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Design of a homodyne photodetector for squeezed states at $2\mu\text{m}$

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Abstract

Next-generation gravitational-wave interferometers will transition to cryogenic crystalline silicon test masses to capitalize on their exceptionally low optical absorption and scattering, thereby reducing critical noise sources. To date, observatories like GEO600, LIGO, and Virgo have successfully pushed past the shot-noise limit by injecting squeezed light states. However, combining the advantages of quantum squeezing with silicon optics dictates a departure from the standard 1064 nm laser wavelength. Because silicon is hindered by severe two-photon absorption at wavelengths shorter than 1500 nm, the 2 μm region has emerged as the targeted operational wavelength for future detector designs.

This thesis describes the design, realization, and characterization of a custom balanced homodyne photodetector explicitly conceived to measure squeezed light at 2 μm . The detector relies on a low-noise transimpedance amplifier and two extended InGaAs p-i-n photodiodes configured in series. This architecture physically subtracts the photocurrents before amplification, thereby minimizing additional electronic noise. The electronics were rigorously simulated for stability and gain prior to being implemented on custom printed circuit boards.

The system was experimentally validated using a 2090 nm Tm-Ho:YAG laser. In the 100 kHz to 200 kHz frequency band, the device successfully resolved the optical shot noise, providing a maximum shot noise clearance over the detector's dark noise of 13.54 dB when operated with a local oscillator power of approximately 2.6 mW. Performance testing confirmed the detector's capability to resolve at least 6 dB of quantum squeezing, validating its potential for future squeezed state detection.

Keywords: Homodyne photodetector, Squeezed vacuum states, Gravitational waves, Shot noise, 2 μm

Abstract in lingua italiana

Gli interferometri per onde gravitazionali di prossima generazione passeranno all'uso di masse di prova in silicio cristallino a temperature criogeniche per sfruttare i loro eccezionalmente bassi livelli di assorbimento e scattering ottico, riducendo di conseguenza le principali fonti di rumore. Ad oggi, osservatori come GEO600, LIGO e Virgo hanno superato il limite dello shot noise iniettando stati di luce squeezed. Tuttavia, combinare i vantaggi dello squeezing quantistico con l'ottica in silicio impone di abbandonare la lunghezza d'onda laser standard di 1064 nm. Poiché il silicio è fortemente limitato dall'assorbimento a due fotoni a lunghezze d'onda inferiori a 1500 nm, la regione dei 2 μm è emersa come lunghezza d'onda operativa di riferimento per i futuri rivelatori.

Questa tesi descrive la progettazione, la realizzazione e la caratterizzazione di un fotorivelatore in configurazione omodina bilanciata concepito per rilevare stati di luce squeezed a 2 μm . Il rivelatore si basa su un amplificatore a transimpedenza a basso rumore e su due fotodiodi p-i-n in InGaAs esteso collegati in serie. Questa architettura sottrae fisicamente le fotocorrenti prima dell'amplificazione, riducendo così al minimo il rumore elettronico aggiuntivo. L'elettronica è stata rigorosamente simulata in termini di stabilità e guadagno prima di essere implementata su circuiti stampati dedicati.

Il sistema è stato validato sperimentalmente utilizzando un laser Tm-Ho:YAG a 2090 nm. Operando con una potenza massima dell'oscillatore locale di 2.6 mW, il dispositivo ha misurato con successo lo shot noise ottico nella banda di frequenza compresa tra 100 kHz e 200 kHz. In questa configurazione ha garantito una clearance massima (o margine massimo) dal rumore elettronico del fotorivelatore pari a 13.54 dB. I test prestazionali hanno inoltre confermato la capacità del rivelatore di misurare almeno 6 dB di squeezing quantistico, dimostrandone il potenziale per future misurazioni di stati squeezed.

Parole chiave: Fotorivelatore omodina, Stati di vuoto squeezed, Onde gravitazionali, Rumore shot, 2 μm

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Introduction

The observation of gravitational waves has profoundly transformed our approach to astrophysics, opening a new window onto the universe. First theorized by Albert Einstein over a century ago as a consequence of general relativity [10, 11], these ripples in spacetime remained elusive until the landmark first detection achieved by the Advanced LIGO and Virgo collaborations in 2015 [2]. Since then, the observation of additional coalescence of binary black holes [1, 3, 4] and neutron stars, such as the multi-messenger event GW170817 [5], has provided unprecedented insights into strong-field gravity, the expansion rate of the universe, and the origin of heavy elements.

As the global network of interferometers prepares for future observing runs and next-generation facilities, increasing detector sensitivity is paramount to expanding the observable horizon. A fundamental boundary to this sensitivity is imposed by quantum noise, which manifests as photon shot noise at high frequencies and radiation pressure noise at lower ones. The theoretical foundation for mitigating this limit was first established by Caves [8], who proposed that injecting squeezed vacuum states into the interferometer could circumvent this boundary. Following the first experimental demonstrations of squeezed light generation [23, 26], this approach has matured into a highly effective and fully implemented strategy in modern gravitational-wave detectors. Over the decades, the technology has advanced remarkably, culminating in the record-breaking detection of 15 dB of quantum squeezing at the standard wavelength of 1064 nm [25] in a controlled laboratory environment. However, translating these table-top achievements to actual kilometer-scale interferometers introduces severe practical challenges, primarily due to optical losses and phase fluctuations. Despite these hurdles, the injection of squeezed vacuum has been successfully integrated into real-world observatories. This has led to substantial milestones, such as the demonstration of 6 dB of quantum noise reduction at GEO600 [19], and has ultimately become a standard, employed to enhance the broadband sensitivity of state-of-the-art detectors like Virgo and Advanced LIGO [12].

To further improve interferometer sensitivity, next-generation observatories are expected to operate at cryogenic temperatures using silicon test masses to minimize thermal noise. This technological shift necessitates a departure from the traditional 1064 nm laser wave-

length, at which silicon is highly opaque. An intense sub-1500 nm wavelength beam on silicon would suffer severe two-photon absorption. Consequently, the 2 μm region has emerged as the targeted operational wavelength for future detector designs [6], making the development of novel optoelectronic components and squeezed light sources at this new wavelength a major research priority.

The transition to the 2 μm regime has driven intense research, particularly concerning squeezed light sources and high-efficiency homodyne photodetectors. The first successful demonstration of squeezed vacuum at this novel wavelength, achieved by Mansell *et al.*, marked a crucial milestone [20]; however, the measured quantum noise reduction was initially limited to 4 dB, heavily constrained by optical losses and the suboptimal quantum efficiency of commercially available extended-InGaAs sensors. Since then, the technology has progressed rapidly. Recent achievements have successfully pushed the boundary of measurable squeezing at 2 μm up to 8 dB [17].

Within this evolving framework, this thesis work aims to design, build, and characterize a balanced homodyne photodetector optimized for the measurement of squeezed light at 2 μm , alongside the development of the optical setup required for the generation of such non-classical states.

In Chapter 1, the theoretical foundations required for the subsequent analysis are laid out. The chapter reviews the normal mode decomposition and the canonical quantization of the electromagnetic field. It then introduces key quantum states, including number, vacuum, coherent, and squeezed states, concluding with a detailed theoretical description of homodyne detection principles.

In Chapter 2, the practical realization of the photodetector is detailed. It begins by defining the specifications required to accurately measure the fluctuations of the input optical signal. The chapter then covers the electronic circuit simulations, the reasons behind the chosen design, and the layout of the experimental setup. It culminates with the presentation and discussion of the experimental results, verifying the detector's capability to accurately measure shot noise at both 1 μm and 2 μm .

In Chapter 3, the experimental setup constructed for the generation of 2 μm squeezed vacuum states is presented. The chapter characterizes the fundamental laser source and details the Second Harmonic Generation (SHG) stage. It then describes the robust four-stage fiber amplification chain implemented to boost the 1045 nm second harmonic signal to power levels sufficient for driving the downstream Optical Parametric Oscillator (OPO).

Finally, Chapter 4 summarizes the core achievements of the project and outlines potential

future developments, including the upcoming integration of the OPO and the design of an upgraded photodetector prototype.

1 | Theoretical concepts

This chapter summarizes the main theoretical concepts that are used in the following chapters of this thesis. It will mainly follow the description given by Grynberg *et al.* in *Introduction to Quantum Optics* [13]. Initially, Section 1.1 presents the normal mode decomposition of the classical electric field in vacuum. Section 1.2 describes the quantization of the electric field using the canonical quantization procedure. Section 1.3 introduces some relevant states of the quantized electric field, such as number states, coherent states, and squeezed states. Finally, Section 1.4 describes homodyne detection.

1.1. Normal modes decomposition

In this section, we derive the normal mode decomposition of the classical electric field in vacuum. Maxwell's equations describing the electromagnetic field in vacuum, thus in the absence of charges or currents, are the following:

$$\nabla \cdot \vec{E} = 0, \quad (1.1)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad (1.2)$$

$$\nabla \cdot \vec{B} = 0, \quad (1.3)$$

$$\nabla \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}. \quad (1.4)$$

These will be used to describe the field dynamics of $\vec{E}$ and $\vec{B}$.

To quantize the electric field, it is useful to rewrite it as a sum of decoupled modes. It will be shown that these modes are simply polarized monochromatic travelling waves. To achieve the normal modes decomposition of the electric field, we start by stating that it can be proven that any spatial configuration of the electric field can be expanded in a Fourier series and thus described in terms of spatial angular frequencies $\vec{k}$. If we consider the electric field to exist within a finite volume, for simplicity a cube of side L so that

$V = L^3$, we can write

$$\vec{E}(\vec{r}, t) = \sum_{\vec{n}} \tilde{\vec{E}}_{\vec{n}}(t) e^{i\vec{k}_{\vec{n}} \cdot \vec{r}} \quad (1.5)$$

where $\vec{k} = k_x \vec{u}_x + k_y \vec{u}_y + k_z \vec{u}_z$ with $\{\vec{u}_x, \vec{u}_y, \vec{u}_z\}$ being a basis of $\mathbb{R}^3$ composed of unit vectors and $k_i = n_i \frac{2\pi}{L}$, $n_i \in \mathbb{Z}$, for $i = x, y, z$. The symbol $\vec{n}$, informally written as a vector, represents the set of the three indices (n_x, n_y, n_z) . $\tilde{\vec{E}}_{\vec{n}}(t)$ is defined as

$$\tilde{\vec{E}}_{\vec{n}}(t) = \frac{1}{V} \int_V \vec{E}(\vec{r}, t) e^{-i\vec{k}_{\vec{n}} \cdot \vec{r}} d^3r. \quad (1.6)$$

It is useful to define $\vec{E}_{\vec{n}}(\vec{r}, t)$ as the single term of the sum of Equation (1.5) thus

$$\vec{E}_{\vec{n}}(\vec{r}, t) = \tilde{\vec{E}}_{\vec{n}}(t) e^{i\vec{k}_{\vec{n}} \cdot \vec{r}}. \quad (1.7)$$

Gauss's law in vacuum, i.e. Equation (1.1), states that $\nabla \cdot \vec{E}(\vec{r}, t) = 0$, which implies $\vec{k}_{\vec{n}} \cdot \vec{E}_{\vec{n}}(\vec{r}, t) = 0$. Since $\vec{E}_{\vec{n}}(\vec{r}, t)$ is parallel to $\tilde{\vec{E}}_{\vec{n}}(t)$, the latter can be written as $\tilde{\vec{E}}_{\vec{n}} = \tilde{E}_{\vec{n}_1} \vec{\epsilon}_{\vec{n}_1} + \tilde{E}_{\vec{n}_2} \vec{\epsilon}_{\vec{n}_2}$, with $\vec{\epsilon}_{\vec{n}_s}$, $s \in \{1, 2\}$, being a pair of unit vectors such that $\vec{\epsilon}_{\vec{n}_s} \cdot \vec{k}_{\vec{n}} = 0 \quad \forall n, s$ and $\vec{\epsilon}_{\vec{n}_1} \cdot \vec{\epsilon}_{\vec{n}_2} = 0 \quad \forall \vec{n}$.

It is possible to order the countably infinite set of (n_x, n_y, n_z, s) using a single index $l \in \mathbb{Z}$. Among all possible ordering rules, we can choose one such that if $l_1 = (n_x, n_y, n_z, s)$ and $l_2 = (-n_x, -n_y, -n_z, s)$, then $l_1 = -l_2$. It follows that $\vec{\epsilon}_l = \vec{\epsilon}_{-l}$ and $\vec{k}_l = -\vec{k}_{-l}$. This allows us to write the electric field as

$$\vec{E}(\vec{r}, t) = \sum_l \vec{\epsilon}_l \tilde{E}_l(t) e^{i\vec{k}_l \cdot \vec{r}}. \quad (1.8)$$

Let us call $\vec{A}(\vec{r}, t)$ the vector potential of the electromagnetic field. In free space, the relations between the vector potential and the electric and magnetic fields as functions of spatial coordinates are

$$\begin{aligned} \vec{B}(\vec{r}, t) &= \nabla \times \vec{A}(\vec{r}, t) \\ \vec{E}(\vec{r}, t) &= -\frac{\partial \vec{A}}{\partial t}(\vec{r}, t) \end{aligned} \quad (1.9)$$

In the Coulomb gauge, i.e. $\nabla \cdot \vec{A}(\vec{r}, t) = 0$, the vector potential can be written in the same form as the electric field, thus

$$\vec{A}(\vec{r}, t) = \sum_l \vec{\epsilon}_l \tilde{A}_l(t) e^{i\vec{k}_l \cdot \vec{r}}. \quad (1.10)$$

The same can be done for the magnetic field, by first defining $\vec{\epsilon}' := \frac{\vec{k}_l}{|\vec{k}_l|} \times \vec{\epsilon}_l$:

$$\vec{B}(\vec{r}, t) = \sum_l \vec{\epsilon}'_l \tilde{B}_l(t) e^{i\vec{k}_l \cdot \vec{r}}. \quad (1.11)$$

Equations (1.9) impose the following conditions on the Fourier coefficients of the fields defined in (1.8) and (1.11):

$$\tilde{E}_l(t) = -\frac{d\tilde{A}_l}{dt}(t) \quad \text{and} \quad \tilde{B}_l = i k_l \tilde{A}_l(t). \quad (1.12)$$

Since all the fields must be real, the Fourier coefficient associated with the l -th mode must be the complex conjugate of the one associated with the $(-l)$ -th mode:

$$\tilde{E}_{-l}(t) = \tilde{E}_l^*(t) \quad \forall t \in \mathbb{R} \quad (1.13)$$

In this way, the sum of complex exponentials becomes a sum of real cosines, and therefore $\vec{E} \in \mathbb{R}^3$.

Similar relations hold for the magnetic field and the vector potential coefficients.

The field dynamics are governed by Maxwell's equations (1.2) and (1.4). Using the fields written in the form of a Fourier expansion, as in (1.8), (1.10) and (1.11), we find that the field dynamics are described by the following conditions on the coefficients of the electric and magnetic fields:

$$\frac{d\tilde{B}_l}{dt}(t) = -i k_l \tilde{E}_l(t) \quad \text{and} \quad \frac{d\tilde{E}_l}{dt}(t) = -i c^2 k_l \tilde{B}_l(t). \quad (1.14)$$

These equations do not couple different l modes, but the reality conditions couple the l -th mode to the $(-l)$ -th mode. It is possible to write fully decoupled equations by defining a new set of modes. Letting $\omega_l := c k_l$, we first define two new complex coordinates:

$$\alpha_l(t) = \frac{1}{2\mathcal{E}_l} \left[\omega_l \tilde{A}_l(t) - i \tilde{E}_l(t) \right] \quad (1.15)$$

$$\beta_l(t) = \frac{1}{2\mathcal{E}_l} \left[\omega_l \tilde{A}_l(t) + i \tilde{E}_l(t) \right] \quad (1.16)$$

where $\mathcal{E}_l$ is an arbitrary constant that will be defined later to simplify the notation.

Equations (1.14), expressed in the new coordinates, become:

$$\frac{d\alpha_l}{dt}(t) + i\omega_l \alpha_l(t) = 0 \quad \text{and} \quad \frac{d\beta_l}{dt}(t) = -i\omega_l \beta_l(t). \quad (1.17)$$

However, the reality conditions, such as (1.13), impose that $\alpha_{-l}(t) = \beta_l^*(t)$. Thus, the β coordinates are simply the complex conjugates of the α coordinates with opposite indices. Therefore, they are redundant, and we can rely solely on the so-called normal modes α_l , whose dynamics are fully described by:

$$\frac{d\alpha_l}{dt}(t) + i\omega_l \alpha_l(t) = 0. \quad (1.18)$$

This equation fully decouples the dynamics of the different l modes, hence the name “normal coordinates” given to α_l . The solution to Equation (1.18) is straightforward to obtain; thus, the time evolution of the classical normal coordinate is:

$$\alpha_l(t) = \alpha_l(0)e^{-i\omega_l t}. \quad (1.19)$$

For simplicity of notation we will hereby use α_l to indicate $\alpha_l(0)$, and $\alpha_l(t)$ when referring to $\alpha_l(0)e^{-i\omega_l t}$.

Inverting definitions (1.15) and (1.16), and keeping in mind that $\alpha_{-l}(t) = \beta_l^*(t)$, one obtains the electric field Fourier coefficient $\tilde{E}_l(t)$ as a function of the normal coordinate $\alpha_l(t)$:

$$\tilde{E}_l(t) = \mathcal{E}_l i [\alpha_l(t) - \alpha_{-l}^*(t)]. \quad (1.20)$$

Therefore, the electric field is given by:

$$\vec{E}(\vec{r}, t) = \sum_l \vec{e}_l \mathcal{E}_l i [\alpha_l(t) - \alpha_{-l}^*(t)] e^{i\vec{k}_l \cdot \vec{r}}. \quad (1.21)$$

Since we are summing over all possible l values, l is a dummy index, and we can substitute $-l$ with l in the complex conjugate of α . Keeping in mind that $\vec{k}_{-l} = -\vec{k}_l$, we get:

$$\vec{E}(\vec{r}, t) = \sum_l \vec{e}_l E_l(\vec{r}, t) = \sum_l \vec{e}_l \mathcal{E}_l i [\alpha_l(t) e^{i\vec{k}_l \cdot \vec{r}} - \alpha_l^*(t) e^{-i\vec{k}_l \cdot \vec{r}}] \quad (1.22)$$

where we defined the single-mode classical electric field E_l as the l -th term of the sum, i.e.:

$$E_l(\vec{r}, t) = \mathcal{E}_l i [\alpha_l(t) e^{i\vec{k}_l \cdot \vec{r}} - \alpha_l^*(t) e^{-i\vec{k}_l \cdot \vec{r}}]. \quad (1.23)$$

Similar results can be obtained for the magnetic field and the vector potential.

The energy of the electromagnetic field, or radiant energy, H , is given by the following integral:

$$H = \frac{\varepsilon_0}{2} \int_V \left[\left| \vec{E}(\vec{r}, t) \right|^2 + c^2 \left| \vec{B}(\vec{r}, t) \right|^2 \right] d^3r. \quad (1.24)$$

Substituting $\vec{E}$ with its expansion in (1.22) and doing the same for the analogous expansion of the magnetic field, the integration yields:

$$H = 2\varepsilon_0 V \sum_l [\mathcal{E}_l]^2 |\alpha_l|^2 = \sum_l H_l \quad (1.25)$$

where we defined

$$H_l = 2\varepsilon_0 V [\mathcal{E}_l]^2 |\alpha_l|^2. \quad (1.26)$$

Therefore, the radiant energy of the system, H , is given by the sum of the energies associated with each single l -th normal mode, H_l , without cross terms. We can now exploit the freedom in choosing the value of $\mathcal{E}_l$ by setting it to:

$$\mathcal{E}_l = \sqrt{\frac{\hbar \omega_l}{2\varepsilon_0 V}}. \quad (1.27)$$

In this way, it is possible to rewrite H_l as:

$$H_l = \hbar \omega_l |\alpha_l|^2 = \frac{\hbar \omega_l}{2} (\alpha_l \alpha_l^* + \alpha_l^* \alpha_l). \quad (1.28)$$

The latter form, although unusual in the classical case, is used to highlight the analogy with the quantum case.

To proceed with the quantization of the electric field, it is necessary to identify the canonical variables of the system. It can be proven that the classical conjugate canonical variables of the system are simply rescaled versions of $\text{Re}(\alpha_l)$ and $\text{Im}(\alpha_l)$. Indeed, the canonical coordinates of the problem are defined as:

$$q_l = \sqrt{\frac{4\varepsilon_0 V}{\omega_l}} \mathcal{E}_l \text{Re}(\alpha_l) = \frac{\sqrt{2\hbar}}{2} (\alpha_l + \alpha_l^*) \quad (1.29)$$

while the conjugate momenta are defined by:

$$p_l = \sqrt{\frac{4\varepsilon_0 V}{\omega_l}} \mathcal{E}_l \text{Im}(\alpha_l) = \frac{-i\sqrt{2\hbar}}{2} (\alpha_l - \alpha_l^*). \quad (1.30)$$

Using these variables, the radiant energy can be rewritten as:

$$H = \sum_l H_l = \sum_l \frac{\hbar \omega_l}{2} (\alpha_l \alpha_l^* + \alpha_l^* \alpha_l) = \sum_l \frac{\omega_l}{2} (q_l^2 + p_l^2). \quad (1.31)$$

Rescaled versions of the conjugate canonical variables, called quadrature components, allow us to rewrite the single-mode electric field in an equivalent, but more intuitive way compared to Equation (1.23). We can define the quadrature components of the electric field as:

$$E_{p_l} = \sqrt{\frac{\omega_l}{\varepsilon_0 V}} p_l = -i \mathcal{E}_l (\alpha_l - \alpha_l^*) \quad (1.32)$$

$$E_{q_l} = \sqrt{\frac{\omega_l}{\varepsilon_0 V}} q_l = \mathcal{E}_l (\alpha_l + \alpha_l^*) \quad (1.33)$$

The electric field of the single mode in Equation (1.23), expressed in terms of E_{p_l} and E_{q_l} , can be rewritten as:

$$E_l(\vec{r}, t) = -E_{p_l} \cos(\vec{k}_l \cdot \vec{r} - \omega_l t) - E_{q_l} \sin(\vec{k}_l \cdot \vec{r} - \omega_l t). \quad (1.34)$$

The field simply is a sinusoidal wave that can be written as

$$E_l(\vec{r}, t) = E_l^0 \cos(\vec{k}_l \cdot \vec{r} - \omega_l t + \varphi_l) \quad (1.35)$$

with

$$E_l^0 = \sqrt{E_{p_l}^2 + E_{q_l}^2} = 2 \mathcal{E}_l |\alpha_l| \quad (1.36)$$

$$\varphi_l = \angle(\alpha_l) + \frac{\pi}{2}.$$

The normal variables description of Equation (1.23), the quadratures description of Equation (1.34) and the phase and amplitude description of Equation (1.35) are all equivalent ways of describing the single-mode electric field. Each of them can reveal itself to be the most convenient description to use in a given situation.

We can define the analytical signal of the single-mode electric field as:

$$E_l^+(\vec{r}, t) = \mathcal{E}_l i \alpha_l(t) e^{i \vec{k}_l \cdot \vec{r}}. \quad (1.37)$$

For uniformity of notation, we will express its complex conjugate as:

$$E_l^-(\vec{r}, t) = -\mathcal{E}_l i \alpha_l^*(t) e^{-i \vec{k}_l \cdot \vec{r}} \quad (1.38)$$

so that the single-mode electric field is:

$$E_l(\vec{r}, t) = E_l^+(\vec{r}, t) + E_l^-(\vec{r}, t) = \text{Re}\{2 E_l^+(\vec{r}, t)\}. \quad (1.39)$$

Using the given definitions one can easily obtain the relation between the analytical signal and the two quadrature

$$2 E_l^+(\vec{r}, t) = (-E_{pl} + iE_{ql}) e^{i(\vec{k}_l \cdot \vec{r} - \omega_l t)} \quad (1.40)$$

Equation (1.40) is very helpful to draw the so called phasor representation of the single-mode electric field:

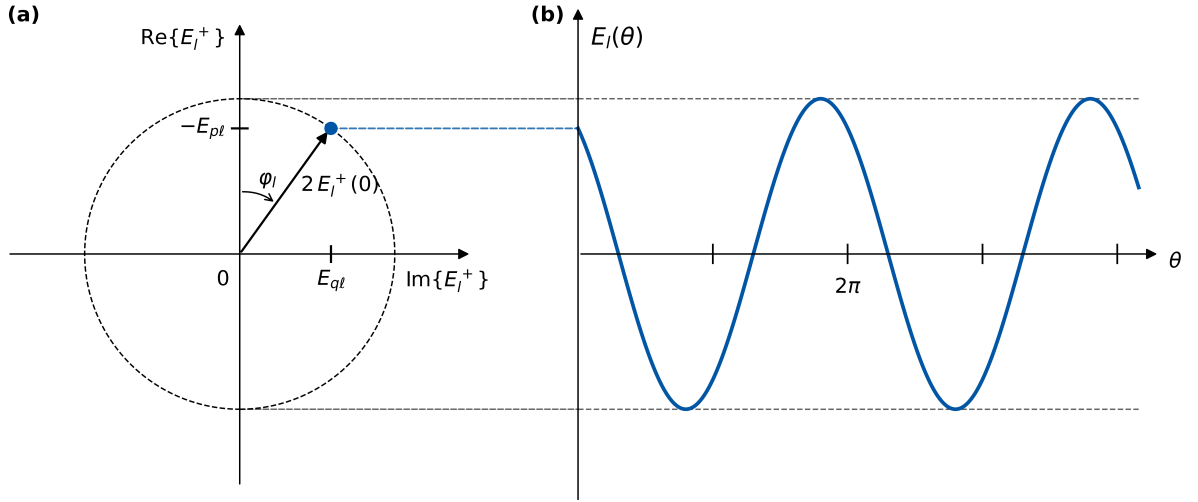


Figure 1.1: Phasor diagram of a classical state.

In Figure 1.1, the variable θ is defined as $\theta = (\vec{k}_l \cdot \vec{r} - \omega_l t)$ and $2 E_l^+(0)$ is used to indicate $2 E_l^+(\vec{r} = 0, t = 0)$. Note that to optimize the page layout, the real and imaginary axes of the complex plane have been transposed, allowing the phasor to be projected horizontally. This convention is applied consistently to all phasor diagrams throughout this thesis.

Using the quadratures description the total electric field is given by:

$$\vec{E}(\vec{r}, t) = \sum_l \vec{e}_l \left[-E_{pl} \cos(\vec{k}_l \cdot \vec{r} - \omega_l t) - E_{ql} \sin(\vec{k}_l \cdot \vec{r} - \omega_l t) \right]. \quad (1.41)$$

Therefore, the electric field has been written as the sum of a countably infinite set of polarized independent modes. Each mode E_l is a sinusoidal, function of space oscillating with wave vector $\vec{k}_l$ and function of time oscillating with angular frequency ω_l .

1.2. Quantization of the electric field

Once the canonical variables have been identified, the canonical quantization method can be applied. Therefore, we associate a quantum operator with each classical canonical variable: $q_l \rightarrow \hat{Q}_l$ and $p_l \rightarrow \hat{P}_l$. The canonical commutation relation is imposed on these operators

$$[\hat{Q}_l, \hat{P}_{l'}] = i \hbar \delta_{ll'} \quad (1.42)$$

and since the modes are decoupled, $[\hat{Q}_l, \hat{Q}_{l'}] = [\hat{P}_l, \hat{P}_{l'}] = 0$. The quantum Hamiltonian of the system is $\hat{H} = \sum_l \frac{\omega_l}{2} (\hat{Q}_l^2 + \hat{P}_l^2)$.

Given the classical definitions of q_l and p_l as functions of α_l , one can introduce the non-Hermitian quantum operator $\hat{a}_l$ corresponding to α_l , and its Hermitian conjugate $\hat{a}_l^\dagger$ corresponding to α_l^* :

$$\hat{a}_l = \frac{1}{\sqrt{2\hbar}} (\hat{Q}_l + i \hat{P}_l) \quad (1.43)$$

$$\hat{a}_l^\dagger = \frac{1}{\sqrt{2\hbar}} (\hat{Q}_l - i \hat{P}_l) \quad (1.44)$$

These two operators satisfy the following commutation relations, which are immediately derivable from the canonical commutation relations of Equation (1.42):

$$[\hat{a}_l, \hat{a}_{l'}^\dagger] = \delta_{ll'} \quad (1.45)$$

$$[\hat{a}_l, \hat{a}_{l'}] = 0 \quad (1.46)$$

The classical electric field defined in Equation (1.22) is associated with the electric field operator $\hat{\vec{E}}(\vec{r})$:

$$\hat{\vec{E}}(\vec{r}) = \sum_l \vec{\epsilon}_l \mathcal{E}_l i \left(\hat{a}_l e^{i \vec{k}_l \cdot \vec{r}} - \hat{a}_l^\dagger e^{-i \vec{k}_l \cdot \vec{r}} \right). \quad (1.47)$$

As already done in the classical case, the single-mode electric field operator $\hat{E}_l(\vec{r})$ is defined as the scalar part of the l -th term of the summation:

$$\hat{E}_l(\vec{r}) = \mathcal{E}_l i \left(\hat{a}_l e^{i \vec{k}_l \cdot \vec{r}} - \hat{a}_l^\dagger e^{-i \vec{k}_l \cdot \vec{r}} \right). \quad (1.48)$$

The classical Hamiltonian of Equation (1.31) has its quantum counterpart in the Hamil-

tonian operator, which can be written in the following equivalent forms:

$$\hat{H} = \sum_l \frac{\omega_l}{2} (\hat{Q}_l^2 + \hat{P}_l^2) = \sum_l \frac{\hbar \omega_l}{2} (\hat{a}_l \hat{a}_l^\dagger + \hat{a}_l^\dagger \hat{a}_l) = \sum_l \hbar \omega_l \left(\hat{a}_l^\dagger \hat{a}_l + \frac{1}{2} \right) \quad (1.49)$$

In the last equivalence, we used the commutator (1.45).

The commutation relations (1.45) and (1.46), together with the expression for the Hamiltonian operator (1.49), form the foundation of quantum optics.

It is worth noting that, in the quantum description as well, the Hamiltonian operator of the entire system, $\hat{H}$, is given by the sum of single-mode terms without any cross terms. By defining

$$\hat{H}_l = \hbar \omega_l \left(\hat{a}_l^\dagger \hat{a}_l + \frac{1}{2} \right) \quad (1.50)$$

as the Hamiltonian operator associated with a single l -th mode, the whole system Hamiltonian is given by $\hat{H} = \sum_l \hat{H}_l$.

Let $|\Psi_l\rangle$ be the eigenstate of $\hat{H}_l$ and $|\Psi\rangle$ be the eigenstate of $\hat{H}$. Then $|\Psi\rangle$ is given by $|\Psi\rangle = |\Psi_1\rangle \otimes |\Psi_2\rangle \otimes \cdots \otimes |\Psi_l\rangle \otimes \cdots$, where $\otimes$ denotes a tensor product.

From here on, the description of the quantum system is analogous to that of a quantum harmonic oscillator.

Given the definitions of $\hat{H}_l$, $\hat{a}_l$, and $\hat{a}_l^\dagger$, one can easily compute:

$$[\hat{H}_l, \hat{a}_l] = -\hbar \omega_l \hat{a}_l \quad (1.51)$$

$$[\hat{H}_l, \hat{a}_l^\dagger] = \hbar \omega_l \hat{a}_l^\dagger. \quad (1.52)$$

These results are sufficient to identify $\hat{a}_l$ and $\hat{a}_l^\dagger$ as ladder operators. In particular, $\hat{a}_l$ is the annihilation operator. Indeed, letting $|\Psi\rangle$ be an eigenstate of $\hat{H}$ with energy E_n (i.e. $\hat{H} |\Psi\rangle = E_n |\Psi\rangle$), the annihilation operator acts on $|\Psi\rangle$ as follows:

$$\hat{H} \hat{a}_l |\Psi\rangle = (E_n - \hbar \omega_l) \hat{a}_l |\Psi\rangle \quad (1.53)$$

Analogously, $\hat{a}_l^\dagger$ is the creation operator. It acts on $|\Psi\rangle$ as follows:

$$\hat{H} \hat{a}_l^\dagger |\Psi\rangle = (E_n + \hbar \omega_l) \hat{a}_l^\dagger |\Psi\rangle \quad (1.54)$$

Therefore, the ladder operators applied to any eigenstate of the system produce a (non-normalized) eigenstate with its energy increased or decreased by a discrete amount $\hbar \omega_l$.

Hence the origin of their names: it is as if the annihilation operator removes a particle of fixed energy $\hbar\omega_l$ from the system, while the creation operator adds one such particle to the system.

To the classical quadrature components of the single-mode electric field (1.32) and (1.33), it is possible to associate the so-called quadrature operators $E_{p_l} \rightarrow \hat{E}_{P_l}$ and $E_{q_l} \rightarrow \hat{E}_{Q_l}$, which are observables and therefore Hermitian operators:

$$\hat{E}_{P_l} = \sqrt{\frac{\omega_l}{\varepsilon_0 V}} \hat{P}_l = -i \mathcal{E}_l (\hat{a}_l - \hat{a}_l^\dagger) \quad (1.55)$$

$$\hat{E}_{Q_l} = \sqrt{\frac{\omega_l}{\varepsilon_0 V}} \hat{Q}_l = \mathcal{E}_l (\hat{a}_l + \hat{a}_l^\dagger). \quad (1.56)$$

The single-mode electric field operator, defined in Equation (1.47), can be rewritten as follows:

$$\hat{E}_l(\vec{r}) = -\hat{E}_{P_l} \cos(\vec{k}_l \cdot \vec{r}) - \hat{E}_{Q_l} \sin(\vec{k}_l \cdot \vec{r}). \quad (1.57)$$

The canonical commutation relation (1.42) implies the following Heisenberg uncertainty relation between the two canonically conjugate variables:

$$\Delta P_l \Delta Q_l \geq \frac{\hbar}{2}. \quad (1.58)$$

Since the quadrature components are simply rescaled versions of these variables, we obtain the following Heisenberg relation for the dispersion of the measurements of the observables $\hat{E}_{P_l}$ and $\hat{E}_{Q_l}$:

$$\Delta E_{P_l} \Delta E_{Q_l} \geq \mathcal{E}_l^2 \quad (1.59)$$

This result is valid for any possible state of the single-mode electric field. When the inequality holds as an equality (i.e. $\Delta E_{P_l} \Delta E_{Q_l} = \mathcal{E}_l^2$), the state is called a minimum uncertainty state.

Finally in complete analogy to what was done in the classical case, given the single-mode electric field operator defined in (1.48), its analytical signal operator is given by:

$$\hat{E}_l^+(\vec{r}) = \mathcal{E}_l i \hat{a}_l e^{i \vec{k}_l \cdot \vec{r}}. \quad (1.60)$$

For uniformity of notation, we will express its Hermitian conjugate as:

$$\hat{E}_l^-(\vec{r}) = -\mathcal{E}_l i \hat{a}_l^\dagger e^{-i \vec{k}_l \cdot \vec{r}}. \quad (1.61)$$

so that the electric field operator is:

$$\hat{E}_l(\vec{r}) = \hat{E}_l^+(\vec{r}) + E_l^-(\vec{r}). \quad (1.62)$$

1.3. Quantum states

In quantum optics, the electromagnetic field is quantized by modelling each optical mode as an independent quantum harmonic oscillator. This theoretical framework naturally gives rise to various fundamental quantum states, each characterizing light with distinct physical and statistical properties.

In this section, we introduce the primary single-mode states of the radiation field. Specifically, we will formalize number (Fock) states, the vacuum state, quasi-classical coherent states, and squeezed states, detailing their algebraic definitions, and characteristic quadrature fluctuations.

1.3.1. Number states

It can be proven that, for every mode, there exists a state $|\Psi_0\rangle$ such that applying the annihilation operator to it yields zero, i.e. $\hat{a} |\Psi_0\rangle = 0$. The energy of the l -th mode in this state is therefore the minimal possible energy, called zero-point energy, E_0^l , given by:

$$\begin{aligned} E_0^l |\Psi_0\rangle &= \hat{H}_l |\Psi_0\rangle \\ &= \hbar \omega_l \left(\hat{a}_l^\dagger \hat{a}_l + \frac{1}{2} \right) |\Psi_0\rangle \\ &= \frac{\hbar \omega_l}{2} |\Psi_0\rangle \\ \implies E_0^l &= \frac{1}{2} \hbar \omega_l. \end{aligned} \quad (1.63)$$

All other eigenstates of the mode can be obtained by iteratively applying the creation operator to this lowest-energy state. Thus, the generic eigenstate is given by:

$$|\Psi_n\rangle = A_n \left(\hat{a}_l^\dagger \right)^n |\Psi_0\rangle \quad (1.64)$$

where A_n is a proper normalization constant. The energy of this state is:

$$E_n^l = \hbar \omega_l \left(\frac{1}{2} + n \right) \quad (1.65)$$

as immediately follows from the application of $\hat{a}_l^\dagger$ n times on $|\Psi_0\rangle$. In terms of energy, it is as if the mode in the $|\Psi_n\rangle$ state is populated by n particles of energy $\hbar\omega_l$, whose energies add to the zero-point energy. These particles are called photons.

It is common practice to write the state in Equation (1.64) in terms of the number of photons such state is populated with, using the notation:

$$|n\rangle = \frac{1}{\sqrt{n!}} \left(\hat{a}_l^\dagger\right)^n |0\rangle \quad (1.66)$$

where $|0\rangle$ is the minimum-energy eigenstate, called the vacuum state, and where we have replaced the normalization constant A_n with its proper value $(\sqrt{n!})^{-1}$. The $|n\rangle$ states are eigenstates of the Hamiltonian H_l , but they are also eigenstates of the so-called number operator $\hat{n} := \hat{a}_l^\dagger \hat{a}_l$:

$$\begin{aligned} \hat{n} |n\rangle &= (\hat{a}_l^\dagger \hat{a}_l) |n\rangle = \left(\frac{\hat{H}_l}{\hbar\omega_l} - \frac{1}{2} \right) |n\rangle = \left(\frac{E_n^l}{\hbar\omega_l} - \frac{1}{2} \right) |n\rangle = n |n\rangle \\ &\implies \hat{n} |n\rangle = n |n\rangle. \end{aligned} \quad (1.67)$$

Indeed, $[\hat{H}_l, \hat{n}] = 0$, and thus $\hat{H}_l$ and $\hat{n}$ share a basis of common eigenstates.

To summarize, the single-mode states $|n\rangle$ are eigenstates of both $\hat{H}_l$ and $\hat{n}$. Thus, they are stationary states characterized by a fixed energy, E_n^l , or equivalently, by being populated by n photons. These states are called Fock states or number states.

1.3.2. Vacuum state

The vacuum state is a Fock state populated by zero photons. It has some interesting properties. As one would classically expect from the vacuum state, the expectation value of the single-mode electric field $\langle \hat{E}_l \rangle$ is zero. Indeed:

$$\begin{aligned} \langle 0 | \hat{E}_l | 0 \rangle &= \langle 0 | i \vec{\epsilon}_l \mathcal{E}_l \left(\hat{a}_l e^{i \vec{k}_l \cdot \vec{r}} - \hat{a}_l^\dagger e^{-i \vec{k}_l \cdot \vec{r}} \right) | 0 \rangle \\ &= i \vec{\epsilon}_l \mathcal{E}_l \left(\langle 0 | \hat{a}_l | 0 \rangle e^{i \vec{k}_l \cdot \vec{r}} - \langle 0 | \hat{a}_l^\dagger | 0 \rangle e^{-i \vec{k}_l \cdot \vec{r}} \right) \\ &= -i \vec{\epsilon}_l \mathcal{E}_l \langle 0 | 1 \rangle e^{-i \vec{k}_l \cdot \vec{r}} = 0. \end{aligned} \quad (1.68)$$

The surprising result, however, is associated with the variance of the field $(\Delta E_l)^2$:

$$\begin{aligned}
 (\Delta E_l)^2 &= \langle \hat{E}_l^2 \rangle - \langle \hat{E}_l \rangle^2 = \langle \hat{E}_l^2 \rangle = \\
 &= -\mathcal{E}_l^2 \langle 0 | \left(\hat{a}_l e^{i\vec{k}_l \cdot \vec{r}} - \hat{a}_l^\dagger e^{-i\vec{k}_l \cdot \vec{r}} \right)^2 | 0 \rangle = \\
 &= \mathcal{E}_l^2 \langle 0 | \hat{a}_l \hat{a}_l^\dagger | 0 \rangle = \mathcal{E}_l^2 \langle 0 | \hat{a}_l | 1 \rangle = \\
 &= \mathcal{E}_l^2 \langle 0 | 0 \rangle = \mathcal{E}_l^2.
 \end{aligned} \tag{1.69}$$

Therefore, even though the expectation value over an ensemble of vacuum states is zero, the electric field is not certainly zero. It is characterized by the so-called vacuum fluctuations.

The vacuum state is a state of minimum uncertainty. Indeed, similarly to what has just been done in (1.68) and (1.69), it is straightforward to obtain that:

$$\langle \hat{E}_{Q_l} \rangle = \langle \hat{E}_{P_l} \rangle = 0 \tag{1.70}$$

but

$$(\Delta E_{Q_l})^2 = (\Delta E_{P_l})^2 = \mathcal{E}_l^2. \tag{1.71}$$

Therefore, in a vacuum state:

$$\Delta E_{Q_l} \Delta E_{P_l} = \mathcal{E}_l^2. \tag{1.72}$$

which implies that it is a minimum uncertainty state.

It is interesting to underline that both $\langle \hat{E}_l \rangle$ and $(\Delta E_l)^2$, as well as $\langle \hat{E}_{Q_l} \rangle$ and $(\Delta E_{Q_l})^2$ (and the same for the other quadrature), are space-independent and time-independent. This latter result may be considered obvious, as Fock states are stationary states. This state does not resemble any classical state, where the field value is either modulated in space and time or identically null (as in the classical vacuum) without carrying any intrinsic uncertainty.

In analogy to the classical field, we can represent the state of the electric field using a phasor diagram. The complex phasor of the quantum state can be plotted on the complex plane as a vector with components $(-\langle \hat{E}_{P_l} \rangle, \langle \hat{E}_{Q_l} \rangle)$. The uncertainties of $\hat{E}_{Q_l}$ and $\hat{E}_{P_l}$ are represented by a blurred region called the uncertainty region. If one takes many measurements of the electric field quadratures over an ensemble of identical states (for example, using homodyne detection) and represents them as dots on the phasor diagram, these dots will spread over the uncertainty region.

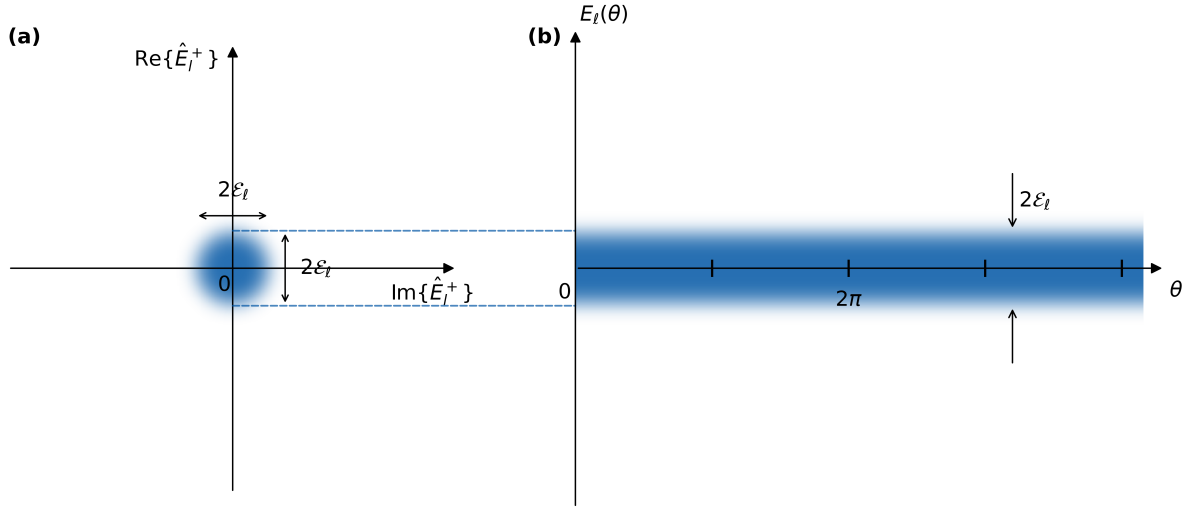


Figure 1.2: Phasor diagram of a vacuum state.

Figure 1.2 shows the phasor diagram for a single-mode vacuum state. In such state $\langle \hat{E}_{P_l} \rangle = \langle \hat{E}_{Q_l} \rangle = 0$ so the phasor is set in the origin. Since $\Delta E_{Q_l} = \Delta E_{P_l} = \varepsilon_l$ the uncertainty region is a disc of diameter $2\varepsilon_l$. In every position $\vec{r}$ and at any time t , thus for any $\theta = \vec{k}_l \cdot \vec{r} - \omega_l t$, the expectation values is null and the uncertainties over the two quadratures are the same.

1.3.3. Coherent states

Coherent states, or quasi-classical states, are states with properties similar to those of classical states. They are defined as the eigenstates of the annihilation operator. The coherent state $|\alpha_l\rangle$ is the eigenstate of $\hat{a}_l$ associated with the eigenvalue α_l , i.e.:

$$\hat{a}_l |\alpha_l\rangle = \alpha_l |\alpha_l\rangle. \quad (1.73)$$

Since $\hat{a}_l$ is not Hermitian, the existence of such eigenstates is not guaranteed. However, one can check that any state of the form

$$|\alpha_l\rangle = e^{-\frac{1}{2}|\alpha_l|^2} \sum_{n_l=0}^{\infty} \frac{\alpha_l^{n_l}}{\sqrt{n_l!}} |n_l\rangle \quad (1.74)$$

satisfies Equation (1.73). This state is a superposition of eigenstates of the Hamiltonian and therefore is not stationary. Letting $|\alpha_l\rangle$ be the coherent state at time zero, the

coherent state at a generic time t can be written as:

$$|\alpha_l e^{-i\omega_l t}\rangle = e^{-\frac{1}{2}(|\alpha_l|^2)} \sum_{n_l=0}^{\infty} \frac{(\alpha_l e^{-i\omega_l t})^{n_l}}{\sqrt{n_l!}} |n_l\rangle. \quad (1.75)$$

Indeed, it satisfies:

$$\hat{a}_l |\alpha_l e^{-i\omega_l t}\rangle = \alpha_l e^{-i\omega_l t} |\alpha_l e^{-i\omega_l t}\rangle. \quad (1.76)$$

which means that a coherent state retains its quasi-classical form over time, but its eigenvalue oscillates as $\alpha_l e^{-i\omega_l t}$.

Using the single-mode electric field operator from Equation (1.48) and the eigenvalue equation above, one can compute the expectation value of the single-mode electric field $\langle \hat{E}_l \rangle$:

$$\langle \hat{E}_l \rangle = \langle \alpha_l e^{-i\omega_l t} | \hat{E}_l(\vec{r}) | \alpha_l e^{-i\omega_l t} \rangle = i \mathcal{E}_l \left(\alpha_l e^{i(\vec{k}_l \cdot \vec{r} - \omega_l t)} - \alpha_l^* e^{-i(\vec{k}_l \cdot \vec{r} - \omega_l t)} \right). \quad (1.77)$$

The expectation value of the electric field in a coherent state oscillates in space and time exactly as the classical electric field of a plane wave of frequency ω_l would do. The frequency of this field is too high to be directly measured. However, one can measure the two quadrature components, whose expectation values (derived in the same way as for $\langle \hat{E}_l \rangle$) are:

$$\langle \hat{E}_{Q_l} \rangle = \langle \alpha_l | \hat{E}_{Q_l} | \alpha_l \rangle = \mathcal{E}_l (\alpha_l + \alpha_l) = 2 \mathcal{E}_l \text{Re}\{\alpha_l\} \quad (1.78)$$

$$\langle \hat{E}_{P_l} \rangle = \langle \alpha_l | \hat{E}_{P_l} | \alpha_l \rangle = -i \mathcal{E}_l (\alpha_l - \alpha_l) = 2 \mathcal{E}_l \text{Im}\{\alpha_l\}. \quad (1.79)$$

Similarly, one can compute the variances of the quadrature components, $(\Delta E_{Q_l})^2$ and $(\Delta E_{P_l})^2$, from which the uncertainties of these two quantities can be derived:

$$\Delta E_{Q_l} = \Delta E_{P_l} = \mathcal{E}_l. \quad (1.80)$$

The uncertainties of the quadrature components are therefore independent of $\vec{r}$ and t . Their product is equal to the minimum imposed by the Heisenberg relation (1.59); thus, any coherent state is a minimum uncertainty state.

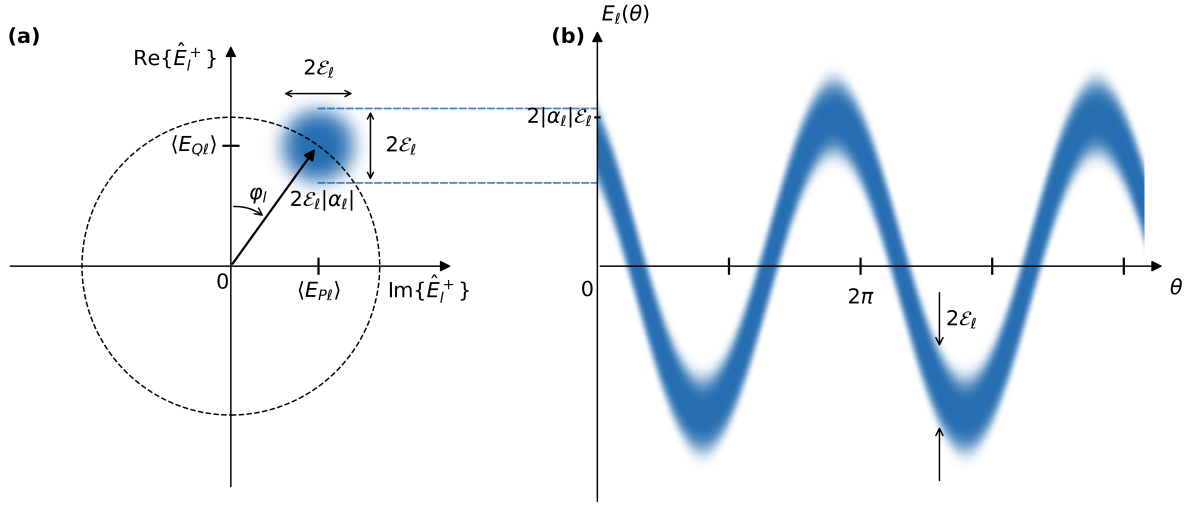


Figure 1.3: Phasor diagram of a coherent state.

Therefore, in a phasor representation like the one of Figure 1.3, the uncertainty region is the smallest possible, and since $\Delta E_{Q_l} = \Delta E_{P_l}$, it has the shape of a disk with diameter of $2\mathcal{E}_l$. The expectation value of the electric field oscillates exactly as expected from a classical mode even though it is characterized by a constant and intrinsic uncertainty.

The vacuum state, besides being the number state associated with 0 photons, can also be viewed as a semi-classical state corresponding to an eigenvalue $\alpha_l = 0$ of the annihilation operator. Thus, $|0\rangle = |(n=0)\rangle = |(\alpha_l=0)\rangle$.

1.3.4. Squeezed states

Talking only of single-mode states we will drop the subscript l for ease of notation. Squeezed states are defined as single-mode eigenstates of the following operator:

$$\hat{A}_R = \hat{a} \cosh R + \hat{a}^\dagger \sinh R \quad (1.81)$$

which is a function of the real variable R . Therefore, its eigenstates will depend on R as well as the associated eigenvalue α . The squeezed states can thus be written as $|\alpha, R\rangle$ and are defined by the following equation:

$$\hat{A}_R |\alpha, R\rangle = \alpha |\alpha, R\rangle. \quad (1.82)$$

These states are not stationary states and, since $\hat{A}_R = \hat{a}$ when $R = 0$, they coincide with coherent states for $R = 0$. Using (1.81), one can express $\hat{a}$ as:

$$\hat{a} = \hat{A}_R \cosh R - \hat{A}_R^\dagger \sinh R. \quad (1.83)$$

This helps us rewrite the single-mode field operator using definition (1.48) by substituting $\hat{a}$ and $\hat{a}^\dagger$:

$$\begin{aligned} \hat{E}(\vec{r}) = & i \mathcal{E}_l \left[\hat{A}_R \left(e^R \cos \vec{k} \cdot \vec{r} + i e^{-R} \sin \vec{k} \cdot \vec{r} \right) + \right. \\ & \left. - \hat{A}_R^\dagger \left(e^R \cos \vec{k} \cdot \vec{r} - i e^{-R} \sin \vec{k} \cdot \vec{r} \right) \right] \end{aligned} \quad (1.84)$$

Omitting the intermediate calculations, the expectation value of the electric field operator in a generic squeezed state is:

$$\langle \alpha, R | \hat{E}(\vec{r}) | \alpha, R \rangle = i \mathcal{E}_l \left[(\alpha \cosh R - \alpha^* \sinh R) e^{i \vec{k} \cdot \vec{r}} - c.c. \right]. \quad (1.85)$$

This is the same expectation value we would obtain for a semi-classical state $|\alpha'\rangle$ with eigenvalue:

$$\alpha' = \alpha \cosh R - \alpha^* \sinh R. \quad (1.86)$$

However, the same does not hold true for the variance of the field or, more interestingly, the variances of the quadrature operators. Substituting (1.83) into (1.56) and (1.55), we get:

$$\hat{E}_Q = \mathcal{E}_l e^{-R} \left(\hat{A}_R + \hat{A}_R^\dagger \right) \quad (1.87)$$

$$\hat{E}_P = -i \mathcal{E}_l e^R \left(\hat{A}_R - \hat{A}_R^\dagger \right) \quad (1.88)$$

which lead to:

$$\left(\Delta \hat{E}_Q \right)^2 = \mathcal{E}_l^2 e^{-2R} \quad (1.89)$$

$$\left(\Delta \hat{E}_P \right)^2 = \mathcal{E}_l^2 e^{2R}. \quad (1.90)$$

We immediately notice that squeezed states are minimum uncertainty states since:

$$\Delta \hat{E}_Q \Delta \hat{E}_P = \mathcal{E}_l^2. \quad (1.91)$$

However, for a fixed value of R (and therefore a fixed value of the squeezing parameter, e^{2R}) the uncertainty in one of the two quadratures will be reduced below that of the vacuum state, while the other will be proportionally increased.

Among all possible squeezed states, the one with $\alpha = \alpha' = 0$ is called the squeezed vacuum state. The expectation value of the electric field operator for this state is the same

as that of the quasi-classical state $|\alpha' = 0\rangle$, which in this case is the vacuum state $|0\rangle$. Thus, the expectation value $\langle \hat{E} \rangle$ for a squeezed vacuum is zero. However, the quadrature uncertainties in this state are governed by (1.89) and (1.90). Thus, a squeezed vacuum state, while sharing the same values of $\langle \hat{E} \rangle$ and $\Delta \hat{E}_Q \Delta \hat{E}_P$ as the vacuum state, is characterized by having one quadrature whose uncertainty is reduced by a factor e^{-R} with respect to the vacuum, while the other quadrature's uncertainty is amplified by a factor e^R . It can be proven that the expectation value of the photon number in this state is not zero (unlike the vacuum state).

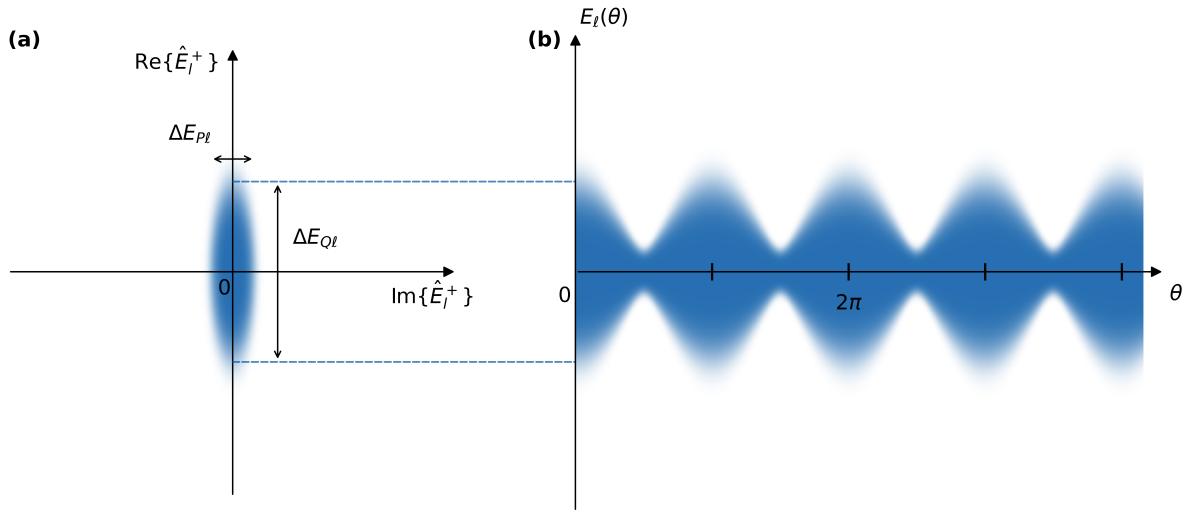


Figure 1.4: Phasor diagram of squeezed vacuum state

In Figure 1.4, a phasor diagram for the squeezed vacuum state is depicted. The expectation value of the electric field is null thus the uncertainty region is centred in the origin of the complex plane. The state is of minimal uncertainty thus the area of the region is the minimal one, the one of a disc of $2\mathcal{E}_l$ diameter. However $\Delta E_{q_1} \neq \Delta E_{q_2}$ and thus the region has the shape of an ellipse. The state is not an eigenstate and thus it evolves in times other than in space as a function of $\theta = \vec{k}_l \cdot \vec{r} - \omega_l t$. The evolution in time or space can be thought as a rotation of the uncertainty elliptic region in the complex plane of the angle θ . As the elliptic region rotates its projection over the real axis changes and thus the uncertainty over the electric field measurement changes. Since in the specific case of Figure 1.4 the uncertainty of $E_l(\theta)$ get much smaller for $\theta = \frac{\pi}{2} + n\pi$, $n \in \mathbb{Z}$ than for $\theta = n\pi$ it is clear that the uncertainty over the sine term of the mode, ΔE_{P_l} , is smaller than the one over the cosine ΔE_{Q_l} . Thus detecting the field for $\theta = \frac{\pi}{2} + n\pi$, $n \in \mathbb{Z}$ one would obtain less fluctuations than the vacuum ones.

1.4. Homodyne photodetector

Currently, no photodetector is fast enough to directly observe the oscillations of the electric field of electromagnetic waves in the optical spectrum. Homodyne detection is a technique that allows us to measure the two field quadratures alternatively and thus obtain equivalent information about the amplitude and phase of the signal. This result is achieved by beating the signal with a stable and known local oscillator field. To beat the electric field of the local oscillator with that of the optical signal, a beam splitter is an essential optical component.

1.4.1. Beamsplitter

Let us assume that all fields are in single-mode states. To simplify the discussion, we will assume that all fields are polarized perpendicularly to the plane of Figure 1.5, which allows us to treat them as scalar quantities.

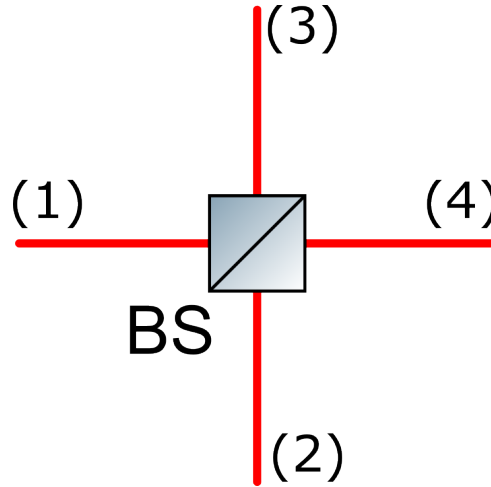


Figure 1.5: Beamsplitter and input-output ports.

Figure 1.5 illustrates a beam splitter with two incident beams and two exiting beams. The two input ports are labelled (1) and (2). The light beam entering from port (1) is transmitted to output port (4) and reflected to output port (3). Symmetrically, input (2) is transmitted to (3) and reflected to (4). The electric field at each port can be modelled as a distinct mode.

It can be proven that, in analogy to classical optics, the analytical signal electric field

operators for each mode transform as follows:

$$\hat{E}_3^+ = \frac{1}{\sqrt{2}} \left(\hat{E}_1^+ + \hat{E}_2^+ \right) \quad (1.92)$$

$$\hat{E}_4^+ = \frac{1}{\sqrt{2}} \left(\hat{E}_1^+ - \hat{E}_2^+ \right). \quad (1.93)$$

Substituting $\hat{E}_1^+$ and $\hat{E}_2^+$ with the definitions given in Equations (1.60) and (1.61), we get:

$$\hat{E}_3^+ = \frac{i}{\sqrt{2}} \mathcal{E}_l e^{i \vec{k}_3 \cdot \vec{r}_3} (\hat{a}_1 + \hat{a}_2) \quad (1.94)$$

$$\hat{E}_4^+ = \frac{i}{\sqrt{2}} \mathcal{E}_l e^{i \vec{k}_4 \cdot \vec{r}_4} (\hat{a}_1 - \hat{a}_2). \quad (1.95)$$

Their Hermitian conjugates are:

$$\hat{E}_3^- = \frac{-i}{\sqrt{2}} \mathcal{E}_l e^{-i \vec{k}_3 \cdot \vec{r}_3} (\hat{a}_1^\dagger + \hat{a}_2^\dagger) \quad (1.96)$$

$$\hat{E}_4^- = \frac{-i}{\sqrt{2}} \mathcal{E}_l e^{-i \vec{k}_4 \cdot \vec{r}_4} (\hat{a}_1^\dagger - \hat{a}_2^\dagger). \quad (1.97)$$

1.4.2. Homodyne detection

A schematic view of the homodyne detection setup is shown in Figure 1.6.

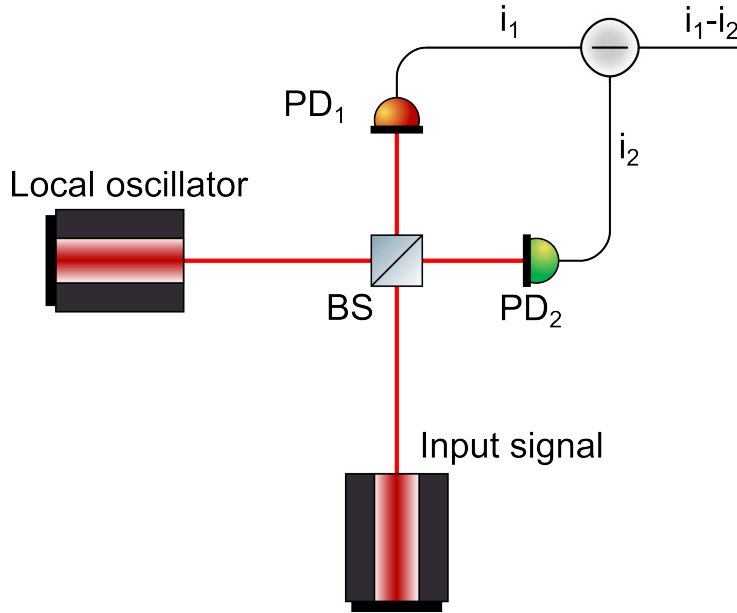


Figure 1.6: Homodyne configuration.

The beam generated by the local oscillator passes through a delay line (not drawn in the simplified scheme of Figure 1.6), allowing us to add a phase delay to the beam. The output ports of the beam splitter are then monitored by two photodetectors, Γ_3 and Γ_4 . The average photocurrent generated by a photodetector is classically proportional to the optical intensity, i.e.:

$$\overline{i(\vec{r}, t)} \propto I(\vec{r}, t) \propto \overline{|E|^2} = \overline{E^- E^+} \quad (1.98)$$

where E^+ is the analytical signal of the classical electric field and E^- is its complex conjugate. Thus, in quantum optics, it is reasonable to assume that:

$$\overline{i(\vec{r})} \propto \langle \Psi | \hat{E}^-(\vec{r}) \hat{E}^+(\vec{r}) | \Psi \rangle. \quad (1.99)$$

The state $|\Psi\rangle$ is the system state describing both mode 1 of the radiation, associated with input port 1, and mode 2, associated with input port 2.

The local oscillator field is usually produced by an intense single-mode CW laser emitting at the same frequency as the incoming optical signal. Therefore, this field can be described as a quasi-classical state, meaning mode 2 can be described as being in the state $|\alpha_2\rangle$ with $\alpha_2 = |\alpha_2| e^{i\varphi_2}$ (see Subsection 1.3.3). The signal field, in mode 1, is in a generic state $|\psi_1\rangle$. The two states are independent; thus, the total state $|\Psi\rangle$ is the tensor product of the two states:

$$|\Psi\rangle = |\alpha_2\rangle \otimes |\psi_1\rangle = |\alpha_2\rangle |\psi_1\rangle. \quad (1.100)$$

The output signal measured by a homodyne detector is the difference between the two photocurrents, i.e. $\overline{i_3 - i_4}$, which is typically amplified to overcome noise issues. From Equation (1.99), we find that the photocurrent difference is proportional to the following term:

$$\begin{aligned} \overline{i_3 - i_4} &\propto \langle \Psi | \left(\hat{E}^-(\vec{r}_3) \hat{E}^+(\vec{r}_3) - \hat{E}^-(\vec{r}_4) \hat{E}^+(\vec{r}_4) \right) | \Psi \rangle \\ &= \langle \Psi | \hat{E}^-(\vec{r}_3) \hat{E}^+(\vec{r}_3) | \Psi \rangle - \langle \Psi | \hat{E}^-(\vec{r}_4) \hat{E}^+(\vec{r}_4) | \Psi \rangle \\ &= \frac{1}{2} \mathcal{E}_l^2 \left[\langle \Psi | (\hat{a}_1^\dagger + \hat{a}_2^\dagger)(\hat{a}_1 + \hat{a}_2) | \Psi \rangle - \langle \Psi | (\hat{a}_1^\dagger - \hat{a}_2^\dagger)(\hat{a}_1 - \hat{a}_2) | \Psi \rangle \right] \\ &= \mathcal{E}_l^2 \langle \Psi | (\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1 \hat{a}_2^\dagger) | \Psi \rangle \\ &= \mathcal{E}_l^2 \langle \psi_1 | \langle \alpha_2 | (\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1 \hat{a}_2^\dagger) | \alpha_2 \rangle | \psi_1 \rangle \\ &= \mathcal{E}_l^2 \left[\langle \psi_1 | \hat{a}_1^\dagger | \psi_1 \rangle \langle \alpha_2 | \hat{a}_2 | \alpha_2 \rangle + \langle \psi_1 | \hat{a}_1 | \psi_1 \rangle \langle \alpha_2 | \hat{a}_2^\dagger | \alpha_2 \rangle \right] \end{aligned} \quad (1.101)$$

To obtain this result, we applied the definition in (1.100) and substituted the transformations of the analytical electric field operators (1.94), (1.95), (1.96), and (1.97). Both $|\alpha_2\rangle$ and $|\psi_1\rangle$ should be understood as time-dependent states, thus $|\alpha_2(t)\rangle$ and $|\psi_1(t)\rangle$.

However, the eigenvalue equation of $\hat{a}_l$, Equation (1.76), implies that:

$$\begin{aligned}\langle \alpha_2 | \hat{a}_2 | \alpha_2 \rangle &= \alpha_2 e^{-i\omega_2 t} \\ \langle \alpha_2 | \hat{a}_2^\dagger | \alpha_2 \rangle &= \alpha_2^* e^{i\omega_2 t}.\end{aligned}\tag{1.102}$$

Similarly, $|\psi_1(t)\rangle$ is a generic state of mode 1, which oscillates at the same frequency as mode 2, thus at frequency ω_2 . Even if the state is generic, we can write:

$$\begin{aligned}\langle \psi_1(t) | \hat{a}_1 | \psi_1(t) \rangle &= e^{-i\omega_2 t} \langle \psi_1(0) | \hat{a}_1 | \psi_1(0) \rangle \\ \langle \psi_1(t) | \hat{a}_1^\dagger | \psi_1(t) \rangle &= e^{i\omega_2 t} \langle \psi_1(0) | \hat{a}_1^\dagger | \psi_1(0) \rangle.\end{aligned}\tag{1.103}$$

These last two results allow us to show that the photocurrent difference in a homodyne configuration is time-independent:

$$\overline{i_3 - i_4} \propto \mathcal{E}_l^2 \left[\alpha_2^* \langle \psi_1(0) | \hat{a}_1 | \psi_1(0) \rangle + \alpha_2 \langle \psi_1(0) | \hat{a}_1^\dagger | \psi_1(0) \rangle \right].\tag{1.104}$$

Writing α_2 as $|\alpha_2| e^{i\varphi_2}$ and making use of the definitions of the quadrature operators (1.55) and (1.56), we can rewrite Equation (1.104) as follows:

$$\overline{i_3 - i_4} \propto \mathcal{E}_l |\alpha_2| \left[\cos \varphi_2 \langle \psi_1(0) | \hat{E}_{Q1} | \psi_1(0) \rangle + \sin \varphi_2 \langle \psi_1(0) | \hat{E}_{P1} | \psi_1(0) \rangle \right].\tag{1.105}$$

This crucial result shows that by using the delay line to set φ_2 equal to either $m\pi$ or $\pi/2 + m\pi$ (where $m \in \mathbb{Z}$), the expectation value of the photocurrents difference becomes proportional to the expectation value of one of the two quadratures, thus enabling us to measure $\langle \hat{E}_{Q1} \rangle$ or $\langle \hat{E}_{P1} \rangle$ alternatively.

Furthermore, Equation (1.105) shows that the local oscillator beam, in addition to selecting the quadrature to be measured via the phase φ_2 , acts as an initial “physical amplification stage.” Indeed, the more intense the local oscillator, the higher $|\alpha_2|$ is, and thus the larger $\overline{i_3 - i_4}$ becomes for a fixed value of $\langle \hat{E}_{Q1} \rangle$ or $\langle \hat{E}_{P1} \rangle$. This amplification effect of the local oscillator is very important when the signal to be measured is so small that the photocurrent difference risks being submerged by the electronic noise of the photodetector.

Homodyne detection also allows one to measure the fluctuations of the two quadratures. Indeed, analogously to what has just been shown, one obtains:

$$\overline{(i_3 - i_4)^2} \propto \mathcal{E}_l^4 \left[|\alpha_2|^2 \langle \psi_1(0) | \left(\hat{a}_1 e^{-i\varphi_2} + \hat{a}_1^\dagger e^{i\varphi_2} \right)^2 | \psi_1(0) \rangle + \langle \psi_1(0) | \hat{a}_1^\dagger \hat{a}_1 | \psi_1(0) \rangle \right].\tag{1.106}$$

If $|\alpha_2|$ is large enough so that the second term is negligible, then we get:

$$\begin{cases} \overline{(i_3 - i_4)^2} \propto \mathcal{E}_l^2 |\alpha_2|^2 \langle \psi_1(0) | \hat{E}_{Q1}^2 | \psi_1(0) \rangle \propto \langle \hat{E}_{Q1}^2 \rangle & \forall \varphi = m\pi \\ \overline{(i_3 - i_4)^2} \propto \mathcal{E}_l^2 |\alpha_2|^2 \langle \psi_1(0) | \hat{E}_{P1}^2 | \psi_1(0) \rangle \propto \langle \hat{E}_{P1}^2 \rangle & \forall \varphi = \frac{\pi}{2} + m\pi \end{cases} \quad (1.107)$$

Therefore, homodyne detection allows one to measure both the expectation value and the fluctuations of one quadrature of an input signal at a time.

The final goal of the research project this thesis is part of, as mentioned in the introduction, is to produce and detect squeezed vacuum states. By “detecting squeezed vacuum states,” we mean measuring the uncertainties of the two quadratures of the purported squeezed vacuum state and proving that one of them is below the standard quantum limit for vacuum states ($\Delta \hat{E}_Q = \Delta \hat{E}_P = \mathcal{E}_l$). Therefore, homodyne detection is the perfect technique to use, as it allows one to measure the quadrature uncertainties. To detect a squeezed vacuum state using the setup in Figure 1.6, the field injected at input port 1 must be the squeezed vacuum state itself. In this case, $|\psi_1\rangle = |0_1, R\rangle$, where $|0_1, R\rangle$ is the squeezed vacuum state in mode 1 with squeezing parameter e^{2R} . In such a setup, it becomes essential to have a sufficiently strong local oscillator so that $|\alpha_2|$ is large enough to amplify the photocurrent difference and overcome the technical electronic noise of the photodetector.

2 | Photodetector

This chapter is dedicated to the description of the detector that has been developed with the goal of measuring the fluctuations of the input optical signal in a balanced homodyne detection configuration.

The chapter is divided into five sections. The first section describes the specifications required for the photodetector to detect squeezed states and outlines some considerations behind the design choices. The second section details the experimental acquisition setup used for the measurements. The third section is devoted to the discussion of the preliminary simulations that helped to assess the optimal configuration. The fourth section describes the chosen design and its technical features. Finally, the fifth section presents and discusses the measurements performed with the developed photodetector.

2.1. Technical requirements

In order to understand the design choices made during the photodetector development, it is useful to identify the requirements that the detector must satisfy to operate properly. Many of these requirements can be immediately derived from purely theoretical considerations.

2.1.1. Theoretical considerations

As described in Chapter 1, a squeezed vacuum state is characterized by having smaller fluctuations of the electric field in one of the two quadratures with respect to those of a vacuum state. When detecting the vacuum state in a homodyne configuration, as described in Section 1.4.2, these fluctuations are observed as shot noise.

The existence of shot noise, and its value, can be derived both from a semi-classical model and from quantum optics.

In the semi-classical approach, the radiation is treated classically, while the photodetector is treated using quantum mechanics. The resulting model states that the photocurrent is

intrinsically affected by white noise, called shot noise, whose spectral density is given by

$$S_{i_{shot}}(f) = 2 e \bar{i}_{ph} = 2 e R_\lambda \bar{P}_{ph} \quad (2.1)$$

where e is the elementary charge and $\bar{i}_{ph}$ is the average photocurrent produced by the photodiode. This latter quantity is equal to the product of the photodiode responsivity R_λ and the average optical power impinging on the photodiode $\bar{P}_{ph}$, [21].

The shot noise current fluctuations can be simply obtained by integrating this spectral density over the full bandwidth of the detector to get the variance of the photocurrent, $(\Delta i_{shot})^2$. Taking the square root of this value yields the standard deviation of the current difference due to shot noise, Δi_{shot} .

The same result can be achieved using a quantum optics description. As shown in Section 1.2, at least one of the two quadratures is intrinsically associated with a certain level of uncertainty. In Section 1.4, it was proven that this quadrature uncertainty is reflected, when using a photodetector in a balanced homodyne configuration, in an intrinsic fluctuation of the photocurrent difference. It can be shown that variance of the photocurrent difference, computed in Equation (1.105) for a non-squeezed vacuum state, yields the same current spectral density obtained semi-classically for shot noise, thus leading to the result of Equation (2.1) [7].

In Subsection 1.3.4, however, we demonstrated that squeezed vacuum states are characterized by having smaller fluctuations than regular vacuum states over one of the two quadratures. This is reflected in the reduced fluctuations of the photocurrent difference produced by a homodyne detector measuring that specific quadrature.

Thus, in the absence of other noise sources, injecting squeezed vacuum states into one of the two ports of the beam splitter in a balanced homodyne configuration will result in a photocurrent difference signal affected by smaller fluctuations than those of shot noise. Therefore, measuring a power spectral density of the photocurrent difference lower than the value for a standard vacuum would serve as empirical proof of the successful injection, and therefore production, of squeezed vacuum states.

Focusing now on the photodetector itself, to perform this kind of measurement, the detector needs to be sensitive enough to measure fluctuations of the photocurrent difference on the order of the shot noise, or even lower in order to detect the injection of squeezed states. Therefore, the technical electronic noise must not mask the shot noise fluctuations. In particular, this requirement translates into the need to have an electronic noise spectral density, at the voltage output of the detector, $S_{V_{electr}}$, much smaller than the

spectral density of the voltage signal associated with shot noise, that is:

$$S_{V_{electr}} \ll S_{V_{shot}}. \quad (2.2)$$

Assuming a constant transimpedance gain of the photodetector (see Section 2.3), $S_{V_{shot}}$ is constant over the full bandwidth of the setup. Therefore, to be able to detect squeezed states, it is sufficient to satisfy Condition (2.2) at least for a range of frequencies (preferably as wide as possible), and not necessarily across the entire bandwidth of the detector.

The electronics of the device is not the only noise source affecting the general experimental setup for homodyne detection (the setup shown in Figure 1.6). In a classical picture, the local oscillator is not only affected by shot noise. It also exhibits fluctuations in the optical signal mainly due to technical noise, which are laser fluctuations caused by environmental, mechanical, or electronic imperfections. This kind of noise could potentially mask the shot noise fluctuations.

However, in an ideal homodyne photodetector, this should not affect the measurement. Indeed, in the classical picture, any noise affecting the local oscillator beam before the beam splitter will equally affect both beam branches impinging on the two photodiodes. Therefore, the fluctuations in the two photocurrents generated by the technical noise of the oscillator will be correlated with each other. If the photodetector is well-balanced and the lengths of the two optical paths after the beam splitter are equal, subtracting the photocurrents will cancel these correlated fluctuations out. In an ideal balanced homodyne photodetector, the only sources of noise affecting the photocurrent difference are those that affect the photocurrent of each photodiode in an uncorrelated manner.

Still within a classical framework, shot noise arises independently from the interaction of each photodiode with its respective branch of the classical electromagnetic wave of the local oscillator. Since these two interaction processes are independent, the noise affecting the two photocurrents can be considered completely uncorrelated. Therefore, the current difference is affected by a noise whose spectral density is:

$$S_{i_{shot}}^{diff} = S_{i_{shot}}^{(1)} + S_{i_{shot}}^{(2)} = 2 S_{i_{shot}}^{(1)} = 2 (2 e R_{\lambda} \overline{P}_{ph}) \quad (2.3)$$

where we assumed a perfectly balanced configuration, such that the average photocurrents produced by the two photodiodes are always equal: $\bar{i}_1 = \bar{i}_2 = \bar{i}_{ph}$, meaning $S_{i_{shot}}^{(1)} = S_{i_{shot}}^{(2)} = 2 e R_{\lambda} \overline{P}_{ph}$.

The previous paragraph highlights several requirements that the photodetector must fulfil to be able to detect squeezed states. The ideal detector should fully reject the common

mode input to the two photodiodes, since this corresponds to the technical noise affecting the local oscillator. To be precise, the local oscillator CW signal itself also acts as a common mode input to the two photodiodes, but since it is a continuous wave, it is irrelevant for shot noise measurements, as we will measure $S_{i_{shot}}^{diff}(f)$ only for frequencies f higher than 100 kHz (see Section 2.5).

However, the photodiode responsivity for a single photodiode model can vary from one unit to another. Furthermore, the responsivity of a single photodiode can slightly vary depending on the incidence point of the focused beam on the active surface. An unbalance in the responsivity of the two photodiodes within the same photodetector may lead to an effectively unbalanced configuration, preventing the full cancellation of the correlated noise at the input. This issue can be mitigated by selecting a pair of photodiodes from the same batch with matched responsivity values (and ideally matching other critical electrical parameters such as junction capacitance or shunt resistance).

Even assuming a perfectly balanced photodiode pair, it is generally very difficult from an electronic standpoint to perform a perfect subtraction of two input signals. To quantify how well two identical input signals cancel out at the output, the Common Mode Rejection Ratio (CMRR) is widely used. It is defined as the ratio, usually in decibels, of the differential gain A_d to the common mode gain A_{cm} :

$$CMRR_{dB} = 20 \log_{10} \left(\frac{|A_d|}{|A_{cm}|} \right). \quad (2.4)$$

Typical values for commercially available photodetector range in the order of 20 – 50 dB.

The Relative Intensity Noise (RIN) represents the power spectral density of the laser optical power fluctuations, normalized to the average optical power squared. It is defined as:

$$RIN(f) = \frac{S_{\delta P}(f)}{\bar{P}^2} \quad (2.5)$$

where $S_{\delta P}(f)$ is the power spectral density of the power fluctuations δP , and $\bar{P}$ is the average optical power. It is typically expressed in dBc/Hz.

The RMS relative noise over a specific frequency band $[f_1, \infty)$ is obtained by integrating the linear RIN spectrum. It represents the standard deviation of the relative power fluctuations and is calculated as:

$$RMS_{noise} = \sqrt{\int_{f_1}^{\infty} RIN(f) df} \quad (2.6)$$

where $RIN(f)$ is given in linear scale Hz^{-1} .

The typical relative intensity noise of a laser source decreases with frequency. At lower frequencies, the laser is dominated by technical noise. As the frequency increases, fewer sources contribute to the noise, until the RIN essentially flattens out at the shot noise limit. Technical noise coming from the local oscillator can generate a common mode photocurrent noise spectral density that is several orders of magnitude higher than the typical photocurrent shot noise spectral density ($S_{i_{shot}}^{diff} \sim 10^{-22} \text{ A}^2/\text{Hz}$ for $\bar{P}_{ph} \sim 1 \text{ mW}$). Furthermore, in some setup configurations, it may not be trivial to perfectly balance the input power at the two photodiodes. Therefore, even when using detectors with a very high CMRR, it is preferable to measure shot noise in a spectral region where the laser's technical noise is inherently lower.

Detecting squeezed states at higher frequencies allows us to operate in a region where the technical noise of the laser is significantly lower. This provides higher sensitivity to shot noise even in the case of imperfect common mode rejection. For this reason, we designed the detector to have at least 1 MHz of bandwidth, with the goal of performing squeezed state detection in the [100 kHz, 1 MHz] frequency band.

2.1.2. General design choices

As previously pointed out, the photocurrent generated by a photodetector is typically an extremely small signal, with a spectral density on the order of $S_{i_{shot}}^{diff} \sim 10^{-22} \text{ A}^2/\text{Hz}$. Before it can be detected, it must be converted into a voltage signal and amplified. For this purpose, a TransImpedance Amplifier (TIA) is an optimal choice. It is a relatively simple circuit that simultaneously converts a current into a voltage and amplifies it. Figure 2.1 shows the schematic of a generic TIA.

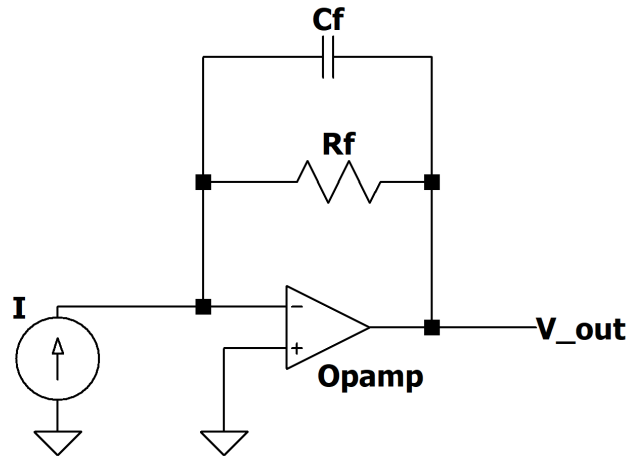


Figure 2.1: Simple TIA electrical scheme.

In general, there are many ways to amplify the photocurrent and convert it into a voltage. However, once the use of a TIA is established, there are two main configurations for implementing a homodyne photodetector: either converting the current from each photodiode into a voltage first and then subtracting the two voltages, or subtracting the photocurrents directly and then amplifying and converting the current difference.

The first approach, suggested, for instance, in a technical note released by LIGO [22], utilizes a circuit with the following general schematic:

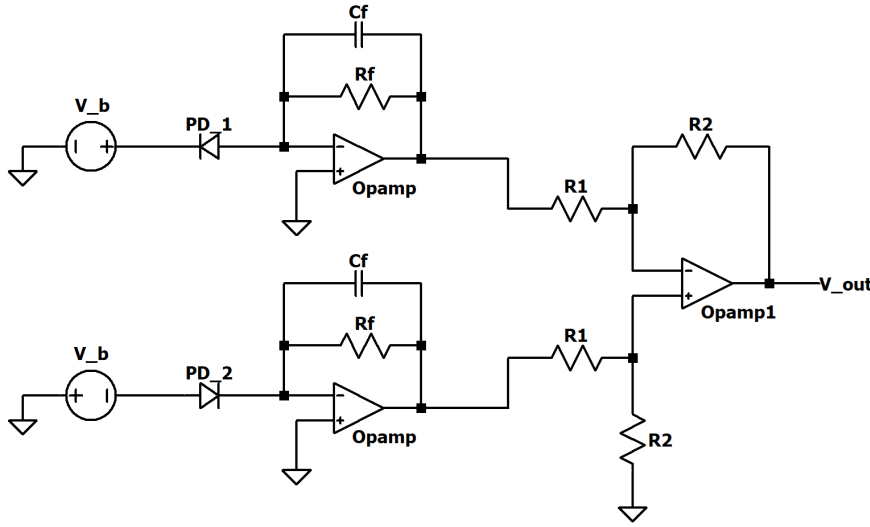


Figure 2.2: Homodyne photodetector general circuit diagram in a voltage difference configuration.

We adopted the convention of using identical labels for components that share the same value.

The second approach, suggested in a different technical note by LIGO [16] and employed by Vahlbruch, Mansell, and Darsow-Fromm in their PhD theses [9, 20, 25], is based on the following general schematic:

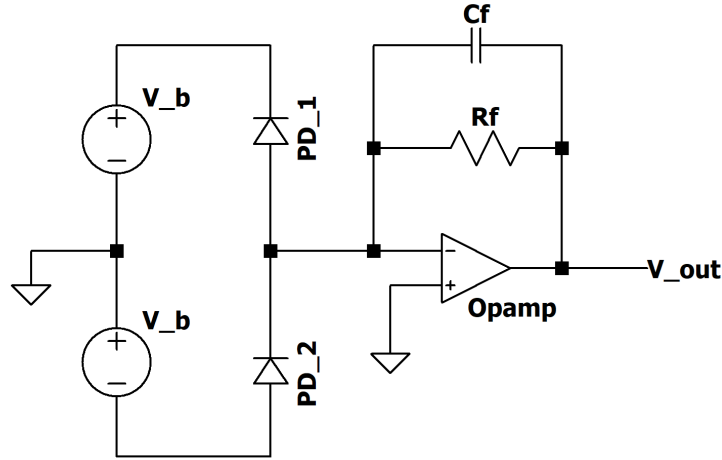


Figure 2.3: Homodyne photodetector general electrical scheme in a current difference configuration.

We chose to design and build a photodetector based on the latter configuration. By utilizing a single fundamental active electronic stage and performing the current subtraction "physically" (by simply placing the two photodiodes in series), the chances of introducing additional electronic noise into the system are minimized. For a first prototype, it also seemed reasonable to simplify the schematic as much as possible to more easily identify the sources of any unwanted behaviour; thus, this design choice proved optimal.

Because the amplification stage incorporates a feedback loop, there is one final technical requirement that the photodetector electronics must satisfy: the circuit must be stable. The analysis of this condition is presented in Section 2.3.

2.2. Acquisition setup

The experimental setup adopted for the shot noise measurements is shown in Figure 2.4. It represents a real implementation of the conceptual configuration illustrated in Figure 1.6.

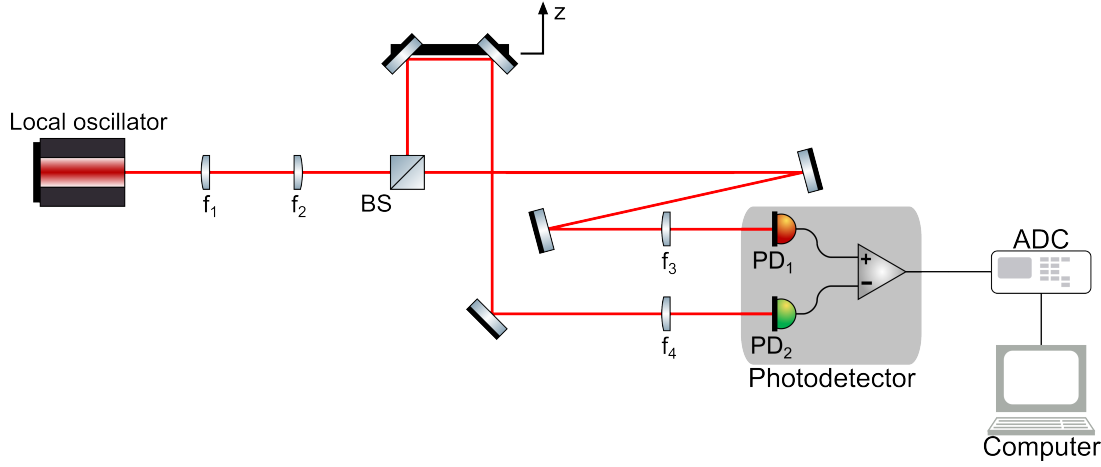


Figure 2.4: Experimental setup for the balanced homodyne configuration.

The local oscillator is a collimated laser beam (collimation optics are not shown). The beam passes through a telescope that enlarges its spot size before being split by a beam splitter. The two split beams are reflected by mirrors, propagate over equal optical path lengths, and finally impinge on the two photodiodes of the photodetector. The current signal generated by the photodiodes is amplified and converted into a voltage signal by the photodetector electronics. This signal is then read by an analog-to-digital converter (ADC) board. The digitized data is analyzed and displayed on a computer.

To ensure that both branches of the local oscillator travel the same optical path length, the pair of mutually perpendicular mirrors (shown at the top of the figure) is mounted on a manual linear translation stage, allowing their position to be finely tuned along the z direction.

The power of the local oscillator was controlled by keeping the laser source near its maximum power output and attenuating it with a Variable Optical Attenuator (VOA) placed just before the lens of focal length f_1 . Since splitting a beam into two perfectly equal powers using a standard beam splitter is not trivial, a second VOA was placed along the optical path of the beam passing through the lens with focal length f_4 . This second VOA was used exclusively to fine-tune the power impinging on photodiode PD_2 so that it matched the power at PD_1 .

The shot noise measurements were performed at two different wavelengths: $\lambda_1 = 1064$ nm and $\lambda_2 = 2090$ nm. For each set of measurements, the overall acquisition setup remained the same, although most of the optical components were replaced. Subsections 2.2.1 and 2.2.2 describe the configurations for each acquisition setup.

However, the ADC board and the data analysis software remained identical for both sets of measurements. The ADC used was an AlazarTech ATS9440. It is a 14-bit analog-to-

digital converter capable of sampling at up to 125 MS/s. It has a fixed input coupling impedance of $50\ \Omega$. Unless otherwise specified, we used the smallest available dynamic range of $\pm 100\text{ mV}$ to minimize the quantization error associated with the 14 bits. We developed a MATLAB script to compute the Fast Fourier Transform (FFT) and, from it, evaluate the power spectral density of the signal. A simple rectangular window was chosen as the windowing function. The acquisition time was fixed at $100\ \mu\text{s}$, yielding a frequency resolution (and thus a resolution bandwidth for the FFT) of 10 kHz . Unless stated otherwise, the signal was sampled at an acquisition frequency of 100 MS/s . Therefore, according to the Nyquist-Shannon sampling theorem, we could reconstruct the signal spectrum up to 50 MHz . The acquired squared voltage spectral densities were averaged across multiple temporal acquisitions to reduce the estimation error of the true power spectral density.

We verified the proper operation of our acquisition setup (both the software and the ADC board) by comparing acquisitions taken with this system against data obtained under identical conditions using an Agilent E4445A spectrum analyser.

The voltage noise spectral density of the acquisition setup itself, originating from the ADC board and its connections to the photodetector (which was kept turned off to avoid injecting any external noise), was measured and is plotted below:

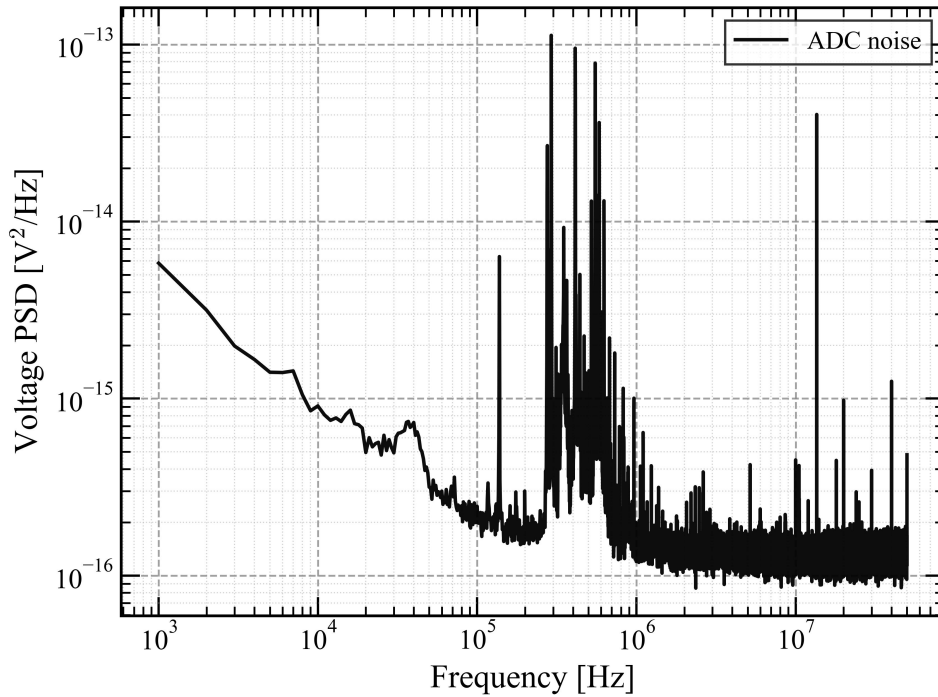


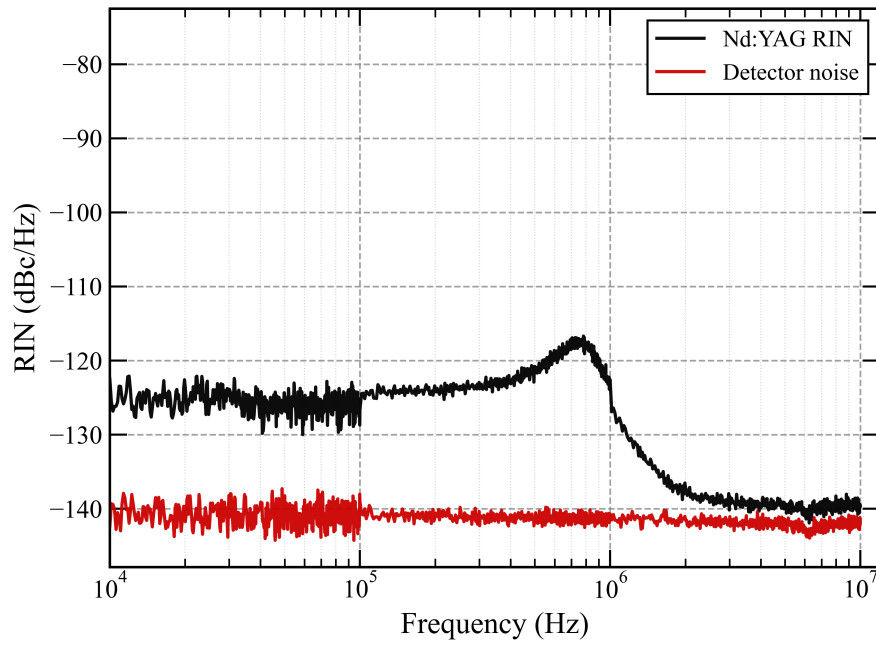
Figure 2.5: Acquisition setup noise.

Even though the noise plotted in Figure 2.5 is actually the noise caused by the ADC board and the disturbances collected by all the connection cables, it is going to be called simply ADC noise as already done in the legend of the graph.

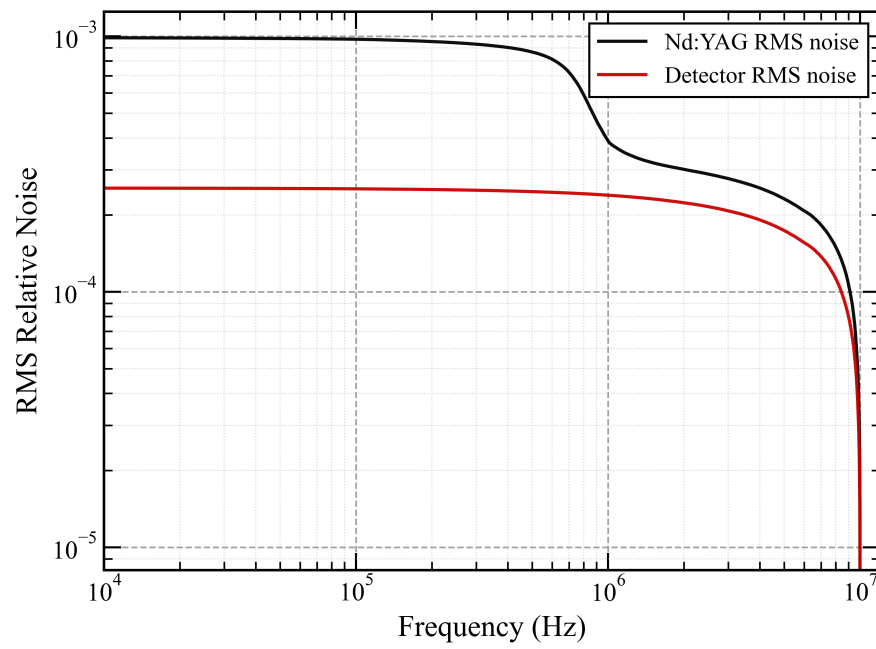
Although the logarithmic plot might suggest otherwise, the noise of the acquisition setup is practically flat over the [100 kHz, 10 MHz] frequency band. In the frequency window used for detection, $f \in [100 \text{ kHz}, 200 \text{ kHz}]$, it maintains an average value of $S_{V_{ADC}} = 2.65 \cdot 10^{-16} \text{ V}^2/\text{Hz}$.

2.2.1. 1 μm configuration

In this configuration, the local oscillator source was a Nd:YAG laser emitting at 1064 nm. The relative intensity noise of this source, is shown in Figure 2.6a. Thanks to the employment of a noise eater built-in with the laser itself, the used Nd:YAG laser is a low noise laser source. The major noise peak is centred around 700 kHz but it is limited to a value of -116 dBc/Hz .



(a) RIN Nd:YAG.



(b) RMS Relative noise Nd:YAG.

Figure 2.6: Relative intensity noise and root mean square relative noise of the Nd:YAG laser.

The telescope stage, composed of two lenses with focal lengths $f_1 = 150$ mm and $f_2 =$

–50 mm, produced a collimated beam with a spot size ($D4\sigma$) of 2.055 mm in the horizontal direction and 2.086 mm in the vertical direction. The spot size was measured using a Spiricon SP620U camera. The acquired intensity profile of the beam is shown below.

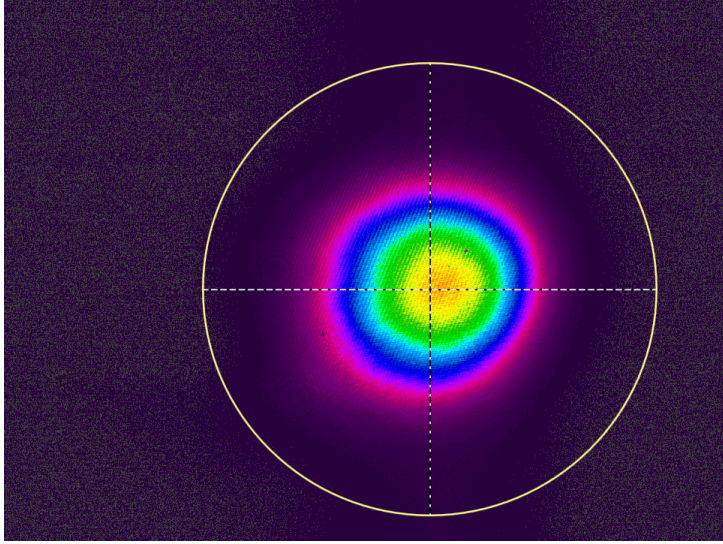


Figure 2.7: Collimated Nd:YAG power profile.

The two lenses used to focus the collimated beams onto the photodiodes had focal lengths of $f_3 = 75$ mm and $f_4 = 100$ mm, respectively. Given a collimated beam waist of approximately 1 mm and the focal lengths of the lenses, we used the approximate formula

$$w_{out} = \frac{\lambda f}{\pi w_{in}} \quad (2.7)$$

(where w is the beam waist) to estimate a maximum focused beam waist of roughly $34 \mu\text{m}$.

This value is much smaller than the photodiodes' active area radius. Therefore, after carefully aligning the optical beams, we could be confident that all the optical power was impinging directly on the active areas of the photodiodes.

For this configuration, two InGaAs p-i-n photodiodes (model G8376-03 from Hamamatsu) were mounted on the photodetector. Their responsivity, extracted from the component datasheet, is plotted below:

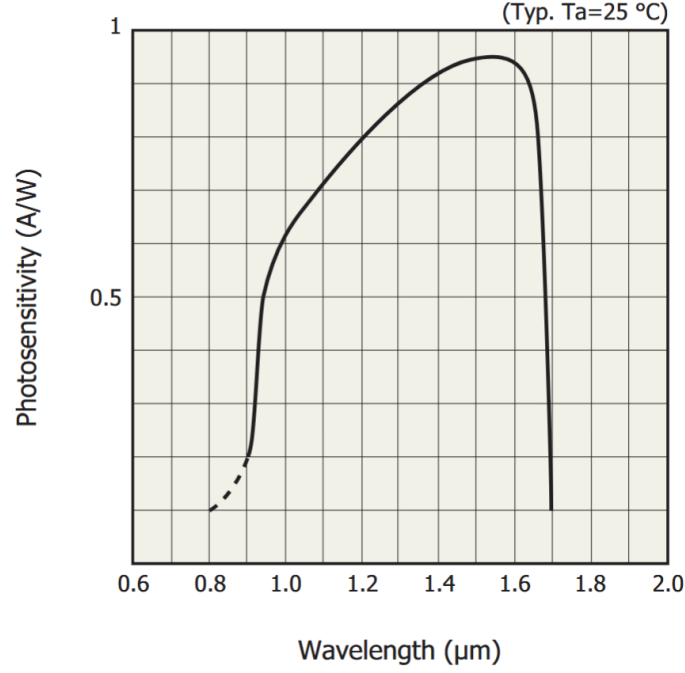


Figure 2.8: Responsivity of the Hamamatsu G8376-03 InGaAs photodiode. Image taken from the manufacturer’s datasheet [14].

As illustrated, the responsivity R_λ is practically zero at 2090 nm but is approximately $R_\lambda = 0.66$ A/W at the Nd:YAG emission wavelength of $\lambda = 1064$ nm. This value was extrapolated from the datasheet plot using a plot digitizing software but it was not measured for the two specific used unit. Therefore, we considered to have a 10% relative error on this estimate. According to the datasheet, these photodiodes feature a photosensitive area with a diameter of 300 μm , a typical junction capacitance of $C_j = 5$ pF at a reverse bias of 5 V, and a shunt resistance of $R_{sh} = 1$ G Ω at a reverse bias of 10 mV.

2.2.2. 2 μm configuration

The local oscillator for the 2 μm measurements was a Tm-Ho:YAG laser emitting at a wavelength of $\lambda = 2090$ nm.

In this case, the spot size of the collimated beam after the telescope stage was measured using the knife-edge technique. The data collected for this measurement is plotted in Figure 2.9.

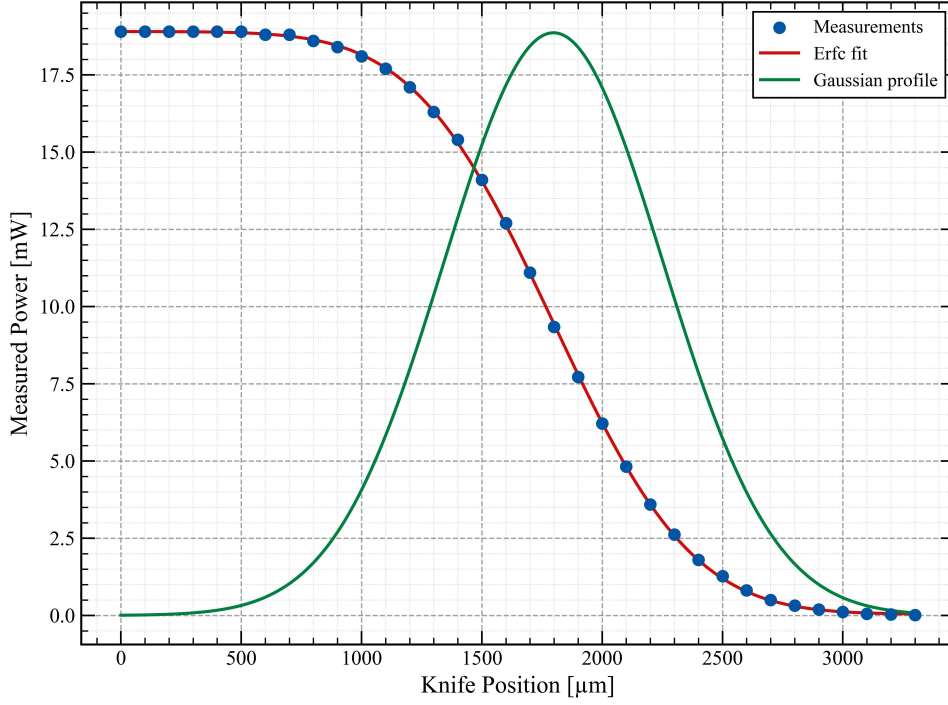
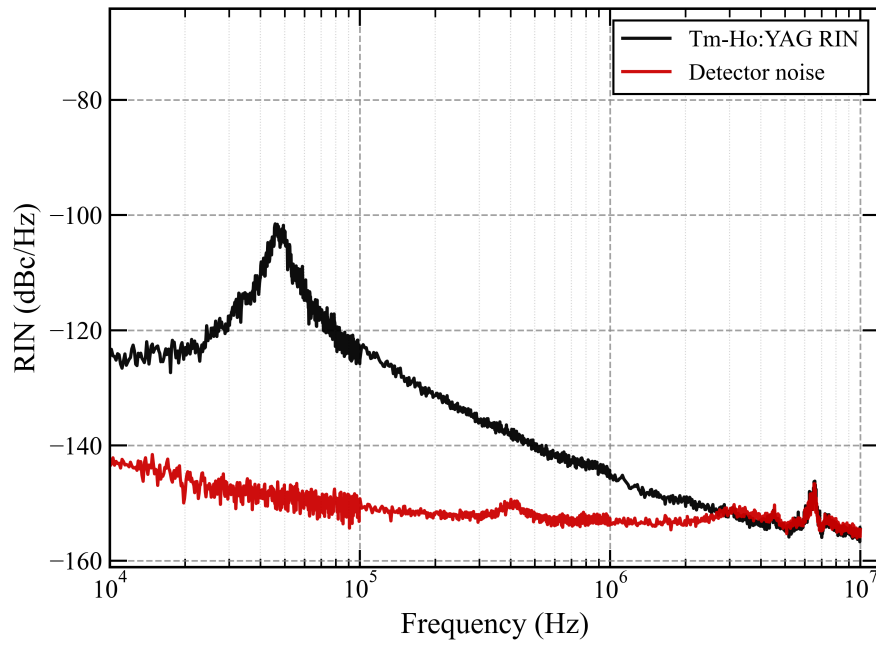


Figure 2.9: Knife-edge measurement of the vertical beam profile of the Tm-Ho laser after the telescope.

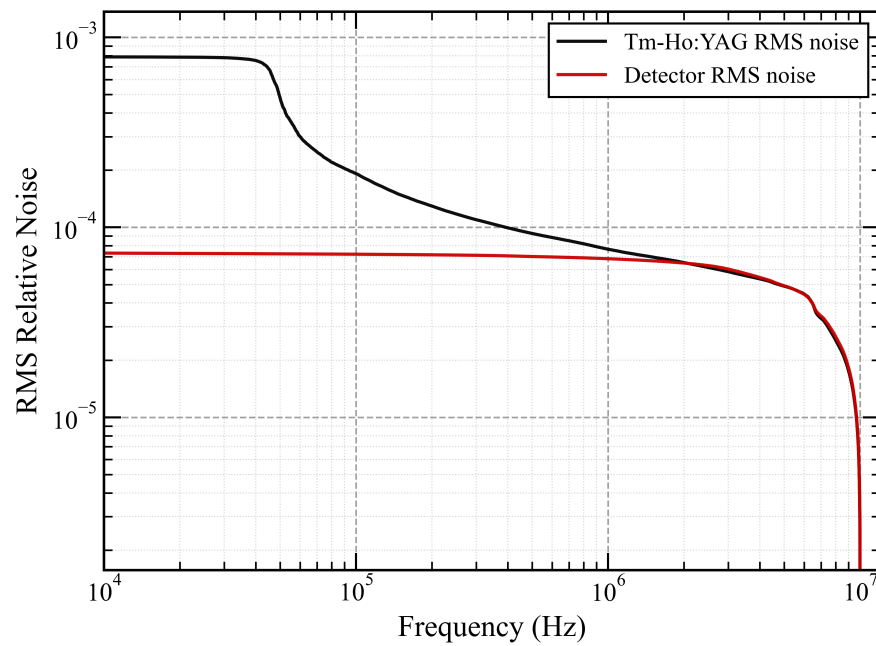
The estimated beam waist is $\omega_0 \simeq 910 \mu\text{m}$. In the horizontal direction, we measured a nearly identical value, with a difference of only a few micrometers.

The collimated beam was focused onto the photodiodes using two lenses with identical focal lengths, $f_3 = f_4 = 100 \text{ mm}$. Using Equation (2.7), the beam radius on the photodiodes was calculated to be around $70 \mu\text{m}$, which is again much smaller than the active area of the devices.

The laser source RIN was measured with the custom made photodetector object of this thesis operated with one of the two photodiodes in the dark. It is shown in the plot of Figure 2.10a.



(a) RIN Tm-Ho:YAG.



(b) RMS Relative noise Nd:YAG.

Figure 2.10: Relative intensity noise and root mean square relative noise of the Tm-Ho:YAG laser.

The RIN shows an important noise peak at 45 kHz at a value of -100 dBc/Hz.

The chosen photodiodes for this configuration were two IG26X500S4i-x extended InGaAs photodiodes from Laser Components. These were supplied without protective windows to minimize reflectivity and thereby maximize efficiency. They feature an active area $500\text{ }\mu\text{m}$ in diameter, a typical junction capacitance of $C_j = 140\text{ pF}$ at a reverse bias voltage of 0 V and a typical shunt resistance of $R_{sh} = 25\text{ k}\Omega$ at a reverse bias of 10 mV .

The responsivity plot taken from the datasheet is shown below:

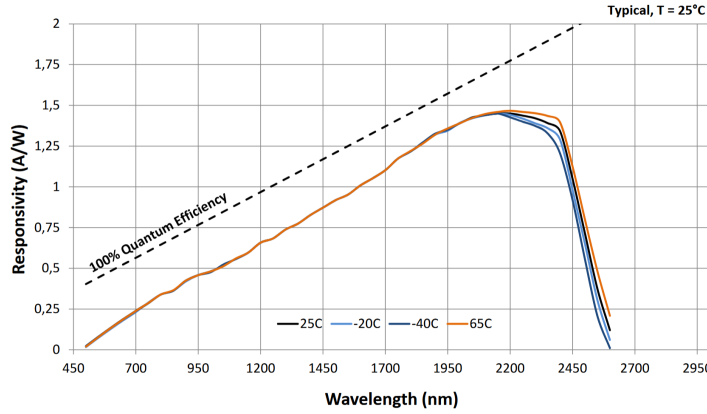


Figure 2.11: Responsivity of the IG26x500S4i-x extended InGaAs photodiode by Laser Components. Image taken from the manufacturer’s datasheet [18].

As done for the $1\text{ }\mu\text{m}$ configuration, the responsivity was extrapolated from the datasheet plot using a plot digitizing software. It was not measured for any of the two specific used units. Therefore, we considered again to have a 10% relative error on the estimated $R_\lambda = 1.44\text{ A/W}$ at $\lambda = 2090\text{ nm}$

2.3. Circuit simulations

Section 2.2 demonstrated that the detection setup has a noise floor of approximately $S_{V_{ADC}} = 2.65 \cdot 10^{-16}\text{ V}^2/\text{Hz}$. Therefore, any photodetector designed to be read by this ADC must have a transimpedance gain high enough to raise the shot noise voltage spectral density above this threshold, ideally by several orders of magnitude.

This requirement imposes a lower bound on the gain. Since the TIA module’s gain should ideally be as high as possible, this condition is not particularly restrictive on its own. The true challenge in designing the photodetector electronics is achieving the highest possible gain while simultaneously maintaining a sufficiently broad bandwidth.

The ideal gain of the TIA shown in Figure 2.1 is straightforward to calculate:

$$G_{id}(s) = \frac{-R_f}{1 + s R_f C_f}. \quad (2.8)$$

The corresponding Bode plot for the values $R_f = 1 \text{ k}\Omega$ and $C_f = 56 \text{ pF}$ is represented by the blue line in Figure 2.16. The gain remains roughly constant and equal to $-R_f$ up to the pole frequency, which defines the circuit's bandwidth:

$$f_p = \frac{1}{2\pi R_f C_f}. \quad (2.9)$$

Beyond this pole, the gain quickly decreases as $1/f$.

Unfortunately, given this ideal gain formulation, the bandwidth requirement discussed in Section 2.1.1 ($\Delta f > 1 \text{ MHz}$) places an upper bound on R_f . For stability reasons (discussed later in this section), C_f must be at least 10 pF . This simple consideration limits the maximum value of R_f to:

$$R_f < \frac{1}{2\pi C_f \Delta f} \simeq 15.9 \text{ k}\Omega. \quad (2.10)$$

A more accurate analysis of the amplification stage requires evaluating the real gain of the TIA,

$$G = \frac{G_{id}}{1 - \frac{1}{G_{loop}}} \quad (2.11)$$

and assessing its stability. Furthermore, we must incorporate the key non-idealities of the components to obtain a realistic description. For example, in any practical transimpedance circuit, stray capacitances are inevitable. One is the input capacitance, C_{in} , present between the input pins of the operational amplifier. In a homodyne photodetector, two other critical parasitic elements are the junction capacitances, C_j , of the two p-i-n photodiodes.

Thus, a more realistic circuit than the one shown in Figure 2.3 looks like the one in Figure 2.12.

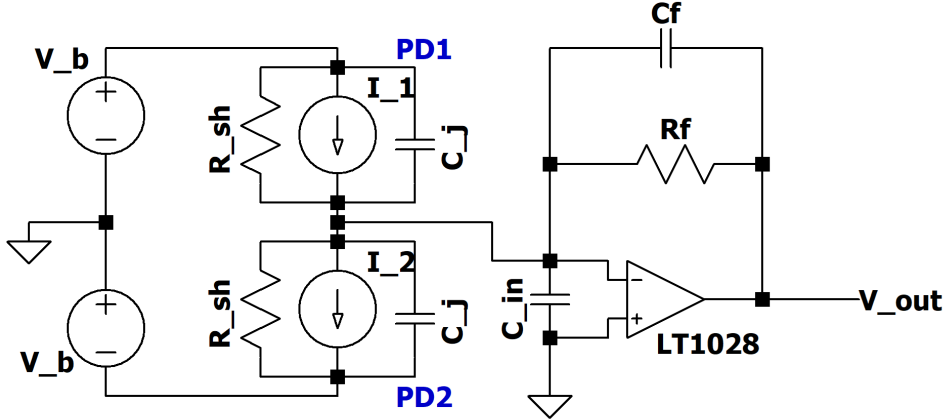


Figure 2.12: More accurate TIA schematic.

The operational amplifier chosen for the TIA stage is the LT1028 from Linear Technology, which specifies a typical input capacitance $C_{in} = 5 \text{ pF}$. Of the two photodiode models described in Subsections 2.2.1 and 2.2.2, the IG26X500S4i presented the more challenging electrical characteristics. Its junction capacitance at a 0 V bias is $C_j = 140 \text{ pF}$, and its typical shunt resistance is $R_{sh} = 25 \text{ k}\Omega$.

The circuit in Figure 2.12 can be redrawn in a much more practical way. For simplicity, the two bias voltage generators can be neglected: because these signals are constant and equal, their common-mode effects on V_{out} cancel out entirely. The only purpose of the bias voltages is to operate the photodiodes in photoconductive mode to enhance their performance.

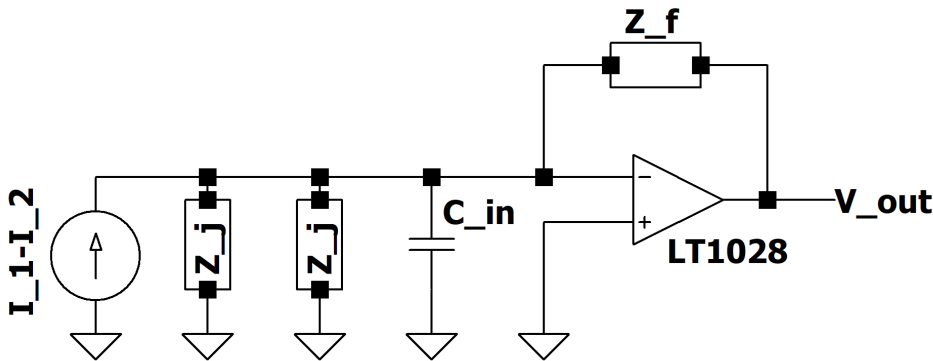


Figure 2.13: Simplified AC model of the realistic TIA.

The rectangular blocks denote complex impedances. Their values are given by:

$$Z_f = \frac{R_f}{1 + s R_f C_f} \quad (2.12)$$

$$Z_j = \frac{R_{sh}}{1 + s R_{sh} C_j} \quad (2.13)$$

In this simplified scheme, it is straightforward to verify that the ideal differential gain seen by the photocurrent difference remains the one expressed in Equation (2.8). To determine the real gain of the amplification stage, we must compute the circuit's loop gain. The loop gain expression is also critical for evaluating stability. It is given by:

$$G_{loop}(s) = -A(s) \frac{R_{sh}}{R_{sh} + 2 R_f} \frac{1 + s R_f C_f}{1 + s \frac{R_f R_{sh}}{R_{sh} + 2 R_f} (2C_j + C_{in} + C_f)} \quad (2.14)$$

where $A(s)$ is the open-loop voltage gain of the op-amp. Assuming a dominant-pole response, it can be approximated as:

$$A(s) = \frac{A_0}{1 + s \frac{A_0}{2\pi GBWP}}. \quad (2.15)$$

According to the LT1028 datasheet, the minimum specifications are $A_0 = 7 \cdot 10^6$ and a Gain-Bandwidth Product ($GBWP$) of 50 MHz.

Therefore, using Equation (2.11), we find:

$$V_{out} = \left[\frac{-R_f}{1 + s R_f C_f} \cdot \frac{G_{loop}(s)}{G_{loop}(s) - 1} \right] (I_1 - I_2) \quad (2.16)$$

where the bracketed term is the (analytically estimated) real transimpedance gain, $G(s)$.

Substituting the known parameters of the electrical components, the only remaining degrees of freedom in the design were R_f and C_f . These values had to be chosen to maximize the differential transimpedance gain while ensuring a bandwidth of at least 1 MHz and maintaining circuit stability.

We wrote a Python script to compute the real gain, the bandwidth, and the phase margin to evaluate stability across a grid of (R_f, C_f) values.

To ensure robust stability, we deemed the circuit stable if the phase margin of the loop gain, φ_m , was greater than 60° ($\varphi_m > \pi/3$).

Furthermore, we required the voltage output to provide a linear, distortion-free replica of the input. Every real op-amp has a finite Slew Rate (SR); for the LT1028, $SR =$

$11 \cdot 10^6 \text{ V/s}$. To have a distortion-free response for any current difference signals whose maximum derivative was smaller than the one of a sinusoidal current at the working frequency of 1 MHz and with amplitude of $1 \mu\text{A}$, we mandated that any acceptable configuration should have a gain such that the maximum $\frac{dV_{out}}{dt}(t)$ given by the upper said current difference signal must satisfy $\frac{dV_{out}}{dt}(t) < 0.8 \cdot SR$.

Finally, to provide a flat frequency response up to 1 MHz, we required the ideal pole frequency to be higher than 2.5 MHz, providing sufficient frequency margin from the pole of the ideal gain.

The program outputs two color maps: one for the phase margin and another for the ideal pole frequency. A color is assigned to any (R_f, C_f) pair that simultaneously satisfies the stability, bandwidth, and slew rate criteria. Combinations failing any of these conditions are left white.

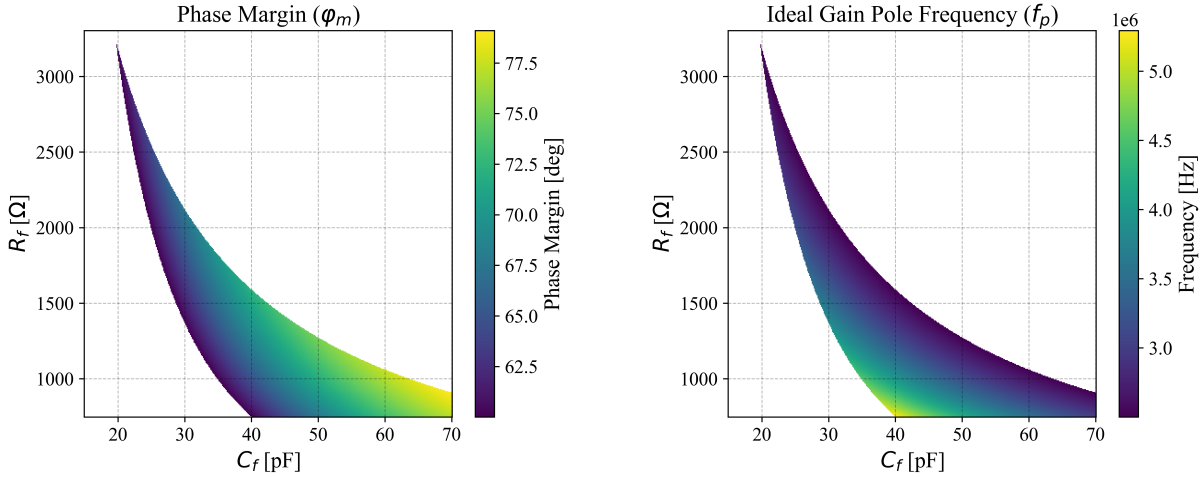


Figure 2.14: Phase margin and ideal pole frequency simulation.

For reference, the figure above represents the output of a 400×400 sweep with $R_f \in [900 \Omega, 2300 \Omega]$ and $C_f \in [10 \text{ pF}, 70 \text{ pF}]$.

Among the sets of (R_f, C_f) that met all conditions, let us consider the values actually implemented in the final photodetector: $R_f = 1 \text{ k}\Omega$ and $C_f = 56 \text{ pF}$.

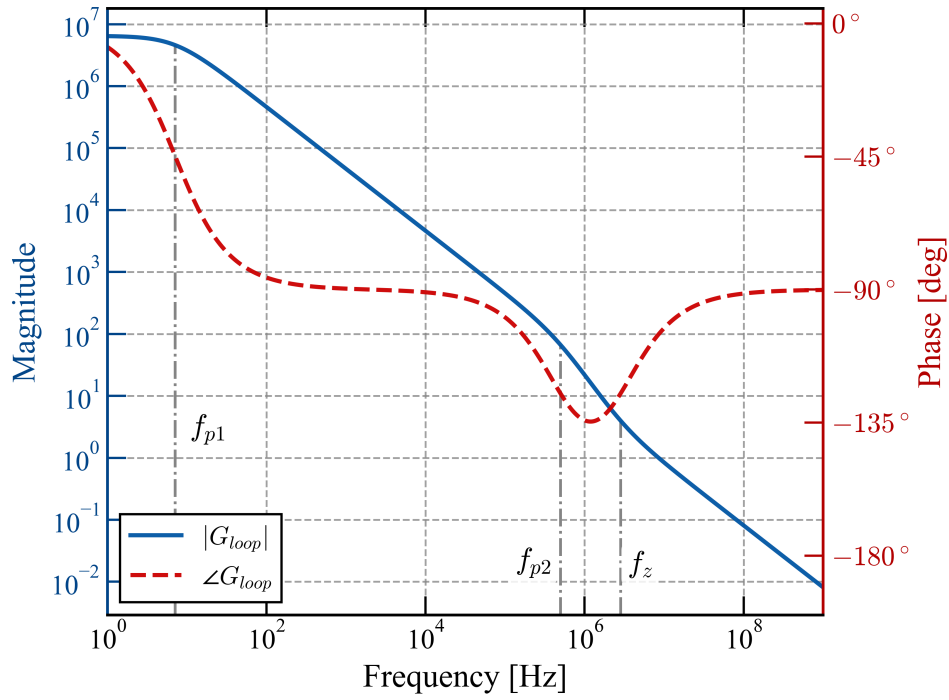


Figure 2.15: Loop gain Bode plot for the chosen (1 k Ω , 56 pF) values.

The loop gain G_{loop} for this configuration is plotted in Figure 2.15, while the open-loop gain G_{OL} , the ideal gain G_{id} , and the real gain G are compared in Figure 2.16.

Looking at Figure 2.15 and Equation (2.14), it should be immediately clear why we previously stated that a feedback capacitor C_f had to be included for stability reasons: C_f adds a zero at the frequency $f_z = \frac{1}{2\pi R_f C_f}$. This prevents the loop gain from crossing the 0 dB axis with a slope of $1/f^2$, which would result in a phase margin of $\varphi_m \approx 0$. Due to the presence of the zero, the loop gain crosses the 0 dB axis with a slope of $1/f$, which guarantees $\varphi_m > 0$ and, consequently, stability.

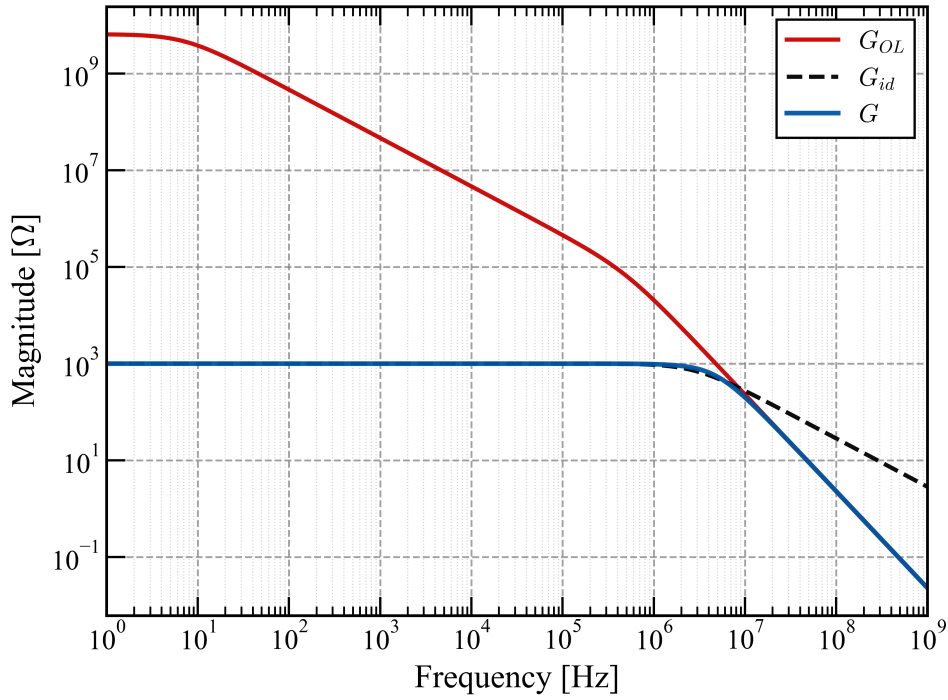


Figure 2.16: Comparison of the ideal and (analytically estimated) real transimpedance gains analytically retrieved for the chosen ($1\text{ k}\Omega$, 56 pF) values.

After evaluating the circuit analytically, we verified the configuration shown in Figure 2.12 using LTspice. Because the LT1028 was available in the standard LTspice library, its macro-model natively accounted for most of the real-world non-idealities not covered by the simplified analytical model.

First, we simulated the frequency response of the circuit. The resulting transimpedance gain is plotted in Figure 2.17.

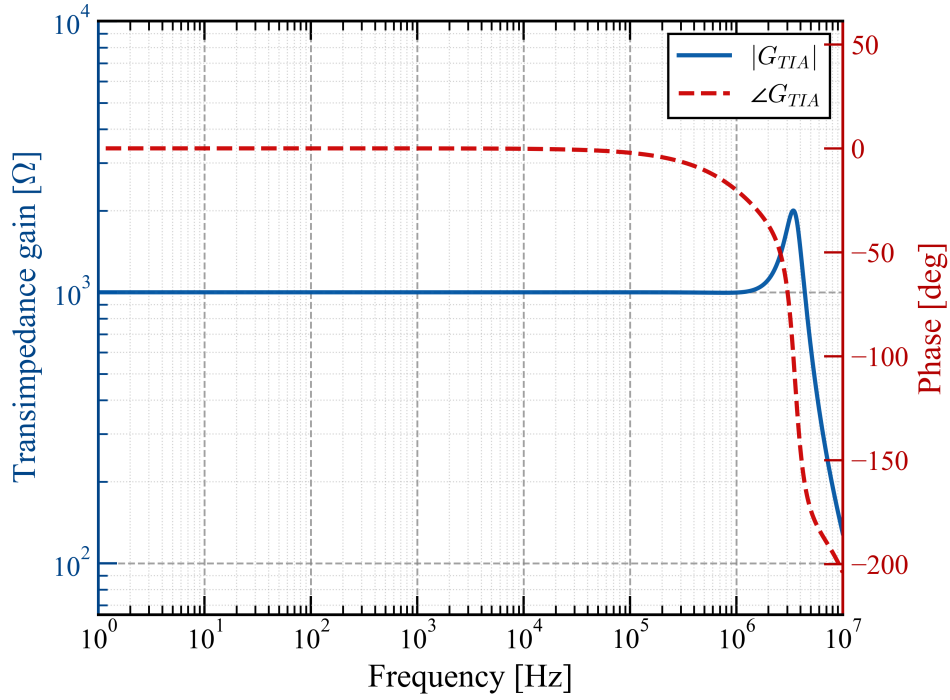


Figure 2.17: LTspice simulation of the transimpedance gain.

This can be compared directly to the real gain curve (the blue line) in Figure 2.16, which was derived analytically. The two graphs match almost perfectly, exhibiting only a minor deviation near the pole frequency. Having confirmed the intended frequency behaviour, we proceeded to transient and noise analyses.

Figure 2.18 displays the circuit's transient response to a sinusoidal differential input current given by $(i_1 - i_2)(t) = (1 \text{ mA}) \sin(2\pi f t) H(t)$ (where $H(t)$ is the Heaviside step function). The output is plotted for four logarithmically spaced frequencies: $f_1 = 100 \text{ kHz}$, $f_2 = 215.44 \text{ kHz}$, $f_3 = 464.44 \text{ kHz}$, and $f_4 = 1 \text{ MHz}$.

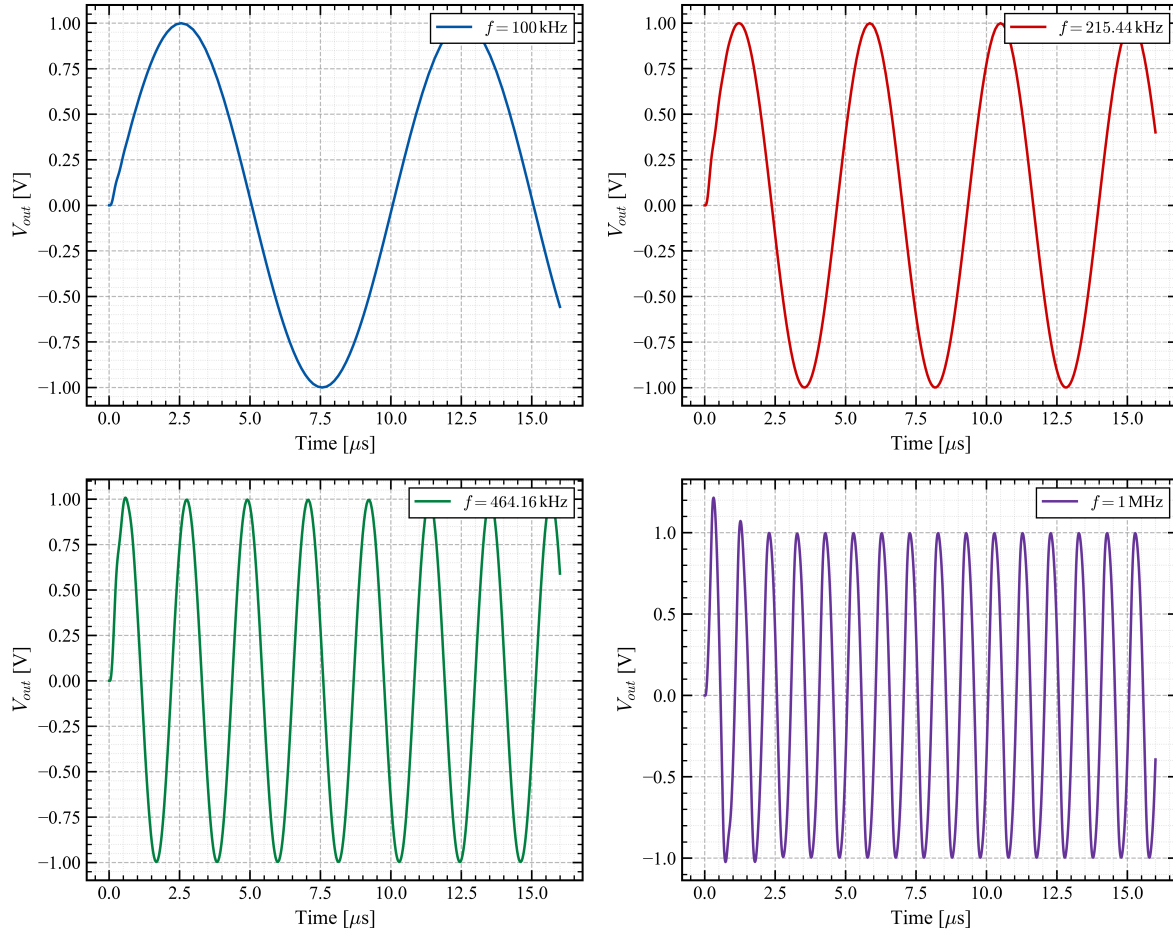


Figure 2.18: LTspice simulation of the transient time response.

The plot shows that all signals reach steady state within a couple of microseconds. All traces are correctly amplified by a factor of 10^3 V/A ($1 \text{ k}\Omega$) and display no visible distortion. Since the simulated circuit behaved exactly as intended, we proceeded to evaluate its most critical property: the output noise spectral density.

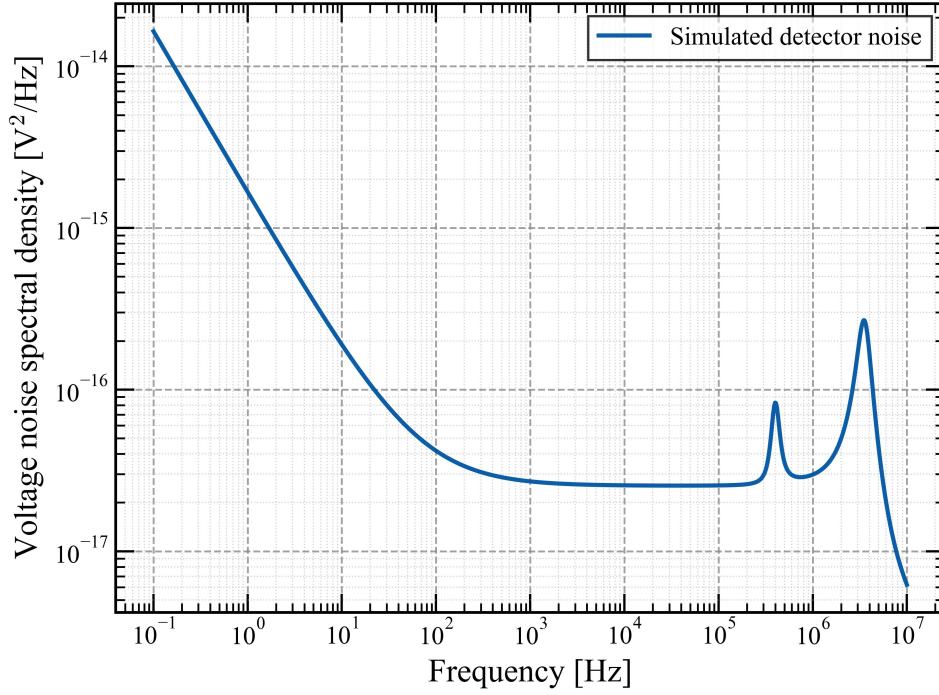


Figure 2.19: LTspice simulation of the output noise spectral density.

The noise spectrum is dominated by $1/f$ flicker noise up to about 100 Hz. Beyond that, it is remarkably flat for nearly 1 MHz at a level of $2.5 \cdot 10^{-17} \text{ V}^2/\text{Hz}$. The first distinct noise peak, at a frequency around 400 kHz, originates from the op-amp's internal voltage noise (as documented in the LT1028 datasheet). The second peak, near 3.5 MHz, correlates with the gain peaking near the pole frequency observed in Figure 2.17.

Thus, the baseline electronic noise of the detector is simulated to be approximately $S_{V_{electr}} \simeq 2.5 \cdot 10^{-17} \text{ V}^2/\text{Hz}$. This result is crucial for predicting the photodetector's performance. With a transimpedance gain of $1 \text{ k}\Omega$ and an expected responsivity of $R_{\lambda=2090 \text{ nm}} = 1.44 \text{ A/W}$ (for the IG26X500S4i photodiode), we can use Equation (2.1) to estimate the output voltage noise spectral density due to shot noise:

$$\begin{aligned}
 S_{V_{out, shot}} &= G^2 S_{i_{shot}}^{diff} = G^2 4 e R_{\lambda} \bar{P}_{ph} \\
 &\simeq 4 (10^3 \Omega)^2 (1.6 \cdot 10^{-19} \text{ C}) (1.44 \text{ A/W}) (10^{-3} \text{ W}) \\
 &\simeq 9.2 \cdot 10^{-16} \text{ V}^2/\text{Hz}
 \end{aligned} \tag{2.17}$$

where we assume an average optical power of $\bar{P}_{ph} = 1 \text{ mW}$ per photodiode. Because $S_{V_{out, shot}}$ is almost two orders of magnitude higher than the simulated electronic noise spectral density, $S_{V_{electr}}$, this photodetector satisfies the requirement $S_{V_{out, shot}} \gg S_{V_{electr}}$ with a comfortable margin of about 16 dB. Consequently, it should be highly capable of

measuring shot noise at 1 mW, and by extension, suitable for detecting squeezed vacuum states using a local oscillator in the same power range.

However, a major issue remains: the squared voltage spectral density of the amplified signal is too low, meaning it would be entirely buried beneath the ADC's noise floor. As detailed in Section 2.2, the measured noise floor of the data acquisition setup is $S_{V_{ADC}} \simeq 2.65 \cdot 10^{-16} \text{ V}^2/\text{Hz}$.

This value must be compared against the effective voltage spectral density actually read by the ADC, which is lower than the value calculated in Equation (2.17). Because the ATS9440 ADC features a 50Ω input impedance, connecting the photodetector output (also 50Ω) forms a 1:2 voltage divider. The actual voltage read by the ADC is half of V_{out} .

We can account for this by defining an effective "measured" transimpedance gain that is half the real gain. Consequently, for $P_{ph} = 1 \text{ mW}$, the measured shot noise spectral density read by the ADC, $S_{V_{shot}}$, becomes:

$$\begin{aligned} S_{V_{shot}} &= S_{i_{shot}}^{diff} \left(\frac{G}{2} \right)^2 = G^2 e R_\lambda \bar{P}_{ph} \\ &\simeq (10^3 \Omega)^2 (1.6 \cdot 10^{-19} \text{ C}) (1.44 \text{ A/W}) (10^{-3} \text{ W}) \\ &\simeq 2.30 \cdot 10^{-16} \text{ V}^2/\text{Hz}. \end{aligned} \tag{2.18}$$

Clearly, $S_{V_{shot}} \simeq 2.30 \cdot 10^{-16} \text{ V}^2/\text{Hz} < S_{V_{ADC}} \simeq 2.65 \cdot 10^{-16} \text{ V}^2/\text{Hz}$, completely failing the condition $S_{V_{shot}} \gg S_{V_{ADC}}$. This analytical finding perfectly mirrors the empirical data presented later in Figure 2.25.

To resolve this, we introduced a second stage of voltage amplification. This raises both $S_{V_{out, shot}}$ and $S_{V_{electr}}$, eventually ensuring that both signals comfortably exceed $S_{V_{ADC}}$. While a second stage inherently injects some of its own noise, slightly degrading the overall Signal-to-Noise Ratio (SNR), it is absolutely necessary to lift the measured shot noise above the ADC's noise floor.

A second stage with a minimum voltage gain of $G_2 = 10$ (20 dB gain) produces a new measured shot noise density $S'_{V_{shot}} = G_2^2 S_{V_{shot}} = 100 S_{V_{shot}}$. This is sufficient to push the signal two orders of magnitude above the ADC noise limit.

Provided that this secondary amplifier does not introduce excessive electrical noise, the simulations confirm that the proposed photodetector architecture is perfectly adequate for detecting squeezed vacuum states.

2.4. Photodetector design

To keep the design process modular and troubleshooting manageable, the detector was split into two separate circuit boards. The first board houses the TIA stage and the two photodiodes; we refer to this as the "homodyne board". The second board houses the power regulation electronics required to supply clean DC voltages to the op-amps on the homodyne board, as well as the filtered bias voltages for the photodiodes. We refer to this as the "supply board".

The schematics and PCB layouts were developed using Autodesk Fusion 360 Electronics. The main design rules adopted for the PCBs are summarized below.

Name	Rule	
Wire - Wire Clearance	10 mil	☒
Wire - Pad Clearance	10 mil	☒
Wire - Via Clearance	10 mil	☒
Pad - Pad Clearance	10 mil	☒
Pad - Via Clearance	10 mil	☒
Via - Via Clearance	10 mil	☒
SMD - Pad Clearance	10 mil	☒
SMD - Via Clearance	10 mil	☒
SMD - SMD Clearance	10 mil	☒
Board Outline Clearance	40 mil	☒
Copper - Restrict Clearance	0 mil	☒
Via - Restrict Clearance	0 mil	☒
Soldermask - Silkscreen Clearance	0 mil	☐
Drill Clearance	6 mil	☒
Component Exclude Clearance	0 mil	☒
Unconnected Lines	N/A	☒
Copper Width	40 mil	☒
Drill Size	13.78 mil	☒

Figure 2.20: PCB design rules.

Once the designs were finalized, the PCBs were manufactured by PCBWay.

The complete schematics for both boards can be found in Appendix A.

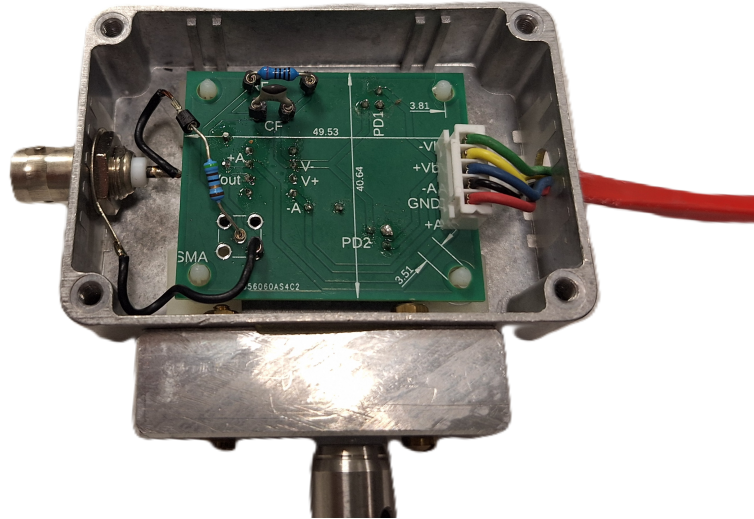


Figure 2.21: Homodyne board layout.



Figure 2.22: Supply board layout.

The supply board operates from raw $\pm 18\text{ V}$ inputs. It also receives the main ground connection and the two unfiltered bias voltages. It then delivers five regulated outputs to the homodyne board via jumpers: the two bias voltages $\pm V_{bias} = \pm 0.25\text{ V}$, the two supplies voltages $\pm A = \pm 15\text{ V}$, and the ground (GND). Because the supply board only handles DC voltages, we utilized OP27 operational amplifiers. These are low-noise op-amps with a $GBWP$ of 8 MHz , which is more than adequate for DC regulation.

The homodyne board receives in input these 5 power lines. Its output is a single voltage proportional to the photocurrent difference (referenced to ground), fed out via a BNC connector. Coaxial cables with a characteristic impedance of $50\ \Omega$ are used to route this signal to the subsequent filtering and amplification stages.

Both PCBs were enclosed in custom aluminium chassis, as we observed that metallic shielding drastically reduced the environmental electromagnetic noise picked up by the ADC. To prevent ground loops, the homodyne board enclosure (which was internally grounded to the PCB) had to be electrically isolated from the optical table. Another crucial factor was the length of the BNC coaxial cable connecting the output: shorter cables consistently yielded cleaner signals, likely by minimizing the antenna effect for ambient electromagnetic interference.

Initially, the supply board was powered by two benchtop power supplies: a Farnell PDD3502A and a Siglent SPD3303X-E. However, empirical testing quickly revealed that these supplies introduced unacceptable levels noise. We isolated the issue to the internal electronics of the power supplies, as the characteristic noise peaks were observable at the ADC even when the power supplies were physically turned off (but still tethered to the supply board). Disconnecting the cables immediately eliminated the noise.

To guarantee a clean power source, we replaced the bench supplies with four 9 V alkaline batteries connected in series. This proved to be a much cleaner voltage supply that did not add too much noise. For long-term convenience, we eventually transitioned to a battery pack consisting of thirty rechargeable 1.2 V Ni-MH cells, which offered similar noise performance to the alkaline batteries.

A secondary issue involved the photodiode bias voltages. These were originally supplied by the benchtop units, routed through a pair of Sallen-Key low-pass filters on the supply board, and finally fed to the homodyne board. Given the insurgence of severe noise when connecting the bench power supplies to the the supply board we observed that the output signal was less noisy when no bias voltage was fed to the photodiodes. Although operating the p-i-n diodes in photovoltaic mode ($V_{bias} = 0\text{ V}$) theoretically sacrifices some of the photodiode performance compared to photoconductive mode (reverse bias applied), the overall noise of the system reduced dramatically without the bias lines. Therefore,

we short-circuited the bias voltage input pins on the homodyne board directly to ground and exclusively operated the photodiodes in photovoltaic mode for all measurements.

Another challenge encountered during testing was aliasing. The "signal" of interest, the shot noise, is theoretically white and spans an infinite bandwidth. While the TIA amplification stage intrinsically limits this to ~ 3 MHz, and our ADC sampling rate of $f_s = 100$ MS/s easily satisfied the Nyquist criterion for the TIA's passband, residual high-frequency noise beyond 50 MHz (the Nyquist frequency) was still present. This out-of-band noise folded back into the baseband spectrum via aliasing.

To stop aliasing, we inserted a low-pass filter (Mini-Circuits BLP-10.7+) immediately after the homodyne board. It features a 3 dB cutoff frequency of $f_c = 14$ MHz and a passband insertion loss of less than 1 dB up to 11 MHz. This filter effectively suppressed any noise above the Nyquist limit, completely resolving the aliasing issue.

This low-pass filter remained in the signal chain for all measurements presented in this thesis.

Despite shielding the boards, using batteries, and installing low-pass filters, the cabling and ADC front-end inevitably introduce a baseline level of electronic noise. To quantify this, we captured the noise floor of the data acquisition system by taking a spectrum with the photodetector fully connected but unpowered (ensuring no active TIA noise reached the ADC). The result is the spectrum previously plotted in Figure 2.5 that was introduced when describing the ADC.

As concluded in the previous section, the raw output of the homodyne board is not large enough to overcome the fixed noise floor of the ADC and its connections. Consequently, we added a final component to the signal chain: a Mini-Circuits ZFL-500LN low-noise amplifier, connected directly after the low-pass filter. It is a broadband amplifier spanning $f_l = 100$ kHz to $f_u = 500$ MHz. Using the frequency response analyser function of a Moku:Lab, we precisely characterized its linear voltage gain. In Figure 2.23 the measured gain.

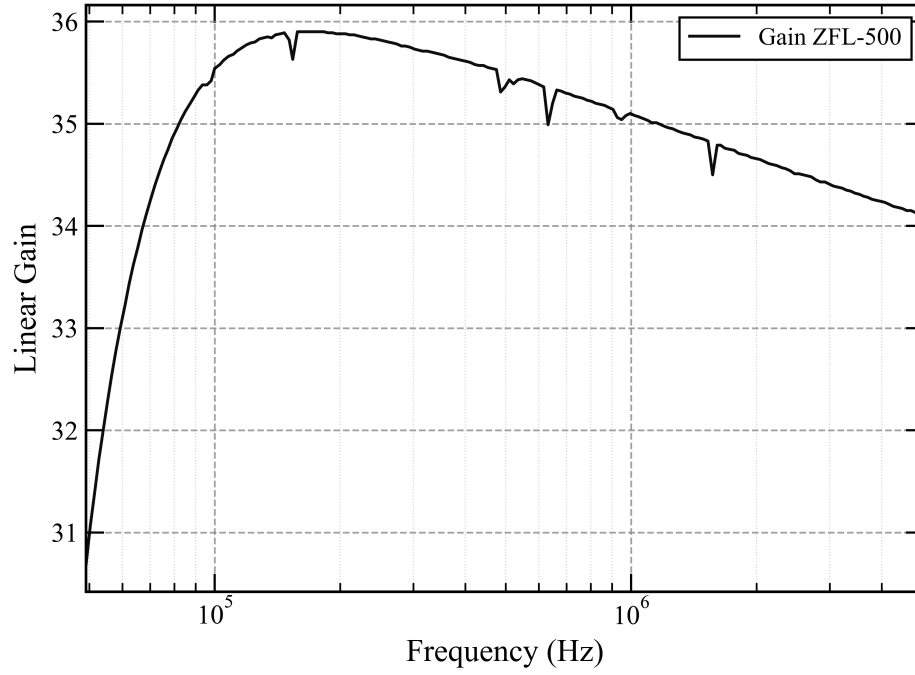


Figure 2.23: Linear gain of the ZFL-500LN amplifier.

The gain is clearly not constant which is not ideal in our set-up. However in the frequency band in which it has actually been used, $f \in [100 \text{ kHz}, 1.5 \text{ MHz}]$, it does not deviate too much from its average value in such band, that is $G_{2nd} = 35.50$. To simplify the study the second stage gain has been considered constant to this average value.

A photograph of the completed photodetector, fully wired and mounted on the optical bench, is provided in Figure 2.24.

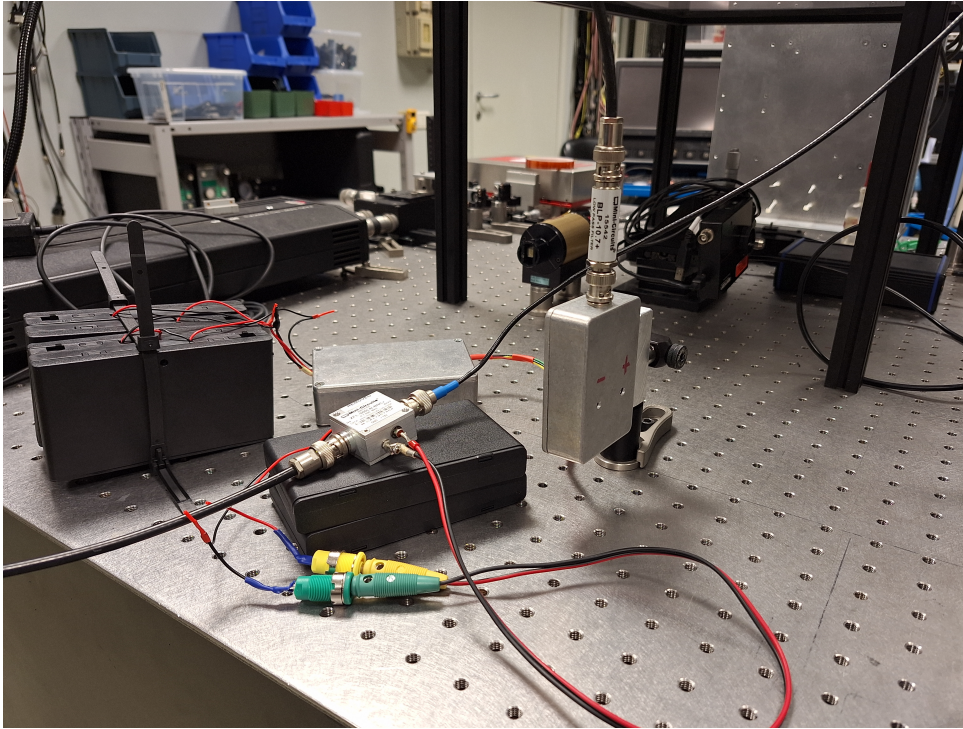


Figure 2.24: Final photodetector setup on the optical bench.

Thanks to this secondary amplification stage, the photodetector's electrical noise floor was lifted approximately 17 dB above the ADC's baseline noise. The differences of having or not having a second amplification stage are illustrated by comparing Figures 2.25 and 2.26. Both figures plot the noise floor of the acquisition setup. The red trace represents the ADC's intrinsic noise floor (detector connected but unpowered). The blue trace represents the active electrical noise of the photodetector (detector powered, but kept perfectly dark with no incident light). Figure 2.25 shows the data taken without the secondary Mini-Circuits amplifier, while Figure 2.26 shows the data with the amplifier in place.

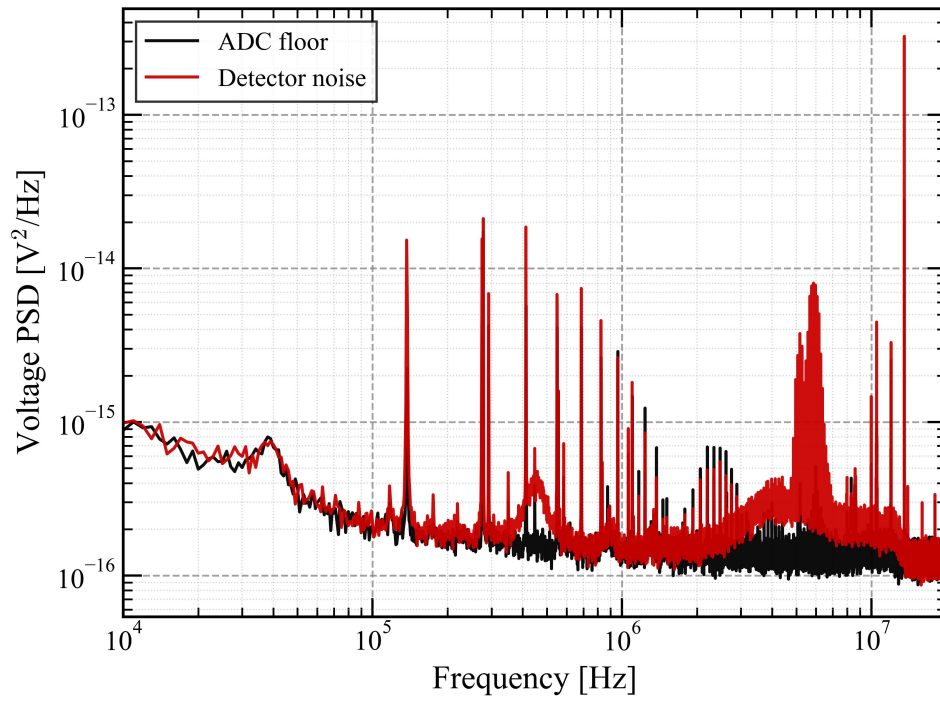


Figure 2.25: Noise spectrum of the unamplified photodetector.

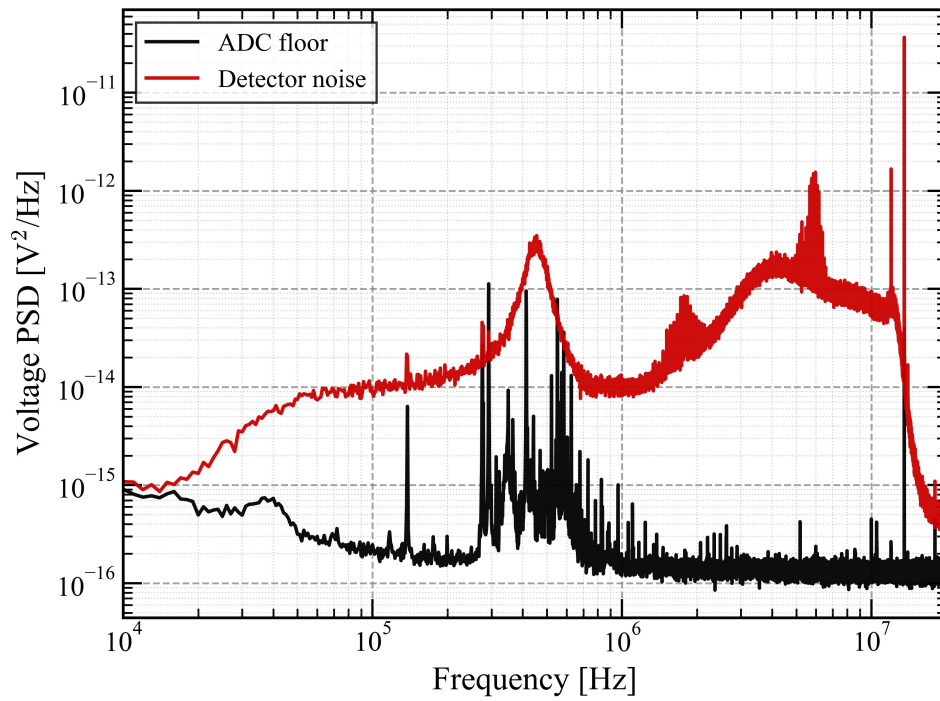


Figure 2.26: Noise spectrum of the amplified photodetector.

In the unamplified case, the photodetector's baseline noise (and eventually also the signal)

is entirely submerged by the ADC noise; only the sharpest photodetector's noise peaks manage to break through (Figure 2.25). With amplification (Figure 2.26), a clear gap is established. The detector's intrinsic dark noise is correctly resolved, sitting nearly two orders of magnitude above the ADC limit.

However, the photodetector's noise spectrum is not perfectly ideal. Rather than being strictly flat, it exhibits several structures. Strictly speaking, this is not a fundamental failure, provided the dark noise remains well below the optical shot noise level. But given our relatively modest noise margin (around one order of magnitude), these peaks are large enough to become comparable to the anticipated shot noise, degrading the measurement quality at those specific frequencies.

Two distinct noise artifacts dominate the spectrum. First, a massive surge in noise occurs near the pole frequency of the TIA stage ($f > 2.5$ MHz). This does not pose a problem, as it resides well outside our 1 MHz target bandwidth. Second, a sharp noise peak appears at roughly 450 kHz. This is an intrinsic characteristic of the LT1028 op-amp, as accurately predicted by the LTspice simulation (Figure 2.19). Unfortunately, this peak sits squarely in the middle of our intended [100 kHz, 1 MHz] operating band. While highly inconvenient, this defect does not completely invalidate the device. We can simply bypass it by conducting measurements in two separate, clean frequency windows: a lower window from $f \in [100 \text{ kHz}, 200 \text{ kHz}]$, and a much wider upper window from $f \in [600 \text{ kHz}, 1.1 \text{ MHz}]$.

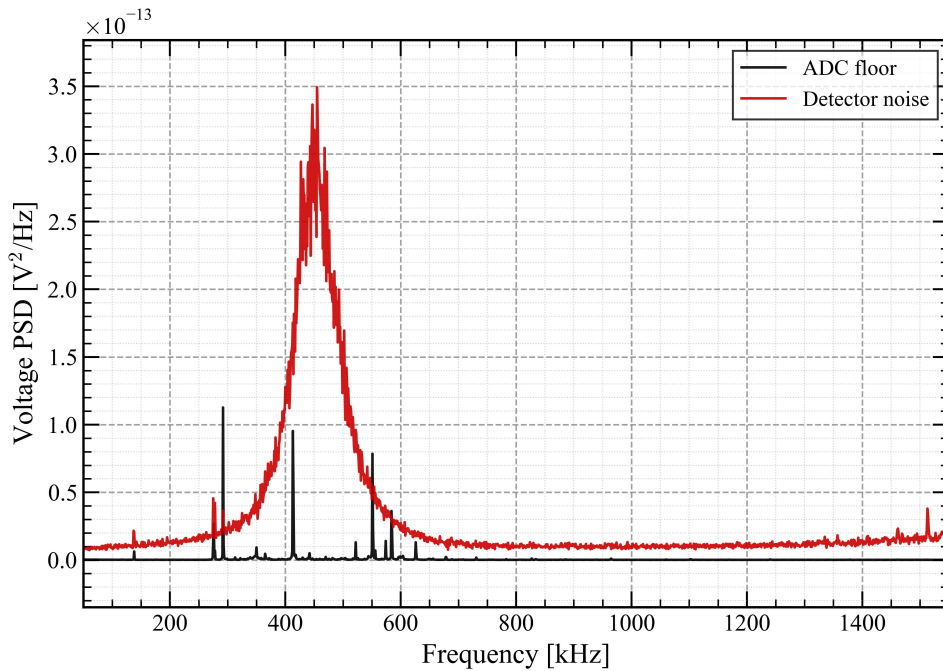


Figure 2.27: Linear plot of the noise spectrum for the amplified photodetector.

It is important to note that the logarithmic frequency axis in Figure 2.26 visually compresses the higher frequencies. When plotted on a linear scale, zooming into the $f \in [50 \text{ kHz}, 1.55 \text{ MHz}]$ range (Figure 2.27), it becomes evident that the noise density is nicely flat across the vast majority of the upper frequency window.

2.5. Measurements

Having confirmed that the detector possesses a sufficiently low noise floor within specific frequency windows, we proceeded to take physical measurements.

The optical setup used was identical to the one described in Section 2.2.1. We injected the local oscillator field into one port of the beam splitter. The other port was blocked, meaning in a quantum optics picture, that we were injecting a standard (non-squeezed) vacuum state. We systematically increased the optical power of the local oscillator to amplify the vacuum fluctuations (Equation (2.18)). If the resulting signal is genuinely optical shot noise, the measured voltage spectral density must scale perfectly linearly with the incident optical power, as dictated by Equation (2.1).

To establish a baseline and test our custom hardware in simpler wavelength spectrum, we opted to first test the detector at 1064 nm before moving to our ultimate goal of 2 μm .

2.5.1. 1 μm measurements

The result of a single acquisition at a specific local oscillator power is shown in Figure 2.28.

With 1080 μW of optical power impinging on the photodetector, the measured noise spectral density clearly detaches from the electronic dark noise. Within the clean frequency windows identified earlier, the measured spectrum provides a robust $\sim 10 \text{ dB}$ clearance above the electronic noise floor. Crucially, the measured spectrum lies completely flat against the theoretical shot noise limit (dashed gold line), which was calculated using Equation (2.17) adapted for the 1064 nm responsivity and an optical power $\bar{P}_{ph} = 1.08 \text{ mW}$.

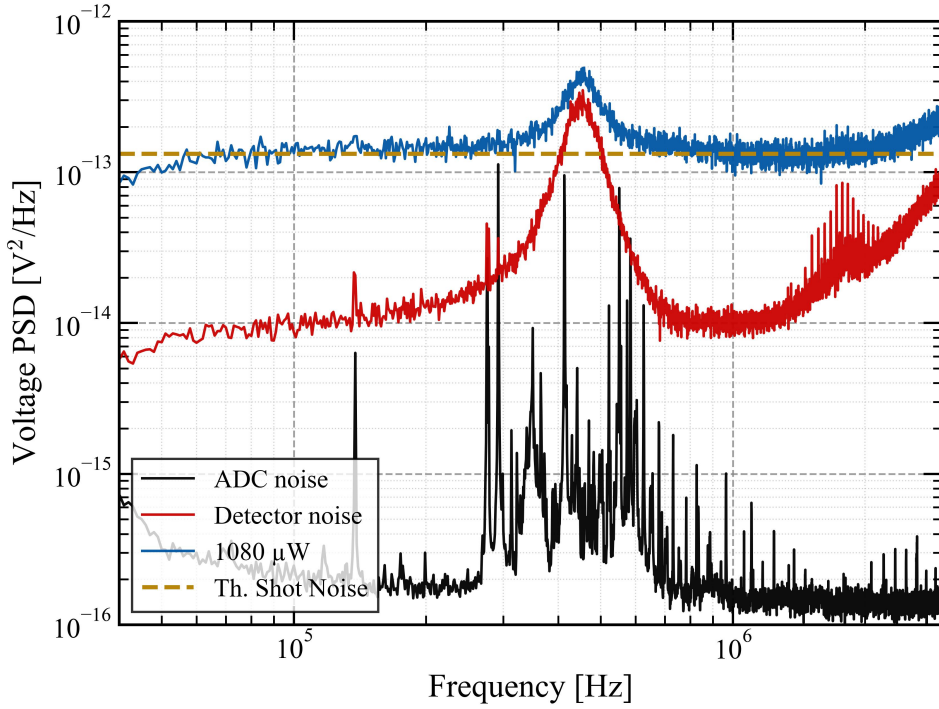


Figure 2.28: Voltage noise spectrum for a single incident optical power (1080 μW).

Since each single plotted spectrum was obtained averaging 100 acquired spectra (as described in Section 2.2) we estimated the measurement error of each single point. The measurement error expressed with a confidence interval of 95%, was in the order of $10^{-14} \text{ V}^2/\text{Hz}$ or lower. The average error for each single point for the $\bar{P}_{ph} = 1080 \mu\text{W}$ measurement was $1.48 \cdot 10^{-14} \text{ V}^2/\text{Hz}$. In these set of plots the error is not indicated to not overfill the figure. Its exact value for each measurement will be however plotted in the final plot 2.32.

This type of acquisition was repeated across a sweep of local oscillator powers. The aggregated spectra are plotted in Figure 2.29:

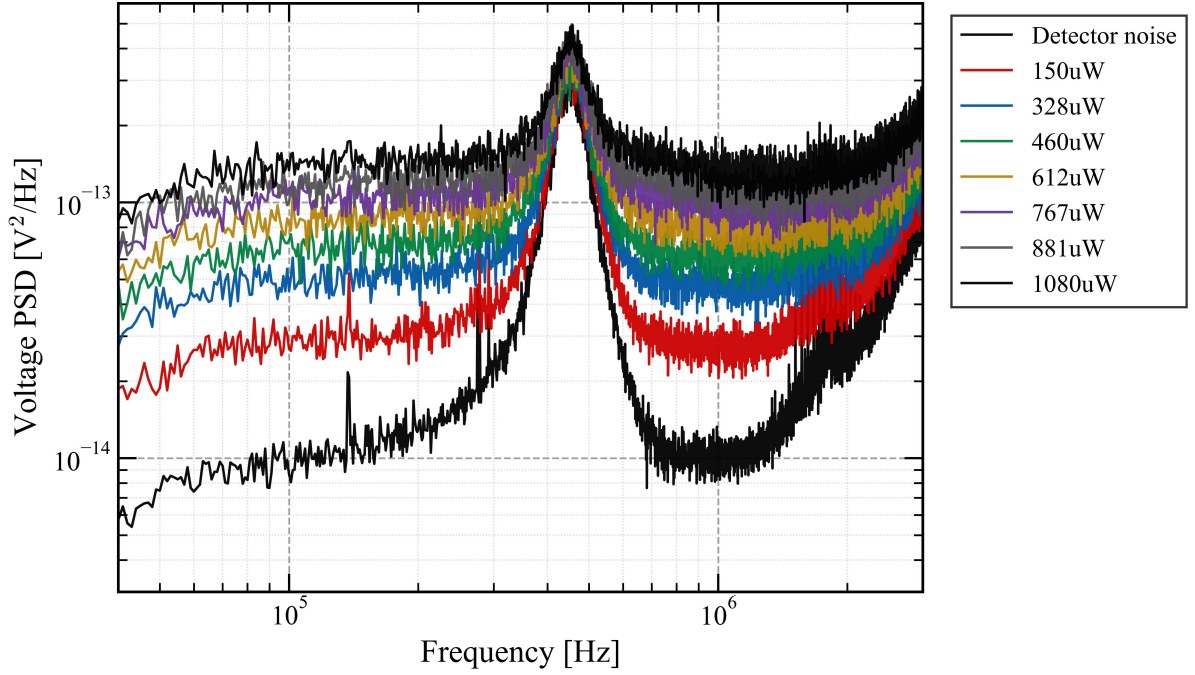


Figure 2.29: Voltage spectra for various incident optical powers.

As expected, the broad-spectrum noise uniformly rises in direct proportion to the local oscillator power.

To be thorough, we performed this entire power sweep twice: once with the Mini-Circuits amplifier in the chain, and once without it. As predicted, the unamplified shot noise measurements failed to clear the ADC noise floor. For reference, an unamplified spectrum is shown below.

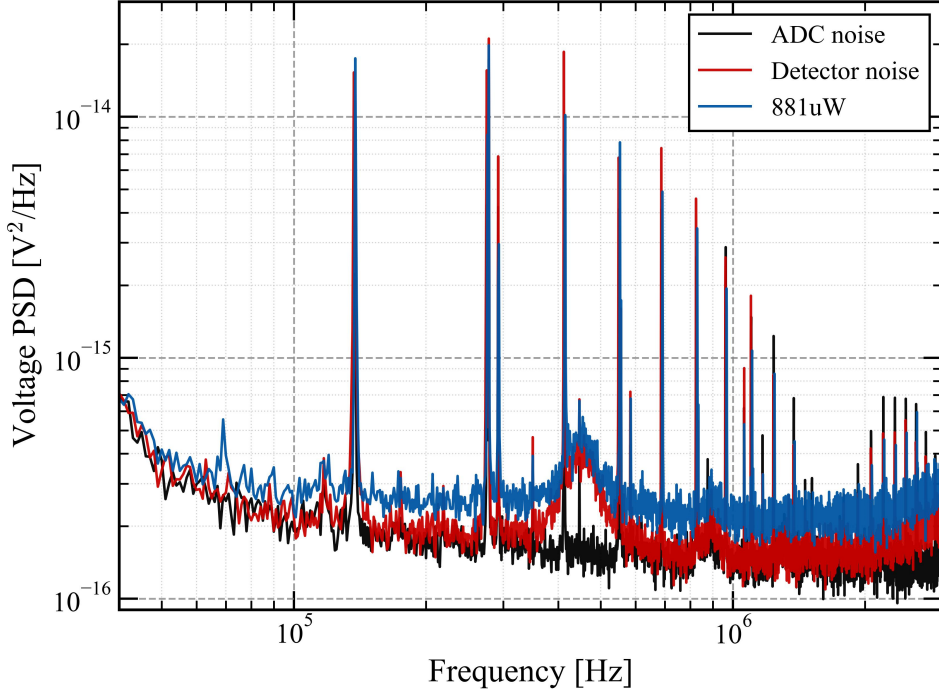


Figure 2.30: Unamplified voltage spectrum for a selected optical power.

Because the unamplified noise traces were compressed together, only one power level is shown in Figure 2.30 for clarity. However, this unamplified data set proved extremely valuable. Because the secondary stage amplifier strictly blocks DC signals, it hides the absolute DC voltage generated by the TIA. By momentarily removing the amplifier and illuminating only a single photodiode, we could measure the raw DC voltage produced by the photodetector at each power level.

This data is summarized in Table 2.1.

Optical Power [μW]	V_{out} [mV]	G_{ph}^{meas} [V/W]
150	51	340.00
328	109	332.32
460	153	332.61
612	202	330.07
767	255	332.46
881	296	335.98

Table 2.1: Output DC voltage and calculated effective gain for different optical powers.

In the table G_{ph}^{meas} is the effective measured transimpedance gain, calculated simply by

dividing the DC measured voltage by the optical input power. This empirical data aligns excellently with our theoretical design. Given a physical TIA resistor of $G_{TIA} = 1 \text{ k}\Omega$ and a responsivity of $R_\lambda = 0.66 \text{ A/W}$, the effective voltage gain seen by a 50Ω matched instrument is:

$$G_{ph} = \frac{1}{2} G_{TIA} R_\lambda \simeq \frac{1}{2} (1000 \Omega) (0.66 \text{ A/W}) \simeq 330 \text{ V/W}. \quad (2.19)$$

The experimental data yields an average effective gain of 333.9 V/W . The minor 1.2% relative discrepancy is likely due to the true quantum efficiency of the photodiodes diverging slightly from the nominal datasheet value and measurement error.

To estimate the common mode rejection ratio we performed two measurements at the same local oscillator power. One in the usual balanced configuration, with both photodiodes illuminated with the same optical power (the red line in Figure 2.31). The other in a single-input configuration, illuminating one photodiode and keeping the other in dark (the black line). The second one is used to see the common mode signal that in the balanced measurement is in input to both photodiodes. This first acquisition should ideally completely reject the common mode signal.

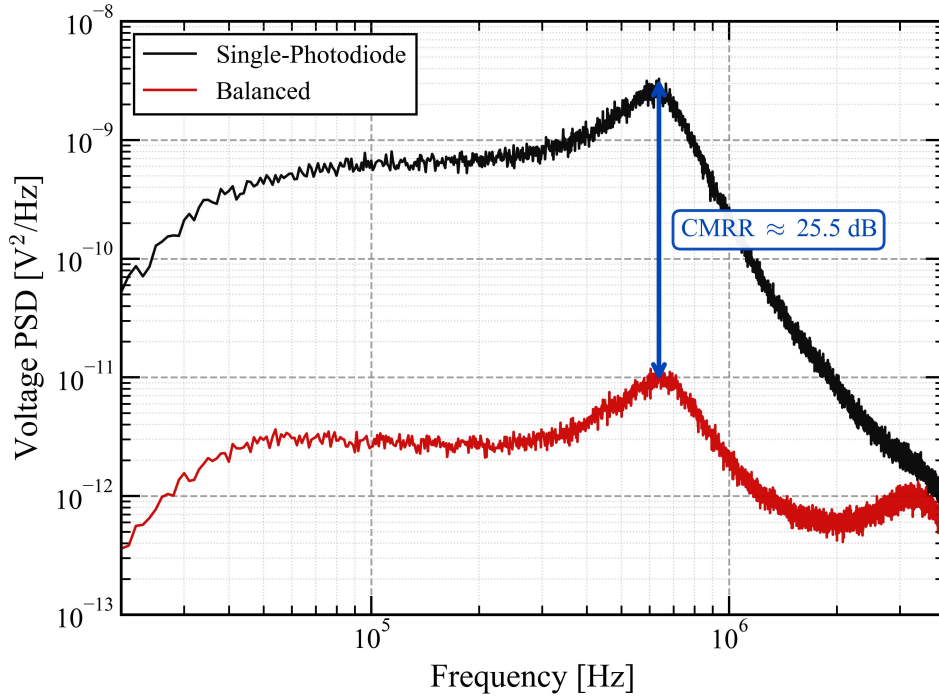


Figure 2.31: CMRR measurement at optical power $\bar{P}_{ph} = 3.4 \text{ mW}$.

However, the real photodetector behaviour is obviously not the ideal one. The value

of CMRR measured from the peaks of the two noise spectral densities is $CMRR_{dB} = 25.45 \text{ dB}$ and given the plot of Figure (2.31) it can be considered constant over the whole operating bandwidth of the photodetector. The measurement was performed at the maximum power the photodiodes could withstand, i.e. $\bar{P}_{ph} = 3.4 \text{ mW}$. It is worth noting that at this power the noise peak appearing in both signals is not the photodetector noise peak due to the opamp, which would be centred at 450 kHz . It is instead the relaxation noise peak of the Nd:YAG laser, centred around 700 kHz and clearly visible in its RIN of Figure 2.6a.

Since the optical power impinging on both photodiodes probably was not exactly equal we can say, that the CMRR of the photodetector itself is at least 25.45 dB .

To definitively prove that the flat wideband noise we are observing is optical shot noise, we calculated the average of the noise spectra in the $f \in [100 \text{ kHz}, 200 \text{ kHz}]$ window for every power level and plotted it.

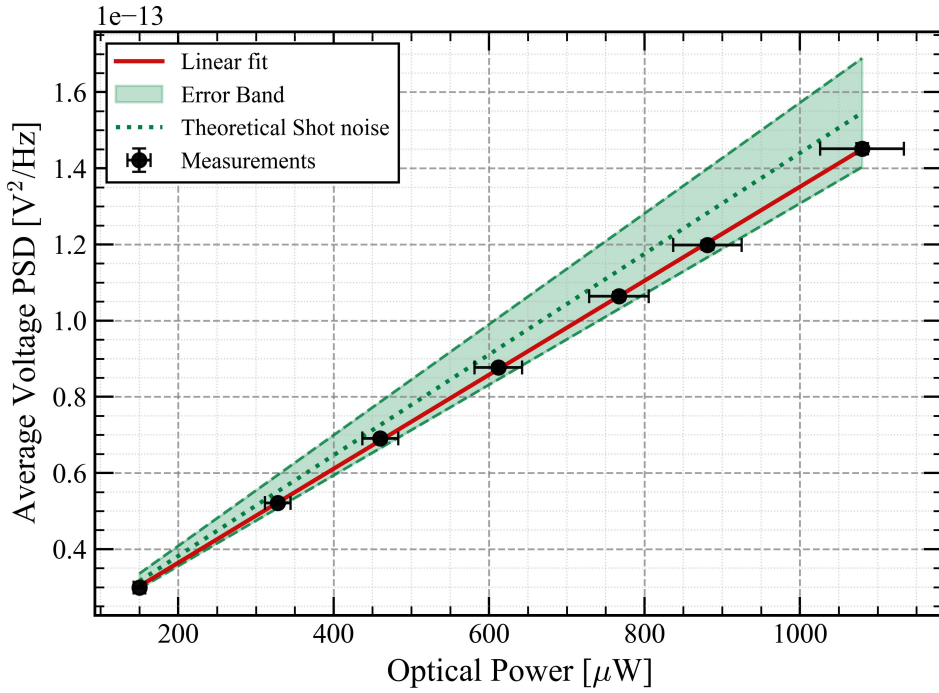


Figure 2.32: Linearity of the noise spectrum against optical power (100 kHz to 200 kHz).

The measured power was assumed to be known with an uncertainty of the 5% due to error in the power meter reading. The error in the average of the voltage spectral density was the measurement error carried from the measurement error in each single point of each spectrum. As the latter error was already said to be in the order of $10^{-14} \text{ V}^2/\text{Hz}$ it is not surprising that the estimated error over the average in the $[100 \text{ kHz}, 200 \text{ kHz}]$ is in the

order of $10^{-15} \text{V}^2/\text{Hz}$. Indeed, in the considered frequency band, given the frequency resolution of 10 kHz, there were 101 points and the error over the average scales as $1/\sqrt{N_{\text{points}}}$. The resulting 95% confidence interval is plotted for each point even though being so small is not easy to see.

The measured points align almost perfectly along a straight line. The linear fit (shown in red) yields an R^2 coefficient of 0.999, characterized by the equation:

$$S_V^{fit} = \left(1.23 \cdot 10^{-10} \frac{\text{V}^2}{\text{W Hz}} \right) P_{ph} + \left(1.18 \cdot 10^{-14} \frac{\text{V}^2}{\text{Hz}} \right). \quad (2.20)$$

Theoretically, the shot noise slope is defined as:

$$\begin{aligned} S_V^{theor} &= \left[2 \left(\frac{G_{TIA} G_{2nd}}{2} \right)^2 2 e R_\lambda \right] P_{ph} = [(G_{TIA} G_{2nd})^2 e R_\lambda] P_{ph} \\ &\simeq [(1 \text{ k}\Omega \cdot 35.5)^2 (1.6 \cdot 10^{-19} \text{ C}) (0.66 \text{ A/W})] P_{ph} \\ &\simeq \left(1.33 \cdot 10^{-10} \frac{\text{V}^2}{\text{W Hz}} \right) P_{ph}. \end{aligned} \quad (2.21)$$

As it was said in Section 2.2.2, the responsivity of the photodiode was assumed to be known with a 10% relative error. The feedback resistance was measured to a value of $R_f = 1000.0 \Omega$ with a relative error of 0.1%. Given the small relative error compared to the one of the responsivity we can consider this value as exact. Also the second stage gain, for simplicity, is assumed exact. This leads to an uncertainty of the theoretical shot noise slope of the 10% of its value. The uncertainty region of this slope is depicted in green in Figure 2.32 and called error band.

The slope of the linear fit is estimated to be $1.23 \cdot 10^{-10} \frac{\text{V}^2}{\text{W Hz}}$ which differs from the theoretical value $1.33 \cdot 10^{-10} \frac{\text{V}^2}{\text{W Hz}}$ of only the 7.5%. Indeed the linear fit line, in red, falls in the green region indicating the error bands of the theoretical shot noise slope.

According to Equation (2.21), pure shot noise should pass from the origin of the axis of Figure 2.32. However, the linear fit displays a y-intercept of $1.18 \cdot 10^{-14} \text{V}^2/\text{Hz}$. This is perfectly consistent with our expectations: the y-intercept is simply the constant electronic dark noise floor of the active photodetector. Assuming the optical noise and electronic noise are uncorrelated, their variances add linearly, which cleanly explains the non-zero intercept.

The detector's behaviour in the frequency window, $f \in [600 \text{ kHz}, 1.1 \text{ MHz}]$, is practically identical. The scatter plot for this wider window is shown below.

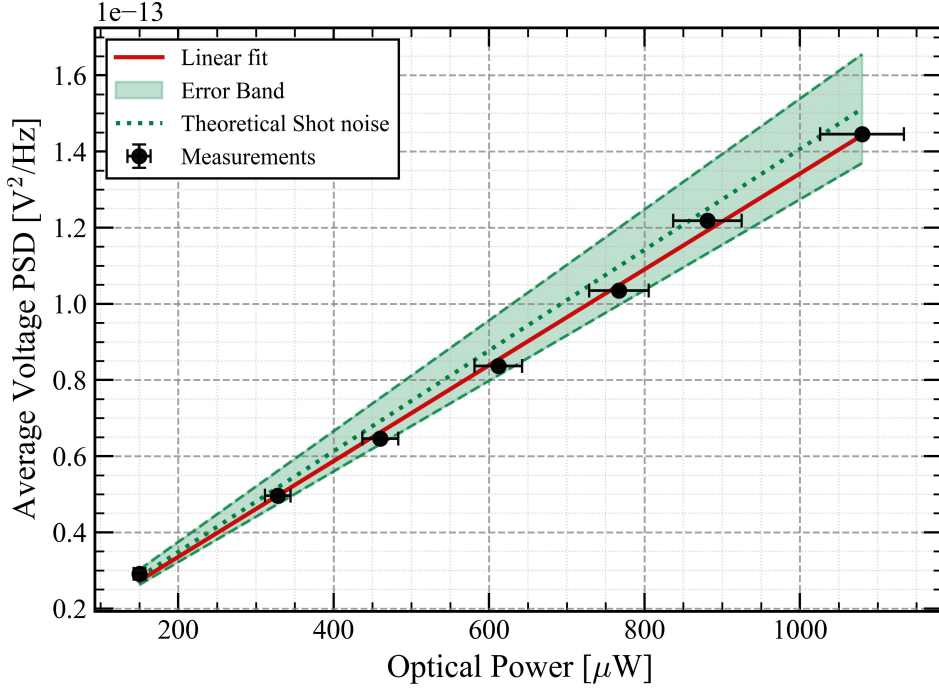


Figure 2.33: Linearity of the noise variance against optical power (600 kHz to 1.1 MHz).

For this region, the linear fit equation is:

$$S_V^{fit} = \left(1.26 \cdot 10^{-10} \frac{\text{V}^2}{\text{W Hz}} \right) P_{ph} + \left(8.39 \cdot 10^{-15} \frac{\text{V}^2}{\text{Hz}} \right) \quad (2.22)$$

with an $R^2 = 0.9982$. The logic regarding the y-intercept remains identical, as $8.39 \cdot 10^{-15} \text{ V}^2/\text{Hz}$ cleanly traces back to the electronic dark noise at those frequencies. The relative error of slope of the fit compared with the theoretical value is roughly 5.6%, which again is small enough to make it fall in the green error band.

2.5.2. 2 μm measurements

Having resoundingly verified that the custom photodetector correctly resolves optical shot noise from a 1064 nm Nd:YAG laser within our specified frequency windows, we were confident enough to transition to the 2 μm regime.

Applying the exact same methodology, we swept the incident optical power using the Tm:Ho:YAG laser. The overlaid noise spectra are shown below.

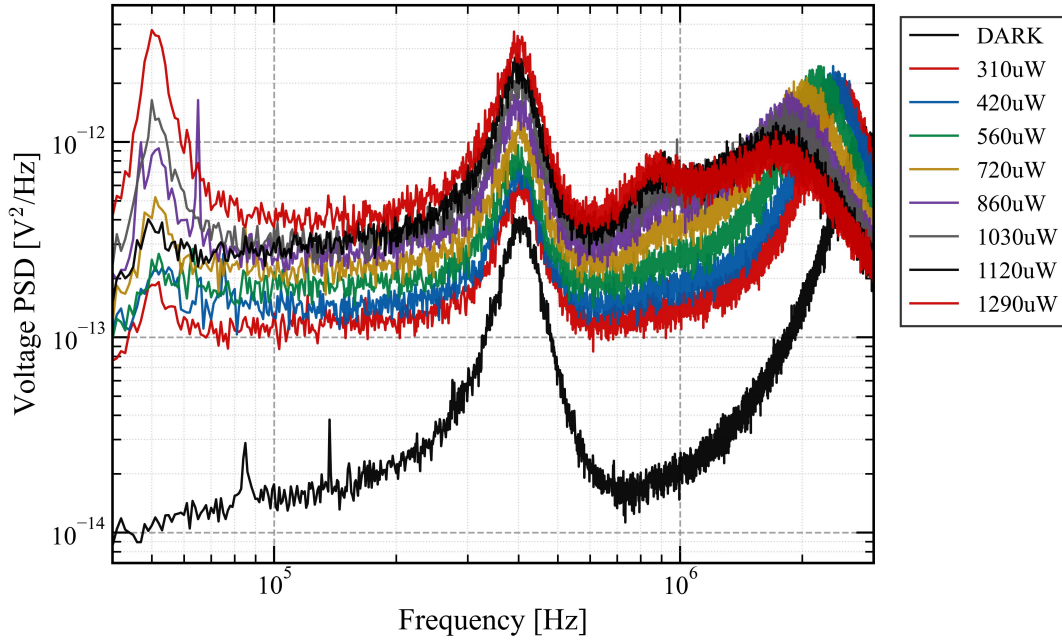


Figure 2.34: Voltage spectra for various incident optical powers at 2 μm .

The governing equation for the shot noise variance (Equation (2.21)) remains true, but we must update the responsivity variable to match the extended InGaAs photodiodes ($R_\lambda = 1.44 \text{ A/W}$). The corrected theoretical slope is:

$$\begin{aligned}
 S_V^{theor} &= [(G_{TIA} G_{2nd})^2 e R_\lambda] P_{ph} \\
 &\simeq [(1 \text{ k}\Omega \cdot 35.5)^2 (1.6 \cdot 10^{-19} \text{ C}) (1.44 \text{ A/W})] P_{ph} \\
 &\simeq \left(2.90 \cdot 10^{-10} \frac{\text{V}^2}{\text{W Hz}} \right) P_{ph}.
 \end{aligned} \tag{2.23}$$

As previously done for the 1 μm configuration in Figure 2.28, a single acquisition trace is isolated below for clarity:

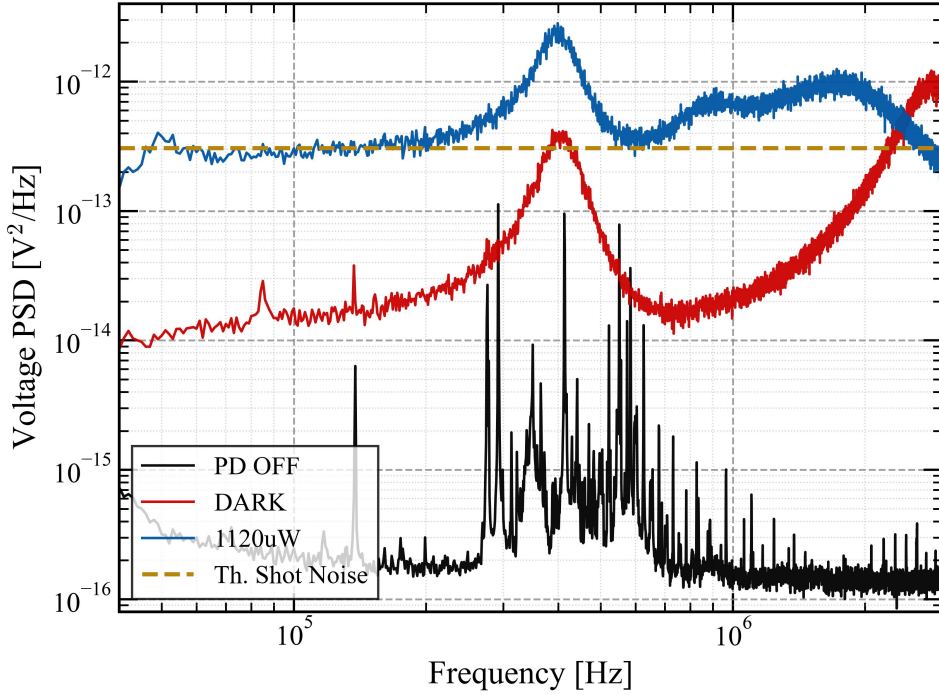


Figure 2.35: Voltage noise spectrum for a single incident optical power (1120 μW) at 2 μm .

The theoretical shot noise level, calculated using Equation (2.23), is overlaid as a dashed gold line. While the overall architecture behaves similarly to the 1 μm configuration, there are critical differences. The raw electronic dark noise and the ADC baseline remain basically identical. However, the active optical trace exhibits several severe new noise peaks. This was expected; the parasitic electrical characteristics (C_j and R_{sh}) of the windowless extended InGaAs photodiodes are markedly worse than the standard InGaAs units, leading to a degraded frequency response.

Notably, a massive low-frequency peak appears around 50 kHz. In some acquisitions, this peak spiked significantly higher than shown in this particular figure. We found that this peak could be somewhat flattened by meticulously adjusting the optical power balance between the two photodiodes. However, it was highly volatile, and it was rarely possible to eliminate it entirely. This erratic behaviour suggests that this peak is driven by a slight optical unbalance in the homodyne setup, manifesting as a failure to completely suppress the relaxation oscillations noise of the Tm-Ho:YAG laser. Fortunately, this 50 kHz peak lies completely outside our target analysis windows, meaning it does not interfere with the actual shot noise measurements.

Similar considerations can be done for the massive noise peak centred around 900 kHz. This severe spike lands directly in the centre of our secondary high-frequency window

([600 kHz, 1.1 MHz]), completely destroying its usability for shot noise analysis. The intensity of this disturbance scales directly with incident optical power, suggesting it is also a common-mode rejection issue.

Consequently, the only viable frequency window for the 2 μm setup was the lower band: $f \in [100 \text{ kHz}, 200 \text{ kHz}]$. The linearity scatter plot for this window is shown below.

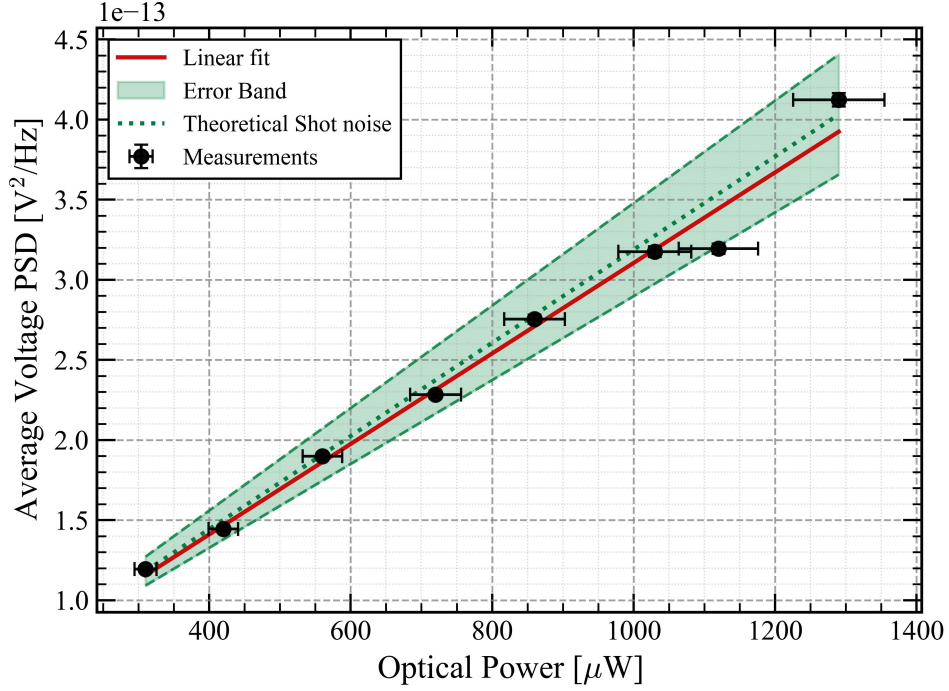


Figure 2.36: Linearity of the noise variance against optical power at 2 μm (100 kHz to 200 kHz).

The above plot was computed in the same way of the one of Figure 2.32. Therefore, the same discussion for the errors holds true.

Each measured point is drawn with confidence interval both in horizontal and vertical axis. The horizontal is a simple 5% error on the measured power. The vertical one is a 95% confidence interval given by the estimated measurement error. In this case its maximum value is $4.2 \cdot 10^{15} \text{ V}^2/\text{Hz}$ which again is so small it makes it difficult to be appreciated in the figure. As explained for the plot of Figure 2.32 the theoretical shot noise slope carries a 10% uncertainty due to the considered error in the responsivity value of the photodiodes.

The linear fit yields an $R^2 = 0.984$ with the equation:

$$S_V^{fit} = \left(2.82 \cdot 10^{-10} \frac{\text{V}^2}{\text{W Hz}} \right) P_{ph} + \left(2.80 \cdot 10^{-14} \frac{\text{V}^2}{\text{Hz}} \right). \quad (2.24)$$

Once again, the y-intercept aligns with the measured electronic dark noise floor. Most importantly, the measured slope matches the theoretical prediction with a relative error of just 2.9%. Therefore, despite the degraded AC performance of the extended InGaAs photodiodes, the photodetector works, providing a clean 100 kHz analysis window centred at 150 kHz.

This result definitively proves that the custom-designed homodyne photodetector successfully resolves optical shot noise from a $\lambda = 2090$ nm source at incident powers ranging from 100 μ W to 1 mW.

The clearance of the shot noise spectral density from the photodetector noise for each power value is described in Table 2.2.

Optical Power (μ W)	Mean PSD (V^2/Hz)	Linear Margin	Clearance (dB)
310.00	1.1941×10^{-13}	6.5390	8.16
420.00	1.4450×10^{-13}	7.9129	8.98
560.00	1.8979×10^{-13}	10.3931	10.17
720.00	2.2834×10^{-13}	12.5042	10.97
860.00	2.7540×10^{-13}	15.0811	11.78
1030.00	3.1749×10^{-13}	17.3862	12.40
1120.00	3.1935×10^{-13}	17.4880	12.43
1290.00	4.1227×10^{-13}	22.5766	13.54
Dark (Reference)	1.8261×10^{-14}	1.0000	0.00

Table 2.2: Mean and Margin Analysis of the shot noise at $\lambda = 2090$ nm in the $[100kHz - 200kHz]$ frequency window.

The highest measured clearance is 13.54 dB and it is obviously obtained for the highest power measurement, $\bar{P}_{ph}^{max} = 1290 \mu$ W. Even the reading with an optical power on the photodiode of $\bar{P}_{ph} = 310 \mu$ W had an acceptable clearance of 8.16 dB. Since one power is more than for time the other thus a factor over 6 dB and we still measured correctly both shot noises spectral densities we can say that the photodetector is capable of detecting up to 6 dB of squeezing given a local oscillator, $\bar{P}_{lo}$, around $\bar{P}_{lo} \sim 2 \bar{P}_{ph}^{max} \sim 2.6$ mW.

3 | Squeezed state detection setup

The development of a homodyne photodetector with sufficient sensitivity to measure shot noise in the 1 mW local oscillator power regime was a foundational component of a broader research initiative. As previously noted, the photodetector was designed to detect the injection of squeezed vacuum state within a balanced homodyne configuration. This is necessary because the ultimate objective of the project is to generate squeezed vacuum at a wavelength of 2 μm using an approach that bypasses the need for a mode cleaner or active cavity stabilisation. During the design and development of the photodetector, parallel efforts were dedicated to constructing the experimental setup required for both the generation and subsequent detection of these squeezed states.

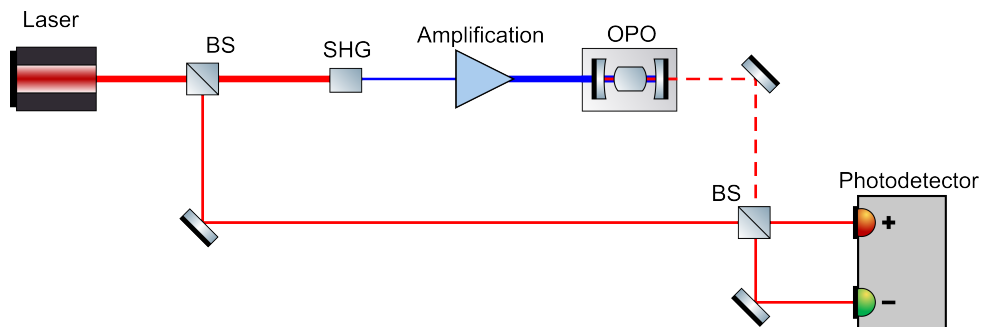


Figure 3.1: Conceptual scheme of the squeezed states experimental setup.

Figure 3.1 illustrates the primary building blocks of the detection setup. The red line represents the 2090 nm laser beam, while the blue line indicates the second harmonic at 1045 nm. This second harmonic is produced in the Second Harmonic Generation (SHG) conceptual block, amplified by an amplification stage, and subsequently injected into a below-threshold Optical Parametric Oscillator (OPO). The OPO is the component responsible for generating the vacuum squeezed state at 2090 nm, depicted by the dashed red line. A fraction of the primary laser source's power is split off to serve as the local oscillator for the balanced homodyne detection, where the injected signal is the vacuum squeezed state.

This chapter mirrors the structure of the block diagram, detailing and characterising each

conceptual block. Section 3.1 characterises the initial laser source. Section 3.2 describes the setup implemented for second harmonic generation, followed by an overview of the system used to amplify this second harmonic signal in Section 3.3. The fourth principal element, the OPO, is currently under development and thus falls outside the scope of this thesis. Finally, while the block diagram includes the homodyne photodetector, this topic was extensively analysed in the preceding chapter and will not be reiterated here.

3.1. Laser source

The laser source employed in the final squeezed state generation and detection setup is the same one used for the $2\mu\text{m}$ photodetector testing. It is a Tm-Ho:YAG laser pumped at 790 nm, featuring a central emission wavelength of 2093 nm with a tunability range of approximately ± 10 nm. The output beam is linearly polarised in the vertical direction.

The relative intensity noise (RIN) and the root-mean-square (RMS) relative noise of this laser were previously presented in Figure 2.10.

The relationship between the pumping diode current and the output optical power of the laser was characterised and is plotted in Figure 3.2.

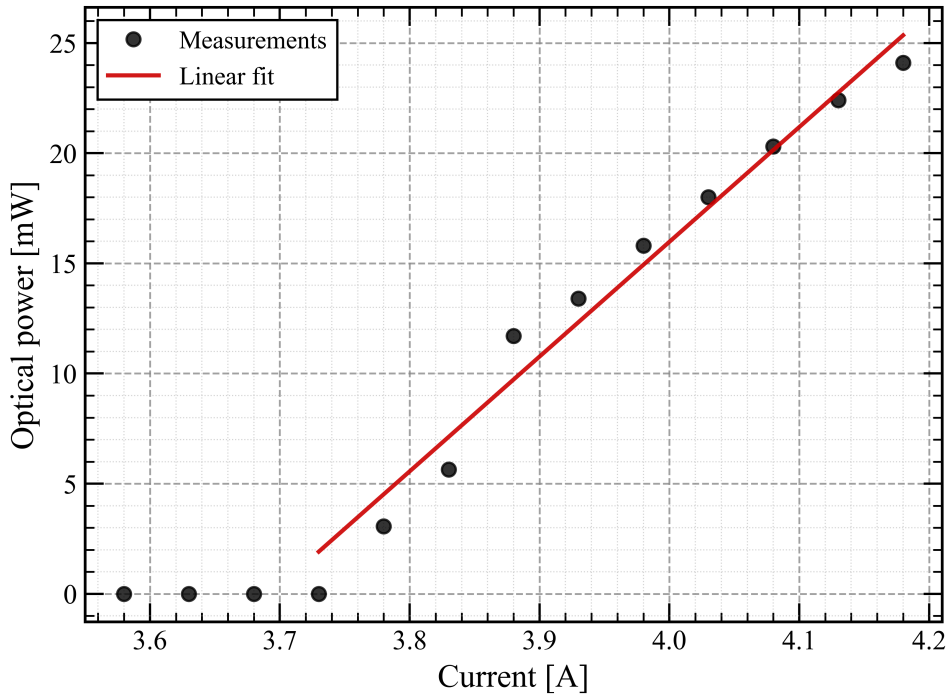


Figure 3.2: Input-output characterization of the Tm-Ho:YAG laser.

The measured data points for the above-threshold regime of the laser were fitted using a

linear function, denoted by the red line. The coefficient of determination is $R^2 = 0.971$, and the resulting fit equation is:

$$P_{out} = (5.21 \cdot 10^{-2} \text{ W/A}) I_{in} - (0.19 \text{ W}). \quad (3.1)$$

3.2. Second Harmonic Generation

The Tm:Ho:YAG laser beam required precise focusing onto a non-linear crystal to efficiently generate the second harmonic beam. Figure 3.3 illustrates the optical setup designed for this purpose.

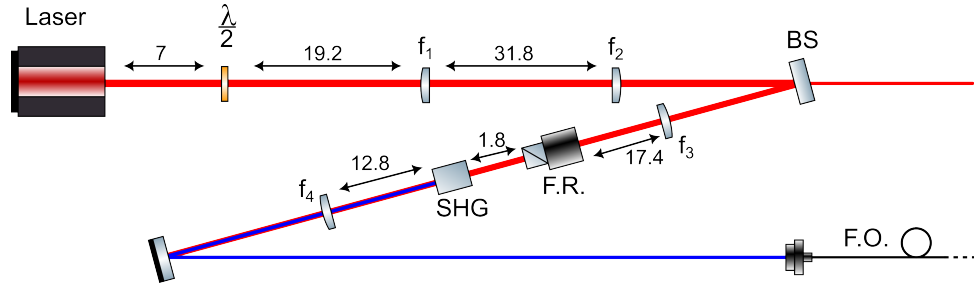


Figure 3.3: SHG optical setup. All lengths are expressed in cm.

All integrated lenses featured anti-reflection coatings to minimise back-reflections and maximise power transmission. The laser beam was collimated using two lenses with focal lengths $f_1 = 50 \text{ mm}$ and $f_2 = 250 \text{ mm}$.

The resulting beam waist was measured applying the knife-edge technique. Figure 3.4 displays the acquired data alongside the estimated Gaussian beam profile. The measured beam waist was $w_0 = 3.62 \text{ mm}$.

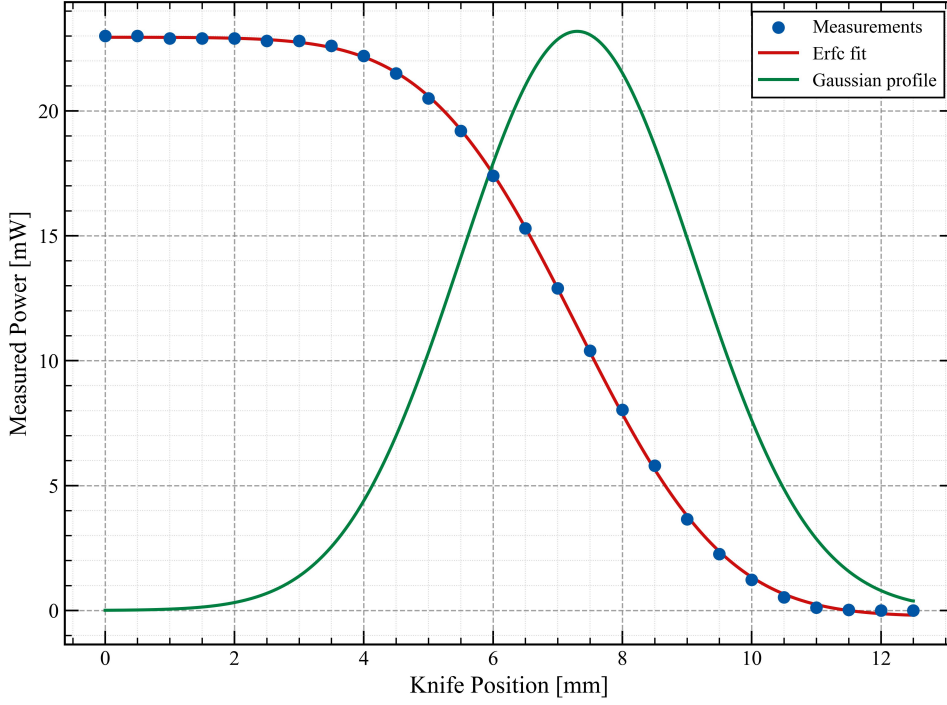


Figure 3.4: Knife edge measurement for beam waist estimation.

The collimated laser subsequently impinged upon a beam splitter, which transmitted 20% of the optical power and reflected the remaining 80%. The laser typically operated near its maximum capacity, delivering an output power of approximately 24 mW. The transmitted beam, holding roughly 4.8 mW, serves as the local oscillator for the balanced homodyne detection. The reflected beam, with approximately 19.2 mW of power, is focused by a third lens with focal length f_3 onto the second harmonic crystal.

The chosen medium is a Periodically Poled Lithium Niobate (PPLN) crystal with a length of 10 mm. To ensure precise alignment, the crystal was mounted on a three-axis positioning stage. To optimise the second harmonic output power, the focusing setup was designed to approach the condition $2z_R \approx L$, where z_R is the Rayleigh length of the focused beam and L is the crystal length. Given the previously calculated beam waist and the radiation wavelength, the focal length f_3 was selected to yield a z_R as close to 5 mm as possible. The most suitable available lens had a focal length of $f_3 = 250$ mm, resulting in $z_R \approx 3.2$ mm.

To finely tune the periodic poling of the non-linear crystal to the exact wavelength of the pump laser, phase matching was achieved by heating the crystal in an oven. The temperature was systematically adjusted to maximise the output power at 1045 nm, with the optimal temperature ultimately determined to be 80 °C.

Prior to entering the non-linear crystal, the focused beam passed through a Faraday

rotator (labelled F.R. in Figure 3.3). Although the lenses were anti-reflection coated, the non-linear crystal itself introduced sufficient back-reflection to destabilise the Tm:Ho:YAG laser, causing random mode-hopping. The combination of the Faraday rotator and a $\lambda/2$ waveplate formed an optical isolator that effectively shielded the laser source from these reflections. The specific placement of this isolator, after the beam splitter, was dictated by power considerations. Since the rotator introduces intrinsic power losses, positioning it here ensured that only the reflected beam was attenuated, preserving the full 20% transmitted power intended for the local oscillator. This is critical, as the local oscillator signal dictates the amplification of the squeezed noise in homodyne detection and must therefore be maximised.

The input-output relationship of the second harmonic generation process was subsequently characterised. Modulating the input optical power by altering the pumping diode current was avoided, as this could induce laser mode-hopping. Instead, the linear polarisation of the laser beam was exploited. Due to the structure of the periodically poled crystal, only the vertically polarised component of the 2090 nm beam contributes to pumping the second harmonic at 1045 nm. Consequently, rotating the $\lambda/2$ waveplate by an angle α rotates the polarisation by $\theta = 2\alpha$, allowing precise control over the vertically polarised power incident on the crystal.

According to Malus's law, the vertically polarised fundamental power is given by $P_{fund} = P_{2\mu m} \cos^2(\theta)$. Since, in low-conversion approximation, the power of the generated second harmonic scales quadratically with the pump power [24], the expected relationship is $P_{1\mu m} \propto P_{fund}^2 \propto \cos^4(\theta)$. The experimental data, plotted in Figure 3.5, confirmed this theoretical model exceptionally well.

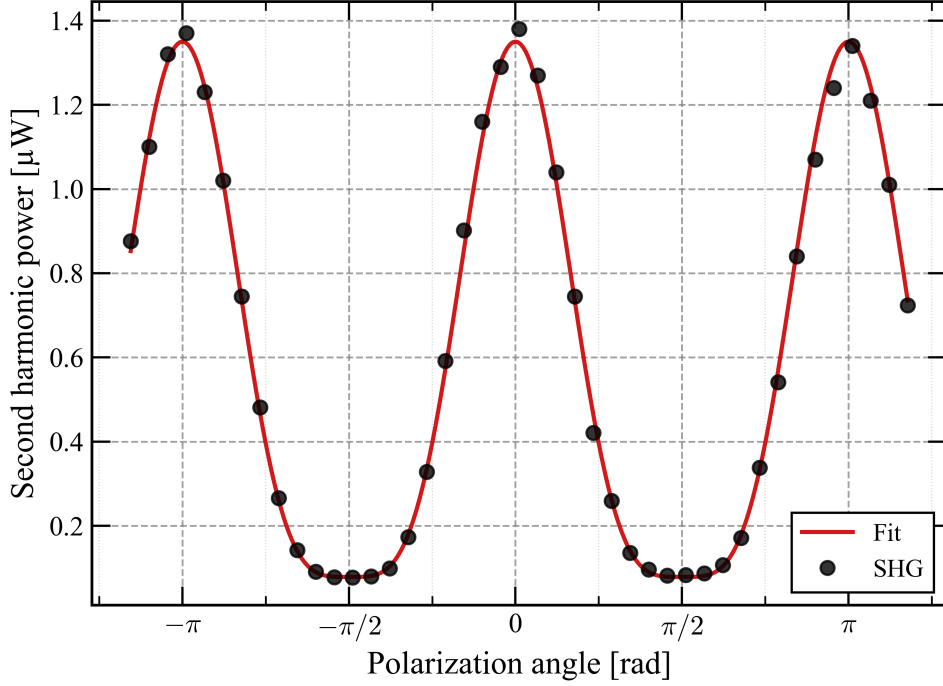


Figure 3.5: Measured second harmonic power as a function of the polarization angle of the 2 μm laser field.

The measured power data points were fitted using the function:

$$P_{1\mu\text{m}}^{\text{fit}} = A \cos^4(\theta) + B \quad (3.2)$$

where A and B are constants; B accounts for a minor non-zero baseline reading of the power meter (the estimated value of B was just 79 nW). The resulting coefficient of determination was $R^2 = 0.998$, validating the expected $\cos^4(\theta)$ proportionality.

Following this, the maximum power incident on the non-linear crystal was measured to be $P_{2\mu\text{m}} = 17.6 \text{ mW}$. Using Malus's law, the vertically polarised fundamental power $P_{\text{fund}}(\theta)$ was estimated for each angle θ . These values were then used to plot the second harmonic power as a direct function of the fundamental pump power, as shown in Figure 3.6.

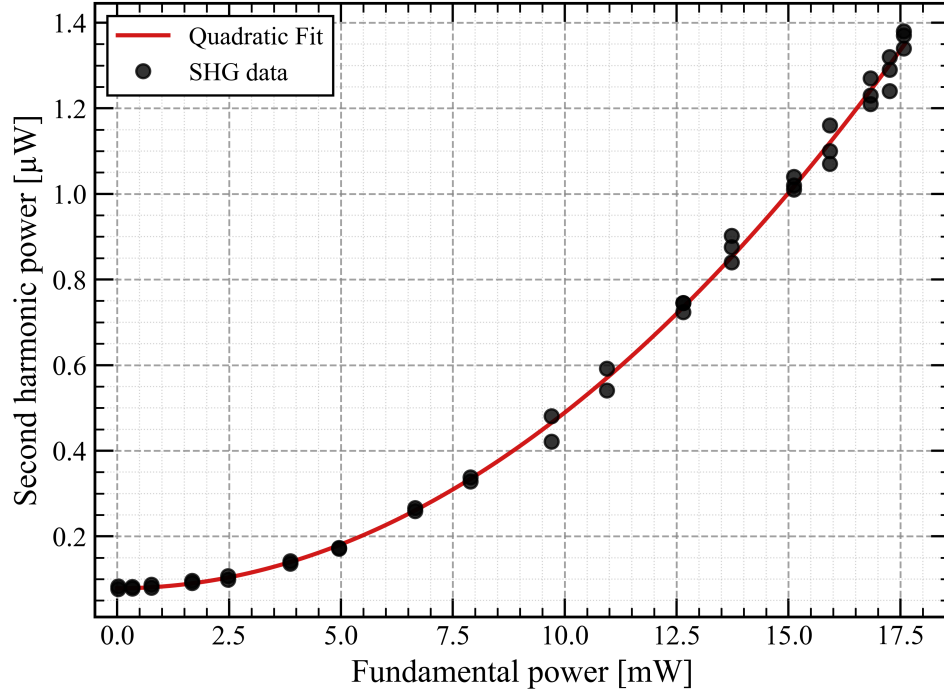


Figure 3.6: Second harmonic power as a function of the fundamental power.

The measured data points align almost perfectly along a quadratic fit curve. The governing equation is:

$$P_{SH}^{fit} = (4.1 \cdot 10^{-3} \text{ W}^{-1}) P_{fundamental}^2 + (7.9 \cdot 10^{-8} \text{ W}) \quad (3.3)$$

where the power offset remains consistent with the previous fit. The R^2 coefficient of 0.998 definitively confirms the quadratic dependence of the second harmonic output on the fundamental pump power.

The second harmonic signal at 1045 nm was then collimated using a lens with a focal length of $f_4 = 150$ mm. The intensity profile of the collimated beam was captured using a Spiricon SP620U camera. Figure 3.7 displays the acquired beam profile.

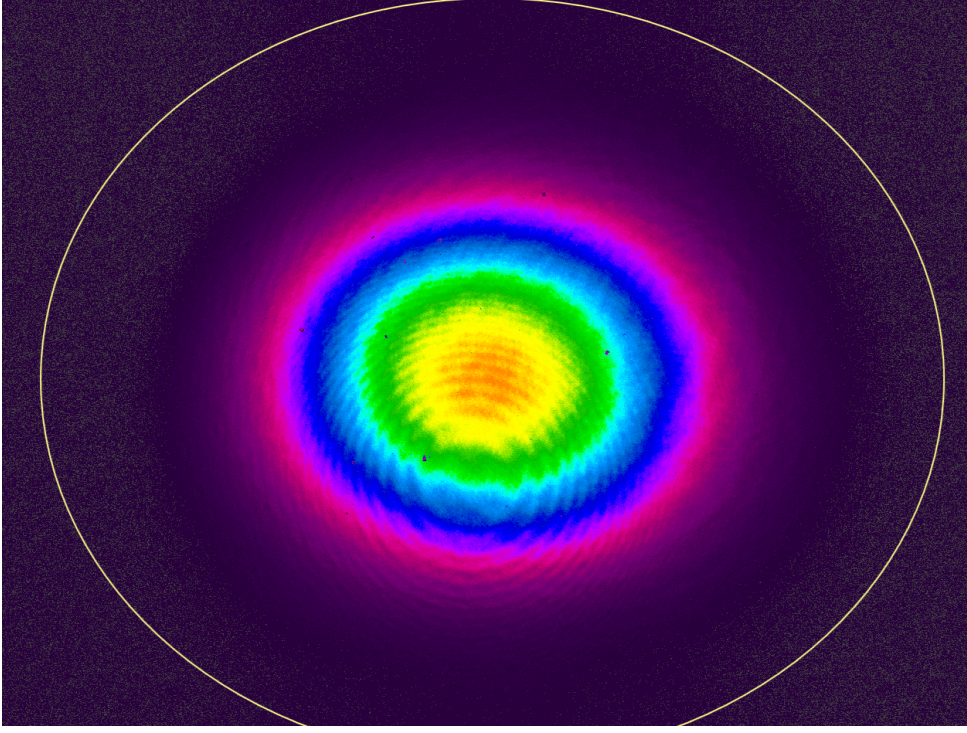


Figure 3.7: Beam intensity profile of the second harmonic beam.

The measured beam diameters using the $D4\sigma$ method were $D4\sigma_x = 3.2$ mm in the horizontal direction and $D4\sigma_y = 2.7$ mm in the vertical direction.

3.3. Amplification

As evident from the plot in Figure 3.6, the maximum obtained second harmonic power was a mere $1.4 \mu\text{W}$. This power level is entirely insufficient to induce any non-linear effects within the OPO. Therefore, substantial amplification was required before the signal could be utilised for squeezed state generation. To achieve this, the second harmonic signal was coupled into an optical fibre using a collimator and fed through a sequence of four fibre-optic amplifiers.

The amplifiers employed were optically pumped ytterbium (Yb) doped fibres.

The first three Yb-doped stages were pumped by three independent II-VI CM97-1010-76 diode lasers, each capable of delivering up to 970 mW of output optical power at a central wavelength of 976.2 nm. In each of these amplification stages, a single integrated module (Wavelength Electronics LDTC 2/2E) was used to simultaneously drive the pump diode and stabilize its temperature via Peltier elements.

The fourth and final amplification stage also utilised a Yb-doped fibre but was pumped

by a high-power nLIGHT-element-e03 diode laser. This diode laser featured a central emission wavelength of 976 nm and could provide up to 15 W of optical power. For this stage, the pump current was adjusted using a Wavelength Electronics PLD12.5K-CH laser diode driver, and the thermal load was managed by Peltier modules paired with a Thorlabs TED350 temperature controller.

To minimise Amplified Spontaneous Emission (ASE) noise, all amplifiers were configured with a counter-propagating pump geometry. Particularly in the first amplification stage, where the seed power was extremely low, this configuration was vital to maximise the gain of the seed signal while suppressing the generation of spontaneous emission. To prevent optical feedback and back-reflections that could destabilise the active medium's gain across the amplification stages, optical isolators (model IO2.5-1064-VLP) were installed between the output collimator of one stage and the input fibre of the next.

At the interface between the first and second amplification stages, an optical filter was introduced to attenuate the ASE noise from the first amplifier, ensuring that primarily the 1045 nm signal seeded the second amplifier. The chosen component was a Semrock TLP01-1116, a tuneable long-pass filter whose transmission profile depends on the angle of incidence. The transmission spectrum at a 45° angle of incidence is presented in Figure 3.8.

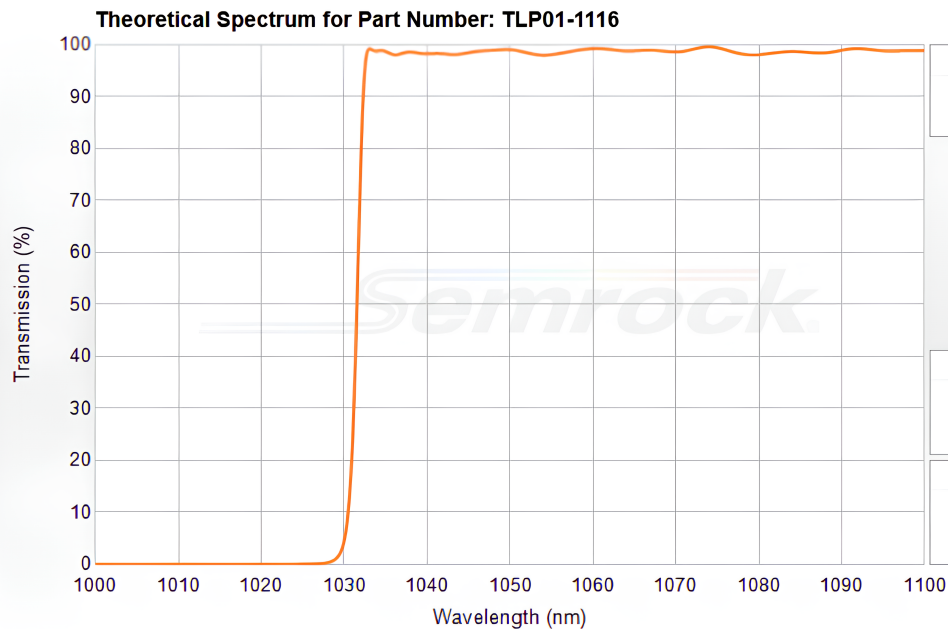


Figure 3.8: Transmission at an incidence angle of 45° for the Semrock TLP01-1116. Image taken from [15]-

This component enabled the effective rejection of most ASE noise, providing a significantly

cleaner seed for the subsequent amplification stage.

Figure 3.9 shows the optical output power (P_{out}) at the end of each amplification stage as a function of the monitor voltage (V_{mon}^i , $i = 1, 2, 3$) of the pumping diode driver for the respective amplification module in the chain. In the top panel, the optical power was measured at the output of the first amplification stage, yielding $P_{out}(V_{mon}^1)$. In the middle panel, the first amplification stage was set to a monitor voltage of $V_{mon}^1 = 0.4$ V, and the optical power at the end of the second module is plotted as a function of V_{mon}^2 . Finally, the bottom panel shows the power at the end of the third stage as a function of V_{mon}^3 . For this measurement, the monitor voltages of the two previous pumping diode drivers were fixed at $V_{mon}^1 = V_{mon}^2 = 0.4$ V. Since all three of these amplification stages shared the same type of pumping diode driver, the relationship between the pumping current sent to the diode and the monitor voltage was:

$$I_{LD} = (1.25 \text{ A/V}) \cdot V_{mon}. \quad (3.4)$$

For each acquisition, the monitor voltages of the previous modules were fixed to the values that maximized the optical gain while keeping ASE noise and power oscillations under control.

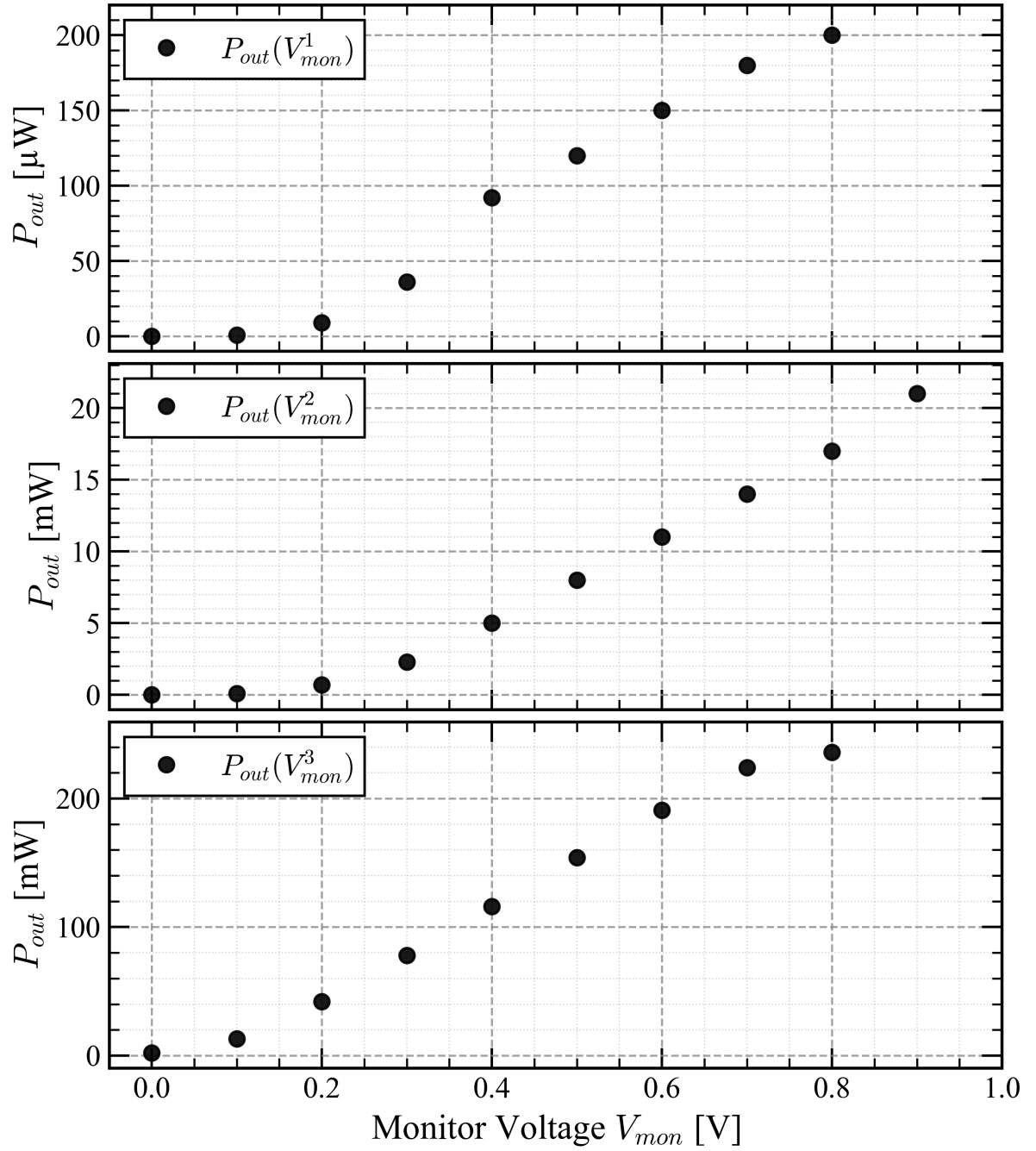


Figure 3.9: Output power as a function of V_{mon} for the first three amplification stages.

As previously mentioned, the fourth stage utilized a different pumping diode and a different diode driver. Its diode current versus monitor voltage relationship was:

$$I_{LD} = (5 \text{ A/V}) \cdot V_{mon}. \quad (3.5)$$

With the monitor voltages of the first three diode laser drivers fixed at $V_{mon}^1 = 0.4\text{ V}$, $V_{mon}^2 = 0.4\text{ V}$, and $V_{mon}^3 = 0.5\text{ V}$, the final 1045 nm output power was measured at the end of the fourth amplification stage. The acquired data are plotted in Figure 3.10 as a function of the last diode laser monitor voltage, V_{mon}^4 .

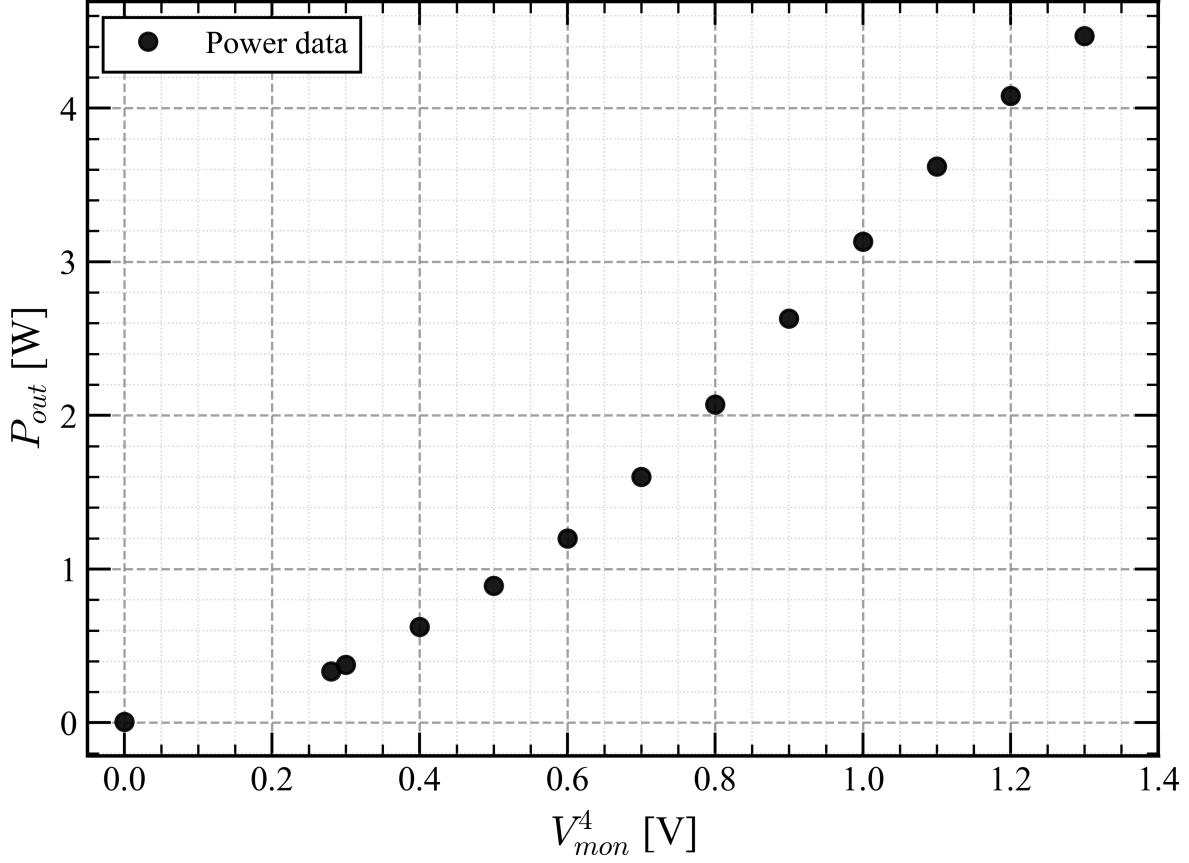


Figure 3.10: Output power as a function of V_{mon}^4 for the last amplification stage.

As shown, the maximum achieved output power for the 1045 nm optical signal was approximately $P_{out} \approx 4.5\text{ W}$. This power level is considered fully sufficient to drive the below-threshold OPO and successfully generate vacuum squeezed states.

4 | Conclusions and future developments

This thesis presented the development and characterization of a highly sensitive balanced homodyne photodetector, alongside the realization of the optical setup required for the generation of squeezed vacuum states at a wavelength of $2\mu\text{m}$. The primary objective of this research was to establish a complete experimental framework capable of producing and detecting these non-classical states of light, pushing towards the sensitivity enhancement of next-generation interferometric gravitational wave detectors.

Main Results

The core achievement of this work was the successful realization and characterization of a low-noise balanced homodyne photodetector. The critical technical challenge was achieving a sufficient clearance to elevate the minuscule optical shot noise above the dominating electronic dark noise floor, particularly in the 1 mW local oscillator power regime. Through a custom transimpedance amplifier (TIA) architecture distributed over two isolated boards, and the integration of a commercial low-noise amplifier as a secondary amplification stage, the required performance was successfully met.

The photodetector was thoroughly tested using a $1\mu\text{m}$ (Nd:YAG) laser source and subsequently characterized using a $2\mu\text{m}$ (Tm:Ho:YAG) one. Operating at $2\mu\text{m}$ with extended InGaAs photodiodes, the detector successfully resolved the optical shot noise in a clean frequency window between 100 kHz and 200 kHz . The measurements exhibited a striking linear dependence of the output voltage variance on the incident optical power ($R^2 = 0.984$), with an experimental slope matching the theoretical prediction for shot noise within a 2.9% relative error. Crucially, the photodetector achieved a clearance of 13.54 dB above the electronic dark noise at a local oscillator power of roughly 2.6 mW . The acquired data confirmed that the developed device is fully capable of detecting up to 6 dB of optical squeezing, fulfilling its primary design objective.

In parallel, most of the optical setup for squeezed state generation was successfully constructed and validated. We demonstrated efficient Second Harmonic Generation (SHG) using a periodically poled crystal, rigorously verifying the quadratic dependence of the 1045 nm output power on the fundamental 2090 nm pump power. To scale the inherently low power generated by the SHG process ($1.4\text{ }\mu\text{W}$) up to levels suitable for parametric down-conversion, a robust four-stage Yb-doped fiber amplification chain was implemented. This system reliably delivered up to 4.5 W of power at 1045 nm, providing an ideal pump field for the downstream Optical Parametric Oscillator (OPO).

Future developments

With both the sensitive homodyne photodetector and the high-power second-harmonic pump field operational, the experimental setup is now ready for the final phase of the project: the integration of the below-threshold OPO.

Immediate future work will focus on injecting the 4.5 W, 1045 nm pump beam into the OPO cavity to drive the parametric down-conversion process. Once the OPO is properly aligned and thermally stabilized, the generated 2090 nm squeezed vacuum will be routed to the signal port of the balanced homodyne detector. By locking the phase of the local oscillator to the squeezed quadrature, we anticipate the direct observation of a noise spectral density falling below the meticulously characterized shot noise limit presented in this thesis.

Concurrently, the development of a second version of the photodetector has been initiated. The printed circuit board has been fully designed, and assembly and testing are pending the arrival of the required components. To address one of the primary limitations of the first prototype, namely, the noise peak of the LT1028 operational amplifier located in the middle of the desired $f \in [100\text{ kHz}, 1\text{ MHz}]$ detection window, the new TIA stage is based on the OPA657. Furthermore, to eliminate the reliance on external equipment, a secondary amplification stage has been directly integrated into the onboard electronics. Finally, the bias voltage supply and power management circuitry have been consolidated, allowing the entire system to be integrated onto a single board powered solely by a $\pm 18\text{ V}$ source. This upgraded device is expected to resolve the main technical issues encountered with the initial prototype. The schematics for this upcoming photodetector are presented in Figure A.4 and Figure A.5.

The successful generation and detection of $2\text{ }\mu\text{m}$ squeezed states without the need for complex mode cleaners or active cavity stabilization will directly contribute to the scalability and robustness of squeezing-enhanced interferometry.

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A | Electronic Schematics

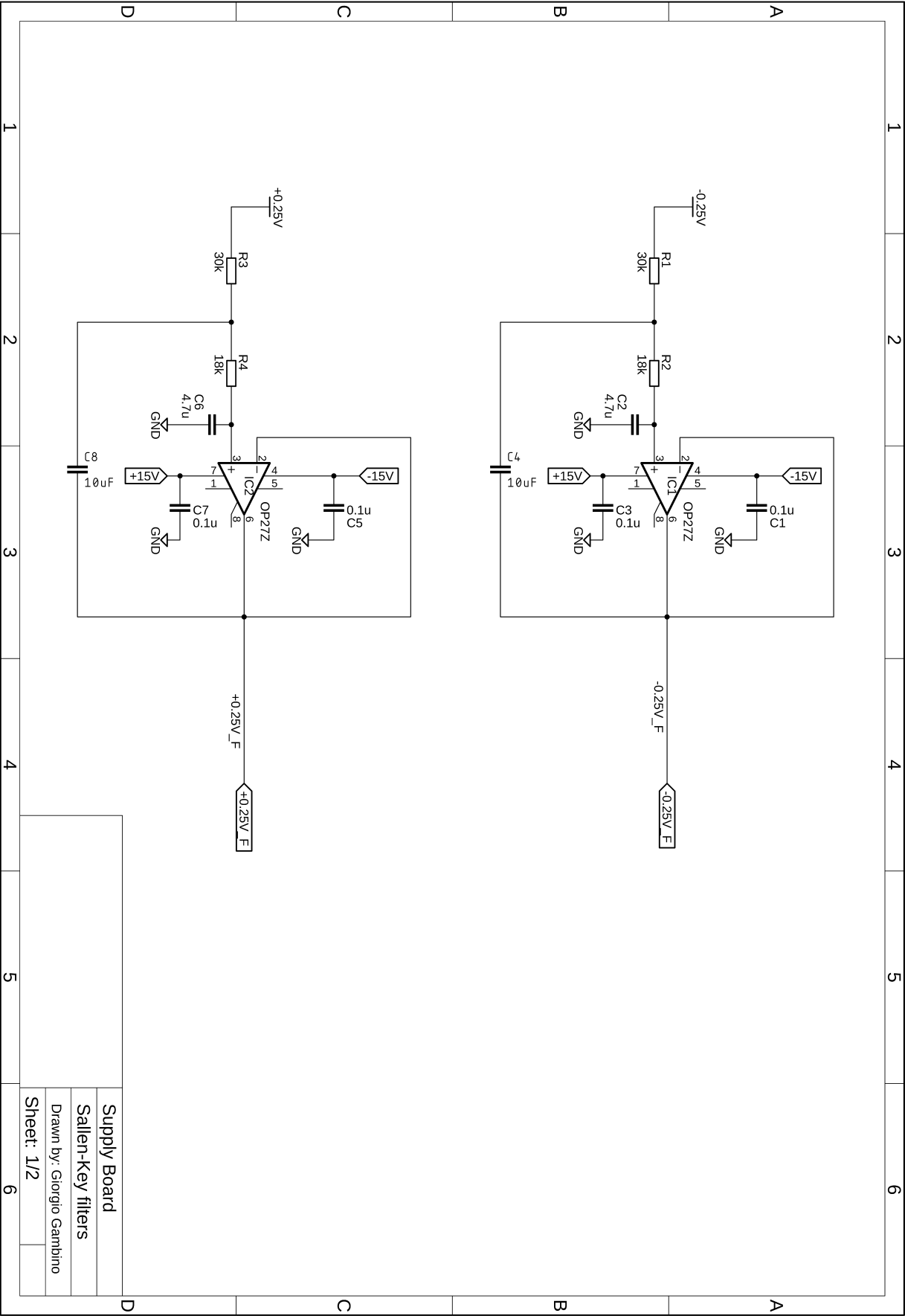


Figure A.1: Supply board Sallen-Key filters.

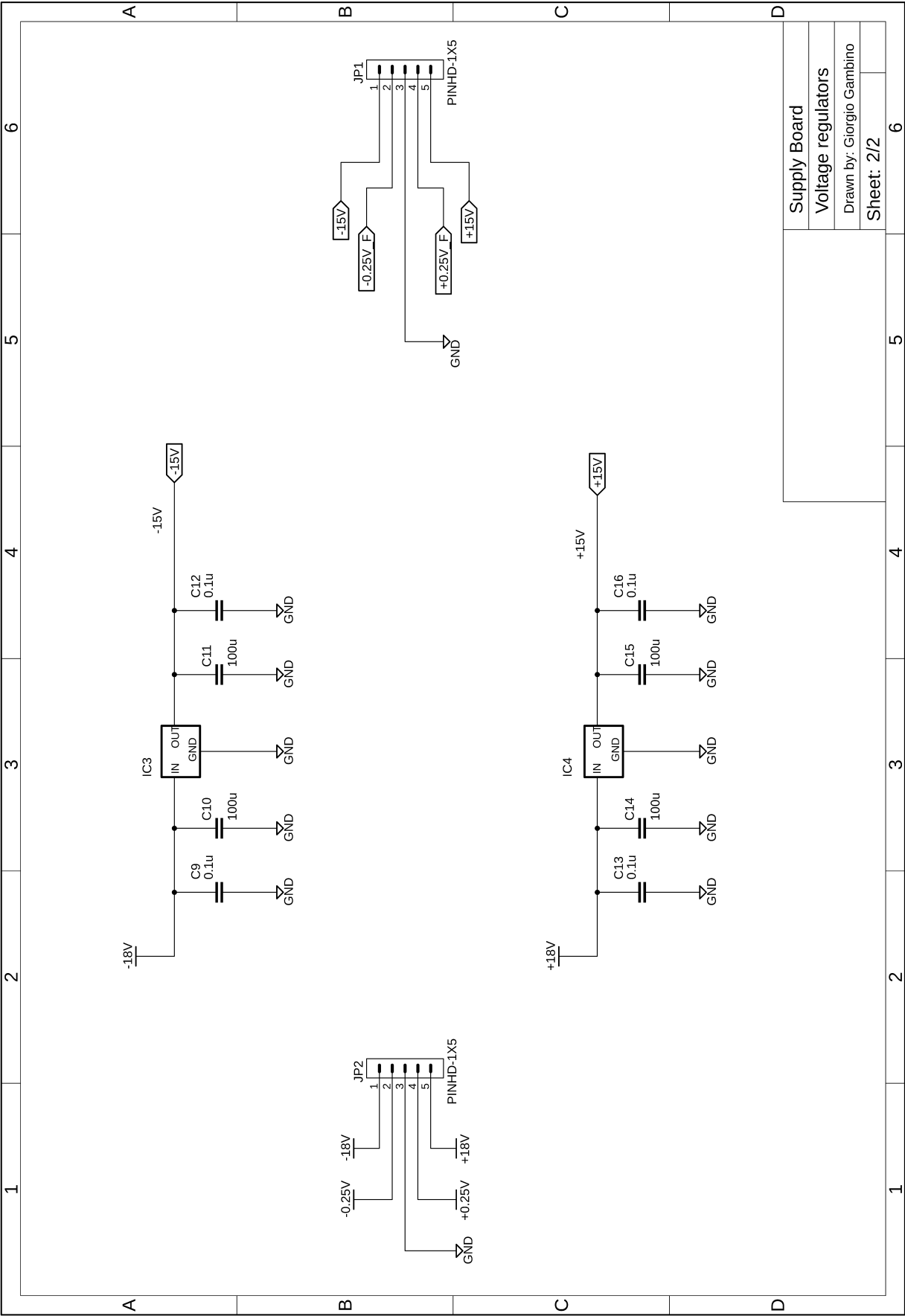


Figure A.2: Supply board voltage regulators.

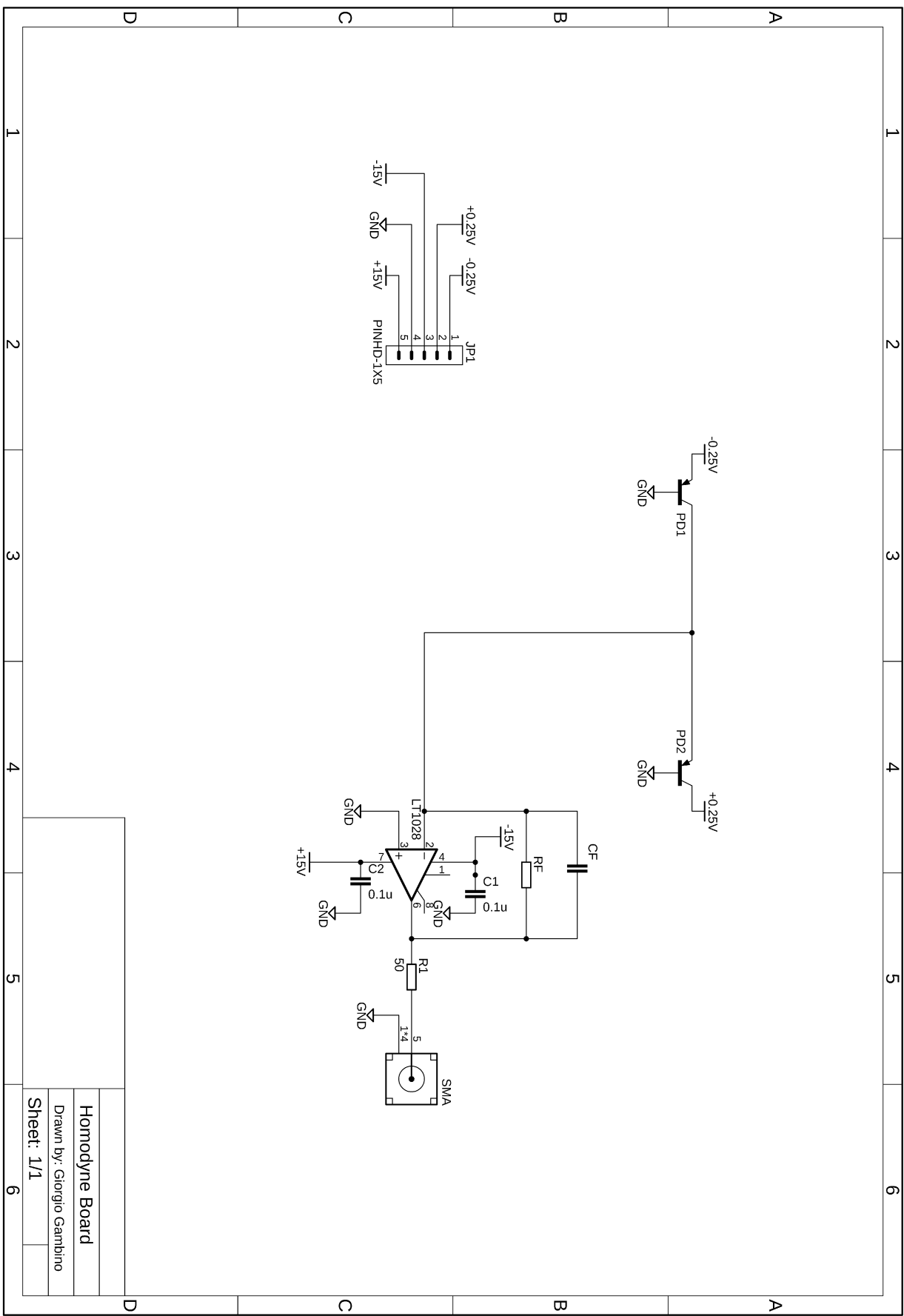


Figure A.3: Homodyne board.

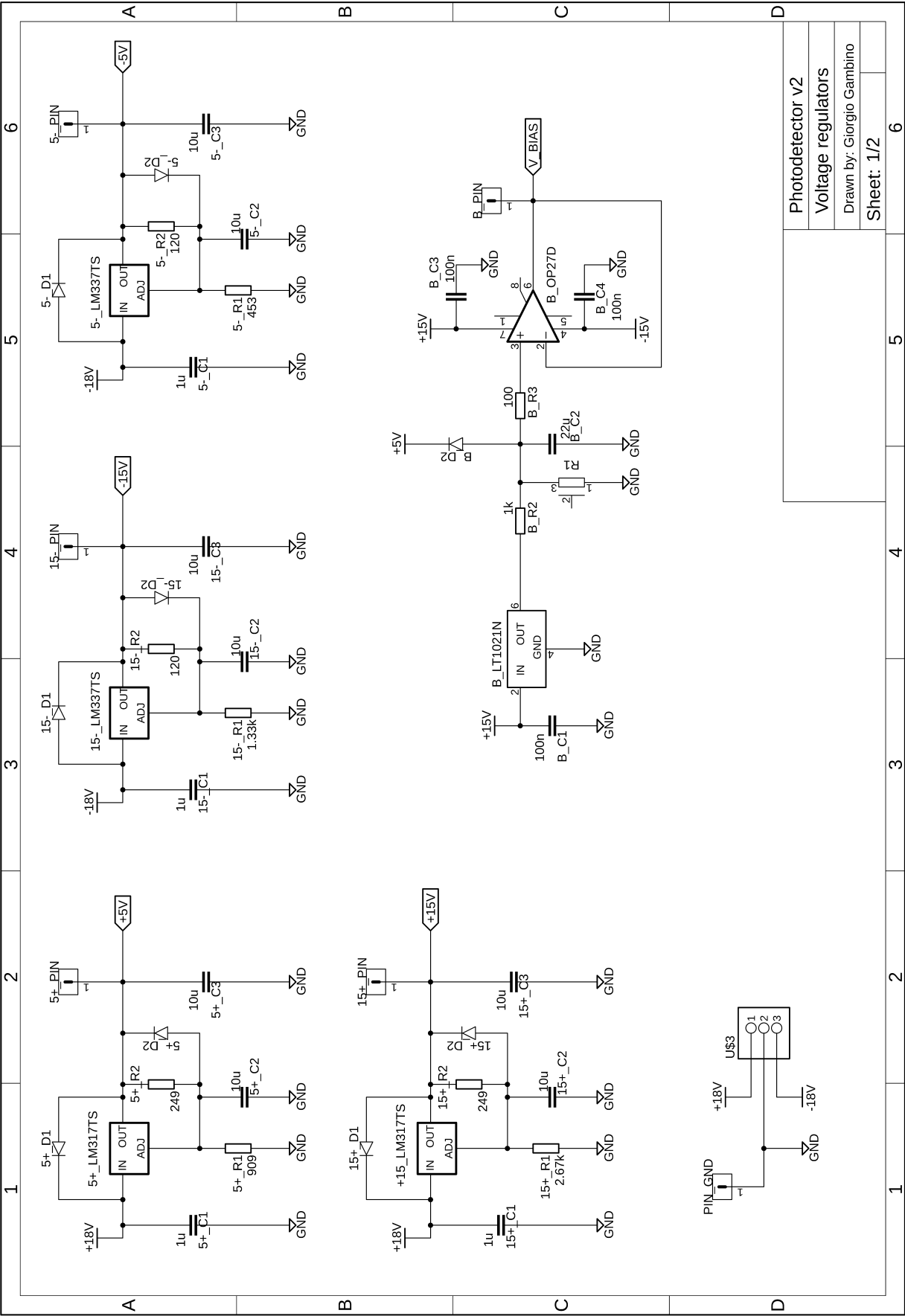


Figure A.4: Second version of the photodetector supply part.

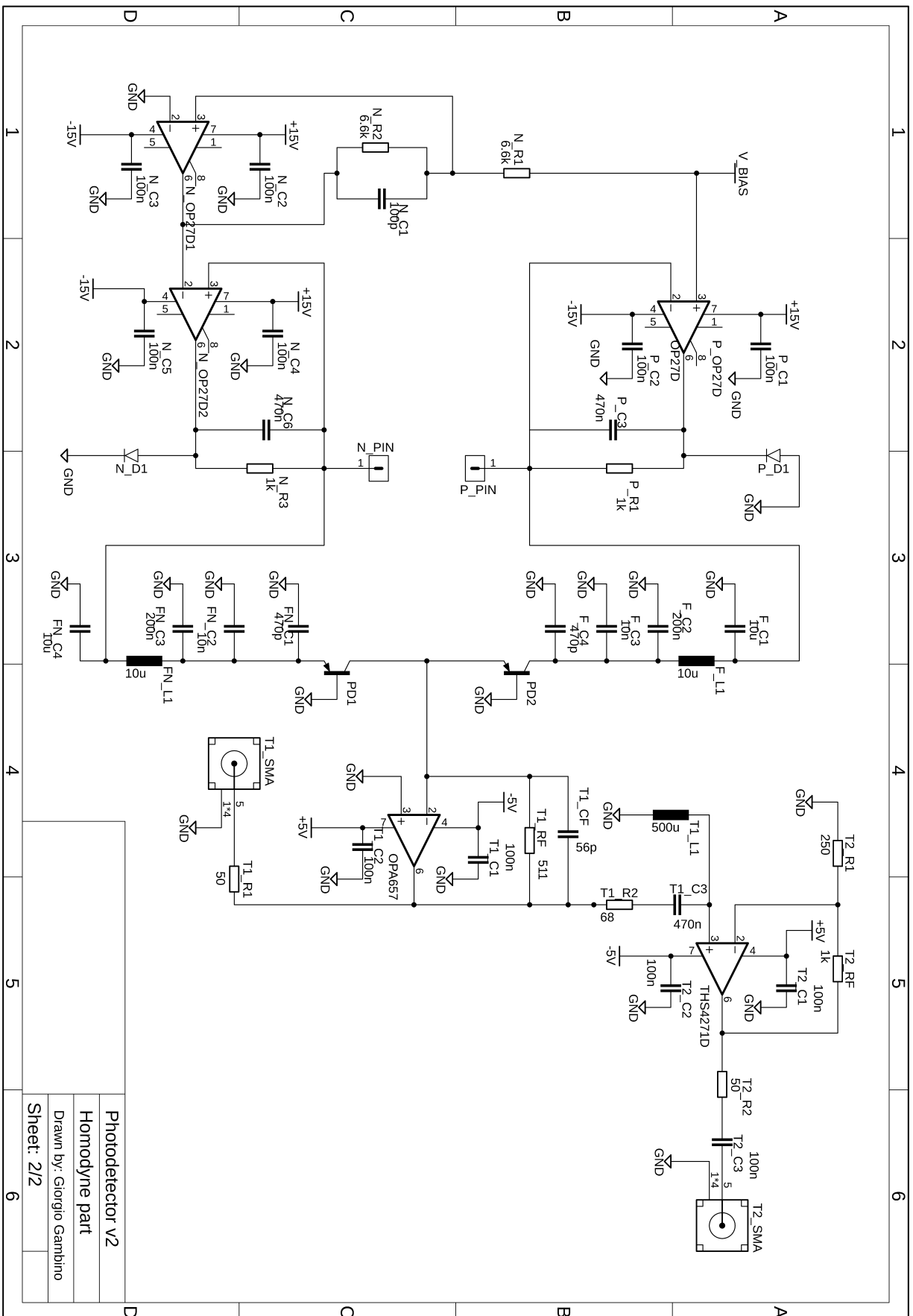


Figure A.5: Second version of the photodetector TIA part.

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List of Symbols

Variable	Description	SI unit
$\vec{A}$	vector potential	V·s/m
$\vec{B}$	magnetic field	T
c	speed of light in vacuum	m/s
$\vec{E}$	classical electric field	V/m
H	radiant energy / classical Hamiltonian	J
$\vec{k}_l$	wave vector of the l -th mode	m ⁻¹
L	non-linear crystal length	m
P_{ph}	optical power	W
t	time	s
V	quantization volume	m ³
w_0	Gaussian beam waist	m
z_R	Rayleigh length	m
α_l	complex normal mode coordinate	-
ε_0	vacuum permittivity	F/m
θ	phase angle / polarization angle	rad
λ	optical wavelength	m
ω_l	angular frequency of the l -th mode	rad/s
$\hat{a}_l, \hat{a}_l^\dagger$	annihilation and creation operators	-
$\hat{E}_l$	single-mode electric field operator	V/m
$\hat{E}_{P_l}, \hat{E}_{Q_l}$	electric field quadrature operators	V/m
$\hat{H}$	quantum Hamiltonian operator	J
$\hat{n}$	photon number operator	-
$\hbar$	reduced Planck constant	J·s
R	squeezing parameter	-

Variable	Description	SI unit
A_d	differential gain	V/V
A_{cm}	common mode gain	V/V
C_f	feedback capacitance	F
C_j	photodiode junction capacitance	F
e	elementary charge	C
f	frequency	Hz
G_{id}	ideal transimpedance gain	Ω (or V/A)
G_{ph}	effective measured transimpedance gain	V/W
i, i_{ph}	photocurrent	A
R_f	feedback resistance	Ω
R_{sh}	photodiode shunt resistance	Ω
R_λ	photodiode responsivity	A/W
$S_{i_{shot}}$	shot noise current spectral density	A ² /Hz
$S_{V_{electr}}$	electronic voltage noise spectral density	V ² /Hz
$S_{V_{shot}}$	shot noise voltage spectral density	V ² /Hz
V_{out}	output voltage	V

