POLITECNICO MILANO 1863

SCUOLA DI INGEGNERIA INDUSTRIALE
E DELL'INFORMAZIONE

Executive Summary of the Thesis

Distributed Networked Predictive Control with application to a laboratory experimental setup

Laurea Magistrale in Automation and Control Engineering - Ingegneria dell'Automazione

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Academic year: 2020-21

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1. Introduction

In the last decades infrastructures and engineering systems have grown widely, both in terms of complexity and geographical dispersion. Modern plants can be often regarded as "networks" of a (often large) number of heterogeneous interacting units, that require the adoption of hierarchical, distributed and decentralized control architectures.

Information between subsystems and controllers is often shared through a communication network. This leads to regard the problems of developing distributed control schemes and to cope with network-induced communication issues as two strictly coupled ones end to the concept of distributed and networked control system.

In this thesis work we will first implement and test a state-of-the-art model Predictive Control-based Distributed algorithm [1], named DPC. The tests will be carried out on a laboratory experimental setup devised specifically for this work, consisting of unicycle mobile robots virtually connected through springs and dampers, moving on a bi-dimensional plane. The realized setup will be characterized by non-idealities typical of networked control systems, like delays and disturbances. These phenomena will be characterized by analyzing the used laboratory equipment, i.e., the E-PUCK mobile robots and the Decawave UWB positioning system. Moreover, a specific calibration strategy for the compensation of the biases affecting the position measurement will be presented.

These undesired phenomena typically deteriorate the performances of control algorithms designed specifically for nominal systems. To overcome this problem, we will present a robust networked variant of the DPC, capable to deal with the delays and the disturbances affecting the experimental setup.

Finally the novel, robust networked DPC algorithm will be tested, and its effectiveness will be discussed.

2. Experimental setup and characterization of delays and disturbances

The devised lightweight experimental setup consists of two E-PUCK mobile robots moving inside an arena of 2x2 meters. The positions of the robots are obtained using the Decawave Real Time Localization System (DRTLS) relying on the Ultra-Wide-Band (UWB) technology. The

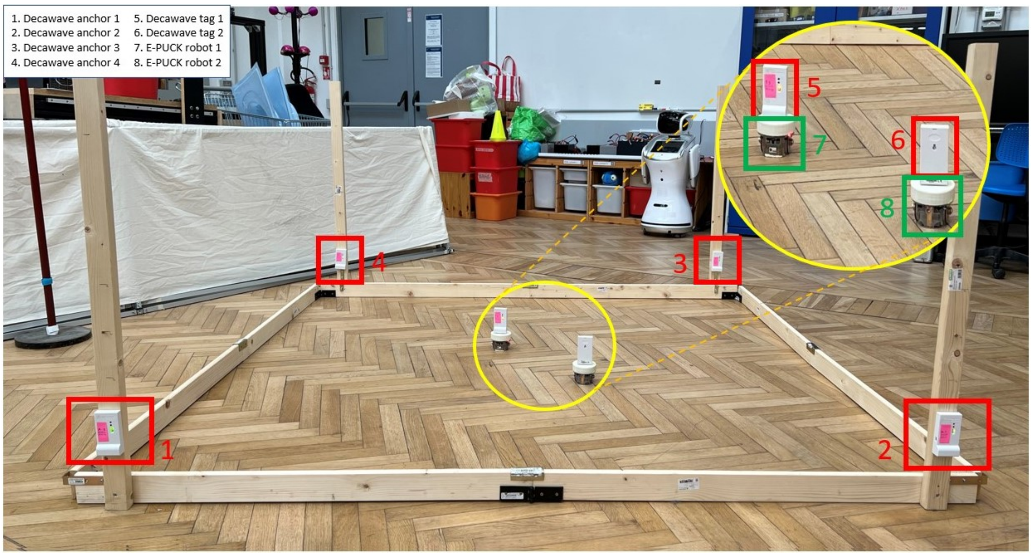
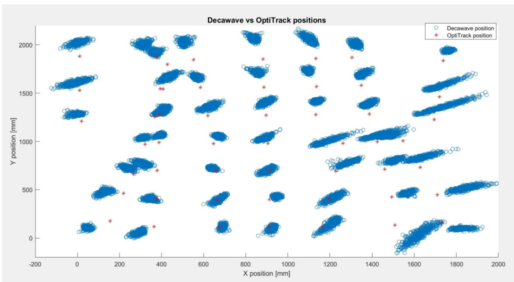


Figure 1: Experimental setup.

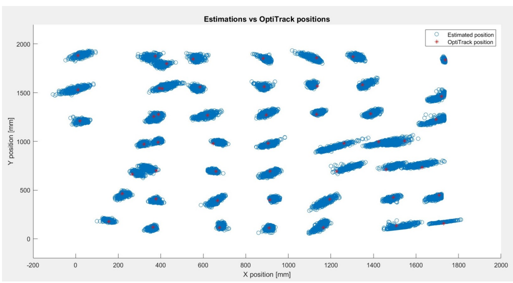
selected devices introduce disturbances and delays. The latter are mainly related to the positioning sensors, while the ones related to Bluetooth transmission, processing, and actuation are negligible. Such setup can be used as a versatile benchmark case study for development and testing of networked control strategies for multi-agent systems.

2.1. Decawave real time localization system characterization

The selected positioning system consists of six DWM1001-DEV UWB modules, four configured as anchors placed at the corners of the arena, and two configured as tags placed on the robots. The position of each tag is computed by the location engine unit equipped on the sensors, solving a trilateration problem. Location measurements are retrieved by the controllers using a Bluetooth radio link.
Several tests determined that the DRTLS is the main source of delays in the setup. Their characterization has been retrieved comparing data with the ones provided by the three axis accelerometer MMA7260QT equipped on the E-PUCK robots. Results show that the UWB sensors introduce a delay of 206.2 ms, with a standard deviation of 41.4 ms.
Preliminary tests and past works [2] show that the location data are affected by significant noise and biases, strongly influenced by multipath and interference in the environment.
These errors influence the disturbance affecting the position measurements, that has been characterized using a high precision, real time tracking system (OptiTrack). Therefore, a suitable calibration strategy based on the polynomial regression technique has been developed to provide better performances, as can be seen from Figure 2. The calibrated DRTLS introduce a zero mean disturbance v_k with a standard deviation of 29 mm along the X axis and 24 mm along the Y axis.



(a) Original data



(b) Manipulated data

Figure 2: Calibration strategy.

2.2. E-PUCK robot modelling and noise characterization

The E-PUCK robots used in this work are developed at the Swiss Federal Institute of Technology in Lausanne (EPFL) and commercially distributed by GCtronic. They are differential drive (i.e., unicycle) vehicles, where the wheels are actuated by two independent stepper motors providing a maximum linear speed of 80 mm/s. They are endowed with communication capabilities. In particular, the Bluetooth connectivity is used.

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{\theta} = \omega \end{cases} \tag{1}$$

The kinematic model of the unicycle robot, described in (1), is non linear. Since these nonlinearities significantly affect the dynamics of the system, it is important to find a way to linearize the model without losing information. This is particularly important because predictive control optimization problems can be formulated in a simpler way provided that the model is linear. Therefore, we decided to apply the feedback linearization technique. This approach, by using an internal loop, is capable to reduce the system to a model represented by a set of simple linear integrators.

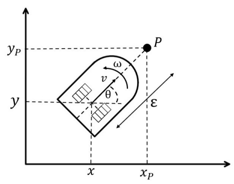


Figure 3: Feedback linearization model.

The applied feedback linearization is non-singular and relies on the choice of a point P along the centre line of the robot chassis as shown in Figure 3, located at arbitrary distance ε from the axle connecting the wheels.
This technique allows to obtain, from an external point-of-view, a linear mathematical model, more suitable to develop a MPC-based regulator, avoiding to resort to the non-linear theory.

$$\begin{cases} \dot{x}_P = v_{xP} \\ \dot{y}_P = v_{yP} \end{cases}$$

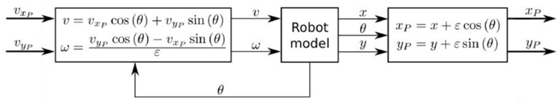


Figure 4: Feedback linearization scheme.

The disturbance w_k acting on the unicycle robot model has been characterized using the OptiTrack system. This disturbance is influenced by errors related to the wheel sliding phenomena and errors coming from the missing use of a dynamical model, e.g., the electro-mechanical interactions, friction, and other unmodeled dynamics. Several tests lead to the characterization of a zero mean disturbance w_k with a standard deviation of 0.65 mm along both the axes. Consequently, the E-PUCK robot state-space model considering the X,Y coordinates of point P as state, which are measured using the DRTLS is

$$\begin{cases} x_{epuck,k+1} = A_{epuck}x_{epuck,k} + B_{epuck}u_{epuck,k} + w_k \\ x_{epuck,k} = C_{epuck}x_{epuck,k} + v_k \end{cases} \tag{2}$$

2.3. The case study

In this thesis work we want to test and develop a distributed algorithm applicable to a system composed of different subsystems, physically coupled between each others. In particular, the setup defined in this thesis will be able to behave as a system, composed of two carts, moving in the 2D plane and connected through a spring and a damper. The linearized discrete model of the i-th subsystem considers carts positions and velocities as state $\boldsymbol{x}^{[i]} = \begin{bmatrix}\delta x_i & \delta\dot{x}_i & \delta y_i & \delta\dot{y}_i\end{bmatrix}^T$ and applied forces as input $\boldsymbol{u}^{[i]} = \begin{bmatrix}F_{i,x} & F_{i,y}\end{bmatrix}^T$, i.e.,

$$\boldsymbol{x}_{k+1}^{[i]} = A_{ii}\boldsymbol{x}_k^{[i]} + B_{ii}\boldsymbol{u}_k^{[i]} + \sum_{j\in\mathcal{N}_i} A_{ij}\boldsymbol{x}_k^{[j]} \tag{3}$$

where $\mathcal{N}_i$ denotes the set containing the dynamic neighbors of the i-th subsystem.

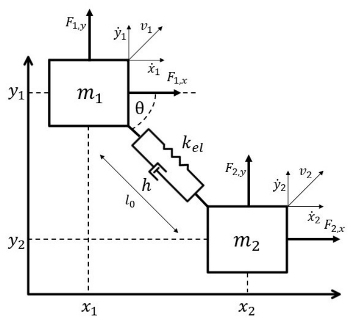


Figure 5: 2D Carts-Spring-Damper system.

The carts will be represented by the unicycle robotic agents, and the connections between them are virtually imposed by software.

3. Robust networked DPC algorithm

The DPC algorithm [1] has been designed specifically for nominal systems. Therefore the presence of network-induced non-idealities typically deteriorate the regulation performances. For this reason, a modified version of DPC is developed, capable to deal with delays and disturbances affecting the experimental setup, and to provide global stability together with satisfactory performances.
The presence of the delay leads to the selection of a sampling time $\tau = 0.2$ s. This entails that the measurement of the E-PUCK robot actual position obtained at time instant k, arrives at the controller with a 1-step delay, at time instant $k+1$. The estimation $\bar{\boldsymbol{x}}_k^{[i]}$ of the current state of the i-th subsystem is obtained using a one-step ahead predictor to retrieve the current positions, while the velocities are available since they correspond to the inputs of the feedback-linearized robots.

$$\bar{\boldsymbol{x}}^{[i]} = \begin{bmatrix} x_{Pi}^{predicted} & v_{xPi} & y_{Pi}^{predicted} & v_{yPi} \end{bmatrix}^T$$

The prediction error e_k is equal to

$$e_k = \boldsymbol{x}_k^{[i]} - \bar{\boldsymbol{x}}_k^{[i]} = P(w_{k-1}^{[i]} - A_{epuck} v_{k-1}^{[i]})$$

where $P \in \mathbb{R}^{4\times 2}$ is a selection matrix associated just to the positions.
Under the assumption that the "virtual connections" between subsystems are defined in an ideal way, we rewrite the model equations (3) as follows

$$\boldsymbol{x}_{k+1}^{[i]} = A_{ii}\boldsymbol{x}_k^{[i]} + B_{ii}\boldsymbol{u}_k^{[i]} + \sum_{j\in\mathcal{N}_i} A_{ij}\boldsymbol{x}_k^{[j]} + P\boldsymbol{w}_k^{[i]} \quad (4)$$

It is possible to define, for each subsystem, the sets $\mathbb{X}_i$ and $\mathbb{U}_i$, as convex neighborhoods of the origin, where the i-th state and input vectors are confined. $\boldsymbol{x}_k^{[i]} \in \mathbb{X}_i \subseteq \mathbb{R}^4$ and $\boldsymbol{u}_k^{[i]} \in \mathbb{U}_i \subseteq \mathbb{R}^2$. This algorithm relies on the fulfillment of the assumption on decentralized stability.

Assumption 1. There exists a block-diagonal matrix $\mathbf{K}^{aux} = \begin{bmatrix} K_1^{aux} & 0 \\ 0 & K_2^{aux} \end{bmatrix}$, with $K_i^{aux} \in \mathbb{R}^{2\times 4}, i = 1,2$ such that:
(i) $\mathbf{F} = \mathbf{A} + \mathbf{B}\mathbf{K}^{aux}$ is Schur.
(ii) $F_{ii} = (A_{ii} + B_{ii}K_i^{aux})$ is Schur, $i = 1,2$.
Where $\mathbf{A} \in \mathbb{R}^{8\times 8}$ and $\mathbf{B} \in \mathbb{R}^{8\times 4}$ are the state space matrices of the complete system.

In order to design a distributed control scheme, assume that at any time instant k, each subsystem transmits to its neighbor its future state and input reference trajectories $\tilde{\boldsymbol{x}}_{k+h}^{[i]}$ and $\tilde{\boldsymbol{u}}_{k+h}^{[i]}$, $h = 0,1,...,N-1$ respectively, predicted over the whole prediction horizon N. In this way, the DPC guarantees that the actual i-th subsystem trajectory lies in a specified time-invariant neighborhood of the reference one, i.e. $\boldsymbol{x}_k^{[i]} \in \tilde{\boldsymbol{x}}_k^{[i]} \oplus \mathcal{E}_i$ and $\boldsymbol{u}_k^{[i]} \in \tilde{\boldsymbol{u}}_k^{[i]} \oplus \mathcal{U}_i$, where $0 \in \mathcal{E}_i$ and $0 \in \mathcal{U}_i$.
In this way Equation (4) can be written as

$$\begin{aligned}\boldsymbol{x}_{k+1}^{[i]} =& A_{ii}\boldsymbol{x}_k^{[i]} + B_{ii}\boldsymbol{u}_k^{[i]} + \sum_{j\in\mathcal{N}_i} A_{ij}\tilde{\boldsymbol{x}}_k^{[j]} + P\boldsymbol{w}_k^{[i]} \\ &+ \sum_{j\in\mathcal{N}_i}(A_{ij}(\boldsymbol{x}_k^{[j]} - \tilde{\boldsymbol{x}}_k^{[j]}))\end{aligned} \quad (5)$$

According to [3], the control law generated using the available datum $\bar{\boldsymbol{x}}_k^{[i]}$ is defined as

$$\boldsymbol{u}_k^{[i]} = \hat{\boldsymbol{u}}_k^{[i]} + K_i^{aux}(\bar{\boldsymbol{x}}_k^{[i]} - \hat{\boldsymbol{x}}_k^{[i]}) \quad (6)$$

where K_i^{aux} satisfies Assumption 1, and the terms $\hat{\boldsymbol{u}}_k^{[i]}$ and $\hat{\boldsymbol{x}}_k^{[i]}$ are the input and the state, respectively, of the nominal model associated with Equation (4), defined as

$$\hat{\boldsymbol{x}}_{k+1}^{[i]} = A_{ii}\hat{\boldsymbol{x}}_k^{[i]} + B_{ii}\hat{\boldsymbol{u}}_k^{[i]} + \sum_{j\in\mathcal{N}_i} A_{ij}\tilde{\boldsymbol{x}}_k^{[j]} \quad (7)$$

Letting $\boldsymbol{z}_k^{[i]} = (\bar{\boldsymbol{x}}_k^{[i]} - \hat{\boldsymbol{x}}_k^{[i]})$, from (5) and (7) we obtain

$$\boldsymbol{z}_{k+1}^{[i]} = (A_{ii} + B_{ii}K_i^{aux})\boldsymbol{z}_k^{[i]} + \bar{\boldsymbol{w}}_k^{[i]} \quad (8)$$

where

$$\begin{aligned}\bar{\boldsymbol{w}}_k^{[i]} =& A_{ii}(w_{k-1}^{[i]} - A_{epuck}v_{k-1}^{[i]}) + P(A_{epuck}v_k^{[i]}) \\ &+ \sum_{j\in\mathcal{N}_i} A_{ij}(\boldsymbol{x}_k^{[j]} - \tilde{\boldsymbol{x}}_k^{[j]})\end{aligned}$$

is a bounded disturbance, $\bar{\boldsymbol{w}}_k^{[i]} \in \bar{\mathbb{W}}_i = A_{ii}\mathbb{W} \oplus A_{ii}(-A_{epuck}\mathbb{V}) \oplus P(A_{epuck}\mathbb{V}) \oplus \bigoplus_{j\in\mathcal{N}_i} A_{ij}\mathcal{E}_j$, where $\mathbb{W}$ and $\mathbb{V}$ are the sets containing the disturbances $w_k^{[i]}$ and $v_k^{[i]}$. Since $\bar{\boldsymbol{w}}_k^{[i]}$ is bounded and F_{ii} is Schur, there exists a Robust Positively Invariant (RPI) set $\mathbb{Z}_i$ for (8) such that, for all $\boldsymbol{z}_k^{[i]} \in \mathbb{Z}_i$, then $\boldsymbol{z}_{k+1}^{[i]} \in \mathbb{Z}_i$. For its computation see [4].
Given $\mathbb{Z}_i$, it is possible to define the neighborhoods of the origin E_i and U_i, such that

$$\begin{aligned}P(\mathbb{W} \oplus (-A_{epuck}\mathbb{V})) \oplus \mathbb{Z}_i \oplus E_i &\subseteq \mathcal{E}_i \\ K_i^{aux}\mathbb{Z}_i \oplus U_i &\subseteq \mathcal{U}_i\end{aligned}$$

At any time instant k, given the future reference trajectories $\tilde{\boldsymbol{x}}_{k+h}^{[j]}$ and $\tilde{\boldsymbol{u}}_{k+h}^{[j]}$, $h = 0,...,N-1$, $j \in \mathcal{N}_i$, it is possible to solve the following *i-DPC problem*

$$\min_{\hat{\boldsymbol{x}}_k^{[i]}, \hat{\boldsymbol{u}}_{[k:k+N-1]}^{[i]}} V_i^N(\hat{\boldsymbol{x}}_k^{[i]}, \hat{\boldsymbol{u}}_{[k:k+N-1]}^{[i]})$$
$$V_i^N = \sum_{h=0}^{N-1} l_i(\hat{\boldsymbol{x}}_{k+h}^{[i]}, \hat{\boldsymbol{u}}_{k+h}^{[i]}) + V_i^F(\hat{\boldsymbol{x}}_{k+N}^{[i]}) \tag{9}$$

subject to, for all $h = 0, ..., N-1$,

$$\bar{\boldsymbol{x}}_k^{[i]} - \hat{\boldsymbol{x}}_k^{[i]} \in \mathbb{Z}_i$$
$$\hat{\boldsymbol{x}}_{k+h}^{[i]} - \tilde{\boldsymbol{x}}_{k+h}^{[i]} \in E_i$$
$$\hat{\boldsymbol{u}}_{k+h}^{[i]} - \tilde{\boldsymbol{u}}_{k+h}^{[i]} \in U_i$$
$$\hat{\boldsymbol{x}}_{k+h}^{[i]} \in \hat{\mathbb{X}}_i \subseteq \mathbb{X}_i \ominus \Big(\mathbb{Z}_i \oplus P(\mathbb{W} \oplus (-A_{epuck}\mathbb{V}))\Big)$$
$$\hat{\boldsymbol{u}}_{k+h}^{[i]} \in \hat{\mathbb{U}}_i \subseteq \mathbb{U}_i \ominus K_i^{aux}\mathbb{Z}_i$$

and to the terminal constraint

$$\hat{\boldsymbol{x}}_{k+N}^{[i]} \in \hat{\mathbb{X}}_i^F \tag{10}$$

At time k, let the pair $\hat{\boldsymbol{x}}_{k|k}^{[i]}, \hat{\boldsymbol{u}}_{[k:k+N-1|k]}^{[i]}$ be the solution to the i-DPC problem and define by $\hat{\boldsymbol{u}}_{k|k}^{[i]}$ the input to the nominal system (7). Then, according to (6), the input to the system (3) is

$$\boldsymbol{u}_k^{[i]} = \hat{\boldsymbol{u}}_{k|k}^{[i]} + K_i^{aux}(\bar{\boldsymbol{x}}_k^{[i]} - \hat{\boldsymbol{x}}_{k|k}^{[i]})$$

4. Experimental results

Here we report the experimental results validating the robust networked DPC applied to our experimental setup.
The experimental results here reported aim to show the ability of the novel version of the DPC algorithm to regulate the positions of the E-PUCK robots to the equilibria, i.e., $\boldsymbol{x}_{equilibria} = [0 \quad 0 \quad 0 \quad 0 \quad 240 \quad 0 \quad -240 \quad 0]^T$, regardless of the presence of undesired phenomena introduced by the usage of a networked control system architecture.
The presence of the delays and the disturbances, together with the other non-idealities affecting the experimental setup (mainly the sliding of the wheels phenomena), result in the fact that the original DPC algorithm couldn't fulfill the control task. On the other hand, the robust networked DPC algorithm proved its effectiveness in these conditions. Indeed, it is capable to correctly regulate the position of the robots at the equilibria, respecting the constraints on the velocities and the accelerations, despite the network-induced non-idealities.

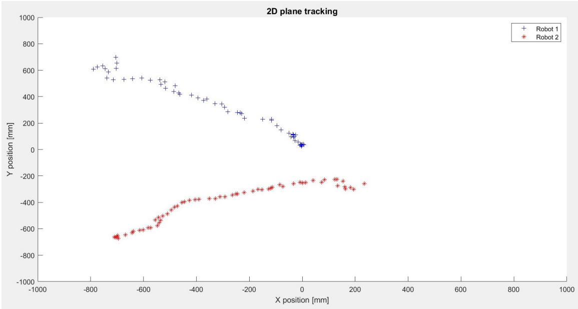


Figure 7: 2D tracking.

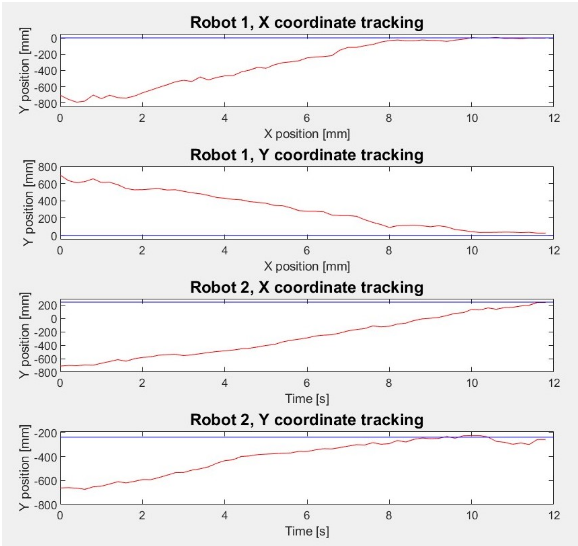


Figure 8: X,Y coordinate tracking.

Figure 6: Experimental test video sequence.

5. Conclusions

In this thesis we developed a lightweight experimental setup subject to non-idealties similar to the ones introduced by the adoption of networked control architectures. This setup can be used as benchmark for testing hierarchical and distributed control strategies in multi-agent applications.

The used laboratory equipment has been analyzed in details and a complete characterization of the noises and delays introduced by each device has been completed. Moreover, a calibration strategy improving the measurements provided by the real time positioning system set up by Decawave UWB sensors is presented.

A state-of-the-art distributed model predictive control strategy, named DPC [1], has been considered.

In particular, we developed a novel, robust, and networked version of the DPC algorithm, capable to deal with delays and disturbances. We equipped the controller with a predictor, to compensate for the effects of the delays. Moreover, the robust positive invariant sets constraining the optimization variables have been re-designed to consider the presence of the noise $w(k)$ affecting the system state, the noise $v(k)$ affecting the measures, and the error due to the predictions computed to compensate for the delay.

Finally, the controller was validated in tests on the experimental setup consisting of carts virtually connected by springs and dampers. The carts are represented by E-PUCK unicycle robots and their position is obtained by UWB positioning sensors. Connections between robots have been virtually imposed by software, enforcing couplings between agents.

The experimental tests witness that the original algorithm was not suitable to deal with network-induced non-idealities. At the same time, they show that the robust networked DPC algorithm here presented was capable to provide good performances also in presence of delays and disturbances.

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