POLITECNICO MILANO 1863

SCUOLA DI INGEGNERIA INDUSTRIALE E DELL'INFORMAZIONE

Executive Summary of the Thesis

Credit Value Adjustment in Reinsurance: A Quantitative Framework and Case Study

Laurea Magistrale in Mathematical Engineering - Ingegneria Matematica

Author: Marco Enrico Baggio

Advisor: Prof. Alessandro Calvia

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1. Introduction

Reinsurance constitutes a fundamental mechanism for risk transfer in the insurance industry. Through this process, insurers mitigate underwriting risk, reduce earnings volatility and optimize capital requirements. However, reinsurance contracts introduce exposure to Reinsurance Counterparty Credit Risk (RCCR), namely the risk that the reinsurer defaults prior to honoring its obligations. Following the 2008 financial crisis, counterparty credit risk became central in financial regulation. In the banking sector, the Credit Value Adjustment (CVA) is now a standard component of derivative pricing and capital calculation frameworks. In contrast, the systematic application of CVA methodologies to reinsurance contracts remains comparatively underdeveloped, despite the explicit recognition of counterparty default risk under Solvency II. The purpose of this thesis is to develop a mathematically rigorous and numerically tractable reduced-form framework for the valuation of RCCR via CVA. The model builds upon the approach introduced in *Value Adjustment and Dynamic Hedging of Reinsurance Counterparty Risk* by Claudia Ceci, Katia Colaneri, Rüdiger Frey and Verena Köck (2020) [2], extending it along two main dimensions, namely introducing both a stochastic mean-reverting claim arrival intensity and a regime-switching default intensity. The thesis combines rigorous theoretical derivations, a characterization of the valuation problem through a system of coupled Partial Integro-Differential Equations (PIDEs), and a detailed numerical comparison between Monte Carlo simulation and finite-difference methods. Within this framework, stochastic claim intensity, regime-switching default dynamics and contagion effects are jointly incorporated, and their quantitative implications for the level and sensitivity of the CVA are rigorously evaluated. The findings highlight the quantitative relevance of dynamic intensity modeling and confirm the numerical robustness and efficiency of the PIDE framework for practical CVA computation.

2. Theoretical Framework for RCCR

The modeling framework is developed on a filtered probability space $(\Omega, \mathcal{G}, (\mathcal{G}_t)_{t\geq 0}, \mathbb{Q})$, where $\mathbb{Q}$ denotes the risk-neutral measure. The reinsurer default time τ^R is modeled in reduced form as the first jump time of a Cox process with stochastic intensity λ^R. Denoting by

$$H_t^R = \mathbf{1}_{\{\tau^R \leq t\}}$$

the associated default indicator process, the survival probability conditional on the reference filtration satisfies

$$\mathbb{Q}(\tau^R > t \mid \mathcal{F}_t) = \exp\left(-\int_0^t \lambda_s^R ds\right).$$

This intensity-based approach ensures analytical tractability and is consistent with the classical credit risk framework developed in *On Cox Processes and Credit Risky Securities* by David Lando [3] and in *Credit Risk: Modeling, Valuation and Hedging* by Tomasz Bielecki and Marek Rutkowski [1].
Aggregate insurance losses are modeled as a compound process

$$L_t = \sum_{i=1}^{N_t} Z_i,$$

where N is a counting process with stochastic intensity λ^L and Z_i are i.i.d. claim severities. This specification allows the claim arrival intensity to evolve stochastically, capturing clustering effects and time-varying underwriting risk.
The default-free value of a reinsurance contract with terminal payoff $\phi(L_T)$ is given by

$$V_t^\phi = \mathbb{E}^{\mathbb{Q}}\left[e^{-r(T-t)}\phi(L_T) \mid \mathcal{G}_t\right].$$

The Credit Value Adjustment is defined as the discounted expected loss due to reinsurer default:

$$\text{CVA}_t = \mathbb{E}^{\mathbb{Q}}\left[\int_t^T \delta^R V_s^\phi e^{-r(s-t)} dH_s^R \,\middle|\, \mathcal{G}_t\right],$$

where δ^R denotes the loss given default. Under Markovian assumptions, the CVA admits the representation

$$\text{CVA}_t = \delta^R(1 - H_t^R) f^{\text{CVA}}(t, L_t, X_t, Y_t),$$

where the deterministic function f^{CVA} satisfies a backward Partial Integro-Differential Equation (PIDE). This characterization provides us with the theoretical foundation of the thesis.

3. Extension of the Baseline Model

The second chapter of the thesis develops the core modeling contribution of the work by extending the framework introduced in Ceci et al. (2020).

The claim arrival intensity is modeled as a function $\lambda^L(X)$ of an Ornstein–Uhlenbeck factor X solving

$$dX_t = \kappa(\theta - X_t)dt + \sigma dW_t.$$

Since the Ornstein–Uhlenbeck process is Gaussian and may attain negative values, the intensity is defined through a functional specification $\lambda^L(\cdot)$ that guarantees non-negativity, ensuring the admissibility of the intensity. This mean-reverting structure captures cyclical patterns in claim frequencies while introducing a diffusion risk factor that directly affects the dynamics of the aggregate loss process and, consequently, the exposure profile underlying the CVA computation. Compared to constant-intensity models, the stochastic specification introduces an additional source of uncertainty in the claim arrival rate. Since the contract value is not linear in the claim intensity, fluctuations in $\lambda^L(X)$ affect the expected discounted loss, thereby modifying the resulting CVA. The reinsurer default intensity is assumed to depend on a two-state continuous-time Markov chain $Y \in \{0, 1\}$, representing normal and stressed market regimes. The intensity is therefore given by

$$\lambda^R(Y_t) = \begin{cases} \lambda_0^R, & Y_t = 0, \\ \lambda_1^R, & Y_t = 1, \end{cases}$$

with transitions governed by a generator matrix $Q = (q_{i(1-i)}), i \in \{0, 1\}$. This regime-switching structure introduces macro-financial variability in credit risk and leads to a system of coupled PIDEs, one for each regime, linked through the transition terms $q_{i(1-i)}(f_{(1-i)} - f_i), i \in \{0, 1\}$.
To account for contagion effects, the loss intensity is allowed to experience a proportional jump at the default time of the reinsurer:

$$dX_t = \kappa(\theta - X_t)dt + \sigma dW_t + \gamma X_{t-} dH_t^R.$$

This mechanism reflects the sudden increase in retained underwriting risk when reinsurance protection disappears and introduces a discontinuity in the state dynamics.
Under this extended framework the CVA at time t is naturally regime-dependent. More precisely, the valuation must be understood conditionally on the current state of the chain, that is, on the event $\{Y_t = i\}, i \in \{0, 1\}$. Consequently,

the CVA admits a regime-indexed representation through deterministic functions f_i, each associated with one state of the credit environment. For each regime i, the CVA function satisfies a backward equation of the form

$$\frac{\partial f_i}{\partial t} + \mathcal{L} f_i + \lambda_i^R v^{\phi}(t, l, x + \gamma x) + q_{i(1-i)}(f_{(1-i)} - f_i) = (r + \lambda_i^R) f_i,$$

where $\mathcal{L}$ denotes the generator of the joint loss-intensity process. This equation incorporates diffusion terms from the Ornstein–Uhlenbeck dynamics, integral terms from claim jumps, and coupling terms from regime switching. Under standard regularity conditions, existence and uniqueness of solutions are established, ensuring well-posedness of the valuation problem.

4. Numerical Implementation

The third chapter addresses the numerical solution of the valuation problem through two complementary methodologies.
The Monte Carlo approach simulates joint trajectories of the Ornstein–Uhlenbeck factor, the regime-switching Markov chain and the default time. Following the Cox process construction, the default time is generated concurrently with the Markov chain Y. The thesis provides a formal proof demonstrating that this joint simulation algorithm is equivalent to the classical inversion of the integrated intensity process. Claim arrivals are simulated conditionally on the stochastic intensity path. The CVA is computed as the empirical average of discounted default losses over simulated paths. Although this approach offers flexibility and conceptual transparency, computational cost increases significantly in the full stochastic-intensity and regime-switching specification.
An alternative deterministic approach is developed by solving the coupled backward PIDEs via finite-difference schemes. Time and state variables are discretized over bounded domains, diffusion components are handled through implicit schemes to guarantee stability, and the integral term associated with claim arrivals is approximated via numerical quadrature. The regime-switching structure yields a block-coupled linear system at each time step. This method exploits the Markovian structure of the model and substantially reduces computational time while preserving numerical accuracy.
A systematic comparison between the two methodologies is conducted to evaluate convergence properties, computational efficiency and robustness with respect to parameter perturbations.

Table 1: CVA Comparison MC and PIDE approach

	MC	CI	PIDE
X const	0.1969	[0.1928 , 0.2010]	0.1953
X OU	0.1969	[0.1928 , 0.2010]	0.1939

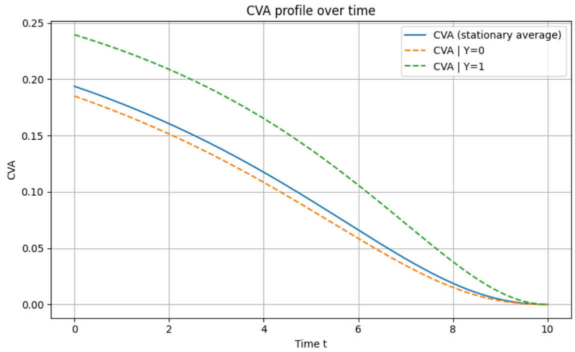

Figure 1: CVA vs time on different initial state of Y

5. Numerical Results and Economic Implications

The fourth chapter presents quantitative results obtained under a benchmark parameter set consistent with realistic underwriting and credit risk characteristics. CVA estimates are computed under both constant and stochastic claim intensity specifications using Monte Carlo simulation and the PIDE approach, and the two methods show strong agreement, with deterministic results lying within the Monte Carlo confidence intervals as in Table 1. In Figure 1 the impact of the initial credit regime is also examined, highlighting higher CVA levels when starting in the stressed state and gradual convergence toward the stationary distribution over time. In light of this consistency and its superior computational efficiency, the PIDE framework with Ornstein–Uhlenbeck loss intensity is adopted for the sensitivity analysis.
The analysis of the OU parameters in Figures 2, 3 and 4 shows that the CVA is primarily driven by the long-term mean θ of the claim in-

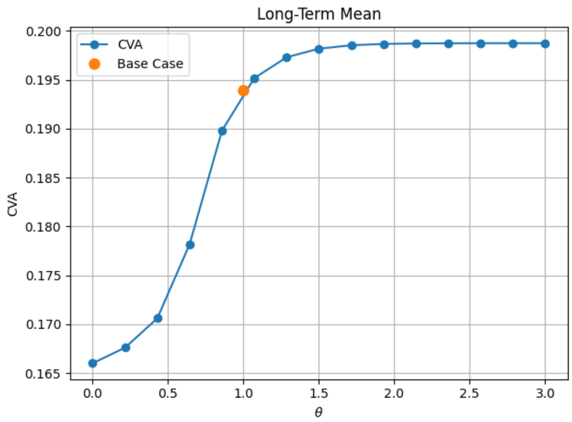

Figure 2: Sensitivity of θ

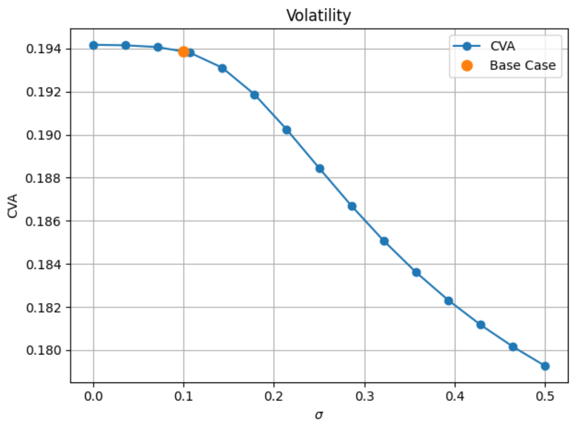

Figure 3: Sensitivity of σ

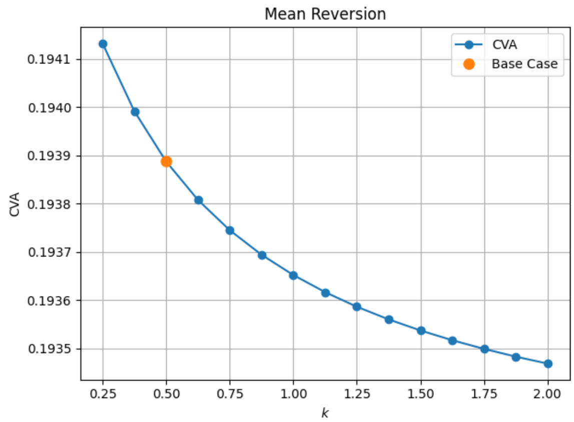

Figure 4: Sensitivity of k

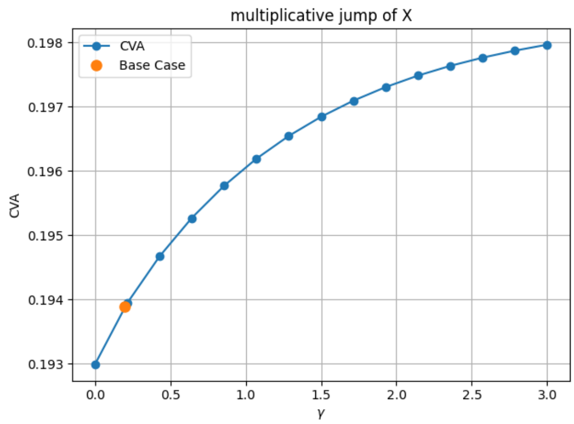

Figure 5: Sensitivity of γ

tensity. An increase in θ leads to a nonlinear rise in the CVA, with diminishing marginal impact for higher parameter values.
A higher structural claim frequency translates into a systematically larger expected exposure at default, thereby increasing the valuation adjustment. In contrast, the volatility parameter σ shows a monotonic decreasing impact on the CVA within the calibrated range, indicating that increased dispersion of the intensity process does not translate into higher average exposure in this specification. The mean reversion speed k also has a negative but quantitatively modest effect, as faster reversion limits the persistence of high-intensity scenarios. Overall, the results confirm that the CVA is more sensitive to structural level parameters, while dynamic parameters primarily influence second-order effects. Regime switching in default intensity produces a marked increase in CVA variability and shifts valuation upward in stressed states characterized by higher default intensity. The persistence of stressed regimes materially influences the overall CVA level, demonstrating that transition intensities play a crucial role in counterparty risk assessment. In Figure 5 it is possible to note that the inclusion of contagion effects further amplifies CVA by introducing discontinuities in exposure at default. Numerical experiments confirm that neglecting contagion leads to systematic underestimation of counterparty risk, particularly when the proportional jump parameter γ is large. The comparison between Monte Carlo and PIDE approaches shows that both methods yield consistent pricing results, while the finite-difference PIDE solution provides a substantial computational advantage in the extended specification.

6. Conclusions

This thesis develops a regime-switching reduced-form framework for the valuation of reinsurance counterparty credit risk via Credit Value Adjust-

ment. By integrating mean-reverting stochastic loss intensity, regime-dependent default intensity and contagion effects at default, the model captures structural interactions between underwriting and credit risk that are typically overlooked in simplified approaches.
Theoretical characterization through a coupled PIDE system ensures mathematical rigor, while the numerical implementation demonstrates practical feasibility. The results indicate that dynamic and regime-aware modeling is quantitatively relevant for RCCR assessment and may materially affect capital calculations under Solvency II.
The proposed framework contributes to bridging the methodological gap between banking CVA modeling and insurance counterparty risk valuation, combining analytical consistency, economic interpretability and computational tractability.

References

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[3] D Lando. On cox processes and credit risky securities. *Review of Derivatives Research*, 2:99–120, 1998.