#### Politecnico di Milano

V Facoltà di Ingegneria

Corso di laurea in Ingegneria delle Telecomunicazioni Dipartimento di Elettronica, Informazione e Bioingegneria



## Heuristic Approaches for a Energy-Aware Management problem in Cellular Networks

Supervisor: Dr. Giuliana CARELLO

Thesis by:

Mattia RACO

Student ID 800630

Session 2013 - 2014

### Ringraziamenti

Non è facile ringraziare, in poche righe, tutte le persone che hanno contribuito allo sviluppo di questa tesi di laurea: chi con una collaborazione costante, chi con un supporto morale o materiale, chi con consigli e suggerimenti o solo con parole di incoraggiamento, sono stati in tanti a dare il proprio apporto a questo lavoro. Innanzitutto desidero ringraziare la Dott.ssa Carello, relatore di questa tesi, per la grande disponibilità e pazienza nel rispondere alle mie domande e per tutto l'aiuto fornito durante questo periodo.

Un sentito ringraziamento alla mia famiglia che, con il suo sostegno morale ed economico, mi ha permesso di completare questo ciclo di studi e di raggiungere questo traguardo. Un ringraziamento speciale a mia nonna e a mia zia che con grande affetto mi hanno sempre sostenuto nel mio percorso scolastico.

Un ultimo ringraziamento a tutti gli amici per essermi stati vicino.

### Contents

Abstract	9
Sommario	10
Executive summary	11

1	Intr	oduction	14
2	Rela	ated Work	23
3	Cell	ular Network Technology	30
	3.1	Cellular Network	30
	3.2	Long Term Evolution	33
		3.2.1 Characteristics	34
		3.2.2 Architecture	35
		3.2.3 Interference	36
	3.3	Channel Attenuation	40
	3.4	Base Station	41
4	Pro	blem Description and Formulation	42
	4.1	Problem Description	42
	4.2	Formulation	48

<b>5</b>	Heu	istic	51
	5.1	Greedy	52
	5.2	Heuristic Concentration	60
	5.3	Geographical Decomposition	63
	5.4	Local search	65
	5.5	Very Large Scale Neighborhood	68
6	Res	lts	73
	6.1	Instances	74
	6.2	Results	84
		6.2.1 Results of Formulation	85
		6.2.2 Greedy Algorithms	88
		6.2.3 Local Search starting from greedy solutions	93
		6.2.4 Very Large Scale Neighborhood	05
		6.2.5 Heuristic Concentration and Local Search	07
		6.2.6 Geographical Decomposition and Local Search 1	14
	6.3	Results for Larger Instances	24
7	Con	clusions 12	27

# List of Figures

GHG and $CO_2$ annual global emissions $\ldots \ldots \ldots \ldots \ldots \ldots$	14
Summary of estimated $GtCO_2e$ abatement in ICT	15
Energy efficiency gains, traffic growth and energy reductions $\ . \ .$	17
Overall mobile data traffic per months	18
A simplified structure of a mobile cellular network	19
Power usage in a cellular network	19
Daily traffic variations of a mobile cellular network $\ldots \ldots \ldots$	20
Weekly traffic variations of a mobile cellular network $\ . \ . \ . \ .$	21
Access and Core network	31
	32
	35
	36
	37
	39
	40
	41
First improvement vs Steepest Descent	67
AMPL and CPLEX scheme	74
Traffic profile of Group A	78
	Summary of estimated GtCO2e abatement in ICT          Energy efficiency gains, traffic growth and energy reductions          Overall mobile data traffic per months          A simplified structure of a mobile cellular network          Power usage in a cellular network          Daily traffic variations of a mobile cellular network          Weekly traffic variations of a mobile cellular network          Access and Core network          Network Solutions from GSM to LTE          Simplified 2x2 MIMO scheme          LTE scheme          Differences between OFDMA e SC-FDMA          Block diagram of a base station transceiver          First improvement vs Steepest Descent          AMPL and CPLEX scheme

6.3	A typical heterogeneous network deployment	80
6.4	Greedy and Local Search Results - Group A	99
6.5	Greedy and Local Search Results - Group B	104
6.6	Heuristic Concentration and Local Search Results - Group A	108
6.7	Heuristic Concentration and Local Search Results - Group B	111
6.8	Geographical Decomposition and Local Search Results - Group A	115
6.9	Geographical Decomposition with Overlapping and Local Search	
	Results - Group B	115
6.10	Results of heuristics for Group A	120
6.11	Time Results of heuristics for Group A	120
6.12	Results of heuristics for Group B $\ldots \ldots \ldots \ldots \ldots \ldots \ldots$	121
6.13	Time Results of heuristics for Group B $\hdots$	121
6.14	Instance $50_{50}_{625}_{1}$	122
6.15	Solution of instance 50_50_625_1 with Geographical approach	122
6.16	Instance $80_220_1296$	124
6.17	Instance 80_220_1296 Result	126

# List of Tables

2.1	Related work on green wireless network	28
4.1	Set description	46
4.2	Parameter description	47
6.1	Instances Parameters	75
6.2	Time intervals duration - Group A	79
6.3	BS configurations and power level - Group A	81
6.4	CPLEX Results - Group A	86
6.5	CPLEX Results - Group B	87
6.6	Greedy Results - Group A	89
6.7	Greedy Results - Group B - Part 1	91
6.8	Greedy Results - Group B - Part 2	92
6.9	Local Search Results from Greedy_1 - Group A	95
6.10	Local Search Results from Greedy_2 - Group A	96
6.11	Local Search Results from Greedy_3 - Group A	97
6.12	Local Search Results from Greedy_4 - Group A	98
6.13	Local Search Results from Greedy_3 - Group B - Part 1	100
6.14	Local Search Results from Greedy_3 - Group B - Part 2	101
6.15	Local Search Results from Greedy_4 - Group B - Part 1	102
6.16	Local Search Results from Greedy_4 - Group B - Part 2	103
6.17	VLSN Results from Greedy_3 - Group A	106

6.18	Heuristic Concentration and Local Search Results - Group A	109
6.19	Heuristic Concentration without time limit Results - Group A	110
6.20	Heuristic Concentration and Local Search Results - Group $\mathrm{B}(1)$ .	112
6.21	Heuristic Concentration and Local Search Results - Group $\mathrm{B}(2)$ .	113
6.22	Geographical Decomposition and Local Search Results - Group A	116
6.23	Geographical Decomposition with Overlapping and Local Search	
	Results - Group A	117
6.24	Geographical Decomposition with Overlapping and Local Search	
	Results - Group B - Part 1	118
6.25	Geographical Decomposition with Overlapping and Local Search	
	Results - Group B - Part 2	119
6.26	CPLEX Results - Group C	125
6.27	Geographical Decomposition with Overlapping and Local Search	
	Results - Group C	125

#### Abstract

In the last period the concern on energy consumption and global warming has significantly increased. Reducing energy consumptions and  $CO_2$  emissions is one of nowadays fundamental challenges. The Information Communication Technology (ICT) area plays an important role in this context; on the one hand it provides an important contribution in offering new services in order to reduce the human impact on nature, on the other hand it is responsible for significant energy consumption. An important part of energy consumption within ICT is due to telecommunications networks and in particular to access part of mobile networks. In this context many projects have been developed with the aim of developing technologies and optimization techniques to minimize power consumption.

One of the most widely used approaches for minimizing wireless network energy consumption is to adapt the transmission power levels in access network to the real needs of users. In this way when the network is less used (e.g. during night) it is possible to save energy.

In this thesis we exploit this idea in order to plan and manage an energy efficient cellular network. The problem can be described as the problem of optimally switching on and off or modulating the power of a set of base stations, while guaranteeing a suitable service.

Since solving the mathematical model of the problem with solvers can be computationally challenging, we propose ILP - based heuristic algorithms and we test them on a large set of instances. The results show that the proposed heuristic techniques provide good quality solutions in reasonable computational times and outperform the formulation solved by a state-of-the-art solver on large size instances.

#### Sommario

Nell'ultimo periodo l'attenzione sui consumi energetici e il surriscaldamento globale ha assunto sempre più importanza. E' un obiettivo globale cercare di ridurre i consumi energetici e le emissioni di  $CO_2$ .

Il settore dell'ICT (Information Communication Technology) ricopre un importante ruolo in questo contesto; se da un lato offre un importante contributo nel garantire nuovi servizi con lo scopo di diminuire l'impatto delle attività umane sulla natura, dall'altro causa un notevole dispendio energetico. Una grande parte dei consumi energetici dell'ICT è legata alle reti di telecomunicazioni ed in particolare alle reti cellulari. In questo contesto sono stati sviluppati molti progetti con lo scopo di sviluppare tecnologie e tecniche di ottimizzazione per minimizzare i consumi.

Uno degli approcci più utilizzati per ridurre i consumi delle reti wireless è quello di adattare i livelli di potenza in trasmissione nella rete di accesso alle reali esigenze degli utenti. In questo modo quando la rete è poco utilizzata (es. durante la notte) è possibile risparmiare molta energia.

In questa tesi sfruttiamo questa considerazione per progettare e gestire una rete cellulare energeticamente efficiente. Il problema di decidere quando attivare le base stations e come modulare la loro potenza può essere descritto e formulato come un problema di ottimizzazione.

Dato che la risoluzione del modello matematico del problema può risultare poco efficiente soprattutto dal punto di vista del tempo necessario per ottenere la soluzione ottima, abbiamo sviluppato algoritmi euristici in grado di produrre risultati soddisfacenti in tempi ridotti. I risultati ottenuti dimostrano che le tecniche euristiche proposte producono risultati migliori rispetto a approcci basati sulla risoluzione di modelli matematici.

### **Executive Summary**

In recent decades, greenhouse gas (GHG) emissions have increased exponentially: climate change is one of the most significant challenges our world is facing today. The concern on energy consumption and global warming has significantly increased.

An important contribution in reducing energy consumptions and  $CO_2$  emissions can be provided by the Information Communication Technology (ICT): in fact it helps addressing the threat of climate change and reducing the human impact on nature. However pervasive information and communication technologies are not without drawbacks and are not environmentally friendly having also a negative impact on electric energy consumption, electromagnetic radiation and global warming. The ICT industry is responsible alone of a percentage that varies between 2% and 10% of the world energy consumption and causes about 2% of global  $CO_2$  emissions.

In this context many projects have been developed with the aim of developing technologies and optimization techniques which can reduce networks power consumption. It has been speculated that it will be possible to reduce by 90% the energy consumption of the sector in 2020 through the combination of technologies, architectures, components, algorithms and protocols, taking into account the increase in traffic and ensuring the same quality of service (QoS) to users.

An important part of energy consumption in ICT is due to telecommunications networks and in particular to the access part of mobile networks.

Wireless and mobile networks will increase exponentially to meet the new demand. This will increase usage emissions and the emissions associated with these infrastructure. Telecommunications networks are not designed with the aim of saving energy, but usually they are over-dimensioned to meet quality constraints in peak-traffic condition and they are under-utilized in low-traffic period. Thus the energy efficiency can be improved when the network is not transmitting at maximum load.

Energy aware management and planning exploits the possibility of reducing energy consumption by switching off unused device or by decreasing their emitted power. In this way it is possible to adapt energy consumption to real users needs and avoid waste of energy.

Based on such idea the aim of this thesis is to develop efficient algorithms for reducing energy consumption and energy related costs in a cellular network.

We consider an energy efficient cellular network planning problem. The problem consists in deciding where to install a set of BSs and how to equip them. Besides, from the operational point of view, the problem asks to adapt the BS power level to the traffic in different time intervals. We consider the planning and managing of a network over a long time horizon: thus beside installation costs (CAPEX), also operative costs (OPEX) are taken into account as both have a strong impact on the overall costs.

We formulate a model for the minimization of costs in the deployment and management of an energy efficient cellular network. We solve the integer linear programming formulation with a state-of-the-art solver; for large instances however the problem can be computationally challenging. To cope with this aspect we developed ILP - based heuristic algorithms. For the best of our knowledge this is the first time that a deep analysis on ILP - based heuristic methods is conducted in relation to this problem. To build a feasible solution four different greedy algorithms have been implemented and a Heuristic Concentration. Besides, we developed a technique based on area network subdivision into smaller areas according to geographical location to exploit the geographical features of the network. Different types of local search algorithms have been implemented to improve the constructive algorithms solutions.

From the results we observe that the heuristics combined with the use of ILP improve upon the use of solvers for solving the mathematical formulation of the problem; the best results are in particular obtained on large instances. The geographical decomposition based method seems to guarantee the best results.

#### Chapter 1

### Introduction

Climate change is one of the most significant challenges our world is facing today. In recent decades, greenhouse gas (GHG) emissions have increased exponentially: in 2011, annual GHG emissions were 48  $GtCO_2e$  and are projected to increase to 55  $GtCO_2e$  by 2020 and to 81  $GtCO_2e$  by 2050 [1] (Fig. 1.1). If nothing is done to stop the rise of emissions, these trends are likely to continue.

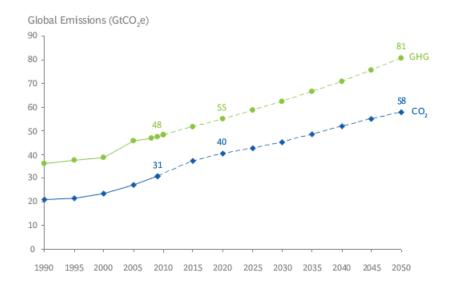


Figure 1.1: GHG and  $CO_2$  annual global emissions ( $CO_2e$  = the amount of  $CO_2$  that would create the same amount of GHG warming) (taken from [1])

It is a worldwide goal to reduce energy consumption and  $CO_2$  emissions. As

reported in [2], human industrial activities emit twice more  $CO_2$  than natural processes can absorb at the moment. The European Union has targeted a reduction of 20% for year 2020. Reducing emissions while maintaining economic growth and improving quality of life for people around the world is one of the fundamental challenges; an important contribution in this direction can be provided by the area of the Information Communication Technology (ICT): in fact it helps addressing the threat of climate change and the sustainable growth of the world's economies, across both developed and developing countries.

ICT has created opportunities to reduce the human impact on nature: in fact it stands to reduce total mileage and the amount of fuel required to transport people and goods. ICT-enabled solutions offer the potential to reduce annual emissions by an estimated 9.1  $GtCO_2e$  by 2020, representing 16.5% of the total emissions in that year (seven times the size of the sector's direct emissions)[1](Fig. 1.2).

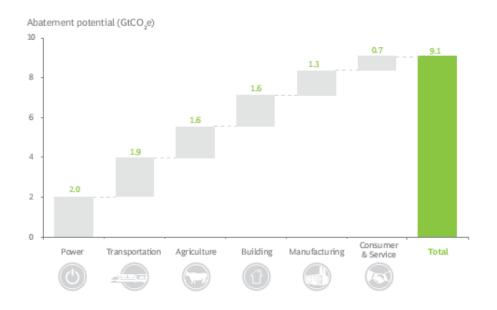


Figure 1.2: Summary of estimated  $GtCO_2e$  abatement in ICT (from [1])

Furthermore, it has developed innovative products and services that are now used in everyday life: for instance the adoption of telecommuting and video conferencing can reduce transportation needs. For such reason it contributes to reduce energy consumption and, accordingly, GHG emissions. The benefits are not limited to the transportation sector, but can be applied to other economic fields (agriculture and land use, buildings, manufacturing, power, and consumer and service).

On the other hand these pervasive information and communication technologies are not without drawbacks and are not environmentally friendly having also a negative impact on electric energy consumption, electromagnetic radiation and global warming. The ICT industry is responsible alone of a percentage that varies between 2% and 10% of the world energy consumption and causes about 2% of global  $CO_2$  emissions (equivalent to the emissions produced by the international air-traffic or by 50 million cars)[1]. It is estimated that ICT energy consumption is rising at 15-20% per year, doubling every five years. In addition widespread adoption of ICT devices has led to a rise in their related GHG emissions; from 2002 to 2011 emissions rose from  $0.53 \ GtCO2e$  to  $0.91 \ GtCO2e$  and are projected to rise up to  $1.27 \ GtCO2e$  by 2020. For these reasons, energy consumption of ICT sector has become a key issue, from ecological, energetic and economic point of view. Within ICT, an important part of the consumes is due to telecommunication networks (including mobile, WLANs, LANs and wired networks): the rapid and continuous development of telecommunications networks will lead soon to a huge increasing of their energy consumption. In Italy for example Telecom network consumes more than 2 TWh a year (1% of the total nation demand) and represents the second major consumer of electricity after the national railways [3]. As a consequence of the development of networks, in the recent years there has been a rapid development especially of mobile communication networks. Wireless and mobile networks will increase exponentially in the future to meet the new demand. This will increase usage emissions and the emissions associated with these infrastructure. However the possibility of energy consumption reduction will also increase (Fig. 1.3).

	Energy Efficiency Improvement Factor (2010 vs. 2020)	Traffic Growth (from 2010 to 2020)	Net Energy Reduction of 2020 Relative to 2010
Mobile Access	1,043x	89x	>90%
Wireline Access	449x	9.6x	98%
Core Network	64x	9.6x	85%

Figure 1.3: Energy efficiency gains, traffic growth and energy reductions that can be achieved in the mobile access, wireline access and core networks. (from [4])

Today there are more than 7 billion mobile subscriptions; this number is expected to double by 2020 [5]. In addition to more users, with the continued dramatic rise of applications, services, devices and machines all being connected to the network, the total Internet traffic in the next decade is expected to grow exponentially. Obviously, this growth is accompanied by an increased energy consumption of mobile networks.

The contribution of mobile communication networks is expected to be of 178 Megatons of  $CO_2$  in 2020, and the total power consumption is expected to grow to 98 TWh in 2020 [6]. In the future with the continuous growth in the market of cellular networks (the global mobile traffic is expected to reach 4.4 exabytes per month by year 2015 and about 20 exabytes in 2020 [5] (Fig. 1.4)), it will be necessary for the operators to optimize the energy efficiency of the networks (energy consumption over amount of traffic delivered).

Statistical analysis and projections suggest that global radio access networks might increase to more than 11.2 million Base Station (BS) sites in 2020 and the average traffic per site will increase up to somewhere between 11 to 18 Mbit/s. This implies that by 2020 the radio access network will have to improve its energy efficiency from about 28 J/kbit to 0.1 - 0.06 J/kbit only to maintain the same consumption.

Then with the continuous and inevitable increase of energy consumption, many institutions have financed and opened research groups in attempt to counter this problem. Several international research projects dedicated to energy efficient

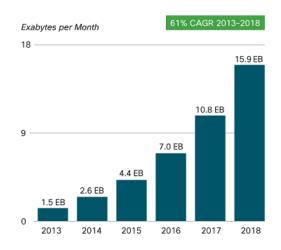


Figure 1.4: Overall mobile data traffic per months (from [5])

wireless communication have been carried out and are still active: GreenRadio, EARTH (www.ict-earth.eu), OPERAnet, eWIN [7]. The various projects designed to reduce energy consumption have pointed out that it will be possible to reduce by 90% the energy consumption of the sector in 2020 through the combination of technologies, architectures, components, algorithms and protocols, taking into account the increase in traffic and ensuring the same quality of service (QoS) to users.

The various components of a network consume energy in different ways; a wireless network is composed of three main parts: core network, access network and end users. The core network provides paths for the exchange of information between different sub-networks; it is able to provide transfer capabilities, multiplexing, switching, management, supervision and fault solving for signals transported. The access part instead is in charge of connecting the location of individual end users and provider: basically it consists of a set of transceivers (Base Station - BS) positioned more or less uniformly over a wide area. Finally all the devices used directly to communicate are called end users. A simplified structure of a mobile cellular network is shown in Fig. 1.5.

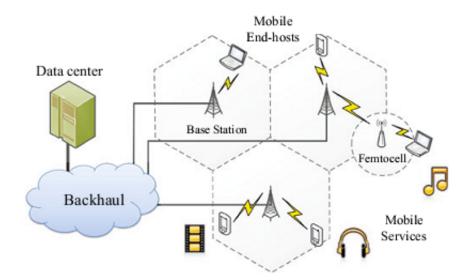


Figure 1.5: A simplified structure of a mobile cellular network (from [7])

In particular, the access part consumes most of the energy [8]: statistical analysis on power consumption in mobile telecommunications show that over 55% of the power is consumed by Base Stations. (Fig. 1.6)

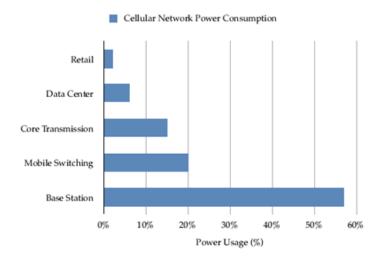


Figure 1.6: Power usage in a cellular network (from [9])

There are several approaches proposed to reduce the energy consumption of the networks. They can be classified into three large groups: those related to the partition of network resources for a more effective energy utilization (virtualization) and to new power saving protocols (software), those using and deploying new technologies in the design and production of network equipment (hardware and renewable resources) and those that deal with rethinking operation and planning features to reduce energy consumption (power management and redesign of the network)[10].

Telecommunications networks are usually not designed with the aim of saving energy, but are over-dimensioned to meet quality constraints in peak-traffic condition and are under-utilized in low-traffic period thus wasting a lot of energy. In particular, there is the possibility of improving the energy efficiency when the network is not transmitting at maximum load, which is a common condition in practice, as a wireless network is primarily providing coverage. For example, it can be easily seen that the traffic profile of the night-time is much lower than that of daytime. It is also observed that there is a slight difference between the traffic profiles of ordinary weekdays and weekend/holiday and between residential areas and urban areas. Since the operators need to deploy their BSs to support the peak time traffic, it is inevitable that the BSs are under-utilized most of the time, especially, at night (Fig. 1.7) and on weekends (Fig. 1.8).

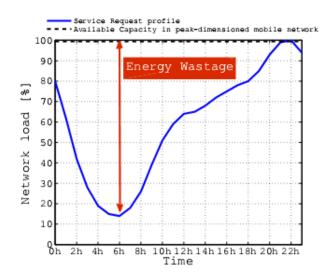


Figure 1.7: Daily traffic variations of a mobile cellular network (from [11])

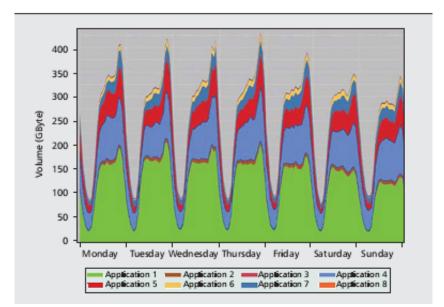


Figure 1.8: Weekly traffic variations of a mobile cellular network (from [11])

Energy aware management and planning exploits the traffic variation to reduce energy consumption by switching off unused device or by decreasing their emitted power.

In this thesis we exploit such idea and we focus on an energy efficient cellular network planning problem, namely the problem of optimally planning and managing networks to reduce energy costs. The problem consists in deciding where to install a set of BSs and how to equip them. Besides, from the operational point of view, the problem asks to adapt the BS power level to the traffic in different time intervals. We consider the planning and managing of a network over a long time horizon: thus beside installation costs (CAPEX), also operative costs (OPEX) are taken into account as both have a strong impact on the overall costs.

In particular the objective of this thesis is to develop efficient optimization algorithms for reducing energy related costs in a cellular network. Since solving ILP formulation of the problem with commercial solvers can be computationally challenging especially on large instances, our contribution is to study and develop heuristics approaches. We explore a large set of heuristic methods in order to figure out which one works better for the problem proposed. We developed in particular ILP - based heuristics. Such approaches combine heuristic features with the solution of ILP formulations and they have proved to be very promising. The proposed algorithms are able to find good solutions for the problem in reasonable computational times.

This work is organized as follows. In Chapter 2 we present an overview and careful analysis of the most innovative researches in green networking in the last years concerning wireless network. We pay particular attention to the various aspects considered by the authors and to the proposed methods.

In Chapter 3 we present an overview of the main characteristic and architecture of a real cellular network, in order to explain some aspects of the considered problem. The standard LTE is also briefly introduced. A propagative attenuation model and the architecture of a BS are also presented.

Chapter 4 focuses on the description of the considered problem. After a detailed description of the main aspects of the problem, we propose a ILP formulation.

In Chapter 5, instead, we present the developed heuristic algorithms.

Finally Chapter 6 is devoted to the computational results. It is explained how instances are generated and the results are reported. The results obtained are analyzed to evaluate the performances of the proposed methods.

Conclusive remarks and some proposals for future possible works and improvements conclude this thesis.

22

### Chapter 2

### **Related Work**

Many studies on green networking have appeared in recent years starting with [12]. An overview of main topics, researches and development opportunities can be found in [8] and [10]. Energy metrics are also proposed.

Due to development of networks, especially mobile communications will have a large expansion to meet the new needs of users. The attention on possible energy savings in wireless networks is therefore increasing. A detailed analysis of this topics can be found in [7] and in [11]. As mentioned, most of the energy consumption in wireless networks (80%) is due to the access part: it is therefore crucial to reduce consumption of access part. Our research is focused on the minimization of the energy in a cellular network.

The literature includes many researches on green networking that depend on the considered technology: typically they exploit some properties of transmission protocols for WLANs [13], or propose innovative developments of new hardware (above all improvements in power amplifiers) and efficient devices [14].

Our research instead deals with the planning and dynamic management of wireless networks and tends to be almost technologically independent. It is based on the minimization of energy consumption via an energy efficient management of network devices. Concerning wireless network management, one of the most promising approach for reducing network energy consumption is the use of sleep modes for BS: in fact it is convenient to shut down the BS that are in idle mode or modulate their power level if they are in low load conditions. It is thus possible to reduce the number of active cells by exploiting traffic variation. The characteristic common to all researches in this field is in fact represented by considering traffic changes over time.

Many approaches have been proposed to reduce energy consumption in wireless network by switching off unused device. The energy consumption management is often formulate as an optimization problem but there are some differences in the problem features and constraints such as the assignment of customers and the coverage of the area. Then the characteristics of BS can vary also: only in some papers for example power modulation is allowed or capacity constraints are imposed. Finally, there are differences in network structure such as the possibility of microcells integration or mechanism for interference-avoidance.

In the following we analyze the main characteristics that the different studies consider and we provide a more detailed description of those works which are more similar to our work.

Concerning assignment constraints, since the approaches proposed are usually applied to real networks, the main priority is to assign all the clients that need to access the network to BS. An exception is represented by [15]: the authors in fact allow some users not to be assigned to any BS; a penalty is assigned to each disconnected client, in order to minimize their number. The problem takes also in account the traffic demand uncertainty and propose a robust formulation and exact solution methods based on cutting planes.

A further division is based on coverage requirements. In some papers [16] only active clients, that generate traffic, must be covered. In [17] consider the possibility of activating the BS in cellular network only when necessary or requested (resource on demand) is considered: the authors propose to go beyond the traditional cellular architecture of wireless access networks, through a complete separation of the signaling infrastructure, that is used to request communication services, and the data infrastructure, that is used to provide these services.

In other papers, instead, such as [18], [19], [20] and [21], the total coverage of the area is imposed: some test points are uniformly distributed in the area and must always receive a minimum signal power.

Many studies consider the possibility of modulating the power transmission of the BS: some works, especially those that consider deterministic temporal traffic variations and offer an optimized solution through ILP ([18], [19], [20], [22]), allow to use only certain pre-defined levels of power assigned to the BS in each time intervals. Also in our research a multi-level configuration for BS is allowed since we believe that by using this strategy significant energy savings can be obtained. Other works consider more scalable power variations together with the change of coverage radius of the cell. The radius can be changed in two ways. First, by increasing the power, it is possible to reach more clients and cover larger areas (cell zooming) [23] allowing to shut down other BSs. Secondly, it may be useful to reduce the power of a cell when it is near congestion and the QoS is low: the radius of the cell decreases and clients are assigned to neighbor BSs (cell breathing) [24]. By adapting the power, 20% of the overall network power can be saved [25].

Another important aspect is the maximum capacity of the BS. In [3] a random choice of the BS to be shut down is applied to limit their number in a UMTS network: first the capacity is evaluated to verify the feasibility of switching off the cells; then the effects of switching off are evaluated taking into account the blocking probability and the electromagnetic exposure.

Other researches instead use a new approach to keep energy-efficient the network even when the traffic increases: the deployment and management of an heterogeneous network. The macrocells in fact are typically good at providing area coverage, but are not as effective in providing high data rates with reasonable energy consumption levels. The basic idea is to reduce the distance between client and BS: above the existing layer of macro-cells some micro-cells are then installed that have a shorter coverage radius (10 - 100 m), but consume less energy and meet the network request of capacity ([26], [7], [27]). Some macros BS in this way can be switched off: a heterogeneous deployment can reduce the total energy consumption by up to 60% in an urban area with today technology.

Despite the importance of reducing the distance between user and BS (most of energy is consumed during transmission and is proportional to the distance), few studies have focused on this aspect. The distance between user and BS has also an important impact on the quality of the received signal. Usually the assignment to the best available BS according to a criteria, such as quality of the received signal or distance, is not imposed (closest assignment) as one of the main constraint, but is evaluated as a secondary condition. Only in [28] the assignment to the nearest BS is forced in the ILP formulation, while [19] considers assignment based on strongest signal received (not always coincides with the nearest BS). In [18] instead, a penalty is introduced in the objective function to push the assignment of each test point to the nearest available BS.

In the problem addressed in this thesis a closest assignment constraint based on signal strength is considered.

Usually the interference between users or between BS is not directly taken into account in the optimization procedure due to assumptions about the protocol used. Some surveys [29] and [30] have highlighted how there is a trade-off between deployment efficiency and energy efficiency (cost-performance). This innovative aspect is analyzed in detail in [31] and [32]: the authors apply a separate approach for minimizing costs at design time and at run time; it is shown how an ideal configuration of the network can save up to 30% of energy. However more energy savings are possible taking into account the two phases simultaneously and incorporating energy management in the planning phase [18]. In [18] the objective function takes into account both installation costs (CAPEX) and operational cost (OPEX): they show that significant energy savings can be reached at the cost of little increase in installation investments. So designing energy efficient networks not only protects the environment but also reduces energy related operational costs. For this reason the aim of this thesis is the minimization of energy consumption based on an efficient design of the network and on a optimal management at the operational level (e.g. by adapting power level of BS according to traffic variations).

Concerning the methods used in the above described studies, a commonly applied approach is to formulate the considered problem as a integer linear programming (ILP) and solve it with solver; it leads to optimal solutions but it has the big disadvantage of requiring long computational times. Such formulations are used for instance for a typical cellular networks [19] and a wireless mesh networks [28]: the authors show that the savings obtained with optimal configurations and a complete knowledge of the traffic is about 45% per month.

To overcome the drawback of the ILP, many efforts have been done to develop algorithms capable of providing a solution in less time and therefore able to adapt quickly to traffic variations. Almost all the proposed algorithms are based on greedy. In [20] an heuristic approach able to obtain a solution close to optimal within a reasonable time is proposed. First they apply a greedy algorithm to build a feasible solution. Then a local search is applied to improve the solution.

Another example of heuristic approach is shown in [33]: they propose a greedy algorithm to adapt cells to the real traffic, thus shutting down some transceivers. Testing their algorithm on a real network topology, they reach savings up to 50% and they evaluate also the impact on QoS perceived by data traffic users.

In [32] the authors initially consider a configuration where all BS are powered on;

then the algorithm checks iteratively if a given BS can be turned off: the leastloaded BS or the most-overlapped cell are considered first (Heuristic algorithm based on Simulation).

Most of the papers focus on centrally managed networks, while in [21] and [34] heuristic algorithms are handled in a distributed manner. The basic idea is to trigger user-specific BS association by designing a BS selection preference function.

Finally, other interesting research topics are proposed in [35] where particular emphasis is given to distances between users and BS, in [36] where a method for switching on/off is proposed similar to the one adopted in wireless sensor networks, in [37] which underlines the importance of renewable energy sources and in [22] where ILP is combined with Benders decomposition.

In Table 2.1 the main characteristics of the problems considered in the above described papers are summarized together with the used approach.

	Switch on/off	Power Modulation	Heterogeneous Deployment	Area Coverage	Total Assignment	Closest Assignment	Capacity
ILP	[18][19] $[28]$	[18][19]	[18]	[18][19]	[28][18] $[19]$	[28][19]	[28][18] $[19]$
Heuristics	[17][32] [34][20] [35],[21] [33][36]	[35][20] [21]	[37][31]	[20][21]	[35][17][32] [33][20][23] [21]	[21]	[35][17][32] [33][34][20] [21]
Exact methods		[22]					[15][22]
Simulation	[3][38]	[3][24] [26]	[27][26]	[3][27]	[3][27] [24]	[24]	

Table 2.1: Related work on green wireless network

Our research aims at developing optimization approaches for energy consumption problem in a cellular network. We formulate the problem of minimizing energy consumption in planning and managing an energy efficient network. Differently from [18] we consider as hard constraint the assignment of each client to the BS that provide the best signal quality. Thus a particular formulation of the closest assignment constraint is introduced. Moreover both CAPEX and OPEX costs are considered in our problem. Since an optimization approach based on ILP methods can be computationally challenging above all on large instances, we developed several optimization approaches which combine heuristics and ILP formulations. Differently from the literature which is mainly focused on greedy and local search algorithms (that we also consider in detail), we propose two innovative approaches that to the best of our knowledge were never considered for solving such a problem. The former is based on heuristic concentration; the latter exploits the geographical characteristics of the problem.

#### Chapter 3

### Cellular Network Technology

Although our research is particularly focused on optimization approaches, it is based on realistic assumptions considerations on technologies used in modern networks. This section is an introduction to the technology that will be used for our tests. We show some of the general characteristics of a cellular network (Section 3.1), then we analyze more specifically how LTE standard works, to describe some of the assumptions made (Section 3.2). Finally we describe the attenuation of a wireless channel and the main components of a Base Station (Section 3.3 and 3.4).

#### 3.1 Cellular Network

A cellular network is a wireless network distributed over land areas called cells, each served by at least one fixed-location transceiver, known as base station (BS). The size of a cell can vary according to the number of users that have to be served in a certain area and the amount of traffic per user. Together such cells provide radio coverage over a wide geographic area.

The main parts of a cellular network are the core network and the access network. The former is the part of a telecommunication networks that transport big user data on geographical scale; it is defined as a set of network elements physically linked that can provide the transfer functionality, multiplexing, switching, management, supervision and fault detection for transported signals.

The access network instead is the part of a telecommunications network which connects subscribers to their immediate service provider. Conceptually, it resides behind a device such as a mobile phone, a computer, or any remotely controlled machine and provides connection with its core network (Fig. 3.1). Depending on the standard, mobile phones and other wireless connected devices are varyingly known as user equipment (UE), terminal equipment, mobile station (MS), etc..

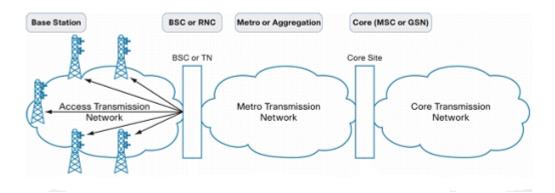


Figure 3.1: Access and Core network (from www.cisco.com)

A cellular mobile-radio network consists of the following elements:

- A network of radio base stations forming the base station subsystem.
- The core circuit switched network for handling voice calls and text.
- A packet switched network for handling mobile data.
- The public switched telephone network to connect subscribers to the wider telephony network.

Such structure is the base of the GSM system network. There are many functions that are performed by such network in order to provide customers the desired service including mobility management, registration, call set up, and handover. GSM was developed to carry real time services, in a circuit switched manner, with data services only possible over a circuit switched connection, with very low data rates. The first step towards an IP based packet switched solution was taken with the evolution of GSM to GPRS, specifically designed to transfer data over packet switched connections.

In order to reach higher data rates in UMTS (Universal Mobile Terrestrial System) a new access technology WCDMA (Wideband Code Division Multiple Access) was developed. Unlike second generation (GSM and GPRS) cellular systems, which were designed mainly for phone calls and low-rate data services, third generation systems are able to support also new multimedia and data services. The access network in UMTS emulates a circuit switched connection for real time services and a packet switched connection for data services.

The Evolved Packet System (EPS) is purely IP based. Both real time services and data services are carried by the IP protocol. (Fig. 3.2)

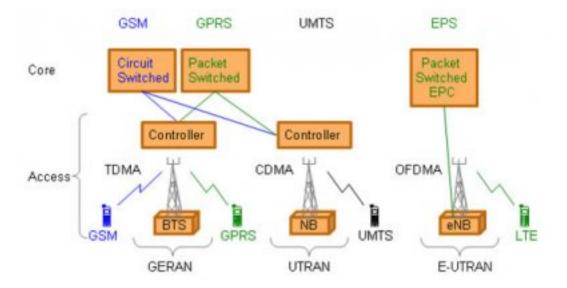


Figure 3.2: Network Solutions from GSM to LTE (from www.3gpp.org)

#### 3.2 Long Term Evolution

The development of new services and diffusion of advanced mobile devices (smartphone, tablet etc.) has revolutionized the concept of mobile phone. No longer terminal only dedicated to voice calls, but also a device that can provide a wide range of additional services.

To meet the ongoing and increasing demands of users, architectures and advanced network protocols that can make the best use of the available frequency band for communication are essential.

The Long Term Evolution (LTE) looks like a viable solution through the implementation of an efficient network. LTE or the E-UTRAN (Evolved Universal Terrestrial Access Network), introduced in 3GPP R8, is the access part of the Evolved Packet System (EPS). In contrast to circuit connection patterns characteristic of previous generations, where data packet communications were handled by dedicated nodes, LTE is designed to support only package services connection. All data, including voice, run on TCP/IP protocols and the connection between the mobile terminal and the external networks are IP type. The unification of all network protocols is one of the major innovations introduced by LTE that reduces costs and latencies.

LTE is continuing evolving to meet the increasingly demands of users and keep up with future wireless systems with the added benefit of enhancements having been introduced in all subsequent 3GPP Releases.

LTE Release 8 (2008) is one of the primary broad-band technologies based on OFDM. It is mainly deployed in a macro/micro-cell layout, it provides improved system capacity and coverage, high peak data rates, low latency, reduced operating costs, multi-antenna support, flexible bandwidth operation and integration with existing systems.

In LTE-Advanced the focus is on higher capacity: the aim in further developing LTE towards LTE-Advanced was to provide higher bit rates in a cost efficient way

and, at the same time, completely fulfill the requirements set by ITU for standard 4G.

Release 10 (2011) significantly enhances the existing LTE Release 8 and supports much higher peak rates, higher throughput and coverage, and lower latencies, resulting in a better user experience. Additionally, LTE Release 10 will support heterogeneous deployments where low-power nodes comprising pico-cells, femtocells, relays, remote radio heads, and so on are placed in a macro-cell layout.

We want to particularly stress the importance of this aspect, because as seen in Chapter 2, one of the key points of our research is the heterogeneity of the network.

Rel-12 (2013/2014) initiates the phase indicated as beyond LTE-Advance (i.e., LTE-B) with the aim of further increase the total capacity of the network and offer new wireless services.

#### **3.2.1** Characteristics

The main characteristics introduced by LTE are:

- Data-rate peaks using spatial multiplexing up to 300 Mb/s in downlink and 75 Mb/s in the uplink
- Scalable bandwidths from 1.25 MHz to 20 MHz,
- 200+ active users per cell (5 MHz)
- Operating bands currently ranging from 700 MHz up to 2.7GHz.
- Multi antenna configurations in both receiving and transmitting (MIMO) to increase the overall bit rate through transmission of two (or more) different data streams on two (or more) different antennas, using the same resources in both frequency and time (Fig. 3.3).

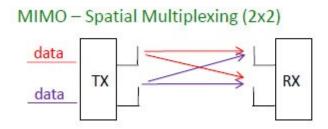


Figure 3.3: Simplified 2x2 MIMO scheme (from www.3gpp.org)

- High mobility support (high levels of quality of service guaranteed up to 15 km/h, with high performance from 15 to 120 km/h, however functional up to 350 km/h)
- Reduced latency (less than 100 ms for the transition from idle to active status, and lower to 5-ms for small IP packets)
- High spectral efficiency (the number of bits per second transmitted per Hz):
  3 times higher than the more advanced version of UMTS, or HSPA

#### 3.2.2 Architecture

LTE has a simpler network architecture than that of third generation networks; it is composed of three interactive domains: the UEs (User Equipments), the Evolved Universal Terrestrial Radio Access Network (E-UTRAN) and the Evolved Packet Core network (EPC).

The access network consist of a single element, the so-called Evolved Node B (eNB) (BS in our research), which includes all those features that in 3G UMTS were handled separately by Node B and Radio Network Controll (RNC). There is no centralized intelligent controller, but all the eNBs are interconnected through standardized interfaces (X2) to allow interoperability between several hardware technology and to enable functionality like mobility management, interference mitigation, energy saving procedures and coordinated transmissions. The reason

for distributing the intelligence among the base stations in LTE is to speed up the connection set-up and reduce the time required for a handover.

Each eNB is then connected to the core network through the S1 interface. The Mobile Management Entity (MME) and the serving Gateway (S-GW) serve as local anchors for the control and data plane respectively. Both interfaces can carry data and signaling information. (Fig. 3.4)

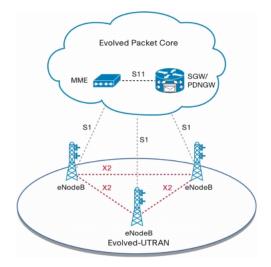


Figure 3.4: LTE scheme (from www.cisco.com)

Heterogeneous eNBs (HeNBs) are introduced mainly to provide coverage indoors, in homes or offices. The HeNB is a low power eNB that is used in small cells – femto cells. Usually it is owned by the customer, deployed without any network planning and connected to the operators EPC (Evolved Packet Core).

#### 3.2.3 Interference

In this subsection we analyze how is possible in a LTE network to avoid interferences between clients and between BSs.

LTE makes use of two innovative medium access schemes, both based on division of the available bandwidth in a set of orthogonal sub-carriers: the Orthogonal Frequency-Division Multiple Access (OFDMA) for traffic in downlink and Single-Carrier Frequency Division Multiple Access (SC-FDMA) for uplink.

Before describing these two multiplexing techniques we briefly recall the strategies used in previous telecommunications networks.

1. Frequency Division Multiple Access (FDMA): this strategy is used in first generation networks (i.e. TACS). Each user is associated with a specific portion of available bandwidth and all users transmit simultaneously occupying different sub-carriers. In the first cellular networks, each cell used a set of frequencies different from those used by neighboring cells, to avoid interference and provide guaranteed bandwidth within each cell. The limited power makes it possible to reuse the same frequency a few cells away without causing interference. In this way a geographic large area can be covered with only a limited number of frequencies, repeating the cluster of cells to cover all the territory. (Fig. 3.5)



Figure 3.5: Frequency reuse in a cellular network (from www.telecomabc.com)

2. Time Division Multiple Access (TDMA): this is the strategy used in second generation networks (i.e. and GSM). Each user has an associated time slots, during which it can transmit its data. All users share the same band, broadcasting at different times so as to not interfere with each other.

3. Code Division Multiple Access (CDMA): multiplexing is introduced in the third generation networks such as UMTS, and represents an evolution with respect to the solutions adopted in previous networks. In fact, both the FDMA and TDMA divide available resources in static way among users, regardless of their actual needs or the state of traffic and network congestion. On the contrary all clients in CDMA can transmit simultaneously on all the available bandwidth. Each user has an associated unique identifier (spreading code) and the separability between individual receiving signals is guaranteed by the use of orthogonal codes. In a CDMA system all cells can operate on the same frequency. The basic concept of a cellular CDMA network remains the same: instead of frequencies, the codes have to be distributed over the cells in such a way that the interference remains beneath a certain threshold.

The access technique implemented in LTE is the Orthogonal Frequency Division Multiplexing Access (OFDMA) which represent a multi-user evolution of the Orthogonal Frequency Division Multiplexing Modulation (OFDM).

In OFDM modulation scheme, the flow of data to be transmitted is split into N sub-streams, each individually modulated first (typically QPSK, 16QAM or 64QAM). Modulated symbols are transmitted in parallel, using a set of orthogonal sub-carriers, through an Inverse Discrete Fourier transformation block (IDFT). The orthogonality of sub-carriers allows not only the separation of individuals received flows, but also guarantees robustness to interference between different flows and of multi-path which are very common in urban and indoor environments. Then data blocks are again converted into a serial stream to which is added a cyclic prefix (CP). The purpose of the cyclic prefix is to mitigate the effects of interference between symbols.

In OFDMA multiplexing, each user has an associated subset of sub-carriers in which the available bandwidth is divided. The scheme is similar to the OFDM modulation in which the sub-carriers are no longer assigned to different blocks of data from the same user, but within different users. This assignment is not static but varies dynamically over time, depending on the users needs and the state of network. One of the main disadvantages of using this modulation is the high peak-to-average ratio (PAR), defined as the ratio of the peak factor and the root-mean-square value. A signal with a high PAR, requires power amplifiers with high linearity for being received correctly, which increase the cost of receivers. For this reason the strategy of multiplexing in uplink is a modified version of OFDMA.

The SC-FDMA access strategy is the technique used for traffic in uplink. The available bandwidth is divided into an array of orthogonal sub-carriers, that are no longer transmitted in parallel, as in OFDMA, but sequentially. (Fig. 3.6)

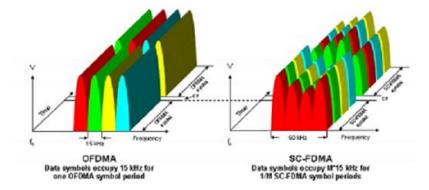


Figure 3.6: Differences between OFDMA e SC-FDMA (from [39])

In OFDMA each carrier is independently modulated, instead in SC-FDMA the transmitted signal on the single sub-carrier is a combination of all transmitted symbols at the same time.

This type of solution allows to reduce considerably the fluctuations of the transmitted signals, resulting in a lower PAR value compared to that found in OFDMA signals. However, the SC-FDMA is often affected by inter-symbol interference and an adaptive equalization systems in the frequency domain is required to cope with this problem.

#### **3.3** Channel Attenuation

We briefly introduce some of the characteristics of a wireless channel, focusing in particular on the attenuative behavior.

A radio propagation wave suffers from three main types of attenuation: path loss, shadowing, fading from multi-path In Fig. 3.7 is shown a typical attenuative behavior (K(dB) in relation to distance (log(d)).

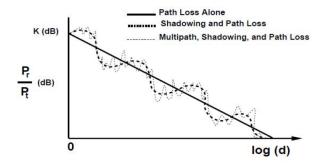


Figure 3.7: Channel attenuation (from www.wireless-communications-systems.it)

Shadowing is characterized by slow variations, also called large scale propagation effects because they occur when the propagation distance d is much larger than the wavelength  $\lambda$  ( $\lambda \ll d$ ). Shadowing is caused by obstacles (e.g. buildings) between the transmitter and the receiver that attenuate signal power through absorption, reflection, scattering, and diffraction.

The fading instead presents fast fluctuations of wave and occurs when  $\lambda >> d$ ; it is also called multi-path attenuation because is caused by various signal components due to reflections: these replicas may be received in phase which leads to strong reception or out of phase which leads to weak signal.

The path loss is finally caused by dissipation of the power radiated by the transmitter in free space as well as effects of the propagation channel; this type of attenuation is characterized by the following formula:

$$PL_{dB}(d) = 10 \ \log_{10} \left(\frac{4\pi d}{\lambda}\right)^2$$

#### **3.4** Base Station

In order to clarify some assumptions and some of the numerical values presented later in this thesis, it seems necessary to explain how a Base Station is built and how it works.

A BS consists of multiple transceivers (TRXs), each serving one transmitting antenna element. A TRX includes a Power Amplifier (PA), a Radio Frequency (RF) small-signal transceiver section, a baseband (BB) interface including a receiver (uplink) and transmitter (downlink), a DC-DC power supply, an active cooling system, and an AC-DC unit for connection to the electrical power grid (Fig. 3.8).

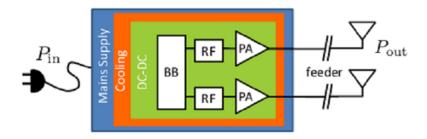


Figure 3.8: Block diagram of a base station transceiver (from [40])

In a conventional BS, the power consumption depends on the traffic load; it is mainly the power amplifier energy consumption that scales down due to reduced traffic load. This mainly happens when the number of occupied sub-carriers is reduced in idle mode operation, and/or there are sub-frames not carrying data. To take into account this aspect, some transmission power levels are then introduced for each device to vary the power according to the needs. Obviously, the higher the power, the greater the radius of coverage. The last level in particular takes into account the consumption, although minimal, that a BS has even when turned off: such minimal consumption is caused by fixed components (AC/DC conversion, filtering, signal processing, cooling) that do not depend on the radiated power and that consume electric power constantly in time.

### Chapter 4

# Problem Description and Formulation

The chapter is devoted to the description of the problem and to its formulation. We first present a the description of the considered problem (Section 4.1). We also explain the reasoning behind the joint design and management of energy-aware cellular networks. Then we describe more in detail the main characteristics of our model and some important assumptions we made (Section 4.2).

#### 4.1 **Problem Description**

The problem of planning a wireless network over a given area consists in selecting the position of Access Points (BS) in order to provide full coverage of the area and satisfy traffic requirements of customers. Moreover, traffic points must be assigned to access points. Service requirements of the network must be satisfied. Concerning green networking aspects, the main aim is to save energy by minimizing the energy consumption of the BSs while satisfying the traffic requirements from users and guaranteeing a suitable area coverage.

In particular our goal is to minimize the cost of deployment and installation

of the network (CAPEX) and the network operating and maintenance costs (OPEX), i.e. the energy consumption of the BSs. We can in particular save energy by adapting the power levels of network devices to real traffic. In fact, a communications network is usually designed to face peak traffic conditions and is thus over-dimensioned: indeed traffic varies over time (and also over space). Significant differences in traffic volumes can be observed on a daily basis (day and night), weekly (weekdays and holidays) or monthly (summer months or not). For instance, during daytime traffic load is generally higher in office areas than in residential areas (and vice-versa during the night).

Configurations that allow a large coverage radius are advantageous for area coverage (allowing to install a small number of BS and keeping low fixed cost per installation site), but consume a lot of energy. On the other hand, smaller cells offer greater flexibility in adapting to changes in local traffic and can cover more isolated areas, ensuring lower cost both of installation and of energy consumption. In [18] it is highlighted that a solution combining different types of configurations can lead to considerable economic savings in a network able to adapt to varying traffic load.

Only daily fluctuations are taken into account in this thesis since traffic behavior remains generally almost unchanged day by day. We then neglect differences between working and week days and between office and residential areas.

The model of a problem must be accurate enough to provide a good realistic solution, but it can not be too complicated because in that case it will be difficult to solve. It is therefore necessary to find a compromise between accuracy and efficiency. However it is useful to develop a problem formulation independent from the applied technology.

We consider a square area in which BSs must be installed.

A set of Candidate Sites (CS) where BSs can be installed is given, represented

by Set J.

For each  $j \in J$  an installation cost is given, that depends for example on the features of the site and on difficulties in installation or future maintenance works: open spaces or buildings, busy or isolated area, presence of particular buildings in the near area (e.g. schools, hospitals, airports may have restrictions on noise, radiation, etc..).

A BS can support different types of devices or configurations represented by the set K; an installation cost is given for each device  $k \in K$ .

The sum of these two costs represents the cost  $(\gamma_{jk})$  of installing a base station in site *j* equipped with a device *k*.

In the area there are two types of test points: the first ones (Set  $I_c$ ) are called covering test points (CTP) and are placed at regular intervals over a grid; by covering such test points the total area coverage is guaranteed. On the other hand, the set  $I_t$  represents the traffic concentration points (TTP). We assume that each traffic point does not represent a single client, but the aggregation of multiple client traffic requests. This approximation is reasonable especially if the density of customers is high (for example in working buildings or meeting spaces).

We consider a time horizon split in time slots to take into account different traffic levels. The set T represents the set of time intervals; parameter  $\delta_t$  represents the length of time slot  $t \in T$ .

Each traffic point  $i \in I_t$  requires certain amount of services  $(p_{it})$  during the day to BSs, consuming part of the capacity. One of the assumptions is that demand in each time slot is known and fixed.

Test points or traffic points can not be served by any BS. A BS can serve a test point or a traffic point only if it provides a suitable service.

The maximum coverage radius for a BS installed in CS j with device k and power level l is given. A CTP or a TTP can be covered by a BS if they are inside its service area. Two Covering matrix (as  $m_{ijkl}$  and  $a_{ijkl}$ ) summarize the coverage information for each CTP or TTP *i*, candidate site *j*, device *k* and power level *l*. The values of covering parameters depend on the quality of the received signal power, which depends on the propagation model and on the power emitted by the BS. Let  $q_{ijkl}$  denote the quality of received signal by *i* from a BS located in *j*, equipped with device *k* and operating at power level *l*.

Regarding the operational level, we assume that BSs are not always active at full power, but can modulate their emitted power level according to real traffic in order to save energy. The set of possible power levels is represented by the set L. For each device k and power level l parameter  $\varepsilon_{kl}$  describes the hourly consumption of a BS equipped with device k and operating at power level l, while  $\Gamma_{kl}$  represents its maximum capacity. Conventionally the capacity remains unchanged by changing the power level, except obviously for the sleep mode level; in sleep mode the capacity is zero.

Finally we need also two parameters  $\beta$  and  $\varphi$ :  $\beta$  is a numerical value used to assign a different weight to the CAPEX or to the OPEX costs in the objective function,  $\varphi$  instead takes into account the cost of energy and the network lifetime.

We consider the problem of planning and managing a cellular network in an efficient way with the aim of saving energy. The problem, called Energy-aware cellular network Management with Closest Assignment (EMCA), consists in choosing in which CSs install the BSs and with which device equip them. Then the power level of each device in each time slot must be decided. The networks have to satisfy some service requirements: all the CTP must be covered, the capacity of each BS can not be exceeded, all TTP must be covered and assigned to the BS that provides the best quality signal (Closest Assignment).

The considered problem can be seen as a particular case of capacitated multifacility (as each BS can be equipped with different devices), multi-level (as each BS can modulate its consumption) and multi-period (as traffic varies along the considered time horizon) facility location problem (CFLP). The CFLP is an optimization problem that considers potential facility sites with different fixed costs for locating a facility. The selected facilities have a given capacity which limits the demand they can serve. The optimal number of facilities to be opened is not known a priori. CFLP is known to be NP-hard (Non-deterministic Polynomial-time hard) to solve. Therefore our problem EMCA is NP-hard, as well as EMCA generalizes CFLP and CFLP can be reduced to EMCA.

Tables 4.1 and 4.2 summarize the sets and parameters used in the problem description.

SETS	
J	Set of the Candidate Sites (CS).
K	Set of available devices (configurations).
L	Set of available power levels.
$I_c$	Set of Coverage Test Points (CTP).
$I_t$	Set of Traffic Test Points (TTP).
Т	Set of time intervals.

Table 4.1: Set description

PARAMETERS	
$\delta_t$	Length of the time interval t
β	Weight parameter for combining CAPEX and OPEX terms in the objective function.
$\varphi$	Cost of the energy consumption over the entire network life.
$\gamma_{jk}$	Installation cost for a BS located in site $j$ with configuration $k$ .
$\varepsilon_{kl}$	Hourly power consumption for a BS with configuration $k$ and power level $l$ .
$\Gamma_{kl}$	Capacity of a BS with configuration $k$ and power level $l$ .
$p_{it}$	Traffic of the Traffic Point $i$ in time slot $t$ .
$m_{ijkl}$	Coverage parameter for test point $i$ , BS $j$ with device $k$ and power level $l$ .
$a_{ijkl}$	Coverage parameter for traffic point $i$ , BS $j$ with device $k$ and power level $l$ .
$q_{ijkl}$	Quality of service for client $i$ served by BS $j$ with device $k$ and power level $l$ .

 Table 4.2: Parameter description

#### 4.2 Formulation

The BS location, the power levels used in each time slot and the assignment of clients in each time slot can be formulated using binary decision variables.

A binary variable is defined for each CS j and each device k, and models the location and equipment decision:

$$z_{jk} = \begin{cases} 1, & \text{if a BS is installed in site } j \in J \text{ with configuration } k \in K, \\ 0, & \text{otherwise} \end{cases}$$

The choice of the power level of each BS j in each time slot t is represented by binary variable  $y_{ijkl}$ :

$$y_{ijkl} = \begin{cases} 1, & \text{if a BS installed in site } j \in J \text{ with configuration } k \in K \\ & \text{is active with power level } l \in L \text{ during time slot } t \in T, \\ 0, & \text{otherwise} \end{cases}$$

Finally we need a set of variables to represent the assignment of the traffic points to installed BSs in each time slot.

$$x_{ijt}^{kl} = \begin{cases} 1, & \text{if TP } i \in I_t \text{ is assigned to a BS installed in site } j \in J \\ & \text{with configuration } k \in K \text{ and power level } l \in L \text{ in period } t \in T, \\ 0, & \text{otherwise} \end{cases}$$

Using the above described sets, parameters and variables, the problem can be formulated as follows:

#### EMCA:

$$\begin{split} \min & \sum_{j \in J} \sum_{k \in K} \gamma_{jk} z_{jk} + \beta \varphi \sum_{j \in J} \sum_{k \in K} \sum_{l \in L} v_{el} \delta_l y_{jklt} & (1) \\ \text{s.t.} & \sum_{j \in J} \sum_{k \in K} \sum_{l \in L} m_{ijkl} y_{jklt} \geq 1 & \forall i \in I_c, t \in T & (2) \\ & \sum_{j \in J} \sum_{k \in K} \sum_{l \in L} a_{ijkl} x_{ijt}^{kl} = 1 & \forall i \in I_t, t \in T & (3) \\ & \sum_{i \in I_t} \sum_{k \in K} \sum_{l \in L} p_{it} x_{ijt}^{kl} \leq \sum_{k \in K} \sum_{l \in L} \Gamma_{kl} y_{jklt} & \forall j \in J, t \in T & (4) \\ & \sum_{j \in J} \sum_{k \in K} \sum_{l \in L} p_{it} x_{ijt}^{kl} \leq 1 - y_{hk'l't} & \forall i \in I_t, h \in J, k' \in K, l' \in L, t \in T & (5) \\ & \sum_{l \in L} y_{jklt} = z_{jk} & \forall j \in J, k \in K, t \in T & (6) \\ & \sum_{k \in K} z_{jk} \leq 1 & \forall j \in J & (7) \\ & x_{ijt}^{kl} \leq y_{jklt} & \forall i \in I_t, j \in J, k \in K, l \in L, t \in T & (8) \\ & z_{jk} \in \{0, 1\} & \forall j \in J, k \in K & (9) \\ & x_{ijt}^{kl} \in \{0, 1\} & \forall i \in I_t, j \in J, k \in K, l \in L, t \in T & (10) \\ \end{split}$$

 $y_{jklt} \in \{0, 1\} \qquad \qquad \forall j \in J, k \in K, l \in L, t \in T$ (11)

The objective function (1) aims at minimizing the overall costs and is composed of two terms: the installation costs (CAPEX) and the operational expenditures (OPEX).

Constraints (2) provide a minimal coverage over all time slots by ensuring that all the CTP are within the service area of at least one installed and active BS.

The assignment constraints (3) guarantee that each traffic point is assigned to only one BS in each time interval, among those which provide enough received power. Constraints (4) guarantee that the amount of traffic assigned to a BS does not exceed the capacity of the installed device.

Constraints (5) represent the closest assignment constraints: in fact they forced the assignment of a traffic point to the BS that provides the best signal quality among the active ones. We formulate this characteristic as a hard constraint: this means that if it is not possible to assign a traffic point to the closest active BS (e.g. due to congestion), the instance is considered infeasible, neglecting the possibility to assign the traffic point to another BS.

Constraints (6) guarantee that each installed base station chooses only one power level for each time interval, while configuration constraints (7) ensures that at most one configuration is chosen for every CS.

Consistency constraints (8) guarantee that if a client is assigned to a BS in a time interval, the BS must be active in that time interval.

Finally constraints (9), (10), (11) impose the binary domain to all variables.

### Chapter 5

### Heuristic

Although an optimization approach based on solving ILP models is a powerful tool such approach is not without drawbacks: some ILP models require high computational resources and above all long computational times to provide the result, in particular when they manage big instances. A long computational time may not be acceptable for instance if the network must react quickly enough and adapt to traffic pattern to guarantee significant energy savings.

To cope with this problem heuristic (from Greek euriskein = to find) methods have to be developed: they provide a good solution without guaranteeing its optimality but ensuring a relatively short computational time. Where the exhaustive search is impractical, heuristic methods are used to speed up the process of finding a satisfactory solution.

Heuristic algorithms may be divided into two main classes: constructive and improving algorithms; the former start from an empty solution and build a feasible solution; the latter instead start from a feasible solution of the problem and try to improve it by partially modifying it.

In this chapter we present the heuristic methods we developed to tackle the considered problem. First we present our constructive algorithms that provide a feasible solution even if not optimal: in Section 5.1 the greedy algorithm is described, Section 5.2 is dedicated to Heuristic Concentration and finally in Section 5.3 we propose an innovative geographical decomposition based approach. Then we discuss our heuristic improving algorithms: in Section 5.4 and Section 5.5 local search algorithms and Very Large Scale Neighborhood algorithms are described, respectively.

#### 5.1 Greedy

Greedy is a constructive algorithm (Alg. 1). It starts from an empty solution and selects the element that, at each iteration, is the most advantageous with respect to the current partial solution according to a specific criterion and adds it to the current solution if it satisfies some feasibility requirements.

The goal is to find a feasible solution, as good as possible: the greedy does not provide always the optimal solution.

_	
Algorithm 1 Generic structure of a greedy algorithm	
1: procedure Greedy	
2:	- $S := \emptyset, X :=$ set of choices/elements;
3:	repeat
4:	- Select the best element E;
5:	-X := X - E;
6:	if $(S \cup E \text{ is a feasible partial solution})$ then
7:	- Add E to the partial solution S;
8:	<b>until</b> (X = $\emptyset \parallel$ S is feasible)

We developed four greedy procedure for EMCA problem. The proposed greedy algorithms aim to find a feasible solution for the considered problem, deciding in which CSs the BSs are installed and with which device (described by variable  $z_{jk}$ in the formulation). The solution is then completed by selecting a power level for each BS in each time interval  $(y_{jklt})$  and by assigning the traffic points  $(x_{ijklt})$ . The main structure of the proposed greedy algorithms is described in Alg. 2. The algorithm is divided into two main parts: in the first one (SET\_COVERING procedure) it selects the best configuration of BSs, namely their location, devices and power levels, needed to cover all the test-points. When a BS with the corresponding device is added to the solution, the associated power level is initially set at the maximum one. The first phase focuses on the covering sub-problem and is based on greedy algorithms for set covering. We have implemented four versions of the SET\_COVERING procedure based on different way of selecting the best configuration thus developing the four greedy algorithms. In the first phase time slots are not considered as test points coverage is time-independent (for simplicity an auxiliary variable  $y'_{jkl}$  is also introduced which has the same meaning of variable  $y_{jklt}$  but does not depend on time slots).

After this phase we consider the solution obtained as a starting point for each time slot (we copy the value of the variable  $y'_{jkl}$  in  $y_{jklt}$  for each  $t \in T$ ): the selected sites, devices and levels are kept for all the time intervals.

In the second step (ASSIGNMENT procedure) all traffic points are assigned in each time interval and additional BSs or devices are installed to meet the traffic demand if needed.

At the end of each stage the possibility to further optimize the partial solution obtained by switching off some BS or by decreasing their transmission power level is evaluated (MINIMUM\_POWER procedure).

Algorithm 2 Greedy algorithm	
1: procedure Greedy Algorithm	
2:	
3:	- $S := \emptyset$ ; $J :=$ set of CS; $K :=$ set of devices; $L :=$ set of power levels;
4:	
5:	- <u>SET_COVERING</u> (Alg. 3);
6:	
7:	- <u>MINIMUM_POWER for SET_COVERING</u> (Alg. 4);
8:	
9:	- $\underline{\text{ASSIGNMENT}}$ (Alg.5);
10:	
11:	- <u>MINIMUM_POWER for ASSIGNMENT</u> (Alg. 6);

The procedure SET\_COVERING (Alg. 3) aims at selecting a set of BSs, with the corresponding device, so as to cover all the CTP. Since the coverage must be guaranteed in each time slot and the test points do not change along time intervals, the solution obtained with the covering phase is the starting point based on which the assignment of traffic points is computed in each time slot.

A preprocessing phase is performed first (Preliminary\_Control, line 5). It tries to reduce the coverage matrices  $(a_{ijkl} \text{ and } m_{ijkl})$  by determining if some BSs must be in the solution, as they are the only one that can cover some test points. It also checks if the problem is infeasible (for example if a CTP can not be covered by any couple of BS and device). Then, at each iteration, the best element is selected, namely the best BS and the corresponding device, to be added to the solution which is represented by the set S of selected BS - device pairs. The power level is initially set to the maximum.

In order to get several different solutions, four criteria for choosing the best element at each iteration (Best\_Choice, line 7) have been implemented each resulting in a different greedy procedure:

1. Minimum installation cost, namely sum of BS cost and device cost:

$$(bestBS, bestDev) = \arg\min_{j \in J, k \in K} (\gamma_{jk});$$

2. Minimum ratio between total installation cost and number of CTP covered:

$$(bestBS, bestDev) = \arg\min_{j \in J, k \in K} (\gamma_{jk}) / \sum_{i \in I_c} m_{ijk1}$$

3. Minimum ratio between total installation cost and number of CTP covered among those not already covered:

$$(bestBS, bestDev) = \arg\min_{\substack{j \in J, k \in K}} (\gamma_{jk}) / \sum_{\substack{i \in I_c \setminus (i \in I_c: \\ \sum m_{ijk1} \ge 1) \\ (i k) \in S}} m_{ijk1};$$

 Minimum installation costs for covering the test points with less chance of coverage (namely that CTP that have only few BSs able to cover them): let I'<sub>c</sub> be the set of uncovered CTPs:

$$i' = \arg\min_{i \in I'_c} \sum_{j \in J} \sum_{k \in K} m_{ijk1}; \qquad (bestBS, bestDev) = \arg\min_{\substack{j \in J, k \in K: \\ m_{i'jk1} = -1}} (\gamma_{jk});$$

#### Algorithm 3 Greedy algorithm - SET\_COVERING Procedure

1: procedure Set\_Covering 2: -  $S := \emptyset, X = J;$ 3: - Preliminary\_Control; 4: 5:repeat - Select Best\_Choice  $\longrightarrow$  (*bestBS*, *bestDev*); 6:  $-S := S \cup \{(bestBS, bestDev)\}; \ z_{BS, Dev} := 1; \ y'_{BS, Dev, 1};$ 7: -  $X := X \setminus \{BS\};$ 8: **until** (all CTP covered  $\parallel X = = \emptyset$ ) 9:

Once we have determined the CSs where to install the BSs and their device, the power level for each BS in the solution is reduced to the minimum value by the procedure MINIMUM\_POWER for SET\_COVERING (Alg. 4). Starting from the BS and device pair with the highest number of covered test points (line 8), we try to gradually reduce the transmission power level as long as it still provide a feasible coverage (checking if each test point is still covered by at least one BS, line 11). The BSs that are put into sleep mode by the procedure (line 13) are removed (line 14). This precisely reflects the heuristic nature of the greedy algorithm: some elements added later in the solution can often "dominate" other elements already in, because they guarantee the same service (in this case coverage) but at a lower cost.

#### Algorithm 4 Greedy algorithm - MINIMUM\_POWER for SET\_COVERING 1: procedure MINIMUM\_POWER FOR SET\_COVERING

2:	
3:	-X := S;
4:	repeat
5:	- Flag := 0;
6:	repeat
7:	- Pick the pair BS-device $(j,k) \in S$ with the maximum coverage
	cardinality:
8:	$(j,k) = \arg \max_{(h,d) \in X} \sum_{i \in I_c} m_{ihd1};$
9:	$(\cdot) = i \in I_c$
10:	${\bf if}$ ( all CTP are still covered) ${\bf then}$
11:	- Reduce power level of $j$ ;
12:	${\bf if}$ ( sleep mode is feasible ) ${\bf then}$
13:	$z_{jk} := 0; \text{ and } y'_{j,k,l+1} := 0$
14:	else Flag:=1;
15:	$\mathbf{until} \ (\mathrm{Flag}==0)$
16:	$X := X \setminus \{j\};$
17:	until (X $\neq \emptyset$ )

The next phase (ASSIGNMENT, Alg. 5) is common to all four variants: starting from the solution provided by the SET\_COVERING and the MINIMUM\_POWER for SET\_COVERING phase, after sorting the time slots in decreasing order of total traffic (line 8), assignment (line 12) is decided in each time slot: each traffic point must be assigned to the base station that provides the best signal (Best\_BS) among those that can cover it, according with the closest assignment constraint. The algorithm selects the TTP i with the highest traffic not vet assigned (line 11). It tries to assign the TTP to the installed BS providing the best power signal (Best\_BS) if the available capacity is enough. If i can not be assigned to Best\_BS, five different procedures are applied in sequence to recover feasibility. First, Procedure 1 changes the device of Best\_BS to increase its capacity. Then it increases the power level of the second closest installed BS  $(BS^*)$  in order to cover the TTP *i*. If capacity of  $BS^*$  is not enough or its quality is not enough the procedure 2 is applied. In procedure 2 the power level of the closest BS from i in sleep mode is increased in order to change the assignment of some clients and satisfy the capacity constraint of Best\_BS. Turning on a BS in fact results in re-evaluating the assignment of all TTP in the radius of coverage of the BS due to closest assignment constraints. If this is useless, in procedure 3 the device of the second closest BS (BS<sup>\*</sup>) is modified to increase its coverage radius, capacity and provided quality. If in this way  $BS^*$  provides a higher quality to *i* than that provided by the former closest BS, TTP i is assigned to BS<sup>\*</sup>. Procedure 4 instead installs a new BS  $(BS_{new})$  in a still available close CS with a device with enough available capacity to satisfy traffic of TTP i; the lowest level that allows to cover TTP i with the best signal is selected; in the other time slots instead  $BS_{new}$  power level is set to sleep mode. Finally if all previous procedures are not able to assign i, procedure 5 installs a new BS in a still available closest CS and with a device with enough available capacity and coverage radius to reassign some clients according to closest assignment constraints in order to

save capacity of the Best\_BS. The idea is the same of that presented in procedure 2.

For each step a single attempt for the assignment is performed: if it is not possible to assign the client at the end of procedure 5, the solution is not feasible. The infeasibility is handled by a local search phase.

When it is necessary to change the device of an already installed BS (to increase the radius of coverage (procedure 3) or capacity (procedure 5)) or to install a new BS (procedure 4), the assignment of some clients is reevaluated, following the same steps. In fact by changing the device of a BS its transmission power and its radius of coverage are changed: so the previous solution may not be feasible due to closest assignment and capacity constraints.

Since the newly selected device is kept for the whole time horizon, assignment must be checked also in other time slots.

The assignment procedure checks also if some BSs could be excluded from the solution: this may happen if a BS signal quality is the highest for a subset of traffic points whose total traffic is greater than the BS capacity; due to the closest assignment constraint all such traffic points should be assigned to the BS thus violating the capacity constraints.

Algo	orithm 5 Greedy algorithm - ASSIGNMENT Procedure
1: p	procedure Assignment
2:	
3: <b>I</b>	nitialization:
4:	- $S := S_{set\_covering};  X := X_{set\_covering};$
5:	$-y_{jklt} := y_{jkl};  \forall t \in T$
6:	
7:	repeat
8:	- Select the Time Slot with the highest traffic not yet analyzed:
	$t = \arg \max_{t' \in T} \sum_{i \in I_t} p_{it'};$
9:	$-C := I_t;$
10:	repeat
11:	- Select the Client with the highest traffic request not yet
	analyzed: $i = \max_{i' \in C} p_{i't};$
12:	- Select Best_BS for $i$ ;
13:	${\bf if}~~({\rm Capacity~of~Best\_BS}~{\rm is~enough}~)$ ${\bf then}$
14:	- Assign $i$ to Best_BS;
15:	<b>else</b> - Procedures for feasibility recovering;
16:	- $X := X \setminus \{j\}; x_{ijklt} := 1; C =: C \setminus \{i\};$
17:	<b>until</b> $(C == \emptyset \parallel X == \emptyset)$
18:	- $T := T \setminus \{t\};$
19:	$\mathbf{until} \ (T == \varnothing)$

Once the assignments are set, MINIMUM\_POWER for ASSIGNMENT procedure (Alg. 6) is applied to reduce the power level of the BSs currently in the solution. The procedure is slightly different from MINIMUM\_POWER for SET\_COVERING: it considers also the traffic characteristics in different time slots. In particular for each time slot the procedure tries to decrease the power level of each BS until the power level reduction does not change the signal quality conditions of traffic points. This means assignment of TTPs to BSs must not be changed decreasing the power level of a BS. All CTP must be covered.

#### Algorithm 6 Greedy algorithm - MINIMUM\_POWER for ASSIGNMENT 1: procedure MINIMUM\_POWER FOR ASSIGNMENT

```
2:
3:
        repeat
            - X := \text{set of } j \in J \text{ in } S;
 4:
            repeat
 5:
                 Flag := 0;
 6:
 7:
                 repeat
                     - Pick the BS j in S with the maximum coverage cardinality:
 8:
                                          j = \arg\max_{h \in X} \sum_{i \in I_t \cup I_c} m_{ihkl} + a_{ihkl};
9:
                     if (All CTP still covered && all TTP assigned) then
10:
11:
                         - Reduce the power level;
                     else Flag := 1;
12:
                 until (Flag == 0)
13:
                 X := X \setminus \{j\};
14:
            until (X \neq \emptyset)
15:
16:
        until (all time slots are not analyzed)
```

#### 5.2 Heuristic Concentration

We developed a two-stage constructive heuristic, the so called Heuristic Concentration (HC) which combines heuristics and solution of ILP models. An overview of the method and of its application is shown in [41] and in [42]. HC can be applied to a wide variety of combinatorial problems. It is particularly suited to location problems. Two steps are combined in HC. The first step aims at building a concentration set G of elements. The second step is an ILP method applied on the concentration set: it solves the considered problem over the subset of variables associated with the concentration set G.

The first stage of the proposed HC algorithm (Alg. 7) applies all the four variants of the greedy algorithm to produce a number of alternative facility location choices. We notice in fact that some CSs are frequently selected in the sub-optimal solutions or in other words the vast majority of CSs are never selected in any heuristic solution. This allows to build a concentration set  $G \subseteq J$ as the union of the sets of BSs selected in different sub-optimal solutions. Among the elements of the concentration set G are likely to be included those sites which are in the optimal solution. To represent the concentration set we need to introduce an auxiliary binary variable  $w_j$ , representing the set of selected CS. Stage two of HC solves the ILP formulation of the problem only on a restricted set of variables based on the concentration set G. The HC formulation is the following:

#### **EMCA**\_Concentrated:

min (1) s.t. (2), (3), (4), (5), (6), (7), (8), (9), (10), (11)  $w_j = \sum_{k \in K} z_{j,k}$   $\forall j \in J$  (12)  $w_j \in \{0, 1\}$   $\forall j \in J$  (13)

The classical approach proposes to look for the optimal solution inside the concentration set: this means that all CS outside G are neglected (line 7). This means that the final solution will not include those CSs which are not in the concentration set ( $w_j = 0$  if  $j \notin G$ ). Sometimes (depending on the characteristics of the problem and of the instance) this leads to an infeasible solution (e.g. if

the size of G is too small, the problem could be infeasible); in this case we force all CS in G to be in the solution  $(w_j = 1 \text{ if } j \in G)$  and we allow the solution of the ILP formulation to include CS which are not in G (line 12): in this way the computational time slightly increases, but the possibility of finding a feasible solution increases too.

## Algorithm 7 HEURISTIC CONCENTRATION 1: procedure HEURISTIC CONCENTRATION

- 2:
- 3: Execute all greedy algorithms;
- 4: Save the chosen CSs;
- 5: Construction of the concentration set  $G \subseteq J$ ;
- 6: repeat

7: - Fix  $w_i = 0$   $\forall j \notin G;$ 

- 8: Solve EMCA\_Concentrated;
- 9: **if** (Feasible solution found) **then**
- 10: Flag := 1;

11: else

12: - Fix  $w_j = 1$   $\forall j \in G;$ 

- 13: Solve EMCA\_Concentrated;
- 14: Flag := 1;
- 15: until (Flag==0)

#### 5.3 Geographical Decomposition

Beside greedy and heuristic concentration algorithms, we developed another constructive algorithm based on the geographical characteristics of the network. In fact the methods seen so far do not fully exploit the geographical characteristics of the network (coordinates of CS, CTP and TTP) even if such information are somehow included in the covering matrices.

Since geographical areas quite distant from each other do not have elements in common and do not interfere with each other, their solutions can be computed independently. Based on this assumption the considered area is divided into smaller sub-areas; if the sub-area are small enough, then the number of CS, CTP and TTP in each sub-area is smaller than the overall number and thus the solving time for EMCA problem applied to the sub-area is significantly smaller. We first solve the sub-problems on the sub-areas. Then we build a set of those CS selected in the sub-problems and solve the EMCA only on such set.

Our algorithm works as follows (Alg. 8): we divide the total area in small square sub-areas (line 3), each with the same size. We build sets  $CS_{Area}$ ,  $CTP_{Area}$ ,  $TTP_{Area}$  as respectively sets of CS, CTP and TTP that are in the considered sub-area. The new formulation of the problem (EMCA\_Restricted) is the same of EMCA with sets J,  $I_c$  and  $I_t$  replaced respectively by sets  $CS_{Area}$ ,  $CTP_{Area}$ ,  $TTP_{Area}$ .

We solve EMCA\_Restricted on each sub-area and save the CS chosen in each subproblems (line 8). We exploit the information obtained about the opened CS  $(w_j)$ to build a set H as the union of all the CS opened. Then the approach is the same seen for the second stage of HC algorithm, considering H as a concentration set (line 12 - 21): first the procedure tries to find a solution within the set H, if it is not possible it expands the search also on CS outside H.

Algorithm 8 Geographically Based Algorithm	
1: procedure Geographically Based ILP	
2:	
3: - Divide the total area in $r$ small sub-areas $A_r$ ;	
4: repeat	
5: - Select sub-areas $A_r$ ;	
6: - Build sets $CS_{Area}$ , $CTP_{Area}$ , $TTP_{Area}$ related to $A_r$ ;	
7: - Solve EMCA_Restricted on $A_r$ ;	
8: - Save values of chosen CSs;	
9: <b>until</b> (All area not analyzed)	
10: $-$ Flag := 0;	
11: - Build set $H \subseteq J$ ;	
12: repeat	
13: - Fix $w_j = 0$ $\forall j \notin H;$	
14: - Solve EMCA_Concentrated;	
15: <b>if</b> (Feasible solution found) <b>then</b>	
16: $- \text{ Flag} := 1;$	
17: else	
18: - Fix $w_j = 1$ $\forall j \in H;$	
19: - Solve EMCA_Concentrated;	
20: $- \text{Flag} := 1;$	
21: <b>until</b> (Flag == 0)	

This approach may have problems especially in the border between the sub-areas: a solution on a small sub-area in fact does not take into account nearest areas. Since BS coverage radius can exceed the size of the sub-areas, adjacent sub-areas could interfere. Thus we build the area so that there is a slight overlap between adjacent sub-areas. In such a way when we compute the set of CS, CTP and TTP in each sub-area (line 6) we include also all the CSs, CTPs and TTPs close to the borders.

#### 5.4 Local search

Beside constructive algorithms, we also developed improving algorithms. A local search has been developed to improve the solution obtained by constructive algorithms.

Local search algorithms are based on the idea of analyzing the neighborhood of the current solution. A neighborhood of a solution is a set of solutions generated by applying a determined move. A move is an operation that generates a set of solutions by slightly modifying the current one. A local search method (Alg. 9) works as follows: it starts from an initial feasible solution S; it builds a neighborhood of S applying a move and returns the best solution found S' among the neighbors. If no neighbor solution improves upon the current one (or other stopping conditions are verified such as maximum number of iteration, maximum time, etc...) the procedure stops, otherwise the current solution is replaced with S'. A detailed analysis of local search algorithms can be found in [43].

Local search is able to find the local optimum starting from the initial solution but it does not guarantees to find the global one.

Generally, we consider moves that cause limited perturbations of a solution. In this way, the neighborhood size remains limited, allowing a rapid evaluation. Clearly, there must be a compromise between the size of the neighborhood (and so the time required to visit it) and its effectiveness, namely its ability to obtain improving solutions and to achieve better local optima.

Algorithm 9 Generic structure of a Local Search algorithm	
1: procedure Local Search	
2:	- Select an initial feasible solution $S$ ;
3:	- $Flag = False;$
4:	repeat
5:	- The neighborhood $N(S)$ is built with a specific move;
6:	- The best neighbor $S' \in N(S)$ is selected;
7:	if (no neighbor solution improves upon $S$ ) then
8:	- $Flag := True;$
9:	else - $S := S';$
10:	$\mathbf{until} \ (Flag = True)$

The choice of the best neighbor solution depends on the exploration strategy of the neighborhood. The classical strategies are:

- <u>Steepest Descent (SD)</u>: all the neighborhood is explored and the best neighbor is chosen; this strategy may require quite high computational times but reaches the best improving solution at each iteration.
- <u>First Improvement (FI)</u>: the exploration of the neighborhood ends as soon as the first improving neighbor is founds; FI does not visit all the neighborhood, resulting in large saving of computational time. However, it may neglect the best improving neighbor.

A representation of the different behavior of the two methods can be seen in Fig. 5.1: FI is usually faster in the first iterations, while SD usually provides, in the first iteration, higher improvements. In the last iteration, however, also FI visits the whole neighborhood.

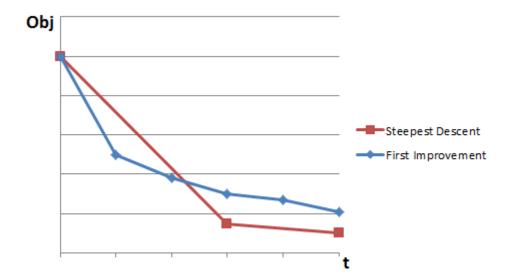


Figure 5.1: First improvement vs Steepest Descent

In our proposed heuristic (Alg. 10) the neighborhood is visited heuristically.

We apply a move that changes BSs location and modifies at least two CSs from the current solution. This means that at each iteration it is possible to remove a BS, to install a new BS or to swap a CS in the current solution with one not in the solution. The neighborhood size is then  $O(|CS|^2)$  and it differs from solution S for at most two BS at time.

Once CS are selected, the space of possible solutions is reduced a lot. Moves based on changes in device or in power levels may results in larger neighborhood. Thus, once the set of selected BSs has been modified by the move, we complete the solution by solving an ILP problem. In particular we formulate the problem of selecting the best devices  $(z_{jk})$ , power levels  $(y_{jklt})$  and assignment  $(x_{ijklt})$  and we solve it with a solver. For each generated neighbor, the best configuration is calculated (Obj); if the neighbor improves upon S (*CurrentCost*) it is saved as best neighbor (line 10); depending on the chosen exploration strategy the neighborhood is visited entirely (SD) or the search is stopped at each iteration as soon as an improving solution is found (FI). The algorithm iterates until no further improvements are possible. In Alg. 10 FI is described.

Algorithm 10 Heuristic Local Search with ILP
1: procedure Heuristic Local Search with ILP
2:
3: - Select initial solution $S \subseteq J$ ; Flag := 0; CurrentCost = cost of S;
4: repeat
5: - Build the neighborhood: $N(S)$ ; Flag_2 := 0;
6: <b>for</b> $(S' \in N(S) \&\& \operatorname{Flag}_2 == 0)$ <b>do</b>
7: - Fix $w_j = 1$ $\forall j \in S';$
8: - Solve EMCA_Concentrated $\rightarrow Obj$ ;
9: if $(Obj \leq CurrentCost)$ then
10: $-CurrentCost := Obj;  S := S';  Flag_2 := 1;$
11: <b>if</b> $(Obj > CurrentCost$ (no improving)) <b>then</b>
12: $- Flag := 1;$
13: <b>until</b> (Flag == 0;)

#### 5.5 Very Large Scale Neighborhood

The neighborhood of the current solution includes a possibly large number of solutions which are similar to the current one. Obviously, the neighborhood might include just those solutions that require a single change from the current solution, or it might include a larger set of solutions. If the neighborhood is not large enough to reach the optimum, the local search gets stuck in a local optimum: in order to visit a wider portion of the feasible region moves that generate neighborhoods of bigger size can be applied. The larger the neighborhood, the better is the quality of the locally optimal solutions, and the greater is the accuracy of the final solution that is obtained. At the same time, the larger the neighborhood, the longer it takes to search the neighborhood at each iteration. Thus very large scale neighborhood (VLSN) approaches are developed to tackle the problem (Alg. 11). At each iteration the algorithm solve an ILP problem and finds the best solution within neighbors.

A survey of this technique is presented in [44] and in [45].

In order to implement this method, we need to introduce in the formulation other constraints that further limit the space of solutions:

#### EMCA\_VLSN:

min (1)

s.t. (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13)  

$$\sum_{\substack{j \in J: \ w'_j = = 1}} (1 - w_j) <= N'; \qquad (14)$$

$$\sum_{\substack{j \in J: \ w'_j = = 0}} w_j <= N; \qquad (15)$$

The neighborhood is given by all the solutions which differ from the current one for at most N CS, i.e. the number of variables  $w_j$ , related to the CS, that change their value is at most N (N' = N). The current solution is represented by variable  $w'_j$ :  $w'_j$  is equal to one.

Equations (14) limit the number of CS that are in the current solution and not in new one. Instead (15) limit the CS entering in the solution.

The neighborhood is visited implicitly, by directly generating the best neighbor solution.

A VLSN approach can be also implemented on other variables to further reduce the size of the neighborhood and so the computational time; this is accomplished by introducing the following additional constrains in the model:

$$\begin{cases} \sum_{j \in J} \sum_{\substack{k \in K \\ z'_{jk} = =1}} (1 - z_{jk}) <= M; \quad (16) \\ \sum_{j \in J} \sum_{\substack{k \in K \\ z'_{jk} = =0}} z_{jk} <= M'; \quad (17) \end{cases}$$

The concept is the same as that seen before in (14) and (15), but in this case we want to limit the number of changed devices. Moreover particular attention should be paid to the values of M and M', since they significantly affect the accuracy of the final solution but also the computational time needed to obtain it.

In order to implement the FI strategy in a VLSN approach we need to modify the formulation of the problem:

#### EMCA\_VLSN\_FI:

s.t. (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13)  

$$\sum_{j \in J: \ w*_j = =1} (1 - w_j) <= N';$$
(14)

$$\sum_{j\in J: \ w*_j==0}^{\infty} w_j <= N; \tag{15}$$

$$\sum_{j\in J}\sum_{k\in K}\gamma_{jk}z_{jk} + \beta\varphi\sum_{j\in J}\sum_{k\in K}\sum_{l\in L}\sum_{t\in T}\varepsilon_{kl}\delta_t y_{jklt} \le Bestcost - \theta;$$
(18)

The objective function is removed and it is added a new constraint that asks for finding an improving solution (18): in fact in *Bestcost* is stored each time the value of the best current solution, instead  $\theta$  is a small enough number (e.g. 0.1). In this way since there is not the objective function, the solution of the problem is the first improving solution found in the branch and bound tree.

Algorithm 11 VLSN Algorithm	
1: procedure VLSN Algorithm $(S_{Greedy})$	
2:	
3: - Flag := 0;	
4: - Select solution $S$ ;	
5: - Set values of $w *_j$ and $BestCost$ ;	
6: repeat	
7: - Solve EMCA_VLSN or EMCA_VLSN_FI $\rightarrow Obj;$	
8: if $(Obj \leq BestCost)$ then	
9: - $BestCost := Obj;  S := S';$	
10: else Flag := 1;	
11: <b>until</b> (Flag == 0;)	

VLSN can be used also for handling infeasible solutions (Alg. 12): the objective function (1\_B) aims at maximizing the number of assigned clients in each time slot. Since intermediate solutions could be still infeasible we have also to relax assignment constraints (3\_B):

#### EMCA\_FEASIBILITY:

$$\max \sum_{i \in I_t} \sum_{j \in J} \sum_{k \in K} \sum_{l \in L} \sum_{t \in T} x_{ijklt}$$
(1\_B)  
s.t. (2), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13), (14), (15)  
$$\sum_{j \in J} \sum_{k \in K} \sum_{l \in L} a_{ijkl} x_{ijt}^{kl} \le 1 \qquad \forall i \in I_t, t \in T$$
(3\_B)

Algorithm 12 VLSN for Feasibility	
1: procedure VLSN for Feasibility $(S_{Greedy})$	
2:	
3: - Select initial solution $S$ ;	
4: - Coverage := $\sum_{i \in I_t} \sum_{k \in K} \sum_{l \in L} \sum_{t \in T} x_{ijklt}$	
5: - Flag := 0;	
6:	
7: repeat	
8: - Solve EMCA_FEASIBILITY $\rightarrow Obj_B := \sum_{i \in I_t} \sum_{k \in K} \sum_{l \in L} \sum_{t \in T} x_{ijklt}$ ;	
9: <b>if</b> $(Obj_B \ge Coverage)$ <b>then</b>	
10: $Coverage := Obj\_B;  S := S';$	
11: <b>if</b> (All TTP are assigned in each time slot) <b>then</b>	
12: $- \text{ Flag} := 1;$	
13: <b>else</b> - Flag := 1;	
$14:  \mathbf{until} \ (Flag == 0;)$	

The algorithm aims at increasing at each iteration the number of TTP assigned, exploiting a VLSN approach. In this case the optimal solution (if found) corresponds to the configuration where all TTP are assigned in each time slot (line 11). The solution is not optimized in terms of energy saving.

# Chapter 6

# Results

In this chapter we present and discuss the computational results. After a brief introduction on the softwares used, we describe in Section 6.1 the instance generator designed to provide realistic data on which apply our tests. In the same section some remarks on BSs configurations, traffic behavior and channel propagation model are also pointed out. In Section 6.2 we discuss the results, focusing on the effectiveness of the heuristic methods proposed in Chapter 5. Each subsection is devoted to the description of the results of one algorithm. Finally in Section 6.3 we discuss the results of the heuristics applied on large instances. All the heuristic approaches have been implemented with AMPL [46] and the ILP models have been solved with CPLEX [47].

The tests have been run on a 2 core machine @ 2GHz and 16Gb RAM.

### AMPL & CPLEX

There are many solvers able to solve an Integer Linear Programming model: generally they take as input an optimization problem description (i.e. model and data) and give as output the optimal solution (if one exists) (Fig. 6.1).

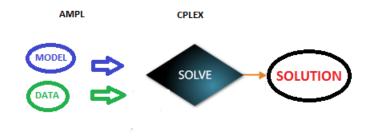


Figure 6.1: AMPL and CPLEX scheme

AMPL, an acronym for "A Mathematical Programming Language", is an algebraic modeling language for describing and solving high-complexity problems for large-scale mathematical computation. Formulation of optimization models is represented through declarative language elements such as sets, scalar and multidimensional parameters, decision variables, objectives and constraints. To support re-use and simplify construction of large-scale optimization problems, AMPL allows separation of model and data.

AMPL does not solve the problems directly; a solver is used to compute solutions. For handling ILP problems, solvers generally apply Branch and Bound based approaches. Branch and Bound is a integer linear programming technique for solving optimization problems and consists of a systematic enumeration of candidate solutions by means of state space search. The algorithm explores subsets of the solution set represented by nodes of the branching tree. Instead of enumerating all the solutions of a subregion, the branch is checked against upper (UB) and lower (LB) estimated bounds on the optimal solution, and it is discarded if it cannot produce a solution improving upon the best one found so far.

## 6.1 Instances

We have tested the approaches on three groups of instances in order to check the effectiveness of the proposed methods on different instance features: the first group (A) consists of 20 instances generated using an instances generator and reflects the characteristics of a network using LTE technology. The second group (B) is instead based on a GSM network and is composed of 60 instances; even if these instances are based on a smaller size network, they are computationally challenging for CPLEX and allow us to stress the methods and observe some particular behaviors of the algorithms. Finally, the last group (C) is composed of few instances proposed in [18] representing very large networks. In Table 6.1 the main features of the three group of instances are summarize: for each group the number of instances, the area size, the number of CS, CTP and TTP in the area, the number of available devices and power levels are reported. Finally parameter  $\Omega$  is reported: it represents the average percentage of CTP that a BS can cover when transmitting at maximum power. For example a value equal to 1 means that all BS can cover all CTP (and also all TTP). Parameter  $\Omega$  is somehow related to the computational challenge of the instances.

	GROUP A	GROUP B	GROUP C
Number of instances	20	60	5
Area size $(m^2)$	2000/6000	10000	6000/10000
Number of CS $( J )$	30 - 40 - 50	10 - 15 - 20	200-220-400-450-500
Number of CTP $( I_c )$	400 - 626 - 900	121	961-1296-1681-2601
Number of TTP $( I_t )$	40 - 50 - 70	20 - 30 - 40 - 50 - 60	70-80-90-150
Number of Device $( K )$	3	3	3
Number of Power Levels $( L )$	4	5	2
Number of Time Intervals $( T )$	6	6	8
Fraction of covered CTP $\Omega$	0.3	1	0.12

#### Table 6.1: Instances Parameters

Each instance is named according to the following notation:  $|I_t| - |J| - |I_c| - index$ (for example the name 40\_30\_400\_1 indicates the first instance of a group with 40 TTP, 30 CS and 400 CTP). Each sub-group of Group A consists of 4 instances (the possible combinations are 40\_30\_400, 40\_60\_625, 50\_30\_400, 50\_50\_625 and 70\_40\_900), while sub-groups of Group B consist of 5 instances each (for all possible combinations). Group C consists of the following instances: 70\_200\_961, 80\_220\_1296, 90\_400\_1681, 150\_500\_1681 and 90\_450\_2601.

We developed an instances generator (IG) in C programming language to generate quite realistic scenarios in the context of a modern cellular network. Based on the values given in input, containing the main characteristics of the instances, the IG generates the TTP traffic in each time slots and the CS in the considered area, it computes the BS costs, the covering and qualities matrices. The CTP are instead uniformly distributed over the area. In presenting the IG we analyze in particular the instances of Group A. Group B instances are generated in a similar way. For the description of instances of Group C we refer to [18].

### Input

The instance generator receives as input the following parameters:

- Area size:  $A [m \times m]$
- Number of CS (|J|);
- Number of TTP  $(|I_t|)$ ;
- Number of CTP  $(|I_c|)$ ;
- Number of available device (|K|);
- Number of available power levels (|L|);
- Number of time intervals (|T|);

Other parameters instead depend on the considered technology:

- Frequency: f [MHz]
- Transceivers and receivers height:  $h_{tx}$  and  $h_{rx}$  [m]
- Minimum and Maximum installation cost for a Candidate Site:  $c\_Inst_{min}$ and  $c\_Inst_{max}$  [€]
- Minimum received power threshold: thr [dBm]
- Installation cost of each device:  $cDev_k \in$
- Power consumption of each device and power level:  $\varepsilon_{kl}$  [W/h]
- Transmitted power of each device and power level:  $pTX_{kl}$  [dBm]
- Capacity of each device and power level:  $\Gamma_{kl}$  [Mb/s]
- Minimum and Maximum traffic that can be generated from a traffic point:  $tr_{min}$  and  $tr_{max}$  [Mb/s]
- Traffic pattern: percentage of traffic with respect to the nominal one in each time slot and duration of time intervals ( $\delta_t$  [h]).

#### Coordinates

Given in input the considered area and the number of CS, CTP and TTP the generator generates the coordinates over the area of each CS and TTP. In particular the abscissa and ordinate value of each CS and TTP are generated in a random way (uniform distribution).

Instead, the CTP are placed according on a regular grid with distance between CTPs equal to  $d_{tp}$ . For Group A  $d_{tp}$  is set equal to 200 m. The lower this value, the higher the guaranteed coverage.

### Traffic

As already pointed out, we can represent the whole day as a set of time slots, each with a different traffic amount.

The IG generates the traffic for each TTP in each time slot. The real traffic in each time slot is a percentage of a nominal value. A low number of time periods reduces approximation accuracy of the traffic pattern and generally results in higher energy consumption. On the other hand, a higher number of time slots increases the needed computational effort. We consider six time intervals as a good trade-off between computational complexity and accuracy. Fig. 6.2 shows the traffic pattern for instances of Group A.

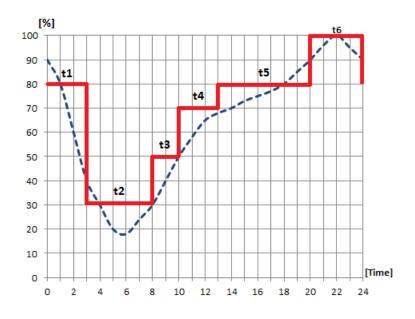


Figure 6.2: Traffic profile of Group A

The dotted line represents the real traffic based on empirical data [11] as a fraction of the nominal one. The straight line is a stepwise approximation. The pick of the traffic is from 20:00 to 24:00. During night (3:00 / 8:00) instead the traffic is very low, so we approximate the real value with a percentage of 30% of the nominal one. Table 6.2 summarizes the length of each time slot ( $\delta_t$ ):

Time Slot	Starting/Ending	Duration $(\delta_t)$	Fraction of
			nominal traffic
t1	0:00 / 3:00	3 hours	0.8
t2	3:00 / 8:00	5 hours	0.3
t3	8:00 / 10:00	2 hours	0.5
t4	10:00 / 13:00	3 hours	0.7
t5	13:00 / 20:00	7 hours	0.8
<i>t</i> 6	20:00 / 24:00	4 hours	1

Table 6.2: Time intervals duration and traffic profile - Group A

The following assumptions are made: the value of traffic remains unchanged during each time slot and there is no transition time between two consecutive time slots. To have a realistic numerical value [5] a nominal value of traffic randomly generated between  $tr_{min} = 15$  Mb/s and  $tr_{max} = 35$  Mb/s is assigned to each TTP: the value is set considering that the average value of traffic generated by a smartphone is 0,25 Mb/s and a traffic point can be consider as representing almost a hundred of clients. Then the real traffic for each TTP in each time slot  $(p_{it})$  is calculated from the nominal value of traffic according to the percentage of usage.

### **Base Station Configurations**

Network flexibility is needed to guarantee energy efficiency. (Fig. 6.3).

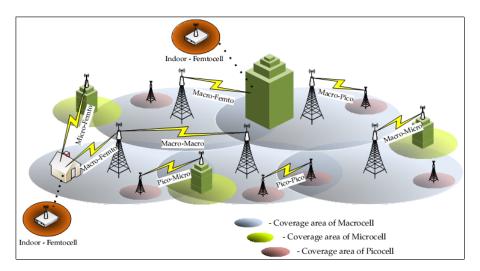


Figure 6.3: A typical heterogeneous network deployment (from [9])

In the context of a cellular network with LTE technology we can distinguish 3 types of device configurations for base stations, which represent a good approximation of reality and variety of installation choices. According to a research led by EARTH Energy Efficiency Evaluation Framework ( $E^3$ F) to map the radiated RF power of LTE base stations [40], we consider the following three types of devices for BS (Set K). The biggest is used for macro-cells and has an emitted power ( $p_T X_{kl}$ ) of 43dBm/19.9W and a consumed power of 1350 W per hour. According to the propagation model its maximum coverage distance is about 1350 m. A smaller device is used for micro-cells: it has a transmitted power of 38dBm/6.3W and a consumed power of 144.6 W per hour. The radius of coverage is about 1000 m. Finally for pico-cells we used another configuration with emitted power of 21dBm/0.1W and a consumed power of 14.7 W per hour. This cell has a small coverage radius of about 350 m.

Each device can also modulate its emitted power according to traffic amount, varying its radius of coverage. Table 6.3 summarizes the numerical information concerning BS.

Device	Power level	Transmitted	Consumed	Capacity	Max. distance
		power [dBm]	power $[W/h]$	[Mb/s]	[m]
	L1 (100%)	43	1350	210	1347
D1	L2 (70%)	41.5	945	210	1193
D1	L3 (50%)	40	675	210	1123
	L4 $(2\%)$	-	27	0	0
	L1 (100%)	38	144.6	70	994
D0	L2 (70%)	36.5	101.2	70	880
D2	L3 (50%)	35	72.3	70	828
	L4 $(2\%)$	-	2.9	0	0
	L1 (100%)	21	14.7	50	353
D9	L2 (70%)	19.5	10.3	50	313
D3	L3 (50%)	18	7.4	50	294
	L4 $(2\%)$	-	0.3	0	0

Table 6.3: BS configurations and power level - Group A

## Propagation

In Chapter 3.3 we have described the effects of the radio propagation and in particular the attenuative behavior of the transmission channel.

Simplifying the channel modeling, we apply the formula of COST-231 Hatamodel (6.1) based on extensive empirical measurements [48]; it represents a good approximation of the attenuation of a radio propagation channel within a generic urban environment, incorporating the effects of diffraction, reflection and scattering caused by city structures:

$$L_{dB} = 46.3 + 33.9 \log(f) - 13.82 \log(h_B) - a_{h_m} + [44.9 - 6.55 \log(h_B)] \log(d) - C_m$$
(6.1)

and for medium sized city:

$$a_{h_m} = (1.1\log(f) - 0.7)h_M - (1.56\log(f) - 0.8)$$
(6.2)

where f[MHz] is the frequency of transmission (for LTE 2600 MHz), d[Km]the distance between the base station and receiver,  $h_B[m]$  and  $h_M[m]$  the height of the base station antenna (12 m) and mobile antenna (1.5 m) respectively. Finally for suburban areas the parameter  $C_m$ , that takes into account other further attenuation due to specific environments, is equal to zero.

We assume, therefore, that the waves propagate uniformly in all directions, so that cells have round shape. In a real scenario the quality of the received power does not depend only on the distance, but it is affected also by other elements: the presence of possible obstacles or possible attenuation due to interference between clients or between cells are already taken into account by the formula (6.1).

#### **Covering and Quality Matrices**

Once all the coordinates of the elements in the network are generated, the IG calculates the covering matrices for CTP and TTP. For each combination of CS, device and power level it calculates the distance from each test point, computes the losses due to propagation and compares the received power to the threshold (*thr* = -102 dBm); if the received power is above the threshold, the TP is covered by the combination of CS, device, power level. The corresponding value of parameter  $a_{ijkl}$  (or  $m_{ijkl}$ ) is set equal to 1. Otherwise it is set equal to 0.

Finally, the quality for each TTP and CS configuration is computed in a similar way, taking into account the real received power.

#### $\mathbf{Costs}$

The installation cost for each CS is generated randomly within the interval  $c\_Inst_{min} = 5000 \in$  and  $c\_Inst_{max} = 20000 \in$ . The cost of installing the different devices is instead fixed and equal to  $30000 \in$  for the macro-cell,  $10000 \in$  for the micro-cell and  $1000 \in$  for the pico-cell.

As described in Chapter 4 the objective function is composed of two terms representing CAPEX and OPEX costs over the time horizon: we set parameter  $\beta$  equal to one, meaning that the two costs have the same weight over the time horizon. The value of  $\varphi$  (cost of energy consumption over the entire network life) is also set equal to one considering a time horizon of 8 years and the energy cost E=0.35  $\in$ /kWh:

$$\varphi = E \cdot n \cdot 365 \cdot 0.001 = 1$$

where n stays for the number of years over which the OPEX costs (daily based) are computed (365 days in a year) and the factor 0.001 is used to convert from Wh to kWh.

We finally apply on the generated instances a feasibility check: we check that the total network capacity is able to meet the traffic demand of customers in any time interval and that each CTP and TTP is within the coverage radius of at least one CS.

## 6.2 Results

The performance of a heuristic technique is evaluated by considering two criteria often in contrast between them: the quality of the solution that can be achieved and the computational time needed to produce it. The quality of a solution is evaluated in terms of percentage difference between the solution obtained and the optimal solution (or a bound if the optimal solution is not known).

$$Gap = \frac{\mid OPT(I) - HEUR(I) \mid}{\mid OPT(I) \mid} \cdot 100$$

The design of an efficient heuristic must find a balance between the computational effort and the quality of the solution.

First we present the results obtained solving the formulation EMCA presented in Section 4.2. This is useful to have reference values to be compared with the results of the heuristics.

We present the results according to the chronological order in which the algorithms were developed. First we analyze the results obtained by using heuristics based on greedy and heuristic concentration. Our analysis evaluates also the improvement obtainable with local search methods. Finally we show the results obtained by the geographical separation approach. On instances of Group A we tested all the proposed methods; on Group B instead we tested only the methods that produced the best results over Group A. Finally Group C is used to study the behavior of the geographical method on large instances and to analyze the features of the obtained solutions. Each table is briefly described highlighting the interesting behaviors. For each method some graphs are provided that summarize the results obtained for the instances tested.

## 6.2.1 Results of Formulation

The EMCA formulation has been solved with CPLEX 12.1.0.0. A one hour time limit is set. We limit the maximum size for the branch and bound tree to 1 Gb and the maximum number of used thread to one.

Tables 6.4 and 6.5 gives CPLEX results on instances of Group A and Group B respectively. They are organized as follows: the first column represents the name of the instance, the second gives the time required to solve it (if time limit is reached it is replaced with the acronym "TL"). Usually the solver is not able to find the optimal solution within the time limit; in that case beside the best integer solution found (upper bound - UB, third column) also the value of the lower bound (LB) is reported in column four. Finally the last column represents the gap between UB and LB. This is useful to understand how far the best integer solution found is from the optimal one. This value is calculated as:

$$Gap = \frac{\mid UB - LB \mid}{\mid LB \mid} \cdot 100$$

For each sub-set of similar instances (same number of CS, TTP, CTP) the average and maximum value of gap between LB and UB is then given.

INSTANCE	[s]	UB	LB	%
40_30_400_1	TL	327703	282690	15.9
40_30_400_2	TL	336081	298349	12.6
40_30_400_3	TL	395091	312639	26.4
40_30_400_4	$\mathrm{TL}$	384714	336977	14.2
average				17.3
max				26.4
40_60_625_1	TL	387745	346190	12.0
40_60_625_2	$\mathrm{TL}$	450419	344641	30.7
40_60_625_3	$\mathrm{TL}$	419985	348451	20.5
40_60_625_4	$\mathrm{TL}$	365725	355348	2.9
average				16.5
max				30.7
50_30_400_1	TL	420253	323057	30.1
50_30_400_2	TL	438898	343891	27.6
50_30_400_3	TL	374526	336984	11.1
50_30_400_4	TL	406851	357107	13.9
average				20.7
max				30.1
50_50_625_1	TL	509710	393450	29.5
50_50_625_2	TL	488733	389164	25.6
50_50_625_3	TL	487904	421412	15.8
50_50_625_4	TL	476101	417399	14.1
average				21.2
max				29.5
70_40_900_1	667.7	729270	729234	0.0
70_40_900_2	TL	748013	726745	2.9
70_40_900_3	TL	705666	669222	5.5
70_40_900_4	TL	723124	713120	1.4
average				2.4
max				5.4

Table 6.4: CPLEX Results - Group A

INSTANCE	[s]	UB	LB	%
20_10_121_1	TL	109428	93377	17.2
20_10_121_2	TL	118386	102345	15.7
20_10_121_3	TL	102063	102063	0.0
20_10_121_4	740	80090	80084	0.0
20_10_121_5	TL	102333	85380	19.9
average				10.5
max				19.9
30_10_121_1	TL	172441	123022	40.2
30_10_121_2	TL	160052	117945	35.7
30_10_121_3	TL	143305	109071	31.4
30_10_121_4	TL	130147	97769	33.1
30_10_121_5	TL	142961	121523	17.6
average				31.6
max				40.2
30_15_121_1	TL	134941	100992	33.6
30_15_121_2	TL	143517	110326	30.1
30_15_121_3	TL	109756	92640	18.5
30_15_121_4	TL	165705	102018	62.4
30_15_121_5	TL	134727	107889	24.9
average				33.9
max				62.4
30_20_121_1	TL	124845	91358	36.7
30_20_121_2	TL	144789	102849	40.8
30_20_121_3	TL	144859	78556	84.4
30_20_121_4	TL	178618	107187	66.6
30_20_121_5	TL	156335	114240	36.8
average				53.1
max				84.4
40_10_121_1	TL	179122	142446	25.7
40_10_121_2	TL	182322	157485	15.8
40_10_121_3	TL	179104	142894	25.3
40_10_121_4	TL	197501	153792	28.4
40_10_121_5	TL	183859	158664	15.9
average				22.2
max				28.4
40_15_121_1	TL	160051	114106	40.3
40_15_121_2	TL	234213	141735	65.2
40_15_121_3	TL	978661	123562	692
40_15_121_4	TL	222150	135874	63.5
40_15_121_5	TL	179033	135438	32.2
average				178.6

INSTANCE	[s]	UB	LB	%
40_20_121_1	TL	160748	107594	49.4
40_20_121_2	TL	194219	123392	57.4
40_20_121_3	TL	760551	102127	645
40_20_121_4	TL	618258	135730	356
40_20_121_5	TL	189764	132945	42.7
average				230.0
max				644.7
50_10_121_1	TL	197501	149737	31.9
50_10_121_2	TL	208385	189262	10.1
50_10_121_3	TL	216950	203562	6.58
50_10_121_4	TL	231779	231779	0
50_10_121_5	TL	229720	186269	23.3
average	11	220120	100200	14.4
max				31.9
50_15_121_1	TL	672770	147744	355
50_15_121_2	TL	616320	172298	258
50_15_121_2	TL	298259	172298 145810	238 105
	TL			
50_15_121_4		654888	172488	280
50_15_121_5	TL	475190	169965	180
average				235.4
max	m	1001068	1 400 00	355.4
50_20_121_1	TL	1001063	148269	575
50_20_121_2	TL	970253	136720	610
50_20_121_3	TL	254538	126947	101
50_20_121_4	TL	260172	162861	59.8
50_20_121_5	TL	860676	152263	465
average				362.1
max				609.7
60_15_121_1	TL	976633	180301	442
60_15_121_2	TL	731415	220802	231
60_15_121_3	TL	774066	177987	335
60_15_121_4	TL	962058	189154	409
60_15_121_5	TL	889064	212248	319
average				347.1
max				441.7
60_20_121_1	TL	368544	163661	125
60_20_121_2	TL	1495405	157938	847
60_20_121_3	TL	1648301	148447	1010
60_20_121_4	TL	1820536	188368	866
60_20_121_5	TL	1527299	186362	720
average				713.7
max				1010.4

Table 6.5: CPLEX Results - Group B

For group A the solver is able to find the optimal solution within the time limit in only one case (70\_40\_900\_1). In almost all other cases the value of the gap from the LB is high. The average gap is in fact about 20% and the maximum value about 30%. Only for sub-group 70\_40\_900 the behavior of CPLEX is different with a smaller gap from LB, on average below 3%.

Despite the smaller sizes of instances of Group B, we notice that the solver has more difficulties handling them. In this case the size of the instances (number of TTP and CS) affects very much the performance of the solver. It never finds the optimal solution within the time limit. When the number of CS is greater than 15 the average gap from LB is very high up to above 200% and for the largest instances (e.g. 60\_20\_121\_3) it reaches also 1010%.

## 6.2.2 Greedy Algorithms

In order to build a feasible solution, we developed four different greedy algorithms, described in Section 5.1.

Table 6.6 shows the results of the greedy algorithms on Group A; the first column gives the name of the instance, then for each greedy algorithm we report the value of the solution, the needed computational time, and the percentage gap from the UB and LB provided by the formulation.

On Group A all the greedy algorithms provide a feasible solution for each instance. None of the greedy algorithms improves upon the UB provided by CPLEX on instances of Group A. Gaps with respect to UB and LB provided by CPLEX are quite high (about 30% - 40%). We must also consider that the greedy algorithm does not take into account the possible different duration of time slots ( $\delta_t$ ); this could have a negative impact on the solution provided.

The computational times instead are very small: the average time is 60 s and the maximum is always below 4 minutes even for the largest instances.

INSTANCE	GREEDY_1	[s]	%UB	%LB	GREEDY_2	$\mathbf{s}$	%UB	%LB	GREEDY_3	[s]	%UB	%LB	GREEDY_4	$\mathbf{s}$	%UB	%LB
40-30-400-1	447113	56	36.4	58.2	426770	26	30.2	51.0	377778	29	15.3	33.6	412090	49	25.8	45.8
40_30_400_2	482136	62	43.5	61.6	494581	18	47.2	65.8	435610	55	29.6	46.0	424283	49	26.2	42.2
40-30-400-3	548868	46	38.9	75.6	477540	24	20.9	52.7	464603	51	17.6	48.6	512153	49	29.6	63.8
40-30-400-4	479672	44	24.7	42.3	600378	31	56.1	78.2	508490	51	32.2	50.9	468105	20	21.7	38.9
average		52	35.9	59.4		25	38.6	61.9		47	23.7	44.8		54	25.8	47.7
max		62	43.5	75.6		31	56.1	78.2		55	32.2	50.9		20	29.6	63.8
40-60-625-1	540999	121	39.5	56.3	590041	55	52.2	70.4	498153	128	28.5	43.9	522506	116	34.8	50.9
40_60_625_2	593174	145	31.7	72.1	707431	75	57.1	105.3	519846	109	15.4	50.8	526640	128	16.9	52.8
40_60_625_3	529851	135	26.2	52.1	594381	61	41.5	70.6	475280	122	13.2	36.4	513641	125	22.3	47.4
40_60_625_4	753011	118	105.9	111.9	58941	57	61.0	65.7	680300	126	86.0	91.4	515700	109	41.0	45.1
average		130	50.8	73.1		62	52.9	78.0		121	35.8	55.6		120	28.7	49.1
max		145	105.9	111.9		75	61.0	105.3		128	86.0	91.4		128	41.0	52.8
$50_{-}30_{-}400_{-}1$	544678	57	29.6	68.6	494815	32	17.7	53.2	554065	86	31.8	71.5	577295	56	37.4	78.7
$50_{-}30_{-}400_{-}2$	541570	65	23.4	57.5	485783	54	10.7	41.3	477217	56	8.7	38.8	463080	63	5.5	34.7
$50_{-}30_{-}400_{-}3$	570871	60	52.4	69.4	514764	41	37.4	52.8	475623	53	27.0	41.1	497212	51	32.8	47.5
$50_{-}30_{-}400_{-}4$	527241	63	29.6	47.6	505396	31	24.2	41.5	501171	45	23.2	40.3	502930	40	23.6	40.8
average		61	33.8	60.8		40	22.5	47.2		09	22.7	47.9		53	24.8	50.4
max		65	52.4	69.4		54	37.4	53.2		86	31.8	71.5		63	37.4	78.7
$50_{-}50_{-}625_{-}1$	575305	135	12.9	46.2	658833	86	29.3	67.5	597128	123	17.2	51.8	565727	133	11.0	43.8
50-50-625-2	570959	114	16.8	46.7	630397	78	29.0	62.0	584804	123	19.7	50.3	58809	107	20.5	51.3
50_50_625_3	649974	122	33.2	54.2	730004	72	49.6	73.2	601320	74	23.2	42.7	603642	86	23.7	43.2
$50_{-}50_{-}625_{-}4$	580627	102	22.0	39.1	651645	53	36.9	56.1	526827	96	10.7	26.2	573362	89	20.4	37.4
average		118	21.2	46.6		72	36.2	64.7		104	17.7	42.7		104	18.9	43.9
max		135	33.2	54.2		86	49.6	73.2		123	23.2	51.8		133	23.7	51.3
$70_{-40_{-900_{-1}}}$	1057935	178	45.1	45.1	1031083	132	41.4	41.4	1024254	173	40.4	40.5	1052515	182	44.3	44.3
$70_{-40_{-900_{-2}}}$	945509	159	26.4	30.1	1169760	137	56.4	61.0	932174	173	24.6	28.3	1013957	210	35.6	39.5
70_40_900_3	861087	135	22.0	28.7	898893	103	27.4	34.3	924151	148	31.0	38.1	877295	173	24.3	31.1
70_40_900_4	977633	190	35.2	37.1	989353	111	36.8	38.7	838703	203	16.0	17.6	853369	168	18.0	19.7
average		166	32.2	35.2		121	40.5	43.9		174	28.0	31.1		183	30.6	33.7
max		190	45.1	45.1		137	56.4	61.0		203	40.4	40.5		210	44.3	44.3

Table 6.6: Greedy Results - Group A

The results show that, even if the different greedy algorithms have quite similar behavior, the best one are the third and fourth version: this for example can be seen comparing the values of average gap from UB; greedy 3 and greedy 4 always improve upon greedy 1 and greedy 2. For this reason we tested only these two versions (3 and 4) of greedy on instances of Group B.

Results on Group B are reported in Tables 6.7 and 6.8. If the greedy is not able to find a feasible solution, as described in Chapter 5, we report the solution (marked with the symbol \*) obtained after a VLSN phase for recovering feasibility described in Section 5.5. However since instances are uniform enough in disposition of TTP and amount of generated traffic as in a real network, this procedure is rarely used (only for instance 50\_10\_121\_4). For Group B when the number of CS increases, the greedy algorithms (both 3 and 4) improve upon the result of CPLEX up to 85%. The average gap from UB in particular is always negative (and so it improves upon CPLEX UB) for instances with 50-60 TTP and 15-20 CS. Greedy 3 improves upon CPLEX in 30 out of 60 instances, while greedy 4 in 33 out of 60. However in the smallest instances they may have significant gaps (up to about 200% in one instance).

The results of the two greedy algorithms are quite similar and sometimes coincide, as in this group of instances the coverage has a small impact. In fact coverage of BS is very large and it is possible to cover all the CTP with a single BS (parameter  $\Omega$  is equal to one). The gap from LB remains fairly high but it is quite difficult to understand the real meaning of this value: we are not able to evaluate how much this value is accurate and how far the LB is from the global minimum of the problem. The computational time are very small: even for largest instances the greedy algorithms solve the problem in less than 30 seconds.

	CDEEDV 2	[_]	ØUD	07 I D	CDEEDV 4	[_]	07 UD	ØID
INSTANCE	GREEDY_3	$[\mathbf{s}]$	%UB	%LB	GREEDY_4	$[\mathbf{s}]$	%UB	%LB
20_10_121_1	112464	6	2.8	20.4	112464	6	2.8	20.4
20_10_121_2	140752	3	18.9	37.5	143521	2	21.2	40.2
20_10_121_3	120259	3	17.8	17.8	176769	4	73.2	73.2
20_10_121_4	103523	4	29.3	29.3	88085	2	10.0	10.0
20_10_121_5	102333	3	0.0	19.9	102333	4	0.0	19.9
average		4	13.8	25.0		4	21.4	32.7
max		6	29.3	37.5		6	73.2	73.2
30_10_121_1	195617	7	13.4	59.0	195617	6	13.4	59.0
30_10_121_2	165303	5	3.3	40.2	169459	6	5.9	43.7
30_10_121_3	155435	5	8.5	42.5	175805	6	22.7	61.2
30_10_121_4	146077	5	12.2	49.4	144705	4	11.2	48.0
30_10_121_5	153996	7	7.7	26.7	153996	6	7.7	26.7
average		6	9.0	43.6		6	12.2	47.7
max		7	13.4	59.0		6	22.7	61.2
30_15_121_1	138414	7	2.6	37.1	138414	7	2.6	37.1
30_15_121_2	147857	7	3.0	34.0	186416	6	29.9	69.0
30_15_121_3	135848	6	23.8	46.6	108045	3	-1.6	16.6
30_15_121_4	153012	8	-7.7	50.0	152919	6	-7.7	49.9
30_15_121_5	195548	11	45.1	81.2	195548	12	45.1	81.2
average		8	13.4	49.8		7	13.7	50.8
max		11	45.1	81.2		12	45.1	81.2
30_20_121_1	121794	8	-2.4	33.3	121794	9	-2.4	33.3
30_20_121_2	168560	7	16.4	63.9	174891	6	20.8	70.0
30_20_121_3	140194	6	-3.2	78.5	137306	5	-5.2	74.8
30_20_121_4	168221	8	-5.8	56.9	203689	7	14.0	90.0
30_20_121_5	164922	9	5.5	44.4	164922	9	5.5	44.4
average		8	2.1	55.4		7	6.5	62.5
max		9	16.4	78.5		9	20.8	90.0
40_10_121_1	201275	7	12.4	41.3	201275	8	12.4	41.3
40_10_121_2	199475	8	9.4	26.7	235274	7	29.0	49.4
40_10_121_3	229464	10	28.1	60.6	202903	7	13.3	42.0
40_10_121_4	604437	21	206.0	293.0	600437	19	204.0	290.4
40_10_121_5	222663	11	21.1	40.3	222663	10	21.1	40.3
average		11	55.4	92.4		10	56.0	92.7
max		21	206.0	293.0		19	204.0	290.4
40_15_121_1	179217	10	12.0	57.1	179217	10	12.0	57.1
40_15_121_2	218616	11	-6.7	54.2	204123	9	-12.8	44.0
40_15_121_3	194469	10	-80.1	57.4	204926	9	-79.1	65.8
40_15_121_4	216634	11	-2.5	59.4	194147	8	-12.6	42.9
40_15_121_5	193409	11	8.0	42.8	193409	11	8.0	42.8
average		11	-13.9	54.2		9	-16.9	50.5
max		11	12.0	59.4		11	12.0	65.8

Table 6.7: Greedy Results - Group B - Part 1

	1				1			
INSTANCE	GREEDY_3	[s]	%UB	%LB	GREEDY_4	[s]	%UB	%LB
40_20_121_1	159487	9	-0.8	48.2	159487	8	-0.8	48.2
40_20_121_2	202883	12	4.5	64.4	176906	6	-8.9	43.4
40_20_121_3	157519	8	-79.3	54.2	153672	8	-79.8	50.5
40_20_121_4	189869	12	-69.3	39.9	216875	12	-64.9	59.8
40_20_121_5	223724	15	17.9	68.3	223724	14	17.9	68.3
average		11	-25.4	55.0		10	-27.3	54.0
max		15	17.9	68.3		14	17.9	68.3
50_10_121_1	210705	10	6.7	40.7	210705	9	6.7	40.7
50_10_121_2	225956	11	8.4	19.4	236069	9	13.3	24.7
50_10_121_3	658906	33	203.7	223.7	262077	11	20.8	28.7
50_10_121_4	372719*	690	60.8	60.8	228929*	320	-1.2	-1.2
50_10_121_5	245704	13	7.0	31.9	245704	13	7.0	31.9
average		151	57.3	75.3		72	9.3	25.0
max		690	203.7	223.7		320	20.8	40.7
50_15_121_1	264872	19	-60.6	79.3	264872	18	-60.6	79.3
50_15_121_2	810687	51	31.5	370.5	262304	12	-57.4	52.2
50_15_121_3	236547	12	-20.7	62.2	224787	14	-24.6	54.2
50_15_121_4	304647	19	-53.5	76.6	254437	15	-61.1	47.5
50_15_121_5	304004	22	-36.0	78.9	304004	22	-36.0	78.9
average		25	-27.9	133.5		16	-48.0	62.4
max		51	31.5	370.5		22	-24.6	79.3
50_20_121_1	191724	13	-80.8	29.3	191724	13	-80.8	29.3
50_20_121_2	206694	13	-78.7	51.2	256008	15	-73.6	87.2
50_20_121_3	193716	12	-23.9	52.6	227920	19	-10.5	79.5
50_20_121_4	253990	23	-2.4	56.0	252133	14	-3.1	54.8
50_20_121_5	253840	21	-70.5	66.7	253840	21	-70.5	66.7
average		16	-51.3	51.2		16	-47.7	63.5
max		23	-2.4	66.7		21	-3.1	87.2
60_15_121_1	318515	27	-67.4	76.7	318515	27	-67.4	76.7
60_15_121_2	358712	27	-51.0	62.5	369015	23	-49.5	67.1
60_15_121_3	282044	29	-63.6	58.5	345689	24	-55.3	94.2
60_15_121_4	280775	22	-70.8	48.4	296372	24	-69.2	56.7
60_15_121_5	311203	31	-65.0	46.6	311203	31	-65.0	46.6
average		27	-63.5	58.5		26	-61.3	68.3
max		31	-51.0	76.7		31	-49.5	94.2
60_20_121_1	252340	21	-31.5	54.2	252340	21	-31.5	54.2
60_20_121_2	238576	17	-84.0	51.1	313669	24	-79.0	98.6
60_20_121_3	224820	17	-86.4	51.4	251375	17	-84.7	69.3
60_20_121_4	250933	21	-86.2	33.2	339130	29	-81.4	80.0
60_20_121_5	328023	32	-78.5	76.0	328023	32	-78.5	76.0
average		22	-73.3	53.2		25	-71.0	75.6
max		32	-31.5	76.0		32	-31.5	98.6

Table 6.8: Greedy Results - Group B - Part 2

## 6.2.3 Local Search starting from greedy solutions

The results obtained with the greedy constructive algorithms can be improved by applying local search methods. In this section we analyze the results obtained applying different local search procedures. The most promising procedures are then applied also to improve the solutions provided by other constructive approaches. We start from the analysis of the results of local search in which each neighbor solution is completed by solving subproblem EMCA\_Concentrated. The neighborhood includes all the possible solutions generated from the starting one by adding, removing one BS or swapping one BS with one element not in the solution. Each neighbor is then completed by solving EMCA\_Concentrated.

As the BS set is selected, EMCA\_Concentrated requires a computational time smaller than EMCA.

In order to further limit the computational time we also limit the number of changes of device in the subproblems EMCA\_Concentrated. The value of M and M' of constraints (16) and (17) described in Section 5.4 are set based on the number of BS in the initial solution provided by greedy algorithms. If the value of M and M' is high the neighborhood may be too large to be explored in short time. On the other hand with a small value the solver might not find an improving solution. After some preliminary tests on these two parameters, we found out that good results can be obtained by using:

 $M = M' = \frac{\text{number of BS in the solution}}{2};$ 

In this way it is possible to change the devices in at most half of the BS selected in the current solution.

We use as time limit for the sub-problems EMCA\_Concentrated 30 seconds; in-

stead, the overall time limit (TL) is 1 hour.

Result of LS starting from greedy solutions for Group A are reported in Tables 6.9, 6.10, 6.11 and 6.12 for greedy 1,2,3 and 4 respectively. For each instance the name of the instance, the UB provided by greedy, the UB provided by LS, the computational times and the gaps from UB and LB provided by CPLEX; in the column *Imp* we also report the local search improvement with respect to the greedy solution. This value is computed as:

$$Imp = \frac{\mid UB_{LS} - UB_{Greedy} \mid}{\mid UB_{Greedy} \mid} \cdot 100$$

If the value of Imp is negative, it means that the local search improves the solution. We report the values of the results obtained with the two neighborhood exploration strategies: steepest descent (SD) and first improvement (FI).

From the results we observe that both the strategies (SD and FI) provide large improvements, on average of about 30% with respect to the solution of the greedy. However LS does not always improve upon CPLEX. In fact starting from greedy 1 it improves upon CPLEX in 7 out of 20 instances if SD is applied and in 9 out of 20 if FI is applied. The gap may be significant as it rises up to about 35% and 16% for SD and FI respectively. The results are in general better if the number of CS is small, and get worse with the increasing number of CS and TTP. FI strategy behaves better than SD both in terms of UB and computational times. The time limit is almost always reached. Similar results are obtained with the other versions of greedy algorithms.

First improvement heuristic local search applied on greedy 3 and greedy 4 solutions provides the best results: starting from greedy 3 it improves upon CPLEX in 11 out of 20 instances and starting from greedy 4 in 10 out of 20.

INSTANCE	GREEDY_1	LS (SD)	$[\mathbf{s}]$	%_UB	%_LB	Imp	LS (FI)	$[\mathbf{s}]$	%_UB	%_LB	Imp
40_30_400_1	447113	330437	TL	0.8	16.9	-26.1	318460	1634	-2.8	12.7	-28.8
40_30_400_2	482136	341998	TL	1.8	14.6	-29.1	366382	671	9.0	22.8	-24.0
40_30_400_3	548868	369007	TL	-6.6	18.0	-32.8	365312	1247	-7.5	16.8	-33.4
40_30_400_4	479672	377771	TL	-1.8	12.1	-21.2	375457	1610	-2.4	11.4	-21.7
average				-1.5	15.4	-27.3		1291	-0.9	15.9	-27.0
max				1.8	18.0	-21.2		1634	9.0	22.8	-21.7
40_60_625_1	540999	448796	TL	15.7	29.6	-17.0	433156	2618	11.7	25.1	-19.9
40_60_625_2	593174	474686	TL	5.4	37.7	-20.0	432812	TL	-3.9	25.6	-27.0
40_60_625_3	529851	444710	TL	5.9	27.6	-16.1	466717	TL	11.1	33.9	-11.9
40_60_625_4	753011	493382	TL	34.9	38.8	-34.5	424611	TL	16.1	19.5	-43.6
average				15.5	33.5	-21.9			8.8	26.0	-25.6
max				34.9	38.8	-16.1			16.1	33.9	-11.9
50_30_400_1	544678	394549	TL	-6.1	22.1	-27.6	384472	874	-8.5	19.0	-29.4
50_30_400_2	541570	408454	TL	-6.9	18.8	-24.6	415838	868	-5.3	20.9	-23.2
50_30_400_3	570871	396591	TL	5.9	17.7	-30.5	393901	704	5.2	16.9	-31.0
50_30_400_4	527241	400499	TL	-1.6	12.2	-24.0	400499	635	-1.6	12.2	-24.0
average				-2.2	17.7	-26.7		770	-2.5	17.2	-26.9
max				5.9	22.1	-24.0		874	5.2	20.9	-23.2
50_50_625_1	575305	501178	TL	-1.7	27.4	-12.9	468071	TL	-8.2	19.0	-18.6
50_50_625_2	570959	461637	TL	-5.5	18.6	-19.1	489946	2920	0.2	25.9	-14.2
50_50_625_3	649974	518121	TL	6.2	22.9	-20.3	498602	3133	2.2	18.3	-23.3
50_50_625_4	580627	503925	TL	5.8	20.7	-13.2	489053	2970	2.7	17.2	-15.8
average				1.2	22.4	-16.4			-1.0	19.8	-18.2
max				6.2	27.4	-12.9			2.7	24.6	-15.0
70_40_900_1	1057935	807433	TL	10.7	10.7	-23.7	762375	TL	4.5	4.5	-27.9
70_40_900_2	945509	805582	TL	7.7	10.8	-14.8	798906	2426	6.8	9.9	-15.5
70_40_900_3	861087	743876	TL	5.4	11.2	-13.6	692449	TL	-1.9	3.5	-19.6
70_40_900_4	977633	774239	TL	7.1	8.6	-20.8	725124	TL	0.3	1.7	-25.8
average				7.7	10.3	-18.2			2.4	4.8	-22.3
max				10.7	11.2	-13.6			6.8	9.9	-15.5

Table 6.9: Local Search Results from Greedy\_1 - Group A

INSTANCE	GREEDY_2	LS (SD)	[s]	%_UB	%_LB	Imp	LS (FI)	[s]	%_UB	%_LB	Imp
40_30_400_1	426770	302312	2088	-7.7	6.9	-29.2	302312	542	-7.7	6.9	-29.2
40_30_400_2	494581	349624	1502	4.0	17.2	-29.3	349624	1762	4.0	17.2	-29.3
40_30_400_3	477540	372385	1367	-5.7	19.1	-22.0	365312	1073	-7.5	16.8	-23.5
40_30_400_4	600378	377771	TL	-1.8	12.1	-37.1	383128	1140	-0.4	13.7	-36.2
average				-2.8	13.8	-29.4		1129	-2.9	13.7	-29.5
max				4.0	19.1	-22.0		1762	4.0	17.2	-23.5
40_60_625_1	590041	481025	TL	24.1	38.9	-18.5	464681	TL	19.8	34.2	-21.2
40_60_625_2	707431	493091	TL	9.5	43.1	-30.3	457460	TL	1.6	32.7	-35.3
40_60_625_3	594381	494426	TL	17.7	41.9	-16.8	474081	TL	12.9	36.1	-20.2
40_60_625_4	588941	424480	TL	16.1	19.5	-27.9	426306	TL	16.6	20.0	-27.6
average				16.8	35.8	-23.4			12.7	30.7	-26.1
max				24.1	43.1	-16.8			19.8	36.1	-20.2
50_30_400_1	494815	382311	1281	-9.0	18.3	-22.7	384472	539	-8.5	19.0	-22.3
50_30_400_2	485783	393644	1284	-10.3	14.5	-19.0	397170	861	-9.5	15.5	-18.2
50_30_400_3	514764	380478	1368	1.6	12.9	-26.1	378414	326	1.0	12.3	-26.5
50_30_400_4	505396	412632	2050	1.4	15.5	-18.4	394429	1784	-3.1	10.5	-22.0
average				-4.1	15.3	-21.5		877	-5.0	14.3	-22.2
max				1.6	18.3	-18.4		1784	1.0	19.0	-18.2
50_50_625_1	658833	474181	TL	-7.0	20.5	-28.0	481958	TL	-5.4	22.5	-26.8
50_50_625_2	630397	496193	TL	1.5	27.5	-21.3	476479	TL	-2.5	22.4	-24.4
50_50_625_3	730004	532643	TL	9.2	26.4	-27.0	508250	TL	4.2	20.6	-30.4
50_50_625_4	651645	469792	TL	-1.3	12.6	-27.9	490060	TL	2.9	17.4	-24.8
average				0.6	21.7	-26.1			-0.2	20.7	-26.6
max				9.2	27.5	-21.3			4.2	22.5	-24.4
70_40_900_1	1031083	754888	TL	3.5	3.5	-26.8	757608	2354	3.9	3.9	-26.5
70_40_900_2	1169760	801947	TL	7.2	10.3	-31.4	763966	1791	2.1	5.1	-34.7
70_40_900_3	898893	692449	TL	-1.9	3.5	-23.0	692449	3124	-1.9	3.5	-23.0
70_40_900_4	989353	745088	TL	3.0	4.5	-24.7	793726	TL	9.8	11.3	-19.8
average				3	5.5	-26.5			-7.2	10.3	-23.0
max				3.5	5.9	-26.0			9.8	11.3	-19.8

Table 6.10: Local Search Results from Greedy\_2 - Group A

INSTANCE	GREEDY_3	LS (SD)	$[\mathbf{s}]$	%_UB	%_LB	Imp	LS (FI)	$[\mathbf{s}]$	%_UB	%_LB	Imp
40_30_400_1	377778	317824	1607	-3.0	12.4	-15.9	318460	983	-2.8	12.7	-15.7
40_30_400_2	435610	341998	2919	1.8	14.6	-21.5	366382	956	9.0	22.8	-15.9
40_30_400_3	464603	365345	2976	-7.5	16.9	-21.4	365345	1620	-7.5	16.9	-21.4
40_30_400_4	508490	396160	TL	3.0	17.6	-22.1	403830	1402	5.0	19.8	-20.6
average				-1.5	15.4	-20.2			0.9	18.0	-18.4
max				3.0	17.6	-15.9			9.0	22.8	-15.7
40_60_625_1	498153	443676	TL	14.4	28.2	-10.9	400392	TL	3.3	15.7	-19.6
40_60_625_2	519846	458964	TL	1.9	33.2	-11.7	446792	TL	-0.8	29.6	-14.1
40_60_625_3	475280	442894	TL	5.5	27.1	-6.8	420883	TL	0.2	20.8	-11.4
40_60_625_4	680300	479872	TL	31.2	35.0	-29.5	437414	TL	19.6	23.1	-35.7
average				13.2	30.9	-14.7			5.6	22.3	-20.2
max				31.2	35.0	-6.8			19.6	29.6	-11.4
50_30_400_1	554065	375815	TL	-10.6	16.3	-32.2	393554	1651	-6.4	21.8	-29.0
50_30_400_2	477217	388728	1092	-11.4	13.0	-18.5	388728	591	-11.4	13.0	-18.5
50_30_400_3	475623	395432	2160	5.6	17.3	-16.9	374793	1600	0.1	11.2	-21.2
50_30_400_4	501171	416304	TL	2.3	16.6	-16.9	405228	1812	-0.4	13.5	-19.1
average				-3.5	15.8	-21.1			-4.5	14.9	-22.0
max				5.6	17.3	-16.9			0.1	21.8	-18.5
50_50_625_1	597128	497432	TL	-2.4	26.4	-16.7	473622	TL	-7.1	20.4	-20.7
50_50_625_2	584804	490051	TL	0.3	25.9	-16.2	465427	TL	-4.8	19.6	-20.4
50_50_625_3	601320	527991	TL	8.2	25.3	-12.2	531261	TL	8.9	26.1	-11.7
50_50_625_4	526827	487474	TL	2.4	16.8	-7.5	475944	TL	0.0	14.0	-9.7
average				2.1	23.6	-13.1			-0.7	20.0	-15.6
max				8.2	26.4	-7.5			8.9	26.1	-9.7
70_40_900_1	1024254	807098	TL	10.7	10.7	-21.2	784719	TL	7.6	7.6	-23.4
70_40_900_2	932174	855327	TL	14.3	17.7	-8.2	798906	TL	6.8	9.9	-14.3
70_40_900_3	924151	751276	TL	6.5	12.3	-18.7	748566	TL	6.1	11.9	-19.0
70_40_900_4	838703	729825	TL	0.9	2.3	-13.0	761130	TL	5.3	6.7	-9.2
average				8.1	10.7	-15.3			6.4	9.0	-16.5
max				14.3	17.7	-8.2			7.6	11.9	-9.2

Table 6.11: Local Search Results from Greedy\_3 - Group A

INSTANCE	GREEDY_4	LS (SD)	$[\mathbf{s}]$	%_UB	%_LB	Imp	LS (FI)	$[\mathbf{s}]$	%_UB	%_LB	Imp
40_30_400_1	412090	302312	2074	-7.7	6.9	-26.6	302312	571	-7.7	6.9	-26.6
40_30_400_2	424283	341998	2243	1.8	14.6	-19.4	361605	851	7.6	21.2	-14.8
40_30_400_3	512153	364745	TL	-7.7	16.7	-28.8	365312	1245	-7.5	16.8	-28.7
40_30_400_4	468105	402405	2830	4.6	19.4	-14.0	375457	1290	-2.4	11.4	-19.8
average				-2.3	14.4	-22.2			-2.5	14.1	-22.5
max				4.6	19.4	-14.0			7.6	21.2	-14.8
40_60_625_1	522506	464947	TL	19.9	34.3	-11.0	432071	TL	11.4	24.8	-17.3
40_60_625_2	526640	444663	TL	-1.3	29.0	-15.6	437704	TL	-2.8	27.0	-16.9
40_60_625_3	513641	435944	TL	3.8	25.1	-15.1	444919	TL	5.9	27.7	-13.4
40_60_625_4	515700	442948	TL	21.1	24.7	-14.1	414026	TL	13.2	16.5	-19.7
average				10.9	28.3	-14.0			6.9	24.0	-16.8
max				21.1	34.3	-11.0			13.2	27.7	-13.4
50_30_400_1	577295	377796	TL	-10.1	16.9	-34.6	389336	1378	-7.4	20.5	-32.6
50_30_400_2	463080	388728	1558	-11.4	13.0	-16.1	388728	781	-11.4	13.0	-16.1
50_30_400_3	497212	379922	2895	1.4	12.7	-23.6	414738	927	10.7	23.1	-16.6
50_30_400_4	502930	408193	TL	0.3	14.3	-18.8	405228	1818	-0.4	13.5	-19.4
average				-4.9	14.3	-23.3			-2.1	17.5	-21.2
max				1.4	16.9	-16.1			10.7	23.1	-16.1
50_50_625_1	565727	485144	TL	-4.8	23.3	-14.2	474410	2838	-6.9	20.6	-16.1
50_50_625_2	588809	479142	TL	-2.0	23.1	-18.6	464336	TL	-5.0	19.3	-21.1
50_50_625_3	603642	487087	TL	-0.2	15.6	-19.3	507707	TL	4.1	20.5	-15.9
50_50_625_4	573362	496206	TL	4.2	18.9	-13.5	482436	TL	1.3	15.6	-15.9
average				-0.7	20.2	-16.4			-1.6	19.0	-17.3
max				4.2	23.3	-13.5			4.1	20.6	-15.9
70_40_900_1	1052515	781410	TL	7.1	7.2	-25.8	732137	2919	0.4	0.4	-30.4
70_40_900_2	1013957	812197	TL	8.6	11.8	-19.9	763697	TL	2.1	5.1	-24.7
70_40_900_3	877295	719463	TL	2.0	7.5	-18.0	753773	TL	6.8	12.6	-14.1
70_40_900_4	853369	759496	TL	5.0	6.5	-11.0	766110	TL	5.9	7.4	-10.2
average				5.7	8.2	-18.7			3.8	6.4	-19.9
max				8.6	11.8	-11.0			6.8	12.6	-10.2

Table 6.12: Local Search Results from Greedy\_4 - Group A

We propose a graph to summarize the results from the tables and figure out which local search method is more advantageous on Group A. In the histogram (Fig. 6.4) a summary of the different performance of LS algorithms applied to variants of the greedy is shown. The column below (green) indicates the number of instances in which LS improves upon CPLEX. The column above (yellow) instead reports the number of instances where LS obtains a slightly worse result compared to CPLEX (gap  $\leq 10\%$ ) but in much less time ( $\leq 2000$  s). We note that the FI performs better than SD starting with all the greedy versions. The best behavior is obtained starting from greedy 3 and greedy 4 solutions.

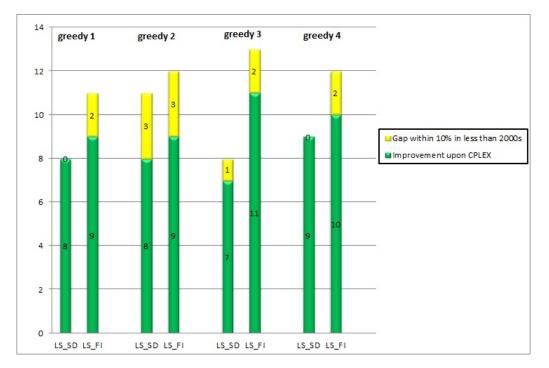


Figure 6.4: Greedy and Local Search Results - Group A

On Group B (results are reported in Tables 6.13, 6.14, 6.15 and 6.16) we applied only those methods that seem to produce the best results: in particular greedy 3 and greedy 4 as told in Section 6.2.2 and first improvement heuristic local search (FI).

INSTANCE	GREEDY_3	LS (FI)	s	%_UB	%_LB	Imp
	GREED I _3		[8]	/0_UD	/0_LD	
20_10_121_1	112464	109428	116	0.0	17.2	-2.7
20_10_121_2	140752	118159	152	-0.2	15.5	-16.1
20_10_121_3	120259	103092	120	1.0	1.0	-14.3
20_10_121_4	103523	91832	52	14.7	14.7	-11.3
20_10_121_5	102333	100753	139	-1.5	18.0	-1.5
average				2.8	13.3	-9.2
max				14.7	18.0	-1.5
30_10_121_1	195617	150919	643	-12.5	22.7	-22.8
30_10_121_2	165303	147200	392	-8.0	24.8	-11.0
30_10_121_3	155435	126601	235	-11.7	16.1	-18.6
30_10_121_4	146077	120477	206	-7.4	23.2	-17.5
30_10_121_5	153996	139511	1247	-2.4	14.8	-9.4
average				-8.4	20.3	-15.9
max				-2.4	24.8	-9.4
30_15_121_1	138414	129589	831	-4.0	28.3	-6.4
30_15_121_2	147857	140605	465	-2.0	27.4	-4.9
30_15_121_3	135848	106723	340	-2.8	15.2	-21.4
30_15_121_4	153012	127232	331	-23.2	24.7	-16.8
30_15_121_5	195548	122737	1174	-8.9	13.8	-37.2
average				-8.2	21.9	-17.4
max				-2.0	28.3	-4.9
30_20_121_1	121794	107540	1180	-13.9	17.7	-11.7
30_20_121_2	168560	126537	863	-12.6	23.0	-24.9
30_20_121_3	140194	89567	347	-38.2	14.0	-36.1
30_20_121_4	168221	124402	1008	-30.4	16.1	-26.0
30_20_121_5	164922	143130	1084	-8.4	25.3	-13.2
average				-20.7	19.2	-22.4
max				-8.4	25.3	-11.7
40_10_121_1	201275	170813	271	-4.6	19.9	-15.1
40_10_121_2	199475	184513	226	1.2	17.2	-7.5
40_10_121_3	229464	175625	271	-1.9	22.9	-23.5
40_10_121_4	604437	186155	462	-5.7	21.0	-69.2
40_10_121_5	222663	181805	298	-1.1	14.6	-18.3
average				-2.4	19.1	-26.7
max				1.2	22.9	-7.5
40_15_121_1	179217	137000	1391	-14.4	20.1	-23.6
40_15_121_2	218616	183704	1594	-21.6	29.6	-16.0
40_15_121_3	194469	147267	840	-85.0	19.2	-24.3
40_15_121_4	216634	169198	884	-23.8	24.5	-21.9
40_15_121_5	193409	168907	1113	-5.7	24.7	-12.7
average				-30.1	23.6	-19.7
max				-5.7	29.6	-12.7

Table 6.13: Local Search Results from Greedy\_3 - Group B - Part 1

INSTANCE	GREEDY_3	LS (FI)	[s]	%_UB	%_LB	Imp
40_20_121_1	159487	139633	2026	-13.1	29.8	-12.4
40_20_121_2	202883	159932	1653	-17.7	29.6	-21.2
40_20_121_3	157519	129665	702	-83.0	27.0	-17.7
40_20_121_4	189869	171401	1627	-72.3	26.3	-9.7
40_20_121_5	223724	163748	TL	-13.7	23.2	-26.8
average				-39.9	27.2	-17.6
max				-13.1	29.8	-9.7
50_10_121_1	210705	176829	375	-10.5	18.1	-16.1
50_10_121_2	225956	204068	645	-2.1	7.8	-9.7
50_10_121_3	658906	216402	702	-0.3	6.3	-67.2
50_10_121_4	372719	372720	830	60.8	60.8	0.0
50_10_121_5	245704	218041	733	-5.1	17.1	-11.3
average				8.6	22.0	-20.8
max				60.8	60.8	0.0
50_15_121_1	264872	264872	2472	-60.6	79.3	0.0
50_15_121_2	810687	225912	2348	-63.3	31.1	-72.1
50_15_121_3	236547	188023	1694	-37.0	29.0	-20.5
50_15_121_4	304647	218284	1260	-66.7	26.6	-28.3
50_15_121_5	304004	304004	2472	-36.0	78.9	0.0
average				-52.7	49.0	-24.2
max				-36.0	79.3	0.0
50_20_121_1	191724	191724	TL	-80.8	29.3	0.0
50_20_121_2	206694	173907	1530	-82.1	27.2	-15.9
50_20_121_3	193716	158390	2272	-37.8	24.8	-18.2
50_20_121_4	253990	208498	1146	-19.9	28.0	-17.9
50_20_121_5	253840	189983	1819	-77.9	24.8	-25.2
average				-59.7	26.8	-15.4
max				-19.9	29.3	0.0
60_15_121_1	318515	318515	2562	-67.4	76.7	0.0
60_15_121_2	358712	358712	2491	-51.0	62.5	0.0
60_15_121_3	282044	211421	TL	-72.7	18.8	-25.0
60_15_121_4	280775	236064	1365	-75.5	24.8	-15.9
60_15_121_5	311203	311203	2490	-65.0	46.6	0.0
average				-66.3	45.9	-8.2
max				-51.0	76.7	0.0
60_20_121_1	252340	252340	TL	-31.5	54.2	0.0
60_20_121_2	238576	196290	TL	-86.9	24.3	-17.7
60_20_121_3	224820	180843	TL	-89.0	21.8	-19.6
60_20_121_4	250933	250933	TL	-86.2	33.2	0.0
60_20_121_5	328023	328023	TL	-78.5	76.0	0.0
average			-	-74.4	41.9	-7.5
max				-31.5	76.0	0.0

Table 6.14: Local Search Results from Greedy\_3 - Group B - Part 2

INSTANCE	GREEDY_4	LS (FI)	[s]	%_UB	%_LB	Imp
20_10_121_1	112464	109428	106	0.0	17.2	-2.7
20_10_121_2	143521	118589	108	0.2	15.9	-17.4
20_10_121_3	176769	111767	97	9.5	9.5	-36.8
20_10_121_4	88085	85238	63	6.4	6.4	-3.2
20_10_121_5	102333	100753	125	-1.5	18.0	-1.5
average				2.9	13.4	-12.3
max				9.5	18.0	-1.5
30_10_121_1	195617	150919	557	-12.5	22.7	-22.8
30_10_121_2	169459	144908	451	-9.5	22.9	-14.5
30_10_121_3	175805	126601	239	-11.7	16.1	-28.0
30_10_121_4	144705	120477	178	-7.4	23.2	-16.7
30_10_121_5	153996	139511	1111	-2.4	14.8	-9.4
average				-8.7	19.9	-18.3
max				-2.4	23.2	-9.4
30_15_121_1	138414	129589	706	-4.0	28.3	-6.4
30_15_121_2	186416	139460	485	-2.8	26.4	-25.2
30_15_121_3	108045	103508	168	-5.7	11.7	-4.2
30_15_121_4	152919	127232	368	-23.2	24.7	-16.8
30_15_121_5	195548	122737	1177	-8.9	13.8	-37.2
average				-8.9	21.0	-18.0
max				-2.8	28.3	-4.2
30_20_121_1	121794	107540	1308	-13.9	17.7	-11.7
30_20_121_2	174891	126537	985	-12.6	23.0	-27.6
30_20_121_3	137306	89567	356	-38.2	14.0	-34.8
30_20_121_4	203689	124402	1040	-30.4	16.1	-38.9
30_20_121_5	164922	143130	1155	-8.4	25.3	-13.2
average				-20.7	19.2	-25.3
max				-8.4	25.3	-11.7
40_10_121_1	201275	170813	279	-4.6	19.9	-15.1
40_10_121_2	235274	184513	235	1.2	17.2	-21.6
40_10_121_3	202903	176107	303	-1.7	23.2	-13.2
40_10_121_4	600437	191303	582	-3.1	24.4	-68.1
40_10_121_5	222663	181805	302	-1.1	14.6	-18.3
average				-1.9	19.9	-27.3
max				1.2	24.4	-13.2
40_15_121_1	179217	137000	1441	-14.4	20.1	-23.6
40_15_121_2	204123	173816	843	-25.8	22.6	-14.8
40_15_121_3	204926	147267	860	-85.0	19.2	-28.1
40_15_121_4	194147	169198	876	-23.8	24.5	-12.9
40_15_121_5	193409	168907	1041	-5.7	24.7	-12.7
average				-30.9	22.2	-18.4
max				-5.7	24.7	-12.7

Table 6.15: Local Search Results from Greedy\_4 - Group B - Part 1

INSTANCE	GREEDY_4	LS (FI)	[s]	%_UB	%_LB	Imp
40_20_121_1	159487	139633	1919	-13.1	29.8	-12.4
40_20_121_2	176906	154170	610	-20.6	24.9	-12.9
40_20_121_3	153672	129665	1163	-83.0	27.0	-15.6
40_20_121_4	216875	171401	1588	-72.3	26.3	-21.0
40_20_121_5	223724	163748	3750	-13.7	23.2	-26.8
average				-40.5	26.2	-17.7
max				-13.1	29.8	-12.4
50_10_121_1	210705	176829	369	-10.5	18.1	-16.1
50_10_121_2	236069	204068	759	-2.1	7.8	-13.6
50_10_121_3	262077	216402	1008	-0.3	6.3	-17.4
50_10_121_4	228929	228929	1065	-1.2	-1.2	0.0
50_10_121_5	245704	218041	736	-5.1	17.1	-11.3
average	236697	208854	787	-3.8	9.6	-11.7
max	262077	228929	1065	-0.3	18.1	0.0
50_15_121_1	264872	264872	2487	-60.6	79.3	0.0
50_15_121_2	262304	209809	2054	-66.0	21.8	-20.0
50_15_121_3	224787	176603	1591	-40.8	21.1	-21.4
50_15_121_4	254437	225947	3012	-65.5	31.0	-11.2
50_15_121_5	304004	304004	2485	-36.0	78.9	0.0
average				-53.8	46.4	-10.5
max				-36.0	79.3	0.0
50_20_121_1	191724	191724	TL	-80.8	29.3	0.0
50_20_121_2	256008	175423	TL	-81.9	28.3	-31.5
50_20_121_3	227920	149269	3000	-41.4	17.6	-34.5
50_20_121_4	252133	204715	3270	-21.3	25.7	-18.8
50_20_121_5	253840	189983	1781	-77.9	24.8	-25.2
average				-60.7	25.1	-22.0
max				-21.3	29.3	0.0
60_15_121_1	318515	318515	2539	-67.4	76.7	0.0
60_15_121_2	369015	369015	2537	-49.5	67.1	0.0
60_15_121_3	345689	345689	2472	-55.3	94.2	0.0
60_15_121_4	296372	296372	2503	-69.2	56.7	0.0
60_15_121_5	311203	311203	2491	-65.0	46.6	0.0
average				-61.3	68.3	0.0
max				-49.5	94.2	0.0
60_20_121_1	252340	252340	TL	-31.5	54.2	0.0
60_20_121_2	313669	313669	TL	-79.0	98.6	0.0
60_20_121_3	251375	185160	TL	-88.8	24.7	-26.3
60_20_121_4	339130	339130	TL	-81.4	80.0	0.0
60_20_121_5	328023	328023	TL	-78.5	76.0	0.0
average				-71.8	66.7	-5.3
max			3787	-31.5	98.6	0.0

Table 6.16: Local Search Results from Greedy\_4 - Group B - Part 2

The LS improves upon CPLEX in 56 over 60 instances starting from greedy 3 and in 57 over 60 starting from greedy 4. The results starting from the solutions provided by the two greedy algorithms are similar. The average improvement of LS is about 15% for smallest instances and it rises up to 50% for largest. However we note that in one instance (50\_10\_121\_4 starting from greedy 3) the gap from UB is very high (60%). The improvements upon greedy due to LS are always significant (average 30%). Unlike Group A, for these instances the overall time limit is rarely reached and the algorithm is often able to find a solution improving upon the one provided by CPLEX.

Fig. 6.5 reports a summary of the results: since differently from Group A, greedy algorithms on these instances have good results, we report also the statistics on greedy algorithms applied to this group of instances. The meaning of the columns is the same as for Group A.

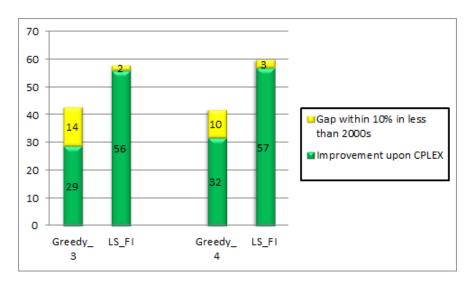


Figure 6.5: Greedy and Local Search Results - Group B

## 6.2.4 Very Large Scale Neighborhood

We also tested the effectiveness of the VLSN approach described in Section 5.5. In this case the time limit for each iteration should be carefully chosen: if too large it guarantees few iterations, if too small the search may be interrupted too early and no improving neighbor is found. After some preliminary tests we set the time limit for each iteration equal to 10 minutes. Obviously also the size of the neighborhood has a great importance: the same considerations hold for this aspect.

In order to compare the quality of the VLSN approach with the results of the local search, we set equal to one parameters N and N' of constraints (14) e (15) from Section 5.5. These parameters represent the number of CSs that can be changed at each iteration. In this way VLSN and local search use the same move to generate the neighborhood. We used the same values of LS also for parameters M and M', representing the number of possible changes of devices. The aim is to evaluate if there are differences in the behavior of the two algorithms with respect to the computational times. Finally we tried to increase values of these parameters (N = N' = 2) to evaluate if larger neighborhood may lead to better results.

Table 6.17 shows the results of VLSN approach starting from greedy 3: in addition to the instance name and the result provided by greedy, VLSN result with SD strategy (VLSN (SD), N = N' = 1), FI (VLSN (FI), N = N' = 1) and FI with N = N' = 2 (VLSN (FI.2)) are reported. For each method the computational time, gaps from UB and LB provided by CPLEX and the percentage of improvement over the greedy result are also reported.

	%-UB
-13.1	0.2 $16.2$
-22.3	0.7 13.5
-16.9	-2.3 23.5
-24.5	-0.2 13.9
-19.2	-0.4 16.7
5 -13.1	0.7 23.5
9.0.0	28.5 43.9
50.8 0.0	15.4 $50$
36.4 $0.0$	13.2 36
26.7 -33.8	23.1 26
4 -8.5	20.0 39.4
.8 0.0	50.8
40.1 -18.3	7.7 40
20.5 -13.2	-5.6 20
17.3 -16.9	5.6 17
27.1 -9.4	11.6 27
26.3 -14.4	
40.1 -9.4	
51.8 0.0	17.2 5
50.3 0.0	
30.4 -8.6	
22.2 -3.2	7.2 22
38.7 -2.9	14.1 38
51.8 0.0	
5 -28.5	0.5 0.5
9.3 -14.8	6.2 9.
9.3 -20.8	3.7 9
1.4 -13.8	0.0
5.1 -19.5	2.6
9.3 -13.8	0 0 0 0

Table 6.17: VLSN Results from Greedy\_3 - Group A

Even if there is often an improvement over the greedy results, it is lower than the one found with the local search. Also in the case of small neighborhood (N=1) the results are not very good: solving the problem with this technique is not efficient; in fact the average gap with respect to CPLEX is around 25%.

We also note that increasing the neighborhood size has a negative effect on the quality of the solution. The time limit is reached very often.

From the results we then realize that is more advantageous explicitly generate the neighborhood within a LS framework. Then complete the solution with ILP. We decided to neglect VLSN methods and use only the local search for the remaining tests.

## 6.2.5 Heuristic Concentration and Local Search

We tested the proposed Heuristic Concentration algorithm described in Section 5.2 on instances of Group A and B. Once obtained the initial solution, we then applied a local search step.

Tables 6.18, 6.20 and 6.21 report the results for Group A and Group B respectively: the second column represents the values of the result obtained using HC algorithm with its computational time and gap from UB and LB provided by CPLEX. The time also takes into account the execution of all greedy algorithms: on large instances, the time required to execute all greedy algorithms can compromise the entire procedure.

The other columns represent the results after the local search phase; the notation is the same described for Table 6.9.

The size of the concentration set has a great impact on the quality of the solution: we must indeed ensure that it is big enough, but without compromising the efficiency and usefulness of the algorithm since the size has a significant impact on the needed computational time. If it is too small a good solution cannot be found.

The time limit for solving EMCA\_Concentrated on the concentration set is set equal to 10 minutes.

The results in Table 6.18 show that HC improves upon CPLEX in 8 out of 20 instances and in 10 instances the solution found is within 10% from CPLEX UB. Even if the time of this first phase include also the time needed for solving all four greedy algorithms, the overall computational time for HC is reasonable and always below 15 minutes.

The HC results are further improved by the local search phase; SD and FI strategy have a similar behavior: both improve upon CPLEX in 15 out of 20 instances. The average improvement over CPLEX is about 5%.

We also tested the possibility to solving EMCA\_Concentrated within a 1 hour time limit neglecting the local search phase. Results are reported in Table 6.19: we conclude that no significant improvement is provided in this way compared to the application of LS. Also the computational times are similar.

We propose a summary graph on HC algorithms in Fig. 6.6: the best performances are provided by the heuristic concentration algorithm followed by a local search phase (SD or FI).

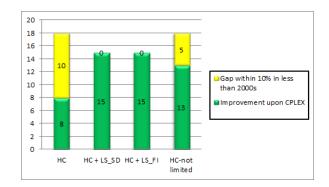


Figure 6.6: Heuristic Concentration and Local Search Results - Group A

								~										r													<u> </u>
	Imp	-0.3	-4.6	-6.1	-6.2	-4.3	-0.3	-0.8	-1.3	0.0	-1.9	-1.0	0.0	-3.4	0.0	-13.6	-1.2	-4.5	0.0	-1.9	-2.3	-1.0	-3.2	-2.1	-1.0	-5.6	-7.1	-8.4	-0.3	-5.3	-0.3
	%_LB	6.9	13.5	18.9	11.4	12.7	18.9	21.8	27.3	15.6	3.6	17.1	27.3	19.7	13.0	11.2	14.2	14.5	19.7	18.2	15.9	24.5	11.6	17.5	24.5	2.4	12.6	3.5	3.8	5.5	12.6
	%_UB	-7.7	0.7	-5.9	-2.4	-3.8	0.7	8.8	-2.6	-4.1	0.7	0.7	8.8	-8.0	-11.4	0.1	0.2	-4.8	0.2	-8.8	-7.7	7.5	-2.2	-2.8	7.5	2.4	9.4	-1.9	2.3	3.0	9.4
	ß	765	1145	1638	1925			TL	$\operatorname{TL}$	$\mathrm{TL}$	$\operatorname{TL}$			1438	1012	1690	1354			ΤL	$\mathrm{TL}$	$\mathrm{TL}$	$\mathrm{TL}$			ΤL	$\mathrm{TL}$	$\mathrm{TL}$	$\mathrm{TL}$		
	LS (FI)	302312	338500	371603	375457			421811	438621	402819	368131			386593	388728	374793	407842			464922	450918	524496	465750			746507	818182	692449	739871		
HC $  \mathbf{j}  $ $\mathbf{j}   \mathbf{k}  $ $  \mathbf{j}  $	imp	-0.3	-5.6	-7.9	-5.6	-4.8	-0.3	-3.1	-1.3	0.0	-1.8	-1.6	0.0	-3.4	0.0	-4.0	-1.2	-2.1	0.0	-5.8	-2.8	-7.2	-3.5	-4.8	-2.8	-5.6	-5.9	-8.4	-0.3	-5.1	-0.3
HC $  5  $ $\%$ LB $\%$ LB $LS$ (SD) $  5  $ 303342         263 $-7.4$ $7.3$ 302312 $671$ 303342         255         18.9         334886         1820           354712         351         5.5         18.9         37771         2327           395839         316         0.2         26.6         364745         1480           395839         316         0.2         26.6         364745         1480           400183         354         4.0         18.8         37771         2327           400183         355         26.6         364745         1480         777           425004         962         9.6         22.8         411764         71           414481         1014         -1.3         29.0         438621         71           414481         1014         -1.3         29.0         438621         71           402819         945         -4.1         15.6         388593         1330           375394         700         2.6         5.6         407842         2759           402819         385533	%_LB	6.9	12.2	16.7	12.1	12.0	16.7	18.9	27.3	15.6	3.7	16.4	27.3	19.7	13.0	23.6	14.2	17.6	23.6	13.5	15.3	16.6	11.2	14.2	16.6	2.4	14.0	3.5	3.8	5.9	14.0
HC $   $ $   $ $   $ $    $ $    $ $      $ $303342$ $263$ $-7.4$ $7.3$ $302312$ $354712$ $351$ $5.5$ $18.9$ $334886$ $354712$ $351$ $5.5$ $18.9$ $334745$ $395839$ $316$ $0.2$ $26.6$ $364745$ $400183$ $354$ $4.0$ $18.8$ $37771$ $400183$ $354$ $4.0$ $18.8$ $37771$ $400183$ $354$ $4.0$ $12.6$ $364745$ $400183$ $354$ $4.0$ $12.6$ $364745$ $40183$ $354$ $4.0$ $12.6$ $4.0$ $42504$ $962$ $9.6$ $22.6$ $411764$ $42404$ $962$ $9.6$ $22.6$ $411764$ $424141$ $1014$ $-1.3$ $29.0$ $438659$ $414481$ $1014$ $-1.3$ $29.0$ $438659$ $414481$ $1014$ $-1.3$ $29.0$ $438659$ $414481$ $1014$ $-1.3$ $29.0$ $438659$ $414481$ $-1.3$ $29.0$ $2.8$ $4116625$ $414044$ $388$ $1.4$ $13.0$ $388728$ $4100056$ $4415$ $-4.1$ $13.0$ $388728$ $4116637$ $3887$ $28.8$ $416675$ $412630$ $388$ $1.4$ $15.4$ $412630$ $388$ $1.4$ $15.6$ $412630$ $388$ $1.4$ $15.6$ $412630$ $388$ $1.4$ $15.6$ $412630$	%_UB	-7.7	-0.4	-7.7	-1.8	-4.4	-0.4	6.2	-2.6	-4.1	0.8	0.1	6.2	-8.0	-11.4	11.2	0.2	-2.0	11.2	-12.4	-8.2	0.7	-2.5	-5.6	0.7	2.4	10.7	-1.9	2.3	3.4	10.7
HC $[s]$ $\%$ UB $\%$ LB $[s]$ $303342$ $263$ $-7.4$ $7.3$ $303342$ $263$ $-7.4$ $7.3$ $354712$ $351$ $5.5$ $18.9$ $395839$ $316$ $0.2$ $26.6$ $395839$ $316$ $0.2$ $26.6$ $400183$ $354$ $4.0$ $18.8$ $395839$ $316$ $0.6$ $17.9$ $400183$ $354$ $4.0$ $18.8$ $400183$ $354$ $4.0$ $18.8$ $440183$ $962$ $9.6$ $22.8$ $444481$ $1014$ $-1.3$ $29.0$ $402819$ $945$ $-4.1$ $15.6$ $402819$ $945$ $-4.1$ $15.6$ $441481$ $1014$ $-1.3$ $29.0$ $442481$ $1014$ $-1.3$ $29.0$ $442404$ $388$ $1.4$ $13.0$ $387728$ $421$ $-11.4$ $13.0$ $434004$ $388$ $1.4$ $15.5$ $442630$ $388$ $1.4$ $15.5$ $442630$ $388$ $1.4$ $15.5$ $4412630$ $388$ $1.4$ $15.5$ $441637$ $873$ $-5.5$ $18.6$ $529795$ $778$ $8.6$ $25.7$ $481077$ $769$ $1.0$ $15.3$ $481077$ $769$ $1.0$ $15.3$ $880419$ $840$ $17.7$ $20.0$ $880419$ $840$ $17.7$ $21.1$ $880419$ $840$ $17.7$ $21.1$ $88041$	[s]	671	1820	1480	2327			TL	TL	2759	TL			1330	689	1110	920			ΤL	2319	$\mathrm{TL}$	$\mathrm{TL}$			ΤL	$\mathrm{TL}$	$\mathrm{TL}$	2582		
HC         [s]         %UB           303342         263         -7.4           303342         351         5.5           354712         316         0.2           355839         316         0.2           395839         316         0.2           395839         316         0.2           400183         354         4.0           400183         354         4.0           400183         354         9.6           414481         1014         -1.3           402819         945         -4.1           402819         945         -4.1           375394         700         2.6           444481         1014         -1.3           402819         945         -4.1           375394         700         2.6           441481         1014         -1.3           4700056         441         0.3           4412630         388         1.4           4712630         388         1.4           4712630         388         1.4           471637         873         -5.5           529795         778         -5.5	LS $(SD)$	302312	334886	364745	37771			411764	438621	402819	368659			386593	388728	416625	407842			446499	448667	491541	464214			746507	828168	692449	739871		
HC     [s]       303342     263       303342     263       354712     351       395839     316       395839     316       400183     354       395839     316       400183     354       395879     945       444481     1014       402819     945       444481     1014       4402819     945       3875394     700       375394     700       375394     700       375394     700       412630     388       412630     388       412630     388       412630     388       412630     388       412630     388       412630     388       412630     388       412630     388       412630     388       481077     769       880419     840       750066     767       741061     912       880419     840       741061     701	%LB	7.3	18.9	26.6	18.8	17.9	26.6	22.8	29.0	15.6	5.6	18.2	29.0	23.8	13.0	28.8	15.5	20.3	28.8	20.5	18.6	25.7	15.3	20.0	25.7	8.5	21.1	13.0	4.0	11.7	21.1
HC 303342 354712 354712 355712 395839 400183 400183 425004 425004 444481 444481 442819 375394 375394 440056 388728 440056 388728 4412630 4112630 4112630 411077 481077 481077 481077 481077 529795 481077 481077 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 529795 52975 529795 529795 529795 529795 529795 52975 52975 52975 52975 52975 52975 52975 52975 52975 52975 52975 52975 52975 52975 52975 52975 52975 52975 52975 52975 52975 52975 52975 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 52075 5	%UB	-7.4	5.5	0.2	4.0	0.6	5.5	9.6	-1.3	-4.1	2.6	1.7	9.6	-4.8	-11.4	15.9	1.4	0.3	15.9	-7.0	-5.5	8.6	1.0	-0.7	8.6	8.5	17.7	7.1	2.6	0.0	17.7
	[s]	263	351	316	354			962	1014	945	700			445	421	388	388			957	873	778	769			912	840	767	890		
STANCE -30-400-1 -30-400-2 -30-400-3 -30-400-3 average max -60-625-1 -60-625-1 -60-625-3 -60-625-3 -60-625-3 average max -30-400-1 -30-400-3 -50-625-3 max max -40-900-2 -40-900-2	НС	303342	354712	395839	400183			425004	444481	402819	375394			400056	388728	434004	412630			473965	461637	529795	481077			791040	880419	756066	741951		
IN         40           40         40           40         40           40         40           50         50           50         50           70         70           70         70	INSTANCE	40_30_400_1	$40_{-30}400_{-2}$	40_30_400_3	40-30-400-4	average	max	40_60_625_1	40-60-625-2	40_60_625_3	40_60_625_4	average	max	$50_{-}30_{-}400_{-}1$	$50_{-}30_{-}400_{-}2$	$50_{-}30_{-}400_{-}3$	$50_{-30_{-400_{-4}}}$	average	max	50-50-625-1	50-50-625-2	50_50_625_3	50_50_625_4	average	max	70_40_900_1	70_40_900_2	70_40_900_3	70_40_900_4	average	max

Table 6.18: Heuristic Concentration and Local Search Results - Group A

INSTANCE	HC (TL)	$[\mathbf{s}]$	%UB	%LB
40_30_400_1	302312	475	-7.7	6.9
40_30_400_2	338500	694	0.7	13.5
40_30_400_3	371603	688	-5.9	18.9
40_30_400_4	375457	1048	-2.4	11.4
average			-3.8	12.7
max			0.7	18.9
40_60_625_1	397756	1873	2.6	14.9
40_60_625_2	420547	3853	-6.6	22.0
40_60_625_3	402819	2196	-4.1	15.6
40_60_625_4	368131	TL	0.7	3.6
average			-1.9	14.0
max			2.6	22.0
50_30_400_1	386593	888	-8.0	19.7
50_30_400_2	388728	602	-11.4	13.0
50_30_400_3	380221	922	1.5	12.8
50_30_400_4	407842	762	0.2	14.2
average			-4.4	14.9
max			1.5	19.7
50_50_625_1	444809	TL	-12.7	13.1
50_50_625_2	448378	2097	-8.3	15.2
50_50_625_3	486916	TL	-0.2	15.5
50_50_625_4	461952	TL	-3.0	10.7
average			-6.0	13.6
max			-0.2	15.5
70_40_900_1	746507	2721	2.4	2.4
70_40_900_2	818182	2283	9.4	12.6
70_40_900_3	692449	3047	-1.9	3.5
70_40_900_4	739871	1936	2.3	3.8
average			3.0	5.5
max			9.4	12.6

Table 6.19: Heuristic Concentration without time limit Results - Group A

On Group B only the best method is tested: HC and local search (FI) algorithm (Table 6.21).

Results on this group of instances show as a great improvement can be obtained both in terms of time and objective function with respect to CPLEX. In fact, HC and a FI local search improves upon CPLEX in 57 out of 60 instances and in the other 3 the gap is anyway within 10% (Fig. 6.7).

On large instances the improvement upon CPLEX rises significantly up to 88%. Moreover almost all the instances are solved within the time limit.

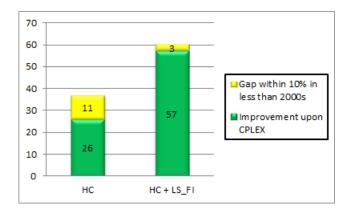


Figure 6.7: Heuristic Concentration and Local Search Results - Group B

INSTANCE	HC	[s]	%UB	%LB	LS (FI)	[s]	%_UB	%_LB	Imp
20_10_121_1	112464	36	2.8	20.4	109428	181	0.0	17.2	-2.7
20_10_121_1 20_10_121_2	112404	32	0.2	15.9	118565	284	0.0	17.2	-2.1
20_10_121_2	115353	45	13.0	13.0	103092	204	1.0	1.0	-10.6
20_10_121_3	80090	49 29	0.0	0.0	80090	299 201	0.0	0.0	0.0
20_10_121_4	102067	44	-0.3	19.5	102067	286	-0.3	19.5	0.0
	102007	44	3.1	13.8	102007	200	0.2	10.7	-2.7
average max			3.1 13.0	13.8 20.4			1.0	10.7	-2.7
30_10_121_1	163362	50	-5.3	32.8	150919	813	-12.5	22.7	-7.6
30_10_121_1	154123	53	-3.7	32.8 30.7	144908	918	-12.5 -9.5	22.1	-6.0
30_10_121_2	147534	53	-3.7	35.3	132241	556	-9.5 -7.7	22.9 21.2	-10.4
30_10_121_3	147354	$\frac{55}{49}$	10.1	46.5	132241 115317	449	-11.4	17.9	
30_10_121_4	145259 150224	$\frac{49}{56}$	5.1	$\frac{40.5}{23.6}$	143915	1824	-11.4	17.9	-19.5 -4.2
	150224	- 00			143913	1624			
average			1.8	33.8 46 E			-8.1 0.7	20.6	-9.5 -4.2
max	F 40070	<u> </u>	10.1	46.5	191150	1040		22.9	-4.2
30_15_121_1	549979	62	307.6	444.6	131150	1046	-2.8	29.9	
30_15_121_2	200596 104004	$59 \\ 55$	39.8 -5.2	81.8 12.3	136829	1307	-4.7 -5.8	24.0	-31.8 -0.6
30_15_121_3					103400	511		11.6	
30_15_121_4 30_15_121_5	158046	61 77	-4.6 37.0	54.9 71.1	127232 122737	$776 \\ 810$	-23.2 -8.9	24.7 13.8	-19.5
	184572	11			122737	810	-0.9		-33.5
average			74.9 307.6	132.9 444.6			-9.1 -2.8	20.8 29.9	-32.3 -0.6
max	500401	60			107540	100			
30_20_121_1 30_20_121_2	599401 654775	69	380.1 352.2	556.1 536.6	107540 125797	$100 \\ 100$	-13.9 -13.1	17.7 22.3	-82.1 -80.8
30_20_121_2	653952	63	352.2 351.4	732.5	89567	100	-38.2	14.0	-86.3
30_20_121_3	769010	71	330.5	617.4	124402	70	-30.2	14.0	-83.8
30_20_121_4	642378	71	310.9	462.3	124402	125	-7.8	26.2	-77.6
average	042010		345.0	581.0	111112	120	-20.7	19.3	-82.1
max			380.1	732.5			-7.8	26.2	-77.6
40_10_121_1	188110	67	5.0	32.1	175295	269	-2.1	23.1	-6.8
40_10_121_2	190229	55	4.3	20.8	184513	351	1.2	17.2	-3.0
40_10_121_3	177235	74	-1.0	24.0	175625	198	-1.9	22.9	-0.9
40_10_121_4	194851	53	-1.3	26.7	186155	246	-5.7	21.0	-4.5
40_10_121_5	193239	65	5.1	21.8	181805	445	-1.1	14.6	-5.9
average			2.4	25.1		-	-1.9	19.8	-4.2
max			5.1	32.1			1.2	23.1	-0.9
40_15_121_1	521291	78	225.7	356.8	148683	637	-7.1	30.3	-71.5
40_15_121_2	547983	80	134.0	286.6	173816	862	-25.8	22.6	-68.3
40_15_121_3	518658	83	-47.0	319.8	147267	218	-85.0	19.2	-71.6
40_15_121_4	537967	79	142.2	295.9	169198	297	-23.8	24.5	-68.5
40_15_121_5	513435	87	186.8	279.1	168907	1124	-5.7	24.7	-67.1
average			128.3	307.7			-29.5	24.3	-69.4
max			225.7	356.8			-5.7	30.3	-67.1
									-

Table 6.20: Heuristic Concentration and Local Search Results - Group B - Part 1

INSTANCE	HC	[s]	%UB	%LB	LS (FI)	[s]	%_UB	%_LB	Imp
40_20_121_1	632034	79	293.2	487.4	136141	758	-15.3	26.5	-78.5
40_20_121_2	653797	81	236.6	429.9	162483	844	-16.3	31.7	-75.1
40_20_121_3	618390	81	-18.7	505.5	133222	652	-82.5	30.4	-78.5
40_20_121_4	643849	90	4.1	374.4	170429	752	-72.4	25.6	-73.5
40_20_121_5	613048	107	223.1	361.1	162926	1506	-14.1	22.6	-73.4
average			147.7	431.7			-40.1	27.4	-75.8
max			293.2	505.5			-14.1	31.7	-73.4
50_10_121_1	194440	76	-1.5	29.9	186564	942	-5.5	24.6	-4.1
50_10_121_2	206550	71	-0.9	9.1	204068	265	-2.1	7.8	-1.2
50_10_121_3	284254	117	31.0	39.6	216402	295	-0.3	6.3	-23.9
50_10_121_4	251087	522	8.3	8.3	251087	630	8.3	8.3	0.0
50_10_121_5	404510	105	76.1	117.2	216318	373	-5.8	16.1	-46.5
average			22.6	40.8			-1.1	12.6	-15.1
max			76.1	117.2			8.3	24.6	0.0
50_15_121_1	551334	123	-18.1	273.2	203826	1129	-69.7	38.0	-63.0
50_15_121_2	550175	108	-10.7	219.3	225912	1066	-63.3	31.1	-58.9
50_15_121_3	546652	99	83.3	274.9	187270	1262	-37.2	28.4	-65.7
50_15_121_4	542824	122	-17.1	214.7	233050	1016	-64.4	35.1	-57.1
50_15_121_5	537100	138	13.0	216.0	201774	1445	-57.5	18.7	-62.4
average			10.1	239.6			-58.4	30.3	-61.4
max			83.3	274.9			-37.2	38.0	-57.1
50_20_121_1	654475	106	-34.6	341.4	185488	1350	-81.5	25.1	-71.7
50_20_121_2	654290	103	-32.6	378.6	176065	1228	-81.9	28.8	-73.1
50_20_121_3	679969	113	167.1	435.6	159164	1233	-37.5	25.4	-76.6
50_20_121_4	684217	138	163.0	320.1	204222	2100	-21.5	25.4	-70.2
50_20_121_5	676949	144	-21.3	344.6	188113	TL	-78.1	23.5	-72.2
average			48.3	364.1			-60.1	25.6	-72.7
max			167.1	435.6			-21.5	28.8	-70.2
60_15_121_1	518657	160	-46.9	187.7	231267	1661	-76.3	28.3	-55.4
60_15_121_2	583975	185	-20.2	164.5	279966	1456	-61.7	26.8	-52.1
60_15_121_3	549685	145	-29.0	208.8	226791	TL	-70.7	27.4	-58.7
60_15_121_4	543875	153	-43.5	187.5	236064	2520	-75.5	24.8	-56.6
60_15_121_5	543688	181	-38.8	156.2	296026	1946	-66.7	39.5	-45.6
average			-35.7	180.9			-70.2	29.4	-53.7
max			-20.2	208.8			-61.7	39.5	-45.6
60_20_121_1	652345	145	77.0	298.6	206304	3846	-44.0	26.1	-68.4
60_20_121_2	682206	140	-54.4	331.9	203813	1879	-86.4	29.0	-70.1
60_20_121_3	650203	126	-60.6	338.0	193338	1107	-88.3	30.2	-70.3
60_20_121_4	688165	165	-62.2	265.3	257877	TL	-85.8	36.9	-62.5
60_20_121_5	686969	200	-55.0	268.6	276609	TL	-81.9	48.4	-59.7
average			-31.0	300.5			-77.3	34.1	-66.2
max			77.0	338.0			-44.0	48.4	-59.7

Table 6.21: Heuristic Concentration and Local Search Results - Group B - Part 2

#### 6.2.6 Geographical Decomposition and Local Search

Finally we tested the heuristic method based on geographical decomposition algorithm described in Section 5.3. Also in this case we apply a local search to improve the initial solution.

The size of the sub-areas has been set 1 km x 1 km. This value is very important in order to obtain good solutions. In fact if the sub-area is to large, solving EMCA\_Restricted may required high computational times. We set as time limit for EMCA\_Restricted for each area 60 s. The time limit for solving EMCA\_Concentrated is set to 10 minutes.

For the technique that includes the overlap between near areas we chose as value for overlapping half area (500 m).

Group A have been tested on both methods to compare them.

The Tables 6.22, 6.23, 6.24 and 6.25 are structured as follows: name of the considered instance, result obtained with the geographical based algorithm with computational time and gap from UB and LB provided by CPLEX, results after local search phase (SD and FI).

The Table 6.22 reports the results of the geographical approach without overlapping between sub-areas for instances of Group A.

From the comparison between Table 6.22 (no overlapping) and Table 6.23 (with overlapping) we notice that this last method gives on average better results.

In fact the algorithm that consider overlapping between areas improves upon CPLEX in 12 out of 20 instances, while without overlapping only 7 times.

The results of both the procedure further improve with the local search phase (FI is better than SD most of all regarding the computational times). Even after this step the better results are provided by the algorithm with overlapping : it improves upon CPLEX in 17 over 20 instances (Fig. 6.8).

The improvements are not very high (on average about 3%) but the time limit is rarely reached (in 7 out of 20 instances). However in two instances the gap from UB provided by CPLEX is high (12% and 25%).

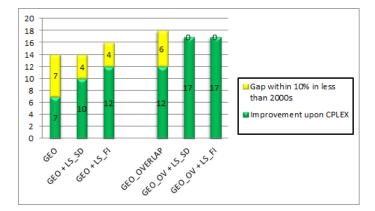


Figure 6.8: Geographical Decomposition and Local Search Results - Group A

For Group B has been applied only the best method: geographical decomposition with overlapping and local search (FI) (Table 6.24 and 6.25). Also for this Group the results are good (Fig. 6.9): the average improvement upon CPLEX is about 20% on smallest instances and about 60% on largest. In 58 instances out of 60 the geographical algorithm with local search improves upon CPLEX.

In most instances the time limit is not reached. However we note that for the remaining instances the algorithm has some difficulties. In 50\_10\_121\_3 for example the gap from CPLEX is still very high (48%) and in instance 50\_10\_121\_4 no solution is found within the time limit.

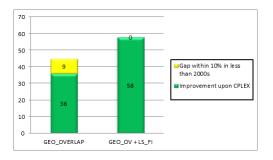


Figure 6.9: Geographical Decomposition with Overlapping and Local Search Results - Group B

																														· ·
Imp	-9.1	0.0	-5.1	0.0	-3.6	0.0	0.0	-0.9	-22.7	-11.1	-8.7	0.0	-2.8	-0.6	-1.4	-1.5	-1.6	-0.6	-3.8	-0.2	-4.6	-21.2	-7.5	-0.2	-0.8	-17.8	-16.8	-17.2	-13.2	-0.8
%_LB	6.9	14.6	16.8	19.4	14.5	19.4	19.6	25.8	19.2	12.1	19.2	25.8	17.5	12.3	9.5	19.3	14.6	19.3	19.0	17.8	17.3	9.0	15.8	19.0	0.3	5.2	7.2	1.5	3.5	7.2
%_UB	-7.7	1.8	-7.5	4.6	-2.2	4.6	6.8	-3.8	-1.1	8.9	2.7	8.9	-9.7	-12.0	-1.4	4.7	-4.6	4.7	-8.2	-6.2	1.3	-4.5	-4.4	1.3	0.2	2.2	1.7	0.1	1.0	2.2
[s]	343	749	888	852			1811	$\mathbf{TL}$	$\mathrm{TL}$	$\mathrm{TL}$			1005	881	763	726			$\mathrm{TL}$	1921	$\mathbf{TL}$	ΤL			1517	$\mathrm{TL}$	2978	2872		
LS (FI)	302312	341998	365312	402405			414207	433428	415402	398303			379506	386026	369153	425986			468151	458469	494299	454859			731092	764298	717567	723488		
Imp	-9.1	0.0	-5.1	0.0	-3.6	0.0	0.0	-2.8	-4.5	-11.3	-4.6	0.0	-2.8	-0.6	-1.4	-4.6	-2.3	-0.6	-3.4	-2.4	-6.1	-4.1	-4.0	-2.4	-0.8	-3.7	-4.7	-5.3	-3.6	-0.8
%_LB	6.9	14.6	16.8	19.4	14.5	19.4	19.6	23.3	47.3	11.9	25.5	47.3	17.5	12.3	9.5	15.5	13.7	17.5	19.6	15.2	15.5	32.7	20.8	32.7	0.3	23.2	22.8	16.1	15.6	23.2
%-UB	-7.7	1.8	-7.5	4.6	-2.2	4.6	6.8	-5.7	22.2	8.7	8.0	22.2	-9.7	-12.0	-1.4	1.4	-5.4	1.4	-7.7	-8.3	-0.2	16.4	0.0	16.4	0.2	19.7	16.4	14.5	12.7	19.7
[8]	1054	693	1782	813			1686	$\mathbf{TL}$	$\mathrm{TL}$	$\mathrm{TL}$			1470	989	966	1653			$\mathrm{TL}$	$\mathbf{TL}$	$\mathbf{TL}$	ΤL			1804	$\mathrm{TL}$	$\mathrm{TL}$	TL		
LS (SD)	302312	341998	365312	402405			414207	424843	513186	397681			379506	386026	369153	412632			470439	448378	486754	554060			731092	895097	821698	828262		
%LB	17.6	14.6	23.2	19.4	18.7	23.2	19.6	26.8	54.2	26.1	31.7	54.2	20.9	12.9	11.1	21.1	16.5	21.1	23.7	18.0	23.0	38.4	25.8	38.4	1.1	27.9	28.8	22.6	20.1	28.8
%UB	1.5	1.8	-2.5	4.6	1.3	4.6	6.8	-2.9	27.9	22.5	13.6	27.9	-7.1	-11.5	0.0	6.3	-3.1	6.3	-4.5	-6.0	6.2	21.3	4.3	21.3	1.1	24.3	22.2	20.9	17.1	24.3
[s]	21	81	231	309			91	137	320	$\overline{96}$			307	308	273	307			314	312	314	319			38	317	316	317		
GEO	332531	341998	385032	402405			414207	437145	537332	448182			390433	388250	374387	432407			486800	459187	518333	577560			736994	929645	862289	874255		
INSTANCE	40-30-400-1	$40_{-}30_{-}400_{-}2$	40_30_400_3	$40_{-}30_{-}400_{-}4$	average	max	40_60_625_1	40_60_625_2	40_60_625_3	40_60_625_4	average	max	$50_{-}30_{-}400_{-}1$	$50_{-}30_{-}400_{-}2$	$50_{-}30_{-}400_{-}3$	$50_{-}30_{-}400_{-}4$	average	max	50-50-625-1	50-50-625-2	50_50_625_3	$50_{-}50_{-}625_{-}4$	average	max	70_40_900_1	70_40_900_2	70_40_900_3	70_40_900_4	average	max

Table 6.22: Geographical Decomposition and Local Search Results - Group A

INSTANCE	GEO_OVERLAP	[s]	%UB	% LB	LS (SD)	[s]	%_UB	%_LB	Imp	LS (FI)	[8]	%_UB	%_LB	Imp
40_30_400_1	302312	166	-7.7	6.9	302312	401	-7.7	6.9	0.0	302312	424	-7.7	6.9	0.0
40_30_400_2	338802	105	0.8	13.6	338500	800	0.7	13.5	-0.1	338500	532	0.7	13.5	-0.1
40-30-400-3	366845	322	-7.1	17.3	366845	664	-7.1	17.3	0.0	366845	722	-7.1	17.3	0.0
40-30-400-4	383557	334	-0.3	13.8	383557	522	-0.3	13.8	0.0	383557	529	-0.3	13.8	0.0
average			-3.6	12.9			-3.6	12.9	0.0			-3.6	12.9	0.0
max			0.8	17.3			0.7	17.3	0.0			0.7	17.3	0.0
40-60-625-1	386635	263	-0.3	11.7	386635	2518	-0.3	11.7	0.0	386635	2727	-0.3	11.7	0.0
40_60_625_2	422130	335	-6.3	22.5	420252	$\mathrm{TL}$	-6.7	21.9	-0.4	420192	TL	-6.7	21.9	-0.5
40-60-625-3	427165	340	1.7	22.6	392737	$\mathrm{TL}$	-6.5	12.7	-8.1	422689	$\mathrm{TL}$	0.6	21.3	-1.0
40-60-625-4	386167	56	5.6	8.7	379416	$\mathrm{TL}$	3.7	6.8	-1.7	376991	$\mathrm{TL}$	3.1	6.1	-2.4
average			0.2	16.4			-2.4	13.3	-2.6			-0.8	15.3	-1.0
max			5.6	22.6			3.7	21.9	0.0			3.1	21.9	0.0
50_30_400_1	382680	328	-8.9	18.5	382680	641	-8.9	18.5	0.0	382680	673	-8.9	18.5	0.0
50_30_400_2	387185	185	-11.8	12.6	387185	492	-11.8	12.6	0.0	387185	557	-11.8	12.6	0.0
50_30_400_3	375873	335	0.4	11.5	375873	616	0.4	11.5	0.0	375873	650	0.4	11.5	0.0
$50_{-}30_{-}400_{-}4$	549245	335	35.0	53.8	511251	TL	25.7	43.2	-6.9	457216	$_{\rm TL}$	12.4	28.0	-16.8
average			3.7	24.1			1.3	21.4	-1.7			-2.0	17.7	-4.2
max			35.0	53.8			25.7	43.2	0.0			12.4	28.0	0.0
50-50-625-1	476881	357	-6.4	21.2	474661	2889	-6.9	20.6	-0.5	474661	2035	-6.9	20.6	-0.5
50_50_625_2	464639	326	-4.9	19.4	464457	1635	-5.0	19.3	0.0	464639	1709	-4.9	19.4	0.0
50_50_625_3	502244	350	2.9	19.2	478838	$\mathrm{TL}$	-1.9	13.6	-4.7	478838	$\mathrm{TL}$	-1.9	13.6	-4.7
$50_{-}50_{-}625_{-}4$	469576	355	-1.4	12.5	459762	$\mathrm{TL}$	-3.4	10.1	-2.1	459762	TL	-3.4	10.1	-2.1
average			-2.5	18.1			-4.3	15.9	-1.8			-4.3	16.0	-1.8
max			2.9	21.2			-1.9	20.6	0.0			-1.9	20.6	0.0
70_40_900_1	729270	280	0.0	0.0	729270	1197	0.0	0.0	0.0	729270	1214	0.0	0.0	0.0
70_40_900_2	939624	325	25.6	29.3	939624	$\mathrm{TL}$	25.6	29.3	0.0	939624	TL	25.6	29.3	0.0
70_40_900_3	724248	322	2.6	8.2	693619	$\mathrm{TL}$	-1.7	3.6	-4.2	693619	2009	-1.7	3.6	-4.2
70_40_900_4	734870	331	1.6	3.0	723124	2787	0.0	1.4	-1.6	723124	2111	0.0	1.4	-1.6
average			7.5	10.1			6.0	8.6	-1.5			6.0	8.6	-1.5
max			25.6	29.3			25.6	29.3	0.0			25.6	29.3	0.0

Table 6.23: Geographical Decomposition with Overlapping and Local Search Results - Group A

INSTANCE	GEO_OVERLAP	[s]	%UB	%LB	LS (FI)	[s]	%_UB	%_LB	Imp
20_10_121_1	112464	305	2.8	20.4	109428	373	0.0	17.2	-2.7
20_10_121_2	121137	305	2.3	18.4	118159	396	-0.2	15.5	-2.5
20_10_121_3	102063	101	0.0	0.0	102063	169	0.0	0.0	0.0
20_10_121_4	80090	200	0.0	0.0	80090	244	0.0	0.0	0.0
20_10_121_5	100321	123	-2.0	17.5	100321	201	-2.0	17.5	0.0
average			0.6	11.3			-0.4	10.0	-1.0
max			2.8	20.4			0.0	17.5	0.0
30_10_121_1	163155	321	-5.4	32.6	156490	818	-9.3	27.2	-4.1
30_10_121_2	147953	311	-7.6	25.4	144908	634	-9.5	22.9	-2.1
30_10_121_3	150420	306	5.0	37.9	132241	484	-7.7	21.2	-12.1
30_10_121_4	119679	308	-8.0	22.4	115317	553	-11.4	17.9	-3.6
30_10_121_5	144834	307	1.3	19.2	139511	781	-2.4	14.8	-3.7
average			-2.9	27.5			-8.0	20.8	-5.1
max			5.0	37.9			-2.4	27.2	-2.1
30_15_121_1	137337	330	1.8	36.0	130674	893	-3.2	29.4	-4.9
30_15_121_2	140574	340	-2.1	27.4	134574	989	-6.2	22.0	-4.3
30_15_121_3	103400	312	-5.8	11.6	103400	475	-5.8	11.6	0.0
30_15_121_4	135130	310	-18.5	32.5	135130	1425	-18.5	32.5	0.0
30_15_121_5	126117	311	-6.4	16.9	122737	888	-8.9	13.8	-2.7
average			-6.2	24.9			-8.5	21.8	-2.4
max			1.8	36.0			-3.2	32.5	0.0
30_20_121_1	110385	351	-11.6	20.8	108297	941	-13.3	18.5	-1.9
30_20_121_2	145804	328	0.7	41.8	128963	773	-10.9	25.4	-11.6
30_20_121_3	102042	316	-29.6	29.9	91929	1183	-36.5	17.0	-9.9
30_20_121_4	151461	320	-15.2	41.3	124402	948	-30.4	16.1	-17.9
30_20_121_5	152437	347	-2.5	33.4	141042	1539	-9.8	23.5	-7.5
average			-11.6	33.4			-20.2	20.1	-9.7
max			0.7	41.8			-9.8	25.4	-1.9
40_10_121_1	215867	336	20.5	51.5	170813	628	-4.6	19.9	-20.9
40_10_121_2	236279	311	29.6	50.0	178868	545	-1.9	13.6	-24.3
40_10_121_3	179961	307	0.5	25.9	175625	501	-1.9	22.9	-2.4
40_10_121_4	226359	314	14.6	47.2	188131	677	-4.7	22.3	-16.9
40_10_121_5	185066	308	0.7	16.6	185066	584	0.7	16.6	0.0
average			13.2	38.3			-2.5	19.1	-12.9
max			29.6	51.5			0.7	22.9	0.0
40_15_121_1	156886	349	-2.0	37.5	141386	1205	-11.7	23.9	-9.9
40_15_121_2	341785	337	45.9	141.1	174780	1661	-25.4	23.3	-48.9
40_15_121_3	170797	317	-82.5	38.2	147267	1075	-85.0	19.2	-13.8
40_15_121_4	217305	333	-2.2	59.9	171067	882	-23.0	25.9	-21.3
40_15_121_5	176244	313	-1.6	30.1	165928	1850	-7.3	22.5	-5.9
average			-8.5	61.4			-30.5	23.0	-19.9
max			45.9	141.1			-7.3	25.9	-5.9

Table 6.24: Geographical Decomposition with Overlapping and Local Search Results - Group B - Part 1

INSTANCEGEO.OVERLAP[a]%IB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB%ILB </th <th></th>										
40.20.121.211171587136111718771L1.1.730.130.140.20.121.31601953407.8.95.6.91305361602-82.82.7.81.8.540.20.121.4333044365-61.1145.4170397171-72.42.5.54.8.840.20.121.5134509378-7.8.338.016617TL-72.42.5.54.8.8average	INSTANCE	GEO_OVERLAP	$[\mathbf{s}]$	%UB	%LB	LS (FI)	[s]	%_UB	%_LB	Imp
40.20.121.3160195340-78.956.91803651602-82.827.8148.840.20.121.5183459378-3.338.0165473TL-72.425.548.840.20.121.5183459378-3.338.0165473TL-12.824.59.8average-77.067.2-77.077.2-78.430.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.010.0	40_20_121_1	168797	385	5.0	56.9	142021	1682	-11.6	32.0	-15.9
40.20.121.1333084365-46.1145.41703771717-72.425.5-48.840.20.121.5183459378-3.338.016473TL-12.824.5-9.8average	40_20_121_2	171587	364	-11.7	39.1	171587	TL	-11.7	39.1	0.0
40.20.121.5183459378-3.338.0165473TL-12.824.5-9.8average max-7067.2-71.637.329.818.6max-75.0145.41795741021-9.119.90.050.10.121.1179574333-9.119.910.250.10-9.119.90.050.10.121.33226401301.111.320505871-9.99.1-7.050.10.121.4Infeasible31950.15.12.1228775363<	40_20_121_3	160195	340	-78.9	56.9	130536	1802	-82.8	27.8	-18.5
average max         -27.0         67.2         -38.3         29.8         18.6           max         5.0         145.4         -11.6         39.1         0.0           50.10.121.1         179574         339         -9.1         19.9         179574         1021         -9.1         19.9         0.0           50.10.121.2         210706         130         1.1         11.3         206550         871         -0.9         9.1         -2.0           50.10.121.4         Infeasible         319         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         -         <	40_20_121_4	333084	365	-46.1	145.4	170397	1717	-72.4	25.5	-48.8
max14.514.514.710.13.910.050.10.121.117957417957410219.19.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.09.0	40_20_121_5	183459	378	-3.3	38.0	165473	TL	-12.8	24.5	-9.8
50.10.121.1       179574       339       -9.1       19.9       1021       -9.1       19.9       0.0         50.10.121.2       210706       130       1.1       11.3       206550       871       -0.9       9.1       -2.0         50.10.121.3       322640       311       48.7       58.5       322640       1155       48.7       58.5       0.0         50.10.121.5       262617       314       14.3       41.0       218041       1001       -5.1       17.1       -17.0         average       -9.0       6.1       -13.3       0.9       -       -       -       3.0.9       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       -       - <t< td=""><td>average</td><td></td><td></td><td>-27.0</td><td>67.2</td><td></td><td></td><td>-38.3</td><td>29.8</td><td>-18.6</td></t<>	average			-27.0	67.2			-38.3	29.8	-18.6
5010121212107061301.11.13206550871-0.99.1-2.050101213132264031148.758.5322640115548.758.50.05010.121.52626173141.341.02180411001-5.11.7.1-1.7.0average-9.06.1-9.1-5.131.41.8.758.5-1.3.30.9-1.5.1max-7.122877536966.054.81840952106-7.2.624.6-1.9.550.15.121.1228775369-66.054.812408230-66.62.6.7-7.3.450.15.121.48223993672.627.6821408230-66.62.6.7-7.3.450.15.121.48223993672.6.33.9.7217162.73-64.32.7.7-70.9average-74734033257.33.9.7217162.71-64.32.7.8-70.950.20.121.185645641.3-14.447.6188542TL-81.22.7.2-70.950.20.121.2282183382-70.9106.42.8.111.4.70.9106.40.050.20.121.487864433-61.163.912.4.73.4.70.9106.40.050.20.121.580784433-61.163.912.4.75.9.8.14.70.9106.4.70.950.20.121.580786433-61.1<	max			5.0	145.4			-11.6	39.1	0.0
50.10.121.1332264031148.758.5322640115548.758.50.050.10.121.526261731414.341.02180411001-5.117.1-17.0average	50_10_121_1	179574	339	-9.1	19.9	179574	1021	-9.1	19.9	0.0
50.10.121.4Infeasible319ImageInfeasible319ImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageImageIm	50_10_121_2	210706	130	1.1	11.3	206550	871	-0.9	9.1	-2.0
50.10.121.522621731414.341.021804110015.1.117.117.0average-9.06.1-8.5-13.30.9-max228775369-60.054.8184092106-7.4.658.550.15.121.271436637715.9314.62251224.5-6.3.331.1-6.8.450.15.121.357471634292.729.21766031816-0.6.626.7-7.3.450.15.121.482239936725.637.6.821.14627.33-6.6.626.3-6.1.350.15.121.574740033257.339.721.11627.33-5.4.327.7-7.0.9average-7.4734033257.339.721.11627.33-5.4.327.7-7.0.950.20.121.185645641.3-14.447.6188542TL-8.1.227.2-7.8.050.20.121.2282183382-7.0916.4282183TL-7.0.916.40.0.150.20.121.391897641.7261.0623.915.2816.1-7.0.916.40.0.150.20.121.4996714443283.151.2021.04-8.3.4-7.0.916.42821811.4-7.0.916.42.0.150.20.121.580784433-6.183.018.8221.6-7.8.13.0.1-7.6.3.0.1-7.6.60.15.121.49674443	50_10_121_3	322640	311	48.7	58.5	322640	1155	48.7	58.5	0.0
average max         -9.0         6.1         -13.3         0.9           max         48.7         58.5         -48.7         58.5           50.15.121.1         228775         369         -60         54.8         184095         2106         -72.6         24.6         -19.5           50.15.121.3         574716         342         92.7         294.2         176603         1816         -40.8         21.1         -69.3           50.15.121.4         822399         367         25.6         37.68         218498         230         -66.6         26.7         -73.4           50.15.121.5         747340         332         57.3         339.7         21716         2773         -54.3         27.7         -70.9           average         25.1         27.60	50_10_121_4	Infeasible	319							
max $48.7$ $58.5$ $48.7$ $58.7$ $50.15.121.2$ $228775$ $369$ $66.0$ $54.8$ $18409$ $210$ $-76.6$ $24.6$ $-19.5$ $50.15.121.2$ $714366$ $377$ $15.9$ $314.6$ $22512$ $245$ $-63.3$ $31.1$ $-66.9$ $50.15.121.3$ $574716$ $342$ $92.7$ $242.2$ $17663$ $1816$ $-40.8$ $21.1$ $-65.9$ $50.15.121.5$ $747340$ $332$ $57.3$ $339.7$ $21716$ $27.5$ $26.3$ $-70.9$ $average$ $-747340$ $332$ $57.3$ $39.7$ $21716$ $27.5$ $-55.9$ $26.3$ $-70.9$ $average$ $-747340$ $332$ $57.9$ $716.8$ $71.5$ $410.4$ $71.6$ $818545$ $71.$ $81.9$ $61.6$ $61.6$ $61.6$ $61.6$ $61.6$ $61.6$ $61.6$ $61.6$ $61.6$ $61.6$ $61.6$ $61.6$ $61$	50_10_121_5	262617	314	14.3	41.0	218041	1001	-5.1	17.1	-17.0
50.15.121.1 $228775$ $369$ $-66.0$ $54.8$ $184095$ $2106$ $-72.6$ $24.6$ $-19.5$ $50.15.121.2$ $714366$ $377$ $15.9$ $314.6$ $225912$ $2245$ $-63.3$ $31.1$ $-68.4$ $50.15.121.3$ $574716$ $342$ $92.7$ $294.2$ $176603$ $1816$ $-40.8$ $21.1$ $-69.3$ $50.15.121.5$ $747340$ $332$ $57.3$ $339.7$ $217116$ $273$ $-54.3$ $27.7$ $-70.9$ average $257.1$ $276.0$ $27.7$ $-70.8$ $-40.8$ $31.1$ $-19.5$ $max$ $-22.7$ $376.8$ $27.7$ $-40.8$ $31.1$ $-19.5$ $50.20.121.1$ $856456$ $413$ $-14.4$ $477.6$ $188542$ $TL$ $-81.2$ $27.2$ $-78.0$ $50.20.121.2$ $282183$ $382$ $-70.9$ $106.4$ $282183$ $TL$ $-70.9$ $106.4$ $0.81.4$ $50.20.121.3$ $918976$ $417$ $261.6$ $63.9$ $152895$ $1961$ $-39.9$ $20.4$ $-83.4$ $50.20.121.4$ $996714$ $443$ $283.1$ $512.0$ $210046$ $3146$ $19.3$ $29.0$ $-78.9$ $50.20.121.5$ $807846$ $433$ $-61.1$ $430.6$ $188749$ $2260$ $-78.1$ $24.0$ $-76.6$ $average$ $295.7$ $41.4$ $263.9$ $126.7$ $-75.9$ $41.4$ $-63.4$ $max$ $265247$ $407$ $65.7$ $49.0$ $285.7$	average			-9.0	6.1			-13.3	0.9	
50.15.121.271436637715.931462259122456.3.331.16.8.450.15.121.357471634292.7294.217660318164.0.821.16.0.350.15.121.574734033257.3339.7217116277354.327.77.0.9average74734033257.3339.7217116277354.327.77.0.9average225.127.627.65.0.57.0.97.0.97.0.9f0.20.121.1856456413-14.447.6188542TL-81.227.2-7.8.050.20.121.2282183382-70.9106.4282183TL-70.9106.420.0.1-30.920.4-83.450.20.121.4996714443283.1512.015280151.6-39.920.4-83.450.20.121.4996714443283.1512.0210463146-19.329.0-7.8.150.20.121.5807846433-6.1430.618874926.0-78.124.0-76.660.15.121.474818239.5-23.4315.02357942156-75.930.8-65.760.15.121.574818239.8-7.8.124.0-76.539.8-65.739.8-65.760.15.121.474818238.9-7.836.937.5511.456.9-66.19.0.9-66.160.15.121.488875	max			48.7	58.5			48.7	58.5	
50.15.121.357471634292.7294.21766031816-40.821.1-69.350.15.121.482239936725.6376.82184982330-66.626.7-73.450.15.121.574734033257.3339.7217116277354.327.7-70.9average25.127.6027.7374.827.770.9-64.831.1-19.550.20.121.1856456413-14.4477.6188542TL-81.227.2-78.050.20.121.2282183382-70.9106.4282183TL-70.9106.4.00.150.20.121.2282183382-70.9106.4282183TL-70.9106.4.01.950.20.121.4996714443283.1512.0210463146-19.320.0.78.150.20.121.5807846433-61.4430.61887492260-78.124.0.76.660.15.121.4996714443283.162.910.6.78.124.0.76.660.15.121.5807846433-61.4430.618874926.0.78.124.0.76.660.15.121.4748182395-23.431.025524729.5.65.749.0.00.160.15.121.5882672419-0.731.545503TL.49.910.9.77.760.15.121.5882672419-0.731.526002<	50_15_121_1	228775	369	-66.0	54.8	184095	2106	-72.6	24.6	-19.5
50.15.121.482239936725.6376.82184982330-66.626.7-70.9average-7434033257.3339.721711627354.327.7-70.9average-25.1276.0-76.0-70.936.8-70.936.8-70.936.8-70.936.9-70.936.9-70.936.9-70.936.9-70.9376.8-70.9106.428218370.9106.428218371.4-70.9106.40.0-70.9106.428218338.2-70.9106.428218314.1-70.9106.4-70.9106.439.9-70.9-70.9-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.6-70.910.610.510.610.510.510.610.510.610.510.610.510.610.510.610.510.510.610.510.510.510.510.510.510.510.510.510.510.510.510.510.510.510.5<	50_15_121_2	714366	377	15.9	314.6	225912	2245	-63.3	31.1	-68.4
50.15.121.574734033257.3339.721711627354.327.77.0.9average25.127.077.60-55.526.3-60.3max92.7376.8-14.831.1-19.550.20.121.1856456413-14.447.6188542TL8.1227.2-78.050.20.121.2282183382-70.9106.4282183TL-70.9106.40.050.20.121.3918976417261.0623.91528951961-9.329.0-78.350.20.121.4996714443283.151.202100463146-19.329.0-78.950.20.121.5807846433-6.1430.618874926078.124.0-76.6average905430.1623.957.941.4-63.4max27.478182395-23.4315.023574215675.930.8-65.760.15.121.281482640911.4269.0308572329.3-57.839.8-67.760.15.121.4887351389-7.8369.137504TL-49.9109.9-49.560.15.121.5882672419-0.7315.944553TL-49.9109.9-49.560.20.121.4859865459133.3425.4206022TL-49.9109.9-77.660.20.121.4798331439-47.2 <td>50_15_121_3</td> <td>574716</td> <td>342</td> <td>92.7</td> <td>294.2</td> <td>176603</td> <td>1816</td> <td>-40.8</td> <td>21.1</td> <td>-69.3</td>	50_15_121_3	574716	342	92.7	294.2	176603	1816	-40.8	21.1	-69.3
average max         25.1         276.0         -59.5         26.3         -60.3           max         92.7         376.8         -40.8         31.1         -19.5           50.20.121.1         856456         413         -14.4         477.6         188542         TL         -81.2         27.2         -78.0           50.20.121.2         282183         382         -70.9         106.4         282183         TL         -70.9         106.4         0.0           50.20.121.3         918976         417         261.0         623.9         152895         1961         -39.9         20.4         -83.4           50.20.121.4         996714         443         283.1         512.0         210046         3146         -19.3         29.0         -78.9           50.20.121.5         807846         433         -6.1         430.6         188749         260         -78.1         24.0         -76.6           average         -27.2         807.8         430.1         -57.9         41.4         -63.4           60.15.121.1         748182         395         -23.4         315.0         235794         2156         -57.8         39.8         -62.1           60.15.121.2	50_15_121_4	822399	367	25.6	376.8	218498	2330	-66.6	26.7	-73.4
max $-40.8$ $-41.3$ $-92.7$ $376.8$ $-40.8$ $-40.8$ $31.1$ $-19.5$ $50.20.121.1$ $856456$ $413$ $-14.4$ $477.6$ $188542$ $TL$ $-81.2$ $27.2$ $-78.0$ $50.20.121.2$ $282183$ $382$ $-70.9$ $106.4$ $282183$ $TL$ $-70.9$ $106.4$ $0.0$ $50.20.121.3$ $918976$ $417$ $261.0$ $623.9$ $152895$ $1961$ $-33.9$ $20.4$ $-83.4$ $50.20.121.4$ $996714$ $443$ $283.1$ $512.0$ $21046$ $3146$ $-19.3$ $29.0$ $-78.9$ $50.20.121.5$ $807846$ $433$ $-6.1$ $4306$ $188749$ $260$ $-78.1$ $24.0$ $-76.6$ $average$ $-78.1$ $2831$ $63.9$ $257.9$ $41.4$ $-63.4$ $max$ $748182$ $395$ $-23.4$ $315.0$ $235794$ $2156$ $75.9$ $30.8$ $-63.4$ $60.15.121.1$ $748182$ $395$ $-23.4$ $315.0$ $235794$ $2156$ $75.9$ $30.8$ $-63.7$ $60.15.121.3$ $265247$ $407$ $-65.7$ $49.0$ $2052$ $-75.9$ $41.4$ $-96.7$ $60.15.121.4$ $887351$ $389$ $-7.8$ $369.1$ $375054$ $TL$ $-61.0$ $98.3$ $-57.7$ $60.20.121.4$ $859865$ $459$ $13.3$ $4254$ $206022$ $TL$ $44.1$ $25.9$ $-76.6$ $60.20.121.4$ $859865$ $459$ $13.3$ $45.4$ </td <td>50_15_121_5</td> <td>747340</td> <td>332</td> <td>57.3</td> <td>339.7</td> <td>217116</td> <td>2773</td> <td>-54.3</td> <td>27.7</td> <td>-70.9</td>	50_15_121_5	747340	332	57.3	339.7	217116	2773	-54.3	27.7	-70.9
50.20.121.1 $856456$ $413$ $-14.4$ $477.6$ $188542$ $TL$ $-81.2$ $27.2$ $-78.0$ $50.20.121.2$ $282183$ $382$ $-70.9$ $106.4$ $282183$ $TL$ $-70.9$ $106.4$ $0.0$ $50.20.121.3$ $918976$ $417$ $261.0$ $623.9$ $152895$ $1961$ $-39.9$ $20.4$ $-83.4$ $50.20.121.4$ $996714$ $443$ $283.1$ $512.0$ $21046$ $3146$ $-19.3$ $29.0$ $-78.9$ $50.20.121.5$ $807846$ $433$ $-61.1$ $430.6$ $188749$ $2260$ $-78.1$ $24.0$ $-76.6$ average $90.5$ $430.1$ $188749$ $2260$ $-78.1$ $24.0$ $-76.6$ $max$ $-748182$ $395$ $-23.4$ $315.0$ $235794$ $2156$ $-75.9$ $30.8$ $-68.5$ $60.15.121.2$ $814826$ $409$ $11.4$ $269.0$ $308572$ $3293$ $-57.8$ $39.8$ $-62.1$ $60.15.121.3$ $265247$ $407$ $-65.7$ $49.0$ $265247$ $2958$ $-65.7$ $49.0$ $0.0$ $60.15.121.4$ $887351$ $389$ $-7.8$ $369.1$ $375054$ $TL$ $-49.9$ $109.9$ $-49.5$ $average$ $-17.2$ $263.6$ $-17.2$ $263.6$ $-14.9.9$ $109.9$ $-49.5$ $average$ $-17.2$ $263.6$ $-16.1$ $49.9$ $109.9$ $-76.0$ $60.20.121.1$ $859865$ $459$ $13.3$ $425.4$ $206022$ <	average			25.1	276.0			-59.5	26.3	-60.3
50.20.121.2282183382.70.9106.4282183TL.70.9106.4.0.050.20.121.3918976417261.0623.9152891961.39.9.20.4.83.450.20.121.4996714443283.1512.02100463146.19.3.29.0.78.950.20.121.5807846433-6.1430.61887492260.78.1.24.0.76.6average	max			92.7	376.8			-40.8	31.1	-19.5
50.20.121.3918976417261.0623.91528951961-39.920.4-83.450.20.121.4996714443283.1512.02100463146-19.329.0-78.150.20.121.5807846433-6.1430.6188749260-78.124.0-76.6average90.5430.1162.9188749260-78.124.0-76.1max	50_20_121_1	856456	413	-14.4	477.6	188542	TL	-81.2	27.2	-78.0
50.20.121.4996714443283.1512.02100463146-19.329.0-78.150.20.121.5807846433-6.1430.61887492260-78.124.0-76.6average90.5430.11887492260-78.124.0-76.6awarage283.1623.9119.3106.463.4max283.1623.9215675.930.868.760.15.121.1748182395-23.4315.02357942156-75.930.868.760.15.121.281482640911.4269.03085723293-57.839.8-62.160.15.121.3265247407-65.749.02652472958-65.749.00.060.15.121.4887351389-7.8369.1375054TL-61.098.3-57.760.15.121.5882672419-0.7315.9445503TL-61.098.3-47.6max	50_20_121_2	282183	382	-70.9	106.4	282183	TL	-70.9	106.4	0.0
50.20.121.5807846433-6.1430.61887492260.78.124.0.76.6average90.5430.1-5.741.4-63.4max283.1623.9-5.791.60.060.15.121.1748182395.23.4315.02357942156.75.930.8.68.560.15.121.281482640911.4269.03085723293.57.839.8.62.160.15.121.3265247407.65.749.02652472958.66.749.00.060.15.121.4887351389.7.8369.1375054TL.61.098.3.57.760.15.121.5882672419.0.7315.9445503TL.44.9109.9.49.5average-17.2263.6-1761.098.3.57.760.20.121.585986545913.3425.4206022TL.49.9109.9.49.560.20.121.2789831439.47.2400.1196199TL.48.5.28.0.75.360.20.121.41122059448.38.4495.7380179TL.78.5.28.5.69.760.20.121.51033619518.32.3454.6234454TL.84.6.25.8.77.360.20.121.41122059448.38.4495.7380179TL.79.1101.8.66.160.20.121.51033619518.32.3454.6 <t< td=""><td>50_20_121_3</td><td>918976</td><td>417</td><td>261.0</td><td>623.9</td><td>152895</td><td>1961</td><td>-39.9</td><td>20.4</td><td>-83.4</td></t<>	50_20_121_3	918976	417	261.0	623.9	152895	1961	-39.9	20.4	-83.4
average         90.5         430.1         -57.9         41.4         -63.4           max         283.1         623.9         -19.3         106.4         0.0           60.15.121.1         748182         395         -23.4         315.0         235794         2156         -75.9         30.8         -68.5           60.15.121.2         814826         409         11.4         269.0         308572         3293         -57.8         39.8         -62.1           60.15.121.3         265247         407         -65.7         49.0         265247         2958         -65.7         49.0         0.0           60.15.121.4         887351         389         -7.8         369.1         375054         TL         -61.0         98.3         -57.7           60.15.121.4         887351         389         -7.8         369.1         375054         TL         -61.0         98.3         -57.7           60.15.121.4         882672         419         -0.7         315.9         445503         TL         -49.9         109.9         -49.5           average         -17.2         263.6         -47.6         -44.9         109.9         0.0           60.20.121.1 <t< td=""><td>50_20_121_4</td><td>996714</td><td>443</td><td>283.1</td><td>512.0</td><td>210046</td><td>3146</td><td>-19.3</td><td>29.0</td><td>-78.9</td></t<>	50_20_121_4	996714	443	283.1	512.0	210046	3146	-19.3	29.0	-78.9
max         283.1         623.9         -19.3         106.4         0.0           60.15.121.1         748182         395         -23.4         315.0         235794         2156         -75.9         30.8         -68.5           60.15.121.2         814826         409         11.4         269.0         308572         3293         -57.8         39.8         -62.1           60.15.121.3         265247         407         -65.7         49.0         265247         2958         -65.7         49.0         0.0           60.15.121.4         887351         389         -7.8         369.1         375054         TL         -61.0         98.3         -57.7           60.15.121.5         882672         419         -0.7         315.9         445503         TL         -49.9         109.9         -49.5           average         -17.2         263.6         -11.4         369.1         -41.1         -44.9         90.0         -60.0           60.20.121.1         859865         459         133.3         425.4         206022         TL         -44.1         25.9         -76.0           60.20.121.4         859865         459         133.3         425.4         206022	50_20_121_5	807846	433	-6.1	430.6	188749	2260	-78.1	24.0	-76.6
60.15.121.1       748182       395       -23.4       315.0       235794       2156       -75.9       30.8       -68.5         60.15.121.2       814826       409       11.4       269.0       308572       3293       -57.8       39.8       -62.1         60.15.121.3       265247       407       -65.7       49.0       265247       2958       -65.7       49.0       0.0         60.15.121.4       887351       389       -7.8       369.1       375054       TL       -61.0       98.3       -57.7         60.15.121.5       882672       419       -0.7       315.9       445503       TL       -49.9       109.9       -49.5         average       -17.2       263.6       -62.1       65.5       -47.6         max       11.4       369.1       -49.9       109.9       0.0         60.20.121.1       859865       459       133.3       425.4       206022       TL       -44.1       25.9       -76.0         60.20.121.2       789831       439       -47.2       400.1       196199       TL       -86.9       24.2       -75.2         60.20.121.3       996274       430       -39.6       571.1       1	average			90.5	430.1			-57.9	41.4	-63.4
60.15.121.2 $814826$ $409$ $11.4$ $269.0$ $308572$ $3293$ $-57.8$ $39.8$ $-62.1$ $60.15.121.3$ $265247$ $407$ $-65.7$ $49.0$ $265247$ $2958$ $-65.7$ $49.0$ $0.0$ $60.15.121.4$ $887351$ $389$ $-7.8$ $369.1$ $375054$ $TL$ $-61.0$ $98.3$ $-57.7$ $60.15.121.5$ $882672$ $419$ $-0.7$ $315.9$ $445503$ $TL$ $-49.9$ $109.9$ $-49.5$ $average$ $-17.2$ $263.6$ $-17.2$ $-62.1$ $65.5$ $-47.6$ $max$ $-11.4$ $369.1$ $-1.5$ $-49.9$ $109.9$ $-49.5$ $60.20.121.1$ $859865$ $459$ $133.3$ $425.4$ $206022$ $TL$ $-44.1$ $25.9$ $-75.2$ $60.20.121.2$ $789831$ $439$ $-47.2$ $400.1$ $196199$ $TL$ $-86.9$ $24.2$ $-75.2$ $60.20.121.3$ $996274$ $430$ $-39.6$ $571.1$ $189965$ $TL$ $-88.5$ $28.0$ $-80.9$ $60.20.121.4$ $1122059$ $448$ $-38.4$ $495.7$ $380179$ $TL$ $-79.1$ $101.8$ $-66.1$ $60.20.121.5$ $1033619$ $518$ $-32.3$ $459.4$ $234454$ $TL$ $-84.6$ $25.8$ $-77.3$ $average$ $-47.4$ $469.4$ $459.4$ $234454$ $TL$ $-76.6$ $41.1$ $-75.1$	max			283.1	623.9			-19.3	106.4	0.0
60.15.121.3 $265247$ $407$ $-65.7$ $49.0$ $265247$ $2958$ $-65.7$ $49.0$ $0.0$ $60.15.121.4$ $887351$ $389$ $-7.8$ $369.1$ $375054$ $TL$ $-61.0$ $98.3$ $-57.7$ $60.15.121.5$ $882672$ $419$ $-0.7$ $315.9$ $445503$ $TL$ $-49.9$ $109.9$ $-49.5$ $average$ $-17.2$ $263.6$ $-17.2$ $-62.1$ $65.5$ $-47.6$ $max$ $11.4$ $369.1$ $-49.9$ $109.9$ $0.0$ $60.20.121.1$ $859865$ $459$ $133.3$ $425.4$ $206022$ $TL$ $-44.1$ $25.9$ $-75.2$ $60.20.121.2$ $789831$ $439$ $-47.2$ $400.1$ $196199$ $TL$ $-86.9$ $24.2$ $-75.2$ $60.20.121.3$ $996274$ $430$ $-39.6$ $571.1$ $189965$ $TL$ $-88.5$ $28.0$ $-80.9$ $60.20.121.4$ $1122059$ $448$ $-38.4$ $495.7$ $380179$ $TL$ $-84.6$ $25.8$ $-77.3$ $60.20.121.5$ $1033619$ $518$ $-32.3$ $459.4$ $234454$ $TL$ $-84.6$ $25.8$ $-77.1$ $average$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.4$ $-44.$	60_15_121_1	748182	395	-23.4	315.0	235794	2156	-75.9	30.8	-68.5
60.15.121.4 $887351$ $389$ $-7.8$ $369.1$ $375054$ $TL$ $-61.0$ $98.3$ $-57.7$ $60.15.121.5$ $882672$ $419$ $-0.7$ $315.9$ $445503$ $TL$ $-49.9$ $109.9$ $-49.5$ average $-17.2$ $263.6$ $-62.1$ $-62.1$ $65.5$ $-47.6$ max $11.4$ $369.1$ $-49.9$ $109.9$ $0.0$ $60.20.121.1$ $859865$ $459$ $133.3$ $425.4$ $206022$ $TL$ $-44.1$ $25.9$ $-76.0$ $60.20.121.2$ $789831$ $439$ $-47.2$ $400.1$ $196199$ $TL$ $-86.9$ $24.2$ $-75.2$ $60.20.121.3$ $996274$ $430$ $-39.6$ $571.1$ $189965$ $TL$ $-88.5$ $28.0$ $-80.9$ $60.20.121.4$ $1122059$ $448$ $-38.4$ $495.7$ $380179$ $TL$ $-79.1$ $101.8$ $-66.1$ $60.20.121.5$ $1033619$ $518$ $-32.3$ $454.6$ $234454$ $TL$ $-84.6$ $25.8$ $-77.3$ $average$ $-47.4$ $469.4$ $-469.4$ $-76.6$ $41.1$ $-75.1$	60_15_121_2	814826	409	11.4	269.0	308572	3293	-57.8	39.8	-62.1
60.15.121.5 $882672$ $419$ $-0.7$ $315.9$ $445503$ $TL$ $-49.9$ $109.9$ $-49.5$ average $-17.2$ $263.6$ $-62.1$ $65.5$ $-47.6$ max $11.4$ $369.1$ $-49.9$ $109.9$ $0.0$ $60.20.121.1$ $859865$ $459$ $133.3$ $425.4$ $206022$ $TL$ $-44.1$ $25.9$ $-76.0$ $60.20.121.2$ $789831$ $439$ $-47.2$ $400.1$ $196199$ $TL$ $-86.9$ $24.2$ $-75.2$ $60.20.121.3$ $996274$ $430$ $-39.6$ $571.1$ $189965$ $TL$ $-88.5$ $28.0$ $-80.9$ $60.20.121.4$ $1122059$ $448$ $-38.4$ $495.7$ $380179$ $TL$ $-79.1$ $101.8$ $-66.1$ $60.20.121.5$ $1033619$ $518$ $-32.3$ $454.6$ $234454$ $TL$ $-84.6$ $25.8$ $-77.3$ $average$ $-44.4$ $-44.8$ $469.4$ $-76.6$ $41.1$ $-75.1$	60_15_121_3	265247	407	-65.7	49.0	265247	2958	-65.7	49.0	0.0
average        17.2         263.6        62.1         65.5         -47.6           max         11.4         369.1         -49.9         109.9         0.0           60_20_121_1         859865         459         133.3         425.4         206022         TL         -44.1         25.9         -76.0           60_20_121_2         789831         439         -47.2         400.1         196199         TL         -86.9         24.2         -75.2           60_20_121_3         996274         430         -39.6         571.1         189965         TL         -88.5         28.0         -80.9           60_20_121.4         1122059         448         -38.4         495.7         380179         TL         -79.1         101.8         -66.1           60_20_121.5         1033619         518         -32.3         454.6         234454         TL         -84.6         25.8         -77.3           average         -48.8         469.4         -76.6         41.1         -75.1	60_15_121_4	887351	389	-7.8	369.1	375054	TL	-61.0	98.3	-57.7
max         11.4         369.1         -49.9         109.9         0.0           60.20.121.1         859865         459         133.3         425.4         206022         TL         -44.1         25.9         -76.0           60.20.121.2         789831         439         -47.2         400.1         196199         TL         -86.9         24.2         -75.2           60.20.121.3         996274         430         -39.6         571.1         189965         TL         -88.5         28.0         -80.9           60.20.121.4         1122059         448         -38.4         495.7         380179         TL         -79.1         101.8         -66.1           60.20.121.5         1033619         518         -32.3         454.6         23454         TL         -84.6         25.8         -77.3           average         -44.1         -44.1         -44.1         -75.1         -75.1         -75.1	60_15_121_5	882672	419	-0.7	315.9	445503	TL	-49.9	109.9	-49.5
60.20.121.1       859865       459       133.3       425.4       206022       TL       -44.1       25.9       -76.0         60.20.121.2       789831       439       -47.2       400.1       196199       TL       -86.9       24.2       -75.2         60.20.121.3       996274       430       -39.6       571.1       189965       TL       -88.5       28.0       -80.9         60.20.121.4       1122059       448       -38.4       495.7       380179       TL       -79.1       101.8       -66.1         60.20.121.5       1033619       518       -32.3       454.6       234454       TL       -84.6       25.8       -77.3         average       -4.8       469.4       -76.6       41.1       -75.1	average			-17.2	263.6			-62.1	65.5	-47.6
60.20.121.2       789831       439       -47.2       400.1       196199       TL       -86.9       24.2       -75.2         60.20.121.3       996274       430       -39.6       571.1       189965       TL       -88.5       28.0       -80.9         60.20.121.4       1122059       448       -38.4       495.7       380179       TL       -79.1       101.8       -66.1         60.20.121.5       1033619       518       -32.3       454.6       234454       TL       -84.6       25.8       -77.3         average       -4.8       469.4       -57.4       -76.6       41.1       -75.1	max			11.4	369.1			-49.9	109.9	0.0
60_20_121_3       996274       430       -39.6       571.1       189965       TL       -88.5       28.0       -80.9         60_20_121_4       1122059       448       -38.4       495.7       380179       TL       -79.1       101.8       -66.1         60_20_121_5       1033619       518       -32.3       454.6       234454       TL       -84.6       25.8       -77.3         average       -4.8       469.4       -57.4       -76.6       41.1       -75.1	60_20_121_1	859865	459	133.3	425.4	206022	TL	-44.1	25.9	-76.0
60.20.121.4       1122059       448       -38.4       495.7       380179       TL       -79.1       101.8       -66.1         60.20.121.5       1033619       518       -32.3       454.6       234454       TL       -84.6       25.8       -77.3         average       -4.8       469.4       -76.6       41.1       -75.1	60_20_121_2	789831	439	-47.2	400.1	196199	TL	-86.9	24.2	-75.2
60_20_121_5       1033619       518       -32.3       454.6       234454       TL       -84.6       25.8       -77.3         average       -4.8       469.4       -76.6       41.1       -75.1	60_20_121_3	996274	430	-39.6	571.1	189965	TL	-88.5	28.0	-80.9
average -4.8 469.4 -76.6 41.1 -75.1	60_20_121_4	1122059	448	-38.4	495.7	380179	TL	-79.1	101.8	-66.1
	60_20_121_5	1033619	518	-32.3	454.6	234454	TL	-84.6	25.8	-77.3
max 133.3 571.1 -44.1 101.8 -66.1	average			-4.8	469.4			-76.6	41.1	-75.1
	max			133.3	571.1			-44.1	101.8	-66.1

Table 6.25: Geographical Decomposition with Overlapping and Local Search Results - Group B - Part 2

In Fig. 6.10 and 6.12 the different behavior of the most promising heuristic algorithms for Group A and Group B are summarized; in Fig. 6.11 and 6.13 the relative computational times are reported: the column below (green) represents the number of instances solved in less than 1000s by the algorithm, the intermediate column (yellow) the instances solved in less than 2000 s, the upper column all the others.

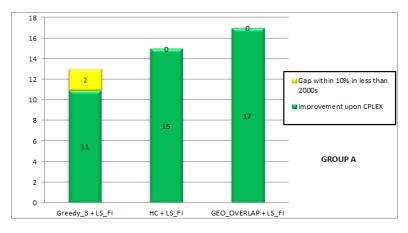


Figure 6.10: Results of heuristics for Group A

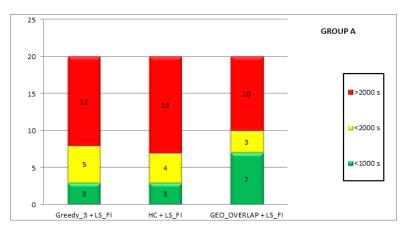


Figure 6.11: Time Results of heuristics for Group A

For Group A the heuristic algorithm based on Geographical Decomposition with Overlapping between sub-areas and a first improvement local search provide the best results: in 17 out 20 instances it provides improvements upon CPLEX (the average improvement is about 3%). Besides, in 10 out of 20 instances the computational times are quite small (below 2000 s).

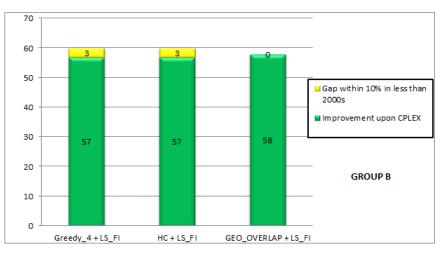


Figure 6.12: Results of heuristics for Group B

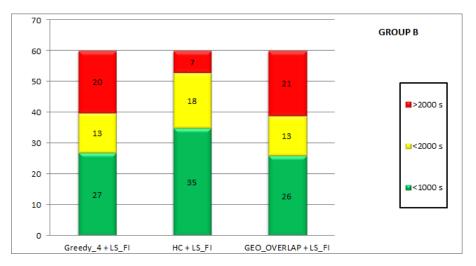


Figure 6.13: Time Results of heuristics for Group B

For Group B all the algorithms tested have good results except for few instances: there is no a clear winner even if the geographical based approach provides the best results (it improves upon CPLEX in 58 out of 20 instances). The improvements are significant especially for largest instances (about 60%). Regarding computational times, instead, Heuristic Concentration algorithm and first improvement local search give the best results.

#### Remarks on one example

We report in Fig. 6.14 the geographical distribution of TTPs for instance 50\_50\_625\_1 and in Fig. 6.15 the solution found with the geographical decomposition algorithm.

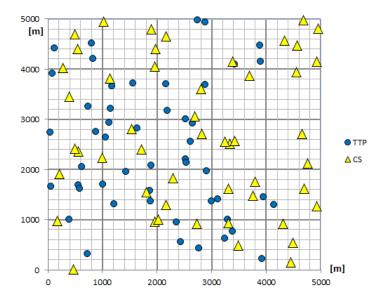


Figure 6.14: Instance 50\_50\_625\_1

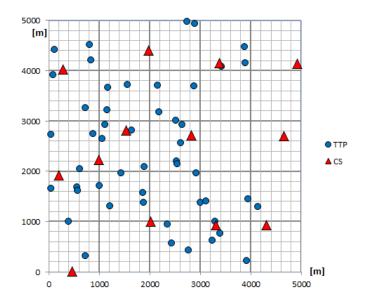


Figure 6.15: Solution of instance 50\_50\_625\_1 with Geographical approach

From Fig. 6.15 we can understand why geographical decomposition gives such good results: if the number of CSs and TTPs in a sub-area is small, solving the sub-problem EMCA\_Restricted is very easy and it requires small computational times. For example in instance 50\_50\_625\_1 in each sub-area there are on average only two CSs and three TTPs.

A further analysis of the obtained solution shows that 13 CSs are chosen to install the BSs. Five BSs are equipped with the most powerful device (macro-cell), seven with the intermediate device (micro-cell) and only one with the smallest one (picocell).

Regarding operative part of the network, five BSs change their power level at least once during the day. If power modulation is not allowed the overall costs rise up to 10%.

#### 6.3 Results for Larger Instances

We finally use the geographical algorithm to solve instances of Group C. These instances do not present a uniform distribution of CS and TTP. An example is shown in Fig. 6.16

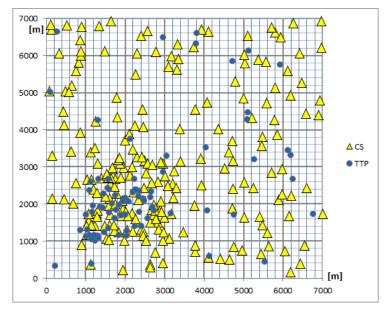


Figure 6.16: Instance 80\_220\_1296

We set the size of each sub-areas to 1 Km x 1 Km; the overlap is still of half cell. Since in some sub-areas the density of CS and TTP is high, the time limit for solving EMCA\_Restricted in this case is increased up to 100 s.

Table 6.26 is related to the results obtained with CPLEX. Since the instances are very large we set the time limit (TL) to 2 hours. Obviously also the heuristic algorithm works with the same TL. In Table 6.27 the results obtained with the geographical based approach and local search (FI) are reported.

INSTANCE	$[\mathbf{s}]$	UB	LB	%
70_200_961	TL	7529764	455410*	_
80_220_1296	TL	7417082	583137	1171.9
90_400_1681	TL	12609752		_
150_500_1681	TL	19006445		_
90_450_2601	TL	14152715	151337	9251.8

Table 6.26: CPLEX Results - Group C

INSTANCE	GEO_OVERLAP	$[\mathbf{s}]$	%UB	%LB	LS (FI)	$[\mathbf{s}]$	%_UB	%_LB	Imp
70_200_961	7180952	1668	-4.6	_	3128665	TL	-58.4	_	-56.4
80_220_1296	7332693	1682	-1.1	1157.5	3581040	TL	-51.7	514.1	-51.2
90_400_1681	12297752	2137	-2.5		12297752	TL	-2.5		0.0
150_500_1681	17732857	2640	-6.7		17732857	TL	-6.7		0.0
90_450_2601	13052842	2146	-7.8	8525.0	13052842	TL	-7.8	8525.0	0.0

Table 6.27: Geographical Decomposition with Overlapping and Local Search Results - Group C

Results show that CPLEX is not able to find the optimal solution within the time limit in any instance. In some cases it also does not find a LB within the time limit. For instance 70\_200\_96 the LB of the continuous relaxation is reported. The gap between UB ad LB are very high (1172% and 9251%). The geographical decomposition based algorithm can bring significant improvements. In fact the heuristic algorithm improves upon CPLEX for all instances even without applying the local search phase.

The time needed to solve the problem is also significantly small (a third of TL). In larger scenarios (when the number of TTP $\geq$ 90 and CS $\geq$ 400) the local search does not have impact on the results. Instead for instance 70\_200\_961 and for instance 80\_220\_1296 the improvements after the local search phase are significant (about 50%). In Fig. 6.17 the solution for instance 80\_220\_1296 is shown.

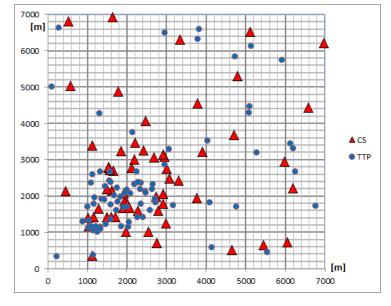


Figure 6.17: Instance 80\_220\_1296 Result

We can note that most of the CS opened are in the area where TTP are concentrated. The other installed BSs helps in providing a basic area coverage.

## Chapter 7

### Conclusions

With the continuous development of new devices and multimedia services, cellular network traffic is expected to grow exponentially in the next years. Network operators have to take into account the increases in energy consumption in order to minimize costs and reduce the human impact on nature. In this context green networking has become one of the most important topics inside the Information and Communication Technology (ICT) area.

An important part of energy consumption within ICT is due to telecommunications networks. On the one hand the scientific telecommunication community tries to design new energy efficient hardware technologies and protocols for network, on the other hand new techniques based on energy-aware planning and management of the networks are developed to limit the telecommunication network consumption. In telecommunication networks traffic has usually significant fluctuations during the day; significant energy savings can be achieved by exploiting this aspect and adapting the transmission power level of BSs to client real needs.

Due to development of networks, especially mobile communications will have a large expansion to meet the new needs of users. The attention on possible energy savings in wireless networks is therefore increasing. This thesis is part of this research field. We consider an energy efficient cellular network planning problem. We proposed methods for ensuring significant energy savings in the access part of a cellular network in which the transmitting power of BS is modulated and adapted to real traffic needs.

The problem consists in deciding where to install a set of BSs and how to equip them. Besides, from the operational point of view the problem asks to adapt the BS power level to the traffic in different time intervals. We consider the planning and managing of a network over a long time horizon: thus beside installation costs (CAPEX), also operative costs (OPEX) are taken into account as both have a strong impact on the overall costs. The problem can be formulated as a ILP problem and solved with a commercial solver. However it becomes computationally challenging especially increasing the size of instances: our main contribution consists in the development of heuristic approaches and in particular ILP - based heuristics in order to find feasible good solutions in reasonable computational times.

We developed constructive heuristics to provide feasible solutions. Four different greedy algorithms have been implemented. Besides, we developed a Heuristic Concentration method in which the ILP formulation of the problem is solved over a subset of variables heuristically selected.

We developed also a constructive technique based on area network division into smaller areas according to geographical location of BSs to exploit the geographical features of the network.

Different types of local search algorithms have been implemented and applied to all constructive algorithms in order to further improve the solutions obtained.

All the developed approaches have been tested on a large set of instances. The instances are realistic and various in order to test the methods on different scenarios.

From the results we observe that the heuristics combined with the use of ILP

provide results better than those obtained using a state-of-the-art solver for solving the ILP formulation in particular on large instances. The geographical decomposition based method seems to outperform other approaches.

Concerning future works it is possible to further improve the algorithms presented in this thesis. Since we developed ILP-based heuristics, more efficient formulations of the problem might provide significant improvements of the proposed algorithms performances. Furthermore it is possible to improve the geographical heuristic algorithm proposed with a more accurate subdivision of areas based on the geographical distribution of clients in the network. Finally, we want suggest the implementation of heuristic algorithms based on population.

# Bibliography

- L. Neves and J. Krajewski. Gesi smarter 2020: The role of ict in driving a sustainable future.
- [2] J. Hansen, M. Sato, P. Kharecha, G. Russell, D. W. Lea, and M. Siddall. Climate change and trace gases. *Philosophical Transactions of the Royal Society* A: Mathematical, Physical and Engineering Sciences, 365(1856):1925–1954, 2007.
- [3] L. Chiaraviglio, D. Ciullo, M. Meo, M. A. Marsan, and I. Torino. Energyaware umts access networks. *Proc. of IEEE W-GREEN*, pages 1–8, 2008.
- [4] Greentouch green meter research study: Reducing the net energy consumption in communications networks by up to 90% by 2020. http://www. greentouch.org/.
- [5] Cisco Visual Networking Index. Global mobile data traffic forecast update, 2013-2018. White Paper, February, 2014.
- [6] O. Blume, D. Zeller, and U. Barth. Approaches to energy efficient wireless access networks. In Communications, Control and Signal Processing (ISCCSP), 2010 4th International Symposium on, pages 1–5, 2010.
- [7] X. Wang, A. V Vasilakos, M. Chen, Y. Liu, and T. Kwon. A survey of green mobile networks: Opportunities and challenges. *Mobile Networks and Applications*, 17(1):4–20, 2012.

- [8] G. Koutitas and P. Demestichas. A review of energy efficiency in telecommunication networks. *Telfor journal*, 2(1):2–7, 2010.
- [9] Z. Hasan, H. Boostanimehr, and V. Bhargava. Green cellular networks: A survey, some research issues and challenges. *Communications Surveys & Tutorials*, 13(4):524–540, 2011.
- [10] H. Mellah and B. Sanso. Review of facts, data and proposals for a greener internet. In Broadband Communications, Networks, and Systems, 2009. BROADNETS 2009. Sixth International Conference on, pages 1–5, 2009.
- [11] A. De Domenico, E. Calvanese Strinati, and A. Capone. Enabling green cellular networks: A survey and outlook. *Computer Communications*, 37:5– 24, 2014.
- [12] M. Gupta and S. Singh. Greening of the internet. In Proceedings of the 2003 conference on Applications, technologies, architectures, and protocols for computer communications, pages 19–26, 2003.
- [13] L. M. Correia, D. Zeller, O. Blume, D. Ferling, Y. Jading, I. Gódor, G. Auer, and L. Van Der Perre. Challenges and enabling technologies for energy aware mobile radio networks. *Communications Magazine*, 48(11):66–72, 2010.
- [14] B. Kim, J. Kim, I. Kim, and J. Cha. The doherty power amplifier. *Microwave Magazine*, 7(5):42–50, 2006.
- [15] G. Claßen, A. MCA Koster, and A. Schmeink. A robust optimisation model and cutting planes for the planning of energy-efficient wireless networks. *Computers & Operations Research*, 40(1):80–90, 2013.
- [16] Amit P Jardosh, Konstantina Papagiannaki, Elizabeth M Belding, Kevin C Almeroth, Gianluca Iannaccone, and Bapi Vinnakota. Green wlans: on-

demand wlan infrastructures. *Mobile Networks and Applications*, 14(6):798–814, 2009.

- [17] A. Capone, I. Filippini, B. Gloss, and U. Barth. Rethinking cellular system architecture for breaking current energy efficiency limits. In Sustainable Internet and ICT for Sustainability (SustainIT), 2012, pages 1–5, 2012.
- [18] S. Boiardi, A. Capone, and B. Sansó. Radio planning of energy-aware cellular networks. *Computer Networks*, 57(13):2564–2577, 2013.
- [19] J. Lorincz, A. Capone, and D. Begušić. Optimized network management for energy savings of wireless access networks. *Computer Networks*, 55(3):514– 540, 2011.
- [20] J. Lorincz, A. Capone, and D. Begušić. Heuristic algorithms for optimization of energy consumption in wireless access networks. KSII Transactions on Internet & Information Systems, 5(4), 2011.
- [21] E. Oh, K. Son, and B. Krishnamachari. Dynamic base station switchingon/off strategies for green cellular networks. 2013.
- [22] B. Gendron, R. G Garroppo, G. Nencioni, M. Scutellà, and L. Tavanti. Benders decomposition for a location-design problem in green wireless local area networks. *Electronic Notes in Discrete Mathematics*, 41:367–374, 2013.
- [23] Y. Qi, M. Imran, and R. Tafazolli. On the energy aware deployment strategy in cellular systems. In *Personal, Indoor and Mobile Radio Communications* Workshops (PIMRC Workshops), 2010 IEEE 21st International Symposium on, pages 363–367, 2010.
- [24] V. Prithiviraj, S. Venkatraman, and R. Vijayasarathi. Cell zooming for energy efficient wireless cellular network. *Journal of Green Engineering*, 3(4):421– 434, 2013.

- [25] X. Weng, D. Cao, and Z. Niu. Energy-efficient cellular network planning under insufficient cell zooming. In Vehicular Technology Conference (VTC Spring), 2011 IEEE 73rd, pages 1–5, 2011.
- [26] F. Richter, A. J Fehske, and G. Fettweis. Energy efficiency aspects of base station deployment strategies for cellular networks. In *Vehicular Technology Conference Fall (VTC 2009-Fall), 2009 IEEE 70th*, pages 1–5, 2009.
- [27] H. Claussen, L. Ho, and F. Pivit. Effects of joint macrocell and residential picocell deployment on the network energy efficiency. In *Personal, Indoor and Mobile Radio Communications, 2008. PIMRC 2008. IEEE 19th International Symposium on*, pages 1–6, 2008.
- [28] S. Boiardi, A. Capone, and B. Sansó. Joint design and management of energyaware mesh networks. Ad Hoc Networks, 10(7):1482–1496, 2012.
- [29] Y. Chen, S. Zhang, S. Xu, and G.Y. Li. Fundamental trade-offs on green wireless networks. *Communications Magazine*, 49(6):30–37, 2011.
- [30] Y. Chen, S. Zhang, and S. Xu. Characterizing energy efficiency and deployment efficiency relations for green architecture design. In *Communications Workshops (ICC)*, 2010 IEEE International Conference on, pages 1–5, 2010.
- [31] K. Son, E. Oh, and B. Krishnamachari. Energy-aware hierarchical cell configuration: from deployment to operation. In *Computer Communications Workshops (INFOCOM WKSHPS), 2011 IEEE Conference on*, pages 289– 294, 2011.
- [32] L. Chiaraviglio, D. Ciullo, G. Koutitas, M. Meo, and L. Tassiulas. Energyefficient planning and management of cellular networks. In Wireless Ondemand Network Systems and Services (WONS), 2012 9th Annual Conference on, pages 159–166, 2012.

- [33] K. Dufková, M. Bjelica, B. Moon, L. Kencl, and J-Y Le Boudec. Energy savings for cellular network with evaluation of impact on data traffic performance. In Wireless Conference (EW), 2010 European, pages 916–923, 2010.
- [34] S. Zhou, J. Gong, Z. Yang, Z. Niu, and P. Yang. Green mobile access network with dynamic base station energy saving. In ACM MobiCom, volume 9, pages 10–12, 2009.
- [35] A. Bousia, A. Antonopoulos, L. Alonso, and C. Verikoukis. Green distanceaware base station sleeping algorithm in LTE-advanced. In *Communications* (ICC), 2012 IEEE International Conference on, pages 1347–1351, 2012.
- [36] I. Tunaru, H. Rivano, and F. Valois. Wsn-inspired sleep protocols for heterogeneous LTE networks. In Proceedings of the 10th ACM symposium on Performance evaluation of wireless ad hoc, sensor, & ubiquitous networks, pages 113–120, 2013.
- [37] T. Pamuklu and C. Ersoy. Optimization of renewable green base station deployment. In Green Computing and Communications (GreenCom), 2013 IEEE and Internet of Things (iThings/CPSCom), IEEE International Conference on and IEEE Cyber, Physical and Social Computing, pages 59–63, 2013.
- [38] M. A. Marsan, L. Chiaraviglio, D. Ciullo, and M. Meo. Optimal energy savings in cellular access networks. In *Communications Workshops*, 2009. *ICC Workshops 2009. IEEE International Conference on*, pages 1–5, 2009.
- [39] E. Guzzon. Long term evolution introduction. http://www.comlab. uniroma3.it, 2011.
- [40] G. Auer, V. Giannini, C. Desset, I. Godor, P. Skillermark, M. Olsson, M. A. Imran, D. Sabella, M. J. Gonzalez, O. Blume, et al. How much energy is

needed to run a wireless network? Wireless Communications, 18(5):40–49, 2011.

- [41] K.E. Rosing and C.S. ReVelle. Heuristic concentration: Two stage solution construction. European Journal of Operational Research, 97(1):75–86, 1997.
- [42] K. E. Rosing and M. J. Hodgson. Heuristic concentration for the p-median: an example demonstrating how and why it works. *Computers & Operations Research*, 29(10):1317–1330, 2002.
- [43] V.J. Rayward-Smith, R. Osman, R. Reeves, G.D. Smith, and Eds. Modern heuristic search methods. Wiley New York, 1996.
- [44] B. Addis, G. Carello, and A. Ceselli. Combining very large scale and ilp based neighborhoods for a two-level location problem. *European Journal of Operational Research*, 231(3):535–546, 2013.
- [45] R. K. Ahuja, O. Ergun, J. B. Orlin, and A. P. Punnen. A survey of very large-scale neighborhood search techniques. *Discrete Applied Mathematics*, 123(1):75–102, 2002.
- [46] http://ampl.com.
- [47] IBM ILOG CPLEX. V12. 1: User's manual for CPLEX. International Business Machines Corporation, 46(53):157, 2009.
- [48] S. S. Kale and A.N. Jadhav. An empirically based path loss models for LTE advanced network and modeling for 4g wireless systems at 2.4 ghz, 2.6 ghz and 3.5 ghz. International Journal of Application or Innovation in Engineering Management, 2013.