z_{is} \ge 0, \ w_{is} \ge 0, \quad \forall i \in \mathcal{O}, \forall s \in \mathcal{S} \tag{3.64}$$

$$y_s \in \{0, 1\}, \ u_s \in Z_N^+, \ \rho_s \ge 0, \quad \forall s \in \mathcal{S}$$

$$r_i \ge 0, \ c_i \ge 0, \ q_i \ge 0, \quad \forall i \in \mathcal{O}$$

$$Q_{min} \ge 0, \ P_{min} \ge 0$$

3.2 MISP mathematical model

The SISP formulation applies to the case when operators invest in a single area which is characterized by a certain size and population. Clearly, if we consider large cities there can be multiple areas that operators find financially attractive for investment. Such areas are not necessarily geographically adjacent or part of the same city. We refer to this potential application scenario as MISP (Multiple area Infrastructure Sharing Problem). The MISP consists in determining the coalitions that are created over different areas and the number of BSs activated for each coalition over each area, given a set of MNOs that want to simultaneously invest in each of them. This section introduces the necessary modifications that have been made to the SISP model in order to obtain the mathematical model for the MISP. Moreover, it states the particular assumptions regarding the second scenario (MISP) in addition to those introduced for the SISP.

In addition to the sets introduced in Subsection 3.1.1, we introduce the set of areas $\mathcal{A}$. The number of areas, *i.e.*, $|\mathcal{A}|$ is denoted by l. Each area is characterized by its size and number of users (N^a). We keep the same share of clients for each operator over all areas, which is justifiable for the case when the distinct areas are part of the same city[2]. The maximum number of BSs per area is still the same and equal to U_{max}, since it does not concern coverage but it allows us to reach rate saturation and therefore explore the effect of different configuration of parameters. The techno-economic parameters δ, γ, g, η and D remain unaltered. Assuming that users are responding the same way to the improved service and on the average have the same activity factor in all areas, both δ and η do not change. Moreover, it is reasonable to think that operators have the same financial targets independently of the area (same γ for all areas). Lastly, D and g are the same for all areas, given that the same technology is being deployed in all of them.

Parameters related to the linearization of the model are all affected by the extension. The reason behind this is that distinct areas have different size and number of users. Both these elements are inputs of the simulation that provides the coalition rate, which is linearized by means of the adaptive piece-wise linearization algorithm. Thus, there will be distinct coalition rate curves for distinct areas. As a result, we add the area dimension to the linearization parameters as follows.

Parameter α^a_{sk} is the gradient of the k^{th} linear piece of user rate for coalition s in area a. U^a_{sk} is the number of BSs for which the gradient changes from α^a_{sk} to α^a_{sk+1}, R^a_{sk} is the user rate of coalition s obtained by activating U^a_{sk} BSs over area a, whereas R^a_{smax} the one obtained by activating U_{max} BSs instead.

We do not alter the binary decision variables (x_{is}) of the SISP model, as we

[2]We generated instances for 5 hotspots in the city of Milan. Therefore, the operators' market share is kept the same for all areas.

consider the case in which operators, if they decide to collaborate, will always join the same coalition in all the independent areas. The reasoning behind this is that it becomes complex, from a practical point of view, for the operators to manage simultaneously several collaboration agreements, such as a different one for each area. Thus, for the MISP, x_{is} equals one if operator i selects coalition s in all areas, otherwise it equals 0.

Since each area is characterized by its size and number of users and, as a result, user density, it is sensible to assume that the number of BSs activated for the same coalition can be different for each area. That is why we introduce variables u_s^a that account for the number of users activated for coalition s in area a. This will affect the coalition user rate which is a function of the number of BSs, and as a consequence, the user rate provided by each operator. The coalition user rate and the one provided by each operator in area a become respectively ρ_s^a and q_i^a. The non-negative continuous variables c_i^a indicate the costs of operators i in area a, given that these costs are a linear function of u_s^a. Similarly, since revenues per area depend on the operator's user rate per area (q_i^a), the corresponding variables are to r_i^a.

Moreover, the expected return on investment applies to the total return on investment and not to the one of each area. This was done keeping in mind the realistic approach of operators, meaning that even though they invest over different areas, they are more interested in the overall return on investment, having no particular financial targets for certain areas.

As far as the objective functions are concerned, the minimum user rate will now be not only over all the operators but also over all the areas. This is in accordance with the reasoning that we made about the *user-oriented* objective functions, keeping in mind that there is a regulatory entity that forces such objective functions. Therefore, for the MISP it will account for the worst served users over all the areas.

The opposite applies to the minimum return on investment, which will be calculated over the global return on investment of the operators and not on the return of the each area, for the same reasons that we calculated a global minimum on return on investment other than one for each area.

These assumptions, apart from being more in line with the operators way of handling the economic aspect of its network, will also allow the outcome of the model to be as different as possible from the case in which all areas are independent. In other words, we want to avoid having non-interfering solutions for the different areas, which would be the same as having different instances and therefore solvable by using the SISP model.

Since both the number of BSs per coalition (u_s^a) and the coalition rate (ρ_s^a) are different for distinct areas, so will the variables that linearize their product with the unaltered binary variable x_{is}. The modified auxiliary variables will therefore be z_{is}^a and w_{is}^a.

A recap of sets, parameters and variables of the MISP model is done in Tables 3.3 and 3.4.

$\mathcal{O}$	Set of operators, $\lvert\mathcal{O}\rvert = n$
$\mathcal{S}$	Set of all subsets of $\mathcal{O}$, $\lvert\mathcal{S}\rvert = m$
$\mathcal{C}_s$	Set of member operators of coalition $s \in \mathcal{S}$
$\mathcal{A}$	Set of areas, $\lvert\mathcal{A}\rvert = l$
N^a	Total number of users in area $a \in \mathcal{A}$
σ_i	Share of clients for operator $i \in \mathcal{O}$, same for all areas
U_{max}	Max number of BSs allowed in any of the areas
δ	User's willingness to pay for 1 Mbps every month [€/month×Mbps]
γ	Expected monthly return from one user [€/month]
g	Cost of a single BS normalized for the investment lifetime [€]
D	Investment lifetime [months]
η	User activity factor
α_{sk}^a	User rate gradient for linear piece $k \in 1..6$, coalition $s \in \mathcal{S}$, area $a \in \mathcal{A}$
U_{sk}^a	Number of BSs for which the gradient changes from α_{sk}^a to α_{sk+1}^a
R_{sk}^a	User rate for coalition $s \in \mathcal{S}$, area $a \in \mathcal{A}$ obtained by U_{sk}^a BSs
R_{smax}^a	Max user rate for coalition $s \in \mathcal{S}$ in area $a \in \mathcal{A}$ obtained by U_{max} BSs

Table 3.3: Sets and parameters of the MISP model

x_{is}	1 if operator $i \in \mathcal{O}$ joins coalition $s \in \mathcal{S}$ in all the areas, 0 otherwise
y_s	1 if coalition $s \in \mathcal{S}$ is created for all the areas, 0 otherwise
u_s^a	Number of BSs installed in coalition $s \in \mathcal{S}$, area $a \in \mathcal{A}$
$\rho_s^{nom,a}$	Nominal user rate for coalition $s \in \mathcal{S}$, area $a \in \mathcal{A}$
ρ_s^a	User rate for coalition $s \in \mathcal{S}$, area $a \in \mathcal{A}$
q_i^a	User rate for operator $i \in \mathcal{O}$, area $a \in \mathcal{A}$
c_i^a	Costs of operator $i \in \mathcal{O}$, area $a \in \mathcal{A}$
r_i^a	Revenues of operator $i \in \mathcal{O}$, area $a \in \mathcal{A}$
P_{min}	Min global return on investment over all the operators
Q_{min}	Min user rate over all areas and operators
z_{is}^a	Auxiliary variable used to linearize $\rho_s^a x_{is}$, $\forall\ i \in \mathcal{O}$, $s \in \mathcal{S}$, $a \in \mathcal{A}$
w_{is}^a	Auxiliary variable used to linearize $u_s^a x_{is}$, $\forall\ i \in \mathcal{O}$, $s \in \mathcal{S}$, $a \in \mathcal{A}$

Table 3.4: Variables of the MISP model

This section also concludes with the linear model for the MISP.

$$\max \sum_{i \in \mathcal{O}} \sum_{a \in \mathcal{A}} q_i^a \tag{3.65a}$$

$$\max\ Q_{min} \tag{3.65b}$$

$$\max \left(\sum_{i \in \mathcal{O}} \sum_{a \in \mathcal{A}} q_i + Q_{min} \right) \tag{3.65c}$$

$$\max \sum_{i \in \mathcal{O}} \sum_{a \in \mathcal{A}} (r_i^a - b_i^a) \tag{3.65d}$$

$$\max\ P_{min} \tag{3.65e}$$

$$\max \left(\sum_{i \in \mathcal{O}} \sum_{a \in \mathcal{A}} (r_i^a - b_i^a) + P_{min} \right) \tag{3.65f}$$

$$\sum_{s \in \mathcal{S}: i \in \mathcal{C}_s} x_{is} \leq 1, \qquad \forall i \in \mathcal{O} \tag{3.66}$$

$$x_{is} = 0, \qquad \forall s \in \mathcal{S},\ \forall i \in \mathcal{O} : i \notin \mathcal{C}_s \tag{3.67}$$

$$x_{is} = y_s, \qquad \forall s \in \mathcal{S},\ \forall i \in \mathcal{O} : i \in \mathcal{C}_s \tag{3.68}$$

$$u_s^a \leq U_{max} y_s, \qquad \forall s \in \mathcal{S},\ \forall a \in \mathcal{A} \tag{3.69}$$

$$\sum_{s \in \mathcal{S}} u_s^a \leq U_{max}, \quad \forall a \in \mathcal{A} \tag{3.70a}$$

$$u_s^a \leq \sum_{i \in \mathcal{C}_s} \sigma_i U_{max}, \quad \forall s \in \mathcal{S}, \forall a \in \mathcal{A} \tag{3.70b}$$

$$\rho_s^a \leq R_{s0}^a + \alpha_{s1}^a (u_s^a - U_{s0}^a) + M(1 - y_s), \quad \forall s \in \mathcal{S}, \forall a \in \mathcal{A} \tag{3.71}$$

$$\rho_s^a \leq R_{s1}^a + \alpha_{s2}^a (u_s^a - U_{s1}^a) + M(1 - y_s), \quad \forall s \in \mathcal{S}, \forall a \in \mathcal{A} \tag{3.72}$$

$$\rho_s^a \leq R_{s2}^a + \alpha_{s3}^a (u_s^a - U_{s2}^a) + M(1 - y_s), \quad \forall s \in \mathcal{S}, \forall a \in \mathcal{A} \tag{3.73}$$

$$\rho_s^a \leq R_{s3}^a + \alpha_{s4}^a (u_s^a - U_{s3}^a) + M(1 - y_s), \quad \forall s \in \mathcal{S}, \forall a \in \mathcal{A} \tag{3.74}$$

$$\rho_s^a \leq R_{s4}^a + \alpha_{s5}^a (u_s^a - U_{s4}^a) + M(1 - y_s), \quad \forall s \in \mathcal{S}, \forall a \in \mathcal{A} \tag{3.75}$$

$$\rho_s^a \leq R_{s5}^a + \alpha_{s6}^a (u_s^a - U_{s5}^a) + M(1 - y_s), \quad \forall s \in \mathcal{S}, \forall a \in \mathcal{A} \tag{3.76}$$

$$\rho_s^a \leq R_{smax}^a y_s, \quad \forall s \in \mathcal{S}, \forall a \in \mathcal{A} \tag{3.77}$$

$$z_{is}^a \leq R_{smax}^a x_{is}, \quad \forall i \in \mathcal{O}, \forall s \in \mathcal{S}, \forall a \in \mathcal{A} \tag{3.78}$$

$$z_{is}^a \leq \rho_s^a, \quad \forall i \in \mathcal{O}, \ s \in \mathcal{S}, \forall a \in \mathcal{A} \tag{3.79}$$

$$z_{is}^a \geq \rho_s^a - R_{smax}^a (1 - x_{is}), \quad \forall i \in \mathcal{O}, \forall s \in \mathcal{S}, \forall a \in \mathcal{A} \tag{3.80}$$

$$q_i^a = \sum_{s \in \mathcal{S}} z_{is}^a, \quad \forall i \in \mathcal{O}, \forall a \in \mathcal{A} \tag{3.81}$$

$$r_i^a = \delta D \sigma_i N^a q_i^a, \quad \forall i \in \mathcal{O}, \forall a \in \mathcal{A} \tag{3.82}$$

$$w_{is}^a \leq U_{max} x_{is}, \quad \forall i \in \mathcal{O}, \forall s \in \mathcal{S}, \forall a \in \mathcal{A} \tag{3.83}$$

$$w_{is}^a \leq u_s^a, \quad \forall i \in \mathcal{O}, \ s \in \mathcal{S}, \forall a \in \mathcal{A} \tag{3.84}$$

$$w_{is}^a \geq u_s^a - U_{max}(1 - x_{is}), \quad \forall i \in \mathcal{O}, \forall s \in \mathcal{S}, \forall a \in \mathcal{A} \tag{3.85}$$

$$c_i^a = \sum_{s \in \mathcal{S}} g \frac{\sigma_i}{\sum_{j \in \mathcal{C}_s} \sigma_j} w_{is}^a, \quad \forall i \in \mathcal{O}, \forall a \in \mathcal{A} \tag{3.86}$$

$$\sum_{a \in \mathcal{A}} (r_i^a - c_i^a) \geq \gamma D \sigma_i \sum_{a \in \mathcal{A}} N^a \sum_{s \in \mathcal{S}} x_{is}, \quad \forall i \in \mathcal{O} \tag{3.87}$$

$$P_{min} \leq \sum_{a \in \mathcal{A}} (r_i^a - c_i^a), \quad \forall i \in \mathcal{O} \tag{3.88}$$

$$Q_{min} \leq q_i^a, \quad \forall i \in \mathcal{O},\, \forall a \in \mathcal{A} \tag{3.89}$$

$$\begin{aligned} x_{is} \in \{0,1\}, \quad \forall i \in \mathcal{O},\, \forall s \in \mathcal{S} \\ y_s \in \{0,1\}, \quad \forall s \in \mathcal{S} \\ u_s^a \in Z_N^+,\ \rho_s^a \geq 0, \quad \forall a \in \mathcal{A},\, \forall s \in \mathcal{S} \\ r_i^a \geq 0,\ b_i^a \geq 0,\ q_i^a \geq 0, \quad \forall i \in \mathcal{O},\, \forall a \in \mathcal{A} \\ z_{is}^a \geq 0,\ w_{is}^a \geq 0, \quad \forall i \in \mathcal{O},\, \forall s \in \mathcal{S},\, \forall a \in \mathcal{A} \\ Q_{min} \geq 0,\ P_{min} \geq 0 \end{aligned} \tag{3.90}$$

Chapter 4

Results

In Chapter 3 we provided a MILP mathematical model for both the SISP and the MISP. Both models were implemented in AMPL [25]. We used CPLEX V12.6.0 as a MILP optimization solver [26]. We tested each model for 20 instances. All tests were run on an Intel Xeon (64 bit x86 architecture) dual socket quad core CPUs @2Ghz. CPLEX was installed within a VirtualBox VM running Ubuntu 10.04.2 LTS (Lucid Lynx) with 15 GB reserved memory and 8 dedicated physical cores. The average execution time required by CPLEX is negligible due to the limited size of the instances and therefore not reported.

In Section 4.1, we report how the different parameters of both the SISP and the MISP models were set in order to generate the instances. We tested several instances in order to explore how both models behave under different configurations of the key parameters of our formulation. The outcomes of the different tests are discussed in detail in Section 4.2 for the SISP and in Section 4.3 for the MISP.

4.1 Instances

We generated two types of instances for both the single (SISP) and multiple area (MISP) scenarios: the first type accounts for a uniform distribution of users among operators, whereas the second for a non-uniform one. Parameters were set either by referring to related literature or by looking at current pricing models of MNOs in the Italian telecom market. Additional parameters, that have been introduced in the linearized version of both models, were obtained for both types of instances by simulating the deployment of picocell BSs in each considered area. These simulations have been carried out in MATLAB environment (Subsection 4.1.3). For the MISP, we considered five dense square areas, referring approximately (in size and population) to five hotspots of the city of Milan. Instead, for the SISP, instances were generated only for one of these areas.

4.1.1 SISP instances

The first parameter that had to be set is the number of coexisting operators over the considered area. The number of operators determines the size of each instance, and has a great impact on the solver's execution time given that the number of variables and constraints of our models grows exponentially with the number of operators. In practice, the number of MNOs in the market is limited to only a few. In this work we considered instances with three MNOs (n equal to 3): A, B and C, which is quite reasonable for the Italian telecom market but also for other European countries. The same number of MNOs has been considered in [18], where the infrastructure sharing problem has been approached by means of game theory.

Since we account for all the possible coalitions that can be created among three MNOs, the corresponding set of coalitions $\mathcal{S}$ is {A, B, C, AB, AC, BC, ABC} and its cardinality m equals 7.

We introduced two key economic parameters in the formulation: δ, which represents the user's willingness to pay for 1 Mbps of service on a monthly basis, and γ, which is the monthly contribution of a single user should have in the minimum return on investment expected by the operator. Since these parameters indicate the user (δ) and the operator (γ) standpoint in the investment, a set of values instead of a single one have been used in our tests. The considered set of values for δ is $\{0.05, 0.2, 0.8, 1, 2\}$, whereas the one for γ is $\{0, 1, 5, 10\}$. The values for δ were deduced from the current pricing models of Italian MNOs. Those for γ, instead, are more intuitive and were chosen to create reasonable lower bounds on the return on investment. Therefore, we generated one instance for each combination of the values of δ and the ones of γ.

In Chapter 2, we explained how the normalized cost (over the investment lifetime) of a single picocell BS (g) is calculated in this work. The CAPEX component of g, that is, g_{capex} was set equal to 3000 € [14]. OPEX annual component (ξ) was obtained from [23], and it is equal to 15%. However, in [14] an OPEX annual percentage equal to 10% has been considered. Since it is not easy to define operational expenditures with high accuracy, in this work we accounted for the largest value of the two.

The investment lifetime (D) was set to 10 years, which is reasonable for investment in a 4G mobile network.

For both models we considered an upper bound on the overall number of new BSs that can be deployed in the area (U_{max}). This parameter was set to 1000, being this the number of BSs for which rate saturation is reached in our simulations. In other words, deploying more BSs neither increases the user rate, nor improves the operator's return on investment. Thus, such limit allows us to correctly analyze the outcome of instances with different values of γ and δ.

The activity factor parameter (η) was set to 0.001, that on the average corresponds to 1.44 minutes per day during which the user obtains the maximum LTE downlink rate.

The SISP instances have been tested for a 4 km^2 square area populated by 20000 users (N), which approximately represents Downtown Milan (referred to as area Z1). As far as the distribution of users (σ_i) among the considered operators is concerned, we defined two cases:

- Uniform distribution of users among operators: $\sigma = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$
- Non-uniform distribution of users among operators: $\sigma = \left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$

This was done in order to capture both the case when there are incumbent operators that have been present in the market for a long enough period, and the case when there is an incumbent operator and two smaller new players.

For each case, we tested all 20 instances that were generated from setting the rest of parameters (all 20 combinations that result from 4 values of γ and 5 values of δ).

4.1.2 MISP instances

For the MISP we considered five areas which approximately represent five dense areas of the city of Milan: Downtown Milan (4 km^2, 20000 users), San Siro

Stadium (0.5 km^2, 20000 users), Stazione Centrale (0.5 km^2, 20000 users), Fiera (1 km^2, 10000 users) and Città Studi (1 km^2, 10000 users).

The remaining parameters were set as described for SISP, since they do not depend on the specific size and population of each area. As a result, we tested 20 instances for each case (uniform and non-uniform user distribution) also for the MISP.

Table 4.1 recaps the values that were given to the common parameters of the SISP and the MISP, whereas Table 4.2 gives the notation and characteristics of the areas that have been considered for the MISP. In order to simplify the presentation of results throughout Sections 4.2 and 4.3, we name each type of objective function for which we investigated our models. Tables 4.3 and 4.4 link this notation with their exact definition (introduced in Chapters 2 and 3) respectively for the SISP and MISP. The same notation has been used for both scenarios since results are presented separately. Therefore the correspondence between notation and objective should be according to Tables 4.3 and 4.4 for the distinct scenarios.

Param.	Description	Value
n	Number of operators	3
m	Number of coalitions	7
γ	Monthly ROI from 1 user	{0,1,5,10} [€/(month×user)]
δ	Monthly price of 1 Mbps	{0.05,0.2,0.8,1,2} [€/(month×user×Mbps)]
η	Activity factor	0.001
ξ	OPEX annual percentage	15 %
g_{capex}	CAPEX of BS cost	3000 [€]
g	Normalized BS cost	7500 [€]
D	Investment lifetime	120 [months] (10 years)
σ_u	Uniform distribution	$\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$
σ_{n-u}	Non-uniform distribution	$\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$

Table 4.1: Common parameters of the SISP and the MISP

Symbol	Area	Number of users	Size
Z_1	Downtown Milan	N_1=20000	A_1=4 km^2
Z_2	San Siro Stadium	N_2=20000	A_2=0.5 km^2
Z_3	Stazione Centrale	N_3=20000	A_3=0.5 km^2
Z_4	Fiera Milano	N_4=10000	A_4=1 km^2
Z_5	Città Studi	N_5=10000	A_5=1 km^2

Table 4.2: Characteristics of the set of areas

Notation	Objective function
TOT_Q	$\max \sum_{i \in \mathcal{O}} q_i$
MIN_Q	$\max \min_{i \in \mathcal{O}} q_i$
$TOT_Q + MIN_Q$	$\max \left(\min_{i \in \mathcal{O}} q_i + \sum_{i \in \mathcal{O}} q_i \right)$
TOT_P	$\max \sum_{i \in \mathcal{O}} (r_i - c_i)$
MIN_P	$\max \min_{i \in \mathcal{O}} (r_i - c_i)$
$TOT_P + MIN_P$	$\max \left(\sum_{i \in \mathcal{O}} (r_i - c_i) + \min_{i \in \mathcal{O}} (r_i - c_i) \right)$

Table 4.3: *User-oriented* and *Operator-oriented* objective functions of the SISP

Notation	Objective function
TOT_Q	$\max \sum_{i \in \mathcal{O},\, a \in \mathcal{A}} q_i^a$
MIN_Q	$\max \min_{i \in \mathcal{O},\, a \in \mathcal{A}} q_i^a$
$TOT_Q + MIN_Q$	$\max \left(\min_{i \in \mathcal{O},\, a \in \mathcal{A}} q_i^a + \sum_{i \in \mathcal{O},\, a \in \mathcal{A}} q_i^a \right)$
TOT_P	$\max \sum_{i \in \mathcal{O},\, a \in \mathcal{A}} (r_i^a - c_i^a)$
MIN_P	$\max \min_{i \in \mathcal{O}} \sum_{a \in \mathcal{A}} (r_i^a - c_i^a)$
$TOT_P + MIN_P$	$\max \left(\sum_{i \in \mathcal{O},\, a \in \mathcal{A}} (r_i^a - c_i^a) + \min_{i \in \mathcal{O}} \sum_{a \in \mathcal{A}} (r_i^a - c_i^a) \right)$

Table 4.4: *User-oriented* and *Operator-oriented* objective functions of the MISP

4.1.3 Simulation of picocell BSs deployment

As stated in Chapters 2 and 3, nominal and therefore coalition rates were obtained through simulation in MATLAB R2014a environment [27]. This simulation provides the coalition user rate for any number u_s of BSs in the range [1, U_{max}]. In this subsection, we briefly describe the simulation and its input parameters.

BSs are added one by one, on a square area, in a random fashion but over predefined points of a square grid. The edge e_a of the square area is one of the inputs of the simulation. Every time a BS is added, the corresponding nominal (and therefore coalitional) rate is calculated. Calculations are carried out as follows.
A fixed number of sample users c are randomly distributed over the square area for each value of u_s. The number of sample users (c) is kept equal to 10 for all the simulations. The downlink SNR of the sample user, whose serving BS is i^{th} one from the deployed u_s, is calculated according to Equations (4.1) and (4.2).

$$SNR_s^a = \frac{P_i}{load_s \times \left(\sum_{j\in 1..u_s,\, j\neq u_s} P_j\right) + T_{noise}} \tag{4.1}$$

$$load_s = 1 - (1-\eta)^{\frac{N_s}{u_s}}. \tag{4.2}$$

T_{noise} is the power of the white gaussian noise derived as the product of the standard power spectral density and system bandwidth, which in this work was set to 10 MHz for any coalition. P_i is the power that sample user receives from its serving BS, that is, the one from which it receives the strongest signal (highest power level). The received power from the serving and non serving BSs is determined by making use of a simplified path loss model starting from the standard downlink transmitted power of a picocell BS. This model has three parameters: transmitted power (P_{tx}), fixed path loss (C_{pl}) and path loss exponent (Γ). The received power is therefore derived according to Equation (4.3), where d is the distance between the sample user and the BS:

$$P_{rx}[dBm] = P_{tx}[dBm] - C_{pl}[dB] - 10\Gamma log(d[km]). \tag{4.3}$$

The power received in downlink from non-serving BSs ($\sum_{j\in 1..u_s,\, j\neq u_s} P_j$) interferes with the transmission of the serving BS when nominal rate is being considered[1]. However, the load factor given by Equation (4.2) reduces the captured interference since users are characterized by an activity factor, that is, a percentage of time during which they demand nominal rate. Once the SNR is calculated, the corresponding rate is determined. Mapping of SNR to LTE nominal rate is done according to a multilevel SNR–to–rate scheme obtained from [28]. The latter provides higher rate to users with a better SNR.

A single value for the nominal rate is obtained by averaging over all 10 users. An additional averaging is obtained by applying 100 iterations every time we calculate the nominal rate for u_s BSs. At this point, the coalition rate that can be achieved by installing u_s BSs, is simply derived as the product between the nominal rate and $1 - load_s$ (referring to the definition of the coalition rate in Equations (3.6) of Chapter 3).

It is important to note that two of the input parameters of this simulation, that is, the edge size e_a and number of users that belong to the coalition s (N_s), vary for each area and each coalition. As a result, we have executed this simulation for each area of the MISP (but only for one in the case of the SISP) and for each coalition of both the uniform and non-uniform cases.

It is also important to note the difference between the sample users (c) and users of a coalition (N_s). Given that the sample users are randomly distributed in the area, they allow us to obtain an average value for the simulated nominal rate (by averaging over all 10 sample users nominal rate). That is why the sample users are not related to the users that make up a coalition, whose number is a variable input parameters of the simulation (and also of our proposed models).

The parameters of the simulation are summarized in Table 4.5. Their values are characteristic for an outdoor propagation scenario and picocell LTE technology.

[1]Given that all LTE resource blocks are needed to reach nominal rate, any other downlink transmission uses a subset or all these resources and unavoidably interferes.

Parameter	Description	Value
N_s	Number of users that belong to coalition s	Depends on $s \in \mathcal{S}$
e_a	Edge size in km of square area a	Depends on $a \in \mathcal{A}$
c	Number of sample users	10
u_s	Variable number of BSs	[1,1000]
I	Number of iterations	100
η	Activity factor	0.001
U_{max}	Max number of BSs for any area	1000
P_{tx}	Picocell BS Downlink emitted power	20 dBm
C_{pl}	Fixed path loss	140.7 dB
Γ	Path loss exponent	3.67
T_{noise}	Thermal noise power for 10 MHz	-94 dBm
p	Number of linear pieces	6

Table 4.5: Simulation input parameters

A second important component in computing model parameters is the adaptive piece-wise linearization algorithm that applies to the coalition rate. This algorithm has only one parameter, that is, the number of linear pieces p. It is adaptive because it chooses the piece width inversely proportional to the gradient of the underlying non-linear function. An example of the nominal rate, coalition user rate and the linearization obtained by the adaptive algorithm was given in Figure 3.1, Chapter 3.

By applying the piece-wise linearization algorithm, we obtain the linearization parameters of both the SISP (U_s^k, α_s^k, R_s^k, R_s^{max}) and the MISP (U_{sk}^a, α_{sk}^a, R_{sk}^a, R_{smax}^a).

4.2 Results of the SISP

This section reviews the main outcomes of each tested instance for the SISP. However, it does not focus on quantitative aspects such as user rate and return on investment. They will be addressed in the next section for the MISP since it is more realistic and gives more insight for a possible practical application of our models.

Results of this section were obtained for area Z1 (Downtown Milan) for both the uniform case (Subsection 4.2.1) and non-uniform one (Subsection 4.2.2). Subsection 4.2.3 provides alternative results for the non-uniform case when the available set of BSs is fairly distributed among operators.

In all subsections, results are presented in tabular fashion; each figure provides the selected coalitions and the corresponding number of activated BSs in each of them, for the 20 different instances related to one of the considered objective functions. Columns show how the increase of the expected return on investment(γ) makes investment unaffordable when users are willing to pay little for an improved service (small δ). Contrarily, rows illustrate how the increase of δ for a fixed value of γ allows operators to install more BSs and improve both user rate and their return on investment.

In the following subsections it will be seen how results for the uniform and non-uniform case are quite similar. The key outcome is that when affordable, there is almost always collaboration among operators. Particular behaviors for the two cases are further addressed in their corresponding subsections.

4.2.1 Uniform user distribution case

Figures 4.2, 4.3, 4.4 and 4.5 summarize the results obtained for the SISP when users are uniformly distributed among MNOs A, B and C. Since MNOs have the same number of users, there are some equivalent outcomes, *i.e.*, the solution in which operators A and B collaborate while C invests by himself is equivalent to the solution in which A collaborates with C instead, and B creates a singleton or alternatively A invests by itself while B and C collaborate. Consequently, there are four distinct outcomes: no investment ($\{\phi\}$), 3 singletons ({A, B, C}), a singleton and a coalition of 2 MNOs (either {AB, C}, {AC, B} or {BC, A}), and the big coalition ({ABC}) which are represented by four distinct colors (Figure 4.1). Numbers reported in each cell represent the number of BSs installed for each coalition. For the big coalition, the total number of BSs that will be shared among operators is given, whereas for the coalition of two and a singleton, the first number refers to the number of BSs activated for the singleton while the second one is the number of BSs installed in the coalition of the remaining two MNOs.

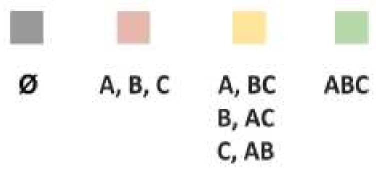

Figure 4.1: Coalitions color legend – Uniform user distribution – SISP

γ / δ	0.05	0.1	0.2	0.8	1	2
0	626	279 721	279 721	279 721	279 721	279 721
1		281 719	279 721	279 721	279 721	279 721
5			279 721	279 721	279 721	279 721
10				279 721	279 721	279 721

Figure 4.2: Selected coalitions and number of activated BSs – TOT_Q – Uniform user distribution – SISP

γ / δ	0.05	0.1	0.2	0.8	1	2
0	626	1000	1000	1000	1000	1000
1		1000	1000	1000	1000	1000
5			1000	1000	1000	1000
10				1000	1000	1000

Figure 4.3: Selected coalitions and number of activated BSs – MIN_Q and $TOT_Q + MIN_Q$ – Uniform user distribution – SISP

γ / δ	0.05	0.1	0.2	0.8	1	2
0	186	100 150	762	279 721	279 721	279 721
1		100 150	762	279 721	279 721	279 721
5			762	279 721	279 721	279 721
10				279 721	279 721	279 721

Figure 4.4: Selected coalitions and number of activated BSs – TOT_P – Uniform user distribution – SISP

γ / δ	0.05	0.1	0.2	0.8	1	2
0	186	427	762	1000	1000	1000
1		427	762	1000	1000	1000
5			762	1000	1000	1000
10				1000	1000	1000

Figure 4.5: Selected coalitions and number of activated BSs – MIN_P and $TOT_P + MIN_P$ – Uniform user distribution – SISP

The first observation that can be made is that, for none of the objective functions, all operators invests by themselves, that is, outcome {A,B,C} is never present. It means that even under *operator-oriented* objective functions that focus on the return on investment, it is still attractive for operators to cooperate.

As far as *user-oriented* objective functions are concerned, MIN_Q (Figure 4.3) leads to the *big coalition* (outcome {ABC}) for any affordable instance (combination of γ and δ). Instead, for the *user-oriented* objective function that maximizes the total user rate (TOT_Q), for most instances, a coalition of two operators and a singleton are created (Figure 4.2). The big coalition is created only for the smallest value of δ, which shows that when users are less willing to pay for an improved service, there is more incentive for collaboration among all operators and not only a subset. Objective function $TOT_Q + MIN_Q$, which was introduced with the purpose of obtaining a more balanced solution among operators with respect to the users rate, gives the same solution as MIN_Q. This comes from the fact that the coalitions of two, created under objective TOT_Q, disadvantage the user rate provided by the operator that invests by himself, even though the total user rate (over all operators) is higher. Under either objective $TOT_Q + MIN_Q$ or MIN_Q, the total user rate is lower but the solution is fair to all the operators[2].

In general, the same behavior is observed also for the *operator-oriented* objective functions, with the only difference that less BSs are installed for the smallest half values of δ (0.05, 0.1 and 0.2) with respect to the number of BSs installed for the corresponding *user-oriented* objective function. For instance, when δ equals 0.1 and γ equals 0, while maximizing TOT_Q, a singleton with 279 BSs and coalition of two with 721 BSs are created (Figure 4.2), whereas when we maximize

[2]That is, all operators provide the same user rate.

TOT_P (Figure 4.4), the coalitions are the same but the number of BSs for the singleton and the coalition of two operators is respectively 100 and 150. As it can be observed from Figure 4.5, under objectives MIN_P and $TOT_P + MIN_P$, only the big coalition is created with again a smaller number of BSs (*e.g.*, when δ=0.05 and γ=0 under MIN_Q there are 626 BSs, whereas under MIN_P only 186 as it can be observed from Figures 4.3 and 4.5).

If we read these figures horizontally, we can see that by increasing the value of δ, that is, the user's willingness to pay for an improved service, more BSs are installed under both *user-oriented* and *operator-oriented* objective functions. On the contrary, if users are willing to pay less they will, as expected, experience a worse service. A particular outcome occurs when δ equals 0.05 and γ equals 0 under *user-oriented* objective functions for whom less than 1000 BSs are installed even when maximizing the total user rate. It means that even if the operators are not prioritizing their most important indicator, that is, the return on investment but the quality provided to their users (user rate), if they want to have a positive return on investment, they need to limit their investment when users are willing to pay very little. The same holds for more than one small value of δ (0.05, 0.1 and 0.2) but under *operator-oriented* objective functions, since they focus on the return on investment instead.

A vertical reading of these figures, shows the effect that the increase of operator's expectations on the return on investment has in the outcome. The combination of high expectations with low user willingness to pay for an improved service leads to no investment at all, *e.g.*, when δ equals 0.1 and γ equals 5. As expected, the unaffordable instances (grey cells) are the same for any objective function, since the constraint that models operators' expectations on the return on investment is always present.

4.2.2 Non-uniform user distribution case

For the non-uniform user distribution case (A having $\frac{1}{4}$ of users, B $\frac{1}{2}$ and C the remaining $\frac{1}{4}$) there are five possible outcomes: no investment ($\{\phi\}$), the big coalition ({ABC}), 3 singletons ({A, B, C}), a coalition between A and C while B selects a singleton ({AC, B}) and finally two equivalent outcomes accounted as one ({BA, C} or {BC, A}). Each outcome is then represented by a distinct color (Figure 4.6).

Figure 4.6: Coalitions color legend – Non-uniform user distribution – SISP

γ / δ	0.05	0.1	0.2	0.8	1	2
0	627	348 652	348 652	348 652	348 652	348 652
1		1000	348 652	348 652	348 652	348 652
5			411 589	348 652	348 652	348 652
10				348 652	348 652	348 652

Figure 4.7: Selected coalitions and number of activated BSs – TOT_Q – Non-uniform user distribution – SISP

γ / δ	0.05	0.1	0.2	0.8	1	2
0	627	1000	1000	1000	1000	1000
1		1000	1000	1000	1000	1000
5			1000	1000	1000	1000
10				1000	1000	1000

Figure 4.8: Selected coalitions and number of activated BSs – MIN_Q and $TOT_Q + MIN_Q$ – Non-uniform user distribution – SISP

γ / δ	0.05	0.1	0.2	0.8	1	2
0	186	427	762	1000	1000	1000
1		427	762	1000	1000	1000
5			762	1000	1000	1000
10				1000	1000	1000

Figure 4.9: Selected coalitions and number of activated BSs – TOT_P and $TOT_P + MIN_P$ – Non-uniform user distribution – SISP

γ / δ	0.05	0.1	0.2	0.8	1	2
0	186	427	762	181 819	181 819	63 937
1		427	762	181 819	181 819	63 937
5			762	181 819	181 819	63 937
10				181 819	181 819	63 937

Figure 4.10: Selected coalitions and number of activated BSs – MIN_P – Non-uniform user distribution – SISP

The outcome of the *user-oriented* objective functions is very similar to the correspondings one for the uniform case. It is important to notice that, for the non-uniform case, the coalition of two created under objective TOT_Q is always between the smallest operators (A and C), whilst B invests by itself. Nevertheless, the number of BSs per coalition is different compared to the uniform case since each operator is accounted for a number of BSs proportional to its customer base. Objectives MIN_Q and $TOT_Q + MIN_Q$ still force the big coalition (Figure 4.8) in order to guarantee fairness among the operators with respect to the quality they offer to their users (user rate).

However, *operator-oriented* objective functions behave slightly different. Both $TOT_P + MIN_P$ and TOT_P lead to the big coalition (Figure 4.9). This result, even though rather counter-intuitive, shows how despite varying the parameters, when affordable, it is more beneficial, from a return on investment perspective, when all the operators invest together.

There are certain particular outcomes in Figure 4.10, observed under the *operator-oriented* objective function that maximizes the minimum return investment among all operators (MIN_P) when δ equals either 0.8, 1 or 2. In this case, the optimal solution is a coalition of the smaller operators and the singleton of the bigger operator with quite a large number of BSs assigned to the coalition and very few to the bigger operator. This is further emphasized if we increase δ from 0.8 to 2. For instance, when δ equals 0.8, the optimal solution assigns 819 BSs to the coalition of A and C and only 181 to the singleton of operator B. For δ equal to 2, this number is further reduced to only 63, while the remaining 937 BSs are activated for coalition AC. The reason behind this is that the big operator has twice as many users and consequently twice as much revenues for equal user rate with the other operators. By assigning more BSs to the smaller operators, MIN_P dramatically reduces the number of BSs for the big operator and consequently its user rate. By halving the user rate of operator B with respect to the one of operators A and C, objective MIN_P evens out the return on investment of the three MNOs.

The rest of observations concerning the effect that varying parameters δ and γ has on the affordability of the instance and on opening all the available BSs, remain valid also for the non-uniform case as it can witnessed from Figures 4.7, 4.8, 4.9 and 4.10.

4.2.3 Fair divison of BSs

The particular case that arises for the non-uniform user distribution case under the *operator-oriented* objective function MIN_P, comes from the fact that we are simply limiting the overall number of activated BSs over a given area. However, if we wanted to avoid such behavior, we could distribute the BSs among operators according to the number of their users, since, in practice, it is very unlikely to install much more BS for the smaller operators with respect to the bigger one in order to improve the return on investment of the operators with a smaller customer base. However, objective MIN_P that maximizes the minimum return on investment over all operators is imposed by a regulatory entity, which justifies the previous unfair division of BSs among operators.

γ / δ	0.05	0.1	0.2	0.8	1	2
0	627	1000	1000	1000	1000	1000
1		1000	1000	1000	1000	1000
5			1000	1000	1000	1000
10				1000	1000	1000

Figure 4.11: Selected coalitions and number of activated BSs – TOT_Q, MIN_Q and $TOT_Q + MIN_Q$ – Fair division of BSs – SISP

γ / δ	0.05	0.1	0.2	0.8	1	2
0	186	427	762	1000	1000	1000
1		427	762	1000	1000	1000
5			762	1000	1000	1000
10				1000	1000	1000

Figure 4.12: Selected coalitions and number of activated BSs – TOT_P, MIN_P and $TOT_P + MIN_P$ – Fair division of BSs – SISP

The key observations that were made in Subsection 4.2.1 and 4.2.2 are valid also for the fair distribution of BSs among operators.

Under this type of constraint, as it can be observed from Figures 4.11 and 4.12, the outcome is the same for all objectives, that is, operators will always join the big coalition when it is affordable. By constraining the distribution of BSs among operators, the outcome becomes indifferent to the different types of *user-oriented operator-oriented* objective functions. In other words, only with the unfair distribution of BSs among operators, it is possible to observe different behaviors for the different objectives and therefore analyze the effect that a regulatory entity can have in the outcome of our models.

4.3 Results of the MISP

Unlike for the SISP, the results of the MISP will be presented in both tabular and graphical fashion.

The tabular results of Subsections 4.3.1 and 4.3.2 have the same notation as before, that is, they give the coalitions that are created for each instance (for each combination of δ and γ) and for each considered objective function.

However, the number of BSs for each coalition is not reported as the trend is always the same: for the smaller half set of values of δ (0.05, 0.1 and 0.2) less than 1000 BSs per area are installed, whereas for the remaining three (δ equal to 0.8, 1, 2) the entire set of 1000 BSs will be shared among the operators in each area.

In Subsections 4.3.3 and 4.3.4 we focus on two main indicators: first, the average rate perceived by a user which depends on the area the user belongs to and the coalition joined by its operator (q_i^a) and second, the operator's total return on investment ($\sum_{a \in \mathcal{A}}(r_i^a - c_i^a)$). Throughout this section, we denote the user indicator by Q, whereas the operator indicator by P. For both uniform and non-uniform cases, we obtained Q_{ave} by averaging Q over all operators and areas and P_{ave} by averaging P over all operators for each considered objective function. Both P_{ave} and Q_{ave} are graphically represented as function of the different values of δ for all the considered values of γ, in order to analyze the effect that the different objective functions have on these important indicators.

In addition, certain instances (*i.e.*, combinations of δ and γ) are selected and the variation of Q with respect to P due to the different objective functions is analyzed for each operator, area and scenario without performing any averaging.

4.3.1 Uniform user distribution case

Since in our formulation operators joined the same coalition over all the areas, it is therefore possible to have a unique representation of the selected coalitions for all areas, exactly as it was done for the SISP (Figure 4.13).

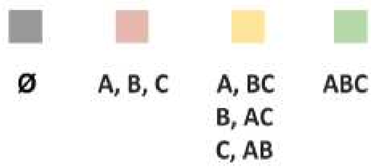

Figure 4.13: Coalitions color legend – Uniform user distribution – MISP

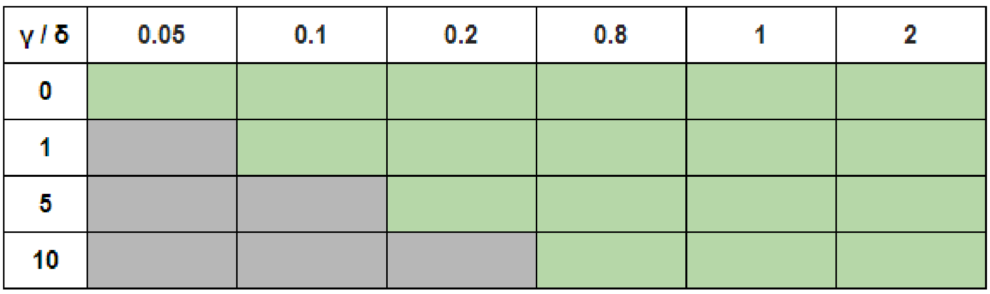

Figure 4.14: Selected coalitions – TOT_Q, MIN_Q and $TOT_Q + MIN_Q$ – Uniform user distribution – MISP

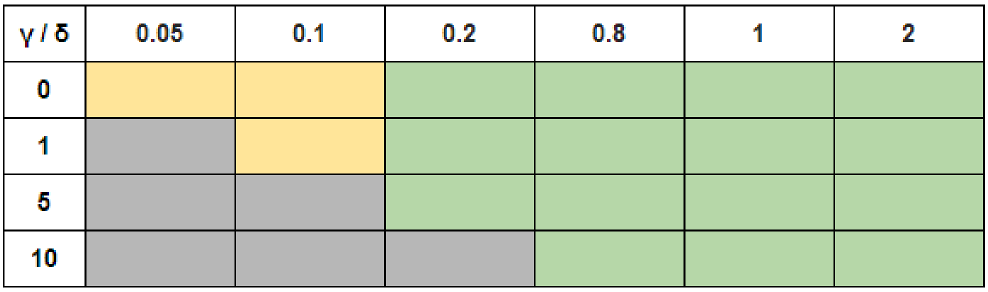

Figure 4.15: Selected coalitions – TOT_P – Uniform user distribution – MISP

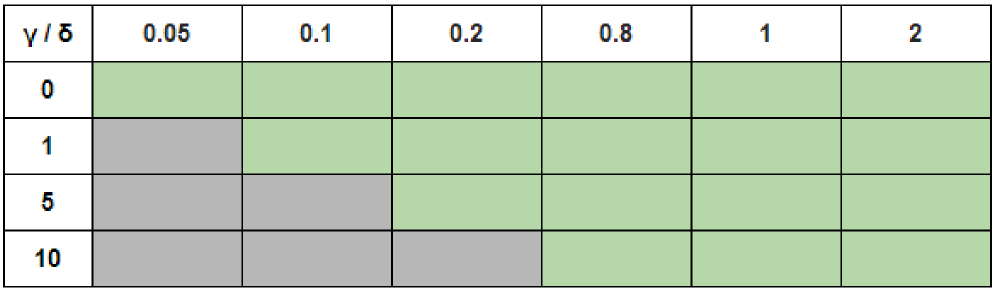

Figure 4.16: Selected coalitions – MIN_P – Uniform user distribution – MISP

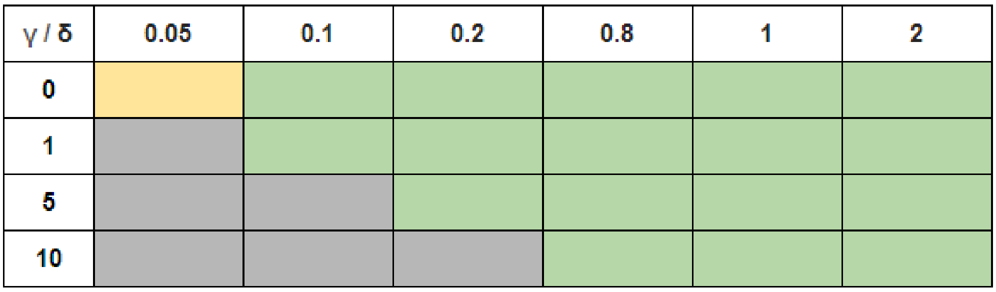

Figure 4.17: Selected coalitions – TOT_P+MIN_P – Uniform user distribution – MISP

Figure 4.14 demonstrates how the *user-oriented* objective functions always result in the big coalition when affordable to invest, even though with a larger number of activated BSs for higher values of δ. In other words, it is always in the users' best interest that all operators join the same coalition.

On the other hand, for the *operator-oriented* objective (Figure 4.15) TOT_P, a coalition of two operators and a singleton result altogether more beneficial for the operators when δ equals 0.05 and 0.1 (for the affordable values of γ). However, for the remaining values of δ, the solution is the one obtained by the *user-oriented* objective functions, that is, operators' and users' best interest provide the same solution. Objective function MIN_P forces the big coalition for all instances (Figure 4.16), in order not to disadvantage the operator that has to invest by itself when objective TOT_P applies (instances with δ 0.05 and 0.1). For the smallest value of δ, objective $TOT_P + MIN_P$ (Figure 4.17) provides the same outcome as TOT_P, otherwise it behaves exactly as MIN_P in order to provide a more balanced return on investment for all operators compared to the one provided by TOT_P.

4.3.2 Non-uniform user distribution case

This subsection summarizes the results of the Non-uniform user distribution case in the following figures.

Figure 4.18: Coalitions color legend – Non-uniform user distribution – MISP

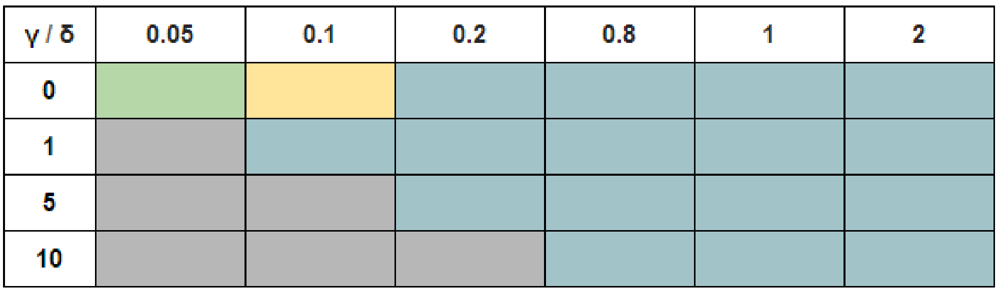

Figure 4.19: Selected coalitions – TOT_Q – Non-uniform user distribution – MISP

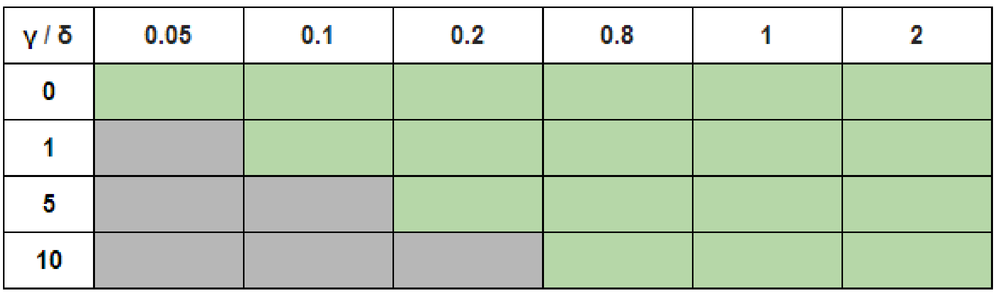

Figure 4.20: Selected coalitions – MIN_Q – Non-uniform user distribution – MISP

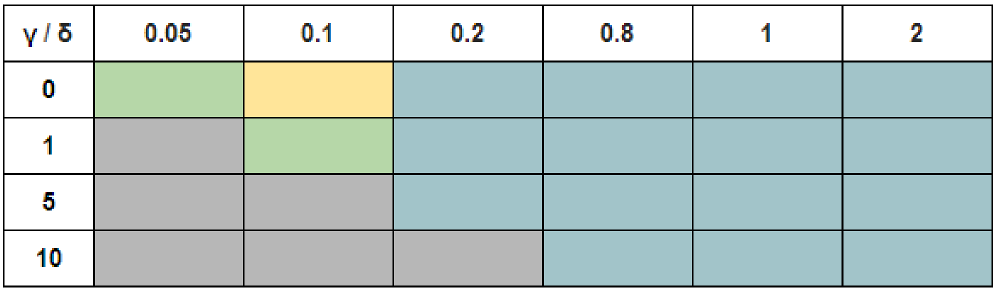

Figure 4.21: Selected coalitions – $TOT_Q + MIN_Q$ – Non-uniform user distribution – MISP

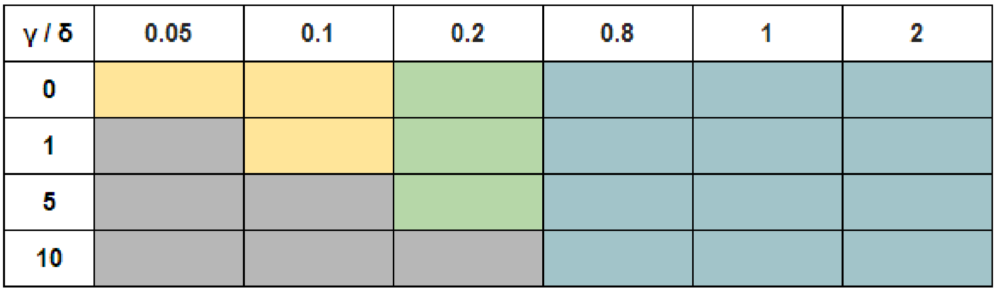

Figure 4.22: Selected coalitions – TOT_P and $TOT_P + MIN_P$ – Non-uniform user distribution – MISP

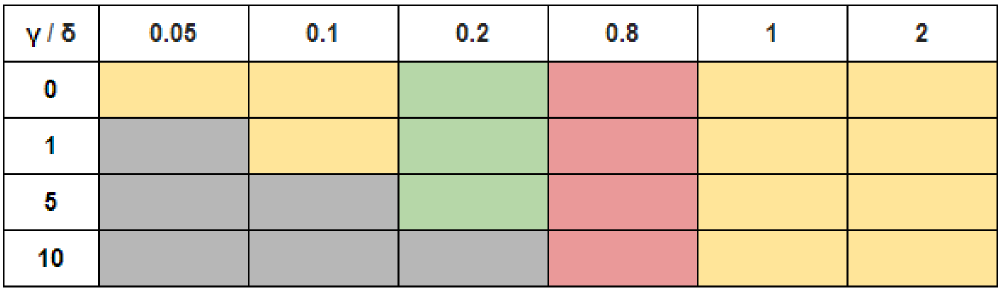

Figure 4.23: Selected coalitions – MIN_P – Non-uniform user distribution – MISP

Contrarily to the uniform user distribution case, for the non-uniform one, the *user-oriented* objective function TOT_Q provides as outcome the big coalition only for one small value of δ (0.05). For larger values, the coalition between a smaller and a bigger operator and a singleton of the remaining operator is persistent (either {BA, C} or {BC, A}). The big coalition is forced only by objective function MIN_Q, whereas $TOT_Q + MIN_Q$ gives in most cases the same solution as TOT_Q (Figures 4.19, 4.20 and 4.21). In other words, only for the objective function that considers the user rate of the worst served users (MIN_Q), operators would have to collaborate all together. Otherwise, either outcome {BA, C} or {BC, A} provides the highest total user rate (over all areas and operators), even though it is unfair to users of the operator that has to invest by itself.

From Figure 4.22 we can see that outcomes for objectives TOT_P and $TOT_P + MIN_P$ coincide. For δ equal to 0.8, 1 and 2, these outcomes are also the same as the one obtained by TOT_Q and $TOT_Q + MIN_Q$, which indicates once more that when users are willing to pay more for the improved service, users' and operators' best interest coincide, that is, users experience the highest rate while operators maximize their return on investment. The particular behavior of the *operator-oriented* objective function MIN_P when the bound on the number of BSs applies to the overall number of BSs, can be observed from Figure 4.23 also for the MISP. For instance, when δ is equal to 0.8 all operators invest by themselves and the number of BSs assigned to the bigger operator in each area is much smaller compared to the other two, which results in a small user rate and therefore reduced return on investment, allowing the smaller operators (who obtain much more BSs) to even out their return on investment with the bigger operator. The same applies to the outcomes for δ equal to 1 and 2, except for the fact that the smaller operators collaborate and equally share the largest part of the 1000 available BSs per area.

4.3.3 User rate and return on investment – Uniform case

For the MISP the user rate depends on the coalition selected by the corresponding operator and on the area the user belongs to (q_i^a). Therefore, we derived the average quality (Q_{ave}) among all areas and operators for all instances and for each objective function ($Q_{ave} = \frac{\sum_{a \in \mathcal{A}, i \in \mathcal{O}} q_i^a}{n \times l}$).

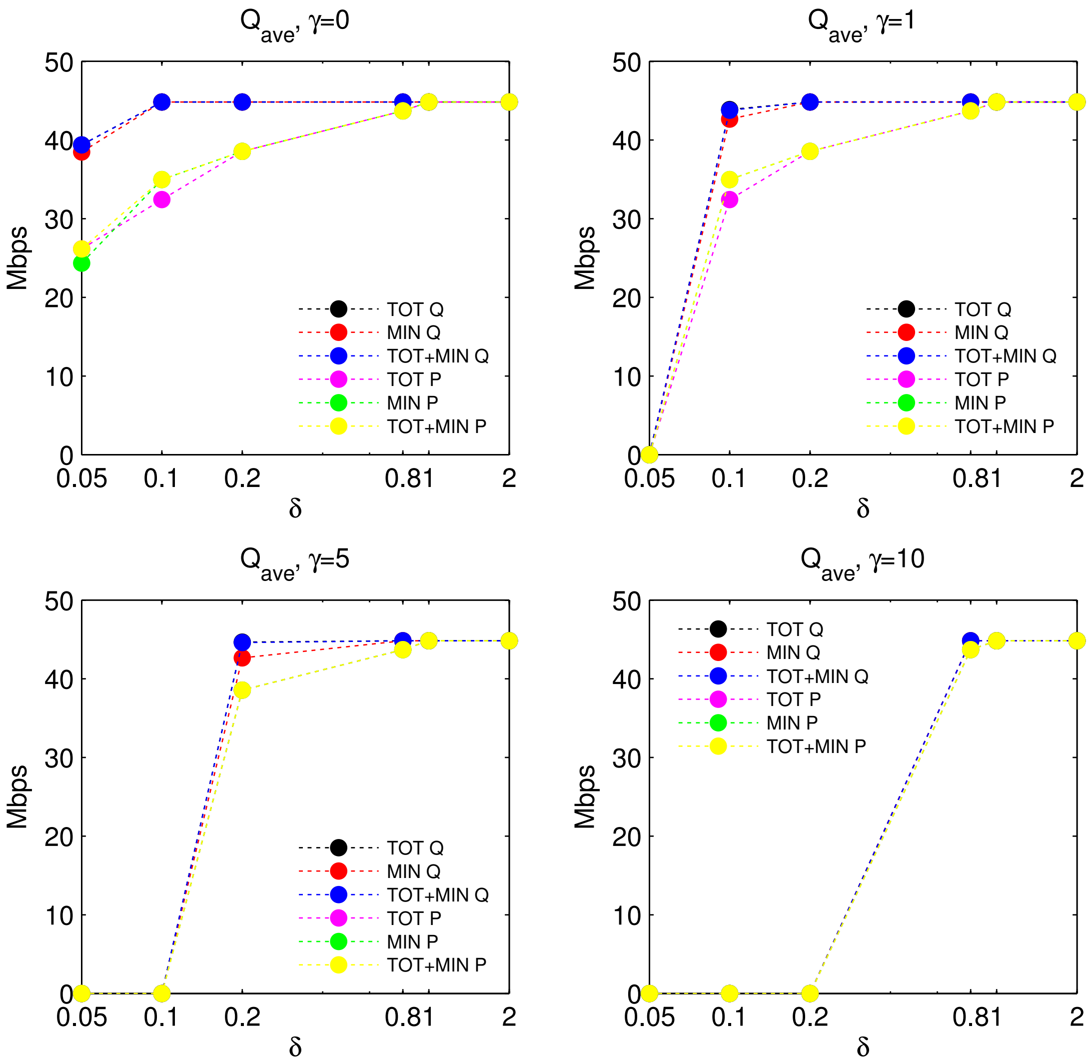

Figure 4.24: Average user rate *vs.* δ for each value of γ and each objective – Uniform user distribution

The average user rates (Q_{ave}) obtained for each objective function have been plotted with respect to the considered values of δ, separately for each value of γ.

Figure 4.24 illustrates how the increase of the operator's expectations on the return on investment (that is, larger value of γ) reduces the number of instances for which investment is affordable while increasing δ (represented by $Q_{ave} = 0$). For instance, when γ equals 10, it is affordable to invest starting only from δ equal to 0.8, otherwise no investment can be made.

For each value of γ, it is possible to examine the effect that different objective functions have on Q_{ave}. The *user-oriented* ones follow more or less the same trend. TOT_Q and $TOT_Q + MIN_Q$ provide the same average user rate, whereas MIN_Q leads, on the average, to a slightly lower user rate for smaller values of δ (0.05, 0.1 and 0.2). Instead, when δ equals 0.8, 1 or 2 the three *user-oriented* objectives provide the same average user rate. This result suggests that, objective MIN_Q improves the rate of the worst served users (minimum user rate) at the cost of lowering the average rate with respect to the one provided by the other two *user-oriented* objectives.

On the other hand, the *operator-oriented* objective functions provide a lower average user rate with respect to the *user-oriented* ones. The gap between the two is approximately 15 Mbps for δ equal to 0.05, but starts shrinking when δ increases. This is due to the fact that higher user rates are achieved by increasing the number of installed BSs and therefore, increasing costs per operator. If users are willing to pay very little for the improved service (*e.g.*, 0.05 € monthly for 1 Mbps), it is not in the operators best interest to invest in more infrastructure, since revenues from its users cannot make up for the costs and as a result, its total return on investment is not maximized. However, for δ equal to 1 and 2, on the average, all objective functions provide the same user rate. In our model, revenues per operator are a linear function of the user rate. That is why, for high values of δ, the increase of revenues due to higher user rate (larger number of activated BSs) dominates the increase of costs that comes with investing in more infrastructure and therefore, generates higher returns of investment, being the latter the subject of the *operator-oriented* objectives. In other words, also through the behavior of Q_{ave}, it possible to confirm that user's and operator's best interest coincide only when users are willing to pay more for an improved service.

The average user rate provided by the *operator-oriented* objective functions is the same in most cases. For the smallest values of δ (0.05, 0.1 and 0.2), objectives TOT_P+MIN_P and MIN_P provide a slightly higher user rate compared to TOT_P. The reason behind this is that the latter objective, in order to maximize the return on investment, limits the investment (number of BSs) and as a result, both costs and average user rate. $TOT_P + MIN_P$ and MIN_P (which improve the return on investment of the operators with the smallest investment gain) on the other hand, result in lower global return on investment with respect to TOT_P, but with slightly higher user rate.

Similarly, the average total return on investment (P_{ave}) has been calculated over all the operators ($P_{ave} = \frac{\sum_{i \in \mathcal{O}, a \in \mathcal{A}} (r_i^a - b_i^a)}{n}$) and plots with respect to δ are obtained for each value of γ and each objective function (Figure 4.25).

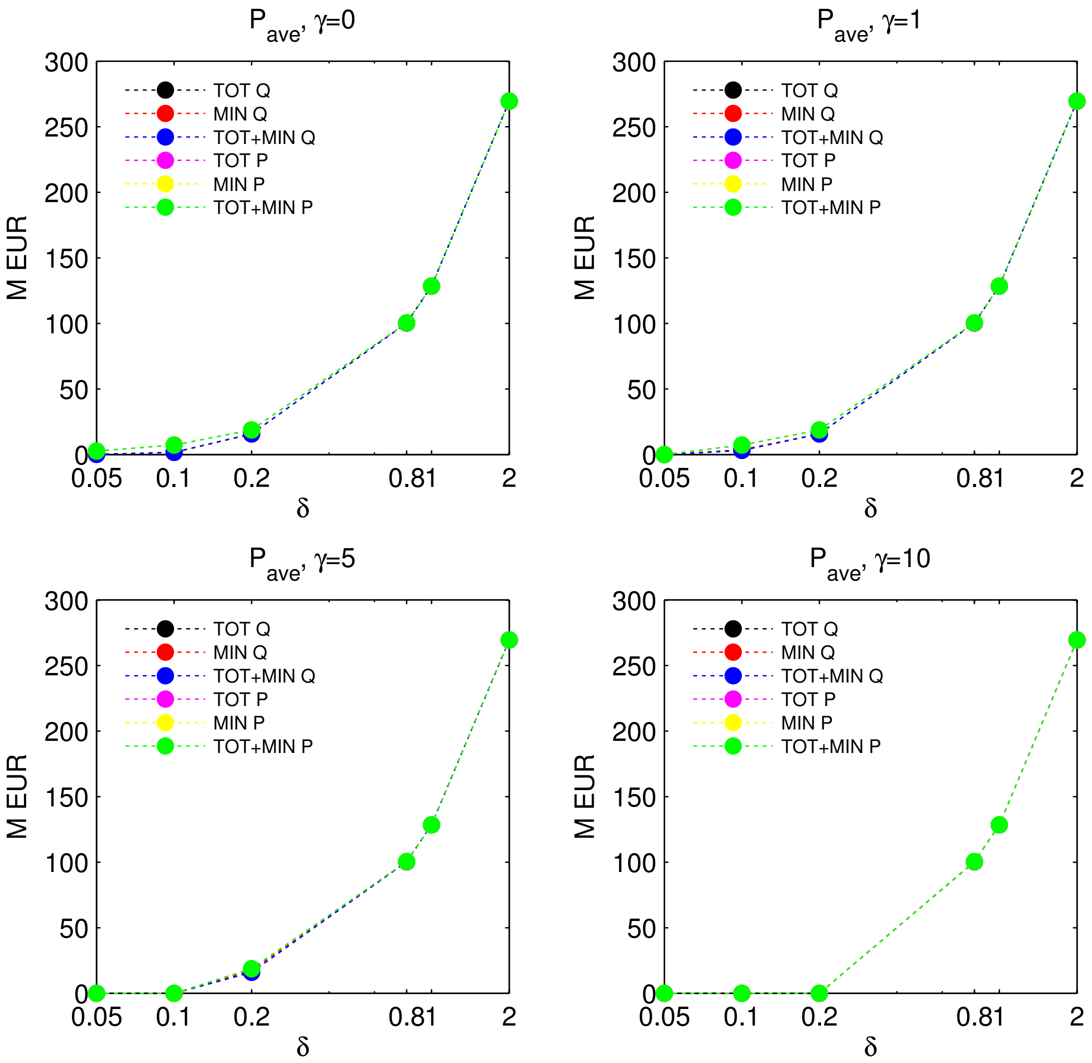

Figure 4.25: Average total return on investment *vs.* δ for each value of γ and each objective – Uniform user distribution

P_{ave} seems to be rather insensitive to the objective function. Differences are observed only for small values of δ. In such cases, the average total return on investment obtained from the group of *user-oriented* objective functions is smaller compared to the one obtained from the *operator-oriented*. For instance, when δ is equal to 0.1 and γ equal to 0, there is a factor of 6 in the difference between the *operator oriented* P_{ave} (7.25 million €) and the *user-oriented* P_{ave} (1.6 million €). For a twice as large δ (0.2), the difference is reduced to only 19.3 %. When δ equals 0.8, the difference is further reduced to 1.55%. For the remaining values of δ (1 and 2), P_{ave} is the same for both types of objectives, since the solution becomes independent from the objective. In other words, when users are willing to pay more, they will perceive a better service even when optimizing the return on investment and not user rate. Therefore, the two families of objective functions tend to provide similar behavior.

The rest of this subsection uses an opposite approach to illustrate the results with respect to the one that has been used so far. Instead of averaging indicators Q and P[3], we graphically represent their exact values for each operator and area only for 3 selected instances (Figures 4.26, 4.27 and 4.28).

The user rate per operator and area is plotted with respect to the corresponding return on investment of the area for two *user-oriented* objective function (TOT_Q and MIN_Q) and two *operator-oriented* objective function (TOT_P and MIN_P). The outcome of objectives $TOT_Q + MIN_Q$ and $TOT_P + MIN_P$ has not been reported since they provide the same solution respectively as TOT_Q and TOT_P for the selected instances. It is important to notice that, for the uniform case, both the user rate and return on investment per area are identical for all operators in case they all collaborate (the outcome is the big coalition), otherwise a different symbol has been used to represent each selected coalition.

[3]For the selected instance, P refers to the return on investment per area and not the total.

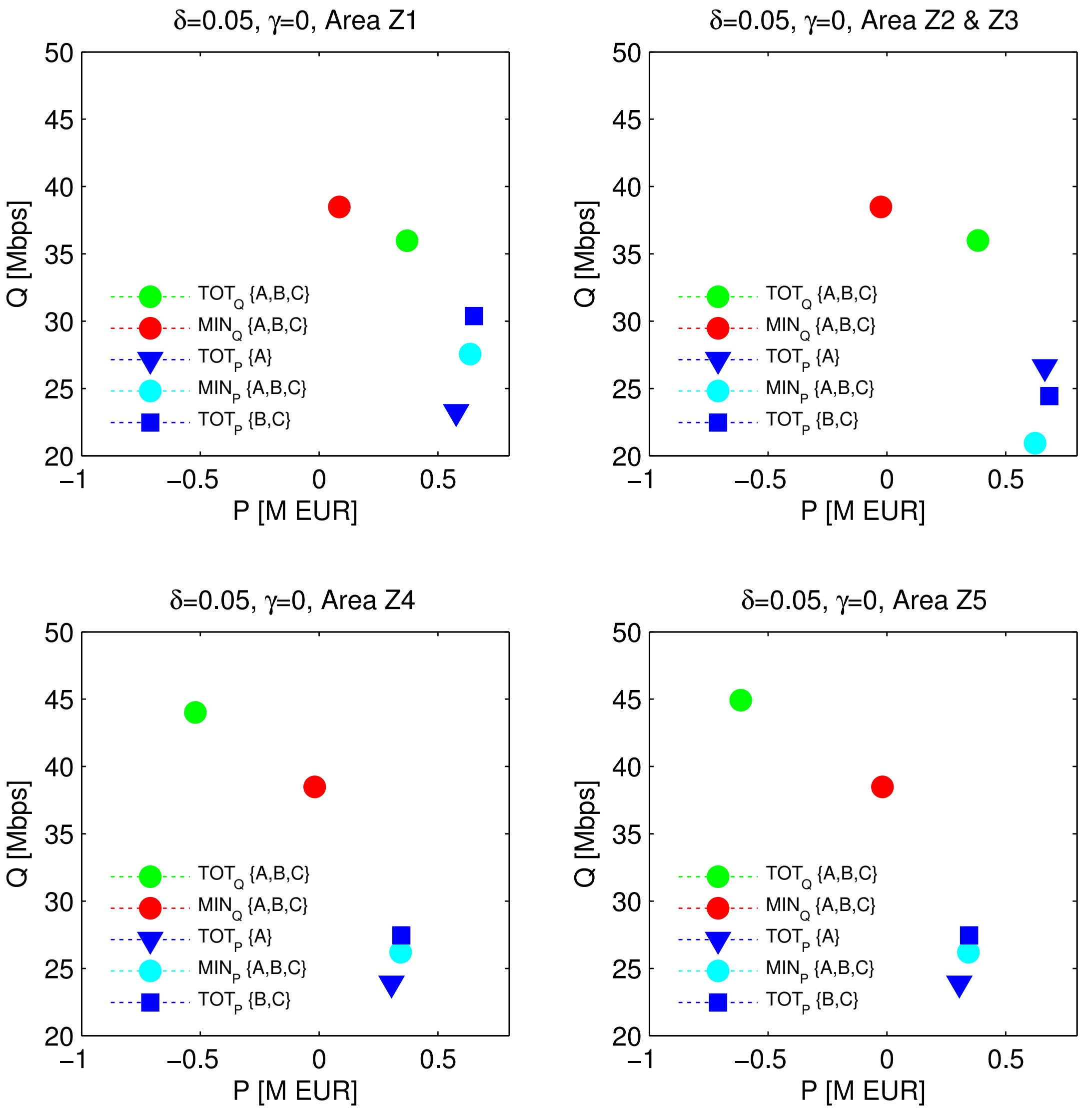

Figure 4.26: User rate *vs.* return on investment for each area and MNO – δ=0.05,γ=0

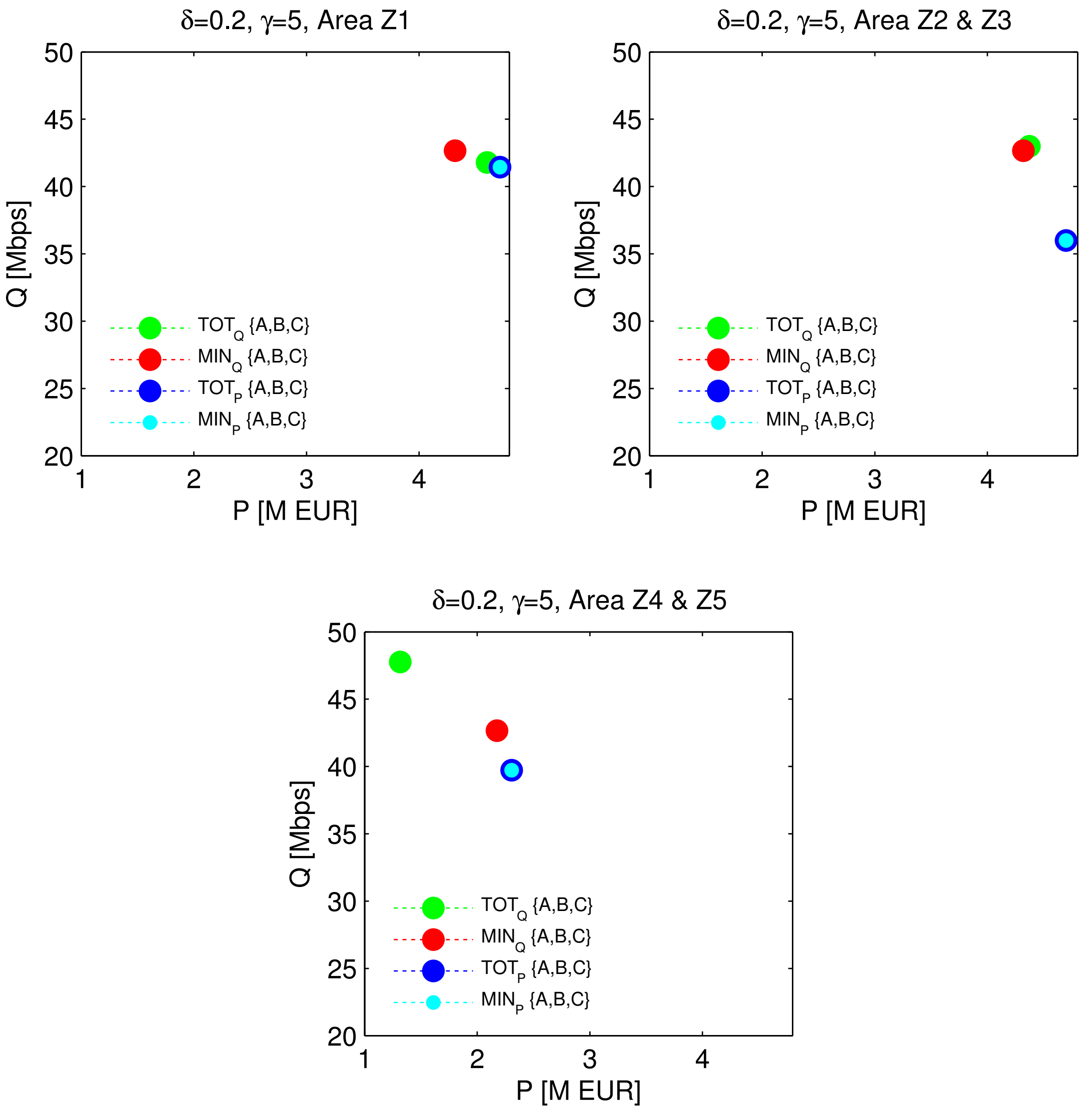

Figure 4.27: User rate *vs.* return on investment for each area and MNO – δ=0.2,γ=5

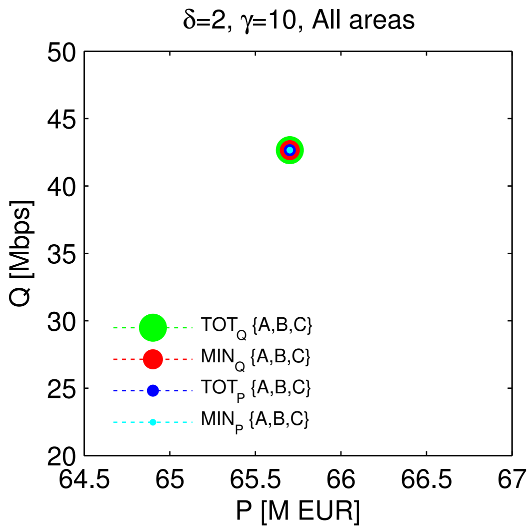

Figure 4.28: User rate *vs.* return on investment for each area and MNO – δ=2,γ=10

For the first two instances (Figures 4.26 and 4.27), the *user-oriented* objective functions provide higher user rate but less return on investment in all the areas compared to the *operator-oriented* ones. As the operator's best interest is to maximize its profit, it will have to face the tradeoff of offering less quality to its users when they are not willing to pay much for an improved service.

Due to our definition of objectives MIN_Q and TOT_Q, we expect the former to lead to a smaller user rate with respect to the latter. Such behavior was verified in the previous plots for the average user rate (Q_{ave}). However, as it can be observed in Figure 4.26 (areas Z1, Z2, Z3) and Figure 4.27 (area Z1), higher user rates are obtained by MIN_Q compared to TOT_Q. For the rest of areas, such expectations are satisfied. This contradictory behavior is related to the fact that different distribution of BSs apply to the different areas under TOT_Q and MIN_Q in order to satisfy a global lower bound on the return on investment (and not one for each area). For instance, in area Z1, 427 BSs are activated for the big coalition under objective TOT_Q, whereas 581 under MIN_Q. Instead, in area Z4 (which behaves as expected), there are 350 activated BSs for the big coalition under MIN_Q and 605 under TOT_Q.

For the first instance (δ equal to 0.05 and γ equal to 0), the *operator-oriented* objective function TOT_P leads to the coalition of two operators and a singleton[4]({BC,A}). For any of the areas in Figure 4.26, it is possible to see how the

[4]Any of the two operators as they are identical.

coalition of two operators (BC) always results in higher return on investment (value of P) compared to the singleton (A), which justifies its selection. Since MIN_P improves the return on investment for operator A at the cost of reducing it for operators B and C with respect to the solution provided by TOT_P, the return on investment under MIN_P [5] lies between the return on investment provided by TOT_P for operator A (singleton) and operators B and C (coalition BC).

Figure 4.26 shows how for areas Z4 and Z5, the *user-oriented* objective functions lead to negative return on investment even though the total positive return on investment is satisfied. This is in line with what can occur in reality, as operators do not focus on particular areas, as long as they are satisfied by the overall return on investment.

The third instance (Figure 4.28) shows how for the highest value of δ that we have considered, the solution is indifferent to the objective function: in all areas the entire set of 1000 BSs is uniformly shared among operators. It highlights once more the similar behavior of the different type of objective functions when users are willing to pay more for an improved service (previously observed in Figures 4.24 and 4.25). Any further increase of user's willingness to pay for improved service would be useless even though counter-intuitive in the first place.

4.3.4 User rate and return on investment – Non-uniform case

Average user rate (Q_{ave}) plots for the non-uniform case are provided in Figure 4.29.

[5] Which is the same for all operators since they are part of the same coalition and have the same number of users.

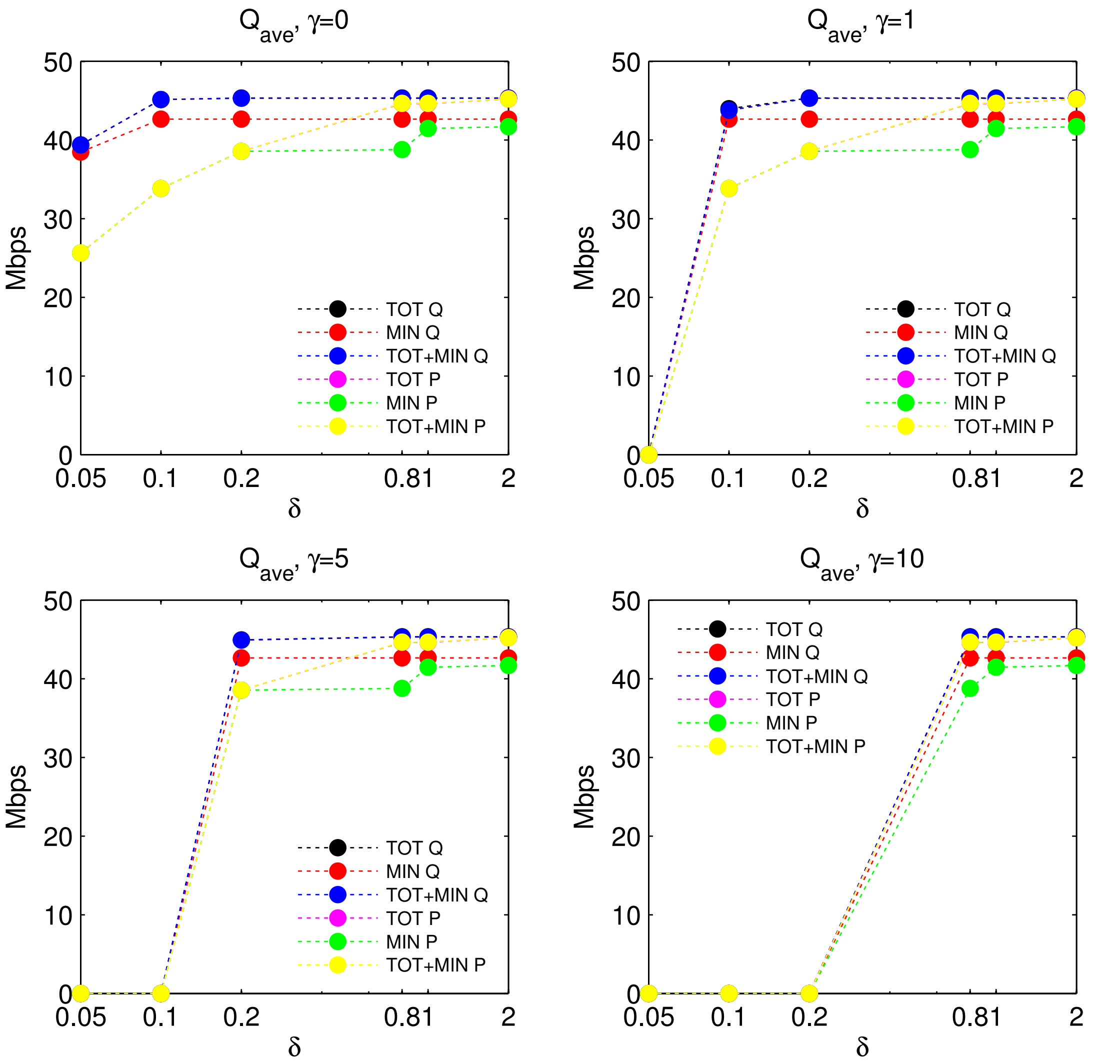

Figure 4.29: Average user rate *vs.* δ for each value of γ and each objective – Non-uniform user distribution

The general observations regarding the behavior of Q_{ave} for the uniform case are valid also for the non-uniform one. For instance, we can still notice a gap of approximately 15 Mbps in the average user rate between the *user-oriented* and *operator-oriented* objectives for δ equal to 0.05 and γ equal to 0. The same instances that are unaffordable for the first case, remain unaffordable also for this one.

However, due to the unbalanced distribution of users among operators, that is translated into different revenues for the same user quality (and, as a result, into a different return on investment), the average user quality provided by different types of objective functions belonging to the same family is quite different. Firstly, TOT_Q and $TOT_Q + MIN_Q$ behave in most cases exactly the same (their Q_{ave} plots overlap). The same observation holds for the corresponding *operator-oriented* ones. However, independently of the increase of δ, the Q_{ave} provided by MIN_Q is about 3 Mbps less compared to the one provided by TOT_Q (or $TOT_Q + MIN_Q$), which is nevertheless only a marginal 7% of difference. This is persistent for all values of δ since the outcome with the largest occurrence for objective TOT_Q is the coalition between the bigger operator and a smaller one and the singleton of the remaining smaller operator ({BC,A} or {BA,C}). MIN_Q on the other hand, achieves fairness by providing the big coalition as the outcome of all instances, which in turn means that on the average the user rate will be lower.

As far as the *operator-oriented* objectives are concerned, contrarily to the uniform case, the difference in Q_{ave} between TOT_P and MIN_P is emphasized for the largest value of δ (0.8, 1 and 2), whereas for the rest they behave the same. This is due to the fact that the user rate of operator B ($\frac{1}{2}$ users) needs to be halved in order to even out its return on investment with the one of the smaller operators. Consequently, the corresponding average user rate (Q_{ave}) will be lower with respect to TOT_P.

Figure 4.30 provides the P_{ave} plots for the non-uniform case.

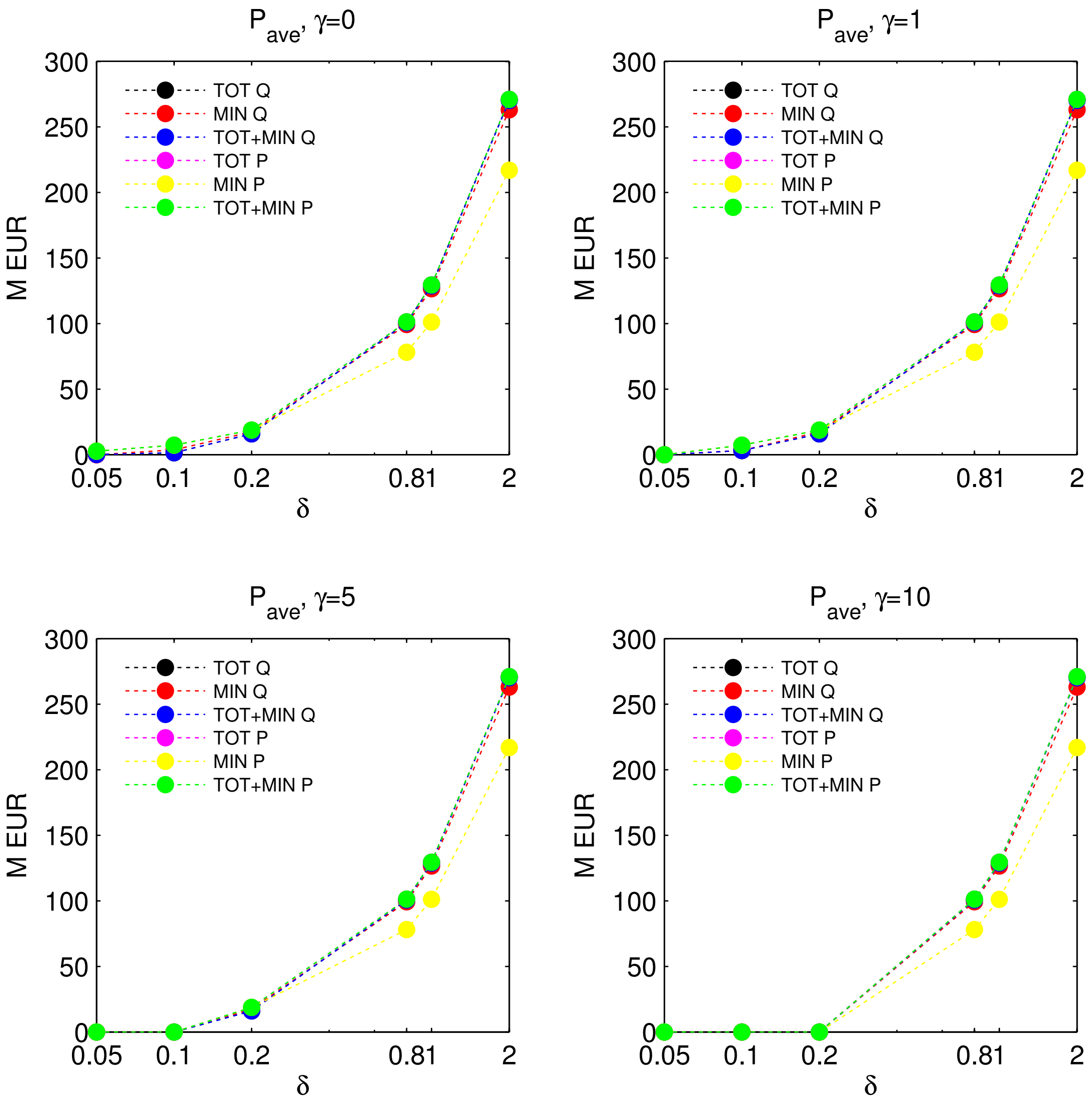

Figure 4.30: Average total return on investment *vs.* δ for each γ and each objective – Non-uniform user distribution

The same observation that was made for the Q_{ave} plots regarding the particular behavior of MIN_P and TOT_P, can be verified also through the average total return on investment (P_{ave}) plots. From the return on investment perspective, the highest gap between the two is approximately 25% (217 million € for MIN_Q and 271 million € for TOT_Q when δ equals 2). On the average, this value indicates the loss in return on investment for the bigger operator when a regulatory entity forces objective MIN_P to improve the financial position of the operator with the minimum return on investment.

For δ equal to 0.05, 0.1 and 0.2, the advantage in P_{ave} of the *operator-oriented* objectives with respect to *user-oriented* ones is noticeable. For δ equal to 0.05 and γ equal to 0, TOT_Q provides on the average only 788 €, whereas TOT_P about 2.7 million €. This suggests that, the *user-oriented* objective merely satisfies the constraint on having a positive return on investment while providing, on the average, a 15 Mbps higher user rate. With the increase of δ the difference in rate becomes negligible, so does the difference in return on investment (only 1.5% for δ equal to 2).

In particular, for δ equal to 0.8, 1 and 2, it is important to notice the difference in return on investment between TOT_Q and MIN_Q. As observed from the Q_{ave} plots, there is a persistent 3 Mbps gap in average user rate, for almost all values of δ, which is translated into about 3% of difference in the average return on investment for δ equal to 2, 1.9% for δ equal to 1 and only 1.5 % for δ equal to 0.8. On the average, these results can be considered as an indicator of the loss experienced by the bigger operator in its global return on investment when a regulatory entity requires that the service level of the worst served users is improved.

The remaining part of this subsection is dedicated to the analysis of three selected instances. For each of them, the user rate per operator and area (Q) is plotted as a function of the corresponding return on investment per area (P)[6].

[6] Also in this subsection, P indicates the return on investment of each area and not the total.

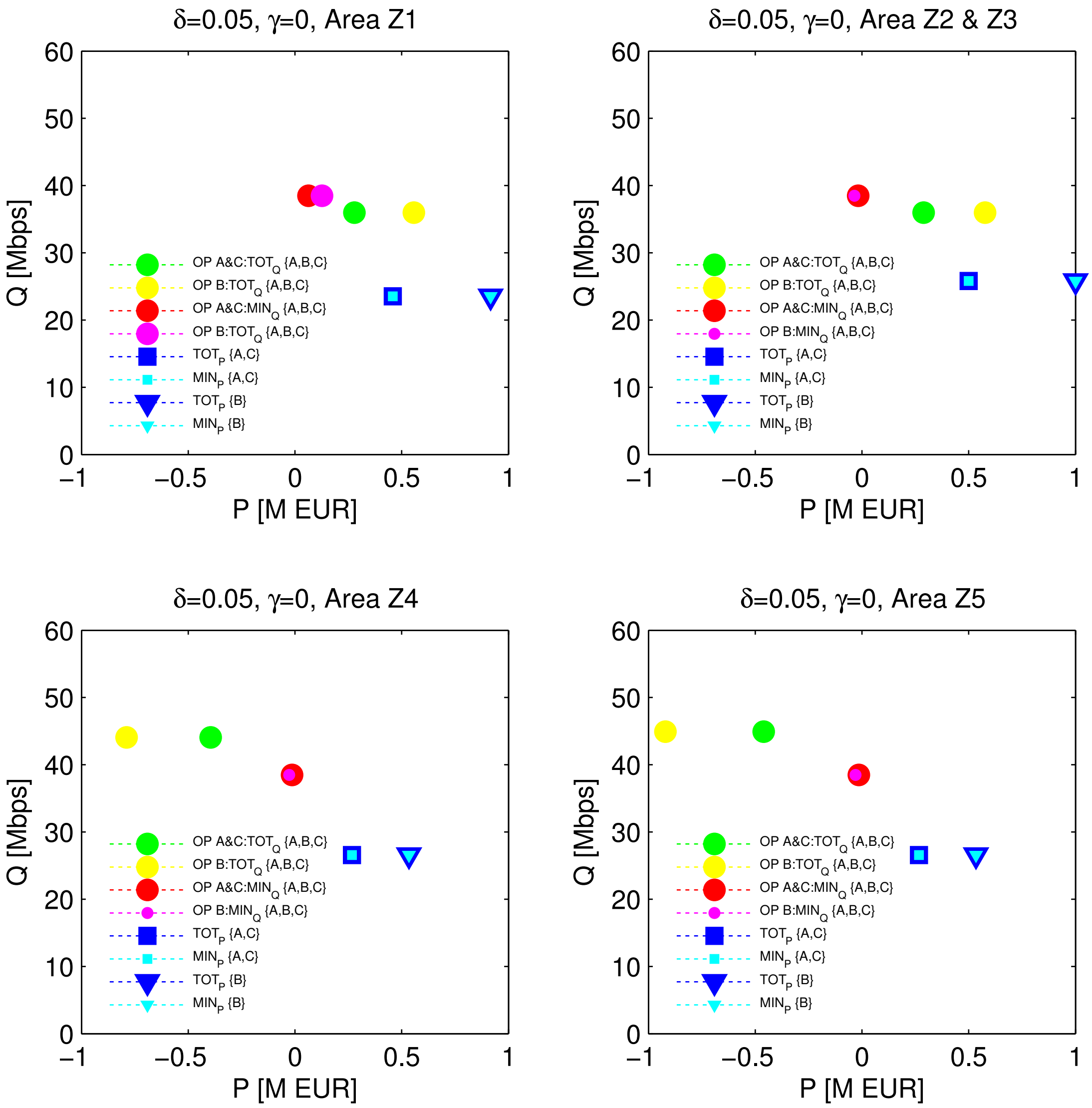

Figure 4.31: User rate *vs.* return on investment for each area and MNO – δ=0.05,γ=0

As far as the first instance (δ equal to 0.05 and γ equal to 0) is concerned, results for all three operators are provided on the same plots since the dominating outcome is the big coalition (represented by a colored circle) and as a result, the user rate is the same for all operators. For outcomes other than the big coalition, different shapes are used (as illustrated in the accompanying legend). Figure 4.31 shows how the big coalition provides the same user quality for all the operators, but twice as much return on investment for the bigger one compared to the other two (objectives TOT_Q and MIN_Q). Similarly for the *operator-oriented* objectives, even though the outcome is not the big coalition but the coalition between the smaller operators and the singleton of the bigger one, user rates are almost the same for all operators, while revenues are twice as much for the bigger operator.

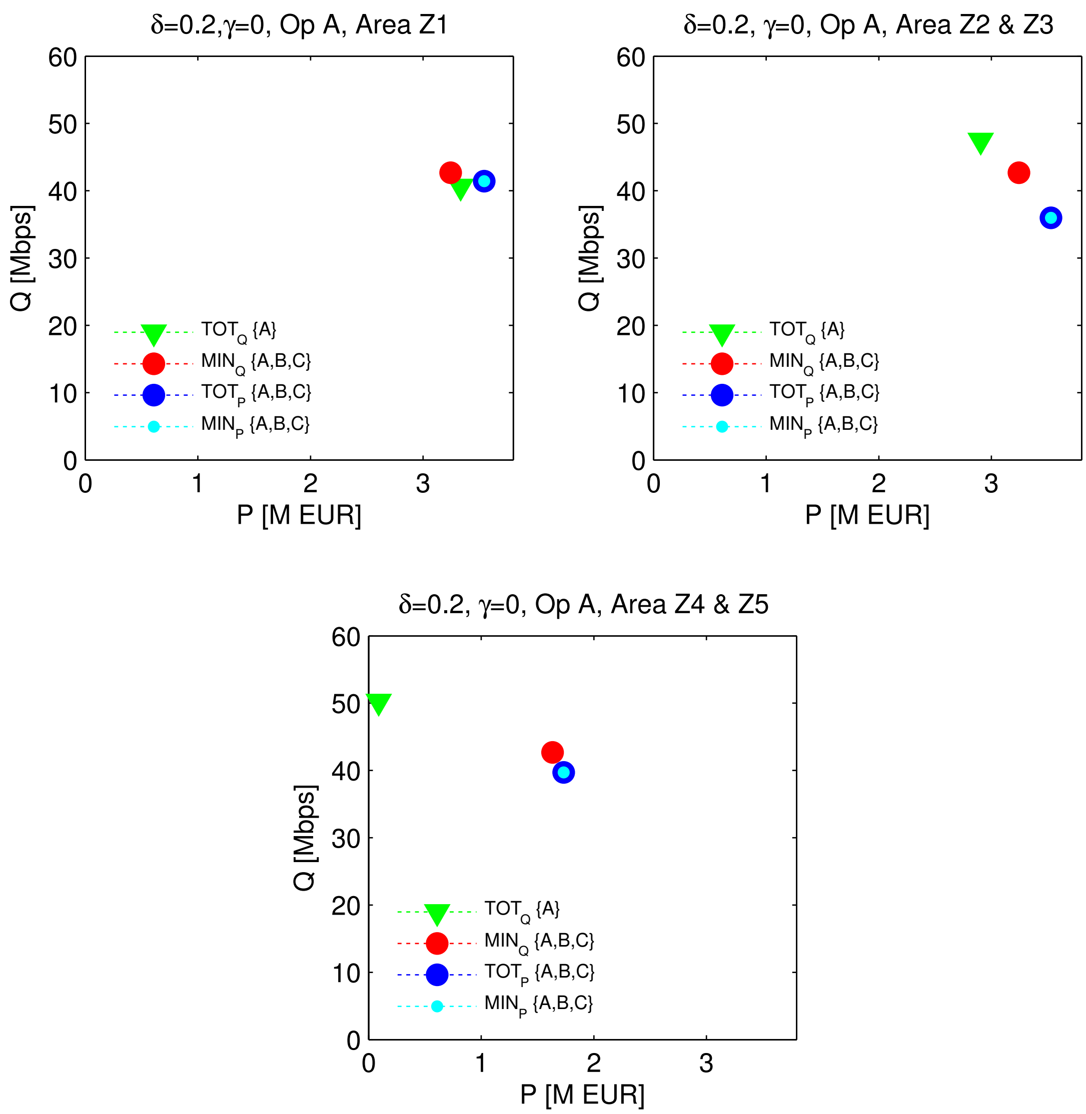

Figure 4.32: User rate *vs.* return on investment for each area – MNO A – δ=0.2,γ=0

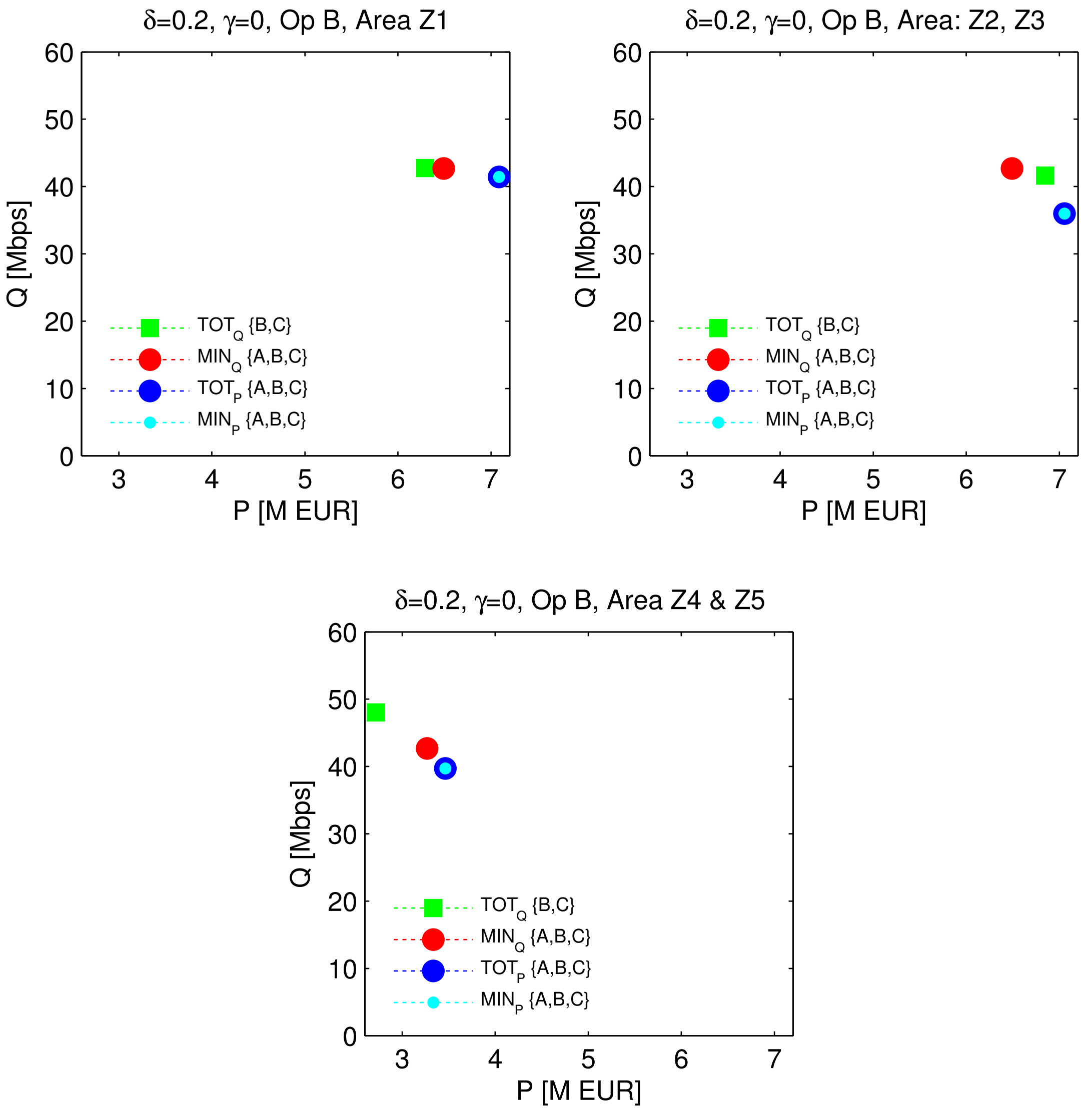

Figure 4.33: User rate *vs.* return on investment for each area – MNO B – δ=0.2,γ=0

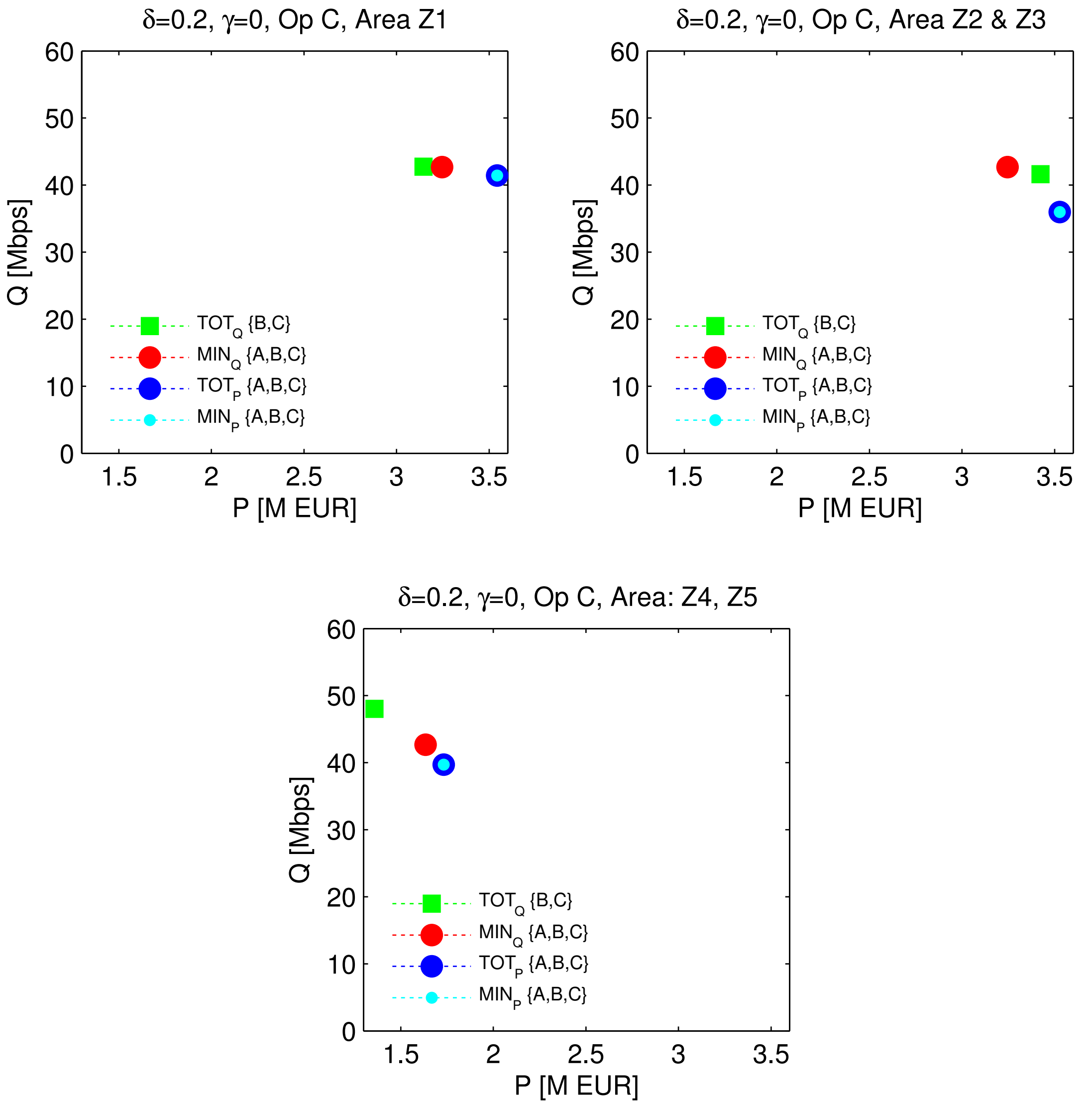

Figure 4.34: User rate *vs.* return on investment for each area – MNO C – δ=0.2,γ=0

A common observation for the first two instances (Figures 4.31, 4.32, 4.33, 4.34), that holds for all areas and operators is that the two *operator-oriented* objective functions give the same solution. However, this is the case when users are not willing to pay much for an improved service.

From the plots of the second instance, we can observe how the user rate for operator A (Figure 4.32) varies from one area to another with a minimum of 40.5 Mbps in area Z1 (20000 users, 4 km^2) and maximum 50 Mbps in areas Z4 and Z5 (10000 users, 1 km^2), even though A invests by itself in all areas (the outcome for all areas is the singleton). Similarly, for operators B and C which join the same coalition and therefore have the same user rate for the same area, but varying user rate over the different areas. (Figures 4.33 and 4.34). This is partly due to the effect that the user density and the size of the area have on the user rate of same coalition (same number of users), and partly due to the fact that the number of BSs activated for the same coalition is also a function of the area, and therefore can be different for each. As expected, higher user rates are achieved for lower user density and smaller area size (areas Z4 and Z5) since the LTE nominal rate is divided among less users and on the average the user its closer to its serving BS.

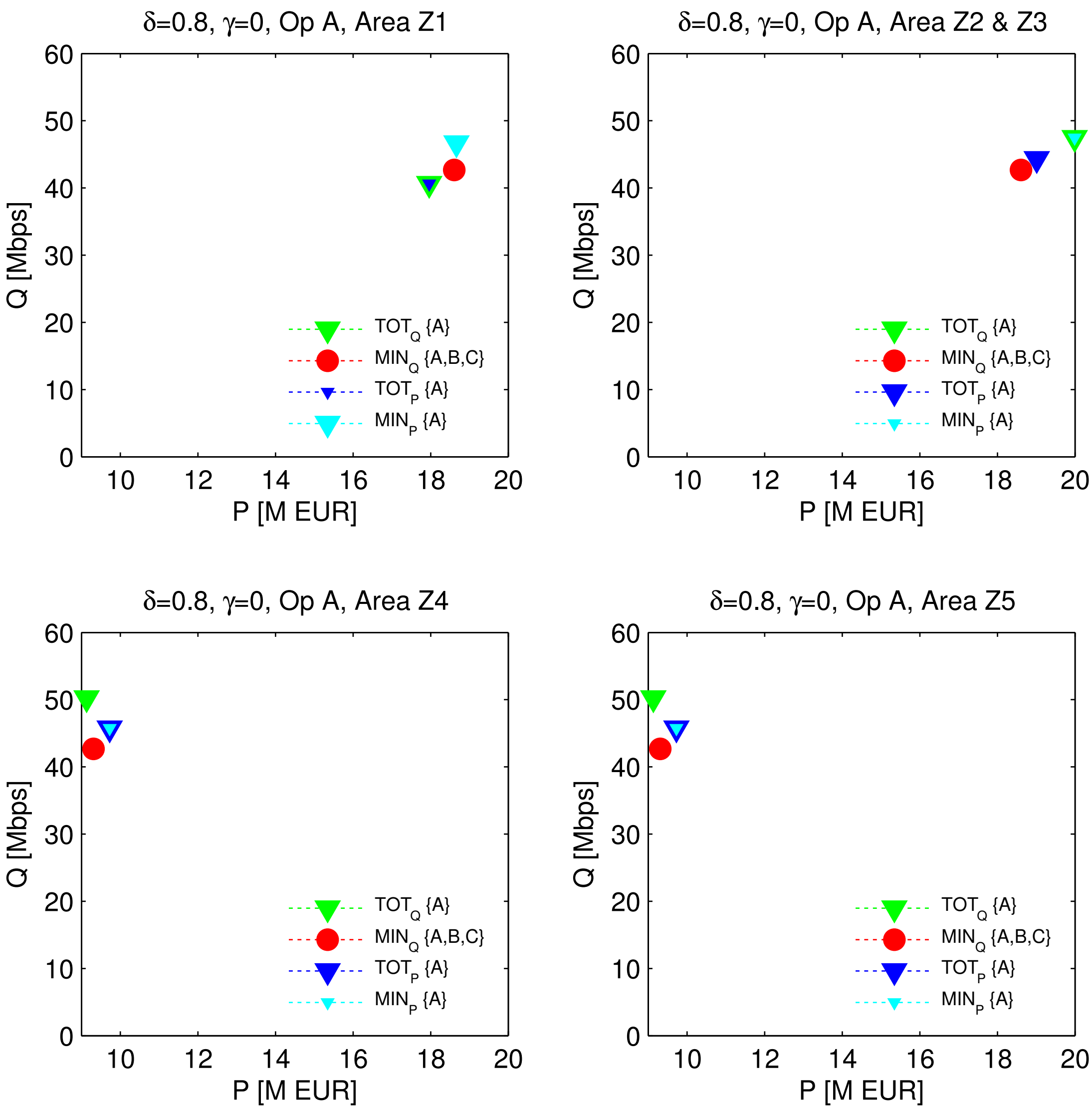

Figure 4.35: User rate *vs.* return on investment for each area – MNO A – δ=0.8,γ=0

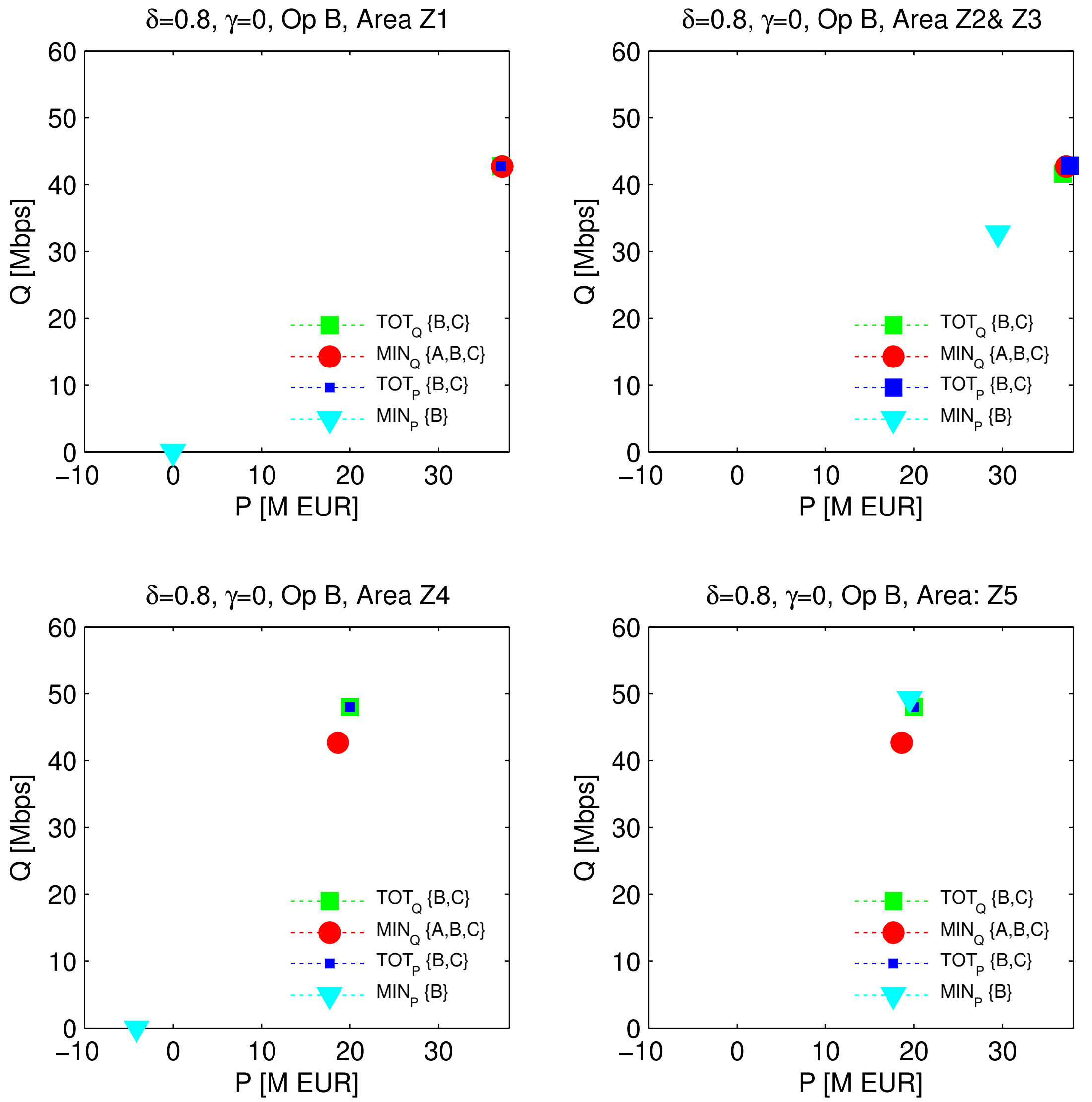

Figure 4.36: User rate *vs.* return on investment for each area – MNO B – $\delta = 0.8,\gamma{=}0$

For δ equal to 0.8 and γ equal to 0 (third instance), as previously emphasized in Subsection 4.3.2, the *operator-oriented* objective function MIN_P forces the outcome that is composed only by singletons, in other words, all operators invest by themselves. In Figure 4.36, we can see how operator B ($\frac{1}{2}$ users) has a negative return on investment and a very low user rate in areas Z1 and Z4 due to the fact that very few BSs are assigned to its singleton and much more to operators A and C in order to level up its global return on investment with the one of other two smaller operators, while its overall return on investment still satisfies the expectations.

Plots for Operator C regarding the third instance conclude this section (Figure 4.37).

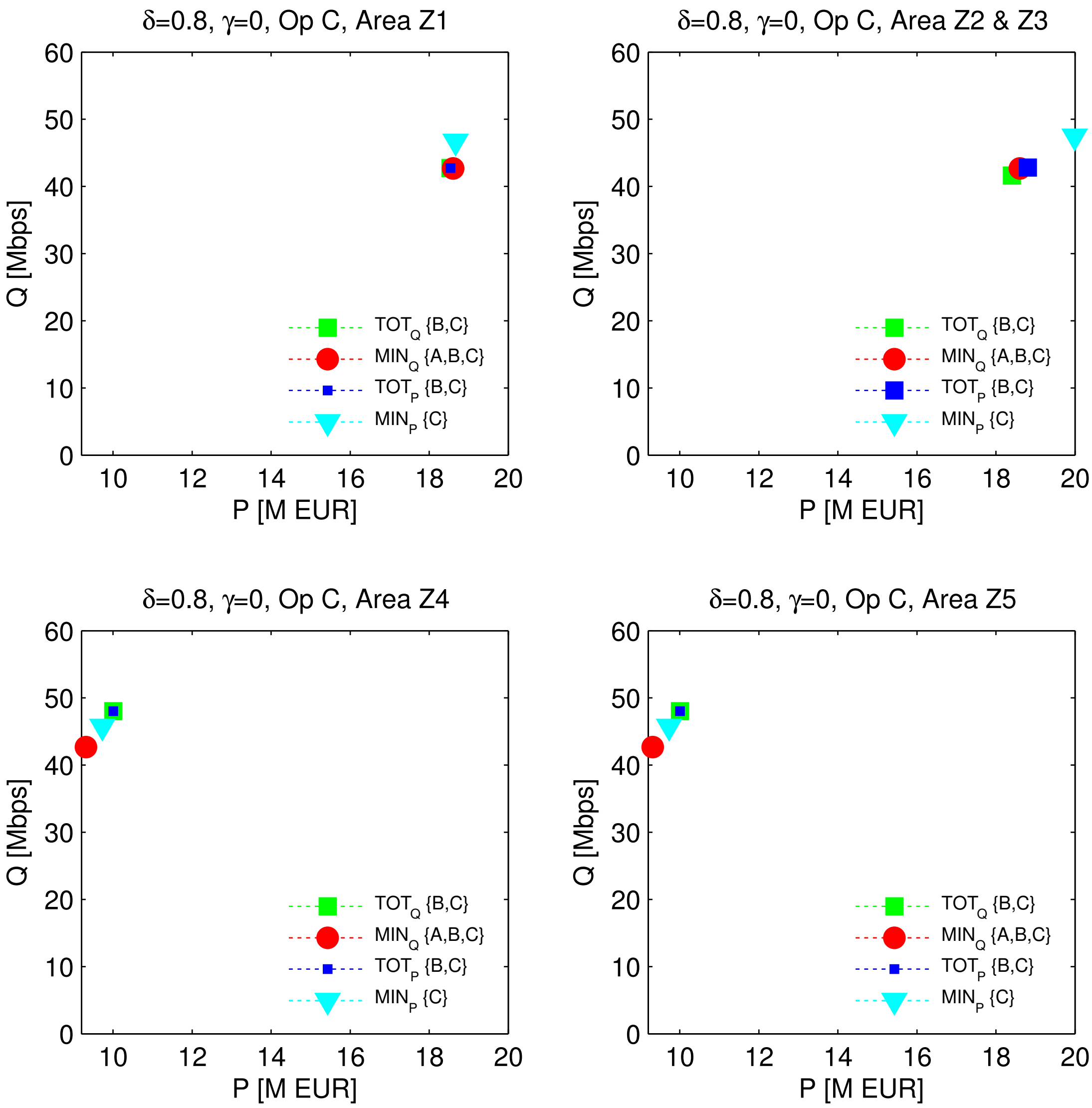

Figure 4.37: User rate *vs.* return on investment for each area – MNO C – $\delta = 0.8$,γ=0

Chapter 5

Conclusions

This last part of the thesis is mainly dedicated to conclusions that have been drawn from analyzing the outcome of the tested instances for all the considered cases and scenarios. In addition, we discuss how it is possible to extend our proposed models for future research.

In order to clearly state the conclusions that have been derived from the numerical results of the previous chapter, we briefly recall the objective of this work and the essential aspects of the proposed model.

We considered 3 MNOs, each with a customer base inherited from its already deployed infrastructure (pre-LTE macrocell BSs). All operators plan to invest in small cell LTE technology in dense urban areas. Our goal is to determine when they decide to make a shared investment, and if they do, which coalitions they select and how much do they invest (number of activated BSs). We modeled the operator standpoint in the investment by introducing its expectations on the return on investment (γ), whereas the user standpoint by considering his/her willingness to pay for the new service (δ) while remaining subscribed to his/her current MNO. The model has been investigated for several instances (combinations of δ and γ), two scenarios (Single and Multiple Area) and two cases (Uniform and Non-Uniform user distribution) under 6 different objectives; three of them focusing on the most relevant technical aspect of the investment (user rate) and the other three on the return on investment as the most important economic indicator. In particular, two of these objectives have been proposed in order to analyze a possible, third party-assisted shared investment, that is, a regulatory entity that intermediates the sharing process in order to guarantee fairness in terms of user rate (or return on investment) among operators. For both scenarios and cases, we analyze how the decision process (coalition selection) and the two main indicators are affected by varying the key parameters (δ and γ) and the objective function.

In broad terms, for all considered scenarios, cases and objective functions, a subset of the tested instances result unaffordable, which quite realistically reflects how there can be no investment if users are willing to pay little while operators have high expectations on their return on investment. Instead, when either users are willing to pay more or operators lower their expectations, investing in the new network infrastructure becomes affordable. Being that the case, the coalitional structure of almost all outcomes consists of either coalitions composed by a subset of the operators (2 operators and a singleton) or all of them (big coalition). This shows that, independently of the focus of the considered objective function, there is always incentive for the operators to jointly invest, and therefore share the LTE access network infrastructure.

We forced the coalitional structure for the MISP to be the same in all the areas, assuming that it would be more preferable from a practical point of view. Thus, the outcomes of the MISP are quite similar with the ones for the SISP when the same instances and user distribution cases are considered. Nevertheless, the characteristics of each area, that is, user density, size and a different number of activated BSs affect the operator's user rate and the return on investment per area, allowing the operator to differentiate the service provided in different areas, even though it joins the same coalition in each of them.

When users are willing to pay little, we are able to observe how the outcome is significantly affected by the type of objective function we investigate. If operators prioritize the rate provided to their users (*user-oriented* objective), the expected return on investment that reflects the selfish behavior of the operator is merely satisfied, since the entire set of allowed BSs is activated in the area(s). This shows how there is little incentive for the operators to prioritize the user (maximize the provided rate) if the latter is not willing to pay much. On the contrary, when operators prioritize their return on investment (*operator-oriented* objectives), even though they decide to collaborate, they will limit their investment by not activating all the available BSs and therefore reasonably provide lower rate to users that show little interest in the improved service.

However, when users are willing to pay more for the new service, all the considered objectives tend to behave the same both from the user rate and the return on investment perspective. This behavior is particularly emphasized for the uniform user distribution due to the symmetry and slightly less for the non-uniform one. Nevertheless, such behavior highlights the fact that user's and operator's best interest coincide only when the former will reasonably respond to the new service that the shared infrastructure provides.

For the uniform user distribution, the regulatory entity objectives, that account for the worst served user or the operator with the smallest return on investment in the market, provide fairness by forcing all operators to collaborate. In such a way, operators will all provide the same user rate and have the same return on investment, at the price of slightly lowering the total user rate (return on investment) compared to the objectives that jointly maximize the user (operator) payoffs.

For the non-uniform user distribution instead, having a regulatory entity intermediate the shared investment is not equally beneficial to all the involved operators. As the smaller operators have half the number of users of the bigger one, only an unfair distribution of the available BSs allows them to level up the return on investment with respect to the bigger operator. In other words, "fairness" is achieved at the cost of significantly degrading both the return on investment and user rate provided by the bigger operator (by allocating more resources to the smaller ones). However, the regulatory entity objective that accounts for the worst served user (maximizes the minimum user rate) forces the big coalition and, as a result, the bigger operator's loss in terms of rate and return of investment is less significant, which makes the role of the regulatory entity more likely to be accepted by all operators.

From a practical point of view, the role of the regulatory entity is more in line with the operators' best interest when they have the same market share (uniform user distribution); otherwise, by improving the payoffs of the smaller operators, it disadvantages the bigger operator, giving the latter less incentive to collaborate.

Recommendations

The MILP models (SISP/MISP), that we proposed to address the infrastructure sharing problem, are strongly coupled with the considered application scenario, that is, they can only be used to address the problem of sharing the RAN part of a mobile network among coexisting MNOs. This is due to the fact that our approach aimed at modeling in detail both the technical aspects of the radio communication at the access interface (transmission rate, user density, area coverage) and economic aspects (revenues, costs, expectations on the return on investment). As a result, the potential extensions of the proposed formulation still rely on our assumptions of technology choice (small cell LTE mobile network) and level of sharing (at RAN). Hereby, we list some possible improvements and extensions for future work.

As stated in Chapter 1, we used a fixed bandwidth (10 MHz) for each base station both when the latter is shared among operators and when it belongs to a single operator (non-sharing scenario). The reason behind this was that, with the current LTE releases, it is not easy to enlarge the base station bandwidth without additional upfront costs. As a result, spectrum sharing is not explicitly included in the model. However, spectrum is a scarce resource and highly underutilized considering the current rigid approach for providing bandwidth to operators. Therefore, it would be interesting to adopt spectrum sharing as a second sharing dimension of our model. Spectrum sharing can be carried out in two ways:

1. Spectrum sharing without carrier aggregation
2. Spectrum sharing with carrier aggregation

Spectrum sharing without carrier aggregation can be done if base stations still operate at a fixed bandwidth (for instance 10 MHz) but they keep dedicated bandwidths (each of 10 Mhz) for different operators when shared.

Carrier aggregation is used in LTE-Advanced [29] in order to increase the bandwidth, and thereby increase the bitrate. Thus, the second way of performing spectrum sharing consists in pulling together the bandwidths of each operator and operating at an enlarged bandwidth in each base station. Clearly, the two types of spectrum sharing require necessary modifications of both the simulations and the problem formulation.

In this work, revenues per operator have been defined as a linear function of the average rate that the operator provides to its users. The proportionality constant, that throughout this work we refer to as user willingness to pay for 1 Mbps in a monthly basis, is the equivalent of the monthly price for 1 Mbps of data service. However, the current pricing models applied by MNOs are more complex since they apply to a bundle of different services (including also voice, text messaging etc.). Clearly, defining pricing models similar to those provided by the operators cannot be done without considering services other than data. We can, however, redefine revenues as a piece-wise linear function of the user rate provided by the operator. In other words, we can account for a user's willingness to pay for an improved service which decreases with the increase of the user rate (use multiple values of δ that apply to different ranges of user rate), that captures in a more realistic way the user standpoint in the investment.

Our formulation of the Multiple Area investment scenario (MISP) is based on the assumption that it is more convenient for operators to select the same coalition in all areas since they do not have to simultaneously manage several collaboration agreements. Alternatively, the model can be easily modified to allow operators to independently choose the coalition that they want to join in each area, in case it is reasonable to do so in practice.

In this work, we considered instances with only 3 MNOs, which implied a negligible solver execution time. It is therefore possible to extend the instances for 4 or 5 MNOs. This would also allow for more diverse user distribution cases, other than the uniform and non-uniform ones that we considered. In addition, it could be interesting to test the MISP for user distribution among operators that varies over different areas instead of keeping the same one over all of them.

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