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EXECUTIVE SUMMARY OF THE THESIS

Analysis of the porkchop plot considering eccentric and inclined planetary orbit

LAUREA MAGISTRALE IN SPACE ENGINEERING - INGEGNERIA SPAZIALE

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1. Introduction

The trajectory design of a space mission needs taking place years in advance with respect to the departure date. An immediate tool which helps the engineers in this process is the porkchop plot. This visualisation tool monitors the variation of specific parameters as a function of departure and arrival date, i.e. the ΔV required by the manoeuvres. Therefore, it is able to track the variation of the cost of the mission.
However, its generation is computationally expensive when performed recursively on large time windows. Hence, the benefit of studying the evolution of porkchop plots under a variety of conditions. This would focus the analysis to limited regions of the epoch space, thus saving computational cost.
Starting from the literature works produced by Menzio [1] and Woolley and Whetsel [2], the focus of this dissertation is characterising the shape, size and location of the generated porkchop plots depending on the orbital elements of the departure and arrival orbits.
To this purpose, an analysis on the particular effect of each parameter to the morphology of the results is carried on independently. Starting from the simplest case of circular and coplanar orbits, the complexities are added one by one up to resemble as much as possible a real life scenario. Artificial ephemerides are generated and results are derived with respect to the Earth-to-Mars direct transfer, varying the parameters of the orbits at will. For this reason when referring to an Earth/Mars orbit it is intended to consider Earth/Mars-like orbits having the proper semi-major axis of the planet but a different eccentricity, inclination or orientation in space.

2. The role of eccentricity

The role of eccentricity is monitored considering one of the two orbits as more and more eccentric. Specifically, the effect of e_2 is tackled, while e_1 is considered initially null.
Consequently, the effect of increased eccentricity is described by a significant variation of the shape of the figure and compared in Figure 1 to the circular case. The three-lobed shaped island stretches and divides in multiple islands, while the internal contours increase their area. At the same time, local peaks emerge from the 3D surface. The minimum ΔV position is no longer identified by the analytical intersection of two curves ($\phi = 180\ deg$ transfer and Hohmann transfer curve) as in the circular and coplanar

case [1]. However, a clear trend shows that for increasing eccentricities the minima align along a vertical line. This line represents a particular date: the date in which Earth is at perihelium. When eccentricity is large enough, this behaviour is removed in favor of a horizontal arranging in correspondence of the date in which Mars transits from its periapsis (where the local peaks emerged).

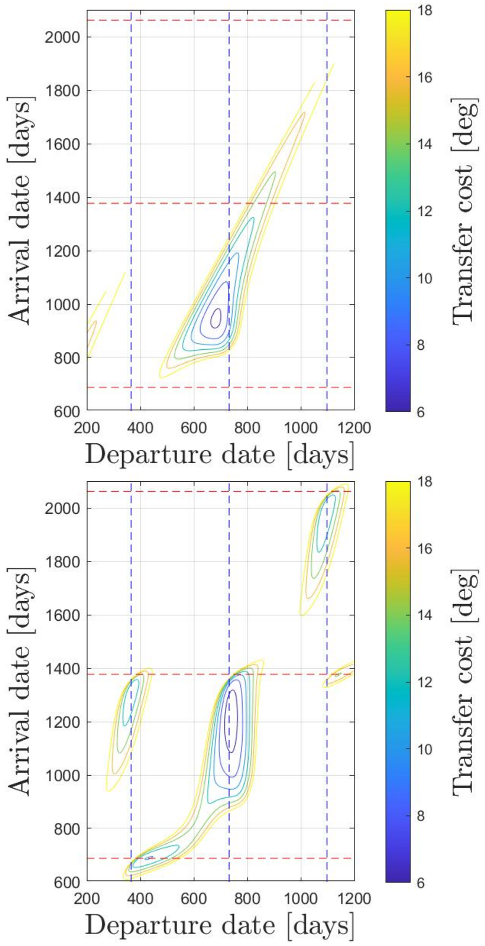

Figure 1: Comparison of porkchop plots arising from the case $e_2 = 0$ (above) and $e_2 = 0.3$ (below).

This shows that the minimum solutions try to emulate a Hohmann-like transfer: $\phi \rightarrow 180\ deg$ from periapsis to apoapsis or vice versa, where ϕ is the transfer angle

$$\phi = \theta_2(t = t_{arr}) - \theta_1(t = t_{dep}) \qquad (1)$$

θ represents the true anomaly as a function of the time t, while its subscripts indicates the planet.
A theoretical but instructive case is the one foreseeing a Mars orbit whose eccentricity is such that the two orbits are tangent, $e_{tang} \approx 0.34$ from calculations. In this case all the inner contours are perfectly located by the line of transits, thus are totally identified a priori from the dates in which the planets cross the apse line. The effect of eccentricity is to resemble more and more this alignment. Analogous results are derived when the eccentricities are switched: the effect of e_1 when $e_2 = 0$ is similar.
In any case, contrarily to the circular case, any successive ΔV island is different to the previous one. From literature it is known that a launch opportunity is found any synodic period. However, when the circularity assumption is removed, any successive alignment is influenced by the phasing, thus involving different positions and velocities which distort the shape of the island.

2.0.1 Phasing and argument of periapsis

Fixing $e_1 = 0$ and $e_2 = \bar{e}$ (or vice versa) the effect of phasing constitutes the variable accounting for an advancement or delay of one planet with respect to the other. Varying the initial phasing means shifting the angular distance at t_0, in this way:

$$\Delta\theta = \theta_2(t = t_0) - \theta_1(t = t_0) \qquad (2)$$

where θ is the true anomaly and t_0 is a reference instant of time.
As a result, for any phasing, the time of transit over the characteristic lines will be different, so the location and shape of the contours will be updated as well. The superimposition of the sequence of contours (at a specific level, i.e. $\Delta V = 6\ km/s$, good approximation of the minimum position) for different phasing generates envelops in the plane. If in the circular case the envelop lays on the Hohmann line (as per Menzio [1]), when $e_2 = \bar{e}$ the envelop assumes an oscillating behaviour with respect to the same line. These oscillations have a frequency dictated by the transit lines (so depending on the semi-major axis) and an amplitude affected by the eccentricity. This defines the region where the contour can be found in the future. Indeed, considering real orbits (so fixed keplerian elements), the evolution in time will provide in a sufficiently large time window any condition of phasing.

The best way to observe the oscillating trend is to consider a parameter y^* which considers the average distance of the envelop from the Hohmann line. Figure 2 shows, as expected, that for increasing eccentricities the contours align towards the transit lines.

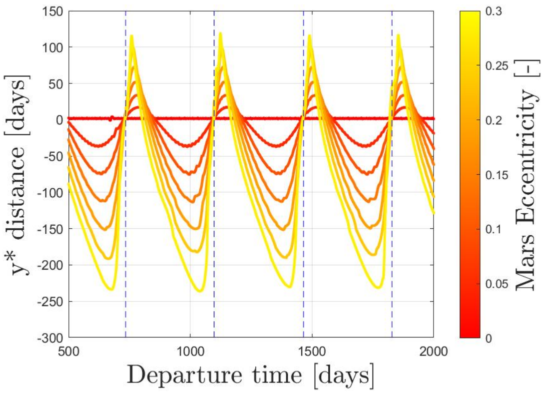

Figure 2: Envelop generated for different phasing conditions of Mars.

Considering now fixed $\theta_1(t_0) = \theta_2(t_0) = 0$, it is possible to verify the effect of modifying the orientation of the eccentric orbit, so the effect of ω_2. Even this time an envelop is produced. This time the frequency is dictated by the transit lines of the planet on a circular orbit. The situation is analogous to the phasing: considering a reference frame fixed with the orbit which has been rotated (due to ω_2), the scenario is the same as imposing an advance or a delay to the other planet (Earth).
A different situation occurs when the orbit is re-oriented but $\theta_1(t_0)$ and $\theta_2(t_0)$ are such that the planets are always aligned at t_0. In this case the contours are modified in shape but they are all found in the same region of the plane. Basically, the modification in shape of the contours is due to the nature of an eccentric orbit, while the sliding along the envelop is caused by the angular distance at t_0 both if due to phasing and ω.

2.0.2 Combined effect

When considering both eccentricities, it is revealed that the ΔV variation due to e_2 are slightly changed since e_1 acts as a disturbance for low values of both. When instead e_1 is higher, the value of e_{tang} rises and more advantageous minima are found. There is not a clear trend expressing most favourable conditions when e_1 and e_2 are varied in a determined way since the phasing introduces a further degree of freedom complicating the analysis.

3. The role of inclination

When considering the effect of inclination the orbits are re-built as circular and an angle Δi is imposed between the two planes.
The porkchop plot obtained from orbits progressively inclined shows a strong reduction of the area of the island. The shape is modified by the presence of the ridge already noticed in literature [2], but all the contours are contained inside the one for coplanar orbits. An important symmetry is present: the same porkchop plots is generated when $i_2 = \overline{\Delta i}$ and $i_2 = -\overline{\Delta i}$. Moreover, identical results are given when inclining the other orbit of the same amount: $i_2 = \overline{\Delta i}$ and $i_1 = \overline{\Delta i}$ are totally equivalent, proving that what matters is just the relative inclination.
Even in this chapter the ΔV islands are not repeated identically and their location is linked to specific dates, particularly the dates of transit of planets from the line of nodes. Moreover, the intersection between proper transit lines and the 180 *deg* transfer line, which overlaps with the ridge, locates the bridge, the only connection between the two lobes generated by the plot. This helps localising the position of the centre of contours simply from the orbit definition. These features are depicted in Figure 3.

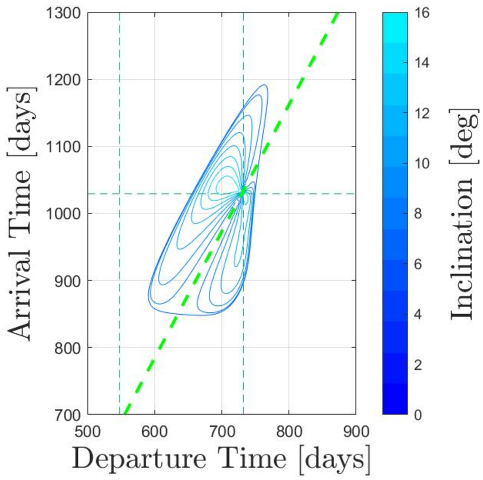

Figure 3: Reduction of contour area with inclination. Bridge located at the intersection of characteristic lines.

An island is located any synodic period, while

in correspondence of the other intersections are present solutions which optimise one of the two manoeuvres but not the overall transfer, so there are no local peaks as occurred in Chapter 2.
As expected, the minima position is attracted by the position of the bridge as the value of inclination grows. In other words, the solutions tend to emulate a Hohmann transfer from node to node. This is confirmed observing the contour plots representing two parameters: the eccentricity of the transfer orbit and the inclination ratio ψ between the transfer orbit inclination and the relative inclination between the orbits. From their analysis emerges that inner contours in both lobes approach the value e_{min} dictated by the eccentricity of the equivalent Hohmann transfer orbit. The ψ plot, instead, behaves differently in the two lobes. The left lobe involves trajectories having $\phi > 180\ deg$ and presents inner contours featuring transfer inclinations close to zero, so performing the change of plane manoeuvre at the arrival point. Contrarily, short transfers belong to the right lobe and imply a tendency to change plane during the first impulsive manoeuvre. In both lobes the inner contours align themselves with the relevant transit lines. In other words this shows the tendency to depart or arrive in correspondence of the line of nodes as soon as the inclination has grown enough.
A further analysis consists in fixing the departure and arrival position and observe the cost of the transfer as a function of the TOF. Imposing the transfer to be from node to node, it is revealed how the TOF which minimise the cost is effectively the Hohmann transfer semi-period. This result is independent from the inclination. Instead, modifying the ϕ from 180 deg the minimum of all the curves shifts. This leads to the definition of an interesting chart which displays on the same graph a family of curves having the departure (or arrival) fixed at the node but different ϕ and their trend is reported with respect to time. Considering that for a fixed departure point the ϕ can be reached only after a specific TOF (which depends on the position of planets), the locus of the admissible solutions is reported for a specific launch opportunity. Given the specific phasing, this allows to design properly the transfer in order to minimise the cost. In Figure 4 an example of the curve of feasible transfers is reported.

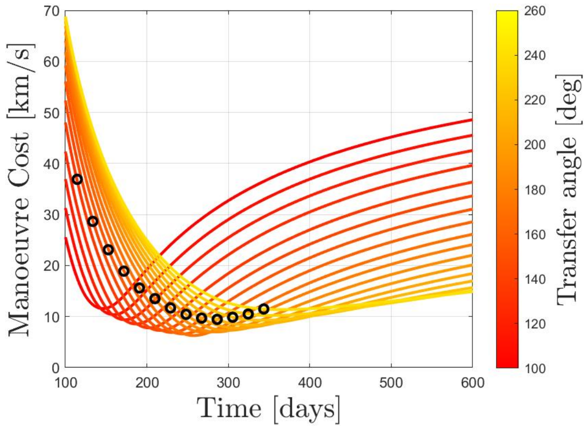

Figure 4: Admissible transfer solutions for a specific phasing.

3.0.1 Phasing and right ascension of ascending node

Even in this case, the effect of a different phasing and the effect of a different orbit orientation are related.
Analogously to Chapter 2, an envelop is produced both varying the phasing and varying the right ascension of ascending node (RAAN) because the initial position of the planets in terms of X, Y coordinates is modified. The circularity of the orbits prevents the envelop to dislocate from the Hohmann line. However, considering a specific contour level (i.e. $\Delta V = 10\ km/s$), some variations in shape are such that the border of the envelop produces slight oscillations depending on the transit lines. On the other hand, the great result of this chapter is that the inclination reduces the size of the contours, but keeping them always enclosed in the envelop obtained for $\Delta i = 0\ deg$. This can be used to reduce the search space just to the belt described by the simplest envelop.
Observing the morphology of the contours a consistent modification in the shape is observable varying phasing or RAAN. The sliding along the fixed Hohmann line is linked to the 180 deg line which translates but always crosses the two lobes. As a result the area and the shape of the two lobes modifies strongly, as well as the position of the bridge which can be found above or below the Hohmann line. The inner contour evolution follows cycles which are linked to the synodic period. Moreover, the effect of inclina-

tion is to concentrate the bridges over the transit lines. Such regions are populated until a jump moves the following bridges to the next transit line. Linked to the sliding effect, this justifies the consequent modification in shape of the islands. Even in this case, when the orbit is re-oriented varying the RAAN but the planets are kept aligned, all the contours are located in the same region of the plane. The sliding along the Hohmann line is substituted by a slight translation along the 180 *deg* line, which is now fixed.

4. The role of eccentricity and inclination

In order to get closer to a real life situation, both the effects of eccentricity and inclination are considered simultaneously. It is possible to observe a superposition of the two aforementioned effects. If an eccentric orbit is progressively inclined the resulting porkchop plot is subject to both the effects: the distortion in shape and location of the island induced by the eccentricity and its miniaturisation caused by the inclination. Moreover, e_1 and e_2 have an analogous but peculiar effect in the evolution of porkchop plots, while i_1 and i_2 are again totally equivalent. Considering all the possible phasing of planets, an envelop is generated for any combination of e_1, e_2, i_1 and i_2. The phasing alters the shape of the contours as expected and the area enclosed in a specific curve is strongly modified due to the angular distance between planets. The main contribution of this area variation is due to eccentricity, while the effect of inclination acts mainly lowering the trend. The combined effect is summarised in Figure 5.

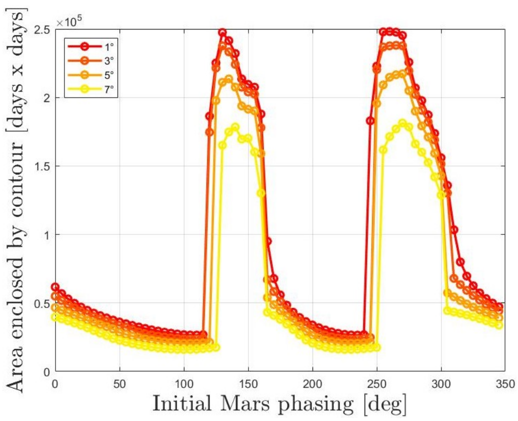

Figure 5: Area enclosed by contours. Effect of i_2 and phasing on eccentric orbits.

In absence of any re-orientation due to ω or RAAN the line of nodes is coincident with the apse line and the transit of planets from these characteristic line modulates the envelops. This results in a slight complication. Even if the ΔV islands generated at higher inclinations are contained in the respective one drawn for coplanar orbits, the presence of an eccentricity different from zero impose a dislocation from the Hohmann transfer line solution. Therefore, it is difficult to define a belt where concentrating the analysis and where applying recursively the algorithms. Moreover, considering many parameters at the same time results in a chaotic behaviour difficult to track.
However, the combination of two results coming from the previous separate chapters results to be productive. From Chapter 2 it is known that the eccentricity distorts the locus of solutions at fixed ϕ due to the presence of e in the equations for θ evolution along the orbit. At the same time, Chapter 3 reveals that the solution sharing $\phi = 180$ *deg* is always crossing the ΔV island defining two lobes. In other words, it is possible to monitor the change of location of the contours under a variety of conditions studying the evolution of the 180 *deg* transfer line which represents the ridge. The definition of a proper corridor which contains the 180 *deg* transfer line and reflects its distortion delimits the meaningful region of the graph.

4.0.1 180 deg transfer line evolution

The shape of the ridge is no more a straight line as in chapter 3 but is affected by eccentricity. Actually, each of the two orbits can influence the shape of the ridge. Contrarily, the relative inclination does not play a role in the locus of solutions. This must not be confused with the shape of the ΔV island which is modified by any of these parameters.
The effect of eccentricity is monitored observing how the locus of solutions distorts starting from the case of two circular orbits in which this is simply a straight line. Varying only e_2 (or e_1) the 180 *deg* line assumes an oscillating trend which resembles a sinusoid. The "amplitude" of this sort of wave is proportional with e_2, while the "frequency" is dictated by the transit dates, therefore is fixed. As a result, a family of curves is generated. The distortion induced by eccen-

tricity is symmetric for all the family and cancels out in correspondence of the transit dates producing stationary points where all the curves intersect with each other. The result visible in Figure 6.

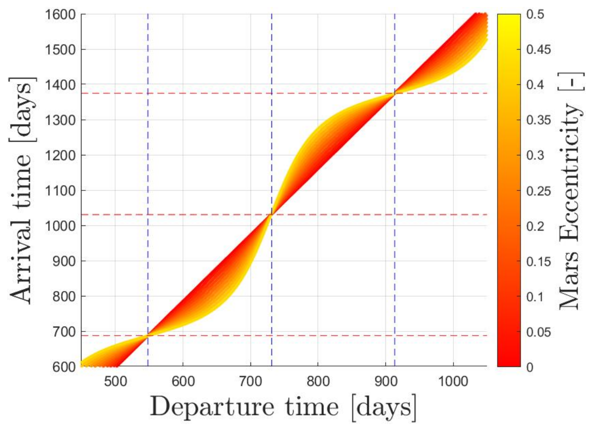

Figure 6: Effect of e_2 on the family of curves sharing $\phi = 180\ deg$.

Considering the simultaneous effect of e_1 emerges how the two eccentricities produce separate distortions which can be summed. As a result the reference straight line is the solution obtained only when both orbits are circular. The amplitude of these oscillations is affected in first approximation just by $(e_1 + e_2)$ and not by their specific values. In order to quantify the value of the distortion, the reference frame is rotated considering the $x\ axis$ laying on the reference straight line obtained for $e_1 = 0$ and $e_2 = 0$. The amplitude of this sinusoidal-like trend as a function of e_1 and e_2 identifies a sort of commutative property: switching the values of e_1 and e_2 produces an equivalent distortion.
The effect of phasing is considered observing the modification of the whole family of curves. The effect is different when imposing an advancement or delay to the planet laying on a circular or elliptical orbit. In the former case the family of curves is subject to a simple translation which is horizontal or vertical depending on the situation. The latter instead consists in a double translation. The main one is analogous to the previous case, so affects indiscriminately the whole family of curves. The secondary one is a translation in the direction perpendicular to the main envelop and affects differently the curves.
Similar effects are generated when the relative orientation of the orbits is modified due to ω or Ω. This is not surprising since it is a particular application of the case of phasing just described. Once again it is possible to decouple the effect of modifying the initial angular distance between planets (responsible for the main translation) and the relative orientation between the orbits (inducing the secondary one).

5. Conclusions

Analysing the specific orbits of any couple of planets it is possible to describe the expected features of the porkchop plot relative to a space transfer between them. A real situation is a composition of the scenarios presented in this dissertation. The prediction of the characteristics of a specific porkchop plot is crucial to save time and focus the expensive algorithms in a specific time window which can be updated time by time due to the orbital dynamics.
Future developments might involve the extension of this analysis to multi-revolution transfers. Particularly, the convenience of performing a direct trajectory rather than a multi-revolution one can be a matter of research.
A deeper analysis on the size of the orbits can unveil new regularities but when considering transfers to the outer Solar System the analysis cannot procede without considering flybys.
All the results obtained are valid in a different solar system or in a particular system of moons just by scaling the results with the peculiar data of the new orbits considered. An interesting scenario involves three of the Galilean moons: Io, Europa and Ganymede. The analysis could be performed in this system considering the effects produced by the orbital resonance phenomenon which alters the gravitational interaction during the conjunctions.

References

[1] Davide Menzio. "Grid search application for trajectory design in presence of flybys". PhD dissertation, Politecnico di Milano, 2019. Supervisor: Camilla Colombo.

[2] Ryan C. Woolley and Charles W. Whetsel. "On the nature of earth-mars porkchop plots". Technical report, NASA/JPL, 2013.