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An efficient algorithm to support recycling systems design

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Ai miei familiari

e a tutti coloro che mi hanno sostenuto

durante questo lungo percorso di studi

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Abstract

Circular economy principles introduced new challenges, moving from traditional "take, make, dispose" concept to a new paradigm, in which wastes re-enter in the loop as raw materials, with the final purpose of reduce waste, increase environmental friendly processes and create new adaptable and resilient manufacturing and demanufacturing systems.

Waste from Electric and Electronic Equipment (WEEE) is the fastest growing waste in European Union. Their recycling is interesting since approximately 30% of their composition is made of valuable materials, such as precious metals and rare earths. The major challenge in this process is the high End-of-Life (EoL) products variability (both in design and materials). In EU, recycling is performed by Small and Medium Enterprises (SMEs) through linear and monolithic lines, with long set-up times and loss of time and material when products under treatment change.

In parallel, a relevant set of technologies (from different industrial sectors) for recycling is available. As a consequence, a proper system design is required.

This work focuses on the development of a computerized tool to support the design phase of recycling systems.

Starting from state of the art model for recycling systems([3]), two different algorithms for buffer allocation and machine choice developed for manufacturing processes ([1][27]) have been adapted to de-manufacturing systems, implementing a software to support design phase in relatively short times.

This IT tool has been tested and validated on a proper recycling line, comparing it with an extensive, time consuming research, giving as result the best machines combination (from a machine set) and the optimal buffer capacities that maximize the profit.

Finally, the developed software has been used on a real case, the recycling cell of the De- and Remanufacturing pilot plant at Institute of Industrial Technologies and Automation of National Research Council (ITIA-CNR) in Milan.

Sommario

Il lavoro di Tesi qui proposto si sviluppa nell'ambito del riciclo di materiali. Questo tema ha acquisito grande rilevanza negli ultimi anni, specie in Europa, considerato il sempre maggiore consumo di materie prime da parte delle industrie e la sempre minore disponibilità di risorse naturali e conseguente aumento del prezzo di mercato. Il riciclo rappresenta l'ultima fase della cosiddetta Economia Circolare: un modello economico finalizzato non agli scarti e allo spreco di materiali da parte delle industrie, ma al disassemblaggio dei prodotti in fin di vita e al riutilizzo dei loro componenti e materie prime. L'Economia Circolare è quindi come una grande opportunità per la riduzione degli sprechi e conseguente guadagno economico per le aziende. Per questo motivo negli ultimi tempi gli Stati Europei si stanno impegnando a promulgare leggi per l'eliminazione delle discariche, favorendo la costruzione e l'uso di impianti in grado di estrarre e separare le materie prime dei prodotti in fin di vita. Di particolare importanza è il riciclo di rifiuti provenienti da prodotti elettronici ed elettrici (WEEE). Essi infatti contengono rilevanti quantità di metalli preziosi come oro, argento, rame, alluminio, ferro e acciaio.

Un tipico impianto di riciclo di materiali è composto d diversi processi, tra cui un processo di Shredding, ovvero sminuzzamento e triturazione dei prodotti iniziali, un processo di Separazione, che separa due o più materiali di interesse in appositi flussi e un processo di Mixing, che mischia due o più materiali provenienti da diversi flussi. Ogni processo può essere svolto tramite diverse tecnologie e presenta particolari caratteristiche e performance di interesse.

In quest'ambito, il lavoro di Tesi propone la creazione di un software, utile in fase di design, in grado di modellare un sistema di riciclo di materiali. Data una linea predisposta di stadi e buffer di stoccaggio, con diverse possibili macchine per ogni stadio, è in grado di scegliere simultaneamente la combinazione ottimale di macchine e la capacità ottimale di ciascun buffer, massimizzando il profitto del sistema in temi brevi. Il programma si appoggia all'algoritmo creato da J. A. Traina e S. B. Gershwin [27] che è sua volta un'estensione del metodo di

Gershwin e C. Shi [1] per la selezione ottimale di buffer e macchine in un sistema di Manufacturing. L'algoritmo di Traina-Gershwin è stato rivisitato su un sistema di riciclo di materiali nella creazione del software.

Il software è stato validato tramite una serie di esperimenti su fittizia linea di riciclo composta da 4 stadi e 3 buffer, con 4 possibili macchine di scelta per ciascuno stadio (un totale di 256 possibili combinazioni), che sono stati svolti utilizzando sia il programma sviluppato, sia un programma che svolge una ricerca esplorativa su tutte le possibili combinazioni di macchine; si è osservato che, per tutti gli esperimenti svolti, le soluzioni ottimali dei due metodi, in termini di combinazione di macchine, dimensione dei buffer e profitto ottimo, coincidono.

Infine il programma è stato applicato su un caso reale: l'impianto pilota di riciclo di materiali dell'Istituto di Tecnologie Industriali ed Automazione del Consiglio Nazionale di Ricerca (ITIA-CNR) di Milano.

Chapter 1

Introduction to Circular Economy

1.1 Circular Economy: paradigm and benefits

Since the first Industrial Revolution, the industrialized countries have developed a "take-make-dispose" pattern of growth – a linear model based on the assumption that resources are abundant, available, easy to source and cheap to dispose of –. But in the last years a significant problem has emerged: natural resources on Earth are gradually running out and valuable raw materials are leaking from European and global industries.

According to the research conducted by Mckinsey Center for Business and Environmental of 2015 [4], the European industries still operate in a "take-make-dispose" model, wasting many resources for the products value creation. In 2012, in Europe, the average pro-capital consume of material has been 16 metric tons [4] with a related waste flow composed by 60% [4] of landfilled or incinerated products against 40% [4] of recycled or reused commodities. In value terms, in Europe, on average, just 5% [4] of material and energy value was recovered from wasted material; the remaining 95% [4] was completely lost. Materials like steel, polyethylene terephthalate (PET), and paper lose 30 to 75 percent [4] of their value in the first-use cycle.

Focusing on mobility, food and built sectors, Ellen Mac Arthur Foundation estimated that this "take-make-dispose" model costs to Europe €7.2 trillion every year [4], subdivided in this way: 1.8€ trillion relates to the actual resources costs; 3.4€ trillion to other related cash costs and 2.0€ trillion to different kinds of externalities, such as traffic congestion, carbon, pollution, and noise [4].

As a consequence, in the last years European industry moved towards a new innovative business model: the so called 'Circular Economy'.

In the article “L'economia: efficiente nell'impiego delle risorse, ecologica e circolare”, the European Environment Agency, affirms: «The expression 'circular economy' indicates a production and consumption system that generates the least possible losses. In an ideal world, almost anything would be reused, recycled or recovered to produce other end-products. The redesign of products and processes production could help to minimize waste and to transform the parties do not used resources» [5]. The key message is clear: the resources kept at the End-of-Life (EoL) of the product could be productively reused several times and therefore create further value, minimizing the resources waste.

The Circular Economy business model involves innovation throughout the value chain, rather than relying on solutions for End-of-Life products. The starting point of this model is a change in the design phase: products can be designed to be used longer, repaired, upgraded, remanufactured or eventually recycled, instead of being wasted. Production processes can be based more on the reusability of products and raw materials, rather than on the exploitation of natural resources in an exhaustive way, allowing to save many expenditures and to deliver more ecological products to the market.

In Figure 1.1 [6] the main phases of a Circular Economy model are shown. Each arrow composing the circle presents opportunities of reducing costs and dependence on natural resources. The initial raw materials are not wasted at the end of the product lifecycle, but they are mainly recycled and reused for a new production loop.

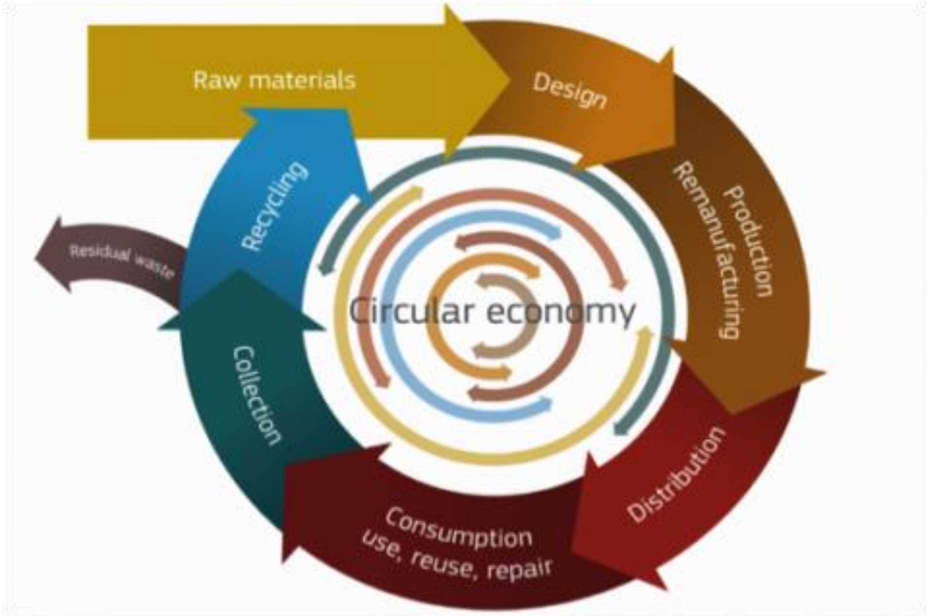
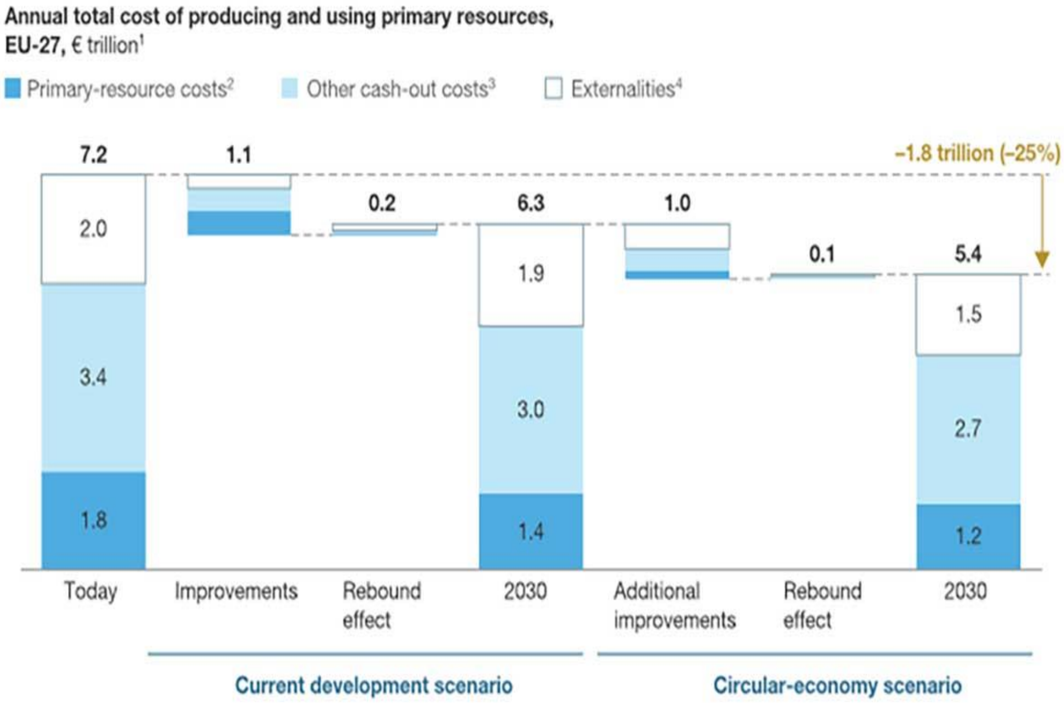


Figure 1.1: Circular Economy model [6]

The Circular Economy model could overall bring many benefits to the European economy and society.

This new business model is enabled by the Fourth Industrial Revolution, also called 'Industry 4.0'. The term 'Industry 4.0' indicates the whole set of industrial systems based on a real-time data exchange given, by the interconnections between humans and machines, so to create a digital virtual copy of the real world [30]. Industry 4.0 involves the use of several growing technologies like artificial intelligence, robotics, drones, the Internet of Things, 3-D printing, nanotechnology, ICT and mobile technology [31]. Industry 4.0 allows therefore the creation of Cyberphisical systems (CPS), which facilitate the transition from a linear production system to a circular production system [30].

The Circular economy could benefit Europe not only for a social and environmental point of view, but also in economic terms, saving up to €1.8 trillion annually by 2030 [4]. In detail, it would allow to grow primary resource productivity up to 3 percent annually, creating a benefit of 0.6€ trillion per year; furthermore, it would generate €1.2 trillion in non-resource and externality benefits. As shown in graph 1.1 [4].



Graph 1.1: Difference in production costs between current and circular scenario [4]

The analysis conducted by McKinsey Center for Business and Environmental of 2015 forecasts, in the next decades, a disruptive impact of digital and broader technology revolution on three particular sectors: mobility, food and housing.

As a matter of fact, in the mobility sector, solutions like car-sharing, autonomous and driverless driving, electric vehicles and the use of better materials could lead to a decrease of the average cost per car-kilometer about 75%. In food, the precision agriculture plays an important role: it could bring benefits in terms of input efficiency of water and fertilizers – an estimated improvement of 20-30% –; this, together with no-tillage farming, could reduce machinery and input costs by up to 75%. In building sector, there could be a reduction of construction costs up to 50% with the use of industrial and modular processes, rather than on-site traditional construction ones and the use of passive houses could even fully eliminate the houses energy need [4].

In Europe, the public sector and policy makers need to be able to integrate these new technologies and business models into the economy, so to minimize the structural waste, in particular in these three sectors.

Furthermore, an increase in resource productivity could bring benefits in sectors like individualized transportation, floor space and food: it has been shown that in these sectors there could be a total annually cost decrease by 0.9€ trillion by 2030, or a reduction of 12% [4].

Studies also suggest that in Europe, the Circular Economy model could bring benefits also in terms of better welfare, GDP, and employment rates than the Take-make-dispose model. According to these studies, in 2030 European households will have a disposable income 11% higher than now, or 7% higher in GDP terms [4].

The Circular Economy also improves the environment and it boosts competitiveness and resilience, by decoupling economic growth from resource use. Just for what concerns the three study sectors described before, the new technologies could lead to an overall decrease of CO_2 emissions by 48 percent by 2030 and by 83 percent by 2050 [4].

Moreover, the recycling activity, the last but not less important phase of the Circular Economy system, creates many jobs at a higher income level than landfilling or incinerating waste. Just in the recycling sector, an increment of the employment rate by 45% from 2000 to 2007 has been estimated; in absolute terms, an increment from 422 per million inhabitants in 2000 to 611 in 2007 [7]. Regarding the future scenario, the European Commission of Bruxelles of 2014 foresees that if all the Member states endeavor to virtually eliminate the landfill of municipal waste by 2030, and successfully implement waste recycling plants, more than 180000 direct jobs will be created in Europe by 2030, in addition to the already estimated 400000 jobs created thanks to the law currently in force [6].

1.2 The role of legislation

The European governments and policy makers play an important role, for the European countries, in the transition to the Circular Economy model, with all the potential improvements in terms of environment, economy and social conditions it can bring. They promote Circular Economy practices and subsidize recycling activities rather than waste landfilling.

European Union laws [EC 2002/96], [EC 2003/108] and [EC 2008/34] obligate the WEEE producers to bear the collection, the recovery and the safe disposal of their products under some recycling targets. The directive [EU 2012/19], published on Tuesday July 24th 2012, encourages Member States to adopt a WEEE separate collection system, to achieve the minimum targets for collection of these waste. Also, important measures have been taken against the phenomenon of illegal export of WEEE in Europe. In particular, in Italy, the Ministerial Decree 185, enacted in 2007 (DM 185/2007), regulates the relationship between WEEE distributors and collection centers, imposing a WEEE collection into five different products groups – refrigeration and air conditioning, large appliances, TV and display, the category of small appliances, consumer electronics, informatics equipment, lighting appliances, and light sources – [3].

Furthermore, the European Commission of Bruxelles seeks to help Member States to boost the several benefits gained from the recycle of municipal waste in the incoming years, proposing different actions:

- Impose a minimum level of waste recycle of 70% by 2030;
- Bring the recycling rate for packaging waste to 80% by 2030;
- Ban the landfills by 2025, and virtually eliminate them by 2030;
- Promote the development of markets of high quality secondary raw materials, including the development of evaluation criteria for certain kinds E-waste materials;

- Ensure a high recycling quality level [6].

1.3 The recycling practice

One of the most important steps in the Circular Economy chain is the last phase: recycling. Basically, it enables the separation of different raw materials by an initial product, so to sell them to the industrial and consumers' market. This activity calls for several stages and several kinds of processes along a recycling chain; we must consider some materials properties, process parameters to set and final performances, that will be all deeply explained in the following chapter.

Materials recycling dates to antiquity. Some proofs show that since Plato's time, in the fourth century BC, some materials (such as ash, broken tools and pottery) were recovered and reutilized for different purposes. In pre-industrial times, in Europe, bronze and other metals were collected and melted for a continuous reuse. In 1031, in Japan, paper was recycled for the first time. In 1813, in Batley, Yorkshire, Benjamin Law was able to transform rags into 'shoddy' and 'mungo' wool. This kind of industry, in the West Yorkshire, lasted at lest to 1914. The industrialization period lead to an increase in demand of affordable materials, in the USA as in Europe. In the 19th and 20th centuries, there was a kind of development in the metals recycling - particularly in relation to the railroad field and the automobile field - and in glass recycling – relation to the recycling of beverages bottles – [29].

The materials recycling practice reached a peak after the1970s because of a significant increase of energy cost; in particular way, the E-waste recycling was born in 1991, with the first recycling system in Switzerland. In 2000 there was a large increase in the sale of electronic devices and, consequently, an increase of E-waste production: this called for new investments in modern automated E-waste recycling facilities, to cope with the old redundant ones [29].

Actually, the recycling activity is still at an early stage of development and currently recycling systems are far to be as “intelligent” as modern production systems (e.g: modern manufacturing systems). Design improvements are needed, ones that take into account market demand, increasing variability of products and materials, and high volatility of prices of secondary materials [12].

1.4 The future of recycling systems

A lot of work has still to be done, in order to develop efficient and effective recycling systems, especially in the design field. Basically, actual recycling systems are affected by rigidity and non-controllability of the process. To now, we can indicate some specific key points which could make the guidelines for a future improvement:

- **Make recycling system modular and reconfigurable**, in order to cope with the variety of material in inputs, requiring different treatment. Of course, the reconfiguration design must be supported with advanced design methodology,
- **Integrate production monitoring in recycling systems**, through advanced online quality control technologies and methods, to better understand the system dynamics,
- **Integrate production planning and control in recycling systems**: it is necessary, considering that the utilization of different resources of the system depends on the incoming material mixture composition [12].

1.5 Focus on WEEE

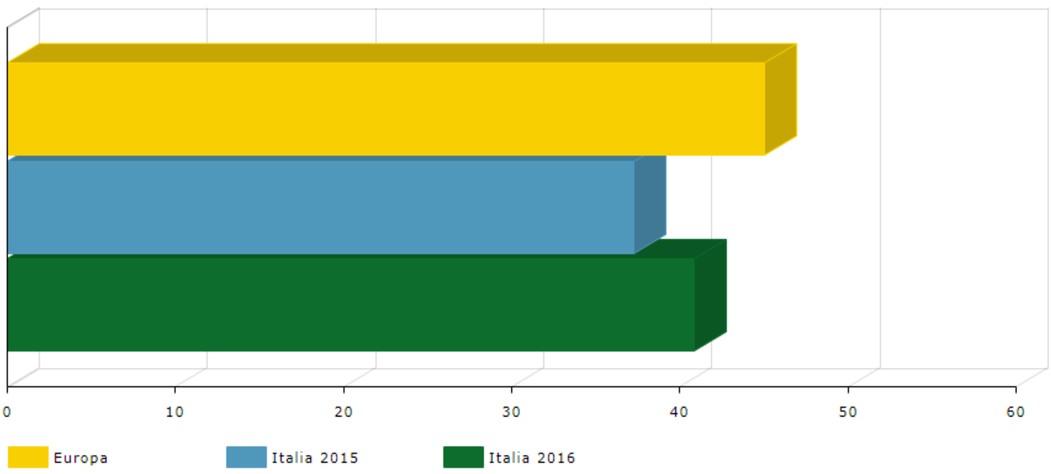
With a growth rate of 10% in the last five years, the recycling market is becoming one of the most relevant industrial sectors. In this field, one of the most important branch is the recycling of WEEE (Waste Electrical Electronical Equipment) - like TVs, PCs, fridges and cell phones; with its growing waste stream, in the EU, which is supposed to reach a value of more than 12 million tons per year by 2020 [8].

WEEE is a complex mixture of materials and components responsible for the major environmental and health problems, because of the intrinsic presence of hazardous components. They cannot be wasted in normal landfills, but they need a typical proper management. The modern electrical products contain a considerable amount of scarce and expensive rare materials, like gold, copper, aluminum and silver. It is therefore necessary that European States encourage the improvement of collection, treatment and recycling of electronics at the end of their lifetime, so to improve the environmental management of WEEE, contribute to a circular economy and enhance resource efficiency [9].

The problem of the disposal of WEEE emerged in the last years, concurrently with the frenetic technological development, the capillary diffusion of portable PCs and the wide diffusion of mobile phones, smartphones and tablets, in addition to the new plasma televisions and LCDs.

According to the WEEE Countering Illegal Trade, in Europe, only 35% of WEEE is recycled properly, the remaining 65% is lost for several reasons. Within Europe, Sweden and Norway have recycled approximately 80% of WEEE in 2015, overcoming the average EU value of 50%; whereas Italy and other countries like Latvia, Spain and Romania are below the average [10].

In graph 1.2 [11] we can see what the Italian WEEE collection rate was in 2015 and 2016, in comparison with the average European one.



Graph 1.2: difference in percentage of WEEE collection rate Europe-Italy, in 2015 [11]

There is an improvement from 2015 to 2016 for what concerns Italy, passing from 37.23% to 40.87%, but still far from the European target level of 45%.

1.5.1 An approach to WEEE management

The amount of WEEE is much increased in the last years and it is supposed to further increase by 3-5% annually [50]. This called for the introduction of Producer Responsibility (PR) directives for WEEE in the EU, with the purpose of involving the manufacturer in the EoL management of their products. However, with the current implementation of this directive, this objective has not been achieved, because many manufacturers refuse to submit to these WEEE directive requirements. As a matter of fact, WEEE recycling facilities has been isolated and they are often developed ad hoc, allowing recovery treatments based only on limited capabilities and available resources. These facilities have now to improve their performances to meet the ever-increasing number of WEEE legislative requirement. To this goal, it has been extensively demonstrated that the adoption of a specific systematic approach for recycling process planning could improve by 20 – 30% the economic and environmental performances of these facilities [13]. The basic idea is the development of a recycling process planner (RPP) working with a variant approach: an initial standard process plan. is set, but it can be modified considering the similarities in the features and attributes among a family of products/parts. The result is a tailored recycling process plan for each electrical/electronic product.

The recycling process planning framework consists of four stages:

- product evaluation,
- legislative compliance monitoring,
- recycling process planning,
- Ecological and Economical (Eco^2) assessment [13],

This is schematized in figure 1.2 [13]:

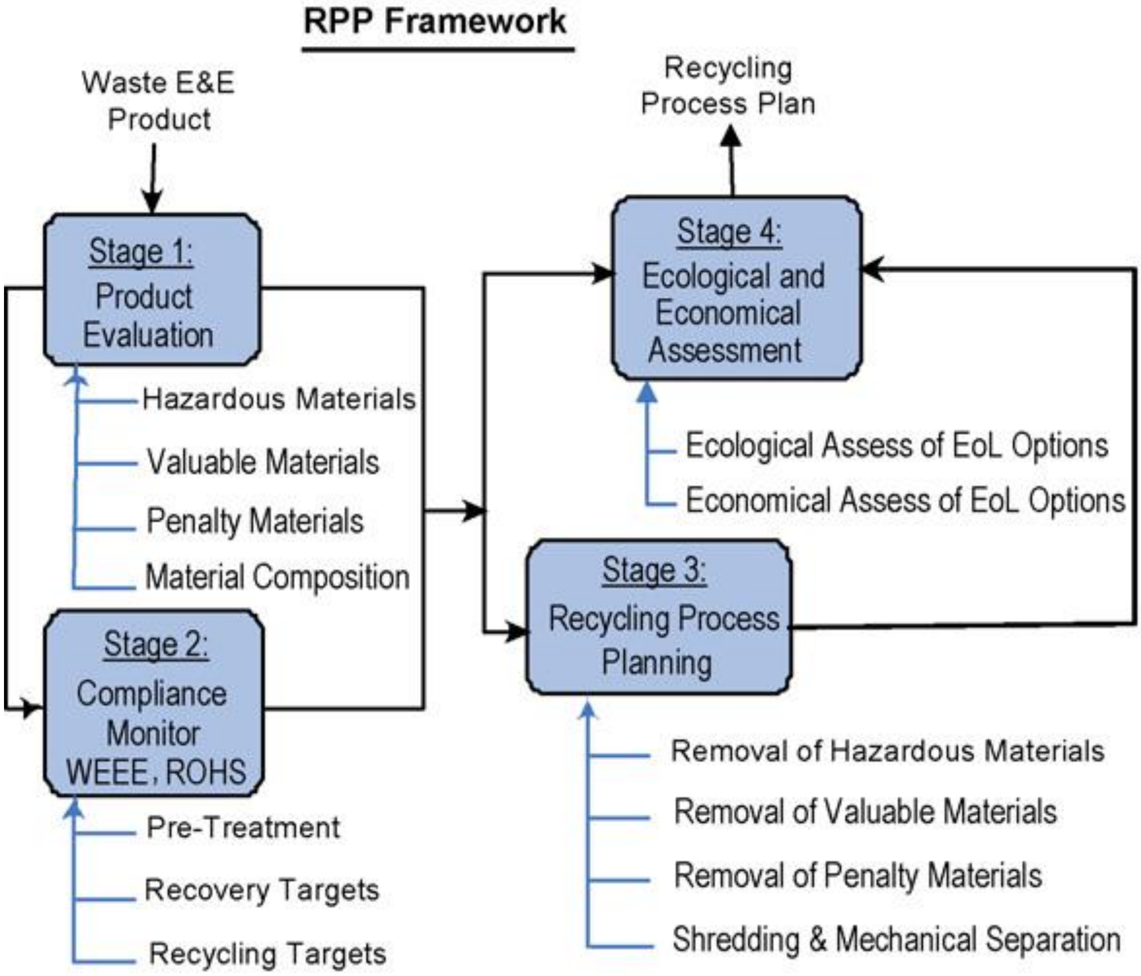


Figure 1.2: The four stages in the RPP framework [13]

In the first stage, the product evaluation one, information is collected about hazardous materials, valuable recoverable materials, penalty materials and material composition, in order to plan the recovery and recycling processes. In the second stage, the information collected is used to check the legislative compliance and so to plan the pre-treatment and to identify recovery and recycling targets: the depollution, recovery and recycling processes of WEEE must be according to the WEEE and Restriction of Hazardous Substances (RoHS), directives.

In the third stage, based on the information coming from the previous stages and from the product evaluation, a specific set of activities is planned in order to prepare the final recycling process: the removal of hazardous materials, valuable materials and penalty materials from components, and their shredding and mechanical separation. In the final stage – the Ecological an economical assessment stage (Eco^2) – a cost-benefit analysis is carried out to assess the ecological and economical impact of the planned processes, different for each EoL product. This impact is essentially based on the different recoverable and recyclable materials present in the product.

First, ecological and economical performance limits are calculated for the different EoL options: upper limits and lower limits.

The upper limit represents the hypothesis that all materials contained in the product are completely recovered and recycled (no reject). On the other hand, the lower limit represents a scenario in which all materials must be rejected. These ecological and economical limits are expressed by equations (1.1) - (1.4) [13].

$$BCS_{ecol} = \sum_i^n (m_i * Eli_{BSC}) \tag{1.1}$$

$$BCS_{econ} = \sum_i^n (m_i * Cli_{BSC}) \tag{1.2}$$

$$WCS_{ecol} = \sum_i^n (m_i * Eli_{WSC}) \tag{1.3}$$

$$WCS_{econ} = \sum_i^n (m_i * Cli_{WCS}) \tag{1.4}$$

Where m_i is the mass of material *i* in the product (kg); Eli_{BSC} is the ecological impact of material *i* in the best case scenario (mPt/kg); Eli_{WSC} ecological impact of material *i* in the worst case scenario; Cli_{WCS} is the ecological impact of material *i* in the worst case scenario; Cli_{BSC} material revenue value of material *i* in the worst case scenario.

Then, based on the cost-benefit analysis, the actual ecological performance and actual economical performance are calculated respectively in equations (1.5) and (1.6) [13]:

$$AP_{ecol} = \sum_i^n (m_i * PE_i * Eli_{AP} * G_i) \tag{1.5}$$

$$AP_{econ} = \sum_i^n (m_i * PE_i * Cli_{AP} * G_i) \tag{1.6}$$

Where: PE_i is the efficiency of the separation process used for material i; Eli_{AP} the ecological impact of material i in an EoL route (mPt/kg); Cli_{AP} the cost impact of material i in a certain EoL route (£/kg); G_i is the grade in which material i is covered.

We compare, therefore, the actual ecological and economical performances, associated with different EoL options for a specific product, and the respective upper and lower performance limits: the closer the actual performance is to the upper performance limit (representing the best-case scenario), the better is the assessed EoL option. However, the combined impact resulting from the evaluation of the actual ecological and economical performances is not always so easy to interpret; it could happen, for specific EoL options, that the ecological performances may be higher, whereas the economical performance may be worse. In most cases, this lead to difficulties in the decision-making process. To cope with this problem, it has been developed a calculation system in which the ecological and economical performance results are combined into 'single ecological and economical ratios'. These ratios range from ‘0’ to ‘1’: the closer a ratio value is to 0, the worse the performance is; on the other hand, the closer a ratio value is to 1, the better the performance is.

Equations (1.7) - (1.9) [13] are used to calculate, respectively, the normalized ecological performance ratio, economical performance ratio and combined Eco^2 performance ratio.

$$ERP_{ecol} = \frac{AP_{ecol} - WCS_{ecol}}{BCS_{ecol} - WCS_{ecol}} \tag{1.7}$$

$$BSC_{econ} = \frac{AP_{econ} - WCS_{econ}}{BCS_{econ} - WCS_{econ}} \tag{1.8}$$

$$CEPR = \frac{EPR_{ecol} + EPR_{econ}}{2} \tag{1.9}$$

The RPP is supported by a Computer Aided Recycling Process Planning (CARPP) that analyzes the big amount of information needed and processes the know-how, in order to plan a bespoke recycling process for each kind of material.

Furthermore, the RPP framework determines the best trade-offs between ecological and economical variables.

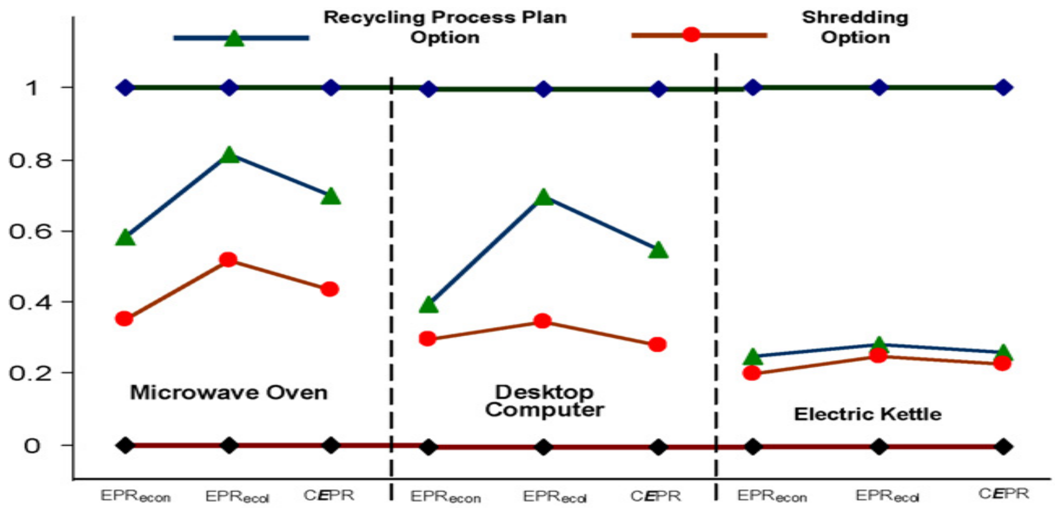
The utility of this particular RPP system is shown in some product case studies results, represented in table 1.1 [13]:

EOL OPTION		Ecological performance ratio (ERP_{Ecol})	Economical performance ratio (ERP_{Econ})	Combined Eco^2 performance ratio (CERP)
Microwave oven	Shredding after de-pollution option	0.51	0.34	**0.43**
	Recycling process plan option	0.82	0.58	**0.7**
Desktop computer	Shredding after de-pollution option	0.35	0.3	**0.28**
	Recycling process plan option	0.7	0.4	**0.55**
Electric kettle	Shredding after de-pollution option	0.25	0.2	**0.23**
	Recycling process plan option	0.28	0.25	**0.26**
Upper limit		**1**	**1**	**1**
Lower limit		**0**	**0**	**0**

Table 1.1: Comparison of different case studies results [13]

It is clear from the table that specific recycling process plans result in higher ecological and economical performances – and even Combined Eco^2 performance ratio – than traditional state-of-the-art recycling practices; there is a potential improvement of about 20-30%.

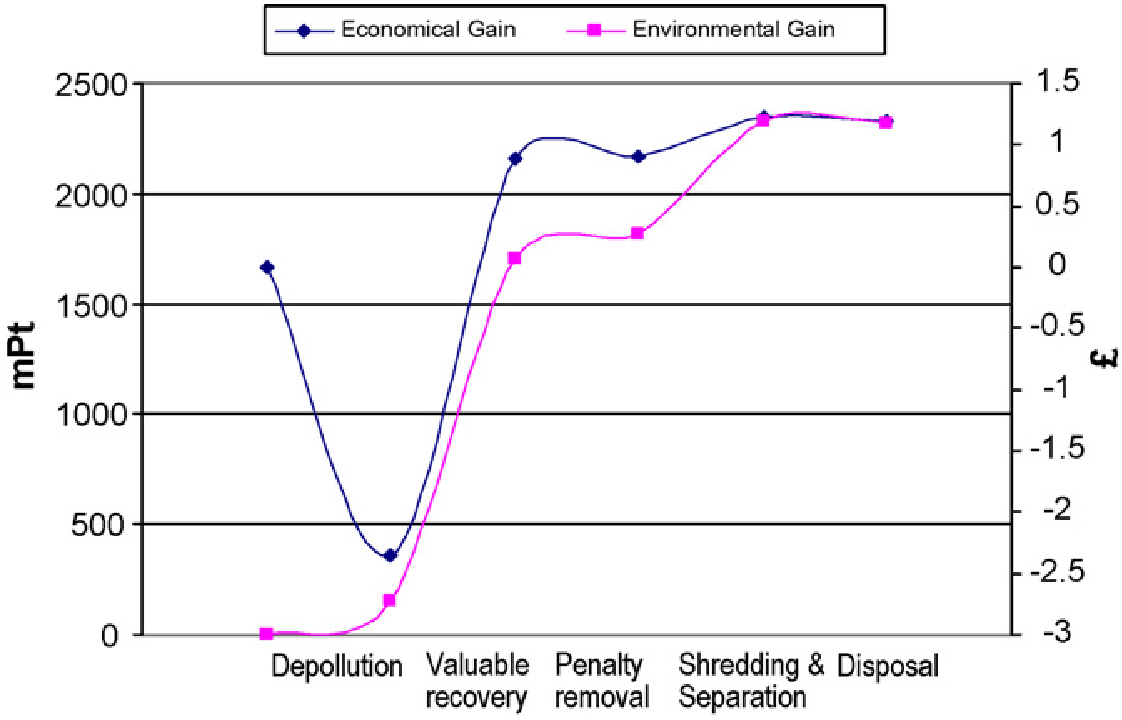
This is also shown also in graph 1.3 [13]:



Graph 1.3: Comparison of different case studies results [13]

As we can see, for each kind of product, two broken lines are shown: one – the one with orange points – represents the economical, ecological and Combined Eco^2 performances achieved with the shredding option, the other broken line – the one with green points – represent performances achieved with the use of the RPP.

Furthermore, in the graph 1.4 [13], an analysis of the operational breakdown economical and ecological gain for the microwave oven is shown. This analysis is useful for the definition of a capital investment strategy decision to improve the recycling facilities. [13].



Graph 1.4: Operational breakdown of ecological and economical gain for microwave oven [13]

1.5.2 The new challenges

There is still a lot to be done, in Italy and Europe, for what concerns the WEEE collection regulations and for what concern E-Waste de-manufacturing systems. Changes are desirable not only from the legislation point of view, but also at a merely technological level, recycling system level and business model level.

Focusing on the recycling process chain, most of the problems come from the mechanical pre-treatment of the WEEE End-Of-Life products: it is indeed the cause of the most material losses - from 40% to 100% depending on material

An improvement in the state-of-the-art mechanical recycling technologies and systems for WEEE recycling is required for the recycling industry.

In Europe, Small Medium Enterprises (SME) form the 85% of the entire recycling industry [8]. They cannot dedicate treatment lines for each specific product type flow, due to space and budget limitations, so, to cope with this limit, they usually adopt batch production and high utilization of a single or few recycling lines. The process parameters don't change for any product type, but they are selected as a compromise between the different EoL product types and they remain constant. For these reasons, the state-of-the-art mechanical recycling systems are extremely rigid and managing product evolution with these kinds of systems is very difficult. Given the high variability of material in input, this rigidity of the system is a big limit for the recycling systems performances. This requires the development of new more adaptable de-manufacturing systems [8].

«'De-manufacturing' can be defined as the set of technologies, tools and knowledge-based methods to recover and re-use materials for industrial waste and postconsumer products», affirms the article "Towards Smart E-Waste Demanufacturing Systems exploiting Cyber-Physical Systems (CPSs) capabilities", of E*lectronic Goes Green 2016+*, Berlin, September 7 – 9, 2016 [8]. As a matter of fact, a de-manufacturing system transforms post-consumer products into valuable materials of secondary use that can ben put on the market. It usually includes different processing stages, among which the mechanical pre-treatments, such as different size-reduction and separation technologies.

The main goal is a transition to new smart de-manufacturing systems, able, ultimately, to adapt process parameters according to the material/product under treatment.

There are several examples of existing technologies supporting this transition. A first example is the set of hardware technologies for the On-line material characterization along the process, such as Inductively Coupled Plasma, Mass Spectrometry (ICP-MS), Inductively Coupled Plasma, Optical Emission Spectrometry (ICP-OES), Scanning Electron Microscopy (SEM), X-Ray Diffraction (XRD) and X-Ray Fluorescence (XRF).

Information is important at all the different stages of the de-manufacturing chain. Before the process, in order to estimate the value and composition of the EoL products to set a proper process flow. During and after the process in order to monitor, control an improve the process quality. For these last two cases, a more advanced HyperSpectral Imaging (HIS) is used; it is able to collect information of the electromagnetic energy at each pixel of the image, therefore identifying the target materials and their spatial location on a surface. However, given a big amount of data, this HIS system requires complex mathematical algorithm to classify treated materials in short time

A limit for changeable de-manufacturing systems is the presence of rigid transportation systems that don't allow a fast reconfiguration of the material flow through the different stages of the process-chain and so they are not adapted to process highly variable products.

To cope with this problem, it is necessary that the hardware components of automated transportation systems support the modification of the material routing with reduced setup times. The hardware should therefore be composed by a combination of rigid transportation modules and (e.g. mechanical conveyors) and flexible routing modules (e.g. based on pneumatic systems) with multiple input and output valves connecting simultaneously multiple processing stages. This allows a dynamic modification of the materials mixture routing with reduced times, by controlling the plant and adjusting the input and output port selection. Ultimately, the adoption of a hybrid transportation system will allow to implement new control logics and to integrate new processing technologies, in order to face significant changes of the incoming products mixture.

Another limitation in changeable de-manufacturing systems is represented by the automatic sorting devices for different kinds of materials: they usually work through arrays of fixed ejectors of compressed air. This system is poorly reconfigurable for different material mixtures and so not suitable in case of big changes in the properties of the material. To solve this problem a new flexible robotic sorting system has been proposed, which can combine different modifications in the robot control and in the robot gripper, so to handle different material sizes and compositions [51].

In conclusion, the adoption of a hybrid transportation system and the implementation of a flexible robotic sorting system, are good enablers towards the smart de-manufacturing systems [8].

1.5.2.1 ICT solutions

The actual de-manufacturing systems are not able to adapt to different products and context conditions. To cope with this limit, new advanced process models must be developed. These models should allow to characterize the interaction between the equipment and the product under treatment, under context conditions, so to predict the recovery and grade of the final separated materials. The separation process is characterized by some specific parameters. There are already specific programming environments (e.g. Chrono::Engine) [28] that are able to simulate systems of millions of interacting particles in very short times; but they are not able to pursue the process parameters optimization without the help of many real life experiments.

A solution to solve this problem is the development of advanced multi-physics process simulation models, that are able to support the process dynamic parameters adaptation during the system operational phase.

Another issue concerning de-manufacturing systems is the need to have a good integration among the different de-manufacturing stages, during the design and reconfiguration phases. This could be achieved and supported using proper modelling and analysis tools. To this purpose, a good idea is the development of a virtual de-manufacturing plant framework, enabling to realize, together with physical plant, its virtual representation, incorporating different process models. This should test alternatively material flow solutions and process-chains before changes in the real plant are implemented and without interfering with its operations, to finally predict the behavior of the process with specific products. There already exists an example of software platform supporting the design of recycling systems; moreover, a new multi-scale model for recycling systems has been proposed. This last model integrates process and system layers and it is characterized by a simulation tool making experiments to capture effect of single processes on the material mixture under treatment.

Another important improvement concerning de-manufacturing systems is related to the process effectiveness. The evolving features of the product under treatment and the ever-changing market demand are very important issues for all de-manufacturing industries. These require for a good system control.

The state-of-the-art control architecture is rigidly based on a central programmable logic controller (PLC) that controls and monitors the critical system parameters. It is not possible to adapt the process parameters and material routings according to the different products under treatment

To solve this problem, in the manufacturing sector, distributed and scalable control solutions based on IEC61499 and Cyber-Physical Systems (CPS) have been proposed as suitable solutions. However, CPS solutions for de-manufacturing are not available on the market yet [8].

1.5.2.2 A real case of CPS design for E-waste De-manufacturing system

The Institute of Industrial Technologies and Automation of the National Council for Research (ITIA-CNR) in Milan (Italy), has developed one of the few smart E-waste de-manufacturing plant, designed to recover WEEE components. It integrates some technologies described in the previous paragraphs and it has been designed and tested for a continuous control of separation processes under variation of the incoming material mixture. The CPS framework is composed by a physical layer (lower layer) and a virtual layer (upper layer). The plant uses a HSI system to gather data about the mixture composition in real-time, so to recognize and classify shredded products in terms of mixture composition.

These data are elaborated by a software platform with monitoring and control functions. Monitoring data is useful to identify statistically relevant changes in the mixture composition that may trigger process parameter adjustment. When the software detects a significant change in the mixture composition, it selects an optimal set of process parameters (electrode voltage, drum speed, feed rate and splitter position), with the help of multi-body process simulation model, to be assigned to the downstream Corona Electrostatic Separator (CES) - eventually adjusted by using the UPC-UA protocol - to optimize the separation of the new mixture.

This new CPS prototype can adapt - thanks to the interconnections between the physical and the virtual system - the parameters of mechanical separation processes to the in-line identified changes of the material mixture.

It has been tested on several electronic mixtures and, according to the results obtained, if put in place, it could bring considerable benefits in terms of the adaptability of the recycling systems. It can be a good enabler for the future de-manufacturing systems [8].

Chapter 2

Recycling framework

2.1 General view

Recycling activity represents the last step of the Circular Economy loop and its aim is to process an incoming product so to obtain output separated flows of pure materials to be re-used as secondary raw materials in the manufacturing processes. A recycling system is a multi-stage system, including multiple-size reduction and separation stages. In figure 2.1 [12] we can see a schematic representation of a recycling system.

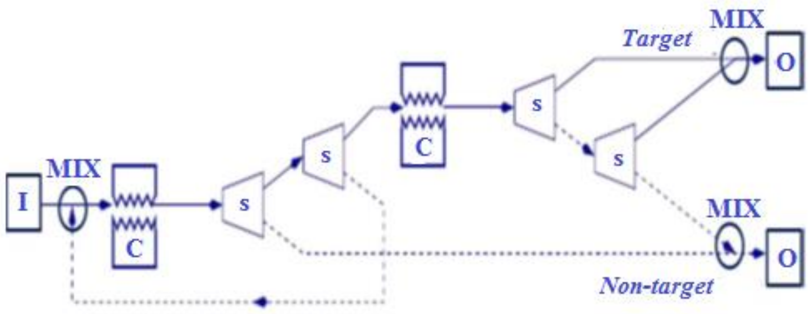


Figure 2.1: Representation of a recycling system [12]

In the image above, the considered system is composed by 7 stages in total (2 mixing stages are put in parallel at the fine of the chain), with an initial input and two final output: the target material output one and the non-target material output one. It includes all the typical processes of any recycling system:

1. Size reduction,
2. Separation,
3. Mixing,
4. Splitting.

2.2 Size reduction process

In the size reduction stage, large material particles are broken into small particles, as we can see in figure 2.2 [12], so to be easier separated with mechanical techniques (e.g: grinding, shredding, pulverizing) [12].

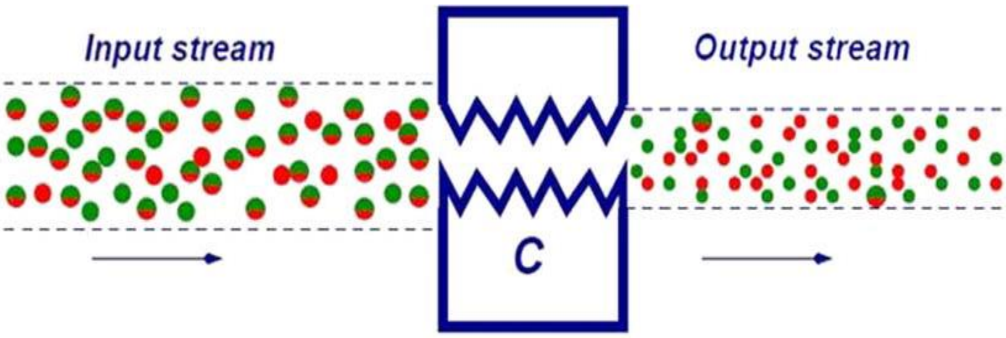


Figure 2.2: Schematic view of Comminution process [12]

The characteristics of the particles in output depends on several parameters affecting both the size reduction process and the quality of the output, like the speed of the fast-turning rotor, where the cutting tools are mounted, the characteristic of the comminution chamber and the particles residence time [3].

To prepare materials for the successive mechanical processes, we must choose the right measure of the particles size reduction. To this purpose, we must take into account some information about the material, like the physical characteristics of the material to be reduced (e.g. particle size, structure, hardness, brittleness and fissionability), the future usage of the material (e.g. is there a following physical or chemical processing of the material) and the required properties of the final material (e.g. particle size, distribution, and average particle size) [32].

Basing on these data, it is possible to select the adequate target for the size reduction.

Particles size reduction process can be carried out in several physical ways, by the effect of Chemical strains or Mechanical strains [33]. The last one is the most common method. There are different kinds of mechanical machines. These have a typical size reduction structure with some specific elements, presented in figure 2.3 [3].

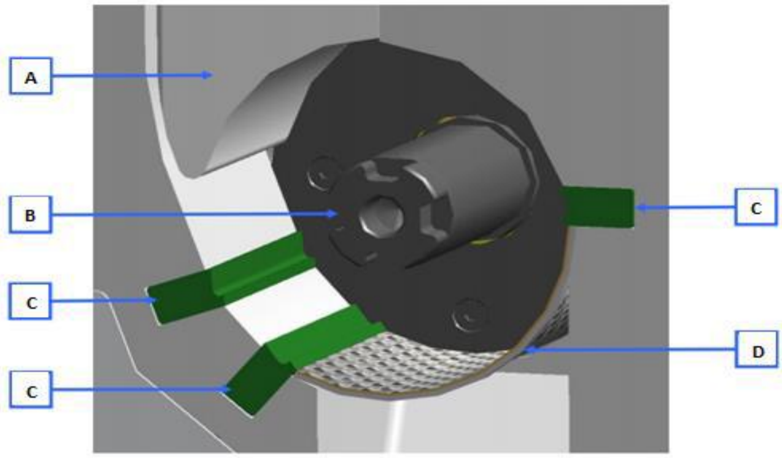


Figure 2.3: Example of a size-reducing unit [3]

In the image above, we can distinguish 4 typical tools: i) The input material feeder (A), ii) the rotor shaft with cutting tools (B), iii) the cutting bars – the counterpart of the cutting tools – (C), iv) the discharge grate, from where the comminuted materials exit (D) [3].

There are several kinds of size reduction machines: ***shearing shredders, hammer mills*** and several types of ***crushers*** for a fisrt size-reduction stage in which components in input are crushed into smaller particles, and ***cutting mills*** for a second stage of reduction, equipped with a cutting rotor, where the remained big material particles are crumbled and cut into finer ones, ready for the following recycling stages.

Shearing shredders are equipped with shears mounted around one or more rotating horizontal shafts working at a speed of 50 ÷ 200 rpm, to tear and cut down the input material [16].

Hammer mills are machines equipped with a series of hammers attached to a rotating shaft, working at a rotation speed about 1,000 rpm, which repeatedly hit the material so to crumble it into smaller particles. The hammers may be mounted to the shredder rotor in a fixed or freely swinging manner [17]. Basically, there are two types of hammer mills, basing on the construction of the rotor on the mill: there are Horizontal hammer mill and Vertical hammer mill [33].

For what about **crushers**, there are 3 types: *Impact crushers, Cascade ball mills* and *Jew crushers*.

Impact crushers are composed of a rotor with blow bars fixed and breaker plates. The operation process is quite intuitive:, the material feed in input, subjected to the centrifugal force caused by the rotating roller, beats against the breaker plates and it breaks up into small pieces; when the material reaches the right size, it can pass through the opening between the rotor and the breaker plates. Breaker plates can be adjusted in terms of distance from the hammers and inclination, so to optimize the performance [33].

A *Cascade ball mill* is a rotation mill on horizontal axis, generating movement of the grinding means inside and so the waste breakage mechanism through the means collision [14].

A *Jaw crusher* is a machinery in which the material enters from an opening above and decreases towards the bottom, between two swinging jaws, one fixed and the other reciprocating. Due to the moving jaws by mechanical pressure, the material is crushed drops down. The size of the jaw influences the final material particles size. Jaw crushers are commonly used in mechanical, metallurgical and allied industries [15].

2.2.1 Process modelling and analysis

The objective of the process level analysis is the estimation of important data about the size reduction process: the transformation matrices[1] as a function of the design and operational process parameters, the input mixture distribution[2] and the input flow rate[3].

In literature, a validated model to describe the material size reduction process doesn't exist yet. However, promising approaches have been developed to model the evolution of the particle population inside the size reduction chamber: these models are the Population Balance Models (PBM) [24]. Basically, we can think to the evolution of the particle population as a continuous *markovian-process*.

In statistic, a Markovian process is an aleatory process in which the transition probability determining the passage from a specific state to a new state, in a specific time unit (e.g: from t_0 to t_1), depends just on the immediately precedent system state (e.g: the state at time t_1 depends just on the state at time t_0) and on nothing else (the Markov property: memoryless). Basing on that, we can define a *Markov-chain* as a discrete state space (with a finite number of different states) markovian process. There are two kinds of markov-chain: *Discrete-time markov-chain* or *Continuous-time markov-chain*.

In Discrete-time markov-chain, state changes happen at a definite time unit, so the transition probability from a particular state to another state, at that particular time unit, can be written as:

$$P[X(t_{k+1}) = x_{k+1} \;/\; X(t_k) = x_k] \tag{2.1}$$

1,2,3: These data will be described in detail in the successive paragraphs

Where $t=\{t_0, t_1, t_2, \ldots\}$ are the different discrete temporal units, $\{x_0, x_1, x_2 \ldots\}$ are the different discrete states.

Whereas in a Continuous-time markov-chain, we consider a continuous flow of time, so that states changes occur within an infinitesimal time span δt. So, the transition probability from a state to another can be finally written as:

$$P[X(t + \delta \mathrm{t}) = x_j \ / \ X(t) = x_i] \tag{2.2}$$

[21]

The PBM is based on the transitions particles classes, regulated by breakage and selection functions. Selection functions express how many particles of a certain class (l, d, s) are broken into smaller particles. The breakage function expresses how many particles of a certain class (l, d, s) generate particles of another class (l', d', s'). This breakage functions is represented by transition matrix *T*. When a particle reaches a size smaller than the grate size d_g, it leaves the chamber according to a probabilistic discharge function, represented by matrix *D(l, d, s)*.

The PBM is described by a system of differential equations:

$$\begin{cases} \frac{dM^{CHAMBER}(t)}{dt} = M^{INPUT}(t) + T * M^{CHAMBER}(t) - \frac{dM^{OUTPUT}(t)}{dt} \\ \frac{dM^{OUTPUT}(t)}{dt} = D * M^{CHAMBER}(t) \end{cases} \tag{2.3}$$

[24]

where M are vectors containing *m(v)* elements, the mass of the material in each particle class *(l(v), d(v), s(v))*. To simulate the stochastic model dynamics, the input masses are generated according to the input flow rate and to the input particle classes. At the end of the simulation, from vector M^{Output} are calculated the processing rate, the transformation matrices and the output particle classes. This model has been tested at ITIA-CNR [24].

2.3 Separation process

In the separation process, a mixed input stream is divided into two or more output streams with improved material concentrations. In figure 2.4 [12] we can have a schematic vision of a separation system.

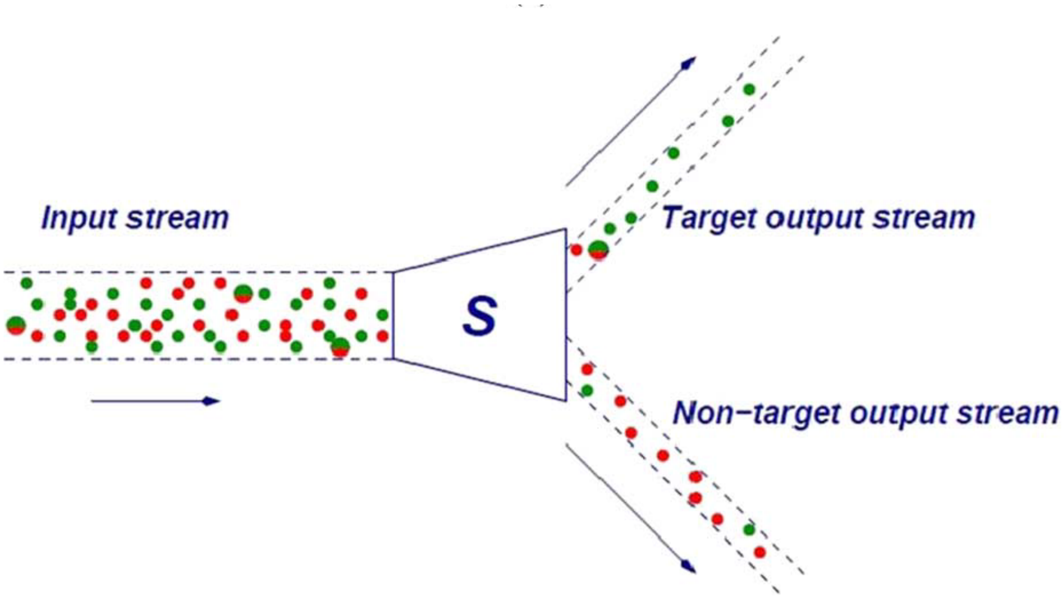


Figure 2.4: Schematic vision of separation process [12]

In the image, we can distinguish very well the mixed input stream and the two 'depurated' output streams of target and non-target material.

Under ideal conditions, each target material concentration in its output flow should be equal to 1. Realistically, this performance value is always less than 1, because of different kinds of random disturbances, variabilities in particle properties, and other effects, that cause particles to be incorrectly diverted. A separation system may include multiple different stages exploiting different properties of materials to separate them, or repeat the same process to increase separation performances of a single target material [12].

Looking more in detail about separation process, it makes use of a material property – such as **conductivity, magnetism, density, color, particle size** – there are therefore different separation process techniques [12].

In this thesis work the aim is to create a software to model a recycling system dealing with WEEE materials, from which are extracted precious metals like copper, aluminum, steel, gold, silver; so, our analysis will focalize on the separation techniques relying on the metallic properties: Conductivity separation process and Magnetic susceptibility separation process. In particular, in our recycling model, separators can be of three kinds: *Eddy Current Separator (ECS)*, *Corona Electrostatic Separators (CES)* or *Density based separator*.

2.3.1 Conductivity based separation: Corona Electrostatic Separator

The conductivity based separation makes leverage on the presence or not of electrostatic current in materials. The most common machine for conductivity based separation is the Corona Electrostatic separation.

The Corona Electrostatic Separator allows to separate Conductive materials (metals) from Non-conductive materials. The most important components are a corona electrode, generating a high voltage electrostatic field, and an earthed electrostatic electrode. Particles, lying on a rotating roll, are first electrically bombarded by the high voltage electrostatic field, then metal particles are discharged by the earthed electrostatic electrode and they detach from the rotating roll, falling into a first hold tank. Non-metal particles, instead, are not discharged and they are transported to another hold tank by the roll [17]. This mechanism is shown in figure 2.5 [18].

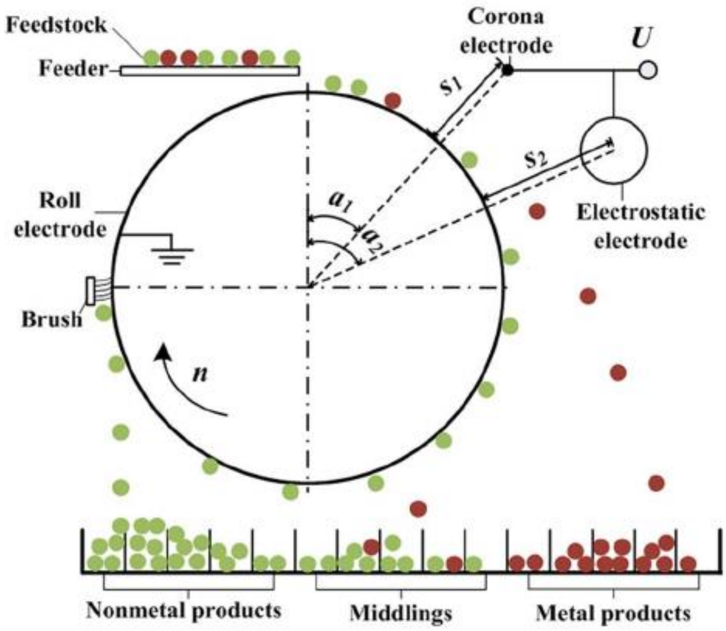


Figure 2.5: Corona Electrostatic Separation process [18]

2.3.2 Magnetic based separation: Eddy Current Separator

The Eddy Current Separator (EDC) makes leverage on the different magnetic intensity of materials. Basically, it separates there three kinds of substances:

- Non-ferrous elements,
- Non-magnetic elements
- Magnetic elements

with the help of internal electrostatic currents of materials. The EDC is formed by a rotating magnetic rotor covered by a conveyor belt. Feed by the top, material is rotated and, at the same time, is bombarded by a magnetic field. Non-ferrous particles are thrown beyond the rotor, because no force blocks them to the rotor, the Non-magnetic particles remain stuck to the conveyor belt and they fall down in compartments slightly beyond the point where the belt curves downward, thanks to the gravity force, while the Magnetic particles fall down at a point under the conveyor in which gravity force is higher than the magnetic attraction [19]. This process is shown is figure 2.6 [19].

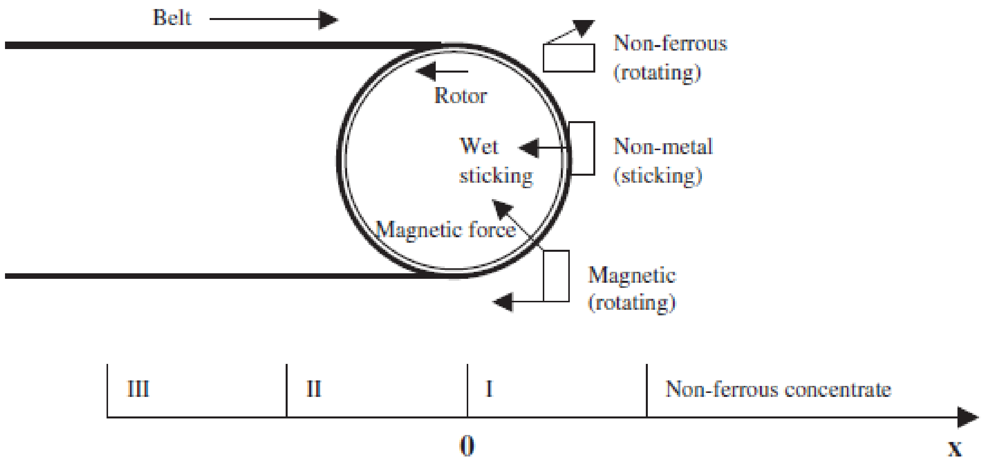


Figure 2.6: Eddy Current Separation process: Non-metals end up in compartments I, II and III, magnetic materials in compartments I and II [19].

2.3.3 Density based separation

The density based separation in mainly used to divide metals from plastic and non-metals materials. The density based separation can be carried out through floatation mechanism, with jigging systems, with air cyclone system or with the use of dry densiometric table.

2.3.4 Process analysis and performances

As we have seen before, the separation process divides an input mixture stream, with two or more materials, into two or more output streams, each one specific of one or more elements, with a high concentration that particular element(s). This means that, once processed the incoming flow, the separator decides in which of the output flows to send the particles according to which material they belong. But the process is not ideal, there are always some random disturbing factors: a target flow in which the concentration of target material is 100% doesn't exist.

To represent the separation process, we will use the Bayesian separation model.

We will suppose now to have a separation stage with one incoming input mixture flow of two products, called A and AC, and two designated output flows, and to do an inspection test on the incoming flow. Our test is a hypothesis test on the incoming particles, in which the assumptions are:

$\boldsymbol{H_0}$: the particle comes from distribution A

$\boldsymbol{H_1}$: the particle comes from distribution AC

We define therefore:

$r=P(A|A)$ as the probability that a particle coming from distribution A belongs to product A,

$q=P(AC|AC)$ as the probability that a particle coming from distribution AC belongs to product AC,

and so:

$1\text{-}r=P(AC|A)$,

$1\text{-}q=P(A|AC)$.

r and q measures, somehow, the process ability to separate the two materials in question. Let's consider to be, the two materials in question, a target one, T, and a non-target one, N; and their respective flow rates are m^T and m^N [22].

This Bayesian separation system is shown in detail in Figure 2.7 [23].

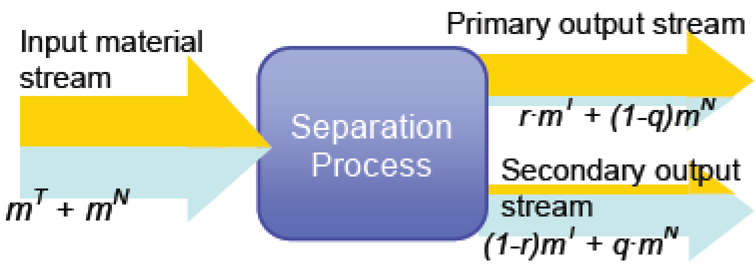


Figure 2.7: Example of Bayesian separation system [23].

It is possible to calculate the mass flow rate of each material in output, for each output stream, designated with 1 (primary) and 2 (secondary):

$$m_1^T = rm^T,$$

$$m_2^T = (1-r)m^T,$$

$$m_1^N = (1-q)m^N,$$

$$m_2^N = qm^N \quad (2.4)$$

[22],

Parameters r and q really represent the *Recovery* of materials T and N respectively – the fraction, for each material, between the material in the designated output flow and the material entering the system –; they can be rewritten like this:

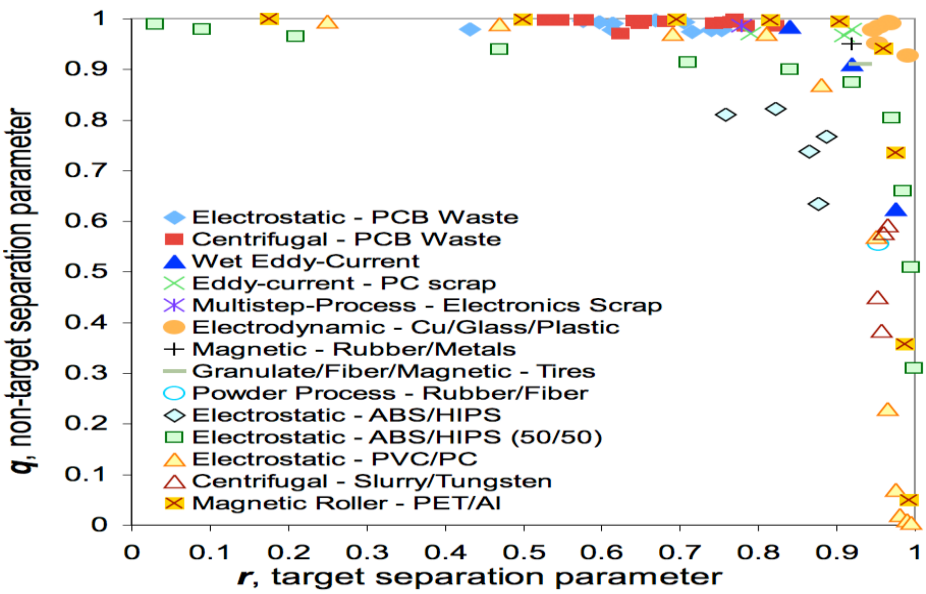
$$r = \frac{m_1^T}{m_{in}^T}, \quad (2.5)$$

$$q = \frac{m_2^N}{m_{in}^N} \quad (2.6)$$

[22],

where *in* denotes the input process input stream.

Parameters q and r depends on the separation process; there is a sort of trade-off between r and q looking at different kinds of separation, as shown in graph 2.1 [22].



Graph 2.1: r and q patterns multiple separation processes [22]

Furthermore, it is also possible to obtain the *Grade* of the target material T as:

$$G = \frac{m_{T_{out}}^{T}}{m_{T_{out}}^{T} + m_{T_{out}}^{N}} \quad (2.7)$$

[23],

where T_{out} represents the designated output flow for the target material *T*. The *Grade* represents the concentration of *T* in the target output flow [23].

Recovery and Grade are two important performances of the material Separation process.

2.3.5 Mixing and splitting processes

The third recycling step is mixing stage: the mixing process allows to merge two or more incoming material flows into an output mixture. Mixers are useful for two purposes: i) to incorporate closed loops with upstream stages, ii) to mix two or more similar incoming flows to have a higher output flow rate. [12]. A simple illustration of this procedure is in figure 2.8.

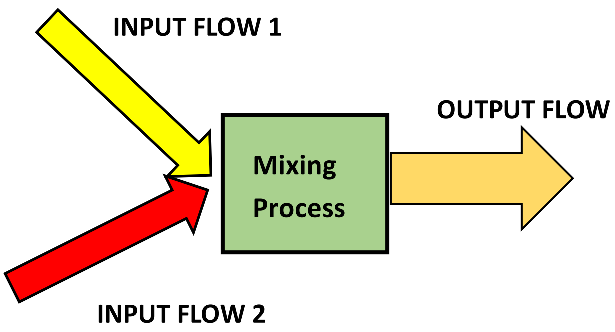


Figure 2.8: Schematic view of Mixing process

Finally, the last step is the splitting stage, that allows to separate one incoming flow of materials into two or more output flows, without changing their concentrations in the output stream. An illustration is in Figure 2.9.

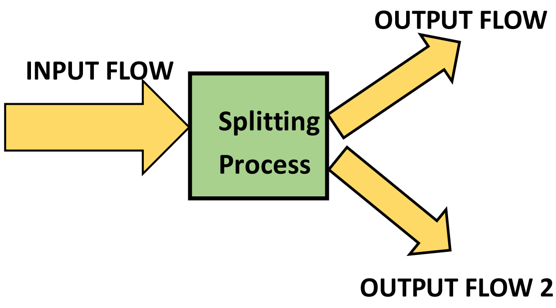


Figure 2.9: Schematic view of Splitting process

Splitters are useful for two purposes: i) to extract samples to be tested in order to check the process quality, ii) to have parallel flows of materials at reduced flow rates to enhance the upstream separation processes performances [12].

2.4 A formal recycling system model

Till now all the recycling processes have been presented, each one with all its peculiarities. But, in reality, an industrial recycling system is composed by may stages, each of these can support, indifferently, one of the 4 processes and they are linked all together. This calls for a way to estimate the final performances of the whole recycling system. It's therefore opportune the presence of a formal model to represent a recycling system, like the one depicted in figure 2.10 [24] below.

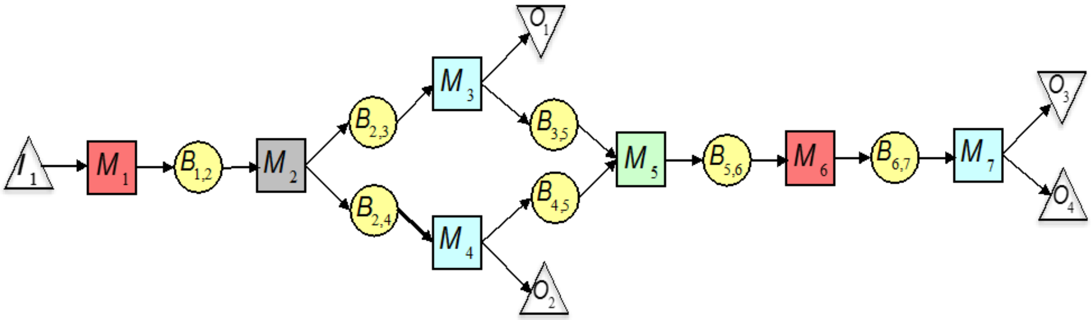


Figure 2.10: Example of formal recycling model [24]

The image shows an example of recycling system model formed by K stages and K-1 buffers of finite capacity. We will denote stages as M_k, $k=1,...,K$, in the set M, and buffers as $B_{i,j}$ where i and j refer to the upstream and downstream stages respectively. We can call $N_{i,j}$ the capacity of each buffer (expressed in Kilograms of material); I should be the set of input stages – the stages where material flows are fed into the system – and O should be the set of output stages – those stages where material flows are collected. We can consider M^+ as $M \cup I \cup O$ and Γ as the set of the system directed connections, or branches, from stage i to stage j. More formally:

$$\Gamma = \{(i,j) | i \in (M \cup I)\, , j \in M^+\} \tag{2.9}$$

Moreover, for each stage M_k, there is a set of branches entering the stage k as Ω_k, such as a set of branches leaving the stage as Δ_k, defined in equations 2.10 and 2.11 respectively.

$$\Omega_k = \{(i,j) \in \Gamma | j = k\} \tag{2.10}$$

$$\Delta_k = \{(i,j) \in \Gamma | i = k\} \tag{2.11}$$

Theoretically, each stage M_k in M can be a size-reduction, separation, splitting or mixing stage - even though initial stages are, more easily, size-reduction stages, and final stages are, more easily, separation stages -. However, each stage M_k in M is characterized by a processing rate μ_k and it is unreliable and subject to multiple operational dependent failures, each of them characterized by a failure rate $p_{k,f}$ and a repair rate $r_{k,f}$, and a failure mode f. The recycling system processes materials $z=1,\dots,Z$, but not all of these are of economical interest for recycling industries. So, we distinguish a subset T of target valuable materials and a subset NT of non-target valuable materials. A material particle is supposed to be characterized of three attributes, i.e. the particle liberation class ($l=1,\dots L$), the particle size class ($d=1,\dots D$) and the shape class ($s=1,\dots.S$). So, in total there are $V=L*D*S$ particle types.

We will introduce the vector $Y_{i,j}$ to describe the particle mixture in each branch (*i*,*j*) in *Γ*, vector such that each element $y_{i,j}(v)$, $v=1,\ldots.V$ represents the weight fraction of the particles mixture belonging to the corresponding class *(l(v), d(v), s(v))*. To describe the correspondence between the elements *v* and the classes (*l*,*s*,*d*), we introduced binary matrices $H^l(v,l)$, $H^s(v,s)$ and $H^d(v,d)$. In this model, for each stage M_k, elements of vector $Y_{i,k}$, change depending on the stage characteristics, thus modifying the mixture characteristics and the routing of particles in the system.

In each recycling system, particular attention should be given to the Separation process. Given the non-ideality of this process, some particles in output flows can be non-homogeneous because materials are not completely liberated. Therefore, it is opportune to introduce, for each liberation class *l*, a vector of concentration *C(l,z)* which reports the fractions of each material *z* in the mixed particle type of class *l*.

In recycling systems, the main performance measure of interest are the average total production rate, denoted with $E_{i,j}$ *[Kg/h]* crossing the branch (*i*,*j*) in *Γ*, the Recovery, the Grade, the Effective production rate, denoted with $E_{i,j}^{eff}$, crossing the branch (*i*,*j)* in *Γ* and the final Profit, denoted with *Π*.

2.5 Outline of the formal recycling model

The formal model described in the chapter before, is based on the approximate analytical method, relying on the decomposition techniques, to evaluate the performance of a production system involving split and merge of production flows, multiple products, buffer with finite capacity and manufacturing/assembly/disassembly operations, described by M. Colledani and T. Tolio [25]. With this method, it’s possible to evaluate any recycling system like the one in figure 2.11.

First, the proposed method is based on the following assumption:

- The system flow is considered as a discrete flow,

- Each machine has a deterministic processing time, equal for all of them. Processing time are measured in specific time units,

- Machines can fail in multiple modes. For each failure mode J_i of machine M_i, the probability of failure in a time unit is p_{ji} , calculated as $1/MTTF_{ji}$, where $MTTF_{ji}$ is the mean time to fail for that particular failure mode, statistically estimated by historical data, and the probability of repairing is r_{ij}, calculated as $1/MTTR_{ji}$, where $MTTR_{ji}$ is the mean time to repair related to that particular failure mode, statistically estimated by historical data. P_{ji} and r_{ji} are transition probabilities of a markovian process; in fact p_{ji} represents the probability to pass from a 'functioning state' of the machine to a 'failure state' of the machine in a time unit, vice versa r_{ji}, and they depend just on the precedent state of the machine.

- There is always a buffer of finite capacity between two machines. The buffers can be in a 'full state' or in an 'empty state'.

- Given the previous two hypotheses, it is possible to both calculate the steady state probabilities of machines of being functioning or down, through the equation (2.1), and consider two more machine states: Blocking and Starvation. The Blocking state is when a machine is functioning but the buffer put after it is full – so the machine cannot proceed anymore, until the buffer remains in a full state –, on the other hand, and the Starvation state is when a machine is functioning but it cannot work because the buffer before it is empty. So, machines have ultimately 4 steady states: Functioning, Not-functioning, Blocking and Starvation (even if it is obvious that the first system machine cannot be in Starvation state and, analogously, the last system machine cannot be in Blocking state). The probabilities related to Blocking and Starvation states are denoted respectively with Pb_{ji}, Ps_{ji}, rb_{ji} and rs_{ji}.

- A machine can produce more than one product type.

Given these assumptions, the basic idea of this method is to perform a two-level decomposition of the original system; more precisely, a machine-level decomposition and a buffer-level decomposition, so to calculate the Blocking and Starvation probabilities and the Failure and Repair probabilities – as in the example depicted in figure 2.11 [25] – to finally evaluate the throughput of the system [25].

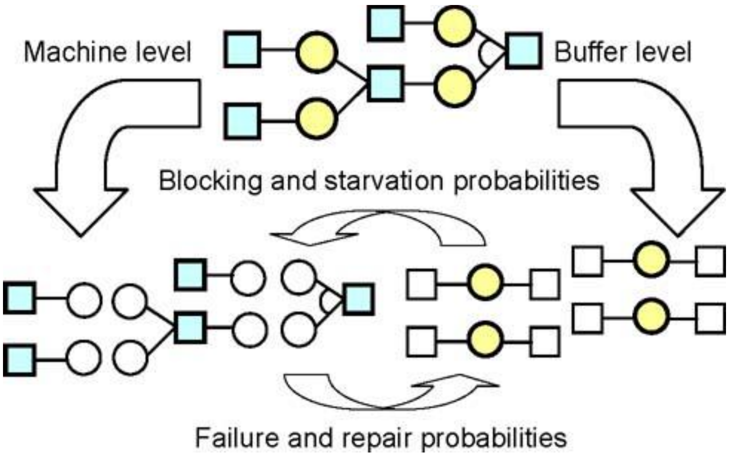


Figure 2.11: Example of two-level decomposition [25]

2.5.1 Statements of the method

The method described is essentially based on the values of all the possible steady state probabilities; to evaluate them, we will rely on the performance evaluation method of production lines, with finite buffers and different products, described by A. Matta, T. Tolio and M. Colledani [26]. We will consider the example of 4-machines line decomposition, in figure 2.12.

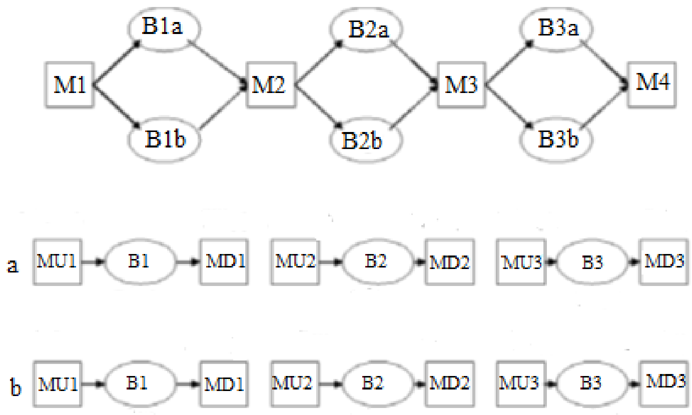


Figure 2.12: Decomposition of a 4-machines line [26]

As we can see, each machine in the system can produce 2 products, A and B, and, for each product, the line is decomposed taking into account each possible building block (2 machines with buffer), in which there is always an Upstream pseudo-machine M^U (the machine put before the buffer) and a Downstream pseudo-machine M^D (the machine put after the buffer). All Upstream and Downstream pseudo-machines have the typical failure and repair rates and the typical functioning and non-functioning steady states. Upstream pseudo machines have also Blocking state and Downstream pseudo machine have also starvation state. Furthermore, Upstream pseudo-machine has also remote failure modes, associated to the interruptions of flow due to starvation. These remote failures have failure probabilities $p_{j,f_1}^{V(A)}$ and $p_{k,f_2}^{V(B)}$ and repair probabilities $r_{j,f_1}^{V(A)}$ and $r_{j,f_2}^{V(B)}$, where $j=1....i-1$, $k=1....i-1$ indicate the machines of the original line that actually failed – so the real responsible ones for the starvation – and $f_1=1....F_j$, $f_2=1...,F_k$ are their remote failure modes. We can suppose that the repair probabilities of all machines are equal to the repair probabilities of the responsible starved machine of the original line. As a matter of fact, the failure probabilities are not known and must be calculated by using decomposition equations.

In addition to the described failure modes, a new failure mode is introduced and assigned to each pseudo-machine of the building blocks to model the interactions between the parts competing for the same machines: the *competition failure*. There is a competition failure when a machine cannot produce a given part type because it is busy producing the other part type. The probability of failure is $p_{j,F_j+1}^{V(A)}$, $p_{k,F_k+1}^{V(B)}$ and the probability of repair is $r_{j,F_j+1}^{V(A)}$, $r_{k,F_k+1}^{V(B)}$, for part type A and B respectively. Furthermore, a machine can produce just one part type and can fail while producing that part type. Therefore, the probabilities of local failures must be adjusted taking into account this situation.

To estimate failure and repair rate of competition failure probabilities, the solution is given by the creation of a model of *a combined pseudo machine* $M^U(i)$ producing two part types. In this model, we will analyze all the possible states in which the combined pseudo-machine can be, as the solutions of the markov-chain of the combined pseudo-machine. All the markov-chain probabilities are obtained by a linear system of equations

The picture 2.13 [26] shows the markov-chain of the combined pseudo machine $M^U(i)$. There are, in total, 14 states and each of them is defined by two variables: one for the pseudo machine of line A and the other for the pseudo machine of line B. Each state variable of the upstream pseudo-machine can assume four values: working (W), down in local mode (R), down in remote mode (V) and blocked (B); there is not a Starving state (S). In the figure two states are actually absent: W^AR^B and R^AW^B, because a combined pseudo machine cannot be both working a part type and being down in local mode for the other part type. The state R^AR^B is renamed as R. We also consider that the combined pseudo machine in state W^AW^B can process A or B depending on the processing rate α_i^A and α_i^B ($\alpha_i^A + \alpha_i^B = 1$), then from state W^AW^B is not possible to go to any Starving state or to state B^AB^B.

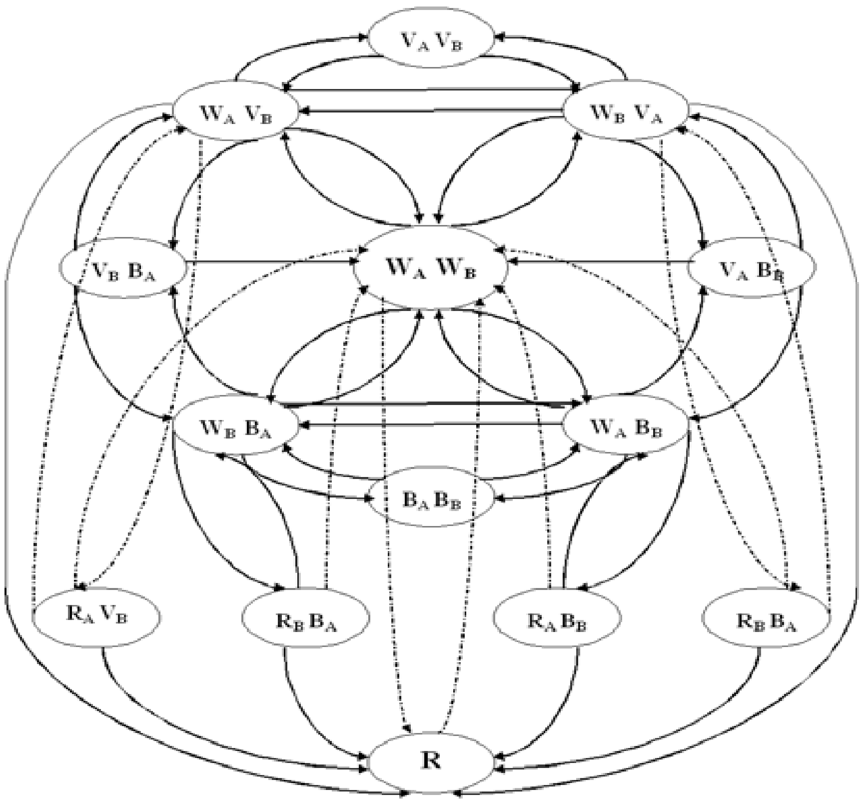


Figure 2.13: Markov chain of the combined pseudo machines [26]

First, we write the equations of conservation of flows, one for line A (2.12) and one for line B (2.13):

$$W^A W^B \alpha_A + \sum_{j=1}^{i-1} \sum_{f=1}^{F_j+1} W^A V_{i,j}^B + \sum_{k=i+1}^{K} \sum_{f=1}^{F_k+1} W^A B_{k,f}^B = E^A(i+1) \tag{2.12}$$

$$W^A W^B \alpha_B + \sum_{j=1}^{i-1} \sum_{f=1}^{F_j+1} W^B V_{i,j}^A + \sum_{k=i+1}^{K} \sum_{f=1}^{F_k+1} W^B B_{k,f}^A = E^B(i+1) \tag{2.13}$$

The following 14 sets of equations are useful to evaluate the probabilities of the various states and the unknown transition probabilities for the upstream combined pseudo machines.

The total probability of being down in remote mode of the combined pseudo machine must be equal to the probability of starvation of the precedent building block:

$$\sum_{k=1}^{i-1} \sum_{f_2=1}^{F_k+1} V_{j,f_1}^A V_{k,f_2}^B + \sum_{k=i+1}^{K} \sum_{f_2=1}^{F_k+1} V_{j,f_1}^A B_{k,f_2}^B + W^B V_{j,f_1}^A + \sum_{g=1}^{F_i} R_{i,g}^B V_{j,f_1}^A = Ps_{j,f_1}(i-1)$$

$j=1,\ldots.i-1$, $f_1=1,\ldots.F_j+1$ (2.14)

$$\sum_{j=1}^{i-1} \sum_{f_1=1}^{F_j+1} V_{j,f_1}^A V_{k,f_2}^B + \sum_{j=i+1}^{K} \sum_{f_1=1}^{F_j+1} B_{j,f_1}^A V_{k,f_2}^B + W^A V_{k,f_2}^B + \sum_{g=1}^{F_i} R_{i,g}^A V_{k,f_2}^B = Ps_{k,f_2}(i-1)$$

$k=1,\ldots.i-1$, $f_2=1,\ldots.F_k+1$ (2.15)

Given that the upstream combined pseudo machine of building block *i* must be coherent with the building blocks of part type A and B, we can write the following equations related to the probability that the combined pseudo machine is blocked:

$$\sum_{k=1}^{i-1} \sum_{f_2=1}^{F_k+1} V_{j,f_1}^A V_{k,f_2}^B + \sum_{k=i+1}^{K} \sum_{f_2=1}^{F_k+1} B_{j,f_1}^A B_{k,f_2}^B + \sum_{g=1}^{F_i} R_{i,g}^B B_{j,f_1}^A + W^B B_{j,f_1}^A = Pb_{j,f_1}(i)$$

$j=1,\ldots.i-1$, $f_1=1,\ldots.F_j+1$ (2.16)

$$\sum_{j=1}^{i-1}\sum_{f_1=1}^{F_j+1} S_{j,f_1}^{A} B_{k,f_2}^{B} + \sum_{j=i+1}^{K}\sum_{f_1=1}^{F_j+1} B_{j,f_1}^{A} B_{k,f_2}^{B} + \sum_{g=1}^{F_i} R_{i,g}^{A} B_{k,f_2}^{B} + W^{A} B_{k,f_2}^{B} = Pb_{k,f_2}(i)$$

k=1,….i-1, f_2=1,….F_k+1 (2.17)

Considering the states R^AV^B, R^AB^B, R^BV^A, R^BB^A, we can write a set of node balance equations: the probability of entering these states must be equal to the probability of exiting the same states.

$$W^{A} V_{j,f}^{B} p_{i,g}\left(1-r_{j,f}^{V(B)}\right) = R_{i,g}^{A} V_{j,f}^{B} r_{j,f}^{V(B)} + R_{i,g}^{A} V_{j,f}^{B} r_{i,g}(1-r_{j,f}^{V(B)})$$

g=1,…F_i, j=1,…i-1, f=1….F_j+1 (2.18)

$$W^{B} V_{j,f}^{A} p_{i,g}\left(1-r_{j,f}^{V(A)}\right) = R_{i,g}^{B} V_{j,f}^{A} r_{j,f}^{V(A)} + R_{i,g}^{B} V_{j,f}^{A} r_{i,g}(1-r_{j,f}^{V(A)})$$

g=1,…F_i, j=1,…i-1, f=1….F_j+1 (2.19)

$$W^{A} V_{j,f}^{B} p_{i,g}\left(1-r_{j,f}^{B(B)}\right) = R_{i,g}^{A} B_{j,f}^{B} r_{j,f}^{B(B)} + R_{i,g}^{A} B_{j,f}^{B} r_{i,g}(1-r_{j,f}^{B(B)})$$

g=1,..F_i, j=1,…K, f=1….F_j+1 (2.20)

$$W^{B} V_{j,f}^{A} p_{i,g}\left(1-r_{j,f}^{B(A)}\right) = R_{i,g}^{B} V_{j,f}^{A} r_{j,f}^{B(A)} + R_{i,g}^{B} B_{j,f}^{A} r_{i,g}(1-r_{j,f}^{B(B)})$$

g=1,…F_i, j=1,…K, f=1….F_j+1 (2.21)

Considering the states V^AV^B, V^AB^B, B^AV^B, B^AB^A, we generate new other node balance equations: the probability of entering these states must be equal to the probability of leaving these states.

$$W^{A} V_{k,f_2}^{B} p_{j,f_1}^{V(A)}\left(1-r_{k,f_2}^{V(B)}\right) + W^{B} V_{j,f_1}^{A} p_{k,f_2}^{V(B)}\left(1-r_{j,f_1}^{V(A)}\right) = V_{j,f_1}^{A} V_{k,f_2}^{B}(r_{j,f_1}^{V(A)} + r_{k,f_2}^{V(B)})$$

j=1,…i-1, f_1=1,…F_j-1, k=1….i-1, f_2=1,….F_k+1 (2.22)

$$W^A B^B_{k,f_2} p^{B(A)}_{j,f_1}\left(1-r^{B(B)}_{k,f_2}\right)+W^B B^A_{j,f_1} p^{B(B)}_{k,f_2}\left(1-r^{B(A)}_{j,f_1}\right)=B^A_{j,f_1} B^B_{k,f_2}(r^{B(A)}_{j,f_1}+r^{B(B)}_{k,f_2})$$

$j=1,\dots i-1,\ f_1=1,\dots F_j-1,\ k=1\dots i-1,\ f_2=1,\dots F_k+1$ (2.23)

$$W^A V^B_{k,f_2} p^{V(A)}_{j,f_1}\left(1-r^{B(B)}_{k,f_2}\right)+W^B V^A_{j,f_1} p^{B(B)}_{k,f_2}\left(1-r^{V(A)}_{j,f_1}\right)=V^A_{j,f_1} B^B_{k,f_2}(r^{V(A)}_{j,f_1}+r^{B(B)}_{k,f_2})$$

$j=1,\dots i-1,\ f_1=1,\dots F_j-1,\ k=1\dots i-1,\ f_2=1,\dots F_k+1$ (2.24)

$$W^A V^B_{k,f_2} p^{B(A)}_{j,f_1}\left(1-r^{V(B)}_{k,f_2}\right)+W^B V^A_{j,f_1} p^{V(B)}_{k,f_2}\left(1-r^{B(A)}_{j,f_1}\right)=V^B_{j,f_2} B^A_{k,f_1}(r^{B(A)}_{j,f_1}+r^{V(B)}_{k,f_2})$$

$j=1,\dots i-1,\ f_1=1,\dots F_j-1,\ k=1\dots i-1,\ f_2=1,\dots F_k+1$ (2.25)

Considering the set of states R, R^AV^B, R^AB^B, R^BV^A, R^BB^A, we will write down the last node balance equations: the probability of entering the states must be equal to the probability of leaving the states.

$$\Big(R_{i,g}+\sum_{j=1}^{i-1}\sum_{f_1=1}^{F_i} R^B_{i,g} V^A_{j,f_1}+\sum_{k=1}^{i-1}\sum_{f_2=1}^{F_k} R^A_{i,g} V^B_{k,f_2}+\sum_{k=i+1}^{K}\sum_{f_2}^{F_k+1} R^A_{i,g} B^B_{k,f_2}+\sum_{j=i+1}^{K}\sum_{f_1=1}^{F_j+1} R^B_{i,g} B^A_{j,f_1}\Big) r_{i,g}=\ p_{i,f_1}(W^A W^B+\sum_{j=1}^{i-1}\sum_{f_1=1}^{F_j+1} W^B V^A_{i,f_1}+\sum_{k=1}^{i-1}\sum_{f_2=1}^{F_k+1} W^A V^B_{k,f_2}+\sum_{j=i+1}^{K}\sum_{f_1=1}^{F_j+1} W^B B^A_{j,f_1}+\sum_{k=i+1}^{K}\sum_{f_2=1}^{F_k+1} W^A B^B_{k,f_2})$$

$g=1,\dots F_i$ (2.26)

To finally calculate the unknown transition probabilities, we take the balance equations for node W^AV^B and, simplifying them, we obtain:

$$p^{V(B)}_{k,f_2}=\frac{Ps^B_{k,f_2}(i-1)}{E^B(i-1)} r^{V(B)}_{k,f_2}$$ $k=1,\dots K;\ f_2=1,\dots F_k+1$ (2.27)

$$p^{V(A)}_{j,f_1}=\frac{Ps^A_{j,f_1}(i-1)}{E^A(i-1)} r^{V(A)}_{j,f_1}$$ $j=1,\dots i-1;\ f_1=1,\dots F_j+1$ (2.28)

Now we need to build two models, one for pseudo-machine $M^{U(A)}(i)$ and another one for pseudo-machine $M^{U(B)}(i)$. The final result is a five state model. The probability of each state is calculated with the values computed through the previous equations. Summing up, we obtain:

$W^{A*} = W^A W^B \alpha^A + W^A V^B + W^A B^B$ $\quad W^{B*} = W^A W^B \alpha^B + W^B V^A + W^B B^A$

$V^{A*} = V^A V^B + V^A B^B + W^B V^A + R^B V^A$ $\quad V^{B*} = V^A V^B + V^B B^A + W^A V^B + R^A V^B$

$R^{A*} = R + R^A V^B + R^A B^B$ $\quad R^{B*} = R + R^B V^A + R^B B^A$

$B^{A*} = W^B B^A + V^B B^A + B^A B^B + R^B B^A$ $\quad B^{B*} = W^A B^B + V^A B^B + B^B B^{\ A} + R^A B^B$

$W^B = W^A W^B \alpha_B$
$W^A = W^A W^B \alpha_A$

With these two models and knowing all the state probabilities, it is possible to calculate local failure parameters for the pseudo machine. For the parameters of the competition failure, we assume that the repair rate of the competition failure must be high enough to allow a frequent switch in the production of the two part types. We can evaluate failure probabilities through balance equation to nodes R^{A*} and W^B:

$$p_{i,g}^{A*} = \frac{R^{A*}}{W^{A*}} r_{i,g} = \frac{R^{A*}}{E^A(i-1)} r_{i,g} \quad g=1,\dots,F_i \tag{2.29}$$

$$p_{i,F_i+1}^{A*} = \frac{W^B}{W^{A*}} r_{i,F_i+1}^{A*} \tag{2.30}$$

We further introduce the constraint that the sum of all the failure probabilities, for the different failure modes, must be lower than 1.

Now, there are two ways to decide the value of p_{i,F_i+1}^{A*} and r_{i,F_i+1}^{A*}:

$$W^B < W^A \quad \rightarrow \; r^{A*}_{i,F_i+1} = 1 \qquad p^{A*}_{i,F_i+1} = \frac{W^B}{W^{A*}} \tag{2.31}$$

$$W^B > W^A \quad \rightarrow \; p^{A*}_{i,F_i+1} = \alpha_B \qquad r^{A*}_{i,F_i+1} = \frac{W^{A*}}{W^B} p^{A*}_{i,F_i+1} \tag{2.32}$$

Similarly, we can calculate $p^{B*}_{i,g}$, p^{B*}_{i,F_i+1} and r^{B*}_{i,F_i+1} for machine $M^{U(B)}(i)$.

Once all the failure probabilities are obtained, they can be used within the building block of product A and the building block of product B [26].

The accuracy of this method depends on the number of machines and buffers by which a line is composed. The more machines and buffers a line has, the less is the precision of calculation. This can be shown with an example of application of this methodology. In this example, we will consider a two machine-two buffer system (CASE 1) and a three-machine/four buffers system (CASE 2), each one producing two product types, A and B, and we will calculate their average throughput and buffer levels.

The initial data and the performance calculation for each product type are reported, respectively, in table 2.2 and table 2.3 [26].

CASE 1	p_i	r_i	$B^A(i)$	$B^B(i)$	α_i^A	α_i^B
i=1	0,23	0,4	4	6	0,6	0,4
i=2	0,37	0,3			0,6	0,4

Table 2.1: Initial parameters of CASE 1 [26]

	$E(i)$	$\bar{n}_b$	$Ps(i)$	$Pb(i)$
TYPE A	0,264	3,223	0,011	0,457
TYPE B	0,183	5,265	0,003	0,537

Table 2.2: Average throughput and buffer levels of CASE 1 [26]

To obtain the results in table 2.3, it was necessary to carry out 9 iterations. We will calculate now the throughput percentages using the results in table 2.3:

$$\alpha^{*A} = \frac{E^A}{E^A+E^B} = 0{,}591 \quad (2.33)$$

$$\alpha^{*B} = \frac{E^B}{E^A+E^B} = 0{,}409 \quad (2.34)$$

It should be noticed that these percentages are quite different from the initial values of $\alpha^A{}_i$ and $\alpha^B{}_i$ adopted in the system. This is due to the different occurrence of blockings.

Let's take now the CASE 2:

CASE 2	p_i	r_i	$B^A(i)$	$B^B(i)$	α_i^A	α_i^B
i=1	0,12	0,35	6	8	0,6	0,4
i=2	0,16	0,3	6	10	0,6	0,4
i=3	0,8	0,5			0,6	0,4

Table 2.3: Initial parameters of CASE 2 [26]

	$E^A(i)$	$E^B(i)$	$\bar{n}_b^A(i)$	$\bar{n}_b^B(i)$	$Ps^A(i)$	$Pb^A(i)$	$Ps^B(i)$	$Pb^B(i)$
i=1	0,381	0,270	4,500	6,861	0,018	0,293	0,001	0,362
i=2	0,381	0,270	0,737	0,565	0,467	0,001	0,556	1,6E-06

Table 2.4: Average throughput and buffer levels of CASE 2 [26]

In CASE 2, the computed α^{*A} and α^{*B} results to be respectively 0,585 and 0,415. It should be noticed that the difference with the initial values used in the system is greater than the one measured in the previous case: this is due to a greater length of the line in CASE 2, and, as a consequence, to an increased occurrence of blocking and starvation phenomena [26].

2.5.2 Example of method application

We will consider now a 4-machines Merge and Split system like the one in Figure 2.14 [25] below.

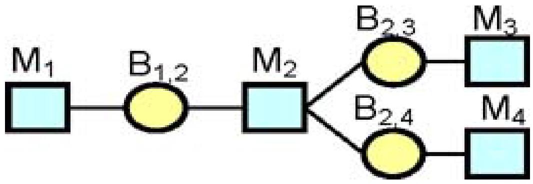


Figure 2.14: Split and Merge system [25]

We want to apply the two-level decomposition technique to calculate all the possible steady state probabilities in which each machine of the system can be; as in the example represented in the previous paragraph, in Figure 2.13. We will consider, for now, machine M_2: in table 2.6 [25] are reported all the machine states, according to the state of the buffers (E=Empty, NF=Not empty, F=Full, NF=Not full).

IN BUFFERS			MACHINE STATE	OUT BUFFERS	PROB
$B_{1,2}$	**$B_{2,3}$**	**$B_{2,4}$**			
E	NF	NF	STARVED (S)	/	/
NE	NF	F	UP ($W^{2,3}B^{2,4}$)	$B_{2,3}$	1
NE	F	NF	UP ($B^{2,3}W^{2,4}$)	$B_{2,4}$	1
NE	NF	NF	UP ($W^{2,3}W^{2,4}$)	$B_{2,3}$	$\alpha^{2,3}$
NE	NF	NF	UP ($W^{2,3}W^{2,4}$)	$B_{2,4}$	$\alpha^{2,4}$
NE	F	F	BL ($B^{2,3}B^{2,4}$)	/	/
E	NF	F	STARVED ($SB^{2,4}$)	/	/
E	F	NF	STARVED ($SB^{2,3}$)	/	/
NE	NF	F	DOWN ($RB^{2,4}$)	/	/
NE	F	NF	DOWN ($RB^{2,3}$)	/	/
NE	NF	NF	DOWN (R)	/	/

Table 2.5: Behaviour of machine M_2 [25]

Then, all the possible states in which the original machine M_2 can be found by solving its specific markov-chain, shown in figure 2.15 [25] below.

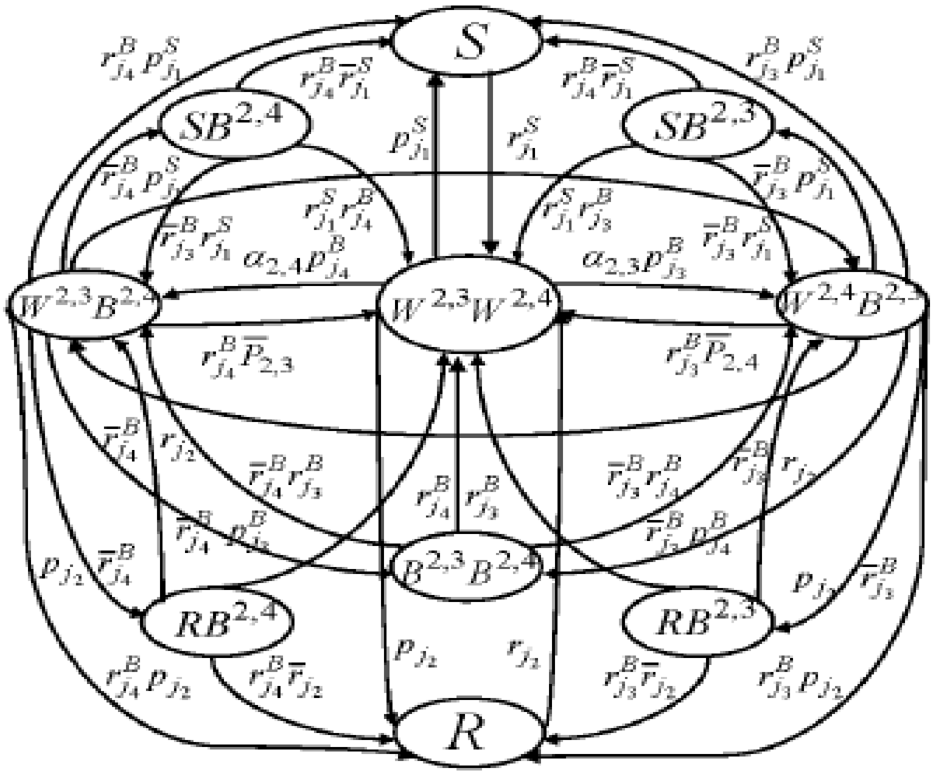


Figure 2.15: Markov chain of machine *M₂* [25]

Once evaluated all the buffer level decomposition probabilities, we pass from the buffer level decomposition to the machine level decomposition, so new equations are needed to calculate the transition probabilities to Starvation states of the downstream pseudo buffers and to Blocking states of the upstream pseudo buffers:

$$Ps_{j_1} = \frac{Ps_{j_1}(1,2)}{E(1,2)} r_{j_1} \tag{2.35}$$

$$Pb_{j_3} = \frac{Pb_{j_3}(2,3)}{E(2,3)} r_{j_3} \qquad Pb_{j_4} = \frac{Pb_{j_4}(2,4)}{E(2,4)} r_{j_4} \tag{2.36}$$

Furthermore, passing from the machine level decomposition to the buffer level decomposition, new decomposition equations are needed to calculate the parameters of the failures of each pseudo machine.

Considering the upstream pseudo machines $M^U(2,3)$ and $M^U(2,4)$, we will compute the probabilities of failure and repair in local mode as:

$$r_{j_2}^{u(2,3)} = r_{j_2} \tag{2.37}$$

$$p_{j_2}^{u(2,3)} = \frac{\pi(R_{f_2}) + \sum_{j_4=1}^{F_4} \pi(R_{j_2} B_{j_4}^{2,4})}{\pi(W^{2,3} W^{2,4})\alpha^{2,3} + \sum_{j_4=1}^{F_4} W^{2,3} B_{j_4}^{2,4}} r_{j_2}^{u(2,3)} \tag{2.38}$$

Consequently, the probabilities of failure and repair regarding the competition failure are:

$$p_{F_2+1}^{u(2,3)} = (1 - P_{2,3})\alpha_{2,4} \frac{\pi(W^{2,3} W^{2,4}) + \sum_{j_4=1}^{F_4} r_{j_4} \pi(W^{2,3} B_{j_4}^{2,4})}{\pi(W^{2,3} W^{2,4})\alpha^{2,3} + \sum_{j_4=1}^{F_4} W^{2,3} B_{j_4}^{2,4}} \tag{2.39}$$

$$r_{F_2+1}^{u(2,3)} = \frac{\pi(W^{2,3} W^{2,4})\alpha^{2,3} + \sum_{j_4=1}^{F_4} W^{2,3} B_{j_4}^{2,4}}{\pi(W^{2,3} W^{2,4})\alpha^{2,4}} p_{F_2+1}^{u(2,3)} \tag{2.40}$$

where $P_{2,3}$ is equal to the sum of all the transition probabilities to local failure states, starvation states and blocking states for machine M_2.

Pseudo machine $M^d(1,2)$ has local failures equal to those of machine M_2 of the original system. Moreover, when M_2 is blocked it interrupts material flow from buffer $B(1,2)$. All this can be described by equations (2.41) and (2.42):

$$p_{j_3 j_4}^{d(1,2)} = \frac{\pi(W^{2,3} B_{j_4}^{2,4}) Pb_{j_3}(1 - rb_{j_4}) + \pi(W^{2,4} B_{j_3}^{2,3}) Pb_{j_4}(1 - rb_{j_3})}{\pi(W^{2,3} W^{2,4}) + \sum_{j_4=1}^{F_4} \pi(W^{2,3} B_{j_4}^{2,4}) + \sum_{j_3=1}^{F_3} \pi(W^{2,4} B_{j_3}^{2,3})} \tag{2.41}$$

$$r_{j_3 j_4}^{d(1,2)} = \frac{\pi(W^{2,3} B_{j_4}^{2,4}) Pb_{j_3}(1 - rb_{j_4}) + \pi(W^{2,4} B_{j_3}^{2,3}) Pb_{j_4}(1 - rb_{j_3})}{\pi(B_{j_3}^{2,3} B_{j_4}^{2,4})} \tag{2.42}$$

The algorithm used to estimate the performances of the whole line, evaluates, alternatively, the machine-level decomposition and the buffer-level decomposition.

The algorithm stops when the pseudo machine failures parameters converge to a unique value [25].

This method has been used to support the reconfiguration of a real system: a washing machine production line. This line is composed by 10 machines and 10 buffer storages. The machines are connected with modular belt conveyor, they act as the buffers as well. In the line, two types of drums are produced and they are both carried on the conveyors by a unique type of pallet. Therefore, just buffers $B_1(4,5)$ and $B_2(4,5)$ are dedicated to a particular part type. It's has been proved that the MTTF and MTTR of machines are geometrically distributed. Below is reported a schematic view of the production system.

The company wants to increment the production rate of the line exploiting the modularity of the transportation system. It has two alternatives:

1) To better reallocate the buffer modules so to increment the production, as shown in figure 2.16 [25], without any change in machines behaviour;
2) To modify the behaviour of machine M_5: given that M_5 is frequently stopped because of buffer B(3,5) is often empty, the idea is a new modified system, as in figure 2.17 [25], in which M_3 can split the material into buffers $B_1(3,5)$ and $B_2(3,5)$, and M_5 can take parts from both the upstream buffers. Moreover, if a part type is not present in the dedicated buffer, machine can process the other one, without wasting time.

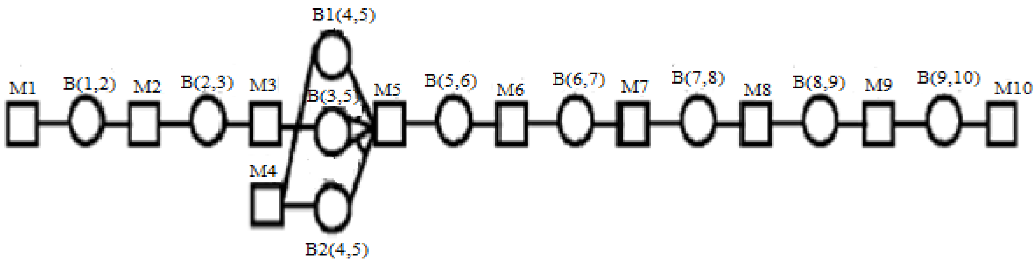


Figure 2.16: Reconfiguration alternative 1 [25]

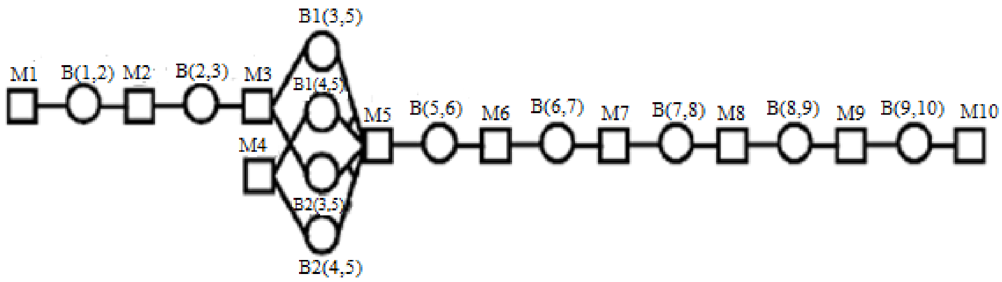


Figure 2.17: Reconfiguration alternative 2 [25]

Table 2.7 shows a comparison between the results of the three system configuration alternatives: line 1 refers to the current production plant, line 2 refers to reconfiguration alternative 1 and line 3 refers to the reconfiguration alternative 2.

CONFIG	AVERAGE THROUGHPUT			AVERAGE WIP			NR MOD
	SIM	AM	e %	SIM	AM	e %	
1	0,662	0,671	1,36	54,39	57,94	4	88
2	0,675	0,676	0,25	52,27	55,7	3,8	88
3	0,681	0,688	1,02	54,8	56,38	1,78	88

Table 2.7: Comparison between different system configuration [25]

The performances are computed through the approximate analytical method and they are quite accurate. It should be noticed by the table that, a modification of the system configuration, could increase the average throughput by about 2.8% [25].

2.6 System level analysis

Once elaborated a proper recycling model, it is necessary to carry out a system level analysis considering the material flow in the recycling system and the logistics issues, such as machine breakdowns, blockages propagation and finite buffers. This analysis level is a completion of the process layer analysis described in the previous chapter, in a final integrated multi-scale and multi-level approach. To accomplish this system analysis, we will introduce a new matrix: the transformation matrix $Q_{k,t}^{i,k}$.

The matrices $Q_{k,t}^{i,k}$ express how the stage M_k transforms the vector $\boldsymbol{Y_{i,k}}$ of input material particles fractions into the vector $\boldsymbol{Y_{k,t}}$ of output material particles fractions. Matrix $\boldsymbol{Q}$ is estimated by process layer, and in case of Size reduction and Separation process, it is in function of the throughput, the input fractions, the process operational parameters (P_C) and the process design parameters (P_D).

$$Q = f(E, \boldsymbol{Y}, P_C, P_D) \tag{2.43}$$

$\boldsymbol{Q}$ has specific properties and characteristics depending on each process type.

- For Size reduction process, there is just one matrix $\boldsymbol{Q_{k,t}}$ and it is a dense matrix V*V (where V is the total number of particle classes) and the sum of the numbers in each columns must be equal to 1. In fact, Size reduction stage has just one output and the entirety (100%, i.e. 1) of particles in input is rejected in the output flow (even though, of course, they are reduced into smaller classes).

- For Separation process, there are as many matrices $\boldsymbol{Q_{k,t}}$ as are the number of output flows; they are diagonal matrices V*V and, taking all the matrices into consideration, for each row, the sum of the numbers in the matrices at a particular row is always 1, for the principle of conservation of input particles flow into the output flows.

- For Splitting process is very similar; what changes is just that in each matrix all the numbers are identical and equal to $\alpha_{k,t}$, the percentage of material sent to output branch *(k,t)*.
- For Mixing stage, there is just one identity matrix $\boldsymbol{Q^{i,k}}$, in fact there's just one output flow and there isn't any kind of change between inputs and output.

The system layer analysis takes in input, from the processes analysis, the updated estimations of the stage processing rates, μ_k, and matrices $Q_{k,t}^{i,k}$; it takes into account the buffer capacity (*N*) of each buffer of the line, the failure rates and repair rates of each machine (p_k and r_k) and it finally gives a result in terms of final throughput (*E*) and final particles fractions (***Y***); all this is shown in the example of figure 2.18 [24] below.

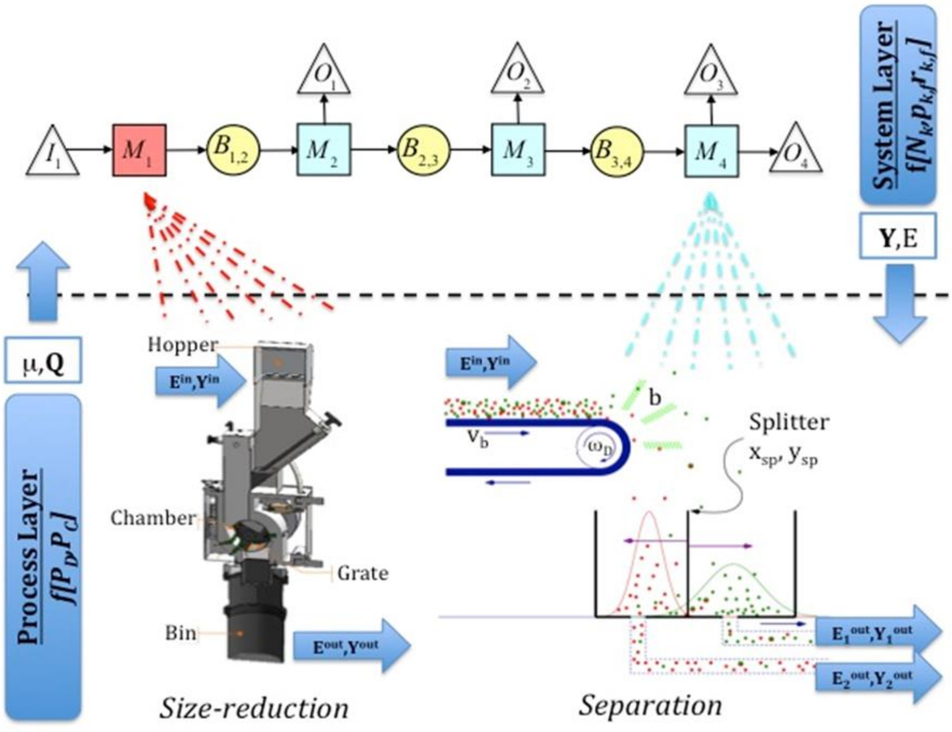


Figure 2.18: Example of multi-scale, multi-level analysis approach [24]

This analysis approach, in couple with the two-level decomposition method explained in the previous chapter, allows to calculate all the performances of interest in the system.

Focusing on the performances equations, the principle of conservation of flow is valid for each stage M_k of any considered process type, as describes equation (2.44):

$$E_{k,t}Y_{k,t} = \sum_{(i,k)\in\Omega_k} Q_{k,t}^{i,k} Y_{i,k} E_{i,k}, \quad \sum_{v=1}^{V} y_{k,t}(v) = 1 \quad \forall(k,t) \in \Delta_k$$

$$\sum_{(k,t)\in\Delta_k} E_{k,t} = \sum_{(i,k)\in\Omega_k} E_{i,k} \tag{2.44}$$

For the Separation process, the equations become:

$$\alpha_{k,t}Y_{k,t} = Q_{k,t}Y_{i,k}, \; \sum_{v=1}^{V} y_{k,t}(v) = 1 \quad \forall(k,t) \in \Delta_k$$

$$\sum_{(k,t)\in\Delta_k} E_{k,t} = E_{i,k}, \quad E_{k,t} = \alpha_{k,t}E_{i,k} \tag{2.45}$$

Once obtained in this way the input material fractions and the throughput for each stage, it is finally possible to calculate the Grade and Recovery for all output branches for each target material is the system:

$$G_{i,j}(z) = \sum_{v=1}^{V} y_{i,j}(v)H^l(v,l)C(l,z) \quad \forall j \in O, \; \forall z \in T$$

$$R_{i,j}(z) = \frac{E_{ij}G_{i,j}(z)}{\sum_{a\in l,b\in M} E_{a,b} \sum_{v=1}^{V} y_{a,b}(v)H^l(v,l)C(l,z)} \tag{2.46}$$

The effective output system flow rates are:

$$E_{i,j}^{Eff} = E_{i,j} \sum_{z\in T} G_{i,j}(z) \quad \forall j \in O \tag{2.47}$$

[30]

Example of performance calculation

We will take now, for calculation, an example of system composed of a size reduction stage and two separation stage, with 3 output branches in total, like the one in figure 2.19.

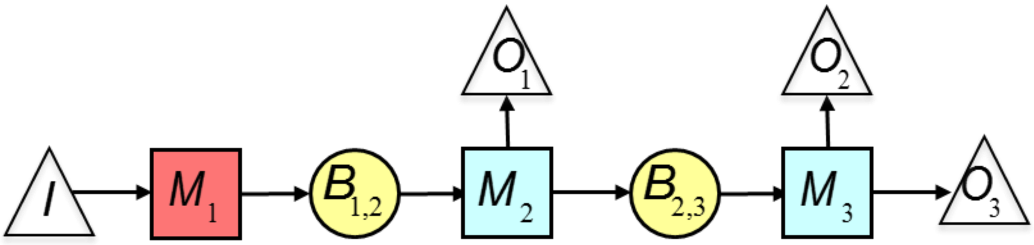


Figure 2.19: 3-stages recycling system

The matrices Q, for each stage, are the following:

Stage 1:

$$Q_1 = \begin{bmatrix} 0.9 & 0 & 0 & 0 \\ 0.1 & 1 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0.8 & 1 \end{bmatrix}$$

Stage 2:

$$Q_{2,O1} = \begin{bmatrix} 0.95 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix} \qquad Q_{2,3} = \begin{bmatrix} 0.05 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0.9 \end{bmatrix}$$

Stage 3:

$$Q_{2,O2} = \begin{bmatrix} 0.09 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0.75 & 0 \\ 0 & 0 & 0 & 0.96 \end{bmatrix} \qquad Q_{2,o3} = \begin{bmatrix} 0.91 & 0 & 0 & 0 \\ 0 & 0.09 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.04 \end{bmatrix}$$

The material mixture is composed of Plastics, Copper and Aluminum and is processed with a flow rate of 70kg/h. The particles are classified into L=2 liberation classes and D=2 size classes, defined as follows:

Size class	**min**	**max**
1	0	2
2	2	10

C(l,z)	**Plastic**	**Al**	**Cu**
1	0.9	0	0.1
2	0	0.8	0.2

HL(v,l)	**1**	**2**
1	1	0
2	1	0
3	0	1
4	0	1

HS(v,s)	**1**	**2**
1	1	1
2	0	0
3	1	1
4	0	0

The concentration of particles in the input flow is as follows $\boldsymbol{Y_{I,1}=[0.6,0,0.4,0]}$.

Making all the calculation, the results in terms of Grade and Recovery are:

Grade:

- Output flow O_1: G(Plastics, Al, Cu)=[0.077, 0.731, 0.191].
- Output flow O_2: G(Plastics, Al, Cu)=[0.47, 0.382, 0.147].
- Output flow O_3: G(Plastics, Al, Cu)=[0.891, 0.008, 0.101].

Recovery:

- Output flow O_1: R(Plastics, Al, Cu)=[0.055, 0.88, 0.526].
- Output flow O_2: R(Plastics, Al, Cu)=[0.078, 0.106, 0.094].
- Output flow O_3: R(Plastics, Al, Cu)=[0.867, 0.013, 0.379].

Chapter 3

A tool for recycling system design

3.1 Introduction to the design tool

In this chapter, we present a new informatic program able to solve the problem of profit maximization. In the first part of the program – given a predefined machines layout, formed by K stages and K-1 buffer storages, and where machines can be of three types, shredder, generic separator and mixer, and given the presence of a certain number of disposable machines for each stage – this tool chooses the best combination of machines to allocate to the stages (each stage has one machine) so that the final system profit is maximized. Each possible machine has a given set of parameters, a matrix Q and a specific hourly cost, and each target material in the system has a specific hourly revenue rate. Given all these data, the program selects, simultaneously, the machine to take for each stage and the optimal buffer capacity for each buffer storage, considering revenues, buffer unitary costs and WIP costs, to finally maximize the total line profit. In order to choose the best machines combinations among all the present ones, the program doesn't analyze each one, but it uses the bisection method: to eliminate all the useless combinations in few iterations and to select the best one in few time, avoiding spending so much time to analyze all of them. It takes inspiration from the Traina-Gershwin algorithm. The latter one is originally applied in manufacturing systems, but it comes in handy also in recycling system design for the optimal machines choice. At the same time, for the calculation of the maximum recycling system profit, the Gershwin-Shi method comes in handy for the computation of the optimal buffer capacity for each two machine line buffer in the system layout; as a matter of fact each recycling system machine is considered to have a typical failure rate and repair rate – exactly as in a traditional manufacturing system –.

Traina-Gershwin model, traditionally applied in manufacturing systems, are now used in the program in new perspective: the implementation design of a de-manufacturing system.

3.2 Assumptions, notations and formulation of the problem

Each possible machine, available for each stage, is characterized by a production rate, one or two failure rate(s) and repair rate(s), the process type chose for that particular stage, a matrix Q dependent on the process type to which it is dedicated to, and an hourly cost. Stages can be set in line each other or can be more stages in parallel. Between two stages is always present a buffer storage, so that any possible system is a K-machine/(K-1)-buffers, so (K-1) two machine lines by which the system can be decomposed, and each buffers storage can have a specific storage capacity; moreover, given a specific storage capacity and production rate computed for each two machine line, is created a certain quantity of Work-in-progress product, which affects the total costs. Each stage can have one or more input and one or more outputs; in detail, Shredding machines can have just one input and one output, Mixers have two or more inputs and just one outputs and Separators have one input and two or more outputs. Separators can have Stage-outputs - if the output flow is directed to another stage - or target output, dedicated to one or more final target materials. In the system, there are two or more target materials, divided into a certain number of Liberation classes and Size classes, therefore there's a total number of particle types given by the product of these two. There is an initial matrix Cl defining the percentage of each material to each liberation class and a binary matrix HL associating the particles types to each liberation class. For all initial stages, there is an initial class concentration vector, with which it's possible to calculate all the successive concentration vectors for the successive stages and also the concentration vectors for target outputs. Whit this last one it is possible to calculate the Grade, for each material, of the target output flows, therefore the effective throughput for each material of the system. The product of this last one with the hourly revenue of materials, minus all the machines costs, it results an estimate of the hourly profit of the system. It doesn't matter if it is just an hourly performance and not an "absolute" value because, among the different possible combinations, the maximum value doesn't change whatever the amount of work hours is.

Following there is a detailed description of all the variables and parameters necessary for the problem, and the problem formulation in equations

Z: set of target materials in the system, *{1,….z,….Z};*

L: set of liberation materials classes, *{1,….l,….L}*

S: set of dimension classes, *{1,…s,..S}*

V: total set of particles classes, according to both the liberation and size, $V = S \cup L$*,*

V={1,….v,….V}

Cl(L,Z): matrix of concentration of material *z* in liberation class *l*

HL(V,L): binary matrix associating, to each particle class *v*, a liberation class *l*

$$\begin{cases} HL(v,l) = 1 \quad if \quad l \in v \\ HL(v,l) = 0 \quad otherwise \end{cases} \tag{3.1}$$

$\boldsymbol{\Omega_k}$*:* set of branches, for each stage M_k*,* entering the stage

Λ_k *:* set of branches, for each stage M_k*,* leaving the stage

Γ *:* set of directed connections, to describe the typology of the system

SET(k): number of disposable machine for each stage *k,* $\{1, \dots .st, \dots ..ST\}_k$

M_k: set of stages $k \in \{1, \dots .k, \dots .K\}$, transforming the input mixture with fractions $Y_{i,k}$, with $i \in \Omega_k$, into an output mixture with fractions $Y_{k,t}$ with $t \in \Lambda_k$*,* according to the matrices $Q_{k,t}^{i,k}$

DECISIONAL VARIABLE-TYPE: t_k

$$\begin{cases} t_k = 1\ , if\ stage\ k\ is\ a\ Shredding\ process \\ t_k = 2\ , if\ stage\ k\ is\ a\ Separation\ process \\ t_k = 3\ , if\ stage\ k\ is\ a\ Mixing\ process \end{cases} \tag{3.2}$$

$Q_{k,t,st}^{i,k}(V,V)$*:* characteristic matrix associated to each stage *k,* of disposable machine *st,* according to the output flow out

($Q^{i,k}_{\Omega_k,st}(V,V)$ for Mixing process*;*

$q^{i,k}_{\Omega_k,st}(v,v)$*:* matrix number for index *(v,v),* for input flow

$Q_{k,t,\Lambda_k;st}(V,V)$ for Size-reduction and Separation processes

$q_{k,t,\Lambda_k;st}(v,v)$*:* matrix number for index *(v,v),* for output flow *)*

I_k*:* set of inputs in the system for each $k \in \{1, \dots . k, \dots . K\}$*,* $\{1, \dots . in, \dots . IN\}$

O_k*:* set of outputs in the system for each $k \in \{1, \dots . k, \dots . K\}$*,* $\{1, \dots . o, \dots . O\}$

$p_{k,st}$*:* probability of failure of stage *k,* for disposable machine st

$r_{k,st}$: reparation rate of stage *k,* for disposable machine *st*

$\mu_{k,st}$*:* production rate of stage *k,* for disposable machine *st [kg/h]*

$n_{k,st}$ *:* hourly operational cost for disposable machine *st* at stage *k [€/h]*

$Y_{k,\Omega_k}(V)$*:* particles input mixture fractions array, for each input branch Ω_k

$Y_{k,1}(V)$ is given

$y_{k,t,\Omega_k}(v)$*:* matrix number for index (v), for output mixture, for input branch Ω_k

$Y_{k,t,\Lambda_k}(V)$*:* particles output mixture fractions array, for each output branch Λ_k

$y_{k,t,\Lambda_k,}(v)$*:* matrix number for index *(v),* for output mixture, for output branch Λ_k

$$\begin{cases} Y_{k,t,\Lambda_k}(V) = Q_{k,t,\Lambda_k;st}(V,V) * Y_{k,\Omega_k}(V) \ , \quad \forall \Lambda_k \in \Delta_k \ , \ if \quad t_k = 1 || t_k = 2 \\ Y_{k,t,\Lambda_k}(V) = Q_{k,t,\Lambda_k;st}(V,V) * \left[\frac{\sum_{\Omega_k \in \Omega_k} Y_{k,\Omega_k}(V) * \sum_{(i,j)\in\Omega_k} E_{i,j}}{\sum_{(i,j)\in\Omega_k} E_{i,j}} \right] \ , \ \Lambda_k = 1, \forall k, \quad if \ t_k = 3 \end{cases} \tag{3.3}$$

$G_{i,j}(z)$*:* grade of target material z in the output flows *(i,j),* with $j \in O$

$$G_{i,j}(z) = \sum_{v=1}^{V} y_{i,j}(v) * HL(V,L) * Cl\,(L,Z) \tag{3.4}$$

$E^{eff}_{i,j}$ *:* effective flow rate of target materials, crossing the branch $(i,j) \in \Gamma$*, [kg/h]*

$$E^{eff}_{i,j} = E_{k,t,st} * \sum_{z\in Z} G_{i,j}(z) \tag{3.5}$$

α_z: hourly revenue from selling a kg of material *z [€*h/kg]*

Boolean variable $f_{k,st}$*:*

$$\begin{cases} f_{k,st} = 1, if\ disposable\ machine\ st\ at\ stage\ k\ is\ chosen \\ f_{k,st} = 0\ , otherwise \end{cases} \tag{3.6}$$

BB: Set of buffers in the system, *{1,….,bb,….,BB}, B=K-1*

N_{bb}: Chose buffer capacity for each line *bb* $\in BB$

WIP_{bb}: Unites of Work-in-progress created in the system for each line *bb* $\in BB$

cn: cost for each unit of buffer capacity

cw: cost for each unit of *WIP* created in the system

OBJECTIVE FUNCTION

$$max \sum_{z\in Z} \sum_{(i,j)\in \Gamma} E_{i,j}^{eff} * \alpha_z - \sum_{k=1}^{K} n_{k,st} * f_{k,st} - \sum_{bb=1}^{BB} cn * N_{bb} - \sum_{bb=1}^{BB} cw * WIP_{bb} \quad \forall st_k \in ST_k, \forall bb \in BB$$

CONSTRAINTS:

- *Existence of variables:*

$f_{k,st} \in \{0,1\}\ , \quad \forall k \in K,\ \forall st \in ST$

$t_k \in \{1,2,3\},\ \ \forall k \in K$

$HL(v,l) \in \{0,1\}\ ,\ \ \forall v \in V,\ \forall l \in L$

- *Just one feasible machine st can be chosen for each stage k:*

$\sum_{st_k=1}^{ST_k} f_{k,st} = 1\ ,\ \forall k \in K$

- *Conservation of flow:*

$$E_{k,t} * Y_{k,t} = \sum_{(i,k)\in\Omega_k} Q_{k,t}^{i,k} * Y_{i,k} * E_{i,k}\ ,\quad \forall(k,t) \in \Lambda_k$$

$$\sum_{v=1}^{V} y_{k,t}(v) = 1\ ,\quad \forall(k,t) \in \Lambda_k$$

$$\textstyle\sum_{(k,t)\in\Lambda_k} E_{k,t} = \sum_{(i,k)\in\Omega_k} E_{i,k} \tag{3.7}$$

Basically, there are no production rate constraints of machines because, given the maximization of profit, the best machines in terms of output flows are already chosen.

3.3 The Traina-Gershwin algorithm

The algorithm developed by J. A. Taina and S. B. Gershwin allows to rapidly choose simultaneously machines combination and optimal buffer capacity for each buffer to finally maximize the profit, considering a big amount of possible machines combinations.

Basically, it gives a solution for this problem:

$$max_{M,N}\ \pi(M,N) = AP(M,N) - \sum_{i=1}^{K-1} b_i \bar{n}_i(M,N) - \sum_{i=1}^{K-1} c_i N_i - \sum_{i=1}^{K} \eta_{i,j_i}$$

$$s.t{:}\qquad P(M,N) > P^*$$

$$N \geq N_{min} \tag{3.8}$$

where $\pi(M,N)$ is the total profit of the line, M should be the vector of chosen machines, N is the vector of chosen buffer capacities, A is the revenue coefficient and $P(M,N)$ is the production rate given the chosen vectors M and N, b_i is the WIP cost, associated to the WIP value $\bar{n}_i(M,N)$ created for each two machine lines by which the system can be decomposed, with the selected N and M; N_i is the chosen buffer capacity for each buffer and c_i is the respective associated unitary cost, and η_{i,j_i} is the hourly revenue machine cost, for each selected machine.

It's assumed that the line production rate *P(M,N)* must be greater than a given constraint production rate P^*, and that, for each buffer storage, the chosen buffer capacity should be at least equal to a minimum value N_{min}.

Considering that the system is composed by *K* stages and, for each stage, there are *s* feasible machines, the number of possible combinations is in total *S=s^K*. This is a very big number and, making an exhaustive research evaluating all the *S* possible combinations requires very much time. To speed things up, the Traina-Gershwin algorithm computes, for each factor influencing the profit π, a mean value between all the possible combinations *s*. For example, for the failure rate, the repair rate and the hourly machine cost, the calculations are:

$$\bar{r}_i^l = \frac{1}{s_i} \sum_j^{s_i} r_{ij} \tag{3.9}$$

$$\bar{p}_i^l = \frac{1}{s_i} \sum_j^{s_i} p_{ij} \tag{3.10}$$

$$\bar{\eta}_i^l = \frac{1}{s_i} \sum_j^{s_i} \eta_{ij} \tag{3.11}$$

and we will denote $\bar{x}_i^l$ as the vector of these three average parameters and x_i as the parameters vector of a specific machine *i*.

Then, we will calculate the profit π^* in function of the machines parameters closest to the average ones $\hat{x}_i = (\hat{r}_i, \hat{p}_i, \hat{\eta}_i)$, and the optimal buffer capacities related to these machines with Gershwin-Shi method.

After that, we take a feasible machine *i* and we compute the gradient of π^* according to x_i:

$$G_i = \frac{\pi^*(x_i+\varepsilon) - \pi^*(x_i)}{\varepsilon} \tag{3.12}$$

Now we will consider the value of G_i: if it is positive, all the parameters minor than $\bar{x}_i^l$ should be discarded (it means that the profit increases directly proportionally with x_i) , on the other hand, if it is negative, all the parameters major than $\bar{x}_i^l$ should be discarded; then a new average $\bar{x}_i^l$ is computed (without the discarded values). New parameters solutions closest to the new $\bar{x}_i^l$ are found and a new gradient G_i is computed anymore. This process is carried on few iterations, until a unique parameters solution remains (it will be the chosen solution), or if G_i reaches the value 0 (in this case, it means that x_i found is the optimal solution).

Empirically, each iteration removes approximately half of the remaining possible machines sets, so the process should be carried out approximately for log2(S) iterations and the profit is calculated log2(S) times, instead of the S times using the exhaustive research method. Therefore, this algorithm makes save very much time. In particular, analyzing two sets of 10000 four-stage random cases, with 4 available machines choices for each stage, it has been calculated that, with this algorithm, the average computation time for the optimal set solution is 35.2 seconds, while the average time for the complete enumeration is 3.54 minutes [27].

3.4 The Shi-Gershwin method

To optimize the buffer capacity of each buffer in the system, taking into account the cost of space and the cost of inventory of each buffer, comes into play the deterministic processing time model of Gershwin and Shi, as mentioned before. First, let's consider a general production line with buffers, like the one in figure 3.1 [1].

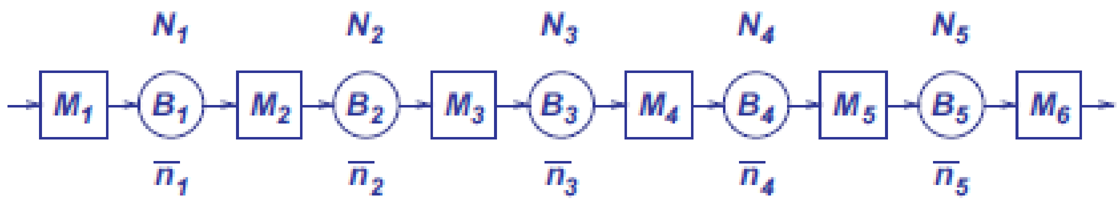


Figure 3.1: Example of a production line [1]

In this model [53], [54], we consider a line formed of k machines and k-1 buffers, called a 'k-machine, k-1 buffer line', or more simply 'k-machine line'. Machines i are denoted as M_i and Buffers i are denoted as B_i. Machines are supposed to have equal, deterministic and constant processing times. Time is discrete: each operation takes one time unit.

The buffers sizes N_i, $\forall i = 1, \ldots, k-1$, are the decisional variables. Machines are unreliable and they have specific failure and repair rates. More precisely, for each M_i, we denote p_i as the probability of failure and r_i as the repair rate. The times to failure and to repair are geometrically distributed and machine parameters are supposed to be fixed.

We denote P as the production rate of a line; although it is a function of machines and their reliability, we assume to vary only buffer sizes, so we write $P=P(N_1, \ldots, N_{k-1})$, or, equivalently, $P(\mathbf{N})$, where $\mathbf{N}$ is a vector $(N_1, \ldots, N_{k-1})$. So, $P(\mathbf{N})$ is a nonlinear function of buffer sizes $\mathbf{N}$, and, for lines having more than two machines, it can be calculated numerically by a two-machine line decomposition method.

The profit of a k-machine, k-1 buffers is given by:

$$profit = AP(N_1, \ldots, N_{k-1}) - \sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i - K \tag{3.13}$$

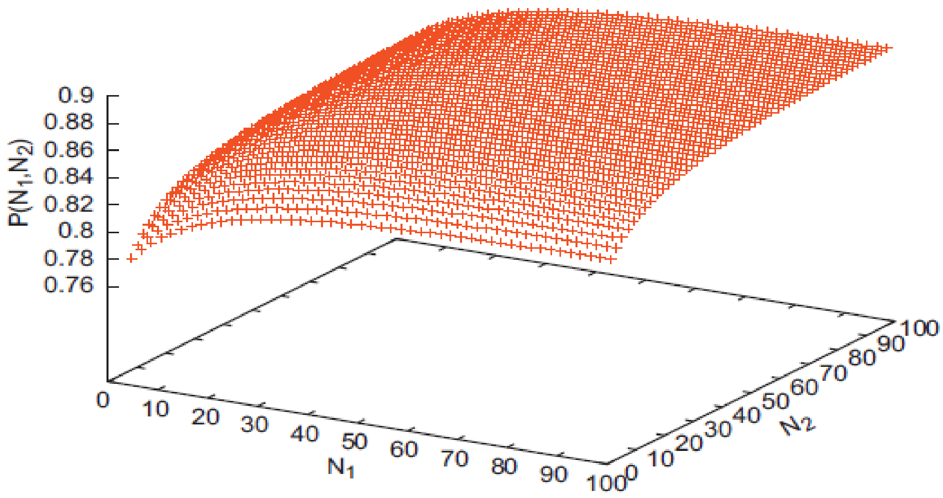
Where $A>0$ is a profit coefficient, b_i and c_i are cost coefficients related to the buffer space and average inventory for the i^{th} buffer, respectively, and K represents all the other types of costs not dependent on the buffer sizes. We can therefore rewrite the formulation in this way:

$$J(N_1, \ldots, N_{k-1}) = AP(N_1, \ldots, N_{k-1}) - \sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i \tag{3.14}$$

[1]

3.4.1 Assumptions, notations and formulations

A key assumption for this method is that P(***N***) is a concave and monotonically function increasing in ***N*****,** as shows graph 3.1 [1], related to an example of three machine line.



Graph 3.1: Function $P(N_1, N_2)$ of a three machine line [1]

The method is implemented in different mathematical models.

In the first model, the main goal is a constrained problem: the maximization of profits of a production line subject to a production rate constraint. The problem formulation is:

$$\max \quad J(N_1, \dots, N_{k-1}) = AP(N_1, \dots., N_{k-1}) - \sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i$$

$$\text{s.t:} \quad P(N_1, \dots, N_{k-1}) \geq P^*,$$

$$N_i > 0\,, \quad \forall i = 1, \dots . k-1, \tag{3.15}$$

where P^* represent a required minimum production rate. The first constraint is the production rate constraint; the second one is the buffers sizes constraint. It comes from the fact that negative buffer sizes are not logical.

In the second model, we add a limitation of buffers sizes. This is because the analytical production rate solution of each two machine line requires buffers sizes to be at least equal to 2. We will denote N_{min} as the minimum buffer size capacity required, and re-write the first constrained problem as:

$$\max \quad J(N_1, \ldots, N_{k-1}) = AP(N_1, \ldots., N_{k-1}) - \sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i$$

$$\text{s.t:} \quad P(N_1, \ldots, N_{k-1}) \geq P^*,$$

$$N_i \geq N_{min}, \quad \forall i = 1, \ldots. k-1, \tag{3.16}$$

Furthermore, there is also a third model: an unconstrained problem, not having the production rate limit, the formulation of which is:

$$\max \quad J(N_1, \ldots, N_{k-1}) = AP(N_1, \ldots., N_{k-1}) - \sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i$$

$$\text{s.t:} \quad N_i \geq N_{min}, \quad \forall i = 1, \ldots. k-1, \tag{3.17}$$

3.4.2 Solution technique

The unconstrained problem is solved through the gradient method, taking advantage of the analytical form of the two machine line evaluation, which enables us to treat N_i as a continuous variables. As a matter of fact, the formulas to calculate the production rate and average inventory of the two machine line do not require N_i to be integers. So, buffers sizes values have no constraints. Consequently, given that our model has deterministic processing time units and discrete state spaces, we can treat N_i as continuous variables. In addition, since we evaluate $P(\boldsymbol{N})$ by the two line decomposition technique, with the continuous variable version of the analytic two machine line evaluation, considering N_i as a continuous variable, we can ultimately treat $P(\boldsymbol{N})$ and $J(\boldsymbol{N})$ as continuously differentiable functions. This allows us to use the gradient method: we calculi the derivate $\frac{\partial P(\boldsymbol{N})}{\partial N_i}, \forall i,$ to solve the unconstrained problem. However, for lines with more than two machines, due to the lack of an analytical expressions of $P(\boldsymbol{N})$, the gradients are computed according to a specific difference formula. Furthermore, the objective function $J(N_1,\ldots,N_{k-1})$ of the unconstrained problem is supposed to have just one maximum point for this optimization method to work correctly.

As regards the unconstrained problem, two scenarios are possible:

1. The unconstrained problem has a solution $(N_1^*, \ldots, N_{k-1}^*)$ such that satisfies $P(N_1^*, \ldots, N_{k-1}^*) \geq P^*$. In this case, this is the final solution of the constrained problem is $(N_1^*, \ldots, N_{k-1}^*)$.
2. $P(N_1^*, \ldots, N_{k-1}^*) < P^*$. In this case, the found solution cannot be considered as the final solution of the constrained problem.

In the last case, we need to consider the following unconstrained problem:

$$\max \quad J(N_1, \ldots, N_{k-1}) = A' P(N_1, \ldots, N_{k-1}) - \sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i$$

$$\text{s.t.} \quad N_i \geq N_{min}\,, \ \forall i = 1, \ldots . k-1 \tag{3.18}$$

in which A of (3.12) is replaced by A'. We will consider now $(N_1', \ldots, N_{k-1}')$, in this case, as the final solution to the problem and $P' = P(N_1', \ldots, N_{k-1}')$.

Then, we make an **Assertion:** It can be demonstrated that the constrained problem

$$\max \quad J(N_1, \dots, N_{k-1}) = A'P(N_1, \dots., N_{k-1}) - \sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i$$

$$\text{s.t:} \qquad P(N_1, \dots, N_{k-1}) \geq P^*,$$

$$N_i > N_{min}\,, \quad \forall i = 1, \dots. k-1, \tag{3.19}$$

has the same solution, for all values of A', such that the solution of the unconstrained problem (3.18) has P'<P^*.

As a matter of fact, the solution of problem (3.19) will satisfy the condition $P(N_1, \dots., N_{k-1}) = P^*$, so the objective function can be rewritten as $A'P^* - \sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i$. Since the first term is not function of N_i, it doesn't change the solution of the problem.

In the assertion, we affirm that, if the optimal solution of the unconstrained problem (3.12) is different from optimal solution of the constrained problem (3.16), then the solution of the constrained problem (3.11), $(N_1^*, \dots., N_{k-1}^*)$, satisfies the condition $P(N_1^*, \dots., N_{k-1}^*) = P^*$. Consequently, to solve problem (3.17), we replace A with A' in both problems (3.16) and (3.17) and we therefore solve problem (3.18) taking different values of A'. Our aim is to find the value of A' such that the solution to problem (3.18) satisfies $P(N_1, \dots., N_{k-1}) = P^*$: this solution should be the same as the solution to problem (3.16).

The assertion is proved by the Karush-Kuhn-Tucker (KKT) conditions [52] of nonlinear programming. As a matter of fact, considering x^* as a regular local minimum of the problem min $\quad f(x)$

s.t: $\quad h_1(x) = 0, \dots, h_m(x) = 0,$

$$g_1(x) \leq 0, \dots., g_r(x) \leq 0, \tag{3.20}$$

where f, h_i and g_i are continuously differentiable functions defined from $\Re^n$ to $\Re$, according to KKT, there exists unique Lagrange multipliers $\lambda_1^*, \dots., \lambda_m^*$ and $\mu_1^*, \dots., \mu_r^*$, satisfying the following conditions:

$$\nabla_x L(x^*, \lambda^*, \mu^*) = 0,$$

$$\mu_j^* \geq 0,\ j = 1, \dots., r,$$

$$\mu_j^* g_j(x^*) = 0,\ j = 1, \dots., r, \tag{3.21}$$

where $L(x, \lambda, \mu) = f(x) + \sum_{i=1}^{m} \lambda_i h_i(x) + \sum_{j=1}^{r} \mu_j g_j(x)$ is called the Lagrangian function.

We will convert now the constrained problem (3.16) into a minimization form:

$$\min \quad -J(N_1, \dots, N_{k-1}) = -AP(N_1, \dots., N_{k-1}) - \sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i$$

$$\text{s.t:} \quad P^* - P(N_1, \dots, N_{k-1}) \leq 0,$$

$$N_i \geq N_{min}\ , \ \ \forall i = 1, \dots. k-1, \tag{3.22}$$

We know that N_i is a continuous variable, and *P(**N**)* and *J(**N**)* are continuously differentiable functions.

Before applying KKT conditions to our problem, we need to consider Slater constraint qualification condition for convex inequalities [52], that guarantees the existence of Lagrange multipliers.

We will take x^* as a local minimum of the problem (3.21), where f and g_j are continuously differentiable functions from $\mathfrak{R}^n$ to $\mathfrak{R}$, and the functions h_i are linear. We assume the functions g_j to be convex and we assume to exist a feasible vector $\bar{x}$ such that $g_i(\bar{x}) < 0,\ \forall j \in M(x^*)$. In this case, x^* is a minimum point that satisfies the KKT conditions.

We will consider now problem (3.22). There are no equality constraints in the problem, but there are k inequality constraints, representing our Slater constraint qualification conditions:

$$g_0(\boldsymbol{N}) = P^* - P(N_1, \dots, N_{k-1}) \leq 0,$$

$$g_i(\boldsymbol{N}) = N_{min} - N_i \leq 0, \quad \forall i = 1, \dots, k-1. \tag{3.23}$$

Given the concavity of *P(**N**)*, $g_0(\boldsymbol{N})$ is a convex function. All other $g_i(\boldsymbol{N})$ are also convex because they are linear. Of course, there exists a sufficiently large $\widehat{\boldsymbol{N}}$ such that $P(\widehat{N}_1, \dots, \widehat{N}_{k-1}) > P^*$ so $g_0(\widehat{N}_1, \dots, \widehat{N}_{k-1}) < 0$. Given $\widehat{\boldsymbol{N}}$, also the condition $g_i(\widehat{N}_1, \dots, \widehat{N}_{k-1}) < 0,\ \forall i = 1, \dots, k-1$ is satisfied because $N_{min} - \widehat{N}_i < 0,\ \forall i = 1, \dots, k-1$. Hence, Slater conditions are satisfied, and there exists unique Lagrange multipliers $\mu_i^*,\ i = 0, \dots, k-1$, for our problem to satisfy the following KKT conditions:

the first one, given by

$$-\nabla j(\boldsymbol{N}^*) + \mu_0^* \nabla\big(P^* - P(\boldsymbol{N}^*)\big) + \sum_{I=1}^{k-1} \mu_i^* \nabla(N_{min} - N_i^*) = 0 \qquad (3.24)$$

or

$$-\begin{pmatrix} \frac{\partial J(\boldsymbol{N}^*)}{\partial N_1} \\ \frac{\partial J(\boldsymbol{N}^*)}{\partial N_2} \\ \vdots \\ \frac{\partial J(\boldsymbol{N}^*)}{\partial N_{k-1}} \end{pmatrix} - \mu_0^* \begin{pmatrix} \frac{\partial P(\boldsymbol{N}^*)}{\partial N_1} \\ \frac{\partial P(\boldsymbol{N}^*)}{\partial N_2} \\ \vdots \\ \frac{\partial P(\boldsymbol{N}^*)}{\partial N_{k-1}} \end{pmatrix} - \mu_1^* \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} - \mu_2^* \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix} - \cdots - \mu_{k-1}^* \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad (3.25)$$

and the other ones that are

$$\mu_i^* \geq 0,\ \forall i = 0, \dots., k-1, \qquad (3.26)$$

$$\mu_0^*\big(P^* - P(\boldsymbol{N}^*)\big) = 0 \qquad (3.27)$$

and

$$\mu_i^*(N_{min} - N_i^*) = 0,\ \forall i = 1, \dots., k-1, \qquad (3.28)$$

in which $\boldsymbol{N}^*$ represents the optimal solution to our constrained problem.

Given all this, it can be demonstrated that finding the Lagrange multipliers $\mu_i^*,\ i = 0, \dots., k-1$, and the optimal solution $\boldsymbol{N}^*$ to satisfy the KKT conditions (3.25) - (3.28) is equivalent to solve the constrained problem (3.16) by our algorithm.

Suppose that $\boldsymbol{N}^*$ is an optimal solution $>N_{min}$ for problem (3.16). By condition (3.28), we know that $\mu_i^* = 0,\ \forall i = 1, \dots., k-1$.

Hence, we can rewrite equations (3.25) - (3.28) as:

$$
-\begin{pmatrix} \frac{\partial J(\boldsymbol{N}^*)}{\partial N_1} \\ \frac{\partial J(\boldsymbol{N}^*)}{\partial N_2} \\ \vdots \\ \frac{\partial J(\boldsymbol{N}^*)}{\partial N_{k-1}} \end{pmatrix} - \mu_0^* \begin{pmatrix} \frac{\partial P(\boldsymbol{N}^*)}{\partial N_1} \\ \frac{\partial P(\boldsymbol{N}^*)}{\partial N_2} \\ \vdots \\ \frac{\partial P(\boldsymbol{N}^*)}{\partial N_{k-1}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \tag{3.29}
$$

$$
\mu_0^* \big(P^* - P(\boldsymbol{N}^*) \big) = 0, \tag{3.30}
$$

where $\mu_0^* \geq 0$.

Since $\boldsymbol{N}^*$ is not the optimal solution of the unconstrained problem, $\nabla J(\boldsymbol{N}^*)/\partial N_i$ are equal to 0. Thus, to satisfy condition (3.29), it's necessary that $\mu_0^* \neq 0$. By condition (3.30), $\boldsymbol{N}^*$ satisfies $P(\boldsymbol{N}^*) = P^*$ and g_0, the only active inequality constraint, so $\boldsymbol{N}^*$ is a regular solution.

μ_0^* and $\boldsymbol{N}^*$ are found by conditions (3.29) and (3.30). For every μ_0^*, condition (3.29) is a system with k-1 equations and k-1 unknowns by which vector $\boldsymbol{N}^*$ is found, and $\boldsymbol{N}^* = \boldsymbol{N}^*(\mu_0^*)$. So, finally we must search for a value μ_0^* such that $P\big(\boldsymbol{N}^*(\mu_0^*)\big) = P^*$, that is basically what our algorithm does.

We will replace μ_0^* by $\mu_0 > 0$ in constraint (3.29):

$$
-\begin{pmatrix} \frac{\partial J(\overline{N})}{\partial N_1} \\ \frac{\partial J(\overline{\boldsymbol{N}})}{\partial N_2} \\ \vdots \\ \frac{\partial J(\overline{N})}{\partial N_{k-1}} \end{pmatrix} - \mu_0 \begin{pmatrix} \frac{\partial P(\overline{N})}{\partial N_1} \\ \frac{\partial P(\overline{\boldsymbol{N}})}{\partial N_2} \\ \vdots \\ \frac{\partial P(\overline{N})}{\partial N_{k-1}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \tag{3.31}
$$

where $\overline{\boldsymbol{N}}$ is the unique solution of the system. $\overline{\boldsymbol{N}}$ is also the solution of the following optimization problem:

$$\min \quad -\bar{J}(\boldsymbol{N}) = -J(\boldsymbol{N}) + \mu_0(P^* - P(\boldsymbol{N}))$$

$$\text{s.t.} \quad N_{min} - N_i \leq 0, \quad \forall i = 1, \ldots, k-1 \tag{3.32}$$

that can be rewritten as:

$$\max \quad \bar{J}(\boldsymbol{N}) = J(\boldsymbol{N}) - \mu_0(P^* - P(\boldsymbol{N}))$$

$$\text{s.t.} \quad N_{min} - N_i \leq 0\,, \quad \forall i = 1, \ldots, k-1 \tag{3.33}$$

or

$$\max \quad \bar{J}(\boldsymbol{N}) = AP(\boldsymbol{N}) - \textstyle\sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i - \mu_0(P^* - P(\boldsymbol{N}))$$

$$\text{s.t.} \quad N_{min} - N_i \leq 0, \quad \forall i = 1, \ldots, k-1 \tag{3.34}$$

or

$$\max \quad \bar{J}(\boldsymbol{N}) = (A + \mu_0)P(\boldsymbol{N}) - \textstyle\sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i$$

$$\text{s.t.} \quad N_i \geq N_{min}, \quad \forall i = 1, \ldots, k-1 \tag{3.35}$$

or, ultimately,

$$\max \quad \bar{J}(\boldsymbol{N}) = A'P(\boldsymbol{N}) - \sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i$$

s.t. $\quad N_i \geq N_{min}, \quad \forall i = 1, \dots, k-1$ (3.36)

that is exactly the unconstrained problem (3.18), and $\bar{\boldsymbol{N}}$ is its optimal solution.

Given that $\mu_0 > 0$, then $A' > A$. In addition, given the KKT condition (3.30), the optimal solution of the constrained problem $\boldsymbol{N}^*$ satisfies $P(\boldsymbol{N}^*) = P^*$. This means therefore that, for every $A' > A$ (or $\mu_0 > 0$), we can find the corresponding optimal solution $\bar{\boldsymbol{N}}$, by solving problem (3.18), that satisfies condition (3.31).

We will denote, now, $\boldsymbol{N}^{A'}$ as the solution to problem (3.18), given a specific A'. What we need to do is to find a value A' satisfying $P\left(\boldsymbol{N}^{A'}\right) = P^*$, so that $\mu_0 = A' - A$ and $\boldsymbol{N}^{A'}$ satisfy conditions (3.29) and (3.30). $\mu_0 = A' - A$ is exactly the Lagrange multiplier satisfying the KKT conditions of our constrained problem, and $\boldsymbol{N}^{A'}$ is the optimal solution of our constrained problem.

So, our **assertion** is essentially proved: solving constrained problem (3.16) with our algorithm, finding therefore the optimal solution $\boldsymbol{N}^{A'}$, is ultimately equal to finding the unique Lagrange multipliers.

Therefore, in our algorithm, we must conduct a one-dimensional search on $A' > A$, so to find a value A' such that $P(N_1, \dots, N_{k-1}) = P^*$, where $\boldsymbol{N}^* = \boldsymbol{N}^{A'}$is the solution of the unconstrained problem and, consequently, it is also the optimal solution of the constrained problem.

The algorithm can be summarized in three steps:

1. Check the feasibility of the problem. This means to ensure that the required production rate, P^*, is feasible for the line to be optimized, that is to say that P^* should satisfy

   $P^* < \min_i \frac{r_i}{r_i + p_i}$, (3.37) where $r_i/(r_i + p_i)$ is the isolated production rate of machine M_i. If this fails, no set of buffers can satisfy the production rate constraint.

2. Solve the unconstrained problem (3.17) and check if the solution $(N_1^*, \dots, N_{k-1}^*)$ satisfies $P(N_1^*, \dots, N_{k-1}^*) \geq P^*$. If this condition is satisfied, the outcoming N^* is the final optimal solution, otherwise, it is necessary to carry out step 3.

3. Do a one-dimensional search on A'>A, until the solution $N^* = N^{A'}$ of the unconstrained problem satisfies $P(N_1^*, \dots, N_{k-1}^*) = P^*$. When this happens, stop: $(N_1^*, \dots, N_{k-1}^*)$ is the final solution.

This third step is carried out using the ***Newton Chord method*** [55], a way to find t' so that $f(t')=0$ for a given function $f(\cdot)$. Thus, in this algorithm, for any particular value of A', $f(A')$ should be defined as:

$$f(A') = P\left(\boldsymbol{N}^{opt_{A'}}\right) - P^* \tag{3.38}$$

where $\boldsymbol{N}^{opt_{A'}}$ is the optimal buffers sizes solution associated with A' and P^* is the required production rate. In our algorithm, the method should consist of 3 steps:

1. Choose an initial value A_0' and $A_1' = A_0' + 1000$. Calculate slope s as

$$s = \frac{f\left(A_1'\right) - f\left(A_0'\right)}{A_1' - A_0'} \tag{3.39}$$

2. Determine A_2' so that

$$f(A_0') + f(A_2' - A_0')s = 0 \tag{3.40}$$

or, equivalently:

$$A_2' = -\frac{f(A_0')}{s} + A_0' \tag{3.41}$$

3. Repeat steps 1 and 2 with $A'_0 = A'_1$ and $A'_1 = A'_2$ until $|f(A'_0)|$ is small enough ($|f(A'_0)| \leq 10^{-4}$) [1].

3.5 Implementation of the program

First, the program computes, for each machine parameter, the combination nearest to the average one. We denote $\widehat{mu}_i$ as the production rate at stage *i* nearest to the mean production rate for stage *i*, $\hat{p}_i$ as the failure rate at stage *i* nearest to the mean failure rate for stage *i*, $\hat{r}_i$ as the repair rate at stage *i* nearest to the mean repair rate for stage *i* and $\hat{\eta}_i$ as the hourly cost at stage *i* nearest to the mean hourly cost for stage *i*.

Once selected the first combinations, the Shi-Gershwin method comes into play to select the optimal buffers sizes for the selected machine line - always considering each parameter -.

Actually, there isn't a practical mathematical way to find which is the function $P(\boldsymbol{N}_1,\dots,\boldsymbol{N}_k)$ for a line with more than 2 machines; so, state that in the function $P(\boldsymbol{N}_1,\dots,\boldsymbol{N}_k)$ the different variables $\boldsymbol{N}_i$ don't influence themselves (as a matter of fact, $P(\boldsymbol{N})$ is supposed to be a concave and monotonically increasing function for each N_i), we can divide the system in many two-machine lines, like the one in figure 3.2, so to calculate the function $P(\boldsymbol{N}_i)$ for each two-machine line *i* and then calculate $J(\boldsymbol{N}_i)$ and set $\frac{\partial J(\boldsymbol{N}_i)}{\partial \boldsymbol{N}_i} = 0$ to find the (positive) optimum values N_i^*(s).

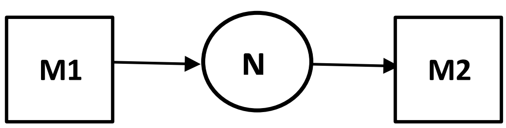


Figure 3.2: Two machine line with buffer

The system in Figure 3.2, has the two typical boundary states and two internal states and their respective steady-state probabilities. We denote r_1, p_1, r_2 and p_2 as the failure and repair rates of machines 1 and 2, and the buffer size is N. the system state is $s = (n, \alpha_1, \alpha_2)$ where n is the buffer level $(0 \leq n \leq N)$ and α_i is the repair state of machine i (i=1,2, $\alpha_i = 0{,}1$). $p(n, \alpha_1, \alpha_2)$ denotes the steady-state probability of the state. For sake of simplicity, we assume that for internal states $2 \leq n \leq N-2$; while for boundary states $n = 0{,}1, N-1$ or N. We denote p_B as the summation of the steady-state probabilities of all boundary states and p_I, as the summation of the steady-state probabilities of just all internal states. So:

$$p_B = \sum_{all\ boundary\ states} p(n, \alpha_1, \alpha_2) = CX\left(\frac{r_1+r_2-r_1r_2-r_1p_2}{r_1p_2}\right) + CX + CXY_2 + \frac{CX}{p_2}\left(\frac{r_1+r_2-r_1r_2-r_1p_2}{p_1+p_2-p_1p_2-r_1p_2}\right) + CX^{N-1} + CX^{N-1}Y_1 + \frac{CX^{N-1}}{p_1}\left(\frac{r_1+r_2-r_1r_2-p_1r_2}{p_1+p_2-p_1p_2-p_1r_2}\right) + CN^{N-1}\left(\frac{r_1+r_2-r_1r_2-p_1r_2}{p_1r_2}\right) \quad (3.42)$$

where:

$$Y_1 = \frac{r_1+r_2-r_1r_2-r_1p_2}{p_1+p_2-p_1p_2-p_1r_2}, \quad (3.43)$$

$$Y_2 = \frac{r_1+r_2-r_1r_2-p_1r_2}{p_1+p_2-p_1p_2-r_1p_2}, \quad (3.44)$$

$$X = \frac{Y_2}{Y_1} \quad (3.45)$$

Furthermore, p_I is given by:

$$p_I = \sum_{all\ internal\ states} p(n, \alpha_1, \alpha_2) = \sum_{n=2}^{N-2} CX^n(1+Y_1)(1+Y_2) = \begin{cases} C\left(\frac{X^{N-1}-X^2}{X-1}\right)(1+Y_1)(1+Y_2) & if\ |X-1| > \delta, \\ C(N-3)(1+Y_1)(1+Y_2) & if\ |X-1| \leq \delta, \end{cases} \quad (3.46)$$

where δ is a very small positive value.

We further denote p_T as:

$$p_T = p_B + p_I = 1 \quad (3.47)$$

To find C, let L_B be p_B/C and L_I be p_I/C. L_B and L_I; so, since p_B and p_I must sum to 1, then:

$$C = \frac{1}{L_B + L_I}. \tag{3.48}$$

The line production rate is obtained as follow:

$$P = \frac{r_1}{r_1 + p_1}(1 - p_b)$$

$$= \frac{r_1}{r_1 + p_1}\big(1 - p(N, 1{,}0)\big)$$

$$= \frac{r_1}{r_1 + p_1}\left(1 - CX^{N-1}\frac{r_1 + r_2 - r_1 r_2 - p_1 r_2}{p_1 r_2}\right), \tag{3.49}$$

where p_b, the probability of blocking, is given by:

$$p_b = p(N, 1, 0) = CX^{N-1}\left(\frac{r_1 + r_2 - r_1 r_2 - p_1 r_2}{p_1 r_2}\right) \tag{3.50}$$

Finally, we must calculate the average line inventory level. First, we compute the average inventory for the internal states, $\bar{n}_J$, as

$$\bar{n}_J = \sum_{all\ internal\ states} np(n, \alpha_1, \alpha_2) = \sum_{n=2}^{N-2} CnX^n(1 + Y_1)(1 + Y_2) = \begin{cases} C\dfrac{X\left(-2X + (N-1)X^{N-2} - \frac{X^{N-1} - X^2}{X-1}\right)}{X-1}(1 + Y_1)(1 + Y_2) & if\ |X - 1| > \delta\,, \\ \frac{1}{2}CN(N-3)(1 + Y_1)(1 + Y_2) & if\ |X - 1| \le \delta\,, \end{cases} \tag{3.51}$$

where δ is a very small positive value. Then, to calculate the total average line inventory level (WIP), we need to consider both internal states and boundary states, whose steady-state probability is different from zero and *n* is non-zero value, in the following way:

$$\bar{n} = \sum_{all\ states} np(n, \alpha_1, \alpha_2) = p(1, 0, 0) + p(1, 0, 1) + p(1, 1, 1) + \bar{n}_J + (N-1)\big(p(N-1, 0, 0) + p(N-1, 1, 0) + p(N-1, 1, 1)\big) + Np(N, 1, 0), \tag{3.52}$$

where:

$p(1,0,0) = CX$, (3.53)

$p(1,0,1) = CXY_2$, (3.54)

$p(1,1,1) = \frac{CX}{p_2}(\frac{r_1+r_2-r_1r_2-r_1p_2}{r_1p_2})$, (3.55)

$p(N-1,0,0) = CX^{N-1}$, (3.56)

$p(N-1,1,0) = CX^{N-1}Y_1$, (3.57)

$p(N-1,1,1) = \frac{CX^{N-1}}{p_1}(\frac{r_1+r_2-r_1r_2-p_1r_2}{p_1+p_2-p_1p_2-p_1r_2})$, (3.58)

$p(N,1,0) = CX^{N-1}(\frac{r_1+r_2-r_1r_2-p_1r_2}{p_1r_2})$. (3.59)

[1]

Once that all the calculations are made, the function $J(N) = AP(N) - bN - c\bar{n}$ for the considered two-machine line is created, where *b* and *c* are, respectively, the hourly cost for each unit of buffer capacity and for each unit of WIP and *A* is calculated as a sort of average hourly revenue per two-machine line and it is in function of *mu* and Grade. Given that *A* comes from initial data, we cannot apply the ***Newton Chord method*** for the One-dimensional search on *A* in case the equation $\frac{\partial J(\boldsymbol{N_i})}{\partial \boldsymbol{N_i}} = 0$ has no positive solution (because initial data cannot be modified); so, we will select values of *A* big enough such that the equation $\frac{\partial J(\boldsymbol{N_i})}{\partial \boldsymbol{N_i}} = 0$ has always a positive solution

We therefore search for the maximum value of *J(N$_i$)* by doing $F(N_i) = \frac{\partial J(N_i)}{\partial N_i} = 0$, for each stage *i*, in order to find a vector N^* of positive optimal buffers solutions.

Once defined N^*, is evaluated the corresponding total profit function $\Pi(N^*) = AP(N^*) - bN^* - c\bar{n} - \sum_i \eta_i$.

Now, for each stage *i*, we suppose the equation $\Pi(N^*)$ to have an unknown represented by a machine parameter. We therefore compute the first partial derivative of $J'(N^*)$ in each machine parameter chosen before: $\frac{\partial \Pi(N^*)}{\partial \widehat{mu}_i}$, $\frac{\partial \Pi(N^*)}{\partial \hat{p}_i}$, $\frac{\partial \Pi(N^*)}{\partial \hat{r}_1}$ and

$\frac{\partial \Pi(N^*)}{\partial \hat{\eta}_i}$. Then, for each parameter, if the derivate is positive, we eliminate all the solutions minor than the first chosen, on the other hand, if it is negative, we eliminate all the solutions greater than the first chosen one, if instead it is equal to 0 it means that the parameter value is already the optimal solution and there is no more to do. Excluding this last hypothesis, A new average with the remaining solutions is recomputed and a new machine is selected. With the parameter of a new chosen machine, all calculations described before – starting from the Shi-Gershwin ones – are made again for a certain number of times, until just a unique parameter solution remains, that is the optimal solution.

Finally, with the selected optimal set of machines, we calculate all the corresponding optimal buffer sizes again with the Gershwin-Shi method.

3.6 Validation

To validate our model, a Design of Experiment plan has been implemented. A 4 machine - 3 buffer line has been taken, with an initial shredding machine, a separator and two other connected separators, like as shown in figure 3.3.

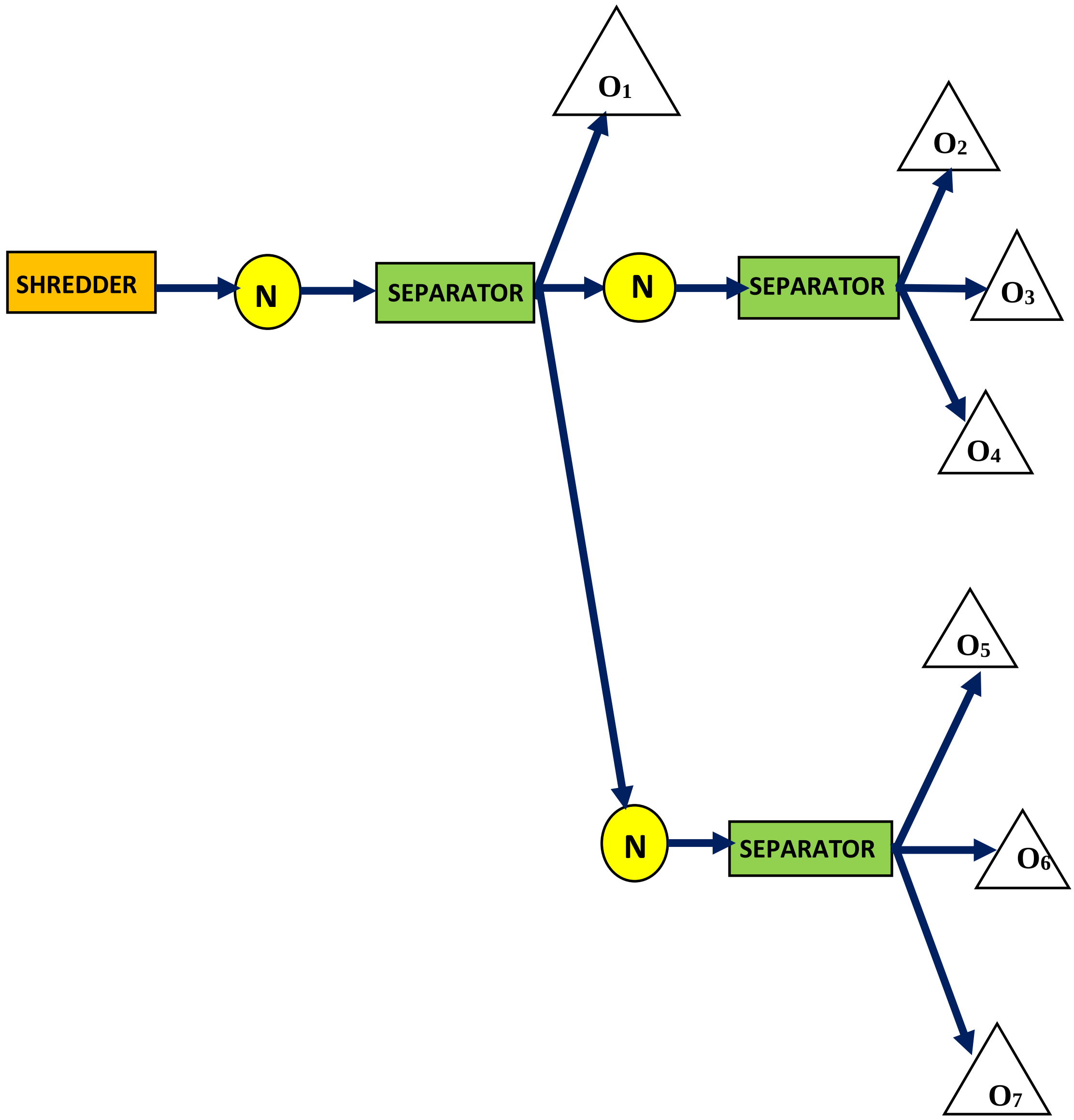


Figure 3.3: 4 machine - 3 buffer line

For each stage, there are 4 alternative machines that can be chosen (so, in total there are 256 possible machines layout combinations), each machine has specific parameters in terms of failure rate, repair rate, hourly production rate, hourly cost and characteristic matrix $\boldsymbol{Q}$. The optimal solution, both in terms of machines combination and in terms of optimal buffer capacities, has been found. This has been carried out with both an exhaustive research methodology program and the bisection algorithm program, finally the outcoming optimal results have been compared.

3.6.1 Program assumptions and notations

Here is a complete list of the notations of all the elements in the program:

Z: total number of target materials in the system,

V: total number of particles classes,

L: total number of liberation classes,

Cl: matrix indicating, for each liberation class *l*, the initial percentage of each material *z*,

HL: binary matrix HL(v, l), associating to each particle class *v* a liberation class *l*,

ST: number of possible machines to choose for each stage,

K: total number of stages in the system,

M(k, st).speed: the production rate, for stage *k*, of possible machine *st*,

M(k, st).lambda: the matrix lambda of the failure rate(s) and repair rate(s),

POS(k): the position along the line of stage *k*, (it can be the first stage, at the beginning of the system, or the second, the third and so on.),

M(k, st).n: the hourly cost, for stage *k*, of possible machine *st*,

M(k).I: the number of inputs stage *k* has (i.e: connections with precedent stages),

M(k).idI: matrix of stages that are inputs for stage *k*,

M(k).O: the number of stage-outputs stage *k* has (i.e: connections with successive stages),

M(k).idO: matrix of stages that are outputs for stage *k*,

M(k).EO: the number of target output of stage *k* (i.e: output flows dedicated to target materials), *M(k).idEO*: matrix of numbers that represent target outputs for stage *k*,

M(k, M(k).idEO(i)).fEO: matrix of the target materials to which is dedicated output flow *M(k).idEO(i)* of stage *k*,

M(k, M(k).idEO(i)).T: total number of target materials to which is dedicated output flow *M(k).idEO(i)* of stage *k*,

M(k).type: type of process of stage *k* (shredder, separator or mixer),

M(k, st, idO(i)).Q: matrix *Q* of stage *k*, for possible machine *st*, directed to stage-output *idO(i)*,

M(k, st, idEO(i)).Q: matrix *Q* of stage *k*, for possible machine *st*, directed to target-output *idEO(i)*,

Y(st): initial particles classes concentration vector for beginning stage *k*, for each possible machine *st,*

A(k): hourly revenue rate for stage *k,*

M(k, st, o).Q: represent matrix *Q* of possible machine *st* at stage *k*, directed to stage-output *o*,

M(k, st, eo).Q: represent matrix *Q* of possible machine *st* at stage *k*, directed to stage-output *eo.*

3.6.2 The selected system data

For what about the recycling system taken for the validation, the detailed data, for each machine, are:

Z=3;

V=4;

L=2;

HL=[1 0; 1 0; 0 1; 0 1];

ST=4;

K=4;

As far as the speeds of the machines are concerned, they are reported in table 3.1:

Speed [Kg/h]				
STAGES →	1	2	3	4
POSSIBLE MACHINES ↓				
1	10	20	34.5	40
2	34	70	70	60
3	20	40	60	30
4	100	100	45	20

Table 3.1: Machines' production rates

Also failure rates, repair rates and hourly costs are reported in table 3.2, table 3.3 and table 3.4 respectively:

Failure rate				
STAGES →	1	2	3	4
POSSIBLE MACHINES ↓				
1	0.2	0.1	0.11	0.13
2	0.3	0.4	0.34	0.01
3	0.1	0.7	0.5	0.6
4	0.5	0.2	0.68	0.1

Table 3.2: Machines' failure rates

Repair rate				
STAGES →	1	2	3	4
POSSIBLE MACHINES ↓				
1	0.9	0.2	0.22	0.34
2	0.45	0.78	0.86	0.071
3	0.3	0.9	0.85	0.7
4	0.76	0.35	0.78	0.8

Table 3.3: Machines' repair rates

Hourly cost [€/h]				
STAGES →	1	2	3	4
POSSIBLE MACHINES ↓				
1	300	100	110	350
2	750	700	600	500
3	400	400	520	210
4	1000	1000	390	100

Table 3.4: Machines' hourly costs

POS(1)=1; *M(1).I=0;* *M(1).idI=0;*

POS(2)=2; *M(2).I=1;* *M(2).idI=1;*

POS(3)=3; *M(3).I=1;* *M(3).idI=2;*

POS(4)=3; *M(4).I=1;* *M(4).idI=2;*

M(1).O=1; *M(1).idO=[2];*

M(2).O=1; *M(2).idO=[3 4];*

M(3).O=0; *M(3).idO=0;*

M(4).O=0; *M(4).idO=0;*

M(1).EO=0; *M(1).idEO=0* *M(2,5).fEO=[1];*

M(2).EO=1; *M(2).idEO=[5];* *M(3,6).fEO=[2];*

M(3).EO=3; *M(3).idEO=[6 7 8];* *M(3,7).fEO=[3];*

M(4).EO=3; *M(4).idEO=[9 10 11];* *M(3,8).fEO=[1];*

M(4,9).fEO=[2];

M(4,10).fEO=[3];

M(4,11).fEO=[1];

M(2,5).T=1; *M(1).type=1;*

M(3,6).T=1; *M(2).type=2;*

M(3,7).T=1; *M(3).type=2;*

M(3,8).T=1;

M(4,9).T=1;

M(4,10).T=1;

M(4,11).T=1;

Regarding matrices ***Q***, they are:

$$M(1,1,2).Q = \begin{bmatrix} 0.9 & 0 & 0 & 0 \\ 0.1 & 1 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0.8 & 1 \end{bmatrix} \qquad M(1,2,2).Q = \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0.2 & 1 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0.7 & 1 \end{bmatrix}$$

$$M(1,3,2).Q = \begin{bmatrix} 0.85 & 0 & 0 & 0 \\ 0.15 & 1 & 0 & 0 \\ 0 & 0 & 0.7 & 0 \\ 0 & 0 & 0.3 & 1 \end{bmatrix} \qquad M(1,4,2).Q = \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0.2 & 1 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0.4 & 1 \end{bmatrix}$$

$$M(2,1,3).Q = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix} \qquad M(2,2,3).Q = \begin{bmatrix} 0.9 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0.0 \end{bmatrix}$$

$$M(2,3,3).Q = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.05 & 0 \\ 0 & 0 & 0 & 0.025 \end{bmatrix} \qquad M(2,4,3).Q = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 01 & 0 & 0 \\ 0 & 0 & 0.05 & 0 \\ 0 & 0 & 0 & 0.025 \end{bmatrix}$$

$$M(2,1,4).Q = \begin{bmatrix} 0.45 & 0 & 0 & 0 \\ 0 & 0.45 & 0 & 0 \\ 0 & 0 & 0.45 & 0 \\ 0 & 0 & 0 & 0.45 \end{bmatrix} \qquad M(2,2,4).Q = \begin{bmatrix} 0.05 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.05 \end{bmatrix}$$

$$M(2,3,4).Q=\begin{bmatrix}0.85&0&0&0\\0&0.8&0&0\\0&0&0.9&0\\0&0&0&0.95\end{bmatrix}\qquad M(2,4,4).Q=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.05&0\\0&0&0&0.05\end{bmatrix}$$

$$M(2,1,5).Q=\begin{bmatrix}0.45&0&0&0\\0&0.45&0&0\\0&0&045&0\\0&0&0&0.45\end{bmatrix}\qquad M(2,2,5).Q=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.1&0\\0&0&0&0.05\end{bmatrix}$$

$$M(2,3,5).Q=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.05&0\\0&0&0&0.025\end{bmatrix}\qquad M(2,4,5).Q=\begin{bmatrix}0.85&0&0&0\\0&0.8&0&0\\0&0&0.9&0\\0&0&0&0.95\end{bmatrix}$$

$$M(3,1,6).Q=\begin{bmatrix}0.9&0&0&0\\0&0.8&0&0\\0&0&0.85&0\\0&0&0&0.95\end{bmatrix}\qquad M(3,2,6).Q=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.05&0\\0&0&0&0.1\end{bmatrix}$$

$$M(3,3,6).Q=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.05&0\\0&0&0&0.1\end{bmatrix}\qquad M(3,4,6).Q=\begin{bmatrix}0.4&0&0&0\\0&0.4&0&0\\0&0&0.4&0\\0&0&0&0.4\end{bmatrix}$$

$$M(3,1,7).Q=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.05&0\\0&0&0&0.025\end{bmatrix}\qquad M(3,2,7).=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.1&0\\0&0&0&0.1\end{bmatrix}$$

$$M(3,3,7).Q=\begin{bmatrix}0.9&0&0&0\\0&0.8&0&0\\0&0&0.85&0\\0&0&0&0.8\end{bmatrix}\qquad M(3,4,7).Q=\begin{bmatrix}0.4&0&0&0\\0&0.4&0&0\\0&0&0.4&0\\0&0&0&0.4\end{bmatrix}$$

$$M(3,1,8).Q=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.1&0\\0&0&0&0.025\end{bmatrix}\qquad M(3,2,8).Q=\begin{bmatrix}0.9&0&0&0\\0&0.8&0&0\\0&0&0.85&0\\0&0&0&0.8\end{bmatrix}$$

$$M(3,3,8).Q=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.1&0\\0&0&0&0.1\end{bmatrix}\quad M(3,4,8).Q=\begin{bmatrix}0.2&0&0&0\\0&0.2&0&0\\0&0&0.2&0\\0&0&0&0.2\end{bmatrix}$$

$$M(4,1,9).Q=\begin{bmatrix}0.9&0&0&0\\0&0.8&0&0\\0&0&0.85&0\\0&0&0&0.95\end{bmatrix}\quad M(4,2,9).Q=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.05&0\\0&0&0&0.1\end{bmatrix}$$

$$M(4,3,9).Q=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.05&0\\0&0&0&0.1\end{bmatrix}\quad M(4,4,9).Q=\begin{bmatrix}0.4&0&0&0\\0&0.4&0&0\\0&0&0.4&0\\0&0&0&0.4\end{bmatrix}$$

$$M(4,1,10).Q=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.05&0\\0&0&0&0.025\end{bmatrix}\quad M(4,2,10)=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.1&0\\0&0&0&0.1\end{bmatrix}$$

$$M(4,3,10).Q=\begin{bmatrix}0.9&0&0&0\\0&0.8&0&0\\0&0&0.85&0\\0&0&0&0.8\end{bmatrix}\quad M(4,4,10).Q=\begin{bmatrix}0.4&0&0&0\\0&0.4&0&0\\0&0&0.4&0\\0&0&0&0.4\end{bmatrix}$$

$$M(4,1,11).Q=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.1&0\\0&0&0&0.025\end{bmatrix}\quad M(4,2,11).Q=\begin{bmatrix}0.9&0&0&0\\0&0.8&0&0\\0&0&0.85&0\\0&0&0&0.8\end{bmatrix}$$

$$M(4,3,11).Q=\begin{bmatrix}0.05&0&0&0\\0&0.1&0&0\\0&0&0.1&0\\0&0&0&0.1\end{bmatrix}\quad M(4,4,11).Q=\begin{bmatrix}0.2&0&0&0\\0&0.2&0&0\\0&0&0.2&0\\0&0&0&0.2\end{bmatrix}$$

3.6.3 Design of Experiments

To design the experiments, 3 factors - external from machines - has been changed: 4 different levels have been chosen for: the matrix *Cl(l,z)*, representing the concentration of materials in the liberation classes, the initial vector *Y(v)* material classes concentration and the vector *A(z)* of the hourly revenue rate for each material *z*. In particular:

$$Y\_1 = \begin{bmatrix} 0.1 \\ 0.4 \\ 0.4 \\ 0.1 \end{bmatrix} \qquad Y\_2 = \begin{bmatrix} 02 \\ 0.4 \\ 0.1 \\ 0.3 \end{bmatrix} \qquad Y\_3 = \begin{bmatrix} 02 \\ 0.35 \\ 0.15 \\ 0.3 \end{bmatrix} \qquad Y\_4 = \begin{bmatrix} 04 \\ 0.05 \\ 0.15 \\ 0.4 \end{bmatrix}$$

$$Cl\_1 = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix} \qquad Cl\_2 = \begin{bmatrix} 0.8 & 0.15 & 0.05 \\ 0 & 0.4 & 0.6 \end{bmatrix}$$

$$Cl\_3 = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.45 & 0.35 \end{bmatrix} \qquad Cl\_4 = \begin{bmatrix} 0.6 & 0.1 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$$

$$A\_1 = [100\ 100\ 100] \qquad A\_2 = [120\ 150\ 70]$$

$$A\_3 = [80\ 50\ 170] \qquad A\_4 = [200\ 40\ 40]$$

Combining all the 4 values for the 3 factors, in total 64 (4*4*4) factors possibilities are created. For each of them, the optimal solution has been found, in relation to the machines combination for optimal profit, the optimal buffer capacities for each two machine line and the respective maximum profit value, both using the exhaustive research program and the **bisection** program; then results have been compared. This procedure is shown in tables 3.5 and 3.6, representing respectively all the possible combinations with the results found by the **exhaustive research** method and the possible combinations with the results found by the **bisection** method.

EXHAUSTIVE RESEARCH METHOD													
COMB	**Y**	**Cl**	**A**		***M1***	***M2***	***M3***	***M4***		***BB1***	***BB2***	***BB3***	**MAX Π**
1	1	1	1	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1762,53
2	2	1	1	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1762,53

3	3	1	1	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1762,53
4	4	1	1	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1762,53
5	1	2	1	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1762,53
6	2	2	1	*SOL*	4	4	1	1	*O.B.*	26,9411	45,0888	27,9612	1848,74
7	3	2	1	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	1803,52
8	4	2	1	*SOL*	4	4	1	1	*O.B.*	25,8424	42,8274	27,1475	1707,48
9	1	3	1	*SOL*	4	4	1	1	*O.B.*	26,6758	44,5400	27,7659	1822,7
10	2	3	1	*SOL*	4	4	1	1	*O.B.*	27,1147	45,4490	28,0886	1875,66
11	3	3	1	*SOL*	4	4	1	1	*O.B.*	26,8973	44,9981	27,9290	1836,51
12	4	3	1	*SOL*	4	4	1	1	*O.B.*	26,4502	44,0745	27,5992	1787,32
13	1	4	1	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1626,53
14	2	4	1	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	1803,56
15	3	4	1	*SOL*	4	4	1	1	*O.B.*	26,4045	43,9805	27,5654	1772,35
16	4	4	1	*SOL*	4	4	1	1	*O.B.*	26,0328	43,2170	27,2894	1731,39
17	1	1	2	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1980,4
18	2	1	2	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1996,72
19	3	1	2	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1996,72
20	4	1	2	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1996,72
21	1	2	2	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1973,66
22	2	2	2	*SOL*	4	4	1	1	*O.B.*	26,9411	45,0888	27,9612	2098,46
23	3	2	2	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	2033,69
24	4	2	2	*SOL*	4	4	1	1	*O.B.*	25,8424	42,8274	27,1475	1901,57
25	1	3	2	*SOL*	4	4	1	1	*O.B.*	26,6758	44,5400	27,7659	1919,37
26	2	3	2	*SOL*	4	4	1	1	*O.B.*	27,1147	45,4490	28,0886	2188,22
27	3	3	2	*SOL*	4	4	1	1	*O.B.*	26,8973	44,9981	27,9290	2138,82
28	4	3	2	*SOL*	4	4	1	1	*O.B.*	26,4502	44,0745	27,5992	2074,94
29	1	4	2	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1858,32
30	2	4	2	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	1920,56
31	3	4	2	*SOL*	4	4	1	1	*O.B.*	26,4045	43,9805	27,5654	1878,01
32	4	4	2	*SOL*	4	4	1	1	*O.B.*	26,0328	43,2170	27,2894	1815,95
33	1	1	3	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1666,73
34	2	1	3	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1666,73
35	3	1	3	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1666,73
36	4	1	3	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1666,73
37	1	2	3	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1701,33
38	2	2	3	*SOL*	4	4	1	1	*O.B.*	26,9411	45,0888	27,9612	1724,06
39	3	2	3	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	1710,8
40	4	2	3	*SOL*	4	4	1	1	*O.B.*	25,8424	42,8274	27,1475	1681,37
41	1	3	3	*SOL*	4	4	1	1	*O.B.*	26,6758	44,5400	27,7659	1621,36
42	2	3	3	*SOL*	4	4	1	1	*O.B.*	27,1147	45,4490	28,0886	1656,12
43	3	3	3	*SOL*	4	4	1	1	*O.B.*	26,8973	44,9981	27,9290	1502,94
44	4	3	3	*SOL*	4	4	1	1	*O.B.*	26,4502	44,0745	27,5992	1609,37
45	1	4	3	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1874,35
46	2	4	3	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	1880,49
47	3	4	3	*SOL*	4	4	1	1	*O.B.*	26,4045	43,9805	27,5654	1866,09

48	4	4	3	*SOL*	4	4	1	1	*O.B.*	26,0328	43,2170	27,2894	1859,66
49	1	1	4	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	2171,73
50	2	1	4	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	2171,73
51	3	1	4	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	2171,73
52	4	1	4	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	2171,73
53	1	2	4	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	2171,73
54	2	2	4	*SOL*	4	4	1	1	*O.B.*	26,9411	45,0888	27,9612	2481,36
55	3	2	4	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	2323,92
56	4	2	4	*SOL*	4	4	1	1	*O.B.*	25,8424	42,8274	27,1475	2007,94
57	1	3	4	*SOL*	4	4	1	1	*O.B.*	26,6758	44,5400	27,7659	2360,37
58	2	3	4	*SOL*	4	4	1	1	*O.B.*	27,1147	45,4490	28,0886	2566,41
59	3	3	4	*SOL*	4	4	1	1	*O.B.*	26,8973	44,9981	27,9290	2452,21
60	4	3	4	*SOL*	4	4	1	1	*O.B.*	26,4502	44,0745	27,5992	2251,25
61	1	4	4	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	2172,27
62	2	4	4	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	2323,92
63	3	4	4	*SOL*	4	4	1	1	*O.B.*	26,4045	43,9805	27,5654	2234,17
64	4	4	4	*SOL*	4	4	1	1	*O.B.*	26,0328	43,2170	27,2894	2099,19

Table 3.5: Combinations and results with **exhaustive research**

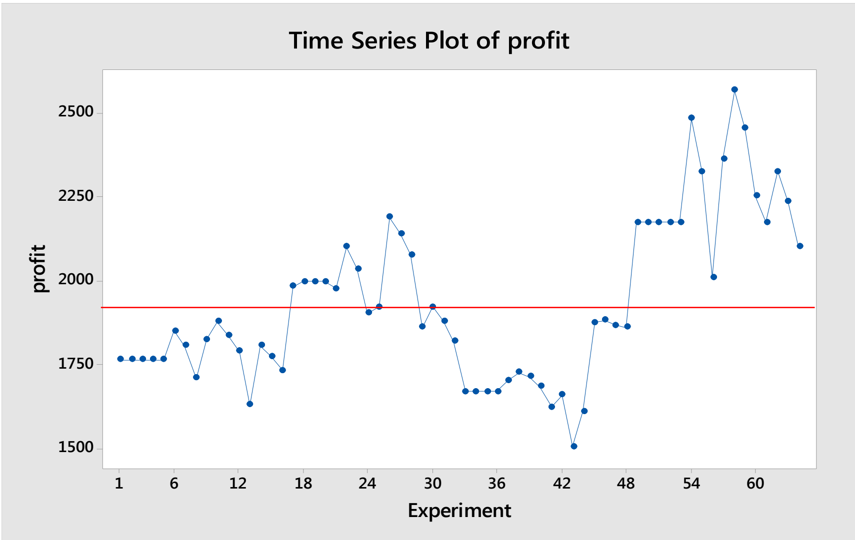
BISECTION METHOD													
COMB	**Y**	**Cl**	**A**		***M1***	***M2***	***M3***	***M4***		***BB1***	***BB2***	***BB3***	**MAX Π**
1	1	1	1	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1762,53
2	2	1	1	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1762,53
3	3	1	1	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1762,53
4	4	1	1	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1762,53
5	1	2	1	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1762,53
6	2	2	1	*SOL*	4	4	1	1	*O.B.*	26,9411	45,0888	27,9612	1848,74
7	3	2	1	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	1803,52
8	4	2	1	*SOL*	4	4	1	1	*O.B.*	25,8424	42,8274	27,1475	1707,48
9	1	3	1	*SOL*	4	4	1	1	*O.B.*	26,6758	44,5400	27,7659	1822,7
10	2	3	1	*SOL*	4	4	1	1	*O.B.*	27,1147	45,4490	28,0886	1875,66
11	3	3	1	*SOL*	4	4	1	1	*O.B.*	26,8973	44,9981	27,9290	1836,51
12	4	3	1	*SOL*	4	4	1	1	*O.B.*	26,4502	44,0745	27,5992	1787,32
13	1	4	1	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1626,53
14	2	4	1	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	1803,56
15	3	4	1	*SOL*	4	4	1	1	*O.B.*	26,4045	43,9805	27,5654	1772,35
16	4	4	1	*SOL*	4	4	1	1	*O.B.*	26,0328	43,2170	27,2894	1731,39
17	1	1	2	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1980,4
18	2	1	2	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1996,72
19	3	1	2	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1996,72
20	4	1	2	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1996,72
21	1	2	2	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1973,66

22	2	2	2	*SOL*	4	4	1	1	*O.B.*	26,9411	45,0888	27,9612	2098,46
23	3	2	2	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	2033,69
24	4	2	2	*SOL*	4	4	1	1	*O.B.*	25,8424	42,8274	27,1475	1901,57
25	1	3	2	*SOL*	4	4	1	1	*O.B.*	26,6758	44,5400	27,7659	1919,37
26	2	3	2	*SOL*	4	4	1	1	*O.B.*	27,1147	45,4490	28,0886	2188,22
27	3	3	2	*SOL*	4	4	1	1	*O.B.*	26,8973	44,9981	27,9290	2138,82
28	4	3	2	*SOL*	4	4	1	1	*O.B.*	26,4502	44,0745	27,5992	2074,94
29	1	4	2	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1858,32
30	2	4	2	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	1920,56
31	3	4	2	*SOL*	4	4	1	1	*O.B.*	26,4045	43,9805	27,5654	1878,01
32	4	4	2	*SOL*	4	4	1	1	*O.B.*	26,0328	43,2170	27,2894	1815,95
33	1	1	3	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1666,73
34	2	1	3	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1666,73
35	3	1	3	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1666,73
36	4	1	3	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1666,73
37	1	2	3	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1701,33
38	2	2	3	*SOL*	4	4	1	1	*O.B.*	26,9411	45,0888	27,9612	1724,06
39	3	2	3	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	1710,8
40	4	2	3	*SOL*	4	4	1	1	*O.B.*	25,8424	42,8274	27,1475	1681,37
41	1	3	3	*SOL*	4	4	1	1	*O.B.*	26,6758	44,5400	27,7659	1621,36
42	2	3	3	*SOL*	4	4	1	1	*O.B.*	27,1147	45,4490	28,0886	1656,12
43	3	3	3	*SOL*	4	4	1	1	*O.B.*	26,8973	44,9981	27,9290	1502,94
44	4	3	3	*SOL*	4	4	1	1	*O.B.*	26,4502	44,0745	27,5992	1609,37
45	1	4	3	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	1874,35
46	2	4	3	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	1880,49
47	3	4	3	*SOL*	4	4	1	1	*O.B.*	26,4045	43,9805	27,5654	1866,09
48	4	4	3	*SOL*	4	4	1	1	*O.B.*	26,0328	43,2170	27,2894	1859,66
49	1	1	4	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	2171,73
50	2	1	4	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	2171,73
51	3	1	4	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	2171,73
52	4	1	4	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	2171,73
53	1	2	4	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	2171,73
54	2	2	4	*SOL*	4	4	1	1	*O.B.*	26,9411	45,0888	27,9612	2481,36
55	3	2	4	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	2323,92
56	4	2	4	*SOL*	4	4	1	1	*O.B.*	25,8424	42,8274	27,1475	2007,94
57	1	3	4	*SOL*	4	4	1	1	*O.B.*	26,6758	44,5400	27,7659	2360,37
58	2	3	4	*SOL*	4	4	1	1	*O.B.*	27,1147	45,4490	28,0886	2566,41
59	3	3	4	*SOL*	4	4	1	1	*O.B.*	26,8973	44,9981	27,9290	2452,21
60	4	3	4	*SOL*	4	4	1	1	*O.B.*	26,4502	44,0745	27,5992	2251,25
61	1	4	4	*SOL*	4	4	1	1	*O.B.*	26,2201	43,6013	27,4287	2172,27
62	2	4	4	*SOL*	4	4	1	1	*O.B.*	26,5861	44,3547	27,6997	2323,92
63	3	4	4	*SOL*	4	4	1	1	*O.B.*	26,4045	43,9805	27,5654	2234,17
64	4	4	4	*SOL*	4	4	1	1	*O.B.*	26,0328	43,2170	27,2894	2099,19

Table 3.6: Combinations and results with **bisection method**

Looking at the results about the machines solution, the optimal buffer capacities for each line and the respective maximum profit, they are equal for each of the two programs used to calculate them, for all the 64 possible combinations. So, the bisection calculation algorithm for the simultaneous machine selection and buffer sizing is verified. The program implemented with the bisection method can be considered reliable.

Looking at the profits, calculated by simulation, they assume different values, ranging from 1502,94€ to 2566,41€. The profits average of all the 64 experiments is 1932,61€, and the standard deviation is equal to 240,64€. In graph 3.2 we have the profits pattern for all the 64 experiments.



Graph 3.2: Profits pattern

With this high variability, we conclude that the maximum profit a system can achieve strongly depends on the external context, whatever is the optimal layout chosen.

The main advantage of the bisection algorithm is the computational speed: the exhaustive research program takes, on average, 3842s for each factors combination to calculate the optimal solutions; while the **bisection** program takes just 165s on average for each combination, around 1/23 of the time taken by the **exhaustive research** one.

Chapter 4

The ITIA-CNR case

4.1 The ITIA Recycling system

In the last part of the work, the program has finally been utilized on a real case: the recycling plant of the Research National Center of ITIA, in Milan. The recycling system in question processes Printed Circuit Boards (PCBs), which are a kind of WEEE. Metals obtainable from a PCB are copper - for the most part -, aluminum, silver and gold. The plant is composed by 4 stages: a single shaft shear shredder - like the one in figure 4.2 [34] -, a vibrating dimensional separator, in figure 4.3, with four output flows: one directed to a cutting mill - like the one in figure 4.4 [35] - for the particles still too big, that need to be shredded again, whose output goes back to the vibrating separator; a scrap output of the smallest particles, containing just 30% of metal material; and two output flows connected to a final Corona Electrostatic Separator - like the one in figure 4.5 [36] - dividing metal particles from non-metal ones. Between each, the material can always be stocked, waiting to be processed; also before the first shredding process the material can wait in the shredder buffer. The layout is schematized in figure 4

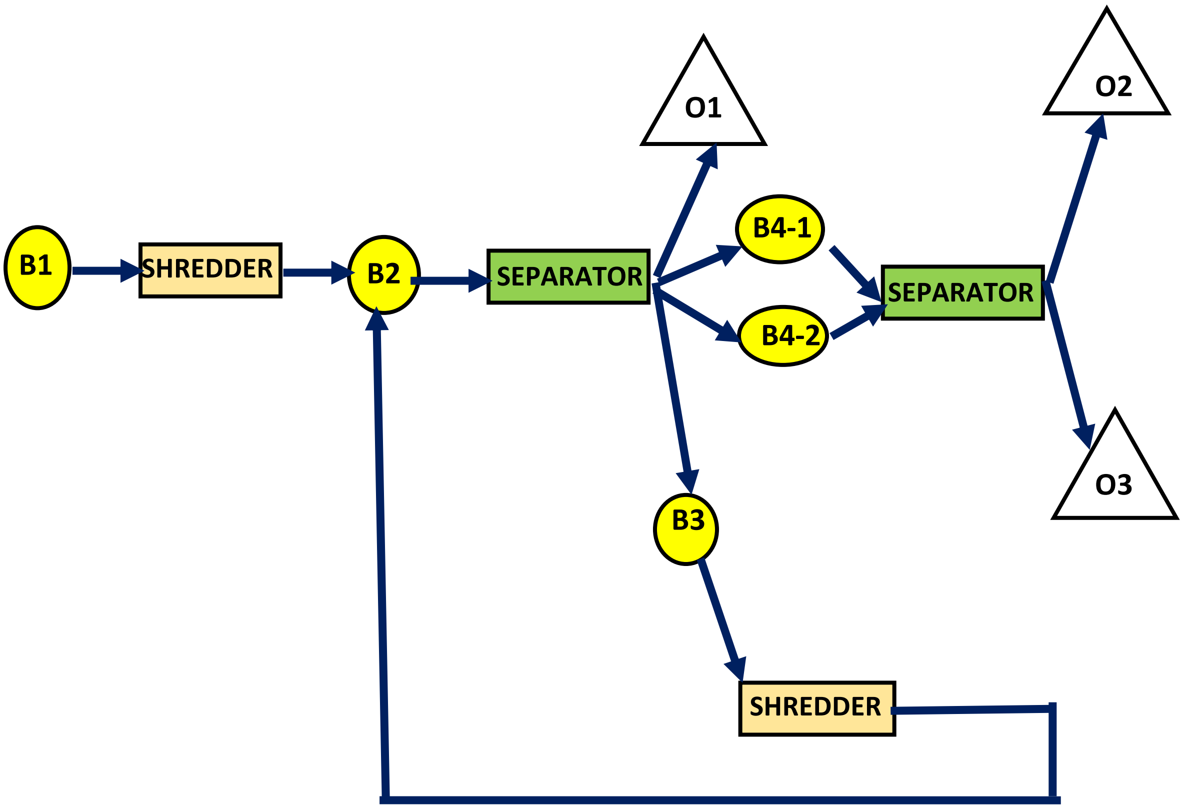


Figure 4.1: ITIA recycling plant scheme

Going into detail, the first single shaft shear shredder breaks PBCs into small particles, creating 4 particles size classes: the first one with particles size bigger than $2mm$, the second one $\epsilon[1.2; 2]mm$, the third one $\epsilon[0.6; 1.2]mm$ and the last class with particles size smaller than $0.6mm$. The particles are directed to the vibrating separator, that divides them according to their size. The first class particles are sent to the second shredder because they must be crumbled again and, once this is done, they are reprocessed by the vibrating separator. The last class particles represent the first output with 30% of metal material, the flows of particles $\epsilon[1.2; 2]mm$ and particles $\epsilon[0.6; 1.2]mm$ are directed to the last separator, which finally creates the two final outputs.

Figure 4.2: Single shaft shear shredder [34]

Figure 4.3: Vibrating separator

Figure 4.4: Cutting mill [35]

Figure 4.5: Corona Electrostatic Separator

4.2 Hypothesis and assumptions

For the software to function, we assumed to have 4 possible machines for each stage - so in total 4^4=256 possible machines combinations -, each machine with specific characteristics. Furthermore, we have 4 particles size classes and 2 particles liberation classes - so in total 8 particles classes -, 2 materials (NON-METALS and METALS) so we assumed to have:

$$Y = \begin{bmatrix} 0.1 \\ 0.8 \\ 0.04 \\ 0.05 \\ 0.003 \\ 0.007 \\ 0 \\ 0 \end{bmatrix}, \qquad HL(v,l) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad C(l,z) = \begin{bmatrix} 0.1 & 0.9 \\ 0.8 & 0.2 \end{bmatrix}$$

and the revenue rates of metals calculated, for each target output, as an average between the different products (*%metal*metal price)*, where the *metal price* is, for each metal, a sort of average current used material price considering the current metals prices [45],[46],[47],[48]; the *%metal* is instead computed, for each target flow, as an average between different percentages of metals present in a PCB, reported in the article "Characterization of Printed Circuit Boards for Metal and Energy Recovery after Milling and Mechanical Separation", by W. A. Bizzo , R. A. Figueiredo and V. F. de Andrade [2]. Furthermore, we assumed a unitary WIP cost and buffer storage cost of 0.05€.

For the first stage, the first possible machine is represented by the single shafts shear shredder present in ITIA. We assumed to have a production rate *mu*=50Kg/h and an energetic power of 5.5Kw, so an estimated hourly cost of 1.33€/h, obtained by doing 5,5Kw*0.241€/Kwh, where 0.241€/Kwh has been the average Italian energy price in 2016 [40]. Regarding failures, it could happen that PCB board fits badly with shaft shear and the shredder doesn't process; a low failure rate *p*=0.5 has been estimated. Remedying this failure is quite easy: we just need to press the emergency button, go upon the machine, unlock the card, go down and press the emergency button again; all this takes just 30 seconds. So, we estimated a quite high *r*, equal to 0.95. These *p* and *r* parameters values are supposed to be the same for all the other possible machines in this stage. For the matrix *Q*, we assumed to have 2 liberation classes that, in couple with the 4 size classes we have, give a total number of 8 particles classes, and so an 8*8 matrix *Q* as follows:

$$Q = \begin{bmatrix} 0.95 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.95 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.05 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0{,}3 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0.7 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.3 & 0 & 0.7 & 0 & 1 \end{bmatrix}$$

For the second possible machine, we considered a *Heavy Duty Granulator*. This particular machine is thought to shred very hard non-homogeneous products, just as PCBs [48]. For this reason, they are supposed to be a little bit more efficient in the shredding process, with a *mu* greater than the first machine, equal to 125Kg/h, and also a more efficient matrix *Q*:

$$Q = \begin{bmatrix} 0.9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0.8 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0.8 & 0 & 1 \end{bmatrix}$$

The computed hourly cost is 7.23€/h.

A detailed illustration of this machine is in figure 4.6 [41]:

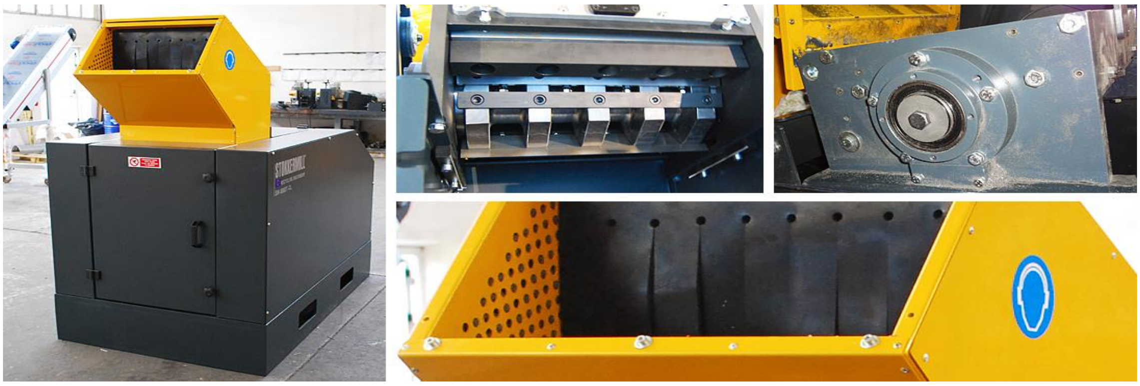


Figure 4.6: HD Granulator [41]

The third choice is supposed to be a two-shafts shear shredder. We assumed it to be a bit less efficient than the single shaft shredder, with *mu*=75Kg/h and matrix Q as follows:

$$Q = \begin{bmatrix} 0.95 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.95 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.05 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0 & 1 \end{bmatrix}$$

The hourly cost is 9.64€/h. An illustration of the two-shafts shredder is in figure 4.7 [36].

Figure 4.7: Two shafts shear shredder [36]

The last choice is a four-shafts shear shredder; it is supposed to be very efficient for shredding products because it has 4 shredding shafts, but slower in terms of throughput (*mu*=75Kg/h). Its characteristic matrix is:

$$Q = \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0.3 & 0 & 0.15 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0.3 & 0 & 0.15 & 0 & 0 \\ 0.1 & 0 & 0.5 & 0 & 0.85 & 0 & 1 & 0 \\ 0 & 0.1 & 0 & 0.5 & 0 & 0.85 & 0 & 1 \end{bmatrix}$$

and its cost is 8.44€/h. There is an illustration in figure 4.8 [38] of this machine.

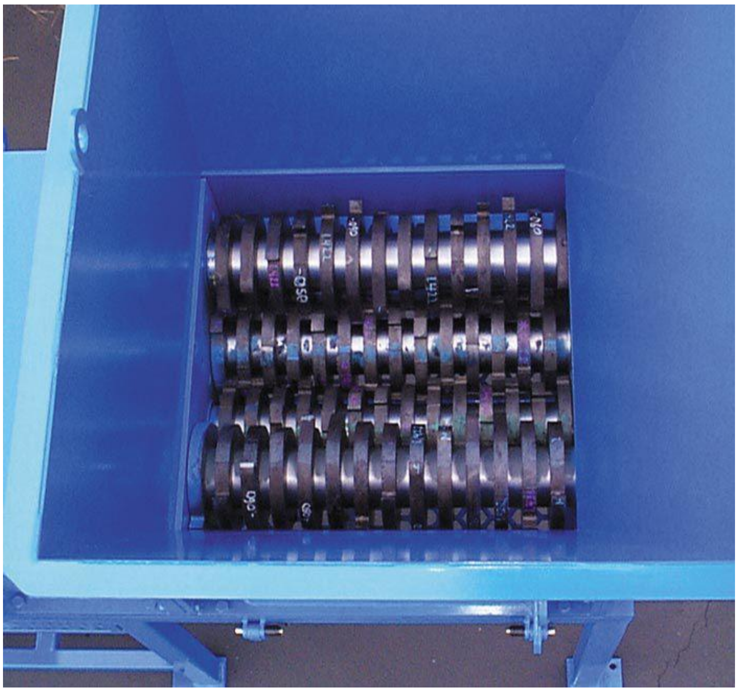


Figure 4.8: Four shafts shear shredder [38]

For the second recycling stage, the first machine choice is the vibrating separator. It has a production rate of 100Kg/h and a low hourly cost, of about 0.27€/h. Within this machine, it can happen that a copper hank gets stuck in any separator tube transporting material. Actually, this happens hardly ever, *p*=0.001; but in case it should be necessary to stop the working separator and to unblock the tube, this requires 5 minutes, so the repair rate is quite low: r=0.2. For the sake of simplicity, we assumed these parameters values to be the same in all the machine choices for this stage.

For each of the separation output flow, we considered a theoretical matrices Q as follow:

$$Q_{flow1} = \begin{bmatrix} 0.61 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.64 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.13 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.13 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.11 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.12 \end{bmatrix},$$

$$Q_{flow2} = \begin{bmatrix} 0.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.12 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.61 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.64 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.13 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.11 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.12 \end{bmatrix},$$

$$Q_{flow3} = \begin{bmatrix} 0.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.12 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.13 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.61 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.64 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.11 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.12 \end{bmatrix},$$

$$Q_{flow4} = \begin{bmatrix} 0.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.12 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.13 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.13 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.64 \end{bmatrix},$$

As a second possible machine, we considered a drum sieve, with a slightly better separation efficiency, but with a lower production rate (*mu*=80Kg/h). Its characteristic matrices are:

$$Q_{flow1} = \begin{bmatrix} 0.67 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.11 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.09 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix},$$

$$Q_{flow2} = \begin{bmatrix} 0.11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.67 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.09 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix},$$

$$Q_{flow3} = \begin{bmatrix} 0.11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.11 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.67 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.09 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix},$$

$$Q_{flow4} = \begin{bmatrix} 0.11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.11 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.73 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 \end{bmatrix}$$

and its hourly cost is 3.62€/h. The machine structure is similar to the first separator.

As third choice, we considered a disc sieve, assuming a *mu* equal to 50Kg/h, an hourly cost of 1.81€/h and less efficient matrices Q:

$$Q_{flow1} = \begin{bmatrix} 0.34 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.37 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.22 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.21 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.22 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.21 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.21 \end{bmatrix},$$

$$Q_{flow2} = \begin{bmatrix} 0.22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.21 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.34 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.37 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.22 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.21 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.21 \end{bmatrix},$$

$$Q_{flow3} = \begin{bmatrix} 0.22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.21 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.22 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.21 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.34 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.37 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.21 \end{bmatrix},$$

$$Q_{flow4} = \begin{bmatrix} 0.22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.21 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.22 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.21 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.22 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.21 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.37 \end{bmatrix}.$$

Also this machine is similar to the vibrating separator.

For the last choice, we considered a Trommel screen, with a *mu* equal to 60Kg/h, an hourly cost of 1.81€/h and hypothetical matrices *Q* as follow:

$$Q_{flow1} = \begin{bmatrix} 0.31 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.34 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.23 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.22 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.23 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.22 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.21 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.22 \end{bmatrix},$$

$$Q_{flow2} = \begin{bmatrix} 0.23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.31 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.34 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.23 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.22 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.21 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.22 \end{bmatrix},$$

$$Q_{flow3} = \begin{bmatrix} 0.23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.23 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.22 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.31 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.34 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.21 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.22 \end{bmatrix},$$

$$Q_{flow4} = \begin{bmatrix} 0.23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.23 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.22 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.23 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.22 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.37 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.34 \end{bmatrix}.$$

An example of Trommel screen machine is depicted in figure 4.9 [32].

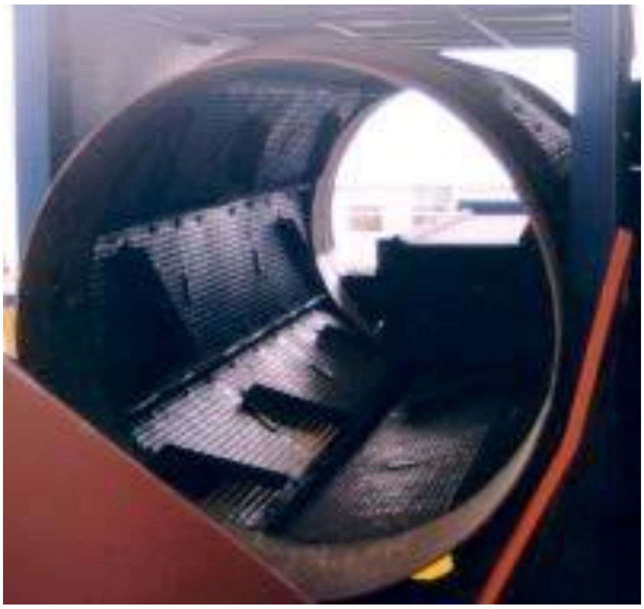


Figure 4.9: Example of Trommel screen [32]

For the third stage, the fisrt and the second machine choices are supposed to be two cutting mills. Within this machine, sometimes it could happen that the grill stops, blocking the whole process. The failure rate for this is supposed to be p=0.05 and its respective repair rate is r=0.2. As a matter of fact, remedying to this failure takes about 5 minutes. The first cutting mill has mu=22Kg/h, η=0.37€/h and a matrix Q as follows:

$$Q = \begin{bmatrix} 0.15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.15 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.15 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.15 & 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0.25 & 0 & 0.2 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0.2 & 0 & 0,2 & 0 & 0 \\ 0.55 & 0 & 0.6 & 0 & 0.8 & 0 & 1 & 0 \\ 0 & 0.55 & 0 & 0.6 & 0 & 0.8 & 0 & 1 \end{bmatrix}.$$

The second one has mu=28.5Kg/h, η=0.73€/h and a Q matrix as follow:

$$Q = \begin{bmatrix} 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0.25 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0.2 & 0 & 0.15 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0.2 & 0 & 0.15 & 0 & 0 \\ 0.4 & 0 & 0.55 & 0 & 0.85 & 0 & 1 & 0 \\ 0 & 0.4 & 0 & 0.55 & 0 & 0.85 & 0 & 1 \end{bmatrix}.$$

The other two alternatives are two horizontal hammermills. The first one has *mu*=20Kg/h and η=2.65€/h, the second one has *mu*=25Kg/h and η=3.61€/h. For these two machines, we assumed to have more efficient matrices Q than those of the cutting mills. In particular, the first one has

$$Q = \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0.1 & 0 & 0.1 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0.1 & 0 & 0.1 & 0 & 0 \\ 0.6 & 0 & 0.8 & 0 & 0.9 & 0 & 1 & 0 \\ 0 & 0.6 & 0 & 0.8 & 0 & 0.9 & 0 & 1 \end{bmatrix},$$

while the second one has

$$Q = \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0.15 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0.15 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0.15 & 0 & 0.15 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0.15 & 0 & 0.15 & 0 & 0 \\ 0.5 & 0 & 0.7 & 0 & 0.85 & 0 & 1 & 0 \\ 0 & 0.5 & 0 & 0.7 & 0 & 0.85 & 0 & 1 \end{bmatrix}.$$

In figure 4.10 [42] there is an example of a horizontal hammermill.

Figure 4.10: Example of horizontal hammermill [42]

Finally, for the last stage, the software chooses between 4 different separation technologies: a Corona Electrostatic Separator (CES), an Eddy Current Separator (EDC), a Densiometric table and a Zig-zag separator.

Within the present CES, if there is too much copper and the voltage is too high, too many electric shocks could be generated blocking the process. This could happen with an estimated probability p=0.2. Remedying to this failure we just have to press two keys to stop the machine and no more, so we considered r=0.99. The assumed p and r values are taken also for all the other alternatives.

For the first three alternatives, we assumed to have *mu*=100Kg/h, the last one has *mu*=75Kg/h. The cost parameters and Q matrices are different for each alternative, even if, among all the possible machines, the best one should be the CES, good for small sizes particles.

In particular, for the CES, η=2.53€/h and, for each output flow:

$$Q_{flow1} = \begin{bmatrix} 0.7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.45 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.45 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4 \end{bmatrix},$$

$$Q_{flow2} = \begin{bmatrix} 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.55 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.55 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 \end{bmatrix}.$$

For the EDC, we supposed to have a hourly cost equal to 0.18€/h and, given that the EDC is better for bigger particle sizes than the 4 sizes obtained by the first shredder, it is supposed to be less efficient for the separation.

$$Q_{flow1} = \begin{bmatrix} 0.42 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.49 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.49 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.68 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.68 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.21 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.21 \end{bmatrix},$$

$$Q_{flow2} = \begin{bmatrix} 0.58 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.58 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.51 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.51 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.32 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.32 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.79 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.79 \end{bmatrix}.$$

A typical EDC is shown in figure 4.11 [43].

Figure 4.11: Eddy Current Separator [43]

The Densiometric table has η=1.81€/h and matrices Q as follow:

$$Q_{flow1} = \begin{bmatrix} 0.76 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.76 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.22 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.22 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.42 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.42 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.75 \end{bmatrix} \text{and}$$

$$Q_{flow2} = \begin{bmatrix} 0.24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.24 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.78 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.78 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.58 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 \end{bmatrix}.$$

An illustration is in figure 4.12 [49].

Figure 4.12: Densiometric table [49]

The last alternative, the Zig-zag separator, is characterized by η=1.33€/h and matrices Q as follow:

$$Q_{flow1} = \begin{bmatrix} 0.55 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.55 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.57 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.57 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.41 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.41 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.32 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.32 \end{bmatrix} \text{ and}$$

$$Q_{flow2} = \begin{bmatrix} 0.45 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.45 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.43 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.43 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.59 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.59 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.68 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.68 \end{bmatrix}.$$

There is a detailed illustration of a typical zig-zag separator in figure 4.13 [44].

Figure 4.13: Zig-zag separator [44]

4.3 Software results

Regarding the machine combination, the software has chosen the following one:

- Stage 1: Heavy Duty granulator,
- Stage 2: Vibrating separator,
- Stage 3: Second Cutting mill, with the greatest production rate,
- Stage 4: Corona Electrostatic Separator.

This is considered a good choice, coherent with the real machines potentialities.

Furthermore, the chosen optimal buffer sizes – rounded to the nearest integers – are:

- B1:1,
- B2: 34,
- B3: 11,
- B4-1: 6,
- B4-2: 6

The average optimal hourly profit is 253,48€/h, with a standard deviation of 2,25€/h.

Chapter 5

Conclusions

Circular Economy introduced relevant challenges: in a society which is characterized by a “Take-Make-Dispose” economy (with damages to the environment and to the economy itself) it should be necessary to find a way to reduce the natural resources consumption. One of the fundamental pillars of Circular Economy is the reintroduction of wastes in the product loop as secondary raw materials. This model brings considerable benefits both in economic and environmental terms, generating new employment and professional figures. In this framework, a relevant role is played by recycling encouraged by European governments and policy makers in the last years.

The most relevant and increasing waste flow in Europe is that o Waste from Electric and Electronic Equipment (WEEE) which EoL products contain high quantity of valuable materials (as precious metals and rare earths), strategical for European Union, due to lack in mines. The major issue for WEEE recycling is the high variability, both on materials and products, of EoL flows. Actual recycling systems are rigid and monolithic, leading to loss of time and material when facing off different flows.

Recycling process are multistage systems based on four key processes (shredding, separation, splitting and mixing) with a wide set of technologies for each process. In this Thesis work, considering a specific multistage layout with finite buffer and different technologies for every stage, a software to choose machine and buffer allocation to maximize the profits have been developed.

Traina and Gershwin bisection method, originally developed for linear manufacturing systems, has been modified and adapted to de-manufacturing systems. The final result is a software for the flow-line design phase, which allows to choose in a very short time the best combination of machines and buffers capacities to maximize the system profit.

This software has been validated using a Design of Experiments with 3 factors and 4 levels for each factors (for a total of 64 experiments), comparing the bisection algorithm results with results obtained through an exhaustive research program. As a result, 100% of the results of both programs coincide and the software can be therefore considered reliable.

Finally, the software has been used on the recycling cell of the De- and Remanufacturing pilot plant at the Institute of Industrial Technologies and Automation of the National Research Council (ITIA-CNR) in Milan, giving the best technologies for each stage.

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Ringraziamenti

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