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School of Industrial an Information Engineering
Master of Science in Mechanical Engineering

ANALYSIS OF UNCERTAINTIES

IN THE POSITIONING OF THE COMBINED LINEAR DELTA

AND AGILE EYE SPHERICAL WRIST ROBOTS

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SUMMARY

This project describes the analysis performed to identifty the uncertainty of the positioning of a linear delta and the agile eye spherical wrist combined robot. Given that the manufacturing tolerances and the uncertainty of the measurement chain component affect the accuracy of the positioning system, their effect on the positioning error of the end effector has been analyzed combining the 2^k factorial design and the monte carlo simulations.

The first step of the 2^k factorial design was the selection of the possibly influencing factors, followed by the kinematics of the linear delta robot and agile eye spherical wrist robot. The investigated factors were the geometrical tolerances of the linear delta motor: for current purposes, the length of the links, the radius of the platform, the distance between the rails' axes and the origin, and the distance between the mobile platform of the linear delta robot and the base platform of the agile eye spherical wrist robot assumed two levels (corresponding to a very accurate manufacturing procedure and a standard one). The numerical models allowed studying the probability density function of the positioning error in the entire working volume of the robot. Results were analysed with the Analysis of Variance technique-ANOVA (Minitab software). Results evidenced that in order to obtain the desired positioning accuracy all the geometrical parameters are important, but the most influencing factors are the lengths of the links that results directly a tilting error of the mobile platform of the linear delta robot and it is obviously related with the position of the end effector.

In a second phase, we have supposed to compensate the geometrical errors of the robot and to investigate the instrumental effects (uncertainty of the position of the sliders given by optical encoders). Results showed that the use of rails with an accuracy of 15 μm allow obtaining a standard deviation of the positioning error of 9 μm.

SOMMARIO

Il presente lavoro di tesi descrive le analisi effettuate per identificare l'incertezza di un posizionatore robot che combina un sistema "Linear Delta" e un sistema "Agile Eye Spherical Wrist". Attraverso l'uso combinato di piani fattoriali 2^k e di simulazioni Monte Carlo si è studiato l'effetto delle incertezze di realizzazione dei vari componenti meccanici (lunghezza dei bracci, geometria della piattaforma) sulla posizione dell'end effector all'interno del volume di lavoro.

Il modello cinematico combinato alle simulazioni Monte Carlo ha permesso di stimare la funzione densità di probabilità degli errori di posizionamento (cartesiani e angolari) all'interno del volume di lavoro del robot. I risultati, analizzati in maniera automatica tramite la tecnica dell'analisi della varianza, hanno evidenziato come il parametro più critico sia la lunghezza dei link. In una seconda fase si è supposto di compensare (tramite taratura iniziale del robot) tutti gli errori sistematici e si è studiato l'effetto dell'incertezza degli encoder lineari sull'errore di posizione. I risultati hanno mostrato che gli encoder scelti (con un'accuratezza di 15 μm) consentono di ottenere uno scarto tipo dell'errore atteso di 9 μm.

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CHAPTER 1

1. Introduction

The main objective of this project is the uncertainty analysis of a linear delta-agile eye spherical wrist combined robot designed for an innovative 3D printing mechanism. The agile eye is mounted on the linear delta robot in order to obtain a five degrees of freedom robot. The three translational degree of freedom are provided by the linear delta robot and two rotational degree of freedom are provided by the agile eye. This new generation robot will be utilized for 3-D printing production of parts and components. To obtain high precision positioning, the analysis of uncertainties on manufacturing and measuring processes has been carried out with a procedure recently published by Tarabini et al. that will be described in the next chapters.

Chapter 2 evaluates the calibration process that will be the part of a further project and the analysis result will be the main component of the calibration for minimizing the uncertainties in order to provide high precision.

In capter 3, the kinematics of the linear delta and agile eye robots are defined. The equations are fundamental in order to understand the mathematical dependence between the geometrical parameter and the end effector. They equations have been used to run Monte Carlo simulations to calculate the uncertainties position of the end effector.

Chapter 4 describes the method that combines the factorial design of experiments and the Monte Carlo simulations to analyze the parameters affecting the method uncertainty. Both the effect of manufacturing tolerances and of the measurement chain uncertainties were analyzed. As later explained a two-level factorial design has been used.

Chapter 5 explains the criteria for the choice of the factors and of their levels. The analysis of uncertainties deriving from the measurement chain elements has been performed with the Monte Carlo method with its classical implementation.

Chapter 6 describes the data analysis, performed with Matlab and Minitab softwares and a graphical approach for the analysing will be performed and commented.

Finally, the thesis conclusions are drawn in Chapter 7, which also includes the results critical discussion.

CHAPTER 2

2. ROBOT CALIBRATION

Robot calibration is a technique which is used for improving robot precision, by use of software instead of modifying the mechanical structure or design of the robot. Moreover, for reducing risk of owning to change application programs that are caused by negligible changes and drifts like wear of parts, dimensional drifts and tolerances, component replacement effects calibration techniques can be carried out. In addition, the more exact relationship between joint transducer indications and real workspace position of the end effector are specified and the robot positioning software is continuously modified between each sequential calibration with exploitation of these specified modifications. Robot calibration techniques eventually set controller parameters and execute model designation permanently according to the specified modifications.

The calibration methods can be classified considering their complications. For instance, the modifications in the kinematic or dynamic model of the robot just are taken into account by some methods; on the other hand, other methods just use joint transducer information. Three different levels of calibration can be identified:

1. Joint level calibration

2. Entire robot kinematic calibration

3. Non-kinematic (non-geometric) calibration

Each calibration level includes four important steps:

A. Modelling

B. Measurement

C. Identification

D. Correction

2.1 Level 1- Joint Level Calibration

The determination of exact relationship between the real joint displacement and the signal is generated by the joint displacement transducer is the main aim of this level. This procedure is done as a part of the construction of the robot. When harm has happened or maintenance, which consists of disassembly of joint, has happened, the repetition of calibration has to be carried out by the user. It must be performed every time the robot is powered up and the robot must be moved to reference position [9].

2.1.1 Modelling

In other words, the indication of the joint sensor and real joint displacement are related each other in the concept of level 1 calibration.

The kinematics of the drive system is contained by this procedure when the transducers are placed on the motor shaft:

$$\theta_i = h_i(\eta_i, \gamma_i)$$

θ_i is the actual joint displacement and h_i represents appropriate input – output functional relationship in explicit form. The signal from the transducer is described by η_i and γ_i is the vector of parameters in the function of $h_i()$. In addtion to these, $h_i()$ is assumed to be linear and can be written as:

$$\theta_i = k_{i1}\eta_i + k_{i2} \text{ where } \gamma_i \text{ will be } [k_{i1}, k_{i2}]^T$$

The determination of values of the vector γ_i in correct way is the aim of calibration.

2.1.2 Measurement

The joint is shifted to some certain configuration and the existing joint angle is defined precisely by some outer measurement device in the measurement process. If there is no feasible visual alignment about the joint angle or the expected level of repeatability is not provided by this alignment, alignment holes should be involved in the joint design or a gripper should be located in a certain position in the workspace or the joint angles should be identified precisely by the outer measurement device.

2.1.3 Identification

The identification step is easy for level 1 calibration when the linear model is considered. The gain of the transducer itself and proportion between the joint motion and the transducer motion are combined to merge parameter k_{i1}. The producer generally identifies transducer gain very precisely. By analyzing the joint design, proportion between the joint motion and the transducer motion is easily learned. This ratio is usually established through a gear train or a similar device [9]. If the joint is located at a certain displacement and later transducer signal is identified, k_{i2} will be found.

2.1.4 Correction

The correction step of level 1 calibration is very trivial. The signal that comes from the joint transducer is converted by the controller into a representation of the actual joint angle in software or through specialized circuitry [9]. The rightness of representations of the parameters are guarenteed by correction step.

2.2 Level 2- Entire Robot Kinematic Calibration

The aim is to enhance the precision of the kinematic model of the manipulator as well as the relationship between the joint transducers and the actual joint displacement. Actually, Level 1 calibration is included by the Level 2 calibration. Some assumptions exist about level 2 calibration; the link of the robots will be rigid and joints of the robot will be perfect and unsought motion about their axes does not exist. The spatial kinematic relationship between the joints and links will be determined in Level 2 calibration:

$$x = g(\eta, \gamma, a) \tag{2.1}$$

x represents the vector that describes the position of the end effoctor in the space(6 vector). The vector of joint transducer readings is represented by η. γ corresponds to the vector of coefficients in the relationships between the joint transducer and the actual joint displacement and a is the vector of coefficients in the kinematic model.

2.2.1 Modelling

The development of the kinematic model can be carried out by the different approaches. The Denavit and Hartenberg have set up the most popular method. The method is based on homogeneous transformation matrices. In addition this, the coordinate systems are formed on each joint axis and each coordinate system is then related to the next corresponds to the particular set of coefficients in the homogeneous transformation matrices.

The robots can be studied by this technique thereby investigating small alteration of their kinematic organization. However, in robot configuration, if two revolute joint axes are parallel, an issue can emerge. When two axes are parallel, an infinite number of common normal exists, they have same length. If one of the axes transforms into misaligned position, the problem will occur. Discontinuities or very large changes can occur and they create various numerical difficulties in the identification step. With changes in the kinematic model, some approaches were developed for managing manipulators.

The adequacy and numerical stability of the representation are the main topics for modelling. The variations in the kinematics of the robot in terms of a finite set of parameters are

characterized by the adequacy of the representation which is the ability of the model in terms of this characterization. On the other hand, stable representation suggests that small variations in the kinematics of the robot will cause similar small changes in the model of the robot.

2.2.2 Measurement

The workspace detection of position of the end effector or tool of the robot is included by the measurement phase. For acquiring the workspace inaccuracy data, a comparision sholud be carried out between the real measured positions of the robot end effector and the positions forecasted by the theoretic model. Moreover, measurement is the most difficult and time-consuming phase of robot calibration.

It is not required to make complete measurement of the end effector position, because the aim of the measurement is to specify the manipulator kinematics. It supplies low cost and elimination of large external measuring instruments.

There are some negative point of views about measurement processes [10]; data collection is fatiguing, time-consuming and difficult to automate.The techniques of measurement are applied for robot calibration in laboratory environment mostly. The human intervention is necessary for the set-up and measurement procedures , which cause some problems for robot on-site calibration in an industrial environment.

2.2.3 Identification

Various models and identification algorithms are used by the identification of the parameters in a robot kinematic model. If two similar robots, A and B are considered, the identification problem will become determining the model of robot B given the model of robot A and some measurements are carried out about robot B. “Perfect” or nominal robot is represented by Robot A**.**

The vectors $\gamma_B \; and \; a_b$ are unknown in level 2 calibration. Estimates $\hat{\gamma}_{\mathrm{B}} and \; \hat{a}_{\mathrm{B}}$ are build based on a set of measurement data. The end effector of robot B is located at m positions within the robot workspace. For each of the m locations the relationship between the joint displacement transducers and workspace position:

$$x_B(j) = g_y(\eta_B(j), \gamma_B, a_B), \qquad j = 1, \cdots, m \tag{2.2}$$

The subset of x_B that is measured or determined from the constraint equations. Then;

$$y_B(j) = g(\eta_B(j), \gamma_B, a_B), \qquad j = 1, \cdots, m \tag{2.3}$$

where the appropriate subset of the kinematic equations is represented by g_y.

The measurement equations are modeled as:

$$y_B(j) = \hat{y}_B(j) + u_B(j), \qquad j = 1, \cdots, m \tag{2.4}$$

where $\hat{y}_B$ is read from the end-point sensors or calculated by using the end-point sensing data and u_B represents measurement error(noise).

Moreover, the joint displacement transducers $\eta_B(j)$ are read at each robot configuration $x_B(j)$. It is suggested that this indication is carried out without error. The measurement data is represented by the parameter identification algorithm:

$$\left(\hat{y}_B(1), \eta_B(1)\right), \cdots, (\hat{y}_B(m), \eta_B(m)) \tag{2.5}$$

Indeed, the identification algorithm may provide for a measure of the estimation error.

The only limited thing regarding the minimum number of measurements m is that sufficient data exist to provide a unique estimate. The ideal situation;

$$\upsilon_B(j) = 0 \text{ and } j = 1,....,m \tag{2.6}$$

The identification problem reduces to the problem of solving nonlinear algebraic equations of the form:

$$y_B(j) = \hat{y}_B(j) = g_y(\hat{\eta}_B(j), \hat{\gamma}_B, a_B) \ , \ j = 1,....,m \tag{2.7}$$

To supply that the number of equations is not smaller than the number of unknowns, the minimum number of measurements m must be:

$$m \dim (y_B) \geq \rho + \zeta \tag{2.8}$$

where ρ is the number of elements in the vector γ and ζ is the number of elements in he vector a.

Since the difference in the two robot models A and B may be expressed as:

$$a_B = a_A + \Delta a \tag{2.9}$$

$$\gamma_B = \gamma_A + \Delta\gamma \tag{2.10}$$

X vector can be defined as:

$$X = (\Delta\gamma^T, \Delta a^T)^T \tag{2.11}$$

X and the measurements $\hat{y}_B$ relationship is related and we can involve a "world coordinate error vector" :

$$e(j) = g_y(\eta_B(j), \gamma_B, a_B) - g_y(\eta_B(j), \gamma_A, a_A) \tag{2.12}$$

This formulation is used for purposes of robot calibration and the robot accuracy estimation and it may be related to the vector of parameter offsets X through a linear transformation. This expression:

$$e(j) = H(j)X \tag{2.13}$$

So, the measurement vector:

$$z(j) = \hat{y}_B(j) - g_y(\eta_B(j), \gamma_A, a_A) \tag{2.14}$$

$$z(j) = H(j)X - \upsilon_B(j), \; j = 1,....,m \tag{2.15}$$

where H(j) is a matrix related with the nominal kinematic parameters and the robot configurations during the robot measurement phase.

Let Z be the vector of all measurements:

$$Z= (z^T(1), \ldots., z^T(m))^T \tag{2.16}$$

$$H= (H^T(1), \ldots., H^T(m))^T \tag{2.17}$$

$$V= -(\upsilon_B{}^T(1), \ldots., \upsilon_B{}^T(m))^T \tag{2.18}$$

Equation (2.15) can be rewritten as:

$$Z=HX+ V \tag{2.19}$$

where V and X are random vectors that have certain probability distribution functions.

The noise V related to the calibration measurement depends on the accuracy and the resolution of the end-point sensors, machining tolerances of the calibration fixtures, axis misalignment, encoder mounting, and quantization noise, etc.
Ignoring the models of both X and V is the simplest approach to the identification problem. A unique least-squares estimate:

$$\hat{X}= (H^TH)^{-1}H^TZ \tag{2.20}$$

under the conditions that dim (Z) ≥ dim (X) and H^TH is nonsingular.

We shall not pay attention to the probabilistic approach to the identification. A possible model for robot calibration is to assume that V and X are both Gaussian with zero mean and covariance matrices $\sum_\upsilon$ and $\sum_x$ respectively. Furthermore, it is assumed that V and X are statistically independent from each other. The minimum-variance estimate X is given by:

$$\hat{X}= (\sum_x{}^{-1} + H^T\sum_\upsilon{}^{-1}H)^{-1}H^T\sum_\upsilon{}^{-1}Z \tag{2.21}$$

The error covariance, $\sum_{\hat{X}} = E[(X - \hat{X})(X - \hat{X})^T]$ is given by:

$$\sum_{\hat{X}} = (\sum_x{}^{-1} + H^T \sum_\upsilon{}^{-1} H)^{-1} \tag{2.22}$$

The calibration error is related to the measurement noise and kinematic parameter offsets uncertainty. The model of X can be ignored by setting $\sum_x{}^{-1} = 0$. A repetitive representation of (2.21) and (2.22) is known as the Kalman filter.

The Kalman filtering formulation provides to study the issues related to number of measurements as well as the effects of robot repeatability. Considering the linear filtering problem:

$$X\ (j+1) = X\ (j)\ ,\ \text{where } X(0) \sim N(O, \textstyle\sum_x) \tag{2.23}$$

$$z\ (j) = H(j)\ X(j) + \tilde{\upsilon}(j)\ ,\ \text{where } \tilde{\upsilon}(j) - N(0, \textstyle\sum_\upsilon(j)) \tag{2.24}$$

where N(-,-) is Gaussian distribution with the indicated mean and covariance and $\tilde{\upsilon}(j) = -\upsilon(j)$. The "process" equation (2.23) belongs to a constant random process. This system is time varying since H (j) , which represents the robot configurations, and $\sum_\upsilon(j)$, which represents the measurement noise, may vary from one measurement to the next.

According to the assumption that X(0) and $\tilde{\upsilon}(j)$ are independent, the problem is being unspecified of total number of measurements m and to solve this problem, we include robot repeatability effects:

$$X\ (j+1) = \Phi(j) X\ (j) + W(j),\ \text{where } W(j) \sim N(0, \textstyle\sum_w(j)) \tag{2.25}$$

Determining suitable ''process dynamics" $\Phi(j)$ and ''process noise" covariance $\sum_w(j)$ is a challenging research task.

To determine the number of measurements, the rate of covergence of the error covariance $\sum_x(j)$ must be studied. We may now order the measurements z(j) without any loss of

generality. Let $K_p \geq K_{min}$ be the number of measurement points and $K_r \geq 1$ be the number of repeated measurements at one measurement point. Then,

$$m = K_p K_r \tag{2.26}$$

The measurements for the filter:

measurements at P_1 :

$$z(1), z(K_p + 1), \ldots, z((K_r - 1)K_p + 1)$$

measurements at P_2 :

$$z(2), z(K_p + 2), \ldots, z((K_r - 1)K_p + 2)$$

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measurements at P_{K_p} :

$$z(K_p), z(2K_p), \ldots, z(K_r K_p)$$

A time-invariant version of the identification problem represented by (2.23) and (2.24) may be formulated by first defining:

$$H_{K_p} = (H^T(1), \ldots, H^T(K_p))^T \tag{2.27}$$

$$\sum_{\upsilon,K_p} = \text{diag}\ \{\sum_{\upsilon}(j)\} \tag{2.28}$$

$$V_{K_p} = (\upsilon^T(1), \ldots, \upsilon^T(K_p))^T \tag{2.29}$$

Then

$$Z_{K_p} = H_{K_p} X(i) + V_{K_p}(i)\ ,\ \text{where}\ \ i = 1,\ldots, K_r \tag{2.30}$$

where i = 1+int((j - l)/ K_p) and "int" represents the "largest integer not greater than."

For an observable system, we require that the rank(H_{K_p})=dim(X), which implies that $K_{min} \geq$1+int (dim (X)/dim z (j)). It means that this time-invariant formulation does not include identification of the model of robot repeatability. Using (2.22) repeatedly it may be shown that:

$$\sum_{\hat{X}}(i) = (\sum_{x}{}^{-1} + iH_{K_p}{}^{T} \sum_{\upsilon,K_p}{}^{-1} H_{K_p})^{-1} \tag{2.31}$$

If H_{K_p} is nonsingular and $\sum_{\upsilon,K_p}$ is positive definite, then $\sum_{\hat{X}}(i)$ goes to 0 as K_r goes to infinity. This can erroneously result that calibration can be infinitely accurate as the number of measurement points increases.

2.2.4 Correction

Correction is the final step and decisive part of calibration. New model is implemented in the position control software of the robot. The two robots A and B, whose kinematic model X=g(η,λ,a) is known perfectly for model A and known with a certain level of uncertainty for model B, are assumed to share the same task. Typically, two types of tasks exist; data driven tasks are described in terms of a sequence of k target world coordinate positions {x(1),...,x(k)} and taught tasks are described in terms of a sequence of k joint level points {η(1),..., η(k)}.

$$x(j) = x_A(j) = g(\eta_A(j), \gamma_A, a_A)$$

$$x(j) = x_B(j) = g(\eta_B(j), \gamma_B, a_B)\ ,\quad j = 1,\ldots,k$$

However, in practice:

$$x(j)=x_A(j)= g(\eta_A(j), \gamma_A, a_A) \tag{2.32}$$

$$x_B(j)= g(\eta_B(j), \hat{\gamma}_B, \hat{a}_B)+\epsilon_{id}(j) \tag{2.33}$$

$$\|x(j) - x_B(j)\| \leq \epsilon_{ac}(j) \tag{2.34}$$

where $\epsilon_{ac}(j)$ is a prescribed accuracy measure attached to every task point and $\epsilon_{id}(j)$ represents world coordinate errors due to imperfect calibration.

In the correction phase, implementing of the identified model of robot B is made to satisfy the necessity expressed in (2.34). In data-driven applications, the model correction phase for robot B (at least conceptually):

Step 1: Substitute $x_B(j) = x(j)$ (i.e., ignoring the unknown errors, $\epsilon_{id}(j)$).
Step 2: From the inverse kinematics of $x(j)= g(\eta_B(j), \hat{\gamma}_B, \hat{a}_B)$, determine the joint commands.
Step 3: Run the application. If the inequality expressed in (34) is not provided and improving the calibration accuracy is meaningless, the task may not be performed.

In step 2, analitic solution of inverse kinematics can be insufficient, so numerical algorithms should be used like Newton-Rhopson method to compansate Cartesian errors [14]. A new closed-form solution which would represent a perturbation about the parallel or intersecting axes solution can be a good solution.
In the case of taught applications; robot A that has been taught an application produced by a large number of task points. Because of wear, part replacement, maintenance, robot replacement, etc., the kinematic structure of the robot is changed and a “new” robot B appears. It must perform the tasks of robot A.

Level 2 is used when a nominal kinematic model is known; but kinematic model of robot A is sometimes not available. Hence, the feasibility of this technique depends on the tasks that are useful for only small portions of the workspace. This approach is applied for the joint level and it will not be discussed under the title of Robot Calibration.

2.3 Level 3-Non-Kinematic Calibration

In Level 2 calibration, the position and orientation of the end effector could be defined as a function of only the joint displacements and the kinematic structure of the robot. This is an assumption and it will be valid in the cases of the links are rigid, the joints are frictionless, no allowance of undesired motion in joint axes, and the robot is not under dynamic control. If these conditions are not provided, level 3 calibration should be performed.

Apart from geometrical errors, non-geometrical errors which are joint and link flexibility, gear transmission error, backlash in gear transmission and temperature effect affect the accuracy of the robot. While the position control software model is being modified, these errors must be considered. If the robot is under dynamic control, the position control software model takes the following form:

$$x = l\,(\eta\,,\,\gamma\,,\,a\,,\,\dot{x}\,,\,\ddot{x}\,,\,\mu) \tag{2.35}$$

where $\dot{x}$ and $\ddot{x}$ represent both the translational and angular velocity and acceleration of the end effector and μ represents the set of coefficients of the dynamic terms in the equations of motion for the robot.

The equation (2.35) represents an extremely complicated functional relationship, under the assumptions perfectly rigid links in the robot and the frictionless joints. For this reason, only a limited amount of work has been done in the area of level 3 calibration. Existing work on level 3 calibration has involved the identification of the mass and inertial parameters for different links in the robot.

There are many researches about calibration methods and they need to be resolved before level 3 calibration can become widely used. For calibration under dynamic control, the velocity and the acceleration of the end effector must be known as well as its position. The complexity of the dynamic model and the uncertainty of the datas from the measurement make both the identification and the correction phases challenging problems.

CHAPTER 3

3. Descriptions and Kinematics of the Linear Delta and Kinematic of the Agile Eye

3.1 Linear Delta Robot

The linear Delta robots are included in the parallel kinematics machine group. Parallel kinematics machines become more and more popular in industrial applications [1]. Their important opportunities over serial manipulators prove this growing attention. Their important properties are better accuracy, lower mass/inertia properties, and higher structural stiffness (i.e. stiffness-to-mass ratio) [2]. Their special kinematics provide these properties. There are parallelogram links which connect to the end effector give always parallel movement to the end effector with respect to the base. Thus, the links work in parallel against the external force/torque, eliminating the cantilever-type loading and increasing the manipulator stiffness [4]. In addition to these, three prismatic joints move separate arms (links) which connect to a single triangular end plate [4] in linear Delta robot mechanisms.

The machine which is responsible of the translational DoFs is a linear Delta. This particular architecture is made of three links of fixed length connecting the mobile platform with three different rails. The actuation is provided by three electrical motors connected to three linear transmission units. Three independent PUS (Prismatic-Universal-Spherical) kinematic chains can be identified. P means that this linear delta robot is driven through prismatic joints. The links are actually three parallelograms; as a matter of fact, this particular architecture ensures that the mobile platform is always parallel to the ground. In order to compute the kinematics of this machine two different reference systems are defined, the inertial frame placed on the ground and the Tool-Center-Point (TCP) which is fixed to the mobile platform. Since the links are parallelograms, the axes of the two frames are always parallel in whatever pose of

the robot. The symbols which will be used in the kinematic problem can be summerized in the following table:

Symbol	Meaning
O-xyz	Global reference system
$\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$	Versors of the global reference system
TCP-xyz	Local reference system, attached to the platform
$\hat{\mathbf{x}}', \hat{\mathbf{y}}', \hat{\mathbf{z}}'$	Versors of the local reference system
TCP	tool center point
A_i	i-th center of the base joint
B_i	i-th center of the platform joint
$\underline{p} = \{x_p, y_p, z_p\}^T$	Coordinates of the TCP
$\underline{b}_i$	coordinates vector of the i-th platform joint
$\underline{l}_i = l_i \hat{n}_i$	length vector of the i-th link (module and versor)
$\underline{s}_i = \{s_{i,x}, s_{i,y}, 0\}^T$	position vector of the i-th rail
q_i	coordinate of the i-th joint
$\hat{u}_i$	orientation versor of the i-th rail

Table 3.1: Symbols used in the solution of the kinematic problem

3.1.1 Inverse Kinematics

For the i-kinematic chain it is possible to write:

$$\underline{l}_i = \underline{d}_i - q_i \hat{u}_i \text{ where } \underline{d}_i = \underline{p} + \underline{b}_i - \underline{s}_i \tag{3.1}$$

and so it is possible to square the previous expression obtaining:

$$l_i^2 = \underline{l}_i^T \underline{l}_i = \left(\underline{d}_i - q_i \hat{u}_i\right)^T \left(\underline{d}_i - q_i \hat{u}_i\right) = \underline{d}_i^T \underline{d}_i - 2\underline{d}_i^T \hat{u}_i q_i + q_i^2 \tag{3.2}$$

By solving this second order equation the relationship between the slider coordinates and the platform position is readily found to be:

$$q_i = \underline{d}_i^T \hat{u}_i - \sqrt{\underline{d}_i^T (\hat{u}_i \hat{u}_i^T - [I]) \underline{d}_i + l_i^2}$$

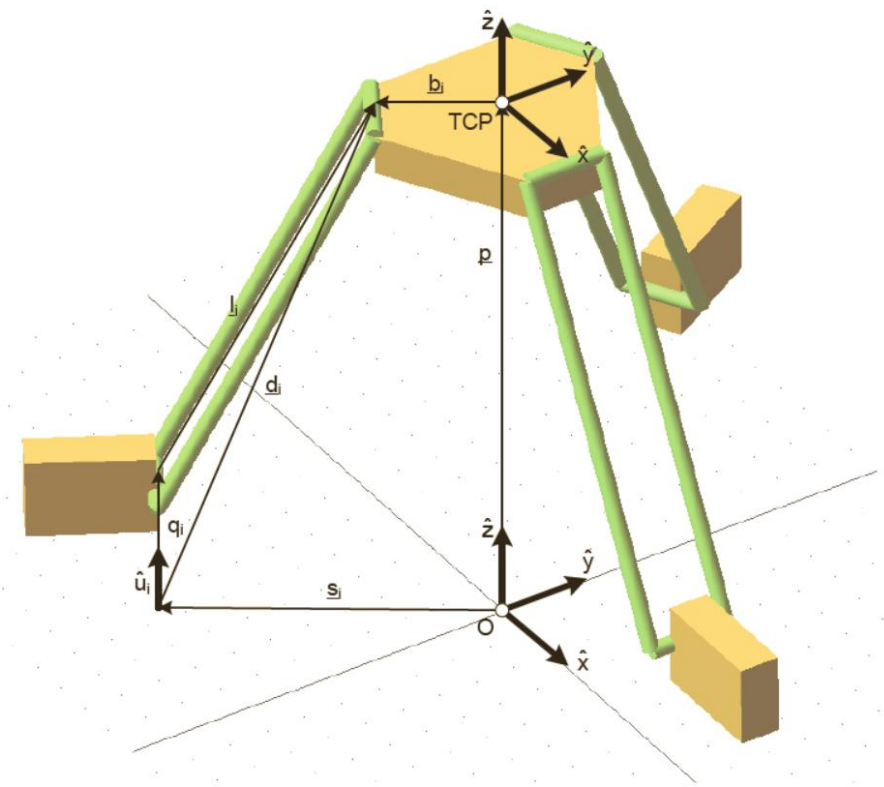

Figure 3.1: Linear Delta Robot

3.1.2 Direct Kinematics

If we consider the second part of the equation 3.1 and 3.2 together, it is possible to write:

$$l_i^2 = \underline{l}_i^T \underline{l}_i = \left(\underline{d}_i - q_i \hat{u}_i\right)^T \left(\underline{d}_i - q_i \hat{u}_i\right) \text{ where } \underline{d}_i = \underline{p} + \underline{b}_i - \underline{s}_i$$

$$l_i^2 = \left(\underline{p} + \underline{b}_i - \underline{s}_i - q_i \hat{u}_i\right)^T \left(\underline{p} + \underline{b}_i - \underline{s}_i - q_i \hat{u}_i\right)$$

$$\underline{p}^T \underline{p} + \underline{b}_i^T \underline{b}_i + \underline{s}_i^T \underline{s}_i + 2\underline{p}^T \underline{b}_i - 2\underline{p}^T \underline{s}_i - 2\underline{b}_i^T \underline{s}_i - 2q_i p_z + q_i^2 - l_i^2 = 0 \qquad (3.3)$$

The all cross production terms between orientation versor of the i-th rail $\hat{u}_i$ and coordinates vector of the i-th platform joint $\underline{b}_i$, position vector of the i-th rail $\underline{s}_i$ are equal to zero. Because $\underline{b}_i$ and $\underline{s}_i$ do not have component at z direction.

Given that both the rails and the platform joints are out of phase by 120° about the z axis, the following expressions hold:

$$\underline{p}^T\underline{b}_i = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \begin{bmatrix} R_{p_x} R_{p_y} 0 \end{bmatrix} = p_x R_p \cos\left(\frac{2(i-1)\pi}{3}\right) + p_y R_p \sin\left(\frac{2(i-1)\pi}{3}\right)$$

$$\underline{p}^T\underline{s}_i = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \begin{bmatrix} s_x s_y 0 \end{bmatrix} = p_x s \cos\left(\frac{2(i-1)\pi}{3}\right) + p_y s \sin\left(\frac{2(i-1)\pi}{3}\right)$$

with $i = 1,2,3$

where Rp and s are respectivelly the modules of $\underline{b}_i$ and $\underline{s}_i$.

If now all the scalar product in equation 3.3 are computed, it could be get:

$$p_x^2 + p_y^2 + p_z^2 + 2(R_p - s)\cos\left(\frac{2(i-1)\pi}{3}\right)p_x + 2(R_p - s)\sin\left(\frac{2(i-1)\pi}{3}\right)p_y - 2q_i p_z + (R_p - s)^2 + q_i^2 - l_i^2 = 0 \quad (3.4)$$

If we consider equation 3.4, with $i = 1,2,3$:

$$p_x^2 + p_y^2 + p_z^2 + 2(R_p - s)p_x + (R_p - s)^2 + q_1^2 - l_1^2 - 2q_1 p_z = 0$$

$$p_x^2 + p_y^2 + p_z^2 - (R_p - s)p_x + \sqrt{3}(R_p - s)p_y + (R_p - s)^2 + q_2^2 - l_2^2 - 2q_2 p_z = 0$$

$$p_x^2 + p_y^2 + p_z^2 - (R_p - s)p_x - \sqrt{3}(R_p - s)p_y + (R_p - s)^2 + q_3^2 - l_3^2 - 2q_3 p_z = 0$$

If you consider 2 times equation 3.4 with i = 1 minus the same equation with i = 2 and i = 3, what it could be get is:

$$6(R_p - s)p_x - 2(2q_1 - q_2 - q_3)p_z + 2q_1^2 - q_2^2 - q_3^2 - 2l_1^2 + l_2^2 + l_3^2 = 0 \quad (3.5)$$

And by subtracting equation 3.4 with i = 3 from the same equation with i = 2 you get:

$$2\sqrt{3}(R_p - s)p_y - 2(q_2 - q_3)p_z + q_2^2 - q_3^2 + l_3^2 - l_2^2 = 0 \quad (3.6)$$

These two equations establish a linear relationship between px and pz and between py and pz. If the expressions of px and py as functions of pz are substituted in equation 3.4 with i = 1 the following second order equation is obtained:

$$k_1 p_z^2 + k_2 p_z + k_3 = 0$$

$$k_1 = \frac{(2q_1 - q_2 - q_3)^2 + 3(q_2 - q_3)^2}{9\left(R_p - s\right)^2} + 1$$

$$k_2 = \frac{3(q_2 - q_3)(q_3^2 - q_2^2 - l_3^2 + l_2^2) - (2q_1 - q_2 - q_3)(2q_1^2 - q_2^2 - q_3^2 - 2l_1^2 + l_2^2 + l_3^2)}{9\left(R_p - s\right)^2} + \frac{2(q_1 - q_2 - q_3)}{3} - 2q_1$$

$$k_3 = \frac{(2q_1^2 - q_2^2 - q_3^2)^2 + (2l_1^2 - l_2^2 - l_3^2)^2 - 2(2q_1^2 - q_2^2 - q_3^2)(2l_1^2 - l_2^2 - l_3^2)}{36\left(R_p - s\right)^2} + \frac{(q_2 - q_3)^2 + (l_2^2 - l_3^2)^2 - 2(q_2 - q_3)(l_2^2 - l_3^2)}{12\left(R_p - s\right)^2} - \frac{(2q_1^2 - q_2^2 - q_3^2)}{3} + \frac{(2l_1^2 - l_2^2 - l_3^2)}{3} + \left(R_p - s\right)^2 + q_1^2 - l_1^2$$

From this equation two solutions are obtained, but one of them provides a negative value of p_z which is not physically admissible, and so the final value of pz will simply be:

$$p_z = \frac{-k_2 + \sqrt{k_2^2 - 4k_1 k_3}}{2k_1}$$

The values of p_x and p_y can be easily computed from equation 3.5 and equation 3.6:

$$p_x = \frac{2(2q_1 - q_2 - q_3)p_z - 2q_1^2 + q_2^2 + q_3^2 + 2l_1^2 - l_2^2 - l_3^2}{6(R_p - s)}$$

$$p_y = \frac{2(q_2 - q_3)p_z - q_2^2 + q_3^2 - l_3^2 + l_2^2}{2\sqrt{3}(R_p - s)}$$

3.2 Agile Eye

The 2-DOF Agile Eye is included in spherical parallel robots. Generally, the mechanism of 2-DOF Agile Eye is composed of 4 links, two rotary actuators connected to the base with two limbs to the end-effector [5]. One limb consists of two links and other limb is a single link. The revolute joints are used in order to connect links and these joints are designed to pass through a common fixed point. This point is the wrist center of the mechanism.

Speaking about the rotational DoFs, they are realized through a spherical agile wrist. This PKM is mounted on the mobile platform of the linear Delta. In order to solve the kinematics of this manipulator the evaluation of the orientation of every circular link is required. The final expressions that relates the rotations of the motors (θ_1; θ_2) to the roll and pitch (α and β respectively) of the platform are:

$$\begin{cases} \theta_1 = \tan^{-1}(\cos\alpha \tan\beta) \\ \theta_2 = \alpha \end{cases} \tag{3.7}$$

According to the inverse kinematics, the direct kinematic is easily computed as:

$$\begin{cases} \alpha = \theta_2 \\ \beta = \tan^{-1}\left(\frac{\tan\theta_1}{\cos\theta_2}\right) \end{cases} \tag{3.8}$$

The angle α represent the rotation of the end effector around the x axis. In addition, the angle β represents therotation of the end effector around the y axis.

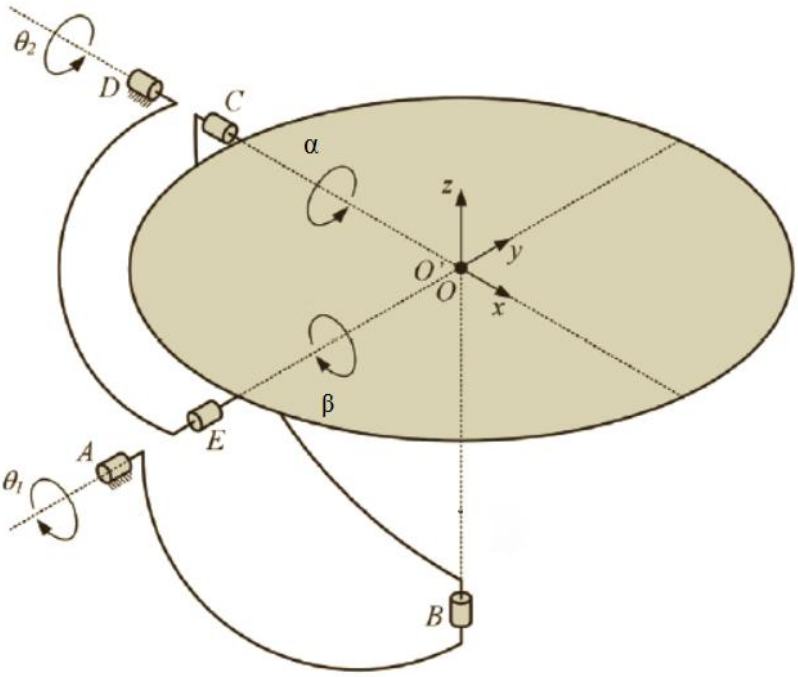

Figure 3.2: The Kinematics of the Agile Eye

3.3 Tilting and Shifting of the Mobile Platform because of the Perturbated Values of Influence Quantites

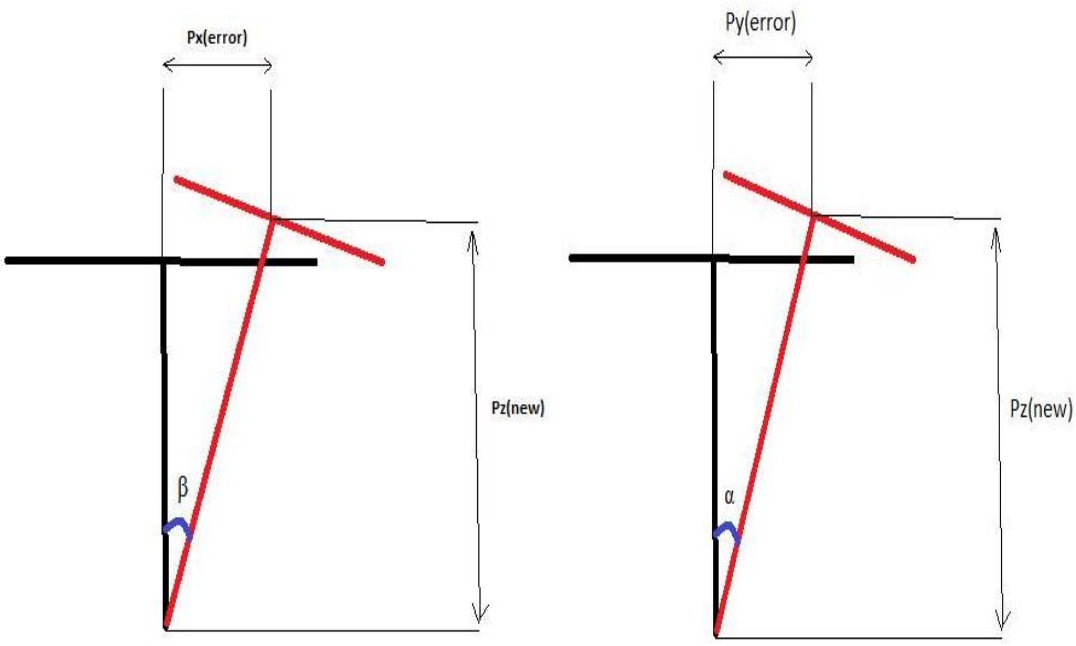

Figure 3.3: Roll and Pitch angle description

The perturbated values of influence quantities by the random noise cause tilting on the mobile platform of the delta robot and consequently on the base platform of the agile eye with the angles β and α. The error at the x direction of the mobile platform results in a tilt around the y axis and it naturally creates a tilt also on the end effector of the agile eye. In addition to this, the error at the y direction of the mobile platform results in a tilt around the x axis and it naturally creates a tilt also on the end effector of the agile eye. The formulas of these angles can be expressed in that way:

$$\beta = \tan^{-1}\left(\frac{P_x(\text{error})}{P_z(\text{new})}\right) \tag{3.9}$$

$$\alpha = \tan^{-1}\left(\frac{P_y(\text{error})}{P_z(\text{new})}\right) \tag{3.10}$$

$$\text{shift= h(new)}\sqrt{\cos^2\alpha + \cos^2\beta} \tag{3.11}$$

where new terms indicate perturbated values and error values indicate perturbation magnitudes.

CHAPTER 4

4. Method Description

When starting to explain the methodology, an ideal single input (x) single output (y) mechanical system should be considered and mathematical relationship f expresses the dependence between y and x.

$$y = f(x) \tag{4.1}$$

If a real mechanical system is considered, we will figure out that not only the input x affect the output $\hat{y}$ but also a set of $k-1$ influence quantities (x_1,x_2,...,x_{k-1}) affect the output $\hat{y}$. Influence quantities can be described as ‘‘the quantities that, in a direct measurement, do not affect the quantity that is actually measured but affect the relation between the indication and the measurement result’’[6]. They are also called as factors. In the following, the mechanical system input and of the influence quantities (IQ) are combined and a generalized input vector can be defined. The mathematical relationship indicates that the real mechanical system behavior $\hat{f}$ is different with respect to the ideal mechanical system and equation 1 can be modified for real mechanical system as following:

$$\hat{y} = \hat{f}(x, x_1, x_2, \dots, x_{k-1}) \tag{4.2}$$

Both the mathematical relationship $\hat{f}$ and the statistical distribution of the influence quantities may change because of different mechanical system design configurations. In any specific configuration, the error e between ideal and real situations is defined as:

$$e = y - \hat{y} = f(x) - \hat{f}(x, x_1, x_2, \dots, x_{k-1}) \tag{4.3}$$

Either deterministic or random variables may define the influence quantities; the PDFs of the generalized input vector and the system model function $\hat{f}$ specify the deriving error statistical

distribution. The uncertainty-driven mechanical system design depends on the identification of the system configuration (i.e. IQ values or PDFs).

For the optimized mechanical system design, offered method is formed by these steps [11]:

1. Mechanical system should be analyzed for the determination of the influence quantities.
2. The instrument model $\hat{f}$ is generated with influence quantities.
3. The ideal input–output model as equation 1 is generated.
4. Mechanical system configurations are generated by the use of design of experiment (DOE) techniques.
5. Monte Carlo simulations are performed for each configuration and the error probability density function PDF_e is identified.
6. A related ‘‘error indicators’’(standart deviations and mean values of obtained error) from PDF_e are identified. They are used as response variable (RV) in step 7.
7. Explorative Data Analysis and Analysis of Variance on RV.

By studying the physical phenomena which is related with the working principle of mechanical system, the environment where the mechanical system has to be used and the system design, the influence quantities of a sytem can be determined. The uncertainy analyses includes influence quantities identification (i.e, the quantities which undesirably affect the system output). The influence quantities may become perturbations or may be defined by the parameters associated to the selections of different design.

In this project, influence quantites are

- l1, l2, l3 length of the linear delta robot links,
- Rp radius of the platform of the linear delta robot,
- s the distance between the rails' axes and the origin,
- h the distance between the center of the mobile platform of the linear delta robot and that of the platform of agile eye.

These factors, in reality, are not represented by a deterministic value, but by a probability density function summarizing the presence of disturbances. The main causes are the clearences and the environmental effects. Consequently they are included with the other inputs of the mechanical system (linear delta robot+agile eye) into the model of the system. In conclusion, they influence the position of the end effector and naturally create a difference between real and ideal situation.

After this step, the system model $\hat{f}$ should be generated. In exactly, the mathematical relationship between the influence quantities and the response is obtained. In this project, the relation ship between influence quantities and position of the end effector at x, y, z directions is represented by direct kinematics of the robots and the system model $\hat{f}$ is obtained by imposing influence quantities with corresponding random errors and slider positions at the z direction as inputs (q1,q2,q3). Thus, for obtaining error between ideal position and real position, in order to get positioning error, in this step modification of the direct kinematics of the system components should be performed.

For the simulation of the calibration method, the ideal input–output model should be generated. This step consists in the identification of the system ideal output with imposing of known inputs to the ideal input-output model. Direct kinematics of the linear delta robot and agile eye represents the ideal input-output model. In addition, generation of the direct kinematics is mentioned in the kinematic of the linear delta and kinematic of the agile eye.

After obtaining input-output model, the set of PDFs or random datas one for each configuration to be used in the MC trials. The 2^k factorial design can be took as the simplest plan for the simulations planning. Every of the k elements of the generalized input vector may have two levels. The influence quantities are described by these levels and they may be value or PDF of influence quantities. The 2^k factorial design will also mentioned in its cahapter in detail.

In our project we have 6 influence quantities and each one have two levels (high and low levels). According to the these levels, 64 configurations are obtained in Matlab software with using datas which are formed by high level or low level situations of influence quantities. In exactly, influence quantities have two distributions, which are formed with introducing

random errors, that one of them has high standart deviation and one of them has low standart deviation.

After the the identification of the IQ levels combination by the DOE, Monte Carlo process is started. Monte Carlo process [11] is used for each obtained configuration with The 2^k factorial design and it determine the difference between real situations and ideal situations about the mechanical system. For 2^6 factorial design, 64 configurations exist, for each configuration positioning error distributions of the end effector are obtained by the Monte Carlo simulations in linear delta robot plus agile eye system.

The result of the MC process [16] is a set of 2^k statistical distributions, PDF_{e1}, PDF_{e2}, . . . $PDFe2^k$ of the errors. Moreover ‘‘error indicators’’ are obtained for evaluation of the results. These may be standart deviations or mean values of obtained error. The obtained PDFs of positioning errors at three directions from 64 configurations were evaluated in terms of standart deviaton and mean values and so 64 standart deviations and mean values were acquired for each configuration in this project. And then, they were used as response values in 2^6 factorial design.

In step 7, the analysis of variance, calculating effects and residual analysis are realized according to the 2^k factorial design on the all responses of the system. Because these method techniques determine whether a influence quantity influence the RV mean or not. Moreover, the presence of specific factor interactions which cause large errors or uncertainty may be exposed by the residual analysis.

In this project, standart deviatons of the positioning error at three directions which are found in step 6 were used in the analysis of variance, calculating effects and residual analysis. For all analysis, Minitab software was used and to try to understand effects of the influence quantities and their interactions on the responses, both from graphs and some results are utilized [12,13]. Which factors or interactions are effective at which positioning error was evaluated. And how does Minitab software work according to the 2^k factorial design and make these analysis depending on some methodology are explained in the 2^k factorial design part of the thesis.

4.1 2^k Factorial Design

Factorial designs are generally implemented in experiments involving several influence quantities where it is necessary to study the interaction effect of the factors on a response. However, some specific cases of the general factorial design are important because they are widely used in research work and they create the basis of other designs of considerable practical value.

The most important thing about these k factors, each factors should be at only two levels. These levels may be both quantitative and qualitative. If these levels are quantitative, they may be two values of temperature, pressure or time. On the other hand, if they are qualitative, they may be two machines, two operators, the “high’’ and “low’’ levels of a factor, or perhaps the presence and absence of a factor. $2 \times 2 \times \cdots \times 2 = 2^k$ configurations are required by a complete replicate of such a design. It is called 2^k factorial design.

The 2^k design is particularly practical in the former stages of experimental work, when many factors are probably to be studied. It supplies the smallest number of runs for which k factors can be studied in a complete factorial design. Because each factor has only two levels, it must be supposed that over the range of the selected factor levels the response is approximately linear.

In this project, in order to determine effects of the influence quantites (factors) (l1,l2,l3 length of the links, Rp radius of the platform, s the distance between the rails' axes and the origin, h the distance between the center of the mobile platform of the linear delta and that of the platform of agile eye) on the response which is positioning errors of the end effector, 2^k factorial design was implemented. Thus, the following questions could be answered; how influence quantities affect the position of the end effector and which influence quantities be effective at which direction of the end effector.

In order to explain the concept of the 2^k factorial design, the simplest design of it which is the 2^2 factorial design can be investiged. There are two factors that A and B, each at two levels. And these two levels represent the low and high level of the two factor. For this spesific situation, four configurations exist and can be called as treatment combinations. A treatment combination can be represented by a serial lowercase letters. If a lowercase letter exist, the corresponding factor is run at the high level in that configuration; if it does not exist,

the factor is run at its low level. For instance, treatment combination a indicates that factor A is at the high level and factor B is at the low level. This notation can be used for any factorial design. For example, when five influence quantities exist, ab lowercase indicates that A and B factors run at high level and remain C, D, E factors run at low level. In the light of such information, 2^k factorial design could be arranged as 2^2 factorial design in this way;

Treatment	A	B
(1)	−	−
a	+	−
b	−	+
ab	+	+

Figure 4.1: 2^2 factorial design

2^k factorial design generally includes calculation of effects of influence quantities and their interactions, also analysis of variance thereby calculating sum of squares of them and residual analysis.

2^k factorial design may be started with calculation of effects of factors and their interactions. For example, in the 2^2 design, the effects of interest are the main effects A and B and the two-way interaction AB. In the figure 4.1, configurations (1), a, b, ab represent the totals of all n observations which were taken at these design points. First of all, in order to estimate main effects of the influence quantities, the observations where an influence quantity is at the high level would be averaged according to the number of observations n and the average of observations where this influence quantity is at low level would be subtracted. For instance to calculate main effects of A and B:

$$A = \bar{y}_{A+} - \bar{y}_{A-} = \frac{a+ab}{2n} - \frac{b+(1)}{2n} = \frac{1}{2n}[a + ab - b - (1)] \tag{4.4}$$

$$B = \bar{y}_{B+} - \bar{y}_{B-} = \frac{b+ab}{2n} - \frac{a+(1)}{2n} = \frac{1}{2n}[b + ab - a - (1)] \tag{4.5}$$

The quantities in brackets in equations (4.4) and (4.5) are called contrast. For example, the contrast for A is:

$$\text{Contrast}_{\text{A}} = [\text{a} + \text{ab} - \text{b} - (1)] \tag{4.6}$$

According to the this explanation, for any 2^{k} factorial design, main effects can be calculated in the following way:

$$\text{Effect} = \frac{\text{Contrast}}{\text{n}2^{\text{k}-1}}$$

Consequently, in order to estimate 2-way interaction of the influence quantities, the difference between the average A effects at low and high level of B would be taken. The AB interaction effect could be represented by the one half of the this average according to the procedure of 2^{k} factorial design:

B	Average A effect
High(+)	$\frac{(\text{ab} - \text{b})}{\text{n}}$
Low(−)	$\frac{(\text{a} - (1))}{\text{n}}$
Difference	$\frac{(\text{ab} + (1) - \text{a} - \text{b})}{\text{n}}$

Table 4.1: The effect of A with respect to B

In the light of these informations, the effect of AB interaction:

$$AB = \frac{(ab + (1) - a - b)}{2n}$$

This method which provide to calculate the effect of interactions could be used for any 2^k factorial design. For instance, if three factors exist in the design, the effect of the ABC interaction will be calculated thereby evaluating the difference between AB interaction at the low and high levels of C. However, for estimating interaction's effect (especially for the high-way interactions), the easier way will be created through forming a table for treatment combinations (configurations) and effects.

The coefficients in the contrasts are always either +1 or -1. A table which includes plus and minus signs can be created to determine the sign of each configurations for a specific contrast.

Treatment Combination	Factorial Effect			
	I	*A*	*B*	*AB*
(1)	+	−	−	+
a	+	+	−	−
b	+	−	+	−
ab	+	+	+	+

Table 4.2: Signs for effects in 2^2 factorial design

While the column headings are established by the effects (A,B main effects, AB interaction, I which represents total), the row headings are formed by configurations (treatment combinations). The easier way to calculate the effects of interactions draws the attention in this table. The signs of contrast of AB is the product of signs from the columns A and B. It means that the coefficients in the contrast of main effects which form interactions could be used to obtain corresponding interactions' contrasts. Once the signs for the main effect columns have been determined, the signs for the remaining columns (interactions) can be obtained by multiplying the appropriate main effect row by row.

As contrasts are used for calculating effects, they also can be used in calculating sum of squares for main effects and interactions. Thus, analysis of variance could be realized through estimating corresponding sum of squares by means of contrasts. The sums of squares formulas for 2^2 factorial design are:

$$SS_A = \frac{[a + ab - b - (1)]^2}{4n}$$

$$SS_B = \frac{[b + ab - a - (1)]^2}{4n}$$

$$SS_{AB} = \frac{[ab + (1) - a - b]^2}{4n}$$

For any 2^k factorial design, the formula of sum of square can be identified in this way:

$$SS = \frac{(\text{Contrast})^2}{n2^k}$$

In addition to them, after calculating the total sum of squares SS_T with 4n-1 degrees of freedom for 2^2 factorial design and 2^kn-1 degrees of freedom for 2^k factorial design and obtaining the error sum of squares SS_E with 4(n-1) degrees of freedom for 2^2 factorial design and 2^k(n-1) degrees of freedom for 2^k factorial design, the analysis of variance will be completed.

The numerical estimates of the effects of the factors and their interactions indicate whether a factor or an interaction has affected corresponding response in an effective manner or not. In exactly, they determine the order of importance among influence quantities and interactions. Moreover, they also show that the direction of the factors and interactions with respect to responses. If a factor has a positve number effect; as this factor increases from low to high level, response also will increase.

The analysis of variance also indicates same things as the effects indicate. It confirms conclusions obtained by examining the magnitude and direction of the effects.

By means of adjustment of a regression model [15] to the data, the residuals from 2^k factorial design could be obtained. The aim of the residual analysis is determining how the observations deviate from the expected values which are calculated by the regression model. For a 2^2 factorial design, the regression model can be created as the following:

$$Y = \beta_0 + \beta_1 x_1 + \in$$

After the effects were calculated and analysis of variance was made: active variables, which are the most effective on the response, among the influence quantites and interactions could be determined. And active variables should be represented in the regression model. The above regression model only contains one active variable. It is represented by a coded variable x_1.

The low and high levels of this active variable are assigned values $x_1 = -1$ and $x_1 = +1$, respectively.

The least square fitted method is obtained through calculating β_0 and β_1 and after that expected value will be estimated. The intercept $\hat{\beta}_0$ is the grand average of all observations and the slope $\hat{\beta}_1$ is one-half the effect estimate for active variable.

Predicted values are calculated at configurations by means of this model. For example, at a configuration where active variable is at low level, x_1 should be equal to -1. After the obtaining predicted values, residuals will be estimated thereby to subtract the value of all n observations from predicted values.

If a 2^k factorial design has large number of factors, regression model can include active variables more than one active variable and it can be modified as the following way:

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \in$$

As is seen from this regression model, there are three active variables. These active variables are two factors and one interaction. This regression model can be also used to obtain the predicted values and residuals.

After the residual analysis, a normal probability plot of the residuals can be drawn in order to analyze whether the residuals lie approximately along a straight line or some residuals deviate. Thus, we can determine any problem with normality in the data.

In addition to all of these, if the number of influence quantities increase, the number of effects also increase in the factorial design. For example, for 2^6 factorial design has 6 main effects, 15 two-way interactions, 20 three-way interactions, 15 four-way interactions, 6 five-way interactions, and 1 six-way interaction. In most situations the sparsity of effects principle applies; that is, the system is usually dominated by the main effects and loworder interactions [7]. It can be supposed that high order interactions can be neglected and when the number of influence quantities is reasonably large, the 2^k factorial design is run at a single replicate and higher order interactions are unified and included in estimate of error.

CHAPTER 5

5. Level Selection

The factorial design is suggested as a powerful technique for determining the effects of factors on a mechanical system or experiment. Generally, experimental trials (or runs) are carried out at all configurations of factor levels in a factorial design. For example, if a chemical engineer is interested in investigating the effects of reaction time and reaction temperature on the yield of a process, and if two levels of time (1 and 1.5 hours) and two levels of temperature (125 and 150° F) are considered important, a factorial experiment would consist of making experimental runs at each of the four possible combinations of these levels of reaction time and reaction temperature [7].

Thus, for 2^k factorial design, the most important two levels should be choosen for each factors and then the analysis of each configuration is performed. Moreover, this design provides the smallest number of runs for which k factors can be studied in a complete factorial design [7,8]. Because just two levels exist for each factor. In this project, a lot of datas for influence quantities are produced in order to analyze positioning error of end effector but two important levels are introduced to the factors among datas to utilize 2^k factorial design's opportunities.

According to our analysis, for manufacturing uncertainty description, there are six influence quantities that influence seven important factors for the calibration of the robot. To calibrate the robot, we must know which main factors are affecting the position of the end effector and how they are perturbated by the random noise. So, the analysis shows us which quantities are the most important for consideration of random errors. These random errors are in the interval of predicted values of components and distances that affect the position of the end effector by the kinematics. We have done a 'two level analysis' and these two levels correspond to low and high levels related to low and high value tolerance intervals that have been set

approximately, because the robot has not been produced yet. The intervals for each influence quantitity that can be seen in the kinematics:

1. The length of the first link (l1)
   Low level: $\pm$ 0.01 mm
   High level: $\pm$ 0.1 mm
2. The length of the second link (l2)
   Low level: $\pm$ 0.01 mm
   High level: $\pm$ 0.1 mm
3. The length of the third link (l3)
   Low level: $\pm$ 0.01 mm
   High level: $\pm$ 0.1 mm
4. The radius of the mobile platform (Rp)
   Low level: $\pm$ 0.015 mm
   High level: $\pm$ 0.15 mm
5. The distance between the rails' axes and the origin (s)
   Low level: $\pm$ 0.02 mm
   High level: $\pm$ 0.2 mm
6. The distance between mobile platform of the delta robot and the base platform of the agile eye spherical wrist robot (h)
   Low level: $\pm$ 0.01 mm
   High level: $\pm$ 0.1 mm

As you see, the intervals of l1,l2 l3 and h quantities have the same values, because three links and h distance are produced by the same way. The manufacturing of circular components or components that are nearly circular is more difficult and as expected, the value of Rp noise is bigger than the values of l1,l2,l3 and h. Finally, s is not a real produced component, it is a imaginary distance, so the arranging of this distance is a more comlex operation and its predicted noise interval has the greatest value.

CHAPTER 6

6. Uncertainty Analysis Results

6.1 Manufacturig Uncertainties Analysis

6.1.1 Effects of Influence Quantities on the Position of the Mobile Platform at x Direction

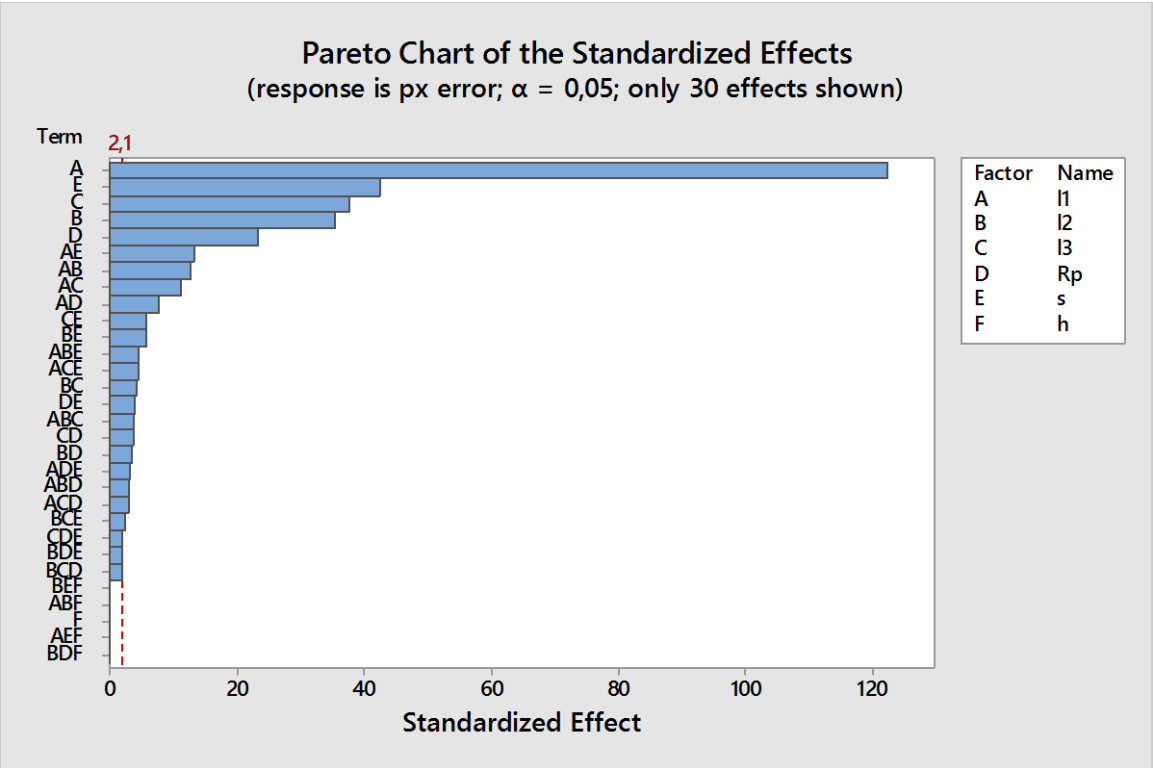

Figure 6.1: Pareto Chart of the Uncertainty Effects on the px Response

Figure 6.2: Interaction Plot of the Uncertainty Effects on the px Response

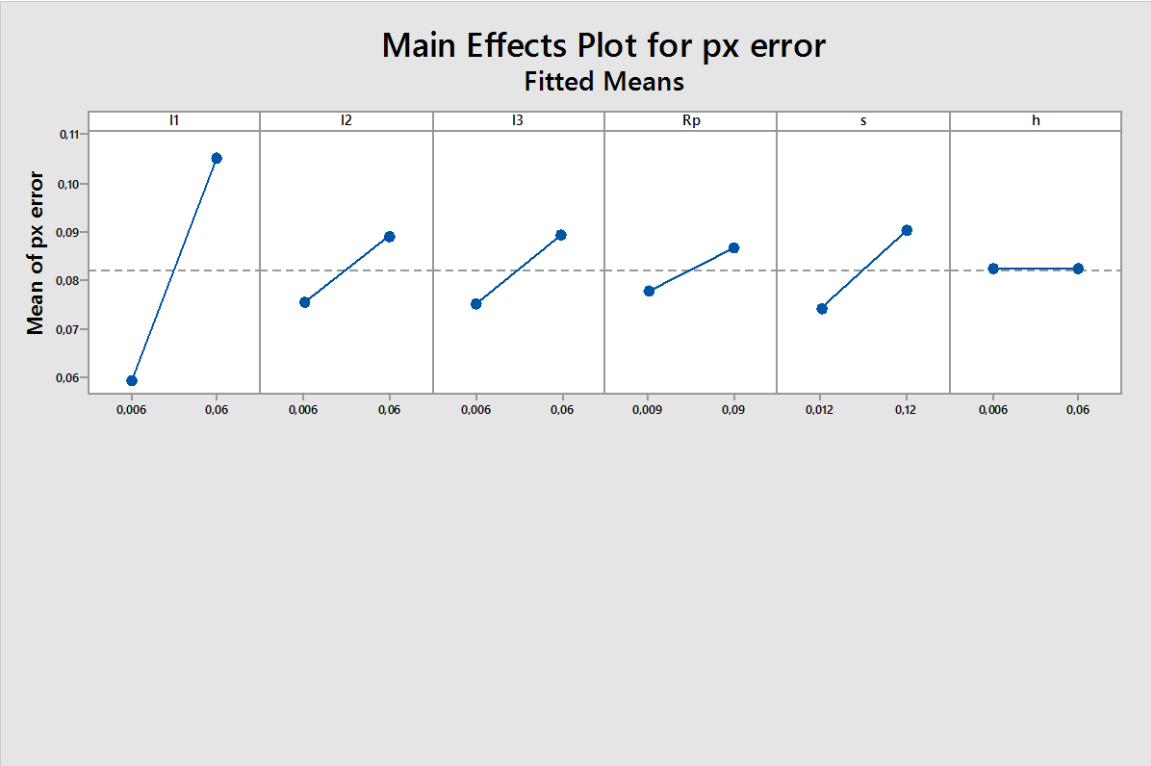

Figure 6.3: Main Effects Plots for the px Response

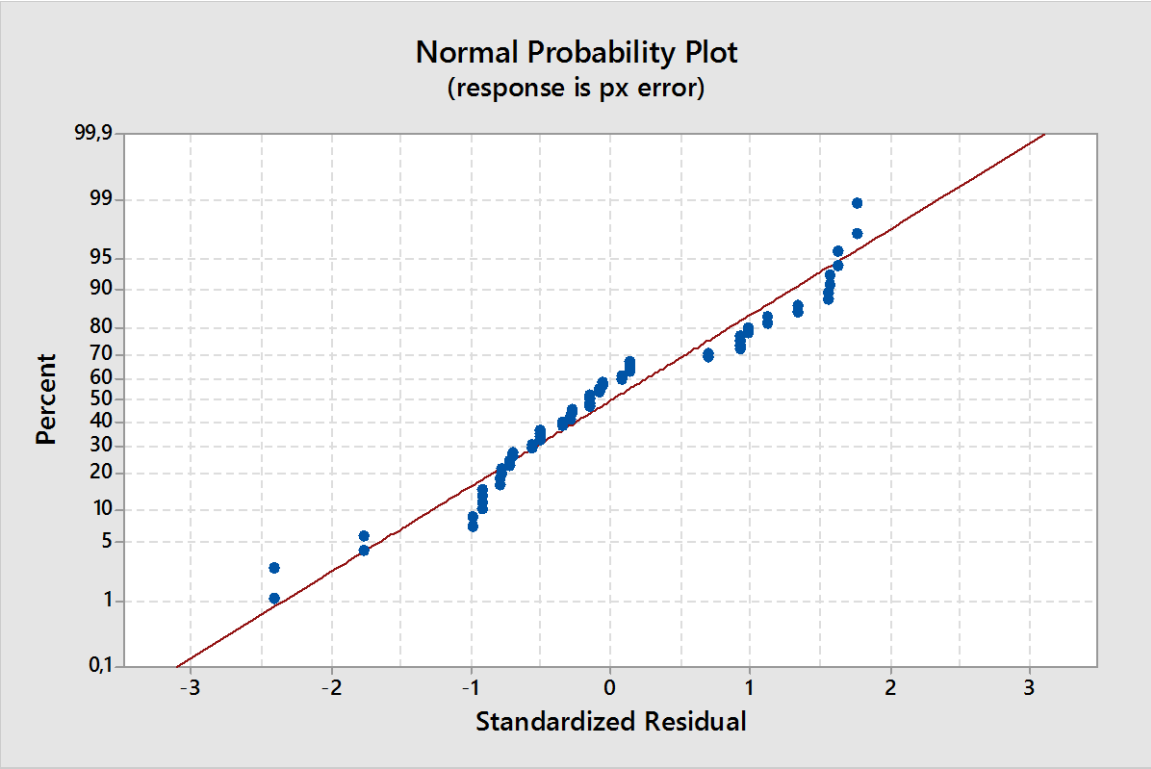

Figure 6.4: Residual Plots for the px Response

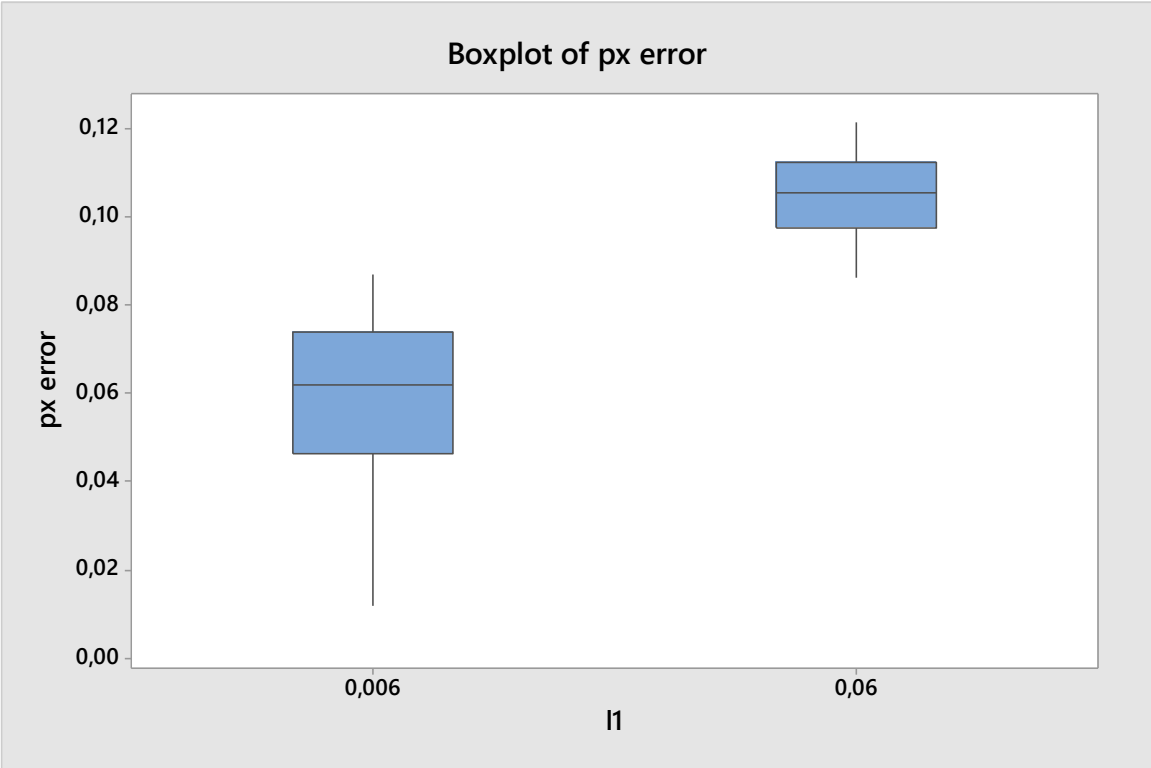

Figure 6.5: Boxplot of px for levels of l1 length

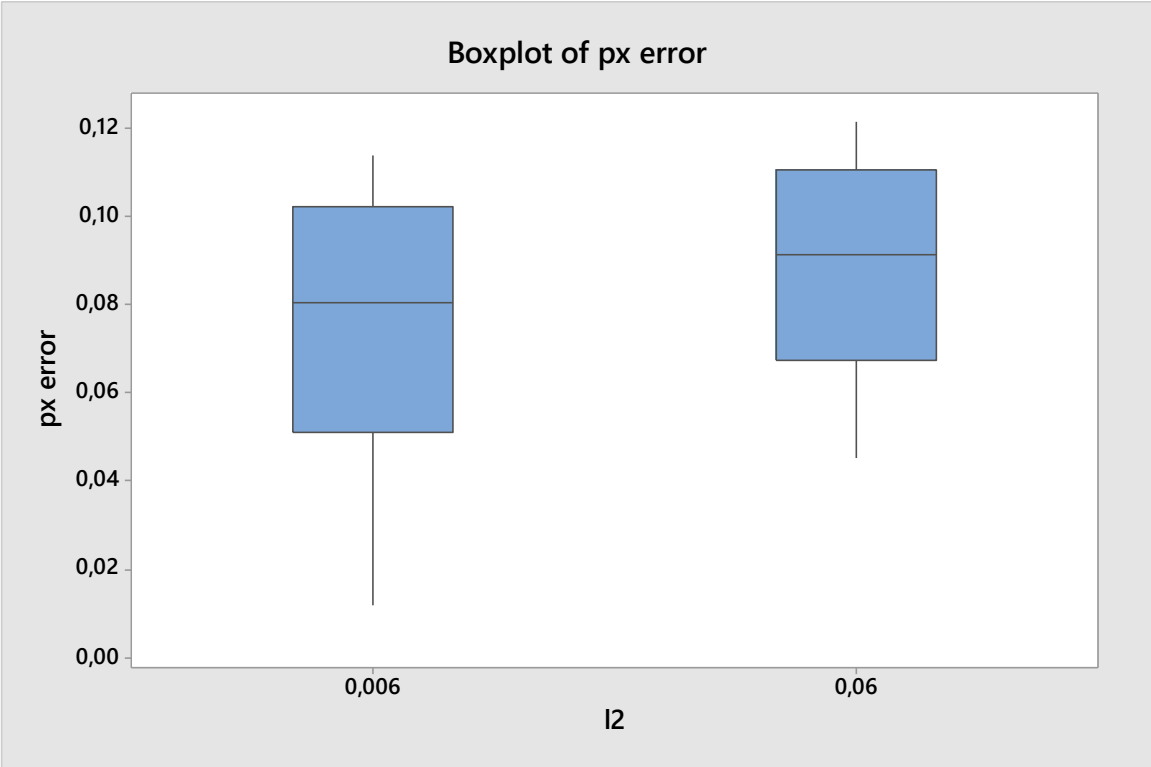

Figure 6.6: Boxplot of px for levels of l2 length

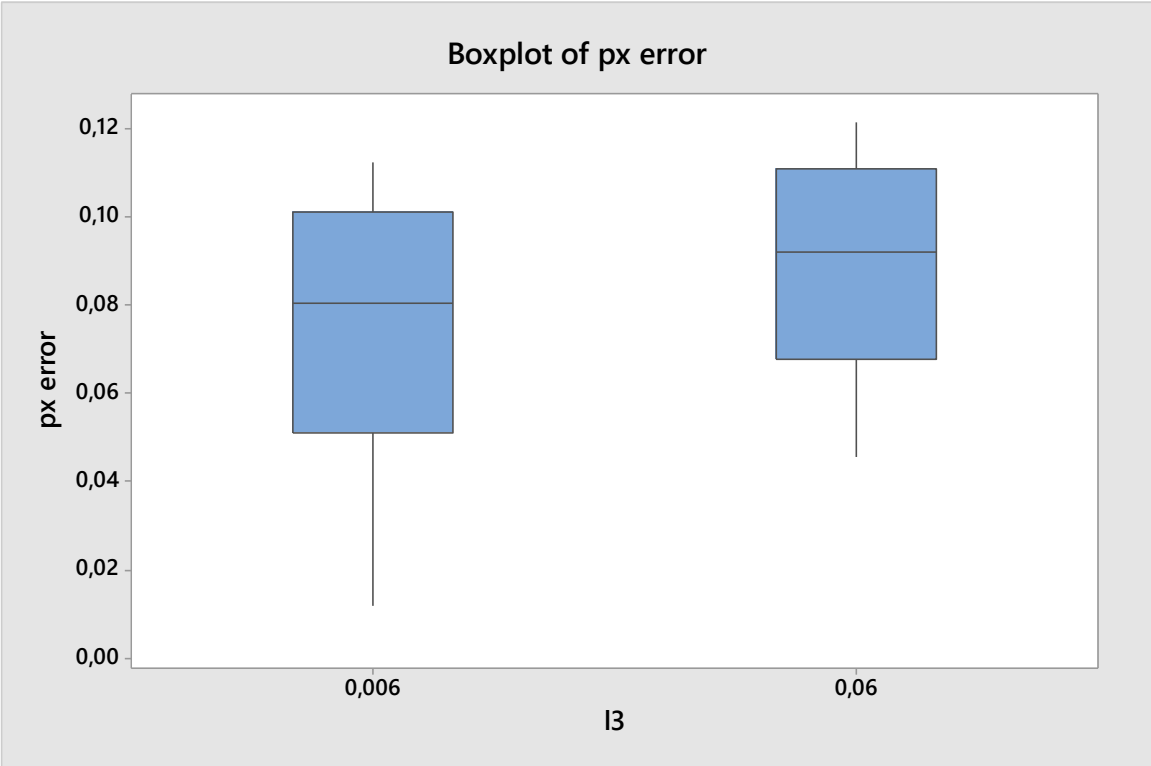

Figure 6.7: Boxplot of px for levels of l3 length

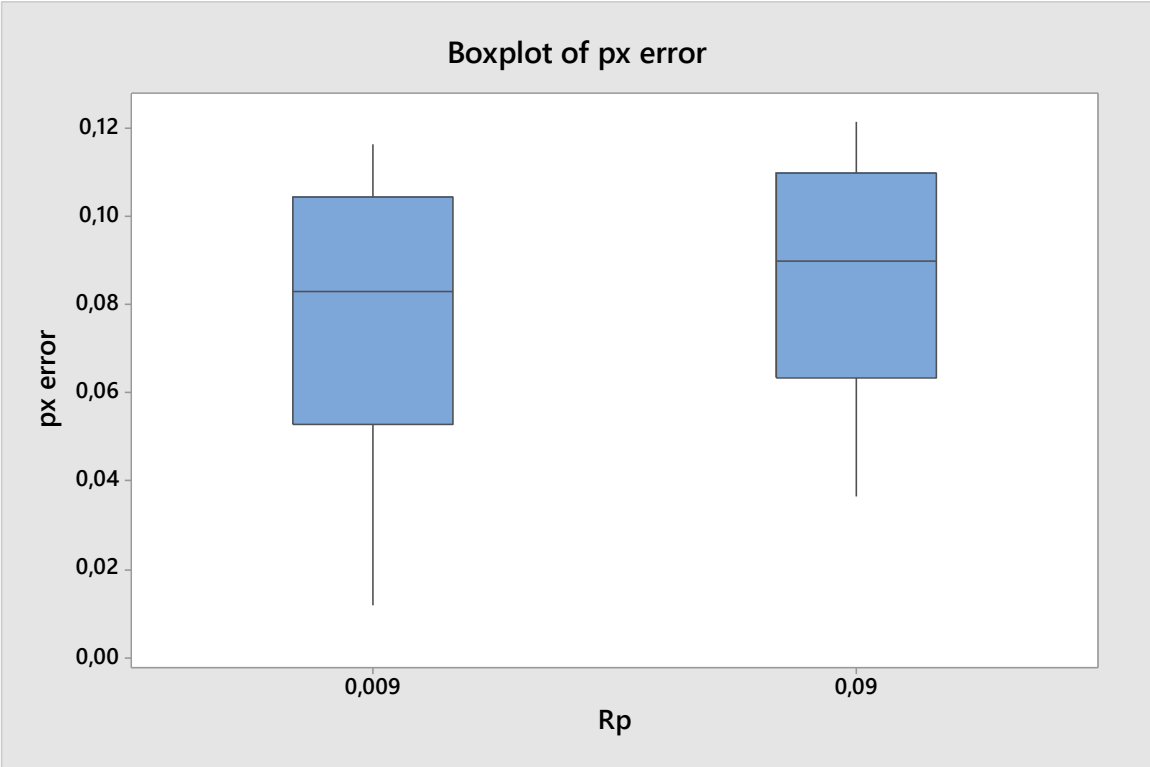

Figure 6.8: Boxplot of px for levels of Rp Radius

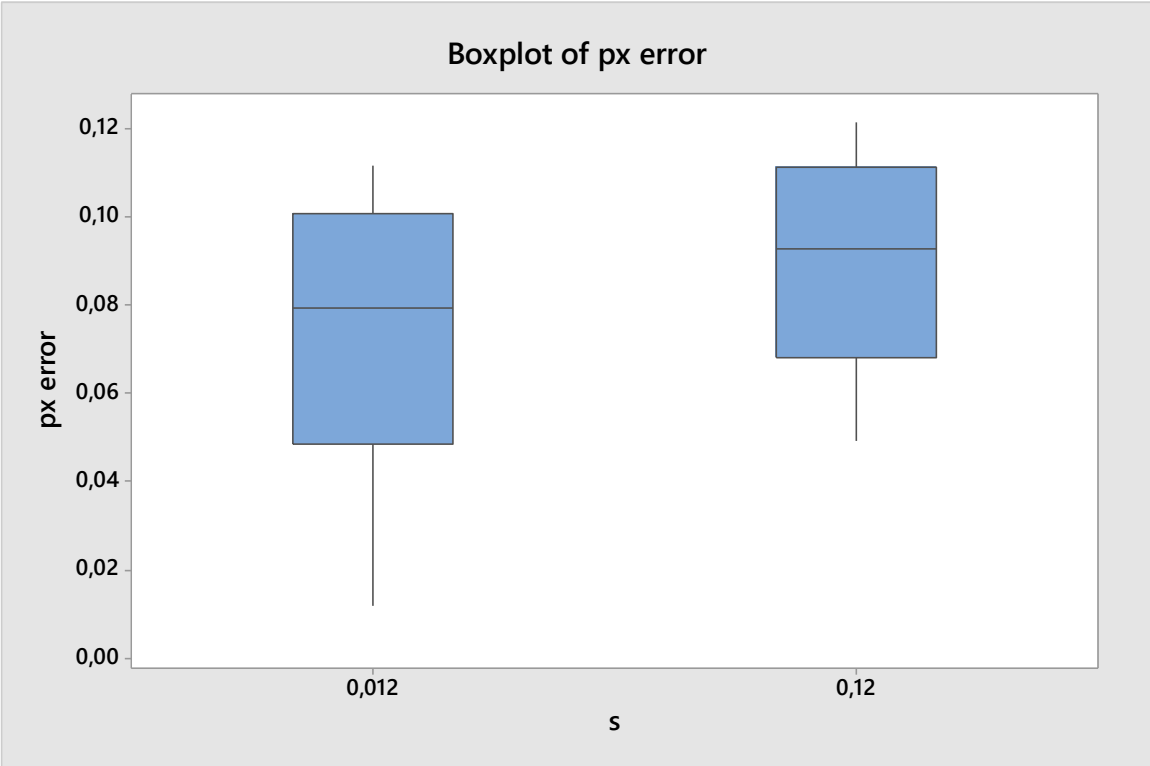

Figure 6.9: Boxplot of px for levels of s length

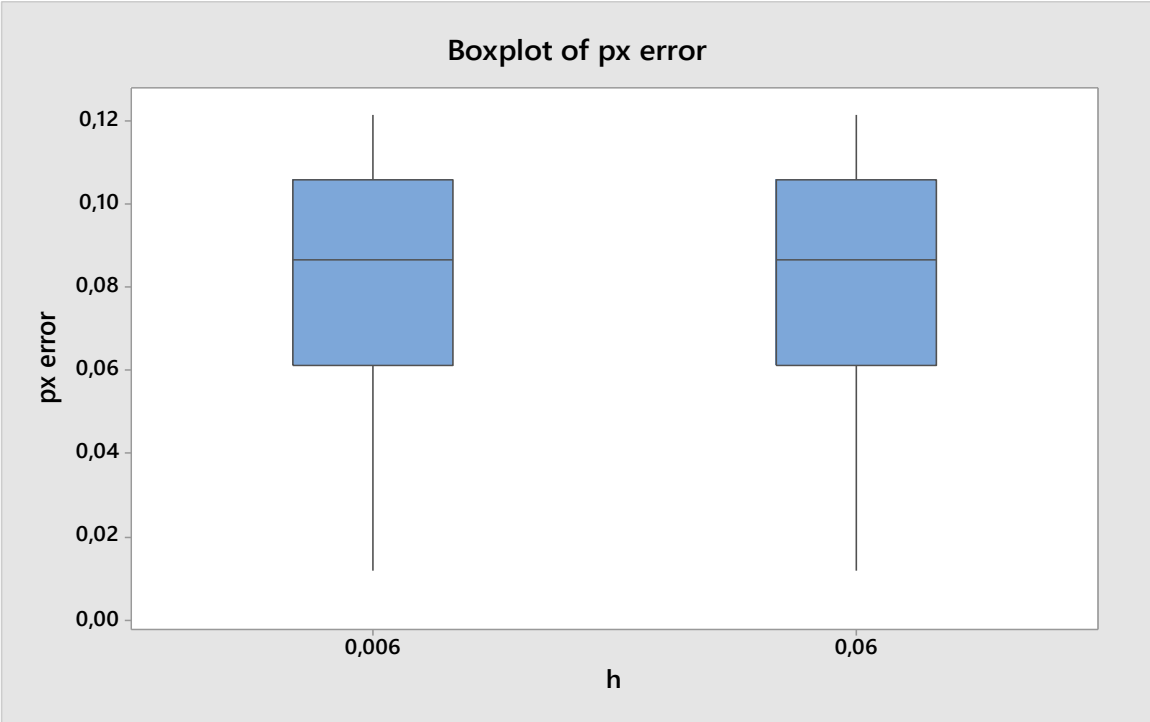

Figure 6.10: Boxplot of px for levels of h length

l1

It has the highest effect on error. Because of having the hihgest effect, its interaction plots with other influence quantities show a great slope on the change of standard deviation of the error of x. In addition, its individiual effect is highest on the standard deviation through the path from low level to high level error values.

l2-l3-s

These influence quantities can be evaluated together. They show lower effect in a comparision with l1. Naturally, they will influence the position from low level to high level, but their interaction plots between each other, have lower slope with respect to interaction plots of influence quantities with l1. Moreover, it can be said that their influences are similar.

Rp

This influence quantity has smallest effect on the position of mobile platform at x direction. Its effect is expected to be similar to s, but its tolerance values are lower than s, so its effect is the smallest one among influence quantities. When interaction plots with other influence quantities are evaluated, low and high level lines are almost coincident as a consequence of its low effect.

h

It does not affect the position of mobile platform at x direction.

Significance Evaluation

If the significance of main effects and interactions are examined, it will be seen that all main effect are significant. Almost, all 2-way interactions are significant, on the other hand 2-way interactions with Rp effect are not significant as expected. In addition to these, only l1l2l3 3-way interaction is significant, because the effects of these main effects are greater. 4-way,5-way and 6-way interactions are not significant.

6.1.2 Effects of Influence Quantities on the Position of the Mobile Platform at y Direction

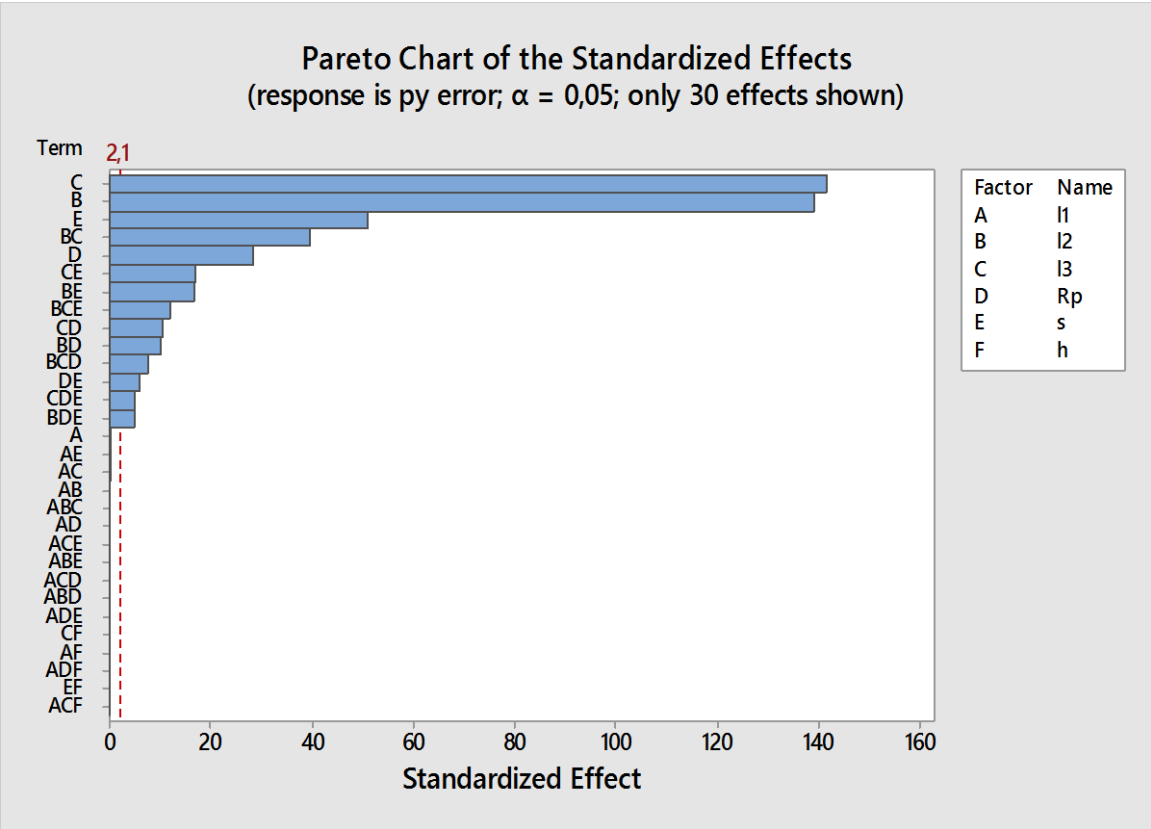

Figure 6.11: Pareto Chart of the Uncertainty Effects on the py Response

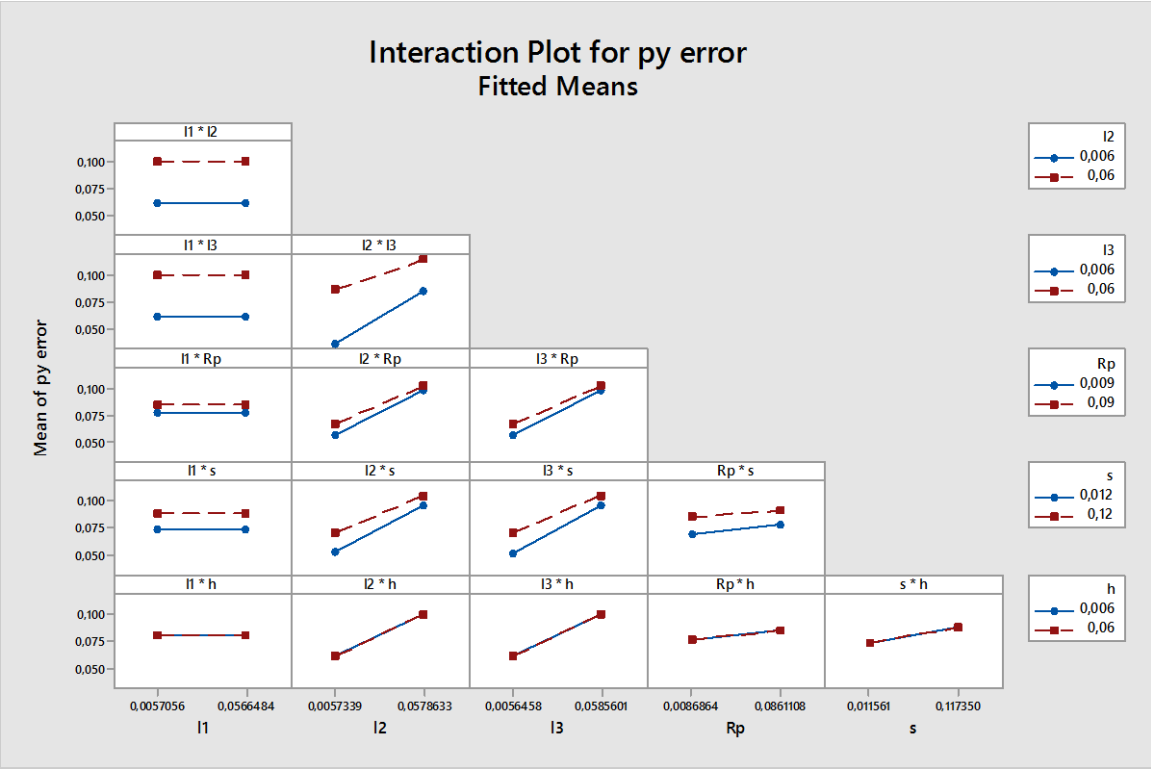

Figure 6.12: Interaction Plot of the Uncertainty Effects on the py Response

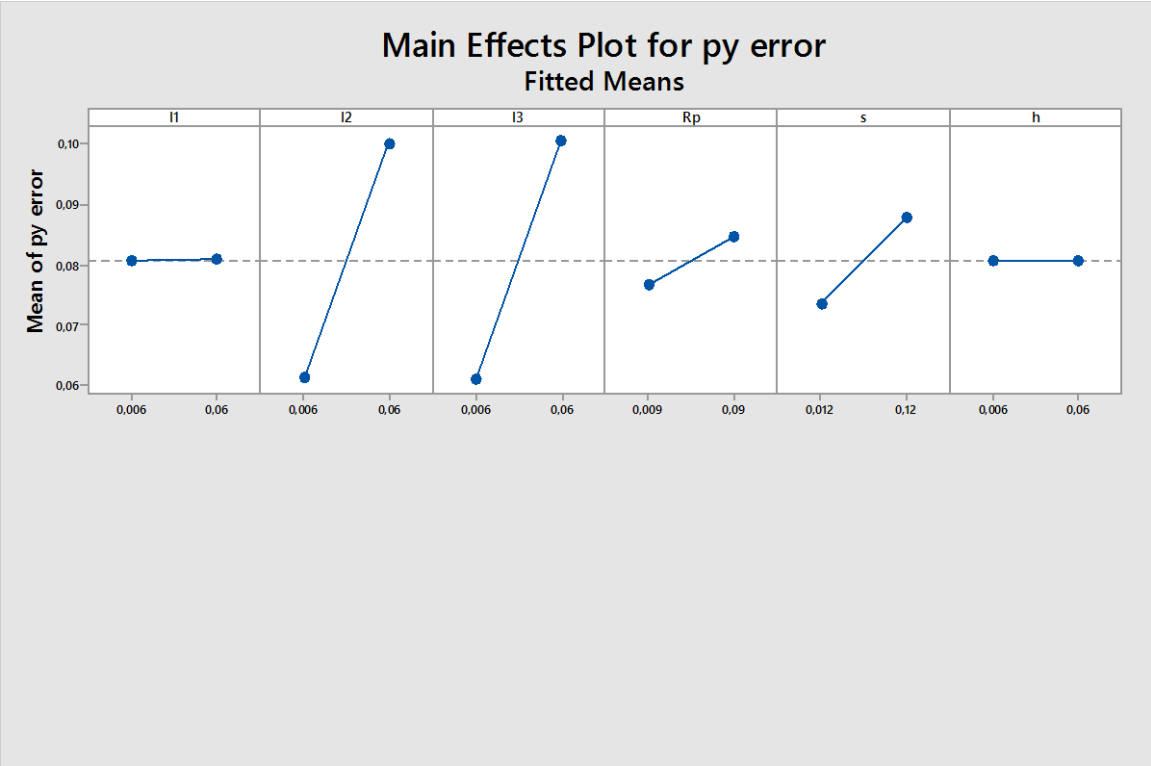

Figure 6.13: Main Effects Plots for the py Response

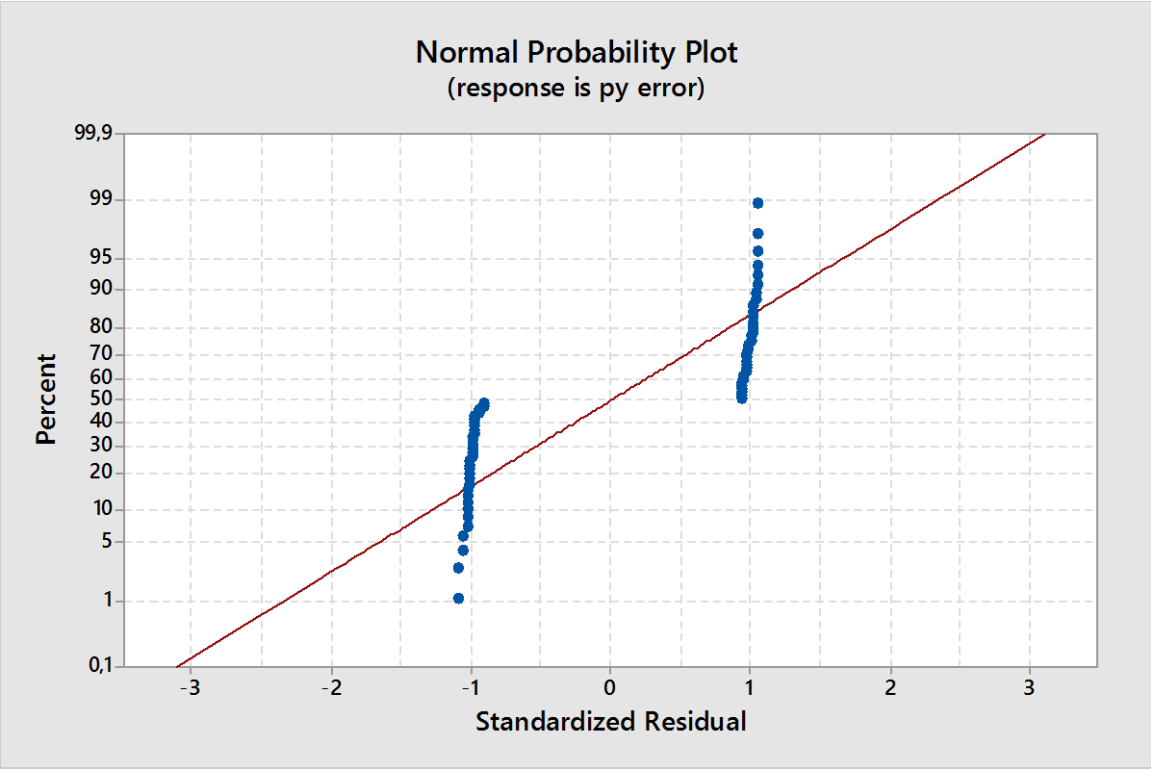

Figure 6.14: Residual Plot for the py Response

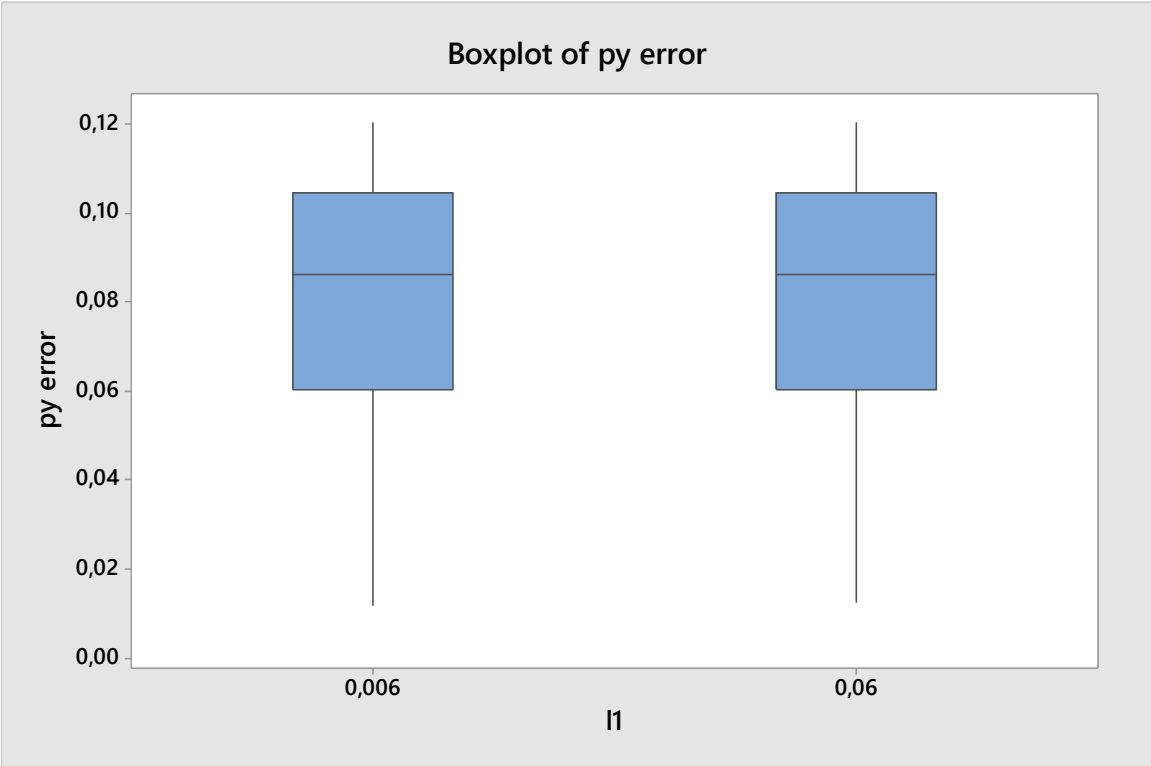

Figure 6.15: Boxplot of py for levels of l1 length

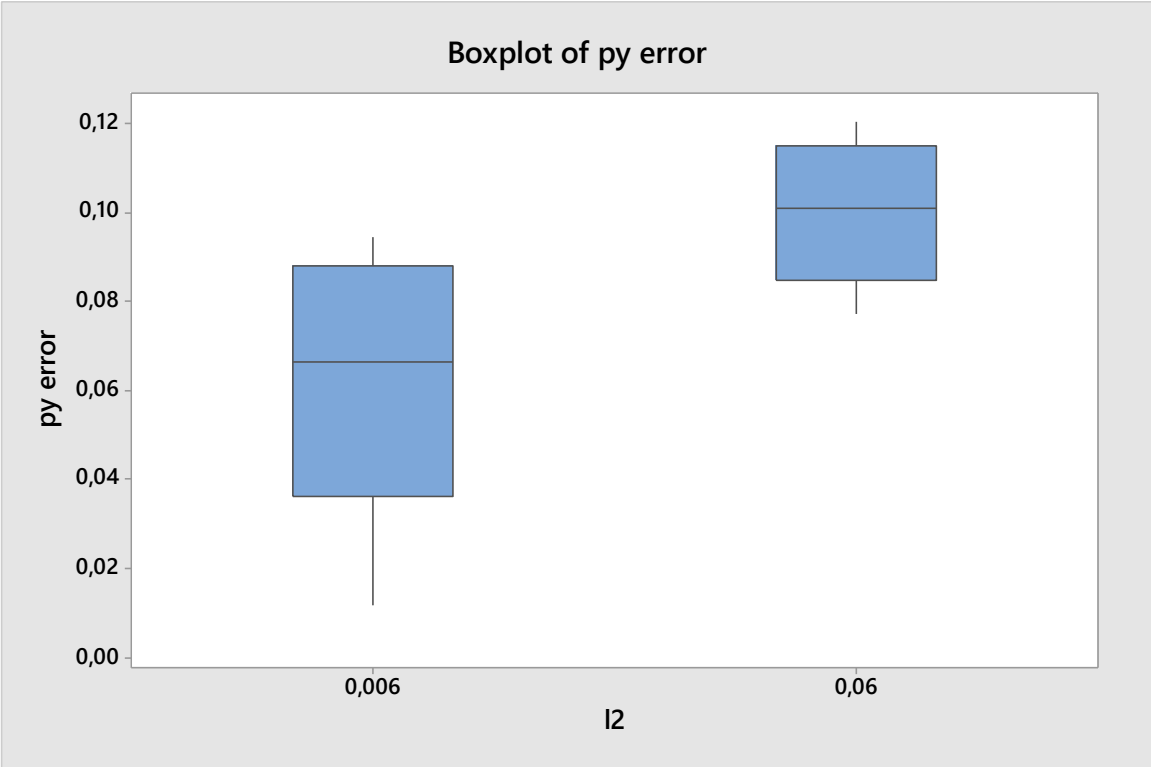

Figure 6.16: Boxplot of py for levels of l2 length

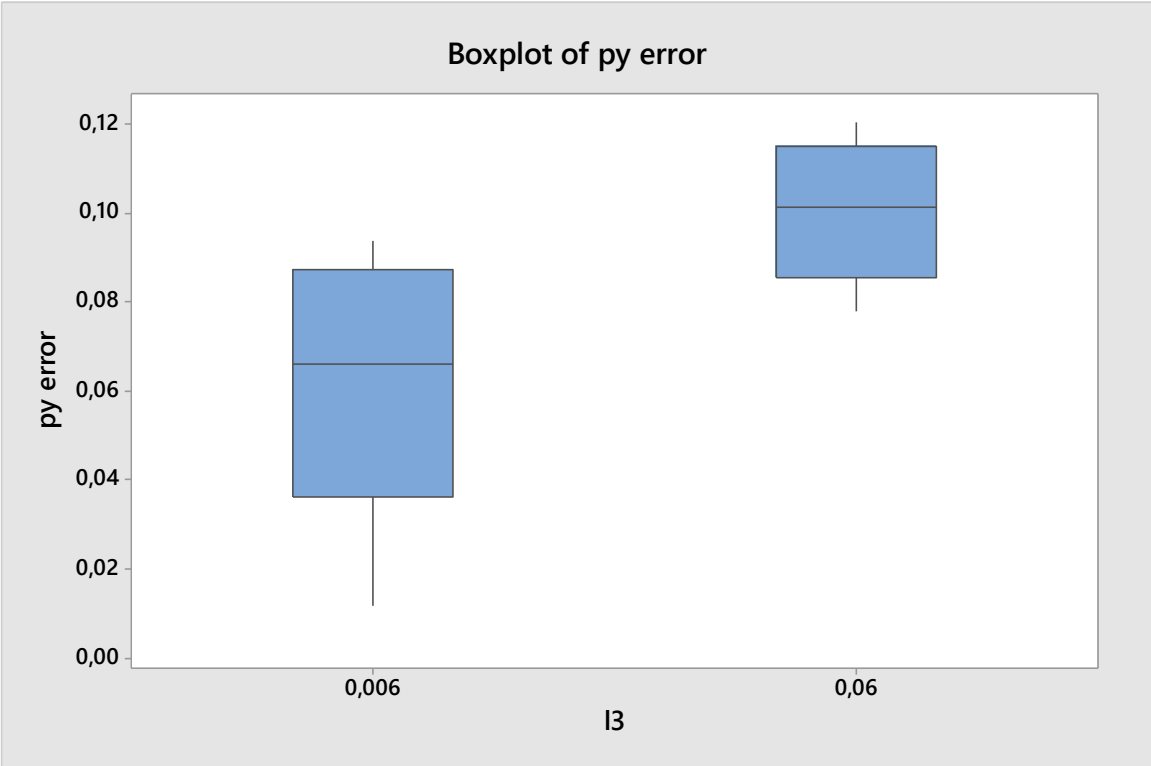

Figure 6.17: Boxplot of py for levels of l3 length

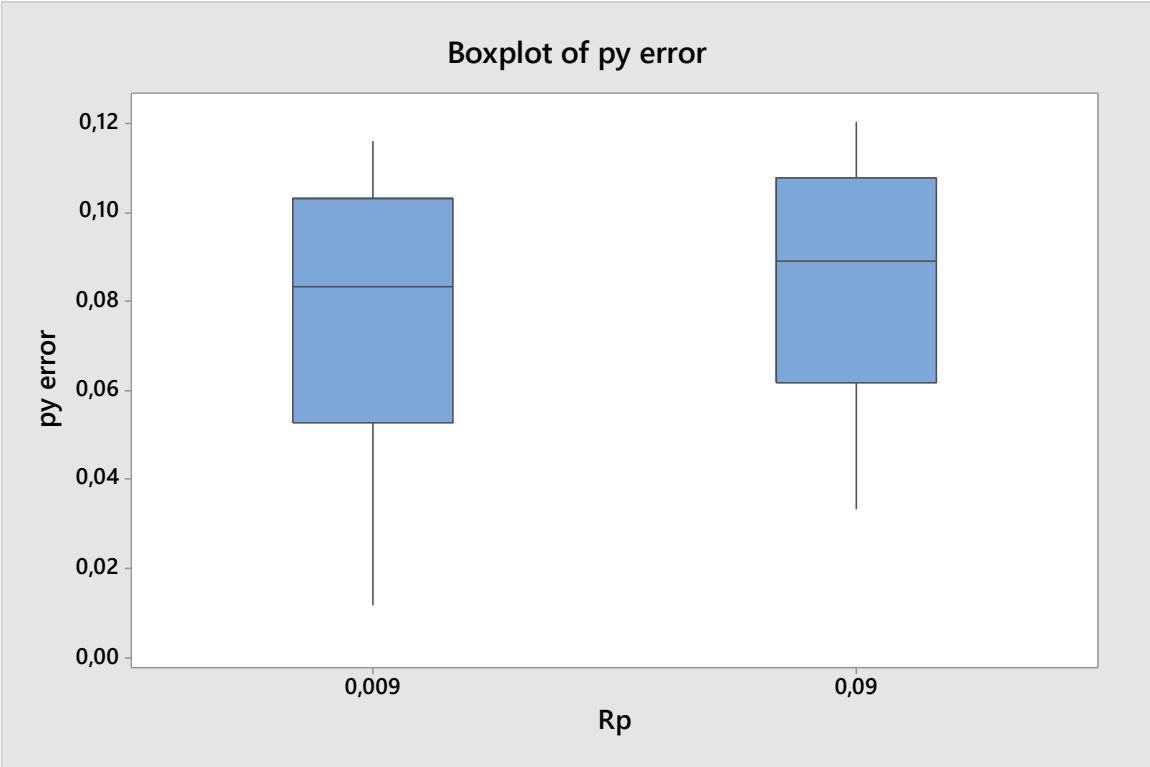

Figure 6.18: Boxplot of py for levels of Rp Radius

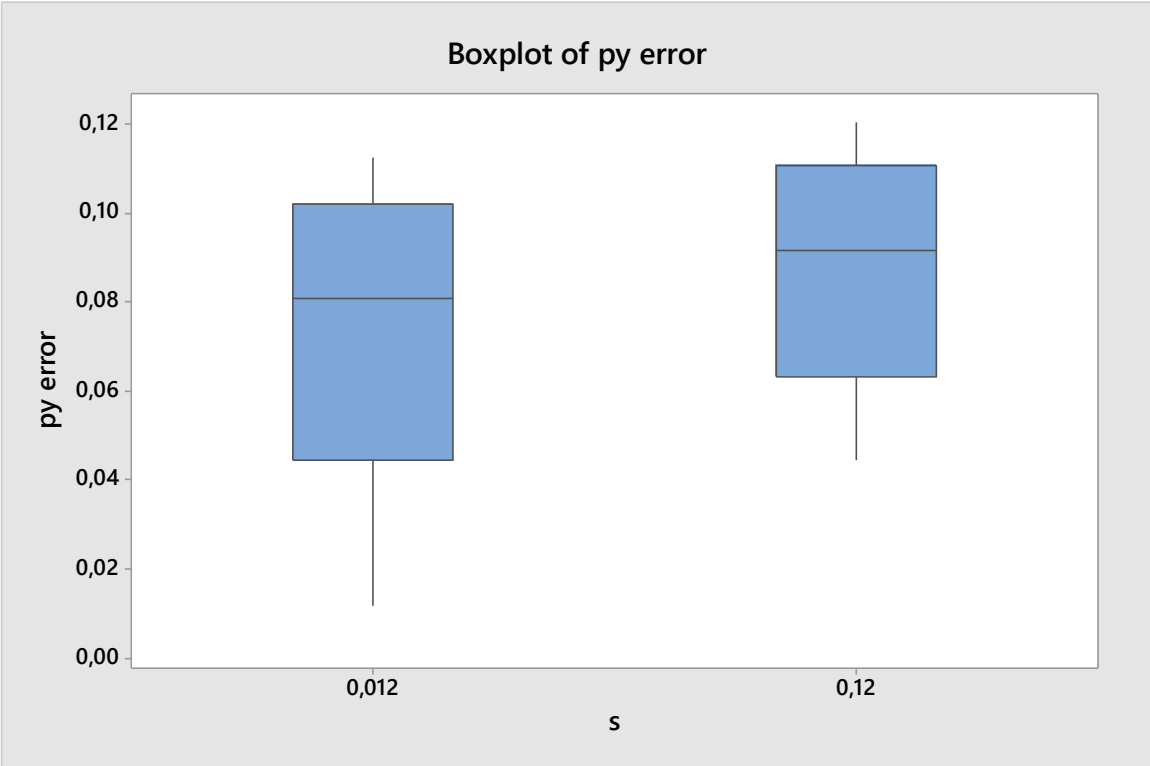

Figure 6.19: Boxplot of py for levels of s length

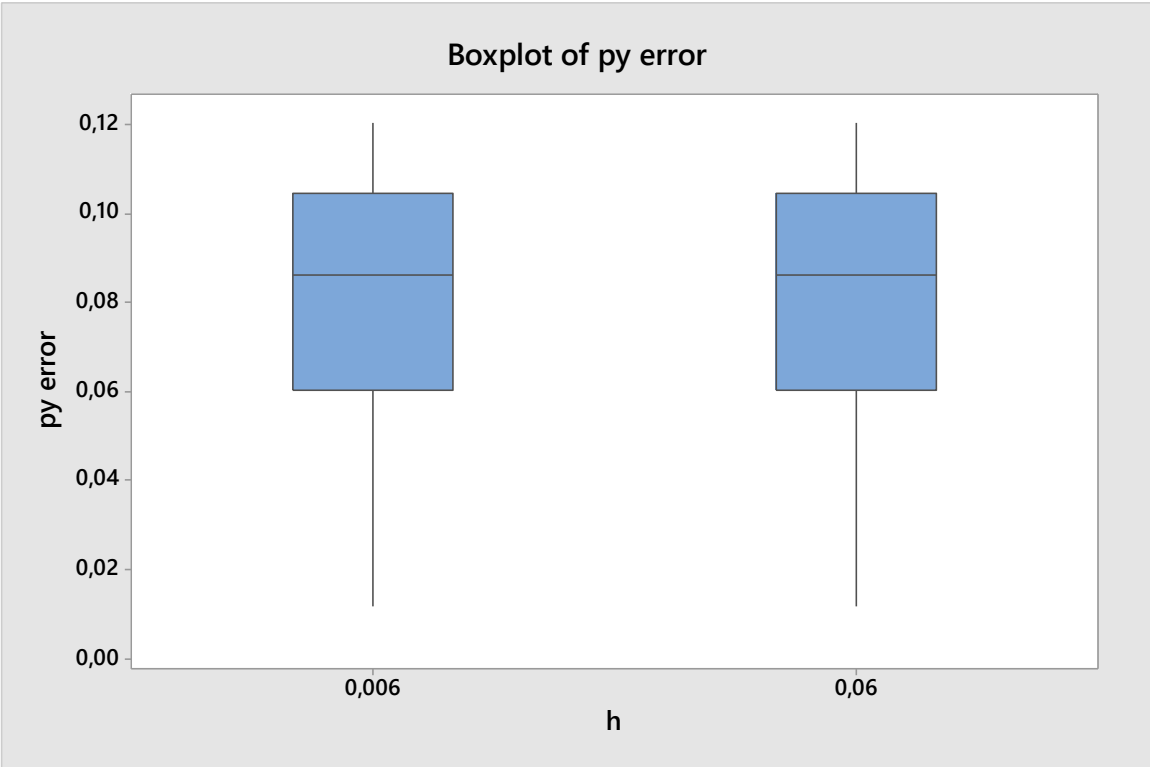

Figure 6.20: Boxplot of py for levels of h length

l1

It has not a significant effect, because of the kinematics of the linear delta robot. So the interaction plots with l1 (low level and high level lines of other influence quantities) are nearly horizontal.

l2-l3

Their effects are greatest, so their interaction plots with other influence quantities have meaningful slope values.

Rp-s

Their effects are lower with respect to the l2 and l3 on the positioning error of y axis. Rp's tolerance values are lower than s, so there is decrease of Rp effect on the error and blue and red lines approach to each other (low and high level lines).

h

It does not affect the position of mobile platform at y direction.

Significance Evaluation

As it is mentioned before, only l1 influence quantity is not significant among main effects. When 2-way interactions are evaluated, as expected, 2-way interactions with l1 are not significant, others are significant, but in reverse direction. 3-way interactions also indicate same behaviour as 2-way interactions indicate. 3-way interactions with l1 terms are not significant. On the other hand, other 3-way interactions are significant. Moreover, 4-way, 5-way and 6-way interactions are not significant.

6.1.3 Effects of Influence Quantities on the Position of the Mobile Platform at z Direction

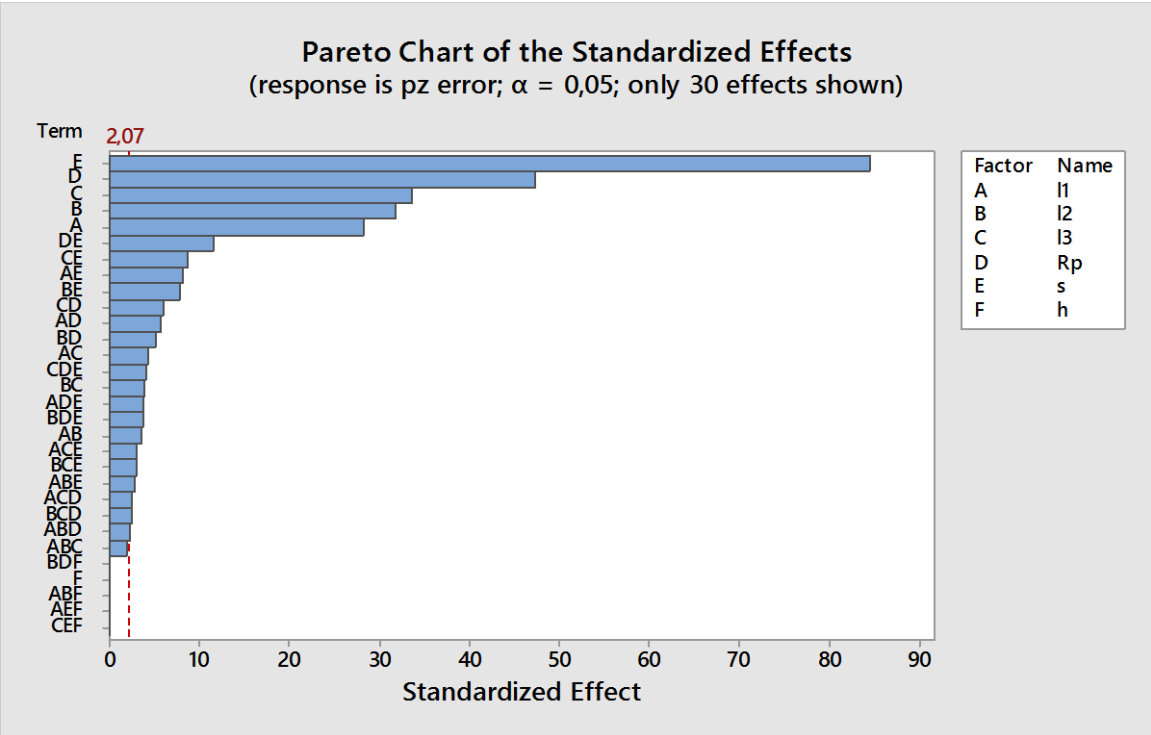

Figure 6.21: Pareto Chart of the Uncertainty Effects on the pz Response

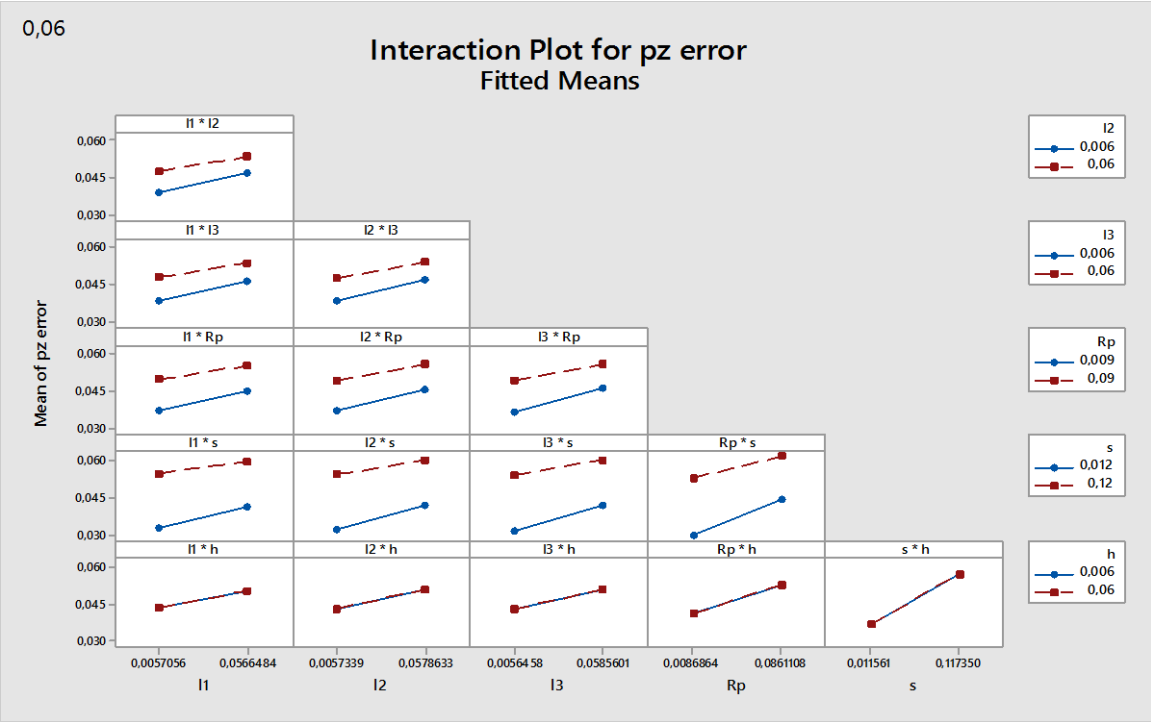

Figure 6.22: Interaction Plot of the Uncertainty Effects on the pz Response

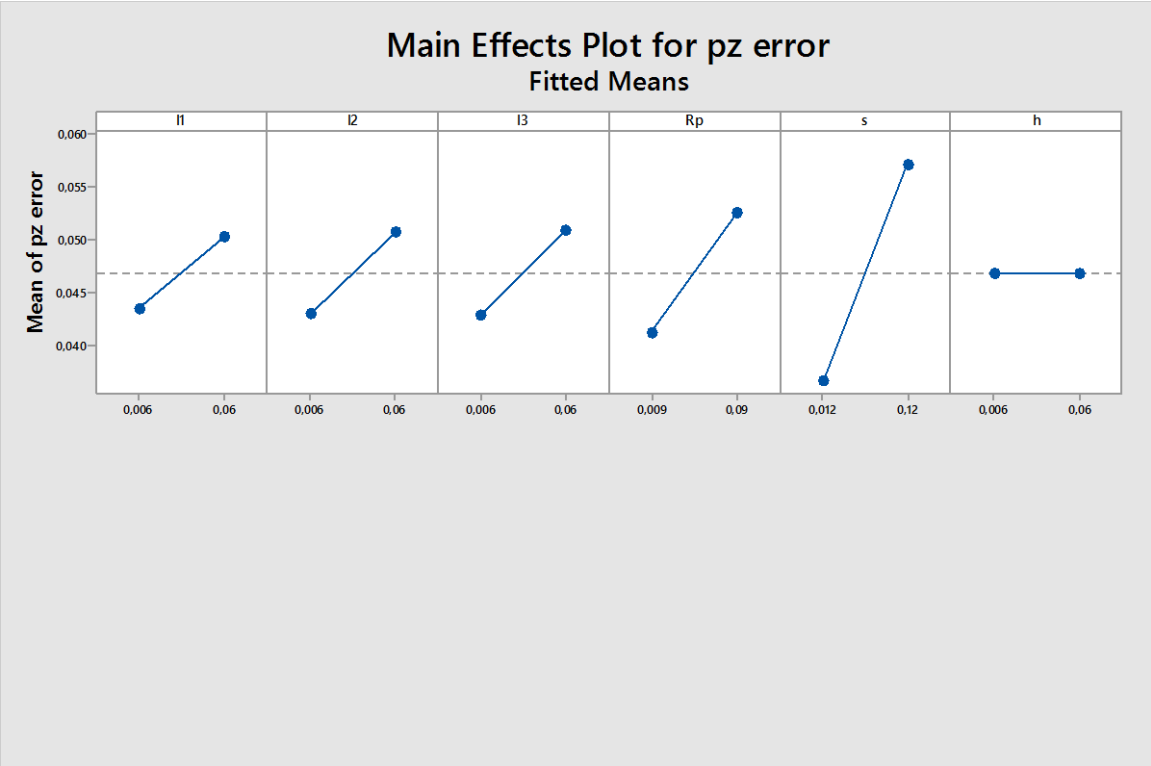

Figure 6.23: Main Effects Plots for the pz Response

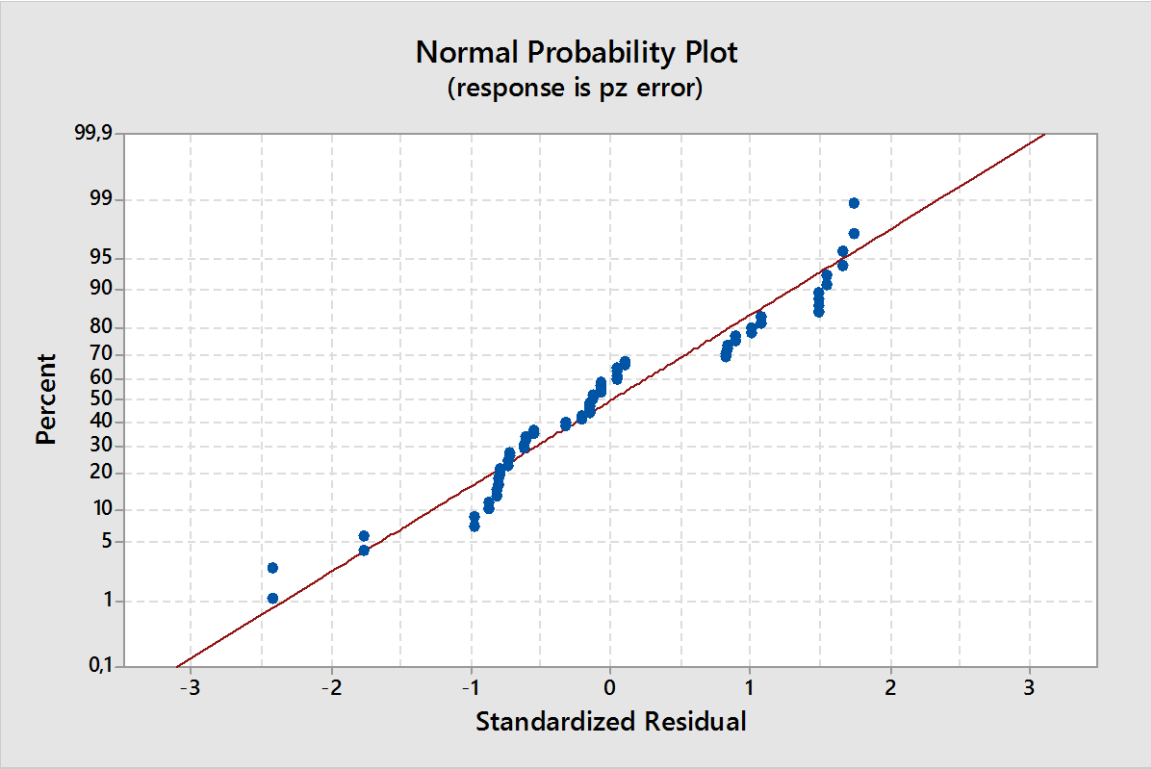

Figure 6.24: Residual Plot for the pz Response

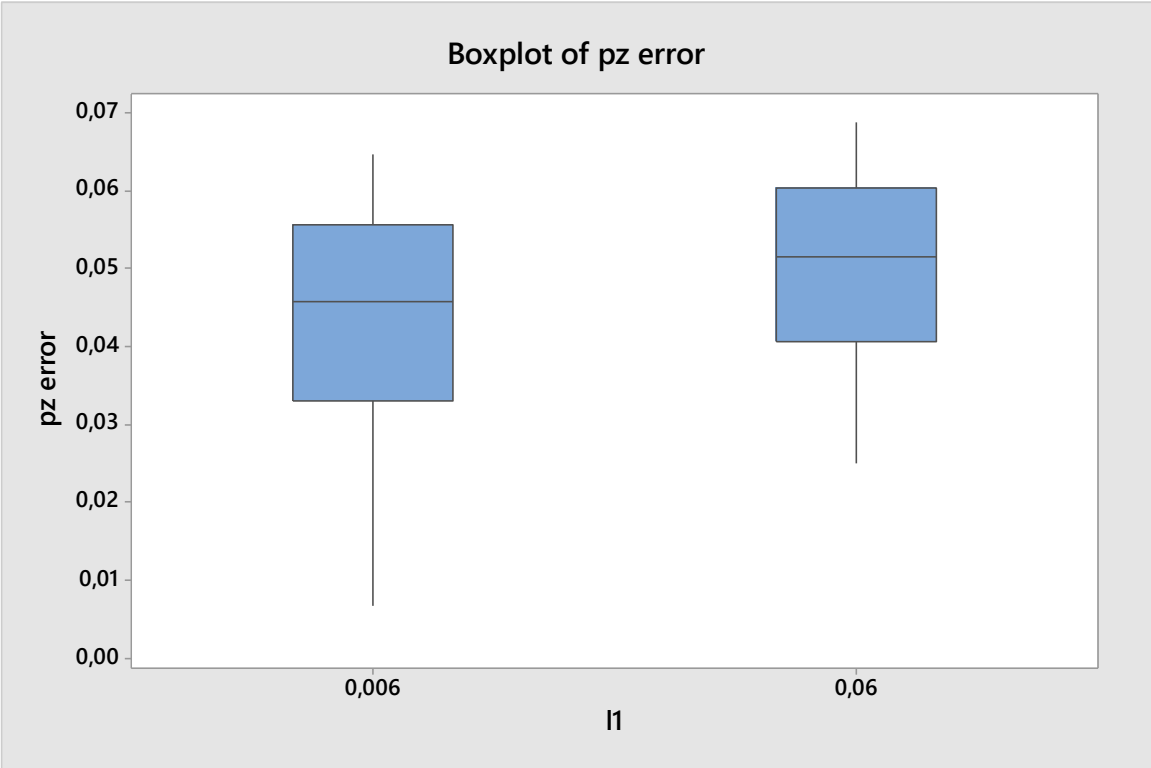

Figure 6.25: Boxplot of pz for levels of l1 length

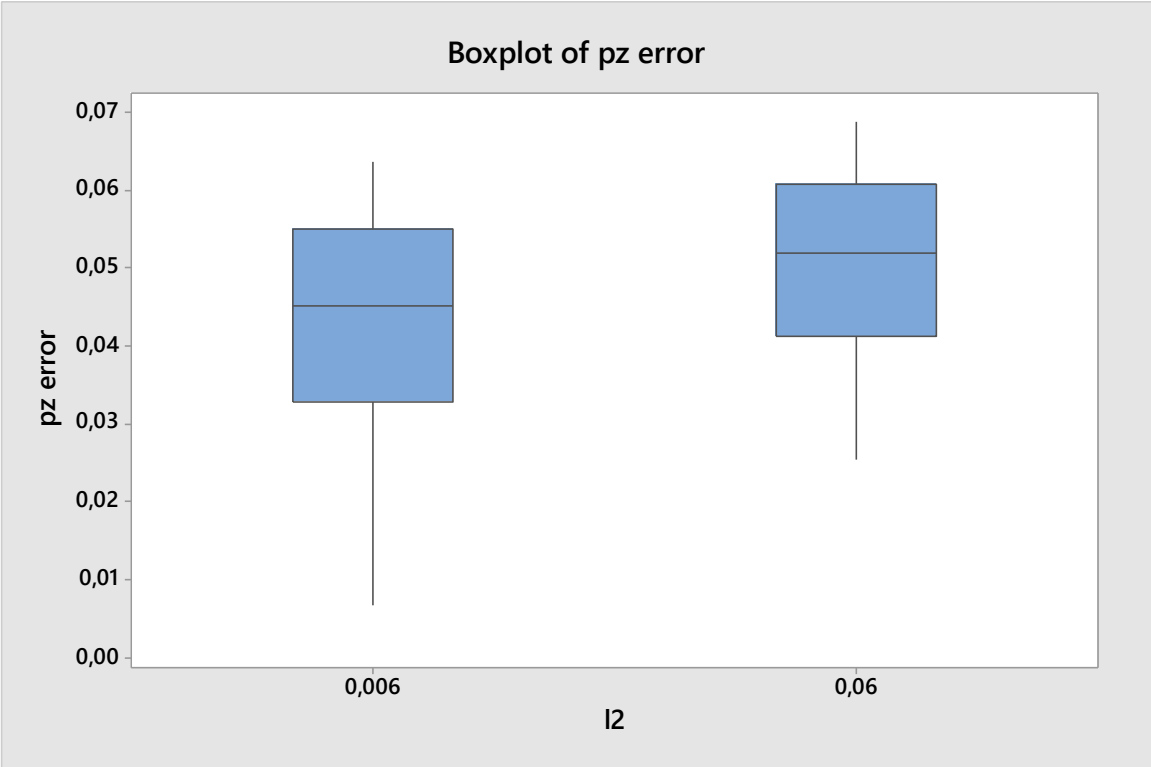

Figure 6.26: Boxplot of pz for levels of l2 length

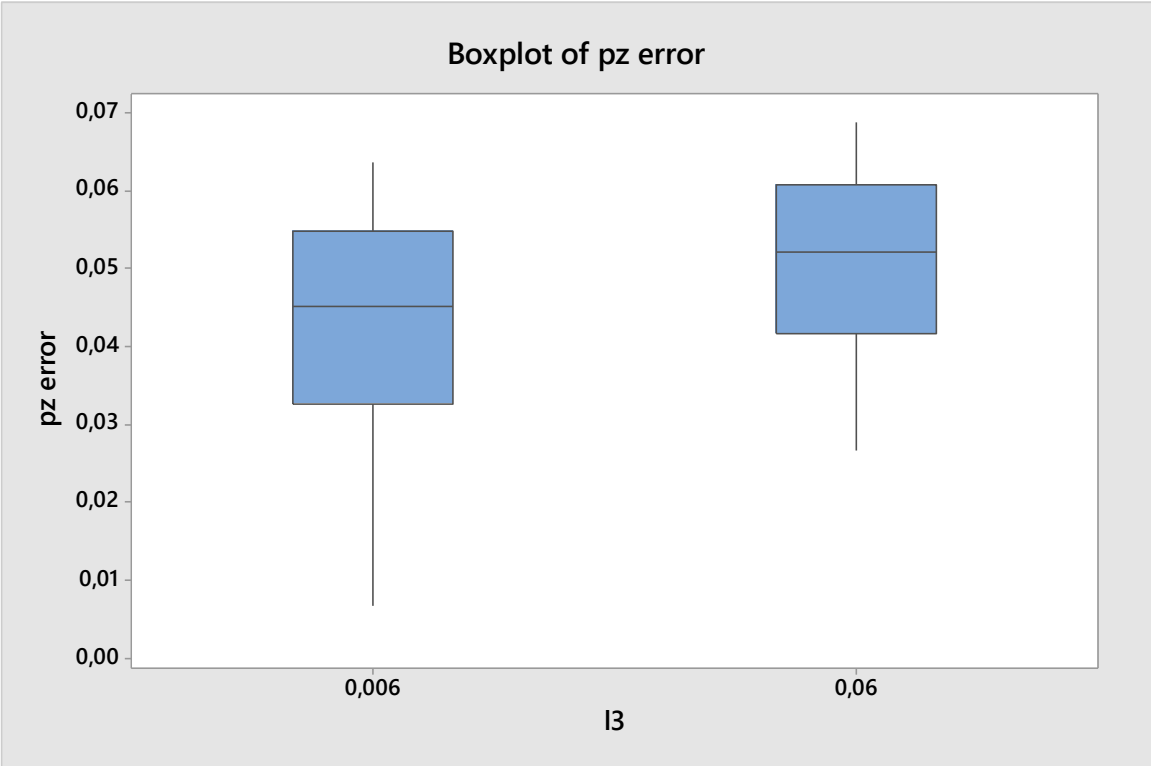

Figure 6.27: Boxplot of pz for levels of l3 length

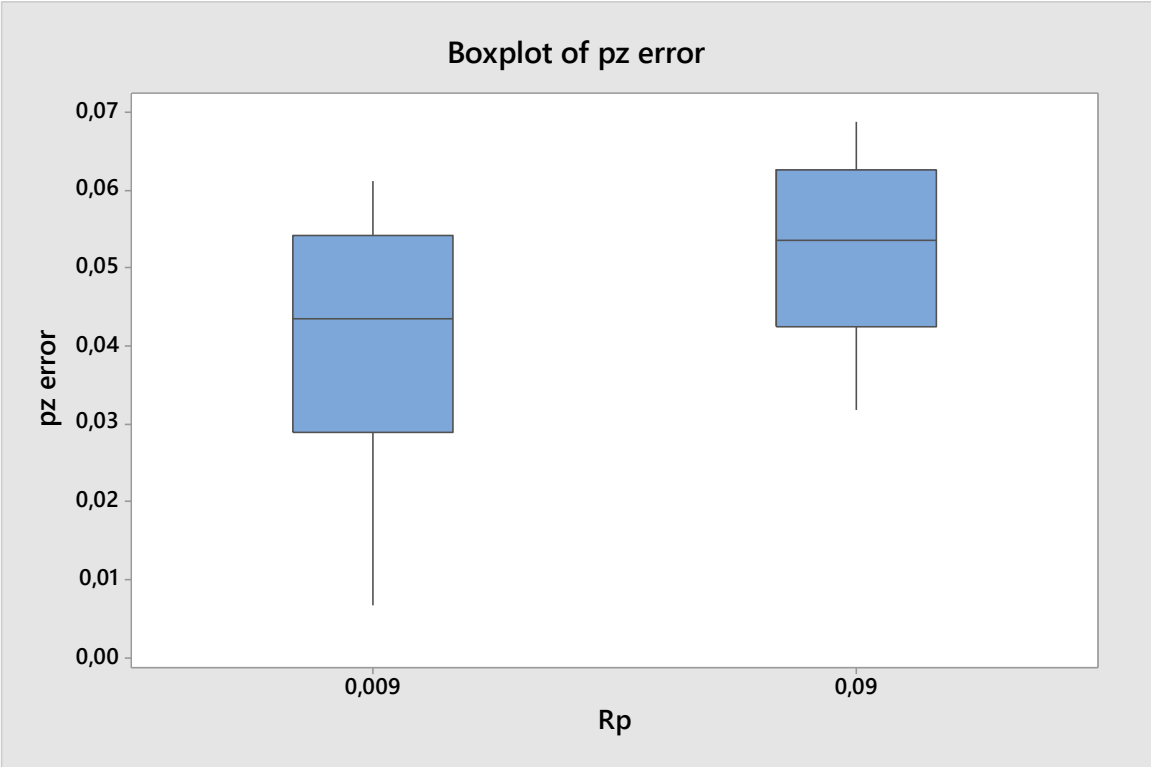

Figure 6.28: Boxplot of pz for levels of Rp Radius

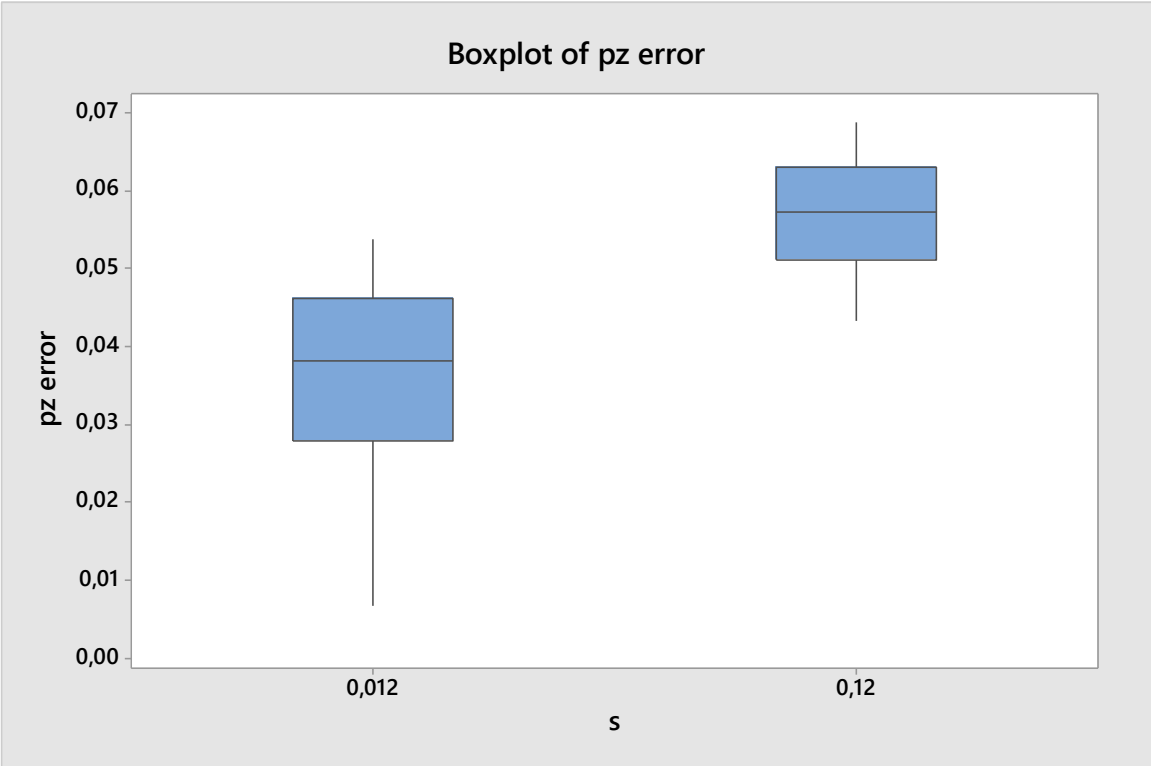

Figure 6.29: Boxplot of pz for levels of s length

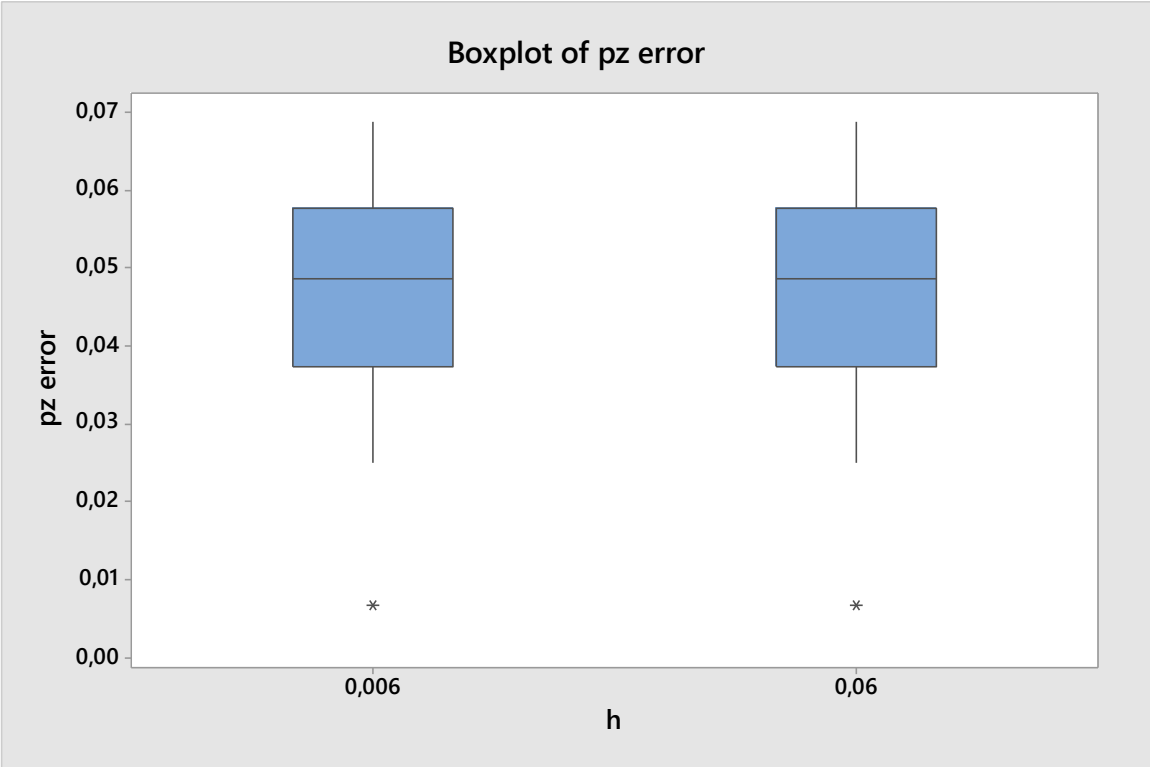

Figure 6.30: Boxplot of pz for levels of h length

l1, l2, l3, s and Rp

All influence quantities are effective, but l1,l2,l3 have lower effects with respect to other directions (x and y). From interaction plots, it can be seen that, their slope are fairly meaningful. However, when interaction plots with s influence quantity are examined, the distance between between high and low level lines are greatest. Because s has the highest effect among influence quantities. In addition to these, main effects plots show us that, s has the highest effect as we mentioned before.

h

It does not affect the position of mobile platform at z direction.

Significance Evaluation

If we look at main effects, all main effects are significant. s and Rp are more significant. Because they influence the z direction directly thereby they affect the angles of the links. While almost all 2-way interactions are significant; 3-way, 4-way and 5-way interactions do not exist in the significant group of interactions.

6.1.4 Effects of Influence Quantities on the Total Error

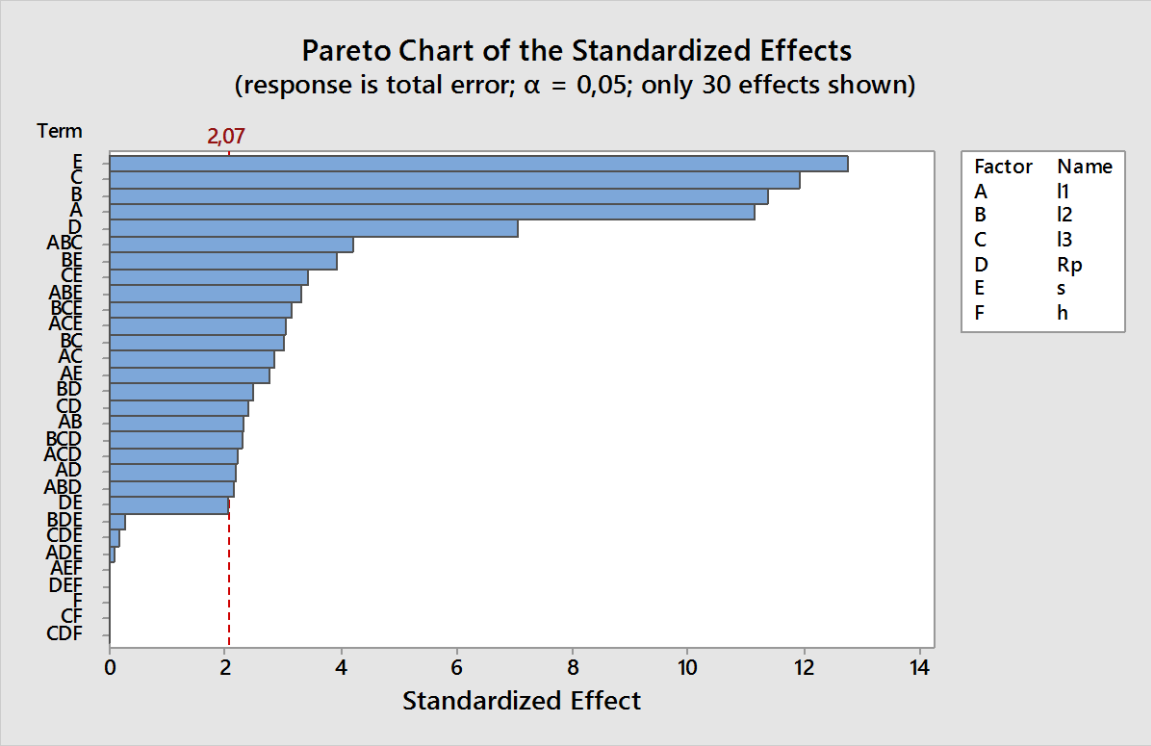

Figure 6.31: Pareto Chart of the Uncertainty Effects on the Total Error Response

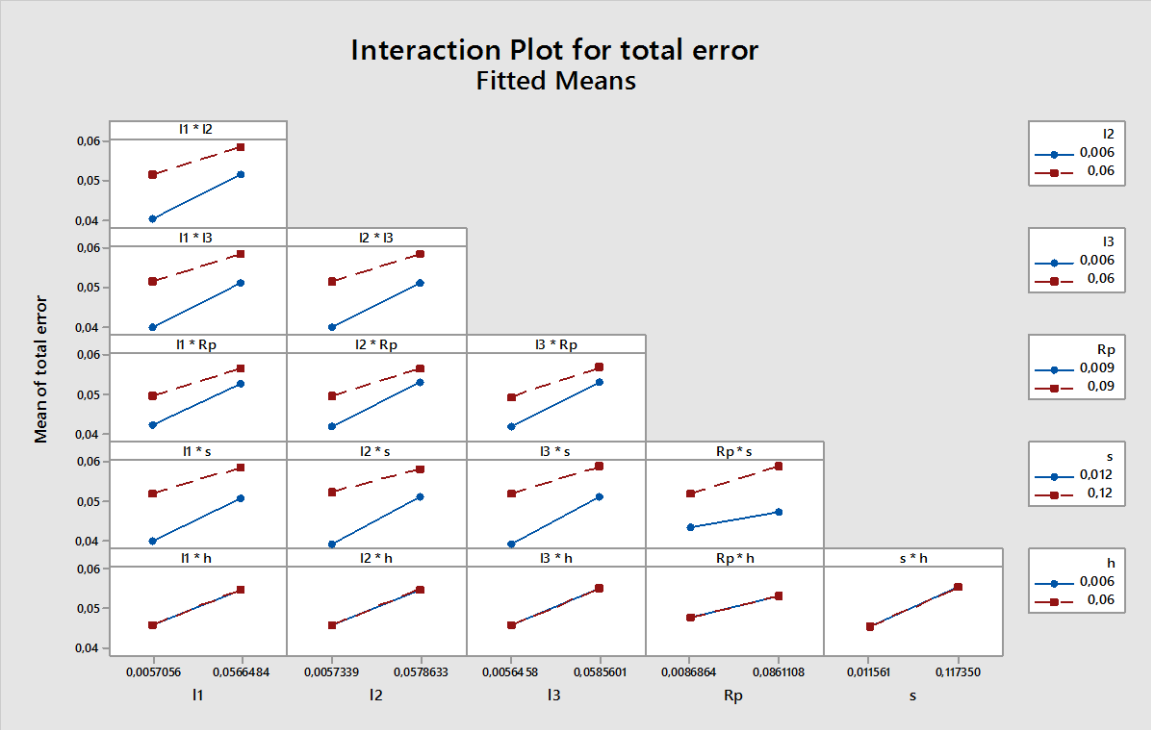

Figure 6.32: Interaction Plot of the Uncertainty Effects on the Total Error Response

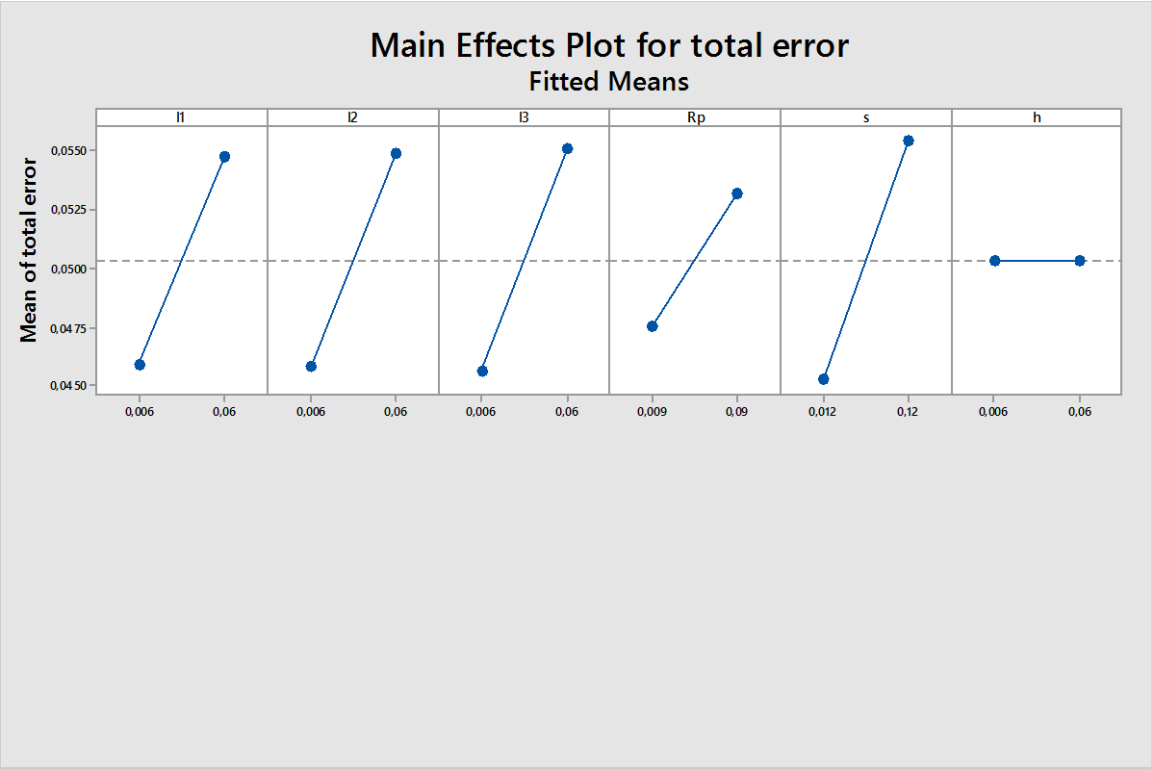

Figure 6.33: Main Effects Plots for the Total Error Response

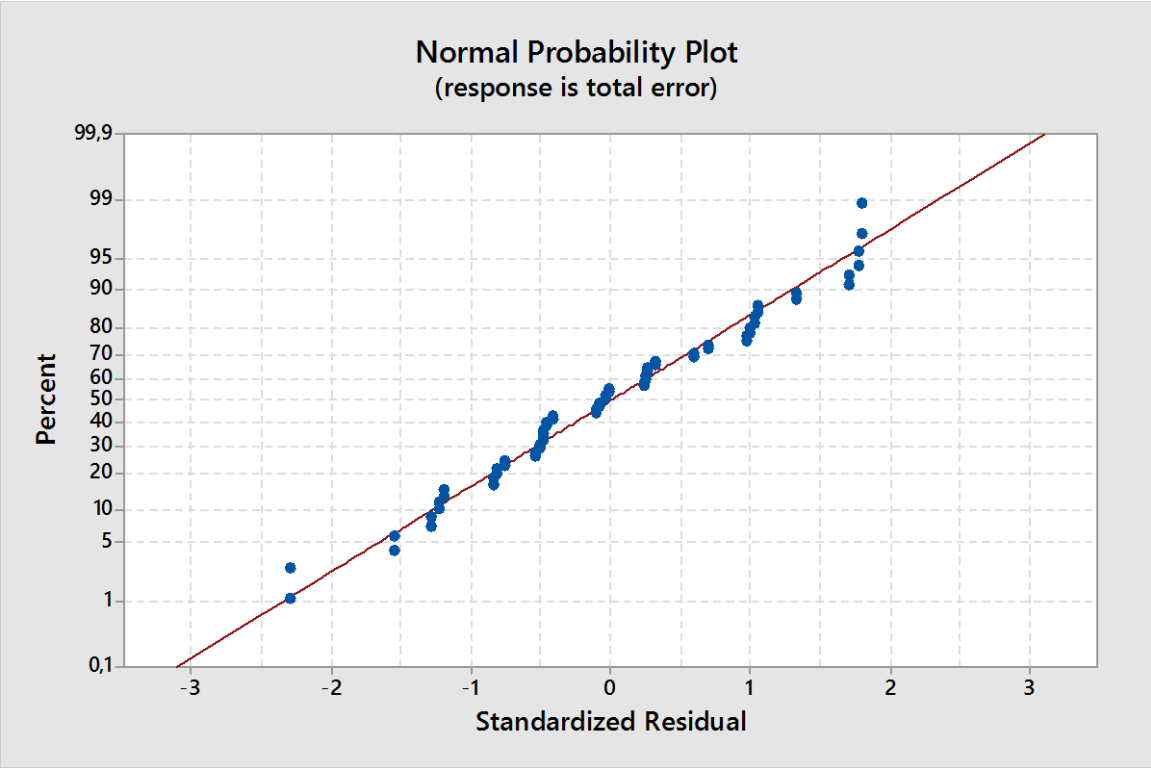

Figure 6.34: Residual Plot for the Total Error Response

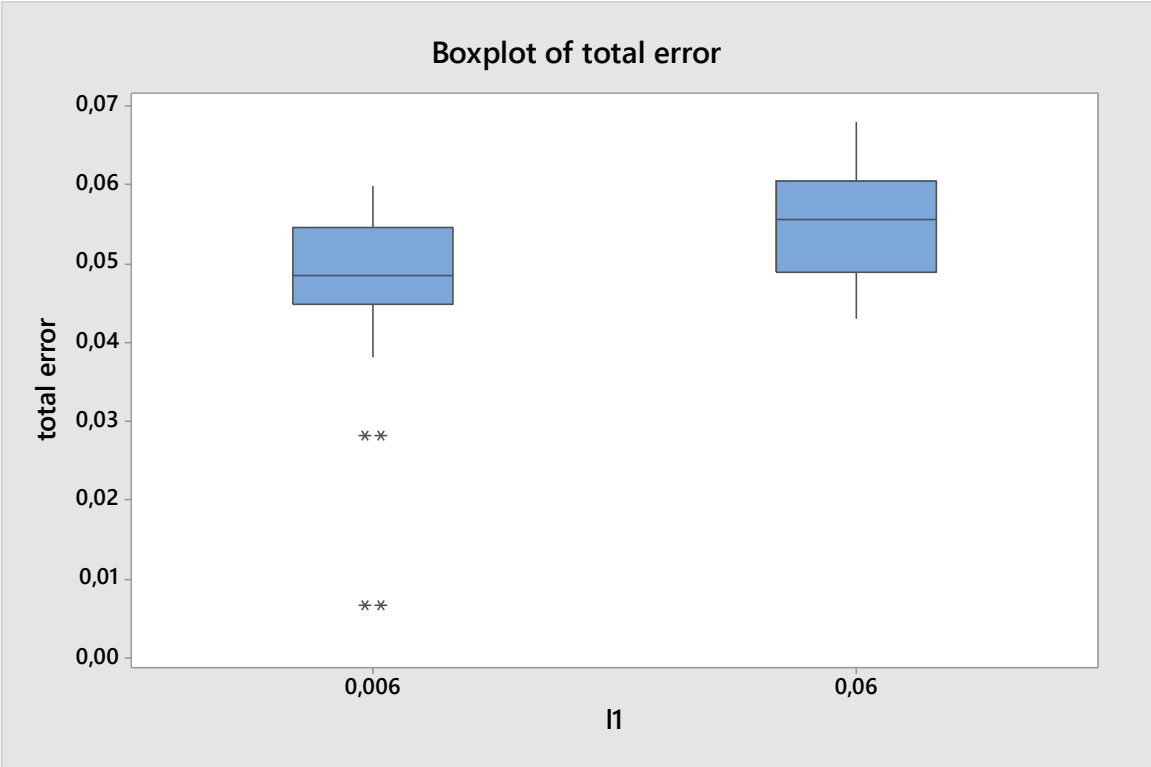

Figure 6.35: Boxplot of Total Error for levels of l1 length

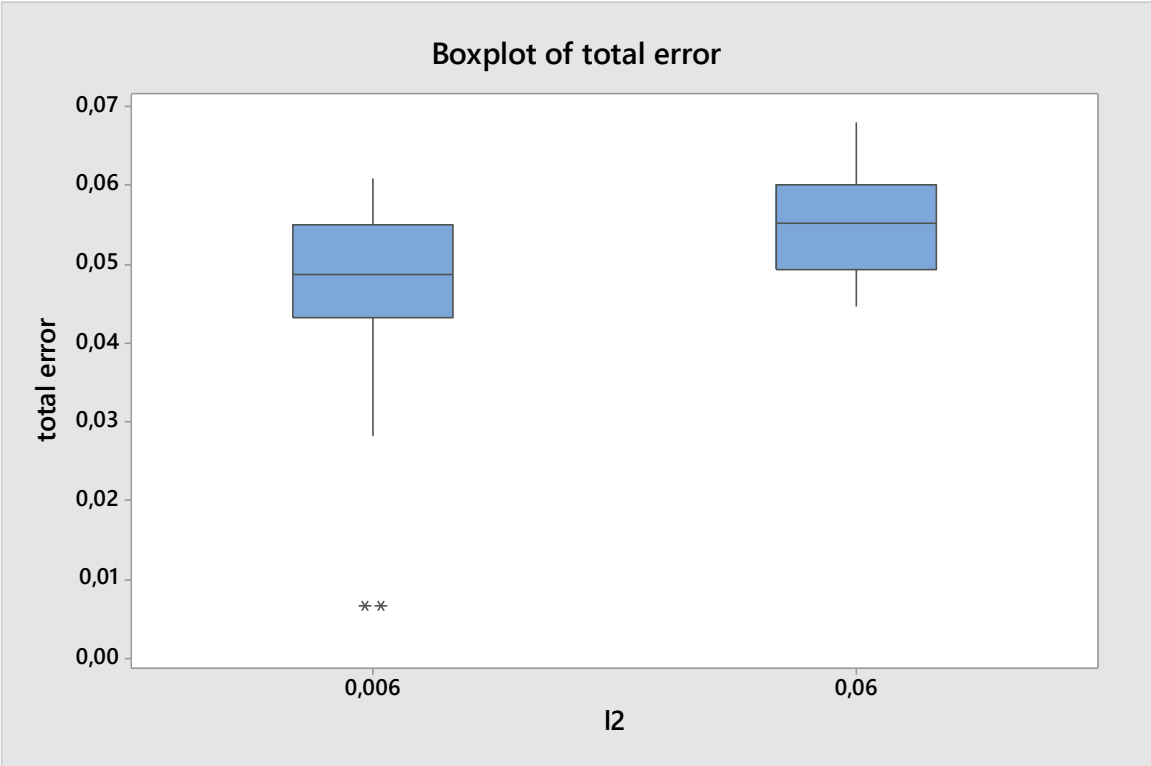

Figure 6.36: Boxplot of Total Error for levels of l2 length

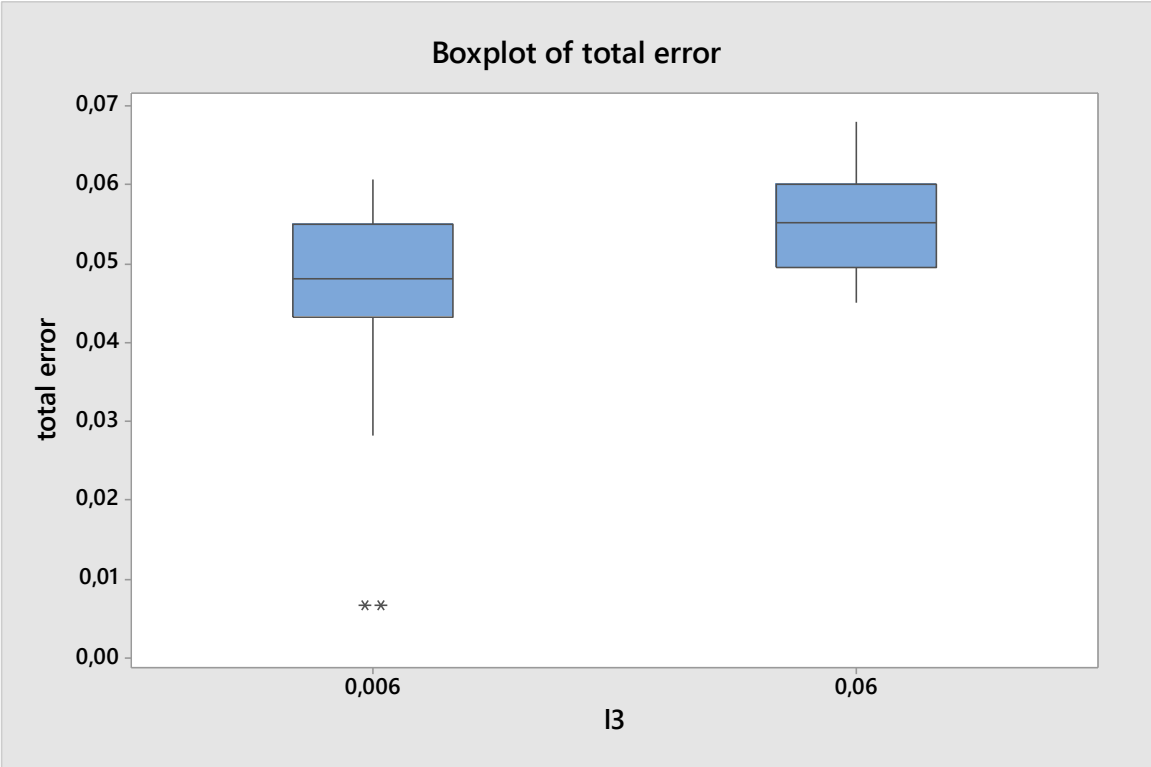

Figure 6.37: Boxplot of Total Error for levels of l3 length

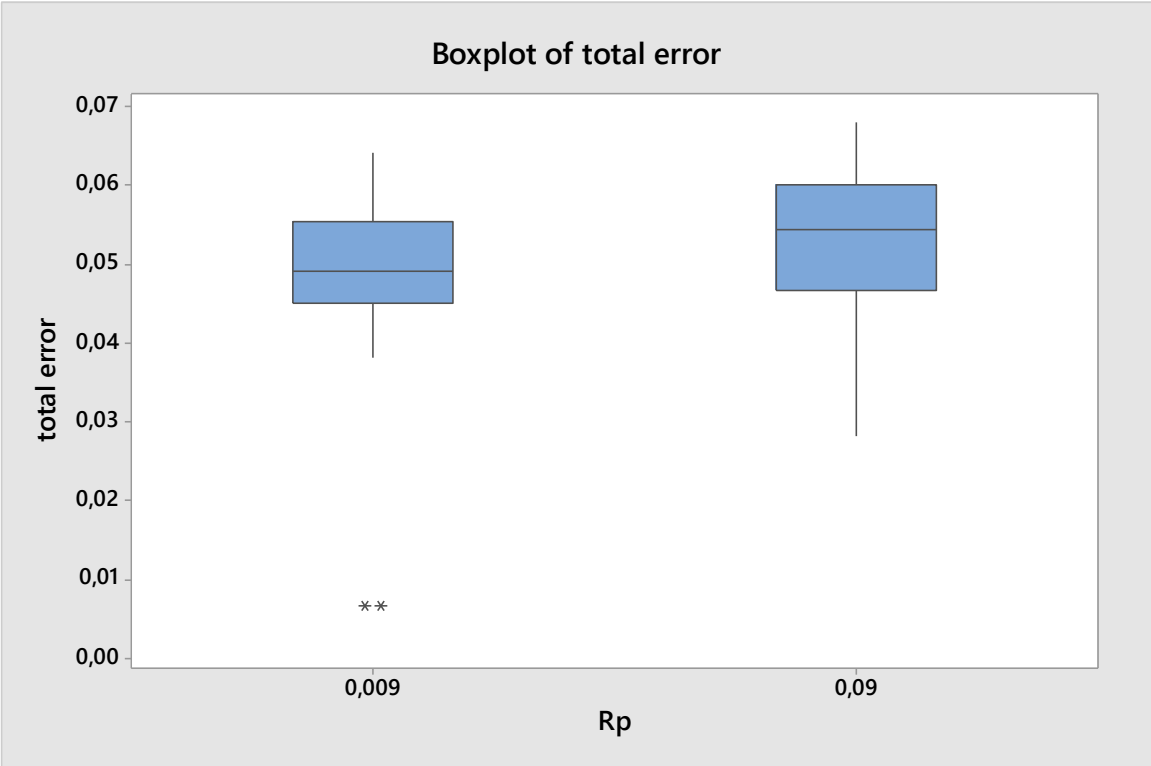

Figure 6.38: Boxplot of Total Error for levels of Rp Radius

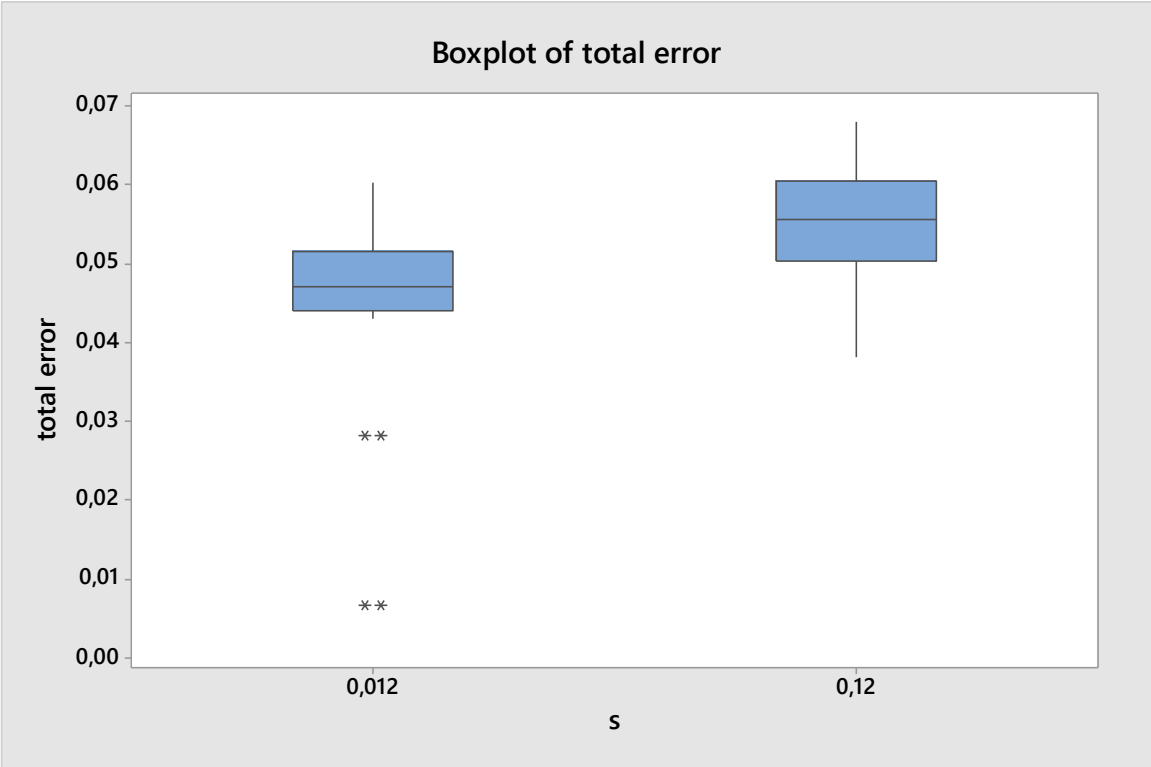

Figure 6.39: Boxplot of Total Error for levels of s length

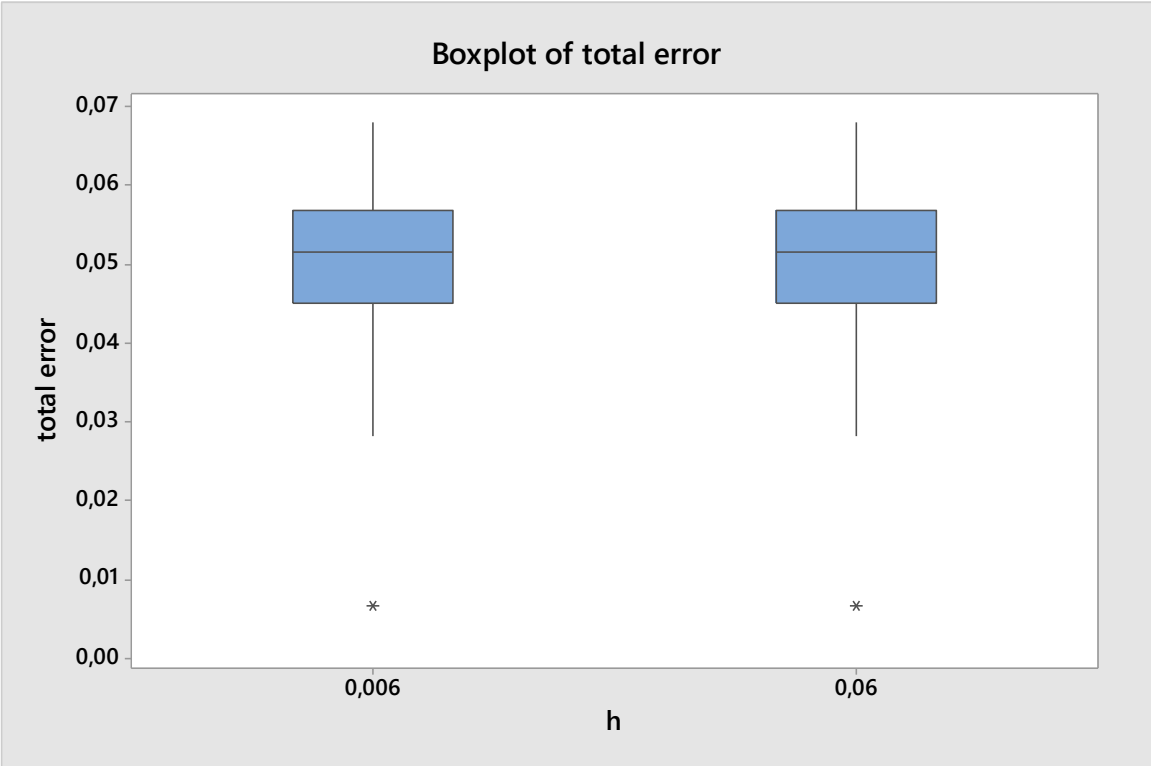

Figure 6.40: Boxplot of Total Error for levels of h length

l1, l2, l3, s and Rp

All influence quantities are effective, but Rp has lower effect with respect to other quantities, because of low tolerance values. From interaction plots, it can be seen that, their slope are fairly meaningful. However, when interaction plots with s influence quantity are examined, the distance between between high and low level lines are greatest. Because s has the highest effect among influence quantities. In addition to these, main effects plots show us that, s has the highest effect as we mentioned before.

h

It does not affect the position of mobile platform at z direction.

Significance Evaluation

If we look at main effects, all main effects are significant. However, h is not significant, because of the kinematics. While some 2-way and 3-way interactions are significant; 4-way, 5-way and 6-way interactions do not exist in the significant group of interactions.

6.1.5 Effects of IQs on the Tilting Roll (α) Angle

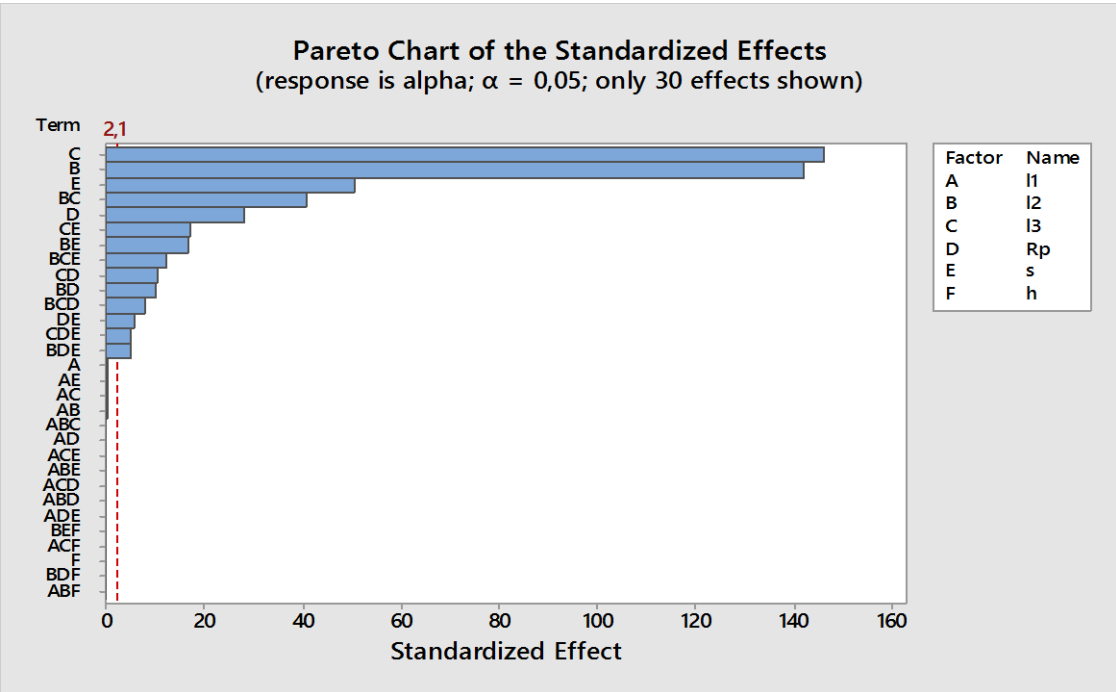

Figure 6.41: Pareto Chart of the Uncertainty Effects on the Roll Angle Response

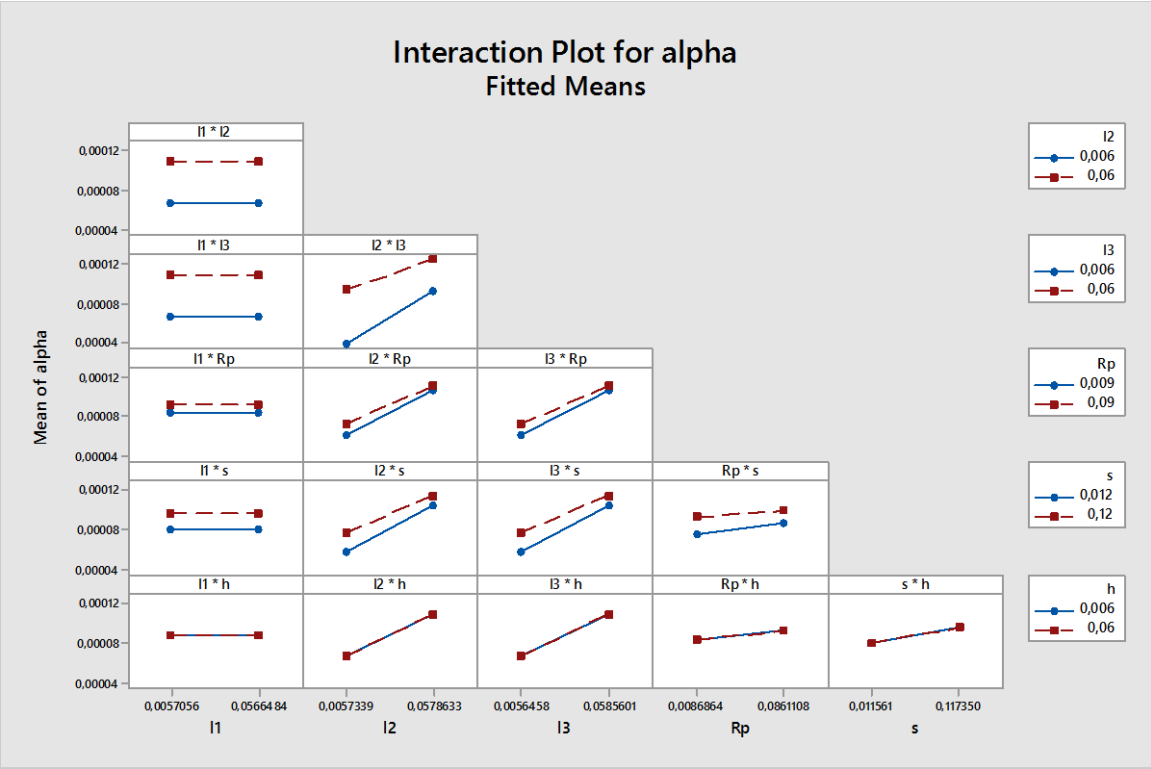

Figure 6.42: Interaction Plot of the Uncertainty Effects on the Roll Angle Response

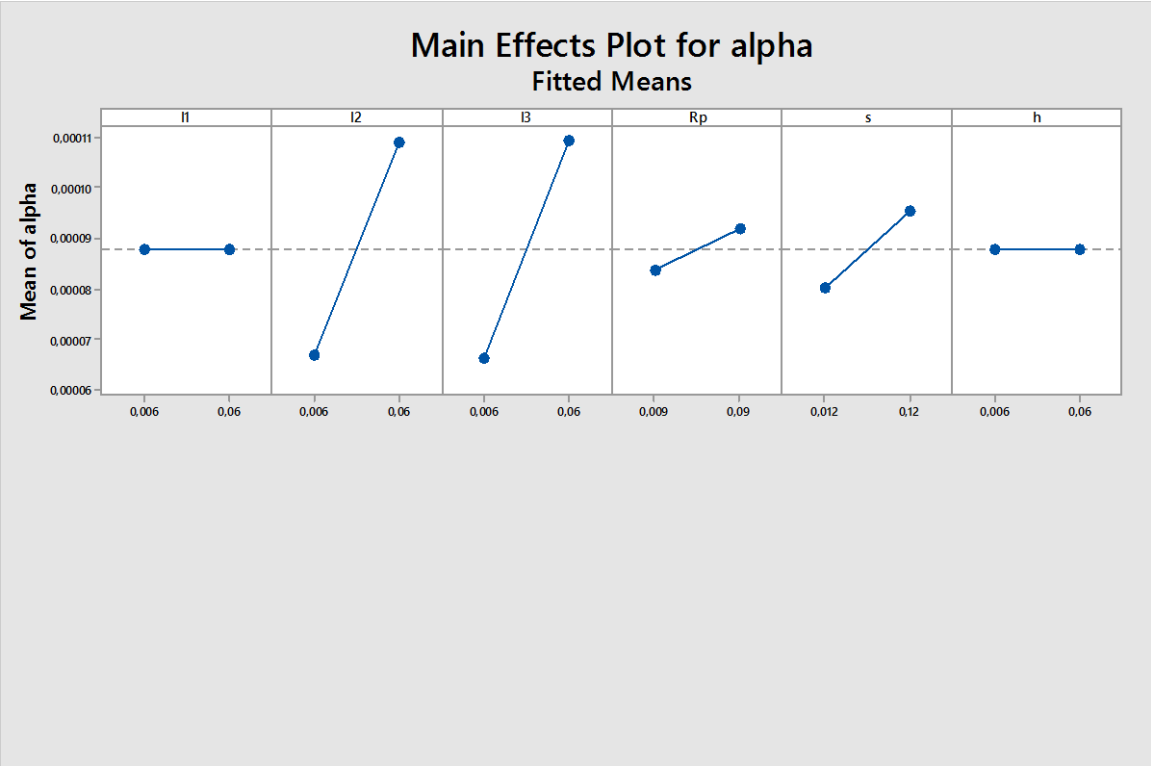

Figure 6.43: Main Effects Plots for the Roll Angle Response

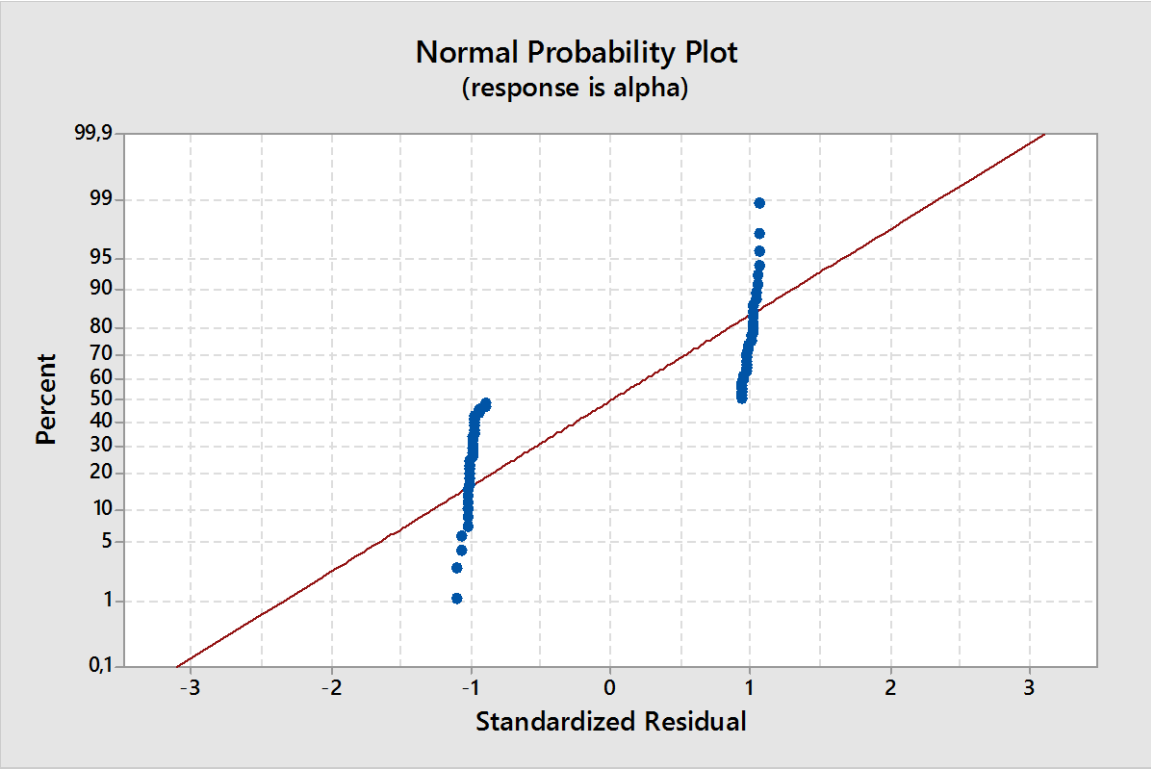

Figure 6.44: Residual Plot for the Roll Angle Response

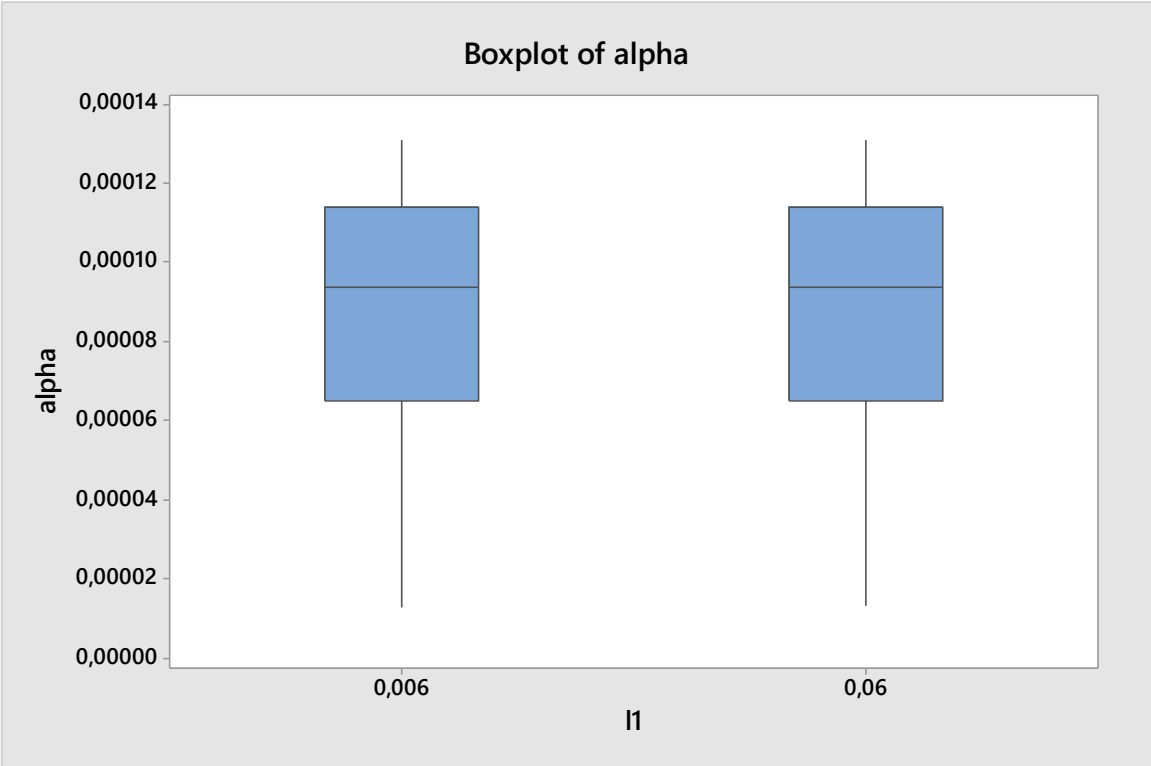

Figure 6.45: Boxplot of Roll Angle for levels of l1 length

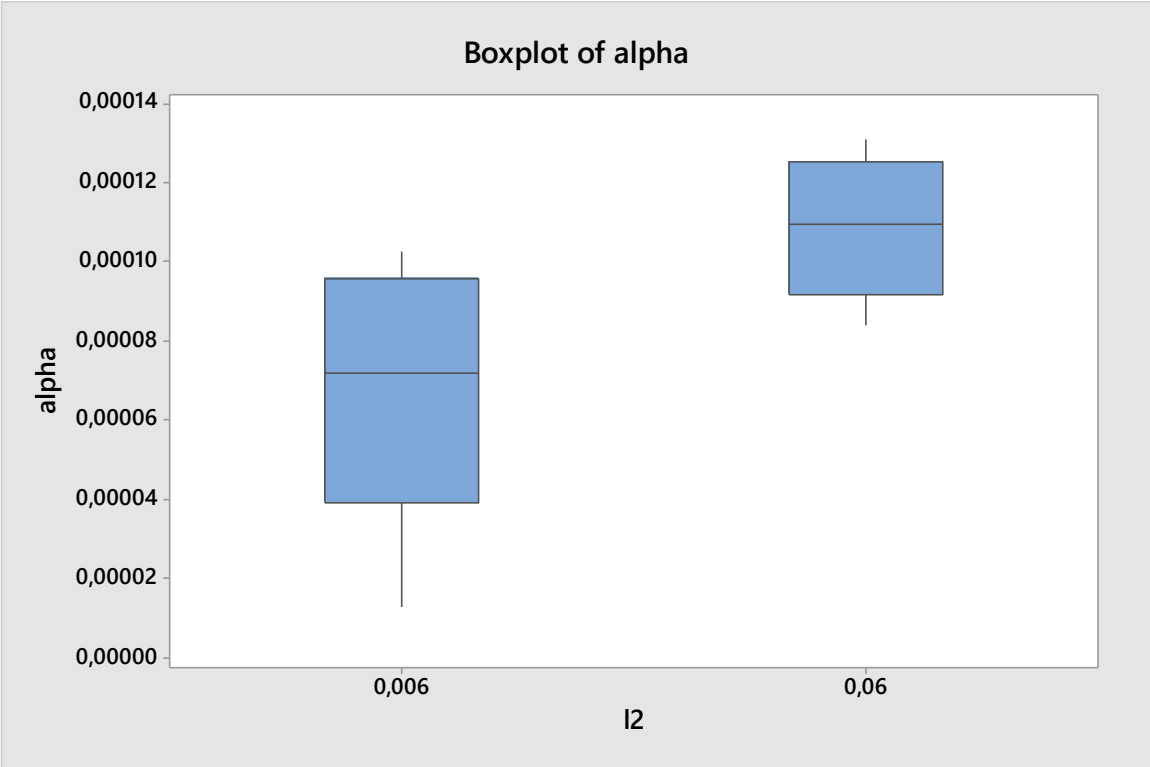

Figure 6.46: Boxplot of Roll Angle for levels of l2 length

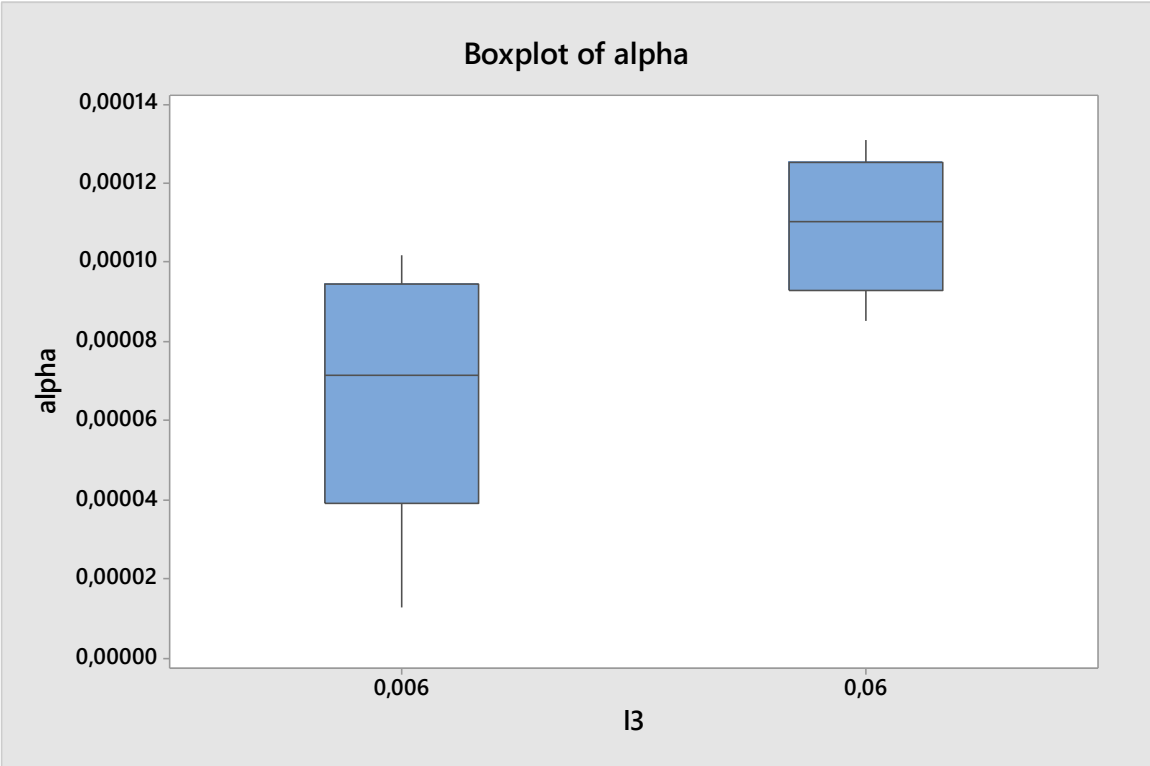

Figure 6.47: Boxplot of Roll Angle for levels of l3 length

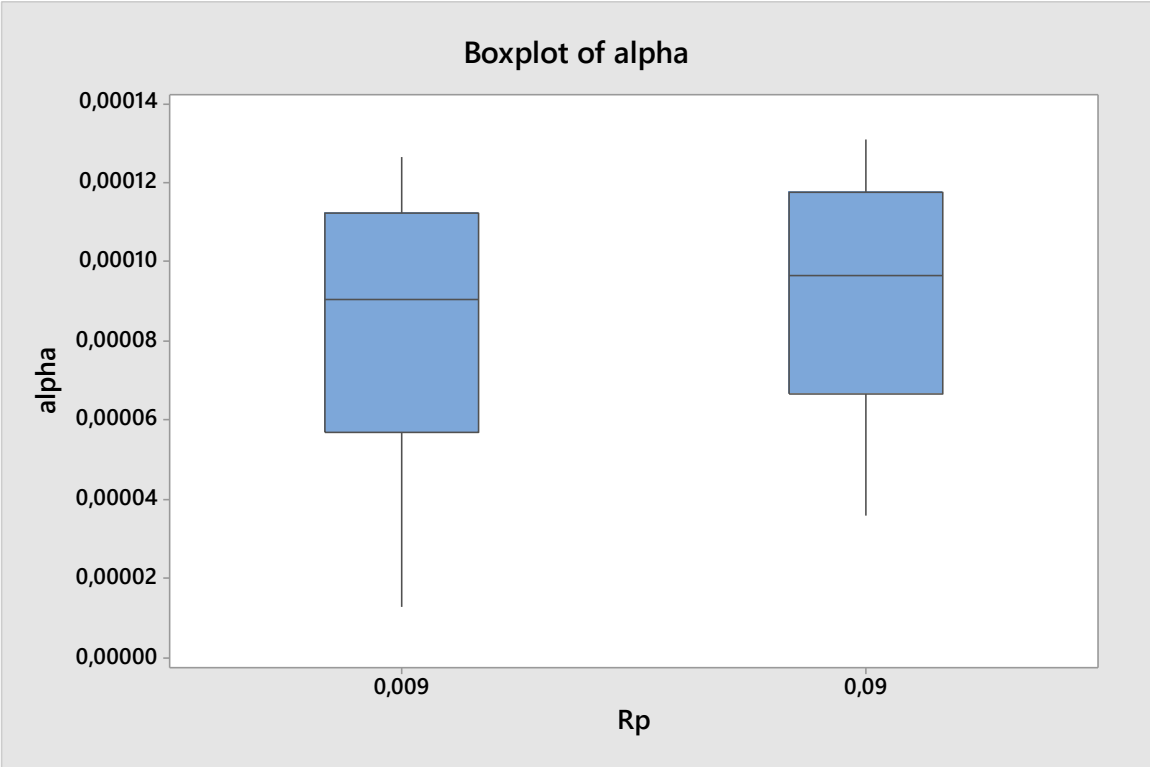

Figure 6.48: Boxplot of Roll Angle for levels of Rp Radius

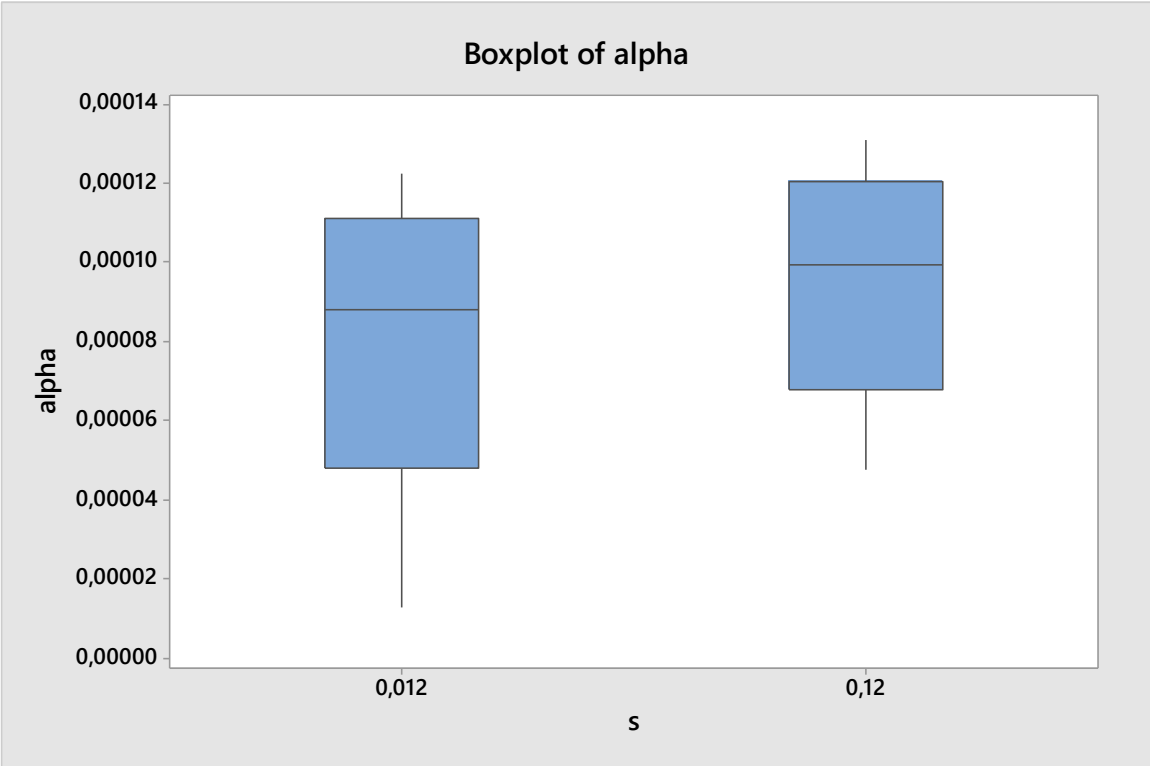

Figure 6.49: Boxplot of Roll Angle for levels of s length

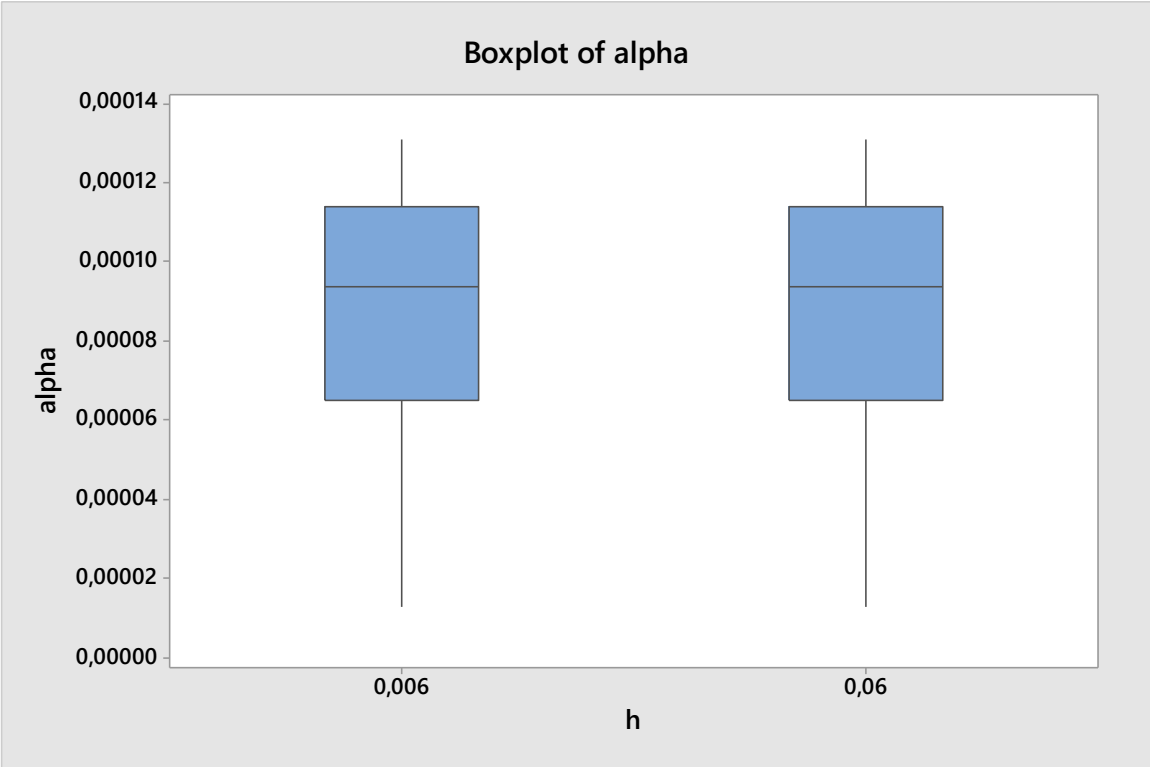

Figure 6.50: Boxplot of Roll Angle for levels of h length

l1, l2, l3, Rp,s and h

Except l1 and h, all other influence quantities are effective, but Rp and s have lower effect with respect to other quantities, because of kinematics. From interaction plots, it can be seen that, their slope are fairly meaningful. However, when interaction plots with l1 and h quantitities are examined, the distance between between high and low level lines are almost coincident. Because l1 and h have no effect among influence quantities. In addition, main effects plots show us that, l2 and l3 have the highest effect as we mentioned before.

Significance Evaluation

If we look at main effects, l2,l3,Rp,s effects are significant. However, h and l1 is not significant, because of the kinematics. While some 2-way and 3-way interactions are significant; 4-way, 5-way and 6-way interactions do not exist in the significant group of interactions.

6.1.6 Effects of IQs on the Tilting Pitch (β) angle

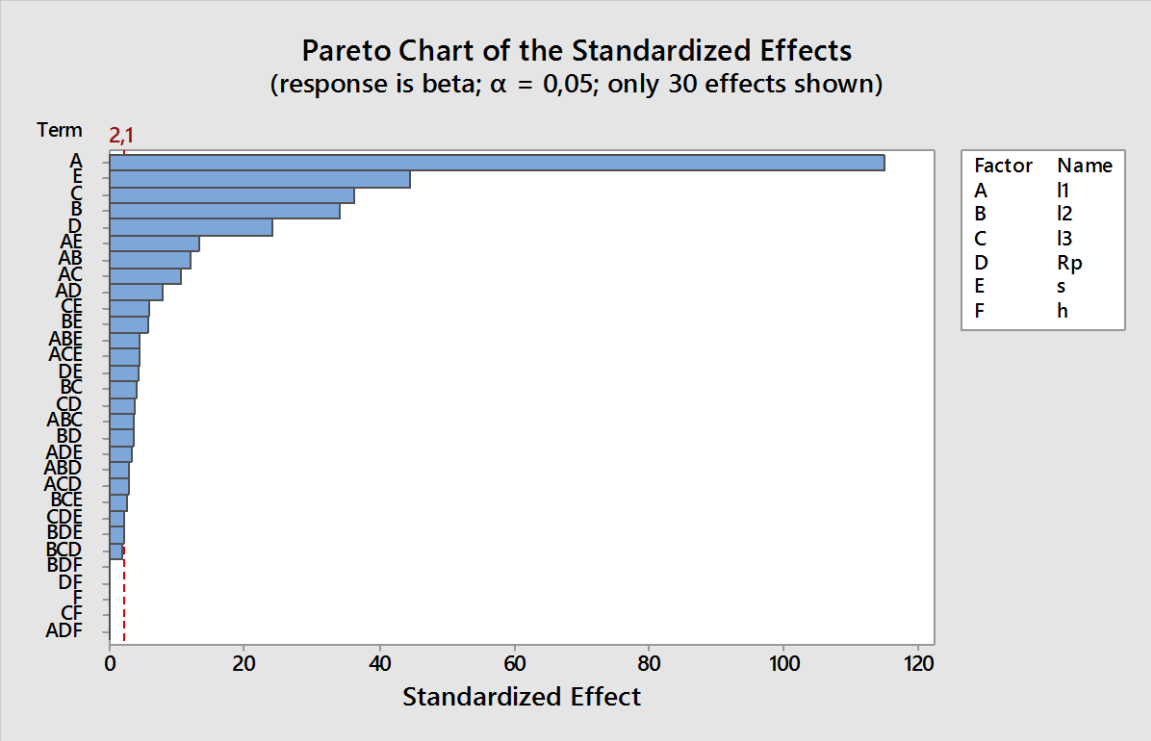

Figure 6.51: Pareto Chart of the Uncertainty Effects on the Pitch Angle Response

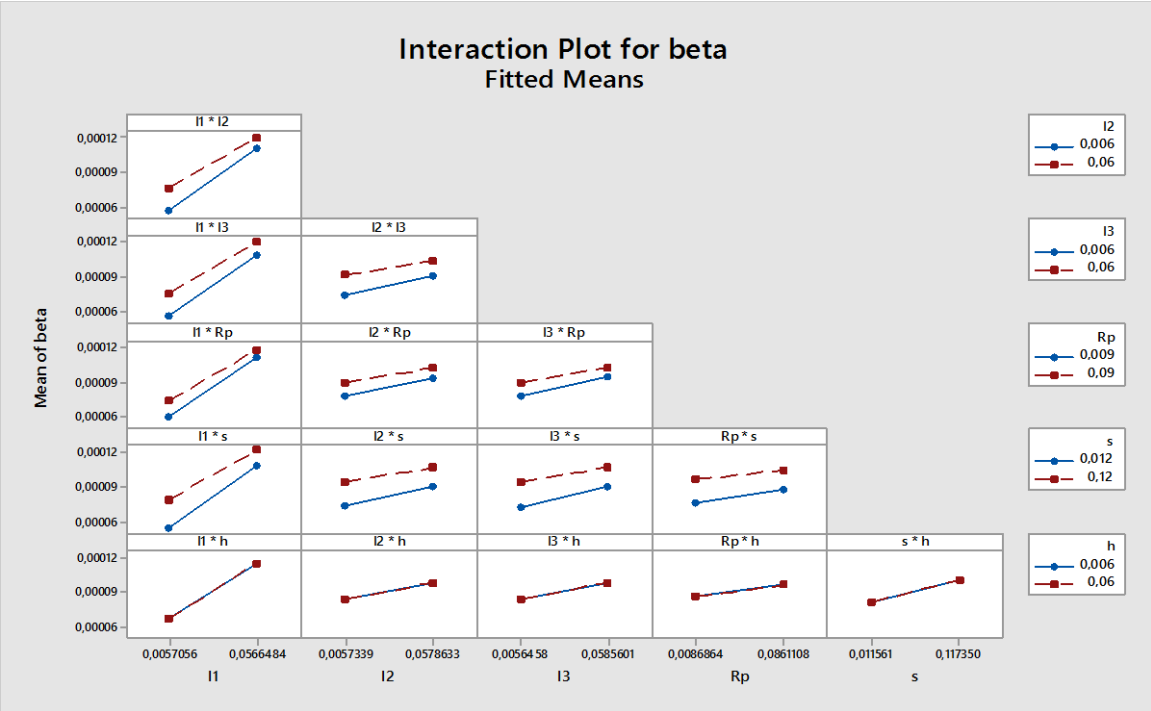

Figure 6.52: Interaction Plot of the Uncertainty Effects on the Pitch Angle Response

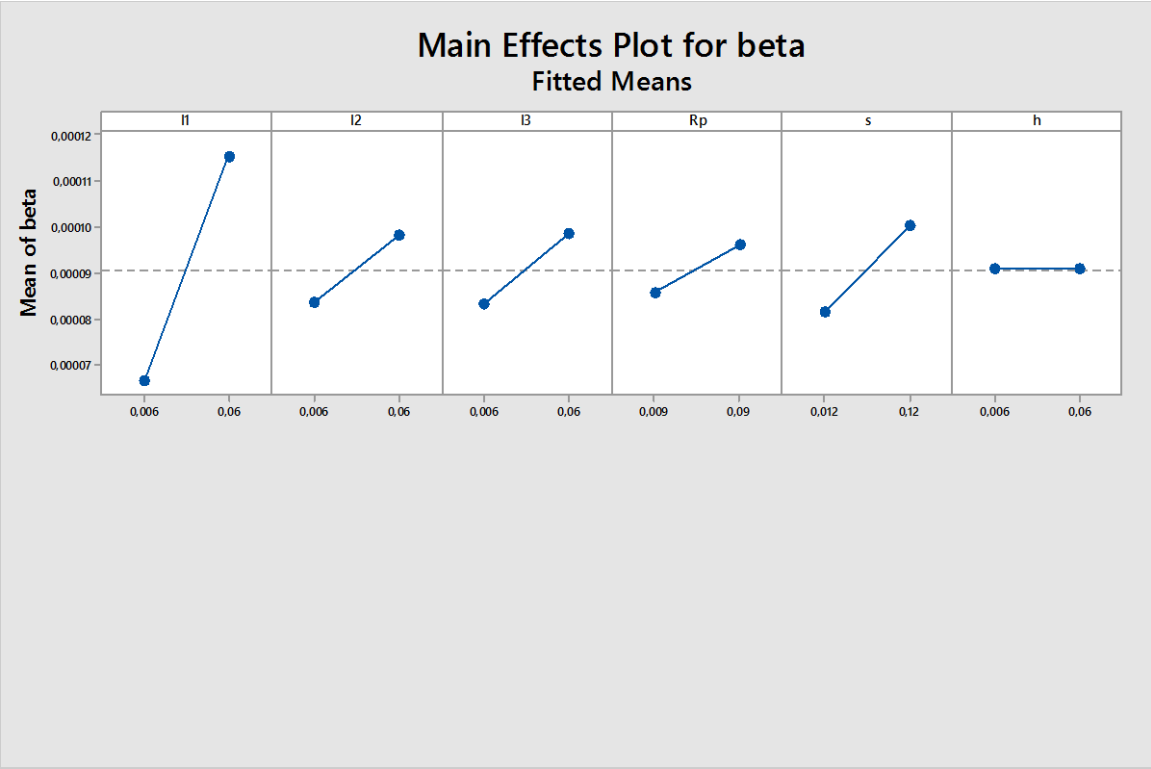

Figure 6.53: Main Effects Plots for the Pitch Angle Response

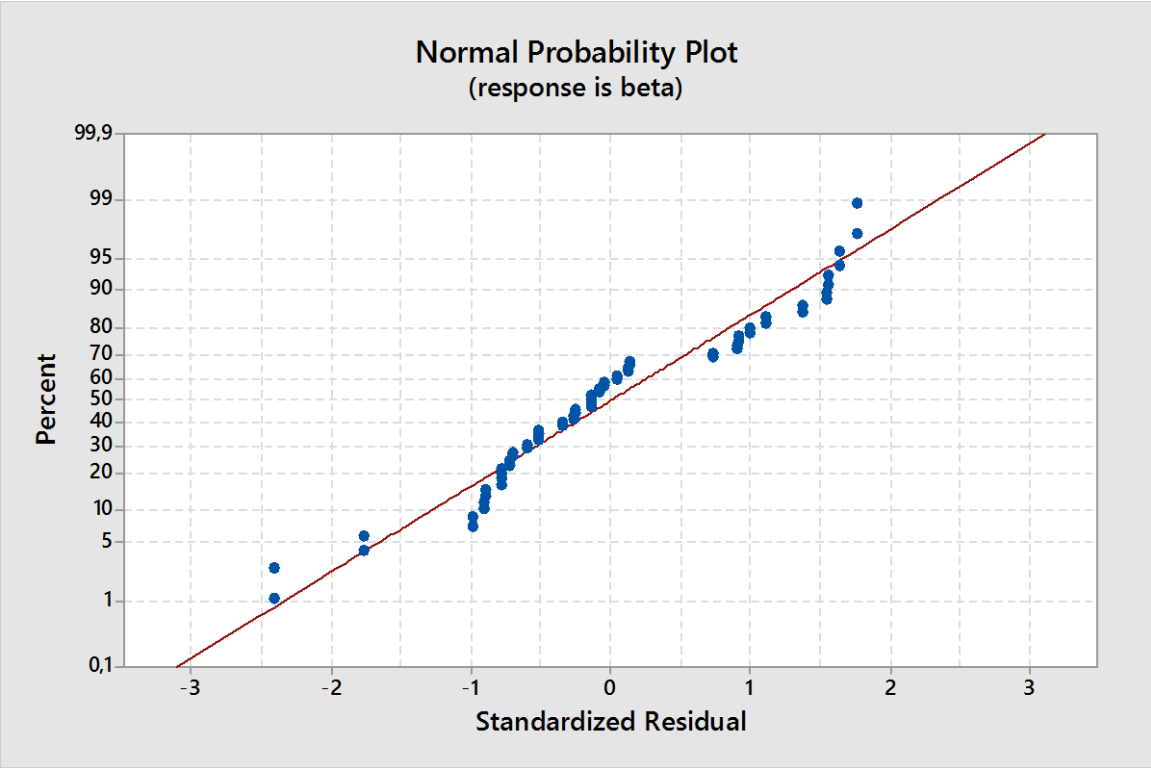

Figure 6.54: Residual Plot for the Pitch Angle Response

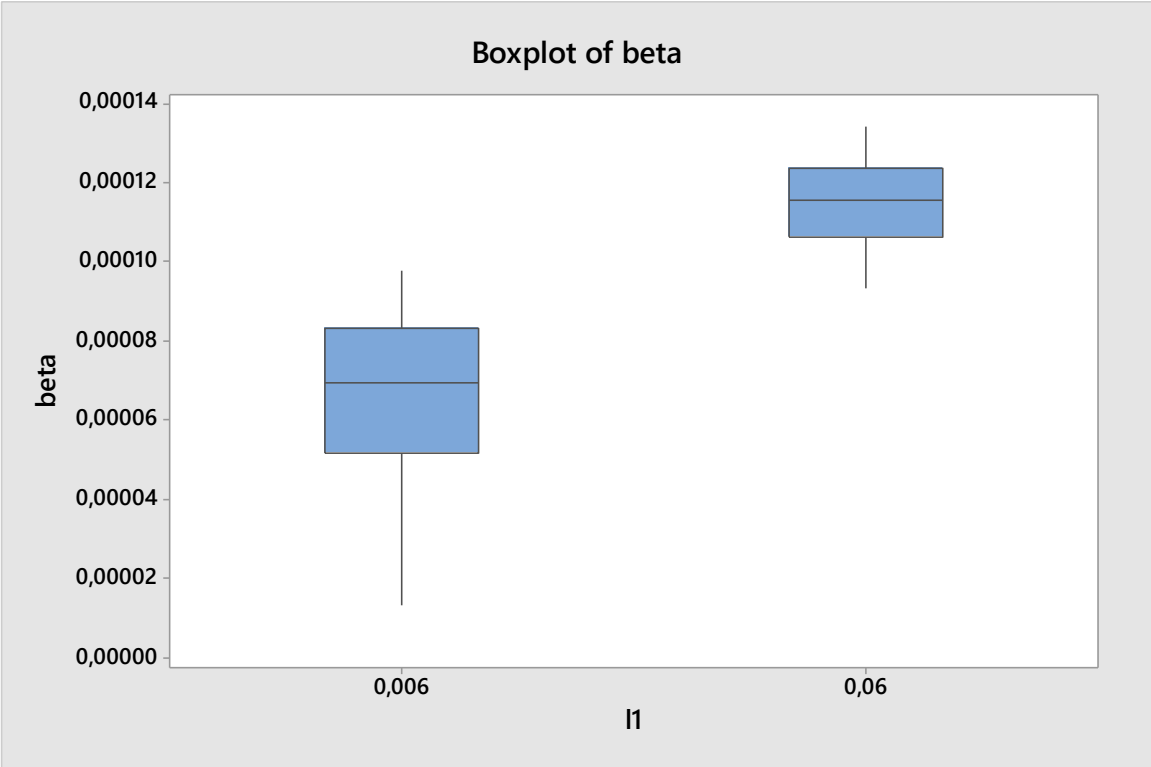

Figure 6.55: Boxplot of Pitch Angle for levels of l1 length

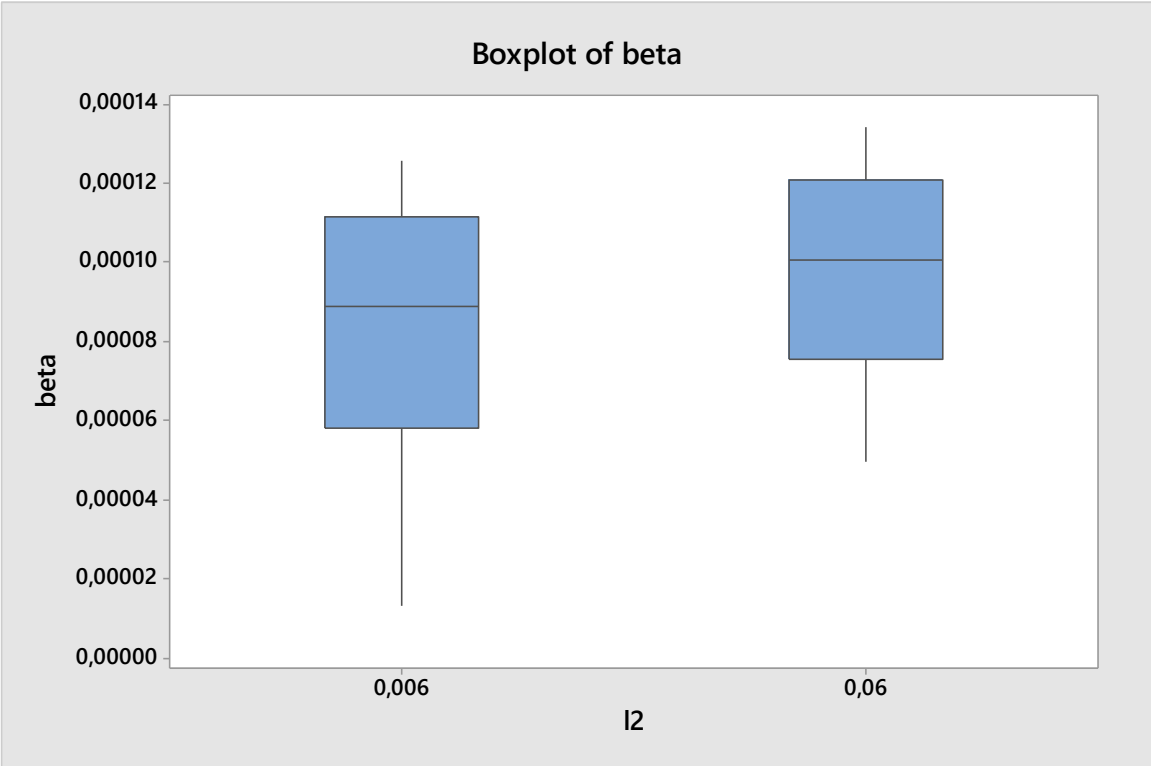

Figure 6.56: Boxplot of Pitch Angle for levels of l2 length

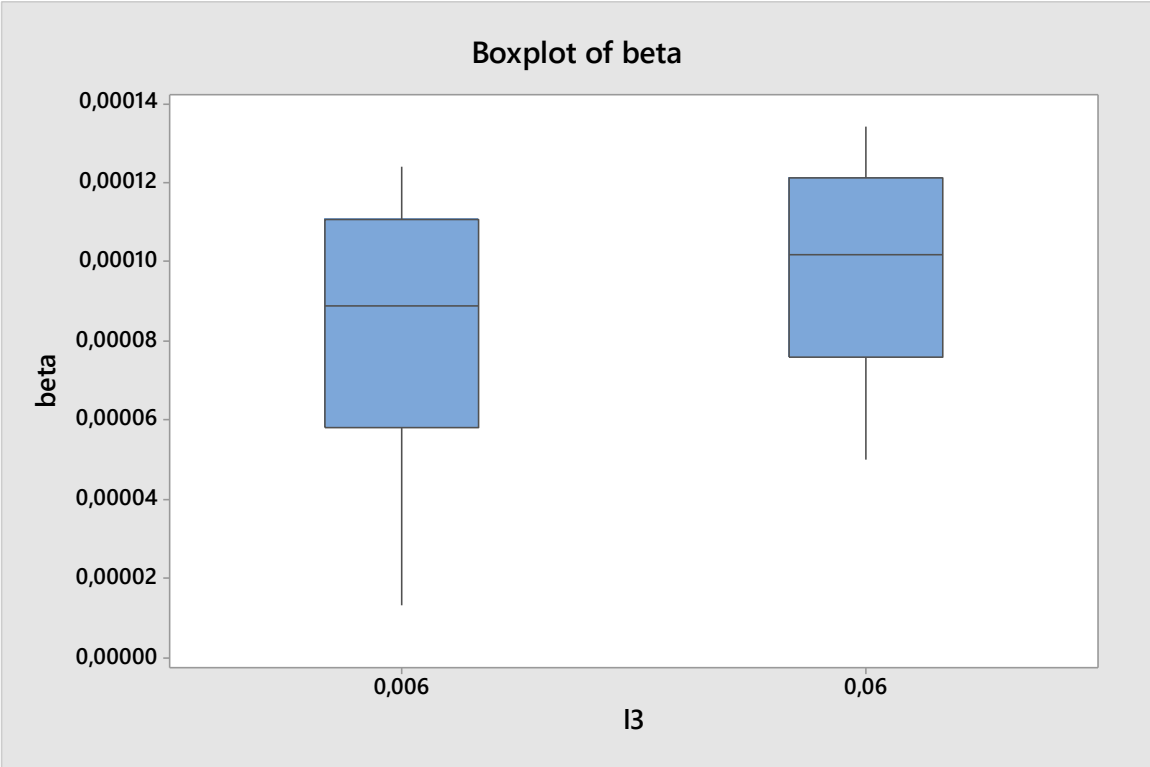

Figure 6.57: Boxplot of Pitch Angle for levels of l3 length

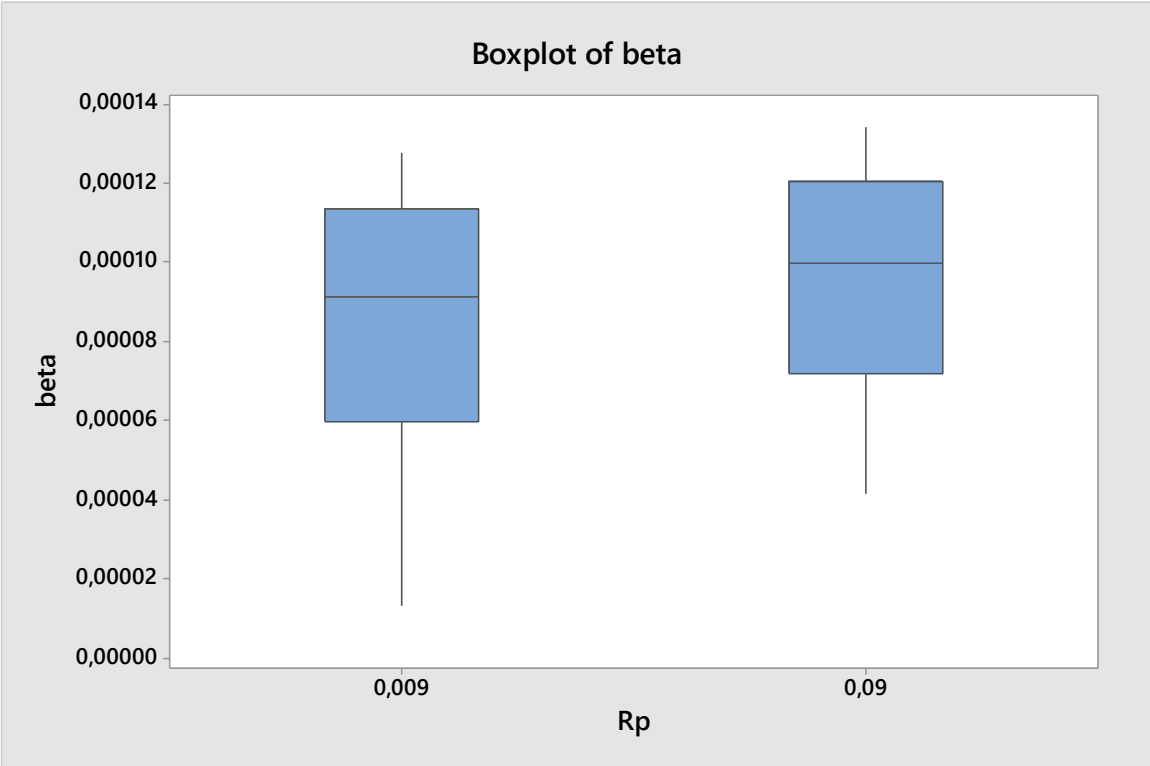

Figure 6.58: Boxplot of Pitch Angle for levels of Rp Radius

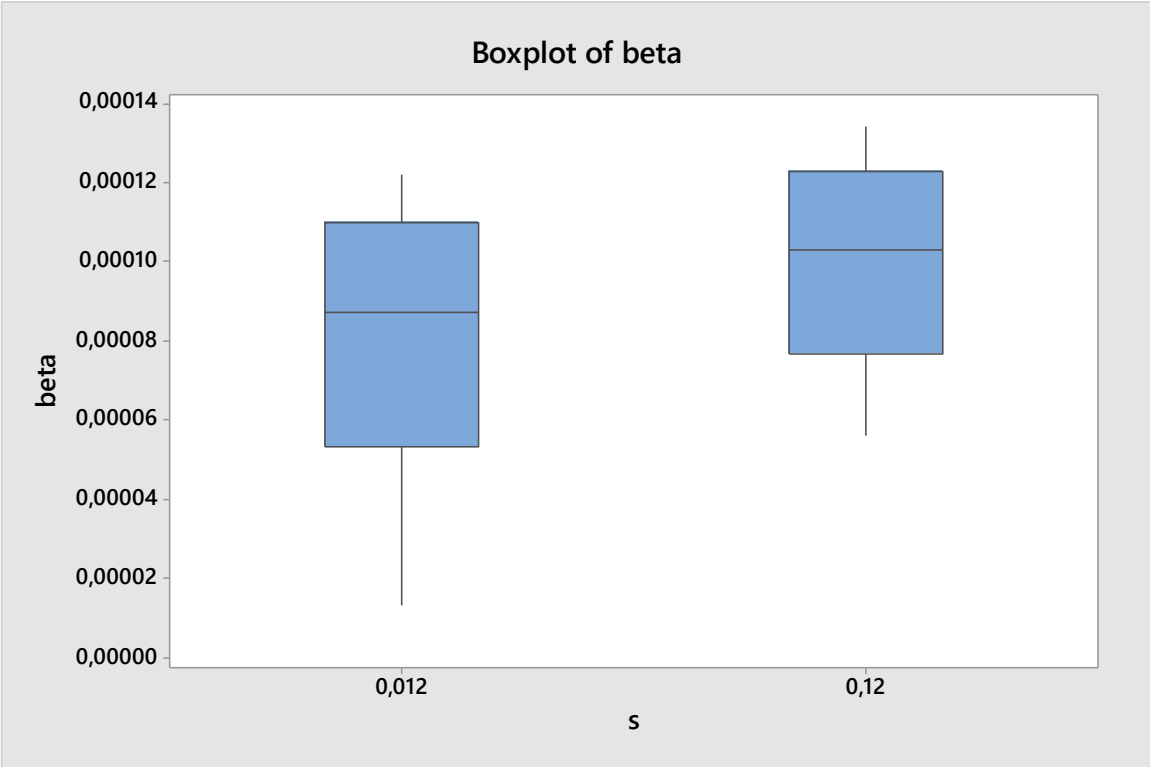

Figure 6.59: Boxplot of Pitch Angle for levels of s length

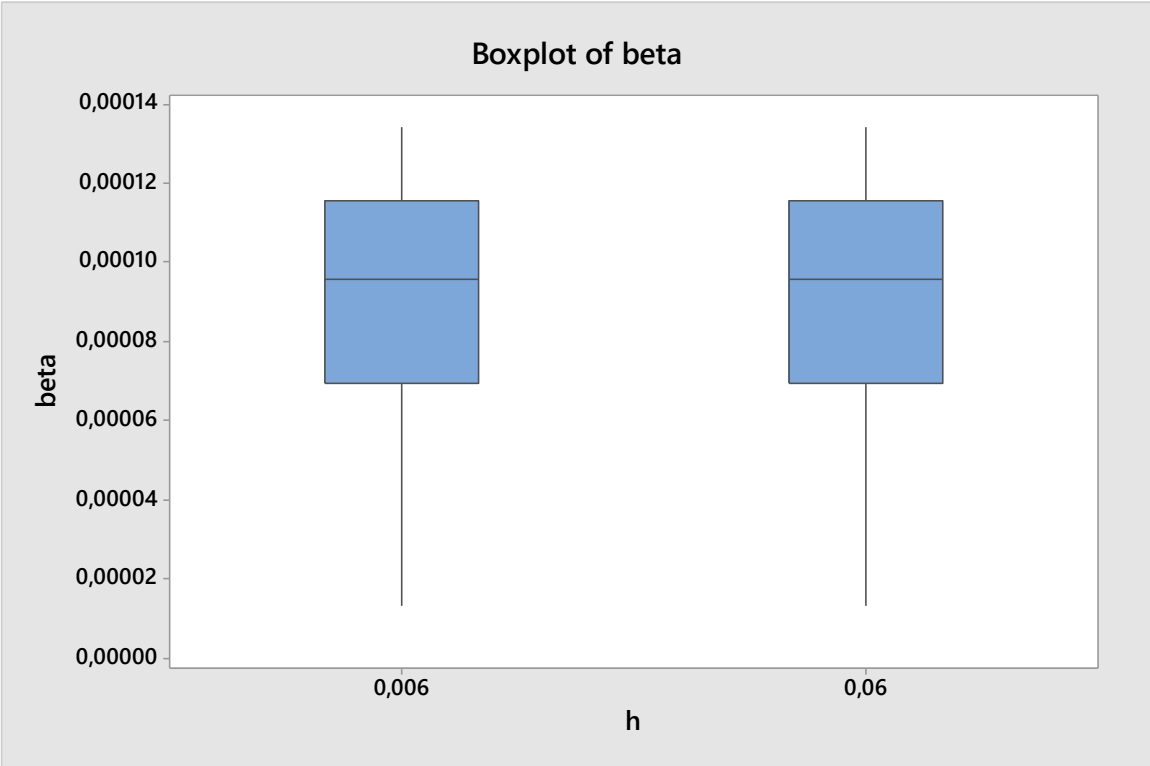

Figure 6.60: Boxplot of Pitch Angle for levels of h length

l1, l2, l3, Rp,s and h

Almost all influence quantities are effective, but only h has no effect because of kinematics. From interaction plots, it can be seen that, their slope with l1 terms have greatest magnitude. However, when interaction plots with h quantitities are examined, the distance between between high and low level lines are coincident. Because h has no effect among influence quantities. In addition, main effects plots show us that, l1 has the highest effect as we mentioned before.

Significance Evaluation

If we look at main effects, l1,l2,l3,Rp,s effects are significant. However, h is not significant, because of the kinematics. While some 2-way and 3-way interactions are significant; 4-way, 5-way and 6-way interactions do not exist in the significant group of interactions.

6.1.7 Effects of IQs on Shift Error

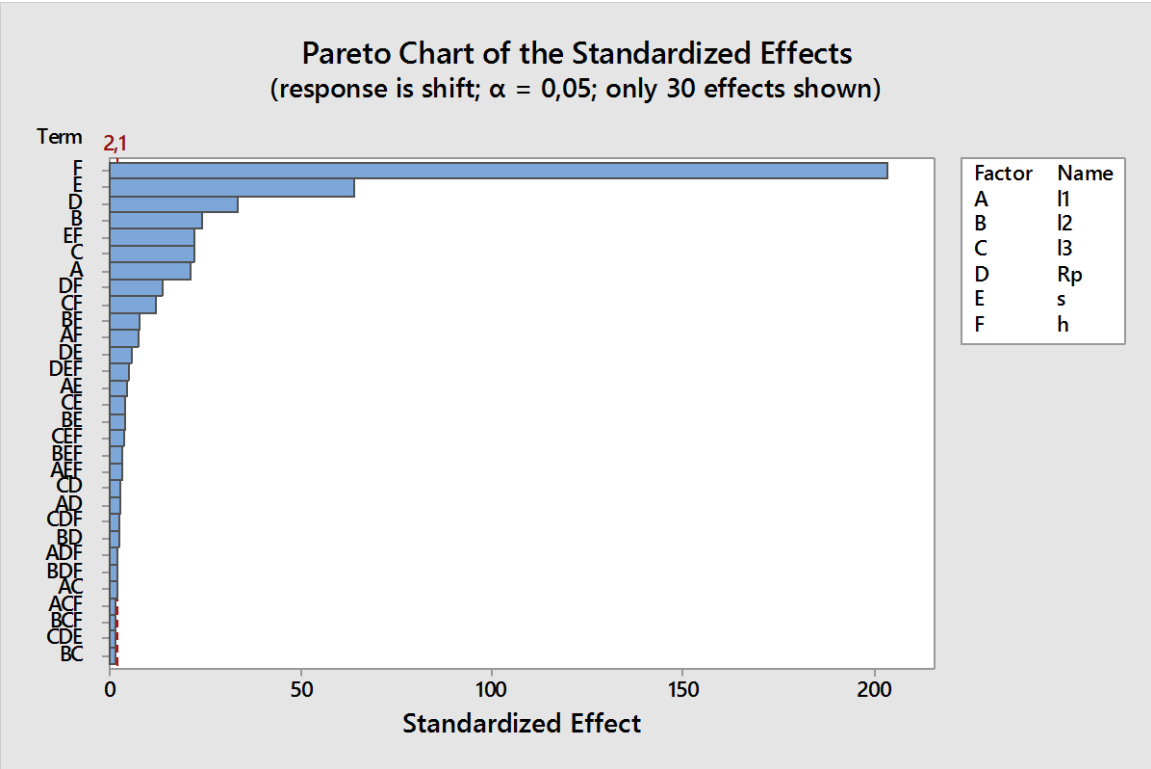

Figure 6.61: Pareto Chart of the Uncertainty Effects on the Shifting Response

Figure 6.62: Interaction Plot of the Uncertainty Effects on the Shifting Response

Figure 6.63: Main Effects Plots for the Shifting Response

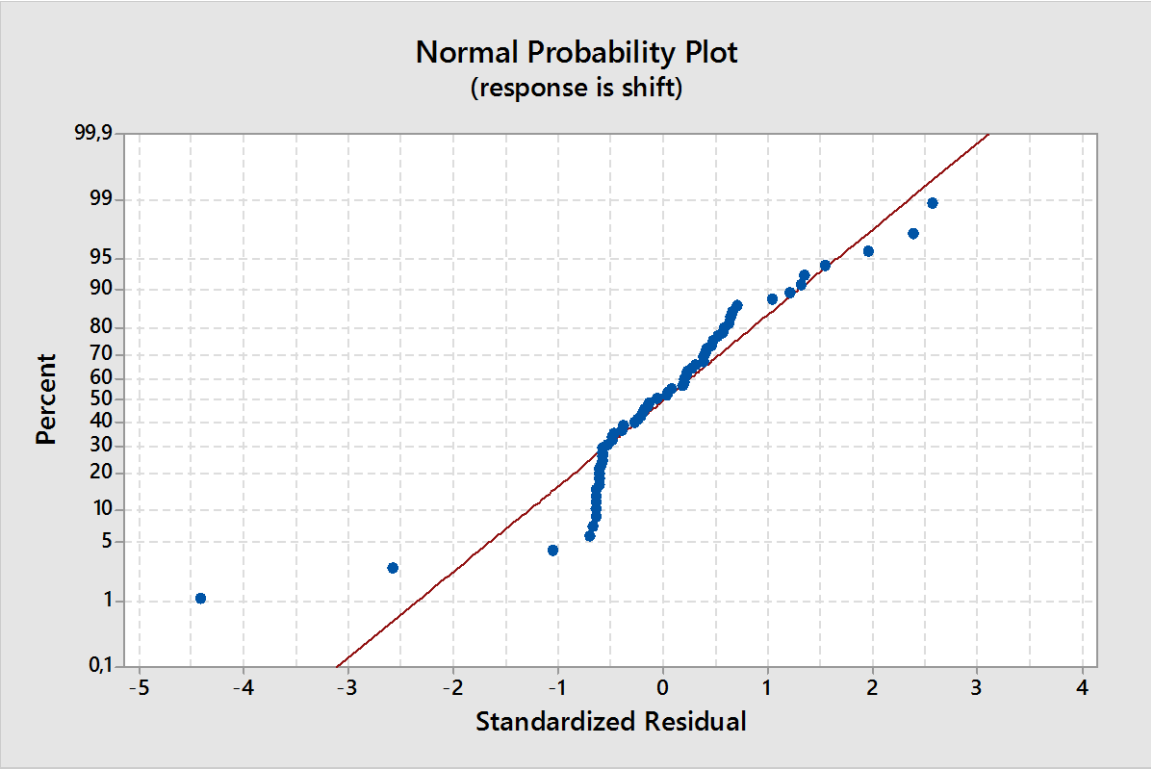

Figure 6.64: Residual Plot for the Shifting Response

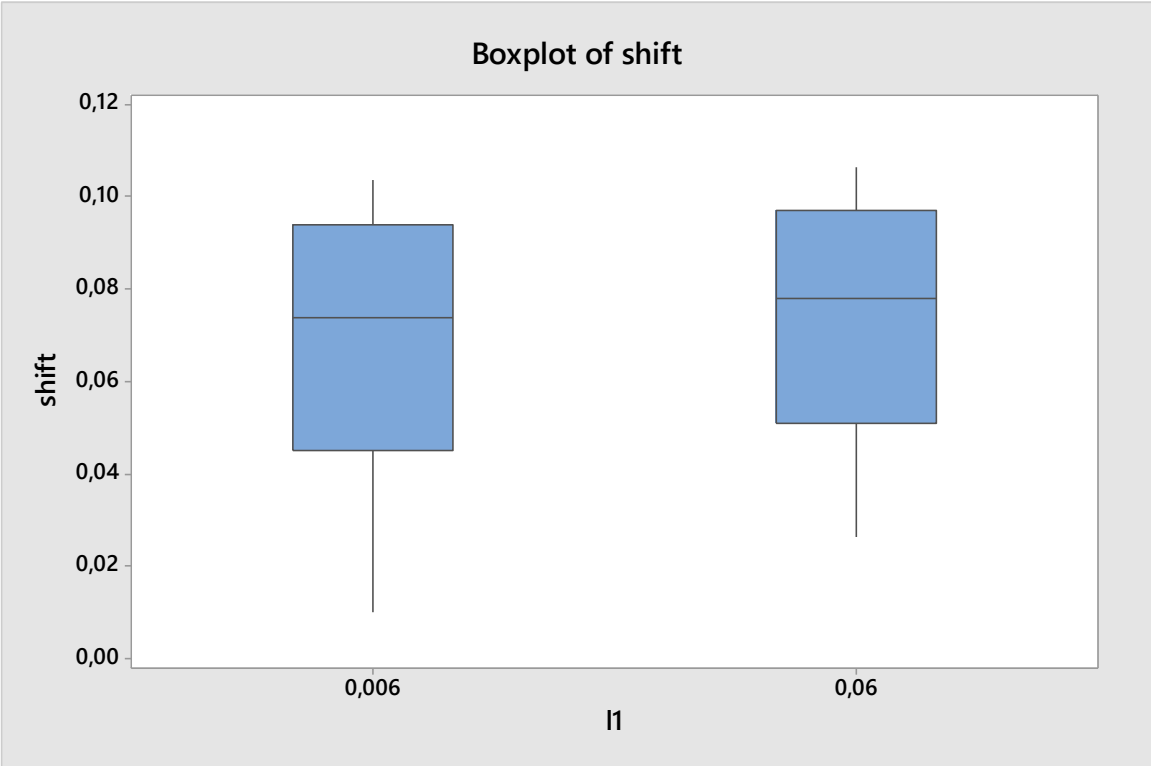

Figure 6.65: Boxplot of Shifting Error for levels of l1 length

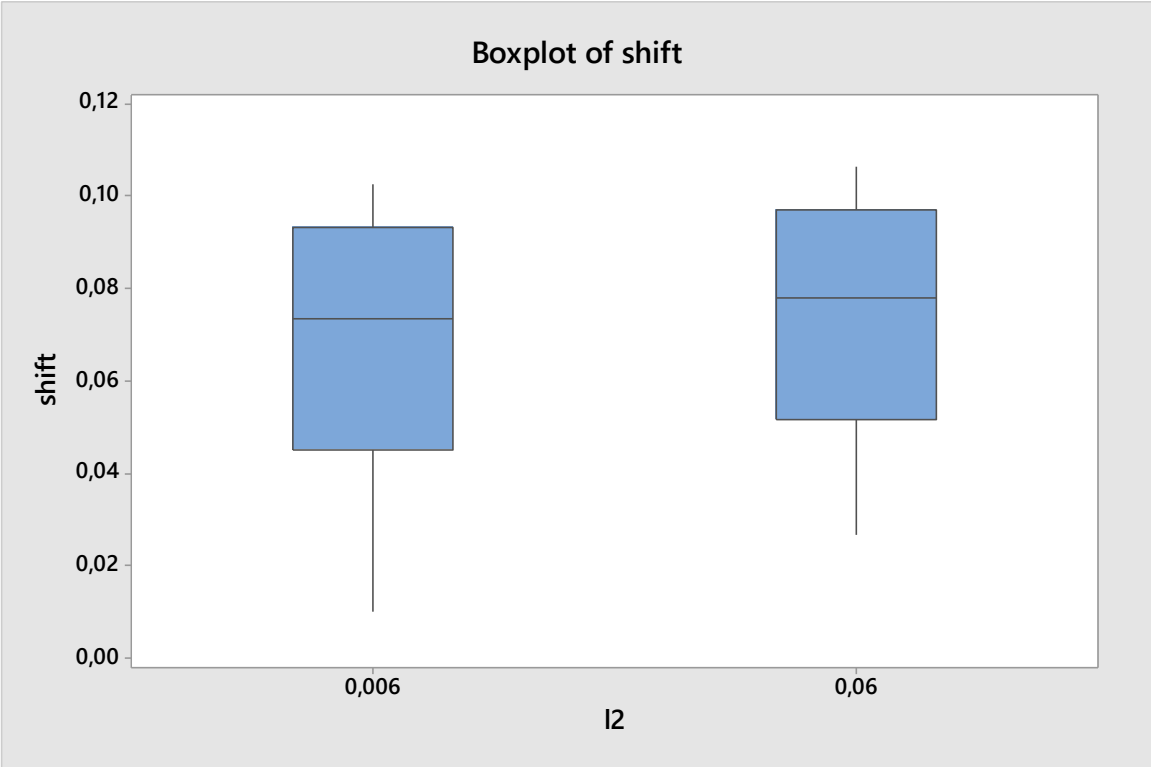

Figure 6.66: Boxplot of Shifting Error for levels of l2 length

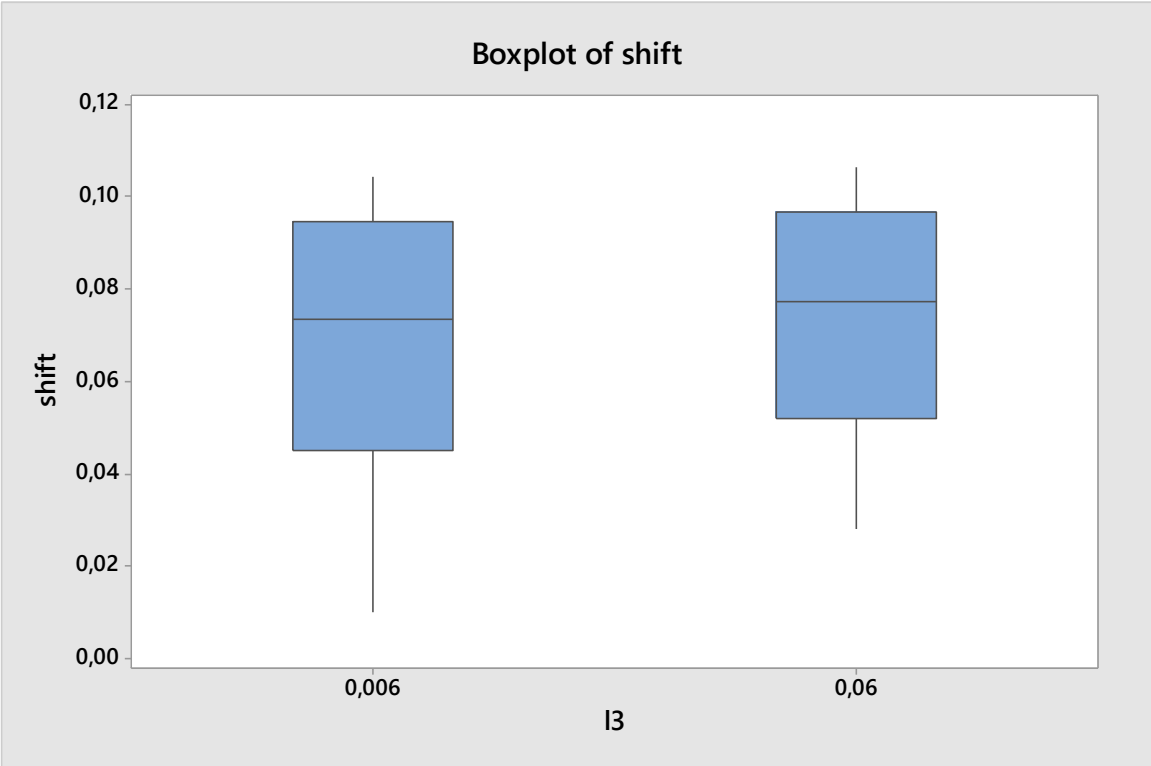

Figure 6.67: Boxplot of Shifting Error for levels of l3 length

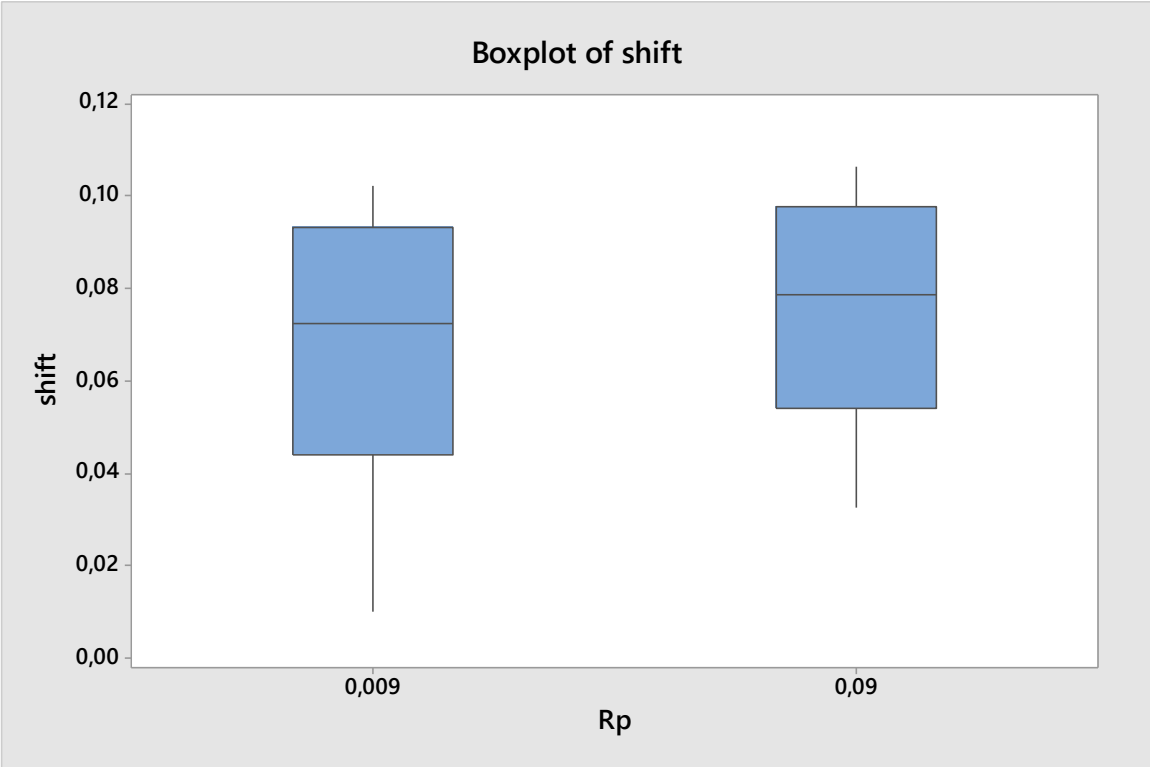

Figure 6.68: Boxplot of Shifting Error for levels of Rp Radius

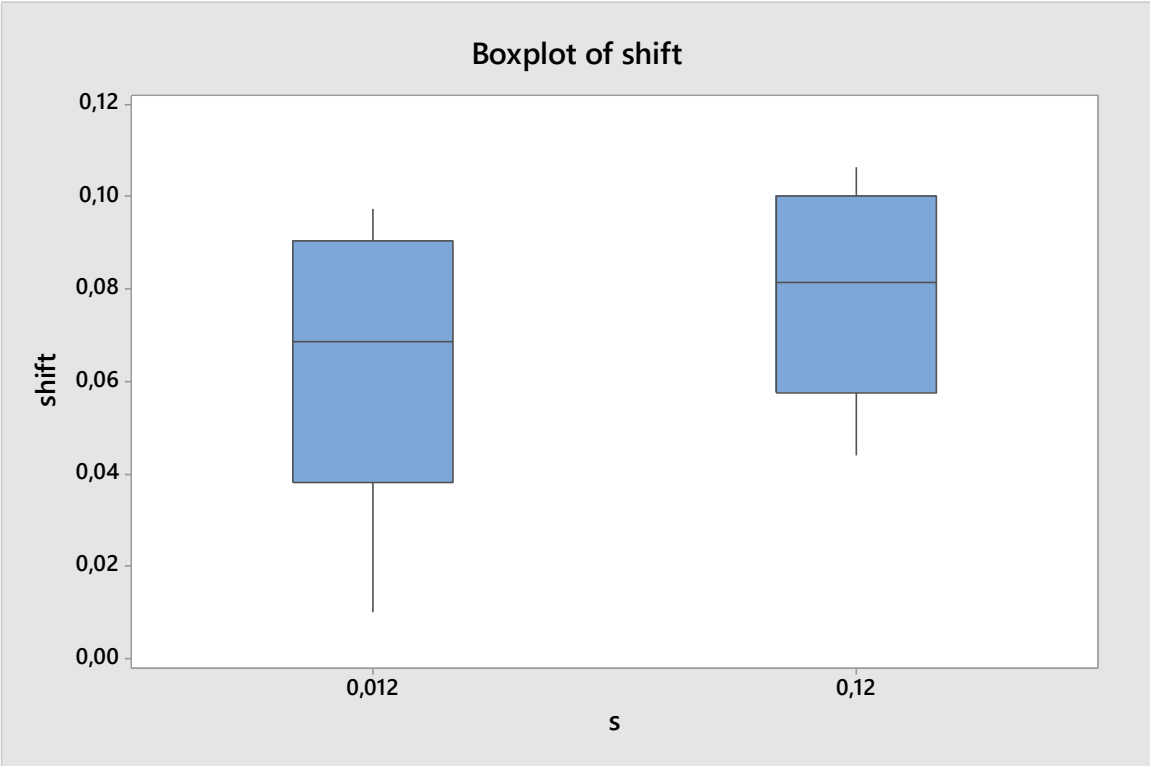

Figure 6.69: Boxplot of Shifting Error for levels of s length

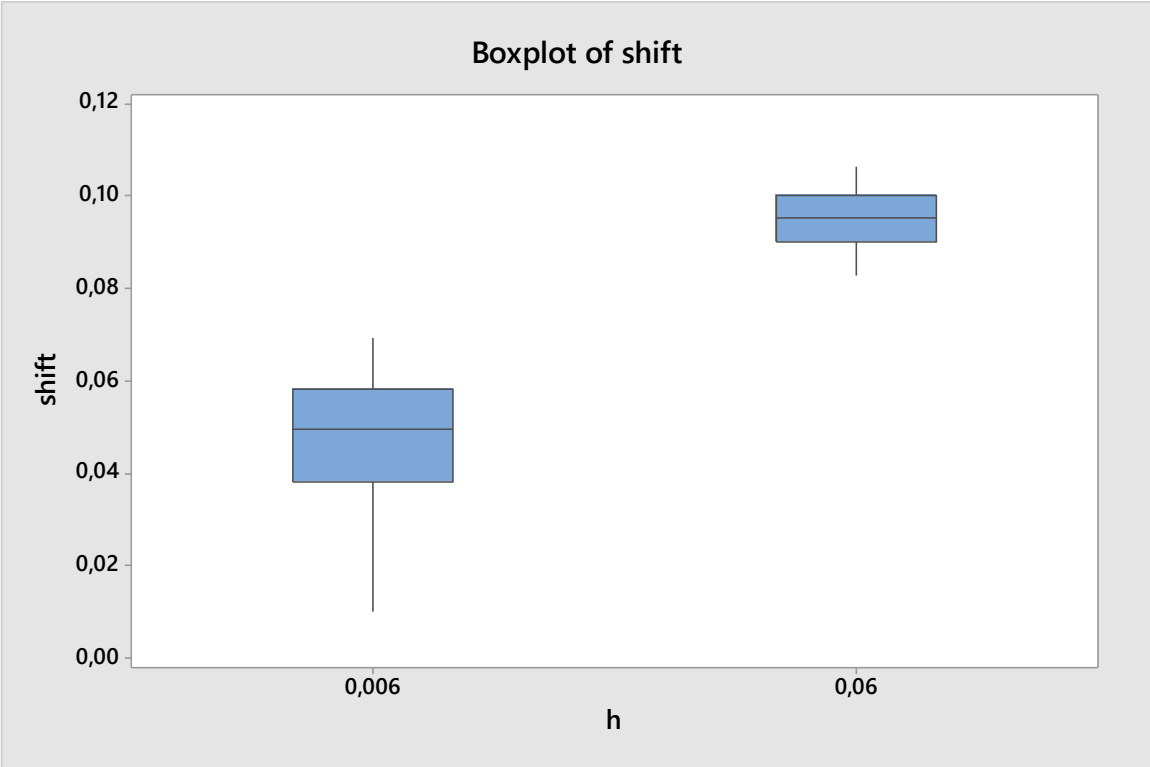

Figure 6.70: Boxplot of Shifting Error for levels of h length

l1, l2, l3, Rp,s and h

All influence quantities are effective, because of kinematics. From interaction plots, it can be seen that, their slope with s and h terms have greatest magnitude and when interaction plots with h terms are examined, the distance between between high and low level lines has the greatest value. Because, main effects plots show us that, h has the highest effect as we mentioned before.

Significance Evaluation

If we look at main effects, all l1,l2,l3,Rp,s,h effects are significant, because of the kinematics. While some 2-way and 3-way interactions are significant; 4-way, 5-way and 6-way interactions do not exist in the significant group of interactions.

6.2 Measurng Uncertainties Analysis

6.2.1 Linear Delta Robot

6.2.1.2 Effects of First Slider Uncertainty

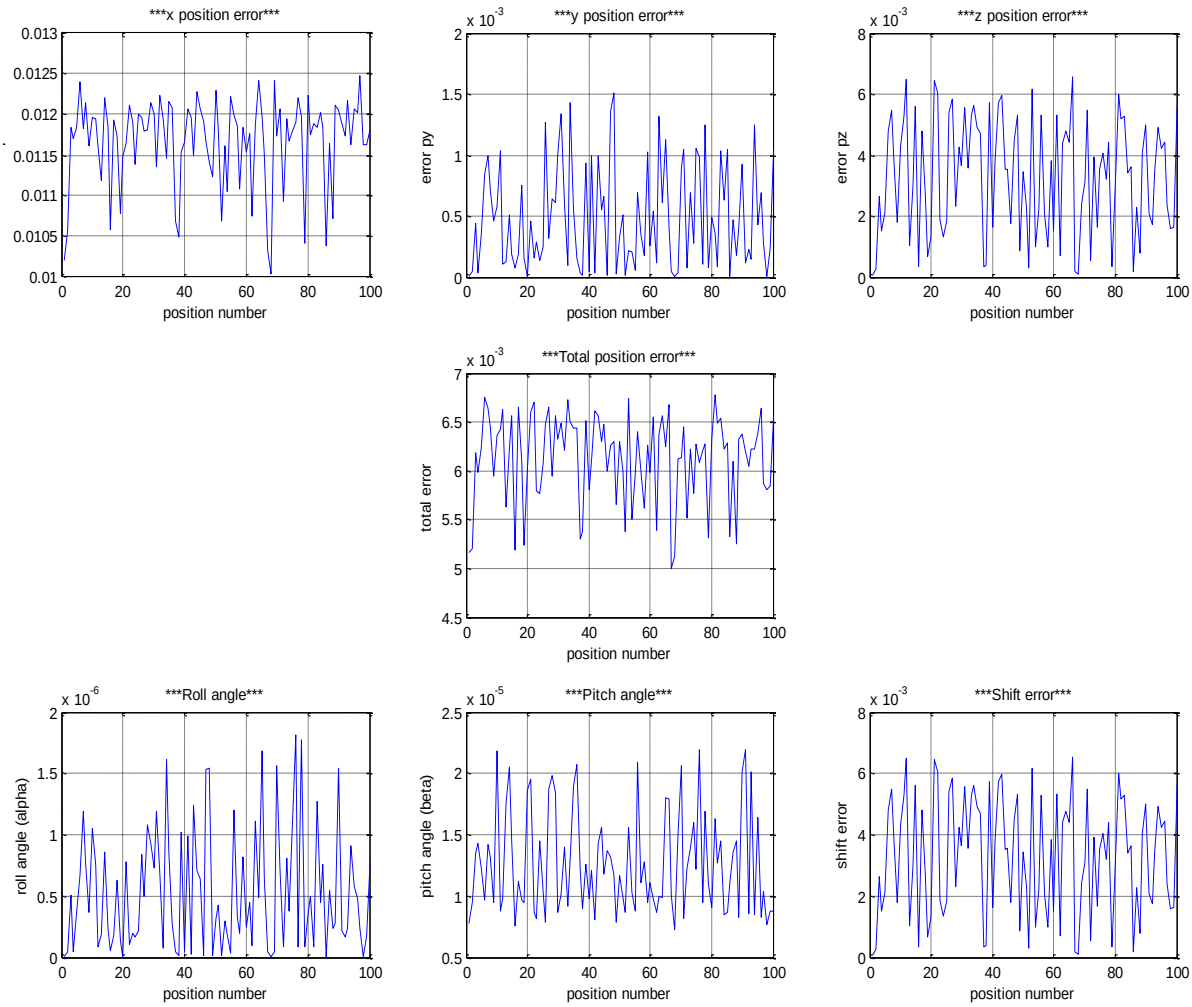

Figure 6.71: Time Histories of the Responses to the Uncertainties on the First Slider

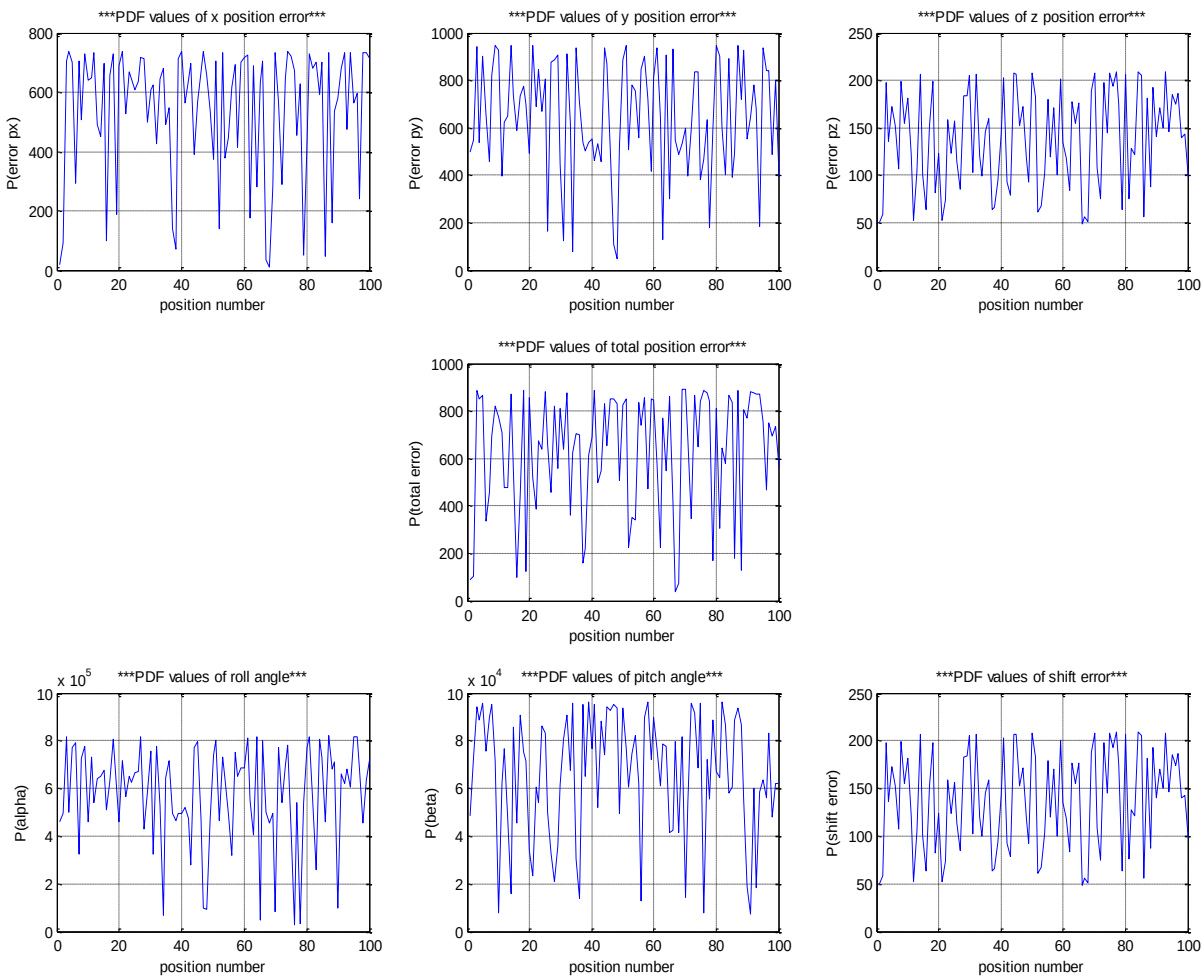

Figure 6.72: PDF Value Graphs of the Responses to the Uncertainties on the First Slider

We are evaluating here the effects of the uncertainty of the first slider's positon on the x,y and z direction of the mobile platform, on the total position error, on the roll and pitch tilt angles, and finally on the shifting error of the base platform of the agile eye robot and $\pm$15μm was used as the uncertainty interval in the working volume. It is assigned accordng to guide of the used motors that excite the sliders' motion.

As we see in the figures, it influences mostly the position of the delta robot's mobile platform in the x direction and the effect is decresing in the y direction with respect to other parameters. The effect on the tilting of the mobile platform illustrates that; it influences mostly the pitch angle.

6.2.1.2 Effects of Second Slider Uncertainty

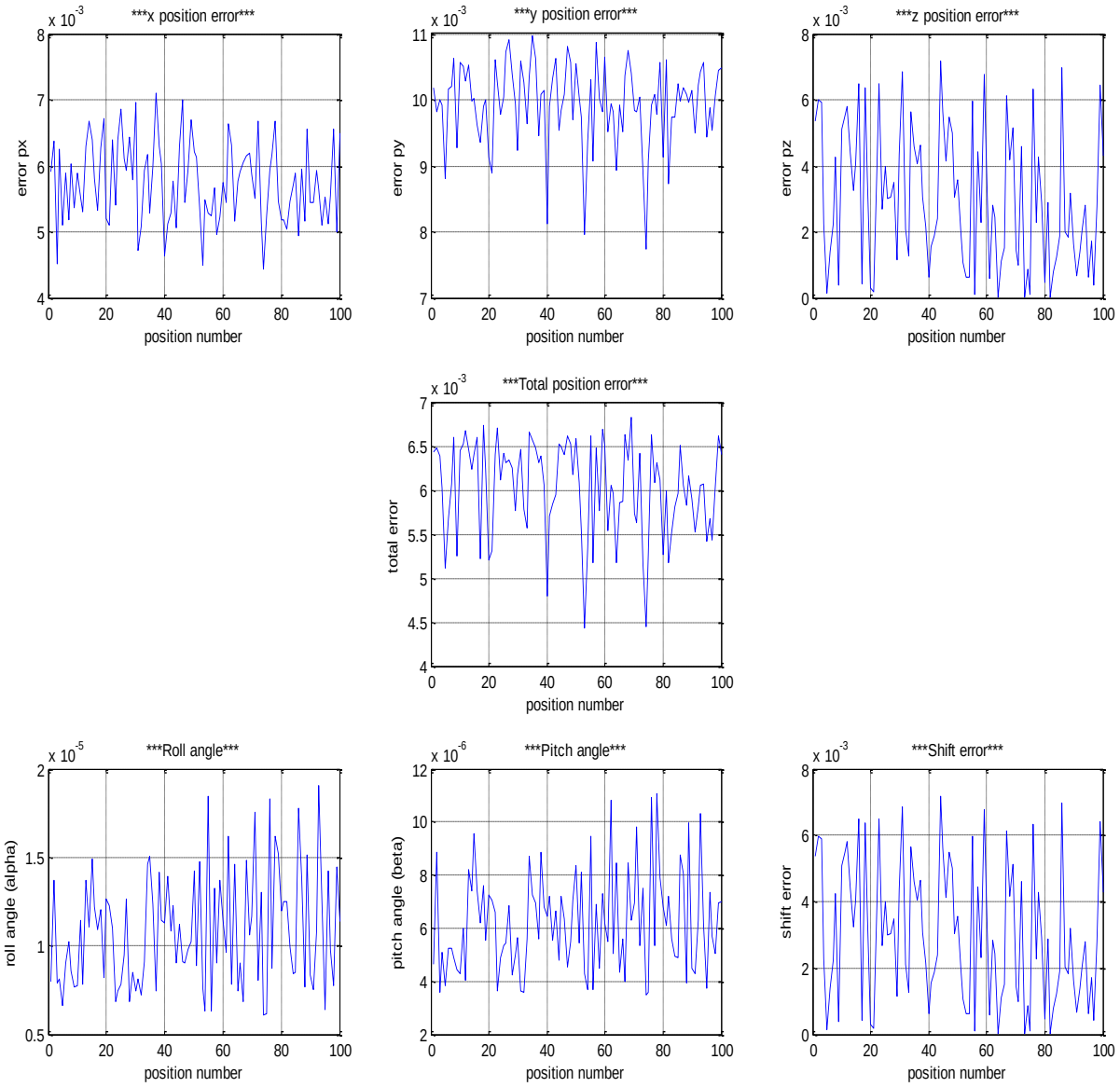

Figure 6.73: Time Histories of the Responses to the Uncertainties on the Second Slider

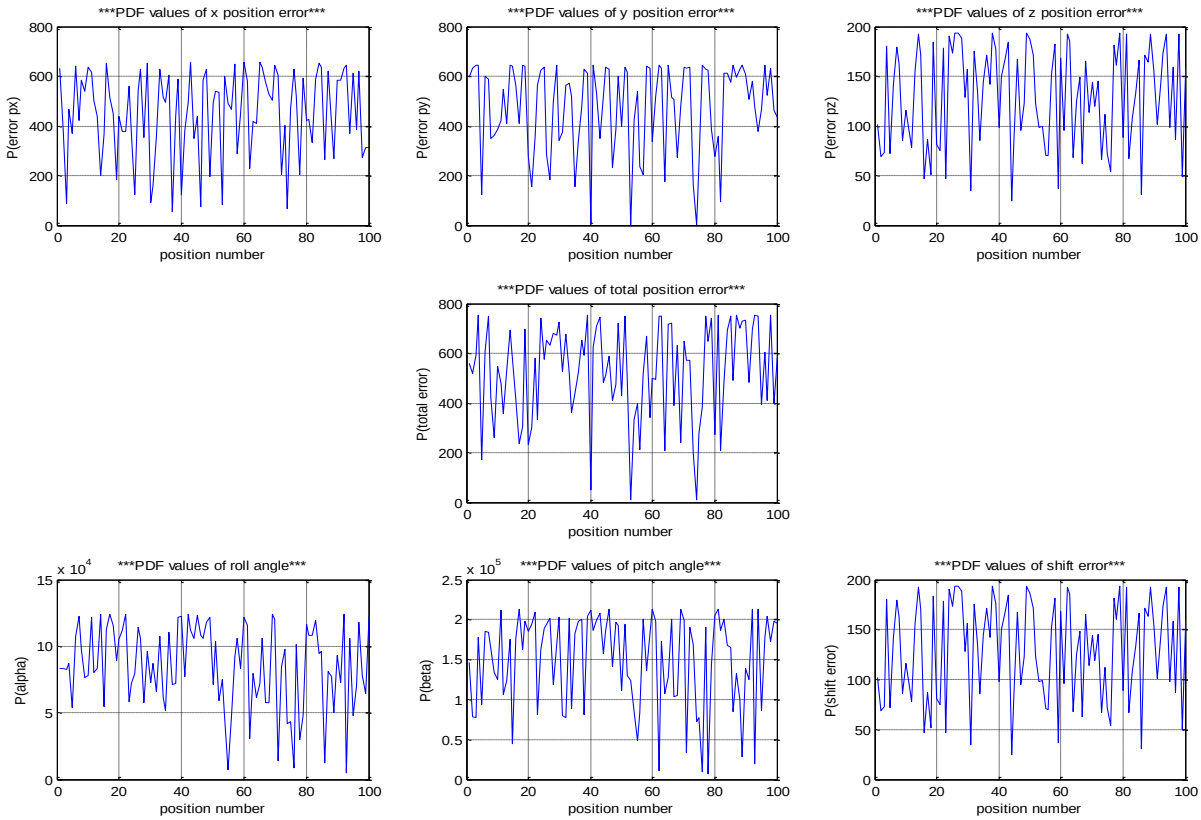

Figure 6.74: PDF Value Graphs of the Responses to the Uncertainties on the Second Slider

It illustrates that, the effects of the uncertainty of the second slider's positon on the x,y and z direction of the mobile platform, on the total position error, on the roll and pitch tilt angles, and finally on the shifting error of the base platform of the agile eye robot and $\pm 15\mu m$ was used as the uncertainty interval again in the working volume. It is assigned accordng to guide of the used motors that excite the sliders' motion.

As we see in the figures, it influences mostly the position of the delta robot's mobile platform in the y direction and the effect is decresing in the z direction a little bit with respect to x direction positioning error, y direction positining error and the shifting error. The effect on the tilting of the mobile platform illustrates that; it influences mostly the pitch angle witth respect to roll angle.

6.2.1.3 Effects of Third Slider Uncertainty

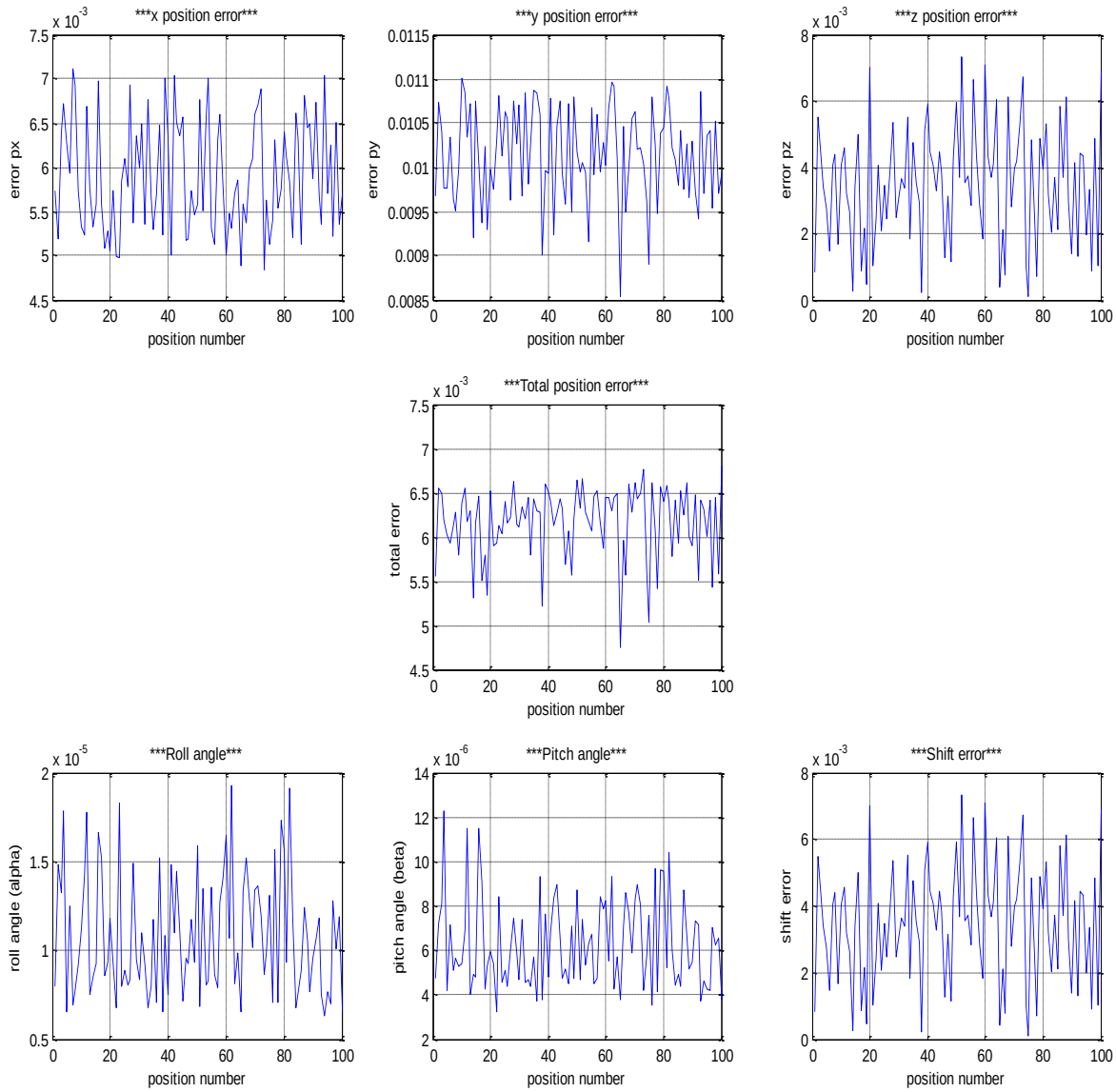

Figure 6.75: Time Histories of the Responses to the Uncertainties on the Third Slider

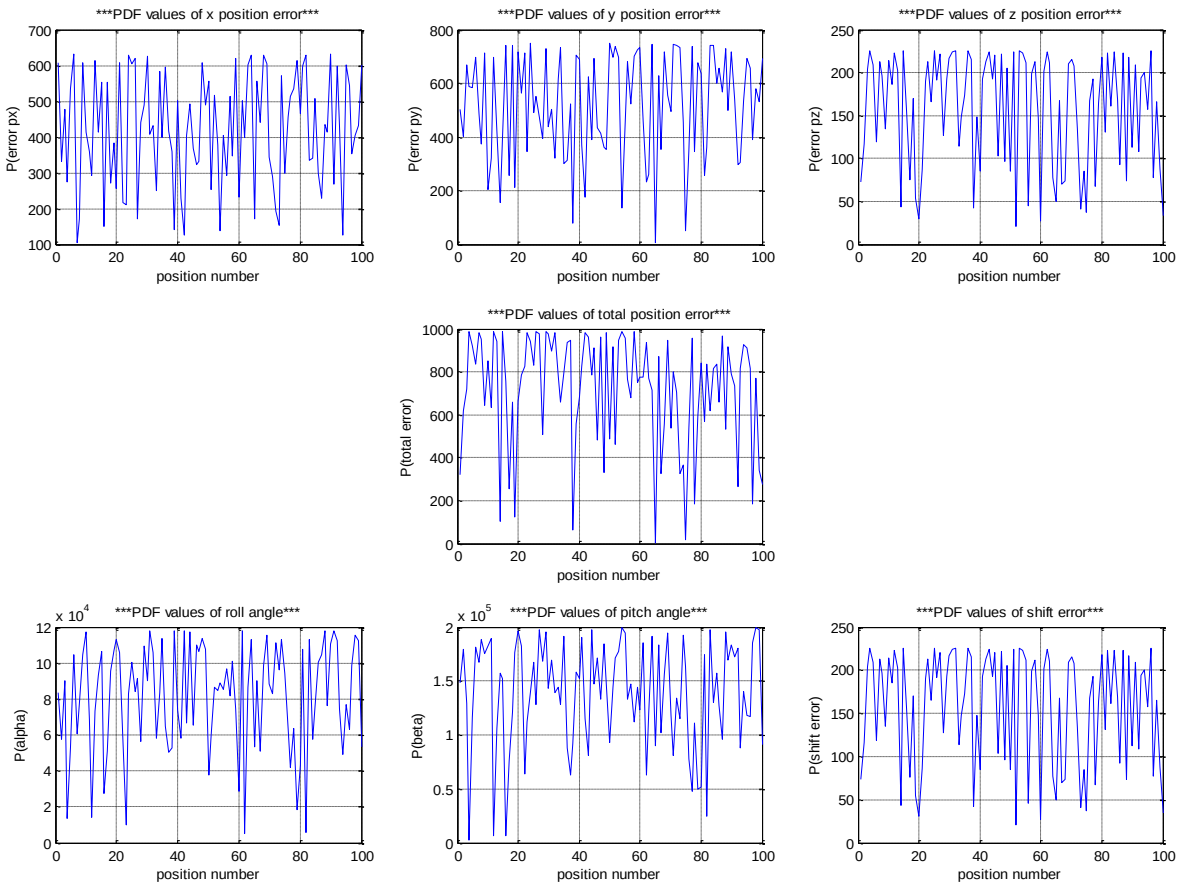

Figure 6.76: PDF Value Graphs of the Responses to the Uncertainties on the Third Slider

It shows that, the effects of the uncertainty of the third slider's positon on the x,y and z direction of the mobile platform, on the total position error, on the roll and pitch tilt angles, and finally on the shifting error of the base platform of the agile eye robot and $\pm 15\mu m$ was used as the uncertainty interval in the working volume. It is assigned accordng to guide of the used motors that excite the sliders' motion. The influences of this type of uncertainty is similar to the second slider's position uncertainty influences.

As we see in the figures, it influences mostly the position of the delta robot's mobile platform in the y direction and the effect is decresing in the z direction a little bit with respect to x direction positioning error, y direction positining error and the shifting error. The effect on the tilting of the mobile platform illustrates that; it influences mostly the pitch angle witth respect to roll angle.

6.2.2 Agile Eye Spherical Wrist Robot

6.2.2.1 Effects of Uncertainty in θ_1 Motor Rotation Angle on α Roll Angle and on β Pitch Angle

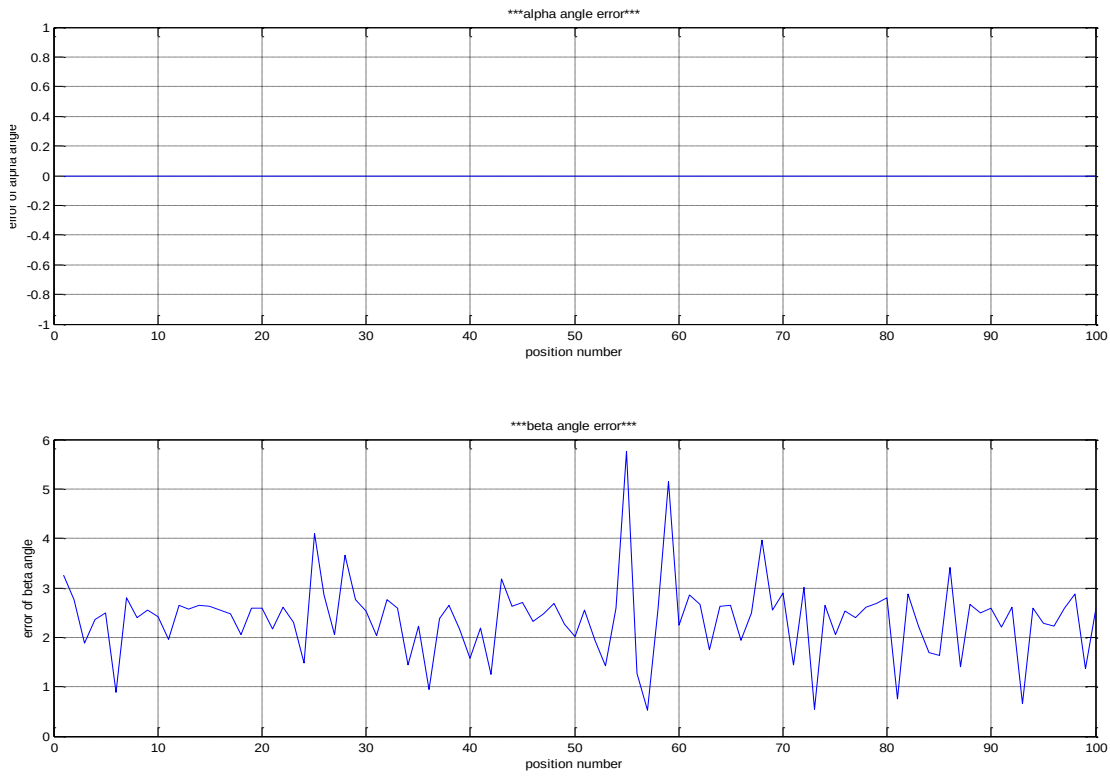

Figure 6.77: Time Histories of the Responses to the Uncertainties on the First Motor

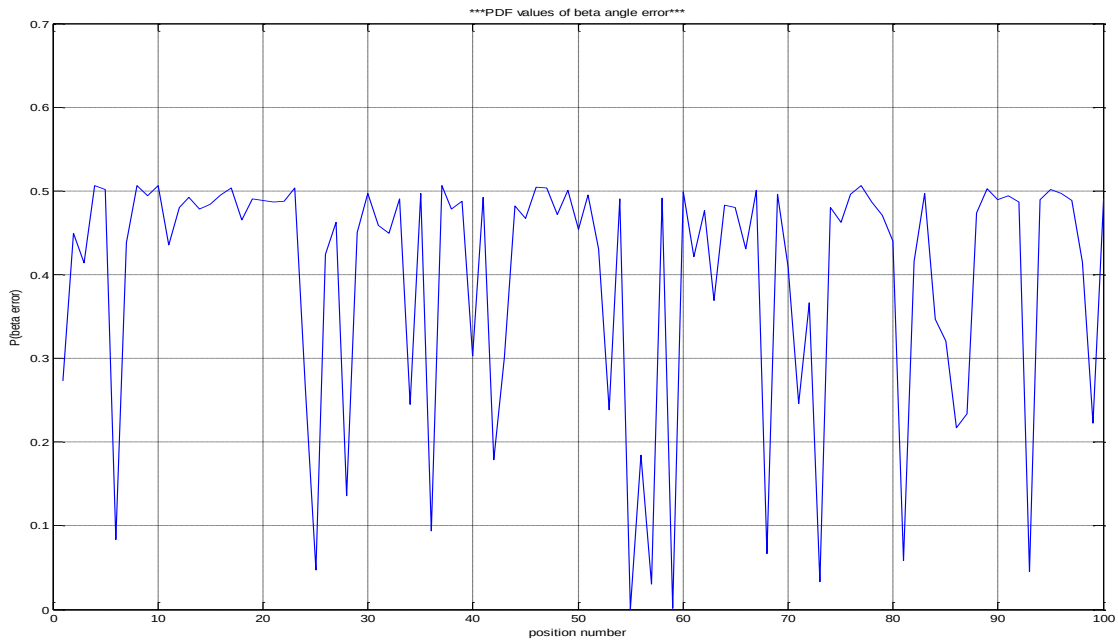

Figure 6.78: PDF Value Graph of the Pitch Angle Response to the Uncertainties on the First Motor

We will illustrate here, the effect of the first driver motor's uncertainty that is in the range of 0°-9° with one thousand uniformly distributed values, on the roll and pitch angles of the agile eye robot's end effector.

As you can also from the agile eye robot kinematics , it is very obvious that, roll angle does not depend on the first motor. Hence, the uncertainty in the first motor rotation only influences the pitch angle and these values are very small.

6.2.2.2 Effects of Uncertainty in θ_2Motor Rotation Angle on α Roll Angle and on β Pitch Angle

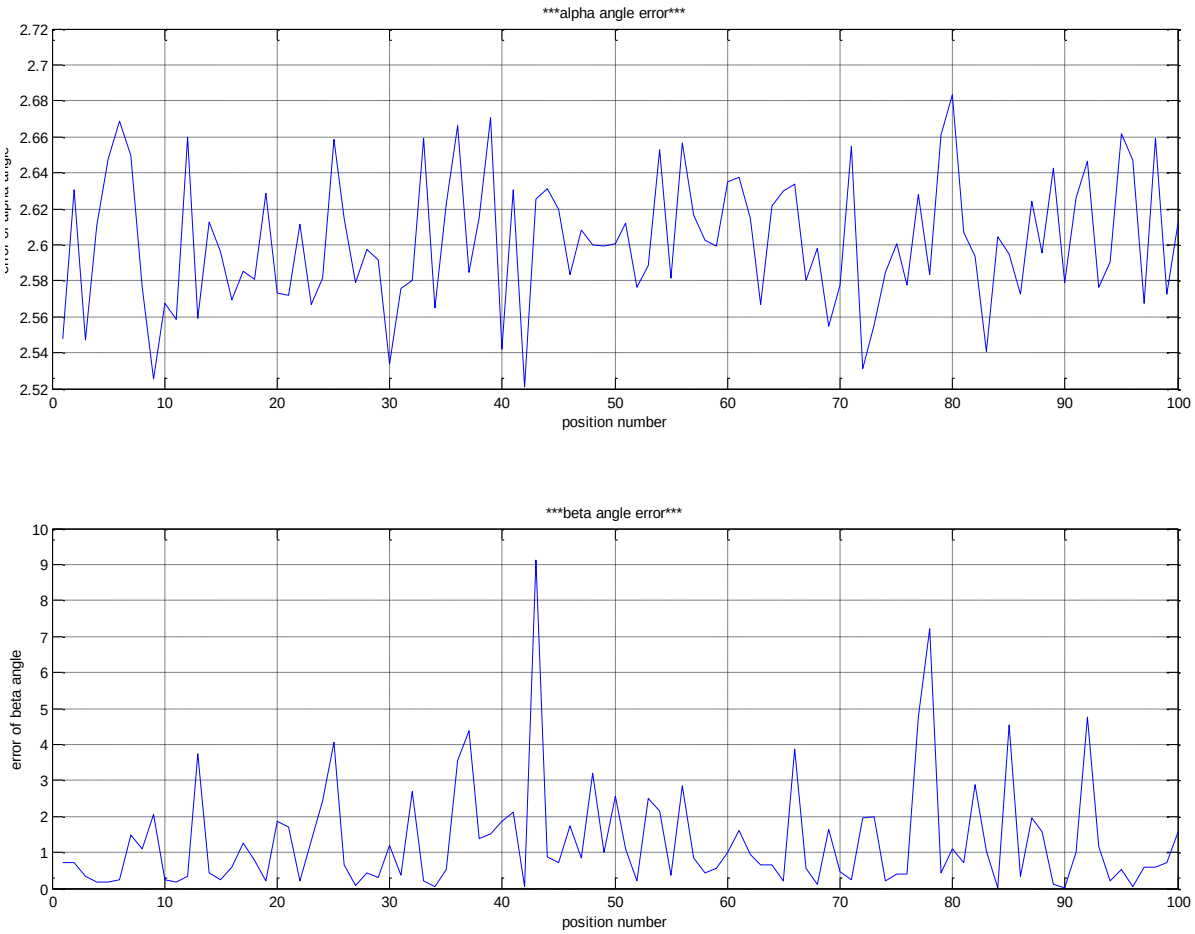

Figure 6.79: Time Histories of the Responses to the Uncertainties on the Second Slider

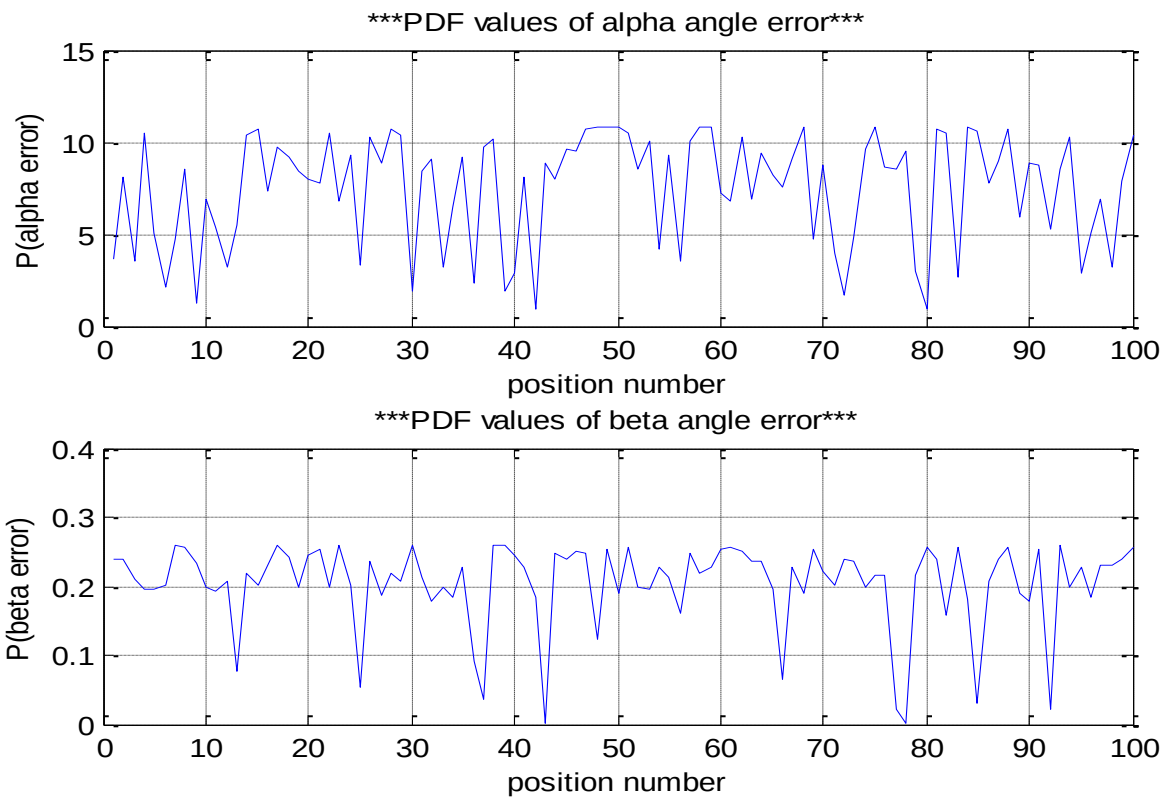

Figure 6.80: PDF Value Graghs of the Responses to the Uncertainties on the Second Motor

We will illustrate here, the effect of the second driver motor's uncertainty that is in the range of 0°-9° with one thousand uniformly distributed values, on the roll and pitch angles of the agile eye robot's end effector.

As you can also from the agile eye robot kinematics , it is very obvious that, roll angle reacts directly to the second motor's uncertainty and it is oscillating around a small error value. If we look at the pitch angle, the error values are in the smaller range with respect to the roll angle, however its peak values are bigger. Consequently, roll angle is the most affected parameter from the uncertainty of the second motor rotation.

CHAPTER 7

7. CONCLUSIONS

In this work, we have described the numerical analyses performed to predict the position uncertainty of a parallel kinematics robot together with a literature review concering the best practices for the robots calibration. The numerical analyses were performed with a method that combines the 2^k factorial design of experiments and the Monte Carlo simulations in order to obtain indications similar to the sensitivity analysis. This particular approach was used to identify the effects of the links' lengths and frame elements' distances on the uncertainty of the combined robot's end effector position.

The kinematics of both of the linear delta and agile eye robots were initially implemented in Matlab and then Monte Carlo trials were used to estimate the effect of the factors'uncertainty in the entire working volume. Globally, more than 1 million configurations were analyzed, including PDF of one thousand elements each at one hundred random position in the working volume for each of the 2^7 configurations of the factorial design. Experimental results were analyzed with MINITAB software. The scatter plots and the main effect plots were used to assess the relative importance of the different factors, in order to understand which quantities are affecting the positioning accuracy. According to analysis, mostly influencing factors are lengths of three links (l1,l2 and l3) for tilting of the mobile platform, the distance between mobile paltform of linear delta robot and the base platform of agile eye robot (h) for shifting error, and lengths of three links (l1,l2 and l3), radius of the linear delta robot's mobile platform (Rp), the distance between the rails' axes and the origin (s) for the total positioning error in linear delta robot's mobile platform as seen obviously.

The literature review outlined the different procedures for the calibration, that should allow recovering the systematic errors due to the mismatches between the nominal and the actual geometrical parameters of the robot. In these conditions, the positioning accuracy should be only provided by the measurement chain (encoders acting as feedback for the position control). The classical Monte Carlo simulations performed in these condition outlined a standard uncertainty lower than 0.01 mm, i.e. met the original project requirements.

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