POLITECNICO MILANO 1863

SCUOLA DI INGEGNERIA INDUSTRIALE E DELL'INFORMAZIONE

Executive Summary of the Thesis

Iterative model-based approach for the estimation of integral length scales from experimental data

Laurea Magistrale in Mechanical Engineering - Ingegneria Meccanica

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Academic year: 2025-2026

1. Introduction

Modern turbomachinery design relies heavily on computational fluid dynamics (CFD) for aerodynamic performance prediction and optimization. Across modeling approaches ranging from Reynolds Averaged Navier Stokes (RANS) to higher-fidelity methods such as Large Eddy Simulation (LES) and Direct Numerical Simulation (DNS), the accurate representation of turbulence and laminar-to-turbulent transition remains a significant challenge. As highlighted by Denton [3], turbulence modeling constitutes a major source of uncertainty. In particular, RANS solvers show strong sensitivity to turbulence and transition model selection and often struggle to accurately capture boundary-layer development and wake formation [9].

These limitations are especially critical in low-pressure turbines (LPTs), where transition and separation phenomena dominate the loss generation mechanisms. Free-stream turbulence intensity and integral length scales strongly influence the boundary-layer state and associated loss mechanisms [6]. Their accurate characterization is therefore essential both for improving CFD predictions and for providing reliable validation data and inflow conditions for higher-fidelity simulations.

However, turbulence measurements are inherently affected by significant uncertainty [2], particularly in the estimation of integral length scales, which are highly sensitive to data processing methodology. Inaccurate length scale determination can compromise numerical validation and the interpretation of transition mechanisms.

Within this framework, the present thesis aims to develop a robust methodology for estimating integral length scales from grid generated turbulence data obtained in a low-speed turbine cascade at the von Karman Institute (VKI). An alternative iterative approach is proposed to overcome the limitations of conventional autocorrelation- and power-spectrum-based methods, enabling more reliable turbulence characterization for future experimental and numerical investigations.

2. Research motivation

The integral length scale represents the characteristic size of the largest energy containing turbulent structures. For the streamwise velocity component, it is formally defined from the longitudinal autocorrelation function as

$$\Lambda_x = U \int_0^\infty \rho(\tau)\, d\tau \tag{1}$$

where U is the mean flow velocity and $\rho(\tau)$ is the normalized temporal autocorrelation coefficient,

$$\rho(\tau) = \frac{\langle u'(t)u'(t+\tau)\rangle}{\langle u'^2\rangle}. \tag{2}$$

Its practical estimation from experimental data is inherently problematic since the integration is performed over a finite time window making the result sensitive to the choice of the upper integration limit. At large lags, the autocorrelation tail is often contaminated by statistical noise rather than physical correlation, leading to biased estimates if fully integrated. Truncation at the first zero-crossing improves robustness, yet this criterion is frequently ill-defined in experimental measurements due to finite acquisition time and low-frequency leakage. Using the spectral equivalence, the integral length scale can instead be expressed as

$$\Lambda_x = \left[\frac{E_{11}(f)\, U}{4\langle u^2\rangle}\right]_{f\to 0} \tag{3}$$

where $E_{11}(f)$ is the longitudinal energy spectrum. Spectral formulations exhibit improved convergence properties [5], but shift the uncertainty to the estimation of the zero-frequency limit, which depends on arbitrary low-frequency averaging choices [1]. These limitations motivate the development of a more robust spectral-based methodology that reduces sensitivity to subjective parameter selection while ensuring stable and reliable integral length scale estimation.

3. Experimental Setup

The experimental data analyzed in this work were acquired in the C-1 Low-Speed Cascade Wind Tunnel at the von Kármán Institute for Fluid Dynamics. The facility is an open circuit, continuous flow wind tunnel designed for detailed turbomachinery investigations. The present campaign was conducted on a low-speed rescaled version of the SPLEEN C1 low-pressure turbine cascade.
Free-stream turbulence was generated upstream of the cascade using jet type turbulence grids previously tested by Bertelli et al. [1], mounted normal to the incoming flow. The grids were operated both in passive and active modes, allowing independent variation of turbulence intensity and integral length scale while maintaining good flow homogeneity. The measurements considered in this study focus on grid-generated turbulence upstream of the cascade leading edge, where the flow is measured to be statistically stationary and nearly isotropic.

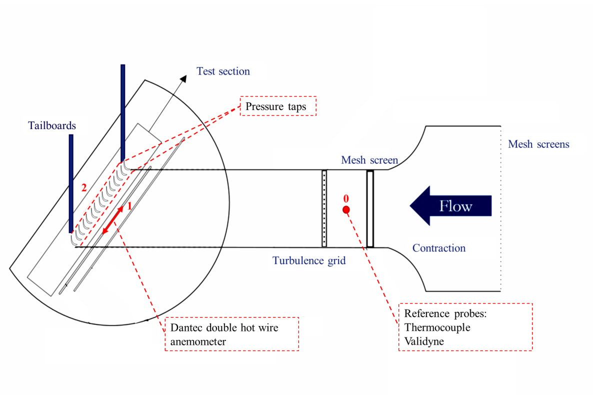

Figure 1: Schematization of instrumentation setup.

Velocity fluctuations were measured using a Dantec Dynamics 55P51 cross-wire hot-wire probe operated in constant-temperature anemometry (CTA) mode. The probe was positioned approximately 1.4 axial chords upstream of the cascade leading edge. Signals were sampled at 100 kHz for a duration of 20 s, ensuring adequate resolution of both inertial and large-scale motions and reducing statistical uncertainty in low-frequency spectral content.

4. Proposed method

To eliminate arbitrary parameter choices in spectral integral length scale estimation, an automated optimization-based framework is developed. The pseudo-algorithm is presented in Algorithm 1. The method treats the integral length scale as an unknown parameter and determines it by minimizing the discrepancy between the experimental energy spectrum and a theoretical turbulence spectrum. The objective function is formulated as a logarithmic least-squares error integrated over a prescribed frequency range, ensuring equal weighting across frequency decades and preventing bias toward high-frequency regions.

Algorithm 1 Optimization-Based Integral Length Scale Estimation

Input: Flow conditions, $[\Lambda_{\min}, \Lambda_{\max}]$, $[f_{\min}, f_{\max}]$
Initialize $\Lambda_x \in [\Lambda_{\min}, \Lambda_{\max}]$
while convergence not satisfied **do**
    Compute theoretical spectrum $E_{11}(f; \Lambda_x)$
    Compute objective function:
    $err(\Lambda_x) = \int_{f_{\min}}^{f_{\max}} [\log E_{\exp}(f) - \log E_{11}(f; \Lambda_x)]^2 \, d(\log f)$
    Update Λ_x using 1D bounded minimization
end while
Output: Optimal Λ_x

The theoretical spectrum adopted in the optimization framework is based on the von Kármán model for homogeneous isotropic turbulence. Starting from its three-dimensional formulation, the spectrum is reduced to the one-dimensional longitudinal form and expressed in the frequency domain using Taylor's frozen turbulence hypothesis, $\kappa_1 = 2\pi f/U$, enabling direct comparison with hot-wire measurements [4]. The resulting frequency-domain expression reads

$$E_{11}(f) = \frac{4\langle u'^2\rangle \Lambda_x}{U} \left[1 + \left(\frac{8\pi f \Lambda_x}{3U}\right)^2\right]^{-5/6}$$

where $\langle u'^2\rangle$ is the variance of the streamwise velocity fluctuations, U is the mean velocity, and Λ_x is the streamwise integral length scale. This formulation explicitly links the spectral shape to Λ_x, which is treated as the unknown parameter in the optimization procedure.
The Pope parametric spectrum model was also implemented [7], to verify the robustness of the optimization framework independently of the specific spectral model adopted.
Experimental spectra are obtained via hot-wire velocity measurements and computed using Welch's method, which provides statistically consistent variance reduction and improved robustness for fitting.
The minimization is performed using both a bounded one-dimensional optimizer and a complementary grid-search approach to ensure convergence toward the global minimum. Sensitivity analyses are conducted to define the appropriate fitting frequency range and ensure adequate low-frequency convergence. This framework provides a systematic, reproducible, and robust spectral-based methodology for integral length scale estimation. An example of the resulting fit is shown in Fig. 2.

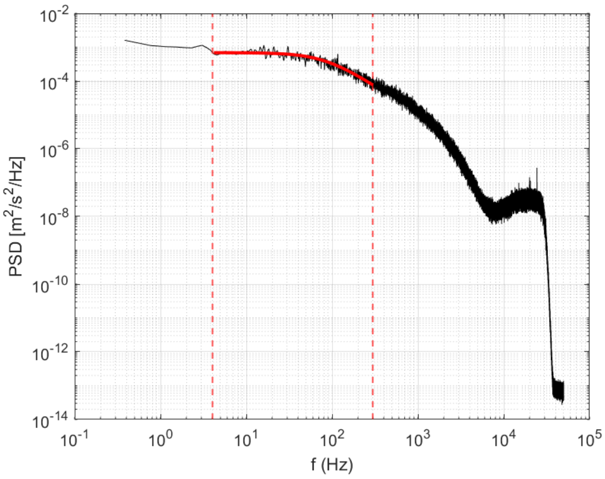

Figure 2: Experimental Power spectral density and fitted von Kármán spectrum with dashed fitting limits

4.1. Fitting limits

The sensitivity analysis of the fitting limits reveals three distinct regions in the $(f_{\min}, f_{\max})$ parameter space (Figure 3). At very low $f_{\min}$, the estimated Λ_x is artificially inflated due to spectral leakage and low-frequency contamination, which bias the minimization toward non-physical energy contributions. Conversely, when $f_{\max}$ extends toward higher frequencies approaching the dissipation range, progressively larger portions of irrelevant high-frequency content are included, leading to a mild but systematic underestimation of the integral scale. This analysis is conducted on a specific reference test case, as the objective is to concretely illustrate how the proposed method is implemented and to demonstrate the associated workflow. Through this example, the robustness and internal consistency of the methodology are assessed.
Between these extremes, an intermediate plateau is identified where Λ_x exhibits minimal sensitivity to variations in both fitting limits. In this physically consistent domain, corresponding to approximately $f_{\min} \in [3, 11]$ Hz and $f_{\max} \in [80, 300]$ Hz, the mean estimated value is $\Lambda_x = 15.01$ and the P5–P95 variability is approximately ± 0.28 mm. The presence of this plateau indicates structural stability. In contrast, applying an analogous sensitivity analysis

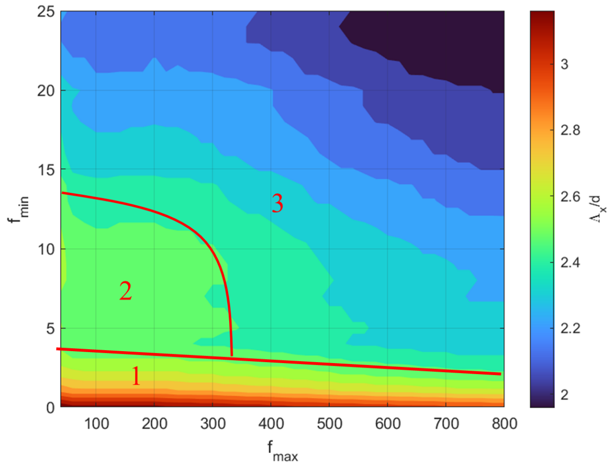

Figure 3: Sensitivity of the estimated integral length scale Λ_x/d ($d = 6$ mm, grid bar diameter) to the fitting limits $[f_{min} f_{max}]$.

to the conventional zero-frequency extrapolation method (Eq. 3) would reveal no such plateau: objective selection of the averaging bounds is not possible therefore any single choice remains subjective.

This confirms that, once the fitting interval are chosen reasonably the optimization provides stable and nearly invariant integral length scale estimates. Within this domain, the proposed methodology yields results consistent with conventional approaches while exhibiting significantly reduced variability.

4.2. Uncertainty analysis

The choice of spectral fitting interval, if not properly selected, represents the dominant source of variability. However, this work provides physically based criteria, via identification of the energy-containing and inertial subranges, to define it consistently. Once fixed, the remaining uncertainty contributions can be quantified. Measurement uncertainty, associated with instrumentation and calibration,is about ± 0.6 mm. Methodological sensitivity, evaluated as the P5–P95 dispersion under reasonable fitting variations, was at most ± 0.3 mm and represents a procedural sensitivity band rather than a strict metrological uncertainty. Statistical uncertainty, arising from the finite duration of the velocity time series [2], was minimized using 20 s recordings at 100 kHz, ensuring adequate convergence and spectral resolution within practical test durations.

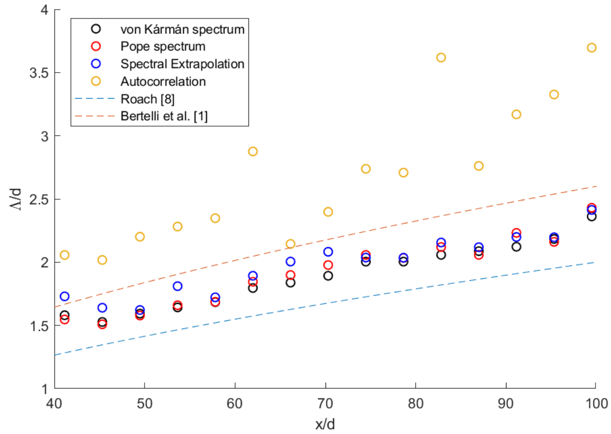

Figure 4: Downstream evolution of the normalized integral length scale Λ_x/d as a function of the normalized distance x/d for grid A (grid bar diameter, d = 6 mm). Results obtained using different estimation approaches are compared with the empirical correlations proposed by Bertelli et al. [1] and Roach [8].

5. Main results

The integral length scale was experimentally determined using the proposed methodology across multiple test cases and compared with conventional formulations to evaluate consistency and physical reliability.

First, the method was assessed on the reference test case and, afterwards, it was applied to independent configurations, including different turbulence grid arrangements and off-design operating conditions. The purpose was to examine its generality and robustness under flow conditions distinct from those used for its development.

Results demonstrate that the proposed iterative spectral methodology provides a robust, repeatable, and physically consistent estimation of the integral length scale across a wide range of turbulence conditions.

For the reference test case (grid A, passive mode, see Fig. 4), autocorrelation-based estimates systematically overpredict Λ_x due to sensitivity to low-frequency content. In contrast, the spectral-based approaches accurately reproduce both the magnitude and decay behavior of Λ_x/d, showing good agreement with established correlations. The iterative implementations based on the von Kármán and Pope spectra yield mutually consistent results, confirming that the methodology is robust with respect to the selected spectral

model. Repeatability analysis conducted considering two tests under identical operating conditions highlights the superior stability of spectral methods compared to the autocorrelation approach. The von Kármán iterative method exhibits the lowest coefficient of variation (4.24%), improving repeatability relative to spectral extrapolation (5.12%).
When applied to alternative grid geometries and to off-design or active-grid configurations, the procedure consistently identifies a stable fitting region in the $(f_{\min}, f_{\max})$ space, enabling an objective selection of fitting limits and a controlled quantification of methodological uncertainty. Across all configurations, the turbulence remained nearly isotropic (isotropy ratio between 1.04 and 1.20), supporting the validity of the spectral framework. Overall, the results confirm that the proposed iterative spectral approach provides a general, stable, and less subjective alternative to classical autocorrelation- and extrapolation-based methods for the estimation of integral length scales in grid-generated turbulence.

6. Conclusions

This work developed and validated an iterative model-based methodology for the experimental estimation of the integral length scale, a key parameter for turbulence characterization in turbomachinery research and CFD validation.
The proposed approach can be interpreted as a structured zero-frequency spectral extrapolation grounded in analytical models (von Kármán and Pope). The method provides a more stable low-frequency spectrum estimation than plain averaging, reducing the arbitrariness of conventional procedures and enabling a controlled and repeatable evaluation of the integral length scale.
It was, moreover, demonstrated that once physically consistent fitting bounds are identified from the energy-containing and inertial subranges, the procedure can be applied systematically across operating conditions without the need for case-by-case retuning, except when significant changes in Reynolds number alter the spectral structure.
The applicability of the method is inherently linked to the validity of Kolmogorov's similarity hypotheses and therefore to the presence of approximately homogeneous, isotropic, and sufficiently high-Reynolds-number turbulence, ensuring the presence of a well-defined inertial subrange.
Overall, the developed iterative framework enhances the physical consistency and practical reliability of integral length scale estimation and establishes a structured and transferable guideline for turbulence characterization in future experimental campaigns.

7. Acknowledgements

The author thankfully acknowledges prof. Paolo Gaetani, prof. Fabrizio Fontaneto, dr. Elissavet Boufidi and Giacomo Pastorino for their significant contribution to this work.

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