Department of Civil, Environmental and Territorial Engineering
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The analysis of bus transit network reliability through scale-free modeling: the case studies of Lecco, Como and Varese

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“A developed country isn’t a place where the poor have cars. It’s where the rich use public transportation.”

Gustavo Petro

DEDICATION

I am dedicating this thesis to the soul of the man who means to me everything, who I take from his soul the power, who am here because of him.

I still remember his words and advice. Thank you, my hero, mentor, guide, and Sheikh, for everything. Thank you, my father.

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My warm and heartfelt thanks go to my friends and brothers in Sudan and Italy for the tremendous support and hope they had given to me. Thank you all for the strength you gave me. I love you all.

Nobody has been more important to me in the pursuit of this life than my family members. I would like to thank my mother, whose love and guidance are with me in whatever I pursue. She is the ultimate role model. Most importantly, I wish to thank my loving and supportive sisters, who provide unending inspiration.

Abstract

Many systems may be described as networks composed of nodes connected by links. Complex natural and man-made systems have connections patterns that closely match a scale-free network structure. Scale-free concepts are commonly described in real-world networks, indicating that nodes with degree k appear with a probability p(k) proportional to $k^{-\varepsilon}$. This pattern has many implications for complex systems structure.

In Italy, the bus transport service plays an important role in moving people within and between cities; therefore, it is significant to study its topological features, network structures, and connectivity as well as analyze its evolution, development, robustness, and resilience.

In this thesis, firstly, the bus transport networks of Lecco, Como, and Varese cities have been modeled as unweighted, directed graphs, and evaluated whether they follow the scale-free feature.

Secondly, the robustness of the three bus systems has been studied under random attack and intentional attack.

Keywords: Urban transit network; Scale-free network; Degree distribution; robustness.

Sommario

Molti sistemi possono essere descritti come reti composte da nodi collegati da lati. Molti sistemi complessi sia naturali, sia creati dall'uomo hanno schemi di connessione che corrispondono strettamente a una struttura di rete scale-free. Le reti scale-free sono comunemente descritte nelle reti del mondo reale, indicando che i nodi di grado k, in tali reti, appaiono con probabilità p(k) proporzionale a $k^{-\varepsilon}$. Questo modello ha molte implicazioni per la struttura di sistemi complessi.

In Italia, il servizio di trasporto con autobus svolge un ruolo importante nello spostamento delle persone all'interno e tra le città; pertanto, è significativo studiarne le caratteristiche topologiche, le strutture di rete e la connettività, nonché analizzarne l'evoluzione, lo sviluppo, la robustezza e la resilienza.

In questa tesi, in primo luogo, modelliamo le reti di trasporto servite da autobus delle città di Lecco, Como e Varese come grafi diretti, non pesati e valutiamo se risultano scale-free.
In secondo luogo, la robustezza delle tre reti viene studiata sotto attacco casuale e attacco intenzionale.

Parole chiave: Rete di trasporto urbano; scale-free network; distribuzione dei gradi; robustezza

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Abbreviations

N number of nodes

E number of edges

V set of nodes

L set of edges/links

A area in km^2

P population in thousands of inhabitants

k degree of a node

d diameter of a graph

k average degree

C average clustering coefficient

r assortative coefficient

l average path length

g(i) betweenness centrality of the node i

LAC Local average connectivity

C_c(i) closeness centrality of the node i

PTN public transport network

xTN x transport network, where x can be the initial letter of any type of transport (Bus, tram, rail, etc.)

BTN Bus transport network

b

Chapter 1

Introduction

In our daily lives, complex systems such as airports, power grids, transportation systems, and disease control systems make our lives more convenient and comfortable. Networks with complex topologies can model these infrastructures.
In the last 20 years, complex network theory has experienced an important growth in popularity. This has been the result of not only many important theoretical insights, e.g., the models introduced by A. L. Barabási, R. Albert, D. J. Watts, or S. H. Strogatz, but also of the wide applicability of network principles in the study of real-world systems. Since the initial works describing the topology of various transportation systems, analyses have become more sophisticated and cover a wide range of domains. Undoubtedly, many systems in nature can be described by models of complex networks, which are structures consisting of nodes or vertices connected by links or edges. (Chen, Xi, Liu, & Li, 2020)

Complex network theory is a science that studies the connection and interaction between components in a system. It is an emerging field, developing at a rapid rate. Particularly in the last decade, the proposing of small-world and scale free models have brought about comprehensive research and interest on this subject from different disciplines, which shows that the structures of networks existing in various domains are more similar than one would have expected. (Van Steen, 2010)

When the Erdos-Renyi method for producing random graphs was initially proposed, network analysis began to gain interest. Nevertheless, it was eventually realized that the topologies and evolution of real-world networks in our daily lives are governed by more advanced concepts in the field of complex networks. A variety of systems can be characterized as complex networks, including PTNs, the Internet, financial systems, social networks, etc. They are applicable to many types of real-world systems.

Two characteristics of complex networks have emerged to be particularly relevant; these are scale-free patterns and small-worlds effects. Small worlds were introduced by Watts and Strogatz; these networks have the particularity of being locally well connected while remaining close. (Watts & Strogatz, 1998)

Scale-free networks, as introduced by Barabási and Albert, follow a power law distribution between the number of nodes and their number of links, i.e., few nodes have many links, and many nodes have few links. (Barabási & Albert, 1999)

Transportation networks are examples of networks in real life. They are among the critical infrastructures underpinning our societies and economies.

In this thesis, we analyze the Bus transport network (BTN) of 3 cities in the northern part of Lombardia, an Italian region: Lecco city, Como city, and Varese city based on the concepts of scale-free network. In order to design and manage the bus system efficiently, it is crucial to understand their topological properties, network structures, and connectivity. Furthermore, the evolution of these cities has an important influence on the development of BTNs.
The work aims to analyze the BTNs in the 3 cities from a complex network perspective, mainly whether they have a scale-free feature. Moreover, we effectively analyze the complexity and robustness of those system.

Chapter 2

Literature Review

This chapter overviews theoretical aspects on networks and graphs that are the base of the analysis tasks we perform. We first introduce general concepts on network theory, properties, and models we are interested at. We then focus on the public transportation network, their topologies, and the bus services they provide, which is the core on this work.

2.1 Network theory

In the context of network theory, a complex network is a graph (network) with non-trivial topological features—features that do not occur in simple networks such as lattices or random graphs but often occur in networks representing real systems. The study of complex networks is a young and active area of scientific research since 2000. (Albert & Barabási, 2002)

There are many definitions describing the theory of network:

- Is the study of graphs as a representation of symmetric or asymmetric relationships among discrete objects.
- Is a branch of mathematics that studies networks in order to represent sets of discrete objects with symmetric or asymmetric pairwise interactions.
- In computer science and network science: a network is a graph with properties on the nodes and/or edges.

Networks are fundamental to the study of large-scale transportation models representing an entire metropolitan area, a state, or multistate regions. They can be applied in many contexts, including alternatives analysis, developing congestion pricing plans, identifying bottlenecks and critical infrastructure,

shipping and freight logistics, multimodal planning, and disaster evacuation planning, to name only a few. The reason network models are so useful, and so broadly applicable, is because a mathematical network is a simple, compact, and flexible way to represent a large, complicated system.
A network is considered reliable if the expected trip costs are acceptable, even when users are extremely pessimistic about the state of the network.

A graph is a representation of a network and its connections in symbols. It entails a reduction of reality to a network of linked nodes. Leonhard Euler, who devised the "Seven Bridges of Konigsberg" problem in 1735, is credited with the invention of graph theory. Someone had to cross all the bridges only once and in a continuous sequence to solve this problem, which Euler proved to have no solution by modeling it as a set of nodes and connections. This paved the way for the development of graph theory and future advancements. Growing effects from studies of social and complex networks have enriched it throughout the last few decades.

Most networks in transportation geography have a clear spatial base, such as road, transit, and rail networks, which are defined more by their links than by their nodes. This isn't always the case with all transit systems. Maritime and air networks, for example, are more defined by their nodes than by their links, which are sometimes ill-defined. A network can also be used to describe a telecommunication system, while its spatial expression may be minor and difficult to depict. Mobile phone networks and the Internet, which may be the most complex graphs to analyze, are examples of networks with a structure that is challenging to represent. Cellphones and antennae, on the other hand, can be depicted as nodes, with individual phone calls serving as linkages. Servers, which are at the heart of the Internet, can be depicted as nodes in a graph, with physical infrastructure, such as fiber optic cables, acting as links. As a result, graph theory can be used to model all transportation networks in some way.

Directed graph is a graph in which edges are directed, i.e. an edge has a starting node and an ending one (sometimes shortened to digraph). In a directed graph, each edge represents a one-way link from one node to another, but not backwards. In graph theory, it is possible to associate a number to each edge. Such a number is called weight, and the graph is said to be weighted. The magnitude of relationships between nodes in a weighted network is essential to the relationship we're researching. The existence of a connection in an unweighted graph is the focus of our attention.

Undirected graphs refer to the graphs in which all edges are bidirectional. In a directed graph, bidirectional interactions are still conceivable (and even common), but they involve two edges rather than just one.

Figure 2.1 An illustration of undirected network (a) and directed network (b).

Understanding graph theory requires the knowledge of the following elements:

➢ ***Graph:*** A graph G is a group of nodes (vertices) linked by edges (links), e.g., G= (V, E).

- ***Vertex (Node***): In transportation networks, a node v is an abstract way to represent a place like a city, a crossroad, or a terminal (stations, terminuses, harbors, and airports).
- ***Edge (Link):*** An edge e connects two nodes. The connection (i,j) connects the starting i and terminal nodes j. A link represents a transport system that allows people to move between nodes. It has a direction, which is usually depicted by an arrow. When no arrow is present, the link is presumed to be bi-directional.
- ***Sub-Graph:*** A sub-graph is a part of a larger graph G. G' = (V', E') is one example of a separate sub-graph of G. Unless the whole global transportation system is evaluated, every transport network is theoretically a sub-graph of another. For example, the road transportation network of a city is a sub-graph of a regional

transportation network, which is itself a sub-graph of a national transportation network.

- ***Buckle (Loop or self-edge):*** A buckle is a connection that makes a node correspond to itself.

- ***Planar Graph:*** A graph in which each vertex represents the intersection of two edges. The topology of this graph is two-dimensional since it lies in a plane. This is common in power grids, road, and railway networks, albeit the definition of nodes must be approached with caution (terminals, warehouses, cities).

- ***Non-planar Graph**:* There is no vertex at the intersection of at least two edges in this graph. Non-planar networks are networks that cannot be represented in a planar manner, such as roadways. This suggests a third dimension in the graph's structure since a movement might "pass over" another movement, such as in air and marine transportation or as an overpass for a road. A non-planar graph might have a lot more connections than a planar network.

- ***Simple graph:*** The term "simple" refers to a graph with no loops and multiple edges. A basic graph is a road or rail network.
  - Emax = N (N - 1)/2.

- ***Multigraph:*** A graph with several sorts of connections between its nodes. Some couples of nodes are connected by a link only, while other couples are connected by many links, active at the same time. A multigraph is a graph that depicts a road and rail network with various relationships between nodes served by one or both modes.

A transportation network facilitates the movement of people, freight, and information through its linkages. Therefore, graph theory should be able to represent movements as links, which can be considered from several perspectives:

***Connection**:* Every node is linked to every other. The knowledge connections allows you to see if you can reach a node from another node in a graph.

Path: A path is a sequence of nodes, pairwise distinct, under the assumption that an edge links two consecutive nodes in a path. In order to measure accessibility and traffic flows, it is necessary to find all feasible pathways in a graph.

Chain: A series of connections that share a common connection. The direction does not matter.

Length of a Link: Connection or Path. A link, connection, or path has a label attached to it. This label might refer to the link's distance, traffic volume, capacity, or other essential features. The number of links (or connections) that make up a route determines its length.

Cycle: Refers to a chain that has the same beginning and terminal node and does not repeat the same path.

Cluster: It refers to a collection of nodes with deeper relationships than with the rest of the network. To find clusters in a network, a variety of approaches are applied, the most common of which are modularity metrics (intra- versus inter-cluster variance). (Rodrigue J.-P. , 2020)

2.2 **Network properties**

➢ *Adjacency matrix*: A graph with N nodes and E edges can be described by its N x N adjacency matrix A, which is defined as

$$A_{ij} = \begin{cases} 1 & \text{if i and j are connected} \\ 0 & \text{Otherwise} \end{cases}$$

The matrix A is symmetric if the graph is undirected. Otherwise, if the graph is directed, its adjacency matrix is not forced to be symmetric.

➢ *Degree:* Node degree is the most straightforward index to quantify the individual centrality. It is believed that the most important node must be the most active one, so the number of edges connecting to a node is indicated by its degree, which is defined based on an adjacent matrix as

$$k_i = \sum_{j \in V} a_{ij}$$

V represents the set of nodes i, and a_{ij} is the entry value in the adjacent matrix. In essence, node degree equals the connectivity in space syntax. If a directed network is considered, then the degree can be extended to in-degree and out-degree, which respectively calculate the number of links ending in or starting from the node. They are written as

$$k_{in}^{i} = \sum_{j \in V} a_{ij}$$

$$k_{out}^{i} = \sum_{j \in V} a_{ji}$$

the well-known scale-free network is based on the degree definitions. If the degree of a graph follows a power-law distribution — that is, $P(k)\ \alpha\ k^{-\varepsilon}$ then the network is referred to as scale-free (Barabási & Albert, 1999). It indicates that there is a huge heterogeneity in the network; a few nodes are highly connected hubs, whereas most of the rest are very poorly connected. This pattern imposes important consequences for dynamic processes on transport networks; therefore, it has attracted much attention since it was proposed.

- *Diameter:* The diameter of the network is determined by the shortest path between all pairs of nodes, i.e.

$$D(G) = \max_{i,j} l_{ij} \qquad (1)$$

l_{ij} = the length shortest path between nodes i and j.

The diameter represents the longest route (number of stations along the longest route) in the network if a passenger uses the optimal routes, which means that they take the shortest route between any two stations. An interesting observation is that the diameter does not correspond with the area of the city.

- *Average path length:* The average path length is defined as

$$(l) = \frac{2}{n(n-1)} \sum_{j \neq i} l_{ij} \qquad (2)$$

It only occurs if the network has no disconnected nodes. If we select these stations at random, the average path length corresponds to how many stations there are between two stations on the shortest route.

- *Eccentricity distribution*: The eccentricity e of a node i is the longest distance between i and any other node in the network; that is

$$e(i) = \max_{j} l_{ij} \qquad (3)$$

here, it tells us how far a stop/station is from the most distant stop/station in the PTN.

- *Degree distribution*: The list of the node degrees is the degree sequence of the network. Degree distribution P(k) is defined as the proportion of nodes with degree k; or, equivalently, as the probability that a uniformly chosen node has degree k. For directed networks, we can examine the in-degree and out-degree distributions.

➢ *Degree centrality:* Basically, it is the degree di of node i (in directed networks, it is the in- and out-degrees) and it indicates how big the neighborhood of i is. According to our earlier observations, the distributions decay with a power law in scale-free networks.

➢ *Local average connectivity*: Let N_i be the set of neighbors of u and $G[N_i]$ be the subnetwork induced by the nodes in N_i. The degree of a node j in the subnetwork $G[N_i]$ is denoted by $\mathrm{d}^{G[N_i]}(j)$. Next, the local average connectivity [20] of node i is defined as

$$LAC_{(i)} = \frac{1}{\mathrm{d}_i} \sum_{j \in N_i} \mathrm{d}^{G[N_i]}(j)$$

and it describes how close its neighbors are. In a public transportation system, it basically means that if a stop/station cannot be used for some reason, the neighboring stops become disconnected from each other. Nodes with high LAC values are the locally central nodes.

➢ *Closeness centrality:* As the name shows, closeness centrality calculates how far it is from a given node to all other nodes in a network

$$C_i^C = \frac{1}{\sum_{j \neq i} l_{ij}} \qquad (4)$$

For a non-weighted graph, it becomes the geodesic distance. However, it works only in a connected graph.

➢ *Betweenness centrality:* Betweenness quantifies the level of intermediate importance of a node in the interaction between other nodes (Freeman, 1977,1979). The node betweenness can be defined as:

$$C_i^B = \sum_{j \neq g} \frac{n_{jg}(i)}{n_{jg}}$$

where $n_{jg}(i)$ is the number of the shortest paths between node $j$and g, which are passing through node i, while n_{jg} is the number of all shortest paths between them. In the same way, edge betweenness can also be determined, which can be used to detect network community structure (Cardillo, Scellato, Latora, & Porta, 2006). The nodes with a high betweenness would impose critical constrains on network security in real transportation systems (Barthelemy, 2004) (Lin & Ban, 2013).

2.3 **Models of complex networks**

In both natural and social sciences, modeling is an effective and easy technique to describe the dynamics of the actual world. In recent decades, many new network models have been suggested in a variety of fields, ranging from simple random models to complex evolving or evolution models.

2.3.1 Random model

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Erdős and Rényi (ER) model is the most famous random model proposed by (Renyi, 1959). This model can be built by connecting randomly two nodes from the initial N nodes with probability p. The ER model exhibits a small path length and a low clustering coefficient as its characteristics.

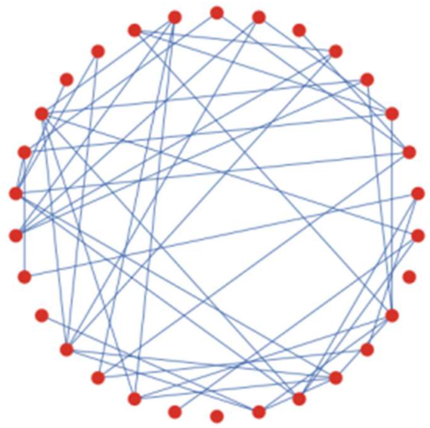


Figure 3.1: The Erdős and Rényi model with NV =30 and p=0.1 (NE =82, C=0.073, L=3.11)

2.3.2 Regular model

Regular models can be generated by providing the number of nodes and average degrees per node. As an example, if there are n nodes, and the degree per node is k, the number of edges in an undirected model equal (n*k)/2. This definition shows a high level of clustering of the network, but the characteristic path length value is large.

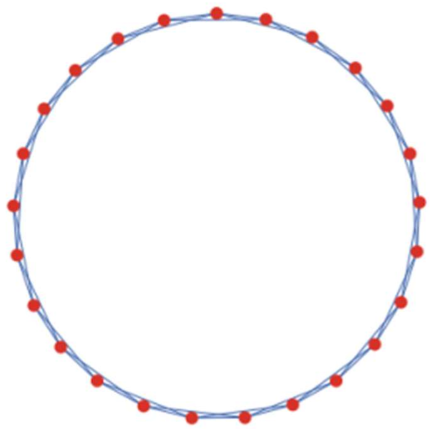


Figure 3.2: The regular circle model with NV = 30 and p=4 (NE =60, C=0.5, L=4.14)

2.3.3 Small-world model

These two models represent two extreme scenarios, neither of which can be compared to real life. Most real network nodes can be reached by passing through a relatively small number of edges. Researchers have been working for a long time to improve and construct another model that more closely replicates the real world from a global perspective with a low average path length and a significant clustering coefficient. In 1998, Watts and Strogatz made a significant breakthrough. They proposed the small-world model, which is also known as the Watts and Strogatz (WS) model. Based on the probability p, the WS model rewires any links in a regular network. The probability p varies from 0 to 1. The two extreme cases correspond to a regular network and to a random one, respectively. Therefore, in a small-world network there are likely to be relatively few intermediate links between two nodes, indicating a small characteristic path length and a large clustering coefficient. In Fig. 3.3, a small-world model is generated based on the regular ring model in Fig. 3.2 and a rewiring probability p.

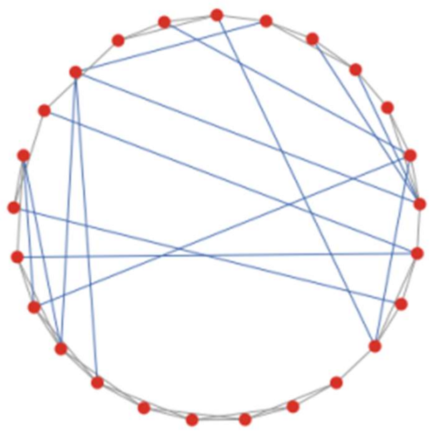


Figure 3.3: A small-world model with NV =30, p=0.30 (NE=60, C=0.322, L=2.717)

2.3.4 Network growth models

Barabási and Albert (BA) model

The understanding of dynamic evolution process has gained much progress in addition to the effort on static topological models. The network growth model is much more complicated because it includes an evolution perspective. According to (Barabási & Albert, 1999), many real networks exhibit preferential connectivity during their evolution. That is to say, a node with a higher connectivity is more likely to be connected with a new node in a network, the big one will only get bigger. In order to quantify the connectivity, the node degree is the most straightforward way. Therefore, the attaching probability of connecting a new vertex to node vi is calculated as follows:

$$\prod_{n \to i} = \frac{k_i^{t-1}}{\Sigma_j k_j^{t-1}}$$

In this example, k_i^{t-1} indicates the degree of node vi at time t-1. It is assumed that the original network only consists of one node, and a new node with c new links are added per step. Then this procedure can be divided into two steps:

- One new vertex is added per time step with c links, thus the number of nodes and edges in the network equal to:

$$N_v^t = N_v^{t-1} + 1 = t + 1$$

$$N_e^t = N_e^{t-1} + c = ct$$

- Preferential attachment: the new vertex connects with c existing nodes according to the probability, then the total degree of the network at time t is:

$$\Sigma_j k_j^{t-1} = 2ct$$

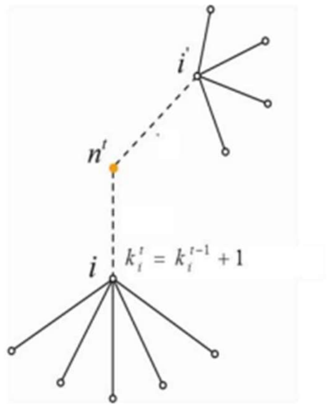


Figure 3.4: The growth mechanism of the BA model. A new node n is connected to vertices i and i' at time t.

Figure 3.4 illustrates the mechanism. It is important to note that the degree distribution of a BA model always follows a scale-free pattern, so it is also called a scale-free model. In conjunction with the small-world model, it is recognized as one of the two most important models to mark the beginning of complex network theory.

2.4 **Transport Network**

Transport (transportation) network refers to a group of centers also known as nodes and linked by routes. Transport network is the overall system consisting of transport route and mode. Networks are connected through roads and streets, railways, pipes, aqueducts, and power lines.

Transport networks enable people and goods to move from one place to another. Street networks, train networks, pedestrian walkway networks, river networks, utility networks, and pipeline networks are only examples of different types of transportation networks. Geographical models of transportation networks consist of linear features and points of intersection. (Rodrigue J. , 2020)

2.4.1 Public Transport

Public transportation is a form of collective transportation for passengers. Unlike privately operated vehicles that charge a posted fee per trip, public transportation is typically scheduled, operated on established routes, and made available to the general public. Furthermore, it is implemented at the local or regional level.

Public transportation systems include a variety of transit options such as buses, light rail, trolleybuses, and subways. Transportation systems help ensure that people can reach everyday destinations, such as jobs, schools, healthy food outlets and healthcare facilities, safely and reliably.

Incorporating public transportation options and considerations into broader economic and land use planning can also benefit a community by facilitating business opportunities, reducing sprawl, and creating a sense of community. By creating a locus for public activities, such development contributes to a sense of community and can enhance neighborhood safety and security. Because of these reasons, areas with good public transit systems are economically thriving communities and are attractive places to work or live.

And, in times of emergency, public transit is fundamental to evacuation safety and efficiency.

The benefits of public transportation system:

○ Reducing motor vehicle travel (air pollution) and traffic congestion.
○ Economic benefits to the community.
○ Cheaper.
○ Public transportation services play an important role for people who are unable to drive, including those without access to personal vehicles, children, individuals with disabilities, and older adults.
○ Public transportation systems provide opportunities for increased physical activity in the form of walking or biking on either end of the trip.

The criteria to measure the usability of different types of public transport are:

○ Speed is calculated from total journey time including transfers.
○ Comfort
○ Safety
○ Cost
○ Proximity means how far passengers must walk or otherwise travel before they can begin the public transport leg of their journey and how close it leaves them to their desired destination.
○ Timelines is how long they must wait for the vehicle.
○ Directness records how far a journey using public transport deviates from the route.

Local public transportation [LPT] refers to the many kinds of public transportation available locally in a city, province, or area. Local public transportation can be provided using a variety of modes of public transportation and a variety of infrastructures, both in a reserved area and in mixed areas. The bus, trolleybus, tram, underground, train, …etc., are the means used in carrying out the LPT service

Some indicators are used in a generic manner to evaluate and compare the different local public transportation systems, such as:

- Network density: length in kilometers of public transport networks per 100 km^2 of municipal surface.
- Density of stops: number of stops per km^2 of municipal surface.
- Demand for public transport: number of passengers transported during the year by public transport in urban areas (buses, trams, trolley buses, underground and funicular).
- Seats-km: total number of seats offered to users during the year. This value is obtained as the product of the cars-km^2 for the average capacity of the cars supplied.

2.4.2 Transport Network topologies

As a specific instance of spatial networks, transportation networks have routes as an essential component of the network topology. Routes are intermediate concepts between paths and edges: a route is a path serviced by a given means of transport. According to the available literature on transportation systems, L-space and P-space are two primary strategies to represent routes. Nodes on an L-space topology are connected if they are consecutive stops on the route. Degree in L-space represents the number of different nodes that can be reached within one segment, and path length represents the number of stops. Two nodes in the P-space are connected if they have a route between them, so the degree of a node indicates how many nodes can be reached, either directly or indirectly, on that route. In P space, a path length corresponds to how many connections/transfers are required to connect one node to another. (von Ferber, Holovatch, Holovatch, & Palchykov, 2007)

L-Space

The graph topology, also called space L, shows each station as a node; the link between nodes indicates at least one route sequentially servicing the two corresponding stations. Furthermore, only one edge can connect two nodes, even when the nodes are directly connected via multiple routes.

P-Space

P-space graphs are particularly useful for analyzing PTNs. Nodes here are stations, just as in L-space, but they are linked if they are serviced by at least one route. Thus, nodes in P-space have neighbors that can be reached without changing modes of transportation, and each route generates a complete P-subgraph.

B-Space

An alternative concept is a bipartite graph, which has proven helpful in analyzing cooperation networks. B-space is a representation that represents both routes and stations as nodes. Each route node is linked to all station nodes that it services. Nodes of the same type are not directly linked.

C-Space

The complementary projection of the B-space graph to route nodes leads to the C-space graph of route nodes, where any two route nodes are neighbors if they share a common station.

2.4.3 Bus services

Buses can be used on regular roads to transport a large number of people over shorter distances. Buses have a lower capacity (than trams or trains) and may run on regular roads with relatively affordable bus stops to service passengers. As a result, buses are usually applied in smaller towns, cities, and rural regions. Local bus routes are typically classified into two categories: urban and suburban.

The level and reliability of bus services are often dependent on the quality of the local road network and levels of traffic congestion, and the population density. Buses are also used for scheduled bus transport, scheduled coach transport, school transport, private hire, or tourism. Buses usually run very frequently.

The statistics are based on a survey of the most used public transport method in Italy in 2018. That year, the most common means were buses and trolleybuses (69 percent), followed by the metro (41 percent). Only 26 percent of passengers stated that the tram was their most used mode of public transportation (Statista Research Department, Nov 5, 2021).

Italy has wonderful natural landscapes that are best enjoyed by taking the bus, which is one of the safest and cheapest ways to travel. Bus connections between Italian cities make it easy to travel to most popular destinations.

Buses are sometimes a better choice in many regions. Moreover, the only way to get to small towns off the rail grid or along Italy's mountain spine will be by bus.

Every province in Italy has its own bus system with independent lines focused just on serving the provincial and neighboring provinces. The bus company offers both "linee urbane" (city bus routes) and "linee extraurbane" (routes out and nearby cities).

Chapter 3

Vulnerability, Resilience and Robustness

Our societies depend enormously on some critical infrastructure, including electric powered power, transport, water supply, sewage handling, information and communication, and banking structures. These structures have grown and grown, and actually are very complex and interdependent systems. If the supply of any of those services stops or is significantly reduced, the established systems will fail or drop at a low level of performance. There are different kinds of disruptions and threats to critical infrastructure, including the transport system, which may require various analysis tools and courses of action for anticipation, prevention, mitigation, and restoration; these threats could be internal or external.

The internal threats may be caused by mistakes and accidents caused by users or staff, technical failures, malfunctioning components, faulty constructions, overloading, etc. However, external threats associated with natural phenomena include numerous degrees of adverse climate and natural disasters, such as: heavy rains, snowfalls, thunderstorms, hurricanes, tornadoes, floods, wildfires, landslides, tsunamis, volcanic eruptions, earthquakes, etc. In the external threats, we include also artificial actions, such as terrorist attacks.

Transport network disruptions have a wide range of impacts. Accidents, infrastructure failures and terrorist strikes all have the potential to result in injuries and deaths, either directly or indirectly. Many ordinary interruptions have less severe consequences: a road may be closed, trains may be forced to stop, or flights may be canceled for a period of time. Passengers' travel periods and goods delivery times will be delayed due to such incidents, and some trips will be canceled. These interruptions will have social and direct economic consequences. The expenses of getting the transportation system reworking and running and repairing or replacing the infrastructure might be significant.

Vulnerability in its different forms is the key concept we use to analyze the network structure. Water distribution systems, transportation systems, the internet, and engineering design provide examples of vulnerability representation and analysis methods.

Measures of vulnerability are estimated as loss of connectivity and efficiency concerning the two different types of disruptive events considered. It permits to prove potential vulnerabilities of the urban networks that should be considered to help the arranging system produce resilient structures. (Candelieri, Galuzzi, Giordani, & Archetti, 2019)

Our work focuses on one specific critical infrastructure system – the transport system. Since a reliable and robust transportation system is very important from an economic and welfare perspective, a lot of studies have been done to figure out what causes its vulnerability, how to make it more robust and resilient, and how to mitigate the consequences of disturbances and disruptions.

This chapter provides network analysis functionalities for vulnerability, resilience, and robustness assessment in public transportation networks regarding disruptive events. The establishment of urban networks with strong connection and interdependence between its components is one of the main scopes of urban transportation planning.

3.1 Network Vulnerability

The idea of vulnerability in a complex network is used to quantify the network's security and stability when it is subjected to various types of failures. The higher the impact of unexpected losses on network performance, the lower the system's resilience and the higher the risk. The concept of risk is represented by a risk curve Fig 3.1, which shows the distinction between transport unreliability (the upper left section of the risk curve) and vulnerability (the lower right section of the risk curve).
In risk analysis, reliability is used to indicate a device's ability to function under specified conditions for the duration of time intended. In the transportation literature, reliability refers to stability, certainty, and predictability. In every case, vulnerability refers to rare events and significant adverse effects, and it describes a network's weakness.

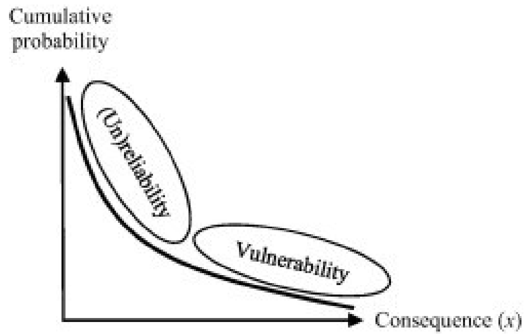


Figure 3.1 . Risk curve showing the cumulative probability of risk scenarios with consequences greater than or equal to x.

To measure the vulnerability of a complex network, many methodologies have been proposed. The vulnerability of complex transportation systems can be linked to their sensitivity to interruptions, resulting in a significant drop in network serviceability. One of the most referenced and typical definitions of vulnerability in a road transportation system is found in (Berdica, 2002): "*Vulnerability in the road transportation system is a susceptibility to incidents that can result in considerable reductions in road network serviceability*." This definition may be generalized to different modes of transportation and emphasizes an initial disruptive incident, that the transportation system's essential function is harmed, and that its consequences are severe. The topological approach is necessary for dealing with transport network vulnerability analysis. An entire transportation network is represented in this method as an abstract graph, with nodes and links that correspond to real-world equivalents. The used graph can be directed or undirected and unweighted or weighted according to the application in mind.

Network efficiency (E) and vulnerability are two related concepts, and when nodes/links are removed are frequently measured as a change in essential metrics. In (Latora & Marchiori, 2001)) the network efficiency is defined as

$$E = \frac{1}{n(n-1)} \sum_{i_i j \in v, i \neq j} \frac{1}{d_{ij}}$$

where d_{ij} represents the distance between nodes i and j. Normalization by n(n−1) ensures that E ≤ 1, When the graph is complete, the maximum value E=1 is achieved. Graphs representing real-world networks, on the other hand, can have large E values. A significant vulnerability of a network will correspond to a significant decrease of E, in case of the removal of a node or some nodes.

The second efficiency indicator is the relative size of the most significant component (S) that considers the greatest connected component's size

$$S = \frac{N_1}{N}$$

N and N1 are the numbers of nodes of the network and its largest component, respectively. As we will see, a vulnerable network will correspond to a significant decrease of S in case of the removal of a node or some nodes.

Another index used to assess vulnerability of a network is operational efficiency. First, we define the operational efficiency from v_i to v_j as the average passenger volume along edges on shortest path from v_i to v_j:

$$\varphi ij(\Delta t) = \frac{\Sigma \omega e(\Delta t)}{dji}$$

where $\Sigma\, \omega e(\Delta t)$ is sum of passenger volume along edges on shortest path from v_i to v_j during a specified time period; dij is shortest distance from v_i to v_j. The smaller the distance between two vertices, the larger the passenger flow is, the higher the operational efficiency from v_i to v_j is.

The average value of all two vertices' operational efficiencies in G, it is defined as,

$$OE_G(\Delta t) = \frac{1}{N(N-1)} \sum_{i,j=1,2,\ldots N}^{i \neq j} \varphi ij(\Delta t)$$

where $OE_G(\Delta t)$ is operational efficiency of G during a specified period.

Based on the physical topology of the transportation network, the operational efficiency varies over time with passenger flows.

The vulnerability of a network can be measured as the decrease of operational efficiency after the failure of a vertex or an edge. It can be measured as follows:

$$\Gamma i(ij)(\Delta t) = OEG(\Delta t) - OEG_{(i/ij)}\,(\Delta t)$$

$$\Gamma(\Delta t) \;= 1/M \sum_{i,j=1,2,\ldots N} \Gamma i\;(ij)(\Delta t)$$

$$TV_G(\Delta t) = \sqrt{\frac{1}{n} \sum_{i_1 j=1,2..N}^{i \neq j} \left(\Gamma_{1(i,j)}(\Delta t) \overline{-\Gamma(\Delta t)}\right)^2}$$

where $TV_G(\Delta t)$ is the vulnerability of G during a specified time; $\Gamma i(ij)(\Delta t)$ is the change of operational efficiency when v_i or e_{ij} fails; $OE_G(\Delta t)$ is normal operational efficiency during a specified time period; $OE_{G(i/ij)}(\Delta t)$ is operational efficiency when v_i or e_{ij} fails; n is the number of vertices or edges;

M is number of edges/links . If $TV_G(\Delta t)$ is large, the degree of failure of vertices and edges on operational efficiency will be greater. A failure of some vertex or edge will lead to a significant drop in operational efficiency, resulting in higher vulnerability and risk. Consequently, the smaller the $TV_G(\Delta t)$ is, the smaller the level of failures on the operational efficiency of its vertices and edges, which translates into lower vulnerability and reduced risk. (Xiao, Jia, & Wang, 2018)

3.1.1 Vulnerability Analysis Using the Degree Distribution

Node degree is the number of links connected to a network node. Node degree distribution is an effective measure of network vulnerability based on the assumption that nodes with higher degrees are more important than those with lower degrees. In a robust degree distribution, removing a small portion of nodes at random will not affect the network's functionality and stability. Based on the distribution of node degrees, networks can be categorized into two types:

- Random networks (or homogeneous networks) in which each node has approximately the same number of links.
- Scale free networks (or inhomogeneous networks) in which the probability of a node having k links $P(k)$ follows a power law. The degree distribution that follows the power-law is a useful property to analyze the vulnerability of a network. It means the fraction $P(k)$ of nodes in the network with a degree d goes from large to low values of d as

$$P(k) \approx k^{-\varepsilon}$$

Where ε is a parameter with a value that is usually in the range $2 < \varepsilon < 3$. Many complex networks have been observed to be scale-free.

In a network, the highly connected nodes serve as "hubs" that maintain the network's topology and function. Hubs are the main characteristic that emerges in the scale-free and are essential in deciding the network's vulnerability. Scale-free networks have the obvious characteristic of being resilient to random node removal, meaning that random attacks on them will not hinder a network's ability to function. Hence, networks work in a stable state. Despite this, scale-free networks are vulnerable to attacks targeting hubs that may result in catastrophic results for the entire network. (Candelieri, Galuzzi, Giordani, & Archetti, 2019).

3.1.2 Vulnerability analysis based on centrality measures

The betweenness centrality measures the centrality and importance of a node in a network. When the shortest path between all network nodes is computed, betweenness centrality is computed as the number of shortest paths that traverse each node. Each node along the shortest paths gets an equal share of betweenness centrality if more than one shortest path connects one pair of nodes. The indicator can be described as follows.

$$C_i^B = \sum_{j \neq g} \frac{n_{jg}(i)}{n_{jg}}$$

Where:
C_i^B= Betweenness Centrality for node i
$n_{jg}(i)$ = the number of shortest paths between node j and g that passes through node i
n_{jg} = the total number of shortest paths between node j and g

There is a correlation between the number of shortest paths that travel through a node and the importance of the node's performance to a network and its location within a network. The larger the number of shortest paths that go through a node, the more important and central the node is in the network.

Betweenness centrality can also be defined for edges. In the same way as for nodes, it is calculated and used to find the most central edges.
Betweenness centrality can also be represented by normalized betweenness centrality. It is calculated by dividing each node's betweenness centrality with the network's total betweenness centrality. As a result, each node gets a betweenness value between 0 and 1, corresponding to the percentage centrality of the whole betweenness network. This means that it is easier to compare the network centrality among different networks with this way of representing the indicator.

The vulnerability of the network can be measured by means of the betweenness centrality as

$$V_{B,P}(G) = \left(\frac{1}{n}\sum_{i\in v} B(i)^p\right)^{1/p}$$

In theory, these p-functions reflect the idea that the distribution of the minimal paths between nodes affects the vulnerability of a network. In this case, the higher $V_{B,P}$ is, the lower its robustness. (Candelieri, Galuzzi, Giordani, & Archetti, 2019)

3.2 Resilience of Networks

Natural and human-caused disruptions, delays, and risks are potential threats to urban transportation networks. To mitigate the effects of disruptions, the concept of resilience has been implemented into urban transportation networks. We are curious about the idea of resilience in transportation systems due to increased exposure to extreme events in our cities, such as heavy traffic jams and natural disasters.
Resilience, from its Latin root "Resilire", means the capability of a system to resist, rebound or spring back in response to endogenous events (e.g., component failures) or exogenous (Natural or man-made) attacks.

Since there are many definitions of resilience in transportation systems, we report some of them:
It's defined as "The ability for a transportation network to absorb disruptive events gracefully and return itself to a level of service equal to or greater than the pre-disruption level of service within a reasonable time frame" (Freckleton, Heaslip, Louisell, & Collura, 2012).
Also, "The concept of resilience is intended to capture a system's capacity to maintain its function after a major disruption or disaster" (Mattsson & Jenelius, 2015).

Resilience is defined as "The ability of a transportation system to absorb disturbances, maintain its basic structure and function, and recover to a required level of service within an acceptable time and costs after being affected by disruptions" (Wan, Yang, Zhang, Yan, & Fan, 2018).
Transportation systems are resilient not only because they avoid a complete or partial system failure in the case of disruption but also because they can adapt, limit their impact, or avoid whole or partial system collapse if necessary.

Resilience of urban transportation systems is defined as "the ability of a system to resist, reduce and absorb the impacts of a disturbance (shock, interruption, or disaster), maintaining an acceptable level of service (static resilience), and restoring the regular and balanced operation within a reasonable period and cost (dynamic resilience). (Gonçalves, L. A. P. J. & Ribeiro, 2020)

Based on our analysis of the conceptual definition of the resilience of transportation systems, the following four main actions are identified: resist, recover, absorb, and transform.

Some studies divided resilience into two categories: static and dynamic. In transportation systems, static resilience is related to its robustness to maintain the system operating after a shock or hazard occurs without immediate system infrastructure restoration. In turn, dynamic resilience aims to re-establish the initial level of performance and operation as quickly as possible after the occurrence of a disaster. This is similar to concept of resilience, which refers to how fast the system returns towards equilibrium after a shock (Pimm's 1984). (Reggiani, Nijkamp, & Lanzi, 2015) further analyze the relationship between resilience, vulnerability, and connectivity by analyzing how they are framed, interpreted, and measured.
These principles suit properly into Fig 3.2, which illustrates the consequences of decision-making on infrastructure resilience (McDaniels, Chang, Cole, Mikawoz, & Longstaff, 2008).

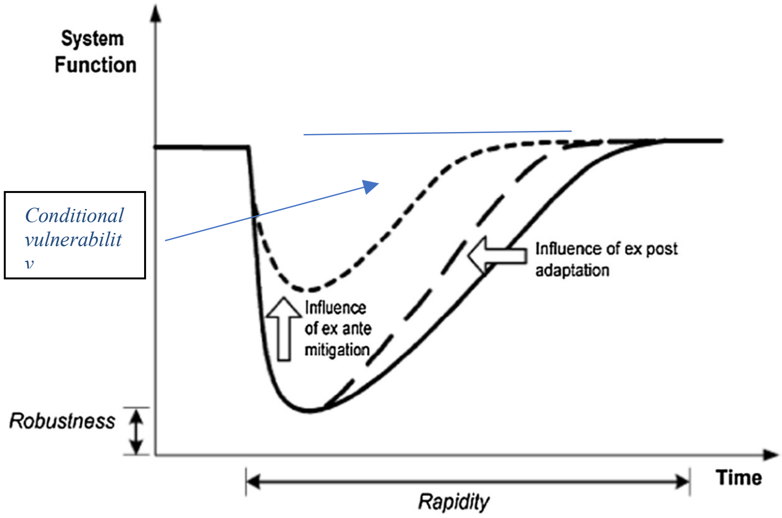


Figure 3.2 Effects of decision-making on resilience

The conditional vulnerability was added to the figure. It can be seen as the aggregate consequences of a disruption scenario represented by the area between the dotted line in the figure corresponding to the whole system function and the relevant curve representing the reduced level of function. The latter curves rely typically on actions of ex-ante mitigation or ex-post adaptation.

The purpose of studying resilience is to determine methods for measuring transportation resilience, to determine what degree of system resilience can be achieved under various disruptions, and to identify critical areas (sections or intersections) from which countermeasures to mitigate consequences can be created, both in a static and/or dynamic framework. (Mattsson & Jenelius, 2015).

3.2.1 Main characteristics of the resilience of urban transportation systems:

There are mainly eight characteristics to assess the resilience of transportation systems, such as redundancy, adaptation, efficiency, robustness, interdependence, preparedness, flexibility, and rapidity. However, some characteristics are more related to the operability of the transportation system, such as interdependence, efficiency, and flexibility. In addition, some have a high level of overlap with other characteristics more used and consolidated in studies on resilience, such as robustness with efficiency and redundancy with flexibility, which may not be considered the main characteristics of the resilience of transportation systems.

i. Redundancy: is the ability of some components of a system to perform the functions of failing components without affecting their performance.
ii. Adaptation: refers to a system's ability to adapt to new demands in a flexible manner.
iii. Efficiency: is the capacity to support disturbances while maintaining a level of service and connection in transportation networks, and it is the positive connection between the service supplied by a static system and the service offered by a dynamic system.
iv. Robustness: A system is robust if it is able to support a certain level of stress or demand without deteriorating or losing its functionality.
v. Interdependence: this term refers to the interconnection of system's components or aspects, as well as the system's network of interactions.
vi. Preparedness: refers to "preparing specific steps before discontinuance" and strengthening a system's resilience by lowering the impact of possible negative consequences of disruptive events.

vii. Flexibility: is known as the ability to reorganize resources in the face of uncertainty. Also, is a system's ability to adapt shocks and adjust to changes following disruptions through contingency planning.
viii. Rapidity: is the ability to fulfill priorities and meet goals in a timely way in order to limit losses and prevent future interruptions.

The oldest and most commonly utilized method in transportation system resilience research is redundancy. According to a review of works and definitions on transportation system resilience, efficiency and adaptability are strongly linked to other resilience characteristics. On the other hand, interdependency is intrinsically linked to the connection and dependence aspects of a transportation system's performance.

In evaluating the resilience of transportation networks, the efficiency, interconnectedness, and preparation are characteristics still under exploration. Efficiency is applied across different dimensions of transportation systems and, more specifically, in freight, railway, and road transportation. Furthermore, it should be used to measure the performance of a transportation system and for that reason must not be considered as a main characteristic of resilience and we can consider that efficiency is already accomplished in the evaluation of robustness. On the other hand, interdependence is mainly applied in studies about connectivity between different modes of transportation in railways, roads and freight transportation systems. Finally, preparedness is transversally associated with almost all transportation infrastructures, freight transportation systems, road transportation systems, railway transportation systems and more broadly in the entire urban transportation system. A transportation system can always be seen as an interconnected system of other transportation subsystems, especially in a multimodal urban context. Thus, interdependency should not be considered a primary characteristic of resilience.

Therefore, we can conclude that there are five main characteristics of the resilience of transportation systems: i) redundancy, ii) adaptation, iii) robustness, iv) preparedness, and v) rapidity.

Based on the following explanation, Fig. 3.3 describe the relationship between the main characteristics and main actions of a transportation system's resilience as follows:
-If a system is robust and prepared, then it can resist most of the disturbances.
-If a system is robust, prepared and with recovering capacity within an acceptable time, it can recover more easily and rapidly to the disturbances.
-If a system is redundant with its subsystems, it can absorb most of the disturbances and impacts.
-If a system cannot recover to the initial stage of operation, it can adapt and transform to a different stage of equilibrium and operation.

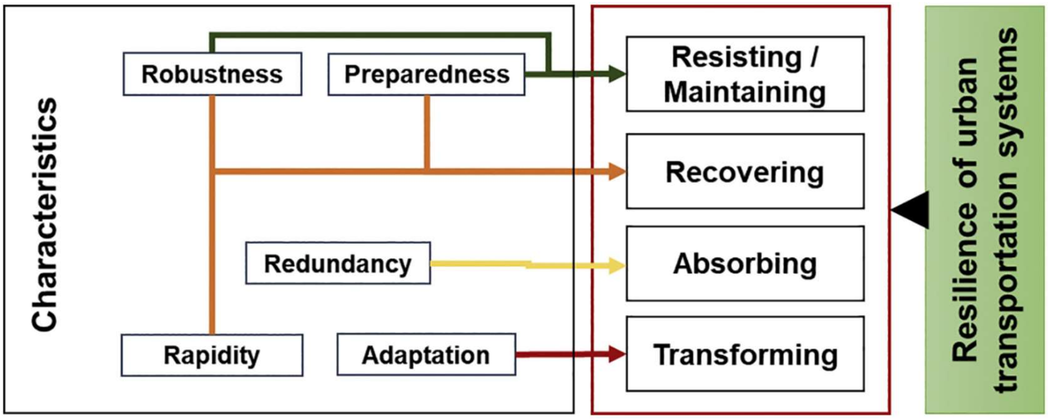


Figure 3.3 The relation between the main characteristics and main actions of the resilience of a transportation system

Transportation resilience system is assessed in terms of its infrastructure/ network and its level of operation and usage. However, it could be observed that there is a minimal number of studies on resilience that simultaneously incorporate both the infrastructure and the operation of the transportation system.

3.2.2 The main variables to assist performance of the resilience:

Currently, the resilience of transportation systems, mainly roads, is assessed in a similar way to the performance analysis commonly used in current mobility studies, i.e., by measuring variables related to supply and demand mismatches, such as delays, travel speed, and traffic flows vs. capacity, among others. Furthermore, many resilience indicators connected to the five key resilience features were presented based on the reviewed studies.
Resilience Indicators are created through the combination of the variables that characterize the system and the performance assessment is most often used in studies on resilience in transport.

I. Variables contributed to resilience studies:

- Travel distance, time, paths, and volume.
- Traffic volumes.
- Geographic location of the elements of the network.
- Capacity of the elements that make up the network
- Bus, Train and Terminal lines
- Number of intermodal stations, the elements that make up the network
- Network size
- Alternative proximity infrastructures
- Number of transportation modes
- Population size and density
- Level of initial damage
- Volatility of traffic flow
- Free flow traffic speed
- Average speed.
- Speed of traffic with the network load.

II. Transportation resilience indicators

Indicators are commonly used tools to illustrate the determinant properties of the study object and thus are essential instruments for decision making. Based on the literature review results, it was sought to identify the significant resilience indicators in transportation and how these could be treated to support mobility planning. Regarding infrastructure, resilience is determined considering network connectivity.

- Multiple routes (travel characteristics - origin – destination)
- Extra infrastructure (links/ nodes) capacity
- Diversity in transportation modes (ground vs. underground, walking/biking vs. motorized transportation, etc.)
- Population data historic/ variation (number of inhabitants, population density, etc.)
- Critical traffic data (flow, capacity, and speed… etc.)
- System performance to disruptions.

According to the literature, resilience analyses of urban transportation systems focus on post-disaster analysis, more specifically after natural disasters such as earthquakes and hurricanes.
The resilience of an urban transportation system mainly applies to: (i) transportation infrastructure, where the structural resilience of infrastructures is analyzed; (ii) operation/use of the transportation system, where the functional performance of the systems and their risks are assessed against a disturbance (interruption, shock, perturbation, disaster). However, studies that incorporate and link the structural and functional aspects of the transportation system are still very limited.
Resiliency analyses in the specific area of road transportation systems have attracted much attention, and the relationship between supply and demand (traffic flow) was the main analytical variable used to evaluate resilience. Finally, the resilience of transportation systems has also been studied on a wider scale, such as an urban system, integrating various surface transport systems such as roads and railways, and these studies are used mainly to

assess: (i) the infrastructure resilience and (ii) the system resilience. (Gonçalves, L. A. P. J. & Ribeiro, 2020)

3.2.3 Methods adopted to measure and/or to improve the resilience of urban transportation systems

To measure and/or enhance the resilience of transportation networks, there are eight main approaches and available strategies.
Identifying methods to measure and/or improve a transportation system's resilience is one of the main challenges and, at the same time, an enigma due to a large number of methods and techniques that have been used for this purpose, ranging from mathematical models to conceptual frameworks, as described below:

• Conceptual framework: a conceptual matrix that serves as the foundation for constructing a logical framework for completing any job and organizing concepts, tasks, and execution stages. This approach is mostly used to identify an issue and methodology before moving on to other strategies to quantify the outcomes.

• System Dynamics: a methodological approach to studying complex systems' behavior throughout time. This method is mostly used in transportation systems to evaluate traffic flow disruptions and infrastructure.

• Stochastic processes: This is a collection of random variables that depict the evolution of a set of values through time. This approach is mainly used to characterize the transportation system's behavior.
• Simulation: is used to study the performance of a system under different scenarios through a calibration and validation process. It is a handy tool to describe and predict system behavior to evaluate the resilience of a transportation system to the hypothetical consequences of different testing scenarios.

• Optimization processes: procedures for selecting the optimal answer from among all possible solutions based on a given objective function depending on whether the variables are continuous or discrete. This method is primarily used to assess the impact of interruptions on transportation network performance.

• Monte Carlo Method: a statistical strategy for approximating actual findings that rely on many random samples. It is a strategy for evaluating the performance of transportation systems using hypothetical scenarios and methodology validation.

• Fuzzy theory: The logical values of the variables can be any actual integer between 0 (FALSE) and 1 (TRUE) in this type of multivariate logic (TRUE). Fuzzy logic has been expanded to include the idea of partial truth, in which the truth value might range from totally true to completely false. Its primary purpose is to assess potentially robust circumstances.

• Network Science is a network technique to construct predictive models that represent physical, biological, and social events. Graph theory is a network science technique that has lately been applied in various publications.

Models for assessing and/or enhancing resilience, like other assessment procedures, can be qualitative or quantitative and include conceptual frameworks, simulation models, and mathematical models. As a result, most of the literature determines resilience using sophisticated mathematical models of theoretical character, which are difficult to use in practical ways. As a result, simply using formulas to represent resilience was not always sufficient, especially in real-life settings. Because it is difficult for transportation authorities and related entities to understand and implement resilience assessment, they are unlikely to be interested in developing more integrated and complex models to measure and assess resilience improvement (Wan et al., 2017). Therefore, there are few studies using models and decision support tools that are easy and friendly to understand and use.

In brief, analyzing the resilience of urban transportation systems while including infrastructures, network operation, and simulation approaches can lead to the development and construction of user-friendly and practical/technical instruments, which represents a future avenue of resilience research. The use of simulation techniques would allow for a predictive assessment of resilience for all transportation systems, identifying the most impacted components, warning organizations to intervene in crucial zones, and other analyses deemed necessary by government officials.

Finally, Fig 3.4 present the interconnectedness of the various concepts, definitions, characteristics, and methods used to measure and improve resilience, which must be a supporting structure of a process for developing new methodologies for evaluating the resilience, i.e., the operability and use, of transportation systems against potential disruptions.

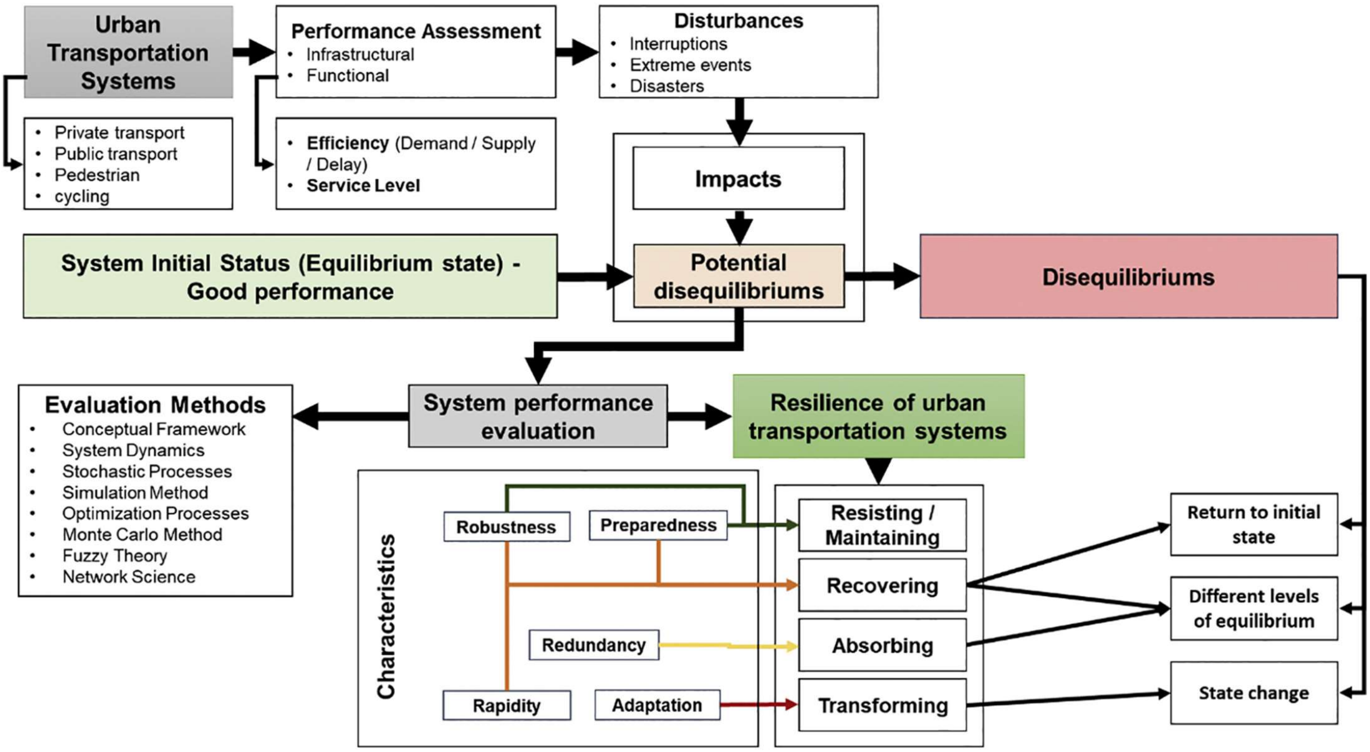


Figure 3.4 The relation between the various concepts of the resilience of urban transportation systems

3.3 **Robustness of Networks**

The robustness of the network refers to the ability to absorb shocks with minimum influence on system performance. Minor disruptions in a component's performance do not significantly impact the overall system performance; hence, a system can be called resilient in terms of disruptions on that component. The fundamental goal of a robustness study is to develop reliability indicators that can be utilized in a variety of domains and can measure changes in network performance throughout the whole range of conceivable capacity decreases. By generating performance curves, two robustness indicators are conceptualized and articulated to assess the absolute change and the initial moment of network performance degradation as a function of link capacity decreases.

The analysis of these indicators allows identifying the extent and the relation between capacity reduction and performance reductions and thus support infrastructure management and capacity allocation.

Most relevant studies on public transportation system safety evaluation concentrates on system-level analysis. Nevertheless, little research has been done on the relationship between capacity reduction and safety. It is crucial to assess the influence of various capacity reduction scenarios on the public transportation system's resilience, not just for efficiency reasons but also for reliability and safety concerns.

Most research in the urban transportation system focuses on big cities' subway systems. However, data reveal that until March 2019, just 212 urban areas in the world had subway systems, and only a few of these cities depended heavily on subways for public transportation. On the other hand, buses are more commonly employed in urban public transportation around the world. In modern cities, the stability and robustness of the transit bus network are critical. Natural catastrophes, road construction, accidents, traffic congestion, and even terrorist acts may represent a threat to this network. Threats to the robustness of a network can be categorized into random failures and intentional attacks.

The scale-free networks are robust under random failures, but they are vulnerable to intentional attack. The robustness of bus networks has been examined primarily with two approaches. The first approach examines a specific type of attack on selected nodes in a bus network, and the second approach probes different types of attacks and their distinct threats to the network robustness.

3.3.1 Robustness measures

G (N, E) is commonly characterized as an undirected network in urban bus network research. The direction of bus routes, particularly in densely populated areas with many one-way streets and tidal traffic flows, has a considerable impact on the strength of the urban bus network. As a result, the UBN is modeled as a directed network, with one undirected edge divided into two directed edges in this research. The degree distribution of an urban bus network, at least asymptotically, follows a power law distribution. The weighted degree of node i (ki) is the sum of weights of all directly connected edges.

$$k_i = \sum_{j \in N} w_{ij}$$

where j is the node directly connected to i, w_{ij} is the number of lines operating between i and j. For a directed network, the degree of a node is defined by the number of different nodes that are directly linked to the given one as in-degree, and vice versa as out-degree, the cumulative degree probability distribution of urban bus network in space L.

Graphs are used to evaluate the network robustness, the vertical axis represents cumulative probability, whereas the horizontal axis represents weighted degree.

A power law distribution is used to explain the probability of node degree in a scale-free network:

$$P(k) = \alpha * \kappa^{-\varepsilon}$$

where the exponent ε is the scaling factor. A larger ε indicates that the hubs of the network are more visible. α is multiplying coefficient.

The complex network theory and attack simulation method are applied to reveal the determinants of urban bus network robustness.

Chapter 4

Scale Free Modelling

A common feature of real world networks is the presence of hubs, or a few nodes that are highly connected to other nodes in the network. The presence of hubs will give the degree distribution a long tail, indicating the presence of nodes with a much higher degree than most other nodes.

Scale-free networks are a type of network characterized by the presence of large hubs. A scale-free network is one with a power-law degree distribution. Networks with power-law distributions are called scale-free because power laws have the same functional form at all scales.

The network technique is helpful not just for reducing and analyzing large amounts of data, but also for identifying the most important elements and their interactions. In addition, various methodologies have been developed to investigate the underlying topological features of a network, such as community structure (Li, Zhang, Di, & Fan, 2008) “Community structure of complex networks”, the core-periphery structure, small-world and scale-free properties.

In this chapter, by adopting the network approach, we analyze and model the bus public transportation systems of 3 Italian cities located in northern Italy in Lombardia region from how close they have the scale free feature. These cities have been chosen as representatives of medium sized cities in terms of population.

The structure and properties of such a transportation system have substantial implications for urban planning for sustainable development.

The transportation network was modeled as a complex network with exact geographical coordinates of its bus stops. Therefore, this study analyzes the structure of the public transportation system through the theory of complex networks in a static approach of network topology.

Previous studies often used undirected networks, one novelty here is to consider directed edges. Our analysis was based on the capacities of the public transport in order to get a detailed picture about the existing public transport networks.

Graph theory was used to describe a PTN using a topological approach. A multi-graph representation of a PTN represents multiple lines/routes between two stations/stops, and specific attributes/labels are applied to distinguish two or more edges between two nodes. The advantage of this approach is that it does not require a large amount of data to build the graph associated with a PTN infrastructure. However, it can nevertheless provide valuable information about the vulnerability of the transport network.

Therefore, to perform a comprehensive network analysis of the public transportation systems of these cities, the first step was to generate the transportation networks (The represented graphs). This was done by modelling stations/stops as nodes and lines that connect them as directed links in MATLAB program. If a line runs between two stops in both directions, as is usually the case, we can decompose the link that represents this line into two directed links due to the orientation.

4.1 Data

Typical network data describe one or two sets of nodes and the relations between them, respectively between the nodes. The adjacency matrix is the square matrix of order n equal to the number of nodes, whose elements equal to 1 if there is an edge between the two given nodes, 0 otherwise.

The dataset (Lines numbers and stops) for Lecco, Como and Varese bus transport network is measured from bus lines companies that service on these cities; “Leccolinee”, “ASF Autolinee” and “Autolinee Varesine srl”, respectively.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
13	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 4.1 Adjacency matrix

In this section, we describe the three public transportation networks of the Lombardia region northern Italy, analyzed in this paper. Besides, the three PTNs are modelled through a directed graph, because more than one route/line may connect two stops. The first PTN considered is the public bus transportation in Lecco Fig 4.2. This touristic city is one of the interesting cities in north Italy, with a population of about 48,131 inhabitants and it lies at the end of the south-eastern branch of Lake Como.

Figure 4.2 Lecco City

The public bus transportation system is done by a single operator, named “LeccoLinee”, and consists of about 20 bus lines (Both direction). Figures 4.3 and 4.4 show the corresponding graph and Map lines, consisting of 233 nodes/stops and 694 edges/links.

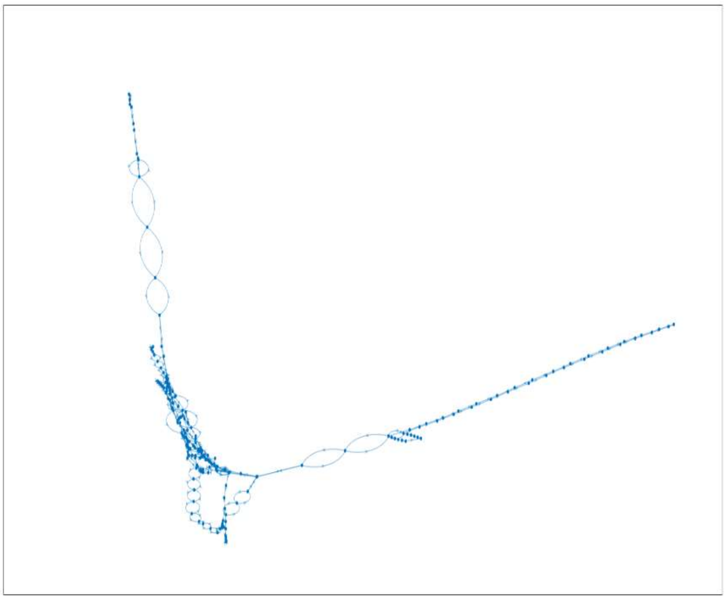


Figure 4.3 Lecco graph

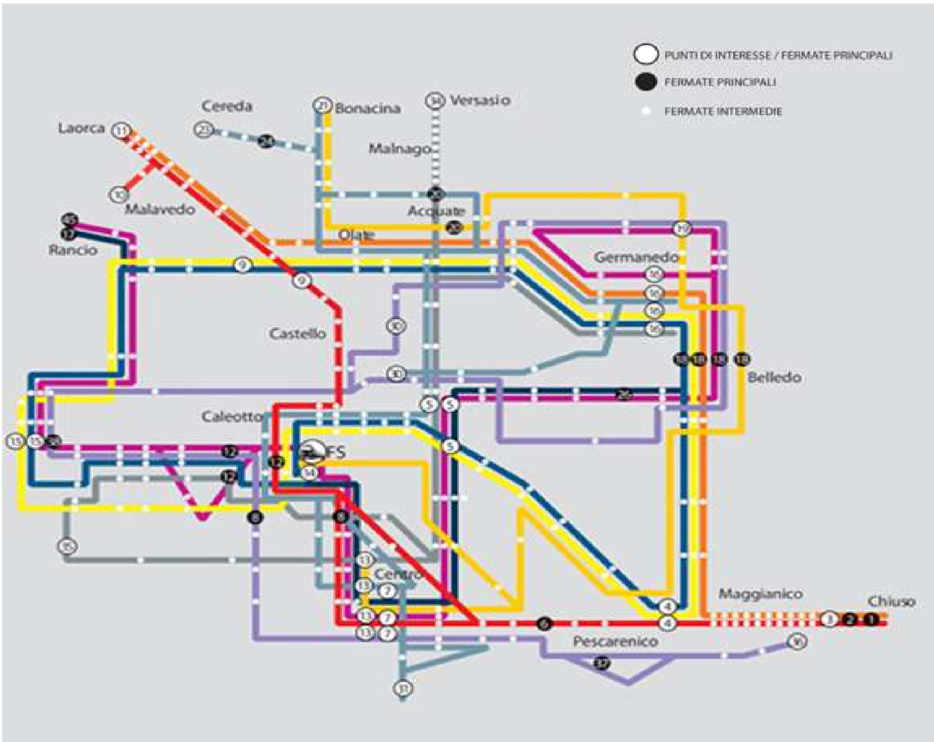


Figure 4.4 Lecco Bus Lines

The second PTN considered is bigger than the first one and consists of the public bus transportation in Como with 84,808 people living in the city. It is located on the southern end of the western branch of Lake Como , in a small basin surrounded by wooded moraine hills in the center of the Lombardy of lakes. The city contains Lake Como, Alps and the city contains numerous works of art, churches, gardens, museums, theatres, parks, and palaces, and so it is a tourist destination.

Buses in Como are sevice by “ASF Lines” with 20 bus lines (Both direction) that cover 237 bus stops and 942 edges/links in the city as illustrate in Fig 4.5.

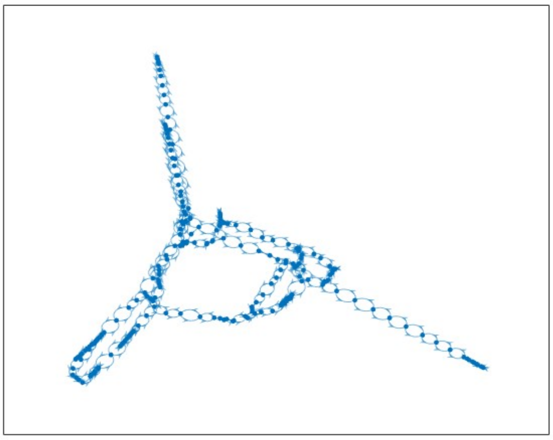


Figure 4.5 Como graph

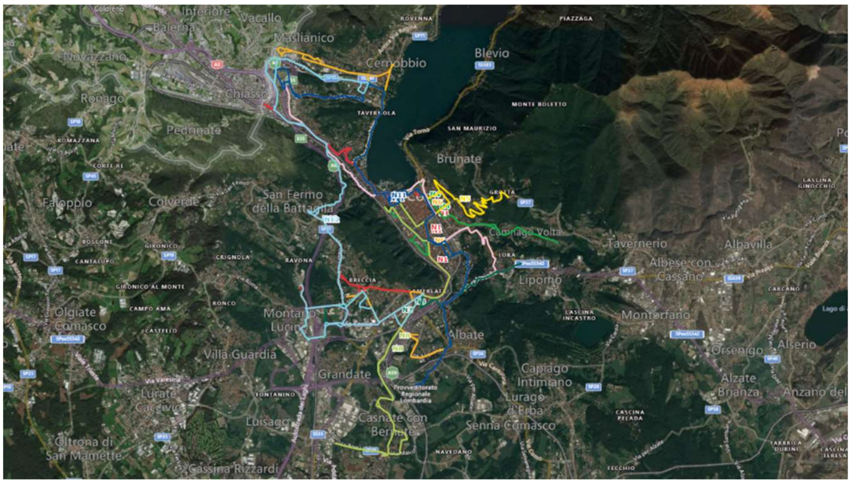


Figure 4.6 Como Bus Lines

Varese is the biggest analyzed city among the three ones, in terms of size with 54km2 and 80,559 inhabitants. It lies among the Alpine foothills descending to the Lake Varese. The public bus transportation system is operated by “Autolinee Varesine,” which service 19 bus lines (Both direction). Figure 4.7 shows the corresponding Map lines, consisting of 215 nodes and 804 edges.

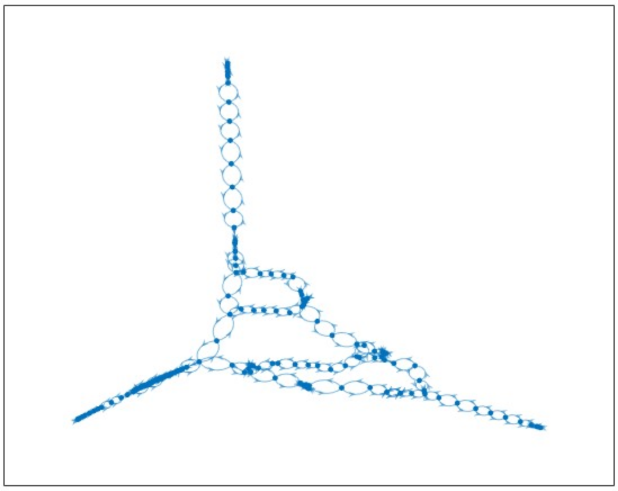


Figure 4.7 Varese graph

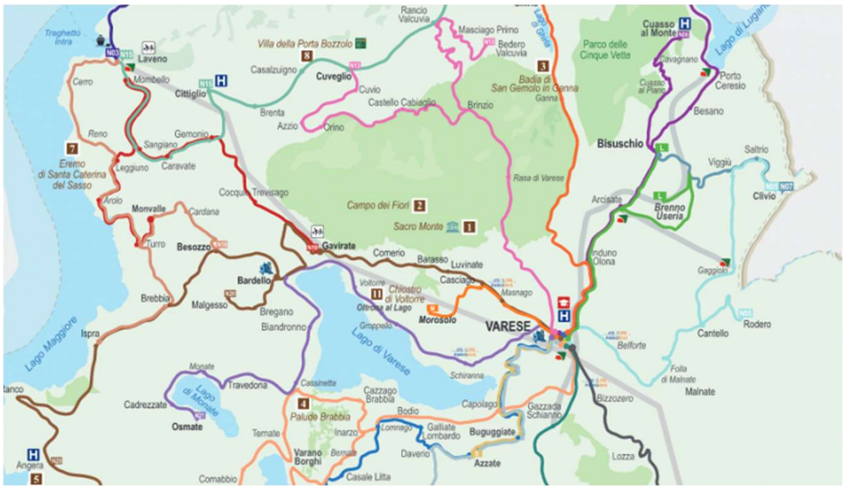


Figure 4.8 Varese Bus Lines

4.2 Model

Degree distribution of a transit stop assumed a central role and appeared in almost every study investigating the topology of public transport networks (Shanmukhappa et al, 2019). For a directed network, the total degree k of a node is the total number of incoming edges and outgoing edges incident at the node. The degree distribution provides the probability of a node having a degree k, mathematically expressed as P(k) = N/sum(N), where sum(N) is the total number of nodes in the network and the N is the number of nodes having a total degree k. The plotting of P(k) versus k for nodes in the topological bus network exhibits a strong power law.

In practice, the methods and techniques we used are carried out by computer programs [MATLAB]. Thus, it is important not only to have a convenient way to theoretically represent a network (the network structure) but a way to store data so that it can be easily accessed and utilized by computer programs.

The dataset collected manually from the bus lines companies is used for the generation of the directed, unweighted graph for the bus transport networks [BTN] in the three cities. The graph has n nodes, where each node represents a bus stop, and e edges which are connected in Space-L (there exists a link between two nodes if there is a bus route connecting them directly).

Degree distribution corresponds to the probability of finding a node with degree k or the probability distribution of node degrees over the complete network. [Barabasi et al.] showed that if the degree distribution of a network follows power law distribution, then the network is scale free. The power law distribution is defined as

$$P(k) = \alpha * \kappa^{-\varepsilon}$$

The analysis is oriented towards the study of the scale-free property. Figures below show a linear scale plot, where the x and y axes represent the degree and cumulative probability density, respectively.
For a digraph, the in-degree is defined as the total number of links in-going to a particular node and the out-degree is defined as the total number of links out-going to the node. Hence, the total degree of a node is total links = "in-going + out-going". By measuring the directed in and out degrees of the network, it is found that every node is typically 2-connected in the cites BTN.

The scale-free behavior of some networks, as defined by [Barabási and Albert], is one of the most significant contributions to the field. Consider a graph $G = \{V, E\}$; it has a scale-free pattern if the probability density function f that a vertex v has b connections (number of edges linked to vertex) or number of lines l passing through a vertex v follows a power law:

$$f(b) = \alpha b^{-\varepsilon}$$
$$f(l) = \alpha\, l^{-\varepsilon}$$

When randomly sampling vertices, the distribution of links and lines are found to follow a power law. The exponent ε is called the scaling factor. If we take all bus stops v of a network and identify those that host one or many lines and if the frequency plot decays following a power law, then it is a scale-free network. This translates into having many stops hosting only one line and few stops hosting more than one line.

The statistical significance of the goodness-of-fit (adjusted R^2), the t-test and the p-test values are examined. We decided to perform a test to validate whether the fitted models followed power law distributions. Essentially, we applied R^2-test that is particularly suited to examine frequency distributions as it is the case under consideration. This test is defined as:

$$R^2 = \sum_i \frac{(Expected_i - Observed_i)^2}{Expected_i}$$

where `Expected' is calculated using hypothesized model, and `Observed' is the data points collected; here, actual values are used, not the log-form.

Goodness-of-fit tests are statistical methods commonly used to interpret observed values. The goodness-of-fit test is used to measure how closely observed data follows expectations. Using goodness-of-fit tests to predict future trends and patterns, we can determine how closely actual values match those predicted by a model. The Chi-square test is the best goodness of-fit test for discrete distributions, which is typically used.

There are different types of goodness-of-fit tests, as noted above. They include the chi-square test, which is the most common, the Kolmogorov Smirnov test, and the Shipiro-Wilk test. These tests are usually conducted using computer software. However, statisticians can calculate these tests using formulas tailored to the particular test.

The Chi-square test determines whether categorical variables are related and if the sample represents the entire group. It measures the degree to which the observed data matches the expected data or how well the two fit together. Observed values are measured, and the frequency is used with expected values and the degrees of freedom to calculate chi-square. (Barceló, 2018)

T-values measure the difference between your sample data and the variation in the data. Alternatively, T is the measured difference, expressed in units of standard error. A larger T indicates stronger evidence against the null hypothesis. This indicates that there is a significant difference. If T is close to 0, it is more likely that there isn't a significant difference.

Given an observed data set and a power-law distribution model from which the data are drawn, we would like to know whether our network is a plausible one to the model. After constructing the networks and identify the ingoing, outgoing, total links and the total lines crossing the nodes as the degree input for analyzing and checking how much our model is identical to the scale free model. Table 1 shows the results (scaling factor "ε", multiplier coefficient "α", the distance between the points in the graph "RMSE" and the R^2-test) of all bus transport lines.

To begin with Lecco Network the graphs follow the scale free model from the entering links, exiting and total lines degree-distribution regarding the adjusted R^2 value is >0.85, the power-law exponent ε is between 2-3 and the multiplier coefficient between 0.76 and 1.

In addition, in Como Network modeling graphs, R^2 values are approximately the same for the entering links and total lines degree-distribution by 0.6 while 0.31 for the exiting links. The power-law exponent ε are 1.96, 1.5 and 0.99 respectively. the multiplier coefficient value between 0.4 and 0.55.

Furthermore, in Varese Network graphs analysis, the entering links and exiting degree-distribution have a typical R^2, ε and the multiplier coefficient values 0.68, 2.6 and 0.75 respectively. At the same the total lines degree distribution values are 0.78, 1.8 and 0.4. Furthermore, fig.4.9 shows the nodes (bus stops) that have the highest crossing lines.

By observing the three networks from the total links degree distribution perspective, we can say they are not following the scale-free features because the R^2 value and multiplier coefficient are less than 0.1, although the multiple factors are less than 0.75.

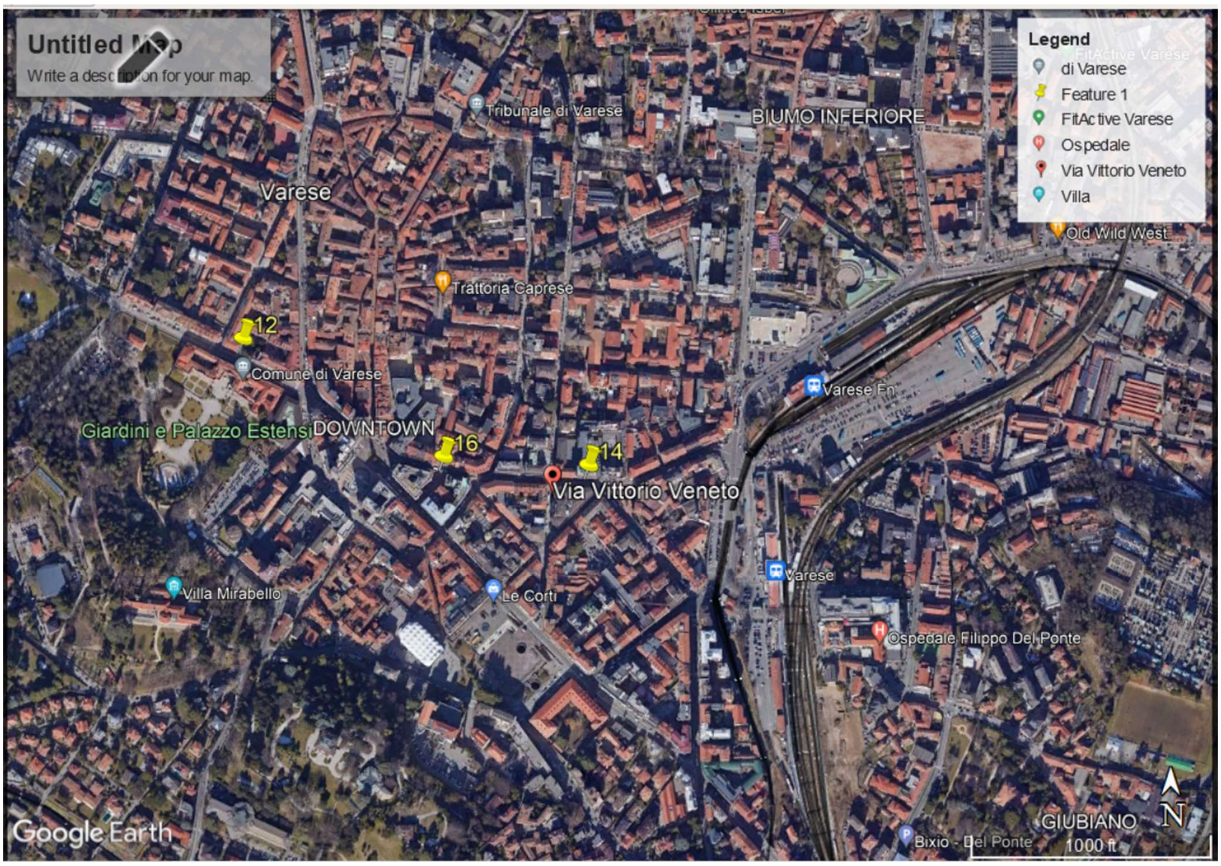


Figure 4.9 Nodes with high bus lines

Modeling Networks Results

Degree	Name	Lecco	Como	Varese
	Number of Stops [Nodes]	233	237	215
	Number of Links [Edges]	694	942	804
	Number of Lines	20	20	19
Total Lines	R^2	0.85	0.61	0.78
	RMSE	0.6617	1.039	0.8365
	α	0.7667	0.403	0.4237
	ε	-1.988	-1.528	- 1.806
Total Links	R^2	0.1	0.048	0.072
	RMSE	1.652	1.339	1.733
	α	0.1147	0.08295	0.02202
	ε	- 0.7359	- 0.3916	0.6197
Entering Links	R^2	0.87	0.59	0.68
	RMSE	0.723	1.193	1.325
	α	0.8845	0.5627	0.7586
	ε	-2.582	- 1.969	-2.634
Exiting Links	R^2	0.89	0.31	0.67
	RMSE	0.7658	1.079	1.339
	α	1.063	0.3584	0.7423
	ε	-3.012	- 0.9879	-2.613

Table 1 Modeling results

Varese Network

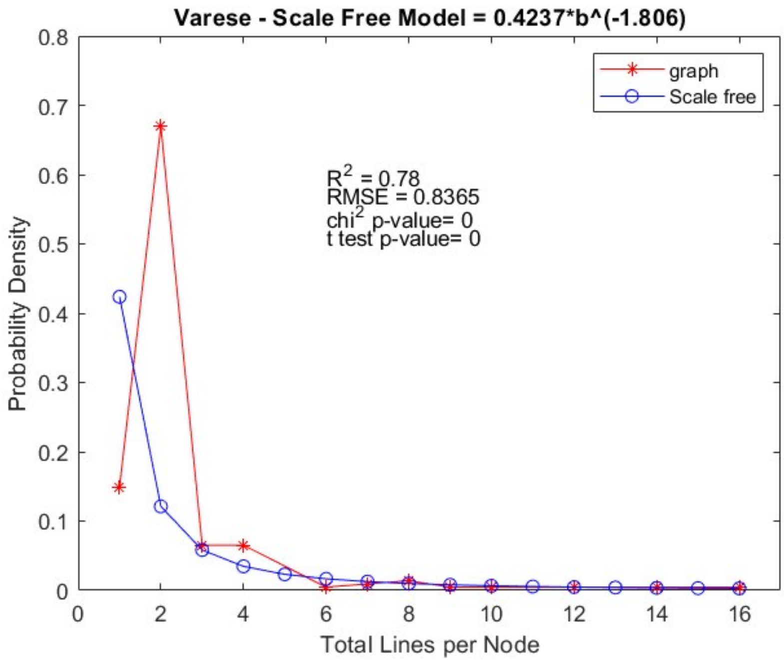


Figure 4.10 Varese -Total lines degree

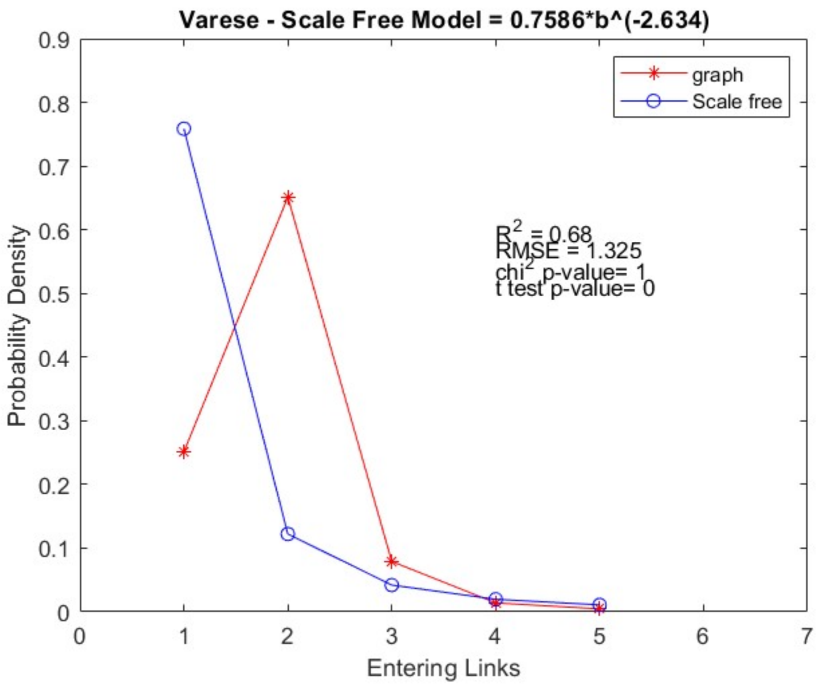


Figure 4.11 Varese - Ingoing links degree

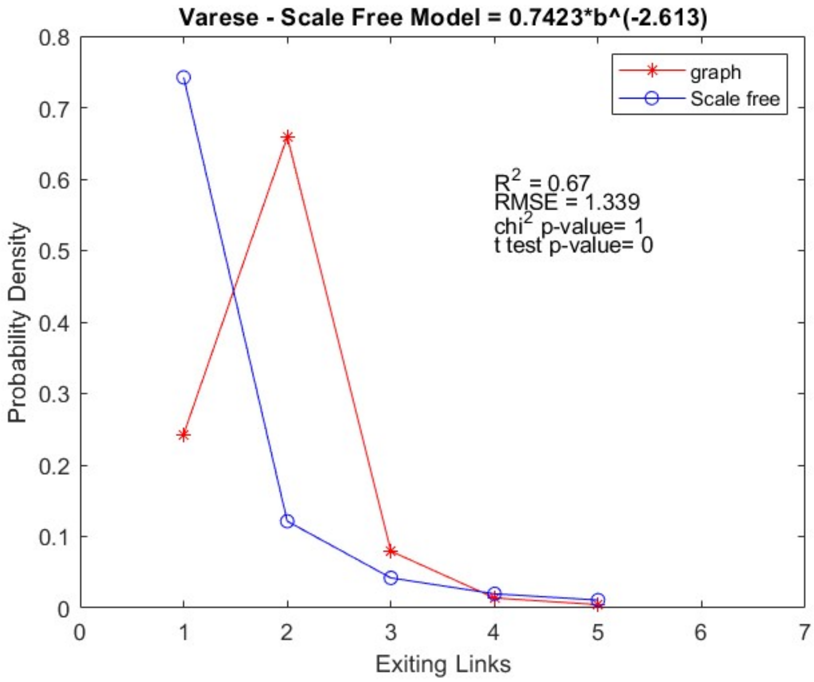


Figure 4.12 Varese-Outgoing links degree

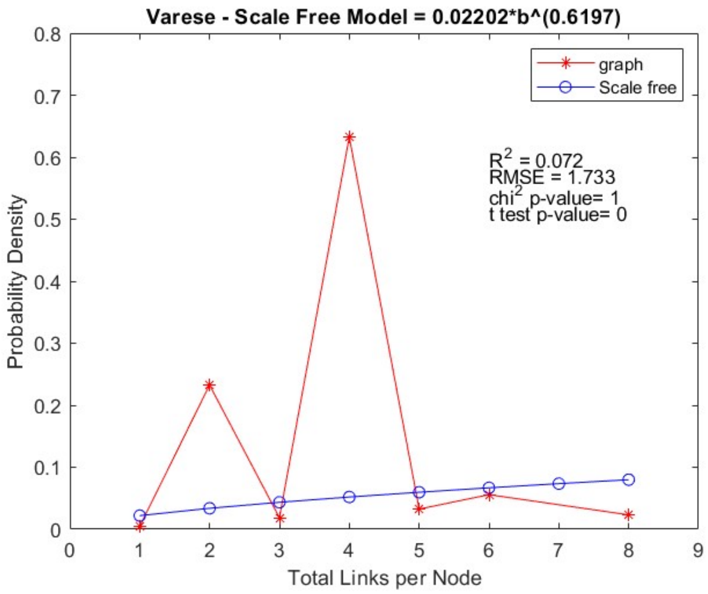


Figure 4.13 Varese - lines degree

Como Network

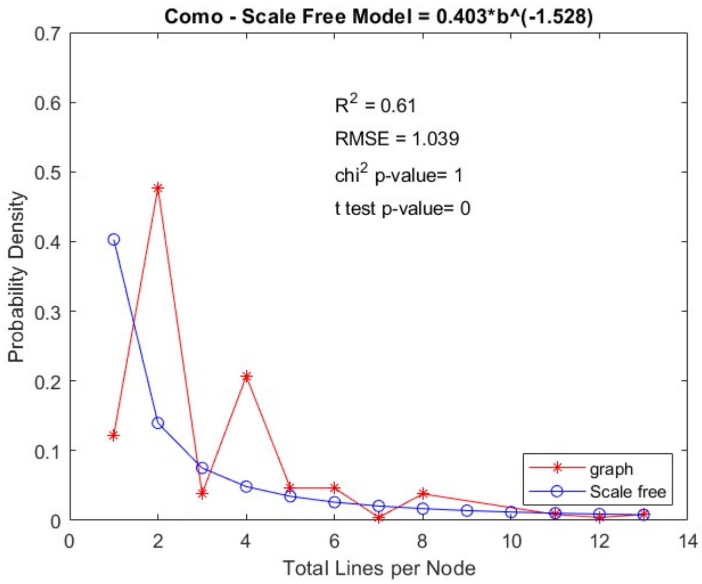


Figure 4.14 Como - Total lines degree

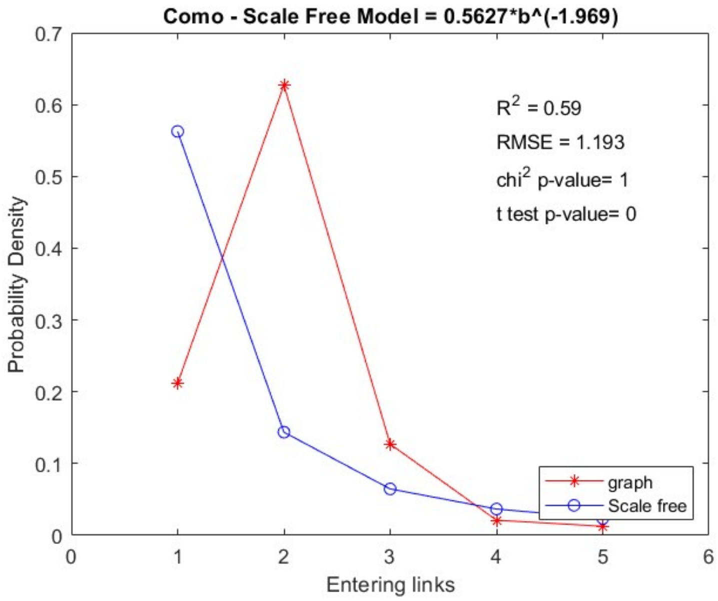


Figure 4.15 Como - Ingoing links degree

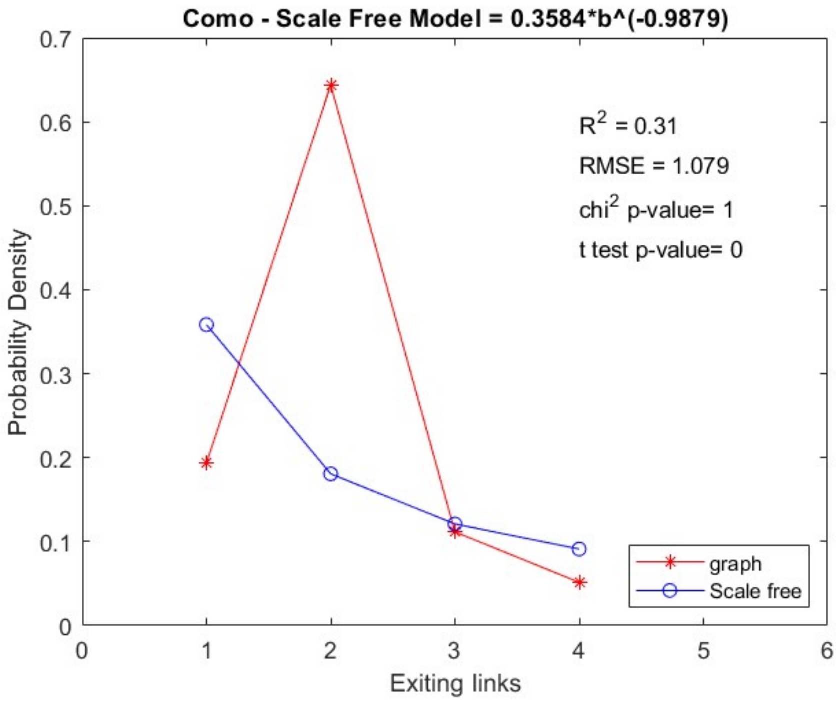


Figure 4.16 Como - Outgoing links degree

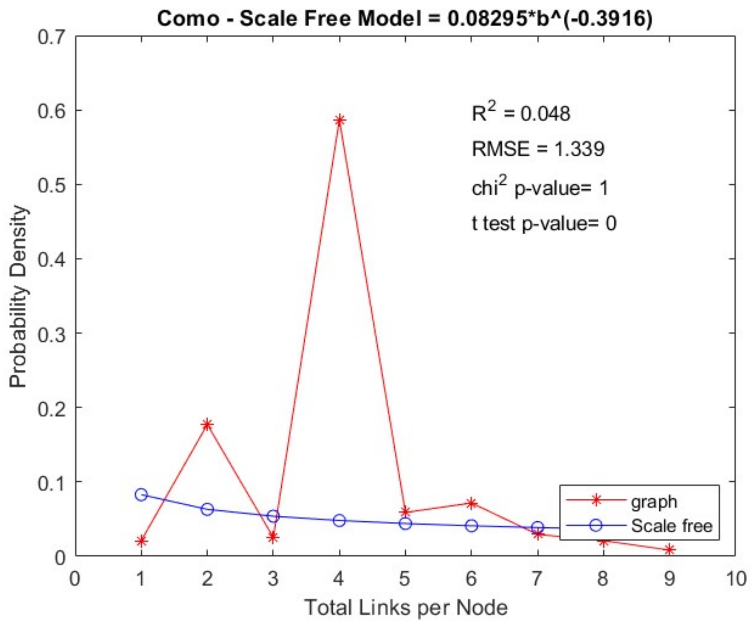


Figure 4.17 Como - Total links degree

Lecco Network

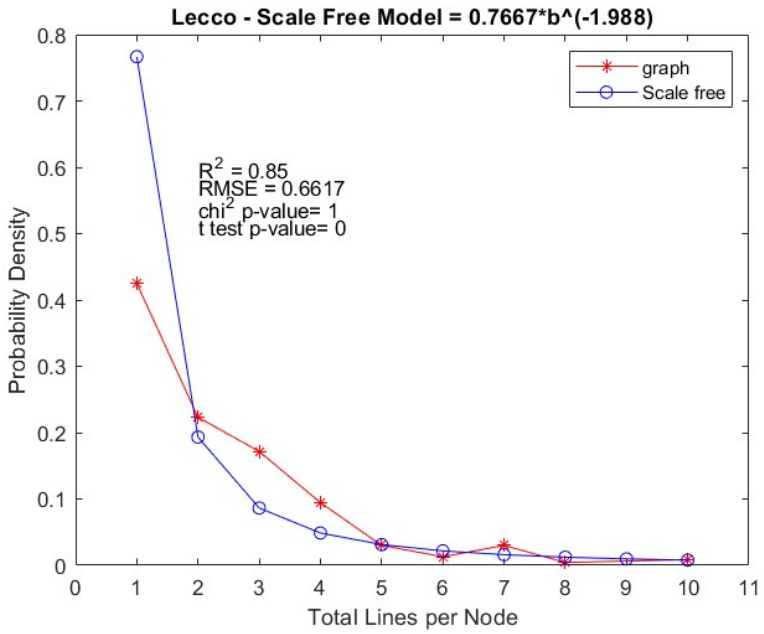


Figure 4.18 Lecco - Lines degree

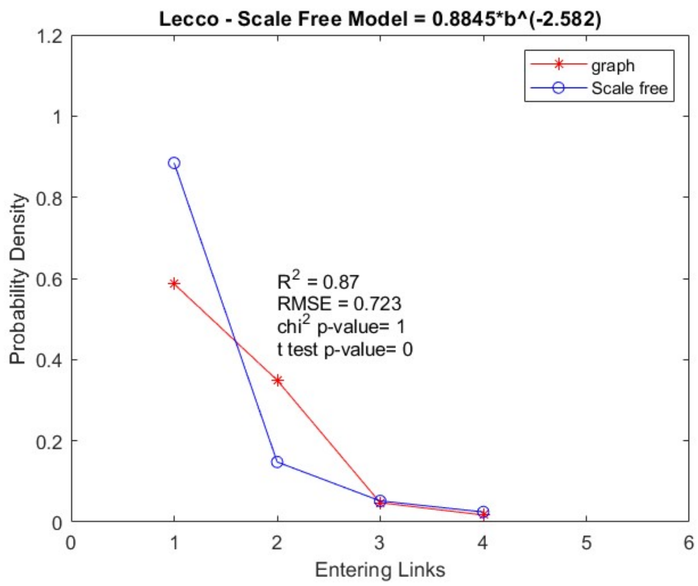


Figure 4.19 Lecco- Ingoing links degree

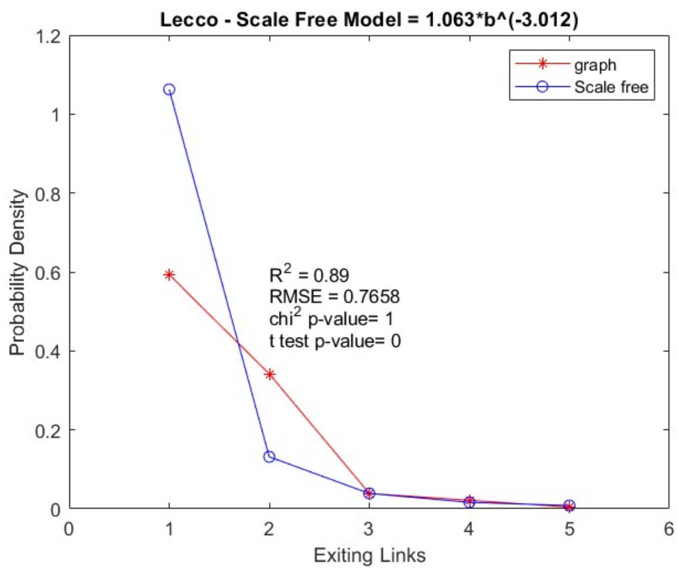


Figure 4.20 Lecco- Outgoing links degree

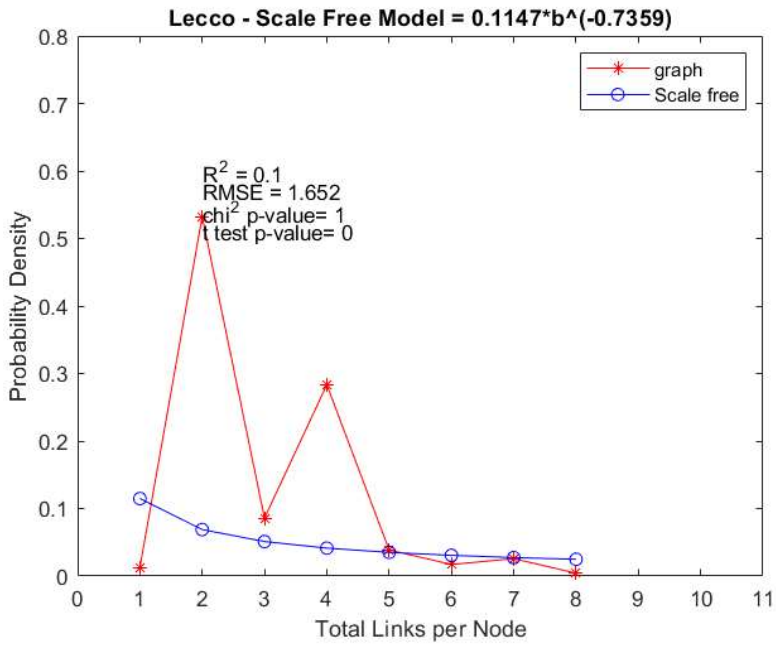


Figure 4.21 Lecco - Total links degree

Chapter 5

Networks Analysis

The robustness of a network topology is a crucial indication for assessing self-organization performance. Understanding how a system responds to failures or perturbations may help you improve or plan for scenarios. Identification and assessment of the most vital connections in a network are critical to transportation maintenance and planning challenges and the overall robustness network (Besinovic 2020). Network topology plays a significant role in its robustness; for example, highly connected networks tend to be more robust (Barabasi 2016). Robustness measurement and system robustness assessment are crucial since the proportion of riders on services increases in highly urban areas. It is highly important to measure and assess the robustness, given that ridership is highly concentrated in dense urban areas.

Multiple connections (multiline, multilayer, or multiplex networks) are the other technique to make networks more robust, and this multiline element is shown in various systems. As example within a same day, information can be exchanged between the same two individuals via various modes of communication (e.g., phone, e-mail, and in person), and transportation networks frequently provide multiple modes of transit (e.g., bus and rail) or alternatives routes between two locations using the same mode of transportation (e.g., roads, railways). The multiline network is able to be more robust or resilient to breakdowns or disturbances than single-line networks, even though these multi-line networks are increasingly being studied (Kivela et al. 2014). Assessing the robustness of multiline transportation systems now faces two key challenges. Firstly, multiline transportation systems are frequently referred to as single-line systems. Their robustness is assessed using network connection criteria, eliminating the added information and value of having several lines (Derrible & Kennedy, The complexity and robustness of metro networks, 2010); (Derrible, Network centrality of metro systems, 2012) (Jun-Qiang, Long-Hai, Liu, & Zhao, 2017). Second, multiline robustness measures have been created for transportation applications, but they are mathematically more challenging to utilize, which may rule them out for usage in planning applications (Boccaletti et al. 2014; Halu, Mukherjee, and Bianconi 2014; De Domenico et al. 2015; (Mattsson & Jenelius, 2015); Auerbach 2018). In general, the percolation process (Barabasi 2016) or disruption simulations (Kim, Kim, and Chun 2016) may be used to verify network robustness. However, these techniques frequently need an additional level of study, potentially reducing the efficiency of the issues or applications.

(Alvarez-Socorro, Herrera-Almarza, & González-Díaz, 2015) developed a global robustness index based on the inclusion-exclusion principle, a line coverage similarity measure comparable to network dissimilarity indices. (Jun-Qiang, Long-Hai, Liu, & Zhao, 2017) The global network index is based on the differences between standard adjacency matrices and new multiline adjacency matrices, which may be used to assess the impact of shared connectivity caused by the system's numerous lines overlapping. This indicator allows you to see how the connectedness of all pairs of lines in a network differs and how it differs. There is a proposal for identifying nodes in multiline systems that significantly impact local robustness measures.

Although this technique has been frequently used and utilized for network analysis due to its ability to generalize complex systems into a basic form, it ignores the additional information provided by many connections between the same pair of nodes in multiline networks. (Derrible & Kennedy, The complexity and robustness of metro networks, 2010)

Our study examines the robustness of multiline networks by modeling them as multiline networks constructed of routes. The directed network comprises nodes (stops) and edges (links). The digraph of these routes represents the whole bus network.

As Kim (2012) and O'Kelly (2015) defined, a hub is a particular facility in transportation networks that is typically regarded as an essential interconnection point where many lines are linked in a facility for reaching, transferring, and departing to destinations. The failure of these specific nodes has been shown to significantly reduce network functionality (Kim, Kim, and Chun 2016; Kim and Ryerson 2017) and influence network vulnerability and robustness to network failure (O'Kelly 2015). Recent research claims that not all hubs are crucial, depending on network topology or node fluxes (Kim 2012; Kovacs and Barabasi 2015). These let to ask what circumstances the best predictors or variables would be affecting network vulnerability and robustness. The robustness of these bus transit systems is related to the position of hubs for transferring passengers, and these multi-lines allow passenger mobility through the transit networks (Kim and O'Kelly 2009; Li and Kim 2014).

Some nodes have a higher priority than others in complicated networks. The more significant a node gets, the more probable it is that its removal would cause the network to collapse; hence identifying key nodes is essential in many situations. To improve network robustness, it is essential to address such crucial nodes (Auerbach, Fitzhugh, and Zavisca, 2021; Auerbach and Kim, 2021).

Adjacency matrices, in which the number of connections between nodes is bidirectional, have been used to quantify connectivity (Wu et al. 2011; Ellens and Kooij 2013).

Because of the kind of stops and the degree of connectivity through lines, the typical binary representation of network connections presents a significant challenge when used in bus transportation systems. As proposed similarly by (O'Kelly 2010) and (Derrible & Kennedy, The complexity and robustness of metro networks, 2010), a graphic representation for transit networks required a distinct understanding of the concepts of nodes and edges. They distinguished between two types of nodes: transfer and end nodes and single and multiple edges. These ideas were proposed to look at the structure of transport systems to figure out how directness and complexity are related (Derrible, Network centrality of metro systems, 2012). On the other hand, intermediate bus stops are critical components that impact a network's robustness since any breakdown between transfer stations might result in the line's immediate loss of connectivity or a stop in system flow. (Kim, Kim, and Chun 2016)

5.1 Robustness networks indicators:

By combining complex network theory with the concept of transit network robustness, network average efficiency and relative size of the giant connected component GCC are chosen as the analysis index:

- Network average efficiency E

When network nodes are attacked, the connections of network changes. E is a metric that measures the degree of connection (Albert and Barabasi, 2002).

$$E = \frac{1}{n(n-1)} \sum_{i \neq j} \frac{1}{d_{ij}}$$

Where d_{ij} is path length. A larger E indicates better network connectivity.

- The relative size of the GCC of the network S

The stability performance can be represented by the relative size of GCC or connectivity probability S:

$$S = \frac{N'}{N}$$

N and N' represent the number of nodes in the network's GCC before and after the attack, respectively. The value of S is between [0, 1]. When S = 1, the network is well connected; when 0< S<1, the network is still relatively integrated; when S=0, the network collapses.

Scale-free networks vary qualitatively from random networks, (Renyi, 1959). These are:

- Scale-free networks are less prone to failure. This means that following the removal of randomly picked nodes, the network is more likely to stay connected than a random network.

- Scale-free networks are more prone to failure under non-random attacks. Therefore, when nodes are dropped in order of their degree, the network rapidly collapses.
- Average path lengths in scale-free networks are generally short. In reality, log N/ log log k is the average length of a path. (Broido & Clauset, 2019).

Attacking methods

The reaction of networks to targeted attacks or random failures is explored to define network robustness. The two procedures for node removal discussed in this study are (i) random node removal and (ii) degree-based node removal.

- Random removal: The node to be eliminated is randomly selected from all the nodes in the network with an equal chance of being removed.
- Removal based on degree: The node to be eliminated in the network has the greatest degree. If more than one node has the highest degree, one is picked at random from all the highest-degree nodes with an equal chance. (Wang, Koç, Derrible, Ahmad, & Kooij, 2015)

Typically, to study the robustness of complex network, one can remove a node and all edges that contain it, and then check if the network is still connected. If it is not, one can compute the size of the largest sub-network or cluster. The node to be removed can be chosen either randomly (error), or according to some principle (attack). Choosing nodes with higher degree is one of the most common criteria for nodes to attack (degree-based attack). Nodes are gradually deleted in this study, starting by eliminating nodes with the greatest degree and remove nodes in decreasing order of degree.

Analyzing the Varese network, in fig 5.1, bus stops [6,7,8,26 and 27] have the highest degree and most lines crossing them. Any attack or maintenance work in these critical stops and lines may lead the drivers to take an alternative road to reach the destinations (the blue and green lines).
As observed on table 2, the network efficiency E is significantly affected after removing these nodes. At the same time, a minor decrease in the connectivity probability S is apparent in table 3.

Moreover, Como network figures 5.2, 5.3, and 5.4 illustrate the most critical nodes and lines that are vulnerable to affect the network robustness due to any natural or target attack. The network efficiency E decreases and the connectivity probability S is between the range 0.97 and 0.99, as shown in table 4 and 5 respectively.

In addition, Fig 5.6 demonstrates the critical nodes, all lines crossing these nodes have alternatives that may be used in case of an attack to Lecco network. The efficiency connectivity E is severely dropped as illustrated in table 6, and connectivity values are minor decreased by 0.05. Furthermore, any attack in the critical nodes 6 and 10 may lead to network collapse due to the remoteness of the alternative ways that buses used to reach their destinations. That makes the network efficiency close to zero, consistent with the theory that a scale-free network is more vulnerable to target attack.

Node	Distance before attack [m]	Alternative distance direction 1 [m]	Alternative distance direction 2 [m]	E before attack %	E alternative 1 %	E alternative 2 %
7	341	674	1190	0.292	0.148	0.084
6 - 7	345	750	-	0.288	0.133	-
8– 27	304	1070	456	0.327	0.093	0.218

Table 2 Varese efficiency connectivity E

	Node number	Total nodes	Total links	S
Before attack	-	215	804	1
After attack	7	215	803	0.995
After attack	6-7	213	800	0.9907
After attack	8-27	213	802	0.9907

Table 3 Varese connectivity probability S

Node	Distance before attack [m]	Alternative distance direction 1 [m]	Alternative distance direction 2 [m]	E before attack %	E alternative 1 %	E alternative 2 %
15-16	470	575	715	0.212	0.173	0.139
17	620	2340	-	0.161	0.042	-
21-22	415	674	1190	0.24	0.148	0.084

Table 4 Como efficiency connectivity E

	Attack Node	Total nodes	Total links	S
Before attack	-	237	942	1
After attack	15-16	235	926	0.9915
After attack	17	236	935	0.9873
After attack	21-22	234	924	0.9873
After attack	All	232	908	0.9789

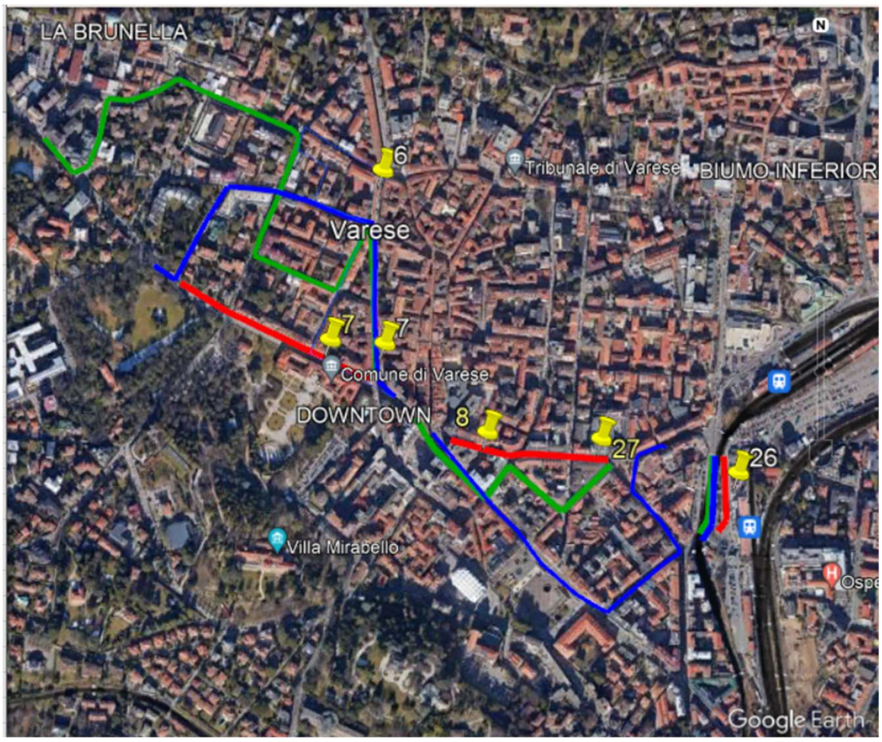
Table 5 Como connectivity probability S

Node	Distance before attack [m]	Alternative distance direction 1 [m]	Alternative distance direction 2 [m]	E before attack %	E alternative1 %	E alternative2 %
5	276	537	531	0.36	0.185	0.187
55	366	560	-	0.272	0.178	-
83	238	605	-	0.42	0.165	-
130	165	485		0.6	0.2	-

Table 6 Lecco efficiency connectivity E

	Node number	Total nodes	Total links	S
Before attack	-	233	694	1
After attack	5	232	687	0.995
After attack	55	232	687	0.995
After attack	83	232	687	0.995
After attack	130	232	686	0.995

Table 7 Lecco connectivity probability S



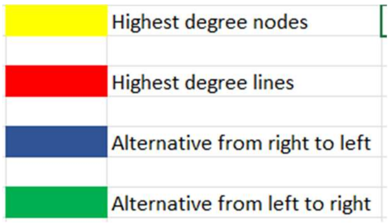


Figure 5.1 Varese network blocks and alternatives lines

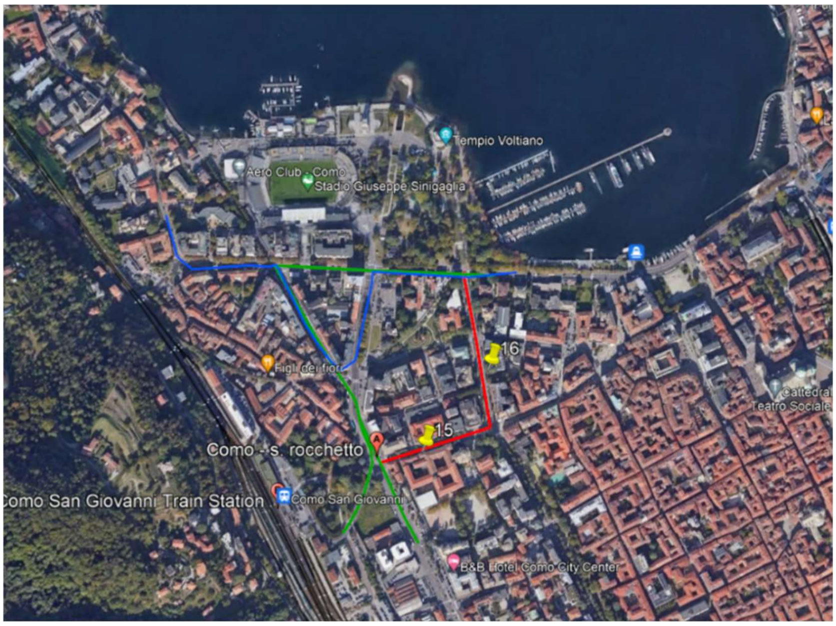


Figure 5.2 Como network block and alternative for node 15 and 16

Figure 5.3 Como network block and alternative for node 17

Figure 5.4 Como network block and alternative for node 21 and 22

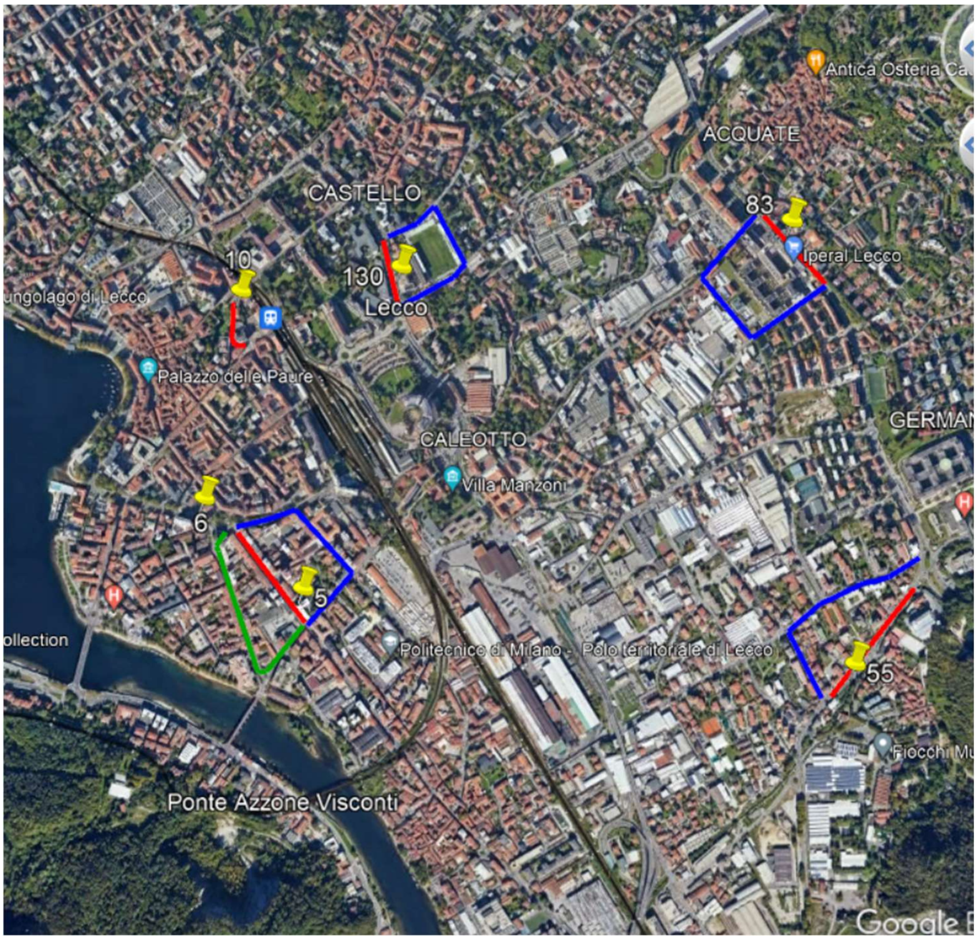


Figure 5.5 Lecco network critical nodes

Chapter 6

Conclusions

This study examined the spatial structure of urban bus networks in medium-sized cities using complex network theory. Based on the operating configuration of bus lines, the bus service network has been modeled through the MATLAB program using the L-space principle to represent the relationship between bus stops by employing directed and unweighted graph-based methods.

Two primary goals drove the analysis of bus transport networks (BTNs). To begin with, defining the scale-free characteristics of Lecco, Como, and Varese BTNs using the goodness-of-fit statistical method to answer the question; is the network under consideration identical to the scale-free model? Then, assessing the robustness of these networks to unpredictable events, namely, random and targeted attacks.

The assessment results from total lines, in-degree and out-degree degree distribution reveal that only the Lecco bus network with R^2 value > 0.85 and the power-law exponent ε between 2-3 follows the scale-free feature, whereas with R^2 value < 0.8 and power-law exponent less than 2 Varese and Como bus network cannot be described as scale-free networks. In addition, the graphs with total links degree distribution for the three cities do not follow the scale-free model.

Also, this research contributes to evaluating the robustness of the urban bus network; the analysis is based on identifying nodes with high clustering and most crossing lines. Scale-free networks are more vulnerable to targeted attacks but less prone to random attacks. In them the connectivity efficiency indicator is lower when the nodes with greater degrees and more lines passing through are removed. Furthermore, the three assessed networks are still relatively integrated into the connectivity probability S index because they are between 0.97 and 0.99.

To finish, our work is just a starting; based on a proper dataset provided by the bus services companies, future research should study the features of these bus transport networks. Furthermore, evaluating the vulnerability and resilience considering other robustness indexes could provide a better indicator of how the networks act under any attack scenario and could also help develop and extend the city.

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