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FUNDING VALUE ADJUSTMENT IN OTC DERIVATIVES MARKET: A QUANTITATIVE STUDY

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Sommario Questo lavoro di tesi analizza il tema del *funding* nel contesto più generale del rischio di controparte e degli xVA, ovvero aggiustamenti al prezzo teorico di uno strumento derivato che considerano una serie di rischi ritenuti trascurabili prima della crisi. In particolare, il Funding Value Adjustment (FVA) misura gli effetti economici derivanti dagli scambi di collateral, strumento diffuso quale mitigazione del rischio di controparte. Negli ultimi 5 anni si è sviluppato in letteratura un vero e proprio dibattito sul tema FVA, alimentato dai contributi degli accademici e degli addetti ai lavori. Per il calcolo di questo aggiustamento sono stati proposti diversi metodi, più o meno complessi. Nella fattispecie, in questo lavoro le analisi quantitative sono condotte sulla base di due diversi approcci: uno più semplice ed immediato (basato su [29]) che inserisce il FVA in un framework di tipo *building blocks* in cui le componenti del prezzo sono considerate additive, e l'altro, più complesso ed elaborato (basato su [15]) che considera il FVA come una componente di prezzo determinabile solamente in maniera ricorsiva, poiché interdipendente con il prezzo stesso dello strumento. Le analisi quantitative si pongono due obiettivi: da un lato valutare l'errore che si commette con l'approssimazione insita nel primo approccio, dall'altro determinare se tale aggiustamento possa essere valutato strumento per strumento, o debba essere calcolato a livello aggregato, considerando in tal senso la possibilità di compensare i costi ed i benefici legati al funding.

Il lavoro è organizzato come segue: dapprima si introduce e si contestualizza il problema del funding nell'ambito delle lezioni apprese in seguito alla grande crisi finanziaria.

Nel **Capitolo 2** viene delineato il framework generale del rischio di controparte dal punto di vista delle best practice operative e a livello di normativa prudenziale.

Nel **Capitolo 3** viene introdotto ed affrontato il nocciolo di questo lavoro di tesi, il FVA, sia dal punto di vista del dibattito accademico, sia dal punto di vista dell'industria, riportando gli esempi delle prassi operative attuali.

Il **Capitolo 4** presenta i *case studies* (basati su portafogli di Interest Rate Swaps e condotti sulla base dei dati di mercato al 30/12/2015) e i risultati ottenuti.

Infine, il **Capitolo 5** conclude la tesi ripercorrendo gli obiettivi, le procedure seguite e risultati principali del lavoro svolto, e delineando alcuni possibili sviluppi futuri dell'argomento.

Abstract This thesis analyzes the *funding* issue in the general context of counterparty risk and xVA (adjustments to derivative contracts' theoretical value to account for a series of risks deemed negligible before the crisis). In particular, the Funding Value Adjustment (FVA) measures the economic effects of collateral exchanges, a common counterparty risk mitigation practice. Over the past five years in literature a lot of debate has emerged on the issue, fueled by the contributions of academics and practitioners. To calculate this adjustment several methods have been proposed over the years, from the simple to the most complex ones. In particular, in this work, quantitative analysis are conducted on the basis of two different approaches: a more simple and immediate one (based on [29]), which considers the FVA as a *building blocks* component of a pricing framework where adjustments are additive, and the other, more complex and elaborated (based on [15]), which considers the FVA as a price component possible to determine only following a recursive way, being interdependent with the price of the instrument itself. Those quantitative analysis have two goals: on one hand to evaluate the error one commits in following the first approach, on the other hand determine whether such an adjustment could be assessed instrument-wise, or has to be calculated at bank-level, considering insofar the possibility to offset funding-related costs and benefits.

The thesis is organized as follows: first the funding problem is introduced and contextualized in the wake of the great financial crisis.

In **Chapter 2** the general counterparty-risk framework is outlined from the point of view of operational best practices and at prudential-regulation level.

In **Chapter 3** we tackle the core of this thesis, the FVA, both from the point of academic debate, and from the industry point of view, providing examples of current operational practices.

Chapter 4 shows the case studies (conducted as of 2015/12/30 market data and based on vanilla IRS portfolios) along with the obtained results.

Finally, **Chapter 5** concludes the thesis recalling aims, followed procedures and main results of the work, and illustrating some possible future developments of the topic.

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Chapter 1

Introduction

The global credit crisis resulted in regulatory and accounting changes regarding derivative valuation. Starting from summer 2007, with the spreading of the credit crunch, market quotes of forward rates and zero-coupon bonds began to violate standard no-arbitrage relationships. This was partly due to the liquidity crisis affecting credit lines, and to the possibility of a systemic break-down triggered by increased counterparty credit risk. Indeed, credit risk is only one facet of the problem, since the crisis started as a funding liquidity crisis, as shown for example by Eisenschmidt and Tapking [25], and it continued as a credit crisis following a typical spiral pattern as described in Brunnermeier and Pedersen [16]. In January 2013, *International Financial Reporting Standards 13* became effective, forcing banks and other derivative dealers to incorporate credit value adjustment (CVA) and debt value adjustment (DVA) in their fair derivative valuations. The credit valuation adjustment (CVA) corrects the price for the expected costs the dealer may incur in case the counterparty defaults, while the so-called debt valuation adjustment (DVA) is a correction for the expected benefits for the dealer due to his own default risk. While the incorporation of CVA and DVA has been widely studied and accepted by both practitioners and academics, the focus is now on the calculation and relevance of funding value adjustment (FVA). The occurrence of FVA is highly linked with the financial crisis. Prior to the crisis, classical derivatives pricing theory has rested on the assumption that one can borrow and lend at a unique risk-free rate of interest, a theoretical risk-free rate that is proxied by a number of market rates. LIBOR was often regarded as a suitable proxy for the risk-free rate. Hence, collateral rates were commonly LIBOR based (Hull & White [32]) and so funding costs were offset. However, with drastically increasing spreads emerging as the crisis took hold, it became apparent that LIBOR is contaminated by credit risk and as such is an imperfect proxy of the risk-free rate. The overnight indexed swap rate (OIS rate) has become benchmark for risk-free rates in the industry. As a result, collateral rates have become mostly OIS based. Thus, as funding has become relatively more costly and the interest received on collateral no longer offsets the funding cost, the FVA has become non-negligible.

Moreover, as far as OTC trades are concerned, collateral agreements have established as a standard market practice among financial institutions, to provide mitigation for credit risk. Accordingly, OTC derivatives have become funded instruments, i.e. they require to raise funds to face the collateral needs. As a matter of fact, the bulk of outstanding OTC derivatives are now regulated by bilateral CSA agreements with daily margin calls, zero thresholds and zero minimum transfer amounts, thus providing nearly perfect collateralization and remarkably shrinking the CVA and the DVA amounts. However there are some counterparties, such as sovereigns, supra-nationals, corporates, pension funds and hedge funds that can trade with banks on uncollateralized basis. The banks usually hedge these derivatives trading the replication strategy under their CSAs with other banks or CCPs. Therefore the main implication of collateral agreements regards non-collateralized transactions: indeed, when a non-collateralized derivative is hedged with a back-to-back deal, which instead is collateralized, funding costs or benefits do occur.

For those reasons, reflecting funding cost into the valuation of derivatives has become a paramount topic in the financial industry: one just has to look at the number of presentations and articles dealing with this topic to realize how much research effort is being put into it. The topic is still very open, and there are no broadly supported standard procedures yet: among other issues, a compromise between the accuracy and ease of implementation at

an industrial-scale level is far from being found.
Hull & White in [30] show in a Black-Scholes framework that implementing FVA conflicts with fundamental derivative pricing theory and believe FVA should not be taken into account when pricing derivatives. However, many practitioners criticize the approach taken by Hull & White, claiming funding costs are real and can dramatically impact banks profit and loss statements. Furthermore, they argue that fundamental assumptions made in the Black-Scholes model do not hold in reality, hence funding costs should be taken into account. Morini and Prampolini in [36], show in a simple modeling setting involving a lender and a borrower, that the crucial variable determining the lender's net funding cost is the bond-CDS basis, leaving extensions to general derivative payouts for future research. Gregory in [29] simply extend to FVA the well-established counterparty-risk methodologies. More comprehensive attempts have been made by Brigo et al. [15] and Crépey [23], both showing that the incorporation of FVA leads to a non-linear recursive pricing problem.

After providing a detailed photography of the FVA in literature and in the industry, in this thesis we take cue from Brigo et al. [15] to provide a recursive framework to model funding value adjustment. Our purpose is quantify the magnitude of the approximation one incurs when modeling the FVA with a simpler linear approach as the one proposed by Gregory in [29]. Moreover, we aim to quantify the effects of collateral rehypothecation on the overall funding costs of an institution. For the sake of simplicity, in this last analysis we adopt the linear approach only.

The rest of the thesis is organized as follows:

In **Chapter 2** we provide a brief excursus on counterparty risk, which is a fundamental basis for xVA-related topic. We describe its concept, measure, common mitigation strategies, and regulatory concerns.

In **Chapter 3** we introduce the *core* of this work, the Funding Value Adjustment and the related debate, from both the point of view of academics and practitioners: we briefly summarize what literature has been offering on the topic for the last years, and how the issue has been practically recognized in the industry.

In **Chapter 4** we present a simpler and a more complex methodological framework to set the problem, and we set up a numerical case study to compare them. Moreover, we try to shed light over the effects of rehypothecation of collateral. At the end of the chapter we discuss the results obtained.

Chapter 2

Counterparty Credit Risk

2.1 Defining Counterparty Credit Risk

2.1.1 Introduction

Financial markets are affected by various non deterministic factors. Counterparty risk is arguably one of the most complex areas to deal with since it is driven by the intersection of different risk types and is highly sensitive to systemic traits, such as the failure of large institutions. Counterparty risk should be considered in the context of other financial risks: market risk and credit risk, which we briefly summarize next.

2.1.2 Market Risk

Market risk is the possibility for an investor to experience losses due to factors that affect the overall performance of the financial markets, such as stock prices, interest rates, foreign exchange rates, commodity prices and so on. Market risk, also called *systemic risk*, cannot be eliminated through diversification, though it can be hedged against. Natural disasters, recessions, political changes and terrorist attacks are all examples of sources of market risk, given the impact they can have on the markets. As we will see, the position whose value is sensitive to interest rates changes take the lion's share in the banking portfolios.

2.1.3 Credit Risk

Credit risk is the risk that a counterparty may be unable or unwilling to make a payment or fulfill contractual obligations, causing a loss in its creditor counterparty. This may be characterised in terms of an actual default or, less severely, by deterioration in a counterparty's credit quality. The former case may result in an actual and immediate loss whereas, in the latter case, future losses become more likely leading to a mark-to-market impact.

2.1.4 Counterparty Risk

Counterparty risk (also called *counterparty credit risk* or *CCR*) is the risk to each party of a contract that the counterparty will not live up to its contractual obligations. In pre-crisis times, many counterparties were given zero or close to zero default probability. But when Lehman Brothers collapsed and other large financial institutions (for example, Bear Stearns, AIG, Fannie Mae, Freddie Mac, Merrill Lynch, Royal Bank of Scotland) needed external support to avoid default, we learned that the *too big to fail* mentality was nothing but an illusion. As its denomination suggest it to be only component of credit risk, yet from its definition we can track down some aspects differentiating it from the "traditional" credit risk:

- The value of a derivatives contract in the future is uncertain, in most cases significantly so. The value of a derivative at a potential default date will be the net value of all future cash flows to be exchanged under that contract, and in many cases the uncertainity extends even to the sign of the value (i.e. the future value can be either positive or negative).

- Since the value of some derivatives contract can be positive or negative (e.g. *Interest Rate Swaps*), counterparty risk is typically bilateral. In other words, in a derivatives transaction, each counterparty has risk to the other.

In figure 2.1 a schematization to better understand those features.

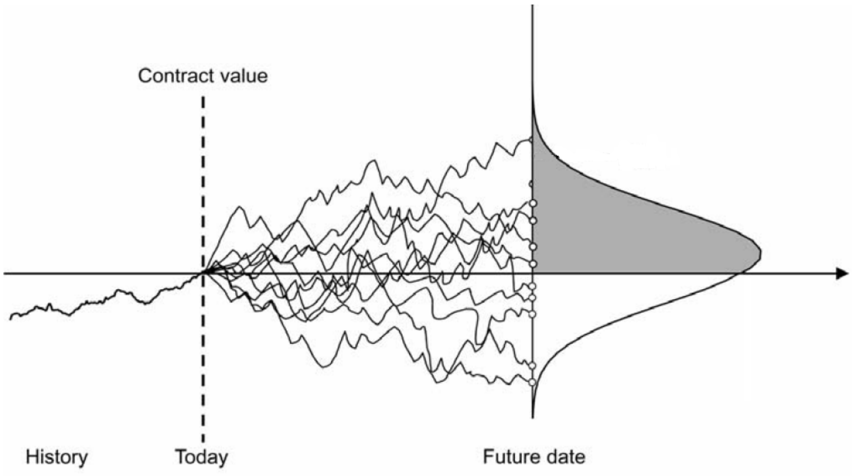

Figure 2.1: Illustration of the randomicity over the future value of a deal.

2.2 Measuring Counterparty Credit Risk

The price of a contract is established at *time zero*, according to market conditions at that time (stock prices, interest rates, et cetera), but as they change with time, the value of the contract can vary consistently in its life (from is start to its maturity). So the market value, or the *mark to market* of the contract is a very time-sensitive measure. At any time, a positive value of the contract represents a *credit* towards the couterparty, while a negative value represent a *debit*.
Credit exposure defines the loss in the event of a counterparty defaults. Exposure is characterised by the fact that a positive value of a financial instrument corresponds to a claim on a defaulted counterparty, whereas in the event of negative value an institution cannot *walk away* from its obligations. This means that if an institution is owed money and their counterparty defaults then they will incur a loss, whilst in the reverse situation they cannot gain from the default as the trustee in bankruptcy will claim their liability.

2.2.1 Preliminary Concepts

Default probability and credit migration When the counterparty risk is concerned, financial reliability of the conterparty has to be considered for a long time in the future.

We define $PD_C(t_i, t_j)$ the probability the counterparty C defaults during the time period (t_i, t_j). The *exponential distribution* is widely used to model such probability, given its absence of memory. In fact the probability a counterparty defauts between t and $t+\Delta t$ and the probability the counterparty defauts between t and $T+\Delta t$ (given the fact that it has survived until T), can be reasonably considered equal. Namely, defining τ_C the (random) default time of the counterparty C we can say:

$$\mathbb{P}(\tau \in (t, t+\Delta t)) = \mathbb{P}(\tau \in (T, T+\Delta t) | \tau > T)$$

Credit migrations or discrete changes in credit quality are important events to consider, as they can seriously affect the future default probabilities.

Exposure Key element in assessing counterparty risk is exposure to credit which defines the potential loss in the event the counterparty defaults. As we said before, when the mark to market of a contract (abb. *MtM*) is positive, the exposure is the MtM itself, as it is the positive amount of money to which one is owed to the counterparty, whilst in case of negative MtM the exposure is null, as the position has to be covered in any case. Therefore, we can define the credit exposure (CE) at a certain time t as

$$CE_t = max(MtM_t, 0)$$

Note that MtM_t and CE_t are random variables, as the future value of a contract is affected by market factors subject to unpredictable variations.

As we said before, an important feature of counterparty risk is the bilaterality, as both parties of a transaction can default and therefore both can experience losses. From an institution's point of view, their own default will cause a loss to any counterparty they are in debt to. This can be defined in terms of negative exposure, which by symmetry is defined as:

$$NE_t = -min(MtM_t, 0)$$

Recovery rate Recovery rate represents the percentage of the outstanding claim recovered when a counterparty defaults. Whilst recovery rates can vary substantially and unpredictably, an average value of 40% is tipically adopted in the industry. Anyway, one cannot always rely on this value: as an example, when Lehman Brothers collapsed, its creditors were able to recover only as much as 12% of their claims (see, ex multibus, [26] and [42]).

2.2.2 Common Risk Figures

When assessing counterparty risk, one cannot simply use VaR-like methods to characterize it. In fact, unlike VaR, exposure needs to be defined over multiple time horizons (often far in the future) so as to fully understand the impact of time and specifics of the underlying contracts. The simplest metric we introduce is the Expected MtM, representing the expected value of the future MtM of the contract

$$ExpMtM_t = \mathbb{E}[MtM_t]$$

This metric is not very useful as does not embody the fact that we have no gain having a negative position in case the counterparty defaults.

Expected Exposure One of the most widely used metric is the *Expected Exposure* defined as the expected value of the future credit exposure:

$$EE_t = \mathbb{E}[CE_t] = \mathbb{E}[max(MtM_t, 0)] \tag{2.1}$$

Negative Exposure By simmetry we can define the Expected Negative Exposure as:

$$ENE_t = \mathbb{E}[NE_t] = \mathbb{E}[-min(MtM_t, 0)] \tag{2.2}$$

Potential Future Exposure In risk management, when we use a VaR measure we ask *what is the worst loss we can incur to in all but a set of scenarios having a probability under a fixed threshold?* The PFE answer this question projected into the future, defining a Credit Exposure that would be exceeded with a probability of no more than $\boldsymbol{\alpha}$%:

$$PFE_t^{\alpha} = inf\{l \in \mathbb{R}^+ : \mathbb{P}(EE_t > l) \leq 1 - \alpha\} \tag{2.3}$$

Here below, in figure 2.2, an illustration of the measure above introduced.

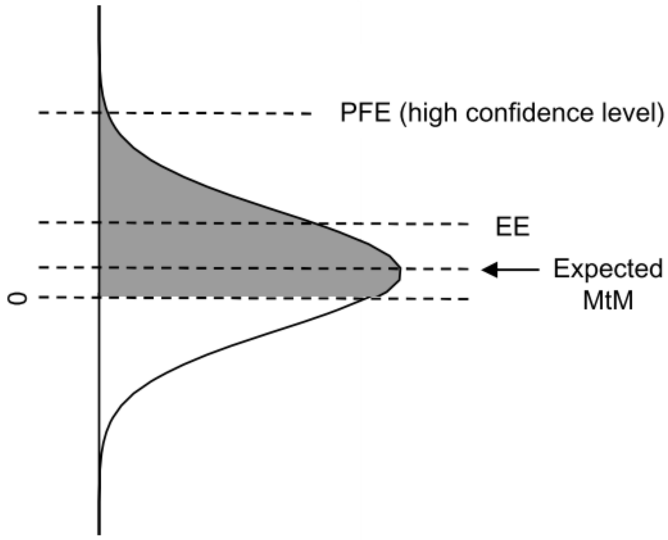

Figure 2.2: Illustration the ExpMtM, EE and PFE snapshotted at a future date, with normally-distributed MtM.

Discounted Measures Losses, like any financial quantity, should be always normalized with respect to their distance into the future. So, discounted "versions" of the risk measured above presented are introduced:

$$DEE_t = DF(t_0, t)EE_t \tag{2.4}$$

$$DENE_t = DF(t_0, t)ENE_t \tag{2.5}$$

$$DPFE_t^{\alpha} = DF(t_0, t)PFE_t^{\alpha} \tag{2.6}$$

where $DF(t_0, t)$ is the spot discount factor between t_0 and t.

2.3 Pricing and Mitigating Counterparty Credit Risk

2.3.1 Credit Value Adjustment

Credit valuation adjustment (CVA) is the difference between the risk-free value of a contract or a portfolio, and the "true" value that takes into account the possibility of a counterparty's default. In other words, CVA is the market value of counterparty credit risk. Until now we have focused separately on credit exposure and default probability. Now we proceed to combine these two components in order to address the pricing of counterparty credit risk.
Accurate pricing of counterparty risk involves attaching a value to the risk of all outstanding positions with a given counterparty. This is important in the reporting of accurate earnings information and incentivising trading desks and businesses to trade appropriately. If counterparty risk pricing is combined with a systematic charging of new transactions, then it will also be hedged generated funds that will absorb potential losses in the event that a counterparty defaults. Counterparty risk charges are increasingly commonly associated with hedging costs.

For our purposes we assume indipendency between credit exposure and default probability, even though the market suggest a positive correlation between them, which leads to a what is called *wrong-way risk.*
Pricing the credit risk for an instrument with one-way payments, such as a bond, is relatively straightforward - one simply needs to account for default when discounting the cash flows and add any default-time payment. However, many derivatives instruments have fixed, floating or contingent cash flows or payments that are made in both directions. This bilateral nature characterizes credit exposure and makes the quantification of counterparty risk dramatically more difficult. In a theoretically rigorous sense, we can

split the risk-free price of a financial transaction into

$$\textit{Risk-free value} \;=\; \textit{Risky value} \;+\; CVA \tag{2.7}$$

Unilateral CVA is given by the risk-neutral expectation of the discounted loss. The risk-neutral expectation can be written as

$$CVA = (1 - R_C) \int_t^T DEE_s \, \mathrm{d}PD_C(t, s). \tag{2.8}$$

where:

- R_C is the recovery rate of the counterparty
- DEE_s is the discounted expected exposure as defined in (2.4)
- PD_C is the default probability for the counterparty

which, for practical purposes, can be discretized into a sum over a finite set of m significant times:

$$CVA \approx (1 - R_C) \sum_{i=1}^{m} DEE_{t_i} PD(t_{i-1}, t_i) \tag{2.9}$$

Given the bilateral nature of counterparty risk, we can introduce the DVA (Debt Value Adjustment) as a mirror image of CVA, representing the pricing of counterparty risk considering an institution's own default:

$$DVA = (1 - R_B) \int_t^T DENE_s \, \mathrm{d}PD_B(t, s). \tag{2.10}$$

where:

- R_B is the recovery rate of the banking institution
- $DENE_s$ is the discounted expected negative exposure as defined in (2.5)
- PD_B is the banking institution own default probability

Its discrete approximation is straightforward from (2.10).

2.3.2 Common Mitigation Methods

One interest point in counterparty risk is the fact that it can be mitigated. A way to achieve this result is the reduction of credit exposure: should the counterparty defaut, the aim is to reduce loss. The most commonly used method of doing this are *netting* and *collateral agreements*. Netting means that in the case of default all transactions with the counterparty are consolidated into a single net obligation. Collateralization of a deal means that the party which is out-of-the-money is required to post collateral - usually cash, government securities or highly rated bonds - corresponding to the amount payable by that party in the case of a default event.
These methods are often bilateral and therefore aim to reduce the risk for both parties. However, any mitigation of counterparty risk is a double-edged sword since it will not necessarily reduce overall risks, and sometimes only burden the trade and converts counterparty risk in other forms of financial risk. Here some examples of the issue one can incur in when mitigating counterparty risk:

- **Operational risks**, whenever collateral has to be posted, the parties have to agree upon the contract value. And this is not trivial, especially when dealing with complex exotic derivatives.
- **Liquidity risk**, in case of non-cash collateral, in case the counterparty defaults one has to *monetize* the collateral, facing liquidity risk.
- **Legal risk**, in case a particular agreement between the parties is not legally recognized, especially when different jurisdictions are concerned.

In view of this, counterparty risk mitigation methods are critical, but it is also important not to overstate their benefits and ignore their dangers.

Netting

Derivatives markets are fast moving, with participants regularly changing their positions and where many instruments offset (hedge) one other. When a counterparty defaults then the market needs a mechanism whereby an institution should be able to offset what it owes to the defaulted counterparty against what they themselves are owed. This is not obvious, since under the law the loss incurred in when a counterparty defaults would be the **sum** of the exposures.

As an example, let's consider the case of a trade (trade 1) being cancelled via executing the reverse transaction (trade 2). Suppose there are two scenarios in that trade 1 and trade 2 can take the values +10 and -10, respectively, or vice versa. Whilst the total value of the two trades is zero (as it should be since the aim was to cancel the original trade), the total exposure is +10 in both scenarios. This means that if the counterparty defaults, in either scenario there would be a loss due to having to settle the trade with the negative MtM but not being able to claim the trade that has a positive MtM.

A "netting set" defines a set of trades that can be legally netted together in the event of a default. A netting set may be a single trade and there may be more than one netting set for a given counterparty. Across netting sets, exposure will always be additive, whereas within a netting set MtM values can be added. More precilely, consider having with n trades with the counterparty C. With no specific netting agreements, the institution's total exposure is:

$$CE = \sum_{i=1}^{n} (max(MtM_i, 0)) \tag{2.11}$$

where MtM_i is the mark to market of the i-th deal.

If the deals are split into m netting sets S_j $(j = 1 \dots m)$, however, the exposure becomes

$$CE = \sum_{i=1}^{m} \left(max \left(\sum_{j \in S_i} MtM_j, 0 \right) \right) \tag{2.12}$$

The fact that the expression (2.12) is less than (2.11) comes from subadditivity.

Notwhistanding the foregoing, the use of netting sets introduces legal risks in cases where a netting agreement cannot be legally enforced in a particular jurisdiction.

Collateralization

Collateralisation (also known as margining) provides a further means to reduce credit exposure beyond the benefit achieved with netting. A reset feature is essentially the periodic payment of collateral to neutralise an exposure. Collateral agreements may often be negotiated prior to any trading activity between counterparties or may be agreed or updated prior to an increase in trading volume or change in other conditions.
Suppose that a netted exposure (sum of all the values of transactions with the counterparty) is large and positive. There is clearly a strong risk if the counterparty is to default. A collateral agreement limits this exposure by specifying that collateral (usually cash, government securities or highly rated bonds) must be posted by one counterparty to the other to support such an exposure. The collateral receiver only becomes the economic owner of the collateral if the collateral giver defaults. Like netting agreements, collateral agreements may be two-way which means that either counterparty would be required to post collateral against a negative mark-to-market value (from their point of view). Both counterparties will periodically mark all positions to market and check the net value. Then they will check the terms of the collateral agreement to calculate if they are owed collateral and vice versa. To keep operational costs under control, posting of collateral will not be continuous and will occur in blocks according to predefined rules. Moreover, usually a *threshold* is agreed, defining the level of MtM above which collateral is posted.

2.3.3 Regulations and Market Standards

Basel Committee Dictates

The Basel III Accord prescribes that banks should compute unilateral CVA by assuming independence of exposure and default. Wrong-way risk is included through one-size-fits-all multipliers. The advanced framework allows banks to compute the effect of wrong-way risk using own models, while the standardized approach accounts for the effect by means of a onesize- fits-all multiplier. Interestingly, the Basel III Accord chooses to ignore the DVA in the calculation for capital adequacy requirements, although consideration of the DVA needs to be included according to accounting standards.

> [...] This CVA loss is calculated without taking into account any offsetting debit valuation adjustments which have been deducted from capital under paragraph 75. (Basel III, page 37, July 2011 release)
>
> [...] The potential for perverse incentives resulting from profit being linked to decreasing creditworthiness means capital requirements cannot recognise [DVA] (Stefan Walter, secretary-general of the Basel Committee)
>
> [...] Because nonperformance risk (the risk that the obligation will not be fulfilled) includes the reporting entities credit risk, the reporting entity should consider the effect of its credit risk (credit standing) on the fair value of the liability in all periods in which the liability is measured at fair value under other accounting pronouncements (FAS 157)

This inconsistency between capital and accounting regulation is sparking much debate in the industry.

The ISDA Master Agreements

The ISDA Master Agreement is the most commonly used master service agreement for OTC derivatives transactions internationally. It is part of a framework of documents, designed to enable OTC derivatives to be fully and flexibly documented. The ISDA master agreement is published by the *International Swaps and Derivatives Association.*

The master agreement is a document agreed between two parties that sets out standard terms that apply to all the transactions entered into between those parties. Each time that a transaction is entered into, the terms of the master agreement do not need to be re-negotiated and apply automatically. The master agreement sets forth all of the general terms and conditions necessary to properly allocate the risks of the transactions between the parties.

Among all the parts the master agreement is made of, for our purposes we report the **Chapter 6**, which contain the netting rules, and the **CSA**, which rule the Collateral Agreements.

Chapter 6 Section 6 of the ISDA Master Agreement contains the provisions which enable a party to terminate transactions early if an Event of Default or Termination Event occurs in respect of the other party and set out the procedure to calculate and net the termination values of those transactions to produce a single amount payable between the parties.The aggregate of the Close-out Amounts and Unpaid Amounts is referred to as the "Early Termination Amount". This is the net amount payable by one party to the other in respect of the Terminated Transactions.

CSA The *Credit Support Annex* is optional but is widely used in most Master Agreements for OTC derivative transactions. The Annex is added if the parties agree that collateral is to be provided by a party if the exposure of the other to it exceeds an agreed amount. The Annex contains provisions concerning the posting and return of collateral, the types of collateral that may be used, and the treatment of collateral by the secured party, specifically whether or not the practice of *rehypothecation* is allowed, that is the allowance for the collateral taker to relatively unrestrictly use the collateral for his liquidity and trading needs until it is returned to the collateral provider. Effectively, the practice of rehypothecation lowers the costs of remuneration of the provided collateral. However, while without rehypothecation the collateral provider can expect to get any excess collateral returned after honoring the amount payable on the deal, if rehypothecation is allowed the collateral provider runs the risk of losing a fraction or all of the excess collateral in case of default on the collateral taker's part. If no rehypothecation is allowed, the collateral is kept safe in a segregated account.

Chapter 3

Framework

3.1 The funding problem

As we said before, as far as OTC trades are concerned, collateral agreements have established as a standard market practice among financial institutions, to provide mitigation for credit risk. Accordingly, OTC derivatives have become funded instruments, i.e. they require to raise funds to face the collateral need. As a matter of fact, the bulk of outstanding OTC derivatives are now regulated by bilateral CSA agreements with daily margin calls, zero thresholds and zero minimum transfer amounts, thus providing nearly perfect collateralization and remarkably shrinking the CVA and the DVA amounts.

However there are some counterparties, such as sovereigns, supra-nationals, corporates, pension funds and hedge funds that can trade with banks on uncollateralized basis. The banks usually hedge these derivatives trading the replication strategy under their CSAs with other banks or CCPs.

Therefore the main implication of collateral agreements regards non-collateralized transactions: indeed, when a non-collateralized derivative is hedged with a back-to-back deal, which instead is collateralized, funding costs or benefits do occur. This is what the funding valuation adjustment (FVA) accounts for. In particular:

- When the dealer is in-the-money in the uncollateralized trade, he will be out-of-the-money in the back-to-back trade, generating a funding cost due to the need to post collateral; this funding cost should be included in the valuation of the uncollateralized trade as funding cost adjustment (FCA). See figure 3.1.

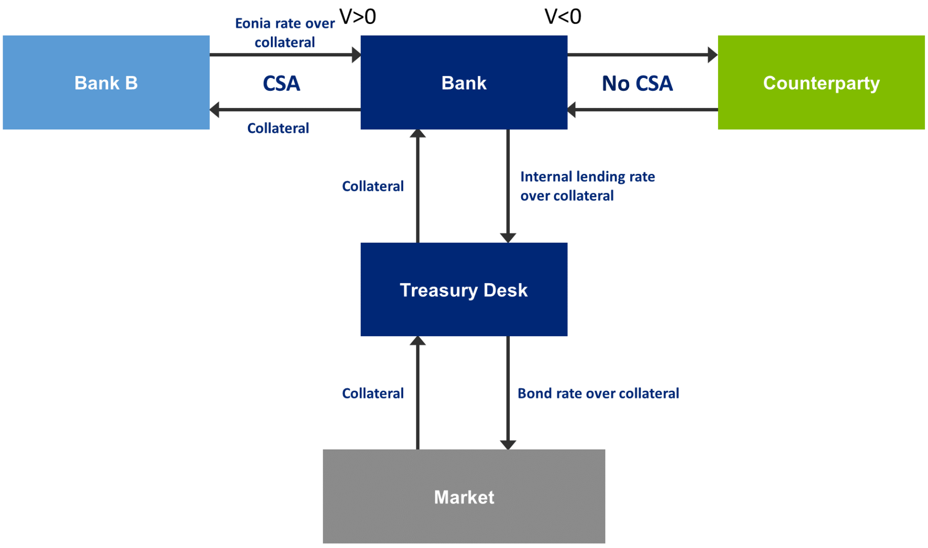

Figure 3.1: Bank institution and counterparty entered in unsecured derivative contract, which has positive value for the bank ($V > 0$). The value of the contract is negative in the secured hedge deal with ($V < 0$). This situation leads to funding costs for the bank.

- When the dealer is out-of-the-money in the uncollateralized trade, he will be in-the-money in the back-to-back trade, generating a funding benefit due to the right to receive collateral; this funding benefit should be included in the valuation of the uncollateralized trade as funding benefit adjustment (FBA). See figure 3.2.

3.2 The FVA debate

In a nutshell, the FVA is the difference between the price of an uncollateralized OTC derivative and the price of the same derivative, but collateralized. This difference may result in a cost or a benefit which ultimately depends on

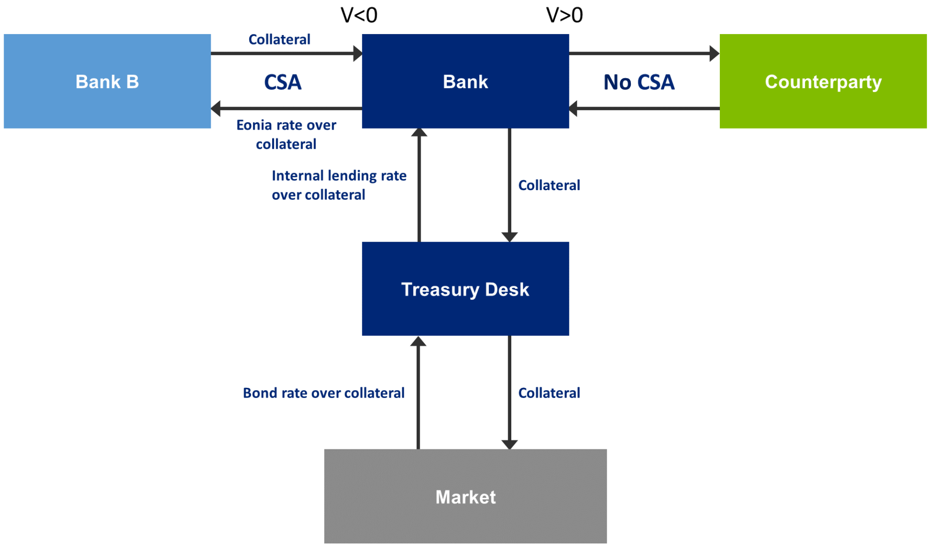

Figure 3.2: Bank institution and counterparty entered in unsecured derivative contract, which has negative value for the bank ($V < 0$). The value of the contract is positive in the secured hedge deal with ($V > 0$). In this situation the bank has no funding costs and hence, and, if rehypothecation is allowed, draw a benefit from posted collateral.

the funding curve of the relevant dealer, apart from the sign of the position. The first question about FVA is whether it should be included in valuation and pricing of uncollateralized derivatives. Even if for almost all international banks the answer is affirmative and straightforward, the well-known position of Hull and White, who still seem to disagree, prevent us to take the answer for granted. To be more precise, in [30], their first work about FVA, Hull and White, exploiting theoretical principles of economics and finance, conclude that the FVA should not be included when an uncollateralized derivative is traded, whereas in their most recent work (2014) they state the inclusion of FVA would be justifiable under certain assumptions, but would have impacts on pricing competitiveness and would give rise to arbitrage opportunities. Even though they have withdrawn the original position, these caveats still seem to persuade to not include FVA in the valuation framework of OTC derivatives.

With the exception of some institutions, the majority disagrees with Hull and White. Some examples of developed FVA business can already be found in the industry and some common practices have emerged anyway, which serve as benchmark for all other banks that are moving towards a FVA management model or are considering to do it in near future. Nevertheless, it is not yet possible to define a FVA best practice.

The issues and open points banks have to solve can be outlined as follows:

- **FVA calculation:** from a methodology perspective, taking advantage of existing CVA models, as FVA and CVA share almost all pricing inputs, would be more advisable; how to deal with funding in valuation and what measure of funding to use are related issues for which banks have found different solutions that are supported or not supported by literature.

- **Internal and external charges:** once the FVA has been somehow measured in pre-deal phase, the dealer indeed is delegating its management to another desk (e.g. treasury desk) according to internal processes, but he is the real owner. Therefore, to correctly assess the performance of the dealer, he should be charged the funding costs or benefits he has generated. Whether to charge the FVA to the client is not clear-cut, mainly because there are materiality issues to consider and the impact on the competitiveness of the dealer could be not trivial.

- **FVA hedge**: as derivatives are not term funded instruments and given the impossibility to derive future funding costs, the hedging of the FVA can only be conducted with some approximations.

- **FVA desk:** there are different possible business solutions regarding the office in charge to measure and manage the FVA, the trading desk itself, the treasury desk or an ad hoc created FVA desk. Each of these business models involves different scopes and responsibilities: the centralised structure creates a powerful new function responsible for determining capital and funding needs, leading to disagreement over whether the desk should be run by traders or part of the treasury department's remit. The opinions are not unanimous even within the same bank, but, in general, the decision to centralize the FVA management would depend upon business volumes. Barclays and JP Morgan were among first to centralise xVA desks, in late 2013.

- **FVA disclosure and accounting:** actually there is no requirement to report FVA in balance sheet, but some major banks either already report the FVA or are committed to report it in future. How the FVA, which is an entity-specific adjustment, can be included in the fair value framework without violating the concept of exit price still remains inconclusive.

- **The law of one price:** On the theoretical side, the dependency of the FVA on an internal variable like a firm's own funding strategies shakes the foundation of the celebrated Law of One Price prevailing in classical derivatives pricing. Clearly, if we assume no funding costs, the dealer and counterparty agree on the price of the deal as both parties can - at least theoretically - observe the credit risk of each other through CDS contracts traded in the market and the relevant market risks, thus agreeing on CVA and DVA. In contrast, introducing funding costs, they will not agree on the FVA for the deal due to asymmetric information. The parties cannot observe each others' liquidity policies nor their respective funding costs associated with a particular deal. As a result, the value of a deal position will not generally be the same to the counterparty as to the dealer just with opposite sign. In principle, this should mean that the dealer and the counterparty would never close the trade, but in practice trades are executed as a simple consequence of the fundamental forces of supply and demand. Nevertheless, as reported in Brigo et al. [15], among dealers it is the general belief that funding costs were one of the main factors driving the bid-ask spreads wider during the recent financial crisis.

3.3 FVA in Literature

In the last luster, academics have investigated how to include the funding risk and what measure of funding risk to adopt. One of the most recurring topics is the warning of double-counting which stems from the interrelation between credit and funding risk. A brief excursus of main contributions in literature is provided in the remainder of this section.

3.3.1 Piterbarg 2010: Rise of the FVA

The funding probem is touched on for the first time in *"Funding beyond discounting: collateral agreements and derivatives pricing"*, an article by Vladimir Piterbarg on *Risk* in 2010 [39]. He suggest that the discount rate for a cash-collateralized derivative should be based on the OIS rate in the currency of the collateral; compared with the collateralized version, the same derivative price but without collateral, would need an adjustment essentially driven by the funding spread of the dealer.

3.3.2 Burgard and Kjaer 2011: the PDE Framework

Burgard and Kjaer in their 2011 work [17] develop a framework that combines funding costs and bilateral counterparty credit risk in a PDE representation of the derivative value. This framework specifies how a positive cash account related to the hedging strategy (back-to-back trade) of an uncollateralized derivative can be used to fund the repurchase of the issuer's own bonds in order to hedge out its own credit risk. In this way the DVA is equivalent to a funding benefit adjustment and its inclusion in the unified framework is justified. However the funding benefit disappears if the rehypothecation of collateral is not allowed and a positive cash account can only provide the risk-free return (OIS rate). If so, there is only the funding cost to consider; even the funding cost could theoretically disappear, being able to use the derivative itself as collateral. The authors show also how the size of the funding cost adjustment and consequently the price of the derivative depend on the funding policy of the dealer. The law "one instrument one price" is broken and the counterparty would enter in derivative position with the dealer with the best funding policy.

3.3.3 Morini and Prampolini 2010: a Deeper Analysis

Morini and Prampolini had shown the same results of Burgard and Kjaer in 2010 [36], trying to solve a different issue, that is the interactions between funding liquidity risk and counterparty credit risk. They stated that the inclusion of funding costs in a bilateral CVA framework by simply modifying the discounting curve, leads to double-counting. Indeed, as the funding spread can be considered the sum of two components, the compensation for credit risk and the compensation for liquidity risk, one can argue that the credit component is already taken into account in the CVA. A consistent unified framework that avoids double-counting can be achieved identifying the liquidity spread in the difference between the whole funding spread of a bank and the spread measuring its risk of default. This argument can be solved also by means of financial variables directly observable in the market: if the issuer's bond spread measures the whole funding spread and the CDS spread is the price of default risk, the use of bond-CDS basis as measure of liquidity risk would avoid double-counting.
In this subsection the findings of Morini and Prampolini paper are briefy discussed. The paper uses a simple modeling setting, considering a deal involving a borrower B and a lender L, where B commits to pay a fixed amount K to L at time T. Assume that party $X \in \{B, L\}$ has a recovery rate R_X. This means that in case of a default of party X, R_X is the percentage of the exposure that is recovered to creditors. The risk-free rate is denoted by r and is assumed to be deterministic. Furthermore, X funds itself in the bond market and is the reference entity in the CDS market. More generally, the CDS spread X is assumed to be deterministic and paid continuously. In Section 3.3.2 of Brigo et al. (2013) it is shown that in this case:

$$\pi_X = \lambda_X LGD_X$$

where X denotes the deterministic default intensity and the loss given default LGD_X equals $1 - R_X$. Recovery is assumed to be zero ($LGD_X = 1$), thus $\pi_X = \lambda_X$. From the exponential distribution assumption for default time τ, it follows that $\mathcal{P}(\tau_X > T) = e^{-\pi_X T}$ (see section 3.3 of Brigo et al. [12]).

Like π_X, the funding spread s_X is also assumed to be instantaneous and deterministic. The cost of funding is commonly measured in the secondary bond market as the spread over a risk-free rate. The difference between the funding spread and the CDS spread of a party X is called the liquidity basis and is denoted by γ_X, hence$s_X = \pi_X + \gamma_X$.
First, the net present value (NPV) for the lender V_L of the above deal is described without including funding cost. If P denotes the premium paid by L at inception, the NPV of the above deal equals

$$V_L = e^{-rT}K - CVA_L - P$$

Where CVA_L is given by

$$CVA_L = \mathbb{E}[e^{-rT}K\mathbb{1}_{\tau_B \leq T}] = e^{-rT}K[1 - e^{-\pi_B T}]$$

To make the value of the contract fair we equate V_L to zero. Therefore, we have $P = e^{-rT}K - CVA_L$. From the perspective of the borrower, the NPV of the deal is

$$V_B = e^{-rT}K - DVA_B + P$$

with $CVA_L = DVA_B$. To make the value of the contract fair we set V_B to zero, thus $P = e^{-rT}K - DVA_B$. Therefore, price symmetry is satisfied$V_B = V_L = 0$ and both parties may agree on the premium of the deal:

$$P = e^{-rT}e^{-\pi_B}K$$

Obviously, funding costs are not implemented in the above derivation. While L needs to finance the claim P until the maturity of the deal at its funding spread s_X, party B can reduce its funding by P. Therefore, B has a funding benefit and party L needs to pay its financing cost and thus has funding costs. Hence, party L should reduce the value of the claim by its financing costs. Besides, we cannot assume both parties having negligible funding cost since we are dealing with possible default risk. To introduce liquidity in the valuation of the deal, the article describes the problem of double counting. In this case, we implement liquidity costs by (only) changing the discount factor. Moreover, the value to the lender is

$$V_L = \mathbb{E}[e^{-(r+s_L)T}K\mathbb{1}_{\tau_B > T}] - P = e^{-rT}e^{-\gamma_L T}e^{-\pi_L T}Ke^{-\pi_B T} - P$$

and the value to the borrower is

$$V_L = -\mathbb{E}[e^{-(r+s_B)T}K\mathbb{1}_{\tau_B>T}] + P = e^{-rT}e^{-\gamma_B T}e^{-2\pi_L T}Ke^{-\pi_B T} + P$$

To discuss this finding, we assume for simplicity that $s_L = 0$, so the lender L is default-free and has no liquidity basis. On the other hand, the borrower B may default, thus $s_B = \pi_B > 0$. In this case we obtain $P_L = e^{-rT}e^{-\pi_B T}K$ and $P_B = e^{-rT}e^{-2\pi_B T}K$, which is a remarkable finding. Firstly, the two parties disagree on the premium of this simple deal. Borrowers can account an immediate profit in all transaction with CVA. And secondly, pricing this deal at fair value to the borrower would involve multiplying the NPV of K twice with its survival probabilities, which is called the problem of double counting in the paper. Both of these aspects belie years of market reality.
In order to solve this puzzle, Morini and Prampolini model the funding strategy explicitly. Following this approach, the deal is split up into two legs. From the lender's perspective, the NPV of the "deal leg" is given by $NPV = \mathbb{E}[-P + e^{-rT}\Pi]$, where Π denotes the pay-off at T with a potential default indicator. The other leg is called the "funding leg" and has $NPV = \mathbb{E}[P - e^{-rT}F]$, where F is the funding payment at T, also including a potential default indicator. Therefore, the total NPV equals

$$V_L = \mathbb{E}[e^{-rT}\Pi - e^{-rT}F]$$

The authors make the assumption funding is made by issuing bonds and excess funds are used to reduce or avoid increasing the stock of bonds. Therefore, the outflow F at T is

$$F = Pe^{-rT}e^{s_L T}\mathbb{1}_{\tau_L>T}$$

In the "deal leg", the lender inflow Π at T is $\Pi = K\mathbb{1}_{\tau_B>T}$. Thus, the total pay-off at T is

$$\Phi = -Pe^{-rT}e^{s_L T}\mathbb{1}_{\tau_L>T} + K\mathbb{1}_{\tau_B>T}$$

Taking the discounted expectation of the previous equation yields

$$V_L = -Pe^{-\gamma_L T} + Ke^{-rT}e^{-\pi_B T} \tag{3.1}$$

Analogously, it can be shown that the NPV of the deal for the borrower is

$$V_B = Pe^{-\gamma_B T} - Ke^{-rT}e^{-\pi_B T} \tag{3.2}$$

where the double counting problem vanished.
It can easily be shown that the break-even premium for the lender is $P_L = Ke^{-rT}e^{-\pi_B T}e^{-\gamma_L T}$ and for the borrower $P_B = Ke^{-rT}e^{-\pi_B T}e^{-\gamma_B T}$. To reach an agreement, the conditions $V_L \geq 0,\ V_B \geq 0$ has to be satisfied, and this implies $P_B \leq P \leq P_L$. Thus an agreement can be found whenever holds

$$\gamma_B \geq \gamma_L \tag{3.3}$$

This shows that, in order to have a positive NPV for both counterparties, the funding cost that needs to be charged in this simplified transaction is just the liquidity basis. The lender's funding cost contains a part that is associated with the lender's probability of default. This part cancels out with the probability of default in the lender's funding strategy, hence the only spread that contributes as a net funding cost to the lender is the liquidity basis.
The conclusions are the same as in Burgard and Kjaer [18]: first, the inclusion of funding costs in the valuation generates an asymmetric market where different dealers give different prices to the same derivatives; then, the impossibility to monetize the DVA, i.e. the impossibility of "selling protection on yourself", can be solved if a bank evaluates the DVA as a funding benefit and uses liquidity generated by the derivatives to buy back its own bonds.

3.3.4 Hull and White 2012: the Denial

In their first work about FVA in 2012 [30], Hull and White conclude that the funding value adjustment should be ignored and not included by dealers in their trading decisions. More precisely they refer to funding cost adjustment as FVA and funding benefit as DVA2. Recalling principles from economics and finance theory, the authors argue that the DVA2 is already taken into account in the whole DVA and that the decision to hedge should not affect the valuation of a derivative; in this sense, FVA can be assimilated to "normal" funding requirements, which are not to be included in the price.

3.3.5 Burgard and Kjaer 2012: a Back Answer

Burgard and Kjaer in [18] promptly provide an answer to Hull and White statements trying to explain the opposite conclusion they achieve: from a replication strategy perspective, the excess funding costs, a dealer should incur in order to hedge an uncollateralized position, matters for pricing and trading decisions. The crux of the matter is that different conclusions about FVA can be reached if the reasoning is driven by different arguments, such as what is theoretically desirable on one side and what is practically achievable on the other side.

3.3.6 Gregory 2012: the CCR Framework

Jon Gregory, in 2012 edition of his widely famous work *"Counterparty Credit Risk and Credit Value Adjustment"* ([29]), to include FVA in his tretatise. Introducing funding cost or benefit by discounting cash flows at the relevant funding rate allows to account for FVA in the simplest way, but there are two drawbacks with this approach: the discount curve method implicitly assumes a symmetry between funding costs and funding benefits and does not allow the treatment of derivatives for which collateralization is not perfect. For this reason, the author deals with FVA exploiting the same formulae of CVA and DVA. In particular he describes FVA as the final missing scenario in the CVA/DVA valuation model: given that CVA accounts for the situation in which the counterparty defaults but the bank survives, DVA accounts for the situation in which the counterparty survives but the bank defaults, the scenario in which funding costs and benefits must be considered is when both the bank and the counterparty survive. The FVA can be expressed as:

$$FVA = \sum_{j=1}^{n} EE(t_j)[1-PD_C(0,t_{j-1})][1-PD_B(0,t_{j-1})]FS_b(t_{j-1},t_j)(t_{j-1}-t_j))+$$
$$+\sum_{j=1}^{n} NEE(t_j)[1-PD_C(0,t_{j-1})][1-PD_B(0,t_{j-1})]FS_l(t_{j-1},t_j)(t_{j-1}-t_j)) \tag{3.4}$$

where:

- $t_j, j = 1...n$ is the time of the j-th cash flow.
- $EE(t)$ is the discounted expected positive exposure at the time.

- $NEE(t)$ is the discounted expected negative exposure at the tie.
- $PD_C(t_i, t_j$ is the probability the counterparty defaults in between the times t_i and t_j.
- $PD_B(t_i, t_j$ is the probability the bank defaults in between the times t_i and t_j.
- $FS_b(t_{j-1}, t_j)$ is the funding spread the bank has to pay when borrows money between the times t_i and t_j (i.e. the spread between the funding rate and the risk-free rate).
- $FS_b(t_{j-1}, t_j)$ is the funding spread the bank can ask when lends money between the times t_i and t_j(i.e. the spread between the lending rate and the risk-free rate).

The first term in the formula is the funding cost adjustment, while the second term is the funding benefit adjustment. According to the definition of FVA as missing scenario, the discounted expected exposure is first adjusted by the survival probability of both bank and counterparty, and then multiplied by the relevant funding spread. This is intuitive, if one thinks that collateral cash flows cease to affect the derivative value in case of default. This formulation allows for funding spreads to be different for borrowing and lending side, and fully captures collateralization features provided that the expected exposure is computed in the same way as for CVA and DVA. Furthermore, the author highlights the importance of the choice of the funding spread to use in the model and addresses possible solutions for the double-counting issue. The choice of the funding spread is problematic as derivatives typically are not term funded. In the opinion of the author the CDS-bond basis would be the proper way to avoid double-counting of DVA and FBA components. Nevertheless two potential frameworks are investigated to provide a consistent valuation model for counterparty risk and funding:

- Symmetric funding and CVA (CVA+FCA+FBA): this framework would ignore the DVA benefit because its monetization is problematic
- Asymmetric funding and bilateral CVA (CVA+DVA+FCA): this framework includes DVA as funding benefit

3.3.7 Castagna 2013: the Replicating Portfolio

Antonio Castagna in his 2013 work ([22]) develops a pricing model for derivatives under CSA agreements, disregarding residual credit risk due to imperfect collateralization. The implication of pricing and the allocation of responsibilities to different desks are discussed as well. The author follows the approach of the replicating portfolio of a contingent claim on a risky asset. The analysis accounts for the possibility that the rate of return paid on collateral account could be different from the risk-free rate, introducing the LVA (liquidity value adjustment). The dynamical replication strategy involves a position in cash to mimic the collateral account, achievable by buying or selling deposits, and the classical delta-hedging position in the risky asset, achievable also by means of the repo market. Therefore, the collateralized derivative value is equal to the value of the non-collateralized plus the LVA, which reflects the cost incurred to finance the collateral, and the FVA, which reflects the cost incurred to replicate the contract and the collateral account:

$$V^C = V^{NC} + LVA + FVA \tag{3.5}$$

The pricing can be performed in two ways, which provide identical results. The first is simply replacing the risk-free rate with the relevant funding rate paid by the bank to finance the replication strategy, while the second is to price separately each component. Though the first way is a very easy approach, the latter can help to allocate components to the different desks of the bank. In this way the trading desk is assigned the V^{NC} component to be hedged only in a market risk environment, where there are no collateral and funding effects to consider, the LVA component would be assigned to the collateral desk, and the FVA component would be splitted to the treasury desk and to the repo desk if there is a repo component.

3.3.8 Brigo et al. 2014: the Recursive Approach

The work we think provides the most reliable pricing methodology so far is [15], the one Brigo, Liu, Pallavicini and Sloth published in 2014, as they brilliantly deal with most of the cumbersome hurdles one incurs when addressing the FVA topic. Based on the risk-neutral pricing principle, they derive a general pricing equation where Credit, Debit, Liquidity and Funding Valuation Adjustments (CVA, DVA, LVA and FVA) are introduced by simply modifying the payout cash-flows of the deal. From this idea we will develop our pricing framework, so the methodology will be later deeply analyzed in **(4.1.5)**.

3.3.9 Recurring themes

The contributions from the academics, though characterized by different levels of complexity, draw the attention to the same recurring themes that can be attributed directly to pricing, but certainly give rise to some issues in the bank internal processes and in the derivatives business profitability. The recurring themes can be summarized as follows:

Double-counting Given the interrelation between funding and credit risk, the correct assessment of funding could be not as straightforward as the FVA definition; the concepts of credit and funding are overlapping, if not the same, as in some cases could be. A correct way to measure the funding spread is addressed, the CDS-bond basis, in order to avoid double-counting of what already included in CVA. Other ways to cope with this issue are to consider the DVA as a funding benefit, discarding the FBA, or to assume that the received collateral can only earn the CSA rate thus providing no funding benefit. The latter would be a prudent assumption, as in practice CSA agreements usually allow the rehypothecation of variation margin.

Bilateral prices The bilateral nature of the deal price is broken with the introduction of the FVA. The dealer and the client would come to different prices, above all because the client does not know the dealer funding policy; this is not trivial considering that among those counterparties that can trade uncollateralized derivatives with banks, some can have a remarkable derivatives expertise (e.g. hedge funds). As warned by Hull and White, this asymmetry, would make the client choose the best price and would yield arbitrage opportunities.

Fair value The introduction of the FVA in the valuation puts another question on the table, and the discordance between price and value is resorted. The point is one comes to different results if the valuation is performed from a pricing perspective on one side, and from an accounting perspective on the other side. Fair value is supposed to represent an exit price, and the exit price is what a third party would pay to buy an asset or charge to assume a liability; the trader valuation instead reflects associated hedging strategies which are entity-specific. The solution would be to make different methodological choices, using the trader's funding curve for internal pricing purpose and an average measure of market cost of funds for accounting purpose.

Pricing approach The discount curve method is a convenient way to include funding in valuation, by simply replacing the risk-free rate with the relevant funding rate and provides the same result as pricing the FVA separately, according to the additive approach. Yet, following this latter approach would help allocating the price components to the relevant desks. As disclosed in Brigo et al. [15], the additive approach works well only under certain assumptions (symmetrical funding and risk-free close-outs), otherwise the pricing problem is non-linear and recursive. Such a feature prevents the possibility to split the adjustments and the clear-cut allocation of components to different offices, without incurring in double-counting.

3.4 Accounting the FVA

3.4.1 CVA accounting

Before discussing funding adjustments in earnest, let us review how accounting rules work for the classical case where a bank's over-the-counter portfolio value is adjusted for credit risk (CVA and DVA).

First, accounting ledger rules for OTC portfolios typically assign trades with positive valuations (i.e., receivables) to an asset account, and trades with negative valuations (i.e., payables) to a liability account. In the absence of credit risk, the portfolio fair valuation (PFV) to the bank holding the positions is then given by the default-free value of assets A minus the default-free value of liabilities L:

$$PFV = A - L$$

Counterparty credit risk adds a few complexities and necessitates the introduction of "contra" accounts as well as a change in the "unit of account" from individual trades to counterparty-specific netting sets. Downward adjustments to asset values from counterparty credit risk are recorded as CVA entries in a "contra-asset" (CA) account. The CA account aggregates CVA against all counterparties and is subtracted from default-free asset values.

In addition to the contra-asset account, there is also a "contra liability" (CL) account that includes DVA entries for each counterparty. The bank's total DVA equals the total CVA recorded by all counterparties against the bank, and ensures the accounting system is symmetric and does not create wealth out of zero-sum bilateral trading. DVA entries are benefits and represent the present value of the bank's option to default on its liabilities.
To summarize, the fair value associated with the derivatives portion of the balance sheet may be written as:

$$PFV = A - L - CA + CL$$

If we include a cash account (Cash), we may complete a simplified balance sheet by writing total assets as $Cash + A - CA$, with the accounting equity defined as:

$$Equity = Cash + PFV = Cash + A - CA - L + CL$$

While DVA is a rational and well-defined component of the bankwide PFV, it should arguably not contribute to regulatory capital, as benefits associated with a bank default are neither loss absorbing nor do they contribute to the wealth of bank equity holders (who are wiped out by a bank default). DVA entries in the CL account are therefore excluded by regulators (see

Basel Committee on Banking Supervision 2012; Federal Register 2014) from CET1. That is:

$$CET1 = Equity - CL = Cash + A - CA - L$$

The interpretation of DVAs not benefiting equity holders will often manifest itself in quotation practices, where it is common for traders to internally de-recognise all or part of DVA benefits in the prices they quote to counterparties, in effect charging the DVA through to the client on top of PFV. If trades are made at the quoted levels, the bank will consequently recognise day-one trading gains that ultimately hit retained earnings and, in turn, contribute to CET1.

3.4.2 FVA accounting

As mentioned in the previous section, the CVA is meaningful for individual netting sets, not for individual transactions, as two trades in a netting set can offset each other's credit risk. At the book level, the CVA is the sum of CVA metrics for each netting set in the bank.
The FVA instead is meaningful only at the "funding set".level, a collection of unsecured trades across which cash received for derivatives funding can be re-hypothecated.
Equivalently, a funding set is a collection of trades for which the variation margin posted on collateralized hedges with dealers may be re-hypothecated. Typically, funding sets include hundreds or even thousands of counterparties and netting sets; funding sets can also cut across netting sets, as some of the trades in one netting set may belong to one funding set and other trades to another. Since 2011's several large banks have been instituting accounting changes aimed at capturing the funding costs for uncollateralised derivatives transactions, but there's no convergence on how FVAs should be calculated, and everybody is doing it differently.

However, in 2014 in three important quants Claudio Albanese, Leif Andersen and Stefano Iabichino, published an article on *Risk* [3] with a revolutionary idea: funding valuation adjustment (FVA) should be appearing in equity, rather than earnings. The three quants argue if a bank borrows unsecured funds to finance uncollateralised derivatives, then this represents a transfer of wealth from shareholders to bondholders if the bank comes into default, limiting claims on the bank's estate and consuming funds that could have been used to generate more income. Under the traditional approach, FVA is the product of a funding cost adjustment (FCA) and a funding benefit adjustment (FBA), but Albanese, Andersen and Iabichino introduce a new term - funding debit adjustment (FDA). When banks take a funding cost, some other entity should be receiving an equal benefit, they argue. FDA mirrors the FVA amount and represents the transfer of wealth. The FDA and FVA terms, being equal, cancel each other out on the balance sheet, eliminating FVA from the income statement. However, on the equity side, FDA does not net with FVA as funding benefits are not reflected in equity. This is because shareholders can't monetise FBA, which exists only for as long as the trade does. As a result, the FCA amount should be written off the bank's common equity Tier I capital (CET1).

Reaction to their statements were contrasting.
As an example, Darrell Duffie, professor of finance at Stanford University in California, agrees with the approach. *"Suppose you decide to buy treasuries and set them aside as collateral for some creditors or derivatives counterparties. That's costly to your shareholders, because you just made your creditors safer and you can't use the money you set aside to buy those treasuries to do something that might have made money for your shareholders. That is effectively what the FVA is - a transfer of wealth from shareholders to creditors. This doesn't mean you should take that cost to shareholders and then assign it to any particular financial instrument,"* he said in [6].

On the contrary, auditors seem not convinced. Amir Kia, a senior manager in the risk and regulation division at Deloitte in London declares in [6]: *"I don't see the rationale behind putting it through CET1. FVA is a cost so should be recognised in the income statement"*. He argues that, in the long term, items reported in the income statement go through retained earnings and will eventually make it to the equity of banks' shareholders. *"In the* *long term, the quarterly fluctuation of FVA charges should cancel each other to a large extent, so it doesn't make any difference what you do".*

3.5 Current market approach

The first bank to ever take account of FVA was *Goldman Sachs* in late 2011 putting pressure to other banks to follow. In the box below, the first 4 FVA disclosures:

- **Barclays: 2012 annual report.** "During 2012, a fair-value adjustment was applied to account for the impact of incorporating the cost of funding into the valuation of uncollateralised derivatives. This was driven by the impact of discounting future expected uncollateralised cashflows to reflect the cost of funding, taking into account observed traded levels on uncollateralised derivatives and other relevant factors. The group continues to monitor market practices and activity to ensure the approach to discounting in derivative valuation remains appropriate."
- **Goldman Sachs: 2011 annual report.** "Valuation adjustments are integral to determining the fair value of derivatives and are used to adjust the mid-market valuations, produced by derivative pricing models, to the appropriate exit price valuation. These adjustments incorporate bid/offer spreads, the cost of liquidity on illiquid positions, credit valuation adjustments and funding valuation adjustments, which account for the credit and funding risk inherent in derivative portfolios."
- **Lloyds Banking Group: 2012 annual report.** "The group has recognised a funding valuation adjustment [of £143 million] to adjust for the net cost of funding certain uncollateralised derivative positions where the group considers that this cost is included in market pricing. This adjustment is calculated on the expected

future exposure discounted at a suitable cost of funds. A ten basis points increase in the cost of funds will increase the funding valuation adjustment by approximately £14 million."

- **Royal Bank of Scotland: 2012 fourth-quarter report.** A footnote on page 115 of the report - relating to valuation reserves for financial instruments - states that the bank has recognised a funding valuation adjustment of £475 million.

At the moment, only about thirty of the world largest banks consider FVA at working level and recognize it in their financial statements. However, there's no convergence on how FVAs should be calculated, and everybody is doing it differently, because at the moment there are no rules to guide them. With no disclosure required, some banks are stitching together FVA numbers from a disparate array of components - their own bond spreads, the basis between bonds and credit default swap (CDS) spreads, and average industry funding costs, for example.
In table 3.1, the FVA losses 29 major banks put in their income statement in Q4 2014.

Some banks are more advanced or sophisticated than others and some may argue that they already have a fair understanding of the current xVA market standards. However, given the level of complexity and details involved, much is to be gained by better understanding the mechanisms, assumptions and processes surrounding the funding implication of counterparty risk management for a broad and relevant set of market participants.

In the last quarter of 2014, the advisory firm *Solum Financial* conducted a survey across a target group of banks to determine market practices with regard to Funding Value Adjustment (FVA). This survey examines in detail current (but evolving) FVA practices, the importance of FVA as a pricing component, its accounting treatment, and how it is calculated and risk managed. Solum surveyed 20 banks and their responses were given as a current

	Loss (millions)	Date Disclosed
Goldman Sachs	Unknown	Q4 2011
Barclays	£101	Q4 2012
Deutsche Bank	€364	Q4 2012
Lloyds Banking Group	£143	Q4 2012
Royal Bank of Scotland	£475	Q4 2012
Macquarie	Unknown	During 2013
Societe Generale	Unknown	During 2013
ANZ	A$ 61	Q4 2013
Bank of Ireland	€36	Q4 2013
JP Morgan	$ 1500	Q1 2014
Nomura	¥10000	Q1 2014
Allied Irish Banks	€15	Q2 2014
BNP Paribas	€166	Q2 2014
Credit Agricole	€167	Q2 2014
Credit Suisse	Sfr. 279	Q3 2014
National Australia Bank	Unknown	Q3 2014
National Bank of Canada	C$ 13	Q3 2014
UBS	Sfr. 267	Q3 2014
Bank of America Merrill Lynch	$ 497	Q4 2014
Bank of Montreal	Unknown	Q4 2014
BCPE	€82	Q4 2014
Canadian Imperial Bank of Commerce	C$ 65	Q4 2014
Citi	$ 474	Q4 2014
HSBC	$ 263	Q4 2014
Morgan Stanley	$ 468	Q4 2014
Royal Bank of Canada	C$ 105	Q4 2014
Scotiabank	C$ 30	Q4 2014
Toronto Dominion Bank	C$ 65	Q4 2014
Westpac	A$ 125	Q4 2014

Table 3.1: Bank accounting for FVA prior to 2014, and reported loss. Source: *Risk Magazine*

state of the situation that existed at that time. The received answers were compiled by Solum and then the results were published (on an anonymous basis). As expected, all participants report CVA, but, among them, only ten report FVA. The number of banks reporting the FVA has grown up since the previous year, when *Risk Magazine* announced only eight banks reporting it. By contrast to the even market split in reporting FVA, only one participant does not believe in FVA at all and therefore does not calculate it, even in pre-deal pricing; the others incorporates an FVA charge/benefit (always or selectively) to the client. In dealing with FVA/DVA overlaps, the stances are even more discordant: one participant classified as "Both are fully accounted for" acknowledges the overlap but deemed the materiality to be small, one participant which currently reports $CVA - DVA + FCA$ is considering FBA as a replacement to DVA, one participant confirmed that despite the theoretical and practical discussions around double counting it will (initially at least) report both DVA and FBA, and, finally, one participant intends to report the equivalent of a full FBA, but has clarified that this is still discussed internally.
Obviously, the situation has slightly changed since the survey, but no revolutions has happened: the stances over FVA pricing and accounting are still very discordant, and even more on technical topics.

The situation will probably remain chaotic until a regulator will decide to set out an international position on FVA.
In that respect, we point out that in mid-2015 The *Basel Committee on Banking Supervision* has allegedly launched a project looking at how banks value their uncollateralised derivatives portfolios. The scopes and aims of the project are not known, but industry sources speculate it may address potential double-counting. At the time of this work, the topic is still very open, even though the general trend towards FVA calculation is day by day more clear.

Chapter 4

FVA Put to the Test

In this chapter we put in place a numerical case study to let emerge the subtleties outlined in the previous chapter.
We first present our methodological frameworks, then we apply them to face FVA computations in a simplified scenario.

4.1 Methodologies

In this work, we consider portfolios of Interest Rate Swaps (IRS), due to the simplicity of their pricing and because their structure made them the perfect derivative contracts to show all the counterparty-risk features. One can argue this assumption as a very limited case. It is, but not so much as it can appear: indeed, interest rate derivatives in general, and interest rate swaps in particular, usually take the lion's share in the banking portfolios, as we can see in figure 4.1.

Interest rate swaps are commonly massively traded around the world both for hedging and speculating.
In an interest rate swap, two counterparties agree to exchange each other, for a specified period of time, a fixed rate S with a floating rate indexed to a reference rate (Euribor in our case) with a specified frequency of payments (possibly different for the fixed-rate and floating-rate payments). By market convention, the counterparty paying the fixed rate is the *payer* (while receiving the floating rate), and the counterparty receiving the fixed rate is

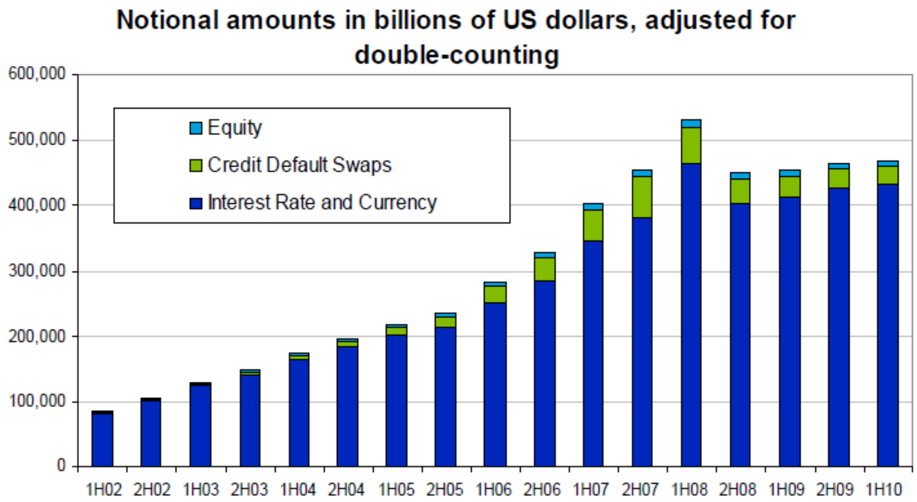

Figure 4.1: Snapshot of the evolution of the volumes of the derivative market.

the *receiver* (while paying the floating rate).
In figure 4.2, a simple scheme of a *receiver* swap.

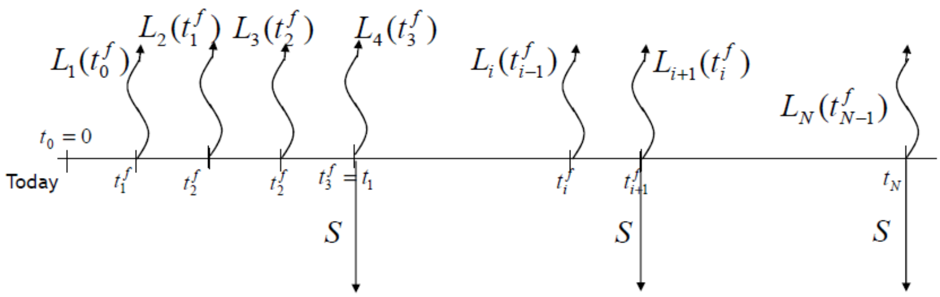

Figure 4.2: Schematization of a plain vanilla IRS.

To determine the raw price of an IRS (prior to any risk charge) one has to price separately the *fixed leg* and the *floating leg*, and then take the sum, namely:

$$V_{fix} = S \sum_{i=1}^{N} \delta(t_{i-1}, t_i) DF(t_0, t_i)) \tag{4.1}$$

$$V float = \sum_{i=1}^{M} L(t_i^s, t_i^f, t_{i+1}^f) \delta(t_{i-1}^f, t_i^f) DF(t_0, t_i^f)) \tag{4.2}$$

$$V_{IRS} = V_{fix} - V_{float} \tag{4.3}$$

where $\delta(t_i, t_j)$ is the year fraction between t_i and t_j in according to a specified day-count convention and $L(t_i^s, t_i, t_j)$ is the forward rate (in our case Euribor) for the period (t_i, t_j) at the time t_s, i.e. the time at which the amount of that floating payment leg is settled.
When the fixed rate S is established so that the value of the swap is zero (i.e. the floating leg and the fixed leg have the same present value), such swap is called *at par*.

4.1.1 The multi-curve approach

The credit crunch crisis started in the second half of 2007 has triggered, among many consequences, the explosion of the basis spreads quoted on the market between single-currency interest rate instruments, swaps in particular, characterized by different underlying rate tenors. In figure 3.1 we show a snapshot of the market quotations as of Dec 30th, 2015 for the some basis swap term structures. As one can see, in the time interval 1Y-20Y the basis spreads are decreasing to around 7-8 basis points. Such very high basis reflect the higher liquidity risk suffered by financial institutions and the corresponding preference for receiving payments with higher frequency (quarterly instead of semi-annually, et cetera.).

There are also other indicators of regime changes in the interest rate markets, such as the divergence between Euribor rates and and Overnight Interest Rates (EONIA based for euro). Indeed, the Libor-OIS spread, once negligible, rose sharply during the crisis, as the Libor quotes began to price a credit risk component. The fact that the Libor-OIS spread was only few basis points before the crisis allowed the dealers to use the Libor curve as proxy of their short-term funding costs, and the theoreticians to use it in valuation models as risk-free rate. But now, this twofold convenience cannot be exploited anymore; all agree that the OIS rate is currently the best proxy for risk-free rate available on the market because of its 1-day tenor.

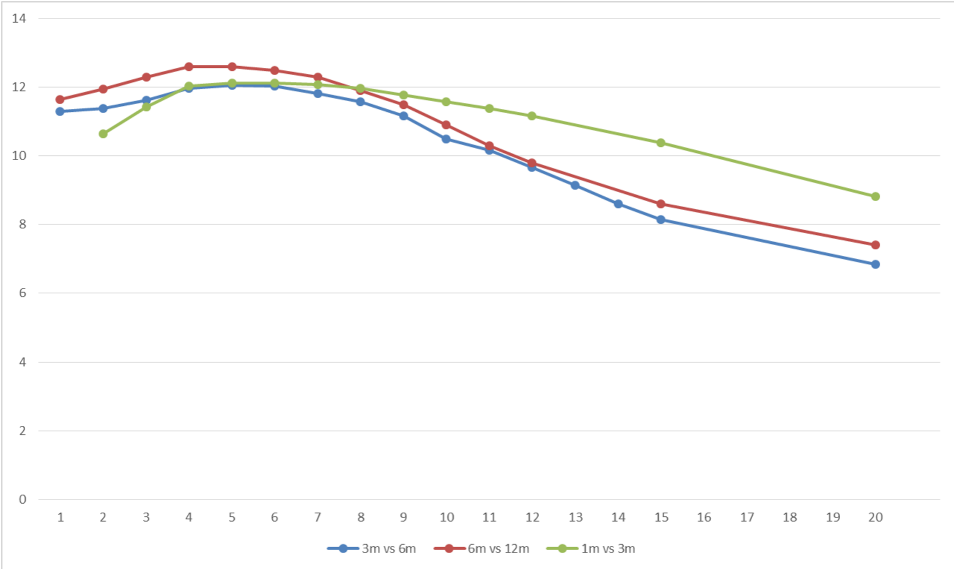

Figure 4.3: quotations (basis points) as of Dec. 30th, 2015 for some EUR basis swap curves. Before the credit crunch of Aug. 2007 the basis spreads were just a few basis points.

The pre-crisis standard market practice for pricing interest rate swaps was based on a single-curve procedure: the same curve simulated to foresee the future floating rates could be used to discount all the cash flows to the valuation time. Unfortunately, due to changes outlined above, the pre-crisis approach is no longer consistent, at least in its simple formulation, with the present market conditions. First, it does not take into account the market information carried by basis swap spreads, now much larger than in the past and no longer negligible. Second, it does not take into account that the interest rate market is segmented into sub-areas corresponding to instruments with distinct underlying rate tenors, characterized, in principle, by different dynamics.

From a modelistic point of view, the simple strategy one can come to mind is to use a interest rate model to simulate both the two curves, possibly considering some correlation between their step-by-step stochastic increments. However, in this case, this strategy can lead to some issue. The most severe problem we would run into is the possibility of EONIA curve overtaking Euribor curve, and this situation would have no sense neither (obviously) has

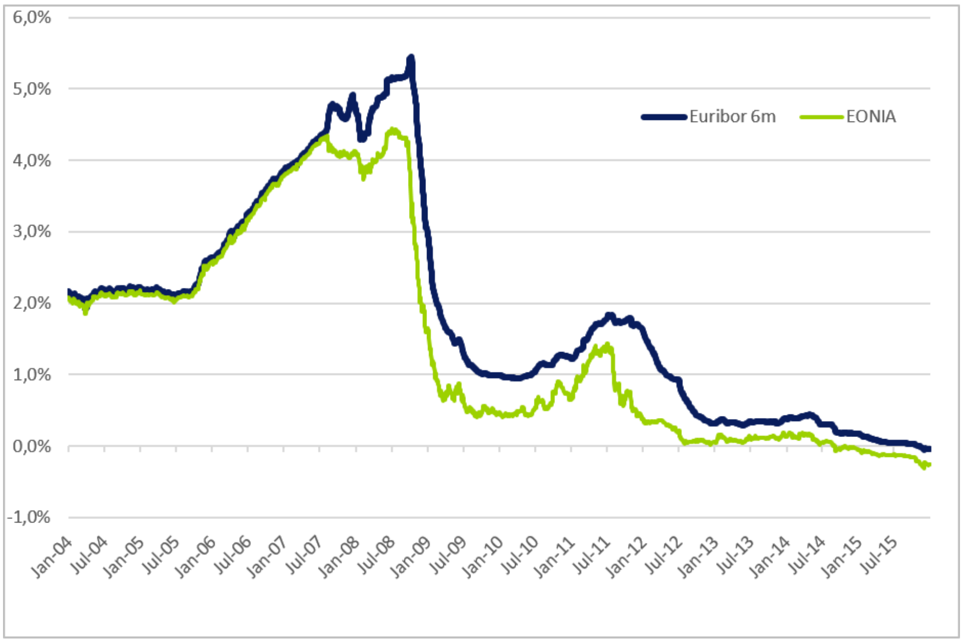

Figure 4.4: Divergence Euribor / EONIA-swap rates: sudden divergence between the 6m Euribor and the 6m EONIA-swap rate that occurred on the first half of Aug 2007.

ever been observed in the market, since as we said before the Euribor rate incorporates a risk premium.

To overcome that obstacle, an elegant solution is to model the evolution Euribor rates using the one-factor Hull&White model, and then, instead of modeling the EONIA rates themselves, EONIA rates are obtained by modelling the evolution of the Euribor-EONIA spread. As the spread should be always a positive quantity (as we said before, OIS rate should be lower than the associated Euribor rate), a straightforward choice is to adopt an *always-positive* Cox-Ingersoll-Ross (CIR) model.
Thereafter, we describe the two models.

4.1.2 Hull&White Model

Most of the theory of interest-rate modeling is based on the assumption of specific dynamics for the instantaneous spot rate process r so that the

Zero Coupon Bond price is equal to

$$P(t,T) = \mathbb{E}_t\{e^{\int_t^T r(s)\,\mathrm{d}s}\} \tag{4.4}$$

Hull and White in 1990 assumed that the instantaneous short-rate process evolves under the risk-neutral measure according to

$$dr(t) = [\theta(t) - a(t)r(t)]dt + \sigma(t)dW(t) \tag{4.5}$$

where θ, a and σ are deterministic functions of time. We used one of the most widely considered assumption, where a and σ are positive constants and θ is chosen so as to exactly fit the term structure of interest rates being currently observed in the market. In fact, as was remarked by Hull and White themselves, the future volatility structures implied by the previous formula are likely to be unrealistic in that they do not conform to typical market shapes. Hence the model (also known as *Extended Vasicek*) becomes

$$dr(t) = [\theta(t) - ar(t)]dt + \sigma dW(t) \tag{4.6}$$

It can be shown that, denoting by $f^M(0,T)$ the market instantaneous forward rate at time 0 for the maturity T, i.e.

$$f^M(0,T) = \frac{\partial lnP^M(0,T)}{\partial T}$$

with $P^M(0,T)$ the market discount factor for the maturity T, we must have

$$\theta(t) = \frac{\partial f^M(0,T)}{\partial T} + af^M(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at}) \tag{4.7}$$

It can be furthermore shown that $r(t)$ conditional on $\mathcal{F}_s$ is normally distributed with mean and variance given respectively by

$$\mathbb{E}[r(t)|\mathcal{F}_s] = r(s)e^{-a(t-s)} + \alpha(t) - \alpha(s)e^{-a(t-s)} \tag{4.8}$$

$$Var[r(t)|\mathcal{F}_s] = \frac{\sigma^2}{2a}\left[1 - e^{-2a(t-s)}\right] \tag{4.9}$$

where $\alpha(t) = f^M(0,t) + \frac{\sigma^2}{2a^2}(1 - e^{-at})^2$

The price at time t of a pure discount bond paying off 1 at time T is given by the expectation (4.4). Such expectation is relatively easy to compute under the dynamics (4.6). Notice indeed that, due to the Gaussian distribution of $r(T)$, conditional on $\mathcal{F}_s$, $t < T$, $\int_t^T r(u)\,\mathrm{d}u$ is itself normally distributed. Precisely it can be shown that

$$\int_t^T r(u)\,\mathrm{d}|\mathcal{F}_t \sim \mathcal{N}\left(B(t,T)[r(t)-\alpha(t)] + ln\frac{P^M(0,t)}{P^M(0,T)} + \frac{1}{2}[V(0,T)-V(0,T)],\, V(t,T)\right)$$

where

$$B(t,T) = \frac{1}{a}\left[1 - e^{a(T-t)}\right]$$

$$V(t,T) = \frac{\sigma^2}{a^2}\left[T - t + \frac{2}{a}e^{-a(T-t)} - \frac{1}{2a}e^{-2a(T-t)} - \frac{3}{2a}\right]$$

We obtain

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)} \tag{4.10}$$

where

$$A(t,T) = \frac{P^M(0,T)}{P^M(0,t)}\left\{B(t,T)f^M(0,t) - \frac{\sigma^2}{4a}(1 - e^{-2at}B(t,T)^2)\right\}$$

4.1.3 CIR Model

This model was introduced by Cox, Ingersoll and Ross in 1985 to meet the then-requirement for a interest rate model to be always-positive. Indeed, a negative interest rate was thought an absurd and illogical scenario at the time.

The model formulation under the risk-neutral measure $\mathbb{Q}$ is

$$dr(t) = k(\theta - r(t))dt + \sigma\sqrt{r(t)}dW(t); \quad r(0) = r_0 \tag{4.11}$$

with r_0, k, θ, σ positive constants. The condition $2k\theta > \sigma^2$ has to be has to be imposed to ensure that the origin is inaccessible to the process (4.11),

so that we can grant that r remains positive.
The solution of the SDE (4.11) is not available in closed form, but its expected value and variance can be calculated, namely

$$\mathbb{E}[r(t)|r_0] = r_0 e^{-kt} + \theta(1 - e^{-kt}) \tag{4.12}$$

$$Var[r(t)|r_0] = r_0 \frac{\sigma^2}{2}(e^{-kt} - e^{-2kt}) + \frac{\theta\sigma^2}{2k}(1 - e^{-kt})^2 \tag{4.13}$$

The fact that $\lim_{t\to\infty} \mathbb{E}[r(t)|r_0] = b$ ensures the *mean-reversion* towards the long-term value b.

4.1.4 Additive Pricing Approach

The simplest way to deal quantitatively with the FVA is as a simple continuation of the classical counterparty risk approach outlined in **Chapter 2**. In [29] FVA is described as the final missing scenario in the CVA/DVA valuation model: given that CVA accounts for the situation in which the counterparty defaults but the bank survives, DVA accounts for the situation in which the counterparty survives but the bank defaults, the scenario in which funding costs and benefits must be considered is when both the bank and the counterparty survive.
The FVA would then end up being the last of the independent *blocks* determining the *total price* of a deal, namely:

$$\hat{V} = V + CVA + DVA + FCA + FBA \tag{4.14}$$

where $\hat{V}$ is the total adjusted price of the deal, V is the raw price without any adjustment, CVA and DVA are Credit and Debt Value adjustments outlined in **2.3.1**, FCA and FBA are the two parts of (3.4), namely:

$$FCA = \sum_{j=1}^{n} EE(t_j)[1-PD_C(0,t_{j-1})][1-PD_B(0,t_{j-1})]FS_b(t_{j-1},t_j)(t_{j-1}-t_j)) \tag{4.15}$$

$$FBA = \sum_{j=1}^{n} NEE(t_j)[1-PD_C(0,t_{j-1})][1-PD_B(0,t_{j-1})]FS_l(t_{j-1},t_j)(t_{j-1}-t_j)) \tag{4.16}$$

where the interpretation of the terms are the same as in (3.4).
According to this approach, the building blocks are independent one from the other, so we can simply follow the procedure commonly used to compute the "classical" counterparty risk measures. And then we can extend the ordinary CVA and DVA computation with FVA as of the formulae (4.15) and (4.16) ($FVA = FCA + FBA$). Finally, we can add all the terms together to produce the xVA-adjusted price.
Using a Monte Carlo method we simulate a large number of scenarios for the underlying risk factors (the two curves as previously explained). We discretize the time-span of the life of the deals into a set of *grid points*: for each scenario we compute the raw value of the deal un every grid point, via a discounted sum of the following cash flows. Obviously the fixed-leg cash flows are all deterministically equal, while to foresee the floating-leg ones we had to determine the forward rates for the payment times at the settlement times (usually the amount of the floating-leg are settled at the time of the previous cash flow).
With a large number of Mark-to-Market profiles, we can uses the formulae in **2.2.2** to determine an exposure profile, the first brick to compute all the xVA add-ons as of this approach.

4.1.5 Recursive Pricing Approach

The additive pricing approach outlined in the previous chapter is simple and straightforward, but its oversimplification can lead to unrealistic results. According to Brigo et al. [15], valuation under funding risk poses a significantly more complex and computationally demanding problem than standard CVA and DVA computations, because FVA does not take the form of a simple additive term as appears to be commonly assumed by market participants.
In that work, they develop a recursive general approach on pricing with collateralization, debit, credit and funding valuation adjustments. They then tailor their method to price a *european option*, instead, we adapt it for dealing with IRS.

Here below, we describe the general framework.

Theoretic Framework

Let $T \in \mathbb{R}$ be the time horizon of a deal and $(\Omega, \mathcal{G}, (\mathcal{G}_t)_{t\in[0;T]}, \mathbb{Q})$ be the probability space on which we define our risk neutral measure, being $\mathbb{Q}$ the risk-free probability measure. The filtration $(\mathcal{G}_t)_{t\in[0;T]}$ is generated both by all the financial-market stochastic processes, including credit, and represents the flow of information of the whole market.

We adopt the notational convention that $\mathbb{E}_t$ is the risk-neutral expectation conditional on the information $\mathcal{G}_t$. In accordance with the classical risk-neutral pricing methodology, we can outline a derivatives deal as a set of non-deterministic cash flows between a banking institution ("B"), on which we set our point of view, and a counterparty ("C"). We denote by $\pi(t, T)$ the sum of all the discounted cash flows happening over the time period $(t, T]$. The classical derivative pricing methods suggest to simply take the risk-neutral expected value of the discounted cash flows. Namely, the price would be given by:

$$V_t = \mathbb{E}_t[\pi(t, T)]$$

Then we have to take into account the default events: one thing we learned with the financial crisis is that *too big to fall* idea was simply an illusion. Let τ_B and τ_C being the default times of the investor and the counterparty respectively, both $\mathcal{G}$ -stopping times by how $\mathcal{G}$ is defined above. Then, we define the time of the first default event between the two parties as $\tau = \tau_B \wedge \tau_C$, and the price becomes:

$$V_t = \mathbb{E}_t[\pi(t, T \wedge \tau)]$$

Present day market practice require to take account also of cash flows other than the ones directly linked with the trade. That is:

1. $\gamma(t, T; C)$: The sum of the discounted cash flows due to collateral margining over the period $(t, T]$, with C being the collateral account.

2. $\theta_\tau(C;\epsilon)$: Cash flows happening in case of a default event occurs. ϵ is the residual value of the deal at the time of default.

3. $\phi(t,T;F)$: Lastly, the sum of the discounted cash flows required to fund the deal, being F the cash account used.

Taking into account the above-listed terms, we can express the price of a derivative deal as:

$$\hat{V}_t(C,F) = \mathbb{E}_t[\pi(t,T\wedge\tau)] + \gamma(t,T\wedge\tau;C) + \mathbb{1}_{\{t<\tau<T\}}DF(t,\tau)\theta_\tau(C;\epsilon)+ \\ +\phi(t,T\wedge\tau;F)] \tag{4.17}$$

where:

- $\mathbb{1}_{\{t_i<\tau<t_j\}}$ is the default event indicator function, i.e. values 1 if a default event occurs between the times t_i and t_j, 0 otherwise.
- $DF(t_i,t_j)$ is the risk-free discount factor between the times t_i and t_j

Being this a general framework, the most general form is acceptable for the close-out amount (in our case study we will consider it deterministic) and the collateralization (which we will nullify, since in our case study we are dealing with uncollateralized corporate counterparties).
A general hedging strategy to replicate a derivative is formed by a position in cash and a position in a portfolio of hedging instruments. If we denote the cash account by F and the risky-asset account by H, we get

$$\hat{V}_t = F_t + H_t$$

hence:

$$F_t = \hat{V}_t - H_t \tag{4.18}$$

It is obvious that if the funding account $F_t > 0$, the dealer needs to borrow cash to establish the hedging strategy at time t. Correspondingly, if the funding account $F_t < 0$, the hedging strategy requires the dealer to invest surplus cash. Specifically, we assume the dealer enters a funding position on a discrete time-grid $\{t_1, ..., t_m\}$ during the life of the deal. Given two adjacent funding times t_j and t_{j+1}, for $1 \leq j \leq m-1$, the dealer enters a position in cash equal to F_{t_j} at time t_j . At time t_{j+1} the dealer redeems the position again and either returns the cash to the funder if it was a long cash position and pays funding costs on the borrowed cash, or he gets the cash back if it was a short cash position and receives funding benefits as interest on the invested cash. We assume that these funding costs and benefits are determined at the start date of each funding period and charged at the end of the period. Let $P_t^{f^+}(T)$ represent the price of a borrowing contract measurable at t where the dealer pays one unit of cash at maturity $T > t$, and let $P_t^{f^-}(T)$ be the price of a lending contract where the dealer receives one unit of cash at maturity. Moreover, the corresponding accrual rates are given by

$$f_t^{\pm}(T) \hat{=} \frac{1}{T-t}\left(\frac{1}{P_t^{f^{\pm}}(T)} - 1\right)$$

In other words, if the hedging strategy of the deal requires borrowing cash, this can be done at the funding rate f^+, while surplus cash can be invested at the lending rate f^-. We define the effective funding rate $\tilde{f}_t$ faced by the dealer as

$$\tilde{f}_t(T) = \hat{=} f_t^-(T)\mathbb{1}_{\{F_t<0\}} + f_t^+(T)\mathbb{1}_{\{F_t>0\}} \tag{4.19}$$

Following this, the sum of discounted cash-flows from funding the hedging strategy during the life of the deal is equal to

$$\phi(t, T; F) = \sum_{i=1}^{m-1} \mathbb{1}_{\{t \leq t_j < T \wedge \tau\}} DF(t, t_j) F_{t_j} \left(1 - \frac{P_{t_j}(t_{j+1})}{P_{t_j}^{\tilde{f}}(t_{j+1})}\right) \tag{4.20}$$

where the zero-coupon bond corresponding to the effective funding rate is defined as $P_t^{\tilde{f}}(T) \hat{=} [1 + (T-t)\tilde{f}_t(T)]^{-1}$. This is, strictly speaking, a discounted payout and the funding cost or benefit at time t is obtained by taking the risk neutral expectation of the above cash-flows.

If we adopt a first order expansion (for small f and r) we can approximate

$$\phi(t, T; F) \approx \sum_{i=1}^{m-1} \mathbb{1}_{\{t \leq t_j < T \wedge \tau\}} DF(t, t_j) F_{t_j} \alpha_j \left(r_{t_j}(t_{j+1}) - \tilde{f}_{t_j}(t_{j+1})\right) \tag{4.21}$$

where with a slight abuse of notation we call $r_{t_j}(T)$ and $\tilde{f}_{t_j}(T)$ the continuously (as opposed to simple) compounded interest rates associated with the bonds P and $P^{\tilde{f}}$.
The particular positions entered by the dealer to either borrow or invest cash according to the sign and size of the funding account depend on the bank's liquidity policy.

Brigo et al. in [15] characterise more deeply the term $\gamma(t, T \wedge \tau; C)$ in (4.17), but for our purposes this term is zero as we are considering uncollateralized corporate deals. Therefore, no further efforts will be done to try to characterize this term in the general situation.
One point to remark is that while the pricing equation (4.17) is conceptually clear - we simply take the expectation of the sum of all discounted cash-flows of the deal under the risk-neutral measure - solving the equation poses a recursive, non-linear problem. The future paths of the effective funding rate $\tilde{f}$ depend on the future signs of the funding account F, i.e. whether we need to borrow or lend cash on each future funding date. At the same time, through the relations (4.18), the future sign and size of the funding account F depend on the adjusted price $\hat{V}$ of the deal which is the quantity we are trying to compute in the first place. One crucial implication of this recursive structure of the pricing problem is the fact that FVA is generally not just an additive adjustment term, in contrast to CVA and DVA. More importantly, the conjecture identifying the DVA of a deal with its funding is not appropriate in general. Only in the unrealistic setting where the dealer can fund an uncollateralized trade at equal borrowing and lending rates, i.e. $f^+ = f^-$, do we achieve the additive structure often assumed by practitioners.

The key now is trying to to turn the recursive pricing equation (4.17) into a set of iterative equations that can be solved by least-squares Monte Carlo methods. Introducing the auxiliary term

$$\tilde{\pi}(t_j, t_{j+1}) \hat{=} \pi(t_j, t_{j+1} \wedge \tau) + \mathbb{1}_{\{t_j < \tau < t_{j+1}\}} D(t_j, \tau) \theta_\tau(\epsilon)$$

which defines the cash-flows of the deal occurring between time t_j and t_{j+1} (if present collateral-margining cash flows should also be insterted). If we then solve pricing equation (4.17) at each funding date t_j in the time-grid $\{t_1, ..., t_m\}$, we obtain the deal price $\hat{V}$ at time t_j as a function of the deal price on the next consecutive funding date t_{j+1}

$$\hat{V}_{t_j} = \mathbb{E}_t[\hat{V_{t_{j+1}}} D(t_j, t_{j+1}) + \tilde{\pi}(t_j, t_{j+1})] + \mathbb{1}_{\{\tau > t_j\}} F_{t_j} \left(1 - \frac{P_{t_j}(t_{j+1})}{P^{\tilde{f}}_{t_j}(t_{j+1})}\right) \tag{4.22}$$

starting from $\hat{V}_{t_m} = 0$.

It can be shown (see [15]) that this recursive scheme can be solved as a set of *backward-iterative* equations on the time-grid $\{t_1, ..., t_m = T\}$. Namely, we have

$$\begin{cases} \hat{V}_{t_j} = 0 & \tau < t_j \\ (\hat{V}_{t_j} - H_{t_j}) = P^{\tilde{f}}_{t_j}(t_{j+1}) \left(\mathbb{E}^{t_{j+1}}_{t_j} \left[\hat{V}_{t_{j+1}} + \dfrac{\tilde{\pi}(t_j, t_{j+1}) - H_{t_j}}{D(t_j, t_{j+1})} \right] \right) & \tau > t_j \end{cases} \tag{4.23}$$

where $\mathbb{E}^{t_{j+1}}_{t_j}$ denotes the expectation taken under the $\mathbb{Q}^{t_{j+1}}$-forward measure.

Montecarlo Simulation

To numerically study this valuation framework a way is a least-square montecarlo algorithm similar to the one proposed by Longstaff and Schwartz in [34] to tackle American options.

We consider a certain number M of scenarios for the underlying risk factors and for the default times. Given the set of simulated paths, we solve the funding strategy recursively in a dynamic programming fashion. Starting one period before T, we compute for each simulated path the funding decision F and the deal price $\hat{V}$ according to the set of backward-inductive equations(4.23). The algorithm then proceeds recursively until time zero. Ultimately it is computed the average of the individual results obtained in each scenario.

The conditional expectations in the backward-inductive funding equations are approximated by across-path regressions based on least squares estimation similar to Longstaff and Schwartz. We regress the present value of the deal price at time t_{j+1}, the adjusted payout cash flow between t_j and t_{j+1}, the collateral account and funding account at time t_j on basis functions ψ of realizations of the underlying risk factors at time t_j across the simulated paths. Namely, the conditional expectations in the iterative equations (4.23), taken under the risk-neutral measure, are equal to

$$\mathbb{E}_{t_j}[\Xi_{t_j}(\hat{V}_{t_{j+1}})] = \beta'_{t_j}\psi(X_{t_j}) \tag{4.24}$$

where we have defined $\Xi_{t_j}(\hat{V}_{t_{j+1}}) \hat{=} D(t_j, t_{j+1})\hat{V}_{t_{j+1}} + \tilde{\pi}(t_j, t_{j+1}) - H_{t_j}$ where β_{t_j} is the least-square estimator computed at each time step. Literature suggest that quadratic polynomials are usually sufficient in these computations, i.e. $\psi(X_{t_j}) = (1, X_{t_j}, X_{t_j})'$.
We obtain, the following system of equations

$$\begin{cases} F_{t_j} - \dfrac{P^{\tilde{f}}_{t_j}(t_{j+1})}{P_{t_j}(t_{j+1})}\mathbb{E}_{t_j}[\Xi_{t_j}(\hat{V}_{t_{j+1}})] = 0 \\ F_t - \hat{V}_t + H_t = 0 \end{cases} \tag{4.25}$$

Each period and for each simulated path, we find the funding requirements and the contract value by solving this system of equations, given the funding and value for all future periods until the end of the deal. Iterating the procedure to *time zero*, we can eventually find the value of the deal.

4.2 Case Studies

To put in practice the above-described methodologies, we set up a series of case studies. In every case study we adopt the point of view of an italian leading bank (Intesa Sanpaolo) entering into uncollateralized transactions with corporate counterparties and hedging those transactions in banks under CSA agreements, but assuming perfect collateralization (i.e with a continuous-time margining procedure). This latter is obviously an approximation, since in a real scenario this operation is done in blocks, however, it is an acceptable approximation, given the usual daily basis on which collateral is posted as today's market practice. The corporates considered are ENEL (energy industry) and Telecom Italia (telecommunications industry). The reference date of our analysis is **30 Dec. 2015**.

Model Calibration In this thesis we take for granted all model calibrations, which were done through specific industrial softwares fed by the market data.

Data We obtained all needed market data from Bloomberg®, observed as of December 30, 2015. First, we obtained the EONIA curve, which represents the market discount, and the 6 month Euribor curve, which is used to determine the floating-leg cash flows of all our swaps. Thereby, we use historical EONIA spot rates and 6 month Euribor spot rates to model the Euribor-EONIA spread. The historical rates are observed between August 1, 2007 and December 30, 2016. Besides, to bootstrap market implied survival and default probabilities, we obtained senior CDS spread quotes for all for all the considered firms.

Default concerns We bootstrap the default probability curves from the CDS prices obtaining values for maturities greater than 10 years (CDS are generally quoted only up to 10 year) via a exponential-distribution extrapolation method. Given those cdf-functions, we are able both to calculate every $PD(t_i, t_j)$ and to create random default times.

4.2.1 Approaches

We firstly adopted the simpler *building blocks* approach. We consider our bank entering into two separate IRS with the two above mentioned corporate counterparties. The swap features are summarized in table 4.2.1 here below.

The fixed rate is chosen so that both the swaps are roughly at par. The choice of "opposite sign" trades allows a further investigation, regarding the rehypothecation of collateral: we want to quantify to which extent the allowance to finance the funding needs with funding surpluses (i.e. offsetting received and posted collateral at a portfolio level) can reduce the overall funding costs.

	1	2
Counterparty	Enel	Telecom Italia
Notional amount	€100	€100
Maturity	10y	20y
Fixed leg tenor	1y	1y
Fixed rate	1%	1.5%
Floating leg tenor	6m	6m
Floating leg	Euribor 6m	Euribor 6m
Swap typology	Payer	Receiver
Day count convention	30/360	30/360

Table 4.1: Swaps features

We simulate **5000** montecarlo scenarios for the risk factors involved (the accruing and discount curves), then, following the models and method in section **4.1**, we compute the present value of each swap on a non-uniform time grid (it is a common practice to thin out grid points more and more going further from *time zero*, to maximize the accuracy in the near future). Once estimated the future exposures, we apply the formulae provided in **4.1.4** to come up with the FVA.

The second thing we want to measure is the approximation one incurs in when computing FVA with a simple linear approach versus a more realistic (yet bundersome) recursive approach as the one displayed in **4.1.5**. This last approach, though, is much more complicated and the least-square montecarlo algorithm involved is over **three times** computationally heavier. Moreover, if the FVA has to be computed for more then one value of the funding spreads, the advantage of the linear approaches increases even more: following the linear approach, the exposures have to be computed only one time (starting from which the FVA can be obtained in a blink of an eye for any value of the funding spreads), while following the recursive approach the computation has to be restarted from scratch every time. Therefore its adoption would be justified only in case it produces much more accurate results.
For sake of simplicity, this analysis is performed only with one counterparty

and one deal, hence we can refer to the first column of table **??** with the typology switched from *Payer* to *Receiver*.

4.3 Results

In this chapter we document the results we obtain in our case studies by applying the described methodologies.
In figure 4.5 and figure 4.6 we plot the exposure profiles of the two counterparties. As we expected, the exposures have "specular" shapes.

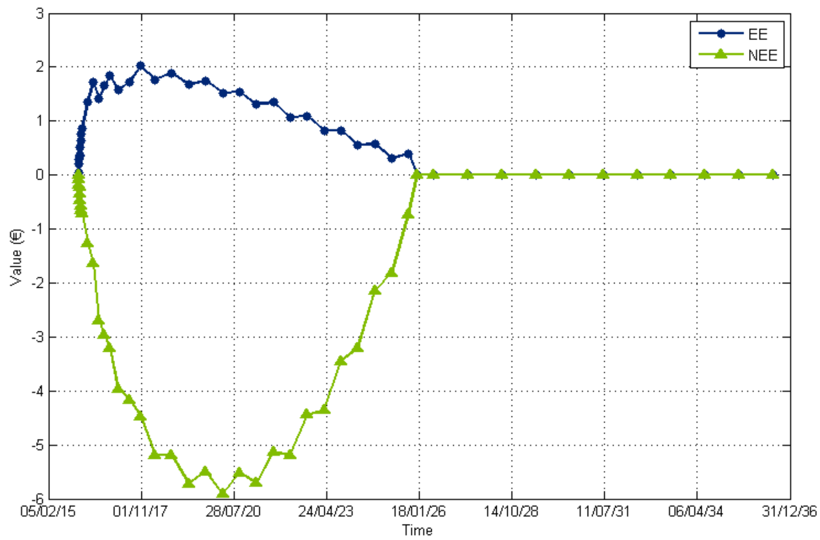

Figure 4.5: Positive and negative exposure towards **counterparty 1**

In the most general situation, the funding rates are not necessarily equal and the same for the funding spreads (i.e. $f^+ \neq f^- \implies FS_b \neq FS_l$). Therefore, in general, FVA has two "degrees of freedom". In the very special case in which not only the funding rates are symmetric (i.e. one can lend and borrow money at the same rate), but they are equal to the risk-free rate, the FVA is obviously null. Indeed, in that case margin calls (which are usually remunerated at EONIA rate) can be financed simply borrowing money at the same rate.

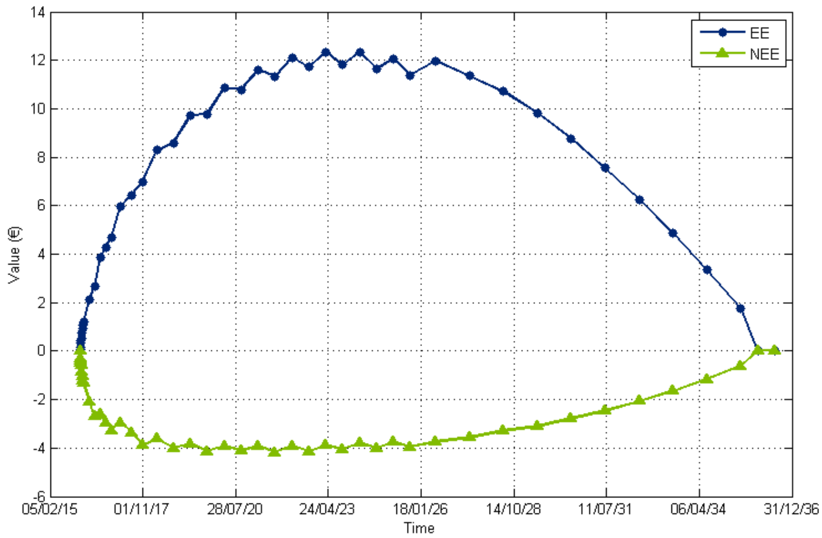

Figure 4.6: Positive and negative exposure towards **counterparty 2**

In figure 4.7, to highlight more the costs than the benefits of funding, we plot the values of FVA charges in case of asymmetric (and flat) funding spread, as function of the borrowing spread, keeping frozen the lending spread to EONIA rate.

Assuming no rehypothecation of collateral we end up with a total FVA which is simply the sum of the ones computed for the two counterparties. In figure 4.8 we plot the total FVA in case of rehypothecation allowance and total FVA in case of no rehypothecation allowance.

As we can see, the FVA is reduced by roughly half, given the fact that the bank can freely use the collateral received for its funding purposes, hence is the total portfolio exposure, which we can see in figure 4.9, that matters.

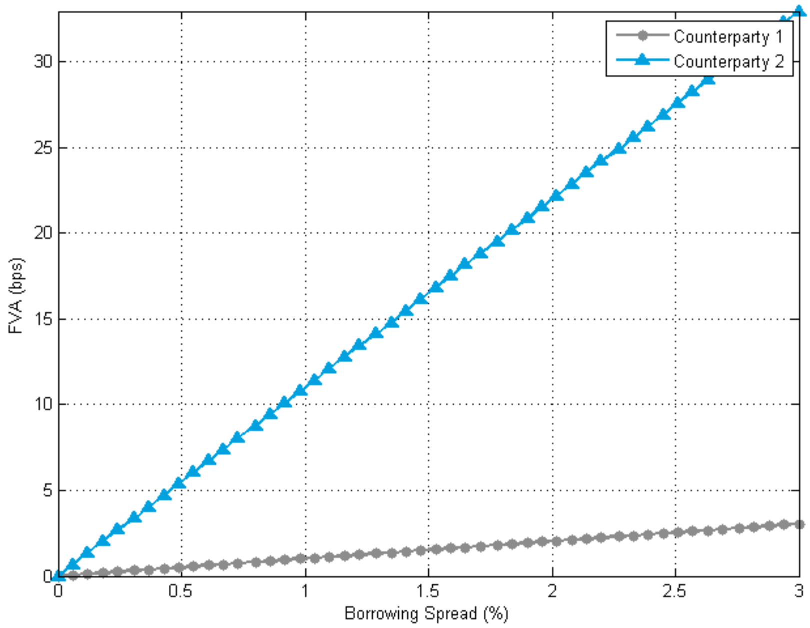

Figure 4.7: FVA computed for the two individual counterparties, as a function of asymmetric funding spread $FS_b = f^+ - EONIA$, i.e. keeping frozen the lending rate $f^- = EONIA$ and varying $FS_b \in [0, 300bps]$

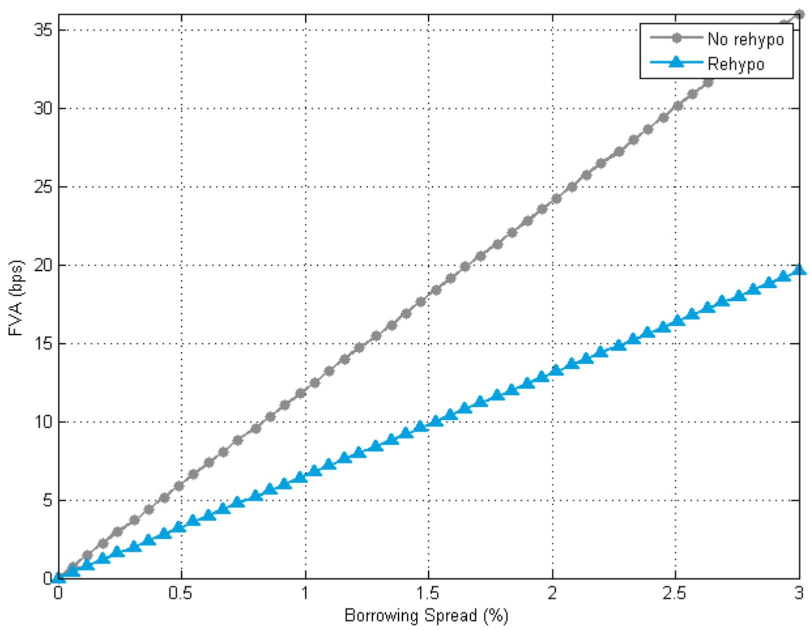

Figure 4.8: FVA at bank-level, allowing and not allowing rehypothecation of collateral. As in figure 4.7 we keep frozen $f^- = EONIA$ and let FS_b vary

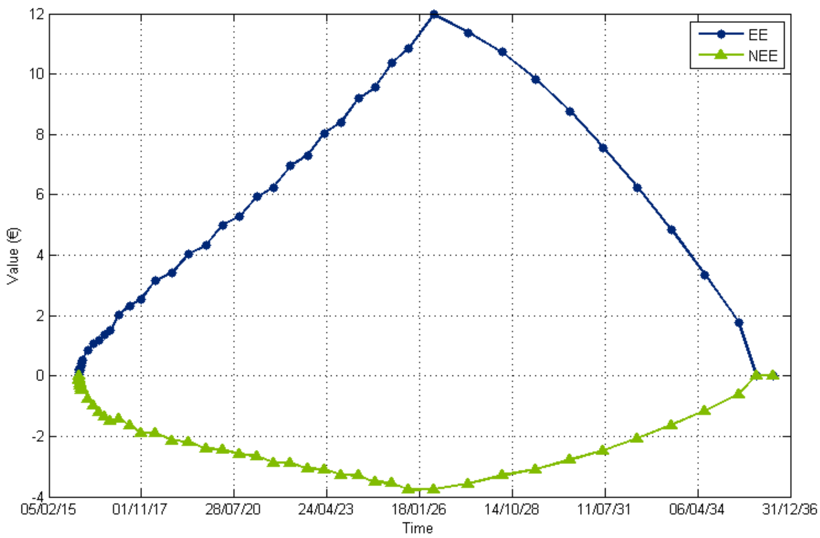

Figure 4.9: Positive and negative exposures of the whole portfolio

Nevertheless, this result is in some sense worrying: while the deal-wise FVA computation is light, to perform it at bank-level all the deal interrelations within the portfolio have to be taken into account, and every time a new deal is entered into, the computation has to be reperformed from scratch. This makes this kind of computation probably unbearable in industry, where porfolios usually contain enormous amounts of deals.

As far as the recursive approach is concerned, in figure 4.10 we show the impact of FVA (defined as the difference between the full funding-inclusive deal price and the full deal price with symmetric funding rates equal to the risk-free rate) as a function of asymmetric funding spread, plotted against the same quantity computed with the linear approach. As we can see the values are very close (at least for realistic ranges of the funding spread), and the nonlinearity is barely noticeable. Therefore, even though the FVA theoretically would not be a simple additive term, the small error we incur in probably does not justify the implementation of such a cumbersome recursive method.

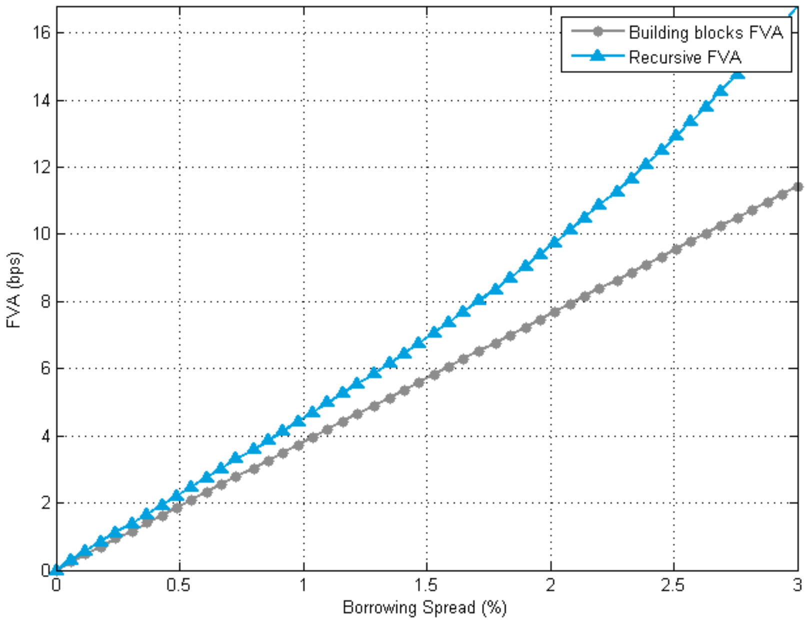

Figure 4.10: FVA for **counterparty 1** swap, computed via *building blocks* approach and *recursive* approach. As before we let vary only FS_b keeping f^- frozen

Chapter 5

Conclusions and Further Developments

The aim of this thesis was to conduct an analysis of FVA debate, from different points of view, giving particular focus to its methodological aspects. We started setting the essence of the problem and the circumstances from which it comes from. Then we provided the variety of points of view that academics and practitioner have been offering for the last years: someone rejects the issue at all, someone provides simple linear models and someone more intricate ones. The FVA issue seems very delicate, giving the magnitude of impacts coming from its introduction in "real world" valuation. And, giving the magnitude of the figures the reception of FVA in "real world" is responsible which, the issue seems very delicate.
Therefore we set up a simple case study to numerically compare two "methodological schools" emerged throughout the debate. We first introduced the basic assumption we would make in the computation. We preferred to include IRS-only portfolios, to focus more on the FVA and not on the pricing issues we would have incurred dealing with more exotic derivatives; we decided to work in a multi-curve framework since the market best practice require it; we assumed a *perfect* back-to-back collateralization, to approximate the daily-basis one (a perfect correlation in the real world is possible only when dealing with CCs, but it is not our case). As far as the model are concerned, we adopted a one-factor Hull&White to simulate the accru-

ing rate (Euribor), and a Cox-Ingersoll-Ross model for the always-positive Euribor-EONIA spread. To simulate the discount scenarios and to bootstrap default probabilities we started with CDS market prices, as the majority of institutions does; in this operation we omitted the *wrong way risk* because the effort needed to take account of it would have taken the thesis off-course. Having set the premises, we moved to the actual FVA computations. Our core analysis consisted in determining whether the implementation of a complex and bundersome methodology is justified or not given the increase in accuracy against a simple linear one. We found a few-bps discrepancy between the two methodologies which suggested that, in our study, a non-linear approach is not justified. There is however the possibility that, adding to the portfolio derivative instruments other than IRS (e.g, cap&floors, options et cetera), the outcome might be different and the adoption of a recursive method justified. Moreover we wanted to estimate to which extent the rehypothecation of collateral can influence the overall FVA magnitude. We found that, with two "opposite sign" swaps partially offsetting received and posted collateral, the total FVA is reduced by more than half. This depends less on the derivarive products and more on exposures offsettability, so in a real bank porfolio with a large number of trades of all kind and moneyness, the rehypothecation of collateral can have a great impact.
Anyway, while the impact of collateral rehypothecation on the FVA magnitude had already been pointed out (for example in C. Albanese and L. Andersen [2]), the main result of this thesis is the fact that, at least in our simplified case, the nonlinearities in FVA calculation are almost negligible.
The proposed framework can be applied, with some arrangements, to many different derivative products. Moreover, there are great opportunities to extend the modeling framework in future research. The academic field of FVA is far from being clean-cut, hence plenty of extensions are possible. For example, one can model the dependence structure between funding spreads and credit spreads (i.e. wrong-way funding risk). Then, a correlation between the treasury's internal lending rate and the bank's credit spread can be considered: does the treasury's internal lending rate rises as the creditworthiness of the bank weakens?

To make things even more complicated, another xVA is emerging: recent changes in the regulatory regime and the increases in regulatory capital requirements have led to theorize the KVA (Capital Value Adjustment) to account for the economic capital a trade consumes over its lifetime. It is not certain KVA will follow the same path of CVA and FVA. It's a safe bet, though, that as capital costs rise, banks will spend more time trying to perfect it.

Yet, this long journey has not reached its destination, since integrating funding costs into the pricing equations leads to a side effect that has relevant consequences for the very notion of "price". Indeed, the funding-inclusive price is different for each institution, since each institution has different funding and investing rates depending on its own funding liquidity policy. Even inside the same bank, the treasury and the trading desk may be applying the equations with different inputs. Even collateralized deals, which have an accrual rate defined by the CSA contract, do not have a unique price, since the underlying risk factors grow at a funding rate that can be different for different calculating parties. Hence an agreement will be reached through negotiation and perhaps an equilibrium approach should be adopted to frame part of the funding costs problem in order to compute the price at which the deal will be actually closed among parties, but this would go beyond the scope of this thesis.

Chapter 6

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****

Terminati i formali ringraziamenti di protocollo, è usanza comune dedicare un piccolo spazio a ringraziamenti più calorosi e informali. Spesso però, per quello che ho potuto leggere spulciando alcune tesi qua e là, questo si traduce in profluvi di dediche melense ad amici e semplici conoscenti, agiografie imbarazzanti di genitori e parenti tutti, ringraziamenti posticci a personaggi randomici, ed altri salamelecchi strappalacrime al cui confronto il libro Cuore sembra la guida Michelin. Ben lungi dal voler canzonare i sentimenti di chicchessia o ergermi a censore degli stessi, tutto questo miele non ha fatto altro che corroborare il mio sopito proposito di tenere pulita questa sezione da invereconde accozzaglie di banalità, lasciandola invece asetticamente focalizzata sulle dovute menzioni a coloro che hanno fattivamente contribuito alla stesura di questo lavoro di tesi. Sperando che nessuno se ne abbia a male, dedico comunque un ringraziamento sui generis allo sparuto lettore capitato tra queste righe.