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Bancassurance: from the organization to the wealth management and Armundia case

Relator: Prof. Laura Grassi

Correlator: Prof. Davide Lanfranchi

Author:

Mattia Dario Andreoli (878239)

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Sommario

Il seguente elaborato si propone di analizzare il contesto normativo in cui sti trova ad operare la bancassurance. Le direttive di riferimento analizzare sono MIFID II e IDD, le quali, nei recenti anni, hanno normato un panorama che prevede che i consumatori siano sempre più protetti e informati sui prodotti che intendono sottoscrivere. Dopo questa fase introduttiva, si procederà ad analizzare le modalità di gestione dei portafogli finanziari, valutando le tecniche utilizzate dai gestori per massimizzare il rendimento. Infine, verrà analizzato come avviene la distribuzione dei fondi una volta ricavati dal cliente. Successivamente, il focus verrà spostato sul ramo assicurativo ed in particolare si partirà ad analizzare prodotti più semplici, fino ad arrivare a quelli più complessi come assicurazioni unit-linked e index-linked. Prima di affrontare questi temi, ci sarà un breve excursus sulla matematica attuariale e la scomposizione del premio, al fine di capire la logica che si trova dietro questi prodotti. Come per la parte finanziaria, verrà spiegato come avviene il processo di distribuzione all'interno delle istituzioni assicurative. Nel quarto capitolo verrà introdotta la bancassicurazione nelle sue peculiarità, le sue forme organizzative e i canali distribuivi. Dopodiché verrà introdotta la realtà di Armundia, piccola-media impresa che propone un servizio omnicomprensivo per supportare le realtà finanziare a svilupparsi e diventare bancassicurazione, grazie all'utilizzo di piattaforme IT che consentono di semplificare la vita degli operatori finanziari. Nel capitolo finale verranno riportate le conclusioni dell'elaborato.

Abstract

The following paper aims to analyze the regulatory context in which the bancassurance is operating. The reference directives to analyze are MIFID II and IDD, which, in recent years, have regulated a panorama that provides that consumers are increasingly protected and informed about the products they intend to subscribe to. After this introductory phase, we will proceed to analyze the financial portfolio management methods, evaluating the techniques used by the managers to maximize the yield. Then, it will be analyzed how the distribution of the funds takes place once they are obtained from the customer. Subsequently, the focus will be shifted to the insurance branch and in particular will start to analyze simpler products, up to the more complex ones such as unit-linked and index-linked insurance. Before addressing these issues, there will be a brief excursus on actuarial mathematics and the breakdown of the award, in order to understand the logic behind these products. As for the financial part, it will be explained how the distribution process takes place within insurance institutions. In the fourth chapter bancassurance will be introduced in its peculiarities, its organizational forms and distribution channels. Then the reality of Armundia, a small-medium enterprise that proposes a comprehensive service to support the financial realities to develop and become bancassurance, will be introduced, thanks to the use of IT platforms that allow to simplify the life of financial operators. The conclusions of the paper will be reported in the final chapter.

Chapter 1: MiFID and IDD, the customer as new focal point

1.1 MiFID II

MiFID II and the accompanying Regulation on Markets in Financial Instruments Regulation (MiFIR) are both pieces of legislation originating from the European Commission, seeking to provide a European-wide legislative framework for regulating the operation of financial markets in the EU.

MiFID II is concerned with the framework of trading venues and structures in which financial instruments are traded, whereas MiFIR focuses on regulating the operation of those trading venues, looking to processes, systems and governance measures adopted by market participants and to their future supervision.

The main impacts can be divided in three areas of interest and are:

Customer Protection	Transparency	Market Infrastructure
•Product •Governance •Remuneration •Distribution •Inducements •Conflict management •Reporting regulations	Reporting requirements: •Best execution •Transaction •Trade	Scope of trading requirements changed to include: •Fixed Income •Derivatives •Foreign Exchanges •Use of dark pools

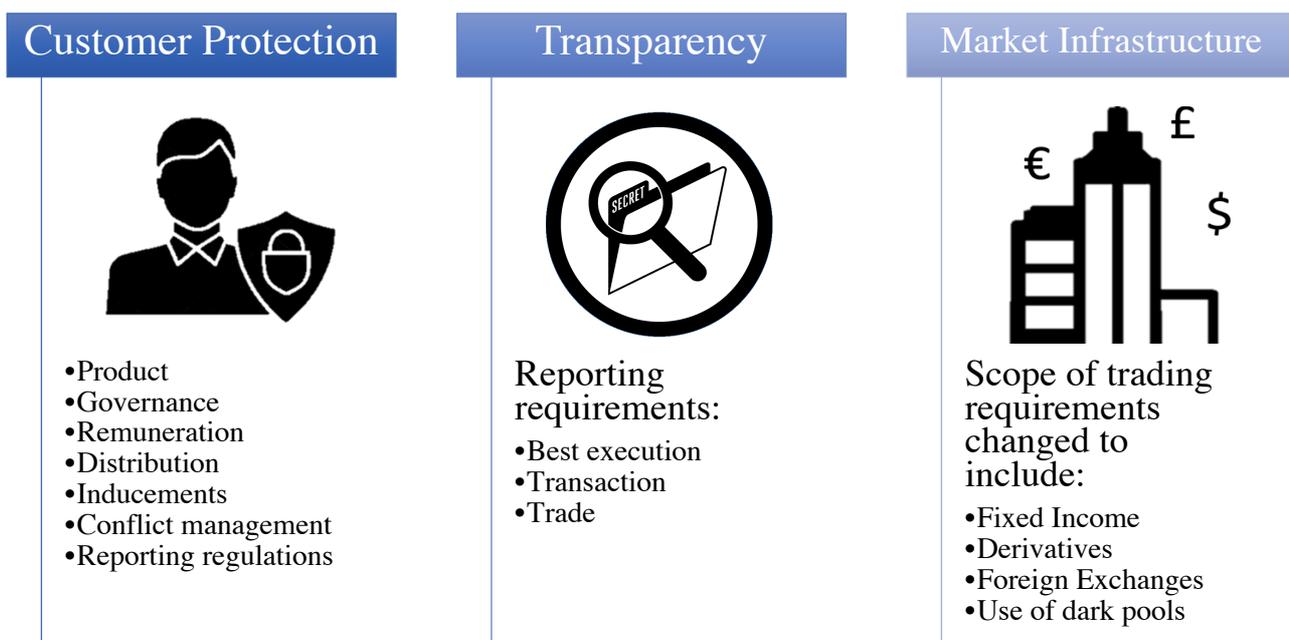*Figure 1: Areas influenced by MiFID*

In this document we will focus on the first section and how banks financial institutions intent to safeguard the customer. As said, the primary requirement of the MiFID II is to profile the investor to understand his degree of risk in order to structure for him the most suitable offer possible on the financial side[1].

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[1] Cfr. art. 24 par. 2 e par. 3, MiFID II.

The principal instrument adopted by banks and financial institutes is constituted by a survey. Currently this instrument is nothing more than a set of questions about the different knowledge and experiences in the financial world, as well as about personal economic situation and the possible returns to get from the investment. The banks then submit the "MiFID questionnaire" to the potential investor in the occasion of the investment proposal, whether they are online trading operations, or even the most common life insurance policies or any other instrument on the securities account.

Differently from IDD survey, which will see further, the questionnaire is not standardized, but give a degree of freedom to the bank which provide it to the customer. What is equal for each test is the structure, that must report three main sections.

The principal points touched by the survey are 3:

1) Objectives of the investment: through simple questions, the customer is requested:
   - Period of time through which it intends to keep a certain investment;
   - Risk appetite;
   - The motivation that leads to investment;
   - If it is a speculative investment or a real capital growth;
   - If the customer invests to maintain and protect his capital avoiding loss, and if he is willing to accept certain levels of risk.

2) Financial situation: in this section, questions will be submitted concerning:
   - Average customer earnings on an annual basis;
   - Primary source of income;
   - Assets held;
   - Any debts and credits.

3) Financial knowledge[2]: in this section all the data concerning the client are indicated and that has to do with the knowledge and experience gained in the financial field in the past. So, it's about:
   - Knowledge of financial products;
   - Investment volume;
   - Frequency;
   - Instruction;
   - Client profession

The MiFID questionnaire has the obligation, for banks and financial intermediaries, to assess the truthfulness and adequacy of the products or services that are offered to the potential new customer, based also on our knowledge. In this way an investor profile is outlined and this allows the bank or

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[2] Cfr. art. 55 par. 1 *Ibidem*

the broker or the financial promoter to proceed in harmony with the client's knowledge of the most appropriate type of investment.

The structure of the survey can be summarized in this way:

In essence, the MiFID questionnaire gathers as much information as possible, to propose appropriate investment information for the trader or the customer.

Then, the promoter company that acquires the client's investments must act honestly, impartially and as professionally as possible, always trying to provide clear and very correct information, but above all relating to the investments required.

1.2 IDD

On the other side, as said before, we can focus on insurance distribution because, from October 2018, a new directive has been released: The Insurance Distribution Directive (IDD). As for the MiFID II, the IDD expressly states that EIOPA (main authority in terms of Insurance at European level) conducts consumer testing before finalising the draft ITS, now the standard in term of survey[3]. The test studied for the IDD is divided in different questions, necessary to understand the needs of the future customer. This test will help not only the client but even the contractor to understand in the correct profiling. One example that can help us understanding the utility of this test is in case of an offer: if the customer does not have the car, probably, in the omni comprehensive package will not appear an insurance against damages with car.

A sample of the test will be reported in the figure below:
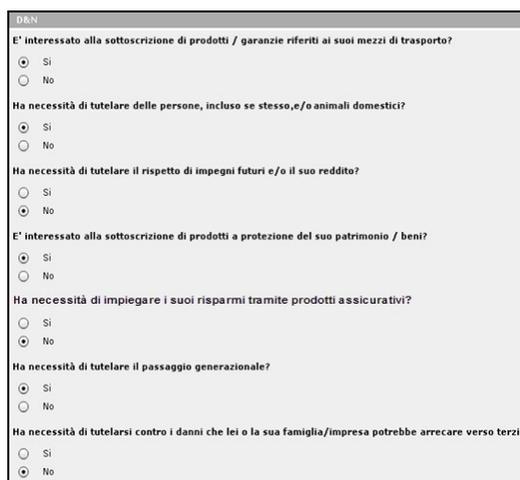
Figure 2: Sample of IDD test

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[3] The reference point of this paragraph is the report "*IPID Consumer Testing and Design Work*" taken from the EIOPA internet site https://eiopa.europa.eu/and the work is collected with the name EIOPA/OP/153/2015

As ween can see, main questions are related to the need of being insured against car incidents or house-damages. This test, as said before, will provide information about the need of the client in order to carve the most suitable offer.

As soon as the result are available, the customer will be put in front of a paper which reports the most suitable offer. In facts, as well as ESMA (The European Securities and Markets Authority is a European Union financial regulatory institution and European Supervisory Authority, located in Paris), EIOPA set a presentation format of an Insurance Product Information Document (IPID) specifying the details of the presentation of the information. The insurance product information document, which will be prepared for non-life insurance products only, needs to contain the following information:

a) information about the type of insurance;

b) a summary of the insurance cover, including the main risks insured, the insured sum and, where applicable, the geographical scope and a summary of the excluded risks;

c) the means of payment of premiums and the duration of payments;

d) main exclusions where claims cannot be made;

e) obligations at the start of the contract;

f) obligations during the term of the contract;

g) obligations in case of a claim made;

h) the term of the contract including start and end date of the contract; and,

i) the means of terminating the contract.

Figure 3: Standard IPID

The IPID will be supplied to the consumer by the insurance distributor prior to the purchase of a non-life insurance product with the goal of assisting the consumer to make an informed decision. As we can see, the IPID is divided in different parts which cover different aspects of the financial situation and the behaviour of the new customer. The IPID could be compared to the KIID, introduced by ESMA, but is more comprehensive because, if KIID is related just to UCITS funds[4], this document must be provided for every insurance product.

This survey, as well as MiFID one, is focused on understand the financial literacy and providing the most suitable products to the clients.

1.3 Points of contact and Italian view

In this chapter we aim to provide an overview of the Italian situation with reference to the investment process and how the consultancy application fits into it. Therefore, some statistical data will be presented regarding the degree of financial literacy in our country and the attitudes of Italians with regard to investment choices - from which a concrete "need for advice" can be deduced - the issue of behavioural biases in which they may incur is addressed. Both investors and consultants, with particular reference to the perception of risk. Finally, investors' attitude towards consulting is described in terms of the variables considered in the professional's assessment. OCSE, in the investigation PISA[5] 2015

The most important international statistical surveys on the level of financial literacy are PISA and the "Standard & Poor's Ratings Services Global Finiteness Survey"[6]. The results that emerge from the most recent editions of these publications do not give an excellent image of our country compared to the other industrialized countries. In particular, on the basis of the S&P survey, Italy is not at the top of the list, neither in the world nor in Europe, as can be seen from the following figures[7]:

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[4] UCITS - The Undertakings for the Collective Investment of Transferable Securities (UCITS) is a regulatory framework of the European Commission that creates a harmonized regime throughout Europe for the management, marketing and sale of mutual funds

[5] PISA, is the acronym of Programme for International Student Assessment, it is an international investigation promoted by the OCSE and it has the scope to evaluate the level of instruction among students within industrial countries.

[6] The S&P FinLit Survey is the most accurate investigation on the financial literacy held at an international level. In the edition of 2015, around 150.000 people have been interviewed in more than 140 countries. The four questions most asked refer to these important aspects: risk diversification, inflation, numeracy(interest), compound interest

[7] Data and figures used are taken from the book: McGraw Hill Financial, Financial Literacy Around the World: Insights from the Standard & Poor's Ratings Services Global Financial Literacy Survey, 2015
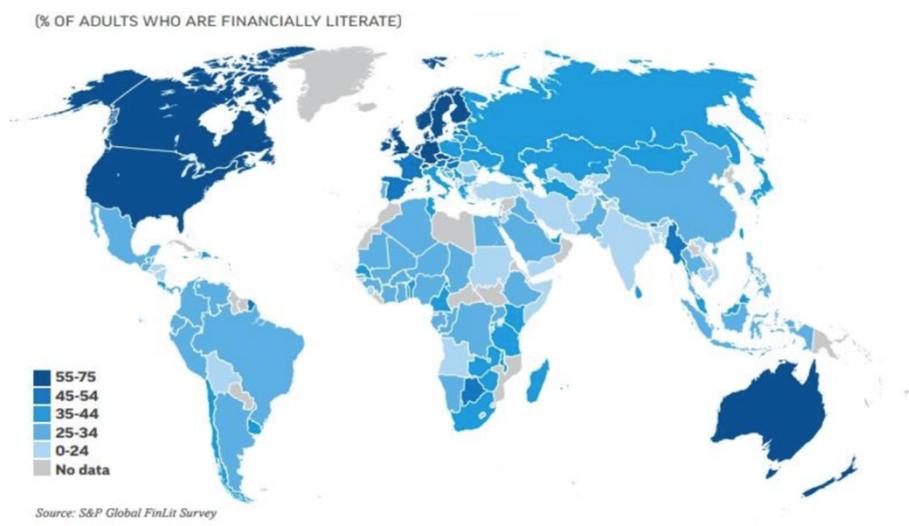
Figure 4: Global variation in financial literacy

In particular, financial literacy rates vary widely across the European Union (Map 29). On average, 52 percent of adults are financially literate, and the understanding of financial concepts is the highest in the northern Europe. Denmark, Germany, the Netherlands and Sweden have the highest literacy rates in European Union: at least 65 percent of their adults are literate in financial terms. But rates are much lower in southern Europe. For example, in Greece and Spain, literacy rates are 45 percent and 49 percent,

Figure 5: European variation in financial literacy

respectively. Italy and Portugal have some of the lowest literacy rates in the south. Financial literacy rates are also low among the countries that joined the EU in 2004 and after. In Bulgaria and Cyprus, 35 percent of adults are financially literate. Then Romania, with 22 percent has the lowest rate in European union.

Present data are referred just to adults. If we look even at young Italians, they rank slightly below the OECD average, placing Italy between seventh and ninth in the ranking of the 15 countries that

participated in the PISA financial literacy survey in 2015[8]. On average, only 6.5% of Italian students reach the highest level (level 5) of performance on the PISA scale relative to financial knowledge[9].

Numerous studies show that a high level of financial literacy has positive repercussions both on an individual and macro-economic level.

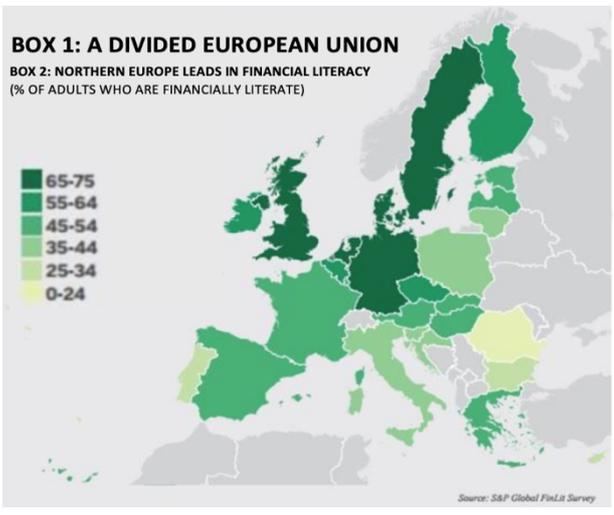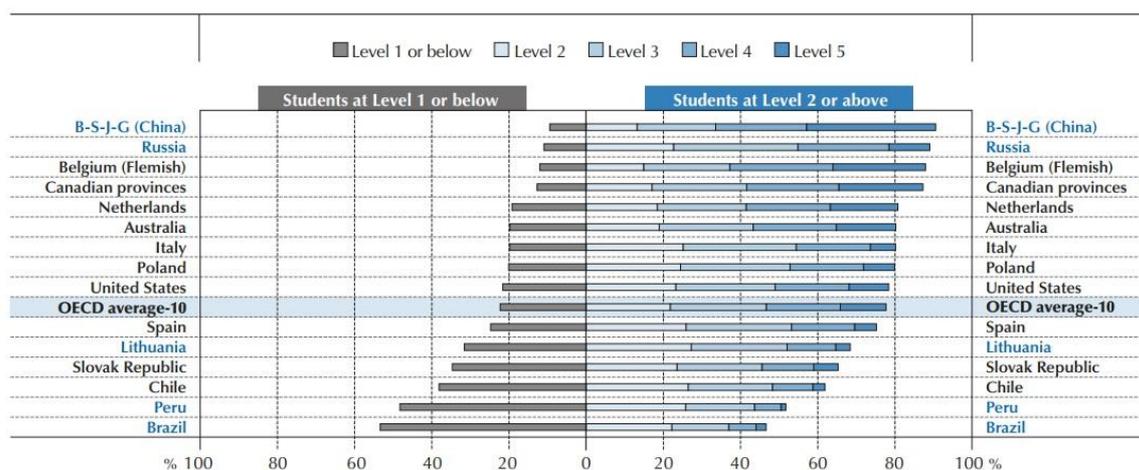Data reported have the purpose to demonstrate that in the Italian optics, not so much people have the right instruments to understand if the investment proposed is good or bad for their wealth. The need to have a survey where to communicate the intentions and the awareness of the customer may represent a way to protect the potential investor.

Countries and economies are ranked in descending order of the percentage of students who perform at or above Level 2.
Source: OECD, PISA 2015 Database, Table IV.3.2.

Figure 6: Financial literacy among students

For this reason, one of the fundamental moments of the consultancy is notoriously that of assessing the adequacy of the investments subject to recommendation with respect to the risk profile and other personal characteristics of the investor.

In almost all cases, the collection of information by the customer is carried out by administering a standardized questionnaire, in which questions are asked to the client about his work situation, his financial situation to assess in particular his tolerance to suffer losses (objective risk), on their knowledge of the main financial concepts such as risk-return and diversification, and finally on his personal risk appetite (subjective risk). As mentioned, in the absence of such information the

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[8] This part of the PISA's survey, related to the financial literacy is not mandatory. In 2015, just 15 countries have joined this option, 10 of these are already members of OCSE (Australia, Belgium, Canada, Chile, Italy, Netherlands, Poland, Slovakia, Spain and USA). In addition to this, it just concerns a few numbers of students. In Italy, this sample refers just to 3.035 students, 2.724 with valid data, within a total of 11.583 students involved.

[9] Elaborations are taken from INVALSI document (Istituto nazionale per la valutazione del sistema educativo di istruzione e di formazione): "Indagine OCSE PISA 2015 - Financial literacy Sintesi dei risultati". The reader can see this document to the following link:
http://www.invalsi.it/invalsi/ri/pisa2015/doc/2017/Sintesi_Financial_literacy_24052017.pdf

intermediary is prohibited from proceeding to provide the recommendation to the client, pursuant to art. 54 par. 8 of the MiFID II Delegated Regulation[10].

This because, on an individual level, people with more financial knowledge are supposed, among other things, to make better decisions about their financial wealth, to save more for their retirement, to manage their balance sheet better, to participate in stock markets, to operate more diversified portfolio choices and, ultimately, to choose investment funds with contained commissions. Not possessing financial notions and skills produces mirror-like results. Those who are financial "illiterate" usually save less and get more debt by paying higher fees and interest.

On a macro-economic level, the most literate people, demanding better quality services, stimulate competition and innovation. They are also able to better tolerate systemic financial shocks, are less likely to make irrational decisions, and therefore also allow governments to help and intervene in support of investors who have taken bad decisions.
One of the fundamental moments of the investment process is notoriously that of assessing the adequacy of the investments subject to recommendation with respect to the risk profile and other personal characteristics of the investor.

In almost all cases, the collection of information by the customer is carried out by administering the survey, object of our analysis, in which questions are asked to the customer about his work situation, his financial situation to assess in particular his tolerance to suffer losses (objective risk), on their knowledge of the main financial concepts such as risk-return and diversification, and finally on his personal risk appetite (subjective risk). As mentioned, in the absence of such information the intermediary is prohibited from proceeding to provide the recommendation to the client, pursuant to art. 54 par. 8 of the MiFID II Delegated Regulation.

It should also be remembered that financial consultants of any kind are required to issue, for each consultancy given, a suitability report that contains the details of the recommendation provided and the reasons why it was deemed adequate to the customer. As clarified by ESMA in a Questions & Answers dedicated[11], among other things, to the assessment of adequacy, this report must be issued to the customer prior to the execution of the order, even if the investment is not followed by the recommendation and also when the recommendation consists of a advice not to buy (not to buy).
The problem that one wants to put in relation to the use of the questionnaire is if it, as it is formulated today in the practice of each credit institution or investment firm, is really able to grasp the information that it aims to obtain from the customer. The empirical researches conducted on this aspect, although not numerous or recent, are in agreement in recognizing wide margins of

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[10] Delegated regulation 2017/565/UE of April 2016

[11] Cfr. Cfr. ESMA, *Consultation Paper Guidelines on certain aspects of the MiFID II suitability requirements*

improvement given the vulnerability of the questionnaire to the cognitive errors and to the behavioural distortions of the clients, of which it was widely discussed in the preceding pages. In particular, as emphasized by ESMA in its "Guidelines on some aspects of adequacy assessment"[12], questions should be formulated to take into account the possible cognitive biases of investors and the questionnaire itself should be unbiased, that is, to contain questions in a way that makes the answers unreliable. Specifically, ESMA suggests to counterbalance self-assessment through objective criteria, such as for example: "a) instead of asking whether a client understands the notions of risk-return trade off and risk diversification, the firm could present some practical examples of situations that may occur in practice, for example by means of graphs or through positive and negative scenarios, asking to choose which one would be correct/real in his opinion; instead of asking a client whether he feels sufficiently experienced to invest in certain products, the firm could ask the client what types of products the client is familiar with and how recent and frequent his trading experience with them is; (c) instead of asking whether clients believe they have sufficient funds to invest, the firm could ask for factual information about the client's financial situation; (d) instead of asking whether a client feels comfortable with taking risk, the firm could ask what level of loss over a given time period the client would be willing to accept, either on the individual investment or on the overall portfolio. "

In a Consob discussion paper of 2012.251 dedicated to the evaluation of risk tolerance through the MiFID I questionnaires, the discrepancy between the indications of the behavioural finance literature and the indications of MiFID I was shown first. The authors also analysed a sample of 20 questionnaires, examining their contents, reliability, structure and methods of administration to customers. This information was collected through an interview prepared by Consob and administered to the intermediaries involved in the research.

The following tables summarize some evidence emerged in the research:

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[12] Cfr. ESMA, *Consultation Paper Guidelines on certain aspects of the MiFID II suitability requirements*

MiFID		Literature	
Items	Variables	Items	Variables
		Socio-demographic characteristics	gender
			Civil state
			Family
			Youth
Experiences and knowledge	Profession	Experiences and knowledge	Profession
	Instruction		Instruction
	Nature, volume and frequency of financial operation		Previous experience in terms of investment and results obtained
	Type of services and instruments traded by the client		Knowledge about financial products / Trade-off between risks and returns
Financial situation	Investments and real estate goods, activities	Financial situation	Investments and real estate goods, activities
	Source of regular income		Source of regular income
	Regular financial liabilities		Regular financial liabilities
Financial goals	Time period by which the customer wants to preserve the investment	Financial goals	Time period by which the customer wants to preserve the investment
	Personal goal		Time preferences, liquidity preferences, investment goals
	Risk preferences	Risk tolerance	Risk profile
	Risk profile		Personal behaviour against risk losses

Table 1: Sample of questions asked in a survey

Another interesting aspect emerging from the Consob reports concern the linguistic and textual profiles of the questions. "[...] Only two of the 20 questionnaires analyzed can be considered sufficiently clear, effective and "valid" because they use precise questions and uniquely identify the quantity to be measured; the remainder indistinctly detect an aptitude for risk, risk capacity, risk tolerance and investment objectives that are missing in lexical and comprehensibility terms "[13].

The questions, particularly those concerning the familiarity to investments, are often double (barrelled) - they refer to several themes simultaneously and so they suggest the answer themselves. The formulation of the questions can also induce the subject to improve his image, knowingly or unknowingly, by providing false answers. Other potential customer biases are the unconscious tendency to respond positively to dichotomous questions (of the type yes, no), to position themselves centrally in the scales for questions involving alternatives arranged in scale (c.d. acquiescence or central tendency).
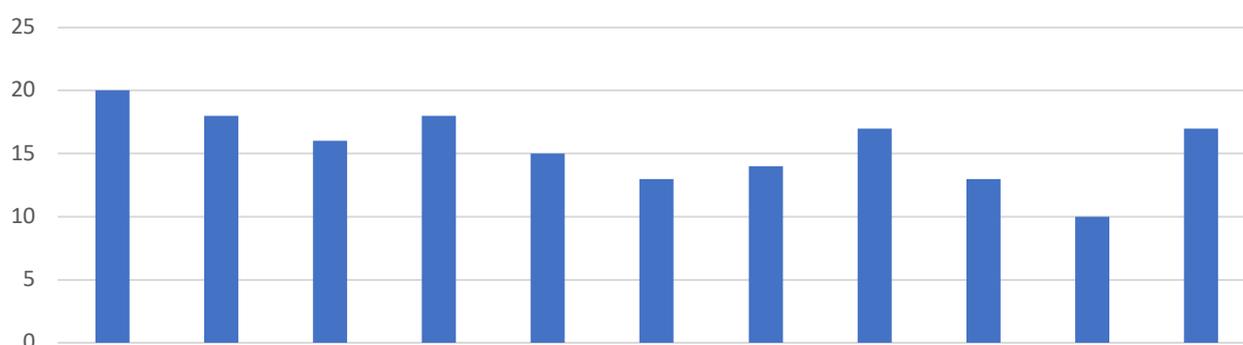
Table 2: N° of questionnaires that contains at least a question for each item
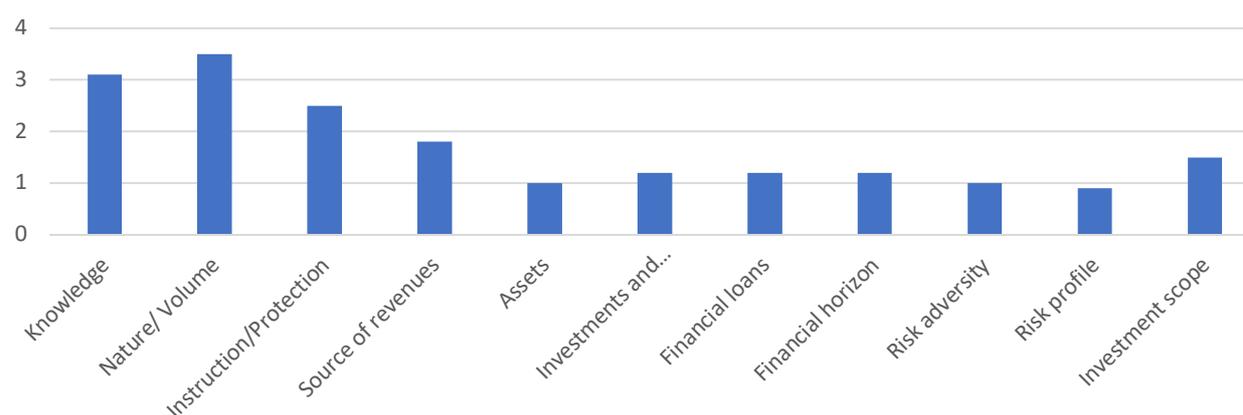
Table 3: Average number of questions for each item

[13] N. Linciano e P. Soccorso, La rilevazione della tolleranza al rischio degli investitori attraverso il questionario, p. 45

Finally, the use of inaccurate terms that can give rise to misunderstandings or difficult to understand technical terms is noted.

In conclusion, I believe that the questionnaire is a suitable tool but that it can be improved by using the measures dictated by "behavioural finance". I also believe that, with regard to the method of administration, the way to go is to encourage the client to go personally to the intermediary and to complete the form with the help of the consultant. In such a way as to be able to respond comprehensively and completely to all the questions and to be guided, in a wise and completely disinterested manner, by an advisor.

Chapter 2: how to create a financial portfolio according to new directives

As already analysed in the previous chapter, a customer can be classified based on objective criteria to which the intermediary will have to refer in order to associate to the new investor the most appropriate portfolio. In fact, after the process of data gathering, usually done with questionnaires, the customer will be defined as:

- Retail if it has low knowledge and experience in the field of investments. This category of investors is subject to strong protection by the MiFID regulation.
- Professional, In the event that the customer has significant knowledge and experience in the field of investments.

As a next step, the client is subjected to an appropriateness test. It represents a valid criterion for verifying that the investor has sufficient knowledge to understand the risks of the transaction. This test can have two results:

- The client has understood the risks associated with the investment activity. Therefore, it will be possible to proceed with the execution of the proposal shown
- The customer has not fully understood the risks of the aforementioned activity. In this case the intermediary must inform the client of this outcome. If the latter wishes to proceed in his activity, he must release the intermediary from any risk not directly attributable to him.

After this brief summary related to financial investment process under MIFID directive, we will go in deep with all the procedures and studies related to the creation of a portfolio, understanding how to create a product for risk-free investor and one for a risk neutral.

2.1 Markovitz and the theory of portfolio selection

In order to obtain a portfolio consisting of the best possible distribution of securities, it is necessary to introduce and describe the study carried out by H. Markowitz, summarized in his article "Portfolio Selection" published in 1952[14]. The starting point of this work is the concept of diversification, i.e. it is proposed to choose securities that are not related to each other. This implies that if one of them were to find a lower return than was reasonable to expect, it will compensate with a higher return of another security. This analysis is then concluded by relating the returns on the investments considered with the relative risk, stating that for each risk value considered there is a maximum possible value of return.

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[14] H. Markowitz, *The Journal of Finance, Vol. 7, No. 1* (March 1952), pp. 77-91

As. Markowitz said, "The process of selecting a portfolio may be divided into two stages. The first stage starts with observation and experience and ends with beliefs about the future performances of available securities. The second stage starts with the relevant beliefs about future performances and ends with the choice of portfolio."

Before proceeding with the effective description of Markowitz's model, it's necessary to expose the fundamental hypothesis that move his studies:

- The timeframe is uni-periodical, this means that it goes from t to t+1;
- Operators are rational and conditioned by a random variable (in our case the return)
- Operator are risk-adverse and wants to maximize their return

Another assumption necessary to move is related to the possible portfolio allocation. As we know, is it possible to invest in different asset classes with different risks, all summarized in the following table:

	Liquidity	Bonds	Equity	Commodities
Risk	Low	Medium-Low	Medium-High	High
Potential revenue	Low	Medium-Low	Medium-High	High
Duration of the investment	Short	3 Years	5-10 Years	>10 Years

Table 4: Types of asset class and investment duration

A good portfolio allocation can take in consideration, especially for diversified funds, more than one asset class, even according to the type of the portfolio. In this work we will take in consideration, at the really beginning, just equities, then even the presence of a bond, as a risk-free asset. In this way we will understand how the portfolio manager can allocate their resources in order to be more or less against the risk.

2.2 Performance and variance analysis

The return on a financial asset ($r_{i,t}$) is defined as the value obtained from the price change ratio in the interval t; t + 1 and the purchase price:

$$r_{i,t} = \frac{P_{i,t} - P_{i,t-1} + D_{i,t}}{P_{i,t-1}} \tag{2.1}$$

Where:

- $P_{i,t}$ represents the actual selling price
- $P_{i,t-1}$ represents the purchase price
- $D_{i,t}$ represents the dividend detached in the period t

The reference rate is defined ex-post and only if it's actually realized, i.e. if it's an historical value. Instead, it is defined ex-ante when you are in the t-1 period and you are making a forecast. In this last case it is not possible to know exactly the value of $P_{i,t}$ and of $D_{i,t}$. This detail gives the investment a random nature, for example it's conditioned by a random variable R[15]. For simplicity of calculation it is assumed that the variable R is discrete (which takes a finite number of values) and that it is described with a probability function that associates each value of Ri to a certain probability pi.

Since Ri is defined with a finite number n of values, it can be represented with a distribution $r_{i1}, r_{i2} \dots r_{in}$ to which is associated a respective probability $p_{i1}, p_{i2} \dots p_{in}$ to, its expected value can be calculated as :

$$E(R_i) = \sum_{j=1}^{n} r_{ij} p_{ij} \tag{2.2}$$

Therefore, the expected value of a security is defined as the average value of the risk-weighted security returns.

The return on a security may however deviate from its expected value. This probability is captured by the concept of Variance. It is defined as the sum of the squares of the deviations from the weighted average for the relative probabilities:

$$\sigma_i^2 = E[R_i - E(R_i)]^2 = \sum_{j=1}^{n} p_{ij} (r_{ij} - E(R_i))^2$$

However, the variance is difficult to be interpreted because, being the average of squares, it is expressed in a different unit of measurement from the one used to calculate the performance. Therefore, it will be necessary to resort to a more representative index, the Standard Deviation. It can be calculated as the square root of the variance:

$$\sigma_i = \sqrt{\sigma_i^2}$$

In order to simplify the calculations, in the present work a further hypothesis will be made it is assumed that the sequences of the historical returns of a security reflect the future performance of the returns since it is not possible to identify the distribution of the probabilities. This implies that

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[15] Definition held from "Statistical introduction", A.Monti, pag.12

the expected return coincides with the historical yield of the security. This assertion is plausible only considering a context of continuous capitalization of interests, i.e. that the interests obtained in each time interval are reinvested countless times. In this way, in a long-term time horizon, the rate of return is equal to the instantaneous rate $\delta_i(t)$:

$$r_{i,t} = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(1 + r_i) = \delta_i(t) \tag{2.3}$$

Taking advantage from the property of the logarithm that allows to transform the logarithm of a ratio into a difference between two logarithms and proceeding with the calculation of the arithmetic average at any time in a delimited interval between t and t-1, it is possible to carry out the following reasoning:

$$\ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(P_t) - \ln(P_{t-1})$$

The average is calculated in each instant of time t as follows:

$$Mean = \frac{\ln(P_2) - \ln(P_1) + \cdots + \ln(P_t) - \ln(P_{t-1})}{t} = \frac{\ln(P_t) - \ln(P_1)}{t}$$

You can also follow an alternative way using the incremental price ratio:

$$\frac{\ln(P_t) - \ln(P_{t-1})}{P_t - P_{t-1}} = \frac{1}{P_{t-1}}$$

$$\ln(P_t) - \ln(P_{t-1}) = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Where $\frac{P_t - P_{t-1}}{P_{t-1}}$ represents the percentage change in the price in our considered time interval.

It is possible to rewrite the formula just obtained as follows:

$$\ln\left(\frac{P_t}{P_{t-1}}\right) = \frac{P_t - P_{t-1}}{P_{t-1}}$$

The use of the formula 2.3 in a context of continuous capitalization of interests is thus justified.

2.3 Composition of a Share Portfolio

An equity portfolio consists of the linear combination of random variables. As already described in the context of individual securities, also for the portfolio the random variable will be its return $(Rptf)$.

Assuming available a certain number n of securities, each of which will have a given return Ri, it is possible to calculate $Rptf$ as the sum of the individual returns that make up the portfolio by weighing them for the relative percentage that was invested in each of them (xi) :

$$R_{pft} = x_1R_1 + x_2R_2 + \cdots + x_nR_n = \sum_{i=1}^{n} x_iR_i$$

Furthermore, it is required that $\sum_{i=1}^{n} x_i$ is equal to 1, since we want to invest all the available capital and do not want to resort to debt transactions. In the same way, only positive values for x_i should be considered in order to avoid short sales (https://www.investopedia.com/terms/s/shortselling.asp).

As already done in the case of individual securities, also for a portfolio the main information is contained in its expected value and in its variance.

The expected value of a portfolio is given by the linear combination of the securities that compose it, in fact:

$$E(R_{pft}) = \sum_{i=1}^{n} x_iE(R_i) \tag{2.4}$$

A similar argument cannot be applied in the case of variance since in this case it will not only be the variability of the returns of individual securities that will define their value but also the way in which the oscillation of the value of a single security can affect the value of the other securities which make up the portfolio. It will therefore be necessary to introduce a statistical tool capable of capturing these relationships in all the individual pairs that make up the reference portfolio: Covariance (σ_{ij}). It can be calculated as:

$$\sigma_{ij} = E\{[R_i - E(R_i)][R_j - E(R_j)]\} \tag{2.5}$$

The study of the sign of covariance is one of the most important argument during the portfolio creation phase. In the event that this value is positive, the pair of securities considered will have a certain concordance, i.e. a positive change in the first security also causes a positive change for the second security. If instead the sign of the covariance is negative, then the couple taken in analysis will have a discordance, that is a positive variation of the first title will cause a negative variation in the second one. However, the value assumed is conditioned by the unit of measurement taken into consideration. Therefore, it will be necessary to resort to another index that can actually give a more correct information: the correlation coefficient (ρ_{ij}). The formula for obtaining it is as follows:

$$\rho_{ij} = \frac{1}{\sigma_i\sigma_j}E\{[R_i - E(R_i)][R_j - E(R_j)]\} = \frac{\sigma_{ij}}{\sigma_i\sigma_j} \tag{2.6}$$

The correlation coefficient can assume values included in the interval [-1; 1] and will have the following interpretation:

- if its value is close to the ends then the two titles have a strong correlation (positive or negative);
- if its value is close to 0 then the two titles are weakly correlated;
- if its value is equal to 0 then the two titles are independent of each other.

Given these premises it is possible to proceed with the calculation of a portfolio's variance. To simplify the presentation, a portfolio case consisting of 2 securities will be presented first and then generalized with a case having n securities.

The variance of a portfolio can be calculated as the product of three matrices:

- Line vector of the relative weights;
- Matrix of the covariances of the titles that compose it;
- Column vector of the relative weights.

So:

$$\sigma_{ptf}^2 = (x_1 \ldots x_n) \begin{bmatrix} \sigma_{11}^2 & \cdots & \sigma_{1n}^2 \\ \vdots & \ddots & \vdots \\ \sigma_{n1}^2 & \cdots & \sigma_{nn}^2 \end{bmatrix} \begin{pmatrix} x_1 \\ \cdots \\ x_n \end{pmatrix}$$

Please, note that the major diagonal of the covariance matrix consists of the variances of the individual titles.

In the event that a portfolio consisting of only 2 securities is available, the variance will be calculated as follows:

$$\sigma_{ptf}^2 = (x_1 \; x_2) \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} =$$

$$= (x_1\sigma_{11}^2 + x_1\sigma_{12} \quad x_1\sigma_{21} + x_1\sigma_{22}^2) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} =$$

$$= x_1^2\sigma_{11}^2 + x_1x_2\sigma_{12} + x_1x_2\sigma_{21} + x_2^2\sigma_{22}^2 =$$

$$= x_1^2\sigma_{11}^2 + 2x_1x_2\sigma_{12} + x_2^2\sigma_{22}^2 \qquad\qquad (2.7)$$

Adding a further title to the portfolio, the same reasoning can be applied, even if with a more complex calculation:

$$\sigma_{ptf}^2 = (x_1 \; x_2 \; x_3) \begin{bmatrix} \sigma_{11}^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}^2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} =$$

$$= x_1^2\sigma_{11}^2 + x_2^2\sigma_{22}^2 + x_3^2\sigma_{33}^2 + 2x_1x_2\sigma_{12} + 2x_1x_3\sigma_{13} + 2x_3x_2\sigma_{32} =$$

$$= \sum_{i=1}^{3}\sum_{j=1}^{3} x_i x_j \sigma_{ij} \tag{2.8}$$

So, it is possible to synthesize the formula of variance in this way:

$$\sigma_{ptf}^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \tag{2.9}$$

2.4 Optimize a Portfolio: Principle of Dominance

Given an equity portfolio, it is possible to obtain different combinations of expected value and variance according to the choices made regarding the placement of one's investments. Therefore, it is necessary to find that combination that allows to obtain the best possible strategy (for example the maximum possible return for a given level of risk). In this regard, the principle of Dominance is introduced. Assuming you have two portfolios A and B available, you can define A efficient and dominant on B if you respect the following properties:

- $E(R_A) \geq E(R_B)$
- $\sigma_A^2 \leq \sigma_B^2$
- One of the two inequalities must be strong (must apply with the greater or with the strictly smaller)

In mathematical terms, to identify the dominant portfolio it is necessary to set a function by imposing a constraint. Indicating with $f(x,y)$ the objective function and with $g(x,y) = c$ the constraint imposed, we proceed following the method suggested by the mathematician Lagrange:

- Max/Min $f(x,y)$
- with constraint $g(x,y) = c$

At this point the LaGrange function is constructed:

$$L(x,y,\lambda) = f(x,y) - \lambda g(x,y) = c \tag{2.10}$$

Where λ represents the Lagrange multiplier. To solve this equation, it is necessary to set up a system consisting of the three partial derivatives and placing them equal to 0.

Alternatively, it is possible to follow a different resolution method suggested by Markowitz himself. It is in fact possible to set up a similar problem by developing two similar alternatives:

- Set up a system that maximizes the return value for a given variance level;
- Set up a system that minimizes variance for a given performance

In both cases a further condition will then be imposed: the total use of the available resources (without resorting to indebtedness). The above can be summarized with the expression $\sum_{i=1}^{n} x_i = 1$. Furthermore, we consider a context in which short sales cannot be carried out, that is, sell shares of which the property does not belong to us by hoping to buy them at a lower price in a second moment at the price agreed upon delivery to the buyer.

The above can be summarized by setting the following two systems:

$$\begin{cases} \min\limits_{x_i} \sigma_p^2 = \min\limits_{x_i} \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \\ E^*(R_{ptf}) = \sum_{j=1}^{n} x_i E(R_i) \\ \sum_{j=1}^{n} x_i = 1 \\ x_i \geq 0 \end{cases} V \begin{cases} \max\limits_{x_i} \sigma_p^2 = \max\limits_{x_i} \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \\ E^*(R_{ptf}) = \sum_{j=1}^{n} x_i E(R_i) \\ \sum_{j=1}^{n} x_i = 1 \\ x_i \geq 0 \end{cases} \quad (2.11)$$

The above has a remarkable application in the geometric field: all the possible portfolios obtainable through a rational choice of yield and variance generate an area. This section is delimited by a set of combinations that constitute the efficient frontier. All the points belonging to this border represent the dominant portfolios.
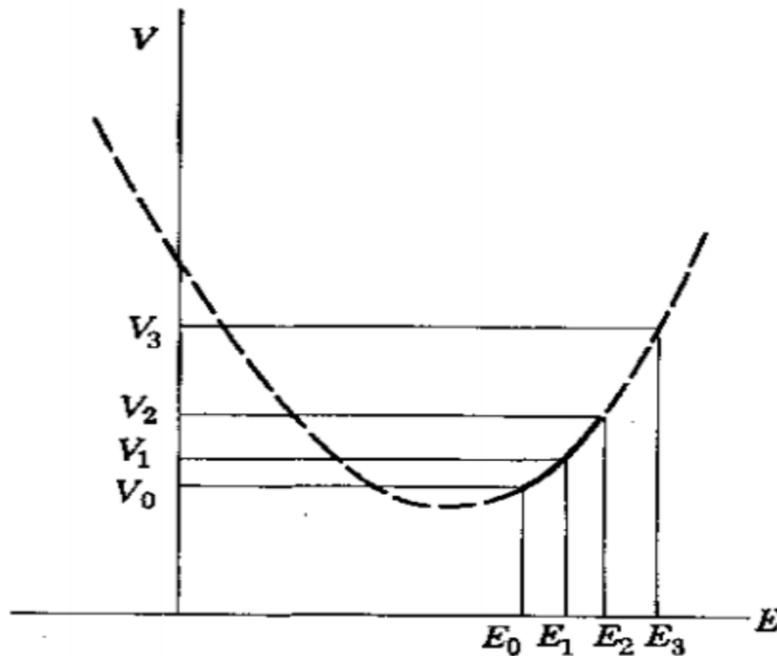

Figure 7: Portfolio Selection

Defined as an iso-mean line the set of all the points obtained by varying the weights of the single titles and keeping constant the yield and iso-variance curves the set of points obtained by varying

the weights of the single titles and keeping constant the variance, it is possible to identify a critical line at their points of tangency. Moving along the critical line it is possible to identify the lower limit of the portfolios selectable by the investor.

At this point it is necessary to understand how to build an efficient frontier and select the combination of portfolio that optimizes the choices of the investor. To simplify the presentation, a portfolio case consisting of only 2 securities will be presented first to then extend the reasoning to a generic valid case for portfolios consisting of n securities.

2.5 Construction of the Frontier Efficient in the case of 2 titles

Suppose you have a stock portfolio consisting of only 2 securities and invest all the available resources in it $x_B = (1 - x_A)$. Knowing the performance of the two titles allows you to calculate the yield of the given portfolio by setting an equation that contains only one unknown:

$$R_{ptf} = x_A R_A + (1 - x_A) R_B \tag{2.12}$$

Since it has been established that the total weight of the securities is equal to one and the formula just presented represents a convex function, it can be stated that the value obtained relating to the portfolio risk is between the risk of the A security and the risk of the title B.

This is also applicable to the expected value and variance of the portfolio:

$$E(R_{ptf}) = x_A E(R_A) + (1 - x_A) E(R_B) \tag{2.13}$$

$$\sigma_{ptf}^2 = x_A^2 \sigma_A^2 + (1 - x_A)^2 \sigma_B^2 + 2x_A(1 - x_A)\sigma_{AB} \tag{2.14}$$

Therefore, the standard deviation will result:

$$\sigma_{ptf} = \sqrt{x_A^2 \sigma_A^2 + (1 - x_A)^2 \sigma_B^2 + 2x_A(1 - x_A)\sigma_{AB}} \tag{2.15}$$

However, as explained in paragraph 2.2, the standard deviation gives an idea of the riskiness of a portfolio but does not take into account the relationship with which the securities which make it up interact with each other. This information is the key to a diversification operation. Therefore, also in this case, it is appropriate to refer to the correlation coefficient (formula 2.5).

Taking the formula (2.6) and isolating the covariance σ_{AB}, it is possible to include ρ_{ij} within the formula 2.15. Therefore, the following formula will be obtained for σ_{ptf}:

$$\sigma_{ptf} = \sqrt{x_A^2 \sigma_A^2 + (1 - x_A)^2 \sigma_B^2 + 2x_A(1 - x_A)\rho_{AB}\sigma_{AB}} \tag{2.16}$$

Resuming what has already been described in the previous paragraph, the correlation coefficient can take values that are limited to the range [-1; 1]. In order to fully understand how the correlation

coefficient affects the construction of an efficient frontier, it is necessary to expose three distinct cases in which ρ_{ij} will assume different values:

- $\rho_{ij} = 1$

In this context, the two stocks are perfectly correlated (positively) and it is not possible to further diversify its share portfolio. Replacing the given value within the formula (2.16) you will get the following expression:

$$\sigma_{ptf} = \sqrt{x_A^2 \sigma_A^2 + (1 - x_A)^2 \sigma_B^2 + 2x_A(1 - x_A)\sigma_{AB}} \qquad (2.17)$$

As it can be easily guessed, having failed ρ_{ij}, this expression represents a simple square root of the square of a binomial. Therefore, it is simplified as follows:

$$\sigma_{ptf} = \sqrt{(x_A \sigma_A + (1 - x_A)\sigma_B)^2} = x_A \sigma_A + (1 - x_A)\sigma_B \qquad (2.18)$$

In geometric terms, to derive a representation of what has been said it is necessary to find the relationship between standard deviation and expected return. To do this the formula (2.13) is taken up again and the common variable x_A:

$$E(R_{ptf}) = x_A E(R_A) + E(R_B) - x_A E(R_B) = x_A E(R_A) + (1 - x_A)E(R_A) \qquad (2.19)$$

So:

$$x_A(E(R_A) - E(R_B)) = E(R_{ptf}) - E(R_B)$$

$$x_A = \frac{E(R_{ptf}) - E(R_B)}{E(R_A) - E(R_B)} \qquad (2.20)$$

Replacing what was obtained in the standard deviation formula (2.18), we obtain:

$$\sigma_{ptf} = \frac{E(R_{ptf}) - E(R_B)}{E(R_A) - E(R_B)}\sigma_A + \sigma_B - \frac{E(R_{ptf}) - E(R_B)}{E(R_A) - E(R_B)}\sigma_B =$$

$$= \frac{E(R_{ptf}) - E(R_B)}{E(R_A) - E(R_B)}\sigma_A + \left(1 - \frac{E(R_{ptf}) - E(R_B)}{E(R_A) - E(R_B)}\right)\sigma_B =$$

$$= \frac{E(R_{ptf}) - E(R_B)}{E(R_A) - E(R_B)}(\sigma_A - \sigma_B) + \sigma_B \qquad (2.21)$$

As we can see from the formula (2.21), an increase in the expected return of the portfolio causes an increase in the standard deviation and, consequently, an increase in the risk. Moreover, as evidenced by the second representation of this formula, the standard deviation can be expressed as a convex combination of the standard deviations of the titles that compose it. In facts, the weight of the standard deviation of the title B is described as the difference between the total weight and the portion associated to the title A.

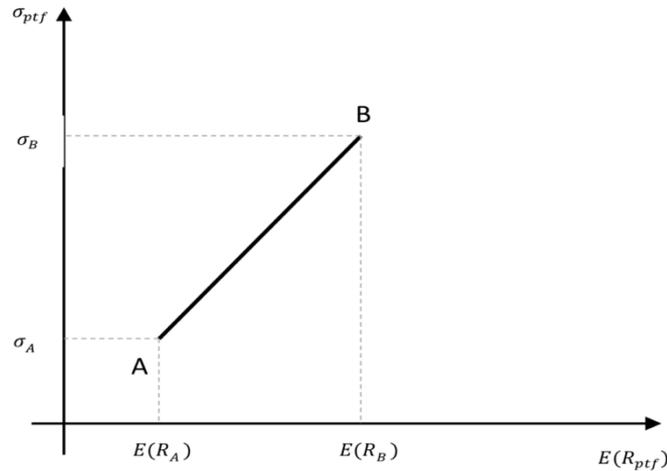Tracing the graph using the expressions just obtained gives the following result:

Figure   8:   Efficient   portfolio

The segment $\overline{AB}$ obtained represents the set of all efficient portfolio combinations. Therefore, it is not possible to identify a combination that guarantees a better performance without further increasing the risk.

- $\rho_{ij} = -1$

This means that the positive change in the yield of a security causes a negative change to the return of the second security. In this context we have the maximum effect of diversification.

Proceeding as already illustrated for the first case, the value -1 of the correlation is replaced in the formula (2.16), obtaining the following result:

$$\sigma_{ptf} = \sqrt{x_A^2 \sigma_A^2 + (1 - x_A)^2 \sigma_B^2 - 2x_A(1 - x_A)\sigma_{AB}} =$$

$$\sigma_{ptf} = \sqrt{(x_A\sigma_A - (1 - x_A)\sigma_B)^2} = |(x_A\sigma_A - (1 - x_A)\sigma_B| \qquad (2.22)$$

Since the variables can take both a positive and a negative value, it is necessary to consider the result as an absolute value.

At this point two distinct results can be obtained:

- If $x_A \sigma_A - (1 - x_A) \sigma_B > 0$, then if $x_A > \frac{\sigma_B}{\sigma_A + \sigma_B}$

    Then the formula (2.22) will result as follows:

$$\sigma_{ptf} = x_A \sigma_A - (1 - x_A) \sigma_B = x_A(\sigma_A + \sigma_B) - \sigma_B \qquad (2.23)$$

- If $x_A \sigma_A - (1 - x_A) < 0$, then if: $x_A < \frac{\sigma_B}{\sigma_A + \sigma_B}$

    Then the formula (2.22) will result as follows:

$$\sigma_{ptf} = \sigma_B - x_A \sigma_A - x_A \sigma_B = \sigma_B - x_A(\sigma_A + \sigma_B) \qquad (2.24)$$

To represent geometrically what has been said we rewrite the formulas (2.23) and (2.24) including within them the expected value of the return using the formula (2.20):

$$\sigma_{ptf} = \frac{E(R_{ptf}) - E(R_B)}{E(R_A) - E(R_B)}(\sigma_A - \sigma_B) - \sigma_B \qquad (2.25)$$

$$\sigma_{ptf} = \sigma_B - \frac{E(R_{ptf}) - E(R_B)}{E(R_A) - E(R_B)}(\sigma_A - \sigma_B) \qquad (2.26)$$

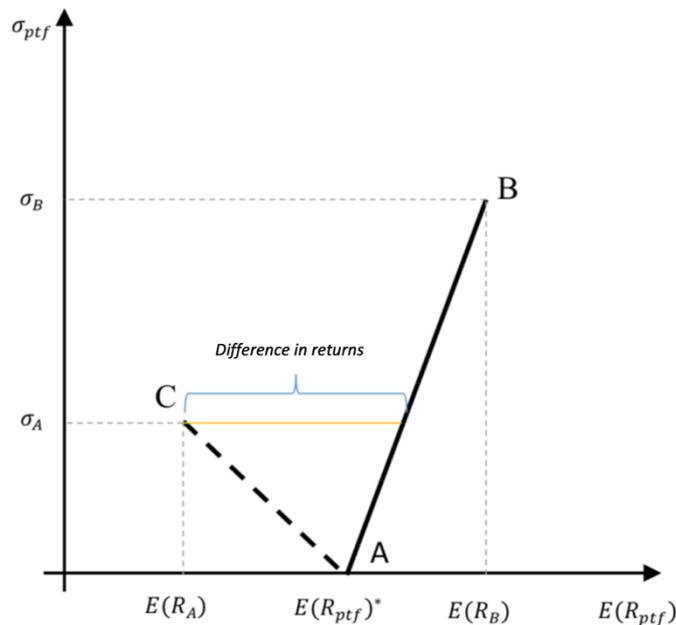At this point it is possible to trace a representation:

Figure 9: Expected value of returns

Segment $\overline{AB}$ represents the set of efficient portfolios that can be obtained by varying the weights of the two securities. Segment $\overline{AC}$ represents instead the set of inefficient portfolios since, as can be seen from the chart, at equal risk it is possible to obtain higher returns.

It is possible to see that the condition of perfect negative correlation turns out to be a case that is difficult to find in real life. In fact, the equity portfolio created at point A would imply the possibility of investing in a combination that guarantees a relevant return at the expense of a zero risk.

In order to identify which is the combination that allows to obtain a portfolio at zero risk, the standard deviation is set equal to 0. Starting from the formula (2.22), we will have:

$$|x_A \sigma_A - (1 - x_A)\sigma_B| = 0$$

$$x_A \sigma_A - (1 - x_A)\sigma_B = 0$$

$$x_A \sigma_A + x_A \sigma_B - \sigma_B = 0$$

$$x_A(\sigma_A + \sigma_B) = \sigma_B$$

Therefore, the proportion will be calculable as:

$$x_A = \frac{\sigma_B}{(\sigma_A + \sigma_B)} \text{ and } (1 - x_A) = \frac{\sigma_A}{(\sigma_A + \sigma_B)}$$

- $-1 < \rho_{ij} < 1$

The peculiarity of this context is that, contrary to the cases of perfect correlation (positive or negative), the expected variance-value relationship does not generate a linear function, but a curvilinear one. This phenomenon is due to the fact that as the value of the correlation coefficient decreases, the possibility of diversification increases:
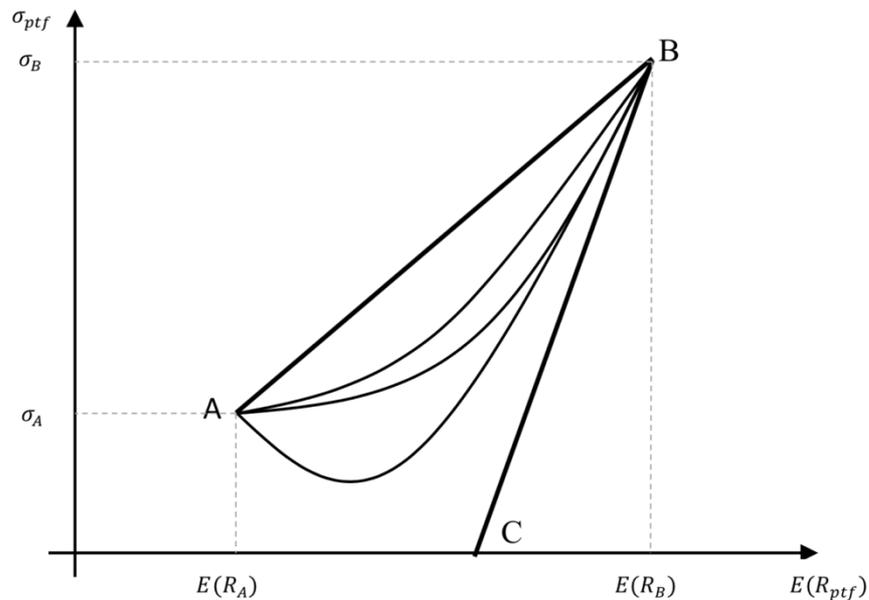

Figure 10: Set of possible combinations

As the correlation coefficient decreases, the curvilinear moves from the segment $\overline{AB}$ to the segment $\overline{BC}$. The area delimited by points A, B and C represents the set of all the combinations that can be obtained using the securities comprising the equity portfolio.

2.6 Construction of the Frontier Efficient in the case of n> 2 titles

Usually an equity portfolio consists of a large number of securities. Without prejudice to the hypotheses of identification of efficient combinations discussed in paragraph 2.3, it is necessary to take into account a further factor: the relationship between the securities. An effective method to solve this problem is provided by the variance-covariance matrix, i.e. a matrix that has the following characteristics:

- It is a matrix having the same number of rows and columns (Square Matrix)
- Its main diagonal is the variance of the securities that make up the portfolio.

The variance-covariance matrix (S) can be calculated with the following formula:

$$S = \frac{A^T x A}{M}$$

(2.27)

Where is it:

- A represents the matrix of refuse of the returns from the average;
- A^T represents the transposed matrix of A;
- M represents the number of returns considered.

As analysed in the previous paragraph, it is possible to graphically represent the realization of an efficient frontier by relating standard deviation and expected value. Contrary to what was observed in the case of a portfolio consisting of only 2 securities, in the case of a larger number of securities it is not possible to obtain a limited area that includes the possible portfolio combinations.

Assuming you have an equity portfolio consisting of three securities (A, B, C) and known their expected return and their standard deviation, it is possible to obtain the following representation:
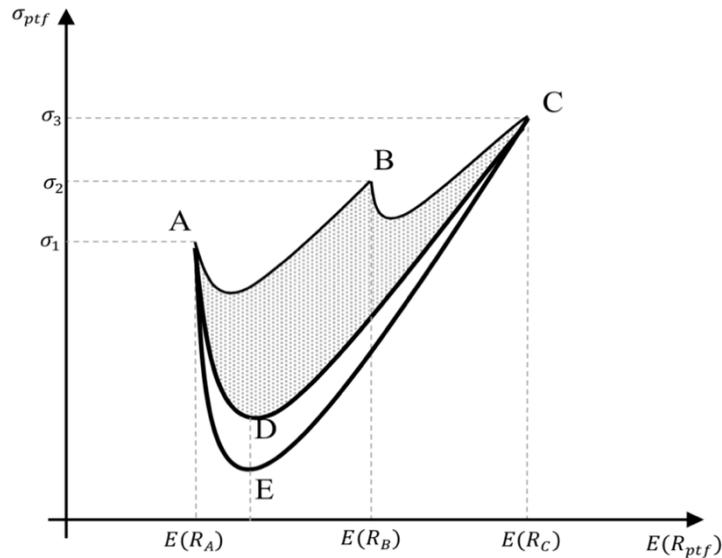
Figure 11: Set of possible combinations

Apparently, the curve defined by the ADC section would seem to represent the set of efficient portfolios, however it is not so since the combinations offered by point B are not taken into account. Therefore, the most appropriate construction is that dictated by the AEC section. All the points contained in this curve outline the set of possible portfolios, while below this sector all the impossible combinations of the titles are placed. Recalling the characteristics of the previously dominant principle of dominance, it can be stated that the efficient frontier is given by the portion of the curve defined by points E and C. Point E also represents the combination of the securities that make up the portfolio with minimum variance.

In order to obtain a portfolio consisting of three securities, it is possible to distinguish the transaction in two distinct phases. The first of these involves obtaining an equity portfolio consisting of only two securities, retracing the steps illustrated in the previous paragraph. The second phase consists of a convex combination from the newly obtained portfolio and the third title taken into consideration.

Assuming to be in a context where the correlation is $-1 < \rho_{ij} < 1$. We can indicate with A and B two titles with standard deviation σ_1 and σ_2 of expected value of the return equal to $E(R_1), E(R_2)$. Remembering that the sum of the weights of the two titles is equal to 1, it is possible to obtain the efficient frontier represented by the following graph:

Point D represented in the figure represents one of the possible border portfolios. It will have a standard deviation of σ_{ptf} and expected return equal to $E(R_{ptf})$.
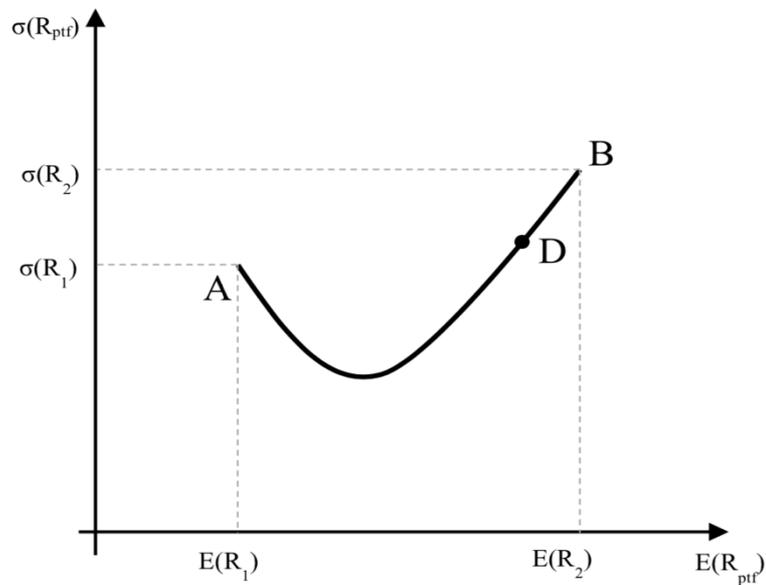
Figure 12: Expected value of returns

Now it is possible to include a third C title with standard deviation equal to σ_c and expected return equal to $E(R_C)$. The introduction of this title in the equity portfolio must guarantee in any case that the total weight of the securities is equal to 1:
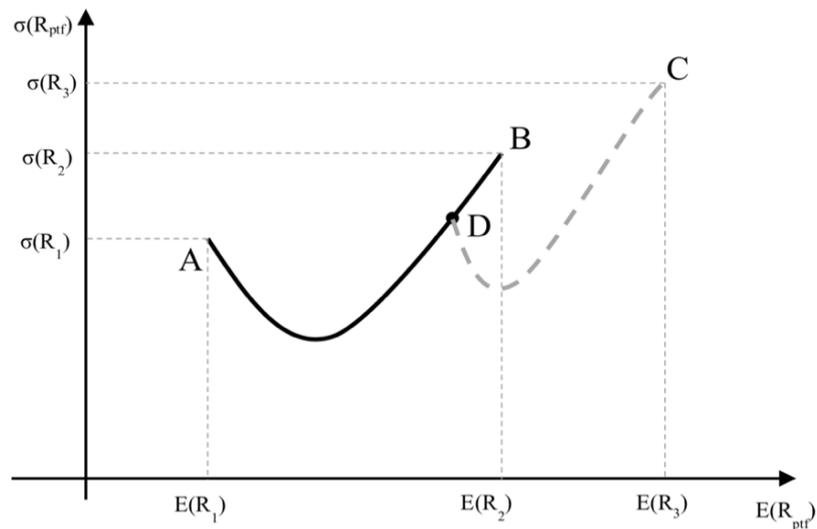
Figure 13: Possible portfolio combinations

Along the curve obtained through points D and C, it is possible to identify all the possible portfolio combinations obtainable by including the title C within the portfolio previously obtained through a convex combination of securities A and B. By indicating with z a generic portfolio obtainable from the border DC, it is possible to obtain the standard deviation using formula (2.17):

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$$\sigma_z = \sqrt{x_1^2 \sigma_{ptf}^2 + (1 - x_1)^2 \sigma_C^2 - 2x_1(1 - x_1)\sigma_{ptf,c}}$$

$\sigma_{ptf,c}$ represents the only unknown of the equation presented. However, it is possible to derive it by analysing the relationship between the returns of the portfolio consisting of two securities and the title C.

Point D represents only one of the possible portfolios obtainable through a combination of securities A and B. Therefore, if one wants to identify every possible portfolio obtainable from the combination of the three titles, it is necessary to take into consideration every single point belonging to the frontier constituted by the titles A and B.

The area obtained in figure 14 represents all the possible portfolio combinations obtainable using the three titles A, B and C.
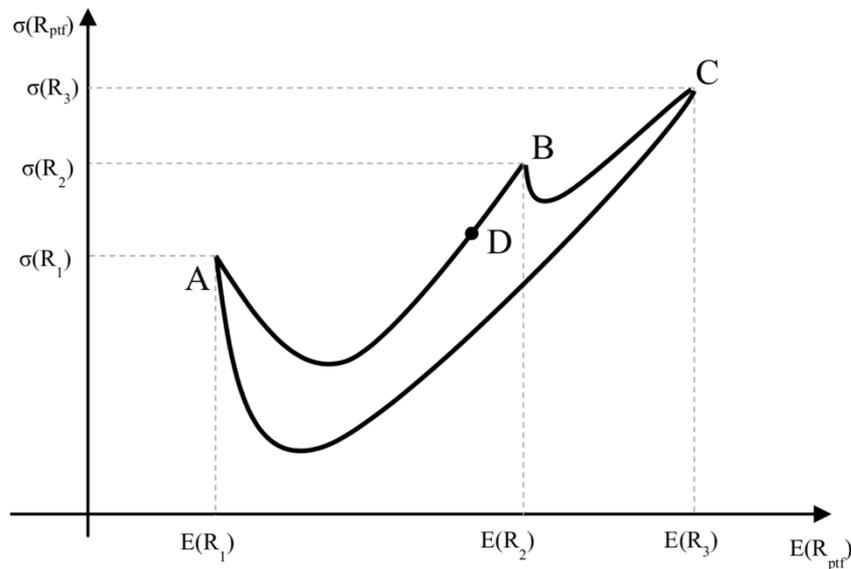
Figure 14: Possible portfolio combinations

As can be seen from the graph, the function obtainable from the combination of the three titles is growing and convex. This implies that an increase in the expected value of the return is accompanied by a more than proportional increase in the standard deviation and, therefore, in the risk.

2.7 Selection of the optimal portfolio

Once the procedure of creation of the efficient frontier has been clarified, it is necessary to make a final statement. The methods analysed in the previous paragraphs allow the investor to have a picture of all the possible combinations that maximize his investment. However, they do not allow to select

the combination that takes into account the preferences of the individual and therefore allows to obtain the risk-return ratio most suitable for its characteristics. To take this into account it is therefore necessary to resort to a subjective variable that captures these elements: the indifference curve.

The indifference curve is the Cartesian representation of the investment choices that provide the individual with the same utility and which therefore make him indifferent to the alternatives obtained. Using a map of all the indifference curves, i.e. a set of curves that summarize for each utility level a set of indifferent choices, it is possible to derive a utility function expected using the expected return as variable. Specifically, the work proposed by H. Markowitz uses an expected squared utility function, which is a function that summarizes the preferences of the individual using two variables (risk and expected portfolio return) and which takes into account his risk aversion. This last yield gives the indifference curves a concave nature since, to compensate for an increase in risk, the individual requires a certain increase in yield.

The expected utility function can be represented as follows:

$$E(U(x)) = E\left(R_{ptf}\right) - \frac{1}{2}\,\lambda\sigma_{ptf}^2 \tag{2.28}$$

Where λ summarizes the degree of risk aversion of the individual.

Graphically it is possible to represent the expected utility function by reporting the mapping of the indifference curves and the efficient boundary ($FEptf$) obtained as described in the previous paragraphs:
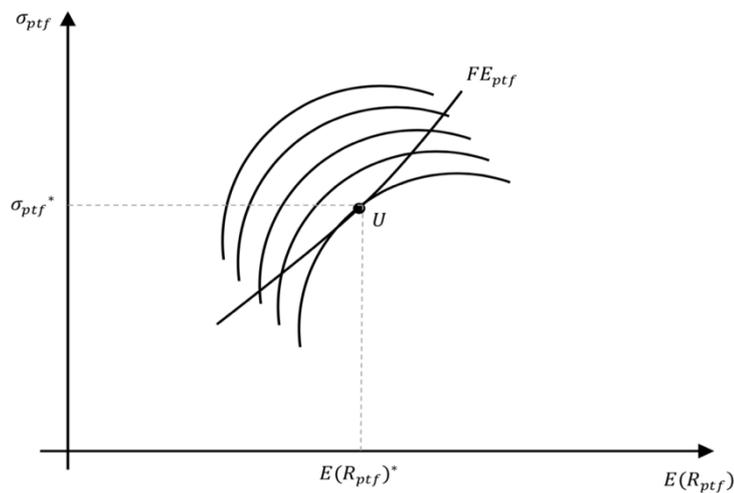

Figure 15: Indifference curves

The point U, obtained as a point of tangency between the efficient boundary and the indifference curve, represents the optimal choice of the investor. This combination takes into account both objective factors (obtaining efficient combinations using the dominance principle) and subjective

factors (recourse to indifference curves to select the portfolio most suited to the characteristics of the individual).

To apply the procedure presented above it is necessary to assign a numerical value to the degree of risk aversion λ. It is therefore necessary for the investor to be able to translate his preferences into analytical terms. Although this is not always easy to implement, the method provided by the expected utility function is certainly the most used in modern finance.

2.8 Limitations of Portfolio Theory

The portfolio allocation theory proposed by H. Markowitz represents the first step of approach to the study of finance. However, there are several limits that make the proposed theory difficult to apply in a real context.

In order to identify the efficient combinations of the securities, only two variables are taken into account (expected return and standard deviation) precluding other possible references. Furthermore, the use of the standard deviation as a risk estimation tool limits the actual forecast of the loss by analyzing only the variations in returns and does not provide a picture of the maximum possible loss.

During the selection phase of a portfolio, only risky securities are considered, making the diversification operation ineffective, since all other investment activities are neglected. Furthermore, a change in the estimate of one of the two parameters (expected return and standard deviation) can cause a noticeable change in the distribution of the securities that make up the share portfolio.

Starting from these factors, various theories have been developed which, although maintaining the starting hypotheses, try to remedy the limits highlighted in the work of H. Markowitz. Some of them will be proposed in the next paragraph.

2.9 Overcoming the limits of Portfolio Theory: the CAPM

As anticipated in paragraph 2.7, the Portfolio Theory proposed by H. Markowitz, although it is of considerable relevance and of great impact during the market analysis, has relevant limits. In order to overcome these limitations, a considerable number of research and studies have been conducted. Among these, the theories presented by three economists stand out for their importance and effectiveness: W. Sharpe, J. Lintner and J. Mossin.

W. Sharpe ("A Theory of Market Equilibrium under Conditions of Risk", The Journal of Finance) collaborated with Markowitz's studies, sharing the 1990 Nobel Prize with him and with M. Miller. He formulated two hypotheses13 additional to those already proposed by the mentor of his theory. It states that all investors have the same expectations on the expected returns and risks of a security and that there is always a security with a certain return and no negotiable risk without being subject to limits.

J.Mossin worked on the stud of W.Sharpe assuming a context in which the uniperiod context proposed by Markowitz was applicable to all investors and that the market was perfect and able to incorporate all the information relating to its financial instruments.

The first step is to include a risk-free security in the equity portfolio. For simplicity of calculation a BOT is used (ordinary Treasury voucher) since it is a title without coupons and has an annual frequency. Having a short duration and being guaranteed by the State (therefore risk-free), it is reasonable to expect a very low return.

It is assumed that a portfolio consisting of only 2 securities is available, one risk-free (BOT) and one risky.

Recalling the formulas (2.13) and (2.15), it is possible to calculate the expected return and the standard deviation of the considered portfolio, remembering that the BOT has zero standard deviation since it is free of risks (consequently also the covariance will be equal to 0):

$$E(R_{prf}) = x_A E(R_A) + (1 - x_A)E(R_f) \tag{2.29}$$

$$\sigma_{ptf} = \sqrt{x_A^2 \sigma_A^2 + (1 - x_A)^2 \sigma_A^2 + 2x_A(1 - x_A)\sigma_{Af}} = \sqrt{x_A^2 \sigma_A^2} = x_A \sigma_A \tag{2.30}$$

Following the same path presented in paragraph 2.3, the relationship between expected return and standard deviation is highlighted. By including the formula (2.20) inside the (2.30) the following result is obtained:

$$\sigma_{ptf} = \frac{\sigma_A}{E(R_A) - R_f} E(R_{ptf}) - \frac{\sigma_A}{E(R_A) - R_f} R_f \tag{2.31}$$

In geometric terms, this function is represented by a semi-straight line:

As can be seen, in this representation x_A can also assume values greater than 1 since it was hypothesized in the context in which it is possible to finance itself unlimitedly at the rate Rf.
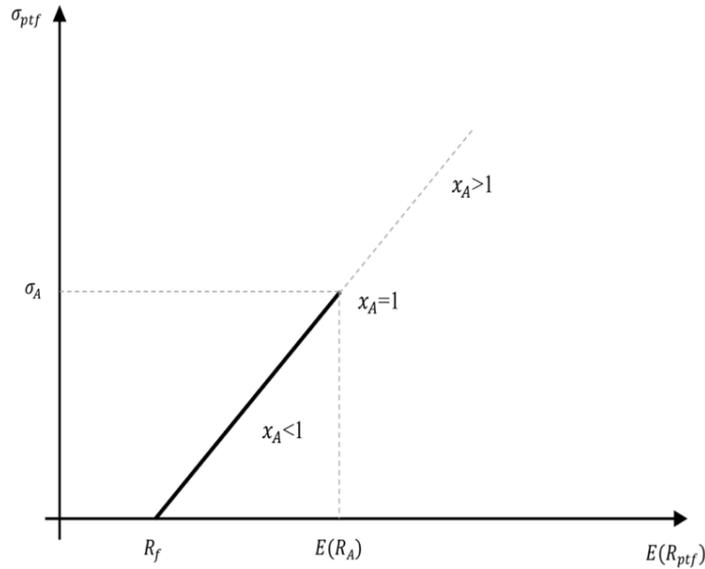
Figure 16: Expected value of returns

The relationship between standard deviation and expected return represents the angular coefficient of the represented line. Therefore, it will assume the value $\frac{\sigma_A}{E(R_A)-R_f}$

Of great importance is the reciprocal of the expression just obtained:

$$\theta = \frac{E(R_A)-R_f}{\sigma_A} \tag{2.32}$$

θ represents the Sharpe Index, i.e. the extra return obtainable from a risky security (with respect to a certain return) related to its risk.

Taking the formula (2.31) it is possible to express the expected return of the portfolio based on the standard deviation:

$$E(R_{ptf}) = R_f + \frac{E(R_A)-R_f}{\sigma_A}\sigma_{ptf} \tag{2.33}$$

From this point, it is possible to state that the expected return can be defined as a combination of the yield of the risk-free security and a portfolio consisting of risk assets.

At this point it is necessary to identify which combinations make the portfolio analysed effective. However, it is not possible to proceed as illustrated by H.Markowitz since in this context the securities are not considered risk-free.

Therefore, it is necessary to identify the relationship between the risk-free security and the efficient frontier obtained by combining risky securities:
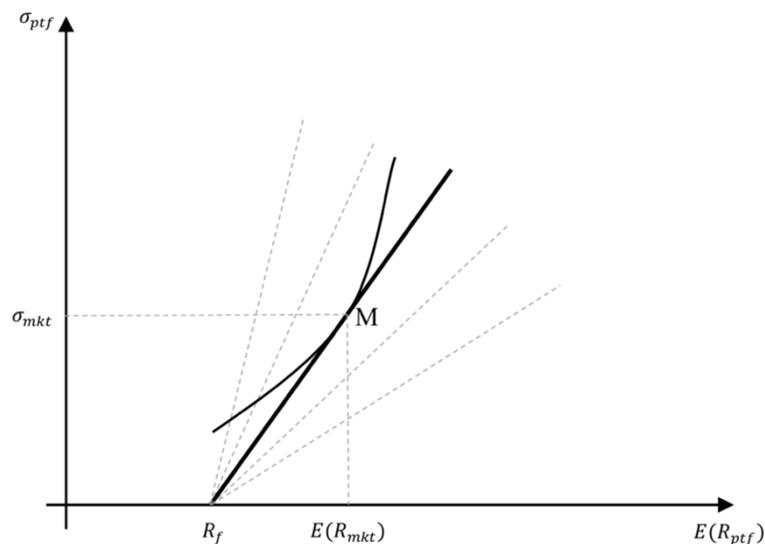
Figure 17: Efficient frontier and risk-free security

Point M represents the market portfolio. It can be defined as the best possible portfolio consisting of risky securities related to the risk-free security.

The plotted curve represents the efficient frontier obtained by combining risky titles according to the model proposed by Markowitz.

The traced half-line represents the Capital Market Line, that is all the possible efficient combinations between the risk-free activity and the market portfolio. In analytical terms it can be calculated by including in the formula (2.33) the expected market return $E(Rmkt)$ and its standard deviation σmkt:

$$E(R_{ptf}) = R_f + \frac{E(R_{mkt}) - R_f}{\sigma_m}\sigma_p \qquad (2.34)$$

Finally, to identify the optimal portfolio for the investor it is necessary to calculate the point of tangency between the CML and the highest indifference curve.

2.10 The distribution and financial mechanism in the financial company

The banking activity, or characteristic activity, takes the form of the so-called "maturity transformation", i.e. the collection of deposits and their transformation into financial assets (e.g. loans) with deferred maturities. It should therefore be noted that this activity consists in the granting of funds to requesting persons (in deficit) deriving from subjects with surplus funds, for this reason

therefore the banking activity is defined as a financial intermediation activity. Deposits represent liabilities for credit institutions, which hold a part invested in financial or real assets, in part it is granted to customers through the subscription of loans or other financing instruments, and finally it is partly held in the form of liquidity to meet its own and customer's operational needs. The remembered characteristics of the liabilities represent a distinctive character of this intermediary and differentiate it from all the others.

the large customer base makes it possible to accumulate personal and exclusive information of the same, for example the degree of risk, the financial position and the ability to manage assets. This allows to eliminate or reduce those information asymmetries at the base of the birth of financial intermediaries (moral hazard and adverse selection), capable of creating many difficulties for the system; the credit institution therefore has a privileged position as regards the containment of agency costs, being able to draw from this information both before the concession of the good / service, and during the contractual process, but also, as already mentioned, can be drawn information concerning the needs of consumers in a perspective of improvement and creation of innovative products capable of responding to different needs.

A further distinctive element of the relationship with the clientele for the institution is the diversification of the risk. Being able to count on a high number of reports, it is possible to bet on certain customers by covering themselves against the risk of insolvency through others, thus exploiting the potential of the market. Finally, the distribution channel appears to be of considerable importance and, as will be explained below, not only for the sale of purely banking products and services, but also for the sale of insurance products and services. In fact, it constitutes one of the most important points with important implications in choosing the partner in the bancassurance sector.

In a perspective of union and collaboration of the banking and insurance intermediary, the characteristics just described cover particular importance, representing peculiarities not found in the business model of the insurance companies and which could constitute a considerable competitive advantage for the latter.

Chapter 3: Insurance and the risk

According to article 1882 of the Italian civil code[16], the concept of insurance is the following: Insurance is the contract with which the insurer, towards payment of a prize, is obliged to re-employ the insured, within the agreed limits, of the damage caused to him by an accident, or to pay a capital or an annuity upon the occurrence of an event pertaining to human life[17].

It is therefore possible, starting from the notion of insurance contract given by the Civil Code, to arrive at a classification of free insurance, that is essentially those stipulated by free choice by individuals, companies or communities:

- Insurance against damages: in this type of contract, the provision of the insurer is a compensation for material damage suffered by the insured (his or other persons) or his assets or situations of civil responsibility. The randomness of the disbursement, which will turn into a cost for the insurer and the premium for the insured, will be a function of two stochastic processes: one concerning the random number of the accidents and the other concerning the randomness of the damage caused by each claim. The duration of an insurance coverage against damages is rather short: it lasts (usually) 1 year. This implies a limited exposure on the markets, therefore a low financial risk for the insurance company but at the same time a high technical risk, due to the risk regarding the claim to be covered.

- Life insurance: in this type of contract the provision of the insurer is a payment of sums upon the occurrence of pre-established events inherent in the life of one or more persons. The randomness of the disbursement will concern the eventuality that the service is provided (if the service must be paid) and the timing of payment (when the service must be provided). As for the quantum, the sums, in a life insurance if they are not already established, are in any case determinable (how much is certain). Unlike the insurance coverage against damages, the duration of a life insurance is medium-long: (usually) minimum 10 - 20 years. This will entail a high financial risk for the insurance company, which will have to manage assets in a medium-long prudential perspective and a substantially limited technical risk.

- Wealth insurance: almost all of these coverages, provided by private insurance companies, do not, unlike insurance against damages, be compensatory, as the insurer's service is almost always commensurate with an amount fixed in the policy and not the actual damage suffered

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[16] Cfr. Art. 1882 from Italian civil code, (R.D. 16 March 1942, n. 262)

[17] This part refers to the first category of private insurance, that is the one against damages (1904 ss. Of the civil code), in which, in addition to the prejudices produced to the insured by a claim, the insurance for damages to third parties also falls. In such cases, the insurer's payment obligation arises when the insured person suffers the damage or when he causes it to the third party.

by the insured (in any case the policies that guarantee a reimbursement of medical expenses are exceptions). The cause of the alteration of the state of health of the insured, which arises determines the performance of the insurer, is given by accident or illness. With regard to the contractual duration, there are single-year policies and multi-year policies.

- Social Insurance: they concern a community, appropriately defined: it can be all the employees of a company, the members of a professional association, the citizens of a State. These policies are in fact provided either by public systems, therefore by the State itself or by social security or health authorities, or are managed directly by private companies, by insurers, banks and other financial intermediaries, in the form of pension funds, with the main purpose of providing to the creation of life annuities payable from retirement.

As we said in the first chapter and according to the new directives, insurances are become more and more important in the risk mitigation activities and in the help of the customer. In particular, this instrument is used to help in facing non-forecasted events that can jeopardize financial wealth of a subject in more than one way.

In this chapter we would like to understand how a financial entity can help a possible investor with the use of insurances instead of derivatives. They both can absolve the function of protection of a particular portfolio but, in the economy of the wealth of a single person or family, this new combination, distributed by a unique supplier, can help the customer in a more integrated way. In particular, the history of life insurance in our system has experienced a phase in which the need to improve the contract's responsiveness to the dynamics of inflation has arisen. The immobilization for long periods of large sums of money entails, at times of high inflation, a relatively rapid loss of value of the capital set aside: in any case a scarce profitability of the same. That saving, that the insured joined in a contract according to his availability, with the prospect of earning the greatest value in a future moment, could not grow according to expectations. So that the life insurance contract was no longer able to guarantee those results that had made it a point of reference for savers, since what had been the main incentive to subscribe was no longer in place.

The offer could not satisfy the demand, oriented elsewhere from the sirens of faster and safer capital increases. They were years in which the stock market knew a consistent development, thanks also to the stories (more or less real) of sudden fortunes realized "in the arc of a night". Insurance companies tried to intercept this new flow of demand, modifying their products to meet market demand: the goal was to create contracts that could offer something more than the mere accumulation in view of a future return. These operations led to an increase in the use of the financial market17. In this sort of "original sin", the configurations of the agreement could include revaluations, equity investments, links to fund results or indices. Indexed policies and evaluable policies were born.

3.1 Concepts of actuarial mathematics

Before focusing on index-linked insurances, we would like to provide some information about policies in general and in particular on the actuarial mathematics that stays behind the different notions that we will be discussed later. With the term actuarial mathematics or "insurance mathematics" we refer to a set of mathematical models related to that particular economic activity consisting in the management of risks transferred to an insurer by economic operators. Different aspects of this activity can be analysed in quantitative terms: in particular it is interesting to study the insurance "demand" by the operators and the analysis of the management of an insurance company. The objective of actuarial mathematics can therefore be summarized in the evaluation of the cost of insurance coverage, a fundamental element in fixing the price, or "premium", of the hedges themselves, in the "management" of this premium over time and in the definition of calculation methods of various types of technical reserves. The various methodologies specific to actuarial mathematics are a consequence of the characteristics of the insurance activity, where financial aspects coexist, arising from the obvious uncertainty of risk management. The probabilistic methods that can be used in actuarial mathematics are very diversified, going from simple discrete models to more complex continuous models, as well as the consideration of stochastic processes.

3.1.1 Insurance contracts and utility's theory

Expected utility theory finds natural application in the insurance field. An insurance policy is essentially a financial operation in which the policyholder reduces or, if possible, cancels the uncertainty of the monetary value of a certain asset exposed to risk. For the sake of simplicity, we will proceed to analyse non-life insurance policies, representing them as uniperiodal contracts and how they would be immediately executed. The case will be that of full coverage policies.

In full coverage policies we consider an individual I, risk averse, whose assets are composed of a certain capital c and an asset exposed to risk, with random value X. Since I is in a financial position exposed to loss, suppose that it stipulates a policy insurance that guarantees full repayment of the damage by an insurance company, against payment of an insurance premium.

- Initial position: $X_1 = c + X$
- Damage: $D = x_m - X$
- Premium: $K = E(D)$

The individual is totally assured: $X_1 \rightarrow X_2$

$$X_2 = c + X + D - K$$

$$= c + X + D - E(D)$$

$$= c + X + (x_m - X) - E(D)$$

$$= c + X + (x_m - X) - [x_m - E(X)]$$

$$= c + E(X) \tag{3.1}$$

The X_2 position is therefore a certain position.

So if we suppose finite x_m, the maximum level of X, that is the value of the good if this was risk-free and its minimum level equal to zero, we can affirm that I, whose patrimony is composed of c and by X, is in a financial position and is exposed to damage of a random amount, which the possible value is between a minimum of zero and a maximum of x_m. Consider the pure premium, that is the cost of the insurance operation: it is now possible to state that, for the individual I, to fully insure against the risk of damage, that is to assume the financial position X_2, means to assume a certain position equal to

$$c + E(X)$$

At this point, can the exchange transaction $X_1 \rightarrow X_2$ be considered fair? Yes, since considering the expected value of the financial position X_1 and the expected value of the financial position X_2 they are identical.

$$X_1 \rightarrow X_2$$

$$E(X_2) = c + E(X) \rightarrow E(X_1) = c + E(X) \tag{3.2}$$

It is a fair operation.

If the exchange transaction $X_1 \rightarrow X_2$ can be said to be fair, the pure premium $K = E(D)$ is called a fair premium. Is the operation however advantageous, disadvantageous or indifferent? The operation, for an individual risk averse, it is advantageous:

Recalling Jensen's Inequality: $E[u(X_1)] < u[E(X_1)]$

If X_2 is a degenerate random variable, then

$$E[u(X_2)] = u(X_2)$$

$$= u[c + E(X)]$$

$$= u[E(X_1)] \tag{3.3}$$

So, if $E[u(X_1)] < u[E(X_2)]$ the operation is beneficial for a risk-averse individual.

This operation, which is advantageous to I, will remain so even if the individual pays a charge or premium $l > 0$, as long as this is lower than the indifference threshold, that is l^*, the value at which

inequality $E[u(X_1)] < u[E(X_2)]$ becomes equality. The maximum acceptable premium l or maximum loading is called risk loading.

3.1.2 Premium loaded

An insurance premium is the amount of money an individual or business pays for an insurance policy. Insurance premiums are paid for policies that cover healthcare, auto, home, life, and others.

Once earned, the premium is income for the insurance company. It also represents a liability, as the insurer must provide coverage for claims being made against the policy. Failure to pay the premium may result in the cancellation of the policy.

The amount of the insurance premium depends above all on the probability that a given event (accident) will occur; to calculate this probability, numerous elements are used, including specific statistical surveys. In addition, the insured must provide the insurer with all the information necessary to enable risk assessment to adequately determine the premium.

Considering l as the premium and l^* as the loading for the risk, $l < l^*$ it is defined, where:

$$l = \eta K \tag{3.4}$$

defining η the percentage loading rate. Therefore, considering the pure premium, the premium and the percentage loading rate, the total premium is defined as the premium charged:

$$\pi := k + l = E(D) + \eta E(D) = (1 + \eta)E(D) \tag{3.5}$$

Therefore, if I pays the loaded premium, the final position follows:

$$X_2' = c + X + D - (1 + \eta)E(D) =$$

$$= c + X + (x_m - X) - (1 + \eta)[x_m - E(X)] =$$

$$= c - [-E(X) + \eta x_m - \eta E(X)] =$$

$$= c + E(X) - \eta E(D) =$$

$$= E(X_1) + \eta E(D) \tag{3.6}$$

given that it is, the individual is assured the certain amount:

$$X_2' = c + E(X) + l$$

Evidently, for I the exchange transaction $X_1 \rightarrow X_2'$ is no longer fair, but not favourable, since:

$$E(X_2') = E(X_1) - \eta E(D) \implies E(X_2') < E(X_1)$$

Therefore, I agrees to exchange a financial position $E(X_1)$ with expected value for a financial position $E(X_2')$ with a lower expected value and is willing to pay a premium l^*, such that:

$$l = E(X_1) - E(X_2')$$

provided the inequality $E[u(x_1)] < E[u(X_2)]$ is satisfied. In this case, therefore, the maximum acceptable loading will be l^* such that:

$$E[u(X_2')] = u(X_2') = E[u(X_1)]$$

Even written as:

$$E[u(X_2')] = u[c + E(X) - l^*] = E[u(X_1)]$$

Applying the inverse function u^{-1} to both members and remembering the definition of a certain equivalent:

$$c + E(X) - l^* = u^{-1}\{E[u(X_1)]\} = m_u(X_1)$$

The maximum acceptable load is defined as:

$$l^* = E(X_1) - m_u(X_1)$$

Furthermore, by expressing as a percentage η of the fair premium, that is in terms of loading rate:

$$c + E(X) - \eta^* E(D) = m_u(X_1)$$

The maximum loading rate is defined as:

$$\eta^* = \frac{l^*}{K} = \frac{c + E(X) - m_u(X_1)}{E(D)} \tag{3.7}$$

Therefore, it can be said that, for $l < l^*$, the not favourable operation is advantageous.

3.1.3 Conclusion about premium

In conclusion, if an individual I risk-averse and with an utility function $u(X)$, is in an uncertain financial position X_1 and wants to exchange it for a certain financial position X_2, he can take out a full coverage insurance policy upon payment of a prize in money. By definition, the fair premium is the one that makes the insurance operation fair. So, if I only paid the fair premium, his final asset position would be X_2. Naturally this operation would be advantageous for I, therefore the individual will be able to accept to pay a loading of the fair premium, assuming the certain final position, provided that he respects the condition of advantage:

$$l \le E(X_1) - m_u(X_1) = \lambda(X_1) \tag{3.8}$$

where $\lambda(X_1)$ represents the indifference premium associated by I to one's risky position.

The maximum acceptable loading to fully ensure a risky financial position is equal to the difference between the expected value of and its certain equivalent; for the concavity of the utility function, this difference is positive[18].

3.2 Index – Linked

The entry into Italy of policies with a financial content is dated around the seventies, but it is an innovation that has far more in time and space: notably the United States of America and the fifties. Even if the reason was the need for insurance companies to re-launch their life products in a period of high inflation, the first contract to be affected by the increase in the financial component was the annual contract (corresponding to our insurance for the life case), in the variable annuity version: part of the prize was allocated to a separate management, merged into separate accounts.
The success of the new formula had also affected life insurance (corresponding to our death insurance policies), which subsequently began to be proposed in the new variable version. Later, a great debate developed about the controls applicable to this new type of insurance: it was questioned whether security (i.e. negotiable instruments representing financial values) and in particular an investment contract, or if they should instead be considered, be classified as normal insurance.

The Index-Linked are life insurance policies which, while maintaining some relevant aspects of traditional life insurance policies, present particular and interesting peculiarities. We are facing with a product that invests in index, in one or more indices, which are today represented by global stock indices, global indices or sector indices.

Index-linked policies are life insurance contracts whose return is linked to the value of a given index or basket of market indices at an agreed maturity. Therefore, these are agreements under which the insurer is obliged to pay a capital or an annuity upon the occurrence of an event concerning human life: this sum is identified on the basis of the proportional relationship between the paid-up capital (premium) and the value that an index or basket of indices assumes at the time of expiry. This is the basic scheme that characterizes the structure of the policy.

In most cases, the event of human life is identified in the survival of the insured on a certain date (life case), with the provision, also, of a benefit for the case of premature death (death case), with which the insurer undertakes to pay a sum that varies according to the value assumed by the index on the day of death. Therefore, as a general rule, index-linked policies are mixed life insurance (with coverage, that is, for both life and death). The relationship between reference entities (index or

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[18] G. Castellani, M. De Felice, F. Moriconi – *Manuale di Finanza*, vol. II, p. 113

basket of indices) and performance, capital or income, can be better understood if we consider both values homogeneously: thus we can quantify the performance by calculating it in units of account equal to those used to define the reference entity. Since in the case of indexes or similar values we consider national monetary units, we will quantify the performance deducted in the contract in national monetary units: on the date foreseen by the policy, we will obtain the amount of the service by multiplying the value of the single unit by the number of the same with which we express the performance.

In this scheme, the extent of the commitment assumed by the insurer will vary according to the value assumed by the reference entity at the time of the death of the insured, in case of death, or at the time of expiry of the fixed term, in case of life. The link between the sum paid by the insured at the time of the subscription (premium) and the number of national monetary units provided by the insurer's performance is therefore a function of the survival hypotheses. Consequently we could define the "demographic risk" at the expense of the insurer as the possibility of deviation between the hypotheses of survival and the actual survival of the insured; the amount of the sum due from the first will vary according to the moment in which the event concerning the life of the second occurs. If this is true, there is equally no doubt that there is a second risk that affects the amount of the benefit, or the value of the reference index. In case of a traditional life insurance policy, the insured undertakes to pay a specific sum upon occurrence of events relating to human life and the business risk is linked to the case in which the occurrence of such events does not coincide precisely with the statistics; in linked policies, unit and index, another variable must be taken into account, namely the fluctuation of the reference value.

The indexes are generally single-premium insurance; this because of the difficulties of having the same title for subsequent issues; for the same reason these policies have a fixed duration.

From the point of view of financial risk, the Index Linked can be pure, guaranteed or partial guaranteed.

In Index Linked, the financial risk is also transferred to the saver, who shares the fate of the investments with the company. If successful, the saver has a gain in the form of an increase in the paid-up capital; otherwise, the saver may also incur in a capital loss. The complete absence of a financial guarantee makes these Index Linked policies pure in the terminology. However, this type is not currently present on the Italian market.

The Guaranteed Index Linked are characterized by the fact that a part of the financial risk is taken by the insurance company. The Guaranteed can guarantee the repayment of the paid-up capital, thus providing a limited coverage of the financial risk, given that only the nominal value is guaranteed. Other forms guarantee the paid-up capital plus a given annual interest rate; the higher the interest rate, the more substantial is the financial guarantee. In this last hypothesis, the company undertakes

to provide a real return, which can be a real return if the rate of return exceeds the rate of inflation. A compromise between the two forms is provided by the Partial Guaranteed Index Linked; here the financial risk borne by the company is further reduced. Basically, this happens if the life company guarantees a part of the paid-up capital at maturity, for example 70%.

In order to be considered an insurance, regardless of the type of assets that distinguish one or the other product, a guarantee must be provided connected to human life. In general, mixed type insurance is provided such as, for example, the payment of a capital in the event of the death of the insured before the expiry of the contract.

3.3 Premium decomposition

We will now see what use is made of the initial premium of the insured to ensure that a certain performance can be guaranteed by the company.

An Index Linked policy is linked to the evolution of an index or a set of reference indexes regardless of the presence or absence of the result guarantee. Normally it is a stock index. We can therefore frame the Index Linked in the range of indexed policies.

The assets covering the commitments assumed with these policies are structured products, as they are constructed through several securities; normally it is an obligation and an option. The dual investment enables the company to count on the contribution of a certain return, at least in nominal terms, of a zero coupon, and in the meantime to enjoy the benefits deriving from having an option in the portfolio for the purpose of obtaining higher return even if not guaranteed.

We will therefore have that the composition in T of the insured guarantee will usually be given by the following term called "call" breakdown:

$$G_T = De^{-\delta T} + N_0 \max\{I_T - K; 0\} \tag{3.9}$$

with N_0 equal to D/I_0, where D is the paid-in price, K is the exercise price of the option quantifiable in $I_0(1 + i)^T$, and where I_T and I_0 are respectively the quotation of the reference security at time T and 0.

The main portion of the premium is for the purchase of a bond; the longer the duration of the contract and the lower this share will be, and similarly the higher the rate provided by the zero coupon and the lower the amount of capital that will be used to pay this obligation. In general, most of the premium amount is used in the purchase of the zero coupon and the residual portion is used to purchase the option.

The purpose of acquiring the obligation is to return the capital at maturity in the event that such a guarantee is provided for the policy.

Once identified:

- the cost of the company (which is made up of commissions, management fees and the cost deriving from the capital immobilization to cover the margin);
- the cost of the death benefit quantifiable with the following expression:

$$P_X^M = \sum_{T=1}^N \frac{l_{X+T-1} - l_{X+T}}{l_X}(1 + i)^{-T+0,5} \tag{3.10}$$

  for a unit of additional capital;

- the profit you intend to obtain with these insurances,

the residual part of the premium is used for investing in options that allow you to bet on stock indices. The option (mostly call type) provides a variable return and therefore cannot be determined before, and grants the right, but not the obligation, to the buyer to buy a specific asset based on a predetermined price within a certain deadline.

Options on the market offer various services linked to particular baskets of shares or stock indexes. The price of the option does not always coincide with the share of the premium that the company has available; the company may find itself in the condition of not being able to buy 100% of the option in which it is interested; it is however possible to take a part of it, in proportion to its financial resources.

By comparing the availability and the cost of the option, the participation rate can be known. In this way a more general form of the breakdown of the prize is obtained:

$$G_T = De^{-\delta T} + N_0 \max\{I_T - K; 0\} \times T.P. \tag{3.11}$$

With:

$$T.P. = \frac{available\ premium}{option's\ price} \tag{3.12}$$

This therefore means that the company commits all its residual in the purchase of a fraction of an option and that the performance will be returned in proportion to that fraction.

We could therefore define the participation rate as the percentage of growth of the index of reference recognized by the Index Bond to the investor.

The retrocession rate, typical of valuable policies, and the typical participation rate of Index Linked and the options associated with them should not be confused. In fact, depending on the amount of the premium available to purchase options, it is possible to obtain a share in the increase in the index

to a greater or lesser extent; such participation in the increase may fluctuate, for example, between 50% and 200%. In any case, if the participation is less than 100%, it does not mean that the life business holds a part of it.

However, there may be a case in which in addition to the participation rate there is also the retrocession rate, just like in the valuable policies.

If this is expected it means that the company subscribes to an index-bond, which provides for example a participation in the increase of the reference index of 150%, but in fact then at the time of transfer of the increase to the insured, it holds back a part, for example 20%.

The lack of financial risk assumed by companies in Index Linked policies explains the close link between the methods of investment of the premium and the services offered to the policyholder; in fact the similarities between the previously analyzed call breakdown and a life event of this type are evident:

$$P.C.V = D \max\left\{\frac{I_T}{I_0}; (1+i)^T\right\} \tag{3.13}$$

in this case the company undertakes to pay at the end of the term T an amount equal to the initial capital plus the change if positive, realized by the reference security.

With regard to the death benefit, the policies all tend to comply, not so much on the coefficients adopted for the calculation, but rather for the flat-rate calculation of benefits:

$$C_T = C_0 q_T (1 + B_X) \tag{3.14}$$

where B_X takes on different values for age groups usually at least five years.

3.4 The distribution and financial mechanism in the insurance company

A lot of economists wondered whether insurance companies can re-enter the financial intermediary branch because, in their typical activity, they are not interposed between two parties and do not relate them, but they represent the counterparty of the insurance contract. By making a parallel with the credit institutions, the latter are identifiable as intermediaries in all respects as they manage exchanges of funds from depositors (in surplus) and requesting parties (in deficit), a nature not found in insurance companies. However, the investment activity, the constitution of the risk portfolio and the mutualisation of the exposures of the insured allow qualifying the insurance companies as institutions that manage an intermediation circuit in the area of risk transfer. The activity in question consists of the preparation of a portfolio of exposures capable of transferring the risk from a single subject to a virtual collective of insured subjects. It should be noted that the represented circuit is

identified as a double circuit of intermediation that winds both between financial exchanges between different states of nature (pure risk transfer), and on intertemporal financial exchanges. For this reason, the activity of the analyzed intermediary can be divided into two different functions, the one called insurance and the investment one.

The distribution of these products is concentrated in different channels, dedicated to different customer targets.

By distribution we do not only indicate the sales channel of insurance products but also the consultative support and the method by which customer relations are maintained. The distribution therefore identifies itself as the moment in which the value proposition is transferred to the customer, formed both by the product itself and by the accessory services typical of the relationship, and by the customer experience.

As aforesaid, given the profitability of the sector, numerous insurance intermediaries have come into being, such as to have to be regulated and listed, in fact legitimizing them, through the transposition of the EU directive 2002/92 / EC. This identifies 5 sections, to which a natural or legal person must be registered to carry out the activity of insurance and reinsurance brokerage[19], listed below:

1. Insurance agents;
2. Insurance or reinsurance brokers (brokers);
3. The direct producers;
4. Banks, securities brokerage companies and Italian post offices;
5. The collaborators of the intermediaries for the brokerage activity carried out outside the premises in which the intermediary operates.

Based on the different categories found in the previous paragraph, we will see how companies collect funds to be returned to insurance products. The insurer collects a series of prizes from the subscribers' public, fees for the obligation to pay a certain amount upon the occurrence of an event concerning human life. It therefore finds itself with current liquidity against a future debt. To guarantee this subsequent and eventual fulfilment, the insurer provides for a series of provisions (some of which are provided for by law), with which to constitute the reserves that will allow it to face the possibility that the insured events will occur, making the payment obligation. These reserves must be distinguished from assets:

- these are part of profits that the company obtains at the end of the year;

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[19] Reinsurance companies are those companies used by insurance companies in order to be insured their selves against big damages or risks for example heart quacking or alluvions.

- the mathematical reserves are, on the other hand, part of the premiums collected with the underwriting of the policies and are set aside to meet future obligations towards the policyholders.

The latter are, therefore, provisions set aside in consideration of the anticipation of revenues with respect to costs. Among the technical reserves, the mathematical reserve is given by the difference between the current value of the commitments to policyholders and the current value of the policyholders' commitments, specific to the life business. In Italy, following the placement of index -- linked policies, insurers covered the related commitments by purchasing "structured" securities, which precisely replicated the services offered to policyholders. Not infrequently, the structured title was not only the asset covering the commitments assumed but corresponded to the reference entity of the services agreed in the policy. Let's take, for example, the hypothesis that the return is linked to the average of the value assumed over ten years by the Dow Jones Index: this means that the capital increases due to the insured will be equal to this value. Therefore, the insurer has an interest in acquiring an asset that at the same maturity will be able to offer the same result: in this way it buys bonds linked to indexed options based on which, in the tenth year, it will be entitled to sell the same at the same average value of the same index to which the policy is parameterized.

In the index-linked, in fact, the premium paid by the insurer (net of taxes and costs) is invested by the company in bonds whose performance is in turn linked to the performance of stock indices, according to the same methods provided for in the policy, and in which there is a guaranteed minimum benefit. It follows that the commitments of the insurer are related to the assets of the company. In fact, the insurer transfers the investment risk entirely to the insured. As long as the yield offered in the policy and the value of the asset purchased to cover remain unconnected, the risk that the value of the asset will not allow the insurer to pay the first one, is entirely charged to the insurer. When the possibility of the asset's loss of value corresponds exactly to the possibility of the loss of value of the yield offered in the policy (because the second is linked to the first), the connected "financial risk" is charged to the insured, which will be the subject engraved by the loss. In fact, there will be no difference between the value of the benefit due and the value of the asset held, because they will be equal; there will be no risk (which would be charged by the insurer) that the asset has a lower value than the benefit due under the policy. This eventuality remains (and therefore the investment risk is at least partially distributed between the insurer and the insured) when the policy provides for a minimum return or at least repayment of the capital: in these cases, in fact, if the asset value, in this case the structured obligation, will be less than the guaranteed minimum or paid-up capital, the difference will be paid by the insurer. The close link between the investment activity of the insurance company and the provision of reserves, in addition to entailing the transfer of the financial risk from the insurer to the insured, entails another reversal, since it is no longer the determination of the commitments to condition the activity of investment, but the

opposite. The investment activity becomes, at this juncture, the conditioning element of the provisioning methods. The assets constituted with the collection of premiums flow into a separate management that invests in structured products, composed as seen by bonds and options, generally devised by specialized companies: not only bonds or fixed-income securities, but also index-linked bonds (Index Bond) or, in any case, similar products already packaged. The insurance company, in fact, often operates in this field almost like an intermediary, collecting savings and investing it in products made by third-party entrepreneurs. The composition of these "packages" can be just as varied and complicated and it is this nature that gives the name of "structured" to the product in which the insurer invests the premium. By virtue of this, the investor (here the insurer) must have the entire amount required at the time of the subscription of the issue of structured products: therefore, the premium collection campaign (i.e. the policy subscription) must take place before of the effective date of the issue. For this reason, the contracts are placed in a time frame prior to the issuance of the policy and only where the necessary sum is reached, will the subscription and (on the same date as the issuance of the structured products) be paid to the premiums. The prize will be paid in a single solution, as is the case for financial investments[56]. It already appears clear here that the mechanism of index – linked policies is articulated, crushed (one might say), on the technicality of the transaction rather than on the formality of the agreement.

4 Bancassurance: a need that comes from the client

The term bancassurance derives from the French neologism bancassurance[20], of which it is the translation in Italian. This term is commonly identified as the phenomenon of selling insurance policies through the banking channel, but as we shall see, this definition limits its meaning.

Over the years, many authors have tried to define bancassurance, Hoschka (1994) defines it as follows:

"This trend towards Bancassurance or Allfinanz refers primarily to banks entering the insurance sector by offering insurance products to their retail customers."[21]

Chatillon, president of the French Association for Credit Institutions, in his definition emphasizes the fact that there are deep connections between banking and insurance, also at an organizational level:

"It is a business strategy - mostly initiated by banks - which aims at associating banking and insurance activities within the same group, with a view to offering common customer service"[22].

A similar definition is presented by Elkington (1993):

"Bancassurance is basically the provision of selling and banking products and insurance products by the same organization under the same roof"[23].

These last definitions appear to be particularly significant as they consider the integration of the activities of the two institutions without focusing solely on the distribution aspect.

As can be seen, this theme has been treated by many authors and economists, but as Molyneux and Genetay point out in their book "Bancassurance", the definition proposed by Swiss Re[24] is the widest and at the same time the most complete. According to this definition

"As a rule, bancassurance can be described as a strategy adopted by banks or insurance companies aiming to operate the financial services market in a more or less integrated manner. In

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[20] The term *Bancassurance* has been deposited by Allegre, Richer Associés (1989)

[21] Genetay & Molyneux, *Bancassurance*, 1998, p. 8

[22] Tribune de l'Assurance, 1993, p.6. This quote came from Genetay & Molyneux, Bancassurance, 1998, p. 8

[23] Genetay & Molyneux, *Bancassurance*, 1998, p. 8

[24] Swiss Re identifies the Swiss reinsurance company Schweizerische Rückversicherungs-Gesellschaft, abroad it is known as Swiss Reinsurance Company Ltd.

practice, the term "bancassurance" is consistently used to describe a new strategic orientation of financial institutions in private customer business"[25]

The implemented strategy can rely on different integrative models depending on the result sought and the characteristics of the companies involved. In fact, therefore, bancassurance is not, as many authors believed, a distribution of insurance products through the banking channel, but rather it is a strategic relationship between the two institutions that operate upstream of the value chain, which inevitably leads to and manifests itself through the distribution channels reaching the final customer.

Bancassurance, as can be seen, emerges from a complex picture dominated by financial intermediaries and their collaboration, seeking risk diversification, product and distribution, thus aiming at customer satisfaction through an approach that is no longer product oriented but market oriented.

The intermediaries mainly involved in the bancassurance relationship are credit institutions and insurance companies. At first glance, two distinctly separate businesses appear, on the one hand, the bank mainly fulfils the monetary functions of financing and deposit, on the other the insurance companies assume a risk and eventually liquidate a certain sum of money established by the contractual conditions. On the other hand, the two intermediaries in question find themselves facing common problems, such as moral hazard, adverse selection, risk assessment and management, customer relationship management, the degree of trust to be preserved and the transparency to be maintained in reference to both the product and the conditions.

The actors involved in the bancassurance relationship are the credit institutions, the insurance companies and the end customer, recipient of the product and service deriving from the collaboration between the two intermediaries.

The integration between the two institutions is certainly an important step towards growth for both intermediaries. Cooperation makes it possible to circumvent the limits of one sector by relying on the strengths of the other, such as the extremely widespread distribution channel of the banking industry, or finding the necessary know-how for the implementation of insurance-social security products, given the customer's increasing need to cover themselves against future economic risks. The collaboration also makes it possible to simplify the processes of the two actors by providing in one place more types of products and services, creating a one stop shopping. Furthermore, the consolidation of the relationship with customers has played an important role in the union between the two institutions, since the client has different interests compared to the experiences of previous years. The bank therefore no longer assumes the mere connotation of a counter with an offer of

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[25] Genetay & Molyneux, *Bancassurance*, 1998, p. 8

"transactions", but increasingly takes on a consulting and support value also at the product level. On the other hand, the insurance company no longer plays only the role of risk management and coverage but also of asset management.

Another fundamental aspect that led to the emergence of bancassurance is certainly the demand for products represented by customers consisted of insurance products very similar to financial products and asset management, therefore with characteristics of both intermediaries, reaching the supply of a global service and defined as wealth management. The response of banks and insurance companies in their union of forces has materialized through the production of highly diversified, simple, standardized and more financial than insurance products such as unit and index-linked policies. The distribution of these products is entrusted to the network of bank branches located in the territory, allowing insurance companies to arrive where it was not possible, except with high infrastructure investments, while for banks the main advantage consisted in being able to reach customers not yet loyal through their own channel. Moreover, this combination of strengths and intent has allowed the customer to find all the answers to financial and insurance needs in one place.

4.1 The distribution and financial mechanism in the bancassurance company

As we said earlier, bancassurance means both the creation of financial-insurance products, thanks to agreements and partnerships between the credit institution and the insurance company, but also the sale of the same through bank branches and other commercials of sales connected to the latter.
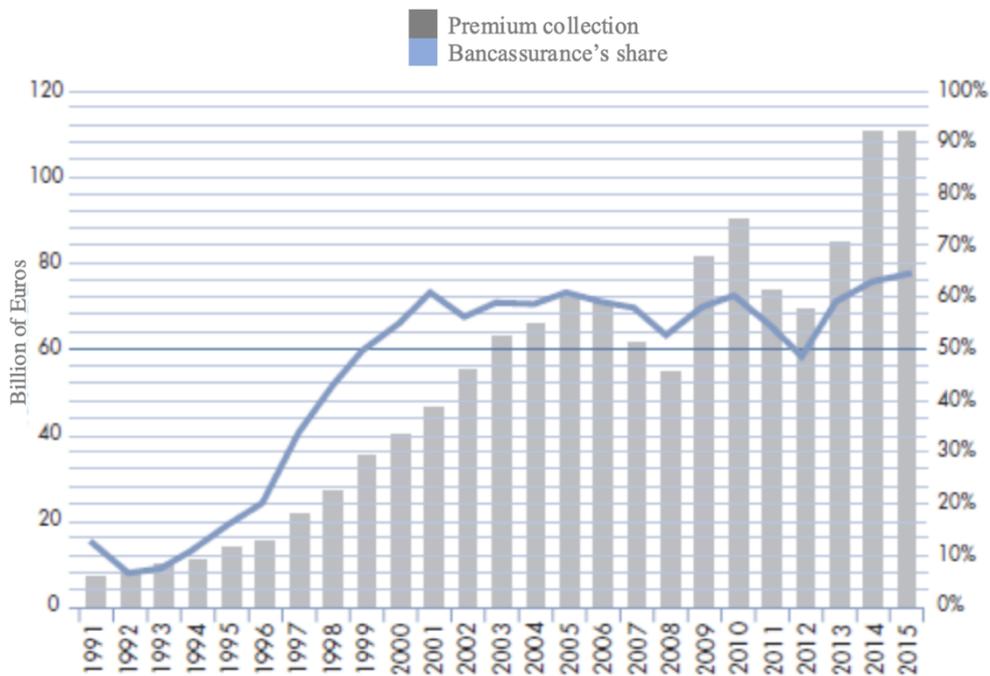

Figure 18: Life premiums and share distributed by banking

Process, product and distribution innovation are running in parallel, drawing each other from the early 1990s to the present day. As can be seen in figure 18, the bancassurance channel has a market share of 63.4% for the sale of life products. It is noted that the percentage held is more than 50% since the 2000s, and as before this level it has recorded exponential growth since the 1990s, the year in which the beginning of the bancassurance phenomenon in Italian soil can be defined, source ANIA.

More specifically, in 2010, in the study conducted by Singhal K. A. and Singh R. "Bancassurance: Leveraging on the Synergy Between Banking and Insurance Industry", three different distribution models were identified in the bancassurance world and are listed below:

- Model of separate sales forces;
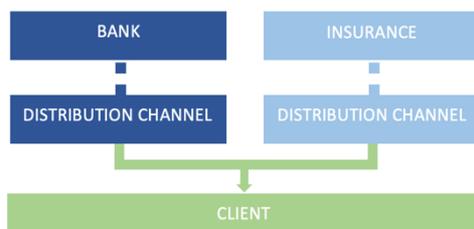- "Hand in glove" model;
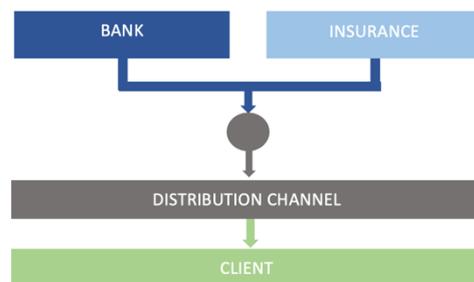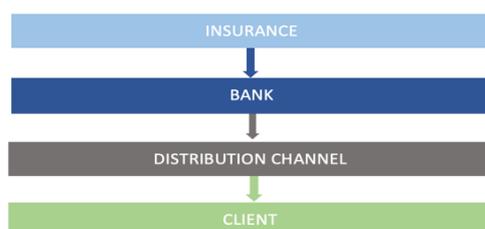- Fully integrated model.

Figure 19: Separate salesforce

The first consists of separate operational management for both companies, without the use of forces or resources belonging to the other institution. This model represents the minimum possible integration in the bancassurance sphere, such as not to share even the information on customers, held by the credit institution.

Figure 20: Hand in glove model

The second model allows the two institutes to integrate on average and is the most used in terms of operational unions and integrations. Unlike the first, in this model the two companies collaborate and work on the same objective together, using and sharing information or resources held individually. For it to be a winning model it must be permeated by a great collaboration, capable of grasping the results of operational synergies.

Figure 21: Fully integrated model

The third and last model categorized by the authors is the fully integrated one. This represents an extreme "hand in glove" model. The collaboration turns out to be very strong in terms of supply production, but as far as the distribution channel is concerned, it is almost absent as it is controlled entirely by credit institutions. It can therefore be identified as a very strong collaboration but

in watertight compartments, which do not communicate with each other. The insurance company is a key player in product creation while the bank in its distribution. This model is very rigid and, for this reason, very risky.

At the moment the "hand in glove" distribution model is the most balanced and therefore the most used and able to capture all the synergies deriving from the collaboration.

However, the bancassurance distribution channel per excellence remains the bank branch. This is a "short three-stage channel on which the control exercisable by the company is medium / low"[26]. The three stages can be identified in the presence of insurance, banking and finally the final customer. The difficult control by insurance companies is mainly caused by the low insurance training of bank employees. However, unlike other distribution methods, the collaboration between the two institutions is of a lasting nature, characterized by unions and company integrations so as to make the organization solid over time.

In the collaboration between the two institutes there are also inherent difficulties mainly due to cultural, operational and training gaps due to different traditional businesses. An example of this is the lack of technical insurance training for bank tellers, a gap that can be filled through a dense training schedule and other training services on bancassurance products. The relationship between the two actors therefore appears to be active and in constant dialogue and evolution, bringing together the missing skills and means to reach the goal of customer satisfaction.

The products sold through this channel to date are mainly the unit and index linked life policies, which almost cover the life segment distributed by this channel. Alongside these products, which are in fact financial in the form of insurance, there are others, which have not had such an important impact on the sector. Reference is made to RC-Cars, to other branches such as home insurance policies, and so on. However, these branches of the bancassurance business have not yet taken hold but certainly represent future market developments, or access to market areas not yet explored through the use of additional distribution channels

4.2 When bank and insurance meet together to create a new entity

The bancassurance agreements represent an extremely important and delicate step for the creation of a more or less intense integration between the two companies where the socio-economic contexts and the regulations in force impact on the decisions and on the variables involved, leading to the creation of different types of contractual agreements. The delicacy of the choice presupposes a

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[26] Maini, *Nuovi spazi per canali distributivi*, Egea Milano, 2008, p. 88.

decision-making and negotiation process between the partners covered by the agreement, which can be deduced from the following analysis.

The areas of contact between the two institutions, which play an important role in defining the collaboration, are represented, as pointed out in Figure 22, by the combined needs of customers, product assortment, distribution channels, logistics and IT services and finally from the management and administration of the entity. The different combinations and intensities of integration of the respective forces lead to the creation of different bancassurance agreements, as we will see later.
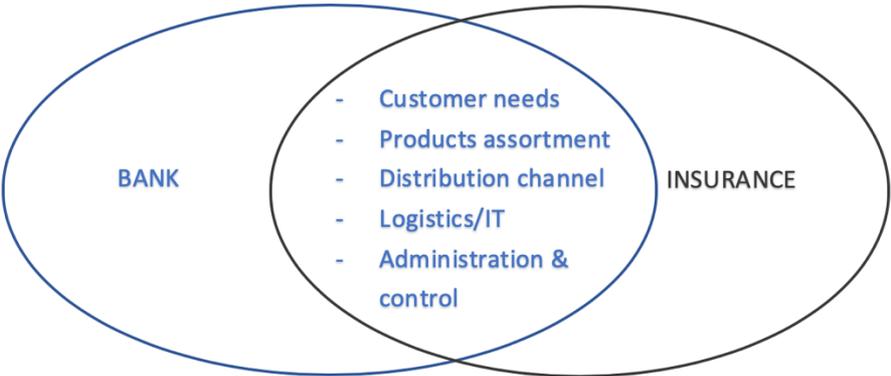

Figure 22: Strategical complementarity in bancassurance's offer

On the demand side, the areas of customer need can be identified in Figure 23. Hybrid needs are present, capable of being filled through collaboration by institutions complementarily. These are attributable to the request for consulting, management and brokerage of the investment.
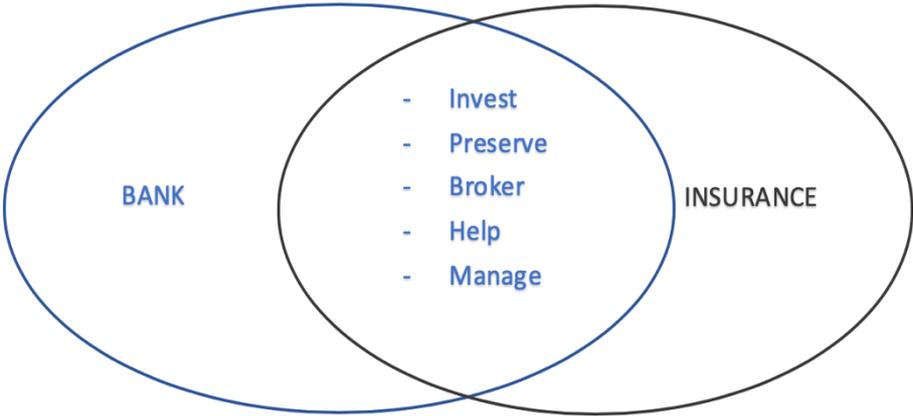

Figure 23: Strategical complementarity in bancassurance's offer

The meeting between supply and demand can be achieved through different paths, aiming at integrations of different strength and complementarity and focusing solely on certain aspects of those mentioned.
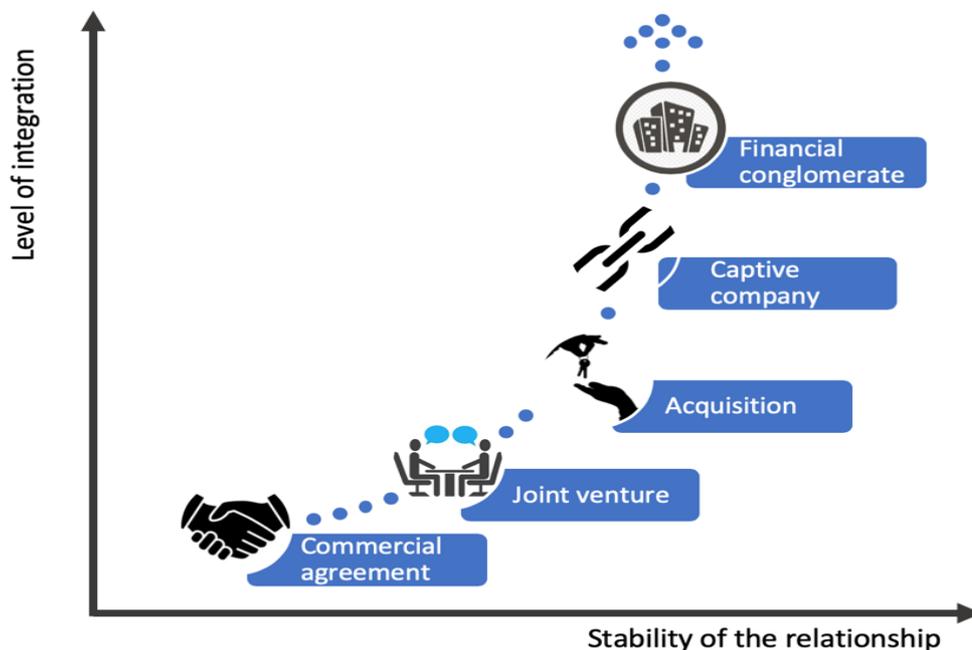

Figure 24: Types of relationship between banks and insurance

As can be seen, therefore, the forms of association and the agreements underlying bancassurance are different, each with peculiar characteristics. The choice of which to implement depends on the willingness of the two partners to face the market.

Each type of agreement considered represents different wills to collaborate, from the simple contractual agreement, to the control by an institution, to the creation of specific companies. The stability of the relationship is proportional to the level of integration and the complexity of the structure. As shown in figure 24, the commercial agreement represents the least possible integration in the bancassurance context, while the financial conglomerate is the standard bearer of maximum stability and integration of the relationship.

4.3 Integrated models to support the business

The birth of bancassurance and the decision to pursue this business model bring with it two obstacles that have an impact in the short and medium term. These two are:
1. The lack of adequate skills to manage a portfolio containing both investment categories (financial and insurance assets);

2. The lack of adequate tools with which to evaluate the convenience, within a portfolio, of an asset class rather than another.

The first problem, which has deep roots, is connected to the heritage of the bank itself, which now, aware of having a long-lasting and consolidated path in the field of investments, suffers from the lack of knowing how to re-orientate and create new skills that are management-oriented combined. This problem can be solved with a strong effort in the area of training, so as to bring valid skills and help to the client that, as we saw in the first chapter, may have difficulty in getting into the world of investment or can provide specific and targeted advice to those clients who know how to navigate.

The second problem, linked to the possibility of having tools capable of helping the manager to carry out his work, has led more and more companies to create innovative and cutting-edge tools in integrated management, composed of banking and insurance elements. The company that proposed an instrument with a high innovative rate is Armundia.

4.3.1 ARMUNDIA Group: a new innovative proposal to bancassurance

Armundia Group is a 100% Italian company specialized in the design and supply of software solutions and specialized consultancy for the banking, financial and insurance ICT sectors. The company is today an international reality in continuous growth, with projects and consultancy in eleven countries divided into two continents and has implementations in more than seventy financial and insurance institutions. The group controls the company Armundia Srl, based in Rome, which deals with "Research and Innovation" in the reference sectors and Armundia Factory, based in Tirana, which deals with providing software development services for the parent company and for the its customers.

4.3.2: Advisory360: the digital platform

What the company promises to propose to the public is the integration of application services in a single advisory framework logic. This therefore entails the possibility of extending the functional coverage of the two contexts through services:

Figure 25: Armundia Advisory360

- Offering multi-branch insurance instruments in the Finance world
- Profiling Regulations for Insurance instruments (MiFID II, IDD)

As we have said before, considering when expressed in this paper, the Banks will be taken as point of sale - Distributors. This is because they exploit their capillary skills and direct relationship with customers.

The solution is a component platform designed to interact and integrate the growth process of a modern reality in a sustainable way. The implemented business model is the cross-advisory in order to support the offering of different assets through a single digital experience. The financial coverage and the target users can be seen in the following scheme:
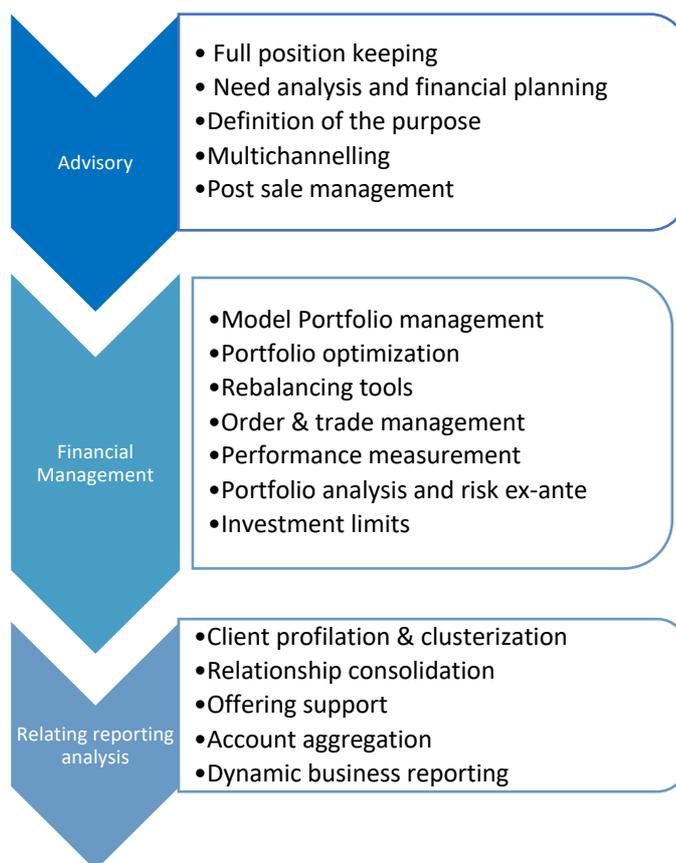

Advisory
- Full position keeping
- Need analysis and financial planning
- Definition of the purpose
- Multichannelling
- Post sale management

Financial Management
- Model Portfolio management
- Portfolio optimization
- Rebalancing tools
- Order & trade management
- Performance measurement
- Portfolio analysis and risk ex-ante
- Investment limits

Relating reporting analysis
- Client profilation & clusterization
- Relationship consolidation
- Offering support
- Account aggregation
- Dynamic business reporting

Figure 26: Financial coverage

Core functionalities are a key focus point for this new platform and are completeley committed to support the end user in providing the most suitable offer to the client. In particular, the two aspects that can be highlighted are:

- Support to the operative workflow of the banker:
  - Management and planning of the meetings with the client, in particular with the integration between systems with the automation
  - Allerting & Trigger events both internal and external
- Analytical support to banking activities
  - Customized banker dashboard and widget

- o Advanced client view
  - ▪ Financial position (consultancy, GPM and "exection")
  - ▪ Insurance position
  - ▪ External positions
  - ▪ Account aggregation and provate view

Every part of this platform is by desing MIFID and IDD compliant, even because it permits to the banker to have a more freedom to choose within the gamma of products provided by the market according to the need specified by the clients. This can be done thanks to:

- Questionnaires
- Calculation and acquisition of profiles and scores
- Advisoryand risk evaluation
- Engine of adequacy, appropriateness and target market

As said before, the analysis of need and the plaaning of meetings can be done thought and algorythm that permits to evaluated the performances and the risks or early warnings related to each single client.

On the other side, the advanced proposition comes thought:

- Multi Modelling (Portfolio management, asset allocation, Optimization and single orders)
- Multi Asset (action plans and Corss- Selling)
- Order Management Integration
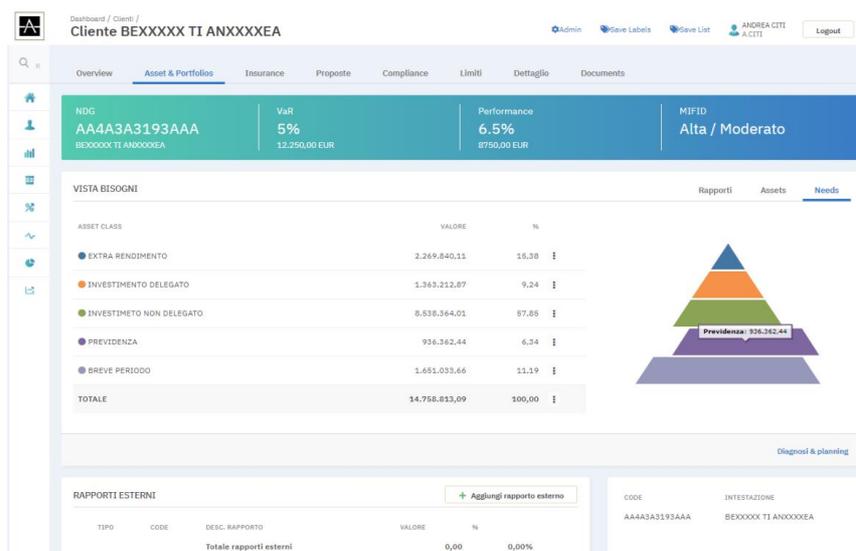- Multi platform Execution (Mobile, Client, Branch session ...)

Figure 27: User view

The part on the banker side permits us to see how the operator can shape the most suitable offer:
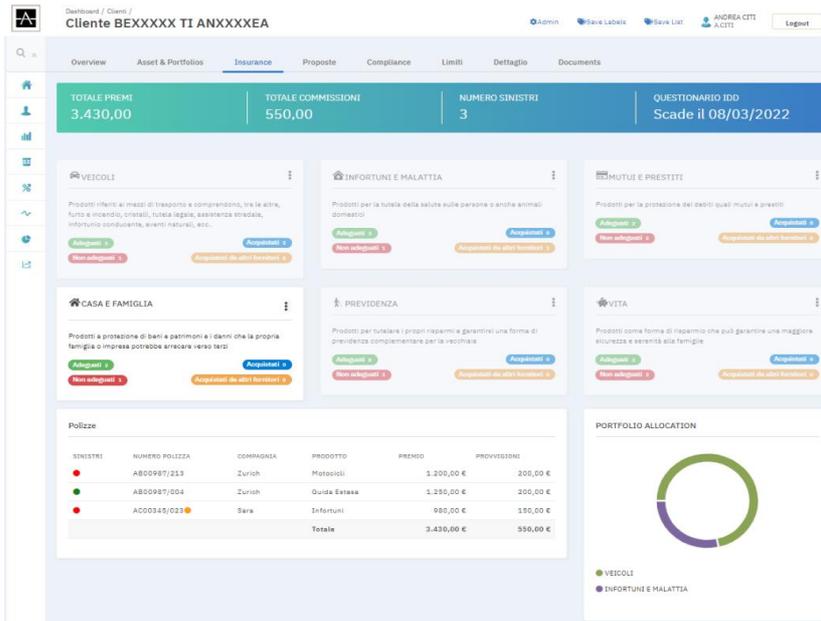

Figure 28: User view

- Banker analytic tool on all dimensions of business and consulting process
- Aggregated view on clients, products and processes
- Advanced reporting tool in order to satisy as better as possible the needs of the clients

As said before, the Armundia's approach to project management is the result of thirty years of management's experience in delivering complex projects, nationally and internationally, filtered through the meshes of the most modern methodologies. All this is accompanied by a deep knowledge of business processes and new technologies that make Armundia a software house and at the same time a management consulting company.

The main objective, which Armundia has always placed in the course of its history, is to adopt a flexible and pragmatic working method, aimed exclusively at putting its customers in a position to benefit from the solution in certain, and above all, short times. The involvement of the customer's resources in the design phases and the continuous iteration between the work teams guarantees the excellent opportunity to acquire Know-How from the first day of the project.

The principles around which the Armundia work method is based are based exclusively on the awareness of the fact that the objectives to be achieved are those of the client and any effort must be aimed at their realization. On this awareness work is organized with the conviction that:

- The relationship between the people of the respective teams is the best resource of a project;

- The close collaboration and the predisposition to a flexible approach, ready to change, increases the realization capacity and the possibility to quickly reach objectives important rather than the constant verification of contractual obligations;
- A strong commitment of customer resources and constant sponsorship of their own management are essential ingredients for the success of the project.

Obviously Armundia has well in mind the types of risks that lurk in each project, especially in those with high strategic value for the customer, on matters of extreme financial sensitivity. Their correct evaluation and the preparation, from day zero, of all the necessary prevention and mitigation actions are basic elements of the design approach that is offered to the customer.

5 Conclusions and suggestions

At the end of this paper we can say that bancassurance has a very recent path and history, this phenomenon, in fact, is starting from the 90s, years from which it began a very rapid process of evolution and innovation, coming to cover at present an important market share in the insurance and economic-financial field.

Altogether the actors identifiable in the bancassurance relationship can be grouped into three figures:

1. the credit institution;
2. the insurance company;
3. the client.

The first is characterized by having an advantageous information position thanks to the active role in managing customers' savings, a great ability to attract customers, credibility, trust, a solid and widespread distribution channel and a relationship with the customer repeated over time.

Insurance companies, in turn, have their own characteristics such as indemnity, the presence of relevant information asymmetries, a relationship with non-lasting customers, the offer of various products such as non-life insurance, life insurance, funds pensions, up to products with highly financial content, such as index and unit linked policies.

The third actor is the customer, representative of the bancassurance application. It appears to be diversified mainly due to three factors: the personal savings trend, the growing technical information and the demographic changes, which impact on its needs. But from his, in the face of the recent financial crises that have compromised his confidence and propensity to invest, he also has new and closer security and prevention criteria.

The similarities of the two intermediaries have created the basis for a possible collaboration. Banking and insurance, although different, have points of contact, such as the nature of the very similar production process. The rapprochement between the two institutions was also made possible thanks to the rapidly evolving economic context, process and product innovation, technological development, the internationalization of markets and the search for a "one stop shopping" .

At the banking level, the rapid acquisition of insurance know-how, the expansion and diversification of the offer is significant, without incurring high costs and greater customer loyalty. In addition to this from the insurance point of view the advantages are many, such as the possibility of distributing products and services on a large scale, reaching unexplored market shares and the use of a solid and

widespread distribution channel. Finally, the customer has access to innovative, differentiated and customizable products, which reflect their needs. Furthermore, the value proposition provided by the insurance segment is represented by index and unit linked policies, by the advice provided by bank branches and Private Centers, by the tax, operational and legal advantages for the insurance instrument and by the contractual options that can be activated at the time of subscription.

As we have seen in the previous chapters, the aggregation between the credit and insurance institution can take place in different ways, the choice depends on the strength and depth of the integrative link sought.

The key resource par excellence is represented by the commercial agreement, with the resulting limits: lack of insurance know-how, lack of customer information, lack of improvement in brand awareness and lack of union of forces and intentions in production and distribution. Consequently, the key activities are purged of training and training sessions. Finally, the cost structure is different above all in the section concerning the distribution channel, using one owned by third parties and therefore more expensive.

In recent years, the race towards the digital world and the Internet of things has had a strong impact on the bancassurance sector, favoring the development of new production techniques, customer relationship management and distribution. The internet channel will be even more protagonist in the bancassurance of the future, not only providing a virtual showcase of products, but giving the possibility to subscribe and manage the insurance policy process independently. In parallel with this development, according to the claims of the big bancassurance players, there will be a concentration of different financial-insurance services in one place, a "one stop shopping", located in any public place and not only through the intermediation of employees banking.

Despite the rapid inclusion of the internet in the bancassurance world, it is believed that traditional channels will not be eliminated but will pursue advisory quality and niche product distribution objectives, which need support for sales.

Future developments also lead to a search for products that are no longer connected to others but stand-alone, especially in the areas of social security and health, a very delicate sector that could act as a driving force for bancassurance.

The links between banking intermediaries and insurance companies are destined to be strengthened, constantly cooperating and innovating; the boundaries between the two intermediaries will be increasingly subtle, becoming confused in certain areas, constantly redefining evolutionary dynamics and strategies.

This will lead to increasingly hybrid institutions integrated with each other that will give the customer more and more innovative and refined products.

The hope is that banks and insurance companies, in deciding to combine their activities to pursue reasonable profit logics, and the Supervisory Authorities that monitor the good conduct of the latter and the market, always look at the real interests of small consumers, increasing the transparency and correctness of their operations, their behaviour and the information on the products / services they are preparing to provide to customers, acting more ethically on the markets and thus setting in motion a virtuous circle where no one will be the only winner or the only loser, where the process of interrelation between markets and sectors of financial intermediation will benefit the entire economic-financial system as a whole, in terms of greater efficiency and stability.

In this case, Armundia will surely play and active role inside the market as a valuable player due to the innovation proposed to the market.

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