POLITECNICO DI MILANO

Scuola di Ingegneria Industriale e dell'Informazione

Corso di Laurea Magistrale in
Ingegneria Gestionale

Reverse Factoring: Benefits and Concerns Addressed with a Formal Method

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Anno Accademico 2013 – 2014

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Acknowledgements

We would like to thank Professor Alessandro Perego, our supervisor, who gave us the opportunity to join Zaragoza Logistics Center and carry out this thesis. A heartfelt thank goes also to Professor Alejandro Serrano and Marta Romero, who warmly welcomed us at ZLC and conceded the opportunity to take the Financing System class.
We also thank Luca Gelsomino, our co-supervisor at Politecnico di Milano, for constantly monitoring our work, supporting and advising us.
We finally thank Spyridon Lekkakos, who has been our co-supervisor at ZLC over six months: not only an outstanding mentor and an excellent professor, but also a true friend.

Carlo and Pietro

I personally thank my parents, for their immeasurable trust, support and love. I thank Lavinia, for being a constant example and a loving sister. I thank Zeno and Giorgio, for the wonderful twenty years spent together. I also thank the rest of my family: my grandmother Silvia, my cousins Antonio, Clelia and Silvia, Aunt Anna and Aunt Bibi.
I thank my oldest friends: Giovanni, Zumi, Anna and Bianca, with whom I grew up in Verona, but also Pit, Coco and Lugo. I thank my friends in Milan, the most brilliant and inspiring people I have ever met: Gianluca, Giuliano and Romolo. I must thank Ali, Jay, Elena and Francesca for the amazing Erasmus in Germany, and Silvia, Martina, Javier, Jorge and Samuele, for the wonderful time in Spain. I thank also Alessia, Gloria and Benedetta, friends and excellent colleagues at Politecnico.
I finally thank Carlo, my thesis-peer, five-year classmate and awesome friend.

Pietro

I am grateful to my parents Pietro and Monica for the love, listening, support and constant encouragement I have received over the years. At the same time I thank Davide and Luigi for their trust and for being friends as well as brothers.
I'd like to thank my housemates in Milan, with whom I grew up in the last five years.
I thank my friends Gianluca, Giuliano and Romolo, who I admire and of whom I’m proud. I also thank my fellow students at Politecnico di Milano, excellent colleagues in numerous projects and friends: Giorgio, Michele, Maria, Mattia, Alessio and Valeria.
I express my gratitude to Javier and Jorge for being true friends since the beginning of my experience in Zaragoza, and Martina, Silvia and Samuele for their patience, understanding and friendship.
Finally, many thanks to Pietro, co-author of this thesis, motivating companion throughout the university years and an amazing friend.

Carlo

Abstract

The recent scenarios of the global economy pictured a situation where financial institutes and large companies started again to have available huge amounts of capital, due to light signals of economic upturn and to the monetary policies implemented by central banks. However, the actors who were most damaged by the crisis, i.e. the SMEs, are the same ones that still do not seem to enjoy these apparent improvements. The reason of this lies in the excessive carefulness that characterizes banks while granting loans and in the strict regulations imposed by Basel III, which push banks to select always the most creditworthy customers. It is in such contexts that Supply Chain Finance tools gain relevance for the economic growth: when there is capital, but it is not equally distributed by financial institutes.

One of the innovative instruments offered by SCF is the evolved Reverse Factoring (RF): a new version of traditional Reverse Factoring, in which the invoice payment-in-advance occurs in presence of more operative information, causing therefore a reduction in both risk and interest rate required by the financial institution.

The academic literature has extensively addressed the Reverse Factoring topic with a qualitative approach: industry surveys, researches, empirical findings that are able to explain the typical benefits and concerns tied up with this SCF tool. However, it is hard to find papers that describe RF with a formal method.

The first objective of this thesis is indeed to address RF with a systemic approach, providing three theoretical models that explain mathematically the decisions made by a firm after the adoption of RF. The second goal is to develop real models that emulate and prove the theoretic work, and to provide different insights on the optimal conditions under which RF works and the scenarios where RF does not perform well. A final testing is carried out with a case study: the model simulates the behavior of two SMEs that operate in Lombardia and tests whether a RF arrangement would add value or not.

Executive Summary

Introduction and objectives

Even though financial institutes recently started to have at disposal relevant amounts of cash and consequently to lower interest rates, thanks to the BCE policies, the general distrust and the strict regulations coming from Basel III still prevent SMEs from proper financing services. This issue worsens considering that SMEs are the ones that mostly need liquidity, since it has been claimed by several findings that in the aftermath of the 2008 financial crisis the deterioration of the SME borrowing capacity has often caused underinvestment problems. In this context, several Supply Chain Finance solutions have been developed, among which Reverse Factoring (RF). RF is initiated by large, investment or almost investment-grade firms as an instrument for soothing their suppliers' financing problems by leveraging on the creditworthiness of ordering parties. The purpose of this thesis is to address RF with a formal method and to elaborate several theoretical models able to explain how RF works and under which conditions it represents a winning solution for both buyers and suppliers.

In particular, the objectives of this work can be expressed through four research questions:

1) *What are the tangible benefits made possible by Reverse Factoring for those large buyers who are supposed to initiate it?*

This thesis seeks to provide a mathematical model able to show that Reverse Factoring (RF), under certain conditions, leads the buyer to a cost reduction. The model shows that the cost reduction is achieved through operational decisions that can be made thanks to the RF agreement. The model also aims to identify the conditions under which the supplier may refuse or accept to join Reverse Factoring, and to verify whether buyers' favorable conditions may jeopardize supplier's benefits.

2) *Focusing on suppliers, what are the benefits induced by the increased financing flexibility that Reverse Factoring grants?*

Reverse Factoring does not merely offer lower interest rates to suppliers, but also higher flexibility when they need to finance themselves. Other SCF tools, as Factoring or Traditional Reverse Factoring (TRF), do not let suppliers choose whether and when to discount invoices: these different conditions, besides lower interest rates, contribute to make RF a value-adding solution. This work aims to propose a mathematical model able to represent correctly these benefits, showing particularly how RF entails different cash dynamics.

3) *What are the operational implications caused by Reverse Factoring and the other SCF tools for suppliers?*

Another purpose is to show how Reverse Factoring may affect operational decisions such as inventory ordering policies, and compare these policies with the ones applied in absence of RF. Each different solution leads to a specific profit function, from which we mathematically derived the optimal replenishment policies. The model aims to show therefore how operating decisions are affected not only by the interest rate on hand, but also by the type of financing solution chosen.

4) *On which conditions Reverse Factoring outperforms the other financing means?*

The final research question that inspired this work concerns the comparison between RF and the other financing means, as Factoring, TRF, Asset Based Lending (ABL) and unsecured Credit Line (CL). The models developed have been extended in order to represent the dynamics implied when other financial tools are used, so that Reverse Factoring can be compared with them. Sensitivity analyses and simulations aim to identify the specific scenarios in which RF constitutes a better choice for suppliers and the ones in which it does not.

Literature review

A significant basis of the literature consists of scientific papers that illustrate the Supply Chain Finance existing models and other academic working papers that suggest further analyses or new perspectives to approach the subject: these provided a solid starting point for deriving the Reverse Factoring-focused models. Journal articles and reports that address Reverse Factoring and the other SCF solutions gave another consistent support in highlighting the main differences between them for the models' development. The references of this work can be thus divided in:

- Journal articles, 34% of total contributions referenced in this work;
- Books or book chapters, 10%;
- Reports, 16%;
- Unpublished contributions, 10%;
- Other sources, 30%.

The first branch of the literature review provides shared definitions of key concepts, such as *supply chain management, supply chain collaboration* and *supply chain finance*.

A second branch illustrates the existing frameworks and classifies the different SCF solutions, with a particular focus on Reverse Factoring, for which are described the major benefits and concerns for each player, as well as enabling factors and barriers.

The conclusive section is focused on the understanding of the current state of the models developed for the evaluation of RF.

Methodology

The methodology followed to carry out each model is similar and constituted by four steps:

1. Review and analysis of the papers that had already addressed the problem and provided a theoretical model to represent it;
2. Analysis of other journal articles, unpublished papers and reports to identify important aspects not yet addressed with a formal method by the literature;
3. Characterization of the assumptions and development of the mathematical model: equations and proofs (where needed);
4. Numerical analysis or simulation.

Models description

This work includes three models, differing from the focus or setting in which Reverse Factoring is evaluated.

Reverse Factoring and Economic Order Quantity in presence of Working Capital Constraints

The first model studies Reverse Factoring from a large buyer's point of view, trying to bridge a gap in the literature. This model is centered on the key concepts of Economic Order Quantity and Working Capital Requirements, i.e. the amount of capital a company must maintain in order to continue to meet costs and expenses, calculated as:

$$WCR = Cash + Accounts\ Receivable + Inventory - Accounts\ Payable$$

It is assumed that the buyer's CFO has set a limit on the Working Capital Requirement, preventing the purchasing department to order as much as the EOQ.

Reverse Factoring is thus used to increase the payment terms and relax the constraint of WCR, allowing the firm to reach the desired order quantity. However, under a collaborating Supply Chain hypothesis, we assume that the buyer would increase her *days payable outstanding* just as long as the supplier is better off or indifferent. The same hypothesis allows to presume that the buyer will try to extend her DPO as much as necessary to reach the EOQ, but not beyond that level. Since we assume a non-worsening condition for the supplier, it follows that the maximum payment extension the buyer may require is a function of the supplier's cost of debt, the current payment terms and the interest rate applied by the bank for RF.

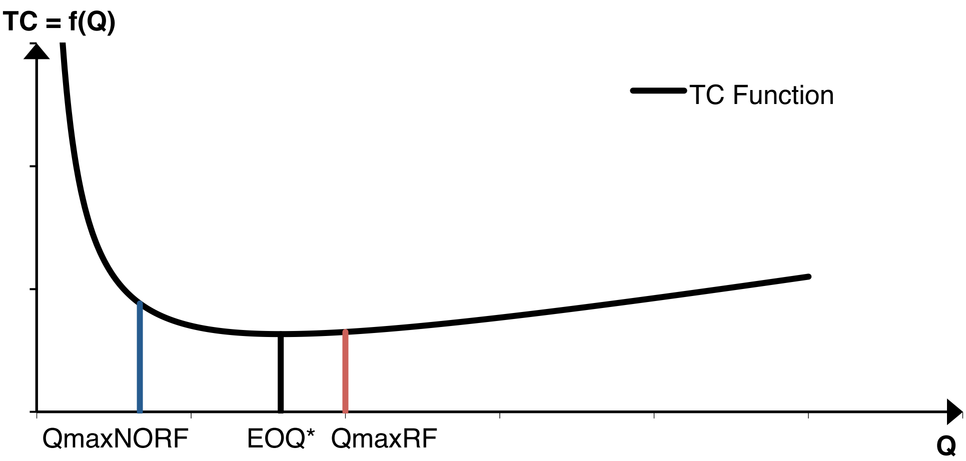


Figure 0.1: Total cost as a function of the order quantity Q.

The buyer's benefit from RF is quantified not only as lower tied-up capital, but especially in terms lower total costs thanks to the affordable EOQ policy. The supplier, on the other hand, is never worse off than the non-RF scenario by definition and he can exploit a lower interest rate to finance his operations.

Figure 0.2 shows that buyer and supplier can save up to 28,3% and 19,2% on total costs and financial costs respectively, while supply chain costs decrease by 23,7%.

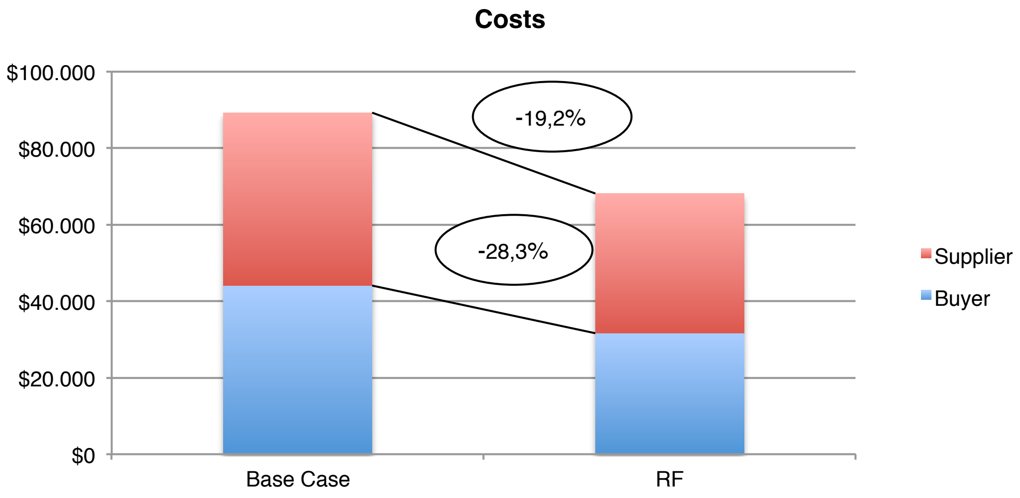


Figure 0.2: Buyer's and supplier's savings thanks to reverse factoring.

The sensitivity analysis proves that buyer's and supplier's savings are negatively correlated, depending on the parameters; i.e., when the WCR constraint is tight, the buyer seizes most of the benefit brought by RF to the supply chain.

Reverse Factoring and Growth

The goal of the second model illustrated in this thesis is to study the impact of RF on the growth ability of a small or medium-sized enterprise (SME). With regard to this focus, we choose a scenario in which the supplier has high growth-potential: a sub-contractor SME that supplies a large firm with a highly required product.

We extend the deterministic model developed by Buzacott and Zhang (2004), which studies the impact of asset based lending on the growth potential of a capital constrained firm that operates over multiple periods in a market with ample demand (never fully satisfied) and upstream and downstream payment delays. Starting from Buzacott and Zhang's model, we adjust it accordingly to capture the dynamics and trade-offs involved with an RF arrangement implementation. Indeed, we believe that Buzacott and Zhang's model is suitable for representing RF for several reasons:

i. It is a multi-period model, thus addressing the RF implementation environment more accurately;
ii. It consider the changes that RF imposes on all three financial statements of the firm, i.e. balance sheet, income statement and cash flow statement;
iii. The decisions of the SME are driven by a growth goal, which has been documented as one of the key motives for the implementation of RF initiatives by large buyers (Klapper 2006).

Then, diverting from Buzacott & Zhang's assumption of unlimited capacity, that allows the firm to grow without the need of any investment, we consider an alternative scenario in which the firm has a production capacity constraint: this capacity can be increased in any period through investments.

The financial state of the SME at the end of each period $t = 1, \ldots T$, is reflected on its three financial reports.. The balance sheet summarizes the SME's assets, liabilities, and shareholders' equity. Since our work concerns SCF solutions and therefore the management of the liquid assets of an SME, for simplicity, we do not consider the firm's Fixed Assets account and Long-term Loans account.

ASSETS	LIABILITIES
Cash	Equity
Accounts Receivable	Retained Earnings
Inventory	Short-term Debt
	Accounts Payable

Table 0.1: SME's balance sheet.

The model is then subdivided in three different cases:

A. The firm has no access to external financing and has available only cash reserves and the cash generated from operations;
B. The firm has access to unsecured loans at a fixed interest rate of r_s, limited by some line of credit ψ that is constant over time;
C. The firm has access to asset-based loans at the same interest rate r_s that are limited by a line of credit ψ_t, which is determined as a linear function of the size of the SME's liquid accounts (i.e., cash, receivables, inventories, and payables).

The measure used by the SME to track performance and compare the different financing solutions is the retained earnings, denoted by $\pi_t, t = 1, \dots, T$, where T is the end of the planning horizon. That is, we assume that the long-term objective of the SME is to maximize π_T. By ignoring the impact of demand uncertainty on the SME's decision making process, the SME's problem reduces to deciding on raw material purchases, production, and loan size (when applicable) to maximize the retained earnings at the end of the planning horizon, subject to the constraints of avoiding bankruptcy and not exceeding the credit limit.

Reverse Factoring

In order to provide more insights about the RF's value added and for consistency, we assume that the firm uses RF as an alternative to the other financing tools. Since the demand is strongly growing and never satisfied, we assume that the firm will discount all the invoices on hand. Consequently, the early payment of the firm's invoices is issued by the buyer's bank, discounted by a rate r_s^{RF} over a period l^{RF}, where $l^{RF} \geq l$, with l representing the former payment term. This last assumption implies that the buyer requires a payment extension in change of RF. Under this scenario, the SME's accounts receivable are factored by the buyer's bank and become zero. That is, whenever a shipment takes place, the respective value is directly transferred to the SME's cash account.

Numerical Analysis

To derive our results, we assume that prices, costs, and all other problem parameters are exogenously defined and constant over time.

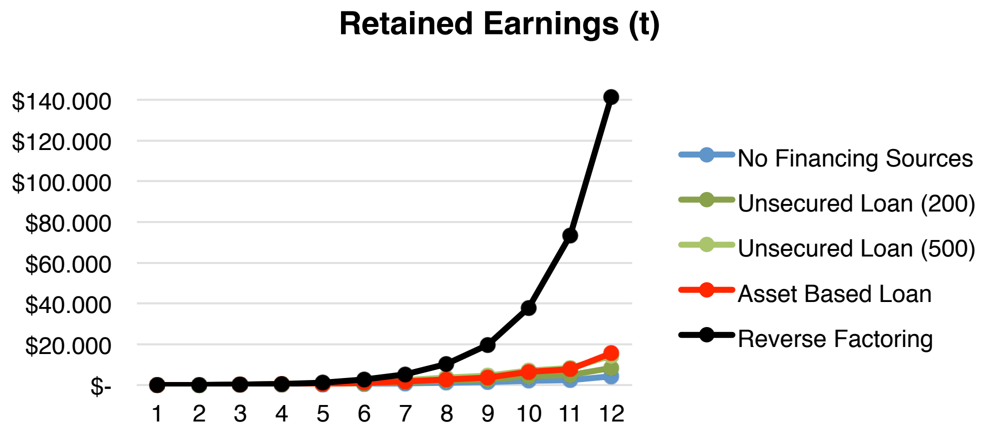


Figure 0.3: Retained earnings achieved thanks to different financing means.

According to the model, RF outperforms the other financing solutions in the long term. The wide difference in the last periods can be explained by the "never-satisfied demand assumption": the firm that uses RF has the possibility to collect a significant amount of cash earlier and re-invest it immediately in raw materials and labor, anticipating the growth. The unlimited capacity assumption also favors the gap spreading: since we suppose a scenario that does not need any investment in capacity even when the production increases by a hundred times, the sooner a firm is able to buy stocks, the faster it grows. This gives an interesting insight about RF potential: RF does not only imply more convenient interest rates, but also better timing of higher quantity of cash. The never-satisfied demand assumption is useful to understand the higher flexibility RF offers in respect to the other traditional financing means. The sensitivity analysis shows that when the discount rate of the RF arrangement, r_S^{RF}, is increased by 100% and exceeds the bank's interest for the SME, retained earnings are equal to $95.819. This means that it is still profitable for the SME to participate in an RF arrangement even if its terms are not so favorable. In other words, the positive impact from financial flexibility, in terms of *timely* access to financing, exceeds the negative impact from the cost of this financing.

Capacity Constraint

We also examine the case of finite capacity for the SME manufacturer. In particular, we assume that at time $t = 0$ the SME manufacturer has some initial capacity, denoted by K_0. Then, at the beginning of each period, the supplier can invest in additional capacity INV_t that becomes available with a one-period delay: $K_{t+1} = K_t + INV_t$. The additional capacity has a unit cost defined by c_K, while by K_T we denote the total capacity investment up to period T. For consistency with the assumptions we still do not represent the firm's fixed assets, meaning that the credit limit imposed by the bank does not count the new investments in capacity.

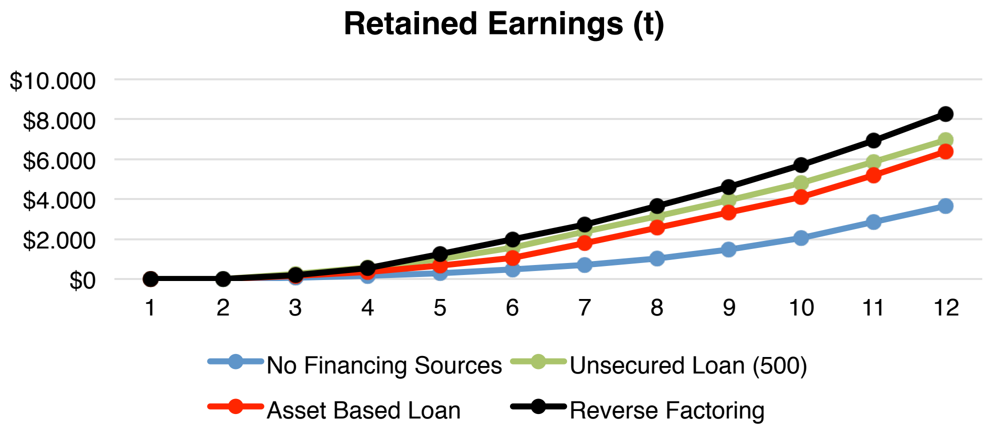


Figure 0.4: Retained Earnings with capacity constraint assumption.

We note that RF still outperforms the other financing means: its performance overcomes ABL by 29,7%. As expected, the gap between different solutions is not as wide as in the unlimited capacity case.

Reverse Factoring and Replenishment Decisions facing Stochastic Demand

The third model addresses the dynamics entailed by the application of Reverse Factoring when a firm is facing unknown and stochastic demand. The first part recalls the working paper *"Reverse Factoring: Implications on SMEs' operational decisions and performance"*, by Lekkakos and Serrano (2014). The authors take inspiration from the multi-period model developed by Luo and Shang (2013), who study an entrepreneurial firm with no access to bank financing that periodically orders inventory to satisfy non-decreasing demand with upstream and downstream payment delays. The authors used the Newsvendor Model framework (Arrow et al., 1951): an expected costs minimization under uncertain and stochastic demand. Lekkakos and Serrano follow a similar approach, but they extend their results by considering a firm's inventory replenishment problem when the firm has the possibility to factor its A/R. To the best of our knowledge, their work is the first attempt to model the inventory management problem of a firm that has access to receivables financing in a stochastic multi-period environment. The key aspect of their paper is an innovative perspective that takes into account a new decision variable: not only how many invoices to discount, but also which percentage of each one. In the Reverse Factoring literature, the authors usually represent this SCF tool as a simple loan, avoiding both the issues about which invoices to sell first (e.g. the most mature or the least mature) and which percentage of the chosen invoices to discount. Lekkakos and Serrano, instead, introduce the vector of the Accounts Receivable accumulated and the decision variable vector of how much of each A/R has to be discounted. Their model is more representative of the actions undertaken by a supplier after the adoption of an RF program.

Lekkakos and Serrano (2014) provide both a model for their base-case, in which the supplier does not have available any financing solution, and a model for Reverse Factoring. Comparing the two cases, they show how Reverse Factoring can improve the supplier's performances in terms of retained profit and demand coverage.

Starting from Lekkakos and Serrano's models, we propose several improvements in order to increase the accuracy of the profit function, of the single-period optimal ordering policy and of the cash dynamics. We also provide three additional extensions for three different scenarios: (i) when the supplier avails himself of Factoring, (ii) of Traditional Reverse Factoring and (iii) when he has access to an unsecured credit line. We run a simulation that compares all the cases illustrated (Base Case, Reverse Factoring, Traditional Reverse Factoring, Factoring and Credit Line) and provides some interesting conclusions about the value brought by each SCF solution. We then proceed testing the models under different assumptions regarding the demand pattern. The simulation both tests the goodness of the theoretic work and provides several insights comparing traditional solutions with Reverse Factoring. A sensitivity analysis examines how each SCF tool performs differently varying the initial cash conditions and gives some conclusive insights on the specific scenarios in which RF has the highest impact and generates most benefits.

Optimal Solutions

The model mathematically proves how a certain category of operational decisions, such as replenishment policies, are affected not only by the cost of capital, but also by the type of SCF tool utilized. Even though in many cases the literature already proved that cost of capital must be included in operating decisions, this model highlights that the interest rate of short-term capital is not the only financing information that matters for the COO: the cash flow dynamics are not less relevant. The fact that under RF cash is collected through a certain process, different from the credit line, and that the relative financing costs are calculated with different methods, have an impact on the *optimal inventory replenishment decision*. As we may notice, each different SCF tool leads to a different formula:

$$S_{TRF} = S_F = S_{NoFin} = F^{-1}\left(\frac{\beta^m \cdot p - c}{\beta^m \cdot p - c \cdot \beta + h}\right)$$

$$S_{CL} = F^{-1}\left(\frac{\beta^m \cdot p - c \cdot (1 + (r_s - r_u) \cdot \beta)}{\beta^m \cdot p + h - \beta \cdot c}\right)$$

$$S_{RF}^{t-n+k-1} = F^{-1}\left(\frac{\beta^n \cdot p - \frac{1}{(1 - r_{RF})^k} \cdot c}{\beta^n \cdot p - c \cdot \beta + h}\right)$$

Simulation

Each simulation compares RF with four other solutions: No Financing (NF), Credit Line (CL), Traditional Reverse Factoring (T-RF) and Factoring (F). The sensitivity analysis tests the results varying three basic assumptions: (i) the payment extension agreement when RF is implemented (no extension vs. 30 days extension), (ii) the demand pattern (Stationary, Growing, Seasonal), (iii) the initial cash on-hand (base case vs. -50% vs. +50%).

The results are summarized in Figure 0.5. We can notice that RF without payment extension outperforms the other solutions, thanks to both a cheaper interest rate and a more efficient use of finance. However, if we assume RF tied up to a 30 days payment extension (Figure 0.6), its performance decreases and the gap between RF and CL is almost null: in a considerable percentage of runs (27,5%) CL overcomes or at least equals RF, without considering the transactional cost of changing platform, provider, bank and similar elements. Therefore, the model suggests that the DPO-increase should not be an automatic request from buyers when they implement RF: sometimes, in fact, when they bear too much upon payment delays, instead of strengthening their supply-base, they weaken it.

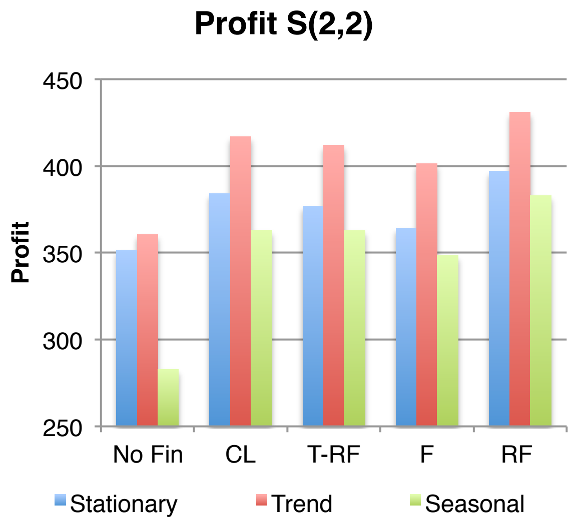


Figure 0.5: SME's profits under different demand assumptions, financing solutions, without DPO extension.

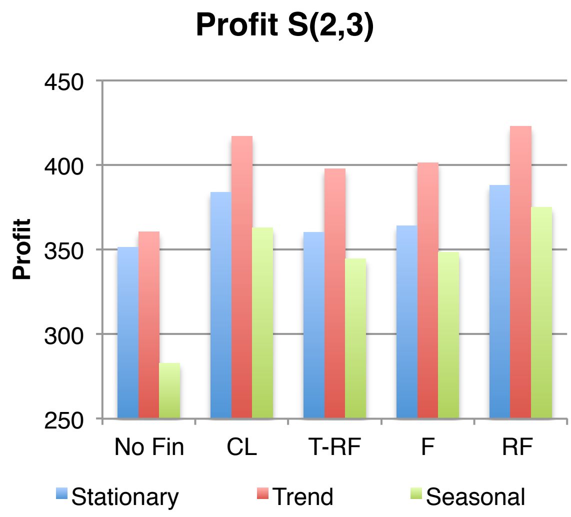


Figure 0.6: SME's profits under different demand assumptions, financing solutions, with DPO extension.

Another interesting insight given by the model concerns the scenarios in which RF performs better and represents a significantly worthwhile solution. For firms that face growing demand, RF creates more value than in presence of stationary demand, and outperforms most of the other existing solutions: however, in this kind of scenarios credit lines are as much as valuable when RF entails a payment lengthening. On the other hand, when a firm faces a strong seasonality in the demand pattern, RF reveals to be a winning solution, even when a DPO increase is implied. The reason lies in the flexibility that a tool such as RF is able to offer: in a variable context as the seasonal-demand's one, RF allows a company to reach remarkably high levels of working capital when needed and to decrease it immediately later.

As illustrated in paragraph 5.6.5 with a further sensitivity analysis, also the initial cash conditions have a relevant impact on the benefits that RF may generate in respect to solutions as Credit Line and No-Financing. At the same time, it is also proved that the value added by Traditional RF and Factoring does not depend on the initial cash conditions, because of the rigidity of auto-discounting agreements.

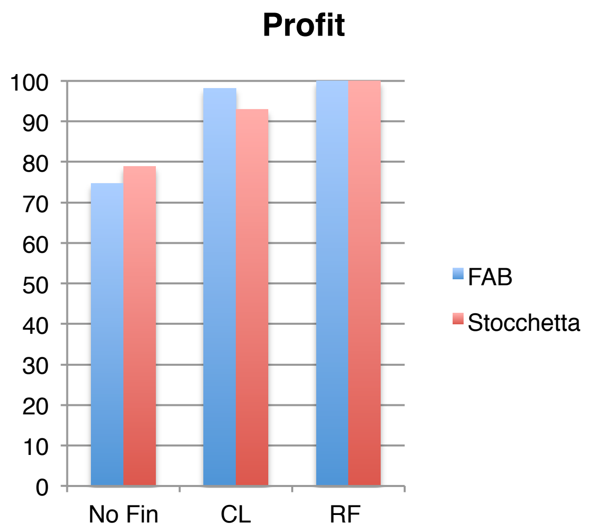
Case Studies

After some numerical analyses on the simulations, the third model has been applied to two Italian SMEs, FAB S.p.A. and Stocchetta Cilindri s.r.l., operating respectively in the personal protective equipment (PPE) and in the hydraulic cylinders industries.

Table X summarizes the most relevant parameters used in the simulations and highlights the differences between the two firms and the respective industries: Stocchetta Cilindri faces a more volatile and growing (+2,289% per month) demand than FAB and has lower margins; on the other hand, FAB is more cash restricted as the initial cash, €6,93, would only cover 0,42% of an average order.

	average D (u/month)	VC (σ/μ)	Initial Cash (€)	Initial Cash (# of orders)	p (€/u)	c (€/u)	Margin %	Holding Cost (€/u per year)	Margin/ holding	annual r_s
FAB	4.000	15%	6,93	0,0042	0,6	0,4174	44%	0,042	4,37	4%
Stocchetta	633	49,6%	151.192	0,6528	440	365,9	20%	36,59	2,03	4,25%

Table 0.2: Parameters used in the simulations.



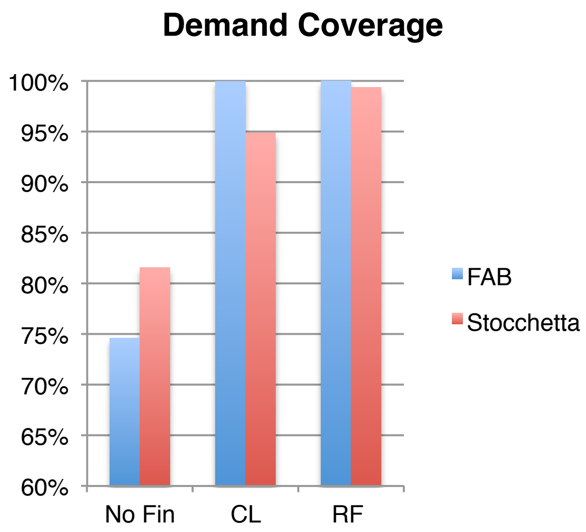


Figure 0.7: Average profits over 1.000 runs.

Figure 0.8: Demand coverage over 1.000 runs.

Figure 0.7 illustrates the retained profit at the end of a 12 months period: in both case studies we set the maximum profit, obtained with RF, to 100 and proportionally calculated the other profits, in order to allow comparisons between different industries. Figure 0.8 shows that FAB has a lower demand coverage in absence of financing, due to the tighter cash conditions. Nevertheless, when accessing to financing, it covers more demand than Stocchetta thanks to the lower volatility.

The two case studies validate the insights gathered from the previous analyses: in both applications the absence of financing considerably affects in a negative manner the ability of the SME to reach the order-up-to level and its service level, with consequences on the retained profit at the end of the horizon T. The second case study also proves that RF is more valuable when demand is growing and remarkably variable, other than showing how an SME can exploit the greater flexibility of RF compared to the credit line.

Conclusions and future research

The models developed and the analyses carried out in this work address several matters and provide some insights about them. The first model argues about the *direct* benefit for buyers when they implement RF, i.e. payments extension, highlighting the scenarios in which these kind of advantages are more incisive and those where instead buyers would not profit much from them. The third model as well suggests some conditions under which buyers actually create value demanding a DPO lengthening, but also the conditions where such an agreement would not be optimal for both suppliers and buyers.

A second argument concerns the value creation for suppliers thanks to a RF agreement and the context in which this SCF tool would not only add value but also outperform other financing means. Our second model shows that RF exceeds significantly different tools such as Asset Based Lending and Unsecured Loans in a context of high-growing demand. The third model, then, suggests that RF usually performs sensibly better than Traditional RF and Factoring, especially in situations of high-variable demand; in such contexts, RF overcomes significantly also Credit Lines. However, it is shown how in other scenarios RF might either not add any value or even worsen supplier's performance. Other analyses show also that RF performance depends on the initial working capital condition set: the lower is the initial liquidity, the higher is the need of cash, the more useful RF turns.

Finally, the third model mathematically proves how different financing solutions may bring to different formulas and operational decisions as the base-stock replenishment level. The model suggests therefore that finance and operations should not be considered as independent functions, they should indeed cooperate since their decisions affect each other's performance.

The models illustrated leave a vast space for further developments:

- Concerning the first model and the benefits for the buyers initiating a RF program, it could be realized a model that evaluates the so-called *indirect benefits*. It could be investigated how much a buyer can gain from a *supply-base risk reduction* or from a *stronger relations (partnership)* and a higher *service level* due to RF.
- The second model can be further extended considering also the up-stream suppliers' capacity: in fact, we assumed the supplier to be able to grow as much as allowed from its liquidity, independently from its environment. A model that considers also the up-stream capacity constraint would be more reliable in terms of supplier's growth-expectations. An additional improvement would regard the demand pattern, which could be set as stochastic and non-infinite, leading to different production launches and purchasing decisions.
- The third model could be improved including payables in the suppliers' working capital requirement. This would bring much more complexity in the myopic profit function and the optimal order policy, but it would provide interesting insight on how payables influence the impact of RF for suppliers. Another interesting improvement would take down the myopic policy assumption, deriving a replenishment policy that maximizes the multi-period profit. A further research should also consider the production lead-time, since WIP inventories do have a strong impact on WCR. Finally, taxes may also be included in the mathematical model in order to consider the interests tax-shield and how they can smooth or exalt the gap between different financing solutions.

Overall, the theoretical work developed in this thesis should be the base for further and more empirical research, e.g. an applicative model that can evaluate the added value of RF for a specific firm, customized on its processes and peculiarities.

Sommario

Introduzione ed obiettivi

Nonostante gli istituti finanziari recentemente abbiano iniziato ad avere a disposizione maggiore liquidità e, di conseguenza, ad abbassare i tassi di interesse, grazie alle politiche economiche della BCE, la sfiducia generale e le severe regolamentazioni di Basilea III impediscono tuttora alle Piccole e Medie Imprese (PMI) di avvalersi di servizi finanziari adeguati. Il problema si fa ancor più grave considerando che sono proprio le PMI ad avere il maggior bisogno di liquidità, dal momento che diversi studi hanno rivelato che il deterioramento della capacità di indebitamento delle PMI causato dalla crisi del 2008 ha spesso portato a problemi di sotto-investimento. In questo contesto sono state sviluppate una serie di soluzioni finanziarie alternative denominate Supply Chain Finance, tra cui il Reverse Factoring (RF). Il RF viene utilizzato dalle grandi imprese caratterizzate da un buon rating come strumento per estendere i termini di pagamento, ma anche per risolvere i problemi finanziari dei loro fornitori (PMI), facendo leva sul proprio merito di credito. Lo scopo di questa tesi è quello di studiare il RF con un metodo formale e di elaborare alcuni modelli teorici in grado di spiegare come funziona questo strumento e sotto quali condizioni rappresenta una soluzione vincente sia per i fornitori che per i clienti.

In particolare, gli obiettivi del presente lavoro possono essere espressi attraverso quattro domande di ricerca:

1) *Quali sono i benefici tangibili resi possibili dal Reverse Factoring per i grandi clienti che lo dovrebbero avviare?*

L'obiettivo è quello di fornire un modello matematico in grado di dimostrare che il RF, sotto determinate condizioni, permette al grande cliente di ridurre i costi. Il modello mette in evidenza che tale riduzione di costo è raggiunta attraverso decisioni operative differenti, rese possibili proprio dal RF. Vengono inoltre studiate le condizioni alle quali il fornitore accetta o rifiuta l'accordo e le condizioni che causano una distribuzione parziale dei benefici tra i due attori, favorendo il buyer.

2) *Focalizzandosi sui fornitori, quali sono i benefici indotti dalla maggior flessibilità di finanziamento ottenibile col Reverse Factoring?*

Il RF non offre solo tassi d'interesse più bassi alle PMI fornitrici, ma anche una maggior flessibilità quando hanno bisogno di raccogliere liquidità. Altre forme come il Factoring o il TRF non permettono di decidere né quali, né quante, né quando scontare le fatture: queste differenze sostanziali, oltre ai tassi d'interesse più vantaggiosi, rendono il RF una soluzione a valore aggiunto. Questo lavoro di tesi mira a proporre un modello che metta in evidenza tali differenze e i benefici derivanti, mostrando in particolare come il RF implichi dinamiche di cash flow diverse dalle altre soluzioni.

3) *Quali implicazioni di tipo operativo comportano per il fornitore il Reverse Factoring e le altre soluzioni di finanziamento?*

Un ulteriore scopo è quello di dimostrare come il RF può influenzare le decisioni di tipo operativo, come la politica di riordino delle scorte, e di comparare tali decisioni con quelle prese con altri metodi di finanziamento. Ogni strumento implica una differente funzione di profitto, dalla quale si derivano differenti politiche di riordino ottimali. Lo scopo è quello di suggerire che tali decisioni non andrebbero prese guardando solo al tasso d'interesse, ma anche al tipo di soluzione finanziaria a cui si ricorre.

4) *Sotto quali condizioni il RF è superiore agli altri metodi di finanziamento?*

L'ultima domanda di ricerca si concentra sulla comparazione del RF con le altre soluzioni sia di SCF, come il factorig, il traditional reverse factoring, l'asset based lending e la linea di credito. A tal fine il modello rappresentante il RF è stato esteso anche alle altre opzioni, con le opportune modifiche sia nella funzione profitto che nelle dinamiche di cassa. Le simulazioni e le analisi di sensitività mirano ad identificare gli scenari specifici in cui il RF rappresenta la miglior soluzione sia quelli in cui non lo è.

Analisi della letteratura

Una parte significativa della letteratura consiste in paper scientifici che illustrano gli esistenti modelli di SCF e in altri working paper che suggeriscono ulteriori analisi o nuove prospettive con cui approcciare al RF: sono questi ultimi ad aver fornito un solido punto di partenza dal quale sviluppare i modelli teorici presenti in questo elaborato. Un altro sostanzioso contributo è stato dato dagli articoli di giornale e dai report riguardanti il RF e le altre soluzioni di SCF, evidenziandone le principali differenze, utili per lo sviluppo dei modelli. Le tipologie di contributi sono quindi così suddivise:

- Articoli scientifici, per il 34% dei contributi;
- Libri di testo, 10%;
- Manuali e report di osservatori, 16%;
- Contributi non pubblicati, 10%;
- Altre tipologie, 30%.

Il primo filone di letteratura definisce ed illustra alcuni concetti chiave come *supply chain management, supply chain collaboration* e *supply chain finance*. Un secondo filone descrive i framework e modelli esistenti e classifica le diverse soluzioni di SCF con un focus particolare sul Reverse Factoring, evidenziandone vantaggi e svantaggi, oltre che fattori abilitanti e possibili barriere. La parte conclusiva dell'analisi della letteratura presenta gli esistenti modelli teorici che mirano a valutare i benefici del RF.

Metodologia

La metodologia seguita per lo sviluppo di ciascun modello è simile ed è costituita da 4 passi fondamentali:

1. Revisione ed analisi dei paper scientifici che hanno già affrontato il problema in questione e fornito un modello teorico per rappresentarlo;
2. Analisi di altri paper, articoli di giornale o materiale non pubblicato al fine di trarre spunto su prospettive o problematiche non ancora rappresentate attraverso un metodo formale;
3. Elaborazione delle assunzioni e sviluppo del modello matematico, con relative dimostrazioni (se necessarie);
4. Analisi numerica e simulazione.

Descrizione dei modelli

Il presente elaborato include tre modelli che differiscono sia per focus specifico che per framework sottostante.

Reverse Factoring ed Economic Order Quantity in presenza di vincoli di Working Capital

Il primo modello analizza il RF dal punto di vista di un grande cliente, provando a colmarne la lacuna nella letteratura. Si tratta di un modello che trae spunto dall'Economic Order Quantity e si fonda sul concetto di

Working Capital Requirement, i.e. l'ammontare di capitale che un'impresa deve avere a disposizione al fine di continuare ad adempire alle spese e ai costi:

$$WCR = Cassa + Crediti\ Commerciali + Rimanenze - Debiti\ commerciali$$

Una delle assunzioni principali è che il CFO dell'azienda cliente abbia imposto alla funzione acquisti un vincolo sul WCR raggiungibile, impedendole in questo modo di raggiungere l'EOQ. Il RF viene quindi implementato dal cliente al fine di poter ottenere un allungamento nei termini di pagamento e rilassare in questo modo il proprio vincolo di WCR, così da poter raggiungere la quantità di riordino desiderata. Tuttavia, sotto un'ipotesi di Supply Chain Collaborativa, assumiamo che il cliente vada ad allungare i propri giorni di pagamento solo se il fornitore non peggiora la sua situazione, e.g. se i suoi costi finanziari non aumentano. La stessa ipotesi consente di presumere che il buyer vada a chiedere l'estensione dei pagamenti fino al livello che gli consente di ordinare l'EOQ, e non oltre. Dal momento che vi è un'assunzione di non-peggioramento del fornitore, ne consegue che l'estensione massima dei termini di pagamento richiedibile dal cliente è funzione del costo del debito del supplier, del precedente termine di pagamento e del tasso d'interesse del RF.

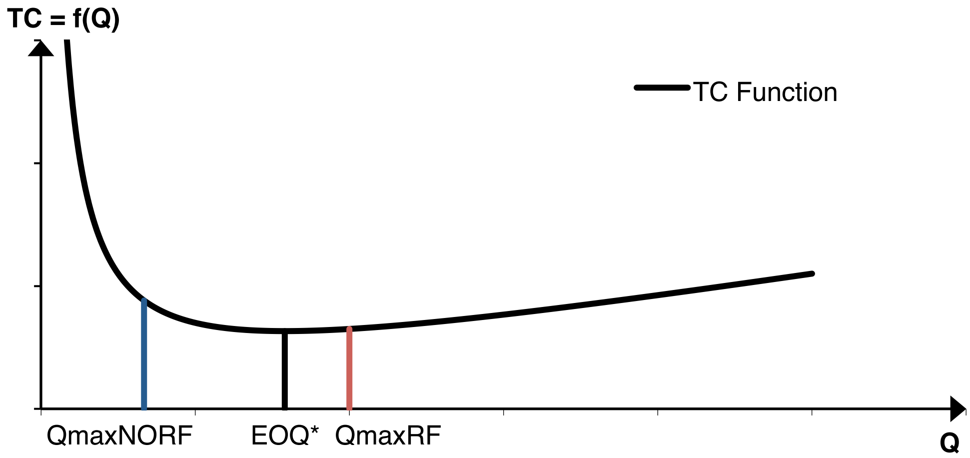


Figura 0.1: Costi Totali in funzione della quantità ordinata Q.

Il beneficio principale per il cliente non è quindi rappresentato semplicemente dal minor capitale "congelato" in WCR, ma risiede soprattutto in una riduzione dei costi totali grazie ad una politica d'acquisto più efficiente. Dall'altro lato il fornitore non peggiora mai la proprio condizione iniziale e, anzi, può beneficiare di un tasso d'interesse più basso con cui finanziarsi. La Figura 0.2 mostra come cliente e fornitore possano risparmiare rispettivamente il 28,3% e il 19,2% dei costi, mentre i costi totali della supply chain calino del 23,7%.

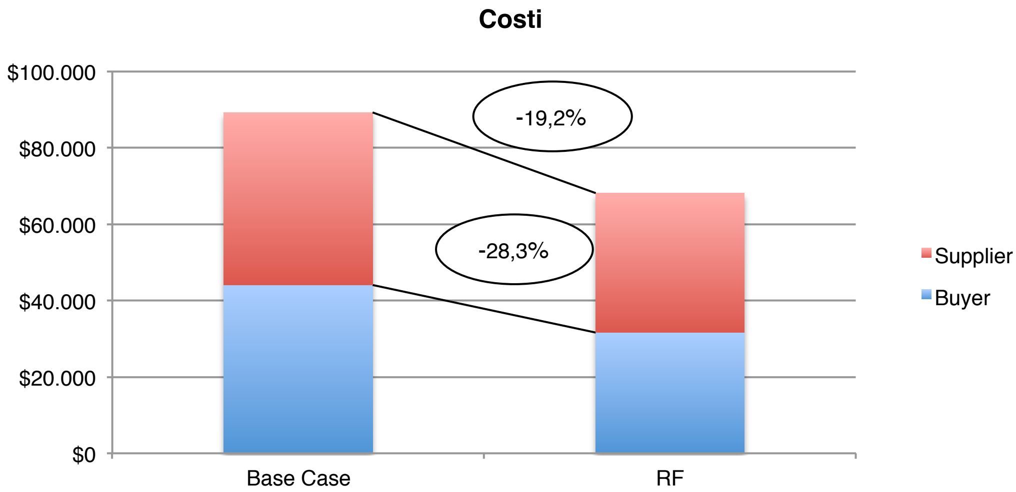


Figura 0.2: Risparmi del cliente e del fornitore grazie al RF.

L'analisi di sensitività prova inoltre che i risparmi dei due attori dovuti al RF sono negativamente correlati; i.e., quando il vincolo di WCR è molto stringente e il cliente ha un grosso margine di benefici da trarre attraverso l'allungamento dei termini di pagamento, la maggior parte dei risparmi è ottenuta da quest'ultimo.

Reverse Factoring e Crescita

L'obiettivo del secondo modello è quello di studiare l'impatto del RF sulla crescita di una PMI fornitrice. Ponendo questo come focus, si è scelto uno scenario in cui il fornitore abbia un alto potenziale di crescita: una PMI captive, che produca esclusivamente per conto di un grande cliente un prodotto altamente richiesto.

A questo scopo, viene proposta nel presente elaborato un'estensione del modello di Buzacott e Zhang (2004), il quale studia l'impatto dell' Asset Based Lending (ABL) sulla crescita potenziale di una PMI. L'impresa opera lungo più periodi, in un mercato caratterizzato da una domanda mai pienamente soddisfabile (ciò che viene prodotto viene sempre consumato), con termini di pagamento allungati sia coi fornitori che coi clienti. Il modello di Buzacott è stato opportunamente corretto in modo da catturare le dinamiche ed i trade-off implicati dal RF. Infatti, si è ritenuto il modello di Buzacott particolarmente adatto a rappresentare le caratteristiche del RF per i seguenti motivi:

i. È un modello multi-period, che quindi rappresenta le dinamiche d'implementazione del RF in maniera più accurata;
ii. Il modello considera i cambiamenti che il RF implica su tutti e tre i documenti del bilancio dell'impresa: conto economico, stato patrimoniale e rendiconto finanziario;
iii. Le decisioni della PMI sono guidate da un obiettivo di crescita, che è stato documentato essere una delle motivazioni chiave per l'implementazione del RF da parte dei grandi clienti (Klapper, 2006).

Inoltre, distaccandosi dalle ipotesi di Buzacott e Zhang riguardanti la possibilità di crescita senza investimenti da parte della PMI, viene presentata un'ulteriore estensione in cui la PMI ha un limite di capacità produttiva, che può essere incrementata all'inizio di ciascun periodo attraverso investimenti. La condizione finanziaria della PMI alla fine di ciascun periodo $t = 1, \ldots, T$ si riflette nei tre documenti di bilancio. Lo stato patrimoniale riporta le attività, le passività e il capitale di rischio. Dal momento che il focus principale è sulle soluzioni di SCF e quindi sulla gestione degli asset liquidi di una PMI, per semplicità e coerenza con le assunzioni del modello base, nello stato patrimoniale non vengono considerati né gli asset fissi né il debito di lungo termine.

ATTIVO	PASSIVO
Cassa	Capitale Sociale
Crediti Commerciali	Utili portati a nuovo
Rimanenze	Debito di breve termine
	Debiti Commerciali

Tabella 0.1: Stato Patrimoniale della PMI.

Il modello viene successivamente diviso in tre diversi casi:

A. La PMI non ha accesso a capitale di debito e ha a disposizione solamente la cassa iniziale e i flussi derivanti dalla gestione operativa;
B. La PMI ha accesso ad una linea di credito ad un tasso di interesse fisso pari ad r_s, limitato da un tetto fisso ψ che è costante nel tempo;
C. La PMI ha accesso all'ABL, allo stesso tasso d'interesse r_s, limitato da un tetto pari a ψ_t che varia nel tempo come una funzione lineare dei suoi asset ed oneri liquidi (cassa, crediti commerciali, rimanenze e debiti commerciali).

L'indicatore utilizzato per misurare la performance della PMI e per comparare le diverse soluzioni finanziarie sono gli utili portati a nuovo, rappresentati da $\pi_t,\ t = 1, \dots, T$ dove T è la fine dell'orizzonte temporale. Assumiamo quindi che la PMI abbia come obiettivo di medio-lungo termine quello di massimizzare π_T. Dal momento che viene trascurato l'impatto dell'incertezza della domanda nel processo decisionale della PMI, quest'ultimo si riduce a stabilire quante materie prime comprare, quante unità lanciare in produzione e quanto capitale prendere a debito (quando è possibile). Le decisioni vengono prese per massimizzare gli utili portati a nuovo dell'ultimo periodo di vita, rispettando i vincoli che impongono di evitare la bancarotta e non eccedere il tetto della linea di credito.

Reverse Factoring

Per coerenza e col fine di trarre maggiori conclusioni dal confronto del RF con le altre soluzioni, assumiamo che l'azienda utilizzi il RF alternativamente ad esse. Essendo la domanda in forte crescita e mai pienamente soddisfabile dalla PMI, assumiamo che quest'ultima sconti sempre tutte le fatture a disposizione, col fine di rendere disponibile al cliente più prodotti possibili. Di conseguenza, il pagamento anticipato delle fatture viene effettuato dalla banca del cliente, che le sconta ad un tasso r_s^{RF} lungo un periodo l^{RF}, dove $l^{RF} > l$, con l che rappresenta i precedenti termini di pagamento. Quest'assunzione considera quindi che il cliente richieda un'estensione dei suoi termini di pagamento in cambio del RF. Sotto queste ipotesi, i crediti commerciali della PMI vengono sempre scontati dalla banca del cliente, annullandosi; i.e. ogni volta che avviene una spedizione di merce, il valore rispettivo viene trasferito nel conto della PMI.

Analisi Numerica

Assumiamo che prezzo, costi unitari e gli altri parametri del problema siano esogeni e rimangano costanti nel tempo.

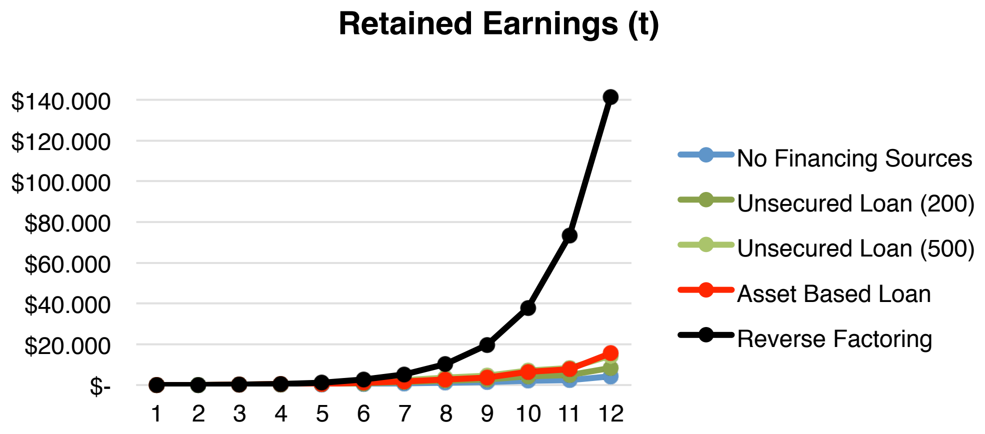


Figura 0.3: Utili portati a nuovo a seconda del tipo di finanziamento.

Notiamo in Figura 0.3 come il RF sia notevolmente superiore agli altri metodi di finanziamento nel lungo termine. La considerevole differenza tra gli utili portati a nuovo degli ultimi periodi è riconducibile all'assunzione di domanda insoddisfacibile: l'azienda che utilizza il RF, infatti, ha la possibilità di raccogliere quantità significative di cassa anticipatamente, potendoli così re-investire prima in materie prime e manodopera, anticipando in questo modo la crescita. L'assunzione di capacità illimitata favorisce ulteriormente il gap: poiché supponiamo non sia necessario alcun investimento in capacità produttiva, nemmeno quando le vendite raggiungono quantità cento volte superiori rispetto a quelle iniziali, prima la PMI compra e produce, più velocemente cresce. Queste assunzioni, estreme, in realtà permettono all'analisi di fornire uno spunto interessante sul RF: esso non è superiore semplicemente per il tasso d'interesse vantaggioso, ma anche perché consente di raccogliere liquidità con maggior tempismo e in quantità superiori, soprattutto in contesti di crescita. È quindi uno strumento caratterizzato da maggior flessibilità. L'analisi di sensitività svolta sul medesimo modello dimostra che in tale contesto, persino aumentando r_s^{RF} del 100% e rendendolo superiore al costo delle soluzioni tradizionali, il RF si confermerebbe la soluzione più conveniente. Questo conferma quanto detto prima, ossia che i benefici del RF in termini di flessibilità e qualità del finanziamento, in contesti di forte crescita, sono di gran lunga superiori all'effetto del tasso di interesse.

Vincolo di capacità

È stato inoltre sviluppato uno scenario a capacità limitata. In particolare, si assume che in $t = 0$ la PMI goda di una certa capacità iniziale, indicata con K_0. All'inizio di ciascun periodo il fornitore può investire in capacità addizionale definita con INV_t, e resa disponibile dal periodo successivo: $K_{t+1} = K_t + INV_t$. La capacità addizionale ha un costo unitario di c_K, mentre con K_T si indica la totale capacità disponibile al periodo T. Per coerenza continuano ad essere trascurati gli asset fissi della PMI, così da rendere ininfluente nel calcolo del tetto della linea di credito il nuovo investimento in capacità.

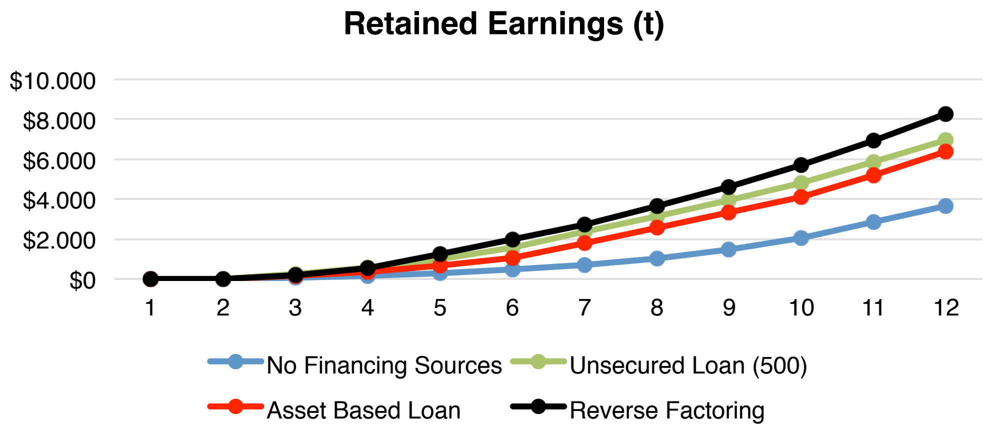


Figura 0.4: Utili portati a nuovo in presenza di vincolo di capacità produttiva.

Si può notare come il RF sia anche in questo caso migliore degli altri strumenti finanziari: gli utili portati a nuovo dell'azienda che si avvale del RF superano del 29,7% quelli dell'azienda con ABL. Tuttavia, naturalmente, la differenza negli utili non è più così pronunciata come nel caso di capacità illimitata.

Reverse Factoring e Decisioni di Ripristino delle Scorte con domanda stocastica

Il terzo modello studia le dinamiche legate al Reverse Factoring nel caso in cui un'azienda debba far fronte ad una domanda stocastica. La prima parte richiama il paper *"Reverse Factoring: Implications on SMEs' operational decisions and performance"*, elaborato da Lekkakos e Serrano (2014). Gli autori traggono ispirazione dal modello multi-periodo sviluppato da Luo e Shang (2013), i quali studiano un'impresa senza accesso a finanziamenti bancari che ordina periodicamente materiali per soddisfare una domanda non-decrescente, con dilazioni di pagamento a monte ed a valle. Alla base vi è il framework del newsvendor problem (Arrow et al., 1951), noto anche con il nome di newsboy problem, volto alla minimizzazione dei costi attesi sotto l'assunzione di una domanda incerta e stocastica. Lekkakos e Serrano seguono un approccio simile ed estendono i loro risultati considerando il problema di ripristino delle scorte nel caso in cui un'azienda abbia la possibilità di finanziarsi cedendo i propri crediti commerciali. Sulla base delle informazioni che abbiamo reperito, si tratta del primo tentativo di modellare un problema di gestione delle scorte contemplando la possibilità di accedere a soluzioni di finanziamento come il reverse factoring in un contesto stocastico e multi-periodo. L'aspetto chiave del loro paper è una prospettiva innovativa, che tiene in considerazione una nuova variabile decisionale: non solo quante fatture, ma anche quale percentuale di ciascuna di esse è opportuno scontare. Nella letteratura riguardante il Reverse Factoring, gli autori solitamente modellizzano questo strumento di SCF come un semplice prestito, evitando di affrontare i problemi: (i) di quali fatture cedere prima (es. dalla più matura alla meno matura), (ii) di quale percentuale scontare. Lekkakos e Serrano introducono un vettore di crediti commerciali accumulati ed un secondo vettore composto dalle variabili decisionali *"quale porzione di ciascuna fattura scontare"*. Il loro modello è quindi più rappresentativo delle decisioni operate dal fornitore dopo aver preso parte ad un programma di RF.

Gli autori mostrano quindi come il RF possa migliorare le performance del fornitore in termini di profitto e livello di servizio, facendo un confronto con il caso in cui non sia possibile accedere ad alcuna fonte esterna di finanziamento del capitale circolante.

Partendo dal modello di Lekkakos e Serrano, proponiamo alcune migliorie che rendono più accurate la funzione del profitto, la politica di riordino ottimale nel singolo periodo e le dinamiche dei flussi di cassa. Proponiamo inoltre tre estensioni che consentano di valutare i seguenti scenari: (i) quando il fornitore si finanzia tramite Factoring, (ii) quando si finanzia tramite il Reverse Factoring Tradizionale e (iii) quando ha accesso ad una linea di credito. Conduciamo una simulazione che confronta i casi illustrati e consente di trarre interessanti conclusioni riguardo al valore di ciascuna soluzione di SCF. Segue il testing dei modelli con diverse assunzioni sulla domanda. Infine è stata svolta un'analisi di sensitività per valutare come variano le performance di ciascuno strumento di SCF al variare delle condizioni di cassa iniziale, consentendo di definire in quali scenari specifici il RF ha un impatto più rilevante e genera maggiori benefici.

Soluzioni Ottimali

Il modello prova matematicamente come alcune decisioni operative, quali la politica di ripristino delle scorte, siano interessate non solo dal costo del capitale, ma anche dalla tipologia dello strumento di SCF scelto. Nonostante sia già stato provato in letteratura che il costo del capitale debba essere incluso nella valutazione delle decisioni operative, il presente modello evidenzia come il tasso d'interesse dei debiti a breve termine non sia l'unica informazione prettamente finanziaria d'interesse del direttore operativo: le dinamiche dei flussi di cassa non sono infatti meno importanti. Il fatto che il processo con il quale il RF crea liquidità per le aziende sia differente rispetto a quello della linea di credito e che gli interessi vengano calcolati in maniera differente, ha impatto sulla *decisione ottimale di ripristino delle scorte*. Come è possibile notare, ciascuno strumento di SCF porta ad una specifica formula di definizione del livello di riordino ottimale:

$$S_{TRF} = S_F = S_{NoFin} = F^{-1}\left(\frac{\beta^m \cdot p - c}{\beta^m \cdot p - c \cdot \beta + h}\right)$$

$$S_{CL} = F^{-1}\left(\frac{\beta^m \cdot p - c \cdot (1 + (r_s - r_u) \cdot \beta)}{\beta^m \cdot p + h - \beta \cdot c}\right)$$

$$S_{RF}^{t-n+k-1} = F^{-1}\left(\frac{\beta^n \cdot p - \frac{1}{(1 - r_{RF})^k} \cdot c}{\beta^n \cdot p - c \cdot \beta + h}\right)$$

Simulazione

Ciascuna simulazione confronta il RF con le altre quattro soluzioni: assenza di finanziamenti (NF), linea di credito (CL), RF tradizionale (T-RF) e factoring (F). L'analisi di sensitività testa i risultati al variare di tre assunzioni di fondo: (i) l'estensione di pagamento eventualmente richiesta dal cliente (nessuna estensione vs. estensione di 30 giorni), (ii) la configurazione della domanda (stazionaria, in crescita o stagionale), (iii) la liquidità iniziale (caso base vs. –50% vs. +50%).

I risultati sono sintetizzati nella Figura 0.5. È possibile notare che il RF senza estensione di pagamento consente di ottenere risultati migliori rispetto alle altre soluzioni, grazie ad un tasso d'interesse più vantaggioso e ad un utilizzo più efficiente dello strumento finanziario. Tuttavia, se assumiamo che per partecipare ad un programma di RF sia necessario estendere i termini di pagamento di 30 giorni (Figura 0.6), le prestazioni del RF peggiorano e la differenza rispetto alla linea di credito è quasi nulla: in un numero considerevole di casi (27,5%) la CL porta a risultati almeno pari a quelli del RF, senza considerare i costi di transizione per attivare la nuova piattaforma, cambiare provider e istituto di credito. Si evince che i clienti iniziatori di un programma di RF non dovrebbero richiedere l'estensione dei termini di pagamento a priori: talvolta, infatti, puntando troppo sulla riduzione del proprio capitale circolante, indeboliscono la propria base di fornitura piuttosto che rafforzarla.

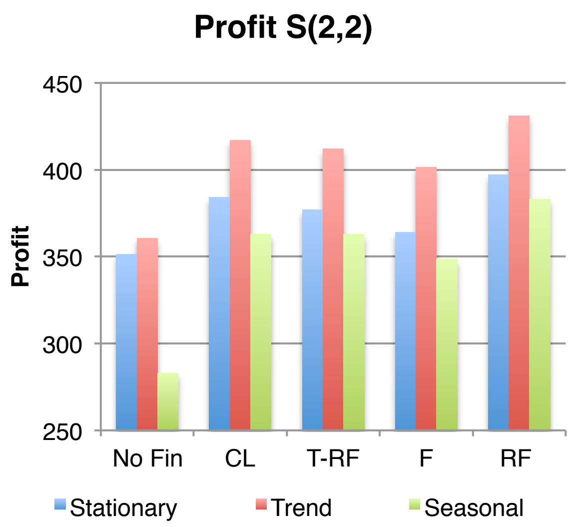


Figura 0.5: profitti delle PMI al variare della tipologia di domanda e della soluzione finanziaria utilizzitata, senza estensione dei giorni di pagamento.

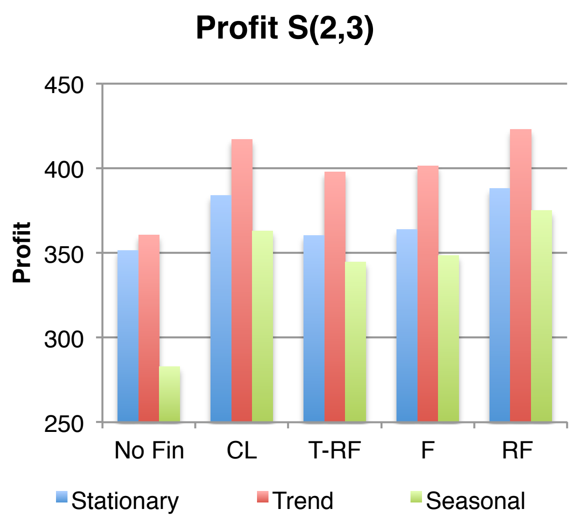


Figura 0.6: profitti delle PMI al variare della tipologia di domanda e della soluzione finanziaria utilizzitata, in presenza di estensione.

Un'altra conclusione interessante riguarda gli scenari in cui il RF risulta maggiormente conveniente. In presenza di domanda crescente, infatti, il RF crea più valore rispetto al caso in cui si debba far fronte ad una domanda stazionaria. In questo caso il RF genera risultati migliori rispetto alle altre alternative; tuttavia, se è richiesta l'estensione dei termini di pagamento, la linea di credito si rivela quasi altrettanto vantaggiosa. Quando invece l'azienda tratta prodotti caratterizzati da domanda stagionale, il RF risulta la soluzione migliore anche quando è richiesta un'estensione dei giorni di credito. In contesti variabili, come in presenza di domanda stagionale, la flessibilità che è in grado di offrire il RF consente di sostenere elevati livelli di capitale circolante in alcuni periodi e di diminuirlo in brevissimo tempo.

Come illustrato in una successiva analisi di sensitività nel paragrafo 5.6.5, anche le condizioni di cassa iniziale hanno un forte impatto sui benefici che può portare il RF rispetto alle altre soluzioni. Allo stesso tempo, è stato provato che il valore aggiunto dal Reverse Factoring Tradizionale e dal Factoring non dipende dalle condizioni di liquidità, a causa della rigidità imposta dalla condizione di sconto automatico delle fatture.

Applicazione del Modello e Casi Studio

A seguito delle analisi numeriche sulle simulazioni, il terzo modello è stato applicato a due PMI italiane, FAB S.p.A. e Stocchetta Cilindri s.r.l., che operano rispettivamente nel settore dei dispositivi di protezione individuale (DPI) ed in quello di produzione dei cilindri idraulici.

La Tabella 0.2 riporta i principali parametri utilizzati nelle simulazioni e mette in luce le differenze tra le due aziende ed i rispettivi settori: Stocchetta Cilindri deve soddisfare una domanda in crescita (ad un tasso mensile del 2,289%) e considerevolmente più volatile rispetto a FAB ed ha margini inferiori; per contro, FAB ha condizioni di cassa molto più stringenti, in grado di soddisfare solo lo 0,42% di un ordine medio.

	D media (u/mese)	CV (σ/μ)	Cassa Iniziale (#ordini)	p (€/u)	c (€/u)	Margine %	Holding Cost (€/u/anno)	Margine/ holding	Costo debito (annuale)
FAB	4.000	15%	0,0042	0,6	0,4174	44%	0,042	4,37	4%
Stocchetta	633	49,6%	0,6528	440	365,9	20%	36,59	2,03	4,25%

Tabella 0.2: Parametri utilizzati nelle simulazioni.

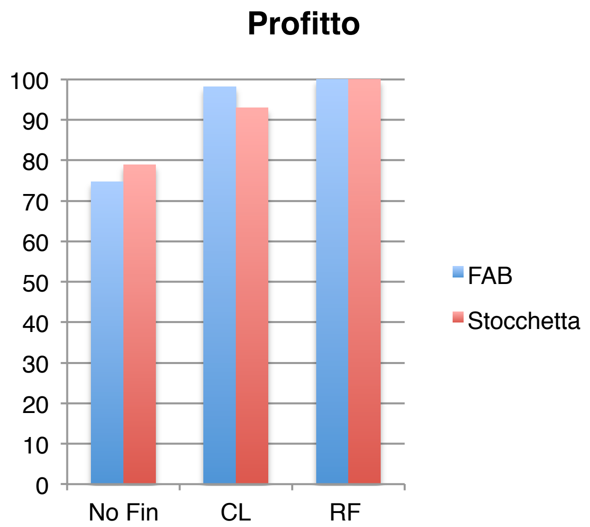


Figura 0.7: Profitto medio in 1.000 repliche.

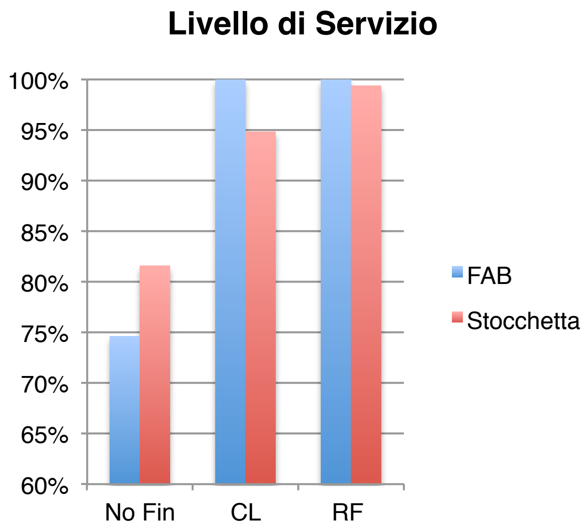


Figura 0.8: Livello di servizio medio in 1.000 repliche.

La Figura 0.7 illustra il profitto alla fine di un periodo di 12 mesi: in entrambi i casi studio i valori sono stati standardizzati in una scala da 0 a 100 per consentire un confronto. La Figura 0.8 mostra che FAB ha un livello di servizio inferiore in assenza di soluzioni di finanziamento, a causa delle condizioni di cassa più stringenti; tuttavia, quando ha accesso al RF o alla linea di credito, è in grado di soddisfare una percentuale maggiore di domanda rispetto a Stocchetta, grazie alla minor volatilità.

I casi studio validano le conclusioni tratte dalle analisi precedenti: in entrambe le applicazioni, l'assenza di finanziamenti esterni ha un effetto rilevante sulla capacità della PMI di raggiungere l'*order-up-to level* definito dalla politica di riordino e, quindi, sul livello di servizio e sul profitto al termine dell'orizzonte temporale T. Il secondo caso studio dimostra inoltre che il RF è di maggior valore quando è la domanda è molto volatile e presenta un trend di crescita, in quanto consente maggior flessibilità rispetto ad una linea di credito.

Conclusioni e ricerca futura

I modelli sviluppati e le analisi svolte in questa tesi affrontano tematiche diverse e forniscono alcuni interessanti spunti. Il primo modello tratta i benefici *diretti* per i clienti iniziatori di un programma di RF, ovvero l'allungamento dei giorni di pagamento al fornitore, indicando in quali scenari i vantaggi sono maggiormente rilevanti ed in quali, invece, le aziende clienti non trarrebbero particolari benefici. Seppur in maniera differente, anche il terzo modello individua le condizioni in cui i clienti possono crear valore richiedendo un allungamento dei termini di pagamento, ma anche alcune situazioni in cui tale accordo non sarebbe ottimale né per il cliente, né per il fornitore.

Una seconda area di ricerca riguarda la creazione di valore per il fornitore grazie all'adozione del RF ed i contesti in cui questa soluzione non solo crei valore, ma risulti più adatta rispetto agli strumenti di finanziamento alternativi. Il secondo modello mostra che il RF consente di ottenere risultati significativamente migliori rispetto all'Asset Based Lending ed alle Linee di Credito in condizioni di forte crescita della domanda. Il terzo modello indica che il RF è sensibilmente più conveniente rispetto al RF Tradizionale ed al Factoring, soprattutto qualora la domanda sia molto variabile; in questo caso, il RF risulta più efficace anche della linea di credito. Tuttavia, viene dimostrato come in altre situazioni il RF possa non portare vantaggi rispetto alla CL o addirittura peggiorare i risultati. Viene inoltre evidenziata l'importanza delle condizioni iniziali di capitale circolante sulle performance: minore è il livello di cassa, maggiore è il livello di cassa e, quindi, il potenziale del RF.

Infine, il terzo modello prova matematicamente che differenti soluzioni di finanziamento portano a diverse formule che regolano le decisioni operative come la politica di ripristino delle scorte. Il modello dimostra quindi che *finance* e *operations* non dovrebbero essere considerate come funzioni indipendenti; al contrario, poiché le rispettive decisioni e prestazioni sono strettamente interconnesse, dovrebbero cooperare.

I modelli illustrati lasciano spazio per sviluppi futuri:

- Per quanto riguarda i benefici portati dal RF ai clienti iniziatori del programma, potrebbe essere realizzato un modello che valuti un secondo tipo di benefici, detti *indiretti*. Si potrebbe analizzare il potenziale beneficio dato dalla *riduzione del rischio* della base di fornitura, dal *rafforzamento delle relazioni (partnership)* o dal miglioramento del *livello di servizio*.
- Il secondo modello può essere ulteriormente esteso considerando la capacità produttiva dei fornitori a monte della PMI: attualmente infatti si assume che l'azienda sia in grado di crescere tanto quanto lo consentono le sue risorse, indipendentemente dalle condizioni esterne. Considerare la capacità degli attori a monte renderebbe più realistiche le aspettative di crescita della PMI. Un ulteriore aspetto riguarda la domanda, che potrebbe essere stocastica e non-infinita, modificando i lanci di produzione e le decisioni d'acquisto.
- Il terzo modello potrebbe essere esteso includendo una dilazione di pagamento concessa dal fornitore della PMI, ovvero la possibilità per la PMI di avere debiti commerciali. Ne risulterebbe una notevole complessità nella definizione della funzione del profitto miope e nel calcolo della politica di riordino ottimale, ma si potrebbero trarre conclusioni interessanti su come i debiti commerciali possano influenzare l'impatto del RF per il fornitore. Poiché le scorte di prodotti semilavorati hanno un forte impatto sul WCR, si potrebbe considerare anche il lead-time di produzione. Un successivo sviluppo futuro sarebbe la minimizzazione della funzione del profitto nel multi-periodo, senza ricorrere ad una politica miope. Infine, si potrebbe includere la tassazione per valutare come lo scudo fiscale possa attenuare o esaltare il gap tra le diverse soluzioni finanziarie.

Complessivamente, il lavoro teorico sviluppato in questa tesi rappresenta la base per un'ulteriore e più empirica ricerca; per esempio un modello applicativo che possa valutare il valore aggiunto dal RF per un'azienda specifica, in grado di rispecchiarne i processi e le peculiarità.

1 Literature Review

This chapter illustrates the most relevant contributions in the literature that represent a fundamental theoretical basis for the development of this work. We first introduce the reader to the concepts of supply chain management, working capital management and supply chain collaboration. Then we focus on supply chain finance, from its definitions and frameworks to the solutions that offers. One of these solutions is Reverse Factoring, which is the core of this thesis and for this reason is illustrated in a section apart. Finally, we propose a review of the most interesting models relating to our scope.

1.1 Introduction

This thesis aims to study the Reverse Factoring arrangement and evaluate its possible value creation for both buyer and supplier. Reverse Factoring is one of the solutions in the field of Supply Chain Finance (SCF), a concept that puts together two fundamental principles of the Supply Chain literature: Supply Chain Management and Working Capital Management. We now introduce these two concepts.

1.1.1 Supply Chain Management

Lummus & Vokurka (1999) define Supply Chain Management as "all the activities involved in delivering a product from raw material through to the customer including sourcing raw materials and parts, manufacturing and assembly, warehousing and inventory tracking, order entry and order management, distribution across all channels, delivery to the customer, and the information systems necessary to monitor all of these activities. Supply chain management coordinates and integrates all of these activities into a seamless process." We choose this wide definition because it is more focused on two main flows between supplier, firm and customer: the goods flow and the information flow. Other authors as Cooper (1993) and Mentzer (2001) include the "financial resources" flow in this definition, e.g. cash, while we prefer to delineate this component to the Working Capital Management. Also Pfohl, Elbert & Hofmann (2003) separate the financial flow from the others within a Supply Chain, naming it "Financial Supply Chain".

1.1.2 Working Capital Management

Working Capital (or Net Working Capital), in a monetary sense, is defined as currents assets less current liabilities (Ross, Westerfield and Jaffe, 2005) and Working Capital Management aims at minimizing the capital tied up in the company's turnover process by reducing current assets and extending current liabilities (Hofmann & Kotzab, 2010). Working Capital also results (1) from lack of synchronization of the formal rising receipts and the real cash receipts from each sale, (2) from divergence during time of rising costs, purchases and the real outflow of cash when the firm pays its accounts payable (Michalski, 2008):

$$WC = CA - CL = AR + INV + G - AP$$

Where CA represents Current Assets, CL Current Liabilities, AR Accounts Receivable, INV Inventory, G cash and AP Accounts Payable. Working Capital is then, by definition, strictly related to the Cash to Cash Cycle (C2C), an indicator that measures how long cash is tied up between procurement and sales (Emery & Finnerty, 1997):

$$C2C = DSO + DIO + DPO$$

Where DSO stands for Days Sales Outstanding, DIO for Days Inventory Outstanding and DPO for Days Payables Outstanding. As stated by Pfhol & Gomm (2009), the C2C cycle is "a key figure to a dynamic and holistic treatment of the net working capital performance – both within the company and within the supply chain".

Generally, from CFOs perspective, management of working capital is a simple matter of guaranteeing the ability of the firm to finance the difference between the current assets and current liabilities (Harris 2005). In reality, working capital management has become one of the most important concerns in the organizations, where many financial executives are trying to identify the basic determinants of the optimal level of working capital (Lamberson 1995). A firm may adopt an aggressive working capital management policy with a low level of current assets as percentage of total assets; in contrast, a conservative investment policy hold a larger proportion of capital in current assets with the opportunity cost of lesser profitability (Afza & Nazir, 2007). Gallaghe & Andrew (2007) provide another perspective: with an aggressive working capital strategy, a firm choose to retain a smaller portion of sales revenue for working capital and invest more on the long-term assets with higher return. More aggressive working capital policies are associated with higher return and higher risk, while conservative working capital policies are concerned with the lower risk and return (Gardner et al. 1986). Therefore, working capital management is of crucial nature because it affects the firm's profitability as well as its risk, and consequently its value (Smith, 1980). Weinraub (1998) also showed that the degree of aggressiveness of the working capital policy varies from industry to industry, depending on both operations characteristics and cost of capital.

1.2 Supply Chain Collaboration

Over the past decade a combination of economic, technology and market forces has compelled companies to examine and reinvent their supply chain strategies. Some of these forces include the globalization of businesses, the proliferation of product variety, increasing complexity of supply networks, and the shortening of the product life cycles (Hau L. Lee and Seungjin Whang 2001). The focus of supply chain management (SCM) shifted from the mere cost reduction in purchasing and logistics activities to value creation, as companies began to see SCM as a source of competitive advantage (Kampstra, Ashayeri and Gattorna 2006).

SCM allows a company to focus on doing exceptionally well a few things for which it has unique skills and advantages, while non-core activities are outsourced to channel members that possess superior capabilities in those areas (Cox, 1999; Laseter, 1998; Quinn, 2000; Rich, 1997; Sheridan, 1999). A perfect example of this trend is given by the automotive industry, which saw fully integrated organizations (such as Ford Motor Company) starting a deverticalization process to focus on their core capabilities.

According to Cooper et al. (1997), the optimization of each single company's results, rather than the integration of their goals and activities with other organizations, leads to sub-optimal performances of the whole chain. Specifically, this results in an inefficient allocation of scarce resources, higher system costs, compromised customer service, and a weakened strategic position (Fugate, Sahin and Mentzer 2005).

The "bullwhip effect" is a typical outcome of poor collaboration along the supply chain and was first discovered by Forrester (1958, 1961). Chen et al. (2000, p. 269) define this effect as the increasing variability of demand faced by upstream stages of the supply chain (i.e. moving away from customer demand). The effects of such phenomenon are excessive stock levels, low demand coverage, production capacity problems, unreliable demand forecasts and extraordinary costs, such as order expediting. Croson and Donohue (2005) categorize the causes of the bullwhip effect in two types: operational (batching, price fluctuations and demand distribution) and behavioral, as the tendency of decision makers to underweight the supply line when making order decisions (shortage gaming, low information sharing).

The basis of integration can be characterized by cooperation, collaboration, information sharing, trust, partnerships, shared technology, and a fundamental shift away from managing individual functional processes to managing integrated chains of processes (Akkermans et al., 1999).

J.B. Rice and R.M. Hoppe (2001) state that "the conventional wisdom is that competition in the future will not be company vs. company but supply chain vs. supply chain. But the reality is that instances of head-to-head supply chain competition will be limited. The more likely scenario will find companies competing –and winning– based on the capabilities they can assemble across their supply networks". Unless a company is completely vertically integrated, indeed, it cannot compete alone: it needs to be part of a broader supply network. Companies may compete at an organization or at a supply chain level, depending on the particular industry in which they operate and on the characteristics of demand and supply systems (market concentration, availability, etc.).

While arm's length relationships imply a zero sum case (i.e. the improved performance of an actor is balanced by an equal loss by an other actor), McClellan (2003) refers to SC collaboration as "a win/win arrangement that is likely to provide improved business success for both parties". It may even be considered a prerequisite for future competitive performance. Indeed, Poirier and Bauer (2001, p. 20) affirm that "future success no longer belongs to a single firm, no matter on what scale it functions. The future belongs to networks of supply".

A survey of supply chain collaboration and management in the UK construction industry (Akintoye, McIntosh, Fitzgerald 2000) observes that the poor understanding of the concept of partnership is the second biggest barrier to collaboration, preceded just by the lack of top management commitment. On the other hand, frequent communication, willingness to share information, use of cross-functional teams, shared expertise with suppliers and common goals are the five most important facilitators identified by Fawcett, Magnan and McCarter (2008). A further survey conducted by Supply Chain Management Review and CSC (2004) observes that collaboration is cited as the single most pressing need, but how to achieve it is not well understood. Holweg, Disney, Holmström and Småros (2005) conclude that the effectiveness of supply chain collaboration relies upon two factors: the level to which it integrates internal and external operations, and the level to which the efforts are aligned to the supply chain settings in terms of the geographical dispersion, the demand pattern, and the product characteristics.

1.3 Supply Chain Finance

In this section we propose several acknowledged definition of Supply Chain Finance (SCF), then illustrate three frameworks and finally describe the current SCF solutions.

1.3.1 Definitions

There are many definitions of Supply Chain Finance. Randall and Farris II (2009) consider it an extended Working Capital Management: "A supply chain financial management approach means smartly extending classic firm-oriented practices dealing with cash-to-cash cycles, cash flow, and weighted average cost of capital (WACC) as they mange their supply chain partnership".

Hofmann (2005) focuses more on the integration and collaboration between the players involved: "Located at the intersection of logistics, supply chain management, collaboration, and finance, Supply Chain Finance is an approach for two or more businesses in a supply chain, including external service providers, to jointly create value through means of planning, steering, and controlling the flow of financial resources on an inter-organizational level".

A perspective that puts in evidence other important aspects of SCF as processes' automation and transactions' transparency is provided by Hurtrez and Salvadori (2010): "SCF integrates the pieces of the financial supply chain from end to end, fully automating the buyers' procure-to-pay and suppliers' order-to-cash cycles. This new level of integration will support event-triggered financial services along the physical supply chain (e.g., purchase order tracking, invoice matching services, e-invoicing, open account payments, import/export financing, reverse factoring) and afford full transparency into each transaction".

We choose the definition provided by Phol & Gomm (2009), which is probably the most complete: "Supply Chain Finance is the inter-company optimization of financing as well as the integration of financing processes with customers, suppliers and service providers in order to increase the value of all participating companies". This definition is interesting because it puts in evidence the concept of "global" optimization: oftentimes a single firm's optimal decision is not optimal for the whole supply chain; SCF, instead, always seeks the global optimum for all participants.

1.3.2 Supply Chain Finance Frameworks

In this section we propose three relevant frameworks that describe supply chain finance solutions and constitute the conceptual base of this thesis.

Pfohl and Gomm (2009)

Pfohl and Gomm developed and published in 2009 one of the first conceptual frameworks and mathematical models of "Supply Chain Finance", addressing the role of financial flows in supply chains and the impact that SCM can have on optimizing such flows in terms of capital cost. The underlying concept is that supply chain information can be used to decrease investment risks and thus capital costs of financing projects within supply chains.

The proposed framework identifies the main dimensions in SCF: which *assets (objects)* are actually financed by *whom (actors)* and on *what terms (levers)*.

- The objects of SCF may be *fixed assets* (production facilities or machines) or *net working capital* (Current assets – Short Term Liabilities);
- The actors of SCF are either *primary members* (i.e. suppliers, customers and the focal company) or *supporting members* (such as logistics service providers, but also financial intermediaries);
- The levers of SCF are the *volume of financing* needed, the *duration of financing*, and the *capital cost rate*.

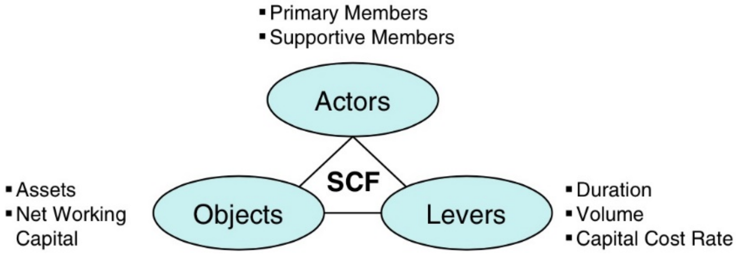


Figure 1.1: The framework of SCF proposed by Pfohl and Gomm.

The three levers, in particular, form the SCF cube (Figure 1.2) and define the Cost of Capital:

$$Capital\ Costs = Volume \cdot Duration \cdot Capital\ Cost\ Rate$$

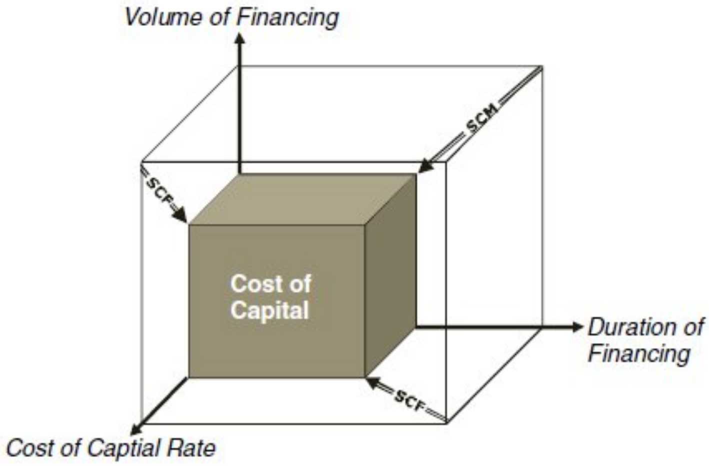


Figure 1.2: The Pfhol and Gomm's finance cube

Pfhol and Gomm (2009) give an interesting interpretation of Supply Chain Finance (SCF) and they clearly differentiate it from Supply Chain Management (SCM): while the latter is able to move the Volume and Duration leverages (e.g. how much inventory a firm needs to finance and for how long), SCF comprehends also the Cost of Capital Rate.

van der Vliet, Reindrop and Fransoo (2013)

A different but equally interesting SCF framework, proposed by van der Vliet, Reindrop and Fransoo (2013), is illustrated in Figure 1.3 and derives from what the authors consider the two main decision dimensions of an SCF implementation: the strategic orientation and the tactical orientation. This framework primarily takes the perspective of an SCF "benefactor" (she), i.e., a firm that uses its financial strength to positively change financing conditions for firm(s) upstream in its supply chain ("beneficiaries"). The authors are more focused on describing the archetypal SCF policies that can be followed by a benefactor when she decides to implement a SCF program with her suppliers.

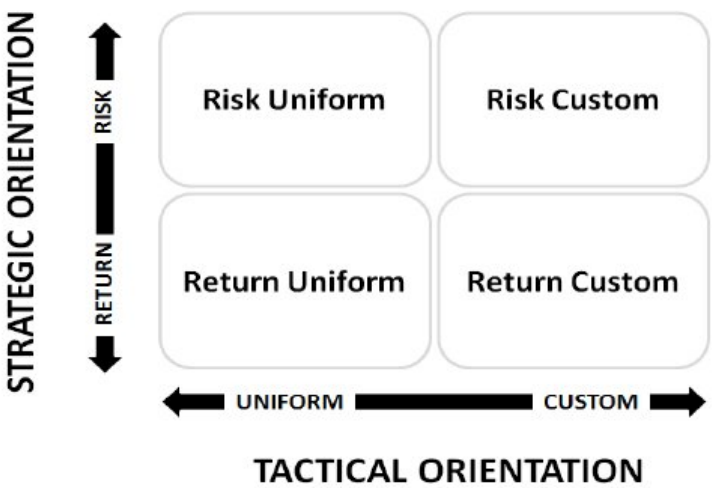


Figure 1.3: van der Vliet, Reindrop and Fransoo's framework

Concerning the strategic orientation, a buyer may adopt either a *return-oriented strategy*, or a *risk oriented strategy*. With the first, the SCF benefactor's objective is to gain direct financial benefits from offering SCF arrangements, i.e., lengthening her payment terms in order to decrease her Working Capital Requirement. With a risk-oriented strategy, instead, the benefactor "aims to benefit indirectly (through the medium of operations), by using the SCF mechanism to mitigate underinvestment problems present in her supply base". Indeed, thanks to a SCF program, a supplier (he) could take operational investments that normally would be out of range because of the prohibitive financing costs (i.e., increasing safety stocks or reducing production lead time). These two possible strategies represent a trade-off from the benefactor's perspective: the more the buyer is willing to earn directly from the SCF arrangement, the less she will benefit from operational improvements in her suppliers base.

The second dimension, tactical orientation, offer as well the benefactor two possible tactics when implementing a SCF program: a "uniformity tactic" and a "customization tactic". Leaning towards uniformity, the benefactor aims to minimize the initial investment cost through standardization; on the other hand, offering customized SCF arrangements might maximize benefactor's long-term returns. Indeed, customization might decrease risk or increase cash flows of SCF arrangement, but it usually needs additional investments in, e.g., legal, information, organizational, or operational processes.

Grounding on these two dimensions and their respective tactics or strategies, van der Vliet, Reindrop and Fransoo identified four archetypal implementation policies for an SCF benefactor: Return-Uniform, Return-Custom, Risk-Uniform and Risk-Custom. They also later distilled five adoption criteria that affect an SCF benefactor's optimal implementation policy: (1) "Expected return from reducing operational underinvestment in the supply base", (2) " Investment risk of reducing operational underinvestment in the supply base", (3) "Required timeframe for collection of benefits", (4) "Expected additional return from customization" and (5) "Investment cost of customization".

Hofmann (2005)

A last insightful framework of Supply Chain Finance is provided by Hofmann (2005), who describes three constitutive elements of SCF: (1) institutional actors, (2) supply chain management characteristics and (3) financial functions.

Institutional actors are divided in two categories, macro-institutional actors and micro-institutional actors. Macro-institutional actors comprehend an industrial or commercial company, its suppliers, its customers and a financial institution (either a banks or an investor). Other actors, defined as "not-genuine members of a supply chain", are Logistics Service Providers. The micro-institutional actors arise because the macro-institutional actors presented above have different organizational and process structures: micro-institutional actors comprehend all departments involved with the operational activities (e.g. purchasing, production, distribution and logistics units) and with the financial activities (e.g. finance).

With *Supply Chain Management Characteristics* Hofmann means all the "collaboration" characteristics in SCF: since the supply chain collaboration is affected by contractual regulations not only concerning the quantity and the delivery times, but also the prices and the payment policy, the direct payments between the FSC-members have to be taken into account. Hofmann cares to highlight this concept because "the financial flows within a collaboration of supply chain actors are determined by strategic decisions taken in a formed cooperation. Contractual agreements about the payment conditions put these strategic decisions into action. The tracking and controlling of payments is therefore an essential part of supply chain collaboration's day-to-day business". This element is fundamental because each actor, in order to decide whether participate in a SCF program or not, evaluates the expected cash inflows and outflows.

The *Financial Functions* are classified in three categories: (1) tracking financial flows and resources, (2) using financial resources and (3) sourcing financial resources. The first activity is fundamental since it is necessary to identify, measure and communicate in real time the cash flows between the organizations in order to set a collaborative supply chain finance program. The cash information relating to payments must visible and always available to all the parties involved in the program. "Using financial resources" refers to the *collaborative investments*: investments taken by two or more parties in order to improve the whole supply chain performance. Finally, "sourcing financial resources" is maybe the most important activity that relates to Supply Chain Finance, since most of the times firms arrange SCF contracts in order to find new ways of financing. Lengthening payment terms for a buyer or obtaining a better discounting rate on customers' invoices factoring are alternatives ways of financing, whose use has been increasing in the last years due to the credit crisis.

1.3.3 Supply Chain Finance Solutions

In this paragraph we illustrate the main SCF tools, illustrated by Politecnico di Milano in the report "Supply Chain Finance: nuove opportunità di collaborazione nella filiera" (2014).

In the report, Caniato et al. (2014) classify the SCF solutions in three categories:

- *Traditional Solutions*
- *Innovative Solutions*
- *Collaborative Solutions*

A common element that arises in each kind of solution is the presence of a technologic platform, sometimes used as a supporting tool (especially in the traditional solutions), other times representing an undeniable factor in order to enable SCF arrangements.

The same report provides a wide description of each solution (February, 2014) that organizes in a clear way the SCF solutions panorama:

Traditional Solutions

- Invoice Advance (or Invoice Discounting): it is a debt incurred by a supplier with a financial institution (typically a bank) against a commercial credit owned by the firm towards a customer, using the invoice as collateral. It is a short-term debt taken in order to increase liquidity and finance working capital requirement. This solution is one of the most broadly used; however, it is becoming harder and costlier to arrange due to the credit crisis and due to the usual high-risk ratings of the small suppliers willing to advance their invoices. Usually, the bank advances 80% of the invoice amount, and requires an annual interest rate between 5% and 10%.

- Factoring: it is a contract between a supplier and a financial institute, named "factor", in which the first sells its accounts receivable to the latter at a discount rate. Sometimes it is set as a framework agreement in which all the invoices considered in the contract are sold to the financial institute; this solution is also named "auto-discounting", since the factoring occurs automatically. The arrangement can involve also other services as credits insurance, invoicing services and credits collection. This kind of agreement is less employed than invoice advance because it requires high transactional volumes (€2 to 5 millions). There are two kinds of factoring: *with recourse*, in which the factor, in case the debtor (the customer) is not able to pay the invoice, has the right to collect the unpaid invoice amount from the supplier (the invoice seller); *without recourse*: in this case the bank must bear the loss in case of customer's payment default. The second solution is obviously costlier than the first one since the seller has to pay not only the interest from the advanced payment of the invoice, but also the invoice insurance costs the financial institution has to bear.

- Reverse Factoring (1ST generation): a firm (A) arranges a contract with a factoring company by which the latter has to buy from the firm's suppliers (B) the invoices they submit to A, anticipating the cash and waiting for B to pay A. Unlike factoring, traditional reverse factoring is initiated by the buyer in order to help her suppliers to finance their receivables and at a lower interest rate than what they would normally be offered. The interest rate is lower because it is negotiated by a firm who is usually large and creditworthy: since the factoring company knows that these invoices are owed by a creditworthy company, the interest rate charged to discount them is not based on the small suppliers risk-rating, but on the initiating firm's one. This solution cannot be considered innovative nowadays, since it was introduced years ago; however, in many countries among whom Italy, it is still not popular. Traditional Reverse Factoring is also frequently an auto-discounting solution, since the financial institution has no information about the status of every single invoice and therefore imposes on the supplier to discount all of them.

Innovative Solutions

- Dynamic Discounting: thanks to this arrangement, a buyer can anticipate the payment of goods to her supplier in return for a discount on the invoice nominal value. This arrangement is usually made between either a buyer and a supplier or a buyer, a supplier and a financial intermediary. The discount varies dynamically and proportionally to the early-payment days in respect to the original payment terms. Theoretically, the maximum discount corresponds to an immediate payment, linearly decreasing day by day, until the discount becomes null if the payment is executed on the pre-arranged date. Possible advantages exist for both buyer and supplier: the first may obtain significant reductions on the purchasing costs when her working capital requirement is low and he can pay early; the latter, on the other hand, has an alternative source of short-term financing that might be convenient when comparing the rate discount with the interest rate on shortages.

- "Evolved" Reverse Factoring: it is an innovative version of Traditional Reverse Factoring (T-RF), in which the invoice payment-in-advance occurs in presence of more operative information, causing therefore a reduction in both risk and interest rate required by the financial institution. In order to implement this SCF program, especially with many SME-suppliers for a single buyer, it is necessary to have available a technologic platform able to support all the processes involved in the business relationship between the parties. The platform must be capable of managing high-volume information, as the trade documentation between buyer and suppliers, or the vendor rating system. This activity allows to define dynamically premium-ratings for the best in class suppliers at a low cost. The benefits are equally shared among all the parties involved in the arrangement: (1) suppliers can obtain lower interest rates through a meritocratic and transparent system; (2) the financial institution has full control on the suppliers' activities, from their operations to their delivery due date, obtaining significant and live updated information that imply a reduction in risk; (3) the buyer on one side may reduce her working capital requirement lengthening payment terms, and, on the other hand, can minimize the possibility that the strategic suppliers face a liquidity crisis. Thanks to the great information availability, this solution allows non-auto-discounting arrangements: suppliers can choose every time which invoices to discount. From now on, we refer to this solution simply as *Reverse Factoring* (RF).

- Clearing House: this solution represents another variation and evolution of Traditional Reverse Factoring and implies a third party (a financial institution) to act as guarantor of the commercial transactions generated by an entire supply chain ecosystem. The Clearing House has full visibility on payments and checks made by the parties and seeks to de couple the postponed payments between firms. The full potential of this tool is expressed in presence of multiplicity of both buyers and suppliers, where the complexity is significantly high. In order to manage the complexity given by this sort of scenario, it is necessary to have available technologic platforms that allow to organize the information in a structured, electronic and computerized way. The interest rates are somehow comparable to the TRF tool.

- Non-Bank Invoice Advance (Non-Bank Invoice Discounting): it derives from the classic Invoice Advance since also in this case a third party anticipates the invoice payment to the supplier, eventually collecting the whole invoice nominal value plus possible interest fees. The main difference is that the third party may be either a bank, other firms, a private investor, a hedge fund, or any other investor. The invoice issued by the supplier to the buyer is turned into a financial instrument, tradable in a specific cloud marketplace and represents a low-risk and early-payback solution for the investors. The player that manages the marketplace platform behaves as guarantor, meaning that he has to evaluate and qualify both the firms who sell their invoices (typically SMEs) and the investors who want to participate to the online-auction. The marketplace manager receives a small fee for each transaction made on the platform, corresponding typically to 1-1,5% of the invoice's nominal value.

Collaborative Solutions

The Supply Chain Collaboration approach has carried out several management models since many years: as the traditional solutions, these models can be re-interpreted with a different approach that aims not only at efficiency in ordering and stocking processes but also at the supply chain financial sustainability.

- Vendor Management Inventory (VMI): the supplier is allowed to monitor his own goods inventory levels among his buyers' warehouses in order to manage independently the replenishment processes. Since the supplier is now his buyers' stocks manager, he has a higher visibility and therefore is able to plan a more efficient replenishment policy, lowering his own inventory level and reducing significantly the possibility of stock-outs. The customer benefits from safety stocks reduction, from lower transportation costs and sometimes from an additional inventory level reduction when the supplier increases the delivery frequency. This decrease in stocks level within the whole Supply Chain corresponds to a general drop in the Supply Chain's Working Capital Requirement: this means that the firms involved in the program need less capital to finance their operations and for this reason also the efficiency tools as VMI can be related to SCF.

- Consignment Stock: the supplier puts his goods at buyer's disposal (i.e. buyer's warehouse), keeping the inventory ownership. The stock is legally owned by the supplier, but held by the buyer, meaning that the risks and rewards regarding to the said stock belong to the former. Ownership of consignment stock is passed only when the goods are used by the buyer. As VMI, this program reduces transportation costs as well as revenue losses due to stock-outs. This solution can also be re-interpreted through a perspective of more financially sustainable Supply Chain: a large supplier may delay the time of sale to his strategic retailers in order to help them. These small firms can always have available their supplies, that are nominally purchased only when effectively picked from the warehouse. In this way suppliers increase their Working Capital Requirement, owning stocks for a longer period, but they also benefit from both minimization of strategic retailers' bankruptcy and possible increase of their market-share, since buyers will be incentivized to push majorly their products down the Supply Chain.

- Collaborative Planning, Forecasting and Replenishment (CPFR): CPFR seeks cooperative management of inventory through joint visibility and replenishment of products throughout the supply chain, on a long-term horizon. Information shared between suppliers and retailers aids in planning and satisfying customer demands through a supportive system of shared information. This allows continuous updating of inventory and upcoming requirements, making the end-to-end supply chain process more efficient. An advanced technologic platform is a minimum requirement to implement CPFR, in order to both manage, analyze and organize significant quantities of data coming from different sources and to make this information visible and sharable to multiple players within the Supply Chain. Forecast mistakes, overstocking and stock-outs costs are vertically reduced, benefiting all the actors involved in the program: looking at final customer demand from an aggregate perspective allows economies of scale and economies of scope. The joint management of forecast, planning and stock replenishment reduces the overall Working Capital Requirement within the Supply Chain, decreasing therefore the liquidity needed to finance it.

1.4 Reverse Factoring

In this section we describe Reverse Factoring: its structure, the dynamics involved, the actors that participate in, and their benefits and concerns. Finally, we analyze the circumstances that favor a RF agreement and the factors that represent an obstacle.

1.4.1 Framework and Actors

The key difference between RF and the other traditional financing solutions is that the bank's risk exposure is fully concentrated on one buyer instead of one buyer and many suppliers (Roumen, 2012). As illustrated in paragraph 1.3.3, in some RF programs, the bank might classify the supplier's receivables based not only on the credit worthiness of the buyer but also on the supplier's performances through a transparent vendor rating (Caniato, Ronchi, Catti and Tumino, 2014). The basic process of RF (Figure 1.4) is well explained by Roumen (2012):

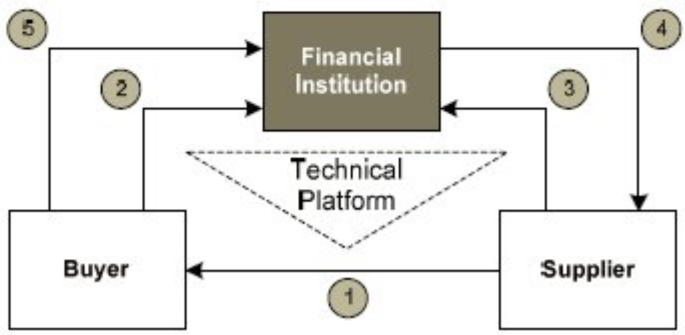


Figure 1.4: The RF basic process

1. A seller sends the buyer an invoice for services / goods delivered;

2. The buyer approves the invoice and creates an irrevocable payment obligation;

3. The supplier is now able to sell the invoice and sends the resulting receivable to the financial service provider in case it desires early payment for services / goods delivered;

4. The financial service provider pays the supplier the value of the invoice less a discount. In turn, the financial service provider takes over all the rights and obligations of the receivable from the seller. The discount rate at which the supplier obtains early payment is more favorable than the supplier's cost of debt;

5. The buyer eventually pays the financial service provider the full invoice amount.

Reverse Factoring is a rare example of tripartite value proposition for banks, buyers and suppliers (Hurtrez, 2010): first, it helps banks to optimize use of capital by reducing the consumption of risk-weighted assets, as the counterparty risk shifts to larger buyers with better risk profile. Second, the rating differential among investment grade buyers and their SME suppliers is wide enough in the current funding market to make the credit arbitrage of reverse factoring an attractive way to improve liquidity for both buyers and suppliers. For example, RF programs allow buyers to extend payment terms from 60 to 120 days while providing suppliers access to better financing rates (e.g., 120 days at 100 bps instead of 60 days at 500 bps). According to industry sources, RF could unlock \$100 billion to \$500 billion of liquidity by accelerating the cash conversion cycle for suppliers and extending days payables outstanding for buyers. Third, the more efficient and automated mechanism of RF strengthens each link of the supply chain, thus decreasing buyers' operations risk. Finally, the elimination of paper processing can reduce processing times of 30 to 60 days to approximately 10 days, enabling suppliers to offer better discounts for early payment.

A fundamental enabling factor of RF is the adoption of a technologic platform. RF technology incorporates both transactional financial supply chain automation and access to financing/credit, extended by the participating financial institution or supply chain partners themselves (Roumen, 2012). Process automation and transaction fee reduction may entail important cost reductions. Most RF Platforms are web-based, making it an ideal low cost interface to the many suppliers involved in a SCF program. Investments in hardware and software are minimal. Buyers generally are linked-up with the platform through secure *straight through processing* connections (STP-connections), which requires limited investments. Once the parties are connected, transactions run with relative ease without manual intervention.

1.4.2 Benefits and Concerns

As illustrated by Van Laere (2012), RF is considered frequently a win-win-win solution for supplier, buyer and bank; however, besides many advantages, we may find also some concerns that regard each actor.

1.4.2.1 Supplier

The first and most important benefit for suppliers lays in lower interest rates (Roumen, 2012). Another consequence given by RF is that the supplier uses a financing tool without affecting his balance sheet debt, so without decreasing his debt capacity (Van Laere 2012, Ras 2013). Third, increasing liquidity suppliers can invest more resources either in their operations (e.g. inventories), improving their service level (Buzacott 2004, Aeppel 2010), or in their fixed assets, increasing their capacity. It is also a zero cost option for them: they only pay for it upon utilization (Ras, 2013). On the other hand, there may also be some concerns. An example occurs when buyers require such longer payment delays in return of lower interest rates that eventually their suppliers face higher financing costs (SCDigest – Editorial Staff, 2013). Honing et al. (2013) also mention the fact that credit ratings of the large firm who initiate RF become *critical* for the whole Supply Chain: some of the weakest small suppliers might become dependent on the cash made available through the advantageous interest rate. In such situations, if the buyer reduces her orders for a long enough period or if she decides to stop offering RF, suppliers might face serious bankruptcy risks. Finally, Sadlovska and Enslow (2006) claim that payables extensions may transfer credit risk and capital costs up the supply chain, pushing them to their weakest point: however, cost shifting to suppliers may result in short-term balance sheet benefit but can boomerang back in the form of a less financially stable and thus higher-risk supply chain base.

1.4.2.2 Buyer

Reduction of working capital is often touted as the buyer's primary gain from RF (Lewis 2007; Milne 2009; Hurtrez and Salvadori 2010; Seifert and Seifert 2011). Due to the longer payment period, the cash conversion cycle (CCC) decreases and thereby the liquidity of the assets increases; furthermore, this extra liquidity does not compromise the buyer's ability to source other forms of financing, and does not impact on its credit rating (Greensill, 2014). Alferink (2010) also argues that buyers may ask for a unit price reduction to their suppliers, in return of letting them join the RF program. More indirect or 'soft benefits' are also argued for better collaboration, partnership and harmonizing ways of doing business with suppliers (Roumen 2012). There may also be a reduction in transactional costs thanks to increased efficiency at both ends (Crowe, 2009). As already mentioned, thanks to the program the supplier could become more reliable and provide a better service level (Seifert and Seifert 2009, Greensill 2014). However, also the buyer may face some negative effects: first, as mentioned previously, the increased risk due to RF-addiction by weak suppliers jeopardizes the whole supply chain, including buyers (Honing et al., 2013). Roumen (2012) highlights that the long-duration contract the buyers have to arrange in order to ensure suppliers to be financed is seen as a disadvantage, since buyers must bind themselves with a certain bank for a certain period while in the meantime better opportunities may arise. Kerle's report (2009) illustrates that for buyers the biggest hurdle is thought to be the *perceived need to change internal processes* followed by the implementation costs and getting the supplier on board. Another issue for buyers regards agency or moral-hazard problem: suppliers may divert funding from RF to personal interests, using them to serve other investments than what was agreed between the parties involved, e.g. increasing service level with inventory investments (Van der Vliet et al., 2013). Finally, Alferink (2010) argues how difficult it is for buyers to involve their most valuable and strategic suppliers in these programs while avoiding to onboard the least reliable ones. Buyers would like to rank suppliers based on their importance and performances, setting corresponding interest rates, while banks would not be intentioned to offer any discount due to better operational performances.

1.4.2.3 Financial Institution

The main reason why banks should join and offer RF programs is that it allows them to access to the SME's market with limited risk (Roumen 2012, Van Laere 2012). Banks also benefit from the reduced price they pay for invoices owed by high-trustworthy customers, while the discount rate they apply is given by the risk-free rate, plus the specific customer credit rating, plus a fee (Tanrisever et al., 2012). Klapper (2006) illustrates how frauds were a concern of traditional factoring in developing countries: suppliers could try to factor accounts receivable of non-existing firms or factor accounts receivable multiple times. Since with RF the buyer uploads her accounts payable on the platform and payment is issued via the bank, the legitimacy of the accounts receivable and the final payment are ensured. Thanks to this, as Van Laere (2012) and Carrodus (2007) highlight, the bank may not only increase her market-share, opening credit to developing countries, but also expand her existing business cross selling to the new SMEs other existing solutions. A final benefit from reverse factoring for the bank might be the compliance with the new Basel III regulations (Roumen 2012, Van Laere 2012). Since the risk of a reverse factoring arrangement is much lower than the risk of a normal loan to a SME, the bank does not need to hold as much capital reserves to back its funding to these high risk firms. On the other hand, offering Reverse Factoring to existing markets may cause cannibalization of the existing and more profitable solutions provided by banks (Jones 2008, Ras 2013). Another negative aspect for banks lays in the technologic platform, which happens very often to be paid by them, as service providers (Hawser, 2010).

1.4.3 Enabling Factors

In this paragraph we address several factors that may enable or facilitate the implementation of RF. Ras (2013) mentions the liquidity constraints created by Basel III: in particular, large companies are concerned that their suppliers will find it difficult to raise capital in the future, which brings potential risk to the supply chain, and thus they start to consider about initiating a RF program. The same author also stresses how the advantages brought by RF push the SMEs (if aware of it) to require it to their customers. When they are willing to offer a unit price discount in exchange of it, the probabilities of implementing RF increase even more (Carrodus, 2007). Carrodus also depicts RF arrangement from the other perspective: how do buyers engage suppliers. According to him, buyers take two routes: the first is to mandate the new arrangement as a condition of continuing to do business with the supplier; the second is to work with the supplier to help it appreciate and quantify the benefits of the new platform. The author finally highlights how the segmentation of the target supply base and the application of supplier onboarding techniques suitable for each segment play a crucial role in RF engagement. Finally, again Ras (2013) draws attention to another important enabling trigger: the fact that major banks have started to sell RF assets to other investors, opening to an investment community to which a company may not normally have access. They found indeed that “investors have a very keen appetite for short-term, investment grade assets that yield a good return, without settlement risk.” This liquidity-boost may open RF to other markets.

1.4.4 Barriers

In this section we illustrate some of the main barriers that may obstacle the implementation of RF. A significant inhibiting factor is that suppliers might lack the required knowledge about finance and Working Capital Management to fully understand RF’s benefits (Van Laere, 2012); due to distrust created by years of exploitation by their buyers, where buyers wrung out their supplier by demanding longer payment terms, price reduction and higher service levels, suppliers feel reluctant to participate in reverse factoring programs. Another obstacle is brought by financial institutions and their willing to implement RF in certain zones: sometimes the specific country risks are too high to set up a long-term RF program with the companies operating there (Hieminga 2012). In addition of that, Hieminga (2012) highlights that there must be a minimum transaction volume and transactions have to be recurrent, otherwise banks would not find it profitable to bear the cost of initiating the program (e.g., the investment in technology). Hurtrez et al. (2010) point out other factors: first, the report that legal and accounting standards in many countries do not recognize e-invoices and other electronic documents as legally binding. Second, they claim that the low cost of capital in the mid-2000s virtually eliminated the marginal advantage of credit arbitrage between large corporate buyers and SME suppliers. Third, they recognize that linking suppliers with banks' proprietary platforms proved to be cumbersome and expensive. Nehk (2012) considers a barrier also the lack of integration between the financial and the purchasing teams and the fact that it is not always clear their division of responsibilities. Honing et al. (2013) add that RF sometimes struggles due to three different factors: platform providers lack network ("Most important challenge for these providers is to build the company network to create a valuable proposition"), logistics providers lack financial know-how and banks are organized in pillars and fear cannibalization. Van der Hooft et al. (2011) consider barriers also the lack of technological capabilities of providers and to the unawareness of suppliers. A last barrier is highlighted by Ras (2013), who found that usually banks seek to involve only the largest and healthiest suppliers in RF programs because of their higher financial value, while buyers are proposing RF in order to help and strengthen the smaller and weaker ones.

1.5 Supply Chain Finance Models

In this paragraph we discuss some of the existing models in the supply chain finance research area. They differ in assumptions and approaches, but all of them are useful in understanding SCF and its benefits for the actors.

1.5.1 Pfohl and Gomm (2009)

Starting from the framework illustrated in paragraph 1.3.2, we now present the model developed by Pfohl and Gomm. This mathematical model assumes the presence of information asymmetries between companies within (insiders) and outside (outsiders) the supply chain and that communicating these pieces of information to external actors is high costly, due to the principal-agent problem between (external) investors and (internal) capital seekers.

The model includes two actors of a supply chain and an external financial market, which offers both actors capital at unequal interest rates because of the differing company risks involved. Within the supply chain, there exists an investment alternative ("project") that can be chosen by one of the two companies, but which could also be financed by the other actor.

Due to their engagement in the supply chain and their connection to the project, both actors have pieces of relevant information that concern the risk related to the project and which are not as easy accessible outside the supply chain.

A borrowing and capital restricted company N wants to finance a project P and has information concerning the return on investment of the project ($r_{project}$); however this information cannot be transferred to an external investor, if not under prohibitively high costs.

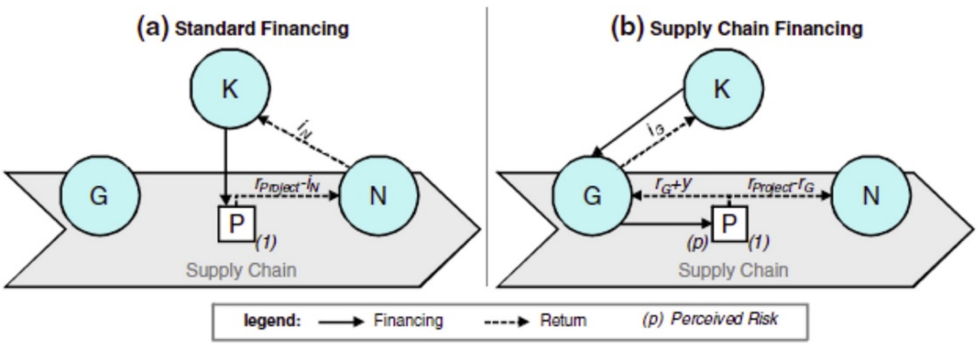


Figure 1.5. Standard Financing and basic model of SCF.

P can be realized completely or not at all, and the only financial alternatives are an external investor K (financial market) or an insider company G, which is directly involved in the project and can evaluate the implementation and the success of the investment. Thanks to this relevant information, G can calculate the success of the project P with a probability $0 < p \leq 1$.

The project P is the only investment alternative for G, which does not have free financial resources and might seek financing from the same investor K. The project P would yield an additional positive effect on G (benefit y) due to its role as an investor.

The external investor K cannot estimate the probability of success of P, but only knows the general risk of the two companies N and G, used to determine the financing interest rates i_N and i_G, where $i_N > i_G$. G claims risk-adequate rate of interest of the project, i.e. $r_G \cdot p + y \geq i_G$.

To simplify matters, y is no longer mentioned in the basic model, which is static, with one financing period and without information costs between G and N. In the basic model:

- $r_{G\_total} = \max(r_G \cdot p - i_G; 0)$;
- $r_{N\_total} = \max(r_{project} - r_G; r_{project} - i_N)$;

SCF is applied if both the conditions are satisfied simultaneously, therefore $i_G < r_G < i_N$ has to apply at least.

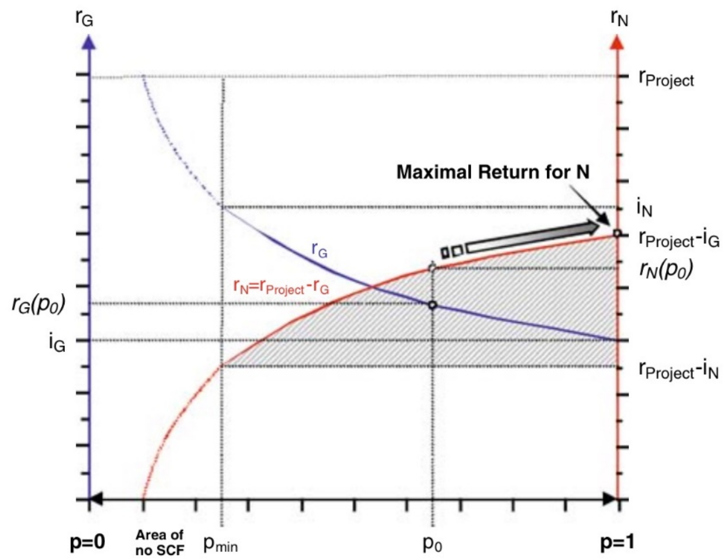


Figure 1.6: Rate of return maximum for N and interest rate arbitrage to realize

The graph in Figure 1.6 shows the returns r_N and r_G as a function of p. G demands a return r_G commensurate with its ability to forecast the success of the project: the more information it has on P, the lower is r_G ($r_G = i_G/p$). For $p < p_{min}$, the information asymmetry between the business partners N and G is too relevant to consider a supply chain finance agreement. On the other hand, when $p \geq p_{min}$ SCF is applied under the condition $i_G < r_G < i_N$ and r_G increases steadily in p. The shaded area in Figure 1.6 represents the interest rate arbitrage to realize in the model without costs of information.

1.5.2 Van Laere (2012)

Van Laere (2012) developed a model in which the buyer approves an invoice in $t = 0$ and the supplier, by the means of auto reverse factoring, immediately proceeds to discount it. For simplicity, the invoice amount is normalized to one and benefits are calculated on a pre-tax basis. A first version of the model investigates into the benefits of the actors and their participation constraints, assuming perfect capital markets. This strong assumption is relaxed in the second part.

Since transaction costs, taxes and information asymmetries do exist, this paragraph only includes the derivation for an imperfect capital market. The financing rate of a firm is composed of the risk-free rate plus a risk premium. Due to the existence of information asymmetry a factor representing the deadweight cost of capital is added in calculating the firm's financing rate. This deadweight cost of capital is caused by a difference in information between the market and a bank. Since a bank invests in its relationship with the buyer and the supplier, it has more information than the market and therefore could charge a lower interest rate. In the model, however, it is assumed that the bank charges the same rate as the market, but the difference between the market and the bank is deadweight cost of capital for the supplier and profit for the bank. This is indicated by epsilon ε_i, with $i \in \{b, s\}$, respectively buyer and supplier. The financing cost is therefore $r_i = r_f + \beta_i + \varepsilon_i$ with $i \in \{b, s\}$, for the buyer and the supplier respectively, where beta is the risk premium and r_f represents the risk-free rate. The benefit is represented by π_i, again with $i \in \{b, s\}$.

This model takes into account the possibility of bankruptcy by the buyer, who pays her outstanding account payables at maturity. Since the buyer can default within the payment period, the payoff at maturity is stochastic. A different terminology in payment term expressed in years without and with RF respectively is given by l_{nrf}, l_{rf}. The SCF premium asked by the bank to the supplier is b. Without bothering about the derivation of the formulas, the normalized benefits are represented accordingly.

Supplier:

$\pi_s = (r_f + \beta_b + \varepsilon_b)l_{nrf} - (r_f + \beta_b + \varepsilon_b + b)l_{rf}$ non capital constrained supplier

$\pi_s = (r_f + \beta_b + \varepsilon_b + \varepsilon_s)l_{nrf} - (r_f + \beta_b + \varepsilon_b + b)l_{rf}$ capital constrained supplier

Buyer

$\pi_b = (r_f + \beta_b)(l_{rf} - l_{nrf})$ non capital constrained buyer

$\pi_b = (r_f + \beta_b + \varepsilon_b)(l_{rf} - l_{nrf})$ capital constrained buyer

Please note that the buyer gains benefit when the condition $l_{rf} > l_{nrf}$ is met, i.e. RF allows a payment term extension. On the other hand the supplier participates in the SCF arrangement only if he can benefit from it, or at least not make a loss. This condition is represented by the following constraint:

$$\frac{l_{rf}}{l_{nrf}} \leq \frac{(r_f + \beta_b + \varepsilon_b + \varepsilon_s)}{(r_f + \beta_b + \varepsilon_b + b)}$$

1.5.3 Cetinay, Tanrisever & Reindorp (2012)

Cetinay Tanrisever & Reindorp provide an analytical model that shows how RF mitigates the information asymmetry between the supplier and the bank, reducing the financing cost and increasing the total value of the supply chain. Analyses in a Make-To-Order (MTO) environment have showed how the spread in external financing costs, operating characteristics and risk free interest rate influence the value of RF. This is the first model in which demand arrival and size are stochastically determined, even if it considers a single period.

The supplier is confronted with stochastic demand arrivals from a corporate buyer and operates a Make-To-Order (MTO) fashion. We assume that demand is realized at a single time-point and production is instantaneous (production lead time is zero). The supplier faces a certain level of liabilities η, such as rents and rates, that are due at end of the planning horizon $t = i = 1$. The arrival time of the demand is denoted by a random variable χ that is uniformly distributed over $[0, i]$. In a first deterministic case, demand size ξ is constant and with value equal to μ. In a second moment, the effect of a stochastic demand size ξ with expectation μ is investigated. Upon an order reception, the supplier purchases raw material at unitary cost c and sells finished products at price p. A demand arrival instantaneously reduces the cash reserves of the supplier by total raw material costs $c\xi$ and increases the accounts receivable (AR) by $p\xi$. However, there is a nominal payment delay from the buyer to the supplier, l_s years.

In case the supplier cannot finance his liabilities as they come due, external resources are available at a cost $r_s = r_f + \beta_s$, where r_f is the risk-free rate and β_s is a premium to compensate the deadweight costs, under the risk neutral measure. Similarly, the buyer's cost of financing is $r_b = r_f + \beta_b$, assuming that $r_s > r_b > r_f$.

In a deterministic demand size setting, the model assumes that the supplier only needs to borrow if the AR are not collected before the liabilities are due at the end of the period.

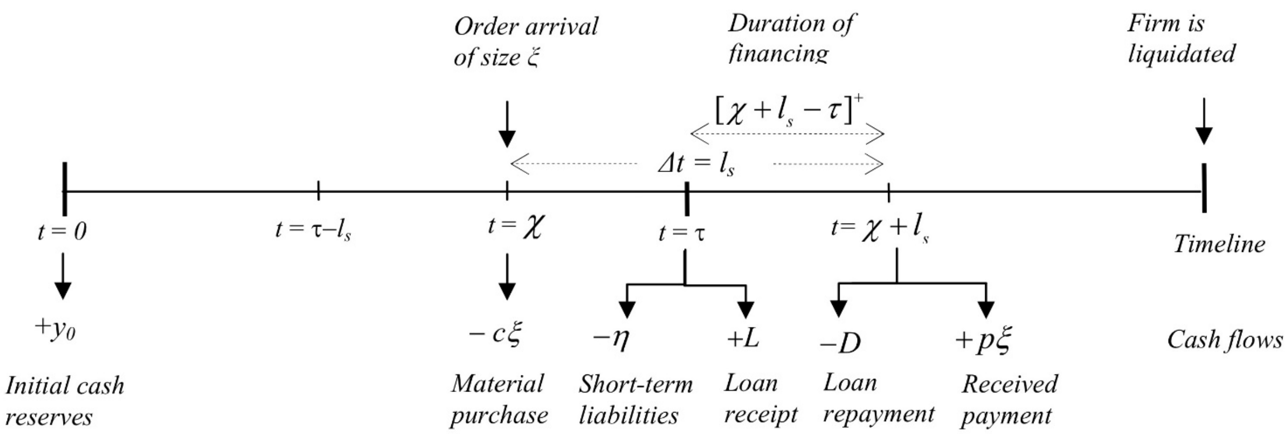


Figure 1.7: The MTO model description with external financing (Cetinay, Tanrisever & Reindrop 2012)

In a Reverse Factoring (RF) arrangement, the supplier may sell his approved invoices (or AR) to the bank and receive a loan maturing at the nominal payment time of the buyer, which from the bank's perspective is assured by the creditworthiness of the buyer. This gives the supplier a funding advantage, as the rate of funding based on creditworthiness of the buyer is cheaper than the one based on his own creditworthiness. The bank may charge an additional premium for providing the RF service, called b. The RF loan is secured by the buyer and together with the financier fee results in the RF rate $r_{rf} = r_b + b$.

The benefit for the supplier under the assumption of deterministic demand can be calculated in Present Value (PV) terms according to the following formula:

$$\beta_s A(i - l_s)L(\mu) - (\beta_b + b)A\big(i - l_{rf}\big)L(\mu) - p\mu r_f(l_{rf} - l_s)$$

where $A(i - l) = E_x[\Delta l I_{\{\chi > i-l\}}]$, i.e. $E[\chi + l - i]^+$ for l_s, l_{rf}, and $L(\mu) = \eta + c\mu - y_0$

The first part of this formula $\beta_s A(i - l_s)L(\mu)$ is the expected external financing cost of the supplier without RF, where $A(i - l_s)$ is the maturity and $L(\mu)$ the nominal value. The second part $(\beta_b + b)A\big(i - l_{rf}\big)L(\mu)$ represents the expected external financing cost with RF. The last part $p\mu r_f(l_{rf} - l_s)$ is the opportunity cost of RF. There is a participation constraint for the supplier: the savings allowed by RF $\beta_s A(i - l_s)L(\mu) - (\beta_b + b)A\big(i - l_{rf}\big)L(\mu)$ must be at least equal to its opportunity cost.

Considering stochastic demand size and timing, the supplier may seek financing upon demand realization or at the end of the period, in order to repay the short-term liabilities. The timing (χ) of incoming demand is crucial:

- If $\chi \leq i - l_{rf}$, AR are collected before the short-term liabilities are due at the end of the period and the supplier has to borrow only when the cost of raw materials exceeds the existing cash reserves;
- If $\chi > i - l_{rf}$, AR are collected after the short-term liabilities are due, and the supplier has to borrow an amount depending on the initial cash reserves and demand size.

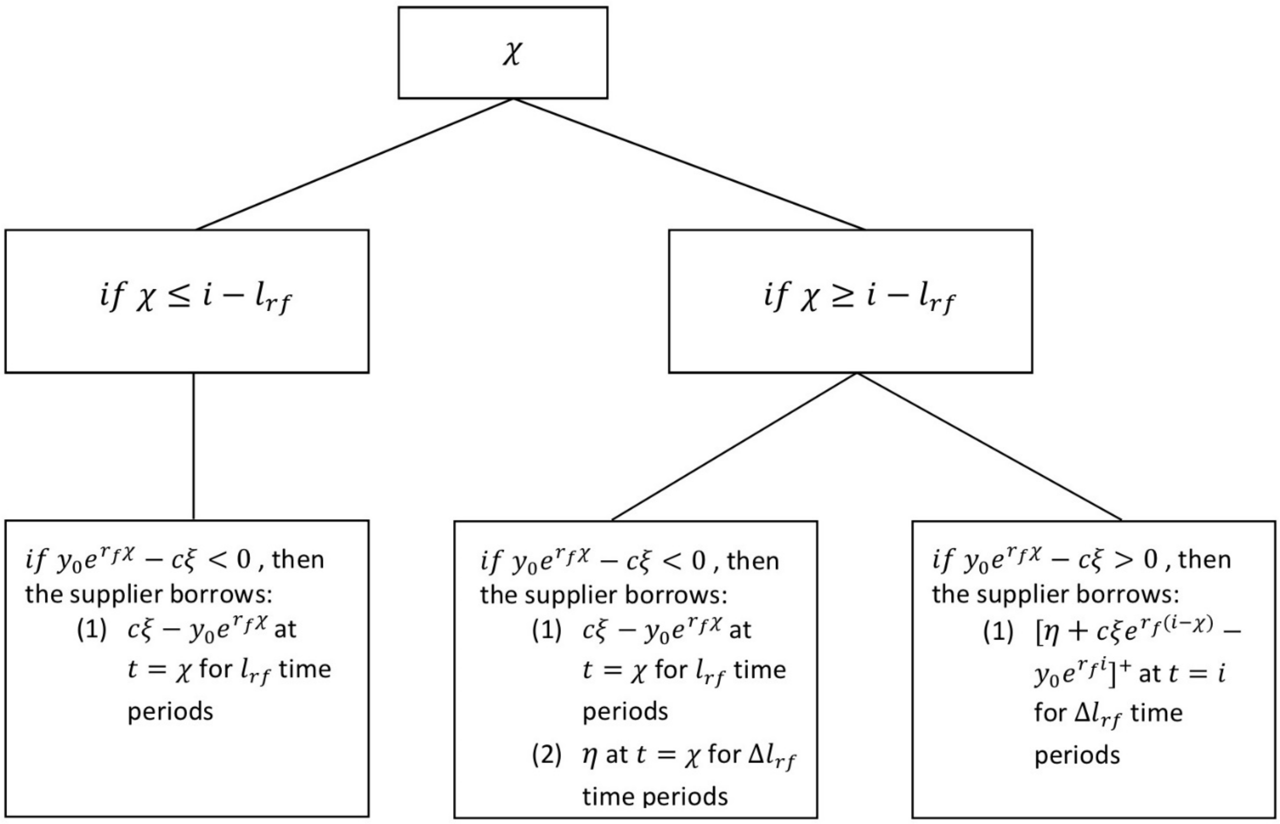


Figure 1.8: Borrowing needs of the supplier in the stochastic MTO model (Cetinay, Tanrisever & Reindrop 2012).

In this case, the supplier's benefits are:

$$\beta_s L(l_s) - (\beta_b + b)L(l_{rf}) - p\mu r_f(l_{rf} - l_s)$$

where, for $l = l_s$ or $l = l_s$,

$$L(l) = E_{\xi,x}\left[(\eta + c\mu - y_0)\Delta l I_{\Omega_1^l} + \eta \Delta l I_{\Omega_2^l} + (c\xi - y_0)\Delta l I_{\Omega_3}\right]$$

$$\Omega_1^l = \left\{\xi, \chi : \xi < \frac{y_0(1 + r_f\chi)}{c}, \eta(1 - r_f i) + c\xi(1 - r_f\chi) - y_0 > 0, \chi > i - l\right\}$$

$$\Omega_2^l = \left\{\xi, \chi : \xi > \frac{y_0(1 + r_f\chi)}{c}, \chi > i - l\right\}$$

$$\Omega_3 = \left\{\xi, \chi : \xi > \frac{y_0(1 + r_f\chi)}{c}\right\}$$

$L(l) = E_{\xi,x}(.)$ represents the financing requirement. If $(\xi, \chi) \in \Omega_3$ the cash reserves are insufficient to procure all needed raw materials and the supplier borrows to close the shortage when demand ξ arrives. Ω_2^l has the same restriction as Ω_3, but in addition to this, the supplier needs to borrow to finance the payment of short-term liabilities at the end of the planning horizon. If $(\xi, \chi) \in \Omega_1^l$ holds, the supplier does not need to borrow to finance the purchase of raw materials, but only to finance the payment of short-term liabilities. These three possibilities of financing requirements are also shown in Figure 1.8. For the proof and derivations of these formulas we refer to the working paper of Cetinay et al. (2012).

1.5.4 Roumen (2012)

No study has so far conducted any research on the benefits of RF under different demand patterns than just constant or normalized demand sizes. Many companies face demand fluctuations, often seasonal, without actually knowing how this might influence their RF contract. Roumen (2012) expands the previous single-period model developed by Cetinay et al., and determines a multi-period analytical derivation, in order to look at the impact of such seasonal patterns in demand size on the benefits generated by a RF arrangement.

Roumen proposes a three period model, which comprehends at least two complete cycles of demand arrival and AR collection, as shown in Figure 1.9.

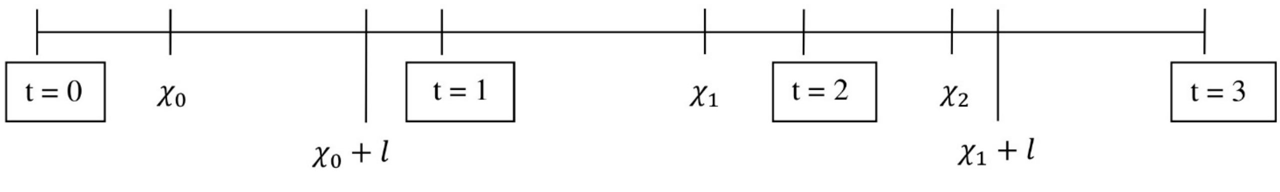


Figure 1.9: Random example of a three period horizon with three demand arrivals χ_i and two AR collections.

Demand arrival and size are stochastic, but it's assumed that the company receives an order once per period. The order reception may be before or after the collection of the AR generated in the previous period. As shown in the third period above, the third demand χ_2 is faced before the collection of the AR generated in the second period, $\chi_1 + l$. Three cases are distinguished, depending on the sequence of the events:

Case 1

AR are collected before liabilities are due:

$$i + \chi_i + l < (i + 1)$$

Case 2

AR are collected after liabilities are due, but before the second demand arrival:

$$(i + 1) \leq i + \chi_i + l < \chi_{i+1}(i + 1)$$

Case 3

AR are collected after liabilities are due and after the second demand arrival:

$$i + \chi_i + l \geq \chi_{i+1}(i + 1)$$

In the three cases, a cash/loan balance is calculated corresponding to each event occurred in the first two periods. The third period is never directly analyzed, since the author infers that there is a constant pattern and that the sequence of the events in the third period will lead back to one of the three possible cases that arose in the second period. Due to the presence of a recurrent pattern, the same three cases arise upon every demand arrival, depending on the stochastic timing of the demand arrival and the payment period. Therefore, it is not necessary to investigate into any further period.

The contribution also includes a simulation, used to investigate different scenarios and the impact of variability in specific input parameters. The focus of this simulation is on the new entity "Dividends", defined as the cash that exceeds a cash limit; in other words, the cash that is not required to run the company is used to pay dividends to its shareholders. This allows studying the impact of seasonality without the possibility that the system builds up too much cash in the long run.

The results indicate that the supplier, other conditions being equal, is better off with a non-auto RF, since it gives the opportunity to decide whether to discount each invoice and to decide upon needs. On the other hand, auto RF contracts are more profitable for the financial providers, as it increases the volume of discounted invoices. The supplier might actually be indifferent between auto and non-auto RF, or even be better off with auto RF, if we take into account other conditions, such as administrative costs (Crowe, 2009).

2 Objectives and Methodology

This chapter is divided in two sections: the first illustrates the research questions behind this work, while the second part provides the methodology followed. In particular, the second part concerns the methods applied to derive the theoretical conclusions, the tools used to verify those conclusions and the approach followed to carry out the case-study.

2.1 Objectives

The objectives can be expressed through three research questions:

1) *What are the tangible benefits made possible by Reverse Factoring for those large buyers who are supposed to initiate it?*

As can be noticed in the Literature Review, the majority of the authors who tried to set a mathematical model that evaluates Reverse Factoring benefits focused mostly on the supplier. When buyer's benefits were considered (e.g. Van Laere 2012), it was a mere calculation of the Accounts Payable's increase. In other cases, buyers' advantages were considered intangible and only laid on suppliers' improvements (Tanrisever et al., 2012). This thesis seeks to provide a mathematical model able to show that RF, under certain conditions, leads the buyer to a cost reduction. The model shows as well that the cost reduction is achieved through operational decisions that can be undertaken thanks to the RF agreement. It also aims to identify the conditions under which the supplier may refuse or accept to join Reverse Factoring, and to verify whether buyers' favorable conditions may jeopardize supplier's benefits.

2) *Focusing on suppliers, what are the benefits induced by the increased financing flexibility that Reverse Factoring grants?*

Reverse Factoring does not merely offer lower interest rates to suppliers, but involves also higher flexibility when they need financing. Other SCF tools as Factoring or Traditional Reverse Factoring do not allow suppliers to choose whether and when discount invoices: these different conditions, besides lower interest rates, contribute to make RF a value-added solution. This work aims to propose a mathematical model able to represent correctly these benefits, showing particularly how RF entails different cash dynamics.

3) *What are the operational implications caused by Reverse Factoring and the other SCF tools for suppliers?*

Another purpose is to show how Reverse Factoring affects operational decisions such as inventory ordering policies, and compare these policies with the ones applied in absence of Reverse Factoring. Each different solution leads to a different profit function, from which we derived mathematically the optimal replenishment policies. The model aims to show therefore how operating decisions are affected not only by the interest on hand, but also by the type of financing solution chosen.

4) *On which conditions Reverse Factoring outperforms the other financing means?*

The final research question that inspired this work concerns the comparison between the 2^{ND} generation Reverse Factoring and the other traditional financing means as Factoring, Traditional Reverse Factoring, Asset Based Lending and unsecured Credit Line. The models developed have been extended in order to represent the dynamics involved when other financial tools are used and allow comparisons with RF. Sensitivity analyses and simulations aim to identify the specific scenarios in which Reverse Factoring constitutes a better choice for suppliers and the ones in which it does not.

2.2 Methodology

The methodology followed to carry out each model is similar and constituted by four steps:

1. Review and analysis of the papers that had already addressed the problem and provided a theoretical model to represent it;
2. Analysis of other journal articles, unpublished papers and reports to identify important aspects not yet addressed with a formal method by the literature;
3. Characterization of the assumptions and developing of the mathematical model: equations and proofs (where needed);
4. Numerical analysis or simulation.

This paragraph illustrates more precisely the methodology followed: the literature review, the mathematical models development, the numerical analyses and the simulations.

2.2.1 Literature Review

Considering the nature of the models developed, rather theoretical than practical, the literature review constituted a fundamental part through the whole process: from the formulation of the Research Questions, to the models developing and the conclusions drawing. A significant basis of the literature consists of scientific papers that illustrate the Supply Chain Finance existing models and other academic working papers that suggest further analyses or new perspectives to approach the subject: these provided a solid starting point for deriving the Reverse Factoring-focused models. Another consistent support was given by journal articles and reports, in particular to address Reverse Factoring and the other SCF solutions, in order to highlight the main differences between them in the models developing. The references of this work can be thus divided in journal articles, books or book chapters, reports, unpublished working papers and theses, and other sources (non-academic articles, industry magazines, web sources). In particular, the journal articles sources were:

- Management Science, (5);
- International Journal of Logistics Management, (2);
- Supply Chain Management: an International Journal, (2);
- Journal of Business Logistics, (2);
- International Journal of Physical Distribution & Logistics Management, (2);
- Seventeen other sources with a single contribution;

Figure 2.1 reports the sources subdivision per category:

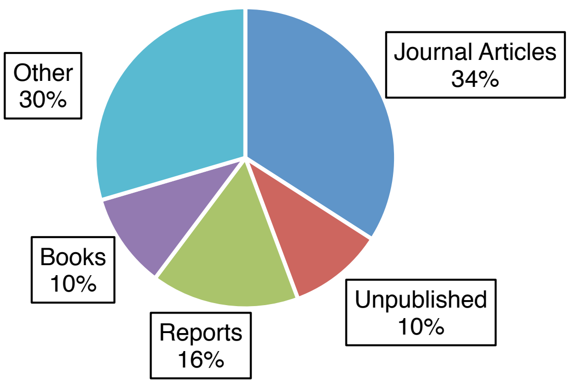


Figure 2.1: Sources divided by category.

2.2.2 Models development

Even though the three models presented in this work have been derived following the same methodology, each one reveals peculiar characteristics.

Reverse Factoring and Economic Order Quantity in presence of Working Capital Constraints

Following from Huang (2007), this model starts from the Economic Order Quantity framework (Harris, Ford W., 1990) and focuses on the buyer's perspective. As in the EOQ model, the main problem consists in a costs minimization under deterministic demand. The additional assumptions regard the WCR fixed constraint and the fixed Days Sales Outstanding. There are two cases characterized by the presence or absence of a Reverse Factoring arrangement. In both cases, the problem turns to a one-variable constrained minimization problem: the solution is therefore given either by the optimal unconstrained solution (if the constraint allows it) or by the boundary-solution, since the function is proven to be concave. We also show and explain the solution graphically. A numerical analysis is provided in order to present different scenarios: when the buyer takes all the advantages and when, instead, the benefits are shared with the supplier.

Reverse Factoring and Growth

The second model consists in an extension of Buzacott's work (2004). His paper addressed the financing problem for high-growing suppliers, which very often happen not to fully capitalize on their growth potential due to lack of liquidity. In particular, it shows how Asset Based Lending (ABL) may enhance suppliers' performances in terms of liquidity and profits in a rapidly growing scenario, in which the demand is always higher than the quantity produced. The problem is then set as a retained earnings maximization problem, with four decision variables and several inventory, production and cash constrains. In order to adapt and extend this model to Reverse Factoring, we modified some equations and introduced some new assumptions. In order to represent the model numerically, we used Microsoft Excel and identified the optimal solution through the Microsoft Excel's tool *Solver*.

t	qRM	qWIP	qFG	I(RM)	I(WIP)	I(FG)	s(t)	K (t)	INV. in K(t)
0	0	0	0	$ -	$0	0	0	100	0
1	25	25	0	$ -	$101	0	0	100	0
2	49	49	0	$ -	$396	0	0	100	0
3	94	94	25	$ -	$763	0	25	100	0
4	100	100	49	$ -	$1.149	0	49	100	0
5	100	100	94	$ -	$1.200	0	94	100	26
6	126	126	100	$ -	$1.303	0	100	126	4
7	130	130	100	$ -	$1.524	0	100	130	17
8	147	147	126	$ -	$1.625	0	126	147	19
9	166	166	130	$ -	$1.839	0	130	166	14
10	180	180	147	$ -	$2.044	0	147	180	0
11	0	0	166	$ -	$1.439	0	166	180	0
12	0	0	180	$ -	$0	0	180	180	0
13	0	0	0	$0	$0	$ -	0	180	0
t	qRM	qWIP	qFG	I(RM)	I(WIP)	I(FG)	s(t)	K (t)	INV. in K(t)
	OK								TOTAL CAP. INVESTMENT
									80

Figure 2.2: screenshot of the MS Excel model setting.

FA (t)	x(t)	w(t)	A'	A	y(t)	z(t)	π(t)
$5.000	$200		$ -	$ -	$ -	$ -	$ -
$5.000	$201	$ -	$ 1	$ -	$ -	$ 101	$ 1
$5.000	$0	$ -	$ 1	$ -	$ -	$ 194	$ 2
$5.000	$0	$ -	$ -	$ 15	$ -	$ 374	$ 189
$5.000	$0	$ -	$ 0	$ 29	$ -	$ 400	$ 549
$5.000	$0	$ -	$ 0	$ 56	$ -	$ 400	$ 1.242
$6.284	$0	$ -	$ -	$ 60	$ -	$ 503	$ 1.983
$6.481	$0	$ -	$ -	$ 60	$ -	$ 518	$ 2.723
$7.354	-$0	$ -	$ 0	$ 75	$ -	$ 588	$ 3.654
$8.279	-$0	$ -	$ -	$ 77	$ -	$ 662	$ 4.614
$8.994	$584	$ -	$ -	$ 88	$ -	$ 720	$ 5.703
$8.994	$1.698	$ -	$ 3	$ 99	$ -	$ -	$ 6.932
$8.994	$4.478	$ -	$ 8	$ 107	$ -	$ -	$ 8.272
$8.994	$ 4.500		$ 22	$ -	$ -3.598	$ -	$ 8.295
FA (t)	x(t)	w(t)	A'	A	y(t)	z(t)	π(t)

Figure 2.3: screenshot of the MS Excel model setting.

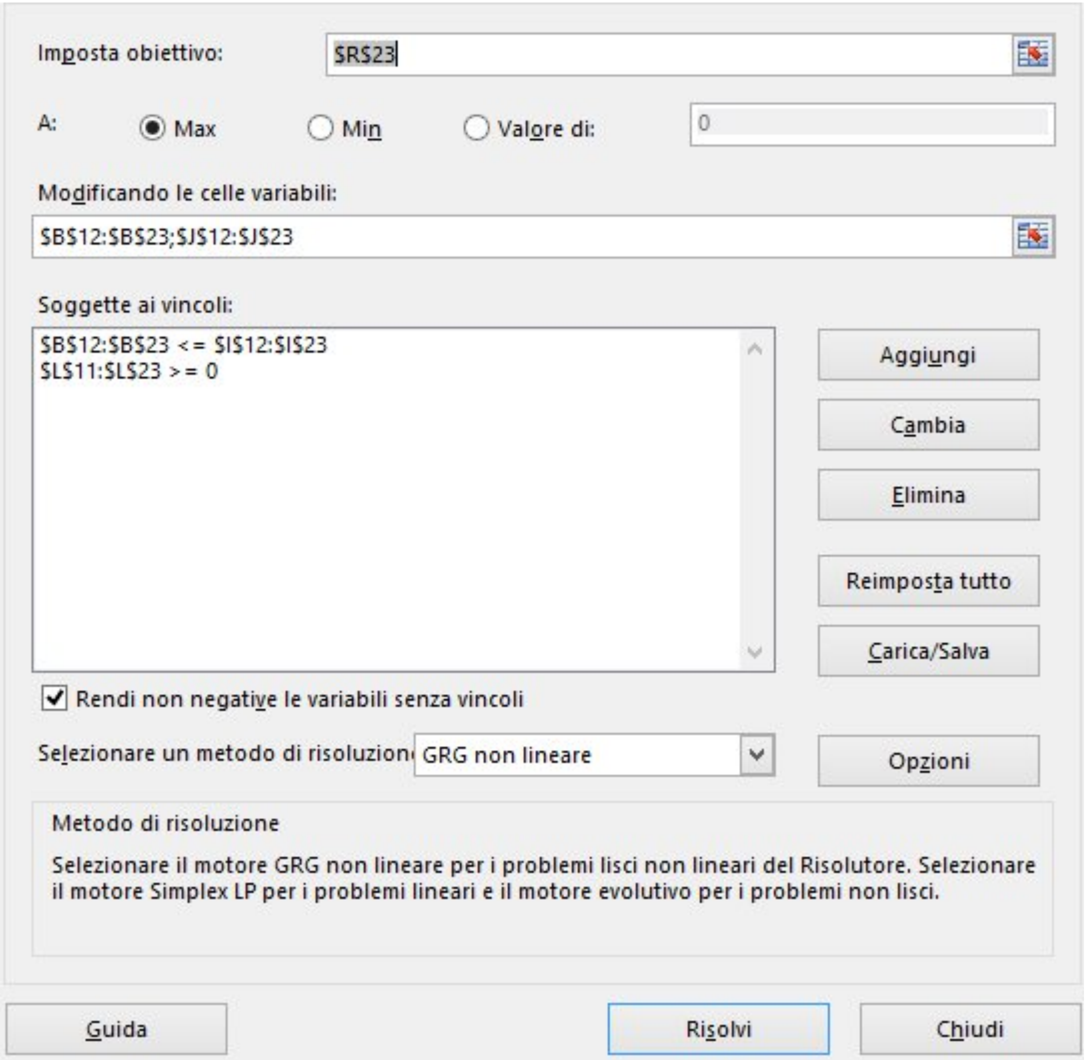


Figure 2.4: screenshot of the MS Excel Solver setting.

The theorems stated by Buzacott were still admissible and provable in the Reverse Factoring case, as it will be shown. In addition, we have also introduced another scenario in which the firm has a capacity constraint: this affected the cash dynamics equations and the production decisions variable, that needed to be modified. The problem solution still followed the same process and brought to similar conclusions.

Reverse Factoring and Replenishment Decisions facing Stochastic Demand

We developed the third model starting from Luo and Shang's working paper (2013): in their work, the authors provide a model to describe a supplier's cash dynamics when he applies trade credit and to evaluate the benefits brought by this SCF solution. The authors used the Newsvendor Model framework (Arrow et al., 1951): an expected costs minimization under uncertain and stochastic demand. We significantly changed Luo and Shang's model in order to represent Reverse Factoring and to find an optimal solution: we introduced new assumptions and rewrote the majority of the equations, since Reverse Factoring behaves extremely differently from Trade Credit. S.D. Lekkakos, our supervisor, had already addressed the same problem: when we compared our model to his, some differences came up in both the profit function and the cash dynamics and the model presented in this work was acknowledged to be more accurate. We then extended it to other assumptions, which implied different equations and new optimal solutions.

In order to run the simulations, we represented the model in Microsoft Excel, as shown in figure 2.5:

Reverse Factoring												
						S_rf	S_t-2	S_t-1				
						112	112	111				
x(t)	z'(t)	R'(t)_2 (oldest)	R'(t)_1 (youngest)	R(t)_2 (oldest)	R(t)_1 (youngest)	z(t)	z(t)_t-2	z(t)_t-1	y	π(t)	π_ret(t)	Demand Coverage %
0	120	60	60	60	60	120	180	239	112	33	33	100%
7						75	135	239	112	-9	23	100%
38						61	165	238	112	34	57	100%
30						83	156	237	112	74	131	63%
0						44	125	235	112	42	172	100%
1						14	124	234	112	-38	134	100%
52						65	174	233	112	96	230	80%
0						62	121	232	112	-2	228	100%
29						38	148	230	112	-24	203	100%
61						97	179	229	112	20	223	100%
53						121	171	229	112	60	283	100%
23						82	140	228	112	64	347	100%
2										2	354	95%

Figure 2.5: Screenshot of the MS Excel model setting.

We then used the Monte Carlo Method to represent the uncertain demand faced by the company. Monte Carlo methods vary, but tend to follow a particular pattern:

1. Define a domain of possible inputs;
2. Generate inputs randomly from a probability distribution over the domain;
3. Perform a deterministic computation on the inputs;
4. Aggregate the results.

The random inputs are generated according to a preset distribution (e.g. Normal, Lognormal, Uniform, Triangular..) and enter the model through the input cells. The model's equations work the inputs and lead to the ending result. Consequently, the Monte Carlo Method allows simulating the behavior of a system in presence of inputs' uncertainty: the more realistic the chosen model is, the more reliable the outcome will be. In particular, the random input cell in our model is the buyer's monthly order (the demand) that we assumed to be normally distributed. A second factor that influences the reliability of the simulation consists in the number of iterations N: each iteration i leads to a different outcome $\rho_{i,j}$ of each KPI j, depending on the demand realization, independent from the previous iterations and equally distributed. The final output of the simulation is given by $\widehat{\mu_j}$, the mean of all the outcomes generated:

$$\widehat{\mu_j} = \frac{1}{N} \cdot \sum_{i=1}^{N} \rho_{i,j}$$

In order to set a proper number of runs, we have to consider the confidence intervals related to the true mean, μ, (Johnson et al., 2002):

$$\left[\hat{\mu} - \frac{s}{\sqrt{N}} \cdot t_{N-1,\frac{\alpha}{2}}, \hat{\mu} + \frac{s}{\sqrt{N}} \cdot t_{N-1,\alpha/2}\right]$$

where $s = \sqrt{\frac{1}{N-1} \cdot \sum_{i=1}^{N-1}(x_i - \hat{\mu})^2}$ is the Corrected Sample Standard Deviation and α represents the level of confidence.

We set $N = 1.000$ in order to have a sufficiently narrow confidence interval, so that when we compare the mean values between different scenarios even small differences may be considered significant.

As will be explained more accurately, we simulated 1000 runs for each possible combination of scenarios: Reverse Factoring vs. Auto-Financing, Reverse Factoring vs. Credit Line, and so on. The behavior of the same company, facing the same random demand, is therefore simulated 1000 times, generating as outcomes the Retained Profit and the Demand Coverage, as shown in Figure 2.6:

	π No_RF	π RF	delta% Profit	Demand Coverage no_RF	Demand Coverage RF
min	246	243	-15,4%	63,2%	84,3%
av	352	398	13,1%	81,3%	95,0%
max	384	517	36,2%	97,8%	100,0%

Figure 2.6: Screenshot of the model's output.

We also derived, for each possible comparison between RF and other SCF tools, an interval of confidence of the differences between the respective profits. When this interval is positive, it means that, according to the confidence level chosen α, the mean profit under a RF program is higher than the profit under the alternative solution. In order to do this, we used the univariate case of the Paired Comparison Method (Johnson et al., 2002):

Let (x_1, y_1), $(x_2, y_2), \dots, (x_{1n}, y_n)$ be the identically distributed (i.i.d.) n samples of the retained profits generated respectively by RF and a generic alternative solution (No financing, Credit Line, Traditional RF or Factoring). We define $d_i = x_i - y_i$ as the difference between x_i and y_i. We then derive

$$\bar{d} = \sum_{i=1}^{n} d_i$$

$$s_d^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (d_i - \bar{d})^2$$

The statistical theory proves that the true mean value of the population d_i is contained within:

$$\left[\bar{d} - \frac{s_d}{\sqrt{n}} \cdot t_{N-1,\frac{\alpha}{2}}, \bar{d} + \frac{s_d}{\sqrt{n}} \cdot t_{N-1,\frac{\alpha}{2}}\right]$$

Case Studies

The testing of the third model, *"RF and Replenishment Decisions facing Stochastic Demand"*, is enhanced developing two case studies, in order to evaluate the actual benefit that RF could bring to a real enterprise that faces a stochastic demand and whose characteristics reflect the assumptions of the model. The first case study is focused on FAB S.p.A., an Italian SME operating in the Personal Protective Equipment (PPE) industry, while the second is realized on Stocchetta Cilindri s.r.l., another Italian SME operating in the mechanical manufacturing and specialized in the production of hydraulic cylinders.

The case studies share a common structure, illustrated in Figure 2.7.

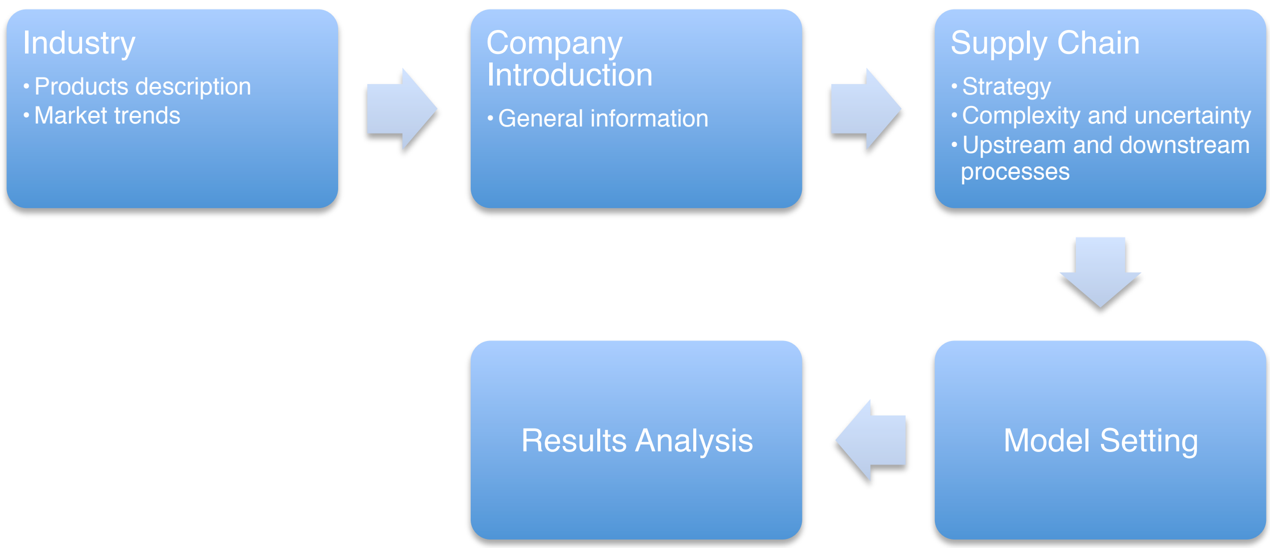


Figure 2.7: Structure of the case studies.

The industry-related information was found in specialized papers and reports, while the information related to each company, its supply chain and the data used to run the model was gathered in interviews carried out in the headquarters. Table 2.1 lists the business contacts:

NAME	ROLE	COMPANY
Paolo Neri	Chief operating officer (COO)	FAB S.p.A.
Roberto Pintossi	Chief financial officer (CFO)	Stocchetta Cilindri s.r.l.
Silvia Archetti	Customer service	Stocchetta Cilindri s.r.l.

Table 2.1: List of interviewed persons.

List of questions

The interviewees were first asked to describe their respective companies and then to answer the following questions.

Supply Chain Related

- What is your supply chain configuration?
    - How is your supply base composed?
    - How is your customer base composed?
    - Which operations are internally managed and which are outsourced?
    - Which supply chain processes are local and which are global?

- Is supply chain complexity relevant for your company?
    - How many items do you manage?
    - Is the technological process complicated?
    - How many suppliers do you do business with?
    - Is your distribution network complex?
    - Are international flows relevant?
- Is supply chain uncertainty relevant for your company?
    - Is the supply base capable to provide the company with the required materials?
    - Is there any productive capacity issue?
    - Is the bullwhip effect relevant?
    - Is lead time considerable?
    - Is the promotion phenomenon relevant? Are customers' promotions jointly planned in advance?
    - Are technologies mature and stable or turbulent?
- Would you define your products as *functional* or *innovative*?
    - Is demand volatile and/or are customers' preferences evolving rapidly?
    - What is the product lifecycle?
    - Is the contribution margin high?
    - What is the product variety?
    - Which is the average forecast error for make-to-stock products?
    - Which is the average stock-out rate for make-to-stock products?
    - Do you clear off unsold inventories (end of season markdown)?
- Would you define your process as *stable* or *instable*?
    - Are technologies and processes mature?
    - Is the supply system rather wide and clear or narrow and unstable?
    - Is production automated?
    - Do you rather use medium or long-term supply contracts or spot supply contracts? Do specifications change frequently?

Model Setting Related

- Do you regularly sell any product managed in MTS strategy to a large customer? If so, which one?
- Could you provide the orders received within the last two years?
- Which is the production or purchasing cost and which the selling price?
- How do you estimate the annual inventory holding cost?
- Which is the average cost of debt for your company?

Subsequently the information collected has been organized and used to run the model.

3 Reverse Factoring and Economic Order Quantity in presence of Working Capital Constraints

The financial crisis and the following credit shortcut have driven the attention of most CFOs to the Working Capital Requirement (WCR) borne by the companies every year. WCR is the amount of capital a company must maintain in order to continue to meet its costs and expenses and it can be defined, referring to a regular balance sheet, as:

$$WCR = Cash + Accounts\ Receivable + Inventory - Accounts\ Payable$$

This definition allows to analyze the balance sheet from a different perspective than the usual one:

ASSETS	LIABILITIES
Cash Accounts Receivable Inventory	Accounts Payable Debt
Fixed Assets	Equity

⇨

ASSETS	LIABILITIES
Working Capital Requirements	Debt
Fixed Assets	Equity

Figure 3.1: Working Capital Requirement from Balance Sheet perspective.

This perspective shows clearly that a company, in order to increase its WCR level, must increase either Debt or Equity by the same amount. With the credit crisis started in 2008, lots of companies had to focus more specifically on their Debt level and the reasons why their capital needs were often higher than the investments planned. This is one of the causes that induced many CFOs to set a WCR constraint to the operating functions of the company, in order to set the next year debt level based on the incremental investments in fixed assets, without worrying about the capital needed by the operations. As already highlighted by Buzacott (2004), this constrained liquidity challenge did not involve only small firms: "even large firms can find their operating decisions constrained by their limited ability to borrow. For example, Crown Books, a bookstore chain, had to return a substantial number of books early in 1998. About $25 million of inventory were returned in addition to its regular seasonal returns in an initiative that was part of Crown's attempt to improve its inventory turns and liquidity. Crown was having liquidity problems partly because of a clause in its credit facility that limited its borrowing limit to $25 million if the company's net worth were under $70 million".

The new focus on WCR restrictions is also shown by the ongoing lengthening of payment terms that large firms (e.g. Procter & Gamble, as reported by Goel and Wohl, Conerly and SCDigest in 2013) are imposing on their suppliers. Lengthening payment term increases Accounts Payable, and therefore lowers WCR. These circumstances suggest that companies are basically trying to free up as much cash as they can, through a better management of their WCR. Reverse Factoring represents nowadays a big opportunity for companies: thanks to RF, a large buyer can lengthen payments without worsening her suppliers' financial position. The payment extension represents for a buying firm a mean of freeing up cash and the opportunity to use this cash in different ways, e.g. increasing the inventory level: "Lack of financial resources means that that operational decisions can be severely constrained. For example, a firm may have to use lot sizes substantially less than the economic order quantity" (Buzacott, 2004).

The first model developed in this paper aspires to evaluate the advantages that a large buyer with a certain WCR constraint can benefit from, applying a RF arrangement with a supplier. The framework chosen to evaluate these benefits is the Economic Order Quantity framework (Harris, Ford W., 1990), as other authors as Huang (2007) already did. In order to do so, we first analyze a basic EOQ model case and how its results can be affected by a WCR constraint. Subsequently, we show how a RF arrangement can bring benefits for both the buyer and the supplier by relaxing WCR constraints.

3.1 The EOQ model with WCR constraint

Consider a facility (buyer) that faces a constant demand for a single item and places orders for the item from another facility (supplier), which is assumed to have an unlimited capacity. The basic EOQ assumptions are the following:

- Demand is constant at a rate of D items per year;
- Order quantities are fixed at Q items per order;
- A fixed set-up cost A is incurred every time the buyer places an order;
- The unit purchasing cost is equal to c;
- A linear inventory holding cost h is accrued for every unit held in inventory per year;
- The lead time is zero;
- Initial inventory is zero;
- The planning horizon is infinite;
- All the demand is satisfied on time.

In addition to the EOQ assumptions, we assume the following:

- The buyer's purchases are made on credit described by a single term t_c, which represents the time after which the buyer makes the order payment to the supplier;
- Due to finance-oriented planning requirements (such as short-term borrowing planning), there is a constraint imposed by the finance department on the average working capital requirements to not exceed a threshold value $\overline{K}$ (described below);
- The supplier is financing his outstanding invoices by using external financing (invoice advance or factoring) which is provided at an annual interest rate of $r_s = r_f + r_{ps}$, where r_f is the risk free rate and r_{ps} is the premium rate required by the bank, based on supplier's risk.

Working Capital Requirements (WCR)

In our model WCR, denoted by K, represents the net capital that is required to finance the firm's investment in current assets. The average WCR is given by $K = R + I - P$, where R is the average value of accounts receivable, I is the average value of inventory, and P is the average value of accounts payable. Specifically:

- *Accounts receivable* (R): Since the demand in our model is assumed constant, R is also constant and equal to the annual sales, D, multiplied by the price p, times the duration of credit in that is offered by the buyer to her customers expressed in years. Therefore: $R = p \cdot D \cdot t_r$ where t_r is the payment term conceded by the company to its own customers, $t_r = \frac{DSO}{365}$.

- *Inventory* (*I*): It can be shown that in the EOQ model the average inventory is equal to $\frac{c \cdot Q}{2}$.
- *Accounts payable* (*P*): The average value of accounts payable depends on the credit duration t_c; in this model, with the assumptions stated above, P is equal to $\frac{c \cdot Q \cdot t_c}{T}$, where T is the cycle time (the time between two consecutive replenishments). This formula can be easily proved using the inductive method (see Appendix A).

Consequently, K can be expressed as follows:

$$K = R + c \cdot \frac{Q}{2} - \frac{c \cdot Q \cdot t_c}{T}$$

And therefore:

$$K = R + c \cdot Q \cdot \left(\frac{1}{2} - \frac{t_c}{T}\right)$$

Optimal solution to the base-case model

The objective in the base-case model is to find the optimal ordering policy minimizing total purchasing and holding cost per unit of time (year) subject to the WCR constraint. Total cost in a cycle of length *T* is

$$A + \frac{h \cdot T \cdot Q}{2} + c \cdot Q$$

Since $Q = T \cdot D$, the average total cost per unit of time is

$$TC(Q) = \frac{A \cdot D}{Q} + \frac{h \cdot Q}{2} + c \cdot D$$

Then, our objective is to minimize $TC(Q)$ subject to the WCR constraint:

$$R + c \cdot Q \cdot \left(\frac{1}{2} - \frac{t_c}{T}\right) \leq \overline{K}$$

Since the second derivative of $TC(Q)$ is always positive, the function is convex, and it has a minimum value where the first derivative is 0, as the Economic Order Quantity theory already suggested; the unconstrained optimal order quantity is given by:

$$Q^* = \sqrt{\frac{2 \cdot A \cdot D}{h}}$$

However, when the WCR constraint is imposed, the optimal solution depends on whether the constraint allows the variable Q to equal the Q^* or not. We have to explore the WCR constraint to understand the relation between Q and the optimal solution:

$$R + c \cdot Q \cdot \left(\frac{1}{2} - \frac{t_c}{T}\right) \leq \bar{K}$$

Since all the parameters besides Q are fixed, setting the constraint as an equation leads to the maximum Q affordable, Q_{max}:

$$Q_{max} = \frac{2}{c} \cdot (\bar{K} - R + \ t_c \cdot D \cdot c)$$

Hence, two cases arise:

$if\ Q_{max} \geq Q^*$ $\qquad Q = \ Q^* \quad and \quad TC\ (Q) = TC(Q^*)_{min}$

Otherwise $\qquad Q = \ Q_{max} \ \ and \quad TC(Q) > \ TC(Q^*)_{min}$

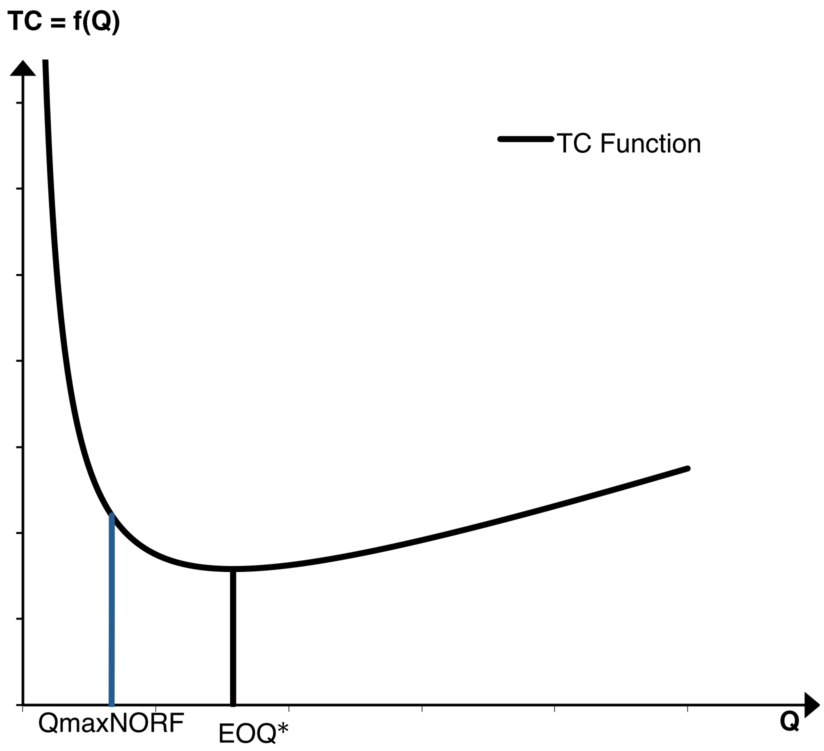


Figure 3.2: Total costs as a function of the ordering policy.

The WCR constraint may affect the optimal solution of the classic EOQ model if the constraint $\overline{K}$ is tight and does not allow the company order the quantity $Q = Q^*$, as shown in Figure 3.2. This situation is quite common for many companies, particularly in the last few years: lack of liquidity puts limitations on a firm's investments in fixed assets or inventory, preventing it from growth opportunities or cost savings. In the next paragraph we evaluate how Reverse Factoring can improve these situations by relaxing the WCR constraint and freeing up cash that can be used to increase the inventory level and save costs.

3.2 The EOQ model with WCR constraint and RF arrangement

Reverse Factoring is a financing solution similar to the traditional "Factoring", in which a supplier has the opportunity to discount his invoices at a lower interest rate, thanks to the better financial ranking of the buyer. It is defined as a "buyer centric" solution as it is the buyer, here, the initiating party that first talks to the bank and proposes to finance the Accounts Receivable of her suppliers. The supplier, who in the base case takes out a debt against his invoices at an interest rate r_s, can now finance himself discounting his Accounts Receivable at a rate $r_{RF} < r_s$. The new rate is given by $r_{RF} = r_b + b$, where $r_b = r_f + r_{pb}$ is the interest rate applied to the buyer and b is the fee that the bank would charge due to the reverse factoring contract. Usually, a buyer who offers RF ties the program with an extension in existing credit terms: we define as t_{RF} the new delay in payments allowed by the supplier with the RF, and as t_c the old one. Assuming the supplier to be a rational player, he will accept RF just if he is not worse off, meaning that his financing costs related to the invoice discounting must be at least equal to the previous ones.

Financing costs can be expressed as the amount provided by the financial institution times the interest rate. For both No-RF and RF cases, this amount equals the invoices the bank discounts, and therefore the Average Accounts Receivable. Thus, financing costs are defined by $\frac{c \cdot Q \cdot t \cdot r_i}{T}$ where $i \in (s, RF)$ and the condition that must be standing is:

$$\frac{c \cdot Q_{RF} \cdot t_{RF} \cdot r_{RF}}{T_{RF}} \leq \frac{c \cdot Q \cdot t_c \cdot r_s}{T}$$

Since the annual demand is deterministic and constant, it can also be expressed as $D = \frac{Q_{RF}}{T_{RF}} = \frac{Q}{T}$

Therefore, the inequality becomes: $t_{RF} \leq \frac{t_c \cdot r_s}{r_{RF}}$

In other words, the cost that the supplier has to bear to obtain early payments with the new solution (RF) must be at least equal or lower than the costs he would have incurred in the base case. In order to assure this condition, it is necessary to focus on t_{RF}, since we can consider all the other parameters exogenous. A new constraint represents the accepting-condition imposed by the supplier, and completes the new optimization problem.

Optimal Solution

$$\min \quad \frac{A \cdot D}{Q} + \frac{h \cdot Q}{2} + c \cdot D$$

$$\text{s.t.} \quad R + c \cdot Q \cdot \left(\frac{1}{2} - \frac{t_{RF}}{T}\right) \leq \overline{K}$$

$$t_{RF} \leq \frac{t_c \cdot r_s}{r_{RF}}$$

$$Q \geq 0$$

Observing the minimization problem, it can be noticed that the new payment term t_{RF} relaxes the WCR constraint, modifying the accounts payable. In order to show it, we can simply express the differential in the Accounts Payable of the buyer with and without the Reverse Factoring: since all the other components of the WCR are independent from the payment terms, any variation in the Accounts Payable results in a different WCR.

$$P_{RF} - P_{NRF} = \frac{c \cdot Q_{RF} \cdot t_{RF}}{T_{RF}} - \frac{c \cdot Q \cdot t_c}{T}$$

Being $D = \frac{Q_{RF}}{T_{RF}} = \frac{Q}{T}$, it follows that

$$P_{RF} - P_{NRF} = c \cdot D \cdot (t_{RF} - t_c)$$

$$P_{RF} - P_{NRF} = c \cdot D \cdot \left(\frac{t_c \cdot r_s}{r_{RF}} - t_c\right)$$

$$P_{RF} - P_{NRF} = c \cdot D \cdot \frac{t_c}{r_{RF}} \cdot (r_s - r_{RF})$$

Since $r_s > r_{RF}$ by the definition of Reverse Factoring, we can conclude that $P_{RF} > P_{NRF}$, and therefore, since the Accounts Payable decrease the Working Capital Requirement by definition, we can conclude that $WCR_{RF} < WCR_{NRF}$. We now derive the optimal solution.

As in the former paragraph, this is a constrained optimization problem, in which the objective function is convex and has a minimum point. However, in this case, there are two constraints instead of one: the new supplier constraint is actually the one that relaxes the WCR constraint. Again here, since the objective function is unvaried, the optimal solution depends on whether the constraint allows the Q to equal the optimal Q^* or not. To verify it, we push the payment term extension to its limit and check whether the constraint for $Q = Q^*$

Maximum payment terms acceptable by the supplier: as stated before, the supplier must be at least even with the previous situation, so

$$t_{RFmax} = \frac{t_c \cdot r_s}{r_{RF}}$$

Maximum Q reachable with the new Reverse Factoring conditions:

$$Q_{RFmax} = \frac{2}{c} \cdot \left(\overline{K} - R + \frac{r_s \cdot t_c \cdot D \cdot c}{r_{RF}}\right)$$

If we compare this Q_{RFmax} with the maximum Q available without RF (Q_{NRFmax}), we can notice that:

Since $r_s > r_{RF}$ and $Q_{NRFmax} = \frac{2}{c} \cdot (\overline{K} - R + t_c \cdot D \cdot c)$,

We can conclude that $Q_{RFmax} > Q_{NRFmax}$

Hence, when the buyer has WCR limitations so tight that she cannot order the optimal quantity (EOQ), she can arrange a Reverse Factoring contract in order to relax this constraint, freeing up some cash and improving her performances:

$$Q_{NRFmax} < Q_{RFmax} \text{ , } Q_{NRFmax} < Q^*$$

$$TC(Q^*) < TC(Q_{RFmax}) < TC(Q_{NRFmax})$$

As we will argue in the next paragraph, the buyer will always relax the constraint up to the point that $Q_{RF} = Q^*$, since exceeding Q^* would just increase the Total Cost, as deducible in Figure 3.3.

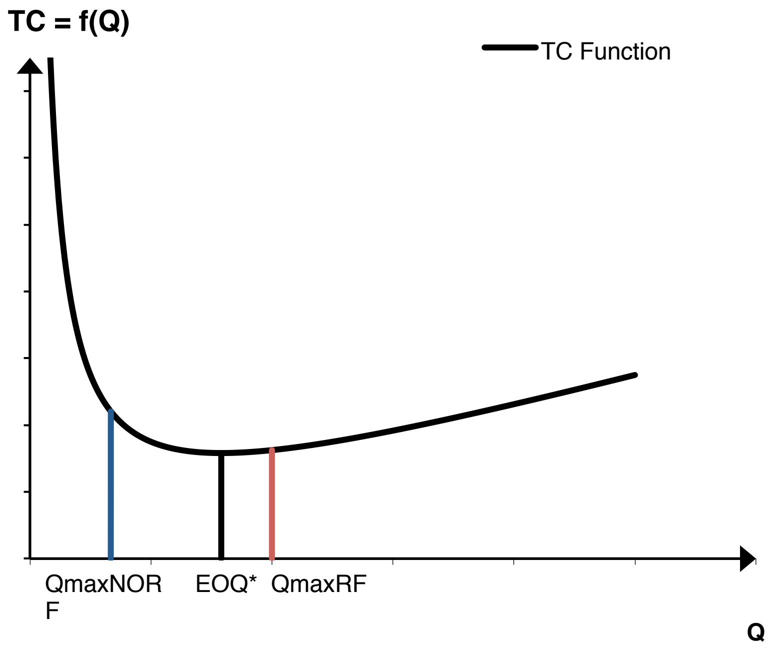


Figure 3.3: the maximum order quantity without RF, with RF and the EOQ

The negotiated payment term

We can then assume that, in the negotiation, the buyer's behavior is not driven by a competitive strategy, but by a collaborative one: the buyer will not receive all the gain from RF but will share it to some extent with the supplier. Paragraph 1.2 about Supply Chain Collaboration supports our assumption. Consequently, it can be gathered that the buyer will always be willing to lengthen the payment term until she reaches the order quantity Q^* that minimizes her costs, but not beyond that $t_{RF(Q^*)}$. We can now easily derive the payments delay, $t_{RF(Q^*)}$, that would lead the buyer to the optimal solution (Q^*):

$$R + c \cdot Q^* \cdot \left(\frac{1}{2} - \frac{t_{RF(Q^*)} \cdot D}{Q^*}\right) = \overline{K}$$

$$t_{RF(Q^*)} = \frac{R + c \cdot \frac{Q^*}{2} - \overline{K}}{c \cdot D}$$

At the same time, the supplier would never accept a payment term higher than $t_{RFmax} = \frac{t_c \cdot r_s}{r_{RF}}$. Hence, if $t_{RFmax} < t_{RF(Q^*)}$, the buyer will be able to extend payment terms only until t_{RFmax}, without reaching the optimal quantity given by the EOQ. If, instead, $t_{RFmax} \geq t_{RF(Q^*)}$, accordingly to the assumptions, the buyer will extend the payment terms up to the optimum value $t_{RF(Q^*)}$, even though she could achieve a higher extension, in order to share some benefits. We can therefore infer that the negotiated payment term t_{RF} results from the minimum between $t_{RF(Q^*)}$ and t_{RFmax} :

$$t_{RF^*} = min\{t_{RF(Q^*)}; t_{RFmax}\}$$

Supplier's Benefits
As the model's assumptions imply, the supplier's benefits can be evaluated in terms of financing costs, since the annual demand is constant as all the other variables besides the payments delay and the interest rate:

$$Supplier's\ Benefits = \frac{c \cdot Q \cdot t_c \cdot r_s}{T} - \frac{c \cdot Q_{RF} \cdot t_{RF} \cdot r_{RF}}{T_{RF}} = c \cdot D \cdot (t_c \cdot r_s - t_{RF} \cdot r_{RF})$$

Therefore, the higher is the difference between $t_c \cdot r_s$ and $t_{RF} \cdot r_{RF}$, the higher are the benefits gained by the supplier. However, since the bargaining power is chiefly in the buyer's hands, the supplier's benefit will depend on the t_{RF} proposed by the buyer. Being $t_{RF^*} = min\{t_{RF(Q^*)}; t_{RFmax}\}$, the supplier's benefit is hence given by:

$$Supplier's\ Benefit = \begin{cases} 0 & if\ t_{RF^*} = t_{RFmax} \\ c \cdot D \cdot \left(t_c \cdot r_s - t_{RF(Q^*)} \cdot r_{RF}\right) & if\ t_{RF^*} = t_{RF(Q^*)} \end{cases}$$

3.3 Numerical Analysis

In this section we illustrate the results obtained implementing the theoretical model on MS Excel.

In Table 3.1 we present the order quantity in three different cases:

i. Unconstrained working capital requirements ($Q = EOQ$);
ii. Constrained working capital requirements;
iii. Constrained working capital requirements with reverse factoring (RF).

The parameters for the base case analysis are: $A = \$500$, $D = 100.000\ u/year$, $c = \$50$, $h = 20\%$, $p = \$80$, $t_r = 0{,}822\ (30\ days)$, $t_c = 0{,}822$, $r_f = 2\%$, $r_{ps} = 9\%$, $r_{pb} = 4\%$, $b = 2\%$, $\overline{K} = \$280.000$.

Unconstrained WCR ($WCR = \$325.632$)	Constrained working capital requirements	Constrained working capital requirements (RF)
3.162	1.337	3.162

Table 3.1: Order Quantity

Note that the WCR constraint for the buyer imposes a tight limit to the maximum orderable quantity, assuming that the firm cannot reduce her accounts receivable, nor increase her accounts payable. This limit shifts the solution from the optimal, causing an increment in total costs, as effectively shown in Figures 3.4 and 3.5.

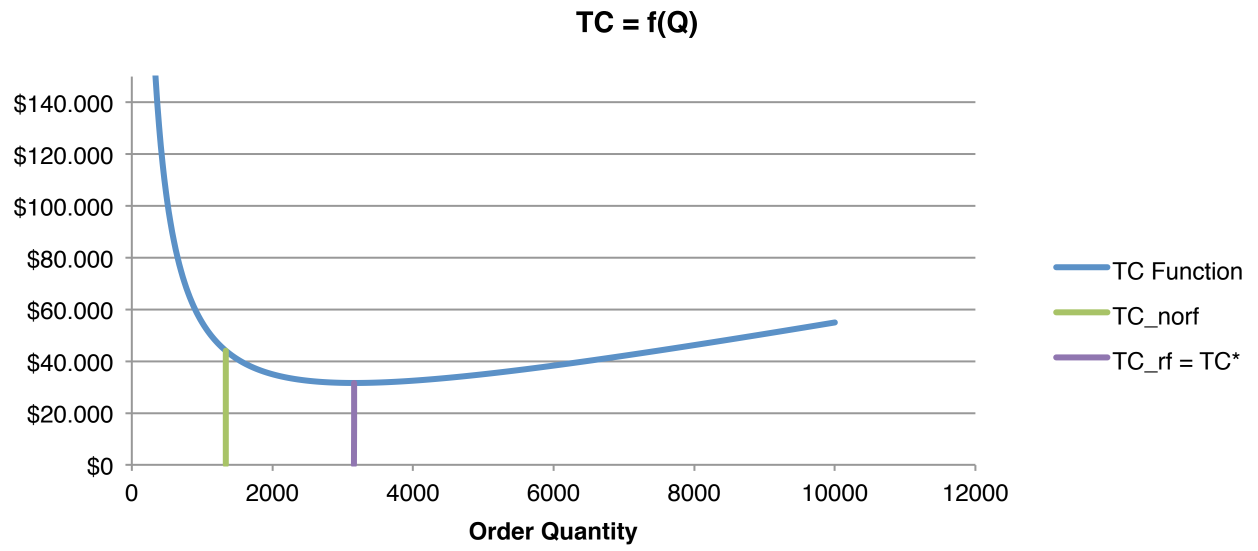


Figure 3.4: Total cost function for the buyer, which depends on order quantity Q.

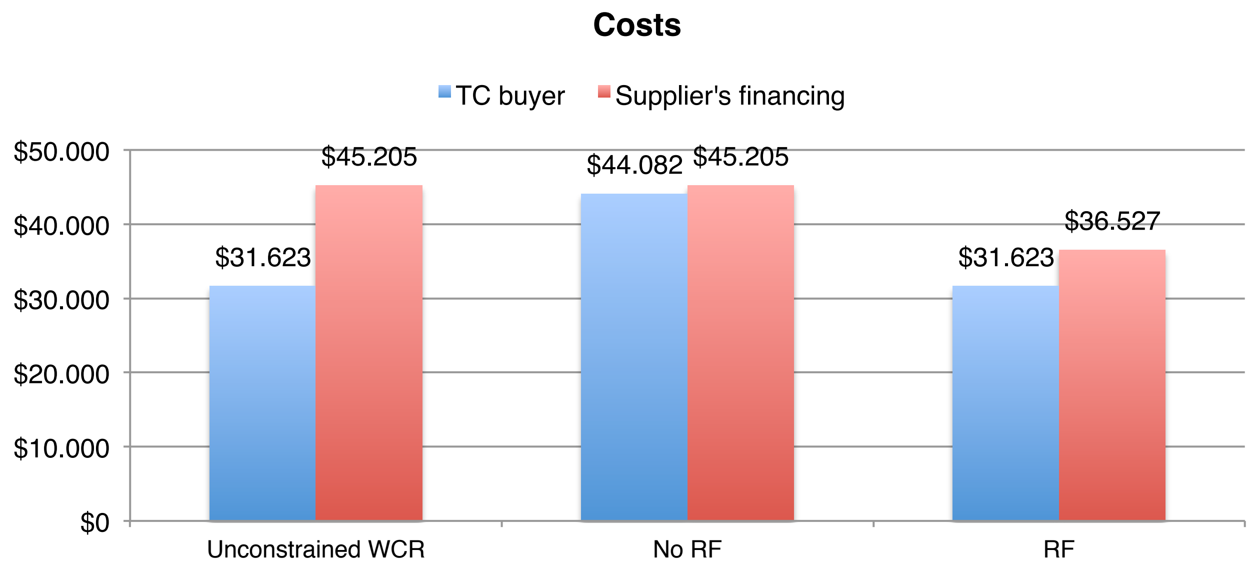


Figure 3.5: Total cost for the buyer and supplier in the three cases.

The maximum extension t_{RFmax} that guarantees a non-negative benefit for the supplier is to 41.3 days (+37.67%). On the other hand, the buyer needs a DPO extension from 30 days to 33.3 days (+11%), in order to obtain a sufficient increase of accounts payable to cope with the WCR constraint. Both the parties are better off signing an agreement to the latter conditions: the buyer saves \$12.460 (−28,27%) in ordering and stock holding costs, while the supplier saves \$8.678 (−19,20%) in financing his own working capital requirements.

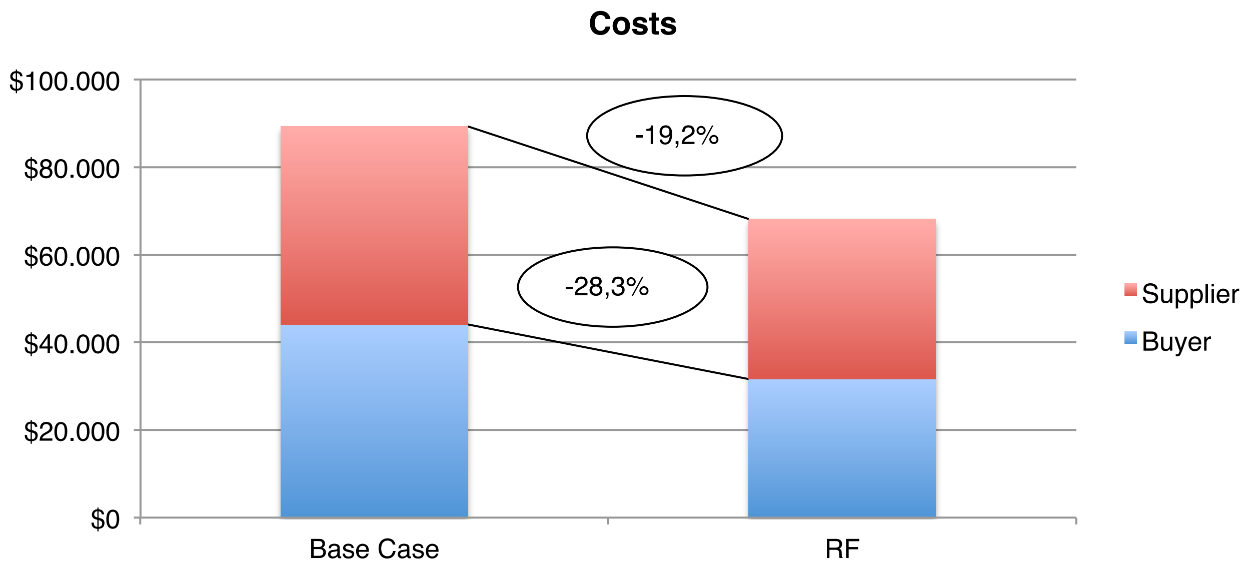


Figure 3.6: Cost reduction allowed by RF.

3.4 Sensitivity Analysis

In this section, we present the sensitivity analysis carried out on the MS Excel model. The analysis is run varying five parameters: the WCR limit granted by the CFO, the initial payment terms t_c, the negotiated payment terms t_{RF}, the ordering cost A and the annual demand D.

The WCR constraint

As we can observe in Figure 3.7, when the WCR constraint is significantly tight, Reverse Factoring generates most of the savings, mostly in favor of the buyer. As this constraint is gradually relaxed, the buyer's need for Reverse Factoring decreases and the supplier can benefit from the arrangement without lengthening payment terms.

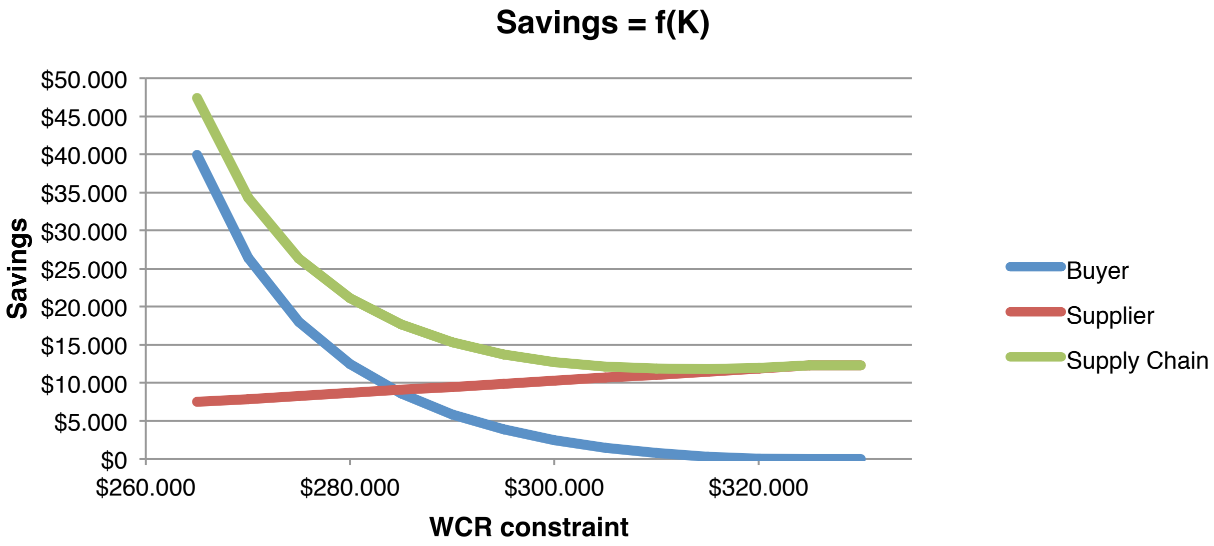


Figure 3.7: benefits as a function of the working capital constraint.

When $\overline{K} > \$325.000$, the constraint is not tight anymore, so the buyer's saving is null, while the supplier's reaches his maximum ($12.329). This behavior is consistent with the initial assumptions: once the buyer obtains the payment deadline t_{RF^*} that minimizes her total cost, she is not interested in extending it further; instead, she prefers her supplier to improve his cash flow thanks to a favorable negotiated payment term lower than t_{RFmax}, and the more advantageous interest rate r_{RF}. This is one of the main aspects of a Collaborative Supply Chain: the stronger party is aware that her own performances are strictly connected to the other supply chain partners. A financial improvement of her suppliers can represent, in the long term, a further benefit to her own performance (e.g. better service level agreements, lower prices or new opportunities thanks to suppliers' higher investments).

The current payment term t_c

The parameter t_c represents the current payment terms between buyer and supplier and is defined as $t_c = \frac{DPO}{365}$; it is also assumed to be exogenous (e.g. given by the market conditions). The more bargaining power suppliers have, the sooner buyers have to issue their payments, and vice versa. Therefore, it can be easily gathered that Reverse Factoring, whose principal effect is to let the buyer lengthen payment terms with her supplier, is more effective in those markets where suppliers managed to be paid relatively soon from the main players.

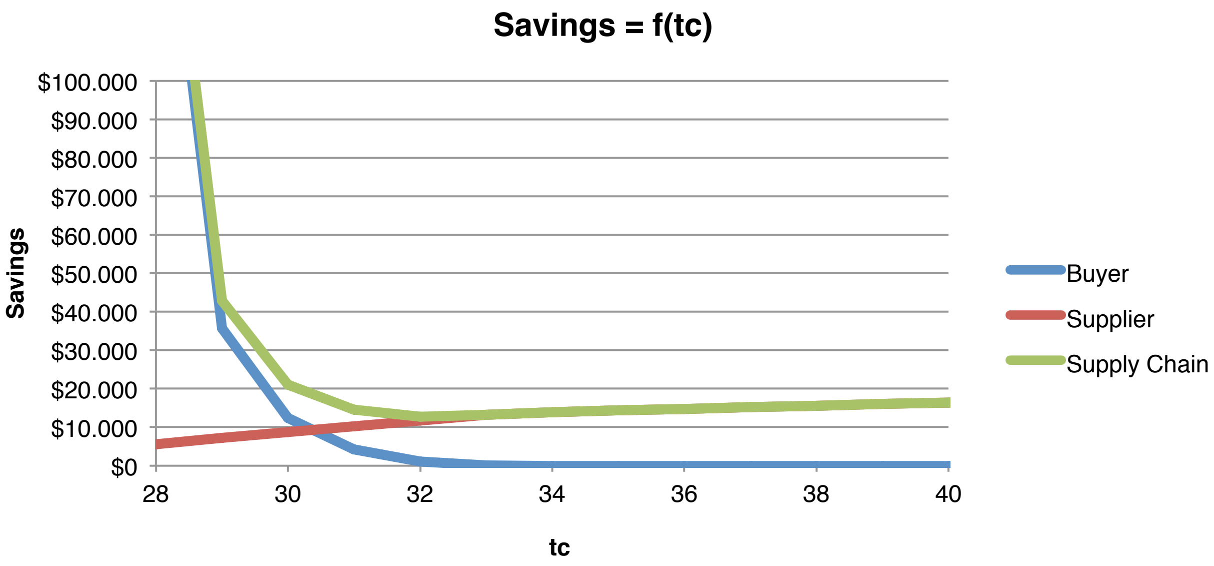


Figure 3.8: base case benefits as a function of the initial payment term.

Figure 3.8, indeed, shows what it has just been inferred: for low t_c values, the benefits are mainly seized by the buyer and they are remarkably high. As the base-case payment terms increase, the buyer's benefits with Reverse Factoring decrease. In fact, the higher t_c is, the higher are the buyer's base-case payables, allowing higher order quantities. A higher base-case order quantity Q is translated in a lower need to turn to RF, since Q is closer to the economic order quantity Q^*. We can also notice that the whole Supply Chain's benefits decrease rapidly, since in this model the main advantage for the Supply Chain is given by the buyer's need to order higher quantities. On the other hand, supplier's savings monotonically increase with t_c, as the buyer is more willing to share the benefits. Indeed, in a base-case scenario with broad payment terms, the buyer seeks a limited payment prorogation, letting suppliers gain more from a better interest rate. After a certain initial t_c value, the buyer's payables in the base-case allow her to order the optimal quantity Q^*, so she does not need to extend the term payment at all. This condition allows the supplier to gain all the benefits given by the possibility to sell his accounts receivable and to pay an interest rate lower than his cost of debt. The basic assumption is still that the buyer behaves in a collaborative manner, instead of a competitive one: there may not be immediate benefits in offering this contract to her supplier, but a more solid supply chain partner represents a certain advantage in the long term.

The negotiated payment term with RF t_{RF}

We proceed with the analysis considering the payment term t_{RF} arranged after the negotiation. Figure 3.9 illustrates buyer's benefits as a function of this parameter, which is not exogenous as the others. Indeed, it is assumed that as long as the buyer needs to extend payments in order to increase inventory and overall performances, she does it, and the supplier, with a much lower bargaining power, is only able to impose his "not-worsening" constraint.

Figure 3.9: Buyer's savings as a function of the new payment term $\mathbf{t_{RF}}$ *negotiated.*

But what if the supplier is actually in condition to bargain? What if the supplier, in order to accept the contract, requires to have some benefits as well? Or, simply, what if the supplier happens to face additional costs, lowering the t_{RF} negotiated from the buyer's optimal t_{RF^*}? The graph shows an interesting insight: a rational buyer, having set the t_{RF} obtainable from the negotiation (that can be either t_{RF^*} or t_{RFmax}), the closer comes to the optimum t_{RF^*}, the less marginal benefit gains. As in Figure 3.9, starting from a 30 days payables condition, having realized that 33,5 days is the maximum and optimum payment delay obtainable from the arrangement, we can see that 65% of the possible benefits of the buyer are reached by lengthening the payment terms by just one day. We can notice therefore that the buyer's benefits do increase with the payment delay, but with decreasing marginal returns. A single day reduction of the buyer's optimal payment term means a low potential loss for the buyer, but most likely a high potential benefit for the supplier and therefore for the whole supply chain.

This intuition is confirmed by Figure 3.10, which shows that the maximum benefit for the supply chain is reached when $t_{RF} = 32{,}4$ days.

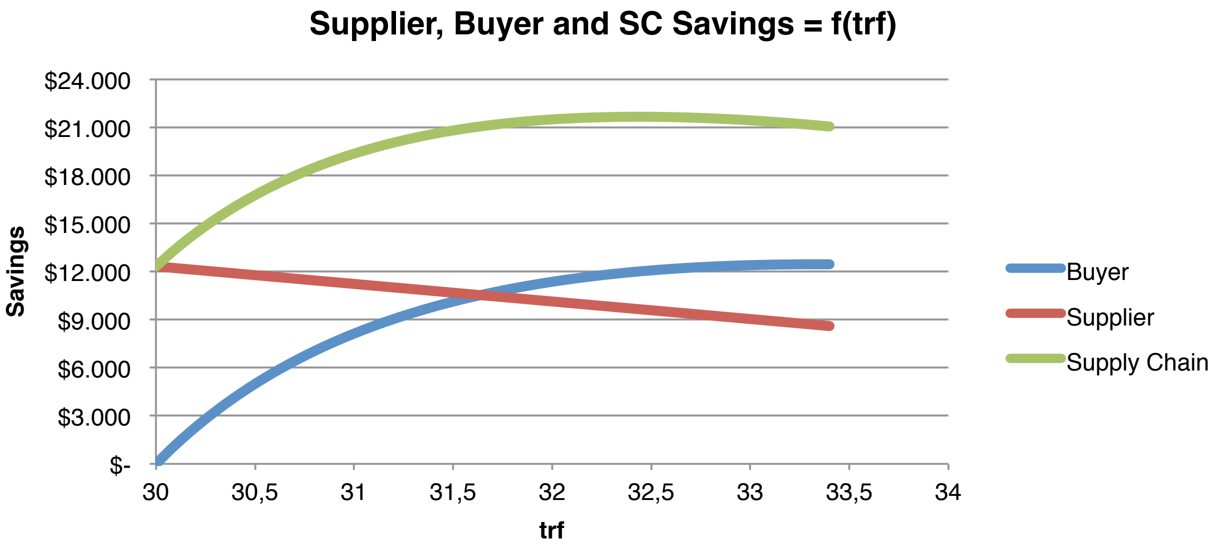


Figure 3.10: savings as a function of the negotiated payment term under a RF agreement.

The fixed cost per order A

Figure 3.11 shows that the global savings for the supply chain obtained by a RF contract are increasing in the fixed cost per order, $\boldsymbol{A}$. This result is not surprising, since the value of $\boldsymbol{A}$ directly affects the $\boldsymbol{EOQ}$ formula: as the cost per order increases, the buyer is inclined to reduce ordering frequency and it is possible to slash the number of transactions by increasing the order quantity ($\boldsymbol{T} = \boldsymbol{Q}/\boldsymbol{D}$). On the other hand, larger batches imply a greater level of inventories and, therefore, WCR.

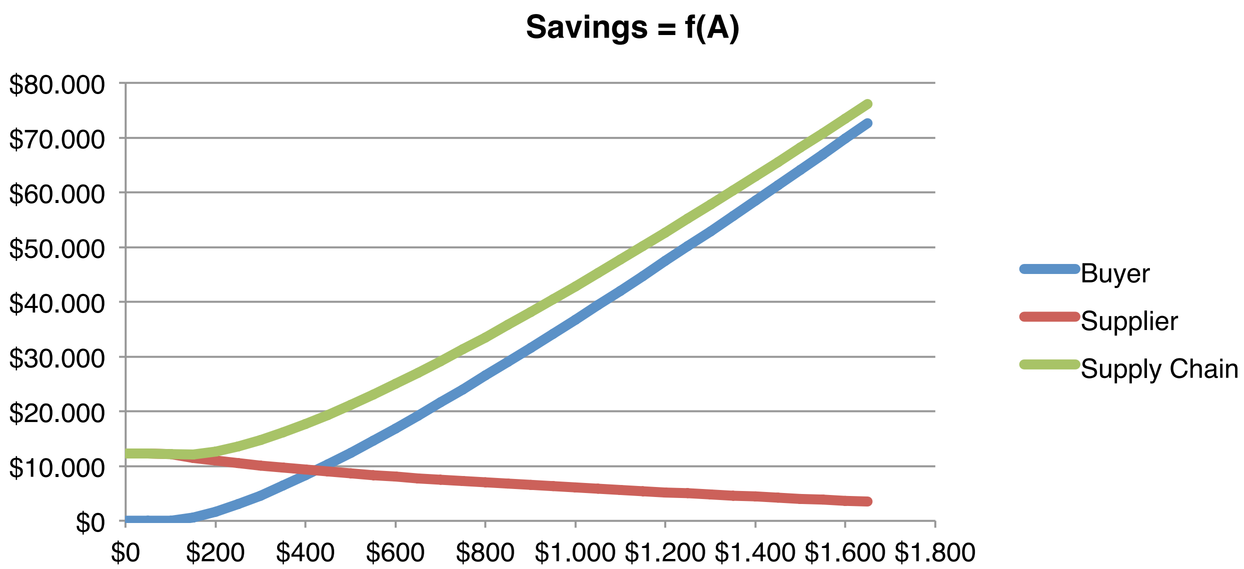


Figure 3.11: Savings as a function of the ordering cost.

The higher A is, the tighter is the constraint (i.e. the higher is the difference between EOQ and $Q_{\max\_norf}$). In this situation, the buyer has to bear higher costs and uses the RF aiming to lengthen the payment terms, in order to free up working capital and increase the purchasing lot size. Savings for the supplier reach their maximum for low values of A (up to $90), since in this situation the WCR constraint is not tight for the buyer, who is not interested into a delay of payment terms.

As we can observe in Table 3.2, the buyer is far more sensitive to the values A, compared to the supplier. The same can be stated about the total supply chain's savings, which are highly dependent on the buyer's ones.

		A	
	$250 (-50%)	$500	$1,000 (+100%)
Buyer	$3.023 (-75,7%)	$12.460	$36.759 (+195%)
Supplier	$10.531 (+21,3%)	$8.678	$6.058 (-30.2%)
Supply Chain	$13.554 (-35,9%)	$21.138	$42.817 (+102,6%)

Table 3.2: Buyer, Supplier and whole SC savings varying the ordering cost A.

The annual demand D

In the previous analyses we considered a deterministic annual demand, equal to 100.000 units per year. Figure 3.12 reveals that the impact of RF on the companies' performance is highly sensitive to this parameter.

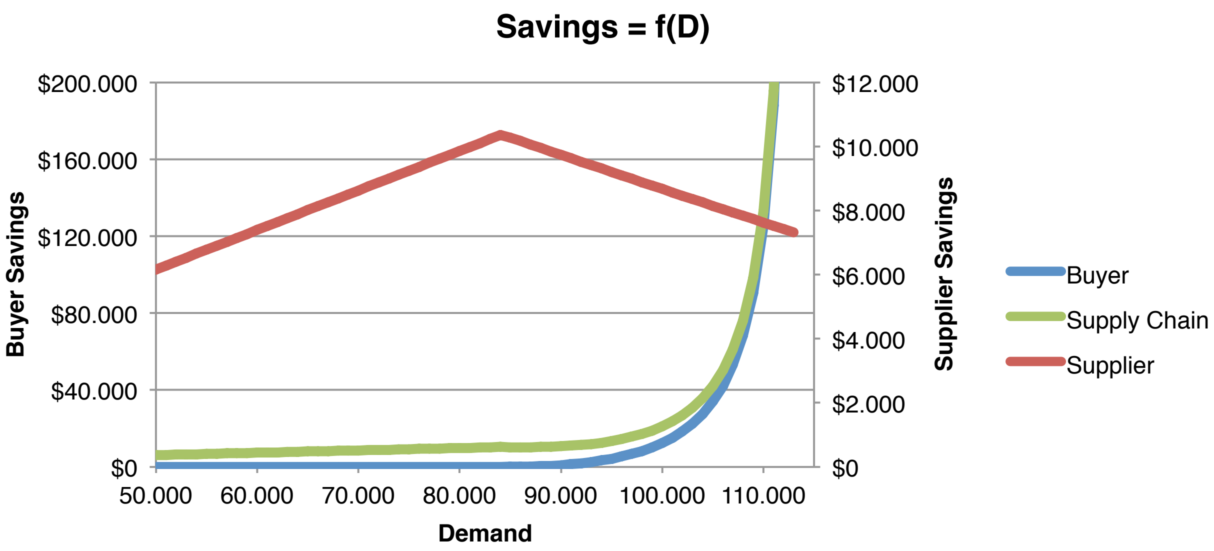


Figure 3.12: Savings as a function of the annual demand.

In order to better understand why savings vary in such a particular way when the demand increases, we need to examine the Working Capital Requirement of the buyer. Given the definition of $WCR = Cash + Inventories + Accounts\ Receivable - Accounts\ Payable$, we expect demand to have an impact on Inventories (+), Accounts Receivable (+) and Accounts Payable (−).

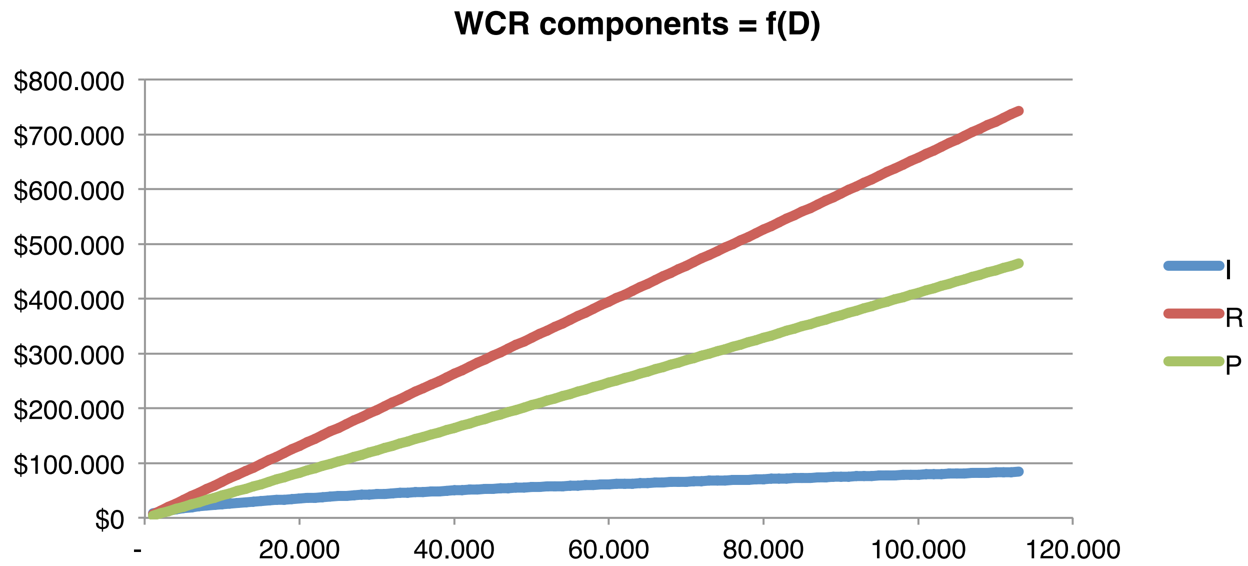


Chart 3.13: The components of WCR as a function of the annual demand.

As we can see in Figure 3.13, $Accounts\ Receivable = t_r \cdot D \cdot p$ increase at a greater rate than $Accounts\ Payable = t_c \cdot D \cdot c$, due to the higher value of the goods transferred between supply chain partners. This trend is responsible for the positive correlation between WCR and D. As D grows, the WCR constraint $\bar{K}$ is more and more tight for the buyer, increasing the positive impact of RF to her. In particular, for low values of D (up to 84.000 units/year) the constraint is not tightening and the supplier gathers all the benefits generated by RF. As D increases, the same happens to the difference between Q^* and $Q_{\max\_norf}$, meaning that the buyer needs to extend the payment terms (t_{RF}). The buyer's benefits grow exponentially, characterizing the supply chain's savings, up to the point in which demand is equal to 113.000 units/year. Beyond this level of demand, savings cannot be assessed, since it is impossible to satisfy all the demand while WCR is constrained and the buyer does not have a RF agreement with her supplier. Of course, in the long term, we expect the WCR constraint $\bar{K}$ imposed by the CFO to increase accordingly with demand. The same is not always true in the short term, when an unexpected order by a customer could cause some troubles to the buyer, who may not be able to satisfy all the demand or faces incredibly high costs in order to do it without a RF agreement.

	D [u/year]		
	90,000 (-10%)	100,000	110,000 (+10%)
Buyer	$986 (-92,1%)	$12.460	$125.423 (+906,6%)
Supplier	$9.742 (+12,3%)	$8.678	$7.630 (-12,1%)
Supply Chain	$10.728 (-49,2%)	$21.138	$133.053 (+529,4%)

Table 3.3: Sensitivity of the savings to the annual demand.

Table 3.3 shows the high sensitivity of the savings to demand, especially for the buyer. RF is extremely valuable also for small increments in demand.

3.5 Conclusions

The model presented evaluates quantitatively the benefits brought by RF from the buyer's point of view. This leads to identifying how much value can be created by a reduction of working capital, i.e., how the buyer can enhance the capital just freed-up. In fact, the other main advantage obtainable through RF, which consists in strengthening her upstream supply-base, is less suitable for being represented by a rigid mathematical model. Even though the model illustrated in this work shows just one side of the coin, it still brings some insights: it shows that there are some scenarios (i.e., companies with an aggressive working capital policy) in which buyers might be more interested in launching such a supply chain finance program, and other in which they might not. The same scenarios might be, on the other hand, the ones in which buyers would initiate RF in order to strengthen their suppliers without *direct* benefits. The model shows also different situations that might bring to different splits of benefits: when the buyer starts from conditions that are already favorable, benefits are majorly absorbed by suppliers, who enjoy a reduction in cost of capital without having to grant longer payment terms. Alternatively, the buyer might start from a very tight-capital situation, in which she aims to free as much capital as possible: in this case, taking advantage of a higher bargaining power, buyers will set RF in such a way that would let them seize all the benefits, leaving the supplier just not worse-off than before the arrangement.

4 Reverse Factoring and Growth

The goal of the second model is to study the impact of RF on the growth ability of a small or medium-sized enterprise (SME). Proposing this focus, we choose a scenario in which the supplier has high growth-potential: an SME sub-contractor that supplies a large firm with a highly required product. This context is met, e.g., in the Retail Industry (Morya and Dwivedi, 2009): Wal-Mart Costa Rica supported around a hundred products manufactured locally by SMEs. Each product, after passing a test based on customers feedback, was immediately launched and spread all over the country, so that it could satisfy a much higher demand than the SMEs would be able to meet without this agreement. Similarly, Carrefour Group seeks local sourcing for a substantial portion of its supplies and long-term partnerships with SMEs. As Carrefour website claims, in France, SMEs manufacture 80% of Carrefour products, accounting around 35% of hypermarkets' turnover.

For this purpose, we consider an adaptation of the deterministic model introduced by Buzacott and Zhang (2004), which studies the impact of asset based lending on the growth potential of a capital constrained firm that operates over multiple periods, in a market with ample demand (never fully satisfied) and upstream and downstream payment delays. Starting from Buzacott and Zhang's model, we adjust it accordingly to capture the dynamics and trade-offs involved by an RF arrangement implementation. Indeed, we believe that Buzacott and Zhang's model is suitable for representing RF for several reasons. First, since it is a multi-period model, it addresses more accurately the RF implementation environment. Second, it considers the changes that RF imposes on all three financial statements of the firm, i.e., balance sheet, income statement, and statement of cash flows. Third, the decisions of the SME are driven by a growth goal, which has been documented as one of the key motives for the implementation of RF initiatives by large buyers (Klapper 2006). In this chapter, we first illustrate the Buzacott and Zhang's framework (No Financing, Asset Based Loan and Unsecured Loan), then we present the Reverse Factoring adaptation. Through a numerical analysis, we compare the different financing solutions and we evaluate the benefits achievable through RF. Then, diverting from Buzacott & Zhang's assumption of unlimited capacity, which allows the firm to grow without the need of any investment, we consider an alternative scenario in which the firm has a capacity constraint: this capacity can be increased every year through investments. Again we study this scenario through a numerical analysis to see how the performances previously illustrated may change when capacity investments are necessary.

4.1 Base Case Framework

We consider a discrete-time, multi-period and finite horizon model of a three-stage supply chain. We focus on a small-medium sized firm (SME) that manufactures a single product. Specifically, the SME purchases a single raw material from a single upstream supplier, converts the raw material into a finished product using some labor, and ships the finished product to a single downstream customer (buyer). Without loss of generality, we assume a zero lead-time for both the delivery of raw materials from the supplier and the shipment of finished goods to the buyer. Once purchased, the raw materials can be either stored in inventory or issued to manufacturing. Similarly, once they are completed, the finished goods either are stored in inventory or shipped to the customer.

Being this a multi-period model, the firm's decisions and performances are monitored at the end of each period t, where $t = 1, \dots, T$ and T represents the end of the planning horizon.

There is a lead time of m periods for the conversion of one unit of raw material into finished product, while to enter manufacturing a unit of raw material must have been ordered and received before the manufacturing

process begins. That is, to enter manufacturing at the beginning of period $t+1$, a unit of raw material must have been issued to manufacturing at period t and, consequently, must have been ordered at or prior period t. Then, by the end of period $t+m$, this unit of raw material will be converted into finished product, which can be stored or instantaneously shipped to the buyer. Once the raw materials are procured, the SME delays the payment to the upstream supplier by k periods. Similarly, the SME receives payment from the buyer with a nominal delay of l periods from the moment the finished goods have been shipped. Also, we assume a manufacturing process without any capacity constraint; this assumption is then relaxed in section 4.4. To avoid the problem of demand uncertainty and since we are addressing the behavior of a rapidly growing firm, we assume that the supplier able to sell as much as he can produce.
We now define the notation for the operating variables, as it follows:

$w =$ finished good unit revenue;

$c_{RM} =$ raw material unit cost;

$c_L =$ total labor cost per unit in manufacture;

$\epsilon_j =$ fraction of the total labor cost per unit incurred at step j of manufacture, $j = 1, \ldots, m$ with $\sum_{j=1}^{m} \epsilon_j = 1$; i.e., m=3, $\epsilon_1 = 0{,}2$, $\epsilon_2 = 0{,}3$ and $\epsilon_3 = 0{,}5$ means that the company needs three periods to manufacture a batch of goods, and that the first, second and third periods cover respectively 20%, 30% and 50% of the total cost of the batch.

The financial state of the SME at the end of each period t is reflected on its three financial reports: balance sheet, income statement, and statement of cash flows. The balance sheet summarizes the SME's assets, liabilities, and shareholders' equity at the end of each period. In our model, the SME's assets consist of cash, accounts receivable, and inventory, while its liabilities consist of accounts payable, short-term debt, equity, and retained profits. Since our work is concerned with the management of the liquid assets of an SME; i.e., with the current portion of the balance sheet, for simplicity, we do not consider the firm's Fixed Assets account and Long-term Loans account. We recognize that an SME manufacturing firm is expected to own fixed assets to support operations. However, there are situations that can justify our simplification. For example, in a labor-intensive industry, the SME's fixed assets (such as building and mostly not specialized equipment) may have been exclusively financed by owner's equity and be fully depreciated. Alternatively, under a zero tax assumption, in which case there is no tax shield from depreciation or from loan interest payments, we can assume that fixed assets have been financed by a combination of equity and long-term loans and serve as security for these loans.

The model then is subdivided in three different cases:

A. The firm has no access to external financing and has available only cash reserves and the cash generated from operations;

B. The firm has access to unsecured loans at a fixed interest rate of r_s, limited by some line of credit ψ that is constant over time;

C. The firm has access to asset-based loans at the same interest rate r_s that are also limited by a line of credit ψ_t, which is determined as a linear function of the size of the SME's liquid accounts (i.e., cash, receivables, inventories, and payables) from the end of period $t-1$. That is, $\psi_t = \gamma_C \cdot x_{t-1} + \gamma_R \cdot R_{t-1} + \gamma_{FG} \cdot I_{t-1}^{FG} + \gamma_{WIP} \cdot I_{t-1}^{WIP} + \gamma_{RM} \cdot I_{t-1}^{RM} - \gamma_P \cdot P_{t-1}$, where x_t, R_t, I_t^{RM}, I_t^{WIP}, I_t^{FG}, and P_t are the accounts for cash, receivables, inventories of raw material, work-in-progress, finished goods, and payables, respectively. Also, γ_C, γ_R, γ_{FG}, γ_{WIP}, γ_{RM}, and γ_P, with $0 \leq \gamma_j \leq 1$ reflect the lender's assessment of the risks associated with each account category.

We assume that the SME has no alternative investment opportunities and can only obtain the risk-free return, r_f, on any cash reserves, with $r_f < r_s$.

In each period t, the SME makes four decisions:

$q_t^{RM} \equiv$ quantity of raw material procured in period t;

$q_t^{WIP} \equiv$ quantity released to manufacturing in period t;

$s_t \equiv$ quantity shipped to the buyer in period t;

$B_t \equiv$ net borrowing in period t, if external financing is utilized (case B and C). If $B_{t+1} > B_t$ the firm has increased the short-term debt in $t+1$ by $(B_{t+1} - B_t)$. If $B_{t+1} < B_t$ the firm has decreased the short-term debt in $t+1$ by $(B_t - B_{t+1})$.

The details of the SME's balance sheet accounts are shown below:

$Accounts\ Receivable$: The size of accounts receivable at the end of period t, with a delay of l periods in the buyer's payment, is given by $R_t = R_{t-1} + w \cdot (s_t - s_{t-l})$.

$Inventory\ Account$: The inventory of raw materials, work-in-progress, and finished goods at the end of period t are denoted by I_t^{RM}, I_t^{WIP} and I_t^{FG}, respectively:

$$I_t^{RM} = I_{t-1}^{RM} + c_{RM} \cdot (q_t^{RM} - q_t^{WIP})$$

$$I_t^{WIP} = I_{t-1}^{WIP} + c_{RM} \cdot q_t^{WIP} + c_L \cdot \sum_{j=1}^{m} \left(\epsilon_j \cdot q_{t-j}^{WIP}\right) - (c_{RM} + c_L) \cdot q_{t-m}^{WIP}$$

$$I_t^{FG} = I_{t-1}^{FG} + (c_{RM} + c_L) \cdot (q_{t-m}^{WIP} - s_t)$$

$Accounts\ Payable$: The size of accounts payable at the end of period t, with a delay of k periods in paying the supplier, is given by $P_t = P_{t-1} + c_{RM} \cdot (q_t^{RM} - q_{t-k}^{RM})$.

$Cash\ Account$: The cash position at the end of each period t is derived by the statement of cash flows, which shows the sources and uses of funds in each accounting period,

$$x_t = x_{t-1} + s_t \cdot (w - c_{RM} - c_L) + \left(r_f x_{t-1} - r_s B_{t-1} + B_t\right) + (R_{t-1} - R_t) + (P_t - P_{t-1}) - (I_t^{RM} + I_t^{WIP} + I_t^{FG} - I_{t-1}^{RM} - I_{t-1}^{WIP} - I_{t-1}^{FG})$$

where $r_f \cdot x_{t-1}$ is the interest received at the beginning of period t from any cash carried from period $t-1$ to period t and $r_s \cdot B_{t-1}$ is the interest payment for the debt carried in period $t-1$.

$Retained\ Profits$: The retained profit up to period t is equal to $\pi_t = \pi_{t-1} + s_t(w - c_{RM} - c_L) + (r_f \cdot x_{t-1} - r_s \cdot B_{t-1})$, where the second term is the income from operations and the third term is the financial income / loss.

Then, in the balance sheet the size of assets must equal the size of liabilities at the end of each period t; i.e.,

$$x_t + R_t + (I_t^{RM} + I_t^{WIP} + I_t^{FG}) = P_t + B_t + x_0 + \pi_t$$

where x_0 is the initial capital (i.e., equity) for operations. The balance sheet is represented in Table 4.1.

ASSETS	LIABILITIES
Cash - x_t	Equity - x_0
Accounts Receivable - R_t	Retained Earnings - π_t
Inventory - $I_t^{RM} + I_t^{WIP} + I_t^{FG}$	Short-term Debt - B_t
	Accounts Payable - P_t

Table 4.1: SME's balance sheet.

The measure used by the SME to track performance is the retained earnings π_t. We assume that the objective of the SME is to maximize the ending period retained earnings, π_T. By ignoring the impact of demand uncertainty on the SME's decision making process, the SME's problem reduces to deciding on raw material purchases, production, and loan size (when applicable) to maximize retained earnings at the end of the planning horizon subject to the constraints of avoiding bankruptcy and not exceeding the credit limit, i.e., $x_t \geq 0$ and $B_t \leq \psi_t$ for all t.

$$\max_{q_t^{RM}, q_t^{WIP}, s_t, B_t} \quad \pi_T$$

subject to:

$$x_t \geq 0 \quad \forall t$$

$$B_t \leq \psi_t \quad \forall t$$

$$q_t^{RM}, q_t^{WIP}, s_t, B_t \geq 0$$

4.2 Reverse Factoring Case

In order to provide more insights about RF value added and for consistency, we assume that the firm uses RF as an alternative to the other financing tools. Consequently, the decision variable B_t does not appear in this scenario. Since the demand is strongly growing and never satisfied, we assume that the firm will discount all the invoices on hand. Consequently, the early payment of the firm's invoices is issued by the buyer's bank, discounted by a rate r_s^{RF}, where $r_s^{RF} < r_s$, over a period l^{RF}, where $l^{RF} \geq l$. I.e. That is, we assume that the buyer requires a payment term extension in return of implementing the RF program. Under this scenario, the SME's accounts receivable are factored by the buyer's bank and become zero. Consequently, whenever a shipment takes place, an amount of $w \cdot s_t \cdot (1 - r_s^{RF})^{l^{RF}}$ is directly transferred to the SME's cash account.

Again, under RF, each period's revenue, ws_t, does not show up in the accounts receivable account, but instead, it is factored by the buyer's bank and directly increases the period's cash account by $ws_t(1-r_s^{RF})^{l^{RF}}$.

Theorems 1 and 2 in Buzacott and Zhang (2004) describe for the base-case framework's settings the features of a firm's optimal operating policy for the cases of asset based lending and no borrowing, respectively. We adapted Buzacott and Zhang's theorems to the Reverse Factoring case, considering the new assumptions, as follows:

"*When operations can only be financed by a reverse factoring arrangement and the objective is to maximize π_t, at an optimal solution:*

(1) $I_t^{RM}=0$, for all t; i.e., raw materials are purchased only when needed.

(2) $I_t^{FG}=0$, for all t; i.e., finished goods are not kept in inventory but are shipped to the customer in all periods.

(3) $q_t^{WIP}=0, t>T-m$; i.e., the production manager will not launch into production any work that will not result in profits by the end of period T.

(4) An optimal schedule can be characterized by $x_t=0$ for $t=m,m+1,\dots,T-1$, provided $w\cdot s_t\cdot(1-r_s^{RF})^{l^{RF}}/(1+r_f)^m>c_L\cdot\sum_{j=1}^{m}\epsilon_j/(1+r_f)^j+c_{RM}/(1+r_f)^k$; that is, one always uses up all cash each period as long as the net present value of purchasing and selling an item is positive."

Note that each period the quantity of raw materials purchased is directly released to manufacturing, i.e. $q_t^{RM}=q_t^{WIP}$, while the last production launch takes place at time $T-m$. Also, in each period $t=m+1,m+2,\dots,T$, all units that finish manufacturing are directly shipped to the buyer; i.e., $s_t=q_{t-m}^{WIP}=q_{t-m}^{RM}$.

4.3 Numerical Analysis

To derive our results, we assume that prices, costs, and all other problem parameters are exogenously defined and constant over time. In table 1, we present the retained profit π_t for the cases of no-borrowing, unsecured loan, asset based loan, and RF. The parameters for the base case example are $c_{RM}=4$, $c_L=8$, $w=20$, $m=2$, $k=1$, $l=1$, $l^{RF}=2$, $r_f=0.005$, $r_s=0.025$, $r_s^{RF}=0.015$, $x_0=\$200$, $\epsilon_1=0.5$, $\epsilon_2=0.5$, $\gamma_{RM}=0.1$, $\gamma_{WIP}=0.1$, $\gamma_{FG}=0.7$, $\gamma_R=0.9$, $\gamma_C=1$, and $\gamma_P=1$. These conditions resemble the operating environment of several SME manufacturers. That is, we consider a profit margin of 40%, while $T=12$ can be seen as a three-year period, i.e., twelve quarters. Initial cash is set as $x_0=200$. Then, we have four downstream orders per year, each of which has three months of payment delay (extended to six when RF is applied). Finally, the annual interest rate translates to around 2% for the risk-free rate, 10% for debt financing, and 6% for the RF discount.

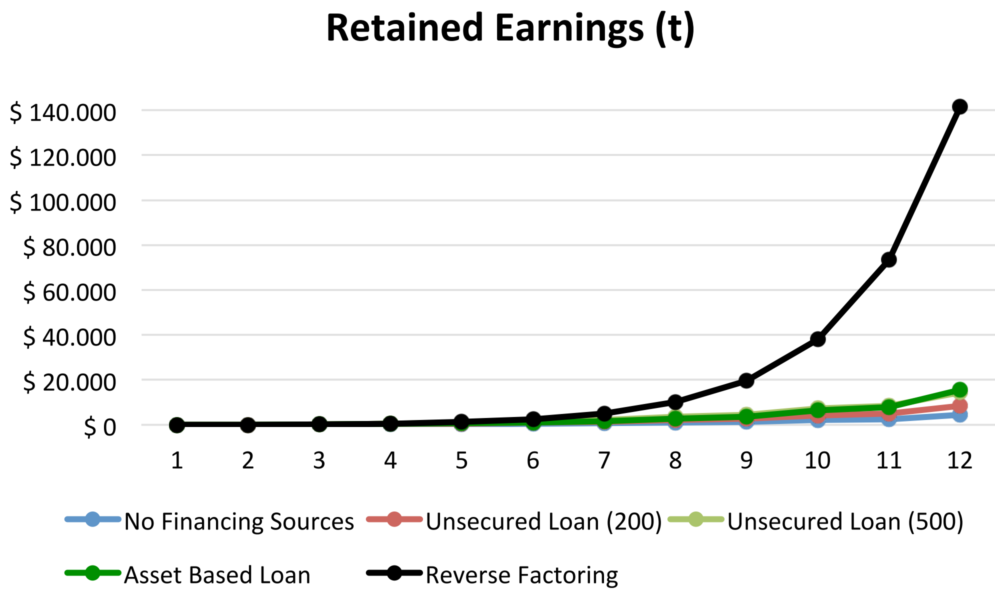


Figure 4.1: Retained earnings achieved thanks to different financing means.

As we may notice, RF outperforms the other financing solutions in the long term. The wide difference that we see in the last periods can be explained by the *never-satisfied demand assumption*: the firm that uses RF has the possibility to collect a significant amount of cash before the others and re-invest it immediately in raw materials and labor, anticipating the growth. The unlimited capacity assumption also helps the gap to spread: since we suppose a scenario that does not need any investment in capacity even when production increases by a hundred times, the sooner a firm is able to buy stocks, the faster it grows. This gives an interesting insight about RF potential: RF does not only mean more convenient interest rates, but also better timing of higher quantity of cash. We now analyze how the retained profit of each solution varies depending on the initial cash conditions:

x_0	No Borrowing	Unsecured $\psi_t = 500$	Unsecured $\psi_t = x_0$	Asset Based Loan	Reverse Factoring
50	1.095	11.153	2.107	3.891	35.340
100	2.189	12.263	4.213	7.782	70.680
200	4.379	14.483	8.427	15.564	141.359
400	8.758	18.877	16.853	31.128	282.719
600	13.137	23.256	25.280	46.692	424.078

Table 4.2: Optimal profit [$] for different types of financing – Sensitivity analysis to initial capital x_0.

Table 4.2 shows that the retained profit in all cases (except unsecured $\psi_t = \$500$) is linear in x_0. Also, the retained profit under RF is greater by about 32 times relative to no borrowing, 17 times relative to unsecured $\psi_t = x_0$, 9 times relative to asset based loans, and from 3 times (poor firm) to 18 times (rich firm) relative to unsecured loan with credit limit $\psi_t = \$500$. This was an expected outcome since we have assumed that there is no capacity constraint and all output can be sold. So, the firm uses the advantage of early payments to speed up production faster and, therefore, to reach a level of q_{t-m}^{RM} that is considerably higher than in the rest of the cases.

In Table 4.3 we compare how our base case result for the RF case is affected when some of the key parameters change. We can see that a 100% increase in the payment extension term required by the buyer, l^{RF} , which is actually the parameter of the RF arrangement that is more directly under buyer's control, results in a profit decrease of 31,81% for the SME. A similar effect is observed when the discount rate of the RF arrangement, r_s^{RF}, is increased by 100%. Note that in the latter case, the RF discount term is greater than the bank's interest for the SME. This means that it still may be quite profitable for the SME to participate in an RF arrangement even if its terms are not so favorable. In other words, the positive impact from financial flexibility, in terms of *timely* access to financing, exceeds the negative impact from the cost of this financing. However, the availability of credit by the upstream supplier, represented in our model through k, appears to have a considerable impact on SME's profitability. In our example, when the SME has no access to credit from its supplier, its profit decreases by 88,99% compared to the base case. This is quite relevant since a typical SME usually lacks negotiating power with their upstream suppliers. The implication for the buyer initiating an RF program to stipulate growth is that focusing only to tier 1 suppliers may not be sufficient for achieving this goal. Finally, the length of the operating horizon also has a considerable impact. In our example, a decrease of 33% in the planning horizon results in a profit decrease by 92,86%. This means that the benefits of financial flexibility granted by RF implementation on the SME's operational efficiency require a long-term perspective to be realized.

k	l^{RF}	r_s^{RF}	T	$\pi(t)$	$\Delta\,\pi(t)$ [%]
1	2	0,015	12	$ 141.359	-
0 (-100%)	2	0,015	12	$ 15.565	-88,99%
1	4 (+100%)	0,015	12	$ 96.386	-31,81%
1	2	0,03 (+100%)	12	$ 95.819	-32,22%
1	2	0,015	8 (-33,33%)	$ 10.097	-92,86%

Table 4.3: Sensitivity of the parameters in the RF arrangement on the SME's profitability.

4.4 Finite Capacity

Finally, to render the problem more realistic, we examine the case of finite capacity for the SME manufacturer. Specifically, we assume that at time $t = 0$ the SME manufacturer has some initial capacity, denoted by K_0. Then, at the beginning of each period, the supplier can invest in additional capacity, INV_t, which becomes available with a one-period delay:

$$K_{t+1} = K_t + INV_t.$$

The capacity constrain is represented by:

$$q_t^{RM} \leq K_t$$

The additional capacity has a unit cost denoted by c_K. The payment to the capacity provider is made in cash and in two consecutive installments of 50% of the total cost, beginning from the period that the order is placed. Accordingly, the cash dynamics are now given by:

$$x_t = x_{t-1} + s_t(w - c_{RM} - c_L) + \left(r_f x_{t-1} - r_s B_{t-1} + B_t\right) + (R_{t-1} - R_t) + (P_t - P_{t-1}) \\ - \left(I_t^{RM} + I_t^{WIP} + I_t^{FG} - I_{t-1}^{RM} - I_{t-1}^{WIP} - I_{t-1}^{FG}\right) - c_K \cdot (0{,}5 \cdot K_t + 0{,}5 \cdot K_{t-1})$$

The maximization problem becomes therefore:

$$\max_{q_t^{RM}, q_t^{WIP}, s_t, B_t, INV_t} \quad \pi_T$$

subject to:

$$x_t \geq 0 \quad \forall t$$

$$q_t^{RM} \leq K_t \ \ \forall t$$

$$B_t \leq \psi_t \ \ \forall t$$

$$q_t^{RM}, q_t^{WIP}, s_t, B_t, INV_t \geq 0$$

We can observe the additional decision variable INV_t (how much to invest in capacity) and the additional constraint $q_t^{RM} \leq K_t$ (the capacity constraint) introduced in the maximization problem.

In our example, we use $K_0 = 100$ and $c_K = \$50$, while by K_T we denote the total capacity investment up to period T. For consistency with what we assumed previously (paragraph 4.1), we still do not represent the firm's fixed assets, meaning that the credit limit imposed by the bank does not count the new investments in capacity. Setting the parameters as in the unlimited capacity analysis we now compare the different financing solutions.

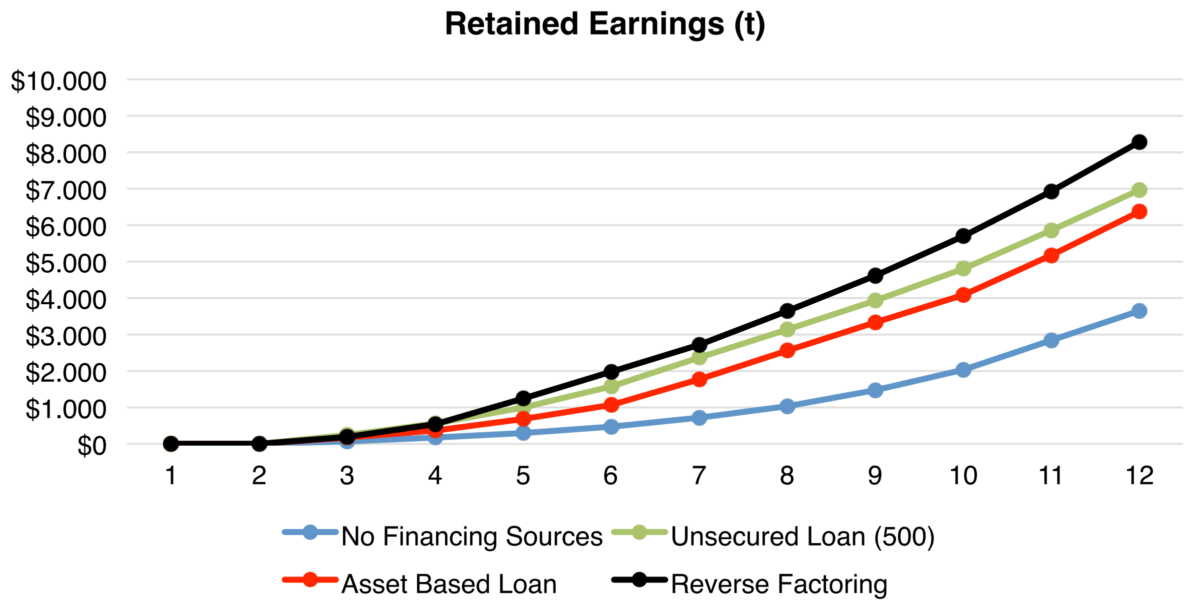


Figure 4.2: Retained earnings achieved thanks to different financing means in a finite-capacity setting.

Figure 4.2 shows how RF still overcomes the other financing tools, even though it does not outstand as in the unlimited capacity case. We can also analyze how much production capacity has increased in each scenario, varying the initial cash conditions, as follows in Table 3:

	No Borrowing		Unsecured $\psi_t = 500$		Asset Based		Reverse Factoring	
x_0	π(t)	K_T	π(t)	K_T	π(t)	K_T	π(t)	K_T
50	$1.095	0	$6.149	32	$3.062	0	$5.704	44
100	$2.117	0	$6.454	32	$4.465	3	$6.947	64
200	$3.644	0	$6.962	39	$6.376	54	$8.272	80
400	$5.441	19	$7.819	54	$8.311	85	$9.692	105
600	$6.723	32	$8.525	58	$9.582	102	$10.414	107

Table 4.4: Optimal profit [$] and capacity investment for the capacity and capital constrained SME.

Table 4.4 provides some interesting insights. First, we can see that the capacity investment under RF is greater compared to all other cases. This is true even for the case of a poor SME (i.e., for $x_0 = \$50$), in which case the generous unsecured credit line of $500 results to higher retained profit compared to the RF, but still to lower capacity investment. Second, both the retained profit and the capacity investment improvement from the implementation of RF, relative to the cases of no borrowing and asset based lending, decreases considerably in regards with the financial health of the SME; i.e., with x_0. The opposite trend is observed compared to the case of unsecured loan. This suggests that even in such an extreme context as a non-stop growing demand, RF adds more value to firms that are short in cash. Interesting is that this finding is indicated also by the third model presented in this work (see paragraph 5.6.5).

	No Borrowing					Unsecured $\psi_t = 500$					Asset Based					Reverse Factoring				
	x_0					x_0					x_0					x_0				
t	50	100	200	400	600	50	100	200	400	600	50	100	200	400	600	50	100	200	400	600
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	28	15
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39	0	0	26	2	10
6	0	0	0	0	0	0	0	0	4	18	0	0	0	35	2	0	24	4	19	19
7	0	0	0	0	0	0	0	10	16	26	0	0	0	0	0	21	6	17	18	16
8	0	0	0	0	32	14	25	21	0	0	0	0	41	14	32	8	16	19	14	19
9	0	0	0	19	0	17	7	7	13	15	0	3	13	36	29	15	19	14	24	23
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	19	32	32	32	39	33	58	0	3	49	85	102	44	64	80	105	117

Table 4.5: Timing of the investments.

Table 4.5 provides a detailed view on the timing of the investments obtained in the numerical analysis, which confirms our insights on the importance of timely access to financing. Under a RF agreement, the SME sells its accounts receivable to the bank as soon as they are generated, freeing up cash. The supplier benefits from this arrangement, and the available cash allows anticipating and increasing investments in production capacity, compared to the alternative financing means. RF therefore does not only promote retained profit growth, but also the total firm's assets.

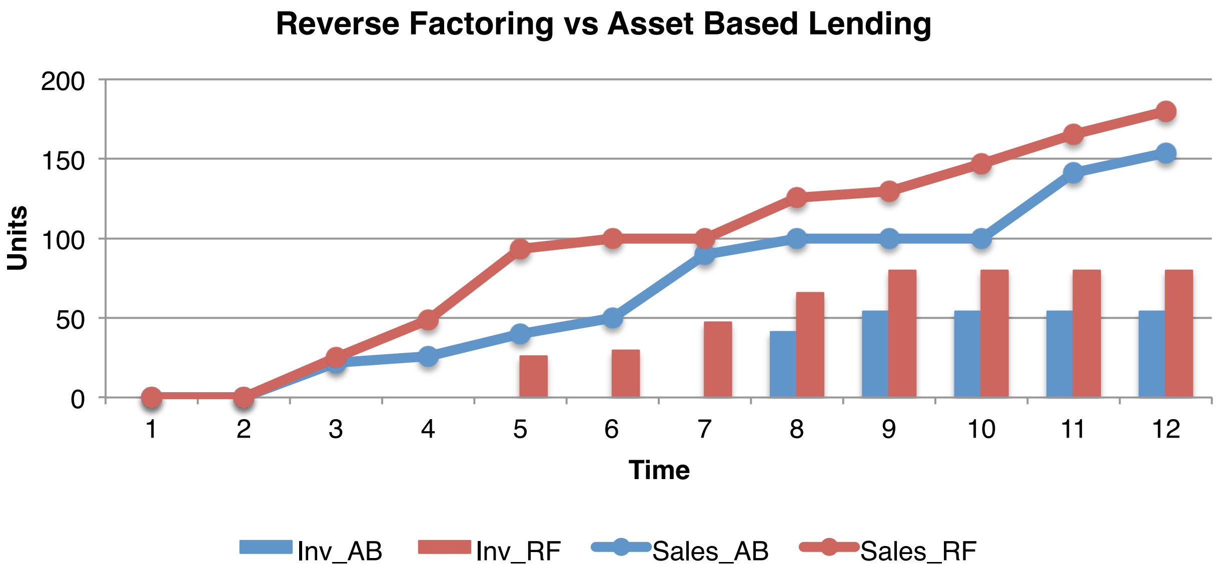


Figure 4.3: Capacity Investments and Sales over time, for $x_0 = 200$.

Figure 4.3 represents the cumulative capacity investments and sales, comparing the performances achieved by the SME when it has access to asset based financing and reverse factoring. Once again, we can note that the supplier takes advantage of the early access to capital and invests it in raw material and fixed assets. The anticipated capacity investments contribute to increase sales volumes earlier, compared to the other financing alternatives (in this case asset based financing).

4.5 Conclusions

The model presented here tries to represent accurately how RF affects the cash flows of a generic supplier and how these dynamics, differently from an asset based lending or an unsecured loan, create value besides the cheaper interest rate. RF, indeed, does not only imply better financing conditions, but it also represents the possibility for a supplier to discount as much as he desires, at any time, depending on the current needs and without bearing on the balance sheet any additional debt, but just collecting cash faster. The chosen framework considers purchasing and labor lead-time, but has the limit of an unlimited increasing and deterministic demand. This allowed the mathematics to be simpler and presented a case in which RF shows all of its potential.

5 RF and Replenishment Decisions facing Stochastic Demand

In this chapter we introduce a third model that addresses the dynamics entailed by the application of Reverse Factoring when a firm is facing unknown and stochastic demand. The first part recalls the working paper *"Reverse Factoring: Implications on SMEs' operational decisions and performance"*, by Lekkakos and Serrano (2014). The authors take inspiration from the multi-period model provided by Luo and Shang (2013), who study an entrepreneurial firm with no access to bank financing that periodically orders inventory to satisfy non-decreasing demand with upstream and downstream payment delays. The authors used the Newsvendor Model framework (Arrow et al., 1951): an expected costs minimization under uncertain and stochastic demand. Lekkakos and Serrano follow a similar approach, but they extend their results by considering a firm's inventory replenishment problem when the firm is able to factor its A/R. To the best of our knowledge, their work is the first attempt to model the inventory management problem of a firm that has access to receivables financing in a stochastic multi-period environment. The key aspect of their paper is an innovative perspective that takes into account a new decision variable: not only how many invoices to discount, but also which percentage of each one. In the Reverse Factoring literature, the authors usually represent this SCF tool as a simple loan, avoiding both the issues about which invoices to sell first (e.g. the most mature or the least mature) and how much of the chosen invoices to discount. Lekkakos and Serrano, instead, introduce the vector of the Accounts Receivable accumulated and the decision variable vector of how much of each A/R has to be discounted. Their model is more representative of the actions undertaken by a supplier after the adoption of an RF program.

Lekkakos and Serrano (2014) provide both a model for their base-case, in which the supplier does not have available any financing solution, and a model for Reverse Factoring. Comparing the two cases, they show how Reverse Factoring can improve the supplier's performances in terms of retained profit and demand coverage.

Starting from Lekkakos and Serrano's models, we propose several improvements in order to increase the accuracy of the profit function, of the single-period optimal ordering policy and of the cash dynamics. We also provide three additional extensions for three different scenarios: (i) when the supplier avails himself of simple Factoring, (ii) of Auto-Discounting Reverse Factoring and (iii) when he has access to an unsecured credit line. We run a simulation that compares all the cases illustrated (base-case, RF, Auto-Discounting RF, TF and Credit Line) and provides some interesting conclusions about the value brought by each SCF solution. We then proceed testing the models under different assumptions regarding the demand pattern (growth and seasonality): these assumptions require deriving additional ordering policies, as will be illustrated. A final simulation both tests the goodness of the theoretic work and provides several insights comparing traditional solutions with Reverse Factoring. We also take into account buyer's perspective, showing under which conditions requiring payment extension in return of RF might be less profitable. An additional analysis examines how each SCF tool performs differently varying the initial cash conditions and gives some conclusive insights on the specific scenarios in which RF has the highest impact and generates most benefits.

5.1 The base case

For coherence, we use the same notation Lekkakos and Serrano propose. Consider a periodic-review inventory control problem where a self-financing supplier (he) sells a single product to a single downstream buyer (she) on credit. The firm is risk neutral and makes inventory replenishment decisions to meet stochastic demand over a T-period planning horizon. Let D_t be the demand in period $t, t = 1, \dots, T$. It is assumed that $D_1, \ldots, D_T$ are independent and identically distributed (i.i.d.) nonnegative random variables. Let $f(\cdot)$ and $F(\cdot)$ be respectively the probability density and cumulative distribution functions of a random variable D with the common distribution. The inventory on-hand at the beginning of each period is represented by x_t. Any unsold inventory in periods $t = 1, \ldots, T-1$ is carried over to be used in subsequent periods while unmet demand in each period is lost. At the end of the planning horizon T, the unit salvage value of any leftover inventory is equal to the unit production cost, meaning that in $T+1$ the firm will earn $c \cdot x_{T+1}$. The supplier has no access to external financing and his production decision is constrained by available cash at the beginning of the period (this assumption is relaxed when the supplier has available RF or other financing means).

Let p, c, and h be, respectively, the unit revenue, the production cost, and the inventory holding cost. Let w' be the unit revenue that is retained for financing operations according to the firm's working capital policy, $0 \leq w' \leq p$. As in Luo and Shang (2013), w' captures the firm's short-term working capital strategy, but also, implicitly, the firm's capital investment and shareholder payout policy. That is, w'adjusts the firm's revenue allocation policy between short-term and long-term investments. Since the focus is on the impact of downstream payment delays on the firm's operational decisions, it is assumed that any purchasing costs are incurred at the beginning of each period. The firm's sales take place under a net term agreement, meaning trade credit is characterized by a single term that specifies the number of days after goods delivery –and invoice approval– within which the outstanding invoice is expected to be paid in full. Another assumption is that the credit term is exogenously determined within the industry in which the SME operates (Klapper et al., 2010). Let m denote the integer number of periods in the trade credit agreement. That is, payment for sales made during period t is due at the beginning of the period $t+m+1$. Without loss of generality, the firm's inventory replenishment and shipping lead times are zero.

The state of the system is summarized by a vector $(x_t, z'_t, \boldsymbol{R'_t})$, where x_t denotes the amount of on-hand inventory, z'_t the available cash, and $\boldsymbol{R'_t} = (R'_{t-m}, R'_{t-m+1}, \ldots, R'_{t-1})$ the m-dimensional vector of the firm's A/Rs still pending. In our notation, R'_{t-m} is the most mature A/R and corresponds to sales for the period $\boldsymbol{t-m}$, the payment for which will materialize in period $t+1$. The sequence of events in each period occurs as follows. At the beginning of each period, after observing the available on-hand inventory, x_t, and cash, z'_t, the firm makes his inventory replenishment decision, q_t. Since the firm is self-financed and all purchasing costs are incurred at the beginning of the period, the replenishment decision satisfies the cash constraint $c \cdot q_t \leq z'_t$. During the period, demand D_t is realized and serviced by on-hand inventory, $x_t + q_t$. Under the lost-sales assumption, the satisfied demand in each period is equal to $\min(q_t + x_t, D_t)$. Finally, any unsold inventory, $(q_t + x_t - D_t)^+$, is carried over to subsequent periods incurring the corresponding holding cost. At the end of the period all costs are calculated.

The dynamics of the Markov decision process under study, for $t = 1, \ldots, T$, are as follows:

$$x_{t+1} = (x_t + q_t - D_t)^+$$
$$z'_{t+1} = z'_t - c \cdot q_t + R'_{t-m}$$
$$\boldsymbol{R'_{t+1}} = (R'_{t-m+1}, \ldots, R'_{t-1}, w' \cdot \min(x_t + q_t \,;\, D_t))$$

Notice that the vector $\boldsymbol{R'_{t+1}}$ captures the portion of accounts receivable that is retained for operations (as this is determined by w') and under the responsibility of the operations manager. This convention is maintained throughout this chapter.

Let β be the single period's discount rate. Then, the expected profit function in each period, $t = 1, \dots, T$, is given by

$$\pi_t(x_t, q_t) = E[\beta^m \cdot p \cdot \min(x_t + q_t ; D_t) - \hat{G}(x_t, q_t)]$$

$$\hat{G}_t(x_t, q_t) = E[H_t(x_t, q_t) + c \cdot q_t]$$

$$H_t(x_t, q_t) = E[h \cdot (x_t + q_t - D_t)^+]$$

Notice that while the inventory holding cost is incorporated in the expected profit function, it is not considered in the cash flow dynamics in (2). Just as Luo and Shang (2013), it is assumed that the firm uses accrual accounting for cost recognition; i.e., costs are recognized when they occur rather than when payment is made. For example, h may represent insurance or obsolescence reserve costs which are settled at the end of the planning horizon. Alternatively, this cost could have been incorporated in the cash flow dynamics by assuming that the revenue from any demand realization is sufficient to cover the inventory holding cost. Since this assumption would not change the insights of the model, h is not included in the cash flows dynamics.

The purpose now is to identify an ordering policy that maximizes the expected profit over the multi-period horizon, given any initial state $(x_1, z'_1, \boldsymbol{R'_1})$.

5.1.1 State Space Reduction

Before deriving the order policy, it is convenient to reduce the state space by introducing new system variables, defining:

$$y_t = x_t + q_t ,$$

$$z_t = x_t + \frac{1}{c} \cdot z'_t,$$

$$\boldsymbol{R_t} = \frac{1}{c} \cdot \boldsymbol{R'_t}$$

$$w = \frac{1}{c} \cdot w'.$$

Notice that now there is a new decision variable that replaces q_t: instead of considering only the ordered quantity q_t, we refer to y_t, i.e. the inventory position (e.g. the inventory policy or up-to-order policy) at the beginning of period t; z_t represents the inventory equivalent of the cash plus available inventory at the beginning of period t. Finally, $\boldsymbol{R_t}$ stands for the inventory equivalent (measured in units) of the A/R still pending. The multi-period profit function becomes $\pi(y_t, x_t) = E[\beta^m \cdot p \cdot \min(y_t, D_t) - h \cdot (y_t - D_t)^+ - c \cdot (y_t - x_t)]$ for $x_t \leq y_t \leq z_t$, and the dynamics of states are given by

$$x_{t+1} = (y_t - D_t)^+$$

$$z_{t+1} = x_{t+1} + R_{t-m} + z_t - y_t$$

$$\boldsymbol{R_{t+1}} = (R_{t-m+1}, \dots, R_{t-1}, w \cdot \min(y_t, D_t))$$

5.1.2 Ordering Policy

We assume that the firm adopts a myopic ordering policy, i.e. a policy that leads to decisions that maximize only the imminent profit, instead of looking at the terminal value of the firm. Different authors (Ignall & Veinott (1969), Schwarz & Schrage (1975), Karush & Dear (1967)) already proved that, in certain multi-period (or multi-stage) systems representing a Markov Decision Process, myopic policies are either optimal or near optimal solutions. This means that if at the beginning of each period (e.g. at each stage) the model maximizes the current period's profit function, also the terminal retained profit is either optimized or very close to the maximum. A myopic policy assumption allows to derive the optimal order-up-to-level with an analytic methodology. Furthermore, besides analytics simplification, a myopic policy is more representative of the usual SMEs' behavior: when SMEs do use an ordering policy, it is usually based on the next period's forecast instead of the long term value maximization.

For consistency, we keep the myopic ordering policy assumption for every case we address.

5.1.3 Single Period Profit Maximization

In order to consider the unit-salvage value of leftover inventory, the single period profit function has an additional term represented by $c \cdot \beta \cdot (y_t - D_t)^+$. Consequently, the single-period profit function $J_t(y_t, x_t)$ is given by:

$$J_t(y_t, x_t) = E[\beta^m \cdot p \cdot \min(y_t, D_t) - h \cdot (y_t - D_t)^+ - c \cdot (y_t - x_t) + c \cdot \beta \cdot (y_t - D_t)^+]$$

In order to maximize this function, we first apply the following rules of transformation:

$$\min(y_t, D_t) = D_t - (D_t - y_t)^+$$

$$y_t = D_t - (D_t - y_t)^+ + (y_t - D_t)^+$$

Accordingly, we write $J_t(y_t, x_t)$ as

$$J_t(y_t, x_t) = E[\beta^m \cdot p \cdot (D_t - (D_t - y_t)^+) - h \cdot (y_t - D_t)^+ - c \cdot (y_t - x_t) + c \cdot \beta \cdot (y_t - D_t)^+]$$

$$J_t(y_t, x_t) = (\beta^m \cdot p - c) \cdot E[D] - E[(D_t - y_t)^+ \cdot (\beta^m \cdot p - c) + (y_t - D_t)^+ \cdot (h + c - c \cdot \beta) - c \cdot x_t]$$

Where we can define $G_t(y_t, x_t)$ as

$$G_t(y_t, x_t) = E[(D_t - y_t)^+ \cdot (\beta^m \cdot p - c) + (y_t - D_t)^+ \cdot (h + c - c \cdot \beta) - c \cdot x_t]$$

So that

$$J_t(y_t, x_t) = (\beta^m \cdot p - c) \cdot E[D_t] - G_t(y_t, x_t)$$

The first part of $J_t(y_t, x_t)$, $(\beta^m \cdot p - c) \cdot E[D_t]$, is the expected profit under perfect demand information. Then, by definition, the second part $G_t(y_t, x_t)$ is the expected cost due to demand uncertainty resulting from mismatching of supply and demand, which can be interpreted as follows. The first part, $(D_t - y_t)^+ \cdot (\beta^m \cdot p - c)$, is the expected underage cost, i.e., the part of demand that in expectation cannot be covered by a given supply y: each unit of shortage represents missing an opportunity of the unit margin $\beta^m \cdot p - c$. The second part $(y_t - D_t)^+ \cdot (h + c - c \cdot \beta)$ is the expected overage cost, i.e., the part of the supply that in expectation cannot be sold. This quantity is multiplied by $(h + c - c \cdot \beta)$: h represents the single unit inventory holding cost that would have to be paid for each extra unit left. The second part $c - c \cdot \beta$ reflects the assumption made previously, i.e., the fact that at the end of the period the supplier will buy-back all the extra units left at a price equal to the unit purchasing cost c. Since there is a time lapse between the purchasing moment and the selling-back one, we have to consider the discount rate within that period; this explains why, even though the supplier is allowed to resell the extra inventory at the same price he bought it, c is not totally erased from the equation, but multiplied by $(1-\beta)$. The last part considers the inventory held at the beginning of each period: the more inventory there is, the less quantity is going to be purchased to reach the optimal level y, decreasing the purchasing cost of that period.

Defining the unit underage cost $u = \beta^m \cdot p - c$ and the unit overage cost as $o = h + c - c \cdot \beta$, we can re-write $G_t(y_t, x_t)$ as

$$G_t(y_t, x_t) = E[u \cdot (D_t - y_t)^+ + o \cdot (y_t - D_t)^+ - c \cdot x_t \ ]$$

The first thing to note is that, observing the profit function $\pi_t(y_t, x_t) = (\beta^m \cdot p - c) \cdot E[D_t] - G_t(y_t, x_t)$, the first part $(\beta^m \cdot p - c) \cdot E[D]$ does not depend on the decision variable y or on the initial inventory level x_t. It follows that maximizing the expected profit $\pi_t(y_t, x_t)$ is equivalent of minimizing the expected opportunity cost $G_t(y_t, x_t)$.

We can re-write $G_t(y_t, x_t)$ by using integrals rather than positive parts:

$$G_t(y_t, x_t) = u \cdot \int_{y_t}^{\infty} (\theta - y_t) \cdot f(\theta) \cdot d\theta + o \cdot \int_{-\infty}^{y_t} (y_t - \theta) \cdot f(\theta) \cdot d\theta - c \cdot x_t$$

Where $f(\cdot)$ is the density function of the demand distribution. As in the classic newsvendor model, this function can be minimized using Leibniz's formula (see Appendix B) and leads to the following *order-up-to* quantity:

$$S = \arg\min_{y} G_t(y_t, x_t) = F^{-1}\left(\frac{u}{u+o}\right)$$

where $F(\cdot)$ is the cumulative distribution function of the demand D.

Substituting the variables, we find that

$$S = F^{-1}\left(\frac{\beta^m \cdot p - c}{\beta^m \cdot p - c \cdot \beta + h}\right)$$

Therefore, optimal order-up-to policy is defined as:

$$\hat{y}_t^{\,*}(x_t, z_t) = \begin{cases} z_t & if\ S \geq z_t \\ S & if\ \ x \leq S < z_t \\ x_t & otherwise \end{cases}$$

Notice that the optimal base-stock level for the myopic problem, S, may not be achievable due to working capital constraint represented by z_t. When $x_t > S$, the optimal decision is to not produce, since inventory on-hand is already enough. When $S \geq x_t$, if there is enough cash to reach the optimal base-stock level (if $S < z_t$), the order-up-to quantity to set is exactly S. Otherwise, the firm will buy as much as affordable with the available cash: the order-up-to quantity becomes therefore z_t.

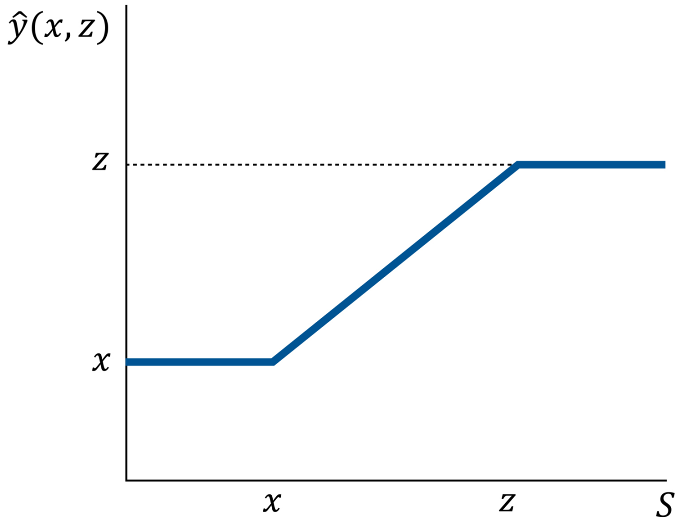


Figure 5.1: The relation between $\hat{\boldsymbol{y}}$ *and* $\boldsymbol{S}$.

5.2 The Reverse Factoring Case

Having addressed the base-case model where the firm is self-financed, in this section we illustrate the case of an SME factoring his receivables to obtain additional funds. We now assume that the buyer initiates a Reverse Factoring (RF) program along with her bank partner to facilitate the supplier's financing of operations. The supplier may choose to factor the invoices from trade transactions with the buyer after the invoices are approved, *i.e.*, at any time after the end of the period when the corresponding sales are made. The buyer's RF program is characterized by two terms, namely r_{RF} and n. The first term, r_{RF}, is the period's interest rate at which the relevant invoices are discounted when factored. We assume that $r_{RF} = r_b + b$, where r_b is the interest rate applicable to the buyer and b is the bank fee for facilitating the program. The second term, n, is the new credit term required by the buyer. In order to participate in the RF program the supplier must agree to extending the credit period from m to n periods, where $n \geq m$.

Following the same notation as in the base-case model, the state of the system at the beginning of each period, $t = 1, \ldots, T$, is summarized by a vector $(x_t, z'_t, \boldsymbol{R'_t})$, where $\boldsymbol{R'_t} = (R'_{t-m}, R'_{t-m+1}, \cdots, R'_{t-1})$ is now the n-dimensional vector of the firm's retained A/Rs, the payment for which is still pending. In the RF model, the firm must not only decide upon the production quantity, q_t, but also whether to factor any of the pending A/Rs. Let L_i^j denote the portion of the A/R generated in period i that is sold in period j, where $j \in [i+1, i+n]$. Then, the factoring decision in period t can be captured by the vector $\boldsymbol{L_t} = (L_{t-n}^t, \ldots, L_{t-1}^t)$, where $L_{t-j}^t \leq R_{t-j}^t$ for all $t = 1, \ldots, T$ and $j = 1, \ldots, n$. Notice that L_i^j represents the nominal value of the invoice to discount, i.e. the value that will be cut off the balance sheet because sold to the bank. This value, though, does not represent the real amount of cash that the company makes available immediately, since L_i^j does not comprehend the discount-factor (i.e. the interests paid off).

Given the firm's decision in period $t = 1, \ldots, T$, the dynamics are as follows:

$$x_{t+1} = (x_t + q_t - D_t)^+$$
$$z'_{t+1} = (z'_t - c \cdot q_t + R'_{t-n} - L'_{t-n})^+$$
$$\boldsymbol{R'_{t+1}} = (R'_{t-n+1} - L_{t-n+1}, \ldots, R'_{t-1} - L_{t-1}, w' \cdot \min(x_t + q_t \, ; \, D_t))$$

The expected profit function in each period, $t = 1, \dots, T$, is then given by

$$\pi_t(x_t, q_t, L_t) = E[\beta^n \cdot p \cdot \min(x_t + q_t\,;\, D_t) - \hat{G}(x_t, q_t, L_t)]$$

$$\hat{G}(x_t, q_t, L_t) = H_t(x_t, q_t) + c \cdot q_t + \sum_{j=1}^{n} L_{t-n+j-1}^{t}\left[1 - (1 - r_r)^j\right]$$

$$H_t(x_t, q_t) = E[h \cdot (x_t + q_t - D_t)^+]$$

for $0 \leq cq_t \leq z'_t + \sum_{k=1}^{n} L_{t-k}^{t}(1 - r_r)^{n-k}$ and $L_{t-j}^{t} \leq R_{t-j}^{t}$, for all $t = 1, \dots, T$ and $j = 1, \dots, n$.

The last term of $\hat{G}(x_t, q_t, L_t)$ corresponds to the interest charged by the bank when the supplier sells some of his pending A/Rs. Notice that when we calculate the cost of RF financing for the period, we consider the interest paid due to factoring the A/Rs, but no discounts for the time-value of money, since the latter has already been accounted for in the cost function through β^n. As in the base case, we now reduce the space of variables to simplify the problem and we derive a myopic optimal ordering policy.

5.2.1 State Space Reduction

As in the base case, it is convenient to reduce the dimension of the problem. In order to do so, we argue that whenever the supplier resorts to factor his A/Rs to finance his operations, it is optimal to do so in decreasing A/R maturity order.

We first explain how a generic invoice-discounting cost should be derived. Consider the situation in which the company needs extra liquidity in a given period and decides to take advantage of reverse factoring to collect an amount $\bar{C}$. For instance, $\bar{C}$ is what the company needs in order to buy more inventory; the question is now how much of the A/Rs should the firm discount in order to receive, after discounting-interests, an exact net amount of $\bar{C}$. Being r_{RF} the invoice-discounting interest rate, for a generic invoice arisen in period k, the net cash amount C the firm would receive discounting the invoice in period t is $C = L'_{t-k} \cdot (1 - r_{RF})^{n-k+1}$. If we already know the amount $\bar{C}$, we can calculate the invoice's nominal value we need to discount:

$$L'_{t-k} = \frac{\bar{C}}{(1 - r_{RF})^{n-k+1}}$$

We can now derive the cost of discounting the general invoice L'_{t-k}, that is given, as in $\hat{G}(x_t, q_t, L_t)$, by

$$Discounting\ Cost = [1 - (1 - r_{RF})^{n-k+1}] \cdot L'_{t-k}$$

Substituting L'_{t-k} in the $Discounting\ Cost$ function, we obtain the cost of discounting a generic invoice arisen in period k in order to free up a certain amount of cash, $\bar{C}$:

$$Discounting\ Cost = \frac{\bar{C}}{(1 - r_{RF})^{n-k+1}} \cdot [1 - (1 - r_{RF})^{n-k+1}]$$

That can be written as

$$Discounting\ Cost = \left[\frac{1}{(1-r_{RF})^{n-k+1}} - 1\right] \cdot \bar{C}$$

This equation also proves that it is always convenient to choose the invoice to discount in a decreasing maturity order. Indeed, for every invoice arisen in each period k, $Discounting\ Cost = \left[\frac{1}{(1-r_{RF})^{n-k+1}} - 1\right] \cdot \bar{C}$, $k \leq n$; the cost is decreasing in k, where $k = n$ indicates the "oldest" invoice, while $k = 1$ indicates the invoice generated in the previous period. As a consequence, whenever extra liquidity is needed, the firm will factor the invoices in decreasing maturity order.

As in the base case, it is convenient to introduce new system variables and definitions as in the base-case model. Define $y_t = x_t + q_t$, $\boldsymbol{R_t} = \frac{1}{c} \cdot \boldsymbol{R'_t}$ and the vector $\boldsymbol{z_t} = (z_t\ , z_t^{t-n}, \dots, z_t^{t-1})$, where $z_t = x_t + \frac{1}{c} \cdot z'_t$, $z_t^{t-n} = z_t + R_{t-n} \cdot (1 - r_{RF})$ and so on up to $z_t^{t-1} = z_{t-2} + R_{t-1} \cdot (1 - r_{RF})^n$. The order-up-to quantity y_t is the new decision variable: as in the base case, at the beginning of each period, instead of deciding how much to order (q_t), the supplier decides at which level he wants to build up inventory. The first element of vector $\boldsymbol{z_t}$ represents the *inventory equivalent* of the cash plus the available inventory at the beginning of t, while the subsequent elements correspond to the same quantity supplemented with the cash that derives from selling each layer of the A/R vector in decreasing maturity order. Fundamentally, this vector answers the question "how much inventory would the firm have available if he factors the oldest invoice? And what if he factors also the second oldest? What if he does not discount any?". $\boldsymbol{z_t}$ is updated every period (as described in paragraph 5.2.3), depending on what happened and how many A/Rs have already been discounted.

5.2.2 Single Period Profit Maximization

Accordingly with the base case model, we assume that also the firm that joins RF program adopts a myopic ordering policy, maximizing just the single period profit function. Considering the new variables introduced and the decreasing maturity order of the A/Rs, the initial problem of $n + 2$ state variables $(x_t, z'_t, \boldsymbol{R'_t})$ and $n + 1$ decision variables $(q_t, \boldsymbol{L_t})$ is now reduced to one with $n + 2$ state variables $(x_t, \boldsymbol{z_t})$ and only a single decision variable (y_t). Accordingly, the single-period profit function $J_t(y_t, x_t, \boldsymbol{z_t})$ can be written as:

$$\begin{aligned} J_t(y_t, x_t, \boldsymbol{z_t}) = {} & E[\beta^n \cdot p \cdot \min(y_t, D_t) - (h - c \cdot \beta) \cdot (y_t - D_t)^+] - c \cdot (y_t - x_t) - 1_{\{z_t < y_t \leq z_t^{t-n}\}} \cdot c \\ & \cdot (y_t - z_t) \cdot \left(\frac{1}{1 - r_{RF}} - 1\right) \\ & - \sum_{j=1}^{n-1} 1_{\left\{z_t^{t-n+j-1} < y_t \leq z_t^{t-n+j}\right\}} \cdot c \\ & \cdot \left[\left(y_t - z_t^{t-n+j-1}\right) \cdot \left[\frac{1}{(1 - r_{RF})^{j+1}} - 1\right] + \frac{(z_t^{t-n} - z_t)}{(1 - r_{RF})} \cdot r_{RF} \right. \\ & \left. + \sum_{i=1}^{j-1} \left(z_t^{t-n+i} - z_t^{t-n+i-1}\right) \cdot \left[\frac{1 - (1 - r_{RF})^{i+1}}{(1 - r_{RF})^{i+1}}\right] \right] \end{aligned}$$

where $1_{\{\cdot\}}$ is the indicator function: if the condition $\{\cdot\}$ is satisfied, the function is worth 1, otherwise 0. The first part of the function is the same as the base-case model. Then, $1_{\{z_t<y_t\leq z_t^{t-n}\}}\cdot(y_t-z_t)\cdot\left(\frac{1}{1-r_{RF}}-1\right)$ represents the cost of discounting only the most mature invoice: as the indicator function suggests, the supplier will face this cost just when he needs to discount only the first invoice, i.e. when the order-up to quantity y_t is higher than z_t (available inventory plus cash-equivalent inventory) but also equal or lower than z_t^{t-n} (available inventory plus cash-equivalent inventory plus cash equivalent inventory available discounting the whole oldest invoice). So, when y_t is in that region, the firm will factor the first invoice in order to free up enough cash to reach the quantity desired y_t: the cash that needs to be freed up is therefore the difference between the quantity desired y_t and the quantity reachable putting together inventories and cash-equivalent inventory on hand, z_t, so y_t-z_t. This amount needs to be multiplied by $\left[\frac{1}{1-r_{RF}}-1\right]$, as already explained (see paragraph 5.2.1). If $z_t\geq y_t$ (i.e. there is enough inventory and cash to reach the order up to quantity), there is no need to discount any invoice, that part of the function is worth zero and the cost disappears. On the other hand, if $z_t^{t-n}<y_t$ (i.e. even discounting the whole oldest invoice y_t is not achievable), again this cost disappears, but appears at the same time in the following part, which is a generalization of what has just been illustrated. Notice that through the indicator function, it is now only the inventory order up to level decision, y_t, that drives the depletion of A/Rs. Consequently, the problem is reduced to one with only a single decision variable.

It can be proved (see Appendix C) that $J_t(y_t,x_t,\boldsymbol{z_t})$ is continuous in y_t,x_t and $\boldsymbol{z_t}$ and concave in $y_t\in[x_t,z_t^{t-1}]$. This suggests that there is an optimal inventory replenishment policy that maximizes the single period profit function.

We now maximize the profit function as we did in the base-case paragraph 5.1.3, so that we obtain:

$$S=F^{-1}\left(\frac{\beta^n\cdot p-c}{\beta^n\cdot p-c\cdot\beta+h}\right)$$

$$s^{t-n+k-1}=F^{-1}\left(\frac{\beta^n\cdot p-\dfrac{1}{(1-r_{RF})^k}\cdot c}{\beta^n\cdot p-c\cdot\beta+h}\right)\quad for\ k=1,2,\dots,n$$

The quantities S and $s^{t-n+k-1}$ represent the optimal inventory replenishment decisions (order-up-to levels), as they are derived from the first-order condition of $J_t(y_t,x_t,\boldsymbol{z_t})$. Totally, they are $n+1$ solutions, because there is a different solution for every decision the supplier makes: if he does not factor any invoice, he orders up to S, the same quantity as in the base-case model, since Reverse Factoring has not been used. If he factors the most mature invoice due to lack of cash (in this case, $k=1$), the quantity ordered has to take into account the cost of factoring, $s^{t-n}=F^{-1}\left(\frac{\beta^n\cdot p-\frac{1}{(1-r_{RF})}\cdot c}{\beta^n\cdot p-c\cdot\beta+h}\right)$: we can notice that $s^{t-n}<S$, since the cost of factoring the first invoice pushes the firm to order less. There is therefore an optimal order-up to level for every situation: when the supplier does not discount, when he discounts the most mature A/Rs, when he discounts the two most mature A/Rs and so on. Notice that the optimal production decisions, $S,s^{t-n},\dots$, and s^{t-1}, are achievable only if $S\leq z_t$, $s^{t-n}\leq z_t^{t-n},\dots$, and $s^{t-1}\leq z_t^{t-1}$, respectively. Next, we formalize the optimal policy for the single period problem. For any given initial state $(x_t,\boldsymbol{z_t})$, the firm will order up to the level $y^*(x_t,\boldsymbol{z_t})$ defined as:

$$
y^*(x_t, \boldsymbol{z_t}) = \begin{cases}
\max(x_t, S), & if\ S \leq z_t; \\
z_t, & if\ s^{t-n} \leq z_t < S; \\
s^{t-n}, & if\ z_t \leq s^{t-n} \leq z_t^{t-n}; \\
z_t^{t-n}, & if\ s^{t-n+1} \leq z_t^{t-n} < s^{t-n}; \\
\dots & \dots \\
s^{t-n+k}, & if\ z_t^{t-n+k-1} \leq s^{t-n+k} \leq z_t^{t-n+k} \quad for\ k = 1, \dots, n-2; \\
z_t^{t-n+k}, & if\ s^{t-n+k+1} \leq z_t^{t-n+k} < s^{t-n+k} \quad for\ k = 1, \dots, n-2; \\
\dots & \dots \\
s^{t-1}, & if\ z_t^{t-2} \leq s^{t-1} \leq z_t^{t-1}; \\
z_t^{t-1}, & if\ z_t^{t-1} < s^{t-1};
\end{cases}
$$

All the cases for the optimal inventory replenishment decision are mutually exclusive and collectively exhaustive; thus, there can only be one optimal decision for any initial state realization. The replenishment decision stands among $n+1$ optimal solutions (S, s^{t-n}, $\dots s^{t-1}$) plus $n+1$ "boundary" solutions (z_t , $z_t^{t-n}, \dots, z_t^{t-1}$). The optimal solutions are straightforward: at the beginning of each period, the firm has a certain amount of inventories and cash-on-hand, z_t; if z_t is enough to reach the optimal quantity S, the firm will order-up to S (or even x_t if the initial inventory overcomes S). Otherwise, the firm will factor the missing cash from the most mature invoice, increasing his potential cash-on-hand from z_t to z_t^{t-n}. However, since the firm will pay some interests to free up that cash, the quantity ordered takes this into account and decreases, from S to s^{t-n}. If the cash potentially freed up with the first invoice discount, z_t^{t-n}, is not enough to reach s^{t-n}, the firm will have to factor also the second most mature invoice, receiving more cash z_t^{t-n+1}, and again the optimal order-up-to level will decrease, considering the extra interests paid (s^{t-n+1}). The boundary solutions, on the other hand, consist in particular cases: these solutions are optimal when the quantity of cash plus inventory available (for instance, z_t) is not enough to reach the respective order-up-to level (S) but it is enough to reach the lower one (s^{t-n}). However, s^{t-n} is optimal just when the firm has actually discounted the most mature A/Rs, but z_t does not discount any. In this case the optimal decision is to order-up-to the cash plus inventory available, z_t. The explanation lies in the marginal profit, i.e. the first order derivative of the profit function $J_t(y_t, x_t, \boldsymbol{z_t})$. An example may help to understand: assume $n = 3$, we therefore have an $(n+1)$-parts piecewise function, since it contains several costs that are "activated" or not depending on the cash level, $\boldsymbol{z_t} = (z_t, z_t^{t-3}, z_t^{t-2}, z_t^{t-1})$. We consider now the profit function first-order derivative in y (Figure 5.2), which represents the marginal profit gained by buying one additional inventory unit.

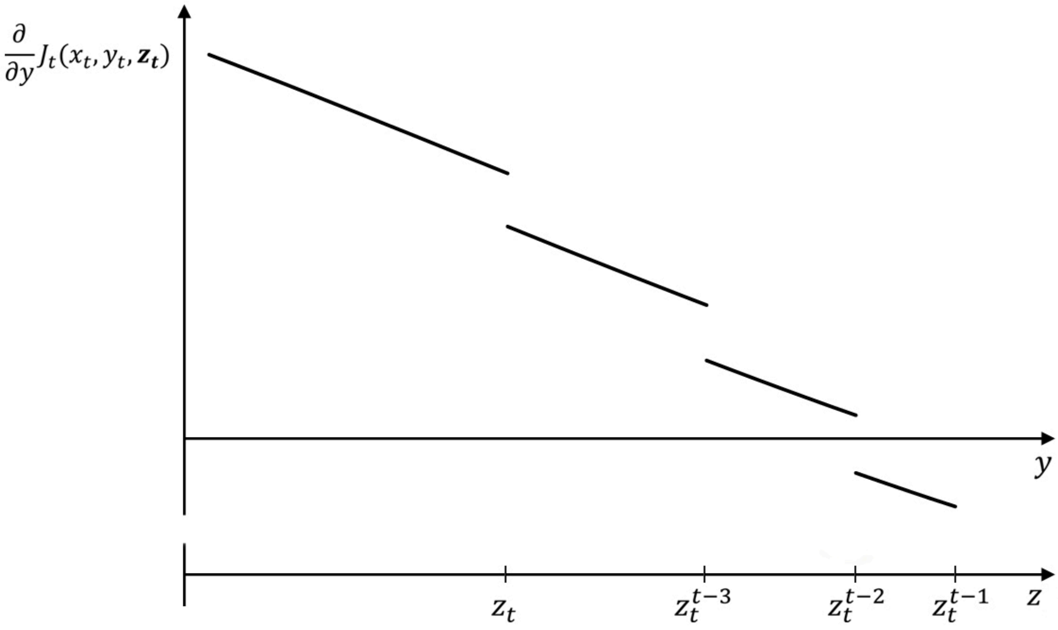


Figure 5.2: Definition of the optimal policy for $\mathbf{n} = \mathbf{3}$.

In this particular case, we see that the marginal profit is positive and decreasing until z_t ; then it is still positive, but lower, since the firm is discounting the first invoice in order to buy additional inventory; when the order up to level overcomes z_t^{t-3}, the marginal profit lowers further, remaining positive, since more interests are paid in order to buy the additional inventory units. Once z_t^{t-2} is passed, the marginal profit becomes negative: this means that it would not be convenient to factor also the last invoice, because the marginal profit would "jump" from positive to negative. To maximize profit, it is necessary to stop before the marginal profit becomes negative: this means stopping exactly in z_t^{t-2}.

5.2.3 Multi-Period Model

With the new variables and the state space reduction, the multi-period profit function is given by:

$$\pi_t(y_t, x_t, \boldsymbol{z}_t) = E[\beta^n \cdot p \cdot \min(y_t, D_t) - h \cdot (y_t - D_t)^+] - c \cdot (y_t - x_t) - 1_{\{z_t < y_t \le z_t^{t-n}\}} \cdot (y_t - z_t) \cdot \left(\frac{1}{1 - r_{RF}} - 1\right) - \sum_{j=1}^{n-1} 1_{\{z_t^{t-n+j-1} < y_t \le z_t^{t-n+j}\}} \cdot c \cdot \left[\left(y_t - z_t^{t-n+j-1}\right) \cdot \left[\frac{1}{(1 - r_{RF})^{j+1}} - 1\right] + \frac{(z_t^{t-n} - z_t)}{(1 - r_{RF})} \cdot r_{RF} + \sum_{i=1}^{j-1} \left(z_t^{t-n+i} - z_t^{t-n+i-1}\right) \cdot \left[\frac{1 - (1 - r_{RF})^{i+1}}{(1 - r_{RF})^{i+1}}\right]\right]$$

For $x_t \le y_t \le z_t^{t-1}$, and the dynamics of states are given by

$$x_{t+1} = (y_t - D_t)^+$$

$$\boldsymbol{z_{t+1}} = (z_{t+1}, z_{t+1}^{t-n+1}, \dots, z_{t+1}^t)$$

Where

$$z_{t+1} = x_{t+1} + (z_t - y_t)^+ + \frac{1}{1-r_{RF}} \cdot (z_t^{t-n} - z_t - \min\left((y_t - z_t)^+, z_t^{t-n} - z_t\right))$$

$$z_{t+1}^{t-n} = z_{t+1} + \frac{1}{1-r_{RF}} \cdot \left(z_t^{t-n+1} - z_t^{t-n} - \min\left((y_t - z_t^{t-n})^+, z_t^{t-n+1} - z_t^{t-n}\right)\right)$$

$$z_{t+1}^{t-n+k} = z_{t+1}^{t-n+k-1} + \frac{1}{1-r_{RF}} \cdot \left(z_t^{t-n+k+1} - z_t^{t-n+k} - \min\left(\left(y_t - z_t^{t-n+k}\right)^+, z_t^{t-n+k+1} - z_t^{t-n+k}\right)\right)$$

for $k = 1, \dots, n-1$, and

$$z_{t+1}^{t} = z_{t+1}^{t-1} + (1-r_{RF})^n \cdot w \cdot \min(y_t, D_t)$$

5.3 The Traditional Reverse Factoring Case

Since the purpose of this model is to compare the evolved Reverse Factoring (RF) with the Traditional one, we keep the same scenario of the base and RF case and most of RF's assumptions. The adjustments needed by T-RF case only concern the interest rate (r_{TRF}) and the auto-discounting constraint. Particularly, we assume a higher interest rate (see paragraph 1.3.3) and remove the supplier's possibility to choose whether or not discount her invoices and therefore also the possibilities to choose which invoice and what percentage of it. In this scenario, every time the supplier issues an invoice, the bank automatically discounts the whole amount it and pays it off. Consequently, except in $t = 1$ because we assume to adopt T-RF from that period on, the A/Rs vector $\overline{\boldsymbol{R}}_{\boldsymbol{t}}$ (see paragraphs 5.1 and 5.2.1) does not contain any invoice, since all of them are cut off once they appear. Accordingly, the initial A/R condition remains $\overline{\boldsymbol{R}}_{\boldsymbol{1}} = (\bar{R}_1, \bar{R}_2, \dots, \bar{R}_m)$, and the dynamics of states are given by:

$$x_{t+1} = (y_t - D_t)^+$$

$$z_{t+1} = x_{t+1} + \sum_{i=1}^{n} R_{t-n+i-1} \cdot (1-r_{TRF})^i + z_t - y_t$$

$$\boldsymbol{R}_{\boldsymbol{t+1}} = (0, \dots, 0, w \cdot \min(y_t, D_t))$$

and the profit function is now defined as:

$$\pi(y_t, x_t) = E[\beta^n \cdot p \cdot \min(y_t, D_t) - h \cdot (y_t - D_t)^+ - c \cdot (y_t - x_t)] - \sum_{i=1}^{n} R_{t-n+i-1} \cdot [1 - (1-r_{TRF})^i]$$

5.3.1 Single Period Profit Maximization

For consistency, we assume the firm adopts a myopic ordering policy, maximizing only the single period profit. The single period profit function is given by:

$$J(y_t, x_t) = E[\beta^m \cdot p \cdot \min(y_t, D_t) - h \cdot (y_t - D_t)^+ - c \cdot (y_t - x_t) + c \cdot \beta \cdot (y_t - D_t)^+] - \sum_{i=1}^{m} R_{t-m+i-1} \cdot [1 - (1-r_{TRF})^i]$$

As we did in the base case model (see paragraph 5.1.3), we can derive the optimal myopic solution:

$$S = F^{-1}\left(\frac{\beta^m \cdot p - c}{\beta^m \cdot p - c \cdot \beta + h}\right)$$

And the consequent ordering policy:

$$\widehat{y_t}^*(x_t, z_t) = \begin{cases} z_t & if\ S \geq z_t \\ S & if\ x_t \leq S < z_t \\ x_t & otherwise \end{cases}$$

5.4 The Factoring Case

The main characteristic that differentiates Factoring from Reverse Factoring lies in the interest rate that the bank adopts to discount the invoices. As illustrated in paragraph 1.3.3, the factoring discounting cost is based on the supplier's credit-worthiness, independently from who and how large the buyer is. Consequently, the first assumption that changes is the interest rate, r_F, which is higher than r_{TRF} and r_{RF}. Another difference is given by the payment term: we assumed that in both RF and T-RF cases the buyer negotiates better interest rates for the supplier but requires a payment lengthening (from m to n with $n \geq m$). In the Factoring Case, instead, the interest rate remains high, but the payment term does not increase: it is the same as in the base case, m. We also assume an auto-discounting policy imposed by the bank, as in the T-RF case.

A further explication of the model would be redundant, since it is developed exactly as the T-RF's one, substituting respectively r_{TRF} and n with r_F and m.

5.5 Credit Line

Bank credit lines are a major source of operations financing for corporations, particularly when considering SMEs. Sufi (2009) found that credit lines account for over 80% of the bank financing provided to U.S. public firms, while Kashyap et al. (2002) found that 70% of bank lending by U.S. small firms is through credit lines. In a credit line offering, a financial institute provides the company with a nominal amount of debt capacity against which the firm draws funds. The firm pays a percentage fee on the unused portion of the debt capacity and an interest rate on the used amounts.

Seeking for more realistic and deeper insights, we adapt our model and evaluate the RF compared to a new base-case, in which the supplier has access to an unsecured credit line (CL). In the new scenario, the company has access to a credit limit equal to $\bar{O}$ and pays an interest rate r_s, while the commitment fee on the unused portion of debt is r_u. We also define o'_t the borrowing decision in t, and $o_t = o_{t-1} + o'_t$ the non-negative outstanding debt in t. Given the constraint $o_t \leq \bar{O}$, we introduce a new parameter $o_t^{max} = \bar{O} - o_{t-1}$ that is an upper limit to the borrowing decision o'_t. Note that o'_t can also be negative, meaning that the firm is paying back the principal. These two conditions are expressed by the constraint $-o_{t-1} < o'_t \leq o_t^{max}$ for $t = 1,2 \ldots, T$. The interest and commitment fee are paid at the end of each period, but do not represent a cash outflow from z'_t since the latter is intended as an operating cash balance. In fact, per each unit sold z'_t increases by w' instead of p, where $0 \leq w' \leq p$, in order to represent a working-capital cash balance, only used to increase or decrease inventory. Consequently, the financial management and the financial risks are not taken into consideration, assuming that there is no bankruptcy.

The new dynamics are as follows:

$$x_{t+1} = (x_t + q_t - D_t)^+$$
$$z'_{t+1} = z'_t - c \cdot q_t + R'_{t-m} + o'_{t-1}$$
$$R'_{t+1} = (R'_{t-m+1}, \dots, R'_{t-1}, w' \cdot \min(x_t + q_t, D_t))$$

while the new profit function is:

$$\begin{aligned}\pi_t(x_t, y_t, o_t) &= \beta^m \cdot p \cdot \min(y_t, D_t) - h \cdot (y_t - D_t)^+ - c \cdot (y_t - x_t) - \beta \cdot [o_t \cdot r_s + (\bar{O} - o_t) \cdot r_u] \\ &= \beta^m \cdot p \cdot \min(y_t, D_t) - h \cdot (y_t - D_t)^+ - c \cdot (y_t - x_t) - \beta \\ &\quad \cdot [(o_{t-1} + o'_t) \cdot (r_s - r_u) + \bar{O} \cdot r_u]\end{aligned}$$

where the last two terms stand for the payment of interests and commitment fee on the used and unused portions of debt, respectively.

The dynamic programming formulation, here omitted, is shared with the previous base case, but considers the new π_t.

5.5.1 Single Period Profit Maximization

We derive the new optimal policy with the method previously presented, assuming that the company uses a myopic policy that optimizes the single period.

It is reasonable to assume that the firm will access to debt only when extra financing is needed, and that the borrowing (pay back) amount is a function of the ordering policy and the state variables. We can therefore remove the new decision variable o'_t and express it in terms of y_t, x_t and z_t, where:

$$y_t = x_t + q_t$$
$$z_t = x_t + \frac{z'_t}{c}$$

The single period profit function is written as:

$$\begin{aligned}J_t(x_t, y_t, z_t) = {} & E[\beta^m \cdot p \cdot \min(y_t, D_t) - (h - \beta \cdot c) \cdot (y_t - D_t)^+] - c \cdot (y_t - x_t) \\ & - \left[1_{\{z_t + o_t^{max} < y_t\}} \cdot o_t^{max} \cdot (r_s - r_u) \cdot \beta + 1_{\{z_t < y_t \le z_t + o_t^{max}\}} \cdot c \cdot (y_t - z_t) \cdot (r_s - r_u) \cdot \beta \right. \\ & - 1_{\{x < y \le z_t\}} \cdot \min[(z_t - y_t) \cdot c; o_{t-1}] \cdot (r_s - r_u) \cdot \beta - 1_{\{y \le x\}} \cdot \min[(z_t - x_t) \cdot c; o_{t-1}] \\ & \left. \cdot (r_s - r_u) \cdot \beta \right]\end{aligned}$$

The optimal inventory replenishment decisions (order-up-to levels), obtained from the first-order derivative of $J_t(x_t, y_t, z_t)$, are:

$$S = F^{-1}\left(\frac{\beta^m \cdot p - c}{\beta^m \cdot p + h - \beta \cdot c}\right)$$

$$S_{CL} = F^{-1}\left(\frac{\beta^m \cdot p - c \cdot (1 + (r_s - r_u) \cdot \beta)}{\beta^m \cdot p + h - \beta \cdot c}\right)$$

For any given initial state, a capital dependent base-stock policy $y^*(x_t, \boldsymbol{z_t})$ is optimal for the myopic problem, where:

$$y_t^*(x_t, \boldsymbol{z_t}) = \begin{cases} z_t + o_t^{max} & z_t + o_t^{max} < S_{CL} \\ S_{CL} & z_t < S_{CL} \le z_t + o_t^{max} \\ z_t & S_{CL} \le z_t < S \\ S & x_t < S \le z_t \quad and \quad o_{t-1} \le z_t - S \\ S_{CL} & x_t < S \le z_t \quad and \quad o_{t-1} > z_t - S \\ x & otherwise \end{cases}$$

As in the single period profit function, we can remove the decision variable o'_t and express it in terms of y_t, x_t and z_t:

$$o'_t = \left\{ 1_{\left\{z_t + \frac{o_t^{max}}{c} < y_t\right\}} \cdot \frac{o_t^{max}}{c} + 1_{\left\{z_t < y_t \le z_t + \frac{o_t^{max}}{c}\right\}} \cdot (y_t - z_t) - 1_{\{x_t < y_t \le z_t\}} \cdot \min\left[(z_t - y_t); \frac{o_{t-1}}{c}\right] - 1_{\{x_t > y_t\}} \cdot \min\left(z_t - x_t; \frac{o_{t-1}}{c}\right) \right\} \cdot c$$

Note that o'_t, as much as o_t^{max}, is a monetary element; they are both initially divided by c to allow comparisons with z_t and y_t in terms of unit, then the whole expression is multiplied by c.

5.5.2 Multi Period Model

We now present the multi-period profit function:

$$\pi_t(x_t, y_t, z_t) = \beta^m \cdot p \cdot \min(y_t, D_t) - h \cdot (y_t - D_t)^+ - c \cdot (y_t - x_t) - \beta \cdot \left[\left(o_{t-1} + 1_{\left\{z_t + \frac{o_t^{max}}{c} < y_t\right\}} \cdot o_t^{max} + 1_{\left\{z_t < y_t \le z_t + \frac{o_t^{max}}{c}\right\}} \cdot (y_t - z_t) \cdot c - 1_{\{x_t < y_t \le z_t\}} \cdot \min[(z_t - y_t) \cdot c; o_{t-1}] - 1_{\{x_t \le y_t\}} \cdot \min\left((z_t - x_t) \cdot c; o_{t-1}\right) \right) \cdot (r_s - r_u) + \bar{O} \cdot r_u \right]$$

5.6 Simulation

In this section we use the model to conduct a simulation and assess the impact of RF on the retained profit at the end of period T, compared to other financing alternatives: respectively, non financing, unsecured credit line, factoring and traditional reverse factoring. The comparison between RF and the other financing means is realized in two different scenarios, reflecting the different nature of the arrangement: in the first case, referred to as $S_{(2,2)}$, $m = n = 2$ and the adoption of RF does not imply any payment term extension for the buyer. In the second case, denoted by $S_{(2,3)}$, $m = 2$ and $n = 3$ representing the situation in which the supplier accepts a payment delay from the buyer. In addition, each scenario breaks into three cases that present three different demand patterns: (i) Stationary Demand, (ii) Growing Demand, (iii) Seasonal Demand. Table 5.1 summarizes the simulations' structure:

$S_{(2,2)}$ Stationary Demand	$S_{(2,2)}$ Growing Demand	$S_{(2,2)}$ Seasonal Demand
$S_{(2,3)}$ Stationary Demand	$S_{(2,3)}$ Growing Demand	$S_{(2,3)}$ Seasonal Demand

Table 5.1: Plan of the simulations.

At the end, we also examine how the results vary when initial cash conditions are modified.

For each of the six scenarios, we implemented 1.000 runs of simulations and extrapolated the mean values of the following key performance indicators (KPIs):

- *Profit ($):* the present value of the earnings from operations at the end of the planning horizon, including the inventory holding cost; we chose this KPI in order to represent the value creation for the supplier.
- *Delta Profit (%):* it is defined as the profit increment (or decrement) given by RF in respect to each alternative solution (Credit Line, No Financing, Traditional RF, Factoring). $Delta\ Profit = \frac{\pi_{RF}-\pi_i}{\pi_i}$, where π_i is the profit achieved recurring to the financing solution i.
- *Demand Coverage (%):* a ratio defined as $C = \min{(y_t/D_t; 1)}$, i.e. the ability of the supplier to provide the customer with the required amount of units (it is 100% if $D_t = 0$, $\min{(y_t/D_t; 1)}$ otherwise); being this a Make-to-Stock context, we chose this KPI to represent the Service Level improvement: indeed, frequently happens that Reverse Factoring is initiated by buyers in order to let their suppliers improve their operating performances.
- *Total Inventory (units):* it refers to the sum of achieved base-stock level over the 12 periods. It represents the capability of the supplier to reach the optimal inventory stock policy and therefore, in the comparison between different solutions, a measure of how much additional liquidity each financing tool is able to free up.

We also calculated the Confidence Interval of the difference between the Profits' mean values, with a confidence level equal to $\alpha = 5\%$;

Each stock unit is replenished at a cost $c = 1$ and sold for $p = 1{,}4$, while the unit revenue that is retained for financing operations is $w' = 1$. The credit limit is $\bar{O} = 180$, as we assume the bank applies a haircut of 50% on the Accounts Receivable and no haircut on the available cash. The single period's discount rate is $\beta = 0{,}9975$ and the interest rates are $r_s = 0{,}02$, $r_u = 0{,}0166$, $r_F = 0{,}015$, $r_{TRF} = 0{,}01$, $r_{RF} = 0{,}05$. The chosen r_{RF} corresponds to an annual rate about 6% (a buyer interest rate of 5% plus 1% as a bank fee), which is in line with the figures provided in Seifert and Seifert's (2009) study of 23 RF programs. The initial inventory, x_1, is fixed to zero under all scenarios, while the initial cash and A/R vector $(z'_t, \boldsymbol{R'_t})$ is fixed to (120,60,60) under all scenarios, with the exception of Traditional RF and Reverse Factoring, in which it is set to (60,60,60,60) when RF entails a payment extension ($S_{(2,3)}$). In each case, the total assets of the firm at the beginning of the simulation are equal to 240, allowing direct comparisons among the results.

5.6.1 Stationary Demand

We consider a supplier that operates over twelve months and at the beginning of each month faces a normally distributed demand from a single buyer, characterized by mean $\mu = 100$ and standard deviation $\sigma = 30$ (truncated to avoid negative realization; i.e. any negative realization is set to zero since demand is a non-negative variable).

$\boldsymbol{S_{(2,2)}}$ Scenario: No Payment Term Extension

		$S_{(2,2)}$				
		No Financing	CL	T-RF	F	RF
Profit	Average	351	384	377	364	397
	Std. dev.	24,50	40,77	43,16	43,43	44,30
	Δ Profit RF (%)	12,5%	2,9%	5,2%	8,4%	
C.I. $(\pi_{RF} - \pi_i)$	Inf.	42,74	6,13	13,09	25,58	
	Average	46,18	13,32	20,57	33,31	
	Sup.	49,62	20,51	28,06	41,03	
Demand Coverage (%)	Average	81,3%	94,8%	95,3%	95,2%	95,1%
	Std. Dev.	5,3%	2,6%	2,5%	2,5%	2,7%
Total Inventory	Average	1.039	1.323	1.348	1.348	1.341
	Std. Dev.	40,00	2,14	0,00	0,00	0,53

Table 5.2: Supplier's average operational performance under different financing alternatives when RF does not entail payment term extension.

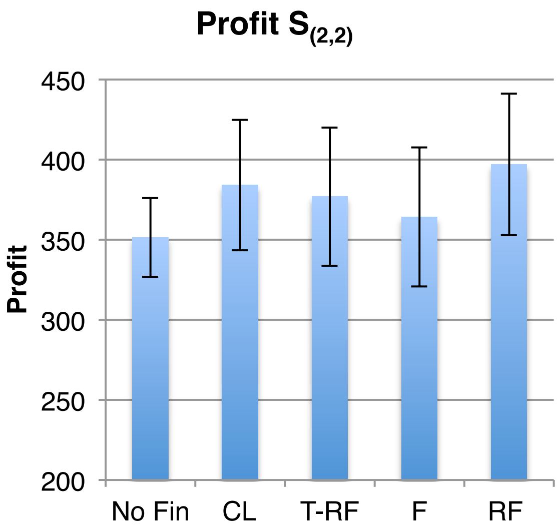


Figure 5.3: Supplier's average Profit for $\boldsymbol{S}_{(2,2)}$

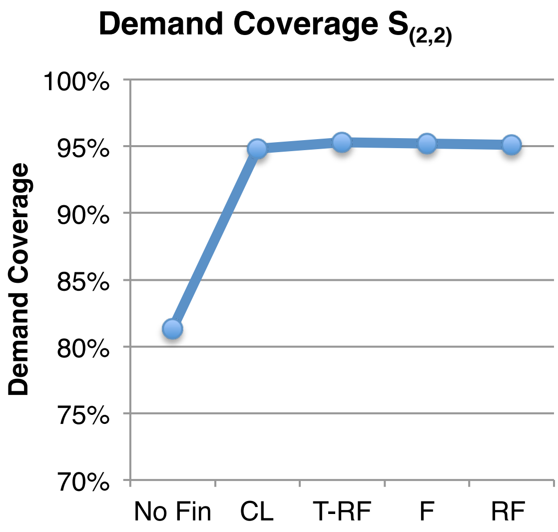


Figure 5.4: Supplier's Demand Coverage for $S_{(2,2)}$

As one would expect, the non-financing scenario is outperformed by all of the alternative scenarios, both in terms of Profit and Demand Coverage. The restriction on working capital expressed by w', indeed, keeps the SME from following the optimal order policy; i.e., the firm uses up all of his cash almost in every period, but can satisfy only 81,3% of demand on average. These stock-outs represent missed sales and, consequently, lead to a lower profit.

Reverse Factoring proves to be the best financing alternative in this scenario, since it allows the SME to discount the invoices only when needed and at an advantageous interest rate. In this scenario, RF dominates Factoring (F) and Traditional Reverse (T-RF) Factoring by definition, since it is characterized by a minor cost and does not entail any payment term extension. F and T-RF perform slightly worse than Credit Line as well, since the auto discounting arrangement forces the SME to pay an interest even when he does not need any anticipated cash collection; the CL, on the other hand, is used only when needed and the interest paid on the unused cash is low (about 2% per year).

Confidence intervals are always positive, implying that RF dominates the other financing means in terms of Profit achieved with a 95% confidence level.

Profit Standard Deviation σ_p increases considerably (+75% on average) when the SME has access to any financing instrument. It is explained by the fact that in absence of financing the firm uses up all of his cash but is not able reach the order policy y; in this situation demand variability affects the Profit milder, since the SME cannot exploit a higher demand to increase his sales. Similarly, when demand is below average, leftover stocks are lower compared to the presence of financing instruments, and so are the holding costs.

It can be noticed that Demand Coverage is imperceptibly lower in the RF program, compared to F and T-RF. The same can be said about Total Inventory, which is strictly linked to the service level and depends on the order policy. When the SME participates in a RF agreement, it has to decide how much to order in each period and, while doing so, it has to consider whether to discount any invoice; as shown in the model description, the order policy y for RF decreases as the firm factors its invoices. On the other hand, F and T-RF deprive the supplier of the discounting decision, which in this case does not affect the order policy. As a result, the SME will always order higher quantities and when it recurs to F and T-RF thus obtaining a 0,1%-0,2% improvement in service level.

$S_{(2,3)}$ Scenario: Payment Term Extension

		$S_{(2,3)}$				
		No Fin	CL	T-RF	F	RF (n=3)
Profit	Average	351	384	360	364	388
	Std. dev.	24,50	40,77	41,81	43,43	42,24
	Δ Profit RF (%)	9,8%	0,5%	7,3%	5,9%	
C.I. $(\pi_{RF} - \pi_i)$	Inf.	33,57	-5,11	20,82	15,56	
	Average	36,81	2,36	27,99	23,15	
	Sup.	40,06	9,83	35,16	30,73	
Demand Coverage (%)	Average	81,4%	94,8%	95,3%	95,2%	94,9%
	Std. Dev.	5,3%	2,6%	2,4%	2,5%	2,6%
Total Inventory	Average	1.039	1.323	1.346	1.348	1.338
	Std. Dev.	40,00	2,14	0,00	0,00	0,06

Table 5.3: Supplier's average operational performance under different financing alternatives when RF entails payment term extension.

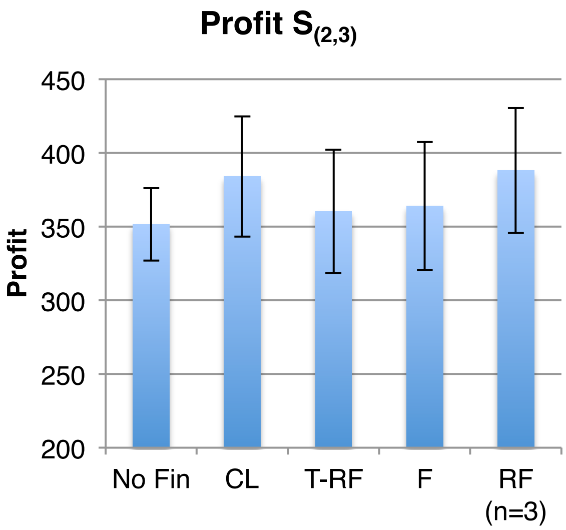


Figure 5.5: Supplier's average Profit for $\boldsymbol{S_{(2,3)}}$.

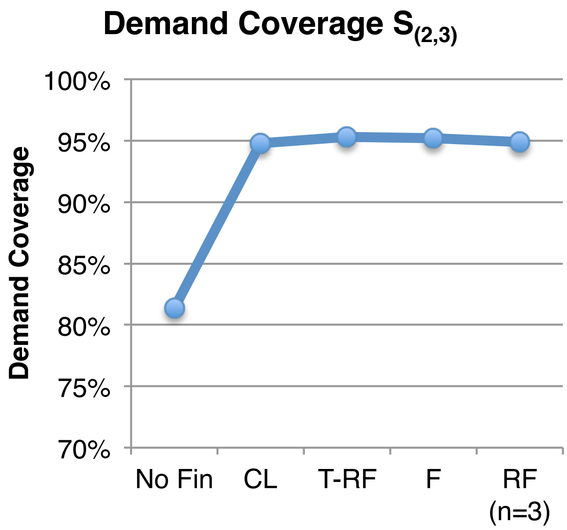


Figure 5.6: Supplier's Demand Coverage for $S_{(2,3)}$.

The previous analysis holds in this scenario, but the payment term extension to $n = 3$ periods involves T-RF and RF causing a slight decrease in their performance: Profit decreases by 4,4% and 2,3% respectively compared to the $S_{(2,2)}$ scenario. In addition, a considerable percentage of runs (27,5%) had Credit Line overcoming or at least equaling RF, without considering the transactional cost of changing platform, provider, bank and similar elements. While the other parameters are unvaried, the new n causes an increase in the cost of financing, due to the higher interest paid to the bank. RF still outperforms other financing means, but profit achieved using a Credit Line is lower by just 0,5% on average and reveals a slightly lower variability.

The Confidence Interval associated to the Credit Line is positive on average, but includes negative values; i.e. RF performs better *on average*, but there is the chance for the SME to obtain a higher profit financing its operations with a Credit Line.

This simulation case illustrates what has already been argued: when buyers insist on lengthening their payments in return of RF, their suppliers will not always be better off, especially if they already benefit from other competitive financing means. Consequently, if a buyer initiates a RF program not only looking at her own firm but also in order to strengthen her supplier-base and benefit from suppliers' performances improvement, she should not insist on increasing too much payment terms.

5.6.2 Trend

In this paragraph, we extend the simulation and study the impact of RF on the same firm facing a systematically increasing demand. The stochastic demand in each period is normally distributed (truncated to avoid negative realizations). Let g be the demand growth rate (trend) and $CV = \sigma/\mu$ the coefficient of variation (or relative standard deviation) that measures its dispersion. For coherence, we assume a constant $CV = \overline{CV}$, i.e. the standard deviation σ increases proportionally with the mean μ. The demand D_t in a general period t is therefore normally distributed with $\mu_t = \mu_{t-1} \cdot (1+g)$ and $\sigma_t = \overline{CV} \cdot \mu_t$.

The myopic single-period optimal policy S is a function of demand and it evolves from period to period, accordingly with μ_t and σ_t.

We define $S_t = F_t^{-1}(\cdot)$ for $t = 1,2,\dots,T$ the new optimal policies for the base-case model, in absence of external financing. Similarly, the optimal policies for the RF and CL models, are obtained applying a different $F_t^{-1}(\cdot)$ in each period.

Simulation

In this section, we discuss the results of the simulation in presence of demand growth. All the parameters are unchanged from the base-case, while growth is $g = 1{,}66\%$ (about 20% per year). The coefficient of variation is $\overline{CV} = 0{,}3$ for every period, so $\mu_t = (1 + 0{,}0166) \cdot \mu_{t-1}$ and $\sigma_t = (1 + 0{,}0166) \cdot \sigma_{t-1} = \overline{CV} \cdot \mu_t$. We assume $\mu_1 = 100$ and σ_1=30.

$S_{(2,2)}$ Scenario: No Payment Term Extension

		$S_{(2,2)}$				
		No Fin	CL	T-RF	F	RF
Profit	Average	361	417	412	401	431
	Std. dev.	20,54	44,59	47,22	47,61	47,00
	Δ Profit RF (%)	20,5%	2,8%	4,7%	7,9%	
C.I. ($\pi_{RF} - \pi_i$)	Inf.	68,00	6,70	11,81	25,20	
	Average	74,29	14,81	20,07	33,36	
	Sup.	80,58	22,93	28,33	41,53	
Demand Coverage (%)	Average	76,3%	94,6%	95,3%	95,2%	95,0%
	Std. Dev.	5,4%	2,7%	2,5%	2,6%	2,5%
Total Inventory	Average	1.026	1.450	1.479	1.479	1.463
	Std. Dev.	41,19	1,78	0,00	0,00	1,16

Table 5.4 : Supplier's average operational performance under different financing alternatives in $\boldsymbol{S_{(2,2)}}$ and in presence of demand growth.

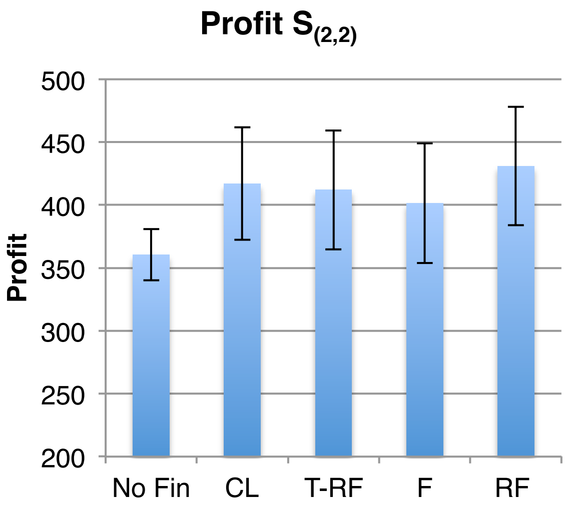


Figure 5.7: Supplier's average Profit for $S_{(2,2)}$ with demand growth.

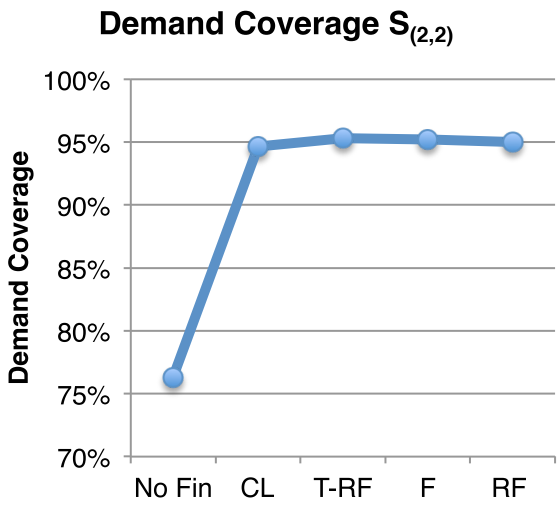


Figure 5.8: Supplier's average Demand Coverage for $S_{(2,2)}$ with demand growth.

In presence of a positive trend in the demand pattern it is natural to expect higher Profit for each financing alternative, since the overall higher level of demand allows an increase in sales. Under these conditions, more than in the base-case, the SME needs an access to financing in order to tap the demand growth and guarantee a sufficient service level. Compared against the base case, Demand Coverage in the non-financing alternative drops from 81,3% to 76,3% (–6,15%), stressing the restriction given by the working capital policy w'. The SME recurs to debt more often and the gap between the performances obtained with the CL and F narrows, as well as the gap between CL and T-RF. Demand growth compels the firm to increase the use of the credit line and factoring, reducing the penalty caused by the auto-discounting arrangement. Nevertheless, confidence intervals are positive and denote the higher performance of the SME recurring to RF.

$S_{(2,3)}$ Scenario: Payment Term Extension

		$S_{(2,3)}$				
		No Fin	CL	T-RF	F	RF (n=3)
Profit	Average	361	417	398	401	423
	Std. dev.	20,54	44,59	46,08	47,61	47,00
	Δ Profit RF (%)	17,5%	1,1%	6,8%	5,4%	
C.I. $(\pi_{RF} - \pi_i)$	Inf.	57,49	-7,50	14,39	10,60	
	Average	61,66	0,31	22,61	18,86	
	Sup.	65,82	8,12	30,84	27,12	
Demand Coverage (%)	Average	76,3%	94,6%	95,1%	95,1%	94,8%
	Std. Dev.	5,4%	2,7%	2,5%	2,6%	3,0%
Total Inventory	Average	1.026	1.450	1.477	1.479	1.457
	Std. Dev.	41,19	1,78	0,00	0,00	1,50

Table 5.5: Supplier's average operational performance under different financing alternatives in $S_{(2,3)}$ and in presence of demand growth.

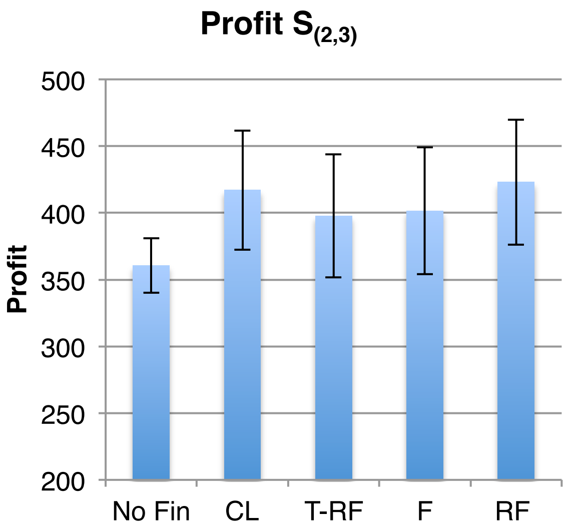


Figure 5.9: Supplier's average Profit for $S_{(2,3)}$ with demand growth.

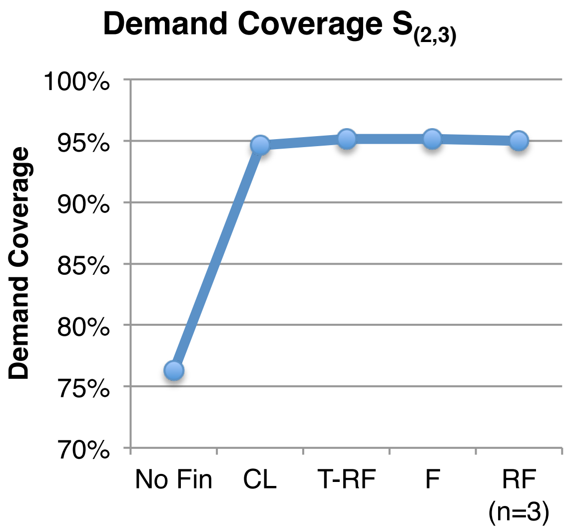


Figure 5.10: Supplier's average Demand Coverage for $S_{(2,3)}$ with demand growth.

As in the base case, the performance obtained in absence of financing, with the CL and F remain unvaried, since these financing means are not involved by the payment term extension. T-RF and F lose respectively 3,5% and 1,9% in terms of Profit, while their demand coverage is not affected by the new scenario; i.e., the buyer is better off, since it has higher days payable outstanding and is guaranteed the same service level. On the other hand, the supplier's convenience of adopting RF is lower and can discourage him to participate, especially if:

i. RF entails higher administrative costs or a relevant initial investment;
ii. He has access to a credit line, which allows to achieve a profit lower just by 1,1% on average and whose confidence interval includes negative values.

5.6.3 Seasonality

In this paragraph we adapt our model to a generic seasonal demand pattern, with positive and negative peaks, and draw insights about its effect on the profit of the firm. Let N be the number of seasons, and θ_i for $i = 1,2, \dots, N$ the coefficients of seasonality, where N is a sub-multiple of T. Once again we assume a constant coefficient of variation $CV = \overline{CV}$ and we choose the coefficients of seasonality θ_i so that $\frac{\sum_{i=1}^{N} \theta_i}{N} = 1$, i.e. the average demand over T is μ. The demand D_t in a general period t is therefore normally distributed with $\mu_t = \theta_i \cdot \mu$ and $\sigma_t = \mu_t \cdot \overline{CV} = \theta_i \cdot \sigma$.

We define the new optimal policies $S_t = F_t^{-1}(\cdot)$ for $t = 1,2, \dots, N$ for the base-case model, in absence of external financing. Similarly, the optimal policies for the RF, T-RF, F and CL models are obtained applying a different $F_t^{-1}(\cdot)$ in each period $t = 1,2, \dots, N$. For example, when $N = 6$ and $T = 12$, the phenomenon is characterized by 6 seasons within T, lasting two periods each; in this case D_t and D_{t+6} share the same coefficient θ_i and are equally distributed.

Simulation

All the parameters are unchanged from the base-case. We assume $N = 6$ seasons, so we have two cycles of seasonality per year. The coefficient of variation is $\overline{CV} = 0{,}3$ for every period and we hypothesize the coefficient of seasonality θ_i as:

$\theta_1 = 0{,}448$ $\theta_4 = 2{,}69$

$\theta_2 = 0{,}538$ $\theta_5 = 0{,}897$

$\theta_3 = 0{,}897$ $\theta_6 = 0{,}538$

$S_{(2,2)}$ Scenario: No Payment Term Extension

		$S_{(2,2)}$				
		No Fin	CL	T-RF	F	RF
Profit	Average	283	363	363	348	383
	Std. dev.	18,59	52,00	40,59	38,21	42,00
	Δ Profit RF (%)	40,0%	5,2%	5,5%	9,3%	
C.I. ($\pi_{RF} - \pi_i$)	Inf.	93,87	9,80	13,60	28,40	
	Average	99,32	17,50	20,69	35,08	
	Sup.	104,77	25,21	27,78	41,77	
Demand Coverage (%)	Average	80,7%	92,0%	93,3%	93,3%	94,9%
	Std. Dev.	3,8%	4,0%	3,0%	2,9%	2,5%
Total Inventory	Average	855	1.271	1.208	1.198	1.364
	Std. Dev.	24,56	70,63	16,36	19,53	53,35

Table 5.6: Supplier's average operational performance under different financing alternatives in $S_{(2,2)}$ and in presence of seasonality.

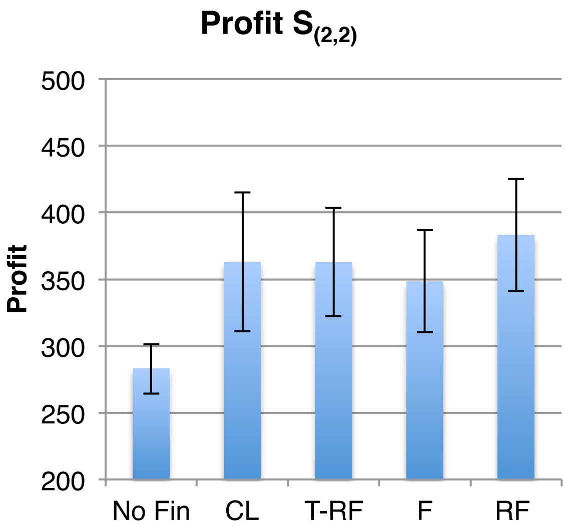


Figure 5.11: Supplier's average Profit for $S_{(2,2)}$ with seasonal demand.

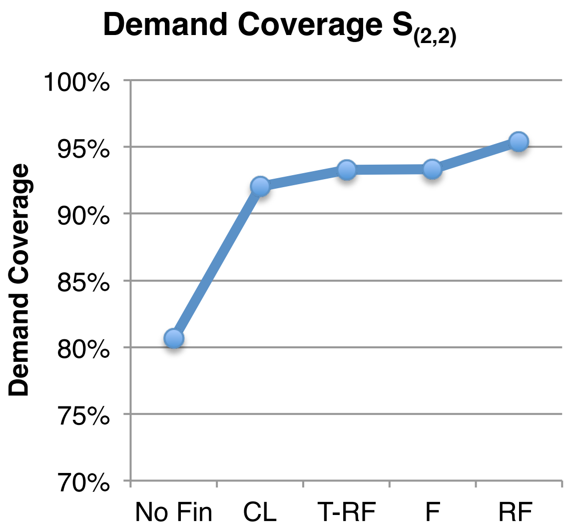


Figure 5.12. Supplier's Demand Coverage for $S_{(2,2)}$ with seasonal demand.

As we can notice, in this scenario RF significantly outperforms the other solutions: an outstanding improvement from the base-case (Mean Profit +35%, average ΔProfit +40%) and a good one from the other financing means (Mean Profit may increase in one year from 5,6% to 9,9%). A second interesting result regards Demand Coverage: this is the only scenario in which Reverse Factoring overcomes not only No-Financing and Credit Lines, but also Traditional Factoring and Automatic-RF. The reason why firms flourish particularly with RF in such a scenario is deducible: not only a lower interest rate, but also the possibility to choose whether to discount invoices or not, have a unique value when the demand (and therefore the inventory replenish decision) changes significantly from period to period. This case requires more often to collect cash due to the demand-boost periods, when demand can be as much as five times higher than the previous period, making it difficult for the firm to accumulate proper inventory just using operating cash flows: the more cash is needed, the better perform low-interest rate solutions (therefore RF outperforms CL). Moreover, when the firm faces demand high peaks, being able to choose whether and how much of each invoice to discount becomes even more relevant (therefore RF outperforms Auto-RF and TF). These insights are confirmed by the confidence intervals, which are all largely positive.

$S_{(2,3)}$ Scenario: Payment Term Extension

		$S_{(2,3)}$				
		No Fin	CL	T-RF	F	RF (n=3)
Profit	Average	283	363	345	348	375
	Std. dev.	18,59	52,00	37,02	38,21	42,00
	Δ Profit RF (%)	32,3%	3,5%	8,9%	7,4%	
C.I. $(\pi_{RF} - \pi_i)$	Inf.	86,17	3,21	23,43	19,41	
	Average	91,96	11,69	30,36	26,44	
	Sup.	97,75	20,16	37,29	33,48	
Demand Coverage (%)	Average	80,7%	92,0%	93,1%	93,3%	94,0%
	Std. Dev.	3,8%	4,0%	3,0%	2,9%	3,0%
Total Inventory	Average	855	1.271	1.195	1.198	1.216
	Std. Dev.	24,56	70,63	18,00	19,53	22,40

Table 5.7: Supplier's average operational performance under different financing alternatives in $S_{(2,3)}$ and in presence of seasonality.

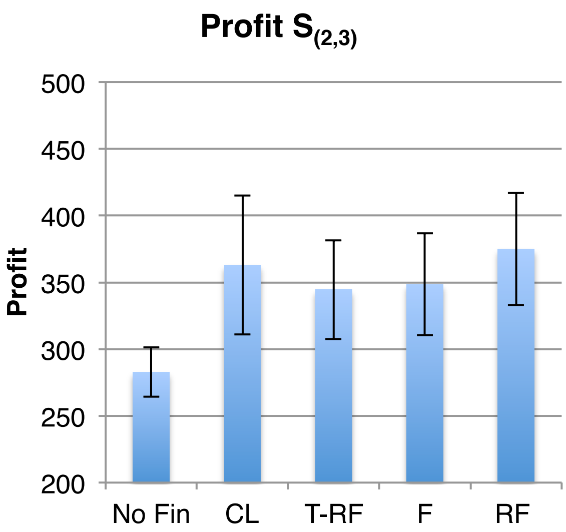


Figure 5.13: Supplier's average Profit for $S_{(2,3)}$ with seasonal demand.

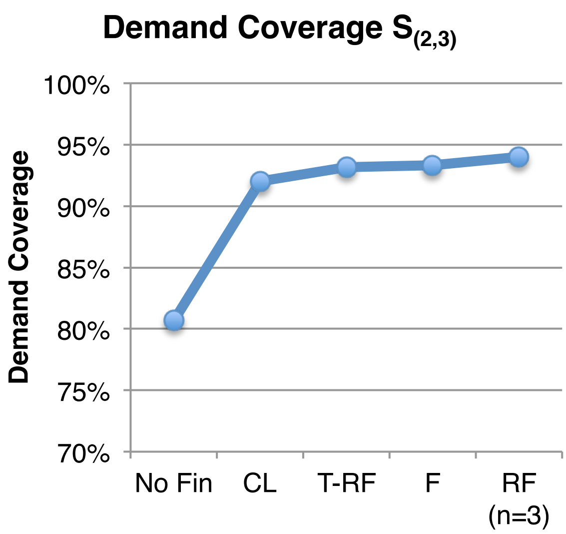


Figure 5.14: Supplier's Demand Coverage for $S_{(2,3)}$ with seasonal demand.

As in the previous case, also here RF outperforms the other financing solutions in both mean profit and demand coverage, even though the differences between the other financing tools' performances and the RF's decrease due to the payment extension. All the of the confidence intervals are positive, denoting the convenience of RF adoption compared to the other financing means, as it provides a better result in terms of profit at a 95% confidence level.

5.6.4 Focus: Is it always convenient for buyers to extend payment terms?

The model presented in this chapter has been totally focused on supplier's benefits and his possible creation of value. We also run a sensitivity analysis over the payment extension, since literature and real cases already acknowledged that buyers are often urging for lengthening their DPO in return of offering RF. We already know from the model's results that payment extension reduces supplier's benefit. In this paragraph, taking the buyer's perspective, we try to argue whether payment extension really represents a favorable option. We only consider the case that compares RF to a non-financing situation.

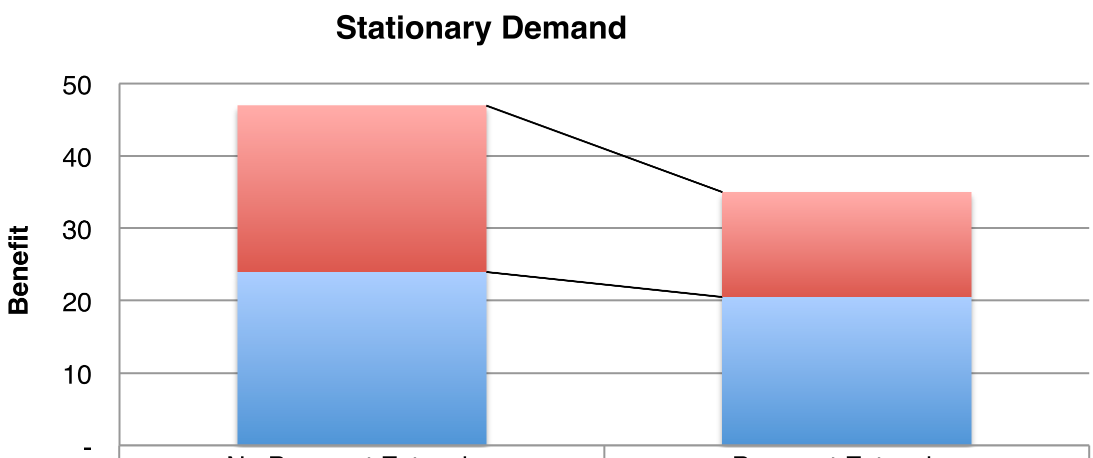
Assume that the buyer, in order involve her supplier in a RF program, demands to share the benefits generated (e.g. through a discount on the next year purchases). Consequently, the supplier's delta-profit coming up in the next year is split with the buyer. We assume this benefit sharing equally distributed. We finally assume that the buyer makes a margin of $\varepsilon = 0{,}4$ for each unit purchased, worked and sold. We leave the rest of the assumptions as section 5.6, which allows to extract the data from the former simulations.
Consequently, each actor's benefits are defined as follows:

$$Supplier's\ Benefit = 0{,}5 \cdot \Delta\pi$$

$$Buyer's\ Benefit\ (No\ payment\ extension) = \ 0{,}5 \cdot \Delta\pi + \ \varepsilon \cdot D \cdot \Delta DemandCoverage$$

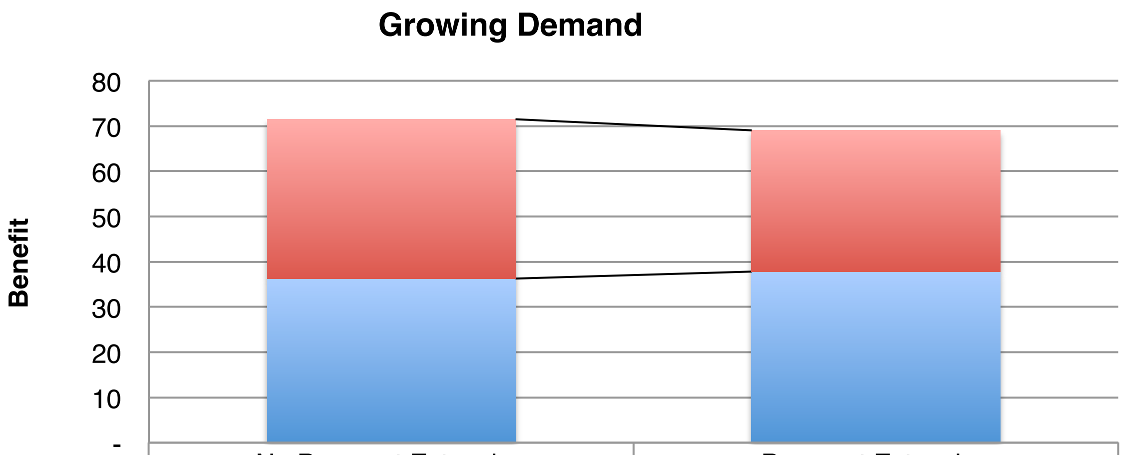
$$Buyer's\ Benefit\ (Payment\ extension) = 0{,}5 \cdot \Delta\pi + c \cdot D \cdot r_b \cdot DPO_{extension}$$

As already noticed in the simulation sensitivity analysis (section 5.6), when there is no payment extension not only supplier's profit is higher, but also his demand coverage. Consequently, the buyer should consider that extending the payment term would jeopardize the service level received. We quantified this service level drop in the unit-contribution margin times the delta-units the buyer would not find available and then sell if she extends payments (contribution margin loss due to service level decrease). On the other hand, when DPO are extended and thus WCR decreases, many advantages may arise: principally debt reduction or alternative investments. For simplicity's sake, we assume buyer will reduce her debt and therefore financing costs (to better understand the assumption and the formula used, see Chapter 3). We run this analysis in the three different demand scenarios.



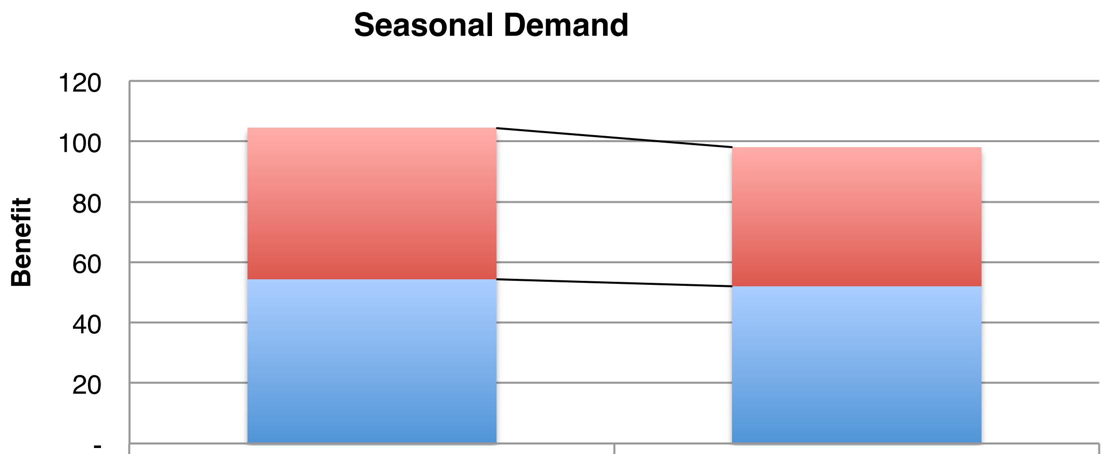
	No Payment Extension	Payment Extension
Supplier Benefit	23	14,5
Buyer Benefit	23,96	20,5

Figure 5.15: Benefit sharing between buyer and supplier when demand is stationary.



	No Payment Extension	Payment Extension
Supplier Benefit	35,22	31,22
Buyer Benefit	36,28	37,82

Figure 5.16: Benefit sharing between buyer and supplier when demand is growing.



	No Payment Extension	Payment Extension
Supplier Benefit	50,03	46,03
Buyer Benefit	54,35	52,03

Figure 5.17: Benefit sharing between buyer and supplier when demand is seasonal.

We notice that in two out of the three cases, stationary and seasonal demand, the buyer would be better off keeping DPO unvaried, while in the growing demand case the buyer should extend them. In the first case, stationary demand, the buyer's benefit with no DPO extension overcomes the alternative solution because of the huge gap standing in supplier's benefit. As already observed, indeed, the payment extension has the highest impact in the stationary demand scenario: since the buyer is now sharing this profit increase, it is convenient for her to keep it as high as possible. In the growing demand scenario, the profit does not decrease so significantly when payments are extended, while the buyer's financing cost savings are increasing since the volume purchased is higher. We may notice that in both scenarios the delta-service level does not have a considerable impact. On the contrary, in a seasonal demand context with high peaks, the service level does change considerably, and the contribution margin lost makes the buyer better off by not extending DPOs.

We may notice as well how in all three cases the value created for the whole supply chain is always higher without extending payments, meaning that even when for the buyer is convenient to demand DPO extension, her delta-benefit would be lower than supplier's loss of value. In a collaborative supply chain context, this should be closely examined.

This analysis does not presume to give a definite insight on the payment extension issue. However, it confirms what in the non-academic literature has already been stated: extending DPOs, sometimes, does not just make suppliers worse off, but also buyers, since the latter would not be able to fully benefit the value created by RF.

5.6.5 Cash initial conditions

From the previous analysis on demand's patterns and DPO extensions, we can conclude that, overall, RF represents a better-off alternative in respect to Traditional RF and Factoring, but not always when compared to Credit Lines. Particularly, when RF is combined with payment extensions, Credit Line might be a better solution. We show here what happens when altering the initial cash level. Since RF loses competitiveness when payments are extended, $S_{(2,3)}$, we take that scenario as base-case.

	Stationary				
CASH Base Case	No Fin	CL	T-RF	F	RF
Profit ($)	351	384	360	364	388
Profit Std. Dev. ($)	24,50	40,77	41,81	43,43	42,24
Δ Profit RF (%)	9,8%	0,5%	7,3%	5,9%	
Demand Coverage (%)	81,4%	94,8%	95,3%	95,2%	94,9%
Demand Coverage Std. Dev. (%)	5,3%	2,6%	2,4%	2,5%	2,6%
CASH (-50%)	No Fin	CL	T-RF	F	RF
Profit ($)	281	348	360	364	382
Profit Std. Dev. ($)	12,00	40,00	41,81	43,43	44
Δ Profit RF (%)	36,7%	9,0%	6,0%	4,7%	
Demand Coverage (%)	64,6%	93,0%	95,3%	95,2%	94,6%
Demand Coverage Std. Dev. (%)	5,3%	3,6%	2,4%	2,5%	3,0%
CASH (+50%)	No Fin	CL	T-RF	F	RF
Profit ($)	392	392	360	364	392
Profit Std. Dev. ($)	39,00	45,00	41,81	43,43	42
Δ Profit RF (%)	-0,5%	-0,5%	8,3%	6,9%	
Demand Coverage (%)	92,7%	95,0%	95,3%	95,2%	95,0%
Demand Coverage Std. Dev. (%)	4,0%	3,6%	2,4%	2,5%	3,0%

Table 5.8: Sensitivity analyses on initial cash level in $S_{(2,3)}$ with stationary demand.

The initial cash sensitivity analysis under stationary demand (Table 5.8) offers several interesting insights. First, we may observe that when the supplier adopts Traditional RF or Factoring, facing a regular and stationary demand, it does not matter how high is the initial cash level, the ending profit remains the same: in fact, since these agreements imply an automatic discounting, no matter the real need of cash, the replenishment decisions do not vary. In addition, this scenario has no peaks in demand, so the firm, discounting all the invoices, has always enough cash to reach the optimal order-quantity. The situation changes for those SCF tools that make a more efficient use of cash: Credit Line and RF. When the initial cash decreases, Credit Line's performance drops because the maximum Credit Limit decreases as well, since we assumed it directly linked to the inventories, Accounts Receivable and initial cash. Having less capacity of collecting cash, the Credit Line is not able to buy the same inventories it used to, so the demand coverage drops and with it the overall profit. RF's performance worsens because with less initial cash there is more need to discount, and therefore more interests: however, the demand coverage does not vary significantly, meaning that the firm was able to buy the same inventory it used to, just paying more interests. The auto-financed firm's retained profit obviously falls down, since initial cash represents the only resource available to them to build up inventory. On the other hand, when initial cash redoubles, Credit Line, RF and Auto-financed firms performances rise sensibly and they end up performing au pair: the very high level of initial cash is enough (for a single year) to smooth demand's variability and to buy the inventory necessary, without the need of SCF tools. Symmetrically, the gap between these solutions and the auto-discounting solutions widens, because the latter ones keep discounting without the need of cash, paying useless interests.

	Trend				
CASH Base Case	No Fin	CL	T-RF	F	RF
Profit ($)	361	417	398	401	423
Profit Std. Dev. ($)	20,54	44,59	46,08	47,61	47
Δ Profit RF (%)	17,5%	2,0%	6,8%	5,4%	
Demand Coverage (%)	76,3%	94,6%	95,1%	95,1%	95,0%
Demand Coverage Std. Dev. (%)	5,4%	2,7%	2,5%	2,6%	3,0%
CASH (-50%)	No Fin	CL	T-RF	F	RF
Profit ($)	282	391	398	401	417
Profit Std. Dev. ($)	9,00	35,00	47,00	47,61	46
Δ Profit RF (%)	47,6%	7,1%	5,7%	4,3%	
Demand Coverage (%)	59,8%	88,9%	95,1%	95,1%	95,0%
Demand Coverage Std. Dev. (%)	5,4%	4,4%	2,5%	2,6%	3,0%
CASH (+50%)	No Fin	CL	T-RF	F	RF
Profit ($)	415	425	398	401	425
Profit Std. Dev. ($)	37,00	47,00	47,00	47,61	47
Δ Profit RF (%)	2,5%	0,0%	7,9%	6,3%	
Demand Coverage (%)	88,6%	95,0%	95,1%	95,1%	95,0%
Demand Coverage Std. Dev. (%)	4,6%	2,5%	2,5%	2,6%	2,5%

Table 5.9: Sensitivity analyses on initial cash level in $S_{(2,3)}$ with growing demand.

The trend scenario (Table 5.9) has an output similar to the precedent. Traditional RF and Factoring still leads to the same profit even though initial cash varies. The reason is that the demand, even if growing, does never reach levels that prevent the firm from making available the optimal inventory. RF outperforms sensibly the no-financing solution and the Credit Line. Notice that even though the demand is higher than the previous one, therefore apparently more difficult to cover, the gap between RF and Credit Line narrows instead of widening. The reason is that Credit Line already used in the previous scenario the whole cash capacity available, and so does in this case: the differential financing costs are therefore null, while the revenues grow. On the other hand, RF's revenues grow as well as the interests in order to maintain the same demand coverage: consequently, the profit grows not proportionally and this is why the gap shrinks.

Seasonal					
CASH Base Case	No Fin	CL	T-RF	F	RF
Profit ($)	283	363	345	348	375
Profit Std. Dev. ($)	18,59	52,00	37,02	38,21	42
Δ Profit RF (%)	32,3%	3,5%	8,9%	7,4%	
Demand Coverage (%)	80,7%	92,0%	93,1%	93,3%	94,0%
Demand Coverage Std. Dev. (%)	3,8%	4,0%	3,0%	2,9%	3,0%

CASH (-50%)	No Fin	CL	T-RF	F	RF
Profit ($)	238	313	312	317	342
Profit Std. Dev. ($)	19,00	23,00	30,00	30,00	31
Δ Profit RF (%)	42,6%	11,2%	8,9%	7,4%	
Demand Coverage (%)	75,4%	84,6%	90,0%	90,0%	90,0%
Demand Coverage Std. Dev. (%)	3,0%	4,2%	3,0%	2,9%	3,0%

CASH (+50%)	No Fin	CL	T-RF	F	RF
Profit ($)	328	375	354	363	383
Profit Std. Dev. ($)	23,00	59,00	53,00	53,00	63
Δ Profit RF (%)	16,9%	2,0%	8,1%	6,6%	
Demand Coverage (%)	84,5%	95,3%	94,9%	95,0%	95,0%
Demand Coverage Std. Dev. (%)	3,0%	2,5%	3,0%	3,0%	3,0%

Table 5.10: Sensitivity analyses on initial cash level in $S_{(2,3)}$ with seasonal demand.

The seasonal scenario (Table 5.10), which has already been proved to be the most favorable for RF, shows that a decrease in the initial cash condition would widen the gap between Credit Lines and RF, for the same reasons stated previously, i.e. a decrease in financing capacity for CL corresponds to a decrease in its profit. The gap between RF and traditional RF or Factoring remains the same even when cash decreases, since both the solutions are equally jeopardized from a drop in the initial cash. Starting with a re-doubled cash, instead, would lead CL and No-Financing cases to resemble RF, but the latter still remains superior because of its capability to cover also the highest peaks of the demand.

5.7 Conclusions

The model here illustrated proposes several insights. First, it shows that RF performs better than certain SCF solutions, like Factoring, thanks to a cheaper interest rate. Then it shows that RF outperforms other solutions, like Traditional RF, thanks to the possibility for the supplier to choose, invoice by invoice, how much to discount: thus, the supplier is able to obtain the cheapest amount of money exactly when he needs it, in the exact amount he requires. It also shows that RF is not always a better off solution, especially when compared to a low-interest Credit Line: when buyers, in order to implement a RF agreement, require their suppliers to lengthen the payment delay, the latter ones might be better off refusing the RF contract and continuing to use the Credit Line to finance their operations. Once again, our model suggests that the DPO increase should not be an automatic request from buyers when they implement this SCF solution: sometimes, in fact, when they bear too much upon payment delays, instead of strengthening their supply-base, they weaken it. Another interesting insight given by the model concerns the scenarios in which RF performs better and represents a significant worthwhile solution. In a demand-growing scenario, as already seen in the second model, RF creates more value than in presence of stationary demand, and outperforms most of the other existing solutions: however, in this kind of scenarios credit lines are as much as valuable, especially when RF entails a payment lengthening. On the other hand, when a firm faces a strong seasonality (and variability) in the demand pattern, RF reveals to be a winning solution, even when DPO increase is implied. The reason lies in the flexibility that a tool such as RF is able to offer: in a variable context as the seasonal-demand's one, RF allows a company to reach very high levels of working capital when needed and to decrease it immediately later. An interest insight is given in the DPO extension analysis that takes in consideration buyers' perspective and shows how it might be convenient for them not to demand for payments lengthening. It is also shown how the initial cash conditions have a strong impact on the benefits that RF may generate in respect to solutions as Credit Line and No-Financing. At the same time, it is proved that the value added of Traditional RF and Factoring does not depend on the initial cash conditions, because of the rigidity of auto-discounting agreements. Finally, the model mathematically shows how different financing solutions may bring to different operational decisions as the base-stock replenishment level: each extension presents a diverse profit function that leads to diverse decisions. The model suggests therefore that finance and operations are not two separated functions, but they constantly condition each other.

6 Case Studies

Despite the insights obtained by the numerical analysis, we test our last model with two real cases, in order to evaluate the actual benefit that RF could bring to a company that faces a stochastic demand and whose characteristics reflect the assumptions of the model.

This chapter illustrates the results of the application of the model to two Italian SMEs operating in different business areas: Personal Protective Equipment (PPE) and mechanical manufacturing. In particular, the first company produces and commercializes PPE and promotional clothing, while the second manufactures hydraulic cylinders.

A description of the industries and their respective supply chains introduces the analysis of the results obtained through the simulation.

6.1 Personal Protective Equipment (PPE)

Personal Protective Equipment (PPE) is used as temporary (until more effective hazard control techniques can be used) or last line of protection for workers against hazards. The PPE used for a certain task depends on the work environment, the work conditions (machine and tools) and the process being performed. Each PPE is appropriate for a specific use and may not be effective in protecting the employee for another use; for example, thick natural rubber gloves will protect the wearer from strong solutions of sodium hypochlorite (bleach) for an 8 hour working day, but not from ammonia hydroxide.

PPE should always come as the last resource, i.e. after engineering, work practice and administrative risk control strategies have been implemented to the greatest extent possible (Mayer et al. 2007, Berry et al. 2008). PPE can be divided into six major categories:

- Eye and face;
- Head;
- Foot and leg;
- Hand and arm;
- Body;
- Hearing.

The types of hazards they can help reduce exposure to are numerous: mechanical, chemical, biological, thermal, electrical, noise, radiations, etc. Most situations generally involve a combination of several types of hazards, thus requiring a multi-risk approach. In addition to protection requirements, functionality and comfort must be carefully considered during PPE selection since it controls the impact of PPE on task performance and even the wearer's health. Unsatisfactory PPE functionality and comfort may also lead people to decide not to wear necessary PPE.

Standards, such as the PPE Directive 89/686/EEC, cover protection parameters for the eye and face, respiratory, head, foot and electrical contact areas. They define legal obligations ensuring that PPE on the market give the highest level of protection against hazards: the CE marking affixed to PPE signals that. Figure 6.1 shows the actors involved organizational scheme and their contribute to the development of the directives.

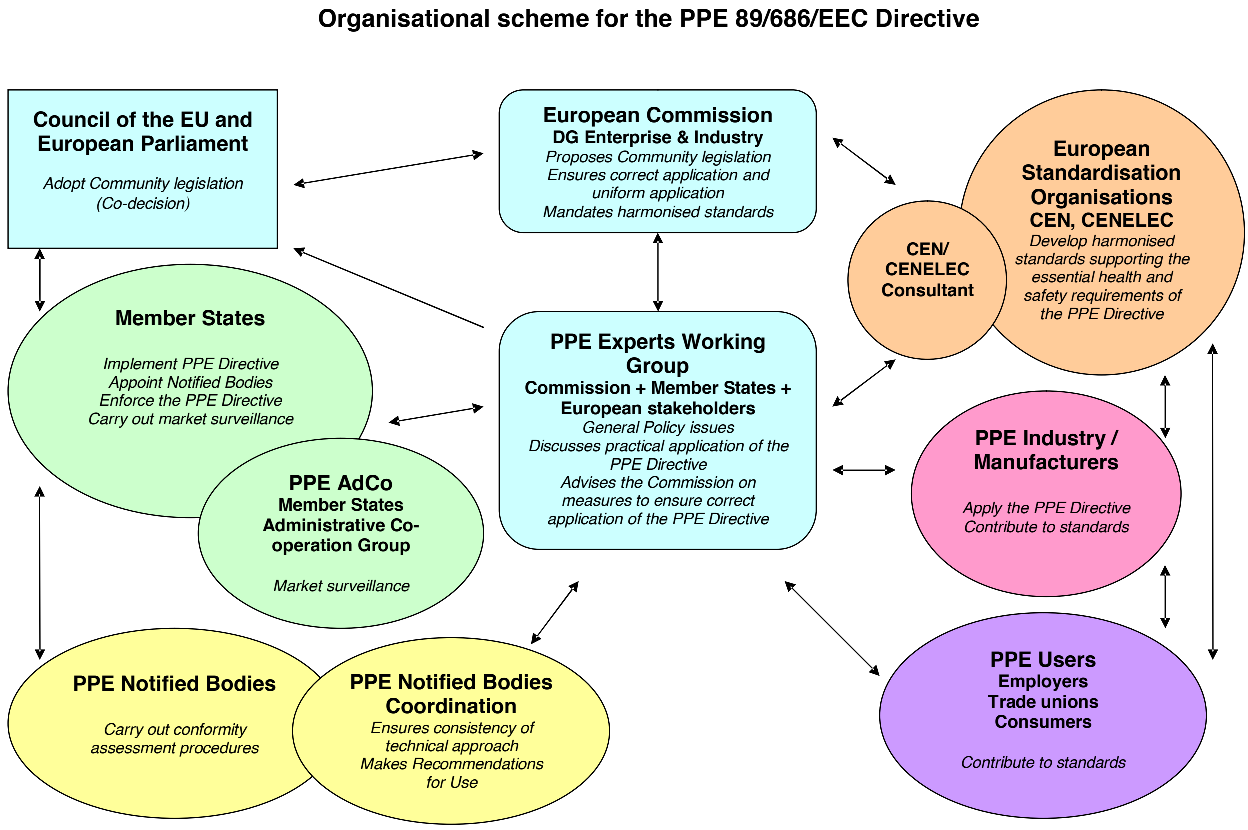


Figure 6.1: Organizational scheme for the PPE 89/686/ECC Directive.

6.1.1 Risk and Hazards

People are just one of the main components of a Working System (Di Giulio, 2014), while the remaining three are the Working Environment, the Working Procedures and the set of Machines and Tools. Hazards are defined as potential sources of lesions and/or damages to health in the Working System; consequently, an Hazardous Situation is any situation in which one or more employees are exposed to hazard.

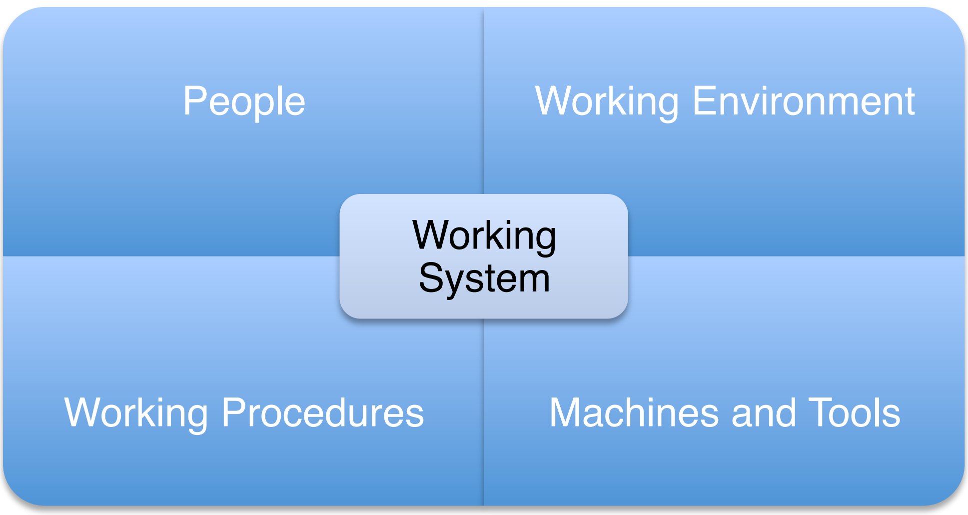


Figure 6.2: Representation of the elements that compose a Working System.

“Risk is a condition in which there is a possibility of an adverse deviation from a desired outcome that is expected or hoped for” (Vaughan, 1997).

Any risk can be expressed as the product between its likelihood L and impact I: $R = L \cdot I$. All the activities that aim to minimize the likelihood and impact of the risks fall under the name of risk reduction, through preventive and protective measures. Prevention and protection are procedures to manage those risks that are classified as “non acceptable” by the company, considering its risk appetite.

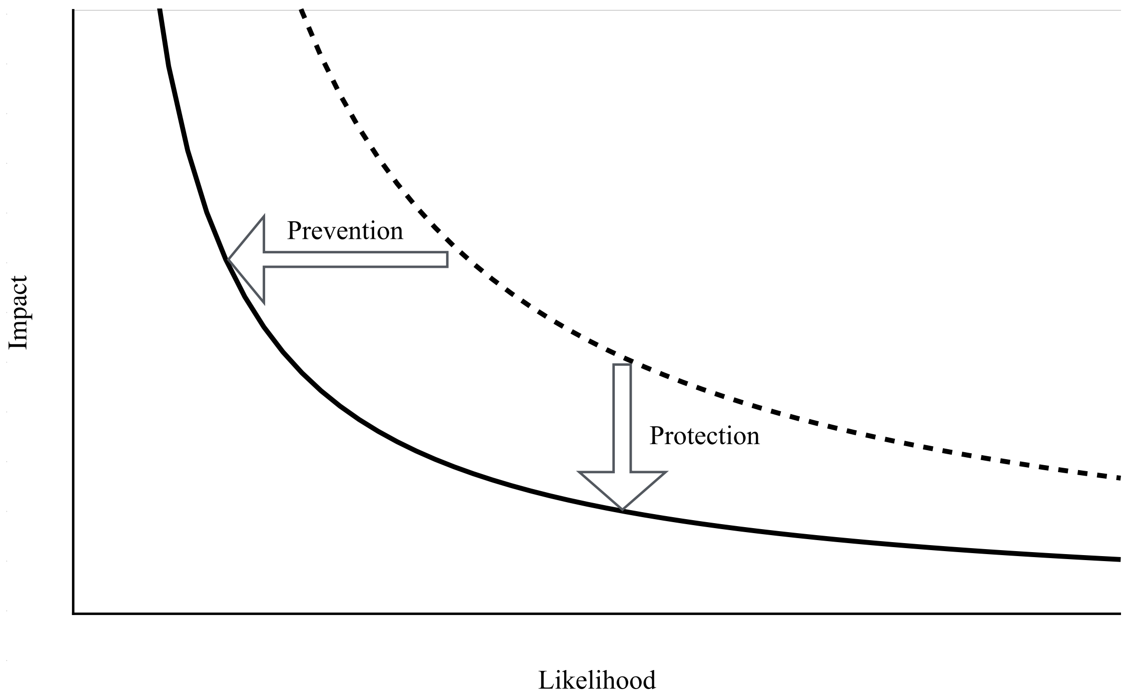


Figure 6.3: Risk reduction measures.

As shown in Figure 6.3, preventive measures alter the probability distribution of risky events, either reducing the likelihood of adverse events or increasing the likelihood of favorable scenarios. Protective measures, like PPE, only aim to reduce the impact of negative events, without any effect on their likelihood.

6.1.2 Trends and Market

The U.S. market for advanced personal protective equipment (PPE) was valued in 2005 at ~2.300 million USD per year (Business Communications Company, 2005). It is expected to reach 3.350 million USD by 2010, with an average annual growth rate (AAGR) of 7.9%. Within that, the textile industry takes a large share, e.g., with the advanced fire protective garments, expected to increase to 628 million USD by 2010, with an AAGR of 12%. In Western Europe, protective clothing and gloves account for ~60% of the total PPE market (Mäkien, 2006).

The PPE Industry relies on end-user segments for its revenues and was affected by the global crisis in 2008. Nevertheless, the industry is showing resilience and is poised for a turnaround, since workplace safety cannot be compromised by economic slowdowns.

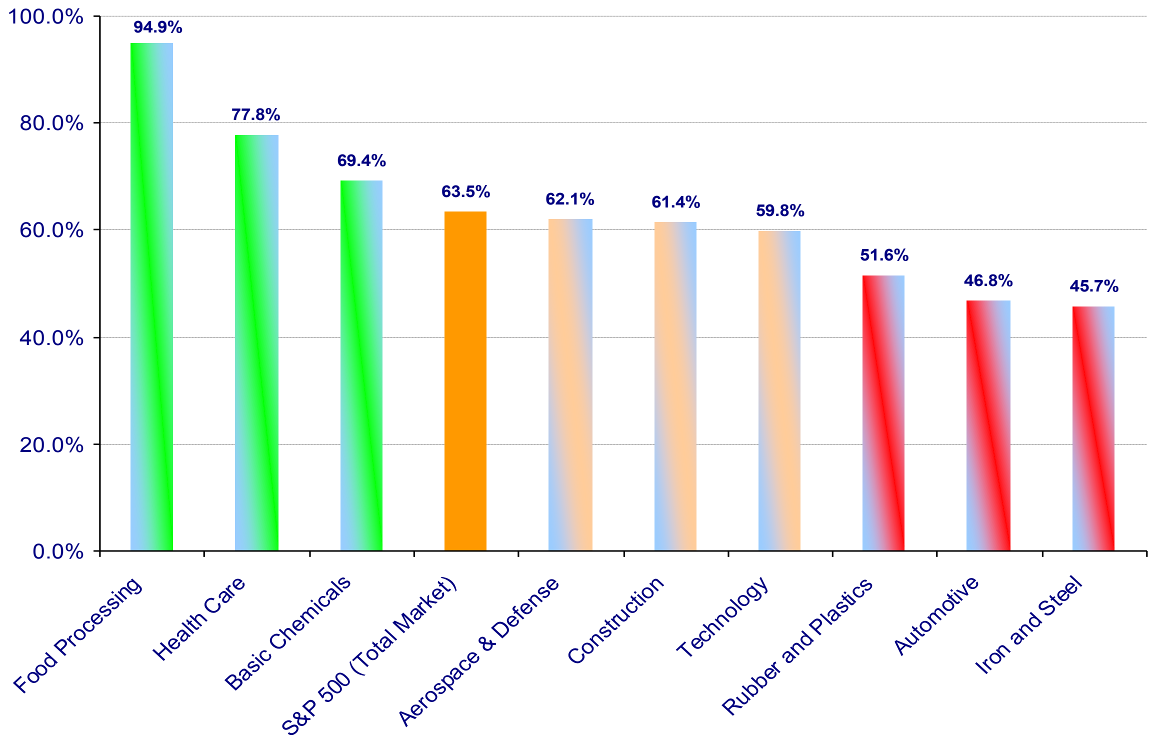


Figure 6.4: The Relative Impact of the Economic Downturn on Specific Industries, values at the beginning of 2009 compared to the previous year. Source: Frost & Sullivan/Google Finance.

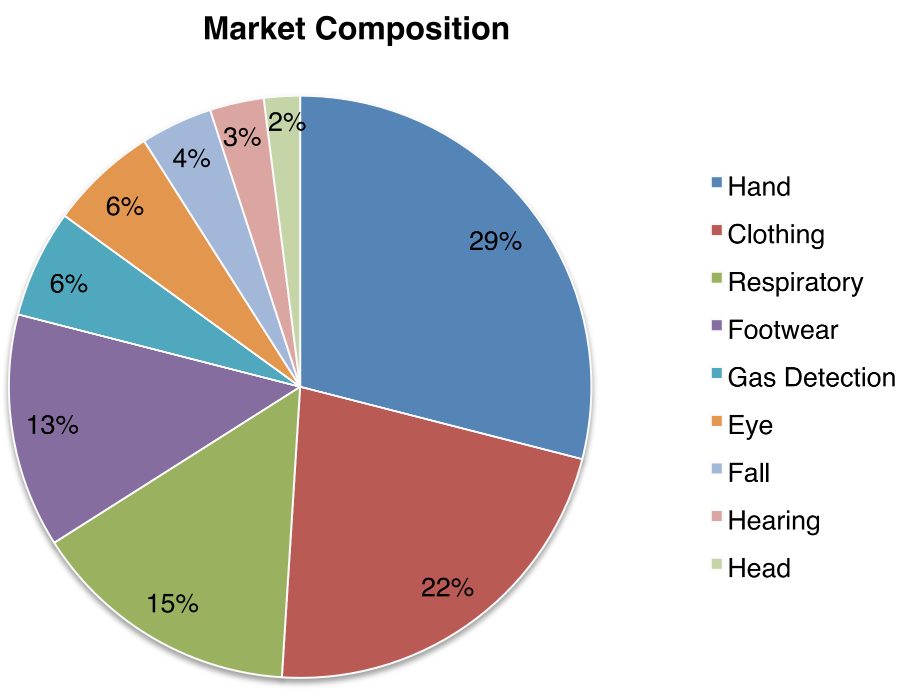


Figure 6.5: Global PPE Market Composition Estimates, 2008. Source: Frost & Sullivan.

Half of the categories account for 79% of the market composition, since they are adopted for most of the tasks in the majority of industries to protect the employees from mechanical, chemical, biological, thermal, and respiratory hazards. A survey of 298 safety professionals in 10 countries representing the Americas, Europe and Asia, conducted by 3M (2012), confirms that these categories are the most popular PPE products. The same study found that the use of PPE varies widely by country, with geographical differences in the specific types of PPE that are used.

Growth Drivers in the PPE Industry	
Positive Effect	Negative Effect
• Niche applications	• Economic downturn
• Alternative distribution channels	• Increased automation and outsourcing in manufacturing facilities
• Product Innovation	• Cost conservation measures (lower replacement rates)
• Increased awareness in the workplace	• Price pressure from generic PPE products
• Geographical expansion in areas with low compliance and penetration	

Table 6.1: Growth Drivers and Restrainers for the PPE industry. Source: Frost & Sullivan.

Table 6.1 shows the main growth drivers and restrainers for the PPE industry. Among the drivers, stand out the increased awareness in the workplace and niche applications. The former translates in more attention to safety, encouraging a greater use of PPE by workers, which not always wear the protection equipment. The latter offers opportunities for higher market penetration. The study by 3M also identifies China as the biggest potential market for PPE, thanks to the booming manufacturing sector and the relatively recent focus on safety. On the other hand, the most relevant restrainer is the economic downturn, which reduced the workforce in most industries, thus affecting negatively the demand for PPE. The increased automation equally affects the workforce, but its effect is less threatening.

Miller Pierce, safety marketing and communications experts, identified the most relevant trends, observing that safety is playing a more significant role in operations and being taken deeply into consideration in operational excellence. Increasingly, safety managers have specialized skill sets, including college degrees in safety, employee health and safety or industrial hygiene. ISHN (2001) also reports that in a recent research project, where more than 450 plant managers were interviewed from a cross section of all the key industrial markets, participants were asked “What are the top five ways you measure operational excellence?” The number one way the majority of participants measured their operations was via an analysis of safety records. Safety records were more important than training records, quality control reports and cost per unit measures. Another research unveiled that safety and plant managers seek durability and reliability rather than price reduction: they are willing to switch to more expensive PPE if the increased life time of each unit can be cost-justified. Manufacturers have to switch from the “cheap, use once and toss” mentality to a “good/durable” one.

Two strictly related capabilities have been identified as critical and potential market winners in the PPE industry: value-added services and Just-in-time supply. PPE products are widely perceived as a commodity and customers, in many cases, do not have the capabilities of storing and managing large amounts of inventory on location, especially when they own multiple plants. If PPE products are not available when needed, the facility could potentially have to shut down operations, costing hundreds of thousands of euros and, in absence of value-adding services, customers do not have any problem in switching to more reliable PPE suppliers. Continuous Replenishment Programs (CRP) include Just-in-time supplies and collaborative solutions such as VMI and consignment stock. These kinds of programs are extremely valuable for the customers that can reduce PPE stocks, reduce managing and administrative costs without renouncing to PPE availability, or even improving it. Extra services like usage controls, dispensing controls, electronic-real time record management and training are also perceived as a value-added. These instruments are used as vendor rating tools on the suppliers themselves and to monitor employees' behaviors, ensuring they use the required PPE but without wasting or losing them in the facility.

6.1.3 FAB S.p.A.

FAB S.p.A. is an SME (turnover in 2014 was €6.590.914) that produces and distributes protective equipment located in Paderno Franciacorta (Brescia, Italy). Over the last 25 years, it has become a reference point in the personal safety market, developing relations with important manufacturers operating in various industries. Its customer base includes:

- A2A S.p.A.
- Acciaieria Arvedi
- Acciaierie Valbruna S.p.A.
- ACI Global S.p.A.
- Arexons S.p.A.
- Bialetti Industrie S.p.A.
- Brembo
- Briko OK
- Ca' del Bosco S.p.A.
- Clariant Italia S.p.A.
- Duferco S.p.A.
- Enel Green Power S.p.A.
- Enel Produzione S.p.A.
- Fabbrica d'Armi P. Beretta S.p.A.
- Fincantieri S.p.A.
- Hera S.p.A.
- Italcementi S.p.A.
- KME Italy S.p.A.
- Leroy Merlin
- Lucchini S.p.A.
- Marcegaglia S.p.A.
- Metra S.p.A.
- Petronas Lubrificants S.p.A.
- Philip Morris Int. S.p.A.
- Pirelli S.p.A.
- Reno De Medici
- Same Deutx Fahr Italia S.p.A.
- Sapio
- Thyssenkrupp

FAB S.p.A. offers customization services in addition to a complete product range that covers:

- Protective Gloves;
- Protective Footwear;
- Protective Headwear and Masks;
- Work Clothing Flame;
- Fireproof Equipment;
- Airway Protection;
- Hearing Protection;
- Fall Prevention Equipment;
- Cleaning;
- Accessories;
- First Aid.

Supply Chain Description

FAB S.p.A. has a global sourcing strategy, mostly to exploit low cost skilled labor and raw materials. A relevant part of the spending is concentrated on wholesalers that import goods –either marked FAB or not– from India, China and Pakistan; some products –such as gloves– are outsourced and imported directly from the manufacturers, which realize the product and label it with FAB's logo. FAB is also a "gold distributor" of 3M, Ansell, Honeywell and Mapa, which are large players and sell their products directly to the company, without intermediaries. Micro enterprises and local artisans complete the supply base, providing special fabrics for niche products or general products in small batches; this channel is not as cheap as far eastern suppliers, but is more flexible and responsive. Downstream operations are local: FAB is physically present only in Paderno Franciacorta and sells its product mostly in Italy, due to regulations aspects regarding safety and health that differ from country to country. Approximately 1.500 firms operating in Italy account for 70% of sales, but Pareto's rule applies to the customer base: roughly one hundred medium companies and large corporations generate most of the company's turnover. Specialized retailers like Self, Briko OK account for nearly 25% of sales and FAB delivers directly to each of their point of sales located in Italy.

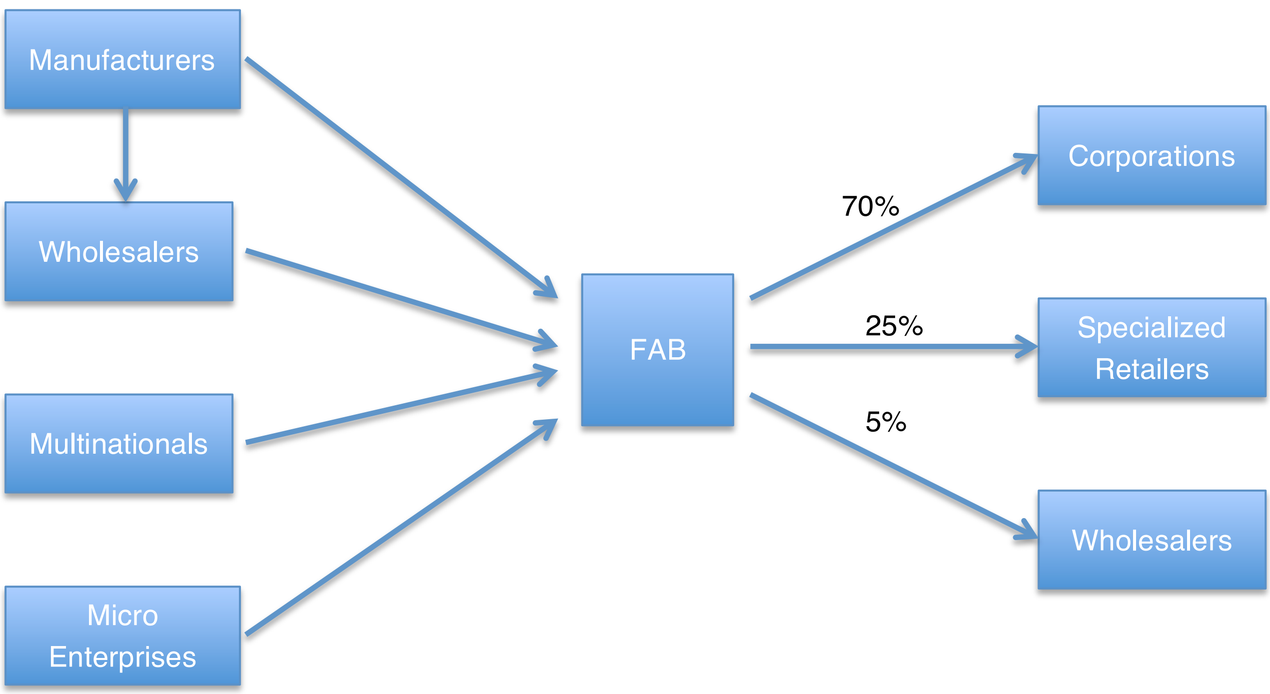


Figure 6.6: Graphic representation of FAB's supply chain.

The major sources of supply chain complexity lie in the vast product range, which covers more than 5.000 items, and in the supply base management. Firstly, the number of suppliers is medium-high and most of them are abroad. Secondly, strategic sourcing and sourcing are critical activities for FAB, which certificates its products and would be prosecuted for any flaw in the protective equipment, especially if it should be cause of any injury or death. Each product is classified with a protection level that ranges from I to III: gloves and generic clothing belong to the first category and protect employees from the majority of injuries; airway protection belongs to the second category, protecting workers from serious harms; fireproof equipment belongs to the third and most critical category, protecting people from deadly damages.

Upstream uncertainty is given by the current crisis that forces companies to decrease their working capital, often slashing stocks. As a consequence, purchasing lead times from wholesalers can easily exceed the canonic 10 days. Lead time for directly imported goods is around 3 months, including 25-35 days to physically ship the products. Downstream uncertainty is given by updates concerning regulations, seen as external factors. Besides the regulatory aspect, demand volatility is limited: from some products it is seasonal, but the average forecast error is low. Specialized retailers frequently offer promotions –averagely one per month– to their own customers, heavily stimulating demand of some products. These promotions are planned together with FAB two months before and thus do not represent any critical factor.

The overall supply chain is physically efficient, consistently with Fisher's (Fisher, 1997) classification of the products as "functional", with a particular focus on regulations-responsiveness.

Fisher Classification: Functional Products	
Demand Uncertainty	Low
Product Lifecycle	Long
Contribution Margin	Low
Product Variety (per category)	Medium-High
Average Forecast Error	Low
Average Stock-out Rate	Depending on wholesalers
End of Season Markdown	No

Table 6.2: Fisher classification of the products.

The company's processes are rather stable, since the internal processes and technologies are mature and contracts guarantee supplies it in the medium-long term. The resulting supply chain strategy is "Lean", as suggested by Hau Lee's framework (Lee, 2002), which is useful to describe the focus of supply chains. That is, FAB focuses on efficiency of total logistic cost through stock control and optimization techniques aiming at distribution and production capacity maximization. As already mentioned, an important focus is on the evolution of the regulatory environment, but this kind of changes is announced in advance.

Process Stability		Demand Variability: Low	Demand Variability: High
	High	*Lean Supply Chain*	Responsive Supply Chain
	Low	Risk Hedging Supply Chain	Agile Supply Chain

Figure 6.7: Hau-Lee's model for supply chain strategies applied to FAB.

6.1.4 Model Setting

The analysis is focused on the product GU2730#10,5, which is a pair of domestic flocked gloves and perfectly adapts to the assumptions of the model; i.e., these gloves are traded by FAB S.p.A. in a Make To Stock (MTS) strategy and are supplied each month to a big customer (client C3932), eligible for offering

Reverse Factoring. For the sake of privacy, the buyer's name cannot be mentioned, but it is a leader in the design, production and commercialization of accessories for automotive and industrial sectors. Founded in 1888 in the United States, it employs 15.000 workers in 80 countries and its turnover is $2,7 billion.

The purchasing cost of GU2730#10,5 is €0,4174 per pair and during 2014 FAB S.p.A. sold 48.900 pairs to C3932 at the selling price agreed: €0,6 per pair. FAB has a framework agreement with C3932, who is expected to order from a minimum of 3.000 to a maximum of 5.000 pairs of GU2730#10,5 each month; i.e. demand is uniformly distributed: $D_t = U(a = 3.000, b = 5.000)$. The payment is due 120 days after the last day of each month in which the order was placed; for example, if C3932 orders 4.000 pairs of GU2730#10,5 on the 6th of May, the invoice is due on September 30th. The estimated annual holding cost is 10% of the purchasing cost, that is €0,04174. The variable w', representing the inventory strategy, is set equal to the purchasing cost to take into account that the company has to bear other costs not considered in this model (e.g. depreciation and taxes).

In the simulation we compare the value of reverse factoring for the company with the use of a credit line, which is the most common financing mean for Italian SMEs. For completeness, the analysis also includes the results obtained in absence of any external financing mean in order to observe the importance of accessing to working capital financing for SMEs. We assume the line of credit to be equal to the maximum amount of short-term debt in the last three years, €2.830.290. While this figure refers to the whole company, the model focuses on the relationship with a single buyer and a single product. For this reason, we multiply that amount by the percentage of revenues generated by the relationship on the company's turnover, i.e. $\bar{O} = 0{,}437\% \cdot €2.830.290 = €12.367$. The annual interest rate is $r_s = 4\%$ and the interest rate paid on the unused portion of debt is $r_u = 1\%$, while the RF annual interest rate is $r_{RF} = 2\%$.

The company's cash is calculated as the average of the figures in the last balance sheet, i.e. $Cash = 0{,}5 \cdot [(Cash\ on\ Hand + Bank\ Deposit)_{2013} + (Cash\ on\ Hand + Bank\ Deposit)_{2012}] = €1.587$. Similarly to the credit line, we relate this amount to the specific relationship and calculate the initial cash $z'_0 = 0{,}437\% \cdot €1.587 = €6{,}93$. The initial inventory is set to zero.

6.1.5 Results Analysis

This paragraph illustrates the results obtained through 1.000 replications of the present scenario. As section 5.6, the indicators used to comment the findings are: Profit, ΔProfit Confidence Interval, Demand Coverage and Total Inventory.

		No Fin	CL	RF
Profit [€]	Average	6.361	8.367	8.519
	Std. dev.	62	357,46	357,71
	Δ Profit RF (%)	34,0%	1,8%	
C.I. (α=0,01)	Inf.	2.127,4	152,4	
	Average	2.156,2	152,7	
	Sup.	2.185,1	153,1	
Demand Coverage (%)	Average	74,6%	100,0%	100,0%
	Std. Dev.	3,2%	0,0%	0,0%
Total Inventory	Average	38.688	59.264	59.367
	Std. Dev.	1.323,00	4,58	12,95

Table 6.3: Results of the simulation over 1.000 runs.

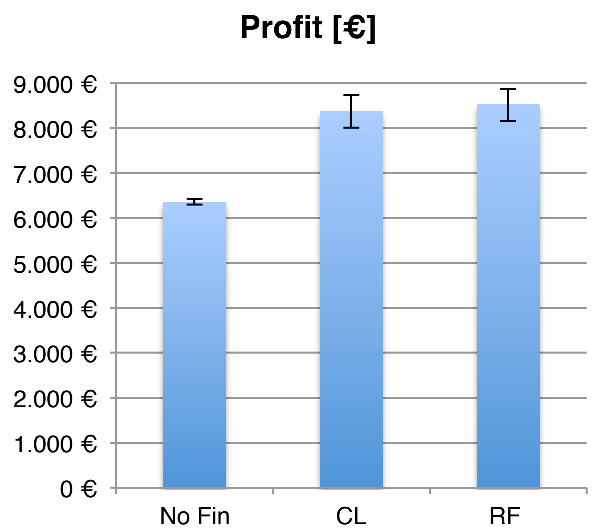


Figure 6.8: Average profit over 1.000 runs.

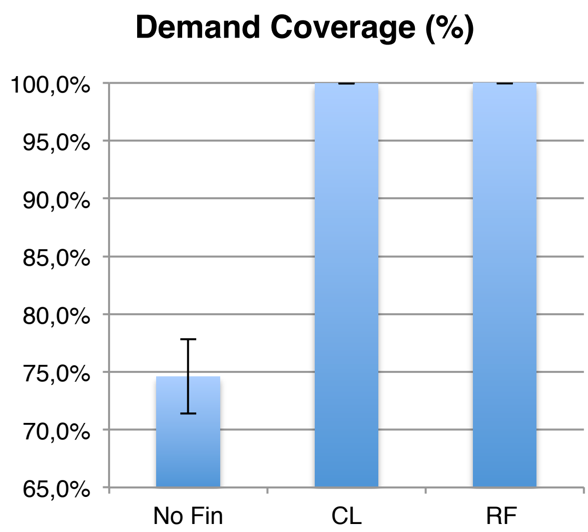


Figure 6.9: Average demand coverage over 1.000 runs.

As in the numerical analysis, the firm is unable to satisfy the demand in absence of financing, which costs 34% of profit compared to RF and –25,4% in service level to the buyer. The particularly low variability of profit and the relatively high variability of demand coverage is explained by the scarce replenishment capability: on average, the annual demand is 48.000 and FAB would be able to cover only 74,6%. When demand is higher, the firm cannot exploit it to increase sales and achieves the same profit as before, while demand coverage decreases. Similarly, when demand is relatively lower, demand coverage improves, but sales are not affected since the SME is still unable to cover the demand.

In this case study, Reverse Factoring yields a slightly higher profit (+1,8%) compared to the use of a Credit Line, thanks to its more flexibility and better conditions negotiated with the bank; on the other hand, the two financing means prove to be equivalent and well performing in terms of demand coverage. Table 6.4 provides some interesting additional information: it can be noted that the average utilization of financing means is low, meaning that an uncovered portion of demand is the outcome of the order policy, rather than the consequence of financial restrictions. For this reason, when demand is higher, the SME can exploit it to increase sales (and therefore profit) and access to financing allows fast replenishment, therefore maintaining demand coverage constant. Vice versa, lower demand leads to lower sales (and profit), while slightly increasing demand coverage. The fairly stable demand is also reflected by the utilization of RF, since most of the times (86,03%) the SME factors the most mature invoice and at least part of the second most mature one, while it is never necessary to factor more than two.

CL	Used Portion of the CL (on average)	€2.323 (18,78%)
	Not Used	0,02 (0,18%)
	Only the most mature invoice is factored (# of times)	1,65 (13,78%)
RF	The firm factors up to the 2^{ND} most mature invoice (# of times)	10,32 (86,03%)
	The firm factors up to the 2^{ND} least mature invoice (# of times)	0,00
	The firm factors up to the least mature invoice (# of times)	0,00

Table 6.4: Average utilization of financing means over 1.000 runs.

Figure 6.10 provides a graphic representation of demand, which is rather steady over the 12-months simulation.

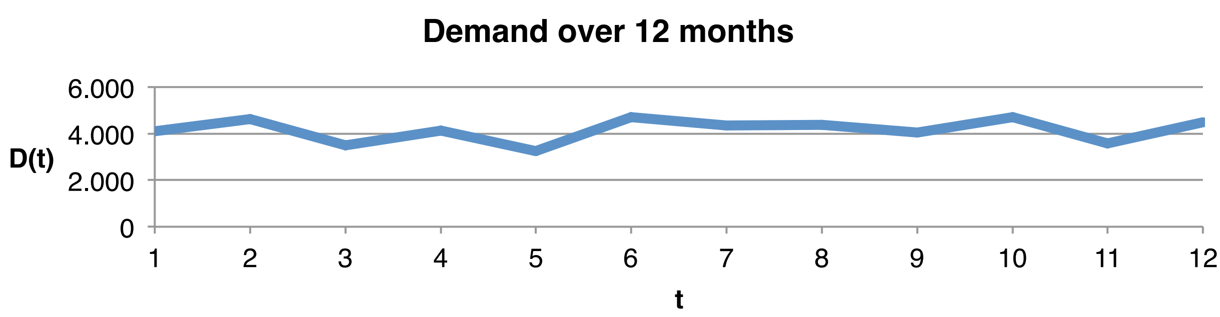


Figure 6.10: Example of demand realization in one of the 1.000 runs.

6.2 Hydraulic Cylinders

A hydraulic cylinder is a mechanical actuator that produces linear motion and unidirectional force by using pressurized hydraulic fluid, typically oil. It converts the energy stored in the hydraulic fluid into a force, allowing the linear motion of the cylinder. A hydraulic cylinder consists of a cylinder barrel, cylinder cap, piston rod, piston, cylinder base, rod gland, cushions, seal, and cylinder base connection. Hydraulic cylinders are classified into two types according to their function: *single-acting* cylinders and *double-acting* cylinders. Single-acting hydraulic cylinders apply hydraulic force in only one direction, usually to extend the cylinder. Double-acting cylinders have two ports for the intake and outtake of hydraulic fluids; the pressure from the fluid can be used in both directions in order to expand and retract the piston and it can be mounted in any direction for lifting, pushing and pulling operations. A further classification, based on the design, distinguishes *tie rod* from *welded* cylinders. In designing tie rod cylinders, a rod is tied to an outer part of the hydraulic cylinder to provide extra stability to the cylinder, while in welded cylinders the barrel is welded to the end caps.

Tie rod cylinders are less complex in terms of design and have limited options in terms of customizability. Generally, they come in fixed bore sizes and stroke lengths, which can reduce one's design options when developing new equipment. Tie rod cylinders are less costly, but also less durable, than welded cylinders.

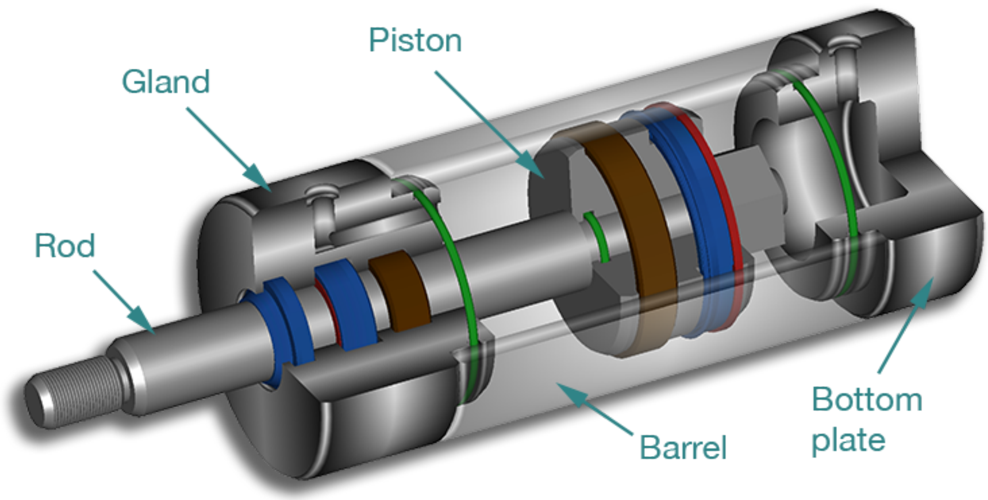


Figure 6.11: Cutaway of a hydraulic cylinder.

6.2.1 Market and Trends

According to a technical market research report (Hydraulic Cylinders: Technologies and Global Markets, www.bccresearch.com), the global hydraulic cylinders market was valued at $9.5 billion for 2012 and $10.1 billion for 2013. BCC Research expects the market to grow to nearly $12.5 billion by 2018 and register a five-year compound annual growth rate of 4.4% from 2015 to 2020. This steady growth is driven by the emerging economies, while the North American and European markets have reached the saturation stage. Although Western markets remain the largest markets in terms of revenue, the Asia-Pacific region is expected to grow enormously at a five-year compound annual growth rate (CAGR) of 5.6%, over 2013 to 2018.

Globally, the demand for hydraulic cylinders is driven by the increasing demand for their application in the equipment used in industries such as construction, mining, manufacturing, agriculture, and aerospace and defense. Additionally, increasing demand from the OEMs due to increasing industrial production and growing global trade also contributes toward growth. Construction, aerospace & defense, and material handling equipment account for more than 60% of the global market for hydraulic cylinders. In the long term, agriculture equipment is expected to drive the market for hydraulic cylinders with increasing demand for agricultural products to feed the growing population.

6.2.2 Stocchetta Cilindri s.r.l.

Stocchetta Cilindri s.r.l. is an Italian SME, founded in 1967 in Brescia, recognized for the design and production of special and large-sized cylinders. Its products include cylinders of many types: oleo-dynamic and pneumatic, with and without tie-rods, telescopic, plunge, multiple, in tandem, clamping, rotary actuators, pressure multipliers, flow dividers, magnetic; the cylinders can be produced with square, round, bolted and welded heads.

The company places great importance on consolidating partnerships with the largest Italian and foreign contractors, especially exploiting the opportunities offered by emerging markets for creating new synergies with foreign partners in both technological and marketing contexts. In 1998, and then again in 2000, the synergy created by a collaboration with a prestigious international constructor in the oleo-dynamic field enabled Stocchetta Cilindri s.r.l. to acquire new skills and know-how in the field of serial design and production of oleo-dynamic cylinders.

Besides its products, the company offers:

- Support for communications in five languages (EN/GE/FR/IT/SP);
- Advice for choosing standard or ad-hoc products;
- Full documentation aids, also specific documentation to order;
- Servicing and technical training courses for external personnel;
- Advanced IT, design and network communication facilities;
- Expert capillary technical and sales assistance network;
- Installation and servicing assistance at Customer site;
- Servicing and revamping of cylinders manufactured either by Stocchetta Cilindri s.r.l. or, in the interests of offering a complete service, by other constructors;
- Supply over long timescales of gasket kits and spare parts, also for components no longer in production.

Supply Chain Description

Stocchetta Cilindri is specialized on cylinders and operates mainly within the Italian market. The company purchases raw materials –typically seamless tubes of steel– and other components, such as seals, sensors and other electronic devices mostly from local suppliers. There are exceptions when a customer requires components produced by a particular manufacturer; when specifically asked, electronic devices are usually imported from Germany. Stocchetta Cilindri is present on the market with a broad and almost infinite range of products, which can be categorized as in Figure 6.12.

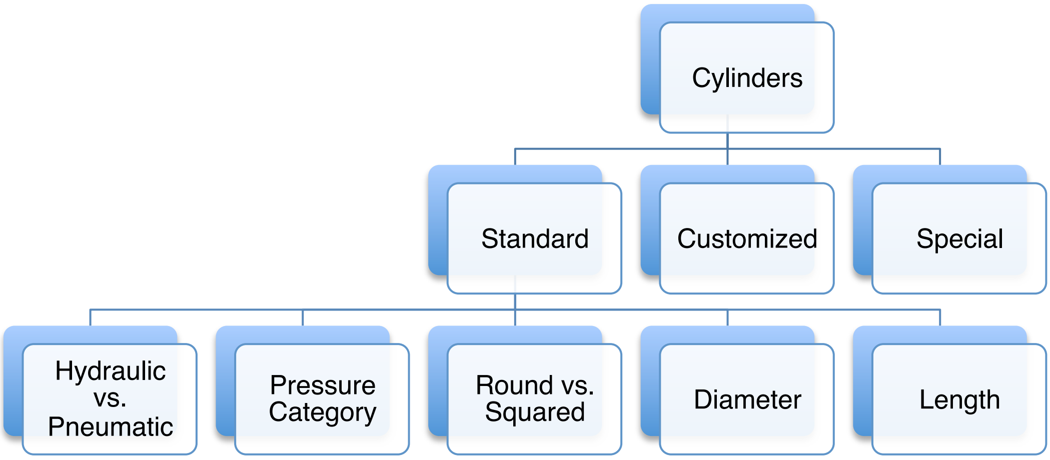


Figure 6.12: Classification of the products realized by Stocchetta Cilindri.

Customized cylinders are standard products for which the customer has specific requirements, while special ones are designed together with the customer starting with a blank sheet. The almost unlimited product range and the impossibility to forecast the characteristics of customized and special products force Stocchetta Cilindri to adopt the MTO strategy, with the only exception of standard oleo-dynamic cylinders ISO 6022/6020/2 that are made-to-stock and on which this thesis is focused.

Production is both internal and external: current production capacity does not cover demand and Stocchetta Cilindri outsources some productive processes or even the whole production of standard cylinders.

- External Contractors treat raw materials and deliver semi-finished products: Stocchetta Cilindri paints, assemblies and tests internally these items;
- CMB s.r.l. owns 40% of Stocchetta Cilindri and supplies the company with finished products, ready for shipping;
- Other suppliers located in the USA supply finished products, usually small cylinders.

The number of suppliers in 2014 was 175, but among these only 75 exceeded €10.000 as transaction volume, while only 17 of them represent 50% of the global spending.

Plant builders like Danieli & C.Officine Meccaniche S.p.A. are the main customers, while end users usually ask for training, technical assistance, revamping, general maintenance and spare parts. Danieli and Eaton Fluid Power generated in 2014 revenues around €2,9mln, equal to 43,5% of the annual turnover. Sales are concentrated in Italy and export is related to Italian companies operating abroad. Figure 6.13 provides a detail of sales by country.

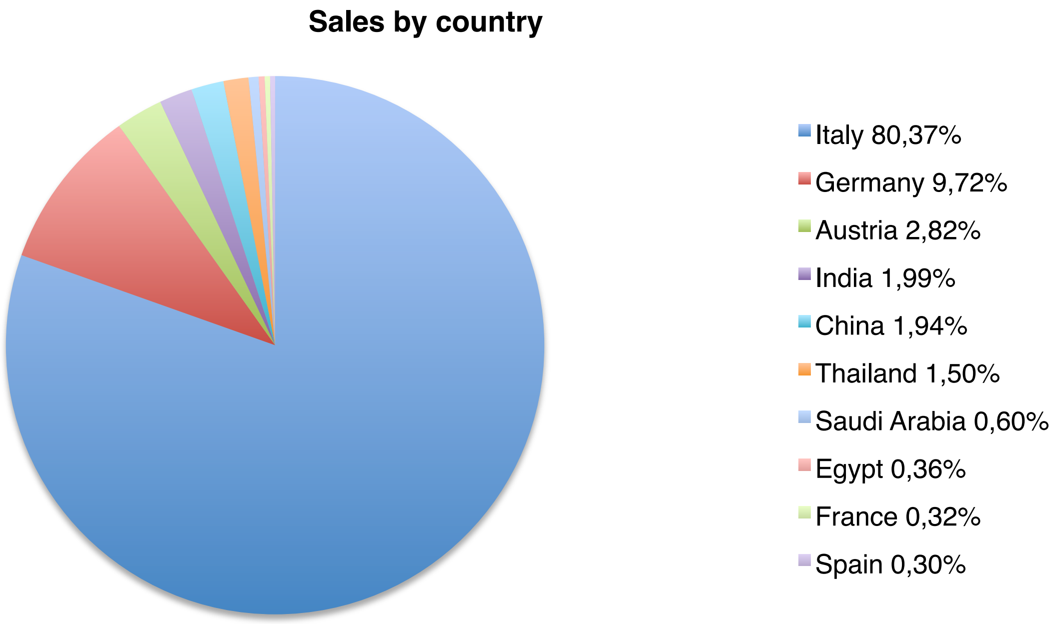


Figure 6.13: Sales by country in 2014.

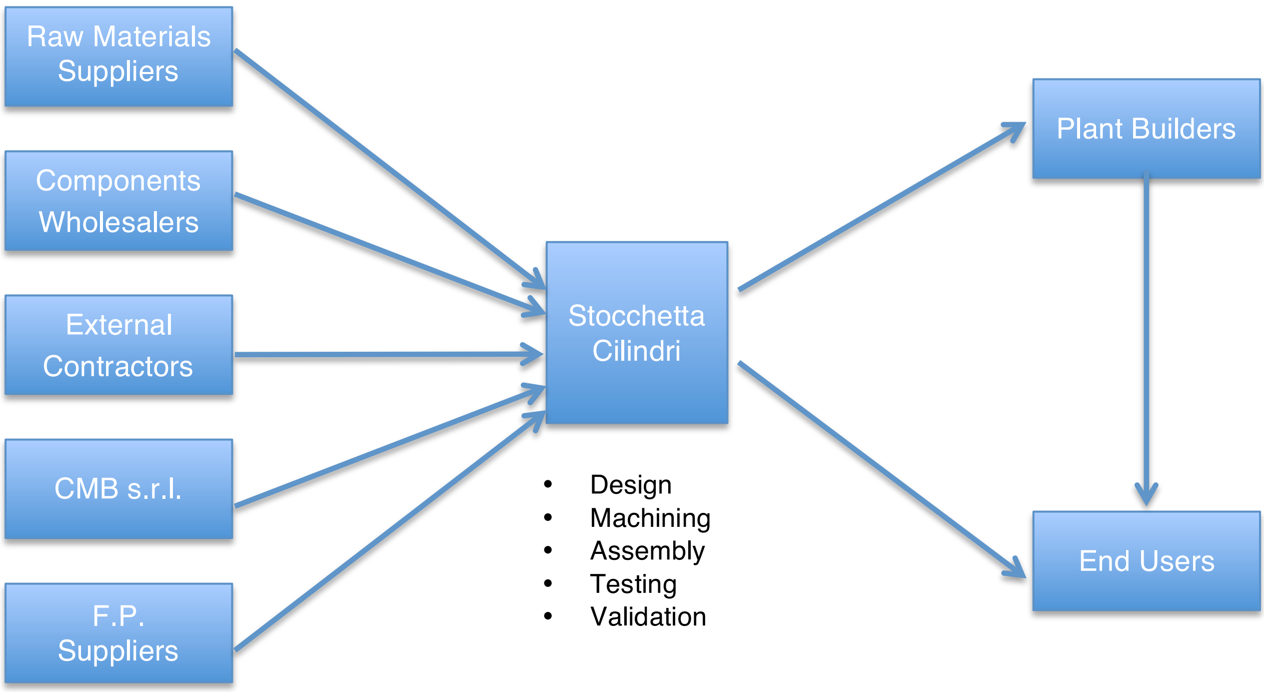


Figure 6.14: Graphic representation of Stocchetta Cilindri's supply chain.

Supply chain complexity is rather low, since business partners are mostly local and the productive process is standard and stable, based on mature technologies such as turning and milling. Managing technical files and machine parameters is a critical activity for Stocchetta Cilindri, since even the smallest variation to a standard product requires the definition of a new item code, technical file and documentation. On the other hand, most of the items share the same work cycle[1], regardless of their specifications.

[1] A sequence of tasks, operations, and processes, or a pattern of manual motions, elements, and activities that is repeated for each unit of work.

Raw material's price volatility is not a source of uncertainty for Stocchetta Cilindri, since variations are usually absorbed by customers. An important risk is given by the scarcity of external contractors and their long lead time that is often the bottleneck of production. An employee manages these suppliers daily and physically visits their plants to expedite production. Besides this, uncertainty is low, since the company is close to end users and does not suffer either the bullwhip effect, or promotions that could generate demand peaks.

Also in this case, the overall supply chain is physically efficient, consistently with Fisher's (Fisher 1997) classification of the products as "functional".

Fisher Classification: Functional Products	
Demand Uncertainty	Medium-High for MTS products
Product Lifecycle	Long
Contribution Margin	Medium
Product Variety (per category)	High
Average Forecast Error	Medium for MTS products
Average Stock-out Rate	Medium for MTS products, but the customer does not "walk away"
Make to Order lead time	4 to 6 weeks for standard products, 8 to 12 weeks for special ones
End of Season Markdown	No

Table 6.5: Fisher classification of the products.

Hau Lee's classification framework (Lee, 2002) once again suggests a lean approach, which reflects Stocchetta Cilindri's supply chain strategy. The company is currently facing supply uncertainty due to low production capacity and chose a backup strategy, increasing stocks and looking for backup supplier. However, capacity investments have already been planned for the current year.

Process Stability	Demand Variability: Low	Demand Variability: High
High	*Lean Supply Chain*	Responsive Supply Chain
Low	Risk Hedging Supply Chain	Agile Supply Chain

Figure 6.15: Hau-Lee's model for supply chain strategies applied to Stocchetta Cilindri.

6.2.3 Model Setting

The analysis is focused on the standard oleo-dynamic cylinders ISO 6022/6020/2, which are produced in a MTS strategy and sold to solid companies that could offer Reverse Factoring arrangements.

Given the characteristics of the product, the timing of the orders received by a single customer is not regular throughout the year. For a more significant application of the model it is opportune to consider the orders received for that product by the entire customer base, which is composed mostly by large players eligible for offering RF, such as Danieli and Eaton Fluid Power. This decision is legitimated by the pursuit of consolidating partnerships with large contractors.

The production cost of ISO 6022/6020/2 cylinders is €365,9 and in 2014 Stocchetta Cilindri sold 7.596 units at the average selling price €440 which is paid, on average, within 4 months. The estimated annual holding cost is 10% of the manufacturing cost, that is €36,59. The variable w', representing the inventory strategy, is once again set equal to the production cost to take into account that the company has to bear other costs not considered in this model (e.g. depreciation and taxes).

The time series of the purchasing orders received in the last two years shows a positive trend $g = 2{,}289\%$ per month. $\mu_1 = 633$ is the average demand over the last year, while $\sigma_1 = 314$ is calculated on the stationary time series D', where each element is calculated as $D'_t = \frac{D_t}{(1+g)^t}$.

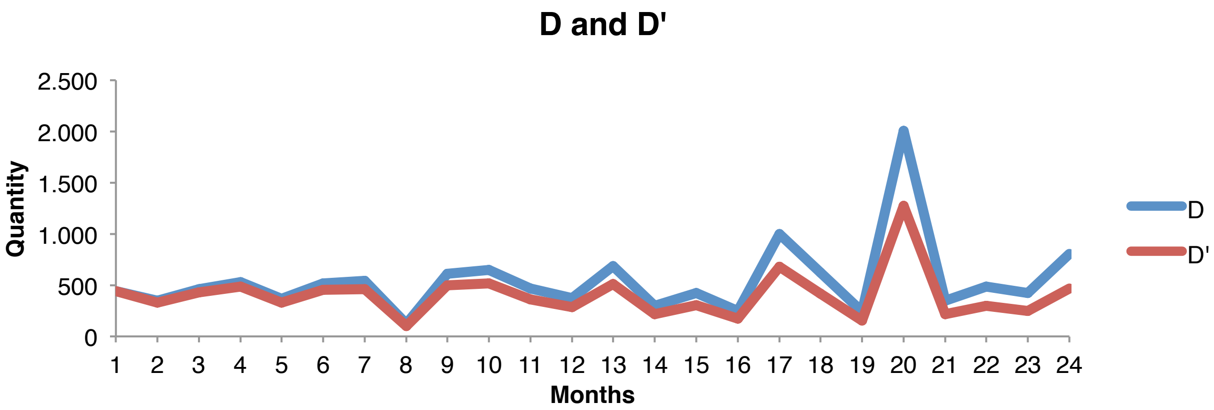


Figure 6.16: $\boldsymbol{D_t}$ and $\boldsymbol{D'_t}$ over the last two years (2013 and 2014).

Also in this case, we compare the value of reverse factoring for the company with the use of a credit line. As in FAB's case, we report the results obtained in absence of external financing and assume the line of credit to be equal to the maximum amount of short-term debt in the last three years, €683.739. While this figure refers to the whole company, the model focuses on a single product line. For this reason, we multiply that amount by the percentage of revenues generated by the product on the company's turnover, i.e. $\bar{O} = 51{,}82\% \cdot €683.739 = €354.298$. The average annual interest rate is $r_s = 4{,}25\%$ and the interest rate paid on the unused portion of debt is $r_u = 1\%$, while the RF annual interest rate is $r_{RF} = 2\%$.

The initial cash level is again calculated as the average of the figures in the last balance sheet multiplied by the percentage of turnover generated by sales of ISO 6022/6020/2, i.e. $z'_0 = 0{,}5 \cdot [(Cash\ on\ Hand + Bank\ Deposit)_{2013} + (Cash\ on\ Hand + Bank\ Deposit)_{2012}] \cdot 51{,}82\% = €151.192$.

The initial inventory is set to zero.

6.2.4 Results Analysis

This paragraph illustrates the results obtained through 1.000 replications of the present scenario. The indicators used are the same presented in FAB's case: Profit, ΔProfit Confidence Interval, Demand Coverage and Total Inventory.

		No Fin	CL	RF
Profit [€]	Average	450.365	530.637	570.507
	Std. dev.	32.930	62.616	84.445
	Δ Profit RF (%)	23,2%	7,2%	
C.I. (α=0,01)	Inf.	111.366	33.528	
	Average	118.461	36.431	
	Sup.	125.556	39.333	
Demand Coverage	Average	81,6%	94,8%	99,4%
	Std. Dev.	6,8%	4,3%	0,9%
Total Inventory	Average	8.567	12.956	15.354
	Std. Dev.	1.141,00	4,58	12,95

Table 6.6: Results of the simulation over 1.000 runs.

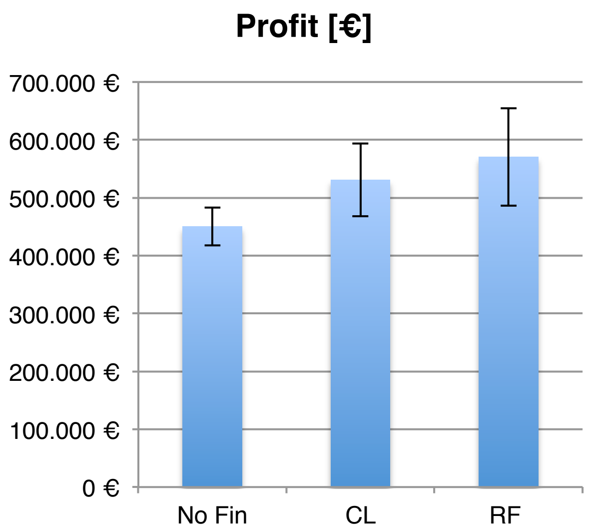


Figure 6.17: Average profit over 1.000 runs.

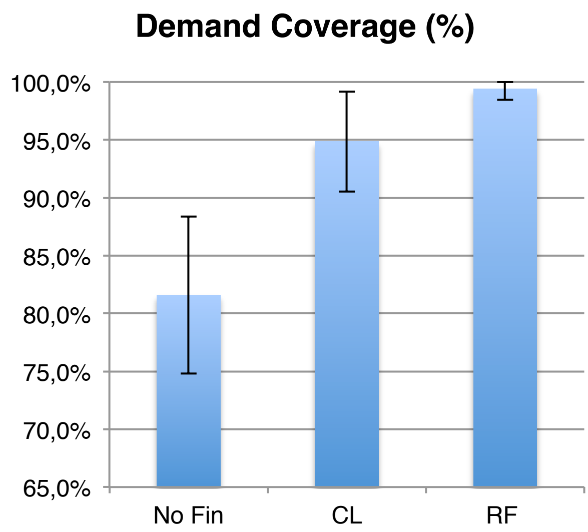


Figure 6.18: Average demand coverage over 1.000 runs.

The characteristics of demand, i.e. growing trend and high coefficient of variation (49,61%), determine more variability in the outcome. Also in this case, we can note that No-Financing leads to lower variability in profit and higher variability in demand coverage, compared to the scenarios in which external financing is available. In the previous case study, RF yields a higher profit thanks to a lower interest rate, while in this case a relevant contribution is given by its flexibility. Indeed, the utilization of financing means is remarkably different in this simulation: the firm uses on average a larger part of its credit line (up to 88,86%) but there is not a clear routine concerning the utilization of RF. Depending on the necessity, the firm may choose to factor up to 4 invoices, freeing up substantial amounts of cash, not achievable through a credit line.

CL	Used Portion of the CL (on average)	€314.844 (88,86%)
RF	Not Used	0,67 (5,61%)
	Only the most mature invoice is factored (# of times)	2,13 (17,75%)
	The firm factors up to the 2ND most mature invoice (# of times)	6,02 (50,13%)
	The firm factors up to the 2ND least mature invoice (# of times)	3,18 (26,52%)
	The firm factors up to the least mature invoice (# of times)	0,00

Table 6.7: Average utilization of financing means over 1.000 runs.

6.3 Conclusions

The results obtained in the two case studies are consistent with each other and with the insights drawn by the simulations. In both applications the absence of financing considerably affects in a negative manner the ability of the SME to reach the order-up-to level and its service level, with consequences on the retained profit at the end of the horizon T. The second case study also proves that RF is more valuable when demand is growing and remarkably variable, other than showing how an SME can exploit the greater flexibility of RF compared to the credit line.

However, the proposed case studies are based on the characteristics of two distinct SMEs operating in different industries and therefore present some differences. It is interesting to notice that demand coverage is 100% in the first case study, while it is 94,8% and 99,4% with the CL and RF respectively in the second, even though the Stocchetta Cilindri does not use the whole credit line and never factors all the pending invoices. The greater demand variability plays a relevant role in this result, but is not the only reason of the lower demand coverage.

Table 6.8 illustrates the parameters that define the optimal order-up-to-level S in the two case studies and Table 6.9 provides the arguments of S that, as proved, are calculated as the underage cost divided by the sum of underage and overage cost per unit.

	β	p (€/u)	c (€/u)	Margin (€/u)	Margin (%)	h (%)	h (€/u/y)	Annual r_s	Annual r_u	Annual r_{RF}	m
FAB	0,9975	0,6	0,4174	0,1826	44%	10%	0,0417	4%	1%	2%	4
Stocchetta	0,9975	440	365,9	74,1	20%	10%	36,59	4,25%	1%	2%	4

Table 6.8: Parameters used in the simulations of the two case studies.

	No-Fin	CL	RF				
	Arg(S)	Arg(S)	Arg ($S_{NO\ FACT.}$)	Arg(S_{t-4})	Arg(S_{t-3})	Arg(S_{t-2})	Arg(S_{t-1})
FAB	0,981	0,969	0,981	0,977	0,973	0,969	0,965
Stocchetta	0,959	0,933	0,959	0,950	0,942	0,934	0,925

Table 6.9: Arguments of the optimal order-up-to level S corresponding to the different financing options.

The arguments of S are always greater than 0,5, meaning that the underage cost is higher than the overage cost; i.e. it's more convenient buy and hold a single unit of inventory for one year than to be unable to sell that unit because of stock-out. Nevertheless, the arguments of S are greater in the first case study, indicating that FAB is more encouraged to overstock. This is comprehensible looking at the indicator $Margin/Holding\ Cost$, which is equal to 4,37 in the first case study and only 2,03 in the second.

7 Conclusions

The conclusive chapter provides the answers to the research questions of this work, summarizing the outcome of the three models. The managerial implications are highlighted, together with the limiting assumptions of the models, which can be released in the suggested future research.

7.1 Answers to the research questions

1) *What are the tangible benefits made possible by Reverse Factoring for those large buyers who are supposed to initiate it?*

The first model presented in this work (Chapter 3) tries to quantify some possible and tangible benefits for a large buyer who should initiate a RF program with his suppliers. As already addressed, the academic literature divides buyers' benefits given by RF in *direct* and *indirect* (Pfohl and Gomm 2009, Van der Vliet 2013): the formers consist in a reduction of Working Capital Requirement and short-term financing costs (Lewis 2007; Milne 2009; Hurtrez and Salvadori 2010; Seifert and Seifert 2011). The latter ones include better collaboration and partnership, suppliers' improvement in reliability and service level, and reduction in upstream supply chain risks and transactional costs (Seifert and Seifert 2009, Roumen 2012, Greensill 2014). Since our research question is about *tangible* benefits, we focused on the direct ones, more suitable to be included in a mathematical model. Starting from an EOQ framework, the model explains how Reverse Factoring allows to lighten Working Capital Requirements thanks to payment extension and how this operational capital might be used by buyers to reduce costs (in this case, investing it in inventory). The model presumes a situation of Working Capital Constraint for the buyer, e.g. a situation in which the COO is free to make decisions over DPO, DSO, Cash and inventory under a certain budget of Working Capital set by the CFO. This represents certainly a limitation for the model, since not all the enterprises may set a WCR limit to the COO. However, this assumption has been useful to interpret the results and to answer to the research question: in fact, the sensitivity analysis over the WCR constraint shows that the *direct* and immediately *tangible* benefits for a buyer depend on her initial conditions. When the buyer starts from a lack of short-term capital (e.g., due to higher needs in long-term capital for investments in fixed assets, or due to a financing-lean policy that pushes on inventory reduction), RF gives the possibility to extend DPO without worsening her suppliers' operating conditions (i.e., service level, prices, risk). This payables extension lets the buyer free-up the capital needed to run the day-by-day operations in a more efficient way, reducing costs. However, the analysis shows also that when buyers start from more favorable conditions (i.e., working capital constraint not much tight), RF would not be so useful in terms of payment extension and the majority of the benefits would be addressed to the supplier. This second scenario does not mean at all that buyers benefits are *less*, it just shows that the *direct* ones are. In fact, arranging a RF contract that would concede to the supplier the majority of the direct savings may lead to the second type of buyers' benefits, the *indirect* ones, such as a less risky up-stream supply chain, a higher service level, a more reliable supplier and better business relationship.

2) *Focusing on suppliers, what are the benefits induced by the increased financing flexibility that Reverse Factoring grants?*

The answer to this question derives from the second and the third model presented in this work (Chapters 4 and 5). The Reverse Factoring and Growth model (Chapter 4) compares RF to other typical SCF solutions, within the assumption of a non-stop growing demand that is never saturated. This scenario was specifically chosen in order to show the full potential of RF for suppliers, and its ability to favor the growth of an SME. An interesting finding is that RF performs better than the other solutions not just thanks to a lower interest rate, but also thanks to a higher flexibility in collecting capital. RF, indeed, represents the possibility for a supplier to discount as much as he desired, at any time depending on the current needs, without bearing on the balance sheet any additional debt, but just obtaining faster what he is supposed to obtain anyway. The model highlights particularly this aspect, emphasizing the importance of the timing and the size of the working capital investments (e.g., inventory), and showing that RF allows collecting more capital in a shorter time. These two aspects play a key-role in a fast-growing environment such as the second model's one and characterize RF as a more flexible solution than the others.

The model presented in Chapter 5 relaxes the infinite demand assumption of the second model and represents the RF dynamics in an uncertain-demand environment. This model enriches the answer to the research question, since it shows that the higher financing-flexibility achievable with RF is valuable not only in a highly growing demand context, but also in others (e.g., stationary or seasonal demand). Particularly, it shows that, in such a variable context as the seasonal-demand's one, RF allows a company to reach remarkably high levels of working capital when needed and to decrease it immediately later. This characteristic allows the supplier to face and cover high peaks of demand, guaranteeing the usual service level. The figure below shows how the value created by RF is not only due to cheaper cost of debt, but also to a wider demand coverage given by the flexibility of this SCF solution (around 20% of the added value).

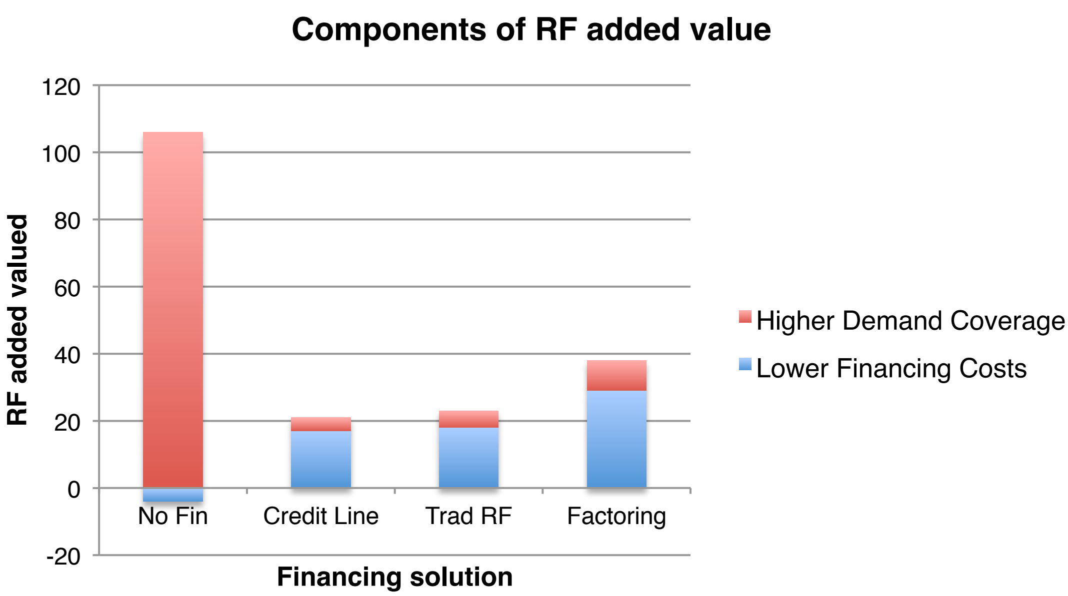


Figure 7.1: RF value added in respect to each alternative solution, subdivided by cause

3) *What are the operational implications caused by Reverse Factoring and the other SCF tools for suppliers?*

The third model shows how a certain category of operational decisions, such as replenishment policies, may be affected not only by the cost of capital, but also by the type of SCF tool utilized. Even though in many cases the literature already proved that cost of capital must be included in operating decisions, this model highlights that the interest rate of short-term capital is not the only financing information that matters for the

COO: the way cash flows come in and go out has an impact as well. The fact that under RF cash is collected through a certain process, different from the credit line, and that the relative financing costs are calculated with different methods, have an impact on the *optimal inventory replenishment decision*. As we may notice, each different SCF tool leads to a different formula:

$$S_{TRF} = S_F = S_{NoFin} = F^{-1}\left(\frac{\beta^m \cdot p - c}{\beta^m \cdot p - c \cdot \beta + h}\right)$$

$$S_{CL} = F^{-1}\left(\frac{\beta^m \cdot p - c \cdot (1 + (\boldsymbol{r_s} - \boldsymbol{r_u}) \cdot \boldsymbol{\beta})}{\beta^m \cdot p + h - \beta \cdot c}\right)$$

$$S_{RF}^{t-n+k-1} = F^{-1}\left(\frac{\beta^n \cdot p - \dfrac{1}{(\mathbf{1} - \boldsymbol{r_{RF}})^{\boldsymbol{k}}} \cdot c}{\beta^n \cdot p - c \cdot \beta + h}\right)$$

The financing cost, when existing, is bold-highlighted. Auto-financed firms (S_{NoFin}) and firms who use Traditional Reverse Factoring (S_{TRF}) or Factoring (S_F) utilize the first ordering policy: we may observe that the formula does not contain any financing cost, since in all the three cases the firm cannot decide how much capital to collect (TRF and Factoring are assumed to be *auto-discounting* solutions). In fact, this framework is inspired to the Newsvendor model: thus, if the firm does not decide whether and how much to collect, this does not count in the replenishment policies. On the other hand, Credit Line and RF work differently from the former solutions: in both cases, the decision maker is able to decide, at the beginning of each period, how much cash to collect. This is why both formulas (S_{CL} and $S_{RF}^{t-n+k-1}$) contain the respective interest rates: ordering more units might cause a higher need of cash and higher interest rates. The formulas show also a difference regarding the way financing costs are calculated, since the two solutions work differently.

Concluding, we may notice how flexible financing solutions such as Credit Line or RF may have an impact also in operating decisions. Obviously, the specific scenario chosen to set the model limits the generality of the answer; however, it is shown how operations and finance should always cooperate and exchange views also in the day-by-day decisions, such as how much inventory to order, and not only in the long-term decisions such as investments.

4) On which conditions Reverse Factoring outperforms the other financing means?

The second model, which is set in a never-saturated demand context, suggests that, in such framework, RF outperforms Asset Based Lending and Unsecured Loan no matter what values assume the other parameters. The only exception regards the scenario in which the firm starts from a very poor cash condition (the lowest) and needs to invest in capacity in order to produce: in this case, Unsecured Loan, even though higher interest rates, offers an investment capacity much higher in the earliest periods and allows the firm to grow sooner. In the other cases, the model shows that in a high-growing demand context RF is the best SCF tool, allowing the firm to invest more and pay fewer interests.

The third model completes the answer including other scenarios such as stationary demand, regularly growing demand and seasonal demand. We also varied the assumptions regarding the DPO extension coming with RF and the initial cash on hand available. The analysis shows that there are some context-conditions that make RF perform better than the other SCF tools, while there are others in which RF does not represent a better-off solution. One of the most interesting insight is that RF outperforms all the other financing solutions when the firm faces a seasonal demand characterized by high peaks. The reason lies in the flexibility that a tool such as RF is able to offer: in a variable context as the seasonal-demand's one, RF allows a company to reach very high levels of working capital when needed and to decrease it immediately later, making the most efficient use of the capital available. Other solutions, e.g. Traditional RF and Factoring, may be able to offer the same cash-availability, but reveal to be more expensive. Firms that choose Credit Lines, on the other hand, do not enjoy the same financing-flexibility and find more difficult to face the high peaks of the demand.

In other scenarios such as regularly growing demand and stationary demand, RF still overcomes Traditional RF and Factoring, but its superiority to Credit Lines is not that remarkable: in fact, when RF carries a DSO extension for the supplier, Credit Lines might be even better. In fact, when the buyer requires DSO extension, RF's performance decreases sensibly. Overall, the sensibility analyses on demand patterns and DSO extensions showed that RF represents a better-off alternative in respect to Traditional RF and Factoring, but not always when compared to Credit Lines.

This situation changes in other contexts, e.g. when the initial working capital is very low. If the firm starts lacking inventory and cash, Credit Lines lose competitiveness: the more cash is needed, the less powerful Credit Lines are when compared to RF, since the latter is able to free up much more cash with a lower cost. At the same time, the higher is the initial working capital, the less is the value added of Reverse Factoring, because the firm already has enough working capital to run properly the day-by-day operations.

7.2 Managerial Implications

The theoretical conclusions provided by this work also suggest some managerial implications:

- Large buyers that decide to implement a RF program with their suppliers should consider accurately the effects of extending their DPO. First, lengthening payment terms always worsen the whole supply chain performance. Second, depending on the specific scenario, buyers might be better off not extending their DPO, but just sharing supplier's benefits through e.g. price discounting. In addition, the DPO's increase provides the buyer a short-term advantage, i.e. the cash freed-up. However, in the long term, suppliers that do not have to concede payment extension grow and improve their service level (directly related to the buyer) much rapidly.

- Suppliers and buyers should consider that RF is quite often a win-win solution, but not always. In particular, the third model presented suggests, as expected, that RF is always a dominant solution in respect to Traditional RF, since they offer the similar interest rates but the former adds much more flexibility and efficiency thanks to the possibility to choose when to discount, which invoices to discount and how much of each. In the same way, RF outperforms Factoring: even though sometimes the former involves a payment extension in order to be arranged, its cheaper interest rate and its higher flexibility makes it always a better off solution. However, other solutions such as Credit Line might match RF's performance or even overcome it. In fact, RF is proved to be superior when its higher efficiency in collecting capital is fully needed, i.e. in scenarios characterized by a high variability and uncertainness. In steadier situations, where demand is stationary, regular and predictable, RF might not add much value. The fundamental managerial insight here is that the true value tied up to RF does not only lie in lower interest rates, that might also depend on the world-wide financial situation, but also on higher capacity of collecting capital and increasing/decreasing WCR in a very short time, without bearing on Liabilities or incurring in new debt. Credit Lines, on the other hand, always entail financial costs, no matter whether a firm is fully using overdrafts or not, offer less flexibility since the credit limit is often subject to *haircuts* and cause higher interests. On top of that, Credit Lines are also not always available to SMEs, e.g. when banks consider them too risky, while it is much easier for large buyers to involve a financial institution to help her supply-base.

- The third model gives also a final insight from the suppliers perspective: COOs should make decisions over production and inventories considering not only interest rates, but also the specific financial mean that the firm is using. Different solutions involve different costs, cash dynamics and capacity. If the day-by-day operations are financed by a Credit Line, a Factoring contract or a RF arrangement, the cause-effect boundaries are not the same. The third model reaffirms a concept already addressed in the literature, i.e. that operations and finance should always be coupled, not only in the long-term decisions such as capacity investments or make-or-buy, but also in the short-term ones.

7.3 Future research

Being this work rather theoretical than empirical, all the models developed are based on diverse assumptions that allow and simplify the mathematical proofs but leave a vast space for further development:

- Concerning the first model and the possible RF's advantages that buyers can benefit from, a further development could be realized, e.g. a model that evaluates the second type of benefits for buyers, the so-called *indirect benefits*. It could be investigated how much a buyer can gain from a *supply-base risk reduction* or from a *stronger partnership* and a *higher service level* due to RF.

- The second model can be further extended considering also the up-stream suppliers' capacity: in fact, we assumed the supplier to be able to grow as much as allowed from its liquidity, independently from its environment. A model that considers also the up-stream capacity limit would be more reliable in terms of supplier's growth-expectations. An additional improvement would regard the demand pattern, which could be set as random and non-infinite, leading to different production launches and purchasing decisions.

- The third model could be improved including payables in the suppliers' working capital requirement. This would bring much more complexity in the myopic profit function and the optimal order policy, but it would provide interesting insight on how payables may influence the impact of RF for suppliers. Another interesting improvement would take down the myopic policy assumption, deriving a replenishment policy that maximizes the multi-period profit. A further research should also consider the production lead-time, since WIP inventories do have a strong impact on WCR. Finally, taxes may be also included in the mathematical model in order to consider the interests tax-shield and how they can smooth or exalt the gap between different financing solutions.

Overall, the theoretical work developed in this thesis should put the base for further and more empirical research, e.g. an applicative model that can evaluate the added value of RF for a specific firm, customized on its processes and its peculiarities. The same stands for the buyer's perspective: working on a real-case, it could be developed a specific *tool* able to evaluate case-by-case the convenience for buyers to initiate a RF program with their suppliers.

List of Abbreviations

List of all abbreviations used in this work.

Acronym	*Extended*
AAGR	Average Annual Grow Rate
ABL	Asset Based Lending
CAGR	Compound Annual Grow Rate
CCC	Cash Conversion Cycle
CE	Conformité Européenne
CL	Credit Line
C2C	Cash to Cash Cycle
DPO	Days Payable Outstanding
DSO	Days Sales Outstanding
F	Factoring
FSC	Financial Supply Chain
ISO	International Organization for Standardization
IT	Information Technology
PMI	Piccola Media Impresa
PPE	Personal Protective Equipment
RF	Reverse Factoring
SC	Supply Chain
SCF	Supply Chain Finance
SCM	Supply Chain Management
SME	Small Medium Enterprise
T-RF	Traditional Reverse Factoring
USD	United States Dollar
WACC	Weighted Average Cost of Capital
WCR	Working Capital Requirement

Appendix A

Average Accounts Payable

The first problem with the Accounts Payable formula is that, as the cycle time T varies, the cumulated Accounts Payable varies as well. The second problem concerns t_c, the payment term established between buyer and supplier, which can be lower than T, meaning that the buyer always pays back her supplier before reordering, or higher, meaning that she will proceed one or even more than one orders before paying the first one. Different scenarios cause different behaviors of the Accounts Payable; however, since the demand D is constant, it is possible to prove that they all reach a steady state in which the Average Accounts Payable (P) converges to the formula shown before.

Demonstration

P is the average accounts payable generated over the year, which, by definition of "average" and since the demand is constant, is equal to the average accounts payable generated over a single cycle time T:

$$P_T = \frac{1}{T} \cdot \int_{T_{n-1}}^{T_n} P(t) \cdot dt$$

Since $P(t)$ is unknown and it varies depending on t_c and T, it is easier to derive its shape with an inductive method, analyzing its behavior in different scenarios:

Scenario 1: $t_c < T$

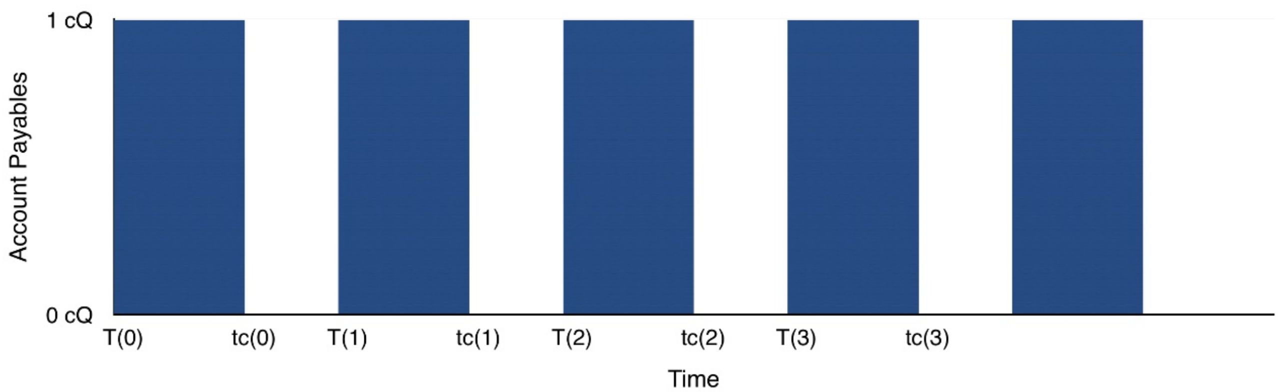


Figure A.1: Accounts payable over time when $\boldsymbol{t_c < T}$.

Assuming that at *t=0* the first order is delivered to the firm, the graph shows that the Accounts Payable immediately raises up to the value of the goods received: $c \cdot Q$. The value of P stays the same until the payment is issued to the supplier, at time t_c; after that point, P's value is 0, since all the goods ordered during the cycle time T have already been paid. Therefore, the Average Value taken on by the Accounts Payables is easily derivable just observing the graph:

$$P_T = \frac{c \cdot Q \cdot t_c + 0 \cdot (T - t_c)}{T}$$

Therefore:

$$P_T = \frac{c \cdot Q \cdot t_c}{T}$$

Scenario 2: $T_1 < t_c < T_2$

Here the payment delay is established longer than a single order cycle time *T,* but lower than two. Hence, it takes just one period to the Accounts Payables to reach a sort of "steady state", as it is shown in the graph:

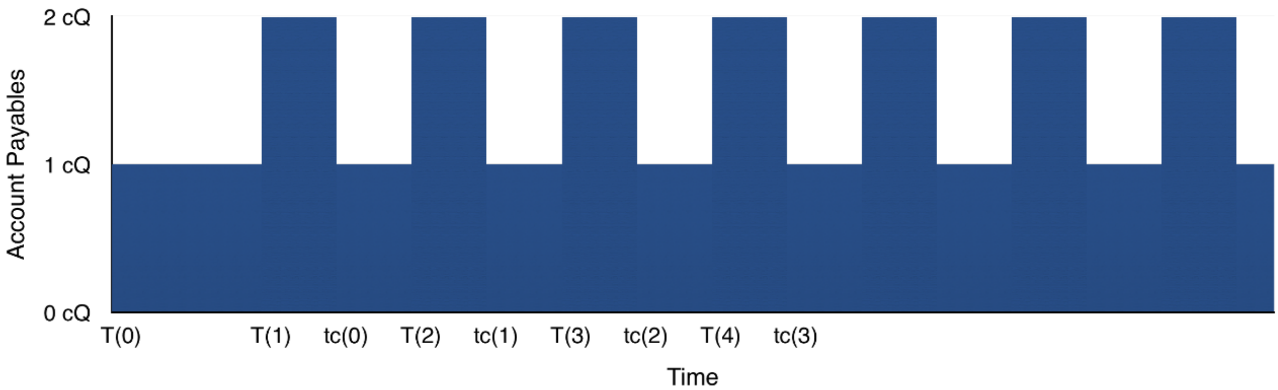


Figure A.2: Accounts payable over time when $\boldsymbol{T_1 < t_c < T_2}$**.**

As we can see, the steady state is achieved after T_1: we can reason about how to consider the Average Value of Accounts Payables from that point on, for a single cycle T. At the beginning of the cycle time, the Accounts Payable level is given by $2 \cdot Q \cdot c$, since there is still the previous period's order left to pay, plus the one just began. After a lapse of time worth $t_c - T_1$, the previous period's order payment is issued and the Accounts Payables level goes down to $Q \cdot c$, since there is still the current period's order to pay. The level stays constant until the successive period T, as the payment of a single period is always issued in the next one. The last thing to notice is that, since the demand is constant and so it is the quantity ordered, the cycle time T is constant as well: this means that $T_1 = T$, $T_2 = 2 \cdot T$, $T_3 = 3 \cdot T$, and so on. The average Accounts Payable can be finally written down:

$$P_T = \frac{2 \cdot c \cdot Q \cdot (t_c - T) + c \cdot Q \cdot (2 \cdot T - t_c)}{T}$$

Therefore, again, we find:

$$P_T = \frac{c \cdot Q \cdot t_c}{T}$$

Scenario 3: $T_2 < t_c < T_3$

In this scenario, the payment of an order is issued after two ordering cycle times but always before the third one. If we assume to start ordering from null stock level, we see that after two cycle-time periods the steady state is reached:

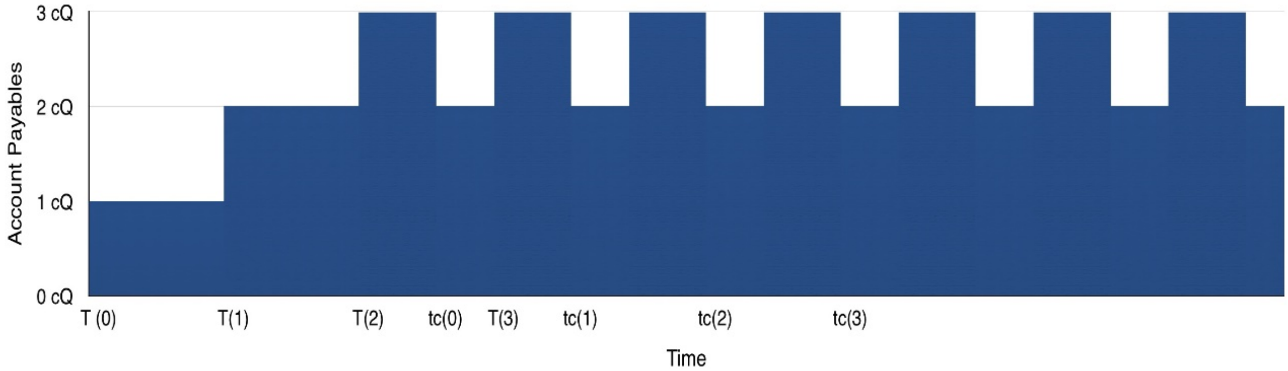


Figure A.3: Accounts payable over time when $\boldsymbol{T_2 < t_c < T_3}$.

As before, we should derive from the graph the average Accounts Payable level of a general steady period *T,* that can be considered, e.g., from T_2 on: in that point, the Accounts Payable are worth $3 \cdot c \cdot Q$, as nobody has yet paid the first order (*t=0)* , the second one ($t = T_1$) and this third one ($t = T_2$). After a time lapse of $t_c - T_2$, the first order payment is issued and the Accounts Payable level is then worth $2 \cdot Q \cdot c$; this value stays until the end of the period. We can therefore derive the Average Value of the Accounts Payable during the general lapse $T = T_3 - T_2$ as:

$$P_T = \frac{3 \cdot c \cdot Q \cdot (t_c - 2 \cdot T) + 2 \cdot c \cdot Q \cdot (3 \cdot T - t_c)}{T}$$

Once again, simplifying the equation we find out that:

$$P_T = \frac{c \cdot Q \cdot t_c}{T}$$

Conclusion

The three scenarios have not only led always to the same conclusion, but also, looking at the formulas generated in each of them, it is notable a recurring pattern, depending on the initial assumptions about t_c and T. More specifically, we can notice that, for a general steady state, not depending whether t_c> T or vice versa:

$$P_T = \frac{u \cdot c \cdot Q \cdot (t_c - T_l) + l \cdot c \cdot Q \cdot (T_u - t_c)}{T_u - T_l}$$

In which:

- *l = cardinal number of the period considered l* ∈ ℕ.
- *u = l + 1, cardinal number of the period immediately successive to l, u* ∈ ℕ.
- T_l *= period in which the order is placed.*
- T_u*= period in which the successive order is placed.*
- $T_u = T \cdot u$ *, by definition.*
- *u = l + 1*
- $T_u = T_{l+1} = (l + 1) \cdot T = T \cdot l + T$
- $T_u - T_l = T$

Developing further the equation above, we obtain:

$$P_T = \frac{(l+1) \cdot c \cdot Q \cdot (t_c - T \cdot l) + l \cdot c \cdot Q \cdot (T \cdot l + T - t_c)}{T}$$

Which leads to:

$$P_T = \frac{c \cdot Q \cdot t_c}{T}$$

We can therefore finally conclude that for this model, considering the assumptions made at the beginning, the average Accounts Payable for a generic period T (once the steady state is reached) are given by the formula: $P_T = \frac{c \cdot Q \cdot t_c}{T}$.

Appendix B

Leibniz's Formula Application

We use Leibniz's formula to minimize the cost function $G_t(x_t, y_t)$, by:

i. Proving that the function is convex;
ii. Finding the stationary point, which is the global minimum.

Since the optimal policy S is not time-dependent, here in the proof we omit the subscript t.

$$\frac{d}{dy} G_t(x_t, y_t) = G'(y)$$

$$G'(y) = u \cdot \left[-1 \cdot 0 \cdot f(y) + 0 + \int_{y}^{\infty} -1 \cdot f(\theta) \cdot d\theta \right] + o \cdot \left[-0 \cdot y \cdot f(-\infty) + 1 \cdot 0 \cdot f(y) + \int_{-\infty}^{y} 1 \cdot f(\theta) \cdot d\theta \right] =$$

$$= -u \cdot \int_{y}^{\infty} f(\theta) \cdot d\theta + o \cdot \int_{-\infty}^{y} f(\theta) \cdot d\theta =$$

$$= -u \cdot [1 - F(y)] + o \cdot F(y) =$$

$$= (u + o) \cdot F(y) - u$$

$G''(y) = (u + o) \cdot f(y) \geq 0 \quad \forall y \quad \Rightarrow \quad G(x, y)$ is convex in y (and $\pi(x, y)$ is concave in y). It follows that the y that satisfies the first order condition $G'(y) = 0$ is the global optimizer:

$$(u + o) \cdot F(y) - u = 0$$

$$F(y) = \frac{u}{u + o}$$

Appendix C

Continuity and Concavity of the Profit Function

The continuity in y_t is straightforward. To show the concavity of the profit function in y_t, notice that despite the fact that the function is not differentiable at the breaking points, we can still consider the first-order and second-order derivatives. Differentiating the single-period RF profit function (paragraph 5.2.2) with respect to y_t yields

$$\frac{\partial \hat{J}_t}{\partial y_t} = \begin{cases} \beta^n p - c - (\beta^n p + h + \beta \cdot c) \cdot F(y), & y_t \le z_t \\ \beta^n p - \left(\frac{1}{1 - r_{RF}}\right) \cdot c - (\beta^n p + h + \beta \cdot c) \cdot F(y), & z_t < y_t \le z_t^{t-n} \\ \beta^n p - \frac{1}{(1 - r_{RF})^{k+1}} \cdot c - (\beta^n p + h + \beta \cdot c) \cdot F(y), & z_t^{t-n+k-1} < y_t \le z_t^{t-n+k} \end{cases}$$

and $\frac{\partial^2}{\partial y_t^2} \hat{J}_t(x_t, y_t, \boldsymbol{z_t}) = -(\beta^n p + h + \beta \cdot c) \cdot f(y)$.

The concavity for all $y_t \in [x_t, z_t^{t-1}]$ follows directly from $\frac{\partial^2}{\partial y_t^2} \hat{J}_t(x_t, y_t, \boldsymbol{z_t}) < 0$.

To show the increasing property and joint concavity in x_t and $\boldsymbol{z_t}$, we first examine the first order derivatives.

$$\frac{\partial \hat{J}_t}{\partial x_t} = c$$

$$\frac{\partial \hat{J}_t}{\partial z_t^{t-n+k}} = \left[\left(\frac{1}{(1 - r_{RF})^{k+2}} - 1\right) - \left(1 - (1 - r_{RF})^{k+1}\right)\right] + \sum_{j=k+1}^{n-2} 1_{\{z^{t-n+j} < y_t \le z^{t-n+j+1}\}} \cdot \left[\left(1 - (1 - r_{RF})^{k+2}\right) - \left(1 - (1 - r_{RF})^{k+1}\right)\right]$$

$$k = 0, .., n - 2$$

The increasing property follows from the fact that all first order derivatives are positive. Taking the second order derivative, we get $\frac{\partial^2}{\partial x_t^2} \hat{J}_t(x_t, y_t, \boldsymbol{z_t}) = 0$, and the second order derivative of for the elements of $\boldsymbol{z_t}$ is zero too. The joint concavity in x_t and $\boldsymbol{z_t}$ follows from the fact that the corresponding Hessian is negative semi-definite.

Bibliography

Aeppel, Timothy. “Report Information from ProQuest: Bullwhip Hits Firms As Growth Snaps Back” *Wall Street Journal* webpage on 27-1-2010, available at http://www.wsj.com/articles/SB10001424052748704509704575019392199662672.

Alferink, H.. “Buyer initiated non-recourse factoring of confirmed payables: A major global corporation case study.” *Ms.C. Theis,* Eindhoven (2010).

Afza, Talat, and Mian Sajid Nazir. "Is it better to be aggressive or conservative in managing working capital." *Journal of quality and technology management* 3.2 (2007): 11-21.

Akintoye, Akintola, George McIntosh, and Eamon Fitzgerald. "A survey of supply chain collaboration and management in the UK construction industry." *European Journal of Purchasing & Supply Management* 6.3 (2000): 159-168.

Akkermans, H., Bogerd, P. and Vos, B., “Virtuous and vicious cycles on the road towards international supply chain management”, *International Journal of Operations & Production Management*, (1999) Vol. 19 Nos 5/6, pp. 565-81.

Arrow, Kenneth J., Theodore Harris, and Jacob Marschak. "Optimal inventory policy." *Econometrica: Journal of the Econometric Society* (1951): 250-272.

Berry C, McNeely A, Beauregard K, Haritos S. A guide to personal protective equipment. Raleigh, NC, USA: N.C. Department of Labor; 2008. Retrieved December 16, 2008, from: http://www .nclabor.com/osha/etta/indguide/ig25.pdf

Bhaskar, S. “Growth Opportunities for Personal Protective Equipment Industry in a Recessionary Economy”, *Frost & Sullivan*, (2009).

Buzacott, John A., and Rachel Q. Zhang. "Inventory management with asset-based financing." *Management Science* 50.9 (2004): 1274-1292.

Carrodus, Mike. “Financial Supply-Chain Management”, *Euromoney*, April-2007.

Chen, F., Ryan, J.K. and Simchi-Levi, D.,“The impact of exponential smoothing forecasts on the bullwhip effect”, *Naval Research Logistics*, (2000) Vol. 47 No. 4, pp. 269-86.

Cooper, M.C., Lambert, D.M. and Pagh, J.D., “Supply chain management: more than a new name for logistics”, *International Journal of Logistics Management*, (1997) Vol. 8, No. 2, pp. 1-14

Conerly B., “Procter & Gamble: Basis Point Wise, Percentage Point Foolish”, *Forbes.com* (April 17, 2013)

Cox, A., “Power, value and supply chain management”, *Supply Chain Management: An International Journal* (1999), Vol. 4 No. 4, pp. 167-75.

Croson, Rachel, and Karen Donohue. "Behavioral causes of the bullwhip effect and the observed value of inventory information." *Management Science* 52.3 (2006): 323-336.

Crowe, T.. Supply chain financing: Full speed in reverse. *Credit management*, 16 (2009).

Di Giulio, A., Sicurezza e Ambiente, *Lecture Notes* (2014).

Dolez, P., and Vu-Khanh, T., "Recent Developments and Needs in Materials Used for Personal Protective Equipment and Their Testing." *International Journal of Occupational Safety and Ergonomics*, (2015).

European Commission website, Personal Protective Equipment - Management of the Directive. Available at: http://ec.europa.eu/enterprise/sectors/mechanical/personal-protective-equipment/directive-management/index_en.htm.

Fawcett, Stanley E., Gregory M. Magnan, and Matthew W. McCarter. "Benefits, barriers, and bridges to effective supply chain management." *Supply Chain Management: An International Journal* 13.1 (2008): 35-48.

Fisher, Marshall L. "What is the right supply chain for your product?." *Harvard business Review* 75 (1997): 105-117.

Forrester, J.W., "Industrial dynamics, a major breakthrough for decision makers", *Harvard Business Review*, July/August, (1958) pp. 37-66.

Forrester, J.W., Industrial Dynamics, *MIT Press*, (1961) Cambridge, MA.

Fugate, Brian, Funda Sahin, and John T. Mentzer. "Supply chain management coordination mechanisms." *Journal of Business Logistics* 27.2 (2006): 129-161.

Gallagher, T., J. Andrew. "Working capital policy." Chapter 17 in *Financial Management: Principles and Practice* (2007). Freeload Press

Gardner M.J., D.L. Mills, and R.A. Pope. "Working Capital Policy and Operating Risk: An Empirical Analysis." *The Financial Review* 21(3), (1986): 31-31.

Goel G., Wohl J., "P&G taking longer to pay suppliers, offers financing.", *Reuters.com* (April 17, 2013)

Greensill, Lex. "Supply Chain Financing Comes of Age", *TMI - Special Report; Citi - A Treasurer's Guide to the Financial Supply Chain*, (2014). Available at http://www.treasury-management.com/article/4/196/1687/supply-chain-financing-comes-of-age.html.

Harris, Ford W. "How many parts to make at once." *Operations Research* 38.6 (1990): 947-950.

Harris A. "Working Capital Management: Difficult, but Rewarding." *Financal Executive* 21-4 (2005): 52-53

Hawser, Anita. "Joining the chain gang, Annual survey, Best supply chain finance providers." *Global Finance Business Module*, (2010), 24, no. 2 (July/August): 28.

Heiminga, Gerben. "Supply chain finance provides Dutch buyers with € 22 billion additional free cash flow", *ING Supply Chain Finance*, June-2012.

Hofmann, Erik, and Herbert Kotzab. "A Supply Chain-Oriented Approach Of Working Capital Management." *Journal of Business Logistics* 31.2 (2010): 305-330.

Hofmann, Erik. "Supply chain finance: some conceptual insights." *Logistik Management – Innovative Logistikkonzepte* (2005): p.149.

Holweg, Matthias, et al. "Supply Chain Collaboration:: Making Sense of the Strategy Continuum." *European Management Journal* 23.2 (2005): 170-181.

Honing H., van Eysden R.A., van den Hengel D., Pelman D. "Step into your clients' shoes and leverage existing knowledge to optimise working capital across the value chain" *Financial Supply Chain Management – Deloitte,* March-2013.

Huang, Y.F., "Economic order quantity under conditionally permissible delay in payments", *European Journal of Operational Research,* Vol. 176, (2007) pag: 911–924

Hurtrez, Nicolas, G. Salvadori M. "Supply Chain Finance: from myth to reality", *McKinsey on Payments* (2010).

Hydraulic Cylinders: Technologies and Global Markets, *BCC Research* (2014).

Ignall E.J., and A. F. Veinott Jr. "Optimality of myopic inventory policies for several substitute products." *Management Science* 15.5 (1969): 284-304.

ISHN – editorial staff, *New study shows differences in PPE use across the globe*, (2012).
Available at: http://www.ishn.com/articles/93693

ISHN – editorial staff, *Top Ten Trends in the PPE Industry*, (2011). Compiled by Miller Pierce www.millerpierce.com 317-228-1262.
Available at: http://www.ishn.com/articles/91741-top-ten-trends-in-the-ppe-industry

Johnson, R.A. e Wichern, D.W., "Applied Multivariate Statistical Analysis (fifth edition)", *Prentice Hall*, (2002)

Jones, Sarah. "Corporate payments: Opportunities for value-added services to be offered alongside payment products." *Journal of Payments Strategy & Systems* 2, (2008) no. 4: 392-399.

Kampstra, R. P., Jalal Ashayeri, and J. L. Gattorna. "Realities of supply chain collaboration." *The International Journal of Logistics Management* 17.3 (2006): 312-330.

Karush W., and R. E. Dear. "Optimal strategy for item presentation in a learning process." *Management Science* 13.11 (1967): 773-785.

Kashyap, A. Rajan, R. and Stein, J., 2002: "Banks as liquidity providers: An explanation for the co-existence of lending and deposit-taking". *Journal of Finance*, 57: 33-73.

Kerle, P. "The Growing Need for Supply Chain Finance." *Demica* (2009).

Klapper, Leora. "The role of factoring for financing small and medium enterprises." *Journal of Banking & Finance* 30.11 (2006): 3111-3130.

Laere, M. v. Modelling international reverse factoring - and the future of supply chain finance. Eindhoven: *TUe MSc Thesis* (2012).

Laseter, T.M., Balanced Sourcing, *Jossey-Bass Publishers* (1998), San Francisco, CA.

Lamberson M. “Changes in Working Capital of Small Firms in Relation to Changes in Economic Activity.” *Mid-American Journal of Business* 10(2), (1995): 45-50.

Lee, Hau L. "Aligning supply chain strategies with product uncertainties." *California Management Review* 44.3 (2002): 105-119.

Lee, Hau L. and Whang, Seungjin. "E-business and supply chain integration." Stanford, CA: *Stanford Global Supply Chain Management Forum*, 2001.

Lekkakos D.S., Serrano A., “Reverse factoring: Implications on SMEs’ operational decisions and performance”, Zaragoza Logistics Center, *Working Paper* (2014).

Lewis, J. “Demand drives supply chain finance,” *Treasury Perspectives*, September-2007 pp. 20-23.

Luo W., Shang K., “Managing Inventory for Entrepreneurial Firms with Trade Credit and Payment Defaults”, IESE Business School – Fuqua School of Business, *Working Paper* (2013).

Mayer A, Garbowsky C. Les vêtements de protection. Choix et utilisation (INRS 995). Paris, France: *Institut National de Recherche et de Sécurité*; 2007. Retrieved December 16, 2008, from: http://www .inrs.fr/inrs-pub/inrs01.nsf/IntranetObject- accesParReference/ED%20995/$FILE/ ed995.pdf

Mäkinen H. Protective clothing— nowadays and vision. In: *Proceedings of the 3rd European Conference on Protective Clothing and Nokobetef* 8 [CD-ROM]. Gdynia, Poland: CIOP-PIB; 2006.

McClellan, M., “Collaborative Manufacturing”, *St Lucie Press*, Delray Beach, FL, (2003).

Milne, R. “Financial Sweetener for Suppliers,” *Financial times*, October 21, 2009.

Morya, K. K., and H. Dwivedi. "Aligning interests of SMEs and a focal firm (MNE) in a global supply chain setup." *The Icfai University Journal of Supply Chain Management* 6.1 (2009): 49-59.

Nehk, Christian. “Supply Chain Financing for the New Trade Finance World”, *TMI - Special Report*, (2012).

Pfohl, Hans-Christian, and Moritz Gomm. "Supply chain finance: optimizing financial flows in supply chains." *Logistics research* 1.3-4 (2009): 149-161.

Poirier, Charles C., and Michael J. Bauer. E-supply chain: using the Internet to revolutionize your business: how market leaders focus their entire organization on driving value to customers. *Berrett-Koehler Publishers*, 2000.

Protective clothing and gear: body/ vehicle armor, fire, chem/bio (Report No. AVM021E). Wellesley, MA, USA: *Business Communications Company*; 2005.

Quinn, F. J. "The clockspeed chronicles." *Supply Chain Management Review* 3.4 (2000): 60-64.

Randall, Wesley S., and M. Theodore Farris II. "Supply chain financing: using cash-to-cash variables to strengthen the supply chain." *International Journal of Physical Distribution & Logistics Management* 39.8 (2009): 669-689.

Ras, Bart. "Make your Supply Chain Finance Programme a Success", *TMI - Special Report; Citi - A treasurer's guide to the financial supply chain*, (2013), available at http://www.treasury-management.com/article/4/236/1964/make-your-supply-chain-finance-programme-a-success.html

Rice, James B., and Richard M. Hoppe. "Supply Chain vs. Supply Chain: The Hype and the Reality." *Supply Chain Management Review*, V. 5, No. 5 (Sept./Oct. 2001), P. 46-54: Ill (2001).

Rich, Nick, and Peter Hines. "Supply-chain management and time-based competition: the role of the supplier association." *International Journal of Physical Distribution & Logistics Management* 27.3/4 (1997): 210-225.

Ronchi, Caniato, Catti, Tumino. "Supply Chain Finance: nuove opportunità di collaborazione nella filiera". *Osservatorio Supply Chain Finance* (2014): 13-40.

Roumen, Roger: "The Impact of Seasonal Demand on the Value of Reverse Factoring". Eindhoven: *TUe MSc Thesis* (2012).

Rudi, Niels, and Netessine S., "Lecture notes on inventory models" (2007).

Sadlovska, Viktoriya and Enslow B. "The New Opportunity to Improve Financial Metrics and Create a Cost-Advantaged Supply Chain", *Supply Chain Finance Benchmark Report - Aberdeen Group,* September-2006.

Salvadori, Nicolas Hurtrez Massimo Gesuasive. "Supply chain finance-From myth to reality." *McKinsey on Payments* 10 (2010): 22-28.

SCDigest – Editorial Staff. "Supply Chain News: Procter & Gamble Late to the Game of Stretching Out Payments to Suppliers, but Ready to Join Party Now" *Supply Chain Digest*, 2-5.2013, available at: http://www.scdigest.com/ontarget/13-05-02-1.php?cid=7006.

Schwarz, Leroy B., and Linus Schrage. "Optimal and system myopic policies for multi-echelon production/inventory assembly systems." *Management Science* 21.11 (1975): 1285-1294.

Seifert, Daniel, and Seifert R. "Supply Chain Finance - What's It Worth?" *IMD (imd.ch)*, October- 2009, Vol. 178.

Seifert, Daniel, and Seifert, R. "Financing the Chain," *International Commerce Review,* (2011), 10(1):32-44.

Sheridan, John H. "Managing the chain." *Industry Week* 248.16 (1999): 50-54.

Smith, Keith. "Profitability versus liquidity tradeoffs in working capital management." *Readings on the management of working capital* (1980): 549-562.

Sufi, Amir. "Bank lines of credit in corporate finance: An empirical analysis." *Review of Financial Studies* 22.3 (2009): 1057-1088.

Supply Chain Management Review (SCMR) and Computer Sciences Corporation (CSC), "The second annual global survey of supply chain progress" (2004), http://www.csc.com/features/2004/55.shtml (data of latest access: December 2005)

Tanrisever, Fehmi, et al. "Value of reverse factoring in multi-stage supply chains." (2012), *Working Paper available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2183991*

Van der Hooft, Steven and Kentgens T. "Trends in Financial Services - Reverse Factoring or Supply Chain Finance", *Capegemini*, (2011), available at http://www.nl.capgemini-consulting.com/resource-file-access/resource/pdf/Trends_in_Financial_Services_0.pdf

Van der Vliet, Kasper, Reindrop M.J. and Fransoo J.C. "Maximising The Value of Supply Chain Finance", *Beta Working Paper,* n.405, January-2013.

Van der Vliet, Kasper, Reindrop M.J. and Fransoo J.C. "Supply Chain Finance: research challenges ahead", *Beta Working Paper*, n.418, November-2013.

Vaughan, Diane. The Challenger launch decision: Risky technology, culture, and deviance at NASA. University of Chicago Press, 1997.

Weinraub, Herbert J., and Sue Visscher. "Industry practice relating to aggressive conservative working capital policies." *Journal of Financial and Strategic Decision* 11.2 (1998): 11-18

Welded Cylinders vs. Tie Rod Cylinders, *Best Metal Products* website. Available at: http://www.bestmetalproducts.com/welded-cylinders-vs-tie-rod-cylinders.