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EXECUTIVE SUMMARY OF THE THESIS

4D Printing of PLA rigid structures onto pre-stretched Lycra fabrics

TESI MAGISTRALE IN MATERIALS ENGINEERING AND NANOTECHNOLOGY – INGEGNERIA DEI MATERIALI E DELLE NANOTECNOLOGIE

AUTHOR: SIMONE BATTISTI

ADVISOR: PROF. STEFANO MARIANI

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1. Introduction

3D printing technology is rapidly growing in the manufacturing world. This technology allows to produce complex shapes reducing production time and material waste. The need to obtain functional products has prompted research to exploit time as a fourth dimension[1]. Environmental factors or electrical stimuli can be used as external stimuli to activate functional structures able to change shape, giving more comfort or generating an advantage in their specific application. The aim of this work is to give insights on this novel approach, focusing on the coupling behavior of rigid structures onto pre-stretched soft tissues[2]. Voron 2.4 has been chosen for its high performances and the possibility to have a fixed bed and an extruder able to move freely in the printing volume. It has been assembled and programmed, before starting with calibrations and tests. Lycra, a polymeric fiber used for garments, is used as the soft substrate. Corrado tessuti, a small business based in Seveso (MB), provided such fabric. SunLu PLA filament is instead used as rigid geometry applied onto the pre-stretched Lycra fabric. The novelty of this approach led us to consider PLA as an elastic isotropic material, while for Lycra fabrics the Arruda-Boyce hyperelastic model has been chosen. This allowed us to obtain a good analogy between the simulations and the tests performed. The first part of this work focuses on Fused Deposition Modelling (FDM) technology, highlighting its advantages, on the 3D printer used and on the Taguchi's method, an unconventional but effective method for tuning the properties, to find the best printing parameters. Sections 3 and 4 give a description of the mechanical models used for this study and the materials. Bolzano's theorem is used to find the fabrics properties by convergence of the tests. Lastly, a comparison between the obtained results and the performed tests is given, together with future applications suggestions for the reader.

2. Fused Deposition Modelling

FDM (Fused Deposition Modelling) is based on the principle of extrusion. A solid thermoplastic filament is fed into a heated die called an extruder. The extruder contains a heating element that melts the filament, while a motor-driven mechanism pushes the molten material through a small nozzle with precise control. The nozzle moves along the X, Y and Z axes, depositing the molten material layer by layer onto a build platform. FDM technology offers several advantages, including affordability, accessibility, and ease of use.

2.1. Voron 2.4

Voron 2.4 (Figure 1) is an open source, high performance and precision 3D printer known for its robust design, modularity, and attention to detail, making it a popular choice for makers who value accuracy and reliability in their prints.

Figure 1: Voron 2.4 3D printer

Voron 2.4 3D printer is typically built using high quality components, including linear rails, CoreXY motion systems and a rigid frame, all of which contribute to the stability of the printer and consistent results. The open-source nature of the Voron project allows users to modify and improve the design, resulting in an active and engaged community that contributes to its ongoing development.

2.2. Taguchi's Robust Design

Taguchi's method, also known as Taguchi Robust Design[3], is a statistical approach developed by Japanese engineer and statistician Dr Genichi Taguchi. The primary objective of Taguchi's method is to minimize the variability in the performance of a process or product, making it more robust and less sensitive to external factors.

Taguchi's method is based on experimental design and statistical analysis. It involves the systematic variation of input factors (also known as control factors) to understand their effects on the response of the output (also known as the quality characteristic). In doing so, it helps to identify the optimal combination of control factors that will lead to the desired output while minimizing the effects of uncontrollable factors (also known as noise factors).

In this work we analyzed the effect of 3 control factors to increase the adhesion of the first layer onto the substrate. The parameters are: wall thickness, first layer speed and top/bottom pattern.

Parameter	Level 1	Level 2	Level 3
Wall thickness (mm)	0	0.4	0.8
First layer speed (mm/s)	20	40	60
Top/Bottom pattern	Lines	Concentric	Zig Zag

Table 1: Taguchi's Robust Design control factors.

The best combination of the control factors is:

- Wall thickness: 0,8 mm
- First layer speed: 20 mm/s
- Top/Bottom pattern: Concentric

Once the best 3D printing parameters are defined, the setup for the experimentation is ready to be tested.

3. Constitutive models

In this work, two constitutive models have been analyzed. Despite the layered nature of 3D printed objects, we considered the rigid structure as an elastic isotropic material due to the very low thickness (only 0,6 mm). Soft tissues are instead considered as hyperelastic materials. We chose the Arruda-Boyce constitutive model for its effectiveness of capturing the time-independent material behavior subjected to large strains.

3.1. Elastic isotropic material

Hooke's law is the fundamental concept of mechanics to describe the behavior of materials under load. It applies to homogeneous and isotropic materials, where mechanical properties are the same in all directions.

The material's isotropic stiffness can be described by the Lamè constants, which relates to the Young's Modulus (E) and the Poisson's ratio (ν) through Equations (1) and (2):

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \tag{1}$$

$$\mu = \frac{E}{2(1+\nu)} \tag{2}$$

where λ represents the bulk modulus, related to volumetric changes under stress and μ represents the shear modulus, related to deformation without volume change.

The elasticity tensor components for an isotropic material are given by Equation (3):

$$C_{ij} = \lambda\delta_{ij}\delta_{kl} + 2\mu\delta_{ik}\delta_{jl} \tag{3}$$

where δ_{ij} is the Kronecker delta (1 if i=j, 0 if i≠j), and δ_{kl} is the Kronecker delta for the other pair of indices.

We can define a Reduced Elasticity Tensor in Equation (4):

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix} \tag{4}$$

The elastic isotropic relation is then shown in Equation (5):

$$\sigma = C\varepsilon \tag{5}$$

3.2. Arruda-Boyce hyperelastic model

The Arruda-Boyce model [4] is a hyperelastic constitutive model commonly used in computational methods for simulating the mechanical behaviour of materials, particularly elastomers and polymers. This model is used to describe the non-linear, time-independent deformation response of these materials under various loading conditions.

The model is typically defined in terms of the strain-energy density function, which characterizes the relationship between the applied deformation and the corresponding stress response. The strain energy density function, W, is a measure of how much energy is stored in the material when it is deformed.

To define the strain-energy density function, we first have to define the first invariant of the deformation tensor, I_1, both for the uniaxial deformation in Equation (6), and for the equibiaxial deformation, in Equation (7):

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = \lambda^2 + \frac{2}{\lambda} \tag{6}$$

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 2\lambda^2 + \frac{1}{\lambda^4} \tag{7}$$

being λ the stretch parameters. Introducing the initial shear modulus, the material's resistance to shear deformation, in Equation (8) and the limiting network stretch parameter, related to the bulk modulus, in Equation (9):

$$\mu = 2C_{11} \tag{8}$$

$$\lambda_m = 2C_{22} \tag{9}$$

We can then define W, the strain-energy density function, in Equation (10):

$$W = \mu\left[\frac{1}{2}(I_1 - 3) + \frac{1}{20\lambda_m^2}(I_1^2 - 9) + \frac{11}{1050\lambda_m^2}(I_1^3 - 27) + \frac{19}{7000\lambda_m^2}(I_1^4 - 81) + \frac{519}{673750\lambda_m^2}(I_1^5 - 243)\right] + \frac{1}{D}\left(\frac{J^2 - 1}{2} - \ln J\right) \tag{10}$$

where D is the compressibility parameter and J the elastic volume ratio. By differentiating Equation (10) with respect to the components of the deformation tensor, we can develop the stress-strain relationship in Equation (11):

$$\sigma = \frac{\partial W}{\partial C}\frac{\partial C}{\partial F} = \left(\frac{\partial W}{\partial C_{11}} + \frac{\partial W}{\partial C_{22}} + \frac{\partial W}{\partial C_{33}}\right)F \tag{11}$$

4. Materials

In this section, a brief description of the material is given.

4.1. PLA

Polylactid acid (PLA) is a thermoplastic polymer that is biodegradable and compostable and is derived from renewable resources such as corn starch, sugar cane or other agricultural crops.

Lactic acid can be derived from fermenting plant sugars or chemically synthesized. The lactic acid monomers undergo a condensation reaction, resulting in the formation of long chains of PLA polymer. It offers a low heat resistance but moderate stiffness and strength, making it suitable for a wide range of applications which do not require a high durability or optimal performance. It's biodegradability make it more suitable for rapid prototyping, packaging, disposable cutlery and textiles.

In this work, despite the anisotropic nature given by the layered 3D printing technology, we considered PLA as an elastic isotropic material due to the very low thickness of just 0,6 mm. The properties considered are:

- $E = 3000$ MPa
- ν = 0,25

4.2. Lycra

Lycra, also known as spandex or elastane, is a synthetic fiber known for its exceptional stretch and recovery properties. Lycra revolutionized the textile industry by introducing a fabric that could stretch significantly without losing its shape.

Lycra is made from polyurethane, which is produced by reacting a polyester with a diisocyanate. The polymer is converted into a fiber using a dry spinning technique.

In this work, the hyperelastic Arruda-Boyce model is exploited to describe the fabric's mechanical behavior. A method for the tuning up of the property values is explained in Section 4.2.

4.3. Materials properties

In this section, the process of tuning up the properties of the materials is shown, starting from a simple beam printed onto a fixed deformation pre-stretched Lycra fabric, fixed PLA Young's modulus and Poisson's ratio.

The tuning of the properties of Lycra fabrics started with a fixed uniaxial deformation applied to the substrate (λ = 1,2), printing on top a small beam of fixed length (L = 50 mm) and width (w = 2 mm). The beam had a thickness of 0,6 mm (3 layers). PLA properties are fixed as stated in Section 4.1.

The resulting deflection was of about 12,6 mm, and the final shape is shown in Figure 2.

Figure 2: PLA beam tuning sample.

To find the Arruda-Boyce properties, the fabric is first treated as an elastic isotropic material with fixed Poisson's ratio at 0,45 to simulate a very high elasticity and rubber-like behavior under stress. Abaqus CAE simulation software was used to perform the convergence. The Theorem of Bolzano (i.e., the Zero Theorem) is then exploited for tuning.

For each step, Young's Modulus was changed to find convergence, as shown in Chart 1.

The difference with the deflection height goal is indicated by the numbers in the chart, while the deflection height goal is indicated by the discontinuous blue line.

The elastic isotropic properties of Lycra are then set to be:

- E = 26 MPa
- ν = 0,45

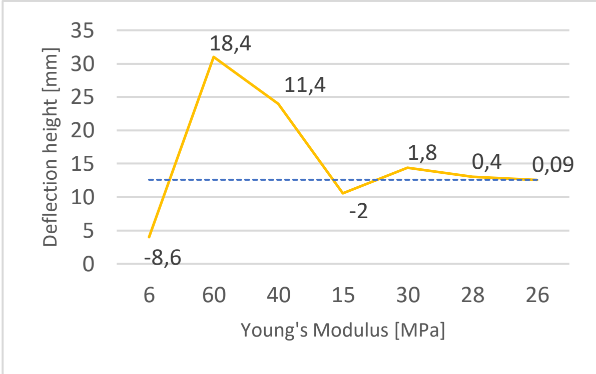

Chart 1: Bolzano theorem for convergence. In yellow, the deflection height for each step, while the deflection height goal is indicated by the discontinuous blue line.

From Equations (1) and (2), we can find then the Arruda-Boyce hyperelastic properties, keeping D, the compressibility parameter, set to 0:

- μ = 8,966 MPa
- λ_m = 86,667 MPa
- D = 0 (completely incompressible material)

being μ the shear modulus and λ_m the bulk modulus.

There's a slight overestimation in the model given by the simulation, with a deflection of 12,81 mm. The 1,4% error is considered small enough to accept the values found from the tuning up.

5. Experimental results

Different geometries have been analyzed in this work to better understand how the beams orientation influenced the final shape. The stretching parameter, λ, was kept fixed at 1,2 for each test. Uniaxial and biaxial tests were performed to better analyze how stresses were redistributed within the contact with the rigid structures. Abaqus CAE is the software used for running the simulations and provide comparative results for this study.

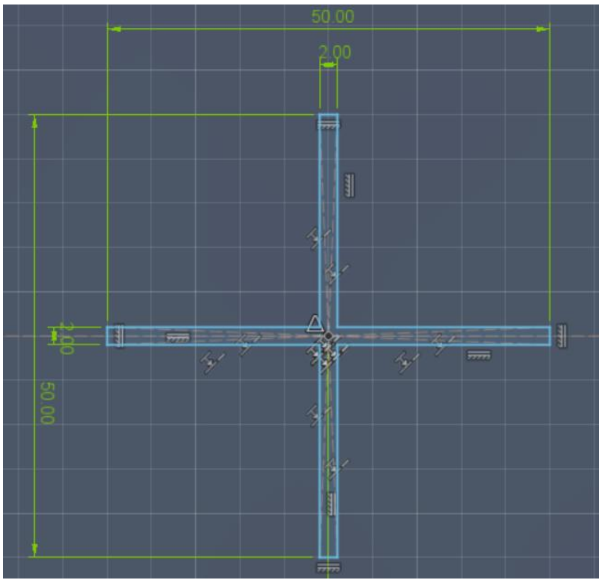

Figure 3: Cross section with the dimensions in mm (Fusion 360 CAD software)

Uniaxial tests gave good evidence between the simulations and the experimental results. The setup was well prepared and the deflections in the simulations followed the experiments.

Biaxial tests resulted in less precise match between the simulations and the results.

The cross section and the HSS (Hollow Structural Section - Square) section are showed, to provide the reader a general overview of the considerations done in this work. The two sections are schematized in Figure 3 and 4. The thickness was fixed to 0,6 mm.

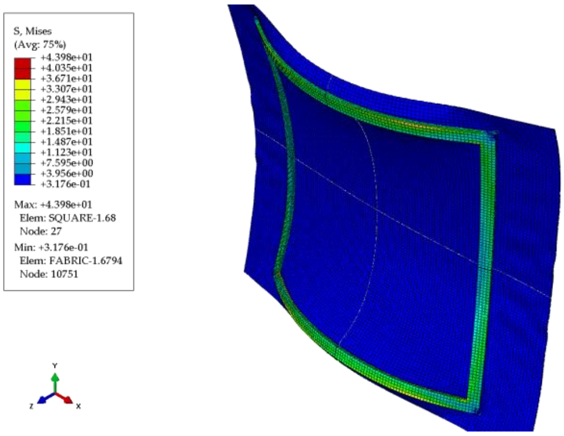

Figure 5: von Mises stresses for the HSS section, uniaxial pre-stretched fabric.

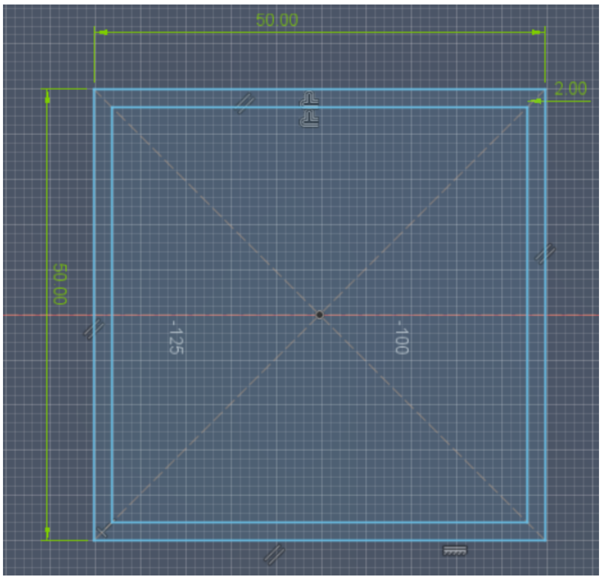

Figure 4: HSS section with the dimensions in mm (Fusion 360 CAD software)

The HSS section results are shown and compared for the uniaxial test. Figure 5 highlight a maximum von Mises stress of 43,98 MPa. The out-of-plane deflection gave 11,2 mm from the simulations, close to the real one of 10 mm, shown in Figure 6.

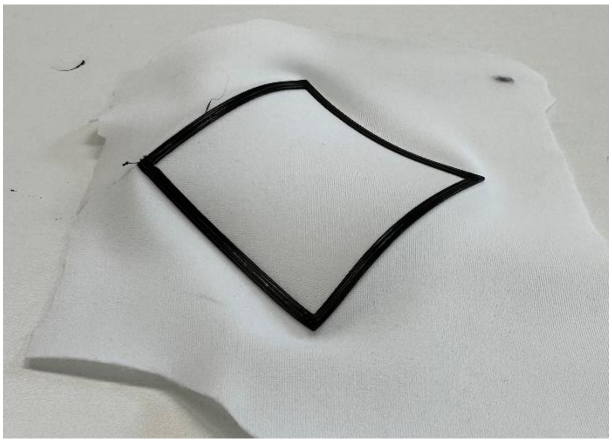

Figure 6: Experimental HSS section subjected to a uniaxial pre-stretching.

Chart 2 shows how the out-of-plane deflections are comparable between the simulations and the experimental tests.

The cross section showed a maximum von Mises stress of 129,9 MPa (Figure 7) and an

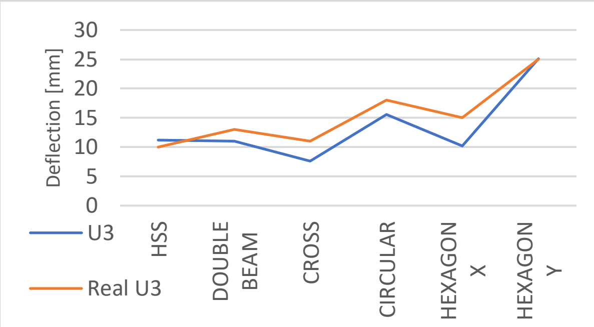

Chart 2: Out-of-plane deflections of the uniaxial tests performed.

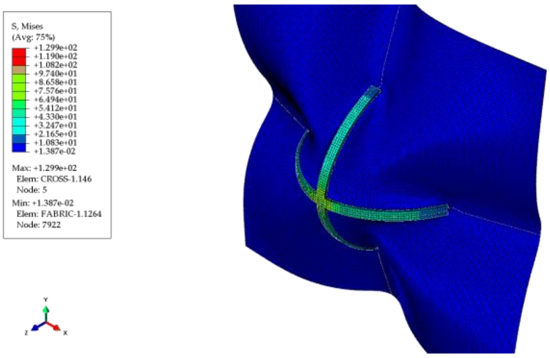

Figure 7: Mises stresses of the cross section, biaxial case.

out-of-plane deflection of 20,6 mm, close to the experimental one of 19 mm (Figure 8).

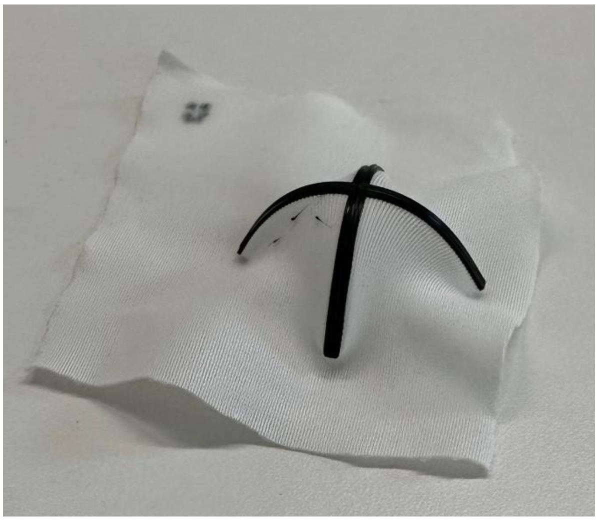

Figure 8: Experimental results of the cross section, biaxial pre-stretching case.

Chart 3 highlights how the out-of-plane deflections of the tested geometries show good evidence for the geometries with perpendicular beams, while for the circle and hexagon sections the error is big.

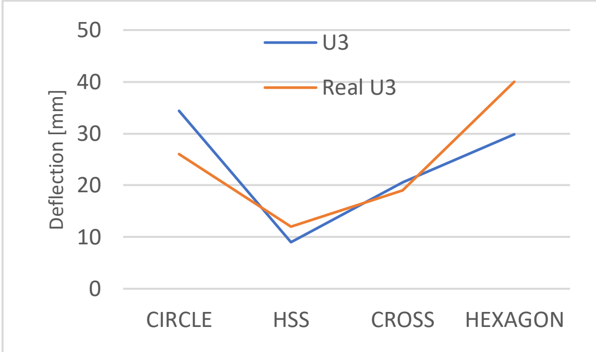

Chart 3: Out-of-plane deflections of the biaxial tests performed.

6. Conclusions

In this thesis, PLA 3D printed geometries were printed onto controlled pre-stretched Lycra fabrics. The coupling behavior of the rigid structures within the stress release of the pre-stretched soft tissues was studied. For large uniaxial strains, good evidence was found between the simulations and the experimental results, giving us positive feedback on the methods used to characterize the materials properties. The orientation of the beams with respect to the stretching direction gave us hints on their structural effect: perpendicular beams allow for a more controlled deflection, helping in assessing the best conformation for functionalize the structures. Biaxial tests showed a non-ideal mechanical model attribution, highlighting the limits of the Arruda-Boyce hyperelastic model to Lycra fabrics. The anisotropic nature of the material is in contrast with the isotropic Arruda-Boyce model: an overestimation of the shear modulus brought to a more rigid behavior of the fabric during the simulations, compromising the final results for geometries with curves or non-perpendicular beams.

This work is meant to be the reference for future development in the field of energy generation and fashion. By understanding how geometries affect the final shape of the objects, one can tune the parameters to create panels able to generate green energy (through the coupling with piezoelectric materials, exploiting, for example, sudden environmental changes such as wind and rain) or to create new original packaging designs or shapes for the fashion industry.

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