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Estimation of cutting forces and tool tip vibrations in milling in stable and unstable conditions

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Abstract

In milling, the regenerative instability phenomenon, also known as chatter, may appear and can lead to a deterioration of the surface quality and to a non-competitive production. This work deals with the problem of monitoring the cutting forces and tool tip vibrations by means of state observers. The aim is to model the real system considering the machine tool dynamics and the information about the cutting process. The additional information on the cutting process is necessary in order to describe the behaviour of the system when it is subject to regenerative instability (chatter). Indeed during chatter, the forces and vibrations of the system grow till the tool, due to the high vibrations reached, detaches from the workpiece. At this point the subsequent detachments and re-enters in cut characterize the system in its unstable condition. The addition of the regenerative term leads to a system of delay differential equations (DDEs). Then a study of state observers for systems of DDEs is performed and a Kalman filter is implemented.

Keywords: chatter; milling process; dynamic model; cutting forces; tool tip vibrations; DDEs; state observers; regenerative effect; detachment of the tool.

Sommario

Il fenomeno di instabilità rigenerativa, noto come chatter, che può verificarsi durante il processo di fresatura, contribuisce in modo significativo al peggioramento della qualità superficiale e ad una produzione non competitiva. Questo lavoro pone l'attenzione sull'identificazione delle condizioni di taglio instabile rigenerativo attraverso gli osservatori di stato per la stima delle forze di taglio e delle vibrazioni in punta utensile. Il sistema considerato si basa sia sulla dinamica della macchina sia sul processo di taglio. Le informazioni circa il processo di taglio sono necessarie al fine di definire il fenomeno rigenerativo del sistema in caso di instabilità. Infatti in presenza di chatter, le forze e le vibrazioni continuano a crescere fino a quando si verifica il distacco dell'utensile dal pezzo in lavorazione. A questo punto il sistema è caratterizzato da successive perdite di contatto e ritorni in presa dell'utensile. L'aggiunta del termine rigenerativo alla classica dinamica della macchina porta ad un sistema descritto da equazioni differenziali ritardate (DDEs). Successivamente uno studio è stato svolto sui metodi di implementazione degli osservatori di stato per sistemi descritti da DDEs e confrontato con un filtro di Kalman.

Parole chiave: chatter; processo di fresatura; modello dinamico; forze di taglio; vibrazioni in punta utensile; DDEs; osservatori di stato; effetto rigenerativo; distacco dell'utensile.

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1. Introduction

1.1 Machine tool sector overview

Nowadays the growing demand for high-quality products has characterized a global competition among manufacturing companies. For this reason, the research of high-performance machine tools, which must in any case satisfy contained costs, is required. Machine tools builders focus on the production of customized, multitasking, efficient and reliable machines and they are projected on the idea of providing technical support to the customer, satisfying the principles of preventive and predictive maintenance. Another important aspect is the reduction of energy consumption; indeed, the industrial sector uses more energy than any other end-user sector, currently consuming about one half of the total energy supplied to the world. The goal in this direction is to produce environmentally friendly machines, according to the binding obligations on industrialized countries to reduce emissions of greenhouse gases as stated by the Kyoto Protocol to the United Nations Framework Convention on Climate Change.

The Italian contribution in the machine tools sector is one of the most important in Europe and in the world.

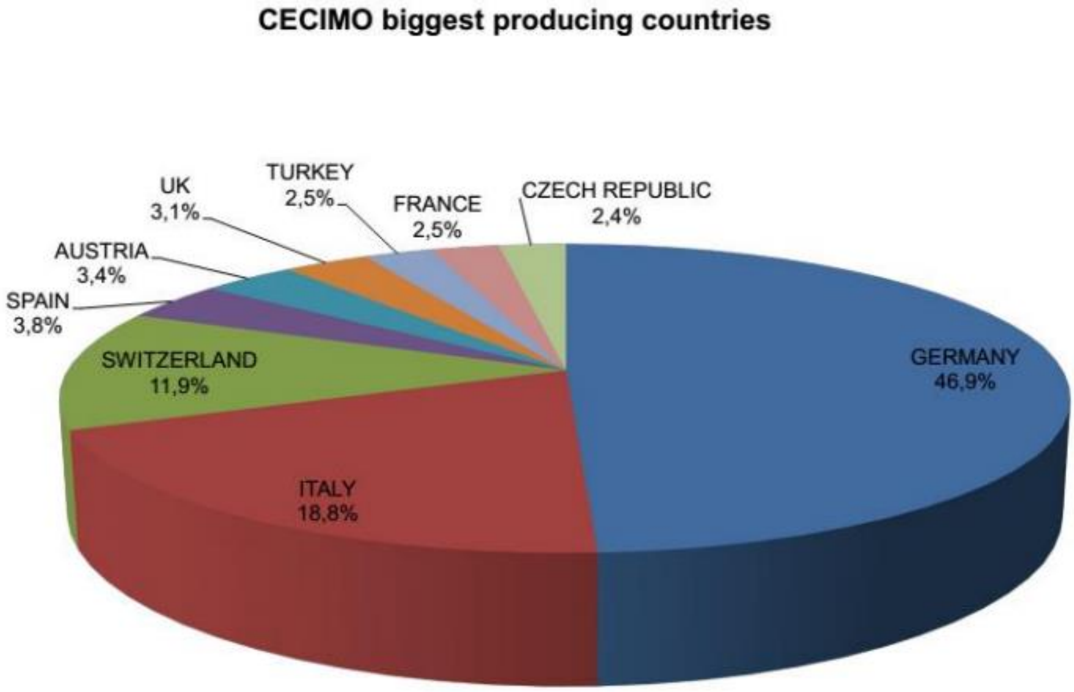


Figure 1.1: Role of Italy in machine tools Europe production (CECIMO 2015)

2018 was the year of record for the Italian machine tool, robot and automation industry (data UCIMU). Indeed in 2018 the production raised to 6900 million € with an increase of the 13.4% compared to 2017. It was the fifth consecutive year of growth and in absolute value the new record for the Italian industry of the sector; 2019 maintains the 2018 trend especially thanks to export. The result was determined both by the excellent performance of deliveries by Italian manufacturers on the internal market, which grew by 21.1%, to 3270 million €, and by the positive trend in exports, up 7.2%, to 3630 million €. According to the UCIMU elaboration on ISTAT data, in the first eight months of the year, the main destination countries of the made in Italy sector were: Germany 246 million € (+ 11.6%), China 237 million € (+ 7.1%), United States 223 million € (+ 9.5%), Poland 143 million € (+ 49.8%) and France 135 million € (-4.6%). In order to maintain the positive trend of results, manufacturers must act towards increasingly efficient use of their machines, both in terms of the minimization of the downtimes periods and the enhancement of their performances. So, the next generation of CNC machine tools need to self-adjusting and organizing, by using reliable and effective condition monitoring solutions, in order to have smart and productive machine tools without operator intervention in accordance with Industry 4.0 principles. Even if advanced sensors are available at quite low cost, monitoring solutions still represent a challenge for industries. The main limitation is due to the incapability of directly measuring physical quantities strictly related to the process in order to control the production, find optimal cutting conditions and have an idea of the cutting quality. Hence the sensors are useful devices that allow to measure the process variables in machining, such as cutting forces, vibrations, surface finishing, temperature etc., but also they can provide information on the machine status for maintenance scheduling, for reducing downtimes and raising productivity.

Among all the machining operations, milling is one of the most important because of the high material removal rate and precision obtainable. It can find application in many different industrial sectors such as automotive, aerospace, aeronautics, precision technology etc., where complex machining are always involved and high quality must be ensured. Cutting quality is strongly linked to machine tool dynamics.

The main variables that have a direct link to the quality of the obtained surfaces are:

- Tooltip displacement
- Tooltip vibration
- Tool deflection
- Cutting forces

1.2 Cutting forces measurements

In milling, the detection of cutting forces can be very useful for process monitoring and controlling. They can provide indications about cutting conditions limits, surface quality of the produced parts, tool wear and other process related information in order to increase machining performances.

However, cutting forces measurements are very difficult to obtain because, during cutting, the tool is engaged in the workpiece.

There are basically two methods to measure the cutting forces:

- Direct measurements
- Indirect measurements

In-process cutting forces measurements can be directly carried out by piezoelectric dynamometers mounted on the work table and above which the workpiece is fixtured as shown in figure 1.2. However, these devices are restricted to the laboratory use because of their limited size, setting difficulties and high costs; so, typically, industrial application is not widespread, except in some niche aeronautical sector. Moreover, these piezoelectric plates depend on the workpiece inertia that can change significantly during machining of the piece.

For these reasons the indirect cutting force measurements are an alternative to the direct ones. They are based on the concept of relate the quantity read by the sensor, not located in the interested zone of detection, even if very close, to the force that must be studied. In particular Spindle Integrated Force Sensors (SIFS) are piezoelectric force sensors placed into the stationary spindle housing [1]; the structural dynamic model between the cutting forces acting on the tool tip and the measured forces at the spindle housing is identified (figure 1.3). A Kalman filter is then designed to filter the influence of structural modes on the force measurements. These types of sensors allow to obtain some advantages compared to direct measurements:

- Saving of machine tool space
- No limited size of the workpiece
- No sensors exposed to chips and coolant in the cutting area

Albrecht and Altintas [2] developed an indirect method for measuring cutting forces from the displacements measurements of rotating spindle shaft with capacitance sensors and Kalman filtering processing, considering the machine tool dynamics.

Figure 1.2: Dynamometer

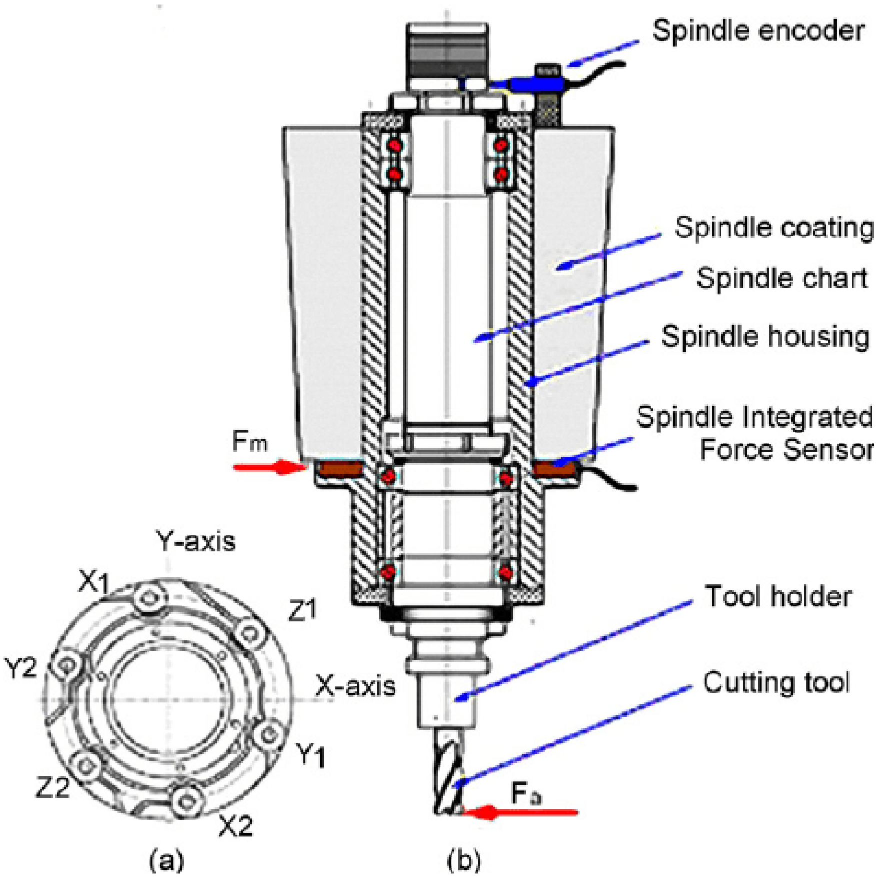


Figure 1.3: Spindle Integrated Force Sensor (SIFS)

An interesting alternative to direct and indirect cutting force measurements is the estimate through a state observer. State observers are systems that provide estimates of the states of a real system from the measurement of input and output. The observer relies on signals coming from sensors, such as triaxial accelerometer and inductive displacement sensors, that can be easily integrated in a commercial electrospindle, thus ensuring an easy industrial diffusion. The advantages related to this approach are:

- Method based on standard built-in spindle sensors
- No devices in contact with chips and coolant
- No modification of the machine tool structure

In order to introduce the principle of operation of the state observer, let's consider the continuous time system (1.1) in state-space form, where x, u and y are the state, input and measurement vectors respectively, while A, B, C and D are the matrices that characterize the system.

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t) \tag{1.1}$$

The state observer (1.2) is designed considering the system (1.1) with the introduction of a corrective term on the measurement residual.

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t))$$

$$\hat{y}(t) = C\hat{x}(t) + Du(t) \tag{1.2}$$

where " ^ " stands for an estimate quantity and L is the gain matrix. Defining the state estimation error as the quantity $e(t) = x(t) - \hat{x}(t)$ and combining the above systems, the dynamic equation of the error can be found as

$$\dot{e}(t) = (A - LC)e(t) \tag{1.3}$$

In order to have a reliable estimation, matrix L should be set to stabilize $(A - LC)$; changing the gains it is possible to rely more on the system (low L, so low residual correction) or on the measurements (high L, so high residual correction).

One of the most widespread state observers is the Kalman filter. It provides an optimal state estimation for linear systems under the hypothesis of gaussian noise. Optimal estimation means that the use of the Kalman filter allows to obtain gains for matrix L that minimize the state covariance matrix.

1.3 Regenerative Chatter and Stability Lobes Diagram

The growing demand for high quality mechanical components leads to search for stable machining processes in which the related vibrations are increasingly limited. Indeed, in a generic cutting process it is possible to observe three types of vibrations:

- Free vibrations
- Forced vibrations
- Self-excited vibrations

Free vibrations are due to a disturbance that perturbs the dynamical system from its equilibrium state and they are damped over time with an oscillatory motion.

In case of a continuous perturbation that changes over time, the excited structure vibrates according to its frequency response function, causing the so-called forced vibrations.

Finally, the self-excited vibrations occur when, from the interaction between workpiece and tool during the cutting process, there is extraction of energy which is stored inside the structure.

The last two types of vibrations may lead to an excessive amplitude of vibration that causes the instability of the process. Indeed, for the case of forced vibration could happen that a component of the force excites the structure in correspondence of a resonance frequency, that implies very high oscillations. Instead the energy stored inside the structure due to the self-excited vibrations, theoretically, causes vibrations with indefinite amplitudes; this phenomenon is the most important one in the study of the dynamic instability during machining and it is known as Chatter. Under these machining conditions the surface quality of the piece is degraded as can be seen in figure 1.4.

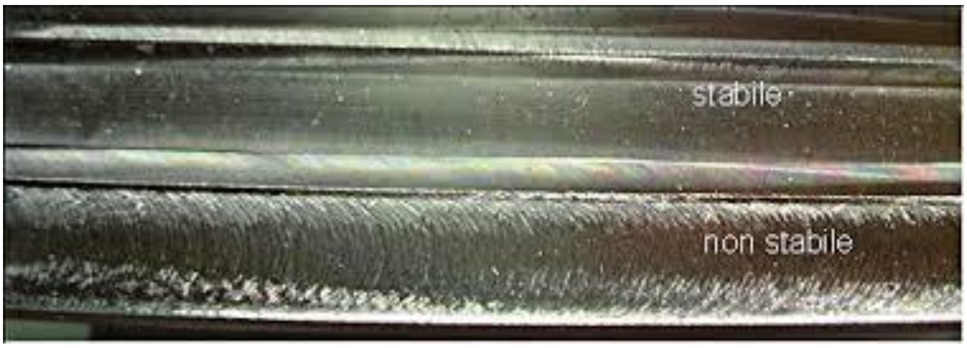


Figure 1.4: Stable vs. Unstable machined surfaces

Figure 1.5 shows schematically what is explained above.

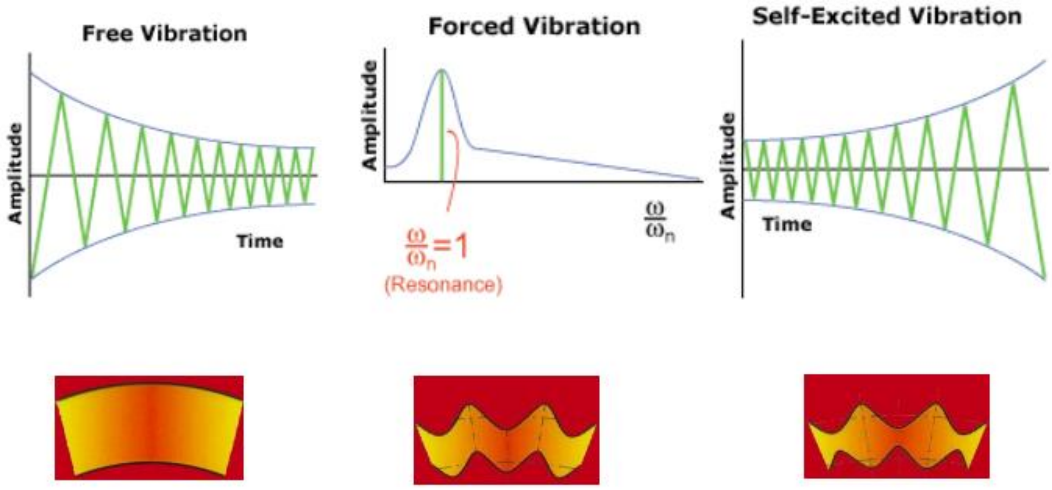


Figure 1.5: Vibrations scheme

Of particular interest is the study of the chatter phenomenon in order to prevent unstable machining conditions. Working in unstable conditions has the following implications:

- High tool wear that could lead to tool breakage
- Damage to spindle bearings
- Poor dimensional accuracy and surface finish of the piece
- High noises (annoying sound) due to high vibrations
- High costs related to material scraps, reworks if the piece is recoverable, etc.

In the present thesis the term chatter identifies the regenerative one, that is the principal among other variants as mode coupling, friction etc. Regenerative chatter occurs when the oscillation of the depth of cut in one pass of the tool leaves waves on the machined surface (called mark or track) that are regenerated in the subsequent passes of cut. This oscillatory motion generates a modulation of the chip thickness (and therefore of cutting forces) that causes an indefinite increase of the amplitude of vibrations; in reality, after a certain value of amplitude, the loss-of-contact between tool and workpiece verifies, determining a sort of limit cycle characterized by subsequent engagements and loss-of-contacts.

In milling the tool has multiple cutting edges and rotates; this means that the cutting forces are intermittent and time-variant quantities. In reason of that the system is described by periodic *Delay Differential Equations* (DDE) which are of fundamental importance in the study of the stability of the process. Various theories have been developed over the years for the study of the DDEs. A very important contribution was provided in 1995 by Altintas and Budak [3]; it is a method for the analytical prediction of stability lobes in milling that requires transfer functions of the structure at the cutter-workpiece contact zone, static cutting force coefficients, radial immersion and the number of teeth on the cutter. Time varying dynamic cutting force coefficients are approximated by their Fourier series components, arrested at zero order (Zeroth Order Approach, ZOA), and the chatter free axial depth of cuts and spindle speeds are computed directly from the proposed set of linear analytic expressions. This methodology allows to obtain the *Stability Lobes Diagram* (SLD), which is a diagram showing on the abscissas axis the spindle speed and on the ordinates axis the axial depth of cut as in figure 1.6, with a good accuracy in case of tools with high number of teeth and for high radial depth of cut, so that the variation of the forces involved in the process is low.

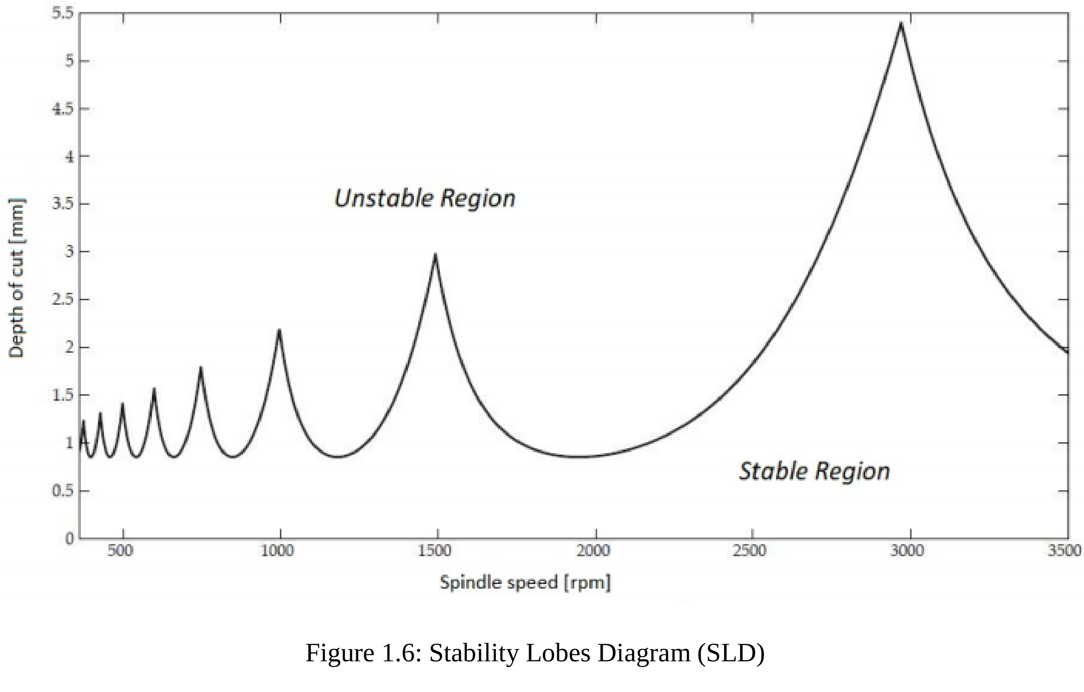


Figure 1.6: Stability Lobes Diagram (SLD)

The above diagram is very useful in order to have a machining process under stable conditions; indeed, by tuning in a proper way the depth of cut and spindle speed parameters by avoiding the unstable region, high quality surface of the piece can be achieved. Moreover, these two parameters are strictly related to productivity and efficiency; to reduce the machining time, according to the SLD, it is possible to select the maximum allowable depth of cut and spindle speed.

1.4 Purpose of the thesis

The importance of monitoring the milling process by the estimation of the cutting forces and tool tip vibrations is due to the growing need in the industrial sector to ensure high quality of the pieces produced, to avoid the breaking of the tools and/or the inserts and to work as much as possible in stable conditions.

The purpose of this thesis is the modelling of the behaviour of the real plant when it is subject to regenerative chatter and the search for implementation methods of a state observer able to estimate cutting forces and tool tip vibrations in milling machining.

Indeed, some cutting force observers have been developed, based on Kalman filter theory, but they are not suitable for working under unstable cutting conditions. One specific reference is the paper of Albertelli et al. [4] that treats about the implementation of a Kalman filter based on the machine dynamic only, to estimate cutting forces and tool tip vibrations. It is shown that the methodology implemented works very well only under stable cutting conditions. For the case of unstable cut, instead, the results are no longer satisfactory; the impossibility to estimate the quantities of interest is due to the not modelled process mechanics. Indeed, process mechanics and machine dynamics, due to the regenerative delay, are mutually coupled, while the implemented observer refers to a dynamical system excited by exogenous cutting forces.

Hence, in order to design an observer that operates even in case of regenerative chatter, in the present discussion the model of the system considers the combination of machine tool dynamics and cutting process. Basically, the process modelling consists in defining the cutting forces as a function of the chip thickness, which depends on the nominal and regenerative contributions. The regenerative one is the most significant in case of chatter and it is expressed in function of the actual and delayed tool tip displacements, therefore it follows that the dynamic model is described by periodic DDEs.

An important contribution in the description of the process mechanics during unstable cutting is provided in this thesis. It consists in the modelling of the detachment phenomenon that occurs between tool and workpiece when high amplitudes of vibrations are reached. This point is an improvement of a previous thesis done by Marzatico [5], in which the detachment of the tool from the workpiece has not been modeled.

The modelled plant turns out to be a hybrid system, namely a system characterized by two modes that are alternatively involved when subject to unstable machining: they correspond to the detachment and engagement conditions as better explained in the next parts. A suitable state estimator for this kind of systems could be a switching observer, that allows to estimate the state of the system in the different modes.

The steps of figure 1.7 describes in a schematic way what explained above.

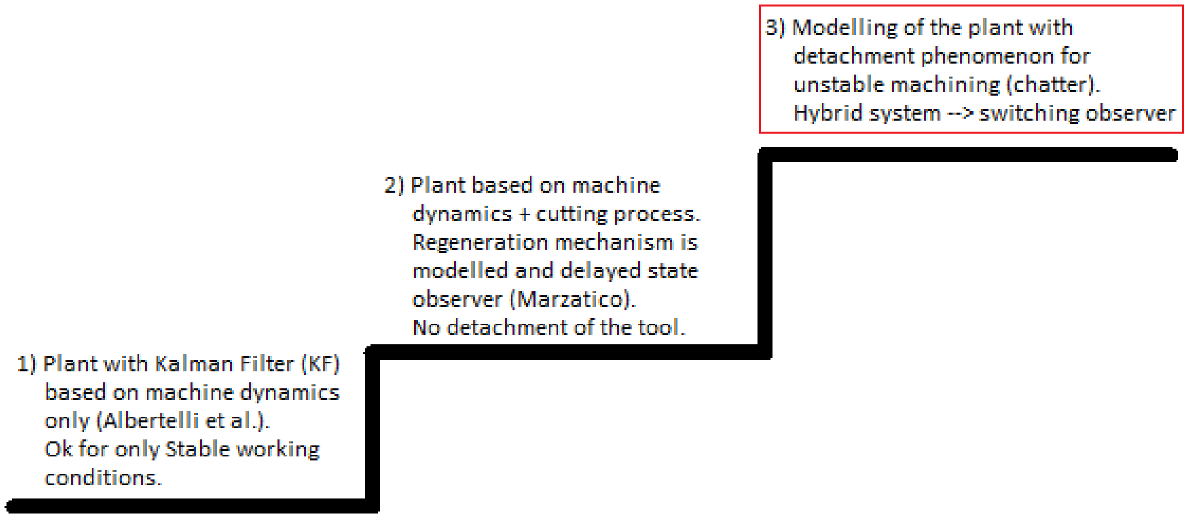


Figure 1.7: Evolution of the thesis

1.5 State observers for state delay systems, state of the art

Different approaches are available in literature for design of observers in time delay systems:

- Spectral decomposition-based
- Prediction-based
- Lyapunov-based
- Continuous pole placement
- Lambert W function-based

However, very few physical applications are present.

Spectral decomposition approaches (infinitesimal generator IG and solution operator SO) are based on the transformation of a finite space problem of DDEs in an infinite dimension problem of abstract ODEs. These approaches allow the introduction of multiple and dependent delays, although the computation of the eigenvalues is a limiting operation.

With Finite Spectral Assignment FSA (prediction-based) it is possible to transform an infinite dimension delayed system in an equivalent finite dimension one and to design a linear controller based on ODEs. Jankovic [6] applied the method to design an observer for an air supply system in a diesel engine with an exhaust gas recirculation valve (EGR) and a turbo compressor with a variable geometry turbine (VGT). The model is characterized by a time delay equal to the interval from the air intake into the cylinder to the exhaust of the combusted mixture.

Lyapunov-based approaches refer to Linear Matrix Inequality (LMI) or Algebraic Riccati Equation (ARE) [7] solutions for finding the observer gains. Particularly the LMI solution ensures the stability and robustness of the designed system, suitable to be described by switching observers [8].

Continuous pole placing is based on a numerical algorithm [9] that modifies the gain matrix when an unstable eigenvalue is found; consequently the eigenvalues change and the process iterates.

Lambert W function approach is an algorithm able to perform eigenvalue placement at desired locations on delayed systems to ensure stability and desired performances with some limitations [10].

2. Plant - machine and milling dynamics modelling

2.1 Dynamic milling model

The dynamic milling model is explained considering the milling cutters to have two orthogonal degrees of freedom in directions X and Y according to figure 2.1. The resulting dynamical system is described by two decoupled differential equations as a combination of mass (m), damping (c) and spring (k) elements:

$$m_x\ddot{x} + c_x\dot{x} + k_x x = F_x$$
$$m_y\ddot{y} + c_y\dot{y} + k_y y = F_y \quad (2.1)$$

Rearranging the terms:

$$\begin{Bmatrix}\ddot{x}\\ \ddot{y}\end{Bmatrix} + \begin{bmatrix}2\xi\omega_x & 0\\ 0 & 2\xi\omega_y\end{bmatrix}\begin{Bmatrix}\dot{x}\\ \dot{y}\end{Bmatrix} + \begin{bmatrix}\omega_x^2 & 0\\ 0 & \omega_y^2\end{bmatrix}\begin{Bmatrix}x\\ y\end{Bmatrix} = \begin{bmatrix}\overline{F_x}\\ \overline{F_y}\end{bmatrix} \quad (2.2)$$

where $\xi = \frac{c}{2\omega m}$ is the damping ratio, $\omega = \sqrt{\frac{k}{m}}$ is the natural frequency and $\bar{F} = \frac{F}{m}$ is the cutting force, all in the considered directions.

The cutter is assumed to have N number of teeth with zero helix angle. The cutting forces excite the structure in X and Y directions, causing x and y dynamic displacements respectively. The total force in the considered direction is the sum of the single forces acting on each tooth. The dynamic displacements are related to rotating tooth number (j) in the radial or chip thickness direction with coordinate transformation

$v_j = -x\ sin\varphi_j - y\ cos\varphi_j$, where φ_j is the instantaneous angular immersion of tooth j and defined as $\varphi_j(t) = \left(\frac{2\pi\Omega}{60}\right)t + j\frac{2\pi}{N}$, where t denotes the dependence with time and Ω is the spindle speed in [rpm].

For the sake of simplicity in the following sections the dependence on time is not reported, but obviously $\varphi_j = \varphi_j(t)$.

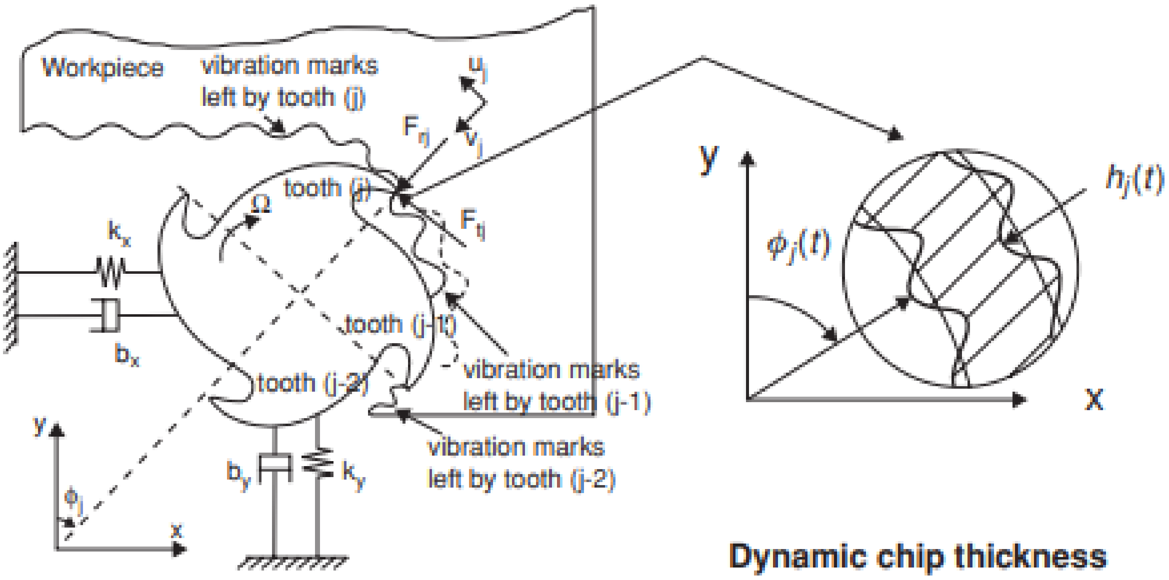


Figure 2.1: Dynamic milling model (2 d.o.f.)

2.2 Chip thickness and cutting forces

The chip thickness consists of two contributions: a static part, which is due to rigid motion of the tool, and a dynamic one caused by the vibrations of the tool at the present and previous tooth periods.

$$h(\varphi_j) = [st\ \sin\varphi_j + (v_{j,0} - v_j)]g(\varphi_j) \tag{2.3}$$

where s_t is the feed rate per tooth and $v_{j,0}$, v_j are the tool tip dynamic displacements at the previous and actual tooth periods, respectively. $g(\varphi_j)$ is a unit step function which determines whether the tooth is engaged in the workpiece, i.e.:

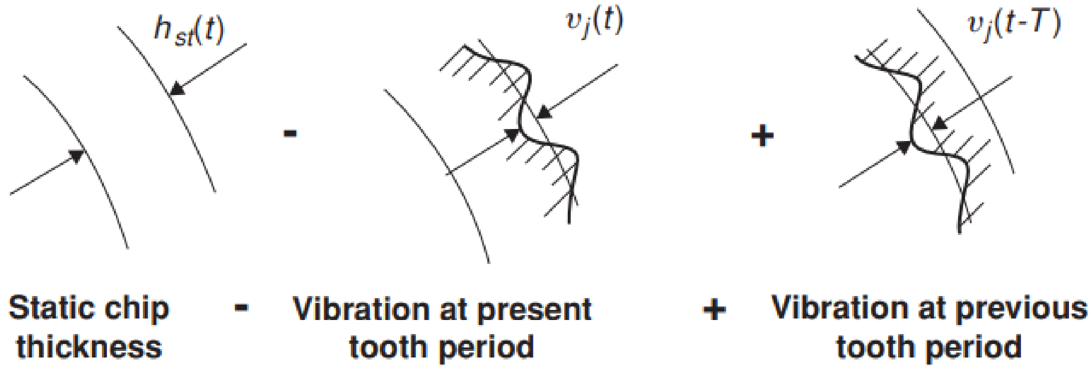
$$\begin{aligned} g(\varphi_j) &= 1 \quad \varphi_{in} < \varphi_j < \varphi_{out} \\ g(\varphi_j) &= 0 \quad \varphi_j > \varphi_{out}\ or\ \varphi_j < \varphi_{in} \end{aligned} \tag{2.4}$$

where φ_{in} and φ_{out} represent the starting and exit immersion angles of the cutter.

Considering the expression of v_j and substituting it in equation 2.3, the chip thickness becomes:

$$h(\varphi_j) = [st\ \sin\varphi_j + \Delta x\ sin\varphi_j + \Delta y\ cos\varphi_j]g(\varphi_j) \tag{2.5}$$

where $\Delta x = x(t) - x(t-\tau)$ and $\Delta y = y(t) - y(t-\tau)$ are the differences between the actual and delayed positions of the tooth passes along x and y directions respectively, with τ that represents the delay between two subsequent teeth passes.



$$h_j(t) = s_j \sin\phi_j(t) - [-x(t)\sin\phi_j(t) - y(t)\cos\phi_j(t)] + [-x(t\text{-}T)\sin\phi_j(t) - y(t\text{-}T)\cos\phi_j(t)]$$

Figure 2.2: Chip thickness contributions

Then the tangential $F_{t,j}$ and radial $F_{r,j}$ cutting forces acting on the j-th tooth are:

$$F_{t,j} = K_t a h(\varphi_j)$$

$$F_{r,j} = k_r F_{t,j} \tag{2.6}$$

where K_t and k_r are constant cutting coefficients, identified performing several cutting tests on the machine tool described in section 2.3 changing the feed per tooth and the cutting velocity, and a is the axial depth of cut.

Projecting the cutting forces in the X and Y directions according to figure 2.1:

$$F_{x,j} = -F_{t,j}\, cos\varphi_j - F_{r,j}\, sin\varphi_j$$

$$F_{y,j} = +F_{t,j}\, sin\varphi_j - F_{r,j}\, cos\varphi_j \tag{2.7}$$

Finally, the total forces acting on the tool are the sum of the cutting forces of equations 2.7:

$$F_x = \sum_{j=0}^{N-1} F_{x,j}$$

$$F_y = \sum_{j=0}^{N-1} F_{y,j} \tag{2.8}$$

Hence, due to the delayed terms of the chip thickness expression, the dynamical system is now described by time varying delay differential equations.

Differently from the paper of Altintas and Budak [3], the cutting forces are here treated without find the matrix of the time varying directional dynamic milling force coefficients $\begin{bmatrix} \alpha_{xx} & \alpha_{xy} \\ \alpha_{yx} & \alpha_{yy} \end{bmatrix}$, further approximated with constant coefficients derived from the application of the Zero-Order Approximation solution, because the aim of this thesis is to describe the detachment of the tool from the workpiece, which verifies during chatter occurrence due to high vibrations. For this reason, in order to impose the condition of the loss-of-contact of the tool from the workpiece, the complete expression of the chip thickness is required; as soon as the chip thickness assumes a negative value, namely the regenerative (dynamic) part becomes greater than the nominal (static) one, it is set equal to zero so that also the cutting forces vanish.

2.3 Machine tool dynamic model

The machine tool dynamic model was obtained through an experimental modal analysis identification on the Mandelli M5 machine present at MUSP lab in Piacenza as stated in [4].

The goal of the experimental modal analysis is to obtain the Frequency Response Function (FRF) and the vibration modes of the system. The experimental session consists of the measure of both the force applied to the tool and the tool tip vibration amplitude. The machine is excited with force pulses applied at the tool tip using different sensorized hammers in order to cover a wide frequency range of excitation, up to 1500 Hz, ensuring to obtain a reliable model.

The schematics of the impact test are highlighted in figure 2.3 for a better understanding of the procedure.

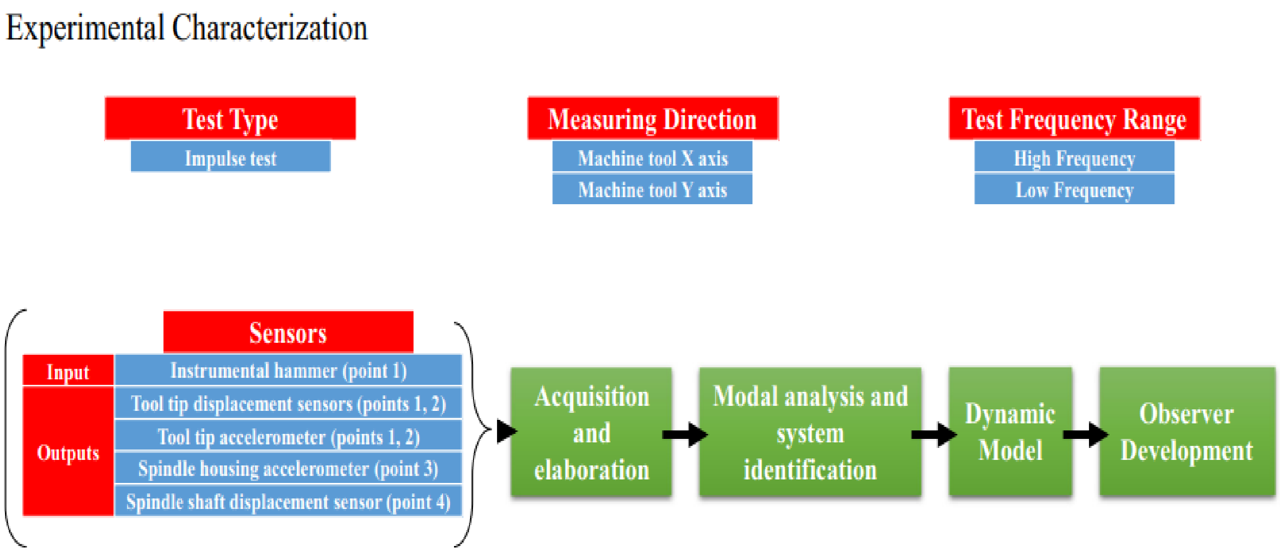


Figure 2.3: Machine tool model identification

The test was carried out considering the following tool, whose parameters are used in the thesis:

- Diameter (D): 80 mm
- Number of teeth (N): 4
- Length (L): 310 mm

For both X and Y machine tool directions, with a sensorized hammer and multiple sensors according to figure 2.4, the following quantities are defined:

- Tool tip force F_c (excitation input)
- Tool vibration at points 1 and 2
- Spindle housing vibration
- Relative radial displacement between spindle shaft and spindle housing

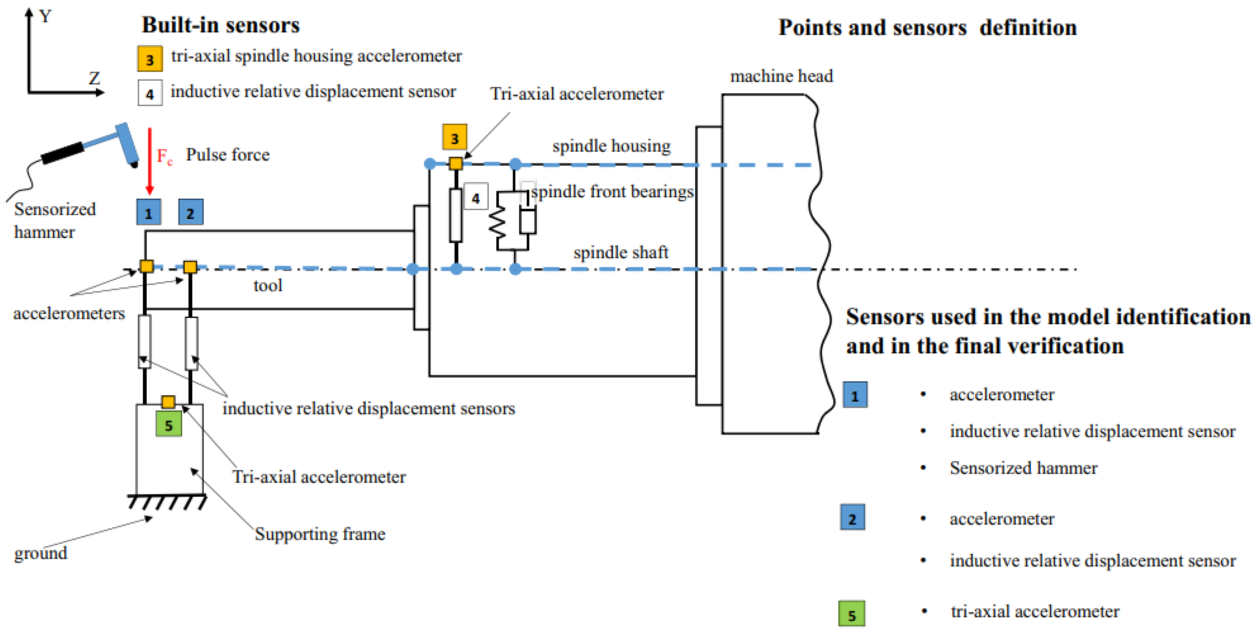


Figure 2.4: Sensors used during the experimental modal analysis

The measured experimental FRFs along both X and Y directions are depicted in figure 2.5. Moreover, the identified FRFs at point 1 (tool tip) are superimposed in the same figure.

In table 2.1, instead, are listed the identified eigenmodes, namely the vibration modes of the structure. As can be seen the total number of identified eigenmodes is $n = 32$, 21 for X direction and 11 for Y direction.

In figure 2.5 it is possible to observe that the low frequency eigenmodes are mainly referred to the machine structure and spindle headstock, while the medium-high frequency eigenmodes are mainly related to the spindle-tool dynamics.

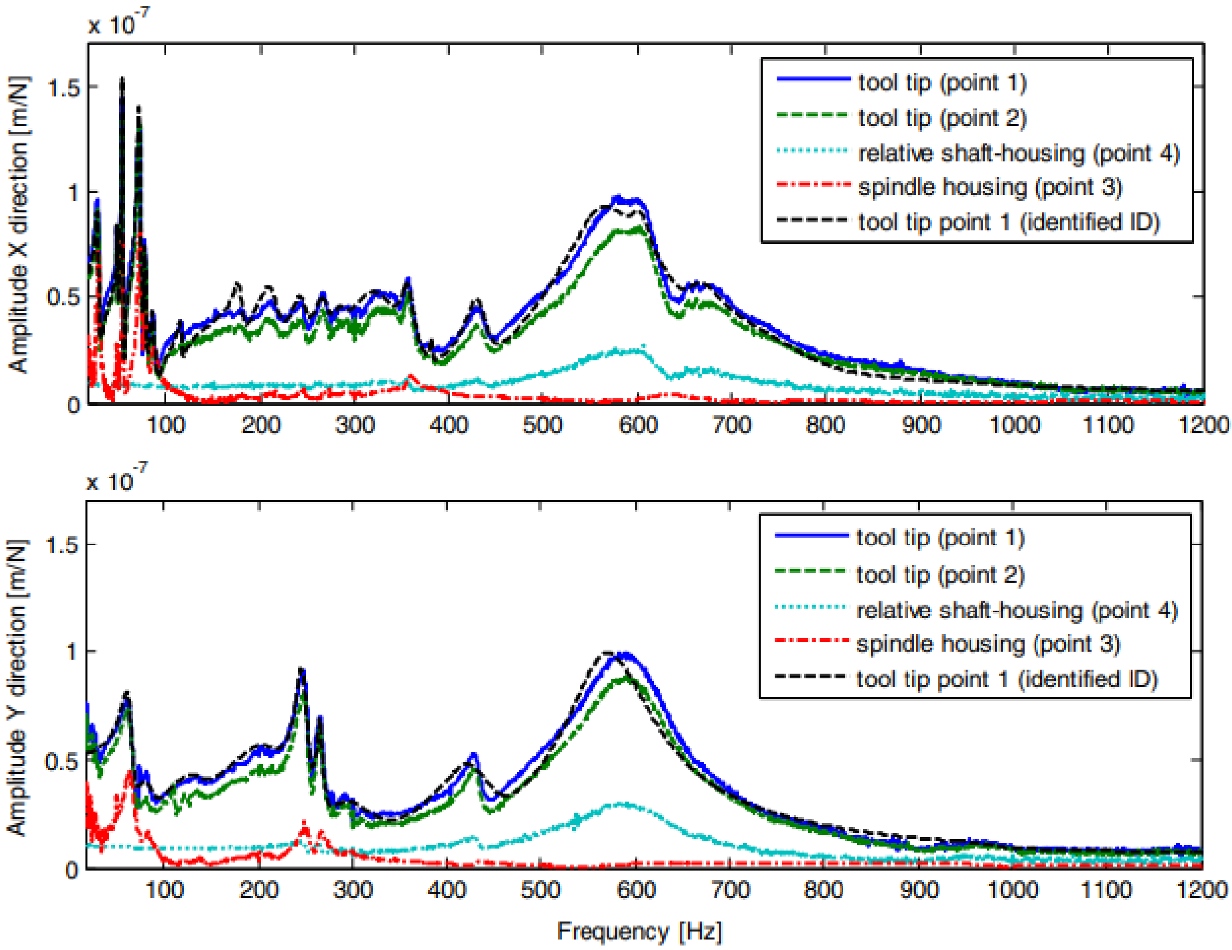


Figure 2.5: Experimental FRFs for X and Y directions and the fitted tool tip dynamic compliances (identified ID)

X direction			X direction			Y direction		
Mode	$\omega_j/2\pi$ [Hz]	ζ_j	Mode	$\omega_j/2\pi$ [Hz]	ζ_j	Mode	$\omega_j/2\pi$ [Hz]	ζ_j
1	28.0	0.069	12	288.0	0.033	1	64.2	0.069
2	48.8	0.006	13	329.7	0.097	2	84.7	0.142
3	54.2	0.008	14	357.8	0.019	3	138.0	0.131
4	72.6	0.027	15	381.9	0.003	4	211.1	0.129
5	78.5	0.013	16	433.0	0.025	5	247.6	0.030
6	86.5	0.035	17	568.2	0.088	6	265.6	0.012
7	116.2	0.015	18	600.8	0.018	7	299.1	0.100
8	179.7	0.025	19	662.9	0.028	8	430.6	0.088
9	214.9	0.054	20	769.6	0.087	9	571.4	0.077
10	245.8	0.032	21	1288.0	0.023	10	960.7	0.031
11	268.9	0.031				11	1214.0	0.026

Table 2.1: Identified eigenmodes for X and Y directions

A high number of vibration modes allows to have a reliable model that can exploit the effect of chatter, which usually verifies in correspondence of high frequencies.

The model obtained by the experimental modal analysis is described by the equations 2.9 and 2.10, that describe the system dynamics and the relationship between the model coordinates and the output respectively.

$$[\boldsymbol{M}s^2 + \boldsymbol{R_d}s + \boldsymbol{K_s}]p(s) = F(s) \tag{2.9}$$

$$y(s) = \boldsymbol{g}p(s) \tag{2.10}$$

where $\boldsymbol{M}$, $\boldsymbol{R_d}$, $\boldsymbol{K_s}$ are mass, damping and stiffness matrices, respectively, $p(s)$ is the model coordinates vector, $F(s)$ is vector of the input forces applied at the tool tip (point 1, fig. 2.4), $y(s)$ is the system output vector that contains the quantities measured by the sensors and $\boldsymbol{g}$ is the output shape matrix.

Performing a modal transformation on the model coordinates, it is possible to get the system modal coordinates $q(s)$:

$$p(s) = \boldsymbol{\Phi}q(s) \tag{2.11}$$

where $\boldsymbol{\Phi}$ is the matrix of the eigenvectors.

The equations 2.9 and 2.10 are then substituted by:

$$[\boldsymbol{I}s^2 + \boldsymbol{\Gamma}s + \boldsymbol{\Omega}^2]q(s) = \boldsymbol{\Phi}^T F(s) \tag{2.12}$$

$$y(s) = \boldsymbol{g\Phi}\text{q(s)} = \boldsymbol{C}'q(s) \tag{2.13}$$

where:

$$\boldsymbol{\Gamma}_{(2nx2n)} = \begin{bmatrix} \Gamma_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Gamma_j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_n \end{bmatrix} = \begin{bmatrix} 2\xi_1\omega_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\xi_1\omega_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\xi_1\omega_1 \end{bmatrix}$$

$$\boldsymbol{\Omega}^2{}_{(2nx2n)} = \begin{bmatrix} \omega_1^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_j^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_n^2 \end{bmatrix}$$

2.4 State Space formulation

Previous works in this field were based on time models characterized by physical behaviour, namely models that considered the actual and previous positions of each insert of the tool in order to describe the regenerative phenomenon.

However, these system models were far from the typical representation of those used for developing state observers, thus a new matrix-based representation of the system, that is most suitable for the state observer implementation, was carried out. This matrix-based representation used in the following is known as state space formulation.

The system 2.12 – 2.13 is converted in state space form in order to design the state observer.

$$\dot{\boldsymbol{x}}(t) = [A]\boldsymbol{x}(t) + [B]\boldsymbol{u}(t)$$

$$y(t) = Cx(t) + Du(t) \tag{2.14}$$

where the matrices A, B, C and D are the same of the article of Albertelli et al. [4], that is the starting point of this research. As already mentioned, this article treats about the development of an observer based on Kalman's theory capable of estimating the in-process cutting forces and tool tip vibrations, properly combining signals from several built-in spindle sensors. The methodology, tested on several stable milling operations, exhibits some limitations in case of unstable cutting. Indeed, in such conditions an accurate estimation of the forces and vibrations is no longer possible, probably since cutting process mechanics and machine dynamics, due to the delay (so called regenerative effect), are mutually coupled, while the observer refers to a dynamic mechanical system excited by exogenous cutting forces. These limitations will be bypassed in the following treatment by developing a milling process-based model that takes into consideration the dynamical coupling between the machine and the process.

Figure 2.6 summarizes the aim of the thesis in a useful way, namely the passage from the system based on machine dynamic only (Albertelli et al.) to that which considers also the cutting process.

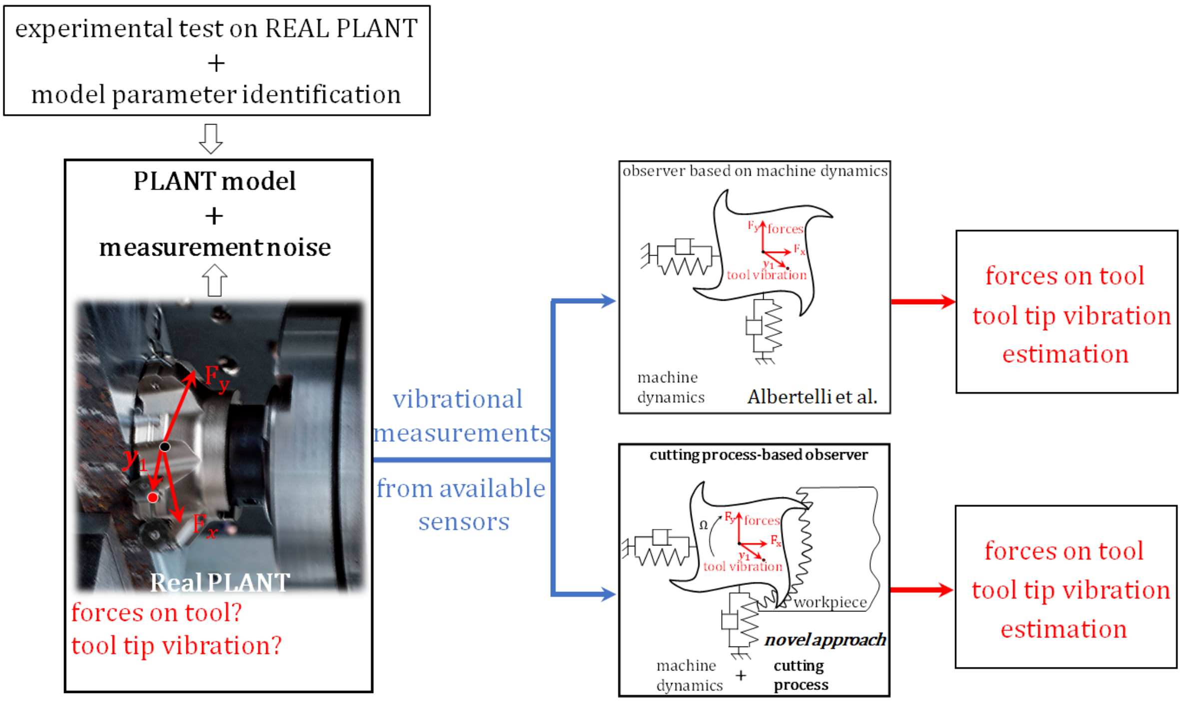


Figure 2.6: Approach description

The real plant model follows the scheme of figure 2.7, where the complex machine dynamics (two main directions X and Y) and the process-machine mutual interaction due to the regenerative phenomenon are depicted.

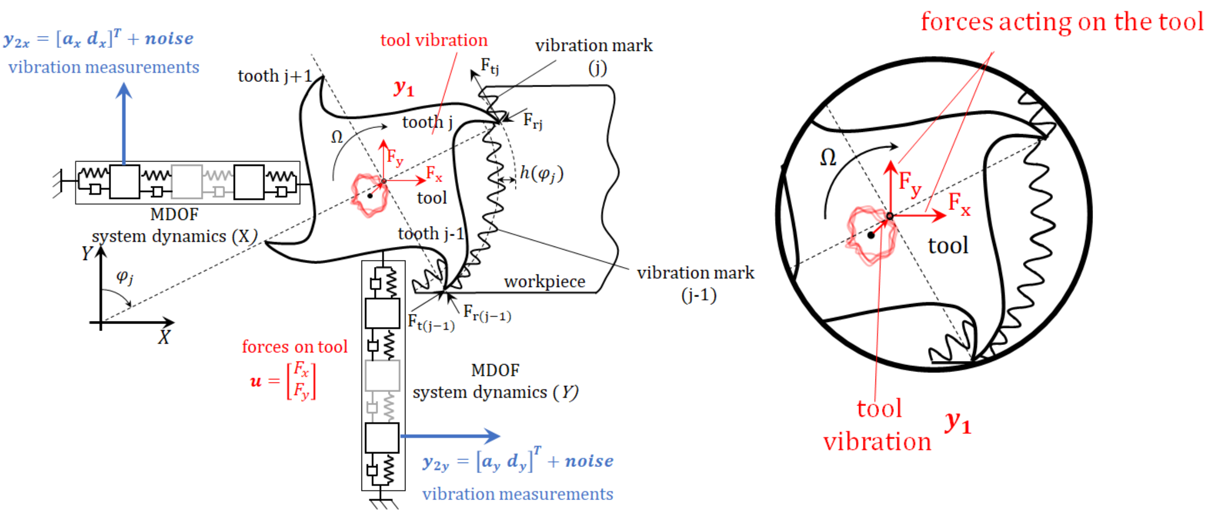


Figure 2.7: Milling dynamics and plant description

According to figure 2.7, the state space formulation 2.14 can be rearranged as in 2.15.

$$\dot{\boldsymbol{x}}(t) = [A]\boldsymbol{x}(t) + [B]\boldsymbol{u}(t)$$
$$\boldsymbol{y_1}(t) = [C_1]\boldsymbol{x}(t)$$
$$\boldsymbol{y_2}(t) = [C_2]\boldsymbol{x}(t) \tag{2.15}$$

where $\boldsymbol{x}$ are the states of the system (machine and milling process), $\boldsymbol{u}$ the system input vector (cutting forces), A is the state matrix of the machine dynamics and B its related input matrix. The output vector $\boldsymbol{y}$ of system 2.14 is here split into two terms: $\boldsymbol{y_1}$ is the tool tip position (not measurable), while $\boldsymbol{y_2}$ is a vector containing all the available measurements from the installed sensors (the spindle shaft-housing relative displacements and the housing accelerations sensors). C_1 and C_2 are the corresponding output matrices.

In the considered reference case, the sensors installed in the spindle can measure both acceleration $\boldsymbol{a}$ and vibrations $\boldsymbol{d}$.

$$\boldsymbol{y_1}(t) = \begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} \qquad \boldsymbol{y_2}(t) = \begin{Bmatrix} \boldsymbol{a}(t) \\ \boldsymbol{d}(t) \end{Bmatrix} + \boldsymbol{noise} \tag{2.16}$$

2.5 Definition of the input vector u(t) and detachment phenomenon

The regenerative contribution is involved in the definition of the $\boldsymbol{u}$ vector, i.e. the vector of the cutting forces.

With reference to equations 2.6 and 2.7, the input vector $\boldsymbol{u}$ of the system 2.15 can be characterized as follows:

$$\boldsymbol{u}(t) = \begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \begin{Bmatrix} \sum_{j=0}^{N-1} F_{x,j} \\ \sum_{j=0}^{N-1} F_{y,j} \end{Bmatrix} = \begin{bmatrix} \sum_{j=0}^{N-1} -F_{t,j}\, cos\varphi_j - F_{r,j}\, sin\varphi_j \\ \sum_{j=0}^{N-1} F_{t,j}\, sin\varphi_j - F_{r,j}\, cos\varphi_j \end{bmatrix} \tag{2.17}$$

$F_{tj} = K_t a h(\varphi_j)$ and $F_{rj} = k_r F_{tj}$ are the tangential and radial forces acting on the jth tooth respectively, as already defined in section 2.2.

Substituting F_{tj} and F_{rj} expressions in equation 2.17 gives:

$$\boldsymbol{u}(t) = \sum_{j=0}^{N-1} K_t a h(\varphi_j) \begin{bmatrix} -cos\varphi_j - k_r\, sin\varphi_j \\ sin\varphi_j - k_r\, cos\varphi_j \end{bmatrix} \tag{2.18}$$

For the sake of comprehension, some contents of section 2.2 will be briefly reported again.

The chip thickness $h(\varphi_j)$ consists of two contributions: a nominal (static) component due to the rigid motion of the tool and a regenerative (dynamic) one due to the tool tip vibrations.

$h_0(\varphi_j) = s_t\, sin\varphi_j$ nominal chip thickness

$h_r(\varphi_j) = \Delta x\, sin\varphi_j + \Delta y\, cos\varphi_j$ regenerative chip thickness

Thus, the expression of the chip thickness for the jth tooth results:

$$h(\varphi_j) = g(\varphi_j)\left[s_t\, sin(\varphi_j) + (x(t) - x(t-\tau))\, sin(\varphi_j) + (y(t) - y(t-\tau)) cos(\varphi_j)\right]$$

where $g(\varphi_j)$ is a unit step function as stated in equation 2.4.

From an expanded point of view, the chip thickness can be depicted as:

$$\boldsymbol{h}(t) = \begin{bmatrix} g(\varphi_1) & 0 & 0 \\ 0 & g(\varphi_j) & 0 \\ 0 & 0 & g(\varphi_N) \end{bmatrix} \left\{ s_t \begin{bmatrix} sin(\varphi_1) \\ sin(\varphi_j) \\ sin(\varphi_N) \end{bmatrix} + \begin{bmatrix} sin(\varphi_1) & cos(\varphi_1) \\ sin(\varphi_j) & cos(\varphi_j) \\ sin(\varphi_N) & cos(\varphi_N) \end{bmatrix} \left[[C_1][\boldsymbol{x}(t) - \boldsymbol{x}(t-\tau)]\right] \right\}$$

(2.19)

For the sake of simplicity, equation 2.19 can be identified with the following notation:

$$\boldsymbol{h}(t) = [\boldsymbol{G}(t)]\{\boldsymbol{H}(t)\} \tag{2.20}$$

where $[\boldsymbol{G}(t)] \in \mathbb{R}^{nxn} \mid \forall i, j \in \{1,2,\ldots,n\}, i \neq j : g_{ij} = 0$ and $\{\boldsymbol{H}(t)\} \in \mathbb{R}^{nx1}$

At this point, in order to describe the real behaviour of the plant when subject to chatter, two conditions must be imposed on $\boldsymbol{G}$ and $\boldsymbol{h}$.

- $[\boldsymbol{G}(t)] \mid \forall i, j \in \{1,2,\ldots,n\}, i = j : \; g_{ij} = 1 \Leftrightarrow \varphi_{in} < \varphi_j < \varphi_{out} \; \wedge$
  $g_{ij} = 0 \Leftrightarrow \varphi_j < \varphi_{in} \vee \varphi_j > \varphi_{out}$

Subsequently, a condition must be imposed also on $\boldsymbol{h}(t)$ in order to model the unstable cutting as a result of chatter. Under this machining mechanism, both cutting forces and vibrations continue to grow until the tool jumps out of the cut. The consequence of that is the vanishing of the cutting forces and a free vibration till the tool re-enters the cut. During the loss-of-contact between the tool and the workpiece, the dynamic chip thickness, that is determined by the displacements of the tool at the actual time and a previous one due to the time delay, is negative: indeed the actual tooth that should be engaged does not cut or it cuts a part of material inferior to that cut in the previous pass. Therefore, the negative value of the chip thickness will be set to zero, so that the cutting forces, which depend on it, will vanish too

- $\boldsymbol{h}(t) \mid \forall i \in \{1,2,\ldots,n\}, h_i < 0 \Rightarrow h_i = 0$

Hence the input vector $\boldsymbol{u}(t)$ expanded assumes the form:

$$\boldsymbol{U}(t) = \begin{bmatrix} F_{x1} & F_{xj} & F_{xN} \\ F_{y1} & F_{yj} & F_{yN} \end{bmatrix} =$$

$$K_t a \begin{bmatrix} -cos\varphi_1 - k_r\, sin\varphi_1 & -cos\varphi_j - k_r\, sin\varphi_j & -cos\varphi_N - k_r\, sin\varphi_N \\ sin\varphi_1 - k_r\, cos\varphi_1 & sin\varphi_j - k_r\, cos\varphi_j & sin\varphi_N - k_r\, cos\varphi_N \end{bmatrix} \begin{bmatrix} h(\varphi_1) & 0 & 0 \\ 0 & h(\varphi_j) & 0 \\ 0 & 0 & h(\varphi_N) \end{bmatrix}$$

(2.21)

Finally the input vector is computed as: $\boldsymbol{u}(t) = \sum_{j=0}^{N} \begin{Bmatrix} F_{xj} \\ F_{yj} \end{Bmatrix} = \sum_{j=0}^{N} \begin{Bmatrix} row_1(\boldsymbol{U}(t)) \\ row_2(\boldsymbol{U}(t)) \end{Bmatrix}$ (2.22)

The description of the detachment of the tool from the workpiece in case of regenerative chatter is an improvement of the plant model implemented in [5] that considers as hypothesis no tool detachment; therefore, in that model, when the regenerative effect starts to affect the system, the forces and the tool tip displacements diverge to infinity.

Figures 2.8 and 2.9 describe the cutting forces trends of the plant modelled without tool detachment and the one that considers it, respectively.

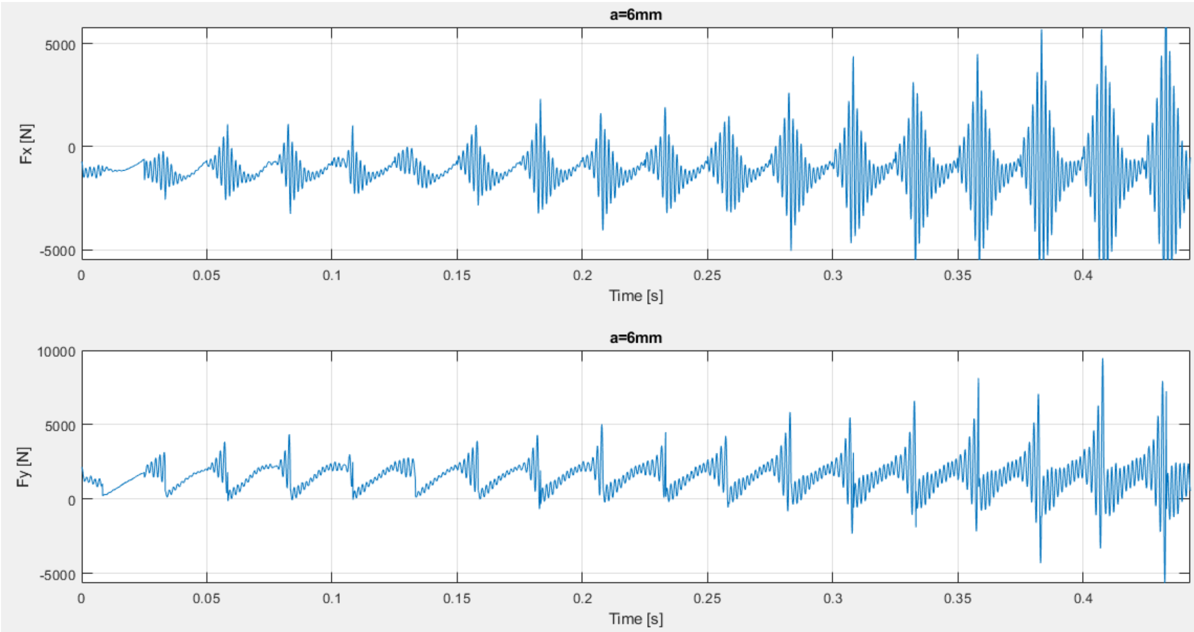


Figure 2.8: Cutting forces with regenerative chatter and without tool detachment

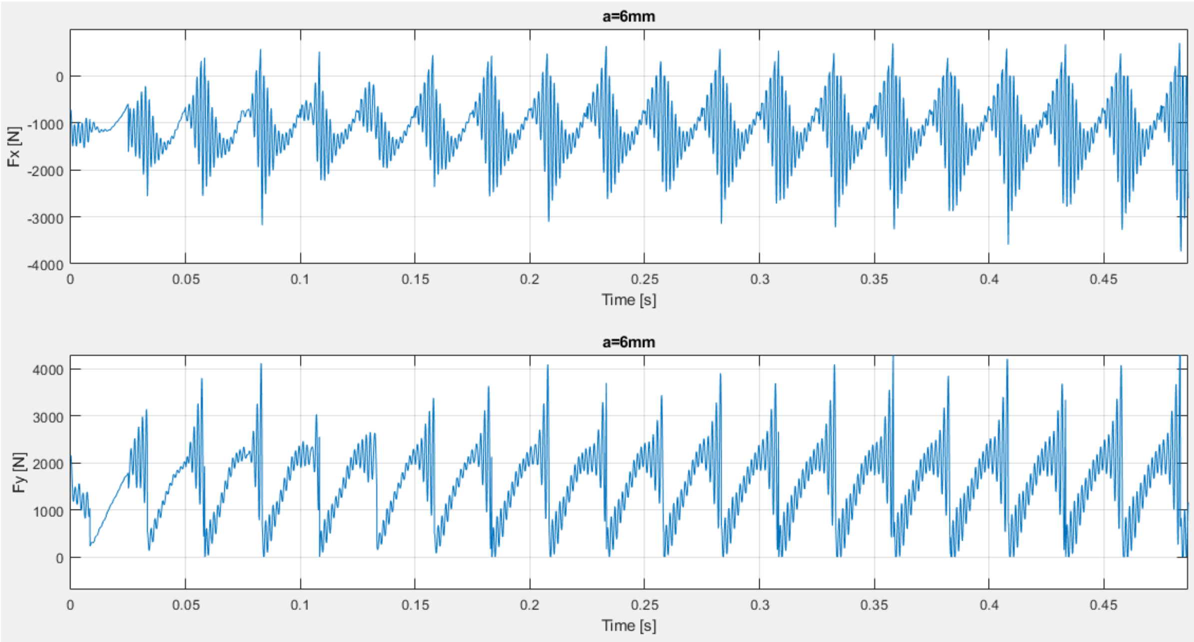


Figure 2.9: Cutting forces with regenerative chatter and with tool detachment

Figure 2.10 clearly shows the differences between the model that doesn't consider the detachment of the tool (blue trends) and the one that considers it (red trends).

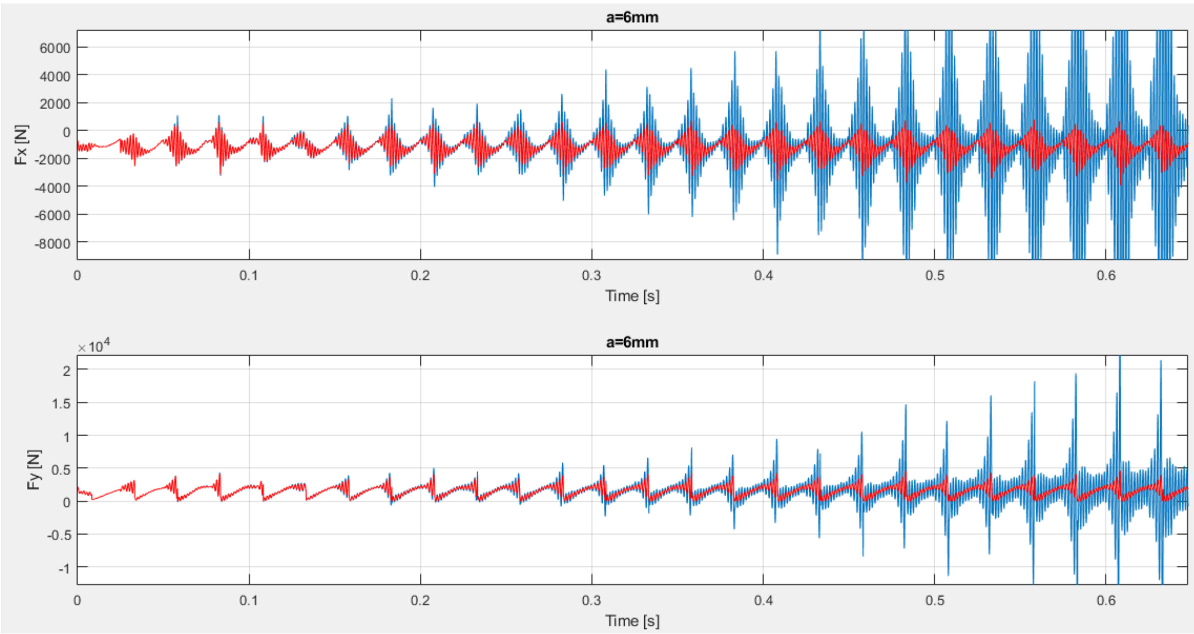


Figure 2.10: Overlap of cutting forces with regenerative chatter with and without tool detachment

2.6 Plant results

This section presents the evolution of the cutting forces and tool tip vibrations in the case of both stable and unstable milling machining.

The configuration adopted for both the cases is characterized by the parameters listed in table 2.2.

Parameter	**Symbol**	**Value**
Tool diameter	D	80 mm
Number of teeth	N	4 [-]
Entrance angle	φ_{in}	0°
Exit angle	φ_{out}	120°
Spindle speed	Ω	600 rpm
Feed rate per tooth	s_t	$0.2\ \frac{mm}{tooth}$
Tangential cutting coeff.	K_t	$1800\ \frac{N}{mm^2}$
Radial non dim. ratio	k_r	0.33 [-]
Axial depth of cut	a	3 mm (stable) 6 mm (unstable)

Table 2.2: Tool and cutting parameters

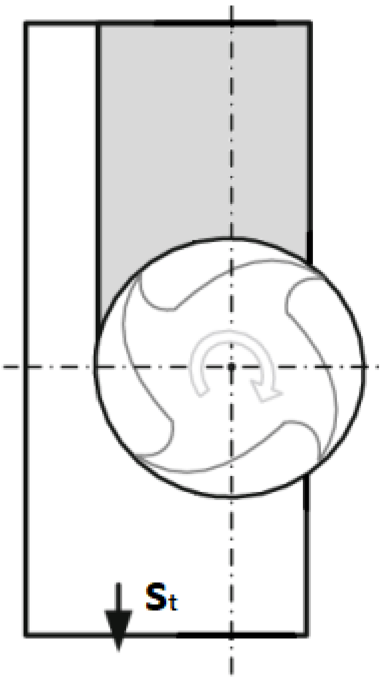


Figure 2.11: Milling machining with 75% radial immersion

2.6.1 Stable cut

The stable cut is achieved by selecting an axial depth of cut equal to 3mm.

Cutting force X direction.

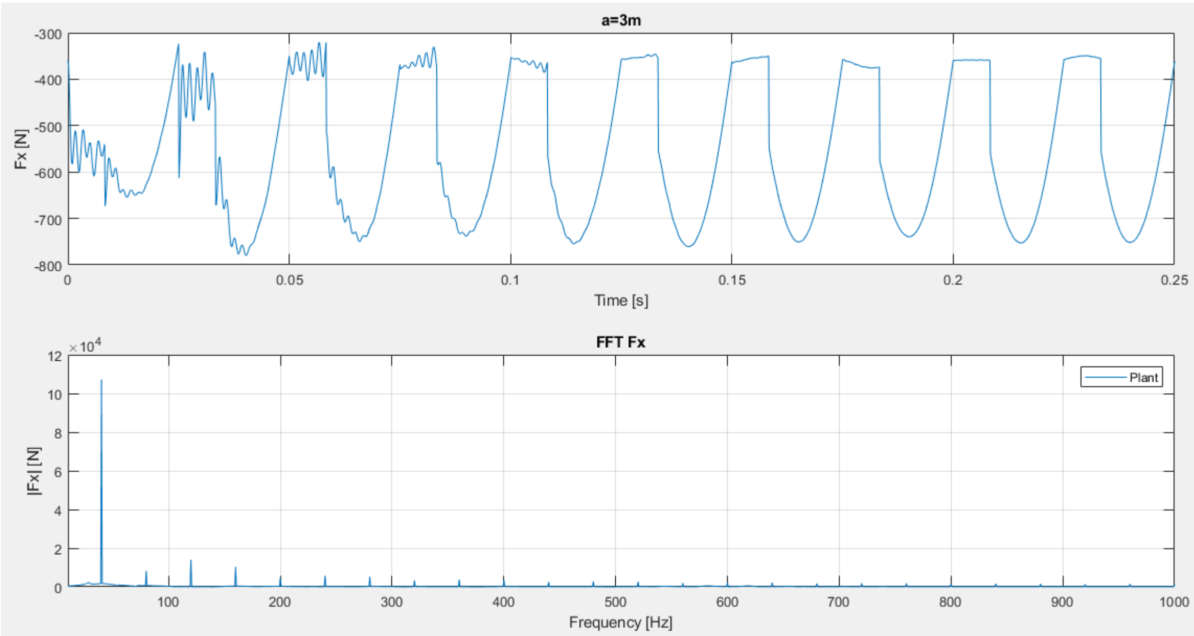


Figure 2.12: Cutting force X direction (stable cut)

Cutting force Y direction.

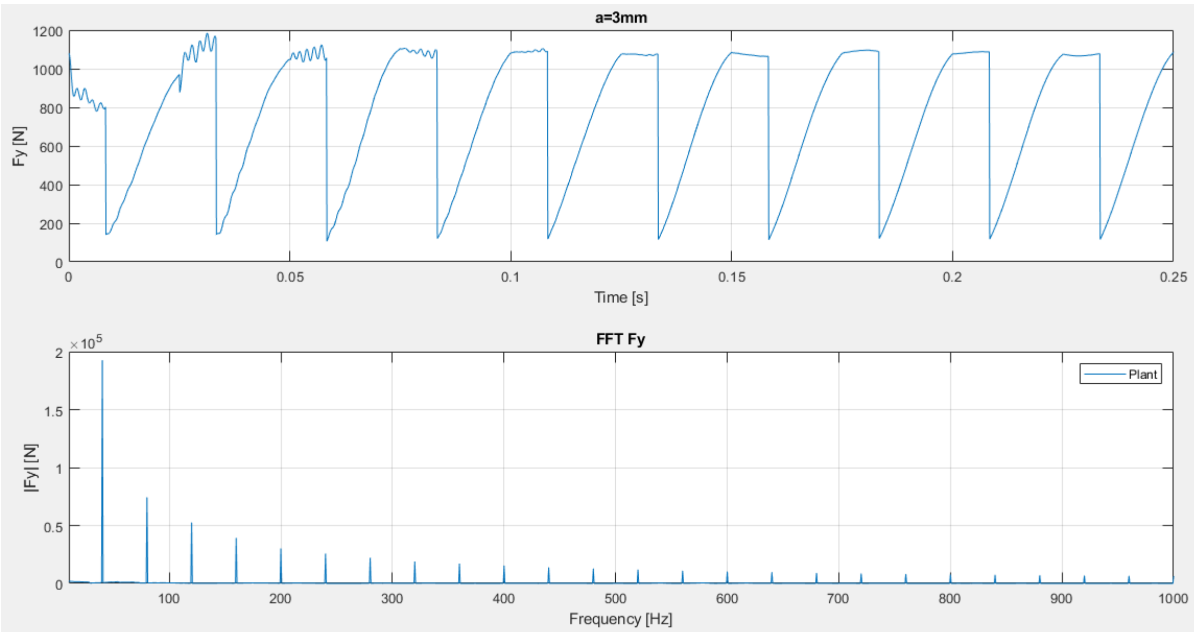


Figure 2.13: Cutting force Y direction (stable cut)

Tool tip vibration X direction.

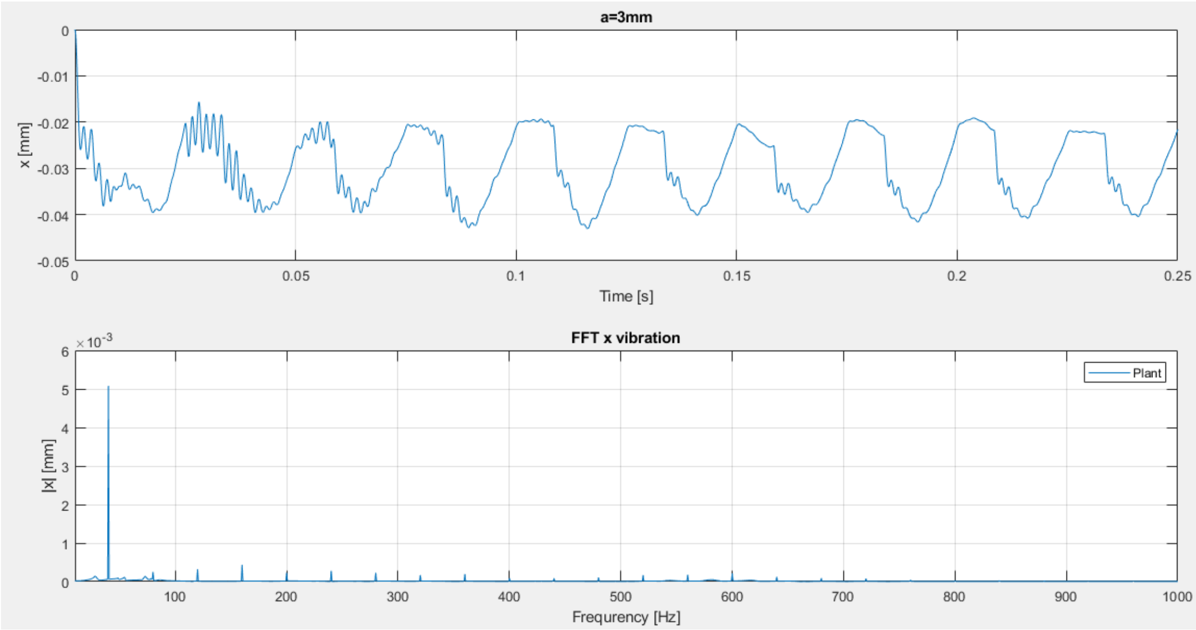


Figure 2.14: Tool tip vibration X direction (stable cut)

Tool tip vibration Y direction.

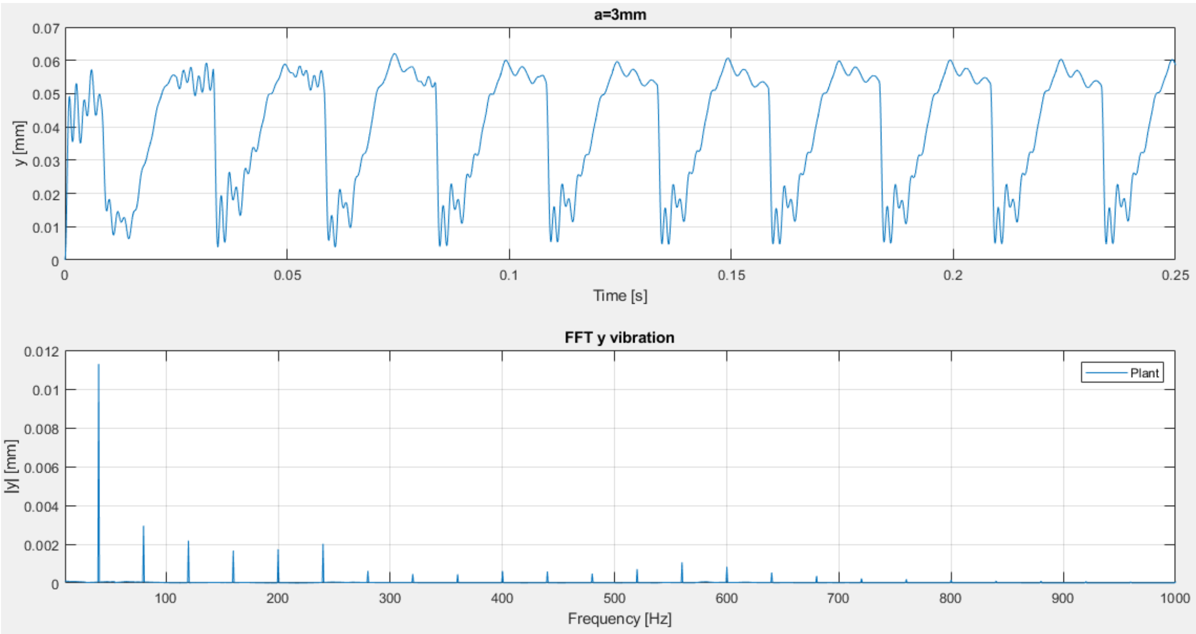


Figure 2.15: Tool tip vibration Y direction (stable cut)

2.6.2 Unstable cut

The unstable cut is achieved by selecting an axial depth of cut equal to 6mm. In this condition the chatter phenomenon with the detachment of the tool is involved.

Cutting force X direction.

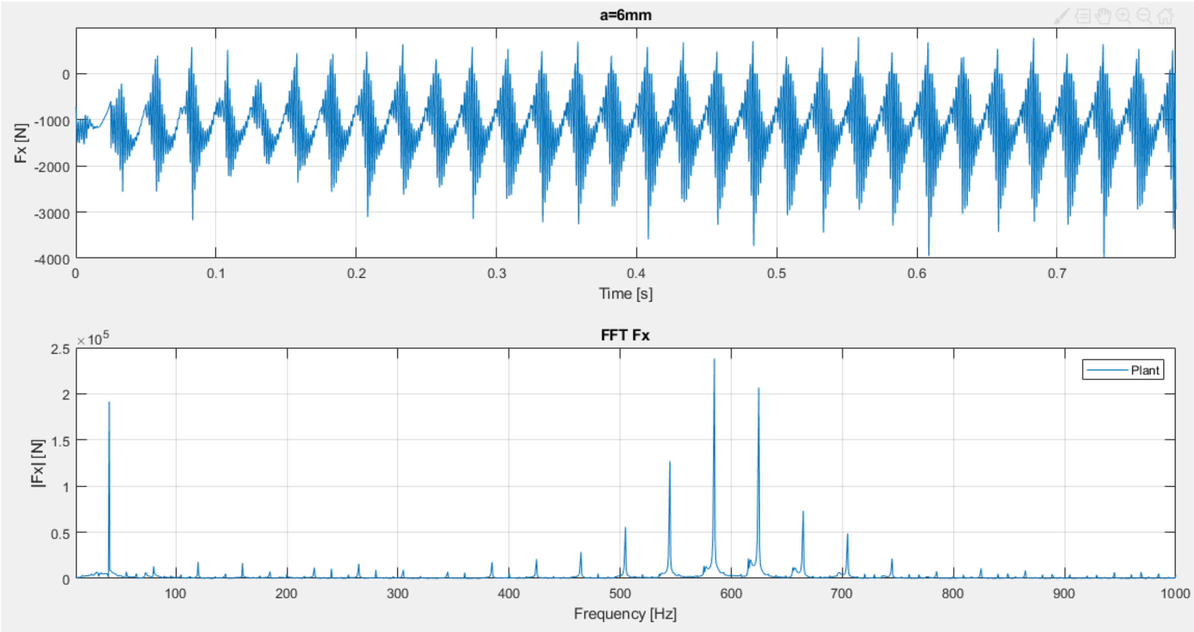


Figure 2.16: Cutting force X direction (unstable cut – detachment)

Cutting force Y direction.

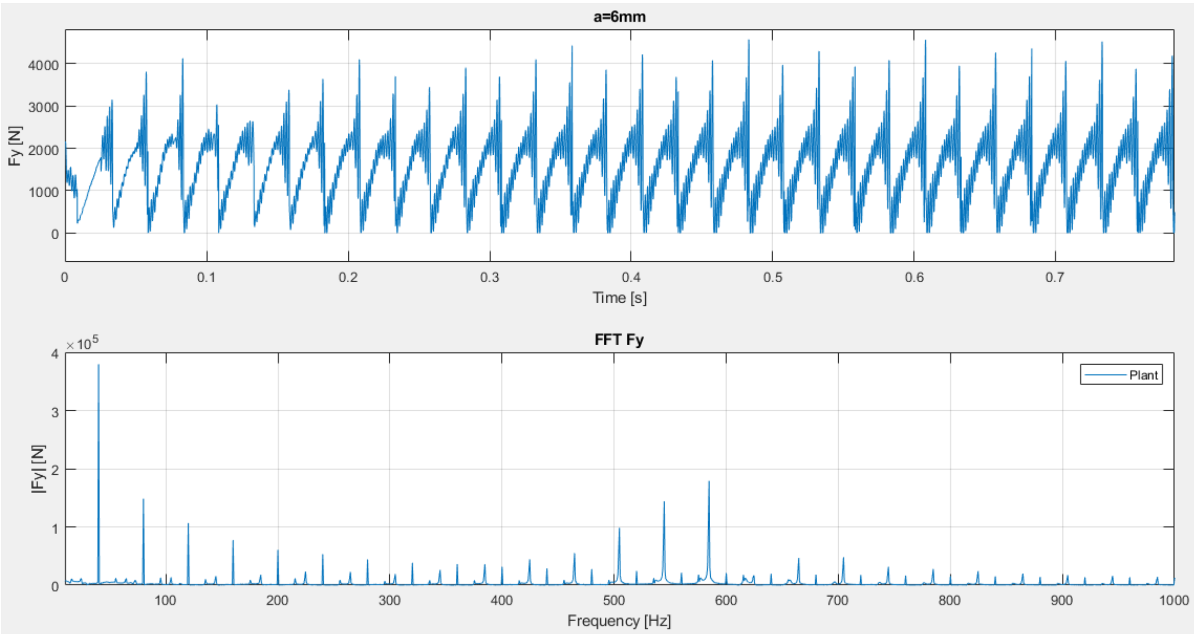


Figure 2.17: Cutting force Y direction (unstable cut – detachment)

Tool tip vibration X direction.

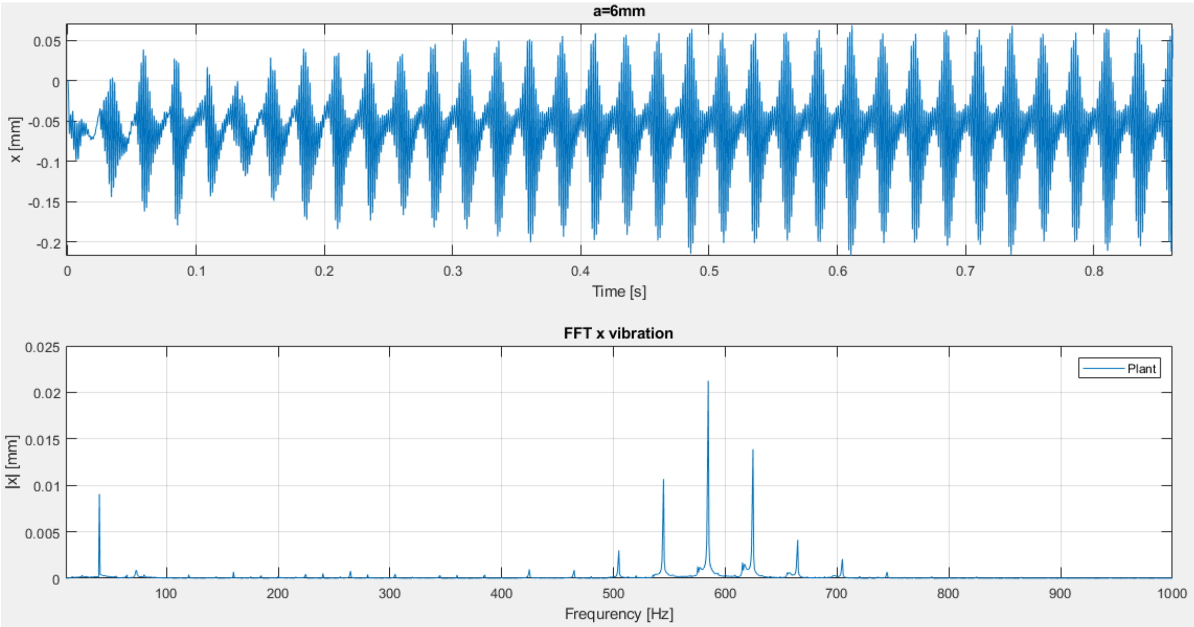


Figure 2.18: Tool tip vibration X direction (unstable cut – detachment)

Tool tip vibration Y direction.

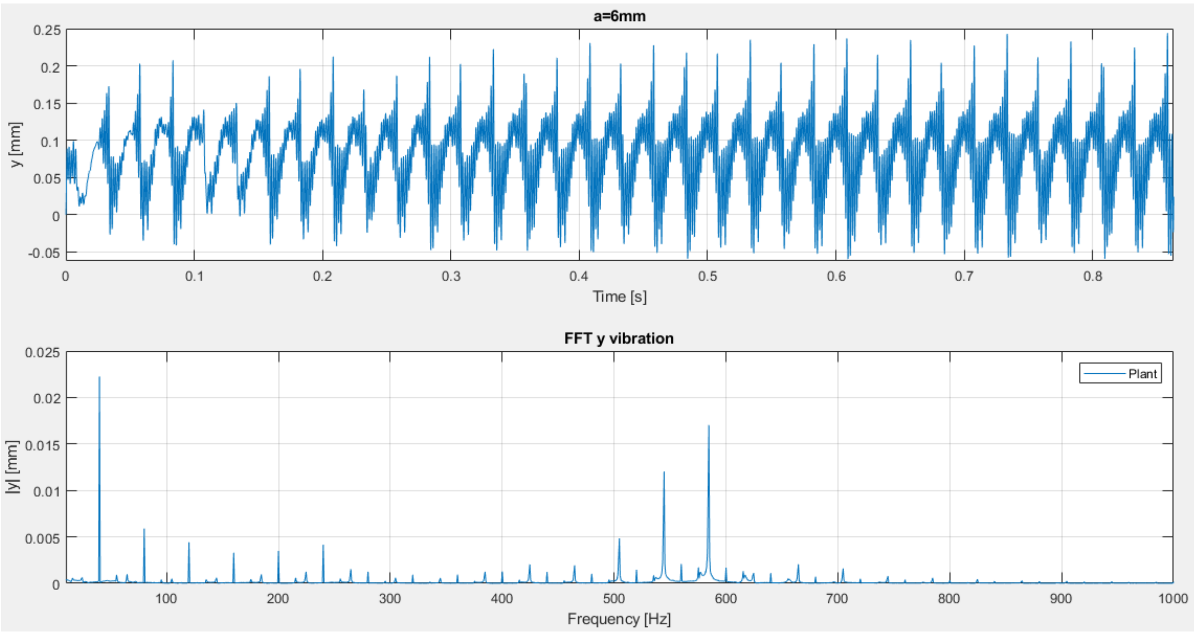


Figure 2.19: Tool tip vibration Y direction (unstable cut – detachment)

3. State observers

The aim of this chapter is to explain the different suitable observers to estimate the cutting forces and tool tip vibrations for the plant under examination.

Particularly a Kalman filter and a state observer for Hybrid System will be considered.

3.1 State observer implementation

In order to perform both input (cutting forces) and state (tooltip displacements) estimation, the system is expanded considering the input as an additional state. The expanded system state vector $\boldsymbol{x_e}(t)$ assumes the form:

$$\boldsymbol{x_e}(t) = \begin{Bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{F}(t) \end{Bmatrix} \tag{3.1}$$

Furthermore, the process noise w and the noise on the measurements $\boldsymbol{z}_1$, $\boldsymbol{z}_2$ are introduced.

The expression for the expanded system results:

$$\begin{aligned} \dot{\boldsymbol{x}}_{\boldsymbol{e}}(t) &= [A_e]\boldsymbol{x_e}(t) + [T]w \\ \boldsymbol{y_1}(t) &= \big[[C_1]\ [0]\big]\boldsymbol{x_e}(t) + \boldsymbol{z}_1 \\ \boldsymbol{y_2}(t) &= \big[[C_2]\ [0]\big]\boldsymbol{x_e}(t) + \boldsymbol{z}_2 \end{aligned} \tag{3.2}$$

where:

$[A_e] = \begin{bmatrix} [A] & [B] \\ [0] & [0] \end{bmatrix}$ and $[T]$ is the system noise matrix.

According to the theory of the state observer, the estimation of the states of the system is updated thanks to the available measurements $\boldsymbol{y_2}$. More specifically, the updating term $(\boldsymbol{y_2}(t) - \widehat{\boldsymbol{y_2}}(t))$ is weighted by the gain matrix [L].

3.2 Kalman filter

A Kalman based observer is an optimal model-based state estimator that is able to estimate the state vector as well as some other non-easily measurable quantities exploiting real-time data coming from the plant, i.e. machine, as stated in [4]. The Kalman filter considers only the machine dynamics and it has no information on the cutting process.

$$\dot{\widehat{\boldsymbol{x}_e}}(t) = [A_e]\widehat{\boldsymbol{x}_e}(t) + [L](\boldsymbol{y_2}(t) - \widehat{\boldsymbol{y_2}}(t))$$

$$\widehat{\boldsymbol{y_1}}(t) = \big[[C_1]\ [0]\big]\widehat{\boldsymbol{x}_e}(t)$$

$$\widehat{\boldsymbol{y_2}}(t) = \big[[C_2]\ [0]\big]\widehat{\boldsymbol{x}_e}(t) \tag{3.3}$$

The Kalman gain matrix $[L]$ is determined by minimization of the error covariance matrix P, obtained by solving the Riccati equation. The tuning procedure of the $[L]$ is done by assigning appropriate quantities to process noise (covariance matrix Q) and measurement noise (covariance matrix R).

Especially in case of instability, when the regenerative effect has an important weight, the gain can allow to compensate the loss of information from the cutting process. However, the gain tuning must deal with the noises that of course change between the stable and unstable cutting conditions. So may happen that a gain suitable for the good estimation of a stable process, could be not suitable if the process becomes unstable.

In the following subsections the performances of the Kalman filter are reported according to various noises that could affect the measurements during the simulations.

3.2.1 Kalman filter - low noises

In this subsection it is supposed that the system is affected by low noises on the measurements sensors during the simulation. The performances of the Kalman filter are tested by tuning the parameter Q. The estimate is the red trend, the plant the blue one.

- $Q = 1e17$ – Stable cut

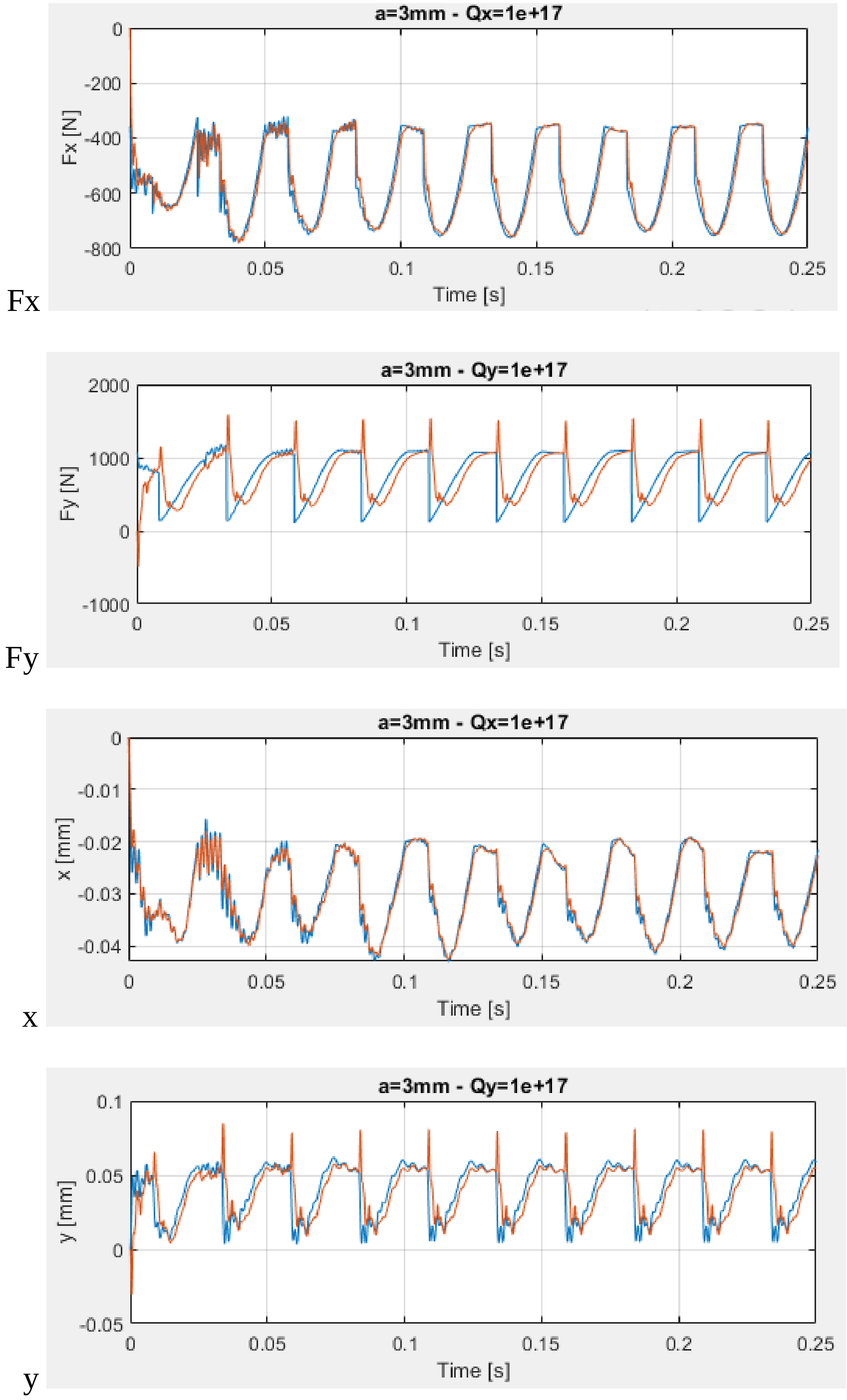
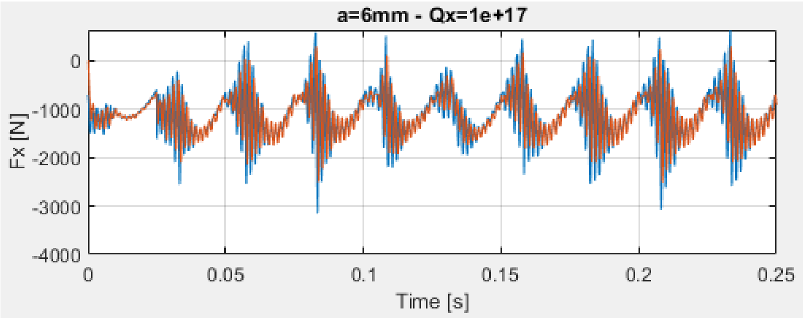
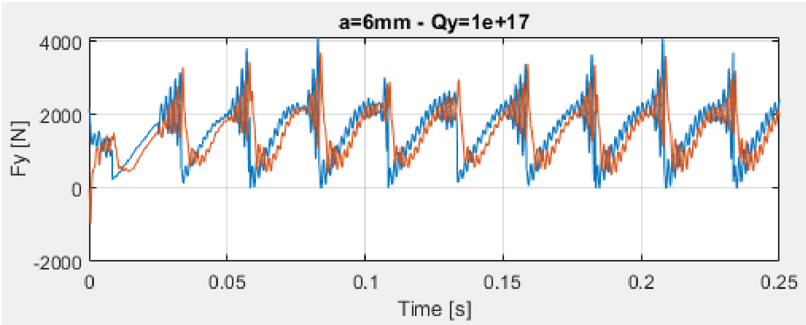


Figure 3.1: Kalman filter – stable cut – Q=1e17 – low noises

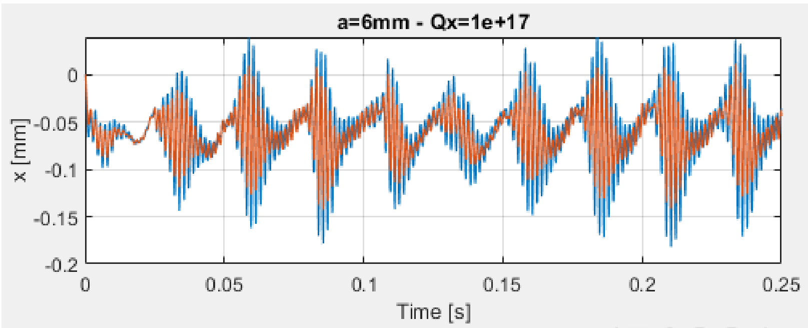
- $Q = 1e17$ – Unstable cut



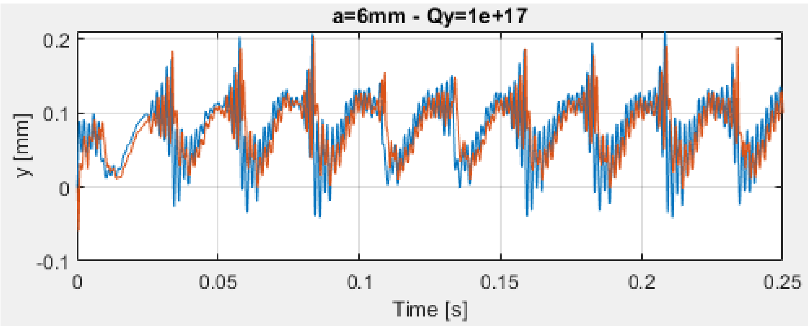
Fx



Fy



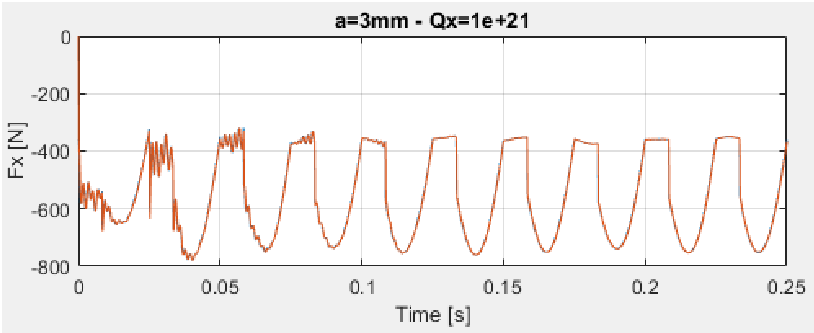
x



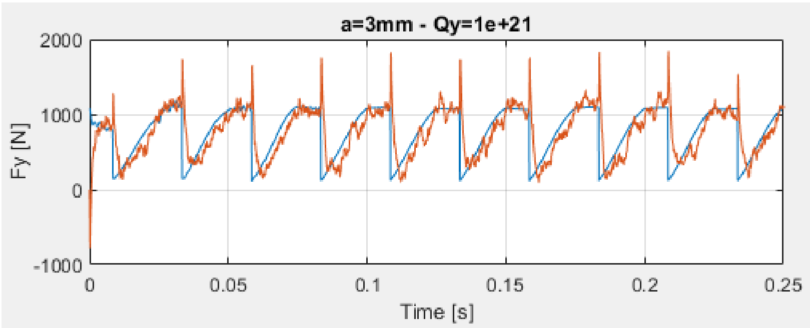
y

Figure 3.2: Kalman filter – unstable cut – Q=1e17 – low noises

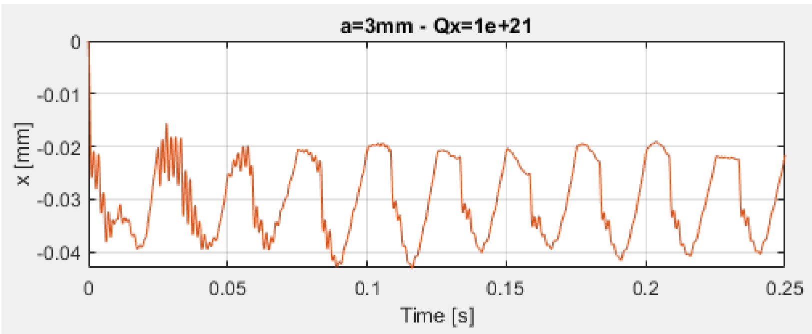
- $Q = 1e21$ – Stable cut



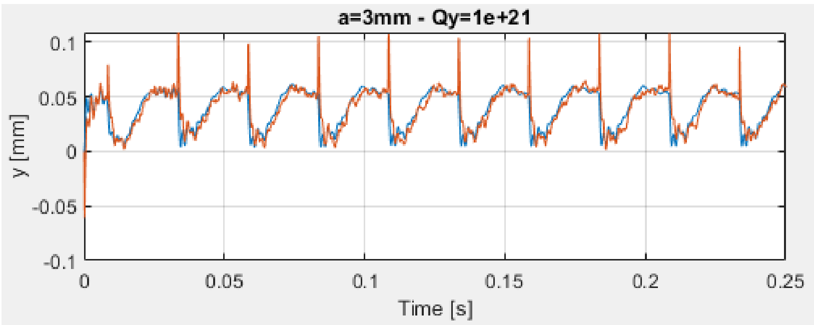
Fx



Fy



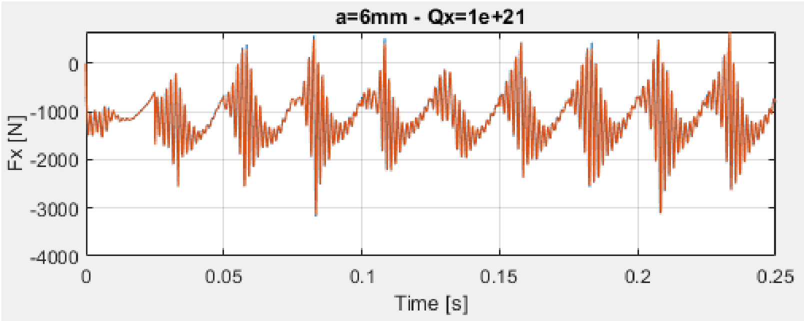
x



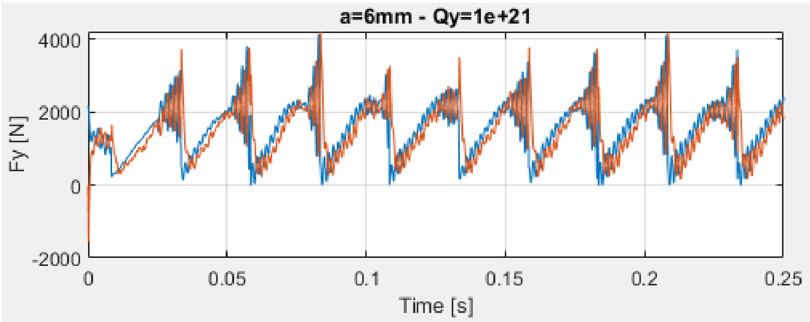
y

Figure 3.3: Kalman filter – stable cut – Q=1e21 – low noises

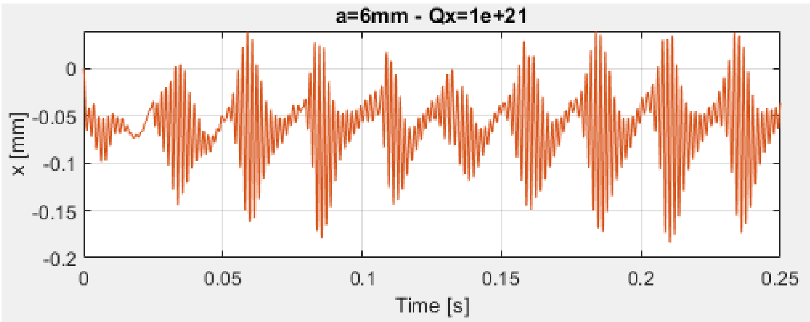
- $Q = 1e21$ – Unstable cut



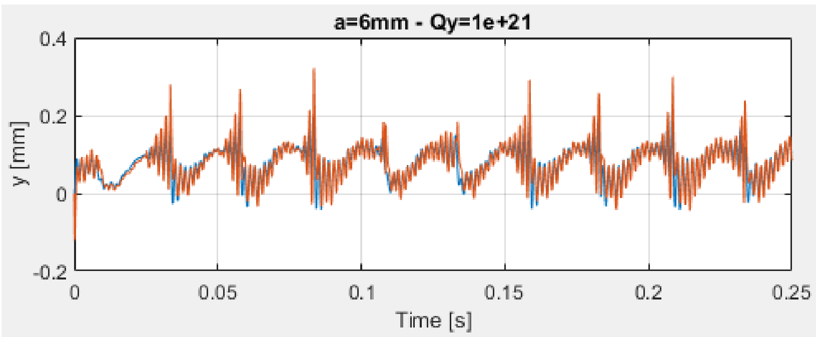
Fx



Fy



x



y

Figure 3.4: Kalman filter – unstable cut – Q=1e21 – low noises

The figures 3.1 to 3.4 show the behaviours of the cutting forces and tool tip displacements in stable and unstable conditions according to the Q parameter.

All the figures represent in order:

- the cutting force along X direction (Fx),
- the cutting force along Y direction (Fy),
- the tool tip displacement along X direction (x),
- the tool tip displacement along Y direction (y).

As can be seen, the performance of the Kalman filter increases if Q is set to $Q = 1e21$ both for the stable and unstable cut, even if the estimate under stable conditions gives quite similar results also using $Q = 1e17$. However, this case is far from the real cutting conditions, since the simulation has been carried out considering low noises.

In Y direction, the estimation seems to be a little less accurate compared to the X direction; this may be due to the configuration of 75% radial immersion of the end mill.

3.2.2 Kalman filter - high noises

By increasing the noises for a more realistic simulation and using the same values of the previous subsection for the tuning parameter Q, the following results can be appreciated.

- $Q = 1e17$ – Stable cut

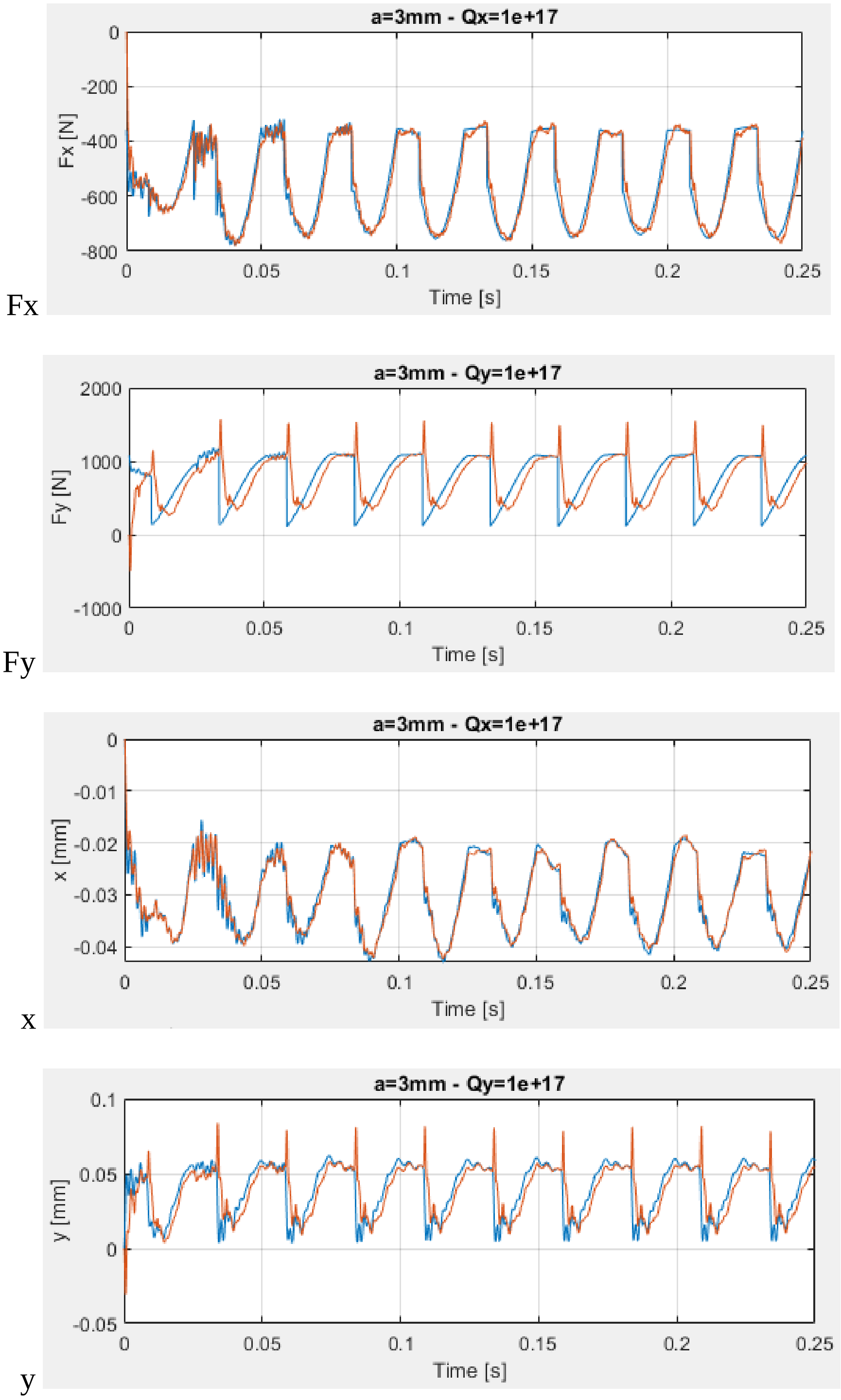
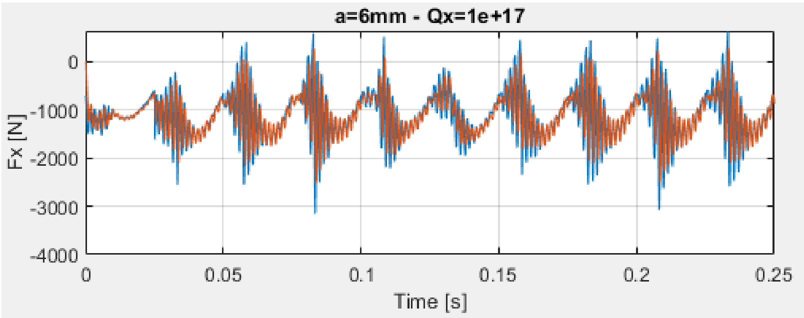
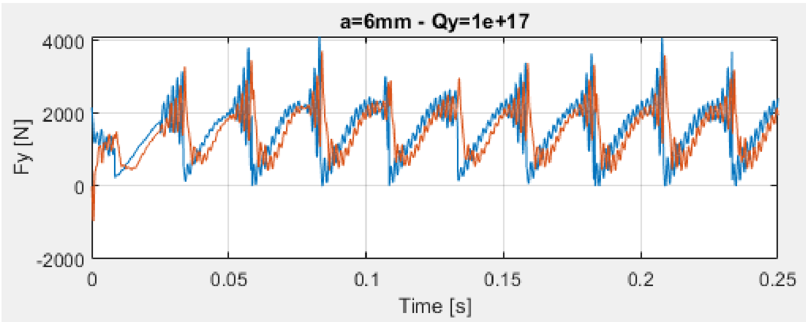


Figure 3.5: Kalman filter – stable cut – Q=1e17 – high noises

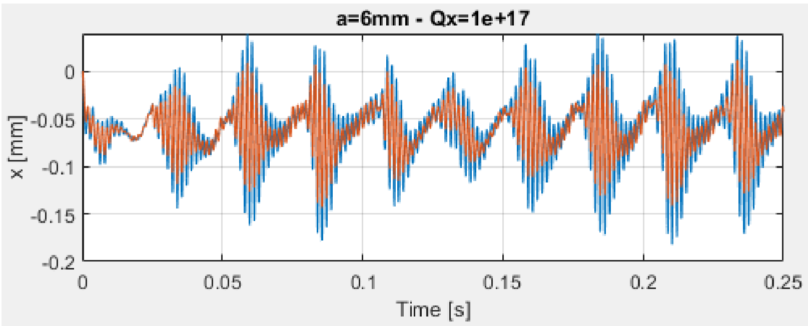
- $Q = 1e17$ – Unstable cut



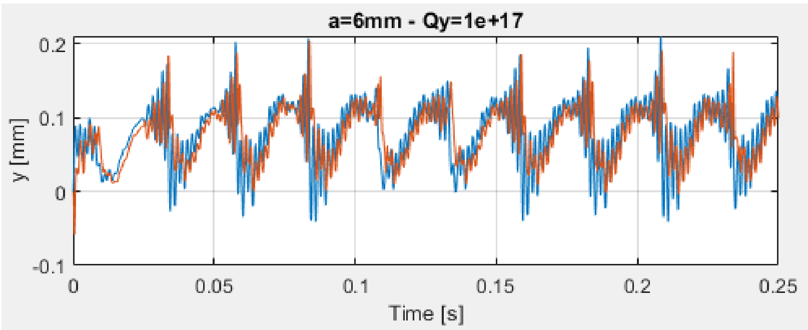
Fx



Fy



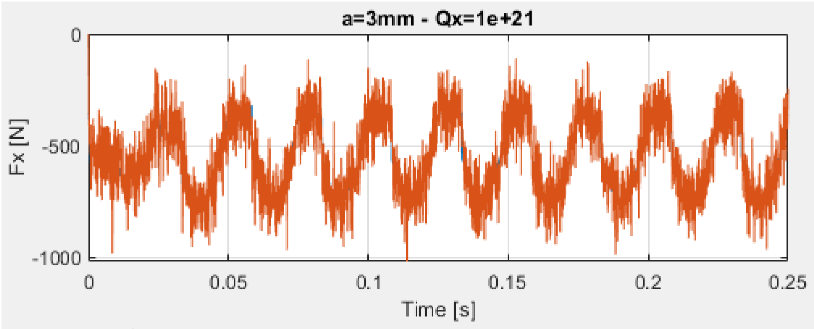
x



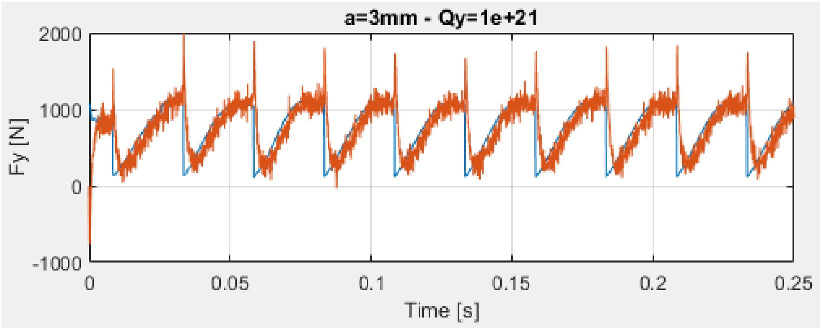
y

Figure 3.6: Kalman filter – unstable cut – Q=1e17 – high noises

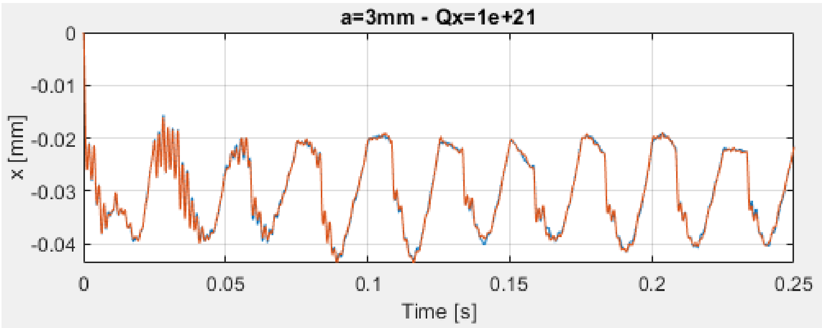
- $Q = 1e21$ – Stable cut



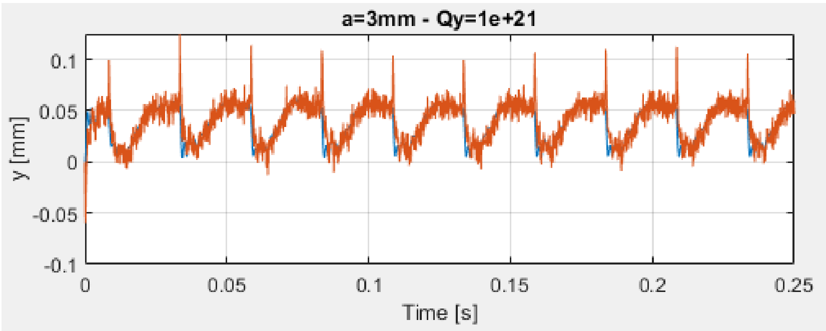
Fx



Fy



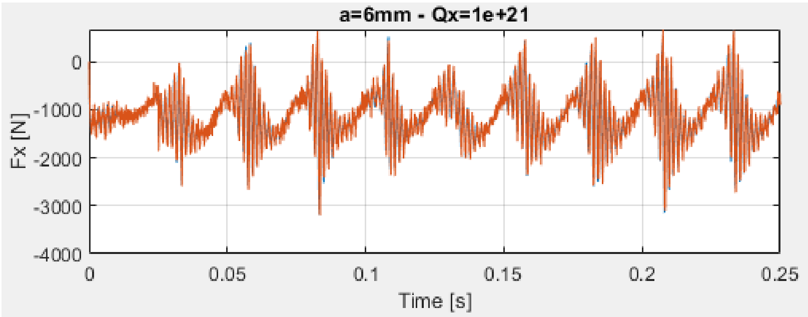
x



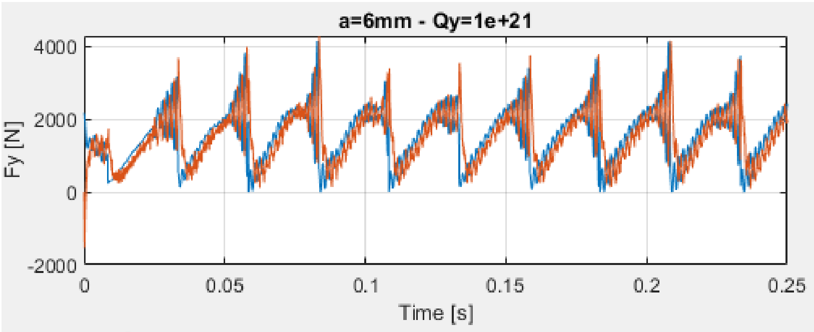
y

Figure 3.7: Kalman filter – stable cut – Q=1e21 – high noises

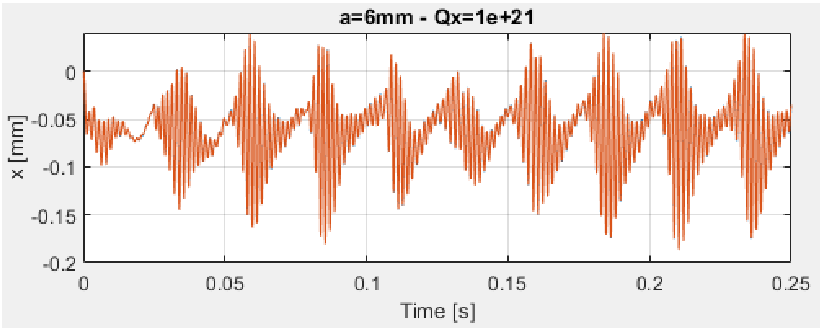
- $Q = 1e21$ – Unstable cut



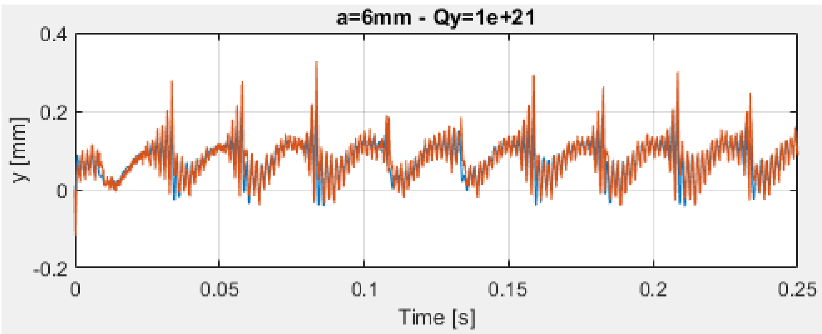
Fx



Fy



x



y

Figure 3.8: Kalman filter – unstable cut – Q=1e21 – high noises

As before, figures 3.5 to 3.8 follows the order:

- cutting force along X direction (Fx),
- cutting force along Y direction (Fy),
- tool tip displacement along X direction (x),
- tool tip displacement along Y direction (y).

In this case the same considerations made for $Q = 1e17$ in the Kalman with low noises are still valid, namely it estimates quite well in stable cut, but it underestimates for unstable cut. By increasing the noises and tuning $Q = 1e21$ causes an improvement of the estimate of unstable cut, but a worsening of the performances of the stable cut. Therefore, with high simulation noises on the measurements, increasing Q allows to good estimate under unstable cutting conditions, but loses the accuracy of the estimate in stable conditions.

In conclusion, the Kalman filter can achieve good performance with low noises by increasing Q, but if the noises increase (more realistic behaviour), increasing Q gives a good estimate only for the unstable cut, losing the precision for the stable cut.

In order to design a more robust observer that can bypass the limitations of the Kalman filter, the following section provides theoretical support for a future development of a state observer.

3.3 Switching observer for Hybrid System

3.3.1 Hybrid System overview

The plant described in section 2 is a hybrid system, which consists of two subsystems that switch according to a condition that depends on the chip thickness value. These two subsystems are alternatively involved under unstable machining and correspond to:

- Detachment condition: loss-of-contact between the tool and the workpiece.
- Engagement condition: tool and workpiece in contact.

When the detachment condition occurs, the cutting forces vanish (input vector u(t) becomes null) until the tool re-enters in cut. Then, in the engagement condition, the cutting forces recur (and so the input vector u(t)) till the next detachment caused by the too high vibration amplitude reached. In the engagement condition the system has memory of the previous state and it is described by DDEs.

The two subsystems, in state space representation, are:

- Detachment condition: no force acting on the system

$$\dot{\boldsymbol{x}}(t) = [A]\boldsymbol{x}(t)$$
$$\boldsymbol{y_1}(t) = [C_1]\boldsymbol{x}(t)$$
$$\boldsymbol{y_2}(t) = [C_2]\boldsymbol{x}(t) \tag{3.4}$$

- Engagement condition: force acting on the system.

$$\dot{\boldsymbol{x}}(t) = [A]\boldsymbol{x}(t) + [B]\boldsymbol{u}(t)$$
$$\boldsymbol{y_1}(t) = [C_1]\boldsymbol{x}(t)$$
$$\boldsymbol{y_2}(t) = [C_2]\boldsymbol{x}(t) \tag{3.5}$$

Since $\boldsymbol{u}(t)$ is the input vector affected by the delayed terms coming from the description of the regenerative effect, it assumes the following form:

$$\boldsymbol{u}(t) = \boldsymbol{u}_{nom} + \boldsymbol{u}_{reg} = \boldsymbol{F_0}(t) + [A_{reg}][C_1](\boldsymbol{x}(t) - \boldsymbol{x}(t-\tau)) \tag{3.6}$$

where $[A_{reg}]$ is a periodic matrix that allows coupling the machine dynamics and the cutting process, according to the tool-workpiece engagement conditions.

So, by substituting the expression 3.6 into 3.5, the engagement condition becomes:

- Engagement condition:

$$\dot{\boldsymbol{x}}(t) = [A_p(t)]\boldsymbol{x}(t) + [A_\tau(t)]\boldsymbol{x}(t-\tau) + [B]\boldsymbol{F_0}(t)$$
$$\boldsymbol{y_1}(t) = [C_1]\boldsymbol{x}(t)$$
$$\boldsymbol{y_2}(t) = [C_2]\boldsymbol{x}(t) \quad (3.7)$$

The switching between the two situations occurs based on a verification of a condition that considers the vibration amplitude. Considering the real plant, it is not easy to know when the switching occurs.

The interaction between the two systems can be seen in figure 3.9 in a simple schematic way.

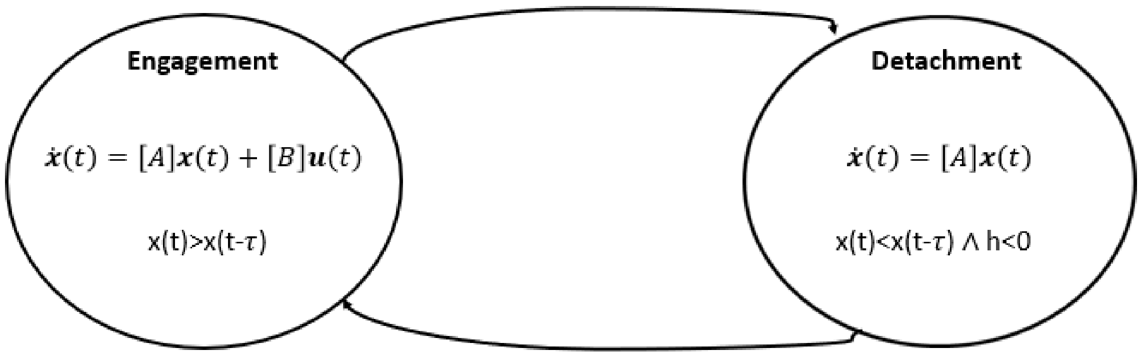


Figure 3.9: Interaction Engagement – Detachment conditions

A suitable observer for this kind of systems is a switching observer that allows to pass from the different modes of the system by means of a switching law.

3.3.2 Switching observer

A switching estimator is used when a system switches to a new configuration, characterized by a different behaviour as stated in [8]. According to the presented plant, a switching observer may be considered as the estimator that is able to better estimate the states of the system.

The goal is still to estimate the tool tip displacements and the cutting forces in a milling process.

By reducing the two subsystems 3.4 and 3.5 to a compact form and neglecting the contribution of the delayed term for an initial simplification, the system is described as follows:

$$\dot{\boldsymbol{x}}(t) = [A_{\sigma(t)}]\boldsymbol{x}(t) + [B_{\sigma(t)}]\boldsymbol{u}(t)$$

$$\boldsymbol{y_2}(t) = [C_{\sigma(t)}]\boldsymbol{x}(t) \tag{3.8}$$

where the matrices $[A_{\sigma(t)}]$, $[B_{\sigma(t)}]$, $[C_{\sigma(t)}]$ consider the definition of the two modes of the system, i.e. detachment and engagement conditions.

The index $\sigma(t)$: $[0\ +\infty] \rightarrow \{1,2\}$ corresponds to the function that allows the switching between the two configurations.

A switching observer for 3.8 is:

$$\dot{\hat{\boldsymbol{x}}}(t) = [A_{\sigma(t)}]\hat{\boldsymbol{x}}(t) + [B_{\sigma(t)}]\boldsymbol{u}(t) + [L_{\sigma(t)}](\boldsymbol{y_2}(t) - [C_{\sigma(t)}]\hat{\boldsymbol{x}}(t)) \tag{3.9}$$

where $[L_{\sigma(t)}]$ is the observer gain matrix.

In order to find the gains, the problem is reduced to the fulfilment of Lyapunov inequalities. Indeed, a Lyapunov function is searched to ensure stability so that the estimation error converges to zero.

The problem is to find the L_i of $[L_{\sigma(t)}]$, $i = 1,2$, such that there exists a symmetric positive definite matrix P solving the Lyapunov inequalities:

$$(A_i - L_iC_i)^TP + P(A_i - L_iC_i) < 0\ , i = 1,2 \tag{3.10}$$

The problem 3.10 is difficult as it involves the combined selection of P and L_i. However, it can be reduced to a simpler form based on the Linear Matrix Inequality (LMI) method. Hence the problem 3.10 can be solved by means of the LMI formulation 3.11:

$$A_i^T P - C_i^T Y_i^T + PA_i - Y_i C_i < 0 \tag{3.11}$$

where the variables to find are P and Y_i, so that $P = P^T > 0$ and Y_i is necessary to find the gains L_i as follows:

$$L_i = P^{-1} Y_i \tag{3.12}$$

Once solved the problem 3.11, it is possible to minimize the upper bound of the quadratic cost function of the estimator error, in order to achieve an optimal design of switching observer. Consequently, the gains of the observer can be selected such that this bound is minimized. Practically, the problem of minimizing the upper bound to the estimation error is reduced to the minimization of the maximum eigenvalue of P: $\lambda I > P$.

Therefore the LMI formulation appears as:

$$S = S^T > 0$$

$$P = P^T > 0$$

$$\lambda I - P > 0$$

$$A_i^T P - C_i^T Y_i^T + PA_i - Y_i C_i + S < 0\,, i = 1,2 \tag{3.13}$$

And the observer gains are obtained as in 3.12, i.e. $L_i = P^{-1} Y_i$.

Since the aim of this work is to estimate the cutting forces and the tool tip vibrations, the states of the system must be expanded as in 3.1, obtaining the expanded system 3.2. For the hybrid system case there will be two matrices $[A_e]$ corresponding to the two conditions (detachment and engagement):

$$[A_e]_{detachment} = \begin{bmatrix} [A] & [0] \\ [0] & [0] \end{bmatrix}$$

$$[A_e]_{engagement} = \begin{bmatrix} [A] & [B] \\ [0] & [0] \end{bmatrix} \tag{3.14}$$

Thus, the switching observer assumes the form:

$$\dot{\widehat{\boldsymbol{x}_e}}(t) = \left[A_{\sigma(t),e}\right]\widehat{\boldsymbol{x}_e}(t) + \left[L_{\sigma(t)}\right](\boldsymbol{y_2}(t) - \widehat{\boldsymbol{y_2}}(t)) \tag{3.15}$$

where the matrix $\left[A_{\sigma(t),e}\right]$ is the expanded matrix that considers the definitions 3.14.

Since the delayed states describing the regeneration phenomenon are involved in the input vector $\boldsymbol{u}(t)$, which must also be estimated, the expanded form is not affected by the previous states of the system. However, this modelling can be a first attempt to simplify the system. The process-based contribution, i.e. the dependence of the system on its delayed states, can be introduced in the observer design as the information extracted from the plant and necessary to impose the condition on the switching between the detachment condition and the engagement one.

Summing up, the simplifications for a first implementation of the switching observer consist in neglecting the delayed terms, related to the input vector and which are responsible to the dynamic behaviour change of the machine tool under unstable cutting conditions, and in extracting the information about the system mode (detachment / engagement) from the plant in order to manage the switching between the two subsystems.

Further robust observer design should count the influence of the delayed term on the state matrix of the machine dynamics ($[A]$) and the possibility to develop and observer able to estimate even the switching between the system mode (detachment / engagement).

4. Conclusions and future works

The present work began with the study of the cutting process in milling with particular attention in the description of the regenerative instability, known as chatter.

The real plant has been modelled considering the complexity of the milling process. Indeed, the cutting forces depend on the actual and delayed positions of the tooth passages, leading to a system described by delay differential equations. The chatter phenomenon has been represented by accounting the detachment phenomenon that verifies between the tool and the workpiece when high vibration amplitudes are reached.

The resulting plant is a hybrid system characterized by two modes which are alternatively involved under unstable machining: detachment and engagement conditions.

In order to estimate the cutting forces and the tool tip vibrations a Kalman filter has been implemented in an expanded version. It is based on machine dynamics only, without information from the cutting process. Therefore, it has been observed that the estimation is good under stable conditions, but it worsens with an unstable cut.

Hence a study for state observer in case of delayed system has been carried out and the information about the cutting process, as well as the machine dynamics, considered. However, the works found in literature provide methods theoretically valid for this kind of problem, but very few of them are tested in real applications. An introduction to an observer for hybrid system, that could be suitable to estimate the cutting forces and the tool tip vibrations for the presented plant, has been shown. It is known as switching observer and it is based on the fulfilment of Lyapunov inequalities by means of Linear Matrix Inequalities (LMI) to find its gains, related to the two modes of the hybrid system, i.e. detachment and engagement. A simplified version has been proposed and future works may try to implement and test it.

Future works could also deal with the implementation of observers that don’t require the knowledge of the process parameters as depth of cut, feed per tooth, cutting coefficients, engagement angles, number of teeth etc., but that are able to provide that information during the processing of the workpiece. Indeed, the need of the process parameters limits the use of the observer according to the tool considered during the acquisition of the data from the experimental modal analysis. Since the experimental modal analysis is a time-consuming activity, by an industrial point of view it is feasible if applied to a limited number of tools, which perform some critical milling operations for which it is useful to monitor the cutting forces and tool tip vibrations. A further step could be the real time evaluation of the surface quality of the workpiece based on the estimated cutting forces and tool tip vibrations.

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