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Executive Summary of the Thesis

Relationships between Interlocking Directorates and governance mechanism: a multiplex network analysis

Laurea Magistrale in Mathematical Engineering - Ingegneria Matematica

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1. Introduction

We want to study if an important trait of the Italian economy, Interlocking Directorates, affects or is affected by the corporate governance of the firms.

The first phenomenon, Interlocking Directorates, is defined as the situation where a person affiliated with one organization sits on the board of directors of another organization [4]. Interlocking is a phenomenon which has been always present in the Italian economy; in the last years there have been limitations to it due to different factors: 2008 financial crisis and legislation decrees as the Interlocking Ban (2011), but it still exercises an important role.

The second phenomenon, corporate governance, is a system of rules, practices and processes by which a firm is directed and controlled [2]. A central role in the corporate governance is exercised by the board of directors and there are several properties of the board which influence the corporate governance of the firm. We concentrate on the following properties: board size, i.e. number of directors who compose the boards; board independence, i.e. percentage of independent directors in the board; 'busy-ness' of corporate directors, i.e. mean of other task that each director has to do besides being in the board; gender diversity, i.e. percentage of women in the board; minority directors, i.e. percentage of minority directors in the board.

To study the relationships between the two phenomena, we use a multiplex network, an instrument which is growing more and more interest in the field of complex networks. Indeed, the multiplex network allows to consider on different layers different properties of the nodes and, analyzing all the layers together, is possible to find out the properties of the network and to understand how the different layers interact one with the other.

Moreover, the Italian market is divided in 11 sectors: Utilities, Luxury goods, Real estate, Industry, Healthcare, Finance, Raw materials, Technology, Commodities, Energy and Telecommunications. We perform the study previously mentioned also for the single sectors and we try to understand if Interlocking Directorates and good governance mechanism are widespread between companies of the same sector or not.

2. Methodology

We introduce the basic definitions about complex networks and multiplex networks.
The definitions given in Section 2.1 have been taken from [5]; the definitions given in Section 2.2 have been taken from [1].

2.1. Basic definitions of complex networks

A network is represented by a graph with N nodes (or vertices) and L links (or edges). Nodes represent individuals, objects, subsystems; links represent the interaction between the nodes. The network could be: directed or undirected, i.e. the links have a direction or not; weighted or unweighted, i.e. the links have a weight or not. In our case, the nodes are all the Italian listed companies; moreover, we have weighted undirected networks and so we state the properties keeping in mind this consideration.
The degree of a node is the number of edges incidental to it and it is equal to:

$$k_i = \sum_j a_{ij} \tag{1}$$

where a_{ij} is an element of the adjacency matrix A and it is equal to 1 if there exists the link between node i and node j and it is 0 otherwise.
Using (1), we can compute the average degree of the network in the following way:

$$\langle k \rangle = \frac{1}{N} \cdot \sum_{i=1}^{N} k_i \tag{2}$$

where N is the number of nodes in the network.
The strength of the node i is computed using the following formula:

$$s_i = \sum_j w_{ij} \tag{3}$$

where w_{ij} is the weight between node i and node j.
Using equation (3), we can compute the average weighted degree:

$$\langle s \rangle = \frac{1}{N} \cdot \sum_{i=1}^{N} s_i \tag{4}$$

where N is the number of nodes in the network.
The graph density measures the density of the links in the network and it is measured in the following way:

$$Density = \frac{\sum_{i,j} a_{ij}}{N(N-1)} \tag{5}$$

where: a_{ij} is an element of the adjacency matrix A and N is the number of nodes in the network.
The giant component is a component of the network which contains a significant proportion of the entire nodes of the network. Usually, it contains a big fraction of nodes.
Regarding the nodes, we can also describe centrality. Centrality is a measure of the importance of every node in the network. There exist different types of centrality, we use the following: degree centrality, more link a node has more it is important; betweenness centrality, more the number of shortest paths which pass through a node is more the node is important; closeness centrality, more a node is close to the others more it is important; eigenvector centrality, more a node relates with many and important nodes more it is important.
Finally, we also compute the modularity coefficient. The modularity is a value in $[-\frac{1}{2}, 1]$ and indicates how good is a given partition of the network. More the value is near 1 more the partition is good.

2.2. Basic definitions of multiplex network

The multilayer network, of which multiplex is a particular case, is a network made up by multiple layers each one representing a network with a given operation mode, social circle or temporal instance. Multiplex is a particular case of multilayer network where all the layers are composed by the same nodes. Indeed, in our case there are the same 201 companies in all the layers of the networks.
In multiplex network, there are two types of links: intra-layer links, i.e. links between nodes of the same layer; inter-layer links, i.e. links between nodes of different layers. In our network there are only intra-layer links.
The first property we analyze in the multiplex network is the global edge overlap. The total overlap $O^{\alpha\beta}$ between layer α and layer β is defined as the number of links present in both the layers:

$$O^{\alpha\beta} = \sum_{i<j} a_{ij}^{\alpha} \cdot a_{ij}^{\beta} \quad (6)$$

where: a_{ij}^{α} is an element of the adjacency matrix A^{α} and it is equal to 1 if there exists the link between node i and node j in layer α otherwise it is 0; a_{ij}^{β} is the same of the previous element but on layer β.

We also generalize the notion of centrality for the multiplex network and finally we compute the projection network of the multiplex. The projection network of the multiplex W is: $proj(W) = (X_W, E_W)$ where X_W is constituted by the same nodes of the layers because we are considering multiplex networks and E_W is equal to the sum of all the links that connect the nodes in the different layers. In the projection network we compute all the properties computed for the single layers, except for centrality, and we also compute the contribution of each layer to the creation of the projection network using the equation:

$$contribution(\alpha) = \frac{\sum_{i=1}^{N} s_i^{[\alpha]}}{\sum_{\beta=1}^{M} \sum_{i=1}^{N} s_i^{[\beta]}} \quad (7)$$

where: α is the considered layer, N is the number of nodes in the network, M is the number of layers in the network and $s_i^{[\alpha]}$ is the weight of node i in layer α computed using equation (3).

3. Data

The dataset is composed by all the Italian companies listed in the Italian market stock exchange Borsa Italiana. We got the firms in our dataset from [3].

We collect data which refer to board of directors and its members for every company. To retrieve these data we search online two documents which each Italian company has to publish: "Relazione sul governo societario e gli assetti proprietari al 31 dicembre 2020" and "Relazione sulla politica in materia di remunerazione e sui compensi corrisposti 2021". In the first document we recover all the data about board of directors and its composition; in the second document we recover all the data about members.

There are some companies for which we are not able to find the two documents previously mentioned, so we decide to not consider these firms in our dataset. This is the reason for which our dataset is composed of 201 companies out of 222.

In our sample of 201 companies there are 1720 directors who occupy 1971 positions. Since the total number of directors is less than the available positions, there are some directors who sit on more than one board and so they create the so called interlocking network. In particular, there are: 1513 directors (about 87,97%) who sit on the board of one company; 171 directors (about 9,94%) who sit on the board of two companies; 30 directors (about 1,74%) who sit on the board of three companies; 4 directors who sit on the board of four companies; 2 directors who sit on the board of five companies. On average, each director occupies 1,15 positions.

Regarding the properties used to create the governance index, board size, board independence, 'busy-ness' of corporate directors, gender diversity and minority directors, we report the maximum, the minimum and the mean value in Table 1.

Prop.	Size	Independence	Busyness
Max	19	90,91%	11,92
Min	2	0,00%	0
Mean	9,80	48,19%	1,76

Prop.	Women	Minority
Max	63,64%	66,67%
Min	0,00%	0,00%
Mean	38,65%	9,06%

Table 1: Properties for each board characteristic

4. Layers' analysis

We proceed building and analyzing the single layers.

The first layer is built using direct interlocking. There is a link between two nodes if there is at least one director who sits on the board of directors of the considered companies. The obtained network is a weighted undirected network where the weight between two nodes is

the number of directors in common between the two boards. However, since we want to compare all the layers, we want to have the weights in $[0, 1]$ and so we normalize all the weights by dividing every weight for the maximum weight.
The second layer is built using indirect interlocking. There is a link between two companies when there are two directors, one on the board of directors of one company and one on the board of directors of the other company, who sit together on the board of directors of a third company. Also this layer is a weighted undirected network where the weight is equal to the number of directors who are indirectly linked. As in the previous case, we want to have all the weights included in $[0, 1]$ and so we normalize by dividing every weight by the maximum weight.
The third layer is built using a governance index. We construct two different third layers and so we will have two multiplex networks to study. The procedure to create the third layers is the following: we construct a similarity index between two nodes using the following formula: $Similarity\_Index(i, j) = 1 - distance(i, j)$ where the value is equal to 1 if the nodes are similar and it is equal to 0 if the nodes are completely different; we select a threshold under which we do not consider the links because we want to compare this layer with the previous ones and so we do not need a completely connected network and because we want to consider only the relevant links; finally, we normalize the network by dividing every weight by the maximum weight. So we obtain two weighted undirected networks with weight in $[0, 1]$. The two third layers differ in the choice we make to compute the distance between two nodes. In particular, for the first multiplex we compute the third layer using the following notion of distance:

$$distance = \frac{1}{5} \cdot Size + \frac{1}{5} \cdot Independence + \frac{1}{5} \cdot Busyness + \frac{1}{5} \cdot Women + \frac{1}{5} \cdot Minority;$$

while for the second third layer we use the following notion of distance:

$$distance = \frac{2}{5} \cdot Independence + \frac{3}{10} \cdot Busyness + \frac{3}{20} \cdot Minority + \frac{1}{10} \cdot Size + \frac{1}{20} \cdot Women.$$

In particular: *Size*, *Independence*, *Busyness*, *Women* and *Minority* are the normalized absolute difference between the respective quantities in the board of the considered companies. In the second notion of distance we give the maximum weight to the variable with the biggest variance, the second maximum weight to the variable with the second biggest variance and so on. We make this choice because we want to understand how the linear combination influences the multiplex network and because we want to study the effect that the variable with the biggest variance has on the network.

Now, we present the main properties of the layers in Table 2. In particular, for each layer we compute the average degree (Avg. Degree) using equation (2), the average weighted degree (Weight Degree) using formula (4), the density of the links (Density) using equation (5) and the percentage of nodes in the giant component of the network (Giant comp.).

Prop.	Layer 1	Layer 2
Avg. Degree	2,69	7,98
Weight Degree	0,34	1,10
Density	0,013	0,04
Giant comp.	75,62%	75,62%

Prop.	Layer3_1	Layer3_2
Avg. Degree	5,43	7,88
Weight Degree	1,43	2,23
Density	0,027	0,039
Giant comp.	70,15%	70,15%

Table 2: Properties for each layer

From Table 2 it is possible to notice how differ the layers we constructed. In particular, the second third layer is slightly different from the first third layer: there are more links in the second third layer and so it is more dense, however the giant component has the same size in both the layers. Also the first two layers are different, in particular the second layer has more links highlighting how there are more indirect interlocks than direct interlocks in the market.

One last notation: the most central nodes in the interlocking layers are different from the most central nodes in the governance layers. This is a first proof of how the two phenomena tend to have different pattern to link the nodes.

Finally, we can make one last consideration about the sectors. For each layer, there are some sectors which do not have link inside: Raw materials, Energy and Telecommunications in Layer 1; Real estate, Healthcare, Finance, Raw materials, Technology, Commodities, Energy and Telecommunications in Layer 2; Utilities, Real estate, Healthcare, Raw materials, Technology, Commodities, Energy and Telecommunications in the first third layer; Real estate, Raw materials and Energy in the second third layer. Moreover, when there are the links inside the single sectors, the properties are very different with respect to the whole network: the average degree and the maximum degree are generally lower and the number of links inside the sector is very low with respect to the total.
All these properties together with the modularity computed in every layer underline that the sectors' partition is not a good partition for the network.

5. Multiplex networks' analysis

To better compare the layers, we consider the multiplex networks.
The first multiplex is composed by the direct interlocking layer, the indirect interlocking layer and the governance layer with the first notion of distance. The second multiplex is composed by the direct interlocking layer, the indirect interlocking layer and the governance layer with the second notion of distance.

We start by computing the edge overlap between every couple of layers in the networks using equation (6). Notice that the first two layers are the same in both the networks and so they have the same properties. In particular, there are 35 overlapping links between the first two layers; they are somehow related because they are both built using the interlocking phenomenon. However, we want to concentrate more on the relations between Layer 1 and Layer 3 and between Layer 2 and Layer 3 of every multiplex because we want to study the relations between interlocking and governance mechanism.
In the first multiplex, there is an interesting relation between Layer 1 and Layer 3. Indeed, we find out that there is a relation between these two layers which there is not in random networks. So, direct interlocking and governance mechanism are somehow related. However, their relation is very weak because there is a little number of overlapping links: only 13 links overlap, they correspond to the $4,81\%$ of all the links in Layer 1 and to the $2,38\%$ of all the links in Layer 3. Instead, there is no particular relation between Layer 2 and Layer 3.
In the second multiplex, there is no interesting relation neither between Layer 1 and Layer 3 nor between Layer 2 and Layer 3. This is the first difference between the two multiplex networks and it is due to the different linear combinations we are using in the third layers.

To better understand if there are some relevant relations between the two phenomena, we create the projection networks of the multiplex and we compute some properties. In particular, we report in Table 3 the same quantities reported in Table 2; in this way we can compare the values obtained in the projection network with the values obtained in the single layers. In the column *Proj* 1 we report all the values of the projection network of the first multiplex while in the column *Proj* 2 we report all the values of the projection network of the second multiplex.

Prop.	Proj 1	Proj 2
Avg. Degree	15,39	17,70
Weight Degree	2,86	3,65
Density	0,077	0,089
Giant comp.	96,02%	98,01%

Table 3: Properties for each projection network

Both the projection networks have properties with values bigger than the single layers. But particularly interesting are the last two properties: density and giant component. The density of both the projection networks is very similar to the sum of the densities of the

layers which compose the multiplex ($0,08$ is the sum of the densities of the first multiplex's layers; $0,092$ is the sum of the densities of the second multiplex's layers). Moreover, both the giant components contain almost all the nodes of the multiplex; in particular, the giant component of the second multiplex contains all the non-isolated nodes, i.e. all the nodes with degree different from 0. Both these considerations allow to understand how the interlocking and the governance mechanism connect the nodes following different patterns and so the two phenomena are somehow complementary, i.e. they connect nodes which are not linked by the other phenomenon. Indeed, if they followed similar pattern, then the density of the projection network would be lower because there would be more overlapping links and also the giant component would have not contained so many nodes.
Finally, we can compute one last quantity for the projection networks: the contribution of each layer. We compute this quantities using equation (7). In the first multiplex, the contribution of each layer is: $15,61\%$ Layer 1, $49,95\%$ Layer 2, $34,44\%$ Layer 3; instead, in the second multiplex the contribution of each layer is: $13,55\%$ Layer 1, $43,00\%$ Layer 2, $43,45\%$ Layer 3. This is a second difference between the two multiplex. In the first multiplex indirect interlocking is the layer which contributes more on the creation of the projection network and the interlocking is the main channel through which the companies communicate; instead, in the second multiplex the governance mechanism has the same influence of the indirect interlocking and the dominance of the interlocking is not marked as before. So, the second notion of distance increases the importance of the governance mechanism.

Finally, we can make some considerations about the multiplex in each sector.
For each sector there is no edge overlap between any couple of layers, neither in the first multiplex nor in the second multiplex. So, the two phenomena are more complementary in the sectors than the whole network.
Moreover, also for the multiplex the partition in sector is not a good partition. Indeed, the modularity coefficient is negative for both the multiplex and the percentages of links inside the network is, respectively, the $5,89\%$ and the $6,46\%$ of all the links in the projection networks.

6. Discussion

In conclusion, there are some differences between the two multiplex introduced by the different weights of the linear combinations: the edge overlap between direct interlocking and governance mechanism and the contribution of the corporate governance in the creation of the projection network. But, in general, the properties of the multiplex networks and of the nodes do not change so much, noteworthy is that the overlap between all the three layers gives the same result in both the multiplex. We can conclude that: the interlocking layers neither affect nor are affected by the governance layer, actually they tend to create link between nodes that are not connected by the other phenomenon; the sectors' partition is not relevant in the market, on the contrary the companies prefer to tie links with firms of different sectors.

References

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