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Executive Summary of the Thesis

Comparison of Uncertainty Quantification and Validation Methodologies on a Civil Tiltrotor Flight Simulation Model

Laurea Magistrale in Aeronautical Engineering - Ingegneria Aeronautica

Author: Matteo Mamino

Advisor: Prof. Giuseppe Quaranta

Co-advisors: Ing. Andrea Ragazzi, Ing. Federico Porchacchia

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1. Introduction

The demonstration of compliance to certification requirements is a fundamental milestone in the development of any rotorcraft and is required to testify that the vehicle meets the safety requirements set by the certification authority. Nevertheless, the compliance demonstration process is generally the most expensive and time demanding part of the certification activity, due to the amount of necessary ground and flight testing. It is predicted that demonstration of compliance through flight simulations may take advantage of the reduction of cost, risk and required time offered by modelling tools. However, in order to deliver these benefits, some effort shall be devoted to the development and the validation of simulation tools of sufficient fidelity. Building on this view, the Rotorcraft Certification by Simulation (RoCS) project aims to explore the challenges and opportunities associated to the use of flight modelling during certification and to provide guidelines [9] for the application of flight simulation to support the compliance demonstration activity for helicopters and tiltrotors.

The objective of the present work is to revise the most widely acknowledged procedures in the field of model Verification and Validation (VV) and to frame them in the Certification by Simulation (CbS) guidelines proposed by RoCS [9], in order to establish what methodology may be overall best suited for an application in the industry and, possibly, implementation into future revisions of the CbS process. Two approaches have been elected to primary reference work: the ASME standard for VV in Computational Fluid Dynamics (CFD) and Computational Heat Transfer (CHT) [4] and the VV approach for scientific computing proposed by Roy and Oberkampf [11]. Once framed in the CbS process, the two standards are applied to a real case scenario and compared in terms of simplicity, fundamental assumptions, computational expense, validation metrics values and suitability to the different uses of model VV tools within the guidelines proposed by RoCS.

The investigation of the research objective is pursued by emulating the implementation of the starting phases (namely, phase 1 and 2a) of the CbS process on a civil tiltrotor, making use of a state-of-the-art Flight Simulation Model (FSM) and a set of experimental flight data both supplied by Leonardo Helicopters Division (LHD). EASA CS.29.143 (d) [3] low speed controlla-

bility and maneuverability certification requirements were chosen for the application presented in this work.
All modelling activities and simulation results presented in this dissertation are obtained with FLIGHTLAB [1]. In addition, all the parametric, optimization, sensitivity and uncertainty quantification analyses presented in this dissertation were performed coupling FLIGHTLAB with Dakota.

2. Verification and Validation Methods Review

2.1. ASME Standard

The foundational idea of ASME VV 20 [4] is to extend the standard approaches adopted for experimental uncertainties to include numerical and model-input uncertainties in a comprehensive validation framework. Error and uncertainty definitions adopted by this standard are directly inherited by [7] and their concepts are extended in order to be applied to the solution variable from a simulation as well. The nomenclature system adopted by ASME standard, together with the proposed verification and validation process, is summarized in fig. 1.

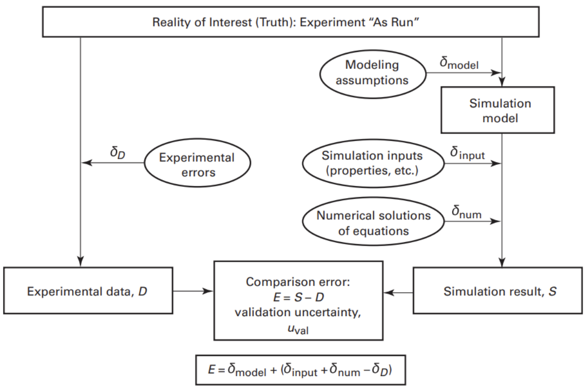

Figure 1: Overview of the sources of uncertainties and model validation process proposed in the ASME VV Standard 20 [4]

The goal of the validation process is to estimate the model contribution δ_{model} to the simulation error δ_S. Such estimate is provided according to eq. (1) as a function of the two validation metrics: the comparison error E and the standard validation uncertainty u_{val}.

$$\delta_{model} = E \pm u_{val} \tag{1}$$

The former is computed according to eq. (2) from the nominal simulation output S and experimental data D. The latter, conversely, is evaluated through equation eq. (3) starting from numerical, model inputs and experimental uncertainties.

$$E = S - D \tag{2}$$

$$u_{val} = \sqrt{u_{num}^2 + u_{input}^2 + u_D^2} \tag{3}$$

As a result, the validation uncertainty u_{val}, whose quantification is a pivotal point of the methodology hereby presented, provides an indication of the dispersion of δ_{model} parent population around the comparison error E.
Upon completion of the validation process, two corollaries follow:

- if $|E| >> u_{val}$, then it is safe to assume that δ_{model} has the same order of magnitude of E and that its estimation is reliable;
- if $|E| \leq u_{val}$, probably δ_{model} is of the same order of magnitude, or even smaller, than $(\delta_{num} + \delta_{input} - \delta_D)$.

Thus, reducing the validation uncertainty is beneficial to obtain a reliable quantification of the modelling error and set up model improvements (i.e. updates to reduce the modelling error). Otherwise, whenever δ_{model} is within the noise level imposed by numerical, input and experimental uncertainty, formulating and measuring modelling improvements is difficult.
Concerning model adequacy assessment and model-form error extrapolation, no explicit procedure is provided in [4]. However, building on the definition of model adequacy assessment delivered in [9], the information that E and u_{val} convey, appropriately extrapolated into the DoP, shall be combined with the quantification of δ_{input} and δ_{num} to obtain a reliable estimate of δ_S on certification-aimed model predictions.

2.2. Roy-Oberkampf Approach

The VV approach proposed by Roy and Oberkampf is summarized in their 2010 conference paper [11]. At the foundation of the Roy-Oberkampf VV framework stands the classification of uncertainties into two categories: aleatory (or irreducible) and epistemic (or reducible). Aleatory uncertainties represent "*the inherent variation in a quantity that, given sufficient samples of the stochastic process, can be characterized via a probability distribution*" and are mathematically characterized with a PDF. Epistemic uncertainties identify the instances "*where there is insufficient information concerning the quantity of interest to specify either a fixed value or a precisely known probability distribution*" and are characterized as intervals with no associated probability distribution. An overview of the sources of uncertainties and verification and validation process proposed by Roy and Oberkampf is presented in fig. 2.

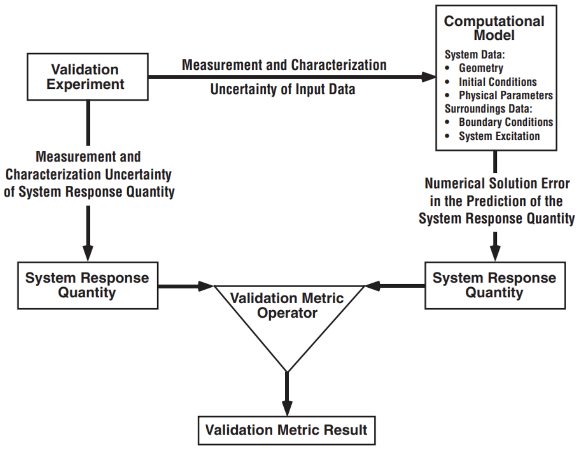

Figure 2: Overview of the sources of uncertainty and model validation process proposed by Roy and Oberkampf in [12]

Whenever both aleatory and epistemic uncertainties are present, the simulation output and the experimental data are represented as *probability boxes*. The probability box is a peculiar type of CDF. It is an interval-valued probability structure which delivers information on both aleatory and epistemic uncertainties but without confounding the two, as reported in fig. 3.
Then, the minimum area of non-overlap included between the two probability structures (the p-box from the simulation and the discrete CDF from experimental data) is identified as the validation metric d (also referred to as *area metric*). The area metric value, dimensional and characterized by the same measurement unit of the SRQ (alike a comparison error), effectively represents the measure of disagreement between the model and the experiment according to the knowledge available to the analyst. Any disagreement is attributed to model-form error, defined as an epistemic uncertainty equal to the validation metric value according to eq. (4).

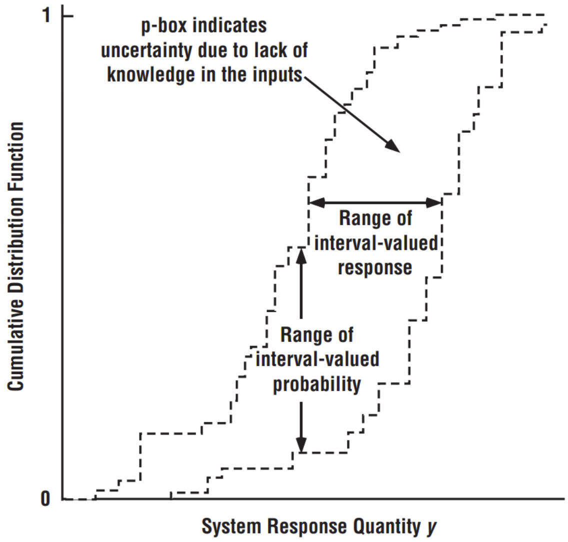

Figure 3: Example of the p-box obtained for a SRQ y (from [11])

$$U_{MODEL} = d \tag{4}$$

Unlike in [4], Roy and Oberkampf provide general guidelines to account for the model-form error also in the domain of application. By "correcting" the area metric value for the increased uncertainty due to the interpolation/extrapolation process, the epistemic model-form uncertainty can be derived and applied to model predictions beyond the validation points. Then, the model adequacy assessment can be carried out directly comparing the uncertain model prediction with the adequacy margins and/or the Applicable Certification Requirement (ACR).
An advantage of the Roy-Oberkampf approach is that the probability box arising from model inputs propagation allows the analyst to quantitatively distinguish the impact of aleatory and epistemic uncertainties on each model SRQ. However, it is predicted that the usage of d as a validation metric (and proxy for the model form uncertainty) may lead to underestimation of the model predictive uncertainty in the domain of application. Moreover, concerning the model extrapolation technique proposed by Roy and Oberkampf, it is observed that a residual disagreement between simulation output and experimental data is still present, in the domain of validation, once the model form uncertainty is accounted for.

2.3. Interval Analysis

The computation of u_{val} (in ASME framework) and the construction of the probability box (in Roy-Oberkampf framework) are costly tasks. At the same time it may be argued that the computation of SRQs statistics may not be strictly necessary for some possible applications of the validation process within RoCS framework. It is in this scenario, hence, that interval analysis might stand out as a viable validation tool to overcome the aforementioned limitations.
Formally speaking, such a framework incorporates elements of both the ASME standard (validation metrics and corollaries) and the Roy-Oberkampf approach (interval-valued uncertain quantities). All model inputs affected by uncertainty are characterized as intervals. Then, the propagation of input uncertainties through the model is performed via global optimization (i.e. interval analysis) techniques. As a result, bounding minimum and maximum values for S, which identify the interval U_{INPUT}, are obtained. Concerning the numerical uncertainty, no difference with respect to Roy-Oberkampf framework is present. Finally, concerning the experiment, the realization(s) of the SRQs of interest are used to derive an interval U_D which bounds all possible contemplated values of the measurand D. Then, upon the quantification of U_{INPUT}, U_{NUM} and U_D, the validation metrics E_{max}, E_{min} and E_{IA} can be evaluated, as reported in fig. 4 and eq. (5).

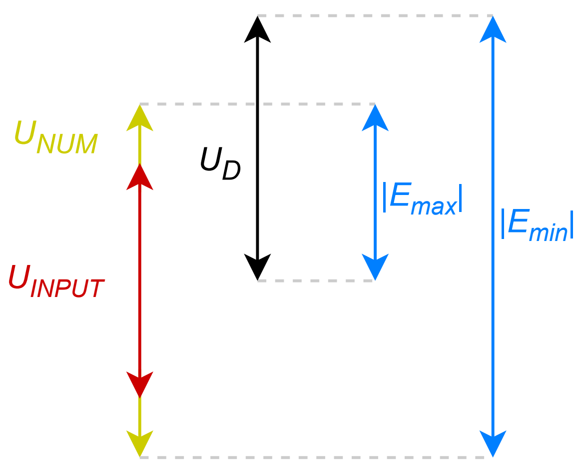

Figure 4: Graphical exemplification interval analysis validation metrics

$$E_{IA} = E_{max} - E_{min} \tag{5}$$

Upon computation of the validation metrics, corollaries based on the ones discussed in the ASME standard can be derived.

- If both $|E_{max}| >> E_{IA}$ and $|E_{min}| >> E_{IA}$, then it is safe to assume that δ_{model} has the same order of magnitude of E_{max} and E_{min} and that its estimation is reliable;
- if either $|E_{max}| \lesssim E_{IA}$ or $|E_{min}| \lesssim E_{IA}$, then the comparison error is dominated by either numerical, input and/or experimental uncertainties and little information can be retrieved and used about the modelling error value.

Fidelity assessments and model improvements can be measured directly on E_{max} and E_{min}.

3. RoCS Guidelines

3.1. Phase 1

Certification specifications for the hereby presented demonstration were extracted from [3]. In particular, CS.29.143 (d) for OGE low speed controllability and maneuverability was considered. The CS requires the applicant to interpret what *loss of control* means in order to be translated into ACRs. For the sake of simplicity, the author decided to exclude any stability and aircraft dynamics related argument, hence rephrasing the controllability requirement as a sole function of the aircraft trim static control margins. As a consequence, model validation is based on the comparison of fidelity metrics at trim and involves the time domain only.
Concerning the fidelity metrics, the preliminary set proposed by RoCS [13] was considered. However, the angle of sideslip and angle of attack were excluded from the set, and the collective control position (xt_{col}) was replaced with the rotors blades collective pitch.
The flight data selected for the guidelines application are represented by a set of measurements carried over four time windows. Each window has a duration of 2 seconds and corresponds to an attempt of horizontal, uniform, rectilinear flight at an assigned ground speed azimuth angle.
Concerning FSM requirements, the model shall be physics-based. Moreover, since all selected validation points and ACRs do not involve

ground effect, its modelling may not be included in the FSM. Nevertheless, considering the low speed environment of the CS, interference aerodynamics are expected to play a crucial role in the aircraft behavior.

3.2. Phase 2 - FSM Development

The FSM is developed in FLIGHTLAB. Within the model, each aircraft functional part is implemented and integrated with other systems in a component-based approach, as exemplified in fig. 5. The model is conceived as a time-marching simulator, capable of the time integration of the aircraft and rotors subsystem equations. All aerodynamic components are modelled as rigid. The rotors are modelled with blade element theory. Fixed lifting surfaces, namely the wing and the tail, are modelled as lifting lines. Finally, fuselage and nacelles aerodynamic loads are managed via specific look-up tables and making use of single control points for airspeed acquisition.

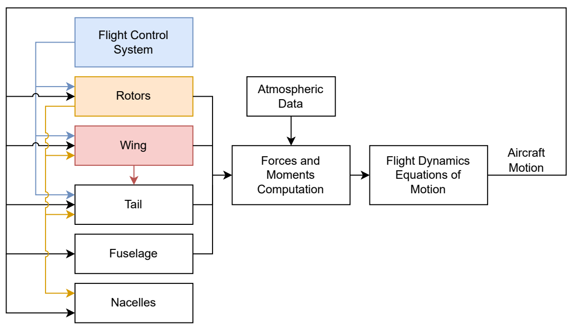

Figure 5: FSM components breakdown

The induced velocity field of the lifting components (namely, blades, wing and tail) is modelled with the Peters-He finite state model ([8], [10]), a physics-based time-domain dynamic wake model originally conceived as a tool for the aeroelasticity and aeromechanics of lifting rotors. The same model is used to account for aerodynamic interference, as outlined in fig. 6.

3.3. Phase 2 - Solution Verification

The widely recognized solution verification practices outlined in existing standards (e.g. [4]) are aimed at CFD and structural analysis problems and cannot be directly applied to a multi-physics flight simulation models. Thus, the author came up with its own operative procedure for the solution verification of the FSM. Nevertheless, the hereby proposed approach was based on several concepts pointed out in [4] and [12].

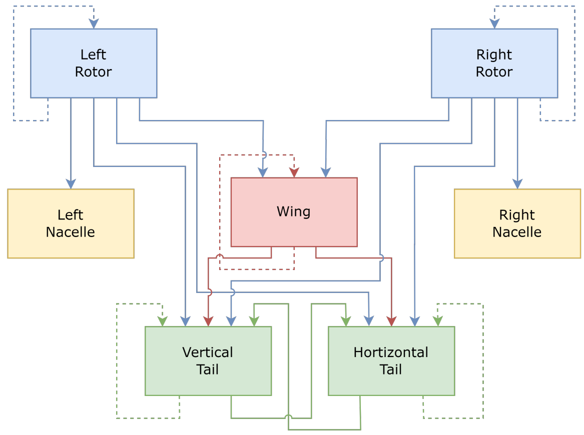

Figure 6: FSM aerodynamic interference outline. Active interference effects are marked by a continuous line. Conversely, dashed lines mark the account for self-induced velocity on a component.

Great variability of all fidelity metrics with respect to the number of inflow (and interference) states was observed, as reported in fig. 7. As a consequence the author decided not to consider the effect of inflow states into the numerical error.

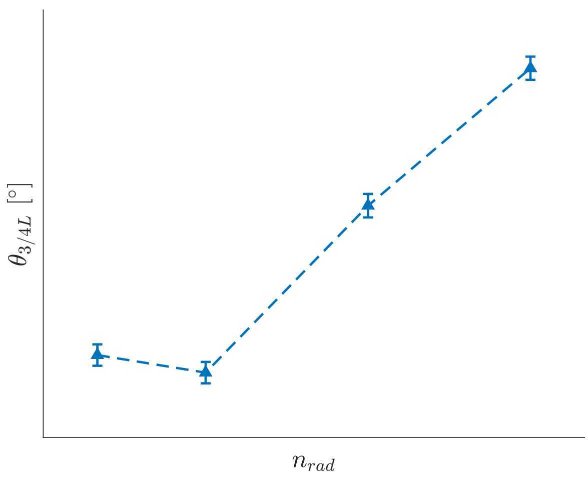

Figure 7: Left rotor collective numerical error estimates. The dashed line represents the variation with respect to inflow states. The errorbars on each point, conversely, represents the sum of all other numerical error contributions estimates accounted for in this study

Concerning the other parameters affecting the numerical error, asymptotic convergence was typically observed. The choice of their value was dictated by a compromise between computational time, confidence on the error estimate and error estimate magnitude. Upon such choice, numerical error estimates (and associated decomposition into the different contributions) were derived for each fidelity metric, as reported in fig. 8.

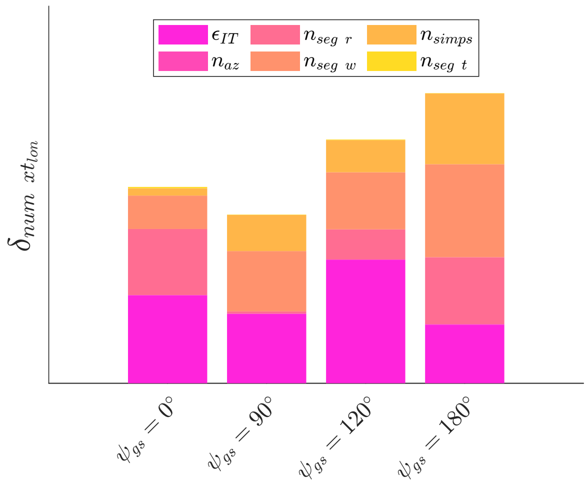

Figure 8: Numerical error estimates contributions on all validation flights for longitudinal control position

3.4. Phase 2 - Model Validation

The first step in solution validation concerns the identification and characterization of FSM input uncertainties. In the present work, based on engineering judgement, it was decided to include uncertainties due to wind speed and direction, treated as epistemic, and aircraft mass and center of gravity position, treated as aleatory. Their mathematical characterization was made building on [2] and the judgement of the author. A preliminary sensitivity analysis was performed with Morris One-at-A-Time (MOAT) global screening method, in order to determine, among the candidate uncertain model inputs, those whose uncertainty results in negligible effects on all the system response quantities of interest. The MOAT method was chosen instead of variance based decomposition method due to its reduced computational cost [6] when a large number of parameters is considered. As a result, the center of gravity waterline was removed from the list of uncertain model inputs, due to the negligible impact exhibited on all fidelity metrics.

To complete the validation process, numerical, experimental and model input uncertainties were derived according to the process provided by each framework, and validation metrics values were computed for each SRQ of interest.

4. Phase 2 - Validation Metrics Comparison

Decent agreement in the trends and values of the metrics was observed, as reported in fig. 9.

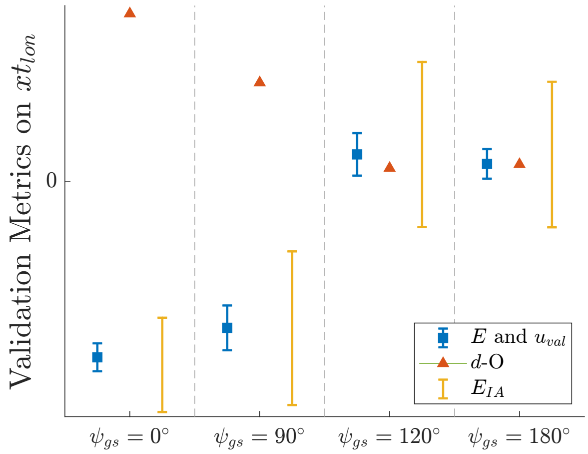

Figure 9: Validation metrics comparison for control positions on all validation flights

Being both based on the comparison error, E_{IA} and $E \pm u_{val}$ are indeed very similar. The yellow band of E_{IA} always include the nominal comparison error E of ASME, as expected. Moreover, the amplitude of the yellow bar is typically of the same order of magnitude of u_{val} and, frequently, 2 to 3 times greater. This is expected considering that u_{val} is a standard uncertainty, while the E_{IA} is extended and directly represents the interval in which the comparison error is expected to fall. The area metric d assumes values which are comparable to the other metrics as well. As a consequence, it is reasonable that the fidelity acceptability margins conceived for the comparison error may also apply sensibly to this validation metrics. In addition, it also typically follows the same trends of ASME and interval analysis. However, whenever the comparison error assumes negative values, considering that the area metric is a positive definite quantity, a difference in the trends emerges, characterized by a pseudo-symmetry with respect to the X-axis of the plot. Thus, whenever E carries the information about the sign of the error, the area metric doesn't. However, this can always be retrieved by the analyst through a visual inspection of the pbox and the area metric. Alike E and u_{val}, also $|E_{max}|$, $|E_{min}|$ and E_{IA}, are typically of the same order of magnitude. Hence, when the corollaries are applied to interval analysis validation metrics, the analyst is supplied with compelled evidence that the uncertainty due to input and flight data is too big to isolate the value of the model error. As a consequence, a reduction of the input and experimental uncertainty is necessary in order to set up well-advised model improvements based on these data, unless the applicant is willing to accept a large uncertainty.

Concerning the area metric, it is straightaway clear that no model improvement can be formulated basing on the instances where $d = 0$. On the other hand, depending on the quality of flight data, model update procedures may be conceived for the cases where $d \neq 0$. Nevertheless, as observed in section 2.2, d does not come with any associated uncertainty (unlike E in the other two frameworks). When the model is extrapolated and the model-form error estimate is exploited in the DoP for certification-aimed prediction or credibility assessments, any information concerning the uncertainties involved in the validation phase is lost. As a consequence, another fundamental difference among the frameworks emerges in regard of model adequacy assessment.

Within the validation methodologies based either on ASME standard or interval analysis, any increase in the uncertainties involved in the model validation phase ends up worsening model credibility. The validation metrics u_{val} and E_{IA} not only draw the attention of the analyst to large uncertainties involved in the validation process but also, and more importantly, are able to transport that information (related to the *goodness* of the model validation) into the DoP. Essentially, any uncertainty associated to model-inputs (u_{input} or U_{INPUT}), numerical approximations (u_{num} or U_{NUM}) and flight measurements (u_d or U_D) in the validation points indirectly affects the model credibility in the DoP through the model-form error uncertainty (either represented by u_{val} or E_{IA}). Hence, the credibility of a FSM is not only affected by the comparison error E and the fidelity of the model itself, but also by the *quality* of the validation which has been carried out to assess its fidelity. As a result, the very same model validated with reduced uncertainties in the validation data can lead to a smaller margin with the same CR when used to support certification (thanks to a reduction on u_{val} or E_{IA}).

Conversely, whenever d is used, this connection does not always emerge. Indeed, as the analyst inadvertently exaggerate either the epistemic input uncertainty or U_{NUM} during model validation, the value of d reduces and, as a consequence, recalling eq. (4), model credibility improves. Thus, apparently, it is possible to conclude that whenever d is used, it becomes critical not to overestimate epistemic uncertainties. However, this might not be trivial at all, considering the fact that epistemic input uncertainties may include inputs of which little information is available and accounting for the difficulties associated to the estimation of U_{NUM}. On the other hand, $E \pm u_{val}$ and interval analysis methods may be considered more resilient in this regard, since, at least, they make available to the analyst an indication of the uncertainty associated to the model-form error.

In regard of uncertainty contributions decomposition, all validation frameworks are able to inform the analyst about what uncertainties dominate model predictions and/or validation metrics.

Another way of comparing the validation metrics relies on their computational cost. Indeed, in the present study, u_{val} and d turned out to be the most computationally intensive metrics to compute, due to the sampling techniques required for the estimation of u_{input} and of the model probability box, respectively. On the other hand, at the cost of sacrificing the information on SRQs statistics, interval analysis proved to be much cheaper, resulting in at least an order of magnitude reduction in terms of required function evaluations per fidelity metrics.

5. Conclusions

In the present work, the ASME VV 20 [4] and Roy-Oberkampf [11] approaches to VV of computational models are revised. Then, emulating the first phases of a partial credit demonstration of compliance to EASA low speed controllability requirements [3], the aforementioned VV

methodologies have been applied to a state-of-the-art FSM developed by LHD. In the course of the application, the CbS guidelines developed by RoCS are strictly followed. FLIGHTLAB and Dakota are employed as tools for flight dynamics modelling and SA/UQ, respectively.
The solution verification process is carried out accounting for several model discretization and solution algorithm parameters. At the time of writing, there is no accepted solution verification procedure suited for the quantification of the numerical error of flight mechanics multi-body models. Hence, the author applies a verification algorithm specifically conceived for the present application, stressing the assumptions, challenges and limits of the currently adopted procedures when applied to complex state-of-the-art multi-body systems. Valuable insight is gained on the behaviour of the Peters-He finite state dynamic inflow model with different number of states. In addition, reliable estimates of the numerical error associated to the FSM solution are obtained.
In the framework of model validation, a preliminary SA with respect to FSM inputs is carried out with the MOAT [5] method implemented in Dakota. Despite a direct comparison with more widely adopted SA methods (e.g. VBD) is not provided, it is predicted that the reduced cost of MOAT may result in a significant reduction of the computational expense of SAs in FSMs with a large number of uncertain input parameters. Model validation is then carried out with both ASME VV 20 and Roy-Oberkampf approaches. Moreover, a validation methodology based on interval analysis is proposed and applied to the FSM.
The three VV procedures are eventually compared in terms of fundamental assumptions, computational expense, validation metric values and suitability to the CbS process developed by RoCS.
ASME and Roy-Oberkampf validation procedures, as expected, proved to be significantly more expensive than interval analysis. Despite this, all validation methodologies lead to similar values of model validation metrics for all SRQs. The VV framework proposed by ASME for CFD and CHT [4] may be best suited for the application to FSMs in RoCS. Indeed, its corollaries provide a powerful tool at disposal of the analyst to critically interpret the outcomes of model validation. Moreover, the standard validation uncertainty u_{val} allows model credibility in the DoP to be affected by *quality* of the FSM model validation (namely, the magnitude of the uncertainties accounted for during validation). This is a powerful feature, especially when, as in the present work, FSM input uncertainties cannot be accurately estimated. Nevertheless, when computational expense is of paramount importance, e.g. when a preliminary guess of affordable FSM input uncertainties shall be provided, interval analysis might turn out to be best suited, thanks to its capability of delivering great proxies of the comparison error E uncertainty band obtained with the ASME approach at a fraction of the cost.

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