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Master degree in Automation Engeneering

Scheduling and Control of Systems Subject to Random Faults

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Abstract

In this thesis we present an application of recent theoretical results regarding Markov Jump Linear Systems to the control of systems affected by random faults. More precisely, we consider a feedback control system where the actuation signal is intermittent, due to the occurrence of faults. The model of faults is described by a discrete-time Markov chain, while the dynamics of the plant and the controller is linear. As an additional control input, a deterministic scheduling signal is considered that can switch among a set of possible controllers. In that case, the model becomes a Dual Switching Linear System.

The main problem addressed in this work is the design of suitable switching feedback strategies able to ensure mean-square stability and the attainment of some guaranteed level of performance in terms of a quadratic cost function.

The design is carried out by using the Matlab LMI-Toolbox under different assumptions on the control scheme (single/multi plant, single/multi controller) and the parameters of the underlying Markov chain. Several simulations are carried out in order to validate the theoretical results, to assess the degree of conservatism of the results on the performance, and to compare different strategies for computing the input applied to the plant when the actuator is faulty (zero-input vs. input-hold).

Keywords: Markov Jump Linear Systems, Dual-switching, random faults, Mean-Square stability, $\mathscr{H}_2$ performance, feedback systems.

Sommario

In questa tesi presentiamo l'applicazione di alcuni recenti risultati teorici sui sistemi lineari a commutazione Markoviana (MJLS) al controllo di sistemi soggetti a guasti. Precisamente, consideriamo un sistema di controllo in retroazione in cui il segnale di attuazione è intermittente a causa della presenza di guasti. Il modello dei guasti è descritto da una catena di Markov a tempo discreto, mentre le dinamiche dell'impianto e del controllore sono lineari. Come ingresso di controllo aggiuntivo si considera un segnale deterministico di schedulazione capace di commutare all'interno di un insieme di possibili controllori. In tal caso il modello diventa un sistema lineare a duplice commutazione (Dual Switching Linear System).

Il principale problema trattato in questo lavoro è il progetto di opportune strategie di commutazione in retroazione capaci di assicurare la stabilità in media quadratica e di ottenere un livello garantito di prestazioni in termini di una funzione di costo quadratica.

Il progetto è realizzato mediante il Matlab LMI-Toolbox formulando diverse ipotesi sullo schema di controllo (impianto singolo/doppio, controllore singolo/doppio) e sui parametri della sottostante catena di Markov. Sono state condotte varie simulazioni allo scopo di convalidare i risultati teorici, valutare il grado di conservativismo dei risultati di prestazione e confrontare diverse strategie per calcolare l'ingresso all'impianto quando l'attuatore è in condizioni di guasto.

Parole chiave: sistemi lineari a commutazione Markoviana, Dual-switching, guasti casuali, stabilità quadratica media, prestazione H2, sistemi retroazionati.

Acknowledgments

First and foremost, we would like to show our deepest gratitude to our supervisor, Prof. Paolo Bolzern, a respectable, responsible and resourceful scholar,who has provided us with valuable guidance in every stage of the writing of this thesis. Without his enlightening instruction, impressive kindness and patience, we could not have completed our thesis. His keen and vigorous academic observation enlightens us not only in this thesis but also in my future study.

We shall extend our thanks to our parents, for their endless supports in these years we studying far away home.

Sincerely

Wang & Zhang

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1 Introduction

1.1 Motivation

One of the most important issues in control systems is their capability of maintaining an acceptable behavior and meeting some performance requirements even in the presence of abrupt changes in the system dynamics. These changes can be due to abrupt environmental disturbances, component failures or repairs, changes in subsystems interconnections and so on. In some situation these systems can be modeled by a set of discrete-time linear systems with modal transition given by a Markov chain. This class is known in the specialized literature as Markov jump linear systems (MJLS). Applications cover diverse fields including economics, biomedicine, networked control, fault tolerant systems, communication networks, aerospace etc.
Switching linear systems are widely studied for their ability to describe the behavior of systems where the dynamics changes abruptly due to jumps in parameters taking values in a finite set. It is well known that the presence of jumps may significantly affect the performance and the stability of the switching system. The switching signal can be modeled as either a deterministic or a random signal taking values in a finite set. In various applications (such as networked control, fault tolerant systems) problems are encountered where the system is jointly affected by two independent external sources, for instance a manipulated switching signal and stochastic jumps. These systems are referred as dual-switching systems. Random packet dropout can cause serious problem in data transitions. Thanks to MJLS, the random faults can be described in a stochastic setting by means of a Markov chain. If we have more than one controller in parallel in a feedback system, then there is a potential application of dual switching framework. In this framework, we consider a scheduling problem for a feedback system with actuator subject to random faults.

1.2 Objective

One of the main topics in the study of MJLS is investigating the notion of stability. Several definitions of stability have been given. The most significant stability notion for the analysis of such systems is Mean-Square (MS) stability.

Besides stability, it is important to endow the control system with other important properties. In this aspect, it is customary to define an index of performance related to a quadratic cost, called $\mathscr{H}_2$ index. This cost represents the energy of the controlled output starting from a given initial state. Keeping this cost small (or even minimizing it) implies a reduced effect of the initial state on the output.

In this thesis we will apply recent mathematic tools from dual-switching MJLS theory to different problems of scheduling and control systems suffered by random faults step by step. Using simulation from Matlab to compare the differences between different feedback systems, and see how the mathematic tools work on those MJLS.

1.3 Organization of thesis

-Chapter2 introduces an overview of the feedback system scheme and the challenges we meet.

-Chapter3 briefly review some notion of mathematic tools that we will implement in this thesis, such as Markov chain, Markov Jump Linear Systems, dual-switching systems , Mean-Square stability and so on.

-Chapter4 precisely shows how we apply the mathematical theory illustrated in Chapter3 to those systems introduced in Chapter2. We will also discuss some results obtained from the simulations and introduce possible extension.

2 Scheduling problem in Control systems subject to random faults

In several modern control systems, a digital communication network is used to support the data transmission between sensors, actuators and controllers. Therefore, this are called Networked control Systems (NCS).

Networked control systems have been applying in a broad range of areas such as mobile sensor networks, remote surgery, and automated highway systems. However, using several dedicated independent connections introduces new challenges to the control system designer as it introduces important limitations and non-idealities which have be considered in order to guarantee stability and performance.

For instance, each communication channel can only carry a finite amount of in information. Moreover, the data transmitted over a network may subject to unpredictable delay. The data also might be lost while travelling along the network.[1]

In this thesis, we will focus on the basic mechanism of packet dropout, which can be described in a stochastic setting by means of a Markov chain.

2.1 Single Plant Single controller Scheme

First we introduce single plant single controller case.

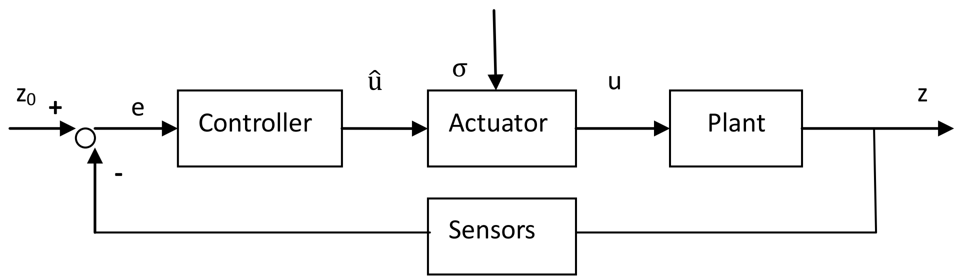

Figure1 scheme of single plant single controller

In this scheme, the channel from controller to plant is affected by the phenomenon of packet dropout. So, in some time instants, the control u which applied to the plant is different from the original $\hat{u}$ delivered by the controller.

In this simplest case, σ is a binary variable describing the fact that the actuator is working properly (healthy mode) or is unable to transmit commands to the plant (faulty mode)

In this thesis, two configurations will be considered:

One is *zero-input* configurations:

When the data is not received by the plant, the input u=0 is applied to the system.

Another one is *input-hold* configurations:

When the data is not received by the plant, the previous value of the input is applied to the system.

2.2 Single Plant Multi-Controller Scheme

Second, we consider the switching case, which contain M different controllers. Instead of one deterministic controller, the input receive data from which controllers will be decided by the switching signal γ (we can also call it scheduling signal).

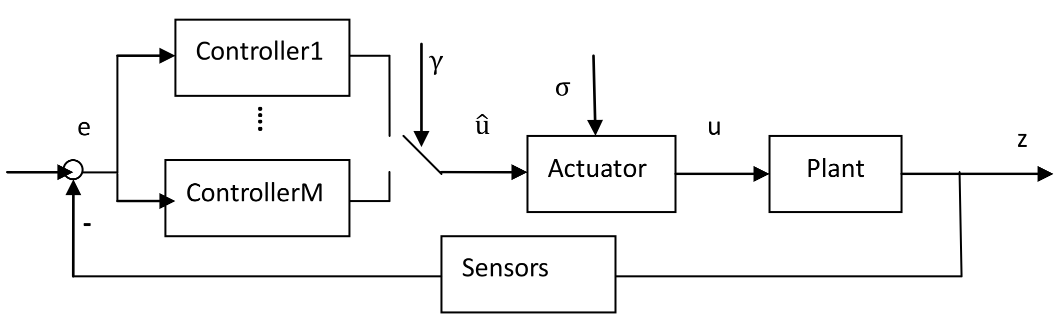

Figure2 scheme of single plant multi-controller

The scheduling signal γ taking values in a set $\mathcal{M}=\{1,2\ldots,\mathrm{M}\}$. Like the single controller case, σ is a binary variable describing healthy mode or faulty mode for plant.

The same, two configurations will be considered: *zero-input* configurations and *input-hold* configurations:

2.3 Multi-Plant Multi-Controller Scheme

Considering a scheduling problem for a multi-loop networked control system subject to packet dropout. Precisely, assume M, a linear (possible unstable) plants have to be controlled by a single regulator exchanging input-output data through a shared network, as described in figure 3. The regulator is allowed to attend only one plant at a time according to the scheduling signal γ taking values in a finite set $\mathcal{M}=\{1,2\ldots\ldots\mathrm{M}\}$. Transmission of actuator data over the net work is subject to random faults modeled by a Markov process σ, σ is a binary variable describing healthy mode or faulty mode.[9]

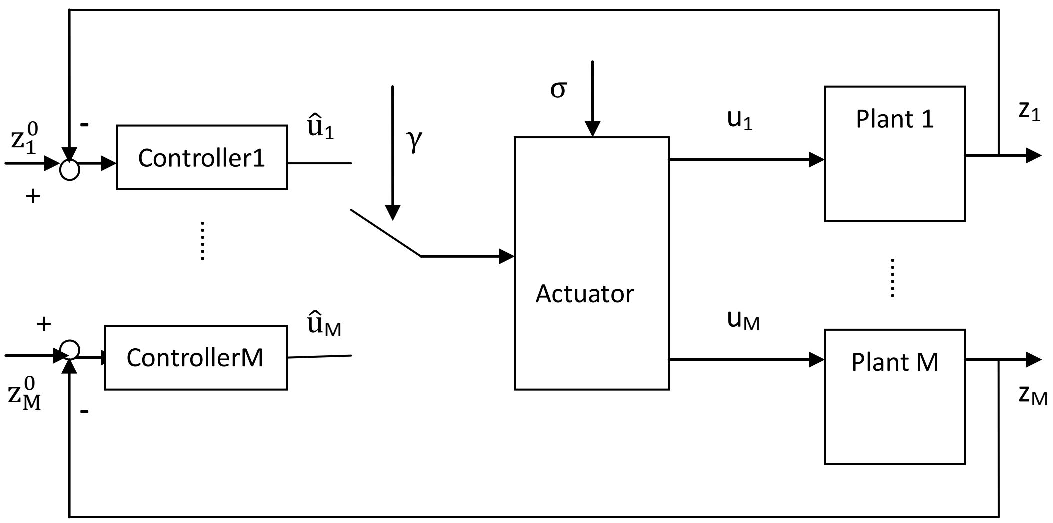

Figure3 scheme of multi-plant multi-controller

The same, two configurations will be considered: *zero-input* configurations and *input-hold* configurations:

In these three schemes, faulty mode indicates packet dropout situations; we will discuss how to guarantee simultaneous stability and the fulfillment of some global $\mathscr{H}_2$ performance specifications for this faulty mode in following chapters

3 Dual-switching Markov Jump Linear systems

In this chapter, we will discuss some mathematical tools that is useful in the analysis and design of systems subject to random faults.

The occurrence of random faults in the system will be described in a stochastic setting in terms of a discrete-time Markov chain. A brief review if the basic properties of Markov Chain are provided in section 3.1.

The next, the class of Markov Jump Linear Systems (MJLS) will be introduced. These models which consist of a finite set of linear subsystems (called modes) and a switching signal, described as a Markov Chain, governing the jumps form one subsystem to the next one. Fundamentally notion related to MJLS, such as stability and performance, will be revealed in section 3.2.

Finally, section3.3 deals with extensions of the MJLS modeling framework which is able to cope with two different switching signals, the switching signal acting as a disturbance is modeled again as a Markov Chain, while the second switching signal is to be regarded as a control variable, used to attain desired properties, like stability and guaranteed performance. This class of models is called Dual-switching Systems.

3.1 Markov Chains in discrete time

A Markov chain is a discrete finite system that undergoes transitions from one state to another on a state space. It is a random process usually characterized as memoryless: the next state depends only on the current state and not on the sequence of events that preceded it. His specific kind of "memoryless" is called the Markov property.Markov chains have many applications as statistical models.[2]

3.1.1 Markov property

To be precise, Let X_k, k = 0, 1, 2 . . ., be a discrete time stochastic process taking values in a finite set $s = \{1,2, \dots N\}$.

The Markov property holds:

$P\{X_k = in \mid X_0 = i_0, \dots, X_{k-1} = i_{k-1}\} = P\{X_k = i_k \mid X_{k-1} = i_{k-1}\}, i_0, \dots, i_k \in S.$

Which says that the probabilities associated with future states only depends on the current state, and not on the full history of the process. Any process X_k, k ≥ 0, satisfying the Markov property is called a discrete time Markov chain.

3.1.2 Transition matrix

The one-step transition probability of a Markov chain from state i to state j, denoted by $\lambda_{ij}(k)$, is

$$\lambda_{ij}(k) = P\{X_{k+1} = j \mid X_k = i\}.$$

Then we can define the transition matrix:

The transition matrix Λ for a Markov chain with state space S = {1, 2, . . . ,N} and one-step transition probabilities λ_{ij} is the N $\times$ N matrix:

$$\Lambda = \left[\begin{pmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1N} \\ \lambda_{21} & \lambda_{22} & & \lambda_{2N} \\ \vdots & & \ddots & \vdots \\ \lambda_{N1} & \lambda_{N1} & \cdots & \lambda_{NN} \end{pmatrix}\right]$$

If the state space S is infinite, then Λ is formally defined to be the infinite matrix with i, jth component λ_{ij}.

the matrix Λ satisfies

$$0 \leqslant \lambda_{ij} \leqslant 1, \quad 1 \leqslant i, j \leqslant N,$$

$$\sum_{j=1}^{N} \lambda_{ij} = 1$$

Any matrix satisfying the above two conditions and is called a Markov or stochastic matrix, and can be the transition matrix for a Markov chain.

3.1.3 State probability distribution

Let $\pi_i(k)$ denote the probability of finding the chain in state I at time k, the vector

$$\pi(k) = \begin{bmatrix} \pi_1(k) \\ \dots \\ \pi_N(k) \end{bmatrix}$$

is called the state probability distribution at time k. Of course, the elements of $\pi(k)$ sum up to 1, $\forall k$.

It can be shown that, starting from an initial state probability distribution $\pi(0)$, the time evolution of $\pi(k),\ k \geq 0$ is the solution of the following equation:

$$\pi(k+1)^T = \pi(k)^T \Lambda$$

Where Λ is the one-step transition matrix. So, at a generic time $k > 0$, it result that

$$\pi(k)^T = \pi(0)^T \Lambda^k$$

It is interesting to understand under which condition the limit of $\pi(k)$ for $k \to \infty$ is constant. If this limit exists, it is called stationary distribution.

3.1.4 Stationary distribution

The n-step transition probability, denoted $\lambda^{(n)}_{ij}$, is the probability of moving from state i to state j in n steps,

$$\lambda^{(n)}_{ij} = P\{X_n = j \mid X_0 = i\} = P\{X_{n+k} = j \mid X_k = i\},$$

Let λ^n_{ij} denote the i, jth entry of the matrix λ^n. For all n ≥ 0 and i, j $\in$ S, $p^{(n)}_{ij} = P^n_{ij}$.

In order to get the stationary distribution for every process X_n, there are some preconditions one the transition matrix.

First, let us introduce some notations.

Reducible

The state $j \in S$ is *accessible* from $i \in S$, and we write $i \to j$, if there is an $n \geq 0$ such that

$$\lambda^{(n)}_{ij} > 0.$$

That is, j is accessible from i if there is a positive probability of the chain hitting j if it starts in i.

States $i, j \in S$ of a Markov chain communicate with each other,

and we write $i \leftrightarrow j$, if $i \to j$ and $j \to i$. We may now decompose the state space using the relation $\leftrightarrow$ into disjoint equivalence classes called *communication classes*.

A Markov chain is *irreducible* if there is only one communication class. That is, if $i \leftrightarrow j$ for all $i, j \in S$. Otherwise, it is called *reducible*.

Periodic

Periodicity helps us understand the possible motion of a discrete time Markov chain.

The period of state $i \in S$ is

$$d(i) = \gcd\{n \geq 1 : \lambda^{(n)}_{ii} > 0\},$$

Where, gcd stands for greatest common divisor. If $\{n \geq 1: \lambda^{(n)}_{ii} > 0\} = \emptyset$, we take d(i) = 1. If d (i) = 1, we say that i is *aperiodic*, and if d (i) > 1, we say that i is *periodic* with a period of d (i).

Stationary distributions characterize the long time behavior of Markov chains.

Consider a Markov chain with transition matrix Λ. A non-negative vector π is said to be an invariant measure if

$$\pi^T \Lambda = \pi^T,$$

Which in component form is

$$\pi_j = \sum_j \pi_j \lambda_{ji}$$

for all i $\in$ S.

If π also satisfies$\sum_k \pi_k = 1$, then π is called *a stationary, equilibrium or steady state probability distribution*.

Thus, a stationary distribution is a left eigenvector of the transition matrix with associated eigenvalue equal to one.

In addition, for a finite Markov chain with transition matrix Λ, to guarantee a unique stationary distribution, Λ should be irreducible and aperiodic.

3.2 Markov Jump Linear Systems

Markov Jump Linear Systems are described by a set of linear subsystems with commutations generated by a finite state Markov chain. Applications cover diverse fields including economics, biomedicine, networked control, fault tolerant systems.

To illustrate MJLS, consider a dynamical system that is, in a certain moment, well described by a model G_1. Suppose that this system is subject to abrupt changes that cause it to be described, after a certain time-instant, by a different model, say G_2. More generally we can imagine that the system is subject to a series of possible qualitative changes that make it switch, over time, among a countable set of models, for example, $\{G_1, G_2, \ldots, G_N\}$. We can associate each of these models to an operation mode of the system or just mode and will say that the system jumps from one mode to the other or that there are transitions between them.[3]

We assume that the jumps evolve stochastically according to a Markov chain, that is, given that at a certain instant k the system lies in mode i, we know the jump probability for each of the other modes, and also the probability of remaining in mode i.(We assume that the jump probability is known)

Switched linear systems are piecewise linear systems evolving according to a finite number of operating modes, subject to an external switching signal that selects, at any time instant, which mode is currently active.

A general form for switched linear systems in discrete time is like the following:

$$x(k+1) = A_{\sigma(k)}x(k),\ \ x(0) = x_0 \qquad (3.1)$$

defined for all $k \geq 0$ where $x(k) \in R^n$ is the state, $\sigma(k)$ is the switching signal and x_0 is the initial condition. Considering a set of matrix $A_i \in R^{n\times n}$,i=1,…..,N be given, the switching signal $\sigma(k)$, for each $k \geq 0$, is such that $A_{\sigma(k)} \in \{A_1, \dots, A_N\}$. $A_{\sigma(k)}$is constrained to jump among the N vertices of the matrix polytope $\{A_1, \dots, A_N\}$. The switching rule of $\sigma(k)$ is characterized by a pre-known $N \times N$ transition matrix Λ.

$$\Lambda = \left[\begin{pmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1N} \\ \lambda_{21} & \lambda_{22} & & \lambda_{2N} \\ \vdots & & \ddots & \vdots \\ \lambda_{N1} & \lambda_{N1} & \cdots & \lambda_{NN} \end{pmatrix}\right]$$

We suppose that Λ is irreducible and aperiodic ,so as to guarantee unique stationary distribution.

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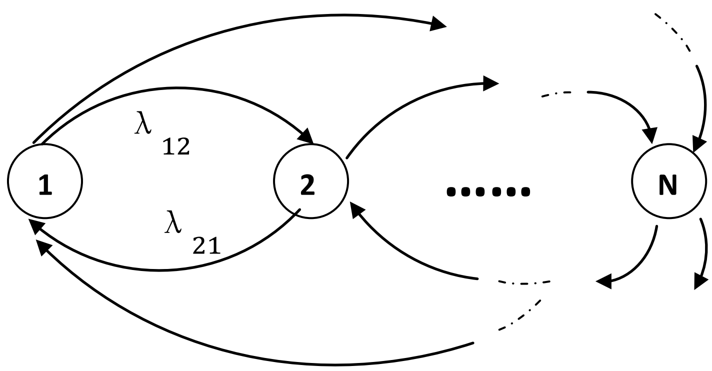


Figure 4

Figure 4 $\boldsymbol{\sigma}$ jumping state

Since $\sigma(k)$ is a stochastic jumping signal, giving a initial state of A_i, we will get a stochastic sequences of A_i and the state itself $x(k)$ is a stochastic process.[4]

Stability

The stability of a class of Markov Jump Linear Systems characterized by constant transition rates and system dynamics is investigated. For these Markov Jump Linear Systems, mean square stability is related to the time evolution of the second-order moment of the state. [5]

For the above switched linear discrete system (3.1), we assume its unique stationary distribution is $\overline{\pi}$, then we get:

$$\overline{\pi} = \Lambda^{T}\overline{\pi}$$

The system is mean-square stable(MS) if,

$$\lim_{t\to\infty} E[\|x(k)\|^2] = 0,$$

for any initial condition X_0, and any initial probability distribution π_0.

The symbol $E[\cdot]$ denotes the expectation with respect to the stationary distribution $\overline{\pi}$.

The system is exponential mean-square stable(EMS-stabel), if $\exists\ \alpha > 0, \beta < 1$:

$$E[\|x(k)\|^2] \leq \alpha\|x_0\|^2\beta,$$

For any $k \geq 0$.

It is obvious that EMS-stability implies MS-stability. The converse implication is also true, so that the two notion are actually equivalent. *[6]*

It is remarkable that a necessary and sufficient condition for MS-stability can be stated in terms of LMI.

THEOREM1. Mean-square stability condition(based on Lyapunov equation):

The MJLS is MS-stable, If and only if, exist a positive-definite matrix P_i satisfying the Inequality:

$$\sum_{j=1}^{N} \lambda_{ij} A_i^T P_j A_i - P_i < 0, \qquad i = 1,2,\dots,N$$

Sketch of proof:

We set $V(x,i) = x^T P_i x$, a stochastic Lyapunov Function, $\forall\, x(k) = x, \sigma(k) = i$, $\varepsilon_{x,i}$ indicates the event $\big(x(k), \sigma(k) = (x,i)\big)$. Computer its expected one-step difference to get:

$$\begin{aligned} &E\big[V\big(x(k+1), \sigma(k+1)\big)\big|\varepsilon_{x,i}\big] - V(x,i) \\ &= E\big[x(k)^T A_i^T P_{\sigma(k+1)} A_i \big|\varepsilon_{x,i}\big] - x^T P_i x \\ &= \sum_j \lambda_{ij} x^T A_i^T P_j A_i x - x^T P_i x < 0 \end{aligned}$$

Since this inequality is negative and V is quadratic, MS-stability follows from standard segments on stochastic Lyapunov functions.

Performance

Extension of system (3.1), consider now the following:

$$\begin{cases} x(k+1) = A_{\sigma(k)} x(k), \\ \quad z(k) = C_{\sigma(k)} x(k) \end{cases}, \qquad x(0) = x_0$$

Then $\mathscr{H}_2$ performance can be defined as the following expected quadratic cost:

$$J_2 = E\left[\sum_{k=0}^{\infty} z(k)^T z(k)\right] = E\left[\sum_{k=0}^{\infty} x(k)^T Q_{\sigma(k)} x(k)\right],$$
$$Q_{\sigma(k)} = C_{\sigma(k)}{}^T C_{\sigma(k)}$$

This cost can be interpreted as the expected energy of the output z when the initial state is x_0.

THEOREM2. If $\exists P_i > 0$ satisfying the equation

$$\sum_{j=1}^{N} \lambda_{ij} A_i^T P_j A_i - P_i + Q_i = 0,$$

Then we have:

$$J_2 = x_0^T \left(\sum_{i=1}^{N} \overline{\pi}_i P_i \right) x_0$$

Or if $\exists P_i > 0$ satisfying the inequality

$$\sum_{j=1}^{N} \lambda_{ij} A_i^T P_j A_i - P_i + Q_i < 0,$$

Then we have:

$$J_2 < x_0^T \left(\sum_{i=1}^{N} \overline{\pi}_i P_i \right) x_0.$$

In other words, either the value of the cost or its upper bound can be obtained.

3.3 Dual-switching systems

In various applications (such as networked control, fault tolerant systems, communication networks) problems are encountered where the system is affected by two independent external sources, for instance a manipulated switching signal and stochastic jumps. These systems are referred to in literature as dual-switching systems. They reveal a complex dynamic behavior due to the interplay between the two switching signals. A typical example of dual-switching systems is a networked control system with deterministically switching control laws and stochastic jumps between levels of network congestion.[7][8]

To define the general form for dual-switching signal, we assume we have two switching signal, one is $\gamma(k)$, which is regarded as a control variable. The other one is $\sigma(k)$, with transition matrix Λ .

The class of discrete-time dual switching linear systems is described below:

$$\begin{cases} x(k+1) = A_{\sigma(k)}^{\gamma(k)} x(k), \\ \quad z(k) = C_{\sigma(k)}^{\gamma(k)} x(k) \end{cases}, \; x(0) = x_0 \qquad (3.2)$$

Where, k is the discrete time index, $x(k) \in R^n$ is the state, $z(k) \in R^p$ is the performance output, $\gamma(k)$ is the switching signal taking values in the finite set M=$\{1,2,\ldots.,M\}$, and $\sigma(k)$ is a time homogeneous Markov process taking values in the set N=$\{1,2,\ldots.,N\}$. As seen already, the entry $\lambda_{ij} \geq 0$ of Λ represents the probability of a transition from mode i to mode j, namely

$$\lambda_{ij} = P\{\sigma(k+1) = j | \sigma(k) = j\},$$

Λ is a right stochastic matrix(row sum to unit and nonnegative matrix).

Let $\pi(k)$ denote by the state probability distribution at time k, its evolution is governed by the equation

$$\pi(k+1)^T = \pi(k)^T \Lambda, \pi(0) = \pi_0$$

In the sequel, we assume that Λ is irreducible and aperiodic, so that the Markov process admits a unique stationary probability distribution $\overline{\pi}$ satisfying $\overline{\pi}^T = \overline{\pi}^T \Lambda$.

The system is subject to both stochastic jumps governed by the form process $\sigma(k)$ and deterministic switches dictated by the control signal $\gamma(k)$.So, the state dynamics of the overall system is characterized by$(A_i^r, C_i^r), i \in N, r \in M$.

Stability

In accordance with standard notions of stochastic stability, like single switching system, for a given deterministic switching signal $\gamma(k)$, system (3.2) is mean-square stable if,

$$\lim_{t\to\infty} E[\|x(k)\|^2] = 0, \;\; \forall x_0, \pi_0,$$

Again, The symbol $E[\cdot]$ will denote the expectation with respect to the stationary distribution $\overline{\pi}$.

Still, we can get the Mean-square stability condition based on Lyapunov equation:

THEOREM3. If there exist positive definite matrix P_i^r , $i \in N, r \in M$ and a right-stochastic matrix $\Phi = [\varphi_{rs}]$ satisfying,$\forall$ i, r

$$\sum_{j=1}^{N} \sum_{k=1}^{M} \lambda_{ij} \varphi_{rk} A_i^{r\,T} P_j^k A_i^r - P_i^r < 0, \qquad (3.3)$$

Then the feedback switching law

$$\gamma^* = \mathrm{argmin}_r x^T P_\sigma^r x$$

Makes the closed loop system MS-stable.

Proof:

Consider the stochastic Lyapunov function $V(x, i) = \min_r x^T P_i^r x$ and compute its expected one-step difference at time k with the position $x(k) = x, \sigma(k) = i$ and $g = \mathrm{argmin}_r x^T P_\sigma^r x$. For brevity, $\varepsilon_{x,i}$ indicates the event $\big(x(k), \sigma(k) = (x, i)\big)$ and ε_i indicates the event $\sigma(k) = i$.

$$E[\Delta V(x, i)] = E\big[V\big(x(k+1), \sigma(k+1)\big)\big|\varepsilon_{x,i}\big] - V(x, i)$$

$$= E\big[\min_r x(k+1)^T P_{\sigma(k+1)}^r x(k+1)\big|\varepsilon_{x,i}\big] - \min_r x^T P_i^r x$$

$$= E\left[\min_r x^T \left(A_i^g\right)^T P_{\sigma(k+1)}^r A_i^g x|\varepsilon_i\right] - x^T P_i^g x$$

Notice that the expected value of the minimum of a function is not greater than the minimum of the expectation. Moreover,

$$E\big[P_{\sigma(k+1)}^r|\varepsilon_i\big] = \sum_j \lambda_{ij} P_j^r$$

Therefore, it follows

$$E[\Delta V(x, i)] \le \min_r x^T \left(A_i^g\right)^T \sum_j \lambda_{ij} P_j^r A_i^g x - x^T P_i^g x$$

Since it holds that

$$\min_r x^T \left(A_i^g\right)^T \sum_j \lambda_{ij} P_j^r A_i^g x \leq x^T \left(A_i^g\right)^T \sum_j \lambda_{ij} \sum_k \varphi_{gk} P_j^k A_i^g x$$

And also holds (3.3) we obtain

$$E[\Delta V(x, i)] < x^T P_i^g x - x^T P_i^g x = 0$$

Mean square-stability follows from standard results on stochastic discrete-time Lyapunov functions.

Performance

By slightly strengthening the conditions of stability, it is possible to yields a guaranteed $\mathscr{H}_2$ performance,

THEOREM4. If $\exists P_i^r > 0, i \in N, r \in M$ and a right-stochastic matrix $\Phi = [\varphi_{rs}]$ satisfying,$\forall$ i, r

$$\sum_{j=1}^{N} \sum_{k=1}^{M} \lambda_{ij} \varphi_{rk} {A_i^r}^T P_j^k A_i^r - P_i^r + {C_i^r}^T C_i^r < 0,$$

then

$$\gamma^* = \operatorname{argmin}_r x^T P_\sigma^r x$$

Makes the closed loop system MS-stable and

$$J_2(\gamma^*) < \overline{J_2} = \min_r x_0^T \left(\sum_{i=1}^{N} \overline{\pi}_i P_i^r \right) x_0$$

It should be notice that in order to implement the switching strategy γ^*, the controller needs to have respect information on both the state x and the stochastic signal σ.

4 Scheduling and Control Design

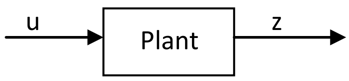
In this chapter, we will apply the mathematical theory on MJLS and Dual-Switching systems to the analysis and design of the control system subject to random faults introduced in chapter2.

First, the models of the plant, the faulty actuator and the controller, along with the evaluation of performance of the control loop in the ideal situation of a fault-free actuator will be presented in section 4.1. Next in section 4.2, analysis of stability and performance of the single controller scheme will be carried out, by using the results of section 3.2. Finally in Section 4.3, the design of stabilizing and suboptimal feedback switching strategies will be reformed with reference to the dual controller scheme.

4.1 The model

4.1.1 Plant

Recall the scheme in chapter 2, the plant part



We define the plant as a double integrator present as state space format

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ z(k) = x(k) \end{cases}$$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B=\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

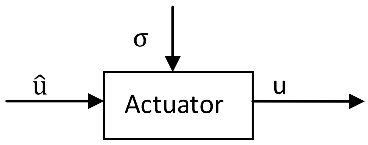
With $x_0 = 0$

It is obvious that the transfer function of plant is

$$G(z) = \frac{1}{(z-1)^2}$$

4.1.2 Faulty Actuator

Recall the scheme in chapter 2, the actuator part



We model signal $\sigma(k)$ as

$$\begin{cases} \sigma(k) = 1 & \text{Healthy} \\ \sigma(k) = 2 & \text{Faulty} \end{cases}$$

So the overall system is jumping between two states according to σ Markov chain, which is determined by transition matrix

$$\Lambda = \begin{bmatrix} 1-\beta & \beta \\ \alpha & 1-\alpha \end{bmatrix}, \quad 0 \le \alpha, \beta \le 1.$$

Λ is a right-stochastic matrix

If the value of β is large, it means system has more chance to jump to faults model.

If the value of α is large, the probability of recovering after a fault is higher.

In the following sections, we will use Matlab to be a simulation tool to find the region of α, β that makes the feedback system mean-square stable.

We will set 2 different configurations for the faulty actuator model

1 zero-input configuration

$$u(k) = \begin{cases} \hat{u}(k) & \sigma(k) = 1 \\ 0 & \sigma(k) = 2 \end{cases}$$

When the actuator is working(σ =1), the system is in closed loop configuration, if the actuator is faulty(σ =2), then the system is in open loop configuration.

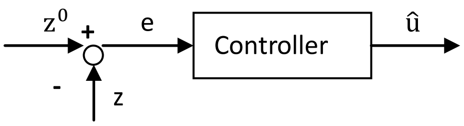
2 input-hold configuration

$$u(k) = \begin{cases} \hat{u}(k) & \sigma(k) = 1 \\ u(k-1) & \sigma(k) = 2 \end{cases}$$

When the actuator is working(σ =1), the system is in closed loop configuration, if the actuator is faulty(σ =2), instead of open loop, we use the previous input $u(k-1)$.

4.1.3 Controller

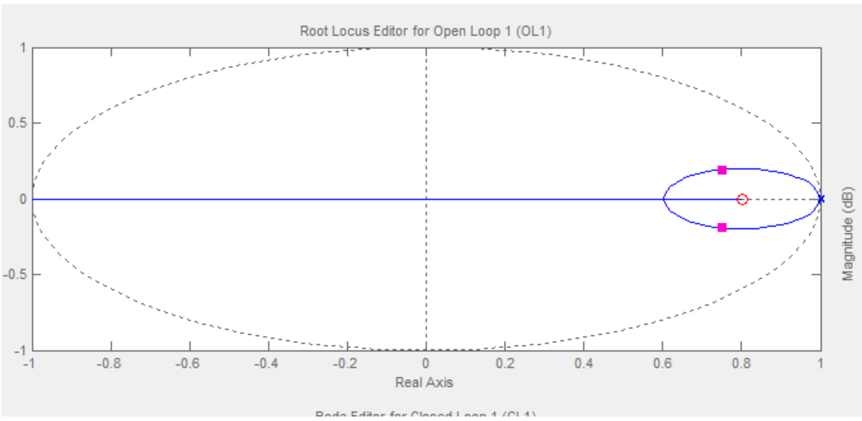
Recall the scheme in chapter 2, the actuator part



The controller is Linear Time invariant (LMI) and at first order.

At single controller scheme we need only one controller, but for dual switching scheme we need two different controllers. So we design an aggressive controller first and then design a moderate controller.

By using the sisotool box in Matlab we get an aggressive controller and a moderate controller to make feedback system stable:



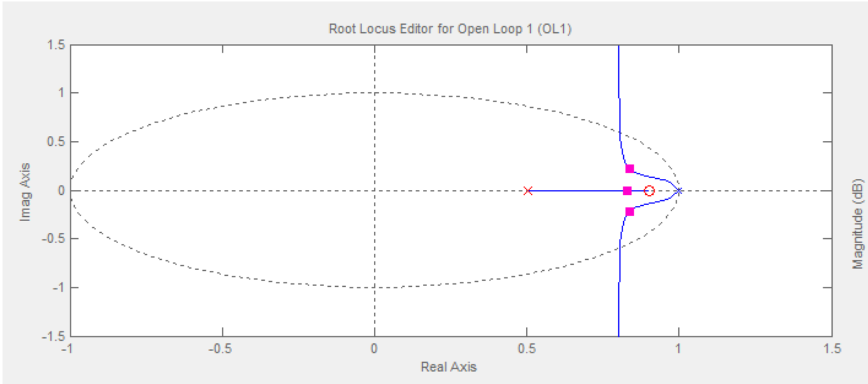
$$R_1(z) = \frac{0.5z - 0.4}{z}$$

State space realization:

$$\begin{cases} x_c(k+1) = A_c x_c(k) + B_c e(k) \\ \hat{u}(k) = C_c x_c(k) + D_c e(k) \end{cases}$$

$$A_c = 0, \qquad B_c = 0.5, \qquad C_c = -0.8, \qquad D_c = 0.5$$

For moderate controller



$$R_2(z) = \frac{0.13(z-0.9)}{z-0.5}$$

State space realization:

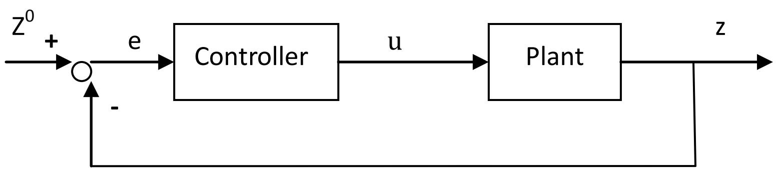
$$\begin{cases} x_{c2}(k+1) = A_{c2}x_{c2}(k) + B_{c2}e(k) \\ \hat{u}(k) = C_{c2}x_{c2}(k) + D_{c2}e(k) \end{cases}$$

$$A_{c2} = 0.5, \qquad B_{c2} = 0.25, \qquad C_{c2} = -0.208, \qquad D_{c2} = 0.13$$

4.1.4 Analysis of the Fault-free control Systems

In order to better understand and analysis the dual-switching feedback system, we now discuss the fault-free situation respect to two different controllers.

That is means without actuator.



The aggressive controller:

The overall feedback system with the aggressive controller can be represent as following state space form:

$$\begin{cases} x_a(k+1) = A_a x_a(k) + B_a z^0 \\ z_a(k) = C_a x_a(k) + D_a z^0 \end{cases}$$

$$A_a = \begin{bmatrix} 1 & 1 & 0 \\ -0.5 & 1 & -0.8 \\ -0.5 & 0 & 0 \end{bmatrix}, \qquad B_a = \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}, \qquad C_a = [1 \quad 0 \quad 0], \qquad D_a = 0$$

We can calculate its transfer function

$$G_a = 0.5\frac{z-0.8}{(z-0.6886\ +\ 0.41i)(z-0.6886-\ 0.4100i)(z-0.6227\ )}$$

We can see, the absolute value of 3 closed loop poles are all smaller than 1, so the feedback system is stable.

As for performance

$$J_2 = E\left[\sum_{k=0}^{\infty} z(k)^T z(k)\right] = 0.6414$$

The moderate controller:

The overall feedback system with the aggressive controller can be represent as following state space form:

$$\begin{cases} x_m(k+1) = A_m x_m(k) + B_m z^0 \\ z_m(k) = C_m x_m(k) + D_m z^0 \end{cases}$$

$$A_m = \begin{bmatrix} 1 & 1 & 0 \\ -0.13 & 1 & -0.208 \\ -0.25 & 0 & 0.5 \end{bmatrix}, \qquad B_m = \begin{bmatrix} 0 \\ 0.13 \\ 0.25 \end{bmatrix}, \qquad C_m = [1 \quad 0 \quad 0], \qquad D_m = 0$$

We can calculate its transfer function

$$G_a = 0.13\frac{z-0.9}{(z-0.8397\ +\ 0.2163i)(z-0.8397-\ 0.2193i)(z-0.8207)}$$

We can see, the absolute value of 3 closed loop poles are all smaller than 1, so the feedback system is stable.

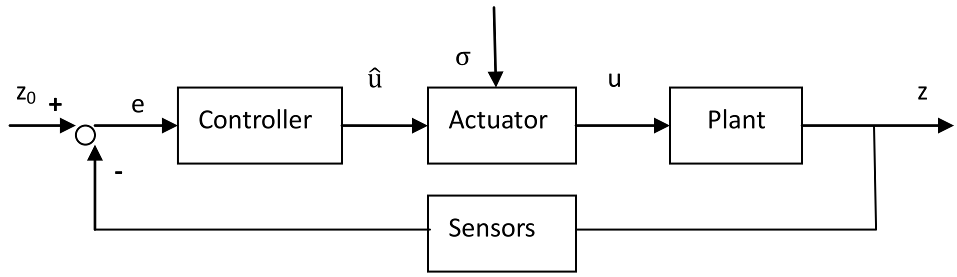
As for performance

$$J_2 = E\left[\sum_{k=0}^{\infty} z(k)^T z(k)\right] = 2.6579$$

4.2 Single Plant Single controller Scheme Analysis

4.2.1 Complete model design

Recall the single controller scheme figure1 in chapter 2



Set sensors to be 1

Specific Plant:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ z(k) = Cx(k) \end{cases}$$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad C = [1 \quad 0], \qquad D = 0$$

The overall system with controller and $\sigma(k)$ present as

$$\begin{cases} \bar{x}(k+1) = A_{\sigma(k)}\bar{x}(k) + B_{\sigma(k)}\, z^0(k) \\ \bar{z}(k) = C_{\sigma(k)}\bar{x}(k) \end{cases}$$

$$\bar{x}(k) = \begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix}, \quad \bar{x}_0(k) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

When system switch to health model, $u(k) = \hat{u}(k)$

$$x(k+1) = Ax(k) + B\big(C_c x_c(k) + D_c e(k)\big)$$

$$x_c(k+1) = A_c x_c(k) + B_c e(k)$$

$$z(k) = Cx(k)$$

Then we have

$$\bar{x}(k+1) = \begin{bmatrix} A & BC_c \\ 0 & A_c \end{bmatrix} \bar{x}(k) + \begin{bmatrix} BD_c \\ B_c \end{bmatrix} e(k)$$

Where $e(k) = z^0(k) - z(k) = z^0(k) - Cx(k)$, we get

$$\bar{x}(k+1) = \begin{bmatrix} A - BD_cC & BC_c \\ -B_cC & A_c \end{bmatrix} \bar{x}(k) + \begin{bmatrix} BD_c \\ B_c \end{bmatrix} z^0(k)$$

When system switch to fault model, $u(k) = 0$

Similarly, we can get

$$\bar{x}(k+1) = \begin{bmatrix} A & 0 \\ -B_cC & A_c \end{bmatrix} \bar{x}(k) + \begin{bmatrix} 0 \\ B_c \end{bmatrix} z^0(k)$$

Then we have

$$A_1 = \begin{bmatrix} A - BD_cC & BC_c \\ -B_cC & A_c \end{bmatrix}, A_2 = \begin{bmatrix} A & 0 \\ -B_cC & A_c \end{bmatrix}$$

$$B_1 = \begin{bmatrix} BD_c \\ B_c \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ B_c \end{bmatrix}$$

$$C_1 = C_2 = [1 \quad 0 \quad 0]$$

4.2.2 Stability and Performance

In this subsection, we will use Matlab to simulate the feedback system, find out which α and β pairs make the system MS-stable and which are not(by using the code 'feasp').Furthermore, we will find out the optimize J_2($\mathscr{H}_2$ performance) of the feedback system.(by using the code 'mincx')

The actuator is subjected to failure modeled by Markov chain with transition matrix Λ and initial distribution π_0, this signal is represented by σ.The plant is a discrete When the actuator is normal(σ =1), the system is in closed loop configuration, if the actuator is faulty(σ =2), then the system is in open loop configuration. So the overall system is jumping between two states according to σ, which in turn is determined by transition matrix Λ =[1- β β ; α 1- α], so as a and α β change from 0 to 1, the system will jump between MS-stable and MS-unstable.

In addition, we will also use Matlab to simulation the real system behavior so that to roughly validate the theorem in chapter 3. Besides, because simulation is not real system, we will also discuss the inconsistencies and consistencies

Region of MS-stability and performance

By using Matlab, we can plot a α, β region that is MS-stable (define in chapter3) and a J_2 surface ($\mathscr{H}_2$ performance) for this system.

First, as we already know, if give a fixed Λ matrix(which means fixed α, β), we can use Matlab LMI tool(code 'feasp') to find if there exist a positive P matrix satisfy the inequality in chapter3(**THEOREM1.**), if exist, the Matlab result shows 'feasible', that is to say the system is MS-stable for this Λ matrix(or α, β pair).

Now we try 400 different α, β pairs (range from 0.05 to 0.95, the resolution is 0.05) to find which pair guarantee MS-stable for system which not, then we can plot an α, β plan shows MS-stable region.

α, β MS-stable region:

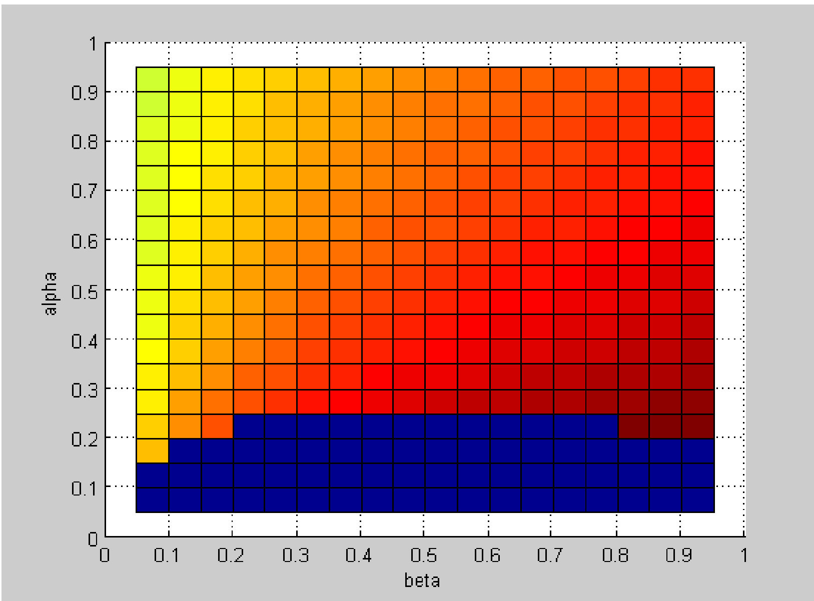


Figure5 single controller MS-stable region

Then we can also calculate performance through the formular （**THEOREM2**）:

$$J_2 = x_0^T \left(\sum_{i=1}^{N} \overline{\pi}_i P_i \right) x_0$$

J_2 is called optimal upper bound in this theorem.

Notice that we can only calculate the MS-stable region, for the reason that the MS-unstable region is infeasible we can't get P matrix, so we set it all to be 0 in this plot.

J2-surface:

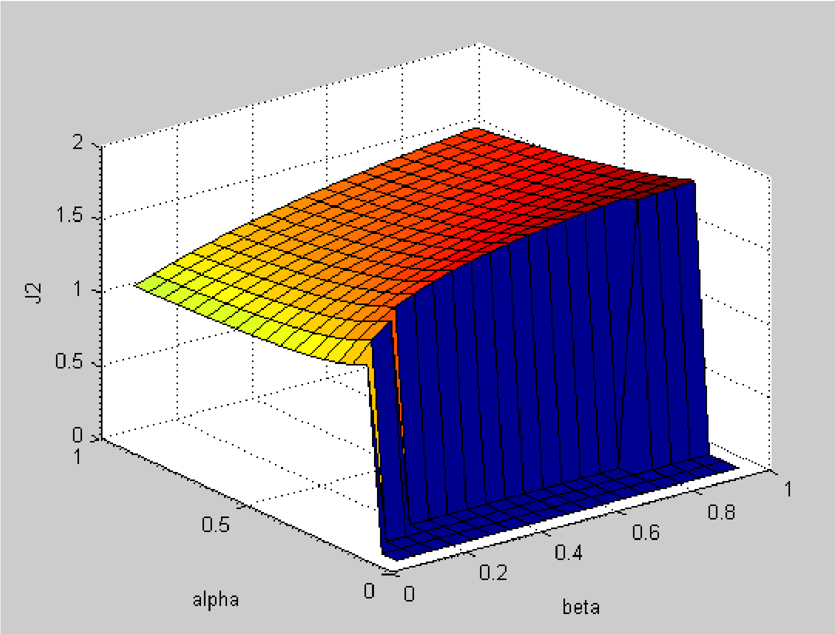


Figure6 single controller upper bound J2-surface

The blue region is MS-unstable region, and the red and yellow region stands for MS-stable region. From the figure4, we can see, α is more important for MS-stable of this feedback system. So in application, in order to guarantee MS-stable of system, the value of α should be large enough.

From figure6, it is obvious that when choose a certain α, the larger the β, the larger the J_2, which means bigger energy theoretically.

For the simulation related to real system realization.

We set the time index k=50, and generate 50(n=50) different $\sigma(k)$ markov chains to simulate $z(k)$, then we can get the value of J_2 by calculating from this simulated output $z(k)$, we call it J_2^*.

$$J_2^* = \left(\sum_{k=1}^{50} z(k)^T z(k)\right) / 50$$

Repeat the 400 pairs of α, β sample; we plot the J_2^* surface respect to α, β plan.

J_2^*-surface of α, β:

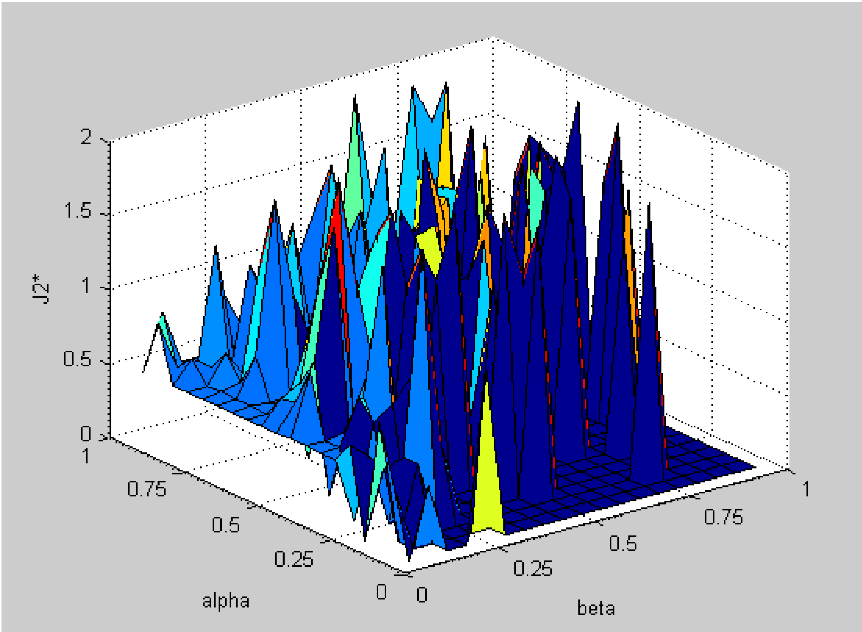


Figure7 single controller simulation J_2^*-surface

From figure7, because of the simulation quantity 400 is not large enough so the surface is not smooth (because $z(k)$ is a little random).But we can still see the trends is similar to figure6 and most of the value of J_2^* is smaller than the value J_2 in figure6, J_2 is the theodicy upper bound.

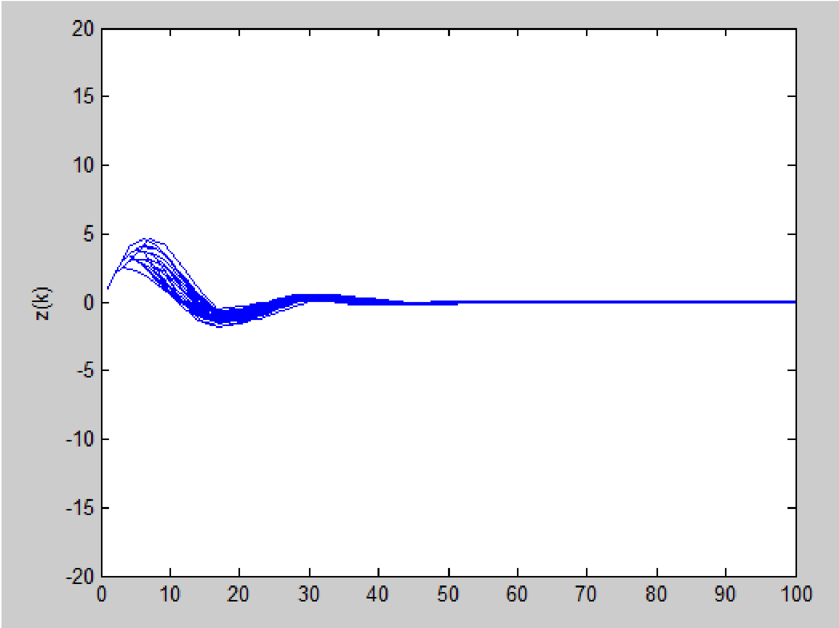
Particular examples for α, β

Now, as we already known the MS-stable α, β region, we choose 3 different points on α, β plan as examples to plot $z(k)$ behaviors. Then use the same simulation method when we calculate J_2^*.

This time we set time index k=100, use Matlab to simulate 50(n=50) different value (view as real data) of $z(k)$ for each point.

We plot both 50 pairs $z(k)$ together and its average value.

$\alpha = 0.9, \beta = 0.9$(well inside the stable region）:



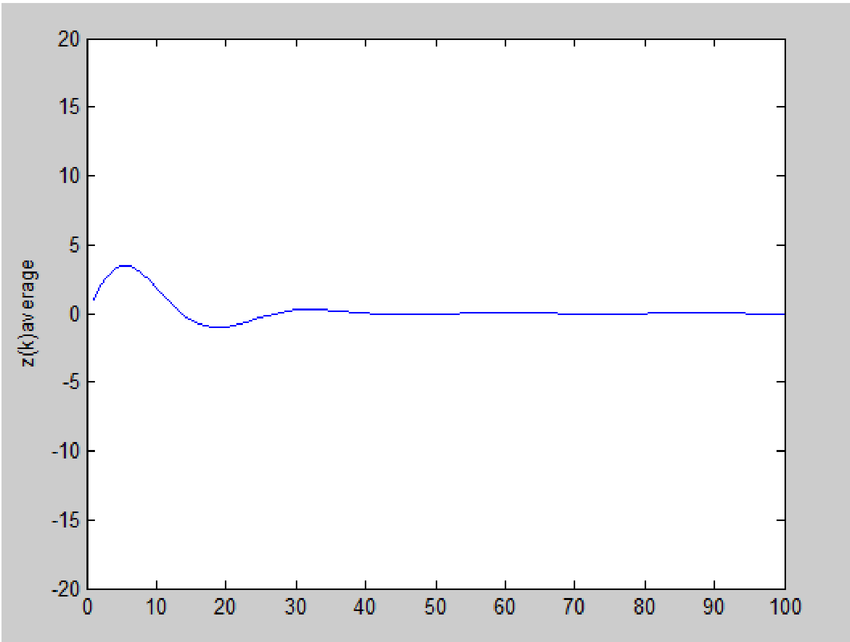
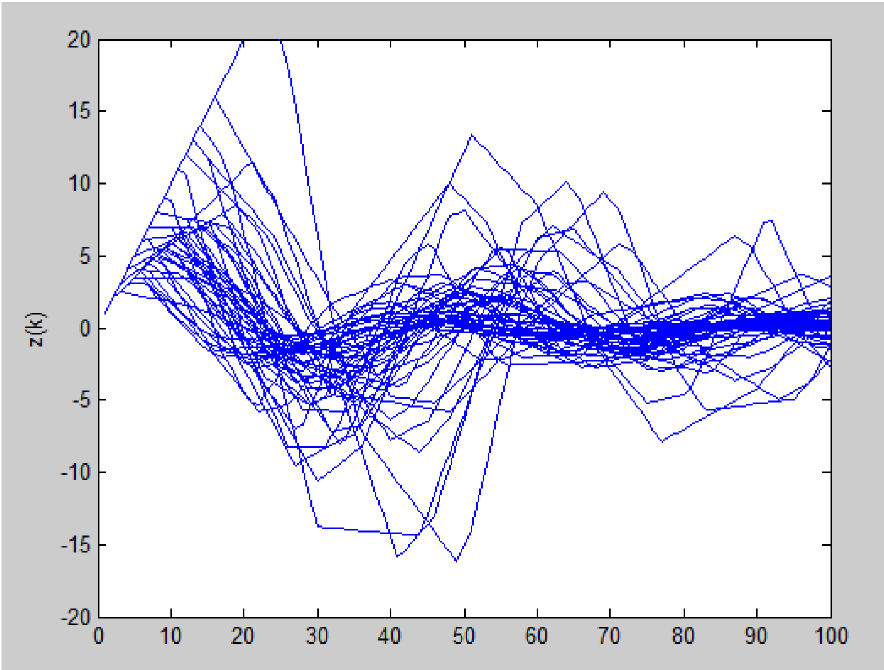


Figure8 particular example z(k) plot for single controller scheme 1

From figure8, we obviously see 50 $z(k)$ converge to steady state quickly, and the average line also shows that $z(k)$ go to steady state fast.

$\alpha = 0.2, \beta = 0.9$(close to stable bound):



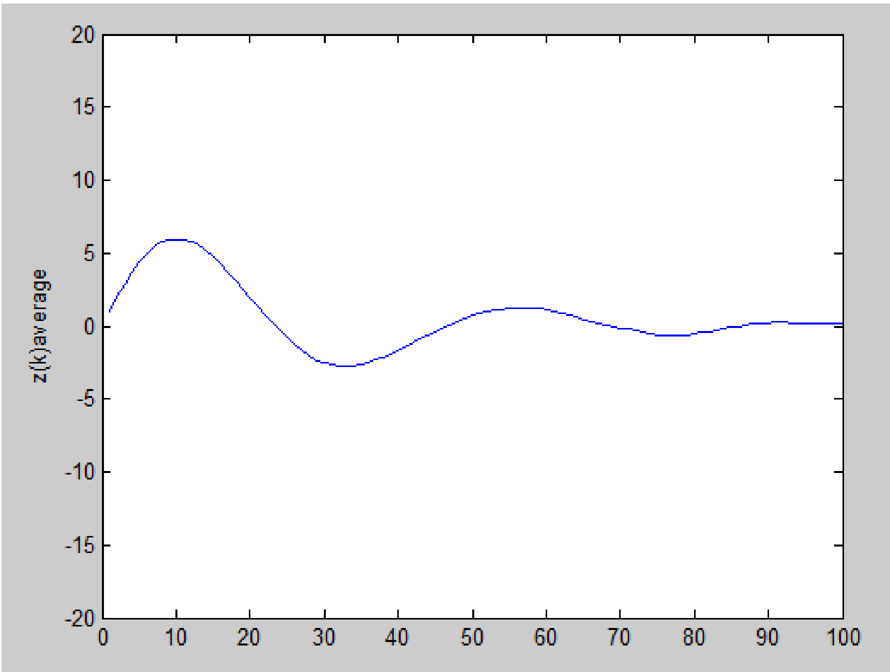
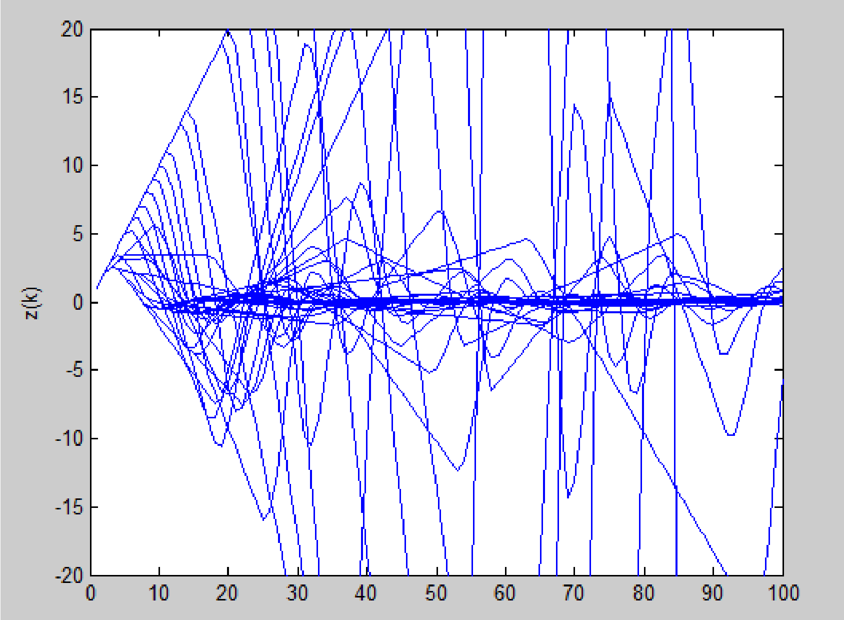


Figure9 particular example z(k) plot for single controller scheme 2

From figure9, some $z(k)$ diverge, but the average line of $z(k)$ might finally go to steady state 0, because this point is near unstable bound, so the time go to steady state is longer.

$\alpha = 0.1, \beta = 0.1$(unstable case):



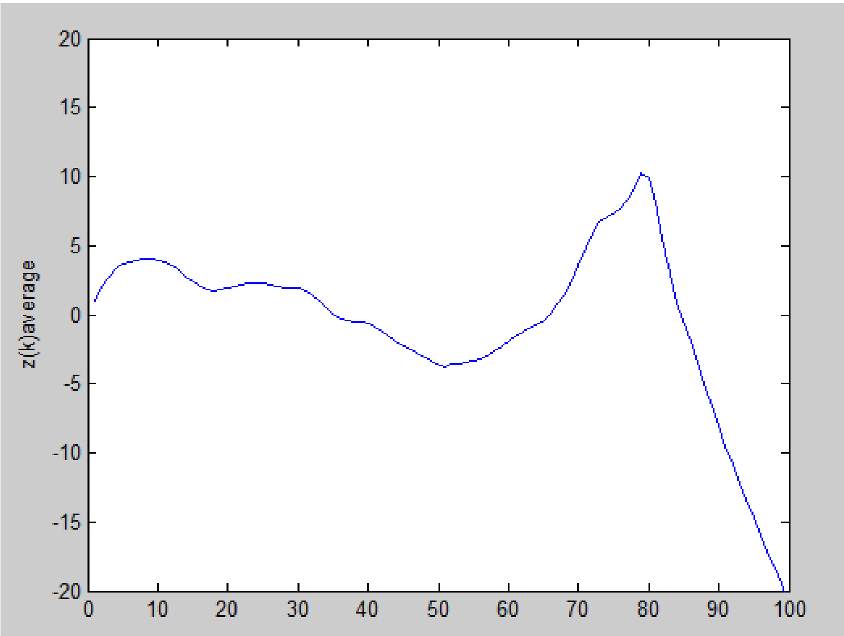


Figure10 particular example z(k) plot for single controller scheme 3

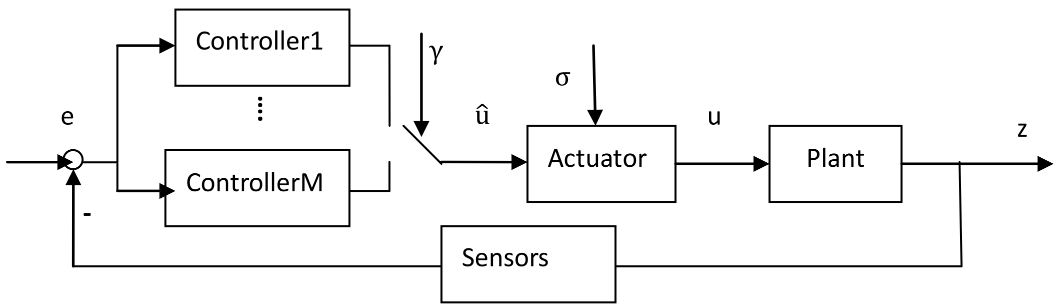
From figure10, many $z(k)$ lines diverge, the average line of $z(k)$ shows that it finally go far away from steady state, so this point must be unstable.

These simulation results of the three points which chosen both from unstable and stable region shows the stable $\alpha\,\beta$ region which calculates from the theorem in Chapter3 is correct.

4.3 Single Plant Dual Controller scheme

4.3.1 Complete model design

Recall the dual-switching scheme in chapter 2



Again assume sensors are 1.

Specific Plant:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases}$$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad C = [1 \quad 0], \qquad D = 0$$

We have two controllers in this system, an aggressive one and a moderate one. So

M=2, and $\begin{cases} \gamma(k) = 1 & \text{controller1} \\ \gamma(k) = 2 & \text{controller2} \end{cases}$

Recall that

Controller1

$$\begin{cases} x_c(k+1) = A_c x_c(k) + B_c e(k) \\ \hat{u}(k) = C_c x_c(k) + D_c e(k) \end{cases}$$

$$A_c = 0, \qquad B_c = 0.5, \qquad C_c = -0.8, \qquad D_c = 0.5$$

Controller2:

$$\begin{cases} x_{c2}(k+1) = A_{c2} x_{c2}(k) + B_{c2} e(k) \\ \hat{u}(k) = C_{c2} x_{c2}(k) + D_{c2} e(k) \end{cases}$$

$$A_{c2} = 0.5, \qquad B_{c2} = 0.25, \qquad C_{c2} = -0.208, \qquad D_{c2} = 0.13$$

Where $e(k) = -z(k)$,

Scheduler signal for controllers:

$$\hat{u}(k) = \begin{cases} \hat{u}_1(k), & \text{if } \gamma(k) = 1 \\ \hat{u}_2(k) & \text{if } \gamma(k) = 2 \end{cases}$$

The signal regulate by actuator here, we have two methods to model the fault state. One is *zero-input configuration,* which set $u(k) = 0$, when fault occurs,

$$u(k) = \begin{cases} \hat{u}(k), & \sigma(k) = 1 \\ 0, & \sigma(k) = 2 \end{cases}$$

Another one is i*nput-hold configuration,* which set $u(k) = u(k-t)$(the last step value):

$$u(k) = \begin{cases} \hat{u}(k), & \sigma(k) = 1 \\ u(k-t), & \sigma(k) = 2 \end{cases}$$

For *zero-input configuration:*

$$\tilde{x}(k+1) = \tilde{A}_{\sigma(k)}^{\gamma(k)} \tilde{x}(k), \quad \tilde{x}(k) = \begin{bmatrix} x(k) \\ x_c(k) \\ x_{c2}(k) \end{bmatrix}$$

$$\tilde{A}_1^1 = \begin{bmatrix} A - BD_cC & BC_c & 0 \\ -B_cC & A_c & 0 \\ -B_{c2}C & 0 & A_{c2} \end{bmatrix}, \qquad \tilde{A}_1^2 = \begin{bmatrix} A - BD_{c2}C & 0 & BC_{c2} \\ -B_cC & A_c & 0 \\ -B_{c2}C & 0 & A_{c2} \end{bmatrix}$$

$$\tilde{A}_2^1 = \tilde{A}_2^2 = \begin{bmatrix} A & 0 & 0 \\ -B_cC & A_c & 0 \\ -B_{c2}C & 0 & A_{c2} \end{bmatrix}$$

Performance output

$$\tilde{z}(k) = \begin{cases} [C - D_cC \quad C_c \quad 0], & \text{if } \gamma(k) = 1 \\ [C - D_{c2}C \quad 0 \quad C_{c2}], & \text{if } \gamma(k) = 2 \end{cases}$$

For i*nput-hold configuration:*

$$\bar{x}(k+1) = \bar{A}_{\sigma(k)}^{\gamma(k)} \bar{x}(k), \quad \bar{x}(k) = \begin{bmatrix} x(k) \\ x_c(k) \\ x_{c2}(k) \\ u(k-1) \end{bmatrix}$$

$$\bar{A}_1^1 = \begin{bmatrix} A - BD_{c2}C & BC_c & 0 & 0 \\ -B_cC & A_c & 0 & 0 \\ -B_{c2}C & 0 & A_{c2} & 0 \\ -D_cC & C_c & 0 & 0 \end{bmatrix}$$

$$\bar{A}_1^2 = \begin{bmatrix} A - BD_{c2}C & 0 & BC_{c2} & 0 \\ -B_cC & A_c & 0 & 0 \\ -B_{c2}C & 0 & A_{c2} & 0 \\ -D_{c2}C & 0 & C_{c2} & 0 \end{bmatrix}$$

$$\bar{A}_2^1 = \bar{A}_2^2 = \begin{bmatrix} A & 0 & 0 & B \\ -B_cC & A_c & 0 & 0 \\ -B_{c2}C & 0 & A_{c2} & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

Performance output

$$\bar{z}(k) = \begin{bmatrix} C & 0 & 0 & \eta\, I \end{bmatrix} \bar{x}(k)$$

Where η is the weight of $u(k-1)$.Greater values of η imply higher cost of the control input, so we should push towards a trade-off between performance and moderation of control.

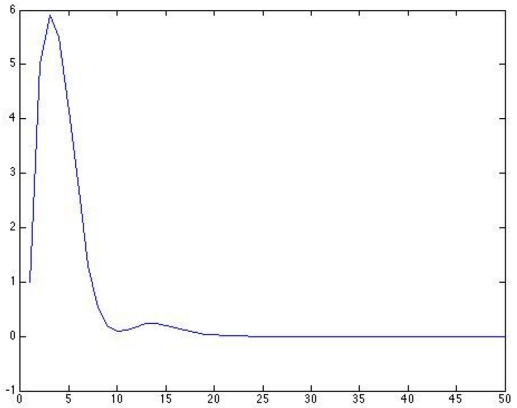
4.3.2 Stability and Performance analysis

We set matrix Φ to be a constant matrix $\begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$, which related to $\gamma(k)$.

still, Λ =[1- β β ;a 1-a] $(|\alpha|, |\beta| < 1)$

Difference on initial state

First, we discuss the influence of different initial state on performance output $z(k)$.

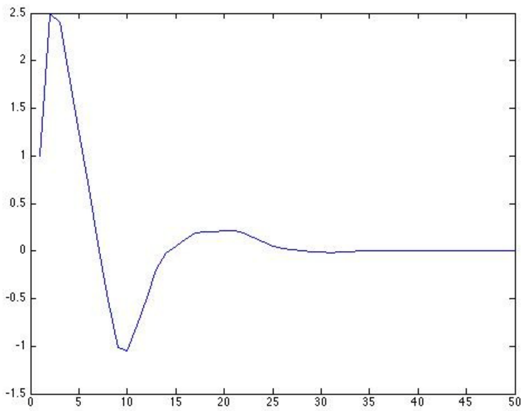


Output $z(k)$ as a function of k

when x_0= [1; 1; 0]

Λ=[0.8 0.2;0.7 0.3]

J_2=205.6214

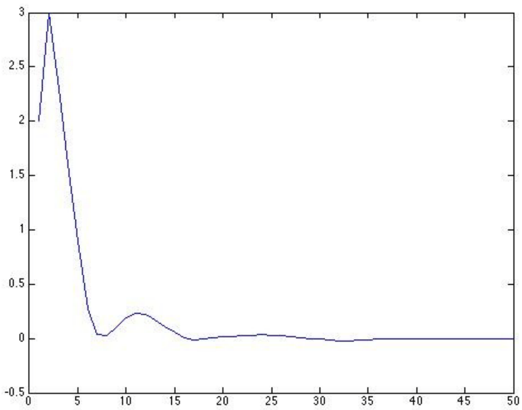


Output $z(k)$ as a function of k

when x_0= [1; 2; 0]

Λ=[0.8 0.2;0.7 0.3]

J_2=803.8743

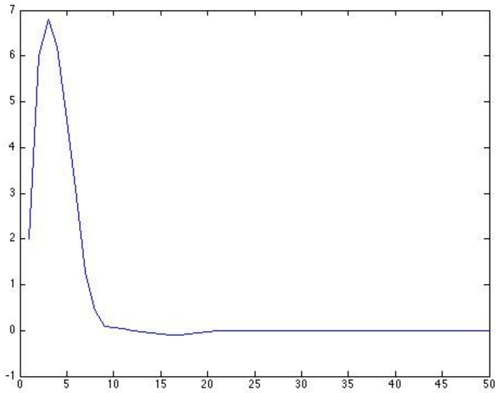


Output $z(k)$ as a function of k

when Λ= [0.8 0.2;0.7 0.3]

x_0= [2; 1; 0]

J_2=268.8563



Output $z(k)$ as a function of k when

x_0= [2; 2; 0]

Λ=[0.8 0.2;0.7 0.3]

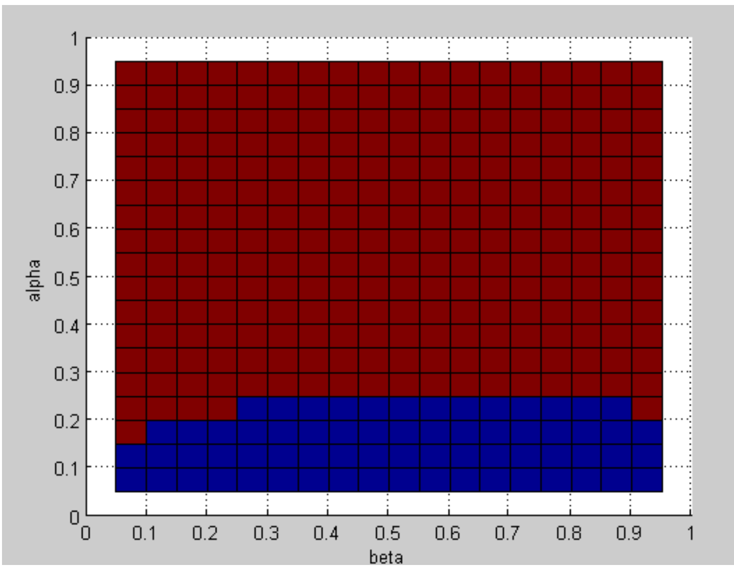
J_2=822.4855

Figure11 difference on initial state

According to the plots above, as long as the α and β pair is chosen from the stability region, the output of the system $z(k)$ always converges. Also because in any cases the output matrix C is always [1 0 0], means the output is actually the first state variable X_1, when the system is MS-stable, the states eventually goes to 0.

Region of MS-stability

Now let's discuss α, β region for Λ matrixes. σ is used to model the stochastic behavior of the actuator, which can take one of the two states either normal or faulty, at each time instant k, when $\sigma(k)$ take value 1, it means the actuator is in normal condition and the system is in feedback configuration, otherwise the actuator is in faulty condition and the system is in open loop configuration.



Feagure12 dual controller MS-stable region

This is the stability region of α and β pair, red region above indicates that this α β pair makes system stable and blue region indicates that this α β pair makes system unstable. Compare to the single controller scheme in last section, the stability region is similar.

Particular examples for stability

To deeply discuss the α β region, now we take 4 pairs of α and β to make simulation, they are 4 point from 4 comers of the stability α β region. When the actuator is faulty the input to the plant is 0(zero-input case).

Since the generation of σ doesn't depend on X_0, let us consider different Λ, we will first simulate the $z(k)$ behavior step by step with the above σ signal series, to check stability of the certain point which we choose in the stable α β region, and second use the law in chapter 3 to generate γ from σ, third calculate $\mathscr{H}_2$ performance J_2 , compare it with simulation output performance J.

① as Λ =[0.8 0.2;0.7 0.3] (well inside stable region).

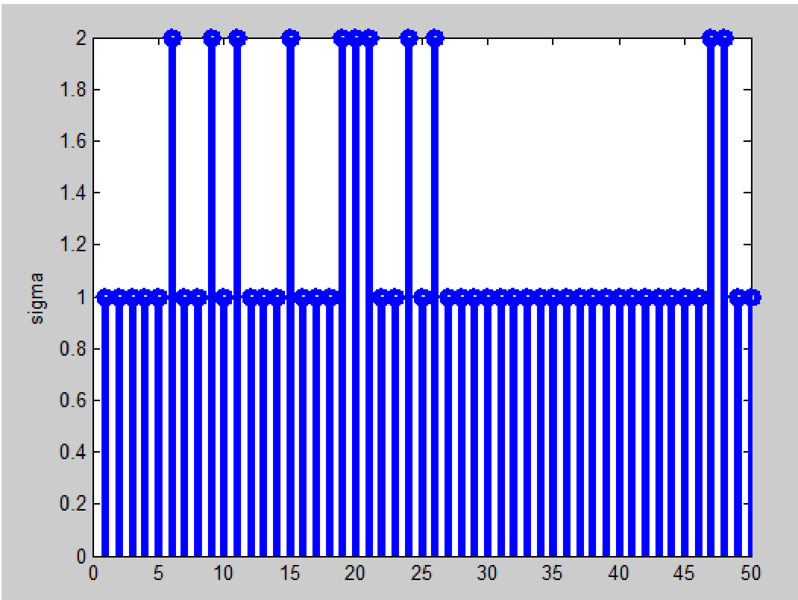


Figure13 particular example for dual controller of σ sample for case 1

This figure shows σ jumps generated from Λ ratio matrix, we have big α, small β , which means it has a high chance be in 1st state, overall the system "spends" more time in state 1 than state 2, which is coincide with the σ plots above. As a result the system should be surely MS-stable.

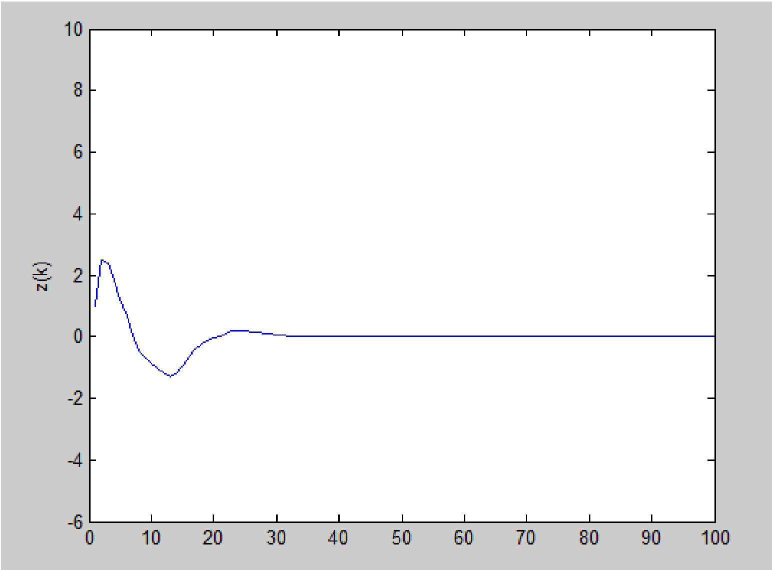


Figure14 particular example for dual controller of z(k) for case 1

The output $z(k)$ as a function of k (=100) in the figure quickly go to steady stead, the good response of $z(k)$ obviously proof that the system is MS-stable. When k is in 0-25 interval, σ jumps frequently, the response of $z(k)$ appears vibrations as well, it says that the jumps cause energy vibration of the system

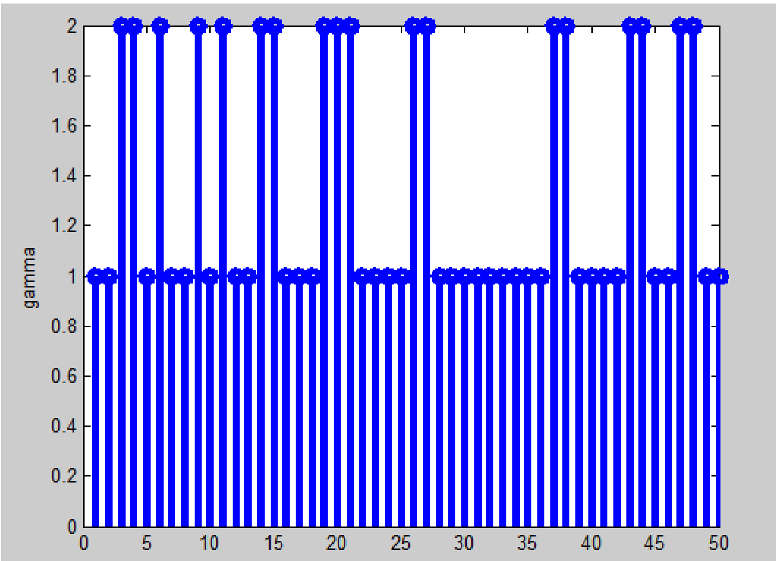


Figure15 particular example for dual controller of γ for case 1

This figure is γ signal jumps generated from the formula

$$\gamma^* = \mathrm{argmin}_r x^T P_\sigma^r x$$

We will discuss, with ②③④, the relationship between γ and σ

J_2= 205.6214, stands for theorem upper bound (according to the theorem in chapter3).

J_bar = 0.2721, stands for classic energy form, which is $E[z(k)^T z(k)]$.

Generally speaking, $J\_bar < J_2$ should be exist, in case ①, it did, we will also check this condition in following cases.

The feasible P matrix related to switching strategies:

$$P_1^1 = \begin{bmatrix} 24.4741 & -33.4072 & 41.1251 \\ -33.4072 & 101.0199 & -77.4483 \\ 41.1251 & -77.4483 & 83.1081 \end{bmatrix} \quad P_1^2 = \begin{bmatrix} 13.4122 & -5.8982 & 20.3861 \\ -5.8982 & 101.4743 & -36.0256 \\ 20.3861 & -36.0256 & 46.7915 \end{bmatrix}$$

$$P_2^1 = \begin{bmatrix} 11.4788 & 11.0953 & 12.1780 \\ 11.0953 & 113.2723 & -8.1485 \\ 12.1780 & -8.1485 & 29.8791 \end{bmatrix} \quad P_2^2 = \begin{bmatrix} 12.9270 & 7.4700 & 15.8007 \\ 7.4700 & 112.7384 & -15.1265 \\ 15.8007 & -15.1265 & 38.4343 \end{bmatrix}$$

② with Λ =[0.8 0.2;0.25 0.75](near unstable bound):

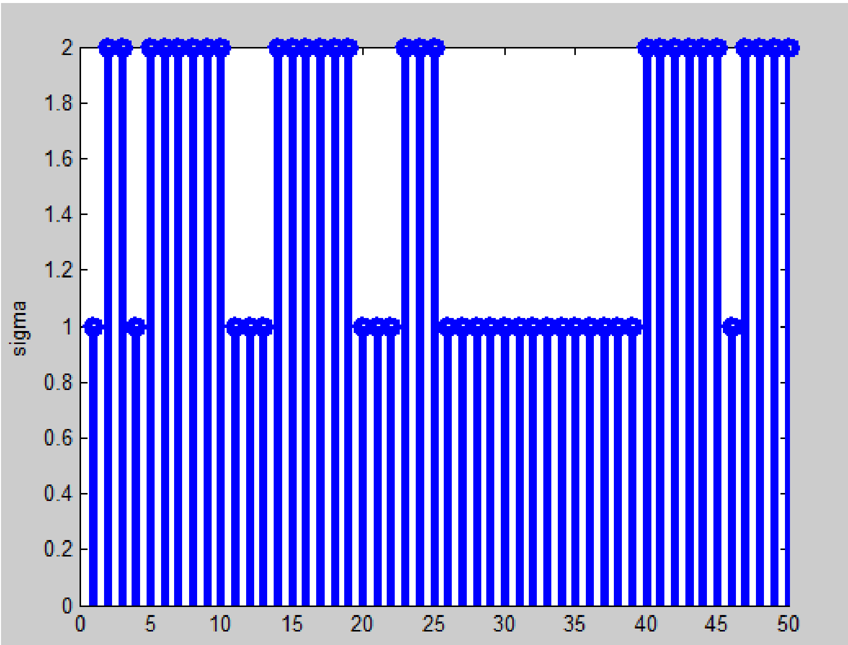


Figure16 particular example for dual controller of σ sample2 for case2

In this case, we have small α, and small β , the state switching frequency will be quiet low, as can be seen from the σ plot, it well reflect the low frequency jump behavior. Since n this case the chosen point is near unstable bound, the MS-stability is not that obvious, so a simulation is necessary to check this property.

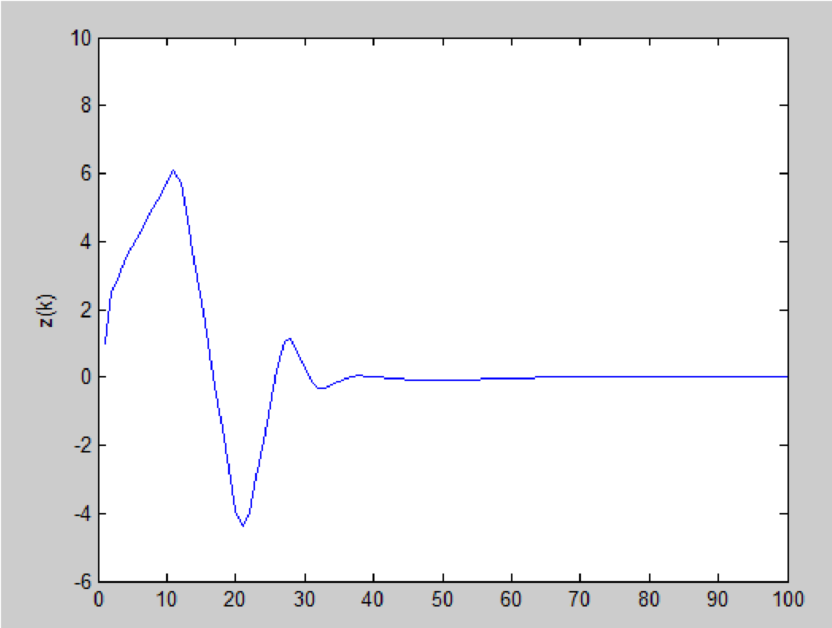


Figure17 particular example for dual controller of z(k) for case 2

The simulation $z(k)$ vibrate amplitude is large but finally go to zero steady state; it indicates the system is still MS-stable. It also proof that this point we chosen from stable α β region really guarantee system stable.

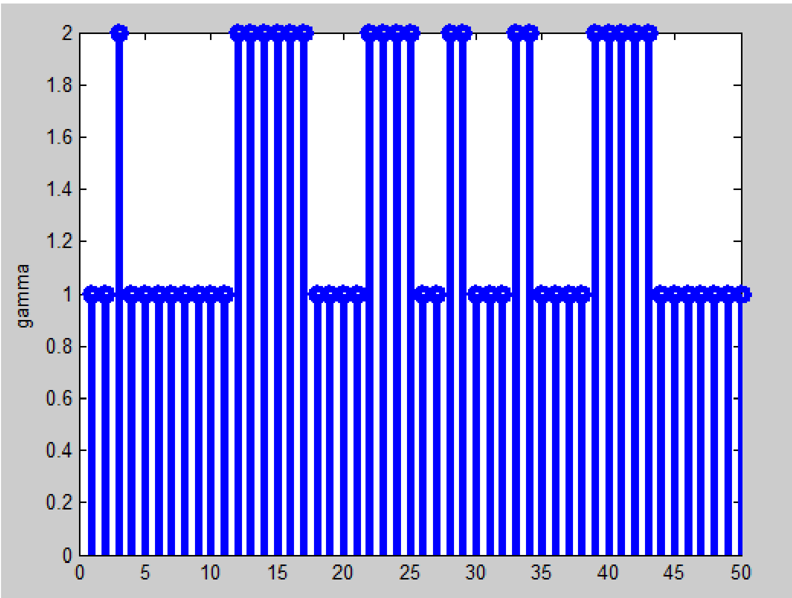


Figure18 particular example for dual controller of γ for case 2

In this case, γ also swichs not frequently.

J_2= 224.0791

J_bar = 3.5331

It satisfied the condition $J\_bar < J_2$, compared with case ①, J_bar is higher because in this case point is near unstable bound, it cost more energy to make system stable, but J_2 is not the same trends.

The feasible P matrix related to switching strategies:

$$P_1^1 = \begin{bmatrix} 25.4934 & -37.8729 & 43.0458 \\ -37.8729 & 90.3932 & -75.9761 \\ 43.0458 & -75.9761 & 79.1414 \end{bmatrix} \quad P_1^2 = \begin{bmatrix} 11.8680 & -10.6231 & 19.3574 \\ -10.6231 & 90.4997 & -34.2603 \\ 19.3574 & -34.2603 & 38.2160 \end{bmatrix}$$

$$P_2^1 = \begin{bmatrix} 6.8081 & 8.5122 & 9.2158 \\ 8.5122 & 160.1058 & -10.7083 \\ 9.2158 & -10.7083 & 19.0769 \end{bmatrix} \quad P_2^2 = \begin{bmatrix} 8.0533 & 7.6857 & 11.4794 \\ 7.6857 & 159.2143 & -12.2187 \\ 11.4794 & -12.2187 & 23.2107 \end{bmatrix}$$

③ Λ =[0.2 0.8;0.3 0.7] (near unstable bound).

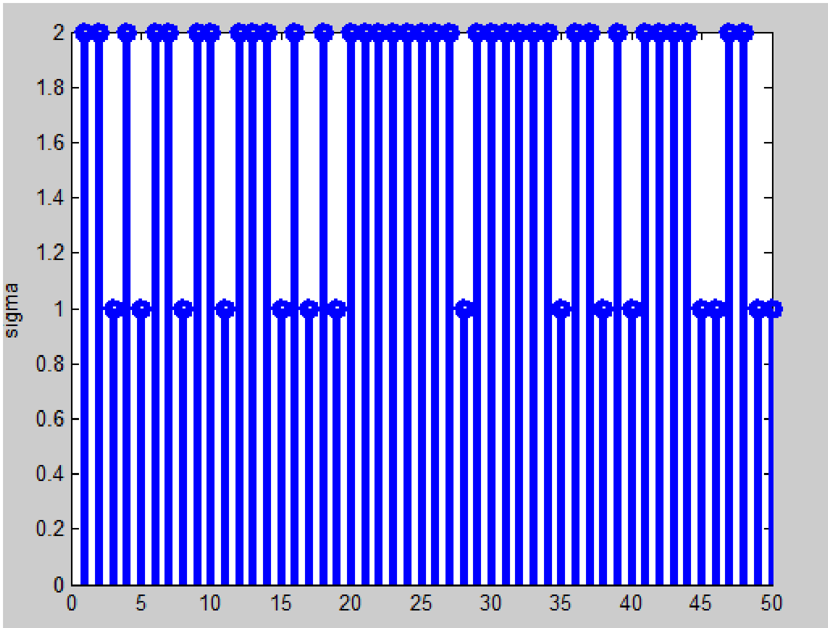


Figure19 particular example for dual controller of σ sample for case 3

In this case, we have small α, and big β, that means system will has more chance to be in 2nd state, the σ plot shows the same trends: state 2 appears more.

The same to case ②, the chosen point is near unstable bound, let's check output $z(k)$.

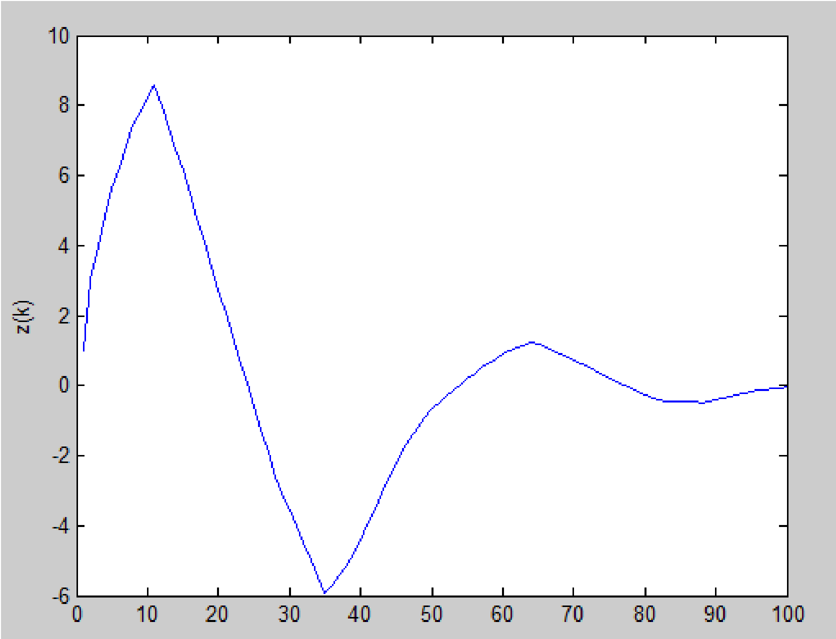


Figure20 particular example for dual controller of z(k) for case 3

The $z(k)$ response's vibration amplitude seems a litter larger (especially at the beginning), though spend more time, eventually, around k=100, it goes to zero steady state, this result proof MS-stable of the system.

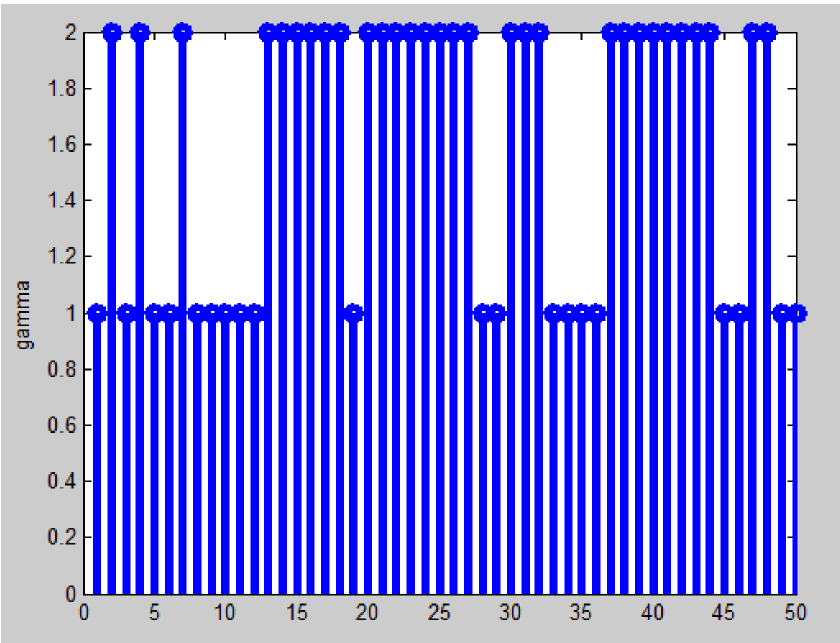


Figure21 particular example for dual controller of γ for case 3

γ jumps more often than case ②.

J_2= 16.6454

J_bar = 10.8688

Condition $J\_bar < J_2$ satisfied.

The feasible P matrix related to switching strategies:

$$P_1^1 = \begin{bmatrix} 2.8619 & -5.0678 & 4.9900 \\ -5.0678 & 11.7312 & -9.4605 \\ 4.9900 & -9.4605 & 9.0854 \end{bmatrix} \quad P_1^2 = \begin{bmatrix} 0.9524 & -1.3426 & 1.6911 \\ -1.3426 & 11.8280 & -3.5987 \\ 1.6911 & -3.5987 & 3.4427 \end{bmatrix}$$

$$P_2^1 = \begin{bmatrix} 0.5289 & 0.1853 & 0.8015 \\ 0.1853 & 11.2884 & -0.9899 \\ 0.8015 & -0.9899 & 1.6487 \end{bmatrix} \quad P_2^2 = \begin{bmatrix} 0.6826 & -0.0020 & 1.0774 \\ -0.0020 & 11.1795 & -1.3584 \\ 1.0774 & -1.3584 & 2.1397 \end{bmatrix}$$

④ Λ =[0.2 0.8;0.9 0.1](well inside stable region).

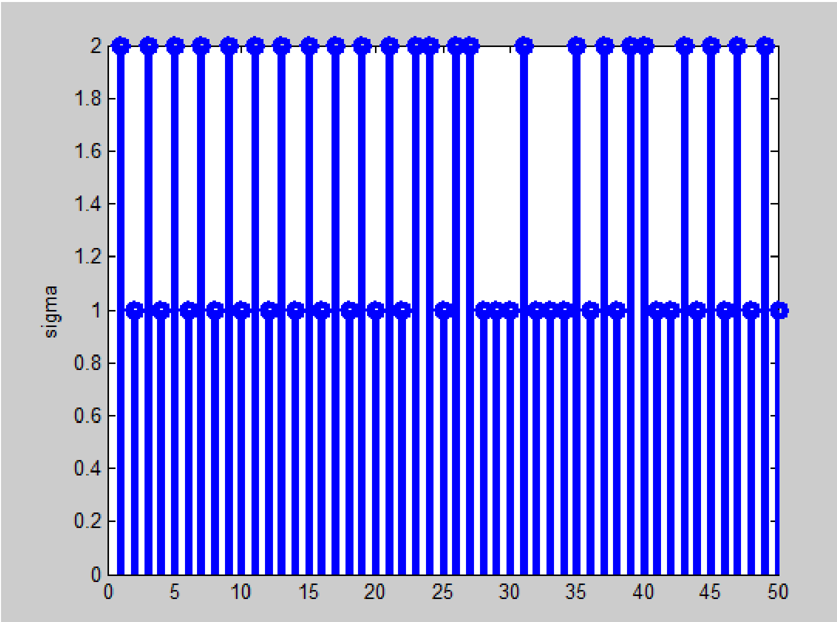


Figure 22 particular example for dual controller of σ sample for case 4

In this case, we have big α, and big β, that is to say there is a high chance to jump to the other one, so as can be seen from the σ plot, the jumping frequency is high.

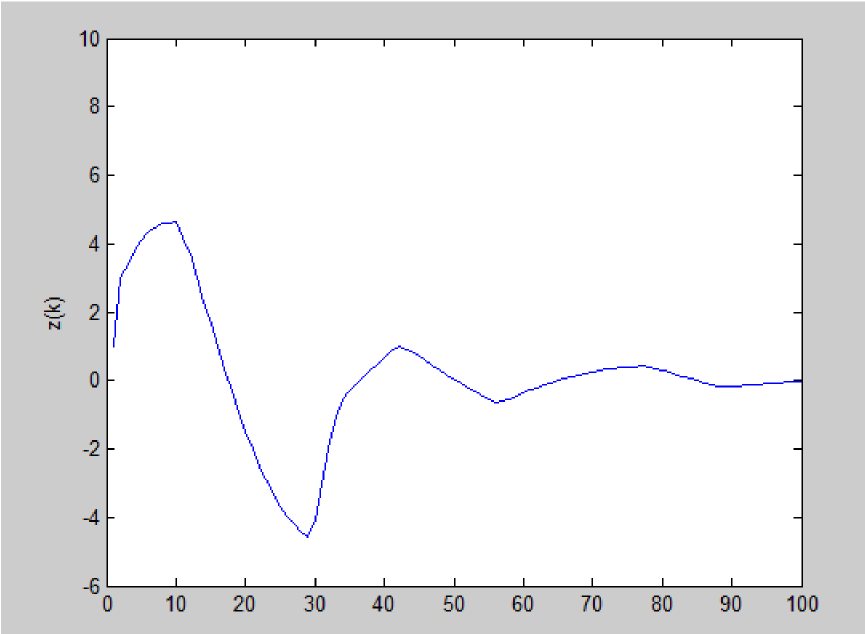


Figure 23 particular example for dual controller of z(k) for case 4

After some little strong vibration, at k=60, $z(k)$ slowly goes to steady state, so the system of this α β pair is MS-stable. In addition, this $z(k)$ shows more oscillation than other case, it may caused by the frequently jumps of σ.

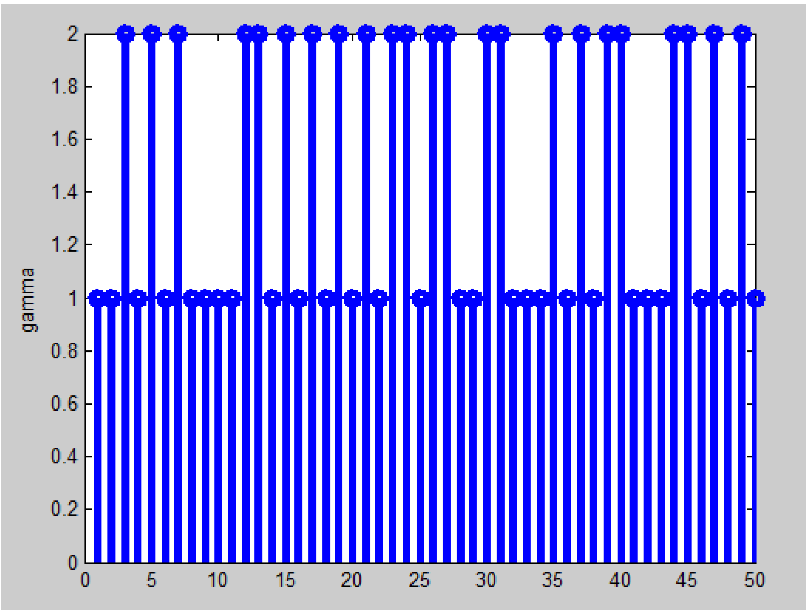


Figure24 particular example for dual controller of γ for case 4

γ jumps frequently, since σ jumps very often.

J_2= 8.2019

J_bar = 3.6508

Condition $\mathrm{J\_bar} < \mathrm{J}_2$ also satisfied.

We can see, the two near unstable bound case ②and③ compared with case①and ④ spend more time and energy to reach steady state.

The feasible P matrix related to switching strategies:

$$\mathrm{P}_1^1 = \begin{bmatrix} 1.1918 & -1.9115 & 2.0458 \\ -1.9115 & 5.3926 & -3.9256 \\ 2.0458 & -3.9256 & 3.9666 \end{bmatrix} \quad \mathrm{P}_1^2 = \begin{bmatrix} 0.5144 & -0.3355 & 0.8135 \\ -0.3355 & 5.4320 & -1.4784 \\ 0.8135 & -1.4784 & 1.8168 \end{bmatrix}$$

$$\mathrm{P}_2^1 = \begin{bmatrix} 0.4070 & 0.1790 & 0.5511 \\ 0.1790 & 4.6755 & -0.4847 \\ 0.5511 & -0.4847 & 1.2307 \end{bmatrix} \quad \mathrm{P}_2^2 = \begin{bmatrix} 0.5156 & -0.0610 & 0.7680 \\ -0.0610 & 4.6306 & -0.9527 \\ 0.7680 & -0.9527 & 1.6666 \end{bmatrix}$$

In conclusion, on one hand, all $\mathrm{z(k)}$ simulation behavior shows that the system is MS-stable at the 4 point (pairs of α β) we chosen from stable α β region. So, the stability theorem in chapter 3 is correct. On the other hand, the vibration frequency of $\mathrm{z(k)}$ and jumping frequency of γ is co react with σ.

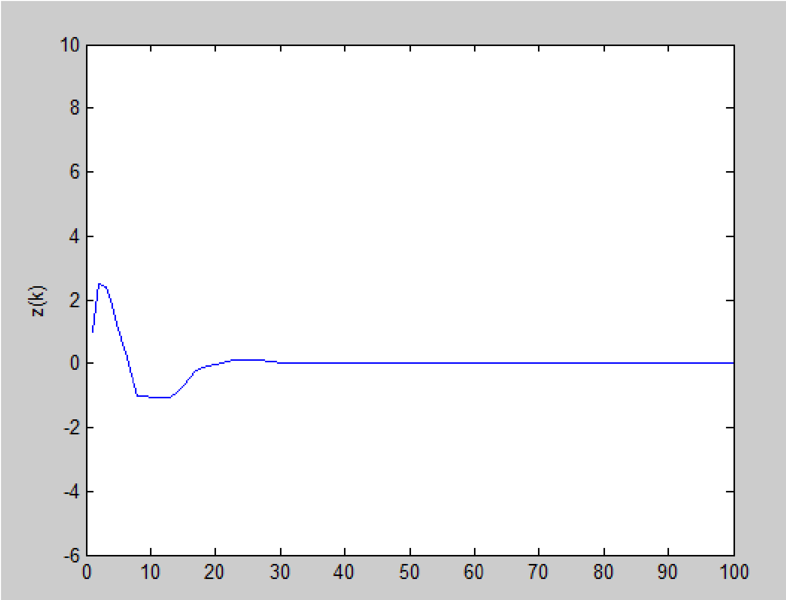
Particular examples for performance (different LMI)

Now begin with the performance problem which includes the $\mathrm{C^TC}$ in LMI terms. Because of the extra $\mathrm{C^TC}$ term and different simulation code (in this case we use optimization 'mincx" code), the optimized P matrix is different from stability case, so the $\mathrm{z(k)}, \gamma$ and J_2 will be little different ether.

Because the code 'mincx' can't directly judge feasibility of LMI, so it's hard for us to get a exactly $\alpha\,\beta$ stable region(when use the code 'feasb' for $\mathrm{C^TC}$ included LIM the region is the same with stability LMI) ,In theory, the stable$\alpha\,\beta$ region for performance LMI should be a little smaller than stability LMI. So, we will repeat the 4 cases step to see the simulation response of $\mathrm{z(k)}$, reproduce γ signal series and calculate J_2 performance with the different P matrix. In order to compare, we use the same σ series with stability case.

First checks the stability of above 4 cases by plotting its output $z(k)$ as a function of k. The initial condition is x_0= [1; 1; 0] in all the cases.

①: Λ =[0.8 0.2;0.7 0.3]



Output $z(k)$ response shows MS-stable for system, and it behavior is similar to the stability case but not completely same.

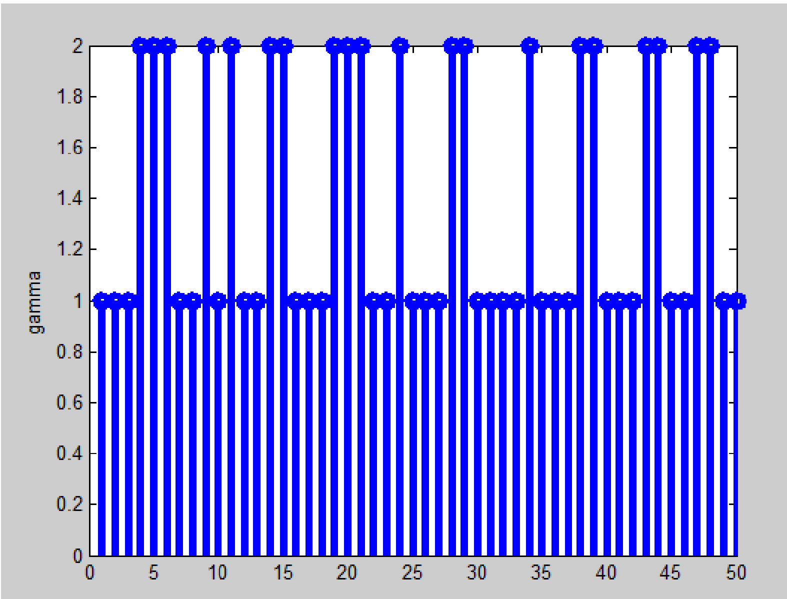


Figure25 dual controller for performance plot of z(k) and γ for case 1

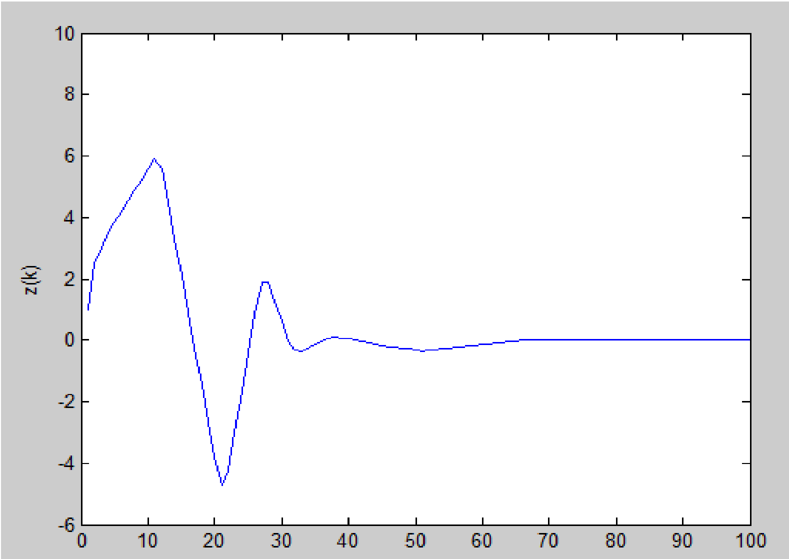
γ series is similar but not same to stability case as well.

J_2= 185.1876

J_bar = 0.2625

Condition $J\_bar < J_2$ satisfied.

②: Λ =[0.8 0.2;0.25 0.75]



Output $z(k)$ response shows MS-stable for system, and it behavior is similar to the stability case but not completely same.

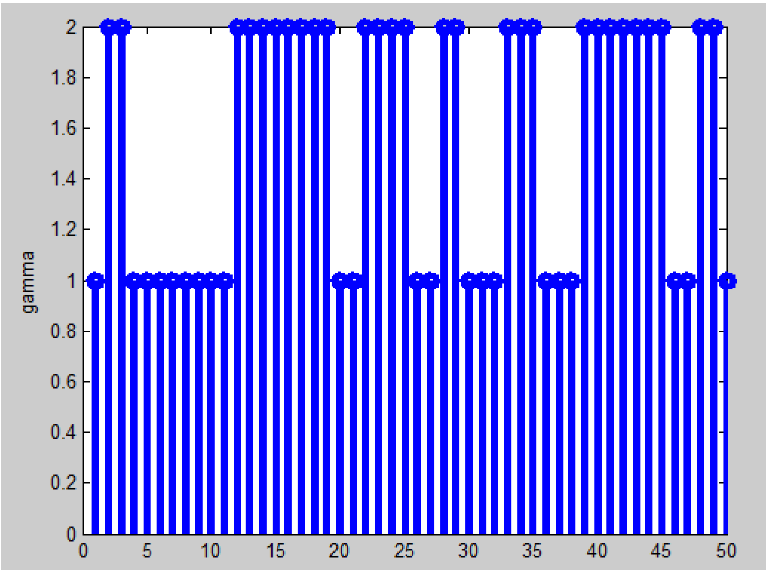


Figure26 dual controller for performance plot of z(k) and γ for case 2

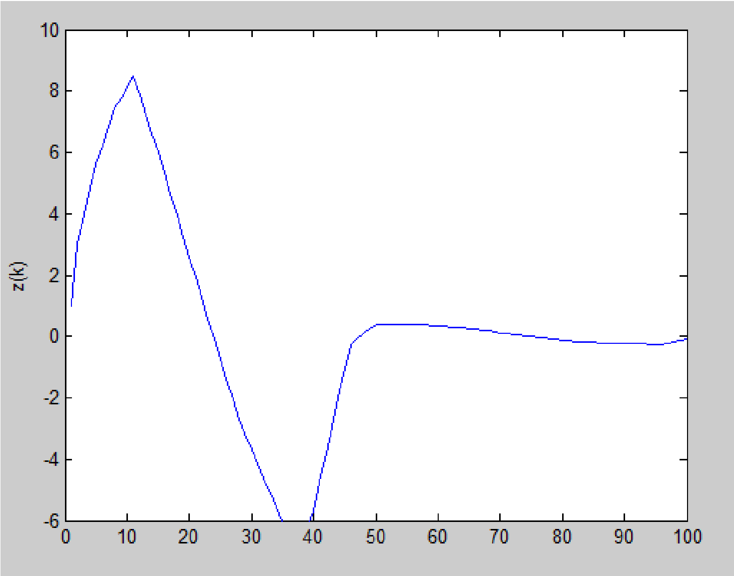
γ series is similar but not same to stability case as well.

J_2= 1.6380e+003

J_bar = 3.5246

Condition $J\_bar < J_2$ satisfied.

③: Λ =[0.2 0.8;0.3 0.7]



Output $z(k)$ response shows MS-stable for system, and it behavior is similar to the stability case but not completely same. In this case the vibration amplitude looks littler larger than stability case, and takes longer time to reach steady state.

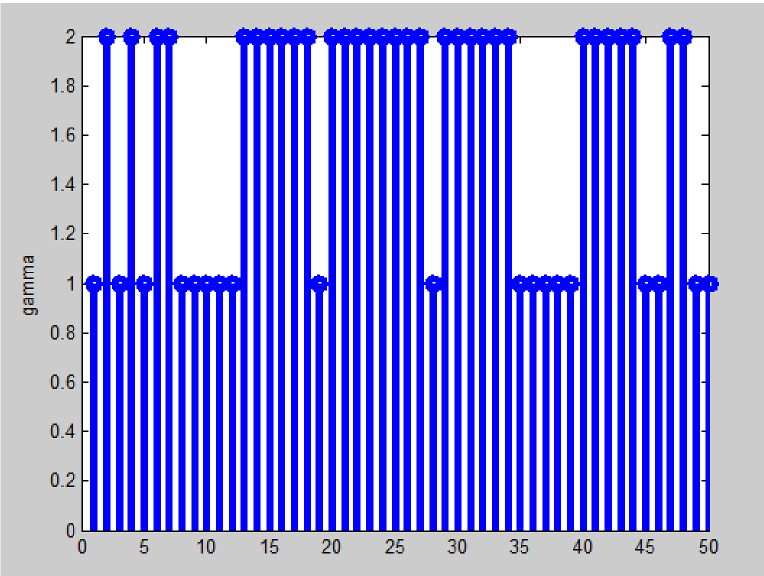


Figure27 dual controller for performance plot of z(k) and γ for case 3

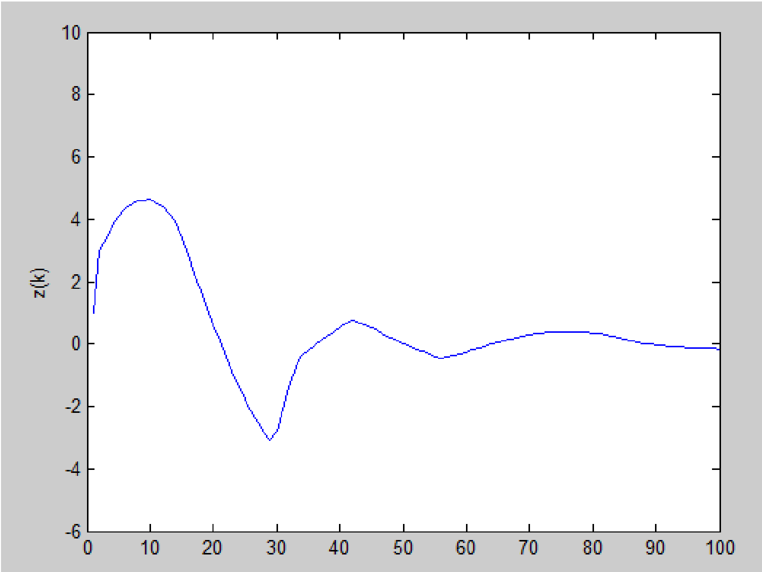
γ series is similar but not same to stability case as well.

J_2= 2.2826e+003

J_bar = 11.2316

Condition $J\_bar < J_2$ satisfied.

④: Λ =[0.2 0.8;0.9 0.1]



Output $z(k)$ response shows MS-stable for system, and it behavior is similar to the stability case but not completely same.

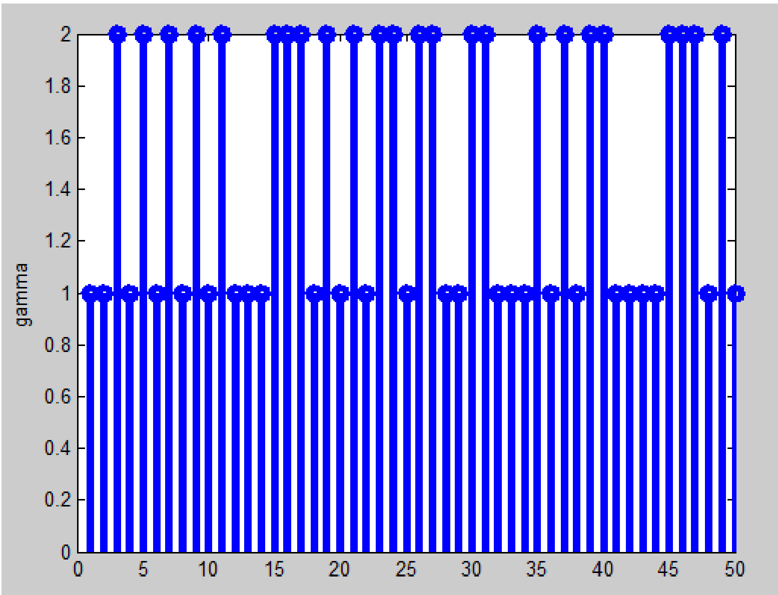


Figure28 dual controller for performance plot of z(k) and γ for case 4

γ series is similar but not same to stability case as well.

J_2= 294.7699

J_bar = 3.1248

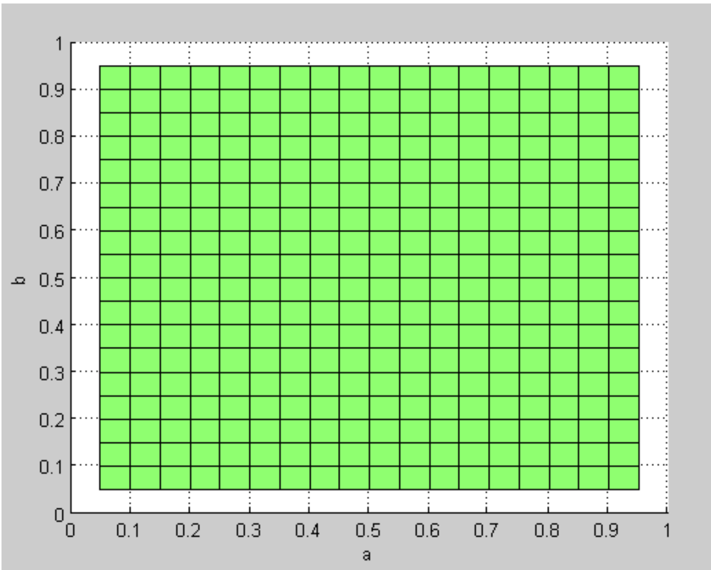
Condition $J\_bar < J_2$ satisfied.

Φ matrix

In this system, Φ matrix influence the switching frequency between two controllers, state1 stands aggressive controller, state2 stands moderate controller. In previous case we fixed γ rate matrix $\Phi = [0.5，0.5；0.5，0.5]$，and change elements of Λ .Now, we are interested in if we fixed Λ matrix, set $\Phi = [1-a，a；b，1-b](|a|,|b| < 1)$, then change the parameter a and b, what will be happen to the feedback system.

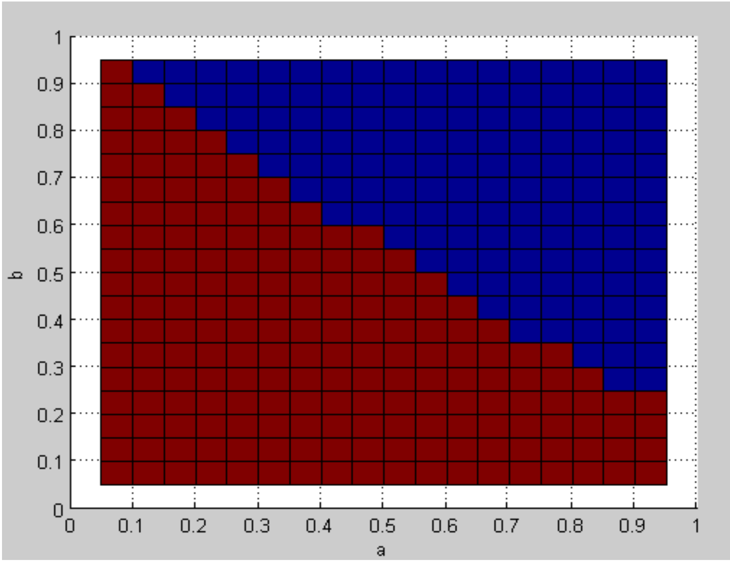
Now we give 5 known Λ matrixes, change a and b ,to find the stable a b region.

①: Λ =[0.8 0.2;0.7 0.3]



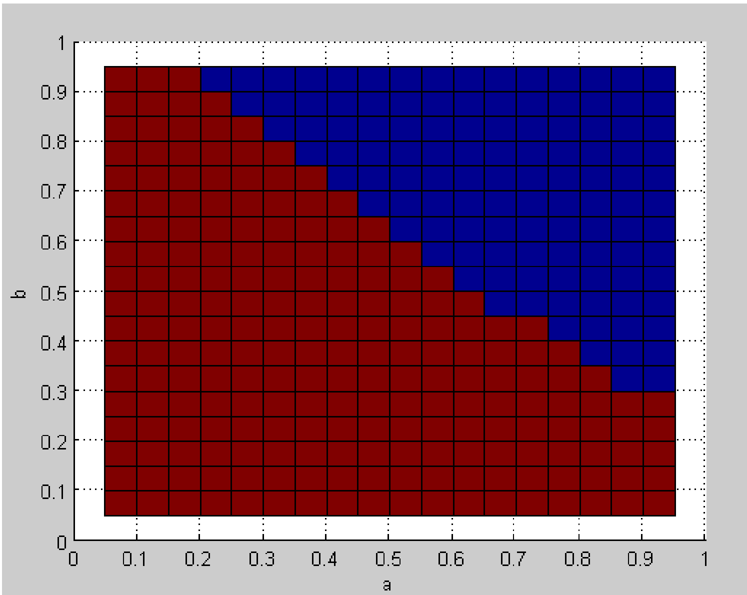
No unstable point, any a and b from 0 to 1 makes system MS-stable

②: Λ =[0.8 0.2;0.2 0.8]



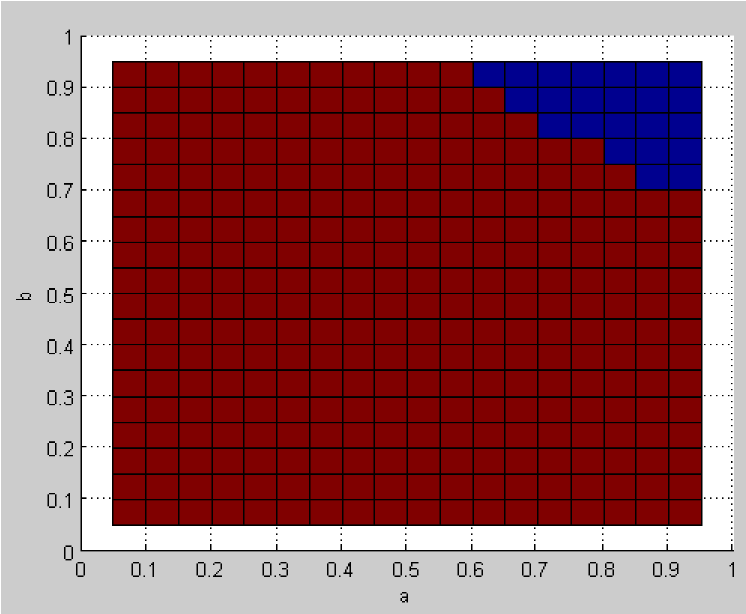
Blue area means unstable, red region means MS-stable.

③: Λ =[0.2 0.8;0.3 0.7]



Blue area means unstable, red region means MS-stable.

④ Λ =[0.2 0.8;0.6 0.4]



Blue area means unstable, red region means MS-stable. Compare to last case, when b become bigger the unstable area becomes smaller.

⑤: Λ =[0.2 0.8;0.9 0.1]

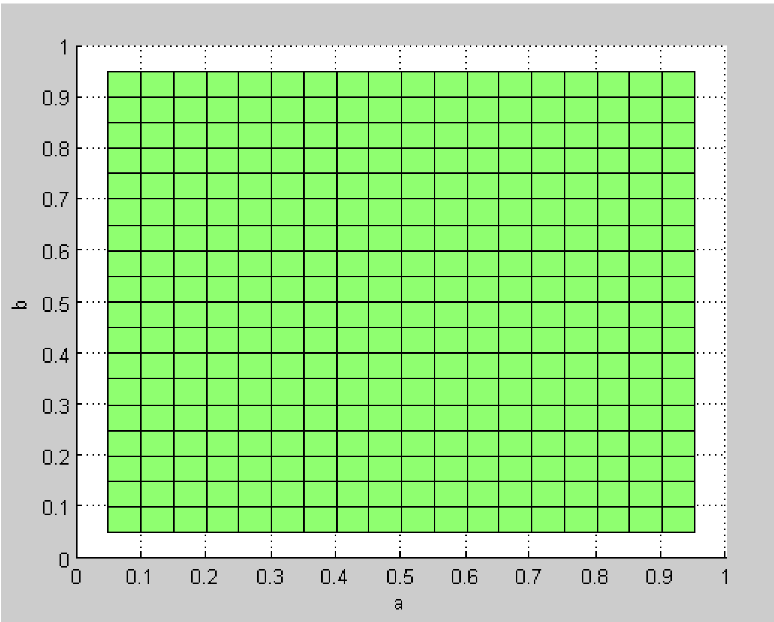


Figure29 Φ matrix MS-stable regions

No unstable point, any a and b from 0 to 1 makes system MS-stable

Conclusion: From the 5 plots, we find that when fixed Λ matrix, different choices of Φ matrixes may influence feedback system MS-stable, and all figure shows that unstable area is in upper right hand area, that is too say when a and b is too bigger, the system might be unstable. Big a and b stands for higher jumping frequency between controllers, so, that's means if we change controller too often, system may be unstable. In addition, there may exist an $\alpha\beta$ region makes any a and b from 0 to 1 guarantee MS-stable for the feedback system.

Input-hold case

Region of MS-stability

Before we assume that when the actuator is in faulty condition, the output of the actuator is 0, however a more realistic situation is when the actuator is faulty; the output takes the previous value. By doing so, the actuator needs to have memory by itself, we model the past value as u(k-1).For simulation, we assume η the weight of $u(k-1)$ to be 0, that is to say past control effort is neglected.

First, let's find the α β stable region for input-hold configuration.

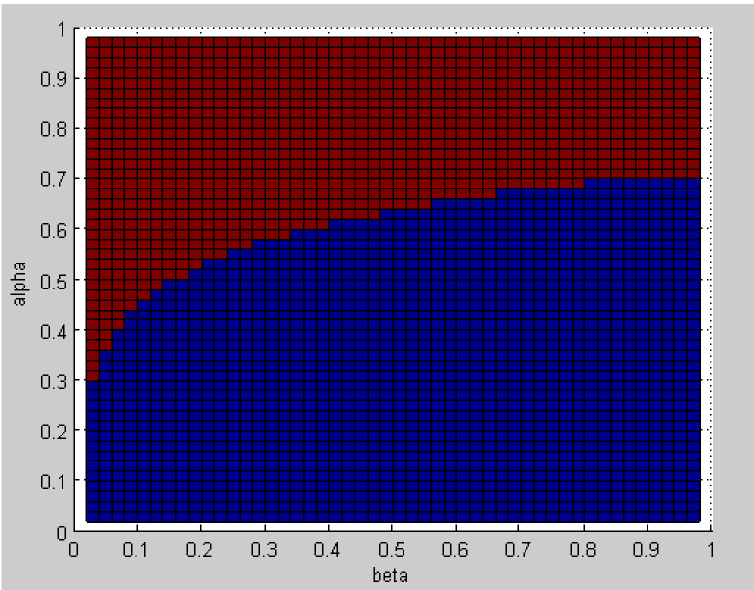


Figure30 input-hold configuration α β MS-stable regions

The stability region of dual-switching input-hold case, compared to the zero-input case before is much smaller.

Particular examples for stability

Again we select 4 α β pairs, taken values from the 4 'corners' of the stable α β region. This time we use performance LMI (containC^TC) to get optimized P matrix, after all the performance LMI is more strictly than stability one. Then, we will compare the result to zero-input case.

① Λ =[0.8 0.2;0.9 0.1]

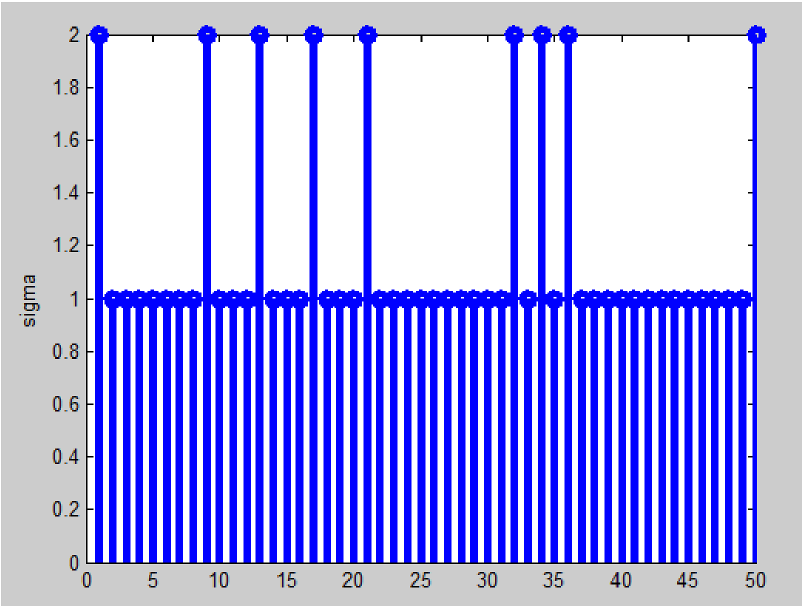


Figure31 input-hold configuration of σ sample for case 1

This is σ series plot; state 2 appears much less than state1.

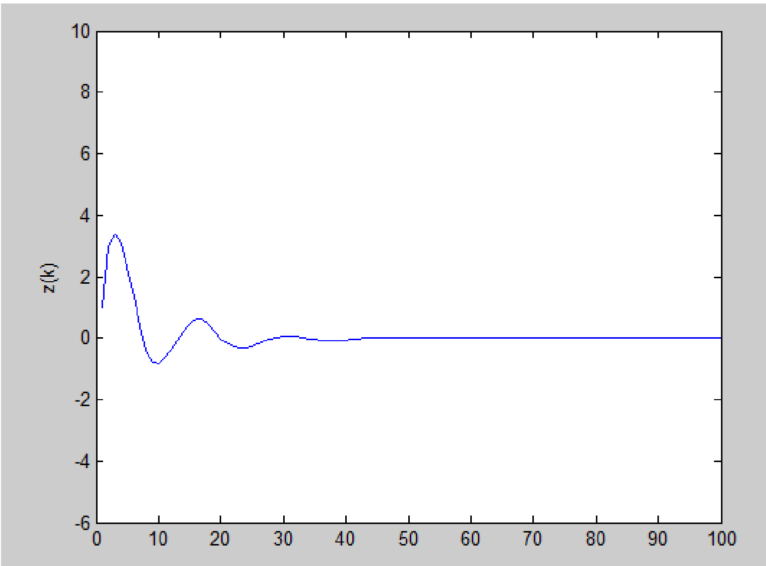


Figure32 input-hold configuration of z(k) for case 1

Output $z(k)$ response goes to steady state quickly indicates MS-stable for system.

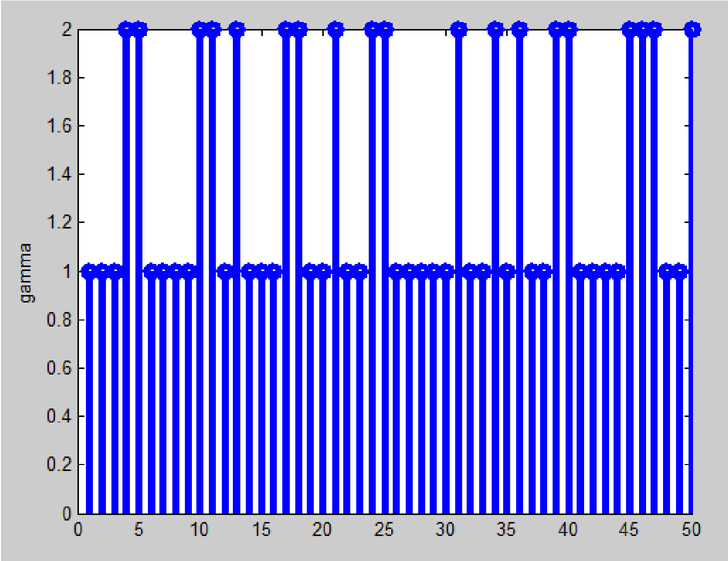


Figure33 input-hold configuration of γ for case 1

J_2= 156.8758

J_bar = 0.4237

Condition $J\_bar < J_2$ satisfied.

② Λ =[0.8 0.2;0.7 0.3](near unstable bound)

In this case, σ series is the same with case① of zero-input model

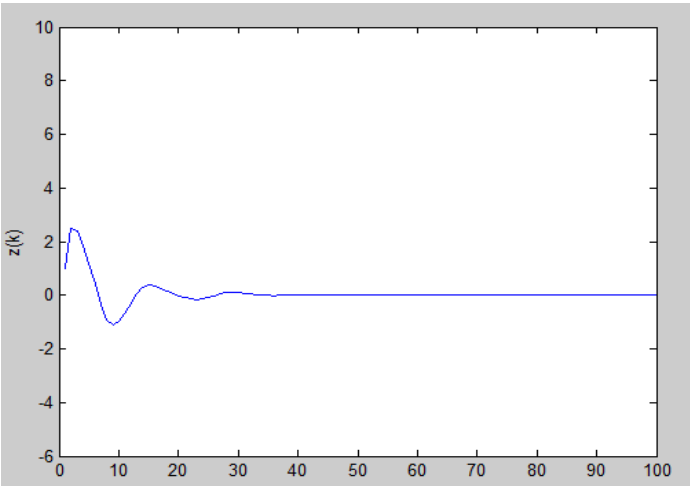
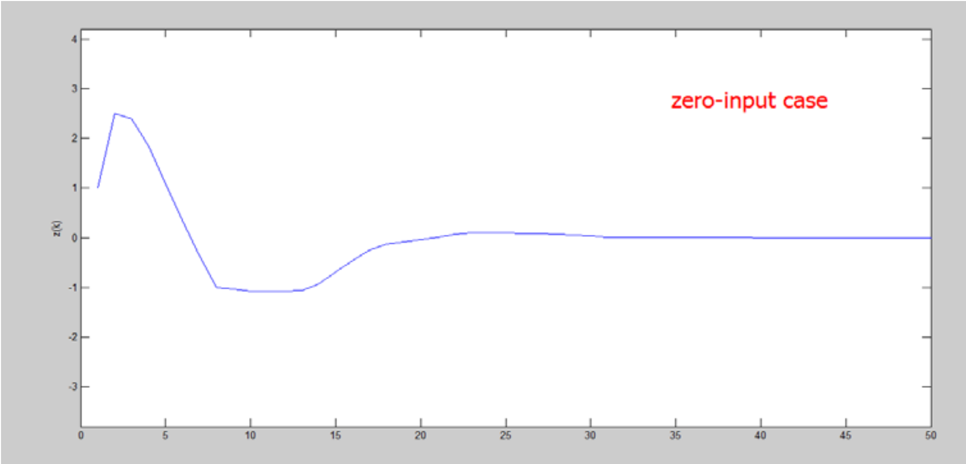
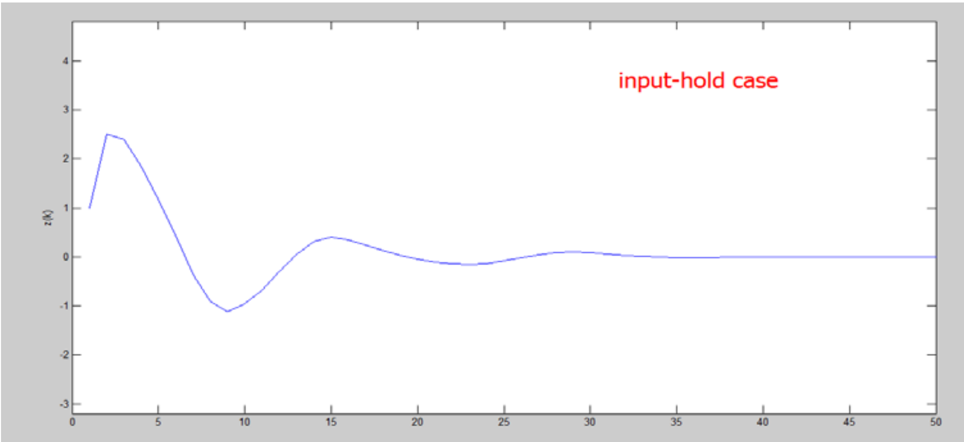


Figure34 input-hold configuration of z(k) for case 2

First the plot indicates MS-stable for system. Second, though this $\alpha\,\beta$ pair is near unstable bound, $z(k)$ response looks smooth not like zero-input case, oscillate a lot.

Zoom out plot





Compared with zero-input case $z(k)$ response, the trends is too similar, even when zoom out the plot, it's still hard to say which one makes more effort, we will see the classic energy form J_bar in the follows sentences.

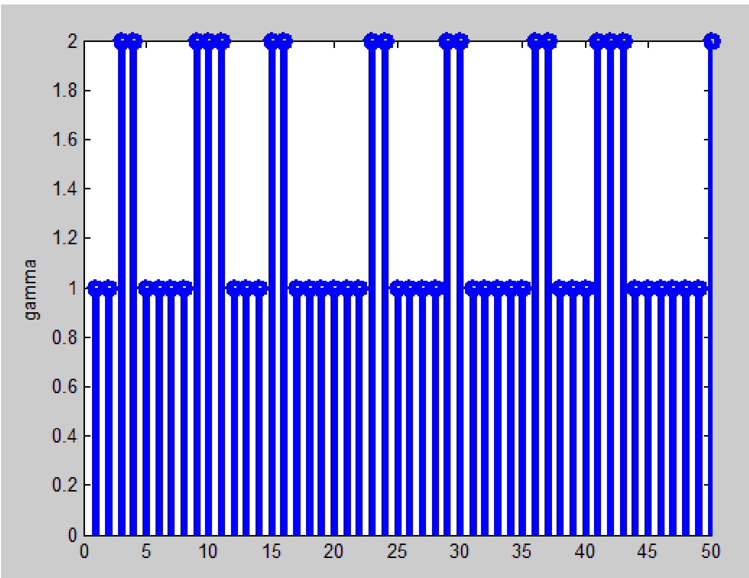


Figure35 input-hold configuration of γ for case 2

Compare γ series plot, the jumping frequency is quiet similar to zero-input case.

J_2= 235.1275

J_bar = 0.2226

Condition $\mathrm{J\_bar} < \mathrm{J}_2$ satisfied.

Zero-input case J_bar = 0.2625, it is little bigger than input-hold case.

③ Λ =[0.3 0.7;0.7 0.3] (near unstable bound)

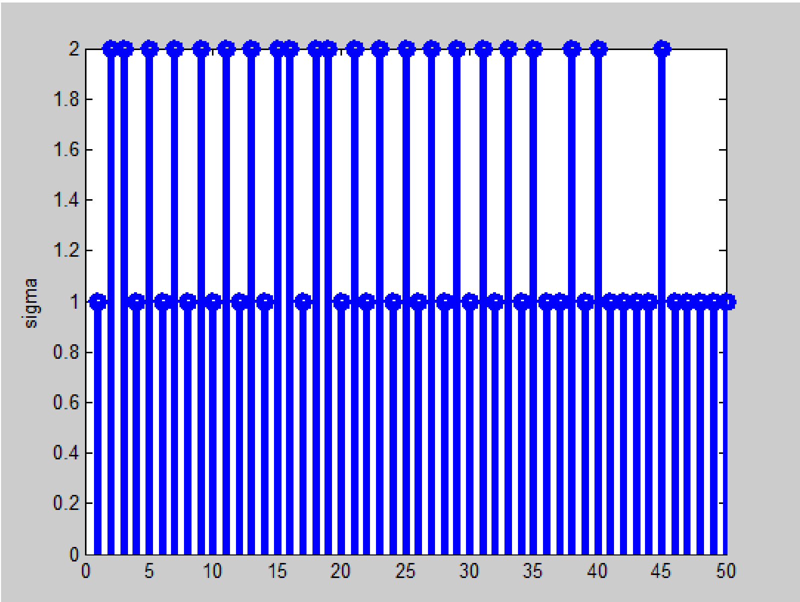


Figure36 input-hold configuration of σ sample for case 3

This is σ series plot; big α and big β makes it jump very often.

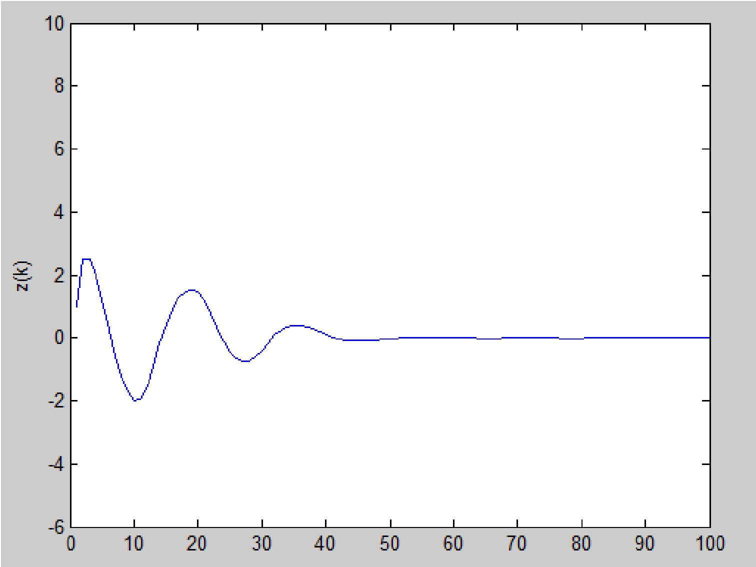


Figure37 input-hold configuration of z(k) for case 3

The plot indicates MS-stable for system. Still, this $\alpha\,\beta$ pair is near unstable bound, $z(k)$ response just vibrate little at beginning and then go to steady state soon, that is much faster than zero-input case when in near unstable bound case.

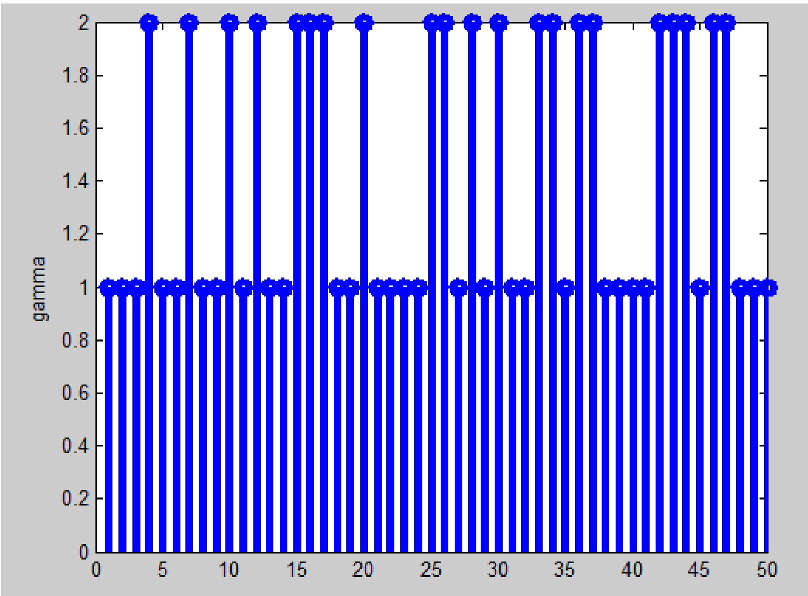


Figure38 input-hold configuration of γ for case 3

γ series plot, γ jumps frequently along with σ.

J_2= 1.1467e+003

J_bar = 0.4909

Condition $\text{J\_bar} < J_2$ satisfied.

④ Λ =[0.2 0.8;0.9 0.1]

In this case, σ series is the same with case④ of zero-input model

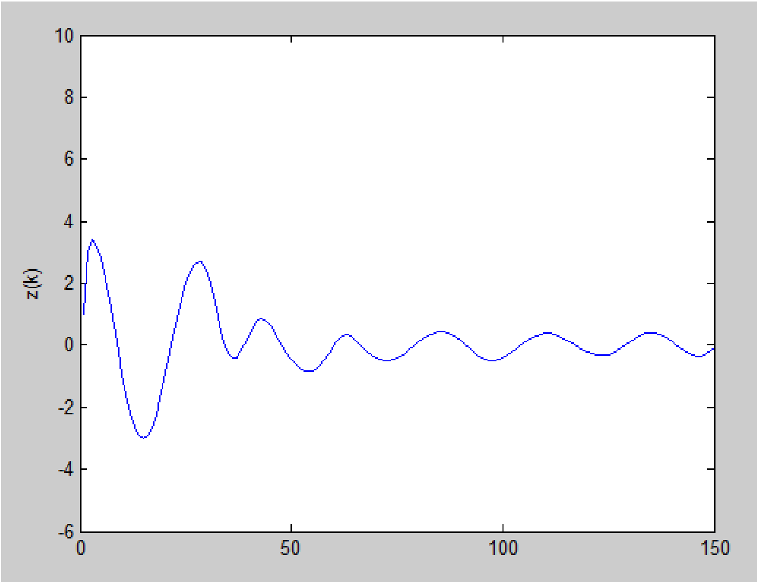


Figure39 input-hold configuration of z(k) for case 4

In this plot, z(k) oscillate frequently along with high switching frequency of σ, though oscillate, we extend k to 150, we can see that after a long time, the energy will become smaller and smaller, it will finally stay in 0, so we still can conclude that the system is MS-stable.

In this case, compare with zero-input case, the vibration amplitude is obviously smaller than zero-input case.

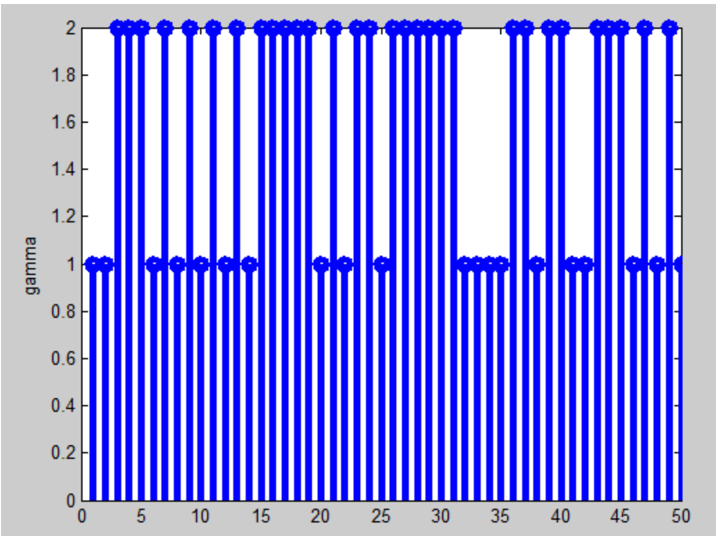


Figure40 input-hold configuration plot of γ for case 4

In this case, γ jumps less frequent than zero-input case.

J_2= 240.9429

J_bar = 1.6267

Condition $J\_bar < J_2$ satisfied.

Zero-input case J_bar = 3.1248 is bigger than this input-hold case one.

In conclusion, after compare zero-input case① to input-hold case②, and zero-input case④ to input-hold case④, we can see that input-hold case is easier and spend less energy to reach MS-stable, that also validate our previous guess: input-hold configuration is behavior better and more reasonable for the dual-switch system.

Balance between output energy and control energy effort:

In previous input-hold case, η the weight of $u(k-1)$ is setting to 0.

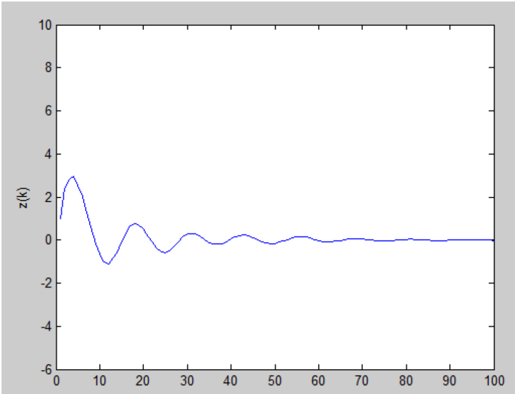
Recall that,

$$\bar{z}(k) = \begin{bmatrix} C & 0 & 0 & \eta\, I \end{bmatrix} \bar{x}(k)$$

we will discuses, when the weight η =1,2,3,4, what will happen to $z(k)$ and classic energy J_bar.

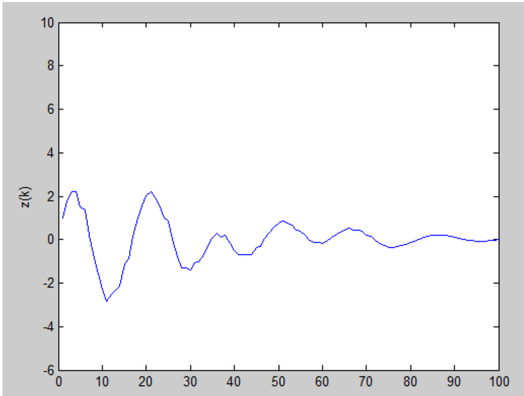
We take case④ Λ=[0.2 0.8;0.9 0.1] as a example, this $\alpha\beta$ pair is surely makes system MS-stable as we proofed before.

when η =1:



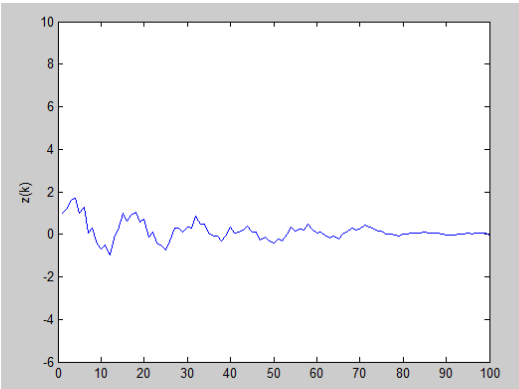
J_bar= 0.4355

when η =2



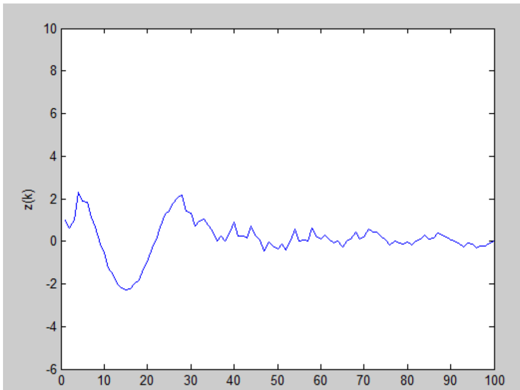
J_bar= 0.8951

when η =3



J_bar= 0.2159

when η =4



J_bar= 0.7804

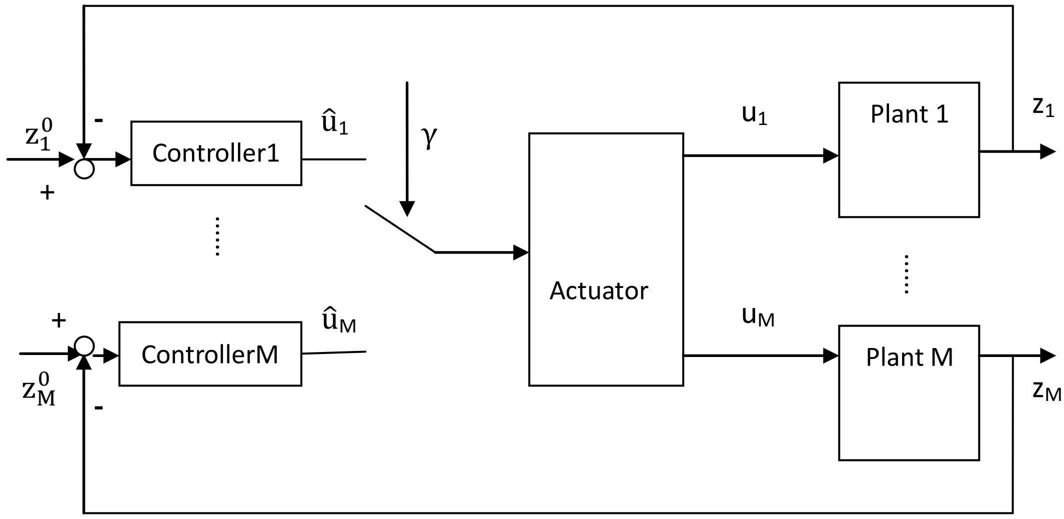
Figure41 different control energy effort weight behavior

From the plots, we can see, when the weight η becomes bigger , $z(k)$ response becomes more serrated, not smooth. J_bar seems no regularity.

4.4 Dual-plant Dual-controller scheme

4.4.1 Complete model design

Recall the Multi-plant Multi-controller scheme in Chapter2



For father applications, we can apply the theorem in chapter 3 also in a scheme with two specific plants and two controllers.

Beside the original plant

$$\begin{cases} x(k+1) = Ax(k) + Bu_1(k) \\ z(k) = Cx(k) \end{cases}$$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 0$$

We can add a plant2

$$\begin{cases} x_2(k+1) = A_2x_2(k) + B_2u_2(k) \\ z_2(k) = C_2x_2(k) \end{cases}$$

$$A_2 = \begin{bmatrix} 1.8 & -0.81 \\ 1 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D_2 = 0$$

$$G_2(z) = \frac{1}{(z-0.9)^2}$$

Plant2 is also a double integrator, but stable.

Then we can design a complete model for the overall dual-plant dual-controller feedback system, and discuss its MS-stable region and performance.

Controller1

$$\begin{cases} x_c(k+1) = A_c x_c(k) + B_c e(k) \\ \hat{u}_1(k) = C_c x_c(k) + D_c e(k) \end{cases}$$

$$A_c = 0, \qquad B_c = 0.5, \qquad C_c = -0.8, \qquad D_c = 0.5$$

Controller2:

$$\begin{cases} x_{c2}(k+1) = A_{c2} x_{c2}(k) + B_{c2} e_2(k) \\ \hat{u}_2(k) = C_{c2} x_{c2}(k) + D_{c2} e_2(k) \end{cases}$$

$$A_{c2} = 0.5, \qquad B_{c2} = 0.25, \qquad C_{c2} = -0.208, \qquad D_{c2} = 0.13$$

Where $e(k) = -z(k)$, $e_2(k) = -z_2(k)$,

Scheduler for controllers:

$$\begin{cases} u_1 = \hat{u}_1(k) \text{ and } u_2 = 0, & \text{if } \gamma(k) = 1 \\ u_1 = 0 \text{ and } u_2 = \hat{u}_2(k), & \text{if } \gamma(k) = 2 \end{cases}$$

The signal regulate by actuator here, we have two methods to model the fault state.

For *zero-input configuration,* we set $u_r(k) = 0$, when fault occurs,

$$u_r(k) = \begin{cases} \hat{u}_r(k), & \sigma(k) = 1 \\ 0, & \sigma(k) = 2 \end{cases}, \quad r = 1,2$$

Then we obtain he overall system:

$$\hat{x}(k+1) = A_{\sigma(k)}^{\gamma(k)} \hat{x}(k), \quad \hat{x}(k) = \begin{bmatrix} x(k) \\ x_c(k) \\ x_2(k) \\ x_{c2}(k) \end{bmatrix}$$

$$A_1^1 = \begin{bmatrix} A - BD_cC & BC_c & 0 & 0 \\ -B_cC & A_c & 0 & 0 \\ 0 & 0 & A_2 & 0 \\ 0 & 0 & -B_{c2}C_2 & A_{c2} \end{bmatrix}$$

$$A_1^2 = \begin{bmatrix} A & 0 & 0 & 0 \\ -B_cC & A_c & 0 & 0 \\ 0 & 0 & A_2 - B_2D_{c2}C_2 & B_2C_{c2} \\ 0 & 0 & B_{c2}C_2 & A_{c2} \end{bmatrix}$$

$$A_2^1 = A_2^2 = \begin{bmatrix} A & 0 & 0 & 0 \\ -B_c C & A_c & 0 & 0 \\ 0 & 0 & A_2 & 0 \\ 0 & 0 & -B_{c2} C_2 & A_{c2} \end{bmatrix}$$

Performance output

$$\hat{z}(k) = [C \quad 0 \quad C_2 \quad 0]\hat{x}(k)$$

4.4.2 Stability and Performance

Region of MS-stability

Again, let's discuss α, β region for Λ matrixes.

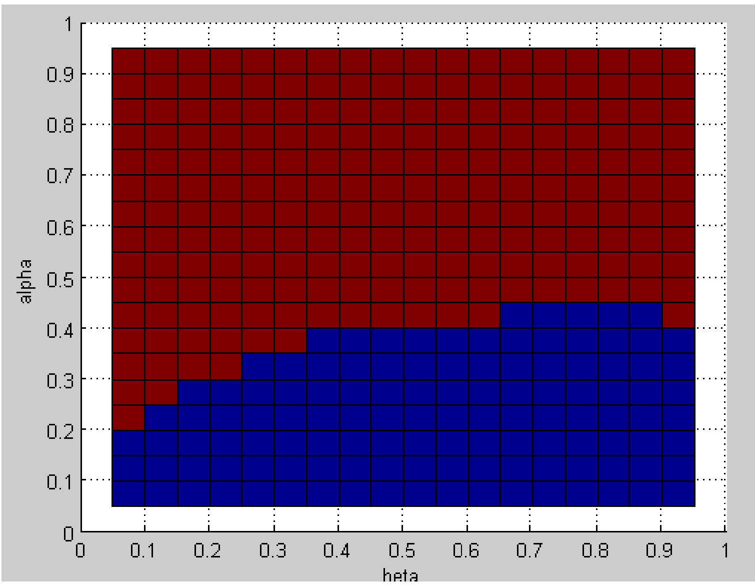


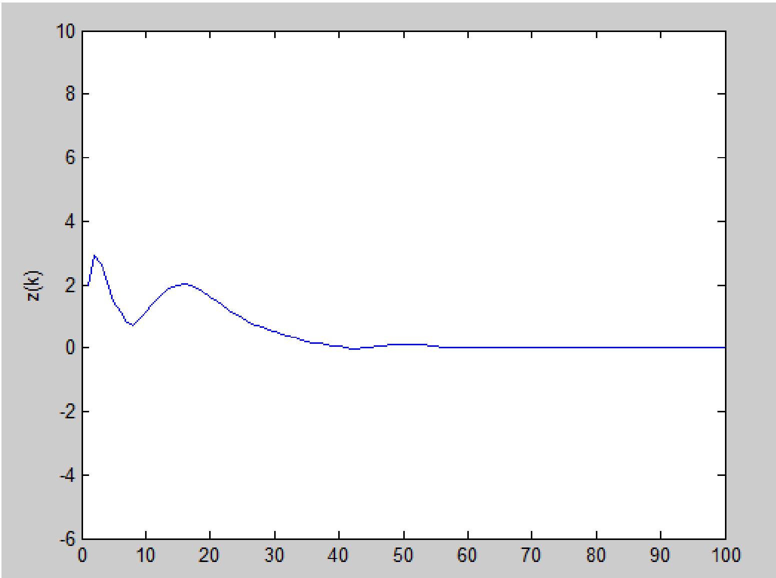
Figure42 MS-stability region for dual-plant dual-controller scheme

This is the stability region of α and β pair, red region above indicates that this α β pair makes system stable and blue region indicates that this α β pair makes system unstable. Compare to the single plant dual-controller scheme in last section, the stability region is smaller though the plant2 is a stable plant.

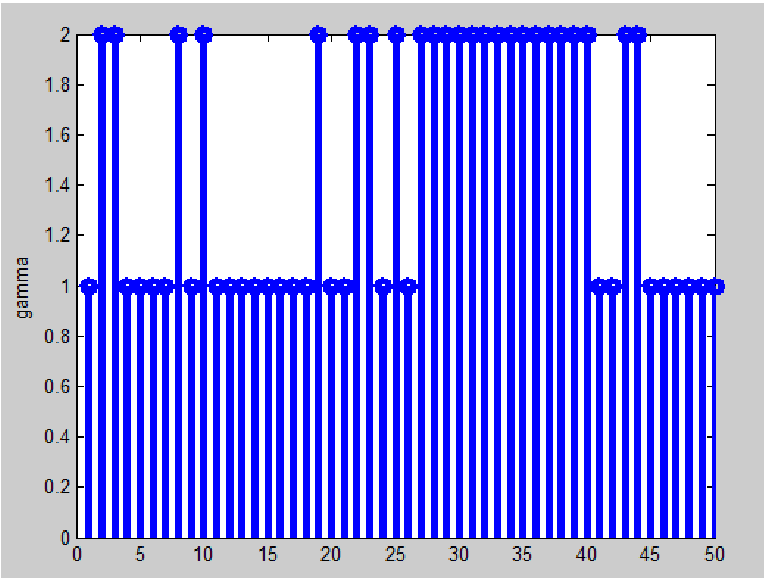
Particular examples for stability

Again we select the same 2 α β pairs with single controller case (which are well inside the stable region), and use the same σ sample sequence to simulate their behaviors of output z(k).

① Λ =[0.8 0.2;0.7 0.3]



The output $z(k)$ as a function of k (=100) in the figure quickly go to steady stead, the good response of $z(k)$ obviously proof that the system is MS-stable. Compared with single-plant scheme, $z(k)$ takes more time to reach steady state.

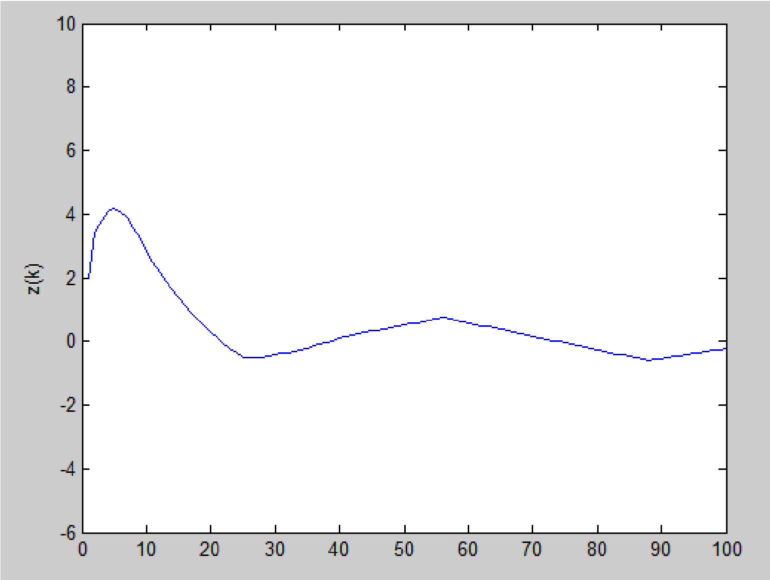


γ signal jumps plot is very different from single-plant case

J_2= 257.7314

J_bar = 0.7331 It satisfied the condition $J\_bar < J_2$

④ Λ =[0.2 0.8;0.9 0.1]



Though this α β pair is well in stable region, $z(k)$ line shows not so obvious at k=100 it will reach steady state or not, it may takes more time to reach steady state, but in single-plant scheme, $z(k)$ already reach steady state at k=100.

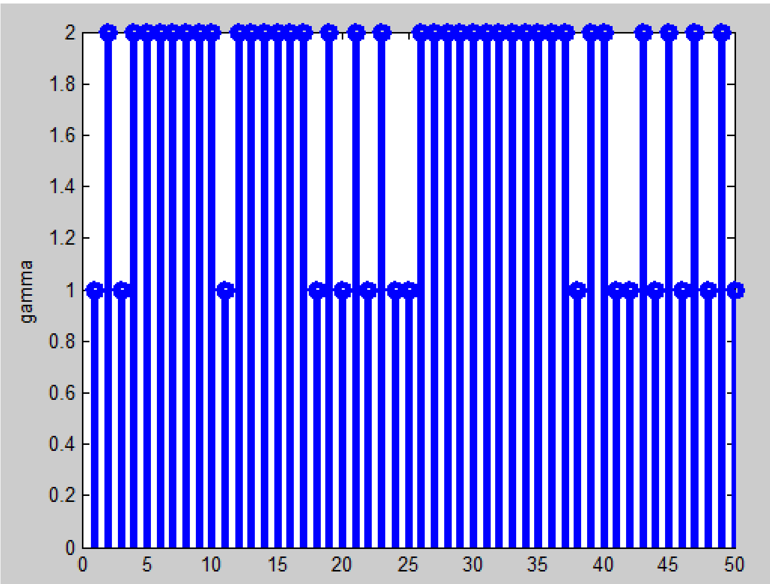


Figure43 particular example plots for dual-plant dual-controller scheme

Different from single plant scheme, γ jumps not frequently while σ jumps very often.

J_2= 324.3988

J_bar = 1.6230 Condition J_bar $< J_2$ also satisfied.

4.5 Possible Extension

In order to expand the range of applications, in the future, we can also consider the system behavior with a deterministic disturbance. We set $w(\cdot) \in \ell_2$, then the overall discrete-time dual switching linear systems can be present like this:

$$\begin{cases} x(k+1) = A_{\sigma(k)}^{\gamma(k)} x(k) + B_{\sigma(k)}^{\gamma(k)} w(k) \\ \quad\quad z(k) = C_{\sigma(k)}^{\gamma(k)} x(k) \end{cases}, \qquad x(0) = x_0$$

Assume $w(k) = \delta(k) e_k$, we may get a new formula which includes $B_{\sigma(k)}^{\gamma(k)}$ yielding a guaranteed $\mathscr{H}_2$ performance.

For the multi-plants situation, each plant has a $w_r(k)$, a new overall system can be designed in the future.

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