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Operation and Control Design of a Modular Multilevel Converter with Integrated Distributed Energy Resources under Unbalanced Grid Conditions

TESI DI LAUREA MAGISTRALE IN
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Author: **Gabriele Liscacci**

Student ID: 247469

Advisor: Prof. Marzio Barresi

Co-advisors: Simone Barcellona

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Abstract

The increasing penetration of renewable energy sources is reshaping modern power systems and accelerating the transition toward converter-dominated electrical grids. In this context, modular multilevel converters (MMCs) are attracting growing interest due to their modularity, scalability, and high-quality voltage synthesis. At the same time, the integration of distributed photovoltaic (PV) sources at submodule level enables advanced operating strategies, including distributed maximum power point tracking (DMPPT), but introduces additional control challenges. In particular, non-uniform irradiance conditions may lead to unequal power availability among phase-legs and between the upper and lower arms of the same leg, while grid-voltage unbalance imposes further constraints on current quality and power regulation, making the coordinated control of MMC-based PV systems significantly complex.

This thesis addresses the control design of a three-phase MMC-based PV system with submodule-integrated distributed energy sources operating under unbalanced grid conditions and internal available power mismatch. The main objective is to develop a control architecture capable of simultaneously: i) ensuring grid-compliant active and reactive power exchange, ii) preserving controllability under negative-sequence grid voltages, and iii) enforcing the internal power redistribution required to satisfy the available arm powers associated with DMPPT operation. Particular attention is devoted to the coordinated generation of circulating currents, which are exploited as control variables for internal power balancing.

To achieve this goal, a multilayer control strategy is formulated and analytically derived. The first layer is a dual-sequence grid controller, which maps the active- and reactive-power references into positive- and negative-sequence current references in synchronous reference frames. Starting from a phasor-based description of grid voltages and currents, the complex-power expression is decomposed into constant and oscillatory terms, and a unified formulation is obtained through a selector parameter $K \in \{-1, 0, 1\}$. This parameter allows the controller to operate in three modes: suppression of active-power oscillations, balanced output currents, or suppression of imaginary-power oscillations. The derivation is then translated into explicit dq -reference equations suitable for implemen-

tation. Since negative-sequence injection affects the interpretation of reactive power, the thesis also clarifies the relationship between the instantaneous imaginary power and the physically meaningful total reactive-power quantity, providing a consistent conversion for practical reference assignment.

The second layer is an AC circulating-current controller designed to manage the vertical power mismatch within each phase-leg, namely the available power difference between upper and lower arms dictated by the DMPPT controller. This work employs three independent single-phase controllers (one per leg) to generate fundamental-frequency circulating-current references that enforce the required arm-to-arm power transfer in order to maintain the operating points dictated by the available arm powers.

The third layer is a DC circulating-current controller, which regulates horizontal power mismatch among phase-legs. This control function operates directly on the DC circulating-current components and generates inter-leg power transfer actions based on mismatch indicators derived from the phase available power distribution. In this way, the converter can simultaneously satisfy the internal available-power constraints and the phase-wise power split required by the grid-side operating point under unbalanced conditions. A final coordination stage, referred to as the arm-voltage reference generator, combines the outputs of the grid controller, AC circulating-current controller, and DC circulating-current controller to produce the upper- and lower-arm voltage references applied to the converter.

The proposed control strategy is validated through time-domain simulations. The results confirm the accuracy of the analytical formulation and the effectiveness of the hierarchical coordination. Under unbalanced grid conditions without internal mismatch, the three K -modes exhibit the expected trade-offs among active-power oscillations suppression, balanced output currents, and imaginary-power oscillations suppression. When internal power mismatch is introduced, the controller maintains proper grid-side power regulation while generating the AC and DC circulating-current components required to redistribute power vertically (between arms of the same leg) and horizontally (among different phase-legs). The simulated arm powers converge to the prescribed available-power references, demonstrating compatibility between grid-side objectives and DMPPT-driven internal constraints.

Overall, this thesis provides a coherent control framework for MMC-based PV systems operating in the simultaneous presence of grid unbalance and non-uniform internal power availability. The main contribution lies in the coordinated integration of dual-sequence grid control with phase-wise AC circulating-current regulation and inter-leg power-balancing

control through DC circulating currents, supported by explicit analytical derivations and simulation-based validation. The developed approach constitutes a solid basis for future extensions toward fully integrated DMPPT supervisory control and more detailed submodule-level modeling.

Keywords: Modular multilevel converter, photovoltaic systems, distributed MPPT, unbalanced grid conditions, circulating-current control.

Abstract in lingua italiana

La crescente penetrazione delle fonti di energia rinnovabile sta rimodellando i moderni sistemi elettrici e accelerando la transizione verso reti elettriche dominate da convertitori. In questo contesto, i convertitori modulari multilivello (MMC) stanno suscitando un interesse crescente grazie alla loro modularità, scalabilità e capacità di sintesi di tensione di elevata qualità. Allo stesso tempo, l'integrazione di sorgenti fotovoltaiche (PV) distribuite a livello di sottomodulo consente strategie operative avanzate, tra cui il distributed maximum power point tracking (DMPPT), ma introduce ulteriori sfide di controllo. In particolare, condizioni di irraggiamento non uniformi possono determinare una disponibilità di potenza differente tra i leg e tra il braccio superiore e quello inferiore dello stesso leg, mentre lo sbilanciamento della tensione di rete impone ulteriori vincoli sulla qualità della corrente e sulla regolazione della potenza, rendendo significativamente complesso il controllo coordinato dei sistemi PV basati su MMC.

Questa tesi affronta la progettazione del controllo di un sistema fotovoltaico trifase basato su MMC con sorgenti di energia distribuite integrate nei sottomoduli, operante in condizioni di rete sbilanciata e di disallineamento tra le potenze interne disponibili. L'obiettivo principale è sviluppare un'architettura di controllo capace di: i) garantire uno scambio di potenza attiva e reattiva conforme ai requisiti di rete, ii) preservare la controllabilità in presenza di tensioni di rete di sequenza negativa, e iii) imporre la ridistribuzione interna di potenza necessaria a soddisfare le potenze disponibili dei bracci associate al DMPPT. Particolare attenzione è dedicata alla generazione delle correnti circolanti, sfruttate come variabili di controllo per il bilanciamento interno della potenza.

Per raggiungere questo obiettivo, viene formulata e derivata analiticamente una strategia di controllo basata su più livelli. Il primo livello è un controllore di rete a doppia sequenza, che mappa i riferimenti di potenza attiva e reattiva in riferimenti di corrente di sequenza positiva e negativa. A partire da una descrizione fasoriale delle tensioni e delle correnti di rete, l'espressione della potenza complessa viene scomposta in termini costanti e oscillatori, e si ottiene una formulazione unificata mediante un parametro selettore $K \in \{-1, 0, 1\}$. Tale parametro consente al controllore di operare in tre modalità: soppressione delle oscillazioni di potenza attiva, correnti di uscita bilanciate oppure

soppressione delle oscillazioni della potenza immaginaria. La derivazione viene quindi tradotta in equazioni nel sistema di riferimento dq , pronte per l'implementazione. Poiché la presenza di correnti e tensioni di sequenza negativa influisce sull'interpretazione della potenza reattiva, la tesi chiarisce inoltre la relazione tra la potenza immaginaria istantanea e la grandezza fisicamente significativa della potenza reattiva totale, fornendo una conversione coerente per l'assegnazione pratica dei riferimenti.

Il secondo livello è un controllore delle correnti di circolazione AC progettato per gestire il disallineamento verticale di potenza all'interno di ciascun leg, ossia la differenza di potenza disponibile tra braccio superiore e inferiore imposta dal controllore DMPPT. In questo lavoro vengono impiegati tre controllori monofase indipendenti (uno per ciascun leg) per generare riferimenti di corrente di circolazione alla frequenza fondamentale che impongano il trasferimento di potenza richiesto tra i due bracci, al fine di mantenere i punti di funzionamento dettati dalle potenze disponibili dei bracci.

Il terzo livello è un controllore delle correnti di circolazione DC, che regola il disallineamento orizzontale di potenza tra i leg. Questa funzione di controllo agisce direttamente sulle componenti DC delle correnti di circolazione e genera azioni di trasferimento di potenza tra leg sulla base di indicatori di disallineamento derivati dalla distribuzione della potenza disponibile per fase. In questo modo, il convertitore può soddisfare simultaneamente i vincoli interni di potenza disponibile e la ripartizione di potenza per fase richiesta dal punto di funzionamento lato rete in condizioni sbilanciate. Uno stadio finale di coordinamento, indicato come generatore dei riferimenti di tensione dei bracci, combina le uscite del controllore di rete, del controllore delle correnti di circolazione AC e del controllore delle correnti di circolazione DC per produrre i riferimenti di tensione dei bracci superiori e dei bracci inferiori applicati al convertitore.

La strategia di controllo proposta è validata mediante simulazioni nel dominio del tempo. I risultati confermano l'accuratezza della formulazione analitica e l'efficacia del coordinamento gerarchico. In condizioni di rete sbilanciata senza disallineamento interno, le tre modalità K mostrano i compromessi attesi tra soppressione delle oscillazioni di potenza attiva, correnti di uscita bilanciate e soppressione delle oscillazioni della potenza immaginaria. Quando viene introdotto un disallineamento interno di potenza, il controllore mantiene una corretta regolazione della potenza lato rete generando al contempo le componenti AC e DC delle correnti di circolazione necessarie a ridistribuire la potenza verticalmente (tra i bracci dello stesso leg) e orizzontalmente (tra phase-leg differenti). Le potenze dei bracci simulate convergono ai riferimenti di potenza disponibile prescritti, dimostrando la compatibilità tra gli obiettivi lato rete e i vincoli interni imposti dal DMPPT.

Nel complesso, questa tesi fornisce un quadro di controllo coerente per sistemi fotovoltaici basati su MMC operanti in presenza simultanea di sbilanciamento della rete e di disponibilità di potenza interna non uniforme. Il contributo principale risiede nell'integrazione del controllo di rete a doppia sequenza con la regolazione delle correnti di circolazione AC e il controllo di circolazione DC finalizzato al bilanciamento della potenza tra leg, supportata infine da derivazioni analitiche e validazione tramite simulazione. L'approccio sviluppato costituisce una solida base per future estensioni che integrino anche il controllo DMPPT e una modellazione più dettagliata a livello di sottomodulo.

Parole chiave: Convertitore modulare multilivello, sistemi fotovoltaici, distributed MPPT, condizioni di rete sbilanciata, controllo delle correnti di circolazione.

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Introduction

Climate change is increasingly recognised as a systemic threat to ecosystems, public health, economic stability, and energy security. In response, international climate governance has set long-term objectives that translate into decarbonisation pathways for energy systems. A central milestone is the Paris Agreement adopted under the United Nations Framework Convention on Climate Change (UNFCCC), in which the Parties commit to keep the increase in global average temperature well below 2°C above pre-industrial levels, while pursuing efforts to limit warming to 1.5°C [1]. Although recent assessments indicate that avoiding an exceedance of this threshold has become increasingly unlikely and suggest that 2024 was the first year with a global mean temperature above 1.5°C [2, 3], urgent mitigation remains essential: sustained emissions reductions can still limit the magnitude and duration of overshoot and, consequently, reduce the risk of irreversible impacts [2]. Achieving these targets requires not only the rapid deployment of low-carbon generation but also a profound transformation of the whole grid.

On this side, global renewable capacity continues to expand at record pace. The International Renewable Energy Agency (IRENA) reports that renewable power capacity grew by about 585 GW in 2024, reaching approximately 4 448 GW worldwide, driven largely by solar photovoltaic and wind additions [4]. This growth is already translating into structural changes: for instance, the EU electricity transition reached a new milestone in 2025, with wind and solar together generating more electricity than fossil fuels, as reported in Ember’s *European Electricity Review 2026* [5]. These developments confirm that the energy transition is no longer a purely prospective scenario but an ongoing operational reality.

However, increasing penetration of converter-interfaced generation changes the physics and control requirements of power systems. Variable renewable energy sources such as wind and solar are inherently weather-dependent, introducing fluctuations and complicating the continuous balance between generation and demand. In addition, the replacement of conventional synchronous generation tends to reduce system inertia and short-circuit strength, making frequency and voltage dynamics more sensitive to disturbances and faults. As a consequence, grids are evolving: renewable plants are increasingly required

not only to inject active power, but also to provide voltage support, frequency response, fault ride-through capability, and power-quality functions.

In this context, power electronic converters become the key interface that transforms renewable generation and storage into controllable resources. However, their role is strongly dependent on the application. Wind power plants, for example, couple aerodynamics and mechanical dynamics with electrical conversion; therefore, converters must coordinate generator-side control with grid-side current control under rapidly changing operating conditions. Photovoltaic plants, instead, are intrinsically DC sources, meaning that the conversion chain must ensure DC-link regulation, implement maximum power extraction strategies, and deliver AC currents that satisfy the already mentioned grid's requirements. Across all these applications, the converter control system becomes central to stability, reliability, and compliance.

When high power and medium-to-high voltage levels are required, conventional two-level or three-level voltage source converters can face practical limitations due to device ratings and the need for bulky filtering and transformer solutions. Modular Multilevel Converters (MMCs) have emerged as a prominent alternative: since the early 2000s, they have progressively established themselves as a leading topology for medium- and high-voltage power conversion, originally driven by the need for scalable, efficient, and high-quality voltage synthesis in HVDC applications [6–8]. Their modular structure, based on cascaded submodules, enables scalable voltage synthesis with high-quality waveforms and reduced harmonic distortion. Beyond HVDC, MMC-based architectures are also being exploited in lower-voltage systems, where distributed energy resources, renewable sources, and energy storage systems are integrated directly at the submodule (SM) level [9, 10]. This approach departs from conventional converter systems, where storage or sources are typically interfaced through dedicated external conversion stages.

Integrating sources and storage directly within MMC submodules offers several potential advantages:

1. distributed energy management with module-level granularity (i.e., energy management at SM level);
2. reduced reliance on external conversion stages (fewer conversion stages and potentially higher efficiency);
3. scalability of voltage and power rating;
4. high output voltage quality with reduced filtering requirements [11, 12].

These features have motivated investigations across a broad set of applications, including

fast-charging infrastructure, transportation powertrains, maritime power systems, and grid-supporting stationary storage.

However, MMCs are more challenging to control than traditional converter topologies because they introduce additional internal dynamics and degrees of freedom. In practical operation, MMC control typically must address different objectives simultaneously:

1. accurate regulation of grid currents to track active and reactive power references;
2. management of internal energy distribution and circulating currents to ensure proper operation of all submodules;

Furthermore, operation in weak or unbalanced grids requires robust synchronisation and the ability to handle negative-sequence components without compromising internal stability. Hence, the design of high-performance control architectures is not an accessory feature but a decisive enabler for MMC-based integration of renewable energy sources and energy storage systems.

Following the MMC-topology classification adopted in [13], the MMC family can be grouped into four canonical arrangements as shown in Figure 1:

- Single-Star Bridge Cell (SSBC),
- Single-Delta Bridge Cell (SDBC),
- Double-Star Chopper Cell (DSCC), and
- Double-Star Bridge Cell (DSBC).

SSBC is a star-connected topology based on full-bridge submodules and, lacking an inherent circulating-current path, it relies on zero-sequence voltage injection for capacitor-voltage balancing, which becomes more constrained under negative-sequence injection and power unbalances. SDBC, instead, employs full-bridge submodules in a delta configuration, providing a circulating current path that enables internal energy exchange and makes it attractive for Static Synchronous Compensator (STATCOM) operation. DSCC is a double-star structure using half-bridge (chopper-cell) submodules in upper and lower arms. DSBC adopts full-bridge submodules in a double-star arrangement, but its higher device count generally makes it less appealing.

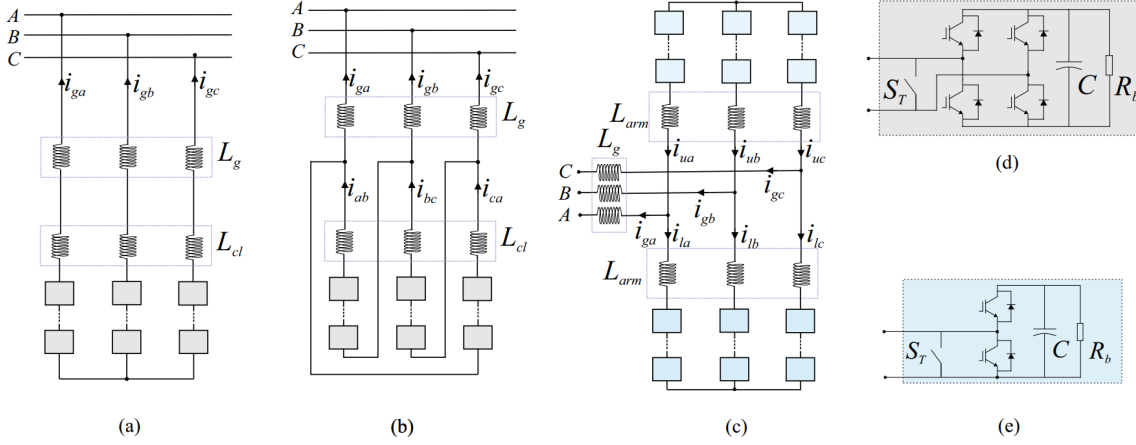


Figure 1: Classification of MMC topologies: (a) SSBC topology; (b) SDBC topology; (c) DSCC or DSBC topology (dependent on the SM employed); (d) Full-bridge SM employed in SSBC, SDBC and DSBC topologies; (e) Half-bridge SM employed in DSCC topology. Source: A. F. Cupertino, J. V. M. Farias, H. A. Pereira, S. I. Seleme and R. Teodorescu, "Comparison of DSCC and SDBC Modular Multilevel Converters for STATCOM Application During Negative Sequence Compensation," in IEEE Transactions on Industrial Electronics, vol. 66, no. 3, pp. 2302-2312, March 2019. © 2019 IEEE

Despite the relevance of all these configurations, this thesis focuses on the DSCC topology, as its features make it well suited for photovoltaic applications and grid power injection, especially when internal energy balancing and circulating-current control are central design requirements.

Based on the above considerations, this thesis investigates the control of a modular multi-level converter as a key enabling technology for renewable-dominated power systems under unbalanced grid conditions. The goal is to validate a coordinated control strategy that guarantees grid-compliant power exchange while preserving stable internal dynamics and energy balance. Particular attention is devoted to operating conditions characterised by unbalanced grid voltages and non-uniform power availability among submodules, which introduce internal energy imbalance and require dedicated balancing and control strategies.

In particular, the study focuses on a three-phase DSCC-based MMC PV system in which photovoltaic arrays are integrated at the submodule level and operated through distributed maximum power point tracking (DMPPT). Under unbalanced grid conditions, the controller must simultaneously satisfy two sets of requirements. On the grid side, it must ensure accurate tracking of the prescribed active and reactive power references and inject currents that comply with grid constraints. Internally, it must generate appropriate circulating-current components to redistribute power within and among phase-legs,

so that the arm-level power availability imposed by DMPPT can be accommodated and each PV array can operate at its maximum power point.

To address these objectives, a mathematical model is derived and used to map measured currents, voltages and power availability constraints into the required arm-level voltage references. Moreover, three alternative operating modes are enabled through a selector parameter K , allowing the controller to enforce suppression of active-power oscillations, suppression of reactive-power oscillations, or balanced output currents.

1 | Double-star configured MMC with Integrated Sources and Storage Systems

The double-star configuration (represented in Figure 1.1) is distinguished by the presence of a common DC bus formed by the parallel connection of the three-phase legs. Each phase-leg comprises an upper and a lower arm, each built from a series connection of N half-bridge submodules and connected to the grid through the arm inductors. Those can be mutually coupled or uncoupled and their aim is to limit the current in case of fault, limit the circulating current among the converter phases, and reduce the current ripple at the switching frequency [9].

Key advantages of this topology include:

1. The separation between phase and circulating quantities, which allows the shaping of internal power flows and energy oscillations without directly affecting the AC-side current [11].
2. Multi-layer energy management, which can be naturally implemented at different scales:
 - (a) Submodule level: voltage control depending on the MPPT requirements;
 - (b) Arm level (vertical balancing): energy redistribution between upper and lower arms of the same leg through controlled fundamental-frequency circulating-current components;
 - (c) Leg level (horizontal balancing): energy redistribution among phase legs through DC circulating-current components.

[14, 15]

3. The ability to support unequal power availability among arms and legs by exploiting DC and AC circulating-current components and by using the DC-side energy buffer

as a natural path for AC circulating homopolar components [14, 15].

Although this thesis focuses on the use of the converter for photovoltaic applications, the modelling framework and the underlying considerations are not PV-specific. In particular, the same principles and control degrees of freedom can be extended to MMC systems integrating other distributed resources at submodule level, such as batteries.

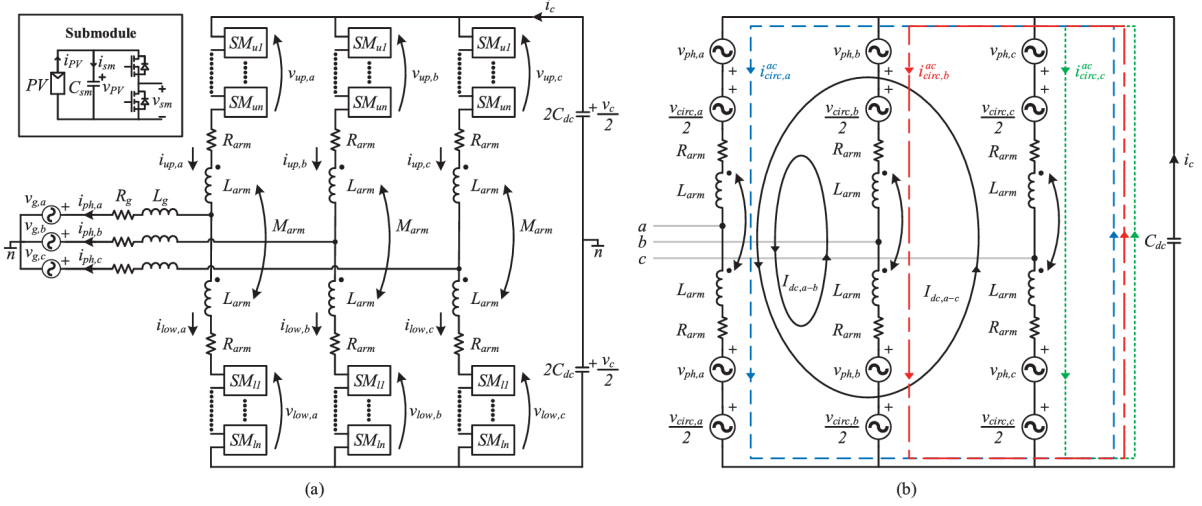


Figure 1.1: Converter topology for PV application: a) MMC three-phase with the dc-side capacitor; b) Equivalent circuit of the MMC with the dc-side capacitor.

Source: S. Barcellona, M. Barresi and L. Piegari, “MMC-Based PV Three-Phase System With Distributed MPPT,” IEEE Transactions on Energy Conversion, vol. 37, no. 3, pp. 1567–1578, Sept. 2022. © 2022 IEEE

1.0.1. Voltage and Current relations

Analysing the model, voltage and current relations can be derived and, in order to generalize, mutually coupled arm inductors have been considered. In fact, to derive the model with uncoupled inductors, it is possible to set $M_{arm} = 0$.

For phase $j \in \{a, b, c\}$, the arm voltages can be expressed as (through KVL):

$$\begin{cases} v_{up,j} = -v_{g,j} - R_g(i_{up,j} - i_{low,j}) - L_g \frac{d(i_{up,j} - i_{low,j})}{dt} + \\ \quad -R_{arm} i_{up,j} - L_{arm} \frac{di_{up,j}}{dt} - M_{arm} \frac{di_{low,j}}{dt} + \frac{v_c}{2} \\ v_{low,j} = v_{g,j} + R_g(i_{up,j} - i_{low,j}) + L_g \frac{d(i_{up,j} - i_{low,j})}{dt} + \\ \quad -R_{arm} i_{low,j} - L_{arm} \frac{di_{low,j}}{dt} - M_{arm} \frac{di_{up,j}}{dt} + \frac{v_c}{2} \end{cases} \quad (1.1)$$

where $v_{up,j}$, $v_{low,j}$, $i_{up,j}$ and $i_{low,j}$ are the upper and lower arms voltages and currents, respectively. Then, v_c is the dc-side voltage, R_{arm} is the arm resistance, L_{arm} is the arm inductance and M_{arm} is the mutual coupling of the arm inductances.

Finally, $v_{g,j}$ and $i_{ph,j}$ are the AC grid voltages and currents.

Moreover, the voltages developed by the upper and lower arms can be decomposed as follows:

$$\begin{cases} v_{up,j} = -v_{ph,j} + \frac{v_{circ,j}}{2} \\ v_{low,j} = v_{ph,j} + \frac{v_{circ,j}}{2} \end{cases} \quad (1.2)$$

where $v_{ph,j}$ is the external AC output terminal voltage and $v_{circ,j}$ is defined as the circulating voltage, associated with internal energy redistribution.

Each arm voltage is synthesised by the N submodules voltages:

$$v_{up,j} = \sum_{i=1}^N v_{SM,up,i,j}, \quad (1.3)$$

$$v_{low,j} = \sum_{i=1}^N v_{SM,low,i,j}. \quad (1.4)$$

It is now possible to introduce grid currents $i_{ph,j}$ and circulating currents $i_{circ,j}$ as:

$$\begin{cases} i_{ph,j} = i_{up,j} - i_{low,j} \\ i_{circ,j} = \frac{i_{up,j} + i_{low,j}}{2} \end{cases} \quad (1.5)$$

Meaning that arm currents can be expressed as:

$$\begin{cases} i_{up,j} = \frac{i_{ph,j}}{2} + i_{circ,j} \\ i_{low,j} = \frac{i_{ph,j}}{2} - i_{circ,j} \end{cases} \quad (1.6)$$

showing that each arm current is the superposition of the circulating current and half of the output current. Moreover, as shown by Equations (1.5), circulating current $i_{circ,j}$ (and voltage $v_{circ,j}$) does not affect the output phase current $i_{ph,j}$ (and voltage $v_{ph,j}$).

The output phase-voltage component is used to drive the output phase currents $i_{ph,j}$, whereas the circulating-voltage component drives the circulating currents $i_{circ,j}$. Therefore, $v_{ph,j}$ and $v_{circ,j}$ are used to regulate the power exchanged with the AC grid and to manage the internal converter balancing, respectively.

From Equations (1.1), (1.2) and (1.6):

$$\begin{cases} v_{ph,j} = v_{g,j} + \left(\frac{R_{arm}}{2}\right) i_{ph,j} + \left(\frac{L_{arm} - M_{arm}}{2}\right) \frac{di_{ph,j}}{dt}, \\ v_{circ,j} = -2 \left[R_{arm} i_{circ,j} + (L_{arm} + M_{arm}) \frac{di_{circ,j}}{dt} \right] + v_c. \end{cases} \quad (1.7)$$

Therefore, it is easy to see that employing two coupled inductors reduces the voltage drop in the output phase voltage and increases the effective inductance of the overall leg.

Defining $v_{circ,j}$ as the sum of an AC term and a DC term, it becomes:

$$v_{circ,j} = v_{circ,j}^{ac} + v_{circ,j}^{dc} \quad (1.8)$$

where, under ideal conditions (homogeneous irradiance conditions), $v_{circ,j}^{ac} = 0$ and $v_{circ,j}^{dc} = V_{dc}$, with V_{dc} defined as the DC voltage component across the terminals of the capacitor C_{dc} .

In contrast, under inhomogeneous irradiance conditions, the circulating quantities may exhibit additional DC and AC components.

Circulating components in MMCs are not only an inherent internal phenomenon, but also a key control lever when the converter operates under non-ideal conditions. Circulating currents are further affected by grid-side unbalance, which alter the instantaneous power exchange and introduce energy oscillations that propagate into internal currents [16].

1.0.2. Half-Bridge Submodule Structure

As shown in Figure 1.2, each DSCC submodule is based on a half-bridge topology, consisting of:

- two switching devices S_{up} and S_{low} (typically IGBTs or MOSFETs) in complementary configuration;
- a capacitor C_{sm} ;
- a power source, in this case PV arrays;

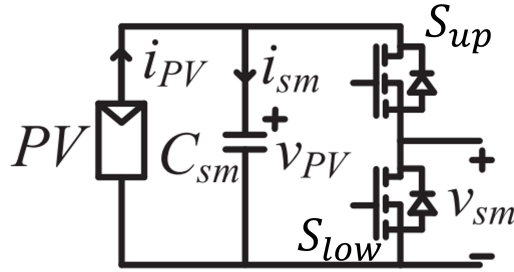


Figure 1.2: Half-bridge submodule topology for PV application.

Source: S. Barcellona, M. Barresi and L. Piegari, “MMC-Based PV Three-Phase System With Distributed MPPT,” IEEE Transactions on Energy Conversion, vol. 37, no. 3, pp. 1567–1578, Sept. 2022. © 2022 IEEE

The operating states of a half-bridge submodule (HB-SM) are summarised in Table 1.1. In the *Inserted* state, the upper switch S_{up} is turned on while S_{low} is kept off, so that the DC source connected to the submodule (here the PV unit, or a battery pack in storage-oriented implementations) is applied to the arm and the submodule output voltage becomes $v_{sm} = v_{PV}$. Conversely, in the *Bypassed* state, S_{up} is turned off and S_{low} is turned on, providing a conduction path that bypasses the source and yields $v_{sm} = 0$. By properly selecting which submodules are inserted or bypassed, the arm voltage is obtained as the sum of the individual submodule contributions, enabling multilevel waveform synthesis.

State	S_{up}	S_{low}	Output voltage v_{sm} [V]
ON (Inserted)	ON	OFF	v_{PV}
OFF (Bypassed)	OFF	ON	0

Table 1.1: Half-bridge SM operating states.

As a direct consequence of this switching structure, the HB-SM can only generate unipolar output voltages (either 0 or a positive level). Therefore, the synthesized arm voltage must

remain non-negative, and the converter cannot inherently produce negative arm voltages, which is a key limitation of half-bridge based MMCs.

While the internal structure of the submodules is introduced for completeness, it is not explicitly considered in the remainder of this work. Each submodule is therefore treated from a macroscopic perspective as an equivalent source capable of providing a given power, and the study concentrates on the resulting arm- and leg-level power flow and energy dynamics.

2 | Unbalanced operation in MMC–PV systems

MMC-based PV systems integrating distributed sources at submodule level can exhibit operating conditions that deviate from the ideal balanced three-phase regime, introducing losses and imbalances.

The purpose of this chapter is therefore to introduce a consistent taxonomy and the main implications of unbalanced operation in MMC–PV systems.

2.1. Motivation and taxonomy of unbalances

In MMC–PV applications, the converter must simultaneously satisfy external objectives (grid-compliant current injection and power control) and internal objectives (stable energy distribution between legs and arms).

Under ideal conditions (homogeneous irradiance and balanced grid), the internal available power can be maintained around a nominal operating point for each phase-leg, while the grid currents remain balanced. In practical PV plants, however, partial shading and non-homogeneous irradiance lead to unequal available power among submodules, arms, and legs, while distribution networks may impose voltage unbalance, increasing internal stress, enlarging RMS arm currents and producing unacceptable voltage drift if not properly managed [11, 14, 17].

From a modelling perspective, it is convenient to distinguish two classes of unbalance:

1. internal power mismatch, due to non-uniform power availability within the converter;
2. grid-side unbalance, due to asymmetric three-phase quantities at the point of common coupling.

This section will now proceed by describing these two classes.

2.1.1. Internal power mismatch

In MMC-based PV systems with DMPPT, the power available to each arm is not necessarily uniform. This is a direct consequence of non-homogeneous irradiance conditions and the fact that each arm (and, at finer granularity, each submodule) is driven toward its own maximum power point [14].

To formalize the problem, consider the average powers available in the six arms:

$$P_{a,\text{up}}, P_{a,\text{low}}, P_{b,\text{up}}, P_{b,\text{low}}, P_{c,\text{up}}, P_{c,\text{low}}. \quad (2.1)$$

Therefore, the available power of each phase leg is

$$P_a = P_{a,\text{up}} + P_{a,\text{low}}, \quad P_b = P_{b,\text{up}} + P_{b,\text{low}}, \quad P_c = P_{c,\text{up}} + P_{c,\text{low}}, \quad (2.2)$$

and the total available power is

$$P_{\text{tot}} = \sum_{j \in \{a,b,c\}} P_{j,\text{up}} + \sum_{j \in \{a,b,c\}} P_{j,\text{low}} = (P_{a,\text{up}} + P_{b,\text{up}} + P_{c,\text{up}}) + (P_{a,\text{low}} + P_{b,\text{low}} + P_{c,\text{low}}). \quad (2.3)$$

Two mismatch patterns are particularly relevant in PV systems and may coexist [14, 15]:

1. mismatch among legs (horizontal mismatch), i.e., $P_a \neq P_b$, $P_a \neq P_c$ or $P_b \neq P_c$;
2. mismatch between arms of the same leg (vertical mismatch), i.e., $P_{j,\text{up}} \neq P_{j,\text{low}}$ for at least one phase $j \in \{a, b, c\}$.

Three indicators of horizontal mismatch are

$$\Delta P_{ab} = P_a - P_b, \quad \Delta P_{bc} = P_b - P_c, \quad \Delta P_{ac} = P_a - P_c. \quad (2.4)$$

If balanced grid currents are required, each phase should deliver the same share of the total power, hence the internal dynamics must enable power transfers among legs. In MMC–PV systems, such inter-leg transfers are enabled by DC components of circulating currents flowing through the legs, while preventing any DC injection into the grid [14, 15].

For vertical mismatch, a useful quantity is the differential arm power

$$P_{\text{diff},j} = P_{j,\text{up}} - P_{j,\text{low}}. \quad (2.5)$$

Even if $\Delta P_{ab} = \Delta P_{bc} = \Delta P_{ac} = 0$ (no horizontal mismatch), a non-zero $P_{\text{diff},j}$ would cause energy drift in one arm and energy depletion in the other unless an internal exchange within the leg is enforced. In PV-oriented MMCs, this exchange is typically implemented through a suitable AC circulating-current component that regulates the arm-to-arm power transfer without directly affecting the grid current [14, 15].

2.1.2. Grid-side unbalance

Grid-side unbalance denotes operating conditions in which three-phase voltages and/or currents deviate from the symmetric set, so they cannot be described as three equal-magnitude sinusoids displaced by 120° . In distribution networks, such conditions may arise from asymmetric faults, unbalanced loads, open-phase conditions, or line asymmetries.

For grid-connected MMCs, negative-sequence voltages and currents are especially relevant because they introduce double-frequency oscillations in synchronous-reference-frame variables and in the instantaneous power exchanged with the grid [11, 17].

An ideal symmetric set of phase voltages can be written as

$$v_a(t) = \hat{V} \sin(\omega t), \quad (2.6)$$

$$v_b(t) = \hat{V} \sin\left(\omega t - \frac{2\pi}{3}\right), \quad (2.7)$$

$$v_c(t) = \hat{V} \sin\left(\omega t + \frac{2\pi}{3}\right), \quad (2.8)$$

where $\hat{V}$ is the peak value of the phase voltages. This implies $v_a(t) + v_b(t) + v_c(t) = 0$ at every instant.

Let $\bar{V}_a$, $\bar{V}_b$ and $\bar{V}_c$ be the three voltage phasors. Under unbalanced conditions, a common analytical tool to be used is the symmetrical-components decomposition (Fortescue Transformation). In phasor form, one can write

$$\begin{bmatrix} \bar{V}_0 \\ \bar{V}_+ \\ \bar{V}_- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix}, \quad a = e^{j\frac{2\pi}{3}}, \quad (2.9)$$

where $\bar{V}_+$ and $\bar{V}_-$ denote the phasors of the positive- and negative-sequence components, respectively, and $\bar{V}_0$ is the zero-sequence component.

The inverse transformation is

$$\begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \bar{V}_0 \\ \bar{V}_+ \\ \bar{V}_- \end{bmatrix}. \quad (2.10)$$

In real three-wire systems, the negative sequence may be present and has a direct impact on converter currents, power oscillations, and internal energy dynamics [11, 17].

2.2. Implications

Although internal mismatch and grid-side unbalance have different causes, they both generate additional dynamics that must be accommodated by the MMC. The main implications can be summarized as follows:

1. Internal imbalances must be managed through internal power transfers enabled by circulating currents;
2. Negative sequence component generates 2ω oscillations in synchronous-reference-frame variables and in instantaneous power;
3. the resulting energy oscillations propagate to arm energies and submodule voltages, increasing internal power imbalances and decreasing the system's stability.

The following sections detail these mechanisms.

2.2.1. Circulating currents

Before introducing any dedicated control law, it is crucial to understand why circulating currents appear and how they are related to the arm quantities.

As already defined in Formula (1.2), for each phase $j \in \{a, b, c\}$, it is convenient to decompose upper and lower arm voltages into a phase component and a circulating component:

$$\begin{cases} v_{up,j} = -v_{ph,j} + \frac{v_{circ,j}}{2}, \\ v_{low,j} = v_{ph,j} + \frac{v_{circ,j}}{2}. \end{cases}$$

Similarly, in Formula (1.5), the output phase current and the circulating current are defined as:

$$\begin{cases} i_{ph,j} = i_{up,j} - i_{low,j}, \\ i_{circ,j} = \frac{i_{up,j} + i_{low,j}}{2}. \end{cases}$$

These definitions highlight that $i_{ph,j}$ is the differential current exchanged with the AC network, whereas $i_{circ,j}$ is an internal current circulating within the phase leg and does not directly contribute to the grid current.

In PV-oriented MMCs equipped with a DC-side capacitor, the circulating voltage drives the circulating current through the internal impedance of the leg. A compact expression for the circulating-voltage relationship is given by Equation (1.7) as follows:

$$v_{circ,j} = -2 \left(R_{arm} i_{circ,j} + (L_{arm} + M_{arm}) \frac{di_{circ,j}}{dt} \right) + v_c,$$

where v_c is the DC-side capacitor voltage. The DC-side dynamics is governed by [14]:

$$v_c(t) = v_c(0) - \frac{1}{C_{dc}} \int_0^t i_c(\tau) d\tau, \quad i_c = i_{circ,a} + i_{circ,b} + i_{circ,c}. \quad (2.11)$$

Under homogeneous irradiance and balanced conditions, circulating quantities are ideally null. Conversely, under inhomogeneous irradiance, circulating components are required to route power internally [14].

Circulating voltage $v_{circ,j}$ has already been defined by (1.8) as:

$$v_{circ,j} = v_{circ,j}^{dc} + v_{circ,j}^{ac}.$$

In partial shading among phase-legs (horizontal mismatch), the average power available in each leg can differ, so an inter-leg active-power transfer is required to reconcile the phase-dependent availability with the phase power split demanded by the grid controller. This is achieved by superimposing leg-dependent DC circulating-current components $i_{circ,j}^{dc}$, driven by the corresponding DC circulating voltages $v_{circ,j}^{dc}$. Since the circulating current is defined as $i_{circ,j} = (i_{up,j} + i_{low,j})/2$, adding a DC offset modifies both arm currents equally and therefore does not directly affect the AC terminal current $i_{ph,j} = i_{up,j} - i_{low,j}$ [14, 15].

Conversely, in partial shading between the upper and lower arms of the same phase-leg (vertical mismatch), the two arm powers become unbalanced and a fundamental-frequency circulating component is needed to exchange the corresponding differential power within the affected leg. In fact, denoting the arm-power mismatch as $P_{diff,j} = P_{j,up} - P_{j,low}$, the

associated instantaneous differential power can be expressed in first approximation as

$$p_{\text{diff},j}(t) \triangleq \frac{p_{j,\text{up}}(t) - p_{j,\text{low}}(t)}{2} \approx v_{ph,j}(t) i_{\text{circ},j}^{ac}(t). \quad (2.12)$$

This approximation holds when taking into account the fundamental-frequency AC component, neglecting harmonic contributions and cross-coupling terms [18, 19]. This shows that an AC circulating current $i_{\text{circ},j}^{ac}(t)$, driven by a generated AC circulating voltage component $v_{\text{circ},j}^{ac}$ and in phase with the leg voltage $v_{ph,j}(t)$, produces the required active arm-to-arm power transfer.

2.2.2. Power oscillations under negative sequence

In unbalanced operation, when the negative component is present, the instantaneous active and imaginary powers exchanged with the grid contain oscillatory terms at 2ω [11]. Introducing the positive- and negative-sequence voltage and current phasors $\bar{V}_g^+$, $\bar{V}_g^-$, $\bar{I}_{ph}^+$ and $\bar{I}_{ph}^-$, the $\alpha\beta$ space vectors are expressed as:

$$\bar{v}_g = \bar{V}_g^+ e^{j\omega t} + \bar{V}_g^- e^{-j\omega t}, \quad (2.13)$$

$$\bar{i}_{ph} = \bar{I}_{ph}^+ e^{j\omega t} + \bar{I}_{ph}^- e^{-j\omega t}. \quad (2.14)$$

Starting from the definition of complex power

$$\bar{S} = \frac{3}{2} \bar{v}_g \bar{i}_{ph}, \quad (2.15)$$

it is possible to write it as

$$\begin{aligned} \bar{S} &= \frac{3}{2} \left(\bar{V}_g^+ e^{j\omega t} + \bar{V}_g^- e^{-j\omega t} \right) \left(\bar{I}_{ph}^+ e^{-j\omega t} + \bar{I}_{ph}^- e^{j\omega t} \right) \\ &= \underbrace{\frac{3}{2} \left(\bar{V}_g^+ \bar{I}_{ph}^+ + \bar{V}_g^- \bar{I}_{ph}^- \right)}_{\bar{S}_{\text{const}}} + \underbrace{\frac{3}{2} \left(\bar{V}_g^+ \bar{I}_{ph}^- e^{j2\omega t} + \bar{V}_g^- \bar{I}_{ph}^+ e^{-j2\omega t} \right)}_{\bar{S}_{2\omega}}. \end{aligned} \quad (2.16)$$

In this formulation, $\bar{S}_{\text{const}}$ denotes the constant term defined as:

$$\bar{S}_{\text{const}} = P + jQ, \quad (2.17)$$

where

$$\begin{aligned} P &= P^+ + P^-, \\ Q &= Q^+ - Q^-. \end{aligned} \tag{2.18}$$

Specifically, P^+ and Q^+ denote the average active and reactive power contributions associated with the positive-sequence voltages and currents, while P^- and Q^- denote the corresponding contributions associated with the negative-sequence ones.

$\bar{S}_{2\omega}$, instead, represents the oscillatory contribution at twice the fundamental frequency originating from cross-coupling between positive and negative sequences.

Grid-side unbalance is therefore intrinsically associated with a trade-off among objectives such as balanced currents and constant instantaneous power.

3 | Control strategy under unbalanced conditions

The operating conditions discussed in Chapter 2 highlight that MMC–PV systems may simultaneously face internal power mismatches, mainly induced by non-uniform irradiance, and grid-side unbalance, associated with negative-sequence components at the point of common coupling. Under these circumstances, a control architecture must guarantee grid-compliant current injection and accurate tracking of the active and reactive power references, while at the same time enforcing the internal power-routing actions required to:

1. maximise the power produced, and
2. redistribute the surplus or deficit among phase legs when the available leg powers are not uniform.

The proposed approach adopts a hierarchical structure, as shown in Figure 3.1. Namely, the grid controller provides the current references required to meet the power objectives and, in parallel, two dedicated circulating-current controllers are introduced:

- an AC circulating-current controller, responsible for arm-to-arm power transfer within each phase leg, and
- a DC circulating-current controller, responsible for power transfer among legs through the DC side.

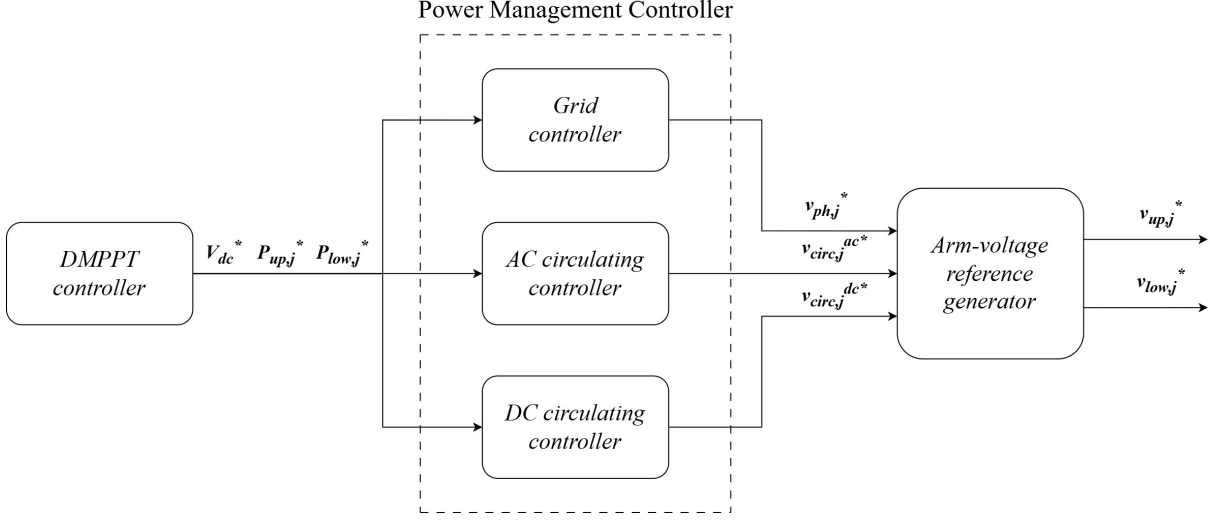


Figure 3.1: Proposed control strategy.

These three control layers are coordinated so that external performance and internal energy management are simultaneously achieved.

Finally, a fourth layer, referred to as the arm-voltage reference generator, is introduced to combine the outputs of the previous three controllers and produce the final upper- and lower-arm voltage references. The variable V_{dc}^* , introduced in Fig. 3.1, denotes the DC-link voltage reference associated with the operating point that maximizes the achievable PV power. $P_{up,j}^*$ and $P_{low,j}^*$, instead, are defined as the arm-level available power references for a general phase $j \in \{a, b, c\}$.

This chapter introduces the three control functions addressed in this thesis and clarifies their objectives and the physical mechanisms they exploit. The presentation follows a progressive approach: it first considers the grid controller, which is designed in line with established solutions reported in the literature, and then moves to the proposed circulating-current controllers, which constitute the main focus and original contribution of this thesis.

Each section concludes with a boxed expression summarizing the final control formulation. In addition, each controller is represented by a block diagram that compacts, in a single view, the sequence of operations described throughout the section.

3.1. Grid controller

The grid controller is responsible for generating the phase-current references and, consequently, the corresponding arm-voltage correction required to enforce them. In doing so,

it regulates the converter–grid power exchange so that the measured active and reactive powers track the prescribed set-points.

Under balanced grid conditions, this task is typically performed in a synchronous reference frame aligned with the positive-sequence voltage, where sinusoidal currents translate into constant dq components and PI regulators can ensure zero steady-state error. In the presence of grid-side unbalance, however, voltages and currents include both positive- and negative-sequence components, and a single $+\omega$ reference frame produces 2ω oscillations in the transformed variables (Chapter 2). Therefore, the grid controller must explicitly account for the negative sequence to guarantee accurate power tracking and grid-compliant current injection [11, 16, 17].

To this end, the controller adopts a dual-sequence structure in which current references are expressed in terms of positive- and negative-sequence components (i_{dq}^{+*} and i_{dq}^{-*}). Therefore, the objective is to generate three-phase current references that achieve the desired average active and reactive powers.

Since the control layer responsible for generating V_{dc}^* is outside the scope of this thesis, the DC-link reference and the associated power references are assumed to be provided by the DMPPT controller and are treated as known inputs.

Specifically, recalling the arm-level available powers already introduced, the active-power reference P^* is set equal to the total available PV power, as defined in (2.3). Therefore:

$$P^* = \sum_{j \in \{a,b,c\}} P_{\text{up},j}^* + \sum_{j \in \{a,b,c\}} P_{\text{low},j}^*. \quad (3.1)$$

This section presents the adopted strategy and analyses it step by step.

3.1.1. Sequence separation and reference frames

As already introduced, the converter control is implemented in a synchronous reference frame obtained through the Park transformation [11]. For a general three-phase signal x , the latter is expressed as:

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos \theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin \theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}. \quad (3.2)$$

Under balanced conditions and when $\theta = \omega t$ is synchronized to the positive sequence, the dq components of sinusoidal three-phase quantities are ideally constant, enabling PI regulation with zero steady-state error. Under grid-side unbalance, the presence of negative-sequence components produces 2ω oscillations when the quantities are expressed in the $+\omega$ synchronous frame.

Denoting positive- and negative-sequence components by $(\cdot)^+$ and $(\cdot)^-$, a common representation is:

$$x_d^{total} = x_d^+ + x_d^- \cos(2\omega t) + x_q^- \sin(2\omega t), \quad (3.3)$$

$$x_q^{total} = x_q^+ - x_d^- \sin(2\omega t) + x_q^- \cos(2\omega t). \quad (3.4)$$

If the same three-phase quantity is expressed in a synchronous reference frame rotating at $-\omega$ (aligned with the negative sequence), the negative-sequence components become constant, whereas the positive-sequence components appear as 2ω oscillations.

A common representation is:

$$x_d^{total} = x_d^- + x_d^+ \cos(2\omega t) - x_q^+ \sin(2\omega t), \quad (3.5)$$

$$x_q^{total} = x_q^- + x_d^+ \sin(2\omega t) + x_q^+ \cos(2\omega t). \quad (3.6)$$

These oscillations deteriorate current tracking if conventional single-frame PI control is applied and may propagate to modulation signals and internal variables [11, 17]. Therefore, the implementation follows a double synchronous reference frame approach based on two parallel transformations and filtering stages.

First, the positive- and negative-sequence voltage and current phasors introduced in Subsection 2.2.2, can be defined as

$$\bar{V}_g^+ = V_g^+ e^{j\phi^+}, \quad \bar{I}_{ph}^+ = I_{ph}^+ e^{j(\phi^+ - \varphi^+)}, \quad (3.7)$$

$$\bar{V}_g^- = V_g^- e^{j\phi^-}, \quad \bar{I}_{ph}^- = I_{ph}^- e^{j(\phi^- - \varphi^-)}, \quad (3.8)$$

where ϕ^+ and ϕ^- are the positive- and negative-sequence phase angles, while φ^+ and φ^- denote the phase angles between the corresponding sequence voltage and current phasors. V_g^+ , V_g^- , I_{ph}^+ and I_{ph}^- are, instead, the corresponding magnitudes.

In complex form, the voltage and current dq components are obtained by rotating the corresponding space vectors previously defined in (2.13) and (2.14):

$$v_{g,d}^+ + jv_{g,q}^+ = \bar{V}_g^+ e^{j\omega t} e^{-j\theta^+}, \quad (3.9)$$

$$i_{ph,d}^+ + ji_{ph,q}^+ = \bar{I}_{ph}^+ e^{j\omega t} e^{-j\theta^+}, \quad (3.10)$$

$$v_{g,d}^- + jv_{g,q}^- = \bar{V}_g^- e^{-j\omega t} e^{-j\theta^-}, \quad (3.11)$$

$$i_{ph,d}^- + ji_{ph,q}^- = \bar{I}_{ph}^- e^{-j\omega t} e^{-j\theta^-}. \quad (3.12)$$

Let ϕ^+ denote the reference angle. Defining the electrical angle θ^+ as the positive-sequence angle estimated by the PLL fed with the measured three-phase voltages $v_{g,abc}$ with fundamental frequency f_g , it can be written as:

$$\theta^+ = \theta_0^+ + \omega t, \quad (3.13)$$

where θ_0^+ represents the Park transformation initial reference angle. Accordingly, two Park transformations are used:

- A Park transformation with angle $+\theta^+$ is applied to the measured three-phase voltages to obtain $v_{g,d}$ and $v_{g,q}$ in the positive synchronous frame. After filtering out the 2ω components, the resulting DC terms are denoted as $v_{g,d}^+$ and $v_{g,q}^+$. Their values depend on the magnitude of the positive-sequence voltage and the phase displacement between the positive-sequence voltage and the Park initial reference angle θ_0^+ .
- A Park transformation with angle $-\theta^+$ is applied in parallel. After filtering, the DC terms provide the negative-sequence voltage components expressed in the negative synchronous frame, denoted here as $v_{g,d}^{-'}$ and $v_{g,q}^{-'}$. Under these conditions, the values of these quantities depend on the magnitude of negative-sequence voltage and the phase displacement between the negative-sequence voltage and the Park initial reference angle θ_0^+ .

The pairs $(v_{g,d}^+, v_{g,q}^+)$ and $(v_{g,d}^{-'}, v_{g,q}^{-'})$ are therefore more conveniently represented in polar form as

$$V_g^+ = \sqrt{(v_{g,d}^+)^2 + (v_{g,q}^+)^2}, \quad \phi^+ - \theta_0^+ = \text{atan2}(v_{g,q}^+, v_{g,d}^+), \quad (3.14)$$

$$V_g^- = \sqrt{(v_{g,d}^{-'})^2 + (v_{g,q}^{-'})^2}, \quad \theta_0^+ - \phi^- = \text{atan2}(v_{g,q}^{-'}, v_{g,d}^{-'}). \quad (3.15)$$

It should be noted that this approach introduces a complex conjugation of the negative-sequence phasor. For this reason, the sign convention used for ϕ^- in (3.15) is chosen consistently with this definition.

For the sake of simplicity, in this work a new negative-sequence frame is defined to make positive- and negative-sequence frames independent from each other. Therefore, a new auxiliary reference angle is introduced as

$$\theta^- = \theta_0^- - \theta^+ = \theta_0^- - \theta_0^+ - \omega t. \quad (3.16)$$

By expressing the positive- and negative-sequence components in their respective synchronous reference frames as described, the values of θ_0^+ and θ_0^- can be freely selected to simplify the formulae employed in the control strategy described in the following paragraphs.

The same dual-frame separation is also applied to the measured phase currents, yielding the positive- and negative-sequence components $(i_{ph,d}^+, i_{ph,q}^+, i_{ph,d}^-, i_{ph,q}^-)$.

The aforementioned method is represented in Figure 3.2, defining f as the measured frequency.

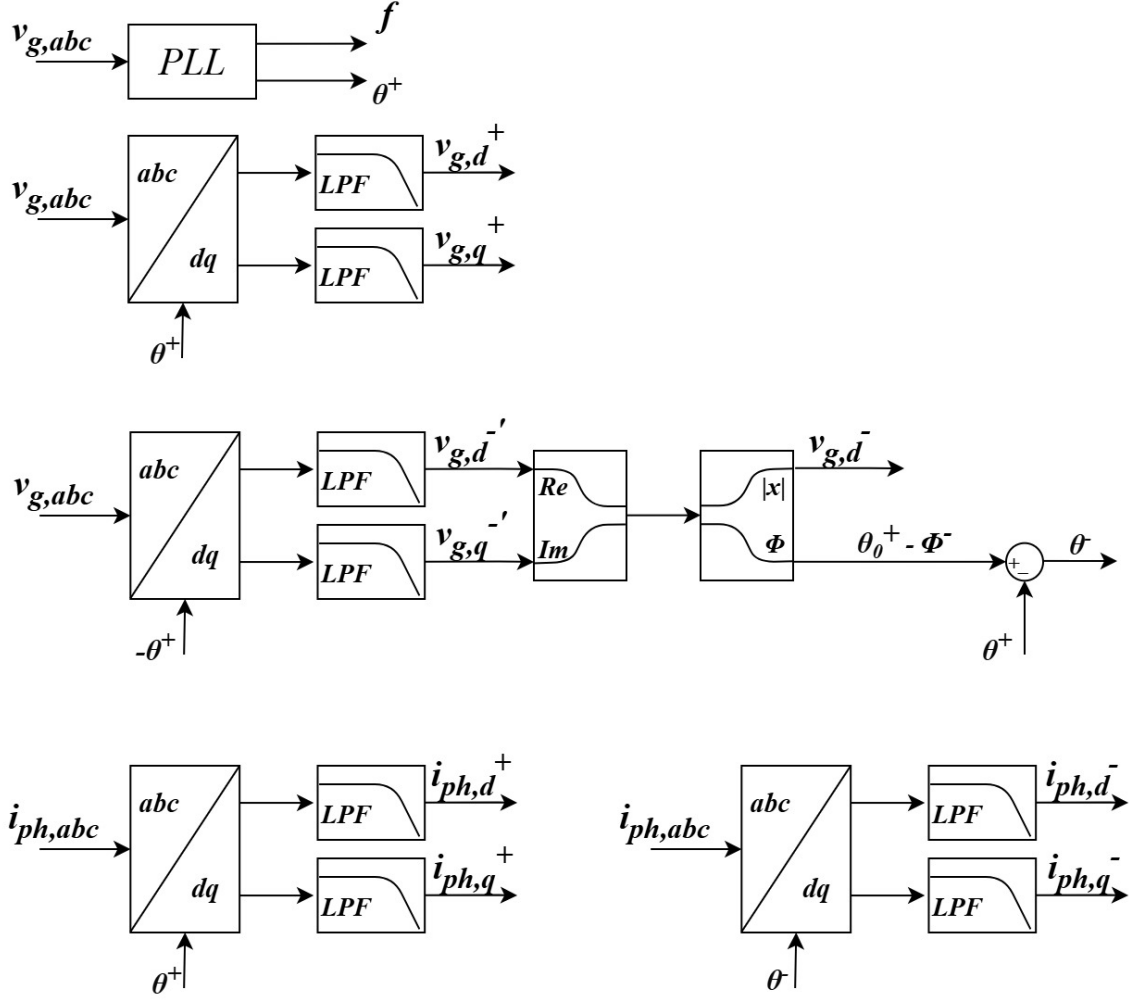


Figure 3.2: Sequence separation strategy.

3.1.2. From power references to current references

Subsection 3.1.1 defined the controller variables, namely

$$\begin{aligned} \text{grid-voltage components: } & v_{g,d}^+, v_{g,q}^+, v_{g,d}^-, v_{g,q}^-, \\ \text{phase-current components: } & i_{ph,d}^+, i_{ph,q}^+, i_{ph,d}^-, i_{ph,q}^-. \end{aligned}$$

The next step is therefore to determine a mapping from the desired power references to the current references $i_{ph,dq}^{+*}$ and $i_{ph,dq}^{-*}$.

To this end, two Park transformations are considered, defined by the reference angles θ^+ and θ^- for the positive- and negative-sequence synchronous reference frames, as already stated in Subsection 3.1.1. Starting from the space-vector complex power definition in

(2.16), the oscillatory contribution $\bar{S}_{2\omega}$ is now rewritten in the compact phasor form

$$\bar{S}_{2\omega}(t) = \frac{3}{2} \left[\bar{A} e^{j2\omega t} + \bar{B} e^{-j2\omega t} \right], \quad (3.17)$$

with

$$\bar{A} \triangleq \bar{V}_g^+ \bar{I}_{ph}^-, \quad \bar{B} \triangleq \bar{V}_g^- \bar{I}_{ph}^+. \quad (3.18)$$

In the following, three alternative operating modes are introduced. Each mode prescribes a different negative-sequence current reference $\bar{I}_{ph}^*$, according to the desired behaviour of the 2ω oscillatory power term.

Mode 1: elimination of reactive-power oscillations ($\bar{S}_{2\omega}$ purely real).

In order to cancel the oscillations in reactive power, the oscillatory term must be purely real.

This is true if $\bar{S}_{2\omega} = \underline{S}_{2\omega}$, namely:

$$\bar{B} = \underline{A}.$$

Substituting $\bar{A}$ and $\bar{B}$ yields

$$\bar{V}_g^- \bar{I}_{ph}^+ = \bar{V}_g^+ \bar{I}_{ph}^-. \quad (3.19)$$

and therefore the negative-sequence current reference becomes

$$\bar{I}_{ph}^- = \frac{\bar{V}_g^-}{\bar{V}_g^+} \bar{I}_{ph}^+ \quad (3.20)$$

Mode 2: elimination of active-power oscillations ($\bar{S}_{2\omega}$ purely imaginary).

To cancel the oscillations in active power, the oscillatory term must be purely imaginary, true if $\bar{S}_{2\omega} = -\underline{S}_{2\omega}$.

This condition is satisfied if and only if

$$\bar{B} = -\underline{A}.$$

Proceeding as above, it is possible to obtain

$$\bar{V}_g^- \bar{I}_{ph}^+ = -\bar{V}_g^+ \bar{I}_{ph}^-, \quad (3.21)$$

hence

$$\bar{I}_{ph}^- = -\frac{\bar{V}_g^-}{\bar{V}_g^+} \bar{I}_{ph}^+. \quad (3.22)$$

Unified formulation and balanced-current mode.

Introducing a selector parameter $K \in \{-1, 0, 1\}$, the three operating modes can be compactly expressed as

$$\bar{I}_{ph}^- = K \frac{\bar{V}_g^-}{\bar{V}_g^+} \bar{I}_{ph}^+, \quad K \in \{-1, 0, 1\}. \quad (3.23)$$

In particular, $K = -1$ enforces a purely imaginary $\bar{S}_{2\omega}$ (active-power oscillations suppressed), $K = +1$ enforces a purely real $\bar{S}_{2\omega}$ (reactive-power oscillations suppressed), whereas $K = 0$ yields balanced output currents by imposing $\bar{I}_{ph}^- = 0$.

Positive-Sequence Current Reference from Power References

$\bar{I}_{ph}^+$ is the component primarily responsible for the average power exchange with the grid. Therefore, once $\bar{I}_{ph}^-$ has been determined, the next step is to compute the positive-sequence current phasor $\bar{I}_{ph}^+$ from the constant complex-power reference, with P^* and Q^* as defined in (2.18).

Starting from

$$\bar{S}_{const} = \frac{3}{2} (\bar{V}_g^+ \bar{I}_{ph}^+ + \bar{V}_g^- \bar{I}_{ph}^-), \quad (3.24)$$

and substituting the unified negative-sequence reference found in Formula (3.23), one obtains

$$\bar{S}_{const} = \frac{3}{2} \left(\bar{V}_g^+ \bar{I}_{ph}^+ + K \frac{|\bar{V}_g^-|^2}{\bar{V}_g^+} \bar{I}_{ph}^+ \right). \quad (3.25)$$

From (3.9)–(3.12):

$$\bar{V}_g^+ = (v_{g,d}^+ + jv_{g,q}^+) e^{-j\omega t} e^{j\theta^+}, \quad (3.26)$$

$$\bar{I}_{ph}^+ = (i_{ph,d}^+ + ji_{ph,q}^+) e^{-j\omega t} e^{j\theta^+}. \quad (3.27)$$

It follows that the first term of (3.25) can be expressed as

$$\begin{aligned} \bar{V}_g^+ \bar{I}_{ph}^+ &= (v_{g,d}^+ + jv_{g,q}^+)(i_{ph,d}^+ - ji_{ph,q}^+) \\ &= (v_{g,d}^+ i_{ph,d}^+ + v_{g,q}^+ i_{ph,q}^+) + j(v_{g,q}^+ i_{ph,d}^+ - v_{g,d}^+ i_{ph,q}^+). \end{aligned} \quad (3.28)$$

Moreover,

$$|\bar{V}_g^-|^2 = (v_{g,d}^-)^2 + (v_{g,q}^-)^2, \quad |\bar{V}_g^+|^2 = (v_{g,d}^+)^2 + (v_{g,q}^+)^2, \quad (3.29)$$

and therefore the second term will be

$$\begin{aligned} \frac{|\bar{V}_g^-|^2}{\bar{V}_g^+} \bar{I}_{ph}^+ &= \frac{(v_{g,d}^-)^2 + (v_{g,q}^-)^2}{(v_{g,d}^+)^2 + (v_{g,q}^+)^2} (v_{g,d}^+ - jv_{g,q}^+) (i_{ph,d}^+ + ji_{ph,q}^+) \\ &= \frac{(v_{g,d}^-)^2 + (v_{g,q}^-)^2}{(v_{g,d}^+)^2 + (v_{g,q}^+)^2} \left[(v_{g,d}^+ i_{ph,d}^+ + v_{g,q}^+ i_{ph,q}^+) - j(v_{g,q}^+ i_{ph,d}^+ - v_{g,d}^+ i_{ph,q}^+) \right]. \end{aligned} \quad (3.30)$$

Hence,

$$\begin{aligned} \bar{S}_{const} &= \frac{3}{2} \left\{ \left[1 + K \frac{(v_{g,d}^-)^2 + (v_{g,q}^-)^2}{(v_{g,d}^+)^2 + (v_{g,q}^+)^2} \right] (v_{g,d}^+ i_{ph,d}^+ + v_{g,q}^+ i_{ph,q}^+) \right. \\ &\quad \left. + j \left[1 - K \frac{(v_{g,d}^-)^2 + (v_{g,q}^-)^2}{(v_{g,d}^+)^2 + (v_{g,q}^+)^2} \right] (v_{g,q}^+ i_{ph,d}^+ - v_{g,d}^+ i_{ph,q}^+) \right\}. \end{aligned} \quad (3.31)$$

By imposing the power reference $\bar{S}_{const} = P^* + jQ^*$ and separating real and imaginary parts, the following relations are obtained:

$$\begin{aligned} P^* &= \Re\{\bar{S}_{const}\} = \frac{3}{2} \left[1 + K \frac{(v_{g,d}^-)^2 + (v_{g,q}^-)^2}{(v_{g,d}^+)^2 + (v_{g,q}^+)^2} \right] (v_{g,d}^+ i_{ph,d}^+ + v_{g,q}^+ i_{ph,q}^+), \\ Q^* &= \Im\{\bar{S}_{const}\} = \frac{3}{2} \left[1 - K \frac{(v_{g,d}^-)^2 + (v_{g,q}^-)^2}{(v_{g,d}^+)^2 + (v_{g,q}^+)^2} \right] (v_{g,q}^+ i_{ph,d}^+ - v_{g,d}^+ i_{ph,q}^+). \end{aligned} \quad (3.32)$$

Therefore, starting from (3.32), the dq components of the positive-sequence current references are expressed as

$$\begin{aligned} i_{ph,d}^{+*} &= \frac{2}{3} \left[\frac{v_{g,d}^+ P^*}{[(v_{g,d}^+)^2 + (v_{g,q}^+)^2] + K[(v_{g,d}^-)^2 + (v_{g,q}^-)^2]} \right. \\ &\quad \left. + \frac{v_{g,q}^+ Q^*}{[(v_{g,d}^+)^2 + (v_{g,q}^+)^2] - K[(v_{g,d}^-)^2 + (v_{g,q}^-)^2]} \right], \\ i_{ph,q}^{+*} &= \frac{2}{3} \left[\frac{v_{g,q}^+ P^*}{[(v_{g,d}^+)^2 + (v_{g,q}^+)^2] + K[(v_{g,d}^-)^2 + (v_{g,q}^-)^2]} \right. \\ &\quad \left. - \frac{v_{g,d}^+ Q^*}{[(v_{g,d}^+)^2 + (v_{g,q}^+)^2] - K[(v_{g,d}^-)^2 + (v_{g,q}^-)^2]} \right]. \end{aligned} \quad (3.33)$$

From (3.23) and (3.9)–(3.12),

$$i_{ph,d}^- - j i_{ph,q}^- = K \frac{(v_{g,d}^- - j v_{g,q}^-)}{(v_{g,d}^+ + j v_{g,q}^+)} (i_{ph,d}^+ + j i_{ph,q}^+), \quad (3.34)$$

hence, the dq components of the negative-sequence current references can be found as

$$\begin{aligned} i_{ph,d}^{-*} &= \frac{2}{3} \left[\frac{K v_{g,d}^- P^*}{[(v_{g,d}^+)^2 + (v_{g,q}^+)^2] + K[(v_{g,d}^-)^2 + (v_{g,q}^-)^2]} \right. \\ &\quad \left. - \frac{K v_{g,q}^- Q^*}{[(v_{g,d}^+)^2 + (v_{g,q}^+)^2] - K[(v_{g,d}^-)^2 + (v_{g,q}^-)^2]} \right], \\ i_{ph,q}^{-*} &= \frac{2}{3} \left[\frac{K v_{g,q}^- P^*}{[(v_{g,d}^+)^2 + (v_{g,q}^+)^2] + K[(v_{g,d}^-)^2 + (v_{g,q}^-)^2]} \right. \\ &\quad \left. + \frac{K v_{g,d}^- Q^*}{[(v_{g,d}^+)^2 + (v_{g,q}^+)^2] - K[(v_{g,d}^-)^2 + (v_{g,q}^-)^2]} \right]. \end{aligned} \quad (3.35)$$

Moreover, under the conditions given by the Park reference angles θ^+ and θ^- defined in Subsection 3.1.1, setting the values of the initial Park reference angles as

$$\theta_0^+ = \phi^+, \quad (3.36)$$

$$\theta_0^- = \theta_0^+ - \phi^- = \phi^+ - \phi^-, \quad (3.37)$$

the quadrature components become $v_{g,q}^+ = 0$ and $v_{g,q}^- = 0$.

Therefore, the general equations (3.33)–(3.35) can be expressed as:

$$\begin{cases} i_{ph,d}^{+*} = \frac{2}{3} \frac{v_{g,d}^+ P^*}{(v_{g,d}^+)^2 + K(v_{g,d}^-)^2}, \\ i_{ph,q}^{+*} = -\frac{2}{3} \frac{v_{g,d}^+ Q^*}{(v_{g,d}^+)^2 - K(v_{g,d}^-)^2}, \\ i_{ph,d}^{-*} = \frac{2K}{3} \frac{v_{g,d}^- P^*}{(v_{g,d}^+)^2 + K(v_{g,d}^-)^2}, \\ i_{ph,q}^{-*} = \frac{2K}{3} \frac{v_{g,d}^- Q^*}{(v_{g,d}^+)^2 - K(v_{g,d}^-)^2}. \end{cases} \quad K \in \{-1, 0, 1\}. \quad (3.38)$$

Again, considering (3.9)–(3.12), (3.7)–(3.8) and the conventions introduced in Subsec-

tion 3.1.1, the latter can be rewritten in phasor form as follows:

$$\begin{cases} \bar{I}_{ph}^{+*} = \frac{2}{3} \bar{V}_g^+ \left(\frac{P^*}{D_P} - j \frac{Q^*}{D_Q} \right), \\ \bar{I}_{ph}^{-*} = \frac{2}{3} K \bar{V}_g^- \left(\frac{P^*}{D_P} - j \frac{Q^*}{D_Q} \right), \end{cases} \quad (3.39)$$

where

$$D_P \triangleq |\bar{V}_g^+|^2 + K |\bar{V}_g^-|^2, \quad D_Q \triangleq |\bar{V}_g^+|^2 - K |\bar{V}_g^-|^2. \quad (3.40)$$

These relations are added to the system through the block represented in Figure 3.3.

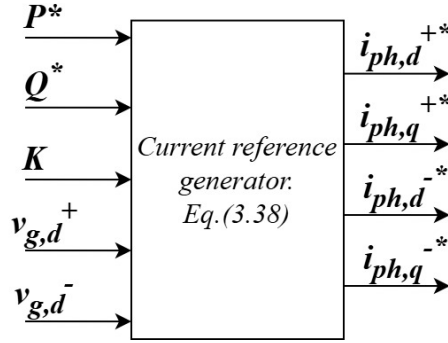


Figure 3.3: Current reference generator.

Total Reactive Power reference definition (Q_T^*)

So far, the grid controller has been formulated by adopting the complex-power definition in the stationary $\alpha\beta$ frame, $\bar{S} = \frac{3}{2} \bar{v} \bar{i}$, and by decomposing voltages and currents into positive- and negative-sequence components. With the conventions introduced in Subsection 2.2.2, the resulting reference mapping leads to the reactive-power definition $Q^* = Q^+ - Q^-$, as in (2.18).

This sign convention is consistent with the adopted definition of the negative-sequence phasor in the space-vector reconstruction, and it is commonly encountered in dual-sequence formulations for unbalanced operation [11, 16].

In practice, however, it is often more meaningful to specify reactive-power requirements in terms of the total reactive power exchanged with the grid, namely

$$Q_T^* \triangleq Q^+ + Q^-. \quad (3.41)$$

This definition differs from (2.18) whenever a negative-sequence current is intentionally

injected, so that $Q^- > 0$. Therefore, in the following a simple conversion is derived to express the controller input Q^* as an equivalent reference corresponding to a desired total reactive power Q_T^* .

From (3.39), the positive- and negative-sequence current references can be written as

$$\begin{aligned}\bar{I}_{ph}^{+*} &= \frac{2}{3} \bar{V}_g^+ \left(\frac{P^*}{D_P} - j \frac{Q^*}{D_Q} \right), \\ \bar{I}_{ph}^{-*} &= \frac{2}{3} K \bar{V}_g^- \left(\frac{P^*}{D_P} - j \frac{Q^*}{D_Q} \right).\end{aligned}$$

Define now the sequence complex powers as

$$\bar{S}^+ \triangleq \frac{3}{2} \bar{V}_g^+ \bar{I}_{ph}^{+*}, \quad \bar{S}^- \triangleq \frac{3}{2} \bar{V}_g^- \bar{I}_{ph}^{-*}, \quad (3.42)$$

so that $\bar{S}^\pm = P^\pm + jQ^\pm$, where

$$\bar{S}^+ = |\bar{V}_g^+|^2 \left(\frac{P^*}{D_P} + j \frac{Q^*}{D_Q} \right), \quad (3.43)$$

$$\bar{S}^- = K |\bar{V}_g^-|^2 \left(\frac{P^*}{D_P} + j \frac{Q^*}{D_Q} \right). \quad (3.44)$$

Therefore,

$$P^+ = |\bar{V}_g^+|^2 \frac{P^*}{D_P}, \quad P^- = K |\bar{V}_g^-|^2 \frac{P^*}{D_P}, \quad Q^+ = |\bar{V}_g^+|^2 \frac{Q^*}{D_Q}, \quad Q^- = K |\bar{V}_g^-|^2 \frac{Q^*}{D_Q}. \quad (3.45)$$

From (3.45), the sequence ratios satisfy

$$\frac{Q^-}{Q^+} = \frac{P^-}{P^+} = K \frac{|\bar{V}_g^-|^2}{|\bar{V}_g^+|^2} \triangleq K_v, \quad (3.46)$$

hence $Q^- = K_v Q^+$.

Using (3.41), the total reactive-power target becomes $Q_T^* = Q^+ + Q^- = Q^+(1 + K_v)$, where

$$Q^+ = \frac{Q_T^*}{1 + K_v}, \quad Q^- = \frac{K_v}{1 + K_v} Q_T^*. \quad (3.47)$$

Finally, recalling that the controller input is defined as $Q^* = Q^+ - Q^-$ in (3.1.2), one

obtains the desired conversion:

$$Q^* = Q_T^* \frac{1 - K_v}{1 + K_v}, \quad K_v \triangleq K \frac{|\bar{V}_g^-|^2}{|\bar{V}_g^+|^2} \quad (3.48)$$

Equation (3.48) shows that $Q^* = Q_T^*$ in balanced conditions ($|\bar{V}_g^-| = 0$) or in balanced-current mode ($K = 0$). Conversely, when negative-sequence injection is enabled ($K = \pm 1$), the controller reference Q^* must be scaled according to the voltage unbalance factor in order to enforce a prescribed total reactive-power target Q_T^* .

This last module is graphically added to the control method as shown in Figure 3.4.

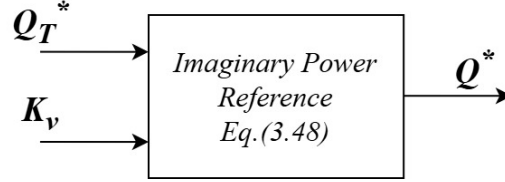


Figure 3.4: Imaginary power reference generator.

3.1.3. Phase-voltage reference generation

Once the current references have been computed in Subsection 3.1.2 and the measured sequence components have been extracted as described in Subsection 3.1.1, an inner current-control layer is employed to generate the converter-side voltage references to be synthesized by the MMC. The role of this stage is to regulate the injected grid currents $(i_{ph,d}^+, i_{ph,q}^+, i_{ph,d}^-, i_{ph,q}^-)$, in order to achieve the values defined by the current references $(i_{ph,d}^{+*}, i_{ph,q}^{+*}, i_{ph,d}^{-*}, i_{ph,q}^{-*})$.

Current dynamics in the synchronous frames.

Let R_{eq} and L_{eq} denote the equivalent series resistance and inductance seen by the phase current and assume $M_{arm} = 0$. Supposing that all the arm resistances and inductances have the same value, they can be expressed as:

$$L_{eq} = L_g + \frac{L_{arm}}{2}, \quad R_{eq} = R_g + \frac{R_{arm}}{2}. \quad (3.49)$$

For each j -th phase, the grid-current dynamics can be written as

$$v_{ph,j} - v_{g,j} = R_{eq} i_{ph,j} + L_{eq} \frac{di_{ph,j}}{dt}. \quad (3.50)$$

Applying the dual synchronous transformations, the corresponding equations in the positive- and negative-sequence dq frames become

$$\begin{cases} v_{ph,d}^+ = v_{g,d}^+ + R_{eq} i_{ph,d}^+ + L_{eq} \frac{di_{ph,d}^+}{dt} - \omega L_{eq} i_{ph,q}^+ \\ v_{ph,q}^+ = v_{g,q}^+ + R_{eq} i_{ph,q}^+ + L_{eq} \frac{di_{ph,q}^+}{dt} + \omega L_{eq} i_{ph,d}^+ \\ v_{ph,d}^- = v_{g,d}^- + R_{eq} i_{ph,d}^- + L_{eq} \frac{di_{ph,d}^-}{dt} + \omega L_{eq} i_{ph,q}^- \\ v_{ph,q}^- = v_{g,q}^- + R_{eq} i_{ph,q}^- + L_{eq} \frac{di_{ph,q}^-}{dt} - \omega L_{eq} i_{ph,d}^- \end{cases} \quad (3.51)$$

The opposite sign of the cross-coupling terms in $v_{ph,d}^-$ and $v_{ph,q}^-$ follows from the complex-conjugate form of the negative-sequence Park transformation.

Dual-sequence PI current regulators.

Two decoupled PI regulators are implemented in each frame. Defining the current errors as

$$e_d^+ \triangleq i_{ph,d}^{+*} - i_{ph,d}^+, \quad e_q^+ \triangleq i_{ph,q}^{+*} - i_{ph,q}^+, \quad (3.52)$$

$$e_d^- \triangleq i_{ph,d}^{-*} - i_{ph,d}^-, \quad e_q^- \triangleq i_{ph,q}^{-*} - i_{ph,q}^-, \quad (3.53)$$

the PI outputs are

$$u_d^\pm = k_{p,g} e_d^\pm + k_{i,g} \int e_d^\pm dt, \quad u_q^\pm = k_{p,g} e_q^\pm + k_{i,g} \int e_q^\pm dt, \quad (3.54)$$

where $k_{p,i}$ and $k_{i,i}$ are the proportional and integral gains of the inner current loop.

Using (3.51), the voltage references are generated by adding

1. a feed-forward of the measured grid voltage,
2. a resistive drop compensation, and
3. cross-coupling terms,

namely

$$\begin{cases} v_{ph,d}^{+*} = v_{g,d}^+ + R_{eq} i_{ph,d}^{+*} - \omega L_{eq} i_{ph,q}^{+*} + u_d^+ \\ v_{ph,q}^{+*} = v_{g,q}^+ + R_{eq} i_{ph,q}^{+*} + \omega L_{eq} i_{ph,d}^{+*} + u_q^+ \\ v_{ph,d}^{-*} = v_{g,d}^- + R_{eq} i_{ph,d}^{-*} + \omega L_{eq} i_{ph,q}^{-*} + u_d^- \\ v_{ph,q}^{-*} = v_{g,q}^- + R_{eq} i_{ph,q}^{-*} - \omega L_{eq} i_{ph,d}^{-*} + u_q^- \end{cases} \quad (3.55)$$

Recombination and injection at the submodule level.

The phase-voltage reference is finally reconstructed by superimposing the two sequence contributions through the inverse Park transformations:

$$\mathbf{v}_{ph,abc}^* = \mathbf{v}_{ph,abc}^{+*} + \mathbf{v}_{ph,abc}^{-*} = \mathbf{T}^{-1}(\theta^+) \mathbf{v}_{ph,dq}^{+*} + \mathbf{T}^{-1}(\theta^-) \mathbf{v}_{ph,dq}^{-*} \quad (3.56)$$

where

$$\mathbf{v}_{ph,dq}^{\pm*} \triangleq \begin{bmatrix} v_{ph,d}^{\pm*} \\ v_{ph,q}^{\pm*} \end{bmatrix}, \quad (3.57)$$

and

$$\mathbf{v}_{ph,abc}^* \triangleq \begin{bmatrix} v_{ph,a}^* \\ v_{ph,b}^* \\ v_{ph,c}^* \end{bmatrix}. \quad (3.58)$$

The phase-voltages $v_{ph,j}^*$ ($j \in \{a, b, c\}$) represent the ac grid components that the MMC must synthesise at its ac terminals.

The grid controller is finally shown in Figure 3.5, following the strategy explained in this section.

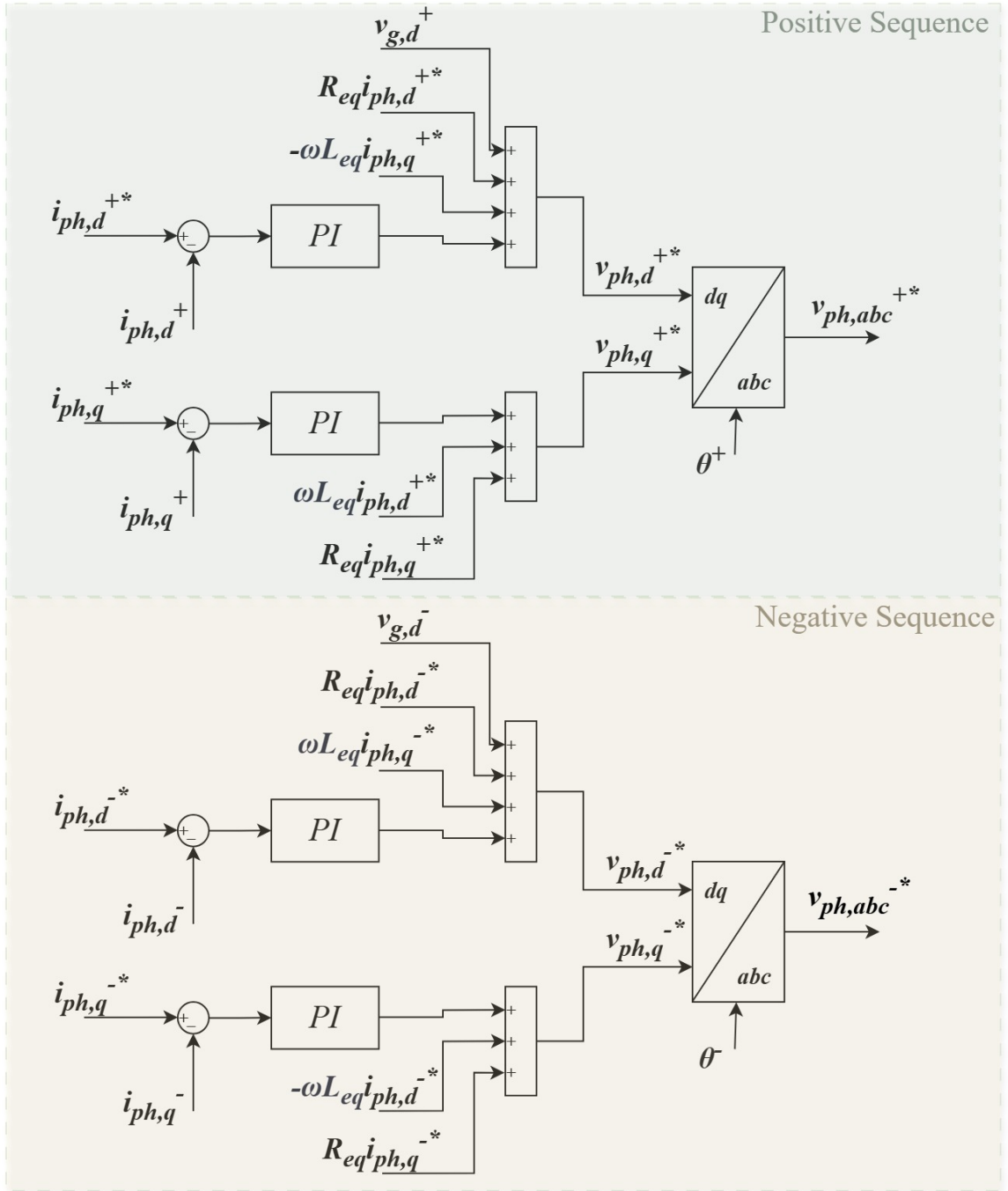


Figure 3.5: Grid Controller.

3.2. AC circulating-current controller

While the grid controller in Section 3.1 regulates the total power exchanged with the grid by shaping the injected phase currents, the AC circulating-current controller governs the internal power redistribution within each phase-leg. Its purpose is to transfer energy be-

tween the upper and lower arms and thus preserve balanced operation when the available arm powers differ, as introduced in Section 2.1.1.

For the j -th phase ($j \in \{a, b, c\}$), the arm power mismatch is defined by (2.5) as

$$P_{\text{diff},j}^* = P_{j,\text{up}}^* - P_{j,\text{low}}^*.$$

A non-zero $P_{\text{diff},j}^*$ represents the required operating condition imposed by DMPPT. The upper and lower arms of the j -th leg should therefore be commanded to generate different active powers according to their local maximum power points, intentionally creating an arm-to-arm imbalance. To realize the corresponding arm-power split, a fundamental-frequency AC circulating-current component is injected within the leg. By enforcing this component to be in phase with the corresponding phase-leg voltage, the circulating-current control enables the controlled transfer of power between the upper and lower arms.

In this work, the circulating-current regulation is implemented independently for each phase. Hence, three single-phase controllers are employed rather than a three-phase vector controller. This choice differs from the three-phase formulation in [14] and follows the single-phase approach of [20], here extended to a three-phase implementation. The main motivation is to obtain a balancing action that is local to each phase-leg, therefore computed only from the measurements and power requirements of that leg, without relying on cross-leg information.

Accordingly, the AC circulating-current controller operates on phase quantities derived from the phase-voltage reference $v_{ph,j}^*$ generated by the grid-current controller (Subsection 3.1.3), and it produces an additional circulating-voltage command $v_{\text{circ},j}^{ac*}$ that is later combined with the arm-voltage synthesis stage.

3.2.1. Phase synchronisation and dq measurements

To regulate the circulating current at the grid fundamental frequency on a per-phase basis, an auxiliary synchronous reference frame is defined for each phase j , so the corresponding electrical angle δ_j is obtained through a single-phase PLL driven by the phase-voltage reference $v_{ph,j}^*$. In order to construct an orthogonal signal to perform the dq transformations for single-phase signals, the β component of the phase voltage is generated via a Hilbert transform:

$$v_{\alpha,j}^* = v_{ph,j}^*, \quad v_{\beta,j}^* = \mathcal{H}\{v_{ph,j}^*\}, \quad (3.59)$$

where $\mathcal{H}\{\cdot\}$ denotes the Hilbert operator.

The resulting analytic signal $v_{\alpha,j}^* + jv_{\beta,j}^*$ is then mapped into the phase synchronous frame through the Park transform with reference angle δ_j . Denoting the corresponding dq components as $v_{ph,d,j}^*$ and $v_{ph,q,j}^*$, the PLL aligns the d axis with the phase voltage, so that $v_{ph,q,j}^* = 0$ in steady state and $v_{ph,d,j}^*$ represents the phase-voltage amplitude in the synchronous frame.

Circulating current measurements

The circulating current of the j -th phase-leg is defined as:

$$i_{\text{circ},j} = \frac{1}{2} (i_{j,\text{up}} + i_{j,\text{low}}). \quad (3.60)$$

In practical operation, $i_{\text{circ},j}$ includes a DC component and multiple harmonic components. Since this controller targets the fundamental-frequency component responsible for arm-to-arm power transfer, first the dc component of the circulating current is removed.

A moving-average operator evaluated over one fundamental period is introduced as

$$\langle x(t) \rangle_T \triangleq \frac{1}{T} \int_{t-T}^t x(\tau) d\tau, \quad T = \frac{2\pi}{\omega}. \quad (3.61)$$

Therefore,

$$i_{\text{circ},j}^{dc} = \langle i_{\text{circ},j}(t) \rangle_T. \quad (3.62)$$

So, the AC circulating-current component at the fundamental frequency is extracted through a band-pass filter centred at ω :

$$i_{\text{circ},j}^{ac} = \text{BPF}_{\omega} \{ i_{\text{circ},j} - i_{\text{circ},j}^{dc} \}. \quad (3.63)$$

This process is graphically represented by Figure 3.6.

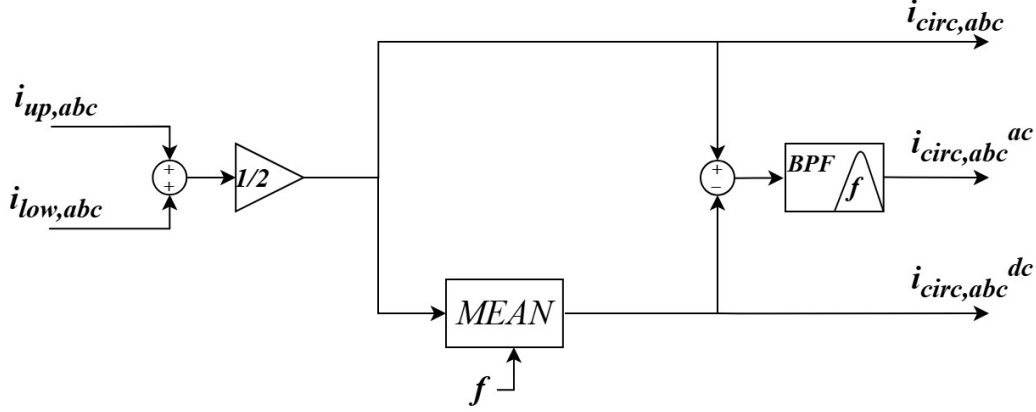


Figure 3.6: Circulating-currents computation.

Similarly to the voltage processing in (3.59), an orthogonal current component is built as

$$i_{\alpha,j}^{ac} = i_{\text{circ},j}^{ac}, \quad i_{\beta,j}^{ac} = \mathcal{H}\{i_{\text{circ},j}^{ac}\}, \quad (3.64)$$

and the Park transform with angle δ_j is applied to obtain the synchronous components

$$i_{\text{circ},d,j} + j i_{\text{circ},q,j} = (i_{\alpha,j}^{ac} + j i_{\beta,j}^{ac}) e^{-j\delta_j}. \quad (3.65)$$

In contrast with the phase-voltage case, $i_{\text{circ},q,j}$ is generally not zero, but it's driven to be zero through the controller to reduce losses. Its measured value is therefore needed to properly perform the control strategy as presented in the next section.

3.2.2. AC circulating-current reference from vertical mismatch

The differential power $P_{\text{diff},j}^*$ must now be transferred between the arms of the j -th phase-leg. Under the adopted single-phase formulation, this transfer is achieved by imposing a fundamental-frequency circulating-current component aligned with the phase voltage in the corresponding synchronous frame.

Since $P_{\text{diff},j}^*$ is purely-active and is driven by the circulating currents, it can be approximated as:

$$\frac{P_{\text{diff},j}^*}{2} = \frac{1}{2} i_{\text{circ},d,j} v_{ph,d,j}^*. \quad (3.66)$$

Therefore, a direct-axis reference for the AC circulating current is generated as

$$i_{\text{circ},d,j}^* = \frac{P_{\text{diff},j}^*}{v_{ph,d,j}^*}. \quad (3.67)$$

The quadrature reference is set to zero,

$$i_{\text{circ},q,j}^* = 0, \quad (3.68)$$

so that the injected circulating current is dedicated to differential active-power transfer rather than to internal reactive-power circulation.

3.2.3. AC circulating-voltage reference generation

The AC circulating-current controller generates the circulating-voltage reference required to enforce the desired dq current components. Defining the current errors as

$$e_{d,j} = i_{\text{circ},d,j}^* - i_{\text{circ},d,j}, \quad e_{q,j} = i_{\text{circ},q,j}^* - i_{\text{circ},q,j}, \quad (3.69)$$

two PI regulators per-phase are applied and the corresponding dq circulating-voltage references are then obtained as

$$v_{\text{circ},d,j}^{ac*} = k_{p,ac} e_{d,j} + k_{i,ac} \int e_{d,j} dt, \quad (3.70)$$

$$v_{\text{circ},q,j}^{ac*} = k_{p,ac} e_{q,j} + k_{i,ac} \int e_{q,j} dt, \quad (3.71)$$

where $k_{p,ac}$ and $k_{i,ac}$ are the proportional and integral gains of the AC circulating-current loop.

In this formulation, the PI outputs directly represent the additional circulating-voltage component required to drive the measured circulating current toward its reference at the fundamental frequency.

Finally, the voltage command is mapped back to the stationary frame through the inverse Park transform,

$$v_{\alpha,j}^{ac*} + j v_{\beta,j}^{ac*} = (v_{\text{circ},d,j}^{ac*} + j v_{\text{circ},q,j}^{ac*}) e^{j\delta_j}. \quad (3.72)$$

Since the implemented circulating-voltage injection is applied along the phase axis, only the α component is retained, yielding the commanded AC circulating voltage for the j -th phase:

$$v_{\text{circ},j}^{ac*} = v_{\alpha,j}^{ac*}. \quad (3.73)$$

The resulting signal $v_{\text{circ},j}^{ac*}$ constitutes the phase-dependent AC circulating-voltage contribution that enables arm-to-arm power transfer according to (2.5).

Therefore the output vector

$$\mathbf{v}_{circ,abc}^{ac*} \triangleq \begin{bmatrix} v_{circ,a}^{ac*} \\ v_{circ,b}^{ac*} \\ v_{circ,c}^{ac*} \end{bmatrix} \quad (3.74)$$

is then combined with the remaining circulating-voltage components and with the grid-voltage synthesis commanded by the grid controller.

Finally, the AC circulating-current controller strategy is shown graphically in Figure 3.7.

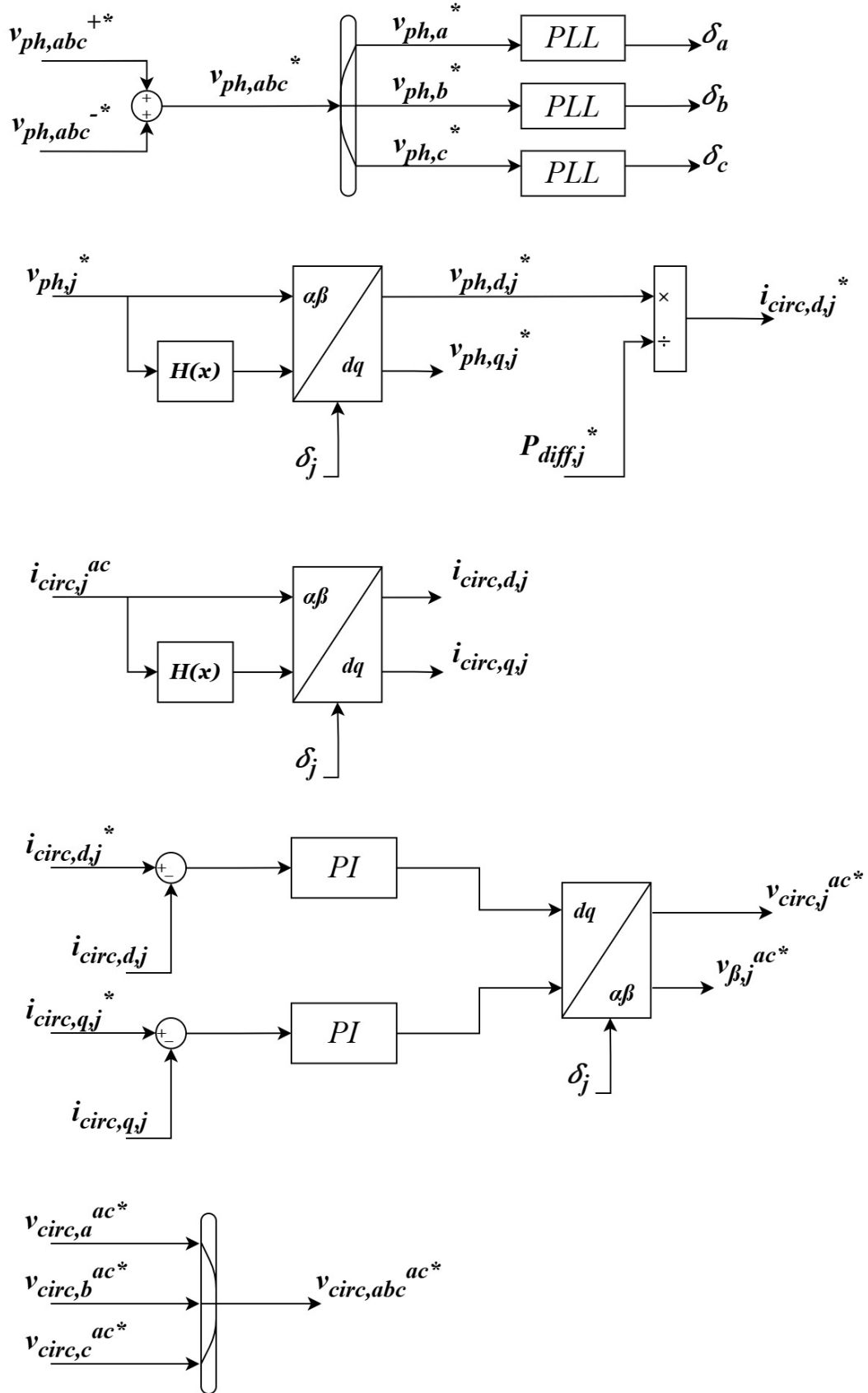


Figure 3.7: AC Circulating-current Controller.

3.3. DC circulating-current controller

As already presented, the grid controller regulates the external power exchange by enforcing the set-points P^* and Q^* , while the AC circulating-current controller redistributes power vertically within each phase-leg through the differential term $P_{\text{diff},j}^*$. The DC circulating-current controller, instead, compensates the horizontal mismatch among phase-legs, which is described here through the indicators introduced in Section 2.1.1. In particular, it is sufficient to define only two mismatch references, namely ΔP_{ab}^* and ΔP_{ac}^* , since the leg DC circulating currents are subject to the inherent zero-sum constraint

$$i_{\text{circ},a}^{dc} + i_{\text{circ},b}^{dc} + i_{\text{circ},c}^{dc} = 0,$$

so that the third independent mismatch is automatically determined.

Since the quantities on which the controller acts are DC by definition, no synchronous transformations are required and the control action is formulated directly in terms of the DC circulating-current component $i_{\text{circ},j}^{dc}$, defined in (3.62).

The objective is to generate DC circulating-current references that enable a controlled power transfer among phase-legs, so that the internal available power distribution can be coordinated with the grid-side operating point. In particular, the controller redistributes power in order to respect the available leg power references P_a^* , P_b^* , and P_c^* imposed by the PV-side constraints, while reproducing the phase-by-phase active-power injection required by the grid controller. This second requirement is non-trivial under voltage unbalance, because the grid-side controller generally imposes unequal injected powers among phases ($P_a^{\text{inj}} \neq P_b^{\text{inj}} \neq P_c^{\text{inj}}$).

The DC circulating-current channel is therefore used as an internal degree of freedom to transfer power among legs and reconcile the available leg powers with the unequal phase power injection demanded by the unbalanced grid conditions.

3.3.1. Injected phase-power split under unbalanced conditions

Under voltage unbalance, the grid controller enforces the sequence current references $\bar{I}_{ph}^{+*}$ and $\bar{I}_{ph}^{-*}$ (Section 3.1.2), which imply a specific phase-by-phase power injection, here denoted as P_a^{inj} , P_b^{inj} , and P_c^{inj} . Therefore, this section aims to provide a methodology to compute such quantities. To this end, the phase current and voltage phasors are first reconstructed from the positive- and negative-sequence components by means of Fortescue's transformation. Defining $a \triangleq e^{j2\pi/3}$, the voltage and current phasors can be

expressed as

$$\begin{cases} \bar{V}_{g,a} = \bar{V}_g^+ + \bar{V}_g^- \\ \bar{V}_{g,b} = a^2 \bar{V}_g^+ + a \bar{V}_g^- \\ \bar{V}_{g,c} = a \bar{V}_g^+ + a^2 \bar{V}_g^- \end{cases} \quad \begin{cases} \bar{I}_{ph,a}^* = \bar{I}_{ph}^{+*} + \bar{I}_{ph}^{-*} \\ \bar{I}_{ph,b}^* = a^2 \bar{I}_{ph}^{+*} + a \bar{I}_{ph}^{-*} \\ \bar{I}_{ph,c}^* = a \bar{I}_{ph}^{+*} + a^2 \bar{I}_{ph}^{-*} \end{cases} \quad (3.75)$$

The injected per-phase complex powers are then defined as

$$\bar{S}_j^{\text{inj}} = \frac{1}{2} \bar{V}_{g,j} \bar{I}_{ph,j}^*, \quad j \in \{a, b, c\}. \quad (3.76)$$

Recalling that, for phase a ,

$$\bar{S}_a^{\text{inj}} = \frac{1}{2} \bar{V}_{g,a} \bar{I}_{ph,a}^*, \quad (3.77)$$

and using Fortescue reconstruction,

$$\bar{V}_{g,a} = \bar{V}_g^+ + \bar{V}_g^-, \quad (3.78)$$

with the current references (3.39), one obtains

$$\bar{I}_{ph,a}^* = \frac{2}{3} (\bar{V}_g^+ + K \bar{V}_g^-) \left(\frac{P^*}{D_P} - j \frac{Q^*}{D_Q} \right), \quad (3.79)$$

where

$$D_P = |\bar{V}_g^+|^2 + K |\bar{V}_g^-|^2, \quad D_Q = |\bar{V}_g^+|^2 - K |\bar{V}_g^-|^2. \quad (3.80)$$

Taking the complex conjugate of (3.79) gives

$$\bar{I}_{ph,a} = \frac{2}{3} (\bar{V}_g^+ + K \bar{V}_g^-) \left(\frac{P^*}{D_P} + j \frac{Q^*}{D_Q} \right), \quad (3.81)$$

and therefore

$$\begin{aligned} \bar{S}_a^{\text{inj}} &= \frac{1}{2} (\bar{V}_g^+ + \bar{V}_g^-) \bar{I}_{ph,a}^* \\ &= \frac{1}{3} (\bar{V}_g^+ + \bar{V}_g^-) (\bar{V}_g^+ + K \bar{V}_g^-) \left(\frac{P^*}{D_P} + j \frac{Q^*}{D_Q} \right). \end{aligned} \quad (3.82)$$

Exploiting the Park components $\bar{V}_g^+ = v_{g,d}^+ e^{j\phi^+}$ and $\bar{V}_g^- = v_{g,d}^- e^{j\phi^-}$, it is possible to define the phase displacement between $\bar{V}_g^+$ and $\bar{V}_g^-$ as $\phi = \phi^- - \phi^+$. Hence, one obtains

$$\bar{S}_a^{\text{inj}} = \frac{1}{3} (v_{g,d}^+ + v_{g,d}^- e^{j\phi}) (v_{g,d}^+ + K v_{g,d}^- e^{-j\phi}) \left(\frac{P^*}{D_P} + j \frac{Q^*}{D_Q} \right), \quad (3.83)$$

where

$$D_P = (v_{g,d}^+)^2 + K(v_{g,d}^-)^2, \quad D_Q = (v_{g,d}^+)^2 - K(v_{g,d}^-)^2. \quad (3.84)$$

Expanding the voltage product and using Euler's identities $e^{\pm j\phi} = \cos \phi \pm j \sin \phi$,

$$\begin{aligned} (v_{g,d}^+ + v_{g,d}^- e^{j\phi})(v_{g,d}^+ + K v_{g,d}^- e^{-j\phi}) &= (v_{g,d}^+)^2 + K(v_{g,d}^-)^2 + v_{g,d}^+ v_{g,d}^- (e^{j\phi} + K e^{-j\phi}) \\ &= (v_{g,d}^+)^2 + K(v_{g,d}^-)^2 + v_{g,d}^+ v_{g,d}^- \left[(1 + K) \cos \phi + \right. \\ &\quad \left. + j(1 - K) \sin \phi \right]. \end{aligned} \quad (3.85)$$

Hence, defining

$$A_a \triangleq (v_{g,d}^+)^2 + K(v_{g,d}^-)^2 + v_{g,d}^+ v_{g,d}^- (1 + K) \cos \phi, \quad B_a \triangleq v_{g,d}^+ v_{g,d}^- (1 - K) \sin \phi, \quad (3.86)$$

(3.83) can be written as

$$\bar{S}_a^{\text{inj}} = \frac{1}{3} (A_a + jB_a) \left(\frac{P^*}{D_P} + j \frac{Q^*}{D_Q} \right) = \frac{1}{3} \left[\left(A_a \frac{P^*}{D_P} - B_a \frac{Q^*}{D_Q} \right) + j \left(A_a \frac{Q^*}{D_Q} + B_a \frac{P^*}{D_P} \right) \right]. \quad (3.87)$$

Accordingly,

$$P_a^{\text{inj}} = \Re \left\{ \bar{S}_a^{\text{inj}} \right\} = \frac{1}{3} \left(A_a \frac{P^*}{D_P} - B_a \frac{Q^*}{D_Q} \right), \quad (3.88)$$

$$Q_a^{\text{inj}} = \Im \left\{ \bar{S}_a^{\text{inj}} \right\} = \frac{1}{3} \left(A_a \frac{Q^*}{D_Q} + B_a \frac{P^*}{D_P} \right). \quad (3.89)$$

By repeating the same procedure for phases b and c , the corresponding injected powers can be expressed in compact form as

$$P_b^{\text{inj}} = \Re \left\{ \bar{S}_b^{\text{inj}} \right\} = \frac{1}{3} \left(A_b \frac{P^*}{D_P} - B_b \frac{Q^*}{D_Q} \right), \quad (3.90)$$

$$Q_b^{\text{inj}} = \Im \left\{ \bar{S}_b^{\text{inj}} \right\} = \frac{1}{3} \left(A_b \frac{Q^*}{D_Q} + B_b \frac{P^*}{D_P} \right), \quad (3.91)$$

$$P_c^{\text{inj}} = \Re \left\{ \bar{S}_c^{\text{inj}} \right\} = \frac{1}{3} \left(A_c \frac{P^*}{D_P} - B_c \frac{Q^*}{D_Q} \right), \quad (3.92)$$

$$Q_c^{\text{inj}} = \Im \left\{ \bar{S}_c^{\text{inj}} \right\} = \frac{1}{3} \left(A_c \frac{Q^*}{D_Q} + B_c \frac{P^*}{D_P} \right), \quad (3.93)$$

where the coefficients (A_b, B_b) and (A_c, B_c) are obtained from (3.86) by replacing ϕ with

$\phi - \frac{2\pi}{3}$ and $\phi + \frac{2\pi}{3}$, respectively:

$$A_b \triangleq (v_{g,d}^+)^2 + K(v_{g,d}^-)^2 + v_{g,d}^+ v_{g,d}^- (1 + K) \cos\left(\phi - \frac{2\pi}{3}\right), \quad (3.94)$$

$$B_b \triangleq v_{g,d}^+ v_{g,d}^- (1 - K) \sin\left(\phi - \frac{2\pi}{3}\right), \quad (3.95)$$

$$A_c \triangleq (v_{g,d}^+)^2 + K(v_{g,d}^-)^2 + v_{g,d}^+ v_{g,d}^- (1 + K) \cos\left(\phi + \frac{2\pi}{3}\right), \quad (3.96)$$

$$B_c \triangleq v_{g,d}^+ v_{g,d}^- (1 - K) \sin\left(\phi + \frac{2\pi}{3}\right). \quad (3.97)$$

Based on the indicators ΔP_{ab}^* and ΔP_{ac}^* already defined in (2.4), two injected active-power unbalance indicators are now introduced:

$$\Delta P_{ab}^{\text{inj}} = P_a^{\text{inj}} - P_b^{\text{inj}}, \quad \Delta P_{ac}^{\text{inj}} = P_a^{\text{inj}} - P_c^{\text{inj}}. \quad (3.98)$$

These quantities represent the horizontal power split target implied by the grid controller under the considered operating point.

3.3.2. DC circulating-current references from horizontal mismatch

The DC circulating-current controller is designed to compensate the discrepancy between the injected split (3.98) and the available split, by transferring DC power among legs. Using V_{dc}^* as the DC-voltage reference, the required differential DC currents are defined as

$$I_{(a-b)}^* = \frac{\Delta P_{ab}^{\text{inj}} - \Delta P_{ab}^*}{V_{dc}^*}, \quad I_{(a-c)}^* = \frac{\Delta P_{ac}^{\text{inj}} - \Delta P_{ac}^*}{V_{dc}^*}. \quad (3.99)$$

In (3.99), the numerator represents the active-power correction that must be exchanged among legs in order to reconcile the internal availability with the grid-imposed phase power split, while the division by V_{dc}^* converts such power correction into a DC current reference.

3.3.3. DC circulating-voltage reference generation

The measured DC circulating-current component for each leg is extracted from the circulating current $i_{\text{circ},j}$ by a moving-average operator over one fundamental period $T = 2\pi/\omega$, as shown in (3.62):

$$i_{\text{circ},j}^{dc} \triangleq \langle i_{\text{circ},j} \rangle_T = \frac{1}{T} \int_{t-T}^t i_{\text{circ},j}(\tau) d\tau.$$

The measured differential currents are then computed as

$$I_{(a-b)} = i_{\text{circ},a}^{dc} - i_{\text{circ},b}^{dc}, \quad I_{(a-c)} = i_{\text{circ},a}^{dc} - i_{\text{circ},c}^{dc}. \quad (3.100)$$

Defining the current errors

$$e_{ab} = I_{(a-b)}^* - I_{(a-b)}, \quad e_{ac} = I_{(a-c)}^* - I_{(a-c)}, \quad (3.101)$$

two PI regulators are employed to generate the corresponding differential DC circulating-voltage commands:

$$V_{(a-b)}^* = k_{p,dc} e_{ab} + k_{i,dc} \int e_{ab} dt, \quad (3.102)$$

$$V_{(a-c)}^* = k_{p,dc} e_{ac} + k_{i,dc} \int e_{ac} dt, \quad (3.103)$$

where $k_{p,dc}$ and $k_{i,dc}$ are the proportional and integral gains of the DC circulating-current loop.

The PI outputs (3.102)–(3.103) represent voltage differences among legs. They are mapped into the leg-wise DC circulating-voltage references $(v_{\text{circ},a}^{dc*}, v_{\text{circ},b}^{dc*}, v_{\text{circ},c}^{dc*})$ by imposing

$$V_{(a-b)}^* = v_{\text{circ},a}^{dc*} - v_{\text{circ},b}^{dc*}, \quad V_{(a-c)}^* = v_{\text{circ},a}^{dc*} - v_{\text{circ},c}^{dc*}, \quad (3.104)$$

together with the zero-sum constraint

$$v_{\text{circ},a}^{dc*} + v_{\text{circ},b}^{dc*} + v_{\text{circ},c}^{dc*} = 0, \quad (3.105)$$

which avoids introducing a common-mode offset in the circulating-voltage command. Solving (3.104)–(3.105) yields

$$\begin{cases} v_{\text{circ},a}^{dc*} = \frac{1}{3} (V_{(a-b)}^* + V_{(a-c)}^*), \\ v_{\text{circ},b}^{dc*} = \frac{1}{3} (-2V_{(a-b)}^* + V_{(a-c)}^*), \\ v_{\text{circ},c}^{dc*} = \frac{1}{3} (V_{(a-b)}^* - 2V_{(a-c)}^*). \end{cases} \quad (3.106)$$

The resulting vector

$$\mathbf{v}_{\text{circ},abc}^{dc*} \triangleq \begin{bmatrix} v_{\text{circ},a}^{dc*} \\ v_{\text{circ},b}^{dc*} \\ v_{\text{circ},c}^{dc*} \end{bmatrix} \quad (3.107)$$

provides the DC circulating-voltage contribution that enforces the desired horizontal power redistribution among phase-legs. This term is finally combined with the AC circulating-voltage contribution $v_{\text{circ},j}^{ac*}$ and with the grid-voltage synthesis commanded by the grid controller, ultimately defining the arm-voltage references used by the modulation and balancing layers.

Finally, the DC circulating-current controller strategy is graphically represented in Figure 3.8.

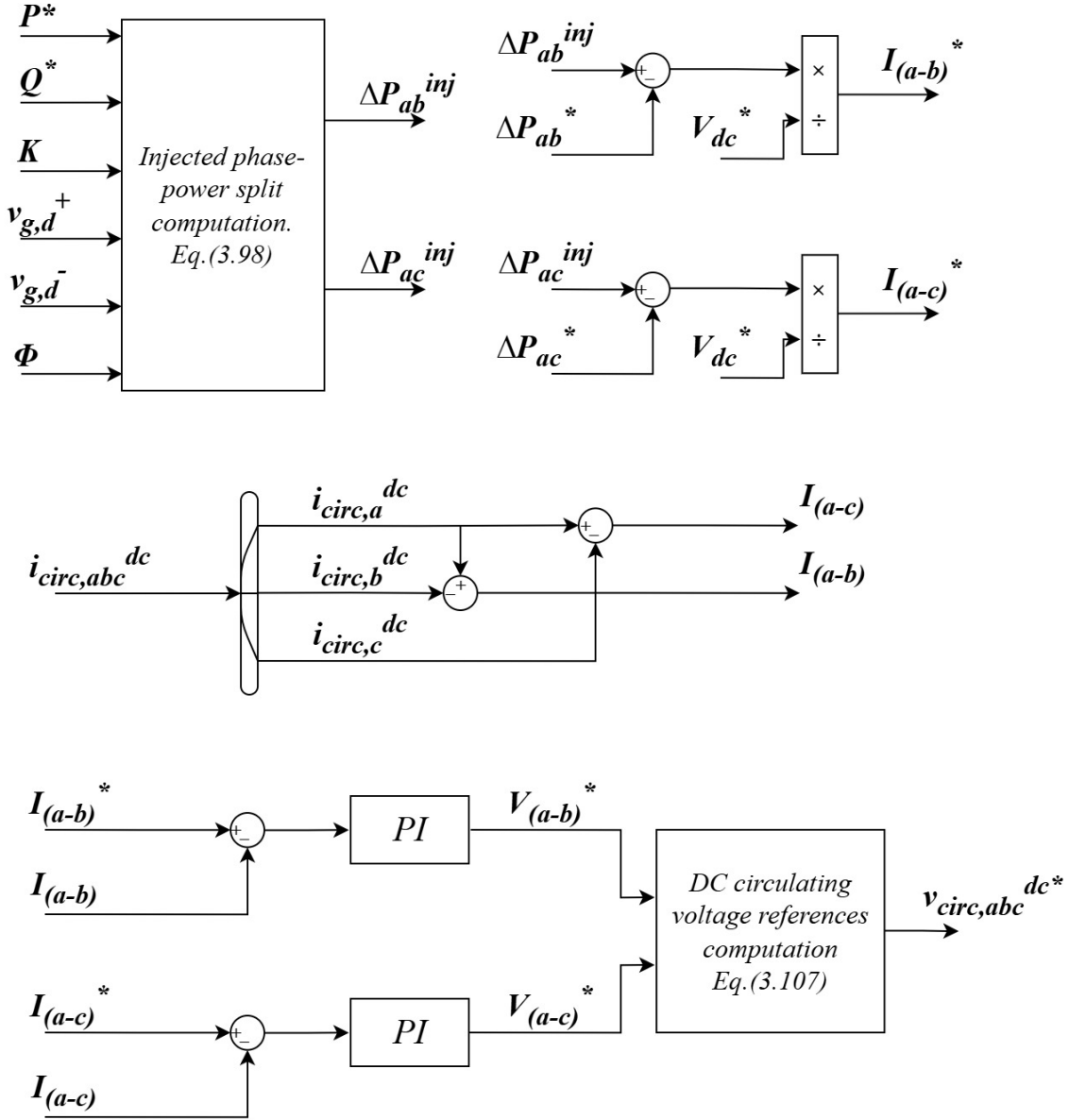


Figure 3.8: DC Circulating-current Controller.

3.4. Coordination and Arm-Voltage Reference Generation

The control architecture described in the previous sections produces three voltage-reference contributions that act on different physical channels:

1. the phase-voltage reference $\mathbf{v}_{ph,abc}^*$ generated by the grid controller,
2. the AC circulating-voltage reference $\mathbf{v}_{circ,abc}^{ac*}$ generated by the AC circulating-current

controller, and

3. the DC circulating-voltage reference $\mathbf{v}_{\text{circ},abc}^{dc*}$ generated by the DC circulating-current controller.

These contributions must be consistently combined into the final upper- and lower-arm voltage references supplied to the modulation and balancing layers. In this work, such coordination is carried out by the arm-voltage reference generator.

The inputs of the latter are $\mathbf{v}_{ph,abc}^*$, $\mathbf{v}_{\text{circ},abc}^{ac*}$, $\mathbf{v}_{\text{circ},abc}^{dc*}$ and the DC-link voltage reference V_{dc}^* . The outputs are the arm-voltage reference vectors

$$\mathbf{v}_{up,abc}^* = \begin{bmatrix} v_{up,a}^* & v_{up,b}^* & v_{up,c}^* \end{bmatrix}^\top, \quad \mathbf{v}_{low,abc}^* = \begin{bmatrix} v_{low,a}^* & v_{low,b}^* & v_{low,c}^* \end{bmatrix}^\top. \quad (3.108)$$

Therefore, introducing the vectors

$$\mathbf{v}_{ph,abc}^* \triangleq \begin{bmatrix} v_{ph,a}^* & v_{ph,b}^* & v_{ph,c}^* \end{bmatrix}^\top, \quad \mathbf{v}_{\text{circ},abc}^* \triangleq \begin{bmatrix} v_{\text{circ},a}^* & v_{\text{circ},b}^* & v_{\text{circ},c}^* \end{bmatrix}^\top, \quad \mathbf{1} \triangleq \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^\top, \quad (3.109)$$

and recalling that $\mathbf{v}_{\text{circ},abc}^* = \mathbf{v}_{\text{circ},abc}^{dc*} + \mathbf{v}_{\text{circ},abc}^{ac*}$, the corresponding arm-voltage references are then obtained by superimposing the external phase-voltage command with the internal circulating-voltage command and the DC-link contribution. In compact vector form,

$$\boxed{\mathbf{v}_{up,abc}^* = -\mathbf{v}_{ph,abc}^* + \frac{1}{2} \left(V_{dc}^* \mathbf{1} - \mathbf{v}_{\text{circ},abc}^{dc*} - \mathbf{v}_{\text{circ},abc}^{ac*} \right)} \quad (3.110)$$

$$\boxed{\mathbf{v}_{low,abc}^* = \mathbf{v}_{ph,abc}^* + \frac{1}{2} \left(V_{dc}^* \mathbf{1} - \mathbf{v}_{\text{circ},abc}^{dc*} - \mathbf{v}_{\text{circ},abc}^{ac*} \right)} \quad (3.111)$$

Equations (3.110)–(3.111) highlight the coordination principle adopted in this thesis: the grid controller sets the required ac terminal voltage through $\mathbf{v}_{ph,abc}^*$ to achieve the prescribed external power exchange, while the circulating-voltage contributions $\mathbf{v}_{\text{circ},abc}^{ac*}$ and $\mathbf{v}_{\text{circ},abc}^{dc*}$ shape the internal energy redistribution among phase-legs. The DC-link reference V_{dc}^* provides the common dc component required by the arm-voltage synthesis.

Figure 3.8 shows the graphical representation of this last module.

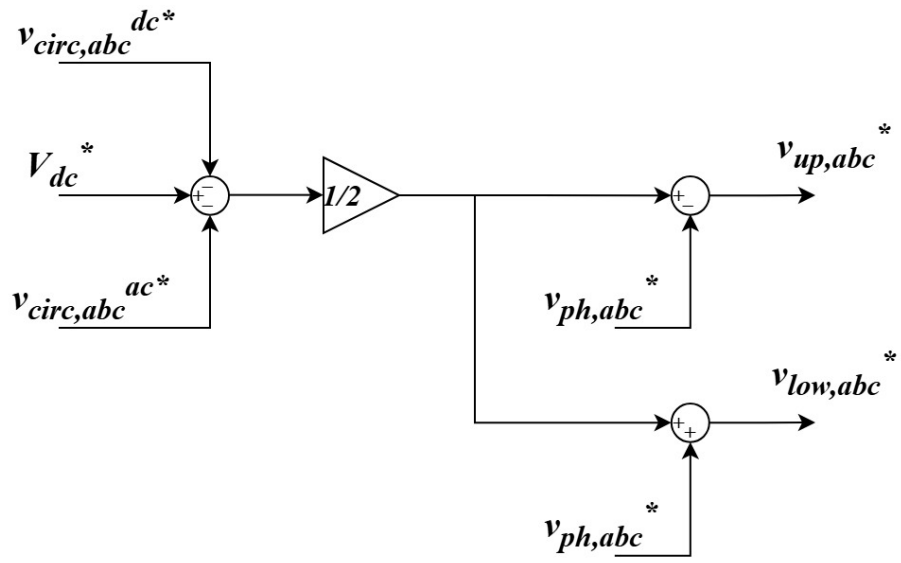


Figure 3.9: Arm-voltage reference generator.

4 | Simulation Results

This chapter presents the time-domain simulation results of the proposed MMC control strategy, with particular emphasis on the combined presence of grid-voltage negative sequence and power availability mismatch among the converter legs.

4.1. Simulation Setup and Control Parameters

This section summarizes the electrical parameters of the simulated MMC system and the tuning values adopted for the PI regulators in the three control functions introduced in Chapter 2.

4.1.1. Electrical Parameters

Table 4.1 reports the key electrical parameters used in the simulations. The grid is modeled at the fundamental frequency f_g with both positive- and negative-sequence components. Moreover, as already introduced in Subsection 3.1.1, the positive-sequence phase displacement has been set to zero.

Table 4.1: Electrical parameters used in the time-domain simulations.

Parameter	Value
Grid fundamental frequency f_g	50 Hz
Positive-sequence grid voltage magnitude V_g^+	326.6 V
Positive-sequence phase displacement ϕ^+	0 °
Negative-sequence grid voltage magnitude V_g^-	65.3 V
Voltage unbalance ratio (V_g^-/V_g^+)	20 %
Negative-sequence phase displacement ϕ^-	30 °
DC-link voltage reference V_{dc}^*	783.8 V
Output resistance R_g	140 mΩ
Output inductance L_g	3.9 mH
Arm resistance R_{arm}	5 mΩ
Arm inductance L_{arm}	1 mH
Mutual arm inductance M_{arm}	0 mH
DC-bus capacitance C_{dc}	5 mF
Total Imaginary Power Reference Q_T^*	1 kvar

4.1.2. PI Controller Gains

The PI controllers are tuned to ensure a stable hierarchy of bandwidths, with the inner current-regulation loops faster than the outer power-management actions. Table 4.2 reports the proportional and integral gains adopted for the main PI regulators, which were selected through iterative trial-and-error tuning.

Table 4.2: PI controller gains used in the simulations.

PI controller	k_p	k_i
Grid controller	1	11
AC circulating controller	3	20
DC circulating controller	5	40

4.2. Results Presentation

The results are presented by progressively analysing the main sources of asymmetry addressed in this thesis.

Two representative scenarios are considered to analyse the effects of grid unbalance and

PV-related power mismatch:

1. *Unbalanced Grid, No Internal Power Mismatch.*

In this scenario a negative-sequence grid voltage component is applied while the available PV power is assumed identical across legs and arms. This case primarily validates the dual-sequence grid controller behaviour and the impact of the selector K on the oscillatory power terms.

2. *Unbalanced Grid, Leg Power Mismatch, and Arm Mismatch.*

Secondly, the most general and realistic scenario is considered by introducing power mismatches within arm and phase-legs. This case requires the coordinated action of all three control functions, including the AC circulating-current controller to transfer differential power within each leg.

For each scenario, the discussion follows the same order to maintain consistency, testing the controller in the three operating modes defined by the selector $K \in \{-1, 0, 1\}$, introduced in Section 3.1.

To complement the waveform-based discussion, a quantitative steady-state assessment is also reported for each operating mode: the plots provide the graphical simulation results together with rough numerical values, which are then analysed more accurately in the discussion. Defining the fundamental period $T_g = \frac{1}{f_g}$, all numerical metrics are extracted over the same $2T_g$ steady-state window used in the corresponding plots. In particular, the mean values $\overline{P}$, $\overline{Q}$, and $\overline{Q}_T$ are computed over that window, while ripple quantities are reported as peak-to-peak values and denoted by $\Delta(\cdot)_{pp}$ (e.g., ΔP_{pp} , ΔQ_{pp}).

4.2.1. Case 1: Unbalanced Grid, No Internal Power Mismatch

This scenario is intended to validate the grid controller under asymmetric steady-state conditions in the absence of internal power constraints. In particular, a negative-sequence component is imposed at the PCC, whose magnitude and phase displacement are reported in Table 4.1.

Since no internal available-power imbalance is present, the arm available powers are uniform and equal across phases, as summarised in Table 4.3. Consequently, recalling (3.1),

$$P^* = \sum_{j \in \{a,b,c\}} P_{j,\text{up}}^* + \sum_{j \in \{a,b,c\}} P_{j,\text{low}}^* = 6000 \text{ W}.$$

The internal mismatch indicators, instead, are identically null, such that $P_{\text{diff},a}^* = P_{\text{diff},b}^* =$

$P_{\text{diff},c}^* = 0$ and $\Delta P_{ab}^* = \Delta P_{ac}^* = 0$. Anyway, it is worth noting that, even in the absence of PV-side power mismatch, an unbalanced grid generally implies a phase-dependent injected active-power split as a consequence of the negative-sequence voltage component. Therefore, the injected power unbalance indicators $\Delta P_{ab}^{\text{inj}}$ and $\Delta P_{ac}^{\text{inj}}$ defined in (3.98) are in general non-zero also in Case 1, and the DC circulating-current controller acts on $i_{\text{circ},j}^{dc}$ to redistribute power among legs and accommodate such phase-wise power split, even when the available powers are uniform.

Table 4.3: Available arm powers used in Case 1.

	Leg a	Leg b	Leg c
Upper arm	$P_{a,\text{up}}^* = 1000 \text{ W}$	$P_{b,\text{up}}^* = 1000 \text{ W}$	$P_{c,\text{up}}^* = 1000 \text{ W}$
Lower arm	$P_{a,\text{low}}^* = 1000 \text{ W}$	$P_{b,\text{low}}^* = 1000 \text{ W}$	$P_{c,\text{low}}^* = 1000 \text{ W}$

In the following, the three operating modes $K \in \{-1, 0, 1\}$ are analysed.

K=-1: Active-power oscillation suppression.

Figures 4.1–4.2 report the transient evolution of P , of the instantaneous imaginary power Q , and of the total reactive power Q_T .

To allow the PLL to settle, the converter is connected to the grid through a three-phase breaker at $t \approx 0.1 \text{ s}$. After the connection, the control loops drive the system to steady state, which is reached at approximately $t \approx 1.5 \text{ s}$. Let $\bar{P}$, $\bar{Q}$ and $\bar{Q}_T$ be the mean values over a period $T_g = \frac{1}{f_g}$ at steady state.

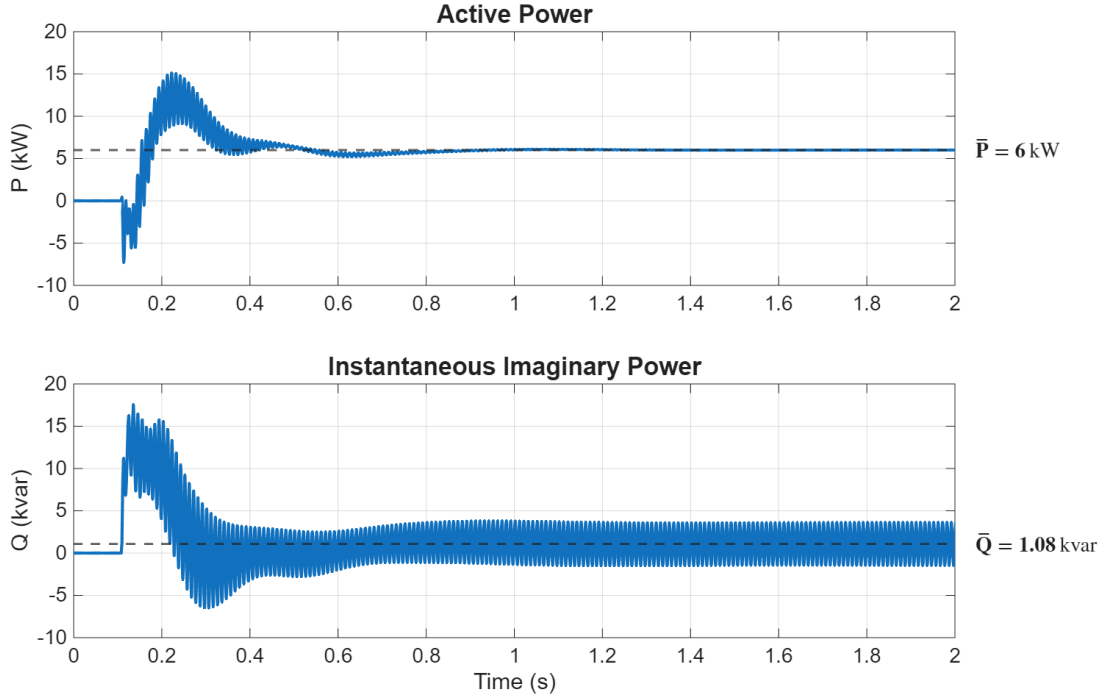


Figure 4.1: Case 1 ($K = -1$) – Transient behavior of P and Q .

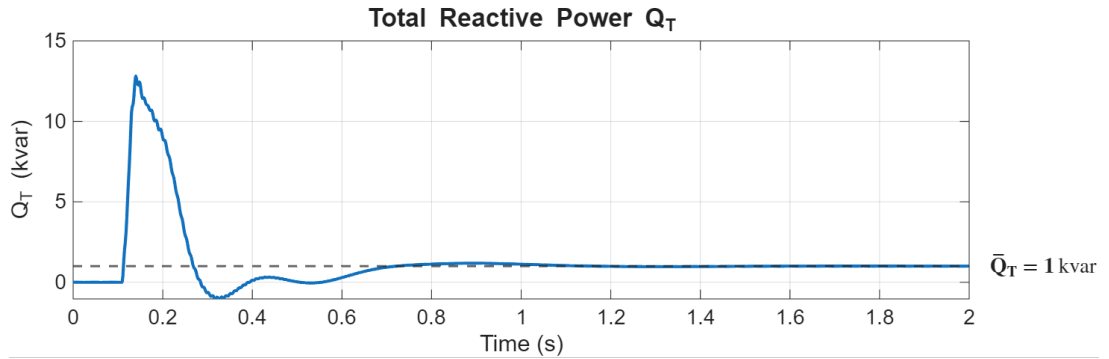


Figure 4.2: Case 1 ($K = -1$) – Transient behavior of Q_T .

Figure 4.3 shows P and Q over a $2T_g$ window at steady state. The results are consistent with the theoretical behavior of the $K = -1$ mode, which is designed to suppress the active power oscillatory term at $2f_g$:

- the active power remains regulated around the reference P^* , with only a small residual ripple attributable to the PI corrective action;
- the instantaneous imaginary power Q exhibits a pronounced 2ω oscillation, which is expected when the negative-sequence current is intentionally injected to shape the oscillatory power term. Moreover, the mean value $\bar{Q}$ does not coincide with $\bar{Q}_T$

because, under dual-sequence operation, Q and Q_T are not equivalent quantities (as introduced in Paragraph 3.1.2). This is consistent with the measured sequence contributions, $\bar{Q}^+ \simeq 1.041$ kvar and $\bar{Q}^- \simeq -0.041$ kvar, whose combination yields a total reactive-power quantity different from $\bar{Q}$.

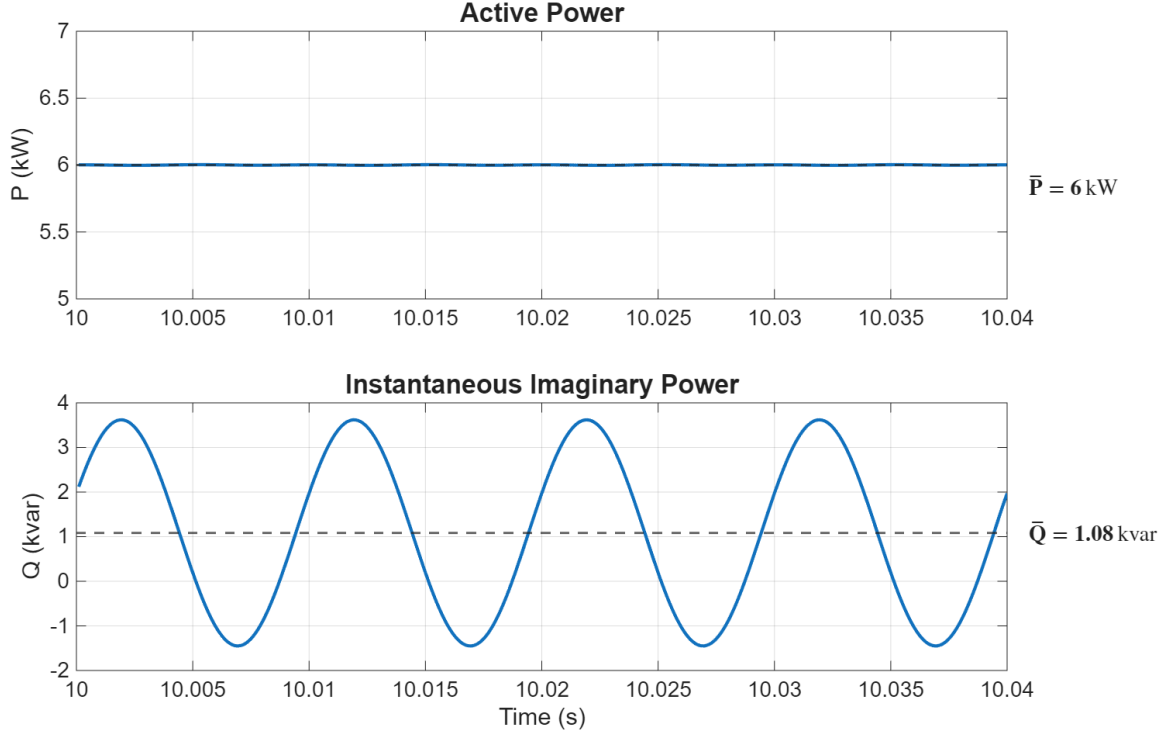


Figure 4.3: Case 1 ($K = -1$) – Steady-state behavior of P and Q over two grid periods.

Figure 4.4 reports the steady-state waveforms of v_g and i_{ph} over the same $2T_g$ window. As expected under unbalanced grid conditions, the voltage contains a negative-sequence component, and the current injection reflects the adopted strategy for $K = -1$, which allows a non-zero negative-sequence current component in order to prioritize active-power ripple suppression.

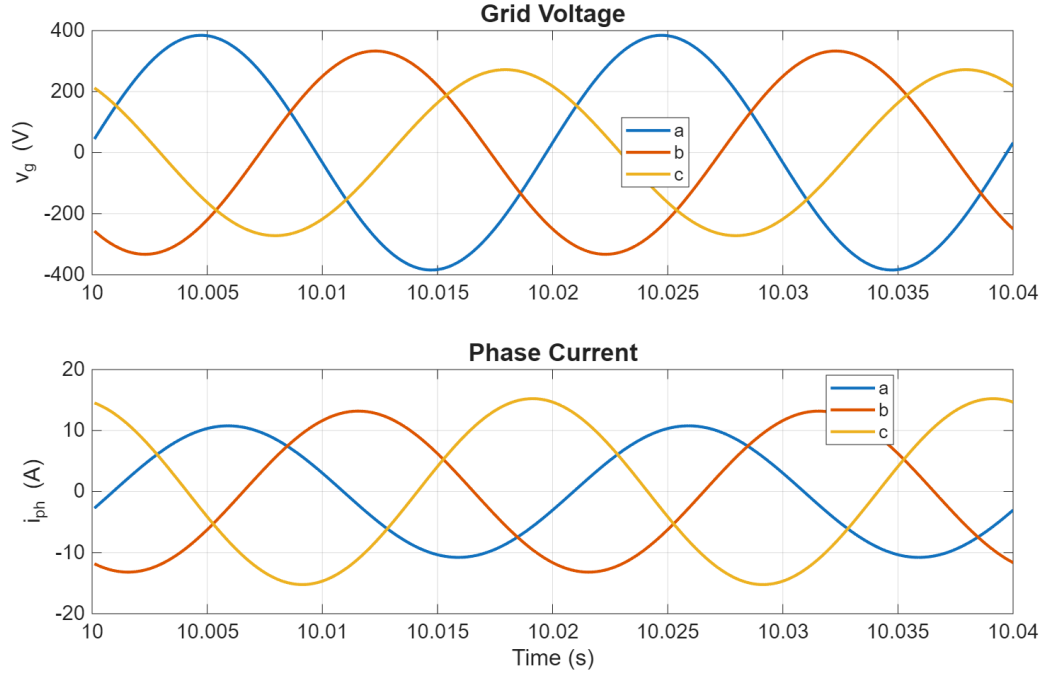


Figure 4.4: Case 1 ($K = -1$): steady-state waveforms of the grid voltage v_g and phase current i_{ph} over two grid periods.

Figure 4.5 shows that all average arm powers converge to the corresponding available-power references in Table 4.3. Accordingly, as shown in Figure 4.6, the AC circulating-current components remain limited, while DC components are injected in order to fulfill injected power requirements in each phase. It is worth noting that $i_{\text{circ},a}$ and $i_{\text{circ},c}$ are completely overlapped.

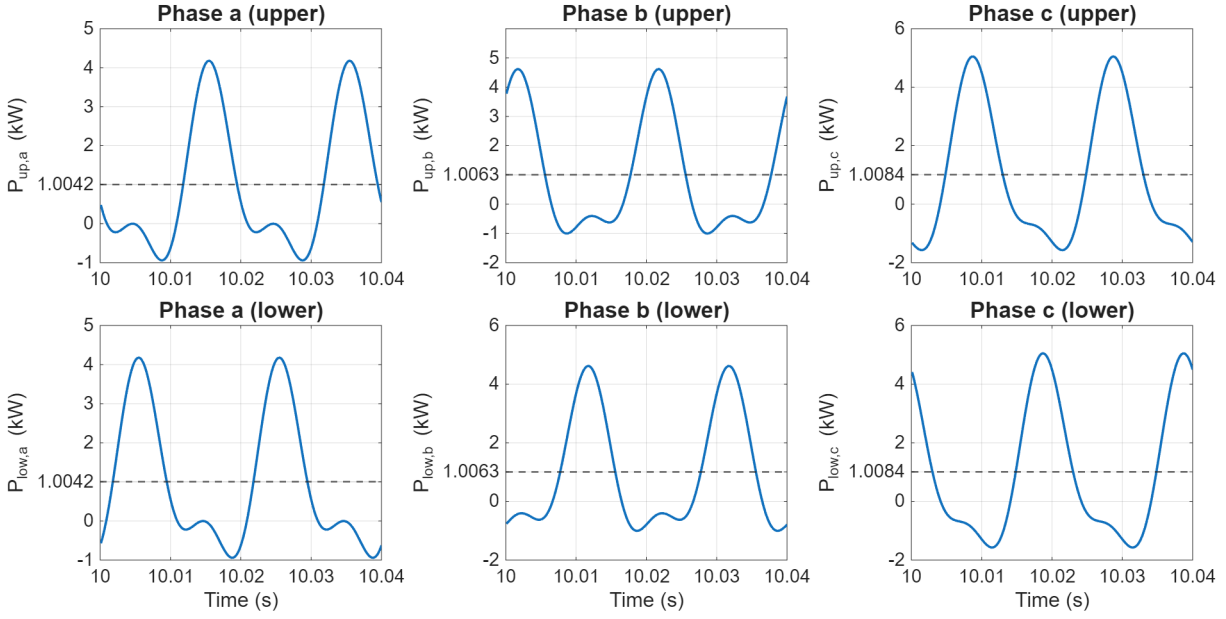


Figure 4.5: Case 1 ($K = -1$) – Generated arm powers over two grid periods.

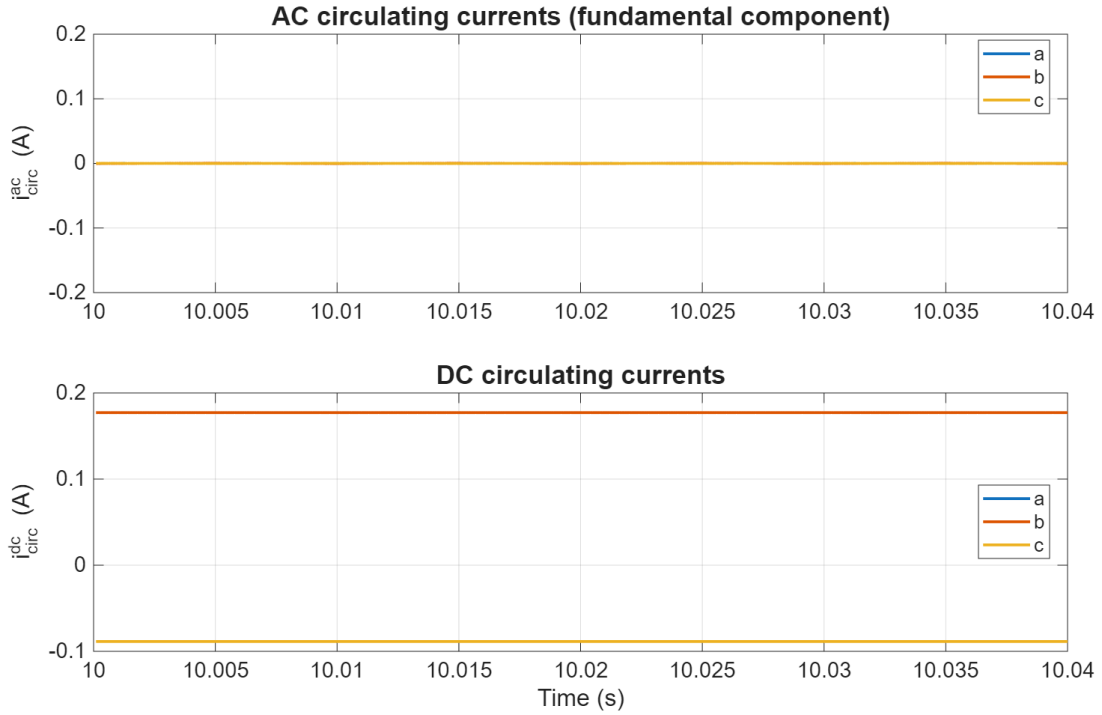


Figure 4.6: Case 1 ($K = -1$) – AC and DC circulating currents over two grid periods.

K=0: Balanced output currents.

Figures 4.7–4.8 illustrate the transient evolution of the active power P , the instantaneous imaginary power Q , and the total reactive power Q_T for $K = 0$.

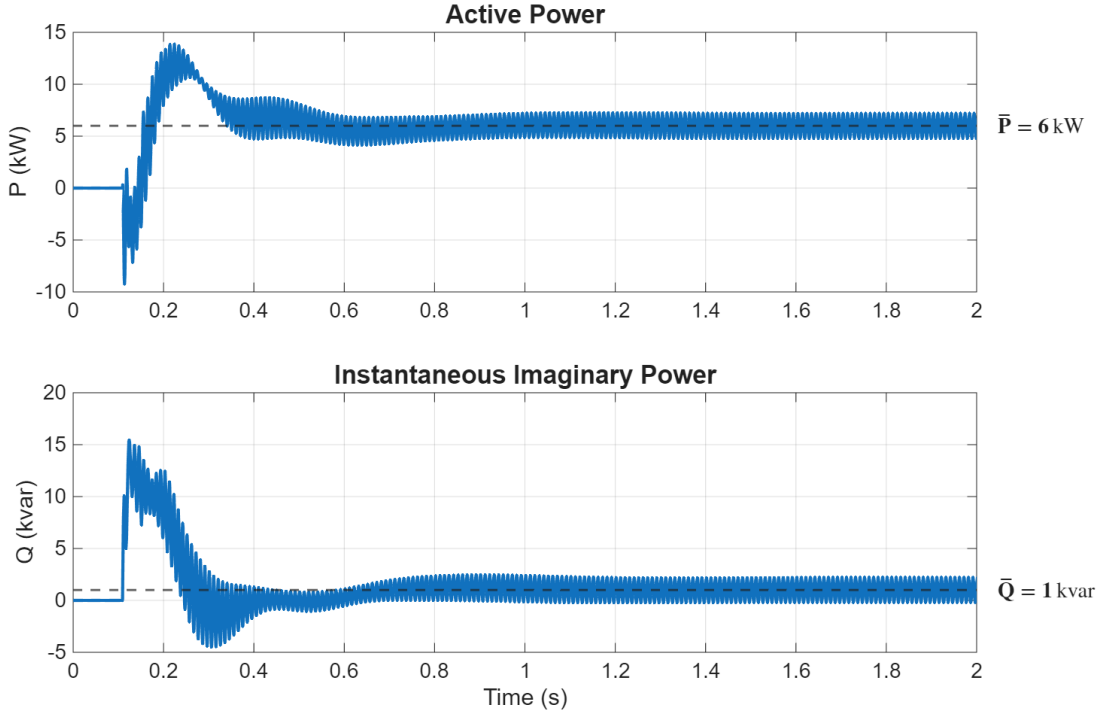


Figure 4.7: Case 1 ($K = 0$) – Transient behavior of P and Q .

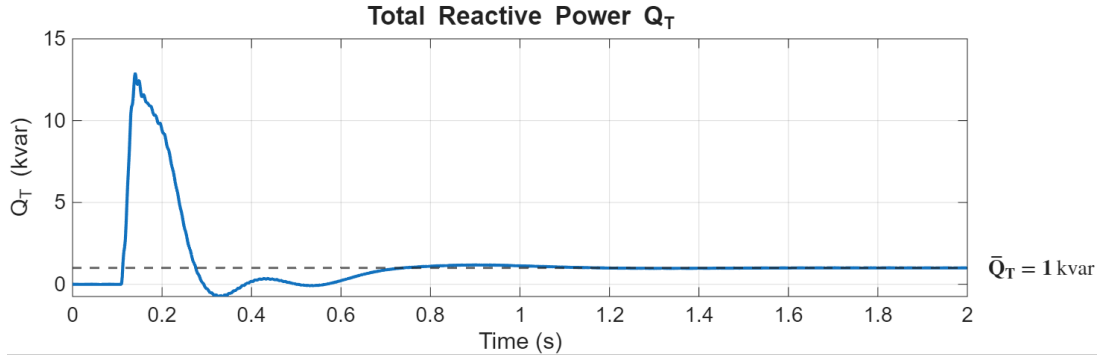


Figure 4.8: Case 1 ($K = 0$) – Transient behavior of Q_T .

Figure 4.9 shows P and Q over a $2T_g$ steady-state window. The behavior is consistent with the defining feature of the $K = 0$ mode, namely $\bar{I}_{ph}^{-*} = 0$: the controller enforces balanced phase currents (no negative-sequence current injection) at the expense of oscillatory power terms. In particular:

- both P and Q exhibit a pronounced 2ω ripple, which originates from the interaction between the negative-sequence voltage component and a purely positive-sequence current injection with no compensating negative-sequence component;
- the average active power still tracks the reference ($\bar{P} \approx P^*$), confirming correct

steady-state power regulation despite the oscillatory exchange;

Moreover, for $K = 0$ the negative-sequence complex power is null and $Q^- = 0$. As a consequence, the distinction between Q and Q_T vanishes in average, since

$$\overline{Q} = \overline{Q}_T,$$

meaning that at steady-state $\overline{Q} = \overline{Q}_T \approx Q_T^*$. This is confirmed by the measured sequence components, namely $\overline{Q}^+ \simeq 1 \text{ kvar}$ and $\overline{Q}^- \simeq 0 \text{ kvar}$.

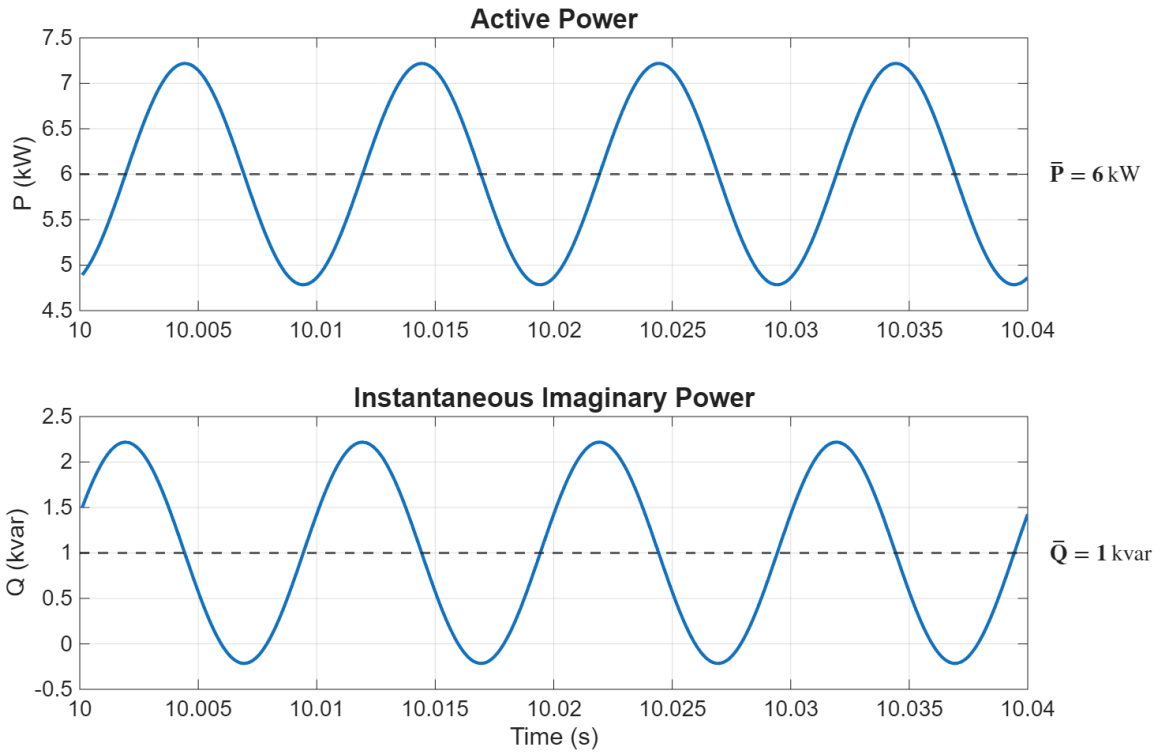


Figure 4.9: Case 1 ($K = 0$) – Steady-state behavior of P and Q over two grid periods.

Figure 4.10 reports the steady-state waveforms of v_g and i_{ph} over the same $2T_g$ interval. As expected under unbalanced grid conditions, v_g contains a negative-sequence component. Conversely, the injected currents remain balanced and sinusoidal across phases, which is precisely the objective of the $K = 0$ mode.

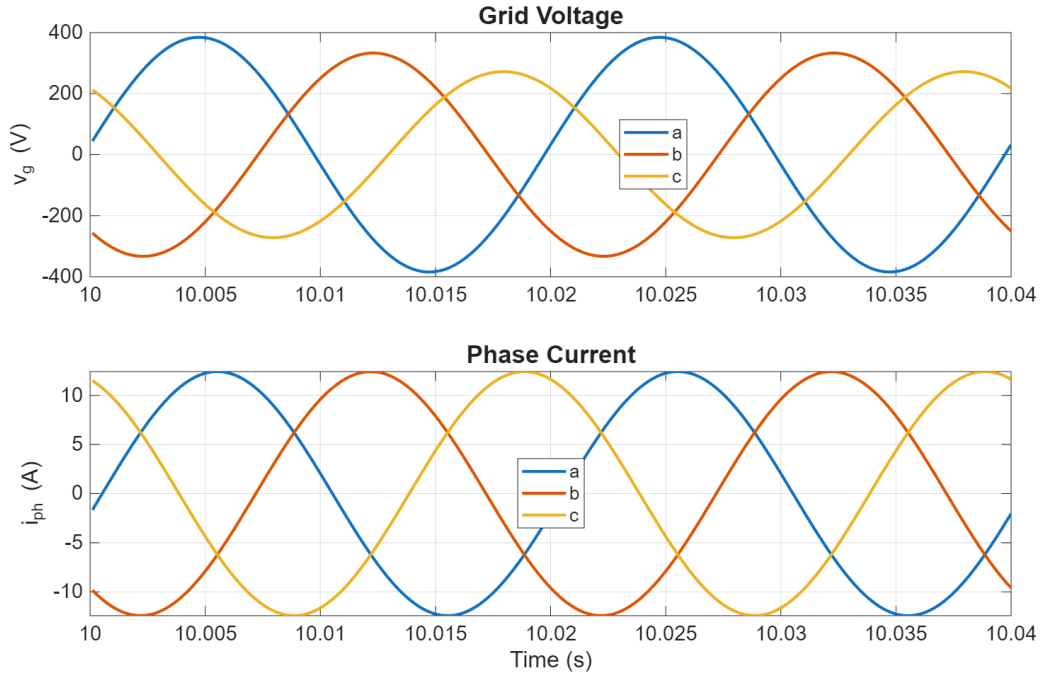


Figure 4.10: Case 1 ($K = 0$): steady-state waveforms of the grid voltage v_g and phase current i_{ph} over two grid periods.

The internal power variables confirm that all arm average powers (reported in Figure 4.11) converge to the references in Table 4.3. Nevertheless, because an unbalanced voltage combined with balanced currents generally implies an unequal phase-wise power split on the grid side, a limited inter-leg redistribution may still be required. This is reflected in Figure 4.12: the AC circulating-current components remain close to zero (no vertical arm-to-arm transfer is demanded, as $P_{\text{diff},j}^* = 0$), whereas DC circulating-current components appear to accommodate the phase-dependent power exchange dictated by the unbalanced grid, while preserving balanced output currents.

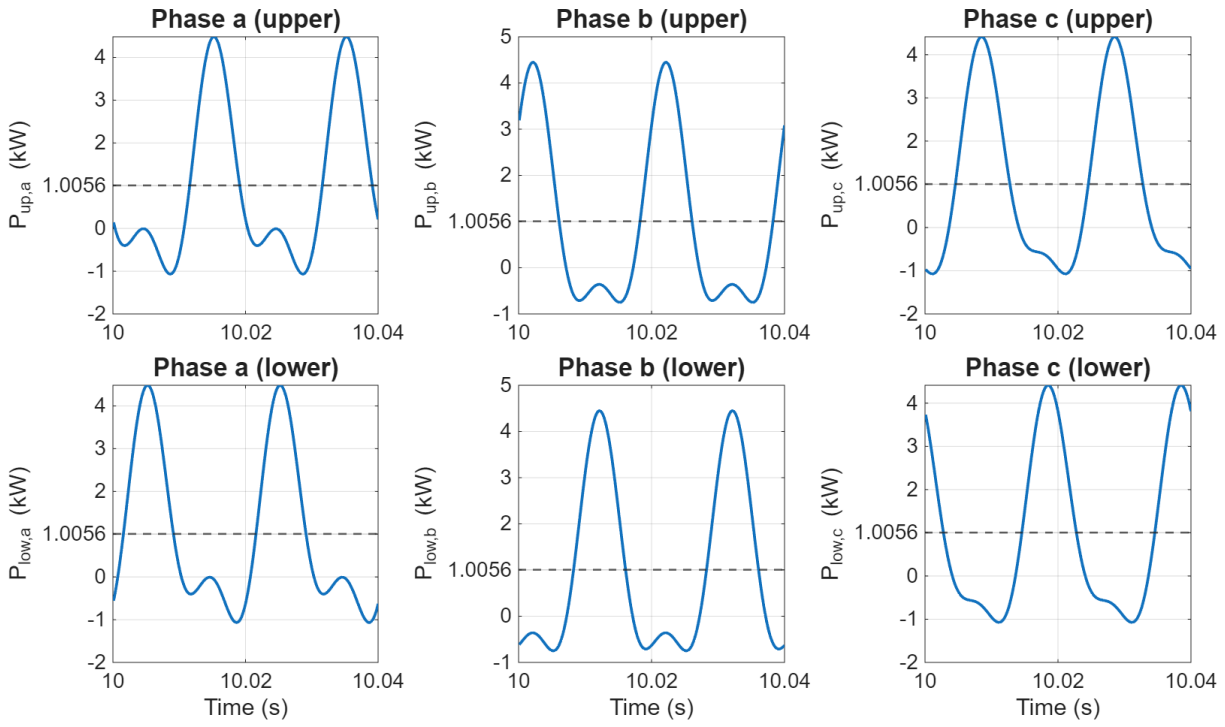


Figure 4.11: Case 1 ($K = 0$) – Generated arm powers over two grid periods.

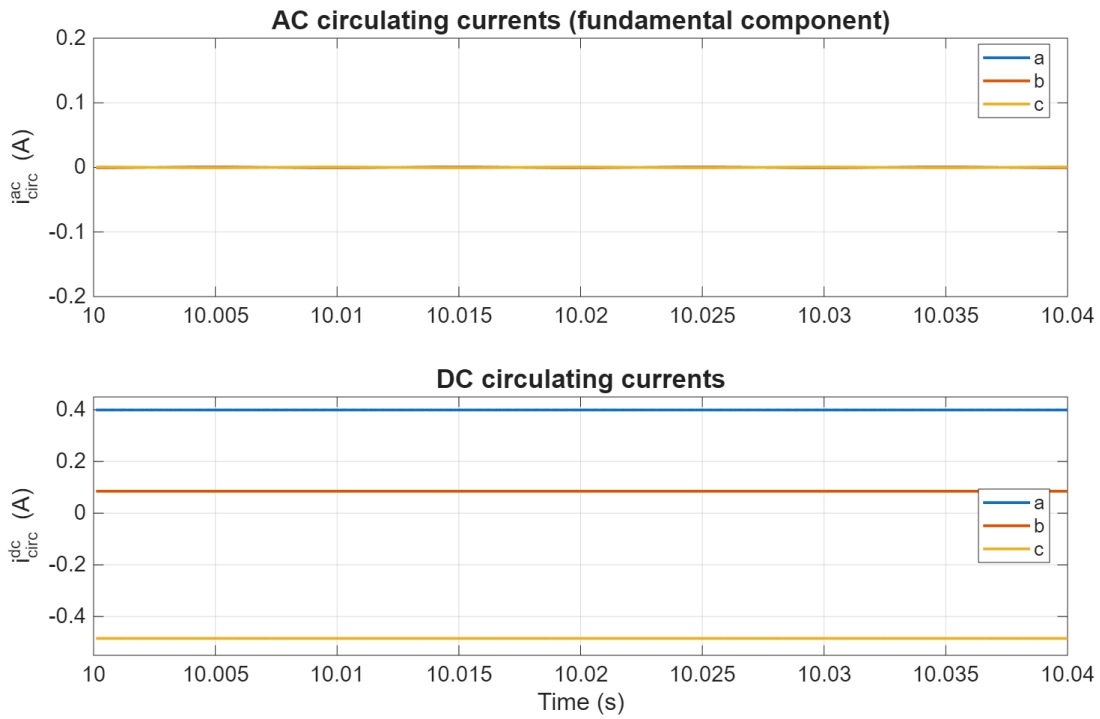


Figure 4.12: Case 1 ($K = 0$) – AC and DC circulating currents over two grid periods.

K=+1: Reactive-power oscillation suppression.

Figures 4.13–4.14 show the transient evolution of the active power P , the instantaneous imaginary power Q , and the total reactive power Q_T when the selector is set to $K = +1$. In this operating mode, the negative-sequence current reference is purposely shaped so that the oscillatory complex-power term becomes real, thereby minimizing the ripple associated with the imaginary power. This effect is clearly visible in Figure 4.13, where the PI regulators progressively attenuate the oscillations in Q .

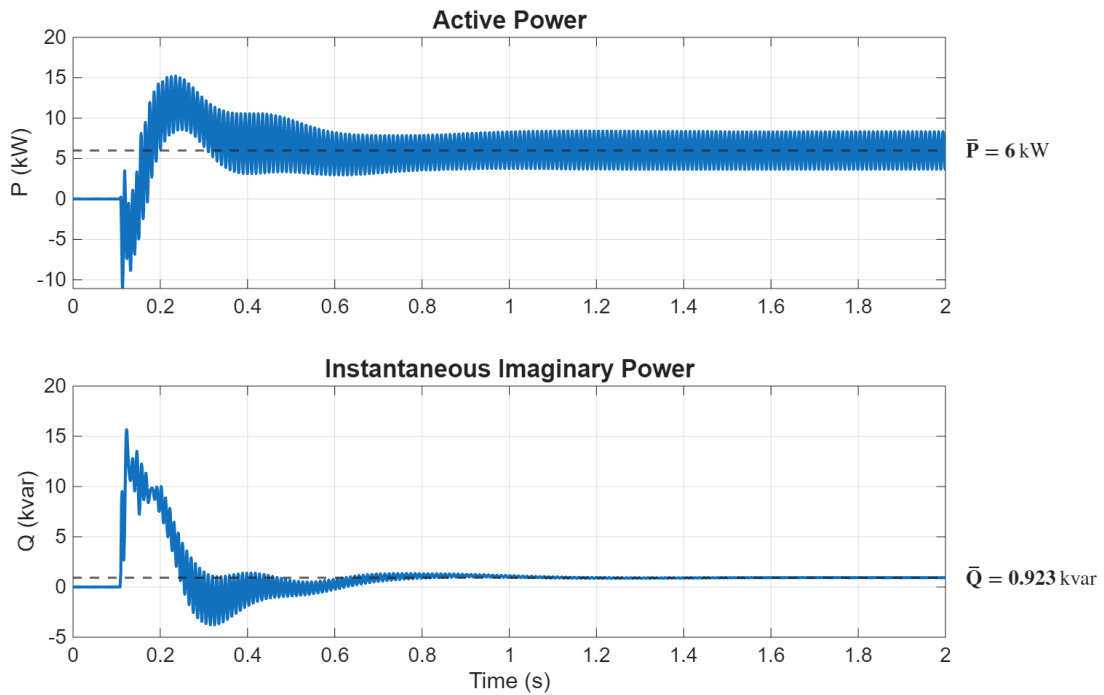


Figure 4.13: Case 1 ($K = 1$) – Transient behavior of P and Q .

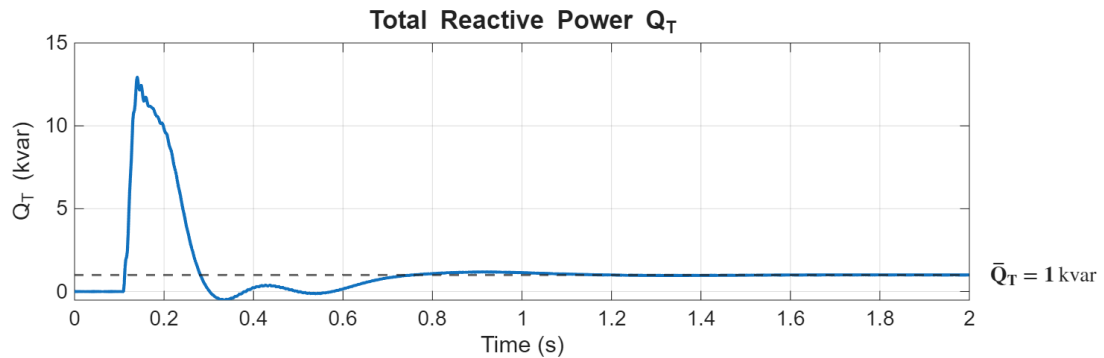


Figure 4.14: Case 1 ($K = 1$) – Transient behavior of Q_T .

The steady-state waveforms over the last $2T_g$ are reported in Fig. 4.15. The results align

with the expected trade-off of the $K = +1$ formulation:

- the instantaneous imaginary power Q becomes nearly constant, with only a small residual ripple, indicating that the controller effectively mitigates the 2ω oscillatory component;
- conversely, the active power P now exhibits the dominant 2ω oscillation. Nevertheless, its average value remains regulated at the prescribed reference P^* , confirming that the grid controller still enforces the desired mean power exchange while redistributing the oscillatory content.

Moreover, since a negative-sequence current is injected, $\bar{Q} \neq \bar{Q}_T$ as expected. The measured sequence components are $\bar{Q}^+ \simeq 0.961$ kvar and $\bar{Q}^- \simeq 0.038$ kvar.

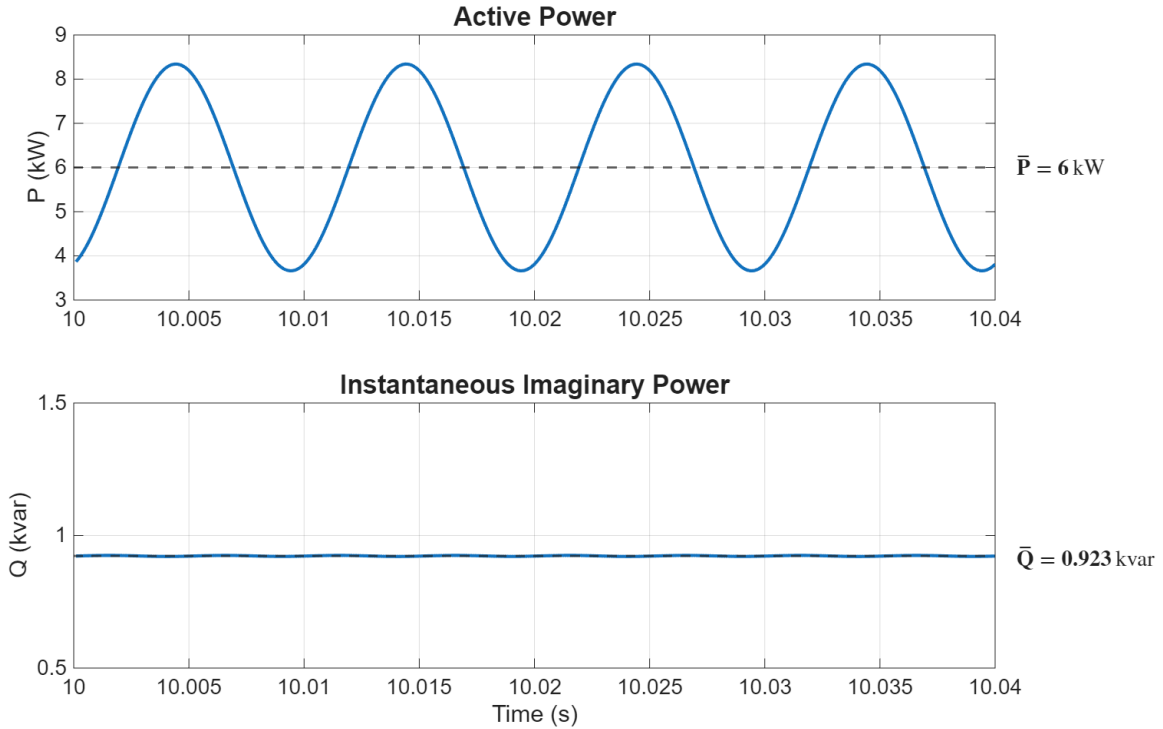


Figure 4.15: Case 1 ($K = 1$) – Steady-state behavior of P and Q over two grid periods.

Figure 4.16 depicts the grid voltage and phase currents over the same interval. While v_g includes a negative-sequence component by construction, the injected current also contains a controlled negative-sequence contribution that is intentionally generated to shape the 2ω power term.

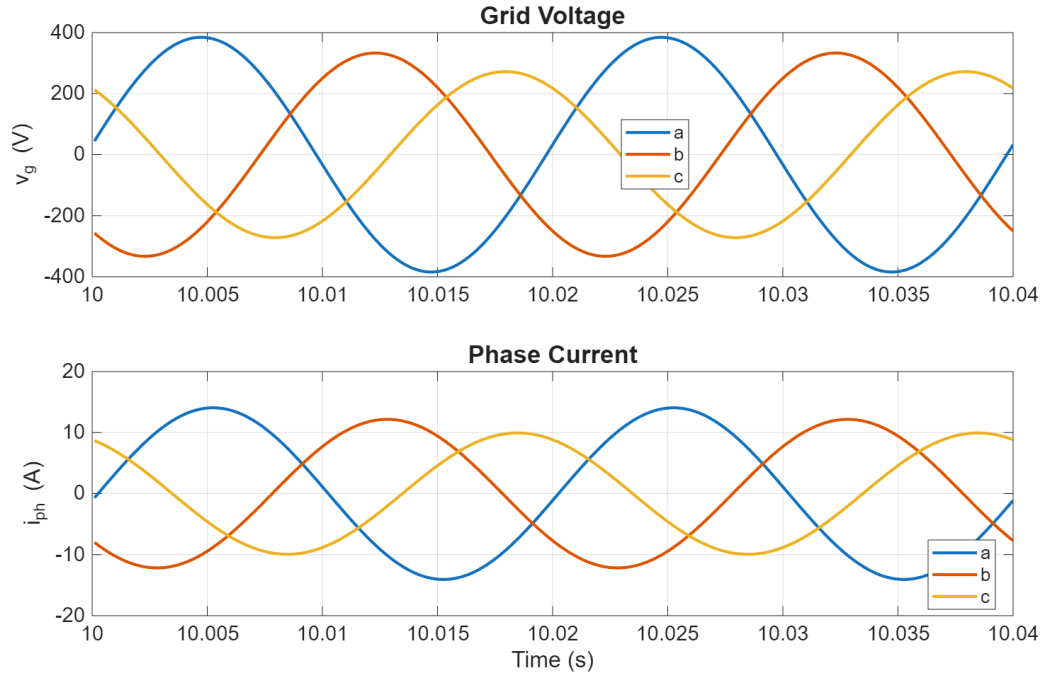


Figure 4.16: Case 1 ($K = 1$): steady-state waveforms of the grid voltage v_g and phase current i_{ph} over two grid periods.

Finally, the internal power variables confirm that, also for $K = +1$, all arm average powers (Figure 4.17) converge to the references in Table 4.3. The circulating-current waveforms in Figure 4.18 remain consistent with the absence of internal mismatch: the AC circulating-current component stays close to zero, whereas DC circulating-current components arise to accommodate the phase-dependent power exchange imposed by the unbalanced grid operating point.

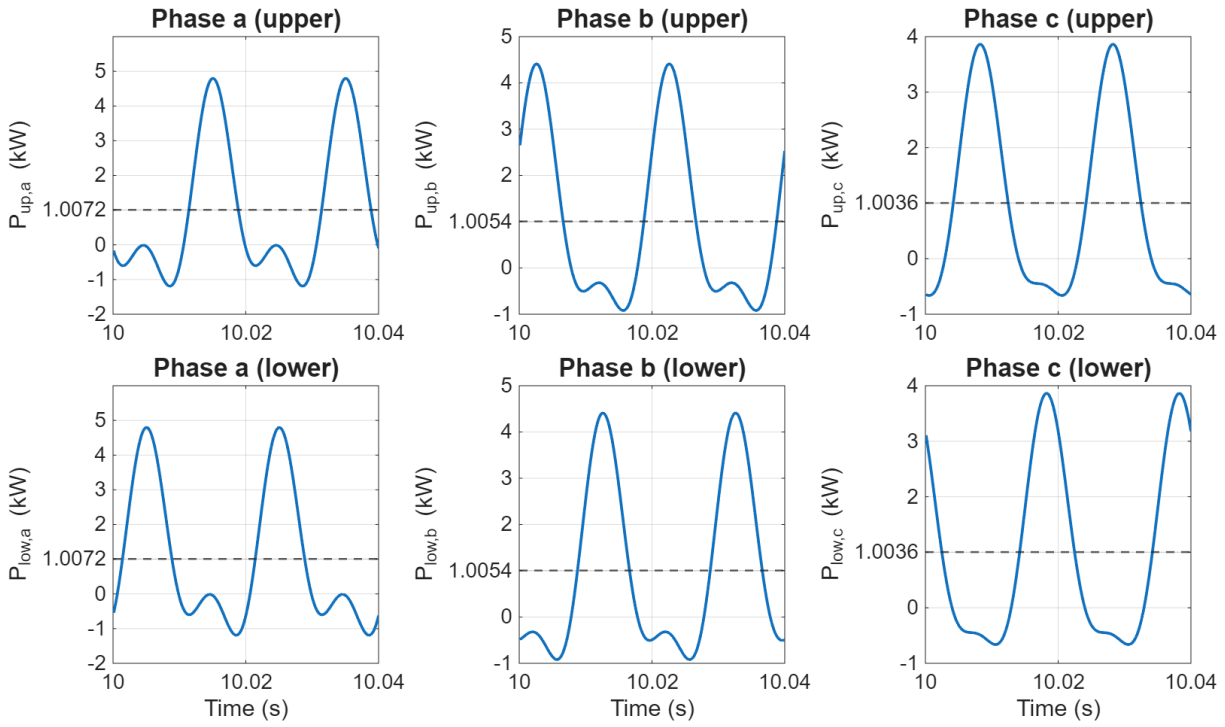


Figure 4.17: Case 1 ($K = 1$) – Generated arm powers over two grid periods.

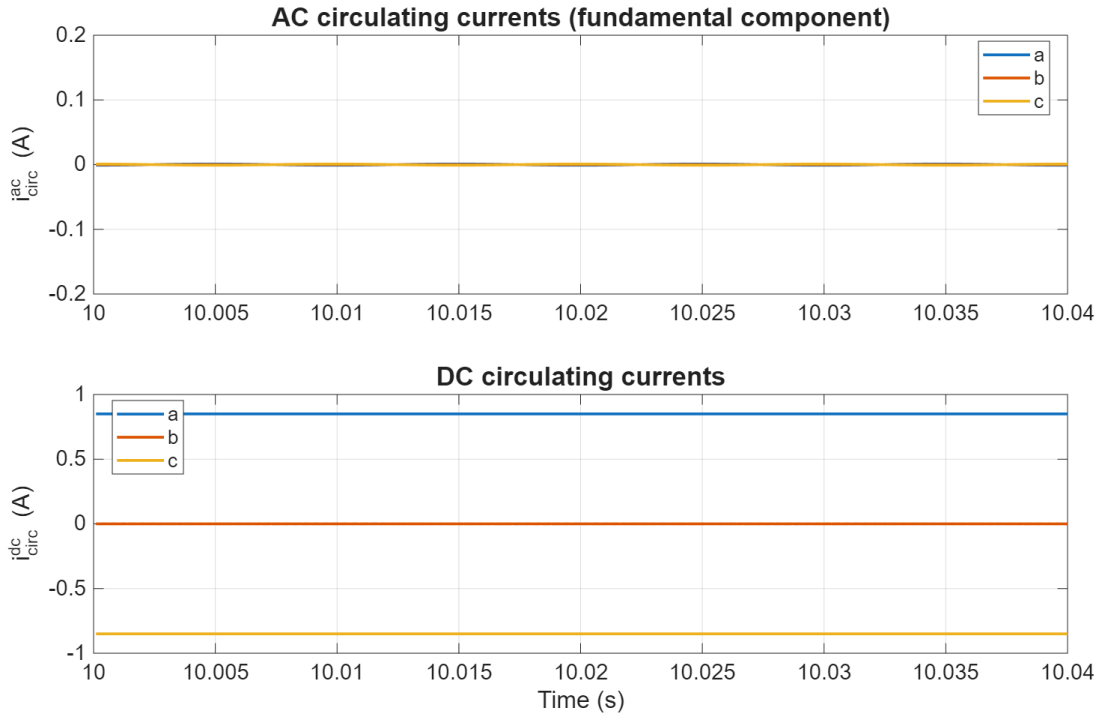


Figure 4.18: Case 1 ($K = 1$) – AC and DC circulating currents over two grids periods.

Quantitative summary of Case 1.

The quantitative results reported in the previous paragraphs are collected in Tables 4.4–4.7 and confirm, in a compact form, the expected selector-dependent trade-off under unbalanced grid voltage with uniform arm available powers. In particular, Table 4.4 shows that all three modes preserve accurate regulation of the commanded average quantities ($\bar{P} \approx 6000$ W and $\bar{Q}_T \approx 1000$ var), while redistributing the 2ω oscillatory content according to the selected objective: primarily to Q for $K = -1$, shared between P and Q for $K = 0$, and primarily to P for $K = +1$.

Table 4.5 clarifies the associated grid-side current-level consequences. The $K = 0$ mode is the only one that keeps phase currents nearly equal, whereas $K = \pm 1$ intentionally introduce current unbalance through a non-zero negative-sequence current component to shape the oscillatory power term. Moreover, the three-phase equivalent RMS phase current is reported as $i_{ph,3\phi}^{\text{rms}}$ for all three cases. Hence, among the three operating modes, $K = +1$ yields the lowest grid-side RMS current, while $K = -1$ gives the highest one, leading to higher losses. Table 4.6 also confirms that, in Case 1, the AC circulating-current amplitudes remain negligible for all modes, coherently with the null vertical mismatch references, while non-zero steady-state DC circulating-current components are still required to accommodate the phase-wise power split imposed by the unbalanced grid operating point.

Finally, Table 4.7 confirms that the internal control objectives are met simultaneously with the grid-side ones: all arm average powers remain close to the imposed 1000 W references in every mode, while the arm-power ripple distribution changes with K consistently with the different grid-side operating conditions.

Table 4.4: Case 1: grid-side steady-state power metrics over the $2T_g$ evaluation window for the three operating modes.

K	$\bar{P}$ [W]	$\bar{Q}$ [var]	$\bar{Q}_T$ [var]	$\bar{Q}^+$ [var]	$\bar{Q}^-$ [var]	ΔP_{pp} [W]	ΔQ_{pp} [var]
-1	6000.51	1083.35	1000.02	1041.685	-41.665	4.95038	5068.65
0	6000.51	1000.01	1000.03	1000.020	0.010	2433.48	2432.96
+1	6000.50	923.078	1000.03	961.554	38.476	4679.12	3.99663

Table 4.5: Case 1: RMS phase-current values over the $2T_g$ evaluation window.

K	$i_{ph,3\phi}^{\text{rms}}$ [A]	$i_{ph,a}^{\text{rms}}$ [A]	$i_{ph,b}^{\text{rms}}$ [A]	$i_{ph,c}^{\text{rms}}$ [A]
-1	9.32744	7.61718	9.32749	10.7694
0	8.78043	8.78042	8.78046	8.78042
+1	8.60992	9.94097	8.60991	7.03127

Table 4.6: Case 1: circulating-current steady-state values over the $2T_g$ evaluation window.

K	$i_{\text{circ},a}^{ac,\text{rms}}$ [A]	$i_{\text{circ},b}^{ac,\text{rms}}$ [A]	$i_{\text{circ},c}^{ac,\text{rms}}$ [A]	$i_{\text{circ},a}^{dc}$ [A]	$i_{\text{circ},b}^{dc}$ [A]	$i_{\text{circ},c}^{dc}$ [A]
-1	7.93985e-05	1.58409e-04	7.90315e-05	-0.0885926	0.177192	-0.0885993
0	3.29228e-04	7.09315e-05	4.00017e-04	0.399425	0.085041	-0.484466
+1	6.46827e-04	8.43317e-06	6.48003e-04	0.849902	-2.124e-05	-0.849881

Table 4.7: Case 1: steady-state arm-power means and ripple over the $2T_g$ evaluation window for the three K -modes.

Arm	Ref. [W]	$K = -1$		$K = 0$		$K = +1$	
		$\bar{P}_{\text{arm}}$ [W]	$\Delta P_{\text{arm,pp}}$ [W]	$\bar{P}_{\text{arm}}$ [W]	$\Delta P_{\text{arm,pp}}$ [W]	$\bar{P}_{\text{arm}}$ [W]	$\Delta P_{\text{arm,pp}}$ [W]
Pup,a	1000	1004.22	5117.04	1005.59	5549.35	1007.16	5984.61
Pup,b	1000	1006.29	5619.96	1005.59	5196.61	1005.38	5328.86
Pup,c	1000	1008.35	6619.23	1005.56	5486.17	1003.58	4522.5
Plow,a	1000	1004.22	5117.04	1005.59	5549.35	1007.16	5984.61
Plow,b	1000	1006.29	5619.96	1005.59	5196.61	1005.38	5328.86
Plow,c	1000	1008.35	6619.23	1005.56	5486.17	1003.58	4522.5

4.2.2. Case 2: Unbalanced Grid, Leg Power Mismatch, and Arm Mismatch

Case 2 extends the previous scenario by introducing a non-uniform distribution of available PV power across legs and arms, while keeping the same grid-side unbalance. Therefore, the negative-sequence magnitude and phase displacement are unchanged from Case 1 and are reported in Table 4.1.

When it comes to the arm available powers, these are now phase-dependent, as summarized in Table 4.8. Consequently, both vertical and horizontal mismatch indicators are non-zero, thus requiring the coordinated action of the AC and DC circulating-current controllers.

Table 4.8: Available arm powers used in Case 2.

	Leg a	Leg b	Leg c
Upper arm	$P_{a,\text{up}}^* = 1000 \text{ W}$	$P_{b,\text{up}}^* = 600 \text{ W}$	$P_{c,\text{up}}^* = 1600 \text{ W}$
Lower arm	$P_{a,\text{low}}^* = 1000 \text{ W}$	$P_{b,\text{low}}^* = 1200 \text{ W}$	$P_{c,\text{low}}^* = 700 \text{ W}$

Accordingly, the total active-power reference is set equal to the total available PV power,

$$P^* = P_{\text{tot}} = \sum_{j \in \{a,b,c\}} P_{j,\text{up}}^* + \sum_{j \in \{a,b,c\}} P_{j,\text{low}}^* = 6100 \text{ W}.$$

The resulting vertical mismatch indicators are

$$P_{\text{diff},a}^* = P_{a,\text{up}}^* - P_{a,\text{low}}^* = 0, \quad P_{\text{diff},b}^* = P_{b,\text{up}}^* - P_{b,\text{low}}^* = -600 \text{ W}, \quad P_{\text{diff},c}^* = P_{c,\text{up}}^* - P_{c,\text{low}}^* = 900 \text{ W},$$

highlighting that legs b and c require an arm-to-arm power exchange to satisfy the MPPT-driven arm power split. Moreover, the leg-wise available powers $P_a^* = 2000 \text{ W}$, $P_b^* = 1800 \text{ W}$, $P_c^* = 2300 \text{ W}$ imply the horizontal mismatch references

$$\Delta P_{ab}^* = P_a^* - P_b^* = 200 \text{ W}, \quad \Delta P_{ac}^* = P_a^* - P_c^* = -300 \text{ W}.$$

In the following, the three operating modes $K \in \{-1, 0, 1\}$ are analysed.

K=-1: Active-power oscillation suppression.

Figures 4.19–4.20 report the transient evolution of the active power P , the instantaneous imaginary power Q , and the total reactive power Q_T for $K = -1$. As in Case 1, the converter is synchronized and connected to the grid through a three-phase breaker at time $t \simeq 0.1 \text{ s}$ to allow the PLL to settle.

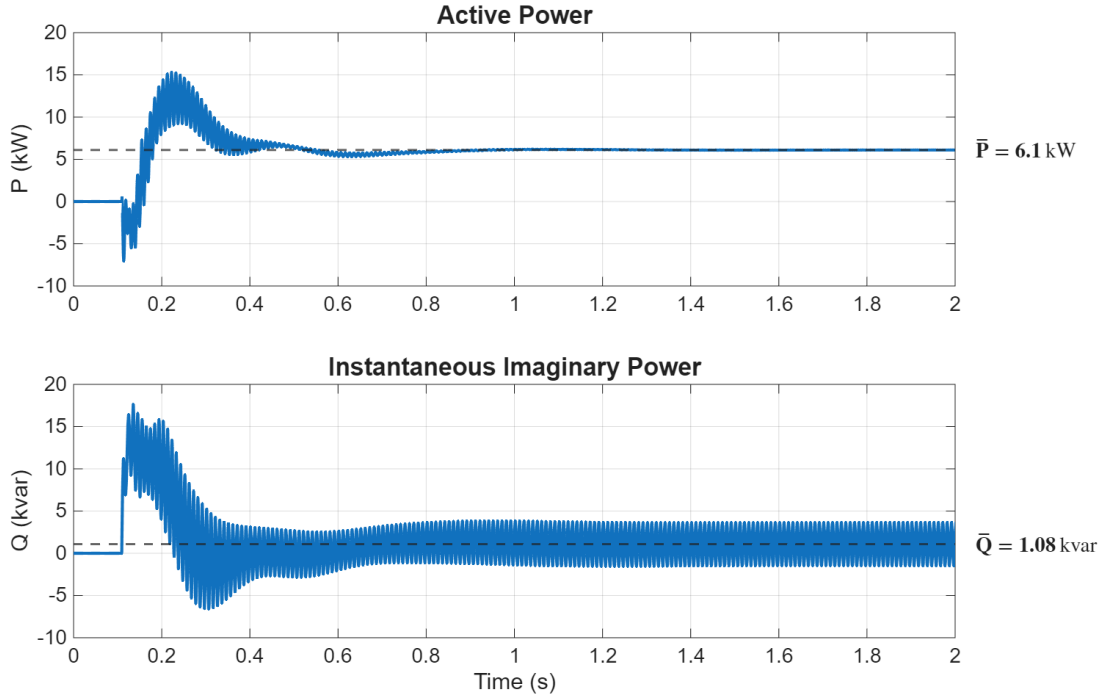


Figure 4.19: Case 2 ($K = -1$) – Transient behavior of P and Q .

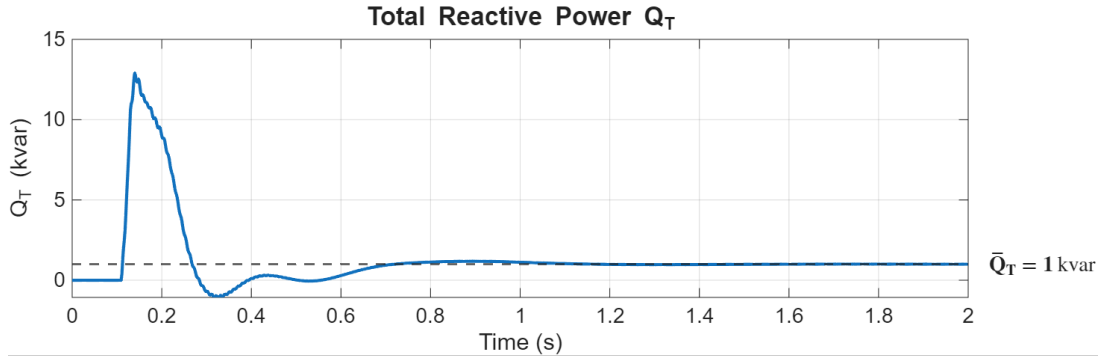


Figure 4.20: Case 2 ($K = -1$) – Transient behavior of Q_T .

Figure 4.21 shows P and Q over a $2T_g$ window under steady-state conditions. The observed behavior reflects the design objective of the $K = -1$ mode, namely suppressing the 2ω oscillatory active-power term. Consequently, P is regulated close to a constant value (with small residual ripple due to finite control bandwidth), while Q exhibits a more pronounced 2ω oscillation. It is also verified that $\bar{Q} \neq \bar{Q}_T$, that is confirmed by the values of the measured sequence components, namely $\bar{Q}^+ \simeq 1.041$ kvar and $\bar{Q}^- \simeq -0.041$ kvar.

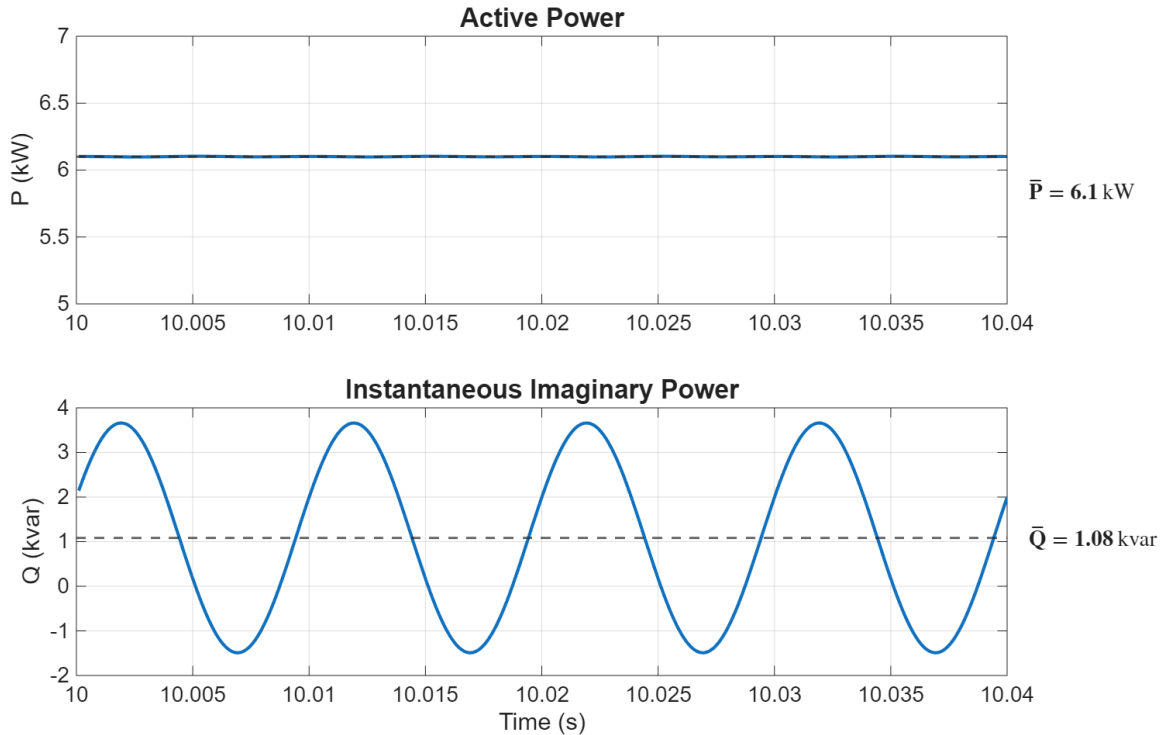


Figure 4.21: Case 2 ($K = -1$) – Steady-state behavior of P and Q over two grid periods.

Figure 4.22 reports v_g and i_{ph} over the same interval. The grid voltage contains the imposed negative-sequence component, and the phase currents include a controlled negative-

sequence contribution.

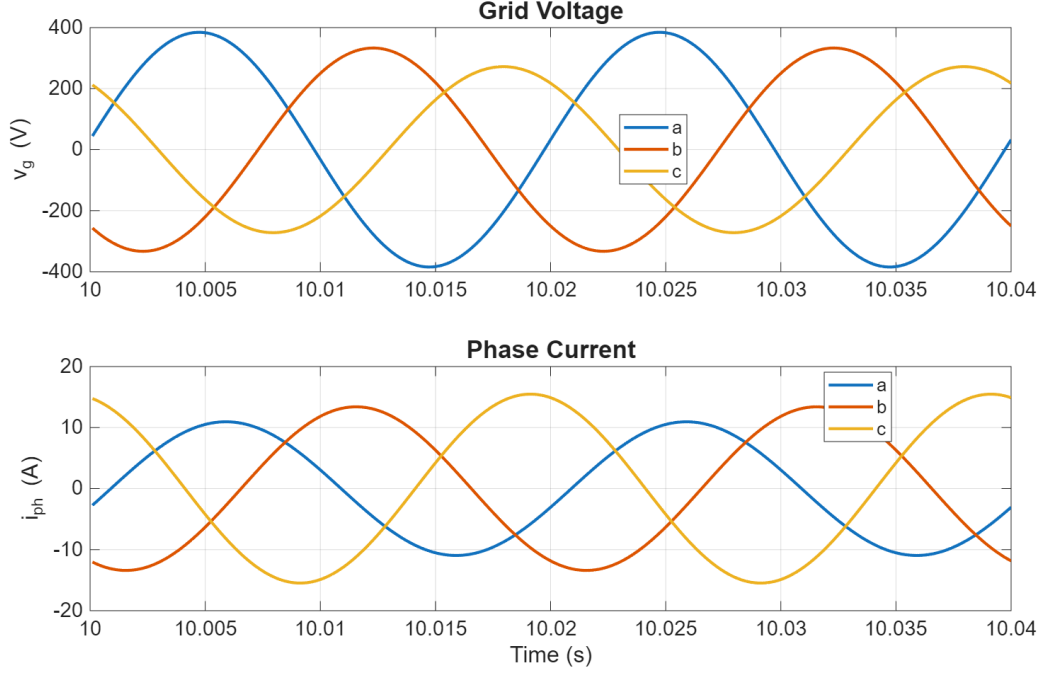


Figure 4.22: Case 2 ($K = -1$): steady-state waveforms of the grid voltage v_g and phase current i_{ph} over two grid periods.

Unlike Case 1, internal power redistribution is required due to both vertical and horizontal mismatch. Figure 4.23 shows that the average arm powers converge to the available-power references in Table 4.8, confirming that the proposed strategy can enforce the MPPT-driven arm power split (non-zero $P_{\text{diff},b}^*$ and $P_{\text{diff},c}^*$). The corresponding circulating-current response is illustrated in Fig. 4.24: AC circulating-current components are generated to satisfy $P_{\text{diff},j}^*$ within each leg, while DC components enable inter-leg power transfer to comply with the horizontal mismatch references (ΔP_{ab}^* , ΔP_{ac}^*) and with the phase-wise power split imposed by the unbalanced grid operating point.

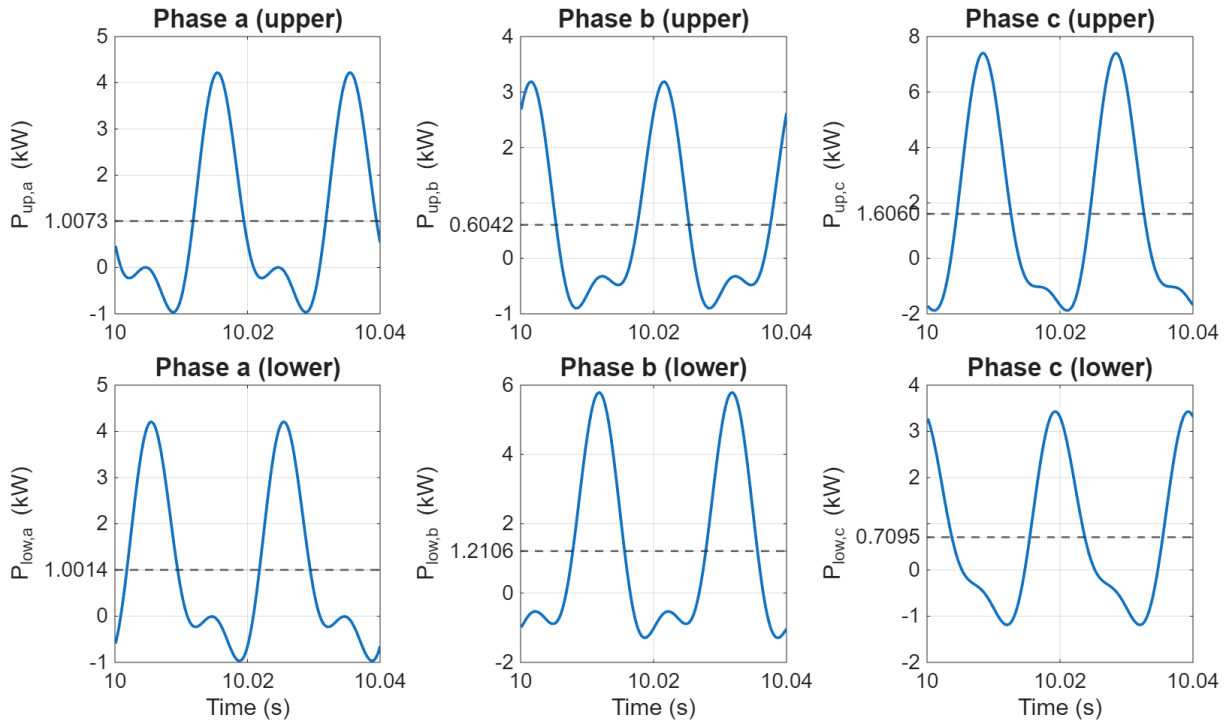


Figure 4.23: Case 2 ($K = -1$) – Generated arm powers over two grid periods.

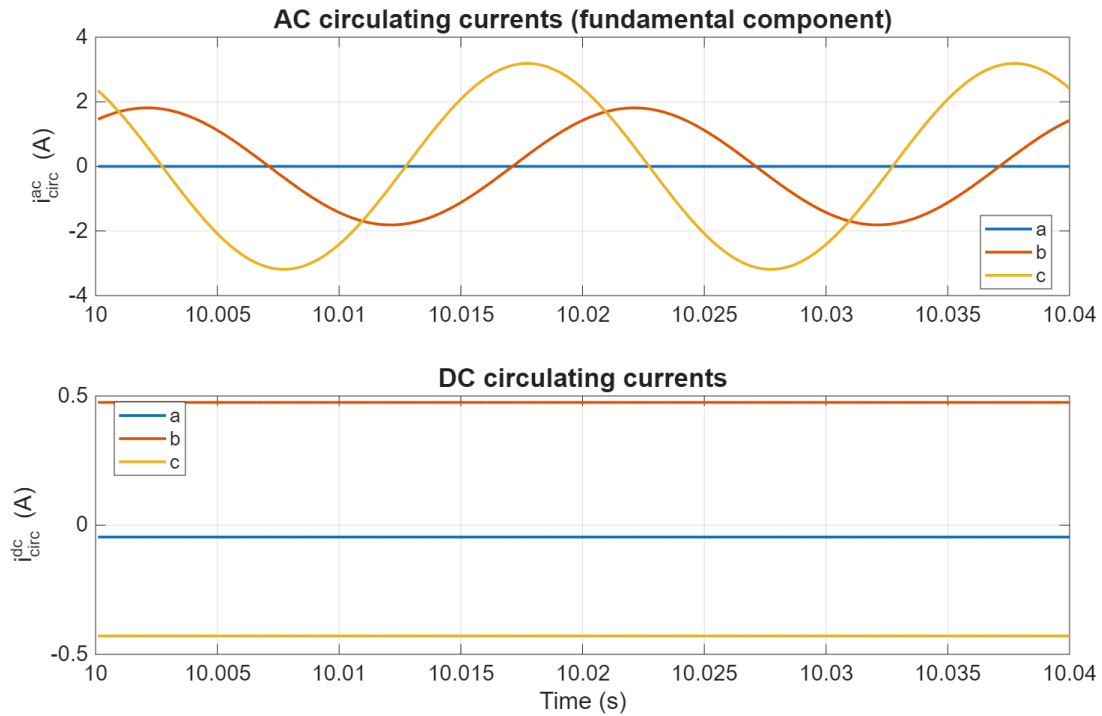


Figure 4.24: Case 2 ($K = -1$) – AC and DC circulating currents over two grid periods.

K=0: Balanced output currents.

Figures 4.25–4.26 show the transient evolution of P , Q , and Q_T for $K = 0$. In this mode the grid controller enforces $\bar{I}_{ph}^* = 0$, hence the phase currents are kept balanced despite the imposed voltage unbalance.

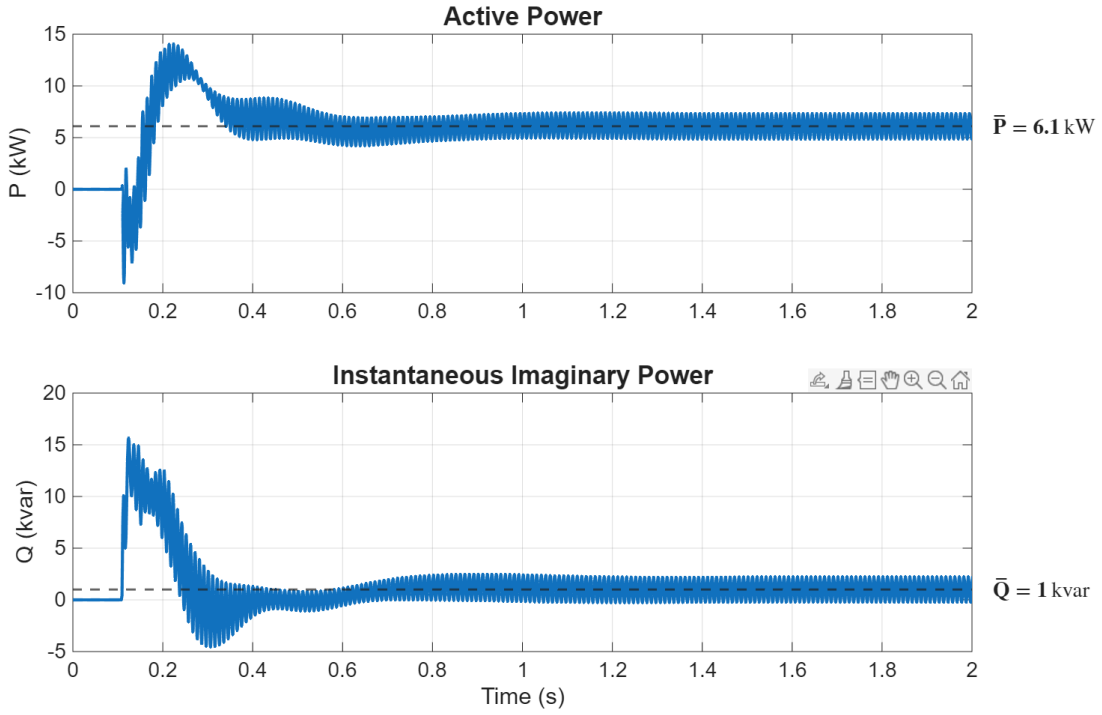


Figure 4.25: Case 2 ($K = 0$) – Transient behavior of P and Q .

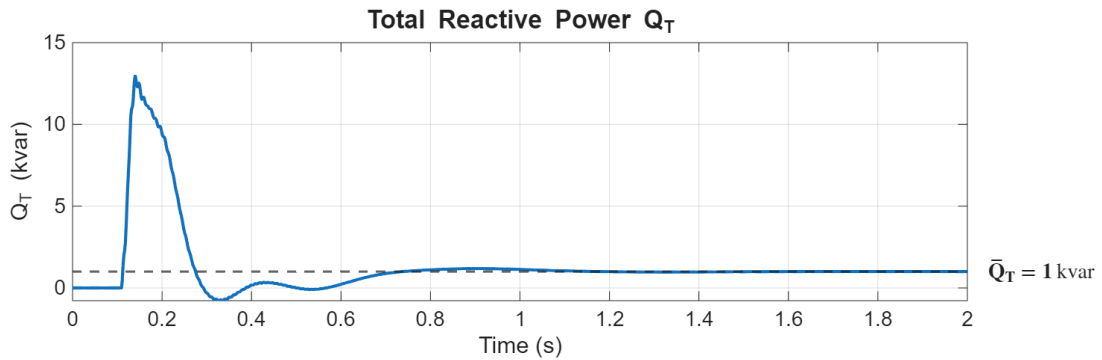


Figure 4.26: Case 2 ($K = 0$) – Transient behavior of Q_T .

Figure 4.27 reports P and Q over the last $2T_g$. Because no compensating negative-sequence current is injected, the interaction between the negative-sequence voltage and the balanced current injection leads to clear 2ω oscillations in both P and Q . In addition,

since $\bar{I}_{ph}^{-*} = 0$, then $Q^- = 0$ and the average values satisfy $\bar{Q} = \bar{Q}_T$. This is consistent with the measured sequence contributions, $\bar{Q}^+ \simeq 1$ kvar and $\bar{Q}^- \simeq 0$ kvar.

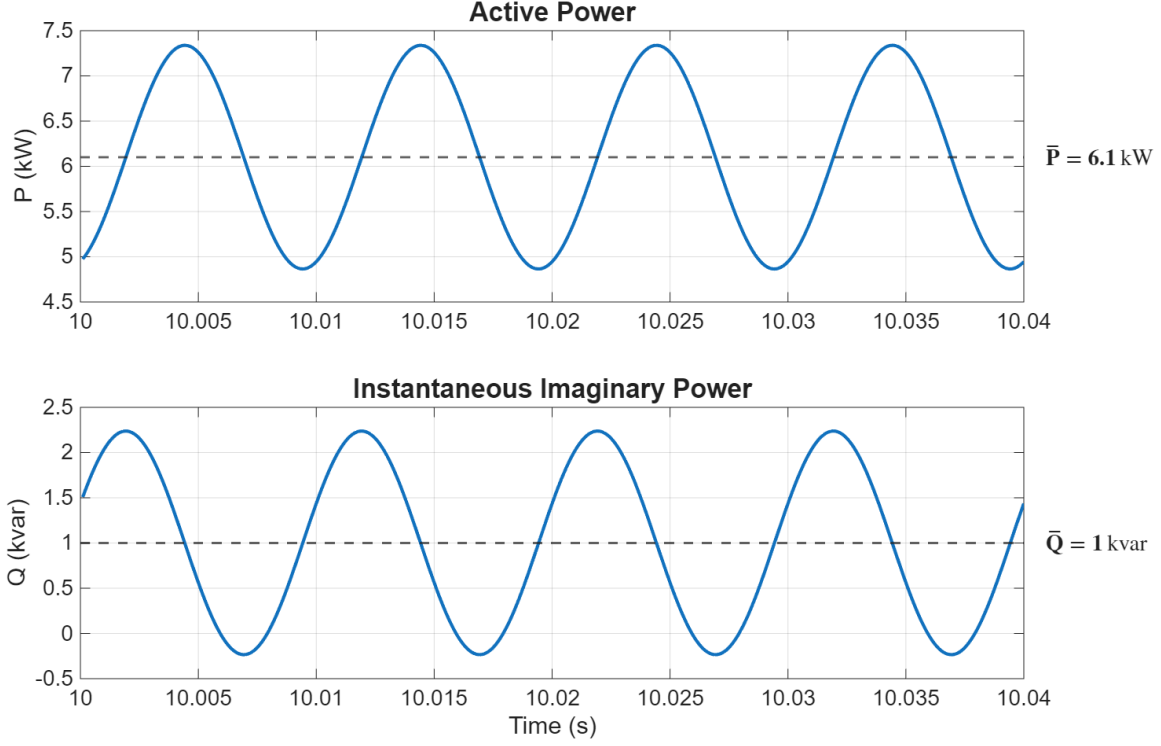


Figure 4.27: Case 2 ($K = 0$) – Steady-state behavior of P and Q over two grid periods.

The steady-state waveforms of v_g and i_{ph} are shown in Fig. 4.28. As expected, the grid voltage contains the imposed negative sequence, while the injected currents remain balanced and approximately sinusoidal, confirming that the negative-sequence current channel is disabled in this mode.

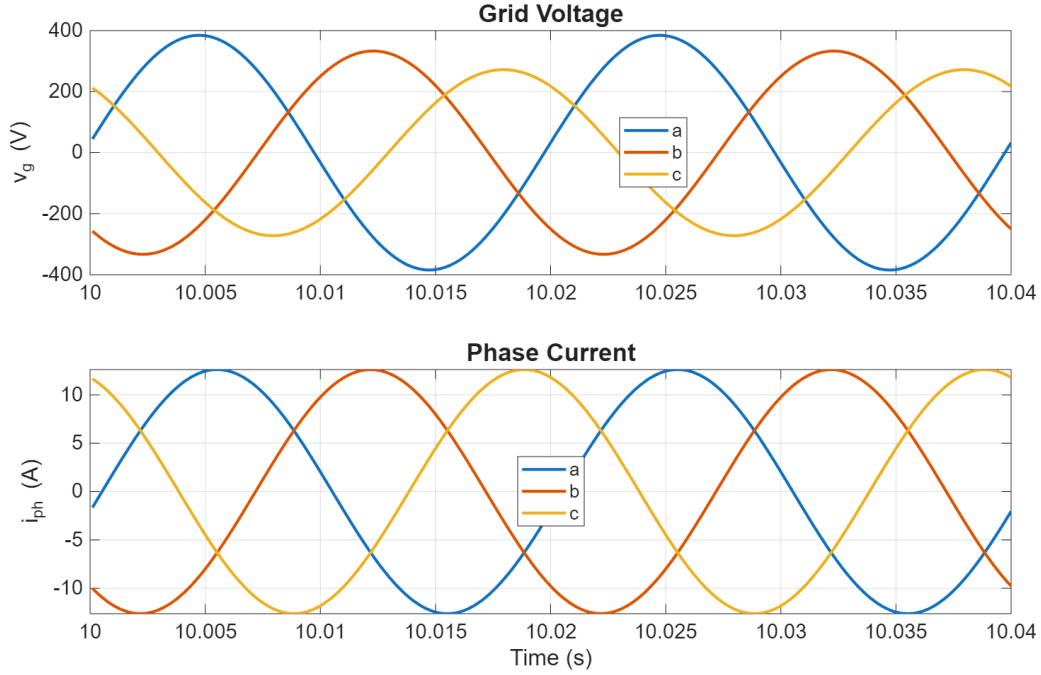


Figure 4.28: Case 2 ($K = 0$): steady-state waveforms of the grid voltage v_g and phase current i_{ph} over two grid periods.

Figure 4.29 confirms that the arm average powers converge to the available-power references, meaning that the AC circulating-current controller successfully enforces the non-zero $P_{diff,j}^*$ where required. Figure 4.30 shows the corresponding circulating-current components: the AC channel is responsible for the arm-to-arm transfer within each leg, while the DC channel enables inter-leg redistribution so that the converter can satisfy the phase-wise power split dictated by the unbalanced grid operating point while maintaining balanced output currents.

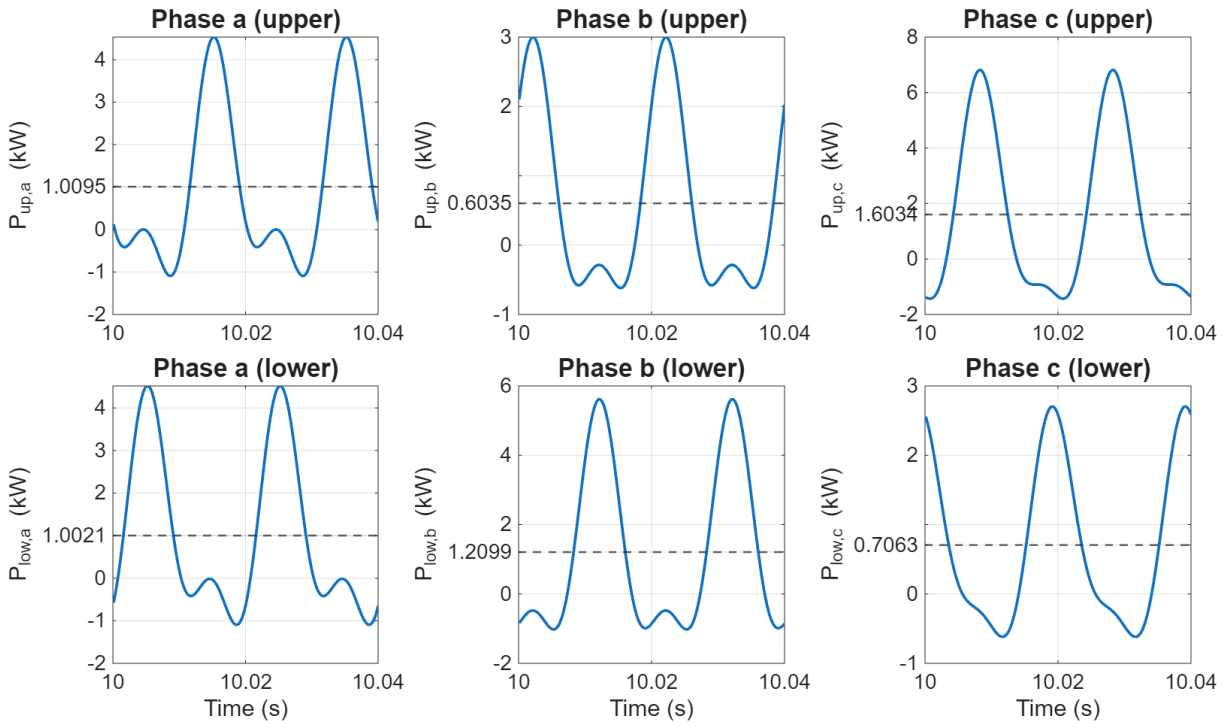


Figure 4.29: Case 2 ($K = 0$) – Generated arm powers over two grid periods.

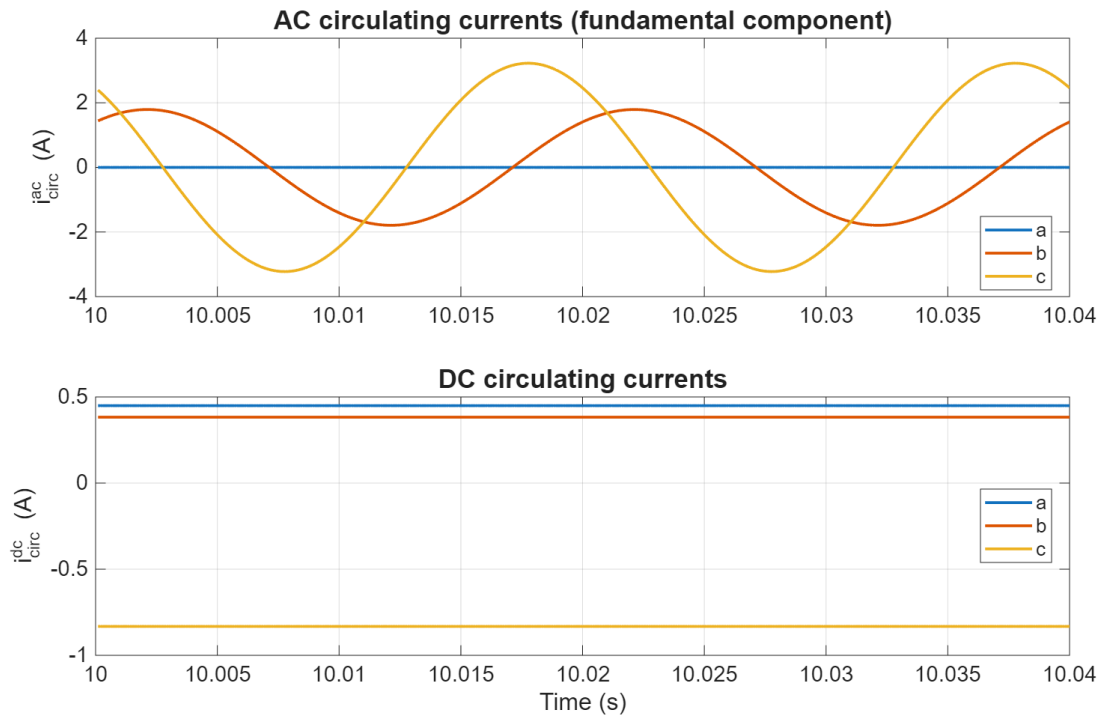


Figure 4.30: Case 2 ($K = 0$) – AC and DC circulating currents over two grid periods.

K=+1: Reactive-power oscillation suppression.

Figures 4.31–4.32 illustrate the transient behavior of P , Q , and Q_T for $K = +1$. In this mode the negative-sequence current reference is shaped to minimize the oscillatory contribution in the imaginary-power channel, mitigating the 2ω oscillations in Q .

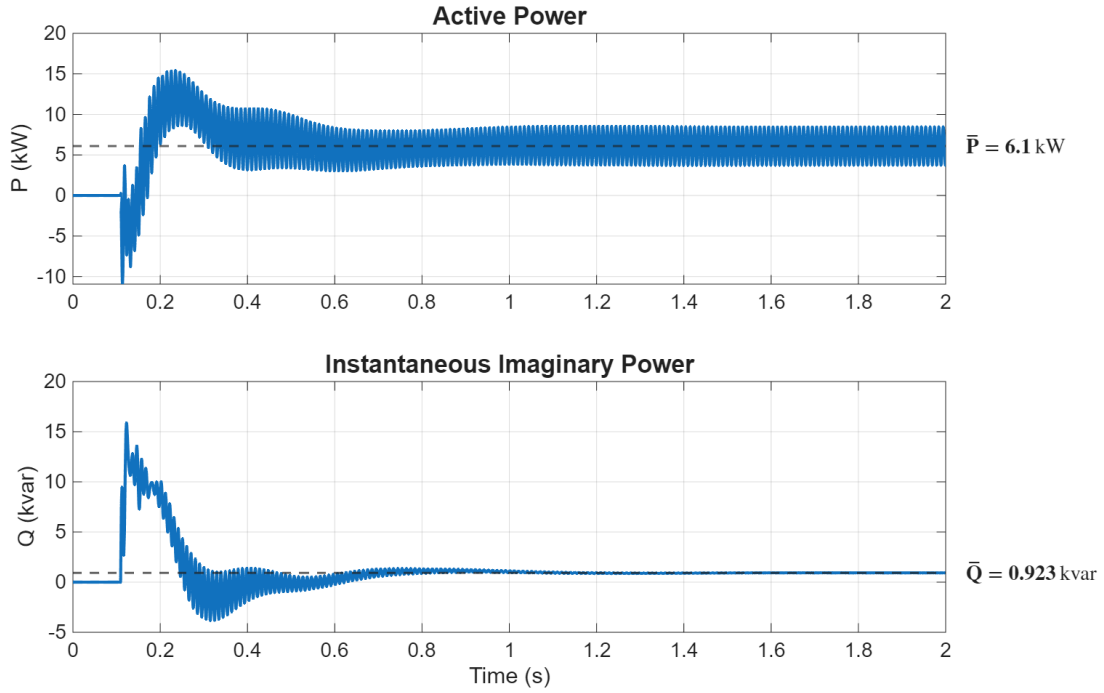


Figure 4.31: Case 2 ($K = 1$) – Transient behavior of P and Q .

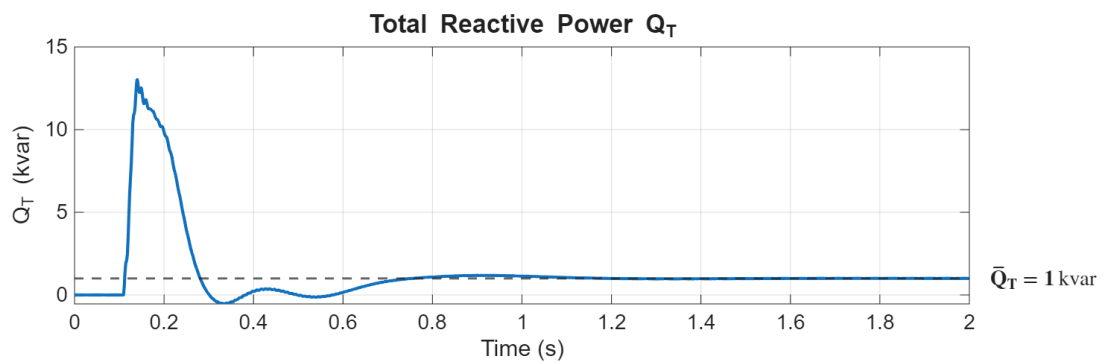


Figure 4.32: Case 2 ($K = 1$) – Transient behavior of Q_T .

Figure 4.33 shows the steady-state waveforms of P and Q over $2T_g$. As expected, Q becomes nearly constant (only a small residual oscillation remains), whereas the oscillatory content is mainly shifted to the active-power channel, so that P exhibits the 2ω ripple

while still satisfying $\bar{P} \approx P^*$. Moreover, it is verified that $\bar{Q} \neq \bar{Q}_T$ due to the injection of a negative-sequence current component, such that $\bar{Q}^+ \simeq 0.961$ kvar and $\bar{Q}^- \simeq 0.038$ kvar.

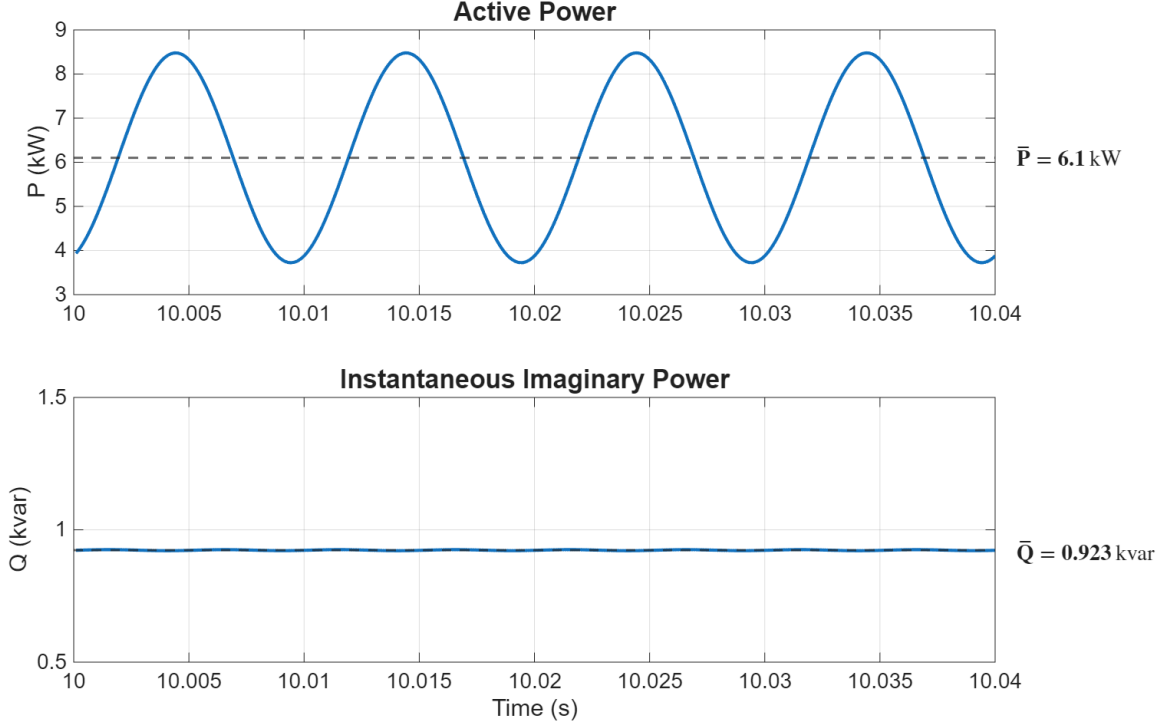


Figure 4.33: Case 2 ($K = 1$) – Steady-state behavior of P and Q over two grid periods.

Figure 4.34 reports v_g and i_{ph} over the same interval. In addition to the imposed negative-sequence voltage, the injected current includes a controlled negative-sequence component ($\bar{I}_{ph}^- \neq 0$), which is exploited to shape the oscillatory power term. Hence, the current waveform is not perfectly balanced, but the instantaneous reactive-power ripple is minimized, as intended.

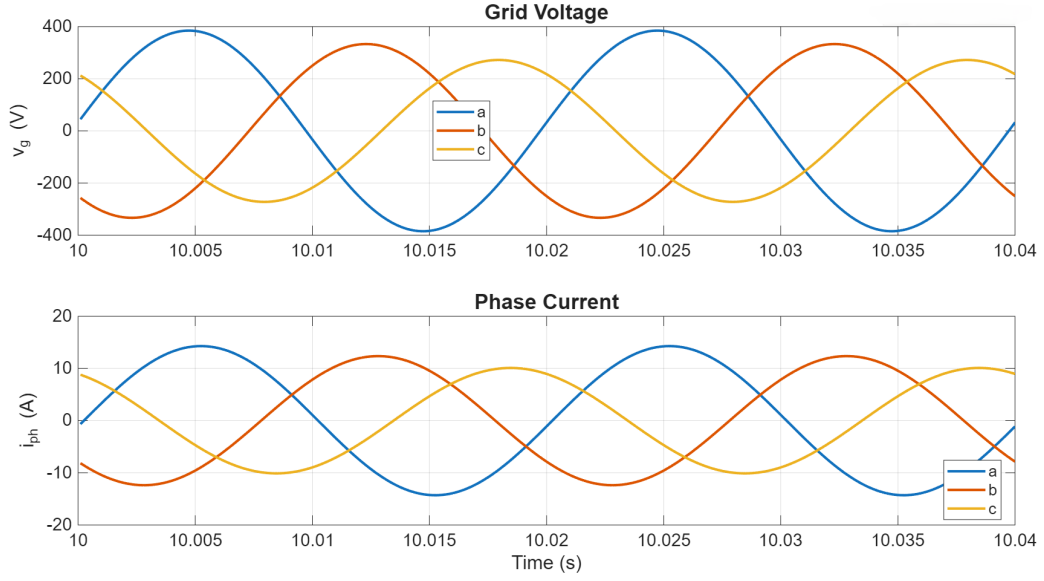


Figure 4.34: Case 2 ($K = 1$): steady-state waveforms of the grid voltage v_g and phase current i_{ph} over two grid periods.

The internal power behavior is summarized in Fig. 4.35, which confirms that the arm average powers match the available-power references despite the simultaneous presence of vertical and horizontal mismatch. The required circulating-current activity is shown in Fig. 4.36: the AC circulating-current components enforce the arm-to-arm power transfer within each leg (as required by $P_{\text{diff},j}^*$), while the DC circulating-current components perform the inter-leg redistribution needed to accommodate the phase-wise power split associated with the unbalanced grid operating point and with the available-power constraints.

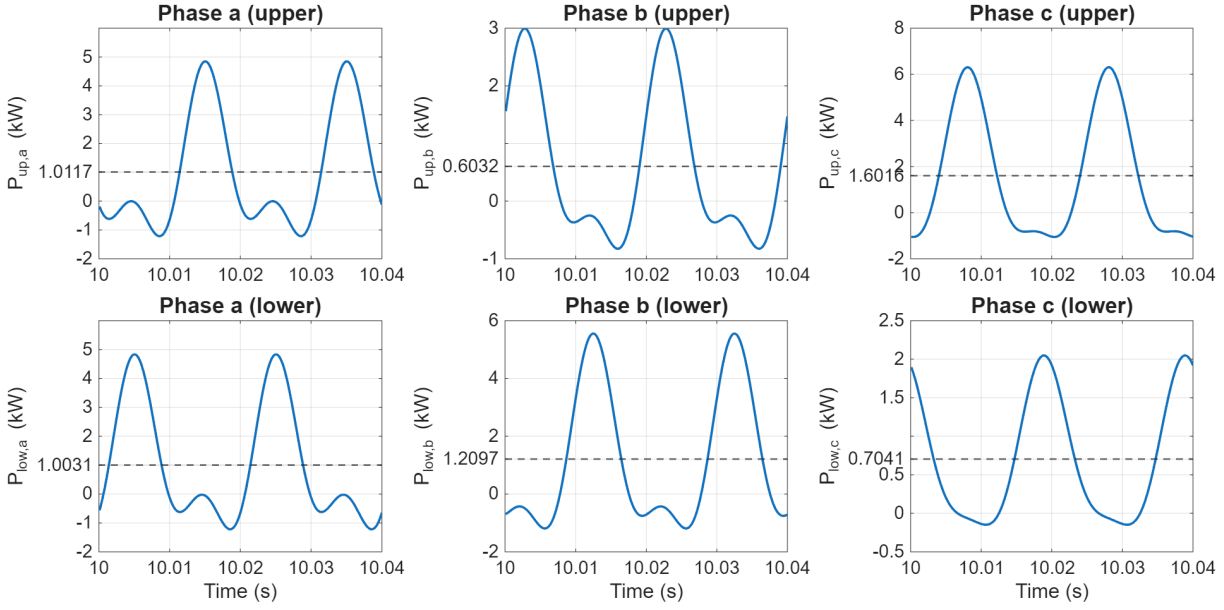


Figure 4.35: Case 2 ($K = 1$) – Generated arm powers over two grid periods.

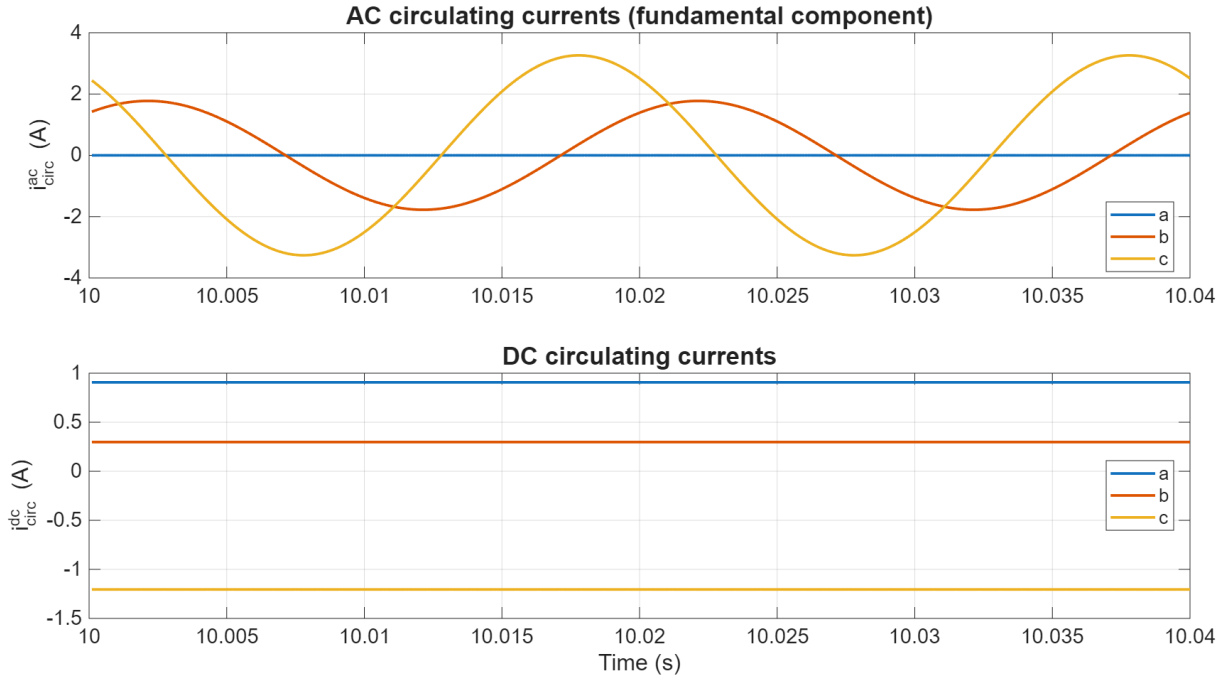


Figure 4.36: Case 2 ($K = 1$) – AC and DC circulating currents over two grid periods.

Quantitative summary of Case 2.

The quantitative results already presented in the previous paragraphs are synthesised in Tables 4.9–4.12. Their joint reading shows that the grid-side selector trade-off is preserved even when both vertical and horizontal internal mismatches are imposed: Ta-

ble 4.9 exhibits the same redistribution of the 2ω oscillatory content among P and Q as in Case 1, while maintaining accurate tracking of the average commands ($\bar{P} \approx 6100$ W, $\bar{Q}_T \approx 1000$ var).

At the same time, Table 4.10 shows the grid-side current values. When it comes to the three-phase equivalent RMS phase current, the same ordering of Case 1 is observed. Therefore, also under internal power mismatch, $K = +1$ minimizes the grid-side RMS current, while $K = -1$ maximizes it. This suggests lower current-dependent losses for $K = +1$ and higher losses for $K = -1$.

Compared with Case 1, as shown in Table 4.11, the AC circulating currents become large in legs b and c , consistently with the non-zero vertical mismatch references $P_{\text{diff},b}^*$ and $P_{\text{diff},c}^*$, while leg a remains close to zero because its vertical mismatch is null. The DC circulating-current components also become more pronounced, confirming the additional inter-leg power redistribution required by the horizontal mismatch and by the phase-wise grid-side power split under unbalanced voltage.

Finally, Table 4.12 confirms that these stronger internal actions effectively enforce the MPPT-driven arm-power constraints for all K -modes: the extracted arm average powers remain close to their individual references, and the expected arm-to-arm differences are preserved in legs b and c , demonstrating compatibility between the grid-side objectives and the internal DMPPT-related power-allocation constraints.

Table 4.9: Case 2: grid-side steady-state power metrics over the $2T_g$ evaluation window for the three operating modes.

K	$\bar{P}$ [W]	$\bar{Q}$ [var]	$\bar{Q}_T$ [var]	$\bar{Q}^+$ [var]	$\bar{Q}^-$ [var]	ΔP_{pp} [W]	ΔQ_{pp} [var]
-1	6100.52	1083.35	1000.02	1041.6850	-41.6650	4.94679	5151.10
0	6100.52	1000.01	1000.02	1000.0150	0.0050	2473.05	2472.53
+1	6100.51	923.077	1000.03	961.5535	38.4765	4755.22	3.99943

Table 4.10: Case 2: RMS phase-current values over the $2T_g$ evaluation window.

K	$i_{ph,3\phi}^{\text{rms}}$ [A]	$i_{ph,a}^{\text{rms}}$ [A]	$i_{ph,b}^{\text{rms}}$ [A]	$i_{ph,c}^{\text{rms}}$ [A]
-1	9.47870	7.74072	9.47878	10.9440
0	8.92285	8.92284	8.92288	8.92283
+1	8.74957	10.1022	8.74956	7.14531

Table 4.11: Case 2: circulating-current steady-state values over the $2T_g$ evaluation window.

K	$i_{\text{circ},a}^{ac,\text{rms}}$ [A]	$i_{\text{circ},b}^{ac,\text{rms}}$ [A]	$i_{\text{circ},c}^{ac,\text{rms}}$ [A]	$i_{\text{circ},a}^{dc}$ [A]	$i_{\text{circ},b}^{dc}$ [A]	$i_{\text{circ},c}^{dc}$ [A]
-1	8.71117e-05	1.28109	2.25352	-0.0460668	0.474873	-0.428806
0	3.37572e-04	1.26755	2.28045	0.449316	0.382722	-0.832038
+1	6.58941e-04	1.25529	2.30569	0.906593	0.297659	-1.204250

Table 4.12: Case 2: steady-state arm-power means and ripple over the $2T_g$ evaluation window for the three K -modes.

Arm	Ref. [W]	$K = -1$		$K = 0$		$K = +1$	
		$\bar{P}_{\text{arm}}$ [W]	$\Delta P_{\text{arm,pp}}$ [W]	$\bar{P}_{\text{arm}}$ [W]	$\Delta P_{\text{arm,pp}}$ [W]	$\bar{P}_{\text{arm}}$ [W]	$\Delta P_{\text{arm,pp}}$ [W]
Pup,a	1000	1007.33	5183.65	1009.45	5622.5	1011.72	6064.53
Pup,b	600	604.208	4086.29	603.461	3615.83	603.215	3820.49
Pup,c	1600	1606.02	9283.82	1603.39	8244.66	1601.57	7359.35
Plow,a	1000	1001.38	5172.11	1002.1	5610.08	1003.06	6051.41
Plow,b	1200	1210.6	7076.4	1209.9	6635.77	1209.71	6745.31
Plow,c	700	709.469	4615.84	706.348	3320.07	704.078	2196.59

5 | Conclusions

This thesis addressed the control of a MMC for a three-phase PV system with submodule-integrated distributed energy sources operating under grid-voltage unbalance and non-uniform internal power availability. The main objective was to develop and validate a control strategy capable of simultaneously ensuring grid-compliant power exchange and coordinated internal power redistribution, so that the converter can satisfy both external grid requirements and internal power constraints imposed by DMPPT operation.

To this end, a hierarchical control architecture was developed and analytically formulated. The first layer is a dual-sequence grid controller, designed to generate positive- and negative-sequence current references from the active- and reactive-power set-points. Starting from the sequence-domain representation of voltages and currents, the derivation led to explicit expressions for the current references in both phasor and dq form, including the unified selector parameter $K \in \{-1, 0, 1\}$. This formulation allows the controller to operate in three distinct modes, corresponding to: suppression of active-power oscillations, balanced output currents, and suppression of imaginary-power oscillations.

The second and third layers addressed the internal converter dynamics through circulating-current control. In particular, the AC circulating-current controller was designed to regulate the vertical power mismatch within each phase-leg, namely the power difference between upper and lower arms. Differently from sequence-based three-phase formulations available in the literature, this thesis adopted a per-phase single-phase control strategy (three independent controllers, one for each leg). The controller generates fundamental-frequency AC circulating-current references to enforce the required arm-to-arm power transfer and maintain the operating points imposed by the available arm powers. The DC circulating-current controller, instead, was formulated to handle horizontal mismatch among phase-legs by regulating inter-leg power redistribution through DC circulating-current components. The corresponding reference-generation logic was derived from phase-wise injected powers and mismatch indicators, leading to a compact and physically interpretable control structure.

A final coordination layer, denoted as the arm-voltage reference generator, was then in-

troduced to combine the outputs of the three control functions, thus providing the final upper- and lower-arm voltage references.

The proposed control strategy was finally assessed by time-domain simulations. The results confirmed the correctness of the analytical derivations and the effectiveness of the coordinated control action in the investigated scenarios. In Case 1 (unbalanced grid, no internal power mismatch), the controller successfully tracked the active-power reference while exhibiting the expected trade-offs among the three operating modes: for $K = -1$, active-power oscillations were strongly mitigated at the expense of imaginary-power ripple; for $K = 0$, balanced output currents were enforced, resulting in oscillatory power terms; for $K = +1$, the oscillations were shifted to the active-power channel while the imaginary-power ripple was almost canceled. In all cases, the average arm powers converged to the prescribed values, confirming consistency between grid-side regulation and internal power management.

In Case 2 (unbalanced grid with non-uniform available arm powers), the simulations further demonstrated the capability of the proposed architecture to manage the simultaneous presence of external asymmetry and internal mismatch. The results showed that the grid controller, AC circulating-current controller, and DC circulating-current controller operate in a coordinated manner: the grid-side power exchange remains compliant with the imposed references and selected K mode, while AC and DC circulating currents are generated to enforce the required internal power distribution. In particular, the arm powers converge to their individual available-power references, thereby confirming the correct operation of the proposed control strategy.

Overall, the thesis demonstrates that the proposed control architecture is a viable solution for MMC-based PV applications operating under unbalanced grid conditions and internal power mismatch.

Despite these positive results, some aspects remain outside the scope of the present work and motivate further developments. First, the DMPPT supervisory controller was treated as an external layer and was therefore not explicitly modeled within the proposed control architecture. A fully integrated framework including these upper-level functions would provide a complete end-to-end control architecture. Second, the simulation model adopted in this thesis does not explicitly include submodule-level switching dynamics and capacitor-voltage balancing, and therefore does not reproduce all intrinsic MMC phenomena, such as submodule-level double-frequency effects and detailed ripple propagation.

In conclusion, this work provides a coherent control-design framework for MMC-based PV systems under combined grid unbalance and internal power mismatch, and it offers

a solid basis for future extensions toward fully integrated DMPPT MMC control and experimental implementation.

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List of Symbols

Variable	Description	SI unit
Indices, operators and general notation		
j	phase-leg index, $j \in \{a, b, c\}$	–
$\overline{(\cdot)}$	phasor notation	–
$\underline{(\cdot)}$	complex conjugate notation (when used in phasor expressions)	–
$\langle x(t) \rangle_T$	moving-average operator over one fundamental period T	–
$\mathcal{H}\{\cdot\}$	Hilbert transform operator	–
$\text{BPF}_\omega\{\cdot\}$	band-pass filter centred at angular frequency ω	–
$\mathbf{1}$	3×1 vector of ones	–
$a = e^{j2\pi/3}$	Fortescue operator for symmetrical components	–
Topology and converter parameters		
N	number of submodules per arm	–
C_{sm}	submodule capacitance	F
C_{dc}	DC-side capacitance	F
R_g	grid-side resistance	Ω
L_g	grid-side inductance	H
R_{arm}	arm resistance	Ω
L_{arm}	arm inductance	H
M_{arm}	mutual inductance between upper and lower arm inductors	H
R_{eq}	equivalent phase-current loop resistance	Ω
L_{eq}	equivalent phase-current loop inductance	H
S_{up}, S_{low}	upper and lower switches of the half-bridge submodule	–
v_{PV}	PV submodule voltage	V
Arm, phase and circulating electrical quantities		

Variable	Description	SI unit
$v_{up,j}, v_{low,j}$	upper- and lower-arm voltages of phase-leg j	V
$i_{up,j}, i_{low,j}$	upper- and lower-arm currents of phase-leg j	A
$v_{SM,up,i,j}, v_{SM,low,i,j}$	i -th submodule voltage in upper/lower arm of phase-leg j	V
$v_{g,j}$	phase grid voltage (phase j)	V
$v_{g,abc}$	measured three-phase grid-voltage vector	V
$i_{ph,j}$	phase current injected into the AC grid (phase j)	A
$v_{ph,j}$	converter AC terminal phase voltage component (phase j)	V
$i_{circ,j}$	circulating current in phase-leg j	A
$v_{circ,j}$	circulating voltage in phase-leg j	V
$i_{circ,j}^{dc}$	DC component of circulating current in phase-leg j	A
$i_{circ,j}^{ac}$	fundamental-frequency AC component of circulating current in phase-leg j	A
$v_{circ,j}^{dc}$	DC component of circulating voltage in phase-leg j	V
$v_{circ,j}^{ac}$	AC component of circulating voltage in phase-leg j	V
v_c	dc-side capacitor voltage (instantaneous)	V
i_c	current flowing into the dc-side capacitor	A
V_{dc}	DC-link capacitor terminal voltage	V
V_{dc}^*	DC-link voltage reference (provided by DMPPT controller)	V
Power quantities and mismatch indicators		
$\bar{S}$	complex power exchanged with the grid	VA
$\bar{S}_{const}$	constant component of complex power	VA
$\bar{S}_{2\omega}$	oscillatory complex-power component at 2ω	VA
P	average active power	W
Q	average imaginary power	var
P^*	active-power reference for grid controller	W
Q^*	imaginary-power reference for grid controller	var
Q_T^*	total reactive-power target	var
P^+, P^-	positive-/negative-sequence active-power contributions	W
Q^+, Q^-	positive-/negative-sequence reactive-power contributions	var
$\bar{S}^+, \bar{S}^-$	positive-/negative-sequence complex powers	VA

Variable	Description	SI unit
$P_{j,\text{up}}, P_{j,\text{low}}$	available average powers of upper/lower arm in phase-leg j	W
$P_{j,\text{up}}^*, P_{j,\text{low}}^*$	arm available-power references (from DMPPT)	W
P_a, P_b, P_c	available phase-leg powers	W
P_a^*, P_b^*, P_c^*	available phase-leg power references	W
P_{tot}	total available converter power	W
$P_{\text{diff},j}$	vertical mismatch (differential arm power) in phase-leg j	W
$P_{\text{diff},j}^*$	reference differential arm power in phase-leg j	W
$\Delta P_{ab}, \Delta P_{bc}, \Delta P_{ac}$	horizontal mismatch indicators among phase-leg available powers	W
$\Delta P_{ab}^*, \Delta P_{ac}^*$	horizontal mismatch references	W
$P_a^{\text{inj}}, P_b^{\text{inj}}, P_c^{\text{inj}}$	phase-wise active powers implied by grid operating point	W
$Q_a^{\text{inj}}, Q_b^{\text{inj}}, Q_c^{\text{inj}}$	phase-wise reactive powers implied by grid operating point	var
$\Delta P_{ab}^{\text{inj}}, \Delta P_{ac}^{\text{inj}}$	injected active-power split indicators from grid controller	W
$A_a, B_a, A_b, B_b, A_c, B_c$	auxiliary coefficients used in phase-wise injected-power expressions	V ²
D_P, D_Q	auxiliary denominators in power-to-current reference mapping	V ²
Symmetrical components, phasors and reference frames		
$\bar{V}_a, \bar{V}_b, \bar{V}_c$	phase-voltage phasors	V
$\bar{V}_0, \bar{V}_+, \bar{V}_-$	zero-, positive-, and negative-sequence voltage phasors	V
$\bar{V}_g^+, \bar{V}_g^-$	positive-/negative-sequence grid-voltage phasors	V
$\bar{I}_{ph}^+, \bar{I}_{ph}^-$	positive-/negative-sequence phase-current phasors	A
$\bar{I}_{ph}^{+*}, \bar{I}_{ph}^{-*}$	positive-/negative-sequence current reference phasors	A
V_g^+, V_g^-	magnitudes of positive-/negative-sequence grid-voltage phasors	V
I_{ph}^+, I_{ph}^-	magnitudes of positive-/negative-sequence phase-current phasors	A
ϕ^+, ϕ^-	positive-/negative-sequence voltage phase angles	rad
φ^+, φ^-	phase displacement between sequence voltage and current phasors	rad
θ^+	positive-sequence synchronous reference angle (from PLL)	rad
θ^-	auxiliary negative-sequence synchronous reference angle	rad

Variable	Description	SI unit
δ_j	phase-voltage angle for phase-leg j (from PLL)	rad
ω	fundamental angular frequency	rad/s
f_g	grid frequency	Hz
f	measured grid frequency	Hz
$T_g = \frac{1}{f_g}$	fundamental period	s
K	operating-mode selector, $K \in \{-1, 0, 1\}$	–
K_v	voltage-unbalance factor used in $Q_T^* \rightarrow Q^*$ conversion	–
dq-frame grid-control variables		
x_d, x_q	generic dq components of a three-phase quantity x	depends on x
$v_{g,d}^+, v_{g,q}^+$	positive-sequence grid-voltage components in $+\omega$ frame	V
$v_{g,d}^-, v_{g,q}^-$	negative-sequence grid-voltage components in $-\omega$ frame	V
$i_{ph,d}^+, i_{ph,q}^+$	positive-sequence grid-current components in $+\omega$ frame	A
$i_{ph,d}^-, i_{ph,q}^-$	negative-sequence grid-current components in $-\omega$ frame	A
$i_{ph,d}^{+*}, i_{ph,q}^{+*}$	positive-sequence current references in dq frame	A
$i_{ph,d}^{-*}, i_{ph,q}^{-*}$	negative-sequence current references in dq frame	A
$v_{ph,d}^+, v_{ph,q}^+$	positive-sequence phase-voltage components	V
$v_{ph,d}^-, v_{ph,q}^-$	negative-sequence phase-voltage components	V
$v_{ph,d}^{+*}, v_{ph,q}^{+*}$	positive-sequence phase-voltage references	V
$v_{ph,d}^{-*}, v_{ph,q}^{-*}$	negative-sequence phase-voltage references	V
$e_d^+, e_q^+, e_d^-, e_q^-$	dual-sequence current-control errors	A
$u_d^\pm, u_q^\pm$	outputs of dual-sequence PI current regulators	V
$k_{p,g}, k_{i,g}$	proportional and integral gains of grid current controller	–
$\mathbf{T}^{-1}(\theta)$	inverse Park transformation matrix	–
$\mathbf{v}_{ph,dq}^{\pm*}$	phase-voltage reference vectors in dq coordinates	V
$\mathbf{v}_{ph,abc}^*$	phase-voltage reference vector generated by grid controller	V
AC circulating-current controller variables (vertical balancing)		
$v_{\alpha,j}^*, v_{\beta,j}^*$	orthogonalized phase-voltage reference components for single-phase dq processing	V
$v_{ph,d,j}^*, v_{ph,q,j}^*$	phase-voltage reference components in phase-synchronous frame	V
$i_{\alpha,j}^{ac}, i_{\beta,j}^{ac}$	orthogonal components of AC circulating current	A

Variable	Description	SI unit
$i_{\text{circ},d,j}, i_{\text{circ},q,j}$	AC circulating-current components in phase-synchronous dq frame	A
$i_{\text{circ},d,j}^*, i_{\text{circ},q,j}^*$	AC circulating-current references in phase-synchronous dq frame	A
$e_{d,j}, e_{q,j}$	AC circulating-current control errors (phase j)	A
$v_{\text{circ},d,j}^{ac*}, v_{\text{circ},q,j}^{ac*}$	dq circulating-voltage references	V
$v_{\text{circ},j}^{ac*}$	commanded phase AC circulating-voltage contribution	V
$\mathbf{v}_{\text{circ},abc}^{ac*}$	vector of AC circulating-voltage references (three legs)	V
$k_{p,ac}, k_{i,ac}$	proportional and integral gains of AC circulating-current controller	–
DC circulating-current controller variables (horizontal balancing)		
$I_{(a-b)}^*, I_{(a-c)}^*$	differential DC circulating-current references among phase-legs	A
$I_{(a-b)}, I_{(a-c)}$	measured differential DC circulating currents among phase-legs	A
e_{ab}, e_{ac}	errors of differential DC circulating-current loops	A
$V_{(a-b)}^*, V_{(a-c)}^*$	differential DC circulating-voltage commands	V
$v_{\text{circ},a}^{dc*}, v_{\text{circ},b}^{dc*}, v_{\text{circ},c}^{dc*}$	leg-wise DC circulating-voltage references	V
$\mathbf{v}_{\text{circ},abc}^{dc*}$	vector of DC circulating-voltage references (three legs)	V
$k_{p,dc}, k_{i,dc}$	proportional and integral gains of DC circulating-current controller	–
Final coordinated arm-voltage references		
$\mathbf{v}_{up,abc}^*$	upper-arm voltage reference vector (all phase-legs)	V
$\mathbf{v}_{low,abc}^*$	lower-arm voltage reference vector (all phase-legs)	V
$v_{up,a}^*, v_{up,b}^*, v_{up,c}^*$	upper-arm voltage references (phase-wise)	V
$v_{low,a}^*, v_{low,b}^*, v_{low,c}^*$	lower-arm voltage references (phase-wise)	V

