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SCUOLA DI INGEGNERIA INDUSTRIALE E DELL'INFORMAZIONE

Executive Summary of the Thesis

Kantian evolutionary game theory

Laurea Magistrale in Mathematical Engineering - Ingegneria Matematica

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1. Introduction

In the realm of evolutionary game theory, classical models often narrow their focus to the singular metric of individual reproductive capacity, overlooking the intricate dynamics of cooperation prevalent in real-world scenarios. Recognizing the inherently social nature of many species as well as human beings [4], this thesis delves into the realm of Kantian game theory.

To rectify the limitations of traditional approaches, the study introduces the so-called **Moralisator equation**. This equation provides insight into how a **Kantian optimizer**, in contrast to a **Nash optimizer**, strategically optimizes decision-making.

By embracing this novel perspective, the research not only acknowledges but also emphasizes the collaborative potential among individuals. Furthermore, the study extends these insights to practical application, exploring the application of these new optimization strategies to classical symmetric two-player games. In doing so, it aims to contribute to a more comprehensive understanding of evolutionary dynamics, highlighting the significance of cooperation in the intricate tapestry of group interaction.

2. Kantian game theory

Kantian game theory constitutes a branch of moral-based game theory that integrates principles from Immanuel Kant's moral philosophy into the analysis of strategic interactions. Here's a concise summary of Kantian game theory, with a broader perspective on moral-based game theory. Kantian game theory departs from traditional game theory by incorporating moral considerations and ethical imperatives into strategic decision-making. Immanuel Kant's moral philosophy, particularly his emphasis on the categorical imperative and the concept of treating individuals as ends in themselves, serves as a foundational framework. In this context, agents are viewed as rational beings capable of moral reasoning and are expected to act in accordance with principles that could be universally applied.

Important contributions in the field include the book "How we cooperate: A theory of Kantian Optimization" by Roemer [2], which has big applications in collaborative economy, and the theoretical article by Istrate in game theoretic [3]. Moreover, in the Kantian game theory landscape, the work of Ingela Alger and Jörgen W. Weibull has been instrumental as well as precursor in introducing the concept of the *homo moralis*, a strategic agent guided by a certain de-

gree of moral principles. One key contribution is the formulation of the utility function of such a player, providing insights into the decision-making process of a Kantian optimizer in contrast to traditional Nash optimizers.
Formally, the utility of a Homo moralis with a degree of morality $k \in [0,1]$, is given in a normal-form game, where each of the n players has the same strategy possibilities. Therefore, in this setting, if player i play strategy x_i against $\boldsymbol{x_{-i}}$ would obtain the utility $u_i(x_i, \boldsymbol{x}_{-i}) = E[\pi(x_i, \tilde{\boldsymbol{x}}_{-i})]$.
$\tilde{\boldsymbol{x}}_{-i}$ is a random vector, which derive from $\boldsymbol{x}_{-i}$ by replacing each of its $n-1$ components by x_i with a probability equal to the degree of morality k. This definition, by construction, embeds the Kant imperative by considering what will the outcome be whether the other act as I do.

3. The Replicator dynamic

A population game is a type of mathematical model used in game theory to study strategic interactions among a large number of individuals or agents within a population. Unlike traditional game theory models that focus on the strategies and payoffs of individual players, population games consider the distribution of strategies in a population and how they evolve over time. These games are particularly useful in understanding the dynamics of strategic interactions in large-scale scenarios, such as biological evolution, social networks, or economic markets.
The assumptions of such a model are that the population has a large amount individuals, who are indistinguishable and can only play a finite number of pure strategies $\{e_i\}_{i \in [|1,n|]}$. each time the game is played, two players are picked randomly within this unique massive population in order to interact with each other. As the population is large enough, we can focus on the evolution of the different strategies' proportion, in a smooth way. Mathematically speaking, we are looking for an Ordinary Differential Equation (ODE) that would describe the evolution of those proportions through time.
As the set of strategies for each player is finite, the payoff function of the game is summarized into a squared matrix $\mathbf{A}$ of size n, where its component $\mathbf{A}_{ij}$ is the payoff of playing strategy e_i against strategy e_j. In addition, the vector $\underline{x} = \begin{pmatrix} x_1 & \dots & x_n \end{pmatrix}'$ is the proportion vector i.e. for all $i \in [|1,n|]$, x_i is the proportion of player that plays the i-th strategy.
In this framework, the **Replicator equation** provides the differential equation that describes the system evolution in time and is given for all component $i \in [|1,n|]$ by

$$\dot{x_i}(\underline{x}) = x_i(\sum_{j=1}^{n} x_j \mathbf{A}_{ij} - \sum_{j=1}^{n}\sum_{l=1}^{n} x_j x_l \mathbf{A}_{jl}) \quad (1)$$

We first observe that the evolution of a proportion is proportional to the proportion itself, and also to the difference between the *fitness* of the strategy with the average fitness of the population, which are natural considerations if we want to simulate the evolution of strategies in an environment ruled by pure natural selection.
Concerning the natural properties we can expect from the equation, on the first hand, the sum of derivatives components is always equal to 0, a sufficient and necessary condition to keep the overall proportion to 1. On the other hand, the fact that the derivative vanishes when the associated proportion becomes 0 ensures that proportions remain positive or null.
Properties are indeed verified by the equation.

4. The Moralisator equation

In the same environment and theoretical population game setting as the one described for the Replicator dynamic, we aim to investigate the evolution of the strategies if the dynamic is ruled by moral-based decisions instead of selfish and personally motivated. In order to do so, we introduce the **Moralisator equation**, which gives the evolution through time of each component $i \in [|1,n|]$ by

$$\dot{x_i}(\underline{x}) = x_i(\mathbf{A}_{ii} - \sum_{j=1}^{n} x_j \mathbf{A}_{jj}) \quad (2)$$

The interpretation of the equation can be done as follows. The rise of a proportion i is proportional to the difference between the i-th common strategy and a reference payoff which is considered to be the weighted mean of common strategies, where the weight associated with each common strategy is equal to the proportion of players currently playing this strategy.
This new way of comparing the potential strategies, from the point of view of a moral player,

makes sense as a player would compare himself to the others, and takes as a moral reference the common output of the mixed strategy profile played in a coordinated way. In some ways, the moral reference is taken as the mean output of a player who knows every time the move of the other and plays like him, as in the 'Eye for an eye' spirit from the law of exact retaliation.
Moreover, the variation of a given strategy e_i is also proportional to the current proportion of players in this strategy x_i, i.e $\dot{x_i} \propto x_i$, as in the replicator equation. This term was indeed necessary in order to fulfill the required equation properties to keep proportions positive and the overall equal to 1, but it could be nicely interpreted as well. As mentioned in [5], this term was originally considered in order to represent reproduction within the populations, of different species of animal for instance. When we deal with strategy choices, the more this strategy is played, the quicker it would evolve, which represents the mimicking comportment within the population, which implies sadly a limit in our modelization.
Indeed, if at the beginning, or at some point, a strategy e_i has vanished then it cannot appear again even if it becomes later on advantageous to be played once again. Therewith, the human capacity to undertake and improve is neglected, which is somehow understandable as those characteristics are by nature stochastic and we are working in a deterministic framework. A stochastic approach might be a further step in order to integrate this aspect, through a Stochastic Differential Equation (SDE). For the moment, as the integration of that undertaken behavior is out of the scope of our studies, we have to be very attentive to the initial state of our system and not let proportions get too close to 0 as we would get unrealistic simulation behaviors.

4.1. Properties of the Replimorator equation

In the realm of full moral-based theory, the quantity $M(\underline{x}(t)) = \sum_{j=1}^{n} x_j(t)\mathbf{A}_{jj}$ can be seen as a moral reference of our system and can be seen as the counterpart of the average fitness given by $\sum_{j=1}^{n}\sum_{l=1}^{n} x_j x_l \mathbf{A}_{jl}$ in the case of the classical evolutionary game theory where the population evolves according to the Replicator equation. Moreover, it is a well-established result that this reference in the case of the Replicator is always growing through time [5]. We establish a similar result when the population evolves according to the Replimorator equation.

Theorem 4.1. *The quantity $M(\underline{x}(t)) = \sum_{j=1}^{n} x_j(t)A_{jj}$ is an increasing quantity through time when $t \to \underline{x}(t)$ follow a moralisator dynamic.*

Concerning the possible equilibriums, in the Replicator framework, those equilibriums are called **ESS**, standing for Evolutionary Stable Strategy, and are associated with mixed strategies **Nash equilibriums** of the associated two players' game. In the case of the Replimorator framework, we have a similar association, with simple **Kantian equilibrium** in this case, that correspond to the strategies associated to the maximum diagonal within the matrix $\mathbf{A}$.

Theorem 4.2. *Let $\mathbf{A}$ be the matrix of the game, where strategies are reordered in order to get a decreased diagonal. Denoting by $r \in [|1, n|]$ the last coefficient such that $\mathbf{A}_{11} = \mathbf{A}_{rr}$, we have that:*

$$\sum_{i=1}^{r} x_i(0) > 0 \implies \sum_{i=1}^{r} x_i(t) \xrightarrow{t\to\infty} 1$$

5. The k-Replimorator equation: the compromise

It is fairly known that in many aspects of life, "things" are not simply black or white but slightly more complex, like a scale of grey. Human interaction is part of those "things", whether it be in the economy, politics, geopolitics, or even at the base scale in business, family, or friendships. In all those fields, there are countless situations where an individual extra-benefit would be in opposition to a solution more suitable for the other parties. In those situations, the outcome in general never ends up in one of both extremes, but in a complex compromise that has been driven by a mix of selfish and moral forces.
Overall, in the symmetric games that are under our studies, players can deeply feel the reciprocity and have the sentiment of being part of the same boat. In this framework, the moral force should be as well considered in addition to the self-interest intrinsic driver. In order to set

what extent a player would be influenced by his morality, we denote by $k \in [0,1]$ the degree of morality, inspired by the Homo moralis utility function. Under those assumptions, proportions of strategies played in the game are then driven by the *k-Replimorator* equation, defined as the convex combination between the Replicator and Moralisator equation, which leads for each proportion x_i the following ODE:

$$\dot{x}_i(\underline{x}) = x_i \times k[\mathbf{A}_{ii} - \sum_{j=1}^{n} x_j \mathbf{A}_{jj}] + x_i \times (1-k)[\sum_{j=1}^{n} x_j \mathbf{A}_{ij} - \sum_{j=1}^{n}\sum_{l=1}^{n} x_j x_l \mathbf{A}_{jl}] \quad (3)$$

As the Replicator and the Moralisator equation did fulfill the basic requirement of such an equation, it directly follows that the convex combination of any kind will fulfill them as well. In fact, for any linear combinations, the requirements would have to stand, which might open the way to the integration of other behavior-driving forces. The new equation would enable us to study a wide range of population behavior according to its degree of morality. The degree of morality is kept constant in the equation to keep things as simple as possible, even though making it time and state path depend would be possible in order to make it more refined. We are now going to simulate numerically the k-Replimorator equation in different classical games.

5.1. The prisoner dilemma

The first game we would like to study is the prisoner dilemma. Indeed, despite the fact that it is the most famous game of the discipline, some very interesting behaviors toward the Replimorator equation can be expected. In fact, this game has a Kantian equilibrium as well as a Nash equilibrium that corresponds to the two different possible common strategies. The Kantian equilibrium corresponds to both convicts collaborating with each other and denying (D), whereas the Nash equilibrium corresponds to the situation where they both confess (C).

		P_2	
		$Confess$	$Deny$
P_1	$Confess$	-10	0
	$Deny$	-15	-1

Figure 1: **A** matrix in the prisoners dilemma setting

According to the theory, if the population converges, it would converge to the Nash equilibrium if it evolves according to the replicator equation, and to the SKE in the case of the Moralisator equation. Therefore, in the case of the k-replimorator equation, we can expect a different comportment according to the morality coefficient k, with an increasing incentive to collaborate between prisoners as k increases.

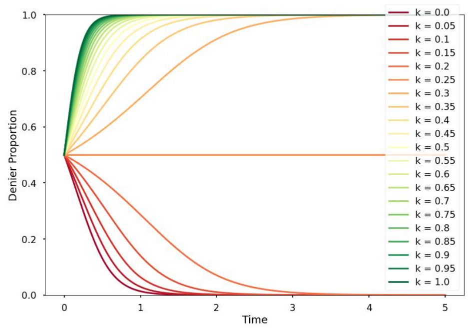


Figure 2: Simulation in prisoners dilemma for different values of k

We observed the rise of a critical value of k, denoted by k_{tip}. Indeed, if $k > k_{tip}$ then the population converges to the Kantian equilibrium and collaborates, and if $k < k_{tip}$ then it converges to the Nash equilibrium. The value of k_{tip} is also very dependent on the initial situation, as if more people collaborate at the beginning, then less morality would be necessary in order to achieve full cooperation.

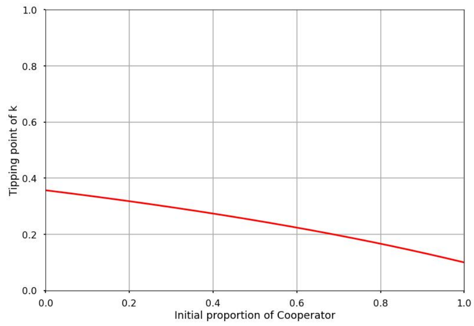


Figure 3: k_{tip} w.r.t the initial proportion of co-operator

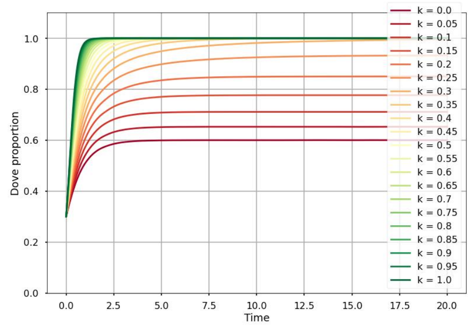
5.2. The hawk and dove game

The Hawk and Dove is another famous and well-studied game. It represent a population of birds where one comportment is aggressive and denoted by Hawk, whereas the other one is docile and characterized by Dove.
Within this population, it often happens that birds find a rival resource that would bring them the payoff V. If they are both doves, they will share the resource and get the payoff $V/2$, but a hawk against a dove would take all the resources. Finally, if two hawks meet in those circumstances, they will escalate the conflict and fight which would cost $C > V$ to the loser. The average payoff, in that final case, is then equal to $(V - C)/2 < 0$. The game is summarized in Figure 4 and the matrix $\mathbf{A}$ is deduced from it by taking the payoff matrix of the first bird.

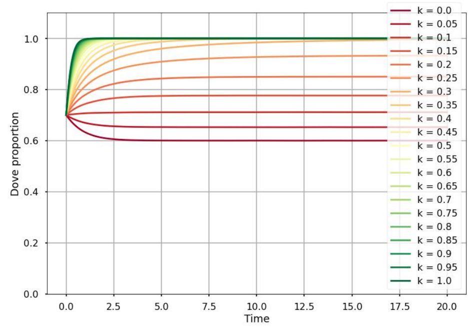
		Bird 2	
		Hawk	*Dove*
Bird 1	*Hawk*	$(\frac{V-C}{2}, \frac{V-C}{2})$	$(V, 0)$
	Dove	$(0, V)$	$(\frac{V}{2}, \frac{V}{2})$

Figure 4: The general hawk and dove game

Moreover, it can be observed that neither the Hawk nor the Dove strategy is Nash equilibrium. Therefore, even if the population composed exclusively of Dove is most optimal in terms of common payoff, as no resources are lost during conflicts, this equilibrium is not reached in a classical evolutionary setting where the Nash equilibrium corresponds to a proportion of hawk of V/C.
We have performed the following simulation with the values $V = 4$ and $C = 10$, and started them for two different proportion settings of $(30\%, 70\%)$ and $(70\%, 30\%)$, we can observe them in Figure 5. We observe, in this game, a convergence of the strategy proportions after a short time for each value of the morality coefficient k. More precisely, the game converges to the Nash equilibrium when $k = 0$, which corresponds to a proportion of $\frac{C-V}{C}$ of doves, and as long as k increases the proportion of dove corresponding to the equilibrium increase as well. We then observe a value critic of k, between 0.25 and 0.3 for the values of V and C previously selected, where the equilibrium is exclusively composed of Doves for values of k higher.



(a) Starting proportion of Dove = 0.3



(b) Starting proportion of Dove = 0.7

Figure 5: Hawk and Dove simulations throw time and according to k

In particular, we empirically observe that the equilibrium reached is independent of the initial proportion of birds, except for the degenerated case where only one kind of bird is initially present.
As we previously discussed, the higher the proportion of Dove, the higher the average payoff within the population, as fewer resources are lost in the fights. As an increasing moral co-

efficient k induces a higher proportion of doves at the equilibrium, we can conclude that an increase in morality would imply an increase in the common well-being of the population at the equilibrium. This is a common feature with the previous game studied, except that in that game, the behavior is achieved in a continuous way until the k_{tip} is reached. Whereas, in the prisoner dilemma the switch of behavior is binary, in the sense that it switches from one extreme equilibrium to the other whether k is under or above k_{tip}.

5.3. The stag and hare hunters

The stag and hare hunters game is another game theory classic, composed of two coordinated Nash equilibriums of different qualities. The common best one, but more risky in case of miss-coordination, is to go hunting the stag, and the other one to go both hunting the hare.

		Hunter 2	
		Stag	*Hare*
Hunter 1	*Stag*	$(1, 1)$	$(-1, 0.5)$
	Hare	$(0.5, -1)$	$(0, 0)$

Figure 6: The stag and hare game

In Figure 7, we have reported the simulation of the game for different initial conditions and different coefficients of morality k. We observe that in the long run, the game ends up either in a *stag hunter* or in an *hare hunter* society.
We also observe that whatever the initial proportion in the game, there exists a morality coefficient k_{tip} that enables the game to converge to the cooperative equilibrium where players are hunting the stag. This is a similar feature that we have observed in the prisoner dilemma, except that in the case the k_{tip} is rather different as we can observe in Figure 8. Indeed, there exists a threshold of cooperator proportion where above, even a morality coefficient $k = 0$ makes the game converge to a fully cooperating population, which is not the case in the prisoner dilemma.

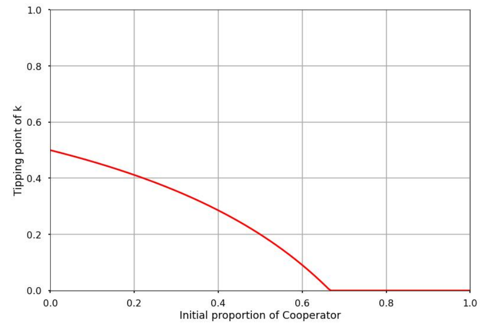
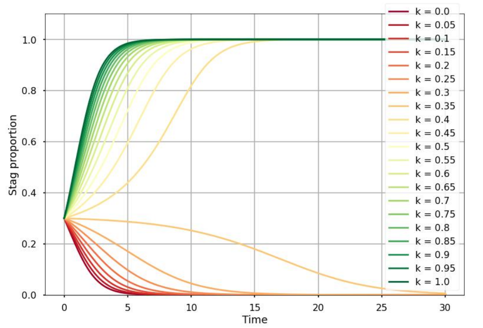
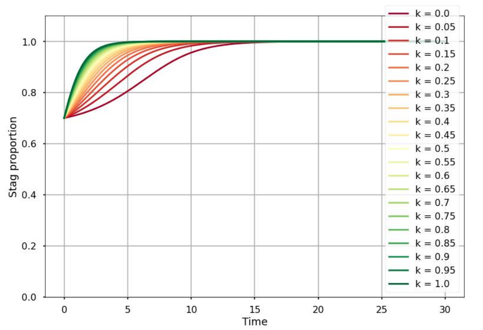


Figure 8: Value of k_{tip} in the Stag and Hare game according to the initial proportion of Hare hunters



(a) Initial proportion of stag hunter = 0.3



(b) Initial proportion of stag hunter = 0.7

Figure 7: Stag and Hare simulations throw time and according to k

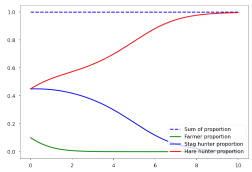
5.4. A 3 strategies coordination game

In the continuity of the stag and hare game, we consider a game with 3 Nash equilibrium, with growing quality in the sense of Pareto, all localized on the diagonal. The payoff matrix of the game, that we named "cow, stag and hare", used for the simulations is represented by 9.
It is, as the stag and hare hunters a coordination

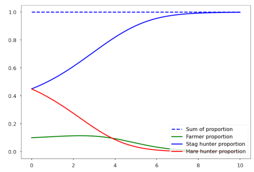
		Player 2		
		Cow	Stag	Hare
Player 1	Cow	$(10,10)$	$(3,6)$	$(0,8)$
	Stag	$(6,3)$	$(5,5)$	$(1,4)$
	Hare	$(8,0)$	$(4,1)$	$(2,2)$

Figure 9: The cow, stag and hare payoff matrix

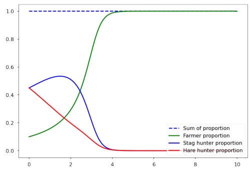
game and we aim to investigate the influence of the morality coefficient on the capacity of the players in order to coordinate themselves.



(a) morality coefficient $k = 0$



(b) morality coefficient $k = 0.15$



(c) morality coefficient $k = 0.2$

Figure 10: Cow, Stag and Hare simulations with initial proportions $(0.1, 0.45, 0.45)$

As we can expect, the higher the morality coefficient k, the better the capacity of the players to coordinate on a higher-quality equilibrium. As justified by the simulation summarised in Figure 10, which are performed with the same initial proportions but with increasing k. In all cases, the system ends up in one of the three possible Nash equilibriums with a bigger incentive for cooperation as long as k increases.

6. Conclusions

In conclusion, the exploration of Kantian game theory within the evolutionary framework marks a distinctive and innovative avenue in the field of game theory. The incorporation of moral considerations through the Moralisator Equation, as introduced by Ingela Alger and Jörgen W. Weibull [1], represents a departure from traditional game theory paradigms. The study of how Kantian optimizers navigate strategic interactions sheds light on the pivotal role morality plays in shaping evolutionary dynamics.

However, this is just the beginning of a fascinating journey. Kantian game theory, being a relatively new theory, presents an open field for theoretical exploration and practical application. The Moralisator Equation provides a foundation, but there remains much to discover both theoretically and empirically.

Moreover, while morality is a central focus, the framework is amenable to expansion. Future research can delve into the incorporation of additional incentives, such as rivalry and altruism, broadening the scope of the evolutionary game theory lens.

In essence, as we venture into this novel intersection of non-Nashian optimization and evolutionary game theory, we find a realm ripe for exploration and discovery. The fusion of morality and strategic interactions provides a platform for a deeper understanding of the dynamics that govern the evolution of cooperation and competition within populations. The journey ahead promises not only theoretical advancements but also practical applications that can contribute to our comprehension of biological, social, and economic systems.

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