

#### Politecnico di Milano Department of Mathematics Doctoral Programme in Mathematical Models and Methods in Engineering

## BIO-POLYMER INTERFACES FOR Optical Cellular Stimulation: a Computational Modeling Approach

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### Abstract

The present thesis concerns with the investigation and the mathematical description of the physical processes underlying the behavior of a particular bio-polymer interface device for optical cellular stimulation. Such device is made of a thin slab of P3HT, a photoactive semiconducting conjugated polymer, sandwiched between a cell and a transparent electrode, the whole system being immersed in a physiological solution to allow cell survival during experiments.

Despite the fact that measured cell membrane current and potential clearly demonstrate that a significant electrical cellular activity is elicited by light mediated stimulation due to absorption by the polymer, the physical processes driving the induced cellular response are still not fully understood. To fill the gap of a lack of a consistent understanding and of a quantitative description of the device working principles, the present thesis proceeds according to a three-step procedure.

In the first step, a systematic classification of performed measurement is carried out, distinguishing between measurements with the sole polymer part of the device and measurements including the cell. In the second step, based on a thorough scrutiny of the two classes of experimental data, an identification of the principal physical phenomena is operated, concluding that electrical and thermal effects play the major role in the overall device function. In the final step, a sound mathematical description is proposed to describe the chain of events that lead from light illumination of the device structure to the generation of a depolarizing signal for the cell membrane.

Electrical characterization and the corresponding modeling results consistently indicate that upon illumination, free charges are generated and displaced by an electric field, resulting from the onset of a depleted region at the P3HT/electrode interface, with characteristic time scales dominated by charge trap dynamics.

Patch clamp measurements and the corresponding modeling results show instead that temperature increase in the system due to light absorption in the polymer, determines the modification of cell membrane properties, most importantly of its capacitance, leading to cell membrane depolarization.

Both phenomena are then compatible with cellular depolarization and this allows us to conclude that while the electric effect is dominant for low illumination intensities, it soon saturates for more intense light stimulation, being replaced by the thermally induced mechanism which instead linearly increases with light intensity.

### Summary

The continuously increasing connection among life sciences, materials science and engineering is the background motivation to the dramatic development of modern neuroscience in our times. In particular, an enormous attention is paid by the world scientific community, as well as by the non-specialized audience, to new discoveries and advancements in the area of bioelectronics and neural prostheses.

In this context, an important branch of medical applications is represented by artificial vision and, specifically, the design, implementation and in vivo implantation of retinal prostheses. These latter are man-made substitutes of dysfunctional photoreceptors, capable of transducing the incident sunlight into an electrical signal that can be ultimately elaborated by the brain to allow normal vision in the patient's eye. To end up with a feasible result that is characterized by a high level of biocompatibility with the human local microenvironment, an appropriate choice of the materials constituting the prosthesis is in order.

In this perspective, the use of semiconducting polymers represents an attractive opportunity because, compared to inorganic semiconductors like silicon, organic materials display superior mechanical properties, allow a more immediate interaction with visible light, and, more importantly, are much more easy and cheap to treat and manage in the production phase.

For these reasons, a strong research activity has been devoted over the last years to investigating polymer material behavior under different stimulation conditions and developing novel structures for possible new prosthetic design. In this context, an important contribution has been given by the group of Prof. Guglielmo Lanzani, Director of Center for Nano Science and Technology (CNST) @Polimi of Istituto Italiano di Tecnologia, in coordinated interaction with the group of Prof. Fabio Benfenati, Department Director of NBT/Synaptic Neuroscience at Istituto Italiano di Tecnologia. These two teams of scientists have demonstrated in a series of recent articles [64, 65] that it is possible to realize a fully biocompatible device based on the use of an organic semiconductor material in which, upon the application of an external input illumination signal, a living cell grown onto the surface of the material can be stimulated to elicit an electrical output signal through the active light mediation of the polymer.

Despite the fact that measured output data, specifically, cell transmembrane current and action potential, clearly demonstrate that a significant electrical cellular activity is elicited by light mediated stimulation of the photoactive polymer, the physical processes driving the induced cellular response are still not fully understood. To fill this gap, the author of the present thesis has invested the three-year doctoral programme participating constantly to the research at CNST in the challenge of providing:

- a rigorous characterization of the main physical processes underlying the functional behavior of a bio-polymer device for light mediated cellular stimulation;
- devising a unified computational framework for self-consistent bio-polymer device simulation;
- conducting an extensive program of numerical experiments with the aim of validating model assumptions and calibrating model parameters by the comparison against available measured data.

The result of the above three-year work is contained into five chapters whose content is briefly describer below.

In the first chapter of the thesis an introduction to the wide scientific area of bioelectronics is overviewed, characterizing the complex interaction between cells, polymers and light, and the translation of such interaction into working devices. Particular attention is paid at discussing the advanced application object of this work, that is, the use of light-mediated cell stimulation with semiconducting polymers.

In the second chapter of the thesis a systematic classification and analysis of performed measurement is carried out, focusing on measurements with the sole polymer part of the bio-polymer device. This choice aims at characterizing the role of electrical effects in determining the overall device performance. To this purpose, a sound mathematical description is proposed to describe the chain of events that lead from input light illumination of the device structure to the development of an output photovoltage signal that can be able to drive cellular depolarization and hyperpolarization. The adopted mathematical picture of the problem is constituted by a system of partial and ordinary differential equations that represent:

- generation, dissociation and motion of excitons in the polymer according to Fick's law of diffusion;
- electric conduction of generated free electrons and holes in the polymer according to the classical Drift-Diffusion formalism;
- trapping and release of charge carriers in localized trap states in the energy gap of the polymer;
- rearrangement of electric field inside the device according to Gauss law.

The resulting nonlinear system of equations is numerically solved by joint adoption of advanced techniques in the modern field of scientific computing, including:

- ★ time advancing with Rothe's method and automatic Backward Differentiation Formulae;
- ★ system linearization with the Newton method;
- ★ spatial discretization with exponentially fitted finite elements in primal-mixed form, incorporating the classical Scharfetter-Gummel stabilization term to avoid the occurrence of unphysical spurious oscillations.

Extensive numerical simulations are conducted to corroborate and support the picture of the device working principles that we draw based on of the obtained experimental evidence:

- A critical role is played by the interface between the P3HT and the ITO electrode, characterized by the presence of a depleted region and of the corresponding electric field, which drives exciton dissociation and determines charge displacement.
- The dynamics of charge trap states distributed in the energy gap of P3HT determines the characteristic time scale of the device response in terms of photovoltage output signal.

In the third chapter of the thesis, an analysis of the response upon photostimulation of cells grown onto such polymer/electrolyte interface devices is carried out using patch clamp measurements. This choice aims at demonstrating that at the considered illumination regimes the observed phenomena are actually determined by a change of the local temperature. The adopted mathematical picture of the problem is constituted by a system of partial and ordinary differential equations that represent:

- heat generation and diffusion in the material and delivery to the cellular environment;
- the electrical response of cell membrane modeled with an equivalent electric circuit;
- the change of membrane capacitance, conductance and resting value due to the temperature increase.

Obtained results are in excellent agreement with measured data and the following picture of the device working principles is supported:

- heat generation and diffusion in the material bulk and subsequent transfer to the surrounding electrolyte cleft are a direct cause of cellular depolarization (light turned on) and hyperpolarization (light turned off);
- temperature-induced variation of the membrane capacitance is identified as the driving force of cellular depolarization.

In the conclusion of the chapter other interpretations of the phenomena previously reported in the literature, based on the classic Guoy-Chapman-Stern theory of double layers, are critically discussed.

In the fourth chapter of the thesis a detailed discussion of the adopted finite element methodologies is carried out, showing how the use of an exponentially fitted stabilized primal-mixed formulation allows for an accurate and robust discretization of problem equations. The analysis of stability and convergence of the proposed methods is performed, and their validation is demonstrated by an extensive series of numerical experiments in singularly perturbed boundary value model problems in the one-dimensional case and in the study of a problem in axisymmetric geometry in the two-dimensional case.

In the concluding chapter of the thesis a summary of the conducted activity and its main results is addressed, together with some indications for future directions of research and model improvement.

The experimental measurements described in the present thesis have been performed by other members of the research group at the CNST @Polimi. In particular, the electrical experiments reported in chapter 2 have been carried out by Sebastiano Bellani, while Nicola Martino performed the patch clamp measurements discussed in chapter 3.

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# CHAPTER **1**

## Introduction: cells, polymers and light

#### **1.1** Bioelectronics and organic semiconductors

Bioelectronics is the field of scientific research in which, as its own name suggests, electronic engineering principles are applied to biology, medicine and health sciences, and in the latest years the hype around it, both in the public opinion and in the scientific community, considerably grew as a consequence of the tremendous amount of achievements in terms of new materials, devices and understanding of living matter.

The birth of bioelectronics is usually dated to the pioneering work of Luigi Galvani in the 18th century, and his famous experiment in which the detached legs of a frog were made move by applying a small voltage. Since then many results have been obtained in directly interfacing artificial materials and biological elements, exploiting the numerous advancements of the rapidly expanding silicon technology. Research activity led to bioelectronic devices that may operate principally in two directions. In one configuration, the biological events alter the interface properties of the electronic elements, thus enabling a readout of the phenomenon by monitoring a particular device property. In a second configuration, instead, the bioelectronic systems use the electronic units to activate the biomaterials towards a particular desired function.

Most activities in this field are related to the development of biosensors, that convert recognition or catalytic processes mediated by cells in the form of electronic signals, and lab-on-chip devices [6,72,76,118]. Another prolific research branch is that dealing with the electrical interfacing of neuronal networks with semiconductor microstructures. Neuronal activity can be monitored with semiconductor devices, opening the way to hybrid systems for dynamic memory and active learning. Conversely, the semiconductor devices can induce cellular response by applying electric signals, the ultimate application being the neural interfaces of prosthetics devices [58,96]. In the latest decades organic semiconductors have emerged as a novel and alternative class of materials in the electronics field, and among this group we name small molecules and conjugated polymers to have been key players of a sort of revolution in electronics and optoelectronics. Research efforts got great impulse and have been directed to explore the possibility for this technology to obtain better performance than the silicon based one and to replace this latter in the fabrication processes. The joint effort in materials chemistry, fundamental research and device engineering led to the demonstration and the commercialization of a new generation of devices, including light emitting diodes, transistors and photovoltaic cells [57, 70, 80].

Compared to inorganic semiconductors, organic materials offer attractive characteristics in terms of mechanical properties, possibility of chemical engineering and interaction with visible light. Importantly, the technology required for processing materials and realizing devices is relatively cheap and easy, and it fits well with transparent, bendable, rollable and lightweight plastic substrates [56]. The counterbalances to be paid back are reduced electronic transport properties and poor environmental stability. Major efforts are currently focusing on improving the performance of solar cells and transistors, which are expected to represent the next applications to be delivered on the market.

At the same time, organic electronics is now experiencing a second birth with the combination with bioelectronics. The term "organic bioelectronics" was used for the first time in 2007 by Berggren et al. in a seminal review [16]. The turning point of conducting and semiconducting polymers for biological applications occurred when researchers started to use them as active, functional materials, opposed to the previous usual adoption limited to the role of passive components for coatings. Since then, the field has been growing at a surprisingly fast rate, as documented by the increasing number of publications and funded projects in the field.

The strong interest in organic bioelectronics manifested by the community stems from the fact that in this field no technology predominance is established so far [116, 122], since traditional technologies fail or represent the limiting factor of the device operation, while organic semiconductors do have specific advantages with respect to inorganic materials, offering innovative and valuable solutions. Most importantly, organic semiconductors, and more specifically conjugated polymers, are characterized by a low degree of toxicity, so they show superior biocompatibility and adaptability to work at the interface with living tissues. As mentioned above, their processing involves simple and cheap steps, usually performed at low temperature and from liquid phase, allowing deposition on virtually any plastic or soft substrate, which suits the non-planar form factors often required for in-vivo applications. At the macroscopic level, soft polymers offer better mechanical properties than traditional (hard) electronic materials and their surface represents an ideal substrate for growing cells on or for interfacing with extremely delicate tissues. At a sub-microscopic level, their peculiar conjugated structure, constituted by alternating single and double carbon bonds, is indeed very similar to the structure found in many biological molecules, and can be conveniently chemically engineered to extend the functionalities and improving performance. Moreover, the weak van der Waals bonding, typical of organic materials, leads to intimate interfaces with electrolytes without dangling bonds or oxide layers [66], so this class of materials offers the unique capability of mixed electronic and ionic conduction [139], thus opening a new interconnection perspective with living matter.

All the above mentioned properties make organic semiconductors extremely attractive for applications in biomedical engineering, neuro-technology and life sciences. In few years, many devices have been developed and demonstrated to work both in-vitro and in-vivo, and in some cases they outperform standard, inorganic technologies, and already reached the necessary development for pre-clinical and clinical application. In the following, we present some examples of the developed devices, mostly intended for cellular electric activity elicitation and/or recording, and we divide them in three classes: conductive organic bioelectrodes, organic electrochemical transistors and organic field-effect transistors. In section 1.2, we describe emerging techniques and devices for optical stimulation of cell activity, especially focusing on hybrid bio-opto-interfaces, sensitive to visible light, which represent the specific topic of this thesis. The organic-based cell photo-activation represents a completely new tool in the neuroscience field and is especially promising for the realization of an allorganic artificial retinal prosthesis.

#### 1.1.1 Conductive organic bio-electrodes

Traditionally, metals and inorganic semiconductors such as gold, platinum and silicon, have been used to fabricate the electrodes in bionic devices for sensing and stimulation tasks. Such elements show considerable limitations which are mainly constituted by their rigidity and sharpness, which damage or exert chronic stress on the tissues and the surrounding environment, leading to inflammatory reactions, rejection or breakage. In addition the characteristic signal-to-noise ratio is not optimal and the materials are not stable in time and prone to rapid degradation.

Semiconducting polymers were first used as coatings of electrodes, and they proved to successfully meet the requirements of an ideal bioelectrode, namely, low impedance and high biocompatibility [116]. Mechanical contact is improved by reducing the mismatch at the interface with the tissue while the lifetime of the implant is increased [73, 90], and, additionally, polymer molecules can be specifically functionalized in order to stimulate cellular outgrowth and minimize the immune response.

Most materials used for this application belong to the class of polythiophenes and biocompatibility has been widely demonstrated in a number of cells. In particular, PE-DOT:PSS, a heavily doped p-type organic semiconductor, has emerged as the reference material for neural interface applications, thanks to its optimal properties of conductivity, chemical and electrical long-term stability, relatively low interface impedance and ease of processability. Biocompatibility of PEDOT was first reported by Martin and coworkers with the fabrication of neuro-electrode interfaces with neuronal cells [137,138]. The reduced impedance of the contact prompted the use of PEDOT:PSS for the fabrication of single electrodes as well as microelectrodes arrays (MEAs) in flexible and bendable configurations [19], see e.g. figure 1.1a. In this way a new family of implantable electrodes able to conform to the curvilinear shapes of organs has been developed. As an example Malliaras and coworkers successfully employed a similar MEA to record electrophysiological activity in cerebral cortex with high accuracy, outperforming plain gold electrodes of similar geometry [90]. This important result will



Figure 1.1: (a) Picture of a PEDOT electrode on a flexible substrate (reproduced with permission from [152]); (b) Schematic diagram of an Organic Field-Effect Transistor (reproduced with permission from [15]).

certainly open the way to other proof-of-concept devices, and since nowadays the actual possibility of progress in neuroscience and medicine strongly relies on finding new materials and available technologies, organic-based bioelectrodes will most probably represent the first field where polymers can find a practical use at the clinical level. Potential applications in neurosurgery have been indeed recently highlighted in an interesting perspective by Von Holst [167], including epilepsy, dysfunctions of central and peripheral nerves, traumatic brain injuries and intracranial tumors.

Besides electrodes intended for stimulation and recording, the use of transistors is emerged as a useful tool to extract small electric voltages generated by cell cultures and tissue slices, providing a better signal-to-noise ratio due to local amplification by the transistor circuitry. However, the use of inorganic transistors in in-vivo configurations has been so far severely hampered by their poor biocompatibility. Organic electronics is rapidly emerging as a valuable alternative, and two kinds of device architectures have been reported, namely, the Organic Field Effect Transistor (OFET) and the Organic ElectroChemical Transistor (OECT).

#### 1.1.2 Organic field-effect transistors

OFETs have been widely used for the realization of biosensors in the latest years, and many applications have been reported, including pH, glucose, cholesterol and enzyme sensing. Several architectures have been considered, e.g. liquid-gated configurations or by using high capacitive gate dielectric materials, in order to let the device able to operate at very low voltages, see figure 1.1b. This is in biological applications a clear advantage over inorganic technology, since it avoids harmful electrochemical degradation and reduces the heat released to the surrounding environment [37]. Among the other advantages of OFETs we mention the possibility of the fabrication of bendable and rollable devices [149,153], using low processing temperatures and simple printing technologies, such as screen printing and ink-jet printing, and, not less importantly, transparency and thermal stability.

The first achievements in using organic transistors for stimulating excitable cell activity are attributed to Stieglitz and coworkers since they used pentacene transistors with silicon oxide as dielectric to stimulate a frog sciatic nerve [53], but in that case a strong applied potential was still required. Very recently, Biscarini et al. [38] showed that a liquid-gated pentacene transistor is able to both stimulate and record the extracellular activity of stem murine cells cultured on top of the active polymer layer. The device operated at very low voltages and the OFET response was monitored during the different phases of the cell differentiation process, and only when stem cells were differentiated into neurons, it was possible to measure electrical signals upon external stimulation. Muccini et al. [15] demonstrated that a transparent organic transistor was able to provide both stimulation and recording of dorsal root ganglion primary neurons. Such device was characterized by a very good signal-to-noise ratio exceeding by 16 times that of standard microelectrode array systems, and the good properties shown in multi-cell activity recording and stimulation allowed the authors to be optimistic for the use of the device even in single cell recording.

#### 1.1.3 Organic electrochemical transistors

Unlike standard OFET structures, where the active material is separated from the electrolyte by an insulating layer, in an OECT device the semiconducting polymer is in direct contact with the electrolyte, see the schematic representation of figure 1.2a. Basically, the OECT device acts as a transconductance amplifier that converts a voltage modulation at the gate to a modulation of the drain current. At present, the most widely used material in OECTs is PEDOT:PSS, thanks to its superior conductivity (both electronic and ionic carriers are transported) and biocompatibility, and the transconductance has been recently reported in the mS range [93] up to frequencies of the order of 1 kHz, a value two orders of magnitude larger than that of planar silicon-based FETs.

OECTs raised interest in the community mainly as biosensors [104], being employed, for instance, in DNA [172], enzymes [176] and cell attachment sensing [20]. The possibility to realize OECT arrays by lithographic processes was first demonstrated in [92], operating at low voltages and showing a response time of the order of 100  $\mu$ s, thus compatible with biological processes recording, and used in-vivo in cortical measurements [91]. Impressively, flexible transistor arrays positively compared with surface electrodes and even with conventional penetrating electrodes, showing superior signal-to-noise ratios, since OECTs locally amplify the signal while conventional electrode recordings can only be amplified in a second moment, increasing also the magnitude of the noise.

Another device that exploits the principle of OECTs, is the Organic Electronic Ion Pump (OEIP) developed by Berggren and his collaborators [83]. The device structure consists in two PEDOT:PSS electrodes patterned on a plastic substrate and connected by a channel made by over-oxidized PEDOT:PSS, which is an electric insulator while preserving the capability of conducting ions. Two different solutions are put in contact with the electrodes and the source electrolyte contains ions to be delivered to the target one, see figure 1.2b. Oxidation of the source electrode (anode) forces ions to enter the anode itself from the source electrolyte. Since the channel allows ionic



Figure 1.2: Schematic diagram of (a) Organic Electrochemical Transistor (reproduced with permission from [93]); (b) Organic Electronic Ion Pump (reproduced with permission from [156]).

conductivity, ions are pumped towards the cathode and finally delivered to the target electrolyte. Since ionic charges transported through the channel are compensated by electronic charges flowing between the two electrodes in an external circuit, the output current represents a measure of the ion delivery rate. In a biological application, an important advantage over devices based on electrical stimulation is that the ions to be delivered can be appropriately selected in order to target only a specific type of cell that expresses the corresponding receptor. Also charged biomolecules relevant for neuronal cell signaling can be delivered with this device and notable examples reported in in-vitro studies include acetylcholine, aspartate, GABA and Glutamate [83]. OEIPs were also fabricated on flexible substrates, suitable for surgical implantation, and the controlled delivery of neurotransmitters was assessed in the cochlear system of guinea pigs, thus demonstrating the possibility to modulate sensory functions in a very specific and quantitative way [156].

#### **1.2** Light mediated cell stimulation

In section 1.1 we reviewed the leading bioelectronics techniques that involve organic materials. Surprisingly, despite their success in photovoltaic and LED technology, the most appealing properties of organic semiconductors, namely their light absorption and emission in the visible spectrum have not been fully exploited in designing new bio-organic interface devices and developing stimulation protocols driven by light inputs instead of electrical ones.

Indeed, only recently organic semiconductors, working in a photodetector-like configuration, have been proposed as photoactive materials for optical excitation of neurons [64], and later extended to astrocytes [14] and retinal explants [62,65]. This new approach to light mediated cell stimulation is very promising since it proved to be highly effective though being based on simple devices structures and working principles, and all the activities described in the present thesis have been focused to identifying and understanding the involved phenomena. However, before proceeding to a more detailed description of the technique, a review of the other already available and competing approaches for light mediated cellular stimulation is in order.

#### 1.2.1 Existing technologies

A first group of methods for light-mediated cellular stimulation is based on direct optical simulation of the cells. Except for few notable exceptions, like the photoreceptors in the retina, mammalian cells usually do not have a specific sensitivity to light, however, interaction of light with biological tissues, especially neurons, has been broadly reported in the literature. In his seminal work in 1971, Fork demonstrated the possibility of photostimulating nerve cells of the marine mollusk Aplysia californica with laser lights both at 488 nm and 515 nm [55], but the mechanism behind the effect was not clear. Since then, several methods have been proposed using light wavelengths ranging a wide part of the spectrum, and aimed at exciting either particular cellular parts or appropriate external absorbing systems used to sensitize the cells. The main advantage of these methods is their simplicity, however, their practical applicability is limited by the high light power usually needed, which can cause mechanical or biochemical stress to the cells.

In this context, the most interesting alternative is Infrared Neural Stimulation (INS), first proposed in 2005 by Wells and coworkers [168, 169]. The technique is based on irradiation of the system with infrared light, below the threshold for tissue damage, and it has been proven that INS is able to stimulate electrical activity in different types of neuronal preparations, both in-vitro and in-vivo [12, 84, 135, 154]. The basic working principle is the IR light absorption by water, which generates a local heating of the preparation. This increase in temperature can have several effects on the cells, and Shapiro et al. proposed a general mechanism based on the variation in the cell membrane capacitance [154]. However, also ion channels have a behavior which is temperature-sensitive and this can take part in the excitation mechanism [3].

A different approach followed by some investigators is to develop particular photoisomerizable compounds that are able to attain conformations with differences in functionality upon light absorption [101]. An example is represented by the acrylamideazobenzene-quaternary ammonium, a kind of azobenzene derivative, that can reversibly switch between a cis- and a trans- configuration, upon light illumination with suitable wavelengths (e.g. UV for trans- to cis-, visible for cis- to trans-) within a microsecond. The trans- configuration binds to cellular K<sup>+</sup> channels, blocking potassium flow and increasing the cell excitability, while in the cis- form cell activity is reduced, and this has been exploited to restore light sensitivity in blind mice retina [129, 163].

The principle of photoinduced change of conformation is also at the basis of molecular cages [30,87,119]. One can imagine this structure as a photostable molecule placed inside a photosensitive cage of molecular dimension. This complex can then be introduced near or within the cell of interest, where light at a particular wavelength can be used to open the cage by inducing a change of its conformation and to release the trapped molecule. In this way, local concentration increase of particular substances is allowed and it is possible to probe the consequent response of living cells. The major advantage of this approach for neuronal stimulation consists in the possibility to stimulate only the region where the compound is activated. However, limitations of the method include the inability to reverse the process, limited tissue penetration of UV light and diffusion of the active compound.

Another strategy is represented by optogenetic stimulation, which consists in the

#### Chapter 1. Introduction: cells, polymers and light

stimulation of specific photoactivable probes expressed in cells upon genetic modification [174]. Up to now, a number of probes can be used to modulate the activity of cells including trans-membrane channels [22], pumps [31], and receptors [2], inhibitors of neurotransmitter release [103] and light-gated protein-protein interaction [102]. Optogenetics represents a powerful technique in neuroscience since it allows the manipulation of the function of specific cell populations of selected regions with high temporal and spatial resolution. However, there are still some issues that have to be solved, in particular, safety concerns regarding the use of viruses for gene expression and the achievement of stable and controlled long-term heterologous protein expression still represent the major challenges.

Optical cell stimulation can also be obtained following indirect pathways, using exogenous photo-absorbers, as an example nanoparticles of semiconducting quantum dots (QDs), The use of cadmium sulfide and cadmium telluride QDs directly binding cells was proposed for the first time by Schmidt and colleagues back in 2001 [170], but effective photostimulation was obtained only several year later using mercury telluride QDs thin films as culturing substrates [123]. Stimulation was attributed to faradaic currents due to charge transfer reactions occurring between the QDs and the extracellular solution, but following work proved that the triggering process is mediated by the local electric field generated by the excited nanoparticles [175]. Additionally, the possibility to control cellular dynamics with nanoparticles of noble metals in in-vitro cultures has been demonstrated [81]. The main concern in the systematic application of QDs regards their possible toxicity, which can arise from different factors, like the particle chemical composition, their geometrical properties or the type of ligands employed. While in some applications the use of QDs has been proven to be safe, there is the need for a standard protocol to address this very important issue.

Finally, a novel use of nanoparticles has been proposed by the group of Shoham [52], exploiting thermal stimulation mediated by specifically designed photoabsorbers, namely, iron oxide microparticles, dispersed in the vicinity of the cells to be activated. With this method, high spatial and temporal resolution have been achieved and similar results are obtained also using carbon microparticles.

To conclude this review, cellular photostimulation can be achieved using semiconductor substrates, and the first approach is attributed to Goda et al. [35]. In their approach cells are grown onto a silicon layer and a voltage is applied between an electrode below the substrate and a reference in the extracellular solution. Upon illumination with intensity usually in the order of few mW mm<sup>-2</sup>, conductivity of minority carriers does increase, and modulating the external bias and illumination intensity, an alternating photocurrent is produced in the device, which is capacitively coupled to the cell membrane. With this approach the selective stimulation of neurons is achievable irrespectively of their location on the substrate [67], but the method is applicable also to non-neuronal cell types, including muscles, secretory cells and cardiac myocytes [28]. Other photoconductive substrates have considered, including single crystal [158] and hydrogenated amorphous silicon [159], and more recently a TiO<sub>2</sub> nanocrystalline film has been used [160].

In general, photoconductive stimulation is a cost-effective, simple and easily implementable technique, which can be coupled in a straightforward way in any standard set-up for electrophysiology. On the other hand, this technique requires high intensity illumination densities and the application of an external bias, both possibly leading to tissue heating and cellular damage. Moreover its applicability in-vivo is difficult, due to limited biocompatibility, the need for electrical wiring, mechanical rigidity and biostability issues. An interesting alternative is represented by silicon-based photovoltaic systems and, as an example, Palanker and his group demonstrated the possibility to realize a wireless, photovoltaic retinal prosthesis, with high pixel density, subretinally implantable and able to induce robust cortical responses in normal and degenerate rats [107, 110].

#### 1.2.2 Light mediated cell stimulation with semiconducting polymers

Since the first experimental studies that coupled organic hybrid solid-liquid photodiodes with light excitation [4, 60, 61], it has been clear that such devices were very suitable for application in a biological framework. However the first demonstration of light-induced modulation of membrane voltage using organic semiconductors can be found in the work by Lanzani and coworkers in [64]. In that work the device structure comprised a layer of a blend of poly(3-hexylthiophene) (P3HT), a photoactive polymer, and phenyl-C61-butyric-acid-methyl ester (PCBM), deposited on an Indium-Tin-Oxide (ITO) covered glass, and immersed in a saline solution, reproducing the extracellular medium. Hippocampal rat neurons were then successfully grown onto the device, previously coated with poly-L-lysine to enhance cell adhesion, see figure 1.3a.

The stability and the biocompatibility of the substrate were proved by the fact that after 28 days in-vitro the optoelectronic properties of the active material were not affected, and the biological functionalities of the neurons, in terms of cell viability and resting membrane voltage, were not significantly different with respect to control cultures. By recording membrane potentials using patch-clamp technique in whole-cell and current-clamp configuration, it was shown that a 20 ms green light pulse (532 nm, 10-15 mW mm<sup>-2</sup>) evoked the generation of action potentials in neurons with great reproducibility, see figure 1.3c. The response time was very rapid (around ten milliseconds to reach the peak of the spike) and the photo-stimulation process was highly spatially selective. Upon moving the light spot just outside the cell body, no significant activity could be elicited in the neuron (figure 1.3b).

The photophysics of the photovoltaic blend used in this study suggests a role for bulk charge photo-generation in the excitation mechanism. However, it has been found that the same transduction process of light pulses into neuronal electrical activity is obtained using a pristine P3HT film deposited on an ITO covered glass substrate [65]. This observation indicates that the working principles of such hybrid interface can be quite different from those of conventional organic photovoltaic devices, and that faradaic currents injected in the cleft between the device and the neuron, which have been successfully recorded with the same device configuration under continuous illumination [99], are not relevant here for the device operation. Indeed, a large body of experimental evidence suggests the presence of a capacitive coupling between the organic layer and the neuron grown on top of it.

It is important to highlight that the working principle of cell photoactivation is not limited to primary neuronal networks, in fact, the possibility of modulating in a controlled way the whole-cell conductance in primary rat astrocytes by means of



**Figure 1.3:** (a) Scheme of the photosensing interface, with the neurons grown on top of the active layer and patch-clamp setup. (b) Spatial selectivity of the photostimulating organic interface: a grid of 9 spots (diameter 20 mm was overlaid to a patched neuron and the elicited spikes counted. (c) Generation of action potentials upon photostimulation by a pulsed light train of 20 ms at 1 Hz (green bars) (From [64]).

optical stimulation, properly mediated by the active polymer film, has been recently demonstrated in [14].

The present approach is very general in its working principles and represents a new tool for neural active interfacing, as a simpler alternative to the existing and widely used neuron optogenetic photostimulation techniques able to avoid potentially hazardous gene transfer protocols. The photostimulation is not specific for selected neuronal populations, as is the case for genetically encoded approaches, but the optical stimulation of neurons could be micrometrically shaped to stimulate selected neuronal populations owing to the high spatial selectivity of the photostimulation interface, and could lead to the development of new artificial optoelectronic neurointerfaces based on biocompatible organic materials.

#### 1.2.3 Possible working principles of the device

The working principles of hybrid organic photodiodes may be considerably different with respect to those of conventional ones, mainly because of the interface phenomena occurring between the polymer film and the electrolyte. Most importantly charge transport may have both electronic and ionic nature, and chemical reactions might occur at the interface.

The experimental evidence obtained so far in the works devoted to analyze this kind of structure [60, 61, 64], suggested the authors to describe the device working principles in terms of phenomena of electric nature. Upon photostimulation, excess charges are generated in the active layer and, due to an unbalance in carrier transport properties, which depends on the particular choice for the photoactive material (donor/acceptor blend, pristine material), they reorganize determining a charge dis-



Figure 1.4: Proposed device working principle based on (a) electric and (b) heating effects.

placement. This in turn is responsible of the establishment of the measured photovoltage and of a consequent reorganization of the ion distributions in the solution cleft between the substrate and the cell. This ultimately determines a modification of the solution configuration close to the extracellular face of the cell membrane, resulting in its depolarization that might elicit an action potential. In figure 1.4a we report a schematic picture of this process.

However, also heat related phenomena might play an important role, as in the case of the INS technique described in section 1.2.1. For high light intensities, light absorption in the polymer layer might induce temperature increase in the solution, which in turn might determine modifications in the membrane properties, see figure 1.4b. Should this picture be confirmed, the main advantage of this stimulation technique over INS would be the fact that light absorption occurs in just a few nanometers (against several millimeters of water) so the temperature gradients would be more localized and less power input would be needed.

Up to now, none of these two possible working principles has been shown to be predominant in determining the device operation yet, and a full and accurate description of all the involved steps is still to be provided. In this picture, the present thesis work is aimed at analyzing the above mentioned electric and thermal phenomena occurring in the hybrid device upon illumination, and to do so we want to address to them singularly, in order to isolate their respective contribution to the overall operation. Using the insight that experimental evidence provides, we then formulate suitable model descriptions of the device working principles and translate them into mathematical terms in order to quantitatively reproduce the measurement outputs using numerical simulations.

#### 1.2.4 Advanced applications: retinal prosthetics

The phenomenon of cell stimulation by polymer photoexcitation, the high spatial and temporal resolution of the device, together with the good biocompatibility properties demonstrated for the organic semiconductors, open the way to potential applications in the field of retinal prosthetics [109].

The final goal is to restore photosensitivity in retinas whose natural photoreceptors are damaged or lost. However, the retinal photoreceptors behave quite differently





**Figure 1.5:** (a) Confocal images of control (left) and degenerate (right) retinal sections (gcl: ganglion cell layer; inl: inner nuclear layer; onl: outer nuclear layer; scale bar 50  $\mu$ m). (b) Schematic representation of the normal and degenerate retina structure. (c) Multi-unit activity recorded upon light stimulation (10 ms, 4 mW mm<sup>-2</sup>) of a control retina over a ITO:glass substrate (left), a degenerate retina over a ITO:glass substrate (middle) and a degenerate retina over a P3HT-coated ITO:glass substrate (right). The bottom histograms show normalized post-stimulus time. Green bars/arrows represent the light stimulus (from [65]).

from primary brain neurons since they do not develop action potentials, and, as a whole, retinal physiology is very complex, so a success of a similar approach is far from being taken for granted.

Nevertheless, Benfenati and coworkers reported that such a bio-organic interface is actually capable of restoring light sensitivity in blind retinas ex-vivo [65]. Acutely dissected retinas from albino rats with degenerate photoreceptors were placed on a single-component organic film of P3HT deposited on a ITO:glass substrate in a sub-retinal configuration, i.e. the degenerate photoreceptor layer contacted the polymer, see figure 1.5a.

Multi-unit activity recordings, performed with an extracellular electrode in the retinal ganglion cell layer and representing the response of the retina to light, showed that a 10 ms light pulse ( $4 \text{ mW mm}^{-2}$ ) was able to stimulate intense activity, at levels indistinguishable from those of control retinas of normal rats, while no significant activity could be recorded in blinded retinas placed on ITO:glass substrates (figure 1.5c). The analysis of the temporal characteristics of the excitation proved that ganglion cell spiking was mediated by the activation of the external cell layer in contact with the polymer.

Dose-response measurements (figure 1.6) revealed a threshold intensity for photostimulation of about  $0.3 \,\mu\text{W}\,\text{mm}^{-2}$ , closely matching the range of retinal irradiance during outdoor activity ( $0.1-10 \,\mu\text{W}\,\text{mm}^{-2}$ ). In contrast, degenerate retinas recorded over ITO:glass displayed low levels of spiking activity only at very high light intensity (threshold  $80 \,\mu\text{W}\,\text{mm}^{-2}$ , figure 1.6). Another interesting feature is related to the linear dynamic range of operation, which spans 2 log units of retinal irradiation (1- $100 \,\mu\text{W}\,\text{mm}^{-2}$ ). This linear operating range allows modulation of the retinal output



Figure 1.6: (a) Comparison of mean peri-stimulus time histograms obtained from: control retinas on ITO:glass (black bars); degenerate retinas on ITO:glass (open bars); degenerate retinas on P3HT-coated ITO:glass (red bars) in response to light illumination (10 ms, 4 mW mm<sup>-2</sup>, green arrow). (b) Doseresponse analysis of the mean retinal ganglion cell firing rate versus light intensity performed in degenerate retinas over P3HT-coated ITO:glass (red circles) or ITO:glass alone (open circles). The dashed line corresponds to the computed maximum permissible radiant power for chronic exposure (from [65]).

depending on the light dose reaching the polymer-retina interface, which is an extremely important requirement in developing artificial prostheses aimed at restoring the physiological function of the retina.

Similar results have been reproduced by Narayan and coworkers [62] with embryonic chick retina at a light insensitive stage of development using moderate light intensities (10-600  $\mu$ W mm<sup>-2</sup>), proving the consistency and the applicability of the stimulation protocol to different types of retinal tissues.

Although the coupling mechanism between neural cells and the semiconducting polymer is still only partially understood, its potential impact is clear-cut. There are however many open issues, regarding in particular the long term stability of the polymeric material, the tolerability over prolonged time in-vivo and the optimization of the device response. For this reason, recently, research is being conducted to further explore this possibility by subretinally implanting these devices in an animal model of retinal degenerative diseases, such as the Royal College of Surgeons rats bearing a mutation in the Mertk gene.

# CHAPTER 2

## Polymer/electrolyte device

In order to study the behavior of the device focusing on the electrical effects that might be responsible of its operation, in the present chapter we consider a simplified device without a cell grown onto it, which consists in a layer of poly(3-hexylthiophene) (P3HT), deposited on a Indium-Tin-Oxide (ITO) covered glass substrate, immersed in a NaCl solution, see section 1.2.2. P3HT is a *p*-type semiconductor conjugated polymer, i.e. it mostly allows the electronic transport of positive charge carriers, which is widely used in photovoltaics due to its remarkable optical properties, most importantly a high absorption coefficient in the visible range of the light spectrum. ITO is instead a conducting oxide which combines high transport properties with optical transparency and ease of processability on thin films.

Two different measurement techniques have been considered, see figure 2.1:

- **Photovoltage measurements**: the change of voltage upon illumination between the ITO contact (1) and a Ag/AgCl reference electrode (5) in the solution is measured using a potentiostat in open circuit configuration.
- Surface potential measurements: the electric potential variation upon illumination in the solution in close proximity of the polymer interface (3) (about  $1 \mu m$ ) is probed using a glass micropipette.

To better understand the role of the components of the device, the measurements are repeated considering different configurations obtained by changing several critical parameters, such as the thickness of the deposited polymer, the ion concentrations of the solution, the light intensity and direction.

In the following sections we report the results of the above mentioned measurements, analyze them to get useful information regarding the working principles of the



**Figure 2.1:** Simplified device structure and performed measurements: (1-5) transient photovoltage; (3-(4)-5) surface potential.

hybrid polymer/electrolyte device, and with this insight we propose a series of models aiming at reproducing the experimental results.

#### 2.1 Photovoltage measurements

The first type of experimental measures that we consider is a particular kind of transient voltage measurement in which the evolution of the difference of the electric potential between the ITO electrode and the bulk solution is tracked upon illumination.

When the transient response of photovoltaic devices upon light stimulation is addressed, the mainstream adopted techniques are transient photocurrent and photo-CELIV (Charge Extraction by Linearly Increasing Voltage) [17]. Transient photovoltage technique is less frequently adopted since photovoltaic devices are typically intended for the generation of charge current. In the configuration considered in the present work, transient photovoltage is instead perfectly fitting the device operation principles, since the final aim of the polymer-electrolyte interface is that of generating a voltage signal to be transduced to the living cell.

For this reason Narayan and coworkers successfully adopted this particular technique for characterizing a hybrid polymer blend/interface in [60], which shares the basic structure with the device currently under study. In our configuration however, the active layer differs by the fact that only a pristine polymer is considered and additionally we adopted a different setup for performing the measurement, that we describe in the following section.

#### 2.1.1 Description of the experimental technique

Electrochemical characterization is carried out by using an Autolab potentiostat/galvanostat (PGSTAT) [9, 113], in a three electrode configuration, see figure 2.2, and a basic diagram of this particular setup is presented in figure 2.3.

The three-electrode cell is the most common setup used in electrochemistry, see figure 2.2, and the three electrode are referred to as Counter (CE), Reference (RE) and Working Electrode (WE), respectively.

The CE (also known as auxiliary electrode), is used to close the current circuit in



Figure 2.2: Schematic view of the three-electrode cell setup (reproduced from [113]).



Figure 2.3: Basic diagram of a potentiostat/galvanostat (reproduced from [113]).

the electrochemical cell, it is made of an inert material (in the present case Pt) and it does not participate in the phenomena to be measured.

As its name suggests, the RE is used as a reference point for the potential control and measurement since it has a stable and well-known potential. High stability of the potential is usually achieved by employing a redox system with constant concentrations of each participants of the redox reaction, and in the current setup the Ag/AgCl couple in a saturated KCl solution is considered.

The WE is that on which the phenomenon under study occurs, and in the current configuration it is represented by the P3HT-coated ITO substrate.

In potentiostatic mode, the PGSTAT accurately controls the potential of the CE against the WE, so that the potential difference between the WE and the RE is well defined and corresponds to the value specified by the user. In galvanostatic mode, the current flow between the WE and the CE is controlled. The potential difference between the RE and WE and the current flowing between the CE and WE are continuously monitored. By using the PGSTAT, the value specified by the user (i.e. applied potential or current) is accurately controlled all the time during the measurement by using a negative feedback mechanism, as illustrated in the circuit scheme of figure 2.3.

In the analysis object of this section, we perform measurements of the voltage at the WE in potentiostatic mode, setting the current between the WE and the CE to zero (open circuit). The measurement is performed starting from dark condition and then



**Figure 2.4:** Typical profile of a photovoltage measurement. The shaded area indicates the period of time when the light is on.

illuminating the device using a collimated white LED (MCWHL2-C4, Thorlabs) source with 50 ms pulses characterized by an abrupt on/off transition, with the possibility of modulating the incident power intensity. Data acquisition and analysis is performed using NOVA 1.10 and Matlab.

Measurements performed on devices which are fabricated in order to have the same specifications result however in slightly different outputs since it is very difficult to have a complete control on the device geometry, and most importantly, in the internal structure of the polymer layer (e.g. impurity concentration, local order, roughness). Nevertheless all the measurements share common features that are well represented by the typical photovoltage profile reported in figure 2.4. Before light is switched on, the instrument measures a constant voltage between the electrodes, which is commonly positive and ranges between a few and one or two hundreds of millivolts. When the device is hit by the light, the polymer absorbs part of it and electrical charges are generated. Consequently an evolution of the measured voltage towards lower values (i.e. the generated photovoltage is negative) with a characteristic time scale of a few milliseconds is observed, due to the rearrangement of the photogenerated charges in the polymer. Since the bath of the solution is taken to be at the reference value of the potential, such change is consistent with a net displacement of negative charge towards the ITO electrode.

Superposed on the fast negative photovoltage transient, the measure shows another slower component characterized by an opposite sign. Such behavior has to be ascribed to a charge movement in the opposite direction with respect to that of the fast component, and since the observed time scales are very different it is likely to be attributed to other kind of phenomena characterized by slower rates (e.g. chemical reactions involving electron transfer). Since on the considered time scale of the measurements that are about to be shown in the next part of the section such feature is less important than the fast negative contribution, we will neglect it in our analysis.

When light is switched off, the initial configuration of the device is recovered and the measured voltage evolves to the values observed before the illumination. A very important fact is that such relaxation has a characteristic time which is longer than that of the light-on transient. A possible interpretation consists in attributing the recovery process to phenomena occurring with a low rate, as charge recombination events involving more than one particle or a large energetic barrier to be overcome.

#### 2.1.2 Changing device thickness and illumination direction

As previously reported, we performed a series of measurements in which we gradually varied either one of the parameters of the device or the characteristics of the illumination protocol. In the first set we consider, some devices have been fabricated by depositing layers of polymer of different thickness, in the range between 26 and 400 nm, and we performed transient photovoltage measurements shining LED light with power density 267.5  $\mu$ W mm<sup>-2</sup> both from the transparent ITO electrode side and the solution side. With these experiments our aim is to understand whether the effects that drive the device behavior are of bulk type or they just occur in the interface regions, and, if this latter is the case, which are the different roles of the two interfaces.

The results are reported in figure 2.5a and 2.5b for the cases of illumination from the ITO electrode and the solution, respectively, and the differences between the two illumination protocols are quite evident. For all the traces the initial offset is removed so that the average of each measurement is equal to zero in the pre-illumination period. In the case of light from the ITO electrode we observe photovoltage transients that are consistently fast for all the considered device thicknesses, see figure 2.6b, whose peak values monotonically increase with larger device thickness. The behavior of this increase is nonlinear, with a steeper increase for thin devices and a certain level of saturation for the thickest devices. The case of illumination from the solution side has instead a richer variety of features. The results for the thinnest devices do not differ significantly with respect to the case of illumination from the electrode side, but thickening further the devices we observe that the photovoltage transients become progressively slower and less intense, see figure 2.5b. The photovoltage peak values and the characteristic rise time reported in figure 2.6 are obtained by fitting the experimental results using a double exponential function of the form

$$V(t) = -\Delta V_1 \left[ 1 - \exp\left(-\frac{t}{\tau_{\text{rise},1}}\right) \right] - \Delta V_2 \left[ 1 - \exp\left(-\frac{t}{\tau_{\text{rise},2}}\right) \right]$$
(2.1)

and by considering the values  $\Delta V_1$  (>  $\Delta V_2$ ) and  $\tau_{\text{rise},1}$  (<  $\tau_{\text{rise},2}$ ) corresponding to the principal fast component of the measurement.

For the interpretation of the results, it is useful to recall here the presumed working principles of the device. When light hits the device, it is in part absorbed by the photovoltaic polymer generating excited states called excitons [69, 70]. These states consist in a strongly bonded electron-hole pair localized on a polymer molecule and for this reason they do not have a net charge. Excitons have a characteristic decay time of a few hundreds of picoseconds and can move in the polymer matrix by diffusion events. The hole and electron that form the exciton may eventually dissociate at heterojunctions of materials having different electronic affinity, and this phenomenon is enhanced by the presence of a local electric field [33,69]. However, in pristine materials like the one used in our device, exciton dissociation usually occurs with a limited rate and only the electric field can increase the dissociation probability to appreciable levels. Once free charges are generated, they can migrate following concentration



**Figure 2.5:** Transient photovoltage measurements obtained with devices characterized by different thickness of the polymer layer (legend reports the values in nanometers).



**Figure 2.6:** Photovoltage peak value and characteristic rise time obtained from the experimental measurements, for both illumination from the ITO side and the solution side.

gradients (diffusion) or under the action of the electric field (drift) as in traditional solar cells. The main difference with these latter is the fact that no charge is extracted, and for this reason a charge displacement is built up that eventually determines the set up of a photovoltage that blocks further charge migration and leads to a condition of dynamic equilibrium.

With this premise, from the results of figures 2.5 and 2.6 we can draw the first conclusion that the phenomena that determine the generation of the photovoltage, most importantly exciton dissociation, are not entirely of bulk type and, instead, interfaces must play a key role. Should this not be the case, for each polymer thickness we would expect to measure the same photovoltage, irrespectively of the illumination direction, and hence obtain increasing signals for thick devices also with light from the solution, which clearly is not observed. Moreover, should the dissociation mechanism be the same at the two interfaces, the photovoltage measurements would have opposite sign for the two illumination protocols. This is a clear indication that only one of the two interfaces plays a role in the operation of the device.

In order to determine which such interface is, it is crucial to consider how exciton

generation occurs and how the density of these latter might look like in the device under illumination. When light hits the polymer the first layers absorb part of the incident photons and the remaining are transmitted through the other layers deeper in the bulk. This applies for the entire material, the result being that less and less light is absorbed in the polymer and hence the exciton generation process has a rate which is a decreasing function of the distance from the incidence interface. Since the excitons have a quite short lifetime, the characteristic diffusion length is limited, and the exciton density profile resembles the generation profile, resulting in a high population at the incidence region which decays in exponential way going into the bulk of the material.

The generation rate of free charges at a particular point of the polymer depends on the local exciton density and if we assume that dissociation occurs mainly in the area close to the ITO/polymer interface, it is possible to interpret the photovoltage results of figure 2.5 after the considerations just drawn above. In the case of light from the solution, thicker devices are expected to have a lower exciton density in the area close to the ITO/polymer interface and for this reason also free charges are generated in lower number, with a slower rate. This eventually results in a less intense photovoltage signal which reaches the peak value in longer time, consistently with the observed measurements.

When light hits the device from the ITO side, the exciton density is such region is independent on the device thickness and for this reason the photovoltage should not be affected. Nevertheless, the measurements show that the photovoltage peak slightly increases with thickness and this might be due to dissociation occurring in the bulk material with a rate lower than that of the interface region.

Moreover, it is possible to provide an estimate of the width of the interface dissociation area from the photovoltage profiles obtained with the thinnest devices, namely those with thickness equal to 26 and 33 nm, since they behave in a very similar way irrespectively of the illumination side. For thin devices the spatial decrease of the exciton generation profile is less pronounced and hence it is possible to consider a constant exciton density in the layer. Hence if the number of locally available excitons is not the limiting factor for the free charge generation rate and if the exciton dissociation is equally efficient all along the polymer, we should expect photovoltage peak values that uniformly increase with the thickness of the polymer. Since the values in figure 2.6a clearly show that this trend is not maintained after thickness values larger than 33 nm, this must mean that the high efficiency dissociation region is not much wider than that. As we already pointed out, with thicker devices in the case of light from the ITO the photovoltage peak still increases because more excitons are generated in a larger polymer bulk domain, still with lower efficiency. With illumination from the solution, instead, the overlap between the high exciton density area and the high dissociation efficiency area is progressively reduced.

#### 2.1.3 Changing illumination intensity

Another set of measurements is performed on a device with a predetermined thickness of about 150 nm, illuminating from the ITO electrode side and changing the LED intensity from  $4.68 \,\mu W \,mm^{-2}$  to the maximum value achievable with the instrument of 267.5  $\mu W \,mm^{-2}$ . The results are reported in figure 2.7.



**Figure 2.7:** Transient photovoltage measurements obtained with light hitting the device from the ITO electrode side with different intensity (legend reports the values in  $\mu W \text{ mm}^{-2}$ ).



**Figure 2.8:** (a) Photovoltage peak value and (b) characteristic rise time obtained from the experimental measurements with different light intensity.

We observe that by increasing the light intensity the device response gets faster and the photovoltage magnitude becomes larger. However this latter dependence is far from being linear, see the fitted values of figure 2.8, where we can spot instead a behavior of the photovoltage peak and rise time like the logarithm of the light intensity  $(\log(I + 1))$  and its inverse  $(1/\log(I + 1))$ , respectively. Similar trends in dependence of light intensity are observed for more conventional experiments like photocurrent measurements, and this is to be ascribed to the nonlinearity of the recombination phenomena, which usually are of second or higher order. Higher light intensity determines more charge generation but since the recombination rate scales with a more than linear order, equilibrium conditions are reached faster and are characterized by free charge density which do not scale linearly. Ultimately since the photovoltage is strictly dependent on charge displacement, this makes the photovoltage to scale accordingly.

#### 2.1.4 Changing solution molarity

Finally, in order to understand whether the polymer/solution interface plays a role in determining the operation of the device, transient photovoltage measurements are



Figure 2.9: Transient photovoltage measurements obtained with light from the ITO electrode side for different solution molarities.

performed on devices that share the configuration of the solid part, namely having the same polymer thickness of about 100 nm, but differ in the solution concentration. We consider solutions with 20 and 200 mM concentrations and, in addition, a device is fabricated using ultrapure water, i.e. water which is highly purified from all type of ion and impurities.

The results are presented in figure 2.9, and notably they do not differ significantly. This means that the processes that determine the generation of the photovoltage do not depend on the used solution. A similar result has been observed also in [61] and this allows us to consider ion conductivity to be negligible with respect to the electric one in our device. Additional discussion about this particular experimental trend is carried out in the next section, while presenting the boundary conditions for the electric potential problem (2.2f).

At this point a recap of the working principles we propose for the device operation is in order, and a schematic picture is reported in figure 2.10. When light hits the device, photons are absorbed by the polymer and excited states are generated following an exponentially decaying profile. Excitons may diffuse in the polymer, and at the interface region close to the ITO electrode they might be dissociated into couples of free holes and electrons. Experimental evidence suggests that electrons remain in the interface region, while holes, that are more efficiently transported in P3HT, diffuse



Figure 2.10: Proposed working principles for the transient photovoltage measurement.

to the bulk of the polymer. In this way a charge displacement is determined and a photovoltage is built-up between the ITO electrode and the solution. When light is switched off, holes migrate back to the ITO interface and recombine with electrons, so the photovoltage vanishes and the initial configuration is recovered.

The experiments presented above provide very valuable information to understand the operation of the device and our discussion is consistent, covering all the observed features and without any appeal to unusual assumptions. The next step for further corroborating our arguments is to provide a mathematical model which is able to reproduce the experiment and to confirm the feasibility of the working principles that we proposed in this section.

#### 2.2 Substrate continuum PDE model

In section 2.1 we highlighted that the transient photovoltage technique, in comparison with transient photocurrent or photo-CELIV techniques, has been significantly less adopted in the characterization of organic photovoltaic devices. As a consequence, to the best of our knowledge, few works are available in the literature in which an accurate mathematical modeling of the transient photovoltage experiment is developed.

Very recently, Pivrikas and coworkers [128] developed a new transient technique called Resistance dependent PhotoVoltage (RPV), in which the photovoltage resulting from a pulse of light is measured using an oscilloscope at various load resistances, and at the same time they also developed a corresponding accurate model. However some differences occur with respect to the case examined in the present thesis. Most importantly, the technique has been applied to solar cells, hence a solid device, while here the electrolyte is expected to behave in a significantly different way as a metal electrode. Moreover the considered active layer is a blend of donor and acceptor materials, hence bearing different characteristics with respect to a pristine polymer. Finally the adopted illumination protocol, although very intense, is short, while the stimulation we apply in the present analysis is prolonged and leads to the generation of a higher amount of charge, eventually reaching a saturation condition. For these reasons, being the mathematical model developed in [128] tailored to the specific case, it might not be suited for a direct application to the one under study in our work.

As reported in section 2.1, Narayan and his group characterized a polymer/solution interface device with a similar structure as that considered in our analysis, performing transient photovoltage measurements [60]. In this latter reference they also proposed a modeling approach which ascribed the generation of the photovoltage to an unbalance in the transport properties of holes and electrons in the photovoltaic blend. However, due to the differences in the active layer composition (we consider a substrate made of a pristine polymer), the observed experimental results therein significantly differ from those reported in section 2.1, and we have not been able to fit them in the modeling picture proposed in [60].

For this reason we develop an original model for the transient photovoltage measurement applied to hybrid polymer/solution interfaces, starting from the state of the art approaches available for organic devices [54, 95, 106, 143] and exploiting the evidences gained with the experimental measurements of section 2.1.
Before starting with the definition of the mathematical model it is important to highlight the peculiarities of the considered system in order to detect the main criticalities and identify the best approaches to face them.

- Since the transient photovoltage measurements are carried out in open circuit configuration, the amount of charges accumulated in the device is quite high. Organic materials are known both to have carrier mobilities which strongly depend on carrier densities and to be characterized by diffusive phenomena which deviate significantly from Einstein relation in case of high carrier densities. In modeling more conventional configurations, such as solar cells in standard operating conditions, such effects can safely be neglected, but here they definitively have to be accounted for in order to accurately describe the observed phenomena.
- The measured transient photovoltage is the macroscopic result of the dynamics of all the charged states that live in the polymer, that can either be free or trapped. In particular, in organic devices, trap dynamics strongly determines the overall temporal response of the device [5], which, moreover, in the considered case is quite uncommon, having time scales of the order of milliseconds, while most devices usually reach stationary conditions after few or tens of microseconds. For this reason particular attention has to be devoted in properly describing all the steps involved in the trapping/detrapping phenomena occurring in the polymer.
- As discussed in section 2.1, the current study is peculiar since both the device and the stimulation protocol are not mainstream, and for this reason the literature lacks an appropriate model with all the desired features. Usually models, and in particular trap models, are formulated either referring to devices with material heterojunctions under illumination (i.e. solar cells and photodetectors) [29, 82, 105, 106] or to pristine materials without illumination (transistors) [94, 95, 130]. The analysis conducted in the present thesis is original in the sense that these two aspects are combined, and as a consequence very few data, if none, are available to compare to and to get information about the values to be attributed to the model parameters.

Since the device has a planar structure we can assume homogeneity along the inplane directions and we can formulate the model in a 1D setting on the ITO electrode/solution direction. However, since the proposed model is in principle appropriate even if this assumption does not hold, we formulate the equations in the general setting of a computational domain in  $\mathbb{R}^d$ , d = 1, 2, 3. Moreover, the photovoltage transients reported in figure 2.9, obtained by changing the solution, show that the liquid part of the device has no impact on the operation of the device under the considered conditions, so we are allowed not to include it in the model and to limit the computational domain to the polymer layer alone. In figure 2.11 we report a schematic picture of the model domain in a one-dimensional setting, where with  $\Omega$  we denote the region occupied



Figure 2.11: 1D computational domain for the PDE model.



Figure 2.12: Flowchart of the phenomena occurring in the device during a transient photovoltage measurement.

by the polymer and with  $\Gamma_{\text{ITO}}$  and  $\Gamma_{\text{int}}$  we denote the interfaces with the ITO electrode and the solution, respectively.

Since the final application of the model is to reproduce the results of the transient photovoltage measurements, it has to appropriately describe the chain of events that has been illustrated in section 2.1 and schematically reproduced in figure 2.10. For this reason the dynamics of each of the introduced quantities, namely exciton and charge densities and electric field, need to be properly described, and for further clarity we summarize the involved phenomena in the flowchart of figure 2.12.

Let us denote by *X*, *p*, *p*<sub>t</sub>, *n*, and *n*<sub>t</sub>, the volumetric densities of singlet excitons, free and trapped holes, free and trapped electrons, respectively, and by  $\varphi$  the electric potential.

Excitation phenomena occurring in the bulk as a consequence of light absorption are described by the parabolic problem

$$\begin{cases} \frac{\partial X}{\partial t} + \nabla \cdot \mathbf{J}_X = G_{\text{light}} - k_{\text{rec}} X - k_{\text{diss}} X & \text{in } \Omega, \\ \mathbf{J}_X = -D_X \nabla X & \text{in } \Omega, \\ \mathbf{J}_X \cdot \boldsymbol{\nu} = 0 & \text{on } \Gamma_{\text{ITO}} \cup \Gamma_{\text{int}}, \\ X(\mathbf{x}, 0) = X_0 & \text{in } \Omega. \end{cases}$$
(2.2a)

The first equation in (2.2a) represents the time rate of change of the exciton density due to the balance between diffusive flux ( $\nabla \cdot J_X$ ), source ( $G_{\text{light}}$ ) and sink terms ( $k_{\text{rec}}X$ and  $k_{\text{diss}}X$ ). Exciton flux  $J_X$  is defined according to the Fick's law,  $D_X$  being exciton diffusivity, since these states do not have a net charge and hence the electric field does not determine a drift on them. The source term  $G_{\text{light}}$  represents the exciton generation rate due to light absorption and further details are presented in section 2.2.1. The quantities  $k_{\text{rec}}$  and  $k_{\text{diss}}$  represent instead the rate constants for the transition of excitons back to the ground state or their dissociation into a couple of free charge carriers. In particular  $k_{\text{rec}}$  is taken to be a constant value while  $k_{\text{diss}}$  is known to be strongly dependent on the module of the local electric field, therefore we adopt the widely used Braun-Onsager model [23, 86], see section 2.2.2 for a more thorough discussion. Finally, the last two equations in (2.2a) are boundary and initial conditions in the device. The boundary conditions are of no-flux type and express the fact that direct dissociation or quenching of excitons at each interface is negligible. The dynamics of the photogenerated free charge carriers in the polymer are described by the two parabolic problems

$$\begin{cases} \frac{\partial p}{\partial t} + \nabla \cdot \mathbf{J}_{p} = k_{\text{diss}} X - \sum_{i=1}^{\overline{N}_{t}^{p}} (r_{\text{hc}}^{p,i} - r_{\text{he}}^{p,i}) - \sum_{j=1}^{\overline{N}_{t}^{n}} (r_{\text{hc}}^{n,j} - r_{\text{he}}^{n,j}) & \text{in } \Omega, \\ \mathbf{J}_{p} = -\mu_{p} p \nabla \varphi - D_{p} \nabla p & \text{in } \Omega, \\ \mathbf{J}_{p} \cdot \boldsymbol{\nu} = j_{p}^{\text{SM}} & \text{on } \Gamma_{\text{ITO}}, \\ \mathbf{J}_{p} \cdot \boldsymbol{\nu} = 0 & \text{on } \Gamma_{\text{int}}, \\ p(\mathbf{x}, 0) = p_{0} & \text{in } \Omega, \end{cases}$$
(2.2b)

and

$$\begin{cases} \frac{\partial n}{\partial t} + \nabla \cdot \mathbf{J}_{n} = k_{\text{diss}} X - \sum_{i=1}^{\overline{N}_{t}^{p}} (r_{\text{ec}}^{p,i} - r_{\text{ee}}^{p,i}) - \sum_{j=1}^{\overline{N}_{t}^{n}} (r_{\text{ec}}^{n,j} - r_{\text{ee}}^{n,j}) & \text{in } \Omega, \\ \mathbf{J}_{n} = +\mu_{n} n \nabla \varphi - D_{n} \nabla n & \text{in } \Omega, \\ \mathbf{J}_{n} \cdot \boldsymbol{\nu} = j_{n}^{\text{SM}} & \text{on } \Gamma_{\text{ITO}}, \\ \mathbf{J}_{n} \cdot \boldsymbol{\nu} = 0 & \text{on } \Gamma_{\text{int}}, \\ n(\mathbf{x}, 0) = n_{0} & \text{in } \Omega. \end{cases}$$
(2.2c)

The first two equations in (2.2b) (and (2.2c)) represent the classical continuity equation and flux definition for holes (and electrons) in the drift-diffusion model,  $\mu_p$  ( $\mu_n$ ) and  $D_p(D_n)$  being the hole (electron) mobility and diffusivity, respectively. The first generation term on the right hand side is the counterpart of that appearing in (2.2a) accounting for exciton dissociation. The remaining terms gathered under the sum signs represent instead the rates of the trapping/detrapping events that involve the carriers. A more detailed description of such rates is provided later on in section 2.2.5. The boundary condition on  $\Gamma_{\rm ITO}$  defines the outflow carrier flux according to the standard model developed by Scott and Malliaras [150] described in section 2.2.3, as the balance of a thermionic injection contribution and a recombination one. The null flux condition on  $\Gamma_{int}$  follows instead from the conclusions we draw from the photovoltage experiments by changing the type of solution. Charge carriers in an excited semiconductor put in contact with an ionic solution might be involved in chemical reactions, like it happens in electrochemical cells and dye sensitized solar cells. As we have experimental evidence that in the device under study such reactions do not take place, at least at the characteristic time scales of the considered measurement, we can assume that electrons and holes do not flow across the polymer/solution interface and hence enforce a no flux boundary condition. The last equation in (2.2b) (and (2.2c)) again represents the initial condition for the free charge density.

The description of charge dynamics is then completed by the equations that govern the evolution of the trapped carrier densities. Let us assume that there exist  $\overline{N}_t^p$  trap levels for holes and  $\overline{N}_t^n$  levels for electrons, so that for the total trapped densities it holds

$$p_{\mathrm{t}} = \sum_{i=1}^{\overline{N}_{\mathrm{t}}^{p}} p_{\mathrm{t}}^{i}$$
 and  $n_{\mathrm{t}} = \sum_{j=1}^{\overline{N}_{\mathrm{t}}^{n}} n_{\mathrm{t}}^{j}$ .

The rate of change of the trapped charge density for each level is given by

$$\begin{cases} \frac{\partial p_{t}^{i}}{\partial t} = r_{hc}^{p,i} - r_{he}^{p,i} - r_{ec}^{p,i} + r_{ee}^{p,i} & \text{in } \Omega, \\ p_{t}^{i}(\mathbf{x},0) = p_{t,0}^{i} & \text{in } \Omega, \end{cases} \qquad i = 1, \dots, \overline{N}_{t}^{p}, \qquad (2.2d)$$

and

$$\begin{cases} \frac{\partial n_{\rm t}^{j}}{\partial t} = r_{\rm ec}^{n,j} - r_{\rm ee}^{n,j} - r_{\rm hc}^{n,j} + r_{\rm he}^{n,j} & \text{in } \Omega, \\ n_{\rm t}^{j}(\mathbf{x},0) = n_{{\rm t},0}^{j} & \text{in } \Omega, \end{cases} \qquad j = 1, \dots, \overline{N}_{\rm t}^{n}, \qquad (2.2e)$$

where on the right hand side the four contributions correspond to the capture (subscript <sub>c</sub>) and release (<sub>r</sub>) rates of holes (<sub>h</sub>) and electrons (<sub>e</sub>), respectively, that also appear in the continuity equations of (2.2b) and (2.2c). A detailed description of the trapping/release phenomenon is presented in section 2.2.5 together with the considered model.

The dependence of the electric potential and field on the space charge density in the device is described by the Poisson problem

$$\begin{cases} \nabla \cdot \mathbf{D} = q \left( p - n - \sum_{i=1}^{\overline{N}_{t}^{p}} \left( z_{t}^{p,i} N_{t}^{p,i} - p_{t}^{i} \right) + \sum_{j=1}^{\overline{N}_{t}^{n}} \left( z_{t}^{n,j} N_{t}^{n,j} - n_{t}^{j} \right) \right) & \text{ in } \Omega, \\ \mathbf{D} = -\varepsilon \nabla \varphi & \text{ in } \Omega, \\ \varphi = 0 & \text{ on } \Gamma_{\text{ITO}}, \\ \mathbf{D} \cdot \mathbf{v} = 0 & \text{ on } \Gamma_{\text{int}}. \end{cases}$$
(2.2f)

where **D** is the electric displacement vector,  $\varepsilon$  is the dielectric permittivity and q is the elementary charge. In the expression for the total charge density on the right hand side of the first equation of (2.2f), in addition to the free and trapped charge carrier densities, also the quantities  $N_t^{p,i}$  and  $N_t^{n,j}$  are involved, representing the available trap state densities for holes and electrons respectively. Such states are determined by material defects or impurities that might either be ionized or have zero net charge. In the model, this is obtained by introducing the valences  $z_t^{p,i}$  and  $z_t^{n,j}$  that can attain the values 1 and 0 to represent the two cases, respectively. In the computation of the total charge density, an additional term might have been added to include a concentration of doping ions that do not also play the role of trapping site. Since in the case under study the ions that are usually referred to as doping ones (superoxides  $O_2^{-}$ ) are actually trap sites, we drop such term.

As for the boundary conditions, we enforce zero electric field on  $\Gamma_{int}$  and we set the value of the electric potential equal to zero on  $\Gamma_{ITO}$ . At first sight, this latter condition

might be in contrast with the fact that the evolution of the electric potential at the ITO/polymer interface is probed with the transient photovoltage measurement, however it represents just a mathematical artifice. Since the elliptic Poisson problem (2.2f) needs two boundary conditions, one for each side of the domain  $\Omega$ , and we already enforce a condition on  $\Gamma_{int}$ , we simply use the fact that the electric potential reference can be set arbitrarily to any point, and we actually do it on  $\Gamma_{ITO}$ .

The other condition to be enforced, namely  $\mathbf{E} = 0$  on  $\Gamma_{\text{int}}$ , represents instead a more challenging issue. It is well known from the literature that high electric fields develop at the interface between semiconducting polymers and ionic solution, due to charge rearrangement in depleted regions in the polymer and double layers in the electrolyte [39]. Since the dielectric permittivity of P3HT is lower than that of the solution, the electric field in the polymer is expected to attain even higher values than in the double layer. However, this latter should in principle depend on the ionic concentration of the solution (and hence modify the value in the polymer accordingly), but no significant modification of the device operation in terms of photovoltage is observed by changing the solution molarity, see section 2.1.4. Moreover, should an electric field, and hence a depleted region, be present in the region of P3HT close to the solution interface, charge dissociation would be significant also on that side of the device, and this would be inconsistent with the proposed working principle, see section 2.4.1 for a more thorough discussion.

For this reason we assume that a surface charge  $\sigma_{int}$  exists on  $\Gamma_{int}$  which completely balances the charge in the electrolyte double layer and allows a flat band configuration in the polymer.

The final output of the model, i.e. the transient photovoltage profile  $\Delta \varphi_{\text{phot}}(t)$ , is simply defined as the difference of the electric potential at the two sides of the domain  $\Omega$ , namely

$$\Delta \varphi_{\text{phot}}(t) = \varphi|_{\Gamma_{\text{ITO}}}(t) - \varphi|_{\Gamma_{\text{int}}}(t) = -\varphi|_{\Gamma_{\text{int}}}(t).$$
(2.2g)

#### 2.2.1 Exciton generation profile - Beer-Lambert model

As pointed out in section 2.1, non-uniformity of exciton generation has a key role in determining the output of the experiments, and for this reason it is mandatory to adopt an appropriate model which takes into account progressive absorption in the material. For this reason we consider the Beer-Lambert model, see [27] section 3.4, which reads

$$G_{\text{light}} = \alpha_{\text{P3HT}} I_0 \exp(-\alpha_{\text{P3HT}} x)$$
  

$$G_{\text{light}} = \alpha_{\text{P3HT}} I_0 \exp(-\alpha_{\text{P3HT}} (L_{\text{pol}} - x))$$
  
in  $\Omega$ , (2.2h)

in the case of illumination from the ITO electrode (i.e. from the left in the computational domain represented in figure 2.11) and from the solution (right), respectively. In (2.2h) we denote by  $\alpha_{P3HT}$  and  $L_{pol}$  the polymer layer absorption coefficient and thickness, and with  $I_0(t)$  the photon incidence rate per unit area which, accordingly to the stimulation protocol, has a step-like shape defined as

$$I_0(t) = \begin{cases} \overline{I} & \text{for } 0 \le t < t_{\text{on}} \\ 0 & \text{otherwise} \end{cases}$$



Figure 2.13: Temporal profile of the photon incidence rate per unit area.

and reported in figure 2.13.

It should be noted that more complex optical models are available in the specialized literature, e.g. those based on the transfer matrix approach [32,74,82]. Nevertheless in the current work we do not intend to investigate in detail such aspect and we consider the Beer-Lambert model to be accurate enough for our scope, leaving the inclusion of such feature for future activities.

## 2.2.2 Braun-Onsager model for exciton dissociation

In modeling exciton dissociation in homogeneous materials, the most widely used model is that developed by Braun [23] which has been obtained by extending Onsager's theory [121]. According to such model, for the exciton dissociation rate constant it holds

$$k_{\rm diss}(\mathbf{E}) = \frac{3q\overline{\mu}}{4\pi\varepsilon a^3} \exp\left(-\frac{\Delta\varepsilon(a)}{V_{\rm th}} \frac{J_1(2\sqrt{-2b})}{-2b}\right), \quad \Delta\varepsilon(a) = \frac{q}{4\pi\varepsilon a}, \quad b = \frac{q|\mathbf{E}|}{8\pi\varepsilon V_{\rm th}^2},$$

where we denote with  $\overline{\mu}$  the average of the carrier mobilities, with  $V_{\text{th}} = k_{\text{B}}T/q$  the thermal voltage,  $k_{\text{B}}$  and T being the Boltzmann's constant and the absolute temperature, and with  $J_1$  the first order Bessel function of the first kind. The additional parameter a is defined as the mean initial separation distance between the positive and negative charge in the dipole represented by the exciton. Such value is the only free parameter, and usually its the order of magnitude is about the mean molecular dimension ( $a \approx 1$  nm). In our approach we opt to restate the model by using the expansion of the Bessel function already proposed by Braun as

$$k_{\rm diss}(\mathbf{E}) = k_{\rm diss,0} \left( 1 + b + \frac{b^2}{3} + \frac{b^3}{18} + \frac{b^4}{180} \right)$$
 (2.2i)

where we condense all the prefactors in the zero-field dissociation rate  $k_{diss,0}$  which now acts as the only free parameter. The exciton dissociation probability is given by

$$\theta_{\rm diss} = \frac{k_{\rm diss}}{k_{\rm diss} + k_{\rm rec}},$$

and for pristine materials with no applied electric field it attains very low values, close to 1-2%. Since the characteristic exciton life time is of the order of 500 ps the recombination rate constant  $k_{\rm rec}$  is approximatively  $2 \times 10^9 \, {\rm s}^{-1} = (500 \, {\rm ps})^{-1}$  and hence  $k_{\rm diss,0} \approx 2-4 \times 10^7 \, {\rm s}^{-1}$ .

## 2.2.3 Contact injection/recombination model

As stated in section 2.2, in our model the polymer/electrode interface allows both charge carriers to flow across it. The net flux is determined by the balance of two competing phenomena, namely thermionic injection from the electrode to the polymer and recombination of free carriers in the polymer with their image charges in the ITO. Such phenomenon has been widely discussed in the specialized literature, and the reference model is that proposed by Scott and Malliaras in [150], which represents an extension to organic materials of the Schottky contact model.

Considering for example the case of electron injection, the energy barrier  $\phi_{B,0}$  that the charge carriers have to overcome in order to pass from the electrode to the semiconductor is given by the difference between the band edge of the LUMO  $E_{LUMO}$  and the work function of the metal  $\phi_{el}$ 

$$\phi_{\rm B,0} = E_{\rm LUMO} - \phi_{\rm el}.$$

The potential energy of such charges near the contact is given by

$$\mathcal{E}(x) = \phi_{\mathrm{B},0} - \frac{q^2}{16\pi\varepsilon x} - qEx,$$

where the second contribution is due to the interaction with the image charges on the contact and the third term is due to the local electric field, which is considered to attain positive values when it is directed towards the electrode. A schematic representation of this energetic profile is reported in figure 2.14.

Scott and Malliaras assume that the recombination at the contact is a Langevintype process, and that all the carriers that have a Coulomb interaction energy with the image charge exceeding  $k_{\rm B}T$ , i.e. closer than  $x_{\rm c}$  in figure 2.14, inevitably recombine. When the applied electric field is zero, it holds  $x_{\rm c} = r_{\rm c}/2$ , where the Coulomb radius  $r_{\rm c}$  is

$$r_{\rm c} = \frac{q^2}{4\pi\varepsilon k_{\rm B}T}$$

and the recombination current is given by

$$j_{\rm rec} = q\mu_n n_0 E(x_{\rm c}) = \frac{16\pi\varepsilon (k_{\rm B}T)^2 \mu_n n_0}{q^2}$$

where we actually approximate  $n(x_c)$  with the value of the charge carrier density at the interface  $n_0$ . The injection current instead, according to the thermionic theory, is given by

$$j_{\rm inj} = AT^2 \exp\left(-\frac{\phi_{\rm B,0}}{k_{\rm B}T}\right)$$

where *A* is the Richardson constant of the thermionic model yet to be determined. In the considered zero-field condition, injection and recombination must cancel out, and if we assume  $n_0$  to be given by a Boltzmann factor

$$n_0 = N_0 \exp\left(-\frac{\phi_{\mathrm{B},0}}{k_{\mathrm{B}}T}\right)$$



**Figure 2.14:** Schematic representation of the electron injection energetic barrier and band profile near the contact.

we conclude that the Richardson constant is

$$A = \frac{16\pi\varepsilon k_{\rm B}^2\mu_n N_0}{q^2}$$

When a positive electric field is applied, as in the case reported in figure 2.14, we observe that the injection barrier is actually lowered to the effective value

$$\phi_{\rm B} = \phi_{\rm B0} - q \sqrt{\frac{qE}{4\pi\varepsilon}} = \phi_{\rm B0} - k_{\rm B}T \sqrt{f}$$

where we introduce the adimensional electric field

$$f(E) = \frac{qEr_{\rm c}}{k_{\rm B}T},$$

and hence the injection is increased. In [150] it is also shown that the recombination current depends on the electric field magnitude as

$$j_{\rm rec} = \frac{16\pi\varepsilon (k_{\rm B}T)^2\mu_n n_0}{q^2} S_E$$

where we define

$$S_E(f) = \frac{1}{4} \left( \frac{1}{\psi^2} - f \right)$$
 and  $\psi(f) = f^{-1} + f^{-\frac{1}{2}} - f^{-1} (1 + 2f^{\frac{1}{2}})^{\frac{1}{2}}.$ 

Such model has been successively extended to the case of negative electric fields [10, 97] by assuming that the recombination length attains the value  $x_c = r_c/4$ . With this hypothesis, the recombination current remains equal to its zero-field value, while the barrier is increased by  $q|E|r_c/4$  and the current is consequently reduced.

At this point we can finally state the boundary conditions in a unified formula for both carrier types, taking into account the fact that the effect of the electric field on holes is reversed. We first define

$$f_p = -f(\mathbf{E} \cdot \boldsymbol{\nu}), \qquad f_n = f(\mathbf{E} \cdot \boldsymbol{\nu}) \qquad \text{and} \qquad B_{\{n,p\}} = \frac{A_{\{n,p\}}T^2}{q\{N_0, P_0\}},$$

so that we can write

$$j_{p}^{\text{SM}} = \begin{cases} B_{p} \left[ pS_{E}(f_{p}) - P_{0} \exp\left(-\frac{\phi_{\text{B},0}^{p}}{k_{\text{B}}T} + \sqrt{f_{p}}\right) \right] & f_{p} \ge 0 \\ B_{p} \left[ p - P_{0} \exp\left(-\frac{\phi_{\text{B},0}^{p}}{k_{\text{B}}T} - \frac{f_{p}}{4}\right) \right] & f_{p} < 0 \end{cases}$$

$$j_{n}^{\text{SM}} = \begin{cases} B_{n} \left[ nS_{E}(f_{n}) - N_{0} \exp\left(-\frac{\phi_{\text{B},0}^{n}}{k_{\text{B}}T} + \sqrt{f_{n}}\right) \right] & f_{n} \ge 0 \\ B_{n} \left[ n - N_{0} \exp\left(-\frac{\phi_{\text{B},0}^{n}}{k_{\text{B}}T} - \frac{f_{n}}{4}\right) \right] & f_{n} < 0. \end{cases}$$
(2.2j)

**Remark.** It is interesting to note that, from a purely mathematical standpoint, each relation in (2.2j) is a Robin boundary condition for p and n of the form

$$j_u^{\rm SM} = \alpha_u u - \beta_u$$

with u = p, n. The fact that  $\alpha_u > 0$  and  $\beta_u > 0$  helps in determining the well-posedness of problems (2.2b) and (2.2c) upon a suitable weak formulation.

## 2.2.4 Extended Gaussian Disorder Model (EGDM)

Conduction in organic semiconductors occurs with mechanisms that strongly differ from the cases of inorganic materials, like silicon or gallium arsenide. While in this latter type of materials the atoms are held together by strong covalent bonds to form a crystal, producing an energetic band structure with electrons delocalized in the whole solid, in organic materials instead molecules interact only by weak van der Waals forces, resulting in a less organized structure. As for the electronic structure, being the superposition of molecular orbitals very limited, the electrons remain localized on the molecules. Since there is a certain level of disorder in molecular organization, energy levels are randomly distributed and can be assumed to have a Gaussian profile, and from this it comes the name Gaussian Disorder Model (GDM). Thus the conduction band is centered on the LUMO level  $E_{LUMO}$ , while the valence band is centered on the HOMO at energy  $E_{HOMO}$ , and the densities of states (DoS) are given by the expressions

$$N_{\text{HOMO}}(E) = \frac{N_0}{\sqrt{2\pi}\sigma_{\text{HOMO}}} \exp\left[-\left(\frac{E - E_{\text{HOMO}}}{\sqrt{2}\sigma_{\text{HOMO}}}\right)^2\right]$$
$$N_{\text{LUMO}}(E) = \frac{P_0}{\sqrt{2\pi}\sigma_{\text{LUMO}}} \exp\left[-\left(\frac{E - E_{\text{LUMO}}}{\sqrt{2}\sigma_{\text{LUMO}}}\right)^2\right]$$

where  $N_0$  and  $P_0$  are the volume density of available transport sites in the conduction and valence band, respectively, and  $\sigma_{\text{HOMO}}$  and  $\sigma_{\text{LUMO}}$  are measures of the spreading of each band, which usually are of the order of 0.1 eV.

Since charge carriers are localized on the molecules, conduction in organic materials occurs by phonon assisted hopping between neighboring states, see figure 2.15.



Figure 2.15: Conduction and valence bands in organic semiconductors. Conduction occurs via phonon assisted hopping between neighboring states.

The hopping rate can be calculated using the theoretical approach first proposed by Miller and Abrahams [114] in which the transition rate from two sites *i* and *j* at energies  $E_i$  and  $E_j$  is given by

$$W_{ij} = \begin{cases} \nu_0 \exp\left[-2\gamma R_{ij} - \frac{E_j - E_i}{k_{\rm B}T}\right] & E_j \ge E_i \\ \nu_0 \exp\left[-2\gamma R_{ij}\right] & E_j < E_i, \end{cases}$$

where  $\gamma$  is the inverse localization length of the wave function of the electron in a localized state, and  $R_{ij}$  is the distance between the sites. The effect of an electric field E is a shift of energy levels that can easily be taken into account with the substitution:

$$E_j - E_i \longrightarrow E_j - E_i - q \mathbf{E} \cdot \mathbf{R}_{ij}.$$

Starting from these rates it is possible to provide an expression for electron and hole mobilities as a function of both electric field and charge densities, using Monte Carlo simulations as it has been done in [124]. Such configuration can be described by writing for each state *i* the master equation

$$\sum_{j \neq i} \left[ W_{ij} p_i (1 - p_j) - W_{ji} p_j (1 - p_i) \right] = 0$$

where  $p_i$  and  $p_j$  are the occupancy probabilities for states *i* and *j*. The equation above means that at steady state the rate of carriers hopping from a site *i* to any other site is equal to the rate of carriers jumping from any other site to the site *i*, i.e. that the hopping current density is constant. Once the master equation is solved for the probabilities  $p_i$ , the mobility of each charge carrier can be computed using

$$\mu_c = \frac{\sum_{i,j} W_{ij} p_i (1 - p_j) R_{ij,x}}{cEV}$$

where  $R_{ij,x}$  is the component of the displacement  $\mathbf{R}_{ij}$  along the direction of  $\mathbf{E}$ , V is the volume of the system considered in the simulation, and the carrier density is computed with  $c = \langle p_i \rangle / a^3$ ,  $a = P_0^{-1/3}(N_0^{-1/3})$  being the lattice constant. By repeating the computations on a large number of configurations, the results can be fitted with appropriate functions in order to obtain the desired expressions for the mobilities, that



**Figure 2.16:** Enhancement factors for a Gaussian DoS with dimensionless widths  $\hat{\sigma} = 3, 4, 5, 6$ .

we report using the formalism proposed in [164]. Each expression can be factorized as follows

$$\mu_p(p, \mathbf{E}) = \mu_{p,0} \, g_{1,p}(p) \, g_{2,p}(\mathbf{E}) \qquad \mu_n(n, \mathbf{E}) = \mu_{n,0} \, g_{1,n}(n) \, g_{2,n}(\mathbf{E}) \tag{2.2k}$$

where  $\mu_{i,0}$  is the mobility of carrier *i* in the limit of a zero carrier density and zero electric field, at a particular temperature *T*, and  $g_1$  and  $g_2$  are the carrier density and field-dependent mobility enhancement factors, respectively. The factor  $g_1$  is defined as a function of the normalized density  $c = p/P_0$  in the case of holes, or  $c = n/N_0$  in the case of electrons, with the expression

$$g_1(c) = \begin{cases} \exp\left[\frac{1}{2}(\hat{\sigma}^2 - \hat{\sigma})(2c)^{\delta}\right] & \text{for } c \le 0.1\\ g_1(0.1) & \text{for } c > 0.1 \end{cases}$$
(2.21)

In (2.2l) the following quantities are used

$$\hat{\sigma} = \frac{\sigma}{k_{\rm B}T}$$
 and  $\delta = 2\frac{\ln(\hat{\sigma}^2 - \hat{\sigma}) - \ln(\ln 4)}{\hat{\sigma}^2}$ ,

and a normalized cutoff carrier concentration  $c^* = 0.1$  is introduced, above which  $g_1$  is taken to be constant, see figure 2.16a, because close to that value the fitting function starts to deviate strongly from the numerical solution of the master equation obtained in [124]. Actually, such value refers to very high carrier concentrations (1 molecule with an excess charge every 10 molecules) which can be hardly achieved in our device and this should not result in an issue for the present study.

The electric field enhancement factor  $g_2$  is defined instead by the formula

$$g_{2}(\mathbf{E}) = \begin{cases} \exp\left\{0.44(\hat{\sigma}^{3/2} - 2.2)\left[\sqrt{1 + 0.8\left(\frac{|\mathbf{E}|}{E^{*}}\right)^{2}} - 1\right]\right\} & \text{for } |\mathbf{E}| \le 2E^{*} \\ g_{2}(2E^{*}) & \text{for } |\mathbf{E}| > 2E^{*} \end{cases}$$
(2.2m)

where

$$E^* = \frac{\sigma}{qa}$$
 and  $a = N_0^{-1/3}$  or  $a = P_0^{-1/3}$ 



**Figure 2.17:** Diffusion constant enhancement factor for a Gaussian DoS with dimensionless widths  $\hat{\sigma} = 3, 4, 5, 6$ .

Again a normalized cutoff field value equal to  $2E^*$  is introduced, above which  $g_2$  is considered to be constant, see figure 2.16b. However since no voltage is applied to the device under study, the internal electric field is the result of charge rearrangement and for this reason it is very unlikely that very high values are reached.

The diffusion coefficients for the two types of carrier  $D_p$  and  $D_n$  are determined with generalized Einstein relations which read

$$D_{p}(p, \mathbf{E}) = V_{\text{th}} \,\mu_{p}(p, \mathbf{E}) \,g_{3,p}(p) \qquad D_{n}(n, \mathbf{E}) = V_{\text{th}} \,\mu_{n}(n, \mathbf{E}) \,g_{3,n}(n), \qquad (2.2n)$$

where  $g_3$  is a dimensionless enhancement factor which depends on the normalized carrier density according to the following complex expression

The quantity  $E_{F_c}$  is the Fermi energy, which is a function of the density *c* but no explicit formula is available for it, so the computation of  $g_3$  requires a suitable iterative approach.

From figure 2.17 we can observe that  $g_3$  is an increasing function of the density c and such behavior follows from the particular shape of the density of states which characterizes organic materials. In fact, unlike the case of inorganic semiconductors, the number of available states is limited by the Gaussian DoS, which vanishes for high energy values. For this reason at high carrier densities, the energy levels of a semiconductor might be filled up to a considerable level, and it might be more energetically favorable for the charge carriers to migrate to the neighboring molecules than to occupy a higher energy local state. As a consequence the diffusion flux, and hence the diffusivity coefficient, increases with c. Once again, a cutoff of the parameter is set at c = 0.5, which is not going to affect the simulations since such carrier density level (1 carrier for each 2 molecules) is hardly achievable.

## 2.2.5 Trapping/release model

The transient response of organic devices is known to be strongly influenced by the dynamics of trap states which are determined by the presence of ions and defects in the material [112]. Such states might be either trapping sites for electrons or holes and are characterized by energy levels between the valence and conduction band edges (HOMO and LUMO). Usually, trap states are considered to be distributed according to Gaussian or exponential continuous DoS but also simpler discrete levels (i.e. the DoS is a Dirac delta distribution) are widely used, see figure 2.18. In our modeling approach we follow the strategy used in [21] and [106], opting to consider a set of multiple trap levels so that it is possible to generalize continuous DoS by approximating them by a sequence of discrete levels, see the bottom-right panel of 2.18. In particular, we divide the trap DoS in  $\overline{N}_t$  discrete trap levels  $E_t^i$  with width  $\Delta E_t^i$ . The density of available trap states for each level  $N_t^i$  is given by the formula

$$N_{\rm t}^i = \int\limits_{E_{\rm t}^i - \Delta E_{\rm t}^i/2}^{E_{\rm t}^i + \Delta E_{\rm t}^i/2} N_{\rm t}(E) dE \qquad (2.2p)$$

where  $N_t(E)$  is the continuous trap DoS.

In figure 2.19 we report a diagram of transport and discretized trap DoS with the trapping and detrapping phenomena involving a generic electron trap at level  $E_t^{n,j}$  and free electrons and holes, which are characterized by rates that we already introduced in section 2.2, by denoting them with the letter r and suitable subscripts. Such rates are given by the following expressions

$$\begin{split} r_{ec}^{n,j} &= c_{e}^{n,j} n \left( N_{t}^{n,j} - n_{t}^{i} \right) \\ r_{ee}^{n,j} &= e_{e}^{n,j} n_{t}^{j} (N_{0} - n) \approx e_{e}^{n,j} n_{t}^{j} N_{0} \\ r_{hc}^{n,j} &= c_{h}^{n,j} n_{t}^{j} p \\ r_{he}^{n,j} &= e_{h}^{n,j} (P_{0} - p) \left( N_{t}^{n,j} - n_{t}^{j} \right) \approx e_{h}^{n,j} P_{0} \left( N_{t}^{n,j} - n_{t}^{j} \right) \end{split}$$

The first equation states that the capture rate of electrons  $r_{ec}^{n,j}$  is proportional to the number of available free electrons n and the number of unoccupied trap states  $N_t^{n,j} - n_t^j$  with a proportionality factor  $c_e^{n,j}$  characteristic of the type of carrier. Conversely the escape rate  $r_{ee}^{n,j}$  is proportional to the number of occupied trap states  $n_t^j$  and the number of free ones in the transport band  $N_0 - n$ , with proportionality constant  $e_e^{n,j}$ . Since the number of available transport states is usually far larger than the number of free charge carriers, it is possible to simplify the expression by neglecting the contribution of the n term. As reported in figure 2.19, also free holes can interact with n-type traps, and the phenomenon consists in the annihilation of the hole with the trapped electron. Hence, according to the third equation above, the hole capture rate  $r_{hc}^{n,j}$  depends on the number of free carriers p and trapped electrons  $n_t^j$  with prefactor  $c_h^{n,j}$ , which can in principle differ from that of electrons. Finally a free hole can be generated by the trapping of an electron of the valence band, and such event occurs with a rate





**Figure 2.18:** Top-left: Gaussian continuous DoS. Top-right: single trap level. Bottom-left: exponential continuous DoS. Bottom-right: exponential DoS approximated by a multiple trap levels.



**Figure 2.19:** A diagram with transport and discretized trap density of states. Electron and hole capture and release processes are shown for the case of an electron trap. Modified from [106].

proportional to the number of free states in the valence band  $P_0 - p$  and the number of traps available to be filled  $N_t^{n,j} - n_t^j$ . Again a proportionality factor  $e_h^{n,j}$  is introduced and the contribution of p can be dropped for the same considerations reported above.

At thermal equilibrium each process has to balance its inverse

$$r_{\rm ec}^{n,j} = r_{\rm ee}^{n,j}$$
 and  $r_{\rm hc}^{n,j} = r_{\rm he}^{n,j}$ 

and it must hold

$$n = N_0 \exp\left(-\frac{E_{\text{LUMO}} - E_{\text{F}}}{k_{\text{B}}T}\right)$$
 and  $p = P_0 \exp\left(-\frac{E_{\text{F}} - E_{\text{HOMO}}}{k_{\text{B}}T}\right)$ 

Moreover, the trapped charge density  $n_t^j$  can be obtained by computing the integral over all energy states of the trap DoS multiplied with the Fermi-Dirac statistics, and using the fact that in our approximation the former are Dirac distributions, we obtain that

$$n_{\rm t} = \frac{N_{\rm t}^{n,j}}{1 + \exp\left(\frac{E_{\rm t}^{n,j} - E_{\rm F}}{k_{\rm B}T}\right)}.$$

Using these results it is possible to provide the relations between capture and escape rate constants that read

$$e_{\rm e}^{n,j} = c_{\rm e}^{n,j} \exp\left(-\frac{E_{\rm LUMO} - E_{\rm t}^{n,j}}{k_{\rm B}T}\right)$$
 and  $e_{\rm h}^{n,j} = c_{\rm h}^{n,j} \exp\left(-\frac{E_{\rm t}^{n,j} - E_{\rm HOMO}}{k_{\rm B}T}\right)$ .

The role of the Arrhenius type term is very intuitive, since the two kinds of escape processes require that the charges overcome energy barriers equal to  $E_{LUMO} - E_t^{n,j}$  and  $E_t^{n,j} - E_{HOMO}$ , respectively. Hence the energetically deeper a trap is, the harder it is for a carrier to escape.

Assuming such relations to hold also in non-equilibrium conditions and repeating analogous computations also for the case of *p*-type traps, we can write the following definition of the trapping/release rates

$$\begin{aligned} r_{\rm hc}^{p,i} &= c_{\rm h}^{p,i} \left( N_{\rm t}^{p,i} - p_{\rm t}^{i} \right) p & \text{in } \Omega \\ r_{\rm he}^{p,i} &= c_{\rm h}^{p,i} P_{0} \exp \left( -\frac{E_{\rm t}^{p,i} - E_{\rm HOMO}}{k_{\rm B} T} \right) p_{\rm t}^{i} & \text{in } \Omega \\ r_{\rm ec}^{p,i} &= c_{\rm e}^{p,i} p_{\rm t}^{i} n & \text{in } \Omega \end{aligned}$$

$$i = 1, \dots, \overline{N}_{\rm t}^{p}, \quad (2.2q)$$

$$r_{\rm ee}^{p,i} = c_{\rm e}^{p,i} N_0 \exp\left(-\frac{E_{\rm LUMO} - E_{\rm t}^{p,i}}{k_{\rm B}T}\right) \left(N_{\rm t}^{p,i} - p_{\rm t}^i\right) \qquad \text{in } \Omega$$

and

$$\begin{aligned} r_{\rm ec}^{n,j} &= c_{\rm e}^{n,j} \left( N_{\rm t}^{n,j} - n_{\rm t}^{i} \right) n & \text{in } \Omega \\ r_{\rm ee}^{n,j} &= c_{\rm e}^{n,j} N_{\rm 0} \exp\left(-\frac{E_{\rm LUMO} - E_{\rm t}^{n,j}}{k_{\rm B}T}\right) n_{\rm t}^{j} & \text{in } \Omega \\ r_{\rm hc}^{n,j} &= c_{\rm h}^{n,j} n_{\rm t}^{j} p & \text{in } \Omega \end{aligned}$$

$$r_{\rm he}^{n,j} = c_{\rm h}^{n,j} P_0 \exp\left(-\frac{E_{\rm t}^{n,j} - E_{\rm HOMO}}{k_{\rm B}T}\right) \left(N_{\rm t}^{n,j} - n_{\rm t}^j\right) \qquad {
m in} \ \Omega$$

## 2.3 Numerical discretization and implementation of the model

In this section we describe the numerical techniques used to solve the mathematical model (2.2) previously described. Such model, in spite of its 1D setting, represents a very challenging problem from the computational point of view, since it is characterized at the same time by several features that deserve particular attention:

- the system equations are strongly coupled by many nonlinear terms and nonlinearities also appear in several parameter models;
- 2. the magnitude of the considered quantities greatly differ within each other;
- 3. also the characteristic time scales of the dynamics of the involved quantities greatly differ and for this reason the system is strongly stiff;
- as it usually occurs in the field of semiconductors, carrier densities and electric potential are expected to show exponential boundary layers, and in order to face this issue appropriate locally refined meshes and numerical discretizations have to be used;
- 5. not all the equations involve time derivatives and for this reason the system that has to be solved is a Differential Algebraic Equation (DAE) system, for which traditional time advancing methods for ODE systems are not appropriate, and other suitable methods have to be employed.

In developing the numerical strategy adopted here, our approach aims to adapt the methods which have been proved to be successful and efficient in previous works on the simulation of semiconductor devices, see e.g. [45,46,131], with particular emphasis on achieving accurate treatment of time transients by applying advanced time-step control techniques. To this end, our chosen approach is based on Rothe's method (also known as method of horizontal lines) which consists of three main steps:

- the time dependent problem is transformed into a sequence of stationary differential problems by approximating the time derivatives by a suitable difference formula;
- the resulting nonlinear problems are linearized by an appropriate functional iteration scheme;

• the obtained linear differential problems are numerically solved using a Galerkin-Finite Element Method (G-FEM) for the spatial discretization.

Before proceeding with the description of the above reported steps, in order to ease the notation, the nonlinear system (2.2) can be written in compact form as

$$\mathbf{F}(t, \mathbf{u}, \dot{\mathbf{u}}) = \mathbf{0} \quad \text{with} \quad \mathbf{F}(t, \mathbf{u}, \dot{\mathbf{u}}) = \begin{bmatrix} f_X(t, X, \dot{X}) \\ f_p(X, p, p_t^i, n_t^j, \varphi, \dot{p}) \\ f_{p_t^i}(p, p_t^i, n, \dot{p}_t^i) \\ f_n(X, p, p_t^i, n_t^j, \varphi, \dot{n}) \\ f_{n_t^j}(p, n, n_t^j, \dot{n}_t^j) \\ f_{\varphi}(p, p_t^i, n, n_t^j, \varphi) \end{bmatrix}, \quad (2.3)$$

where we define the vector of the degrees of freedom **u** as  $\mathbf{u} := [X, p, p_t^i, n, n_t^j, \varphi]^T$ . In the definition of **F** in (2.3) we also evidence the strong coupling between the equations we referred to at point 1 of the listing at the beginning of the section.

## 2.3.1 Time discretization

In order to transform the time dependent problem (2.3) into a sequence of stationary problems, we replace the partial time derivative with suitable finite difference approximations, specifically, the Backward Differencing Formulas (BDF) of order  $m \le 5$ , see e.g. [7]. If we let  $t_{init} = t_0 < \ldots < t_{K-1} < t_K = t_{end}$  be a strictly increasing, not necessarily uniformly spaced, finite sequence of time levels, and assume that the quantities  $X, p, p_t^i, n, n_t^j$  and  $\varphi$  are known functions for every  $t_K, k = 0, \ldots, K - 1$ , solving (2.3) is equivalent to solve K stationary problems

$$\mathbf{F}(t_k, \mathbf{u}_k) = \mathbf{0}$$
 for  $k = 0, \dots, K - 1$ , (2.4)

where  $\tilde{F}$  is obtained from F by inserting the expression for  $\dot{u}$  provided by the BDF formula.

As we already anticipated, system (2.3) is very stiff since the time scales of dynamics of the unknowns are expected to span a wide range of values. In fact, excitons and free carrier dynamics are usually of the order of few tens of nanoseconds and some microseconds, respectively, while the photovoltage experimental signals, which are most probably determined by trap state dynamics, have characteristic time scales of the order of milliseconds. For this reason, in our implementation, the choice of the next time level  $t_k$  and of the order m of the BDF formula is performed adaptively to minimize the time discretization error while minimizing the total number of time steps, via the solver IDA, a part of the software library Sundials [78, 79], which is specifically designed to solve DAE systems.

## 2.3.2 Linearization

The sequence of problems (2.4) includes several nonlinearities and for this reason the use of linearization techniques is in order. The functional iteration technique used in the present study for the linearization and successive solution of problems (2.4) is the Newton-Raphson method, see [132] chapter 7 and reference cited therein. One step

of this scheme consists in updating the current approximation of the solution  $\mathbf{u}^{(k)}$  by adding the increment  $\Delta \mathbf{u}$  which is obtained by solving

$$\widetilde{\mathbf{J}}(\mathbf{u})\Delta\mathbf{u} = -\widetilde{\mathbf{F}}(\mathbf{u})$$

where  $\widetilde{J}(\mathbf{u})$  is the Jacobian matrix

$$\widetilde{\mathbf{J}}(\mathbf{u}) = \begin{bmatrix} \partial_X(f_X) & 0 & 0 & 0 & 0 & 0 \\ \partial_X(\widetilde{f_p}) & \partial_p(\widetilde{f_p}) & \partial_{p_t}(\widetilde{f_p}) & 0 & \partial_{n_t}(\widetilde{f_p}) & \partial_{\varphi}(\widetilde{f_p}) \\ 0 & \partial_p(\widetilde{f_{p_t}}) & \partial_{p_t}(\widetilde{f_{p_t}}) & \partial_n(\widetilde{f_{p_t}}) & 0 & 0 \\ \partial_X(\widetilde{f_n}) & 0 & \partial_{p_t}(\widetilde{f_n}) & \partial_n(\widetilde{f_n}) & \partial_{n_t}(\widetilde{f_n}) & \partial_{\varphi}(\widetilde{f_n}) \\ 0 & \partial_p(\widetilde{f_{n_t}}) & 0 & \partial_n(\widetilde{f_{n_t}}) & \partial_{n_t}(\widetilde{f_{n_t}}) & 0 \\ 0 & \partial_p(\widetilde{f_{\varphi}}) & \partial_{p_t}(\widetilde{f_{\varphi}}) & \partial_n(\widetilde{f_{\varphi}}) & \partial_{n_t}(\widetilde{f_{\varphi}}) & \partial_{\varphi}(\widetilde{f_{\varphi}}) \end{bmatrix}_{(X,p,p_t,n,n_t,\varphi)}$$

 $\partial_a(f)$  denoting the Frechét derivative of the operator f with respect to the function a, and  $\Delta \mathbf{u} = [\Delta X, \Delta p, \Delta p_t, \Delta n, \Delta n_t, \Delta \varphi]^T$  being the increment of the Newton iteration. In the computation of the Jacobian matrix the dependence on the electric field and the carrier densities of certain model parameters, such as the carrier mobilities and the dissociation rate constant, are neglected so the adopted linearization technique should actually be referred to as a quasi-Newton method [132]. This simplification is due the fact that the expressions of such derivatives are quite complex and we do not expect their inclusion to improve substantially for the convergence of the iterative method. Moreover, should new coefficient models be implemented in the solver code, this latter would require extensive modifications.

Alternative approaches for the solution of (2.4) are represented by fixed point map schemes consisting in suitable modifications of the classic Gummel map algorithm [85]. In such iterative procedures, starting from a provided initial guess of the solution, the system equations are decoupled and successively solved freezing all but one of the unknowns until convergence is reached. These approaches are very popular in the semiconductor simulation context since they are simple to be implemented, they are well known to be more robust as compared to the fully coupled Newton approach with respect to the choice of the initial guess and also less memory consuming. However in this particular study we can rely on the knowledge of the system variables at previous time levels to construct a reasonable initial guess and, as we are dealing with an intrinsically one-dimensional problem, memory occupation is not likely to be a stringent constraint. A final aspect that made us prefer to adopt a Newton method approach over a Gummel map like one, is that the former has the further advantage of facilitating the use of a standard software library like Sundials for time advancing, being fully compatible with the input structures needed by the library algorithm to work.

#### 2.3.3 Spatial discretization

Once the linearization described in the previous section is applied, the resulting linear system of PDEs is numerically approximated with a suitable G-FEM. To avoid instabilities and spurious oscillations that may arise when either the drift or the reaction

terms become dominant, we employ an exponential fitting finite element discretization [8, 59, 100, 171]. This formulation is based on the classic 1D Scharfetter-Gummel difference scheme [26, 145] and ensures that the computed carrier concentration is strictly positive under the condition that the right hand side of the linear system is non negative. Details and analysis of the G-FEM are fully reported in chapter 4.

## 2.3.4 Algebraic system balancing

As discussed in the beginning of the section, the unknowns of the system have different physical nature and their values range on a very wide interval. As a result, the final algebraic problems to be solved may be badly scaled and therefore ill-conditioned, and, in turn, the accuracy and efficiency of the algorithm can be negatively affected. To work around this issue, we introduce two sets of scaling coefficients, denoted by  $\{\sigma_X, \sigma_p, \sigma_{p_t}, \sigma_n, \sigma_{n_t}, \sigma_{\varphi}\}$  and  $\{\overline{X}, \overline{p}, \overline{p_t}, \overline{n}, \overline{n_t}, \overline{\varphi}\}$  and restate problems (2.4) as

$$\begin{cases} \frac{1}{\sigma_X} \widetilde{f}_X(t, \bar{X}\hat{X}) = 0\\ \frac{1}{\sigma_p} \widetilde{f}_p(\bar{X}\hat{X}, \bar{p}\hat{p}, \bar{p}_t\hat{p}_t, \bar{n}_t\hat{n}_t, \bar{\varphi}\hat{\varphi}) = 0\\ \frac{1}{\sigma_{p_t}} \widetilde{f}_{p_t}(\bar{p}\hat{p}, \bar{p}_t\hat{p}_t, \bar{n}\hat{n}) = 0\\ \frac{1}{\sigma_n} \widetilde{f}_n(\bar{X}\hat{X}, \bar{p}_t\hat{p}_t, \bar{n}\hat{n}, \bar{n}_t\hat{n}_t, \bar{\varphi}\hat{\varphi}) = 0\\ \frac{1}{\sigma_{n_t}} \widetilde{f}_{n_t}(\bar{p}\hat{p}, \bar{n}\hat{n}, \bar{n}_t\hat{n}_t) = 0\\ \frac{1}{\sigma_{\varphi}} \widetilde{f}_{\varphi}(\bar{p}\hat{p}, \bar{p}_t\hat{p}_t, \bar{n}\hat{n}, \bar{n}_t\hat{n}_t, \bar{\varphi}\hat{\varphi}) = 0, \end{cases}$$
(2.5)

where the generic quantity  $\hat{a}$  is defined as  $\hat{a} = a/\bar{a}$ . Solving (2.5) equals to solve the original Newton method step where the rows of the Jacobian matrix  $\tilde{J}$  and of the residual  $\tilde{F}$  are multiplied by the factors  $\{1/\sigma_X, 1/\sigma_p, 1/\sigma_{p_t}, 1/\sigma_n, 1/\sigma_{n_t}, 1/\sigma_{\varphi}\}$  while the columns of  $\tilde{J}$  are multiplied by the factors  $\{\bar{X}, \bar{p}, \bar{p}_t, \bar{n}, \bar{n}_t, \bar{\varphi}\}$ . Computational experience reveals that such coefficients have a strong impact on the performance of the algorithm and a proper choice of them is necessary to guarantee the convergence to a solution. As an example, to obtain the results of section 2.4 we set  $\sigma_X = \sigma_{p_t} = \sigma_n = \sigma_{n_t} = \sigma_{\varphi} = 1$ ,  $\sigma_p = 10^4, \overline{X} = 10^{18}, \overline{p} = \overline{p_t} = \overline{n} = \overline{n_t} = 10^{21}$  and  $\overline{\varphi} = V_{\text{th}}$ .

## 2.3.5 Initial condition

When solving a DAE problem in the form (2.3) we need to provide to the IDA solver a couple of initial condition vectors  $\mathbf{u}_0$  and  $\dot{\mathbf{u}}_0$  that are required to satisfy (2.3) itself at  $t = t_0$ , in order to be consistent. To this purpose, IDA provides a routine that in one of its operation modes computes  $\mathbf{u}_0$  given the vector  $\dot{\mathbf{u}}_0$  [79]. Since if a stimulus like that of figure 2.13 is applied at  $t_0 < 0$  the problem under study is at steady state, we can set  $\dot{\mathbf{u}}_0 = 0$  and get the initial value for  $\mathbf{u}$ .

#### 2.3.6 Well-posedness of the problem

The well-posedness analysis of problem (2.2) has not been addressed in the present thesis work, however some reasonable considerations can be drawn.

The model proposed here is very similar to that presented in [46] where an existence result for the solution has been obtained in both stationary and time dependent case. Two main differences in the model proposed here arise that prevent an immediate extension of such results. As first, the description of trap dynamics is included in the present model by the introduction of the corresponding ODEs (2.2d) and (2.2e). However in the stationary case such ODEs are just algebraic constraints, and this issue can be faced by applying the same technique used in [46], consisting in the elimination of the corresponding variables from the model. An additional difference consists in the fact that exciton diffusion is considered and equation (2.2a) is a PDE so that the elimination approach of the variable X is no longer applicable. Nevertheless, such equation is linear in X and weakly coupled to the remainder of the model, so it is expected that an extension of the existence result obtained in [46] should be achievable without particular complications. This aspect has been scheduled for future activity.

As it is well known in the specialized literature, developing an existence theorem for the time dependent case represents a much more challenging task. A possible approach is to follow that adopted in [36], in which discrete time steps are considered by using the backward Euler method, and the obtained system equations are linearized by means of an iterative solution process, in a Gummel map like fashion. By checking the solvability requirements of the resulting semidiscrete systems and identifying an invariant region for the iterative map, it would be in principle possible to prove the existence of a solution also ++in the time dependent case.

## 2.3.7 Implementation details

Model (2.4) and all the techniques described in this section have been implemented using GNU Octave [50, 120] (version 4.1.0+), an open source programming language mostly compatible with MATLAB. The Octave Forge packages msh [44] (version 1.0.10) and bim [43] (version 1.1.4) are employed for mesh handling and finite element matrix assembly, respectively.

#### 2.4 Numerical results

In this section we present the results of the numerical simulations performed using the model presented in section 2.2. A reference configuration is identified and the obtained results are analyzed in order to confirm the working principles we devised from the experimental evidence in section 2.1. Then, the robustness of the model is tried out by reproducing the photovoltage trends observed by changing device thickness and light illumination intensity and direction. Finally a sensitivity analysis of the model is performed and the impact on the photovoltage output by the most important model parameters is discussed.

## 2.4.1 Reference configuration and confirmation of the proposed device working principles

Model (2.2) has a large number of parameters and for most of them it is difficult to provide very reliable values either because they refer to properties which are hard to access, and consequently to measure, or they are little debated in the literature. Hence, before analyzing in detail the model response in order to corroborate the working principle picture that we propose and to verify the trends we observe with the experimental results presented in section 2.1, it is necessary to accurately determine the values to be assigned to the parameters. This task is necessarily accomplished by performing a sequence of trial simulations in which the parameter values are successively modified, in order to detect increasingly appropriate sets of values. In this regard, very valuable information about standard values and admissible ranges of variation for the parameters can be obtained from previously available literature in the field of organic semiconductor modeling.

In the following we consider a reference device configuration with a polymer layer approximatively 150 nm thick immersed in a 200 mM NaCl solution.

In table 2.1 we report the values we consider for the model parameters, with the corresponding source, which could either be an appositely performed measurement or a literature reference. The values attained by some of the listed quantities are quite ascertained by now. As an example, P3HT material properties as the relative permittivity or the peak absorption coefficient, are usually considered to be 3-4 and  $10^5$  cm<sup>-1</sup>, respectively, but also exciton dynamics are quite known from the literature, so we consider values for the model parameters  $D_X$  and  $k_{rec}$  which are consistent with characteristic life time and diffusion length equal to 500 ps and 5 nm, respectively. As for the exciton dissociation rate constant  $k_{diss,0}$ , there exists extensive literature about solar cell modeling in which such parameter is discussed [86], however in most cases exciton dissociation occurs in a blend of two materials which is specifically developed to enhance the phenomenon. In the case under study instead, the device is made by a layer of pristine material, and the dissociation probability should be very low, around 1-2%. We then consider  $k_{diss,0}$  to be a fitting parameter with a reference value given by  $4 \times 10^7$  s<sup>-1</sup>, as obtained in section 2.2.2.

Organic semiconductors are soft materials and many of their properties depend on the processes they undergo. As an example the internal morphology is strongly affected by the type of deposition process used in fabricating the device, and this reflects in a pronounced variability of the values of the charge carrier mobilities. In P3HT, hole mobility is known to depend on the degree of regioregularity, and a huge amount of literature is available about this aspect. Being P3HT a *p*-type semiconductor, performing a measurement of electron mobility is a very challenging task since traditional techniques are suitable to estimate only the mobility of the majority carrier and as a consequence electron transport is usually neglected. One possible approach to do that, is to consider electron mobility to be much smaller than the hole one, in the limit of approaching zero ( $\mu_p \gg \mu_n \approx 0$ ). In this analysis we instead consider that conduction in P3HT occurs mainly by means of hole migration since electrons are trapped. For this reason a model that properly accounts for this phenomenon would result in a low free electron density and hence the choice for the value of the

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Parameter	Value	Unit	Source
E	$3.4\varepsilon_0$	-	[146]
$\alpha_{ m P3HT}$	10 <sup>5</sup>	$\mathrm{cm}^{-1}$	[130, 131]
$k_{ m rec}$	$2\cdot 10^9$	$s^{-1}$	[134, 155]
$k_{ m diss,0}$	$2\cdot 10^7$	$s^{-1}$	
$D_X$	$5\cdot 10^{-4}$	$\mathrm{cm}^2\mathrm{s}^{-1}$	[134, 155]
$\mu_{n,0}$	$10^{-4}$	${ m cm}^2{ m V}^{-1}{ m s}^{-1}$	
$\mu_{p,0}$	$10^{-4}$	${ m cm}^2{ m V}^{-1}{ m s}^{-1}$	[130]
$\phi^p_{ m B}$	0.45	eV	
$\phi_{\mathrm{B}}^{\overline{n}}$	1.5	eV	
$\tilde{N_0}$	$10^{21}$	$\mathrm{cm}^{-3}$	[18]
$P_0$	$10^{21}$	$\mathrm{cm}^{-3}$	[18,95]
$\sigma_{ m HOMO}$	$4k_{\rm B}T\simeq 0.1$	eV	[18]
$\sigma_{ m LUMO}$	$4k_{\rm B}T\simeq 0.1$	eV	[18]
$\overline{N}_{ ext{t}}^{n}$	1	-	
$\overline{N}_{t}^{p}$	1	-	
$N_{t}^{\dot{p}}$	$10^{17}$	$\mathrm{cm}^{-3}$	measured
$N_{t}^{n}$	$10^{20}$	$\mathrm{cm}^{-3}$	
$\Delta E_t^p = E_t^{p'} - E_{HOMO}$	0.2	eV	[95]
$\Delta E_{\rm t}^n = E_{\rm LUMO} - E_{\rm t}^n$	0.5	eV	
$c_{\rm h}^p$	$10^{-20}$	$m^3 s^{-1}$	[95]
$c_{\rm e}^{{\baselinethat}}$	$10^{-17}$	$m^3 s^{-1}$	
$c_{\rm h}^n$	$10^{-20}$	$m^3 s^{-1}$	[95]
$c_{\rm e}^{\rm in}$	$10^{-17}$	$m^{3} s^{-1}$	

**Table 2.1:** Model parameter values that define the reference configuration.

corresponding mobility would have very little impact, so we can motivate the ansatz  $\mu_{n,0} = \mu_{p,0}$  reported in table 2.1.

Organic materials are characterized by a certain variability in the position of the conduction and valence energy levels and this makes it difficult to determine their relative position with respect to the levels of other materials or electrodes they might be contacted with. Typical values for P3HT HOMO and LUMO band edges are -5 eV and -3 eV, while disorder parameters are usually quantified in the order of 0.1 eV. The work function of the ITO contact has been reported to span a range of values depending on the treatments which it is exposed to, varying from -4.5 eV (as shipped) and -5 eV (after O<sub>2</sub> plasma treatment) [24]. In the present case no particular operations are performed on the ITO electrode before the deposition of the polymer, hence considering values in the lowest part of such range (4.5-4.7 eV) might be appropriate. After these considerations, hole ( $\phi_B^p$ ) and electron ( $\phi_B^n$ ) injection barriers heights can be estimated to be around 0.45 eV and 1.5 eV respectively. As it will be shown later in the current section, knowing the value of  $\phi_B^p$  with good reliability is important since it plays a key role in determining the working principle of the device.

As for the trap model, in order to reduce the dimension of the free parameter space, we consider just one discrete level for each kind of trap ( $\overline{N}_t^p = \overline{N}_t^n = 1$ ), so to ease the notation we can drop all the superscripts <sup>*i*</sup> and <sup>*j*</sup> since they are no longer necessary. In the description of section 2.2.5 we already reported that *p*-type trap states are determined by the contamination due to environmental oxygen which determines the negatively charged ions in the polymer layer. These ions are the result of the reac-

tion of oxygen molecules, which have high electron affinity, with the polymer to form complexes of P3HT<sup>+</sup> :  ${}^{3}O_{2}^{-}$  in which the electron is localized on the oxygen molecule, while the polymer remains positively charged [13]. For this reason the trap number density  $N_t^p$  is given by the oxygen doping concentration experimentally measured for the device under study in which has been found to be of the order of  $10^{17}$  cm<sup>-3</sup>. Since P3HT is widely used in organic devices, a good number of modeling approaches are available in the literature, see e.g. [95, 106], and we can get the values for trap energy depths and capture rate coefficients therein. However for *n*-type traps, collecting appropriate model parameter values is a bit more complicated. In fact, if P3HT is used blended with some acceptor material, as in solar cells or light emitting diodes, the average properties of the blend do differ from those of the pristine material, and hence we are not allowed to use the same values used in other modeling approaches, such as in [106]. If instead pristine P3HT alone is used in fabricating the devices, e.g. in organic transistors, usually unipolar transport is considered and no negative carriers are included in the modeling, hence no *n*-type trap information is needed. In this setting, we derived some guesses for the missing parameters starting from experimental evidence, and we later improved the choice of such values by fitting the photovoltage measurements. Being P3HT a *p*-type material, hole transport is quite efficient, while electron contribution to charge currents is negligible. This can be reinterpreted as the electrons being trapped with high yield on the polymer molecules, which is equivalent to consider values of the trapping coefficient  $c_e^{\{n,p\}}$  higher than that attained by  $c_{\rm h}^{\{n,p\}}$ . Since this effect is not related to defects or contamination ions, in principle it can occur on any polymer molecule so the number density of trap states  $N_t^n$  might be quite high, approaching that of the molecules itself. Moreover we also expect that once trapped, electrons do experience a considerable barrier to detrapping, otherwise P3HT would not be a *p*-type material and would allow high densities of negative free charge carriers. In our formalism this is obtained by considering high values of the trapping depth  $\Delta E_t^n$ .

As already stressed above, all the quantities that have been addressed to as unknown or little characterized in the literature have been treated as fitting parameters and the reference values reported in table 2.1 are obtained by performing the sensitivity analysis discussed in section 2.4.4, where we also evaluate their impact on the model response.

## 2.4.1.1 Dark conditions

The primary model response to be analyzed is that obtained in dark condition, before the illumination stimulus is applied, i.e. at t < 0 in figure 2.13. Such solution consists in the initial condition to be provided to the time advancing solver and in figure 2.20 we report the computed electric potential and charge carrier densities. We observe that the potential is flat in most of the polymer and it bends just in a limited region (about 40 nm) close to the interface with the ITO contact, determining an electric field directed from the ITO to the bulk of polymer. This behavior is a typical example of band bending, as occurring at metal-semiconductor contacts in the case of both inorganic [115,117] and organic materials [18,98,146]. In the bulk of the polymer the hole density is such that  $p \approx N_t^p$  so that the fixed charge due to the oxygen doping is



**Figure 2.20:** (a) Electric potential and (b) charge carrier density profiles in the polymer layer in dark condition (t < 0).

counterbalanced. In the bending region, instead, both free and trapped hole profiles decrease exponentially towards the electrode, meaning that to reach the equilibrium condition, in which thermionic injection and recombination currents balance each other, some holes diffuse in the electrode and a negatively charged depleted region is generated in the polymer. Electron profiles are not visible in the figure because the attained density values are smaller than the lower bound of the y-scale of the graph, since no light stimulation is applied to the device.

On the contrary, in [146] for an apparently similar ITO/P3HT system, it is clearly reported the occurrence of a band bending of the opposite sign with respect to the obtained one, that is attributed to a transfer of electrons from the tail states of the HOMO of the polymer to the metal, and thus to a positive space charge in the semiconductor interface region. Even though it has the same structure of the device under study, in that work the ITO electrode was cleaned with oxygen plasma and for this reason an interface characterized by a reduced hole injection barrier ( $\phi_B^p = 0-0.1 \text{ eV}$ ) was obtained. Moreover the device was not directly exposed to air so the contamination level, and consequently the oxygen doping, reached much lower levels. For this reason, unlike the case under study, the polymer is almost empty of holes in dark condition and some electrons transfer to the electrode leaving in the polymer a space charge region of positive sign.

Anyway, occurrence of space charge regions is widely documented in doped organic semiconductor devices and it is often indicated as the responsible of the working principle of the devices themselves, see e.g. [48,151]. In the present case, the presence of a depleted region with a negative excess charge in the polymer, is consistent with the working principle proposed in section 2.1. Keeping in mind the behavior of exciton dissociation, modeled by the Onsager-Braun model presented in section 2.2.2, an electric profile which is non zero in the interface area and then vanishes in the bulk region would explain the inhomogeneities of dissociation efficiency we hypothesized. In addition, the dimension of the depleted region (read: the thickness of the area where the field is nonzero) matches the threshold length after which the photovoltage measurement have a switch in their behavior. Finally, the electric field is directed from the electrode to the polymer, and hence it would help holes to migrate towards the bulk



**Figure 2.21:** (a) Transient photovoltage resulting from an on-off step-like illumination protocol and (b) electric potential profile evolution in the polymer during illumination (t = 0 ms, 0.5 ms, 1 ms, 1.5 ms and 50 ms).

of the polymer, generating a charge displacement and hence a photovoltage.

#### 2.4.1.2 Device response upon illumination

The confirmation to such considerations comes from the analysis of the transient response of the model when the light stimulus is applied. We consider an illumination protocol as described in figure 2.13 using a white LED light source with power  $267 \,\mu\text{W}\,\text{mm}^{-2}$ , which amounts to a photon flux  $\bar{I} = 7.15 \times 10^{20} \,\text{photon}\,\text{m}^{-2}\,\text{s}^{-1}$ , with light-on time  $t_{\text{on}} = 20 \,\text{ms}$ .

In figure 2.21a we report the computed transient photovoltage obtained with the reference configuration and we observe a very good agreement with experimental measurements in terms of attained peak voltage, characteristic rise time and light-off relaxation dynamics. Figure 2.21b displays instead some snapshots of the evolution of the electric potential in the polymer, showing the effect of the photogenerated charges in partially screening the built-in voltage.

Such screening effect is achieved with a charge displacement characterized by a net negative charge concentrated in the area close to the ITO electrode and a positive charge distributed in the bulk of the material, see figure 2.22. More in detail, electrons are trapped on the entire length of the polymer, note that the continuous blue line (which corresponds to the free electron density) is always below the dashed one, but a much higher density is obtained in a few nanometers of the interface region where many of the available trap states are occupied. Holes instead are mostly free and, if we exclude the interface area where a steep boundary layer is observed due to recombination at the electrode with the corresponding image charge, the density profile is characterized by a smooth gradient, typical of a diffusive process.

In figure 2.22a a sequence of snapshots of the evolution of the trapped electron density is displayed, and it is possible to observe that the characteristic time scale is the same as that of the photovoltage. Even though free hole relaxation time is usually of the order of the microsecond, their dynamics is characterized by a millisecond time scale (see figure 2.22b). This is due to the fact that the slowest occurring process, i.e.



**Figure 2.22:** Evolution of trapped electron (a) and free hole (b) densities. (c) Zoom of the charge carrier density profiles in the region of the polymer close to the ITO interface, at stationary state during the illumination period ( $t \approx t_{on}$ ).

electron trap dynamics, dominates the characteristic time of the device response and holes can be considered to instantaneously balance any modification in the distribution of trapped electrons. As a consequence, also the transient photovoltage during the light-on period is dominated by the dynamics of trap states and a stationary condition is reached only when the traps reach a dynamic equilibrium condition.

As for the light off transient, the evolution of trapped electron and hole densities are displayed in figure 2.23. The relaxation process of the device is driven by the recombination of charges, and as in our model this latter occurs between trapped and free charges with opposite sign, the most probable recombination pathway is represented by the mutual annihilation of a filled electron trap when a free hole is captured. As soon as light is switched off, charge populations dramatically drop towards the equilibrium levels and so does the photovoltage, meaning that in the regions where both carrier coexist, recombination occurs efficiently and the polymer gets depleted therein. After that, the residual charges are displaced in the polymer with very low overlap, and since electrons cannot move because they are trapped, the recombination mechanism is only supplied by free holes that reach the trapped electrons by diffusion processes. However, given that the system is evolving back to the initial condition, the built-in electric field in the depleted region is gradually restored and



**Figure 2.23:** (a) Evolution of trapped electron density (zoom in the region of the polymer close to the ITO interface) and (b) of free holes (right) during the light-off period (t = 50 ms, 51 ms, 55 ms and 200 ms).

since it is directed against the concentration gradient, it opposes to the diffusion of holes towards the interface region. This results in a low rate for the recombination process which ultimately determines the long lasting tail of the photovoltage. In addition, since we assumed electron traps to be deep, electron release process is very slow, so we are allowed not neglect in our interpretation the alternative recombination pathway involving free electrons and trapped holes.

In the case of incident light from the solution side, the profiles of charge carriers, potential and electric field do not change significantly, and the comments reported above about the operation of the device still apply. The main differences occur instead for exciton dynamics and this aspect will be accurately discussed in section 2.4.2, while reproducing the numerical results obtained by changing device thickness and light direction.

At this point, the model provided us valuable information to support the picture we draw about the working principle of the device. Among the many aspects that concur in determining the device response we highlight that:

- A significant band bending at the interface between the polymer and the electrode is needed to allow both an efficient exciton dissociation and the build up of a charge displacement which determines a photovoltage which partially screens the built-in voltage.
- No particular trap distribution is needed to drive charge displacement. Even if we assume a trap distribution which is constant in space, electrons tend to accumulate close to the ITO interface due to the presence of the built-in electric field.

## 2.4.1.3 Decreasing the hole injection trap $\phi_{\rm B}^p$

While discussing the choice of the reference values for the model parameters of table 2.1, we mentioned that a certain degree of uncertainty exists in the determination of the relative position of the polymer band edges and the electrode work function. For this reason, since we identified the height of the injection barrier for holes to be



**Figure 2.24:** (a) Electric potential and (b) charge carrier density profiles in the polymer layer in dark condition (t < 0) in the case a reduced hole injection barrier  $\phi_{\rm B}^p = 0.35$  eV is considered.



**Figure 2.25:** (a) Transient photovoltage resulting from an on-off step-like illumination protocol and (b) charge carrier density profiles in the interface region of the polymer at stationary state during the illumination period ( $t \approx t_{on} = 50 \text{ ms}$ ).

a fundamental parameter in the determination of the operation of the device, it is important to evaluate the impact of changing such value.

In this respect, we consider the case of a slightly reduced hole injection barrier  $\phi_{\rm B}^p = 0.35 \, {\rm eV}$ . In figure 2.24 we report the dark equilibrium configuration of the electric potential and the charge densities. As expected, the band bending is reduced and the interface region is less depleted with respect to the reference case. In particular the built-in voltage is about 110 mV and, as observed in the reference case, the photogenerated carriers are only able to partially screen it. In fact, the computed transient photovoltage attains a peak value of about 85 mV, see figure 2.25a.

Given this, we can conclude that the maximum photovoltage magnitude  $\Delta \varphi_{\text{light}}$  is always smaller than the built-in voltage  $V_{\text{bi}}$  determined by band bending, according to

$$\Delta \varphi_{\text{light}} < V_{\text{bi}}$$

Hence, being the built-in voltage in this last configuration strictly smaller than the experimental photovoltage peak (> 100 mV), the considered reduced value of 0.35 eV



**Figure 2.26:** Simulated transient photovoltage with light incident from the ITO side and different device thickness values.



**Figure 2.27:** (a) Maximum photovoltage peak and (b) characteristic rise time with light incident from the ITO side and device length ranging from 20 nm to 400 nm.

for the hole injection barrier is too small to be compatible with the measurements. A lower bound for the  $\phi_{\rm B}^p$  can be obtained as the value determining a built-in voltage equal to the maximum magnitude of the experimental photovoltage.

Figure 2.25b shows instead the computed carrier densities at the end of the transient under illumination condition, and consistently lower charge accumulation is observed with respect to the reference case presented above.

# 2.4.2 Changing device thickness and illumination direction - Comparison with experiments

In section 2.1 we described a set of experimental measurements aimed at analyzing the impact of device thickness and illumination direction, which have also been fundamental in drawing conclusions on the actual working principle of the device. Now we want to further strengthen the proposed picture by reproducing the experimentally observed trends of the photovoltage by numerically simulating the device response using the developed mathematical model.

In this respect we consider a configuration with the model parameters set at the values of table 2.1, light intensity equal to  $267 \,\mu W \, mm^{-2}$  and we let the device length

vary in the range from 20 nm to 400 nm. Changing the incidence direction of the light amounts to using either of the two forms of the exciton generation profile of (2.2h).

The computed transient photovoltages are reported in figure 2.26, whose most important information can be condensed in the maximum photovoltage value and the characteristic rise time, obtained from the principal component of the exponential function that better fits the numerical results, which are displayed in figure 2.27 together with the experimental values.

Most importantly the change of trend in the photovoltage peak around 40-50 nm is correctly reproduced, although some discrepancy exists in the absolute values. As discussed in section 2.4.1, such distance is approximatively equal to the thickness of the depleted region, in which the electric field is significantly different from zero and the exciton dissociation is enhanced. In thin devices, dissociation efficiency is high and the device dimension limits the amount of photogenerated charges and hence the photovoltage, whose peak increases steeply with the thickness. After the threshold value of 40 nm, device thickness is no longer a limiting factor, while exciton dissociation efficiency is. The residual increment of the photovoltage peak is the result of the combined action of a still nonzero bulk dissociation and of the additional space that holes have to diffuse to generate more charge displacement. As for the characteristic rise time, an approximately constant value is obtained for all the considered configurations, see figure 2.27b, consistently with the experimental findings, with the only exception of the thinnest devices.

The case of illumination from the solution side represents however the most important test for the model, as the corresponding measurements have been crucial in identifying the working principle of the device. Results are organized again in two figures, as used for the previous configuration, with transient photovoltage profiles in figure 2.28 and photovoltage peak value and rise time in figure 2.29.

The model is able to satisfactorily reproduce the non-monotonic trend of the PV peak, attaining a maximum around the threshold thickness value of 40 nm, and the interpretation provided in section 2.1 is confirmed. Since in the thinnest considered configurations the exciton diffusion length is comparable with the device thickness, the overall effect of exciton generation does not depend on the illumination direction, being the stationary exciton densities quite similar for the two cases, see figure 2.30. For this reason, being the generation of charge carrier determined by the built-in electric field, which is a property of the device only, the photovoltage evolution response necessarily evolves in a similar way.

When thick devices are considered, instead, the number of excitons available in the interface region is clearly reduced in the case of illumination from the solution, see figure 2.30b, and this results in the already discussed slower and weaker response of the device.

## 2.4.3 Changing light intensity - Comparison with experiments

In section 2.1 we presented another series of measurements carried out on a single device with a specified thickness in which we illuminated it from the ITO interface and changed the illumination intensity in the range from  $4.68 \,\mu W \,mm^{-2}$  to  $267.5 \,\mu W \,mm^{-2}$ . The resulting transient photovoltages showed a nonlinear response of the device, typ-



**Figure 2.28:** Simulated transient photovoltage with light incident from the solution side and device length ranging from 20 nm to 400 nm.



**Figure 2.29:** (a) Maximum photovoltage peak and (b) characteristic rise time with light incident from the solution side and device length ranging from 20 nm to 400 nm.



**Figure 2.30:** Exciton density profile at stationary condition ( $t \approx 50 \text{ ms}$ ) for devices with different thickness, with (a) illumination from the ITO and (b) the solution side. Zoom on the ITO interface region.

ical of organic photovoltaic devices and of saturation effects due to multi-particle phenomena, like electron-hole recombination.

As a final step of the verification process of our model, we now aim at reproducing



**Figure 2.31:** Simulated transient photovoltage with light incident from the ITO side and intensity ranging from  $4 \mu W \text{ mm}^{-2}$  to  $267 \mu W \text{ mm}^{-2}$ .



**Figure 2.32:** (a) Maximum photovoltage peak and (b) characteristic rise time with light incident from the ITO side and intensity ranging from  $4 \mu W \text{ mm}^{-2}$  to  $267 \mu W \text{ mm}^{-2}$ .

such trend by simulating the transient photovoltage response of a device with thickness equal to 150 nm. In this process the choice of a hole injection barrier  $\phi_{B,0}^p = 0.4 \text{ eV}$  in place of the value of 0.45 eV previously used in section 2.4.2 was necessary to fit the results, most probably due to the fact that the considered device was slightly different from that with same thickness we used instead in the such set of measurements.

Simulated photovoltages presented in figure 2.31 reproduce in very convincing way the trend of the experimental results of figure 2.7, and photovoltage peak value and rise time follow the  $\propto \log(I + 1)$  and  $\propto (\log(I + 1))^{-1}$  behavior, respectively. Such results are motivated by considering the quadratic nature of the terms that appear in the trap and release model presented in section 2.2.5 acting as recombination pathways, i.e.  $n_t p$  and  $np_t$ . Since the exciton dynamics is linear and is almost uncoupled with respect to the rest of the system (the only indirect coupling being through the dissociation rate that depends on the electric field), we can reasonably assume that doubling the light intensity would result in a doubled generation rate for the charge carriers. This would in principle determine a stationary state with a higher number of charge carriers, but since recombination quadratically increases and the self consistently determined electric field is also enhanced, the equilibrium carrier densities increase as the logarithm of the light intensity. The characteristic rise time, i.e. the time needed by the system to reach such equilibrium configuration, is determined by the superposition of two effects. Since the generation rate scales as I, the transients would instead correspondingly scale as  $I^{-1}$ , but, since the equilibrium density to be reached scales as  $\log(I + 1)$ , the overall behavior of the rise time is proportional to  $I^{-1} \cdot \log(I + 1) \propto (\log(I + 1))^{-1}$ , where we used the fact that  $I \propto (\log(I + 1))^2$ .

#### 2.4.4 Sensitivity analysis

In the previous sections we showed that our model is able to successfully reproduce the experimental measurements and the trends observed by changing in an appropriate manner some device parameters. As a consequence we could conclude that the picture of the device working principles we propose is consistent, since the mathematical model is derived starting from such considerations.

The most difficult and time consuming step of such procedure has been to identify the proper set of parameter values that constitutes the reference configuration reported in table 2.1. Such process consisted in performing a large number of simulations in which each parameter has been singularly addressed to and its value has been varied in a range of admissible values. By doing so, it has been possible to understand the dependence of the system response to the variation of its parameters, in terms of the photovoltage output.

In this section we report the result of such analysis focusing on the parameters that have the strongest impact on the device response.

## 2.4.4.1 Changing the zero-field exciton dissociation rate constant $k_{diss,0}$

Changing the zero-field exciton dissociation rate constant  $k_{\text{diss},0}$  amounts to considering different probabilities for the exciton dissociation process. Reasonable values are in the range from  $10^5 \text{ s}^{-1}$  to  $5 \times 10^9 \text{ s}^{-1}$ , which correspond to a dissociation probability from 0.005 % to 70 %.

The results presented in figures 2.33 and 2.34 show that the photovoltage response time monotonically decreases with  $k_{diss,0}$ , but the peak value has a maximum in the considered range. This behavior is counterintuitive since one would expect that a higher exciton dissociation rate would determine the generation of more charges, that in turn would establish a more intense photovoltage signal. However this apparent inconsistency can be explained in the framework of our model. If  $k_{\text{diss},0}$  is high, the dissociation probability attains non negligible values also where the electric field is low, i.e. in the bulk of the polymer, so charges are generated also far from the interface. This configuration does not allow for a maximization of the charge displacement (and hence of the photovoltage) because electrons are trapped almost as they are generated, due to the fact that P3HT is a *p*-type semiconductor, while holes are in principle free to diffuse both towards the ITO or the solution, following the local concentration gradient. In the case of small (but not too small)  $k_{diss,0}$ , as in the reference case discussed in section 2.4.1, charge displacement is instead enhanced by the fact that, being generated mostly at the ITO interface, free holes can only diffuse in the polymer towards the solution.



**Figure 2.33:** Model sensitivity on the zero-field exciton dissociation rate constant  $k_{diss,0}$ : transient photo-voltage.



**Figure 2.34:** Model sensitivity on the zero-field exciton dissociation rate constant  $k_{diss,0}$ : (a) photovoltage peak value and (b) characteristic rise time.

## 2.4.4.2 Changing the electron trap density $N_{+}^{n}$

In section 2.4.1 we stressed the fact that the nature of the two kinds of carrier trap sites is different, and since electron trapping is not related to defects and/or dopant ions, the corresponding density parameter  $N_t^n$ , can attain a value close to that of the molecule number density, i.e.  $10^{21} \text{ cm}^{-3}$ . However since few information is available for such parameter, in our approach we consider it as a fitting parameter considering values in the range from  $10^{17} \text{ cm}^{-3}$  to  $10^{21} \text{ cm}^{-3}$ .

As reported in figure 2.35, higher values of  $N_t^n$  allow more intense photovoltage peaks, with equilibrium configurations being reached in longer times. By increasing the trap state density we are actually increasing the capacity of the material in locally trapping negative charges, making it possible to have a more important accumulation of electrons at the interface. If  $N_t^n$  is low, instead, trap states close to the ITO interface are filled more quickly, and it happens that both detrapping occurs with higher rate and electron trapping spreads more in the bulk. These facts bring to a reduced charge displacement and an enhanced recombination, so the photovoltage magnitude gets dampened. Some saturation occurs for very high values of the trap density, approaching the 1:1 ratio with polymer molecule density.



**Figure 2.35:** Model sensitivity on the electron trap density  $N_t^n$ : (a) photovoltage peak value and (b) characteristic rise time.

## 2.4.4.3 Changing the electron capture rate constants $c_e^p$ and $c_e^n$

As for  $N_t^n$ , the capture coefficients rates  $c_e^p$  and  $c_e^n$  represent free parameters for our model since no information is available in the literature about them, see the empty cells in the corresponding lines of table 2.1.

If we change the capture coefficients rates  $c_e^p$  and  $c_e^n$  in the model, we are actually considering trap states which have different ability in trapping and releasing electrons. Since the material is a *p*-type semiconductor, electrons are likely to be more efficiently trapped than holes, so a lower bound for the capture coefficient rate is that attained by  $c_h^p$  and  $c_h^n$ , i.e.  $10^{-20}$  m<sup>3</sup> s<sup>-1</sup>, which is way more documented in the organic semiconductor modeling literature [95]. Numerical trials are performed for increasing values of the parameters until no appreciable changes in the model response are observed, see figure 2.36.

If we consider electron capture values comparable with those used for electrons, the device response is quite weak. In that case a considerable number of electrons remains



**Figure 2.36:** Model sensitivity on the trap electron capture rate constants  $c_e^p$  and  $c_e^n$ : (a) photovoltage peak value and (b) characteristic rise time.



**Figure 2.37:** Model sensitivity on the hole trap density  $N_t^p$ : transient photovoltage.



**Figure 2.38:** Model sensitivity on the hole trap density  $N_t^p$ : (a) photovoltage peak value and (b) characteristic rise time.

in the free state and, due to the choice of considering the same value for the mobilities of both carriers, they diffuse in a similar way as holes do. As a consequence there is more overlap for the carrier distributions, resulting in an enhanced recombination and hence a limited charge displacement. If we progressively increase the value of  $c_h^p$  and  $c_h^n$ , the number of trapped electrons is increased and configurations with more charge displacement are obtained with faster transients, see 2.36b. Eventually a saturation is observed for values 5 to 6 orders of magnitude higher than the hole capture value, since further increasing the trapping efficiency does not determine changes in the charge distribution because most of the photogenerated electrons are already trapped.

## 2.4.4.4 Changing the hole trap density (oxygen doping concentration) $N_{+}^{p}$

We reported in section 2.4.1 that experimental measurements revealed a doping level  $N_t^p$  of the order of  $10^{17}$  cm<sup>-3</sup>. As for any other measure, some degree of uncertainty exists on the obtained value, so here we perform simulations for values in the range around the reference one to evaluate the effects on the photovoltage output.

Results show that upon changing the doping density, a maximum for the photovoltage peak value is clearly observed, see figure 2.37 and 2.38a. Modulating the value of the doping concentration has the effect of changing the number of holes in the de-
vice in dark condition, and as a consequence the extent of the band bending. With low doping, band bending is reduced and hence the built-in electric field is low. So exciton dissociation efficiency is lower, fewer charges are generated and the photovoltage magnitude is limited. Conversely, increasing the doping leads to more intense band bending and exciton dissociation efficiency. On the other side this effect is counterbalanced by an increased occurrence of recombination events, which are enhanced by the higher number of positive charges available due to the more intense doping level and, as reported in section 2.2.5, this effect scales quadratically, so for very high doping values the photovoltage peak decreases and a maximum is observed.

We instead observe that both the light-on characteristic rise time (figure 2.38b) and, most importantly, the recovery transient (see the photovoltage tails in figure 2.37) become significantly faster. Such result is motivated by the fact that both the above described phenomena due to the change of doping density, namely the higher exciton dissociation and recombination rates, determine a speed up of the response of the device, so no competition between them exists.

#### 2.4.4.5 Changing the charge carrier mobilities $\mu_{p,0}$ and $\mu_{n,0}$

In section 2.4.1 we motivated our choice of considering electron mobility to be equal to the hole one using the argument that since most electrons are trapped, electron transport is negligible irrespectively of the mobility value.

Here we want to quantify the role that the mobility value plays in determining the device response by varying  $\mu_{p,0}$  and  $\mu_{n,0}$  in the range of values that are usually reported in the literature.

Although charge displacement is determined by a built-in electric field, no ohmic currents flow in the device and dynamics are determined mainly by trap evolution, with mobile charges that just adjust their configuration to balance that of trapped ones. Consistently, the photovoltage output is scarcely affected by the value attained by the mobility parameter, and the peak value and characteristic rise time do not vary significantly over the range of the considered values.

To gain further confidence on the validity of our assumption  $\mu_{n,0} = \mu_{p,0}$ , we per-



**Figure 2.39:** Model sensitivity on the hole and electron zero-field/zero-density mobilities  $\mu_{p,0} = \mu_{n,0}$ : (a) photovoltage peak value and (b) characteristic rise time.



**Figure 2.40:** Model sensitivity on the electron zero-field/zero-density mobility  $\mu_{n,0}$  with hole mobility  $\mu_{p,0}$  fixed at the reference value  $10^{-4}$  cm<sup>2</sup> V<sup>-1</sup> s<sup>-1</sup>: (a) photovoltage peak value and (b) characteristic rise time.

formed a similar analysis by changing  $\mu_{n,0}$  only and considering  $\mu_{p,0}$  to be fixed at the reference value  $10^{-4}$  cm<sup>2</sup> V<sup>-1</sup> s<sup>-1</sup>. Numerical results confirm our hypothesis since both the photovoltage peak value and the rise time do not vary appreciably as a function of  $\mu_{n,0}$ .

# 2.4.4.6 Changing the trap depths $\Delta E_t^p$ and $\Delta E_t^n$

Finally we investigate the sensitivity of the model response on the depth of both types of traps we consider, namely  $\Delta E_t^p$  and  $\Delta E_t^n$ . As reported in section 2.4.1, hole trap states are usually believed to be quite shallow, so we consider the values 0.1 eV, 0.2 eV and 0.3 eV, 0.2 eV being the reference value. In our understanding, electron traps are instead deeper states, since they are responsible of the *p*-type nature of the polymer and must retain electrons efficiently. The reference value selected in section 2.4.1 is 0.5 eV, so in this analysis we let  $\Delta E_t^n$  range from 0.3 eV to 1 eV.

Results in figure 2.41 show that the outputs of the cases of  $\Delta E_t^p = 0.1 \text{ eV}$  and 0.2 eV do not differ significantly, and very little differences exist also with the case of 0.3 eV. Hence, in the range of its reasonable values,  $\Delta E_t^p$  has limited impact on the device operation, and this can be motivated by the fact that in any of these cases the number of trapped positive charges is significantly smaller than the free ones, at least by a factor of 10.

As expected,  $\Delta E_t^n$  has a much more important effect on the photovoltage response. With low trap depths, electrons are more likely to escape from the traps, resulting in a limited occupancy of such states. As extensively discussed in the present section, charge displacement, and hence photovoltage, is strictly related to the number of electrons trapped in the ITO interface area, so less intense signals are obtained. Increasing the trap depth allows for higher photovoltage peaks but saturation occurs at 0.6 eV. In that configuration, most of the photogenerated electrons are efficiently trapped and further deepening the traps cannot determine an increase of charge displacement.



**Figure 2.41:** Model sensitivity on the trap depths  $\Delta E_t^p$  and  $\Delta E_t^n$ : transient photovoltage as a function of  $\Delta E_t^n$ .

#### 2.5 Lumped model for the photovoltage measurements

After having thoroughly validated the performance of model (2.2), in this section we address the reduction of the full PDE formulation into a simpler approach based on lumped parameters. The obtained equivalent circuit model will be later used in section 2.6 as the fundamental element for the modeling of another kind of experimental measurement presented therein, in which the evolution upon illumination of the surface voltage in the solution close to the polymer layer is probed with an appropriate electrode.

In analyzing the measured photovoltage outputs in section 2.1 we found that good fits for the light-on rise transient are obtained with the exponential function (2.1). In view of a description of the photovoltage measurements using a lumped parameter circuit model, we recall that the characteristic voltage response of RC circuits to a steplike current source is an exponential transient. For this reason a possible circuit to be considered could be that reported in figure 2.42, where with  $V_{\rm ITO}$  we denote the value of the electric potential at the ITO electrode with respect to the bath, which is taken as the reference of the system. The effect of charge separation due to light absorption is represented by the current source  $i_{\rm ph}$  and the lumped parameters  $c_{\rm B}$  and  $g_{\rm B}$  denote the characteristic polymer/ITO interface specific capacitance and conductivity. Notice



Figure 2.42: Circuit model for the photovoltage measurements.

that, since the photovoltage measurement is performed in open-circuit configuration, no current flows to the external circuit, so the  $V_{\text{ITO}}$  node is not connected to the ground.

However we also evidenced that the photovoltage recovery transient occurs with a characteristic time which is significantly different from that of that of the rise time, for this reason the purely linear RC circuit of figure 2.42 would be unfit to reproduce the correct behavior, and some modifications are to be applied.

A very good fit of the photovoltage recovery transient is obtained using functions of the following type

$$V(t) = V_{\rm on}^{\rm end} \left[ 1 - \frac{t - t_{\rm off}}{\tau_{\rm off} + (t - t_{\rm off})} \right]$$
  
$$= V_{\rm on}^{\rm end} \frac{\tau_{\rm off}}{\tau_{\rm off} + (t - t_{\rm off})},$$
  
(2.6)

where  $V_{\text{on}}^{\text{end}}$  denotes the photovoltage value at time  $t = t_{\text{off}}$  when the light stimulation is switched off and  $\tau_{\text{off}}$  is the characteristic recovery time. The asymptotic behavior of (2.6) is O(1/t), and this is another indication that the most appropriate model circuit must have a form other than the linear one reported in figure 2.42.

In order to obtain an appropriate relation for describing the evolution of the potential  $V_{\text{ITO}}$ , we apply a model reduction procedure to (2.2). As a consequence of the choice of the values for the model parameters discussed in the preface of section 2.4.1, numerical results showed that the density of trapped holes is negligible with respect to that of free holes, and the same applies for free electrons with respect to trapped electrons ( $p_t \approx 0$  and  $n \approx 0$ ). As a consequence the behavior of the device is mostly determined by the dynamics of free hole (p) and trapped electron ( $n_t$ ) densities. Moreover, the lumped model derivation can be equivalently performed starting from the equation corresponding to either of these two densities, so we just report the mathematical procedures adopted to reduce the electron trap rate equation.

The rate equation for the trapped electrons (2.2e) with the trapping/release rates as defined in (2.2r) reads

$$\frac{\partial n_{t}}{\partial t} = \underbrace{c_{e}^{n}(N_{t}^{n} - n_{t})n}_{\approx k_{diss}X} - \underbrace{c_{e}^{n}N_{0}\exp\left(-\frac{E_{LUMO} - E_{t}^{n}}{k_{B}T}\right)n_{t}}_{\approx 0} \\ - c_{h}^{n}n_{t}p + \underbrace{c_{h}^{n}P_{0}\exp\left(-\frac{E_{t}^{n} - E_{HOMO}}{k_{B}T}\right)(N_{t}^{n} - n_{t})}_{\approx 0}.$$
(2.7)

The second and the fourth term on the right hand side of (2.7) are negligible since the trap energy level  $E_t^n$  is deep in the band gap of P3HT. In fact  $E_{LUMO} - E_t^n = 0.5 \text{ eV}$  and  $E_t^n - E_{HOMO} = 1.5 \text{ eV}$ , so the corresponding exponential terms equal  $4.45 \cdot 10^{-9}$  and  $8.75 \cdot 10^{-27}$ , respectively. Moreover, since in P3HT most of free electrons generated from exciton dissociation are rapidly trapped, we can assume a direct transition from two states, thus equating the first term on the right hand side of (2.7) to the dissociation rate  $k_{\text{diss}}X$ . Hence (2.7) can be approximated with

$$\frac{\partial n_{\rm t}}{\partial t} = k_{\rm diss} X - c_{\rm h}^n n_{\rm t} p.$$
(2.8)

Integrating (2.8) over the domain  $\Omega$  and multiplying both sides by the ratio  $q/|\Omega|$  we obtain

$$q\frac{\partial\bar{n}_{\rm t}}{\partial t} = \frac{q}{|\Omega|} \int_{\Omega} k_{\rm diss} X \, dx - \frac{q}{|\Omega|} \int_{\Omega} c_{\rm h}^n n_{\rm t} p \, dx, \tag{2.9}$$

upon introducing the integral averages

$$\bar{n}_{t} = \frac{1}{|\Omega|} \int_{\Omega} n_{t} dx$$
 and  $\bar{p} = \frac{1}{|\Omega|} \int_{\Omega} p dx$ 

Then if we further assume that

$$\overline{n_{\rm t}p} = \frac{1}{|\Omega|} \int_{\Omega} n_{\rm t}p \, dx \approx \frac{1}{|\Omega|} \int_{\Omega} n_{\rm t} \, dx \cdot \frac{1}{|\Omega|} \int_{\Omega} p \, dx = \bar{n}\bar{p}$$

and we use the argument that the device has approximatively zero overall charge, i.e.  $\bar{n}_{t} \approx \bar{p}$ , equations (2.9) turns out to be equivalent to

$$q\frac{\partial\bar{n}_{\rm t}}{\partial t} = q\bar{G}(t) - qc_{\rm h}^n\bar{n}_{\rm t}^2, \qquad (2.10)$$

 $\overline{G}(t)$  being a suitable approximation of the dissociation contribution  $1/|\Omega| \int_{\Omega} k_{\text{diss}} X \, dx$ . An appropriate treatment of such term will be discussed in at the end of the section. If we finally introduce the simple constitutive relation for the photovoltage  $\sigma = c_{\text{B}} V_{\text{ITO}}$ , and we assume that the surface charge density  $\sigma$  is linearly related to the average volumetric excess carrier densities according to

$$\sigma = qk\bar{n}_{t}$$

where k is a suitable proportionality constant (in length units, [k] = m), equation (2.10) can be restated as

$$c_{\rm B}\frac{\partial V_{\rm ITO}}{\partial t} - qk\overline{G}(t) + \frac{c_{\rm h}^n c_{\rm B}^2}{qk}V_{\rm ITO}^2 = 0.$$
(2.11)

In (2.11) we can spot the structure of Kirchhoff's current law for the circuit reported in figure 2.42, where we can interpret the second term as the current density source  $i_{\rm ph}$ . In addition, by defining the conductivity  $g_{\rm B}$  as

$$g_{\rm B}(V_{\rm ITO}) = g_{\rm B}^* |V_{\rm ITO}| = \frac{c_{\rm h}^n c_{\rm B}^2}{qk} |V_{\rm ITO}|,$$
 (2.12)



**Figure 2.43:** Approximation of y = tanh(x) by the exponential function y = 1 - exp(-3x/2).

the last term in (2.11) can be ascribed to a voltage-dependent resistive circuit element and the equation can be formulated in the form

$$i_{\rm ph}(t) + c_{\rm B} \frac{\partial V_{\rm ITO}(t)}{\partial t} + g_B(V_{\rm ITO})V_{\rm ITO}(t) = 0.$$
(2.13)

Once a constitutive relation for  $g_{\rm B}$  has been obtained, a consistency check of the voltage response of the nonlinear model is in order. During the light-on period, the problem has the following form

$$\begin{cases} \dot{V} = f - aV^2 \\ V(0) = 0 \end{cases}$$

which is a particular version of the Riccati's equation whose solution is

$$V(t) = \sqrt{\frac{f}{a}} \tanh\left(\sqrt{fa} t\right).$$

This, in turn, can be well approximated by an exponential function as reported in figure 2.43 and this is consistent with the fitting function (2.1) successfully used in section 2.1.

In the light-off period, instead, the generation term is zero and (2.11) is equivalent to

$$\begin{cases} \dot{V} = -au^2 \\ V(t_{\text{off}}) = \overline{V} \end{cases}$$

whose solution is

$$V(t) = \frac{\overline{V}}{\overline{V}a(t - t_{\text{off}}) + 1}$$

which has an asymptotic behavior as  $\sim t^{-1}$ , similarly to the fitting function (2.6) that has been proven to successfully fit the recovery tail of the photovoltage signal.

With this evidence we are then allowed to modify the model circuit describing the transient photovoltage measurements as reported in figure 2.44 by introducing a nonlinear resistive element representing the device conductivity  $g_{\rm B}(V_{\rm ITO})$ . From (2.13)



**Figure 2.44:** Nonlinear circuit model for the photovoltage measurements. The conductance  $g_B$  is considered to be a linear function of  $V_{\text{ITO}}$ .

the stationary state of  $V_{\rm ITO}$  and the characteristic rise time are given by

$$\overline{V_{\text{ITO}}} = -\sqrt{rac{i_{ ext{ph}}}{g_{ ext{B}}^*}} \quad ext{and} \quad au \simeq rac{c_{ ext{B}}}{g_{ ext{B}}^*|V_{ ext{ITO}}|} = rac{c_{ ext{B}}}{\sqrt{g_{ ext{B}}^*i_{ ext{ph}}}}$$

which allow to recover a dependence of the photovoltage peak and its characteristic rise time on  $\sqrt{i_{\text{ph}}}$  and  $(\sqrt{i_{\text{ph}}})^{-1}$ , respectively, which quite nicely fit the  $\log(I+1)$  and  $(\log(I+1))^{-1}$  behavior observed both experimentally in section 2.1 and from numerical simulations of the continuum model in section 2.4.

# **2.5.0.7** Approximation of $\int k_{diss} X$

An approximation of the exciton distribution at stationary condition  $\bar{X}(x)$  can be obtained using simple considerations from the corresponding continuity equation. This latter reads

$$\underbrace{\frac{\partial X}{\partial t}}_{\approx 0} + \underbrace{\nabla \cdot \mathbf{J}_X}_{\approx 0} = G_{\text{light}}(x) - k_{\text{rec}}X - k_{\text{diss}}(\mathbf{E})X,$$

and at stationary regime the time derivative is zero and also the divergence term can be neglected. Hence we obtain  $\bar{X}(x)$  as

$$\bar{X}(x) = \frac{G_{\text{light}}(x)}{k_{\text{diss}}(\mathbf{E}) + k_{\text{rec}}},$$
(2.14)

where  $G_{\text{light}}(x)$  is defined as (2.2h), according to the light incidence direction. In figure 2.45 we report a schematic representation of such exciton profile in the case of incident light from the ITO side. As evidenced from the experimental results and confirmed by the numerical simulations, the polymer layer (0, L) can be divided into two regions, characterized by different values of the dissociation rate constant, namely  $k_{\text{diss}}^{\text{high}}$  and  $k_{\text{diss}}^{\text{low}}$ . The dividing line has been shown to be considered as coinciding with the end of the depletion region, namely at  $x = L_{\text{depl}} \approx 40$  nm.

Following the Braun-Onsager model described in section 2.2.2, the dissociation parameter can be expressed as

$$k_{\rm diss} = \begin{cases} k_{\rm diss,0} \ \overline{f_{\rm BO}} & x \le L_{\rm depl}, \\ k_{\rm diss,0} & x > L_{\rm depl}, \end{cases}$$
(2.15)

where

$$\overline{f_{\rm BO}} = f_{\rm BO}(\overline{E})$$
 and  $\overline{E} = \frac{\varphi_{\rm bi}}{L_{\rm depl}}$ 



**Figure 2.45:** Approximation of the exciton density profile in stationary condition in case of incident light from the ITO electrode.  $L_{depl}$  denotes the width of the depletion region, dividing the polymer in two regions characterized by different values of the dissociation rate constant, namely  $k_{diss}^{high}$  and  $k_{diss}^{low}$ .

 $f_{\rm BO}$  being the enhancement factor of Braun-Onsager formula (2.2i) and  $\phi_{\rm bi}$  the built-in voltage, determining the mean electric field  $\overline{E}$  in the depleted region.

Hence using (2.14) and (2.15) we can compute the generation term  $k\overline{G}$  in (2.11) as

$$\int_{0}^{L} k_{\text{diss}}(x) X \, dx = \int_{0}^{L_{\text{depl}}} k_{\text{diss}}^{\text{high}} \bar{X}_{\text{depl}} \, dx + \int_{L_{\text{depl}}}^{L} k_{\text{diss}}^{\text{low}} \bar{X} \, dx$$
$$= \int_{0}^{L_{\text{depl}}} k_{\text{diss},0} \overline{f_{\text{BO}}} \frac{G_{\text{light}}(x)}{k_{\text{diss},0} \overline{f_{\text{BO}}} + k_{\text{rec}}} \, dx + \int_{L_{\text{depl}}}^{L} k_{\text{diss},0} \frac{G_{\text{light}}(x)}{k_{\text{diss},0} + k_{\text{rec}}} \, dx$$
$$= \frac{k_{\text{diss},0} \overline{f_{\text{BO}}}}{k_{\text{diss},0} \overline{f_{\text{BO}}} + k_{\text{rec}}} \int_{0}^{L_{\text{depl}}} \alpha \bar{I} e^{-\alpha x} \, dx + \frac{k_{\text{diss},0}}{k_{\text{diss},0} + k_{\text{rec}}} \int_{L_{\text{depl}}}^{L} \alpha \bar{I} e^{-\alpha x} \, dx$$
$$= \frac{k_{\text{diss},0} \overline{f_{\text{BO}}} \bar{f}_{\text{BO}}}{k_{\text{diss},0} \overline{f_{\text{BO}}} + k_{\text{rec}}} \left[ -e^{-\alpha L_{\text{depl}}} + 1 \right] + \frac{k_{\text{diss},0} \bar{I}}{k_{\text{diss},0} + k_{\text{rec}}} \left[ -e^{-\alpha L} + e^{-\alpha L_{\text{depl}}} \right],$$

where  $\overline{I}$  is the photon flux density defined as

$$\bar{I} = \frac{P_{\text{light}}}{E_{\lambda}},$$

 $P_{\text{light}}$  and  $E_{\lambda}$  being the incident light power density and the electron energy of wavelength  $\lambda$ , respectively.

Should  $L < L_{depl}$ , the above expression is modified by considering just the first contribution to give

$$\int_0^L k_{\rm diss}(x) X \, dx = \frac{k_{\rm diss,0} \, \overline{f_{\rm BO}} \, \overline{I}}{k_{\rm diss,0} \, \overline{f_{\rm BO}} + k_{\rm rec}} \left[ -e^{-\alpha L} + 1 \right].$$

Similar computations can be performed in case of light incident from the solution side.

#### 2.5.1 Numerical results

We now test the capability of the above presented model in reproducing the experimental trends described in section 2.1 upon modification of the polymer layer thickness and light intensity.

The characteristic capacitance of the P3HT/ITO interface can be estimated using the value of the depleted region width  $L_{depl}$  obtained in section 2.4, namely 40 nm, with the simple relation

$$c_{\rm B} \approx \frac{\varepsilon_0 \varepsilon_{\rm P3HT}}{L_{\rm depl}} = 6.2 \times 10^{-2} \,\mu {\rm F} \,{\rm cm}^{-2}.$$

With this result, the conductance parameter  $g_{\rm B}^*$  defined in (2.12) attains the value of 0.48 mS V<sup>-1</sup> cm<sup>-2</sup>. These and all the other model parameter values that determine the reference configuration are reported in table 2.2.

We first assess the ability of the model in reproducing the photovoltage trends observed by changing the polymer thickness in case of illumination from the ITO side. The considered values of the thickness range between 20 nm and 400 nm, and the numerically computed transient photovoltages are reported in figure 2.46a. Even though such model is simple, it catches the dependence of the photovoltage peak on the polymer thickness, see figure 2.46b. Consistently with the experimental results, the characteristic rise time is around 1 ms for all the thickness configurations, and a significantly slower recovery process is reproduced. However the decay time scale still appears to be quite faster than that characterizing the measurements. In section 2.4 we identified the reason of the slow recovery in the fact that the diffusion of free holes from the bulk of the polymer back to the ITO interface where electrons are trapped is made difficult by the built-in electric field. As in the current description we neglect these effects, the model consequently cannot behave in a completely accurate way.

If we instead modulate the light intensity between 4 and 267  $\mu$ W mm<sup>-2</sup>, with a fixed polymer layer thickness of 150 nm, we obtain the photovoltage profiles of figure 2.48a.

As expected from the considerations we outlined while deriving the model, the stationary level reached by the photovoltage follows a square root like profile as a function of the illumination intensity, see figure 2.48a. The characteristic rise time follows a trend which is qualitatively in accordance with the experimental evidence,

Parameter	Value	Unit
$c_{\mathrm{b}}$	0.062	$\mu F  cm^{-2}$
$g^*_{ m h}$	0.5	${ m mS}{ m V}^{-1}{ m cm}^{-2}$
$P_{\text{light}}$	267	$\mu W  mm^{-2}$
$k_{\rm diss,0}$	$4 \cdot 10^7$	$s^{-1}$
$L_{ m pol}$	150	nm
$\overline{f_{\rm BO}}$	3	-
$\alpha_{ m P3HT}$	10 <sup>5</sup>	$\mathrm{cm}^{-1}$
λ	520	nm
$L_{depl}$	40	nm

Table 2.2: Reference model parameter values used in the simulations of the photovoltage measurements.



**Figure 2.46:** (a) Computed transient photovoltage using the lumped model, for different values of the polymer thickness (left), and (b) extracted photovoltage peak value as a function of the polymer thickness.



**Figure 2.47:** Computed transient photovoltage using the lumped model, for different values of the illumination intensity.



**Figure 2.48:** (*a*) Photovoltage peak and (*b*) characteristic rise time extracted from the numerical results as a function of the polymer thickness.

however in the cases with low illumination intensity the computed values underestimate the measured value by a factor of about 4. Again, this is most probably due to the fact that transport phenomena are not described by the simplified model and shorter times are needed by the model to reach a stationary condition.

# 2.6 Surface potential measurements

The second set of experimental measures that we consider to analyze the operation of the hybrid polymer/solution device, is a particular probing of the electric potential transient occurring upon photostimulation in the electrolytic solution, close to the polymer interface. Such measurements are performed using a setup which is typically used in electrophysiology, consisting in a glass pipette filled with an electrolytic solution, and an electrode contacted with this latter and connected with an operational amplifier, see figure 2.1. More details about the instrumentation and the technique will be provided in section 3.1.2, while analyzing the membrane voltage modulation response of cells attached on the hybrid device surface. For the current purpose it is important to know just that the operational amplified keeps the potential of the solution inside the pipette at the reference value, that of the bulk of the solution far from the interface, and measures the current flowing into it, due to a difference of electric potential in the bath solution just outside the tip of the pipette. Knowing the value of the pipette resistance, it is then possible to obtain the value of the electric potential in the regions probed by the pipette tip.

In the configuration used for the photovoltage measurements, the ITO layer on the glass substrate was contacted with the operational amplifier in order to apply a dynamic feedback so that an open-circuit condition was achieved and no current flowed in the device. In this case instead, the instrumentation operates in passive way, so the ITO electrode is free to "float" in terms of electric potential and current can flow in the device.

The adopted illumination protocol follows the same step-like profile as that described in section 2.1 and represented in figure 2.13. A cyan LED is used as light source with the possibility of modulating its intensity.

A typical profile of this kind of measurement is reported in figure 2.49. The considered device has a P3HT layer about 100 nm thick, the saline solution has a concentration of 100 mM and the stimulation consists in a 50 ms pulse with power density set to  $20.48 \text{ mW mm}^{-2}$ , incident from the ITO electrode side of the device.

Before the onset of the illumination stimulus, the pipette records zero current, so the electric potential in the solution close to the interface with the polymer is equal to the value attained in the bulk of the liquid, meaning that the potential is homogeneous in the solution. When the LED is switched on, we expect the active layer to behave in the exact way as discussed in section 2.1. Free charges are generated, a displacement is determined with a negative excess close to the ITO interface and a positive charge distributed in the polymer bulk, and a photovoltage is established. As a consequence the potential in the region close to the polymer/solution interface is increased with respect to the initial condition, and such variation of about some tens of millivolts is probed by the pipette. Since current can flow in the solution, after this abrupt transient, the ion distributions reorganize towards a new configuration, charging the interface double layers and restoring the equilibrium in the electrolytic bath between the tip of the pipette and the bulk. When light is switched off, the phenomenon is reversed. The photovoltage generated in the active layer decays, thus determining



**Figure 2.49:** Typical voltage profile probed by the pipette (P3HT layer thickness 100 nm, illumination with cyan LED 20.48 mW mm<sup>-2</sup> for 50 ms, 100 mM NaCl solution).

a sudden decrease of the potential at the tip of the pipette. Then ions in the solution evolve back to the initial configuration and a double layer discharge transient is measured. Interestingly the voltage peak value in the off transient is significantly lower than that observed after the illumination onset, and the characteristic time scale is longer. This might be ascribed to the corresponding difference in the light-on and light-off transients of the photovoltage measurement, which is the driving force of the phenomenon under study, with the on-transient more similar to a step profile while the decay is characterized by a more gentle slope towards the zero level.

As previously done in section 2.1 for the transient photovoltage measurement, in the following we present the results of a series of measurements in which we systematically changed the device configuration, namely varying the solution molarity, the illumination intensity and the polymer layer thickness.

#### 2.6.1 Changing solution molarity

In the first group of surface potential measurements we consider configurations that differ by the molarity of the saline solution, namely, NaCl concentration is set to the values of 1, 10, 100 and 500 mM. In addition a device with ultrapure water is also tested. Polymer thickness is set to be about 100 nm and light stimulation is provided with a 50 ms pulse at  $8.36 \text{ mW mm}^{-2}$  from the ITO electrode side. With this set of measurements we want to assess the impact of the solution ionic conductivity on the evolution of the voltage in the surrounding of the polymer layer upon photoexcitation.

In figure 2.50a we report the obtained voltage transients, and notably we observe that the ion concentration in the solution, and hence the conductance of this latter, plays a very significant role in the device dynamics. The signal with the highest magnitude is registered for the case of ultrapure water (which is to be regarded as a poor conductor due to the intrinsically low ion concentration) and the peak voltage value is inversely proportional to the solution concentration, i.e. its conductance, see figure 2.51a.

If we normalize the measured signals with respect to the light on peak value, we observe that also the characteristic decay time decreases with the solution concentration, see the extracted values in figure 2.51b, but the values change only by a factor of



**Figure 2.50:** Electric potential transient in the solution close to the polymer interface for different values of the solution molarity (left). In the inset we report the potential profiles that are not distinguishable in the main panel due to the magnitude of the larger signals. Normalized values with respect to the light-on peak (right).



**Figure 2.51:** Peak value (left) and characteristic decay time (right) of the transient voltage measurements for different values of the solution molarity.

4 over 4 orders of magnitude of change in solution molarity.

#### 2.6.2 Changing incident light power density

In a second set of measurements we modulate the power density of the photostimulation in a range between approximatively 1 and 20.5 mW mm<sup>-2</sup>, fixing device thickness to 100 nm and NaCl solution molarity to 100 mM.

Interestingly, the measured transients of figure 2.52a show voltage peaks that linearly scale with the light intensity, see the extracted values of figure 2.53a. Conversely, the characteristic time scale of the system response is inversely proportional to the magnitude of the light stimulus, see the normalized profiles of figure 2.52b and the extracted values in figure 2.53b. This behavior can be explained by considering the fact that the measured signal is a response of the system to the perturbation determined by the onset of the photovoltage in the polymer bulk. In section 2.1.3 we observed that with increasing light intensities, polymer response becomes more intense ( $\propto \log(I+1)$ )



**Figure 2.52:** Electric potential transient in the solution close to the polymer interface for different values of the incident light power density (a). Normalized values with respect to the light-on peak (b).



**Figure 2.53:** (a) Peak value and (b) characteristic decay time of the transient voltage measurements for different values of the incident light power density.

and significantly faster ( $\propto 1/\log(I+1)$ ). As a consequence it is most likely that also the voltage response in the surrounding of the polymer layer exhibits a similar behavior. A more thorough discussion of this aspect is conducted in section 2.6.5.

### 2.6.3 Changing polymer layer thickness

In this last group of surface potential measurements we consider, some devices have been fabricated by depositing P3HT layers with different thickness, in the range between 17 and 170 nm, while light power density (incident from the transparent ITO electrode) and solution molarity have been fixed at  $8.36 \text{ mW mm}^{-2}$  and 100 mM, respectively.

When analyzing in section 2.1.2 the variation of the photovoltage response of the device upon modifications of the polymer layer thickness, we observed a increasing trend of the photovoltage peak as a function of the device thickness. The experimental results displayed in figure 2.54a present a similar behavior, with increasing values of the voltage peak value, see figure 2.55a. The decay time instead, roughly increases for thicker devices, see figures 2.54b and 2.55b.



**Figure 2.54:** (a) Electric potential transient in the solution close to the polymer interface for different values of the P3HT layer thickness. (b) Normalized values with respect to the light-on peak.



**Figure 2.55:** (a) Peak value and (b) characteristic decay time of the transient voltage measurements for different values of the P3HT layer thickness.

# 2.6.4 Lumped parameter circuit model

In this section we aim at describing the system response using a compact lumped parameter model as we have previously done in section 2.5 for the photovoltage measurement. This approach is widely exploited in the literature, and in our analysis we follow the approach used in [147, 148], and we appropriately adapt the model circuits to the current configuration.

Ideally blocking electrodes, i.e. electrodes where no reactions occur at, are usually modeled using a capacitive element which represents the capacitance of the double layer in the ionic solution. In fact, upon a change in the electric potential between the electrode and the bulk of the solution, the ionic composition of the double layers is modified, i.e. a capacitive charging occurs. However, interfaces between ionic solutions and metals or semiconductors most likely have a non-ideal behavior, and the occurrence of faradaic reactions is to be taken into account. For this reason the



Figure 2.56: Circuit model for the surface potential measurements.

most widely adopted way to model the electric behavior of such interfaces consists in using a parallel RC circuit, in which a charge transfer resistive element is added to the double layer capacitance.

In the present configuration, two main interfaces are to be considered, namely that between the polymer layer and the ionic solution and that between the ITO electrode and the solution. The corresponding RC loops are constituted by the specific capacitances  $c_{int}$  and  $c_{ITO}$  and conductances  $g_{int}$  and  $g_{ITO}$ , respectively, and in figure 2.56 they are connected at the opposite sides of the block already used to model the photovoltage response of the device, see figure 2.44. Since the bulk solution is a conductor, it is accounted for in the model circuit by adding a corresponding conductance element  $g_{sol}$ . With  $V_{ITO}$  and  $V_{int}$  we denote the values of the electric potential attained at the ITO electrode and in the polymer at the interface with the solution, respectively. Finally with  $V_{sol}$  we indicate the value of the potential in the solution close to interface, which is probed by the pipette in the considered experiment.

Notably, in the configuration of the photovoltage measurement a voltmeter (i.e. an open circuit) is inserted between the solution conductivity  $g_{sol}$  and the ground. In that case, no current can flow in the RC loops corresponding to the polymer/solution and ITO/solution interfaces, so the circuit consistently simplifies to that reported in figure 2.44.

The current densities flowing in each of the loops of the circuit in figure 2.56 are obtained with  $i_{1} = - - V$ 

$$i_{\text{sol}} = g_{\text{sol}} V_{\text{sol}}$$

$$i_{\text{int}} = c_{\text{int}} \frac{d(V_{\text{int}} - V_{\text{sol}})}{dt} + g_{\text{int}} (V_{\text{int}} - V_{\text{sol}})$$

$$i_{\text{B}} = i_{\text{ph}} + c_{\text{B}} \frac{d(V_{\text{ITO}} - V_{\text{int}})}{dt} + g_{\text{B}} (V_{\text{ITO}} - V_{\text{int}})$$

$$i_{\text{ITO}} = -c_{\text{ITO}} \frac{dV_{\text{ITO}}}{dt} - g_{\text{int}} V_{\text{ITO}}$$
(2.16a)

and the following Kirchhoff current laws hold at the nodes  $V_{sol}$ ,  $V_{int}$  and  $V_{ITO}$ , respec-



**Figure 2.57:** (a) Reduction of the model circuit for the surface potential measurements. Interface conductances are neglected and (b) introducing the series capacitance  $c_{int}^*$  the circuit is further reduced.

tively

$$i_{\text{int}} = i_{\text{sol}}, \qquad i_{\text{B}} = i_{\text{int}} \qquad \text{and} \qquad i_{\text{ITO}} = i_{\text{B}}.$$
 (2.16b)

#### 2.6.5 Numerical results

Since measurements are performed with fast characteristic times, it is very likely that reactions possibly occurring at the interfaces do not play an important role in the current configuration, so we neglect them and we assume  $g_{\text{ITO}} = g_{\text{int}} = 0$ , see figure 2.57a. In that configuration the capacitances  $c_{\text{ITO}}$  and  $c_{\text{int}}$  are in series and since they both are to be considered as fitting parameters, it is impossible to distinguish their single contribution in a fitting procedure. For this reason we further reduce the model circuit by introducing the series capacitance  $c_{\text{int}}^*$  as reported in figure 2.57b.

The above described approximations are further supported by the fact that the typical surface potential measurement profile, see figure 2.49, resembles the response of a series RC circuit (as the  $c_{int}^*$ - $g_{sol}$  branch of the reduced circuit in figure 2.57b) to a voltage step.

System (2.16) has been implemented in Matlab, using the ode15i routine which provides a time advancement method which is recommended for stiff problems, as the current one in the possible case of very different characteristic RC times, and implements an adaptive time step strategy. A charge-oriented approach is adopted in implementing the equations in order to obtain a more accurate conservation of the charge [71].

In section 2.5 we provided some estimates of the lumped parameters  $c_b$  and  $g_b^*$  and demonstrated that with such choices, the lumped model is capable of reproducing in satisfying way the main features of the device response in the transient photovoltage experiment. For this reason, we keep such values fixed in the set of simulations described in the current section, and we define a reference configuration characterized by the values reported in table 2.3. Notably, some parameters which refer to characteristic of the active layer, such as  $k_{diss,0}$ ,  $L_{pol}$  and  $\alpha_{P3HT}$ , slightly differ from those of

Parameter	Value	Unit
$g_{ m sol}$	400	mS cm <sup>-2</sup>
$c_{\rm int}$	1	$\mu F  cm^{-2}$
$c_{ m b}$	0.062	$\mu F  cm^{-2}$
$g^*_{ m b}$	0.5	${ m mS}{ m V}^{-1}{ m cm}^{-2}$
$P_{\text{light}}$	20	$ m mWmm^{-2}$
$k_{\rm diss,0}$	$10^{7}$	$s^{-1}$
$L_{\rm pol}$	100	nm
$f_{\rm BO}$	3	-
$\alpha_{ m P3HT}$	$7\cdot 10^4$	$\mathrm{cm}^{-1}$
λ	470	nm
$L_{depl}$	40	nm

 Table 2.3: Reference parameter values used in the simulations of the surface potential measurements.

table 2.2. This modification has been necessary during the fitting procedure, since the measured voltage transient appears to be determined by a photovoltage signal sensibly smaller than those usually registered in the measurements of section 2.1. This is most probably due to the fact that the presently considered active layers are characterized by lower optical performance, so the parameter values are modified accordingly.

The polymer/solution interface capacitance  $c_{int}$  is set to a value of  $1 \,\mu\text{F}\,\text{cm}^{-2}$ , which is typical for this kind of interface and has also been measured experimentally. The resistance of the 100 mM NaCl solution layer has been quantified as 50  $\Omega$ , and the device has a surface of approximatively 1 cm<sup>2</sup>, so the corresponding conductance per unit area is obtained as

$$g_{\rm sol} = \frac{1}{R_{\rm sol}A} = 20 \,{\rm mS}\,{\rm cm}^{-2}.$$

However, the fitting procedure determined that higher values are more appropriate for describing the results, so we used 400 mS cm<sup>-2</sup> as the reference value for  $g_{sol}$ .

In figure 2.58 we report the surface voltage profile resulting from a simulation of the reference configuration. The signal magnitude fits in a good way the values usually obtained in this kind of experiment, and also the light-on and light-off transients are properly reproduced. Most importantly, the model returns a light-on peak which is significantly higher and faster than that in the recovery transient, consistently with the motivation proposed in the previous section.

The next group of simulations are performed by changing some model parameter values in systematic way, in order to assess the ability of the circuit model to reproduce the observed experimental trends.

#### 2.6.5.1 Changing solution concentration

Changing the ionic concentration in the solution has the most important result of modifying the solution conductance. For this reason we consider  $g_{sol}$  to attain the values 4, 40, 400 and 4000 mS cm<sup>-2</sup>, to mimic a change of molarity between 1 mM and 1 M.

The computed transient voltages of figure 2.59a are in good agreement with the experimental traces of figure 2.50. The computed peak voltage scales as the inverse



**Figure 2.58:** Computed transient voltage profile using the lumped model reproducing the surface potential measurement.



**Figure 2.59:** (a) Computed potential transient for different values of the solution molarity and (b) zoom on the smaller signals. Conductance values are reported in  $mS \text{ cm}^{-2}$ .



**Figure 2.60:** (*a*) *Extracted peak value and* (*b*) *characteristic decay time of the computed transient voltages for different values of the solution molarity.* 

of the solution molarity, and this can be motivated by observing the model circuit in figure 2.57b. The characteristic time  $c_{int}^*/g_{sol}$  of the external RC branch ranges from 250 ns to 250 µs, and it is significantly smaller than that of the internal loop  $c_{bulks}/g_{bulk}$ 



**Figure 2.61:** Computed potential transient for different values of the incident light power density reported in  $mW mm^{-2}$ .



**Figure 2.62:** (a) Extracted peak value and (b) characteristic decay time of the computed transient voltages for different values of the incident light power density.

(0.31 ms). For this reason the overall dynamics of the circuit is mainly determined by the inner loop representing the photovoltage generation, while the external one has little impact. Hence the current flowing in the capacitance  $c_{int}^*$  (and the conductance  $g_{sol}$ ) only depends on the time derivative of the voltage  $V_{int}$ , so an increase of the conductance  $g_{sol}$  determines a reduction of the peak value of  $V_{sol}$  ( $i_c = g_{sol}V_{sol}$ ), while the decay time is unaffected, see figure 2.60b. This latter result is not consistent with experimental measurements, however the observed reduction is small over the entire range of variation of  $g_{sol}$ , so it might be due to minor modifications of the system.

#### 2.6.5.2 Changing light power density

In this second set of simulations we modulate the light power density  $P_{\text{light}}$  over the range of values considered in the experiments, i.e. from 1 to 20 mW mm<sup>-2</sup>.

Again, numerical results of figure 2.61 are in excellent agreement with the corresponding measurements, see figure 2.52. The interpretation follows from the considerations just drawn in the previous point. The current flowing in  $c_{int}^*$  only depends on the time derivative of  $V_{int}$ , which follows the photovoltage profile. This latter in turn,

in the light-on transient has a functional form as

$$V_{\rm int}(t) = V_{\rm int}^{\infty} \left(1 - \exp\left(-\frac{t}{\tau}\right)\right)$$

so its derivative amounts to

$$\dot{V}_{\rm int}(t) = rac{1}{ au} V_{
m int}^{\infty} \exp\left(-rac{t}{ au}
ight)$$

Since  $V_{\text{int}}^{\infty}$  and  $\tau$  scale as  $\log(P_{\text{light}} + 1)$  and  $1/\log(P_{\text{light}} + 1)$ , respectively, the maximum value of  $\dot{V}_{\text{int}}$ , and hence of  $i_{\text{c}}$  and  $V_{\text{sol}}$ , scales as

$$\frac{1}{1/\log(P_{\text{light}}+1)} \cdot \log(P_{\text{light}}+1) = \left(\log(P_{\text{light}}+1)\right)^2 \propto P_{\text{light}}$$

as reported in figure 2.62a. The characteristic time scale of  $V_{int}$  is  $\tau$ , so the decay time of  $V_{sol}$  gets the same dependence on the light power density, see figure 2.62b.

#### 2.6.5.3 Changing polymer thickness

To conclude, we change the polymer thickness in the range between 20 and 150 nm, obtaining the results in figure 2.63.

The experimental trend of the the peak value of  $V_{sol}$  as a function of the active layer thickness is properly caught by the model, since it follows that of the photo-voltage measurement, see section 2.5. Instead, the extracted trend of the decay time is incorrect, see figure 2.64b, and it might be ascribed to the overestimation of the photovoltage characteristic rise time for thin devices, which is consistently observed in all the presented modeling approaches.



Figure 2.63: Computed potential transient for different values of the P3HT layer thickness.



**Figure 2.64:** (a) Extracted peak value and (b) characteristic decay time of the computed transient voltages for different values of the P3HT layer thickness.

# CHAPTER 3

# Thermally induced effects

In chapter 1 we identified thermally induced modifications of the cell membrane properties, due to heating resulting from light absorption by the polymer, to be one of the possible effects that are responsible of the working principles of the neuronal photostimulation device of [64, 65]. In the current chapter, we carry out an analysis of the response upon photostimulation of cells grown onto different device architectures, in terms of change of membrane voltage using patch clamp measurements. The selected configurations have been specifically designed to demonstrate that at the considered illumination regimes the observed phenomena are actually determined by a change of the local temperature. The temperature changes occurring in the system are then quantified, and an appropriate model is proposed to provide a detailed description of heat generation and transfer. Finally patch clamp experimental results are rationalized by developing a phenomenological electric circuit model, and other interpretations of the phenomena previously reported in the literature are critically discussed.

# 3.1 Patch clamp measurements

In this section we report the measurement that we performed in order to prove that heat generation due to polymer light absorption has a key role in determining the properties of the cells that are grown onto the substrates described in chapter 1. However, to draw a consistent and comprehensive picture of the setting, a brief introduction to the cell membrane structure and to the characteristics of the experimental technique is in order.



Figure 3.1: Schematic representation of the cell membrane structure.

### 3.1.1 Cell membrane

The cellular membrane is a complex structure that separates the cell cytoplasm from the external environment, and it is responsible for the cell vitality since it carries out a protection task, provides mechanical stability and regulates the flux of substances in and out of the cell.

The main constituents of cell membranes are a particular kind of amphiphilic lipids, called phospholipids, which are composed by a hydrophilic polar head, represented by a phosphate group, and a hydrophobic tail, which is usually a hydrocarbon with a variable number of carbon atoms. Since aqueous solutions are polar fluids, such molecules spontaneously rearrange into bilayer structures so that the tails remain isolated and the polar head are directed towards the water phase, see figure 3.1. As previously mentioned, the cell membrane represents a barrier for the flux of water molecules and dissolved ions and makes it possible to maintain the different compositions of cytoplasm and extracellular solution.

However the cell membrane is not a completely impermeable structure, on the contrary, it is selectively able to regulate what enters and exits the cell, both to provide the materials that are necessary for the cell vitality and to regulate its behavior. Such transport of substances can occur directly through the membrane, as in the case of carbon dioxide ( $CO_2$ ) and oxygen ( $O_2$ ), or be mediated through specialized structures called ion channels, that are essentially proteins adsorbed to the lipid membrane, see figure 3.1. Several types of ion channel exist, each devoted to the transport of a specific substance, and they can be further divided into active or passive ones, depending on whether they require the use of energy to function. Their operation is highly influenced by several parameters, such as the local temperature, the electric voltage across the cell and the ion concentrations on both sides of the channel.

Due to the fact that biological solutions are ionic and that ions are transported through the channels, it is straightforward to interpret the membrane cell behavior in terms of electrical equivalents. Since the membrane shows a divalent nature acting both as an insulator and a (selective) conductor, it can be characterized by particular values of electric capacitance  $C_{\rm m}$  and resistance  $R_{\rm m}$ , and its behavior can be assimilated to that of the RC circuit depicted in figure 3.2. All living cells experience an electric voltage drop  $V_{\rm m}$  between the inside and the outside defined as

$$V_{\rm m} = V_{\rm in} - V_{\rm out},$$

which is referred to as the membrane voltage. This voltage might change in time if the cell is stimulated or the environmental conditions are changed, and the value  $V_r$  that it attains at the equilibrium is called resting voltage and it is usually negative (between -70 and -30 mV). When the membrane voltage becomes higher that the resting one  $(V_m > V_r)$  the membrane is said to be depolarized. Conversely, if the membrane voltage attains more negative values ( $V_m < V_r$ ) the membrane is hyperpolarized. A typical example in which depolarization and hyperpolarization occur is during the propagation of an action potential in neurons. To conclude, the current flowing through the membrane is denoted with  $I_m$ .



Figure 3.2: Electric circuit representation of a cell membrane.

#### 3.1.2 Patch clamp technique

The patch clamp technique is a very popular method used in electrophysiology to measure the activity of almost any kind of biological cell, and in particular to track the changes of the membrane potential and the current flowing through its ion channels [51, 77, 88].

The experimental setup needed for a patch clamp measurement is composed principally by the following instruments, see figure 3.3:

- a glass micropipette,
- a chlorided silver (Ag/Cl) electrode,
- a highly sensitive electronic amplifier.

The glass pipette is a micrometer-sized electrode with an open tip (one can picture it as a very tiny straw), and it is the terminal through which the cell environment is accessed. The interior part of the pipette is filled with an ionic solution, and this latter might change according to the kind of measurement that one wants to perform. An Ag/Cl electrode is then put in contact with the pipette solution and connected to the amplifier. Another electrode immersed in the electrolytic bath that surrounds the cell is used to set the zero level.



Figure 3.3: Schematic picture of the patch clamp technique and electric scheme of the instruments. Reproduced from [166].



Figure 3.4: Whole cell configuration for the patch clamp technique. Reproduced from [166].

In order to perform the measurement, the micropipette is pressed against the cell membrane and a high resistance seal (of the order of  $1 \text{ G}\Omega$ , from which the name "gigaseal" is derived) between this latter and the glass is obtained by applying some suction, see figure 3.4. The high resistance of the seal most importantly allows for measurements less affected by noise, but it also provides some mechanical stability to the pipette-membrane system.

Several configurations are available for such kind of measurement but here we only discuss the protocol that has been used in the experiments performed in this activity, namely the whole-cell configuration. In such protocol the suction applied from the pipette is increased until the membrane patch is broken, see figure 3.4, so the pipette gets complete access to the cytoplasm of the cell. In this way it is possible to record the current flowing through the entire cell membrane. A disadvantage of this technique is that, after a certain amount of time, the liquid content of the cell is replaced by that of the micropipette, since the volume of this latter is much larger, and this inevitably determines the modification of the cell properties. This effect can be minimized by filling the electrode with a solution which reproduces the cytoplasmic one, however the available time slot for performing the measurement is limited to about ten minutes.

The described setup allows to perform measurements in two different configurations, namely the voltage-clamp mode, in which the voltage is kept constant and current is recorded, or the current-clamp mode, where the current is kept constant and changes in the membrane potential can be observed.

#### 3.1.3 Patch clamp measurements

In this work we perform patch clamp measurements to study the photomodulation of the membrane voltage in non-excitable cells grown onto the substrates presented in section 1.2.3. As a valuable experimental model, we select Human Embryonic Kidney 293 cells (HEK-293), since, being non-excitable, they allow to isolate the direct effect of photostimulation of the cell membrane, that would be otherwise superposed with the self-induced cellular activity, should an excitable type of cell be considered instead. HEK cells have also other advantages over different types of culture cells since this cell line is very easy to grow, extensive literature exists about it, and the single cells have a limited size which is good for reducing the measurement errors due to space-clamp artefacts [161].

The devices used for the cellular photostimulation reproduce the structure described in section 1.2.3 and used in [14,64,65], and consist in a layer of P3HT of about 150 nm thickness deposited on an Indium-Tin-Oxide (ITO)-covered glass substrate. In addition, in the present analysis we also consider devices in which the P3HT is deposited on a bare glass substrate, without any ITO layer. The devices are then thermally sterilized (120 °C for two hours) and treated with fibronectin to promote cell adhesion [144]. Afterwards cells are grown directly on the device surface and maintained in an extracellular solution with the following composition [mM]: 135 NaCl, 5.4 KCl, 5 HEPES, 10 glucose, 1.8 CaCl<sub>2</sub>, 1 MgCl<sub>2</sub>. A schematic picture of the device is reported in figure 3.5. Cell proliferation and viability on the polymer substrate have been measured and no significant differences are evident with respect to the control samples. Further details on the preparation of the substrates and about the culture and plating of HEK cells are reported in [108].

The effect of photostimulation of the active substrate on the HEK-293 cells was assessed by measuring the light induced changes in the membrane potential with a standard patch clamp setup. Whole-cell recordings were carried out in current-clamp configuration by fixing the membrane current to zero, in response to 20 ms pulses of light. Representative cellular responses are reported in figures 3.6a and 3.6b for illumination performed with blue laser ( $\lambda = 475$  nm) at intensities ranging from 7.7 to 47 mW mm<sup>-2</sup>.

Upon photostimulation a comparable depolarization  $\Delta V_{\rm m}$  of the cell membrane is



**Figure 3.5:** Schematic picture of the considered device with a cell grown on top and of the stimulation and measuring setup.



**Figure 3.6:** Measured membrane potential variation in HEK-293 cells cultured on (a) P3HT layers deposited on ITO-covered glass and (b) bare glass substrates. The traces refer to four increasing light intensities (7.7, 15, 35 and 47 mW mm<sup>-2</sup>). In the boxes in the left panel, the fast spikes attributed to the photovoltage generation at the ITO/polymer interface are highlighted. Each trace is the mean of 25 consecutive sweeps.

observed in the two cases, with magnitude which increases for more intense incident light power densities. When light is switched off instead, the change in membrane voltage is reversed and develops into a hyperpolarization which is eventually recovered before a new stimulation pulse is applied (measures in figure 3.6 are averaged over 25 sweeps.) Notably, in the device architectures comprising the ITO-covering layer on the glass substrate, sharp spikes appear just after light is switched on or off, see the boxes in figure 3.6a. As demonstrated in section 2.6, such fast spikes are not correlated to the presence of the cell, while they are attributed to the capacitive charging of the polymer/solution interface resulting from the build up of a photovoltage in the polymer substrate. In particular this signal does not have any significant effect on cell dynamics, since the subsequent variations on the membrane voltage are very similar to those measured in the ITO-less configuration.

From this evidence we can conclude that for the systems under study and the considered illumination regimes, the evolution of the membrane voltage is independent of the charge generation phenomenon in the polymer. In order to further corroborate this statement, we consider an alternative device configuration in which cells are grown onto a layer of photoresist (MicroPosit<sup>®</sup> S1813<sup>®</sup>) deposited on a glass coverslip with 100 nm thickness, that has been determined so that the optical density is the same as in the P3HT devices. In the photoresist the absorbed photons are expected to create neutral states that decay non-radiatively, without any charge generation, so any measured output would be for sure unrelated to charge generation effects. The devices are then excited at  $\lambda = 435$  nm with the same intensity values reported before, and a representative response in terms of change of membrane voltage is reported in figure 3.7a. Notably the same behavior as for the other device architectures is observed, thus confirming that the considered phenomena are not charge generation related. As a consequence, a photo-thermally induced effect due to the heating of the bath in proximity of the device surface is the most plausible origin of the observed phenomena, similarly to what happens with infrared neural stimulation upon absorp-



**Figure 3.7:** Measured membrane potential variation in HEK-293 cells cultured on (a) a photoresist layer deposited on glass substrate and (b) control configuration with cells grown onto glass directly. The traces refer to four increasing light intensities (7.7, 15, 35 and 47 mW mm<sup>-2</sup>). Each trace is the mean of 25 consecutive sweeps.

tion by water [168]. This response is necessarily due to light absorption by the P3HT, since it cannot be ascribed neither to endogenous chromophores present in the cells nor to absorption by water [136, 168], as demonstrated by the absence of any effect in cells cultured on glass substrates without the polymer, see figure 3.7b. Temperature change effects are specifically addressed in section 3.2.

Since the depolarization component in the traces is qualitatively the same in all the device configurations, in the following experiments we focus on P3HT-on-glass devices. The responses of four representative HEK-293 cells to 20 ms and 200 ms light pulses at 57 mW mm<sup>-2</sup> are reported in figure 3.8a. As shown above, cells undergo a transient depolarization of the membrane voltage upon illumination, which turns into a hyperpolarization when light is turned off.

It is also evident that the response obtained upon photostimulation has a large variability from cell to cell, in fact the peak depolarization value  $\Delta V_{\rm m}^{\rm peak}$  and the characteristic time of the transient  $\tau^{\rm peak}$ , i.e. the time needed by the depolarization signal to reach its maximum value, are very different among the four reported traces. Since the membrane response to external inputs can be modeled as that of an RC circuit,  $\Delta V_{\rm m}^{\rm peak}$  and  $\tau^{\rm peak}$  necessarily depend on the membrane electric parameters, namely resistance (conductance) and capacitance. These quantities are characterized by an intrinsic variability and they are experimentally determined for each of the considered samples (n = 51 cells) by applying a sinusoidal voltage-clamp paradigm to the cell and performing impedance measurements, see [108] suppl. inform. for further details. Then, in figure 3.9 the peak depolarization value and the time to peak are reported as a function of the RC time, clearly showing that cells characterized by a longer RC time experience a higher depolarization following a slower evolution of the signal.

Looking at the patch clamp results for the 200 ms stimulation of figure 3.8b, we clearly observe that after the peak value has been reached, the depolarization starts to decay and the membrane voltage attains a minimum value  $\Delta V_{\rm m}^{\rm min}$  which is lower



**Figure 3.8:** Measured membrane potential variation upon illumination (20 ms and 200 ms, 57 mW mm<sup>-2</sup>) for four HEK-293 cells cultured on P3HT/glass.



**Figure 3.9:** (a) Correlation between peak depolarization value and (b) time to peak with the time constant of the membrane equivalent circuit.

than the initial resting value. Again when light is switched off the signal is reversed, hyperpolarization occurs, and eventually the membrane voltage relaxes to the initial configuration. As in the case of the peak depolarization  $\Delta V_{\rm m}^{\rm peak}$ ,  $\Delta V_{\rm m}^{\rm min}$  is variable among individual cells, however, in contrast with the previously analyzed quantities, no evident correlation exists with the RC characteristic time, nor with other electric properties of the membrane, see [108].

To get a better insight on this effect, we analyze the membrane electrical characteristics extracting the cell I-V curves around the resting voltage  $V_r$  in dark conditions and at the end of a 200 ms illumination at 57 mW mm<sup>-2</sup> (figure 3.10a). The cell is held at  $V_r$  and a series of voltage steps (from -5 mV to 5 mV in 1 mV steps) are applied in voltage-clamp configuration, see the inset of figure 3.10a. From the recorded current traces we extract the membrane I-V curves in dark and upon illumination and obtain the reversal voltage (the voltage where the I-V crosses the *x*-axis) and the membrane resistance (as the inverse of the I-V curve slopes at the reversal voltage).



**Figure 3.10:** (a) Membrane response to a voltage-clamp step protocol (holding value  $V_r$  with 1 mV steps from -5 mV to 5 mV) with a 200 ms light pulse stimulation (57 mW mm<sup>-2</sup>, cyan rectangle). The gray and magenta boxes represent the regions from which the membrane characteristics were calculated for dark and light conditions, respectively. (b) I-V characteristics in dark and light.



**Figure 3.11:** (a) Correlation between  $\Delta V_{\rm m}^{\rm min}$  measured in current-clamp experiments and  $\Delta V_{\rm r}$  obtained from the I-V characteristics. (b) Correlation between the membrane resistance at the end of the light stimulus and its value in dark, with linear fitting (slope  $0.804 \pm 0.017$ ).

The reversal potential in light condition might be regarded as the membrane resting voltage of the system in the perturbed condition, and it exhibits a shift  $\Delta V_r$  towards more negative values ranging from few hundreds of  $\mu V$  up to 1 mV (n = 17 cells), which correlates well with the  $\Delta V_m^{\min}$  value measured on the same cell in current-clamp measurements, see figure 3.11a. Such strict correlation is a clear hint that for long illumination pulses, even if the membrane voltage attains values lower than the initial resting voltage, the membrane is not hyperpolarized yet, the actual resting membrane voltage most probably being modified by the change of local temperature. The membrane resistance decreases as well upon illumination, see figure 3.11b, with a variation which is consistently of about 20% for all the considered samples. This is in accordance with the fact that ion channels usually increase their conductance with increasing temper-



**Figure 3.12:** Time evolution of (a) normalized membrane capacitance and (b) resistance during illumination (200 ms,  $57 \text{ mW mm}^{-2}$ ), measured for n = 39 cells. The blue line represents the average over the samples while the width of the shaded area indicates the value of the standard deviation.



**Figure 3.13:** (a) Peak depolarization  $\Delta V_{\rm m}^{\rm peak}$  and (b) resting voltage variation  $\Delta V_{\rm r}$  resulting from stimulation with different light intensities (7.7, 15, 34, 47 and 57 mW mm<sup>-2</sup>.)

ature [77].

For further addressing this last evidence, membrane capacitance and resistance have been monitored during illumination (200 ms light pulse at 57 mW mm<sup>-2</sup>) by periodically repeating the impedance measurements used to determine the equilibrium (dark) values of  $C_{\rm m}$  and  $R_{\rm m}$ . In figure 3.12 the time evolution of the normalized membrane capacitance and resistance are reported. The blue continuous lines correspond to the average computed over n = 39 samples, while the shaded areas delimit the values at a distance of one standard deviation from it. Membrane capacitance  $C_{\rm m}$  is found to increase of about 2%, and this is consistent with results previously available in the literature [154]. Consistently with the results obtained from the analysis of the I-V characteristic lines, membrane resistance  $R_{\rm m}$  decreases by  $18\pm4\%$ .

To conclude the presentation of the patch clamp measurements we report in figure 3.13 the extracted values of  $\Delta V_{\rm m}^{\rm peak}$  and  $\Delta V_{\rm r}$  obtained under increasing intensity of the illumination. Since the samples are characterized by a high variability, a boxplot representation is used, which allows to observe a clear increasing trend with higher illumination intensity. This again is consistent with a temperature mediated phenomenon, since higher light energy input would determine a more intense temperature variation.

# 3.2 Temperature measurements

In section 3.1.3 we demonstrated that the recorded cellular activity is a consequence of the local change of temperature due to light absorption by the polymer. For this reason a detailed assessment of the temperature dynamics in the device is in order.

When light is absorbed by some material, an energy transfer occurs between the incident photons and the material molecules. Then such energy can either be stored in alternative forms, e.g. electrostatically or chemically, or be lost by dispersive effects, as heat production of radiative emission. As evidenced in chapter 2, the device under study is not designed to either store electromagnetic energy in efficient way or to transfer it to an external circuit in the form of an electric current, as it happens in photovoltaic devices. If we exclude the amount of charge that is responsible for the generation of the photovoltage signals, the rest of the absorbed energy necessarily ends up following one of the above mentioned loss pathways, and since no light emission is observed, we can conclude that most of the energy must be transformed into heat. Moreover in section 3.1.3 we also show that electric effects become negligible if the polymer is deposited onto a ITO-less glass plate, so we can safely assume heat generation to have a 100% yield.

The generated heat eventually diffuses into the rest of the device, determining a change in the local temperature across its components. In order to quantify the magnitude of such temperature modification in a configuration that is as close as possible to that of the experiments of section 3.1.3, we consider a device structure fabricated according to the same specifications, without a cell attached on the top. Precisely, a layer of P3HT with a thickness of approximatively 150 nm is deposited onto a bare glass substrate and then immersed in a bath of 200 mM NaCl solution. Should a ITO-covered glass be used instead, results would not change significantly, since as previously reported, only a minor part of the absorbed energy is stored in electrostatic form.

Illumination of the device is performed using a laser at wavelength 475 nm, with a temporal profile like that reported in figure 2.13, with illumination duration of either 20 or 200 ms. The laser is focused on a circular spot with radius equal to 270  $\mu$ m, see figure 3.14, and the intensity is modulated with a system of lenses between 3.68 and 56.6 mW mm<sup>-2</sup>.

Temperature change of the bath in proximity of the interface with the polymer layer is accessed with a pipette using the method of the calibrated pipette resistance [173]. In such technique a patch pipette is micromanipulated at about 1-2  $\mu$ m from the interface, and filled with the same solution as that of the bath whose temperature is to be measured. When the temperature in the bath increases, the resistance of the pipette decreases accordingly, since the mobility of the ions in the solution is enhanced. Then if the values attained by the pipette resistance are tabulated at controlled levels of the bath temperature, it is possible to extract the actual temperature just by keeping track



Figure 3.14: Schematic picture of the device and the illumination spot.



Figure 3.15: Bath temperature transient measurements in proximity of the interface with the polymer for several values of the laser intensity. Values are reported in  $mW mm^{-2}$ .

of the evolution of the pipette resistance.

The obtained temperature profiles are reported in figure 3.15 for both the considered illumination durations. We observe that during the light-on period the temperature increases monotonically with time, with a slope which is progressively reduced. This means that immediately after light is switched on, the generated heat determines a pronounced increase of the temperature close to the interface, while after a spatial temperature gradient is established, heat starts to diffuse more and more to the bulk of the solution. The temperature increments scale linearly with the intensity of the laser beam, and with the maximum considered value a 3 °C increment is obtained even with the shortest illumination protocol. The light-off recovery is instead characterized by slower rates, with long lasting tails after an initial sudden decrease occurring just after the light input is interrupted. In view of the use of the device in a practical application in which the stimulation most likely would consist in a repeated on/off switch of the light input, this asymmetry in the characteristic transients would result in an incomplete recovery before the following illumination cycle starts. A memory effect would most likely occur and it would take a certain number of cycles to reach a condition in which the system oscillates periodically between two fixed temperature values.

Notably, the shape of the temperature traces of figure 3.15b closely resembles that of the membrane electrical parameters in figure 3.12, and this represents an additional confirmation of the fact that the membrane parameters change as a consequence of

local temperature variations.

#### 3.3 Heat diffusion modeling

At this point of the analysis, an attempt to numerically reproduce the temperature measurements is in order, so that we can check in quantitative way the compatibility of the working operation discussed above.

We opt to describe the phenomenon using the framework of the standard heat diffusion equation [41] and assuming that:

- 1. all the absorbed energy is converted into heat;
- 2. no heat flux occurs between the glass substrate and its supports;
- 3. the bath domain is large enough to consider the solution far from the absorption area to be unperturbed.

Assumption 1 has been widely discussed above, while assumption 2 is motivated by the fact that the supports are made of plastic, which is a good thermal insulator. Assumption 3 is justified by the fact that the stimulation protocol is quite fast, and most likely the temperature perturbation in the domain does not reach the walls that contain the solution. As a consequence we can study the diffusion effect in just a portion of the actual domain, chosen in such a way to include the regions where the local temperature is significantly modified with respect to the initial condition.

A good accordance between experimental and numerical results would confirm the suitability of the above expressed assumptions, most importantly assumption 1, proving that the amount of energy eventually stored in electromagnetic form is negligible.

Let  $\Omega \in \mathbb{R}^3$  denote the computational domain of the problem, reproducing the structure represented in figure 3.14. A section of  $\Omega$  along the diametrical direction of the laser beam spot is reported for better clarity in figure 3.16, with its partition in subdomains which reflect the device components.  $\Omega_{pol}$  represents the part of the domain occupied by the polymer layer, and is further divided into two parts,  $\Omega_{pol}^{light}$  and  $\Omega_{pol}^{dark}$  denoting the part which is passed through by the laser beam and the remaining one, respectively. With  $\Omega_{glass}$  and  $\Omega_{bath}$  we denote instead the glass substrate layer and the solution bath. As for the domain boundaries,  $\Gamma_{sub}$  represents the contact area of the glass substrate with the device supports, and  $\Gamma_{eq}$  is an artificial boundary which is considered to be far enough from the stimulation region  $\Omega_{pol}^{light}$ , so that assumption 2 holds.

With this setting, the heat diffusion problem can be formulated as: find the temperature profile  $T = T(\mathbf{x}, t)$  which satisfies

$$\begin{cases} c_p \rho \frac{\partial T}{\partial t} - \nabla \cdot k \nabla T = G & \text{in } \Omega \\ -k \nabla T \cdot \boldsymbol{\nu} = 0 & \text{on } \Gamma_{\text{sub}} \\ T = T_{\text{eq}} & \text{on } \Gamma_{\text{eq}} \\ T(\mathbf{x}, 0) = T_{\text{eq}} & \text{in } \Omega, \end{cases}$$
(3.1a)



**Figure 3.16:** Section of the 3D computational domain  $\Omega$  along the diametrical direction of the laser beam spot and its partition into subdomains representing material parts.

where  $c_p$ ,  $\rho$  and k denote the material specific heat capacity at constant pressure, the density and the thermal conductivity, respectively, which in principle attain different values in the several device components. The continuity equation in (3.1a) states that heat is transported in the materials by diffusion events only, and the source term G on the right hand side represents the heat generation contribution due to light absorption. Such term reads

$$\begin{cases} G = \alpha_{P3HT} I_0 \exp\left(-\alpha_{P3HT} d(\mathbf{x})\right) & \text{ in } \Omega_{pol}^{\text{light}} \\ G = 0 & \text{ in } \Omega \setminus \Omega_{pol}^{\text{light}} \end{cases}$$
(3.1b)

where  $\alpha_{P3HT}$  denotes the optical absorption coefficient of the polymer,  $I_0$  the incident laser beam power density and  $d(\mathbf{x})$  the distance of  $\mathbf{x}$  from the incidence surface, i.e. the polymer/solution one. Since light absorption occurs with an exponential profile in the P3HT layer, see section 2.2.1, in the definition (3.1b) we employ the same functional form as in (2.2h), the only difference being the use of  $I_0$  as the profile of the incident light power instead of the incident number of photons, which can be defined again as

$$I_0(t) = \begin{cases} \overline{I} & \text{for } 0 \le t < t_{\text{on}} \\ 0 & \text{otherwise,} \end{cases}$$

where *I* is the laser power density value, see figure 2.13.

As for the boundary equations, the second and third equations in (3.1a) are the mathematical equivalent of assumptions 2 and 3, where  $T_{eq}$  denotes the temperature equilibrium level, which is also used as initial condition of the problem.

Problem (3.1) can be restated in an alternative and more efficient way, using cylindrical coordinates  $(r, z, \theta)$  in place of the Cartesian ones, and exploiting the fact that the configuration is characterized by a rotational symmetry around the laser beam axis. In fact, the beam spot is circular, heat diffusion is isotropic and we can choose the artificial boundary  $\Gamma_{eq}$  to determine a cylinder with the same axis as the beam


**Figure 3.17:** 2D computational domain  $\Omega$  used in the axisymmetric formulation of the problem. The radial and longitudinal coordinates r and z are highlighted.

laser. Using the symmetry argument, we can drop the dependence of the temperature profile on the angular coordinate and consider the computational domain  $\Omega$  to be half of the diametrical section of the original one, as reported in figure 3.17. In the new setting we introduce the boundary  $\Gamma_{symm}$ , being the rotational axis where symmetry conditions have to be enforced.

Problem (3.1) stated in cylindrical coordinates with rotational symmetry reads

$$\begin{cases} c_p \rho \frac{\partial T}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left( rk \frac{\partial T}{\partial r} \right) - \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) = G & \text{in } \Omega \\ -k \nabla T \cdot \boldsymbol{\nu} = 0 & \text{on } \Gamma_{\text{symm}} \\ -k \nabla T \cdot \boldsymbol{\nu} = 0 & \text{on } \Gamma_{\text{sub}} \\ T = T_{\text{eq}} & \text{on } \Gamma_{\text{eq}} \\ T(\mathbf{x}, 0) = T_{\text{eq}} & \text{in } \Omega, \end{cases}$$
(3.2)

where the continuity equation is modified with respect to that of (3.1a) as a consequence of the change of coordinates. Symmetry of the solution on the axis is enforced by using homogeneous flux condition across it with the second equation of (3.2). The remaining equations are not modified with respect to (3.1a) and since the definition of *G* just depends on the longitudinal coordinate *z*, which is unaffected by the change of reference, its expression does not change and (3.1b) still applies.

#### 3.3.0.1 Numerical implementation

From the mathematical point of view, problems (3.1) and (3.2) do not represent a highly challenging task, since heat equation is a very well established problem both analytically and numerically. Since they are equivalent, it is in principle the same to either solve the former or the latter, however, when it comes to numerically implement them, they present peculiarities that have to be considered before choosing which of the two should be be addressed to.

As a premise, in this process we want to use as much as possible the code framework already developed for the transient photovoltage problem, see section 2.3. For this reason we decide to adopt the IDA solver which implements a time advancing algorithm which uses BDF schemes and allows for an adaptive determination of the time step size. Notably, both problems do not have any nonlinear term, so linearization techniques are no longer needed.

Should we consider to implement model (3.1), the octave package bim [43] still provides the routines for the matrix assembly resulting from the application of the Edge Averaged Finite Element (EAFE) method in a three-dimensional framework, see section 2.3. The features of the device geometry displayed in figure 3.14 need to be accurately reproduced to generate a mesh which is suitable for the application of the finite element method. An appropriate mesh refinement is needed in the very thin polymer region  $\Omega_{pol}$  and in the neighboring parts of the glass and solution bath, while the number of elements can be reduced while approaching  $\Gamma_{eq}$  in order to keep the computational cost affordable for a standard laptop.

Problem (3.2) is instead formulated in a two-dimensional framework, hence it is intrinsically less expensive from the computational point of view and meshes with much higher degree of refinement can be considered. On the other side, the original version of the bim package (version 1.1.3) did not include routines specifically designed for the discretization of diffusion-advection-reaction problems in coordinate systems other than the Cartesian one, such as polar, cylindrical and spherical ones. For this reason we extended the EAFE method to the case of cylindrical coordinates with axial symmetry which have been included in the most recent version of the package (1.1.4). The mathematical details about the modified EAFE scheme as described in detail in section 4.7.

#### 3.3.0.2 Numerical results

The materials that compose the device under study have very different characteristics from the point of view of heat transport, and the values attained by the model parameters are reported in table 3.1. In accordance with the experimental specifications, the thickness of the polymer and glass layers are set to 150 nm and 170  $\mu$ m respectively, and the laser beam radius is set to 270  $\mu$ m. Following assumption 3, the dimension of the considered computational domain  $L_{\text{bath}}$  is taken to be big with respect to the laser spot, so we opted for a value of 2 mm.

In the numerical simulations we set the incident laser power density to the same values considered for the the experimental results, and in figure 3.18 we compare the

Parameter	Solution	P3HT	Glass
$c_p  [\mathrm{Jkg^{-1}K^{-1}}]$	$4.1813 \times 10^{3}$	$1.4 \times 10^3$ [125]	$0.84 \times 10^{3}$
$\rho  [\mathrm{kg}\mathrm{m}^{-3}]$	1000	1100 [11]	2500
$k  [{ m W}{ m m}^{-1}{ m K}^{-1}]$	0.6	0.2 [49]	1.0
$\alpha  [\mathrm{cm}^{-1}]$		$10^{5}$	
$\kappa \; [\mathrm{m}^2  \mathrm{s}^{-1}]$	$1.435 \times 10^{-7}$	$1.2987 \times 10^{-7}$	$4.7619 \times 10^{-7}$

Table 3.1: Values of the model parameters for the different materials.



**Figure 3.18:** Bath temperature transient measurements in proximity of the interface with the polymer for several values of the laser intensity. Numerical versus experimental results. Intensity values are reported in  $mW mm^{-2}$ .

value of the computed temperature profile in a node above the center of the laser spot at a distance of 1 nm from the interface (continuous line), with the experimental results (dots). The agreement between the two sets of data is excellent, and such robustness of the model is a strong confirmation that our description of the working principles is appropriate. In particular, assumption 1 has been proven to hold for the range of considered intensities and we can successfully neglect the accumulation of electromagnetic energy in our analysis.

#### **3.4** Patch clamp measurement model

In this section our aim is to exploit the valuable insights that we obtained from the experimental measurements presented in section 3.1.3 and 3.2, to provide a consistent mathematical description of the phenomena that determine the membrane voltage dynamics observed with the patch clamp measurements.

In section 3.1.1 we reported that, since the membrane possesses both a capacitive and a conductive character, its operation can be described with a RC circuit model, see figure 3.19, where with  $C_m$ ,  $R_m$  and  $V_r$  we denote the membrane capacitance, resistance and resting voltage, respectively. We denote with  $V_c$  the value of the potential in the interior of the cell with respect to the extracellular one, which is considered to be the reference value (in the experimental setting it is the value of the potential at the bath reference electrode), while  $I_p$  represents the current flowing through the entire cell membrane, since we are considering a whole-cell configuration. The additional voltage source  $V_{\sigma}$  has been added to take in account the asymmetry between the inner and outer membrane surface charges that usually occur in biological membranes [63, 154].

Kirchhoff's law for the considered circuit simply reads

$$\frac{\partial}{\partial t} \left[ C_{\rm m} \left( V_{\rm c} - V_{\sigma} \right) \right] + \frac{V_{\rm c} - V_{\rm r}}{R_{\rm m}} = I_{\rm p}, \qquad (3.3a)$$

where in principle both  $V_c$  and  $I_p$  are quantities to be determined. However in voltageclamp and current-clamp configurations either one is determined by the experimen-



Figure 3.19: Lumped parameter circuit model for the patch clamp configuration.

talist, so it is a known function of time, and the problem is closed. In the present case, since current-clamp with zero membrane current has been considered, we fix  $I_p = 0$ .

In representing the model circuit in figure 3.19 we used the symbols for nonlinear capacitance and resistance since we want to include in the model the observed effect of heating the cell membrane:

- the increase in capacitance *C*<sub>m</sub>;
- the decrease in membrane resistance *R*<sub>m</sub>;
- the variation of the resting voltage V<sub>r</sub>.

Variation of the membrane capacitance has been recently suggested as the main mechanism for the depolarization observed under thermal stimulation in different types of cells [126, 127, 154]. Based on such existing literature and on the measurements in figure 3.12, we assume that the increase in the membrane capacitance is proportional to the increase in temperature, according to the following relation

$$C_{\rm m}(T) = C_{\rm m}(T_0) \Big[ 1 + \alpha_C (T - T_0) \Big],$$
 (3.3b)

with  $\alpha_C$  representing the relative increase for 1 °C temperature change and  $T_0$  the baseline room temperature. Relation (3.3b) is purely phenomenological and the mechanisms that determine the capacitance variation are not fully understood yet. The underlying phenomenon could have either an electrostatic origin, namely a variation in the size of the diffuse layers at the two sides of the cell membrane, as proposed in [154], or depend on a temperature driven modification of the structural properties of the membrane lipid bilayer. Cell membranes, in fact, under physiological conditions, are close to a transition between an ordered gel phase and a more disordered liquid one [75], and a slight temperature increase might significantly change the membrane thickness, determining an overall increase in its electric capacitance ( $C_m = \varepsilon/d$ ). A deeper discussion about these different approaches is presented in section 3.5.

The temperature increase also affects the conductances of the ion channels spread on the cell membrane, that are in turn responsible of the other two items of the list above, namely the decrease of the membrane resistance and the variation of the cell resting voltage. The effect on membrane resistance is due to the fact that ion transport through membrane channels is enhanced with increasing temperature, and this effect is usually formulated in the biological framework using a  $Q_{10}$ -relation [162] that reads

$$R_{\rm m}(T) = R_{\rm m}(T_0) Q_{10}^{-\frac{1-t_0}{10\,{\rm K}}},$$
(3.3c)

where the coefficient  $Q_{10}$  is a measure of the rate of change of  $R_m$  as a consequence of increasing the temperature by 10 °C.

However, temperature induced modifications of ion channel conductivities do not only affect  $R_{\rm m}$ , but also result in a different equilibrium condition for ion transport across the membrane. According to the Goldman-Hodgkin-Katz (GHK) equation [89], the resting membrane potential is given by

$$V_{\rm r}(T) = \frac{RT}{F} \ln\left(\frac{P_{\rm Na^+}[{\rm Na^+}]_{\rm out} + P_{\rm K^+}[{\rm K^+}]_{\rm out} + P_{\rm Cl^-}[{\rm Cl^-}]_{\rm in}}{P_{\rm Na^+}[{\rm Na^+}]_{\rm in} + P_{\rm K^+}[{\rm K^+}]_{\rm in} + P_{\rm Cl^-}[{\rm Cl^-}]_{\rm out}}\right),$$
(3.3d)

where *R* is the ideal gas constant, *F* is the Faraday constant and  $P_X$  and [X] denote the membrane permeability and concentration for the ionic species *X*. The subscripts in and <sub>out</sub> indicate that the concentration values are to be considered at the internal and external side of the membrane, respectively. In (3.3d) temperature is directly involved in the prefactor RT/F and, since usually  $V_r(T_0) < 0$ , its increase results in a contribution towards more negative resting values, which is qualitatively similar to what we observed in the measurements, see figure 3.13b. In addition, there is also an implicit dependence on *T* through the permeabilities, as they depend on ion channel conductivities. Since the several ion channel types are in principle characterized by specific  $Q_{10}$  temperature coefficients to be identified, and also their spatial distribution participates in affecting the permeability value, the actual behavior of the logarithmic term in (3.3d) is difficult to determine in reliable way. We thus approximate this dependence with the following empirical power law

$$V_{\rm r}(T) = V_{\rm r}(T_0) \left(\frac{T}{T_0}\right)^{\alpha_V},\tag{3.3e}$$

where the fitting parameter  $\alpha_V$  is a measure of the deviation of the realistic case from the "ideal" behavior. In fact, the case for  $\alpha_V = 1$  corresponds to the configuration in which all the permeabilities have the same functional behavior, and  $V_r$  scales linearly with *T*.

#### 3.4.0.3 Numerical results

We use model (3.3) to fit all the available patch clamp measurements presented in section 3.1.3 (n = 51 cells) for both 20 ms- and 200 ms-long illumination protocols.

The temperature dynamics T(t) under illumination, obtained from the heat diffusion numerical simulations, see section 3.3, is provided as input to the model to reproduce the effect of the light pulses. For each cell, the values of  $C_m(T_0)$  and  $R_m(T_0)$ are known, however slight modifications have been considered to obtain better fit of the model response over the experimental data. Since the temperature increase at the end of a 200 ms light pulse at 57 mW mm<sup>-2</sup> has been measured to be of the order of 7.5 °C, see figure 3.15b, and the ratio  $R_m(T_{200 \text{ ms}})/R_m(T_0)$  is about 0.804, see the fit in figure 3.11b, plugging such values into (3.3c), we can estimate  $Q_{10}$  to be around 1.32, which is in the range of literature data for many different ion channels [47]. The best fits were obtained by fixing the value of  $V_{\sigma}$  to 160 mV, which is comparable to those used in [154].

The adopted fitting procedure consists in suitably modifying the above introduced parameters  $\alpha_C$  and  $\alpha_R$  to better fit the particular membrane voltage measurement under study, while keeping the normalized transient variation of  $C_{\rm m}$  and  $R_{\rm m}$  at a distance



**Figure 3.20:** Statistical distribution of the values for the parameters  $\alpha_C$  and  $\alpha_V$  obtained from the fitting of the experimental measurements (n = 54 cells).

of at most one standard deviation from the average of the experimental data presented in figure 3.12.

The problem has been numerically implemented in Matlab, using the ode15i routine for time advancement.

The distributions of the values of  $\alpha_C$  and  $\alpha_V$  obtained by applying the fitting procedure to all the available membrane voltage traces (n = 51 cells) are reported in figure 3.20. The fitted values for  $\alpha_C$  have a sharp Gaussian distribution with mean value 0.0031 K<sup>-1</sup> and a standard deviation of 0.0004 K<sup>-1</sup>, which is in excellent agreement with data reported in [154]. In particular, the low value attained by the standard deviation is a clear indication that the obtained estimate for  $\alpha_V$  is reliable, and that the phenomenon that determines the capacitance variation is consistently occurring in all the samples, with limited influence of the particular cell configuration.

A different trend is instead observed for the fitted values of  $\alpha_V$ , which show a more pronounced variability in the range of values from 0 to 2. However most samples are characterized by values between 0.2 and 0.6, indicating a generally sublinear dependence of the variation of the resting voltage  $\Delta V_r$  on the temperature. The higher variability of the results can be ascribed to a more pronounced dependence of the phenomena on the local configuration of the cell-substrate interface, and a possible interpretation of this evidence is proposed in the following section.

Typical simulated temporal traces of the normalized membrane capacitance and resistance are reported in figure 3.21 ( $\alpha_C = 0.0032$ ), and they prove to be in good agreement with the experimental data.

Finally, the simulated traces of the membrane voltage transient are reported for two representative cells in figures 3.22a and 3.22b, for 20 ms and 200 ms light pulses, respectively, denoting an excellent agreement of the model with experimental data. The competing contributions to the overall response of the membrane voltage due to the temperature induced changes in capacitance and resting voltage are isolated, so a clear picture of the phenomenon can be drawn.

• Light absorption by the polymer leads to the generation of excited states which



**Figure 3.21:** Simulated time evolution of the normalized membrane capacitance and resistance for a typical cell under 20 ms illumination, compared to the experimental values.



**Figure 3.22:** Comparison between the membrane voltage experimental measurements and numerical simulation for typical cells under photoexcitation with 20 ms and 20 ms pulses. Dashed lines represent the contributions due to the variation in membrane capacitance and resting voltage.

eventually recombine releasing thermal energy, that in turn determines a temperature increase in the bath.

- As a consequence membrane capacitance increases, but since the charge stored on the cell membrane cannot change instantaneously, at short times cell depolarization is observed. In formulas, since  $Q_m = C_m V_c$ , if  $Q_m \approx$  constant and  $C_m$ increases, then  $V_c(< 0)$  has to decrease in absolute value, meaning that the cell membrane is depolarized.
- As a result of the change of the membrane voltage, on a longer time scale, currents start to flow in the ion channels to restore the equilibrium condition. However the change of temperature also affects the resting membrane voltage, so the system relaxes to a configuration different from the initial one.
- The time scale of this process is determined by the membrane time constant  $\tau = R_{\rm m}C_{\rm m}$  and the maximum depolarization value results from the competition between the time constant  $\tau$  and the slope of the temperature increase.

• When light is switched off, the capacitive contribution and that due to the variation of the resting voltage switch their role and a hyperpolarization is observed. Since the bath temperature eventually decays to the equilibrium value, the membrane voltage relaxes to the level observed before the stimulation.

#### 3.4.0.4 Interpretation of the high variability of the fitted values of $\alpha_V$

In the previous section the fitting procedure led to the determination of a distribution of values of the resting voltage parameter  $\alpha_V$  characterized by a broad interval of variation, ranging from  $\alpha_V = 0$ , i.e. constant resting membrane voltage irrespectively of the temperature, to  $\alpha_V = 1$  and even  $\alpha_V = 2$ , meaning linear and quadratic dependence. Such behavior can be in part interpreted by addressing the motivations that pushed us in adopting (3.3e) in place of (3.3d).

As already discussed, ion permeabilities depend on the density distribution of ion channels on the membrane, that might vary among the several cells and can even be inhomogeneous on the same cell, but a quantification of this effect is difficult since we have little chance to get useful data from experimental values.

Conversely, a quantification of the impact on the variation of  $V_r$  due to the implicit dependence of the permeabilities on *T* through the ion channel conductances is possible. In developing the model for the patch clamp measurements, we considered the simplified circuit in figure 3.19, while a more accurate description of the membrane behavior is provided by that reported in figure 3.23, where an individual conductance has been introduced for each kind of ion channel.

We can assume the conductance of each kind of ion channel to be described by a specific version of (3.3c), possibly characterized by different values of the  $Q_{10}$  parameter. If we further assume that

$$P_i \propto g_i = g_{i,0} Q_{10,i}^{\frac{T-T_0}{10K}} \qquad i = Na^+, K^+, Cl^-,$$

we can numerically evaluate the behavior of  $V_r$  as determined by (3.3d) for different choices of the  $Q_{10}$  values.

Let  $[Na^+]_{in}$ ,  $[K^+]_{in}$  and  $[Cl^-]_{in}$  be equal to 50, 400 and 40 mM and  $[Na^+]_{out}$ ,  $[K^+]_{out}$ and  $[Cl^-]_{out}$  be 460, 10 and 560 mM, respectively, let  $T_0 = 20$  °C and assume a  $P_{Na^+}$  :  $P_{K^+} : P_{Cl^-}$  ratio equal to 0.25:1:0.1. We consider a reference configuration in which all the specific  $Q_{10}$  parameters equal 1.35 and we gradually modify it either by increasing the Na<sup>+</sup>-specific value (1.3, 1.35, 1.4, 1.45) of by decreasing the K<sup>+</sup>-specific one (1.4, 1.35, 1.3, 1.25). The computed trends are reported in figure 3.24, where with the black line we denote the reference configuration, while the series of red and blue curves



**Figure 3.23:** Lumped parameter circuit model for the patch clamp configuration with separate conductances for each ion channel type.



**Figure 3.24:** Resting voltage value as predicted by the GHK equation, as a function of the temperature increase. The black line denotes the reference configuration, while for the red and blue curves the  $K^+$  and  $Na^+$  specific parameters are reduced and increased, respectively.

refer to the configurations in which the K<sup>+</sup> and Na<sup>+</sup> specific parameters are modified, respectively.

These results clearly show that even a very small variation on just one of the ionspecific  $Q_{10}$  parameters might determine significant modifications in the behavior of  $V_r$  as a function of the temperature increase. Since our measurements are not able to distinguish the contributions to the membrane current to due the different ion channels, we do not have valuable information either about the temperature dependence of their conductance or their spatial distribution on the membrane. In such condition the inclusion of (3.3d) in the patch clamp model to determine  $V_r$  is not reliable due to the high number of free parameters, and using (3.3e) represents a more effective alternative.

#### **3.5** Approaches to describe temperature dependent capacitance

In section 3.1.3 we experimentally observed an increase in membrane capacitance in correspondence of an increment of the temperature of the surrounding solution, and to account for this behavior we used the phenomenological model (3.3b). However the true motivation of this phenomenon is still not fully understood and is currently under debate. Recently Shapiro and coworkers [154] claimed that the capacitance increase is consistent with classic theories of double layers [63], and this description is rapidly getting the credit of the involved scientific community.

In this section we describe and analyze the model considered in the previously mentioned work, highlighting some critical aspects that should be taken into account before such description is fully acknowledged and becomes mainstream in the field. Then we propose an alternative picture to motivate the thermally induced increase in membrane capacitance, based on the thermodynamics of the lipid bilayer, which represents the main constituent of the cell membrane.

#### 3.5.1 Critical discussion of Shapiro-Bezanilla model

The work by Shapiro and coworkers [154] represents a breakthrough in the understanding of thermally mediated cell stimulation techniques, since it provides clear experimental evidence that the sole increment of membrane capacitance due to the change of the bath temperature can determine alone cellular excitation. In addition the authors also provide a theoretical description of the phenomenon, claiming that the proposed working principles are fully consistent with classic models based on the Guoy-Chapman-Stern theory of double layers in diluted solutions [9,68].

The model is an extension of a previously available description of the electric potential across the cell membrane based on the simplified Gouy-Chapman theory [63]. In that work, each of the two sides of the cell membrane is described with such electrostatic theory based on the nonlinear Poisson-Boltzmann equation. Since the considered solution are not characterized by an extreme value of the ionic concentrations, i.e. they are referred to as "diluted", ions are considered as point charges and the solvent as a continuous dielectric with uniform permittivity. The novelty of the approach consists in introducing the coupling of the two sides of the membrane, unlike all the previously available works in literature, in which they were treated as independent electrodes.

The additional contribution by Shapiro et al. consists instead in the further refinement of the model, by including the effect of Stern layers on the electric potential profile, as we describe below.

We consider a cell membrane with thickness  $\delta_b$  and dielectric constant  $\varepsilon_b$ . Since the lateral extension of the membrane is significantly larger than its thickness, we can consider it to be infinite and set the model in a 1D framework, as reported in figure 3.25. The membrane separates two semi-infinite domains of electrolyte solutions, whose dielectric permittivity can be identified with that of water  $\varepsilon_w$ . Water dielectric permittivity is well known to decrease as a function of temperature [40], and we



**Figure 3.25:** Profile of the electric potential  $\phi$  around a cell membrane of thickness  $\delta_b$  separating two electrolyte solutions. The surface charge densities on the two sides of the membrane are denoted with  $\sigma_i$  and  $\sigma_o$ . Stern layers of thickness  $\delta_S^i$  and  $\delta_S^o$  are accounted for, characterized by voltage drops  $\Delta \phi_S^i$  and  $\Delta \phi_S^o$ .

describe this effect with the phenomenological expression

$$\varepsilon_{\rm w}(T) = (175.34892 - 0.32214 \,{\rm K}^{-1} \cdot T) \,\varepsilon_0,$$

 $\varepsilon_0$  being the vacuum permittivity, which is obtained by fitting with a linear function the experimental values  $88\varepsilon_0$ ,  $80.1\varepsilon_0$  and  $55.3\varepsilon_0$  measured at 0, 20 and 100 °C, respectively. The internal and external sides of the membrane respectively bear intrinsic surface charge densities  $\sigma_i$  and  $\sigma_o$ , which are usually negative and result either from the dissociation of the membrane constituents or from adsorbed species from the solution [77, 111]. We denote with  $\phi_i$  and  $\phi_o$  the values of the electric potential at the interfaces between the membrane and the inner and the outer solution, respectively, i.e.  $\phi_i = \varphi_i(-\delta_b)$  and  $\phi_o = \varphi_o(0)$ .

The Stern layers are regions with width  $\delta_{S}^{i}$  and  $\delta_{S}^{o}$ , characterized by zero volumetric charge, determined by the hydration of the ions in the solution and of the polar heads of the phospholipids. For this reason the electric potential is linear therein, and the potential differences  $\Delta \phi_{S}^{i}$  and  $\Delta \phi_{S}^{o}$  are defined as

$$\Delta \phi_{\rm S}^{\rm i} = \phi_{\rm i} - \varphi_{\rm i} (-\delta_{\rm b} - \delta_{\rm S}^{\rm i})$$
 and  $\Delta \phi_{\rm S}^{\rm o} = \phi_{\rm o} - \varphi_{\rm o} (\delta_{\rm S}^{\rm o}).$ 

The dielectric permittivity of the Stern layers is usually assumed to be significantly lower than that of the bulk solution, and here we take

$$\varepsilon_{\rm S}^{\rm i}(T) = \varepsilon_{\rm S}^{\rm o}(T) = \frac{\varepsilon_{\rm w}(T)}{10}.$$

The following nonlinear Poisson-Boltzmann equations hold for the electric potential profiles  $\varphi_i(x)$  and  $\varphi_o(x)$  in  $(-\infty, -\delta_b - \delta_s^i)$  and  $(\delta_s^o, \infty)$ 

$$\frac{d^2\phi_{\rm i}}{dx^2}(x) = -\frac{F}{\varepsilon_{\rm w}} \sum_{j=1}^n z_j^{\rm i} c_j^{\rm i}(-\infty) \exp\left(-\frac{z_j^{\rm i} F}{RT} \left((\phi_{\rm i} - V_{\rm m}) - \Delta\phi_{\rm S}^{\rm i}\right)\right),\tag{3.4a}$$

$$\frac{d^2\phi_{\rm o}}{dx^2}(x) = -\frac{F}{\varepsilon_{\rm w}} \sum_{k=1}^n z_k^{\rm o} c_k^{\rm o}(\infty) \exp\left(-\frac{z_k^{\rm o} F}{RT} \left(\phi_{\rm o} - \Delta\phi_{\rm S}^{\rm o}\right)\right),\tag{3.4b}$$

where with *z* and  $c(\pm \infty)$  we denote the valence and the bulk concentrations of the *n* ion species in the inner and outer solutions, with the boundary conditions

$$\lim_{x \to -\infty} \phi_{i}(x) = V_{m}, \qquad \lim_{x \to -\infty} \frac{d\phi_{i}}{dx}(x) = 0, \qquad (3.4c)$$

$$\lim_{x \leftarrow \infty} \phi_{i}(x) = 0, \qquad \lim_{x \leftarrow \infty} \frac{d\phi_{o}}{dx}(x) = 0.$$
(3.4d)

Continuity of the electric displacement field is enforced at the interfaces between the

membrane and the solution, and between the Stern layers and the diffuse ones

$$-\varepsilon_{\rm w} \frac{d\varphi_{\rm i}}{dx} (-\delta_{\rm b} - \delta_{\rm S}^{\rm i}) + \varepsilon_{\rm S}^{\rm i} \frac{\Delta \phi_{\rm S}^{\rm i}}{\delta_{\rm S}^{\rm i}} = 0$$
(3.4e)

$$-\varepsilon_{\rm S}^{\rm i} \frac{\Delta \phi_{\rm S}^{\rm i}}{\delta_{\rm S}^{\rm i}} - \varepsilon_{\rm b} \frac{\phi_{\rm tm}}{\delta_{\rm b}} = -\sigma_{\rm i}$$
(3.4f)

$$\varepsilon_{\rm b} \frac{\phi_{\rm tm}}{\delta_{\rm b}} - \varepsilon_{\rm S}^{\rm o} \frac{\Delta \phi_{\rm S}^{\rm o}}{\delta_{\rm S}^{\rm o}} = -\sigma_{\rm o} \tag{3.4g}$$

$$\varepsilon_{\rm S}^{\rm o} \frac{\Delta \phi_{\rm S}^{\rm o}}{\delta_{\rm S}^{\rm o}} + \varepsilon_{\rm w} \frac{d\varphi_{\rm o}}{dx} (\delta_{\rm S}^{\rm o}) = 0$$
(3.4h)

where the transmembrane voltage  $\phi_{\rm tm}$  is defined as

 $\phi_{\rm tm} = \phi_{\rm i} - \phi_{\rm o}.$ 

Integrating (3.4a) and (3.4b) over the computational domain, and enforcing the boundary and interface conditions, with few steps it is possible to obtain the following equations

$$\left(\sigma_{\rm i} - \frac{\varepsilon_{\rm b}}{\delta_{\rm b}} \left(\phi_{\rm i} - \phi_{\rm o}\right)\right)^2 = 2\varepsilon_{\rm w}(T)RT \sum_{j=1}^n c_j^{\rm i}(-\infty) \left[\exp\left(-\frac{z_j^{\rm i}F}{RT} \left(\left(\phi_{\rm i} - V_{\rm m}\right) - \Delta\phi_{\rm S}^{\rm i}\right)\right) - 1\right],\tag{3.5a}$$

$$\left(\sigma_{\rm o} + \frac{\varepsilon_{\rm b}}{\delta_{\rm b}} \left(\phi_{\rm i} - \phi_{\rm o}\right)\right)^2 = 2\varepsilon_{\rm w}(T)RT \sum_{k=1}^n c_k^{\rm o}(\infty) \left[\exp\left(-\frac{z_k^{\rm o}F}{RT} \left(\phi_{\rm o} - \Delta\phi_{\rm S}^{\rm o}\right)\right) - 1\right], \quad (3.5b)$$

which relate the intrinsic and the transmembrane capacitive charges on both sides of the bilayer to the potential at each bilayer surface, corrected by the Stern potential drops at each side. These latter quantities are computed as

$$\Delta \phi_{\rm S}^{\rm i} = \frac{\delta_{\rm S}^{\rm i}}{\varepsilon_{\rm S}^{\rm i}} \left( \sigma_{\rm i} - \frac{\varepsilon_{\rm b}}{\delta_{\rm b}} \left( \phi_{\rm i} - \phi_{\rm o} \right) \right) \tag{3.5c}$$

$$\Delta \phi_{\rm S}^{\rm o} = \frac{\delta_{\rm S}^{\rm o}}{\varepsilon_{\rm S}^{\rm o}} \left( \sigma_{\rm o} + \frac{\varepsilon_{\rm b}}{\delta_{\rm b}} \left( \phi_{\rm i} - \phi_{\rm o} \right) \right) \tag{3.5d}$$

and the thicknesses are given by

$$\delta_{\rm S}^{\rm i} = \delta_{\rm lipid}^{\rm i} + \frac{\sum_{j=1}^{n} r_j^{\rm i} c_j^{\rm i}(-\infty) \exp\left(-\frac{z_j^{\rm i} F}{RT} \left((\phi_{\rm i} - V_{\rm m}) - \Delta \phi_{\rm S}^{\rm i}\right)\right)}{\sum_{j=1}^{n} c_j^{\rm i}(-\infty) \exp\left(-\frac{z_j^{\rm i} F}{RT} \left((\phi_{\rm i} - V_{\rm m}) - \Delta \phi_{\rm S}^{\rm i}\right)\right)}, \qquad (3.5e)$$

$$\delta_{\rm S}^{\rm o} = \delta_{\rm lipid}^{\rm o} + \frac{\sum_{k=1}^{n} r_k^{\rm o} c_k^{\rm o}(\infty) \exp\left(-\frac{z_k^{\rm o} F}{RT} \left(\phi_{\rm o} - \Delta \phi_{\rm S}^{\rm o}\right)\right)}{\sum_{k=1}^{n} c_k^{\rm o}(\infty) \exp\left(-\frac{z_k^{\rm o} F}{RT} \left(\phi_{\rm o} - \Delta \phi_{\rm S}^{\rm o}\right)\right)}. \qquad (3.5f)$$

The capacitive charges  $\sigma_{dl}^{i}$  and  $\sigma_{dl}^{o}$  stored in the double layers are given by

$$\sigma_{\rm dl}^{\rm i} = -\sigma_{\rm dl}^{\rm o} = \frac{\varepsilon_{\rm b}}{\delta_{\rm b}} \left(\phi_{\rm i} - \phi_{\rm o}\right)$$

and they are related to the membrane overall specific capacitance  $C_{\rm m}$  by

$$\sigma_{\rm dl}^{\rm i} = C_{\rm m} V_{\rm m}$$

In addition to the above mentioned dependence of the dielectric constant  $\varepsilon_w$  of the temperature *T*, and hence also that of  $\varepsilon_S^i$  and  $\varepsilon_S^o$ , the temperature appears also in (3.5a), (3.5a), (3.5e) and (3.5f). For this reason model (3.5) can be used to investigate the role of temperature in determining the electric capacitance of a cell membrane, as an example by computing a *C*-*T* characteristic curve of a particular membrane or by plugging-in the temperature profiles computed in section 3.3 to observe the corresponding variation in the capacitance.

In their work, Shapiro and coworkers followed this latter approach considering the simplified configuration of an artificial lipid bilayer, which, unlike biological membranes, does not have ion channels and exhibits an almost purely capacitive behavior  $(g_m \approx 0)$ . As a consequence, transmembrane ion currents can be neglected and any measured current must be attributed to capacitive effects. Voltage clamp configuration has been considered by fixing the membrane voltage  $V_m$  to well defined values, and measuring the current resulting from light stimulation. The results showed inward (negative) currents for negative values of the bias  $V_m$ , consistent with the also measured increase of membrane capacitance  $C_m$ .

Model (3.5) has been then numerically solved to support the experimental evidence, and the obtained results have been claimed to be in accordance with the experimental measurements within a 25% difference. The used parameter configuration has been reported in table 3.2.

Parameter	Value	Unit
$\sigma_{\rm i}$	-0.006	$C m^{-2}$
$\sigma_{ m o}$	-0.006	$C m^{-2}$
$\varepsilon_{\mathrm{b}}$	$2.5\varepsilon_0$	$\mathrm{F}\mathrm{m}^{-1}$
$\varepsilon^{i}_{S}$	$\varepsilon_{ m w}(T)/10$	$\mathrm{F}\mathrm{m}^{-1}$
$\varepsilon_{\rm S}^{\rm o}$	$\varepsilon_{ m w}(T)/10$	$\mathrm{F}\mathrm{m}^{-1}$
$\check{\delta_{\mathrm{b}}}$	3	nm
$\delta^{ m i}_{ m lipid}$	0.45	nm
$\delta^{\mathrm{o}}_{\mathrm{lipid}}$	0.45	nm
$c_i^i(-\infty)$	(140, 140)	mМ
$z_i^i$	(1,-1)	-
$r_i^i$	(0.355, 0.335)	nm
$c_k^{\rm o}(\infty)$	(140, 140)	mМ
$z_k^{o}$	(1,-1)	-
$r_{L}^{o}$	(0.355, 0.335)	nm

At this point we also numerically implement the nonlinear equation system (3.5) in order to reproduce the simulation results presented in [154]. In voltage clamp config-

**Table 3.2:** Model parameter for the lipid bilayer configuration.



**Figure 3.26:** (a) Computed membrane current for a lipid bilayer resulting from a temperature change of about 20°C in 10 ms, for three different voltage clamp configurations ( $V_m = -60$ , 0 and 60 mV). (b) Membrane capacitance variation with respect to the initial value  $V_m = -60$  and 60 mV.

uration the value of  $V_{\rm m}$  is fixed to a predetermined value, so the number of unknowns of the system, namely  $\phi_{\rm i}$ ,  $\phi_{\rm o}$ ,  $\Delta \phi_{\rm S}^{\rm i}$ ,  $\Delta \phi_{\rm S}^{\rm i}$ ,  $\delta_{\rm S}^{\rm i}$  and  $\delta_{\rm S}^{\rm i}$ , is equal to the number of available equations an the problem is closed. The corresponding equation systems is strongly nonlinear and we solve it using the fsolve routine of the Matlab suite.

We consider the same lipid bilayer configuration analyzed by Shapiro et al. by setting the parameter values to those in table 3.2, and providing to the code a temperature profile which reproduces that reported in their work (about 20 °C increase in 10 ms). The obtained values of  $\phi_i$  and  $\phi_o$  are then used to compute the capacitive charge densities  $\sigma_{dl}^i$  and  $\sigma_{dl}^o$  from which we obtain the membrane current density  $i_m = \sigma_{dl}^i$  and the profile of the membrane capacitance  $C_m$  reported in figure 3.26, for the case of a clamp voltage  $V_m = -60$  mV.

Notably, the results are in contrast with those reported in [154]. For negative applied values a positive, i.e. outward, current is obtained, whereas in positive membrane polarization conditions, the current sign is switched, the crossover value being 0 mV. These results are consistent with a decrease in the membrane capacitance, which is also numerically observed and amounts to a 1% variation with respect to the initial value, see figure 3.26.

In order to rule out the chance of the above results being determined by either an implementation mistake or a numerical issue in the solution of nonlinear system (3.5), we crosscheck the obtained results by implementing also a solver for the Poisson-Boltzmann boundary value problem (3.4). Similarly to the approach adopted in section 2.3, the problem is linearized using the Newton method and the primal-mixed finite element method is used for spatial discretization. A sequence of problems of the form (3.4) characterized by different values of the temperature T(t) are solved to reconstruct the temporal evolution of the charge profile, the membrane current transient and the membrane capacitance variation.

The domain in which problem (3.4) is set is, in principle, unbounded and hence it is not amenable for direct numerical treatment. For this reason we consider instead a subset  $(-\bar{L} - \delta_b, \bar{L})$  of the original one, the length  $\bar{L}$  being chosen large enough so that at  $x = -\bar{L}$  and  $x = \bar{L}$  the system can be considered to be at the inner and outer bath



**Figure 3.27:** Computed electrical potential (a) and charge profile (b) for a lipid bilayer, before t = 0 ms and after t = 10 ms a temperature increase. Model parameters are set at the values of table 3.2, but the surface charge densities  $\sigma_i$  and  $\sigma_o$  are set to zero. The membrane voltage is clamped at -60 mV.



**Figure 3.28:** Computed variation in the charge profile between the configurations before t = 0 ms and after t = 10 ms a temperature increase. Model parameters are set at the values of table 3.2 and the membrane voltage is clamped at -60 mV.

equilibrium configurations, respectively.

Moreover, since we have numerical evidence that the Stern layer thicknesses  $\delta_{S}^{i}$  and  $\delta_{S}^{o}$  do not change significantly, especially if the *r* parameters attain similar values for all the ions, we consider them to be constant values equal to

$$\delta_{\mathrm{S}}^{\mathrm{i}} = \delta_{\mathrm{lipid}}^{\mathrm{i}} + \frac{1}{n} \sum_{j=1}^{n} r_{\mathrm{j}}^{\mathrm{i}}$$
 and  $\delta_{\mathrm{S}}^{\mathrm{o}} = \delta_{\mathrm{lipid}}^{\mathrm{o}} + \frac{1}{n} \sum_{k=1}^{n} r_{\mathrm{k}}^{\mathrm{o}}$ .

At first we consider a configuration with all the parameters values set in accordance with table 3.2, the only exception being the interface surface charges  $\sigma_i = \sigma_o$ , which are set equal to zero in order to reduce the complexity of the configuration, allowing for a simpler comprehension of the involved phenomena.

In figure 3.27 we report the obtained profile of the electric potential and the selfconsistently determined charge distribution across the membrane. The electric potential is flat in the bulk of the solution, up to the Stern layers, and a negative charge is accumulated in the diffuse layer in the inner side of the membrane, with a corresponding positive excess on the other side. The red and the blue lines, corresponding to the



**Figure 3.29:** Computed electrical potential (a) and charge profile (b) for a lipid bilayer, before t = 0 ms and after t = 10 ms a temperature increase. Model parameters are set at the values of table 3.2 and the membrane voltage is clamped at -60 mV.



**Figure 3.30:** Computed variation in the charge profile between the configurations before t = 0 ms and after t = 10 ms a temperature increase. Model parameters are set at the values of table 3.2 and the membrane voltage is clamped at -60 mV.

configurations before (t = 0 ms) and at the end (t = 10 ms) of the temperature increase, are almost overlapped since the variation is very limited. In figure 3.28 we show the charge variation between such configurations, which amounts to a small decrease in the absolute value of the charge stored in the double layers, and hence to a decrease of the capacitance value.

If the voltage clamp value is set at 0 mV, the resulting profile is obviously flat and no charge accumulation occurs, while with  $V_{\rm m} = 60$  mV specular results are obtained with respect to the above considered case. Interestingly, the obtained current profiles and the membrane capacitance values are exactly the same as those computed for the case with  $\sigma_{\rm i} = \sigma_{\rm o} = -0.006 \text{ C/m}^2$ .

We now consider the original case considered by Shapiro and coworkers, see table 3.2, with a polarization  $V_{\rm m} = -60$  mV, and report the obtained results in figure 3.29. The surface charges densities  $\sigma_{\rm i}$  and  $\sigma_{\rm o}$  strongly alter the ionic distribution in the neighborhood of the membrane, as they attract charged species of opposite sign and repel those of the same sign. This is clear from the charge distributions of figure 3.29b, which show that in both the diffuse layers positive charges are stored. However the

density peak on the inner side of the membrane is sensibly lower than that on the opposite side, so with the contribution of the interface surface charges the overall result is again a negative excess in the interior of the bilayer.

Again the charge displacement between the initial and the post-stimulus configurations is very small, so we report it in figure 3.30. In the present case the computed quantities show a picture richer in features since the displacement does not have a defined sign in each side. Upon temperature increase, there is a migration of positive charge towards the membrane (or, equivalently a migration of negative charge towards the two solution bulk regions). However, as previously highlighted, the overall charge variation in the inner (left) side of the bilayer is positive, as evidenced by the different heights of the peaks, and this is consistent with the decrease of membrane capacitance previously obtained. Notably also with this approach, the same graphs of figure 3.26 are obtained, further excluding the chance of possible bugs in our code.

At this point a critical discussion of the picture proposed by Shapiro and coworkers to motivate the capacitance increase experimentally measured is in order. They claimed that such variation is in accordance with classic theory of double layers applied to membrane structures. The overall capacitance  $C_{\rm m}$  of the system is the series of the core lipid bilayer and those of the diffuse layers, according to

$$\frac{1}{C_{\rm m}} = \frac{1}{C_{\rm dl}^{\rm i}} + \frac{1}{C_{\rm b}} + \frac{1}{C_{\rm dl}^{\rm o}}$$

Since in this description  $C_b$  is a constant value, the system capacitance enhancement is necessarily due to an increase of the double layer capacitances. According to the Gouy-Chapman-Stern theory [9], the double layer capacitance is given by (for the particular case of the outer side of the membrane)

$$\frac{1}{C_{\rm dl}^{\rm o}} = \frac{\delta_{\rm S}^{\rm o}}{\epsilon_{\rm w}} + \frac{1}{\left(2\epsilon_{\rm w}(z^{\rm o})^2 q^2 c^{\rm o}(\infty)/k_{\rm B}T\right)^{1/2} \cosh\left(zq\varphi_{\rm o}(\delta_{\rm S}^{\rm o})/2k_{\rm B}T\right)}.$$

The first contribution corresponds to the capacitance of the Stern layer, which decreases with *T* since  $\varepsilon_w$  does. The second term is instead attributed to the diffuse layer and it depends in a complex way on the system temperature,  $\varepsilon_w$  being also a function of *T*. However this particular expression is a decreasing function of the temperature so the overall capacitance  $C_{dl}^o$  decreases for increasing value of the temperature.

With this said, let us consider again the lipid bilayer configuration corresponding to the parameter values of table 3.2, and slightly modify the value of the intrinsic surface charge  $\sigma_i$  to  $-0.007 \text{ Cm}^{-2}$ . Figure 3.31 reports the computed values of the membrane current and, unexpectedly, negative currents are registered also for case of the negative bias and, correspondingly, a small increase of the capacitance of the system is observed.

This fact further supports our opinion that the model provided by Shapiro and coworkers, still needs more investigation and verification, since it has not been possible to reproduce the numerical results presented in [154] and also since it is not robust enough, showing a strong sensitivity on the model parameters, as it occurs in the last example, in which just a slight modification of one of the parameters causes the sign switch of system response.



**Figure 3.31:** Computed membrane current for a lipid bilayer resulting from a temperature change of about  $20^{\circ}C$  in 10 ms, for three different voltage clamp configurations ( $V_{\rm m} = -60$ , 0 and 60 mV). Membrane capacitance variation with respect to the initial value  $V_{\rm m} = -60$  and 60 mV. Model values are those of table 3.2 with  $\sigma_{\rm i} = -0.007 C m^{-2}$ .

# 3.5.2 Liquid crystal model and membrane thickness variation

In section 3.1.1 we provided a brief description of cell membranes, as composed of phospholipids organized in a bilayer, which provides the mechanical structure and determines its insulating properties, and of particular proteins, that constitute the channels that let the interior and exterior sides communicate with each other.

However such heterogeneous structure is not static, i.e. proteins and other particles possibly included in the lipid matrix are not characterized by a fixed position, but rather they are free to diffuse over the entire cell membrane. Moreover, the phospholipids forming the bilayer are in turn not strongly bonded with the neighboring elements, therefore they resemble a liquid crystal structure since they can flow in the two dimensions defined by the planar membrane, while preserving the double layered configuration, see the seminary work by Singer and Nicolson [157] in which they introduced the so called "fluid-mosaic" model of cell membranes.

Focusing on the microscopic structure formed by the lipid molecules, it is well known that changes of configuration are observed at different temperature values since the bilayers can undergo a change of phase defined as melting [75], see figure 3.32. In particular the most important configurations are:

- the crystalline lipid phase. At temperatures below the melting value  $T_{\rm m}$ , lipids are organized according to a three-dimensional order, see the schematic representation in the left panel of figure 3.32. Polar heads are arranged on a triangular lattice, and the chains are predominantly in trans- configuration.
- the fluid phase, also called liquid disordered phase. At  $T > T_m$  the lattice order of the polar head groups is lost and the hydrocarbon chains show isomerization of the C-C bonds resulting in a disordered stacking, see the right panel of figure 3.32.

In artificial lipid bilayers composed by a single type of lipid, such transition occurs in a very narrow temperature interval around the melting temperature value  $T_{\rm m}$ . However, since cell membranes are composed by several types of lipids which are in principle



**Figure 3.32:** Schematic picture of lipid melting from a solid-ordered to a liquid-disordered phase. Top: the order within the lipid chains is lost upon melting. Bottom: The crystalline order of the lipid head groups is also lost and the matrix undergoes a solid-liquid transition. Reproduced from [75].

characterized by different melting temperatures, the phase transition might occur in a more gradual way on a wider temperature range. An additional contribution in this direction is given by the presence of proteins and cholesterol in biological membranes, so the extension of the temperature window in which structural modifications are expected might be significant.

For most biological membranes the characteristic temperature value  $T_{\rm m}$  at which the transition occurs is in the range between 10 and 25 °C, which includes the temperature value we fixed in the experiments of section 3.1.3. For this reason we might consider that in such configuration cell membranes are right in the middle of such transition and that an increase in the system temperature might drive the structure to attain a more disordered phase, while temperature reduction might determine a partial recovery of the internal order.

What it is mostly important to us is that, while experiencing such melting transitions, the phospholipids change their effective lipid chain length, so the overall thickness of the cell membrane  $t_m$  is reduced. Since the electric specific capacitance of a membrane is given by

$$C_{\rm m} = \frac{\varepsilon}{t_{\rm m}},$$

a reduction of  $t_{\rm m}$  resulting from an increase of the system temperature *T* would result in an increase of  $C_{\rm m}$ , consistently with the experimental observation reported in section 3.1.3.

These speculations do not come with the claim of being a conclusive description of the observed phenomenon of membrane capacitance increase upon local heating, conversely they aim at giving a preliminary indication of possible mechanisms alternative to those proposed by Shapiro et al. Since an appropriate mathematical modeling of this systems would require to describe the mutual interaction between the cell membrane constitutive elements, i.e. phospholipids and proteins, and those of the surrounding environment, i.e. water molecules and dissolved ions, a continuum-based model approach is no longer appropriate, due to the very small scales involved. One should instead resort to using different techniques that are better suited for addressing these

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characteristic scales, such as molecular dynamics or Monte Carlo simulations, and this might be object of future activity.

# CHAPTER 4

# Finite element methods

In this chapter we focus on the description of the Primal Mixed (PM) finite element scheme that we use for the discretization of each PDE illustrated in chapters 2 and 3. For sake of generality, we consider an advection-diffusion-reaction (ADR) boundary value model problem (BVP) in one and two spatial dimensions in conservation form. The primal-mixed finite element method, in its lowest order form, provides an approximation to both flux and scalar dependent variables, and, in particular, enforces weak continuity of the flux across interelement boundaries with a piecewise linear continuous approximation of the scalar unknown that makes it structurally similar to standard displacement-based (SD) finite element schemes. An interesting feature that distinguishes the PM method from the SD formulation is that the former replaces the diffusion coefficient of the model by its harmonic average over each element of the grid while the latter replaces the diffusion coefficient of the model by its integral mean over each element of the grid. While the two approaches lead to similar results in the case of a slowly varying solution, the behavior of the two methods may be quite different in the presence of internal and/or boundary layers. In particular, in the case of the drift-diffusion model for the flux adopted in the present thesis in (2.2b) and (2.2c), it turns out that the PM discretization scheme automatically recovers the classical exponentially fitted Scharfetter-Gummel (SG) difference scheme [145] which is well-known to be an optimally accurate and robust method for the treatment of singularly perturbed advective-diffusive problems in one spatial dimension. We refer to [141] for the analysis of the SG method and for a complete survey of stable and accurate approximations of ADR equations. We also refer to [140] for an introduction and analysis of the PM finite element method in the case of BVPs with diffusive and reactive terms while for the case of BVPs where the flux is in drift-diffusion format we refer to [25, 26]. For the treatment of BVPs with possible degeneracy of the diffusion coefficient, leading therefore to a markedly advective character, we refer to [142] where an upwind stabilization mechanism is introduced in the PM formulation to avoid the onset of spurious unphysical oscillations in the computed solution.

# 4.1 The 1D model boundary value problem

We consider the following BVP for the dependent variable u = u(x)

$$\frac{\partial J}{\partial x} + \sigma u = f$$
 in  $\Omega = (0, L)$  (4.1a)

$$J = \upsilon u - \mu \frac{\partial u}{\partial x}$$
 in  $\Omega = (0, L)$  (4.1b)

$$\gamma J \cdot \boldsymbol{\nu} = \alpha u - \beta$$
 on  $\partial \Omega = \{0, L\}$ . (4.1c)

The differential system (4.1) is an advection-diffusion-reaction BVP in one spatial dimension and in conservation form and is representative of each of the equations introduced in section 2.2. In the flux balance equation (4.1a),  $\sigma$  and f are two given non-negative functions over [0, L] such that  $f - \sigma u$  represents the net generation term at each point of the domain. In the constitutive equation (4.1b) for the flux J, the quantities  $\mu$  and v are the diffusion coefficient and the particle velocity, respectively. Relations (4.1c) represent the boundary conditions and allow to completely close (4.1). In particular,  $(\alpha, \beta) : \partial \Omega \rightarrow (\mathbb{R}^+ \times \mathbb{R})$ , while  $\gamma$  may be equal to 1 or 0 at x = 0 and x = L. If  $\gamma = 1$ , the corresponding relation (4.1c) is a Robin condition, while if  $\gamma = 0$ , relation (4.1c) degenerates into the Dirichlet condition  $u = \beta/\alpha$ . In the case of a Robin condition, if  $\alpha = 0$  then the (outward) flux density  $J = -\beta$  is given at the boundary and relation (4.1) becomes a Neumann condition.

# 4.2 Unique solvability and maximum principle

In what follows we assume that the BVP (4.1) admits a unique (sufficiently smooth) solution pair (u, J). Let us then introduce the second-order linear differential operator

$$\mathcal{L}u := \frac{\partial J}{\partial x} + \sigma u = \frac{\partial}{\partial x} \left( -\mu \frac{\partial u}{\partial x} + v \, u \right) + \sigma u : \Omega \to \mathbb{R},$$

with  $\mu(x) \ge \mu_0 > 0$  and  $\sigma(x) \ge 0$  for all  $x \in \Omega$ . The following result holds.

**Theorem 4.2.1** (Inverse-monotonicity). Let  $u \in C^2(\Omega) \cap C^0(\overline{\Omega})$  be such that:

$$\mathcal{L}u(x) \ge 0 \qquad \qquad \text{for all } x \in \Omega \\ u(x) \ge 0 \qquad \qquad \text{for all } x \in \partial\Omega.$$

Then, we have

$$u(x) \ge 0$$
 for all  $x \in \Omega$ . (4.2)

In such an event, we say that  $\mathcal{L}$  is inverse-monotone or, equivalently, positivity preserving.

Unless otherwise stated, we assume throughout that  $\mathcal{L}$  is inverse monotone.

**Remark** (Physical interpretation of inverse monotonicity). Inverse monotonicity has an important physical significance, because it mathematically expresses the obvious fact that the dependent variable of the problem, say, a concentration, a temperature or a mass density, cannot take negative values. This is exactly the case of each equation in the mathematical model introduced in section 2.2.

#### **4.3** The primal-mixed finite element approximation

In this section, we are going to study the numerical approximation of the BVP (4.1) using the Primal Mixed Finite Element Method (PM-FEM). The word "mixed" is related to the fact that besides the primal variable u, also the variable J is treated as an independent unknown, while the word "primal" is referred to the fact that the PM formulation is a modification of the standard displacement (or primal)-based approach.

# 4.3.1 Finite element spaces

We start by introducing a partition (triangulation)  $\mathcal{T}_h$  of  $\overline{\Omega}$  into a number  $M = M(h) \ge 2$  of 1-simplex (open intervals)  $K_i = (x_i, x_{i+1}), i = 1, ..., M$ , in such a way that  $x_1 := a$  and  $x_{M+1} := b$ . We denote by  $h_i := x_{i+1} - x_i$  the length of each interval and set  $h := \max_{\mathcal{T}_h} h_i$ . We set N := M + 1. We associate with  $\mathcal{T}_h$  the following pair of function spaces

$$Q_h := \{ q_h \in Q : q_h | _{K_i} \in \mathbb{P}_0(K_i) \; \forall K_i \in \mathcal{T}_h \}$$
$$V_h := \{ w_h \in C^0(\overline{\Omega}) \subset V : w_h | _{K_i} \in \mathbb{P}_1(K_i) \; \forall K_i \in \mathcal{T}_h \}$$

 $Q_h$  is the vector space of piecewise constant polynomials defined over  $\Omega$ , while  $V_h$  is the vector space of piecewise linear continuous polynomials over  $\overline{\Omega}$ .  $Q_h$  and  $V_h$  are the finite element spaces of degree 0 and 1 associated with  $\mathcal{T}_h$ . The dimension of  $Q_h$  is simply equal to the number of elements M, while the dimension of  $V_h$  is equal to the number of vertices N = M + 1.

# 4.3.2 Basis functions

By definition of vector space, any function  $q_h \in Q_h$  and  $w_h \in V_h$  can be written in the form

$$q_h(x) = \sum_{k=1}^{M} q_k \psi_k(x)$$
 and  $w_h(x) = \sum_{j=1}^{N} w_j \varphi_j(x),$  (4.3)

where  $\psi_k$ , k = 1, ..., M and  $\varphi_j$ , j = 1, ..., N are the basis functions of  $Q_h$  and  $V_h$ , while the real numbers  $q_k$  and  $w_j$  are the degrees of freedom of  $q_h$  and  $w_h$ , that is, the coordinates of  $q_h$  and  $w_h$  with respect to the bases  $\{\psi_k\}$  and  $\{\varphi_j\}$ , respectively. We use for  $V_h$  Lagrangian basis functions, in such a way that the quantities  $w_i$  are the nodal values of  $w_h$  at the nodes of  $\mathcal{T}_h$ , i.e.

$$w_h(x_i) = \sum_{j=1}^N w_j \varphi_j(x) = \sum_{j=1}^N w_j \delta_{ji} = w_i \qquad i = 1, \ldots, N.$$

For  $Q_h$  we take as basis functions the characteristic function of  $K_k$ , for k = 1, ..., M

$$\psi_k(x) = \chi_k(x) := \begin{cases} 1 & \text{if } x \in K_k \\ 0 & \text{elsewhere.} \end{cases}$$

# 4.3.3 Finite element formulation

Given a bounded function  $g : \Omega \to \mathbb{R}$ , we indicate by  $g|_k$  the restriction of g over the element  $K_k$ , k = 1, ..., M, and we set

$$\overline{g}|_k := \frac{1}{h_k} \int_{K_k} g|_k(x) \, dx.$$

The (stabilized) primal mixed FE approximation of (4.1) reads:

Find  $(J_h, u_h) \in (Q_h \times V_h)$  such that:

$$\int_{0}^{L} \mu_{h}^{-1} \left( J_{h} - \upsilon u_{h} \right) q_{h} dx + \int_{0}^{L} \frac{\partial u_{h}}{\partial x} q_{h} dx = 0 \qquad \forall q_{h} \in Q_{h}$$

$$(4.4a)$$

$$\frac{1}{\gamma(0)}\alpha(0)u_{h}(0)w_{h}(0) + \frac{1}{\gamma(L)}\alpha(L)u_{h}(L)w_{h}(L) - \int_{0}^{L} J_{h}\frac{\partial w_{h}}{\partial x} dx + \int_{0}^{L} \sigma u_{h}w_{h} dx \quad (4.4b)$$
$$= \frac{1}{\gamma(0)}\beta(0)w_{h}(0) + \frac{1}{\gamma(L)}\beta(L)w_{h}(L) + \int_{0}^{L} f w_{h} dx \quad \forall w_{h} \in V_{h},$$

where  $\mu_h$  is the modified diffusivity defined as

$$\mu_h|_k := \mu|_k (1 + \Phi(\mathbb{P}e|_k)) \qquad k = 1, \dots, M,$$
(4.4c)

 $\Phi$  being a yet undefined nonnegative stabilization function of the local Péclet number

$$\mathbb{P}\mathbf{e}|_{k} := \frac{|\upsilon|_{k}|h_{k}}{2\mu|_{k}} \qquad k = 1, \dots, M.$$

$$(4.4d)$$

The modified diffusion coefficient  $\mu_h$  is the sum of the original diffusivity  $\mu$  and of the additional (local) contribution  $\mu|_k \Phi(\mathbb{P}e|_k)$ , k = 1, ..., M. This artificial diffusivity is to be properly selected in the discrete PM finite element model to enforce the non-negativity property (4.2) also for the solution  $u_h$  of system (4.4).

By definition of basis of a vector space, and using the expansions (4.3), system (4.4) becomes

$$\sum_{l=1}^{M} J_l \int_0^L \mu_h^{-1} \psi_l \, \psi_k \, dx - \sum_{j=1}^{N} u_j \int_0^L \mu_h^{-1} \upsilon \varphi_j \, \psi_k \, dx + \sum_{j=1}^{N} u_j \int_0^L \frac{\partial \varphi_j}{\partial x} \, \psi_k \, dx = 0$$

$$k = 1, \dots, M \tag{4.5a}$$

$$\frac{1}{\gamma(0)}\alpha(0)u_{1}\varphi_{1}(0) + \frac{1}{\gamma(L)}\alpha(L)u_{N}\varphi_{N}(L) - \sum_{k=1}^{M}J_{k}\int_{0}^{L}\psi_{k}\frac{\partial\varphi_{i}}{\partial x}\,dx$$
$$+ \sum_{j=1}^{N}u_{j}\int_{0}^{L}\sigma\varphi_{j}\varphi_{i}\,dx = \frac{1}{\gamma(0)}\beta(0)\varphi_{1}(0) + \frac{1}{\gamma(L)}\beta(L)\varphi_{N}(L) + \int_{0}^{L}f\,\varphi_{i}\,dx$$
$$i = 1, \dots, N.$$
(4.5b)

The PM system (4.5) can be written in matrix form as

$$\begin{bmatrix} \mathbf{A} & (\mathbf{B}^T + \mathbf{C}) \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{pmatrix} \mathbf{j} \\ \mathbf{u} \end{pmatrix} = \begin{pmatrix} \mathbf{g} \\ \mathbf{f} \end{pmatrix}$$

where  $\mathbf{A} \in \mathbb{R}^{M \times M}$  is the flux mass matrix,  $\mathbf{B} \in \mathbb{R}^{N \times M}$  is the matrix corresponding to the weak form of the divergence operator,  $\mathbf{C} \in \mathbb{R}^{M \times N}$  is the advective term matrix and  $\mathbf{D} \in \mathbb{R}^{N \times N}$  is a matrix accounting for boundary conditions, while  $\mathbf{u} \in \mathbb{R}^{N \times 1}$ ,  $\mathbf{j} \in \mathbb{R}^{M \times 1}$  is the unknown vector pair.

The *M* finite element equations (4.5a) constitute a diagonal system for the approximate flux  $J_h$ . In matrix terms we have, since matrix **A** is symmetric positive definite

$$\mathbf{j} = \mathbf{A}^{-1} \left( \mathbf{g} - (\mathbf{B}^T + \mathbf{C}) \mathbf{u} \right),$$

from which the flux densities  $J_k$ , k = 1, ..., M, can be computed independently over each mesh element  $K_k$ . To do this, we replace  $v|_k$  with its integral mean value  $\overline{v}|_k$  to obtain

$$J_k = -\mathcal{H}_k(\mu_h) \frac{u_{k+1} - u_k}{h_k} + \overline{v}|_k \frac{u_k + u_{k+1}}{2} \qquad k = 1, \dots, M$$
(4.6)

where

$$\mathcal{H}_k(\mu_h) := ((\mu_h)^{-1})^{-1}$$

is the harmonic average of  $\mu_h$  over the element  $K_k$ . Comparing (4.6) with the exact expression (4.1b), we see that the following approximations have been introduced over the element  $K_k$ :

$$\mu(x)|_{k} \quad \Rightarrow \quad \mathcal{H}_{k}(\mu|_{k} \left(1 + \Phi(\mathbb{P}e|_{k})\right)), \tag{4.7a}$$

$$\frac{\partial u(x)}{\partial x}|_{k} \quad \Rightarrow \quad \frac{u_{k+1} - u_{k}}{h_{k}}, \tag{4.7b}$$

$$(\upsilon(x)\,u(x))|_k \quad \Rightarrow \quad \overline{\upsilon}|_k \frac{u_k + u_{k+1}}{2} \equiv \overline{\upsilon}|_k \,\overline{u}_h|_k. \tag{4.7c}$$

Approximation (4.7a) states that the exact diffusion coefficient  $\mu$  has been replaced by the harmonic average of the modified diffusivity  $\mu_h$ . Approximation (4.7b) states that the exact diffusive flux  $\partial u(x)/\partial x$  has been replaced by the first-order incremental ratio  $(u_{k+1}-u_k)/h_k$ . Approximation (4.7c) states that the exact advective flux v(x)u(x)has been replaced by its average  $\overline{v}|_k \overline{u}_h|_k$ .

Having eliminated the variables  $J_k$ , k = 1, ..., M, in favor of the sole variables  $u_j$ , j = 1, ..., N, we can now replace the former variables into the finite element equations (4.5b). By doing so, we obtain the tridiagonal system

$$\mathbf{K}\mathbf{u} = \mathbf{b} \tag{4.8}$$

for the *N* unknowns  $u_i$ , i = 1, ..., N, where

$$\mathbf{K} = \left[ \mathbf{D} - \mathbf{B}\mathbf{A}^{-1}(\mathbf{B}^{T} + \mathbf{C}) \right]$$
$$\mathbf{b} = \mathbf{f} - \mathbf{B}\mathbf{A}^{-1}\mathbf{g}.$$

In the system (4.8),  $\mathbf{K} \in \mathbb{R}^{N \times N}$  is the stiffness matrix,  $\mathbf{u} \in \mathbb{R}^N$  is the column vector of nodal unknowns and  $\mathbf{b} \in \mathbb{R}^N$  is the (column) load vector. The stiffness matrix can be

expressed as the sum of four matrices

$$\mathbf{K} = \mathbf{K}^{\mathrm{d}} + \mathbf{K}^{\mathrm{a}} + \mathbf{K}^{\mathrm{r}} + \mathbf{K}^{\mathrm{bc}}$$

where  $\mathbf{K}^{d}$  is the diffusion matrix,  $\mathbf{K}^{a}$  is the advection matrix,  $\mathbf{K}^{r}$  is the reaction matrix and  $\mathbf{K}^{bc}$  is a matrix accounting for boundary conditions. The entries of the first and last rows of  $\mathbf{K}^{d}$ ,  $\mathbf{K}^{a}$  and  $\mathbf{K}^{r}$  are equal to zero, while  $\mathbf{K}^{bc}$  is the null matrix except for the first and last rows. The expressions of the entries of  $\mathbf{K}^{d}$ ,  $\mathbf{K}^{a}$  and  $\mathbf{K}^{r}$  corresponding to the internal nodes are, for i = 2, ..., N - 1

$$\begin{split} K_{i,i-1}^{d} &= -\frac{\mathcal{H}_{i-1}(\mu_{h})}{h_{i-1}}, \quad K_{i,i}^{d} &= \frac{\mathcal{H}_{i-1}(\mu_{h})}{h_{i-1}} + \frac{\mathcal{H}_{i}(\mu_{h})}{h_{i}}, \quad K_{i,i+1}^{d} &= -\frac{\mathcal{H}_{i}(\mu_{h})}{h_{i}}, \\ K_{i,i-1}^{a} &= -\frac{\overline{\upsilon}|_{i-1}}{2}, \quad K_{i,i}^{a} &= -\frac{\overline{\upsilon}|_{i-1}}{2} + \frac{\overline{\upsilon}|_{i}}{2}, \quad K_{i,i+1}^{a} &= +\frac{\overline{\upsilon}|_{i}}{2}, \\ K_{i,i-1}^{r} &= \int_{K_{i-1}} \sigma \varphi_{i-1} \varphi_{i} \, dx, \quad K_{i,i}^{r} &= \int_{K_{i-1}} \sigma \varphi_{i}^{2} \, dx + \int_{K_{i}} \sigma \varphi_{i}^{2} \, dx, \\ K_{i,i+1}^{r} &= \int_{K_{i}} \sigma \varphi_{i+1} \varphi_{i} \, dx. \end{split}$$

The entries of  $\mathbf{K}^{bc}$  corresponding to node  $x_1$  and node  $x_N$  are

$$\begin{split} K_{1,1}^{\mathrm{bc}} &= \alpha(0) + \gamma(0) \left[ \frac{\mathcal{H}_{1}(\mu_{h})}{h_{1}} + \frac{\overline{\upsilon}|_{1}}{2} + \int_{K_{1}} \sigma \varphi_{1}^{2} dx \right] \\ K_{1,2}^{\mathrm{bc}} &= \gamma(0) \left[ -\frac{\mathcal{H}_{1}(\mu_{h})}{h_{1}} + \frac{\overline{\upsilon}|_{1}}{2} + \int_{K_{1}} \sigma \varphi_{2} \varphi_{1} dx \right] \\ K_{N,N-1}^{\mathrm{bc}} &= \gamma(L) \left[ -\frac{\mathcal{H}_{M}(\mu_{h})}{h_{M}} - \frac{\overline{\upsilon}|_{M}}{2} + \int_{K_{M}} \sigma \varphi_{N-1} \varphi_{N} dx \right] \\ K_{N,N}^{\mathrm{bc}} &= \alpha(L) + \gamma(L) \left[ \frac{\mathcal{H}_{M}(\mu_{h})}{h_{M}} - \frac{\overline{\upsilon}|_{M}}{2} + \int_{K_{M}} \sigma \varphi_{N}^{2} dx \right] \end{split}$$

Also the load vector can be decomposed into the sum of two contributions,  $\mathbf{b}^{in}$  and  $\mathbf{b}^{bc}$ , accounting for internal nodes and boundary nodes, respectively. The expressions of the entries of  $\mathbf{b}^{in}$  are, for i = 2, ..., N - 1

$$b_i = \int_{K_{i-1}} f\varphi_i \, dx + \int_{K_i} f\varphi_i \, dx$$

while the entries of  $\mathbf{b}^{bc}$  are

$$b_1 = \gamma(0) \int_{K_1} f \varphi_1 \, dx + \beta(0)$$
 and  $b_N = \gamma(L) \int_{K_M} f \varphi_N \, dx + \beta(L)$ 

# 4.4 Stability analysis

In section 4.2 we assumed that the BVP (4.1) admits a unique solution, such that  $u(x) \ge 0$  for all  $x \in \overline{\Omega}$ . Here below, we provide sufficient conditions for the solution of the discrete counterpart (4.8) to enjoy analogous properties. For ease of presentation, we

consider the special and simple case where the coefficients  $\mu$ , v,  $\sigma$  and f are positive constants and homogeneous Dirichlet boundary conditions for u are applied, which amounts to setting  $\gamma = \beta = 0$  and  $\alpha = 1$  in (4.1c). The number of degrees of freedom for  $u_h$  is in this case equal to n := N - 1. We remark that completely similar conclusions and properties for  $u_h$  hold in the general case where problem coefficients are given positive functions of the position x and Robin boundary data are imposed.

# 4.4.1 Stabilization in the reaction-diffusion case

Set a = 0, b = 1,  $\mu = 10^{-4}$ ,  $\sigma = 1$ , v = 0 and f = 1. In this case  $\sigma \gg \mu$  and the BVP is strongly reaction-dominated. The stabilization approach consists of using the trapezoidal quadrature rule to compute the entries of the reaction matrix **K**<sup>*r*</sup>. The resulting modified linear algebraic system to be solved is

$$\widetilde{\mathbf{K}}\mathbf{u} = (\mathbf{K}^{\mathrm{d}} + \widetilde{\mathbf{K}}^{r})\mathbf{u} = \mathbf{b}$$

where  $\widetilde{\mathbf{K}}^r = \sigma h\mathbf{I}$ , I being the identity matrix of order *n*. Application of the discrete comparison principle (see [141]) allows to conclude that  $\widetilde{\mathbf{K}}$  is an M-matrix (see [165]) for all h > 0 from which it follows that  $u_h \ge 0$  in  $\overline{\Omega}$ . This latter property is often referred to as Discrete Maximum Principle (DMP).

**Remark** (Reduced integration and lumping). The use of a trapezoidal quadrature rule is called reduced integration, because instead of computing exactly the entries  $K_{ij}^{r}$ , we are deliberately introducing a quadrature error. The advantage of reduced integration is that the reaction matrix becomes diagonal. The diagonalization of the reaction matrix is often called lumping because the entries  $\tilde{K}_{ii}^{r}$  can be interpreted as obtained by summing by row the matrix  $\mathbf{K}^{r}$ . This is equivalent to "lump" the weight of the reaction term into each mesh node  $x_{i}$ .

The effect of the use of the trapezoidal numerical quadrature can be seen in figure 4.1, which shows the computed solution  $u_h$  over a mesh of M = 10 elements. In the left panel no lumping stabilization is used and the solution exhibits spurious oscillations in the boundary layer regions at x = 0 and x = 1. In the right panel the lumping technique is used on  $\mathbf{K}^r$  and oscillations completely disappear. It can be checked that  $0 \le u_h(x) \le 1/\sigma = 1$  for all  $x \in [0, 1]$  and for all h > 0, so that the DMP is satisfied (irrespectively of the mesh size h).

# 4.4.2 Stabilization in the advection-diffusion case

Set  $a = 0, b = 1, \mu = 5 \cdot 10^{-3}, \sigma = 0, v = 1$  and f = 1. In this case  $v \gg \mu$  and the BVP is strongly advection-dominated.

**Proposition 4.4.1.** Let the artificial diffusion  $\Phi(\mathbb{P}e)$  be chosen in such a way that

$$\Phi(\mathbb{P}\mathbf{e}) \ge \mathbb{P}\mathbf{e} - 1 \qquad \forall K_k \in \mathcal{T}_h.$$
(4.9)

Then, K is an irreducible diagonally dominant M-matrix with respect to its colums [165].

As a consequence of proposition 4.4.1, the solution  $u_h$  of the PM finite element system (4.8) is such that  $0 \le u_h(x) \le 1/v = 1$  for all  $x \in [0, 1]$  and for all h > 0, so



**Figure 4.1:** Reaction-diffusion problem. Exact and approximate solutions in the case  $\mu = 10^{-4}$ ,  $\sigma = 1$  and M = 10. (a) No lumping stabilization. (b) Lumping stabilization.

that the DMP is satisfied (irrespectively of the mesh size h). The simplest choice that allows to satisfy (4.9) is the upwind stabilization

$$\Phi(t) = t \qquad t \ge 0. \tag{4.10}$$

Another, more elaborate, choice is the so called Scharfetter-Gummel (SG) stabilization

$$\Phi(t) = t - 1 + \mathcal{B}(2t) \qquad t \ge 0, \tag{4.11}$$

where  $\mathcal{B}(x) := x/(e^x - 1)$  is the inverse of the Bernoulli function. This latter choice is also known as exponential fitting [141, 145]. The two above stabilizations tend to the same limit as the Péclet number increases. However, their behavior is quite different as the mesh size *h* decreases, because (4.10) introduces an artificial diffusion of O(h)as  $h \to 0$  while (4.11) introduces an artificial diffusion of  $O(h^2)$  as  $h \to 0$ . For this reason, the SG stabilized PM formulation is preferable as far as accuracy is concerned, and is the one implemented in the simulations reported in section 2.4.

The effect of the use of the SG stabilization can be seen in figure 4.2, which shows the computed solution  $u_h$  over a mesh of M = 10 elements. In the left panel no stabilization is used and the solution exhibits spurious oscillations all over the computational domain. In the right panel the SG technique is used on K<sup>d</sup> and oscillations completely disappear. It can be checked that  $0 \le u_h(x) \le 1/v = 1$  for all  $x \in [0,1]$  and for all h > 0, so that the DMP is satisfied (irrespectively of the mesh size h).

#### 4.5 Convergence analysis

Let us provide below a quantitative measure of the accuracy of the computed solution pair  $(J_h, u_h) \in (Q_h \times V_h)$  as a function of h in the case of homogeneous Dirichlet boundary conditions u(0) = u(L) = 0. In this case, the variable u is sought in the subset of the functions  $w \in H^1(\Omega)$  vanishing on the boundary  $\partial \Omega$ , i.e.

$$V \equiv H_0^1(\Omega) = \left\{ w \in H^1(\Omega) \text{ such that } w(0) = w(L) = 0 \right\}.$$

Such space is a Hilbert space endowed with the following norm

$$\|w\|_{V} = \|w\|_{H^{1}_{0}(\Omega)} := \left\|\frac{\partial w}{\partial x}\right\|_{L^{2}(\Omega)}$$



**Figure 4.2:** Advection-diffusion problem. Exact and approximate solutions in the case  $\mu = 5 \cdot 10^{-3}$ , v = 1 and M = 10. (a) No stabilization. (b) SG stabilization.

In the remainder of the discussion, we denote by C a positive constant independent of h, but possibly depending on u, J and on problem data, whose value is not necessarily the same at each occurrence.

**Theorem 4.5.1** (Convergence of the primal-mixed FE approximation). Assume that the solution pair of the BVP (4.1) with homogeneous Dirichlet conditions is such that  $u \in (H^2(\Omega) \cap H^1_0(\Omega))$  and  $J \in H^1(\Omega)$ , and let h be sufficiently small. Then, there exists a constant C > 0 independent of h such that

$$||u - u_h||_V + ||J - J_h||_Q \le Ch(||u||_{H^2(0,L)} + ||J||_{H^1(0,L)}).$$

Thus, the FEM (in non-stabilized and stabilized forms) is convergent with order 1 with respect to h.

For each continuous function  $\eta : [0,L] \to \mathbb{R}$ , we introduce the so-called discrete maximum norm associated with the triangulation  $\mathcal{T}_h$  and defined as

$$\|\eta\|_{\infty,h} := \max_{x_i \in \mathcal{T}_h} |\eta(x_i)|.$$

Then, the following results can be proved to hold.

**Theorem 4.5.2** (Uniform convergence for the reaction-diffusion problem). Assume that  $\mu \in C^1([0,L])$  and that  $\sigma$  and f are given continuous functions on [0,1], and let h be sufficiently small. Then, the FEM (in non-stabilized and stabilized forms) is convergent in the discrete maximum norm and we have

$$\|u-u_h\|_{\infty,h} \le Ch^2.$$

**Theorem 4.5.3** (Uniform convergence for the advection-diffusion problem). Assume that  $\mu \in C^1([0,L])$ ,  $v \in C^1([0,L])$  and f is given a continuous function on [0,1], and let h be sufficiently small. Then, the non-stabilized, the UP and SG stabilized FEMs are all convergent in the discrete maximum norm and we have:

$  u-u_h  _{\infty,h} \le Ch^2$	Non stabilized FEM
$\ u-u_h\ _{\infty,h} \le Ch$	Upwind stabilization
$\ u-u_h\ _{\infty,h} \le Ch^2$	SG stabilization.

The corresponding estimate shows that the upwind stabilization is far less accurate than the SG stabilization as h becomes asymptotically small. An even better result can be proved for the SG method in the special case of a constant coefficient advection-diffusion BVP.

**Theorem 4.5.4** (Nodal exactness of the SG scheme). Assume that  $\mu > 0$ , v and f are given constants (with f = 1), and that  $\mathcal{T}_h$  is a uniform partition of  $\overline{\Omega}$ . Then the solution  $u_h$  of the SG FEM satisfies the following relation

$$u_h(x_i) = u(x_i) = \frac{1}{v} \left[ x_i - \frac{e^{vx_i/\mu} - 1}{e^{v/\mu} - 1} \right]$$
  $i = 1, \dots, N$ 

Therefore, the SG scheme is nodally exact in the special case of constant coefficients and uniform grid.

#### 4.6 Conservation properties of the primal-mixed method

We conclude the characterization of the PM FEM adopted in the present thesis by illustrating its conservation properties with respect to those of the exact solution of the BVP (4.1).

#### 4.6.1 Conservation for the continuous problem

Let *x* be the spatial coordinate of a point belonging to the domain  $\Omega = (a, b)$ , let *t* be a time instant, and let  $\mathcal{V}_x \in \mathbb{R}^3$  denote an arbitrary time-invariant volume centered at *x* (see figure 4.3). Let us consider the conservation law (4.1a) for the quantity u = u(x, t) in the general time-dependent case

$$\frac{\partial u}{\partial t} + \frac{\partial J}{\partial x} + \sigma u = f \qquad \text{in } \Omega = (0, L), \qquad \forall t > 0.$$
(4.12)

The number of particles, associated with u = u(x, t), contained in the volume at time t is

$$N_u^x(t) = \int_{\mathcal{V}_x} u(x,t) \, dx \, dy \, dz.$$

**Theorem 4.6.1** (Conservation of mass). Let  $x^-$  and  $x^+$  denote the abscissae of the boundary of the volume  $V_x$  that surrounds point x (see figure 4.3). Then, we have

$$\frac{\partial N_u^x(t)}{\partial t} = \int_{V_x} \left( f(x,t) - \sigma(x,t)u(x,t) \right) dx dy dz - S \left( J(x^+,t) - J(x^-,t) \right).$$

**Figure 4.3:** Volume  $V_x$  for the calculation of particle number. S denotes the cross-section area of  $V_x$  in the plane y-z.



**Figure 4.4:** Electrical interpretation of flux balance at point x. The quantities  $I_{in}$  and  $I_{out}$  represent the nodal (particle) current entering and leaving node x. Left:  $\mathcal{P}_x = 0$ ; center:  $\mathcal{P}_x > 0$ ; right:  $\mathcal{P}_x < 0$ .

*Proof.* The result follows by integrating (4.12) over the volume  $V_x$  and using the fundamental theorem of Calculus.

Based on the above fundamental property, the BVP (4.1) expresses the physical fact that, in stationary conditions, the net production rate of u inside the volume (given by  $f - \sigma u$ ) exactly balances the net flux of u across the volume boundary (given by  $J(x^+) - J(x^-)$ ). Then, in absence of net production inside the volume  $(f = \sigma u)$ , the flux of u crossing the volume is constant  $(J(x^+) = J(x^-))$ . This is, for instance, what happens in the case of an ideal non-reacting fluid (for which the density is a constant in both time and space). If  $f - \sigma u > 0$  then the outflow flux is greater than the inflow flux  $(J(x^+) > J(x^-))$ , while the opposite situation occurs if  $f - \sigma u < 0$ . These three cases are summarized in figure 4.4 where the black dot represents point x, each black arrow represents a nodal (particle) current  $I := J \cdot S$  while each red arrow represents the volumetric net production term

$$\mathcal{P}_x := \int_{\mathcal{V}_x} (f - \sigma u) \, dx \, dy \, dz.$$

In equivalent electrical terms, we see that the mass conservation principle 4.6.1 is nothing but the classical Kirchhoff's current law at node x.

#### 4.6.2 Conservation for the the discrete primal-mixed problem

Let us consider the discretized equilibrium equation (4.5b) at each internal node  $x_i$ , i = 2, ..., N - 1. We get the following system of nodal conservation laws

$$J_i - J_{i-1} = (f_i - \sigma u_i) \left( \frac{h_{i-1} + h_i}{2} \right)$$
  $i = 2, ..., N - 1.$ 

The above equation expresses the fact that at each internal node of the partition the output flux  $J_i$  is equal to the sum of the input flux  $J_{i-1}$  plus the nodal net production term  $P_i := (f_i - \sigma u_i)(h_{i-1} + h_i)/2$ , in complete analogy with the conclusions of the previous section. In particular, if  $f = \sigma = 0$ , we get strong flux conservation at each internal node  $x_i$ , i = 2, ..., N - 1, otherwise flux is conserved in a weak sense, i.e., its jump at the node  $x_i$  is balanced by the nodal net production rate.

In conclusion, the PM finite element scheme reproduces in a remarkable manner on the discrete level the conservation properties enjoyed by the solution of the BVP (4.1) at the continuous level: exact current conservation across a control volume in the case of no production, weak current conservation across a control volume in the case of non-zero net production. In the former case, the adoption of the SG stabilization in the PM formulation allows to obtain current exactness at the barycenter of each mesh element unlike the SD FE approach. In the latter case, computational evidence shows that the accuracy of the flux  $J_h$  in the  $L^2$  norm is of order 2 with respect to the mesh size h, which shows that the SG-stabilized PM scheme is superconvergent with respect to both primal and mixed variables unlike the SD method.

For further consideration of these properties in the multi-dimensional case, we refer to [25, 26].

# 4.7 Finite element approximation in axisymmetric geometries

In this section we turn on attention to the solution of the ADR model problem in two spatial dimensions in the special case of axisymmetric geometry as that considered in section 3.3. To this purpose, we use the Edge Averaged Finite Element method (EAFE), a multidimensional extension of the Scharfetter-Gummel one-dimensional difference scheme [8, 42, 59, 100, 171]. The advantage of the EAFE method is that if a maximum principle holds for the problem on the continuous level, then the discrete counterpart holds too, giving rise to a "monotone scheme". A well-known sufficient condition for a scheme to be monotone is that the corresponding stiffness matrix is an M-matrix, and it can be shown that the stiffness matrix obtained with the EAFE method is an M-matrix under the sole assumption that the triangulation of the domain is of Delaunay type [171]. This result is very important, since applying the EAFE method to the ADR model problem ensures that the computed solution is strictly positive if the corresponding source term and boundary data are non negative.

# 4.7.1 Model problem in an axisymmetric configuration

For the sake of clarity, we introduce a model continuity problem for a function u, on a domain  $\Omega \subset \mathbb{R}^3$  with a Lipschitz boundary  $\partial \Omega = \Gamma_D \cup \Gamma_N$  such that  $\Gamma_D \cap \Gamma_N = \emptyset$ , as follows

$$\begin{cases}
-\operatorname{div} \mathbf{J} (u) + cu = f & \text{in } \Omega \\
\mathbf{J} (u) = \mu (\nabla u - \mathbf{b}u) & \text{in } \Omega \\
u = 0 & \text{on } \Gamma_D \\
-\mathbf{J} (u) \cdot \mathbf{n} = j_N & \text{on } \Gamma_N.
\end{cases}$$
(4.13)

Here  $\mu \in C^0(\overline{\Omega})$  is a strictly positive real function such that  $\mu = \mu(x) \ge \mu_0 > 0, \forall x \in \overline{\Omega}$ . The drift field can be written as  $\mathbf{b} := \nabla \psi, \psi$  being a continuous piecewise linear function over  $\overline{\Omega}$ , the reaction coefficient  $c \in L^{\infty}(\Omega), c \ge 0$  a.e. in  $\Omega$  and  $f \in L^2(\Omega)$ . Regarding the boundary conditions, for ease of presentation we consider homogeneous Dirichlet boundary conditions on  $\Gamma_D$ , while on  $\Gamma_N$  Neumann conditions are enforced, having  $j_N \in L^2(\Gamma_N)$  as a given datum.

Under these assumptions, we can reformulate the flux expression in a way that will be useful when studying the spatial discretization of the flux. Introducing the following change of variable

$$u := n e^{\psi} \tag{4.14}$$

and replacing (4.14) into the definition of the flux, yields

$$\mathbf{J}(n)=\mu e^{\psi}\nabla n.$$



**Figure 4.5:** Schematics of an axisymmetric configuration: the dependence on the  $\phi$ -coordinate can be neglected and the problem can be solved only in the  $\Omega_{as}$  part, obtaining a three dimensional domain with the rotation of  $\Omega_{as}$  around the symmetry axis.

The change of variable (4.14) is usually referred to as the Slotboom variables, see [85]. Using (4.14), problem (4.13) reads as follows

$$\begin{cases} -\operatorname{div} \mathbf{J}(n) + c e^{\psi} n = f & \text{in } \Omega \\ \mathbf{J}(n) = \mu e^{\psi} \nabla n & \text{in } \Omega \\ n = 0 & \text{on } \Gamma_D \\ -\mathbf{J}(n) \cdot \mathbf{n} = j_N & \text{on } \Gamma_N. \end{cases}$$
(4.15)

For our purpose, one can then rewrite problem (4.15) with the use of cylindrical coordinates  $(r, \phi, z)$  and obtain the boundary value problem

$$\begin{cases} \frac{1}{r}\frac{\partial}{\partial r}\left(rJ_{r}\right) + \frac{1}{r}\frac{\partial}{\partial\phi}J_{\phi} + \frac{\partial}{\partial z}J_{z} + ce^{\psi}n = f & \text{in } \Omega\\ n = 0 & \text{on } \Gamma_{D}\\ -\mathbf{J}\left(n\right) \cdot \mathbf{n} = j_{N} & \text{on } \Gamma_{N}, \end{cases}$$
(4.16)

where the operator  $\mathbf{J} = \mathbf{J}(n)$  is defined as

$$\mathbf{J} = \begin{bmatrix} J_r \\ J_\phi \\ J_z \end{bmatrix} = \begin{bmatrix} \mu e^{\psi} \frac{\partial n}{\partial r} \\ \mu \frac{1}{r} e^{\psi} \frac{\partial n}{\partial \phi} \\ \mu e^{\psi} \frac{\partial n}{\partial z} \end{bmatrix}, \qquad (4.17)$$

and the outward unit normal vector in cylindrical coordinates is  $\mathbf{n} = |n_r, n_{\phi}, n_z|$ .

At this point, we choose to study problem (4.16) in an axisymmetric configuration, for example the domain  $\Omega_{as}$  schematically depicted in figure 4.5. This two dimensional

domain, if rotated around its symmetry axis, defines a three-dimensional rotational solid. Rotational symmetry implies that the configuration is invariant with respect to the  $\phi$ -coordinate and then we can take  $\frac{\partial}{\partial \phi} = 0$ , ending up with the following model problem to be solved in  $\Omega_{as}$ 

$$\begin{cases} \frac{1}{r}\frac{\partial}{\partial r}\left(rJ_{r}\right) + \frac{\partial}{\partial z}J_{z} + ce^{\psi}n = f & \text{in }\Omega_{as}\\ n = 0 & \text{on }\Gamma_{D}\\ -\mathbf{J}\left(n\right) \cdot \mathbf{n} = j_{N} & \text{on }\Gamma_{N}, \end{cases}$$
(4.18)

where

$$\mathbf{J} = \begin{bmatrix} J_r \\ J_z \end{bmatrix}, \qquad \mathbf{n} = \begin{bmatrix} n_r \\ n_z \end{bmatrix}$$

and, for ease of notation, the boundaries  $\Gamma_N \cap \partial \Omega_{as}$  and  $\Gamma_D \cap \partial \Omega_{as}$  of the new domain are simply denoted by  $\Gamma_N$  and  $\Gamma_D$ , respectively.

To numerically solve system (4.18), we need to introduce a spatial discretization on the domain  $\Omega_{as}$ . Dealing with radial and cylindrical coordinates is a little different from the usual cartesian case, because the operators assume a different form. Besides, when writing the weak formulation, the integration procedure leads to the introduction of a new scalar product. The cylindrical test volume of integration is  $d\omega = rdr d\phi dz$ , which can be reduced to  $d\omega = rdr dz$  thanks to axial symmetry, having  $\int_0^{2\pi} d\phi = 2\pi$ . Therefore, for a given open set  $\Omega \subseteq \mathbb{R}^2$  we can define on  $L^2(\Omega)$  a new scalar product  $\langle \cdot, \cdot \rangle_{\omega}$  as

$$\langle f,g \rangle_{\omega} := \int_{\Omega} f(\omega)g(\omega)d\omega$$

$$= \int_{0}^{Z} \int_{0}^{R} f(r,z)g(r,z)rdrdz$$

$$= \int_{0}^{Z} \int_{0}^{R} \widetilde{f}(r,z)\widetilde{g}(r,z)drdz = \left\langle \widetilde{f},\widetilde{g} \right\rangle,$$

$$(4.19)$$

where  $\tilde{f} := \sqrt{r}f$  and  $\tilde{g} := \sqrt{r}g$ . Expression (4.19) shows that  $\langle \cdot, \cdot \rangle_{\omega}$  inherits all the properties of the usual scalar product. With the use of an analogous argument, one can introduce a "cylindrical measure"  $\omega$  starting from the Lebesgue measure  $\lambda$ , to actually measure manifolds using cylindrical coordinates and also to build the  $L^p$  spaces. In this way, all the usual properties are inherited and the usual results valid in the standard cartesian orthogonal case easily follow. For example, the new norm in  $L^2(\Omega)$  is defined on the scalar product (4.19) as

$$\|w\|_{\omega,L^{2}(\Omega)} = \langle w, w \rangle_{\omega}^{1/2} \quad \forall w \in L^{2}(\Omega).$$
(4.20)

Proceeding with the derivation of the weak formulation of problem (4.18), we can now integrate it against a test function v = v(r, z) and obtain

$$\int_{0}^{Z} \int_{0}^{R} \frac{1}{r} \frac{\partial}{\partial r} (rJ_{r}) vrdrdz + \int_{0}^{Z} \int_{0}^{R} \frac{\partial}{\partial z} J_{z} vrdrdz + \int_{0}^{Z} \int_{0}^{R} \frac{\partial}{\partial z} J_{z} vrdrdz = \int_{0}^{Z} \int_{0}^{R} fvrdrdz.$$

Applying Gauss theorem we end up with

$$-\int_{\Omega_{as}} J_r \frac{\partial v}{\partial r} d\omega + \int_{\partial \Omega_{as}} J_r v n_r ds_\omega - \int_{\Omega_{as}} J_z \frac{\partial v}{\partial z} d\omega + \int_{\partial \Omega_{as}} J_z v n_z ds_\omega + \int_{\Omega_{as}} c e^{\psi} n v d\omega = \int_{\Omega_{as}} f v d\omega.$$

Here we introduced the curvilinear abscissa in radial coordinates  $ds_{\omega} = rds$ , where ds is the usual curvilinear abscissa. At the end of this procedure we can regroup the terms and come to the usual weak formulation: find  $n \in V$  such that:

$$a_{\omega}(n,v) = F_{\omega}(v) \quad \forall v \in V$$

$$a_{\omega}(n,v) = -\left\langle \mu e^{\psi} \nabla n, \nabla v \right\rangle_{\omega} + \left\langle c e^{\psi} n, v \right\rangle_{\omega}$$

$$= -\int_{\Omega_{as}} \mathbf{J} \cdot \nabla v d\omega + \int_{\Omega_{as}} c e^{\psi} n v d\omega$$

$$F_{\omega}(v) = \left\langle f, v \right\rangle_{\omega} + \int_{\Gamma_{N}} j_{N} v ds_{\omega}$$

$$= \int_{\Omega_{as}} f v d\omega + \int_{\Gamma_{N}} j_{N} v ds_{\omega},$$

$$(4.21c)$$

where the vector  $\mathbf{J}$  is defined in (4.17). The functional space is defined on the new measure space in the usual way

$$V := H^{1}_{\Gamma_{D}}(\Omega_{as}) = \left\{ v \in H^{1}(\Omega_{as}) : v|_{\Gamma_{D}} = 0 \right\},$$
(4.21d)

with the norm defined as follows, thanks to the Poincaré inequality [133]<sup>1</sup>

$$\|w\|_{V} := \|\nabla w\|_{\omega, L^{2}(\Omega_{as})}.$$
(4.22)

The following theorem can be proved, as in the usual case of cartesian orthogonal coordinates.

**Theorem 4.7.1.** The bilinear form (4.21b) is continuous and coercive on V and the functional (4.21c) is continuous on V. Therefore the application of the Lax-Milgram Lemma ensures that problem (4.21) has a unique solution and that the following a-priori estimate holds

$$\|n\|_{V} \leq \frac{C_{\Omega_{as}} \|f\|_{L^{2}(\Omega_{as})} + C_{T} \|j_{N}\|_{L^{2}(\Gamma_{N})}}{e^{\psi_{m}} \mu_{o}}.$$

This automatically implies that (4.18) admits a unique weak solution  $u \in V$ .

*Proof.* Thanks to the hypotheses introduced for problem (4.13), we define the maximum and the minimum values of the functions  $\psi$  and  $\mu$  over  $\overline{\Omega}$  in the following way

$$\psi_M := \max_{\mathbf{x}\in\overline{\Omega}}\psi(\mathbf{x}), \qquad \psi_m := \min_{\mathbf{x}\in\overline{\Omega}}\psi(\mathbf{x}) \text{ and } \mu_M := \max_{\mathbf{x}\in\overline{\Omega}}\mu(\mathbf{x}).$$

<sup>&</sup>lt;sup>1</sup>The Poincaré inequality  $\|w\|_{L^2(\Omega_{as})} \leq C_{\Omega_{as}} \|w\|_{H^1_{\Gamma_D}(\Omega_{as})} = C_{\Omega_{as}} \|\nabla w\|_{L^2(\Omega_{as})}$  holds in this case because we are considering  $w \in H^1_{\Gamma_D}(\Omega_{as})$ , having homogeneous boundary conditions on  $\Gamma_D$ .

Using the norms defined in (4.20) and (4.22) and omitting the subscript  $\|\cdot\|_{\omega}$  for ease of notation, we start proving that the bilinear form  $a_{\omega}(\cdot, \cdot)$  is continuous on *V*. Thanks to the Hölder and Cauchy-Schwarz inequalities, the first integral in (4.21b) can be upper bounded as follows

$$\int_{\Omega_{as}} \mu e^{\psi} \nabla n \cdot \nabla \upsilon d\omega \bigg| \leq \mu_M e^{\psi_M} \|\nabla n\|_{L^2(\Omega_{as})} \|\nabla \upsilon\|_{L^2(\Omega_{as})} = \mu_M e^{\psi_M} \|n\|_V \|\upsilon\|_V.$$

Using the Poincaré inequality on the second term, we obtain

$$\left|\int_{\Omega_{as}} c e^{\psi} n \upsilon d\omega\right| \leq e^{\psi_M} \|c\|_{L^{\infty}(\Omega_{as})} C^2_{\Omega_{as}} \|n\|_V \|\upsilon\|_V.$$

We have then proved the continuity of  $a_{\omega}(\cdot, \cdot)$  on *V*, namely that there exists a constant M > 0 such that

$$|a_{\omega}(n,v)| \leq M ||n||_{V} ||v||_{V} \quad \forall n,v \in V, \qquad M = e^{\psi_{M}} \left( \mu_{M} + C_{\Omega_{as}}^{2} ||c||_{L^{\infty}(\Omega_{as})} \right).$$

The coercivity can be easily proved by considering only the first term of (4.21a), thanks to the hypothesis on the reaction term that  $c \ge 0$  a.e. in  $\Omega$ . Using the definition of the norm on V, we obtain

$$a_{\omega}(v,v) \geq \mu_0 e^{\psi_m} \|\nabla v\|_{L^2(\Omega_{as})}^2 = \mu_0 e^{\psi_m} \|v\|_V^2.$$

The bilinear form is therefore coercive with a coercivity constant  $\alpha$ , as follows

$$a_{\omega}(v,v) \geq \alpha \|v\|_{V}^{2} \quad \forall v \in V, \qquad \alpha = \mu_{0} e^{\psi_{m}}.$$

At last, we need to prove the continuity of the functional (4.21c). Using again the Poincaré inequality, we can find an upper bound for the first term as

$$\left| \int_{\Omega_{as}} f \upsilon d\omega \right| \leq \|f\|_{L^2(\Omega_{as})} \|\upsilon\|_{L^2(\Omega_{as})} \leq C_{\Omega_{as}} \|f\|_{L^2(\Omega_{as})} \|\upsilon\|_V$$

On the boundary integral, after the Cauchy-Schwarz inequality, we apply a trace inequality<sup>2</sup> in the following way

$$\left|\int_{\Gamma_N} j_N v ds_\omega\right| \leq \|j_N\|_{L^2(\Gamma_N)} \|v\|_{L^2(\Gamma_N)} \leq C_T \|j_N\|_{L^2(\Gamma_N)} \|v\|_V.$$

The linear functional is then continuous, with a continuity constant  $\Lambda > 0$  such that

$$|F_{\omega}(v)| \leq \Lambda ||v||_{V} \quad \forall v \in V, \qquad \Lambda = C_{\Omega_{as}} ||f||_{L^{2}(\Omega_{as})} + C_{T} ||j_{N}||_{L^{2}(\Gamma_{N})}.$$

Therefore, the assumptions of the Lax-Milgram lemma are verified for problem (4.21), and the following a-priori estimate holds

$$\|n\|_{V} \leq \frac{\Lambda}{\alpha} = \frac{C_{\Omega_{as}} \|f\|_{L^{2}(\Omega_{as})} + C_{T} \|j_{N}\|_{L^{2}(\Gamma_{N})}}{e^{\psi_{m}} \mu_{0}}.$$

<sup>&</sup>lt;sup>2</sup>For a function  $w \in L^{p}(\Gamma)$ , with  $\Gamma \subseteq \partial \Omega$ , the trace inequality states  $||w|_{\Gamma}||_{L^{p}(\Gamma)} \leq C_{T} ||w||_{H^{1}(\Omega)}$ .
#### 4.7.2 Spatial discretization

We are now able to apply the EAFE method [34,42] to the model problem in cylindrical coordinates introduced in section 4.7.1. In order to do that, we use piecewise linear finite elements on a regular triangulation  $T_h$  of the domain  $\Omega_{as}$  such that

$$\overline{\Omega}_{as} = \bigcup_{K \in T_h} K$$

with the following properties:

- $int(K) \neq \emptyset;$
- int  $(K_1) \cap$  int  $(K_2) = \emptyset$  for each distinct  $K_1, K_2 \in T_h$ ;
- if  $F = K_1 \cap K_2 \neq \emptyset$  (with  $K_1$  and  $K_2$  distinct elements of  $T_h$ ) then F is a common side or vertex of  $K_1$  and  $K_2$ ;
- diam $(K) \leq h, \forall K \in T_h$ .

For the purpose of simplifying the presentation, we assume that the triangulation covers  $\Omega_{as}$  exactly. Given  $K \in T_h$ , we introduce in figure 4.6 a local notation for the triangles in such a way that the vertices  $v_i$ , i = 1, 2, 3, are labeled in counterclockwise order, and we denote with  $\mathbf{e}_i$  the edge opposite to  $v_i$ , orienting it in such a way that it connects  $v_{i+1}$  to  $v_{i-1}$ . The cylindrical coordinates of the vertices are  $(r_i, z_i)$ ,  $l_i$  denotes each edge length,  $\mathbf{t}_i$  is the unit tangent vector oriented in the same direction as  $\mathbf{e}_i$  and  $\mathbf{n}_i$  is the unit outward normal vector to edge  $\mathbf{e}_i$ . As last, the segment from the midpoint of  $\mathbf{e}_i$  to the intersection of the perpendicular edge bisectors is denoted by  $s_i$ . We will also need a difference operator along  $\mathbf{e}_i$ , defined, for each continuous function  $\eta$ , as

$$\delta_i(\eta) := \eta(v_{i-1}) - \eta(v_{i+1}). \tag{4.23}$$

Let then

$$V_h = \{ \upsilon \in C^0(\overline{\Omega}_{as}) : \upsilon|_K \in \mathbb{P}^1(K), \ \forall K \in T_h \} \subset H^1_{\Gamma_D}(\Omega_{as})$$

be the piecewise linear finite element space (subspace of the functional space V defined in (4.21d)) and denote by  $\varphi_i$  the nodal basis function which is equal to one at  $v_i$  and to zero at the other vertices.

The equation associated with the generic test function  $\varphi_h$  over an element *K* for problem (4.21a) reads as follows

$$-\int_{K} \mathbf{J}(n_{h}) \cdot \nabla \varphi_{h} r dr dz + \int_{K} c e^{\psi} n_{h} \varphi_{h} r dr dz$$

$$= \int_{K} f_{h} \varphi_{h} r dr dz + \int_{\partial K \cap \Gamma_{N}} j_{N} \varphi_{h} ds_{\omega}$$
(4.24)

for  $n_h, \varphi_h \in V_h$ .

We start analyzing the first integral, namely we build an approximation  $J_h$  of the flux. This problem is dealt in the present work with the EAFE method, characterized by the approximation of the diffusion coefficient of the flux differential operator with



Figure 4.6: Parameters associated to a generic triangle K of the triangulation.

the harmonic average along the triangle sides  $\mathbf{e}_i$ . Given a function  $\eta \in C^0(\overline{K})$ , such average is defined as

$$\widehat{\eta_i} := \left(\frac{1}{l_i} \int_{\mathbf{e}_i} \eta^{-1} ds\right)^{-1}.$$
(4.25)

The first term in (4.24) becomes then

$$\int_K \mathbf{J}_h^{EA}(n_h) \cdot \nabla \varphi_h \, r dr dz,$$

where we introduce the discretized EAFE expression for the flux, defined with the use of relation (4.25) in the following way

$$\mathbf{J}_{h}^{EA}(n_{h}) = \sum_{j=1}^{3} J_{j}^{EA}(n_{h}) \, \mathbf{j}_{j}.$$
(4.26)

In this latter relation,  $\mathbf{j}_j$  is a vector-valued shape function associated with edge  $\mathbf{e}_j$  to be suitably defined, while  $J_j^{EA}$  is the associated degree of freedom for the flux. Since  $n_h$  is a piecewise linear function, using  $\delta_i$  as defined in (4.23), each component of the first term in the last relation can be explicitly written as

$$J_j^{EA} = \widehat{a}_j \nabla n_h \cdot \mathbf{t}_j = \widehat{a}_j \frac{\delta_j(n_h)}{l_j}, \qquad (4.27)$$

where  $\hat{a}_i$  is the harmonic average of  $\mu e^{\psi}$  along  $\mathbf{e}_i$ , in this case equal to

$$\widehat{a}_{j} := \left(\frac{1}{l_{j}} \int_{\mathbf{e}_{j}} \left(\mu e^{\psi}\right)^{-1} ds\right)^{-1}$$
$$= \mu e^{\psi_{j-1}} \mathcal{B}(\psi_{j-1} - \psi_{j+1}).$$

The choice of piecewise linear finite elements for the approximation of *n* is crucial, since in this way the flux projection  $J_j^{EA}$  along each triangle edge a constant. The basis function set for the flux approximation along the edge is defined as follows

$$\mathbf{j}_j = \frac{l_j s_j}{|K|} \mathbf{t}_j \qquad j = 1, 2, 3.$$

The above description therefore shows that  $J_h^{EA}(n_h)$  is a constant approximation of J(n) over the element K. In particular, (4.26) is a linear operator that allows to reconstruct a vector field over K starting from its tangential components along the triangle edges [8]. To numerically implement the method and to analyze its monotonicity, we construct the stiffness matrix associated with the generic element K. Substituting the test function  $\varphi_h$  with the basis function  $\varphi_i$ , i = 1, 2, 3 defined on the triangle K (see figure 4.6), we obtain

$$\int_{K} \mathbf{J}_{h}^{EA}(n_{h}) \cdot \nabla \varphi_{i} r dr dz = \sum_{j=1}^{3} J_{j}^{EA}(n_{h}) \int_{K} \mathbf{j}_{i} \cdot \nabla \varphi_{i} r dr dz$$

$$= J_{i-1}^{EA}(n_{h}) \int_{K} \frac{l_{i-1}s_{i-1}\mathbf{t}_{i-1}}{|K|} \cdot \nabla \varphi_{i} r dr dz$$

$$+ J_{i}^{EA}(n_{h}) \int_{K} \frac{l_{i}s_{i}\mathbf{t}_{i}}{|K|} \cdot \nabla \varphi_{i} r dr dz$$

$$+ J_{i+1}^{EA}(n_{h}) \int_{K} \frac{l_{i+1}s_{i+1}\mathbf{t}_{i+1}}{|K|} \cdot \nabla \varphi_{i} r dr dz.$$
(4.28)

For the computation of the integrals, we recall that the following relations hold in each K

$$\nabla \varphi_i = -\frac{\mathbf{n}_i}{h_i}$$
$$l_i \mathbf{t}_i \cdot \nabla \varphi_i = 0$$
$$l_{i \pm 1} \mathbf{t}_{i \pm 1} \cdot \nabla \varphi_i = \pm 1.$$

Using these properties, (4.28) becomes

$$\int_{K} \mathbf{J}_{h}^{EA}(n_{h}) \cdot \nabla \varphi_{i} \, r dr dz = \frac{J_{i+1}^{EA}(n_{h})s_{i+1}}{|K|} \int_{K} r dr dz - \frac{J_{i-1}^{EA}(n_{h})s_{i-1}}{|K|} \int_{K} r dr dz \qquad i = 1, 2, 3.$$
(4.29)

So far, the procedure is identical to that valid in the Cartesian case, but here we also need to account for the presence of *r* and to compute the term  $\int_{K} r dr dz$ . We adopt a mid-point quadrature rule, which in this case allows to compute esactly the integral as

$$\int_{K} r dr dz = \overline{r_{K}} |K| = \frac{r_{1} + r_{2} + r_{3}}{3} |K|, \qquad (4.30)$$

where  $\overline{r_K}$  is the radial coordinate of the center of mass of the element *K*.

Combining the use of relations (4.27) and (4.30) in (4.29), the explicit form of the flux integration over K can be easily written, highlighting the contribution of each

basis function:

$$\begin{split} \int_{K} \mathbf{J}_{h}^{EA}(n_{h}) \cdot \nabla \varphi_{1} \, r dr dz &= J_{2}^{EA}(n_{h}) s_{2} \overline{r_{K}} - J_{3}^{EA}(n_{h}) s_{3} \overline{r_{K}} \\ &= \left( \left( \widehat{a}_{2} \frac{s_{2}}{l_{2}} + \widehat{a}_{3} \frac{s_{3}}{l_{3}} \right) n_{1} - \widehat{a}_{3} \frac{s_{3}}{l_{3}} n_{2} - \widehat{a}_{2} \frac{s_{2}}{l_{2}} n_{3} \right) \overline{r_{K}} \\ &\int_{K} \mathbf{J}_{h}^{EA}(n_{h}) \cdot \nabla \varphi_{2} \, r dr dz = J_{3}^{EA}(n_{h}) s_{3} \overline{r_{K}} - J_{1}^{EA}(n_{h}) s_{1} \overline{r_{K}} \\ &= \left( \widehat{a}_{3} \frac{s_{3}}{l_{3}} n_{1} + \left( \widehat{a}_{1} \frac{s_{1}}{l_{1}} + \widehat{a}_{3} \frac{s_{3}}{l_{3}} \right) n_{2} - \widehat{a}_{1} \frac{s_{1}}{l_{1}} n_{3} \right) \overline{r_{K}} \\ &\int_{K} \mathbf{J}_{h}^{EA}(n_{h}) \cdot \nabla \varphi_{23} \, r dr dz = J_{1}^{EA}(n_{h}) s_{1} \overline{r_{K}} - J_{2}^{EA}(n_{h}) s_{2} \overline{r_{K}} \\ &= \left( -\widehat{a}_{2} \frac{s_{2}}{l_{2}} n_{1} - \widehat{a}_{1} \frac{s_{1}}{l_{1}} n_{2} + \left( \widehat{a}_{1} \frac{s_{1}}{l_{1}} + \widehat{a}_{2} \frac{s_{2}}{l_{2}} \right) n_{3} \right) \overline{r_{K}} \end{split}$$

The last expression in algebraic form reads as follows

$$A^K \mathbf{n}^K = \mathbf{f}^K,$$

where

$$A^{K} = \begin{bmatrix} \hat{a}_{2} \frac{s_{2}}{l_{2}} + \hat{a}_{3} \frac{s_{3}}{l_{3}} & -\hat{a}_{3} \frac{s_{3}}{l_{3}} & -\hat{a}_{2} \frac{s_{2}}{l_{2}} \\ -\hat{a}_{3} \frac{s_{3}}{l_{3}} & \hat{a}_{3} \frac{s_{3}}{l_{3}} + \hat{a}_{1} \frac{s_{1}}{l_{1}} & -\hat{a}_{1} \frac{s_{1}}{l_{1}} \\ -\hat{a}_{2} \frac{s_{2}}{l_{2}} & -\hat{a}_{1} \frac{s_{1}}{l_{1}} & \hat{a}_{1} \frac{s_{1}}{l_{1}} + \hat{a}_{2} \frac{s_{2}}{l_{2}} \end{bmatrix} \overline{r_{K}} \quad \text{and} \quad \mathbf{n}^{K} = \begin{bmatrix} n_{1} \\ n_{2} \\ n_{3} \end{bmatrix}.$$

Summing the above local contributions over each mesh triangle K, we can assemble the global stiffness matrix A of the problem. It is immediate to check that A is a symmetric and positive definite M-matrix (as in the cartesian case), because its entries satisfy the following conditions:

$$A_{jj} > 0 \quad \forall j; \qquad A_{ij} \le 0 \quad \forall i,j : i \ne j; A_{jj} \ge \sum_{i=1,i\ne j}^{N_h} |A_{ij}| \quad \forall j; \qquad A_{jj} > \sum_{i=1,i\ne j}^{N_h} |A_{ij}| \quad \text{for at least one } j.$$

$$(4.31)$$

With the considered regularity properties of the the problem coefficients and if the triangulation is of Delaunay type, then A is an irreducible M-matrix with respect to its columns, and the discrete maximum principle holds for the EAFE method. For a more general diffusion-convection equation, if we assume that the coefficients are piecewise smooth functions and the triangulation is weakly acute, then the stiffness matrix is still an M-matrix [83]. Returning to the original u variable, we need to invert (4.14) at each mesh node, obtaining

$$A^{K}\mathbf{n} = \begin{pmatrix} A^{K} \begin{bmatrix} e^{-\psi_{1}} & 0 & 0 \\ 0 & e^{-\psi_{2}} & 0 \\ 0 & 0 & e^{-\psi_{3}} \end{bmatrix} \mathbf{u}^{K} \qquad \mathbf{u}^{K} = \begin{bmatrix} u_{1} \\ u_{2} \\ u_{3} \end{bmatrix},$$

for each  $K \in T_h$ .

For the discretization of the reaction term  $\int_{K} ce^{\psi} n_h \varphi_h r dr dz$  and of the source term  $\int_{K} f_h \varphi_h r dr dz$  in (4.24), we adopt the same approximation used for the cartesian case in [42] to build the corresponding local reaction matrix and load vector. For the reaction term we can invert (4.14) and return to the original variable *u* before starting with the discretization procedure, having

$$\int_{K} c e^{\psi} n_h \varphi_h r dr dz = \int_{K} c u_h \varphi_h r dr dz.$$

We introduce the discretization of the function u using its nodal values  $u_j$  over the generic triangle K as  $u = \sum_{j=1}^{3} u_j \varphi_j$ . Once again, we substitute the test function  $\varphi_h$  with the basis function  $\varphi_i$ , i = 1, 2, 3, defined on the triangle K, and, in order to approximate the integrals, we use a trapezoidal quadrature rule, which yields

$$\int_{K} c u_{h} \varphi_{h} r dr dz = \sum_{j=1}^{3} \int_{K} c_{j} u_{j} \varphi_{j} \varphi_{i} r dr dz$$
$$= \frac{|K|}{3} \sum_{i=1}^{3} c_{i} u_{i} r_{i}.$$

Since we approximate the integral of a third order polynomial (result of the product of three first-order polynomials), we introduce a quadrature error of the order of  $h^2$ , which however does not degrades the accuracy of the formulation (which is O(h) in the  $H^1$ -norm and  $O(h^2)$  in the  $L^2$ -norm). By applying the same procedur to the source term, we have

$$\int_{K} f_h \varphi_h r dr dz = \frac{|K|}{3} \sum_{i=1}^{3} f_i r_i.$$

Regarding the boundary term in (4.24), we approximate the integral  $\int_{\partial K \cap \Gamma_N} j_N \varphi_h ds_\omega$ using the same argument as before. We need to account for all the edges **e** lying on the Neumann boundary  $\partial K \cap \Gamma_N$  of the considered triangle *K*, therefore we obtain

$$\sum_{\mathbf{e} \in \partial K \cap \Gamma_N} \sum_{i \in \mathbf{e}} \int_{\mathbf{e}} j_N \varphi_i r ds = \sum_{\mathbf{e} \in \partial K \cap \Gamma_N} \frac{|\mathbf{e}|}{2} \sum_{i \in \mathbf{e}} j_{N,i} r_i.$$

At the end of the above discretization procedure, we are able to write the local mass matrix over a triangle *K* as

$$M^{K} = \begin{bmatrix} c_{1}r_{1} & 0 & 0\\ 0 & c_{2}r_{2} & 0\\ 0 & 0 & c_{3}r_{3} \end{bmatrix} \frac{|K|}{3},$$

and the local load vector as

$$\mathbf{F}^{K} = \begin{bmatrix} f_{1}r_{1} \\ f_{2}r_{2} \\ f_{3}r_{3} \end{bmatrix} \frac{|K|}{3} + \sum_{\mathbf{e} \in \partial K \cap \Gamma_{N}} \frac{|\mathbf{e}|}{2} \sum_{i \in \mathbf{e}} j_{N,i}r_{i}.$$

Therefore, the discretized formulation of the system for the variable u derived above reads as follows

$$\left(A^{K}\begin{bmatrix}e^{-\psi_{1}} & 0 & 0\\0 & e^{-\psi_{2}} & 0\\0 & 0 & e^{-\psi_{3}}\end{bmatrix} + M^{K}\right)\mathbf{u}^{K} = \mathbf{F}^{K}$$

Defining the local matrix of the system as

$$\Sigma^{K} = A^{K} \begin{bmatrix} e^{-\psi_{1}} & 0 & 0 \\ 0 & e^{-\psi_{2}} & 0 \\ 0 & 0 & e^{-\psi_{3}} \end{bmatrix} + M^{K},$$

we can sum the above local contributions over each mesh triangle K and assemble the global matrix  $\Sigma$  of the entire system. In an analogous manner we assemble the global load vector **F**, so that the linear algebraic system associated with the EAFE discretization in axisymmetric configuration of (4.21) reads

$$\Sigma \mathbf{u} = \mathbf{F}.\tag{4.32}$$

As already pointed out above, the global stiffness matrix A is an M-matrix, and it is easy to extend properties (4.31) to  $\Sigma$  since the mass matrix is diagonal. One can then prove that for this discretized system the following theorem applies.

**Theorem 4.7.2.** The global matrix  $\Sigma$  of system (4.18) is an irreducible M-matrix with respect to its columns and the discrete maximum principle holds for the EAFE method under the regularity assumptions on the problem coefficients and if the triangulation is of Delaunay type. Thus, as a consequence, if  $\mathbf{F} \ge 0$ , then the solution of (4.32) is such that  $\mathbf{u} \ge 0$ .

#### 4.7.3 Convergence analysis on a test case for the Octave library

The EAFE scheme in axisymmetric configuration discussed above has been implemented as an extension of the already available Octave package bim, which originally allowed for the discretization of ADR problems in Cartesian coordinates in a one-, two- or three-dimensional framework. For this reason we introduced a set of routines in 1D and 2D to to implement the FE scheme in radial and cylindrical coordinates, respectively. For the validation of the code developed in the present work and in [1], we performed a broad set of test cases, and in particular here we discuss the results of a convergence analysis carried out on a two-dimensional ADR (we refer to the model continuity problem (4.13)), defined on the square domain  $\Omega_{as} = [1,2] \times [0,1]$ in the *r*-*z* plane, being the *z*-axis the rotational symmetry axis. We consider the case with  $\mu = 1$ , the drift field  $\mathbf{b} = \nabla \psi = [1,1]^T$ , c = 1, and the source term *f* and the Dirichlet boundary conditions enforced on  $\Gamma_{as}$  are suitably chosen in such a way that the exact solution is

$$u_{ex}(r,z)=z^2\ln r.$$

Figure 4.7 reports the values of the inf-,  $L^2$ - and  $H^1$ -norm of the error  $u_{ex} - u_u$  as a function of the mesh size h. It is remarkable to notice that the numerical solution



**Figure 4.7:** Convergence analysis:  $||u_{ex} - u_h||_{\infty(\Omega)}$ ,  $||u_{ex} - u_h||_{L^2(\Omega)}$  and  $||u_{ex} - u_h||_{H^1(\Omega)}$  as a function of the mesh size h.

 $u_h$  shows the same convergence properties as for the case of the EAFE method in Cartesian coordinates, as proved in [42, 100, 171].

# CHAPTER 5

# **Conclusions and future research perspectives**

### 5.1 Conclusions

The present PhD thesis is devoted to the experimental investigation and numerical description of the physical processes that underlie the functional behavior of a biopolymer interface device for light mediated cellular stimulation. Such device consists in a thin slab of poly(3-hexylthiophene) (P3HT), a photoactive semiconducting polymer, that is deposited on an Indium-Tin-Oxide (ITO) coated glass substrate. A cell is grown on top of the polymer layer, and the overall system is immersed in a physiological solution that allows cell survival during experiments.

This thesis aims at filling the gap of the lack of a consistent understanding of biopolymer interface devices for optical cellular stimulation and to provide a mathematical description of the involved phenomena. Two types of phenomena are identified as possible driving forces of the device operation, both resulting from light absorption by the polymer, namely:

- electric effects, determined by free charge carrier generation and subsequent displacement;
- thermal effects, determined by heat generation and diffusion, eventually modifying the cell membrane properties.

For this reason we performed a systematic program of experiments which are specifically designed to singularly address each of the two types of phenomena.

In order to analyze the impact of charge displacement in the device, a simpler configuration without the cell attached on top of the polymer has been considered. Transient photovoltage results are performed by varying several device parameters, such as active layer thickness, illumination direction and intensity, and solution molarity. Experimental results evidenced that the P3HT/ITO interface has a fundamental role in the generation of the photovoltage signal, since the magnitude and the characteristic time scale of the device response dramatically change upon switching the illumination direction, namely from the ITO or the solution side.

Starting from this evidence, we proposed a computational model for the transient photovoltage measurement which is equipped with all the features that characterize state of the art approaches in the field of organic photovoltaics modeling. More in detail the adopted model consists in a system of partial and ordinary differential equations representing:

- 1. generation, dissociation and motion of excitons in the material bulk according to Fick's law of diffusion;
- 2. electrochemical conduction of generated free electrons and holes in the polymer according to the Drift-Diffusion formalism;
- 3. trapping and release of charge carriers in localized trap states in the energy gap of the polymer;
- 4. electric field dependent exciton dissociation rate, (Braun-Onsager model);
- 5. energetic disorder induced effects according to the Extended Gaussian Disorder model;
- injection and recombination carrier dynamics at the ITO/P3HT interface, including electric field induced barrier lowering;
- 7. rearrangement of electric field inside the device according to Gauss law.

The original contribution of the present modeling approach consists in two main aspects:

- the extension of this type of model (usually adopted for standard photovoltaic devices) to a hybrid solid/liquid device;
- the inclusion of light excitation in the context of a pristine *p*-type semiconductor material (instead of an acceptor/donor blend). This leads to a bipolar transport description with the inclusion of electron dynamics that is instead usually neglected.

The resulting nonlinear system of equations is numerically solved by the adoption of several advanced techniques in the modern field of scientific computing. Time advancing is dealt with Rothe's method in conjunction with the use of automatic Backward Differentiation Formulae to allow for adaptive time stepping, which is crucial for the problem at hand because of widely varying temporal scales affecting the dynamical behavior of the device. Then, system linearization is carried out using the Newton method because of its quadratic rate of convergence once a suitable initial guess is available that is sufficiently close to the exact fixed point of the nonlinear problem. Finally, spatial discretization is dealt with by resorting to the use of exponentially fitted finite elements in primal-mixed form, incorporating the classical Scharfetter-Gummel stabilization term to avoid the occurrence of unphysical spurious oscillations in the computed solution. Notably, current conservation across interelement boundaries is enforced in a weak sense in the presence of net production terms while exact current conservation is automatically achieved in the case of equilibrium conditions.

Numerical results showed an excellent agreement with experimental trends and the following conclusions can be drawn:

- the device is characterized by the presence of a depleted region at the ITO/P3HT interface. This depletion region determines the onset of a electric field that promotes exciton dissociation and charge displacement, which in turn results in the measured photovoltage transient;
- trap dynamics plays a key role in the evolution of the photovoltage transient and is responsible for the long time scales (of the order of ms) which are experimentally observed.

In addition, the device has also been characterized using a particular kind of measurement in which the surface potential in close proximity of the polymer/solution interface is probed. Experimental results have been successfully described in terms of an equivalent electric circuit model and allowed to claim that

 the generation of a photovoltage in the polymer layer determines a capacitive signal in the surrounding electrolyte solution, which is consistent with cellular depolarization. However, since the photovoltage signal saturates for high illumination intensities, this mechanism cannot be enhanced in great extent by increasing the stimulus input.

In order to investigate whether heat-related mechanisms, that are well-known to be able to induce cellular excitation, can have an important impact also in the present configuration, patch clamp measurements upon photostimulation have been carried out on cell deposited on the active layer. By changing the substrate structure, (namely, P3HT/ITO/glass, P3HT/glass and photoresist/glass configurations have been considered) it has been possible to determine that, at the considered level of light excitation, the electric effect contribution does not play a significant role and that the measured cellular depolarization has to be ascribed to modifications of system temperature.

The adopted mathematical picture of the problem is constituted by a system of partial and ordinary differential equations that represent:

- heat generation and diffusion in the material and delivery to the cellular environment;
- the electrical response of cell membrane modeled with an equivalent electric circuit;
- the change of membrane capacitance, conductance and resting value due to the temperature increase.

Obtained results are in excellent agreement with measured data and support the proposed picture in which light absorption related heat generation determines a temperature increase in the electrolyte, which in turn modulates the cell membrane properties, ultimately resulting into cellular stimulation.

For this reason, the following conclusions can be claimed:

- temperature-induced modulation of the membrane capacitance is identified as the driving force of cellular depolarization;
- the developed optical stimulation protocol benefits of some specific advantages with respect to the mainstream infrared-based techniques. Since heat is generated only in the polymer region where light is absorbed, a higher spatial resolution can be achieved and lower power input is needed; moreover heat can be dissipated with a faster rate since the temperature of the water is lower than that of the excited polymer, so a higher stimulation frequency can be achieved.

To conclude we also analyzed an interpretation of the phenomenon of the cell membrane capacitance increase previously reported in the literature, based on the classic Guoy-Chapman-Stern theory of double layers. Since we have not been able to reproduce the reported model results and these latter suffer of a strong dependence on small modifications of the parameter values, more careful investigation is in order to draw definitely significant conclusions in terms of fundamental physical driving principles. A possible alternative mechanism based on temperature driven structural modification of the cell membrane due to a phase change of the lipid bilayer is proposed, which might be object of future experimental and simulation activity.

## 5.2 Future perspectives

As for the analysis of the electric effects occurring in the operation of the bio-polymer active interface, future activity could be devoted in further expanding the model capabilities.

- Most importantly, the coupling with the electrolytic solution would represent a major improvement, allowing for a fully consistent treatment of the whole system. Up to date double layer models should be employed, along with an accurate description of charge adsorption and of possibly occurring faradaic reactions.
- Another improvement that we envisage is the implementation of a more consistent boundary condition for the ITO/P3HT contact, accounting for the energetic disorder, which is known to further decrease the barrier height, allowing more intense injection current.
- Since the framework in which we defined problem (2.2) is not limited to the 1D setting that we employed in the current analysis, the implementation of a 2D/3D computational code might allow new investigations, as an example, the impact of a nanostructured polymer/solution interface might be assessed.
- From a purely mathematical point of view, a thorough existence and uniqueness analysis of the solution of problem (2.2) is a challenging task, due to the huge number of involved nonlinearities. A possible approach might be that of defining an iterative solution map similar to the classic Gummel-map and to prove its contractivity.
- In this activity, we assumed that ionic conductivity in P3HT could be efficiently neglected. Further experimental and modeling activity might be devoted to investigate in more detail this aspect, in order to support the approximation here adopted.

Also the analysis of the temperature dependent phenomena can be significantly extended, and especially from the experimental point of view, many possible investigations can be identified.

- A crucial activity would be that of repeating the patch clamp, capacitance and resistance measurements by changing the initial temperature level. Should the results be compatible with those already obtained, it would be a decisive demonstration that the observed phenomena depend only on the temperature variation and not on a the particular initial state of the system.
- In this analysis we considered passive cells in order to eliminate any active response and to focus on the effects due to the temperature induced modification of the membrane properties. Further work should be devoted to repeat the analysis using different kinds of cells, e.g. neurons, in order to understand if this stimulation protocol is universally effective.
- Correspondingly, numerical simulations might be performed using appropriate models developed for each specific type of cell.
- In our analysis of section 3.5.1 about the capacitance model proposed in [154], we evidenced that it has not been possible to reproduce the numerical results presented therein, and that the model sensitivity on the choice of the model parameters is quite pronounced. For these reasons in our opinion a more thorough investigation of the model is essential, in order to definitively assess the effective-ness of the model. However a more appropriate model approach would consist in a description of the membrane dynamics up to the molecular scale, using appropriate tools such as molecular dynamics simulations.

As the last point, since we determined that electric and thermal phenomena drive the device behavior for different regimes of the light illumination input, it would be extremely interesting to couple the two descriptions in a unique model, in order to investigate the device response in threshold regimes of light illumination density, in which the two effects have comparable impact on the device operation and might concur in determining the cellular depolarization.

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