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Executive Summary of the Thesis

An improved cross-entropy method for the water pump scheduling optimization problem

Laurea Magistrale in Automation and Control Engineering - Ingegneria dell'Automazione e Controllo

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1. Introduction

In urban areas facing rising demand due to population growth, water is a crucial resource. While expanding water distribution systems is necessary, the challenge of high-power consumption and energy waste in water pump stations must be addressed. Research highlights that a substantial portion of industrial electricity usage is due to electric motors [1], with water pumps accounting for a significant fraction [2]. Efficient water pump scheduling is vital for optimizing distribution systems. Traditionally, manual methods for pump activation often result in suboptimal scheduling, leading to increased costs and negative impacts on resource management and water quality. Researchers aim to find optimal operating conditions for pumps, focusing on minimizing power consumption and maximizing system efficiency. However, achieving consistently optimal solutions in complex scenarios remains a challenge for existing methodologies.
In the context of water pump scheduling, the Cross-Entropy (CE) method is a niche approach. Introduced by Rubinstein in 1997 [3], the CE method iteratively refines the probability distribution of solutions, ultimately leading to optimal or near-optimal solutions. A key strength of the CE method is its capacity to analyze solutions without relying on derivative functions, distinguishing it from traditional optimization methods. However, since the classic version of the CE method is better suited for system models with relatively simple constraints, an improved version of the CE method proves more suitable for addressing the problem of water pump station optimization.
This work makes three primary contributions. First, it expands the research on the CE method applied to the optimization of water pump scheduling. Second, it introduces the use of an asymmetric smoothed updating step to enhance the algorithm's performance, with the ultimate aim of finding the most efficient operating point for the pumps and thus minimizing pump station costs. Third, it empirically demonstrates the practical effectiveness of the improved optimization method in reducing costs for a real-life water pump station, achieved through the implementation of an adaptive cost function.

2. The pump station model

The model considers the simultaneous operation of multiple pump types to meet system requirements while minimizing operational costs. It considers the total power consumption for each pump, hourly electricity price variations, and introduces penalty functions to address complex constraints. Ultimately, by solving the model, we can determine the optimal states of the pumps, including their head, flow, power, and speed.

2.1. The affinity laws

The affinity laws govern the behavior of water pumps. They describe the fundamental relationships between the pump's head, flow, power, and speed.
Considering the following measures:

- Flow, Q (m^3/h): liquid volume.
- Head, H (m): liquid force measured.
- Speed, S (rpm): shaft speed.
- Power, P (kW): energy needed to pump a liquid.

the affinity laws state that:

$$\frac{Q_1}{Q_2} = \frac{S_1}{S_2}; \quad \frac{H_1}{H_2} = (\frac{S_1}{S_2})^2; \quad \frac{P_1}{P_2} = (\frac{S_1}{S_2})^3 \tag{1}$$

Eqs. (1) show how a 3% increase in speed results in a significant 9% surge in power consumption. In the industrial sector, this translates to a substantial rise in costs, highlighting the importance of optimal solutions.

2.2. The system and pumps curve

In water distribution systems, water pumps respond to the system's operational requirements to meet the supply demands. Changes in the system's characteristics, such as an increased flow demand affect the total dynamic head (THD). The THD is a parameter associated with the system's requirements and is composed by the static head (Hs), friction head (Hf), velocity head (Hv), and pressure head (Hp).

$$TDH = Hs + Hp + Hf + Hv \tag{2}$$

Eq. (2) characterizes the dynamics of the system and plays a fundamental role in establishing the best efficiency point (BEP) of water pumps, which is located by intersecting the system curve with the pump family curve.

Fixed-frequency pumps maintain a constant speed ratio. Their performance curves for head and flow, as well as power and flow, can be defined with:

$$\begin{aligned} H &= h_1 + h_2 Q + h_3 Q^2 \\ P &= p_1 + p_2 Q + p_3 Q^2 \end{aligned} \tag{3}$$

Where $h_1, h_2, h_3, p_1, p_2, p_3$ are fitting parameters. Combining the pump affinity laws and the equations (3), the expressions for the variable frequency pumps are found.

$$\begin{aligned} H &= s^2 h_1 + h_2 Q + h_3 Q^2 \\ P &= s^3 p_1 + s^2 p_2 Q + s^3 p_3 Q^2 \end{aligned} \tag{4}$$

Where s is the speed ratio of the pump.

2.3. The objective function

The objective function used for optimizing the water pump scheduling problem takes various factors into account. The total power consumption is calculated as the sum of contributions from each operational pump.

$$\begin{aligned} J = min \Bigg\{ &\sum_{i=1}^{m} \omega_i (s_i^3 p_1 + s_i^2 p_2 Q_i + s_i p_3 Q_i^2) \\ &+ \sum_{i=m+1}^{m+n} \omega_i (p_1 + p_2 Q_i + p_3 Q_i^2) \Bigg\} \end{aligned} \tag{5}$$

The status coefficients ω_i represent the on-off status of each pump and are pivotal in determining the optimal combination of pumps within the scheduling solution.

2.4. The constraints

Constraints establish the boundaries and limitations within which a solution must operate.

2.4.1 Speed ratio constraint

Variable speed pumps can be controlled to change their speed ratio in the interval $s \in [0, 1]$. However, to increase efficiency and prevent issues like cavitation and shortened life cycles, the speed ratio of each pump has been constrained.

$$s_{min,i} \leq s_i \leq 1, \quad \text{for } i = 1, 2, ..., m+n \tag{6}$$

Where m, n are, respectively, the number of variable speed pumps and fixed speed pumps.

2.4.2 Flow balance equation

The balance equation for the water pump flow states that the output flow of the system Q_s must be equal to the sum of the input flow generated by the pump station.

$$Q_s = \sum_{i=1}^{m} Q_i + \sum_{j=m+1}^{m+n} Q_j \tag{7}$$

2.4.3 Parallel operation of pumps

Pumps operated in parallel will increase the flow but not the head. When this requirement is not met, there is the risk of water flowing backward toward the pump with lower head, potentially causing significant damage. As a result, this condition is imposed as a constraint.

$$H_s = H_1 = H_2 = ... = H_{m+n} \tag{8}$$

2.4.4 High-efficiency area

It is crucial to operate the pumps such that the control variables maximize the machine's efficiency. For fixed frequency pumps, this corresponds to an interval defined by a maximum value, $Q_{max,i}$, and a minimum value, $Q_{min,i}$, of pump flow.

$$Q_{min,i} \leq Q_i \leq Q_{max,i}, \quad \text{for } i = 1, 2, ..., n \tag{9}$$

In the case of variable frequency pumps, the speed parameter extends the interval to encompass a 'best efficiency area' (BEA). The BEA is delimited by flow and speed constraints, which are indicated by the curves connecting the four points A, B, E, and F.
This region determines the area where the pump efficiency, η is equal to or greater than 85%. The curves AB and EF are the head-flow curves of the pump, respectively, at the maximum speed ratio and the minimum speed ratio. The parabolic curve EA corresponds to an efficiency of $\eta = 85\%$, while the FB curve to an efficiency of $\eta = 89.2\%$.
The two boundary curves OA and OB, where O is the origin and which contain the curves EA and FB, are described by the equations:

$$\begin{aligned} H_{OA} &= k_1 Q^2 \\ H_{OB} &= k_2 Q^2 \end{aligned} \tag{10}$$

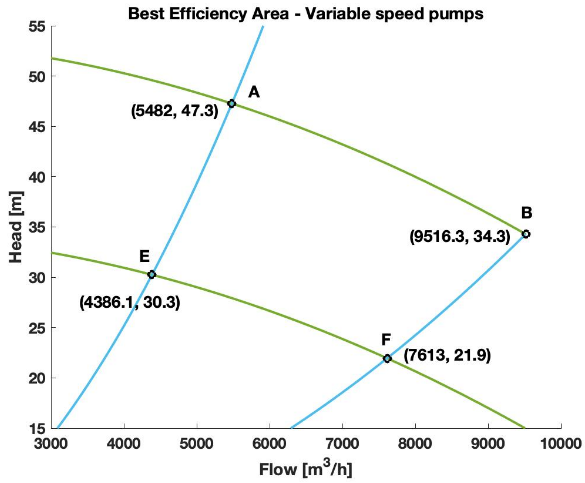


Figure 1: BEA delimited by A, B, E, F.

Where k_1, k_2 are fitting parameters.
Finally, the maximum and minimum flow values are found using equations (1), (9) and (10).

$$\begin{aligned} Q_{max,i} &= \begin{cases} \sqrt{\frac{h_1 - H_s}{h_3}} Q_A & H_s \geq H_B \\ \sqrt{\frac{H_s}{H_B}} Q_B & H_s < H_B \end{cases} \\ Q_{min,i} &= \begin{cases} \sqrt{\frac{H_s}{H_A}} Q_A & H_s \geq H_E \\ \sqrt{\frac{h_1 s_{min}^2 - H_s}{h_3}} & H_s < H_E \end{cases} \\ &\text{for } i = 1, 2, ..., m+n \end{aligned} \tag{11}$$

where H_S is the system's operating head value.

2.5. Summary

The general model is characterized as a non-linear, multi-objective, multi-variable combinatorial problem, encompassing both equality and inequality non-linear constraints.

$$J = min \left\{ \sum_{i=1}^{m} \omega_i (s_i^3 p_1 + s_i^2 p_2 Q_i + s_i p_3 Q_i^2) + \sum_{i=m+1}^{m+n} \omega_i (p_1 + p_2 Q_i + p_3 Q_i^2) \right\}$$

such that

$$\begin{aligned} & Q_s = \sum_{i=1}^{m} Q_i + \sum_{j=m+1}^{m+n} Q_j \\ & H_s = H_1 = H_2 = ... = H_{m+n} \\ & s_{min,i} \leq s_i \leq 1, & i = 1, ..., m+n \\ & Q_{min,i} \leq Q_i \leq Q_{max,i}, & i = 1, ..., m+n \end{aligned}$$

Therefore, in addressing such a problem, the CE method proves to be suitable, whereas classical methods frequently falter when faced with intricate models.

3. The improved cross-entropy method

The Cross-Entropy (CE) method derives from the Monte Carlo method and was introduced by Rubinstein in 1997 [3]. Its central concept involves iteratively refining the probability distribution of solutions, starting from an initial parameter distribution. This refinement process directs the focus toward the most promising regions within the solution space, ultimately leading to convergence towards an optimal or near-optimal solution.

3.1. The population

The initial step in the CE method is to define the probability density function (pdf) that characterizes the set of solutions and from which the initial population is sampled. A multivariate normal distribution is employed to sample the initial population.

3.2. Multivariate normal distribution

We can define each population member with a k-dimensional random vector $\boldsymbol{X} = (x_1, x_2)^T$ sampled from the initial multivariate normal distribution N.

$$\boldsymbol{X} \sim N\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \tag{12}$$

$$\boldsymbol{\mu} = \begin{bmatrix} \bar{s} \\ \bar{h} \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} \left(\frac{s_{max}-\bar{s}}{3}\right)^2 & 0 \\ 0 & \left(\frac{head_{max}-\bar{h}}{3}\right)^2 \end{bmatrix}$$

$\bar{s}, \bar{h}$ are the mean values between the upper and lower bounds of the speed and head limitations given by the constraints. This ensures the proper coverage of the search region.

3.3. Asymmetric smoothed update

In the CE method, the update step is responsible for retaining a record of the best solutions found and for the refinement of the pdf used in the subsequent iteration.
In the classic version of the CE method this step is straightforward. The parameter vector $\hat{v}_t$, which comprises the mean and covariance matrices, is updated at each iteration according to $\hat{v}_{t-1} = \hat{w}_t$, where t is the iteration and $\hat{w}_t$ is the new parameter vector that describes the mean and standard deviation of the elite samples. This update method shows to be inefficient and renders the algorithm vulnerable to local minima.
Decomposing the parameters vector into its its mean and covariance components and introducing a second parameter β, the previous issue is addressed by providing an asymmetric smoothed update.

$$\begin{aligned} \hat{v}_{\mu,t} &= \alpha \hat{w}_{\mu,t} + (1-\alpha)\hat{v}_{\mu,t-1} \quad && 0.4 \leq \alpha \leq 0.9 \\ \hat{v}_{\sigma,t} &= \beta \hat{w}_{\sigma,t} + (1-\beta)\hat{v}_{\sigma,t-1} \quad && 0.3 \leq \beta \leq 0.7 \end{aligned}$$

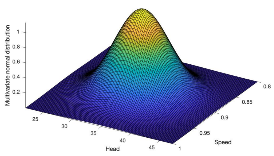
In optimization problems, the factor $(1-\alpha)$ is commonly employed in smoothed update steps. In this context, our focus extends beyond the introduction of a second parameter β, which enables an asymmetric smoothed update. The CE method samples from a Gaussian pdf, which, in 2 dimensions, forms a bell-shaped curve (Fig. 2). When using a single parameter α, in each update step, the bell-shaped curve 'moves towards the best solution' as the mean matrix updates and 'reduces in size' when the covariance matrix updates. Unfortunately, this simultaneous movement and scaling can lead the algorithm to converge into local minima, compromising accuracy. By applying two parameters instead, we enable the Gaussian pdf to 'move' and 'scale' in an asymmetric manner, as shown in Fig. 2. This results in faster convergence, reduced susceptibility to local minima, and an improved ability to explore better solutions, particularly when specific directions offer a higher probability of yielding superior results.

3.4. Adaptive objective function

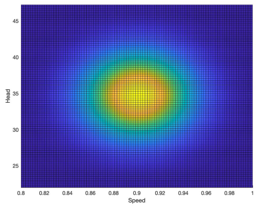
To achieve a faster and more accurate optimization a different parametrization has been applied to the expression (5) by reformulating the flow values Q_i in terms of H_s and s_i.

$$Q_i = \sqrt{\frac{h_1 s_i^2 - H_s}{h_3}}$$

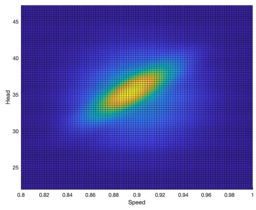
The constraints and the choice of their mathematical representation can significantly impact the feasibility and quality of the solution. Therefore, two penalty functions Pe_1 and Pe_2 that take into account the constraints expressed by the equations (7) and (9) are introduced in the model.
The penalty function Pe_1 represents the BEA



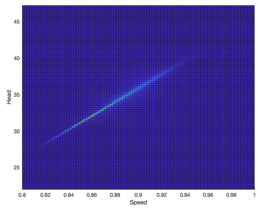
(a) Frontal representation.



(b) Iteration 1.



(c) Iteration 5.



(d) Iteration 15.

Figure 2: 'Moving' and 'resizing' in the update step.

constraint.

$$
\begin{aligned}
Pe_1 &= \sum_{i=1}^{n+m} (\Delta Q_i)^2 \\
&= \begin{cases} \sum_{i=1}^{n+m} (Q_i - Q_{min,i})^2, & Q_i < Q_{min,i} \\ 0, & Q_{min,i} < \quad Q_i < Q_{max,i} \\ \sum_{i=1}^{n+m} (Q_i - Q_{max,i})^2, & Q_i > Q_{max,i} \end{cases}
\end{aligned}
$$

The penalty function Pe_2 represents the flow balance constraint.

$$
Pe_2 = (\sum_{i=1}^{n+m} \omega_i \sqrt{\frac{h_1 s_i^2 - H_s}{h_3}} - Q_s)^2
$$

Therefore, the problem's mathematical formulation to be solved with the CE method is summarized in:
Where σ is a weight coefficient that decreases with the iterations, giving less importance to the penalty functions as the solutions converge to the optima. It is characterized by an initial value σ_0 and a cooling parameter γ.

$$
\sigma = \sigma_0 \frac{1}{\gamma T}, \quad \gamma \in [0, 1]
$$

4. Comparative analysis of performance

The improved CE method has introduces enhancements that improve best existing solutions.

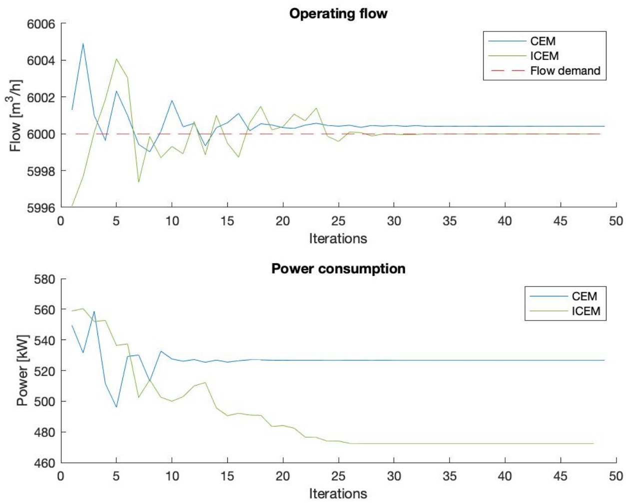


Figure 3: Comparison of the improved CE method in a single pump case.

Table 1: Comparative analysis

	CEM	ICEM
Flow demand (m^3/h)	6000	6000
Computed flow (m^3/h)	6004.0	6000
Power (kW)	526.77	472.43
Iterations	30	32

Its performance is thoroughly analyzed from various perspectives to highlight its contributions and improvements. Fig. 3 shows the comparison between the classic CE method (CEM) and improved CE method(ICEM). Furthermore, Tab. I shows the their operational parameters under the condition of system flow demand equal to $6000 m^3/h$.
The improved CE method performs better compared to the classic CE method, as shown in Tab. 1. The improved CE method is able to converge to the global minima in power consumption, achieving a remarkable reduction of approximately 11%, in the $6000 m^3/h$ case. Furthermore, it demonstrated superior effectiveness in refining its search for the optimal working point, resulting in a very accurate flow value. The differences in convergence observed can be directly attributed to the fundamental principles underlying the improved CE method. By smoothing the update of the multivariate normal distribution, the algorithm doesn't rapidly converge toward the best-found solution but al-

lows room for exploration in the proximity of the current optimal point. This approach enhances the algorithm's capability to search for global minima and improves its overall accuracy.

5. Practical implementations

The model has been adapted to optimize a real-life-inspired scenario featuring the water pump station of a megalopolis equipped with four pumps, including two variable frequency pumps (pump 1, 2) and two fixed frequency pumps (pump A, B). Their real performance curves have been incorporated, and the operation of the water pump station over several months has served as the foundation for the database in use. With the application of the improved CE method, effective working points for these set of pumps have been defined, ensuring compliance with all constraints, especially their operation within the BEA (Fig. 2). Additionally, the method has provided insights into the pumps' statuses (on-off), indicating the optimal combination of pumps that should be operated to minimize the costs of the pump station.
Since the system flow demand is constantly varying, a key flow demand value of $25000m^3/h$ have been chosen. The respective performance values are detailed in Table II and the optimal operating condition for the variable-speed pumps are indicated with a star symbol in Fig. 4. The efficiency of the variable-speed pumps consistently exceeds 85%, and the head constraint is respected. The implementation of the resulting scheduling system effectively reduces the operational and maintenance costs of the water pump station.
In conclusion, the application of the improved CE method results in the successful operation of the pumps within their best efficiency area, while satisfying the water system constraints and requirements of head and flow.

6. Conclusions

The optimization of the water pump station scheduling was explored using an improved CE method. The algorithm was developed and simulated in MATLAB, and its performance was tested in a real-life-inspired scenario, representing a megalopolis water pump station. The results of this study were highly satisfactory, with the system and its components operating

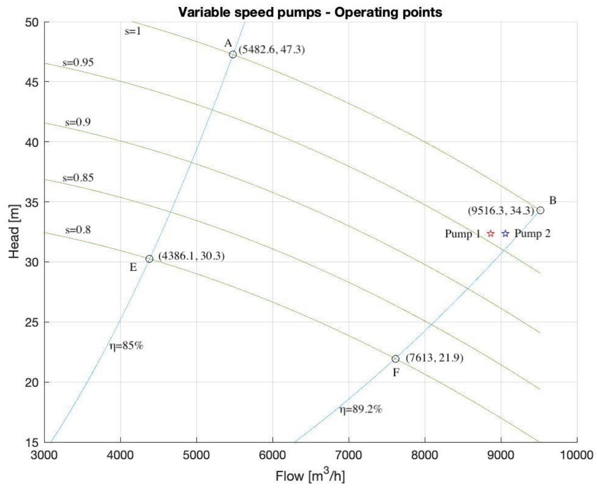


Figure 4: Working points when $Q_s = 25000m^3/h$.

Table 2: Pump station parameters when $Q_s = 25000m^3/h$.

Pump	**A**	**B**	**1**	**2**
Status	On	On	On	On
Output flow	24969.7 (m^3/h)			
Power	2394.05 (kW)			
Head	32.33 (m)			
Flow (m^3/h)	4196	3902	8299	8570
Speed ratio	0.92	0.87	0.87	0.89
Efficiency (%)	75.83	75.83	90.95	90.51

at their most efficient points while adhering to all specified constraints.

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