



POLITECNICO

MILANO 1863

Department of Electronics, Information and Bioengineering
Master degree in Automation and Control Engineering

DEVELOPMENT OF A HYBRID MPC TECHNIQUE TO INCREASE THE ENERGY EFFICIENCY OF A BUILDING HEATING SYSTEM

Thesis presented by:
Federico Ceruti, 884079

Supervisor:
Prof. Luca Ferrarini

Correlator:
Ing. Riccardo Babini

2019/2020

PAGE INTENTIONALLY LEFT BLANK

INDEX

INDEX	3
RINGRAZIAMENTI	5
SOMMARIO	7
ABSTRACT	9
1-INTRODUCTION	11
1.1 Thesis objective	11
1.2 State of the art.....	11
1.3 Document structure.....	15
2-MODELING	17
2.1 Heat Exchange	18
2.2 Thermal models	20
2.2.a Radiator.....	20
2.2.b 3-States Model of the Basement.....	22
2.2.c 4-States Model of the First Floor.....	24
2.2.d 4-States Model of the Second Floor	27
2.2.e 9-States Model of the Entire Building	30
2.4 Chapter 2 Conclusions.....	34
3-PARAMETERS IDENTIFICATION	37
3.1 Grey-Box Models.....	38
3.2 Data analysis.....	42
3.3 Parameters estimation.....	44
3.3.a First floor	44
3.3.b Second floor.....	47
3.3.c Basement.....	50
3.4 Analysis of the Results	52
3.5 Chapter 3 Conclusions.....	54
4-HYBRID MODEL	57
4.1 Hybrid Systems and Mathematics	57
4.1.a Propositional Calculus and Linear Integer Programming	61
4.1.b Mixed Logic Dynamical Systems.....	64
4.2 HYSDEL-Hybrid System Description Language-	67

4.2.a Language Description	70
4.2.b Hybrid Model of the Building	72
4.2.c Simulation of the MLD and PWA systems	75
4.3 Chapter 4 Conclusions	78
5-HYBRID MPC	79
5.1 Model Predictive Control	79
5.1.a MPC of Linear Systems	80
5.1.b Industrial MPC Algorithms	85
5.1.c Predictive Control of Non-Linear System	87
5.1.d MPC Algorithm and Stability System	87
5.2 Model Predictive Control of Hybrid Systems	89
5.2.a Convergence of Hybrid MPC	90
5.2.b Computational Aspects of Optimal Control of Hybrid Systems	91
5.3 Case of Study: Simulations, Comparisons and Analysis	92
5.3.a Base Case: System Controlled through Logic On-Off	94
5.3.b Hybrid MPC Simulation-1: $T_s=60s$, $N=2$	97
5.3.c Hybrid MPC Simulation-2: $T_s=100s$, $N=6$	98
5.3.d Hybrid MPC Simulation-3: $T_s=600s$, $N=3$	99
5.3.e Hybrid MPC Simulation-4: $T_s=900s$, $N=4$	100
5.3.f Analysis and Comparison	101
5.4 Performances Analysis with Different Constructive Features of the Building	101
5.5 Performances Analysis with Different Temperature Set-Point	103
5.6 Robustness Analysis	104
5.7 Chapter 5 Conclusions	106
6-CONCLUSIONS	107
6.1 Main Achievements	107
6.2 Future Developments	108
LIST OF TABLES	111
LIST OF FIGURES	112
BIBLIOGRAPHY	114

RINGRAZIAMENTI

Un ringraziamento particolare va al Prof. Luca Ferrarini, per avermi permesso di svolgere questo progetto di tesi ed essersi mostrato sempre molto disponibile. Un sentito e profondo grazie a Riccardo, ormai considerato un amico, punto di riferimento fondamentale in questi mesi tramite i suoi suggerimenti preziosi, la sua pazienza e la sua comprensione.

Un grazie di cuore a papà Agostino e mamma Monica, per non avermi mai fatto mancare il loro sostegno e il loro affetto in tutti questi anni, soprattutto nei momenti più difficili, per il loro esempio, per conoscermi meglio di chiunque altro, per avermi reso estremamente felice nel vederli soddisfatti ed orgogliosi, per avermi reso la persona che sono.

Un grazie a Benny e Samu, essenziali nella mia quotidianità, per il nostro legame dal valore inestimabile.

A tutto il resto della mia famiglia, per l'appoggio datomi e mostratomi e per avermi sempre spronato e spinto a raggiungere i miei obiettivi.

A Case e Ema, compagni di avventure durante quest'ultimo anno e con cui ho stretto un rapporto di amicizia speciale.

Ai compagni di Liceo: Andre, Bosa e Jappy, indispensabili per la mia crescita.

Agli amici più cari e autentici, a quelli di sempre e a quelli più recenti: Civo, Fainal, Facchettino, Marchino, Moz, Picco, Riz, Simo, Ste e Topo. Da ognuno di voi ho ricevuto un contributo particolare per arrivare fin qui, con le vostre parole, con i vostri consigli, con la vostra capacità di ascoltarmi e di capirmi, con i vostri abbracci, con le risate e i sorrisi condivisi. Siete una spalla su cui poggiarmi nei momenti più difficili ma, soprattutto, presenza fondamentale con cui condividere quelli più felici.

Al nonno Giuseppe.

PAGE INTENTIONALLY LEFT BLANK

SOMMARIO

La presente tesi è dedicata allo sviluppo di un MPC ibrido nel contesto del controllo dell'impianto di riscaldamento a radiatori di un edificio a 3 piani. Questa strategia di controllo è implementata con il fine di minimizzare la potenza termica scambiata dai radiatori con la stanza garantendo allo stesso tempo il comfort termico dell'utente. Si è scelto di generare un MPC ibrido, poiché il modello, in grado di descrivere le dinamiche che regolano gli scambi termici che avvengono all'interno delle diverse zone, è ibrido. È stata considerata una sola zona per piano, quindi 3 zone totali, ed un radiatore equivalente per zona. Il modello termico è stato sviluppato a parametri concentrati ed è sufficientemente dettagliato da riuscire a descrivere le dinamiche dominanti del sistema; inoltre questa scelta modellistica permette una maggiore rappresentatività fisica dei risultati sulla base dei quali la strategia di controllo predittiva sarà sviluppata. Inizialmente, il modello è sviluppato in ambiente Matlab in cui è stata effettuata un'identificazione di parametri ignoti. I parametri identificati sono quelli relativi alle caratteristiche termiche di pareti, pavimenti, tetto e i coefficienti correttivi della potenza termica emanata dai radiatori. L'identificazione è fatta con un modello Grey-Box, analizzando i 3 piani separatamente, per poi riportare i parametri stimati nel modello complessivo dell'edificio. Successivamente, in ambiente Hysdel è descritto, tramite proposizioni logiche, il comportamento ad isteresi caratterizzante l'andamento della temperatura di ogni zona e quindi il modello è reso ibrido a tutti gli effetti. Nel descrivere il comportamento ad isteresi, sono sfruttate variabili booleane ed espressioni relative al calcolo proposizionale, messe poi in relazione con le dinamiche termiche continue per poter creare un modello di tipo MLD (Mixed Logical Dynamical) sfruttabile dal MPC ibrido.

Dopo aver sviluppato l'MPC ibrido, vengono svolte alcune simulazioni, ed i risultati ottenuti sono confrontati con quelli ricavati dallo stesso sistema ibrido regolato da una semplice logica ON-OFF e vengono evidenziati i vantaggi, in termini di risparmio energetico, raggiunti con la strategia del controllo predittivo.

Infine, vengono effettuate due analisi delle performance basate su differenti caratteristiche costruttive dell'edificio e su differenti set point di temperatura imposti. Per concludere viene effettuata un'analisi della robustezza del sistema di controllo predittivo sviluppato.

PAGE INTENTIONALLY LEFT BLANK

ABSTRACT

This thesis is focused on the development of a Hybrid MPC for the control of the heating system with radiators of a 3-floors building. This control strategy is implemented to minimize the thermal power exchanged by the radiators with the room, guaranteeing the user's thermal comfort. To generate the hybrid MPC, it is required to build a hybrid model, able to describe the dynamics that govern the thermal exchanges that occur in the different zones. For the sake of simplicity and for data availability, only one zone per floor is considered, 3 total zones, and one radiator per zone. The thermal model is developed with lumped parameters and it is sufficiently detailed to describe the prevailing dynamics of the system, moreover, this modelling choice allows a stronger physic representativeness of the results, on which the predictive control strategy will be developed. At first, the model is implemented in Matlab environment, in which a parameters identification is carried out. The identified parameters are those relative to the thermal characteristics of walls, pavements and ceiling and the remedial coefficient of the thermal power issued by the radiators. The identification is performed through a Grey-Box model, by analysing the 3 floors separately, to then report the estimated parameters in the whole model of the building. Subsequently, in Hysdel environment, through logical propositions, the hysteresis behaviour, that characterizes the progress of the temperature of each zone, is described and the model is made hybrid. To describe the hysteresis behaviour, Boolean variables and expressions related to the propositional calculus are exploited, and then are connected to the continuous thermal dynamics to generate an MLD model (Mixed Logical Dynamical), useful to the hybrid MPC.

Once the hybrid MPC is developed, some simulations are performed, and the results obtained are compared with those got from the same hybrid system but regulated by a simple logic ON-OFF, and the advantages gained with the strategy of the predictive control are shown.

Finally, two performances analysis are carried out, the first based on different constructive features of the building while the second based on different temperature set points imposed to the hysteresis. To conclude a robustness analysis, of the predictive control developed, is performed.

PAGE INTENTIONALLY LEFT BLANK

1-INTRODUCTION

1.1 THESIS OBJECTIVE

The main objective of this thesis is the development of a hybrid MPC able to minimize the thermal power produced by the radiators in a 3-floors building, guaranteeing the user's comfort. The process required for the creation of a controller is followed, starting from the study of the building and the analysis of the available data, passing through the modelling, the identification of the unknown parameters, the definition of the logical propositions that governing the hybrid behaviour of the system and finally the implementation of the predictive controller. This work is based on a lumped parameters model that has been developed so to capture the main dynamics of the system to control while maintaining the physical representativeness of the quantities involved. During the identification of the unknown thermal parameters are founded the values that allow to the simulated system to reproduce the real progress of the temperature of each zone, whether the radiators are turned OFF or ON. In the development of the logical propositions that make the system hybrid, the aim is to recreate the hysteresis behaviour to regulate the switch on and the switch off of the radiators. Once the hybrid model, in MLD (Mixed Logical Dynamical) form, is created, the hybrid MPC is implemented, with the purpose of improve the energy efficiency of the heating system of the entire building.

1.2 STATE OF THE ART

Energy consumption of the residential sector averages approximately 40% in Europe, and a relevant part of those energy is spent in heating systems. According to the regulation in force, from the 1 January 2021 all new buildings in the EU will have to consume a quantity of energy for heating, cooling and hot water equal to zero or very small. The current regulations also introduce an energy certification to compare and verify the energy performance.

Furthermore the member states will have to set up long-term national strategies to facilitate restructuring, so that by 2050 all buildings in the EU will have almost zero energy consumption and will have to adopt measures to encourage the use of smart technologies to reduce the energy consumption. [1].

These considerations have made sure that the problem of buildings thermal efficiency was highlighted and that they began to look for solutions that controlled and minimized consumptions.

To address this problem, it is useful to have the model of the considered building's thermal behaviour. A correct model can provide information useful in different fields, like the prediction of the energy consumption, the evaluation of the building thermal efficiency, the thermal comfort that the building can guarantee, and it can be used for control optimization purposes.

Various modeling techniques are already present in the literature to capture the thermal behaviour of a building. Here some techniques are classified, without entering into details, but highlighting advantages and disadvantages of each one.

- **Lumped Parameters Modeling:** this technique is usually the most adopted when studying the main dynamics of a thermal system. With this approach, the state variables of our system are grouped in a small set of representative variables, i.e. a single state variable is adopted to describe the thermal behaviour of an entire wall, another state variable is used to describe the air temperature and so on. Of course, this approximation doesn't consider perfectly the material property of the wall, of the pavement or of the ceiling and introduces some inaccuracy with respect to the real plant behaviour, but it allows to establish physical/logical connections between the thermal phenomena. Moreover, is possible to generate a RC equivalent circuit for a simple thermal system by adopting a lumped parameter model. This is to simplify the analysis of the system: a thermal capacity will be associated with a capacitor and a thermal transmittance with a resistor, obtaining an electric circuit whose passive components are resistances and capacitors only.

Once a lumped parameters model is developed, it can also be insert in a grey box model to identify some unknown parameters if necessary.

- **Simulation software:** a lot of simulation software are existing. Each software is different from the others for the complexity of the model that is used. These tools

can reach a remarkable precision with respect to the real plant through a high number of variables and parameters. For this reason, the simulation software is very helpful during the phase of the building design to perform a sensitivity analysis with respect to different constructive alternatives. One of the most famous software packages in built environment is Energy Plus, a tool that can include all the main aspects of the plant-building system, from indoor environment thermal loads to HVAC plants and heating and cooling generation system. However, this modeling due to its excessive complexity is rejected in the phase of the planning of the control strategy: in fact, during the control designing, it is recommended to trust only on a restricted set of important quantities in order to easily verify its performance properties.

- Black box modeling: this type represents one of the simplest modeling approaches existing in literature. This kind of technique tries to reproduce the thermal behaviour of the building by creating ARX/ARMAX/Artificial Neural Network models from data [2], [3], [4]. The main advantage of this method stands on its easy implementation and low computational effort thanks to its long-affirmed literature and algorithms. These techniques cannot be adopted in our thesis because the obtained model lacks physical representation and we do not have a sufficient number of available data.

In this thesis the lumped parameters modeling is adopted thanks to its functionality to the aim, i.e. creation of a hybrid model and control it with a hybrid MPC.

Hybrid systems compose the class of systems that take into account continuous valued and discrete valued variables as well as their interaction. Here again, many modelling approaches are available, and they put more emphasis either on the continuous valued or on the discrete valued component. Here are listed some of these different approaches:

- Hybrid Automata: a hybrid automaton is a finite state machine where a continuous dynamic is associated to each discrete state.
- Petri Nets: a Petri Net is a model of a discrete event system where transition can occur asynchronously. Moreover, with Petri Nets can be modelled actions as concurrency, synchronization and resource sharing.
- Generalized Hybrid Dynamical Systems.
- Linear Complementarity (LC) System: LC model are usually adopted to model mechanical system with inequality constraints, but it can be extended to a general

class of hybrid systems. With LC systems can be faced the question of existence and uniqueness of solution trajectories. [5]

- Extended LC
- Mi-max-plus-scaling
- Piecewise Affine (PWA) System: PWA systems are the “simplest” extension of linear systems that can still model nonlinear processes with accuracy and are capable of handling hybrid phenomena. PWA systems are suggested for the question of the stability of the system. [5]
- Mixed logical dynamical (MLD) System: MLD systems are an integration of logic, dynamics, and constraints. Control and verification techniques are proposed for MLD systems. [5]

For the purpose of this thesis, through the software HYSDEL a PWA model and an MLD model (equivalent to each other) have been created and then exploited to create a hybrid MPC.

As already said, for control of systems that are generally subject to constraints, MPC allows to obtain great performances in terms of constraint-satisfactory and efficiency.

It is possible to divide the hybrid optimal problem in two categories. The first includes those cases in which the number and switches is determined a priori.

Application of this method can be found in [6], in which a variable time transformation method is exploited for a mixed-integer optimal control problems, in [7] in which an economic model predictive control of switched nonlinear systems is developed, and in [8] in which a sequential linear quadratic optimal control approach is implemented for nonlinear switched systems.

The second, more problematic, includes problems in which the sequence and number of switches are not known a priori but have to be determined. In this case, the problem is conducted to the class of Mixed-Integer Optimal Control Problems.

In the literature, several approaches for solution of mixed-integer programs within MPC applications are presented. For some applications, direct use of general solvers can be sufficient, as shown in [9] and [10] where corresponding Mixed-Integer Linear Problems are solvable using CPLEX and Gurobi, respectively.

In [11] is presented an approach for modeling and predictive control of hybrid systems referred to as Mixed Logical Dynamical (MLD) systems and solves the resulting Mixed-Integer Optimal Control Problem using a general Mixed-Integer Quadratic Program solver. Some customized algorithms that want to exploit special aspects of Mixed-Integer Optimal Control Problems for MLD systems have been presented, e.g., by [12]

This is exemplary for several approaches that can be found in the literature that aim towards decreasing solution times of mixed integer problems for real-time applications by tailored solution methods. [13] presents a tailored Branch-and-Bound method incorporating a search space reduction to save computation time. In [14] is solved a mixed-integer MPC problem by application of a linear model-predictive controller with a longer prediction horizon in combination with a mixed-integer MPC problem with shorter horizon.

Regarding Mixed-Integer Non-Linear Problems, harder to solve, in literature they can be found analysis like [15] in which is adopted the Mixed-Integer Non-Linear Problem decomposition solution method within a warm-started algorithm for MPC of switched nonlinear systems under combinatorial constraints.

In this thesis is presented a modeling and predictive control problem of a hybrid system expressed in MLD and PWA form and then solved with a Mixed-Integer Quadratic Program.

1.3 DOCUMENT STRUCTURE

In the chapter 2, the modeling adopted in this thesis is presented. Initially the 3 floors are considered separately, then is considered the whole building. In the chapter 3, the procedure followed in the identification of the unknown parameters is described, starting from the first floor, going through the second floor and finally with the basement. The Hysdel language exploited to implement the hybrid model of the building and the simulation of the MLD and of the PWA system created, are explained and shown in the chapter 4. In the chapter 5 the develop of the hybrid MPC is presented, followed by the comparison with the “base case” of the system, by the performances analysis and by the robustness analysis.

PAGE INTENTIONALLY LEFT BLANK

2-MODELING

This case of study has been referred to the Power Flexhouse 03 facility, a 150 m² 3-floor house with several rooms, at DTU Risø, built in 1954. Flexhouse 03 is well sized for parallel operation on the SYSLAB power grid. All loads can be controlled by the central control room which gets all data from bus controlled, sensors and switches. The house is fully equipped with remotely temperature sensors in rooms and stairways. In the building are installed 4 different type of radiators in each floor. (construction company [16]). For each radiator there is a controllable electro valve.



Fig. 2.1: front view (left) and rear view (right) of Flexhouse_03

For a sake of simplicity, each floor is considered as a single room where all the radiators are seen as one input for each floor. Here a schematic picture of the house, with relative dimensions.

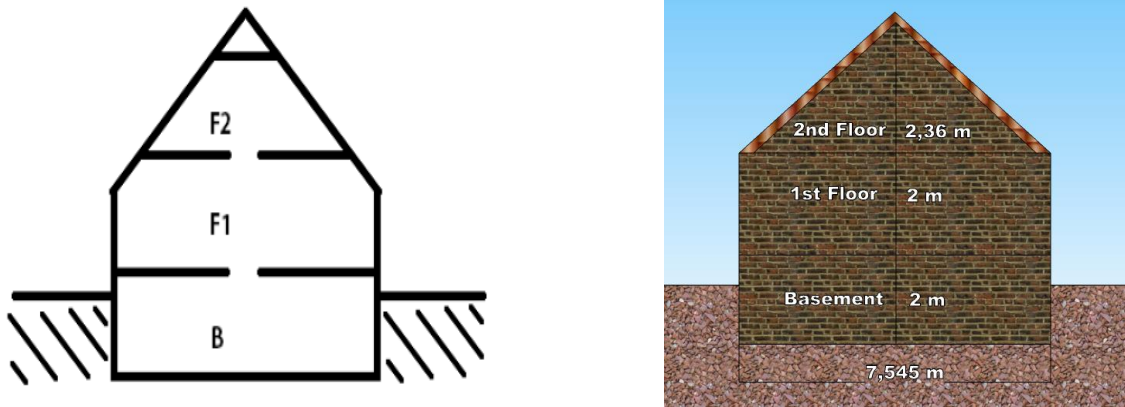


Fig. 2.2: schematic picture of the house, with relative dimensions

In this chapter, four different thermal models are shown and described. At first heat exchange equations are explained, then a complex model and a simple model of the radiator are presented and a model of the basement, a model of the first floor and a model of the second floor are formulated, independent of each other. Secondly, a model of the entire building is expressed.

This differentiation will be helpful for the next sections of this thesis. From the firsts three independent models, the unknown parameters will be obtained through an estimation of the parameters, while the model of the entire building will be useful during the formulation of the hybrid model of the system.

2.1 HEAT EXCHANGE

The energy moves from a hotter system to a colder one. This energy is known as heat. The transfer or the dispersion of the heat can happen through three main processes [17]:

- Conduction;
- Convection;
- Radiation;

CONDUCTION: It is the flow of heat through solids and liquids by vibration and collision of molecules and free electrons". Is the heat transfer from a hot end of an object to the cold

end and this type of transfer can occur in solids, in liquids and in gasses. The formula to calculate the conductivity for a given system is:

$$q = -kA(\Delta T/\Delta n)$$

Where $\Delta T/\Delta n$ is the temperature gradient in the direction of area A . K is the thermal conductivity constant in W/mK .

CONVECTION: is the type of heat exchange that interests us, because is the type of heat transfer that occur in the room model (between walls, pavements and zones). This phenomenon happens with fluids or gasses. When a heated fluid is caused to move away from the source of heat, then the energy is also carried with it. The formula of the convection is:

$$q = hA(T_s/T_\infty)$$

Where h is the convective heat transfer coefficient, A is the area implied in the heat transfer process, T_s is the temperature of the system and T_∞ is a reference temperature.

RADIATION: It is heat transfer by electromagnetic waves or photons. It does not need a propagating medium. For example, the heat radiated by the Sun and that reaches the Earth surface is transferred by radiation. The formula to know the amount of heat transferred by radiation is:

$$q = \varepsilon\sigma A(\Delta T)^4$$

Where ε is the emissivity of the system, A is the area involved in the heat transfer by the radiation, σ is the constant of Stephan-Boltzmann ($5.6697 \times 10^{-8} W/m^2K^4$).

In this thesis the contribution of the solar radiation has been neglected because not significant, while the formula used for the phenomenon of convection is:

$$C_X \dot{T}_X = U_{XY}(T_Y - T_X)$$

Where \dot{T}_X is the variation of temperature, C_X (J/K) is the thermal capacity, the ratio of the change in heat to the change in temperature, U_{XY} (W/K) is thermal conductance,

2.2 THERMAL MODEL

The model of each floor and of the entire building can be seen as a RC-network where the temperature is considered as a voltage on a node in the electric circuit, the heat flow is handled as electric current and thermal capacitances and thermal conductances can be seen as capacitors and resistors and respectively.

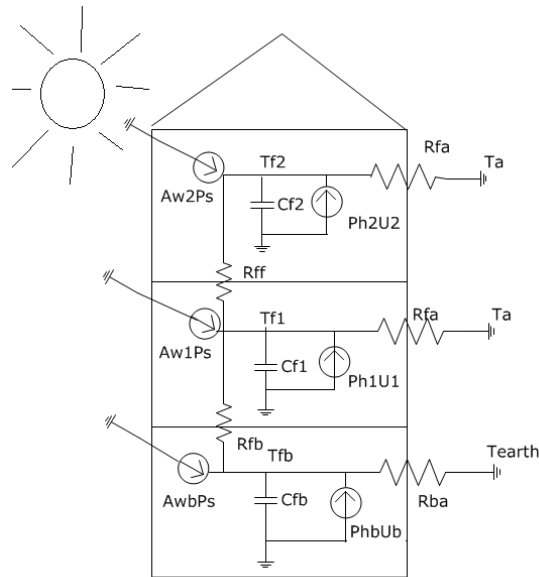


Fig. 2.3: RC-network that models the thermal behaviour of the building

As said before, during the development of this thesis, the contribution of the solar radiation has been neglected. With T_a is identified the outside temperature (measured) and with T_{earth} the temperature of the ground (kept fixed and equal to 7°C).

2.2.a RADIATOR

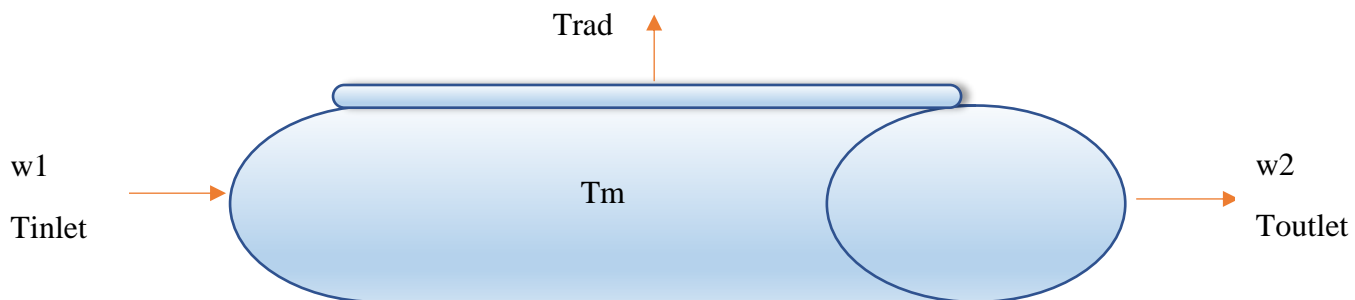


Fig. 2.4: complex model of the radiator

T_{inlet} represents the temperature of the inlet water, T_{outlet} is the temperature of the outlet water, T_m stands for the temperature of the medium and T_{rad} is the temperature of the radiator. The heat passes from the medium to the external part of the radiator, and then is exchanged with the zone.

During the development of the model of the radiator, some assumptions has been made:

- $w_1 = w_2 = w$ flowrate [L/s]
- $T_m = (T_{inlet} + T_{outlet})/2$ [°K]
- $T_{outflow} = (2 T_m - T_{inlet})$ [°K]

The model obtained is:

$$m_w c_w \dot{T}_m = w c_w (2T_{inlet} - 2T_m) + \gamma_1 (T_{rad} - T_m)$$

$$m_{rad} c_{rad} \dot{T}_{rad} = \gamma_1 (T_m - 2T_{rad}) + \gamma_2 (T_z - T_{rad})$$

- m_w = mass of water inside the radiator [Kg];
- c_w = specific heat of water [J/(Kg*K)];
- m_{rad} = mass of radiator [Kg];
- c_{rad} = specific heat of radiator [J/(Kg*K)];
- γ_1 = conductance between radiator and the medium [W/K];
- γ_2 = conductance of the radiator [W/K];

With this model, the number of parameters to be identified increases , so that , for a sake of simplicity, the contribution of the radiators in each floor and in the entire building, is substituted with a simpler expression, represented by a thermal power signal , to insert in the equation of heat exchange of the zone: $P_{rad} = \gamma w c_w Q_{rad}$, as made in [18]. This because the data available regarding the radiators are only the inlet and the outlet temperature of the water.

In this way, the single parameter to be identified is γ a remedial coefficient of heat transfer between the radiator and the zone and has to be as close as possible to 1.

2.2.b 3-STATES MODEL OF THE BASEMENT

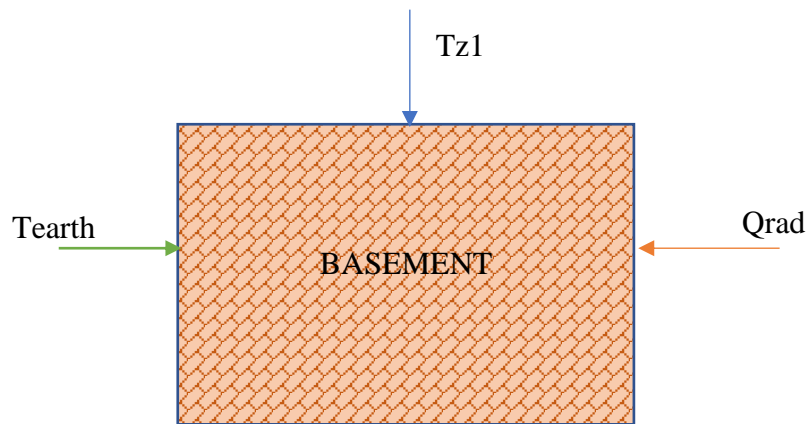


Fig.2.5: model of the basement

The basement is the only zone not in contact with the outside temperature, but with the temperature of the ground, this because is considered underground. In the below part there is no heat exchange due to fact that there are the foundations of the building, so there is a lot of space which acts as insulating, between the pavement and the ground.

The inputs considered in this zone are:

- T_{earth} , the ground temperature;
- T_{z1} , the temperature of the zone of the first floor;
- Q_{rad} , thermal power, only if the radiator is ON (is the only controllable input).

Following the lumped modeling strategy, the model of the basement is characterized by 3 states variables:

- T_z : zone temperature. The temperature of the controlled thermal volume assumed to be uniform.
- $T_{pav_b_1}$: pavement temperature. Is the temperature between the basement and the first floor, assumed to be uniform.
- T_{wall_b} : walls temperature. The temperature of external walls (in this work, there is no difference between internal and external walls, and is neglected the contribution of windows, this to simplify the model).

Equation of the zone temperature:

$$C_z \dot{T}_z = U_{wall_b}(T_{wall_b} - T_z) + U_{pav}(T_{pav_b_1} - T_z) + \gamma_1 W C_w Q_{rad}$$

Equation of the temperature of the walls:

$$C_{wall_b} \dot{T}_{wall_b} = U_{wall_b}(T_z - T_{wall_b}) + U_{wall_b}(T_{earth} - T_{wall_b})$$

Equation of the temperature of the pavement between basement and first floor:

$$C_{pav_b_1} \dot{T}_{pav_b_1} = U_{pav_b_1}(T_z - T_{pav_b_1}) + U_{pav_b_1}(T_{z1} - T_{pav_b_1})$$

C_z is the zone air mass thermal capacity [J/K] given by the mass of the air included in the zone multiplied by the specific heat of the air, it will be explained in detail in the chapter 3 (parameters identification). This parameter depends also on the number of appliances, furnitures and objects present in that specific zone.

C_{wall} is the thermal capacity of the walls [J/K].

$C_{pav_b_1}$ is the thermal capacity of the pavement between the basement and the first floor.

U_{wall} and $U_{pav_b_1}$ represent the convective conductance of the walls and of the pavement respectively [W/K], and they are obtained multiplied S_{wall} (walls' surface) by u_{wall} and S_{pav} (pavement's surface) by u_{pav} .

Has been created a state-space representation of the model, setting up the matrices A, B, C, D.

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

$$A = \begin{bmatrix} \frac{(-U_{wall_b} - U_{pav_{b_1}})}{C_z} & \frac{U_{wall_b}}{C_z} & \frac{U_{pav_{b_1}}}{C_z} \\ \frac{U_{wall_b}}{C_{wall_b}} & -2\frac{U_{wall_b}}{C_{wall_b}} & 0 \\ \frac{U_{pav_{b_1}}}{C_{pav_{b_1}}} & 0 & -2\frac{U_{pav_{b_1}}}{C_{pav_{b_1}}} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\gamma_1 w C_w}{C_z} & 0 & 0 \\ 0 & \frac{U_{wall_b}}{C_{wall_b}} & 0 \\ 0 & 0 & \frac{U_{pav_{b_1}}}{C_{pav_{b_1}}} \end{bmatrix}$$

$$C = [1 \quad 0 \quad 0]$$

$$D = [0 \quad 0 \quad 0 \quad 0] \text{ no disturbances}$$

Vector of the states

$$x = \begin{bmatrix} T_z \\ T_{wall_b} \\ T_{pav_{b_1}} \end{bmatrix}$$

Vector of the inputs

$$u = \begin{bmatrix} Q_{rad} \\ T_{earth} \\ T_{z1} \end{bmatrix}$$

2.2.c 4-STATES MODEL OF THE FIRST FLOOR

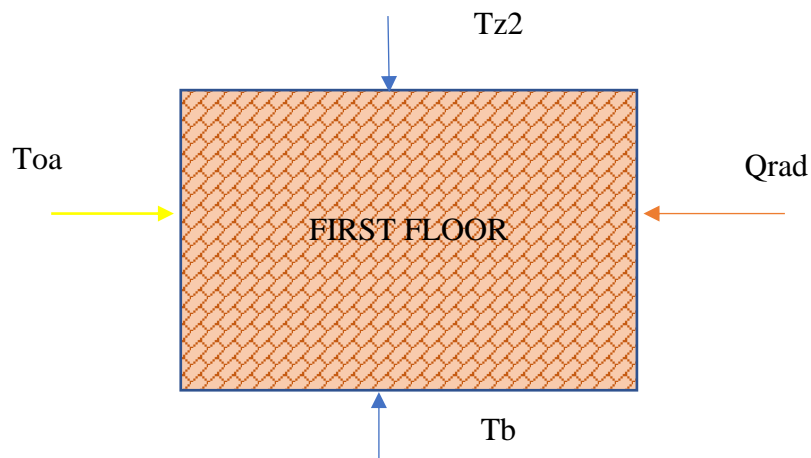


Fig. 2.6: model of the first floor

The main difference between the basement model is that here there is one more state variable, this because there is heat transfer through both the pavements, the one that divides the basement from the first floor and the one between the first and the second floor. Another difference concerns the set of inputs. Instead of the temperature of the ground, here there is the impact, on the external walls, of the outside temperature.

Summing up, the inputs for the model of the first floor are:

- T_{oa} , the outside temperature;
- T_b , the temperature of the zone of the basement;
- T_{z2} , the temperature of the zone of the second floor;
- Q_{rad} , thermal power, only if the radiator is ON (is the only controllable input).

As anticipated, the state variables here are 4:

- T_z : zone temperature. Explained in the basement model.
- $T_{pav_b_1}$: pavement temperature. Explained in the basement model.
- T_{wall_1} : walls temperature. Explained in the basement model.
- $T_{pav_1_2}$: pavement temperature. Is the temperature of the pavement that divides the first and the second floor.

Equation of the zone temperature:

$$C_z \dot{T}_z = U_{wall_1}(T_{wall_1} - T_z) + U_{pav_b_1}(T_{pav_b_1} - T_z) + U_{pav_1_2}(T_{pav_1_2} - T_z) + \gamma_2 w C_w Q_{rad}$$

Equation of the temperature of the walls:

$$C_{wall_1} \dot{T}_{wall_1} = U_{wall_1}(T_z - T_{wall_1}) + U_{wall_1}(T_{oa} - T_{wall_1})$$

Equation of the temperature of the pavement between basement and first floor:

$$C_{pav_b_1} \dot{T}_{pav_b_1} = U_{pav_b_1}(T_z - T_{pav_b_1}) + U_{pav_b_1}(T_b - T_{pav_b_1})$$

Equation of the temperature of the pavement between first floor and second floor

$$C_{pav_1_2} \dot{T}_{pav_1_2} = U_{pav_1_2}(T_z - T_{pav_1_2}) + U_{pav_1_2}(T_{z2} - T_{pav_1_2})$$

The parameters here used and not yet explained are:

$C_{pav_{1,2}}$: thermal capacity of the pavement between the first and second floor [J/K];

$U_{pav_{1,2}}$: represent the convective conductance of the pavement between first and second floor [W/K] again obtained by multiplying S_{pav} (pavement's surface) by u_{pav} .

Also for this model a state-space representation has been created, setting up the matrices A, B, C, D.

$$A = \begin{bmatrix} \frac{(-U_{wall_1} - U_{pav_{b_1}} - U_{pav_{1,2}})}{C_z} & \frac{U_{wall_1}}{C_z} & \frac{U_{pav_{b_1}}}{C_z} & \frac{U_{pav_{1,2}}}{C_z} \\ \frac{U_{wall_1}}{C_{wall_1}} & -2 \frac{U_{wall_1}}{C_{wall_1}} & 0 & 0 \\ \frac{U_{pav_{b_1}}}{C_{pav_{b_1}}} & 0 & -2 \frac{U_{pav_{b_1}}}{C_{pav_{b_1}}} & 0 \\ \frac{U_{pav_{1,2}}}{C_{pav_{1,2}}} & 0 & 0 & -2 \frac{U_{pav_{1,2}}}{C_{pav_{1,2}}} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\gamma_2 W C_w}{C_z} & 0 & 0 & 0 \\ 0 & \frac{U_{wall_1}}{C_{wall_1}} & 0 & 0 \\ 0 & 0 & \frac{U_{pav_{b_1}}}{C_{pav_{b_1}}} & 0 \\ 0 & 0 & 0 & \frac{U_{pav_{1,2}}}{C_{pav_{1,2}}} \end{bmatrix} \quad C = [1 \quad 0 \quad 0 \quad 0]$$

$$D = [0 \quad 0 \quad 0 \quad 0] \quad \text{again, no disturbances}$$

Vector of the states

$$x = \begin{bmatrix} T_z \\ T_{wall_1} \\ T_{pav_{b_1}} \\ T_{pav_{1,2}} \end{bmatrix}$$

Vector of the inputs

$$u = \begin{bmatrix} Q_{rad} \\ T_{oa} \\ T_b \\ T_{z2} \end{bmatrix}$$

2.2.d 4-STATES MODEL OF THE SECOND FLOOR

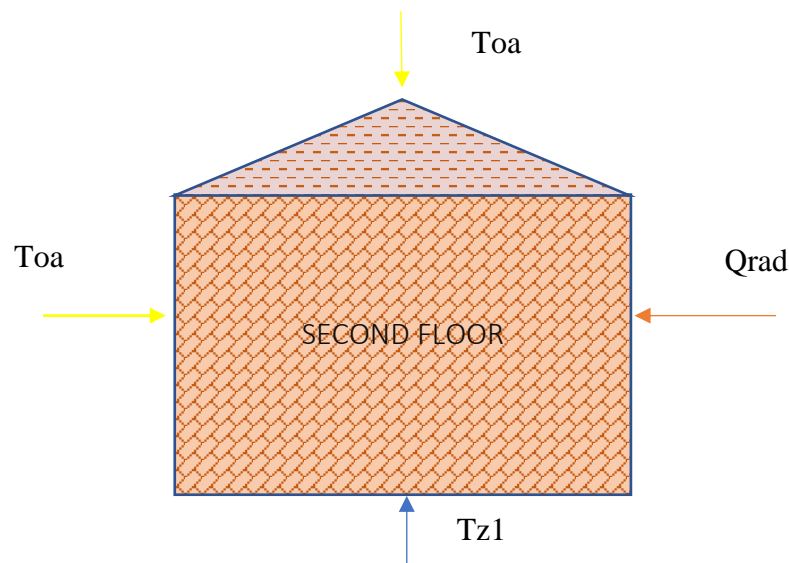


Fig 2.7: model of the second floor

The model of the second floor is characterized by 4 state variables. The new state variable, with respect to the previous model is that regarding the ceiling. Here, the input relating to the outside temperature occurs two times. One time affects the external wall, like the previous model of the first floor, the second time affect the ceiling. The ceiling is considered with its own value of thermal capacity and conductance, since the material with which it is made is different from that used for the walls.

The inputs considered in this model are:

- T_{oa} , the outside temperature;
- T_{z1} , the temperature of the zone of the first floor;
- Q_{rad} , thermal power, only if the radiator is ON (is the only controllable input).

The 4 state variables here are:

- T_z : zone temperature. Explained in the basement model.
- $T_{pav_{1,2}}$: pavement temperature. Explained in the first floor model.
- T_{wall_2} : walls temperature. Explained in the basement model.
- T_{ceil} : ceiling temperature. Is the temperature of the ceiling of the building.

Equation of the zone temperature:

$$C_z \dot{T}_z = U_{wall_2}(T_{wall_2} - T_z) + U_{pav_1_2}(T_{pav_1_2} - T_z) + U_{ceil}(T_{ceil} - T_z) + \gamma_3 w c_w Q_{rad}$$

Equation of the walls temperature:

$$C_{wall_2} \dot{T}_{wall_2} = U_{wall_2}(T_z - T_{wall_2}) + U_{wall_2}(T_{oa} - T_{wall_2})$$

Equation of the pavement temperature:

$$C_{pav_1_2} \dot{T}_{pav_1_2} = U_{pav_1_2}(T_z - T_{pav_1_2}) + U_{pav_1_2}(T_{z1} - T_{pav_1_2})$$

Equation of the ceiling temperature:

$$C_{ceil} \dot{T}_{ceil} = U_{ceil}(T_z - T_{ceil}) + U_{ceil}(T_{oa} - T_{ceil})$$

Parameters exploited but not yet explained are:

C_{ceil} : is the thermal capacity of the ceiling [J/K];

U_{ceil} : described the thermal convective conductance of the ceiling [W/K], obtained by multiplying

S_{ceil} (pavement's surface) by u_{ceil} .

As did in the previous two sections, the model is represented in state space form, with the respective matrices A, B, C, D.

$$A = \begin{bmatrix} \frac{(-U_{wall_2} - U_{pav_{1,2}} - U_{ceil})}{C_z} & \frac{U_{wall_2}}{C_z} & \frac{U_{pav_{1,2}}}{C_z} & \frac{U_{ceil}}{C_z} \\ \frac{U_{wall_2}}{C_{wall_2}} & -2\frac{U_{wall_2}}{C_{wall_2}} & 0 & 0 \\ \frac{U_{pav_{1,2}}}{C_{pav_{1,2}}} & 0 & -2\frac{U_{pav_{1,2}}}{C_{pav_{1,2}}} & 0 \\ \frac{U_{ceil}}{C_{ceil}} & 0 & 0 & -2\frac{U_{ceil}}{C_{ceil}} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\gamma_2 W C_w}{C_z} & 0 & 0 \\ 0 & \frac{U_{wall_1}}{C_{wall_1}} & 0 \\ 0 & 0 & \frac{U_{pav_{1,2}}}{C_{pav_{1,2}}} \\ 0 & \frac{U_{ceil}}{C_{ceil}} & 0 \end{bmatrix} \quad C = [1 \quad 0 \quad 0 \quad 0]$$

$$D = [0 \quad 0 \quad 0] \quad \text{as before, no disturbances}$$

Vector of the states

$$x = \begin{bmatrix} T_z \\ T_{wall_2} \\ T_{pav_{1,2}} \\ T_{ceil} \end{bmatrix}$$

Vector of the inputs

$$u = \begin{bmatrix} Q_{rad} \\ T_{oa} \\ T_{z1} \end{bmatrix}$$

2.2.e 9-STATES MODEL OF THE ENTIRE BUILDING

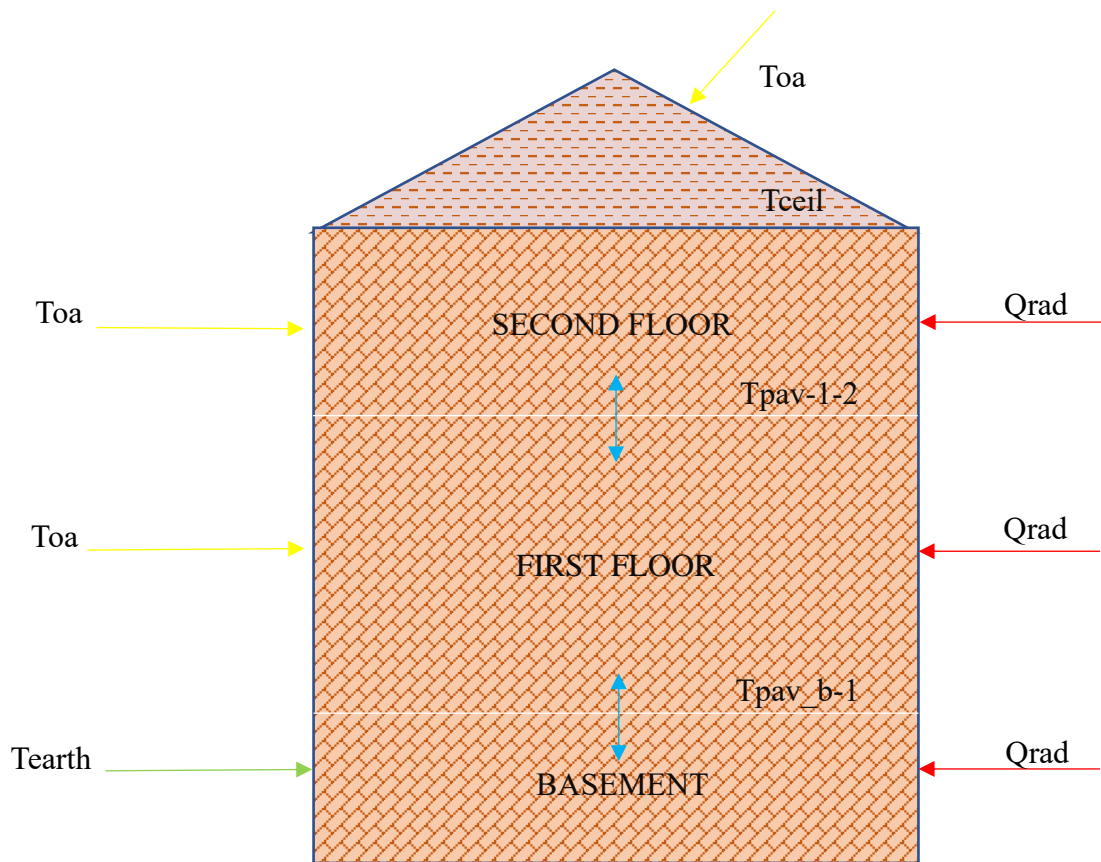


Fig. 2.8: model of the entire building

In this section is presented and described a 9-states model of the entire building. This will be useful, as starting point, for the creation of the hybrid model in the next chapters. In this case the three floors are not seen as independent from each other, but they are connecting among them. The temperature of the different zones do not cover again the role of input for the other zones (i.e. in the model of the basement, the temperature of the zone of the first floor was considered as input), but they are states variables of the model, and the 3 outputs of the system. The variables that link the different zones are the pavement, through which the heat exchanges occur.

In this model the input variables are 3, the temperature of the ground, that affects the walls of the basement, the outside temperature that affects the walls of the first floor, of the second floor and the ceiling, and the thermal power derived from the radiator.

The different values that this last input can assume in each floor according to the flowrate value, will be explain in the chapter in which the hybrid model will be developed.

Summarizing, here the set of inputs is made by:

- T_{oa} , the outside temperature;
- T_{earth} , the temperature of the ground;
- Q_{rad} , thermal power, only if the radiator is ON (is the only controllable input).

While the state variables are:

- T_{zb}, T_{z1}, T_{z2} : temperature of the zones;
- $T_{wall_b}, T_{wall_1}, T_{wall_2}$: temperature of the walls;
- $T_{pav_b_1}, T_{pav_1_2}$:temperature of the pavements;
- T_{ceil} : temperature of the ceiling;

Equation of the zone temperature of the basement:

$$C_b \dot{T}_{zb} = U_{wall_b}(T_{wall_b} - T_{zb}) + U_{pav}(T_{pav_b_1} - T_{zb}) + \gamma_1 w c_w Q_{rad}$$

Equation of the walls temperature of the basement:

$$C_{wall_b} \dot{T}_{wall_b} = U_{wall_b}(T_{zb} - T_{wall_b}) + U_{wall_b}(T_{earth} - T_{wall_b})$$

Equation of the pavement temperature between basement and first floor:

$$C_{pav_b_1} \dot{T}_{pav_b_1} = U_{pav_b_1}(T_{zb} - T_{pav_b_1}) + U_{pav_b_1}(T_{z1} - T_{pav_b_1})$$

Equation of the zone temperature of the first floor:

$$C_{z1}\dot{T}_{z1} = U_{wall_1}(T_{wall_1} - T_{z1}) + U_{pav_b_1}(T_{pav_b_1} - T_{z1}) + U_{pav_1_2}(T_{pav_1_2} - T_{z1}) + \gamma_2 W C_w Q_{rad}$$

Equation of the walls temperature of the first floor:

$$C_{wall_1}\dot{T}_{wall_1} = U_{wall_1}(T_{z1} - T_{wall_1}) + U_{wall_1}(T_{oa} - T_{wall_1})$$

Equation of the pavement between first and second floor:

$$C_{pav_1_2}\dot{T}_{pav_1_2} = U_{pav_1_2}(T_{z1} - T_{pav_1_2}) + U_{pav_1_2}(T_{z2} - T_{pav_1_2})$$

Equation of the zone temperature of the second floor:

$$C_{z2}\dot{T}_{z2} = U_{wall_2}(T_{wall_2} - T_{z2}) + U_{pav_1_2}(T_{pav_1_2} - T_{z2}) + U_{ceil}(T_{ceil} - T_{z2}) + \gamma_3 W C_w Q_{rad}$$

Equation of the walls temperature of the second floor:

$$C_{wall_2}\dot{T}_{wall_2} = U_{wall_2}(T_{z2} - T_{wall_2}) + U_{wall_2}(T_{oa} - T_{wall_2})$$

Equation of the ceiling temperature:

$$C_{ceil}\dot{T}_{ceil} = U_{ceil}(T_{z2} - T_{ceil}) + U_{ceil}(T_{oa} - T_{ceil})$$

Also for this 9-states model of the building is produced a state-space representation, through the matrices A, B, C, D.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{31} & 0 & a_{33} & a_{34} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} & a_{46} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{54} & a_{55} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{64} & 0 & a_{66} & a_{67} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{76} & a_{77} & a_{78} & a_{79} \\ 0 & 0 & 0 & 0 & 0 & 0 & a_{87} & a_{88} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_{97} & 0 & a_{99} \end{bmatrix}$$

$$a_{11}: (-U_{wall_b} - U_{pav_{b_1}}) / C_b$$

$$a_{12}: U_{wall_b} / C_b$$

$$a_{13}: U_{pav_{b_1}} / C_b$$

$$a_{21}: U_{wall_b} / C_{wall_b}$$

$$a_{22}: -2 U_{wall_b} / C_{wall_b}$$

$$a_{31}: U_{pav_{b_1}} / C_{pav_{b_1}}$$

$$a_{33}: -2 U_{pav_{b_1}} / C_{pav_{b_1}}$$

$$a_{34}: U_{pav_{b_1}} / C_{pav_{b_1}}$$

$$a_{43}: U_{pav_{b_1}} / C_{z1}$$

$$a_{44}: (-U_{wall_1} - U_{pav_{b_1}} - U_{pav_{1_2}}) / C_{z1}$$

$$a_{45}: U_{wall_1} / C_{z1}$$

$$a_{46}: U_{pav_{1_2}} / C_{z1}$$

$$a_{54}: U_{wall_1} / C_{wall_1}$$

$$a_{55}: -2U_{wall_1} / C_{wall_1}$$

$$a_{64}: U_{pav_{1_2}} / C_{pav_{1_2}}$$

$$a_{66}: -2U_{pav_{1_2}} / C_{pav_{1_2}}$$

$$a_{67}: U_{pav_{1_2}} / C_{pav_{1_2}}$$

$$a_{76}: U_{pav_{1_2}} / C_{z2}$$

$$a_{77}: (-U_{wall_2} - U_{pav_{1_2}} U_{ceil}) / C_{z2}$$

$$a_{78}: U_{wall_2} / C_{z2}$$

$$a_{79}: U_{ceil} / C_{z2}$$

$$a_{87}: U_{wall_2} / C_{wall_2}$$

$$a_{88}: -2 U_{wall_2} / C_{wall_2}$$

$$a_{97}: U_{ceil} / C_{ceil}$$

$$a_{99}: -2U_{ceil} / C_{ceil}$$

$$B = \begin{bmatrix} \frac{\gamma_1 W C_w}{C_b} & 0 & 0 \\ 0 & \frac{U_{wall_b}}{C_{wall_b}} & 0 \\ 0 & 0 & 0 \\ \frac{\gamma_2 W C_w}{C_{z1}} & 0 & 0 \\ 0 & 0 & \frac{U_{wall_1}}{C_{wall_1}} \\ 0 & 0 & 0 \\ \frac{\gamma_3 W C_w}{C_{z2}} & 0 & 0 \\ 0 & 0 & \frac{U_{wall_2}}{C_{wall_2}} \\ 0 & 0 & \frac{U_{ceil}}{C_{ceil}} \end{bmatrix}$$

$$C = [1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]$$

$$D = [0 \quad 0 \quad 0] \text{ No disturbances}$$

Vector of states:

Vector of inputs:

$$x = \begin{bmatrix} T_{zb} \\ T_{wall_b} \\ T_{pav_b_1} \\ T_{z1} \\ T_{wall_1} \\ T_{pav_1_2} \\ T_{z2} \\ T_{wall_2} \\ T_{ceil} \end{bmatrix} \quad u = \begin{bmatrix} Q_{rad} \\ T_{earth} \\ T_{oa} \end{bmatrix}$$

2.4 CHAPTER 2 CONCLUSIONS

This chapter explains the lumped parameters models exploited to describe the thermal dynamics of the 3 floors separately and of the entire building. Initially also an analysis on the model of the radiator was made, and, due to its complexity and its various unknown parameters, its contribution was modelled as thermal power. For the basement a 3-states model was implemented, and it was noticed that in this case there were not heat exchange between the pavement and the ground since there are the foundations of the building, so

there is a lot of space which acts as insulating. Moreover, in this zone the outside temperature did not affect the model, due to the fact that the basement is considered totally underground. For the first floor and for the second floor a 4-states model was developed. Finally, a 9-states model was generated to describe the behaviour of the entire building.

PAGE INTENTIONALLY LEFT BLANK

3–PARAMETERS IDENTIFICATION

In this chapter will be explained the procedure adopted to estimate the unknown parameters of the model developed previously. At first will be made a description of the functioning of the grey-box model and of the theory that stands behind it. Then will be listed the data sets recorded and that were available during the estimation process. In the final sections will be described how the sequence of the operation that have been made and the results obtained. As said in [19] model must be able to predict the future evolution of the system, capturing the nonlinear behaviour of such systems and providing tools to handle the noise in the form of process noise caused by approximation errors or unmodelled inputs and measurement noise due to imperfect measurements. A white box models, as explain in [18] allows to define a complete description of the system which means that the prior knowledge of the physics is essential for the model. In the building sector, a number of specified equations are adopted to create the deterministic physical model of the building, so it is possible to understand in a clear way the heat dynamics of the building, but on the other hand is difficult to handle the unknown parameters that might assume different values. Therefore, many physical systems can only be described by complex sets of equations which can make this approach not so efficient. A black-box model considers the system as a box with inputs and outputs, without any prior knowledge of the system. The physical description of the procedure is not available, and the physical parameters are hidden in the discrete parametrisation. The black box model is discrete system opposite to continuous white-box model. Grey-box models, which consist of a set of stochastic differential equations (SDEs), are able to describe the dynamics of the system in continuous time and a set of discrete time measurement equations, and can provide a way of combining the advantages of both model types by including a prior physical knowledge of the system and applying statistical methods for parameter estimation.

3.1 GREY-BOX MODELS

As said again in [19] the main benefit of grey-box model with respect to the black-box is the possibility of including directly in the model the physical knowledge and other prior information. As opposed to white-box, parameter estimation can give a more reproducible results and less bias because random effects caused by the process and the measurements noise, are not absorbed into the parameter estimates but considered by the diffusion and the measurement noise terms.

Here are listed some mathematical concepts that stand behind the grey-box models that refer to models consisting of a set of nonlinear, discretely, partially observed stochastic differential equations with measurement noise:

$$dx_t = f(x_t; u_t; t; \theta)dt + \sigma(u_t; t; \theta)d\omega_t;$$

$$y_k = h(x_k; u_k; t_k; \theta)dt + e_k;$$

$t \in R$ and is the time variable ($t_k, k=0, \dots, N$, sampling instants); $x_t \in X \subset R^n$ vector of state variables; $u_t \in U \subset R^m$, vector of input variables; $y_k \in Y \subset R^l$ vector of output variables; $\theta \in \Theta \subset R^p$ vector of possibly unknown parameters; $f(\cdot) \in R^n$, $\sigma(\cdot) \in R^{n \times m}$ and $h(\cdot) \in R^l$ represent nonlinear functions; $\{\omega_t\}$ is an n-dimensional standard Wiener process and e_k stands for an l-dimensional white noise process with $e_k \in N(0; S(u_k; t_k; \theta))$.

A possible method useful for the parameters' estimation, is the Maximum likelihood. It can find the parameters θ , that maximize the likelihood function of a given sequence of measurements $y_k, y_{k-1} \dots y_0, y_N$. Introducing $Y_k = [y_k, y_{k-1} \dots y_0, y_1]$, likelihood function is the joint probability density $L(\theta; Y_n) = p(Y_n|\theta)$ or, another equivalent expression, is

$$L(\theta; Y_n) = \left(\prod_{k=1}^N p(y_k | Y_{k-1}; \theta) \right) p(y_0 | \theta)$$

Where $P(A \cap B) = P(A|B)P(B)$ is applied to create a product of conditional density.

To obtain an exact value from the likelihood function, a general non-linear filtering problem must be solved.

As previously said, stochastic differential equations depend on a Wiener process, and the increments of a Wiener process are Gaussian. So, it is possible to assume, under some regularity under some regularity conditions, that the conditional density can be approximated by Gaussian densities and then a method based on the extended Kalman filter (is linear), can be applied.

As known the Gaussian density is characterized by its mean and covariance:

$$\hat{y}_{k|k-1} = E\{y_k | Y_{k-1}; \theta\}$$

$$R_{k|k-1} = V\{y_k | Y_{k-1}; \theta\}$$

$$\varepsilon_k = y_k - \hat{y}_{k|k-1}$$

So, the likelihood function can be rewritten in this way:

$$L(\theta; Y_n) = \left(\prod_{k=1}^N \frac{\exp\left(-\frac{1}{2} \varepsilon_k^T R_{k|k-1}^{-1} \varepsilon_k\right)}{\sqrt{\det(R_{k|k-1})} (\sqrt{2\pi})^l} \right) p(y_0 | \theta)$$

And parameter estimates can be determined by conditioning on y_0 and solving the optimization problem

$$\hat{\theta} = \arg \min\{-\ln(L(\theta; Y_n | y_0))\}.$$

For each set of parameters θ in the optimization, ε_k , innovation terms, and the covariance $R_{k|k-1}$ are computed in a recursive way using the Extended Kalman Filter, that consists of the output prediction equations:

$$\hat{y}_{k|k-1} = h(\hat{x}_{k|k-1}; u_k; t_k; \theta),$$

$$R_{k|k-1} = CP_{k|k-1}C^T + S,$$

the innovation equation:

$$\varepsilon_k = y_k - \hat{y}_{k|k-1}$$

and the Kalman gain equation:

$$K_k = P_{k|k-1}C^TR_{k|k-1}^{-1}$$

the updating equation:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\varepsilon_k$$

$$P_{k|k} = P_{k|k-1} - K_kR_{k|k-1}K_k^T$$

and the state prediction equations:

$$\frac{d\hat{x}_{t|k}}{dt} = f(\hat{x}_{t|k}; u_t; t; \theta),$$

$$\frac{dP_{t|k}}{dt} = AP_{t|k} + P_{t|k}A^T + \sigma\sigma^T$$

solved for (t_k, t_{k+1}) .

In the previous equation, the following notation has been applied:

$$A = \left. \frac{\partial f}{\partial x_t} \right|_{\hat{x}_{k|k-1}, u_k, t_k},$$

$$B = \left. \frac{\partial h}{\partial x_t} \right|_{\hat{x}_{k|k-1}, u_k, t_k},$$

$$\sigma = \sigma(u_k; t_k; \theta),$$

$$S = S(u_k; t_k; \theta).$$

Initial condition for the Extended Kalman Filter, $\hat{x}_{t|t_0} = x_0$, can be pre-specified or estimated as part of the overall problem, while $P_{t|t_0} = P_0$ can be computed in this way:

$$P_0 = P_s \int_{t_0}^{t_1} e^{As} \sigma \sigma^T (e^{As})^T ds$$

it represents the integral of the Wiener process and the system dynamics over the first sample.

Since the Extended Kalman Filter is sensitive to non-linear effects, a better approximation with respect to the previous equations, is to divide in subsamples the time interval (t_k, t_{k+1}) and the equations are linearized at each subsampling instant.

In this way you can get:

$$\frac{d\hat{x}_{t|j}}{dt} = f_0 + A\hat{x}_t - \hat{x}_j + B(u_t - u_j)$$

$$\frac{dP_{t|j}}{dt} = AP_{t|j} + P_{t|j}A^T + \sigma\sigma^T$$

Solved for $t \in (t_j, t_{j+1})$.

The notation applied corresponds to:

$$A = \left. \frac{\partial f}{\partial x_t} \right|_{\hat{x}_{j|j-1}, u_j}, \quad B = \left. \frac{\partial f}{\partial u_t} \right|_{\hat{x}_{j|j-1}, u_j, t_j}$$

$$f_0 = f(\hat{x}_{j|j-1}, u_j, t_j, \theta) \quad \sigma = \sigma(u_j; t_j; \theta)$$

And the analytical solutions are:

$$\hat{x}_{j+1|j} = \hat{x}_{j|j} + A^{-1}(\Phi_s - I)f_0 + (A^{-1}(\Phi_s - I) - I\tau_s)A^{-1}B\alpha$$

$$P_{j+1|j} = \Phi_s P_{j|j} \Phi_s^T + \int_0^{\tau_s} e^{As} \sigma \sigma^T (e^{As})^T ds$$

Where:

$$\tau_s = t_{j+1} - t_j,$$

$$\Phi_s = e^{A\tau_s},$$

$$\alpha = \frac{u_{j+1} - u_j}{t_{j+1} - t_j}$$

And α has been introduced to allow assumption of zero-order hold ($\alpha = 0$) or first-order hold ($\alpha \neq 0$).

The Maximum a posteriori estimation is a variant with respect to the previous method. Without entering in the mathematical details, if prior information regarding the parameters is available in terms of a prior probability density function $p(\theta)$ for the parameters, Bayes' rule can be applied obtaining an improvement in the estimation process, by creating the posterior probability density function.

3.2 DATA ANALYSIS

A various set of data was available during the parameters' estimation. First of all, the temperature of the three zone, registered from the 31st October 2018 to the 7th November 2018, sampled each second so the total number of samples is 690000 (8 days). During this period, the radiators in the building were turned on in each floor for the first day, then were turned off for the entire duration of the process of collecting data in the first and second floor, while in the basement zone they were returned on in the last day, as can be seen in the following plot.

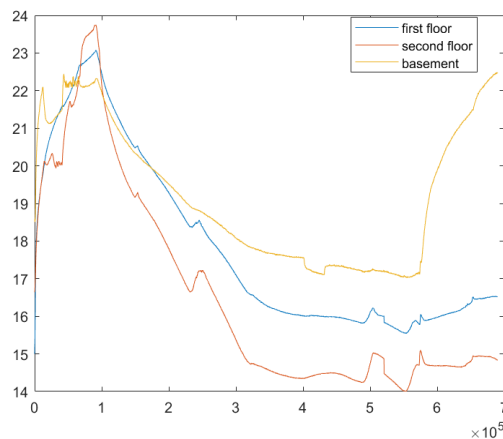


Fig. 3.1: real temperatures of the 3 floors

Other available data, are those regarding the outside temperature and the ground temperature, again registered in the same period of the previous data. They behave as input

on the system, since affect the walls' temperature of the first and second floor and of the basement respectively. As simplification, the ground temperature has been considered constant at 7°C during the entire duration of the collection process. Outside temperature and ground temperature are represented in the following graph.

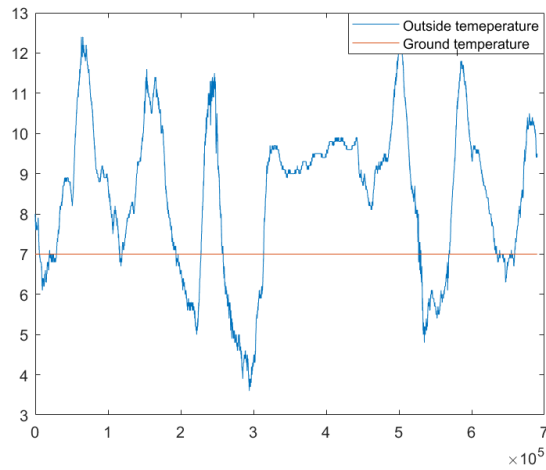


Fig. 3.2: outside and ground temperature

The last available set of data is that concerning the thermal power Q_{rad} exchanged by the radiators with the three zones again from the 31st October to 8th November. Clearly, as anticipated previously, the thermal power has a meaningful contribution only when the radiators in turned ON, so in the first and last day. This is clear in the following plot, in which Q_{rad} is represented.

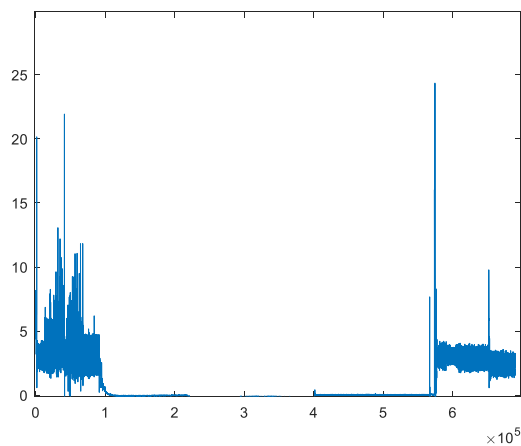


Fig. 3.3: thermal power

3.3 PARAMETERS ESTIMATION

The implemented method for the identification of the unknown parameters of the building is the following. At first the first floor was analysed considering the situation of radiator turned OFF, and the thermal capacity and thermal conductance of the pavement (considered with the same thermal characteristics in each floor) and of the external walls were identified. Then the radiator was considered turned ON and the remedial coefficient γ was estimated, paying attention that its value was as close as possible to one. Subsequently, maintaining the values of the thermal parameters of the pavement already found, the thermal capacity and thermal conductance of the external walls of the second floor and of the ceiling were estimated, considering the radiator OFF at first and then the remedial coefficient γ of the radiator was found like before and with the same considerations regarding its value. Lastly the same operation was made for the basement zone, at first, considering the radiator OFF, the thermal parameters of the external walls in contact with the ground were identified, then, with the radiator turned ON, γ was estimated. Since, it has been verified that the fitting obtained is high, it is possible to consider the interconnection between the different floors, when the models of the 3 floors are developed separately, by defining the zone temperature of one specific floor as one of the inputs of the model of one of the other floors, while in the model of the entire building, the interaction between floors was defined in the state equations.

3.3.a FIRST FLOOR

RADIATOR OFF

The model exploited is that built in the previous chapter. The inputs used for this case are the outside temperature, the temperature of the basement zone and the temperature of the second floor, while the outputs are the zones temperature. The parameters estimated here are the thermal capacity and the thermal transmittance of the external walls and the thermal capacity and the thermal transmittance of the pavements (between first floor and basement and between first and second floor, considered as having the same thermal characteristics).

- The average wall thermal capacity C_{wall} : this parameter is defined as the ratio between the heat exchanged between the body and the environment and the consequent temperature variation. It is obtained by multiplying the walls mass by the walls specific heat c_{wall} .
- The average walls thermal conductance U_{wall} : this parameter is a physical quantity that measures the amount of thermal power exchanged by a material or a body per unit area and unit of temperature difference. It is given by multiplying the conductive surface of the walls S_{wall} by the average convective heat transfer coefficient u_{wall} .
- The average pavement thermal capacity C_{pav} : the same considerations, regarding the pavement mass and its specific heat c_{pav} , made for the walls can be valid also for the pavement.
- The average pavement thermal conductance U_{pav} : the same considerations, regarding the pavement surface and its average convective heat transfer coefficient u_{pav} , made for the walls can be valid also for the pavement.

C_z is a known and found by multiplying the heat capacity of the air by the volume of the zone (available data).

These four parameters define the thermal properties of the walls and of the pavements of the first floor.

To identify these 4 parameters, a grey-box model has been developed with Matlab R2019a establishing a sample time T_s of 1 second and defining 4 unknown variables. The data selected for the identification cover the period from the 2nd November to the 6th November, about 450000 seconds, paying attention to the fact that the system in the final portion was in steady-state condition to capture all the dynamics that characterize the system itself. In the following plot the zone temperature of the first floor is represented. The red line stands for the real and recorded temperature, while the blue one depicts the temperature of the simulated system with the parameters identified by the grey-box model. The values of the parameters obtained are the following:

$C_{wall} \left[\frac{kJ}{K} \right]$	$U_{wall} \left[\frac{W}{K} \right]$	$C_{pav} \left[\frac{kJ}{K} \right]$	$U_{pav} \left[\frac{W}{K} \right]$
828	19.2	6050	890

Tab. 3.1: thermal capacity and thermal conductance of pavement and walls (first floor)

Considering that the conductive surface of the pavement is $S_{pav} = 50 \text{ m}^2$ and the conductive surface of the walls is $S_{wall} = (7.545 * 2) * 2 + (7.44 * 2) * 2 = 60 \text{ m}^2$

$c_{wall} \left[\frac{kJ}{m^2K} \right]$	$u_{wall} \left[\frac{W}{m^2K} \right]$	$c_{pav} \left[\frac{kJ}{m^2K} \right]$	$u_{pav} \left[\frac{W}{m^2K} \right]$
13.8	0.32	121	17.8

Tab. 3.2: specific heat and convective heat transfer coefficient of pavement and walls (first floor)

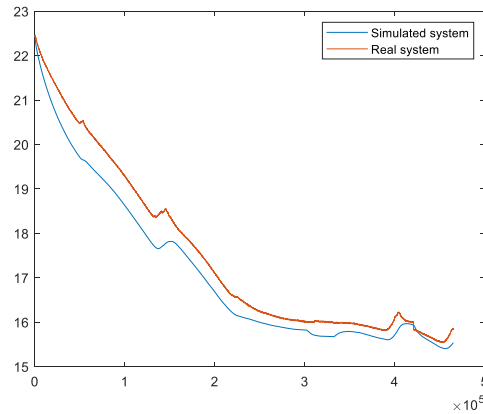


Fig. 3.4: simulation of the first floor temperature with the parameters identified (radiator OFF)

RADIATOR ON

Once the previous parameters have been estimated, the focus was put on the situation characterized by the radiator turned ON to try to identify the regulation coefficient γ , the only unknown parameter remained, the flowrate w considered constant and equal to $0,2 \text{ L/s}$. Another data window has been selected, shorter than the previous since the system with radiator ON is distinguished by a faster dynamic. As data have been considered those

collected during the 31st October and a few hours of the 1st November, about 100000 seconds. Again, as before, in the following plot the zone temperature, affected by the behaviour of the radiator tuned ON, is depicted. The red line represents the real temperature while the blue one the temperature of the zone simulated by the grey-box model implemented and with the regulation coefficient identified equal to $\gamma = 0.89$. As expected, its value is as close as possible to 1.

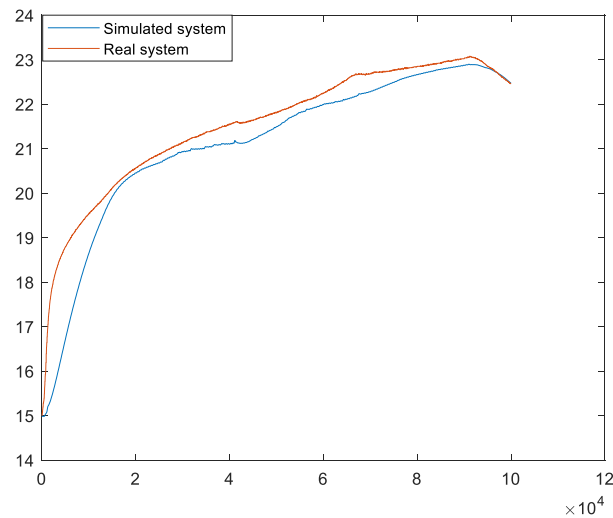


Fig. 3.5: simulation of the first floor temperature with the parameters identified (radiator ON)

3.3.b SECOND FLOOR

RADIATOR OFF

During the identification of the unknown parameters of the second floor, the thermal parameters concerning the pavement have been considered the same found previously. The set of the inputs is made by the outside temperature and the temperature of the first floor. In this case the unknown variables here are:

- The average wall thermal capacity C_{wall} : explained in the previous section;
- The average walls thermal conductance U_{wall} : explained in the previous section;

- The average ceiling thermal capacity C_{ceil} : given by the ceiling mass multiplied by its specific heat c_{ceil} ;
- The average pavement thermal conductance U_{ceil} : obtained by multiplying the ceiling surface and its average convective heat transfer coefficient u_{ceil} ;

The data considered was those that covered the same period of the identification made for the first floor, from the 2nd November to the 6th November. Again, a grey-box model has been implemented, considering a sample time equal to 1 second and with 4 parameters to identify.

$C_{wall} \left[\frac{kJ}{K} \right]$	$U_{wall} \left[\frac{W}{K} \right]$	$C_{ceil} \left[\frac{kJ}{K} \right]$	$U_{ceil} \left[\frac{W}{K} \right]$
912	24.6	819	15

Tab.3.3: thermal capacity and thermal conductance of ceiling and walls (second floor)

Total convective surface of the walls = 60 m².

Surface of the ceiling : through the Pythagorean theorem the oblique side is computed =

$$\left(\sqrt{\left(\frac{7,54}{2}\right)^2 + 2,36^2} \right) = 4,4 \text{ cm. Total convective surface of the ceiling} = (4,4 \times 7,4) \times 2 = 65 \text{m}^2.$$

$c_{wall} \left[\frac{kJ}{m^2K} \right]$	$u_{wall} \left[\frac{W}{m^2K} \right]$	$c_{ceil} \left[\frac{kJ}{m^2K} \right]$	$u_{ceil} \left[\frac{W}{m^2K} \right]$
15.2	0.41	12.6	0.23

Tab.3.4: specific heat and convective heat transfer coefficient of ceiling and walls (second floor)

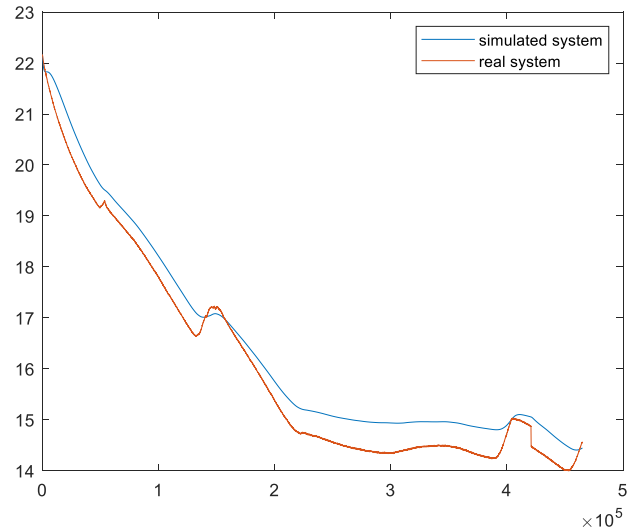


Fig. 3.6: simulation of the second floor temperature with the parameters identified (radiator OFF)

RADIATOR ON

The parameters estimated by considering the radiator turned OFF have been exploited to identify the residual coefficient γ present in the thermal power formula. This coefficient was the only unknown variable, since again the flowrate was considered constant and equal to $0,2 \text{ L/s}$.

The set of data was a little bit different from that utilised for the first floor, in fact the period covered is almost the entire day of the 31st October. Since the dynamics of the system, when the radiator is turned ON, is faster than the dynamics of the system in the condition of radiator OFF, the set of data can be shorter than the previous case. The value of γ estimated through the grey-box model is 0.96, again very close to 1.

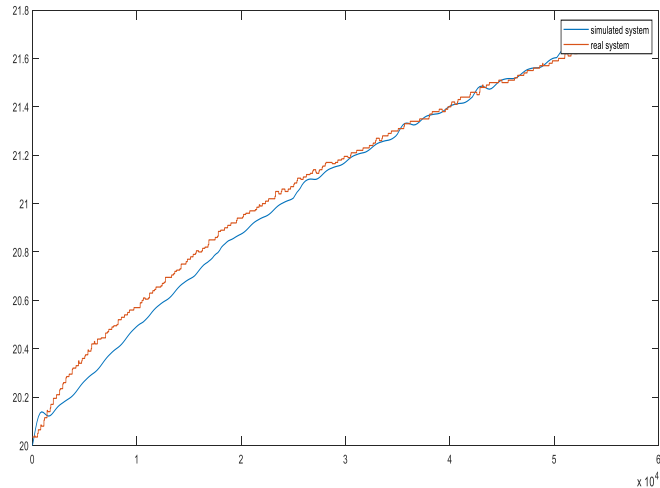


Fig. 3.7: simulation of the second floor temperature with the parameters identified (radiator ON)

3.3.c BASEMENT

RADIATOR OFF

The basement is the last zone considered during the identification process. The main issue found during the analysis of this zone is deriving from the fact that the basement is located under the ground level so that there is no heat exchange between the external walls and the external environment but the thermal transfer happens with the earth, considered at constant temperature and equal to 7°C . Therefore, as said during the development of the physical model, there is not thermal exchange between the pavement below and the ground, since there is a big layer of air, foundations and other constructive elements that guarantee the almost thermal insulation. Following these considerations, the only parameters to identify are those regarding the thermal characteristics of the external walls, since those regarding the pavement that divides the basement from the first floor have been identified previously. The set of inputs is composed by ground temperature and by the temperature of the first floor.

The thermal parameters here estimated are:

- The average wall thermal capacity C_{wall} : explained in the previous sections;
- The average walls thermal conductance U_{wall} : explained in the previous sections;

The data considered cover the period that goes from the 2nd November to the first 12 hours of the 5th November, about 300000 seconds, and again a grey-box model has been developed with a sample time of 1 second and considering 2 unknown variables.

$c_{wall} \left[\frac{kJ}{K} \right]$	$U_{wall} \left[\frac{W}{K} \right]$
882	21.6

Tab. 3.5: thermal capacity and thermal conductance of walls (basement)

Total convective surface of the walls = 60 m².

$c_{wall} \left[\frac{kJ}{m^2K} \right]$	$u_{wall} \left[\frac{W}{m^2K} \right]$
14.7	0.36

Tab. 3.6: specific heat and convective heat transfer coefficient of walls (basement)

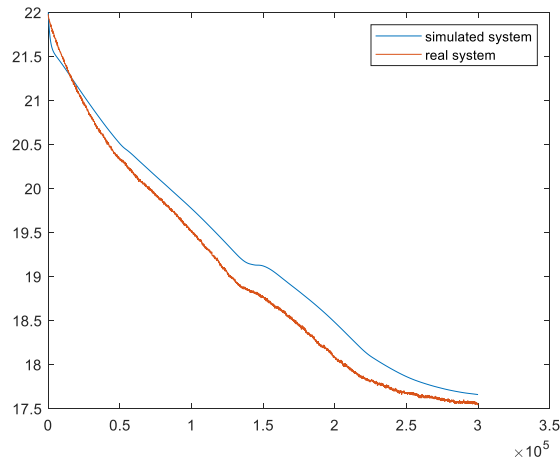


Fig. 3.8: simulation of the basement temperature with the parameters identified (radiator OFF)

RADIATOR ON

Following the procedure performed in previous 2 cases, once the radiator allocated in the basement zone has been turned ON, the remedial coefficient γ , which influence the thermal

power, has been identified, while the flowrate w is considered again constant and equal to $0,2 L/s$.

The data considered belong to the last day of recording, 7th November (shorter period with respect to the case of radiator turned OFF due to the preceding considerations). The value of γ identified is equal to 1.03, acceptable.

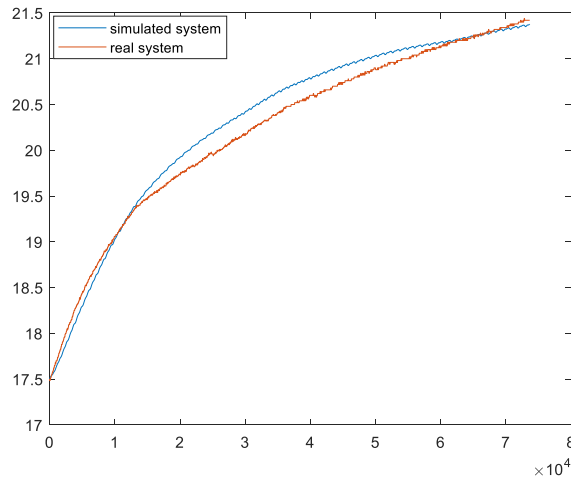


Fig. 3.9: simulation of the second floor temperature with the parameters identified (radiator ON)

3.4 ANALYSIS OF THE RESULTS

In the following table are reported the identified parameters for each floor:

	BASEMENT	FIRST FLOOR	SECOND FLOOR
$c_{wall} \left[\frac{kJ}{m^2K} \right]$	14.7	13.8	15.2
$u_{wall} \left[\frac{W}{m^2K} \right]$	0.36	0.32	0.41
$c_{pav} \left[\frac{kJ}{m^2K} \right]$	-	121	-
$u_{pav} \left[\frac{W}{m^2K} \right]$	-	17.86	-
$c_{ceil} \left[\frac{kJ}{m^2K} \right]$	-	-	12.6
$u_{ceil} \left[\frac{W}{m^2K} \right]$	-	-	0.23
γ	1.03	0.89	0.96

Tab.3.7: parameters identified

Referring to [20] the values of the thermal parameters obtained through the identification, has been compared with those found on a summary document drafted by the “Comitato Termotecnico Italiano Energia e Ambiente” and which are based on the constructive features of the walls. This comparison has been made to verify that the values obtained from the estimate were effective and reasonable. In that document the main parameters characterizing the walls are divided in three different time period, since each of them are associated to typical features adopted for the walls.

The three ranges are:

Old walls (1930-1975): $c_{wall} \in [57;65]$ and $u_{wall} \in [0.98;1.61]$;

Modern walls (1975-2005): $c_{wall} \in [17;18]$ and $u_{wall} \in [0.594;0.796]$;

New walls (2005-today): $c_{wall} \in [13;13]$ and $u_{wall} \in [0.28;0.28]$;

Summing up, reasonable values for c_{wall} and u_{wall} are those included in these intervals:

$c_{wall} \in [13;65]$;

$u_{wall} \in [0.28;1.61]$;

The values estimated through the grey-box model belong to walls with constructive features identifiable as modern or new.

Regarding the pavement, four different type of screed have been considered from different market brands and materials. They have been divided according to their height and, as range values, an average of the four different types, for each height, has been chosen, similarly to what was done with the three ages of the walls.

The intervals are the following:

Small screed (2-4 cm height): $c_{pav} \in [35;70]$ and $u_{pav} \in [31;62]$;

Medium screed (5-7 cm height): $c_{pav} \in [87;122]$ and $u_{pav} \in [18;25]$;

Big screed (8-10 cm height): $c_{pav} \in [140;174]$ and $u_{pav} \in [12;16]$;

The interval of values for the pavement is limited by: $c_{pav} \in [35;174]$;

$u_{pav} \in [12;62]$;

Analysing the values of the identified parameters, can be stated that the pavement is equipped with a medium screed.

3.4 CHAPTER 3 CONCLUSIONS

In this chapter was performed the identification of the unknown thermal parameters of the model, through a Grey-Box model. First, an analysis of the available data was made, and, for each floor, two cases were studied, when the radiator was considered turned OFF and when considered turned ON. In the first case, the thermal parameters relative to pavements, walls and ceiling were estimated, while in the second one, the remedial coefficient γ , of the thermal power formula, was identified. Initially the first floor was considered, in the condition of radiator turned OFF, since it was the floor with available data more clear than other floors. The parameter estimated were the average thermal capacity C_{pav} and thermal conductance U_{pav} of the pavement and the average thermal capacity C_{wall} and thermal conductance of the walls U_{wall} . Subsequently, the radiator was considered switched, and the remedial coefficient γ was estimated, taking fixed the parameters previously identified. Then, the second floor was analysed, in condition radiator OFF, and in this case, as thermal parameters of the pavement (C_{pav}, U_{pav}), were considered those identified for the first floor. The parameters estimated were the average thermal capacity C_{wall} and thermal conductance of the walls U_{wall} and the average thermal capacity C_{ceil} and thermal conductance of the ceiling U_{ceil} . Once these parameters were identified, again the radiator was considered turned ON and the remedial coefficient γ was estimated with the same procedure followed with the first floor. The last zone considered was the basement, and here the parameters that had to be identified were those relative to the walls in contact with the ground, since this zone was considered to be underground. Again, as thermal parameters of the pavement (C_{pav}, U_{pav}), were considered those identified for the first floor. Finally, also for the radiator in the basement, the remedial coefficient was estimated. Regarding the three remedial coefficients, was checked the fact that their values were as close as possible to 1.

The last step of this chapter was an analysis of the results obtained for the different parameters and they were compared to those found on a summary document drafted by the “Comitato Termotecnico Italiano Energia e Ambiente” and it was concluded that the walls of the building studied had constructive features identifiable as modern or new, while for the pavement a medium screed was adopted during the construction.

PAGE INTENTIONALLY LEFT BLANK

4-HYBRID MODEL

This chapter is intended to explain how the hybrid model, of the building studied, has been developed. First of all, a description of the different way to express a hybrid model will be stated, with particular interest on MLD and PWA models. Subsequently the focus will be put on the software Hysdel, exploited during the work to create the MLD model. Then will be explained how the continuous part of the model has been linked to the discrete one in this particular case of study and will be described the hysteresis behaviour with the logic that stand behind it. Finally, the analysis and the simulation of the MLD and the PWA trajectories will be performed.

4.1 HYBRID SYSTEMS AND MATHEMATICS

Hybrid systems compose the class of systems that takes into account continuous valued and discrete valued variables and their interaction in one model.

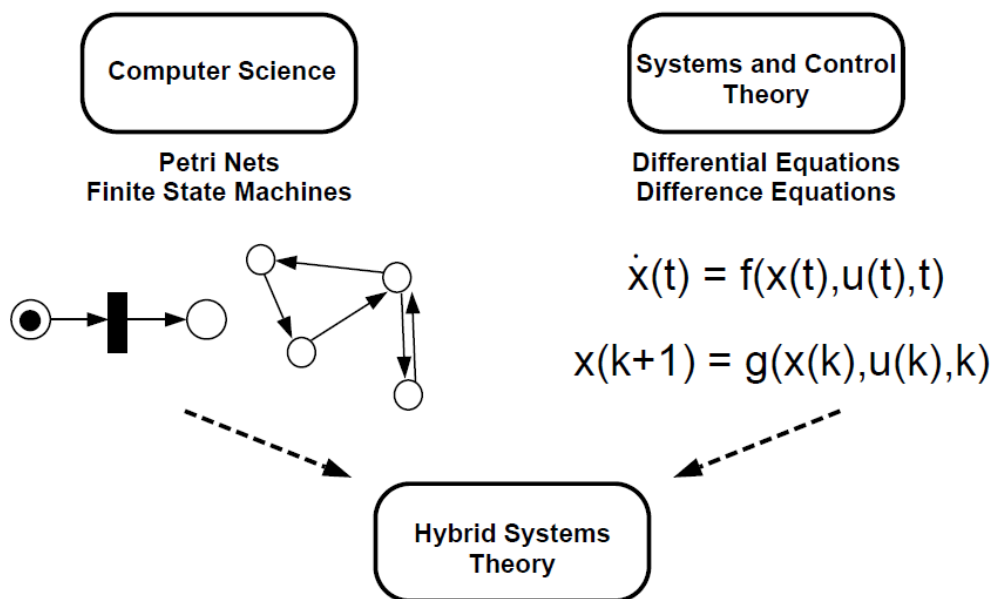


Fig. 4.1: hybrid systems theory

Different ways to modelling a hybrid system are available in literature but is a challenge to decide which adopt and which is more suitable for a particular case. Two basic requirements for a model of a hybrid system are [21]:

- The model must be descriptive enough to represent with accuracy the behaviour of the system focusing on the interdependence between the discrete part and the continuous dynamics;
- The model must allow to formulate problems found during the system design and operation, such that the controller synthesis or the fault detection. These problems must be solvable by exploiting numerical tools.

The main modelling approaches are listed here, in a more detailed way with respect to the description made in the “State of the art” of this work:

Hybrid automata – are finite state machines in which a continuous dynamic is associated to each discrete state. A hybrid automaton is made up by seven components:

- A set L of discrete states or locations;
- A set X of continuous states;
- A set E of edges or transitions;
- A set A of symbols labelling the edges;
- A set W of the continuous external variables;
- A mapping Inv from L to the subsets of X . Whenever the system is at location l , the continuous state has to satisfy $x \in Inv(l)$. All sets $Inv(l)$ are called location invariants;
- A mapping Act assigning to each discrete state l a differential algebraic equation system F_t . The solution to those differential algebraic equations is called the activities of the system.

A switch is a change of the discrete location of a hybrid automaton and the time instant at which it takes place is called event time. A jump is a resetting of the continuous state of a hybrid automaton when a switch takes place. The switch and the jump are two phenomena associated with each event.

Petri Nets – is a model of a discrete event system in which the transition can occur out of a fixed time. With this approach is possible to model behaviours including concurrency, synchronization and resource sharing. This type of model can be fluidified by including continuous dynamics.

Generalized Hybrid Dynamical Systems – In [22] is explained how to model framework in a way that includes a number of model classes in a unified set.

Linear Complementary Systems – is an approach used originally to model mechanical systems in which there was inequality constraints, but it can be extended to a general class of hybrid system. Within this framework it can be possible to formulate the question about the existence and uniqueness of solutions. An extension of this approach is the Extended Linear Complementary System.

Min-Max-Plus-Scaling - class of discrete event systems that can be modelled using maximization, minimization, addition and scalar multiplication [23].

Piecewise Affine Systems - this class of hybrid systems is strongly connected to the Mixed linear dynamical systems. Assuming that the state space is partitioned into separated i cells. In each state-space cell the dynamics is governed by a different affine dynamical system. The state space equations that describe PWA systems are:

$$x_{k+1} = A_i x_k + B_i u_k + a_i$$

$$y_k = C_i x_k + D_i u_k + c_i, \text{ for } \begin{bmatrix} x_k \\ u_k \end{bmatrix} \in X_i$$

$$(x, u) \in \mathbb{X}$$

$$X_i \text{ is a cell}$$

$\mathbb{X} \subset \mathbb{R}^{n_c} \times \{0,1\}^{n_l} \times \mathbb{R}^{m_c} \times \{0,1\}^{m_l}$ is a polyhedron that contains the origin $\{X_i\}_{i=1}^S$ is a polyhedral partition of \mathbb{X} and a_i and c_i are constant vectors of suitable dimension. PWA and MLD are equivalent systems, they can describe the same model behaviour. Given them the same inputs and the same initial conditions, the system trajectories are identical. PWA systems are suggested for the question of the stability of the system.

Mixed Linear Dynamical – systems characterized by linear dynamic equations subject to linear mixed integer inequalities that regard both continuous and discrete variables. The latter are introduced in accordance to logical techniques exploited in operations research. Through MLD is possible to extrapolate a various set of models such as constrained linear systems, finite state machines interacting with dynamic systems, some classes of discrete event systems, piecewise linear systems and systems with discrete inputs. In the next section the MLD will be explained in a more detailed way since is the approach selected and exploited during this thesis.

There are various types of system characterized by a specific behaviour or that evolve following a particular logic that can be expressed through a hybrid model. Some of these are the multiple model systems, the systems with switching components, the adaptive systems, systems with modelled failures or the systems involving synchronization.

Multiple model systems have a complex evolution regulated by various sub models, either by dividing the state space into cell or by modifying the variables of the system following a known signal (e.g. switched systems or systems with operating mode changes).

The systems with switching components are composed by switching elements like relays, dead-zones or hysteresis. A practical example is the real case that has been developed during this work.

The adaptive systems can be considered as hybrid systems due to switching rules given by finite state machines manipulating the adaptation law.

The systems with modelled failures are characterized by sudden or abrupt faults that can be modelled as a switching signal.

Systems involving synchronization signals are common systems in communication networks.

In a model, the logic part is represented by the propositional logic and it is possible to express the logical propositions in the form of linear constraints on binary variables. In this way a powerful modelling framework is guaranteed for hybrid dynamical systems is guaranteed.

4.1.a PROPOSITIONAL CALCULUS AND LINEAR INTEGER PROGRAMMING

The Boolean algebra allows statements to be exploited to create compound statements or logic propositions through operators, named connectives, such as:

- \wedge logical conjunction, “and”;
- \vee logical disjunction, “or”;
- \neg logical negation, “not”;
- \rightarrow logical implication;
- \leftrightarrow logical equivalence, if and only if;
- \oplus logical “exclusive or”;

The connectives satisfy some properties that can be exploited to reformulate compound statements into equivalent statements using different connectives to simplify them. It is possible to define and to link with a literal X_i a binary variable $\delta_i \in \{0,1\}$, that is equal to 1 if X_i is true or 0 if X_i is false. X_i is used to express statements. Through the linear integer programming, it is possible to solve a propositional logic problem in which the statement X_1 has to be proven to be true given a set of statements including literals X_2, \dots, X_n . A linear integer program transforms and reformulates the original statements into integer linear inequalities implying the binary variable δ_i . Here are listed the conversions of basic logic relations into integer inequalities:

relation	Logic proposition	Integer inequalities
AND: $X_1 \wedge X_2$	$[\delta_1 = 1] \wedge [\delta_2 = 1]$	$\delta_1 = 1, \delta_2 = 1$
OR: $X_1 \vee X_2$	$[\delta_1 = 1] \vee [\delta_2 = 1]$	$\delta_1 + \delta_2 \geq 1$
NOT: $\overline{X_1}$	$\overline{\delta_1 = 1}$	$\delta_1 = 0$
XOR: $X_1 \oplus X_2$	$[\delta_1 = 1] \oplus [\delta_2 = 1]$	$\delta_1 + \delta_2 = 1$
IMPLY: $X_1 \rightarrow X_2$	$[\delta_1 = 1] \rightarrow [\delta_2 = 1]$	$\delta_1 - \delta_2 \leq 0$
IFF: $X_1 \leftrightarrow X_2$	$[\delta_1 = 1] \leftrightarrow [\delta_2 = 1]$	$\delta_1 - \delta_2 = 0$

Tab.4.1: from basic logic relation to integer inequalities

The main challenge is to try to understand how to build statements starting from operating events that interest the continuous dynamics. One approach might involve the mixed integer linear inequalities, in which are present both continuous variables $x \in R^n$ and logical variables $\delta \in \{0,1\}$. In the following tables are listed equivalences that enable to perform the conversion into a set of inequalities to be exploited as constraints in mathematical optimization problems.

Set: $X \triangleq [f(x) \leq 0]$

relation	Logic proposition	Mixed integer inequalities
$[f(x) \leq 0] \rightarrow X$	$[f(x) \leq 0] \rightarrow [\delta = 1]$	$f(x) \geq \varepsilon + (m - \varepsilon)\delta$
$X \rightarrow [f(x) \leq 0]$	$[\delta = 1] \rightarrow [f(x) \leq 0]$	$f(x) \leq M - M\delta$
$[f(x) \leq 0] \leftrightarrow X$	$[f(x) \leq 0] \rightarrow [\delta = 1]$	$f(x) \leq M - M\delta$ $f(x) \geq \varepsilon + (m - \varepsilon)\delta$
IF X THEN $z = f(x)$ ELSE $z = 0$	$z = \delta f(x)$	$z \leq M\delta$ $-z \leq -m\delta$ $z \leq f(x) - m(1 - \delta)$ $-z \leq -f(x) + M(1 - \delta)$

Tab.4.2: from logic propositions to mixed integer inequalities

M and m in R are upper and lower bounds of the linear function $f(x)$ for $x \in X$, $\varepsilon > 0$ is a small tolerance. Thereby $f : R^n \rightarrow R$ is linear, $x \in X$ and X is a given bounded set, and define

$$M \triangleq \max f(x)$$

$$m \triangleq \min f(x)$$

Through the association of δ with X is possible to convert $X \triangleq [f(x) \leq 0]$ in:

$$f(x) \leq M - M\delta \quad (\text{a})$$

$$f(x) \geq \varepsilon + (m - \varepsilon)\delta \quad (\text{b})$$

In the following table are listed the resulting implications to check the equivalence of the logical statement to the given set of inequalities. Some assumptions of fixed quantity are made. For each fixed quantity, one inequality is always fulfilled.

Fixed quantity	Inequality(a)	Inequality(b)	
$\delta = 0$	$f(x) \leq M$	$f(x) \geq \varepsilon$	$f(x) \geq \varepsilon$
$\delta = 1$	$f(x) \leq 0$	$f(x) \geq m$	$f(x) \leq 0$
$f(x) \leq 0$	$\delta \in \{0,1\}$	$\delta = 1$	$\delta = 1$
$f(x) \geq \varepsilon$	$\delta = 0$	$\delta \in \{0,1\}$	$\delta = 0$

Tab.4.3: the resulting implications to check the equivalence of the logical statement to the given set of inequalities

4.1.b MIXED LOGIC DYNAMICAL SYSTEMS

This approach to the hybrid systems is based on three main ideas and observations, that are the representation of logical propositions in the form of linear inequalities through binary variables, the representation of propositions coupling logic relations to continuous variables as linear inequalities including continuous and binary variables and the last idea is the exploitation of binary variables and linear constraints in a description suitable for continuous system. Again, with reference to [21]

The general form of a mixed logic dynamical system is:

$$x(k+1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k)$$

$$y(k) = Cx(k) + D_1u(k) + D_2\delta(k) + D_3z(k)$$

$$E_2\delta(k) + E_3z(k) \leq E_1u(k) + E_4x(k) + E_5$$

The first equation represents the state update function, the second stand for the output the last is the set of the constraints on the system, in form of inequalities, obtained from the logic proposition in the previous tables. The variables are:

$$x = \begin{bmatrix} x_c \\ x_l \end{bmatrix}, x_c \in R^{n_c}, x_l \in \{0,1\}^{n_l}$$

continuous and binary states;

$$y = \begin{bmatrix} y_c \\ y_l \end{bmatrix}, y_c \in R^{p_c}, y_l \in \{0,1\}^{n_l}$$

continuous and binary outputs;

$$u = \begin{bmatrix} u_c \\ u_l \end{bmatrix}, u_c \in R^{m_c}, u_l \in \{0,1\}^{m_l}$$

continuous and binary inputs;

$$z \in R^{r_c}, \delta \in \{0,1\}^{r_l}$$

auxiliary continuous and binary variables and are introduced when the logic propositions are translating into linear inequalities;

$A, B_1, \dots, B_3, C, D_1, \dots, D_3, E_1, \dots, E_5$ are real matrices. In A, B_1, B_3 some entries have to be excluded, if binary states are present in x . This because the binary components of the state x_l cannot depend directly on continuous variables x_c, u_c, z through the first equation of the system, and their entries in A, B_1, B_3 have to be zero for a valid MLD model [24].

The state update matrix A has the form:

$$A = \begin{bmatrix} A_{cc} & A_{cd} \\ 0_{n_l \times n_c} & A_{dd} \end{bmatrix}$$

The system expressed in the general form of MLD model is considered to be completely well posed, for a given state $x(k)$ and a given input $u(k)$ the last equation, composed by inequalities, have a unique solution for $\delta(k)$ and $z(k)$. At each time instant, the maps $(x(k), u(k)) \rightarrow \delta(k)$ and $(x(k), u(k)) \rightarrow z(k)$ are single-valued. In this way $x(k+1)$ is uniquely defined and is possible to find a unique trajectory of the state $x(\cdot)$ given an initial

state $x(0)$ and an input sequence $u(0) \dots u(N)$. Thanks to the well-posed definition, it is allowed to define the algorithm of simulation of MLD systems:

1. Set $k = k_0$ and choose $x(k)$.
2. Given $x(k)$ and $u(k)$, obtain $z(k)$ from a feasibility test over the set of the constraints on the system.
3. Determine $x(k + 1)$ from the first equation of the system.
4. Set $k = k + 1$ and go to point 2.

It is good to specify that for simulation of large systems, more efficient algorithms exist.

A various set of methods that allow to translation the logic propositions into inequalities by decreasing the number of variables needed, since this process is fundamental to determine the size of resulting MLD model.

Before listing them, some general remarks on the Boolean algebraic are reported here.

A Boolean formula $F(X_1 \dots X_n)$, with X_i Boolean variable, is an arbitrary combination of variables X_i with the logical connectives explained in the previous section ($\wedge, \vee, \neg, \rightarrow, \leftrightarrow, \oplus$).

If a Boolean formula F is written in the following form

$$F = \bigwedge_{i=1}^n \varphi_i$$

with $\varphi_i = \bigvee_{j=1}^m X_{ij}$, F is a conjunctive normal form (CNF) or product of sums. The Boolean formula φ_i are named terms of product or maxterms, and X_{ij} are the terms of the sums. Through the laws of distribution, negation and DeMorgan, any Boolean formula can be transformed into a CNF.

A Boolean formula is called Boolean function if it is possible to define a variable X_n as function of $X_1 \dots X_{n-1}$ as $X_n \leftrightarrow f(X_1, X_2, \dots, X_{n-1})$, where f is an arbitrary Boolean formula. So, generally, it can be possible to define relations between Boolean variables $X_1 \dots X_n$ thanks to Boolean relation

$$F(X_1, X_2, \dots, X_{n-1}) = T$$

The satisfiability problem for a Boolean relation consists to determine if the variables of a given Boolean formula can be consistently replaced by the values TRUE or FALSE in such

a way that the formula can be evaluated to TRUE. A truth assignment for a Boolean formula is called valid point.

Methods that allow to translation the logic propositions into inequalities are the substitution method, the analytical method and the geometrical methods.

The first method provides that one recognizes basic logic relations and substitutes them with additional Boolean variables. The advantage of this method is its easy applicability, in fact it is based on a recursive execution of the basic logic relation, but its disadvantage consists in the introduction of additional variables.

The second method consists in converting $X_n \leftrightarrow f(X_1, X_2, \dots, X_{n-1})$ and $F(X_1, X_2, \dots, X_n) = T$ in conjunctive normal form (CNF), that can be performed automatically through one of several techniques available. Subsequently it is needed that the feasible points satisfy all inequality simultaneously, by setting up a set of constraints in an optimization problem. With this method is not necessary to adding other variables, therefore is simple, once the CNF of the specific Boolean function is known. One disadvantage is the effort in computing symbolically the CNF from a general Boolean formula.

The last method derives from a geometrical interpretation and, through its, it is possible to translate the truth table representing Boolean formula, into linear integer inequalities, automatically. The truth table for a Boolean formula $F(X_1, X_2, \dots, X_n)$ is the complete list of valid point for F . Again the main advantage of this method is that it is no necessary the adding of new variables, the disadvantage is that if the dimension of the truth table is large, it requires an effort in its construction and computation.

4.2 HYSDEL -HYBRID SYSTEM DESCRIPTION LANGUAGE-

Deriving the MLD formulation by hand is a trivial task. In this work it has been exploited the software Hysdel (Hybrid System Description Language).

Hysdel is a high-level modelling language for the description and the implementation of hybrid systems representable by discrete hybrid automata (DHA) [25]. As already mentioned, DHA come from the connection of a finite state machine, that represents the

discrete part of the hybrid system, with a switched affine system, that stands for the continuous part of the hybrid dynamics.

The following scheme represents the structure of the discrete hybrid automata and more in detail the role of the event generator, of the switched affine system, of the finite state machine and of the mode selector are explained.

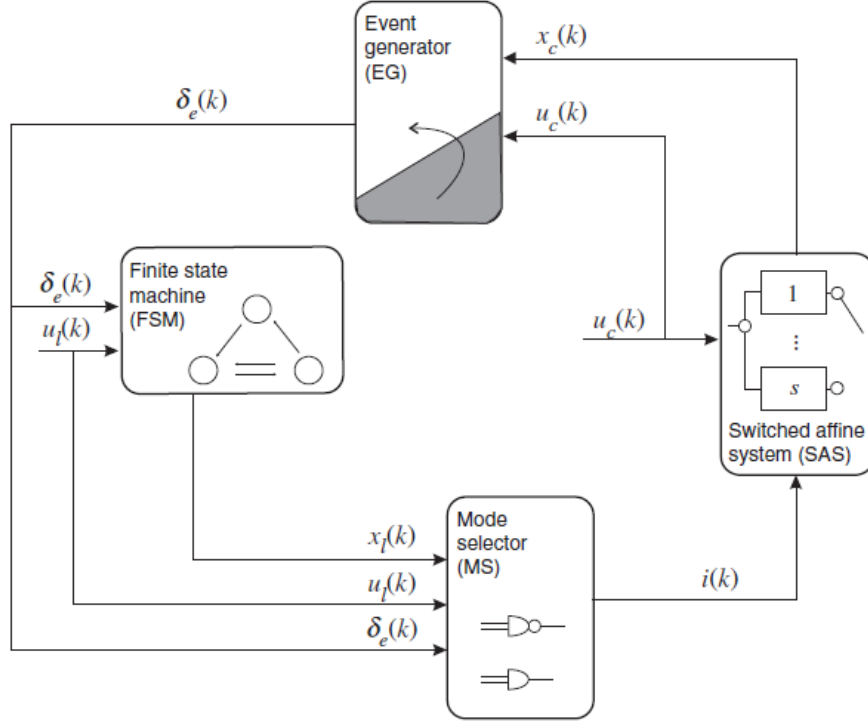


Fig 4.2: structure of the discrete hybrid automata

The switched affine system is a composition of linear affine system:

$$x_c(k+1) = A_{i(k)}x_c(k) + B_{i(k)}u_c(k) + f_{i(k)}$$

$$y_c(k) = C_{i(k)}x_c(k) + D_{i(k)}u_c(k) + g_{i(k)}$$

Where $k \in \mathbb{Z}^+$ stands for the time indicator, $x_c \in X_c \subseteq \mathbb{R}^{n_c}$ is the continuous state vector, $u_c \in U_c \subseteq \mathbb{R}^{m_c}$ exogenous continuous input vector, $y_c \in Y_c \subseteq \mathbb{R}^{p_c}$ represents the continuous output vector $\{A_i, B_i, f_i, C_i, D_i, g_i\}_{i \in Q}$ is a set of matrices, and $i(k) \in Q$ is the mode, an input signal that determines the update dynamics of the affine state. A switched affine state can be also rewritten as combination of linear terms combined with if-then-else condition.

The event generator is a mathematical operator that produces a logic signal such that satisfies the linear affine constraint

$$\delta_e(k) = f_H(x_c(k), u_c(k), k)$$

$f_H: X_c \times U_c \times Z_{\geq 0} \rightarrow D \subseteq \{0,1\}^{n_e}$ represents a vector of descriptive functions of a hyperplane, and $Z_{\geq 0} \triangleq \{0,1,\dots\}$ is the set of non-negative integers. The threshold events are modelled as $[\delta_e^i(k) = 1] \leftrightarrow [a^T x_c(k) + b^T u_c(k) \leq c]$, with i we refer to the i -th element of a vector. It is possible to model time events as $[\delta_e^i(k) = 1] \leftrightarrow [t(k) \geq t_0]$, with $t(k+1) = t(k) + T_s$ that represents the time, T_s sampling time and t_0 is a given time.

The finite state machine is a discrete dynamical process that progresses following an update function relative to a logic state:

$$x_l(k+1) = f_B(x_l(k), u_l(k), \delta_e(k))$$

$x_l \in X_l \subseteq \{0,1\}^{n_l}$ stands for the discrete state, $u_l \in U_l \subseteq \{0,1\}^{m_l}$ represents the exogenous Boolean input $\delta_e(k)$ is the endogenous input derived from the event generator, $f_B: X_l \times U_l \times D \rightarrow X_l$ deterministic logic function. Might be present also an associated Boolean output equation

$$y_l(k) = g_B(x_l(k), u_l(k), \delta_e(k))$$

with $y_l \in Y_l \subseteq \{0,1\}^{p_l}$ and $g_l: X_c \times U_c \times D \rightarrow Y_l$.

The mode selector is a Boolean function $f_M: X_l \times U_l \times D \rightarrow Q$ by which the logic state $x_l(k)$, the Boolean input $u_l(k)$ and the events $\delta_e(k)$ choose the dynamical mode $i(k)$ of the switched affine system. The output of the mode selector function is:

$$i(k) = f_M(x_l(k), u_l(k), \delta_e(k))$$

and is called the active mode. A mode switch happens if at step k $i(k) \neq i(k-1)$. In discrete time a mode switch can occur only at sampling instants, while in the continuous-time hybrid models it can happens at any time.

The discrete hybrid automata are linked to the hybrid automata with the main difference that stands in the time model. For the discrete hybrid automata are accepted natural numbers, for the hybrid automata the real numbers. Another difference is that discrete hybrid automata are not admitted instantaneous transitions and are deterministic, while with the hybrid automata any enable transition can happen in zero time. Moreover, working with the discrete

hybrid automata, the mode selector selects the continuous dynamics in line with discrete inputs and events.

In Hysdel, the representation of the system is on an abstract level (Hysdel file.hys), and a tool (hysdel compiler) takes and manages this abstract description to create, computational models in the form of mixed-logic dynamical (MLD) or piecewise-affine (PWA), through Matlab, that can be exploited during the system optimization, the verification and the design of the control.

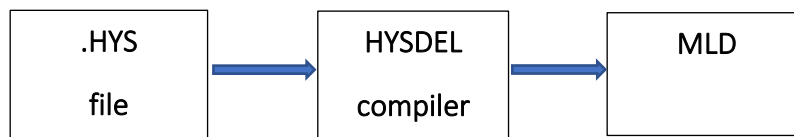


Fig. 4.3: generation of the MLD system

4.2.a LANGUAGE DESCRIPTION

The main two sections of a Hysdel file, that is developed on C language, are the interface and the implementation [26]

In the interface section are declared all the variables useful for that specific system. In detail, this section is divided in input variables, state variables and output variables. Other variables that do not belong to these sets, are supposed to be defined in the section parameter.

- The input section is useful to define the input variables of the system that can be of type real $u_r \in R$ and bool $u_b \in \{0,1\}$. The general syntax of the input section is:

```
INPUT {input_item}
```

And input item for real variables are anticipated by the syntax REAL while the Boolean variables are anticipated by BOOL. Moreover, the variables might be limited by a lower and an upper bound. In that case the name of the variable has to be followed by this syntax: [var_min, var_max], where var_min and var_max stand

for the lower and upper bound respectively. The variables might also be real vector with a specified dimension.

- The state section allows to declare state variables of the system that can be of type real $x_r \in R$ and bool $x_b \in \{0,1\}$. The syntax of this section follows the rules that govern the syntax of the input section.
- In the output section can be possible to declare the output variables of the system. These types of variables can be real $y_r \in R$ and bool $y_b \in \{0,1\}$. Again, the syntax of this section is the same of that presented in the input section. The only difference stands in the fact that for the output variables are not allowed to define lower and upper bound, since they are automatically inferred, being the output considered as an affine function of states and inputs.
- In the parameter section are declared variables which will be considered as constants as through the whole Hysdel structure.

In the implementation section are created and determined the different relations between variables. This section can be seen as the core of the entire Hysdel file and it can be divided into subsections.

- AUX section allows to define auxiliary variables useful for creations of other relations. The definition of these variables is followed by the specification of the type of the variable, real or bool. Here are not define lower or upper bonds on the variables, since their values are automatically computed through the variables declared in the interface section.
- In the continuous section are defined the equations concerning the state update. In this part only variables of type real and declared in the state section can be assigned.
- In the automata section, it is possible to describe the equations regarding the state transition for variables of type bool. Again, only variables of type bool and defined in the state section can be assigned.

- Linear section has its utility in the declaration of additional variables, created through affine expressions of states, inputs, parameters and auxiliary variables of type real.
- In the logic section is possible to express and to define additional relations between variables of type bool.
- AD section allows to define relations between variables of type real to Boolean, through the logical operator equivalence (\Leftrightarrow). In a simpler way, in this section are defined the conditions for which the Boolean variables assume the true value, equal to 1.
- In the DA section are expressed continuous variables following some if-then-else conditions. The variables here defined, are those declared in the AUX section. Often the if-then-else condition are based on the Boolean variables defined in the AD section.
- In the must section are declared the constraints on the variables of the inputs, of the states and of the outputs.
- In the output section, the last section, are defined the output variables for the overall mixed logical dynamical system. These variables can be both of type real and bool. In this section are accepted only the variables declared in the output section in the interface part.

4.2.b HYBRID MODEL OF THE BUILDING

The thermal model of the building that has been developed during this work, can be considered a hybrid system thanks to the hysteresis behaviour that governed the progress of the zone temperature and the functioning of the radiators. Through the hysteresis behaviour the value assumed by some dependent variables derives not only from the instantaneous value of these specific variables but also from the values that they had covered previously.

In a simpler way, the hysteresis is the characteristic of a system to be able to react to the solicitations that receives with a delay and in dependence on the previous state.

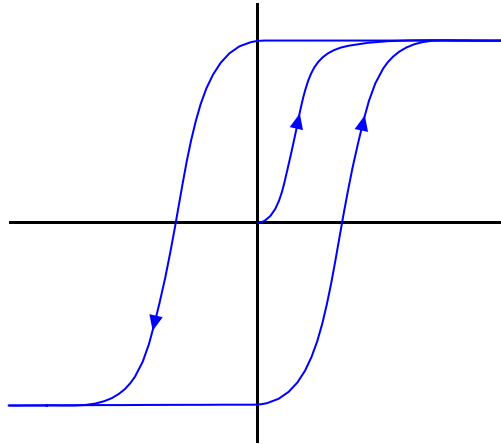


Fig. 4.4: hysteresis diagram

At graphic level, the curve of the response of a system, characterized by a hysteresis behaviour, to a solicitation, is closed. The system progresses in a different way if the curve is followed from right to left or in the opposite direction. The 2 horizontal lines represent the upper and lower saturation limits of the system. In a practical way, if the desired temperature of the zone is 20°C is possible to set to turn on the radiator when the registered temperature results below 18°C and turn it off when the temperature exceed 22°C.

To develop and implement the hybrid model of the building studied, Hysdel software has been exploited. The starting point is the model of the entire building developed and identified in the chapter 2 and chapter 3 of this thesis. The number of states is 9, the inputs are 3 and the outputs 3. In addition, have been considered 3 other states variables with the role of memory variables and exploited in the development of the hysteresis logic. The Boolean variables defined are 9, 3 for each radiator (each floor has considered to be equipped with 1 radiator), and are listed below:

```
AD { alfa1 = T1 <= Tcold1;
    alfa2 = T1 <= Thot1;
    alfa3 = T10 > 0;
    beta1 = T4 <= Tcold2;
    beta2 = T4 <= Thot2;
    beta3 = T11 > 0;
    delta1 = T7 <= Tcold3;
    delta2 = T7 <= Thot3;
    delta3 = T12 > 0;
```

Where T_{cold} stands for the lower limit of the zone temperature, T_{hot} is the upper bound of the zone temperature, T_1 , T_4 , T_7 are the states relative to the basement zone, the first floor zone and the second floor zone respectively. T_{10} , T_{11} and T_{12} are the state that cover the role of the memory variables. The variables α_1 , α_2 , β_1 , β_2 , δ_1 and δ_2 assume the TRUE value if the inequality condition expressed for each one is verified. α_3 , β_3 and δ_3 assume the TRUE value if at the previous step the radiator of that specific floor was turned ON.

Through these logic variables, it has been possible to describe the hysteresis behaviour for each radiator in the following way:

If $((\alpha_1 \text{ OR } \alpha_3) \text{ AND } \alpha_2)$ then RADIATOR1 is ON else is OFF

If $((\beta_1 \text{ OR } \beta_3) \text{ AND } \beta_2)$ then RADIATOR2 is ON else is OFF

If $((\delta_1 \text{ OR } \delta_3) \text{ AND } \delta_2)$ then RADIATOR3 is ON else is OFF

Only the basement zone will be taken in consideration during the explication of these 3 logic proposition, since they are identically. That logic proposition means that if the zone temperature is below the lower limit T_{cold} , the radiator is turned ON and it is kept in ON condition even when the lower limit is exceeded during the increase of temperature, this because of the memory variable α_3 that has assumed the TRUE value (in the previous step the radiator was turned ON). This situation persists until the zone temperature exceeds the upper limit T_{hot} . When the temperature is registered over the upper bound the radiator is turned OFF and consequently the zone temperature starts to decrease. When it touches and passes below the upper limit (α_2 inequalities verified), the radiator remains in OFF condition because the α_1 condition is not verified and the memory variable (α_3) assumes the value FALSE (in the previous step the radiator was turned OFF) giving information to the system to maintain the radiator in OFF condition until the zone temperature is registered below the lower bound again.

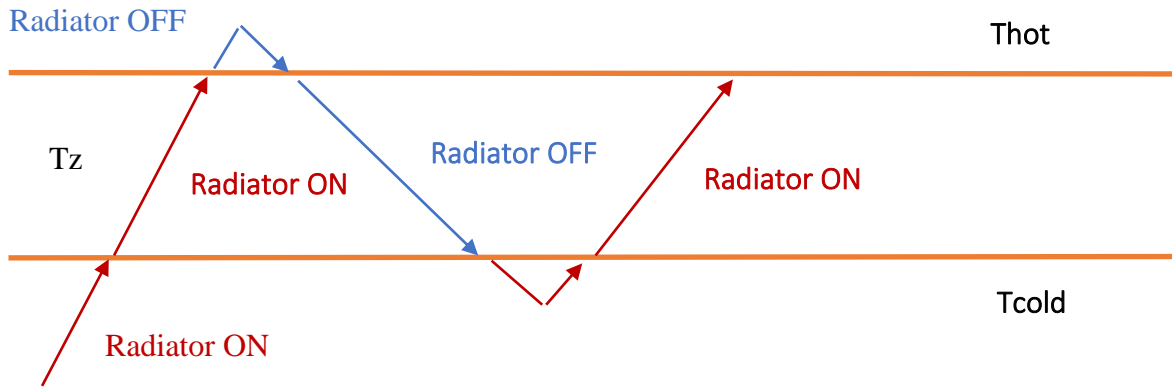


Fig. 4.5: hysteresis behaviour of the zone temperature

To insert the contribution of each radiator in the model support variables are introduced, for each floor and depending to the logical proposition defined previously, which can assume or 0 value or the value of the thermal power ($\gamma w c_w Q_{rad}$). For a sake of simplicity, as mentioned, the flowrate w is considered constant to 0,2 L/s for each radiator and with no difference if the number of radiators turned ON are 1 ,2 or 3.

Lastly the memory variables are explained. They simply assume the value of the support variables (0 or $\gamma w c_w Q_{rad}$) of the previous step, since the Hysdel model is expressed in discrete time, so that the system recognizes if the radiator were in ON or in OFF condition.

4.2.c SIMULATION OF THE MLD AND PWA SYSTEMS

Through Matlab has been implemented a script to create the Mixed logic dynamical system and, consequently, the Piecewise affine system, starting from the Hysdel file created and compiled previously. Once they have been created, their trajectories have been simulated. The simulation has been performed with a sampling time equal to 1 second and with a duration of 84600 seconds. The upper and lower bounds chosen for the hysteresis behaviour are the following:

- T cold basement = 20°C
- T cold first floor = 20°C
- T cold second floor = 20°C
- T hot basement = 22°C

- $T_{\text{hot first floor}} = 22^{\circ}\text{C}$
- $T_{\text{hot second floor}} = 22^{\circ}\text{C}$

In the following plot are represented the 3 inputs that act on the system, the outside temperature, the thermal power and temperature of the ground (constant). As can be seen, from the instant 150000 to the instant 625000, the thermal power is not available, so the zones temperature is expected to decrease even if is registered below the lower limit. On the contrary instead, in the last section, it is expected to increase.

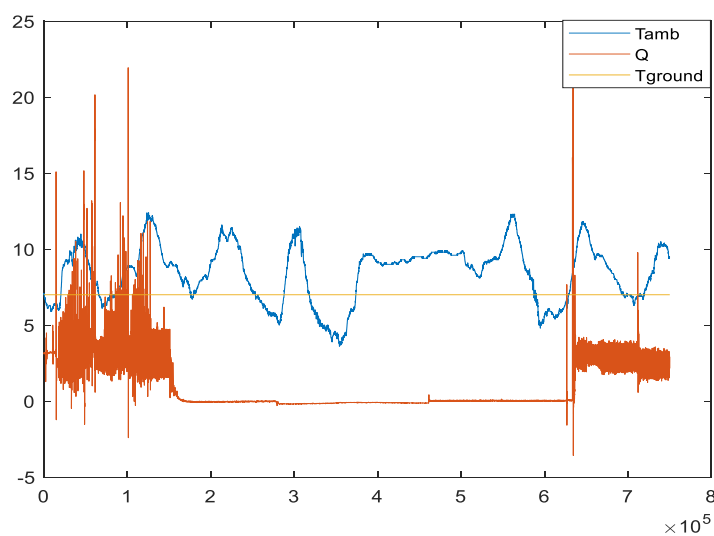


Fig. 4.6: inputs of the MLD model (outside temperature, ground temperature, thermal power)

Here are depicted the trajectories of the MLD and PWA systems. In the plot are represented all the 9 states, the first three lines, purple, light blue and red one stand for the basement zone, for the first floor zone and for the second floor zone respectively, while the other lines refer to walls, pavements and ceiling.

The green line visible in the neighbourhood is linked to the states that cover the role of memory variables, as explained previously.

As expected, when the thermal power is not available, there is a drop of the temperature.

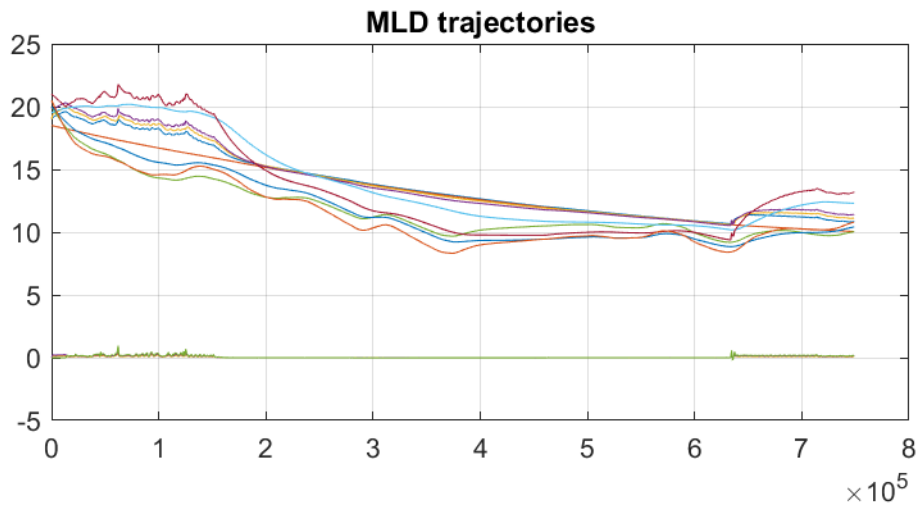


Fig. 4.7: MLD trajectories



Fig. 4.8 PWA trajectories

In this last plot, the difference between MLD and PWA trajectories is depicted, and as we expected is almost nothing. This because giving the same initial conditions and the same inputs to both systems, they result identical.

Every well-posed PWA system can be rewritten as an MLD system assuming that the set of feasible states and inputs is bounded. [27]

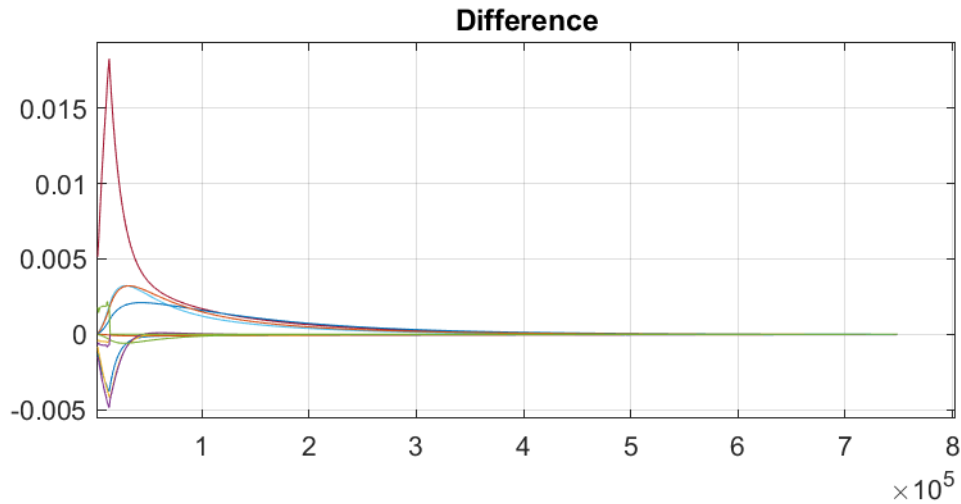


Fig 4.9: difference between MLD trajectories and PWA trajectories

4.3 CHAPTER 4 CONCLUSIONS

This chapter describes the hybrid model of the building and the steps made to generate it. Initially was given importance to the propositional calculus and to the linear integer programming, then, between the different types of hybrid modelling approaches, the MLD (Mixed Logical Dynamical) was chosen for this thesis and its properties were investigated here. The building model could be defined hybrid thanks to its hysteresis behaviour, and the logical expressions that stands behind it, were expressed through the Hysdel language by defining some Boolean variables, some upper and lower limits of temperature and some inequalities. These expressions were then put in relation with the continuous equations of the model in order to create the MLD model, exploitable by the hybrid MPC. Finally, the model was created also in PWA (Piecewise Affine) form and both the MLD and PWA models were simulated with the same inputs (thermal power, outside temperature and ground temperature) and same initial conditions, to check their correct behaviour and their equivalence, as expected.

5-HYBRID MPC

Once the hybrid model has been created and simulated, the attention has been put on the control of the system. It has been chosen to follow an approach related to the control based on the optimization, more specifically a hybrid MPC has been developed.

First of all, in this chapter the basic principles of the Model Predictive Control will be explained, subsequently the focus will be switched to the Hybrid MPC and to the theory on which it is based. Then, the hybrid model, that has been implemented during this work, will be taken in consideration and a hybrid MPC will be developed, based on the hybrid building model. Some simulation will be performed at different sampling times and different prediction horizon and the results obtained will be compared with those derived by a model of the system, implemented in Simulink, not managed by an MPC but governed only by a simple logic ON-OFF and an hysteresis behaviour (base case).

Then an analysis of the performances will be carried out by running the hybrid MPC on different buildings 'quality and with different thresholds of temperature.

Finally, a robustness analysis will be performed going through the comfort and the consumption.

5.1 MODEL PREDCTIVE CONTROL

Model predictive control (MPC) is a clustering of advanced control algorithms greatly exploited in the industrial world in the last thirty years and today. MPC is the most used control strategy in the process industry, since it is very flexible and applicable to a huge number of systems. Selecting an MPC approach it is possible to formulate the control problem as an optimization one, with different goals, to consider some constraints to be respected in the control problem formulation, to develop the controller based on model of the process implemented experimentally.

Referring to [28] The most important elements considering during the develop of an MPC algorithm are:

- The model of the process, possibly in discrete time;
- Constraints on the inputs, on the outputs and on the states;
- The cost function J defined, at every time instant k , over a finite horizon $[k + 1, k + N]$;
- An algorithm that can compute the future optimal control sequence;
- The principle of the Receding Horizon : at any time instant k , based on the available process information , solve the optimization problem with respect to the future control sequence $[u(k), \dots, u(k + N - 1)]$ and apply only its first element $u^0(k)$. At the next time instant $k + 1$, a new optimization problem is solved, based on the process information available at time $k + 1$, along the prediction horizon $[k + 1, k + N]$.

Following this strategy, a time invariant feedback control law is obtained if, at any time instant, a finite horizon optimization problem is solved at any time instant.

5.1.a MPC OF LINEAR SYSTEMS

Considering:

$$x(k + 1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k)$$

$x \in R^n$ assumed to be measurable, $u \in R^m$ control variable and $y \in R^p$ output variable.

At each time instant k , the control sequence $u(k), u(k + 1), \dots, u(k + N - 1)$ is computed, in order to minimize the finite horizon quadratic function

$$J(x(k), u(\cdot), k) = \sum_{i=0}^{N-1} \left(\|x(k + i)\|_Q^2 + \|u(k + i)\|_R^2 \right) + \|x(k + N)\|_S^2$$

With $Q = Q' \geq 0$, $R = R' > 0$, $S = S' \geq 0$ matrices of suitable dimensions. N is the prediction horizon of the MPC, and it is a positive integer.

It is possible to differentiate the solution in closed-loop or in open-loop. Considering the following time-varying optimal control law:

$$u^0(k+i) = -K(i)x(k+i), \quad i = 0, 1 \dots N-1$$

it is a closed-loop control law, since the control variable $u(k+i)$ is in dependence of the state at $x(k+i)$. $K(i)$ is defined as follows:

$$K(i) = (R + B'P(i+1)B)^{-1}B'P(i+1)A$$

And $P(i)$ is the solution of the difference Riccati equation

$$P(i) = Q + A'P(i+1)A - A'P(i+1)B(R + B'P(i+1)B)^{-1}B'P(i+1)A$$

with boundary condition $P(N) = S$. Respecting the receding horizon principle, only the first element of the optimal future control sequence is applied, so the MPC control law is state-feedback, time invariant and given by:

$$u^{MPC}(k) = -K(0)x(k)$$

Regarding the open loop solution, it is possible to fix, as starting point, the optimization problem defined above and to notice that it can be obtained by retrieving that, in view of Lagrange equation:

$$x(k+i) = A^i x(k) + \sum_{j=0}^{i-1} A^{i-j-1} B u(k+j) \quad i > 0$$

Setting:

$$X(k) = \begin{bmatrix} x(k+1) \\ x(k+2) \\ \vdots \\ x(k+N-1) \\ x(k+N) \end{bmatrix}, \quad A = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^{N-1} \\ A^N \end{bmatrix}, \quad U(k) = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+N-2) \\ u(k+N-1) \end{bmatrix}$$

$$B = \begin{bmatrix} B & 0 & 0 & \dots & 0 & 0 \\ AB & B & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ A^{N-2}B & A^{N-3}B & A^{N-4}B & \dots & B & 0 \\ A^{N-1}B & A^{N-2}B & A^{N-3}B & \dots & AB & B \end{bmatrix}$$

It is obtained:

$$X(k) = Ax(k) + Bu(k).$$

Then, define the matrices Q and R as follows, with N elements on the diagonal:

$$Q = \begin{bmatrix} Q & 0 & \dots & 0 & 0 \\ 0 & Q & \dots & 0 & 0 \\ \dots & \dots & \ddots & \dots & \dots \\ 0 & 0 & \dots & Q & 0 \\ 0 & 0 & \dots & 0 & S \end{bmatrix}, \quad R = \begin{bmatrix} R & 0 & \dots & 0 & 0 \\ 0 & R & \dots & 0 & 0 \\ \dots & \dots & \ddots & \dots & \dots \\ 0 & 0 & \dots & R & 0 \\ 0 & 0 & \dots & 0 & R \end{bmatrix}$$

And notice that

$$\arg \left(\min_{u(\cdot)} J(x(k), u(\cdot), k) \right) = \arg \left(\min_{U(\cdot)} \bar{J}(x(k), U(\cdot), k) \right)$$

With

$$\bar{J}(x(k), U(\cdot), k) = X'(k)QX(k) + U'(k)RU(k)$$

Since $x'(k)Qx(k)$ in $J(x(k), u(\cdot), k)$ is not in dependent on $u(k+j), j \geq 0$, it can be neglected.

With reference to $X(k) = Ax(k) + Bu(k)$, the following equation is obtained:

$$\begin{aligned} \bar{J}(x(k), U(\cdot), k) &= (Ax(k) + Bu(k))' Q (Ax(k) + BU(k)) + U'(k)RU(k) \\ &= x'(k)A'QA x(k) + 2x'(k)A'QBU(k) + U'(k) (B'QB + R) U(k) \end{aligned}$$

Since $R > 0$, this is a positive definite quadratic form, and is possible to compute its minimum by deriving it with respect to $U(k)$ and setting its derivative equal to zero. In this way, the following control sequence $U^0(k)$ is obtained:

$$U^0(k) = -(B'QB + R)^{-1}B'QA x(k)$$

Setting:

$$\mathcal{K} = (B'QB + R)^{-1}B'QA = \begin{bmatrix} \mathcal{K}(0) \\ \mathcal{K}(1) \\ \dots \\ \mathcal{K}(N-1) \end{bmatrix}, \quad \mathcal{K}(i) \in R^{m,n}$$

The previous equation becomes:

$$U^o(k) = - \begin{bmatrix} \mathcal{K}(0) \\ \mathcal{K}(1) \\ \dots \\ \mathcal{K}(N-1) \end{bmatrix} x(k)$$

Or

$$u^o(k+i) = -\mathcal{K}(i)x(k), \quad i = 0, 1, \dots, N-1$$

In this case, the solution depends on the prediction of the state, along the considered horizon, based on the current state $x(k)$, and it is said to be in open loop for $i > 0$ at least.

Applying the receding horizon principle also in this case, a state feedback control law is obtained:

$$u^{MPC}(k) = -\mathcal{K}(0)x(k)$$

With the open loop approach, it is allowed to include in the optimization problem the presence of state, input and output constraints. Assuming that the future inputs, outputs and states satisfy the following constraints:

$$u_m \leq u(k+i) \leq u_M \quad i = 0, \dots, N-1$$

$$x_m \leq x(k+i) \leq x_M \quad i = 1, \dots, N$$

$$y_m \leq y(k+i) \leq y_M \quad i = 1, \dots, N$$

and denoting the following vectors made by N elements:

$$U_m = \begin{bmatrix} u_m \\ u_m \\ \dots \\ u_m \end{bmatrix}, \quad U_M = \begin{bmatrix} u_M \\ u_M \\ \dots \\ u_M \end{bmatrix}, \quad X_m = \begin{bmatrix} x_m \\ x_m \\ \dots \\ x_m \end{bmatrix}, \quad X_M = \begin{bmatrix} x_M \\ x_M \\ \dots \\ x_M \end{bmatrix}$$

$$Y_m = \begin{bmatrix} y_m \\ y_m \\ \dots \\ y_m \end{bmatrix}, \quad Y_M = \begin{bmatrix} y_M \\ y_M \\ \dots \\ y_M \end{bmatrix}, \quad Y(k) = \begin{bmatrix} y(k+1) \\ y(k+2) \\ \dots \\ y(k+N-1) \\ y(k+N) \end{bmatrix}$$

The optimization problem on the finite horizon can be reformulated in this way:

$$\bar{J}(x(k), U(\cdot), k) = (Ax(k) + Bu(k))' Q (Ax(k) + BU(k)) + U'(k)RU(k)$$

constrained by:

$$X_m \leq X(k) = Ax(k) + BU(k) \leq X_M$$

$$U_m \leq U(k) \leq U_M$$

$$Y_m \leq Y(k) \leq Y_M$$

The constraints on $X(k)$ and on $Y(k)$ are soft constraints while those defined on $Y(k)$ are said to be hard constraints. $Y(k)$ is obtained from $X(k)$ through the output transformation.

This constrained optimization problem does not admit an explicit solution, but by exploiting quadratic programming methods, the optimal vector $U^0(k)$, made by the future values of the control variable along the prediction horizon, can be obtained.

Two consideration on the closed loop and on the open loop solutions:

- without disturbances or modeling errors , the closed loop solution and the open loop solution are the same, while in presence of disturbances or modeling errors, only the first elements $u^o(k+i) = -\mathcal{K}(i)x(k)$ and $u^o(k+i) = -K(i)x(k+i)$ are equal

$$\mathcal{K}(0) = K(0)$$

- The open loop solution has been computed through the prediction of the future states based on the current state and this approach can be generalized to compute the model predictive control law starting from different model representation.

5.1.b INDUSTRIAL MPC ALGORITHMS

Through these algorithms are managed systems affected by disturbances or are faced tracking problems. Defining the system as:

$$x(k+1) = Ax(k) + Bu(k) + Md(k)$$

$$y(k) = Cx(k) + d(k)$$

Where $d \in R^p$ is the disturbance, known at the current instant k and, if possible, in the future prediction horizon.

The aim is to track a reference signal y^0 in the future prediction horizon, with the output variable. First, it is possible to define:

$$x(k+i) = A^i x(k) + \sum_{j=0}^{i-1} A^{i-j-1} (Bu(k+j) + Md(k+j))$$

$$y(k+i) = Cx(k+i) + d(k+i)$$

and, during the formulation of the quadratic cost function must consider including a term that weights the future error variable.

$$J(x(k), u(\cdot), k) = \sum_{i=0}^{N-1} (\|y^0(k+i) - y(k+i)\|_Q^2 + \|u(k+i)\|_R^2) + \|y^0(k+N) - y(k+N)\|_S^2$$

With $Q = Q' \geq 0$, $R = R' > 0$, $S = S' \geq 0$ matrices of suitable dimensions.

Now, based on the previous formula:

$$Y(k) = A_c x(k) + B_c U(k) + M_c D(k)$$

Where A_c, B_c, M_c matrices of suitable dimensions.

Setting:

$$Y^0(k) = \begin{bmatrix} y^0(k+1) \\ y^0(k+2) \\ \dots \\ y^0(k+N-1) \\ y^0(k+N) \end{bmatrix}$$

Defining matrices Q, R, S as in the previous section, it is obtained:

$$\arg \left(\min_{U(\cdot)} J(x(k), u(\cdot), k) \right) = \arg \left(\min_{U(\cdot)} \bar{J}(x(k), U(k), k) \right)$$

$$\begin{aligned} \bar{J}(x(k), U(k), k) &= (Y^0(k) - Y(k))' Q (Y^0(k) - Y(k)) + U'(k) R U(k) = \\ &= (Y^0(k) - A_c x(k) - B_c U(k) - M_c D(k))' Q (Y^0(k) - A_c x(k) - B_c U(k) \\ &\quad - M_c D(k)) + U'(k) R U(k) \end{aligned}$$

To be minimized with respect to the future values of the control variable $u(k+i)$, $i \geq 0$ and under suitable constraints on x, u, y . It now depends on the future reference signals and on the disturbances.

Focusing on the disturbances, they are often unknown, so they have to be estimated in this way, to solve the optimization problem.

Considering a system affected by unknown constant disturbance d and characterized by:

$$x(k+1) = Ax(k) + Bu(k) + Md(k)$$

$$d(k+1) = d(k)$$

$$y(k) = Cx(k) + Fd(k)$$

To represent the interconnection between the state, output and disturbance, different choice for matrix M and F ($M = B, F = 0$ to model input disturbances, $M = 0, F = I$ to express disturbances on the output). The disturbance and the state (if possible) can be estimated by exploiting an observer, if the enlarged system results observable.

If the system is observable, it is possible to compute through a proper observer and at any time instant k , the estimate $\begin{bmatrix} \hat{x}(k) \\ \hat{d}(k) \end{bmatrix}$ and obtain the pair composed by the input and the state $\begin{bmatrix} \bar{x}(k) \\ \bar{u}(k) \end{bmatrix}$ corresponding to the equilibrium specified by a future constant disturbance. Finally, with $\bar{x}(k)$ and $\bar{u}(k)$ is possible to formulate the previous control problem.

Regarding the control horizon N , often is selected so that it includes in the prediction horizon all the main process dynamics. In this way the optimization variables are in large number, but the computational load linked to the solution of the optimization problem results consequently heavy. To obtain a less aggressive control action, in many industrial algorithms

is possible to define a control horizon $0 < N_u < N$ and consider the additional constraints $u(k) = u(k + i - 1)$, $i = N_u, \dots, N$.

5.1.c PREDICTIVE CONTROL OF NONLINEAR SYSTEMS

The approach of the Model Predictive Control can also be used to control nonlinear systems of the form

$$x(k + 1) = f(x(k), u(k))$$

With the performance index that can be selected as a generic non-linear function of the state and control variables over the prediction horizon, and the optimization problem that can be expressed as follows: $\min_{U(k)} J = \sum_{i=0}^{N-1} l(x(k + i), u(k + i))$ and that can include constraints on the state and on the input.

5.1.d MPC ALGORITHMS AND STABILITY SYSTEMS

Here, the closed loop stability for linear and non-linear systems is analysed, through some formulation of MPC. Only the state regulation problem is addressed. The system under control is seen to be expressed by:

$$x(k + 1) = f(x(k), u(k))$$

with $f(x, u)$ continuously differentiable with respect to its arguments, $f(0,0) = 0$, and with the constraints $x \in X, u \in U$ on the state and on the input, X and U are closed and bounded sets including the origin.

For the system previously defined, consider to know this auxiliary control law $u = \kappa_a(x)$, such that $\kappa_a(0) = 0$ and $X_f \subset X$ a positively invariant set, including the origin, defining in the way that for the closed loop system

$$x(k + 1) = f(x(k), \kappa_a(x(k)))$$

it results that $x(\bar{k}) \in X_f$ involves

$$x(k) \in X_f, \quad k \geq \bar{k}$$

$$u(k) = \kappa_a(x(k)) \in U, \quad k \geq \bar{k}$$

So, starting from an initial state $x(\bar{k}) \in X_f$, the evolution of the state of the closed loop, under the auxiliary control law, does not exceed the boundaries of X_f , satisfying the constraints on the input and on the state.

Consider now to find the control sequence $u(k), u(k+1), \dots, u(k+N-1)$ that minimizes the cost function over the finite horizon:

$$J(x(k), u(\cdot), k) = \sum_{i=0}^{N-1} (\|x(k+i)\|_Q^2 + \|u(k+i)\|_R^2) + V_f(x(k+N))$$

$$x(k+i) \in X, u(k+i) \in U, x(k+N) \in X_f.$$

V_f is a terminal penalty, $V_f(x) \geq 0$, $V_f(0) = 0$ and $Q = Q' > 0, R = R' > 0$.

It is possible to derive the implicit state feedback time invariant control law, through the solution of the optimization problem on the finite horizon and thanks to the receding horizon principle:

$$u = \kappa^{MPC}(x)$$

Is defined the optimal value of the cost function $J^0(x(k), k)$, with J^0 continuous function of its arguments. The following theorem stands, under all the previous assumptions:

If, for any $x(\bar{k}) \in X_f$,

$$V_f(f(x(k), \kappa_a(x(k)))) - V_f(x(k)) + (\|x(k)\|_Q^2 + \|\kappa_a(x(k))\|_R^2) \leq 0$$

then the origin $x = 0$ is an asymptotically stable equilibrium point of the closed loop system obtained with the MPC control law.

5.2 MODEL PREDICTIVE CONTROL OF HYBRID SYSTEMS

All the previous statements and the Model Predictive Control approach can be extended also to the control of hybrid systems, expressed in Mixed Logical Dynamical form or in Piecewise Affine form. Through the MLD model (or the PWA), the future behaviour of the hybrid system is predicted, and, on this prediction, the optimization is performed. The aim is the control of the hybrid system.

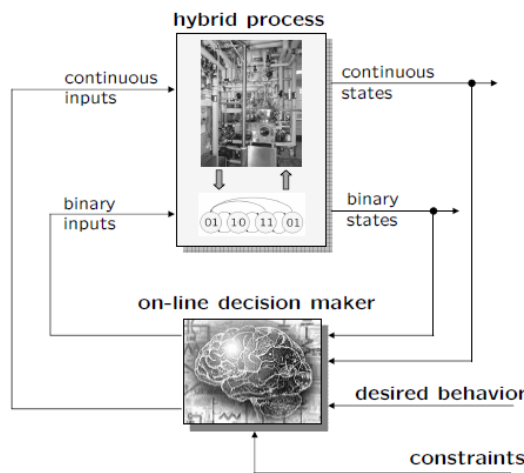


Fig. 5.1: model predictive control of the hybrid systems

With reference to [29] continuous and binary states are sent to the on-line decision maker by the hybrid system. Once the optimization problem is solved, the decision maker forwards to the hybrid process the control sequence in binary and continuous form. The constraints and the desired behaviour can be included in the process.

MLD SYSTEM:

$$x(k + 1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k)$$

$$y(k) = Cx(k) + D_1u(k) + D_2\delta(k) + D_3z(k)$$

$$E_2\delta(k) + E_3z(k) \leq E_1u(k) + E_4x(k) + E_5$$

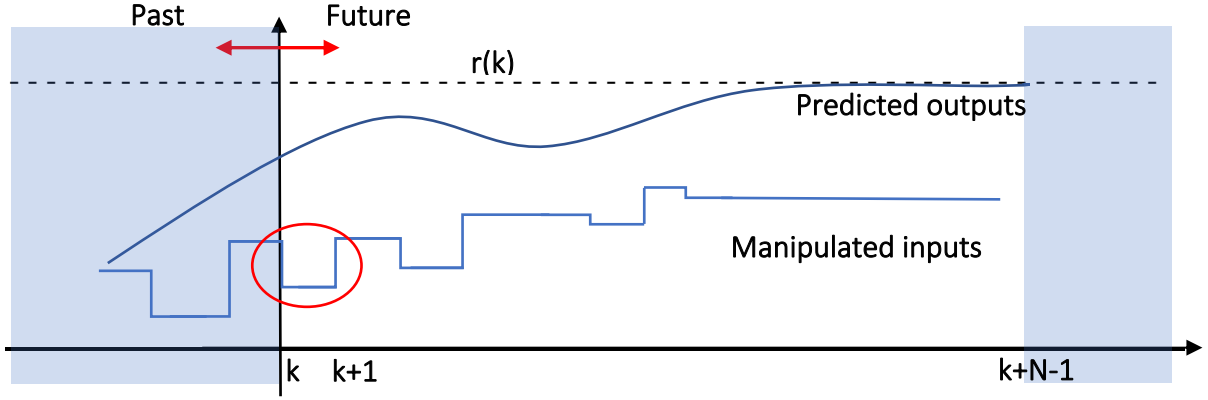


Fig. 5.2: model predictive control scheme

As mentioned, at time t the optimal control problem is solved, on the finite horizon N , with respect to $U \triangleq \{u(k), u(k+1), \dots, u(k+N-1)\}$ and apply only the first element $u^0(k)$:

$$\min_{u(\cdot)} \sum_{i=0}^{N-1} (\|y(k+i) - r(k)\| + \|u(k+i) - u_r\| + \sigma(\|\delta(k+i) - \delta_r\| + \|z(k+i) - z_r\| + \|x(k+i) - x_r\|))$$

Where $z \in R^{r_c}$, $\delta \in \{0,1\}^{r_l}$, are the auxiliary continuous and binary variables, x_r, u_r, δ_r, z_r represents the equilibrium values corresponding to the set point r .

5.2.a CONVERGENCE OF HYBRID MPC

Regarding the closed loop convergence and stability, a theorem exists:

Let $(x_r, u_r, \delta_r, z_r)$ be the equilibrium values corresponding to the set point r , and assume $x(0)$ is such the MPC problem is feasible at time $t=0$. Then $\forall Q, R > 0, \forall \sigma > 0$

$$\lim_{t \rightarrow \infty} y(t) = r$$

$$\lim_{t \rightarrow \infty} u(t) = u_r$$

$$\lim_{t \rightarrow \infty} x(t) = x_r$$

$$\lim_{t \rightarrow \infty} \delta(t) = \delta_r$$

$$\lim_{t \rightarrow \infty} z(t) = z_r$$

and all the constraints are fulfilled.

5.2.b COMPUTATIONAL ASPECTS OF OPTIMAL CONTROL OF HYBRID SYSTEMS

The problem of the optimal control can be solved through Mixed Integer Quadratic Program (MIQP) or by exploiting the Mixed Integer Linear Program (MILP).

The formulation of the first is the following:

considering the optimal problem, represented by

$$\begin{aligned} \min_{\xi} J(\xi, x(0)) &= \sum_{i=0}^{N-1} y'(k+i)Qy(k+i) + u'(k+i)Ru(k+i) \\ \text{subject to} &\begin{cases} x(k+1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) + B_5 \\ y(k) = Cx(k) + D_1u(k) + D_2\delta(k) + D_3z(k) + D_5 \\ E_2\delta(k) + E_3z(k) \leq E_1u(k) + E_4x(k) + E_5 \end{cases} \end{aligned}$$

And the optimization vector

$$\xi = [u(0), \dots, u(N-1), \delta(0), \dots, \delta(N-1), z(0), \dots, z(N-1)]'$$

The optimization problem results: $\min_{\xi} \frac{1}{2} \xi' H \xi + x(0)' F \xi$

$$\text{Subject to } G\xi \leq W + Sx(t)$$

$u \in R^{n_u}, \delta \in \{0,1\}^{n_\delta}, z \in R^{n_z}$ and it derives $\xi \in R^{(n_u+n_z)T} \times \{0,1\}^{n_\delta T}$, has both real and binary components.

Regarding the formulation of the second one:

the optimal control problem is represented by

$$\min_{\xi} J(\xi, x(0)) = \sum_{i=0}^{N-1} \|Qy(k+i)\|_{\infty} + \|Ru(k+i)\|_{\infty}$$

subject to the MLD model.

In this case slack variables are introduced $\min_{x, \epsilon} \epsilon$ such that $\epsilon \geq x$ and $\epsilon \geq -x$.

$$\begin{cases} \epsilon_i^x \geq \|Qy(k+i)\|_{\infty} \\ \epsilon_i^u \geq \|Ru(k+i)\|_{\infty} \end{cases}$$

The optimization vector is made by:

$$\xi = [\epsilon_1^x, \dots, \epsilon_N^x, \epsilon_0^u, \dots, \epsilon_{N-1}^u, u(0), \dots, u(N-1), \delta(0), \dots, \delta(N-1), z(0), \dots, z(N-1)]'$$

and the optimization problem is described by:

$$\min_{\xi} J(\xi, x(0)) = \sum_{i=0}^{N-1} \epsilon_i^x + \epsilon_i^u$$

$$\text{such that } G\xi \leq W + Sx(0)$$

again ξ has both real and binary components

5.3 CASE OF STUDY: SIMULATIONS, COMPARISONS AND ANALYSIS

During this thesis project, a hybrid MPC has been developed in order to minimize the thermal power exchanged by each radiator with the zone, guaranteeing the thermal comfort in each floor at the same time. The reference model, exploited to implement the MPC, is that in form Mixed Logical Dynamical formulated in the chapter 4 of this work. The control action, as already said, has been applied on the thermal power, imposing, as hard constraint, that it was always greater or equal to zero. To define the thermal comfort as the cost function, were imposed a lower and an upper bound for each floor between which the zone temperature had

to remain (soft constraint). Different simulations have been performed with different sampling time and different prediction horizon and a comparison has been made with the result obtained through a base case. In the base case is considered the same building model, controlled, through the software Simulink, with a simple ON-OFF logic and characterized by a hysteresis behaviour. Its functioning is the following: when one of the three floor needs thermal power, the system is considered ON. The thermal power in the base case assumes the zero value if the system is in OFF condition or the constant average value of the real thermal power signal measured if is in ON condition.

The simulations were performed with these different sampling times and prediction horizons:

- Sampling time = 60 s, prediction horizon = 2;
- Sampling time = 100s, prediction horizon=6;
- Sampling time = 600s, prediction horizon = 3;
- Sampling time = 900s, prediction horizon = 4;

In the following sections the base case scheme is described, then the different results obtained from the simulation of the hybrid MPC are reported, with the comparison with the base case in terms of total consumption.

5.3.a BASE CASE: SYSTEM CONTROLLED THROUGH LOGIC ON-OFF

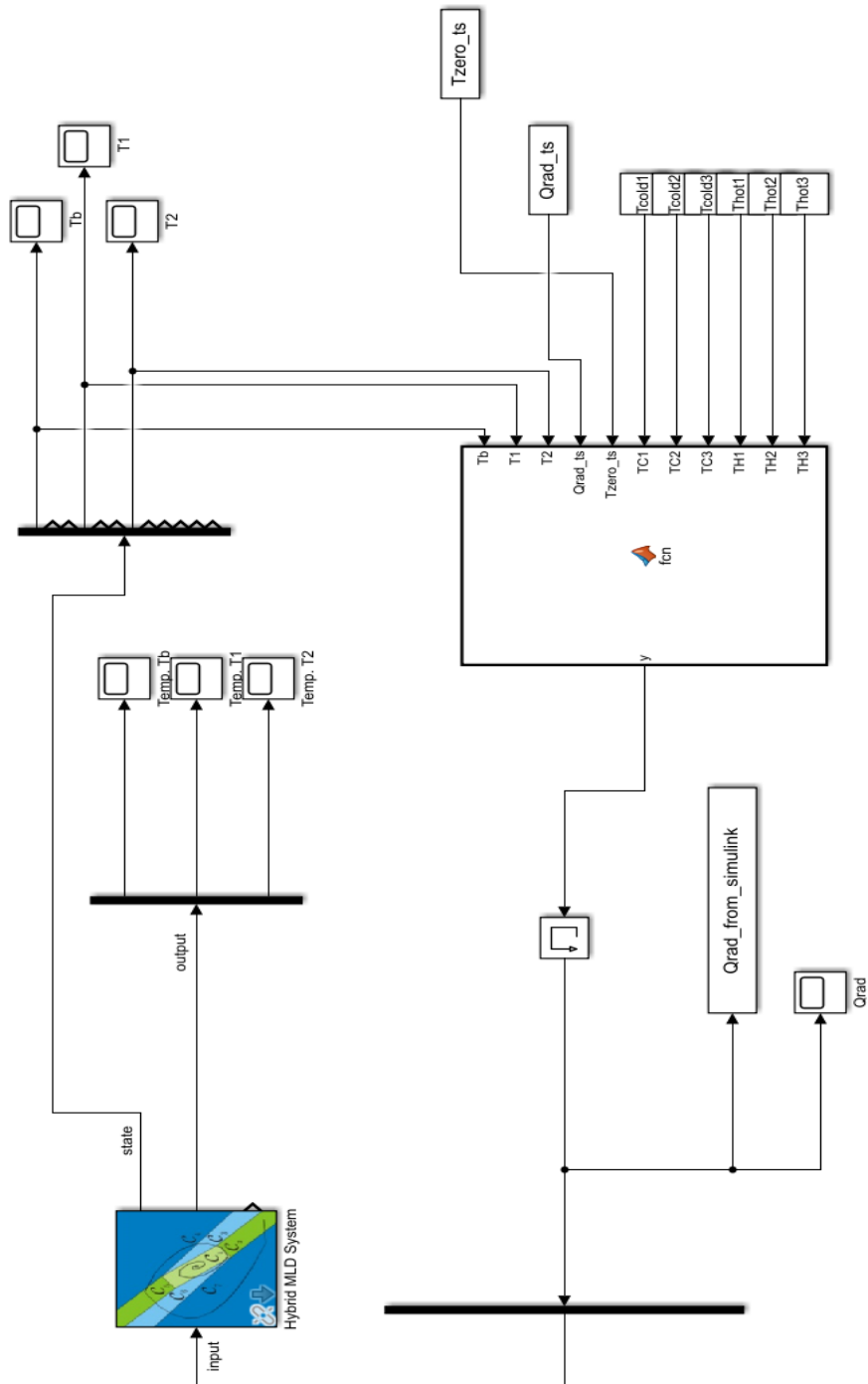


Fig. 5.3: Simulink scheme of the base case

As mentioned, in the base case the MLD system is controlled and regulated by a simple logic ON-OFF and it is characterized by a hysteresis behaviour. The lower and the upper limits imposed for the hysteresis are the same of those selected for the hybrid MPC algorithm, for each zone. The value of the thermal power, input of the system, can assume only two values: equal to 0 when the heating system is considered OFF or equal to 4, the average value of the real and measured thermal power, when the heating system is considered ON. It is sufficient that only one floor needs thermal power that the heating system is turned ON. There is not difference if the radiators turned ON at the same time are 1, 2 or 3, for a sake of simplicity the flowrate is considered constant and equal to 0.2 L/s. Here are represented the zone temperatures of the basement, of the first floor and of the second floor, obtained through the simulation of the base case. This simulation has been performed for 86400 seconds and with a sample time of 1 second. Moreover, also the progress of the thermal power is represented in the last plot. To obtain an approximated value of the energy consumption, the mean value of the thermal power is considered and is multiplied by the length of the time of the simulation.

As can be seen, the zone temperature of each floor is limited between 20°C and 22°C, boundaries imposed previously, and their progress is characteristic of a hysteresis behaviour. Also the thermal power has a trend expected, with the plot that touches only the value 0 and 4kW.

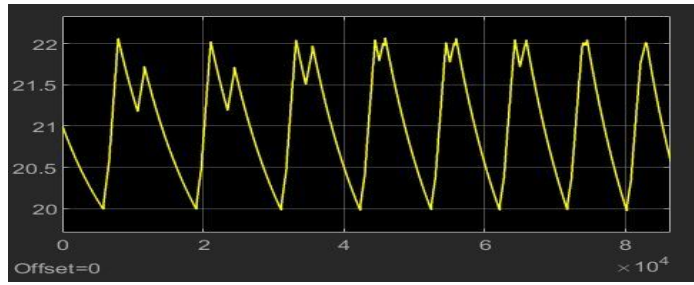


Fig. 5.4: basement temperature obtained through logic ON-OFF

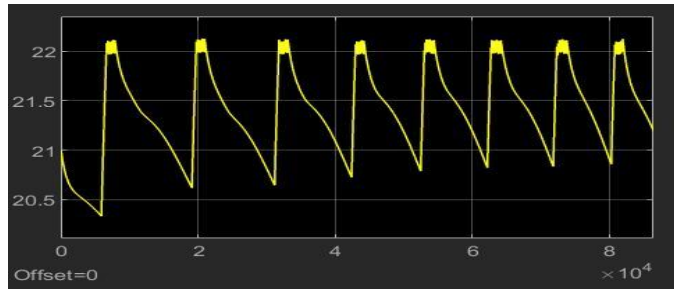


Fig. 5.5: first floor temperature obtained through logic ON-OFF

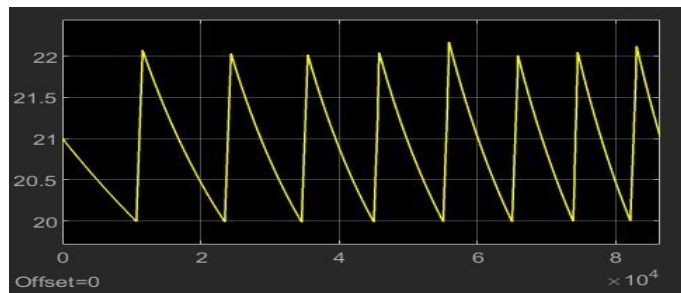


Fig. 5.6: second floor temperature obtained through logic ON-OFF

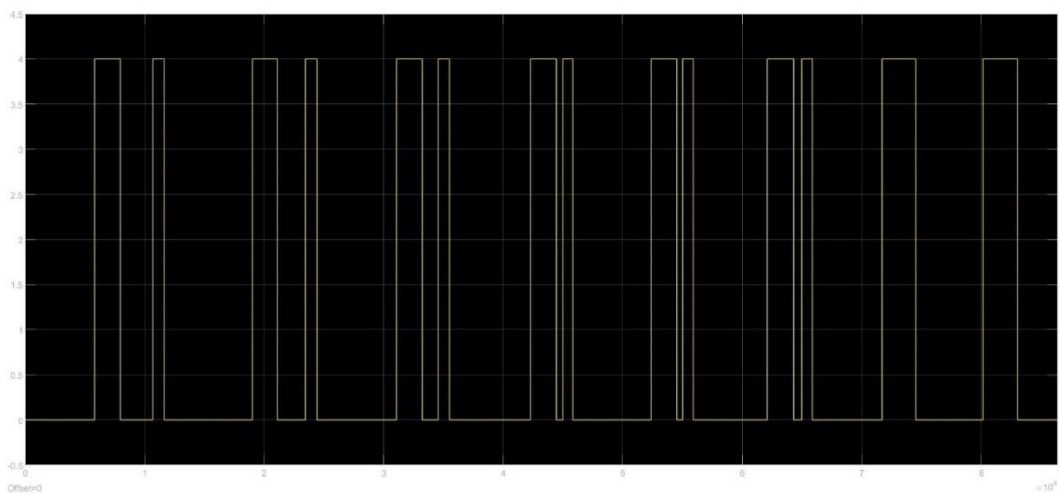


Fig. 5.7: thermal power obtained through logic ON-OFF

5.3.b HYBRID MPC SIMULATION-1: $T_s=60s$; $N=2$

With a low sampling time and a small prediction horizon the MPC can keep the temperatures of the three zones perfectly within the pre-established bounded. The basement temperature is kept near to the lower limit, since this is the coldest zone. The prediction is performed each 2 minutes, so there are not errors due to disturbances caused by the long term of the prediction.

Regarding the thermal power, the MPC tends to minimize it. To compute the energy consumption again the mean value of the thermal power is considered and then is multiplied by the length of the time of the simulation. The energy consumption results lower than that obtained through the base case. Here a represented the progresses of the temperatures oh the 3 zones and the progress of the thermal power.

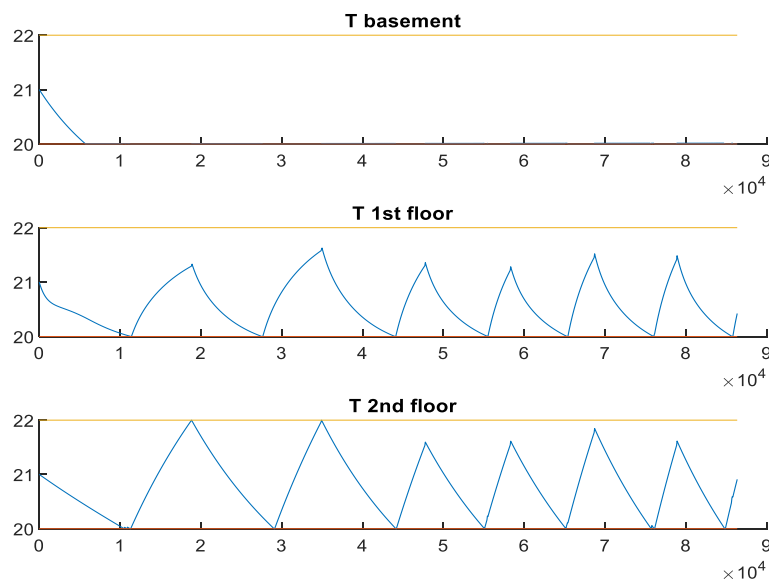


Fig. 5.8: temperatures of the 3 floors obtained through hybrid MPC ($T_s=60s$, $N=2$)

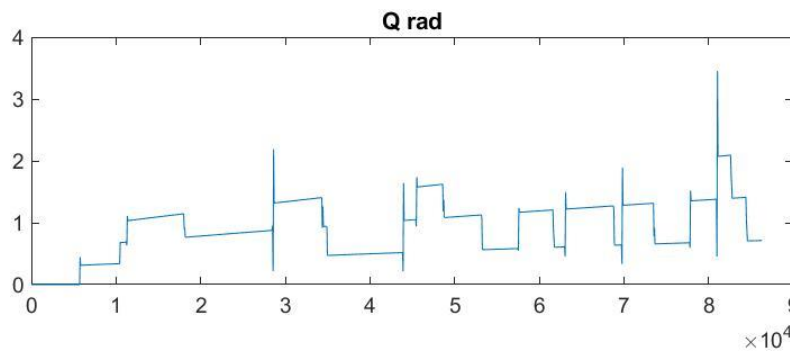


Fig. 5.9: thermal power obtained through hybrid MPC ($T_s=60s$, $N=2$)

5.3.c HYBRID MPC SIMULATION-2: $T_s=100s$; $N=6$

The same simulation is performed with a different sampling time and different prediction horizon. There is a little difference between the previous case, since the prediction is performed every 10 minutes instead of 2 minutes. In terms of energy consumption there is a slight improvement. Here are reported the plots of the zone temperature of the three floor and of the thermal power.

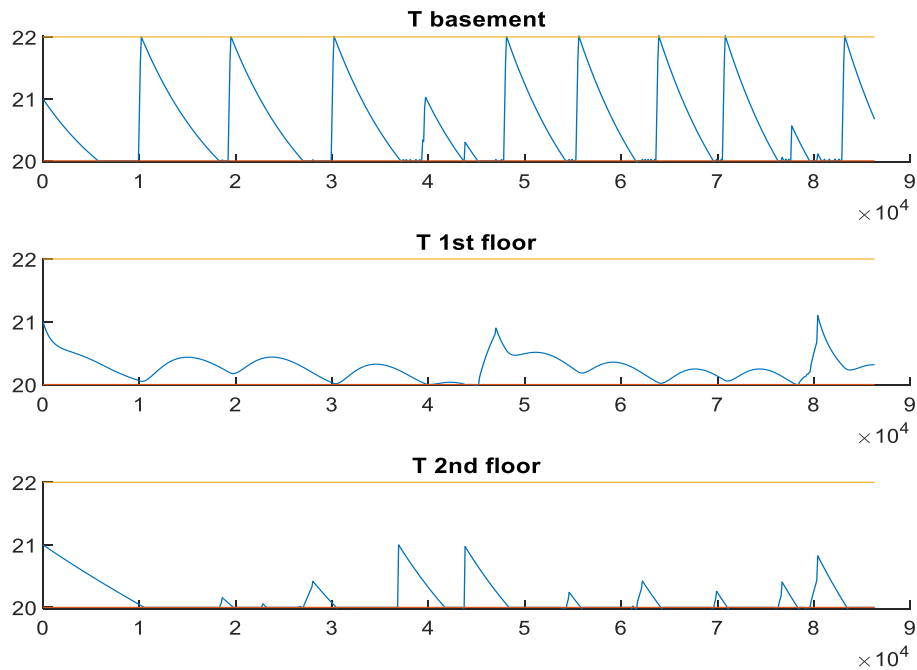


Fig. 5.10: temperatures of the 3 floors obtained through hybrid MPC ($T_s=100s$, $N=6$)

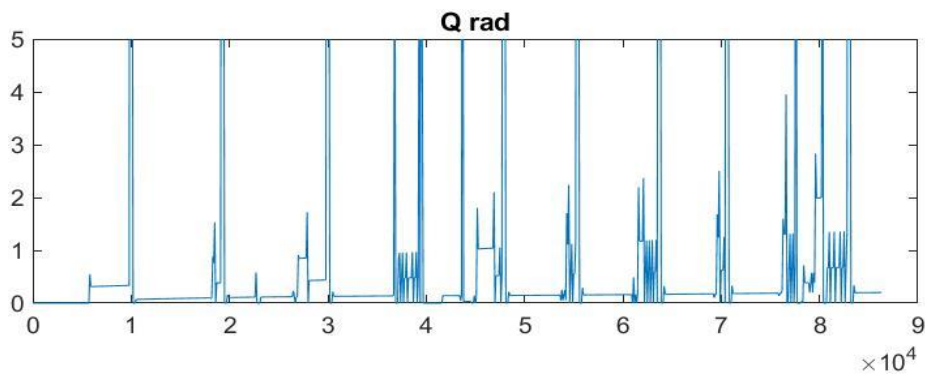


Fig. 5.11: thermal power obtained through hybrid MPC ($T_s=100s$, $N=6$)

5.3.d HYBRID MPC SIMULATION-3: $T_s=600s$; $N=3$

When the sampling time is augmented and the prediction is performed for the next 30 minutes, there is a further improvement regarding the energy consumption, since the controller knows in advance the progress of the system, but in this case are present some instants in which the zones temperature exceed the boundaries imposed as soft constraints, due to the fact that the sampling occurs each 10 minutes. Here below are represented the progress of the temperature of each floor and the thermal power

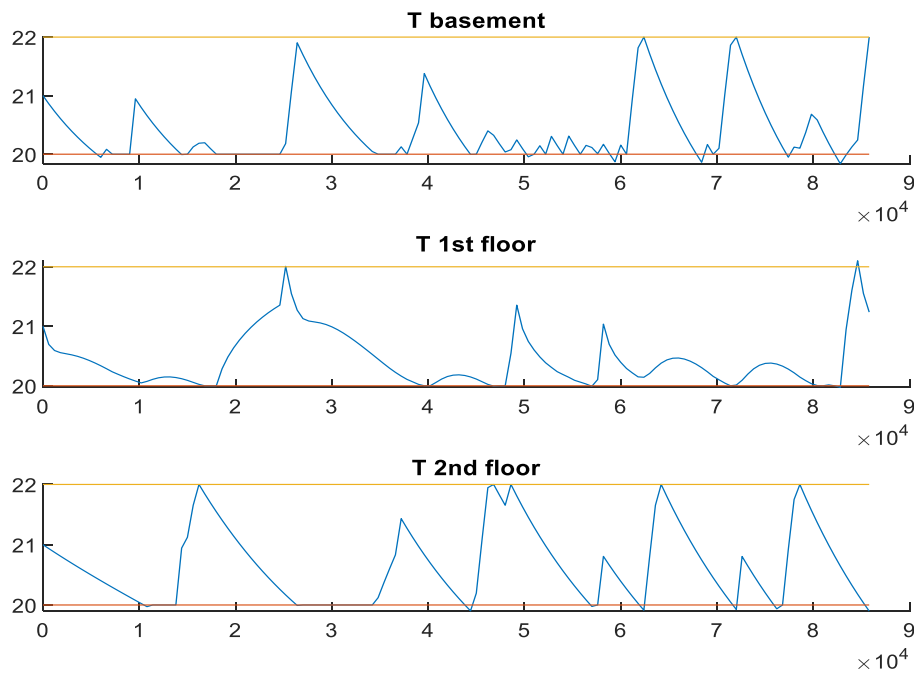


Fig. 5.12: temperatures of the 3 floors obtained through hybrid MPC ($T_s=600s$, $N=3$)

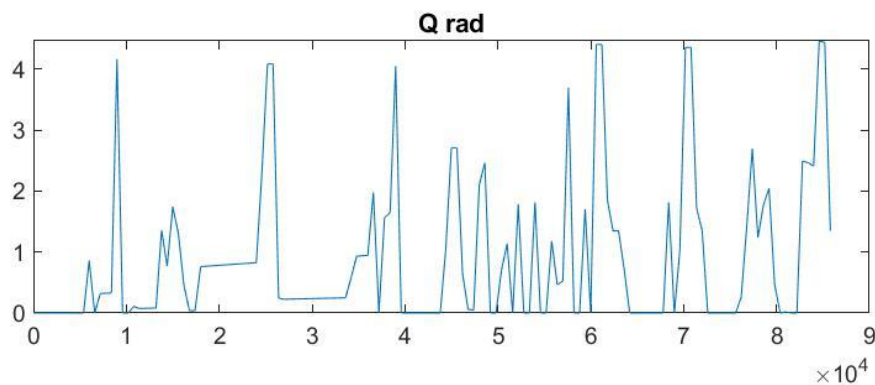


Fig. 5.13: thermal power obtained through hybrid MPC ($T_s=600s$, $N=3$)

5.3.e HYBRID MPC SIMULATION-4: $T_s=900s$; $N=4$

In this last case, the prediction happens every hour. The MPC knows the progress of the system and can be more conservative from the point of view of the energy consumption, but, again, the temperatures exceed, in some instants, the comfort limits previously imposed.

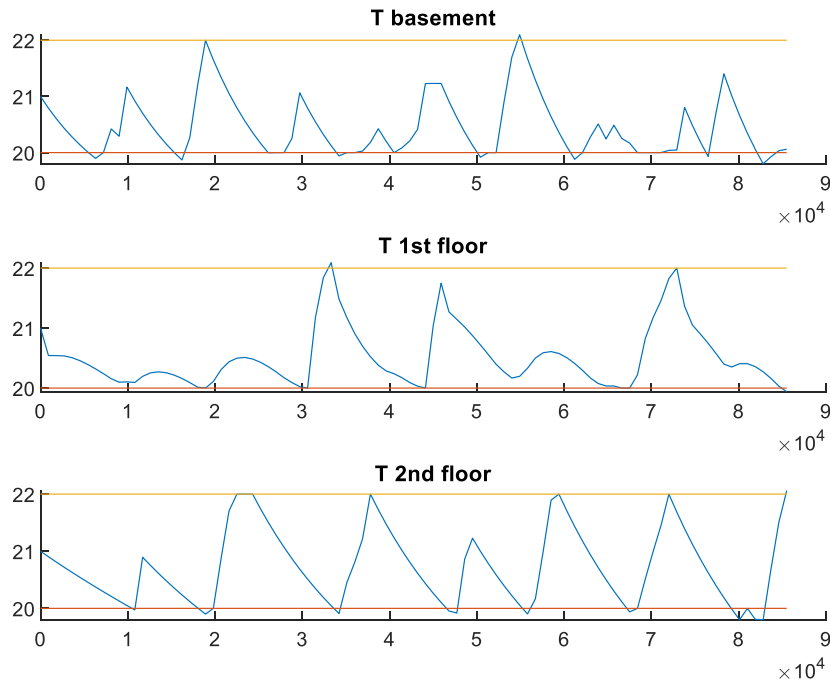


Fig. 5.14: temperatures of the 3 floors obtained through hybrid MPC ($T_s=900s$, $N=4$)

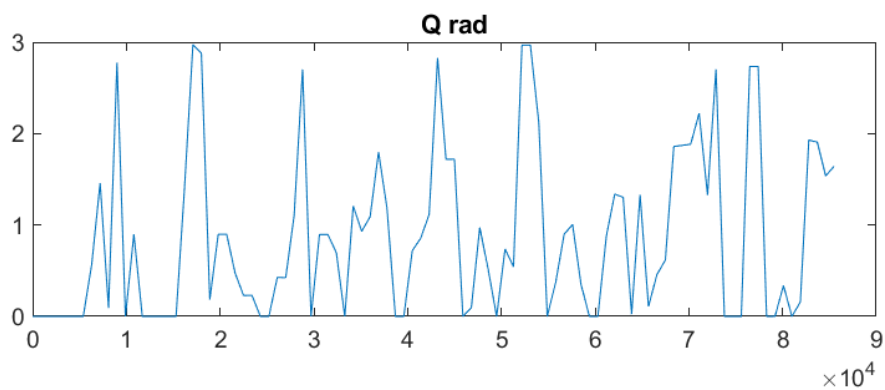


Fig. 5.15: thermal power obtained through hybrid MPC ($T_s=900s$, $N=4$)

5.3.f ANALYSIS AND COMPARISON

In the following table are reported the different values of the energy consumption obtained through the different simulations performed. As can be seen, all the simulation made by considering the system controlled by the hybrid MPC, have registered a lower energy consumption then that obtained in the base case, controlled by the logic ON-OFF. In particular, is notable an improvement with the increasing of the sampling time, this at the cost of the non-total respecting of the comfort limits.

	Base case	Hybrid MPC Ts=60s,N=2	Hybrid MPC Ts=100s,N=6	Hybrid MPC Ts=600s,N=3	Hybrid MPC Ts=900s,N=4
Thermal power mean (kW)	1,119	0,91	0,88	0,85	0,82
Energy consumption (kJ)	96682	78624	76032	73440	70848

Tab.5.1: comparison of energy consumption obtained in the different simulation

5.4 PERFORMANCES ANALYSIS WITH DIFFERENT CONSTRUCTIVE FEATURES OF THE BUILDING

A performances analysis is performed by creating the hybrid MPC with a model in which the thermal parameters relating to the walls vary in ranges explained in the section 3.4 of this work. In this way are considered different qualities of the building, starting from values that characterize more modern constructions up to the typical values of older structures.

The intervals considered for the specific heat and for the convective heat transfer coefficient are the following:

$$c_{wall} \in [13;65];$$

$$u_{wall} \in [0.28;1.61];$$

Where $13 \frac{kJ}{m^2K}$ and $0.28 \frac{W}{m^2K}$ are typical values of modern buildings (more thermally insulated), while $65 \frac{kJ}{m^2K}$ and $1.61 \frac{W}{m^2K}$ characterize ancient buildings (thermally dispersive). A cycle is implemented so as to be able to evaluate and record the energy consumptions obtained with the MPC created through models with 30 different values of c_{wall} and u_{wall} included in the previous intervals. The sampling time selected during the simulations is 900 seconds and the prediction horizon is equal to 4.

The same procedure is carried out for the base case, and again the energy consumptions are recorded to be plotted.

In the following plot are represented the energy consumptions obtained through the hybrid MPC and through the base case, considering, as explained, different building's qualities.

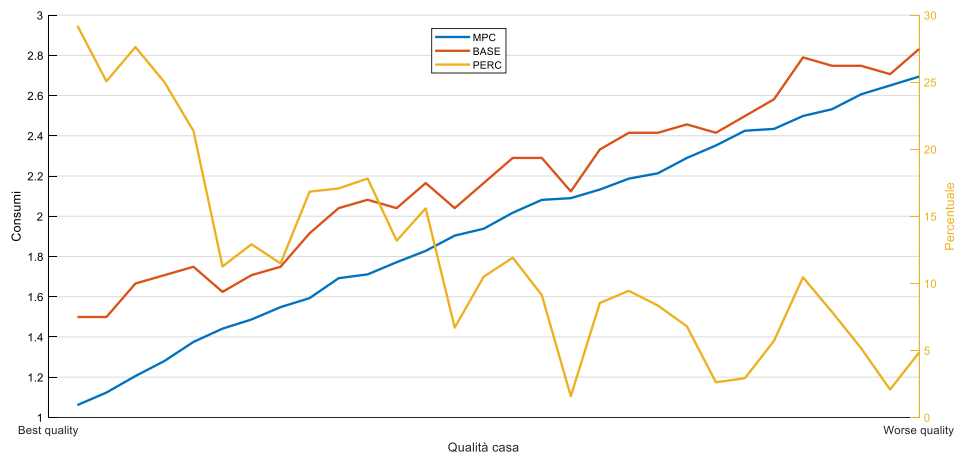


Fig. 5.16: consumption obtained with respect to different building qualities

As can be noticed from the yellow line (that represents the percentage difference between the consumption obtained through the base case and those obtained through the MPC), the hybrid MPC scheme is very convenient, with respect to the logic ON-OFF, in the modern buildings, where the difference is wide. Regarding the more ancient buildings, the energy savings is not so clear, so in that case the hybrid MPC is not so helpful.

5.5 PERFORMANCES ANALYSIS WITH DIFFERENT TEMPERATURE SET-POINT

Another performances analysis is carried out with the same procedure performed in the previous section, but in this case the parameter that varies is the temperature set point for the switch-on and switch-off of the radiators. The interval of temperatures in which this parameter changes is from 19°C to 24°C.

The analysis is made both for the system controlled through the hybrid MPC and for the system regulated by the simple logic ON-OFF.

Here is reported the plot that represents the progress of the energy consumption of the systems regulated through logic ON-OFF and through MPC. As expected, with the MPC the saving is always greater than the control through logic ON-OFF, and, clearly, when the set point is growing, the consumption increases. The yellow line represents again the percentage difference between the two control methods.

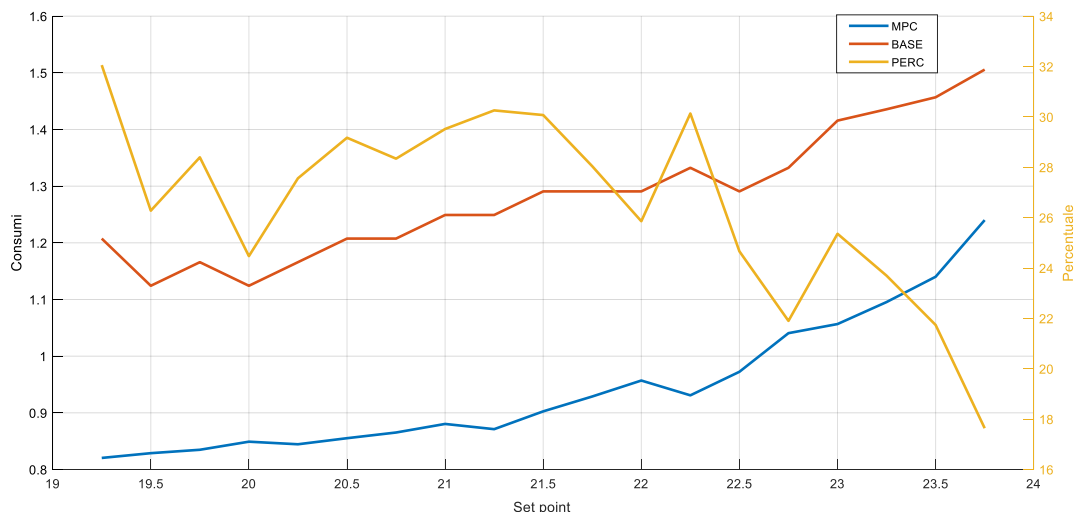


Fig. 5.17: consumption obtained with respect to different temperature set point

5.6 ROBUSTNESS ANALYSYS

The last analysis conducted is that relating to the robustness of the control system implemented. To perform it, 2 different hybrid MPCs are developed. The first (MPC_1) is created with an MLD model of the building called W . In the model W , the building's thermal parameters vary from values that are reduced of the 30%, with respect to the identified values, to values increased of the 30%, still with respect to the estimated values. Then this MPC is simulated with the model W as reference model, so it knows perfectly the system to control.

The second hybrid MPC (MPC_2) is created with the MLD model of the building S , developed in the chapter 4 of this work, and in which the thermal parameters are fixed and equal to those identified in the section 3 of this thesis. Then, this second MPC, is simulated giving it, as reference model, the model W , previously described and with the thermal parameters that vary. In this case, the MPC does not know perfectly the model, due to the mismatch introduced for the thermal parameters of the building.

In the following plots are reported the results, in terms of energy consumption and comfort, obtained in the two cases explained and described previously.

ENERGY CONSUMPTION

Regarding the energy consumption the results obtained are the following. As expected through the MPC_1, that knows perfectly the reference model, the energy use is always lower than the case in which the simulation is performed through the MPC_2, that does not know perfectly the reference model. Moreover, the value of the energy consumption is smaller when the thermal parameters are reduced with respect to the original ones, this because the building is more heat-insulated in both cases, and consequently both the MPCs can be more conservative.

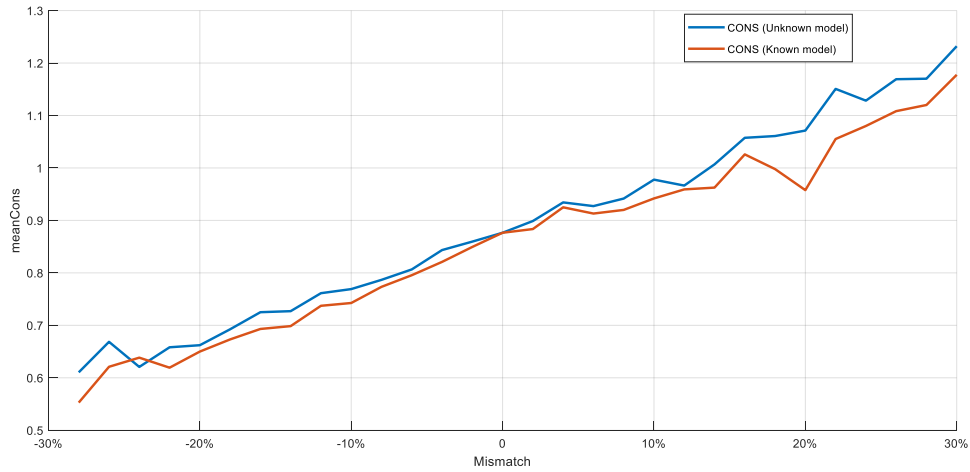


Fig. 5.18: consumption obtained with respect to model mismatch

COMFORT

In addition to the energy consumption, also the thermal comfort is evaluated. Again, the two MPCs are simulated in the same way previously explained, and all the times that, over the period of the entire simulation, the temperature of one of the three zones exceeds the lower or the upper bound, imposed as soft constraints, a counter variable is increased. In the following plot, is reported the value of this counter variable, for both the MPCs, with respect to the mismatch model. Again, as expected, when the MPC knows perfectly its reference model, the number of exceeding bounds is lower than when the reference model is not known by the MPC. Moreover, when the values are reduced, both the MPCs can be more conservative, in fact, as visible in the plot, the number of crossing bounds rises considerably when the values of thermal parameters are increased with respect the original ones, in particular when the mismatch goes from the 20% to the 30% .

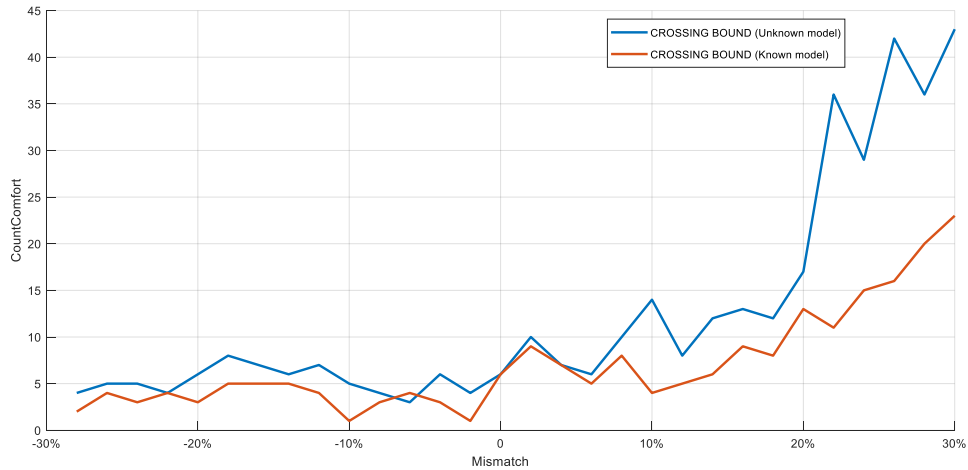


Fig. 5.19: comfort obtained with respect to model mismatch

5.7 CHAPTER 5 CONCLUSIONS

In this chapter, the hybrid MPC was developed and then simulated with different sampling times and different prediction horizon. The objective was to minimize the thermal power of the radiators, guaranteeing, at the same time, the thermal comfort of the user. The results obtained during the simulations were compared to those obtained through a base case, developed in Simulink environment, in which the same model was regulated through a logic ON-OFF. The simulation was performed at $T_s=60s$ and $N=2$, at $T_s=100s$ and $N=6$, at $T_s=600$ and $N=3$ and $T_s=900$ and $N=4$. The energy consumption obtained through the hybrid MPC was always lower than that obtained through the base case. Moreover, it was noticed that increasing the sampling time, the consumption decreased slightly, but not always the limits of thermal comfort were respected. Subsequently, a performances analysis based on different constructive features of the building was performed. It was obtained that the hybrid MPC gave more benefits with modern building with respect to the results obtained through the base case, while, with more ancient building, the difference between hybrid MPC and base case is not so visible. Another performances analysis was carried out, based on different temperature set point, and, as expected, with lower temperature the consumptions were reduced. Finally, a robustness analysis of the predictive system control developed was made, and, again as expected, when the MPC knew perfectly its reference model, during the simulation, the results were better in terms of energy consumption and thermal comfort.

6-CONCLUSIONS

This final chapter is dedicated to the conclusion of the whole thesis. The main contribution of this work will be described along with its limitations and suggestions for a future development. The main achievements of this research are presented in the first section 6.1, followed by the limitations and future developments section 6.2.

6.1 MAIN ACHIEVEMENTS

Through this thesis, a thermal model of a building was developed, made hybrid by the hysteresis behaviour of the temperatures of the different zones. A crucial part of this work was performed in the chapter 3, the identification of the unknown parameters. Through some available data sets and by exploiting the real progress of the temperatures of the 3 floors of the building, the thermal parameters of walls, pavements, ceiling and the remedial coefficients of the radiators, were estimated via Grey-Box model. Subsequently, the logical propositions that govern the hysteresis behaviour of the building were expressed and, thanks to Hysdel software, they were connected to the dynamical equations of the system to create the MLD model of the building. The last step was the implementation of the hybrid MPC, in order to minimize the thermal power, maintaining, at the same time, the thermal comfort of the user. The results obtained were compared to those given by the same MLD system, controlled by a simple logic ON-OFF. Finally, a performance analysis highlighted that the system of predictive control developed, guaranteed a clear saving, in terms of consumption, when applied to building with modern constructive features, and, as expected, with a lower temperature set point. Concluding, a robustness analysis of the hybrid MPC was performed.

During the development of this project, it was possible to understand, in a practical way, the importance of creating a model that describes correctly the thermal dynamics of the system in a sufficiently detailed way. This guaranteed a stronger physic representativeness of the results, on which the predictive control strategy was developed. Moreover, it was possible to verify the advantages that the predictive control gave, in terms of energy consumption, with respect to a simple control based on the logic ON-OFF.

6.2 FUTURE DEVELOPMENTS

There are many possible future developments that can be carried out in order to verify the real-world efficiency of such idea. These future developments are listed in this section.

During the modeling of the radiator, it was chosen a simplified model to express the thermal power. A possible development could be the implementation of a model of the radiator that specifically considers the internal thermal dynamics and not only the inlet and outlet temperature of the water.

Another in-depth analysis could be to consider the flow rate not constant and not equal for each radiator, but divided between them according to the need, modeling a distribution system.

Then, it could be possible to consider the heating system of the building connected to a heat pump and modeling the whole plant.

Finally, this type of control could be extended to other existing heating systems, different from the system with radiators.

PAGE INTENTIONALLY LEFT BLANK

PAGE INTENTIONALLY LEFT BLANK

LIST OF TABLES

Tab. 3.1: thermal capacity and thermal conductance of pavement and walls (first floor)...	46
Tab. 3.2: specific heat and convective heat transfer coefficient of pavement and walls (first floor)	46
Tab. 3.3: thermal capacity and thermal conductance of ceiling and walls (second floor)...	48
Tab. 3.4: specific heat and convective heat transfer coefficient of ceiling and walls (second floor).....	48
Tab. 3.5: thermal capacity and thermal conductance of walls (basement).....	51
Tab. 3.6 specific heat and convective heat transfer coefficient of walls (basement)	51
Tab. 3.7: parameters identified	52
Tab. 4.1: from basic logic relation to integer inequalities	62
Tab. 4.2: from logic propositions to mixed integer inequalities.....	63
Tab.4.3: the resulting implications to check the equivalence of the logical statement to the given set of inequalities	64
Tab. 5.1: comparison of energy consumption obtained in the different simulations	101

LIST OF FIGURES

Fig. 2.1: front view and rear view of Flexhouse_03	17
Fig. 2.2: schematic picture of the house, with relative dimensions.....	18
Fig. 2.3: RC-network that models the thermal behaviour of the building	20
Fig. 2.4: complex model of the radiator	20
Fig. 2.5: model of the basement	22
Fig. 2.6: model of the first floor	24
Fig. 2.7: model of the second floor.....	27
Fig. 2.8: model of the entire building	30
Fig. 3.1: real temperatures of the 3 floors.....	42
Fig. 3.2: outside and ground temperature	43
Fig. 3.3: thermal power.....	43
Fig. 3.4: simulation of the first floor temperature with parameters identified (radiatorOFF).....	46
Fig. 3.5: simulation of the first floor temperature with parameters identified (radiatorON).....	47
Fig. 3.6: simulation of the second floor temperature with parameters identified (radiatorOFF).....	48
Fig. 3.7: simulation of the second floor temperature with parameters identified (radiatorON).....	50
Fig. 3.8: simulation of the basement temperature with parameters identified (radiatorON).....	51
Fig. 3.9: simulation of the basement temperature with parameters identified (radiatorON).....	52
Fig. 4.1: hybrid system theory	57
Fig. 4.2: structure of the discrete hybrid automata	68
Fig. 4.3: generation of the MLD model.....	70
Fig. 4.4: hysteresis diagram.....	73
Fig. 4.5: hysteresis behaviour of the zone temperature	75
Fig. 4.6: inputs of the MLD model (outside temperature, ground temperature, thermal power).....	76
Fig. 4.7: MLD trajectories	77
Fig. 4.8: PWA trajectories	77
Fig. 4.9: difference between MLD trajectories and PWA trajectories	78

Fig. 5.1: model predictive control of the hybrid systems	89
Fig. 5.2: model predictive control scheme	90
Fig. 5.3: Simulink scheme of the base case	94
Fig. 5.4: basement temperature obtained through logic ON-OFF	96
Fig. 5.5: first floor temperature obtained through logic ON-OFF.....	96
Fig. 5.6: second floor temperature obtained through logic ON-OFF	96
Fig. 5.7: thermal power obtained through logic ON-OFF	96
Fig. 5.8: temperature of the 3 floors obtained through hybrid MPC (Ts=60s, N=2)	97
Fig. 5.9: thermal power obtained through hybrid MPC (Ts=60s, N=2).....	97
Fig. 5.10: temperature of the 3 floors obtained through hybrid MPC (Ts=100s, N=6) ..	98
Fig. 5.11: thermal power obtained through hybrid MPC (Ts=100s, N=6).....	98
Fig. 5.12: temperature of the 3 floors obtained through hybrid MPC (Ts=600s, N=3) ..	99
Fig. 5.13: thermal power obtained through hybrid MPC (Ts=600s, N=3).....	99
Fig. 5.14: temperature of the 3 floors obtained through hybrid MPC (Ts=900s, N=4)	100
Fig. 5.15: thermal power obtained through hybrid MPC (Ts=900s, N=4).....	100
Fig. 5.16: consumption obtained with respect to different building qualities	102
Fig. 5.17: consumption obtained with respect to different temperature set point	103
Fig. 5.18: consumption obtained with respect to model mismatch	105
Fig. 5.19: comfort obtained with respect to model mismatch	106

BIBLIOGRAPHY

- [1] [www.europarl.europa.eu/News/Efficienza energetica: nuove regole per case ecologiche e intelligenti](http://www.europarl.europa.eu/News/Efficienza_energetica:_nuove_regole_per_case_ecologiche_e_intelligenti), 2018
- [2] G.J.Ríos-Moreno, M.Trejo-Perea, R.Castañeda-Miranda, V.M.Hernández-Guzmán, G.Herrera-Ruiz , "Modelling temperature in intelligent buildings by means of autoregressive models", *Automation in Construction*, pp. 713-722, 2007
- [3] SiyuWu, Jian-Qiao Sun, "A physics-based linear parametric model of room temperature in office buildings", *Building and Environment*, pp.1-9, 2012
- [4] K. Amarasinghe, D. Wijayasekara e M. Manic, "Neural Network Based Downscaling of Building Energy Management System Data", in *IEEE International Symposium on Industrial Electronics (ISIE 2014)*, Istanbul, Turkey, 2014.
- [5] A. Bemporad, M.K. Camlibel, W.P.M.H. Heemels, A.J. van der Schaft, J.M. Schumacher, B. de Schutter, "Further switched systems", *Handbook of hybrid systems control : theory, tools, applications*, pp. 139-192, 2009
- [6] M.Gerdts, "A variable time transformation method for mixed-integer optimal control problems", *Optimal Control: Applications and Methods*, pp. 169-182, 2006
- [7] M. Heidarinejada, J. Liub, P.D. Christofides, "Economic model predictive control of switched nonlinear systems", *Systems and Control letters*, pp.77-84, 2013
- [8] F. Farshidian, M. Kamgarpour, D. Pardo, J.Buchli, "Sequential Linear Quadratic Optimal Control for Nonlinear Switched Systems", *IFAC-PapersOnLine*, pp.1463-1469, 2017
- [9] K. Kobayashia, J. Imurab, "Deterministic finite automata representation for model predictive control of hybrid systems", *Journal of Process Control*, pp.1670-1680, 2012
- [10] A. Khakimova, A. Kusatayeva, A. Shamshimova, D. Sharipova, A. Bemporad, Y. Familiant, A. Shintemirov, V. Ten, M. Rubagotti, "Optimal energy management of a small-size building via hybrid model predictive control", *Energy and Buildings*, pp. 1-8, 2017
- [11] A. Bemporad, M Morari, "Control of systems integrating logic, dynamics, and constraints", *Automatica*, pp. 407-427,1999
- [12] D. Frick, A. Domahidi, M. Morari, "Embedded optimization for mixed logical dynamical systems", *Computers and Chemical Engineering*, pp.21-33, 2015

- [13] [B. Mayer, M. Killian, M. Kozek “A branch and bound approach for building cooling supply control with hybrid model predictive control”, *Energy and Buildings*, pp.552-566,2016
- [14] E. Luchini, A. Schirrer, M. Kozek, “A hierarchical MPC for multi-objective mixed-integer optimisation applied to redundant refrigeration circuits”, *IFAC-PapersOnLine*, pp.9058-9064, 2017
- [15] A.Bürger, C.Zeile, A. Altmann-Dieses, S.Sager, M.Diehl, “Design, implementation and simulation of an MPC algorithm for switched nonlinear systems under combinatorial constraints”, *Journal of Process Control*, pp.15-30, 2019
- [16] PURMO - www.purmo.com
- [17] http://www.biocab.org/Heat_Transfer.html
- [18] K.Tsapralidis, "Modeling, Identification and Control for Heat Dynamics of aThree-Storey Brick House", Master Thesis, 2014.
- [19] N. R. Kristensen, H. Madsen, S.B. Jørgensen, "Parameter estimation in stochastic grey box models", *Automatica*, pp.225-237, 2004.
- [20] R.Babini, "Self-Tuning Algorithms For Outside Temperature Rejection For Radiant Panel Heating Systems", Master Thesis, 2016
- [21] D.Mignone, "Control and Estimation of Hybrid Systems with Mathematical Optimization", A Dissertation Submitted to the Swiss Federal Institute of Technology (ETH) Zurich, 2002
- [22] M.S. Branicky, V.S. Borkar, S.K. Mitter, "A Unified Framework for Hybrid Control: Model and Optimal Control Theory", *IEEE Transactions on Automatic Control*, pp. 31-45,1998
- [23] B. De Schutter, T.J.J. van den Boom, “Model predictive control for max-min-plus scaling systems” *Proceedings of the 2001 American Control Conference*, Arlington, Virginia, pp. 319–324, 2001.
- [24] G. Ferrari-Trecate, F. Cuzzola, M.Morari, D.Mignone," Analysis of discrete-time piecewise affine and hybrid systems", *Automatica*, pp.2139-2146, 2002

- [25] A.Bemporad, F.D.Torrise, "HYSDEL—A Tool for Generating Computational Hybrid Models for Analysis and Synthesis Problems", IEEE Transactions on Control Systems Technology, pp. 235-249,2004
- [26] HYSDEL Manual
- [27] W.P.M.H.Heemels. B.De Schutter, A.Bemporad "Equivalence of hybrid dynamical models", Automatica, pp.1085-1091,2001
- [28] L. Magni, R. Scattolini, "Advanced and Multivariable Control", Automation Theory, Pitagora Editrice Bologna, 2014.
- [29] A.Bemporad, "Model Predictive Control of Hybrid Systems", Advanced Process Control Applications for Industry Workshop, Vancouver, British Columbia, 2007