SCUOLA DI INGEGNERIA INDUSTRIALE
E DELL'INFORMAZIONE

Executive Summary of the Thesis

Mass evacuation planning: infrastructure management approaches and behavioural models

Laurea Magistrale in Mobility Engineering - Ingegneria della Mobilità

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1. Introduction

Mass evacuation planning represents an important challenge for Transportation and Safety Engineers, due to the inherent complexity of such tragic situations: congestion and panic make the flow on the road graph extremely difficult to be dispersed. The transportation planning is involved both in the simulation of the crowd movement and in the management of resources allocation on the emergency area (e.g. police corps, military enforcement, emergency healthcare, etc.).

In this work, we focus mainly of the first problem, the flow simulation, in order to get some summary information to optimize resources allocation. After discussing the general framework for the mathematical modelling of evacuations, we present three different approaches to infrastructure management, split depending on the degree of State intervention, power and coercion. The simulation algorithms have been tested on two small dummy graphs and, then, applied to the real graph of Sioux Falls, South Dakota.

2. Scenarios tree

Firstly, we outlined the general modelling framework for these kind of transportation problem. The structure employed is the so-called *Scenarios tree*, where every level is defined by the choice of a mathematical modelling feature or a physical/practical issue of the emergency scenario. These tree levels are:

- Predictability;
- Coercion level;
- Flow and time interval discretization;
- Game theory applications;
- Operations research applications.

The applications of mathematics are discussed thoroughly, following the example given by [3]. The relationship among these modelling choices and the third level (*Flow and time interval discretization*) is dealt with, in order to define the most suitable framework to set our work within. Predictability is the most defining feature of the classification tree. In real evacuation practice, we split between predictable emergencies, such as hydrological phenomena or unexploded ordnances defusing, and unpredictable ones, e.g. earthquake or relevant accidents in chemical plants. The main difference underlined in this work is the convenience to shift the computation of flows and costs towards a probabilistic approach, to consider their expected values. For example, the general formula for the number of people entering the road graph at discrete time t is:

$$N_t = \alpha * N_{TOT} * P(t) \tag{1}$$

α is the percentage of the total evacuees N_{TOT} who decided to leave the affected area and $P(t)$ is the probability of getting into the graph at time t (coming from a discrete probability distribution).

For the coercion level, we distinguished 3 approaches: the *Complete Coercion*, the *Simple Information* and the *Partial Prescription*. All the scenarios have been modelled and compared for the three graphs.
Summing up, we described an ideal *predictable* emergency within a macroscopic, discrete framework, resorting to an *instrumental* use of Game Theory and to the so-called *MILP* branch of Operations Research (Mixed-Integer Linear Programming). We refer to an ideal flood in urban area, where evacuees have to collect in a *safe point*. After the zoning phase of the city/town, we divide the area in three sections, S_1, S_2, S_3, from the most to the least dangerous. The save point corresponds to a node in S_3, labelled as the last one of each graph, n. The framework for the emergency is an intense flood: thus, we can see S_1, S_2, S_3 as areas of different altitude, with S_1, the most dangerous area, as the lowest part of our ideal city/town, S_3 as the highest and reasonably safe one and S_2 as the intermediate section, which should be evacuated as fast as possibile, though safer than S_1.

3. Complete Coercion

The first approach is also the least suitable for a Western, democratic country; nevertheless, real emergencies can never be faced without a minimum amount of freedom limitation even in our part of the world; on the opposite part, even in dictatorships, it is really difficult to intervene with absolute precision on the movement of every single citizen. So, we can evaluate the results of the first two approaches as extreme values, taking into account that each real situation will correspond to some sort of Partial Prescription. Basically, this part of the paper is an expansion of the model of [1], where Achrekar and Vogiatzis developed a MILP formulation. This approach allows to complete two tasks at the same time:

1. To define the exact number of citizens, i.e. the flows, to have on each arc and to direct to each node, $\forall t$;
2. To determine which arcs should be closed or inverted to accelerate the evacuation.

Basically, we impose that the number of arcs exiting a node must be constrained by a function of the number of arcs entering the node and the quantity m_i, the "number of divergences allowed" for the node i. The model variables, objective and costraints follow.

m_i = divergences allowed for the node i

$$x_{ij} = \begin{cases} 1 & \text{if the arc } i \to j \text{ is employed} \\ 0 & \text{else} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if the arc } i \to j \text{ is inverted} \\ 0 & \text{else} \end{cases}$$

f_{ij}^t = flow about to leave i to go to j at time t

Δq_i^t = users entering the graph from node i, at time t

$$\min \sum_{i=1}^{n} \sum_{j \in FS(i)} \sum_{t=1}^{T} r_j^t f_{ij}^t \tag{2}$$

$$s.t.$$

$$x_{ij} \geq y_{ji}, \quad \forall i = 1, 2, \ldots, n, \quad j \in FS(i) \tag{3}$$

$$\sum_{v \in FS(i)} x_{iv} \leq \sum_{k \in BS(i)} x_{ki} \quad \forall i = 1, 2, \ldots, n \tag{4}$$

$$\sum_{i=1}^{n} \hat{C}_i m_i \leq \hat{B} \tag{5}$$

$$\sum_{i=1}^{n} \sum_{j \in FS(i)} \overline{C}_{ij} y_{ij} \leq \overline{B} \tag{6}$$

$$\sum_{j \in FS(i)} x_{ij} = 1 + m_i, \quad \forall i = 1, 2, \ldots, n-1 \tag{7}$$

$$\sum_{v \in FS(i)} f_{iv}^t - \sum_{k \in BS(i)} g_{ki}^t - \Delta q_i^t = 0 \tag{8}$$

$$\sum_{\tau=\max\left\{0,\quad t-w_{ij}+1\right\}}^{\min\left\{t,\quad T-w_{ij}\right\}} f_{ij}^{\tau} \leq U_{ij}x_{ij} + U_{ji}y_{ji} \tag{9}$$

$$f_{ij}^{t} \geq 0 \tag{10}$$

$$g_{ij}^{t} = \begin{cases} 0 & \text{se } t \leq w_{ij} \\ f_{ij}^{t-w_{ij}} & \text{se } t > w_{ij} \end{cases}$$

$$\Delta q_i^t \geq 0 \quad \forall i = 1, 2, \ldots, n-1, \tag{11}$$

$$x_{ij} \in \{0, 1\} \tag{12}$$

$$y_{ij} \in \{0, 1\} \tag{13}$$

$$m_i \in N \quad \forall i = 1, 2, \ldots, n-1 \tag{14}$$

We intentionally avoided to declare exactly the indexes of constraints for the long equations, for the sake of readability. The linear parameters of the objective function, r_j^t, are called *danger factors*. They take into account the difference between early and late arrivals at safety and can be chosen in several ways to penalize more or less those who need more time to get to the save point. In our work, we put:

$$r_j^t = \begin{cases} t & \forall j \in S_1 \\ r_2(=1) & \forall j \in S_2 \\ r_3(=0) & \forall j \in S_3, j \neq n \end{cases}$$

$$r_n^t = t - T$$

Essentially, this MILP problem returns us the exact optimal schedule of departure from each user's origin node, their movement on each arc of their own optimal path towards the final node, and the updated topology after inverting some streets traffic directions and forbidding the access to some other. The intervention on these features of the graph is subject to budget constraints (5, 6), both for practical feasibility and to avoid unnecessary confusion to road users.

4. Simple Information

This section can be considered the core of this work, since it relies heavily on Game Theory and behavioural models, melting them with the classic *4 phase model* formulation for flows assignment (e.g. see [2]). The model presented hereafter is intended to be iterative: we compute arc and path flows/costs at every time interval t. In our case, t is equivalent to 1 minute, to get a proper compromise between the realism of a small discretization timestep and the time scale of users decisions.

4.1. Demand generation

The first step is the generation of evacuation demand in each node. We decided to follow a setting similar to the work of Achrekar and Vogiatzis, describing the time distribution of the inlet of road users into the graph. Our assumption is that users decide to enter the graph from each starting node following a parabolic curve. If the total horizon of the evacuation is τ, $T < \tau$ will be the window available for citizens to get off their house. Every user shall leave their origin before αT, with $\alpha \in (0, 1)$ (here, $\alpha = 0.8$). This modelling choice has been formulated likewise to the well-known *Cournot's monopoly*, a Game Theory establishment.

$$\begin{cases} \Delta q_i^t = c_i + b_i t - a_i t^2, t = 1, 2, ..., \alpha T \\ \Delta q_i^t = 0, t \geq \alpha T + 1 \end{cases}$$

Δq_i^t is the demand leaving its origin node i at time t, while Q_i is the overall demand from that node. The parameters a_i, b_i and c_i have been chosen according to the following equations (curve simmetry and conservation of demand):

$$b_i = \alpha T a_i \tag{15}$$

$$\sum_{t=1}^{\alpha T} \Delta q_i^t = Q_i \tag{16}$$

Together, they lead to:

$$\frac{Q_i}{\alpha T} = c_i + \frac{1}{6}(\alpha T + 1)(\alpha T - 1)a_i \tag{17}$$

Imposing the equality between the two components (arbitrarily, for sake of simplicity), we get:

$$c_i = \frac{Q_i}{2\alpha T} \tag{18}$$

$$a_i = \frac{6c_i}{\alpha^2 T^2 - 1} = \frac{3Q_i}{\alpha T(\alpha T - 1)(\alpha T + 1)} \tag{19}$$

$$b_i = \frac{3Q_i}{(\alpha T - 1)(\alpha T + 1)} \tag{20}$$

4.2. Route choice model

This subsection can be considered the core of this model. In short, we assume that users will choose their own path depending on the *forecasted* cost of the arcs composing the path. This virtual arc costs, w_{ij}, are proportional to the real base costs, $\tilde{c}_{ij}$ (i.e. c_{ij} when arcs are completely empty). The proportion factor is the danger parameter, r_{ij}^t, depending on the zone of the next node of the path; this parameter is non-decreasing with respect to time, to account for the increasing probability of accidents and congestion and, at the same time, to describe a *risk-averse* users' approach. We underline that the other, aforementioned set of danger parameters has a rather different meaning, though quite related.

$$w_{ij}^t = r_{ij}^t \tilde{c}_{ij} \tag{21}$$

$$r_{ij}^t = \begin{cases} x & \text{se } j \in S_3, \forall i \\ x + yt & \text{se } j \in S_2, \forall i \\ x + yt + zt^2 & \text{se } j \in S_1, \forall i \end{cases}$$

The other major assumption in our context is that the reconstructed forecast of path cost comes from the sum of two contributions:

1. the next forecasted arc cost;
2. and the *Dijkstra-optimal* path cost.

This modelling choice allows to represent the individualistic behaviour of evacuees during such delicate situations, since we consider them to consider only the fastest way from the actual node to the save point. Moreover, we are considering the chance of a *mixed preventive-adaptive path choice*, or *re-routing*, allowing the users to change the selected path dynamically (as in real situations).

$$h_{ij}^t = w_{ij}^t + \sum_{(u,v) \in DJ(j,t)} w_{uv}^t \tag{22}$$

$DJ(j,t)$ is the ordered succession of arcs belonging to the Dijkstra-optimal path from the node j to the save point n at time t.

The last equation of this model subset is the established Logit distribution for the path choice, which, in our case, is rather a "arc-related" path choice. The Logit input argument is the set of forecasted path costs towards n.

$$p_{ij}^t = \frac{\exp(-\frac{1}{\theta} h_{ij}^t)}{\sum_{v \in FS(i)} \exp(-\frac{1}{\theta} h_{iv}^t)} \tag{23}$$

4.3. Proximity flows

The following two parts of the model deal with the flow distribution among the streets. We identify two different kinds of flow variables:

- The so-called *Arc flows*, F_{ij}^t, i.e. the actual users' flow on each arc at time t, with the usual meaning;
- The *Proximity flows*, f_{ij}^t, a representation of the number of people headed towards the node j, leaving node i between times t and $t+1$.

The relationship between these variables is explained in the next subsection, 4.4. Essentially, we need to balance the flows through each node in order to apply the Logit probability and get the mean number of users on the arcs. Therefore, we set the conservation of people at each node, taking into account the time necessary to leave an arc:

$$\sum_{v \in FS(i)} f_{iv}^t = \Delta q_i(t) + \sum_{k \in BS(i)} f_{ki}^{t - z_{ki}^t} \tag{24}$$

The quantity z_{ki}^t is discussed thoroughly in the thesis and is defined as follows:

$$z_{ki}^t = \lfloor c_{ki}^{t - z_{ki}^t} \rfloor \tag{25}$$

Afterwards, we can split these flows according to the path choice model.

$$f_{ij}^t = p_{ij}^t \Big(\sum_{v \in FS(i)} f_{iv}^t \Big) \tag{26}$$

4.4. Arc flows and costs update

We determine the arc flows as a linear combination of proximity flows of users who started their own motion on the arc *in the past*. In analogy to z_{ki}^t, we skip the deep explanation of properties of the number $n(t)$; we just write the definition of this fundamental variable.

$$F_{ij}^t = \sum_{k=1}^{n(t)} f_{ij}^{t-k} \tag{27}$$

$$n(t) = \max \Big\{ k | c_{ij}^{t-k} \geq k \Big\} \tag{28}$$

Finally, we can update the real arc costs with the *BPR-like* function:

$$c_{ij}^t = \tilde{c}_{ij} \Big[1 + \beta_1 \Big(\frac{F_{ij}^t}{u_{ij}} \Big)^{\beta_2} \Big] \tag{29}$$

5. Partial Prescription

The third and last approach could be interpreted in a variety of ways; in this work, it can be seen as an application of the Simple Information on a different graph, that comes from the results of the Complete Coercion. Basically, we look for a performance enhancement of the evacuation by intervening of the road graph topology.
The procedure applied in this work is the following:

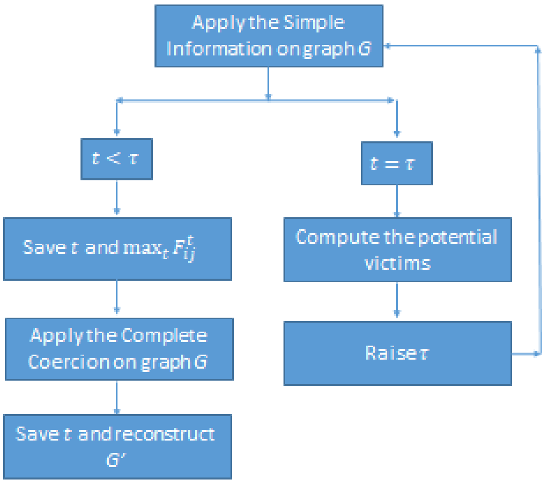

Figure 1: Procedure to compute time and victims and to build the second graph.

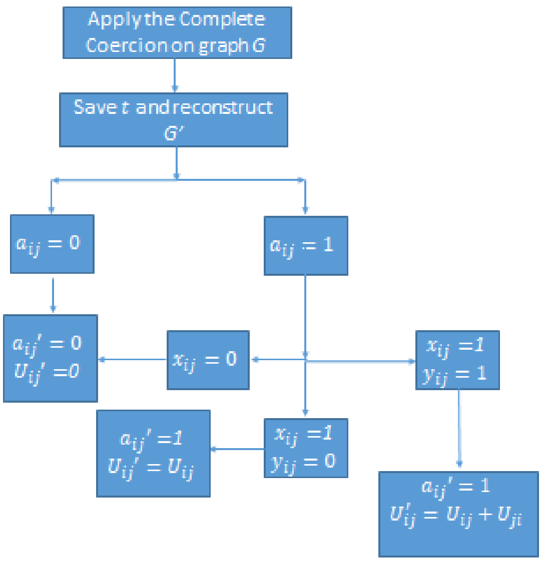

Figure 2: Procedure to compute the new incidence and capacity matrices.

The function adopted to estimate (as an upper bound) the likely victims is the following:

$$\textbf{potential\_victims}(t) = \sum_{i=1}^{n-1} \sum_{j \in FS(i)} F_{ij}^t \quad (30)$$

6. Application: dummy graphs

These two dummy graphs have been created and used in order to evaluate the correctness of the models and to debug the related MATLAB algorithms. We omit to discuss the parameters employed in this simulations, referring to the Chapters 4 and 5 and Appendix B of the thesis. We show each graph together with its twin, arising from the procedure explained in figure 2.

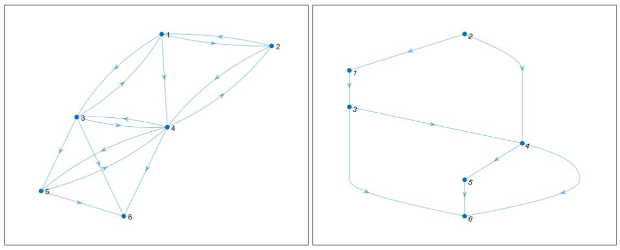

Figure 3: 6-nodes dummy graphs and its optimized counterpart.

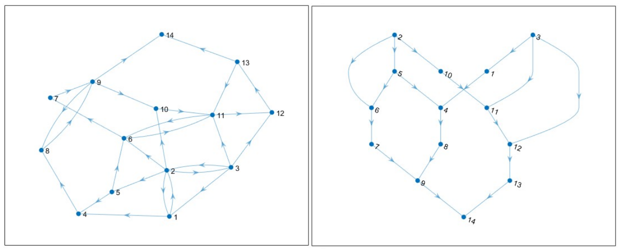

Figure 4: 14-nodes dummy graphs and its optimized counterpart.

We report in a table the results of these two simulations, run with such features that allow to show the differences in victims, time and efficiency among the three approaches. For the smallest graph, the time window is $\tau = 50$ minutes. We got:

	S. I.	C.C.	P.P.
6-nodes	48 mins	25 mins	48 mins

Table 1: Time results for the 6-nodes graph.

The evacuation is completely successful; the coercitive approach is extremely more efficient than the other two, requiring about half time of both the prescriptive approach and the free one.

	S. I.	C.C.	P.P.
14-nodes	5405	3998	5172

Table 2: Victims results for the 14-nodes graph.

The other evacuation process, with an insufficient time window of 200 minutes and a demand of 14000 evacuees, can not be completed in any scenario, since we got an elevated number of victims. The Complete Coercion proves again to be the most useful, despite its difficulty of application.

7. Application: real graph

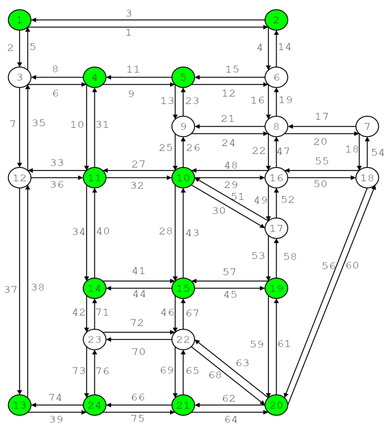

Figure 5: Sioux Falls road graph. Source: **Github**.

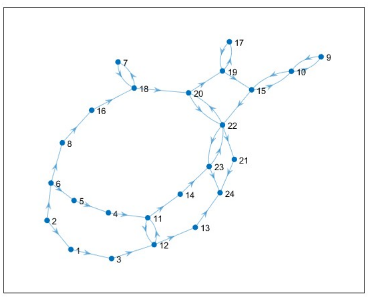

Figure 6: Sioux Falls road graph optimized counterpart.

For the real application, we decided to use this city in South Dakota due to the availability of graphs attributes and data about the road transport. For an evacuation demand of 200.000 citizens, we set a time window of 20 hours, 1200 minutes. The evacuation is perfectly successful, with a huge advance:

	S. I.	C.C.	P.P.
Sioux Falls	321 mins	300 mins	321 mins

Table 3: Time results for the Sioux Falls graph.

The C.C. MILP simulation required several attempts to find a suitable time window; the last one, referred in the table, required a run-time of approximately 21 hours.

8. Conclusions

Even though the Complete Coercion reveals the highest level of efficiency, it is extremely difficult to be applied in a real, Western scenario. However, the difference with the other, free approaches is so small to be completely irrelevant. Between these two, we suggest to adopt the Simple Information for the sake of simplicity, since it does not require any form of direct intervention of resources on the graph topology, allowing to employ them in other circumstances. Also, this helps to avoid confusion in road users.
In conclusion, we suggest to try other applications of these approaches, e.g. changing parameters, providing more than one save point, etc..

References

[1] Omkar Achrekar and Chrysafis Vogiatzis. *Evacuation Trees with Contraflow and Divergence Considerations.* Springer, 2018.

[2] Ennio Cascetta. *Transportation Systems Analysis: Models and Applications.* Springer, 2009.

[3] Y. Hollander and J. N. Prashker. The applicability of non-cooperative game theory in transport analysis. 2006.