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Master of Science in Energy Engineering

"Development and application of an importance measure for predictive maintenance in oil and gas offshore installations"

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Abstract:

A Prognostics and Health Management System (PHMS) provides information on the degradation state of an industrial component and the prediction of its Remaining Useful Life (RUL). In this thesis work, we consider a complex industrial plant made by several components which can be in multiple states. The main objectives are:

1) To propose a method to identify which components should be equipped with a Prognostics and Health Management System;
2) To obtain the best impact on the overall plant performance and to quantify the expected impact.

To this aim, we proposed a modification of the risk achievement worth importance measure which allows prioritizing components with respect to the expected benefits of equipping them with a PHMS. The computation of the proposed importance measure has required:

1) The definition of a novel method based on the simulation of the Prognostics and Health Management System predicting Remaining Useful Life and characterized by a given performance for estimating the average component availability;
2) The embedding of the method within a Monte Carlo simulation approach for the computation of the overall plant performance

An application to an oil and gas offshore installation is proposed. The components are prioritized according to the expected benefits of equipping them with a Prognostic and Health Management System taking into account variabilities in Prognostic and Health Management System performance. Furthermore, the benefits of equipping a component with a Prognostic and Health Management System in term of overall plant performance is discussed.

Symbols and acronyms:

RUL	residual useful life
RUL^*	ground truth residual useful life
PHMS	prognostics and health management system
n	number of components within the system
i	refers to i-th component
N	number of production levels of the plant
RUL^*	ground truth RUL
γ	PHM predicted RUL
T_{start}	time at which the component begins operation
T_f	time at which the component fails
$\lambda = \frac{T-T_{start}}{T_f-T_{start}}$	component specific time or time window modifier.
$\hat{\gamma}_\lambda$	mean value of PHM predicted RUL at specific time λ
FP	false positive
$E[\underline{RA_\lambda}]$	mean value of the relative accuracy in the case that the PHMS under-estimates the ground truth of RUL
$E[\overline{RA_\lambda}]$	mean value of the relative accuracy in case of PHMS over-estimates the RUL
$R_s(t)$	system reliability
$R_i(t)$	component i reliability
U	unavailability
$\overline{U_\iota}^{\leq\alpha}(W^*)$	mean unavailability of the multi-state system when the performance of i-th component is restricted to be below or equal to α in t ∈ $[0, \tau]$
$\overline{U_\iota}^{>\alpha}(W^*)$	mean unavailability of the multi-state system when performance of i-th component is restricted to be above to α in t ∈ $[0, \tau]$.
$\overline{W_\iota}^{\leq\alpha}$	mean performance of the multi-state system when the performance of i-th component is restricted to be below or equal to α in t ∈ $[0, \tau]$.

$\overline{W_i}^{>\alpha}$ mean performance of the multi-state system when performance of i-th component is restricted to be above to α in t $\in [0, \tau]$.

$MTTR(i)$ mean time to repair for component i where it is not equipped with PHMS.

X instantaneous degradation level as a percentage ratio to the failure time degradation level of the component

$\hat{\gamma}_\lambda$ mean value of PHM predicted RUL

$\lambda_{predicted}$ predicted specific time

$(1 - \lambda_h)$ predictive maintenance specific time threshold

Chapter (1) Introduction:

Offshore drilling is the leading extraction method for oil and gas which are the main resources to fulfill global energy demand. Since year 2000, contribution of offshore production to global oil and gas production has been 30% and 27% respectively [1]. Since 1950, the average water depth of offshore oil and gas platforms increased from 650 ft to 13000 ft, causing logistic and technical challenges in operating these platforms. In practical, plant components are forced to work in harsh environment conditions which accelerate the component degradation and increase the failure probabilities. Such challenges not only increase the economic burden in terms of capital investment and operational costs but also increase the associated risk and the complexity of decision making for routine operations such as those performed to keep equipment, machinery, and supporting utilities in operable condition and to prevent breakdowns, or to restore the plant in operating condition in case of failures. Such operations are called maintenance and can be broadly classified into two categories - preventive maintenance and corrective maintenance. Preventive maintenance can be further classified into two categories, condition-based predictive maintenance and periodic-based predictive maintenance. Achieving a high plant availability requires a well-designed maintenance process. An ill-planned maintenance strategy can lead to frequent outages or breakdowns and can have a severe impact on the plant profitability. Periodic-based predictive maintenance is widely used for industrial systems characterized by high levels of risk and complexity like oil and gas plants. Although periodic-base predictive maintenance can be effective in decreasing the risk of unexpected failure, it leads to increase the level of unnecessarily maintenance actions and interventions and in [2], it is shown that moving from corrective maintenance policy to periodic-based predictive maintenance policy leads to a reduction of average plant production of oil and gas. Decrease the probabilities of unexpected failure, decrease level of the risk, decrease the unnecessarily maintenance actions and enhance components availability and system performance are some of the reasons to move from corrective maintenance and periodic maintenance to condition-based predictive maintenance.

It should be noticed that, although, preventive maintenance helps prevent multiple failures, it might lead to increased unavailability due to frequent interventions, if not properly planned.

Condition monitoring is now increasingly employed to supplement maintenance decisions, minimize system downtimes and maximize the process efficiency.

The main advantages from moving to preventive maintenance strategy can be defined as:

- Reduce number of failures (but number of organized stops for maintenance will increase)
- Decreasing the repair time which lead to decrease unavailability of the component
- Lower maintenance cost
- Less spare parts necessary
- With decreasing unavailability of the component, availability of the system will increase and the average useful life time
- And with reducing number of failure, level of safety of the system will be increased

Offshore platforms, particularly those in harsh environmental conditions or in remote locations, can lead to an increased difficulty in maintenance operations due to transportation challenges. A corrective maintenance strategy in such plants can have a severe economic impact. As offshore plants have little spare inventories, a breakdown can lead to large costs incurred not only in bringing in new parts but also in terms of the time that the platform remains out of service. A preventive maintenance strategy coupled with condition monitoring becomes even more impactful in such cases [3].

The effect of condition-based predictive maintenance on component availability were studied in [4] and [5]. Although, [4] and [5] show that condition-based predictive maintenance is able to increase system availability and decrease system down-time. in both [4] and [5], it was assumed that the condition of the system, degradation level, at time t can be summarized by a scaler aging variable X_t which increases as the system deteriorate. From a practical point of view, the previous assumption can't be applied easily as the degradation state of the component can be more complicated to be represented in a single number depending on a single signal from one of the sensors installed on it. As an example, for a gas turbine, detecting an abnormal condition related to increase of exhaust gases temperature isn't enough to perform a diagnosis or detect the degradation level as it could be because of failure of fuel control valve or a blow-off valve on the compression stage. A Prognostics and Health Management System (PHMS) is needed to aggregate and process all the signals from all the sensors installed on the component to perform a component diagnosis and prognosis. Degradation level can be expressed using PHMS output which is residual useful life (RUL). A model able to describe the effect of equipping the component with PHMS on its availability is introduced in [6]. Although the model in [6] was able to evaluate the effect of equipping the component with a PHMS considering a specific maintenance rule, it is not applicable in general cases wherein predictive maintenance is performed based on degradation level, or state, of the component.

The first novelty in this thesis work is modifying the model in [6] and develop a model able to evaluate the effect of equipping the component with a PHMS considering a general maintenance policy based on degradation level of the component which can be represented or described using the PHMS outputs.

Allocating scarce resources and investments creates a need to prioritize the components within the system with respect to the benefits resulting from improvement activities and moving to condition-based predictive maintenance is one of those activities. Because of that, there is need for a metrics able to measure the importance of the components within the system. These metrics are called importance measures and there are five classical importance measures, Birnbaum importance, criticality importance, Reliability or Risk Achievement worth (RAW), Reliability Reduction Worth (RRW) and Fussell-Vesely importance [7] [8]. Classical importance measures apply to systems made up of binary components and characterized also by binary states. This hypnosis does not fit of the real functioning of many systems. Modification on classical importance measures have been made in [7] and [8] allowed them to be used for multi-state components within multi-state system. The previous modifications cannot be used to quantify the impact of equipping the component with a PHMS as they made to quantify the impact of component reliability improvement on the overall system performance while installing a PHMS improve mainly maintenance operation performance to decrease component downtime.

Further modification of classical importance measures is required and, in this thesis work, risk achievement worth (RAW) is chosen to be used as it quantifies the impact of improvement activities on the component.

The second novelty of this thesis work is modifying RAW importance measure to quantify the impact of installing PHMS on a multi-state component within a multi-state system.

At the end, an application to an oil and gas offshore installation consists of 6 components and characterized by 7 production levels is proposed. Benefits of PHMS installation for each component is quantified. By applying the modified RAW importance measure, prioritize the component has been made to identify the component which is more convenient to install a PHMS.

Chapter (2): Problem statement

In this thesis work, we considered an industrial plant made by n components. Each component i can be in N_i different states which corresponding to different level of degradation and different production level. the overall industrial plant can provide N different levels of production that result from the different possible configuration of the individual component states.

The final objective are:

1- Prioritize the components according to the expected benefits in term of overall plant performance of equipping them with a PHMS.

2- Quantify the plant performance improvement when a component is equipped with a PHMS.

In order to answer the previous questions, we assumed to have available:

a) a stochastic multi-state model of the individual component degradation and failure process;

b) a stochastic model of the duration of the maintenance interventions which take into account the component degradation state;

c) information on the performance of the PHMS;

d) a model which associates to the different plant component configurations the corresponding plant production state;

With respect to a), given the complexity of the components used in complex industrial plants which typically renders unfeasible the use of physic-based models[9].we consider a multi-state degradation model based on the discretization of the degradation process in three or more states characterized by different values of suitable degradation indicators or different performance level or different symptoms. The main advantage of this approach over the binary model which only considers the two states of the component “operating” and “failing” is that it provides a more accurate description of the sequential component degradation phases. Transition time from one state to another are assumed to be distributed according to exponentially probability distribution.

With respect to d), we consider a stochastic model which integrates at the plant level the individual component degradation and failure process modeled in a) taking into account the component process capacities, the functional and operation dependencies and the effects of component availability on plant production. The model will be evaluated by Monte Carlo simulation to provide the expected production of the plant. [2].

With respect to b), we consider a stochastic model which represent the duration of the maintenance interventions as a random variable distributed according to a probability distribution whose repair rate is a function of the degradation state of the component. The model is based on the assumptions of [5] and [4].

With respect to c), PHM performance metric introduced in [6] are considered and it is assumed that the PHMS developer provides their values.

Chapter (3): prognostics and health management system

The main objective of Prognostic and Health Management System (PHMS) is to detect and diagnose the abnormal behaviors of industrial equipment, predict its future behaviors, and opportunely schedule the maintenance actions before catastrophic failure. ISO13381-1 defines prognostics as “an estimation of time to failure, which is the Residual Useful Life (RUL) of the component, and risk for one or more existing and future failure modes” [10]. This definition highlights the fact that a failure mode may initiate other failure modes and those other failure modes can cause the component failure. RUL is defined as the period during which the component is expected to be usable for the purpose it was acquired while it may or may not correspond with the item's actual physical life. According to [11] prognostics has the following characteristics:

- it is typically carried out at component and sub-component level.
- it provides predictions of the component RUL and of the progression of the failure mode.
- It requires an estimation of the future operational conditions which will be expected for the component.

Prognostics is typically preceded by the identification of the onset of the degradation process (fault detection) and diagnosis of the type of degradation, i.e. the identification of the degrading component / subcomponent (fault isolation) and the estimation of the degradation state.

Refer to Appendix A, for further information on the PHM.

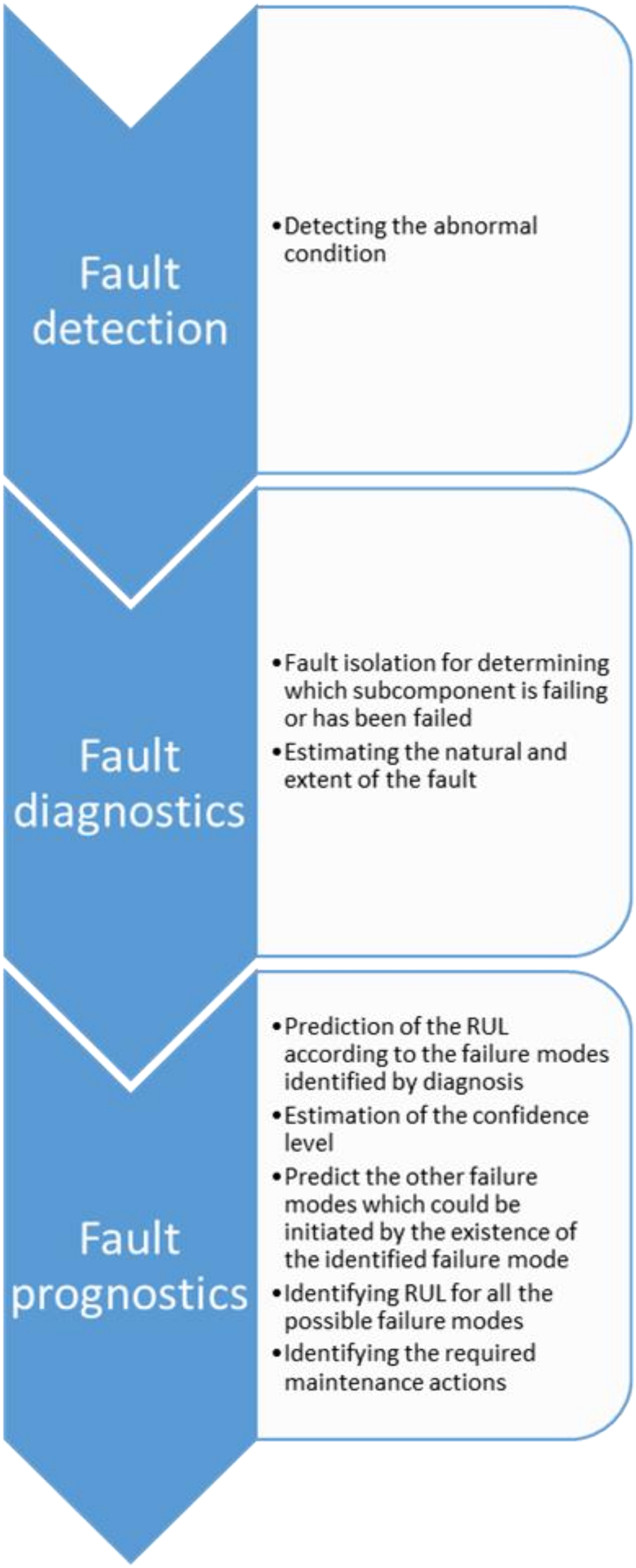


Figure 1 the main activities proposed by PHMS

3.1 Prognostics performance metric:

Metrics for evaluating of PHMS are classified into three main groups considering the type of information they provide [12] [13] [14]:

1- Algorithmic performance metrics, they evaluate the capability of the algorithms of predicting the future evolution of the component degradation. This group of metrics can be further sub- classified into:
    - Accuracy based metrics which evaluate the closeness of the predicted RUL to the ground truth RUL.
    - Precision based metrics which evaluate the variability of the estimated RUL, i.e. the variability of the RUL predicted error with respect to its mean value.
    - Robustness based metrics which evaluate the ability of the PHMS to afford perturbation.
2- Computational based metrics; they evaluate the computational load and time needed to the algorithm for predicting the RUL. They are important for applications needing real time data monitoring for taking critical safety decisions
3- Cost benefit based metrics; they evaluate the expected benefits of equipping plant component with PHMS in terms of economic indicator such as life cycle cost or return on investment of investing capital cost a PHMS [6].

Algorithmic performance metrics are considered in this work to investigate the effect of equipping components with PHMS. The idea is that more efficient in the detection, diagnosis and prognosis of the failure, larger are expected to be the benefits in terms of plant performance improvement.

In this thesis work, we consider the following the PHM performance metrics:

1- False Positive (FP); it represents the probability that the PHMS under-estimates the ground truth RUL. And it is defined by:

$$FP_\lambda = E[\phi P_\lambda], \phi P_\lambda = \begin{cases} 1, \ if \ \gamma_\lambda - RUL_\lambda^* < threshold \\ 0, \ otherwise \end{cases} \qquad \textbf{[1]}$$

And we will set this threshold is 0

while RUL^* is the ground truth RUL, γ is the PHM predicted RUL and λ is component specific time or time window modifier.

2- The mean value of the relative accuracy in the case that the PHMS under-estimates the ground truth of RUL. It is defined by:

$$E[\underline{RA}_\lambda] = 1 - \frac{E[RUL_\lambda^* - \gamma_\lambda \,|RUL_\lambda^* > \gamma_\lambda]}{RUL_\lambda^*} = \frac{[RUL_\lambda^* - \hat{\gamma}_\lambda | RUL_\lambda^* > \hat{\gamma}_\lambda]}{RUL_\lambda^*} \qquad \textbf{[2]}$$

$\hat{\gamma}_\lambda$ is the mean value of PHM predicted RUL at specific time λ

3- Standard deviation of relative accuracy in case of RUL underestimation.

4- The mean value of the relative accuracy in case of PHMS over-estimates the RUL and the following equation represent it:

$$E[\overline{RA}_\lambda] = 1 - \frac{E[\gamma_\lambda - RUL_\lambda^* | \gamma_\lambda > RUL_\lambda^*]}{RUL_\lambda^*} = \frac{[\hat{\gamma}_\lambda - RUL_\lambda^* | \hat{\gamma}_\lambda > RUL_\lambda^*]}{RUL_\lambda^*} \qquad \textbf{[3]}$$

5- Standard deviation of relative accuracy in case of RUL overestimation.

Our motivation for choosing the previous PHMS performance metric can be defined as:

1. PHMS is installed on a component within an industrial system which characterized by component with relatively long mean time to fail. In the case study considered in this thesis work, the main time to fail for the components are in the range of thousands. Comparing to mean time to fail for the components, time needed to the algorithm for running and predicting RUL is neglected. For that reason, we are not using computational based metrics.
2. Not only economic benefits but, also managing the risk and decrease unexpected failure of the components are also needed to be evaluated. Also, the proposed PHMS performance metric is use to quantify system performance improvement which can be used to quantify the economic benefits for installing PHMS. For that reason, we are not using cost benefit based metric.
3. We decided to choose algorithmic performance metrics as they meet our requirements. We choose relative accuracy from accuracy based metric and to represent its uncertainty,

we use its standard deviation which belong to precision based metrics and false positive to evaluate the PHMS tendency to overestimate or underestimate RUL.

3.2 Effects of equipping a component with a PHMS:

A PHMS provides a real-time information on the component degradation state and prediction on its RUL. Although this information can be used to many purposes such as condition informed management of the production, in this thesis work we focus on the use of the PHMS outcomes for maintenance planning. The main effect of installing a PHMS is the shift from corrective-based and schedule-based maintenance to predictive maintenance. This means that the number of component faults consequent unexpected component downtimes are expected to dramatically decrease. At the same time, the fraction of useless maintenance interventions periodically performed when scheduled maintenance applied can be avoided.

In this work, we model the effect of equipping a component with a PHMS as:

1) No scheduled maintenance interventions are applied. As maintenance interventions derive from an informed decision which consider the RUL prediction.
2) Corrective maintenance interventions are performed only in case of under performance of the PHMS
3) The time necessary to repair the component is reduce.

This is due to the fact that the overall repair time is formed by many contributions:

- time necessary for failure causes and failure modes investigation
- time necessary for organizing the maintenance operation considering logistic issues. This can be very critical for offshore oil and gas platforms
- The physical time needed to perform component repair/replacement
- Time wasted because of uncomplete or unsuccessful maintenance operation.

Notice that the use of a PHMS can reduce time required for failure causes and modes investigation and wasted in unsuccessful maintenance because of diagnosis. Also, earlier knowledge about component failure time can reduce time needed for maintenance operation organizing as the maintenance preparations start early before component stopped for maintenance. Furthermore, as the degradation evaluation dynamics become faster and more complex as the component get closer to its end of life, physical time needed to perform repair can be reduced by early stop of the component at a low degradation state.

With respect to the model of the time required to perform maintenance operation, we assumed that it is proportional to component degradation according to [5][4].

In Liao et al. [5], a relationship between repair time and degradation state was proposed as:

$$E\left[T_{(repair)}\right] = \gamma_0 X \, exp(i \, \gamma_1 X) \qquad \textbf{[4]}$$

where X represent the component degradation state and ϒ0 and ϒ1 are constants and i is number of maintenance action already performed on the component.

In Berenguer et al. [4], the relationship between repair time and degradation state was proposed as:

$$E\left[T_{(repair)}\right] = \rho_1 + \rho_2 E[X] \qquad \textbf{[5]}$$

where ρ_1 and ρ_2 are known parameters.

Chapter (4): importance measures for multi-state multi-component system

Importance measure aims at quantifying the contribution of components to the measure of interest of the system performance [15] [8] [7] [19]. Since components of oil and gas plants can be typically repaired after a failure, one of most useful performance indicator is the plant availability. Indeed, the identification of which components mostly determine the overall system availability allows systems designers and managers to trace system bottlenecks and provides guidelines for effective actions of system improvement.

Appendix (b) provides a detailed description of the most used importance measures that are Birnbaum importance, criticality importance, Reliability Achievement worth (RAW), Reliability Reduction Worth (RRW) and Fussell-Vesely importance.

In this thesis work, we are going to consider among the different possible importance measures RAW which defined by:

$$RAW_i = \frac{R_s(t; R_i(t) = 1)}{R_S(t)}$$

where $R_s(t)$ is system reliability and $R_s(t; R_i(t) = 1)$ is system reliability knowing that component i reliability is equal to 1.

Since equipping the component with a PHMS is expected to lead to an improvement of system performance, choice of RAW as importance measure of interest is motivated by the fact that it quantifies the impact of component performance improvement on overall system performance whereas the other importance measures (IMs) quantify the effect of the component losing its performance on the overall system performance.

The above-mentioned IMs have been developed for application characterized by components whose state is modeled to be either functioning or faulty. This hypothesis does not apply to components of many industrial plants, such as, for example, manufacturing production, power generation, transportation systems and chemical process plants [8] [7]. Components of oil and gas plants are often represented by multi-state models with different states such as "healthy" with associated maximum production, partially performance, degraded with associated reduced production performance and faulty.

Two modifications of the classical IMs have been introduced to customize them to multi-state multi-component systems [8] [7].

In [8], individual components of electricity transmission networks are characterized by their sustained outage rate λ_s^i instead of their failure probability. Then, the definition of IMs is based on setting upper u_s^i and lower l_s^i limits for each component λ_s^i, representing worst and best cases for the component outage rate, respectively. The obtaining definition of RAW is:

$$I_i^{RAW} = \frac{U_S(\lambda_S,\mu_S)}{U_S(\lambda_S,\mu_S|\lambda_S^i = l_S^i)} \quad \textbf{[6]}$$

where U_s is the system unavailability, u_s^i and l_s^i are the upper and lower limits of the i-th component sustained outage rate, and λ_s^i is component i sustained outage rate.

Although, the upper and the lower limits can be set to ∞ and 0 respectively to represent the extreme cases of the component with availability equal to 0 (fully unavailable) or 1 (always functioning), these two values may not be close to reality.

The modification of the classical IMS proposed in [8] can be very useful in this thesis work to measure importance of repairable component since upper and lower limits can be set to μ_i instead of λ_i.

Zio and Podofillini in [7] has introduced further modifications to adapt importance measures to multi-state components in multi-state systems.

In [7], the definition of availability has been generalized for application to multi-state system. The main idea is to determine whether the system performance is higher or lower with respect to a minimum system performance level $W^*(t)$required at time t. Thus, considering a system which can be in N different states characterized by a production level W_j, $j = 1, \dots\dots, N$, the system availability is defined by:

$$A(W^*, t) = \sum\nolimits_{j:Wj \geq W^*(t)} \mathbb{P}_j(t)$$

Where $\mathbb{P}_j(t)$ is the probability that the system is in state j. Thus, the mean multi-state system unavailability over a period of time τ is given by

$\overline{U}(W^*) = \frac{1}{\tau}\int_0^{\tau}[1 - A(W^*, t)]dt$

In order to define the RAW importance measures for multi-component multi-state systems, the following notations has been introduced:

> Γ_i^{α} : the set of those states of component i characterized by a performance level bellow or equal to α;
>
> $\overline{\Gamma}_i^{\alpha}$: the set of those states of component i characterized by a performance level above α.

It is now possible to introduce the main availability of the multi-state system considering the special case in which the component i is restricted to the states Γ_i^{α} ($\overline{\Gamma}_i^{\alpha}$) characterized by performance below (above) α:

> $\overline{U_i}^{\leq\alpha}(W^*)$ = $\overline{U}(W^*|j_i \in \Gamma_i^{\alpha} in\ [0, \tau])$: mean unavailability of the multi-state system when the performance of i-th component, (j_i is the state of the i-th component), is restricted to be below or equal to α in t ∈ $[0, \tau]$
>
> $\overline{U_i}^{>\alpha}(W^*)$ = $\overline{U}(W^*|j_i \in \overline{\Gamma}_i^{\alpha} in\ [0, \tau])$: mean unavailability of the multi-state system when performance of i-th component, (j_i is the state of the i-th component), is restricted to be above to α in t ∈ $[0, \tau]$.

Then, the modified RAW importance measure is defined by [7]:

RAW of α-level: $I_i^{RAW} = \frac{\overline{U}}{\overline{U}_i^{>\alpha}}$ **[7]**

The above-mentioned IMs is applied to multi-state systems which required to meet a minimum system performance at time t, (i.e. power production plants). This condition is not necessarily considered in other multi-state systems in which a specific minimum system performance at time t is not required but overall mean system performance is the main criteria (i.e. oil and gas plants in which system mean performance is represented by plant average production).

It is possible to introduce the main performance of the multi-state system considering the special case in which the component i is restricted to the states Γ_i^α ($\bar{\Gamma}_i^\alpha$) characterized by performance below (above) α as:

$\overline{W_i}^{\leq\alpha}$= $\bar{W}(j_i \in \Gamma_i^\alpha in\ [0,\tau])$: mean performance of the multi-state system when the performance of i-th component, (j_i is the state of the i-th component), is restricted to be below or equal to α in t ∈ $[0,\tau]$.

$\overline{W_i}^{>\alpha}$= $\bar{W}(j_i \in \bar{\Gamma}_i^\alpha in\ [0,\tau])$: mean performance of the multi-state system when performance of i-th component, (j_i is the state of the i-th component), is restricted to be above to α in t ∈ $[0,\tau]$.

And importance measures will be:

RAW of α-level: $I_i^{RAW} = \frac{\overline{W_i}^{>\alpha}}{\overline{W}}$ **[8]**

In this thesis work, we consider the idea in [7] of computing the RAW considering the mean performance of the multi-state system since for oil and gas plants, the main criteria is average production of the plant. The way the numerator of equation (8) is calculated should be modified as is case of using preventive maintenance transition rates of component (from healthy to failure, healthy to degraded or degraded to failure) increases due to component early stops but, also, lower expected repair time is achieved. Therefore, the performance of i-th component is not restricted to be above a specific threshold but transition rates from degraded or faulty states to healthy state are increased due to equipping the component with PHMS. So, further modifications on importance measures should be done considering PHMS performance and relationship between degradation state and repair time.

Since importance measures of a component are based on identifying the impact of changing specific criteria of the component on the overall system performance, According to the analysis in chapter (3), one of the main effects of equipping the component with a PHMS is to reduce the duration of the repair time and, thus, to reduce the downtime of the component.

Thus, the objective is to measure the modification of overall system performance obtained by reducing in order to quantify the effectiveness of the PHMS in the repair time reduction.

In order to assess the expected benefits of equipping the component i with a PHMS, the rule is to substitute in equation (8) the numerator with a term indicating the system performance when component i is equipped with the PHMS and the denominator with the system performance when component i is not equipped with PHMS.

we introduce the parameter ε defined by:

$$\varepsilon = MTTR\,(considering\ PHM)/MTTR \qquad \textbf{[9]}$$

For oil and gas plants, considering N production levels:

$Y_s^n(\lambda_s,\mu_s)$=$\frac{1}{\tau}\int_0^{\tau}\mathbb{P}_n(t)$ dt : the average availability of the system in the production level n at $t \in [0,\tau]$.

$W_s^n(\lambda_s,\mu_s)$: system production in n-th production level

$W_s(t,\lambda_s,\mu_s)$ = $\sum_{n=1}^{N} W_s^n(\lambda_s,\mu_s) * \mathbb{P}_n(t)$: the value of the expected system instantaneous performance, production, at time t

$\overline{W_s}(\lambda_s,\mu_s) = \frac{1}{\tau}\int_0^{\tau} W_s(t,\lambda_s,\mu_s)dt$: average system performance production, in $[0,\tau]$

$\overline{W_s}(\lambda_s,\mu_s) = \sum_{n=1}^{N}(W_s^n(\lambda_s,\mu_s) * Y_s^n(\lambda_s,\mu_s))$

Using the symbols in equation (6), the importance measure can be rewritten as where $\mu_i = l_i$ represents the lower limit of repair rate achieved when the component i is not equipped with the PHMS, whereas $\mu_i = u_i$ represent the upper limit of the repair rate achieved when component i is equipped with the PHMS and here modeled as:

$u_i = \frac{1}{\varepsilon * MTTR(i)}$: $MTTR(i)$ is mean time to repair for component i where it is not equipped with PHMS.

Therefore:

$\overline{U}_i^{PHMS}(\lambda_i, \mu_i | \mu_i = u_i)$: component i mean unavailability when it is equipped with a PHMS and u_i is the upper limit for μ_i and it depends on PHMS performance.

$\overline{U}_i(\lambda_i, \mu_i)$: component i mean unavailability when is it not equipped with a PHMS

And, RAW for component i defined by:

$$I_i^{RAW} = \frac{\overline{U}_i^{PHMS}(\lambda_i, \mu_i | \mu_i = u_i)}{\overline{U}_i(\lambda_i, \mu_i)}$$

It should be mentioned that, for multi-state component, component availability is calculating depending on if the component is functioning or not.

To extend the previous importance measure of i-th component to be used for multi-component system, we introduce the following notation:

$\overline{W}_s(\lambda_s, \mu_s | \overline{U}_i^{PHMS})$: system average performance for $t \in [0, \tau]$ known that component i is equipped with a PHMS.

So, RAW, or can be called performance achievement worth, of the i-th component within the system will be equal to:

$$I_i^{RAW} = \frac{\overline{W}_s(\lambda_s, \mu_s | \overline{U}_i^{PHMS})}{\overline{W}_s(\lambda_s, \mu_s)} \quad \textbf{[10]}$$

And RAW is a function of ε which depends on degradation state of the component which is a function of specific time λ at which component has been stopped and the performance of the PHMS.

Chapter (5) will describe how to compute $\overline{U}_i^{PHMS}(\lambda_i, \mu_i | \mu_i = u_i)$.

Chapter (5): availability and performance model of a component equipped with a PHMS:

Consider a degrading component, within a system, which is monitored every Δt unit of time and subject to a continuous gradual random degradation. Prediction of the RUL will start from the beginning of the life time of the component at specific time equal to zero. T_{start} is the time at which the component begins operation and T_f is the time at which component fails. Specific time λ is equal to $\frac{T-T_{start}}{T_f-T_{start}}$ where T is the current time. Degradation level is a function of specific time λ. Figure (2) shows an example [16] of the relationship between degradation and specific time.

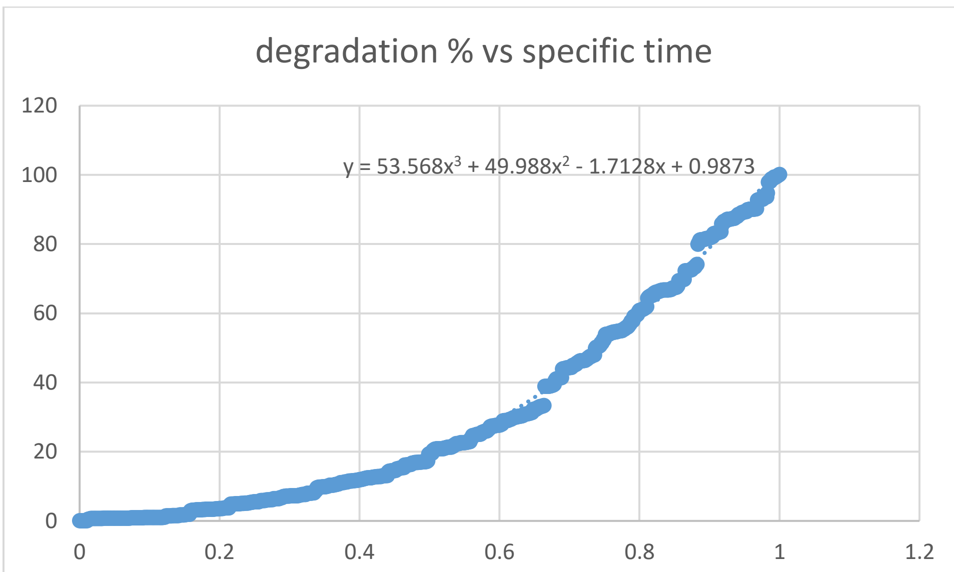


Figure 2 an example degradation vs specific time

As shown, time rate of changing of degradation level, time derivative of degradation level, increases by time. In other words, degradation occurs faster when the component is close to failure. The duration of maintenance operation depends on the degradation state of the component at the beginning of maintenance action. Does, preventive maintenance policy aims to remove the component from operation before it reaches a high level of degradation. Stopping the component at low specific time increases unnecessarily maintenance operation while stopping at high specific time decreases the benefits of using preventive maintenance. For the model, here, times at which degradation and failure transitions occur and the duration of maintenance operation are assumed to be exponentially distributed.

In this work, we assume that the equation describing the relationship between degradation level and specific time λ is a polynomial equation of degree 3 as:

$$X = a\lambda^3 + b\lambda^2 + c\lambda \qquad \textbf{[11]}$$

where a, b and c are known parameters, X represent instantaneous degradation level as a percentage ratio to the failure time degradation level of the component and λ is the specific time of the component when it stops.

As mentioned in chapter (3), the maintenance duration is a function of current degradation level and is represented as:

$$E(MTTR) = d + fX \qquad \textbf{[12]}$$

where d and f are known parameters and X the percentage level of degradation [4].

In Compare and Bellani [6] a model describing the availability of PHM equipped component has been introduced. They assumed that PHMS makes a prediction every Δt unit time and the purpose of the model is to stop the component before it fails knowing that there is a need of $j * \Delta t$ hours to stop the component. They stop the component if the predictive RUL is smaller than $h * \Delta t$ hours.

Their model is based on calculating stop probability of the component at each prediction which represents the probability that predicted RUL fail below the threshold $h * \Delta t$ considering the PHMS performance metric as long as the component isn't removed from operation or fails. Monte Carlo approach of stop probability is in appendix (E).

The maintenance strategy introduced by Compare and Bellani model is based on the hypothesis that stopping the component before failure is enough to take advantages of preventive maintenance. This hypothesis does not fit with the real functioning of many systems. For such systems, taking advantages of preventive maintenance depends on component degradation state at the beginning of maintenance operation. As mentioned before, degradation state is a function of specific time λ. So, for such systems, maintenance strategy should be built on the concept of predicted specific time based on predicted RUL. In other words, if the actual failure time is known, preventive maintenance should always perform at specific time $\lambda_{optimum}$ which depends on the degradation model.

Compare and Bellani's model has been modified in this thesis work and an optimum maintenance strategy has been developed. This strategy takes into account the component stop specific time, degradation state and maintenance operation performance.

The flowchart in appendix (c) explains the mean framework of the mathematical model and in appendix (D) the algorithm is explained.

For the model built in this work, same inputs as in Compare and Bellani are used, which are:

- FP; false positive which represent the probability of PHMS predicts underestimated RUL.
- $\hat{\gamma}_{\lambda}$= (2-$E[\overline{RA_{\lambda}}]$)*$RUL^{*}(t_{\lambda})$ **[13]**

  where $\hat{\gamma}_{\lambda}$ is mean value of PHM predicted RUL in case of RUL overestimation and $RUL^{*}(t_{\lambda})$ is the ground truth RUL at specific time λ.
- $\hat{\gamma}_{\lambda}$= $E[\underline{RA_{\lambda}}]$*$RUL^{*}(t_{\lambda})$ **[14]**

  where $\hat{\gamma}_{\lambda}$ is mean value of PHM predicted RUL in case of RUL underestimation and $RUL^{*}(t_{\lambda})$ is the ground truth RUL at specific time λ.
- Standard deviation of predicted RUL which can be calculated by $\sigma^2_{\hat{\gamma}_{\lambda}}$=$\sigma^2_{\overline{(RA)}\lambda}$* $RUL^{*}(t_{\lambda})^2$ by applying quadratic property Var[aX]=a^{2}Var[X], in case of RUL overestimation, while in case of RUL underestimation $\sigma^2_{\hat{\gamma}_{\lambda}}$=$\sigma^2_{\underline{(RA)}_{\lambda}}$* $RUL^{*}(t_{\lambda})^2$
- They used the well-known one-side Chebyshev's inequalities [20] for sake of generality and for not making any parametric assumptions about the distribution function of the PHM predicted RUL

$$\mathbb{P}(X \geq \mu + a) \leq \frac{\sigma^2}{\sigma^2+a^2} \quad \textbf{[15]}$$

$$\mathbb{P}(X \leq \mu - a) \leq \frac{\sigma^2}{\sigma^2+a^2} \quad \textbf{[16]}$$

To provide an estimate of the minimum benefits achievable from a PHMS with a known performance metric, equations 15 and 16 are taken as equality:

For RUL is underestimated:

$$\mathbb{P}(\gamma_\lambda \leq \hat{\gamma}_\lambda - a) \cong \frac{\sigma^2_{\underline{(RA)}_\lambda} * RUL^*(t_\lambda)^2}{\sigma^2_{\underline{(RA)}_\lambda} * RUL^*(t_\lambda)^2 + a^2} \qquad \textbf{[17]}$$

$$\mathbb{P}(\gamma_\lambda \geq \hat{\gamma}_\lambda - a) \cong 1 - \frac{\sigma^2_{\underline{(RA)}_\lambda} * RUL^*(t_\lambda)^2}{\sigma^2_{\underline{(RA)}_\lambda} * RUL^*(t_\lambda)^2 + a^2} \qquad \textbf{[18]}$$

And for RUL is overestimated:

$$\mathbb{P}(\gamma_\lambda \geq \hat{\gamma}_\lambda + a) \cong \frac{\sigma^2_{\overline{(RA)}\lambda} * RUL^*(t_\lambda)^2}{\sigma^2_{\overline{(RA)}\lambda} * RUL^*(t_\lambda)^2 + a^2} \qquad \textbf{[19]}$$

$$\mathbb{P}(\gamma_\lambda \leq \hat{\gamma}_\lambda + a) \cong 1 - \frac{\sigma^2_{\overline{(RA)}\lambda} * RUL^*(t_\lambda)^2}{\sigma^2_{\overline{(RA)}\lambda} * RUL^*(t_\lambda)^2 + a^2} \qquad \textbf{[20]}$$

Where γ_λ is PHM predicted RUL

The main deference between the two models is while in [6] stop probability is calculated and then compared with a stochastically random number within Monte Carlo methods, the model proposed here is based on the following steps within a Monte Carlo simulation:

- Comparing false positive, equation (3), with a stochastically random number to identify if the predicted RUL is overestimated or underestimated.
- If predicted RUL is overestimated, mean value of predicted RUL is calculated using equation (13) and for calculating predicted RUL, stochastically random number is compared to the left-hand side of equation (20) to calculate parameter a and predicted RUL γ_λ will be equal to $(\hat{\gamma}_\lambda + a)$.
- If predicted RUL is underestimated, mean value of predicted RUL is calculated using equation (14) and for calculating predicted RUL, stochastically random number is compared to the left-hand side of equation (18) to calculate parameter a and predicted RUL γ_λ will be equal to $(\hat{\gamma}_\lambda - a)$.

Then predicted specific time is calculated by:

$$\lambda_{predicted} = \frac{T - T_{start}}{(T - T_{start}) + \gamma_\lambda} \qquad \textbf{[21]}$$

- $\lambda_{predicted}$ is compared with a specific time threshold equal to $(1 - \lambda_h)$. The component is stopped for preventive maintenance if $\lambda_{predicted}$ is higher than the threshold set otherwise the component keeps working until the next prediction after Δt unit of time in which the model calculation will be repeated.

Incomplete information about the distribution function of relative accuracy in overestimated and underestimated cases is the reason for using possibility theory instead of probability theory [52].

In the possibility theory, for the even (A), $\mathbb{P}$(A) represents event A probability and according to possibility theory, the previous quantity is lower than the possibility of event A, $\pi(A)$, and higher than the necessity of event A, N(A),

$$N(A) \leq \mathbb{P}(A) \leq \pi(A)$$

In this thesis work, we use Chebyshev's inequalities to build the possibility distribution. To estimate the minimum benefit achievable, possibility function of the event represents the worst case, which represented by predicted RUL farther from the ground truth RUL, is built.

The two events in equations 15 and 16, $A = (X \geq \mu + a)$ and $A = (X \leq \mu - a)$, figure (3), shows the shape of the probability, possibility and necessity cumulative of event A, so seeking for building a conservative model, calculations based on possibility cumulative function.

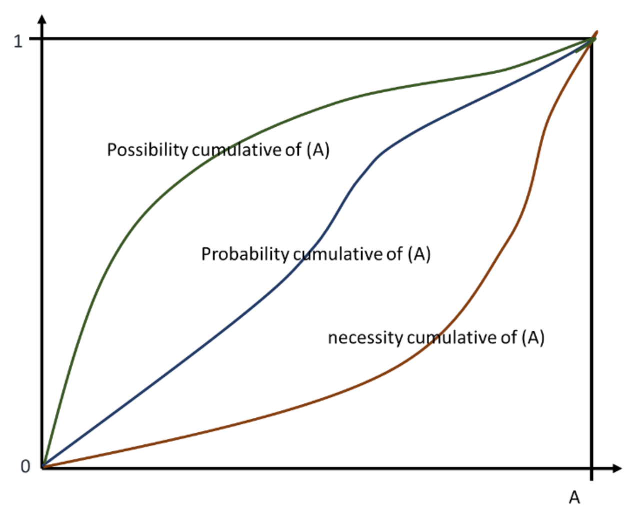


Figure 3 shape of cumulative probability, possibility and necessity functions

Notice that, this stop condition aims at removing the component at a specific time close to $\lambda_{optimum}$ and as the PHMS performance increases, $\lambda_{optimum}$ value will get closer to $(1-\lambda_h)$.

When the component is stopped for preventive maintenance the actual specific time λ is calculated and accordingly, component degradation state and mean time to repair are calculated using equations 11 and 12 respectively.

This model identifies the optimum λ_h to maximize component availability and system performance and then importance measure of the component equipped with PHMS considering PHM performance metric can be calculated.

Chapter (6): case studies:

In this chapter, two case studies are presented. The first one, which consists of one component, aims mainly to validate the availability model introduced in the chapter (5) by comparing it with that one developed by Copmare and Bellani [6] under the same maintenance policy. The second case study is multi-state multi-component system, offshore oil and gas installation, and aims to study the impact of introducing predictive maintenance and prioritize and rank the components based on their criticality.

First case study, Compare and Belani, [6]:

The first case study is a component affected by a fatigue degradation mechanism which is simulated according to the Paris Erdogan model [16] [17].

The crack length x_i reaches the first threshold x=1mm according to the equation:

$$x_{i+1} = x_i + a * e^{\omega_i^1}$$

Where a = 0.003 is the growth speed parameter and $\omega_i^1 \sim N(-0.625,1.5)$ model the uncertainty in the speed values.

Then the crack length follow the next equation to reach to the second threshold, failure threshold $x = 100$mm

$$x_{i+1} = x_i + C * e^{\omega_i^2}(\eta * \sqrt{x_i})\text{^}n$$

where C = 0.005 and n=1.3 are parameters related to the component material properties and are determined by experimental tests; η=1 is a constant related to the characteristics of the load and the position of the crack and $\omega_i^2 \sim N(0,1.7)$ is used to describe the uncertainty in the crack growth speed values.

After running N=100000 Monte Carlo simulations, the mean availability of the component over the whole life time is equal to 0.7943

The time needed to stop the component is 30 hours and if the component is stopped before failure, the repair time $T_{pred} = 100$ hours while if the component fails, the repair time will be

$T_{cor} = 200$ hours. The time Δt between two successful predictions is 10 hours. And the PHM performance metric values are shown in table (1):

SPECIFIC TIME	**0 – 0.25**	**0.25 – 0.5**	**0.5 – 0.75**	**0.75 – 1**
$\boldsymbol{E[\underline{RA}]}$	0.75	0.8	0.85	0.9
$\boldsymbol{\sigma^2_{\underline{RA}}}$	0.04	0.0225	0.01	0.0025
$\boldsymbol{E[\overline{RA}]}$	0.75	0.8	0.85	0.9
$\boldsymbol{\sigma^2_{\overline{RA}}}$	0.04	0.0225	0.01	0.0025
FP	0.3	0.4	0.5	0.6

Table 1 PHM performance metric

Finally, the maintenance rule followed is that the component is stopped if the PHM predicted RUL is lower than 50 hours and the two models in [6] and in this thesis work, which is referred by thesis model, are used.

After running N=100000 Monte Carlo simulations considering simulate a component life time per each Monte Carlo simulation:

- The mean availability of the component over the whole life time using thesis model equals to 0.8531 while in case of using model in [6] is equal to 0.8566.
- The estimated probability distribution function of specific time λ at which the PHM model stops the component are compared in figures (3) and (4). Figure (3) shows the results coming from applying thesis model while figure (4) shows the results coming from applying model in [6].

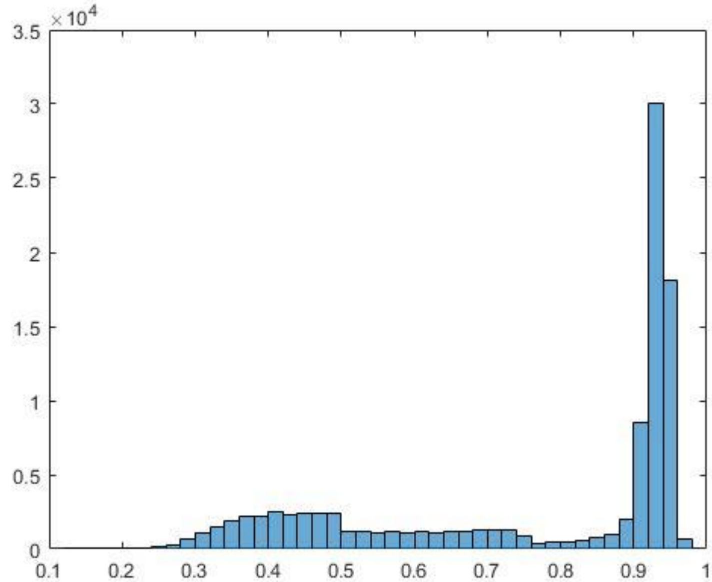


Figure 4 distribution of λ when PHM stopped the component, model developed in this thesis work

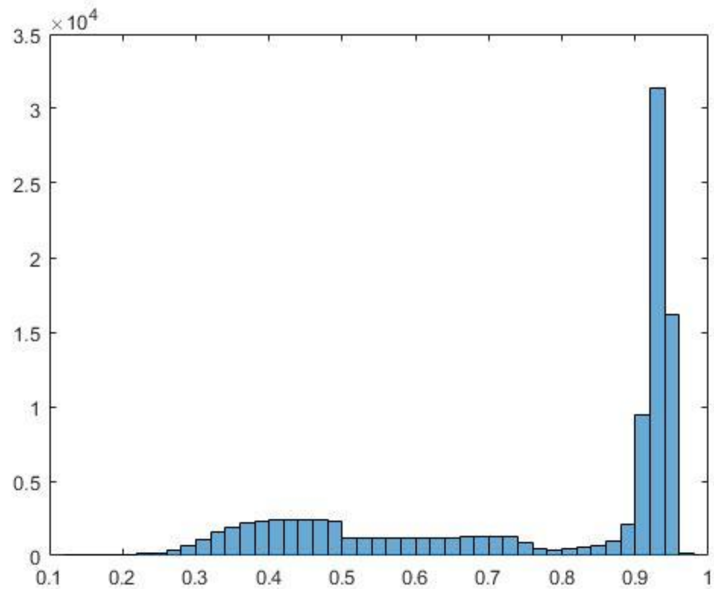


Figure 5 distribution of λ when PHM stopped the component, model developed by Compare & Bellani

Also, after running N=100000 Monte Carlo simulations considering the same component life time per all of Monte Carlo simulations:

- The mean availability of the component over the whole life time using thesis model equals to 0.8348 while in case of using model in [6] is equal to 0.8396 noticing that the failure time consists of 89 hours from the component started working till the detection threshold and 675 hours from the detection threshold to the failure threshold.
- The estimated probability distribution function of specific time λ at with the PHM model stops the component are compared in figures (5) and (6). Figure (5) shows the results coming from applying thesis model while figure (6) shows the results coming from applying model in [6].

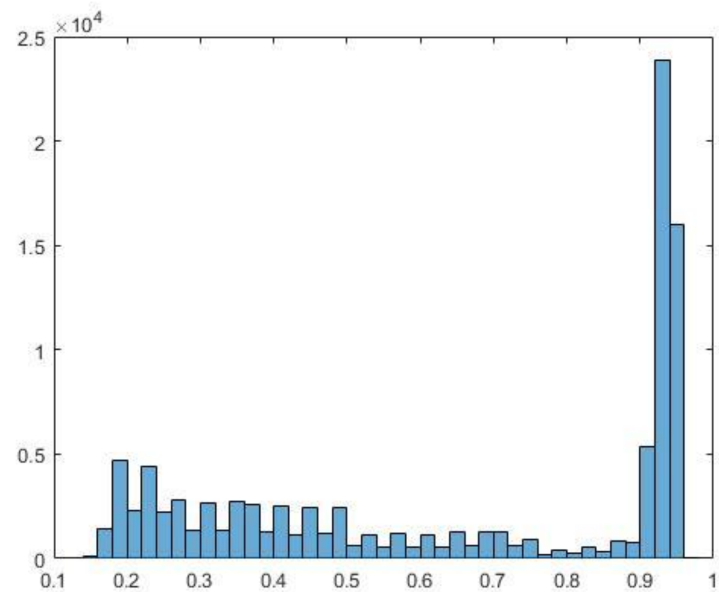


Figure 6 distribution of λ when PHM stopped the component, 1 simulated lifetime, model developed in this thesis work

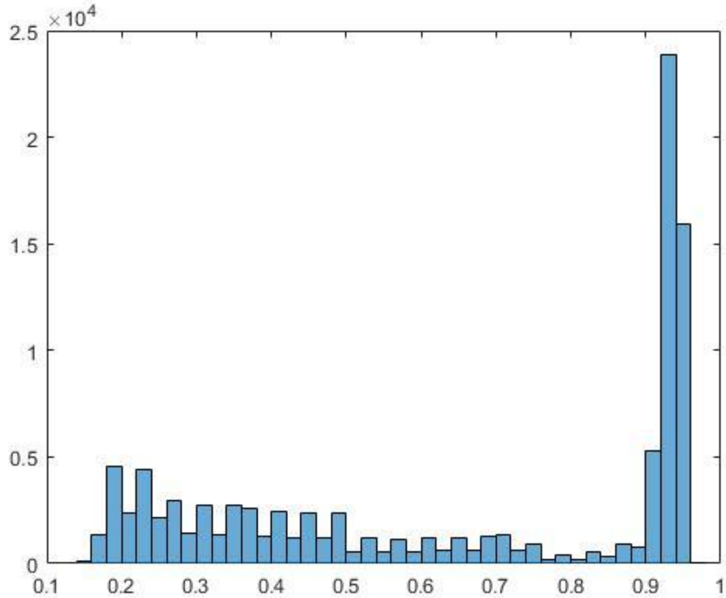


Figure 7 distribution of λ when PHM stopped the component, 1 simulated lifetime, model developed Compare & Bellani

As expected, the figures in the previous two situations are identical as the two methods are the same under the same maintenance strategy or maintenance rule.

The next part of this case study is applying the maintenance strategy introduced in chapter (5), which is built on early stopping the component if the predicted specific time exceeds the threshold $(1-\lambda_h)$ while expected component repair time is considered proportionally increase with component degradation state.

The following steps are considered: -

- Predicted RUL, γ_λ, is simulated using the availability model introduced in the chapter (5).
- Calculating the predicted specific time $\lambda_{\text{predicted}} = \frac{\text{T}-\text{T}_{\text{start}}}{(\text{T}-\text{T}_{\text{start}})+\gamma_\lambda}$
- Comparing predicted specific time to the predictive maintenance threshold $(1-\lambda_h)$. Component is stopped if the predicted specific time exceed the predictive maintenance threshold while in case of predicted specific time lower than the threshold $(1-\lambda_h)$, the component goes for another prediction after Δt hours.
- If the component is stopped, calculate the actual specific time λ and determine the actual degree of degradation.

- Repair time proportionally increases with component degradation level as shown in equation 12 and in this case study parameters d and f in equation 12 are assumed to be 40 and 1.6 respectively. Therefore, predictive maintenance time is:

  $T_{pred} = 40 + 1.6 * X(\lambda)$ (hours) where 40 hours represents the minimum repair time required and $X(\lambda)$ is degradation level at specific time λ and ranging from 0 to 100.
- The final step is to find the optimum value of λ_h that maximize the availability of the component.

After running N=100000 Monte Carlo simulations:

The mean availability of the component over the whole life time vs the value of λ_h is shown in figure (7) and the optimum value of $\lambda_h$is 0.45 which can achieve mean component availability equal to 0.8697.

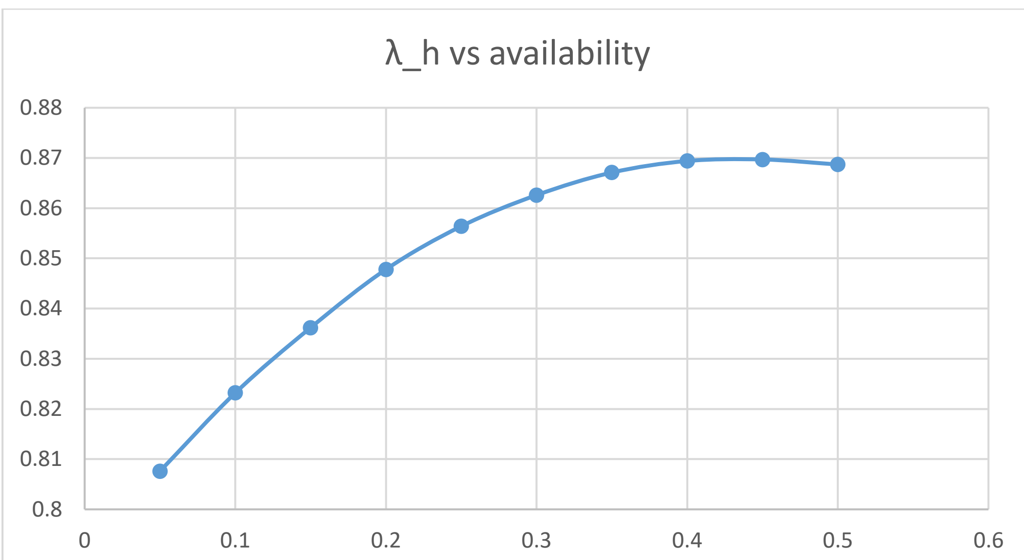


Figure 8 preventive maintenance threshold λ_h vs availability of the component

<u>The second case study </u>[2]<u>:</u>

In the previous case study, the system was consisting of only one component, in this case study we will introduce equipping with PHM model and preventive maintenance to a multi-state multi-component system.

This multi-state multi-component system case study was studied in Zio and Baraldi [16] to calculate system availability and average production in case of using corrective maintenance policy and periodic maintenance policy and here we will study the effect of preventive maintenance policy on system availability and average production but before presenting the results a brief description of the system will be presented. Figure (8) shows the scheme of the offshore production plant.

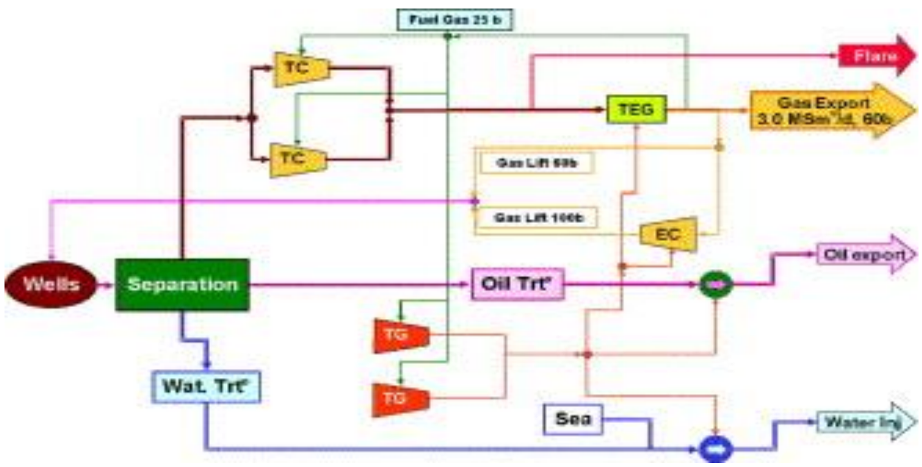


Figure 9 scheme of the offshore production plant

The first step or the process is the separator in which oil, gas and water coming from the production well are separated. The gas from the separator is then compressed by two turbo-compressor in which each one of them has a maximum lad of 2.2*10^6 Sm^3/d then the gas will go to tri-ethylene glycol dehydration unit which have a maximum capacity of 4.4*10^6 Sm^3/d then the gas will be exported with a maximum capacity of 3*10^6 Sm^3/d at pressure 60 bar. The oil coming from the production well which is separated from gas and water in the separator is then go the oil treatment unit and then exported through pumping unit. The maximum productivity of the gas well is 26500 m^3/d of oil while the maximum capacity of the oil treatment unit and pumping unit is 23300 m^3/d. part of the gas produced are used to enhance the productivity of the production well by gas lifting technique to achieve the nominal production level. the gas needed for gas lifting is 1*10^6 Sm^3/d from the output of the gas dehydration unit and then compressed with an electric compressor to reach the pressure 100 bar. Although gas lifting with pressure 60 bar can be considered but the productivity of the production well will be decreased to 80% and in case of no gas lifting the productivity of the production well will decrease to 60%. There is a consumption of 0.4*10^6 Sm^3/d of gas to two gas turbo compressors and two turbo generators. The two turbo generators are responsible of generating 26 MW of electricity. The production and consumption of electricity are shown in table (2):

ELECTRICITY PRODUCTION AND CONSUMPTION OF THE COMPONENTS OF THE SYSTEM

COMPONENT	Electricity production (MW)	Electricity consumption (MW)
TG	13	–
EC	–	6
EXPORT OIL	–	7
WATER INJECTION	–	7
TEG	–	6

Table 2 electrical production and consumption

The following diagrams show the scheme of lifting gas (figure (9)), generation and distribution of fuel gas (figure (10)) and electricity power production and distribution (figure (11)):

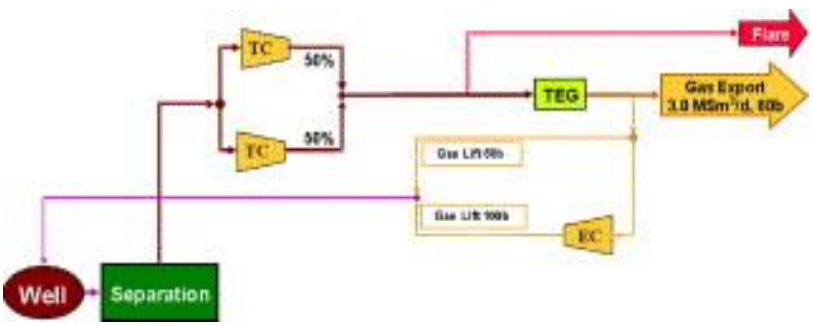


Figure 10 scheme of lifting gas

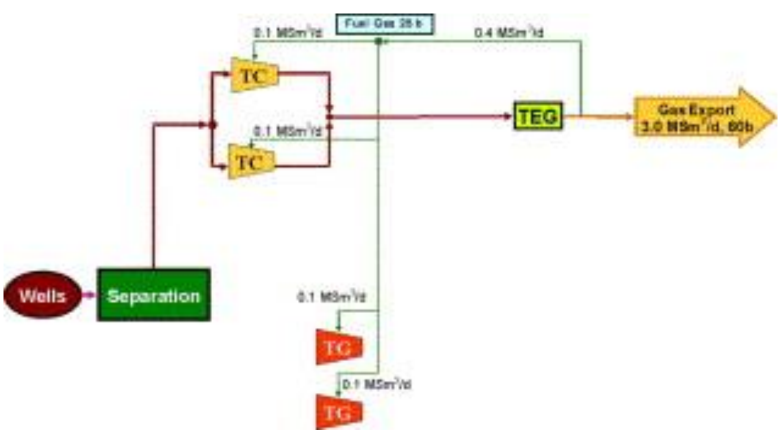


Figure 11 generation and distribution of fuel gas

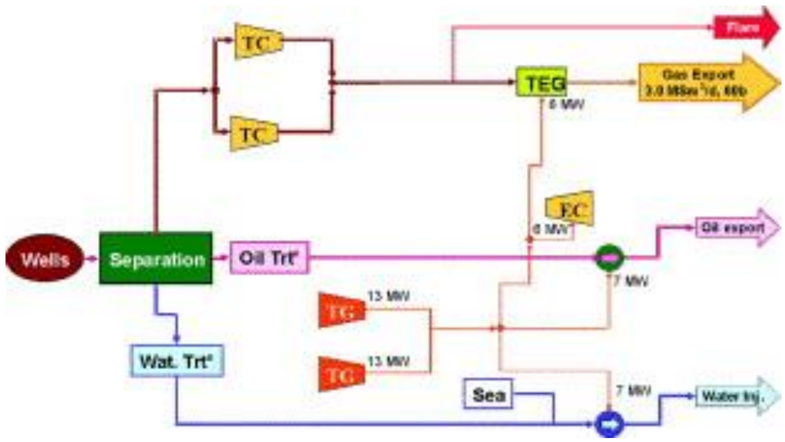


Figure 12 electricity power production and distribution

Only the failure of six components will be considered in this case. Four components out of these six components, two gas turbo compressors and two turbo generators, can be in three different states as shown in figure (12), state (0) which represent as good as new, state (1) which represent degraded but still functioning and state (2) which represent failed.

Two components, dehydration unit and electric compressor, can be in two different states as shown in figure (13), state (0) which represent as good as new and state (2) which represent failed.

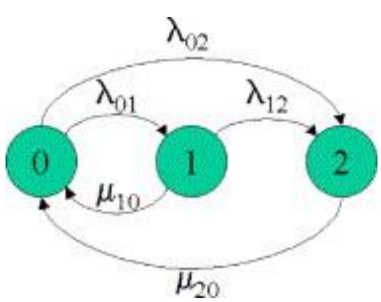


Figure 13 component states for TGs and TCs

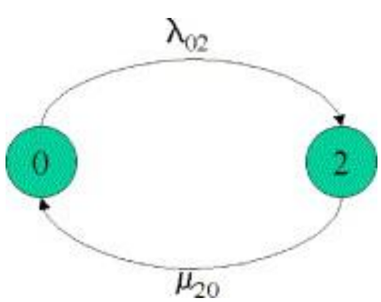


Figure 14 component states for TEG and EC

The failure and repair rates for the six components are shown in table (3):

FAILURE AND REPAIR RATES OF THE COMPONENTS

TRANSITION	Rate (1/h)			
COMPONENT (NUMBER)	TEG (1)	EC (2)	TG (3&4)	TC (5&6)
0→1	–	–	0.79×10−3	6.70×10−4
1→2	–	–	1.86×10−3	2.12×10−3
0→2	5.70×10−5	0.17×10−3	0.77×10−3	7.40×10−4
1→0	–	–	3.20×10−2	3.30×10−2
2→0	3.33×10−1	3.20×10−2	3.80×10−2	4.80×10−2

Table 3 failure and repair rates for the 6 components

It should be mentioned that there is only one maintenance team which means there is a priority level for each component in case of two components are in failed state in the same time while the maintenance team didn't start the repair process on any one of them yet and table (4) shows the priority level of each component. It should be noticed that once the maintenance team start working on repair of a specific component, the maintenance team does not leave the component until it is fixed whatever the components fail during the repair process.

COMPONENTS REPAIR PRIORITY LEVEL

PRIORITY	Component	System conditions
1	TEG	–
1	TG	Other TG failed
1	TC	Other TC failed
2	EC	–
2	TC	Other TC not failed
3	TG	Other TG not failed

Table 4 components priority level

According to the previous description of the system, there are 7 production levels or 7 possible states of the system. Table (5) shows the 7 different states of the system and oil, gas and water production of each state and the minimum cut sets of each state of them.

Minimal cut sets (mcs) and maximum cut sets (MCS) of the different production levels

P. level	Gas (kSm3/d)	Oil (km3/d)	Water (m3/d)	mcs	MCS
0	3000	23.3	7000	–	–
1	900	23.3	7000	X5 or X6	X5 or X6

2	2700	21.2	0	X3 or X4	X2X3 or X2X4
3	1000	21.2	0	X3X5 or X3X6 or X4X5 or X4X6	X2X3X5 or X2X3X6 or X2X4X5 or X2X4X6
4	2600	21.2	6400	X2	X2
5	900	21.2	6400	X2X5 or X2X6	X2X5 or X2X6
6	0	0	0	X1 or X3X4 or X5X6	X1X2X3X4X5X6

Table 5 production level with mcs and MCS

The average annual productivity of the system per year considering corrective maintenance policy is shown in table (6):

ANNUAL PRODUCTIVITY OF THE SYSTEM		
AVERAGE OIL ANNUAL PRODUCTION	8449409.11	m^3 /year
AVERAGE GAS ANNUAL PRODUCTION	1060794158	Sm^3/year
AVERAGE WATER ANNUAL PRODUCTION	2437045.29	m^3 /year

Table 6 average system production using corrective maintenance

The main production of the system is the oil and the gas which have different economical values and to be able to compare the annual production of the system using corrective maintenance, there is a need to represent oil and gas production of the system in one number and there is two ways to achieve that. Firstly, convert gas production to the equivalent barrels of oil and add this value to oil production. Although, this way ignores the fact that oil has more economical value than gas, it still can be used as a way of simplified. The other way is to convert the oil and gas production into their economic value and show the annual production as the total revenue or total income of the system. Oil price in this thesis work is taken as 53.5 US$ per barrel while gas price is taken as 3.5 US$ per MMBTU. According to the prices mentioned before the total production of the system will be 2980290631 US$ per year.

First of all, the importance of the 6 components are calculated using [8] and [7]. The annual gas production is converted to barrel of oil equivalent, barrel of oil equivalent is equal to 5800 standard cubic feet of natural gas, and summed to annual oil production as a simplification to have one umber represent the total performance of the system.

The results are shown in table (7)

	Coit		Zio	
	RAW	rank	RAW	rank
component 1	1.000303	6	1.000303	6
component 2	1.000462	5	1.000462	5
component 3	1.003203	1&2	1.002902	1&2
component 4	1.003203	1&2	1.002902	1&2
component 5	1.002162	3&4	1.002005	3&4
component 6	1.002162	3&4	1.002005	3&4

Table 7 importance measures

From the previous results, it can be identified that components 3 and 4 are the most important component, the two components are identical components, and these components are the two turbo generators.

In the following part, the preventive maintenance rules introduced in chapter (5) and the availability model for PHM-equipped component introduced and developed in this thesis work will be used to identify the impact of preventive maintenance policy on component 3 and the optimum value of the preventive maintenance threshold $(1 - \lambda_h)$. Also, a sensitivity analysis of the system performance to the PHM performance metric parameters will be made.

The PHMS that will be equipped to the component 3 assumed to have a performance metric as shown in table (8) knowing that Δt, between two successive RUL predictions, is equal to 1 hour:

SPECIFIC TIME-Λ	0 – 0.25	0.25 – 0.5	0.5 – 0.75	0.75 - 1
$E[\underline{RA}]$	0.8	0.85	0.9	0.95
$\sigma_{\underline{RA}}$	0.15	0.1	0.08	0.03
$E[\overline{RA}]$	0.8	0.85	0.9	0.95
$\sigma_{\overline{RA}}$	0.15	0.1	0.08	0.03
FP	0.3	0.4	0.5	0.6

Table 8 PHM performance

The following steps are considered:

- Monte Carlo simulation method is used to simulate failure and repair time for the components within the system knowing their failure and repair rates.
- For component 3, the time T_{start} till stopping of the component or its failure, a prediction of the RUL is made every Δt interval if the maintenance team is available using the model introduced in chapter 5.
- After simulating the predicted RUL, the predicted specific time is calculated and compare with the preventive maintenance threshold $(1-\lambda_h)$ and if the condition of $\lambda_{predicted} \geq (1-\lambda_h)$ is <u>achieved twice in a row</u>, it is decided to achieve the maintenance rule twice in a row to increase level of confident about PHM predictions and avoid the rare spurious predictions, the component will be stopped for preventive maintenance.
- When the component stopped for preventive maintenance, actual specific time is calculated and the degradation level percentage ratio in equation 11 is calculating, parameters a, b and c are assumed to be 53.568, 49.88 and -1.7128 respectively.

  $$X(\lambda) = 53.568 * \lambda^3 + 49.988 * \lambda^2 - 1.7128\lambda$$

  Then ε with represent

  $$\varepsilon = \frac{MTTR_{preventive\ maintenanc}}{MTTR_{corrective\ maintenance}}$$

  Then ε can be calculated, considering equation 12, by the following equation:

  $$\varepsilon = \left(0.2 + 0.8 * \frac{X(\lambda)}{100}\right) * 0.7$$

  For the previous equation, we assumed that 30% of the MTTR can be saved in case of successful predictions of PHMS due to saving the time needed for logistic preparations and diagnosis especially the system is an offshore oil and gas installation which characterized by logistic difficulties combined with small footprint and complex component.

  We also considered that 20% of the 70% of the MTTR is needed regardless the degradation level and the ε is proportionally related to $X(\lambda)$

After running N=10000 Monte Carlo simulations for 50000 hours of lifetime for each simulation the results shown as the economic value of the total average production, US$, for different λ_h is shown in table (9) and shown in figure (14):

λ_h	0.1	0.2	0.3	0.4	0.45	0.5
PRODUCTION	2983865884	2984718970	2985311388	2985531902	2985564343	2985531322
PRODUCTION + STANDARD ERROR OF THE MEAN	2983904587	2984755704	2985346903	2985567085	2985598676	2985565696
PRODUCTION - STANDARD ERROR OF THE MEAN	2983827182	2984682236	2985275873	2985496719	2985530009	2985496948
STANDARD ERROR OF THE MEAN	38702.27013	36734.21522	35515.2254	35182.71644	34333.3792	34373.93499

Table 9 system performance as a function of λ_h

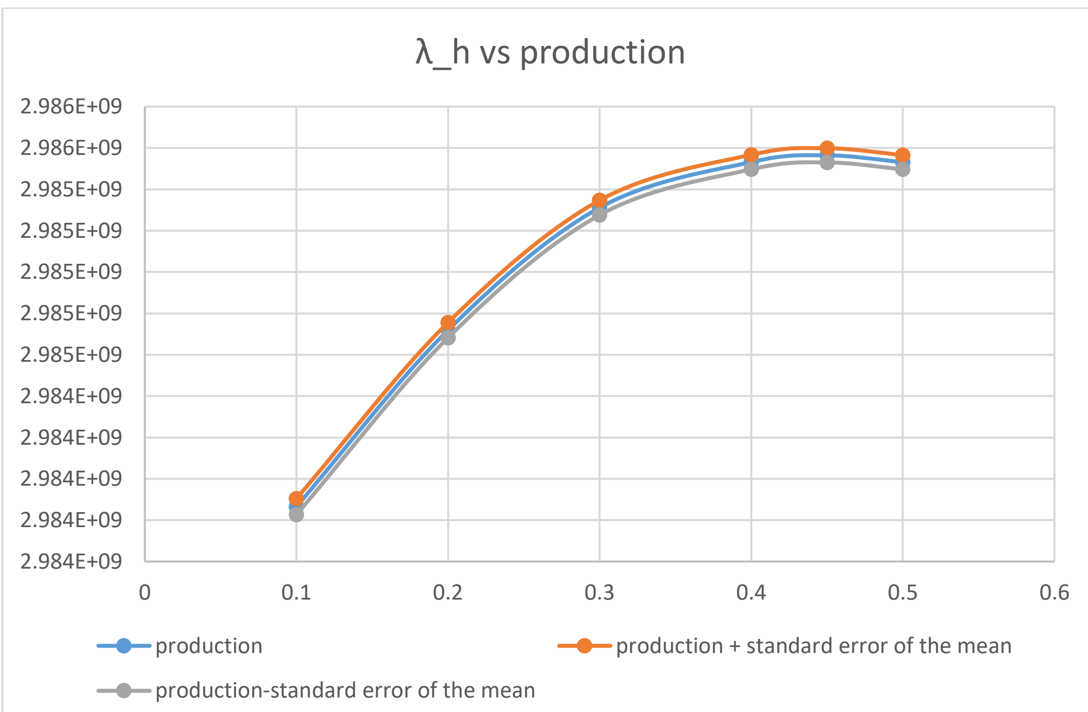


Figure 15 λ_h vs system performance

From the previous result, it is shown that the optimum λ_h is equal to 0.45

For component (3), for λ_h equal to 0.45, the estimated probability distribution function of specific time λ at with the PHM model stops the component is shown in figure (15) while figure (16) shows the estimated cumulative distribution function of the same specific times.

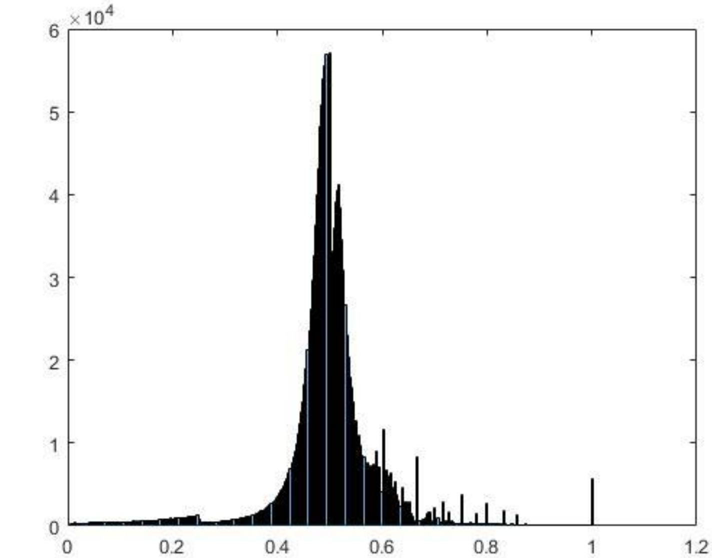


Figure 16 distribution of λ_t at which PHM stopped the component

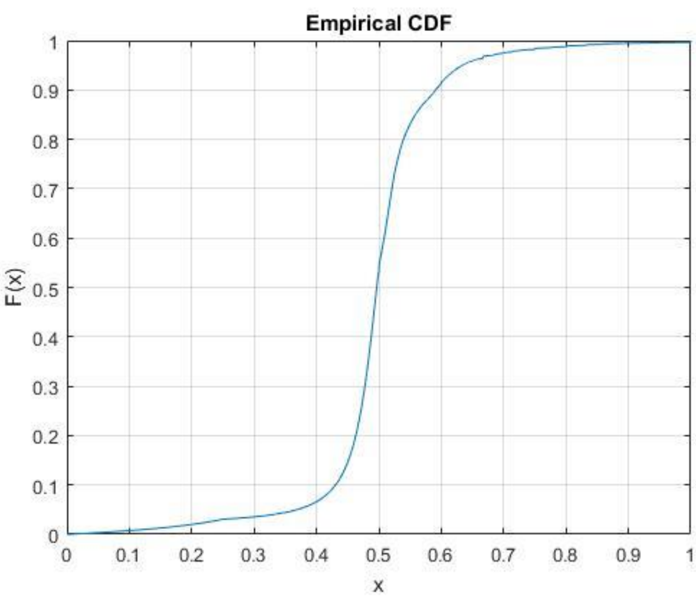


Figure 17 cumulative distribution of λ_t at which PHMS stopped the component

Table (10) shows the results of applying the previous steps for each component individually on the performance of the system with preventive maintenance threshold equal to 0.45:

COMPONENT	1 [TEG]	2 [EC]	3 [TG]	4 [TG]	5 [TC]	6 [TC]
PRODUCTION	2980134103	2980261826	2985645416	2985622385	2982603230	2982686567
PRODUCTION + STANDARD ERROR OF THE MEAN	2980181542	2980310588	2985679630	2985656806	2982646438	2982728996
PRODUCTION - STANDARD ERROR OF THE MEAN	2980086665	2980213063	2985611201	2985587964	2982560022	2982644137
STANDARD ERROR OF THE MEAN	47438.1791	48762.92971	34214.43359	34421.37353	43208.07379	42429.37208

Table 10 system performance when preventive maintenance applied for each component individually

Table (11) shows importance measures of the components using the equation:

$$\mathrm{I}_{\mathrm{i}}^{\mathrm{RAW}} = \frac{\overline{\mathrm{W}}_{\mathrm{s}}(\lambda_{\mathrm{s}}, \mu_{\mathrm{s}} | \overline{\mathrm{U}}_{\mathrm{i}}^{\mathrm{PHMS}})}{\overline{\mathrm{W}_{\mathrm{s}}}(\lambda_{\mathrm{s}}, \mu_{\mathrm{s}})}$$

COMPONENT	1 [TEG]	2 [EC]	3 [TG]	4 [TG]	5 [TC]	6 [TC]
IMPORTANCE MEASURE	0.99994748	0.999990335	1.001796732	1.001789005	1.000775964	1.000803927

Table 11 components importance measure

It should be mentioned that, although the previous equation gave the same ranking as [7] and [8], different relationships connecting λ_h, $X(\lambda_h)$, ε and the performance of the PHMS that can be achieved for the component can lead to different importance and different ranking.

In the next part of this case study, we perform a sensitivity analysis to investigate how PHM performance metric parameters affect the performance of the system and to do that, only one parameter at a time will be changed for analyzing the corresponding changes of system performance.

From figures (15) and (16), we can notice that in more than 90% of the cases the PHMS able to stop the component before actual specific time equal to 0.6 which means that the sensitivity of system performance to the values of relative accuracy at the last 40% of the component life time can be neglected.

The sensitivity of system performance to the value of $\underline{RA}_1$ is shown in figure (17) and as can be noticed the lower the value of $\underline{RA}_1$the higher the number of unnecessarily stops and the lower the value of system performance.

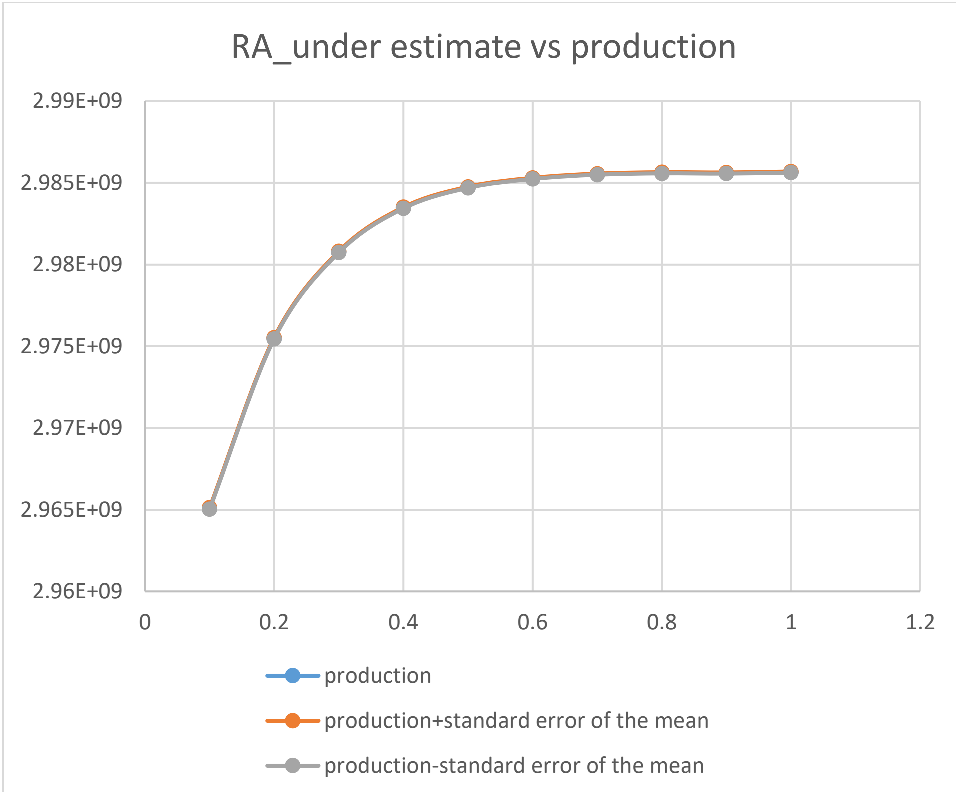


Figure 18 $\underline{RA}_1$ *vs system performance*

The sensitivity of system performance to the value of $\underline{RA}_2$ is shown in figure (18) and it is noticed that the rate of change system performance per changing $\underline{RA}_1$before $\underline{RA}_1$ reach to 0.6 is much higher than the rate change system performance per changing $\underline{RA}_2$.

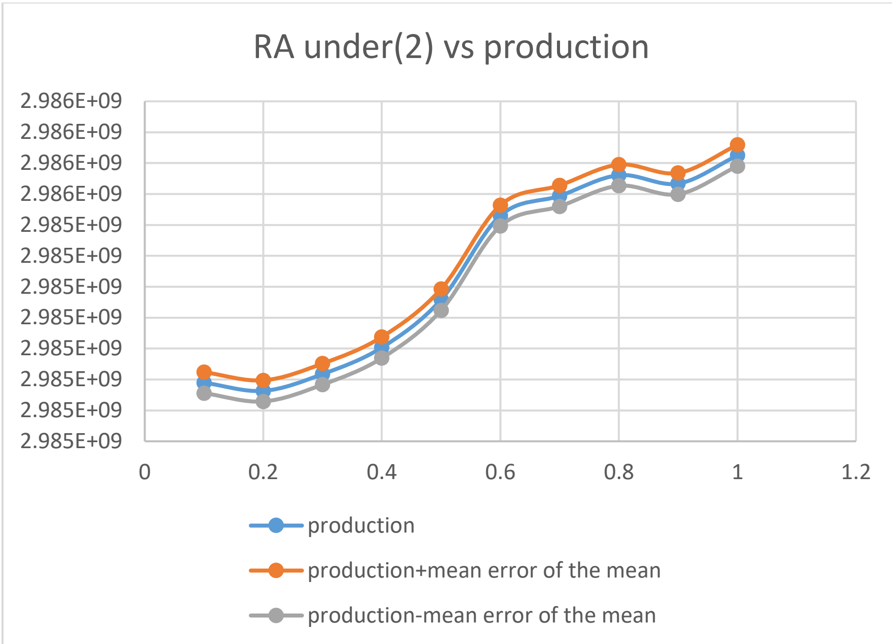


Figure 19 $\underline{RA}_2$ *vs system performance*

The sensitivity of system performance to the standard deviation of the value of $\underline{RA}_1$ is shown in figure (19)

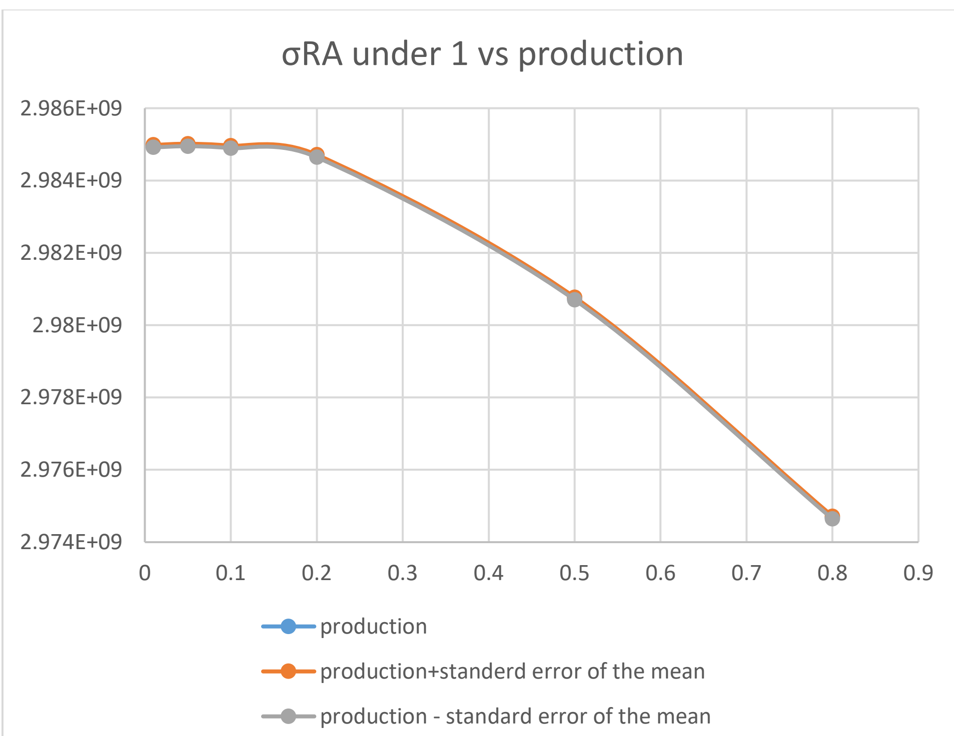


Figure 20 $\sigma_{\underline{RA}_1}$ *vs system performance*

As well, figure (20) shows the sensitivity of the system performance to standard deviation of the value of $\underline{RA}_2$

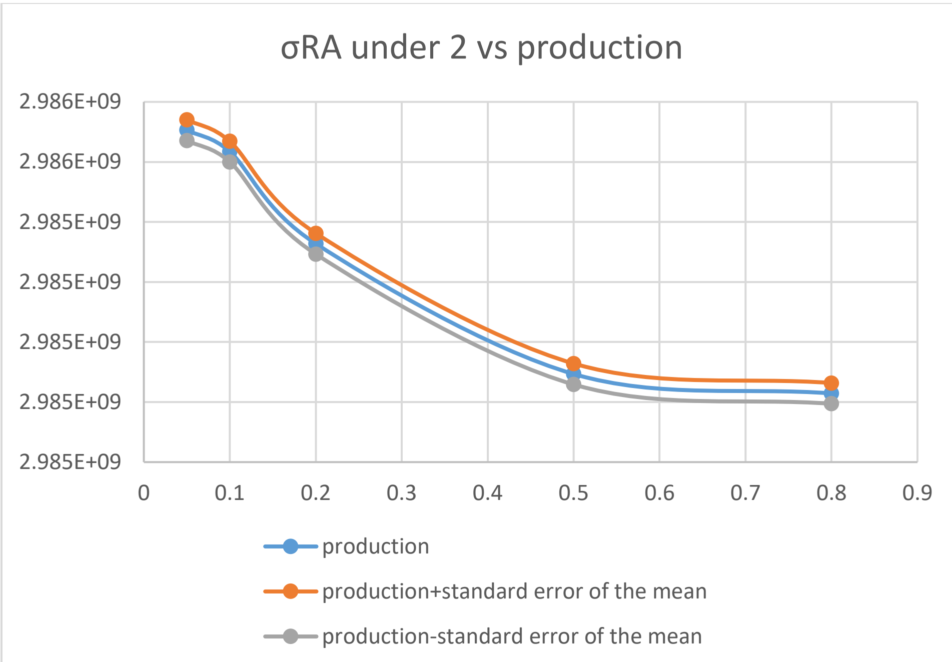


Figure 21 $\sigma_{\underline{RA}_2}$ *vs system performance*

After showing the sensitivity of the system performance to relative accuracy in case of RUL underestimation and to its standard deviation, sensitivity of system performance to Δt has been studied and it is shown in figure (21) and as expected by increasing Δt, number of predictions will decrease and hence system performance will also decrease.

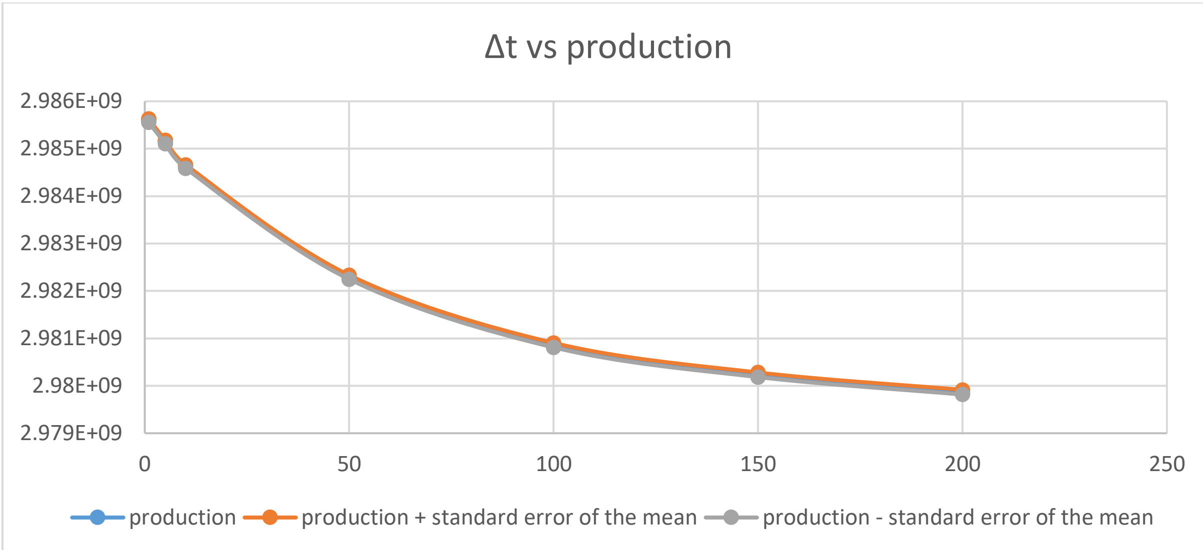


Figure 22 Δt vs system performance

On the other hand, system performance sensitivity to relative accuracy in case of RUL overestimation in the specific time between 25% to 75% of component life time is shown in figure (22) and figure (23). And the figures show that there is a very little sensitivity to the value of $\overline{RA}_2$ and $\overline{RA}_3$. This effects are because the system performance is much affected by the early stop of the component and in case of over estimation of the RUL this probability will be eliminated. Also, because of the preventive maintenance threshold which supposed to stop the component after 55% of life time so even if the RUL is over estimated there is still long time for early stop the component for preventive maintenance.

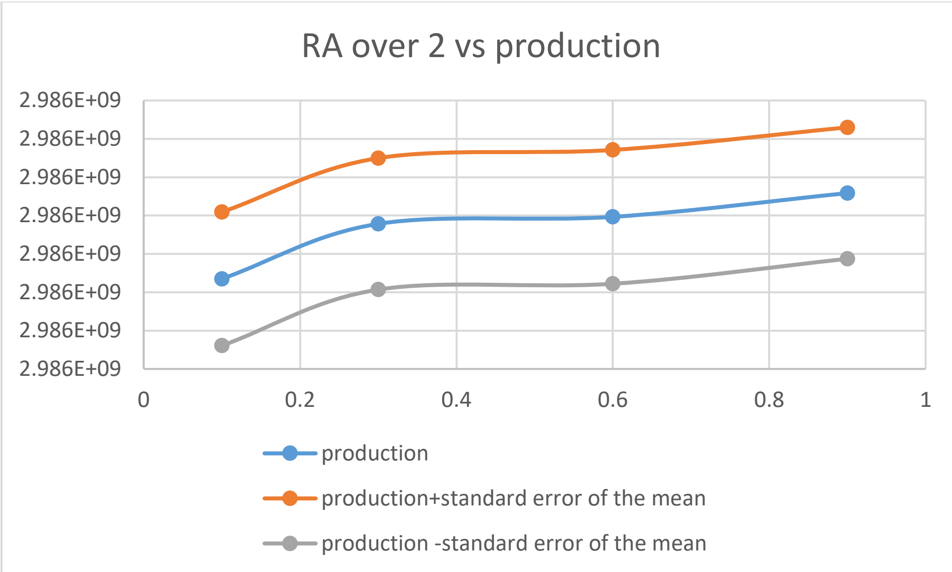


Figure 23 $\overline{RA}_2$ *vs system performance*

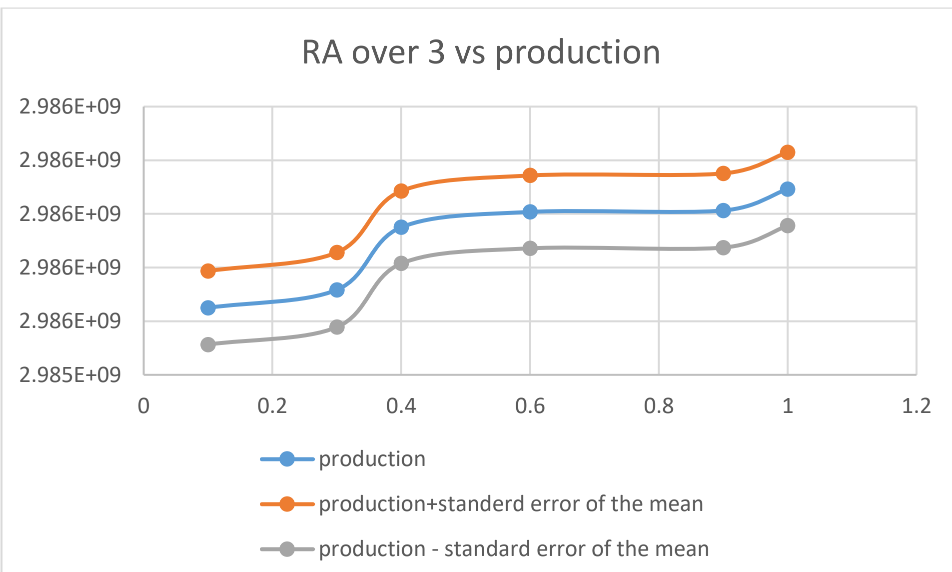


Figure 24 $\overline{RA}_3$ *vs system performance*

The other important result is the reduction of the maintenance cost after using preventive maintenance. We, also, will consider that the preventive maintenance cost will increase proportional to degradation state and the parameter $\phi = \frac{preventive\ maintenance\ cost}{correcrive\ maintenance\ cost}$ and the relationship between the parameter ϕ and degradation state $X(\lambda)$ will be like:

$$\phi = 0.2 + 0.8 * \frac{X(\lambda)}{100}$$

So the results shown in table (12) :

WITH PHM		WITHOUT A PHM	
ORGANIZED STOPS	164.7162	organized stops	0
TOTAL STOPS	166.1108	total stops	71.43923
ORGANIZED STOP COST	58.65028	organized stop cost	0
NON-ORGANIZED STOP COST	1.39458	non-organized stop cost	71.43923
TOTAL MAINTENANCE COST	60.04486	total maintenance cost	71.43923

Table 12 total maintenance cost with-PHM vs without-PHM

The “cost” cells must be multiplied by a factor equal to corrective maintenance cost in order to obtain the true cost.

From the previous results, it can be figured out that the ratio of overall maintenance cost of the component using PHMS to overall maintenance cost of the component without equipping with a PHMS is equal to (60.04486/71.43923) = 0.840503 which means that the overall maintenance cost of the component decreased by 15% because of equipping with PHMS. Also, it can be noticed that the unexpected failure of the component decreased by 98%.

Chapter (7): conclusion

In this work, a general framework to compute the availability of a PHMs-equipped component within a multi-state multi-component system has been proposed based on a prognostics and health management system monitoring and predicting the RUL periodically every Δt unit of time. The performance of the PHMS is defined by a metric able to describe the uncertainties in RUL prediction and mainly depends on (FP, $\overline{RA}$ and $\underline{RA}$). For simulating the PHM predicted RUL, one-side Chebyshev's inequalities have been used and that was mainly to estimate the least achievable performance benefits.

In the developing model, the component will be removed for maintenance once the predicted specific time fall below a predictive maintenance threshold. By early stop of the component, degradation state will be lower and time needed for repair process will be lower also and by choosing the optimum threshold, higher component availability can be achieved.

The previous model evaluating PHMS-equipped component availability is then integrated into the model evaluating the availability and the performance of the multi-state system containing this component. The aim is to evaluate the impact of component availability improvement on the overall system performance.

Finally, modification of the risk achievement worth (RAW) importance measure has been done in which it allows prioritizing components with respect to the expected benefits of equipping them with a PHMS.

The model has been applied to two case studies, the first one was for a single component and the goal was to achieve the maximum availability of the component while the second one was for a component within a multi-state multi-component system and the goal was to achieve the maximum system performance possible and to rank the components according to their importance regarding overall system performance.

A sensitivity analysis has been done to identify the parameters of PHM performance metric which the system is more sensitive to.

In the first case study, it was shown that equipping the component with PHMS increase its availability by 9.5%. this increase in availability can be attributed to the effectiveness of predictive maintenance based on installing of PHMS.

In the second case study, equipping the most important component with a PHMS and moving from corrective maintenance to predictive maintenance improves system performance by 0.18 %.

condition-based predictive maintenance based on installing a PHMS on the most component decreases the unexpected failure by 98% which means it is effective to decrease level of unexpected failure as periodic-based predictive maintenance without decreasing the overall system performance as periodic-based predictive maintenance.

Also, condition-based predictive maintenance was proved to be costly effective from overall component maintenance cost point of view as using predictive maintenance based on installing a PHMS decreased the overall component maintenance cost by about 15%.

Possible interesting development of this thesis work include developing a model connecting PHMS performance with capital cost of developing a PHMS and a model describing PHMS operation cost. Therefore, an optimum economic PHMS performance can be identified.

Another possible development of this thesis work, is to develop a framework able to identify the optimum PHMS model that can be used for every specific component.

Appendix (A): A Review on Prognostics and Health Management System (PHMS)

PHM is an innovative, interdisciplinary and relatively young research field, which is quickly developing thanks to the continuous development of sensors and monitoring systems, techniques for signal processing and machine learning, and the increase of computational capability of modern computers [21]. The main objective of PHM is to detect (i.e., by a fault detection system) and diagnose the abnormal behaviours of industrial equipment (i.e, by a fault diagnosis system), predict its future behavior (i.e., by a fault prognosis system), and opportunely schedule the maintenance actions before catastrophic failure [21]. This will indeed increase system availability and safety and reduce maintenance costs by provide optimized recommendations and prioritized operational actions [22].

1) The fault detection system

A fault detection system monitors the health state of the equipment and aids the decision maker to decide whether it is working in normal or abnormal conditions. A typical fault detection module is based on: 1) a signal reconstruction model and 2) a decision tool that supports the decision maker [23].

The signal reconstruction (empirical) model receives in input the sensor measurements of signals representative of the equipment behaviour and provides in output the signals values (called reconstructions) as if it was working in normal conditions. This is usually an auto-associative model, such as Auto-Associative Kernel Regression (AAKR) [24], Artificial Neural Networks (ANNs) [25], Principal Component Analysis (PCA) [26], Support Vector Machines (SVMs) [27] and Fuzzy Similarity [28], trained with data collected during the equipment operation in normal conditions.

The decision tool typically analyzes the residuals between the measured signals and the reconstructions: basically, if they are similar to each other, i.e., residuals are close to zero, then the equipment is recognized to be working in normal conditions and no alarm is triggered, and vice versa [29]. Traditional techniques, like threshold-based methods (i.e., an abnormal condition is detected when the residuals exceed a prefixed threshold) [30,31] and the Sequential

Probability Ratio Test (SPRT) (i.e., by considering the residual as a random variable whose statistical distribution is to be analyzed) [32,33], have been widely developed and applied with success in practice.

2) The fault diagnostics system

A fault diagnostics system typically solves a supervised classification problem [34]. The basic idea is to build an empirical classification model (classifier) whose parameters are tuned through an iterative process, called training, based on a set of examples that consist in pairs of measurements taken from monitored signals (temperature, pressure, vibration, etc.) and the label of the class of anomaly, that has led to the measurements [35]. A number of empirical diagnostics models based on various classification techniques such as K-Nearest Neighbor (KNN) [36], ANNs [37], SVMs [38], have been developed for the sake of the classification task. However, their practical application is often limited because, even though there might be plenty of monitored data, the information on the class of anomaly that is occurred is usually not available [39].

For these reasons, unsupervised fault diagnostics methods (also called clustering methods) partition the monitored data into dissimilar groups (whose number is a priori unknown), such that data belonging to the same group are more similar than those belonging to the other groups. In particular, one can distinguish, among the groups, different anomalous behaviors and relate them to specific root causes [40]. Several clustering algorithms have been proposed, like K-Means [41], Self-Organizing Maps (SOM) [42], Fuzzy C-Means (FCM) [43,44], and Hidden Markov Models (HMMs) [45].

3) The fault prognostics system

Prognostics systems rely on different sources of information and data to estimate the RUL (for example, a physical degradation model or run-to-failure data are available). To adapt to the variety of sources, a wide range of approaches has been developed, based on different modeling/computational schemes and processing algorithms. In general, the prognostics approaches can be categorized into model-based and data-driven [46].

Model-based approaches typically rely on comprehensive physical degradation models to describe the behavior of the equipment for the estimation of the RUL [47]. For example, Cadini et al. [48] used Particle Filtering (PF) method for estimating the RUL of an equipment subjected to a fatigue crack growth; in all cases, the RUL estimations are, then, to be embedded within an optimal policy of condition-based equipment replacement. Despite that these approaches lead to accurate prognostics results, uncertainty arising due to the assumptions and simplifications of the adopted physical models may pose limitations on their practical deployment [49,50].

Contrarily, data-driven prognostics approaches do not use any explicit physical model, but rely exclusively on the availability of process data related to equipment health to build (black-box) models that capture the degradation and failure modes of the equipment [47,50]. For example, Di Maio et al., [49] introduced a data-driven fuzzy similarity-based prognostics approach for estimating the RUL of equipment subject to fatigue cycles. In spite of the recognized potential of these data-driven approaches, challenges still exist for their practical applications [47,49], for example, to build the models, data-driven approaches usually require abundant complete run-to-failure data which, in some practical cases, might be expensive or impractical to obtain; for this reason, these data-driven approaches are, usually, applied for RUL estimation of equipment characterized by short life, rather than safety-critical and slow degrading equipment for which complete run-to-failure trajectories are very difficult or even infeasible to be acquired [47,51] and these approaches are computationally burdensome [47];

Appendix (B):

Importance measure was firstly introduced by Birnbaum [19] which measure the rate of system reliability changing to the change of component reliability and components with higher values are with higher importance measures. The following equation shows Birnbaum importance measure:

$$I_i^B(t) = \frac{\partial R_S(t)}{\partial R_i(t)} = R_s(t; R_i(t) = 1) - R_s(t; R_i(t) = 0)$$

$I_i^B(t)$ is the Birnbaum importance of component i, $R_s(t)$ is the reliability of the system at time t, $R_i(t)$ is the reliability of the component i at time t, $(R_s(t;\ R_i(t) = 0)$ is the reliability of the system knowing that component i reliability is equal to 0 and $R_s(t;\ R_i(t) = 1)$ is the system reliability knowing that component i reliability is equal to 1.

Birnbaum importance ranking represents the maximum difference in system reliability when component i transfers from perfect functioning ($R_i(t) = 1$) to certain failure ($R_i(t) = 0$). The weak point in Birnbaum importance is that it does not depend on the component reliability. Therefore, two components can have the same Birnbaum importance but different level of reliabilities and in practice less reliable component is considered more important.

As an extinction of Birnbaum importance but including the unreliability into consideration is criticality importance and identified by the following equation:

$$I_i^{CR}(i) = I_i^B(t)\frac{1 - R_i(t)}{1 - R_s(t)}$$

While $I_i^B(t)$ is Birnbaum importance, $(1 - R_i(t))$ is component unreliability and $(1 - R_s(t))$ is system unreliability.

Other two types of importance measure that are commonly used to rank the importance of components within a system are the reliability reduction worth (RRW) and reliability achievement worth (RAW) [15]. RRW calculate the impact of component i losing reliability on the reliability of the system while RAW calculate the impact of increasing component i reliability on the reliability of the system and the equations describing RRW and RAW are:

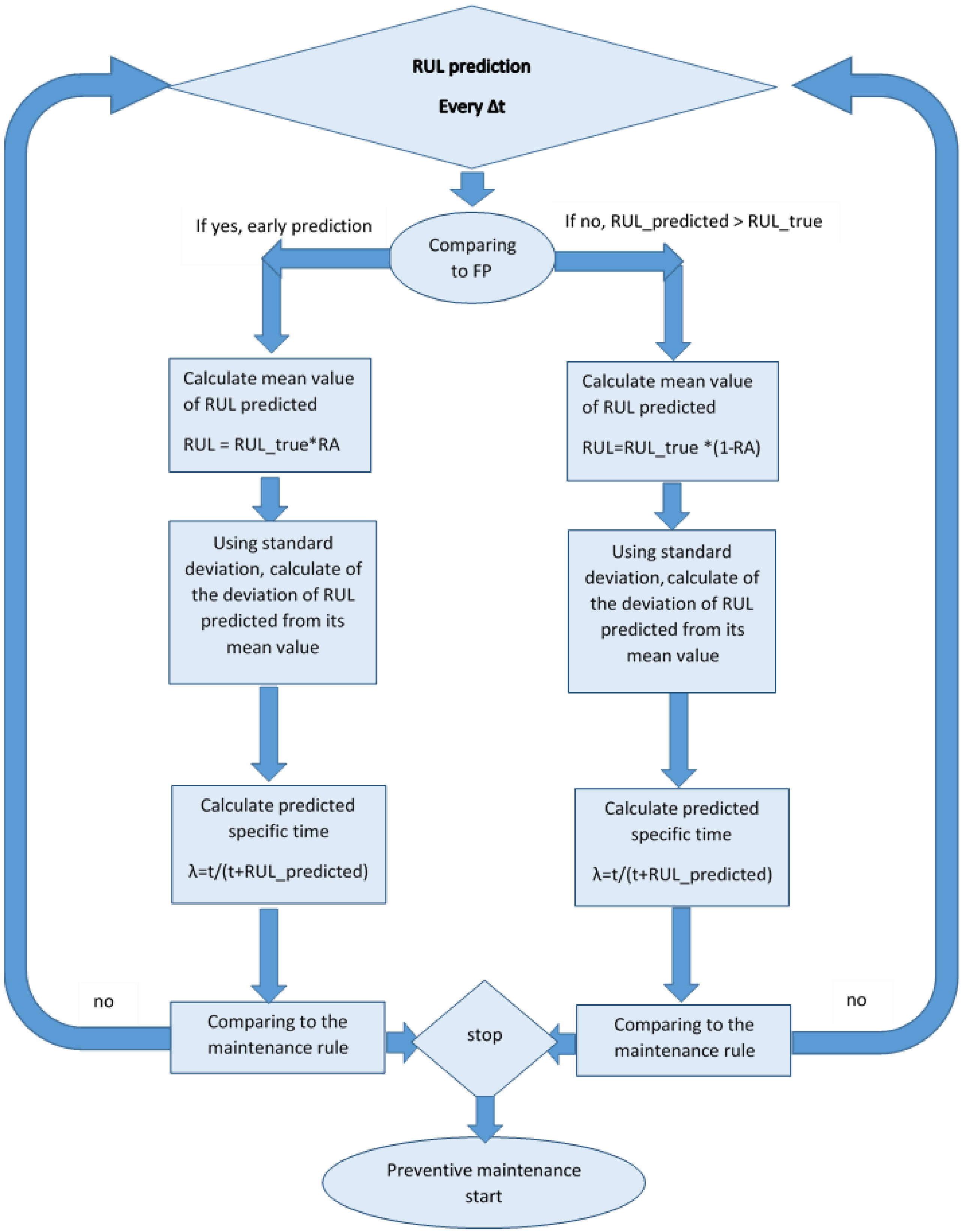
$$RRW_i = \frac{R_S(t)}{R_S(t; R_i(t) = 0)}$$

$$RAW_i = \frac{R_s(t; R_i(t) = 1)}{R_S(t)}$$

Also, Fussell and Vesely proposed an importance measure metric and it depends on the number and order of the cut-sets in which the component appears and can be described by the following equation:

$$I_i^{FV}(t) = \frac{R_S(t) - R_S(t; (t) = 0)}{R_S(t)}$$

Appendix (C)


RUL prediction
Every Δt
If yes, early prediction
Comparing to FP
If no, RUL_predicted > RUL_true
Calculate mean value of RUL predicted
RUL = RUL_true*RA
Calculate mean value of RUL predicted
RUL=RUL_true *(1-RA)
Using standard deviation, calculate of the deviation of RUL predicted from its mean value
Using standard deviation, calculate of the deviation of RUL predicted from its mean value
Calculate predicted specific time
λ=t/(t+RUL_predicted)
Calculate predicted specific time
λ=t/(t+RUL_predicted)
no
Comparing to the maintenance rule
stop
Comparing to the maintenance rule
no
Preventive maintenance start

Appendix (D)

Data: $\overline{RA}$, FP, $\underline{RA}$, $\sigma_{\overline{RA}}$, $\sigma_{\underline{RA}}$, Δt, MTTR, T_start, T_failure

Results: average annual oil production, average annual gas production and total revenue

Repeat for every life time of the component inside the Monte Carlo model simulating the all system

for $\frac{T_{start}-T}{\Delta t}$ **to** $\frac{T_{failure}-T}{\Delta t}$

If maintenance team is available

If rand

$\hat{\gamma}_\lambda = E[\underline{RA_\lambda}]*RUL^*(t_\lambda)$

$$\mathbb{P}(\gamma_\lambda \geq \hat{\gamma}_\lambda - a) \geq 1 - \frac{\sigma^2_{(\underline{RA})_\lambda} * RUL^*(t_\lambda)^2}{\sigma^2_{(\underline{RA})_\lambda} * RUL^*(t_\lambda)^2 + a^2}$$

$$\lambda_{predicted} = \frac{\gamma_\lambda}{\gamma_\lambda + t_\lambda}$$

Else

$\hat{\gamma}_\lambda$= (2-$E[\overline{RA_\lambda}]$)*$RUL^*(t_\lambda)$

$$\mathbb{P}(\gamma_\lambda \geq \hat{\gamma}_\lambda + a) \leq \frac{\sigma^2_{(\overline{RA})\lambda} * RUL^*(t_\lambda)^2}{\sigma^2_{(\overline{RA})\lambda} * RUL^*(t_\lambda)^2 + a^2}$$

$$\lambda_{predicted} = \frac{\gamma_\lambda}{\gamma_\lambda + t_\lambda}$$

End

If maintenance rules applied

Component stop

$$X(\lambda) = 53.568 * \lambda^3 + 49.988 * \lambda^2 - 1.7128\lambda$$

$$\varepsilon = \frac{MTTR_{preventive\ maintenanc}}{MTTR_{corrective\ maintenance}} = \left(0.2 + 0.8 * \frac{X(\lambda)}{100}\right) * 0.7$$

end

End

End

Appendix (E):

Data: $\underline{ra}_\lambda, \overline{ra}_\lambda, \sigma_{\underline{RA}_\lambda}, \sigma_{\overline{RA}_\lambda}, fp_\lambda\ \forall\lambda \in [0;1]$; $T_{cor}, T_{pred}, f_{T_f}, f_{T_d}, f_{DTD}, \Delta t, h, j, T$

Result: $U(t) = 1 - A(t)$

Repeat B times the following procedure:

Initialize $T_{es} \leftarrow 0$, MC counter $W_b(0:T) \leftarrow 0$

while $T_{es} < T$ **do**

$\quad$ $stop \leftarrow 0$, $T_d \sim f_{T_d}(t) + T_{es}$, $DTD \sim f_{DTD}(t)$, $T_{pr} \leftarrow (\lfloor \frac{T_d + DTD}{\Delta t} \rfloor + 1)\Delta t$, $T_f \sim f_{T_f}(t) + T_d$, $N \leftarrow \lfloor \frac{T_f - T_{pr}}{\Delta t} \rfloor$, $T_{main} \leftarrow T_{cor}$, $T_s \leftarrow T_f$

$\quad$ **if** $N > j$ **then**

$\quad\quad$ **for** k *in* $0 : N - h - 1$ **do**

$\quad\quad\quad$ $\lambda \leftarrow \frac{k}{N}$

$\quad\quad\quad$ **if** $\underline{ra}_\lambda(N-k)\Delta t \leq h \cdot \Delta t$ **then**

$\quad\quad\quad\quad$ $stop \sim Be(fp_\lambda)$

$\quad\quad\quad$ **else**

$\quad\quad\quad\quad$ $a \leftarrow h \cdot \Delta t - \underline{ra}_\lambda(N-k)\Delta t, \quad stop \sim Be\left(\frac{\sigma^2_{\underline{RA}_\lambda}[(N-k)\Delta t]^2}{\sigma^2_{\underline{RA}_\lambda}[(N-k)\Delta t]^2 + a^2} \times fp_\lambda\right)$

$\quad\quad\quad$ **end**

$\quad\quad\quad$ **if** $stop = 1$ **then**

$\quad\quad\quad\quad$ $t_\lambda \leftarrow T_{pr} + k \cdot \Delta t$, $T_{main} \leftarrow T_{pred}$, $T_s \leftarrow t_\lambda + j \cdot \Delta t$, go to $\star$

$\quad\quad\quad$ **end**

$\quad\quad$ **end**

$\quad\quad$ **for** k *in* $N - h : N - j$ **do**

$\quad\quad\quad$ $\lambda \leftarrow \frac{k}{N}$

$\quad\quad\quad$ **if** $(2 - \overline{ra}_\lambda)(N-k)\Delta t \leq h \cdot \Delta t$ **then**

$\quad\quad\quad\quad$ $a \leftarrow h \cdot \Delta t - (2 - \overline{ra}_\lambda)(N-k)\Delta t, \quad stop \sim Be\left(fp_\lambda + (1 - fp_\lambda) \times \frac{\sigma^2_{\overline{RA}_\lambda}[(N-k)\Delta t]^2}{\sigma^2_{\overline{RA}_\lambda}[(N-k)\Delta t]^2 + a^2}\right)$

$\quad\quad\quad$ **else**

$\quad\quad\quad\quad$ $stop \sim Be(fp_\lambda)$

$\quad\quad\quad$ **end**

$\quad\quad\quad$ **if** $stop = 1$ **then**

$\quad\quad\quad\quad$ $t_\lambda \leftarrow T_{pr} + k \cdot \Delta t$, $T_{main} \leftarrow T_{pred}$, $T_s \leftarrow t_\lambda + j \cdot \Delta t$, go to $\star$

$\quad\quad\quad$ **end**

$\quad\quad$ **end**

$\quad$ **end**

$\quad$ $\star$: $T_{es} \leftarrow T_s + T_{main}$, $W_b(T_s : T_{es}) \leftarrow 1$

end

After B times

$$U(t) = 1 - A(t) \simeq \frac{\sum_{b=1}^{B} W_b(t)}{B}$$

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