The reduced velocity V^* and the angle of attack α are the main keywords to understand the wind interaction, and are described in the following:

- The reduced velocity is defined as:

$$V^* = \frac{V}{fB}$$

(**Error! No text of specified style in document.**-1)

It represents the ratio between the period $T = 1/f$ associated to the deck's oscillation and the time B/V needed by a fluid particle to move through the deck width B. A conjugate parameter can be defined as the inverse of the reduced velocity V^* called the reduced frequency f^*:

$$f^* = \frac{fB}{V}$$

(**Error! No text of specified style in document.**-2)

The frequency f in the reduced frequency expression can also represent (in case that the turbulence effect is considered) the frequency of fluctuation of the wind turbulence spectrum.

- The angle of attack is defined with respect to a reference axis (Figure **Error! No text of specified style in document.**.2) as:

$$\alpha = \theta + \psi = \theta + \tan^{-1}\left(\frac{w - \dot{z} - B_1\dot{\theta}}{V + u - \dot{y}}\right)$$

(**Error! No text of specified style in document.**-3)

It represents the angle between the incoming flow and the deck's position. If small displacements and velocities (of the deck) are considered, equation (**Error! No text of specified style in document.**-3) is linearized as:

$$\alpha = \theta + \psi = \theta + \frac{w - \dot{z} - B_1\dot{\theta}}{V + u - \dot{y}}$$ (**Error! No text of specified style in document.**-4)

B_1 is furtherly defined.

Problems related to the wind action on a bridge can be distinguished into static and dynamic:

- Static problems are related to the static loads exerted by the average wind speed. Such loads are function of the angle of attack α.
- Dynamic problems are related to the turbulence of the incoming wind and to the aerodynamic forces (equation **Error! Reference source not found.**) and is also referred to as the aeroelastic problem.

Static Problem

The average wind speed produces a static load that acts on all the components of the bridge. For very long bridges, the load applied on the deck is the most important. In the case of suspended bridges, the loads on the deck are transferred through the hangers to the main cables and to the top of the towers, thus producing a very high bending moment that has a strong impact on the tower and on the overall bridge design. In consequence, drag on the deck is one of the parameters that must be minimized for long-span bridges.

For the sake of the analysis, the crucial aspect of the static problem stands in the definition of the configuration of static equilibrium, around which it is possible to linearize the aerodynamic forces. In practice, what really matters is the static rotation of the deck θ that determines the aerodynamic parameters that must be used to describe the aeroelastic phenomena and the forces due to the turbulence of the incoming wind.

Aeroelastic Problems

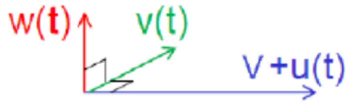

Figure **Error! No text of specified style in document.**.1. Wind turbulence components.

The incoming wind is characterized by turbulent components in three directions, *u(t)*, *v(t)* and *w(t)* (Figure **Error! No text of specified style in document.**.1), to be added to the average wind velocity . Equation (**Error! No text of specified style in document.**-4) shows that turbulence components have an impact on the definition of the wind angle of attack and consequently on the aerodynamic forces, that, therefore, change randomly in time. This variation of the aerodynamic forces produces a bridge motion induced by turbulence, called *buffeting*. If the deck or any part of the bridge is moving with a given velocity in the wind flow, the forces applied to the body are functions of the relative velocity V_{rel} of the incoming wind with respect to the body and the same expressions can be applied introducing V_{rel} instead of V.

As observed in equation (**Error! No text of specified style in document.**-3) and in Figure **Error! No text of specified style in document.**.2, the motion of the bridge and its position have an effect on the angle of attack α. More specifically, the rotation of the deck θ and the deck velocities $\dot{z}$ and $\dot{\theta}$ concur in the definition of α. The body motion has therefore an impact on the aerodynamic forces. Depending on the shape of the deck, if the motion-dependent - also called self-excited - aerodynamic forces act in favor of the motion, they introduce energy in the system and the oscillations magnitude is amplified. In other words, in such situation the bridge becomes unstable. Different kinds of instability can be identified: one degree of freedom instability in the vertical or torsional mode, or two degrees of freedom instability, which results from the coupling of vertical and torsional motions. The second type of instability is also known as *flutter instability*.

As it will be discussed in Section 1.1, self-excited forces can be seen as equivalent damping and stiffness terms that modify the structural properties of the bridge. Typically, long-span bridges behave in the following way:

- At zero wind speed the overall damping is due only to the structural part (usually structural damping assumes values of about 3-5‰).
- As the wind speed increases, the contribution of the aerodynamic forces becomes very important and the overall damping becomes slightly higher.
- A further increase in wind speed causes a reduction in the overall damping. The wind speed at which the overall damping becomes negative is defined as the flutter velocity.

Section **Error! Reference source not found.** studies the above behavior for the cross section of the BB3 with a scaled train, which is the object of study of this thesis.

The prediction of the buffeting response to turbulent wind is generally secondary to the question of aerodynamic stability. However, when the bridge is proved to be stable, the bridge response to wind gusts is important for the design of the superstructure and the assessment of the user comfort by predicting the acceleration levels. Moreover, the large vibrations reached by the structure may give rise to fatigue problems. Vibration amplitudes associated with buffeting can be controlled by increasing the aerodynamic damping or equivalently, by increasing the stability of the bridge. From this point of view, it is important to have a high critical flutter velocity not only to be conservative on stability conditions, but also to increase the aerodynamic damping so as to reduce the turbulence-induced motions.

1.1 Definition of Aerodynamic Forces

Different analytical approaches are available to model the aerodynamic behavior of bridges. In the following, reference is made to a single isolated deck section of length L with three degrees of freedom: the horizontal displacement y, the vertical displacement z, and the rotation θ. An illustration of the problem is depicted in Figure **Error! No text of specified style in document.**.2.

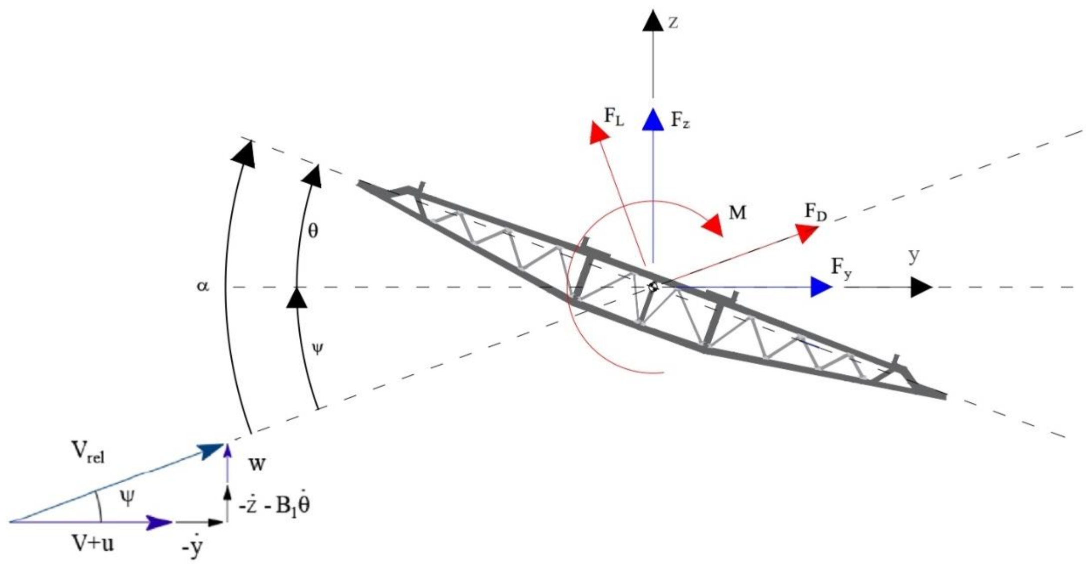

Figure **Error! No text of specified style in document.**.2. Aerodynamic forces on a generic deck section and its sign conventions.

1.1.1 Quasi-steady Theory

The quasi-steady theory (QST) is the most suitable approach to better understand the physics of the aeroelastic problem (M. F. G. Diana 1995), (F. C. G. Diana 1991). The QST well reproduces the aerodynamic forces on a deck for high values of the reduced velocity V^*>15. High reduced velocity means that the time needed by a particle to cross the body is very small compared to the period of oscillation of the body. Hence, the QST assumes that the aerodynamic forces acting on the bridge deck are not influenced by the frequency of the deck's motion. Consequently, the QST allows to match the aerodynamic forces to the static forces measured on still sectional models via static tests in the wind tunnel (furtherly discussed in Section **Error! Reference source not found.**), and are expressed as:

$$F_D = \frac{1}{2}\rho B L V_{rel}^2 C_D(\alpha)$$

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$$F_L = \frac{1}{2}\rho B L V_{rel}^2 C_D(\alpha)$$

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$$M = \frac{1}{2}\rho B^2 L V_{rel}^2 C_M(\alpha)$$

(**Error! No text of specified style in document.**-7)

Where ρ is the air density, V_{rel} is the relative wind velocity, B is the deck width, L is the length of the generic bridge deck, α is the angle of attack and $C_D(\alpha)$, $C_L(\alpha)$, $C_M(\alpha)$ are respectively the drag, lift and moment aerodynamic coefficients.

In accordance to Figure **Error! No text of specified style in document.**.2, the square of the relative velocity V_{rel}^2 can be expressed as follows:

$$V_{rel}^2 = (V + u - \dot{y})^2 + \left(w - \dot{z} - B_1\dot{\theta}\right)^2$$

(**Error! No text of specified style in document.**-8)

And the angle of attack α was previously defined by equation (**Error! No text of specified style in document.**-4).

The equations of motion of the deck written in its degrees of freedom y, z and θ are:

$$m_y\ddot{y} + r_y\dot{y} + k_y y = F_y = F_D\cos(\psi) - F_L\sin(\psi)$$

(**Error! No text of specified style in document.**-9)

$$m_z\ddot{z} + r_z\dot{z} + k_z z = F_z = F_D \sin(\psi) + F_L \cos(\psi)$$ (**Error! No text of specified style in document.**-10)

$$I_\theta\ddot{\theta} + r_\theta\dot{\theta} + k_\theta\theta = F_\theta = M$$ (**Error! No text of specified style in document.**-11)

Where $m_{y,z}$ and I_θ are the effective inertias of the deck for the horizontal, vertical and torsional degrees of freedom, $r_{y,z,\theta}$ are the effective viscous damping, and $k_{y,z,\theta}$ the effective stiffness.

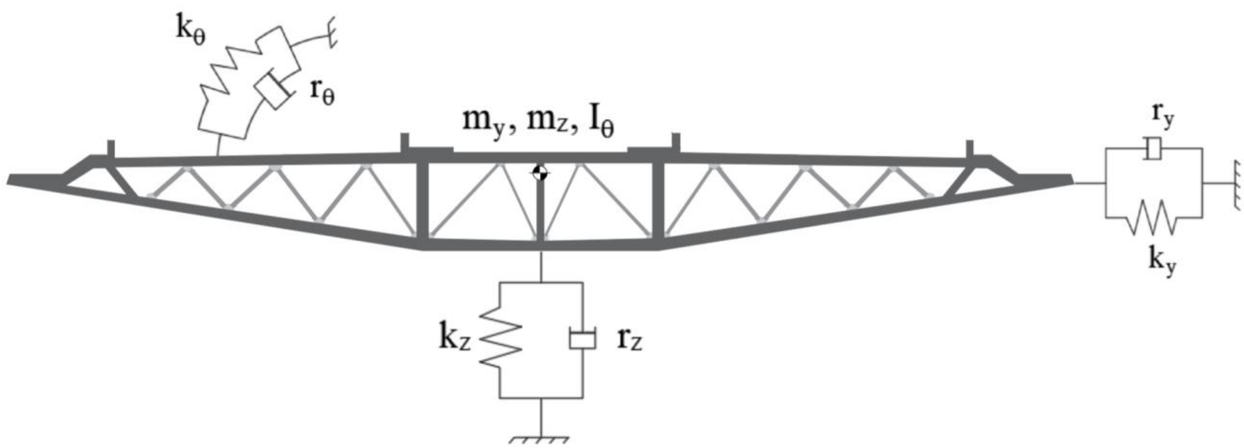

Figure **Error! No text of specified style in document.**.3. Elastically suspended deck section.

Substituting equation (**Error! No text of specified style in document.**-8) into equations (**Error! No text of specified style in document.**-5)-(**Error! No text of specified style in document.**-6)-(**Error! No text of specified style in document.**-7), the equations of motion are rewritten as:

$$m_y\ddot{y} + r_y\dot{y} + k_y y = \frac{1}{2}\rho BL\left((V + u - \dot{y})^2 + \left(w - \dot{z} - B_{1y}\dot{\theta}\right)^2\right)(C_D(\alpha)\cos(\psi) - C_L(\alpha)\sin(\psi))$$ (**Error! No text of specified

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$$m_z\ddot{z} + r_z\dot{z} + k_z z = \frac{1}{2}\rho BL\left((V+u-\dot{y})^2 + \left(w-\dot{z}-B_{1z}\dot{\theta}\right)^2\right)(C_D(\alpha)\sin(\psi) + C_L(\alpha)\cos(\psi))$$

(**Error! No text of specified style in document.**-13)

$$I_\theta\ddot{\theta} + r_\theta\dot{\theta} + k_\theta\theta = \frac{1}{2}\rho B^2 L\left((V+u-\dot{y})^2 + \left(w-\dot{z}-B_{1z}\dot{\theta}\right)^2\right)C_M(\alpha)$$

(**Error! No text of specified style in document.**-14)

The reference body dimensions $B_{1y,z,\theta}$ contain information of the *flutter derivatives* (Section **Error! Reference source not found.**) and are aimed at 'correcting' the quasi-steady approach by introducing a slight dependence of aerodynamic forces at on the reduced velocity V^*. Hence, the above formulation is referred to as Corrected Quasi-steady Theory (QSTC).

Introducing the displacement vector:

$$\underline{x} = \begin{bmatrix} y \\ z \\ \theta \end{bmatrix}$$

The equation of motion of the system may be written employing a matrix formulation:

$$[M_s]\underline{\ddot{x}} + [R_s]\underline{\dot{x}} + [K_s]\underline{x} = \underline{F}_{aero}(\underline{x}, \underline{\dot{x}}, V_m, \underline{b})$$

(**Error! No text of specified style in document.**-15)

Where $\underline{b} = \begin{bmatrix} u/V \\ w/V \end{bmatrix}$ contains the turbulence components of the