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Efficient energy management in a building with storages via sporadic model predictive control

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Abstract

The problem of optimally controlling a building equipped with energy storages is considered and formulated in order to be naturally compatible with a predictive control approach. The focus is restricted to thermal systems, since they are among the most relevant and influential.

A general form to state the addressed problem is introduced and its complexity (coarsely) quantified with respect to the intended applications. In particular, a formal statement of the problem under analysis is presented, and the required mathematical notation is established.

Our effort consists in trying to lighten the burden that is required to solve the complex optimization problem considered. In this view, an approach, named "Sporadic Model Predictive Control", is presented. Thanks to this control technique, the optimization process should not be carried out at each sampling time, but only when considered necessary. The proposed control scheme is analysed in a view to outline its tuning.

A proof-of-concept case is addressed and solved, showing the capabilities of the approach. A real numerical optimization system is in place and, specifically, the application is carried on using the Modelica modelling language and the GenOpt optimization tool. This allows to additionally demonstrate that the approach can effectively bring engineering models into play also for control and management purposes, thereby joining and streamlining the design and the operation phases of a project. Finally, the use of Modelica models positions this thesis as a contribution to a long-term research that is being carried out on building energy efficiency. The proposed approach, applied to an illustrative thermal system, has been proved working.

As a result, a method and a procedure are now available to use optimization tools in conjunction with modeling and simulation environments. Therefore, the addressed method and procedure allow the resolution of the presented optimization problem, that, otherwise, could not have been solved in a reasonable computation time.

Sommario

In questa tesi si considera il problema di controllare in modo ottimale ed efficiente, da un punto di vista energetico, un edificio dotato di accumuli di energia. In questo studio ci si focalizza strettamente su sistemi termici, dal momento che sono tra i più rilevanti e influenti, e il problema di controllo considerato viene formulato in modo tale da essere compatibile con un approccio di tipo predittivo.

Si introduce una generica formalizzazione del problema in analisi e se ne quantifica a grandi linee la complessità, in base alle possibili applicazioni a cui questo si presta. In particolare, si forniscono tutte le notazioni matematiche necessarie alla sua formulazione e utili per introdurre le possibili operazioni che il sistema può compiere.

Il nostro intento principale consiste nel provare ad alleggerire il pesante carico computazionale richiesto dalla risoluzione del problema di ottimizzazione considerato che, in base al numero di vincoli a cui è sottoposto, può rivelarsi molto complesso. In quest'ottica, si presenta un approccio da noi denominato "Sporadic Model Predictive Control". Grazie a questa tecnica di controllo, il processo di ottimizzazione non deve più essere risolto a ogni passo di campionamento dell'orizzonte di predizione, ma solo quando considerato necessario. Lo schema di controllo proposto viene quindi analizzato in modo da delinearne le sue principali caratteristiche e la sua messa a punto.

Per validare le capacità dell'approccio proposto per la risoluzione del problema in analisi, si definisce un esempio applicativo e lo si risolve in modo da mostrarne il funzionamento. A questo fine viene messo a punto un sistema reale di ottimizzazione numerica e, nello specifico, l'applicazione viene portata a termine usando il linguaggio di modellazione Modelica e utilizzando GenOpt come strumento di ottimizzazione. In questo modo si dimostra come l'approccio in esame può efficace-

mente far entrare in gioco modelli di tipo ingegneristico anche per scopi di gestione del controllo, unendo quindi e ottimizzando le fasi di design e di funzionamento di un progetto. Inoltre, si sottolinea che l'utilizzo di modelli di tipo Modelica rende questa tesi un contributo a una ricerca a lungo termine finalizzata al conseguimento di efficienza energetica negli edifici. L'approccio proposto, applicato a un sistema termico esemplificativo, viene quindi validato per quanto riguarda il suo effettivo funzionamento.

In conclusione, sono ora disponibili un metodo e una procedura che permettono di usare strumenti di ottimizzazione calati nello scenario di ambienti di modellazione e simulazione. Pertanto, il metodo e la procedura considerati consentono la risoluzione del problema di ottimizzazione presentato, il quale, altrimenti, non sarebbe stato risolvibile in un tempo computazionale ragionevole.

Chapter 1

Introduction

The aim of this thesis consists in addressing a specific but very relevant problem in the management of energy systems in buildings. In particular, we would like to focus our attention on how to control in an optimal and computationally efficient manner a system composed by buildings, with specific (but not exclusive) reference to the case with energy storages.

Since the optimization problem could be very complex, also due to the presence of constraints, in this work we specifically focus on proposing a way to lighten the consequent computational effort. To this end, we first analize a typical optimization problem concerning a thermal system equipped with energy storages, showing how much such a thermal problem could be complex. Then we propose a control strategy to be applied at our system and we formally analize it.

Now we present the structure of our work, focusing on the content of each chapter.

- In Chapter 2 we introduce and discuss some background material concerning the optimal management of energy storage systems in buildings and the importance of an efficient energy use. From the proposed literature review, three dominant research trends arise out. The first one concerns the maybe most used control strategy for building systems (Model Predictive Control), the second one is related to studies on demand side control and on techniques like peak shaving, load shedding and load shifting, while the last one is referred to an economic issue concerning dynamic energy tariffs and cost function definitions for optimization problems. After this literary discussion, in order to explain the purpose of this thesis, we locate our work inside the context of the optimal control of energy systems.

- In Chapter 3 the problem of optimally controlling a building equipped with energy storages is considered. A general form to state the addressed problem is introduced, and its complexity (coarsely) quantified with respect to the intended applications. In particular, a formal statement of the problem under analysis is presented, and the required mathematical notation is established. Since we consider a heating problem, it is based on a thermal model, formalized by a system of dynamic equations. Afterwards, we provide the model with convenient decision variables, in order to define some operations on the system. The considered operations are related to storages and loads management. They concern storage charging and discharging, load partialization and shifting. Finally, in order to prove that the problem, even if formulated in a general way, can be fitted in a solution framework, we propose the problem formulation using the KKT-conditions method.

- In Chapter 4 an approach, named "Sporadic Model Predictive Control", is presented. This control technique is aimed at solving the complex optimization problem considered, in such a way to reduce the computation time required by the optimization process. Thanks to this approach, the optimization process has not to be carried out at each sampling time, but only when considered necessary. In order to decide when a new optimization is required, some guidelines concerning the optimization retrigger criterion are presented. The proposed control scheme is analysed in a view to outline its tuning. The approach is later put to work by using a well assessed Model Predictive Control strategy as the Optimization technique of the control scheme proposed. The strategy used at this point was chosen simple enough to allow for the required formal analysis, which would not be possible - and even if possible, not so informative - with a real numeric optimiser in place.

- In Chapter 5 a proof-of-concept case is addressed and solved, showing the capabilities of the approach. A real numerical optimization system is here in place and, specifically, the application is carried on using the GenOpt optimization tool. The proposed approach is applied to an illustrative thermal system, in order to demonstrate its operation. Hence, we show that the addressed method and procedure allow the resolution of the presented optimization problem, that, otherwise, could not have been solved in a reasonable computation time.

- Chapter 6 concludes the thesis by drawing some conclusions and sketching out future research.

Chapter 2

Background and motivations

The initial aim of this chapter is to provide a sort of literature review about the management of energy storage systems in buildings.
Afterward, considering the existing research trends, we would like to fill some gaps related to aspects that have not been examined up to now. In particular, we want to focus on the optimal control of a building equipped with energy storages.

2.1 Preliminaries

Efficient energy use, sometimes simply called energy efficiency, is the goal to reduce the amount of energy required to provide products and services. There are many motivations to improve energy efficiency, since reducing energy use reduces energy costs. This fact may result in a financial cost saving to consumers if the energy savings offset any additional costs of implementing an energy efficient technology. Improving energy efficiency is a goal desired in a lot of sectors, like in the appliances sector, in the building design, in industry and in the automotive sector. In particular, energy efficiency has proved to be a cost-effective strategy for building economies without necessarily increasing energy consumption. As a result, the optimal control of a building for heating is relatively straightforward.

The need for control in buildings usually resides in the mechanical and electrical systems that are installed to maintain a comfortable and safe indoor environment. A wide range of these systems can be found in buildings including heating, ventilating, air-conditioning (HVAC), lighting, security, elevators, escalators, fire detection and abatement. All these systems use energy and, in the case of HVAC, energy is used to maintain temperature, humidity, and air quality at levels in accordance with the building purpose. For this reason the building sector is the largest energy consumer in the world. Therefore, it is economically, socially, and environmentally significant to reduce the energy consumption of buildings. Achieving substantial energy reduction in buildings may require rethinking the whole processes of design, construction, and operation of a building.

In this work we will focus on the specific issue of the thermal problem aimed at obtaining energy efficiency in buildings. Energy use and utility cost can be reduced significantly by distributing thermal energy more efficiently and by more closely meeting the needs of building occupants.

This thermal problem is very complex due to the overcoming of stricter requirements. For example, the use of diversified energy sources involves systems increasingly interacting. In particular, we focus on the rising role of the thermal energy storages. Their optimal control results in a valuable energy efficiency achievement. To this reason, in the following section, we are going to discuss the importance of thermal energy storages.

2.2 The importance of energy storage

We start introducing the reader to the importance of energy storage and its control, as strongly assessed in literature.
The development and implementation of different types of storage technologies, each one used in a specific application field, has led to the emergence of storage as a crucial element in the management of energy from renewable sources, allowing energy to be released during peak hours when it is more valuable.

Thermal energy storage (TES) is considered one of the most important advanced energy technologies and increasing attention has recently been paid to the utilization of this essential technique for thermal applications, ranging from heating to cooling, particularly in buildings. Several studies have revealed that TES systems can be practically employeed in a wide range of industrial applications. In this regard, they have a considerable high potential for more effective use of thermal energy equipment and for facilitating large-scale energy substitutions from the economic point of view. TES appears to be the only solution to correct the mismatch between the supply and demand of energy, significantly contributing to meet society's needs for more efficient and environmentally benign energy use.

We can surely state that TES is a key component of any successful thermal system in buildings and a good TES should allow minimum thermal energy losses, leading to energy savings, while permitting the highest possible extraction efficiency of the stored thermal energy. Although TES is used in a huge variety of applications, the benefits achieved by the systems fulfill the same purposes, as increasing generation capacity (energy demand is seldom constant over time and the excess generation available during low-demand periods can be used to charge a TES in order to increase the effective generation capacity during high-demand periods), shifting energy purchases to low cost periods (this is the demand-side application of the previous purpose and allows energy consumers subject to time-of-day pricing to shift energy purchases from high to low cost periods) and increasing system reliability. It is important to highlight that the selection of a TES system depends on a lot of critical factors as the storage period required, economic viability and operating conditions, which in turn are influenced by several parameters.

Since substantial energy savings can be realized by TES, nowadays the development of these systems is considered as an advanced energy technology; their use has been attracting increasing interest in several thermal applications (active and passive solar heating, water heating, cooling and air-conditioning) and they are presently identified as the most economic storage technology for building heating, cooling, and air-conditioning applications.

2.3 Background and literature review

In this section the state of the art in the context of energy optimization in building systems is presented. Our research domain has been very flourishing and this shows that there is a lot of interest in this issue. For example, at the time of this writing, the scholar search with this key words "energy storage + building" produces 105.000 hits, with "energy storage + building + optimization" produces 30.800 hits, with "energy system + control + building" produces 38.200 hits.

For the purposes of our work, three dominant research trends arise out of our literature review. The first one is related to the Model Predictive Control (MPC), the most used control strategy for building systems and storage management. The second one concerns studies on demand side control and correlated techniques like peak shaving, load shedding and load shifting. The last trend is referred to an economic issue with respect to dynamic energy tariffs and cost function definitions for optimization problems.

In the following paragraphs we analyze the presented trends on the basis of papers coming from literature.

2.3.1 Model Predictive Control

The Model Predictive Control is one of the most recent control strategy applied to thermal systems in buildings. We underline that in the past different control approaches were used. The control strategy was totally decentralized or, at most, there was an integration only between local systems. As already stated, in our work we analize the use of the MPC approach in buildings.

According to any generic process control problem, the objective of MPC is to satisfy the output requirements in the most efficient way, hence with the least amount of input (i.e. energy). Because energy is a cost, control in buildings, as in most other applications, can be translated to an economic optimization problem: the problem can be stated as the minimization of the integral of the energy usage subject to constraints on the measured variables. To this end, the vast majority of the considered papers focuses on the model predictive control method of thermal energy storage in building systems. The main idea of predictive control is to use the model of a plant to predict the future evolution of the system. At each sampling time, starting at the current state, an open-loop optimal control problem is solved over a finite horizon. The optimal command signal is applied to the process only during the following sampling interval. At the next time step a new optimal control problem, based on new measurements of the state, is solved over a shifted horizon. Applications of MPC have become increasingly prevalent due to their ability to handle the multivariable/nonlinear nature of the dynamics, constraints and optimality in an integrated fashion.

In paper [14], to minimize energy consumption while satisfying the unknown but bounded cooling demand of a campus building and operational constraints, a Model Predictive Control (MPC) for the chillers operation is designed in order to optimally store the thermal energy in a tank by using predictive knowledge of building loads and weather conditions. The goal of finding the optimal control sequence that satisfies the required cooling load and minimizes electricity usage is achieved by solving the following optimization problem: the optimization cost function J is given by the minimum of the energy consumption price over the considered prediction horizon (24hours with a control sampling time of 1 hour). The minimization problem is solved with respect to the control variables of the model and the simulation results shown are very promising: the daily electricity bill can be significantly reduced of 24.5%, compared to the current heuristic manual control sequence.

Also the simulation results in paper [17] show how MPC control method is a way to reduce energy costs in buildings. The economic objective function designed is intented to minimize the total electricity expense, seen as a combination of energy and demand costs; this function is subject to the dynamic model constraints and the temperature comfort constraints, in order to achieve the comfort level of the building. This optimization procedure is repeated and a new program is solved in subsequent time steps, when new measurement data are available.

In paper [16], in order to compare the different tariff schemes and investigate the need for an additional storage device, the optimal building response is computed by applying MPC. As in the previous works, the optimal control input to the building is computed by solving an MPC problem that minimizes the cost of electricity consumption.

The approach utilized in [4] is to apply dynamic optimization techniques to computer simulations of buildings and their associated cooling systems for a range of conditions in order to determine the maximum possible savings. In this paper two different optimization problems are proposed. The first one is applied to the system model and it is used to determine the minimum operating costs, assuming that future ambient conditions and internal gain inputs are known. The true optimal performance results provided a basis for identifying the potential savings as compared with conventional control strategies. The optimal control of the considered cooling system, that takes advantage of the thermal capacitance of the building, involves minimizing an integral of operating costs over a day while satisfying required constraints; the optimal solution is a trajectory of controls throughout the specified optimization period. The other optimization problem proposed tries to minimize the peak electrical demand over a day and so the minimized cost function is the maximum total building electrical use for the day. Results of this study show that, using optimal control, both energy costs and peak electrical use can be significantly reduced through proper control of the intrinsic thermal storage within building structures.

In paper [15] is highlighted as MPC is a control methodology that can naturally and systematically be used to improve building thermal comfort, decrease peak demand and reduce total energy costs. A simple example of a single thermal mass model is considered and a MPC problem is formulated with the objective of minimizing total heating and cooling energy consumption, minimizing the peak power consumption and maintaining the building zones within a desired temperature range despite predicted load changes. In this paper the Model Predictive controller obtained is compared to a proportional controller designed to reject the load without predictive information and it inputs zero power when the space temperature is within the comfort range, otherwise it uses a proportional control law.

Comparing model predictive and proportional control, it emerges that the two controllers use the same energy for the same amount of constraint violation and increased comfort violation corresponds to a lower energy use for both controllers. In closed-loop simulation it comes to light that the peak power consumption is reduced by 89% relative to the proportional controller when the MPC is used. This behavior is obtained by taking advantage of the predictive knowledge of the disturbance and using the space thermal storage.

Also paper [9] strongly deals with storage management using MPC control and, in particular, taking into account the use of active and passive energy storages. An active thermal storage is a system that requires an additional fluid loop to charge and discharge the storage tank or to deliver cooling to the existing chilled water loop. Instead, a passive thermal storage, like a building thermal capacitance, requires no additional heat exchange fluid in addition to the conditioned air stream. The main idea of this work is to evaluate the merits of combined optimal control of both passive building thermal capacitance and active thermal energy storage systems to minimize an objective function of choice including total energy consumption, energy cost, occupant discomfort or a combination of these. In particular, a consecutive time block optimization (CTBO) is employed and it means that the predictive optimal controller carries out an optimization over a predefined planning horizon and the complete generated optimal strategy is executed. Then an other proposed option is to use a closed-loop optimization (CLO), in which the predictive optimal controller carries out an optimization over a predefined planning horizon and only the first action of the generated optimal strategy is executed; at the next time step the process is repeated. In the case of perfect forecasts, both CLO and CTBO will produce identical results, while, when the future is subject to uncertainty, CLO should exhibit superior performance. In paper [9] it is assumed a perfect prediction and so CTBO is used. It is highlighted that there is a casual relationship from the passive to the active storage, which requires to solve firstly the passive storage and then the optimization of the active thermal storage inventory on the basis of the previous outcomes. The final results show that the combined use of active and passive thermal storages under optimal control allows significant operating cost savings (18%) and electrical demand reduction. The obtained results are much better than the promising savings potentials when building operation has been optimized in buildings without storage.

2.3.2 Demand Side Control and correlated techniques

Demand side management is the modification of consumer demand for energy through various methods such as financial incentives. The goal of demand side management is to encourage the consumer to use less energy during peak hours or to move the time of energy use to off-peak times. The related technique, called load shifting, consists of shaping the energy profile delivered to a building, exploiting the possibility of storing energy for later use. Peak demand management does not necessarily decrease total energy consumption, but could be expected to reduce the need for investments in networks or power plants for meeting peak demands. An example, on which we will focus in our thesis, is the use of energy storage units to store energy during off-peak hours and discharge them during peak hours.

In this respect, the authors of paper [17] focus on the fact that in the United States about 70% of electricity is consumed in commercial and residential buildings and, to make it worse, the peak demands of building cooling or heating usually occur around the same time period during the day. This situation make the electricity consumption at the peak time, known as demand, extremely high relative to the average consumption level; the high peak demand dictates that the power generation capacity has to be at least equal to the peak demand, or a blackout would occur. If the peak demand can be reduced by properly making use of storage capacity or managing the consumption pattern to be more friendly to the power generation, the efficiency of existing power plants is improved. Therefore, there is great interest to reduce the peak demand by shifting part of the peak load away from the peak time, implementing storage systems, such as using the building thermal capacity, while always keeping thermal comfort as the ultimate goal. In conclusion, it is stated that a desirable demand response control strategy should accomplish the following objectives simultaneously: optimize the trade-off between the energy consumption and demand cost by taking advantage of the time of use price difference, make use of the building thermal storage to store and release cooling dynamically and handle real-time and predicted changes of load disturbances, weather and price changes.

Also in the investigation of paper [16], shifting thermal loads like heating and cooling in buildings is used in building control for shaping demand profiles, using additional information in the form of dynamic electricity tariffs, if possible. As first approach, the thermal capacity of a building alone is used for shifting electricity demand; then an electric storage, in addition to the building's thermal capacity, is considered for the same purpose. As result, it is assessed that increasing thermal or electric storage capacities markedly improves the building controller's demand shifting capabilities and hence adds to the reduction of electricity demand during peak load hours.

In paper [4] shifting cooling loads from daytime to nighttime is seen as an oppurtunity to reduce peak electrical demands, take advantage of low nighttime electrical rates, offset mechanical cooling with free cooling at night and enhance equipment operation at more favorable part-load conditions. It is shown as the use of a building's thermal storage for load shifting can significantly reduce operational costs, even though the total zone loads may increase.

Also in paper [15] load shifting, as well active storage mechanism, is seen as a technique that allows strong performance improvement, using forecasted information, if available.

2.3.3 Economic issues

This trend of research takes into account a fundamental aspect related to energy, that is its integration with economic issues related to energy costs and tariffs.

As highlighted in paper [14], the development of highly efficient heating and cooling systems is necessary to reduce the building energy consumption and this goal is really important from both an environmental and an economical point of view. The enhanced efficiency for a wide range of innovative heating and cooling systems depends on the active storage of thermal energy, the theme we will later analyze in our work. As further proof of the fact that the economic issue should be strongly considered in all optimization problems concerning energy, the cost function J used in paper [14] is given by the minimum of the energy consumption price over the considered prediction horizon.

Also in paper [17] it is stated that, since energy is a cost, control in buildings can be translated to an economic optimization problem which results in the minimization of the integral of the energy cost subject to constraints on the measured variables. It is highlighted how, employing control logic at a high enough level, even an economic signal, like energy price, is measurable and this is an important improvement over the current state of the art: the proposed control strategies incorporate economic optimization as well as setpoint regulation. It is stated that a desirable demand response control strategy should necessarily accomplish the objective of optimizing the trade-off between the energy consumption and demand cost by taking advantage of the time of use price difference. To this end, as already mentioned before, the economic objective function designed in paper [17] is intented to minimize the total electricity expense, seen as a combination of energy and demand costs; this function is subject to the dynamic model constraint and the temperature comfort constraint, in order to achieve the comfort level of the building.

In paper [16] the results from a proof-of-concept study combining modern building automation systems (BAS) with dynamic electricity tariffs are presented, proposing the use of a BAS that optimizes, in a fully automated fashion, the electricity demand of a retail end-consumer, while managing a local battery unit and respecting all comfort constraints on room temperature, illuminance and indoor air quality. In this investigation it is clearly shown how additional information in the form of dynamic electricity tariffs, a strong economic incentive, can be used in building control for shaping demand profiles of retail end-consumer groups, which consists in shifting thermal loads like heating and cooling in buildings. The optimal building response is computed by applying Model Predictive Control, in order not only to investigate the need for an additional storage device, but also to compare the different tariff schemes considered; of course, the MPC problem minimizes a function based on the cost of electricity consumption. In conclusion, the authors highlight as the presented study shows that building automation systems can effectively be combined with dynamic electricity tariffs for reducing peak electricity demands; the result is an electric load demand profile that behaves price-responsive within the given operation framework, which is set by constraints, like room temperature bounds and others.

2.4 Conclusion and Motivations

Some common aspects clearly come to light from the few samples presented above from a vast literature. We notice that the focus on the control strategies has moved toward demand side control, causing a decentralization of the building control system. In the given scenario the energy storage is mainly seen as a subsystem provided with its own control, it means there is a hierarchical intregration with the Building Management System (BMS). The strong predominance of studies concerning MPC problems has been carried out at a building control level (BMS), while there is not much emphasis on the lower levels of the control system. In particular, we notice a somehow reduced investigation effort concerning model predictive optimization aimed at the local control of storage components in buildings. As final consideration, we highlight there is not a particular attention to possible revolutionary technological changes in terms of materials and components.

Given the presented scenario, we can now introduce the aspects in which we want to concentrate our efforts and studies. The aim of this work consists in trying to cover some visible lacks of the actual researches in the application of energy storage systems in buildings. According to the literature review and the relative gaps which could need to be filled, we would like to focus our attention on a possible strategy to optimally structure and control an energy storage in a building in a coordinated manner with respect to the overall management problem.

The importance of this study is due to the fact that a wrong sizing of the storage or its unsuitable control could lead to serious energy problems as substantial thermal losses, peak demands that can either not be fulfilled but with the necessity of accessing "precious" energy sources, and consequently high costs.

Since the optimization problem could be very complex, and is surely made more complex by storage, in this work we specifically focus on proposing a way to lighten the computational effort of this problem. To this end we shall proceed as follows. First we analize a typical optimization problem concerning a thermal system equipped with energy storages. This discussion is mainly aimed at showing how much such a thermal problem could be very complex. Then we propose a control strategy to be applied at our system and we formally analize it in the assumption of a Linear Time Invariant system. In this way an MPC strategy can be applied.

Chapter 3

Problem statement and first analysis

In this chapter, a general and formal statement of the problem under analysis is presented, and the required mathematical notation is established.

The problem is formulated in order to be naturally compatible with a predictive control approach. Focus is here restricted to thermal systems because they are the most relevant and influential systems in a building. In comparison with electric loads, the thermal ones are more significant: by optimizing the trend of the building thermal components, the most of the optimization problem concerning the energy system is solved. Furthermore, during their service life, thermal energy storages are less subject to intrinsic variability than electric ones. Due to the previous motivations, and in order to keep the study complexity at a level compatible with the scope of the thesis, hereafter we only deal with thermal energy systems.

3.1 System

We consider a system characterised by one or more buildings and composed by a set of users, loads, energy storages, and external energy sources. To give the reader an idea about the possible kinds of sources related to our problem, we mention gas energy, electric energy, solar energy, and thermal energy transferred by a fluid. In this work energy storages and their control play a key role, since they enhance the system by providing the possibility of operations that, once defined and properly managed, enhance the flexibility of the loads management. The possibilities offered by a thermal storage will be discussed in the following sections, to highlight what a storage makes possible and what is not achievable without its presence.

In order to simplify the notation, we introduce some hypothesis, that can be easily relaxed, without jeopardising the generality of our problem.
These are our hypothesis:

- we consider a heating problem, so we are interested in thermal energy. Dealing with a heating problem, all powers involved have positive values; we highlight as the same problem could be easily transformed in a cooling one just by changing the powers sign;

- we consider one storage for each user;

- each user has only one HVAC (Heating Ventilating Air Conditioning) component. In particular, we consider a heater.

3.1.1 Thermal Model

This problem is based on an energetic model. Since we consider a heating problem, the model is a thermal one and can be represented by the scheme in Figure 3.1. This scheme shows the possible power flows between the three detected subsystems and coming from the external enviroment. Due to the fact that we deal with a thermal model, each subsystem is characterized by its temperature and each quantity has a thermal connotation.

T_a: ambient temperature
T_h: HVAC component temperature
T_s: storage temperature
P_d: disturbance power (e.g. loads, room occupancy)
P_e: external power (e.g. solar radiation)
T_e: external temperature
P_{ha}: power from the HVAC component to the ambient
$P_{l_{as}}$: loaded power from the ambient to the storage
$P_{l_{hs}}$: loaded power from the HVAC component to the storage
$P_{u_{sa}}$: unloaded power from the storage to the ambient
$P_{u_{sh}}$: unloaded power from the storage to the HVAC component
P_s: power supplied to the storage from external energy sources
P_h: power supplied to the HVAC component from external energy sources

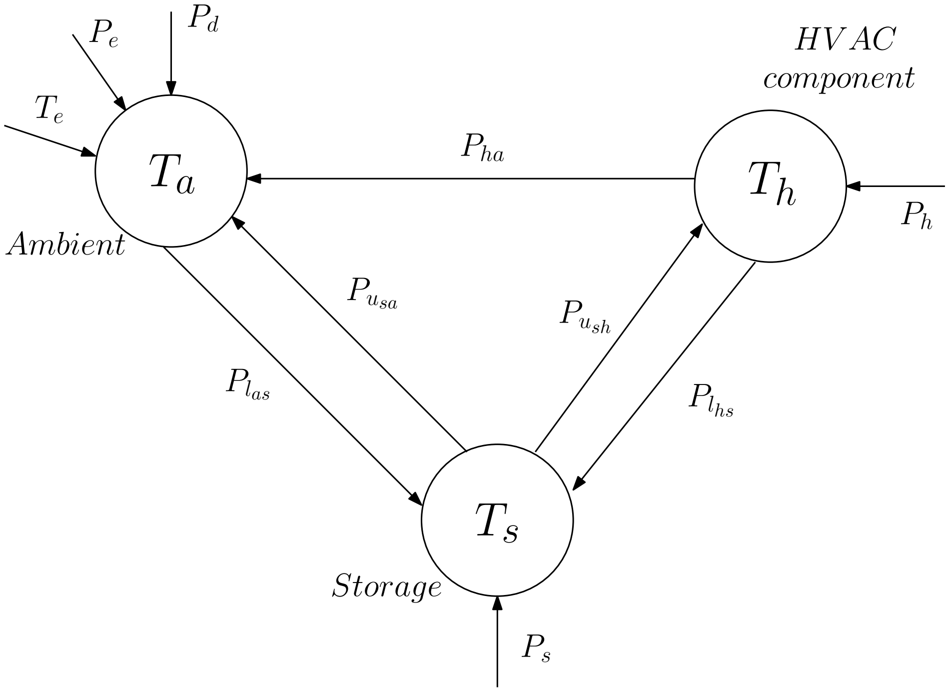


Figure 3.1: Thermal Model.

The state vector of the system can be represented as follows:

$$x = \begin{bmatrix} T \\ T_s \end{bmatrix} = \begin{bmatrix} T_a \\ T_h \\ T_s \end{bmatrix}, T = \begin{bmatrix} T_a \\ T_h \end{bmatrix}$$

The thermal model to be used for the optimization problem can be formalized by the following system of dynamic equations. Each equation clearly shows the relationships between the system states and the other variables.

$$\begin{cases} \dot{T}_a = f_a(T_a, P_{ha}, P_e, P_d, T_e, P_{l_{as}}, P_{u_{sa}}) \\ \dot{T}_h = f_h(T_h, P_h, P_{ha}, P_{l_{hs}}, P_{u_{sh}}) \\ \dot{T}_s = f_s(T_s, P_s, P_{l_{as}}, P_{u_{sa}}, P_{l_{hs}}, P_{u_{sh}}) \end{cases}$$

where the detailed aspect of functions f_a, f_h and f_s depends basically on the chosen heat transfer correlations, which are highly inessential for the purpose of this chapter.

3.1.2 Storage State of Charge

A generic model for the energy storage is given as the following equation:

$$C_s(i)SOC(i,k+1) = \alpha C_s(i)SOC(i,k) + \eta_l P_l(i,j,k) - \eta_u P_u(i,k) \tag{3.1}$$

where:

$SOC(i,k) \in [0,1]$: State of Charge of the storage used by the i-th user at the k-th stage
$C_s(i)$: the i-th user's storage capacity
$P_l(i,j,k)$: loaded power to the storage, equal to the sum of $P_{l_{as}}$ and $P_{l_{hs}}$
$P_u(i,k)$: unloaded power from the storage, equal to the sum of $P_{u_{sa}}$ and $P_{u_{sh}}$
α: coefficient referred to the storage internal losses
η_l: storage charging efficiency
η_u: storage discharging efficiency

The storage model allows to highlight the State of Charge, a variable that plays a fundamental role in all the decisions concerning the storage use and control. The coefficients appearing in this equation make the model more detailed. However, since they increase the complexity of the problem, for the purpose of the study here presented, they can be set to a fixed value without modifing the problem structure. For example, we could consider an ideal case where $\alpha = 0$ and $\eta_l = \eta_u = 1$.

3.2 Problem Structure

This section can be considered the heart of the chapter: we present the problem statement to the reader. The formulation of this problem naturally induces to a predictive control approach over a finite prediction horizon.

3.2.1 Cost Function

The cost function proposed in our problem, in a view to maintain a sufficient level of generality, is an economic objective function. It mainly depends on the trend of energy price and on the power consumption over the considered prediction horizon.

$$J = \sum_{h=1}^{N_L} \sum_{i=1}^{N_U} \sum_{j=1}^{N_S} \sum_{k=1}^{N} c(i,j,k) P(h,i,j,k) \tag{3.2}$$

where:

N_L: number of loads
N_U: number of users
N_S: number of energy sources
N: prediction horizon
$c(i,j,k)$: energy price for the i-th user of the j-th source at the k-th stage
$P(h,i,j,k)$: power consumption of the h-th load of the i-th user from j-th source at the k-th stage. It is equal to the sum of P_s and P_h

If we assume there are not different energy contract for each user, the previous cost function formulation is simplified considering the same energy price for all the users: $c(j,k)$.

Furthermore, if we want to focus our attention on the peak electrical demand over the entire prediction horizon, a possible extension of the presented cost function can be proposed as:

$$J = \sum_{h=1}^{N_L} \sum_{i=1}^{N_U} \sum_{j=1}^{N_S} \sum_{k=1}^{N} c(i,j,k,P(h,i,j,k)) P(h,i,j,k) \tag{3.3}$$

In this case the energy price directly depends on the amount of power consumption. Many other variations could be thought of, apparently, but this would stray from the scope of this work.

3.2.2 Control Inputs

The control inputs are the decision variables for the optimization problem. The optimal trend of these unconstrained variables, coming from the optimization, will be later used by the control action.

- $P_l(i,j,k)$
- $P_u(i,k)$
- P_{ha}

3.2.3 Problem Statement

The most general optimization problem that we consider in this thesis can be stated as follows:

$$\min_{P_l, P_u, P_{ha}} J \quad s.t. \tag{3.4}$$
$$T_{a_{min}}(i,k) \leq T_a(i,k) \leq T_{a_{max}}(i,k)$$
$$P_{min}(i,j,k) \leq \sum_{h=1}^{N_L} P(h,i,j,k) \leq P_{max}(i,j,k)$$
$$T_{h_{min}}(i,k) \leq T_h(i,k) \leq T_{h_{max}}(i,k)$$

The first constraint is due to comfort requirements, as it keeps the ambient temperature within a certain accetable range, according to the limits established by standards like the ANSI/ASHRAE Standard 55 [1]. Similar constraints could be introduced e.g. for humidity or some of the numerous comfort indices in the literature, but the matter would be conceptually the same. The other two constraints are related to physical issues: one limits the power required to each source, the other keeps the heater temperature within a feasible range as per its design and admissible operating conditions.

In this work a single prediction horizon is considered, however the presented optimization problem could be solved over several prediction horizons, in order to identify the best achievable result. Over the selected prediction horizon, time granularity is an essential ingredient to be carefully defined. During each step, all variables are supposed to be constant and the step choice strongly influences the possible operations realizable on the system.

3.3 Operations

In this section further decision variables will be added to our problem, in order to define some operations on the system. The introduction of these additional variables let the corresponding operations be part of the problem, without altering its structure. Later on, these decision variables will let us formulate further constraints related to these operations and connected to the way the control can act on the system. In other words, the considered operations can be seen as negotiable contraints since can be viewed as prescribing of forbidding the system to do something (at some time). It will be shown how the introduction of these decision variables is useful to present the problem to the optimizer, since they stand for possible actions to be applied to the system.

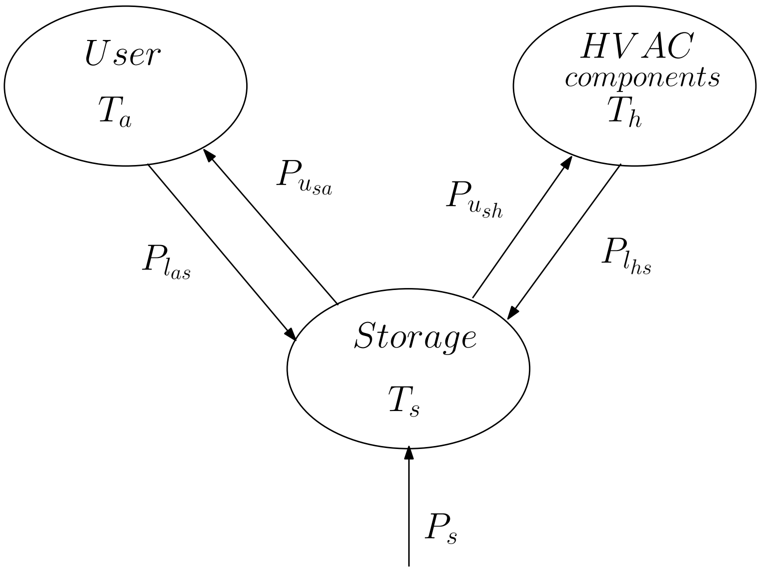


Figure 3.2: Storage Charging and Discharging.

3.3.1 Storage Charging and Discharging

The most important operations related to the storage use are the charging and the discharging ones. In Figure 3.2 the charging and discharging modes are clearly outlined.

We formalize these operations by listing the following conditions related to the charging and discharging modes:

$$SOC_{min}(i,k) \leq SOC(i,k) \leq SOC_{max}(i,k)$$

The values of SOC_{min} and SOC_{max} can be modified according to how much we want to let the storage charge or discharge; for example, we could set $SOC_{min} = 0$ and $SOC_{max} = 1$ to allow a complete storage discharging and charging.

$$SOC(i,k) = SOC_{min}(i,k) \Rightarrow P_u(i,k) = 0$$

$$SOC(i,k) = SOC_{max}(i,k) \Rightarrow P_l(i,j,k) = 0$$

These conditions state that the system can not unload the storage if its SOC is at the minimum value and, on the contrary, the storage can not be further loaded if it is already charged at its maximum level.

$$P_u(i,k)P_l(i,j,k) = 0$$

This constraint avoids the simultaneous charging and discharging of the storage.

$$P_{l_{as}}(i,k)(T_a(i,k) - T_s(i,k)) \geq 0$$

$$P_{l_{hs}}(i,k)(T_h(i,k) - T_s(i,k)) \geq 0$$

$$P_{u_{sa}}(i,k)(T_s(i,k) - T_a(i,k)) \geq 0$$

$$P_{u_{sh}}(i,k)(T_s(i,k) - T_h(i,k)) \geq 0$$

In these last four conditions, since we consider a thermal problem, the energy state of the storage is represented by a difference of temperatures. These constraints determine the feasible thermal power flows during the charging and discharging operations.

3.3.2 Load Partialization and Load Shifting

In this problem load demands could be distinguished between those that must be fulfilled at a fixed time and those that, on the contrary, could be shifted in time and positioned in the most suitable way for the purpose of optmization.
To this end, we assume all load demands known at the beginning of the optimization period, except for the time scheduling of the shiftable ones. Demands appearing during the prediction horizon are considered unforeseen disturbance for the current optimization horizon, and taken into account in the subsequent one.
As shown in Figure 3.3, there are some loads, not involved in the decision block, that directly affect the thermal model; while, other disturbance powers are related to loads that can be shifted in time or partialised through the use of convenient decision variables. The decision block is used to optimize a cost function that just relies on the disturbance powers of the fixed loads.

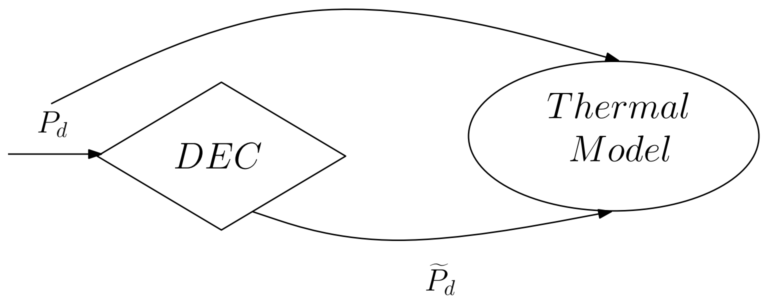


Figure 3.3: Load Partialization and Load Shifting.

Load Partialization

We introduce $\gamma(k) \in [0, 1]$ as the decision variable related to the parzialization operation: to each negotiable load corresponds one of these variables. If $\gamma(k) < 1$ the considered disturbance power has been partialized by the optimizer in the prediction horizon. We assume that the residual power demanded by a partialized load will not be supplied in the corresponding optimization horizon, unless a further demand occurs. The following equation clearly catches the load partialization operation just discussed.

$$\widetilde{P}_d(h, i, j, k) = \gamma(k) P_d(h, i, j, k)$$

Load Shifting

Dealing with the shifting operation, the total power demanded by the negotiable load, even if shifted in time, must be guaranteed. To this end, the following equation must be satisfied:

$$\sum_{k=1}^{N} \widetilde{P}_d(h, i, j, k) = \sum_{k=1}^{N} P_d(h, i, j, k)$$

We consider q as the decision variable that allows the optimizer to shift in time the loads demanded by a disturbance power. Each negotiable load is associated to a q variable that can assume values in the range $[0, N - D]$, where D is the shifted load duration. We assume that any shiftable load can not be fractioned and so it must be shifted in time in its entirety. The operating principle of this operation can be clearly shown by the following equation:

$$\widetilde{P}_d(h, i, j, k) = P_d(h, i, j, k - q)$$

3.4 KKT-conditions

Given the general optimization problem above and considering the various methodologies for its solution, we propose the problem formulation using the KKT-conditions method. In this way it will be shown as a standard methodology, like the KKT one, can be easily applied to our problem. It is so proved to the reader that the problem, even if formulated in a general way as stated before, can be fitted in a solution framework. Also, expressing the problem in one of the possible forms that are suitable for its symbolic (if possible) or numerical solution, allows to immediately envisage how complex and computationally heavy that solution can be, paving the way to the approach proposed in Chapter 4 and put to work in the subsequent one.

According to the first step of KKT-method, the real function to be minimised is written in the form:

$$f = \sum_{h=1}^{N_L} \sum_{i=1}^{N_U} \sum_{j=1}^{N_S} \sum_{k=1}^{N} c(i,j,k)P(h,i,j,k) \tag{3.5}$$

Hereafter we state the problem constraints, following the KKT-condition procedure. These equality and inequality constraints can be classified in three main groups, according to their origin and role in the optimization problem. The first one is made up of physical constraints. The second one is related to the solution feasibility: these constraints limit the set of admissible solutions to those that do not violate them. The last group concerns control constraints principally related to comfort issues.
Rearranging for convenience by constraint type instead of origin, we get:

- Equality constraints :
  $g_1 : P_u(i,k)P_l(i,j,k) = 0$
  $g_2 : \widetilde{P}_d(h,i,j,k) - \gamma(k)P_d(h,i,j,k) = 0$
  $g_3 : \widetilde{P}_d(h,i,j,k) - P_d(h,i,j,k-q) = 0$
  $g_4 : C_s(i)SOC(i,k+1) - \alpha C_s(i)SOC(i,k) - \eta_l P_l(i,j,k) + \eta_u P_u(i,k) = 0$
- Inequality constraints :
  $h_1 : P_{l_{as}}(i,k) \geq 0$
  $h_2 : P_{l_{as,max}}(i,k) - P_{l_{as}}(i,k) \geq 0$
  $h_3 : P_{l_{hs}}(i,k) \geq 0$
  $h_4 : P_{l_{hs,max}}(i,k) - P_{l_{hs}}(i,k) \geq 0$
  $h_5 : P(h,i,j,k) \geq 0$
  $h_6 : P_{u_{sa}}(i,k) \geq 0$
  $h_7 : P_{u_{sa,max}}(i,k) - P_{u_{sa}}(i,k) \geq 0$
  $h_8 : P_{u_{sh}}(i,k) \geq 0$
  $h_9 : P_{u_{sh,max}}(i,k) - P_{u_{sh}}(i,k) \geq 0$

$h_{10} : T_a(i,k) - T_{a_{min}}(i,k) \geq 0$

$h_{11} : T_{a_{max}}(i,k) - T_a(i,k) \geq 0$

$h_{12} : \sum_{k=1}^{N_L} P(h,i,j,k) - P_{min}(i,j,k) \geq 0$

$h_{13} : P_{max}(i,j,k) - \sum_{k=1}^{N_L} P(h,i,j,k) \geq 0$

$h_{14} : T_h(i,k) - T_{h_{min}}(i,k) \geq 0$

$h_{15} : T_{h_{max}}(i,k) - T_h(i,k) \geq 0$

$h_{16} : SOC(i,k) - SOC_{min}(i,k) \geq 0$

$h_{17} : SOC_{max}(i,k) - SOC(i,k) \geq 0$

$h_{18} : P_{l_{as}}(i,k)(T_a(i,k) - T_s(i,k)) \geq 0$

$h_{19} : P_{l_{hs}}(i,k)(T_h(i,k) - T_s(i,k)) \geq 0$

$h_{20} : P_{u_{sa}}(i,k)(T_s(i,k) - T_a(i,k)) \geq 0$

$h_{21} : P_{u_{sh}}(i,k)(T_s(i,k) - T_h(i,k)) \geq 0$

According to the previous classification, we now briefly discuss the real meaning of the constraints listed above.
First of all, we can look at the equality contraints group. The first one g_1 is an admissibility constraint, that do not allow to load and unload at the same time the storage. The second and third ones g_2, g_3 are related to the partialization and shifting operations, that we have already discussed in section 3.3.2. The last one g_4 is a physical constraint involving the balance of the storage SOC.
Now we concentrate on the inequality constraints group. The constraints h_1, h_3, h_5, h_6, h_8 are physical contraints that set the corresponding powers to positive values. Also h_2, h_4, h_7, h_9 are physical constraints, but they limit the maximum feasible value for the corresponding powers. In the matter of control constraints we have h_{10}, h_{11}, that are related to comfort requirements of the ambient temperature. Subsequently there are physical constraints h_{12}, h_{13}, that limit power consumption in a fixed range. Also h_{14}, h_{15} are physical constraints that limit the heater temperature in a fixed range. In order to keep the state of charge of the storage in a range, we introduce h_{16}, h_{17}, that are considered as physical constraints. Finally, the physical constraints h_{18}, h_{19} and h_{20}, h_{21} are related to the storage charging and discharging operation, based on the second law of thermodynamics.

3.5 Conclusions

Based on the considerations reported so far, it can be quite straightforwardly concluded that the problem illustrated in this chapter is very complex, since it is stated - consistently with the addressed economic cost function - for a compound system and not only for a particular subsystem.
Moreover, we highlight the idea of introducing decision variables to represent operations that can be applied on the system. In particular, these variables are associated to a large number of constraints, in order to guide the optimization process. The optimal solution of the problem does not omit any kind of possible energy transfer between the components of the system.

Furthermore, expressing our problem through the KKT equations, it is clear how it can be formally stated and that it can be represented at a deep level of detail, appropriate to the solution we want to get. However, it is also easy to see that we are dealing with a very complex problem of dynamic optimization and, even by expressing it through a different approach from the KKT one, used here to highlight the mentioned complexity, its solution results very hard from a computational point of view.

Taking into account this important aspect, in the next chapter our effort will consist in trying to lighten the burden that will be required to solve this optimization problem. In this view, we will present a way to avoid the computation of the optimization problem under analysis at each sampling time of the prediction horizon. In this way the optimization will be done only if strictly necessary and, obviously, the computational effort required by the resolution of this complex problem, could be strongly reduced.

Chapter 4

Sporadic Model Predictive Control

The main idea of this chapter is to present a peculiar application of the Model Predictive Control technique, the most used control strategy for building systems and storage management, in order to solve a complex optimization problem, as the one analyzed in the previous chapter.

We name our strategy Sporadic Model Predictive Control, since our aim is that the optimization process should not be carried out at each sampling time, but only when this is considered necessary, in the sense we are going to expose. In this way, the computational time required by the optimization process could be strongly reduced. To this end, our proposal is to proceed through a batch MPC that is able to compute a vector of future control signals and outputs over a finite predictive horizon length through the resolution of a complex optimization problem. The vector of optimal command signals and outputs is applied to a feedback loop where a regulator is ready to act in the way that will be explained afterwards. Finally, we are going to discuss about the choice of the moment when a new optimization process is needed.

4.1 The proposed Approach

In this analysis we consider a control system able to compute a vector of future control signals and outputs over an assigned horizon length through the resolution of a complex optimization problem, that could clearly be the one presented in the previous chapter. We want to underline as this computation can be done not only by every Model Predictive Control system, but also by a generic optimal control system and even by a simple model reference controller, that does not have any kind of predictive connotation (although the interest of our approach clearly emerges in the presence of a computationally heavy optimization).

The important assumption to be considered, as shown in the previous chapter, is that this computation may indeed be really heavy, independently of the way we choose to do it.

The traditional Model Predictive Control consists in using the model of a plant to predict the future evolution of the system. At each sampling time, starting at the current state, an open-loop optimal control problem is solved over a finite horizon. The optimal command signal is applied to the process only during the following sampling interval. At the next time step a new optimal control problem, based on new measurements of the state, is solved over a shifted horizon.

Considering a system in nominal conditions and without unpredicted disturbances, our purpose is instead to apply the vector of optimal command signals not only for the following time step, but for the entire prediction horizon. In this way it is clear that we can strongly reduce the computation time required by the MPC optimization. In our study we therefore consider the scheme in Figure 4.1.

In this scheme an optimization technique is used to calculate the vectors u_{opt} and $\widehat{y}$, over a horizon of length N_{opt}. The former vector contains the optimal values for future controls, the latter holds the corresponding trajectory of the controlled variable. The element named H, since it can be viewed as a generalized holder, has the role of acquiring the N_{opt} future controls and predicted outputs and applying them in sequence, one at each control step, to the feedback loop below. The controller R_d of this loop receives the forecast (optimal) controlled variable as set point, and the optimal control signal as additive bias, summed to its output u_R to produce the control u applied to the process.

One can immediately notice that the role played by the regulator R_d is totally irrelevant if the process under control is in nominal conditions and there are no unforeseen disturbances, as in this case u_{opt} causes the controlled system to output $\widehat{y}$, thus u_R is zero. In the opposite case, R_d will conversely exert some action, with the twofold role of counteracting the possible variations of the model nominal conditions, and the presence of disturbances. In these cases the regulator effort consists in forcing the output to follow in the best possible way the predicted output coming from the optimization block; this prediction is the best we could get with respect to the set point signal w. The regulator action, when nominal conditions are lost and disturbances are not too “strong”, in a sense to be qualified,

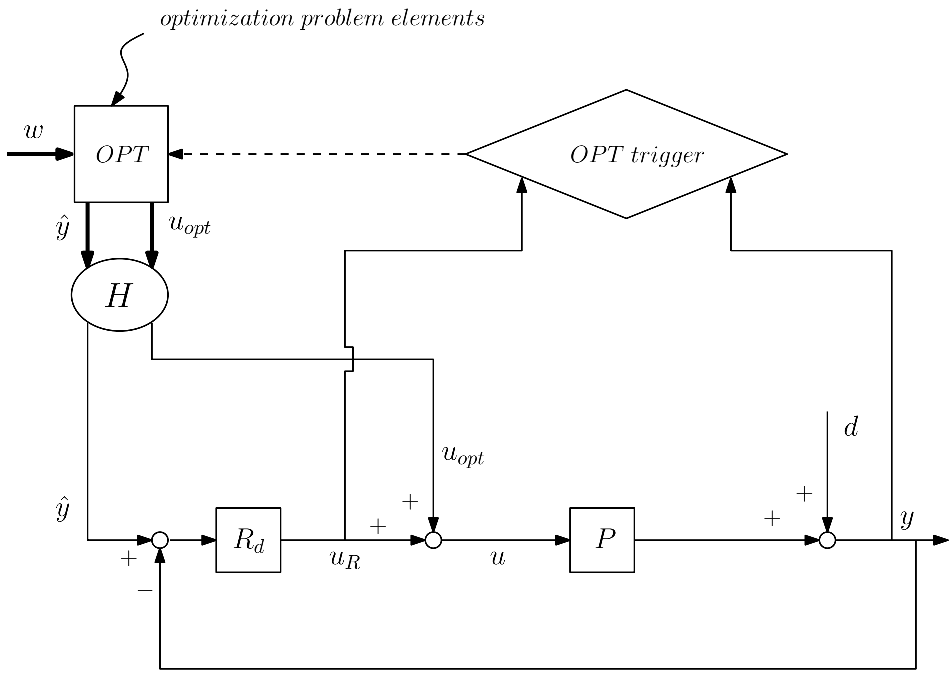


Figure 4.1: Introductory scheme for the proposed approach.

should prevent the optimization block from the recomputation of the optimization problem at each step of the control horizon. Anyway, if the control signal u_R results too strong or the output y deviates too much from its reference $\widehat{y}$, we should consider the possibility to carry out the optimization again, in order to obtain the new vectors of future controls and outputs.

As shown in Figure 4.1 by the decision block named "*OPT trigger*", the behavior of u_R and y is supervised at each sampling time in order to decide when a new optimization is needed. If a new optimization is required, a retrigger signal alerts the optimization block, as shown by the dotted arrow.

In the light of what we have shown up to this point, our purpose in this chapter, without considering the optimization problem itself and assuming the model predictive controller as given, is to reduce the number of the necessary optimization computations. In other words, and worth stressing for clarity, here we are not concerned with the synthesis of the MPC, which therefore we take as given. Our focus is conversely on the use of that MPC in the scheme of Figure 4.1, for the reasons discussed so far. Hence, in order to apply the proposed approach, we need to analyze and discuss:

- the structure of regulator R_d, its role in the system, and how to tune it;
- how the scheme can decide when a new optimization should be done before all the lastly computed N_{opt} controls are applied.

We underline that also the prediction horizon has an important role in this study, since a new optimization is obviously required at its end. Anyway, we do not concentrate on it, assuming that it has been already determined with some literature technique when setting up the MPC.

4.1.1 Analysis of the proposed approach

In the light of the problem overview shown before, we now propose a possible application of our idea, considering the scheme in Figure 4.1. In order to analyze the control scheme, the Optimization block will be represented by a predictive control structure. The predictive controller calculates the future control signals and outputs over the settled prediction horizon. These signals feed the Holder block H, whose fundamental role is to give them one by one, at each sampling time, to the control scheme below. If the optimization has to be redone before the end of the prediction horizon, it discards the residual signals and it is ready to receive the new optimal ones.

We now study the scheme using for OPT a very simple predictive controller, so that a formal analysis is straightforward. The same would not be true, apparently, with a complex enough OPT to evidence the advantages of our approach. We however conjecture that if the scheme works with the expected computational advantages with a simple OPT (where it can be assessed), the same will happen with a complex one.

Coming back to the main topic, assumed a Linear Time-Invariant System in the Optimization block and solved the optimization problem, the result of the optimal predictive control can be described as a LTI system and so the optimization block can be examined through the scheme in Figure 4.2. In this way the predictions coming from a predictive controller, like for example a Generalized Minimum Variance one, can be expressed as inputs of the transfer functions represented by R_w and R_y. In particular, if the process is in nominal conditions and no disturbances occur, we can better rearrange the scheme as shown in Figure 4.3.

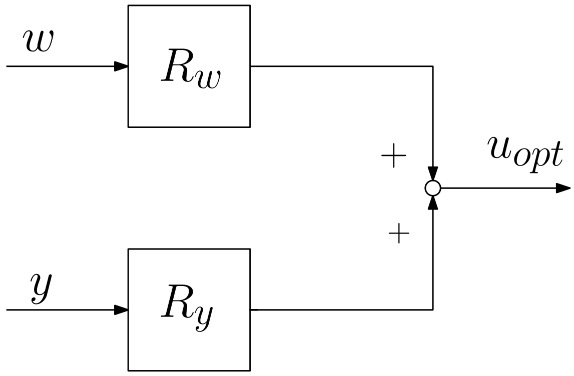


Figure 4.2: The MPC expressed as two transfer functions (in the LTI case).

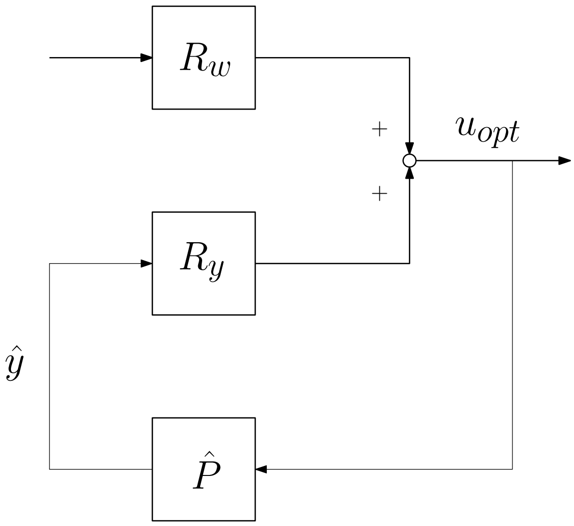


Figure 4.3: Obtaining the inputs for H from the MPC.

The expression of the optimal control signals and the optimal predicted outputs is:

$$U_{opt} = \frac{R_w}{(1 - R_y \widehat{P})} W$$
$$\widehat{Y} = \widehat{P} U_{opt}$$

We underline how these signals can be expressed in function of R_w and R_y, independently of the transfer functions nature and structure.

On the basis of these formulations, we propose an analysis of the feedback scheme, starting by the computation of the output y and the control signal u_R.

$$\begin{aligned} Y &= \frac{R_d P}{(1+R_d P)}\widehat{Y} + \frac{P}{(1+R_d P)}U_{opt} + \frac{1}{(1+R_d P)}D = \\ &= \frac{R_d P}{(1+R_d P)}\frac{R_w \widehat{P}}{(1-R_y\widehat{P})}W + \frac{P}{(1+R_d P)}\frac{R_w}{(1-R_y\widehat{P})}W + \frac{1}{(1+R_d P)}D = \\ &= \frac{P R_w(1+R_d\widehat{P})}{(1+R_d P)(1-R_y\widehat{P})}W + \frac{1}{(1+R_d P)}D \end{aligned}$$

$$\begin{aligned} U_R &= \frac{R_d}{(1+R_d P)}\widehat{Y} - \frac{R_d P}{(1+R_d P)}U_{opt} - \frac{R_d}{(1+R_d P)}D = \\ &= \frac{R_d}{(1+R_d P)}\frac{R_w \widehat{P}}{(1-R_y\widehat{P})}W - \frac{R_d P}{(1+R_d P)}\frac{R_w}{(1-R_y\widehat{P})}W - \frac{R_d}{(1+R_d P)}D = \\ &= \frac{R_d R_w(\widehat{P}-P)}{(1+R_d P)(1-R_y\widehat{P})}W - \frac{R_d}{(1+R_d P)}D \end{aligned}$$

The key factor of this problem formulation is that it does not rely on the kind of predictive controller used for the optimization process. The only requirement is that the chosen predictive technique can be described as a LTI system fed by a setpoint and providing future controls and outputs, as illustrated in Figure 4.3.

From the expressions of y and u_R we can analyze the behavior of the system, focusing on some particularly relevant cases of study. First of all, we consider the process in nominal conditions, i.e., $P = \widehat{P}$:

$$\begin{aligned} Y &= \frac{\widehat{P} R_w}{1-R_y\widehat{P}}W + \frac{1}{1+R_d\widehat{P}}D = \\ &= \widehat{Y} + \underbrace{\frac{1}{(1+R_d\widehat{P})}D}_{\Delta Y} \\ U_R &= \frac{R_d R_w(\widehat{P}-\widehat{P})}{(1+R_d\widehat{P})(1-R_y\widehat{P})}W - \frac{R_d}{(1+R_d\widehat{P})}D = \\ &= \underbrace{-\frac{R_d}{(1+R_d\widehat{P})}D}_{\Delta U_R} \end{aligned}$$

If no disturbances occur, it is clearly visible that the output perfectly follows the optimal prediction and the control signal u_R is zero, since no action of the feedback regulator is required, as stressed above. Otherwise, in the presence of a disturbance, the feedback action tries to reject it as showed by the term ΔU_R, while ΔY shows a deviation of the output from the predicted one. The deviation terms ΔU_R and ΔY can be calculated as functions of the chosen regulator R_d, which is the only unknown element if we assume as known the nominal process $\widehat{P}$ and some characterisation (e.g. as a class of signals) of the disturbance d.

As further analysis, we underline that the structure of R_d can be related to some requirements on the system response to a specified d, so that everything comes to depend on them. For example, if we have a step disturbance and we want to reject it in ν steps, we set R_d in order to obtain a zero ΔY in ν steps and so the only unknown parameter results ν itself.

In conclusion, if the system is in nominal conditions and if the predictive technique applied can be described through the transfer functions R_w and R_y, assuming a certain disturbance d acting on the system, given some requirements on its rejection by means of R_d, the trends of ΔU_R and ΔY are perfectly known, since we can compute their Z-Transforms.

Now we release the hypothesis of being in nominal conditions as for the process, i.e., we assume $P \neq \widehat{P}$ but still that no disturbances occur.

$$
\begin{aligned}
Y &= \frac{PR_w(1+R_d\widehat{P})}{(1+R_dP)(1-R_y\widehat{P})}W \\
\Delta Y &= Y - \widehat{Y} = \\
&= \frac{PR_w(1+R_d\widehat{P})}{(1+R_dP)(1-R_y\widehat{P})}W - \frac{R_w\widehat{P}}{(1-R_y\widehat{P})}W = \\
&= \frac{PR_w(1+R_d\widehat{P}) - \widehat{P}R_w(1+R_dP)}{(1+R_dP)(1+R_y\widehat{P})}W \\
U_R &= \frac{R_dR_w(\widehat{P}-P)}{(1+R_dP)(1-R_y\widehat{P})}W = \\
&= \Delta U_R
\end{aligned}
$$

In order to further analyse the previous formulation, we consider two standard cases of the possible process variations with respect to its nominal state.
Considering an additive variation $P = \widehat{P} + \Delta P$ we obtain:

$$
\begin{aligned}
\Delta Y &= \frac{(\widehat{P}+\Delta P)R_w(1+R_d\widehat{P}) - \widehat{P}R_w(1+R_d(\widehat{P}+\Delta P))}{(1+R_d(\widehat{P}+\Delta P))(1-R_y\widehat{P})}W = \\
&= \frac{\widehat{P}R_w + \Delta P R_w + \widehat{P}^2 R_d R_w + \Delta P R_w R_d \widehat{P} - \widehat{P}R_w - \widehat{P}^2 R_w R_d - \widehat{P} R_w R_d \Delta P}{(1+R_d(\widehat{P}+\Delta P))(1-R_y\widehat{P})}W = \\
&= \frac{\Delta P R_w}{(1+R_d(\widehat{P}+\Delta P))(1-R_y\widehat{P})}W \\
\Delta U_R &= \frac{R_d R_w(\widehat{P} - \widehat{P} - \Delta P)}{(1+R_d(\widehat{P}+\Delta P))(1-R_y\widehat{P})}W = \\
&= -\frac{\Delta P R_d R_w}{(1+R_d(\widehat{P}+\Delta P))(1-R_y\widehat{P})}W
\end{aligned}
$$

Otherwise we can look at a multiplicative deviation $P = \widehat{P}(1+\Delta P)$:

$$
\begin{aligned}
\Delta Y &= \frac{\widehat{P}(1+\Delta P)R_w(1+R_d\widehat{P}) - \widehat{P}R_w(1+R_d\widehat{P}(1+\Delta P))}{(1+R_d\widehat{P}(1+\Delta P))(1-R_y\widehat{P})}W = \\
&= \frac{\widehat{P}R_w + \widehat{P}\Delta P R_w + \widehat{P}^2 R_w R_d + \widehat{P}^2 \Delta P R_w R_d - \widehat{P}R_w - \widehat{P}^2 R_w R_d - \widehat{P}^2 \Delta P R_w R_d}{(1+R_d\widehat{P}(1+\Delta P))(1-R_y\widehat{P})}W = \\
&= \frac{\widehat{P}\Delta P R_w}{(1+R_d\widehat{P}(1+\Delta P))(1-R_y\widehat{P})}W \\
\Delta U_R &= \frac{R_d R_w(\widehat{P} - \widehat{P}(1+\Delta P))}{(1+R_d\widehat{P}(1+\Delta P))(1-R_y\widehat{P})}W = \\
&= \frac{R_d R_w \widehat{P} - R_d R_w \widehat{P} - R_d R_w \widehat{P}\Delta P}{(1+R_d\widehat{P}(1+\Delta P))(1-R_y\widehat{P})}W = \\
&= -\frac{R_d R_w \widehat{P}\Delta P}{(1+R_d\widehat{P}(1+\Delta P))(1-R_y\widehat{P})}W
\end{aligned}
$$

Analysing the obtained deviations of the output, i.e., ΔY, and of the control signal u_R, i.e., ΔU_R, we observe that, both in case of an additive and a multiplicative process variation, the loop transfer functions are the terms $R_d\widehat{P}$ and $R_y\widehat{P}$. In particular, the bigger is their modulus, the more the deviation terms are attenuated. In other words, the deviation terms are strongly influenced by the structure of the regulators R_d and R_y. In this regard, we highlight that R_d is the real regulator of the final system provided with the real numerical optimizer, while R_y does not really exist in the system since its role (that of producing u_{opt} and $\hat{y}$) is in fact played by the optimiser itself. For this reason the regulator R_d can be directly tuned according to the behavior of the output of the system, while the tuning of

R_y actually consists of specifying the optimisation problem and its solution policy and thus requires further investigations. In particular, one can intuitively state that the tuning of R_y has surely a correlation with the terms appearing in the optimization cost function. Hence, for example, the desired control action of R_y can be get by conveniently balancing the control error penalization in the optimazion cost function. However, there is here no doubt on the necessity and convenience of an analysis that extends far beyond the scope of this work.

4.1.2 Regulator Synthesis

In this part we focus on the design of the feedback regulator R_d, and we outline a synthesis method suitable for its role in the presented scheme. We first recall that its action is mainly required for two reasons:

- to reject an unpredicted disturbance that occurs in the system under control;
- to constrast possible variations of the nominal process.

We underline that R_d should not be involved in the setpoint tracking, since this is a task of the optimal control signal u_{opt} coming from the MPC. In this regard, we first consider a direct synthesis completely focused on the rejection of a disturbance acting on the output signals. Therefore our aim is to minimize the transfer function between the disturbance d and the output y, the so called Sensitivity function $S(z)$. In this way we reject the disturbance by imposing $S(z)$ equal to an opportune reference transfer function $S^o(z)$. So, given the process model, we find the regulator such that $\frac{Y(z)}{D(z)} \simeq S^o(z)$.

We consider a process $P(z)$ asymptotically stable, having a relative degree $\nu_P > 0$, and we define:

$$P(z) = \frac{P_N(z)}{P_D(z)} \qquad R_d(z) = \frac{R_N(z)}{R_D(z)} \qquad S^o(z) = \frac{S^o_N(z)}{S^o_D(z)} \tag{4.1}$$

where $P_N(z), P_D(z), R_N(z), R_D(z), S^o_N(z), S^o_D(z)$ are polynomials.
The transfer function $S(z)$ has the following form:

$$S(z) = \frac{1}{1 + R_d(z)P(z)} = \frac{R_D(z)P_D(z)}{R_N(z)P_N(z) + R_D(z)P_D(z)}$$

Imposing $S(z) = S^o(z)$ we obtain the structure of R_d, as follows.

$$R_d(z) = \frac{1}{P(z)}\frac{(1 - S^o(z))}{S^o(z)} = \frac{P_D(z)}{P_N(z)}\frac{(S^o_D(z) - S^o_N(z))}{S^o_N(z)}$$

Hence, in order to obtain a feasible regulator, we should consider the following guidelines:

- the degree of $S^o_D(z)$ ($\#S^o_D$) should be the same of $S^o_N(z)$ ($\#S^o_N$) otherwise the relative degree of the term $\frac{1-S^o(z)}{S^o(z)}$ results negative and the regulator not feasible, since the degree of $\frac{1}{P(z)}$ is surely negative;
- given ν_P and $\#S^o_D = \#S^o_N$, in the polynomial $S^o_D(z) - S^o_N(z)$ the terms having degree $\#S^o_N, \#S^o_N - 1, \ldots, \#S^o_N - \nu_P + 1$ should be null.

In the following simulation results, the regulator considered is designed in the way illustrated above.

However, in this section we want to show the usefulness of a second guideline aimed at the regulator design. This is focused on the process variations with respect to the nominal conditions. The main idea is to lighten the optimization process not only by doing it sporadically, but also by reducing the complexity of the model under control. In this regard, we suppose to have at our disposal models of different complexity to do the optimization. Clearly, the simpler the model, the lighter the optimization process, typically because there are less parameters and some nonlinearities may not be considered. Since, reducing the order of the model, we obviously release the hypothesis of being in nominal conditions, the regulator R_d in such a situation will surely be required to act. Thus, the way the regulator is tuned is aimed at deciding things like how many non nominalities it should hide to the outer control layers through its action. If the regulator does not hide some non nominalities, a new optimization process is required.

In order to give some guidelines for the regulator tuning, we need to examine its control action in presence of a non nominality. Hence, we are going to analyze how a process variation is reflected in a variation of the loop dynamic, depending on the regulator structure. The most significant analyses are shown below.

We consider a system not affected by any disturbance, in order to focus only on the process variations. In particular, we consider two possible kind of variations, the additive and the multiplicative one.

1. Multiplicative process variation $P = \widehat{P}(1 + \Delta P)$.

   - We consider the effect of a normalized process variation $\frac{\Delta P}{P}$ on the normalized control action $\frac{\Delta U}{U}$, where $\Delta U = U - U_{nom}$:

$$\frac{\Delta U}{U} = -P \frac{R_d \widehat{P}}{1 + R_d \widehat{P}} \frac{\Delta P}{P} = -P\widehat{F}\,\frac{\Delta P}{P}$$

Analysing the induced normalized perturbation of U, i.e., $\frac{\Delta U}{U}$, we observe that it is given by the product of the normalized process variation, the nominal Complementary Sensitivity function $\widehat{F}$ and the real process P. P is the only non nominal contribution, however it is just a matter of scale. The typical shape of the frequency response magnitude of the Complementary Sensitivity function allows to state that the process variations are filtered at high frequency, while at low frequency they directly influence the control action. Hence, the action of the regulator R_d depends on the frequency band related to the process variation.

- We consider the effect of a normalized process variation $\frac{\Delta P}{P}$ on the normalized output signal $\frac{\Delta Y}{Y}$, where $\Delta Y = Y - \widehat{Y}$:

$$\frac{\Delta Y}{Y} = \widehat{P}\frac{1}{1 + R_d\widehat{P}}\frac{\Delta P}{P} = \widehat{P}\widehat{S}\,\frac{\Delta P}{P}$$

Analysing the induced normalized perturbation of Y, i.e., $\frac{\Delta Y}{Y}$, we observe that it is given by the product of the normalized process variation, the nominal Sensitivity function $\widehat{S}$ and the nominal process $\widehat{P}$. The typical shape of the frequency response magnitude of the Sensitivity function allows to state that low-frequency variations of the process are hidden to the output for the filtering action of $\widehat{S}$, while the high-frequency ones directly affect the normalized output index.

2. Additive process variation $P = \widehat{P} + \Delta P$.

- We consider the effect of a normalized process variation $\frac{\Delta P}{P}$ on the normalized control action $\frac{\Delta U}{U}$, where $\Delta U = U - U_{nom}$:

$$\frac{\Delta U}{U} = -P\frac{R_d}{1 + R_d\widehat{P}}\frac{\Delta P}{P} = -P\widehat{Q}\,\frac{\Delta P}{P}$$

Analysing the induced normalized perturbation of U, we observe that it is given by the product of the normalized process variation, the nominal Sensitivity function of the control $\widehat{Q}$ and the real process P. Since $\widehat{Q}$ represents the regulator at high frequency, this result is really useful if we are interested in high-frequency response of the control action to a process variation and if we use $\widehat{Q}$ to design the regulator.

- We consider the effect of a normalized process variation $\frac{\Delta P}{P}$ on the normalized output signal $\frac{\Delta Y}{Y}$:

$$\frac{\Delta Y}{Y} = \frac{1}{1 + R_d\widehat{P}}\frac{\Delta P}{P} = \widehat{S}\,\frac{\Delta P}{P}$$

Analysing the induced normalized perturbation of Y, we observe that it is given by the product of the normalized process variation and the nominal Sensitivity function $\widehat{S}$. According to the typical shape of the frequency response magnitude of the Sensitivity function, this allows to state that low-frequency variations of the process are hidden to the output for the filtering action of $\widehat{S}$, while the high-frequency ones directly affect the normalized output index.

Based on these analyses, the synthesis of the regulator, following the second approach, strongly influences the frequency response of our system through the shaping of the Sensitivity functions. Hence, the Sensitivity functions should be properly shaped by the regulator tuning.

On this point, we suggest a matter for a future reflection. We could think to integrate the frequency shaping of the frequency responses analyzed before into the model order reduction technique applied to the system. In particular, this integration should be done before giving the reduced model to the MPC optimizer. In this way, we could try to find the model order reduction technique that is specifically aimed at the best functioning of our system. Even if this final remark will no longer be discussed in our work, it can be of interest if we want to reduce the model order to immediately lighten the computational effort of the MPC optimizer.

4.1.3 Retrigger Criterion

Now, given the presented scenario, we present some guidelines concerning the optimization retrigger criterion.

The retrigger consists in forcing the MPC to do a new optimization process in order to calculate the new N_{opt} controls and outputs. The most general idea is to evaluate in real-time a certain index to verify whether or not it exceeds a fixed threshold. In this way we can monitor the behavior of the system in order to take a decision about the need to do a new optimization process. The choice of the threshold plays a fundamental role in this decision, because it influences the time steps after which the optimization should be retaken.

Since our aim is to reduce the computation time of the optimization process, we focus our attention on indices not too complex to calculate. For example, the index could concern the deviation of the output y from its reference $\widehat{y}$ or the amount of the control signal u_R coming from the feedback regulator. Then, if the chosen index exceeds the settled threshold, it is necessary to consider other aspects before retaking the optimization problem. For example, we can compare the value of the cost function with the optimal one and, obviously, if it gets worse, the optimization problem has to be redone. Alternatively, we could evaluate if some constraints risk to be violated, or if the control signal gets too close to certain limits. From the computational point of view, the calcutation of any possible indices should be far easier than the computation of the optimization process. For example, considering the process cost function as index, it is obvious that its computation is much lighter than its minimization.

In our analysis the retrigger criterion to decide when a new optimization process is needed consists in the two steps listed below. For the following discussion, we need to preliminary define $J(k)$ and $J_{opt}(k)$, for which we mean, respectively, the value of the cost function J and the value of the optimization cost function J_{opt} computed at each time interval. The steps mentioned before are:

1. evaluation of the regulator control energy through a convenient index. If the regulator action results too strong with respect to a fixed threshold, a new optimization process should be probably considered.

2. evaluation of the behavior over time of the cost function $J(k)$ in comparison with the optimal one $J_{opt}(k)$. If $J(k)$ is getting worse than $J_{opt}(k)$, a new optimization should be done.

As already discussed, another interesting retrigger index could be referred to the behaviour of the output y. In this case we should monitor the deviation of y from its reference $\widehat{y}$. This means that we are disposed to use more control action in order to obtain a perfect behavior of y with respect to its reference. Otherwise, monitoring the behavior of u_R, as done in our work, we are more interested in saving as much control energy as possible.

Retrigger Indices

In order to evaluate the regulator control energy, we use an integral index calcutated as follows:

$$\Phi_{u_R}(k) = \sqrt{\sum_{i=0}^{k} u_R^2(i)} \qquad k \in [0, N_{opt}] \tag{4.2}$$

Imposing a threshold to the value of this index, we determine when to consider the possibility of retaking the optimization. This threshold is chosen in accordance with the features desired for the system. Of course, stricter is the threshold, bigger is the number of times in which a new optimization is needed.

To further analyze this topic, we propose to evaluate, through the Final-Value Theorem, the response of the control variable u_R to the disturbance d. We examine two cases concerning two different kind of disturbance, the step and the impulse. We suppose to have an asymptotically stable process P and a regulator R_d in the form $\frac{(1-k)+kz}{z-1}$, where k is an arbitrary constant.

1. we consider a step disturbance with area ρ.

$$
\begin{aligned}
&\lim_{t\to\infty} u_R(t) = \lim_{z\to 1}(z-1)G(z)D(z) = \\
&\lim_{z\to 1}(z-1)\left(-\frac{R_d}{1+R_dP}\right)\left(\rho\frac{z}{z-1}\right) = \\
&\lim_{z\to 1}\left(-\frac{(1-k)+kz}{z-1+(1-k+kz)P}\right)\rho z = -\frac{\rho}{P}
\end{aligned}
$$

   The response of the single control action u_R to a step disturbance is constant. Hence, the integral index considered above diverges. It follows that a new optimization is surely needed, independently of the set threshold, if assuming a very large prediction horizon N_{opt}.

2. we consider an impulse disturbance with area μ.

$$
\begin{aligned}
&\lim_{t\to\infty} u_R(t) = \lim_{z\to 1}(z-1)G(z)D(z) = \\
&\lim_{z\to 1}(z-1)\left(-\frac{R_d}{1+R_dP}\right)\mu = \\
&\lim_{z\to 1}(z-1)\left(-\frac{(1-k)+kz}{z-1+(1-k+kz)P}\right)\mu = 0
\end{aligned}
$$

   The response of the single control action u_R to an impulse disturbance is null. This means that the proposed index does not diverge and so the decision about retaking the optimization depends only on the chosen threshold.

After considering this integral index, we look at the cost function $J(k)$. In accordance to its behavior, we take the final decision about the need of a retrigger. Applying an MPC strategy, we can compute the optimization cost function $J_{opt}(k)$ at each time step, in order to evaluate its behavior as compared to the one of $J(k)$. It is important to explain the meaning of $J(k)$. Thinking of the result of the optimization, the optimizer has found the optimal values of all the future controls as for the minimization of the cost function J. Now if we apply those controls one by one and compute the formula for J, only up to the current instant, we obtain the behavior of $J(k)$. If everything goes as the optimization expected, that signals will reach the final optimal value at the end of the horizon N_{opt}. However, if we

observe deviations of $J(k)$ as a signal during the application of the control, we can infer that something is not going in the same way the optimizer expected.
To evaluate the behavior of $J(k)$ with respect to $J_{opt}(k)$, we consider the following normalized index.

$$J_{norm}(k) = \frac{J(k)}{J(k) + J_{opt}(k)} \tag{4.3}$$

A new optimization is needed if $J(k)$ is getting worse than $J_{opt}(k)$, on the basis of desired achievements.

4.2 An application with the GMV controller

In this section we put our approach to work (under the simplificative hypotheses outlined at the beginning of section 4.1.1 to motivate the analysis of this chapter) by using a well assessed MPC technique in the Optimization block of the control scheme proposed in Figure 4.1. In this way, we get the scheme proposed in Figure 4.4. This scheme is valid in between two executions of OPT. In particular, we use the Generalized Minimum Variance control because, allowing a direct penalization of the control in the optimization process, it seems to be the most suitable choice in an energy-related scenario.

The GMV controller minimises a generalised minimum variance cost function which includes the control variables, allowing to improve the stability of the closed-loop control system by penalising large control actions. The GMV control focuses on how to design a dynamic time-invariant controller that acts in feedback. This controller is aimed at the minimization of the following quantity:

$$J = E[(P(z)y(t+k) + Q(z)u(t) - y^o(t))^2] \tag{4.4}$$

The terms $P(z)$ and $Q(z)$ play a key role in this cost function. They weigh the output and the control action, allowing to constrain the considered control problem.

There are two main strategies used in the project of a GMV controller. It is possible to design a Model Reference Control, settling $Q(z) = 0$ and leaving $P(z)$ arbitrary. Alternatively, a Penalized Control strategy can be adopted just setting $P(z) = 1$, leaving $Q(z)$ arbitrary. In our analisys we will consider this second approach, since it allows us to directly penalize the control action, a fundamental issue in an energy-related optimization problem.

We underline as the GMV control system can perfectly suit the rearranged scheme proposed in Figure 4.2, because the optimal control variables and the predicted outputs supplied can be expressed as fuctions of the only setpoint signal. In particular, the transfer functions R_w and R_y directly depend on the weights $P(z)$ and $Q(z)$. Therefore all the formulations and considerations stated before in the analisys of the proposed approach still hold good.

4.2.1 Example

We consider an ARMAX model in the form (4.5):

$$A(z)y(t) = B(z)u(t-k) + C(z)e(t) \qquad e(t) \sim WN(0, \sigma^2) \tag{4.5}$$

As said before, we apply to the process a GMV control in order to obtain the optimal control vector u_{opt} and the forecast outputs $\widehat{y}$ of the system. Using the Penalized Control strategy, we set $P(z) = 1$ and $Q(z) = \lambda\frac{(1-z^{-1})}{1-\lambda z^{-1}}$. The feedback control scheme is fed by $\widehat{y}$ and u_{opt} and, in particular, the control variables coming from the GMV optimization process bias the ones computed by the regulator R_d. A crucial point, in the implementation of the overall system, is the design of the regulator R_d, whose details will be exposed in the following part.

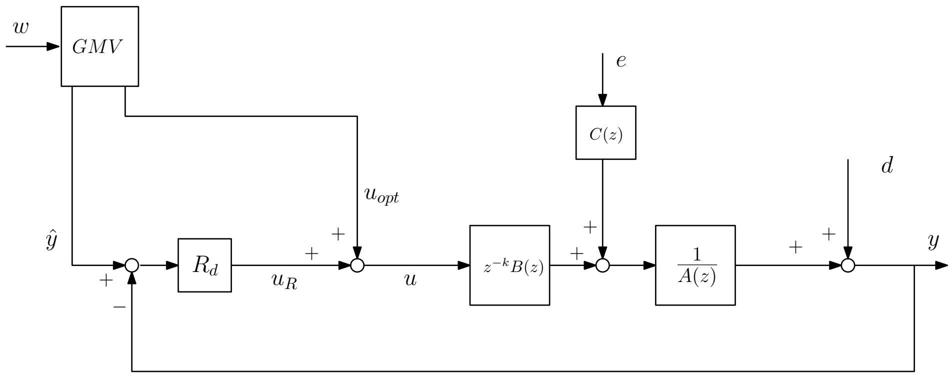


Figure 4.4: The scheme of Figure 4.1 specialized to the GMV+ARMAX case (the H block can be thought of as part of the GMV one).

Simulation Results - Sporadic GMV

At this point, to further analyze the proposed example, we show the main results obtained through simulations.

In the simulations we assume that the white noise variance is almost negligible, as in this context it just represents measurement noise. We first consider an LTI asympotically stable system with a reference set point in the form of a ramp followed by a step signal and affected by a step disturbance on the output. The feedback regulator has been designed, following the guidelines shown before, in order to reject this disturbance in two steps. As already highlighted, the control action of the feedback regulator u_R is not necessary if no disturbance occurs; the overall control action of the scheme, u, is the one coming from the GMV controller u_{opt}, as shown in Figure 4.5.

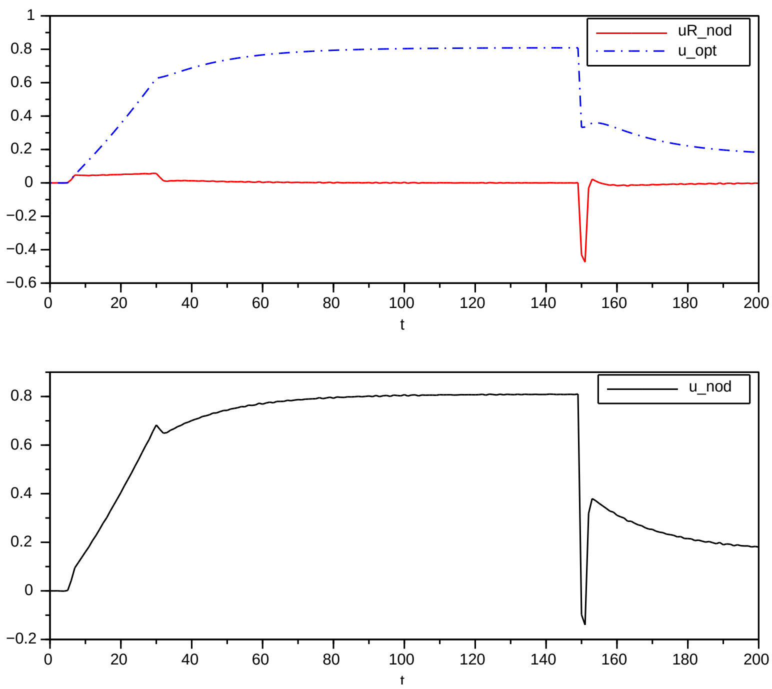


Figure 4.5: Control actions u_R, u_{opt} and u if no disturbance occurs in the system.

In Figure 4.6 we illustrate the trend of the output y with respect to the set point $\widehat{y}$, the prediction coming from the GMV optimization.

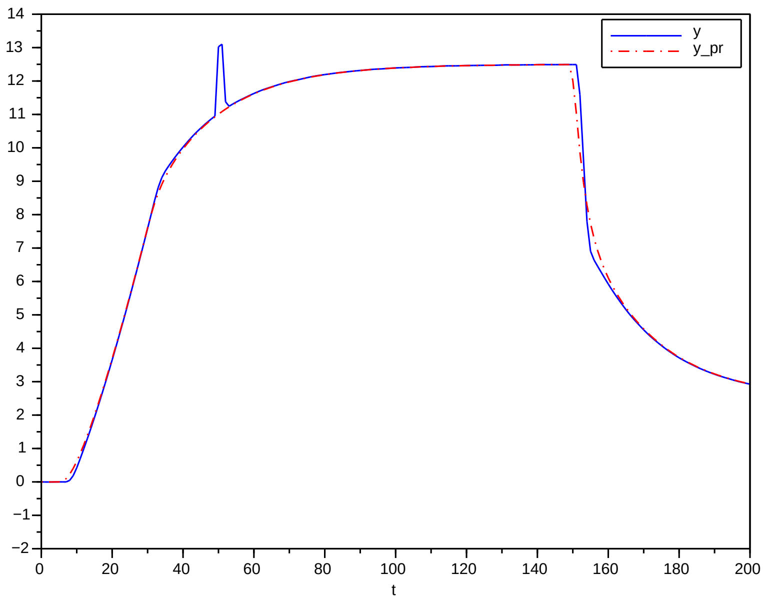


Figure 4.6: Output y and its set point $\hat{y}$ if a step disturbance occurs in the system.

The result shows how the trend of the output perfectly follows the predicted one, except for a peak related to the disturbance incoming. As designed, the regulator rejects the undesired disturbance in about two steps.

The control action u_R is depicted in Figure 4.7 and it is clearly visible that its work is directly aimed at the disturbance rejection.

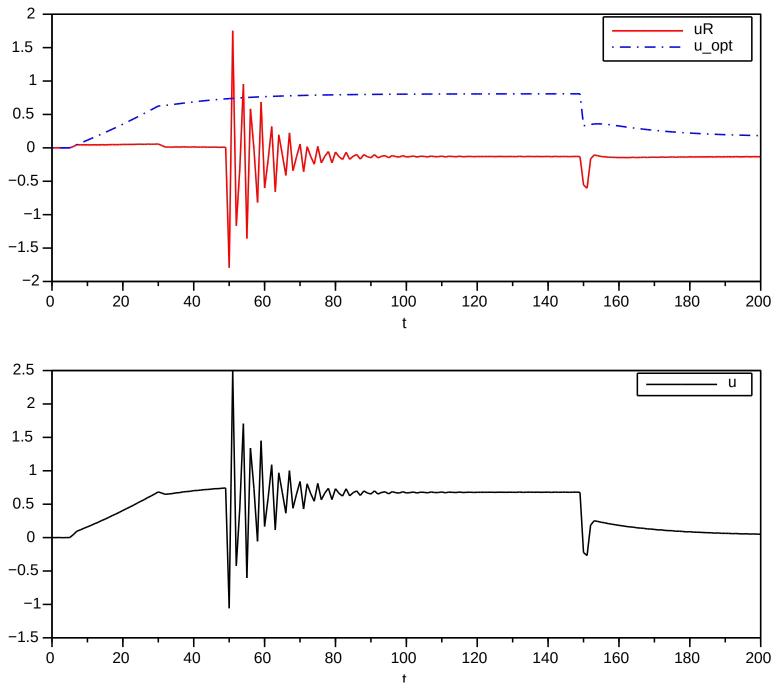


Figure 4.7: Control actions u_R, u_{opt} and u if a step disturbance occurs in the system.

In order to better compare the trend of the outputs and of the control signals in a nominal case with the ones simulated in presence of the disturbance, we show the Figure 4.8.

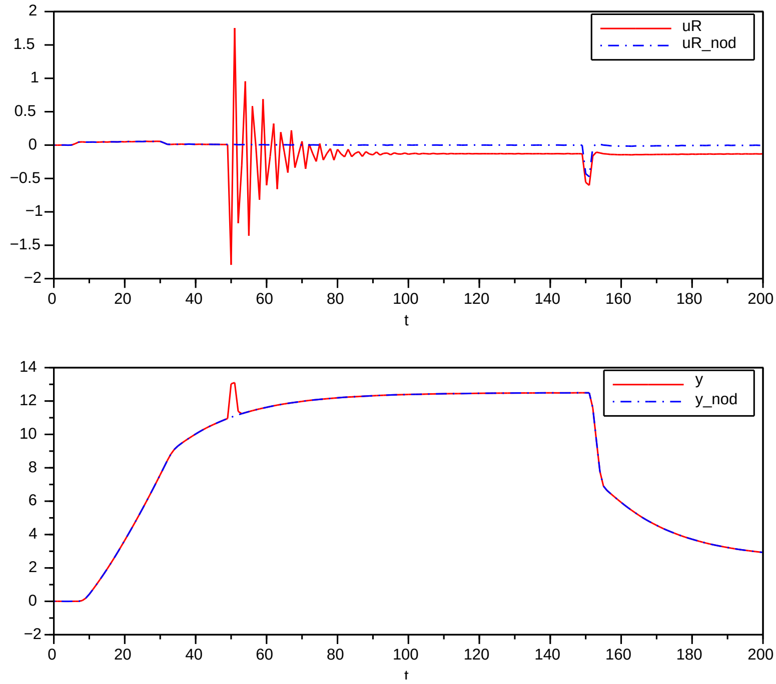


Figure 4.8: Comparison of the feedback control action u_R and the output y in the cases of the system with and without disturbance.

In nominal case the trend of the output perfectly follows the set point and u_R is nearly zero. Otherwise, if a disturbance occurs, the strong swinging control action lets the output y follow its reference by rejecting the disturbance in two steps.

In order to further examine the proposed example, now we consider the same system affected by an impulse disturbance and with a simple step set point. We have verified that all the previous considerations hold also in this case, except for those about the disturbance rejection time. The regulator rejects the disturbance in three steps causing a double peak in the output trend, as shown in Figure 4.9.

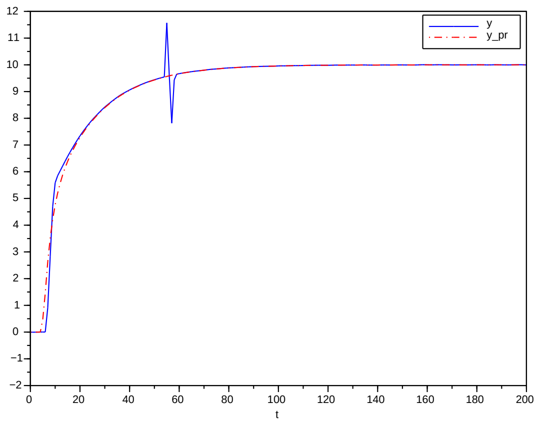


Figure 4.9: Output y and its set point $\hat{y}$ if an impulse disturbance occurs.

As for the control action u_R, its behaviour is depicted in Figure 4.10 and we can notice that, after the swing due to the disturbance incoming, u_R tends to zero as supposed.

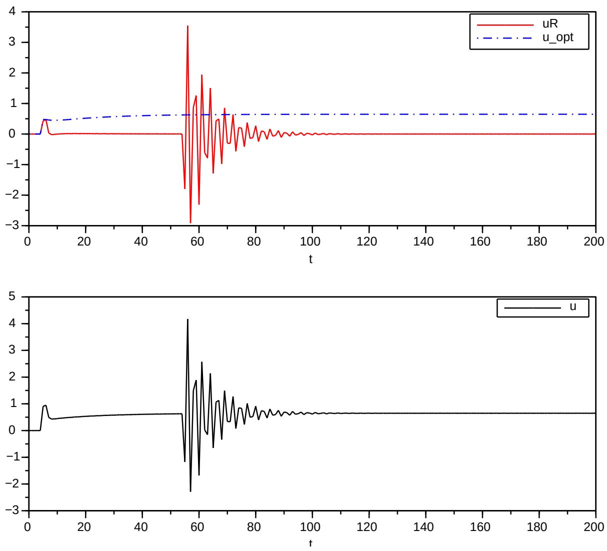


Figure 4.10: Control actions u_R, u_{opt} and u if an impulse disturbance occurs.

Retrigger Indices

In this part we show the simulations concerning the index for the retrigger.

Figure 4.11 shows the integral index $\Phi_{u_R}(k)$ (see Equation (4.2)) obtained in the case of the system affected by a step disturbance d.

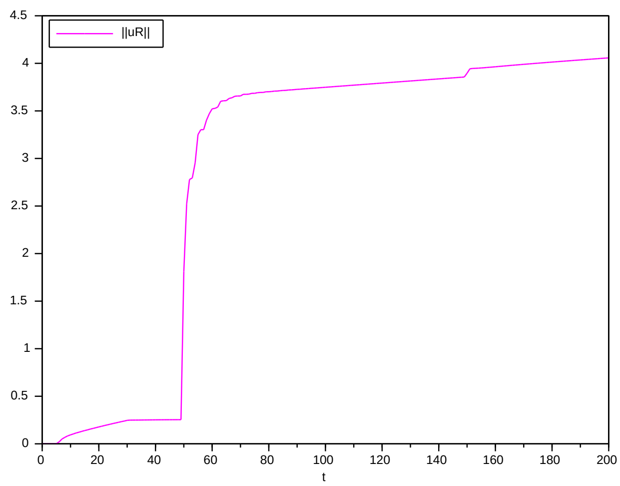


Figure 4.11: $\Phi_{u_R}(k)$ if a step disturbance occurs in the system.

We notice that it diverges, increasing more and more, as expected from the considerations already discussed.

In Figure 4.12 it is illustrated the comparison between the cost functions $J(k)$ and $J_{opt}(k)$, while in Figure 4.13 we can look at the normalized index $J_{norm}(k)$ (see Equation (4.3)).

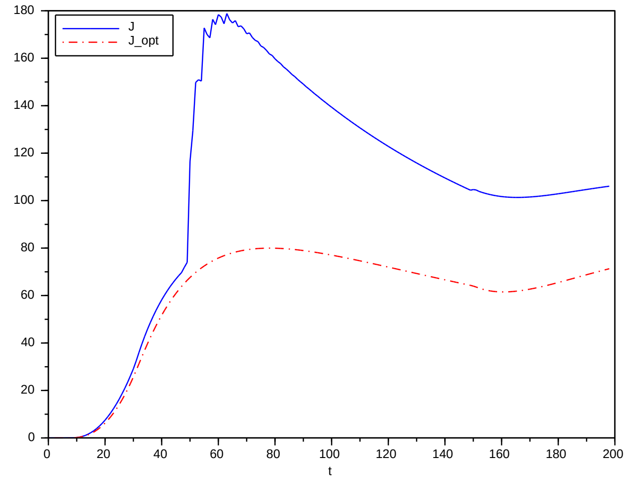


Figure 4.12: Cost functions $J(k)$ and $J_{opt}(k)$ if a step disturbance occurs in the system.

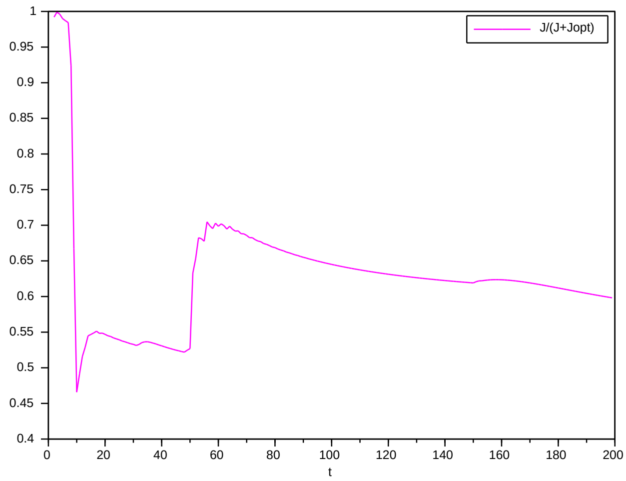


Figure 4.13: $J_{norm}(k)$ if a step disturbance occurs in the system.

These figures are fundamental to decide when a new optimization is needed. As we can see, when the disturbance occurs at the fiftieth time step, $J(k)$ increases faster than $J_{opt}(k)$ and then it keeps above $J_{opt}(k)$ until the end of the prediction horizon N_{opt}. Looking at Figure 4.13, this consideration is proved by the fact that $J_{norm}(k)$ strongly outdistances the value 0.5, that represents a perfect equality of $J(k)$ and $J_{opt}(k)$.

Now we consider the system affected by an impulse disturbance d. In Figure 4.14 we present the integral index $\Phi_{u_R}(k)$ and we can notice that it strongly increases when the disturbance occurs, but after a few time steps it goes to a constant value, as expected.

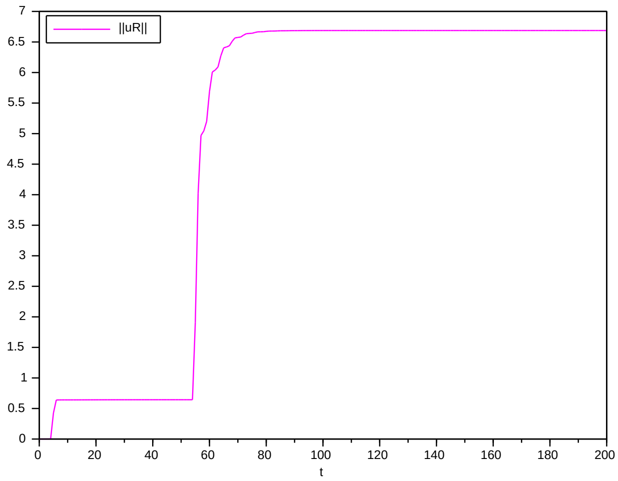


Figure 4.14: $\Phi_{u_R}(k)$ if an impulse disturbance occurs in the system.

Looking at Figures 4.15 and 4.16, we notice the difference between the two cost functions, which increases from the moment of the disturbance incoming.

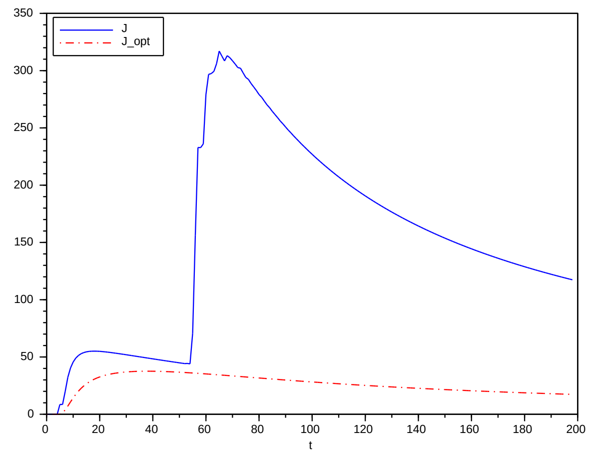


Figure 4.15: Cost functions $J(k)$ and $J_{opt}(k)$ if an impulse disturbance occurs in the system.

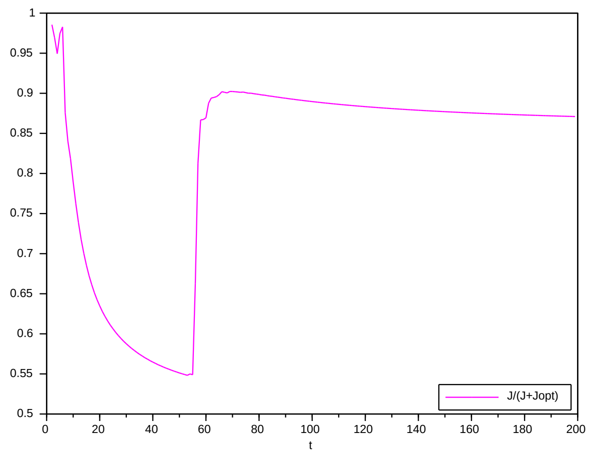


Figure 4.16: $J_{norm}(k)$ if an impulse disturbance occurs in the system.

Simulation Results with Retrigger - Sporadic GMV

In this part we show the simulation results obtained when a retrigger is needed in our system. As already discussed, the retrigger consists in a new optimization process that supplies new optimal control signals and outputs over the finite horizon N_{opt}.

In this example, we suppose that the system under control is affected by two step disturbances, at time t=50 and at time t=180. The control variable u_R has to reject them and so the integral index related to the control energy will surely increase. As already discussed in the previous sections, we need to set a threshold on the integral index $\Phi_{u_R}(k)$ and on the normalized cost function $J_{norm}(k)$, in order to evaluate whether or not a retrigger is needed.

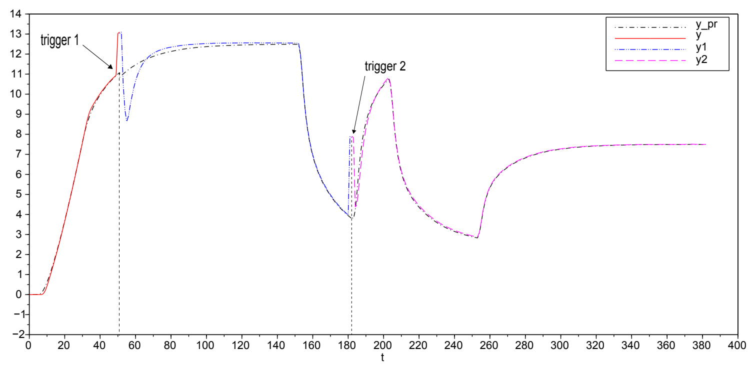
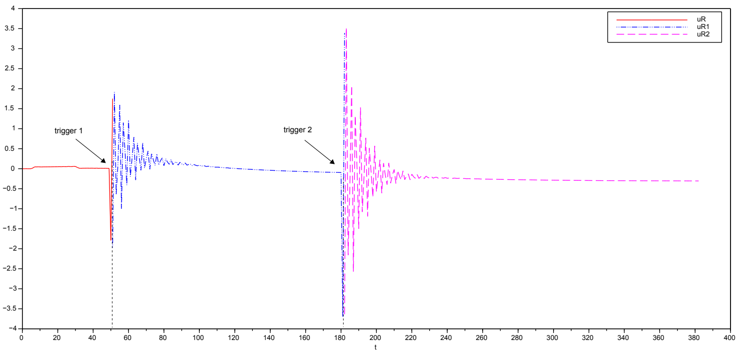


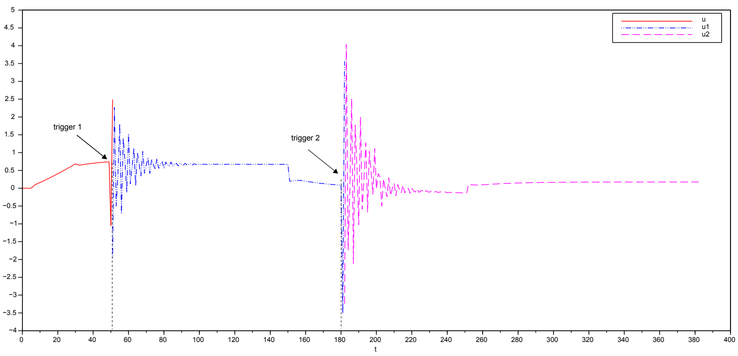
Figure 4.17: Output behavior when two retriggers are needed.

In the example used for the simulations, two retriggers are needed, as showed by the arrows in Figures 4.17 and 4.18. For the first one we consider a threshold for $\Phi_{u_R}(k)$ equal to 3 and for $J_{norm}(k)$ equal to 0.55. When the threshold concerning the control energy is exceeded, at time t=51, also the value of $J_{norm}(k)$ exceeds its threshold. This means that a new optimization is needed. According to the same criterion, a second retrigger is necessary at time t=182, since a new disturbance has appeared in our system at time t=180. For the second retrigger we change the threshold value for $\Phi_{u_R}(k)$ by setting it equal to 20. In Figure 4.17 we can see the system output and we can clearly notice the two peaks related to the presence of the two step disturbances. The overall output of the system traverses three phases in time, denoted as y, $y1$, $y2$ and distinguished with different colours and line styles. The first one, y, is the system output before the first retrigger time, while $y1$ and $y2$ are the outputs obtained after the first and the second retrigger, respectively. In Figure 4.17 is also shown the output predicted by the optimization process. Since this one represents the feedback for the closed-loop system, we can state that the output signals perfectly follows their reference, except for some oscillations due to the disturbances occurance.

The control signals u and u_R are shown in Figure 4.18.



(a) u_R



(b) u

Figure 4.18: Control actions when two retriggers are needed.

In the upper and lower part of Figure 4.18 we can see, respectively, the three phases in time of the overall control variables u (u, $u1$, $u2$) and u_R (u_R, u_{R1}, u_{R2}), the one coming from the feedback regulator R_d. These phases are distinguished with different colours and line styles. The control signals u and u_R are the ones before the first retrigger time at t=51; $u1$ and u_{R1}, $u2$ and u_{R2} are the control variables required after the first and the second retrigger, respectively. We can notice how the control action is strongly required when the two disturbances occur; this is the reason why the two simulated retriggers are needed.

Simulation Results - Classic GMV

To quantify the control quality degradation incurred in by adopting our technique, we now compare the previous results to those obtained by using the MPC in the classical receding horizon setting (i.e., generalising for the purpose of this work, performing the optimization at every control step). To this end, we propose a comparison between the simulation result obtained applying our Sporadic GMV techinque and the one obtained applying a classic GMV technique. In the latter case, at each sampling time the control variable u_{opt} applied to the system is the first one of the control vector computed by the optimizer. This means that the optimization is done at each sampling time of the control horizon N_{opt}.

We focus our attention on the output y. Its behavior is analyzed with respect to its reference $\hat{y}$. In Figure 4.19 we can see the output obtained when a retrigger is needed. According to the Sporadic GMV procedure, two optimizations are needed: the first one at the beginning of the control horizon, the second one in order to reject the disturbance occurred. The deviation of y from its reference $\hat{y}$ is visible only when the disturbances occur.

Now we compare this behavior of y to the one depicted in Figure 4.20, obtained when a classic GMV procedure is applied to the system. In this second case N_{opt} optimizations have been done. We want to highlight as the deviation of y from its reference is around the same depicted in the other figure, except for the decreasing pick related to the first disturbance. We can conclude that the computation time spent for taking the all N_{opt} optimizations is not worth. Hence, the simulation result obtained when the sporadic GMV is applied can be surely accepted.

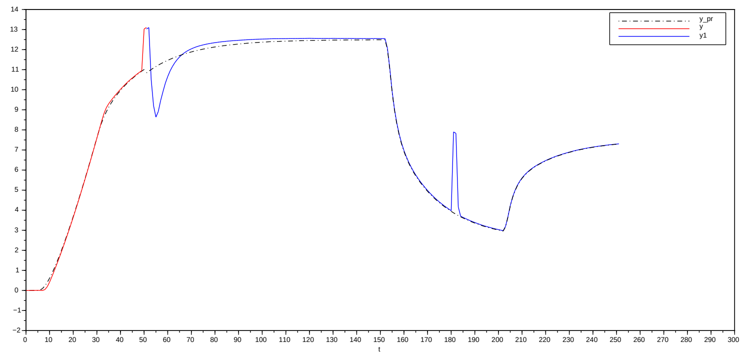


Figure 4.19: Output behavior when a retrigger is needed.

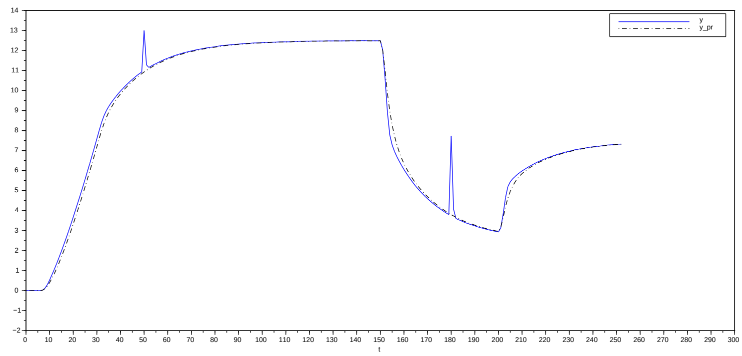


Figure 4.20: Output behavior applying a classical GMV procedure.

Chapter 5

Application

In this chapter we present an application of the approach proposed in Chapter 4, in which the simplified setting with standard LTI predictive controllers is abandoned, and a real numerical optimization system is in place. Specifically, the application is carried on using the GenOpt optimization tool. The addressed problem falls in the category defined by the general statement illustrated in Chapter 3, although for a better interpretability a quite simple situation is considered, as this allows to illustrate the required concepts and thus correctly serves the purpose of this chapter in the overall organisation of the thesis.

5.1 GenOpt

In order to solve the optimization problem discussed in Chapter 3, we have chosen to use the optimization program GenOpt.

Nowadays, the use of system simulation for analyzing complex engineering problems is increasing. Usually, a lot of time is spent on specifying the problem for a computer simulation. Once this has been done, the analyst seldom can afford the time to optimize the design. One reason for this is the lengthy process of varying the input data, running lots of simulations and comparing the various results. However, in recent years, tools have become available to do automatic optimization using search techniques that require little effort and time. Another reason for the difficulty just mentioned, is that systems are often so complex that determining the optimal design parameters is not feasible without using an optimization algorithm. Optimization tools like GenOpt are being developed to overcome these difficulties.

From now on, for apparent reasons, we refrain from discussing optimization in general, and we focus on the adopted tool.

Since one of the main application fields of GenOpt is the optimization of cost functions that are evaluated by building simulation programs, it considers the special characteristics of simulation problems in this area.

GenOpt is an optimization program for the minimization of a cost function that is evaluated by an external simulation program, such as EnergyPlus, TRNSYS, Dymola, IDA-ICE or DOE-2. It has been developed for optimization problems where the cost function is computationally expensive and its derivatives are not available or may not even exist. GenOpt can be coupled to any simulation program that reads its input from text files and writes its output to text files. The independent variables can be continuous (possibly with lower and upper bounds), discrete, or both. Constraints on dependent variables can be implemented using penalty or barrier functions.

GenOpt is written in Java, hence it is platform independent. The platform independence and the general interface make GenOpt applicable to a wide range of optimization problems.

GenOpt has a library with local and global one and multi-dimensional optimization algorithms, as well as algorithms for doing parametric runs. By using GenOpt's algorithm interface, new optimization algorithms can be added to GenOpt's algorithm library without knowing the details of the program structure. Hence, the user can select an appropriate optimization algorithm from a library or implement a custom algorithm without having to recompile and understand the whole optimization environment. This contributes in making GenOpt a widely applicable program.

To perform the optimization, GenOpt automatically writes the input files for the simulation program. The generated input files are based on input template files, which are written for the simulation program in use. GenOpt then starts the simulation program, checks for possible simulation errors, reads the value of the function being minimized from the simulation result file and then determines the new set of input parameters for the next run. The whole process is repeated iteratively until a minimum of the function is found. During the optimization, GenOpt's graphical user interface displays intermediate results.

The optimization algorithm we chose for in our example is the Discrete Armijo Gradient one. It is a gradient line search algorithm for multi-dimensional optimization and it can be used to solve a problem where the cost function is continuously differentiable. This algorithm approximates gradients by finite differences and uses the Armijo line search algorithm. It can be used for problems where the cost function is evaluated by computer code that defines a continuously differentiable function, but for which obtaining analytical expressions for the gradients is impractical or impossible. For all these reasons, this algorithm is well suited for the problem according to the features of our model, which in the general case could be of much larger size, but quite invariantly could be formulated so as to make it devoid of discontinuites. These discontinuites could in fact mostly derive from the dynamic equations concerning the thermal balance of the system and, tipically, they are related to On/Off systems. If the used solver can efficiently handle stiff systems, a viable solution is to smooth out these discontinuites by introducing "fast"" state variables.

Hence, summarising the main features of GenOpt, we clearly get the reasons why we have chosen it:

- it is very flexible since it is suitable for different simulation programs; in this work we use Dymola;

- it is suited to different kind of problems since it offers a lot of optimization algorithms and does not require any particular characteristic on the problem, such as convexity;

- it is an open program and so it allows for a better dissemination of the obtained results and their possible improvement.

The joint use of GenOpt and Dymola programs allows to use the models coming from the engineering for control purposes. This means that, even if in this work we have chosen a simple model, also a complex model of a building, like the ones coming from engineering, could have easily been considered.

5.2 Problem Characterization

In this section we state the problem taken into account for this application. This problem contains a subset of the components and features defined in Chapter 3. In the problem setting we consider one user provided with one energy storage and one heater, since we focus on a heating problem.

This problem is represented by a thermal model, shown by the scheme in Figure 5.1.

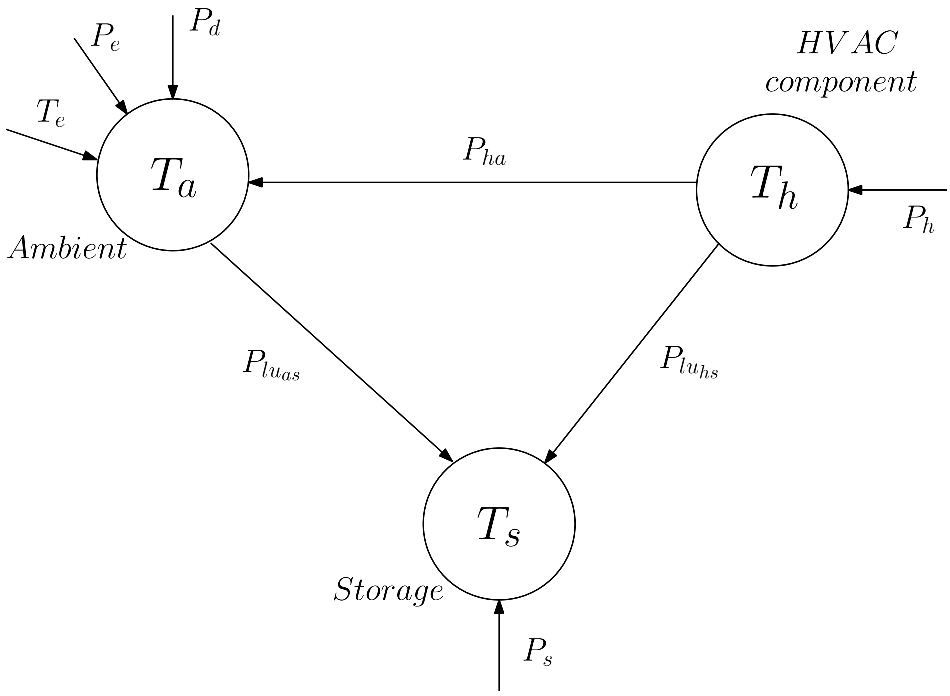


Figure 5.1: Thermal Model considered for this application.

T_a: ambient temperature
T_h: HVAC component temperature
T_s: storage temperature
P_d: disturbance power (e.g. loads, room occupancy)
P_e: external power (e.g. solar radiation)
T_e: external temperature
P_{ha}: power from the HVAC component to the ambient
P_s: power supplied to the storage from external energy sources
P_h: power supplied to the HVAC component from external energy sources
$P_{lu_{as}}$: loaded or unloaded power between the ambient and the storage
$P_{lu_{hs}}$: loaded or unloaded power between the HVAC component and the storage

We further specify the meaning of the loaded and unloaded powers. If $P_{lu_{as}}$ and $P_{lu_{hs}}$ are positive, they represent the loaded power from the ambient to the storage and from the heater to the storage. On the contrary, if they are negative, they are the unloaded power from the storage to the ambient and to the heater, respectively.

Now that the problem has been illustrated from a graphical point of view, we further analyse it by using a mathematical notation. The thermal model used for the optimization problem can be formalized by the following system of dynamic equations. Each equation clearly shows the relationships between the system states and the other variables:

$$\begin{cases} \dot{T_a} = f_a(T_a, P_{ha}, P_e, P_d, T_e, P_{lu_{as}}) \\ \dot{T_h} = f_h(T_h, P_h, P_{ha}, P_{lu_{hs}}) \\ \dot{T_s} = f_s(T_s, P_s, P_{lu_{as}}, P_{lu_{hs}}) \end{cases}$$

where the detailed aspect of functions f_a, f_h and f_s depends basically on the chosen heat transfer correlations, which are highly inessential for the purpose of this chapter.

Since this model is used for an optimization problem, as discussed in Chapter 3, we present the cost function that has to be minimized. The cost function is set up in order to weigh the squared error of the ambient temperature with respect to its reference and the cost represented by the powers coming from external sources, i.e., P_h and P_s. The cost function J is therefore formulated as follows:

$$J = \int_{t=0}^{N_{opt}} \mu_{T_a}(\widehat{T_a} - T_a)^2 + \mu_{P_h} P_h + \mu_{P_s} P_s \, dt$$

where:

- μ_{T_a} weighs the squared error represented by the difference between the set point $\widehat{T_a}$ and the state variable T_a;
- μ_{P_h} weighs the power required by the heater to an external energy source;
- μ_{P_s} weighs the power required by the storage to an external energy source;
- N_{opt} is the control horizon considered in this application. It has been set equal to one day, i.e., 86400 seconds.

The weights listed above can be fixed in such a way to obtain the desired effect of the control action acting on the system. In this application, in order to obtain the ambient temperature as much as possible equal to its reference, we set μ_{T_a} three times bigger than μ_{P_h} and μ_{P_s}.

The aim of the optimization problem is to minimize the cost function J. To this end, the considered optimization variables are the following ones:

- c_{P_h}: a command variable defined in the interval $[0, 1]$. It allows to decide the optimal value of the power required by the heater to external sources;

- c_{P_s}: a command variable defined in the interval $[0, 1]$. It allows to decide the optimal value of the power required by the storage to external sources;

- $c_{P_{luas}}$: a command variable defined in the interval $[-1, 1]$. It allows to decide the optimal value of the power loaded or unloaded from the ambient to the storage;

- $c_{P_{luhs}}$: a command variable defined in the interval $[-1, 1]$. It allows to decide the optimal value of the power loaded or unloaded from the heater to the storage.

In accordance with the approach proposed in Chapter 4, we consider the control scheme shown in Figure 5.2.

The vector c_P of the command variables and the vector $\widehat{T}$ of the optimized temperatures are:

$$c_P = \begin{bmatrix} c_{P_h} \\ c_{P_s} \\ c_{P_{luas}} \\ c_{P_{luhs}} \end{bmatrix}$$

$$\widehat{T} = \begin{bmatrix} \widehat{T_a} \\ \widehat{T_h} \\ \widehat{T_s} \end{bmatrix}$$

In this scheme the role of the optimization is to calculate the vectors c_P and $\widehat{T}$, over a horizon of length N_{opt}. We recall that the former vector contains the optimal values for future control commands, the latter holds the corresponding trajectory of the controlled variables. The element named H, since it can be viewed as a generalized holder, has the role of acquiring the N_{opt} control commands and predicted outputs and applying them in sequence, one at each control step, to the feedback loop below. The controller R of this loop receives the optimal

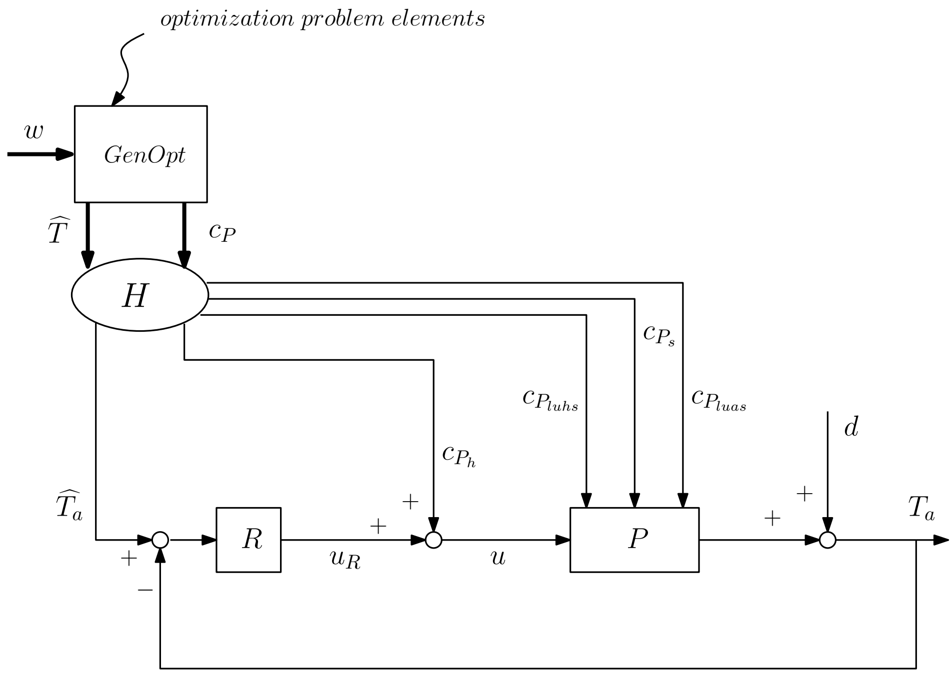


Figure 5.2: The scheme of Figure 4.1 specialized to this application (the optimization retriggering mechanism discussed later on, is not shown to improve readability).

temperature $\widehat{T_a}$ as set point, and the optimal control command c_{P_h} as additive bias, summed to its output u_R to produce the control u applied to the process.

Control commands, not directly involved as set-points or control signals in the feedback loop, are computed in the optimization process and fed directly to the process under control.

We highlight that there could be numerous ways to set the feedback loop. In this application, as clearly shown in the scheme, we focus on the control of the ambient temperature T_a. The regulator R will act on the command variable c_{P_h} that is related to the power required by the heater to an external source. This source represents the most expensive energy source, whose consumption has to be minimized.

According to what we stated about the retrigger criterion in section 4.1.3, in order to decide when a new optimization is needed, we look at the normalized index $J_{norm}(t)$. This index is calculated as follows:

$$J_{norm}(t) = \frac{J(t)}{J(t) + J_{opt}(t)} \tag{5.1}$$

Imposing a threshold to the value of this index, we determine when to consider the possibility of retaking the optimization. This threshold is chosen in accordance with the features desired for the system. Of course, the stricter the threshold, the bigger the number of the required optimization operations.

5.3 Simulation Results and brief discussion

In this section we show the results obtained by the simulation of the problem characterized before.

The simulation results are obtained using Dymola (Dynamic Modeling Laboratory) [6], a commercial modeling and simulation environment based on Modelica. The simulation environment plays a role of analysis and manipulation of the symbolic and numerical equations of the model, in order to produce highly efficient simulation code.

The control horizon N_{opt} considered for the simulation is an entire day, i.e., 86400 seconds.

In Figure 5.3 we consider the output T_a with respect to its reference $\widehat{T_a}$, the result of the optimization process.

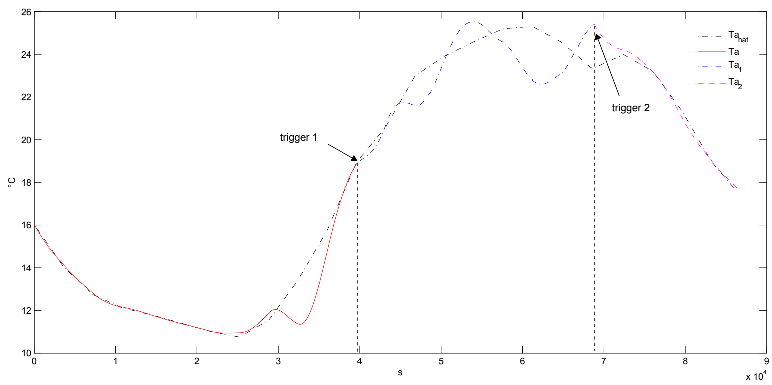


Figure 5.3: Ambient temperature during a day, when two triggers are needed.

The controlled output traverses three phases in time, denoted as T_a, T_{a_1}, and T_{a_2} and distinguished in Figure 5.3 with different colours and line styles. This is due to the fact that the optimization process has been carried on three times. The first time, related to the output T_a, has been computed at the beginning of the control horizon. The second and the third ones, related to T_{a_1} and T_{a_2}, have been computed after the first and the second retriggers, respectively. Both retriggers are caused by unforeseen disturbances on the output of the system. In particular, we consider disturbances represented by unexpected variations of P_d, the disturbance power (e.g. loads, room occupancy). Due to the presence of these disturbances, in Figure 5.3 we can see that the output does not perfectly follow its reference. In this respect, it should be remembered that in the cost function to be minimized there is not only the error represented by the difference of the output and its reference, but also the terms related to the cost of the energy supplied to the system. This means that the reference $\widehat{T_a}$ could be better followed by T_a, but not at a minimum cost, related to the power P_h and P_s supplied respectively to the heater and the storage.

The retrigger instants are chosen according to the criterion already discussed in the previous section. In order to better show when the retriggers are needed, we provide in Figures 5.4 and 5.5 a detailed view of the normalised index J_{norm} (see Equation (5.1)) and the corresponding threshold.

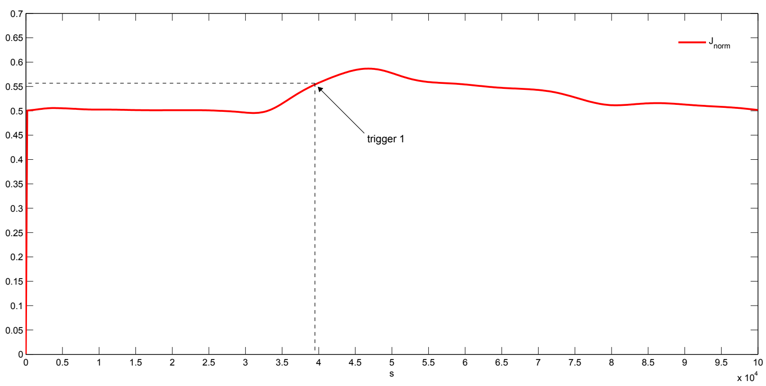


Figure 5.4: $J_{norm}(t)$ before the first trigger.

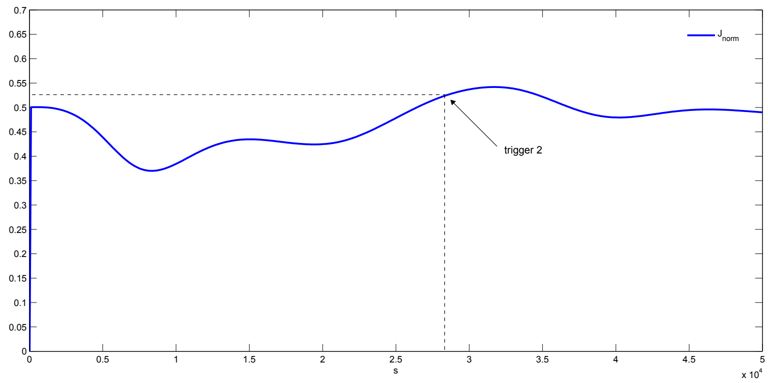


Figure 5.5: $J_{norm}(t)$ before the second trigger.

The two Figures just mentioned represent respectively the retrigger index $J_{norm}(t)$, before the first and the second trigger. The arrows indicate when the determined threshold on J_{norm} has been trespassed, i.e., when a new trigger is necessary. In particular, for the first trigger we have chosen a threshold on J_{norm} equal to 0.55411, that is overtaken at time t=39600 s. For the second one the threshold has been fixed to 0.5345 and it is overtaken at time t=68400 s. The overtaking of the threshold is due to the incoming disturbances that affect the system. Concerning the meaning of J_{norm}, we remind that the value of J_{norm} equal to 0.5, represents a perfect equality of the cost function $J(t)$ and the optimal cost function $J_{opt}(t)$.

Contrary to what we have done in the previous chapter, in this section it would make little (if any) sense to compare the previous result to the one obtained by using the predictive control in the classical receding horizon setting, performing the optimization at every control step as this would be practically infeasible for computational burden reasons, even in a small case like this. As a matter of fact, the optimization can not be done at each sampling time of the control horizon N_{opt}. This is due to the fact that the optimization process carried on by the program GenOpt lasts too much time. Moreover, we remember that this happens even if the problem considered is very simple, with respect to the possible problems that can be derived from the statement of Chapter 3. In this regard, we highlight that, using an optimization tool, there is a default overhead time (concerning the reading and writing of files, the simulation phase etc.) even if the problem is very simple. This fact highlights that our approach enables to solve this problem without incurring in an excessive computation time. In order to quantify the control quality degradation incurred in by adopting our technique, we now compare the previous results to those obtained in the nominal case, when no disturbances affect the system under control. Figure 5.6 shows the nominal output T_a with respect to its reference $\widehat{T_a}$. As expected, the nominal output perfectly follows the set point. This result, as already discussed before, is the best we can get in this proof of concept.

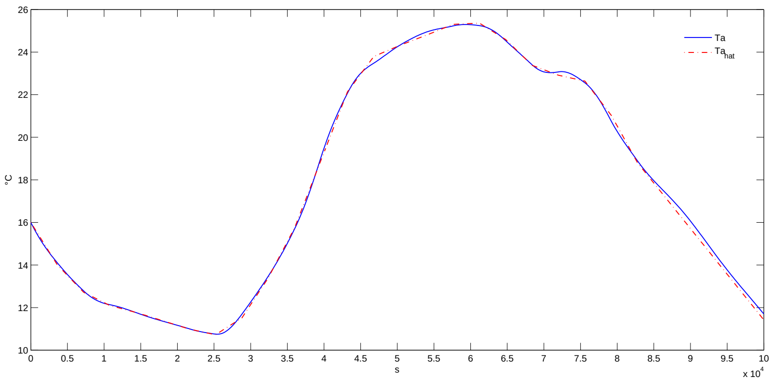


Figure 5.6: Ambient temperature during a day in nominal conditions.

In conclusion, we can state that the reported proof of concept has been successful. Applying our approach to an illustrative thermal system, we have proved that it works. Hence, it allows the resolution of the optimization problem, that, otherwise, could not have been solved. The thermal model considered in this application can easily be get more complex, since the optimization program GenOpt allows to analyze complex engineering problems.

Chapter 6

Conclusions and Future Perspectives

The problem of optimally controlling a building equipped with energy storages was considered. The control problem was formulated in order to be naturally compatible with a predictive control approach and the focus was restricted to thermal systems, since they are the most relevant and influential systems in a building. The considered system was characterised by one or more buildings and composed by a set of users, loads, energy storages, and external energy sources.

A general form to state the addressed problem was introduced in Chapter 3, and its complexity (coarsely) quantified with respect to the intended applications. In particular, a formal statement of the problem under analysis was presented, and the required mathematical notation was established. Since we considered a heating problem, the addressed problem was based on a thermal model.

An approach, named "Sporadic Model Predictive Control", was presented in Chapter 4. This control technique is aimed at solving the complex optimization problem considered, in such a way to reduce the computational time required by the optimization process. Thanks to this approach, the optimization process should not be carried out at each sampling time, but only when considered necessary. The proposed control scheme was analysed in a view to outline its tuning. The approach was later put to work by using a well assessed Model Predictive Control strategy as the Optimization technique of the control scheme proposed.

A proof-of-concept case was addressed and solved in Chapter 5, showing the capabilities of the approach. A real numerical optimization system was in place and, specifically, the application was carried on using the GenOpt optimization tool. The proposed approach, applied to an illustrative thermal system, has been proved working.

As a result, a method and a procedure are now available to use optimization tools in conjuction with modeling and simulation environments, like Dymola. Therefore, the addressed method and procedure allow the resolution of the presented optimization problem, that, otherwise, could not have been solved in a reasonable computation time.

We would like to end our dissertation by spending a very few words on some future perspectives that can derive from our work. Once the proposed approach has been demonstrated viable, as we did here by a convenient proof of concept, future research could be devoted to investigate optimization problems of higher complexity. Since our analysis of the problem was aimed exclusively at the validation of our approach, an additional methodological analysis is in order to further assess the property of the system. In particular, it would be interesting to create a complete methodological assessment by, for example, providing a robustness and sensitivity analysis of the approach or identifying its applicability limits. An other idea would be to compute how many optimization executions can be saved using our approach with respect to a classical predictive approach, based on the receding horizon technique. Finally, since the validation of our work has been carried on only in simulation, we suggest its experimental assessment on some laboratory apparatus.

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