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Executive Summary of the Thesis

Extended Weighted Least Squares (EWLS) Estimator Performance Analysis

Laurea Magistrale in Electrical Engineering - Ingegneria Elettrica

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1. Introduction

Electric power system control relies heavily on an accurate evaluation of system states to maintain the reliability and continuity of the service. In order to evaluate such actual state of a working electric power system, it is necessary to use a State Estimation (SE) approach [4] based on a redundant set of data, often characterized by a certain degree of uncertainty, which the method then proceed to address through links between available data and state variables, derived from the given model itself. In this way, the redundancy of the data can be exploited to reduce the impact of its inaccuracy on the computed state.

Typically, the data given to an SE algorithm are either measurements directly from telemeters installed on the network, or pseudo-measurements from prior knowledge. However, regardless of the source, the presence of uncertainty can often be found in both categories: on-site measurement data suffer from the inaccuracy of the instruments, while those pre-defined in the design may possess error due to the geometrical placement of power lines, temperature deviation due to weather or loading conditions, or even potential unbalance in the system that were overlooked by the mathematical model. Therefore, the known parameters can be just inaccurate as measurement data are.

The classical Weighted Least Squares (WLS) approach gives an uncertain analysis of the measurement data based on a static power system model assumed to be exact (as in perfectly certain). This assumption risks a significant bias on the state, which can even rise further as the accuracy of measurement improves, as the proportion of parameters' contribution will increase. Though many effective network parameter estimation (PE) methods are proposed to improve SE performance [3] [5], they still do not directly account for the problem of including uncertain parameters in an SE method.

In this thesis, a WLS method proposed in [2] for SE is considered, accounting for uncertainty in both measurements and network parameters. A complex system model with artificially controlled uncertainty is adopted to extensively test the proposed method and a detailed comparison with the classical WLS is provided. With IEEE14 bus system served as an example, a Monte Carlo trial is performed to verify the effectiveness and robustness of aforementioned method, analyzed both graphically and numerically.

2. Problem Modelling

2.1. System Model

The system modelled established in this thesis uses a two-port π-model for transmission lines, describing the properties of line parameters by a combination of a series impedance $R + jX$ and the total line charging susceptance as shown in figure 1. Those line data are considered the primary uncertainty within the parameters in this work.

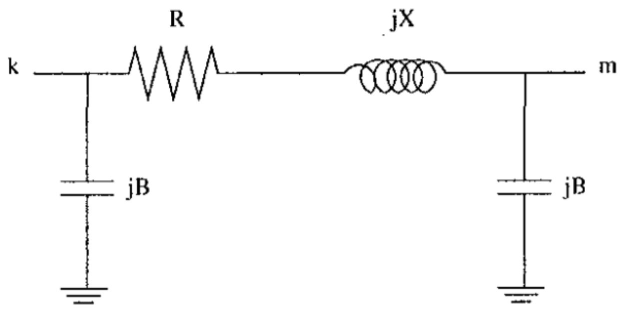

Figure 1: Transmission line model

Different measurement as well as pseudo-measurement data are computed and stored in a M-vector y, related to the state x and parameter π by their relationships expressing the structure of the power system:

$$y = f(x_t; \pi_t) \in \mathbf{R}^{M\times 1} \tag{1}$$

Taking into account the uncertainty present in known data the equation would become:

$$y - \Delta y = f(x_t; \pi - \Delta\pi) \tag{2}$$

where Δy, $\Delta\pi$ are the respective errors, and the subscript t indicates the true state.

Using a first-order Taylor series expansion it is possible to isolate the error contribution resulted from $\Delta\pi$:

$$y - \Delta y \approx f(x_t; \pi) - \left.\frac{\partial f}{\partial \pi^T}\right|_{x_t,\pi} \cdot \Delta\pi \tag{3}$$

which can then be reassembled into the following form used in the estimator:

$$r(x_t; d) \approx A(x_t; \pi) \cdot \Delta d \tag{4}$$

in which r is the misfit of measured y and actual y_t, Δd is the combined error of both data, and A is the intermediate matrix:

$$r(x_t; d) = y - f(x_t; \pi) \tag{5}$$

$$\Delta d = \begin{bmatrix} \Delta y \\ \Delta\pi \end{bmatrix} \tag{6}$$

$$A(x_t; \pi) = [I_M \quad -\frac{\partial f}{\partial \pi^T}] \in \mathbf{R}^{M\times(M+P)} \tag{7}$$

2.2. WLS Method

The classical weighted least square (WLS) method ignores the contribution of parameter errors, and may be formalized as an overdetermined problem as following:

$$\begin{aligned} &\underset{x,\Delta y}{argmin} |\Delta y|^2_{\Sigma y} \\ &s.t. \Delta y = y - f(x; \pi) \end{aligned} \tag{8}$$

where Σ_y is the variance-covariance matrix of the random measurement errors.

The problem is then solved using a Newton-Gauss iterative method

$$\tilde{x}_{k+1} = \tilde{x}_k + \Delta\tilde{x}_k \tag{9}$$

$$\Delta\tilde{x}_k = G^{-1}_{\tilde{x}_k} \left(\frac{\partial f}{\partial \tilde{x}^T}\right)_{\tilde{x}_k,\pi} \Sigma_y^{-1} \tilde{r} \tag{10}$$

$$G_{\tilde{x}_k} = \left(\frac{\partial f}{\partial \tilde{x}^T}\right)^T_{\tilde{x}_k,\pi} \Sigma_y^{-1} \left(\frac{\partial f}{\partial \tilde{x}^T}\right)_{\tilde{x}_k,\pi} \tag{11}$$

2.3. EWLS Method

The extended WLS method [2] on the other hand considers the error present in parameters and redefines the minimization problem as such:

$$\begin{aligned} &\underset{x,\Delta d}{argmin} |\Delta d|^2_{\Sigma d} \\ &s.t. r = A(x; \pi)\Delta d \end{aligned} \tag{12}$$

The variance-covariance matrix Σy of the data vector Δd is constructed from those of the measurements Σy and parameter data $\Sigma\pi$ as following, assuming measurements and parameter data are entirely uncorrelated:

$$\Sigma_d = \begin{bmatrix} \Sigma_y & 0_{M\times P} \\ 0_{P\times M} & \Sigma_\pi \end{bmatrix} \tag{13}$$

It is then possible to solve the redefined problem in the same way, only replacing the Σ_y by an intermediate matrix Q addressing both groups of uncertain data with the links associating them:

$$Q = A\Sigma_d A^T \tag{14}$$

3. Case Studies

A MATLAB simulation is run using the IEEE14 bus system [1] as an example, though may be easily extended to other similar systems. The test recognizing the official data as true values, firstly performs a regular load flow to obtain the actual state as well as perfect measurements of the system. Afterwards, it perturbs both measurement and parameter data with a stated variance, then feed the now inaccurate data set, along with a sufficiently good initial guess to the different estimators discussed in the previous chapters. A 1000-times Monte Carlo trial is performed for each case under its specified circumstances.

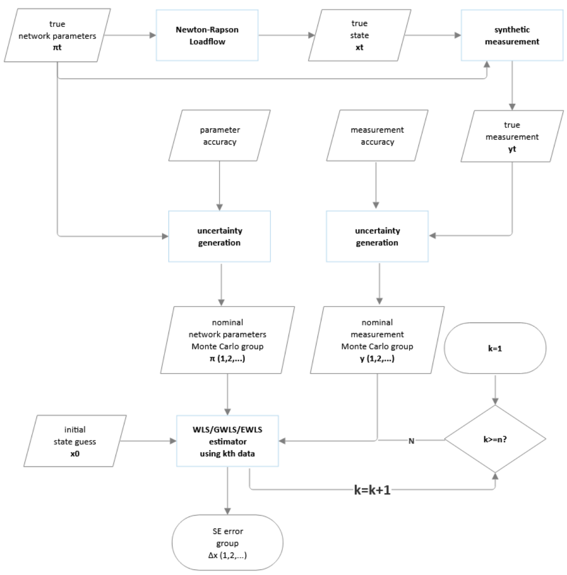

Figure 2: Flowchart of the simulation test

In one case where there is a nominal measurement error and almost no parameter errors involved, it is reported both methods can give reliable and similar results estimating the state. An example of estimate error on bus 2 state is shown in figure 3, with the red ellipse indicating the predicted state variance.

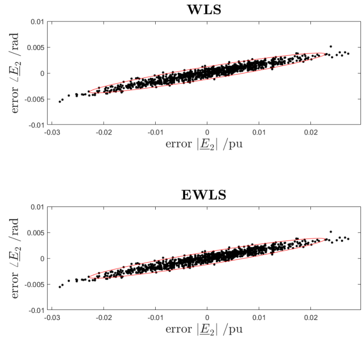

Figure 3: Voltage-phase Error of Bus 2 -Case 1

However, as the errors in parameters grow, the WLS method will become increasingly biased and its estimate error much more spread out. On the other hand, the EWLS method is capable of properly managing the newly-added uncertainty, having both a good prediction on state variance as well as a thinner spread of the errors.

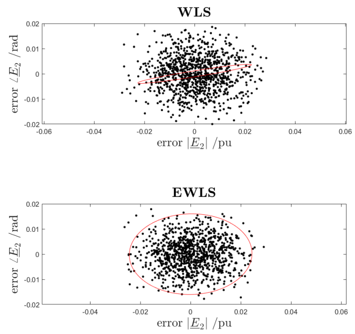

Figure 4: Voltage-phase Error of Bus 2 -Case 2

This difference is even more significant when dealing with the bus voltage phases as shown in figure 5. This is because while voltage magnitudes may be measured directly with only minimal impact from parameter errors, phases require more complex derivation, often from power meters which depends a lot on accurate parameter information.

The abundance, positions and types of measurements also have a role in the precision of aforementioned estimators. In circumstances where an heavy overabundance, an optimal placements and various measurement types of instruments are available in the network, with only a small parameter uncertainty present, or if the errors

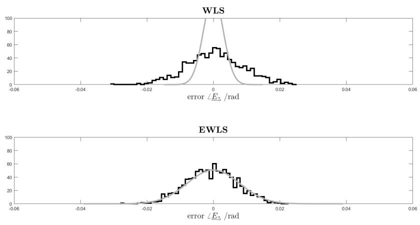

Figure 5: Phase Error PDF of Bus 6 -Case 2

in measurements are much more impactful than those from parameters, the results provided by WLS aren't greatly outclassed by EWLS. However, with more accurate measuring instruments and less abundance, the advantages of EWLS will begin to emerge. As such, EWLS may be of particular interest to circumstances where certain system states are indirectly or sparsely monitored, in which case parameter uncertainty can play a heavy role in the final estimate.

4. Conclusion

This thesis performs a detailed examination, both through theoretical analysis and a simulation test based on MATLAB, on the previously proposed extended WLS method used in power system state estimation. A system model as well as the algorithm for realizing the introduced estimators are established in a extensible way and results obtained using IEEE14 bus system as an example are analyzed.

The EWLS method, though more complicated than the original WLS as it requires an additional Jacobian matrix to be computed in the iteration, is shown to be capable of better utilizing the overabundance of measurement data, and as a result, gives a more accurate estimate than WLS could. Furthermore, EWLS is also able to properly formulate the error estimation from measurements and parameters which WLS cannot, providing an opportunity to examine bad data that may be present in them even further.

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