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EXECUTIVE SUMMARY OF THE THESIS

Data-based control for linear systems with stability guarantees

TESI MAGISTRALE IN AUTOMATION AND CONTROL ENGINEERING – INGEGNERIA DELL'AUTOMAZIONE

AUTHOR: Onur OKCU

ADVISOR: Prof. Marcello FARINA

CO-ADVISOR: Ing. William D'AMICO

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1. Introduction

The design of controllers based on data is of great importance in practical applications and many data-driven control methods have been recently developed. These methods can be classified into two groups: direct and indirect methods. The main difference is that, for the former group, an identification phase of the mathematical model of the plant is not required.

Virtual reference feedback tuning (VRFT) is a well-known non-iterative direct data-driven control design approach. VRFT is advantageous for several aspects. However, the resulting closed-loop system is not guaranteed to be stable. In this work, we propose an approach based on VRFT and set membership (SM) identification which provides stability guarantees. An alternative direct approach based on controller unfalsification with stability guarantees is proposed in [2].

The work is organized as follows: firstly, the theoretical background is provided. Secondly, by combining the SM and VRFT methodologies, a novel data-based control design technique for linear systems with stability guarantees is proposed in three different configurations. Then, the proposed approach is tested in simulation and the results are compared with the direct control design based on controller unfalsification with stability guarantees that is proposed in [2]. Finally, conclusions and hints for future developments are provided.

2. Theoretical background

In this section we refer to [1] for a full description of the SM identification. The main steps are recalled.

A linear time-invariant system of order n described by the autoregressive exogenous (ARX) structure is considered:

$$\mathcal{S}:\begin{cases} z(k) = \theta^{o^T}\varphi(k) & (2.1a) \\ y(k) = z(k) + d(k) & (2.1b) \end{cases}$$

where $z(k) \in \mathbb{R}$ is the output, $y(k)$ is the output measure, $d(k)$ is a bounded additive measurement noise, $\theta^o \in \mathbb{R}^{n_a+n_b}$ are the system parameters and $\varphi(k) \in \mathbb{R}^{n_a+n_b}$ is the regressor defined as $\varphi(k) = [z(k-1) \quad \dots \quad z(k-n_a) \quad u(k-1) \quad \dots \quad u(k-n_b)]^T$.

The goal of SM identification is to define the set of unknown parameters $\hat{\theta}$ of the predictor

$$\hat{\mathcal{S}} : \hat{z}(k) = \hat{\theta}^T\hat{\varphi}(k) \tag{2.2}$$

which are compliant with the data, where $\hat{\theta} \in \mathbb{R}^{n_a+n_b}$ and $\varphi(k) = [y(k-1) \quad \dots \quad y(k-n_a) \quad u(k-1) \quad \dots \quad u(k-n_b)]^T$.

The following set of assumptions are required for the consistency of the work:

Assumption 2.1.

a. *The system $\mathcal{S}$ is asymptotically stable*

b. $u(k) \in \mathbb{U} \subset \mathbb{R}, \forall k \in \mathbb{Z}$, *where $\mathbb{U}$ is compact*

c. $|d(k)| < \bar{d}, \forall k \in \mathbb{Z}$, *where $\bar{d} > 0$ is known*

The global error bound is defined in such a way to provide the maximum possible error with respect to all regressors and noise realizations. However, with a finite-length data set the latter can only be estimated with the following linear program

$$\underline{\lambda} = \min_{\theta\in\Omega,\lambda\in R^+} \lambda \qquad s.t. \quad \left|y - \hat{\theta}^T\hat{\varphi}\right| \le \bar{d} + \lambda \quad \forall \begin{bmatrix}\hat{\varphi}\\ y\end{bmatrix} \in \tilde{\mathcal{J}}^N \tag{2.3}$$

where $\tilde{\mathcal{J}}^N := [\hat{\varphi}(k)^T \quad y(k)]^T$ for $k = 1, \dots, N$ and $\Omega \subset \mathbb{R}^{n_a+n_b}$ is a compact set where the set of all parameters lies. Moreover, according to [1] the uncertainty is compensated with the positive inflation parameter $\alpha > 1$

$$\hat{\bar{\varepsilon}} = \alpha\underline{\lambda} \tag{2.4}$$

Notably, with a sufficiently large dataset and sufficiently exciting signals, it is expected that $\alpha \approx 1$. Now, we can define the feasible parameter set (FPS) $\tilde{\Theta}$ as follows

$$\tilde{\Theta} = \left\{\hat{\theta} \in \Omega : \left|y - \hat{\theta}^T\hat{\varphi}\right| \le \bar{d} + \hat{\bar{\varepsilon}} \quad \forall \begin{bmatrix}\hat{\varphi}\\ y\end{bmatrix} \in \tilde{\mathcal{J}}^N\right\} \tag{2.5}$$

3. The proposed approach

The problem we address is the tuning of the controller based on the available experimental input-output dataset $\mathcal{D}$ obtained from the system (2.1). The objective is to provide stability guarantees and the desired closed-loop performance, simultaneously, even if the system parameters are unknown.

3.1. Feasible state-space models

The system $\mathcal{S}$ in (2.1) can be rewritten in the following form

$$y(k) = \theta^{o^T}\hat{\varphi}(k) + w(k) \tag{3.1}$$

where $w(k)$ accounts both for the measurement noise $d(k)$ and for the prediction error $\varepsilon(k)$.

In the following, we assume that the real parameter $\theta^o \in \tilde{\Theta}$. Notably, if $\tilde{\Theta}$ is bounded, it can be represented as a convex hull with N_V vertices θ^i, $i = 1, \dots, N_V$. More specifically, we can write, for any $\theta \in \tilde{\Theta}$, that

$$\theta = \sum_{i=1}^{N_V} \lambda_i\theta^i \tag{3.2}$$

where $\sum_{i=1}^{N_V} \lambda_i = 1$ and $\lambda_i > 0$. (3.1) admits the following state-space representation:

$$\mathcal{S}: \begin{cases} x(k+1) = F^o x(k) + G^o u(k) + G_w w(k) \\ y(k) = Hx(k) \end{cases} \tag{3.3}$$

with the state vector $x(k) = [y(k) \quad \dots \quad y(k-n_a+1) \quad u(k-1) \quad \dots \quad u(k-n_b+1)]^T$ and the system, input and output matrices are

$$F^o = \begin{bmatrix} \theta_1^o & \cdots & \theta_{na}^0 & \theta_{na+2}^o & \cdots & \theta_{na+nb}^o \\ I_{n_a-1} & 0_{(n_a-1)x1} & & 0_{(n_a-1)x(n_b-1)} & & \\ & 0_{1\,x\,n_a} & & 0_{1x(n_b-1)} & & \\ & 0_{(n_b-2)x1} & & I_{(n_b-2)x1} & 0_{(n_b-1)x1} & \end{bmatrix} \tag{3.4}$$

$$G^o = [\theta_{na+1}^o \quad 0_{1x(n_a-1)} \quad 1 \quad 0_{1x(n_b-2)}]^T$$

$$H^T = G_w = [1 \quad 0_{1x(n_a+n_b-2)}]^T$$

Recalling that, in view of Equation (2.1a), we can write the unknown $\theta^o \in \tilde{\Theta}$ as $\theta^o = \sum_{i=1}^{N_V} \lambda_i\theta^i$, also the unknown matrices F^o and G^o can be expressed as convex combinations of known matrices F^i,G^i with $i = 1, \dots, N_V$.

More specifically, we can write

$$[F^o \quad G^o] = \sum_{i=1}^{N_V} \lambda_i\,[F^i \quad G^i] \tag{3.5}$$

where F^i and G^i are constructed in the same way as (3.4) for all i with the corresponding vectors θ^i.

3.2. Condition for robust stability

In this section we introduce the conditions required for robust asymptotic stability for a simple state-feedback regulator.

We consider a state-feedback controller of the type $u(k) = Kx(k)$, $K \in \mathbb{R}^{1\times(n_a+n_b-1)}$.

The closed-loop system dynamics is

$$x(k+1) = (F^o + G^oK)x(k) + G_w w(k) \tag{3.6}$$

where F^o and G^o are uncertain but, according to the SM identification, are convex combinations of matrices F^i,G^i, known for all $i = 1, \dots, N_V$ and defined according to Section 3.1.

According to [4], the Schur stability of $F^o + G^oK$ is guaranteed if $\exists P = P^T > 0$ and K such that

$$(F^i + G^iK)P(F^i + G^iK)^T - P < 0 \tag{3.7}$$

For $L = KP$ and in view of the Schur complement, an equivalent linear matrix inequality (LMI) to equation (3.7) can be obtained as follows:

$$\begin{bmatrix} P - F^iPF^{i^T} - F^iL^TG^{i^T} - G^iLF^{i^T} & G^iL \\ L^TG^{i^T} & P \end{bmatrix} > 0 \tag{3.8}$$

$\forall i = 1, \dots, N_V$. Therefore, if such P and L exist, then $K = LP^{-1}$ is guaranteed to provide asymptotic stability to the closed-loop system for all possible parametrizations of the model compatible with the available data.

3.3. Control schemes for tracking reference signal

In this section, we show the necessary steps to tune a controller for the system in (3.1) that, besides guaranteeing closed-loop stability, provides the desired control system performance with three alternative possible configurations. The general form of such control system is displayed in Figure 3.1.

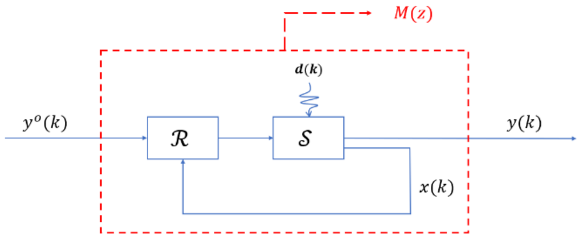

Figure 3.1: General form of tracking scheme

3.3.1. Case I

In this part, the controller is derived in the simplified case where the system gain μ is assumed to be known (or a-priori identified).

3.3.1.1. Stability condition

We consider the following control law

$$u(k) = \rho y^o(k) + K\left(x(k) - x^o(k)\right) \tag{3.9}$$

where $x^o(k) = \mu_r^o y^o(k) = [1_{1\times n_a} \quad \rho 1_{1x(n_b-1)}]^T y^o(k)$ corresponds to desired steady-state values for the input and the output, and $\rho = \mu^{-1}$.

By means of simple computations, the closed-loop stability condition for Case I is obtained as (3.8) since the closed-loop state matrix is equal to (3.6) if we set $y^o(k) = 0$.

3.3.1.2. VRFT-based cost function

The following VRFT cost function can be defined for any state-feedback controller parameter vector K

$$J_1(K) = \sum_{k=1}^{N-n_r} (u(k) - \rho y^o(k) - (x(k) - \mu_r^o y^o(k))^T K^T)^2 \tag{3.10}$$

where $y^o(k)$ is the virtual reference, according to the VRFT algorithm, obtained as

$$y^o(k) = M^{-1}(z) y(k) \tag{3.11}$$

The alternative cost function to (3.10) in the variables P and L can be written in compact form as follows

$$J_{12} = \left\| \mathring{\mathbb{x}}_1^{+} \mathring{\mathbb{u}}_1 - K^T \right\|_P^2 \tag{3.12}$$

where $K = LP^{-1}$, $\mathring{\mathbb{u}}_1 = [u(1) - \rho y^o(1) \quad \cdots \quad u(N-n_r) - \rho y^o(N-n_r)]^T$, $\mathring{\mathbb{x}}_1^{+}$ is the pseudo-inverse of $\mathring{\mathbb{x}}_1 = [x(1) - \mu_r^o y^o(1) \quad \cdots \quad x(N-n_r) - \mu_r^o y^o(N-n_r)]^T$ and $\|\cdot\|$ denotes the Euclidean norm. The motivation of the alternative cost function is that it has the same optimal solution in unconstrained case of J_1 and it allows to include stability constraints. Note that, minimizing J_{12} is equivalent to minimizing a scalar σ such that $\sigma - \mathring{\mathbb{u}}_1^T \left(\mathring{\mathbb{x}}_1^{+}\right)^T P \mathring{\mathbb{x}}_1^{+} \mathring{\mathbb{u}}_1 - LP^{-1}L^T + 2\mathring{\mathbb{u}}_1^T (\mathring{\mathbb{x}}^{+})^T L^T \geq 0$ and the inequality can be rewritten, thanks to the Schur complement, as

$$\begin{bmatrix} \sigma - \mathring{\mathbb{u}}_1^T \left(\mathring{\mathbb{x}}_1^{+}\right)^T P \mathring{\mathbb{x}}_1^{+} \mathring{\mathbb{u}}_1 + 2\mathring{\mathbb{u}}_1^T (\mathring{\mathbb{x}}^{+})^T L^T & L \\ L^T & P \end{bmatrix} \geq 0 \tag{3.13}$$

3.3.1.3. Algorithm

The steps of the algorithm for Case I are the following

1. Given $\mathcal{D}$, M , $\bar{d}$ and the inflation parameter α
2. Compute $\hat{\bar{\varepsilon}}$ according to (2.4).
3. Find the N_V vertices of FPS $\widetilde{\Theta}$ as in (2.5) and construct the corresponding (F^i,G^i pairs) according to Section 3.1 for $i = 1, \dots, N_V$
4. Compute $y^o(k)$, construct $\mathring{\mathbb{u}}_1$ and $\mathring{\mathbb{x}}_1$, and compute $\mathring{\mathbb{x}}_1^{+}$
5. Solve the optimization problem
$$\min_{\sigma,P,L} \sigma$$
$$subject\ to$$
$$-LMI\ (3.13)$$
$$-LMI\ (3.8)\ \forall i = 1, \dots, N_V$$
6. Set $K = LP^{-1}$ in case of a feasible solution.

3.3.2. Case II

The main objective in this part is coping with the unknown system gain.

3.3.2.1. Stability condition

The control law in (3.9) can be restated as

$$u(k) = \gamma y^o(k) + Kx(k) \tag{3.14}$$

where γ is the necessary additional scalar unknown parameter to be tuned for compensate the unknown gain. With such control law, by setting $y^o(k) = 0$, the stability condition (3.8) is obtained.

3.3.2.2. VRFT-based cost function

The following VRFT cost function can be defined for any state-feedback controller parameter vector K and scalar parameter γ

$$J_2(K, \gamma) = \sum_{k=1}^{N-n_r} (u(k) - \gamma y^o(k) - Kx(k))^2 \tag{3.15}$$

The alternative cost function to (3.15) in the variables P and L can be written in compact form

$$J_{22}(\gamma, P, L) = \left\| \mathring{\mathbb{x}}_2^{+} \mathring{\mathbb{u}}_2 - \begin{bmatrix} \gamma \\ K^T \end{bmatrix} \right\|_{\begin{bmatrix} 1 & 0 \\ 0 & P \end{bmatrix}}^2 \tag{3.16}$$

where $K = LP^{-1}$, $\mathring{\mathbb{u}}_2 = [u(1) \quad \cdots \quad u(N-n_r)]^T$ and $\mathring{\mathbb{x}}_2^{+}$ is the pseudo-inverse of $\mathring{\mathbb{x}}_2 = \begin{bmatrix} y^o(1) & \cdots & y^o(N-n_r) \\ x(1) & \cdots & x(N-n_r) \end{bmatrix}^T$. We can introduce a scalar σ to be minimized in order to write an equivalent minimization problem; where $\sigma - \left(\mathring{\mathbb{x}}_2^{+}\mathring{\mathbb{u}}_2\right)^T \begin{bmatrix} 1 & 0 \\ 0 & P \end{bmatrix} \mathring{\mathbb{x}}_2^{+}\mathring{\mathbb{u}}_2 + [\gamma \quad LP^{-1}] \begin{bmatrix} 1 & 0 \\ 0 & P \end{bmatrix} \mathring{\mathbb{x}}_2^{+}\mathring{\mathbb{u}}_2 + \left(\mathring{\mathbb{x}}_2^{+}\mathring{\mathbb{u}}_2\right)^T \begin{bmatrix} 1 & 0 \\ 0 & P \end{bmatrix} \begin{bmatrix} \gamma \\ P^{-1}L^T \end{bmatrix} - [\gamma \quad L] \begin{bmatrix} 1 & 0 \\ 0 & P^{-1} \end{bmatrix} \begin{bmatrix} \gamma \\ L^T \end{bmatrix} \geq 0.$

Thanks to the Schur complement:

$$\begin{bmatrix} \sigma - (\mathbb{x}_2^{\circ +}\mathbb{u}_2^{\circ})^T \begin{bmatrix} 1 & 0 \\ 0 & P \end{bmatrix} \mathbb{x}_2^{\circ +}\mathbb{u}_2^{\circ} + 2(\mathbb{x}_2^{\circ +}\mathbb{u}_2^{\circ})^T \begin{bmatrix} \gamma \\ L^T \end{bmatrix} & [\gamma \quad L] \\ \begin{bmatrix} \gamma \\ L^T \end{bmatrix} & \begin{bmatrix} 1 & 0 \\ 0 & P \end{bmatrix} \end{bmatrix} \geq 0 \tag{3.17}$$

3.3.2.3. Algorithm

The steps of the algorithm for Case II are the following

1. Given $\mathcal{D}$, M , $\bar{d}$ and the inflation parameter α
2. Compute $\hat{\bar{\varepsilon}}$ according to (2.4).
3. Find the N_V vertices of FPS $\tilde{\Theta}$ as in (2.5) and construct the corresponding (F^i,G^i pairs) according to Section 3.1 for $i = 1, \dots, N_V$
4. Compute $y^o(k)$, construct $\mathbb{u}_2^{\circ}$ and $\mathbb{x}_2^{\circ}$, and compute $\mathbb{x}_2^{\circ +}$
5. Solve the optimization problem
$$\min_{\sigma,\gamma,P,L} \sigma$$
$$subject\ to$$
$$-LMI\ (3.17)$$
$$-LMI\ (3.8)\ \forall i = 1, \dots, N_V$$
6. Set $K = LP^{-1}$ in case of a feasible solution.
7. Set $\gamma = \gamma^*$ after performing
$$\gamma^* = \operatorname{argmin}_{\tilde{\gamma}} J_2{}^*(\tilde{\gamma}) = \left\| \mathbb{x}_2^{\circ +}\mathbb{u}_2^{\circ} - \begin{bmatrix} \tilde{\gamma} \\ K \end{bmatrix} \right\|^2$$

3.3.3. Case III

In this part, we investigate the case in which the controller is equipped with an explicit integrator.

3.3.3.1. Stability condition

The control law can be defined using the following state-space realization

$$\begin{cases} \eta(k+1) = \eta(k) + e(k) \\ u(k) = Kx(k) + g\big(\eta(k) + e(k)\big) \end{cases} \tag{3.18}$$

where g is the additional scalar parameter to be tuned together with the vector K and $e(k) = y^o(k) - y(k)$ is the error term. After neglecting $y^o(k)$ and $w(k)$ that do not affect the stability, the alternative closed-loop system dynamics can be written as the following state-space realization by defining the new state variable $\zeta(k) = [x(k)^T \quad \eta(k)]^T$

$$\zeta(k+1) = D^o\zeta(k) \tag{3.19}$$

where

$$D^o = \begin{bmatrix} F^o + G^oK - gG^oH & gG^o \\ -H & 1 \end{bmatrix} = A^o + B^oJ \tag{3.20}$$

$A^o = \begin{bmatrix} F^o & 0_{(n_a+n_b-1)\text{x}1} \\ -H & 1 \end{bmatrix}$, $B^o = \begin{bmatrix} G^o \\ 0 \end{bmatrix}$ and $J = [K - gH \quad g]$, respectively. Notably, (3.5) is also valid for the pair A^o and B^o and the following relation is verified for $i = 1, \dots, N_V$

$$[A^o \quad B^o] = \sum_{i=1}^{N_V} \lambda_i\, [A^i \quad B^i] \tag{3.21}$$

Considering the closed-loop system given in (3.19), the Schur stability of D^o is guaranteed if $\exists P = P^T > 0$ and $t > 0$ such that

$${D^o}^T P^{-1} D^o - P^{-1} + tI_{(n_a+n_b)} \leq 0 \tag{3.22}$$

Consistently, (3.22) holds if and only if $\exists P = P^T > 0$ such that

$$P\big({D^o}^T P^{-1} D^o - P^{-1} + tI_{(n_a+n_b)}\big)P \leq 0 \tag{3.23}$$

After substituting $D^o = A^o + B^oJ$, with basic computations, and using the Schur complement, we obtain

$$\begin{bmatrix} P & P & (A^iP + B^iL)^T \\ P & t^{-1}I_{(n_a+n_b)} & 0_{(n_a+n_b)\text{x}(n_a+n_b)} \\ (A^iP + B^iL) & 0_{(n_a+n_b)\text{x}(n_a+n_b)} & P \end{bmatrix} \geq 0 \tag{3.24}$$

where $L = JP$ and $i = o$. Considering (3.21), if $\exists P = P^T > 0$ and L such that for $i = 1, \dots, N_V$ and $t > 0$ (3.24) holds, then the closed-loop asymptotic stability for (3.19) is guaranteed for $[K \quad g] = LP^{-1}E^{-1}$ with $E = \begin{bmatrix} I_{(n_a+n_b-1)} & 0_{(n_a+n_b-1)\text{x}1} \\ -H & 1 \end{bmatrix}$.

3.3.3.2. VRFT-based cost function

The following VRFT cost function can be defined for any state-feedback controller parameter vector K and scalar parameter g

$$J_3(K, g) = \sum_{k=1}^{N-n_r} \left(u(k) - [K \quad g] \begin{bmatrix} x(k) \\ \bar{v}(k) \end{bmatrix} \right)^2 \tag{3.25}$$

where $\bar{v}(k)$ the integrated virtual error. On the other hand, after obtaining $y^o(k)$ as in (3.11), the integrated error $\bar{v}(k)$ can be derived by computing recursively the equation set listed below

$$\bar{e}(k) = y^o(k) - y(k) \tag{3.26a}$$

$$\bar{v}(k) = \bar{v}(k-1) + \bar{e}(k) \tag{3.26b}$$

The alternative cost function to (3.25) in variables P and L can be written in compact form

$$J_{32}(L, P) = \left\| E^T \mathbb{x}_3^{\circ +}\mathbb{u}_3^{\circ} - J^T \right\|_P^2 \tag{3.27}$$

where $J = LP^{-1}$, $\mathbb{u}_3^{\circ} = [u(1) \quad \cdots \quad u(N - n_r)]^T$ and $\mathbb{x}_3^{\circ +}$ is the pseudo-inverse of $\mathbb{x}_3^{\circ} = \begin{bmatrix} x(1) & \cdots & x(N-n_r) \\ \bar{v}(1) & \cdots & \bar{v}(N-n_r) \end{bmatrix}^T$. A scalar σ is introduced and minimized in order to write an equivalent minimization problem to J_{32}, where $\sigma - {\mathbb{u}_3^{\circ}}^T \left(E^T \mathbb{x}_3^{\circ +}\right)^T P E^T \mathbb{x}_3^{\circ +}\mathbb{u}_3^{\circ} - LP^{-1}L^T + 2{\mathbb{u}_3^{\circ}}^T \left(E^T \mathbb{x}_3^{\circ +}\right)^T L^T \geq 0$. Such inequality can be rewritten, thanks to the Schur complement, as

$$\begin{bmatrix} \sigma - {\mathbb{u}_3^{\circ}}^T (E^T \mathbb{x}_3^{\circ +})^T P E^T \mathbb{x}_3^{\circ +}\mathbb{u}_3^{\circ} + 2LE^T \mathbb{x}_3^{\circ +}\mathbb{u}_3^{\circ} & L \\ L^T & P \end{bmatrix} \geq 0 \tag{3.28}$$

3.3.3.3. Algorithm

The steps of the algorithm for Case III are the following

1. Given $\mathcal{D}$, M , $\bar{d}$, t and inflation the parameter α
2. Compute $\hat{\bar{\varepsilon}}$ according to (2.4).
3. Find the N_V vertices of FPS $\tilde{\Theta}$ as in (2.5) and construct the corresponding (A^i,B^i pairs) for $i = 1, \dots, N_V$ according to (3.20) and Section 3.1

4. Compute $y^o(k)$ and $\bar{v}(k)$, construct E, $\mathring{\mathbb{w}}_3$ and $\mathring{\mathbb{x}}_3$, and compute $\mathring{\mathbb{x}}_3^{+}$
5. Solve the optimization problem

$$\min_{\sigma,P,L} \sigma$$
$$subject\ to$$
$$-LMI\ (3.28)$$
$$-LMI\ (3.24)\ \forall i = 1, \dots, N_V$$

6. Set $[K \quad g] = LP^{-1}E^{-1}$ in case of a feasible solution.

4. Simulation Example

In this part, the proposed approaches are validated in MATLAB/Simulink and the results are compared with the results of the algorithm proposed in [2].

We consider a system with transfer function

$$G(s) = \frac{Z(s)}{U(s)} = \frac{1}{s^3 + 3s^2 + 3s + 1} \tag{4.1}$$

The system is discretized with a sampling time T_s of 0.5 s with the zero-order hold method and the corresponding real parameter vector $\theta^o = [1.82 \quad -1.104 \quad 0.2231 \quad 0.01439 \quad 0.03973 \quad 0.006794]^T$ is obtained.

The batch of data consists of 10000 input-output pairs collected with an open-loop experiment on the simulated system. The system is fed by an input signal that is a multilevel pseudo-random signal (MPRS) uniformly selected in the range $[-1,1]$. Furthermore, the first half of the dataset has switching period of T_s, i.e., 0.5 s and the other half has switching period of $50T_s$, i.e., 25 s. The output measurement is affected by a measurement noise which varies uniformly at each time step in the range [-0.005,0.005]. The corresponding signal to noise ratio (SNR) is 42.616 dB.

Thanks to the *lcon2vert* function [3] in MATLAB, the feasible parameter set $\tilde{\Theta}$ is obtained with 3469 vertices. The projection of the FPS in three-dimensional spaces is represented in Figure 4.1. Note that the real parameters are included in the FPS.

The desired complementary sensitivity function (i.e., the reference model $M(z)$) is expressed with a first order transfer function. The desired input sensitivity function $Q(z)$ is chosen under the assumption that the system gain equals to 1 is known.

$$M(z) = \frac{0.4z^{-1}}{1 - 0.6z^{-1}} \tag{4.2a}$$

$$Q(z) = \frac{1}{\hat{P}(1)} \frac{(1 - 0.02z^{-1})}{1 - 0.6z^{-1}} \frac{0.4}{0.98} \tag{4.2b}$$

$K = [K_1 \quad K_2 \quad K_3 \quad K_4 \quad K_5]^T$ is the parameter vector to be tuned for proposed approach. On the other hand, for the approach proposed in [2] the controller is parametrized as follows

$$C(z,\theta) = \frac{1}{1 - z^{-1}} \frac{s_0 + s_1 z^{-1} + s_2 z^{-2} + s_3 z^{-3}}{1 + r_1 z^{-1} + r_2 z^{-2} + r_3 z^{-3}} \tag{4.3}$$

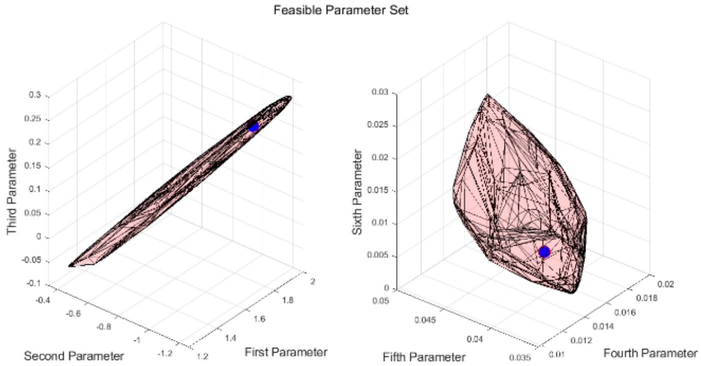

Figure 4.1: Feasible Parameter Set (blue spheres correspond to the real parameters)

The algorithm in [2] and the proposed approach are both performed with the available data. For the "Unfalsification Method", the optimal solution that guarantees closed-loop stability is obtained for $\delta = 0.8$. For the proposed approach, Yalmip and MOSEK solvers are used to perform the algorithms in Section 3.

For the validation phase, the reference signal used is specified in Table 4.1.

Reference	Interval
0	[0,20)
1	[20,40)
-1	[40,60)
2	[60,80)
1	[80,...]

Table 4.1: Reference input values

Moreover, to evaluate the closed-loop performances, the following performance index is used

$$FIT(\%) = 100\left(1 - \frac{\|y - \hat{y}\|}{\|y - \bar{y}\|}\right) \tag{4.4}$$

where y is the real system output vector, $\hat{y}$ is the desired output vector and $\bar{y}$ is the vector that has the same size of the real system output vector and all the elements equal to the mean value of the real output vector.

Figure 4.2 and Figure 4.3 depict the resulting input and output trajectories, respectively. Moreover, Table 4.2 displays the validation results and the spectral radius of the closed-loop systems. Case I and Case II display almost the same input and output trajectories. They have better fitting with respect to the reference model; however, they require more reactive control actions. On the other hand, Case III and the unfalsification method have lower $FIT(\%)$, but the control effort required for these cases is smaller.

	Spectral Radius $\rho(F)$		Fit Percentage $FIT(\%)$
Case I	0.8409		93.3384
Case II	0.8417		93.3437
Case III	0.6935		76.0497
	ETFE of $\|Q(d)\Delta_Q(\theta,d)\|_\infty$	$\rho(F)$	$FIT(\%)$
$\delta = 0.8$	0.9532	0.7616	78.5574

Table 4.2: Performance indexes for validation

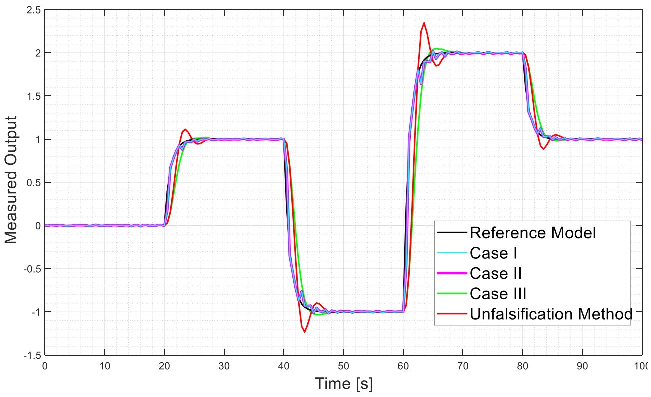

Figure 4.2: Output trajectories of Reference Model, Case I, Case II, Case III and UF with $\delta = 0.8$

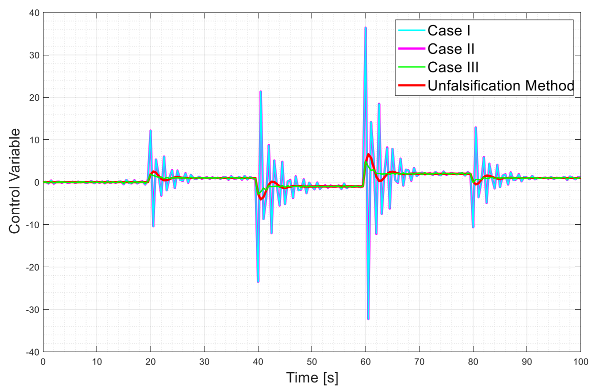

Figure 4.3: Input trajectories of Case I, Case II, Case III and UF with $\delta = 0.8$

5. Conclusions

The purpose of the thesis was to develop a data-based control method for single-input-single-output systems with stability guarantees inspired by VRFT and SM. These two methodologies have been combined, allowing to enforce the closed-loop stability under suitable conditions during the control design phase.
Firstly, SM identification have been recalled with its main steps from a theoretical point of view. Secondly, a data-based control design technique for linear systems with stability guarantees has been proposed in three different configurations. Lastly, the proposed approach has been validated on the simulation environment and the results have been compared with the results of [2].
Future extensions will include the development of the algorithm on more challenging systems such as time-varying, nonlinear, or multi-input-multi-output ones. The large control effort in Case I and Case II, and the high frequency components on the control variable even at steady-state in Case III will also be investigated. Also, the reference model optimization could be combined with the proposed approach.

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