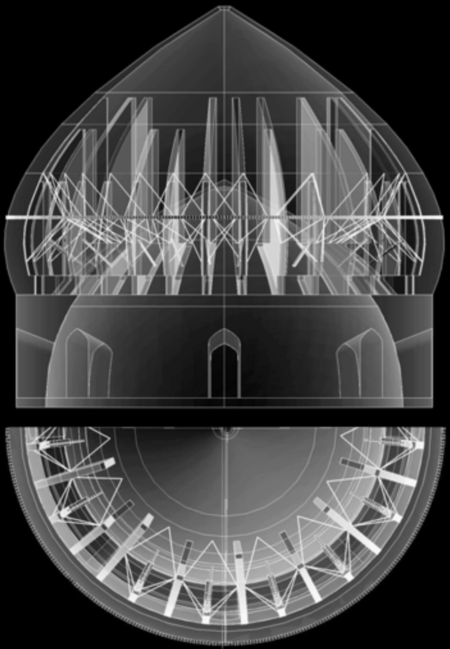
POLITECNICO
MILANO 1863

Hybrid Double Dome

Building performance and cable-net strengthening of Esfahan Shah Mosque's dome

Ali Tavakoli Dinani

Supervisor: **Prof. Lorenzo Jurina**
Co-Supervisor: **Prof. Paulo B. Lourenço**



Doctoral Thesis in

Preservation of Architectural Heritage

at the

Polytechnic University of Milan

March 2019

Hybrid Double Dome

Building performance and cable-net strengthening of Esfahan Shah Mosque's dome

by

Ali Tavakoli Dinani
B. Sc in Architectural Engineering
M. Arch. in Architecture and Urban Design
alitavakoli.dinani@polimi.it , atd@interessestudio.com

Supervisor: **Prof. Lorenzo Jurina**

Associate professor of Construction Engineering, Dep. ABC,
Politecnico di Milano

Co-Supervisor: **Prof. Paulo B. Lourenço**

Full professor of Structural Engineering, Dep. Civil Engineering, ISISE
University of Minho

Scientific - Disciplinary Sector: Structural Engineering
Administrative department: Department of Architecture and Urban Studies
Scientific department: Department of Architecture, Built Environment and Construction Engineering
Thematic Classification: Preservation of Architectural Heritage
PhD Program Coordinator: Prof. Carolina Di Biase (2014-2018), Prof. Maria Cristina Giambruno (Since Nov. 2018)
Thesis readers: Prof. Michele Paradiso, University of Florence
Prof. Eva Coisson, University of Parma
Final exam commission: Prof. Massimiliano Bocciarelli, Politecnico di Milano
Prof. Francisco Javier Gallego, University of Granada
Prof. Federica Ottoni, University of Parma
Prof. Sergio Pace, Politecnico di Torino

Hybrid Double Dome

Building performance and cable-net strengthening of Esfahan Shah Mosque's dome

To Hossein Moarefi
Who understood what the dome needs

Esfahan Shah Mosque, Moarefian strengthening of South eyvan
Photo by author, 2016

ABSTRACT

In 1611, the construction of the Esfahan Shah Mosque, Masjed-e Shah, became a part of the re-modeling of the Safavid capital (the Safavids ruled from 1501 to 1722 a large area of Western Asia, with Esfahan being the third and last capital). It is a building that adopts a glazed bulbous curvature as a hybrid double dome (HDD) of brick and wooden elements. This is one of the enormous masonry double domes (MDD) which suffers from the lack of knowledge in their building technology and structural behavior. This research aims to present the building techniques, construction sequences and conservation actions of the complex HDD of Shah Mosque through historical evidence, onsite surveys as well as structural analysis. This work also proposes a cable-net strengthening of the dome, based on its intrinsic capacities and an inspired pattern of HDD in another major dome, the Chahar Bagh (1704), to reach a reversible, cost-effective, optimized and sustainable intervention that responds to the original concept and construction technique of the structure.

For these purposes, an overview on the development of MDD in Persia is provided. The different typological categories of MDD and HDD allow extending the discussion about constructive components and techniques, basic principles of form, structure and equilibrium of domes. In particular, the investigation explores the geometry, interventions and consequences of both Safavid domes, the Shah and Chahar Bagh HDDs, during the last century, from the *Salnameh* report (1934–36) up till the present.

In the case study, the construction sequences and structural features constitute fundamental knowledge to recognize the role of each wooden element of Shah's HDD as well as any deflection in intermediate configurations of the dome and existing damage. The exploration of the construction sequences of the Shah mosque HDD is an indispensable part of this research that can be complemented by the early history of the structure, both of which support in diagnosing the causes of damage. To assess the dome's safety, the investigation examines subsequently the structural behavior of the Shah Mosque's HDD, through finite element (FE) modeling and analysis with Midas FX+ and DIANA 10.2, including non-linear analysis due to vertical and horizontal loading. The study allows the development of a crack pattern to explore redistribution of the force flow.

Historical studies indicate that economic, social and aesthetic reasons required a lofty double dome structure in Naghsh-e Jahan *meidan*, which had to be optimized and hybridized by walls and wooden elements. Time limitations of the initial construction, the earthquake of 1844 and the modern strengthening of 1932 are particularly relevant historical events. The techno-poetic solution for strengthening the frontal semi dome proposed by Moarefi in 1932 used an integrated structural system that gives an appropriate lesson for the proposal developed in this work.

The structural analysis reveals that the radial walls and the wooden ties prevent horizontal thrust of the bulbous dome and increase the capacity of the structure due to vertical and horizontal loading. However, the drum is the most vulnerable part of the double dome as it includes 8 openings. The discussion on the crack patterns and the structural condition leads to plausible strengthening alternatives which would enhance the structural capacity of the existing Shah's HDD by tie-encircling components and deploying the pattern of wooden tie system in Chahar Bagh's HDD in order to join the walls to the domes.

All in all, this investigation primarily echoes that on-going conservation requires structural analysis that deeply considers historical evidence and performance. An attempt to bring together the contribution of the structural knowledge of Chahar Bagh and scientific measures of cable-net system has been made to achieve a more durable, flexible and compatible strengthening. As such, the obtained result is a combination of local practices and global modern codes.

Key words: Shah Mosque’s double dome, hybrid structure, building technology, construction sequences, building performance, FE analysis, cable-net strengthening

RIASSUNTO

[tradotto dall'inglese] Nel 1611 La realizzazione della Moschea di Esfahan Shah, Masjed-e Shah, è stata iniziata come parte della rimodellazione della capitale Safavide. Dal 1501 al 1722 i Safavidi hanno governato una vasta area dell'Asia occidentale, di cui Esfahan è stata la terza ed ultima capitale. La moschea presenta una copertura a doppia cupola ibrida (HDD), in laterizio e legno, che adotta una geometria con curvature a bulbo. La copertura è annoverabile tra le più grandi cupole doppie in muratura (MDD) delle quali, tuttavia, si ha ancora una scarsa conoscenza, per quanto riguarda la tecnologia costruttiva eppure di comportamento strutturale. La presente ricerca si rivolge ad illustrare e commentare le tecniche di esecuzione, le sequenze delle costruzioni e le operazioni di consolidamento della cupola della Moschea di Shah, attraverso testimonianze storiche e rilievi in loco, nonché analisi strutturali. La ricerca è finalizzata con una proposta di intervento che consiste nello rinforzo della cupola mediante l'uso di una rete di cavi, intervento ispirato da quello realizzato sull'HDD della grande cupola di Chahar Bagh (1704). L'obiettivo quindi è individuare un sistema di rinforzo reversibile, economico, ottimizzato e sostenibile, tenendo conto dei concetti costruttivi e delle tecniche di costruzione originarie della struttura.

A tal fine nella parte iniziale della tesi viene fornita una panoramica sullo sviluppo storico delle MDD in Persia. Le differenti tipologie di MDD e HDD hanno consentito infatti di ampliare la conoscenza dei componenti e delle tecniche costruttive, oltre ai principi di base della forma, della struttura e dell'equilibrio delle cupole. In particolare, l'indagine approfondisce la geometria e gli interventi realizzati su entrambe le cupole safavidi (quella di Shah e quella di Chahar Bagh) nel corso dell'ultimo secolo.

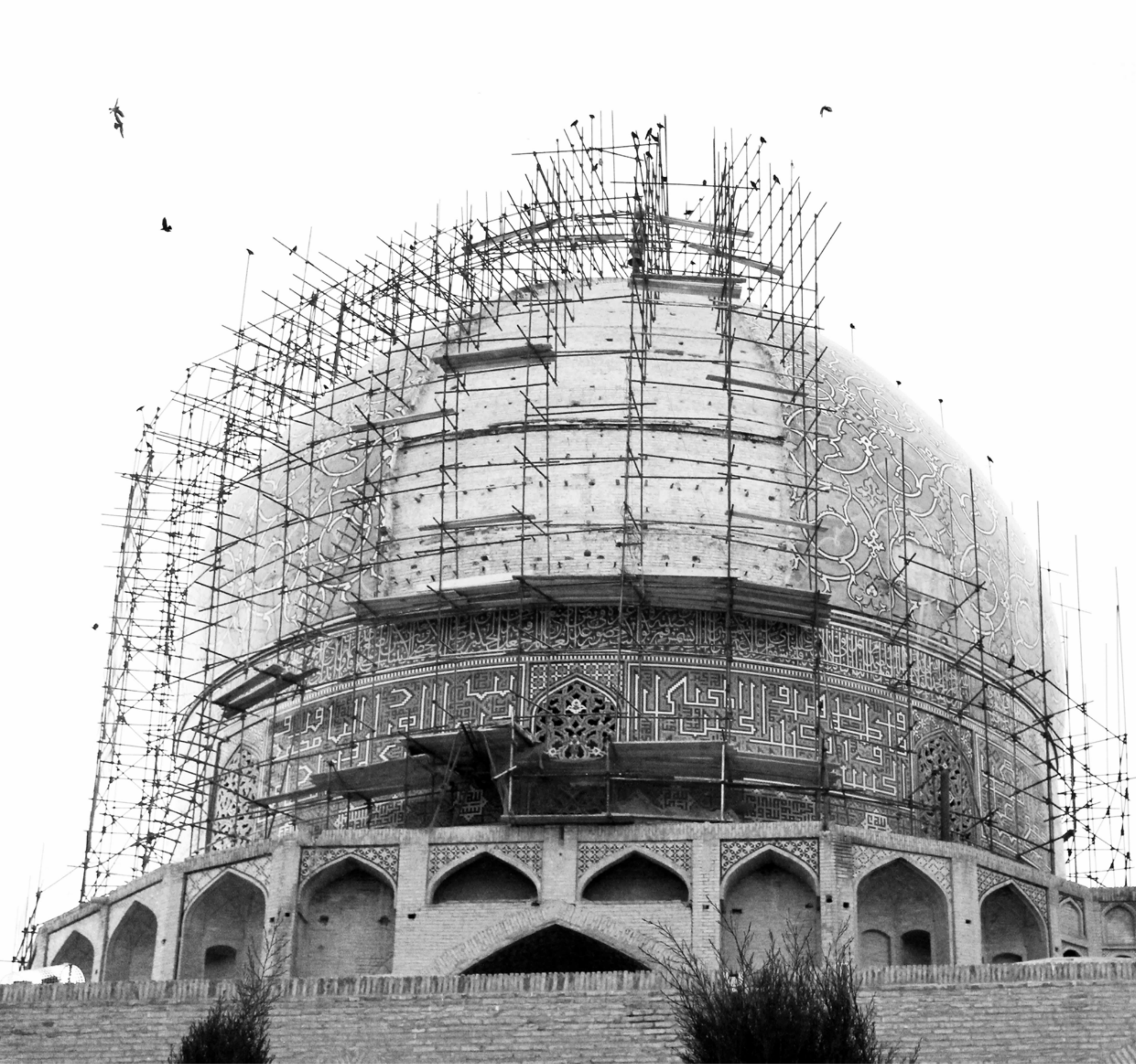
Nei casi analizzati, le sequenze delle costruzioni e le caratteristiche strutturali costituiscono una base di conoscenza fondamentale per individuare il ruolo degli elementi in legno dell'HDD di Shah. L'indagine delle sequenze di costruzione dell'HDD della moschea di Shah rappresenta un componento indispensabile per la presente ricerca, sulla quale viene integrata dalla storia recente della struttura, allo scopo di indagare le cause dei dissesti. Per valutare la sicurezza della cupola si è esaminato il comportamento strutturale dell'HDD della Moschea di Shah attraverso modellazione ed analisi ad elementi finiti (FE) condotte con Midas FX + e DIANA 10.2, includendo anche l'analisi non lineare sotto carichi verticali e orizzontali. Lo studio ha consentito di seguire lo sviluppo del quadro fessurativo, così da analizzare le modifiche del flusso delle tensioni all'interno della cupola.

La ricerca storica individua in ragioni economiche, sociali ed estetiche la necessità di adottare una struttura a doppia cupola in Naghsh-e Jahan Meidan, la quale doveva essere ottimizzata strutturalmente affiancando muri ed elementi in legno. I limiti temporali della costruzione, il terremoto del 1844 eppure il moderno intervento di rinforzo del 1932 sono eventi storici particolarmente rilevanti che hanno fornito spunti alla ricerca. Ad esempio, la soluzione tecnica adottata per rafforzare la semi cupola frontale, proposta da Moarefi nel 1932, utilizza un sistema strutturale integrato che offre uno spunto appropriato per la proposta sviluppata nel presente lavoro. L'analisi strutturale rivela che le pareti radiali interne ed I tiranti in legno contrastano la spinta orizzontale della cupola a bulbo e aumentano la capacità della struttura, sia sotto carico verticale che orizzontale. Il tamburo, che include 8 aperture, resta tuttavia come la parte più vulnerabile della doppia cupola.

La discussione relativa al quadro fessurativo e alle differenti condizioni strutturali delle cupole analizzate, hanno condotto ad alternative di consolidamento possibili in grado di migliorare la capacità strutturale dell'HDD di Shah mediante cerchiature con cavi e che, allo stesso tempo, tengono conto del sistema di tirantatura in legno utilizzato nell'HDD di Chahar Bagh per unire tra loro le pareti radiali e la cupola.

La ricerca condotta evidenzia come i criteri attuali di restauro e consolidamento necessitino di una preventiva ed approfondita conoscenza degli aspetti storici. Nella tesi si è tentato di legare tra loro la conoscenza strutturale di Chahar Bagh ai moderni sistemi di consolidamento a rete di cavi così da ottenere una soluzione il più possibile duratura, flessibile e compatibile. Il risultato ottenuto costituisce quindi una combinazione di antiche pratiche locali e moderni criteri strutturali.

Key words: Doppia cupola di Shah Mosque, struttura ibrida, tecnologie di costruzione, sequenze di costruzione, performance degli edifici, analisi FE, consolidamento con rete di cavi



Previous image:
Esfahan Shah Mosque, Restoration workshop
of tile work. Photo by author, January 2018

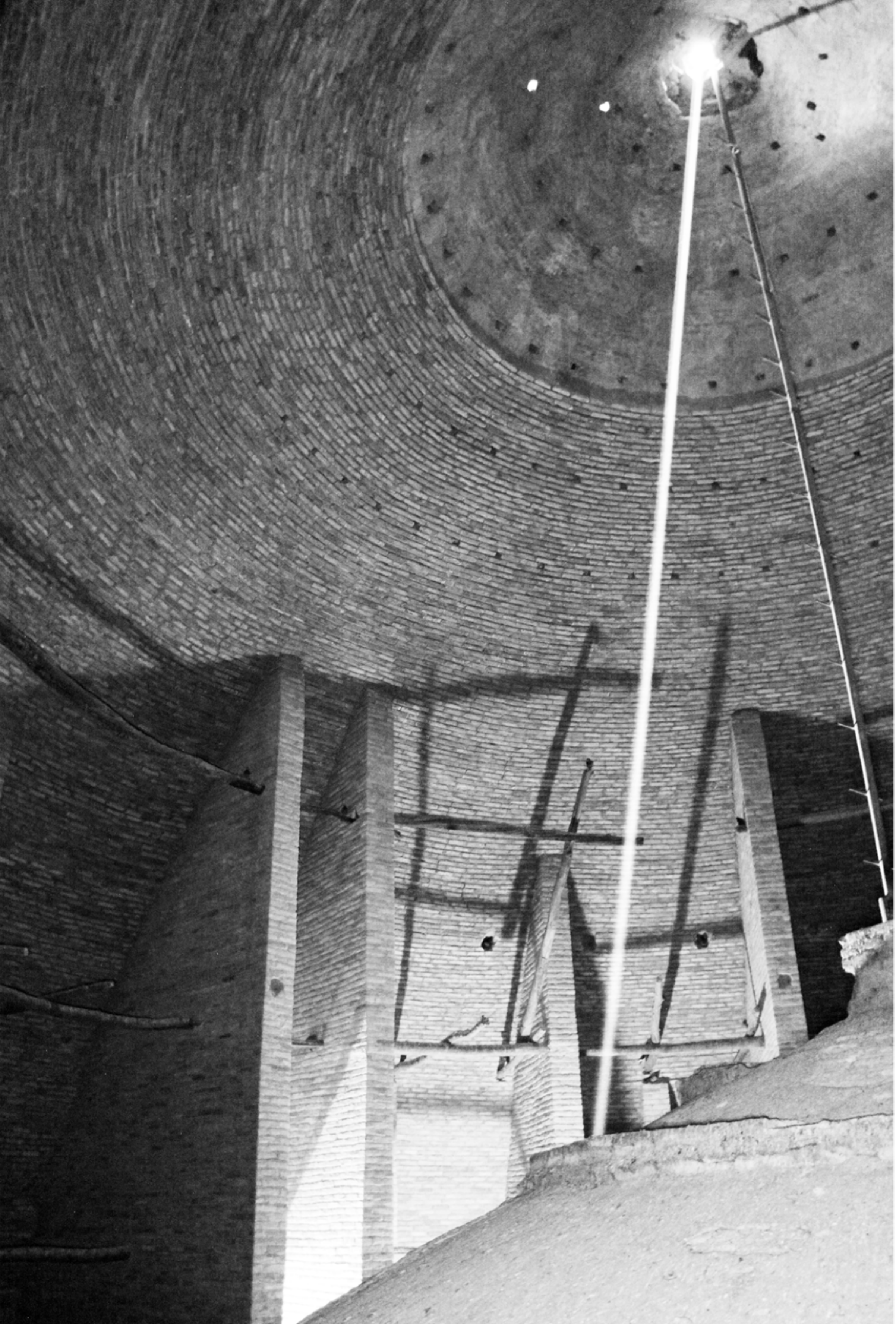
[Chapter I]

INTRODUCTION

1.1 Motivation

"*Iranian originality is clearer when one turns to architectural forms, for regardless of the function of a monument, the same small number of formal units appears constantly at all times, in almost any setting, and in almost every area. Five of these are particularly notable: the dome, the eyvan, the court, the tower and the wall…. dome is one of the 'true glories' and was the source of exciting technical developments of Iranian architecture.*" (O. Grabar, 1986: 344)

The tradition of masonry construction extends back thousands of years, to the mud brick and stone buildings of our earliest ancestors, in the wide range of types and forms with unique structural behavior for each individual building. Maintaining these monuments is profoundly linked with the recognition of the original construction process, historical performance, and structural capacity, which is the most challenging and crucial step for their strengthening, when applicable. The archetype of the Safavid dome (the Safavids ruled from 1501 to 1722 a large area of Western Asia) is an ambiguous form taken as the subject of this research, as it presents a bulbous shape that still needs a genuinely analytical identification through both historical studies and cutting-edge digital tools.



Previous image: The space in-between the domes, Shah mosque, Photo by author, 2016

[Chapter II]

LITERATURE REVIEW

2.1 Basic structural principles and equilibrium of domes and arches

"*In most cases, masonry structures fail due to instability rather than a lack of material strength*" (Heyman 1995).

2.1.1 Structural analysis, a historical approach

Structural analysis is a method to better understand the stability of historical structures for their conservation and restoration. It can be used not merely to define structural safety for gravity and horizontal loading such as wind and earthquake, but also for soil settlement actions which can help to determine the actual cause of existing damage (Roca et al., 2010). The spatial character of curved elements such as domes, vaults and arches have been always considered in structural analysis applications from classical methods to the most advanced ones. Studies have tried to bring awareness of existence and to discuss effects of forces on masonry construction, which have been as a significant reference from diagnosis to intervention stages.

Analysis of the complicated geometry of historical structures, made with a weak material in tension prone to cracking, has demanded a computational tool for simulating material and geometry to characterize the structural behavior (Roca et al., 2013). The first use of computer for structural analysis in building project for shells might have been Jorn Utzon's Sydney opera house (Fig. 2-3) by Arup in1958 (Brazil, 2016). Among examples of cutting-edge computational processes for historical structures, the studies on Brunelleschi Dome by Chiarugi et al. (1993), Pisa Tower by Macchi et al. (1993), St Mark's Basilica by Mola et al. (1995) and Colosseum by Croci (1993, 1995) are significant (Fig. 2-1, 2-4), which contrast with the scaled models from Gaudi (Fig. 2-2).

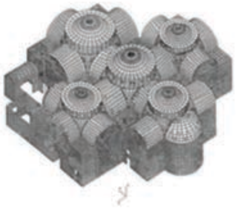
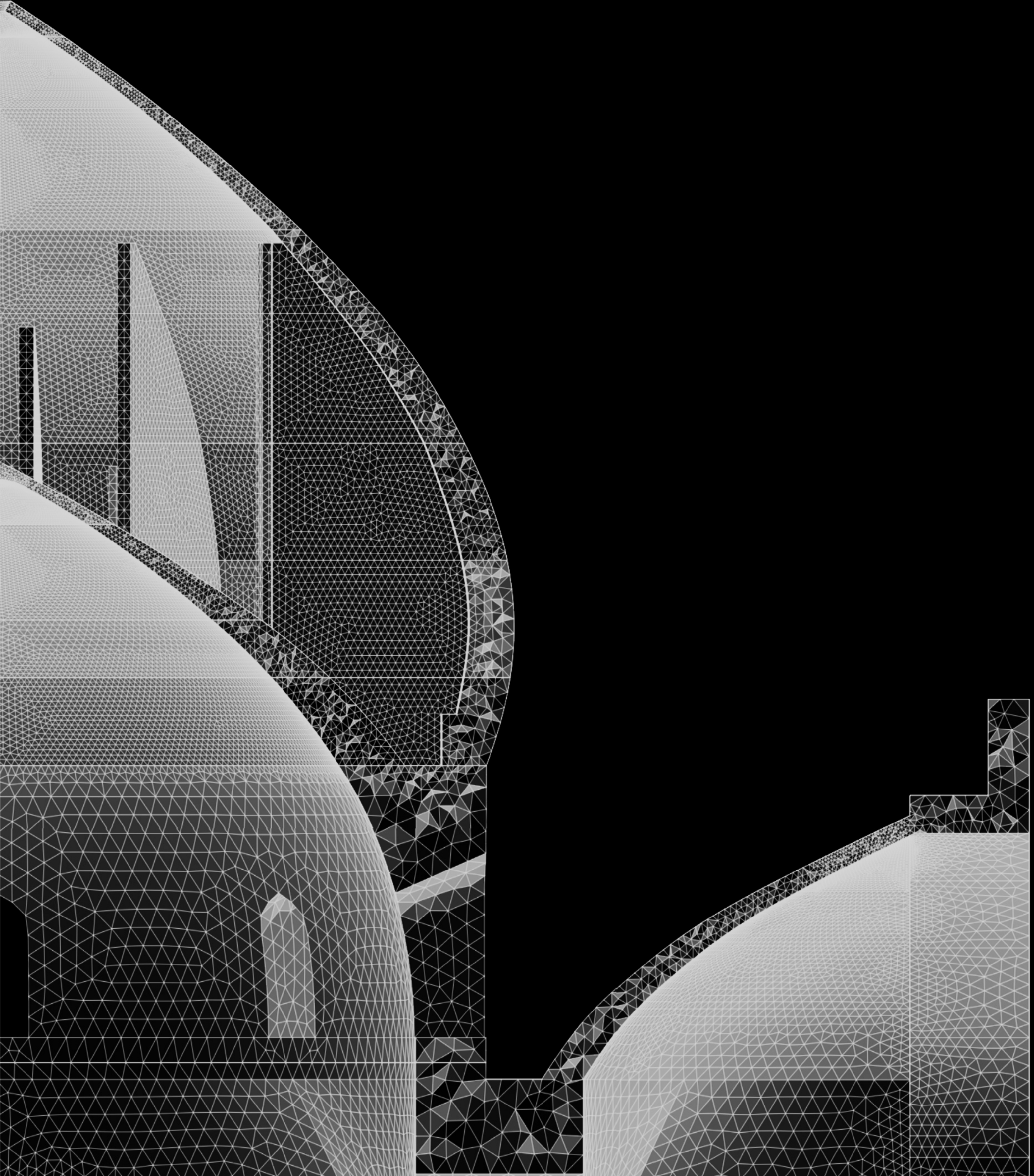


Fig. 2-1 Spatial structural model for FE analysis of Basilica San Marco by Mola et al. (1995)



Previous image:
Shah Mosque's HDD and north eyvan, section, 3D model and the mesh in Midas FX+ for DIANA, Model by author, 2017

[Chapter III]
RESEARCH METHODOLOGY

3.1 Conceptual model and methodological approach

Historical evidence, inspections, and structural analysis constitute the fundamental categories of our conceptual model for surveying the Shah Mosque's HDD. These aspects are inextricably linked to each other in order to reach an appropriate recognition, interpretation, and solution in each individual step of the structural investigation, which the last diagram (Fig. 1-3) of the chapter 1 undoubtedly attempts to illustrate.

Given the historical evidence, the research should revisit facts, events, and actions through modeling, analyzing and evaluating the Shah's HDD. For example, the *Kashanian* geometrical pattern and IsMeo's surveys are a key element in modeling the dome. As another example, the Esfahan 1844 earthquake recorded in *Salnameh*, induced serious damage and Moarefi's plan of the Shah dome's strengthening is also much relevant, which is one of the neglected parts by previous researches. Then any crack analysis nearby the Eyvan should be widely discussed in this respect. The other major ignored evidence is Pirnia's claim about radial wood elements and *Shahang* that was cut off following the dome completion, which should contribute to the discussion on the future strengthening plan.

This research has not monitored the case studies by means of professional instruments, yet carefully inspections revealed concealed facts about the existing conditions, construction process and building system of the Shah's HDD. For example, the wooden tie-rod system around the bulbous dome and precise crack pattern of the structure are outstanding issues that should be examined and measured in structural analysis and acknowledged by the relevant authorities.

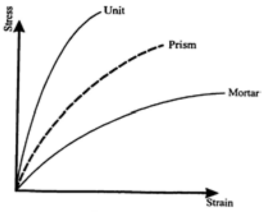
decrease uncertainties. Moreover, experimental tests, onsite surveying, monitoring and inspections can enhance the quality of the model. The performance of the dome is mainly studied under gravitational load, horizontal loading replicating an earthquake and soil settlement, with ample discussion about the double dome performance.

3.2 Modeling strategy for the masonry structure

3.2.1 Brick masonry as a composite material

Masonry is a composite material that contains brick and mortar. The properties and strength of the masonry are strongly dependent upon the properties of the constituents and construction technique (Binda, 1988; Lourenco, 2002), albeit needless to mention the role of workmanship in the mechanical properties of the masonry. The strength of the mortar also depends on the properties of the components and mixture such as lime, sand, water and etc. (Paulay, 1992). For example, Binda through an experimental testing evaluates the mechanical properties of a masonry prisms with three different types of mortar. The result shows that the characteristics of masonry components in terms of compressive and tensile strength are different as well as strength (Binda, 1988; Lourenco, 2006). Both brick and mortar not only have different strengths but also different deformation characteristics. In general, the uniaxial compressive strength and the modulus of elasticity of the mortar are considerably lower than the corresponding values of the bricks. Therefore, if the mortar could deform freely, its lateral strains would be larger than the strains in the brick (Fig. 3-2). However, because of bond and friction between brick and mortar, the mortar is confined (Hilsdorf, 1969).

“Masonry is a material which exhibits distinct directional properties due to the mortar joints which act as planes of weakness” (Lourenco, 1998: 67). Mortar is more flexible than brick and inelastic behavior of the mortar begins much earlier that brick. Nonlinear deformation typically occurs in the joins before failure (Page, 1978). For the purpose of analysis and level of accuracy, this precise observation in the literature of the masonry science, concludes that the analysis could be based on the distinctive geometry of bricks and mortars in micro-modeling, as well as, masonry as a homogenous composite material in macro- modelling.



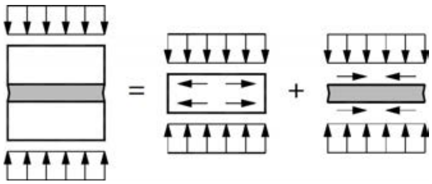


Fig. 3-1 (Left) Schematic diagram of stress-strain curves of masonry unit, mortar and prism assemblage. Two different materials with vastly different properties are combined to form structural element.
Fig. 3-2 (Right) Stresses in the masonry components (Lourenco, 2006)

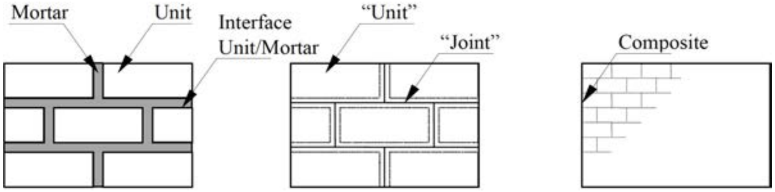


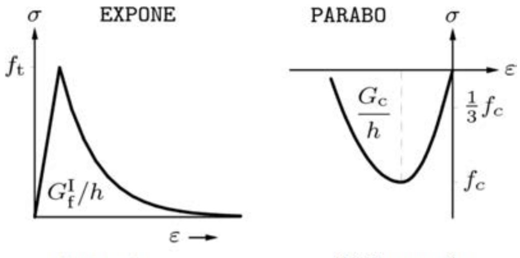
Fig. 3-3 Modeling strategies for masonry structures: detailed micro-modeling; simplified micro-modeling; macro- modeling; Lourenco, 1998

There are three approaches for modeling a masonry structure. Lourenco defines them as micro-modeling, simplified micro-modeling and macro modeling (1998) (Fig. 3-3). He discusses that "one modeling strategy cannot be preferred over the other because different application fields exist for micro and macro-models". In the first approach micro-modeling oriented, the properties of brick, mortar, and interface constitute essential data to explore the local behavior of the structure. However, in the second approach each joint is lumped into an average interface and the accuracy is lost. In the macro modeling single brick and joint are not distinguished, but it "treats masonry as a homogeneous anisotropic continuum," which is applicable for larger structures.

For the reasons related to the mesh generation and obtaining an acceptable size of the analysis file, which become challenging due to time saving in running analysis for such a mega case of the research, macro modeling has been presumed, as this strategy balances accuracy and efficiency (Lourenco, 1998: 68). Yet, the material properties of this homogeneous unit are derived from the properties of simple brick and mortar joint. For this purpose, the basic cell in the Shah Mosque needs to be clarified by on site surveying.

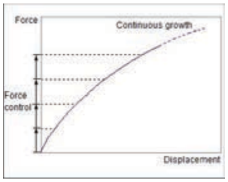
3.2.2 Total strain rotating crack model (TSCM)

For the non-linear analysis in DIANA, the total strain crack model (TSCM) has been adopted. TSCM describes the tensile and compressive behavior of a material with one stress-strain relationship. This makes it an appropriate model for analysis, which is qualified by cracking or crushing of the material (TNO DIANA BV 2014). Firstly, the input data for the TSCM contains the basic properties of linear elasticity such as density, modulus of elasticity and Poisson's ratio, and secondly, the definition of the behavior in compression and tension related non-linear material properties. The tensile behavior is identified by an exponential softening curve and the compressive behavior by a parabolic curve based on the definition of the tensile and compression fracture energy (Fig. 3-4) (Lourenco, 1997).

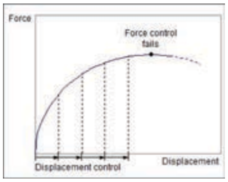


(a) Tension (b) Compression

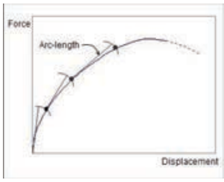
Fig. 3-4 Stress and strain curve presents tensile and compression behavior of the masonry with exponential softening and parabolic curve (DIANA, 2014)



Load control



Displacement control



Arc-length method

Fig. 3-5 Three types of load incremental procedures in DIANA, diagram © TNO DIANA BV, 2013

3.2.3 Load increments and Iteration procedure

In the nonlinear FE analysis, the correlation between load and displacement vectors is not linear anymore. The non-linear analysis needs an incremental-iterative procedure to reach the equilibrium at the end of the increment by using an iterative solution algorithm (TNO DIANA BV, 2017). More details behind the incremental-iterative solution and the equations can be found in the DIANA Finite Element Analysis User's Manual Release 10.2 that are not the subject of this research.

Three types of load incremental procedure have been defined in the DIANA based on the structure response curve, such as a gravity load which is progressively applied (Fig. 3-5). The force control, the displacement control and the arc-length control method (Palacio, 2013). The load control analysis is not recommended for the model with softening behavior and when the load is applied more than the load capacity of the structure. The displacement control method is endorsed in the curve experiencing the snap-through (Palacio, 2013) (Fig. 3-6). The last solution for the load increments is arc-length method that adapts the step size derived from the outcomes of the present step (TNO DIANA BV, 2017). The three available iterative solution methods in DIANA are the Regular Newton-Raphson where the tangent stiffness matrix is derived at every iteration; the Modified Newton-Raphson where the tangent stiffness matrix is derived at the start of every load increment; and the Quasi-Newton where the secant stiffness matrix is derived at every iteration (Palacio, 2013).

The non-linear analysis for the vertical and horizontal loading are considered in the analysis part. The load factor increases based on a certain size to reach its self-weight and until the failure point. For the nonlinear response, the regular Newton-Raphson method is adopted as an iteration method and the arc-length method is used for an incremental procedure (Fig. 3-7). Newton-Raphson method is converging with a fewer iterations, which is time consuming for every iteration (TNO DIANA BV, 2017). For example, in the gravity loading the load factor increases 10% of dead load in 10 steps and the analysis for the vertical overloading is a series of concatenated analysis with certain size of load factor and iteration method to attain the highest possible load. Then the load displacement diagram presents the linear or/and nonlinear behavior of selected nodes and any fluctuations during the load increment. The maximum capability to carry the load appears in the peak point of the graph and the failure may accrue after the peak.

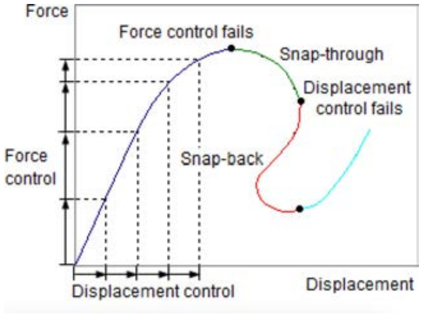


Fig. 3-6 Schematic load displacement diagram for the horizontal and vertical overloading. Diagram © TNO DIANA BV, 2013

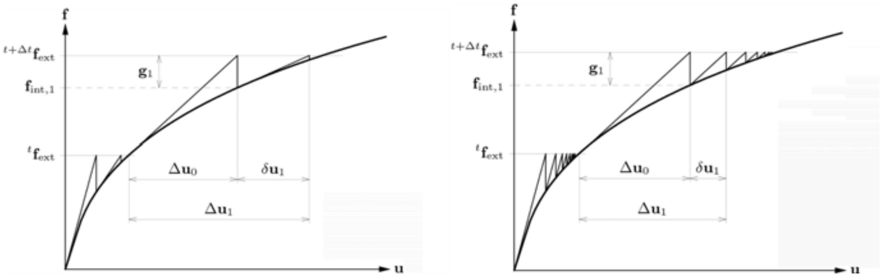


Fig. 3-7 Regular Newton-Raphson iteration and Modified-Raphson iteration method. Diagram ©TNO DIANA BV, 2013

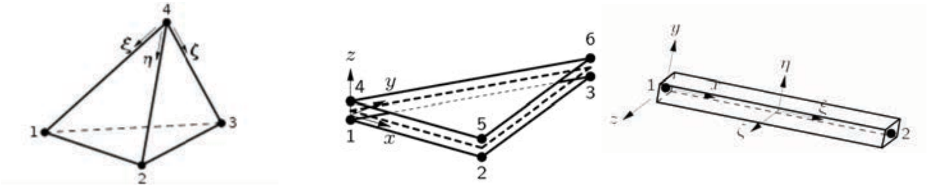


Fig. 3-14 Type of elements used in the FE model: TE12L, 4-node, tetrahedron element, three-side isoperimetric solid pyramid (3D solid element); L12BEA, 2-node three-dimensional class III (1D beam element); T18IF, plane triangle, 3+3 nodes, (2D interface element). TNO DIANA BV 2014)

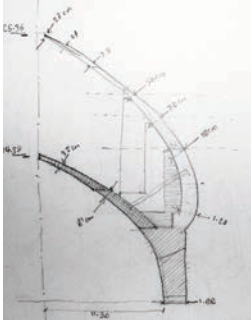


Fig. 3-15 variable thickness of the dome © sketch by author, 2017

The full FE model contains 621,496 nodes and 2,998,689 elements with the different mesh sizes from 50cm in the body of the building and drum to 10cm in the upper part of the domes (Fig. 3-13, 3-17). Different sizes are used to have at least four nodes in the thickness of each part of the model, with variable thickness from 1.88 m to 0.27 m (Fig. 3-15). Besides, the mesh size control is used during the mesh setting to control the auto mesh in appropriate parts of the geometry such as the vault of the *eyvan* and openings. This automatically converts from the mesh size of 50cm to 5cm. This action reduces the general size of mesh to the individual size of the proposed element. For the beam elements, the auto mesh with 20 cm length is applied. The mesh needs to be checked through the mesh quality command and the nodes should be tested for connectivity (Fig. 3-16).

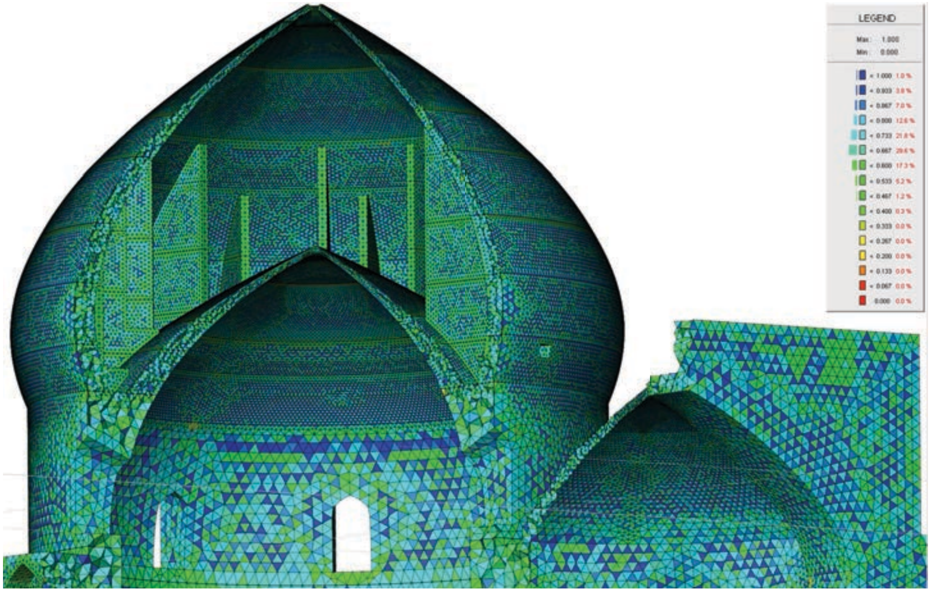


Fig. 3-16 Quality mesh for the dome in relation to the aspect ratio of elements. © Model by author, 2018

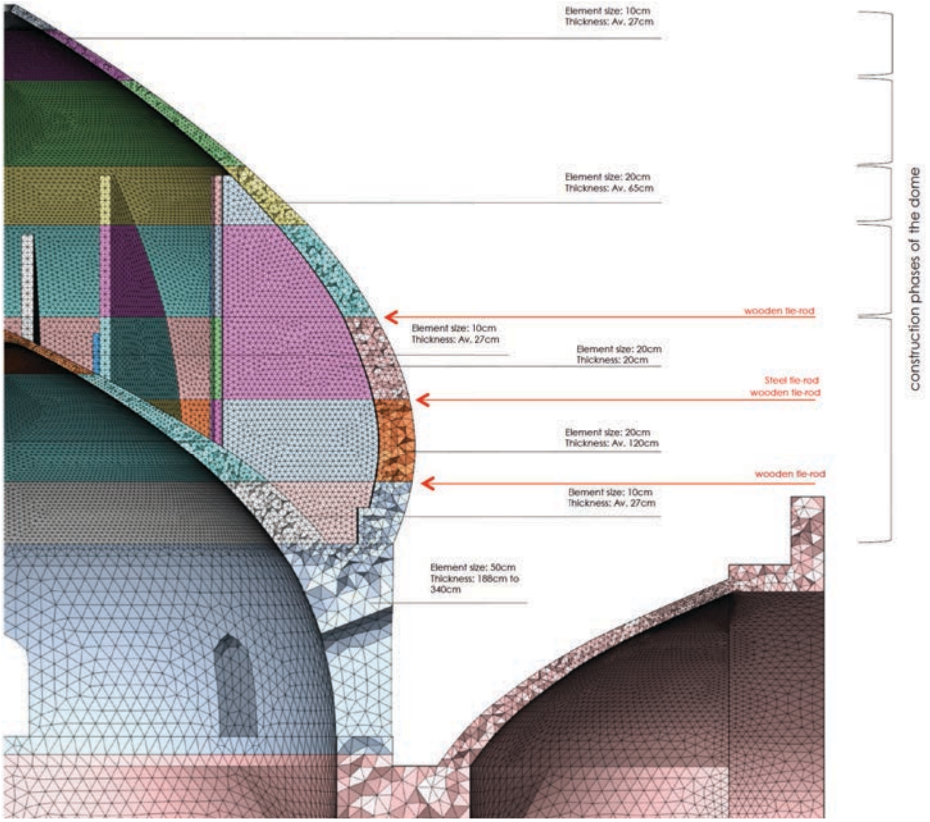


Fig. 3-17 3D model and the mesh by considering the construction phases and encircling ties system.
© Model by author, 2018

3.3.4 Boundary conditions and soil structure interaction

Degrees of freedom of the selected nodes at the base and the end of the beams should be restrained through boundary condition setting in the Midas FX for DIANA. Restraining displacement of all the 1549 nodes at the base (pinned) option is considered for the linear analysis and when the damages on the dome are studied. This option is accessible in Midas FX+ under the section of the analysis, BC, constraint. Additionally, interface elements have been generated at the base when the simulation of the soil structure interaction is contemplated. The soil behavior under horizontal and vertical stresses needs to be considered, adopting normal (Kn) and shear stiffness modulus (Ks) (Fig. 3- 18).

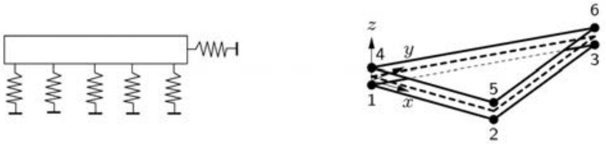
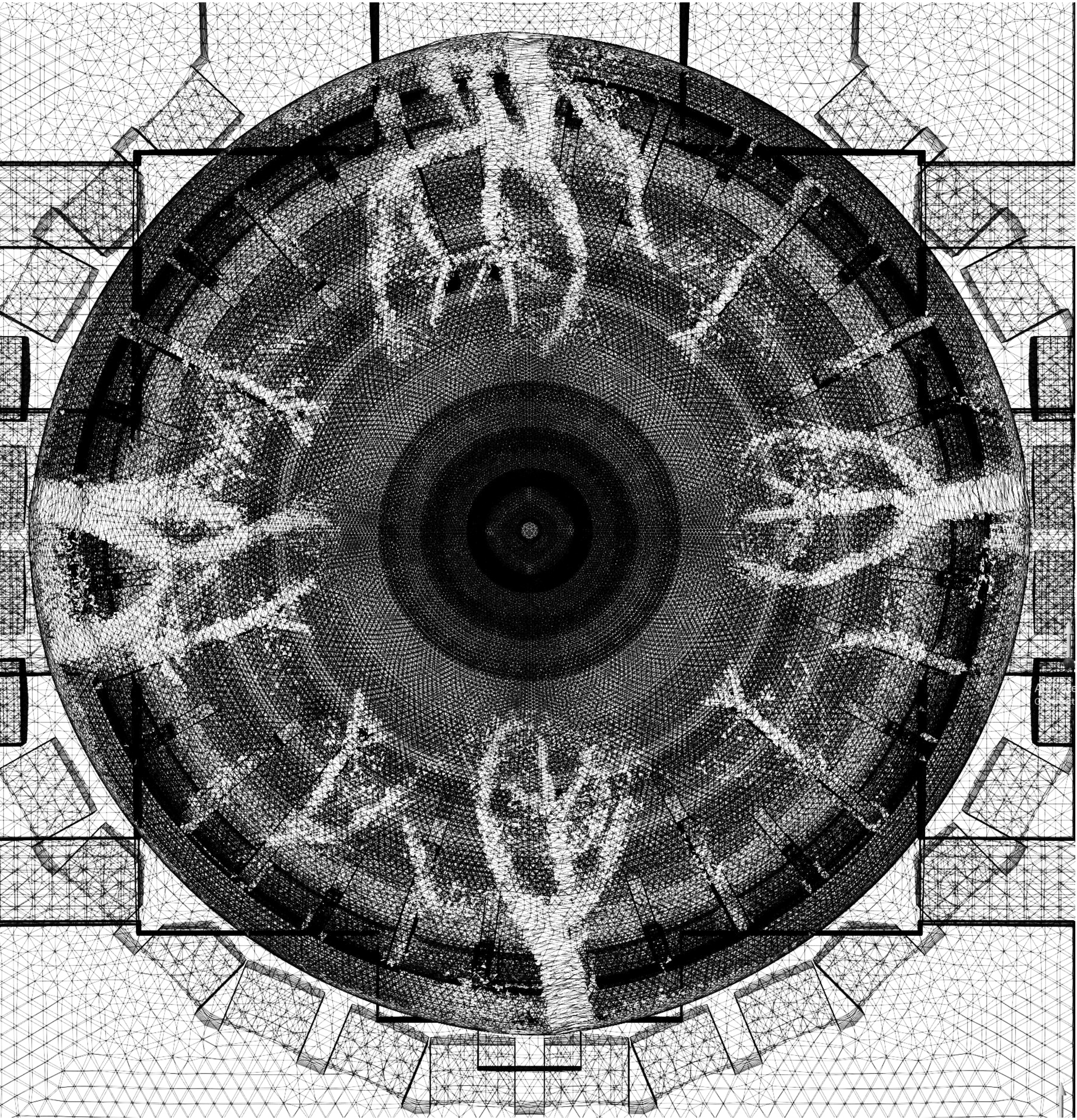


Fig. 3-18 (Left) Soil structure interaction, Winkler model (system of identical but mutually independent, closely spaced, discrete, linearly elastic springs). (Right) interface model



Previous image:
Crack distribution of the dome due to the vertical load of 4.7 of self-weight at the end of the capacity curve
Image by author, 2017

[Chapter IV]

CASE STUDY: Structural Analysis

Shah Mosque's HDD presents more unknown scientific points than we have recognized. This matter then requires a series of quantitative analyses to complete the discussion on all historical evidences that were qualitatively studied in the chapter 3.

To fill the gap between historical studies and modern codes, firstly we provide structural features, crack pattern, forces and construction sequences of Shah Mosque's HDD based on historical data, observation to reach logical responses. Subsequently assigning material properties to make a platform, to study the dome deeply, through analytical plan consisting of linear static analysis, besides, non-linear static analysis comprising gravity loading and vertical overloading.

4.1 Structural features of Shah Mosque's HDD

4.1.1 Domes


Fig. 4-1 Aerial photo of Shah Mosque's dome. © Photo by Barati 2007

The domes of Shah Mosque are erected on top of each other, which reach upwards to 38m and 52m in height and 26.3 m in the outer diameter, covered by faience tiles outwards (Fig. 4-1). The dome has the thickness of almost 1/13 of its internal diameter. A square base of 22.6m serves as a groundwork of HDD, which transitioned into the circular base by squinches. The space between the two shells provides not only a spatial structure but also works as insulation absorbing radiant heat (Tavassoli, 1974). The inner dome, Ahiyaneh, is a pointed dome with 8 openings, with the thickness from 170 cm to 25cm (Fig. 2-53), carries all the outer dome, radial walls and wooden elements. The geometry follows the rotated arch type II described by Jamshid al-Kashani in 15th century (Kashani, 1967), who estimated geometrical properties of the arches such as length of intrados, area of the façade, rise of intrados, height of convexity and empty area underneath the arch, for building construction and providing the required building materials and scaffolding (Kashani, 1967; Memarian,

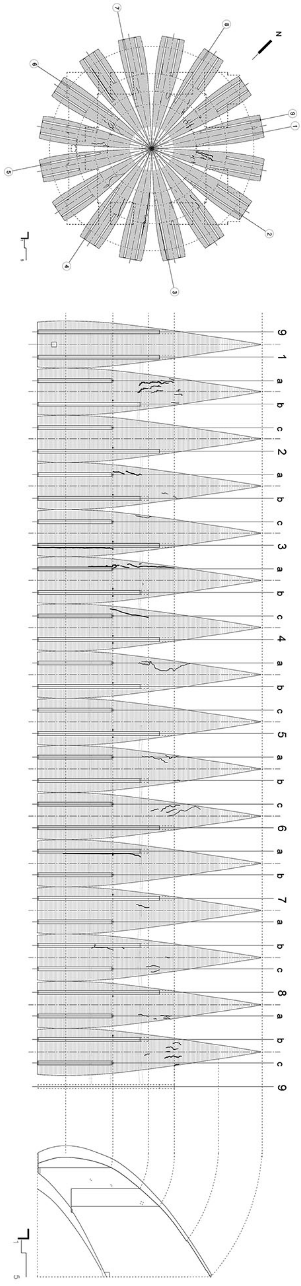


Fig. 4-11 Distribution of the cracks on the outer dome recorded on 24, August 2016. © Inter_Esse Studio (Sadeghi-Dinani)

4.2 FE Analysis

In this section, firstly the material characteristics of the masonry are derived from micro- model (Lourenco,1998). Secondly, the linear and nonlinear analysis are applied on the FE model due to the dead load as well as horizontal loading for different scenarios.

4.2.1 Material properties

4.2.1.1 Basic cell in the Shah dome

The properties and strength of the masonry brick of the Shah's dome are derived from a basic cell of masonry surveyed from the building as it strongly depends upon the properties of the constituents of brick and mortar and construction technique (Binda, 1988; Lourenco, 2002). As described in chapter 3, the presumption for Shah Mosque case study is to consider both Micro and Marco modeling (Lourenco,1998) for defining the material properties and for modeling in structural analysis part, respectively.

The basic cell in the Shah mosque contains bricks and mortar with dimension of 270x140x250 mm where the size of the brick, as the unit, is 250x 250x50 mm with almost 2cm for the head and bed joints. So the density, modulus of elasticity, tensile strength and compressive strength of masonry are the important properties for design and evaluation of masonry structure, which are relating to all its components, brick unit and mortar (Fig. 4-27).

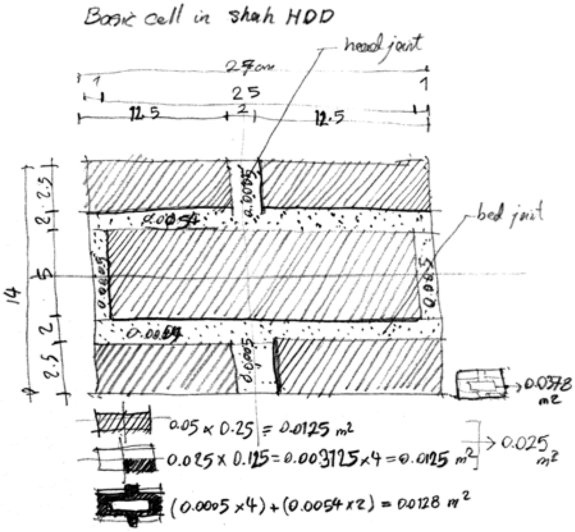


Fig. 4-27 Basic unit of the Shah dome. by author, 2018

4.2.1.2 Density

The density of material is presented as material mass per unit volume with SI unit of Kg/m^3. Density of brick wall or shell calculates based on the brick in combination with mortar in a basic cell. The mass of brick and mortar are divided by the total volume, giving the density of the masonry:

$$\rho = \frac{m}{v}$$

eq. 4-1 Density of the material: where ρ is the density [kg/m^3], m is mass [kg], v is the volum [m^3]

On the 2D of the basic cell the total area of the brick and mortar are 0.025 m^3 and 0.0128 m^3. Then the volume with the depth of 0.25 m is 0,00625 m^3 for brick and 0,0032 m^3 for mortar. Density [ρ] is 1330 kg/m^3 for brick and 1600 kg/m^3 for mortar. So the density based on the combination of the mortar and brick in a basic cell is 1421 kg/m^3.

4.2.1.3 Modulus of elasticity

Modulus of elasticity defines the stiffness of material and its relation to elastic deformation under the applied upon tensile or compressive load which is unique to the substance. For example, the mortar has a lower elastic modulus than brick. Modulus of elasticity is identified from the simple equation bellow. Moreover, the change in the length divided by the original length defines the amount of strain. On the other hand, the total change in length of the masonry is equal to the sum of the changes in length of brick and mortar.

$$\mathrm{E} = \frac{\text{strees}}{\text{strain}} = \frac{\sigma}{\varepsilon} \qquad \varepsilon = \frac{\Delta \mathrm{L}}{\mathrm{L}} \qquad \Delta \mathrm{L} = \frac{\sigma \times \mathrm{L}}{\mathrm{E}}$$

eq. 4-2 Where E is modulus of elasticity in Pascal, ε is the strain, (ΔL) the change in length per (L) original length of element and (σ) is the stress.

$$\Delta_{y,M} = \Delta_{y,u} + \Delta_{y,m}$$

eq. 4-3 Vertical displacement of a masonry prism, (Lourenço & Pina-Henriques, 2006)
Where $\Delta_{y,M}$ Vertical displacement of a masonry prism, $\Delta_{y,u}$ Vertical displacement of the unit and $\Delta_{y,m}$ Vertical displacement of the mortar joints

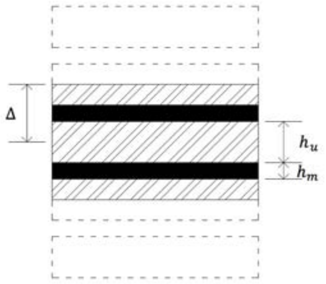


Fig. 4-29 Side view of a basic cell illustrates vertical displacement and the height of the joint and unit. Drawing by author 2018

4.2.1.5 Poisson's ratio

In the elastic limit, when a material is loaded, the ratio of the lateral strain on the longitudinal strain is identified as a Poisson's ratio which remains constant. Its value for the brick and mortar is considered 0.2.

4.2.1.6 Tensile strength

The safety against cracking of masonry depends on its stress by influence of exterior loading, temperature and humidity as well as on the strength and deformation properties. These are essentially determined by the properties of the components, brick and mortar, and their bonding properties. The restraint of deformation causes tensile stress, which means an increased cracking risk because of the relatively low tensile and shear strength. Tensile stress parallel to the bed joints causes vertical or zigzag cracks as a common damage. (Backes, 1985: 779). One rule says the tensile strength of a masonry material is not more than 10% of its compressive strength (Calzada and Ruiz, 2009).

The calculation of the tensile strength for the masonry material in the Shah mosque referred to the tensile test that Backes proposed (Fig. 4-30). Two failure modes have been considered for calculation (Fig. 4-31). First, the vertical crack which passes along the head joints and units. Second, failure occurs as a stepped crack. It passes along the head joints and the length of the bed joint. As the configuration of the failure modes present in the tensile bond strength of mortar joints, shear bond strength of joints and tensile strength of brick need to be considered with amount of 0.13, 0.3 and 1.25 MPa respectively (Table. 4-4, 4-5).

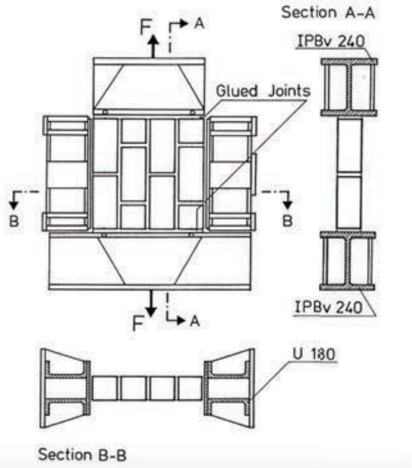


Fig. 4-30 Tensile testing of masonry and its behavior, parallel to the bed joints (Backes, 1985)

The stress, tensile strength f_t is force per area. The sum of the forces on each area of the crack through the specimen in mortar or brick is the total tensile force for masonry. Finally, the tensile strength is assumed here as the average between two failure modes, given the uncertainty involved.

$$f_t = \frac{F}{A} \qquad\qquad F = f_t \times A$$

eq. 4-6 Stress [MPa] is force [N] per area [mm^2]

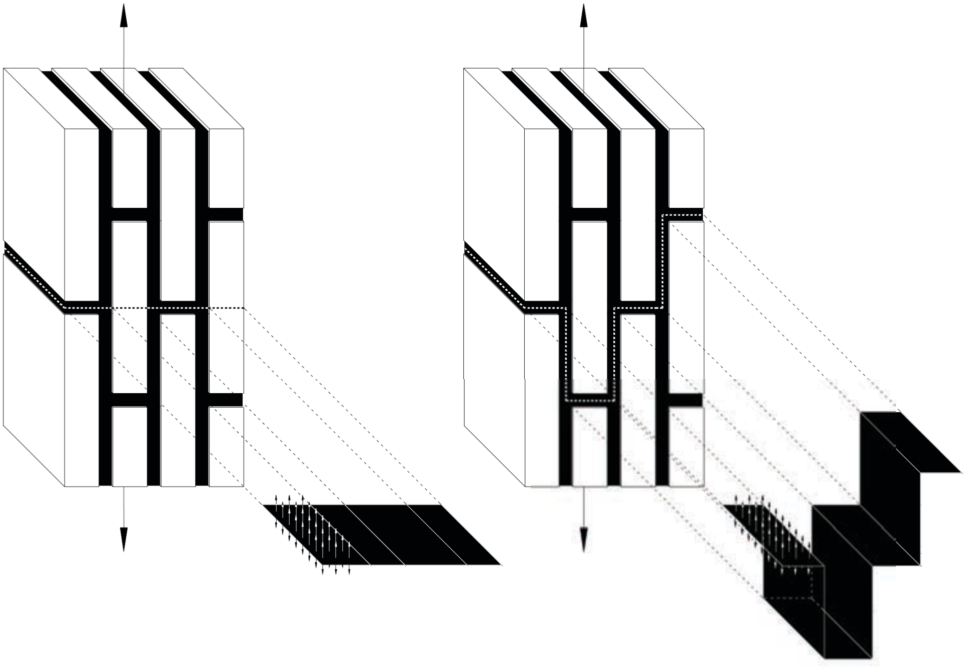


Fig. 4-31 two failure modes could cause vertical crack in the Shah Mosque masonry specimens in the dimension of 26x52 x52 cm based on the Backes' tensile test. Diagram by author, 2018

4.2.1.8 Geotechnical properties of the soil

There is no individually geotechnical investigation for the Shah Mosque, which oblige us to refer to the survey of the Esfahan metro project by Zamin Fanavaran Co. in 2006. The boring point BH-70415 located 180 m southwest of the dome, is the nearest mechanical drilling. However, a comparison with the BH-70420 located south part of the *meidan* indicates that almost the same layers and geotechnical properties are valid for the mosque area. Soil boring of BH-70415 then serves as soil properties of the case (Fig. 4-35).

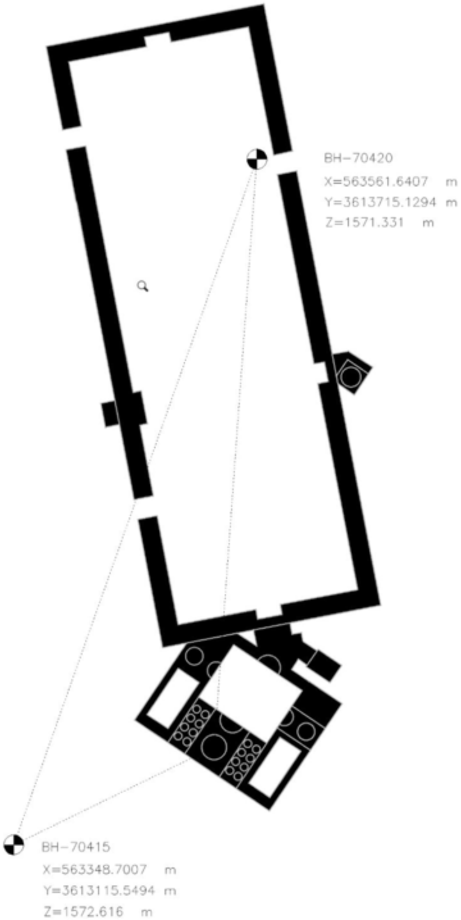


Fig. 4-35 Shah Mosque and nearby soil boring points, BH-70415 located 180 m of the mosque. Boring test by Zamin Fanavaran Co. 2006. Drawing by author, 2017

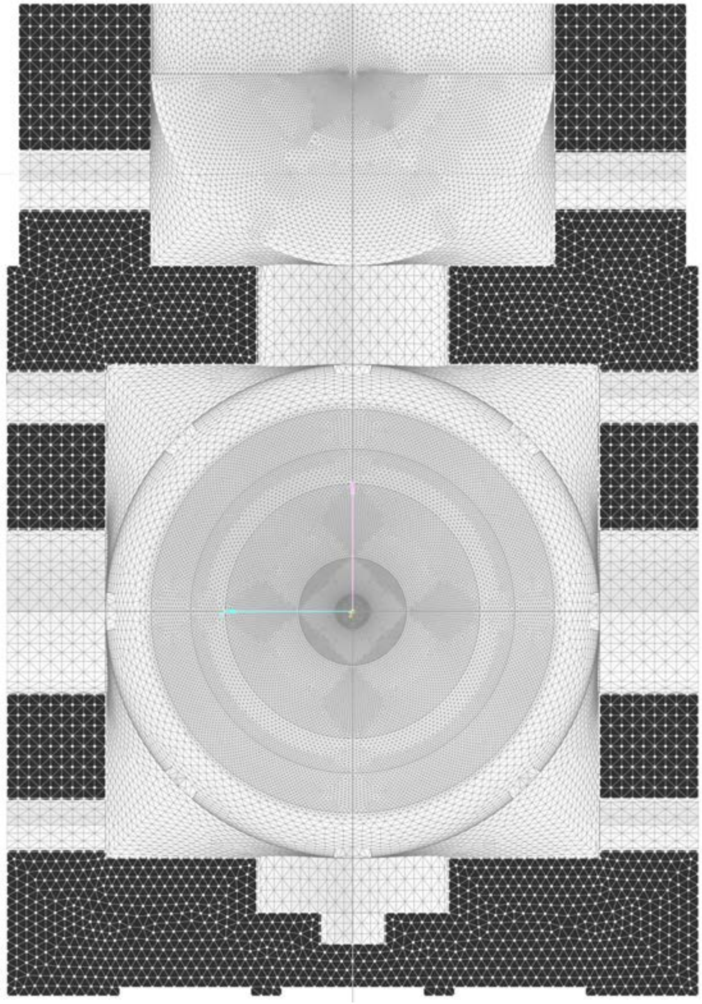


Fig. 4-39 Arrangement of the nodes at the base point, Diagram by author, 2018

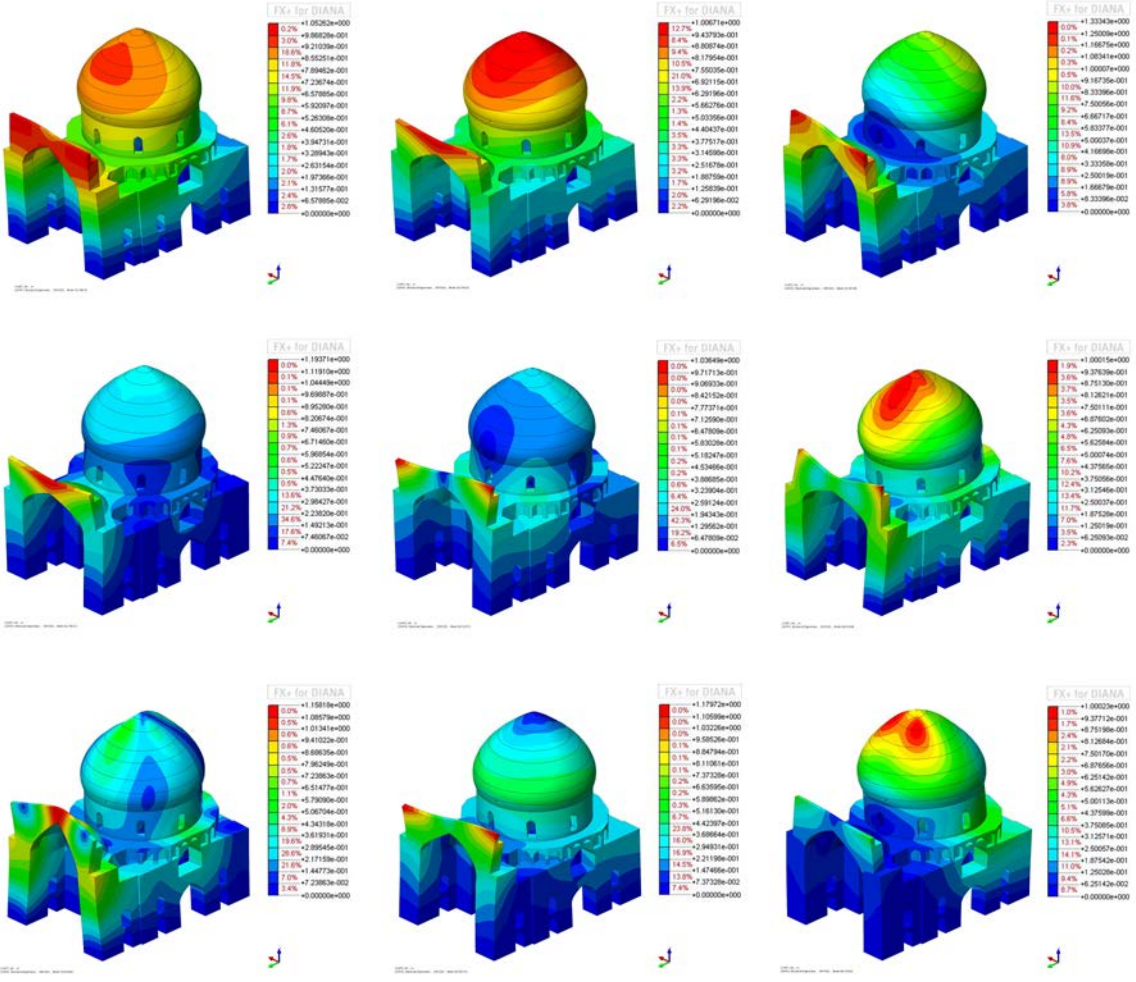


Fig. 4-44a Mode shape configuration of modes 1 to 9, with natural frequencies. Image by author 2018

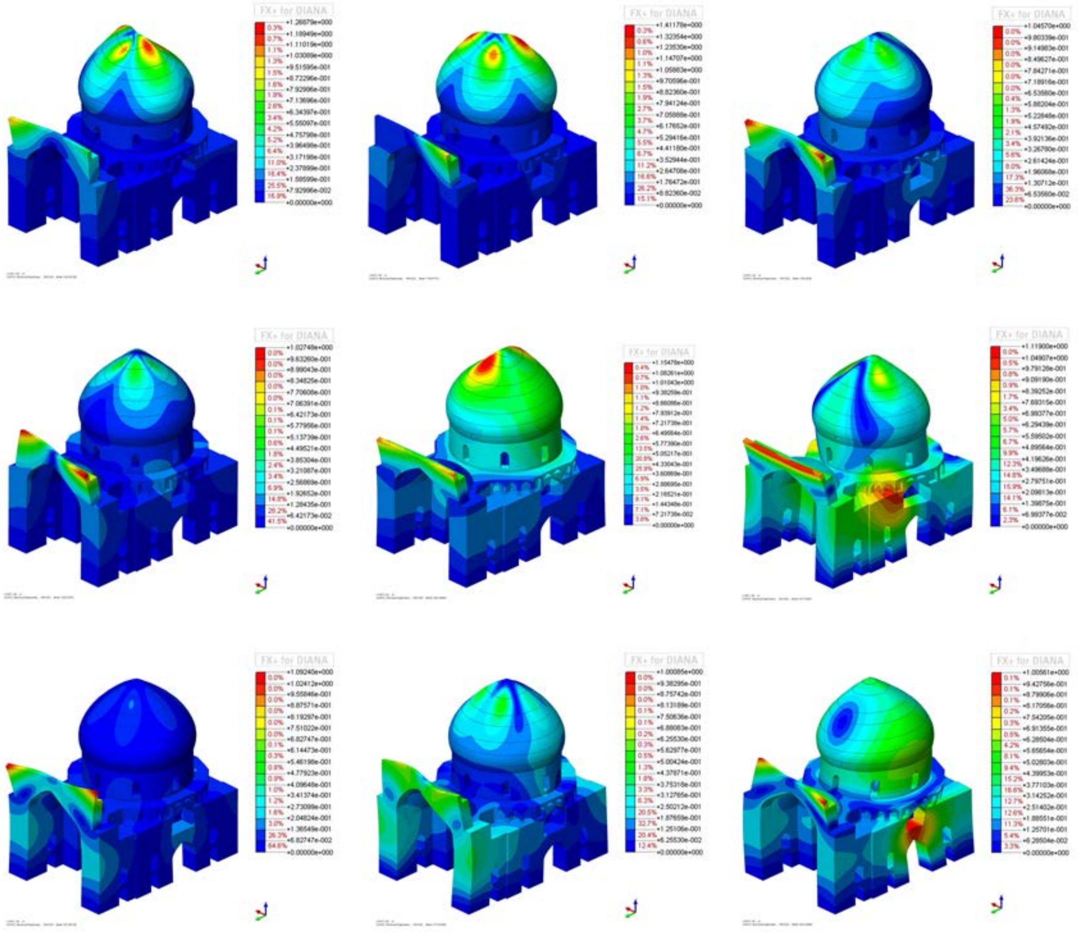


Fig. 4-44b Mode shape configuration of modes 10 to 18, with natural frequencies. Image by author 2018

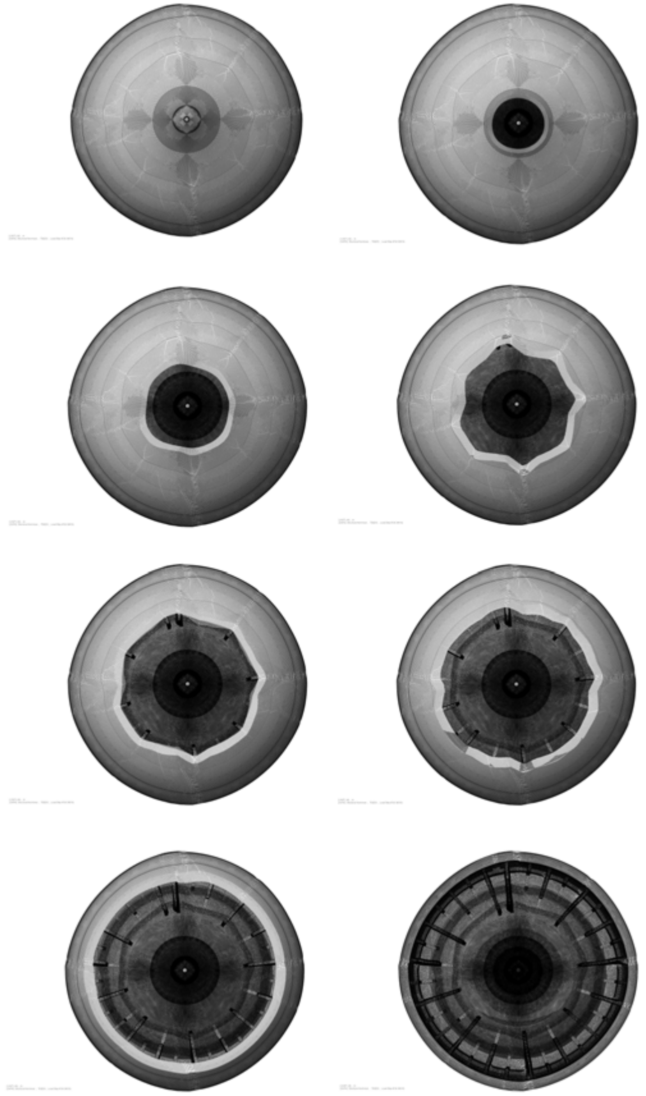


Fig. 4-64 Deformed-shape study through the sequential sections of the dome due to the vertical load of 4.7 of self- weight at the end of the capacity curve

Fig. 4-65 Superimposition of the crack distribution and deformed shape of the dome due to the vertical load of 4.7 of self-weight at the end of the capacity curve

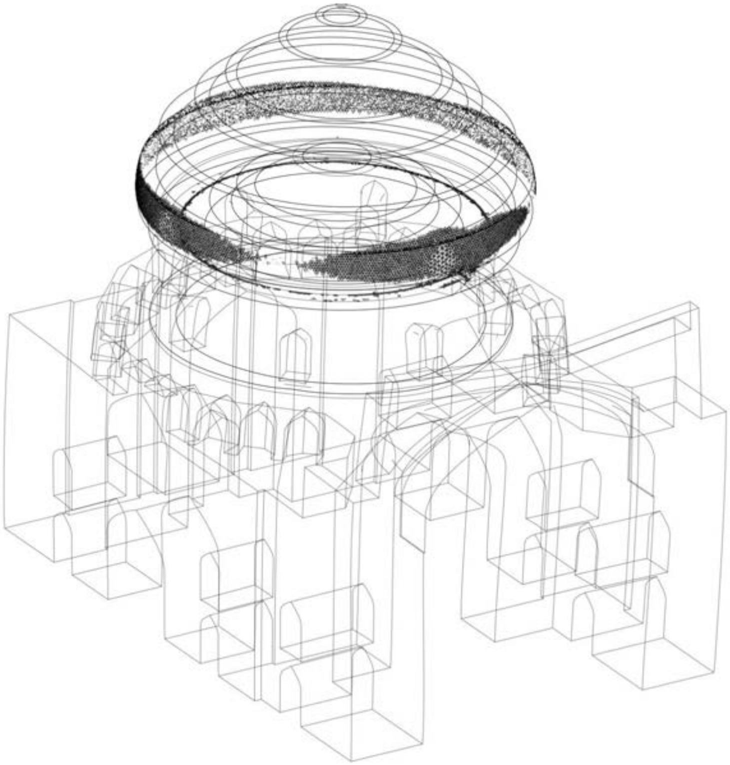


Fig. 4-67 Normal strain crack (Eknn) due to the vertical load of 2.2 of self-weight

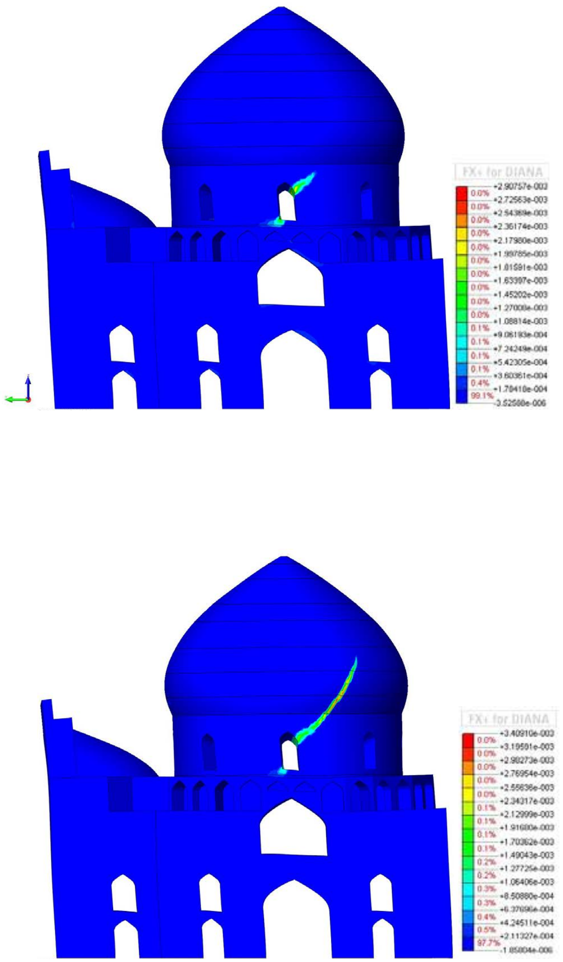
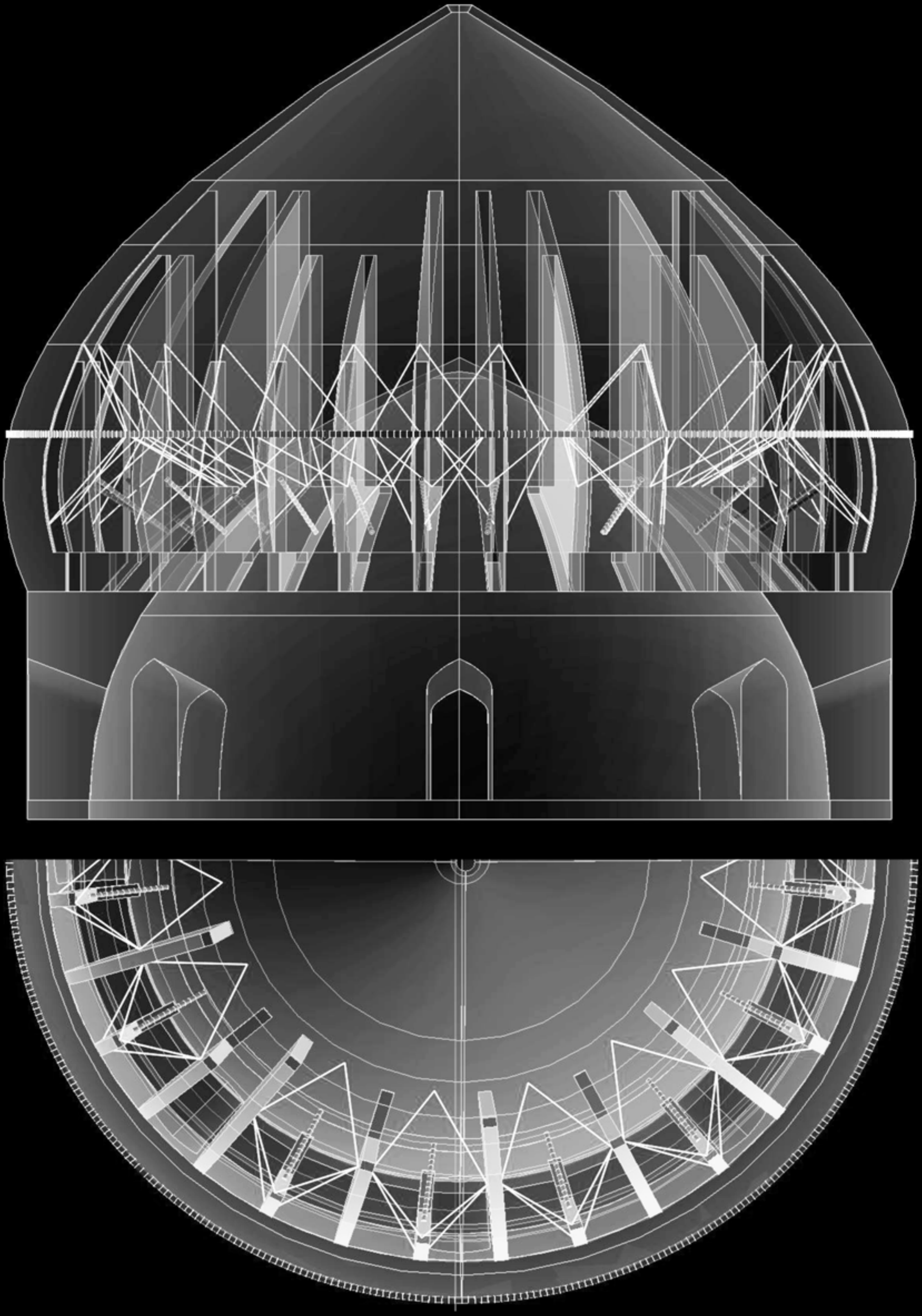


Fig. 4-70 Pushover analysis +Y, maximum principal strain at peak (left) and post peak of capacity curve (right)



Previous image:
Cable-net strengthening for the double dome
Image by author, 2018

[Chapter V]

NET STRENGHTENING: Experimentations and results

"Each intervention should, as far as possible, respect the original concept and construction technique and historical value of the structure and of the historical evidence that it provides." (ICOMOS/ISCARSAH, 2003)

Previous image:
Double dome of Shah mosque
Photo by author, 2016

[Chapter VI]
CONCLUSION

6.1 Conclusion, contributions and future works

The strengthening of the Shah Mosque's HDD is a challenge born between historical studies and modern codes. A study of the Persian double dome is not merely to understand the essential transformations for the structural performance, but also to analyze the structural aptitudes by means of hybridization of the masonry in the past, which leads to minimum intervention with sustainability in the present.

This section attempts to provide a dynamic debate on the achievements of the research regarding historical performance, construction process, and structural analysis, while considering real conditions of the monument, local policy of the structural restoration, and local facilities to conclude the final reflections on what can be as an appropriate plan for strengthening of Shah's HDD. Subsequently, the research's contribution simultaneously addresses limitations of the recent model study and analysis process as much as requests for further scientific studies.

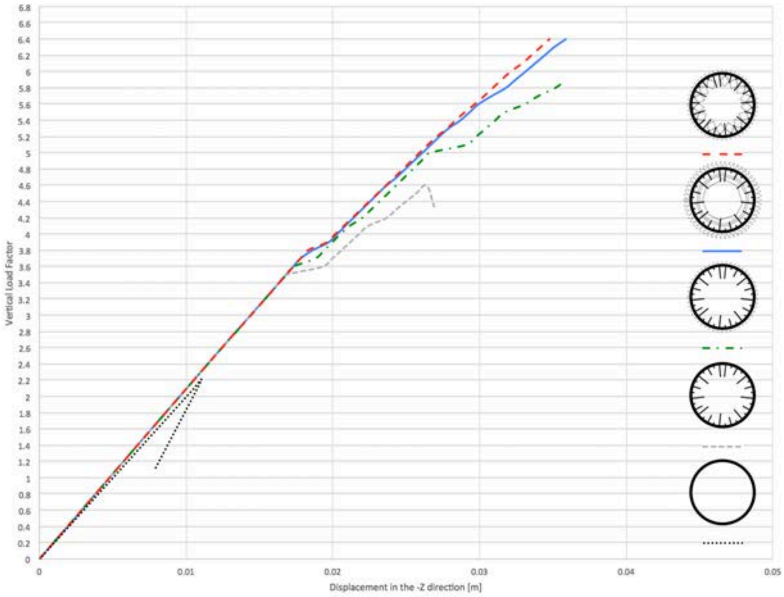


Fig. 6-1 Capacity curve due to the vertical loading for the MDD with and without walls, MDD with steel tie-road and two strengthening proposals. Controlled node: 77477

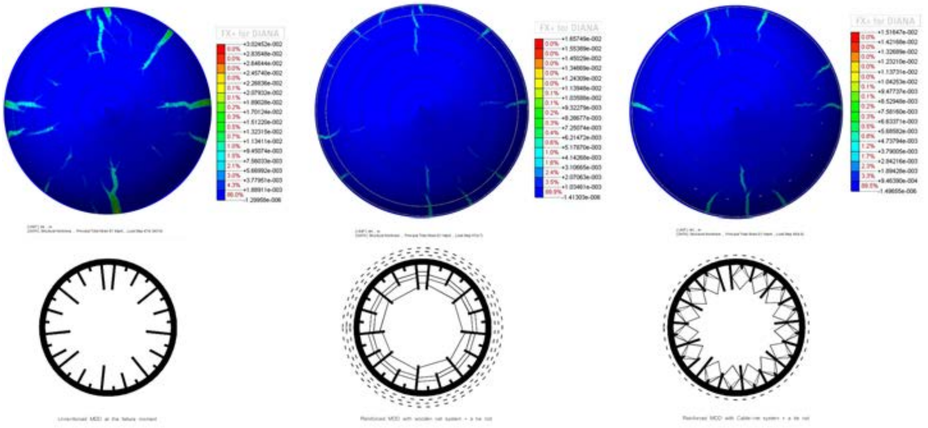


Fig. 6-2 Maximum principal strains E1 due to the vertical loading, damage pattern at end of the capacity curve, Load 6.4 of self-weight for unreinforced dome compared with two strengthening proposals.

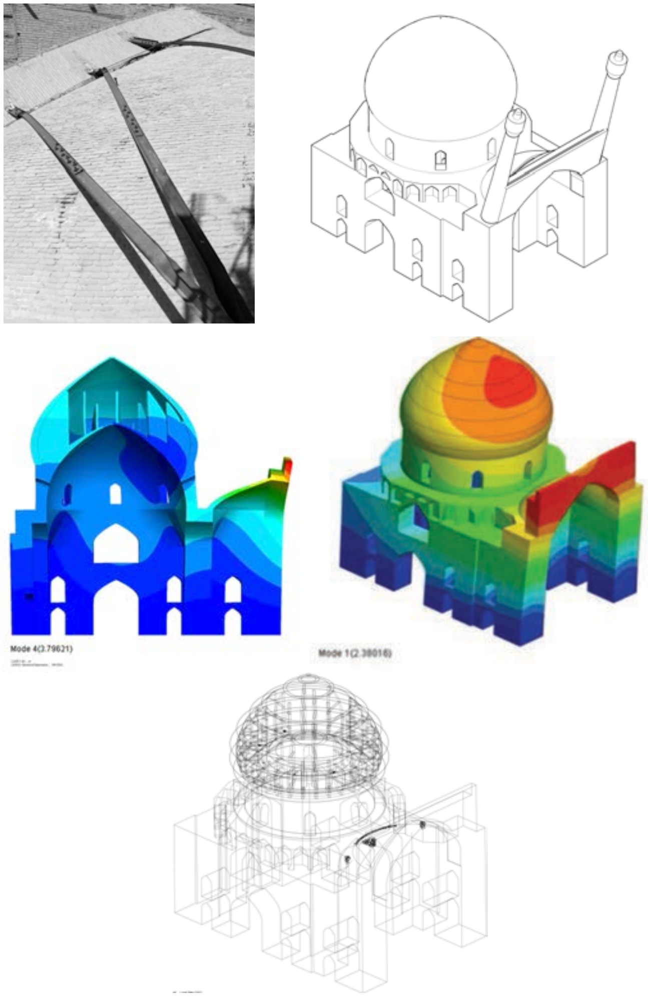


Fig. 6-4 An integration between historical evidences, inspection and analytical model's results: The semi-dome of the *Eyvan* presenting the restrained deformation of a crack during the inspection, which was reported in *Salnameh* of 1937. Meanwhile, the mode shape configuration of both the 1st and 4th modes, proves the possibility overturning façade and anticipating the crack pattern which is so akin to the current situation of the semi-dome. Images by author

Printed in Venice
Feb. 2019