Executive Summary of the Thesis

Strategic Behaviour Study of Flexible Generator in Hybrid DSO and P2P Markets - A Three-Phase Unbalanced Distribution Network Case

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Abstract

DSO ancillary services (AS) market as a price-setter calculates the distribution locational marginal prices (DLMPs) based on linearized AC-OPF. The DLMPs contribute to the loss reduction in the DN, support the voltage along the feeder, and manage the line congestion. The essential novelty of this work is to model the strategic behaviour of a large-scale strategic generator (SG). Accordingly, a game-theoretic bi-level programming approach is employed to model the problem where SG in the upper-level problems tries to maximize its revenue. On the contrary, DSO clears the AS market in the lower level problem by minimizing the losses and costs in the DN. Then, the equivalent single-level problem is attained in terms of equivalent linearized MPEC formulation.

Index Terms-DSO AS market, Strategic DER, Bi-level programming, P2P market, ADMM.

Notation

Scalars are small letters, e.g., x. Vectors and matrices are in bold letters, i.e., $\mathbf{x}, \mathbf{X}$. All vectors are column vectors. Entrywise matrix multiplication is denoted by $\mathbf{X} \circ \mathbf{X}$. The optimal solutions for an optimization variable $x, \mathbf{x}$ are denoted by $x^*, \mathbf{x}^*$. The transpose of a vector or matrix is denoted by $()^\intercal$.

1. Introduction

The AC-OPF problem in DN is highly non-convex and non-linear [1]. To this end, the approximated AC-OPF is proposed where the constraints' non-linear terms are linearised by their sensitivity to the DER's active and reactive power nodal injection in the radial DN [2].

Considering a DSO AS market, the perception is that the market organizer has already participated in the wholesale TSO day-ahead market (DAM) by anticipating its local power demand pattern [3]. On the following day, the DSO AS market clearing takes place with the objective of minimizing the losses in the downstream grids, minimize the power procurement from real-time or the reserve wholesale market, and to ensure the safe and optimal operation of DN without voltage violation or line thermal overloading. In addition, DSO determines DLMPs to motivate to proactively take part in the AS program by maximizing the social welfare [4]. Promising works have been conducted to explore the feasibility of DSO local market by ensuring the functionality in reducing the operational costs and incentivizing the DERs to participate in the market. These works are mainly implemented by considering the domination of DSO on the market as a price-setter and designate the role of DERs to be price-taker. However, considering the characteristics of three-phase DN, it is likely for a potential FG with relatively high nominal power to change the DSO market's output to his benefit [5]. To this end, the latest relative works are reviewed in the following.

A) Literature Review

A[6], a bi-level programming method is proposed to motivate DERs to participate in the market proactively. The model adopts a novel undirected SOC form of AC OPF to calculate DLMPs under sufficient assumptions and conditions. To solve the bi-level programming, mixed-integer semidefinite programming is employed. Literature [7] has provided A GNB-based market mechanism to settle the energy transaction between DERs. Also, relaxation is used to transform non-convex AC OPF to SOC programming, and bilinear terms are linearized by the linear outer approximation method. The mechanism of cooperation is mutual trust and truthful sharing of information. Similar to the work we will propose, In [8], bi-level demand management is presented for an industrial zone, including heat and power infrastructure based on DLMP derived from SOCP AC power flow. J. Conejo in [9], analyses the equilibrium point for the strategic prosumer and DSO market using SOC relaxed AC-OPF in a single-phase DN. Eventually, it concludes that the equilibrium point

obtained from bilevel programming coincides with the results of the market fully controlled by DSO. Similarly, in [10], a game-theoretic bi-level programming for modelling the prosumer in the DSO market; however, a transparent methodology for the network constraints is not illustrated. Eventually, the paper [11] obtains the GNE problem of the strategic DER and DSO market by considering the SOC of AC-OPF and the GNE point is obtained based on iterative approaches.
In the majority of the works introduced above the SOCP is adopted to solve the AC-OPF in DN. The theoretical drawback of this approach is the risk of obtaining a solution from the relaxed problem, which does not coincide with the original solution set [12]. Moreover, mainly the relaxed AC-OPF results in a recursive formula making the interpretation of DLMPs in radial DN counterintuitive. Moreover, to deal with the non-linearities, approximations are applied. Furthermore, the three-phase unbalanced configuration of DN is not taken into account as the most critical characteristic of DN.

B) Contribution

As a comprehensive study to clear the research gaps by considering the realistic characteristic of DN, the essential contribution of this work can be summarized as follows:

1. Three-phase DN is modelled,
2. The strategic behaviour of large-scale FG is studied by adopting the Stackelberg game-theoretic approach.
3. The MPEC of the bi-level problem is obtained as an equivalent mixed-integer linear programming (MILP).
4. The decentralized P2P market is coordinated with the bi-level market problem

The following sections are allocated to develop the methodologies and discuss the optimization case study and results.

2. DER State-Space Modelling

The flexible DERs, including the FGs and FLs, are subject to operational limits in the DSO markets. For a FL, the minimum and maximum curtailing over a given time can be determined by eq. (1) $\forall t \in \mathcal{T}$.

$$-e_d^{pu}\,|p_d^{nm}|\,\Delta t \leq e_{d,t} \leq e_d^{pu}\,|p_d^{nm}|\,\Delta t \qquad (1a)$$
$$-p_d^{-,pu}p_{d,t}^{bas} \leq -p_{d,t} \leq -p_d^{+,pu}p_{d,t}^{bas} \qquad (1b)$$

The symbol e_d^{pu} is the maximum curtailing period of the flexible load d and its unit is hours. The scalars $p_d^{-,pu}$ and $p_d^{+,pu}$ stand for the per-unit minimum and maximum power curtailing factor with respect to the baseline power $p_{d,t}^{bas}$. Since the demand value is negative, eq. (1b) is multiplied by minus to model the curtailing limit properly.

Similarly, for a FG the operation limit is indicated in terms of maximum and minimum power dispatch as in eq. (2).

$$p_d^{-,pu}p_{d,t}^{bas} \leq p_{d,t} \leq p_d^{+,pu}p_{d,t}^{bas} \qquad \forall t \in \mathcal{T} \qquad (2)$$

Unlike eq. (1b) for FL, for the FG, this limitation is not multiplied by minus since the power injection is positive.
The state-space equations introduced in eq. (3) eencapsulate the operational limits of the DERs by the flavour of control-oriented model [13].

$$\mathbf{x}_{t^0} = \hat{\mathbf{x}} \qquad : \boldsymbol{\lambda}^{x0} \qquad (3a)$$
$$\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{u}_t + \mathbf{v}_t^x \qquad : \boldsymbol{\lambda}_t^x\ \forall t \in \mathcal{T}/\{t^n\} \qquad (3b)$$
$$\mathbf{y}_t = \mathbf{C}\mathbf{x}_t + \mathbf{D}\mathbf{u}_t + \mathbf{v}_t^y \qquad : \boldsymbol{\lambda}_t^y\ \forall t \in \mathcal{T} \qquad (3c)$$
$$\mathbf{p}_t = \hat{\mathbf{p}}_t^{fixed} + \mathbf{M}^{p,y}\mathbf{y}_t \qquad : \boldsymbol{\lambda}_t^{p,y}\ \forall t \in \mathcal{T} \qquad (3d)$$
$$\mathbf{q}_t = \hat{\mathbf{q}}_t^{fixed} + \mathbf{M}^{q,y}\mathbf{y}_t \qquad : \boldsymbol{\lambda}_t^{q,y}\ \forall t \in \mathcal{T} \qquad (3e)$$
$$\underline{\mathbf{y}}_t \leq \mathbf{y}_t \leq \overline{\mathbf{y}}_t \qquad : \underline{\boldsymbol{\mu}}_t^y, \overline{\boldsymbol{\mu}}_t^y\ \forall t \in \mathcal{T} \qquad (3f)$$

Matrix $\mathbf{A}_t$ and $\mathbf{B}_t$ are the state and control matrices. Whereas $\mathbf{c}_t$ and $\mathbf{D}_t$ represent the output and feed-through matrices, respectively. $\mathbf{v}_d^{x/y}$ indicate the disturbance vectors for state and output equations that are considered as parameters in our model. Moreover, $\mathbf{M}^{p,y}$ and $\mathbf{M}^{q,y}$ are the mapping matrices, with binary values for its arrays, mapping the flexible DERs to the output vector.

3. Linear AC-OPF

The three-phase DN linear AC-OPF model is emerged from [14, 15]. Where the state variables, i.e. active and reactive power losses (p/q_t^{loss}), the absolute value of the nodal voltages ($|\mathbf{u}_t|$), and the line power flow in "From" and "To" directions ($|\mathbf{s}_t^{f/t}|$) are written in terms of theirs sensitivity with respect to active and reactive power injection from DERs. Therefore $\forall t \in \mathcal{T}$ we can write,

$$p_t^{loss} = \hat{p}_t^{loss} + \mathbf{M}_p^{p^{loss}}\mathbf{p}_t + \mathbf{M}_q^{p^{loss}}\mathbf{q}_t \qquad : \lambda_t^{p^{loss}} \qquad (4a)$$
$$q_t^{loss} = \hat{q}_t^{loss} + \mathbf{M}_p^{q^{loss}}\mathbf{p}_t + \mathbf{M}_q^{q^{loss}}\mathbf{q}_t \qquad : \lambda_t^{q^{loss}} \qquad (4b)$$
$$|\mathbf{u}_t| = |\hat{\mathbf{u}}_t| + \mathbf{M}_p^{|u|}\mathbf{p}_t + \mathbf{M}_q^{|u|}\mathbf{q}_t \qquad (4c)$$
$$|\underline{\mathbf{u}}_t| \leq |\mathbf{u}_t| \leq |\overline{\mathbf{u}}_t| \qquad : \underline{\boldsymbol{\mu}}_t^{|u|}, \overline{\boldsymbol{\mu}}_t^{|u|} \qquad (4d)$$
$$|\mathbf{s}_t^f| = |\hat{\mathbf{s}}_t^f| + \mathbf{M}_p^{|s^f|}\mathbf{p}_t + \mathbf{M}_q^{|s^f|}\mathbf{q}_t \qquad (4e)$$
$$|\underline{\mathbf{s}_t^f}| \leq |\mathbf{s}_t^f| \leq |\overline{\mathbf{s}_t^f}| \qquad : \underline{\boldsymbol{\mu}}_t^{|s^f|}, \overline{\boldsymbol{\mu}}_t^{|s^f|} \qquad (4f)$$
$$|\mathbf{s}_t^t| = |\hat{\mathbf{s}}_t^t| + \mathbf{M}_p^{|s^t|}\mathbf{p}_t + \mathbf{M}_q^{|s^t|}\mathbf{q}_t \qquad (4g)$$
$$|\underline{\mathbf{s}_t^t}| \leq |\mathbf{s}_t^t| \leq |\overline{\mathbf{s}_t^t}| \qquad : \underline{\boldsymbol{\mu}}_t^{|s^t|}, \overline{\boldsymbol{\mu}}_t^{|s^t|} \qquad (4h)$$

In eq. (4) the $\mathbf{M}_{p/q}^{(\cdot)}$ are the sensitivity matrices of the state variables of AC-OPF to the active and reactive power injections of DERs ($\mathbf{p}/\mathbf{q}_t$) obtained based on [14, 15]. Accordingly, $\mathbf{M}_p^{p/q^{loss}} \in \mathbb{R}^{1\times\mathcal{D}}$ with $\mathcal{D}$ as the size of DER power column vectors indicating the sets of DERs. About the voltage sensitivity matrix $\mathbf{M}_{p/q}^{|u|} \in \mathbb{R}^{3\mathcal{N} \times \mathcal{D}}$ with $\mathcal{N}$ indicating the three-phase node index. For line power flow in both direction the size of matrix is defined as $\mathbf{M}_{p/q}^{|s^{f/t}|} \in \mathbb{R}^{3\mathcal{H} \times \mathcal{D}}$ with $\mathcal{H}$ denoting the three-phase lines sets. Moreover, the Lagrangian variables corresponding to the equality and inequality constraints are give in by $\boldsymbol{\lambda}_t^{(\cdot)}$ and $\boldsymbol{\mu}_t^{(\cdot)}$, respectively. The variable's maximum and minimum range are given by overline and underline notation of the same variable.

4. Bi-level problem of SG in DSO AS Market

As discussed earlier, it is possible for a potential producer to behave strategically and submit an offer comparatively higher than its actual marginal cost, untruthfully. Considering the DSO AS market as a case, for such a strategic market player, it is necessary to anticipate the market-clearing price (DLMP) and the network structure to maximize its revenue by submitting offers that are not necessarily equal to their actual marginal cost. On the other hand, the DSO receives the offers/bids and, by noting the energy price at the GSP, clears the market to determine DLMPs by minimizing the costs and losses in the DN. Meanwhile, DSO ensures the constraints related to the nodes' voltage and the lines' loading are met. Therefore, the pool energy markets require sequential decision-making. By considering the SG owner and DSO as independent decision-makers with contradicting objectives, the problem constitutes a sequential leader-follower game so-called Stackelberg [5]. The bi-level structure of such a game is depicted in fig. 1 and formulated in eq. (5).

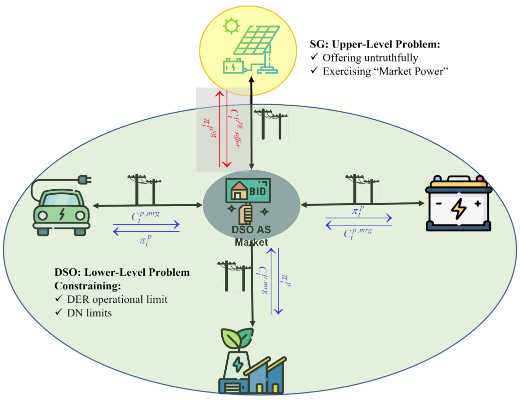

Figure 1: Structure of bi-level programming of SG in DSO market.

As it regards, in the upper-level, SG anticipates the DLMP (π_t^{sg}) for his node as a variable and attempts to submit offers ($c_t^{offer,p^{sg}}$), which leads to its maximum revenue. On the other hand, DSO considers a linear cost and utility objective function for DERs and takes all the offers as the parameter ($\mathbf{c}_t^{p,DER*}$) to clear the market and determine the DLMPs.

$$\underset{\substack{\mathbf{x}_t,\mathbf{u}_t,\mathbf{y}_t\\ p_t^{loss},q_t^{loss},\mathbf{p}_t,\mathbf{q}_t\\ c_t^{offer,p^{sg}},\pi_t^{sg},\boldsymbol{\lambda}_t,\boldsymbol{\mu}_t}}{Max} \quad \sum_{t\in\mathcal{T}} \left\{ p_t^{sg}\pi_t^{sg} - p_t^{sg}c_t^{mrg,p^{sg}} \right\} \tag{5a}$$

$$s.t. \quad c_t^{offer,p^{sg}} \geq 0 \qquad : \mu_t^{c^{offer}} \; \forall t \in \mathcal{T} \tag{5b}$$

$$\underset{\substack{\mathbf{x}_t,\mathbf{u}_t,\mathbf{y}_t\\ p_t^{loss},q_t^{loss}\\ \mathbf{p}_t,\mathbf{q}_t}}{Min} \sum_{t\in\mathcal{T}} \Big\{ c_t^{p,0}p_t^{loss} - c_t^{p,0}\mathbf{1}^{\mathrm{T}}\mathbf{p}_t + \left(\mathbf{c}_t^{p,DER*}\right)^{\mathrm{T}}\mathbf{p}_t + \left(\mathbf{c}_t^{q,DER}\right)^{\mathrm{T}}\mathbf{q}_t + c_t^{q,0}q_t^{loss} - c_t^{q,0}\mathbf{1}^{\mathrm{T}}\mathbf{q}_t \Big\} \tag{5c}$$

$$s.t. \qquad eqs.\ (3) and\ (4) \tag{5d}$$

The bi-level problem given in eq. (5) is strictly non-convex and non-linear. Therefore, First, it needs to be written as a single-level problem. Then, the non-linearities should be tackled. To this end, an important property of the lower-level DSO problem is that it is a convex equilibrium problem [3, 14].

Theorem 4.1. *Providing that an equilibrium problem is convex and its equality constraints are affine functions, then the solution to its equivalent joint-KKT condition will be identical to the Nash equilibrium solution of the original problem as a global optimum.*

By adopting theorem 4.1, the lower level problem of DSO can be replaced by its KKT conditions to constitute a mathematical problem with equilibrium constraints (MPEC) given in the following eq. (6).

$$\underset{\substack{\mathbf{x}_t,\mathbf{u}_t,\mathbf{y}_t\\ p_t^{loss},q_t^{loss},\mathbf{p}_t,\mathbf{q}_t\\ c_t^{offer,p^{sg}},\pi_t^{sg},\boldsymbol{\lambda}_t,\boldsymbol{\mu}_t}}{Max} \quad \sum_{t\in\mathcal{T}} \left\{ p_t^{sg}\pi_t^{sg} - p_t^{sg}c_t^{mrg,p^{sg}} \right\} \tag{6a}$$

$$s.t. \quad c_t^{offer,p^{sg}} \geq 0 \qquad : \mu_t^{c^{offer}} \; \forall t \in \mathcal{T} \tag{6b}$$

$$KKT\ conditions\ of\ the\ lower\text{-}level \tag{6c}$$

The complete derived KKT conditions of the DSO problem can be found in the thesis. However, as the important case in point, the KKT conditions for the SG and active power losses are developed in eq. (7).

$$\frac{\partial L(\cdot)}{\partial p_t^{loss}} = c_t^{p,0} + \lambda_t^{p^{loss}} = \mathbf{0} \quad \forall t \in \mathcal{T} \tag{7a}$$

$$\frac{\partial L(\cdot)}{\partial q_t^{loss}} = c_t^{q,0} + \lambda_t^{q^{loss}} = \mathbf{0} \quad \forall t \in \mathcal{T} \tag{7b}$$

$$\frac{\partial L(\cdot)}{\partial p_t^{sg}} = c_t^{offer,p^{sg}} + \lambda_t^{p^{sg},y} - c_t^{p,0} - \left(M_{p^{sg}}^{p^{loss}}\right)\lambda_t^{p^{loss}} - \left(M_{p^{sg}}^{q^{loss}}\right)\lambda_t^{q^{loss}} + \left(\mathbf{M}_{p^{sg}}^{|u|}\right)^{\mathrm{T}}\left(\overline{\boldsymbol{\mu}}_t^{|u|} - \underline{\boldsymbol{\mu}}_t^{|u|}\right) + \left(\mathbf{M}_{p^{sg}}^{|s^f|}\right)^{\mathrm{T}}\left(\overline{\boldsymbol{\mu}}_t^{|s^f|} - \underline{\boldsymbol{\mu}}_t^{|s^f|}\right) +$$

$$+\left(\mathbf{M}_{p^{sg}}^{|s^t|}\right)^{\mathrm{T}}\left(\overline{\boldsymbol{\mu}}_t^{|s^t|}-\underline{\boldsymbol{\mu}}_t^{|s^t|}\right)=\mathbf{0}\quad\forall t\in\mathcal{T}\qquad(7c)$$

Let the DLMP be eq. (8) from the DER perspective [14]. Then, by noting the KKT conditions in eq. (7), the DLMP from the grid perspective for the SG yields as eq. (9).

$$\pi_t^{p^{sg},DER}=c_t^{offer,p^{sg}}+\overbrace{\overline{\mu}_t^{p^{sg}}-\underline{\mu}_t^{p^{sg}}}^{\lambda_t^{p^{sg},y}}\quad\forall t\in\mathcal{T}\qquad(8a)$$

$$\begin{aligned}\pi_t^{p^{sg},grid}=\;&c_t^{p,0}\quad\mathbf{a)}\\&\mathbf{b)}-\left(M_{p^{sg}}^{p^{loss}}\right)c_t^{p,0}+\left(M_{p^{sg}}^{q^{loss}}\right)c_t^{q,0}\\&\mathbf{c)}-\left(\mathbf{M}_{p^{sg}}^{|u|}\right)^{\mathrm{T}}\left(\overline{\boldsymbol{\mu}}_t^{|u|}-\underline{\boldsymbol{\mu}}_t^{|u|}\right)\\&\mathbf{d)}-\left(\mathbf{M}_{p^{sg}}^{|s^f|}\right)^{\mathrm{T}}\left(\overline{\boldsymbol{\mu}}_t^{|s^f|}-\underline{\boldsymbol{\mu}}_t^{|s^f|}\right)\\&\mathbf{d)}-\left(\mathbf{M}_{p^{sg}}^{|s^t|}\right)^{\mathrm{T}}\left(\overline{\boldsymbol{\mu}}_t^{|s^t|}-\underline{\boldsymbol{\mu}}_t^{|s^t|}\right)\quad\forall t\in\mathcal{T}\qquad(9a)\end{aligned}$$

Evidently, the DLMP is formed by four main components. a) The energy price at root node (π_t^e), b) the negative contribution of the losses (π_t^{loss}), c) the voltage component ($\pi_t^{|u|}$), and d) the line congestion component in "From" and "To" directions ($\pi_t^{s^{f/t}}$). Up to this point, the MPEC formulation of the strategic DSO market is obtained. The mixed complementarity slackness KKT conditions in MPEC that are associated with the inequality constraints are bi-linear terms which are linearized by adopting Fortuny-Amat (also known as Big-M) method [16]. However, the term $p_t^{sg}\pi_t^{sg}$ in the objective function MPEC problem is a bi-linear term that is problematic. By multiplying the explicit formulation of DLMP from the DER prospective, eq. (8), and summing them up the bi-linear term can be retrieved in other forms as in eq. (10)

$$\sum_{t\in\mathcal{T}}\pi_t^{p^{sg},DER}p_t^{sg}=\sum_{t\in\mathcal{T}}\Big\{c_t^{offer,p^{sg}}p_t^{sg}+\left(\overline{\mu}_t^{p^{sg}}-\underline{\mu}_t^{p^{sg}}\right)p_t^{sg}\Big\}\qquad(10a)$$

Term $c_t^{offer,p^{sg}}p_t^{sg}$ belongs to the objective function of DSO problem in vector product form, i.e. $(\mathbf{c}_t^{p,DER*})^{\mathrm{T}}\mathbf{p}_t$. Knowing the convexity of the DSO problem, the strong duality theorem discloses that the solution for the primal problem's objective function is equal to the dual problem objective function at the optimum point [5]. Moreover, the complementarity slackness for the SG's operation limit as a part of the DSO problem's KKT conditions can be written as eq. (11).

$$\underline{\mu}_t^{p^{sg}},\overline{\mu}_t^{p^{sg}}\geq 0\quad\forall t\in\mathcal{T}\qquad(11a)$$

$$\left(\underline{p_t^{sg}}-p_t^{sg}\right)\underline{\mu}_t^{p^{sg}}=0\quad\forall t\in\mathcal{T}\qquad(11b)$$

$$\left(-\overline{p_t^{sg}}+p_t^{sg}\right)\overline{\mu}_t^{p^{sg}}=0\quad\forall t\in\mathcal{T}\qquad(11c)$$

Considering MCP eqs. (11b) and (11c) it turns out that at the optimum point, the relationship denoted in eq. (12) holds.

$$p_t^{sg}\underline{\mu}_t^{p^{sg}}=\underline{p}_t^{sg}\underline{\mu}_t^{p^{sg}}\quad\forall t\in\mathcal{T}\qquad(12a)$$

$$p_t^{sg}\overline{\mu}_t^{p^{sg}}=\overline{p}_t^{sg}\overline{\mu}_t^{p^{sg}}\quad\forall t\in\mathcal{T}\qquad(12b)$$

By involving the findings in eqs. (10) and (12) and engaging them with the dual objective function of DSO problem, the MPEC problem with the equivalent linearized objective function can be formulated as:

$$\begin{aligned}&\underset{\substack{\mathbf{x}_t,\mathbf{u}_t,\mathbf{y}_t\\p_t^{loss},q_t^{loss},\mathbf{p}_t,\mathbf{q}_t\\c_t^{offer,p^{sg}},\pi_t^{sg},\boldsymbol{\Lambda}_t,\boldsymbol{\Phi}_t,\boldsymbol{\Psi}_t}}{Maximize}\quad\sum_{t\in\mathcal{T}}\Big\{p_t^{sg}\pi_t^{sg}-p_t^{sg}c_t^{mrg,p^{sg}}\Big\}=\\&-\sum_{t\in\mathcal{T}}\Big\{c_t^{p,0}p_t^{loss}-c_t^{p,0}\mathbf{1}^{\mathrm{T}}\mathbf{p}_t+\left(\mathbf{c}_t^{p,DER*}\right)^{\mathrm{T}}\mathbf{p}_t\\&\qquad+c_t^{q,0}q_t^{loss}-c_t^{q,0}\mathbf{1}^{\mathrm{T}}\mathbf{q}_t+\left(\mathbf{c}_t^{q,DER}\right)^{\mathrm{T}}\mathbf{q}_t\Big\}\\&+\sum_{t\in\mathcal{T}}\Big\{\overline{p_t^{sg}}\overline{\mu}_t^{p^{sg}}-\underline{p_t^{sg}}\underline{\mu}_t^{p^{sg}}\Big\}\\&+(-\hat{\mathbf{x}})^{\mathrm{T}}\boldsymbol{\lambda}^{x0}\\&+\sum_{t\in\mathcal{T}-\{t^n\}}(-\mathbf{v}_t^x)^{\mathrm{T}}\boldsymbol{\lambda}_t^x+\sum_{t\in\mathcal{T}}(-\mathbf{v}_t^y)^{\mathrm{T}}\boldsymbol{\lambda}_t^y\\&+\sum_{t\in\mathcal{T}}\Big\{\left(-\hat{\mathbf{p}}_t^{fixed}\right)^{\mathrm{T}}\boldsymbol{\lambda}_t^{p,y}+\left(-\hat{\mathbf{q}}_t^{fixed}\right)^{\mathrm{T}}\boldsymbol{\lambda}_t^{q,y}\Big\}\\&+\sum_{t\in\mathcal{T}}\Big\{\left(\underline{\mathbf{y}}_t\right)^{\mathrm{T}}\underline{\boldsymbol{\mu}}_t^y+(-\overline{\mathbf{y}}_t)^{\mathrm{T}}\overline{\boldsymbol{\mu}}_t^y\Big\}\\&+\sum_{t\in\mathcal{T}}\Big\{\left(-\hat{p}_t^{loss}\right)^{\mathrm{T}}\lambda_t^{p^{loss}}+\left(-\hat{q}_t^{loss}\right)^{\mathrm{T}}\lambda_t^{q^{loss}}\Big\}\\&+\sum_{t\in\mathcal{T}}\Big\{(|\underline{\mathbf{u}}_t|-|\hat{\mathbf{u}}_t|)^{\mathrm{T}}\underline{\boldsymbol{\mu}}_t^{|u|}+(-|\overline{\mathbf{u}}_t|+|\hat{\mathbf{u}}_t|)^{\mathrm{T}}\overline{\boldsymbol{\mu}}_t^{|u|}\Big\}\\&+\sum_{t\in\mathcal{T}}\Big\{\left(|\underline{\mathbf{s}_t^f}|-|\hat{\mathbf{s}}_t^f|\right)^{\mathrm{T}}\underline{\boldsymbol{\mu}}_t^{|s^f|}+\left(-|\overline{\mathbf{s}_t^f}|+|\hat{\mathbf{s}}_t^f|\right)^{\mathrm{T}}\overline{\boldsymbol{\mu}}_t^{|s^f|}\Big\}\\&+\sum_{t\in\mathcal{T}}\Big\{\left(|\underline{\mathbf{s}_t^t}|-|\hat{\mathbf{s}}_t^t|\right)^{\mathrm{T}}\underline{\boldsymbol{\mu}}_t^{|s^t|}+\left(-|\overline{\mathbf{s}_t^t}|+|\hat{\mathbf{s}}_t^t|\right)^{\mathrm{T}}\overline{\boldsymbol{\mu}}_t^{|s^t|}\Big\}\qquad(13a)\\&s.t.\quad\textit{constraints of MPEC, eqs. (6b) and (6c)}\qquad(13b)\end{aligned}$$

The linearized MPEC (13) constitute a MILP problem that can be solved by off-the-shelf solvers.

5. Integrating P2P Market via ADMM

Thanks to the advancements in IoT and blockchain sector, prosumers can enjoy sharing their excess energy in their own community. In this way, they will have the opportunity to liberally trade energy in a bilateral P2P manner while preserving their local information. Therefore, they will pay more premium on investments in green energy [17].

Even though the energy transaction in the P2P market takes place independently in a distributed manner, the DSO has to supervise the grid operational limits and assign grid usage price (GUP) for the peers in the DN [3]. The GUP can be derived from the DLMPs as the difference in the energy price (DLMP) for the buyer and the seller in the DN. Hence, the whole GUP matrix for the buyer and seller peers in the DN can be assigned by DSO as:

$$\mathbf{\Pi}_t^p = \mathbf{1}_{s,t}(\boldsymbol{\pi}_{b,t}^p)^{\mathrm{T}} - \boldsymbol{\pi}_{s,t}^p(\mathbf{1}_{b,t})^{\mathrm{T}} \qquad \forall t \in \mathcal{T} \tag{14a}$$

where $\mathbf{\Pi}_{s,b,t}^p \in \mathbb{R}^{\mathcal{S}\times\mathcal{B}}$, and $\mathcal{S}$ indicates the sets of seller peers and $\mathcal{B}$ represents the buyer ones.

$$\begin{aligned}
\underset{\mathbf{p}_t^s,\mathbf{p}_t^b,\mathbf{E}_t}{Max} \; & -\sum_{t\in\mathcal{T}}\Big\{(\mathbf{c}_t^s)^{\mathrm{T}}\mathbf{p}_t^s - (\mathbf{c}_t^b)^{\mathrm{T}}\mathbf{p}_t^b \\
& +\mathbf{1}_s^{\mathrm{T}}\cdot(\mathbf{\Pi}_t \circ \mathbf{E}_t)\cdot\mathbf{1}_b\Big\} && \text{(15a)}\\
\text{s.t.} \quad & \mathbf{E}_t\mathbf{1}_b = \mathbf{p}_t^s \quad : \boldsymbol{\lambda}_t^s \in \mathbb{R}^{\mathcal{S}} \quad \forall t \in \mathcal{T} && \text{(15b)}\\
& (\mathbf{E}_t)^{\mathrm{T}}\mathbf{1}_s = \mathbf{p}_t^b \quad : \boldsymbol{\lambda}_t^b \in \mathbb{R}^{\mathcal{B}} \quad \forall t \in \mathcal{T} && \text{(15c)}\\
& \underline{\mathbf{p}_t^s} \le \mathbf{p}_t^s \le \overline{\mathbf{p}_t^s} \quad : \underline{\boldsymbol{\mu}_t^s}, \overline{\boldsymbol{\mu}_t^s} \in \mathbb{R}^{\mathcal{S}} \quad \forall t \in \mathcal{T} && \text{(15d)}\\
& \underline{\mathbf{p}_t^b} \le \mathbf{p}_t^b \le \overline{\mathbf{p}_t^b} \quad : \underline{\boldsymbol{\mu}_t^b}, \overline{\boldsymbol{\mu}_t^b} \in \mathbb{R}^{\mathcal{B}} \quad \forall t \in \mathcal{T} && \text{(15e)}\\
& \mathbf{E}_t \ge \mathbf{0} \quad : \mathbf{\Omega}_t \in \mathbb{R}^{\mathcal{S}\times\mathcal{B}} \quad \forall t \in \mathcal{T} && \text{(15f)}
\end{aligned}$$

The optimization problem given in eq. (15) denotes the enteric optimization problem for the P2P market. In this problem, the matrix $\mathbf{E}_t \in \mathbb{R}^{\mathcal{S}\times\mathcal{B}}$ denotes the energy transaction among the sellers and buyers. This particular variable links the optimization problems of the buyers and sellers as a complicating variable. Therefrom, if it is relaxed, the problem can be decomposed into multiple optimization problems to be solved by ADMM iteratively [18]. Accordingly, the seller and buyer ADMM Lagrangian subproblems can be solved in coordination with DSO by adopting algorithm 1.
where considering a generic seller node $i \in \mathcal{S}$ the given augmented Lagrangian function for this particular peer can be formulated as eq. (20).

$$\begin{aligned}
L_{i,t}^{s,admm}\left(p_{i,t}^s, \mathbf{e}_{i,*,t}\right) = \sum_{t\in\mathcal{T}} \Big\{ & c_{i,t}^s p_{i,t}^s + \frac{1}{2}\mathbf{e}_{i,*,t}\mathbf{\Pi}_{i,*,t}^{\mathrm{T}} \\
& +\mathbf{\Lambda}_{i,*,t}\left(\mathbf{e}_{i,*,t} - \mathbf{e}_{i,*,t}^{+}\right)^{\mathrm{T}} \\
& +\frac{1}{2}\rho\left(\mathbf{e}_{i,*,t} - \mathbf{e}_{i,*,t}^{+}\right)\left(\mathbf{e}_{i,*,t} - \mathbf{e}_{i,*,t}^{+}\right)^{\mathrm{T}}\Big\}
\end{aligned} \tag{20a}$$

In a similar way, the augmented Lagrangian Function for the buyer DERs can be developed by:

$$\begin{aligned}
L_{j,t}^{b,admm}\left(p_{j,t}^s, \mathbf{e}_{*,j,t}\right) = \sum_{t\in\mathcal{T}} \Big\{ & -c_{j,t}^b p_{j,t}^b + \frac{1}{2}\mathbf{e}_{*,j,t}\mathbf{\Pi}_{*,j,t}^{\mathrm{T}} \\
& +\mathbf{\Lambda}_{*,j,t}\left(\mathbf{e}_{*,j,t} - \mathbf{e}_{*,j,t}^{+}\right)^{\mathrm{T}} \\
& +\frac{1}{2}\rho\left(\mathbf{e}_{*,j,t} - \mathbf{e}_{*,j,t}^{+}\right)\left(\mathbf{e}_{*,j,t} - \mathbf{e}_{*,j,t}^{+}\right)^{\mathrm{T}}\Big\}
\end{aligned} \tag{21a}$$

Algorithm 1 ADMM solution algorithm for P2P market problem.

1: DSO Announces an initial GUP to the peers.
2: **while** *ADMMconverged* **do**
3: Maximize the social welfare sub-problems of the seller and buyer peers in parallel:
for seller i:

$$\begin{aligned}
\hat{p}_{i,t}^{s(k+1)}, \hat{\mathbf{e}}_{i,*,t}^{(k+1)} &:= \text{argmin } L_{i,t}^{s,admm}\left(p_{i,t}^s, \mathbf{e}_{i,*,t}\right) \\
\text{s.t.} \quad \mathbf{e}_{i,*,t}\mathbf{1}_b = p_{i,t}^s & \quad : \lambda_{i,t}^s \quad \forall t \in \mathcal{T} \\
\underline{p}_{i,t}^s \le p_{i,t}^s \le \overline{p}_{i,t}^s & \quad : \underline{\mu}_{i,t}^s, \overline{\mu}_{i,t}^s \quad \forall t \in \mathcal{T} \\
\mathbf{e}_{i,*,t}^{\mathrm{T}} \ge \mathbf{0} & \quad : \mathbf{\Omega}_{i,*,t} \quad \forall t \in \mathcal{T}
\end{aligned}$$

for buyer j:

$$\begin{aligned}
\tilde{p}_{j,t}^{b(k+1)}, \tilde{\mathbf{e}}_{*,j,t}^{(k+1)} &:= \text{argmin } L_{j,t}^{b,admm}\left(p_{j,t}^b, \mathbf{e}_{*,j,t}\right) \\
\text{s.t.} \quad \mathbf{e}_{*,j,t}^{\mathrm{T}}\mathbf{1}_s = p_{j,t}^b & \quad : \lambda_{j,t}^b \quad \forall t \in \mathcal{T} \\
\underline{p}_{j,t}^b \le p_{j,t}^b \le \overline{p}_{j,t}^b & \quad : \underline{\mu}_{j,t}^b, \overline{\mu}_{j,t}^b \quad \forall t \in \mathcal{T} \\
\mathbf{e}_{*,j,t} \ge \mathbf{0} & \quad : \mathbf{\Omega}_{*,j,t} \quad \forall t \in \mathcal{T}
\end{aligned}$$

4:
- Seller i broadcasts $\hat{\mathbf{e}}_{i,*,t}^{(k+1)}$ to all buyers and $\hat{p}_{i,t}^{s(k+1)}$ to the DSO;
- Buyer j broadcasts $\tilde{\mathbf{e}}_{*,j,t}^{(k+1)}$ to all sellers and $\tilde{p}_{j,t}^{b(k+1)}$ to the DSO;

5: DSO solves AC-OPF by acquiring $\hat{p}_{i,t}^{s(k+1)}$ and $\tilde{p}_{j,t}^{b(k+1)}$ and updates GUP.
6: Peers update the local copies for their energy transaction:

$$\begin{aligned}
\mathbf{e}_{i,*,t}^{+(k+1)} &= \frac{1}{2}\left(\hat{\mathbf{e}}_{i,*,t}^{(k+1)} + \tilde{\mathbf{e}}_{i,*,t}^{(k+1)}\right) \quad \forall t \in \mathcal{T} \\
\mathbf{e}_{*,j,t}^{+(k+1)} &= \frac{1}{2}\left(\hat{\mathbf{e}}_{*,j,t}^{(k+1)} + \tilde{\mathbf{e}}_{*,j,t}^{(k+1)}\right) \quad \forall t \in \mathcal{T}
\end{aligned}$$

7: Update Lagrangian multipliers:

$$\begin{aligned}
\mathbf{\Lambda}_{i,*,t}^{(k+1)} &= \mathbf{\Lambda}_{i,*,t}^{(k)} + \rho\left(\hat{\mathbf{e}}_{i,*,t}^{(k+1)} - \mathbf{e}_{i,*,t}^{+(k+1)}\right) \quad \forall t \in \mathcal{T} \\
\mathbf{\Phi}_{*,j,t}^{(k+1)} &= \mathbf{\Phi}_{*,j,t}^{(k+1)} + \rho\left(\tilde{\mathbf{e}}_{*,j,t}^{(k+1)} - \mathbf{e}_{*,j,t}^{+(k+1)}\right) \quad \forall t \in \mathcal{T}
\end{aligned}$$

8: **end while**

For the seller problem $\mathbf{e}_{i,*,t}, \mathbf{\Pi}_{i,*,t} \in \mathbb{R}^{\mathcal{S}}$ stand for i-th row of the energy transaction and GUP matrices, respectively. The factor $\frac{1}{2}$ in $\frac{1}{2}\mathbf{e}_{i,*,t}\mathbf{\Pi}_{i,*,t}^{\mathrm{T}}$ term is due to dividing the GUP into two parts for seller and buyer peers. The equality constraints' dual variables of the whole problem are encapsulated in $\mathbf{\Lambda}_t \in \mathbb{R}^{\mathcal{S}\times\mathcal{B}}$ which characterises the information regarding the energy trade price and it should be identical for all seller and buyers upon the convergence. In order to solve the ADMM subproblems iteratively a container parameter for storing the energy transaction results at each iteration is designated to be $\mathbf{E}_t^{+}$, the energy transaction local copy, which upon the consensus it is going to be equal to $\mathbf{E}_t$. Last but not least, ρ is the penalty factor constant adopted for the ADMM approach.

6. Case Study and Results

To validate the theoretical findings, the IEEE 34-node DN test feeder with multi-phase unbalanced configuration, shown in fig. 2 is adopted as the case study [19]. Accordingly, the location and capacity of the PV generators and FL emerged from the same reference.

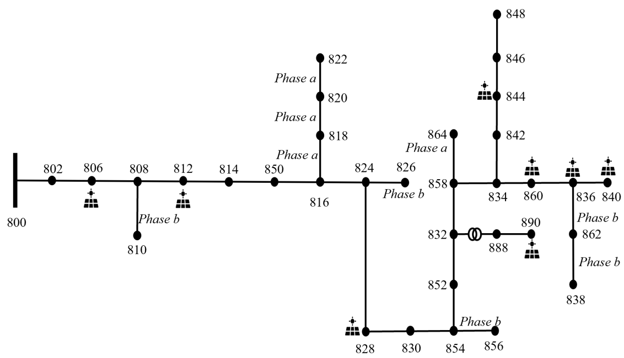

Figure 2: IEEE 34-node unbalanced DN test feeder.

As a potential FG downstream of the feeder, the strategic behaviour of the PV generator located at node 860 will be investigated. The problem was formulated in the Python environment via the MESMO tool, and the adopted solver was GUROBI [20]. Overall, the computational burden of the MPEC problem was enormous and to deal with that, the simulation was conducted for a limited time steps during the peak demand and generation hours, i.e. 1-3 PM.

Comparing the strategic and non-strategic scenarios, the following results were obtained for the SG's behaviour fig. 3.

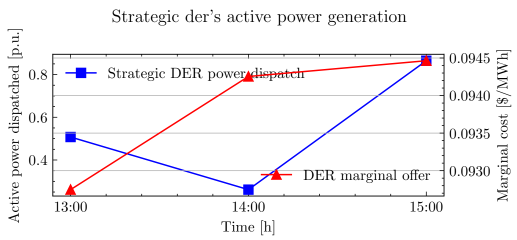

Figure 3: Strategic behaviour of SG.

Genuinely, the marginal cost of SG is defined as 0.09 \$/kWh. However, by adopting a strategic behaviour the respective DER can submit offers higher than its actual marginal cost, untruthfully. On the other hand, the DSO as the market organizer reduces its dispatch level. Nevertheless, for DSO to keep the system operating in the optimal condition, it is obliged to increase the dispatch regardless of the higher offers. According to fig. 4 which is reported for 2 PM, it is evident that such behaviour will also impact the output of other DERs.

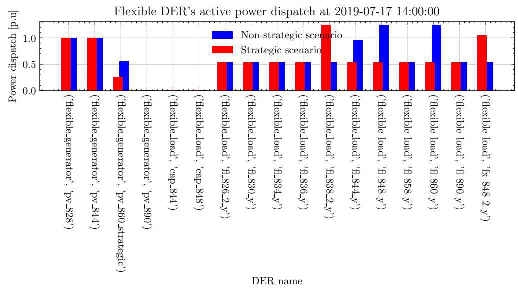

Figure 4: Flexible DERs' active power dispatch at 2 PM.

Since the voltage profile and line loading for the DN depend on the DERs' power dispatch, they are expected to take different values in each scenario.

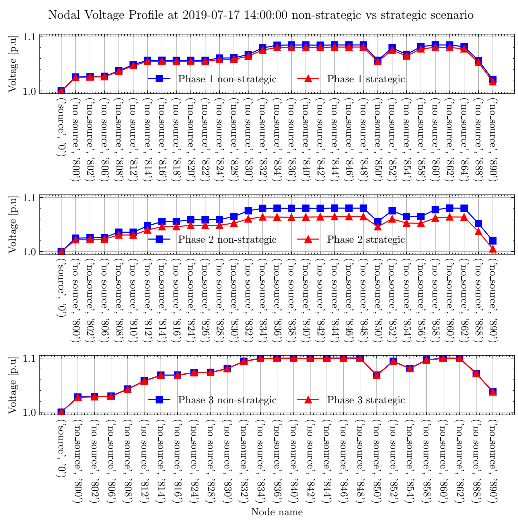

Figure 5: Voltage profile of different phases at 2 PM.

As regards in fig. 5, in the strategic scenario, the voltage profile has reduced throughout the feeder. Noting the DLMP formulation, eq. (9), since the voltage is binding to its maximum level over the feeder in the non-strategic scenario, it will lead to a negative contribution to the DLMP. In the strategic scenario, the voltage is drooped to its normal range; therefore, it is expected that the DLMP will increase in favour of SG.

Since the thermal capacity of lines is high, no congestion happens in the feeder. Hence, it will not impact the DLMPs. So, the three-phase line loading comparison is not provided in the manuscript.

The component of the nodal DLMP for the third phase is provided in fig. 6. Respectively, in the non-strategic scenario, its contribution to the DLMP has been negative due to the over-voltage. However, in the strategic scenario, the voltage is dropped, and

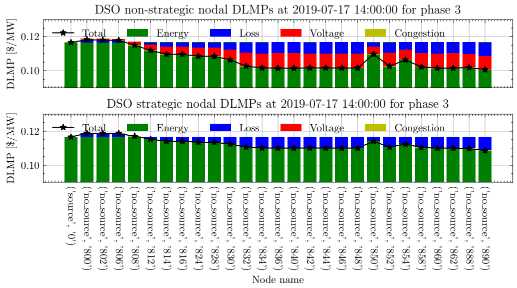

Figure 6: DLMP components phase 3 at 2 PM.

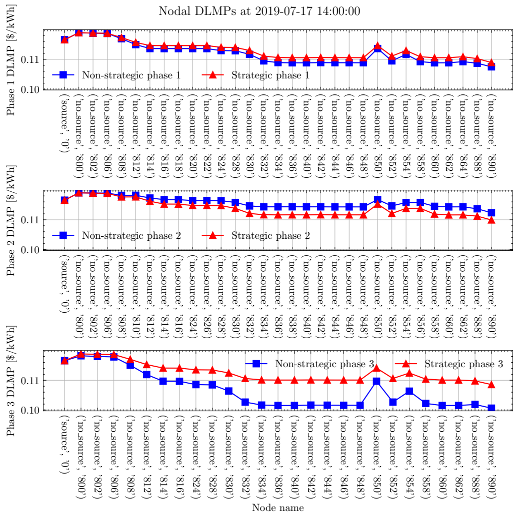

Figure 7: Nodal DLMPs at 2 PM.

the DLMP is increased to benefit the SG. Moreover, it is clear that the losses have subjected the DLMPs negatively. The nodal DLMPs for the rest of the phases are given in fig. 7 where in the second phase, the strategic behaviour has reduced the DLMP slightly, but since the strategic DER has a three-phase Y connection to the grid, it will enjoy the overall high DLMP.

With regards to the impact of the strategic behaviour of SG in the DSO market on the P2P market it is essential to note that the GUP is a derivative of the DLMPs. Therefore, any manipulation in DLMP will be projected to the GUP. To this end, the GUP in both scenarios is demonstrated in fig. 8.

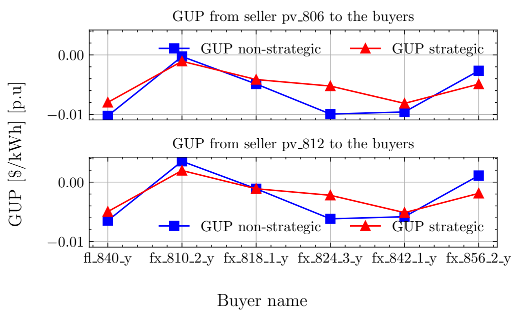

Figure 8: GUP for P2P market at 2 PM.

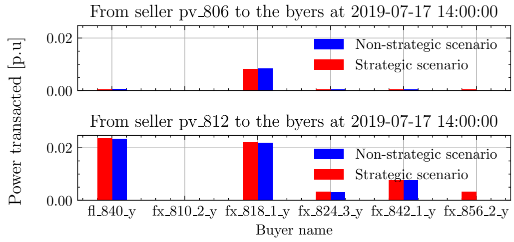

Figure 9: Energy transaction in P2P market at 2 PM.

The most significant fact to conclude from the GUP and the energy transaction plots, figs. 8 and 9, are that since the DLMP is lower downstream of the feeder, the respective GUP from the buyer's perspective will be lower. Therefore, the buyer nodes tend to procure energy from "pv_812" which has lower GUP from the buyer perspective. Moreover, the strategic behaviour of SG has led to a noticeable difference in energy transactions among the peers, especially for "fx_865_2_y" DER, which is located well in the tail of the feeder.

As explained in algorithm 1, the P2P market is coordinated with the DSO market in a fully distributed manner via ADMM. Figure 10 demonstrate how the ADMM has converged to the obtained results in 40 iterations in strategic and non-strategic scenarios.

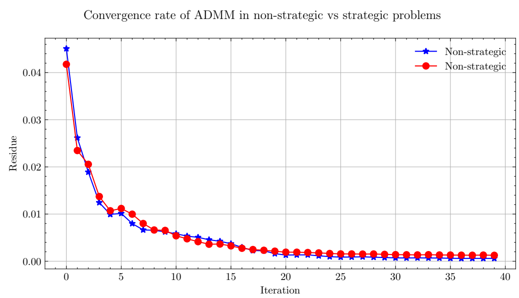

Figure 10: ADMM overall convergence rate.

Accordingly, since the problem of the strategic market is a mixed-integer linear MPEC, the ADMM convergence ratio is slightly lower compared to the DSO non-strategic linear problem. Overall, the problem in the strategic scenario takes longer to solve due to its complexity and the significant number of real and integer variables.

7. Conclusion

Since the R/X ratio is higher in the DN, we saw that the active power injection of DERs will significantly impact the voltage magnitude and loading of the lines. Therefore, in this work, the objective was to investigate the strategic behaviour of an FG downstream feeder in three-phase unbalanced DN via bi-level programming. We saw that the SG submits offers higher than its real marginal cost in the upper-level problem. The DSO, as the lower-level problem, cleared the market and calculated DLMPs with an increase in its level compared to the non-strategic scenario. The main component impacting the level of DLMPs in both scenarios was the voltage drop along the feeder. Accordingly, we also investigated the effect of such strategic behaviour on the P2P market, which is in coordination with the DSO AS market through the GUP, which is the derivative of DLMP. Hence, it was proved that any deviation on the DLMP was projected on the GUP. Consequently, the energy transaction among the peers was also affected.

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